WORLD-WIDE CHANGES IN THE GEOMAGNETIC FIELD

by

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1. Nishida, A. and N. Fukushima. Three dimensional consideration for current-system of geomagnetic variations (II) Sq-field, Report of Ionosphere and Space Research in Japan 13, 273-282, 1959.

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ABSTRACT

The geomagnetic field is found to change quite frequently on a world-wide scale. In the three months' period near sunspot maximum, such changes are found on 90 per cent of all days. Most of these changes are not registered either as sudden commencements or as sudden impulses, and are tentatively called in this thesis 'world-wide changes'. The frequent occurrence of world-wide changes seems to be consistent with the idea that world-wide features of the geomagnetic field are always influenced by a permanently flowing corpuscular stream from the sun. The physical state of the corpuscular stream may be as variable as that of the solar atmosphere, and sudden changes in it will give rise to sudden, world-wide changes in the geomagnetic field.

The morphology of world-wide changes is studied, and the form of the change, the distribution of magnitude and the mode of spreading over the earth are clarified. It is found that world-wide changes can be classified into two groups according to the sign of the main part of the change which appears all over the world. Those with an increase in the total force are called positive changes and those with a decrease are called negative changes. Except for the sign of the change, negative changes are morphologically indentical to positive changes. Since the morphology of sudden commencements and sudden impulses is the same as that of world-wide changes, they must be produced by a common mechanism, and an explanation of negative changes is a new, fundamental requirement imposed upon any theory of these changes.

The observed change in the geomagnetic field may originate at the magnetospheric boundary where the solar corpuscular stream interacts with the geomagnetic field. The change may be modulated by the screening effect of the ionosphere before it is observed at ground level. Although this effect has been shown to be negligible for changes with a time scale of the order of world-wide changes, incorrect assumptions have been made in existing theories. More accurate calculations show that this effect is actually significant for a certain mode of the incident field.

From these results, a physical picture is obtained of events taking place at the magnetospheric boundary at the time of world-wide changes, and the cause of such events is considered. It seems that positive and negative changes correspond to a sudden increase and a sudden decrease respectively in the intensity of the solar corpuscular stream. The main part of the change which is observed all over the world may result from the sudden change in the impact pressure of the stream on the magnetosphere, and the reverse change which precedes the main part of the change on the afternoon side of the Earth may be due to a sudden change in the shear stress exerted on the matnetosphere as the stream passes by.

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CHAPTER I

INTRODUCTION

It is known that there are sudden changes in the geomagnetic field which appear almost simultaneously all over the world with certain features in common. When such a change marks the beginning of an interval of increased activity, it is called a sudden commencement, which is classified as ssc or ssc* depending on its detailed features. (The present official classification of sudden commencements seems to present certain difficulties. These are discussed in Appendix I). Chapman and Ferraro (1931) related a sudden commencement to the approach of a solar corpuscular stream towards the earth. The existence of a solar corpuscular stream was presented by them as a hypothesis, but later observations have testified the correctness of this hypothesis. These observations are: (1) The existence of a solar corpuscular stream at about 1 A.U. was inferred by Biermann (1957) from observations of comet tails. He showed that the form and the degree of ionization of a comet tail could not be explained solely by electromagnetic radiation from the sun and that at 1 A.U. there might exist corpuscular radiation from the sun with a density of 10^2 cm^{-3} and a velocity of 500 km sec^{-1} when the sun is quiet, and with a density of 10^4 cm^{-3} and a velocity of 1500 km sec^{-1} when the sun is active. (2) Those sudden commencements which are preceded by an increase in the absorption of radio waves over the polar cap can be correlated with solar flares accompanied by type IV radio bursts (Sinno and Hakura, 1958), and (3) Direct observation of a corpuscular stream has been obtained by space vehicles, and an increase in its intensity has been observed at the time of a sudden commencement (IGY Bulletin, 1962).

The main feature of a sudden commencement is an increase in the horizontal component of the geomagnetic field. This occurs with a time scale of the order of a few minutes and appears all over the world. Chapman and Ferraro originally demonstrated that this can be produced when a highly ionized corpuscular stream is propagated from the sun across the empty space between the sun and earth. The additional magnetic field due to the induction current produced on the surface of the stream by the intrusion of the highly conductive corpuscular stream into the geomagnetic field can be represented approximately, outside the stream, by an image of the geomagnetic field with respect to the surface of the stream, thus resulting in an increase in the horizontal component of the magnetic field as observed at the surface of the earth. This theory has been modified since it has now been shown that the space inside the geomagnetic field is filled with sufficient plasma to be a good electrical conductor (Storey, 1953: Pope, 1961). It is now considered that the geomagnetic field is always confined to a region of finite dimensions by the impact pressure of a continuously flowing corpuscular stream. The space occupied by the geomagnetic field is compressed when the impact pressure of the corpuscular stream is increased. The resulting increase in the geomagnetic field is transmitted to the surface of the earth as a magnetohydrodynamic wave (Dungey, 1954; Hoyle, 1956; Parker, 1958; Obayashi, 1958; Dessler and Parker, 1959; Francis, Green and Dessler, 1959; Piddington, 1960; Dessler, Francis and Parker, 1960; and Beard, 1960, 1962).

