DIRECT MEASUREMENTS OF STRESS AND SPECTRA OF TURBULENCE
IN THE BOUNDARY LAYER OVER THE SEA

by

HENRY SVEN WEILER

B.Sc. University of Toronto 1962

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

in the
Department of Physics

We accept this thesis as conforming to the
required standard

THE UNIVERSITY OF BRITISH COLUMBIA

JULY , 1966

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Department of Physics

The University of British Columbia
Vancouver 8, Canada.

Date July 11, 1966

THE UNIVERSITY OF BRITISH COLUMBIA

FACULTY OF GRADUATE STUDIES

PROGRAMME OF THE
FINAL ORAL EXAMINATION
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

of

HENRY SVEN WEILER

B.Sc., University of Toronto, 1962

MONDAY, JULY 11, 1966, at 3:30 P.M.

IN ROOM 301, HENNINGS BUILDING

COMMITTEE IN CHARGE

Chairman: J. R. Adams

R. W. Burling	R. D. Russell
G. V. Parkinson	R. W. Stewart
G. L. Pickard	N. H. Thyer

External Examiner: S. Corrsin

Department of Mechanics
The Johns Hopkins University
Baltimore, Maryland

Supervisor: R. W. Burling

DIRECT MEASUREMENTS OF STRESS AND SPECTRA OF TURBULENCE IN THE BOUNDARY LAYER OVER THE SEA

ABSTRACT

Fluctuations in the vertical and horizontal components of wind velocity in the boundary layer over the sea were measured with an X-array of hot wires. Special techniques were developed to mount and calibrate the wires, and to measure directly their responses to the two velocity fluctuations. Analog techniques were developed to analyze the hot wire signals, to give the spectra of the two velocity fluctuations, and their cospectrum, over a range of mean wind speeds from 140 to 1000 cm/sec, and at frequencies between 0.016 to 60 Hz. Three runs with U-wire probes mounted vertically were analyzed to provide additional checks on the similarity theory of turbulence.

Ten X-wire runs were analyzed. The measurements showed that X-wire techniques can be used successfully to measure velocity fluctuations in two directions to give spectral and cospectral estimates within ± 30 and $\pm 50\%$ or better, respectively.

Spectral and cospectral estimates were obtained, which showed that anisotropy existed in turbulence to much smaller scales than theoretical predictions indicated. The observed stress had maximum contributions between about .01 to 10 Hz, and for most runs analyzed, the largest proportion of the stress was present at frequencies lower than the estimated frequencies of the dominant waves. Ten direct estimates of stress were obtained, which gave drag coefficients (corrected to the 5 m. height) which were apparently invariant with mean wind speed. The average value was 1.5×10^{-3} .

Indirect estimates of stress, using the mean wind profile method generally underestimated it. Indirect estimates of the stress using the wavenumber spectrum in the inertial subrange overestimated the stress by about 40%. Of the two indirect estimates, the spectral method gave more consistent results.

Measurements by the three U-wires spaced vertically, provided confirmation of the validity of the Monin-Obukhov similarity theory at heights below about 5 m.

GRADUATE STUDIES

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PRIZES

- 1958-59 - University College Alumni Scholarship
- 1960-62 - University College Alumni Scholarship
- 1962-66 - National Research Council Studentships

PUBLICATION

Weiler, H. S., Convergence Predictions Using Trilinear
Gradient Approximation, August 1962,
Defence Research Board Lab. Note No. 62-14.

ABSTRACT

The work carried out for this thesis forms part of the air-sea interaction program, which has been under way since 1961 at the Institute of Oceanography of the University of British Columbia.

Measurements of fluctuations in the vertical and horizontal components of air velocity were made using hot wires in an X-array, in order to study the spectra of the fluctuations, and their co-spectrum over a range of mean wind speeds from 140 - 1000 cm./sec. in the boundary layer over the sea.

In order to use the X-wire probe properly in the field, special techniques were developed to mount and calibrate the wires, and to measure directly their responses to the two velocity fluctuations. Analog techniques were developed to analyze the hot wire signals, and final calculations were made by digital computer. Single (U-wire) hot wire probes were used to measure the horizontal velocity fluctuations to check the behaviour of X-wires, and to provide additional checks on the similarity theory of turbulence.

Measurements showed that X-wire techniques can be used successfully to measure velocity fluctuations in two directions in the field. Hot wires have responses which give spectral levels which are accurate only within about 35%, but comparison of the horizontal velocity spectrum measured

simultaneously with the X- and U-wire probes showed that their spectral shapes were similar, giving confidence in the X-wire measurements.

In the high frequency range, the observed spectra of the two velocity fluctuations did not conform to the theoretical predictions. The observed behaviour is believed to be real.

The cospectrum gives a direct estimate of contributions to the Reynolds' stress by fluctuations in small ranges of frequency.

The stress observed between the frequency limits 0.016 to 10 Hz had significant contributions over about one frequency decade, which apparently lies entirely within these extremes. Estimates of the frequencies of dominant waves at the experimental site fell between about 0.2 to 0.5 Hz. Significant stress was present in this interval, but the largest proportion of the observed stress was present at lower frequencies.

Ten direct estimates of stress were obtained with the X-wire. Values estimated indirectly from the wind profiles tended to give low estimates and were poorly correlated with the direct estimates. Values determined indirectly using the inertial subrange appeared to be consistently related to the directly estimated stress, but overestimated it by about 40%. Drag coefficients corrected to the 5 m height were near

1.5×10^{-3} for wind speeds between 1.4 and 10 m.sec⁻¹.

Measurements by three U-wires spaced vertically, provided confirmation of the validity of the Monin-Obukhov similarity theory at heights below about 5 m.

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ACKNOWLEDGEMENTS

This investigation formed part of the Institute of Oceanography of the University of British Columbia research program which is supported by the Defence Research Board of Canada, Grant No. 9550-09. Support was also obtained from the National Research Council of Canada, Grant No. BT-100, and from the Canada Department on Transport, Meteorological Branch, Grant No. 5920-0. Further support was also provided by the Office of Naval Research (U.S.A.), Grant No. N00014-66-C0047.

I would like to thank the Mechanical Engineering Department of the University of British Columbia for permitting us to use their low speed wind tunnel facilities.

In the course of this work I was given support by the National Research Council of Canada of Canada.

I wish to acknowledge my debt to Dr. R. W. Burling, and Dr. R. W. Stewart for their encouragement and guidance throughout the course of this work. My thanks are due also to all of the Institute staff and graduate students for their help and cooperation. In particular I wish to thank Mr. F. W. Dobson, who often went out of his way to help my project along. In preparation of my thesis, my thanks go to Mr. H. M. Inostroza, who gave invaluable help in preparing the original diagrams. Lastly, my heartfelt thanks go to my

wife, who often helped me during instrument calibrations
and who has rendered invaluable aid in preparing my thesis.

1. INTRODUCTION

Since 1961, a program to study air-sea interaction has been in progress at the Institute of Oceanography of the University of British Columbia. This program has been concerned with processes occurring in the layer of air just above the sea surface, and in the water itself.

Past work has been concerned with the mean wind velocity profile, fluctuations of the wind velocity component along the mean wind direction, some preliminary measurements of the temperature fluctuations, and with the two dimensional wave-spectrum.

The measurements of Pond (1963, 1965) showed that hot wire techniques can be used successfully to measure horizontal velocity fluctuations over a wide frequency range in the turbulent boundary layer over the ocean. My first objective was to demonstrate that an X-array of two hot wires can be used to measure velocity fluctuations in two directions, one vertical, and the other horizontal in the direction of the mean wind velocity. This entailed developing special techniques to mount the wires on the probes, to calibrate these wires and measure directly the dependence of their responses on both velocity components. Another objective is to analyze the recorded analog data representing wind fluctuations in order to extract the spectra of both velocity

components and their cospectrum throughout a frequency range from 0.016 to 60 Hz. These objectives were met, and the methods and techniques developed will aid the future collection and analysis of sufficient amounts of X-wire data for adequate statistical examination of their relation to other phenomena.

Of the spectra measured, the cospectrum between horizontal and vertical components of wind velocity is the most important. Besides furnishing a greater insight into turbulent structure in the boundary layer, the integral of this spectrum also provides a direct numerical estimate of the stress close to the air-sea boundary; this is an important parameter in theories of wind-driven ocean circulations.

Until recently, estimates of surface stress over a water surface were obtained by indirect methods. My own direct stress measurements allow comparison with two independently measured indirect stress determinations. One is based on the mean velocity profile and the other on the nature of the spectrum of horizontal wind velocity fluctuations in the inertial subrange. These stress measurements also allowed me to determine the drag coefficient C_D .

Since mean winds are frequently and fairly easily measured over extensive regions on the oceans, and since C_D

multiplied by air density and the square of the mean wind speed provides an estimate of the stress, the proper evaluation of C_D is potentially most useful. Another parameter which could perhaps also be measured relatively easily over the oceans, compared with the difficulty of observations on wind profiles or on covariances, is the level of the wind spectrum in the inertial subrange. This parameter is, physically, quite closely related to the stress. Thus it could be used extensively to estimate stress over the oceans, and to extend the evaluation of C_D to a wide range of even more rigorous conditions over the ocean.

The measurements of the vertical and horizontal kinetic energy spectra provide a test of theoretical predictions of their behaviour at high frequencies. The effect of the introduction of a horizontal transverse mean wind component, resulting from the deviation of the mean wind direction in the field measurements from the plane of the X-wires, is used in an attempt to account for the observed deviations from theoretical predictions.

To test the Monin-Obukhov similarity theory, measurements are made with three single hot wire probes mounted in a vertical line at different heights from the sea surface. The high frequency end of the spectra of the velocity components is used to indirectly estimate the frictional velocity (u_*) at the three heights to see if the values

remain invariant with the height.

The characteristics of turbulence are strongly influenced by the prevailing temperature structure. Unfortunately, sufficiently adequate temperature measurements are not available, so that no discussion of stability effects is possible except in a very general sense.

In order to maintain continuity of the first part of the thesis, in which some aspects of the theory of turbulence, observations and results are discussed, my work on the theory, manipulation and calibration of hot wires in an X-array (Section A1.A, A1.C, A1.D) and also my work on the techniques for analysis of hot wire data (Appendix IV), have been written as appendices.

II. THEORETICAL DISCUSSION

A. Velocity Fluctuations

A.1. Spectral Representation of Data

To describe the turbulent velocity field, a system of axes, x_1, x_2, x_3 is chosen such that x_3 is positive upwards from a boundary. The wind velocity is $\underline{v} = (v_1, v_2, v_3)$, where $v_i = U_i + u_i$ is in the direction of x_i . U_i is the component of mean velocity; u_i is the fluctuating velocity component. By this definition, $\overline{v_i} = U_i$, so that $\overline{u_i} = 0$, where the bar denotes a time average.

For convenience the x_1 - axis is taken in the direction of the mean wind velocity U , so that $\underline{v} = (U + u_1, u_2, u_3)$.

Spectral representations are used to describe the velocity field. Spectral densities ϕ_{ij} represent the contribution to the covariance $\overline{u_i u_j}$ in a unit frequency or wave number interval and are so defined that

$$\overline{u_i u_j} = \int_0^{\infty} \phi_{ij}(f) df = \int_0^{\infty} \phi_{ij}(k) dk \quad (II A 1.1)$$

where f is frequency in cycles per second (Hz) and k is the one-dimensional radian wave number (Grant et al, 1962; Pond, 1963.)

$\overline{u_i u_j}$ represents a time average at one point for a stationary state, and $\phi_{ij}(k)$ represents the contributions to

the covariance in a unit range of wave numbers centered at k . The equality of the two spectral integrals in eqn. II A 1.1 is possible by defining the scale length k in such a way as to achieve this. However, in order to have k represent a space scale of the turbulence, Taylor's hypothesis must hold. The validity of physical interpretations of the k spectra and of the scales thus depends on the validity of Taylor's hypothesis and limitations on it; these matters are discussed in Section A.2.

The ergodic hypothesis in general implies that under certain assumptions the ensemble average (e.g. the probability average of values from different places but in otherwise the same environment) of any function of random variables e.g. moments, can be estimated by an average over one or more coordinates in space or time. This means, for example, that if $F(x)$ is a function of a continuous random variable x which has the probability density $f(x)$ which is stationary, in time then

$$\int_{-\infty}^{\infty} F(x)f(x)dx \approx \frac{1}{T} \int_0^T F(t)dt$$

where $F(t)$ is the value of F at time t .

In geophysics, time averages such as $\overline{u_i u_j}$ are averages over one unique time interval at one point in space. In order to generalize from such a single record, one always must

assume that the ergodic hypothesis holds.

From eqn. II A 1.1, the relationship

$$\overline{u_i u_j} = \int_{-\infty}^{\infty} f \phi_{ij}(f) d(\ln f) = \int_{-\infty}^{\infty} k \phi_{ij}(k) d(\ln k) \quad (\text{II A 1.2})$$

follows. It is common practice to plot $f \phi_{ij}(f)$ against $\log f$ or $\log k$ (where the base is 10, so that $\ln x = 2.303 \log x$).

The reasons for presenting the data in this fashion are:

1. $k \phi_{ij}(k) = f \phi_{ij}(f)$ everywhere,
2. The scale of $\log k$ is the same as for $\log f$ but the origin is shifted (from eqn. II A 2.2)
 $\log k = \log \frac{2\pi}{U} + \log f,$
3. The area under a segment of a curve representing $f \phi_{ij}(f)$ (or $k \phi_{ij}(k)$) represents the contribution to the covariance in the corresponding frequency or wave number interval,
4. A wide range of frequencies and wave numbers can be represented without crowding the data in the low frequency and low wave number regions.

A correlation coefficient $R_{ij}(f)$ is also defined, such that

$$R_{ij}(f) = \frac{\phi_{ij}(f)}{[\phi_{ii}(f) \cdot \phi_{jj}(f)]^{1/2}} \quad (\text{II A 1.3})$$

The repeated indices i and j are not summed in this formula. This coefficient does not represent the coherence.

The physical interpretation of the covariances is that $\overline{u_1^2}$ and $\overline{u_3^2}$ represent twice the contributions to the turbulent kinetic energy per unit mass of fluctuations in the x_1 and x_3 directions respectively. The covariance $\overline{u_1 u_3}$ represents the vertical flux of horizontal momentum per unit mass. (Lumley and Panofsky, 1964, p 63; Hinze, 1959, pp 19-21).

A.2. Taylor's Hypothesis

In Section A.1, a one-dimensional wave number k was introduced. This "wave number" is related to the physical space wave number by using Taylor's hypothesis. This states that the turbulence structure is carried past a given point at the mean velocity U of the flow (see Hinze, 1959, pp 40-42). The relationship between time and space scales is then

$$\frac{\partial}{\partial x} = -\frac{1}{U} \frac{\partial}{\partial t} \quad (11 A 2.1)$$

$$k = \frac{2\pi f}{U} \quad (11 A 2.2)$$

However, in a shear flow, this hypothesis is a good approximation only as long as

$$kx_3 \gg 1 \quad (11 A 2.3)$$

In general, Taylor's hypothesis is a good one as long as $5\overline{u^2}/U^2 \ll 1$. (In our experiments the upper limit of this is about 0.1)

(Lin, 1953). For small scales (large k), the spatial structure or turbulence is quite well approximated in terms of functions of k . For large scales (small k), this physical interpretation of k is no longer tenable, since the large eddies are distorted in the shear flow near the boundary. Thus it is not possible to give a physical interpretation to k over all scales of interest.

To avoid these difficulties of physical interpretation, one can take the attitude that a k is defined by eqn. II A 2.2, by a linear transformation from a frequency parameter to a length^{-1} parameter. Further discussion of Taylor's hypothesis is found in Section II E.1.

A.3. The Shear Flow Model and Assumptions

In the model which the actual shear flow above the boundary is assumed to fit, the following assumptions are made:

1. The shear flow is two-dimensional and has horizontal homogeneity in the $x_1 - x_2$ plane,
2. The mean wind velocity is in the x_1 direction,
3. The direction of the mean wind velocity is invariant with height,
4. A stationary state exists,

5. No local convergences or divergences exist (x_1, x_2 plane)
6. The only body force is gravity,
7. Transport by molecular action is negligible compared with eddy transports.

Use of these assumptions leads to simplified equations for conservation of mechanical and thermal energy (see Lumley and Panofsky, 1964, p. 72, eqn. 2.38). Analysis of the energy budget equation shows the following facts. Energy is fed from the mean flow into that part of the turbulent kinetic energy per unit volume contained by the x_1 -component, due to working of the Reynolds stress ($-\rho \overline{u_i u_j}$, ρ is the density) on the mean flow (the production term is $-\rho \overline{u_1 u_3} \partial U / \partial x_3$). Buoyancy forces feed energy into or out of the x_3 -component of turbulence; pressure gradients redistribute this energy along all velocity components; inertial forces redistribute energy along scales of the same component, and viscous forces dissipate the energy in all three components. The existence of anisotropy is necessary to sustain the turbulence. This model gives a reasonably good approximation over boundaries remaining fixed in space.

It is not yet certain whether the model gives a reasonable approximation over a water surface which has simultaneous spatial and temporal variations in shape. In this case, some of the above assumptions appear questionable,

even in the absence of heat sources or sinks.

The validity of some assumptions of this model will be discussed in detail in Section E.1.

A.4. Anisotropy and Isotropy

Measurements in a wind tunnel by Favre et al (1957) showed that for large "eddies" in a horizontal shear flow, the scales in the direction of the mean wind were much larger than those in the lateral and vertical directions. The latter two scales were of the same order. The asymmetry of the transverse and vertical scales decreased with increasing distance from the boundary.

The nature of the mean shear flow and of convection determine the structure of the largest eddies which are necessarily anisotropic. According to the postulates of Kolmogoroff (1941) the turbulent energy present in these anisotropic eddies is passed down to smaller scales by inertial transfer in an "energy cascade", and at the same time is redistributed among velocity components by pressure fluctuations. These tend to decrease anisotropy. The anisotropy of the energy feeding process influences the energy less and less at smaller and smaller scales. Since the energy is "handed down", the turbulent structure thus tends to become more and more isotropic as the wave number increases.

A lower limit for isotropy has been estimated to be at $kx_3 > 4.5$ (Pond et al, 1963).

A.5. The Inertial Subrange

The "inertial subrange" region is discussed fully in Hinze (1959, pp 181-200) and Lumley and Panofsky (1964, pp 79-85 and 162-3).

Kolmogoroff (1941) made the hypothesis that if the Reynolds number were large enough, then there exists a range of scales, called the "inertial subrange", in which the properties of turbulence are determined entirely by the dissipation rate. Dimensional analysis gives the following form to the $\phi_{11}(k)$ spectrum in this region:

$$\phi_{11}(k) = K' \epsilon^{2/3} k^{-5/3} \quad (11 A 5.1)$$

where K' is the universal Kolmogoroff constant, having a measured value of 0.48 ± 0.055 (Pond et al., 1963), and ϵ is the rate of energy dissipation per unit mass.

In this region, no significant energy production or dissipation takes place. The turbulence is here isotropic. From analysis of the special form of the energy equations appropriate to the isotropic region (Hinze, 1959, pp 165-7), the relationship

$$\phi_{33}(k) = \frac{4}{3} \phi_{11}(k) \quad (11 A 5.2)$$

is derived.

A.6. The Two-Dimensional Reynolds Stress "Tensor"

The (negative) Reynolds stress tensor in kinematic units is $\overline{u_i u_j}$. As seen in Section 11 A.1, the tensor has the elements $\overline{u_1^2}$, $\overline{u_3^2}$ and $\overline{u_1 u_3}$ in the $x_1 - x_3$ plane. In isotropic turbulence, it must be possible to express this tensor as $\overline{u^2} \delta_{ij}$, where δ_{ij} is the Kronecker delta.

However, if the entire velocity component is not considered, but only the contribution of a single wavenumber of a one-dimensional spectrum, then a necessary condition for isotropy is

$$\begin{aligned} \phi_{13} &= 0 \\ \phi_{33} &= \frac{1}{2}(\phi_{11} - k \frac{\partial \phi_{11}}{\partial k}) \end{aligned} \quad (11 A 6.1a)$$

Thus where $\phi_{11} \propto k^{-5/3}$, $\phi_{33} = (4/3)\phi_{11}$ if the field is isotropic at the wavenumber k .

In order to express the degree and nature of the anisotropy of the turbulence at the wavenumber k , we introduce the "pseudo-tensor"

$$\begin{pmatrix} \phi_{11} & \sqrt{\frac{3}{4}} \phi_{13} \\ \sqrt{\frac{3}{4}} \phi_{13} & \frac{3}{4} \phi_{33} \end{pmatrix}$$

In isotropic turbulence this would be given by

$$\phi_{ij} \delta_{ij}$$

The orientation of the principal axes of the "pseudo-tensor", θ , which gives the "direction" of the anisotropy, can be estimated using the relationship

$$\tan(2\theta) = \sqrt{3}\phi_{13} / (\frac{3}{4}\phi_{11} - \phi_{33}) \quad (11 A 6.1)$$

This process is analogous to that used by Corrsin, (1956, pp 377-9). If at small scales, θ remains approximately constant, then it is probable that the observed anisotropy is real; while if θ varies significantly, then in all probability the measured spectral values (especially ϕ_{13}) are uncertain enough that no conclusion can be drawn with respect to anisotropy. If, on the other hand, the small scales are isotropic, the value of θ should become indeterminate. The measured values of θ thus give a good indication as to the existence or non-existence of anisotropy at small scales.

B. The Monin-Obukhov Similarity Theory

B.1. The Theory

In this theory it is assumed that the structure of the turbulence is determined by the turbulent flow itself except for scaling factors. Hence, any dimensionless grouping is universal. Then there exist near the boundary (and outside the "viscous boundary layer") a velocity u^* , a length L (the Monin-Obukhov length), and a temperature T^* which are essentially invariant with height. When velocities, lengths and temperatures are expressed as fractions of these quantities, non-dimensional equations can be formed which are of universal validity in the boundary layer.

B.2. Definitions of u^* and L

Since the present work will not be dealing with temperature distributions, T^* will not be discussed.

The quantities u^* and L describing the turbulence are defined as follows:

$$u^* = \sqrt{-\overline{u_1 u_3}}$$

$$L = - \frac{u^{*3}}{K(g/\tau)(\theta u_3)} \quad (11 B 2.1)$$

where $K = 0.4$ is the von Karman constant, g the gravitational acceleration, T the mean temperature, and $(\overline{\theta u_z})$ the average temperature flux, where θ is the potential temperature.

The non-dimensional grouping usually used for the length scale is x_z/L .

B.3. The Behaviour of L with Stability

The flux Richardson number R_f is defined by Lumley and Panofsky (1964, eqn. 2.39, p 72). It has the form

$$R_f = \frac{g}{T_o} \frac{\overline{\theta u_z}}{(\overline{u_1 u_z})(du/dx_z)} \quad (11 B 3.1)$$

For large positive R_f , characteristic of extremely stable density gradients, no turbulence exists. L is indeterminate.

For the neutral case of $R_f = 0$, no heat flux is present and $L = \infty$. The characteristic length is then x_z , the height above a boundary.

For negative R_f , L is finite and decreases as instability increases and convection grows. The value of x_z/L can then become of the order of unity.

In the extreme cases of pure convection, mechanical turbulence is not present. Here $L = 0$, so that the characteristic length is x_z .

Physical conditions encountered in atmospheric shear flows, except for some very stable conditions are usually neutral, near-neutral or unstable conditions. It is thus not correct to use x_3 as a characteristic length unless the value of L is known and is very much larger than x_3 .

C. The Mean Velocity Profile and Drag Coefficient

C.1. The Mean Velocity Profile

The mean velocity profile over a solid boundary under neutral conditions is logarithmic. It has the form

$$U = \frac{u^*}{K} \ln(x_3/z_0) \quad (11 C 1.1)$$

where z_0 is a characteristic length and $K \approx 0.4$ is von Karman's constant. A derivation of this expression is found on p. 103, Lumley and Panofsky (1964).

For near neutral conditions, L is finite but x_3/L is small since the rate of production of convective energy is small when compared to the rate of production of mechanical energy.

For this condition, under the Monin-Obukhov assumption, the mean velocity profile becomes partly logarithmic and partly linear. The form of the profile is "log-linear"

$$U = \frac{u^*}{K} \left[\ln(x_3/z_0) + \alpha (x_3/L) \right] \quad (11 C 1.2)$$

(pp 106-107 of Lumley and Panofsky, 1964). In this case, x_3/z_0 must be much larger than one and x_3/L much less than one. α is a constant which must be empirically determined.

In the eqn. 11 C 1.2, L appears. This contains the temperature flux which is rarely available. A modified L' is then commonly used, where

$$L' = \frac{u^*(dU/dx_3)T}{Kg(dT/dx_3)} \quad (11 C 1.3)$$

This gives the modified log-linear profile

$$U = \frac{u^*}{K} \left[\ln(x_3/z_0) + \alpha' (x_3/L') \right] \quad (11 C 1.4)$$

The value of the constant α' has been estimated to be anywhere from 4 to 7. (See Lumley and Panofsky, 1964, pp 107-108.)

