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#### YUN-KWONG SEBASTIAN TAM

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COMMITTEE IN CHARGE

Chairman: B. N. Moyls

B. AhlbornR. NodwellA.J. BarnardL. de SobrinoM.M.Z. KharadlyJ.H. Williamson

Research Supervisor: F. L. Curzon.

External Examiner: P. Savic, National Research Council, Ottawa.

#### LIMITATIONS OF MAGNETIC PROBE MEASUREMENTS IN PULSED PLASMAS

#### ABSTRACT

A "gradient probe" consisting of two search coils has been developed to measure the current density in a pulsed plasma. This probe measures both the magnitude and the gradient of the magentic field simultaneously enabling more accurate measurements than the conventional magnetic probe which has only one It has been used to measure the current coil. densities and the magnetic fields in z-pinch discharges in helium at pressures between 500  $\mu$  and 4 mmHg. The collapse curves obtained agreed with the predictions of a modified snowplow equation which allowed for the loss of particles from the collapsing current shell.

The flow of current in the plasma is distorted by the presence of a probe. Such an effect spoils the spatial resolution so that the measured values of the current density  $J_p$  are averages of the true current density  $J_o$  over a finite region. To investigate this, a correction formula which relates  $J_p$  to  $J_o$  has been developed. Our error analyses showed that any scatter in  $J_p$  due to experimental errors was magnified twenty times in  $J_o$ . For a pulsed plasma, therefore, one should try to reduce the perturbation of the probe instead of relying on the correction procedure.

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Field of Study: Plasma Physics

Plasma Physics L. de Sobrino Advanced Plasma Physics A. J. Barnard Plasma Dynamics Elementary Quantum F. L. Curzon Mechanics W. Opechowski Advanced Quantum Mechanics H. Schmidt R. M. Ellis Waves G. M. Volkoff Electromagnetic Theory Special Relativity Theory P. Rastall Statistical Mechanics R. Barrie

### AWARDS

- 1957-60 Hong Kong Government Bursary
- 1961-62 Mrs. Eng Wong Yuk-Mui's Scholarship
- 1964-65 National Research Council Studentship

#### PUBLICATIONS

F.J. Jankulak, L.G. de Sobrino and Y.K.S. Tam, "A Solution of the Fokker-Planck Equation for Homogeneous Plasmas", Can.J.Phys. <u>42</u>, 1743 (1964).

# LIMITATIONS OF MAGNETIC PROBE MEASUREMENTS IN PULSED DISCHARGES

by

### YUN-KWONG SEBASTIAN TAM

B.Sc. (Special Honours), University of Hong Kong, 1962M.Sc., University of British Columbia, 1965

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Department of PHYSICS,

The University of British Columbia Vancouver 8, Canada

Date \_\_\_\_\_ July 18, 1967\_\_\_\_\_

#### ABSTRACT

A "gradient probe" consisting of two search coils has been developed to measure the current density in a pulsed discharge. This probe measures both the magnitude and the gradient of the magneitc field simultaneously enabling more accurate measurements than the conventional magnetic probe which has only one coil. It has been used to measure the current densities and the magnetic fields in z-pinch discharges in helium at pressures between 500  $\mu$  and 4 mmHg. The collapse curves obtained agreed with the predictions of a modified snow-plow equation which allowed for the loss of particles from the collapsing current shell.

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Chapter 1

#### INTRODUCTION

In this thesis, we attempt to improve the method of measuring the current density in a pulsed plasma by using a magnetic probe, and to investigate in detail how the perturbation of a probe on the plasma affects the measured current density.

The local current densities and magnetic fields of a plasma can, in principle, be derived (a) from induced voltages in search coils or (b) from the deflection of However, the interinjected beams of charged particles. pretations of the second technique are usually too complicated to be of practical importance (Huddlestone, 1965, p.69). So far the only tool for measuring such quantities is still the magnetic probe in the form of a search coil. Ever since it was first reported by a group of Russian scientists (Artsimovich et al., 1956), the magnetic probe has been extensively used for measuring the magnetic field. To determine the current density, the gradient of the field has to be computed numerically after a mapping of the field in the plasma has been obtained. It is obvious that such a method is tedious and inaccurate. We therefore attempted to develop a gradient probe which directly measures both the magnetic field and its gradient, and gives much more accurate measurements of current densities in a plasma.

The gradient probe we used consists of two search coils connected in such a way that its output can give the difference  $\triangle B$  between the magnetic fields measured by the two coils. When the separation  $\triangle r$  between the coils is accurately known, the gradient  $\frac{dB}{dr}$  of the magnetic field can be taken as  $\frac{\triangle B}{\triangle r}$ . The gradient probe we use in this thesis is 7 mm in diameter (see Fig. 23). It has a frequency response which is flat up to 1 MHz and a spatial resolution better than 3 mm (see §2.4.3)

Any probe disturbs the current flow by its mere presence in the plasma. If we know the nature of this disturbance, then in principle, we can correct for it, and obtain the undisturbed current distribution. Apart from the work of Ecker et al. (1962), Malmberg (1964) and Daughney (1966), very little has been done to develop such a correction procedure for the probe perturbation. We find it important to investigate this point before we can obtain conclusive results from probe measurements. We therefore study the perturbation caused by the probe, its effects on the measured values of the magnetic fields and the current densities of a plasma, and the limitations of such a method of probing.

Based on the steady state theory, Ecker et al. (1962) have done some computer calculations to see how a probe perturbs the current flow in linear discharges. They studied how the measured probe signals could be used to calculate the

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true current density. In their calculations, the discharge is split up into a set of current layers. The perturbed magnetic field due to the distorted current flow in each current layer is first calculated. By adding the fields of the different layers, the resultant magnetic field is obtained. will then give the value of the field that would be mea-This sured by a magnetic probe. By correcting the measured probe result, the true current density and magnetic field in the discharge can be computed. Their theory is based on time independent calculations and has not considered moving currents such as those observed in z-pinch discharges. Nevertheless, we shall show that their theory also applies to time dependent cases and moving currents provided that the current is not moving too quickly. The condition under which the theory applies is  $|\vec{v}| \ll \omega \delta$  where  $\omega$  and  $\delta$  are the frequency and the skin depth of the current respectively and  $\vec{v}$  is the velocity at which the current layer collapses radially.

Malmberg (1964) has obtained an analytic solution for the diffusion of magnetic field through the hole of a plane current sheet of infinite electrical conductivity  $\sigma$ . This model is discussed in the thesis because it helps in the understanding of the problem and it is relevant to high frequency current flows in highly conducting material such as the return conductor of the discharge vessel. However, for plasmas, it is generally not applicable because the conductivity is too low.

The most extensive experimental study of the perturbation of a probe on the plasma in a z-pinch was carried out by

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Daughney (1966). By modifying Malmberg's formula and splitting the plasma current into a number of current shells, he has developed an integral equation relating the measured magnetic fields to the values of the field occurring in the absence of the probe. However, the curvature of the cylindrical current sheets was not properly accounted for in his treatment. As a result, his integral equation has a singular kernel. In the present thesis this particular error has been eliminated from Daughney's treatment of the problem. Besides, he has not made a quantitative analysis of the effects of experimental errors.

To investigate how experimental errors affect the corrected values of the current density, we employ the Monte Carlo method in which random errors are added to the measured values of the magnetic field (B) and its gradient  $(\frac{dB}{dr})$ . The results of the investigation show that the probe greatly smooths out the details of the true current density. Thus the probe measurements have to be extremely accurate (within  $\frac{1}{4}$ ) in order to obtain the fine structure of the true current since any small errors in the probe measurements will be magnified by a factor of 20 in the correction procedure. The perturbation on a thin current sheet is severe and the apparent current density distribution (i.e. no correction made ) is broadened and greatly reduced in amplitude. However, for smooth current density distributions, the correction procedure is unnecessary.

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The magnetic probes described in this thesis have been employed to study the initial collapse of cylidrical current layers in z-pinch discharges. This work has been carried out in helium over the pressure range from 500 µHg to 4 mmHg.

By equating the magnetic force to the rate of change of momentum of the collapsing cylindrical current sheet, the radius of the sheet as a function of time can be predicted (the so-called collapse curve). The theoretically predicted collapse curves agree very well with the experimental observations, provided that the initial mass of the shell is taken into account. Inefficient trapping of gas by the shell is also accounted for in the theory.

We shall discuss the theoretical aspects of the magnetic probe measurement in Chapter 2 and the experimental results in Chapter 3. In the theoretical part, we first introduce the correction formulae using the models by Ecker et al. (1962) ( $\S$ 2.2.1), and Malmberg (1964) ( $\S$ 2.2.2) respectively. In each case, we derive an integral equation which relates the gradient probe measurements to the true current density of a z-pinch allowing for the perturbation of the probe. The limitations of the model of Ecker et al. are then given ( $\S$ 2.2.3).

To convert the equation to a form suitable for numerical computations, we transform it into a matrix equation ( $\oint 2.3$ ) which can be used to calculate the true current density from the probe signals using the IBM 7040 digital

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computer. Subsequently, we study the stability of the equation with the help of the Monte Carlo method (Fox, 1962, p. 425) by simulating experimental errors with random errors ( $\{2,4\}$ ).

In the experimental part, we give details about the construction ( $\S 3.3$ ), calibration ( $\S 3.3.1$ ), the frequency response ( $\S 3.3.2$ ) and the reproducibility of the gradient probe signals ( $\S 3.3.$ ). The measurements of the current density and magnetic field in z-pinch discharges in helium are then obtained ( $\S 3.4.1$ ). The results are used to calculate the accelerating magnetic force on the collapsing current shell in the modified snow-plow model ( $\S 3.4.2$ ).

#### Chapter 2

MEASUREMENTS OF THE CURRENT DENSITY OF A PLASMA

IN A PULSED DISCHARGE

### 2.1 Introduction

We are interested in the dynamics of a plasma. Since magnetic forces play the dominant role in the motion of such systems, it is important to be able to measure the magnetic field accurately. The simplest device to accomplish such measurements is a magnetic probe. The most convenient probe is the search coil which is introduced into the plasma in an insulating shield.

A conventional magnetic probe consists of a small coil enclosed in a cylindrical insulating jacket. When placed in a time varying plasma, an emf is induced which is proportional to the rate of change of the total magnetic flux threading through the sensing coil. To obtain the magnetic flux, the output signals from the probe are integrated by an RC circuit as shown in Fig. 1.

This kind of magnetic probe was first used extensively by a group of Russian scientists (Artsimovich et al., 1956). Apart from the investigations of Malmberg (1964) and Ecker et al. (1962), little has been done to improve the techniques and interpretations of such probe measurements. For this reason, we attempt to investigate in detail the various problems in magnetic probing and develop techniques for more accurate mea= surements of current densities.

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# FIG. 1 A TYPICAL ARRANGEMENT FOR MAGNETIC PROBING

Ö -

## (i) <u>Inaccuracy in the Gradient of the Measured Magnetic Field</u>

One great difficulty in measuring the current density with a magnetic probe is the inaccuracy in obtaining the gradient of the magnetic field. In a pulsed plasma with good reproducibility, the magnetic field of the plasma is mapped by taking measurements for different discharges with the probe at different positions. For a linear pinch, the discharge is assumed to be axially symmetric. In cylindrical coordinates, the current density  $J_z(r)$  in the axial direction at a radius r is related to the azimuthal magnetic flux density  $B_{p}(r)$  by the (Maxwell) equation

(1) 
$$J_{z}(r) = \frac{1}{\mu} \left[ \frac{B_{\beta}(r)}{r} + \frac{dB_{\beta}(r)}{dr} \right].$$

Here  $\mu$  is the magnetic permeability of the plasma and equation (1) is in mks units. Therefore to determine the current density  $J_{z}(r)$ , we should measure both  $B_{\phi}(r)$  and  $\frac{dB\phi(r)}{dr}$ .

A conventional probe measures the magnetic flux density  $B_{\phi}(r)$  of a discharge with reasonable accuracy. However, the values of  $\frac{dB_{\phi}(r)}{dr}$  are usually obtained by differentiating  $B_{\phi}(r)$  numerically. This gives unreliable values of  $\frac{dB_{\phi}(r)}{dr}$  especially when experimental values of  $B_{\phi}(r)$  always have a scatter about an unknown curve. This can be seen as follows.

A magnetic probe which measures  $B_{\phi}(r)$  at intervals  $\Delta r$  gives the measured current density

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(2) 
$$J'_{z}(r) = \frac{1}{\mu} \left( \frac{B_{2}+B_{1}}{r_{2}+r_{1}} + \frac{B_{2}-B_{1}}{r_{2}-r_{1}} \right),$$

at  $r = \frac{r_1 + r_2}{2}$ , where  $B_1$  and  $B_2$  are the respective fields at  $r_1$  and  $r_2$  which are separated by a distance  $\Delta r$ . If terms involving second and higher derivatives of  $B_{\phi}$  (see § 2.4.3) are negligible, equation (2) gives the true value  $J_z(r)$  when  $B_1, B_2, r_1$  and  $r_2$  are known exactly.

We now denote the standard errors in the measurements of  $J_z$ ,  $B_{\phi}$  and r by  $\sigma_J$ ,  $\sigma_B$  and  $\sigma_r$  respectively. Under typical experimental conditions, we have  $\sigma_r \sim \bigtriangleup r$  and  $r \gg \bigtriangleup r$ . Standard calculations with the help of (2) gives

(3) 
$$\frac{\sigma_{J_z}}{J_z} \sim 2^{\frac{1}{2}} (\frac{r}{\Delta r}) \left[ \left( \frac{\sigma_{B_{\phi}}}{B_{\phi}} \right)^2 + \left( \frac{\sigma_r}{r} \right)^2 \right]^{\frac{1}{2}} \sim \frac{r}{\Delta r} \frac{\sigma_B}{B_{\phi}} \phi$$
, for  $\frac{B_{2}-B_1}{r_2-r_1} \sim \frac{B_{\phi}}{r}$ ,

corresponding to gradual changes in  $B_{a}$  and

(4) 
$$\frac{\sigma_J}{J_z} \sim 2^{\frac{1}{2}} \left[ \frac{\sigma_{B\phi}}{B_{\phi}} \right]^2 + \left( \frac{\sigma_r}{\Delta r} \right)^2 \right]^{\frac{1}{2}} \sim 1$$
, for  $\frac{B_2 - B_1}{r_2 - r_1} \sim \frac{B_{\phi}}{\Delta r}$ ,

corresponding to sharp changes in  $\underset{\not \phi}{\overset{B}{\beta}}$ . Here the symbol " ~ " means "of the same order of magnitude as ". From (3) and (4), we immediately see that the fractional error in the computed value of  $J_z$  is either  $(\frac{r}{\Delta r})$  times the larger of the fractional errors in the measurements or comparable to unity.

In order to avoid this difficulty, the current density may be measured by a method reported by Wright and Jahn (1965). They use a miniature Rogowski coil enclosed in a small toroidal insulating tube (see Fig. 2(a) and (b)). However, such a probe not only has a poor spatial resolution, but also its toroidal geometry perturbs the plasma current severely. Besides, it is always difficult to determine what fraction of the true current passes through the loop.



Fig. 2(a). Sketch of miniature Rogowski Coil Probe.



I RUDE JACKET

Fig. 2(b). Cross-section of Rogowski coil probe.

In an alternative approach, Ohkawa et al. (1963) use a probe containing fourteen coils separated at intervals of 1 cm. The probe is placed across the discharge tube along a minor diameter of a stabilized toroidal pinch. The coils are arranged in seven pairs. Each pair has one coil coupling the axial stabilizing field and the other coupling the transverse field due to the pinch. Such an arrangement gives consistent results when taking measurements in a system of poor reproducibility. However, the magnetic field is determined only

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at seven positions across the discharge tube which has a minor diameter of 14 cm. The spatial resolution of the system is therefore not very satisfactory.

To improve the spatial resolution of Ohkawa's system, we have followed Lovberg's suggestion (Huddlestone et al., 1965, p. 79) by developing a miniature probe which measures both the flux density and its radial gradient in one measurement, with comparable accuracies. In future, to distinguish it from the conventional magnetic probe, we shall call it the gradient probe. It consists of two small identical search coils wound in the same direction and connected as shown in Fig. 3(a),(b) (see Fig. 23,24 also). After passing through an external balancing circuit, the output signals are fed into the differential amplifier of an oscilloscope. The balancing circuit ensures a zero signal for a uniform field while the differential amplifier helps to eliminate common mode signals.





Fig. 3(a). Sketch of gradient probe.

Fig. 3(b). Connections in gradient probe and balancing circuit. The gradient probe has several advantages. If a dual beam scope is used, one beam can measure the difference of the signals from the two coils and another beam picks up signals from one of them. This enables simultaneous measurements of both  $B_{g}$  and  $\frac{dBg}{dr}$ . Using equation (1), the local current density  $J_{z}(\mathbf{r})$  can be calculated from the information of one discharge only. This avoids the tedious procedure of mapping  $B_{g}(\mathbf{r})$ , and in addition, the values of  $\frac{dBg}{dr}$  obtained have comparable accuracy as that of  $B_{g}(\mathbf{r})$ . The spatial resolution is better than that of the Rogowski coil probe (Wright and Jahn, 1965).

#### (ii) Interaction of Probe with the Discharge

Another difficulty which limits the use of a probe is that it perturbs the plasma appreciably. A magnetic probe not only cools the plasma in its neighbourhood (thereby contaminating the plasma), it also distorts the current flow and the magnetic field by its geometry and by the reaction of the sensing coil on the field. The contamination and cooling of the plasma essentially increase the effective radius of the probe. For a plasma in a linear pinch having a temperature of 10 eV or less, the increase in the effective radius is small (Huddlestone, 1965, p. 103). Also since the current flowing in the coil is negligible compared with the discharge current, the reaction of the coil can be ignored. The most important correction in measurements using magnetic probes is therefore the distortion of the current flow.

If we can obtain the changes in  $\mathbf{B}$  caused by the probe, we can in principle allow for them, and deduce the value of which would occur in the absence of the probe. Ecker et al. (1962) have made a computer study of the probe perturbation on the steady current flow around a cylindrical probe, and Malmberg (1964) has given an analytic expression for the perturbed magnetic field near the hole in a thin plane infinite conductor. In Malmberg's calculations, he has assumed that the magnetic field  $\mathbf{B}$  is tangential to the conducting surface. This assumption is valid for high frequency alternating currents. However, for a low frequency field in which the skin depth δ is much greater than the thickness of the conducting sheet, such an assumption requires further refinements. For example, at a frequency of 1 MHz, the skin depth of brass is about .1 mm whereas the skin depth of a plasma in a fast linear pinch can be larger than 1 cm (see Table I). It is therefore not correct to assume that  $\overrightarrow{\mathbf{B}}$  is tangential to the metal surface without further justifications.

### Table I

	Conductivity tivity or (mhos/m)	Erequency ω (rad/sec)	Magnetic permeability µ (H/m)	Skin depth S (mm)
Brass	107	10 <sup>6</sup>	$4\pi \times 10^{-7}$	. 5
Plasma in a z-pinch	$10^4$ to $10^5$	10 <sup>6</sup>	$4\pi \times 10^{-7}$	14 to 5

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The previous models apply rigorously only to the current flow around the hole of an infinite plane conducting sheet. However, the results can also be used for current flows around a circular hole in a cylindrical sheet (see Fig. 4(a), (b), (c), and (d)), provided that the radius of the cylinder is much larger than that of the hole (see appendix I). In the case of a z-pinch discharge, we will approximate the plasma current by a. superposition of coaxial cylindrical current sheets of different radii.

In what follows, we shall briefly introduce the E-K-Z model (Ecker et al., 1962) and Malmberg's model (Malmberg,1964). In each case, an integral equation for calculating the true unperturbed current from the probe measurements is obtained ( $\S$  2.2.1 and  $\S$  2.2.2). We then consider the limitations of the E-K-Z model for time dependent currents in moving plasmas ( $\S$  2.2.3). In order that the integral equation can be solved numerically, it is transformed into a matrix equation suitable for a digital computer ( $\S$  2.3). Monte Carlo techniques are then applied to investigate how the experimental errors in the probe measurements affect the solution for the unperturbed current density ( $\S$  2.4). This is done by adding random errors to known current distributions and studying their effects on the solution of the integral equation employed in the correction procedure.

Before we introduce the various models, we define the following symbols and explain their physical meanings:

- 15 -



REGIONS WHERE CURRENT FLOWS ARE DISTORTED



Fig. 4(a). Cylindrical current sheet with a probe passing through it diametrically. Fig. 4(b) Approximating cylindrical surface by planes.



CYLINDRICAL CURRENT SHEETS THICKNESS OF CURRENT SHEET

Fig. 4(c). Cross-section of current sheets perpendicular to to axis of cylinder.

Fig. 4(d). Representation of plasma current by cylindrical current sheets. the unperturbed magnetic flux density -- the actual flux density in the absence of probes,

- the perturbed magnetic flux density -- the observed flux density produced by the plasma whose current flow pattern has been distorted by probe,
- ΔB,

J<sub>o</sub>,

в,

B<sub>n</sub>,

the change in magnetic flux density due to the presence of the probe, i. e.  $\vec{B} = \vec{B}_p - \vec{B}_0$ , the true current density -- the current density in the

41 S.

absence of a probe,

 $\vec{J}_p$ , the measured current density -- the computed value of the current density taking  $\vec{B}_p$  as the true flux density,  $\Delta \vec{J}$ , the change in current density due to the presence of the probe.

In the future, we shall denote the value of a quantity f at a coordinate r by f(r). Unless otherwise stated, this does not mean that f(r) is a function of r only. In the application of the correction procedure of this thesis, we are mainly interested in the axial current density and the azimuthal magnetic flux density in a linear discharge with an axial symmetry. Therefore, for convenience, we shall use the symbols  $B_0$ ,  $B_p$ and  $J_0$ ,  $J_p$  to stand for these components of the corresponding fields.

2.	2	•	1	The	Mode	1 by	7 Ec	cke	r,	Krö	11	and	ZÖ	ller	(E	- K	$-\mathbf{Z}$	model	)
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We now study the perturbation of a cylindrical probe of radius a on the plasma current flow in a linear discharge inside a cylindrical enclosure of radius R. We shall derive a formula which relates the perturbed current density to the true current density. The approach we use here differs from that used by Ecker et al. (1962). However, the result is essentially the same. We shall therefore call it the E-K-Z model.

For convenience, we first consider the time independent case. The problem will be extended to time dependent cases in & 2.2.3. We now assume the following.

(a) In the absence of the probe, the system is axially symmetric and the discharge current is in the axial direction,
 i.e.

(1) 
$$\vec{J}_0 = J_0(\mathbf{r})\vec{k}$$
,

k being a unit vector in the axial direction (see Fig.5).
(b) There is no radial flow of current, i.e.

$$(2) \qquad (\vec{J}_{o})_{r} = 0.$$

(c) The probe passes through the enclosure diametrically.
 (This is the arrangement we use in our experiment; see § 3.1)

From Maxwell's equations and their linearity, we readily obtain the change in azimuthal field along the axis of the probe as

(3) 
$$\Delta \vec{B}_{\phi}(r) = \frac{\mu}{4\pi} \int ds \frac{\left[\Delta \vec{j}(\vec{s}) \times (\vec{r} - \vec{s})\right]_{\phi}}{|\vec{r} - \vec{s}|^3}$$

Here  $\vec{r}$  and  $\vec{s}$  refer to the field point and the source point respectively. The rest of the symbols have their usual meanings.



Fig. 5. Perturbation of probe on plasma current

Since there is no radial flow, we can split up the discharge current into cylindrical layers and the current in each layer will be confined within the layer itself. If the radius of the cylinder is much larger than both the thickness of the layer and the radius of the hole (see Appendix I), the layer may be regarded as infinitely thin and the region where the distortion of current is significant will be small compared with the whole layer. Using the fact that the arc and the tangent subtended by a small angle differ by a quantity of second order in the angle, the region of the curved layer over which  $\Delta \vec{J}$  is significant can be replaced by the tangent plane at the centre of the hole.  $\Delta \vec{J}$  can now be solved with the help of Ohm's law, Maxwell's equations and their linearity, assuming the region of flow to be planar. For a cylinder of radius s and infinitesimal thickness ds,  $\Delta \vec{J}$  satisfies the equations

$$(4) \qquad \operatorname{div} (\Delta \vec{J}) = 0,$$

$$(5) \qquad \operatorname{curl}(A\vec{J}) = 0,$$

for  $\frac{1}{2}$  ds  $\langle \zeta \rangle \leq \frac{1}{2}$  ds and  $\rho \geq a_{\downarrow}$  (see Fig. 6). The boundary conditions for  $\Delta \vec{J}$  are

$$(6) \qquad \Delta \mathbf{J} = \mathbf{0}, \quad \mathbf{J}$$

far away from the probe and the normal component

(7) 
$$(\Delta \vec{J})_n = -(\vec{J}_o)_n$$

at the boundary of the hole punctured by the probe. Equations. (6) and (7) do not specify the problem uniquely because we do not know the tangential component of  $\Delta \vec{J}$  at the boundary. We can resolve this difficulty by using the symmetry of the problem and assumption (b) which corresponds to zero flow across the plane (see §2.2.3). Using the cylindrical coordinates ( $\rho$ ,  $\theta$ ,  $\varsigma$ ) defined in Fig. 6, standard calculations give

(8) 
$$\Delta \vec{J} = (\Delta J_{\rho}, \Delta J_{\theta}, \Delta J_{\zeta})$$
  

$$= J_{o}(s) \vec{F} (\rho, \theta, a)$$

$$= J_{o}(s) (-(\frac{a}{\rho})^{2} \cos\theta, +(\frac{a}{\rho})^{2} \sin\theta, 0), \qquad \rho \geqslant a,$$

a being the radius of the probe. Inside the hole, we have



Fig. 6. Approximating a curved surface by a plane

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From equation (8), it is clear that

(10) 
$$\left| \Delta \vec{J} \right| \sim \left( \frac{a}{\rho} \right)^2 \left| \vec{J}_0 \right|, \qquad \rho > a.$$

Therefore  $\Delta \vec{J}$  is significant for  $\rho < a'$ , where  $a' \sim a \ll s$ and the region where  $\Delta \vec{J}$  is significant is small compared with the cylinder.

Substituting (8) and (9) into (1) and integrating over  $\rho$  and  $\theta$  (see Appendix I), we obtain

(11) 
$$\Delta B_{\phi}(\mathbf{r}) = \frac{1}{2} \mu \mathcal{P}_{o} ds J_{o}(s) C'(\mathbf{r}, s, a) \frac{s}{r}$$

where

(12) 
$$C'(r,s,a) = C(\frac{r-s}{a}) - C(\frac{r+s}{a}),$$

and C(u) is defined by the functional relation

(13) 
$$C(u) = -\left[sgn(u) - u(1 + u^2)^{-\frac{1}{2}}\right].$$

The symbol  $\mathcal{P}_{o}^{\infty}$  ds denotes the principal value of the integral. We have two terms containing  $C(\frac{r+s}{a})$  and  $C(\frac{r-s}{a})$  respectively in the integral of (11) because there are two holes in each cylinder.

In deriving equation (11), we have assumed that a circular hole distorts the current flow in its neighbourhood in a cylindrical layer in the same way as it distorts the flow in a plane current sheet. This is a good approximation for

cylinders of radii large compared with that of the hole, but it does not hold for cylinders of small radii. However, if we are interested in  $\Delta B_{\phi}(r)$  far away from the axis such that  $(\frac{a}{r}) \ll 1$ , the predominant contribution to the integral of equation (11) comes from current shells of large radii for which the approximation holds. Besides, in a z-pinch discharge, the current density near the axis is negligible before the current shell has collapsed to the axis. Equation (14) should therefore be valid for a z-pinch discharge before the formation of the first pinch.

By the definition of  $\Delta B_{\phi}(\mathbf{r})$  (see §2.1), the perturbed azimuthal magnetic field  $B_{p}(\mathbf{r})$  along the axis of the probe is given by (14)

(14) 
$$B_{p}(r) = B_{o}(r) + B_{o}(r)$$
$$= \mu \int_{0}^{r} \frac{s}{r} J_{o}(s) ds + \frac{1}{2} \mu \int_{0}^{\infty} J_{o}(s)C(r, s, a)\frac{s}{r} ds.$$

The measured axial current density  $(\vec{J}_p)_z$  is then given by

$$(\vec{J}_p)_z = (curl \vec{B}_p)_z$$

or

(15) 
$$J_{p}(r) = \frac{1}{\mu} \begin{bmatrix} \frac{\partial B_{p}(r)}{\partial r} + \frac{B_{p}(r)}{r} \end{bmatrix}.$$

Substitution of (14) into (15) gives

(16) 
$$J_{p}(r) = \int_{0}^{\infty} J_{0}(s) K(r,s,a) ds,$$

where

(17) 
$$K(r,s,a) = \frac{s}{r} \frac{\partial C'(r,s,a)}{\partial r} .$$

From equation (11) and (16), the values of  $\Delta B_{\phi}(\mathbf{r})$  and  $J_{p}(\mathbf{r})$  due to a cylindrical layer of radius s and infinitesimal thickness ds can be written as

(18) 
$$f(r,s,a) ds = \frac{1}{2} \mu J_{o}(s) ds C'(r,s,a),$$

(19) 
$$g(r,s,a) ds = J_0(s) ds \frac{\partial C'(r,s,a)}{\partial r} \frac{s}{r}$$
,

respectively. The plots of C'(r,s,a), f(r,s,a)ds and g(r,s,a)ds are given in Fig. 7(a),(b) and (c).




## 2.2.2 <u>Malmberg's Model</u>

A separate calculation of the perturbation of the magnetic flux density by a hole in a plane infinite conductor is given by Malmberg. In his calculation, he assumes that the magnetic flux density near the conductor is tangential. This is valid in the case of a conductor in a field of such a high frequency that the skin depth is negligible compared with the thickness of the sheet. However, in the case of a plasma sheet in a field of 1 MHz which is typical in a fast linear pinch, such a condition is usually violated. In fact, for a typical plasma in a z-pinch having a conductivity of the order of  $10^4$ - $10^5$  mhos/m (Tuck, 1958), the skin depth  $\delta$  at this frequency is of the order .5 to 1.4 cm (see table I). Therefore, for a plasma sheet of thickness 1 to 2 mm, Malmberg's model is no longer applicable.

His calculation is essentially as follows. Consider a thin infinite plane conducting sheet in the  $\xi \eta$ -plane. A uniform current of current density  $\vec{J}_0$  flowing in the positive  $\xi$  direction in the sheet producing a magnetic flux density  $B_0(\zeta)$  given by

(1) 
$$\overrightarrow{B}_{0}(\varsigma) = \frac{1}{2} \mu J_{0} d\varsigma \overrightarrow{a}_{l}, \qquad \varsigma < 0,$$

(2) 
$$\vec{\mathbf{B}}_{0}(\boldsymbol{\xi}) = -\frac{1}{2} \mu J_{0} d\boldsymbol{\xi} \vec{\mathbf{a}}_{\eta}, \qquad \boldsymbol{\xi} > 0,$$

where  $d\zeta$  is the thickness of the current sheet and  $\vec{a}_{\eta}$  a

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unit vector in the  $\eta$  direction.