The relationships between L and L' , and α and α' are

$$L' = \frac{K_h}{K_m} L$$

$$\alpha' = \frac{K_h}{K_m} \alpha \quad (11 C 1.5)$$

where K_h is the eddy conductivity and K_m the eddy viscosity. The value of this ratio of eddy coefficients remains in dispute; in neutral air it is considered to be near unity.

C.2. The Drag Coefficient C_D

At a uniform solid surface in neutral conditions, the stress is found to increase linearly with the square of the mean wind speed. This allows a "drag coefficient" to be defined, such that

$$C_D = \frac{u_*^2}{U^2} \quad (11 C 2.1)$$

C_D must always be referred to a fixed height.

A discussion of the drag coefficient is found in Priestley (1959, pp 21-22.)

D. The Indirect Determination of $u_*^2 = -\overline{u_1 u_3}$

D.1. Using $\phi_{11}(k)$ in the Inertial Subrange

The eqn. (11 A 5.1) contains $\mathcal{E}$, the rate of energy dissipation. If it is now assumed that the logarithmic wind profile holds (neutral conditions), and that the rate of production of turbulent energy equals the dissipation rate

[i.e. $\mathcal{E} = \frac{u_*^2}{dx_3} \frac{dU}{dx_3}$] then from 11 C 1.1, and 11 A 5.1.

$$u_*^2 = \left(\frac{K^{2/3}}{K'} \right) (k x_3)^{2/3} k \phi_{11}(k) \quad (11 D 1.1)$$

where the value of the constant bracketed term is around 1.13.

Use of the above eqn. allows one to estimate u_*^2 from measurements of the spectrum $\phi_{11}(k)$, of horizontal fluctuations, in the inertial subrange, provided the above conditions hold.

D.2. Using the Mean Wind Velocity Profile

Both eqns. 11 C 1.1 and 1.2 contain u_* . By measuring the mean wind profile under neutral or near-neutral conditions, an estimate of u_* can be obtained. If 11 C 1.1 holds, this can simply be estimated from the slope of a graph of U versus $\log x_3$.

E. The Results of Other Workers

E.1. The Shear Flow Model

The model and its assumptions are discussed in Section 11 A.3.

Although the two-dimensional shear flow model has been widely used to describe boundary layer flows in the atmosphere, recent studies by Fleagle et al (1958), Faller (1963), and Kraus (1966) have brought some of its assumptions into question.

Faller (1963) studied boundary layer instabilities generated in an Ekman flow produced in a rotating tank. He

found that the laminar boundary layer became unstable above a critical Reynolds number, and that the initial instability had a banded form with the band at a characteristic and quite small angle to the basic flow. These instabilities were a boundary layer phenomenon, and persisted even after the onset of turbulence.

Since Ekman-type flows occur in the atmospheric boundary layers to heights of many hundreds of meters, then such instabilities are possible, although the analogy must not be drawn too far since the boundary layer is turbulent and can exhibit thermal instabilities. If such non-homogenous elongated cellular structure occurs, the structure may extend to the surface so that divergences and convergences could be present in the turbulent boundary layer. The distribution of the downwards rate of momentum transfer would thus be spatially non-homogeneous, since vertical advections associated with convergences also transfer momentum. Also, phenomena associated with large cellular structures, elongated almost downwind, would change slowly with time, so that the distribution in one localized area would change slowly and be non-stationary. Estimation of the average stress acting over a very large area, by measurements of any kind at one point only, might then give erroneous estimates of the momentum flux, unless averages are taken over sufficiently long time periods.

Measurements of vertical wind profiles by Fléagle et al (1958) pointed toward the possible existence of large-scale ordered roll vortices with axes perpendicular to the mean wind direction. Kraus' (1966) observations off Aruba showed that a systematic three-dimensional variation in the velocity field occurred. The variations appeared to be of a cellular nature, and did not average out over reasonably short averaging times.

The above results show that the assumptions of large scale horizontal homogeneity in a two-dimensional shear flow, are suspect. Until further field measurements resolve this question, the assumptions can be accepted only with reservations.

The assumption of stationarity in time is one which can be imposed approximately on the field data by analyzing only those sections which reasonably exhibit this condition (see Appendix IV B.).

In the boundary layer over the sea the horizontal velocity scales are of the order of a few meters per second, with length scales (at a height of a few meters) of a few hundred meters as an upper limit. Analysis of the equations of motion and conservation, for situations not involving roll vortices or similar phenomena, shows that the Coriolis force has negligible effects on the variation of stress in the

lowest few meters from the surface. This leaves gravity as the only body force, and allows one to neglect the variation in direction of the mean wind with height in this bottom boundary layer.

E.2. The Inertial Subrange

Wind tunnel turbulence measurements to very small scales have been made by many workers (see Gibson, 1962; Nature, 195, pp 1281-1283). The data agree quite well with the Kolmogoroff spectrum described by II A 5.1. Measurements by Grant et al (1962) in a tidal channel also show very good agreement. Pond's (1963, 1965) measurements in the turbulent boundary layer over the sea further confirm the $k^{-5/3}$ behaviour. He also summarizes values of the Kolmogoroff constant K' measured in the ocean, an air jet, water tunnel, and the atmosphere, obtaining an average value of 0.48 ± 0.055 over a wide range of Reynolds number. This agreement among values found in different media and different types of flow demonstrates that K' is indeed approximately universally constant.

The measured value of K' clearly allows the dissipation rate ϵ to be inferred (from II A 5.1) from the measurements of the one-dimensional energy spectrum in the inertial subrange.

E.3. The Mean Velocity Profile over a Water Surface

The mean velocity profile over a water surface has been measured by many workers. (Including Charnock, 1955; Deacon et al, 1956; Sheppard, 1963; Brocks and Hasse, 1963; Fitzgerald, 1963; Takeda, 1963; Hamblin, 1965.) The majority of the profiles have been approximately linear in U versus $\log x_3$, but large scatter in slope and about the best fit line has been observed, except for Fitzgerald's data over a water surface in a wind tunnel. Negative profile curvatures were found by Hamblin (1965) which could not be related to any observed phenomenon. He also found that the profile slopes, at the same mean wind speed, tended to be higher when the measured wind speed had been increasing just before the profile was measured, and vice versa when following a decrease.

No proven criterion exists for determining the best averaging time for mean velocity profiles. The averaging periods used have thus depended on the worker. Hamblin (1965) found that 10 minute averages gave significant correlation between the slope and average speed, at a fixed height, and that shorter periods gave smaller correlations. The matter is still unresolved.

E.4. The Drag Coefficient

The drag coefficient C_D is defined by eqn. 11 C 2.1. It has been common practice to refer it to winds at a height of 10 m, although 5 m and lesser heights have been used. Values used in the following discussion have all been adjusted to a 5 m height, since my mean wind data does not extend any higher.

The behaviour of C_D with increase in the mean wind speed is in dispute. Brocks (1963) obtained a constant value of 1.5×10^{-3} for all wind speeds measured (up to about $20 \text{ m} \cdot \text{sec}^{-1}$), but his mean wind profiles used for the stress determinations are disputed by some. Sheppard (1963) obtained a linear increase with mean wind up to about $18 \text{ m} \cdot \text{sec}^{-1}$, from $C_D = 1.1 \times 10^{-3}$ at $2 \text{ m} \cdot \text{sec}^{-1}$ to 1.8×10^{-3} at $8 \text{ m} \cdot \text{sec}^{-1}$, again using wind profiles. Fitzgerald in 1963 observed the profile over water in a wave tank and found a constant drag coefficient (2.24×10^{-3} at a height of 2 cm) at wind speeds below about $4 \text{ m} \cdot \text{sec}^{-1}$, above which it slowly increased with increasing wind speed. The results of Zubkovskii and Timanovskii (1965) included stress measurements made by other workers which were compared with their own profile and direct stress measurements (using a sonic anemometer). They found that the drag coefficient at a height of 200 cm increased with increasing wind speed, from a value of 0.6×10^{-3} at $3 \text{ m} \cdot \text{sec}^{-1}$ to a value of 1.1×10^{-3} at $5 \text{ m} \cdot \text{sec}^{-1}$. Hamblin (1965) obtained average values of 2.1×10^{-3} , although scatter was very large, e.g. near

one wind speed it varied from about 1×10^{-3} to 4×10^{-3}

Hamblin (1965) also suggested that a large amount of the scatter could be due to inherent weaknesses of the wind profile method of stress measurement. The results of Fleagle et al (1958), Faller (1963), and Kraus (1966) (described in Section II E.1) would seem to indicate this. Also, Stewart (1961) pointed out that over an ocean boundary which is not stationary in space or time, besides the height, another length parameter determined by wave characteristics may also be important in determining turbulence characteristics.

III EXPERIMENTAL METHODS

The data were collected at a site located about 0.5 km north of Spanish Banks Beach (see Fig. III.1). A hut on a platform raised 20 ft. above the sandy bottom housed the recording instruments, and a rotatable aluminum mast 6 m high and 40 m N.N.W. of the platform supported the sensors. The sensors were attached to a movable frame on the mast, so that they could be moved up or down together. Electrical cables carried signals from the sensors to the recording equipment.

The sensing instruments were hot wire probes, cup anemometers, and relative and absolute wind direction indicators. These are discussed in Appendices I to III. Measurements were also made simultaneously with a thrust anemometer, bead thermistors, and a capacitance wave probe; these form parts of various projects within the air-sea interaction program and are not discussed in this thesis.

Fig. III.2 shows a photograph of the experimental site. It should be noted that the apparent lean of the main mast is an optical effect and is not real. Also, the equipment on the horizontal arm observed at the left of the main mast was not part of the regular air-sea interaction program. It belonged to the University of Washington, and was used in a joint attempt to obtain simultaneous and independent

measurements of various statistics of turbulence.

All outputs from the equipment measuring fluctuating quantities and spoken information were recorded as analog signals on an Ampex CP-100 14-channel magnetic tape recorder.

Digital data from the cup anemometers, the wind direction indicator and a current meter were recorded on photographic film using a camera with an automatic timer. To ensure the availability of mean wind speed data, pulses from the cup anemometers were also recorded on the magnetic tape as multiplexed F.M. signals.

To check on possible interference to wind flows caused by instruments, supports, cables and boxes mounted on the mast, a single, well-exposed cup anemometer called a "Rover" was used to attempt to calibrate the cups in situ. This was at the top of a separate 1 inch diameter aluminum mast 10 m cross-wind from the instrument mast. It was raised and lowered hydraulically; a scale attached to the Rover mast rode past a fixed pointer to indicate the relative height of the cup anemometer.

Cup anemometer data from the electro-mechanical counters, wind direction readings, and tidal current readings and other relevant information were entered in data log sheets at regular intervals and also read into the voice channel.

The descriptions of these instruments and of their electronics are found in Appendix III, Section A.

The data collected on photographic film and magnetic tape were analyzed later using analysis techniques discussed in Appendix IV.

Times are quoted in Pacific Daylight Saving Time , written on a 24 hour basis as hour/day/month/year. Logarithms are all to the base 10.

IV. EXPERIMENTAL RESULTS

A. General Information on Each Run, and Wind Direction and Cup Anemometer Data

With the exception of run 1555-1629 /20/9/1965, wind direction and mean speed data were obtained from data log sheets (see Section III, and Appendix IV). Numerical results are summarized in Tables IV A.1 and A.2, and are arranged approximately in order of increasing wind speed. Multiple runs recorded during a single long time interval are listed in the order they were recorded. Runs with three vertically spaced U-wires are grouped at the end. Mean wind speeds quoted are those at the height at which the X-wire probe was mounted above the mean surface. For the three U-wire runs, the speed quoted is that measured near the central hot wire probe. Errors to be expected in numerical results are discussed in Appendix VI.

The run 2010-2034 /21/6/1965 (Fig. IV. A.1) was the only run with an easterly wind, and also had the lowest wind speed. At the end of the run, the wind swung from NE to due east, and dropped to almost zero speed. The tide was rising to a maximum. Conditions were slightly unstable, with water temperature 17.7°C and air temperature 17.7°C about 360 cm above the mean water surface. The fetch was about 7 km.

The run 1555-1620 /20/9/1965 (Fig. IV A.2) was recorded

in a westerly wind during slack water. Conditions were unstable, with water temperature 15.0°C and air temperature 14.5°C about 40 cm above the surface. Temperatures were estimated only to about the nearest one-half degree for this run. The fetch was about 40 km.

Runs 2039-2122 and 2122-2206 /29/6/1965 (Figs. IV A.3 and A.4) were recorded as one piece during slack water, and then divided into two sections for analysis. During the runs, the mean speed of the westerly wind dropped and, after the end of the recorded sections, veered southward.* Conditions were strongly stable, with water temperature of 16.6°C and air temperature of 18.4°C about 240 cm above the mean surface. Fetch was about 40 km.

For the day /22/7/1965, four runs were analyzed (Figs. A.5, A.6, A.7 and A.8). After the first run, the instrument mast was turned slightly to point it better into the westerly wind, and the last three runs were recorded as one piece during a slowly ebbing tide; this piece was divided for analysis. At the end of the final run, the waves suddenly turned the mast, ending the recording for the day. Conditions

* Later analysis using one-minute averages showed that for the last 5 minutes of the run 2122-2206, the mean wind dropped from near the mean speed quoted to about half the value.

during the four runs were unstable, with the water at 19.0°C and the air at 18.0°C about 350 cm above the mean surface. The fetch was about 40 km.

Runs 2230-2258 and 2305-2336 /26/6/1965 (Figs. IV A.9 and A.10) were recorded during the highest useful wind speeds of the summer, about $10 \text{ m}\cdot\text{sec}^{-1}$. During the evening of 26/6/1965, a north-westerly gradient wind had been building up speed, reaching the maximum before midnight. From about 2220-2235, the speed remained almost constant, and dropped rapidly thereafter. Recording was terminated at 0010, when heavy waves turned the mast about 60° . The tide was almost slack during the analyzed runs. Conditions during the end of the first run were slightly stable, with the water temperature 15.7°C and the air temperature 16.0°C , about 340 cm above the surface. Midway during the second run, conditions were slightly unstable with the water temperature 15.7°C and the air temperature 15.2°C about 330 cm about the surface. The fetch was about 70 km.

Runs 1550-1605 and 1607-1645 /24/7/1965 (Figs. IV A.11 and A.12) were recorded as one piece in a westerly wind during a slowly rising tide.

Three U-wire probes at different heights were used. The mean wind remained approximately in the same direction, but decreased slightly for the second run, and continued

blowing with decreasing wind speed after the last recording. Conditions for the two runs were stable, with water temperature at 19.3°C , and air temperature at 21.0°C , about 300 cm above the mean surface. The fetch was about 40 km.

The run 1501-1525 /25/7/1965 (Fig. IV A.13) was the first section of a piece recorded using 3 U-wires in a westerly wind during a rapidly rising tide. Only the first section could be usefully used, since after it, the wind speed increased about 15% in the next 15 minutes, and then dropped rapidly below the speed of the first section. The mean wind direction remained almost constant throughout. Conditions during the run were effectively neutral, with the water at 19.6°C and the air at 19.7°C , about 320 cm above the mean surface. Fetch was about 40 km.

Profiles for 1448-1525 /22/7/1965 (Fig. IV A.5), 1650-1715 /22/7/1965 (Fig. IV A.8) and 2230-2258 /26/6/1965 (Fig. IV A.9) are anomalous in that the speeds measured by the bottom cup anemometer exceed those of the cup just above it. No explanation has been determined for this condition. Rover data were not included since the instrument was not operated properly during the periods of the measurements.

Wind direction data for all the above runs are summarized in Table IV A.1 below. Direction recording methods are described in Appendix III. The averages $\overline{\theta_s}$ are

given for N readings for the run by

$$\overline{\theta'_s} = \frac{1}{N} \sum_{i=1}^N \theta'_i \quad (\text{IV A.1})$$

with the standard error σ_s , where

$$\sigma_s^2 = \frac{1}{(N-1)} \sum_{i=1}^N (\theta'_i - \overline{\theta'_s})^2 \quad (\text{IV A.1})$$

The subscripts "r"* stand for direction relative to the X-wire probe at the instrument mast and "a"* for the direction relative to Pt. Atkinson indicated by the Casella wind vane at the platform. The latter vane was lined up on the Pt. Atkinson lighthouse ($\theta'_a = 0$). True north lies 19° east of the 'absolute' zero direction i.e. $\theta'_{\text{true}} = \theta'_a - 19^\circ$. The correlation coefficient ρ' is defined by

$$\rho' = \frac{(\overline{\theta'_r - \overline{\theta'_r}})(\overline{\theta'_a - \overline{\theta'_a}})}{\sigma_r \times \sigma_a} \quad (\text{IV A.2})$$

where the overbars in this case denote the averages of similar form to eqn. II A.1.

* In order to make these angles clearer, explanatory diagrams are provided below:

θ_r θ_a

Table IV A.1: Wind Direction Data. (Angles in degrees):

1965	Time	N_r	$\bar{\theta}_r$	σ_r	N_a	$\bar{\theta}_a$	σ_a	ρ'
Date	of Run							
21/6	2010-2034	7	9.6	-	3	55	-	-
20/9	1555-1629	41	-2.7	6.9	41	264	7.3	0.83
29/6	2039-2122	37	3.5	4.4	36	264	5.3	0.24
29/6	2122-2206	49	-3.5	4.7	49	258	7.2	0.54
22/7	1448-1525	36	6.8	5.1	36	273	5.7	0.51
22/7	1553-1625	33	3.5	4.7	33	270	5.2	0.60
22/7	1620-1652	30	9.9	3.4	30	276	2.7	0.11
22/7	1650-1715	21	11.3	3.5	21	274	5.4	0.38
26/6	2230-2258	17	1.7	7.1	17	304	4.1	0.38
26/6	2305-2336	25	-2.8	7.9	25	305	4.8	0.35
24/7	1550-1605	16	8.2	5.0	16	260	-	-
24/7	1607-1645	35	7.3	4.2	35	250	-	-
25/7	1501-1525	24	1.8	4.4	24	273	3.9	0.83

For the run 2010-2034 /21/6/1965, no variances were computed since there is an insufficient number. The average absolute direction was judged to be about 55° . No correlation coefficient was calculated. For the two runs on 24/7/1965, absolute direction data were for the most part read to the nearest 5° , so that computed variances and correlation coefficients would have little statistical significance.

The unreliability of estimates of ρ' is well known. The distribution of ρ' is discussed by Cramer (1946, p 397); the observed values have reasonable significance only when $N \geq 20$ and $\rho' > 0.5$ (approximately). Nevertheless, since observed values of ρ' are all of one sign, it can be asserted that the two separated instruments observe dominantly the same fluctuations.

Wind profile data for all runs are summarized in Table IV A.2 below. Linearity of the profile, implying that a reasonable straight line can be fitted through the measured points for the U versus $\log x_3$ plots (Fig. IV.A and see Section II C) is denoted by the letter L in the appropriate column. N denotes that a curve with negative curvature gives the best fit. These values of u_*^2 are computed from the observed profiles using eqn. II C1.1 (see also Section II D and Appendix III). For the negative curvature cases, the local slope between the second and third cups is used, because the X-wire probe is normally positioned there. Stability during runs is denoted by: U - unstable, N' - neutral and S - stable. When the air temperature minus the water temperature (measured just under surface) was positive, conditions were considered stable, and if negative, unstable. If the difference was zero, or a few tenths of a degree, conditions were considered neutral.

The last two columns of Table A.2 are described in Section IV F.

Table IV A.2: Mean Wind Profile (and Temperature) Data:

1965 Date	Time of Run	Line- arity	Stabil- ity	U^*^2 cm.sec ⁻²	L' cm.	$\times 3$ $\overline{L'}$
21/6	2010-2034	N	U	1.7	-200	-0.7
20/9	1555-1629	L	U	112	-	-
29/6	2039-2122	L	S	356	300	0.6
29/6	2122-2206	L	S	151	(200)	(1.)
22/7	1448-1525	N	U	2.8	-600	-0.3
22/7	1553-1625	L	U	128	-900	-0.2
22/7	1620-1652	L	U	109	-1×10^3	-0.1
22/7	1650-1715	L	U	47.1	-800	-0.2
26/6	2230-2258	N	S	1320	1×10^4	0.03
26/6	2305-2336	N	U	645	-6×10^3	-0.05
24/7	1550-1605	L	S	428	$S_4: 700$ ($S_3: 500$)	0.2 (0.5)
24/7	1607-1645	N	S	122	$S_4: 700$ ($S_3: 500$)	0.2 (0.5)
25/7	1501-1525	L	N'	126	$S_4: 1 \times 10^4$ ($S_3: 6 \times 10^3$)	0.01 (0.03)

B. X-and U-Wire Data

The construction and calibration techniques developed for field measurements of turbulence by the X- and U-wire probes are described fully in Appendix I. Electronics of the hot wire system are described in Appendix II.

The construction developed for mounting the wires onto probes (see Appendix I, Section B.1 and C.1) proved to be successful. Wire breakages were negligible, and the wires remained rigid as demonstrated by the constancy of the measured wire angles during repeated calibrations (see Appendix I, Section B.2 and C.3). For example, for the X-wire

probes used, the measured wire angles of the two wires changed 1° or less between repeated calibrations.

The calibration procedure for the U-wires was straightforward and did not give rise to any difficulties. It is described in Appendix I, Section B.2. Repeated calibrations of the same U-wire probe showed that the sensitivity to relative velocity fluctuations (the constant B of the King's Law eqn. A 1.12) changed by 4.4% at the most between calibrations with the usual change being less than 4%. This was considered adequate for field measurements. However, the absolute calibration level (the constant A of eqn. A 1.12) changed up to about 10% between calibrations. This would cause differences in determinations of the mean velocity U only of about 20%, illustrating the common opinion that hot wire probes are not reliable instruments for measuring absolute winds.

Section C.3 in Appendix I describes the new calibration procedure used for X-wire probes. Repeated calibrations for the determination of the constants in King's Law, gave a similar usual change of less than 4% in the constant B of eqn. A 1.12 (the maximum change was 4.2%). Changes in A, between repeated calibrations were again less than 10% between calibrations.

The empirical determination of the wire constant b' (which determines the wire's sensitivity to the vertical wind velocity component, eqns. A 1.10 and A 1.14) for each wire of the X-wire probes used for data collection in 1965 is

described in Appendix I, Section C.4. The empirical determination of b' was accomplished once during the fall of 1965 for each wire of the X-wire probes used, and the method worked successfully. The only drawback to this empirical technique is that it is very time consuming; even under the very best of conditions it takes the best part of a day to measure the wire angle, calibrate the wire responses and empirically determine the wire constant b' for each of the two wires of an X-wire probe.

The procedures used to record, rerecord and analyze hot wire analog data are discussed in Appendix IV.

The analysis technique used for U-wire analog data is straightforward, and is discussed in Section C of Appendix IV.

The analysis technique developed for the X-wire analog data is discussed in Section D of Appendix IV. Eqns. A IV.9 to A IV.14 outline the derivation of the values of the spectra $f\phi_{ij}(f)$ and $\phi_{ij}(k)$ from the raw results of the analog analyses. These determinations were carried out using the University's I.B.M. digital computer.

A normal analysis of a single section of data gave about two dozen spectral points over a frequency range from 0.016 to 60 Hz, and took about two days to accomplish. Serially analyzed sections took less time, since one setting

of the filters was necessary to analyze two to four runs at the same frequency. Section E of Appendix IV outlines the effect on spectral values of varying β , the angle the mean wind direction makes with the vertical plane parallel to the X-wires.

Sample calculations for analog analysis of X- and U-wire data are outlined in Appendix V. Errors to be expected are discussed in Appendix VI.

Figs. IV B.1 to B.10 show $f\phi_{ij}(f)$ (defined in Section II A.1) versus $\log f$. For comparison, values of $f\phi_{11}(f)$ measured both with the U-wire and the X-wire during each run are shown. It should be noted that a variety of scales are used along the $f\phi_{ij}(f)$ axis to obtain the optimum display of data. Scales of $\log f$ remain the same for all runs. The origins for $\log k = \log f + \log \left(\frac{2\pi}{U} \right)$ and $\log (kx_3) = \log f + \log \left(\frac{2\pi x_3}{U} \right)$ are indicated (see Figure Captions p 84). The data are arranged in the same order as the mean wind data (Section IV A. Figs. A.1 to A.10).

Four representative plots of $\log \phi_{ij}(k)$ versus $\log k$ (Figs. IV B.11 to B.14) are included to demonstrate the behaviour of the $\phi_{ij}(k)$ spectra of high wave numbers (see Section II A.5 and eqn. II A 5.1). The straight line on each plot has a $-5/3$ slope.