If a circular hole of radius a is now bored at the origin, the flow of the current near the hole will be much distorted (see Fig. 8(a)). The perturbed field  $\vec{B}_p$  outside the sheet is given by the Maxwell's equations

$$(3) div \vec{B}_p = 0,$$

$$(4) \qquad \operatorname{curl} \overline{B}_{p} = 0,$$

assuming that  $\vec{B}_p$  is tangential at the boundary surface S of the current layer, i.e.

(5) 
$$\vec{B}_{p} \cdot \vec{n} = 0,$$

where  $\overrightarrow{n}$  is the outer normal of S (see Fig. 8 (a) and (b)). The boundary condition at positions far away from the hole is

(6) 
$$\vec{B}_{p} = \pm \frac{1}{2} \mu J_{c} d\xi \vec{a}_{\eta}, \quad \xi \leq 0.$$

Since  $\overrightarrow{B}_p$  is irrotational as can be seen from (4), it can be regarded as the gradient of a scalar function  $\mathcal{Y}$ , or writing explicitly

(7) 
$$\vec{B}_p = \text{grad } \gamma$$
.

Substitution of this into (3) gives a Laplace equation for  $~~\gamma$ 

(8) 
$$\nabla^2 \varphi = 0$$
.

Now if we choose the oblate spheroidal coordinates ( $\alpha$ ,  $\beta$ ,  $\theta$ ) (Morse and Feshbach, 1953, p. 1292) taking the  $\zeta$  axis as the axis of symmetry, the boundary surface S coincides with the coordinate surface  $\beta = 0$  (see Fig. 8(b)) and the Laplace equation (8) is separable. Using the boundary conditions (5) and (6), the solution of (8) is

(9) 
$$\mathcal{Y}(\alpha,\beta,\theta) = -\frac{1}{2}\mu J_0 d\xi a \sin\theta \left[ (\alpha^2 + 1)(1-\beta^2) \right]^{\frac{1}{2}} \left\{ 1 + \frac{2}{\pi} \left[ \frac{\alpha}{1+\alpha^2} + \tan^{-1}\left(\frac{1}{\alpha}\right) \right] \right\},$$

for 
$$0 \le \beta \le 1$$
,  $0 \le \alpha < \infty$ ,  $0 \le \theta \le 2\pi$ , and

(9a) 
$$\psi(\alpha,\beta,\theta) = -\frac{1}{2}\mu J_0 d\xi a \sin\theta \left[ (\alpha^2 + 1) (1 - \beta^2) \right]^{\frac{1}{2}} \left\{ 1 + \frac{2}{\pi} \left[ \frac{\alpha}{1 + \alpha^2} + \tan^{-1} \left( \frac{1}{|\alpha|} \right) \right] \right\},$$

for  $0 \le \beta \le 1$ ,  $-\infty < \ll \le 0$ ,  $0 \le \theta \le 2\pi$ . Here the inverse tangent is defined for a positive quantity  $\gamma$  in the branch

(10) 
$$0 \leq \tan^{-1}\gamma \leq \frac{1}{2}\eta$$
.

The  $\mathcal{N}$  component of the perturbed field  $\overrightarrow{B}$  at the particular of the perturbed field  $\overrightarrow{P}$  at the  $\zeta$ -axis is then given

$$(11) \quad (\overrightarrow{B}_{p}(\varsigma))_{\eta} = -\frac{1}{2}\mu J_{o}d\varsigma \left[1 + \frac{2}{\eta} \left\{\frac{\varsigma}{1 + (\frac{\varsigma}{a})^{2}} - \tan^{-1}(\frac{a}{\varsigma})\right\}\right], \qquad \varsigma \geqslant 0,$$

$$= \frac{1}{2}\mu J_{o}d\varsigma \left[1 + \frac{2}{\eta} \left\{\frac{\varsigma}{1 + (\frac{\varsigma}{a})^{2}} + \tan^{-1}(\frac{a}{\varsigma})\right\}\right], \qquad \varsigma \leqslant 0.$$



circular hole 2

Fig. 8(a). Distorted current Fig. 8(b). Cross-section in flow in current sheet due to  $\xi\zeta$ -plane of the oblate spherjoidal coordinates

Substituting (1) and (2) into (11), we obtain

(12) 
$$\Delta B_{\eta}(\zeta) = -\frac{1}{2} \mu J_{o} d\zeta C(\frac{\zeta}{a}) ,$$

where C(u) is a correction factor defined by the functional

relation

(13) 
$$C(u) = \frac{2}{\eta} \left[ \frac{u}{1+u^2} - \text{sgn}(u) \tan^{-1}(\frac{1}{|u|}) \right].$$

To obtain a correction formula similar to (11) of  $\oint 2.2.1$  (p. 22) for a linear discharge in a cylindrical enclosure, we again split up the discharge current into cylindrical current layers. The contribution to  $\Delta B_{g}$  due to a current cylinder of an average radius s and an infinitesimal thickness ds is then of the form

(14) 
$$f(r,s,a) ds = \frac{1}{2} \mu J_0 ds C'(r,s,a) \frac{s}{r}$$

where we have put

(15) 
$$C'(r,s,a) = C(\frac{r-s}{a}) - C(\frac{r+s}{a})$$
.

We have added a factor  $\frac{s}{r}$  in (14) (compare (14) with (12)) to allow for the curvature of the layer (see Appendix I). The two terms  $C(\frac{r-s}{a})$  and  $C(\frac{r+s}{a})$  arise from the fact that there are two holes. Combining the effects of all such layers, the resultant change in azimuthal field gives

(16) 
$$\Delta B_{\phi}(r) = \frac{1}{2} \mu \mathcal{P}_{0} ds J_{0}(s)C'(r,s,a)\frac{s}{r}$$
, (compare (11) of §2.2.1).

With the same procedure as that used in  $\begin{cases} 2.2.1, we again obtain \end{cases}$ 

(17) 
$$B_{p}(r) = \mu \int_{0}^{r} \frac{s}{r} J_{0}(s) ds + \frac{1}{2} \mu \int_{0}^{\infty} J_{0}(s) C'(r, s, a) \frac{s}{r} ds,$$

(18) 
$$J_{p}(r) = \int_{0}^{\infty} J_{o}(s) K(r,s,a) ds,$$

where

(19) 
$$K(r,s,a) = \frac{s}{r} \frac{\partial C'(r,s,a)}{\partial r}$$
,

and

(20) 
$$g(r,s,a) ds = J_0(s) ds \frac{\partial C'(r,s,a)}{\partial r} \frac{s}{r}$$

The plots of C'(r,s,a), f(r,s,a)ds and g(r,s,a)ds are given in Fig. 7(a), (b) and (c).

The main difference between Malmberg's model and the  $\vec{E}-K-Z$  model lies in the fact that the former requires  $\vec{B}_p$  to be tangential everywhere at the surface of the current sheet while the latter requires the current to be tangential at the boundary of the hole.

In our experiment, we do not have all the information about the boundary conditions of the fields. To correct for the perturbation of a hole on the flow of electric current in a current layer, we therefore choose the simplest model appropriate to the experimental conditions. To do so, we first consider the following limiting cases.

(a) Low frequency limit.

For a low frequency magnetic field such that the skin depth  $\delta$  is comparable with the dimensions of the layer, the field will diffuse into the layer giving rise to a normal component at the surface and increasing the effective radius of the hole. The boundary condition in Malmberg's model is therefore not valid. In this case the E-K-Z model is applicable.

(b) High frequency limit.

If the frequency of the field is so high that  $\delta$  is negligible compared with the dimensions of the layer, the magnetic field at the surface of the layer is tangential. Malmberg's model is expected to be more suitable if the layer is not too thick.

A convenient test of the models is to measure  $(\overline{J}_{p})_{Z}$ for the return conductor of the z-pinch discharge. In our apparatus (see Fig. 22), the return conductor is 1 mm thick and is made of a grid of brass wires for convenience in taking pictures of the discharge. The holes in the grid are 1.5 mm × 1.5 mm square. Therefore both the thickness and the holes are small compared with size of the circular hole (7 mm in diameter) punctured by the probe. The squares surrounding the probe are filled up with solder so that the neighbourhood of the probe becomes a current sheet instead of a grid (see Fig. 9). Under such conditions, we may approximate the return conductor by a current sheet.

DIAMETER OF WIRE = 5mm DIST. BETWEEN AXES\_ OF WIRE = 2 mm SOLDER DIAMETER OF HOLE = 7mm

Fig. 9. Neighbourhood of hole in return conductor.

We have measured  $J_{p}(r)$  for the return conductor of the z-pinch discharge ( $\omega \sim 10^6$  rad/sec) and found that the most significant part of the experimental values of  $J_{p}(r)$  lies between the values predicted by Malmberg's model and the E-K-Z model respectively (see Fig. 10). From Table I, we realize that for the return conductor (brass),  $\delta \sim .5$  mm which is comparable with the thickness (~1.5 mm) of the conductor. The experimental conditions therefore lie somewhere between the low and the high frequency limits respectively. Under such conditions, the restriction that the magnetic flux be tangential at the hole is unrealistic. We therefore choose the E-K-Z model which requires a more realistic boundary condition, i.e.  $J_n = 0$  at the hole.



Fig. 10.

Plot of -  $J_p(r)$  at return conductor of z-pinch

### 2.2.3 Limitations of the E-K-Z model

In the derivation of the correction formulae (14) and (16) of  $\oint 2.2.1$  (p.§23), we assumed that the discharge was made up of a series of current layers. For the time independent system, the E-K-Z model is valid. However, if the current is moving, the Ohm's law on which the calculations are based contains extra terms which are not considered in the steady state calculations. In addition, for time dependent currents, electric fields depending on the rate of change of the magnetic. field also occur. These are not included in the steady theory either.

The calculation presented below is intended to establish the limiting speed and frequency of the discharge current such that the steady state E-K-Z theory remains valid.

# (i) Moving sheet

Consider a current layer carrying a current density  $\vec{J}$ and moving with a velocity  $\vec{v}$ . Ohm's law for such a system is

(1) 
$$\vec{J} = \sigma' (\vec{E} + \vec{v} \times \vec{B})$$

where  $\sigma$  is the electrical conductivity of the conducting medium,  $\vec{E}$  and  $\vec{B}$  are the electric and magnetic fields respectively. Since the model proposed by Ecker et al. (see § 2.2.1) uses the Ohm's law for a stationary current, i.e.

$$(2) \qquad \vec{J} = \sigma \vec{E} ,$$

we would like to investigate under what conditions equation (2) can replace (1). To do so, we compare the magnitudes of  $\vec{E}$  and  $\vec{v} \times \vec{B}$ .

From Maxwell's equations, we have

(3) 
$$\operatorname{curl} \vec{B} = \mu \vec{J}$$
.

If 1 is the characteristic length over which the fields change appreciably, equation (3) gives

(4) 
$$\left| \overrightarrow{B} \right| \sim \mu \mathbf{1} \left| \overrightarrow{J} \right|.$$

Assuming that  $\vec{v} \times \vec{B}$  is at the most of the same order of magnitude as the other terms in (1), we have

$$(5) \qquad |\vec{\mathbf{E}}| \sim \frac{1}{\sigma} |\vec{\mathbf{J}}|.$$

The various terms in equation (1) therefore bear the approximate ratios

(6) 
$$\left| \overrightarrow{J} \right| : \left| \overrightarrow{\sigma E} \right| : \left| \overrightarrow{\sigma v} \times \overrightarrow{B} \right| = 1 : 1 : \mu \sigma v l.$$

Therefore if

the term  $\sigma \overrightarrow{v} \times \overrightarrow{B}$  can be neglected.

Condition (7) is satisfied at the initial stage of a fast linear pinch discharge where  $\sigma \sim 10^4$  mhos/m,  $v \sim 10^3$  m/sec,  $1 = \delta \sim 10^{-3}$  to  $10^{-2}$  m,  $\mu \sim 10^{-6}$  H/m, and  $\mu \sigma v 1 \sim v (\delta \omega)^{-1} \sim 0.01$  to .1, 1 being taken as the skin depth  $\delta$ ,  $(\delta = \sqrt{\frac{2}{\mu \sigma \omega}})^4$  since the gradient of the field inside the current layer is governed by the skin depth (see Table I, p 14). At the later stage of the discharge where the collapsing current shell has been accelerated and has come closer to the discharge axis, this condition is quite marginal since both the velocity  $\vec{v}$ and the conductivity will have increased.

In principle, it is possible to include the effect of the term  $\vec{\nabla} \propto \vec{B}$  in equation (1). The change in current  $\Delta \vec{J}$ in a plane current sheet due to a hole will then be different from that obtained by Ecker et al. (see equation (8) in §2.2.1). However, the calculations will be very complicated. In fact it is obvious that both  $\vec{\nabla}$  and  $\vec{B}$  depend on  $\vec{J}$  and equation (1) will give a nonlinear equation in  $\vec{J}$ .

We must realize that in equation (1) we have neglected gravity, the kinetic pressure of the ions and the time variations in the Generalized Ohm's Law written in the form  $(1a) \frac{\vec{J}}{\sigma} = (\vec{E} + \vec{v} \times \vec{B}) - \frac{m_e}{n_e q_e} \frac{\partial \vec{J}}{\partial t} - \frac{1}{n_e q_e} \{ \nabla p_i - n_i m_i (\nabla \phi + \frac{\partial \vec{V}}{\partial t}) \}$ (see Rose and Clark, 1961 and Spitzer, 1962) where  $\nabla \phi \sim g$  is the gravitational field,  $p_i$  the kinetic pressure due to the ions only and the rest of the symbols have their usual meanings. Under our experimental conditions, we have  $n_e = n_i \sim 10^{22} \text{ m}^{-3}$ ,  $kT_i \lesssim .1eV$ ,  $m_e \sim 10^{-3} m_i \sim 10^{-31} \text{kg}, \nabla \phi \sim 10 \text{ N kg}^{-1}$ . The various terms in (1a) therefore bear the approximate ratios

$$1:1:\mu \text{ov} 1: \frac{m_e \sigma w}{n_e q_e^2} : \frac{\sigma kT_i}{|J| q_e 1} : \frac{\sigma m_i |\nabla \theta|}{|J| q_e} : \frac{\sigma m_i w v}{|J| q_e} = 1:1:10^{-1}:10^{-5}:10^{-2}:10^{-13}:10^{-3}$$

showing that our approximation is valid.

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We now consider time varying fields in a plane current layer of infinitesimal thickness ds having a circular hole of radius a (see Fig. 11). If the unperturbed current density is

(8) 
$$\overrightarrow{J}_{o}(\varsigma,t) = J_{o}(\varsigma,t) \overrightarrow{a}_{\xi}$$
,

 $\vec{a}_{\xi}$  being a unit vector in the  $\xi$  direction, we will show that, under restrictive conditions to be given below, the hole changes the current density by an amount

(9) 
$$\Delta \vec{J}(\rho, \theta, \xi, t) = J'_{0}(\xi, t) \vec{F}(\rho, \theta, a), \qquad \rho \ge a,$$
  
=  $-\vec{J}'_{0}(\xi, t), \qquad \rho < a,$ 

where  $\vec{F}(\rho, \theta, a)$  is the correction factor already defined in equation (9) of §2.2.1 (p. 21).



Fig. 11. Distorted current flow in plane layer with hole.