Numerical results obtained from the X- and U-wire data

are shown in Table IV B.1. The data are grouped in the same order as before. The mean velocity quoted is the one interpolated at the probe height. Directly measured values of $u^{*2} = -\overline{u_1 u_3}$ were obtained by approximating eqn. II A1.2 by summation of measured spectral densities. Indirect estimates of u^{*2} from the values of $\phi_{11}(k)$ measured in the inertial subrange (see theory in Section II D) were obtained using eqn. II D1.1. X-wire values of $\phi_{11}(k)$ were used, since use of data from the U-wire unnecessarily introduces variations associated with changes in mean response of the U-wires.

Table IV B.1: X- and U-Wire Spectral Data:

1965 Date	Time of Run	U cm.sec ⁻¹	x_3 cm.	u^{*2} :X-wire cm ² sec ⁻²	u^{*2} :from $\phi_{11}(k)$ cm ² sec ⁻²
21/6	2010-2034	138	160	30.8	55.9
20/9	1555-1629	247	202	131	190
29/6	2039-2122	334	168	179	220
29/6	2122-2206	289	192	127	190
22/7	1448-1525	336	195	178	308
22/7	1553-1625	431	195	265	405
22/7	1620-1652	379	200	384	375
22/7	1650-1715	328	210	254	343
26/6	2230-2258	983	270	1420	2000
26/6	2305-2336	975	265	1240	1510

C. Data from Three Vertically Spaced U-Wires:

To check the Monin-Obukhov similarity theory, three U-wires were mounted at heights -13, 67 and 340 cm relative to the bottom cup anemometer. From each of the three

simultaneously recorded responses, values of $kx_3 \phi_{11}(kx_3) = f\phi_{11}(f)$ were obtained at each frequency analyzed, as were indirect estimates of $u_*^2 = -\overline{u_1 u_3}$ (from ϕ_{11}).

Figs. IV C.1, C.3 and C.5 show the plots of $kx_3 \phi_{11}(kx_3)$ against $\log(kx_3)$. The data were plotted in this fashion for reasons given in the discussion in V C.

The Monin-Obukhov similarity theory is discussed in Section II B. Since eqn. II D1.1 used to estimate u_*^2 is valid only in the inertial subrange, the quantity

$$S = \left[\frac{k}{k_1} \right]^{2/3} (kx_3)^{2/3} k\phi_{11}(k) \quad (\text{IV C.1})$$

is plotted as a function of $\log(kx_3)$ for each run in Figs. IV C.2, C.4 and C.6. At wave numbers high enough to be in the inertial subrange, $S = u_*^2$, provided all the assumptions of Section II D.1 hold.

Procedures for recording, rerecording and analyzing the analog data are discussed in Chapter III and Appendix IV. Sample calculations are outlined in Appendix V. Errors to be expected in the quoted values are discussed in Appendix VI.

Wind profiles for the three runs are shown in Figs. IV A.11 to A.13.

Numerical values for these three runs are given in Table IV C.1. Units of u^{*2} are $\text{cm}^2\text{sec}^{-2}$. The mean wind speed at the 5 m height is denoted by U_5 .

Table IV C.1: Data for Three Vertically Spaced U-Wires:

1965 Date	Time	U_5 cm sec ⁻¹	u^{*2} U-wire	u^{*2} Profile
24/7	1550-1605	456	330	428
24/7	1607-1645	408	300	122
25/7	1501-1525	464	170	126

D. Summary of Profile and Hot Wire Numerical Results

Values of u^{*2} (in units of $\text{cm}^2\text{sec}^{-2}$) determined directly from X-wire measurements, indirectly from the $\phi_{11}(k)$ spectrum in the inertial subrange, and from the mean wind profile are summarized in Table IV D.1. As before, the three vertically spaced U-wire runs are included last. Mean velocities and probe heights are quoted for each of the three vertically spaced U-wire probes (labelled S4, S3, and S9) for each run using this probe array.

These results, excluding the three vertically spaced U-wire ones, are displayed graphically in Fig. IV D.1. A line sloping at 45° is shown for each graph.

Table IV D.1: Values of u^2 Determined for All Hot Wire Runs:

Date		U	x_3	u^2	u^2	u^2
1965	Time	cm sec ⁻¹	cm	X-wire	$\phi_{11}(k)$	Profile
21/6	2010-2034	138	160	30.8	55.9	1.7
20/9	1555-1629	247	202	131	190	112
29/6	2039-2122	334	168	179	220	356
29/6	2122-2206	289	192	127	190	151
22/7	1448-1525	336	195	178	308	2.8
22/7	1553-1625	431	195	265	405	128
22/7	1620-1652	379	200	384	375	109
22/7	1650-1715	328	210	254	343	47.1
26/6	2230-2258	983	270	1420	2000	1320
26/6	2305-2336	975	265	1240	1510	645
24/7	1550-1605	S4: 395 S3: 416 S9: 457	157 237 511	-	330	428
24/7	1607-1645	S4: 376 S3: 388 S9: 409	147 227 501	-	300	122
25/7	1501-1525	S4: 413 S3: 432 S9: 459	80 160 433	-	170	126

Values of the drag coefficients C_D , determined from the eqn. II C 2.1 for each run, are summarized in Table IV D.2. All coefficients are referred to a standard 5 m height where the mean wind speed is denoted by U_5 .

These values are arranged in the same order in Table IV D.1. Values of C_D as a function of the mean wind speed at the 5 m height are shown in Fig. IV D.2, and are annotated according to the measurement technique used to determine them.

Table IV D.2: Values of C_D Determined for All Hot Wire Runs:

U_5 cm sec ⁻¹	X-wire	$C_D \times 10^3$ $\phi_{11}(k)$	Profile
145	1.62	2.65	0.2
267	1.62	2.66	1.57
388	1.14	1.46	2.36
328	1.18	1.76	1.40
370	1.30	2.24	0.0
455	1.29	1.95	0.61
408	2.32	2.25	0.65
345	2.11	2.88	0.39
1060	1.26	1.77	1.17
1057	1.11	1.35	0.57
456	-	1.58	2.05
408	-	1.80	0.73
464	-	0.78	0.58

E. The Diagonalization Angle θ of the Reynold's Stress Tensor:

The two-dimensional Reynold's stress "tensor" is discussed in Section II A.6.

Fig. IV E.1 shows the values of θ determined from eqn. II A 6.1 as a function of frequency for three X-wire runs on /22/7/1965. This figure demonstrates typical and atypical behaviour of θ . In runs 1620-1652 and 1650-1715, which are typical, θ increases as frequency increases, to an almost constant value for the frequencies corresponding to wave numbers in the inertial subrange. The run 1553-1625 was the one atypical run; in this case θ does not increase with frequency but lies between about 1° and 10° , with no systematic variation.

F. The Monin-Obukhov Length:

The Monin-Obukhov length L is defined by eqn. II B 1.1. Values of L' , the modified Monin-Obukhov length, are calculated for each run using eqn. II C 1.3 and summarized in the last two columns of Table IV A.2 (on page 35). In eqn. II C 1.3, the approximation

$$\frac{dT}{dx_3} = \frac{T_{\text{air}} - T_{\text{water}}}{\Delta x_3}$$

is used, where Δx_3 is the height above the mean surface at which T_{air} was measured. T_{air} and T_{water} (the air and the water temperatures respectively), are given in Section IV A.

Temperatures for the run 1555-1629 /20/9/1965 were estimated only to the nearest one-half degree Celsius; thus the estimate of L' would be extremely questionable and was not included. For the run 2122-2206 /29/6/1965, the estimated value of L' was of about the same magnitude as the probe height x_3 . For the runs with three vertically spaced U-wires, L' was computed for the two lower U-wire probes, S4 and S3; temperature data did not extend any higher.

Although for most runs, x_3/L' is of the order of 0.3 or less, the ratio is sometimes near unity. However, since the temperature measurements were of a rather crude nature, the numbers quoted cannot be considered more than order of

magnitude estimates.

G. The Effect of $\beta \neq 0$:

An analysis of the effect on measured spectral values, of a deviation β , in the direction of the mean wind from the vertical plane parallel to the X-wires, is given in Appendices A I.D and A IV.D.

To illustrate this effect, a run was chosen (from Table IV A.1) with a small average relative deviation angle $\overline{\theta}_r$ (= estimated value of β) and a small standard error σ_r . The run 2122-2206 /29/6/1965 with $\overline{\theta}_r = -3.5^\circ$ and $\sigma_r = 4.7^\circ$ best met these criteria.

Estimates of the relative changes, from spectral levels $f\phi_{ij}(f)$ to levels $f\phi_{ij}'(f)$ appropriate to $\beta = \pm 5^\circ, \pm 10^\circ$, and $\pm 15^\circ$ are shown in Fig. IV G.1. The $f\phi_{ij}'(f)$ values were calculated from eqns. A IV.16 and A IV.17, using the $f\phi_{ij}(f)$ values as normally computed from eqns. A IV.10 to .13. The ratio $\phi_{ij}'(f) / \phi_{ij}(f)$ was calculated and plotted as a function of f for each of the three values of β .

From Fig. IV G.1, it is seen that the proportional change in level for each $\phi_{ij}(f)$ is almost constant at frequencies higher than about 0.2 Hz ($\log f \sim -0.7$). The apparent scatter in the ratios at $f > 10$ Hz was introduced by the lower signal-to-noise ratio in the recorded data at these

frequencies. At frequencies higher than 0.2 Hz, the effect of $\beta \neq 0$ is to overestimate $\phi_{11}(f)$ values, and to underestimate $\phi_{33}(f)$ values, in each case by almost a constant percentage which depends on β . At frequencies lower than this, the effect on $\phi_{33}(f)$ is frequency-dependent. The effect on values of $\phi_{13}(f)$ is negligible at frequencies higher than about 0.2 Hz, but below this frequency, $\phi_{13}(f)$ is overestimated, with the amount again being frequency-dependent.

The proportional changes for $\beta = \pm 5^\circ$, $\pm 10^\circ$ and $\pm 15^\circ$ are given in Table IV G.1. These proportional changes are expressed in terms of the ratios $\phi_{ij}'(f)/\phi_{ij}(f)$ at frequencies higher than 0.2 Hz.

Table IV G.1: The Effect of $\beta \neq 0$:

β	ϕ_{11}'/ϕ_{11}	ϕ_{33}'/ϕ_{33}	ϕ_{13}'/ϕ_{13}
± 15	1.10	0.89	1.01
± 10	1.05	0.95	1.00
± 5	1.01	0.99	1.00

H. The Ratio of ϕ_{33}/ϕ_{11} for All X-Wire Runs

The ratio ϕ_{33}/ϕ_{11} in the "isotropic" wave number range was calculated for all X-wire runs. This was done to check the theoretical predictions for the numerical value of this ratio, as outlined in Section II A.5.

Table IV H.1 outlines the measured values of the ratio for each X-wire run. In order to determine the minimum, average and maximum values, the ratio was computed for the frequency range which corresponded to $kx_3 \gg 10$. This was above the 'isotropic limit' estimated by Pond et al (1963). An upper frequency limit of about 20 Hz was set because of noise considerations (see Section V B). Since it was suspected that the effect of $\beta \neq 0$ (see Section IV G) may produce discrepancies between the theoretically predicted value and the value actually observed, the measured angular deviation $\overline{\theta}_r = \beta$ and standard error σ_r were included in the table.

Table IV H.1: The Ratio $\phi_{33}(f) / \phi_{11}(f)$ for X-Wire Runs:

1965 Date	Time of Run	ϕ_{33}/ϕ_{11}				
		$\overline{\theta}_r$	σ_r	Minimum	Average	Maximum
21/6	2010-2034	9.6	-	0.86	0.95	1.05
20/9	1555-1629	-2.7	6.9	0.97	1.04	1.15
29/6	2039-2122	3.5	4.4	1.06	1.12	1.18
29/6	2122-2206	-3.5	4.7	1.08	1.14	1.24
22/7	1448-1525	6.8	5.1	0.75	0.82	0.88
22/7	1553-1625	3.5	4.7	0.62	0.69	0.82
22/7	1620-1652	9.9	3.4	0.87	0.95	1.02
22/7	1650-1715	11.3	3.5	0.96	1.00	1.09
26/6	2230-2258	1.7	7.1	0.55	0.67	0.75
26/6	2305-2336	-2.8	7.9	0.58	0.68	0.82

V. DISCUSSION AND CONCLUSION

A. Wind Direction and Cup Anemometer Data

Mean wind data are described in Section IV A., and displayed in Figs. IV A.1 to .13. Numerical results are summarized in Tables IV A.1 and .2.

The F test (Fraser, 1958, pp 144-146) was applied to the standard errors σ_r and σ_a (see Table IV A.1) of the wind directions measured at the mast and instrument hut respectively. Of the ten runs tested, the F ratio ($= \sigma_1^2 / \sigma_2^2$) exceeded that allowed at the 5% level (95% confidence level) in 4 cases; of the 4, 2 just exceeded that allowed at the 1% level. Since the vanes have different response times, and have different exposures to the wind, such a difference in their variances can reasonably be expected; however, no conclusions regarding spatial homogeneity can be drawn from these data.

It is clear from Table IV A.2 that there is no obvious relationship between the linearity of profiles (denoted by L), nor of the slopes of approximately linear profiles, and stability conditions. This is in keeping with the results of Hamblin (1965). None of the measured profiles, with perhaps the exception of the run 2305-2336/26/6/1965, exhibit two linear sections as found reasonably often by Hamblin (1965) at the same site. Also, no obvious relationship exists between

the slopes of the linear profiles and mean wind speeds.

Much of the large scatter in values of u^2 estimated from non-linear profiles (marked N in Table IV A.2), is probably due to errors inherent in determining small speed differences between adjacent cups. For example, an error of only 1% in determining the speed of each cup, can produce an error approaching 100% of the difference in speed between the second and third cups. This is the case, for example, in a mean wind near $4 \text{ m}\cdot\text{sec}^{-1}$, and a speed difference of $10 \text{ cm}\cdot\text{sec}^{-1}$ between the cups.

One must conclude, in concord with earlier workers, that the estimates of stress (and hence drag coefficients C_D) from non-linear profiles are very unreliable. Estimates from the linear profiles should be more reliable; these are compared with direct estimates of stress from X-wires in Section V D.

B. X-and U-Wire Data

The X- and U-wire data are described in Section IV B, and displayed in Figs. IV B.1 to .14. Numerical results are summarized in Table IV B.1. The techniques of analyzing the hot wire signals are described in Appendix IV.

B.1. Performance of X- and U-Wires

Comparisons of cup anemometer and U-wire spectral values

were made by Pond et al (1965). These showed that spectral values derived from U-wire measurements at low frequencies differed by up to 35% from the spectral values derived from much more precise cup anemometer data. Since my U-wire calibration and measuring techniques were similar to those used by Pond, and since my spectral values of $\phi_{11}(k)$ conform to the $k^{-5/3}$ behaviour in the inertial subrange (see Figs. IV B.11 to .14 and Section II A.5) which has been confirmed by Pond (1963, 1965) and others (see Section E.2), one can have confidence that my U-wire spectral measurements give spectral estimates for the downwind fluctuations which are within about 35% of the precise ones.

It is also instructive to consider the reliability of the X-wire spectral measurements. As can be seen from Figs. IV B.1 to .10, the spectra of the downwind fluctuations ($f\phi_{11}(f)$) obtained from X-wire measurements agree well in shape with those obtained from U-wire measurements for all runs. The spectral levels obtained from both probes are essentially the same for half the runs. For the other runs, the level measured by the U-wire may be higher or lower than that measured by the X-wire. The maximum difference between levels is about 35%, for the run 1555-1629/20/9/1965 (Fig. IV B.2). As was noted in Section IV B, this behaviour is to be expected, since hot wires are not good instruments for measuring absolute winds. The agreement in spectral shapes

(for $f\phi_{11}$) from the two types of hot wire probes gives one confidence that the X-wire probes operated properly over the whole frequency range of interest, since the U-wires have already been shown to give reliable spectral estimates.

The measured spectral levels of $\phi_{11}(k)$ (of the downwind fluctuations), at scales small enough to be in the inertial subrange, again exhibited the predicted $k^{-5/3}$ behaviour (see Figs. IV B.11 to .14 and Section II A.5). This provides added confidence in the reliability of X-wire measurements.

All the measured spectral values of $f\phi_{13}(f)$ are negative over the total frequency range analyzed. As will be seen from the discussion in Section V B.3, changes in the wire coefficients (b') and calibration factors (K) would have to be unreasonably large in order to produce zero or positive values of $f\phi_{13}(f)$, so that the negative values must be considered as being real. This conformed to the expectations of the downward flux of momentum and gives added weight to the confidence in X-wire measurements.

An added, but indirect indication of the reliability of X-wire measurements at high frequencies is provided by the behaviour of θ , the angle of orientation of the principal axes of the "pseudo-tensor" described in Sect. II A.6. As was seen in Sect. IV E (see also Fig. IV E.1), θ reached an almost constant value in the inertial subrange for all runs, excepting one where significant variation was noted. This gives indirect confirmation of

the proper operation of the X-wires, with the exception of the one questionable run. In that run, only the behaviour of θ was anomalous; in all other respects the run appeared to behave normally.

It appears then, that we can have confidence in the operation of the X-wire equipment, and that the shape of each spectrum is properly measured, both by the U- and the X-wires. But the relative levels are only reasonably close (within to at most 35%, and most probably within about 25%) to the precise ones. The calibration procedures for the hot wires outlined in Appendix I, and the procedures of analysis of the rerecorded data outlined in Appendix IV, thus have proven to be suitable for the spectral measurements which were made.

Spectral values at low frequencies from the much more precise data derived from the cup anemometers can in principle be used to obtain field calibrations for the hot wire responses; the relative levels obtained from both X- and U-wires can then be adjusted to give precise spectral values at all frequencies. However, since spectra are not yet available from the cup anemometer recordings, it will simply be assumed (see Section VI), that the spectral levels and values of $u^*{}^2$ may be in error by at the most about 35%, at a probability level approximating 0.95.

B.2. Spectra of Kinetic Energy ($f\phi_{11}$ and $f\phi_{33}$)

Both the spectral values $f\phi_{11}(f)$ and $f\phi_{33}(f)$ are defined by eqn. II A 1.2, while $\phi_{11}(k)$ is defined by eqn. II A 1.1. "Low frequencies" are considered those under 1 Hz.

As seen from Figs. IV B.1 to .10, the spectra $f\phi_{11}(f)$ have a general upward trend towards low frequencies, so that one can expect significant contributions to the value of $\overline{u_1^2}$ (see eqn. II A 1.2) from frequencies lower than those analyzed. Possible exceptions to this general behaviour are the three runs displayed in Figs. IV B.3, .7, and .10, which tend to have decreasing values at the lowest frequencies analyzed. But this fall-off may not be too significant, since the statistical stability of the lowest frequency spectral estimates is low, and the spectral values may be increasing again at frequencies lower than those analyzed. At high frequencies, the spectra all exhibit the $k^{-5/3}$ behaviour as predicted for the inertial subrange (see Section II A.5). Figures IV B.11 to .15, showing four plots of $\log \phi_{11}(k)$ against $\log k$ to demonstrate the $k^{-5/3}$ law, show that the $k^{-5/3}$ behaviour extends to much larger scales (smaller k), than expected from the estimated isotropic limit of Pond (1963); that is $kx_3 > 4.5$. This feature conforms to the shapes of spectra observed by other workers and will be discussed further in Section V B.4.

As seen from Figs. IV B.1 to .10, the curves of $f\phi_{33}(f)$ versus $\log f$ exhibit the smoothest shapes of the three measured curves of $f\phi_{ij}(f)$, decreasing monotonically toward zero values toward both ends of the frequency range analyzed. The decrease in these curves toward both ends of the frequency range indicates that most of the contribution to the vertical fluctuations occurs between about 0.016 and 60 Hz. The maximum contribution to the fluctuations in u_3 -velocity component occurs between about 0.1 and 10 Hz*, and at lower frequencies $\phi_{33} \ll \phi_{11}$.

The behaviour of the 'maxima' of the curves of $f\phi_{11}(f)$ and $f\phi_{33}(f)$ against $\log f$ is thus in keeping with the wind tunnel results of Favre et al (1957) (Section II A.4), which showed that eddies larger than the distance to the boundary have scales in the direction of the mean wind much larger than those in either the lateral or vertical directions. Since in both cases the observations were made in turbulent shear flows near a boundary, this is hardly surprising.

B.3. Spectrum of the Momentum Transfer ($f\phi_{13}$)

As seen from Figs. IV B.1 to .10, (and noted in Section V B.1), all of the observed spectral values of $f\phi_{13}(f)$ were negative, even at the smallest scale sizes observed. This implies that momentum is transferred downwards, at least by fluctuations in the range of frequencies observed.

*The recent boundary layer measurements made at $Re = 2.3 \times 10^5$ in a wind tunnel by Compte-Bellot (1965) had $k\phi_{33}$ peaking at the same value of $kx_3 \sim 2-3$, as did my measurements.

It was decided to check whether all of the observed negative and small, but non-zero, values of contributions to the stress at the lowest and highest frequencies were real, or whether they may be biased from zero as a result of errors (discussed in Appendix VI) introduced in the estimate of the wire coefficient b' and the calibration factor K (see eqn. A 1.10), or because of misalignment of the X-wires in the mean wind. The latter feature is discussed in Section IV G, and, from Fig. IV G.1, it is evident that no significant change in ϕ_{13} could be introduced by improper alignment. The value of b' depends on the inclination of each wire to the horizontal. Checks made throughout the summer of 1965 showed that the geometry of the wires on the X-wire probes used remained essentially unchanged (see Appendix 1.C). Calculations on the data of run 2101-2034/21/7/1965 showed that a change as large as 15% in one wire coefficient b' of one of the wires of an X-array led to 10% changes (or less) in the spectral level at high frequencies (above 1 Hz), and to changes up to 15% at low frequencies (below 1 Hz). A similar calculation for the run 1448-1525/22/7/1965, showed that a 13% change in one wire coefficient b' led to a maximum change of 10%, and an average change of 5%, in spectral levels over all frequencies analyzed. The change in one calibration factor K of one wire, required to reduce some spectral estimates to zero, was examined by doing simple calculations on observed data at each frequency for the run

2230-2258/26/6/1965. The least change in the calibration factor K required was over 20%. Since errors in b' are less than $\pm 3\%$ and of K less than $\pm 11\%$ at the 95% level (see Appendix VI), the above changes in b' and K appear unreasonable in the light of experience. These calculations thus show that it is not unreasonable to consider all of the spectral values of $f\phi_{13}(f)$ as being significantly non zero and negative.

The spectra $f\phi_{13}(f)$ of the kinematic stress remain fairly flat and have broad maxima in the frequency range between about 0.01 to 1 Hz for most runs, showing that significant amounts of momentum are transferred downwards over a wide range of frequencies of almost two decades (see Figs. IV B.1 to .10). There is very little contribution to the stress at frequencies higher than 10 Hz. Since the turbulence is measured just above the sea surface, it is instructive to note that the estimated frequencies of dominant waves fall between about 0.2 to 0.5 Hz (equivalent to $\log f = -0.7$ to -0.3) for the range of wind speeds and fetches encountered. From Figs. IV B.1 to .10, it is seen that no obvious peaks occur in the $f\phi_{13}(f)$ spectra in this frequency range, with the exception of two runs on /29/6/1965 (Figs. IV B.3 and .4) which have more pronounced maxima near these frequencies. Significant stress is present in this frequency interval in every run; however, the larger proportion of the totally observed stress is present at frequencies which are lower than wave frequencies.

The spectral curves for the $f\phi_{13}(f)$ values show no general regularity in shape, but all drop toward zero toward the highest frequencies and in most cases towards the lowest frequencies, giving some confidence that most of the significant contributions to the Reynolds' stress arise from fluctuations with frequencies in the interval analyzed. However, spectral estimates are the least reliable statistically at lower frequencies, and in runs 1448-1525, 1620-1652, and 1650-1715 /22/7/1965, (Figs. IV B.5, .7, and .8) the drop-off is not well established. Moreover, in all runs the rate of drop-off is slow and may decrease at still lower frequencies. Thus the possibility that there may be significant contributions from still lower frequencies requires further investigation.

At frequencies higher than about 10 - 20 Hz, the spectral values began to show scatter because the recorded signal was contaminated by a larger proportion of noise. Values of $f\phi_{13}(f)$ at high frequencies are obtained by subtraction of two almost equal parts derived from the signals from both wire channels, both of which are contaminated by noise which may be coherent.

The spectra of Reynolds' stress observed in unstable conditions (Figs. IV B.1, .2, .5 to .8 and .10) have broad maxima extending to larger scale sizes (smaller k) than those for runs where near-neutral or stable conditions exist

(Figs. IV B.3, .4, and .9). This conforms to the general prediction that contributions to the turbulent stress, produced by convective effects, occur at larger scales than those of mechanically produced stress.