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To prove (9), we assume the following:

- (a) All the fields involved are simple harmonic and are proportional to  $e^{j\omega t}$ ,  $(j = \sqrt{-1})$ . This does not restrict problem to fields varying with e<sup>jwt</sup> only-since thea more general time varying field can be regarded as a more superposition of such harmonics.
- (b) The hole does not affect the field at  $\rho \rightarrow \infty$ , requiring

(10) 
$$\Delta \vec{J}(\rho \rightarrow \infty, \theta, \zeta, t) = 0.$$

- (c) The conductivity  $\sigma$  is constant across the layer.
- (d)  $\vec{\mathbf{v}}$  is small such that Ohm's law takes the simple form  $\vec{J} = \sigma \vec{E}$ .
- (e) There is no accumulation of charges inside the current layer, i. e.

$$(11) \qquad \qquad \operatorname{div} \vec{J} = 0.$$

- (f)  $\omega$  is so small that the displacement current is negligible.
- (g) There is no current flow across the layer,

$$(12) J_{\zeta} = 0.$$

This is a direct consequence of the assumption  $J_r = 0$ for a current cylinder in the E-K-Z model (see assumption (b) of (2.2.1, p. 18).

With these assumptions, the unperturbed current density  $\vec{J}_{o}^{i}(\varsigma,t)$ 

for a plane current layer is given by

(13) 
$$\vec{J}_{o}(\xi,t) = J_{o}Z(\xi,\omega) e^{j\omega t} \vec{a}$$

where  $Z(\varphi, \omega)$  is a solution of the equation

(14) 
$$\frac{d^2 z}{d \varsigma^2} = j \mu \sigma \omega z,$$

(see Landau and Lifshitz, 1960, p. 190). The explicit form of  $Z(\zeta,\omega)$  is given by

(15) 
$$Z(\zeta, \omega) = \cosh\left[\left(\frac{1+j}{\delta}\right)\zeta\right]$$
.

We may now set up a boundary value problem to solve  $\Delta \vec{J}$  in terms of  $\vec{J}'_0$ . From Maxwell's equations, Ohm's law and their linearity in the fields, the above assumptions require

(16) 
$$\operatorname{div}(\Delta \vec{J}) = 0$$
,

(17) 
$$\nabla^2(\Delta \vec{J}) = \lambda^2(\Delta \vec{J})$$
,

(18) 
$$\lambda^2 = j \mu \sigma \omega$$
.

For the boundary conditions, we know

(19) 
$$(\Delta \vec{J})_n = -(\vec{J}_0)_n$$

at the hole and

(20) 
$$\overset{\rightharpoonup}{J}(\rho \rightarrow \infty, \theta, \zeta, t) = 0$$

at large distances from the hole. Equations (19) and (20) do

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not completely specify the boundary conditions for a unique solution of  $\Delta \vec{J}$  since we do not know the tangential component of  $\Delta \vec{J}$  at the surface of the layer. To overcome such a difficulty, we use assumption (g), i. e.  $J_{\zeta} = 0$ , and the symmetry of the problem. This will greatly simplify the calculations.

In cylindrical coordinates  $(\rho, \theta, \beta)$ , equations (12), (16) and (17) give 2 2 2

(21) 
$$\left(\frac{\partial}{\partial \rho^2} + \frac{\partial}{\rho}\frac{\partial}{\partial \rho} + \frac{1}{\rho^2} + \frac{1}{\rho^2}\frac{\partial}{\partial \theta^2} + \frac{\partial}{\partial \varsigma^2}\right)\Delta J_{\rho} = \lambda^2 \Delta J_{\rho}$$

(22) 
$$\frac{\partial}{\partial \rho} \left( \rho \Delta J \rho \right) + \frac{\partial \Delta J \rho}{\partial \sigma} = 0 ,$$

$$\Delta J_{\zeta} = 0 .$$

The boundary conditions at  $\rho \rightarrow \infty$  and at the boundary of the hole give, respectively,

(24) 
$$\Delta \vec{J}(\rho \rightarrow \infty, \theta, \zeta, t) = 0$$

(25) 
$$\Delta J_{\rho}(\rho=a,\Theta,\zeta,t) = -(\vec{J}_{o}')_{\rho}.$$

By symmetry about the  $\xi$  axis and about the central plane of the layer, we also have

(26) 
$$\Delta J_{\Theta}(\rho, \theta, \zeta, t) = \Delta J_{\Theta}(\rho, -\theta, \zeta, t) ,$$

(27) 
$$\Delta \vec{J}(\rho, \theta, \xi, t) = \Delta \vec{J}(\rho, \theta, -\xi, t)$$
.

We now solve equation (21) by separation of variables (Morse and Feshbach, 1953). Putting

(28) 
$$\Delta J_{\rho}(\rho, \theta, \xi, t) = \underline{\Phi}(\rho, \theta) Z'(\xi) e^{j\omega t}$$

equation (21) gives

(29) 
$$\frac{1}{\underline{\Phi}}\left(\frac{\partial^2}{\partial \rho^2} + \frac{3}{\rho}\frac{\partial}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \theta^2}\right) \underline{\Phi}(\rho, \theta) = -m,$$

(30) 
$$\frac{\mathrm{d}^{-} \mathrm{Z}'(\zeta)}{\mathrm{Z}' \mathrm{d} \zeta^{2}} = \mathrm{m} + \lambda^{2} \quad .$$

Here m is an undetermined constant and may be obtained by considering the low frequency limit of the problem. At steady state, i. e.  $\lambda^2 = j \mu \sigma' \omega = 0$  (see (18), p. 38),  $\Delta J$  must be constant across the thickness of the layer and  $\frac{d^2 Z'(\zeta)}{Z' d\zeta^2} = 0$ . Equation (30) therefore requires m to be zero.

Rewriting (29) and (30), we have

$$(31) \qquad \left(\frac{\partial^2}{\partial\rho^2} + \frac{3}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2}\right) \not \equiv (\rho, \theta) = 0 ,$$

$$(32) \qquad \qquad \frac{\mathrm{d}^2 z'(\zeta)}{z' \mathrm{d} \zeta^2} = \lambda^2 .$$

Standard calculations using (24), (25), (26), (27), (28), (31) and (32) give

(33) 
$$\Delta \vec{J}(\rho,\theta,\zeta,t) = J_0 Z(\zeta,\omega) e^{j\omega t} \vec{F}(\rho,\theta,a), \quad \rho \ge a.$$

Here we have put

(34) 
$$Z(\zeta, \omega) = Z'(\zeta)$$

to stress on the  $\omega$  dependence. Substituting (8) and (13) into (33), we finally obtain

(35) 
$$\Delta \vec{j}(\rho,\theta,\zeta,t) = J_{0}(\zeta,t) \overrightarrow{F}(\rho,\theta,a), \quad \rho \geqslant a.$$

The solution of  $\Delta \vec{J}$  for  $\rho < a$  is trivial and is given by

(36) 
$$\Delta \vec{J}(\rho, \theta, \zeta, t) = -\vec{J}_{0}(\zeta, t), \qquad \rho < a,$$

because the original current  $\vec{J}_{o}(\zeta,t)$  in the hole is removed when the hole is introduced.

It is important to realize that for a current layer of finite thickness, the unperturbed current density  $\vec{J}_0$ defined in (8) for a plane differs from the unperturbed current density  $\vec{J}_0$  in a cylinder in their spatial variations over the thickness even if their average magnitudes are the same. In fact, for the cylindrical layer,  $\vec{J}_0(\varsigma,t)$  should be a linear combination of the Hankel functions of the first and the second kinds. However, at low frequencies such that the thickness of the layer is small compared with the skin depth  $\delta$ , the variations of  $\vec{J}_0$  and  $\vec{J}_0$  over the thickness become insignificant and their difference vanishes. Under such a condition, equation (8), (13) and (33) give

(31) 
$$\Delta \vec{J} = \Delta \vec{J}(\rho, \theta, t) = J_0 e^{j\omega t} \vec{F}(\rho, \theta, a)$$
.

Equation (31) can now be applied to time dependent

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cases in the E-K-Z model if we replace  $J_0(s)$  in equation (8) of §2.2.1 by  $J_0(s) e^{j\omega t}$  corresponding to the unperturbed time dependent current density in a thin current cylinder of radius s.

2.3 <u>Transformation of the Integral Equation into a Matrix</u> Equation Suitable for Solution with a Digital Computer

In principle, equation (16) of  $\S$  2.2.1 (p. 23), i. e.

(1) 
$$J_{p}(r) = \int_{0}^{\infty} ds J_{0}(s) K(r,s,a)$$

where

(2) 
$$K(r,s,a) = \frac{1}{2} \frac{s}{r} \frac{\partial C'(r,s,a)}{\partial r}$$

can be used to solve for  $J_o(r)$  analytically if  $J_p(r)$  is a known function. However, since  $J_p(r)$  is only given as a set of measured values at different values of r, we need to solve the problem numerically. One standard technique is to convert the integral to a summation and solve the integral equation as a matrix equation (Fox, 1962, p. 159). The range of integration is taken from 0 to a value R corresponding to a range over which  $J_p(r)$  is appreciable.

We divide the interval [0,R] into K equal intervals each of a width DX. The value  $s_N$  of s at the Nth interval  $(N \leq K)$  is then taken as the value of s at the centre of the interval, i. e.

(3) 
$$s_N = (N - .5) DX.$$

If r is the value of s at the midpoint of the Mth interval, we can easily see that

(4) 
$$r - s = (M - N) DX$$
,

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and

(5) 
$$r + s = (M + N - 1) DX.$$

If we put,

(6) 
$$u(M) = J_0(r) |_{r=(M-.5)DX}$$

(7) 
$$v(M) = J_p(r) |_{r=(M-.5)DX}$$

and  $\mathbf{and}$ 

(8) 
$$A(M,N) = K(r,s,a) DX | r=(M-.5)DX s=(N-.5)DX$$

equation (1) can be approximated by a set of equations of the form

(9) 
$$\sum_{N=1}^{K} A(M,N) u(N) = v(M), \qquad M = 1,2,3,\ldots,K.$$

In matrix notations, (9) takes the form

$$(10) \qquad \qquad A u = v$$

Here we have defined

(11) 
$$A = \begin{pmatrix} A(1,1) & A(1,2) & \dots & A(1,K) \\ A(2,1) & A(2,2) & \dots & A(2,K) \\ \dots & \dots & \dots & \dots \\ A(K,1) & A(K,2) & \dots & A(K,K) \end{pmatrix}$$

and u and v are column vectors whose Mth components are u(M) and v(M) respectively.

In order to eliminate unnecessary round off errors in the computation, we reduce the matrix A to a sparse matrix by putting small matrix elements equal to zero. In the matrix A, the diagonal elements are of the same order of magnitude and the nondiagonal matrix elements decrease rapidly as we go away from the diagonal. In fact, with the matrix that we are using, A(M,N) is reduced to a value smaller than .1% of A(M,N)as the difference of M and N is larger than 10. We therefore put

(12) 
$$A(M,N) = K(r,s,a) DX | , M-N \le 10,$$
  
 $r = (M-.5) DX$   
 $s = (N-.5) DX$   
(13)  $A(M,N) = 0,$   $M-N > 10.$ 

The shape of the matrix is shown in the following figure.



Fig. 12. Diagram showing form of matrix A Here the shaded area represents nonvanishing elements. The dummy indices M and N give the changes in r and s respectively.

A 3-dimensional figure of the magnitudes of the matrix

elements is shown in Fig. 13. It consists of a ridge along the diagonal of the matrix. A vertical cross-section of this ridge parallel to the r axis is of the shape shown in Fig. 14.



Fig. 13. Figure showing magnitudes of matrix elements of A.





The explicit form of A(M,N) depends on the form of the correction factor C'(r,s,a) and hence the model we choose for the correction procedure. The solution of (10) is then given by

$$\mathbf{u} = \mathbf{A}_{\mathbf{u}}^{-1} \mathbf{v}.$$

Therefore we need only invert the matrix A once and the solution for different v's obtained from experiments may then be calculated.

In our calculations, the  $50 \times 50$  matrix A obtained using the E-K-Z model has a condition number of 7.4 (Turing, 1948). The inverted matrix  $A^{-1}$  has the largest matrix elements near the diagonal. A similar cross-section for  $A^{-1}$ is shown in Fig. 15.



Fig. 15. Diagram showing cross-section of  $A^{-1+}$  parallel to r axis (a = 3.5 mm).

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## 2.4 Error Analysis

The purpose of this analysis is to show under what condition the correction procedure described above is likely to be useful. In order to do this, we first establish that the computer solutions of equation

(1) 
$$J_{p}(r) = \int_{0}^{\infty} ds J_{0}(s) K(r,s,a)$$

are not affected by round-off errors in the computer program (see  $\S2.4.1$  below).

Since the computer program is satisfactory, the next step is to determine the dependence of  $J_p$  on  $J_o$  for the kind of current distribution expected in the z-pinch. This analysis (see Fig. 16(a), (b)) shows that if  $J_o$  is a smooth function of r, the difference between  $J_p(r)$  and  $J_o(r)$  is very small (see Fig. 16(a)). However, if  $J_o(r)$  has the flimiting form of a  $\delta$ -function, the corrections become very large (see Fig. 16(b)). For practical purposes, if  $J_o(r)$ is a  $\delta$ -function, we shall assume it to be the current density distribution due to a current sheet of thickness equal to the mesh interval of the numerical calculation.

The next stage in the calculation ( $\S2.4.2$ ) is to investigate how fluctuations in  $J_p$  affect the calculated values of  $J_0$ . There are two sources of fluctuations in  $J_p$ ; one is due to actual variations in  $J_0$ ; the other can be ascribed to the processes employed in measuring  $B_p$ . In general, it is not possible to distinguish between these two possible sources of fluctuations in  $J_p$ . However, by considering ideal cases it is shown that measuring errors are more significant than errors which might be expected from fluctuations of reasonable magnitude in  $J_p$ .



Fig. 16(a). Computer result for a typical  $J_p(r)$  and its corrected value  $J_0(r)$ .



Fig. 16(b). Plot of  $J_{p}(r)$  and  $J_{p}(r)$  versus r assuming  $J_{p}(r)$  to be a S-function.

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## 2.4.1 Round-off Errors due to Numerical Calculations

Since we solve the integral equation (see (1) of  $\S2.4$ , p. 48) by transforming it into a matrix equation (1)A u = v, (see (10) of §2.3, p. 44) where A is a  $50 \times 50$  matrix, we have to make sure that round-off errors are insignificant. To do so, we use a known current density distribution  $J_{\alpha}(\mathbf{r})$ and calculate  $J_{p}(r)$  from (1) numerically. With the calculated values of  $J_p(r)$ , the solution  $J_o^*(r)$  of equation (1) is obtained. The result of  $J^*_{o}(r)$  is then compared with  $J_{n}(r)$ . If the calculation is accurate, the difference  $J_{o}^{*}(r) - J_{o}(r)$  should be negligible compared with  $J_{o}(r)$ . Such a procedure is done numerically using equation (1). If  $J_{\alpha}(r)$  is represented by the vector u, and v is calculated from equation (1), then  $J_0^{\star}(\mathbf{r})$  is the vector  $A^{-1}Au$ . We therefore have to compare  $A^{-1}Au$  with u. Computer results for typical trapezoidal current distributions with fluctuations

are obtained (see Fig. 17). In all cases, we find that the fractional errors  $(A^{-1}Au - u)/u \leq \overline{10}^{-6}$  showing that round-off errors in the numerical calculations are negligible. We have tried different mesh intervals from 1 mm to 4 mm, taking the range of integration over r from 0 to 10 cm corresponding to the radius over which the measured values of  $J_p$  are not zero. The condition numbers (Turing, 1948) of the matrices corresponding to the various mesh intervals are given in

Table II.