At low frequencies (below about 1 Hz say), observed values of the correlation coefficients $R_{13}(f)$ (defined by eqn. II A 1.3) are all negative and almost all numerically greater than 0.3. Two exceptions are the smaller values near -0.2 at low frequencies in the runs 2010-2034 /21/7/1965, and 1448-1525/22/7/1965 (Figs. IV B.1 and .5). The extreme value for all runs is -0.73 (Fig. IV B.3), and values near -0.5 to -0.6 are common. For all runs $|R_{13}|$ decreases to quite small values ($< .15$) as frequency increases to 5 or 10 Hz, but for more than half the runs, the value increases towards still higher frequencies. This increase is believed to be due to the imprecise compensation of the two hot wires under field conditions.

B.4 Evidence for Non-Existence of Isotropy at Small Scales

A discussion of the structure of turbulence and of anisotropy is given in Sections II A.3 to .5. Theoretical predictions in general have been based on the assumption that the turbulence at very small scales is isotropic. In isotropic eddies, it is predicted that $\phi_{13} = 0$ and that equation II A 6.1a holds, so that if $\phi_{11}(k) \propto k^{-5/3}$, then

$\phi_{33} = (4/3)\phi_{11}$. In a shear flow near a boundary an 'isotropic limit' has been estimated to be near a wavenumber k such that the eddies are unlikely to be isotropic unless $kx_3 \gtrsim 4.5$ (Pond et al, 1963). At scale sizes larger (smaller k) than that estimated for the 'isotropic limit', the turbulence is anisotropic and it is expected that contributions to the stress, $f\phi_{13}$, will be non-zero at these lower wavenumbers. The present observations are well in accord with these predictions.

My measurements of the $f\phi_{13}(f)$ (stress) spectrum over a range of frequencies (or scale sizes) of almost four decades, are described in Section IV B, displayed in Figures IV B.1 to .10 and discussed in Section V B.3.

From Figs. IV B.1 to .10, it can be seen that all the values of $f\phi_{13}(f)$ for all runs are non-zero. Calculations discussed in Section B.3 above showed that the negative non-zero values must be considered as being real, even at the highest frequencies analyzed. It thus appears that isotropy is not reached even at the smallest scale sizes, (largest k or f) observed. However, at wavenumbers higher than the 'isotropic limit', the amount of anisotropy is relatively small; as is seen from $R_{13}(f)$ in Figs. IV B.1 to .10, values of $f\phi_{13}(f)$ are about 5 to 10 or more times smaller than values of $f\phi_{11}$ or $f\phi_{33}$.

The data in Table IV G.2, which were chosen from the region for which $kx_3 \geq 10$ and which might be expected to be isotropic so that $\phi_{33} = (4/3)\phi_{11}$, show that of the 10 X-wire runs analyzed, only four runs had average ratios of ϕ_{33}/ϕ_{11} greater than, or equal to, one. For three runs, the average ratio was just less than 0.7. The maximum ratio of a single estimate was 1.24 and ratios as low as around 0.6 were observed. In all cases, the above ratios were computed from observations in the frequency range $kx_3 \geq 10$ (which was chosen as a more stringent requirement than Pond's $kx_3 > 4.5$) and where contamination of signal by noise was not too serious. (Signal-to-noise ratio had to be greater than 10.)

Calculations were made in order to check whether the observed behaviour of this ratio was real, or whether it merely reflected effects introduced by the techniques of measurement and analysis. The effect of changes in the wire coefficients (b' , in Eqn. A 1.10) were considered. Calculations of the effect of a 15% change in b' , (partly discussed in Section V B.3), also showed that the ratio ϕ_{33}/ϕ_{11} changed by 8% at the most at high frequencies. Furthermore, analysis of the equations A IV.15 for $\phi_{33}/\phi_{11} \sim 1$ and $f\phi_{13} \sim 0$, shows that a change of P% in a calibration factor K_1 of K_2 (Eqn. A IV.1), produces only a change of 0.5P% in the ratio of ϕ_{33}/ϕ_{11} . Since estimates of b' are better than $\pm 11\%$ at about

the 95% level of probability (see Appendix VI), then the changes necessary in b' and K in order to bring the observed ratios of ϕ_{33}/ϕ_{11} (see Table IV G.2) to the theoretical level of $4/3$ (Section II A.5), appear unreasonable in the light of experience.

As seen in Section IV G (and Fig. IV G.1), the effect of misaligning the X-wire in the mean wind so that $\overline{\theta_r} = \beta$ was not equal to, or very close to zero, would be to decrease the ratio ϕ_{33}/ϕ_{11} at the high frequencies. For $\beta = 15^\circ$, the observed ratio would be reduced almost 20%. However, the observed deviations of the direction of the mean wind from the plane of the X-wires ($\overline{\theta_r}$) cannot account for all of the values of the actual ratios observed. This is seen from Table IV H.1, which shows that the smaller observed values of the ratio ϕ_{33}/ϕ_{11} (those close to, or less than 1.0) were associated with quite small (5°) observed values of $\overline{\theta_r} = \beta$ in over half the cases.

Indeed, if one took a liberal view in estimating the maximum probable values of β from those observed for each run, in estimating maximum adjustments to the ratios ϕ_{33}/ϕ_{11} , and also if one allowed a liberal adjustment ($\sim 15\%$) to allow for possible changes in wire coefficients (b') and calibration factors (K), it is found that for only two runs (both on /29/6/1965) is it reasonable to expect that the observed ratios might conform with the predicted value of $4/3$. The

observed behaviour of the ratio in all other runs thus must be interpreted as being real. This suggests that the structure of the turbulence differs in some way from that assumed for the prediction. Since the only assumption required for the prediction that the ratio is $4/3$ was that of isotropy and $\phi_{11} \propto k^{-5/3}$, then we must interpret the observations as showing that isotropy is not reached in the observed field of fluctuations within the range of wavenumbers examined.* This view is definitely supported by the observed behaviour of the spectra $\phi_{13}(f)$, (see Sect. V B.3), where it is seen that spectral values of the stress at high frequencies never became zero.

From Figs. IV B.11 to .14, which are representative plots of $\log \phi_{ij}(k)$ against $\log k$, it can be seen that the $\phi_{11}(k)$ spectra follow the $k^{-5/3}$ law to much larger scales (smaller k), than those allowed by considerations of isotropy. This behaviour has also been noticed by earlier workers (Grant et al, 1962; Pond et al, 1963; Pond, 1965). As was seen in Sect. II A.5, the $k^{-5/3}$ law for the inertial subrange (Eqn. II A5.1) was derived using the assumption of isotropy (plus one other). My measurements indicate that anisotropy is never absent (at any scales), so that the assumption of isotropy appears to be an unnecessarily stringent criterion for the

* The measurements of ϕ_{33} of Comte-Bellot (1965) were compared to those calculated from ϕ_{11} (assuming isotropy). For $Re = 2.3 \times 10^5$, measured values were less than calculated ones for $kx_3 > 4$ by about 35%. This gave $\phi_{33}/\phi_{11} \sim 0.9$. Other runs at lower Re gave similar results.

prediction of the $k^{-5/3}$ law.

C. Direct and Indirect Estimates of the Kinematic Stress u_*^2 ,
and the Drag Coefficient C_D

C.1. Direct and Indirect Estimates of the Kinematic Stress u_*^2

Directly and indirectly estimated values of u_*^2 for all hot wire runs are described in Section IV A to IV C. Values from profile data are summarized in Table IV A.2, those from X- and U-wire data in Table IV B.1, and those from the three vertically spaced U-wires in Table IV C. All values of u_*^2 are listed for comparison in Table IV D.1. Comparisons between estimates are shown in Fig. IV D.1. In this figure, values estimated indirectly from the wind profile (Section II D.2) and those from the $\phi_{11}(k)$ spectrum (Section II D.1) are plotted separately against values measured directly using the X-wire (Section IV B).

There appears to be little correlation between values of u_*^2 determined from the wind profiles and those from the X-wire (Fig. IV D.1). With the exception of the two runs (the two points above the line at 45°), the profile method underestimates the directly measured stress by about 40% on the average. The results thus demonstrate that estimates based on wind profiles are not very reliable, as was noted in Section V A.

As can be seen from Fig. IV D.1, values estimated indirectly from the observed values of the spectrum $\phi_{11}(k)$ in the inertial subrange, are evidently strongly correlated with, but (except for one run) are all greater than, the directly measured kinematic stresses. The mean value of the ratio of the former estimates (from ϕ_{11}) to the direct estimate is 1.42, with standard error 0.42. If only those runs which exhibited linear profiles (marked L in Table IV A.2) were used, the average ratio was 1.34, with standard error 0.53. Application of the t test to the two average overestimations, (Moroney, 1962, pp 227-233), and of the F test to the two standard errors (Fraser, 1958, pp 144-146), showed that the two average overestimations were not significantly different, so any effects due to linearity of the profiles are not distinguished. It is clear that, of the two indirect methods, the technique based on the Kolmogoroff form of the spectrum gives more consistent values than the profile method, despite the fact that it overestimates the stress.

What could cause such an overestimation? According to the assumptions of Section II D.1, the eqn. II D 1.1 used to estimate the values of u_*^2 is valid when (a) the logarithmic wind profile holds and (b) the rate of production of mechanically produced turbulent energy equals the rate of dissipation of turbulent energy (ϵ). Considering assumption (b) first, one could argue that the lack of balance between

mechanical production and dissipation leads to the over-estimation. However, it can be shown from eqns. 11 A 5.1 and 11 C 1.1 that turbulent energy equal to about half of the dissipation rate would have to be produced or advected from some other source besides the mechanically produced turbulent energy source. However, estimates of energy budgets in both neutral and non-neutral conditions have shown that, in general, it appears that mechanically energy production and energy dissipation are not significantly different from each other (see Lumley and Panofsky, 1964, pp 120-125; and Hess and Panofsky, 1966). From this, one can conclude that assumption (b) above is a fairly good one. Also it has been shown above that the degree of overestimation is not biased by the linearity of the profile of U plotted against $\log x_3$, so that conformity or not to assumption (a) must be excluded as a cause of the overestimation. However, as was noted before, (see also Fig. IV D.1, left-hand figure), the profile method has not estimated u_*^2 well; it has consistently underestimated it. Hence the probable cause of the over-estimation of u_*^2 from the ϕ_{11} spectral values is the general underestimation by the profile method of measuring stress. (The logarithmic profile appears in eqn. 11 D 1.1.)

C.2. The Drag Coefficient C_D

Values of the drag coefficients $C_D = u_*^2 / U_5^2$ adjusted to a height of 5 m are shown in Table IV D.2, and are plotted

against wind speeds (extrapolated if necessary to 5 m) in Fig. IV D.2. Values calculated from wind profiles show large scatter. Values estimated from the $\phi_{11}(k)$ are generally greater than those calculated from direct estimates of stress.

Values C_D from directly estimated stress appear to have no discernible trend with increasing wind speed; however, only two values were observed at wind speeds higher than 4.6 m sec^{-1} . The mean value is 1.45×10^{-3} , and the standard error of the observations is 0.42×10^{-3} ($\sim 35\%$ of the mean) for ten values. A substantial part of this error probably arises from the 35% range expected in spectral values derived from hot wires, (see Section V.B). For wind speeds near 4 m sec^{-1} , the total range in values is about a factor of two. These direct estimates of the drag coefficient using the X-wire are still too few (ten runs) to give reliable statistical estimates of the mean or of any trend with mean wind speed.

Indirectly estimated values of C_D , using the wind profile method, and those from the spectral values of $\phi_{11}(k)$ in the inertial subrange (see Section II D), also showed no obvious trend with increasing wind speed for the ten X-wire runs. The profile method gave a mean value of 0.89×10^{-3} with standard error of 0.66×10^{-3} ($\sim 75\%$ of the mean) and the $\phi_{11}(k)$ spectral values gave a mean value of 2.10×10^{-3} with a standard error of 0.53×10^{-3} ($\sim 25\%$ of the mean).

It was then decided to apply statistical tests to determine whether these observed differences in the three values of C_D were statistically significant. First, the t test was applied to the mean values. (A description of this test can be found in M. J. Moroney, pp 227-233.) It was found that all cross comparisons of directly and indirectly observed means for C_D were significantly different, at the 0.1% level. As a check, values of C_D estimated from linear profiles, and also those estimated from profiles obtained under near-neutral conditions (chosen by the criterion $|x_3/L| \leq 0.1$), were tested against values of C_D estimated from the directly measured stress. The result of these tests was the same as before. Application of the F test to the three standard errors, showed that the three standard errors were not significantly different at the 5% level. (A description of the F test can be found in D.A.S. Fraser, 1958, pp 144-146.) From these two tests, one must assume that the three methods of determining u_*^2 give statistically significant differences between the corresponding determinations of C_D . Hence, one must exercise care when interpreting results from the two indirect methods of determining stress.

The directly measured drag coefficients are higher at the lower wind speeds than those found by most other workers (see Section II E.4), and seemingly do not exhibit the increase

with wind speed found by some other workers. Values of the drag coefficient measured directly (using a sonic anemometer) by Zubkovskii and Timanovskii (1965), show C_D (referred to the wind speed at a height of 2 m) increasing with wind speed from 0.6×10^{-3} at 3 m sec^{-1} to 1.1×10^{-3} at 5 m sec^{-1} . This is contrary to my directly estimated results, but in keeping with the profile measurements of other workers (see Section II E.4).

D. Data from Three Vertically Spaced U-Wires

Results from the three runs in which three vertically spaced U-wire probes were used to check the Monin-Obukhov similarity theory are described in Section IV C, and numerical values are summarized in Table IV C. The theory itself is discussed in Section II B.

The wind profiles observed (Figs. IV A.11 to .13) are approximately linear. Conditions were close to neutral for the two runs on /24/7/1965, and neutral for the run on /25/7/1965. This indicates that the wind profiles may be reasonably represented by the logarithmic expression (eqn. II C 1.1).

From Table IV A.2, it is seen that the ratios x_3/L' are approximately 0.2 for the two runs on /24/7/1965, and 0.01 for the run on /25/7/1965. From remarks in Section II B.3, it is appropriate to take the height x_3 , which has the

dimensions of length, as the characteristic length scale. Since k , the wave number, has the dimensions of inverse length, then a dimensionless grouping involving length scales can be formed using the product kx_3 .

How appropriate is $1/k$ in representing actual scales of turbulence? As discussed in Section II A.2, Taylor's hypothesis is valid as long as $kx_3 \gg 1$; that is when the scales of turbulence are small as compared to the height, and anisotropy is small. At these small scales, the spatial structure is quite well approximated by $1/k$, so that here kx_3 can be considered a dimensionless grouping which has an actual physical basis.

According to similarity theory, the structure of turbulence is determined by the turbulent flow, except for scaling parameters. If one then scales velocities by a velocity characteristic of the turbulence, then any functions of the scaled velocities will be universal functions of the scaled heights, whenever they describe similarity properties. In a given run, the corresponding function of an unscaled velocity will be independent of the height. Thus, a plot of $kx_3 \phi_{11}(kx_3) (= \phi_{11}(f))$, which has the dimensions of (velocity)², against the non-dimensional wave number kx_3 , should give a single curve independent of height for a given run, at scale sizes where $1/k$ represents actual scales of turbulence; that is at scales where Taylor's hypothesis is valid.

This wave number range should encompass the inertial subrange, and from Figs. IV C.1, .3, and .5 it is seen that in this range the levels of the vertical spectra at the three heights (the three curves of $kx_3\phi_{11}(kx_3)$ against $\log(kx_3)$) are almost the same for a given run. Differences in shape of the curves are most marked at values of $\log(kx_3)$ less than about 1, where turbulence is expected to be influenced by the sea boundary, and hence be strongly anisotropic. Here, Taylor's hypothesis lacks validity, and k is a poor measure of spectral scales. At higher values of kx_3 , where $kx_3 \gg 10$, which is greater than Pond's (1963) 'isotropic limit', the three curves essentially coincide. This provides strong evidence for the correctness of the similarity hypothesis.

Considering the similarity theory from another point of view, it was seen in Section II B.1. that the theory predicted that there exists a velocity $u^* = \sqrt{-\overline{u_1 u_3}}$, which is essentially invariant with height outside the "viscous boundary layer". This velocity u^* is used as a suitable scaling velocity for other velocity scales.

As was seen in Section IV C, the quantity S , defined by eqn. IV C.1, estimates the kinematic stress indirectly by using the measured spectral densities $\phi_{11}(k)$ in the range of scales appropriate to the inertial subrange (compare eqns. II D 1.1 and IV C.1). However, as was seen in Section V C.1, these indirect estimates overestimated the actual stress

present (u_*^2) measured directly with the X-wire, by about 42% on the average. Thus, $S \approx 1.4u_*^2$, in this range of scales, so that S should be constant with frequency in this range. This implies that if the measured value of S in the inertial subrange remains invariant with height for a given run, then u_* also remains invariant within the limits of experimental error.

Figures IV C.2, .4, and .6 show the values of S (defined by eqn. IV C.1) for each of the three U-wire probes, as a function of the non-dimensional wave number kx_3 , for each of the three runs. Values of S were calculated from the spectra at the three heights (Figs. IV C.1, .3, and .5) using eqn. IV C.1. It is seen from the figures that the values of S for the three heights converge to the same average value within about $\pm 15\%$ (total range) for a given run, in the wave number range in which scales are small enough to be in the inertial subrange. At the highest wave numbers ($\log kx_3 \gg 2$), the scatter in the values of S increases, since the signal-to-noise ratios of the signals decrease to 10 or less, and is least for the run on /25/7/1965 which displays the greatest scatter; these parts of the curves are therefore ignored. The measurements thus demonstrate that, within the limits of experimental error, S is invariant over the height range of the measurements.

The measurements made with the three vertically spaced

U-wires thus provide strong support for the validity of the Monin-Obukhov similarity theory (Section II B.1) for the range of heights (about 1 to 5 m) above the sea surface in which the measurements were made.

VI. SUMMARY

1. The X-wire method has been used successfully to measure velocity fluctuations in the x_1 and x_3 directions over a range of wind speeds from about 140 to 1000 $\text{cm} \cdot \text{sec}^{-1}$.
2. Special techniques were developed to mount the X-wires on the probes, to calibrate the wires, to directly measure their response to both horizontal and vertical components of velocity, and to analyze the tape-recorded analog data in order to extract spectral and cospectral densities over a frequency range from 0.016 to 60 Hz.
3. The work of Pond et al (1963) and Pond (1965) showed that U-wire probes can be used successfully to measure fluctuations in the u_1 -wind velocity component over the ocean. Comparison of the shapes of downwind velocity spectra $f\phi_{11}$ measured with both X- and U-wires, the behaviour of the $\phi_{11}(k)$ spectrum in the inertial subrange, the existence of only negative spectral values of $f\phi_{13}$, and the behaviour of the diagonalization angle of the Reynolds' stress tensor (θ), all showed that the X-wire technique provides reliable information on velocity fluctuations in the x_1 and x_3 directions.
4. The measured kinetic energy spectral density, $f\phi_{11}$, showed a general upward trend towards the lowest frequencies analyzed. The spectral density, $f\phi_{33}$, of the fluctuating

velocity component, decreased monotonically toward zero at the lowest and highest frequencies analyzed, and had its maximum contribution in the frequency range from 0.1 to 10 Hz. At low frequencies, (< 1 Hz), $\phi_{33} \ll \phi_{11}$, which is in keeping with the wind tunnel results of Favre et al (1957), which showed that the scales of turbulence in the direction of the mean wind were very much larger than those perpendicular to it.

5. The spectrum of momentum transfer, $f\phi_{13}$, had negative values of spectral densities at all frequencies analyzed. Checks made by varying the value of the wire coefficients (b') and the sensitivity factor (K), showed that even the assumption of unacceptably large errors in b' and K did not change this result. The spectra ($f\phi_{13}$) were fairly flat, with broad maxima between about 0.01 to 1 Hz. For only two runs did the maxima roughly coincide with the estimated frequency range of the dominant waves at the experimental site. For the other runs, the larger proportion of the totally observed stress, was present at frequencies which were lower than the estimated wave frequencies. The spectral curves had no regularity in shape: they all dropped^P toward zero at the highest frequencies, and for most cases at the lowest frequencies. This gives confidence that most of the contributions to the Reynolds' stress occurred from fluctuations in the

frequency range analyzed. In general, the spectra measured in unstable conditions had their broad maxima at larger scale sizes than for those in neutral, near-neutral, or stable conditions, showing that convective effects occur at larger scales than those of mechanically produced turbulence.

6. The correlation coefficients $R_{13}(f)$ all had negative values of magnitude greater than 0.3 for frequencies less than 1 Hz. Two runs were the exceptions, with values around -0.2. The maximum value was -0.73.
7. My measurements show that anisotropy is apparently never absent, even at the smallest scales (highest frequency 60 Hz observed). This is shown by the negative and non-zero values of stress $f\phi_{13}$ at all frequencies, and by the behaviour of the ϕ_{33}/ϕ_{11} ratio in the range of scales usually assumed to be 'isotropic' according to the criterion ($kx_3 > 4.5$) of Pond et al (1963). The average values of the ratio ϕ_{33}/ϕ_{11} in this frequency range, were greater than or equal to 1.0, for only four runs. Checks of the effect of larger errors than expected in b' and K , and in the angle of deviation (β) of the mean wind direction from the plane of the wires, showed that in only two cases could the observed ratio be considered as being close to the theoretically predicted value of $4/3$. Since the $4/3$ ratio is based on the assumption of isotropy,

then the observed ratios must be interpreted as showing the existence of anisotropy.

8. The observed Kolmogoroff spectrum, $\phi_{11}(k)$, in the inertial subrange shows the $k^{-5/3}$ behaviour to much larger and greatly anisotropic turbulence scales than the original assumptions, which include that of isotropy, would predict. This behaviour has also been noted by other workers (Grant et al, 1962; Pond et al, 1963; Pond, 1965), and indicates that the assumption of isotropy is an unnecessarily stringent criterion for the prediction of the $k^{-5/3}$ law.
9. Indirect estimates of the kinematic stress, u_*^2 , from the wind profile showed little correlation with direct estimates using the X-wire, and on the average underestimated the stress by about 40%. Indirect estimates of u_*^2 , using the $\phi_{11}(k)$ spectrum in the inertial subrange, were strongly correlated with the direct estimates, and overestimated the stress by about 40%, on the average, irrespective of stability conditions, or the non-existence of logarithmic wind profiles. The overestimation is probably due to the general underestimation of stress by the profile method. Of the two indirect methods, the $\phi_{11}(k)$ spectral method gives more consistent results.
10. The directly estimated values of the drag coefficient

showed no discernible trend with mean wind speed. The average value of C_D , corrected to the 5 m height, was 1.45×10^{-3} , with standard error of 0.42×10^{-3} ($\sim 35\%$ of the mean). The indirect estimate from the profiles was 0.89×10^{-3} ($\sigma = 0.66 \times 10^{-3}$) and that from the values of ϕ_{11} was 2.10×10^{-3} ($\sigma = 0.53 \times 10^{-3}$). Statistical tests showed that the differences in the mean values of C_D from the three methods were significant. The observed direct estimates are in general higher than those measured by other workers.

11. The measurements made with the three vertically spaced U-wires provide strong support for the validity of the Monin-Obukhov similarity theory for the range of the heights (about 1 to 5 m) above the sea surface, in which the measurements were made.

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FIGURE CAPTIONS

(All logs are to base 10.)

Fig. III	.1	Experimental Site for 1965		88
	.2	Photograph of Experimental Site		88
Fig. IV	A.	Mean Wind Data		89
		Mean Wind Profile and		
		Mean Wind Direction (θ_a and θ_r):		
		θ_a -direction relative to		
		Pt. Atkinson		
		θ_r -direction relative to probe.		
		Relative to the lowest cup, the		
		cup heights were 0, 30, 80, 150,		
		241, and 351 cm. Height of bottom		
		cup for each run is included in		
		brackets.		
	A.1	Run 2010-2034/21/7/1965 (95)		89
	.2	1555-1629/20/9/1965 (137)		90
	.3	2039-2122/29/6/1965 (103)		91
	.4	2122-2206/29/6/1965 (127)		92
	.5	1448-1525/22/7/1965 (130)		93
	.6	1553-1625/22/7/1965 (130)		94
	.7	1620-1652/22/7/1965 (135)		95
	.8	1650-1715/22/7/1965 (145)		96
	.9	2230-2258/26/6/1965 (205)		97
	.10	2305-2336/26/6/1965 (200)		98
	.11	1550-1605/24/7/1965 (170)		99
	.12	1607-1645/24/7/1965 (160)		100
	.13	1501-1525/25/7/1965 (95)		101

Fig. IV B. X- and U-Wire Data: 102

Spectrum : $f\phi_{11}(f)$ of Downwind
Fluctuations

$f\phi_{33}(f)$ of Vertical
Velocity
Fluctuations

Cospectrum: $f\phi_{13}(f)$ of Horizontal and
Vertical Velocity
Fluctuations

Correlation
Coefficients: $R_{13}(f)$ of Horizontal and
Vertical Velocity
Fluctuations.