### Table II

<u>Mesh Interval</u>	Dimension of Matrix	Condition Number
4 mm	25×25	1.4
2 mm	50x 50	7.4
1 mm	100x100	10 <sup>12</sup>

Since the spatial resolution of the probe is about 3 mm, we choose the 2 mm mesh interval. The  $50 \times 50$  matrix thus obtained is reasonably well-conditioned.

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## 2.4.2 The Influence of Fluctuations

From a measured set of values of  $J_p$ , it is not possible to ascertain whether any scatter about a mean value at a given point corresponds to real fluctuations in  $J_o$ , or whether the fluctuations are merely due to inaccuracies in measurements. If we can be sure there are no instabilities, then fluctuations in  $J_o$  (the true current density) are unlikely. However, if this is not the case, we need some method of determining the causes of fluctuations in the measured values  $J_p$ . Some assessment of the relative importance of the two contributions can be obtained by considering idealized cases.

Suppose for example that fluctuations in  $J_p$  can only arise from fluctuations in  $J_o$ . A current spike is greatly smoothed and reduced in magnitude by the perturbation of a probe. We therefore expect that any fluctuations in  $J_o(r)$ will be much smoothed out also (see Fig. 17).



Fig. 17. Computer result of  $J_p(r)$  showing smoothing effect of probe on fine structure of  $J_p(r)$ .

To investigate the smoothing caused by the probe, we consider the effects of random errors added to a trapezoidal distribution for  $J_o(r)$  (typical of z-pinch discharges). The errors have a normal distribution with a standard deviation  $\sigma$  of 30% of  $J_o(r)_{max}$ , i. e.

 $\sigma = .3 |J_0(r)|_{max}$ , (see Table III).

They are assigned to various points r, using tables of pseudo-random numbers generated by the computer (Shreider, 1966). These calculations show that the standard deviation  $\sigma_p$  of the errors produced in  $J_p(r)$  is given by the equation

$$\sigma \sim 3 \sigma_p$$

Here we use the  $J_{o}(r)$  given in Fig. 16(a).

$$\frac{\text{Free III}^{\circ} + \text{Fluctuations in } J_p(r) + \text{due}}{\text{to Fluctuations in } J_o(r)}$$

$$J_{0}(r)_{max} \qquad \sigma \qquad \gamma_{p} \qquad \frac{\sigma}{\sigma_{p}} \qquad (10^{7} \text{ Amp m}^{-2}) \qquad 20. \qquad 1.0 \qquad .35 \qquad 2.9 \\ 20. \qquad 2.8 \qquad 1.1 \qquad 2.6 \\ 20. \qquad 2.8 \qquad 1.1 \qquad 2.6 \\ 20. \qquad 4.8 \qquad 1.3 \qquad 3.7 \\ 20. \qquad 7.4 \qquad 3.1 \qquad 2.4 \\ 20. \qquad 9.3 \qquad 3.7 \qquad 2.5 \end{cases}$$

 $C^{\gamma}$ 

The reduction in the standard deviation arises because the probe "correlates" random errors occurring at neighbouring points. It might at first sight appear therefore that to determine  $J_{\alpha}$  to an accuracy  $\sigma'$ , we only need to evaluate J to an accuracy of  $\sigma/3$ . Unfortunately this is not the case, because the errors in  $J_p$  ( arising from random errors in J have a particular spatial distribution, i.e. the errors at neighbouring points are correlated (see Fig. 17). However,  $J_{p}$  will also have measuring errors which will be truly random --- i. e. no spatial correlations. These errors should be removed from the values of  $J_{p}$  before using the correction procedure. In principle, this could be done by measuring a whole set of  $J_{n}(r)$  several times and using the mean. However, this procedure is generally impracticable, and in any case evidence for instabilities might also be lost.

The practical solution is therefore to examine what happens if the measuring errors are fed through the correction program. We again employ a Monte Carlo method, starting with a  $J_p$  corresponding to a trapezoidal distribution for  $J_o$ (Fig. 16(a)). Normal errors with a standard deviation  $\sigma'_p$ are assigned at random to the values of  $J_p$ , and the resulting deviations in  $J_o(\sigma'_p)$ , the standard deviation  $\sigma'$ , are computed from equation (1) of §2.4,

(1) 
$$J_{p}(r) = \int_{0}^{\infty} ds J_{o}(s) K(r,s,a)$$
.

A typical computer result is shown in Fig. 18.

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Fig. 18. Errors in 
$$J_{o}(\mathbf{r})$$
 due to errors in  $J_{p}(\mathbf{r})$ .  
 $\sigma'_{p}$  of  $J_{p}(\mathbf{r}) \sim .005 \times |J_{p}|$ ;  $\sigma'$  of  $J_{o}(\mathbf{r}) \sim .1 \times |J_{o}|$ .

Repeating the calculation for different assignments of the errors leads to the following results.

Table IV Fluctuations in J (r) due to Fluctuations				
	in J <sub>p</sub> (r)			
$J_{o max}$ (10 <sup>7</sup> Amp m <sup>-2</sup> )	$\sigma^{\prime}$ (10 <sup>7</sup> Amp m <sup>-2</sup> )	$\sigma_{\rm p}^{\prime\prime}$ (10 <sup>7</sup> Amp m <sup>-2</sup> )	ڡؚؠ ڡ	
20.	2.0	.1	20.	
20.	5.0	.28	18	
20.	9.7	.48	20.	
20.	14.	.74	19.	
20.	15.	• 93	16.	

From Table IV, it is obvious that the standard deviation in the measured results is increased by an order of magnitude in the correction process. In fact, we have

This result has very serious implications as can be seen by the following argument. Let  $J_o$  be the true current which would be calculated if  $J_p$  could be measured accurately, and let  $\hat{J}_o$  be the value of the "true" current calculated when  $J_p$  has a measuring error with a standard deviation  $\sigma'_p$ . We have from above for a trapezoidal current distribution

(2) 
$$\left| J_{o} - J_{p} \right| \lesssim .05 \left| J_{p} \right|.$$

However, from the above calculations

(3) 
$$\left| \hat{J}_{o} - J_{p} \right| \sim 20 \sigma'_{p}$$
.

If  $\sigma'_{p}$  is less than 1% of J (a rather unrealistic assumption), then we have

(4) 
$$\left| \hat{J}_{o} - J_{p} \right| \lesssim \cdot 2 \left| J_{p} \right|$$

Equations(2) and (3) indicate that

(5) 
$$\left| J_{o} - J_{p} \right| \lesssim \cdot 2 \left| J_{p} \right|$$

If we compare the inequalities (4) and (5), we see that the measured current  $J_p$  (with errors) is probably a better approximation to the true current  $J_o$  than the values  $\hat{J_o}$  computed

from  $J_n$  by means of equation (1).

Equation (4) therefore indicates that unless one has reason to believe that measuring errors are less than  $\frac{1}{4}$ , the measured current is normally the best approximation to the current which would flow in the absence of the probe. However, for "spiky" current distributions, the correction technique for calculating J<sub>o</sub> is still of value since the correction is several times larger than both J<sub>p</sub> and the errors it introduces (see Fig. 19 for a typical computer result).





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## 2.4.3 Spatial Resolution

A final problem which must be considered is the spatial resolution of the probes. In our experiment section, we define the values of  $B_p$  and  $\frac{dB}{dr^p}$  at r by the expres- $B_{p} = \frac{1}{2}(B_{1}+B_{2}), \frac{dB_{p}}{dr} = \frac{B_{1}+B_{2}}{2b}, \text{ (see § 3.3), where}$ sions 2Ъ is the spacing between the sensing coils in the gradient probe. If we denote the defined quantities by asterisks, then we have

(1) 
$$B_{p}^{*}(r) = \frac{1}{2} \left[ B_{p}(r+b) + B_{p}(r-b) \right],$$

(2) 
$$\frac{dB_{p}^{*}(r)}{dr} = \frac{1}{2b} \left[ B_{p}(r+b) - B_{p}(r-b) \right],$$

where  $B_p^{\star}(r)$  is the measured values of  $B_p(r)$  at the points. Expanding these equations yields the following results,

(3) 
$$B_{p}^{*}(r) = B_{p}(r) + \frac{b^{2}}{2!} \frac{d^{2}B_{p}(r)}{dr^{2}},$$

(4) 
$$\frac{dB_{\mathbf{p}}^{\star}(\mathbf{r})}{d\mathbf{r}} = \frac{dB_{\mathbf{p}}(\mathbf{r})}{d\mathbf{r}} + \frac{b^2}{3!} \frac{d^3B_{\mathbf{p}}(\mathbf{r})}{d\mathbf{r}^3},$$

dr

neglecting higher order terms in the Taylor expansions. Hence for the defined values to be equal to the values which would actually be measured at r, we must have

For comparison, we compute the values of  $B_p(r)$ ,  $\frac{dB_p(r)}{dr}$ ,  $\frac{b^2}{2!} \frac{d^2 B_p(r)}{dr^2}$  and  $\frac{b^2}{3!} \frac{d^3 B_p(r)}{dr^3}$  for the return conductor using the E-K-Z model (see §2.2.1). The computed results show that for the gradient probe used in this thesis, we have

(7) 
$$\left| B_{p}(r) - 1\left(\frac{b^{2}}{2!} \frac{d^{2}B_{p}(r)}{dr^{2}}\right) \right| < .001.$$

Therefore we may assume that  $B_p^{\star}(r) = B_p(r)$ .

For the gradient, however, we obtain

(8) 
$$\frac{dB_{\rm p}(\mathbf{r})}{d\mathbf{r}}^{-1} \left(\frac{b^2}{3!} \frac{d^3 B_{\rm p}(\mathbf{r})}{d\mathbf{r}^3}\right) < .1,$$

showing that  $\left| \frac{dB_{p}}{dr} - \frac{1}{r} \left( \frac{dB_{p}}{dr} - \frac{dB_{p}}{dr} \right) \right| < 10\%$ . Therefore we plot both  $\frac{dB_{p}^{*}(r)}{dr}$  and  $\frac{dB_{p}(r)}{dr}$  against r in Fig. 20. These results show that the gradient probe we use should reproduce true values of  $B_{p}(r)$  and  $\frac{dB_{p}(r)}{dr}$ .

Hence the major influnce on the computed values of  $J_{o}$  is the radius of the probe, and not the coil spacing. This result together with the observed form of the correction factor indicates that the spatial resolution of the probe is approximately 3 mm (~ a).



Fig. 20. Graphs showing comparison between  $\frac{dB_p^*}{dr}$  and  $\frac{dB_p}{dr}$  (assuming a cylindrical current shell of radius 8.5 cm).

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## 2.4.4 Conclusions from Error Analysis

From the error analysis presented in previous several sections, we have shown that magnetic probe destroys information about the fine structure of the current in a plasma. The errors in the unperturbed current density  $J_{o}(r)$  are about twenty times the errors in the measured values of  $J_{p}(r)$ . Since in most cases, the difference  $J_0 - J_p(r)$  is about 5% of  $J_p(r)$ , except where  $J_{0}(r)$  is a sharp current spike (see equation (2), p. 56), the correction is not meaningful unless  $J_p(r)$ ismeasured with an accuracy of better than  $\frac{1}{4}$ %. Such an accuarcy is quite impossible in most cases. In addition, highly unstable pulsed discharges present further difficulties due to a lack of reproducibility in the probe signals. However, if we are certain that the current distribution has a sharp "spike", we must correct the measured value of  $J_{p}(r)$  to obtain  $J_{0}(r)$ . For this special case, the correction is usually several times larger than both  $J_{p}(r)$  and the possible errors introduced by the correction process.

## Chapter 3

## EXPERIMENTAL RESULTS

### 3.1 Introduction

In this chapter we describe the experimental technique used in measuring the current density of a linear pinch discharge in helium. We choose helium because several workers in our laboratory have been studying z-pinch discharges in helium using such techniques as time resolved spectroscopy, Stark broadening measurements, and laser interferometry, and it is profitable to compare our current density measurements with their results. This will provide a better picture for the structure and the dynamics of the z-pinch.

The apparatus for the z-pinch discharge used for this experiment consists of a Pyrex tube of 15 cm i.d., a vacuum system and a 5 KJ condenser bank having a natural oscillation period of about 20 µsec. Details of this equipment appear in Table V. For current density measurements, we use a gradient probe which has already been briefly described in §2.1. The theory, properties and noise problems of the gradient probe are outlined in §3.3. Because the sensitivity of a magnetic probe is usually low, i.e. it can only detect strong fields such as those produced by pulsed discharges, experimental calibration requires strong pulsed fields and is itself an important technique. This will be explained in §3.3.1. §3.3.2 describes the calibration and the frequency response of the

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probe and  $\S3.3.3$  explains why reproducibility of probe signals is necessary.

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In §3.4.1, we give the experimental results of the current density measurements for z-pinch discharges in helium at initial pressures of 500  $\mu$ , 1 mm, 2 mm, and 4 mmHg respectively. To explain the results, a modified snow-plow equation which considers a loaded, thick current sheet is developed. Using the magnetic pressure measured from the probe, the equation is solved numerically with the help of the IBM 7040 computer. The computed radius of the collapsing current shell in the discharge is then compared with experimental findings.

### 3.2 Apparatus

The apparatus used in this experiment consists mainly of a linear pinch discharge system and a gradient probe circuit. In this section, we only give details of the linear pinch device and shall leave the detailed description of the probe circuit in the next section ( $\S$ 3.3).

The discharge system consists of a 5 KJ condenser bank power supply, a triggering system, a vacuum system and a 15 cm i.d. pyrex discharge tube. The condenser bank and the triggering system are conventional in design (Medley, 1965a). The basic specifications are listed in Table V, and the circuit diagram appears in Fig. 21. The detailed design of the discharge tube is shown in Fig. 22, because it differs slightly from the usual discharge tube. The main difference is that the probe moves in a guide-tube mounted across the diameter of the discharge vessel. In a normal system, the probe is introduced into the vessel through an o-ring seal in the wall.

## Table V Apparatus

#### (A) En

# Energy Source for Discharge

(1) Condenser Bank

Total capacity (5×(10 ± .1) μF N.R.G. Low Inductance<br/>Storage Capacitors in parallel)53 μFTotal Inductance.12±.01 μHCharging Voltage10.0±.2 kVMaximum Discharge Current200 k AmpRinging Frequency100 kHz

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(2) Triggering System (Medley, 1965a)

Pulse generator (produces -9 kV spike voltage of a rise time of 40 nsec and a duration of 6 psec)

Theophanis unit (doubles spike voltage from pulse

generator)

Ultra violet trigger Main triggering spark gap

- (3) Voltage Measurement Convoy Microameter A.V.O. Multiplier (25 kV d.c.)
- (4) Current Measurement Rogowski Coil (Sensitivity:(1.86±.11) 10<sup>5</sup>Amp Volt<sup>-1</sup>) Integrator (RC passive, integration time constant: 24 msec)

## (B) <u>Discharge System</u>

1)	Discharge tube (pyrex)		
	Inner diameter	15	cm
	Outer diameter	17	cm
	Length	61	$\mathbf{cm}$
	Electrodes	Bra	iss
	Electrode separation	59	cm

(2) Vacuum System

Type 17 Balzers Oil Diffusion Pump Hyvac 14 Cenco Backing Pump Vacuum attainable 1 µHg Leak Rate 7 µHg/hr Macleod Gauges 0-1 mmHg 0-10 mmHg

Pyrani Gauge (Type GP-110 Pirani Vacuum Gauge)



Fig. 22. Cross-section of discharge tube.