◆ Origin of log k scale

▲ Origin of log (kx_3) scale

(Scales of both are the same as
for log f; f is in cycles per
second, k is in radians per cm)

B.1	Run	2010-2034/21/7/1965		102
.2		1555-1629/20/9/1965		103
.3		2039-2122/29/6/1965		104
.4		2122-2206/29/6/1965		105
.5		1448-1525/22/7/1965		106
.6		1553-1625/22/7/1965		107
.7		1620-1652/22/7/1965		108
.8		1650-1715/22/7/1965		109
.9		2230-2258/26/6/1965		110
.10		2305-2336/26/6/1965		111

Log-log plots of Wave Number Spectra
 $\phi_{11}(k)$ and $\phi_{33}(k)$ from X-Wire Data,
 and of $\phi_{11}(k)$ from U-Wire Data.

B.11	Run	2010-2034/21/7/1965		112
.12		2039-2122/29/6/1965		113
.13		2122-2206/29/6/1965		114
.14		1448-1525/22/7/1965		115
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C.1	Run	1550-1605/24/7/1965		116
.2		1550-1605/24/7/1965		117
.3		1607-1645/24/7/1965		118
.4		1607-1645/24/7/1965		119
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FIG. III.1 EXPERIMENTAL SITE FOR 1965

Main instrument mast

Platform

Rover mast

FIG. III.2 PHOTOGRAPH OF EXPERIMENTAL SITE

FIG. IVA.1 MEAN WIND DATA 2010-2034/21/7/1965

FIG. IVA.6 MEAN WIND DATA 1553-1625 / 22/7/1965

FIG. IV A.9 MEAN WIND DATA 2230-2258 /26/6/1965

FIG. IVA.10 MEAN WIND DATA 2305 - 2336 / 26 / 6 / 1965

FIG. IVA.12 MEAN WIND DATA 1607-1645/24/7/1965

FIG. IV A.13 MEAN WIND DATA 1501-1525/25/7/1965

FIG. IV B.10 X- and U-wire data 2305-2336/26/6/1965

FIG IV C.2 DATA FOR THREE VERTICALLY SPACED U-WIRES 1550-1605/24/7/1965

FIG. IV C.3 DATA FOR THREE VERTICALLY SPACED U-WIRES 1607-1645/24/7/1965

FIG. IV C.4 DATA FOR THREE VERTICALLY SPACED U-WIRES 1607-1645/24/7/1965

FIG. IV C.5 DATA FOR THREE VERTICALLY SPACED U-WIRES 1501-1525/25/7/1965

FIG. IV C.6 DATA FOR THREE VERTICALLY SPACED U-WIRES 1501-1525/25/7/1965

FIG. IV D.1 COMPARISON OF DIRECTLY AND INDIRECTLY DETERMINED
VALUES OF U_*^2 FOR ALL X-WIRE RUNS: (U_*^2 in cm^2/sec^2)

FIG. IV D.2 DRAG COEFFICIENT AT 5M HEIGHT

FIG. IV E. I. THE DIAGONALIZATION ANGLE Θ OF THE REYNOLD'S STRESS TENSOR
AS A FUNCTION OF FREQUENCY FOR THREE RUNS ON /22/7/1965

FIG. IV.G.1 THE EFFECT ON SPECTRAL VALUES OF $\beta \neq 0$

APPENDICES

APPENDIX 1: THE CONSTRUCTION AND CALIBRATION OF HOT WIRE PROBES

A. The Wind Tunnel and Manometer System

In order to calibrate hot wire probes and to measure the wire coefficients of an X-wire, a wind tunnel with a low turbulence level ($\ll 1\%$) is necessary.

Initially, the low speed wind tunnel in the Department of Mechanical Engineering was employed, but increasing use of the tunnel by graduate students in that department allowed such infrequent and irregular use of the tunnel that it became necessary to obtain a wind tunnel for use in the Institute of Oceanography.

With this in view, a small wind tunnel (Flow Corporation Model WT4) was purchased together with a micromanometer (Flow Corporation Model MM3 Micromanometer) to determine mean wind speeds in the 3" x 3" test section. However, the tunnel as supplied proved to have sharp leading edges on the intake, so that very high and unacceptable turbulence levels were present. To lower the turbulence level, a two dimensional intake of plexiglass was constructed, in the form of a scaled-down version of the intake to a non-return wind tunnel at McGill University. The final design (see Fig. A 1.1 and .2) has an areal reduction ratio of 11.9:1, and a fine nylon mesh of 5

meshes/mm stretched across the intake to minimize turbulence still further. This design proved to be satisfactory.

Pressure taps for the micromanometer were in the middle of the test section and just behind the screen.

The standard "velocity equation" for relating the mean velocity U (in $\text{m} \cdot \text{sec}^{-1}$) in the test section of the Flow Corporation wind tunnel to a pressure difference Δp (in 10^{-3} in. of butylalcohol) is given by

$$U = K \sqrt{\frac{460 + T}{535}} \sqrt{\frac{d}{0.8166}} \sqrt{\frac{29.92}{P_a}} \sqrt{\Delta p} \quad (\text{A 1.1})$$

T is the manometer housing temperature in $^{\circ}\text{F}.$, $d = 0.8176 - 0.0004 (T - 50)$ is the specific gravity butyl alcohol in the manometer system, and P_a is the atmospheric pressure in inches of mercury. K is a constant, having a value of 0.5822 (in units of $\text{m} \cdot \text{sec}^{-1} \text{in.}^{-\frac{1}{2}}$) with the reference pressure tap at atmospheric pressure.

Since in the modified tunnel the reference pressure tap was placed just behind the screen, it was no longer at atmospheric pressure. Hence the constant $K = 0.5822$ was no longer valid. To obtain the appropriate value of K , the Flow Corp. wind tunnel was calibrated as follows with a single hot wire probe. First, the probe was carefully calibrated in the Mechanical Engineering wind tunnel to obtain the relationship between the probe's wire current and the mean wind velocity

in the tunnel; this is given by King's Law (eqn. A 1.12). Next, the probe was immediately taken to the Flow Corporation wind tunnel, and the wire currents and micromanometer pressure differences (suitably corrected for temperature and atmospheric pressure) were both observed at several different wind velocities. This complete calibration procedure was performed twice.

The numerical value of K was calculated as follows. For each Δp observed in the Flow Corporation tunnel, a value for U was transferred (assuming no changes in the hot wire system) from the Mechanical Engineering tunnel. Then K was calculated for each Δp from A 1.1, and the results averaged. Combination of all values from the repeated calibration gave a weighted average value of

$$K = 0.5606$$

$$\sigma = 0.0179$$

for 31 paired values of Δp , where σ is the standard deviation.

The error is discussed in Appendix VII, Section A.

B. The Single, or U-Wire Probe

B.1. Construction

The probes used for all measurements were standard Flow Corporation Model HWP-B probes using Wollaston platinum wire

of 0.0003 in. (about 7.5 microns) core diameter. The method of etching was similar to that used by Pond (1965).

Initially wires were mounted by pulling them taut and soldering them across the ends of the stainless steel prongs. Experience showed that this technique was unsuitable for two reasons: (1) the small area of the tip was insufficient for good solder joints (2) the taut etched parts of the wires were readily broken by prong vibrations.

To remedy these defects, the wires were mounted in 1965 in a way suggested by H. Grant of Pacific Naval Laboratory (pers. comm.), and shown in Fig. A 1.3. After soldering the ends of a short length of wire bent in the form along the prongs, kinks in the wire were removed by manipulation with a tweezer, and then a section about 1.5 mm long was etched in the central portion. This configuration had adequate rigidity while minimizing breakages.

B.2. Calibration

During 1965, most of the calibrations of wires were performed in the Flow Corporation tunnel.

The calibration procedure consisted of visually placing the single wire vertically in the test section. Comparison of measurements of the angles showed that such alignment was correct to within about 1.5° . (The method of measuring wire

angles is outlined in Flow Corporation Bulletin 16-A, Feb. 1962.) This accuracy was considered sufficient for field use.

Calibration points were obtained at about 10 mean wind speeds as measured by the micromanometer. The calibration procedure is outlined in the manual provided with the HWB-2 hot wire bridge. The results were plotted as I^2 against $U^{\frac{1}{2}}$, where I was the wire current in ma., and U the mean wind speed in m.sec⁻¹. A straight line on this plot represents the standard form of King's Law (eqn. A 1.12).

Frequent repetition of calibration was desirable, but since the small tunnel had not been set up at the beginning of the summer of 1965, this was not always possible. For later runs, calibrations were performed before and after a set of one to a few runs.

C. The X-Wire Probe

C.1. Construction

The X-wire probes used were standard Flow Corporation Model HWP-X probes 24 in. long; the same length as the U-wire probes.

The wires were attached and etched in a similar fashion to the U-wire probes; the etched parts were not quite in the center of each wire but on the upwind half of each to

minimize interference. A probe tip with a finished array is shown in Fig. A 1.4.

Although this method of construction at first seemed to lack rigidity, it was found by repeated calibrations that with careful handling the wire angles did not change during use.

C.2. Theory

When a single hot wire is placed in a turbulent flow at an angle θ to the horizontal, it becomes sensitive to velocity fluctuations in both the vertical and horizontal directions. If the plane parallel to the two tilted wires is parallel to the direction of the mean flow (see A1.D for a discussion on misalignment), the X-wire array can be used to measure two velocity components at the same time.

If it is assumed that the wire is cooled only by the velocity component perpendicular to the wire (we will return to this assumption later), then the effective instantaneous cooling velocity C in a turbulent flow of mean horizontal velocity U is given by

$$C = \{[(U + u) \sin\theta + w \cos\theta]^2 + v^2\}^{\frac{1}{2}} \quad (\text{A } 1.1)$$

where u , v and w are the fluctuating velocity components in the x , y and z directions respectively. The angle θ that the wire makes with the horizontal, is normally about 45° .

The above expression can also be written as

$$C = U \sin \theta \left[1 + (2/U)(u + w/\tan \theta) + \frac{1}{U^2} (u^2 + w^2 \cot^2 \theta + u^2 \csc^2 \theta + 2uw \cot \theta) \right]^{\frac{1}{2}} \quad (\text{A } 1.2)$$

In the atmosphere, rms turbulence levels are about 10% or less of the mean flow, so that contributions of the quadratic terms are of the second order, and will be neglected, giving, by binomial expansion

$$C = U \sin \theta \left[1 + (1/U)(u + w/\tan \theta) + \dots \right] \quad (\text{A } 1.3)$$

The bridge network in a hot-wire anemometer system is used to balance out the contribution of the mean velocity, $U \sin \theta$, so that the fluctuating velocity component is approximately

$$c \doteq \sin \theta \cdot u + \cos \theta \cdot w \quad (\text{A } 1.4)$$

which is a linear combination of the velocity components in the plane of the tilted wire.

However, unpublished experimental measurements of the temperature distribution along a heated wire made by F. H. Champagne at Boeing Corporation (personal communication to R. W. Stewart, in 1965)* have shown that the cooling effect

* More recently, these results have appeared in Report No. 103, Flight Sciences Laboratory, Boeing Scientific Research Laboratories; and as a Ph.D. dissertation, U. of Wash. (Dec. 1965).

of the velocity component along the wire cannot be neglected. As a result, the above expression for c can no longer be considered valid, and a more empirical relationship has to be employed.

If C is the cooling velocity of the wire, U is a steady horizontal wind velocity taken to be in the plane of the wire and a vertical line, and θ is the angle of the wire to the horizontal, then a generalization of the case above allows us to write

$$C = f(\theta, P)U$$

where $f(\theta, P)$ is an unknown function of the wire angle θ and other geometrical wire parameters P .

A small change dC in the cooling velocity C associated with a change and tilt in U is thus represented by

$$c = dC = f(\theta, P)dU + \frac{df(\theta, P)}{d\theta}Ud\theta \quad (A 1.5)$$

In a fluctuating wind c represents the change in cooling velocity where we assume that as in eqn. A 1.3, transverse fluctuations have only a second order effect. For small turbulence levels, the vertical fluctuating wind velocity component is

$$\begin{aligned} w &= (U + u)\tan\theta \\ &\doteq U d\theta \end{aligned} \quad (A 1.6)$$

where $d\theta$ is in radians. Hence, since $u = dU$,

$$c = f(\theta, P) u + \frac{df(\theta, P)}{d\theta} w \quad (\text{A } 1.7)$$

For a properly balanced hot wire bridge, the voltage E across the wire depends on the wire current I , while the latter is a function of the cooling velocity C . Hence, if there is a fluctuation dC in the cooling velocity, a voltage fluctuation e is produced across the wire of magnitude

$$e = dE = \left(\frac{dE}{dI} \right) \left(\frac{dI}{dU} \right) \left(\frac{dU}{dC} \right) c \quad (\text{A } 1.8)$$

Use of the regular calibration procedure (Flow Corporation Bulletin 37B) gives

$$\frac{dE}{dI} = m_s / i \quad (\text{A } 1.9)$$

where m_s is the rms square wave calibration voltage for square wave current amplitude of i . Putting $I' = dI/dU$, $dU/dC = f(\theta, P)$, and using eqn. A 1.7 in eqn. A 1.8 gives

$$e = K(u + b'w) \quad (\text{A } 1.10)$$

where

$$K = (m_s / i) I' \\ b' = \frac{I}{f(\theta, P)} \frac{df(\theta, P)}{d\theta} \quad (\text{A } 1.11)$$

The quantities determining K can be easily obtained.

A normal calibration of the wire at an angle θ in the wind tunnel shows that King's Law remains true (Fig. A 1.5), so that

$$I^2 = A + B\sqrt{U}. \quad (\text{A } 1.12)$$

Thus

$$I' = \frac{B}{4I\sqrt{U}}. \quad (\text{A } 1.13)$$

I , the mean wire current, is determined in setting up for each run, the quantities m_s and i are obtained from the corresponding square wave calibration in the field, and B from the wind tunnel calibrations. The empirical determination of b' is described in Section C.4.

C.3. Calibration of the X-Wire

The calibration of each wire of the X-wire was made similarly to that for a single wire, with the following additions.

A brass rectangle was fastened, by bolts and a locking screw, to the probe arm near its connector end. Its one smooth, flat face could be aligned vertically, using a carpenter's level, to serve as a reference surface.

The probe was turned before its first calibration until both wires made visually approximately the same angle to the

horizontal and then it was permanently locked to the vertical surface. Vertical alignment of the surface and horizontal adjustment of the probe arm ensured that the wire geometry was the same during calibrations and field runs, provided a plane parallel to both wires also contained the mean wind direction.

In the Flow Corporation wind tunnel, the probe was supported by a clamp to a retort stand, on which was fastened a standard protractor, centered on the probe arm. A removable pointer, fastened to the probe, was used to measure angular rotations of the probe about its long axis.

The angle that each wire made with the horizontal was determined according to the standard procedure outlined in Flow Corporation Bulletin 16A. This angle was determined each time the wires of the probe were calibrated to ensure that the wire geometry had not changed since the last calibration. During 1965, variations in θ between calibrations before and after a period of use were of the order of 1° or less. Since this was of the order of accuracy obtained in successive determinations during one calibration, it was assumed that the wire geometries remained unchanged during all runs.

Each wire was then calibrated at this angle θ to the horizontal in order to obtain A and B of eqn. A 1.12 (an example is shown in Fig. A 1.5).

C.4. Empirical Determination of the Wire Constant b'

If a hot wire probe is placed in a low turbulence level wind tunnel, so that one of its wires is at θ to the horizontal, then the voltage E across the wire will remain a constant as long as the mean velocity U does not change. However, if the wire is tilted through a small angle $d\theta$, then the voltage across the wire will change an amount e as determined from eqn. A 1.10. (In this case $u = dU = 0$, and $w \doteq U d\theta$). Hence, by calibrating the probe as outlined before to determine K , the value of b' can be determined from the voltage variations $dE = e$ across the etched part of the wire which are produced by turning the wire in measured angular increments. This can be seen from the relation

$$b' = \frac{K}{U} \left(\frac{\Delta E}{\Delta \theta} \right) \quad (\text{A 1.14})$$

derived from eqn. A 1.10 and eqn. A 1.6.

For these measurements, the Flow Corporation wind tunnel was used. The procedure was as follows. First, K was determined (as described following eqn. A 1.11); then the wind speed was set at a constant calibration velocity U_c (in the range 5 to 7 m.sec⁻¹ in order to obtain reasonably high voltage sensitivities). This was followed by a square wave calibration. Then the wire was tilted successively through the angles $\Delta\theta = \pm 3^\circ, \pm 5^\circ, \pm 8^\circ, \pm 10^\circ$ from θ , which were

measured on the protractor using the detachable pointer. The DC voltage output was recorded on a Sanborn dual-channel recorder, with a zero line ($\Delta\theta = 0$) being established after each successive turn $\Delta\theta$. The procedure was repeated about 20-50 times for each increment of $\Delta\theta$, and the calculated values of b' for each angular increment $\Delta\theta$ were averaged and the averages plotted as $\overline{b'}$ versus $\Delta\theta$. A curve was drawn visually through the points, as shown in Fig. A 1.6.

The value used for computational purposes was the one appropriate to $\Delta\theta = 0$. This was obtained by fitting a least-squares line or quadratic function to the values of $\overline{b'}$ versus $\Delta\theta$. Each value of $\overline{b'}$ was weighted according to the number of values of b' that were used to determine the average.

The change of $\overline{b'}$ with $\Delta\theta$ as shown in Fig. A 1.6 appears quite large. Since w is typically much smaller than u , and $\sqrt{u^2}/u < 0.1$, then $\Delta\theta$ can reach at most about 5° occasionally, and will normally be much less. Hence the value of $\overline{b'}$ for $\Delta\theta = 0$ was used for all computations, thus avoiding use of a linearization procedure.

It must be noted at this point that all the calibrations were made with the probe in the mean stream, and the mean stream direction was assumed to lie in the plane of the X-wires. This last assumption is in reality unverified, since the wire configuration was set by eye and not checked by

measurement. During field measurements also, the direction of the mean wind also varied, so that a horizontal variation of the wind vector direction also must have affected the measurements. This effect, however, was not compensated for, but exists nevertheless, and will be discussed in the next section.

D. Effect of Deviation in the Direction of Mean Velocity from the Plane of the X-Wires

If the direction of a horizontal line in the plane containing one sloping X-wire and a vertical plumb line makes an angle β with the direction of the mean velocity U , then the velocity components in the plane are

$$(U + u)\cos\beta + v\sin\beta, \quad \text{horizontally}$$

and $w, \quad \text{vertically}$

with β positive from the x - axis. The component perpendicular to the plane is

$$v\cos\beta - (U + u)\sin\beta$$

If the wire slopes at angle θ to the horizontal, then the total cooling velocity C is given by (compare with eqn. A 1.1)

$$C = \left\{ [(U + u)\cos\beta + v\sin\beta]\sin\theta + w\cos\theta \right\}^2 + [v\cos\beta - (U + u)\sin\beta]^2 \quad (\text{A 1.15})$$

By expanding eqn. A 1.15 and neglecting all terms of order $u^2/U^2, v^2/U^2$ etc., then the equation reduces to

$$c^2 \doteq U^2 \sin^2 \theta \left\{ \left(\cos^2 \beta + \frac{\sin^2 \beta}{\sin^2 \theta} \right) + \frac{2}{U} \left[\left(\cos^2 \beta + \frac{\sin^2 \beta}{\sin^2 \theta} \right) u + \frac{\cos \beta w}{\tan \theta} - \frac{\sin \beta v}{2 \tan^2 \theta} \right] \right\} \quad (\text{A 1.16})$$

Use of the Binomial expansion (as in the derivation of eqn.

A 1.4) reduces eqn. A 1.16 to

$$\begin{aligned} \frac{c}{U \sin \theta} &\doteq \left(\cos^2 \beta + \frac{\sin^2 \beta}{\sin^2 \theta} \right)^{\frac{1}{2}} u + \frac{\cos \beta w}{\left(\cos^2 \beta + \frac{\sin^2 \beta}{\sin^2 \theta} \right)^{\frac{1}{2}} \tan \theta} \\ &\quad - \frac{\sin(2\beta) v}{2 \left(\cos^2 \beta + \frac{\sin^2 \beta}{\sin^2 \theta} \right)^{\frac{1}{2}} \tan^2 \theta} \end{aligned} \quad (\text{A 1.17})$$

for the fluctuating component of the cooling velocity. By following the procedure leading to eqn. A 1.10 we derive expressions for the two fluctuating wire voltage signals

$$e'_1 = K_1 \left[M_1 u - a \frac{\cos \beta}{M_1} w - \frac{N_1 v}{M_1} \right] \quad (\text{A 1.18})$$

$$e'_2 = K_2 \left[M_2 u + b \frac{\cos \beta}{M_2} w - \frac{N_2 v}{M_2} \right] \quad (\text{A 1.19})$$

where

$$M_r = \left[\cos^2 \beta + \frac{\sin^2 \beta}{\sin^2 \theta_r} \right]^{\frac{1}{2}} \quad (\text{A 1.20})$$

$$N_r = \frac{1}{2} \frac{\sin(2\beta)}{\tan^2 \theta_r} \quad (\text{A } 1.21)$$

where r equals 1 or 2.

By putting $\beta = 0$, the original form of the equation A 1.10 is obtained.

It may be noted that for β as high as 15° , and θ_r in the range 30° to 70° , the value of M_r changes only from 0.97 to 1.07.

E. The Effect of Temperature on Hot Wire Measurements

Changes in mean temperature and temperature fluctuations can produce spurious contributions to measurements of velocity fluctuations, since the hot wire technique utilizes changes in the wire temperature to indicate changes in wind speed. For measurements of turbulent velocity, the constant current technique uses the fact that a change in wind speed causes a change in wire temperature due to a change in cooling rate, which then causes a change in wire resistance, which in turn produces a fluctuation in the voltage across the wire. However, a change in wire temperature is also produced by a change in the temperature of the fluid surrounding the wire. The resultant voltage fluctuation shows up in the same way as a "velocity" contribution to the u_1 component.

The effect of temperature fluctuations and changes in mean temperature on the measured spectra of fluctuating velocities can be estimated as outlined below.

The resistance R of a wire at a temperature T is given by the expression

$$R = R_0 [1 + \sigma(T - T_0) + \sigma' (T - T_0)^2 + \dots] \quad (A1.22)$$

where R_0 is the wire resistance at the reference temperature T_0 (Hinze, 1959, p 78). (The magnitude of the squared term is only about 3% of the linear term for conditions encountered and will be neglected.) For the platinum wires used, $\sigma = 3.5 \times 10^{-3} (^\circ\text{C})^{-1}$, so that the overheat for the normally used resistance ratio $R/R_0 = 1.8$ was about 230°C .

A common way of expressing King's Law is by the equation

$$I^2 R = (a + b\sqrt{U})(T - T_0) \quad (A1.23)$$

for a wire temperature T (Hinze, 1959, p 78, Eqn. 2-7).

Both a and b are constants and comparison of equations A1.12 and .23 show that

$$B = b(T - T_0)/R \quad (A1.24)$$

The change in wire temperature produced by a change dT is equivalent to a change in mean wind speed dU since ambient temperature fluctuations show up as variations in U . Then

from equations A1.22, .23, and .24

$$\frac{dT}{dU} = -\frac{B(T-T_0)(1+\sigma(T-T_0))}{212\sqrt{U}} \quad (A1.25)$$

For measurements I made, R/R_0 was either 1.8 or 2.0 and calculations from data showed that for all ten runs analyzed $dT/dU \sim -0.2$ to -0.3 . Then θ' , the rms fluctuation of wire temperature generated by an rms velocity fluctuation u' is about $\theta' \sim 0.2u'$. Hence, comparison of the value of θ' estimated from the measured turbulence level in the shear flow to the estimated rms temperature fluctuation θ of the atmosphere, allows one to gauge the importance of the latter fluctuations in 'contaminating' the velocity signal.

Since the temperature fluctuation measurements were not available, θ was estimated as follows. Taking the Monin-Obukhov length $L \sim L'$ and using equation 11 B 2.1 gives

$$\frac{dU}{d(\ln x_3)} = \frac{u_*^*}{K} + \frac{u_*^*}{K} \alpha' \frac{x_3}{L'}$$

so that at the two heights x_3' and x_3

$$\left(\frac{dU}{d(\ln x_3)} \right)' - \left(\frac{dU}{d(\ln x_3)} \right) = \frac{u_*^* \alpha'}{KL'} (x_3' - x_3) \quad (A1.26)$$

where $\alpha' \sim 5$ (Lumley and Panofsky, 1964, pp 107-108). Then from equations 11 B2.1 and A1.26 an approximate expression for estimating the temperature flux can be obtained:

$$\overline{\theta u_3} \approx \frac{u_*'^2 T}{2.3 \alpha' g} \Delta \left(\frac{dU}{d(\log x_3)} \right) \quad (A1.27)$$

The Δ expresses a difference. From the data shown in Figures IV B.1 to .10, $\sqrt{u_3'^2} \sim u_*'$, so that $\overline{\theta u_3} \sim \theta u_*'$, allowing θ to be estimated from equation A1.27.