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## 3.3 The Gradient Probe and the Delay Line

A gradient probe consists of two small search coils connected in such a way that they not only measure the magnetic field, but also the difference of the fields at the respective positions of the coils. For use in current density measurements of a pulsed plasma, the probe should have

- (a) small size for minimum perturbations on the plasma,
- (b) small sensing coils for good spatial resolution (this requires small areas and small numbers of turns),
- (c) large bandwidth response so that it gives true information from high frequency fields (this requires small inductances or small coils with very few turns),
- (d) good sensitivity for accurate measurements and large signal to noise ratios.

Conditions (a), (b) and (c) are compatible with each other since they all require coils with small cross sections and a low number of turns. However, these all help to violate condition (d) which requires exactly the opposite. To meet all these requirements, therefore, a compromise has to be made.

In order to measure  $\frac{dB_{p}(r)}{dr}$ , it is often necessary to record accurately the difference between two large signals which are almost equal to each other. This requires that the two probe circuits be as closely identical to each other as possible. In particular they must be balanced so that in a

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uniform magnetic field, the measured value of  $\frac{dB_{p}}{dr}$  is zero (i. e. the difference between typical output signals from the two coils is 100 times smaller than the signal from each coil).

Accurate values of  $\frac{dB_{\rm p}}{dr}$  also requires the elimination of common mode signals from the probe. This is accomplished by the differential amplifier in the recording oscilloscope, which is carefully adjusted to give a common mode rejection of 10<sup>4</sup>.

A design which meets the above requirements is shown in Fig. 23. It consists of two small coils  $L_1$  and  $L_2$  attached to the tip of a small glass tube. Each coil is made of 100 turns of AWG No. 42 enamelled copper wire wound on a P.V.C. insulating tube of 1.8 mm o.d. The coils are held in place side by side by means of Scotch tape. The leads from each coil are twisted tightly together so that the only emf induced is produced by flux changes in the coils themselves. They are then connected to a resistor  $R_{2}$  and a miniature potentiometer  $\infty$  $R_1$ , which are enclosed in a metal casing (see Fig. 24).  $L_1$ and  $L_2$  are connected in such a way that they produce signals of the same sign when the probe is placed in a uniform field. To screen out noise signals, the probe outputs are fed to an oscilloscope through a screened twin-feeder. By delaying the signals, they can be displayed after "hash" produced in the scope by the discharge has disappeared from the oscillogram (Medley, 1965b). The delay is produced by two delay units (General Radio 314886, characteristic impedance 220 ohms,

maximum delay .5 µsec --- see Fig.24 ). Since we are interested in values of the magnetic field B<sub>p</sub>, the probe outputs must be integrated with respect to time. This is accomplished by a passive RC integrator (see Fig. 23).  $\frac{dB_{n}}{dr}$  can be calculated from the equation  $\frac{dB_p}{dr} = \frac{B_2 - B_1}{\Delta r}$ , where  $B_1$  and  $B_2$  are the measured fields obtained from the two integrated probe signals, and  $\Delta r$  is the distance between the coils (see Fig. 23).  $\frac{B_2-B_1}{\Delta r}$ is taken to be the value of  $\frac{dBp}{dr}$  midway between the two coils.  $B_2 - B_1$  is measured directly by the differential amplifier in the oscilloscope (Tektronix Differential Amplifier Type W; Tektronix Scope 551). The output from one coil of the probe is also amplified by another amplifier (Tektronix Type G), and displayed on the other beam of the oscilloscope. This signal is proportional to  $B_1$ . The value of  $B_p$  midway between the two coils is taken to be

 $B_{p} = B_{1} + \frac{1}{2}(B_{2} - B_{1})$ 

where  $B_1$  and  $(B_2-B_1)$  are both recorded signals.

The integrator has a time constant of 100 µsec and  $R_3$ ,  $R_4$ ,  $R_5$  and  $R_6$  are terminating resistors (Fig. 24) of 220 ohms each.  $R_7$  is a 2.2 k ohm resistor and  $R_8$  is composed of a 1.0 k ohm resistor in series with a 1.5 k ohm miniature potentiometer. The circuit is made as symmetrical as possible in order to obtain a good balance for the two probe coils. The signals at the screen of the scope are recorded with a polaroid camera. The essentials of the gradient probe and the accompanying measuring devices are shown in Table VI.

### Table VI Gradient Probe and Measuring Devices

(1) Gradient Probe and Loading Circuits Sensitivity in gradient measurements (see Table VII) Sensitivity in flux density measurements (see Table VII) Transmission line (10 meters RG 880 twin-feeder) belay Unit (two General Radio 314886 delay units; characteristic resistance: 220 ohms; delay

time:.5 µsec)

Integrator (passive RC network; integration time constant: 100 µsec)

(2) Oscilloscope and Accesories

Oscilloscope (Tektronix Type 551 Dual Beam Scope) Plug-in Units:

> Tektronix Type W Differential Amplifier Rejection ratio: 10<sup>4</sup>

Pass band: dc to 20 MHz at 1 mV/cm dc to 8 MHz at 1 mV/cm Tektronix Type G Differential Amplifier Rejection ratio: 10<sup>3</sup>

> Pass band: dc to 18 MHz from .5 V/cm to 20 V/cm)



Fig. 23. Design of gradient probe (to be inserted into a guide tube across the discharge tube; see Fig. 32(a), and (b) also).



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3.3.1 Balancing of Probe and Calibration Circuits

To balance and calibrate the gradient probe, a uniform magnetic field is generated by discharging the condenser bank through two parallel leads connected to a resistance as shown below.



Fig. 25. Calibration circuit (b = width of current leads = 20 cm; h = separation between leads = 1 cm ).

With its complete loading circuit connected (see Fig. 24), the probe is first balanced by adjusting the potentiometer  $R_1$  so that the probe gives zero output when the condenser bank is discharged (see Fig. 26). The uniform field  $B_1$ between the copper strips is  $\mu$  I/b, where b is the width of the strips and I is the current. Since the signal  $V_1$  from each coil of the probe is proportional to the field  $B_1$ , we have  $V_1 = k_b B_1$  where  $k_b$  is a constant. If I is measured,  $B_1$  is known. Therefore the above equation serves to calibrate the probes.

To complete the calibration, we have to know I. This is measured by a Rogowski coil. The signal from the Rogowski coil is fed into the scope through an integrator having a time constant of 2.4 msec which is much longer than the significant time constant of the discharge (20 psec). The output from this integrator  $(V_r)$  is proportional to the current in the circuit, i. e.  $V_r = k_i I$ , where  $k_i$  is a constant. Integrating this eqaution with respect to time gives

(1) 
$$\int_{0}^{\infty} \mathbb{I} dt = Q = k_{1}^{-1} \int_{0}^{\infty} V_{r} dt$$

where Q is the total charge discharged from the capacitor. This is equal to  $CV_c$ , the product of the capacity and the charging potential. Since  $\int_{o}^{\infty} V_r dt$  can be measured and C and  $V_c$  are known, equation (1) enables  $k_i$  to be evaluated. The probe sensitivity  $k_b$  is then given by the final equation

$$k_{b} = \frac{\mu V_{r}}{b k_{i} V_{1}}$$

(2)

In our experiment,  $\int V_r dt$  is measured on an enlarged oscillogram of  $V_r$  (see Fig. 26) with the help of a planimeter.

We have made two gradient probes (Probe No.1 and No.2). Their sensitivities are given in Table VII.



### 3.3.2 Frequency Response with Loading Circuits

In principle, the frequency response of a magnetic probe could be obtained by inserting the probe into a timevarying, strong magnetic field. However, it is difficult to design such systems which give fields greater than  $10^3$ gauss at frequencies higher than 1 MHz. Instead of using a strong magnetic field to produce an emf in the coil, we therefore use a signal generator connected in series with the coil (see Fig. 27(a)). In order to minimize stray signals, all connections are kept as short as possible and a special cover (see Fig. 27 (b)) is made to replace the bottom cover of the balancing unit of the probe (see Fig. 23, p. 71). The grounded lead of one of the coils is now disconnected from the casing and soldered on to the central pin of the connector T (see Fig. 27(a) and (b)).



Fig. 27(a). Circuit for determining frequency response of probe and loading circuit (see Fig. 24 also).

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Fig. 27(b). Special cover to replace bottom of casing of balancing unit.

To obtain a sine wave response, the signal from a sine wave signal generator (Tektronix Type 190B Constant-Amplitude Signal Generator) is input at T. The output from the probe through the loading circuit is then observed. The ratio and the phase shift between the input and output signals are shown in Fig. 29. In the same figure, we also plot the frequency response of the probe and the induced emf in one coil due to the other. The frequency response of the integrator is given in Fig. 28.



Fig. 28. Frequency response of integrator (  $A^{\pm} = amplication;$  $f^{\pm} = frequency; \Phi = phase shift).$ 



Fig. 29. Frequency response of probe (with complete loading circuit except integrator;  $|e_0/e_1|$  = ratio between signals from coils).

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3.3.3 Reproducibility of Probe Signals

There are many factors that would affect the probe signals. If their effects are significant, the measured signals might be different from one discharge to another even though experimental conditions are not varied in successive discharges.

Changes in experimental conditions arise from errors in the initial charging voltage of the condenser bank and errors in the filling pressure of gas in the discharge vessel. These can be reduced by performing the experiments with greater care and patience. However, there is one intrinsic difficulty in pinch discharges which is difficult to allow for, if it is present. This is the problem of instabilities. They arise spontaneously from small perturbations during the discharge (Rose and Clark, 1961), and often cause variations in the probe signals from successive discharges. It is therefore most important in magnetic probe measurements to ensure that the observed signals are reproducible.

In our experiment, we found that the signals from the probe were reproducible until the first pinch occurred. Fig. 30 shows a typical record of several superposed signals from various discharges under the same experimental conditions.

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20R0 5 TRACES OPERIMPOSED)

TIME (1 usec/cm) in

Fig. 30. Reproducibility of probe signals; for z-pinch discharge in He at 4 mm initial pressure (upper trace:  $\frac{dB_{p}}{dr} \text{ at .02 V/cm; lower trace: } B_{p} \text{ at.2 V/cm; sweep:}$ 1 µsec/cm).

One other characteristic which was checked before embarking on measurements was whether common mode signals were appreciable. This was done by recording the probe signal and rotating the probe through 180° about its stem. The signals obtained for the two orientations were mirror images of each other, indicating the absence of common mode signals (i.e. no spurious "pick-up") (see Fig. 31).

EKTICAL DEFLECTION CRC

TIME (I useckm)

diger 50mV/cm

By (r) 200 mV/cm

Fig. 31. Probe signals showing absence of common modes, for z-pinch discharge in He at 4 mm initial pressure.

## 3.4.1 Experimental Results

In this section we explain how the gradient probe is used for finding the current density of a linear pinch discharge. We first describe the arrangements of the probe and explain how measurements are taken and recorded. Then we give the measured current densities of discharges in helium at 500  $\mu$ , 1 mm, 2 mm and 4 mmHg initial pressures respectively. Finally we present graphs of the discharge radius as functions of time.

The location of the probe in the discharge tube is shown in Fig. 32(a) and (b). To map the current density of a linear pinch discharge, probe signals for B<sub>p</sub> and  $\frac{dB_p}{dr}$  are obtained at different probe positions for different discharges under the same experimental conditions. In order to reduce contamination, each discharge is produced in fresh gas (except for measurements in helium at 4 mmHg initial pressure when the effect of contamination is apparently negligible----the probe signals at a given position are identical in several successive discharges without changing gas).

The measured values of  $B_p$  and  $\frac{dB_p}{dr}$  are then employed to calculate the perturbed current density  $J_p(r)$ . By applying the results of Chapter 2, the unperturbed current density  $J_o(r)$  and magnetic field  $B_o(r)$  can be evaluated.

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To test the accuracy of the probe measurements, we also compute the total current of the discharge and the total current through the return conductor. Since these should be equal, the difference of the computed values should give an idea of the accuracy of the probe measurements. An independent check is obtained by comparing these with the current measured by a Rogowski coil (see Fig. 33). This check shows that in most cases calculated values of  $J_p(r)$  are consistent with values measured by the Rogowski coil to within 10%. The experimental results obtained with the gradient probe are summarized in figures in the following pages. For each pressure, we plot  $J_p(r)$  and  $B_p(r)$  respectively (Fig. 34(a) to Fig. 37(d)).





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# 3.4.2 Comparison of Dynamics with Theory

In this section, we study the dynamics of the z-pinch with the information from the measured profiles of the current density and the magnetic field. To obtain the physical picture of the collapsing current layer, we develop a modified snow-plow equation taking into account the finite conductivity of the plasma and the thickness of the shell.

If we assume that the z-pinch discharge consists of a thin collapsing cylinder of infinite conductivity and infinitesimal thickness, and that the sheet sweeps all the particles on its way, then the snow-plow equation can be written as

(1) 
$$\frac{d}{dt} \left[ \rho_0 \pi \left( r_0^2 - r^2 \right) \frac{dr}{dt} \right] = -F(t)$$

where r(t) is the radius of the collapsing cylinder,  $r_0$ its initial value,  $\rho_0$  the initial density of the gas and - F(t) the net magnetic force acting on a unit length of the cylinder (see Rose and Clark, 1961,p. 335). Equation (1) is too simple to account for all the effects that govern the dynamics of the discharge. Nevertheless, despite its simplicity, it predicts the collapse of the current layer with surprising accuracy under favourable conditions (Curzon et al., 1962). The discrepancy of the snow-plow equation given in (1) usually arises from the fact that the cylindrical conducting layer in the discharge does not have an infinite conductivity and it is by no means "thin". Consequently magnetic fluxes can diffuse through it and the layer may expand during the collapse. To take into account these effects, a modified equation of (1) is now used to obtain the rate of collapse of the current layer in the discharge.

For simplicity, we now make the following assump-

- (a) The current layer has an initial thickness d, and hence an initial mass  $m_0$ , as confirmed by the measured probe signals.
- (b) The inner surface of the layer moves with the same velocity as that of the centre of mass across the layer. This is introduced to allow for the expansion of the current layer due to Joule heating.
- (c) The kinetic pressure outside the current layer is negligible. The theory therefore applies only before the current layer has collapsed so close to the discharge axis that the kinetic pressure becomes significant.
- (d) The amount of gas escaped from the current layer due
   to inefficient trapping is always a constant fraction
   \$\nu\$ of the total mass in the layer.

(2) 
$$m_{o} = \rho_{o} \pi (R^{2} - r_{o}^{2}),$$

by

where R is the radius of the inner wall of the discharge vessel,  $r_0$  the initial radius of the inner surface of the layer  $(r_0 = R - d)$ . Since this mass will be carried by the current layer as the latter collapses, it should be added to equation (1) such that

(3) 
$$\frac{d}{dt} \left[ \nu \left\{ m_{o} + \rho_{o} \eta \left( r_{o}^{2} - r^{2} \right) \right\} \frac{dr}{dt} \right] = -F(t),$$

where the constant  $\mathcal{Y}$  is introduced to allow for the inefficient trapping of particles.

The accelerating force -F(t) is mainly due to the magnetic force acting on the current layer and is given by

(4) 
$$F(t) = 2 \eta \left( \frac{r \cdot B_p^{+2}}{2 \mu} - \frac{r \cdot B_p^{2}}{2 \mu} \right)$$

(see Fig. 38). Here  $B_p$  and  $B'_p$  are the magnetic flux densities at the inner and outer surfaces respectively, and r' is the radius of the outer surface. Substituting (2), (4) into (3), the modified equation is written as

(5) 
$$\frac{d}{dt} \left[ \nu \rho_0 \eta \left\{ (R^2 - r_0^2) - (r_0^2 - r^2) \right\} \frac{dr}{dt} \right] = - \frac{\eta}{\mu} (r' B_p'^2 - r B_p^2),$$

6) 
$$\frac{\mathrm{d}}{\mathrm{dt}} \left[ \nu \left( \mathrm{R}^2 - \mathrm{r}^2 \right) \frac{\mathrm{dr}}{\mathrm{dt}} \right] = - \frac{1}{\mu} \frac{\rho_{\mathrm{o}}}{\rho_{\mathrm{o}}} \left( \mathrm{r'B_p}^2 - \mathrm{rB_p}^2 \right).$$

From our probe measurements, we have obtained the current density profiles and the magnetic flux density at different stages of the collapse. We can therefore obtain the experimental values of r',  $B_p$  and r,  $B_p$  corresponding to the radii and flux densities at the inner and outer surfaces of the collapsing sheet (see Fig. 38(a) and (b)). The inner and outer edges of the shell are taken as the radii at which the current density  $J_p(r)$  drops to half of its peak values.





We have solved numerically the modified snow-plow equation in the form of (6) for the collapse curve of the current layer and compared the solutions with those measured by the gradient probe. By varying the parameter  $\mathcal{V}$ , we obtain an idea of the efficiency of trapping of particles by the magnetic flux. The best fit between experiment and theory is obtained with the following values for  $\mathcal{V}$ :

 $\mathcal{Y} = 0.9$ , for helium at 1, 2, 4 mmHg initial pressures,  $\mathcal{Y} = 0.75$ , for helium at 500 µHg initial pressure.

There are two possible explanations for the above calculated results: inefficient trapping of particles by the current shell and experimental bias. A consistent bias of 5% in the current measurements would change the accelerating force by 10%. This margin of error is reasonable for the experimental set-up and could explain the discrepancy between theory and experiment at 1, 2 and 4 mmHg in helium. The fact that  $\nu$  does not vary with pressure over this range strongly suggests that the discrepancy is due to errors in measurements rather than imperfect trapping. However at 500 µHg, the value for  $\nu$  is 0.75, which suggests that at this pressure not all the gas is swept up by the collapsing current shells, i. e. trapping is inefficient (Liebing, 1963). The theoretical and experimental collapse curves are shown in Fig. 39-42.

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• Outer radius of shell • Position of current peak • Inner radius of shell • Theory (y = .75)

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Outer radius of shell
Position of current peak
Inner radius of shell
Theory (*V* = .9)

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Outer radius of shell
 Position of current peak
 Inner radius of shell
 Theory ( \nu = .9)

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Fig. 42. Collapse curves (He 4 mmHg)

- Outer radius of shell
- + Position of current peak
- $\triangle$  Inner radius of shell
- X Theory  $(\nu = .9)$

CONCLUSIONS AND PROPOSALS FOR FUTURE WORK

4.1 <u>Conclusions</u>

The main contributions of this thesis to the measurements of the current densities using a magnetic probe are the following:-

- (1) We have examined the limitations of the E-K-Z model for the current flow round a hole in a cylindrical current sheet. Our investigations show that the model is valid:
  - (a) for time dependent cases provided that the conductivity  $\sigma'$  is independent of time,
  - (b) for a moving current layer with a velocity  $\vec{\mathbf{v}}$ , if  $\vec{\mathbf{v}} \ll \omega \delta$ , where  $\omega$  is the characteristic frequency of the current and  $\delta$  is its skin depth for the layer, and
  - (c) the skin depth  $\delta$  is much larger than the thickness of the sheet.

This model is used to obtain a correction formula relating the true current density of a plasma to the apparent current density (i. e. without correction for the perturbation of the probe ) measured by a magnetic probe.

(2) In the formula which corrects for the perturbation of a magnetic probe on the true current distribution,

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we have removed the discontinuity in  $\frac{dB_{D}}{dr}$  (appearing in Daughney's thesis) by taking into account the curvature of the current sheet. This is an importantimprovement because it eliminates the singularity from the integral equation from which the current is computed.

- (3) We have obtained quantitative results of the corrections necessary for the measured current density  $J_p$  to give the true current density  $J_o$ . In general for smooth current distributions  $J_o'$ ,  $|J_p-J_o'/J_p| \leq 5\%$ . However, if  $J_o$  is due to a thin current sheet, a probe will smear out  $J_p$  and measure a much thicker current. In such cases,  $|J_o-J_p|$  can be several times larger than  $|J_p|$ .
- (4) We have assessed the accuracy of the correction procedure for practical applications. Employing the Monte Carlo method, we have simulated experimental errors with random numbers and have found that for smooth current distributions the errors in the corrected current density J<sub>o</sub> were usually twenty times the experimental errors in the measured current density  $J_p$ . Therefore, if the experimental errors in J<sub>p</sub> are larger than  $\frac{1}{4}$ %, the errors in the calculated values of J<sub>o</sub> will be larger than 5% of J<sub>p</sub>. Thus for a smooth current density distribution J<sub>o</sub>, such a correction

procedure is unnecessary. However, if  $J_0$  is known to be a current "spike" (i. e.  $J_0$  is due to a thin current sheet), a probe measures a much more broadened current several times reduced in magnitude. In such cases, correction is important. Due to the difficulty in reproducibility, the correction of the measured  $J_p$ in a pulsed plasma cannot be achieved since the scatter in experimental parameters is usually larger than 2%.

- (5) We have developed a gradient probe using two search coils. It has all the advantages of a magnetic probe in the form of one single coil and it measures both the magnetic field and the gradient of the field directly. This enables much more accurate measurements of plasma current densities to be made.
- (6) Measurements of the current densities  $J_p$ , the magnetic flux densities  $B_p$  in z-pinch discharges in helium at pressures between 500  $\mu$  and 4 mmHg have been obtained.
- (7) A modified snow-plow equation has been used to study the dynamics of the z-pinch discharges in helium at initial pressures ranging from 500  $\mu$  to 4 mmHg. This equation has allowed for the thickness of the collapsing current shell and the escape of gas from it. The accelerating forces on the shell due to magnetic pressures have been taken from the probe measurements.

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The comparison between experiment and theory suggests that at pressures between 4 mmHg and 1mmHg in helium, trapping is better than 90% and that the observed discrepancy is due to a systematic bias in the calibrations of the measuring equipment. In helium at 500  $\mu$ Hg, there is evidence that some of the discharge gas escapes through the collapsing current shell.

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#### 4.2 Proposals for Future Work

The probe was originally designed to give enough signals for a Tektronix Type G differential amplifier which has a much smaller amplification than the Tektronix Type W differential amplifier now being used. It is therefore possible to reduce the size of the probe to about 3 mm in diameter and about 10 turns in each of the sensing coils (the gradient probe used in this thesis is 7 mm in diameter and has 100 turns in each of the coils). This will probably reduce the probe signals to about a few millivolts, but the frequency response will probably be flat up to 5 MHz. Using such a smaller probe, the perturbation on the plasma current will be reduced and better frequency response and spatial resolution should be achieved.

In the study of the perturbation of a magnetic probe on the current flow of a plasma, we have not considered other possible "dynamical effects" on the current layer. Apart from the distortion of current flow round the probe as discussed in the present thesis, the probe might split up the current layer or induce instabilities. Boundary layer effects will also be appreciable. Further experimental and theoretical work should be done to investigate such effects.

For the z-pinch discharges, our results show that the trapping of particles by the collapsing shell depends on the filling pressures. Experiments may be done to investigate the amount and the nature of gas left behind by the collapsing current shell.

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Appendix I

CALCULATION OF  $B_{\phi}$  FOR THE E-K-Z MODEL

In this appendix, we derive equation (11) of  $\oint 2.2.1$ (p. 22). We will first show that to first order in  $\frac{a}{s}$ ,  $\Delta \vec{J}$ is the same for both the plane current and the current cylinder of radius s and infinitesimal thickness ds. Then we will prove that the corresponding values of  $\Delta \vec{R}$ differ by a quantity of the order  $\frac{a}{s}$ .

From equation (9) of  $\oint 2.2.1$  (p. 21), the change in current density  $\Delta \vec{J}$  in a plane current layer due to a circular hole of radius a is given by

(1) 
$$\Delta \vec{J} = (J_0 (\frac{a}{\rho})^2 \cos \theta, +J_0 (\frac{a}{\rho})^2 \sin \theta, 0), \quad \rho \ge a,$$
  
=  $-\vec{J}_0, \qquad \rho < a.$ 

This immediately shows that

(2) 
$$\left|\Delta \vec{J}\right| \sim \left(\frac{a}{\rho}\right)^2 \left|\vec{J}_{o}\right|, \qquad \rho \geqslant a.$$

 $\Delta \vec{J}$  is therefore significant over a finite region with  $\rho < a', a'$  being comparable to a. In fact we have  $\left|\frac{\Delta \vec{J}}{\vec{J}_{o}}\right| \lesssim .1$  for  $\rho > a' \sim 3a$ .

Consider a current cylinder of radius s and infinitesimal thickness ds and the tangent plane at the centre of the hole. Using the fact that the arc and the tangent subtended by a small angle differ by a quantity of second order in the angle, the region of the current cylinder over which  $\Delta \vec{J}$  is significant can be replaced by

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the tangent plane if we retain quantities up to the first order in  $\frac{a}{s}$ . It is therefore obvious that the solution of  $\Delta \vec{J}$  given in (1) for a plane can be used as the solution in the current cylinder ( $(\frac{a'}{s})^2 \sim .1$  for  $s \gtrsim 3a' \sim 9a$ ).

To show that  $\Delta B'_{\phi}$  and  $\Delta B_{\phi}$  for the plane and the cylindrical current layers respectively differ by a first order quantity in  $\frac{a}{s}$ , we consider equation (3) of §2.2.1 (p. 19), i. e.

(3) 
$$\Delta B_{\phi}(\mathbf{r}) = \frac{\mu}{4\pi} \int d\vec{s} \frac{\left[\Delta \vec{j}(\vec{s}) \times (\vec{r} - \vec{s})\right]_{\phi}}{\left|\vec{r} - \vec{s}\right|^{3}}$$

In the same coordinates systems (see Fig. 43(a) and (b)) as those of Fig. 6 (p. 21), the required component  $\Delta B_{\phi}$  due to the current cylinder is given by

(4) 
$$\Delta B_{\phi}(\mathbf{r}) = -\Delta B_{\eta}(\varsigma)$$
  
=  $\frac{\mu ds}{4\pi} J_{0}(s) \int_{0}^{2\pi} d\theta \left( \int_{0}^{a} d\rho \mathbf{I}_{1} + \int_{a}^{a'} d\rho \mathbf{I}_{2} \right)$ 

where

(5) 
$$I_{1} = \frac{\left[\zeta + \frac{(\rho \sin \theta)^{2}}{a} \epsilon\right] \rho}{\left[\left\{\zeta + \frac{(\rho \sin \theta)^{2}}{a} \epsilon\right\}^{2} + \rho^{2}\right]^{\frac{3}{2}}},$$

(6) 
$$I_2 = I_1 \cos 2\theta \left(\frac{a}{p}\right)^2$$
,

and

(7) 
$$\epsilon = \frac{a}{2s}$$
.









Fig. 43(b). Geometrical relations between  $\Delta B_{\eta}(\varsigma)$  and  $\Delta \vec{j}(\varsigma)$ ( $\varsigma < 0$ ).

In (4), we have taken a' as the upper limit of  $\rho$ . Since the contribution to the integral for current elements lying beyond a' is negligible, we can replace the upper limit by  $\rho \rightarrow \infty$ .

It is difficult to evaluate the integrals of (4) directly because sin0 appears in the complicated expression in the denominators of the integrands. However, we can evaluate the integrals approximately by expanding the integrals in power series of  $\epsilon$  such that

$$(5a) = I_{1} = \frac{\rho}{(\varsigma^{2} + \rho^{2})^{3/2}} \left\{ \varsigma + \varepsilon \frac{\rho^{2} \sin^{2} \theta}{a} \left[ 1 - 3 \frac{\varsigma^{2}}{(\varsigma^{2} + \rho^{2})^{2}} \right] + O(\varepsilon^{2}) \right\}$$

and

(6a) 
$$I_{2} = \frac{\rho \cos^{2} \Theta}{(\zeta^{2} + \rho^{2})^{3/2}} \left\{ \zeta(\frac{a}{\rho})^{2} + \epsilon a \sin^{2} \Theta \left[ 1 - 3 \frac{\zeta^{2}}{(\zeta^{2} + \rho^{2})} \right] + O(\epsilon^{2}) \right\}$$

Substituting (5a) and (6a) into (4) and integrating over  $\rho$  and  $\Theta$ , we obtain

(8) 
$$\Delta B_{\phi}(\mathbf{r}) = -\Delta B_{\eta}(\varsigma)$$

$$=-\frac{1}{2}\mu J_{0}(s)ds\left\{\left(sgn(\zeta)-\frac{\zeta}{(\zeta^{2}+a^{2})^{\frac{1}{2}}}\right)\left(1-\frac{\zeta}{s}\right)\right.\\\left.-\frac{a}{8s}\left[\frac{a}{(\zeta^{2}+a^{2})^{\frac{1}{2}}}+\frac{3a^{2}}{(\zeta^{2}+a^{2})^{3/2}}\right]+o(\varepsilon^{2})\right\}.$$

For values of  $|\zeta| \leq 2a$ , the expansion procedure gives accurate values for  $\Delta B_{\phi}$  whereas for  $|\zeta| \geq 2a$ , the approximations are not accurate but the absolute value of  $|\Delta B_{\phi}|$ which decreases rapidly with  $|\zeta|$  and  $\Delta B_{\phi}$  becomes The corresponding quantity  $\Delta B_{l}(\zeta)$  for a plane current layer is obtained from (8) by letting  $s \rightarrow \infty$ , i.e.

(9) 
$$AB_{\eta}'(\zeta) = -\frac{1}{2}\mu J_{0}(s)ds \left\{ \left[ sgn(\zeta) - \frac{\zeta}{(\zeta^{2} + a^{2})^{\frac{1}{2}}} \right] + O(\zeta^{2}) \right\}$$

Comparing (8) and (9), we therefore have

(10) 
$$\Delta B(\zeta) = -\frac{1}{2}\mu J_0(s) ds \left\{ \left[ sgn(\zeta) - \frac{\zeta}{(\zeta^2 + a^2)^{\frac{1}{2}}} \right] (1 - \frac{\zeta}{s}) + 0(\epsilon^2) \right\}$$
  
=  $\Delta B_{\eta}'(\zeta) \left\{ \left[ \frac{1}{1 + \frac{\zeta}{s}} \right] + 0(\epsilon^2) \right\}$ 

or neglecting  $O(\epsilon^2)$ ,

(11) 
$$\Delta B_{\phi}(\mathbf{r}) = \frac{1}{2}\mu J_{o}(\mathbf{s}) d\mathbf{s} C(\frac{\mathbf{r}-\mathbf{s}}{\mathbf{a}}) \frac{\mathbf{s}}{\mathbf{r}}$$
  
=  $\Delta B_{\phi}'(\mathbf{r}) \frac{\mathbf{s}}{\mathbf{r}}$ ,

where  $C(\frac{r-s}{a})$  has been defined in (13) of §2.2.1. We have neglected the second term inside the brackets { } of (8). This means that equation (11) is only correct provided terms of  $O(\frac{a}{8s})$  are negligible. Equation (11) now gives us the value of  $\Delta B_{\phi}(r)$  due to a current cylinder of radius s and infinitesimal thickness ds, having an "atial current density  $J_{0}(s)$ . It also enables us to obatin  $\Delta B_{\phi}(r)$  from the corresponding value of a plane current.