For all runs, $\theta \sim .07^\circ\text{C}$ or less, except in the two runs shown in Figures IV C.9 and .10, where it was found that $\theta \sim 0.9^\circ\text{C}$. For these two worst cases, $u_*' \sim 130-140 \text{ cm}\cdot\text{sec}^{-1}$, which gives rise to an rms wire temperature $\theta' \sim 26-28^\circ\text{C}$. Hence, for these worst cases, it is seen that temperature fluctuations would produce an error in u_1 of at the most about 4%, or an error of at the most 8% in ϕ_{11} . For all other runs recorded, the percentage was very much less than this. The neglect of the effect of turbulent temperature fluctuations thus does not lead to very large errors in spectral estimates.

The mean air temperature is usually different when the probe is calibrated and when it is used in the field. This would necessitate a systematic correction to spectral levels. However, the Flow Corporation constant current anemometer utilizes the constant resistance ratio method. As is pointed out in Flow Corporation Bull. 25, p 5, no such temperature corrections are necessary since changes in ambient mean temperature are automatically corrected for if the anemometer

is used properly. Also, the same bulletin points out (p 19) that proper square wave alignment nearly completely compensates for the "damping" effect on velocity fluctuations produced by conduction to the wire supports. This "damping" becomes important for wires with length/diameter ratios of 200 or less. (My wires had this ratio about 200.) Hence effects due to changes in ambient mean temperature can be neglected, provided the anemometer is used properly.

APPENDIX II: THE ELECTRONICS OF THE HOT WIRE SYSTEM

A block diagram of the circuit used in each channel is shown in Fig. A 11.1. During each set of observations, there were either two X-wire channels and one U-wire channel, or three U-wire channels.

The output from the bridge led to a DC potentiometer to remove the DC across the probe. The resulting signal was amplified by a Honeywell Accudata V DC amplifier and then recorded on a separate F.M. record channel on a 1 in. magnetic tape.

Compensation to adjust for the thermal inertia of the hot wire was added to the output stage of the DC amplifier. The compensation circuit added a proportion of the differentiated signal to the probe output, and provided filtering above 1 kc. (It was the same as that used by Pond (1965), except that the capacitance C was increased from 16 to 26 μ f to lower the attainable compensation frequencies.) Typical response curve with compensation is shown in Fig. A 11.2. Without the compensation circuit, the amplifier responses were flat within $\pm 1\%$ from 0 to 10 kc.

To avoid saturating the tape recorder amplifiers, care had to be taken to remove as much DC as possible with the potentiometer and to use a sufficiently low voltage gain. Although this procedure resulted in quite low signal-to-noise

ratios, noise was sufficiently small at frequencies of interest (0.02 to 60 sec^{-1}).

The response of the recording system for each channel was effectively limited by the 0 to 625 sec^{-1} bandwidth of the F.M. record system of the magnetic tape recorder. In this range, the voltage linearity was better than 1% . At frequencies higher than about 60 sec^{-1} , the contribution to the total velocity correlations was small, and at somewhat higher frequencies (in which there was no interest for this thesis) the signal was usually lost in the noise. For some shorter runs made during very steady wind conditions, higher signal levels were possible and the signal-to-noise ratio improved.

APPENDIX III: AUXILIARY MEASURING EQUIPMENT

Section III, pp25-27, contributes an introduction to this Appendix.

A. The Cup Anemometer System

A.1. The System

The profile of the mean wind speed was measured by an array of six sensitive Thornthwaite cup anemometers (C. W. Thornthwaite Associates 1961, Hamblin, 1965). The cups were mounted vertically on the vertically movable frame on the instrument mast at heights 0, 30, 80, 150, 241 and 351 cm relative to the lowest cup.

Normally the X-and U-wire probes were mounted at a level 15 cm below the cup at 80 cm, and about 16 cm cross-wind (to the edge of the cups). The cup anemometer arms extended 35 cm from the frame, and the cup assembly diameter was 17 cm.

Each time the cup assembly rotated once, a pulse was generated from a photocell. This pulse was amplified and used to activate an electro-mechanical counter, which was photographed using an automatic camera. These pulses were also recorded via a multiplexing system on the magnetic tape. In this system, the pulses in each channel fire a Schmitt trigger which gates an oscillator. The seven oscillator frequencies, 6 for the anemometers on the instrument mast and one for the Rover, were separated by 20% steps. The 7

gated outputs were summed and recorded on one Direct Record Channel of the magnetic tape.

To ensure that cup anemometer data would be always available, the counter readings were entered into the data log. A cup reading was entered at least once every minute. On most occasions the counters were read in turn (from the lowest to the highest cups), thus each counter was read at least once every seven minutes.

The height of the bottom cup anemometer relative to a base level on the instrument mast was determined for each run by visually sighting its level against reflective tape markers on the mast. The expected error, using 25 cm marker spacings, was 2 to 3 cm.

A.2. Calibration of the Cup Anemometers

The cup anemometers were calibrated individually in the low speed wind tunnel in the Department of Mechanical Engineering, at 4-5 different speeds between 2 to 13 m.sec⁻¹.

Calibrations were carried out at the beginning, middle and end of the summer of 1965. For all cups each calibration gave an expected linear relationship between cpm and average wind speed. However, fairly large discrepancies of up to 3.5% between calibrations of the same cup anemometer occurred. It was found that a slow leak had been present in the

manometer system of the tunnel prior and up to mid-summer. As a result, the end of the summer calibrations of October 19, 1965, which followed the repair of the leak, were used. From general experience in use of these cups in our program, the expected errors are no more than 2%.

B. Wind Direction Indicators

At the platform, a Casella wind vane was mounted about 3 feet above the top of the instrument hut. On the mast a Thernthwaite light-weight potentiometric wind vane was mounted on the movable frame about a meter and a half above the ends of the hot wire probes. This vane was so adjusted that the "N" on the display dial corresponded to a wind direction perpendicular to the probe shafts. Deviations from this direction were taken as deviations from the plane of the X-wires.

The meter dials of the direction indicators were photographed and also their readings recorded in the data log sheets once per minute.

C. Temperature Measurements

It was hoped that the temperature profile equipment would have been operational by the summer of 1965, but various difficulties prevented its proper operation. Reliance thus

had to be placed on a commercial thermometer using a bead thermistor, with a dial readout. With this, the air temperature was measured at various heights above the water surface, plus the water temperature just below the surface.

This instrument was far from accurate as an absolute instrument. However temperature differences between different heights measured at a given time are probably significant. The temperatures were estimated to 0.1°C on the readout dial.

Temperature data were obtained at least once during each run. For long runs, determinations were made at about 40-60 minute intervals.

D. Tide Measurements

Tidal streams were measured by a Savonius rotor current meter raised or lowered by a winch. The meter was about 3 m from one corner of the platform. Current speed and direction were displayed on two panel meters which were photographed and also recorded in the data log sheet once every minute.

Tidal heights were estimated to within ± 5 cm from averages of wave heights read against reflecting tape markers on the main instrument mast.

APPENDIX IV: ANALYSIS OF HOT WIRE DATA

A. Introduction

As seen from equations A 1.10, the voltage signals from each of the wires of an X-wire array are of the form

$$e_1 = K_1(u - aw)$$

$$e_2 = K_2(u + bw)$$

e_1 refers to the wire sloping downward into the wind; e_2 refers to the wire sloping upward. Both contain u and w .

In former X-wire analysis work, the values of $\overline{u^2}$, $\overline{w^2}$ and $\overline{uw}$ were obtained by one of two general methods:

1. Precise knowledge of the constants K_1 , K_2 and a and b allow the u and w portions of the signal to be separated by addition and subtraction. From these the correlations are formed. The wires of an X-array need not be at the same angle to the horizontal. This method demands that the constants be known very accurately. This is not always possible and never easy to achieve. If changed values of the constants are necessary the data must be completely reanalysed.
2. If both wires of an X-array are at the same angle to the horizontal and have the same sensitivity to the

w fluctuations (i.e. $a = b$), the signals are readily manipulated to give the correlations without extracting u and w separately. This equality is often assumed, and the observed sensitivity of the outer wire is used for both. (See Flow Corporation Bulletin 16A, 68.)

This method demands that the wires be geometrically similar and be set at the same angle to the horizontal. This is very difficult to achieve. Furthermore, the assumption of similar sensitivities to w of both wires is at best a crude approximation.

Both of these methods have serious disadvantages which are well known to those who work in wind tunnels, and which make their application to atmospheric observations extremely dubious. It was thus decided to find an alternative method of analysis. After a suggestion by R. W. Stewart, a third method was worked out and tried. It proved to be successful, and applicable to an X-array whose wires were not geometrically similar or equally sensitive to w fluctuations. Furthermore, checks designed to investigate the effects of changes in the parameters did not necessitate repetition of the analog analysis.

This method for analysis of X-wire data is fully described in Section D of this appendix.

B. Selection and Rerecording of Data

Normally a continuously recorded section of data longer than 20 minutes was broken into one or more subsections for analysis.

The criteria used to accept a subsection for analysis were as follows:

1. A reasonably steady mean wind speed was present throughout the interval. A net change in speed of 15% between the beginning and end of the run was considered as an upper limit. (It is noted, however, that during all of the present runs, excepting three, means over seven minutes only were available, instead of the normally preferred 1 minute means.)
2. The wind direction was reasonably steady throughout the interval. Wind directions which were steady within $\pm 10^\circ$ of the plane of the X-wires were considered acceptable. In order to extend observations over a wider range of meteorological conditions, one run in which observed directions varied up to 15° from the mean was used.
3. The subsection contained at least 15 complete cycles of the lowest frequency to be analysed to provide reasonable statistical reliability at low frequencies.

This frequency was normally 0.0156 sec^{-1} corresponding to a period of 64 sec . Thus the subsection had to be at least 16 minutes long.

4. The subsection contained no "spikes" or other discontinuities, which would cause electronic filters used during analysis to ring and introduce spurious results.
5. For an X-wire analysis a signal existed on both the wire channels throughout the subsection.

The subsections chosen for analysis were rerecorded, to provide a well-defined beginning and end to the wire signals. For X-wires, both reproduced signals could be made coincident within about $\pm 0.1 \text{ sec}$. Since several subsections could be serially analysed for one setting of the analysis equipment, and since the analyses were performed at rates of 16 to 64 times real time speeds, the time required for analysis was substantially shortened.

Rerecording of data was done as follows. Each reproduced signal, speeded up either 4 or 16 times from the original tape recording, was passed through a Krohn-Hite filter on "all-pass" (0.02 sec^{-1} to 2000 sec^{-1}) to remove any DC present in the original signal. These filtered signals were simultaneously switched into F.M. recording channels of a second tape recorder, after at least 5 sec of signal from

the start of each selected run had passed, to allow the filters to settle down. The lengths of the rerecorded intervals were timed with a stopwatch, and ended by shorting the filter outputs. The U-wire signal was recorded over the same interval as the two X-wire signals.

Ahead of each rerecorded run, itself consisting of several separated subsections, an identification sine wave was recorded. This was followed by a short section of square-wave signal (for calibration) from the original data. Subsections were separated by a zero-signal of reasonable length. Following each rerecorded run, square waves from the end of the original run were recorded, followed by another sine wave identification.

Transfer curves of the Krohn-Hite filters, set at all-pass with actual loads, are shown in Fig. A IV.1. Measurements made at the beginning of 1964 and the end of 1965 are indistinguishable. The sections between 1 and 100 sec^{-1} were not included since the response curves were flat in that interval.

Recording procedures in the field, and rerecording procedures at the laboratory, introduced noise which contaminated the turbulent signal. In order to correct the turbulent signals for noise, the noise signals recorded

in the field* were rerecorded at the end of the sections containing rerecorded turbulent signals or else between two serially rerecorded sections. Later analysis showed that nearly all of the noise present in the rerecorded noise interval was introduced during the rerecording procedure. This was shown by comparing the noise levels rerecorded from the field data, and the noise levels produced by shorting the inputs of the Krohn-Hite filters used for rerecording, and recording the output of the filters as a noise signal.

* The noise signal recorded in the field was produced by shorting the hot-wire probe, and recording all the noise generated beyond that point by the measuring equipment. Pond's (1965) analysis of the noise generated by the equipment, showed that the hot-wire itself generated negligible noise.

C. Single, or U-wire Analysis

The fluctuating signal on the U-wire essentially consists only of the horizontal down-wind components, provided the total turbulence level is not too large. This can be seen from eqn. A 1.3 with $\theta = \pi/2$. In this case, $\frac{dU}{dC} = 1$ and $b' = 0$ in eqn. A 1.10, so that a change in velocity u produces a change in voltage e of magnitude

$$e = Ku . \quad (A \text{ IV.1})$$

The rerecorded voltage signal e_s contains a noise voltage e_n which is added after sensing the wind, and which is uncorrelated with the wind signal. A squared and time averaged signal $\overline{e_s^2}$ contains then signal and noise, so that

$$\overline{e_s^2} = \overline{(e + e_n)^2} = \overline{e^2} + \overline{e_n^2}$$

The true mean square signal level is

$$\overline{e^2} = \overline{e_s^2} - \overline{e_n^2} \quad (A \text{ IV.2})$$

so that

$$\overline{u^2} = C(\overline{e_s^2} - \overline{e_n^2}) \quad (A \text{ IV.3})$$

where

$$C = K^{-2} \quad (\text{A IV.4})$$

and K is given by equation A 1.11.

To find a spectral density, each signal must be filtered, amplified, squared and then integrated to find an average. A block diagram of such an analysis system is shown in Fig. A IV.2.

The rerecorded signal was played back and passed through a Krohn-Hite filter set at $\frac{1}{2}$ octave band-pass (i.e. with central frequency FF , the bandwidth to 3 db points is $FF/2$). The analysis (central) frequencies were successively separated by the ratio $\sqrt{2}$ in the range of 0.25 to 1024 sec^{-1} , these being 16, 32 or 64 times the real signal frequencies. Twenty-four frequencies per data run were normally sampled. The Accudata V DC amplifier and Philbrick band-pass pre-amplifier were used to boost the signal level to an average amplitude of 30-40 volts. Lower amplitudes are undesirable, since the first break-point of the piecewise linear approximation to the square law in the Donner Square is at 16 v. Also, if the average signal level exceeds 30-40 v, some clipping of signal peaks occur through saturation of the operational amplifiers at ± 100 v.

The effect of the Donner squaring circuit on the output

signal was discussed by Pond (1965, pp. 136-137). His measured values of output voltages were in good agreement with the ideal response for inputs between about 15 and 105 volts.

The squared signal was integrated and the integral (designated by X) recorded on a Varian paper chart recorder. A reset circuit was used to reset the integrator automatically to zero on reaching 100 volts (corresponding to the full scale on the chart paper). An integrator offset allowed computer noise to be balanced out under a zero-signal condition (i.e. input to analysis system shorted). The noise level was treated similarly, to obtain an integrated noise signal, XN.*

The total durations, (T and TN), of the hot wire and noise level signals, were timed with a stopwatch to ± 0.1 sec.

The combined squared gain $(XK)^{-1}$ of the amplifiers and multiplying circuits, for each setting of the system, was obtained by feeding a measured calibration sine wave of frequency 25 Hz and rms voltage $\sqrt{e_c^2}$ into the Accudata V input, and obtaining the average voltage level $\overline{V_c^2}$ from the timed integral.

* It is noted that the terminology is that used for instructions to the digital computer so that XN is a single signal.

Thus:

$$\overline{e_c^2} = (XK) \overline{v_c^2} \quad (A. IV.5)$$

A filter transfer term A was obtained by measuring the ratio (amplitude of output/amplitude of input) for the filter when it was set at ' $\frac{1}{2}$ octave bandpass'. This was done for each desired central frequency FF, for each of the 25 central (analysis) frequencies normally used. The ratio of the area under the curve of the square of this response ratio, plotted against frequency, to that of an ideal "unit filter", is denoted by A. A "unit filter" is defined to have unit response between 0.5 FF and 1.5 FF, and zero response elsewhere.

A "% bandwidth" may be defined, such that if F is the relative amplitude response of the filter at the central frequency, then

$$A = F^2 \left(\frac{\% \text{ bandwidth}}{100\%} \right) \quad (A. IV.6)$$

(It is noted that F^2 is the maximum value of the curve under which the area, when plotted against frequency, is A.) Measured values of the "% bandwidth" were all between 50 and 60% for the Krohn-Hites used.

The measured values of A varied by as much as 5% between

different days, at a given central frequency, while the "% bandwidth" showed little variation ($<2\%$). As a result, for each filter setting F was measured, and the "% bandwidth" of the filter measured beforehand at each filter frequency was used to compute individual values for A .

The analysis system (i.e. multipliers, squarers, integrators etc.) could be used over a frequency range from zero to at least 2 kHz, without loss of rated performance. The gains of the Accudata V amplifiers used to boost the signal levels were flat from zero to about 5 kHz. The Philbrick preamplifiers, however, had flat (measured) gains from 0.1 to about 300 Hz (see Fig. A IV.3). Above 300 Hz, their amplitude response dropped off in order to reduce noise. Corrections based on these curves were applied to the values of A used at central frequencies greater than 300 Hz.

If the output from the filter is represented by $e_s(f)$, then the combined result of amplification, squaring and integration over time T is to produce a recorded signal X , which from eqn. A IV.5 has the mean value of

$$\overline{e_s^2(f)} = (XK) \frac{X}{T} \quad (\text{A IV.7a})$$

A similar analysis of the 'noise' voltage e_n , results in a recorded signal XN during the time T_N , which has the mean value

$$\overline{e_n^2}(f) = (XK) \frac{XN}{TN} \quad (A IV.7b)$$

The means $\overline{e_s^2}(f)$ and $\overline{e_n^2}(f)$ are those parts of the signal and noise spectra which pass through the filter, which (from the discussion leading to eqn. A IV.6) passes the fraction A of that part of a spectrum in a frequency band of width equal to the central frequency. It follows from eqns. A IV.1 to .4, and from the definition of $\phi_{11}(f)$ (eqn. II A1.1) that the contribution to the downwind velocity spectrum in the band of width f, centered at f, is

$$\overline{u_1^2}(f) = f\phi_{11}(f) = \frac{C}{A} \left[\overline{e_s^2}(f) - \overline{e_n^2}(f) \right] \quad (A IV.8)$$

and from A IV.7

$$f\phi_{11}(f) = \frac{C(XK)}{A} \left[\frac{X}{T} - \frac{XN}{TN} \right] \quad (A IV.9)$$

so that

$$\phi_{11}(k) = \frac{f\phi_{11}(f)}{k}$$

from Section II A.1.

Computations:

Since the expression for $f\phi_{11}(f)$ was of a straightforward nature, some computations were carried out using a Friden 130 electronic calculator.

\$JOB 70045 WEILER UWIRE SPECTRA
\$TIME 5
\$IBFIC FREQ

	DIMENSION	AT(12)
	PI=3.1415926	
	READ(5,1) AT	
	WRITE(6,2) AT	
1	FORMAT(12A6)	
2	FORMAT(1X,12A6/)	
	READ(5,3) C,U,S,Z	
3	FORMAT(E10.3,3F10.3)	
	WRITE(6,4) C,U,S,Z	
4	FORMAT(1X,2HC=,E14.4,1X,2HU=,E14.4,1X,2HS=,E14.4,2HZ=,E14.4/)	
	KO=0	
2001	READ(5,5) FF,A,XK,X,T,XN,TN	
5	FORMAT(2F10.3,E10.3,4F10.3)	
	WRITE(6,77) FF,A,XK,X,T,XN,TN	
77	FORMAT(1X,7E10.3/)	
	RF=FF/S	
	RFLG=ALOG10(RF)	
	WVNBR=2.0*PI*RF/U	
	WLOG=ALOG10(WVNBR)	
	WRITE(6,6) FF,RF,RFLG	
6	FORMAT(1X,3HFF=,E14.4,1X,3HRF=,E14.4,1X,5HRFLG=,E14.4/)	
	WRITE(6,7) WVNBR,WLOG	
7	FORMAT(1X,7HWVNBR=,E14.4,1X,5HWLOG=,E14.4/)	
	ZK=WVNBR*Z	
	ZKLOG=ALOG10(ZK)	
	WRITE(6,12) ZK,ZKLOG	
12	FORMAT(1X,3HZK=,E14.4,2X,6HZKLOG=,E14.4/)	

FORTRAN IV PROGRAM FOR U-WIRE ANALYSIS

	FPHIU=(C*XK/A)*(X/T-XN/TN)
	PHIUK=FPHIU/WVNBR
	PULOG=ALOG10(PHIUK)
8	WRITE(6,8) FPHIU FORMAT(1X,6HFPHIU=,E14.4/) WRITE(6,9) PHIUK,PULOG
9	FORMAT(1X,6HPHIUK=,E14.4,1X,6HPULOG=,E14.4/) SQKPHI=WVNBR*WVNBR*PHIUK WRITE(6,10) SQKPHI
10	FORMAT(1X,7HSQKPHI=,E14.4/) R=((ZK)**0.667)*FPHIU UW=1.13*R
13	WRITE(6,13) R, UW FORMAT(4X,3HR =,E14.4,4X,4HUW =,E14.4/) KO=KO+1
11	WRITE(6,11) KO FORMAT(1X,32HNUMBER OF FREQUENCIES ANALYSED =,I6//) GO TO 2001
	END
	\$ENTRY

FORTRAN IV PROGRAM FOR U-WIRE ANALYSIS (Cont'd.)

To speed up the computation for some necessary checks, and for future U-wire analysis, a computer program was written in Fortran IV language. This program is included on the following pages. A glossary of the printed outputs is given below:

FF	filter frequency	sec^{-1}
RF	real frequency f	sec^{-1}
RFLG	$\log f$	
WVNB	wave number k	cm^{-1}
WLOG	$\log (k)$	
ZK	kx_3	
ZKLOG	$\log (kx_3)$	
FPHIU	$f\phi_{11}(f)$	$\text{cm}^2\text{sec}^{-2}$
PHIUK	$\phi_{11}(k)$	$\text{cm}^3\text{sec}^{-2}$
PULOG	$\log (\phi_{11}(k))$	
SQKPHI	$k^2\phi_{11}(k)$	$\text{cm}\cdot\text{sec}^{-2}$
R	$(kx_3)^{2/3}f\phi_{11}(f)$	$\text{cm}^2\text{sec}^{-2}$
UW	u^*2	$\text{cm}^2\text{sec}^{-2}$

All input constants, plus all values used for each filter frequency, were printed to facilitate the location of errors.

D. X-Wire Analysis

D.1. Introduction

The two signals from the two wires of an X-wire array can

be represented by

$$e_1 = K_1(u - aw)$$

$$e_2 = K_2(u + bw) \quad (\text{A IV.10})$$

using eqns. A 1.10 (see also Section A IV.A). The voltage signal e_1 arises from the wire sloping downward into the wind, and e_2 arises from the wire sloping upward. K_r ($r=1,2$) is given by eqn. A 1.11.

The object of the present analysis is to find the contributions,

$$\overline{u^2}(f) = f\phi_{11}(f); \quad \overline{w^2}(f) = f\phi_{33}(f); \quad \overline{uw}(f) = f\phi_{13}(f) \quad (\text{A IV.11})$$

to the various spectra in the frequency bands of width f centered at the frequency f . By analogy with the operations on signals e_s (in eqns. A IV.2 and .3) from a U-wire, to $f\phi_{11}(f)$ in eqn. A IV.9, we can operate on the signals e_1 and e_2 in eqn. A IV.10, to find the three simultaneous equations

$$A(1) = \overline{u^2}(f) + a^2\overline{w^2}(f) - 2a\overline{uw}(f)$$

$$A(2) = \overline{u^2}(f) + b^2\overline{w^2}(f) + 2b\overline{uw}(f)$$

$$A(3) = \overline{u^2}(f) - ab\overline{w^2}(f) + (b-a)\overline{uw}(f). \quad (\text{A IV.12})$$

Thus the two voltage signals in eqn. A IV.10 must be filtered at the same central frequency, in such a way that

both active filters produce an identical phase change at every frequency within the band pass interval. In this way they can be correctly multiplied without introducing phase errors into the product.

Using the two Krohn-Hite filters set at $\frac{1}{2}$ octave band pass and carefully matched in phase, a filtered signal from each wire is obtained. Each filtered signal is then squared and integrated, and their product also integrated. From the 3 integrated outputs $XI(I)*$, proceeding as in finding eqn. A IV.9, we find the three numbers $A(I)*$, for $I = 1, 2, 3$

$$A(I) = \frac{C(I) \times XK(I)}{XA(I)} \left[\frac{XI(I)}{T(I)} - \frac{XN(I)}{TN(I)} \right] \quad (A \text{ IV.13})$$

The definitions of the $C(I)$ are, from eqn. A IV.10, (compare eqns. A IV.1 and .4)

$$C(1) = (K_1)^{-2}; \quad C(2) = (K_2)^{-2}; \quad C(3) = (K_1 K_2)^{-1}. \quad (A \text{ IV.14})$$

Every other symbol on the right hand side of eqns. A IV.13 (e.g. $XA(I)$) has a meaning identical to that of a corresponding symbol (e.g. XA) in eqn. A IV.9.

* Since in the X-wire analysis, two signals are used a subscripted notation is used. Although the symbol I is used elsewhere to denote the mean wire current (see eqn. A 1.12), it will also be used in brackets in following sections, as an identifying symbol. This makes it compatible with the digital computer. The difference should be noted to avoid confusion.

The solutions to eqns. A IV.12 are,

$$f\phi_{11}(f) = B'(b^2A(1) + a^2A(2) + 2abA(3))$$

$$f\phi_{33}(f) = B'(A(1) + A(2) - 2A(3))$$

$$f\phi_{13}(f) = B'(-bA(1) + aA(2) + (b-a)A(3)) \quad (A IV.15)$$

where $B' = (a + b)^{-2}$.

Use of the definition of k (Section II A.1) gives the spectral value

$$\phi(k) = \frac{f\phi(f)}{k} \quad (A IV.16)$$

where $k = 2\pi f/U$.

D.2. Treatment of the Rerecorded Data

The X-wire signals recorded in the field were rerecorded as explained in Section A IV.B.

Fig. A IV.4 shows the block diagram of the analog system used to analyze X-wire signals.

The $\frac{1}{2}$ octave filters were matched as follows. A sine wave was set at the filtering frequency by using a counter, and was first fed from a signal generator into a filter input. The input signal to the filter, and the output signal from the Philbrick preamplifier following the filter (see Fig.

A IV.4), were fed into an X-Y oscilloscope. The controls for upper and lower cutoff frequencies (3-db points) were maintained at a common frequency to obtain correct band pass, while both were adjusted until the Lissajou figure indicated a zero phase shift. The second filter was matched in the same manner. The signal generator was then fed to both filter inputs and swept through a wide frequency band to test for zero phase difference between Philbrick preamplifier outputs across the pass band. Initial measurements of the filter transfer curves showed that, although the filter dial readings could differ as much as 5% between the two filters (2.5% from the central frequency on both sides), the transfer curves were centered about the same frequency, as far as it could be visually observed. The input and output loads of both filters and preamplifiers were the same as during analysis.

Next, the signal generator was again set to the filtering frequency and the filter responses measured (see eqn. A IV.6), to obtain the filter corrections $XA(I)$ ($I = 1, 2$) for each filter, and $XA(3) = (XA(1) \cdot XA(2))^{\frac{1}{2}}$.

The signals were then analysed, ensuring that the signal inputs to the squaring and multiplying circuits were large enough, but not so large as to produce clipping (see Section B.2). For each piece of data, three signal and three noise integrals were obtained at each filter frequency from 0.25 to 1024 sec^{-1} , to obtain values of $XI(I)$ and $XN(I)$. Their

respective time intervals $T(I)$ and $TN(I)$ were measured with a stopwatch to within ± 0.1 sec.

The squared gain of the system, $XK(I)^{-1}$, was obtained as for the U-wire analysis. For this see eqn. A IV.5.

D.3. Calculations from X-Wire Analysis

Initially, the three values of $A(I)$ in eqn. A IV.13 and thence the solutions for eqn. A IV.14 were computed from the results of the analog analysis, using an electronic desk calculator.

This method proved to be unduly laborious and time-consuming, as well as prone to human errors. As a result, for the I.B.M. 7040 digital computer, a program was written by a member of the staff of the University's Computing Center. Analog results were entered on punched cards, and more than one-half of the solutions were computed by means of this program.

The detailed final version of the Fortram IV program is outlined on the following pages. A glossary of the printed outputs is given below:

FF	filter frequency	sec^{-1}
RF	real frequency f	sec^{-1}
RFLG	$\log f$	
WVNR	wave number k	cm.^{-1}
WLOG	$\log k$	
ZK	kx_3	
ZKLG	$\log (kx_3)$	
FSQU	$f\phi_{11}(f)$	$\text{cm.}^2\text{sec}^{-2}$
FSQW	$f\phi_{33}(f)$	$\text{cm.}^2\text{sec}^{-2}$
FUW	$f\phi_{13}(f)$	$\text{cm.}^2\text{sec}^{-2}$
RUW	$R_{13}(f)$	
PHIUK	$\phi_{11}(k)$	$\text{cm.}^3\text{sec}^{-2}$
PHIWK	$\phi_{33}(k)$	$\text{cm.}^3\text{sec}^{-2}$
PHIUWK	$ \phi_{13}(k) $	$\text{cm.}^3\text{sec}^{-2}$
PULOG	$\log \phi_{11}(k)$	
PWLOG	$\log \phi_{33}(k)$	
PUWLOG	$\log \phi_{13}(k)$	
SQKPHU	$k^2\phi_{11}(k)$	$\text{cm.}^2\text{sec}^{-2}$
PWOVPU	ϕ_{33}/ϕ_{11}	
TAN2T	$\tan (2\theta)$	
THETA2	$\theta/2$	

All gains and filter transfer functions for each central filter frequency, and all values such as $X(I)$, $XN(I)$, $T(I)$, etc. were printed out along with spectral values, to facilitate the location of any errors. A typical run for 25 filter frequencies used about 1.5 minutes of computer time.

\$JOB 70045 WEILER XWIRE SPECTRA
 \$TIME 5
 \$IBFTC FREQ

```

    DIMENSION C(5),T(3),AT(12),XA(3),XK(3),XI(3),XN(3),TN(3),A(3)
    PI= 3.1415926
    ASSIGN 2000 TO M
    CALL EOF(5,M)
1    FORMAT (12A6)
2    FORMAT(3E10.3,5F10.3/3F10.3)
500  FORMAT (2F10.4)
3    FORMAT (1X,12A6/)
4    FORMAT(F10.3,E10.3,3F10.3)
5    FORMAT(1X,3HU =,E14.4,3X,3HS =,E14.4,3X,3HZ =,E14.4/)
700  FORMAT(1X,5HC(1)=,E14.4,1X,5HC(2)=,E14.4,1X,5HC(3)=,E14.4/)
800  FORMAT(1X,5HC(4)=,E14.4,4X,5HC(5)=,E14.4/)
6    OFORMAT (1X,4HFRFR,E14.4,4HREFR,E14.4,7HLOGREFR,E14.4,5HWVNR,E14.4
1,10H LOG WVNR,E14.4/)
7    FORMAT (1X,4HFSQU,E14.4,4HFSQW,E14.4,4H FUW,E14.4,4H RUW,E14.4/)
9    FORMAT (1X,6HSQKPHU,E14.4,6HPWCVPU,E14.4/)
8    OFORMAT (1X,5HPHIUK,E14.4,8HLOGPHIUK,E14.4,5HPHIWK,E14.4,8HLOGPHIWK
1,E14.4/1X,6HPHIUWK,E14.4,10H LOGPHIUWK,E14.4/)
2000 READ(5,1) AT
    WRITE(6,3) AT
    KO =0
    READ(5,2) C, T, U, S, Z
    WRITE(6,700) C(1), C(2), C(3)
    WRITE(6,800) C(4), C(5)
    WRITE(6,38) T(1),T(2),T(3)
38   FORMAT(1X,5HT(1)=,F14.4,1X,5HT(2)=,F14.4,1X, 5HT(3)=,F14.4/)
    WRITE(6,5) U,S,Z
    C45SQ = 1./((C(4)+C(5))*2
    C4SQ  = C(4)*C(4)
    C5SQ  = C(5)*C(5)
  
```

FORTAN IV PROGRAM FOR X-WIRE ANALYSIS

2001	READ (5,500) FF DO 101 I=1,3 READ(5,4) XA(I), XK(I),XI(I),XN(I),TN(I) WRITE(6,11) XA(I), XK(I),XI(I),XN(I),TN(I)
11	FORMAT (1X,5E20.5/)
101	A(I)=(C(I)*XK(I))/XA(I)*(XI(I)/T(I)-XN(I)/TN(I))
555	FORMAT(1X,5HA(1)=,E14.4,2X,5HA(2)=,E14.4,2X,5HA(3)=,E14.4/) WRITE(6,555) A(1),A(2),A(3) RF=FF/S RFLG = ALOG10(RF) KO= KO+1 FSQU = C45SQ*(C5SQ*A(1)+C4SQ*A(2)+2.0*C(4)*C(5)*A(3)) FSQW = C45SQ*(A(1)+A(2)-2.0*A(3)) FUW = C45SQ*(-C(5)*A(1)+C(4)*A(2)+(C(5)-C(4))*A(3)) RUW = FUW/SQRT(FSQU*FSQW) WVNR= 2.*PI*RF/U WLOG =ALOG10(WVNR) ZK=WVNR*Z ZKLG=ALOG10(ZK) WRITE(6,6) FF, RF, RFLG, WVNR, WLOG
41	FORMAT(1X,3HZK=,E14.4,2X,5HZKLG=,E14.4/) WRITE(6,41) ZK,ZKLG WRITE(6,7) FSQU, FSQW, FUW, RUW

FORTAN IV PROGRAM FOR X-WIRE ANALYSIS (Cont'd.)

```

PHIUK= FSQU/WVNBR
PULOG= ALOG10(PHIUK)
PHIWK= FSQW/WVNBR
PHIWK=ABS(PHIWK)
PWLOG= ALOG10(PHIWK)
PHIUWK=FUW/WVNBR
PHIUWK=ABS(PHIUWK)
PUWLOG= ALOG10(PHIUWK)
WRITE(6,8) PHIUK, PULOG, PHIWK, PWLOG, PHIUWK, PUWLOG
SQKPHU= WVNBR*WVNBR*PHIUK
PWOVPU= PHIWK/PHIUK
WRITE(6,9) SQKPHU, PWOVPU
TAN2T=(1.7321*FUW)/(0.75*FSQW-FSQU)
THETA2=0.5*ATAN(TAN2T)
THETA2=180.0*THETA2/PI
600  FORMAT(1X,5HTAN2T,E14.4,2X,6HTHETA2,E14.4/)
WRITE(6,600) TAN2T, THETA2
WRITE(6,10) KO
10   FORMAT (1X,32HNUMBER OF FREQUENCIES ANALYSED = I6//)
GO TO 2001
END
$ENTRY

```

FORTRAN IV PROGRAM FOR X-WIRE ANALYSIS (Cont'd.)

E. Changes in Spectral Values for $\beta \neq 0$

In Appendix I, Section D, it was seen that the fluctuating signal from each wire is modified when the mean wind direction makes a horizontal angle β to the vertical plane containing the sloping wire. (The modified voltage signals are given by eqns. A 1.18 and .19.)

If both wires are assumed to be parallel to the same vertical plane, the same angle β enters into the expressions (see eqns. A 1.18 and .19) for both signals. For actually constructed X-wire arrays, this assumption is unwarranted, since it is extremely difficult to ensure such precise geometry during construction. However, the probes are calibrated, and their sensitivities to $u(=u_1)$ and $w(=u_3)$ velocity fluctuations measured, under conditions where the mean wind direction is always in the vertical plane perpendicular to the probe arm (long axis of the probe). As long as the vertical planes containing the actual two wires of the X-array do not make too large angles with the plane perpendicular to the probe arm (less than 5° say, which can be achieved in practice), their actual horizontal directions can be ignored. In this case, β represented angular deviations from the plane perpendicular to the probe arm.

It is now instructive to calculate how the spectral values $f\phi_{ij}(f)$, which are measured for $\beta = 0$, are modified to give the

spectral values $f\phi_{ij}(f)$ when $\beta \neq 0$. Proceeding as in Section A IV.D, one can derive the expressions $A'(1)$ (equivalent to eqns. A IV.12) for the filtered and averaged signals for the case where $\beta \neq 0$. By using eqns. A I.19, one obtains the values

$$\begin{aligned}
 A'(1) &= M_1^2 \overline{u^2}(f) + a \frac{2 \cos \beta}{M_1^2} \overline{w^2}(f) - 2a \cos \beta \overline{uw}(f) + \frac{N_1^2}{M_1^2} \overline{v^2}(f) \\
 A'(2) &= M_2^2 \overline{u^2}(f) + b \frac{2 \cos \beta}{M_2^2} \overline{w^2}(f) + 2b \cos \beta \overline{uw}(f) + \frac{N_2^2}{M_2^2} \overline{v^2}(f) \\
 A'(3) &= M_1 M_2 \overline{u^2}(f) - ab \frac{\cos \beta}{M_1 M_2} \overline{w^2}(f) + \left[b \frac{M_1}{M_2} - a \frac{M_2}{M_1} \right] \cos \beta \overline{uw}(f) \\
 &\quad + \frac{N_1 N_2}{M_1 M_2} \overline{v^2}(f) \quad (A IV.17)
 \end{aligned}$$

where $\overline{u^2}(f)$, $\overline{w^2}(f)$ and $\overline{uw}(f)$ are defined in eqn. A IV.11 ($\beta = 0$), and where $\overline{v^2}(f) = f\phi_{22}(f)$. Eqns. A IV.17 reduce to eqns. A IV.12 when $\beta = 0$. The extra contributions produced by having $\beta \neq 0$ is thus given by

$$\delta A(1) = A'(1) - A(1) \quad (A IV.18)$$

using eqns. A IV.17 and .12. These values are given below

$$\delta A(1) = (M_1^2 - 1) \overline{u^2}(f) + a^2 \left(\frac{\cos^2 \beta}{M_1^2} - 1 \right) \overline{w^2}(f) - 2a(\cos \beta - 1) \overline{uw}(f) \\ + \frac{N_1^2}{M_1^2} \overline{v^2}(f)$$

$$\delta A(2) = (M_2^2 - 1) \overline{u^2}(f) + b^2 \left(\frac{\cos^2 \beta}{M_2^2} - 1 \right) \overline{w^2}(f) + 2b(\cos \beta - 1) \overline{uw}(f) \\ + \frac{N_2^2}{M_2^2} \overline{v^2}(f)$$

$$\delta A(3) = (M_1 M_2 - 1) \overline{u^2}(f) - ab \left(\frac{\cos^2 \beta}{M_1 M_2} - 1 \right) \overline{w^2}(f) \\ + \left[b \left(\frac{M_1}{M_2} \cos \beta - 1 \right) - a \left(\frac{M_2}{M_1} \cos \beta - 1 \right) \right] \overline{uw}(f) \\ + \frac{N_1 N_2}{M_1 M_2} \overline{v^2}(f). \quad (A \text{ IV.19})$$

The spectral values are similarly modified when $\beta \neq 0$. Substitution of the values $A'(1)$ (eqn. A IV.18) into eqns. A IV.15, gives the new spectral values $f\phi'_{ij}(f)$, such that

$$f\phi'_{ij}(f) = f\phi_{ij}(f) + \delta[f\phi_{ij}(f)]. \quad (A \text{ IV.20}) \\ (\beta \neq 0) \quad (\beta = 0) \quad (\beta \neq 0)$$

The values of $\delta[f\phi_{ij}(f)]$ are easily obtained by replacing the values of $A(1)$, $A(2)$ and $A(3)$ in eqns. A IV.15 by their respective values of $\delta A(1)$, $\delta A(2)$ and $\delta A(3)$.

In order to obtain numerical estimates of the effect of $\beta \neq 0$, a run must be chosen such that the measured angle between the mean wind direction and the vertical plane perpendicular to the probe arm is quite small. The modified spectral values $f\phi'_{ij}(f)$ can then be calculated from the actual measured values $f\phi_{ij}(f)$ by introducing non-zero values of β into eqns. A IV.19 and .20. However, X-wire arrays can measure the spectral values $f\phi_{11}(f)$, $f\phi_{33}(f)$ and $f\phi_{13}(f)$ only; hence, in order to use eqns. A IV.19, a value for $f\phi_{22}(f)$ must be assumed or estimated. According to spectra estimated using a thrust anemometer, (S. Smith, 1966, pers. comm.), a fair approximation would be

$$f\phi_{22}(f) = (\frac{1}{2})(f\phi_{11}(f) + f\phi_{33}(f)) . \quad (\text{A IV.21})$$

To carry out numerical computations for the effect of $\beta \neq 0$, an additional two parts were written for the Fortran IV programs. These parts (A and B) are outlined on the following pages.

Part A includes all calculations which produce values of the constants formed from terms containing β and θ . The 'effective' value of θ is estimated using the eqn. $\tan \theta = 1/b'$. This relationship is obtained by putting $f(\theta, P) = \sin \theta$ in eqn. A 1.11, by analogy with arguments leading to eqn. A 1.1 in Appendix I, Section C.2. This section is placed just before the first step of the computation loop; that is just before the I.B.M. card reading.

2001 READ(5,500) FF

Part B includes all calculations which depend on the computed spectral values ($f\phi_{11}(f)$ etc.) for each filtering frequency FF. This section is placed just before the last card of the loop; that is, just before the I.B.M. card reading

GO TO 2001.

A glossary of the extra printed outputs is given below

T1	θ_1	°
T2	θ_2	°
BETA	β	°
D(I)	$\delta A(I)$	$\text{cm} \cdot \text{sec}^{-2}$
DELTA 1	$\delta\phi_{11}(f)$	$\text{cm} \cdot \text{sec}^{-2}$
DELTA 2	$\delta\phi_{33}(f)$	$\text{cm} \cdot \text{sec}^{-2}$
DELTA 3	$\delta\phi_{13}(f)$	$\text{cm} \cdot \text{sec}^{-2}$
FP11	$f\phi_{11}'(f)$	$\text{cm} \cdot \text{sec}^{-2}$
FP33	$f\phi_{33}'(f)$	$\text{cm} \cdot \text{sec}^{-2}$
FP13	$f\phi_{13}'(f)$	$\text{cm} \cdot \text{sec}^{-2}$
RA1	$f\phi_{11}'(f)/f\phi_{11}(f)$	
RA2	$f\phi_{33}'(f)/f\phi_{33}(f)$	
RA3	$f\phi_{13}'(f)/f\phi_{13}(f)$	
RATIO	$f\phi_{33}'(f)/f\phi_{11}'(f)$	

```

TNT1= 1.0/C(4)
TNT2= 1.0/C(5)
T1= ATAN(TNT1)
T2= ATAN(TNT2)
T1= (180.*T1)/PI
T2= (180.*T2)/PI
52 WRITE(6,52) T1, T2 , BETA
   FORMAT(2X,3HT1=,F14.4,2X,3HT2=,F14.4,2X,5HBETA=,F14.4//)
   T1=(PI*T1)/180.
   T2=(PI*T2)/180.
   BETA=(PI*BETA)/180.
   CSB=COS(BETA)
   SNB=SIN(BETA)
   SNT1=SIN(T1)
   SNT2=SIN(T2)
   TNT1=TAN(T1)
   TNT2=TAN(T2)
   SN2B=SIN(2.0*BETA)
   P(1)= CSB*CSB + (SNB*SNB)/(SNT1*SNT1)
   P(1)' = P(1)**0.5
   P(2)= CSB*CSB + (SNB*SNB)/(SNT2*SNT2)
   P(2) = P(2)**0.5
   P(3)=(0.5*SN2B)/(TNT1*TNT1)
   P(4)=(0.5*SN2B)/(TNT2*TNT2)
   P(5) = (CSB*CSB)/(P(1)*P(1)) - 1.0
   P(6) = (CSB*CSB)/(P(2)*P(2)) - 1.0
   P(7) = (CSB*CSB)/(P(1)*P(2)) - 1.0

```

FORTTRAN IV PROGRAM FOR β EFFECT: PART A.

Q(1)= P(1)*P(1) - 1.0

Q(2)= P(2)*P(2) - 1.0

Q(3)= P(1)*P(2) - 1.0

R(1) = C(4)*C(4)*P(5)

R(2) = C(5)*C(5)*P(6)

R(3) = C(4)*C(5)*P(7)

E(1) = 2.0*C(4)*(CSB - 1.0)

E(2) = 2.0*C(5)*(CSB - 1.0)

E(3) = C(5)*(P(1)*CSB/P(2)-1.0) - C(4)*(P(2)*CSB/P(1)-1.0)

F(1) = (P(3)/P(1))*2.0

F(2) = (P(4)/P(2))*2.0

F(3) = (P(3)/P(1))*(P(4)/P(2))

FORTRAN IV PROGRAM FOR β EFFECT: PART A (Cont'd.)

```

FSQV = 0.5*(FSQU + FSQW)
D(1) = 0(1)*FSQU + R(1)*FSQW - E(1)*FUW + F(1)*FSQV
D(2) = 0(2)*FSQU + R(2)*FSQW + E(2)*FUW + F(2)*FSQV
D(3) = 0(3)*FSQU - R(3)*FSQW + E(3)*FUW + F(3)*FSQV
WRITE(6,57) D(1), D(2), D(3)
57  FORMAT(1X,5HD(1)=,E14.4,2X,5HD(2)=,E14.4,2X,5HD(3)=,E14.4/)
DELTA1=C45SQ*(C5SQ*D(1) + C4SQ*D(2) + 2.*C(4)*C(5)*D(3) )
DELTA2=C45SQ*( D(1) + D(2) - 2.0*D(3) )
DELTA3=C45SQ*( -C(5)*D(1) + C(4)*D(2) + (C(5)-C(4))*D(3) )
WRITE(6,55) DELTA1, DELTA2, DELTA3
55  FORMAT(1X,7HDELTA1=,E14.4,2X,7HDELTA2=,E14.4,2X,7HDELTA3=,E14.4/)
FP11 = FSQU + DELTA1
FP33 = FSQW + DELTA2
FP13 = FUW + DELTA3
WRITE(6,56) FP11, FP33, FP13
56  FORMAT(3X,5HFP11=,E14.4,2X,5HFP33=,E14.4,2X,5HFP13=,E14.4 //)
RA1 = FP11/FSQU
RA2 = FP33/FSQW
RA3 = FP13/FUW
WRITE(6,99) RA1, RA2, RA3
99  FORMAT(1X,4HRA1=,F10.3,2X,4HRA2=,F10.3,2X,4HRA3=,F10.3//)
RATIO = FP33/FP11
WRITE(6,98) RATIO
98  FORMAT(4X,6HRATIO=,F10.3)

```

APPENDIX V: SAMPLE CALCULATIONSA. Glossary of Terms Used for Calculations

(i) Terms used for calculations

R.R.	Resistance ratio used for probe.
B.N.	"Bridge Null" reading for "Cold Balance" of the hot wire bridge circuit.
I	Wire current for probe in ma.
i	Rms square wave current for square wave calibration of wire in ma rms.
ms	Rms square wave voltage in volts rms.
B	Slope of the I^2 versus $\sqrt{U}$ calibration line.
S	Analysis speed-up rate $= \left(\frac{\text{Real time}}{\text{Analysis time}} \right)$.

(ii) Equations:

$$I' = \frac{B}{4 IU^2} \quad (\text{eqn. A 1.13})$$

$$i = \frac{3.05}{1 + \frac{(B.N.)(R.R.)}{400}} \quad \text{anemometers 1 and 2}$$

$$i = \frac{2.50}{1 + \frac{(B.N.)(1 + 4(R.R.))}{1600}} \quad \text{anemometer 3}$$

$$C = \left(\frac{i}{m_s} \times \frac{1}{I'} \right)^2 = K^{-2} \quad \text{U-wire (eqn. A 1.11)}$$

$$C(I) = K_I^{-2} \quad \text{X-wire (eqn. A 1.11)}$$

$$C(2) = K_2^{-2} \quad \text{X-wire (eqn. A1.11)}$$

$$C(3) = (K_1 K_2)^{-1} \quad \text{X-wire (eqn. A 1.11)}$$

B. Data of 1550-1629/29/6/1965: U-wire Calculation:

$$U = 289 \text{ cm. sec}^{-1}$$

$$x_3 = 192 \text{ cm.}$$

Probe S3

(i) Values of Constants

$$R.R. \quad 1.8$$

$$B.N. \quad 143$$

$$m_s \quad 0.351(6) \quad \text{volts rms}$$

$$4 I \quad 347 \quad \text{ma}$$

$$B \quad 2.00 \times 10^2 \quad \text{ma}^2 (\text{cm/sec})^{-\frac{1}{2}}$$

$$S \quad 16$$

(ii) Coefficient C

$$I' = \frac{2.00 \times 10^2}{347 \sqrt{289}} = 0.0338(8) \text{ ma} (\text{cm/sec})^{-1}$$

$$i = \frac{2.50}{1 + \frac{143(1 + 4 \times 1.8)}{1600}} = 1.443 \text{ ma rms}$$

$$C = \left(\frac{1.443}{0.3516} \times \frac{1}{0.03388} \right)^2 = 1.46(7) \times 10^4 \text{ cm}^2 (\text{volts}^2 \text{sec}^2)$$

(iii) Data for Filtering Frequency (FF) = 2.8 Hz.