For a radial distribution of current, equation (11) gives

(12) 
$$\Delta B_{\rho}(\mathbf{r}) = \frac{1}{2}\mu \mathcal{P}_{\rho} \left( \frac{\mathrm{ds}}{\mathrm{ds}} J_{\rho}(\mathbf{s}) C\left(\frac{\mathbf{r}-\mathbf{s}}{a}\right) \frac{\mathbf{s}}{\mathbf{r}} \right)$$

We have shown in equation (11) that  $\Delta B_{p}$  and  $\Delta B_{p}$ for a curved and a plane layers respectively differ by a multiplication factor  $\frac{s}{r}$ . If  $|r-s| \ll s$ , the difference is small and we might be tempted to replace  $\frac{s}{r}$  by unity. However, we now show that this factor is essential in the calculation of the gradient of  $B_{p}(r)$ , and hence  $J_{p}(r)$ , for a curved layer.

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Consider the same current cylinder of radius s and infinitesimal thickness ds as described previously. The perturbed azimuthal field along the axis of the probe is

(13) 
$$B_{p}(\mathbf{r}) = B_{o}(\mathbf{r}) + \Delta B_{o}(\mathbf{r})$$

$$= \mu J_{o}(s) ds \left[ 1 + \frac{1}{2} C\left(\frac{r-s}{a}\right) \right] \frac{s}{r} , \qquad r > s$$
$$= \frac{1}{2} \mu J_{o}(s) ds C\left(\frac{r-s}{a}\right) \frac{s}{r} , \qquad r < s$$

Differentiating  $B_{p}(r)$  with respect to r, we obtain

$$(14) \quad \frac{\partial B_{p}(r)}{\partial r} = \mu J_{0}(s) ds \left[ -\left\{ 1 + \frac{1}{2}C\left(\frac{r-s}{a}\right) \right\} \frac{s}{r} 2 + \frac{s}{r} \frac{\partial C\left(\frac{r-s}{a}\right)}{2\partial r} \right], r > s,$$
$$= \mu J_{0}(s) ds \left[ -\frac{1}{2}C\left(\frac{r-s}{a}\right) \frac{s}{r} 2 + \frac{s}{r} \frac{\partial C\left(\frac{r-s}{a}\right)}{2\partial r} \right], r < s,$$

which is continuous at r=s. However, if the factor

1

\$ r in  $\Delta B_{p}(r)$  is neglected, the gradient of  $B_{p}(r)$  becomes

(15) 
$$\frac{\partial B_{\rm p}(\mathbf{r})}{\partial \mathbf{r}} = \mu J_{\rm o}(s) ds \left[ -\frac{s}{r^2} + \frac{\partial C(\frac{\mathbf{r}-s}{a})}{2\partial r} \right], \qquad r > s,$$
$$= \frac{1}{2} \mu J_{\rm o}(s) ds \frac{\partial C(\frac{\mathbf{r}-s}{a})}{2\partial r}, \qquad r < s,$$

which gives

(16) 
$$\frac{\partial B_{\mathbf{p}}(\mathbf{r})}{\partial \mathbf{r}} = \mu J_{\mathbf{0}}(\mathbf{s}) d\mathbf{s} \left[ -\frac{1}{\mathbf{s}} + \frac{\partial C(\mathbf{0}^{+})}{2\partial \mathbf{r}} \right], \qquad \mathbf{r} = \mathbf{s}^{+},$$
$$= \mu J_{\mathbf{0}}(\mathbf{s}) d\mathbf{s} \quad \frac{\partial C(\mathbf{0}^{-})}{2\partial \mathbf{r}}, \qquad \mathbf{r} = \mathbf{s}^{-}.$$

Since  $\frac{C(\frac{r-s}{a})}{r}$  is continuous, it is easy to see that  $\left(\frac{\partial B_{p}(r)}{\partial r}\right)'$  is discontinuous at r=s. Such a discontinuity cannot occur in a region where there is no current flow. Further, a discontinuity in  $\left(\frac{\partial B_{p}}{\partial r}\right)'$  creates a singularity in the integral equation (see equation (16) of §2.2.1, p. 23) used for calculating the current density. It is therefore important to include the factor  $\frac{S}{r}$ .

#### Appendix II

### FORTRAN IV PROGRAMME FOR SOLVING

### THE INTEGRAL EQUATION

In this appendix, we give the complete Fortran IV computer programme for solving the integral equation

(11) 
$$\int_{0}^{\infty} J_{o}(s)K(r,s,a)ds = J_{p}(r)$$

which was already transformed into a set of equations

(2) 
$$\sum_{N=1}^{K} A(M,N) u(N) = v(M), M = 1,2,...,K,$$

(see equation (1) and (9) of  $\S2.3$ ). We use the following notation in the computer programme :-

DX = width of mesh interval,

K = number intervals used in the calculation,

RHO = probe radius,

AMU = magnetic permeability,

GPSENS = sensitivity of probe for measuring the gradient of the magnetic field,

PBSENS = sensitivity of probe for measuring the magnetic field,

x(M) = r = radial coordinate of the centre of the Mth interval,

$$AJ(M) = u(M) = J_{o}(r),$$
$$PJ(M) = v(M) = J_{p}(r),$$

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$$B(M) = B(r),$$
  

$$BP(M) = B_{p}(r),$$
  

$$DB(M) = \frac{B_{p}(r)}{r},$$
  

$$C(M) = C(\frac{r-s}{a}), \text{ for } r-s = M.DX \text{ and } a = RHO,$$
  

$$DC(M) = \frac{1}{2}DX - \frac{B_{p}(r)}{r}, \text{ for } r-s = M.DX \text{ and } a = RHO.$$

In the programme, the matrix is first inverted, and then multiplied by different sets of input data to give the output values of AJ(M).

		\$\$ \$ 10B	79046	Y-K S- TAM	······································	· · · · · · · · · · · · · · · · · · ·					· .	MA860	
	* . · · · ·	\$PAGE	E 7,040	30		•	•			•		•	
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	· ·	\$IBFT	ГС ТАММ	· ·				N	•	÷ 4			
	•	С	PROGRAM	ME NO. 5C.	E-K·	-Z MODEL		•					
	· · · · · · · · · · · · · · · · · · ·	C	NO PLOT	OUTPUT.							<u>,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,</u>		
	•	Ç	TO COMPL	JTE J(X) AND	B(X) FR	OM MEASURE	D VALUE:	S OF BP	(X) AN[	D DB(X	).		
	·	C	PJ(I) IS	S SMOOTHED I	N THE PRO	DGRAMME •			•		· · · · ·		
		. <b>C</b>			• •	•							•.
		Ç	K=NO• O	F MESH INTER	VALS								
_		<u> </u>	B(I)=UN	PERTURBED MA	<u>GNETIC FI</u>	LUX DENSIT	· Y						
	·. ·	C	BP(I) = PI	ERTURBED B.									• •
		C	DB(I) = GF	RADIENT OF B	P(I)								
	· · · · · · · · · · · · · · · · · · ·	C	C(I) = COI	RRECTION FAC	TOR .								
		C	DC(I)=•!	5*DX*(GRADIE	NT OF CI	I))					•		
		C	AJ(I)=U	NPERTURBED J			· ·					· .	
		C .	PJ(I)=PI	ERTURBED J	•								
	· · ·	C	X(I) = RAI	DIAL COORDIN	ATE OF CI	ENTRE OF J	TH INTE	RVAL			· .	· .	
		C -	-A(M,N)=1	MATRIX ELEME	NT						. : •		·
		C .	AMU=MAGI	NETIC PERMEA	BILITY				والمحدو معهم الوالم المريو المحا ووحور		1.	· ·	1 <b></b>
-		C	DX=WIDT	H OF MESH IN	TERVAL				•		•		
		. C .	SC=SCAL	ING FACTOR F	OR MATRI	X ELEMINIS	5			·	· · · .	· ·	Ň
		С	RHO=PRO	BE RADIUS			•						· · ·
		С	PBSENS=	PROBE SENSIT	IVITY FO	R MEASURIN	IG B			· .	-		1
		C	GBSENS=1	PROBE SENSIT	IVITY FO	R MEASURIN	IG GRADI	ENT OF	В				
		C				×			•		1 - A	:	
			DIMENSI	ON A(100,100	),BP(100	),DB(100),	C(200) •!	C(200)	,AJ(100	5)	· • · · · · · · · · · · · ·		
			DIMENSI	ON B(100),PJ	(100),X(	100)				· • .		· · .	
	: · · · ·	C .	TO EVAL	JATE C(I) AN	D DC(I)		•					• •	
	······································		KD=100										
			READ (5	,201) AMU,DX	•K								
		201	FORMAT	(2E20.8,I3)		• •						· .	
			READ (5	\$202) RHO			•	·.				· ·	
		202	FORMAT	(E20•8)									
			WRITE (	6,301) RHO,D	X,AMU,K								
	*****	301	FORMAT	(1H1,5H RHC=	,E20.8,5	X,3HDX=,E2	20.8,5X,	4HAMU=,	E20.8,	5X,2HK	= , I		
		• •	13)								· ·		
			R=DX/RH0	С <u>,</u> С					· .·	,	• • •		
			KK=K+K	· · · · · · · · · · · · · · · · · · ·		· · ·							
			PIJ=•5*1	R				. •					· •
	•		DO 11 M	-1 10									

· · · · ·		· · ·	
		· · ·	AM=M
	· · ·		AMR=AM*R
			<u>SS=1•/(1•+AMR*AMR)</u>
			SR = SQR (1SS)
		1 7	$C(M) = 65^{\circ}(-1) + AMR + SR$
	·	11	$DC(M) = 0.7 \times 10^{-11} \times 10^{-11}$
	· · ·	•	C(1)=0.
* .		997	DC(T)=0.
			K1=K-1
;		c	TO EVALUATE A(M.N)
	,		DO 122 M=1,K
. '			MM=M+M
1			AM=M
			AM=1•/(AM-•5)
			DO 121 N=1,K
	•		IF(N.EQ.M) GO TO 121
•			AN=N
			AN=AN-•5
		2.	MN=IABS(M-N)
	•	•	NM=M+N
· .			A(M,N) = AM*AN*(DC(MN) - DC(NM))
1. X .	· · ·	121	CONTINUE
		122	A(M,M) = PIJ-DC(MM)
		C	TO SCALE A(M,N) SO THAT NO NUMBER CAN BE LARGE THAN 1.E38
'		•	SC=1./PIJ
			DO 15 M=1•K
	•		
١.		15	$A(M_{9}N) = S(*A(M_{9}N))$
			TO INVERT A
			WRITE (6.202) DET COND
		202	WRITE (0) $JUZ$ ) DETICUND EORMAT (6Y, AUDET = E20, R, EUCOND = E20, R)
÷.	<u></u>		$\frac{1}{1} = \frac{1}{1} = \frac{1}$
			$IF (DET_{0}GT_{0}(1 + E38)) STOP$
¥			READ (5.203) PRSENS.GRSENS
1 12	Baarran 1999	203	EORMAT (2E10-5)
<u> </u>		205.	$DO(1111) M = 1 \cdot K$
2 10			DO 1111 N=1.K
· 3. 0		1111	A(M,N) = SC*A(M,N)
		C	TO READ DATA
4 6	· · ·	1	READ (5,207) GAS, P, TIME
. U S			
5 4	•	•	
3		•	

	<b>N</b> . 14				
• .			207	FORMAT (A6,2F10.5)	
	<u>,</u>			IF (P.EQ.O.) STOP	
		•		READ (5,204) (DB(I),I=1,K)	
			••••••••••••••••••••••••••••••••••••••	READ (5,204) (BP(I),I=1,K)	
	C.		204	FORMAT (8F10.5)	
	C.	· .		KM1=K-1	
1.2.3				DX1=.5*DX	
	C	· · ·		DO 998 I=1.K	
•	•	· · · · · ·	998	BP(I) = BP(I) + DX1 * DB(I)	
•				DDX=1./DX	
	(		•	AAU=1./AMU	
		· · · · · · · · · · · · · · · · · · ·	·	DO 205 I=1.K	
	·		·.	CI=I	
	· .			CII=CI-•5	
;				X(I) = CII * DX	
				BP(I)=BP(I)*PBSENS	
• .	(	· ·		DB(I)=DB(I)*GPSENS	
			205	PJ(I)=AAU*(BP(I) *DDX/CII+DB(I))	
		•	C	TO CALCULATE AJ(I)	
-	(			CALL MATVEC(A,PJ,AJ,K,KD)	
			<u> </u>	TO CALCULATE B(X)	
		•		AXU=•5*AMU*DX	
	(			DO 17 M=1.0K	1
		••••••••••••••••••••••••••••••••••••••			
	٠				N
	Ę.			$ \mathbf{v}  = \mathbf{v} + \mathbf{I}$	Ci.
	۰.	• · · · · · · · · · · · · · · · · · · ·			
				DO IO N-I = M - N	
	(·	,	18	SUM = SUM + A I (N) * C (MN)	
. •					
	1 .	, 1		NM=N-M	
			19	SIIM = SIIM + A I (N) * C (NM)	
		·	17	$B(M) = BP(M) - AXU \times SUM$	
	1		- · · ·	WRITE (6,500) GAS, P, TIME	
	1	2	500	FORMAT (1H1+1X+A6+3HAT +F10+5+11HMICRONS /	AND . E 10.5.6HU SEC.)
	1	1		WRITE (6,501) PBSENS, GPSENS	
	2 € 1	0	501	FORMAT (1X,21HM. PROBE SENSITIVITY=,E20.8	<pre>&gt;21HWEBERS/CU.METER/VOLT.</pre>
				1,2X,21HG. PROBE SENSITIVITY=,E20.8)	
	3 8	}		WRITE (6,502) (X(I),BP(I),DB(I),PJ(I),B(I	),AJ(I),I=1,K)
	( .)	,	502	FORMAT (/4X,6HX (M.),9X,11HBP (W/SQ.M),9X	<pre>&gt;11HDB (W/CU.M),8X,12HJP(</pre>
	4 6	i		1AMP/SQ.M),2X,18HCOMPUTED B(W/SQ.M),2X,20HC	COMPUTED J(AMP/SQ.M)/
	9		····· <u></u> ·····		
	5 ( - 1	i			
	3	l		·	

С	۰.			· · ·						· _ · · · · · · · €
- <u>C</u> .		C C	2(F10.5,4E20.8, TO CHECK EXPER TO CALCULATE D	E22.8)) IMENTAL RESUL ISCHARGE CURF	TS. Rent sum1	AND CURF	RENT IN T	He return		
. –		C	CONDUCTOR SUM2 SUM1=0. SUM2=0.			· · · · · · · · · · · · · · · · · · ·		· · · ·		
		· · ·	DO 505 I=1,K IF (AJ(I).LĨ.O SUM1=SUM1+AJ(I	•) GO TO 507 )*X(I)						
	• •	507 505	GO TO 505 SUM2 =SUM2+AJ( CONTINUE	I)*X(I)						
· · -			DI=3.14159265* SUM1=SUM1*DI SUM2=SUM2*DI	2•*DX					· · ·	
		508	WRITE (6,508) FORMAT (//24H 124H TOTAL NEGA	SUM1,SUM2 TOTAL POSITIN TIVE CURRENT=	/E CURRENT =,E20.8,6H	=,E20.8, AMPS.)	6H AMPS.	/		
		601	WRITE (7,601) FORMAT (A6,2X, WRITE (7,600)	GAS,P,TIME F10.5,2X,F10. SUM1,SUM2	.5)					
	· ·	600 602	FORMAT (2E20.8 WRITE (7,602) FORMAT (5E16.8	) (DB(I),BP(I), )	B(I),PJ(I	),AJ(I),	I=1,K)	۰.,	. •	I
(	· · ·	\$ENTR	GO TO 1 END Y							
					· .			······································		ſ
		· · · · · · · · · · · · · · · · · · ·							· · · · ·	
1 12 _								· · · · · · · · · · · · · · · · · · ·		
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