(See Sect. A IV.C)

$$X = 321 \quad \text{volts}^2 \text{ sec.} \quad (\text{signal}^2 \text{ integral})$$

$$T = 114.9 \quad \text{sec.} \quad (\text{signal}^2 \text{ integral duration})$$

$$\begin{aligned}
 XN &= 0.9 \text{ volts}^2 \text{ sec.} && (\text{noise}^2 \text{ integral}) \\
 TN &= 60.0 \text{ sec.} && (\text{noise}^2 \text{ integral duration}) \\
 XK &= 3.942 \times 10^{-4} && (\text{gain factor}) \\
 A &= 0.1592 && (\text{filter response})
 \end{aligned}$$

(iv) Calculations

$$\begin{aligned}
 f\phi_{11}(f) &= \frac{C(XK)}{A} \left(\frac{X}{T} - \frac{XN}{TN} \right) && (\text{eqn. A IV.9}) \\
 &= \frac{1.467 \times 10^4 \times 3.942 \times 10^{-4}}{0.1592} \left(\frac{321}{114.9} - \frac{0.9}{60.0} \right) \\
 &= 101 \text{ cm}^2 \text{ sec}^{-2}
 \end{aligned}$$

$$\phi_{11}(k) = \frac{f\phi_{11}(f)}{k} = 2.65 \times 10^4 \text{ cm}^3 \text{ sec}^{-2} \quad (\text{eqn. A IV.15})$$

$$f = \frac{FF}{S} = \frac{2.8}{16} = 0.175 \text{ Hz.}$$

$$k = \frac{2 \times \pi \times .175}{289} = 0.00381 \text{ cm}^{-1}$$

$$kx_3 = 0.00381 \times 192 = 0.731$$

$$\log f = -0.757$$

$$\log k = -2.419$$

$$\log kx_3 = -0.136$$

$$\log \phi_{11}(k) = 4.423$$

C. Data of 1555-1629/29/6/1965: X-wire Calculations:

Run 2: 2122-2206

$$U = 289 \text{ cm sec}^{-1}$$

$$x_3 = 192 \text{ cm.}$$

Probe X3

(i) Values of Constants: Inner Wire (Signal 1)

R.R.	1.8	
B.N.	98	
m_s	0.310(1)	volts rms
I	409	ma
B	$2.09(8) \times 10^2$	$\text{ma}^2 (\text{cm/sec})^{-\frac{1}{2}}$
S	16	
b'	1.06(8)	(= a in eqn. A IV.11)

(ii) Values of Constants: Outer Wire (Signal 2)

R.R.	1.8	
B.N.	94	
m_s	0.351(6)	volts rms
I	424	ma
B	$2.38(2) \times 10^2$	$\text{ma}^2 (\text{cm/sec})^{-\frac{1}{2}}$
S	16	
b'	0.825(2)	(= b in eqn. A IV.11)

(iii) Coefficient C(1): Inner Wire

$$I' = \frac{2.098 \times 10^2}{409 \sqrt{289}} = 0.030(5) \quad \text{ma} (\text{cm/sec})^{-1}$$

$$i = \frac{3.05}{1 + \frac{98 \times 1.8}{400}} = 2.117 \quad \text{ma rms}$$

$$C(1) = \left[\frac{2.117}{0.3101} \times \frac{1}{0.03015} \right]^2 = 0.512(8) \times 10^5 (\text{cm sec}^{-1} / \text{volt})^2$$

(iv) Coefficient C(2): Outer Wire

$$I' = \frac{2.382 \times 10^2}{424 \times 289} = 0.0330(2) \text{ ma(cm/sec)}^{-1}$$

$$i = \frac{3.05}{1 + \frac{94 \times 1.8}{400}} = 2.143 \text{ ma.rms}$$

$$C(2) = \left[\frac{2.143}{0.3516} \times \frac{1}{0.03302} \right]^2 = 0.340(7) \times 10^5 (\text{cm sec}^{-1}/\text{volt})^2$$

(v) Coefficient C(3)

$$C(3) = (C(1) \times C(2))^{\frac{1}{2}} = 0.418(0) \times 10^5 (\text{cm sec}^{-1}/\text{volt})^2 (\text{eqn. A IV.13})$$

(vi) Data for Filtering Frequency (FF) = 4.0 Hz.

Inner Wire:

$$XI(1) = 986 \text{ volts}^2 \text{ sec.} \quad (\text{Signal}^2 \text{ integral})$$

$$T(1) = 159.8 \text{ sec.} \quad (\text{signal integral duration})$$

$$XN(1) = 2.0 \text{ volts}^2 \text{ sec.} \quad (\text{noise}^2 \text{ integral})$$

$$TN(1) = 74.1 \text{ sec.} \quad (\text{noise integral duration})$$

$$XK(1) = 9.80(5) \times 10^{-5} \quad (\text{gain factor})$$

$$XA(1) = 0.123(6) \quad (\text{filter response})$$

Outer Wire:

$$XI(2) = 308 \text{ volts}^2 \text{ sec.} \quad (\text{signal}^2 \text{ integral})$$

$$T(2) = 159.8 \text{ sec.} \quad (\text{signal integral length})$$

$$XN(2) = 0.5 \text{ volts}^2 \text{ sec.} \quad (\text{noise}^2 \text{ integral})$$

$$TN(2) = 74.1 \text{ sec.} \quad (\text{noise integral length})$$

$$XK(2) = 1.02(5) \times 10^{-4} \quad (\text{gain factor})$$

$$XA(2) = 0.083(1) \quad (\text{filter response})$$

Multiplied signal:

$$XI(3) = 340 \text{ volts}^2 \text{ sec.} \quad (\text{signal}^2 \text{ integral})$$

$$\begin{aligned}
 T(3) &= 159.8 \text{ sec.} && (\text{signal integral length}) \\
 XN(3) &= 1.0 \text{ volts}^2 \text{ sec.} && (\text{noise}^2 \text{ integral}) \\
 TN(3) &= 74.1 \text{ sec.} && (\text{noise integral length}) \\
 XK(3) &= 1.00(5) \times 10^{-4} && (\text{gain factor}) \\
 XA(3) &= 0.103(0) && (\text{filter response})
 \end{aligned}$$

(vii) Calculations: (eqn. A IV.11)

$$\begin{aligned}
 A(1) &= \frac{0.5128 \times 10^5 \times 9.805 \times 10^{-5}}{0.1236} \left(\frac{986}{159.8} - \frac{2.0}{74.1} \right) \\
 &= 250 \text{ cm}^2 \text{sec}^{-2}
 \end{aligned}$$

$$\begin{aligned}
 A(2) &= \frac{0.3407 \times 10^5 \times 1.025 \times 10^{-4}}{0.0831} \left(\frac{308}{159.8} - \frac{0.5}{74.1} \right) \\
 &= 80.7 \text{ cm}^2 \text{sec}^{-2}
 \end{aligned}$$

$$\begin{aligned}
 A(3) &= \frac{0.4180 \times 10^5 \times 1.005 \times 10^{-4}}{0.1030} \left(\frac{340}{159.8} - \frac{1.0}{74.1} \right) \\
 &= 86.2 \text{ cm}^2 \text{sec}^{-2}
 \end{aligned}$$

$$(a + b)^{-2} = 0.2790$$

$$f\phi_{11}(f) = (a + b)^{-2} (b^2 A(1) + a^2 A(2) + 2ab A(3))$$

(eqn. A IV.14)

$$\begin{aligned}
 &= 0.2790 (0.6810 \times 250 + 1.141 \times 80.7 + 1.763 \times 86.2) \\
 &= 116 \text{ cm}^2 \text{sec}^{-2}
 \end{aligned}$$

$$f\phi_{33}(f) = (a + b)^{-2} (A(1) + A(2) - 2A(3)) \quad (\text{eqn. A IV.14})$$

$$\begin{aligned}
 &= 0.2790 (.250 + 80.7 - 2 \times 86.2) \\
 &= 44.1 \text{ cm}^2 \text{sec}^{-2}
 \end{aligned}$$

$$\begin{aligned}
 f\phi_{13}(f) &= (a + b)^{-2}(-bA(1) + aA(2) + (b-a)A(3)) \text{ (eqns. A IV.14)} \\
 &= 0.2790(-0.8252 \times 250 + 1.068 \times 80.7 - 0.2428 \times 86.2) \\
 &= -39.1 \text{ cm}^2 \text{sec}^{-2}
 \end{aligned}$$

$$f = \frac{FF}{S} = \frac{4.0}{16} = 0.25 \text{ Hz}$$

$$k = \frac{2 \times \pi \times 0.25}{289} = 0.00544 \text{ cm}^{-1}$$

$$kx_3 = 0.00544 \times 192 = 1.04$$

$$\phi_{11}(k) = \frac{f\phi_{11}(f)}{k} = \frac{116}{0.00544} = 2.13 \times 10^4 \text{ cm}^3 \text{sec}^{-2} \text{ (eqn. A IV.15)}$$

$$\begin{aligned}
 \phi_{33}(k) &= \frac{f\phi_{33}(f)}{k} = \frac{44.1}{0.00544} = 8.12 \times 10^3 \text{ cm}^3 \text{sec}^{-2} \\
 &\text{ (eqn. A IV.15)}
 \end{aligned}$$

$$\begin{aligned}
 \phi_{13}(k) &= \frac{f\phi_{13}(f)}{k} = \frac{-39.3}{0.00544} = -7.24 \times 10^3 \text{ cm}^3 \text{sec}^{-2} \\
 &\text{ (eqn. A IV.15)}
 \end{aligned}$$

$$R_{13}(f) = \frac{f\phi_{13}(f)}{[f\phi_{11}(f) \times f\phi_{33}(f)]^{\frac{1}{2}}} = \frac{-39.3}{(116 \times 44.1)^{\frac{1}{2}}} = -0.551$$

$$\log f = 0.620$$

$$\log k = 2.264$$

$$\log kx_3 = 0.146$$

$$\log \phi_{11}(k) = 4.328$$

$$\log \phi_{33}(k) = 3.910$$

$$\log |\phi_{13}(k)| = 3.860$$

APPENDIX VI: THE ERRORS TO BE EXPECTED IN NUMERICAL RESULTS

In estimating errors, the standard error (σ) was calculated where sufficient data existed to allow this estimate to be made. For a given expression involving several independent factors, the standard errors were treated normally i.e. $\sigma^2 = \sum_{i=1}^N \sigma_i^2$. In such cases the error quoted is expressed as 2σ , which corresponds to the 95% confidence level.

When standard errors are not available, an error equal to about 2σ (or greater) was estimated. This is called a "maximum error", and expressed as a percentage of the mean value. The "maximum error" is often estimated by halving the maximum observed range (which is expected to be larger than 4σ).

For an expression which contained many factors, each of which had a "maximum (percentage) error", the total error for the expression was obtained by simply adding all the "maximum (percentage) errors" estimated for the factors. The final estimate of error should thus have a probability level in excess of 95%. In other words, they are (often substantial) overestimates of the error at the 95% confidence level.

A. Mean Wind Data

The mean wind data is described in Section IV A and discussed in Section V A.

Calibrations of the cup anemometers followed the method outlined by Hamblin (1965). Calibration curves relate the counts per minute of each individual cup assembly, to the mean velocity measured in the wind tunnel. The standard error in estimates of wind speed is less than 1%. For most runs, individual cup counters were read once in every 7 minutes. The averages from different cups over multiples of 7 minutes overlapped by less than 4 minutes with each other and with the run duration. This procedure is expected to contribute a "maximum error" less than about 2% of the mean wind. The maximum error of the estimate of the difference in speed between cups is taken to be about 4% of the mean wind (see Section V A).

Since the wind tunnel in the Mechanical Engineering Department was used to calibrate the cup anemometers (and also the Flow Corporation tunnel), a systematic error of about 1% appears in the wind speeds determined in that wind tunnel.

B. Calibration of the Flow Corporation Wind Tunnel

Calibration of the Flow Corporation wind tunnel is described in Appendix I, Section A.

The constant K in the "velocity equation" (eqn. A 1.1) relating the mean wind speed in the test section to the corrected manometer pressure difference readings, was

determined to be 0.5606. The standard error (of 31 paired values) was 0.0179 or 3.2%. The accuracy of reading the quantities in eqn. A 1.1, introduces an additional standard error of less than 1.5%. The standard error in one mean wind determination in the wind tunnel is thus less than 3.5%. The systematic error at the 95% confidence level is thus less than 7%.

C. U-Wire Data

The U-wire data is described in Section IV B and discussed in Section V B. Calibration procedures are outlined in Appendix I, and analysis procedures in Appendix IV.

King's Law relates the hot wire (balance) current I to the measured velocity U , by the relationship

$$I^2 = A + B\sqrt{U} \quad (\text{A } 1.12)$$

Measured differences in B between two successive calibrations of the same wire showed that B would be known to about 5% at the worst.

The equation used to determine spectral values $f\phi_{11}(f)$ at each frequency analyzed, is given by

$$f\phi_{11}(f) = \frac{C(XK)}{XA} \left[\frac{X}{T} - \frac{XN}{TN} \right] \quad (\text{A } IV.9)$$

where

$$C = \left[\frac{i}{m_s} \times \frac{1}{I'} \right]^2 \quad \text{and} \quad I' = \frac{B}{4 I \sqrt{U}} \quad (\text{A 1.11}), (\text{A 1.13})$$

The maximum percentage error in I' is tabulated below:

B	5. % (from eqn. A 1.12)
I	0.5 (for field measurements)
U (from cup anemometers)	2. (for field measurements)

Maximum total error in $I' = 7.5\%$ (eqn. A 1.13).

The systematic error in B would be 3.5% at the 95% confidence level.

.....

The maximum percentage error in C is tabulated below:

i	0.5 %
m_s	3. (each measured from 10 or more consecutive square waves)
I'	7.5

Total maximum error in C = $2 \times 11. \% = 22\%$ (eqn. A 1.11)

The maximum error in the calibration factor K (eqn. A 1.11) is thus about 11%.

.....

The maximum percentage error in the individual spectral values of $f\phi_{11}$ is tabulated below:

(XK)	2%	(see Section A IV.C)
(XA)	2	(see Section A IV.C)
(X/T)	1	(see Section A IV.C)
C	22	

Total maximum error in $f\phi_{||}(f) = 27\%$.

The systematic errors in $f\phi_{||}(f)$ are 7% in a given run due to mean velocity measurements, plus 4% due to errors in using the Mechanical Engineering wind tunnel.

The maximum error in $k = 2\pi f/U$ is 4%, so that the total maximum error in values of $\phi_{||}(k) = f\phi_{||}(f)/k$ is about 31%. These maximum errors in $f\phi_{||}(f)$ and $\phi_{||}(k)$ thus roughly conform with the differences obtained by comparing cup anemometer and hot wire spectral values (see Section V B.1).

D. X-Wire Data

X-wire data is described in Section IV B and discussed in Section V B. Procedures used to calibrate X-wire probes, and to measure their wire coefficients b' , are outlined in Appendix I. Analysis techniques are outlined in Appendix IV.

The largest difference in the constant B of King's Law (eqn. A 1.12) was found to be 3%, for a wire which was calibrated and recalibrated within the same day.

The value of the wire coefficients b'_i for each wire is

obtained from the equation

$$b'_i = \frac{IK}{KU} \cdot \left(\frac{\Delta E}{\Delta \theta} \right) = \frac{i}{m_s} \cdot \frac{4I}{B} \cdot \frac{1}{\sqrt{U}} \cdot \left(\frac{\Delta E}{\Delta \theta} \right) \quad (\text{A 1.14})$$

which is obtained by substituting eqns. A 1.11 and A 1.13 into eqn. A 1.14. The maximum percentage range of random errors in each term is tabulated below. Note that no systematic errors appear, since b' measures the relative response to two velocity components.

i	0.5%
m_s	2. (estimated from standard error for 10 or more square waves)
I	0.5 (wire current in field use)
B	3. (using calibration with 15 or points for King's Law calibration)
$\sqrt{U}$	1. (using 2 or more determinations per wire calibration)
$\frac{\Delta E}{\Delta \theta_i}$	3. (from 20 or 30 readings at each θ_i)

Total maximum error in $b'_i = 10. \%$

b' was calculated from a least-square line or quadratic function fitted to 10 estimates of the mean b'_i of 20 or 30 readings at 10 different values of θ_i ($\pm 7^\circ$ was also included with the other angles mentioned in Section A 1.C.4) each of which has maximum error 10%. The maximum error in b' is thus $10/\sqrt{10}$, or less than $\pm 3.2\%$.

In order to determine the maximum errors in the spectral values $f\phi_{ij}$, computer calculations were made using the analog data for the run 1448-1525/22/7/1965. Four separate calculations were made to study the effects of errors in various constants used in eqns. A IV.13 to .15. The calculations were to study the effect of:

- (1) changing the calibration factor (K) of one wire by 11% equal to the estimated maximum error,
- (2) changing the wire coefficient (b') of one wire by 3% equal to the estimated maximum error,
- (3) rotating one wire of the X-array by 3° , and
- (4) rotating the whole X-array by 3° .

The results are summarized in Table A IV.1. The calculations showed that the percentage error introduced by the above four effects were frequency dependent. As a result, the estimated errors were divided into three rough groups according to the following convention. The letter L covers errors in the frequency range from 0.01 to 0.05 Hz; the letter M, those in the range from 0.05 to 1 Hz, and the letter H, those in the range from 1 Hz upward. The average maximum percentage errors in the spectral values $f\phi_{ij}$, and in the ratio ϕ_{33}/ϕ_{11} are outlined in the table below.

As noted in Section A I.C.3, it was felt that the X-wire array was set up on the mast at low tide within an angle of

1° at the worst. This angle was then doubled to include any extra tilting of the instrument mast due to tidal streams, and an extra degree added as a further margin for error.

It may be noted from the table that the proper alignment of the X-array in the field is quite crucial, since a tilt of the array of 3° can produce large changes in spectral levels of the stress. It is believed, however, that this is an overestimation, since a tilt of the mast of $\pm 2^\circ$ was not noticed, but would be easily observed visually.

Table A VI.1: Maximum Percentage Errors in Spectral Values:

Effect			Percentage		Errors		In:
			$f\phi_{11}$	$f\phi_{33}$	$f\phi_{13}$		ϕ_{33}/ϕ_{11}
(1) 11% change in K	L		16	22	65		-
	M		16	15	38		4
	H		16	10	65		5
(2) 3% change in b'	L		1	5	0		-
	M		1	5	3		5
	H		1	5	8		5
(3) Tilting one wire 3°	L		5	8	30		-
	M		5	7	15		1
	H		5	6	35		1
(4) Tilting X-array 3°	L		5	0	65		-
	M		6	4	35		7
	H		8	7	70		7

From Table A VI.1, it can be seen that spectral values of $f\phi_{11}$ are affected little by errors, and the ratio ϕ_{33}/ϕ_{11} least of all. Spectral values of $f\phi_{13}$ are most sensitive to large errors; however no change in sign (from negative to

positive) was produced at high frequencies.

The systematic errors in the spectral values $f\phi_{ij}$ are as before: $\pm 7\%$ due to the calibration of the Flow Corporation wind tunnel, and $\pm 1\%$ due to errors in the speed determinations in the wind tunnel in the Mechanical Engineering Department.

E. Maximum Errors in Determination of $u^*{}^2$

In estimating $u^*{}^2$ directly, the integral II A 1.1 is obtained from about 25 measured values of $f\phi_{13}$. Since most of the contribution to the kinematic stress ($u^*{}^2$) occurs in the frequency range from about 0.05 to 1 Hz (the range labelled M in Table A VI.1), it is reasonable to consider the errors in that frequency range to be representative of the errors encountered in the estimates of the stress $u^*{}^2$. The errors estimated in the table are maximum estimates, which doubtlessly overestimate the errors one normally expects to encounter. Also, the care exercised in setting up the probes and calibrating them, and self-consistency of the results described in Section IV, points toward errors in $u^*{}^2$ which are smaller than that indicated by a simple summing of the errors introduced by maximum errors in the K's, the b's, and the angular tilt of the whole X-array. Taking these considerations into account, it seems then, not too unreasonable to consider values of $u^*{}^2$ to be known say to

within about $\pm 50\%$.

For the logarithmic profile, the value of the mean velocity U is available within a maximum error of 4% at each cup. The height of the bottom cup is known to within ± 5 cm. This is also the maximum error in an individual height, but the distance between cups is known to ± 1 cm. A best-fit line is drawn through the experimental points, and the greatest and least slopes are drawn within the limits of error of U (4%) and x_3 (± 5 cm). The maximum difference in slope of lines drawn visually to fit the points, in the worst cases in which the points exhibited a large amount of scatter, is of the order of 35% . For the profiles with less scatter, differences in slope are of the order of 25% as an appropriate upper limit. This gives the total range of estimation of u_*^2 (from the logarithmic profile), of about 80% for the worst cases, and 60% for typical cases. This is equivalent to maximum errors of 40% , and 30% respectively.

For the runs with non-linear profiles, the velocity differences between cups 2 and 3 from the bottom were used to estimate the slope $dU/d(\log x_3)$ of eqn. 11 C 1.1. Runs which had very small slopes in the wind profile at this height (see run 1448-1525/22/7/1965 for example), give differences in slope of a factor of two to five, making estimates of u_*^2 completely unreliable.

The maximum error in S (eqn. IV C.1) determined from the $\phi_{11}(k)$ spectrum in the inertial subrange is about 30% from the average of 10 or more individual values, taking into account the errors in various terms of the eqn. II D1.1.

F. Maximum Errors in Determining the Drag Coefficient

Considering the discussion at the start of Section E above, an error of about 50% in determinations of u^2 would give an error of about 60% in the values of the drag coefficient determined directly from X-wire measurements. This does not appear unreasonable, since the observed range in C_D as noted from Figs. IV D.2 is about $\pm 50\%$ from the mean value 1.5×10^{-3} (i.e. near $U \sim 4$ m/sec, C_D takes on values from about 1×10^{-3} to about 2×10^{-3}).

The indirect estimates of the drag coefficient from linear wind profiles have errors of 48% to 38%. Use of non-linear profiles gives estimates of drag coefficients which can differ by factors of about 4 to 25.

The indirect estimates of the drag coefficient from measurements of the $\phi_{11}(k)$ spectra in the inertial subrange, have a standard error of 19% and an error of 38% at the 95% confidence level.

FIGURE CAPTIONS FOR APPENDICES

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of X-Wire Signals.....A78

$R = 24.0$ in

AREAL REDUCTION RATIO 11.9 : 1

FIG. A1.1 ENTRANCE FOR FLOW CORP. WIND TUNNEL

SCALING FACTOR 0.395

X (in)	Y (in)
0	1.58
.91	1.59
1.90	1.63
2.39	1.67
2.68	1.70
2.98	1.75
3.28	1.84
3.48	1.88
3.57	1.92
3.67	1.97
3.77	2.02
3.87	2.09
8.10	6.37

FIG-A12 SCALED THROAT DESIGN FOR
FLOW CORP. WIND TUNNEL

- (a) PROBE BODY
- (b) PRONGS
- (c) SOLDER JOINT
- (d) WOLLASTON WIRE
- (e) ETCHED PORTION OF WIRE WITH SLIGHT BOW
DIAMETER 0.00075 cms

FIG. A1.3 U-WIRE PROBE

- (a) PROBE BODY
- (b) PRONGS
- (c) SOLDER JOINT
- (d) WOLLASTON WIRE
- (e) ETCHED PORTION OF WIRE WITH SLIGHT BOW
DIAMETER 0.00075 cms

FIG. A1.4 X-WIRE PROBE

PROBE X N°3

CALIBRATION DATE: SEPT. 13-14, 1965

FOR $\Delta\theta = 0^\circ$

[	$\overline{b'} = 1.058$	INNER WIRE
	$\overline{b'} = 0.825$	OUTER WIRE

FIG. A1.6 VALUE OF $\overline{b'}$ AS A FUNCTION OF $\Delta\theta$

FIG. A II.1 BLOCK DIAGRAM OF ONE. CHANNEL
 OF HOT WIRE CIRCUIT

$$\left(\frac{\text{Volts Output}}{\text{Volts Input}} \right)$$

160

140

120

100

80

60

40

20

0

AMPLIFIER GAINS: $\times 10$
COMPENSATION FRE-
QUENCIES: ~ 270 Hz.

0

100

200

300

400

500

Frequency (sec^{-1})

FIG. A II. 2 THE AVERAGE GAIN OF THE THREE
ACCUDATA V D.C. AMPLIFIERS WITH COMPENSATION

FIG. AIII.1 : BLOCK DIAGRAM OF INSTRUMENT MAST CUP ANEMOMETER AND ROVER CUP ANEMOMETER ELECTRONICS

FIG.AIV.2: BLOCK DIAGRAM OF ANALOG SYSTEM FOR ANALYZING U-WIRE SIGNALS

FIG.AIV.3 SQUARED VOLTAGE RESPONSE OF PREAMPLIFIERS AT HIGH FREQUENCIES, RELATIVE TO RESPONSE AT LOW FREQUENCIES

FIG. A IV. 4 BLOCK DIAGRAM OF ANALOG SPECTRAL ANALYSIS OF X- WIRE SIGNALS
(FILTERS 1 AND 2 WERE PHASE - MATCHED)