The overall increase of the horizontal component (which will be called the main change) is accompanied, in particular regions of the earth determined by latitude and local time, by two kinds of decreases in the horizontal component (Nagata, 1952; Oguti, 1956; Obayashi and Jacobs, 1957; Abe, 1959; and Matsushita, 1960). The first type (which will be called a preceding reverse change) precedes the main change and occurs mostly on the afternoon side of the earth. The second type (which will be called a following reverse change) follows the main change in high latitude regions in the morning hours. These have been attributed largely to electric currents flowing in the ionosphere, and an atmospheric dynamo action (Obayashi and Jacobs, 1957) or the separate injection of positive and negative particles into the ionosphere at high latitudes (Vestine and Kern, 1962) have been suggested as possible causes. Singer (1957), on the other hand, considered as a possible cause of the first type of decrease an extension of the auroral zone magnetic tubes of force by the intrusion of the corpuscular stream into the geomagnetic field. Wilson and Sugiura (1961) studied the sense of rotation of the horizontal magnetic vector instead of noting the change in the horizontal component, and attributed the resulting regularity in the sense of rotation to the incidence of a transverse magnetohydrodynamic wave due to the deformation of the magnetosphere by the solar corpuscular stream. So far, no general agreement has been reached concerning the mechanism responsible for these reverse changes.

When a sudden change appears almost simultaneously all over the world, but not followed by an interval of increased activity, it is called a sudden impulse. Except for the lack of association with an interval of increased activity, sudden impulses have the same characteristics as sudden commencements (Jackson, 1952; Ferraro, Parkinson and Unthank, 1951; Matsushita, 1962), and are attributed also to a solar corpuscular stream. The difference between sudden commencements and sudden impulses may result from the difference in the energy spectrum of the corpuscular stream.

* * * * *

Thus, two kinds of sudden, world-wide changes in the geomagnetic field (sudden commencements and sudden impulses) have been known and studied. In the geophysical and solar data of the Journal of Geophysical Research several sudden commencements and sudden impulses are listed each month from reports obtained from stations distributed all over the world. From the viewpoint of theories of these changes summarized above, the reported number of occurrences seems to be rather too small. Through the medium of the continuously flowing solar corpuscular stream, world-wide features of the geomagnetic field are always related, with some time lag, to the physical state of the sun. Since the physical state of the solar atmosphere is known to be quite variable, it seems reasonable to expect world-wide changes in the geomagnetic field to occur quite frequently.

Hence magnetograms have been examined to find hitherto unnoticed changes recorded almost simultaneously all over the world. Data were obtained over a three-month period during the IGY from an extensive network of stations. Results are presented in Chapter II. It was found, as was expected, that world-wide changes in the geomagnetic field take place almost every day, at least around sunspot maximum. Sometimes it was found that magnetograms from all over the world showed similar traces for several hours.
These changes are morphologically identical to sudden commencements (regardless of the activity in the following interval) and sudden impulses. However, many of these

changes are not listed as sudden commencements or sudden impulses by a single station. These have been left unnoticed, possibly because their intensities are small (mostly less than a few tens of gammas in middle and low latitudes), and because they do not give a strong impression of an impulse since they occur so frequently that they seldom appear isolated. Partly to distinguish these newly-found changes from the already known and confusingly defined sudden impulses and partly to stress their morphological, instead of their configurational, characteristics, these changes will be called in this thesis 'world-wide' changes.

* * * * *

The frequent occurrence of world-wide changes supports the idea that sudden changes in the geomagnetic field appear on a world-wide scale as a result of a sudden and sharp change in the physical state of the solar corpuscular stream. Moreover, the present study of world-wide changes has revealed another fact that imposes a new requirement on any theory of all types of world-wide changes. This fact is that not only a sudden increase in the horizontal component, but also a sudden decrease in it, is found to occur on a world-wide scale. Changes will be called 'positive' or 'negative' according to the sign of the main change. Except for the sign of the overall change, negative changes are morphologically identical to positive changes. In regions where reverse changes appear as a decrease in the horizontal component in association with positive changes, an increase in it is found in the case of negative changes. This indicates that positive and negative changes are produced by a mechanism which can work equally well for both senses of the change. Since sudden commencements, sudden impulses and world-wide changes show identical features (regardless of the activity in the following interval),these must originate from the same mechanism. Thus any theory of sudden commencements and sudden impulses must also be able to explain world-wide changes, both positive and negative.

Suggested theories of sudden commencements are now examined from this point of view. Two regions have been considered as possible places where the change in the electromagnetic field originates. These are the magnetospheric boundary and the ionosphere. In cases where the former is assumed, the change in the electromagnetic field is considered to result directly from the interaction between the solar corpuscular stream and the magnetosphere. This idea is adopted in all theories of the main change but only by Wilson and Sugiura (1961) for the preceding reverse change. Mechanisms described by them may be applied to negative changes merely by replacing an increase by a decrease in the intensity of the solar corpuscular stream; but in theories where the ionosphere is considered as the position of the source of the change in the electromagnetic field, an extension to negative changes involves a problem. An ionospheric source is assumed in most theories of the preceding reverse change and the following reverse change, and according to them an essential role is played by the intrusion of an aggregate of charged particles into the ionosphere at high latitudes. This aggregate of charged particles is assigned certain properties which vary depending on the author. Thus it is polarized in a certain way (Vestine and Kern, 1962), or it is able to create a sufficient increase in the ionospheric conductivity (Obayashi and Jacobs, 1957), or it is intense enough to distort the configuration of the auroral zone lines of force (Singer, 1957). With these properties, this aggregate of charged particles has been shown to produce the ionospheric current system which gives rise to the observed change in the geomagnetic field. Hence to explain a negative change by an extension of this type of theory, one of the following mechanisms must be proved to be possible. These are: (1) An outflow, instead of an inflow, of particles from the ionosphere to the outside; (2) The creation of a polarization opposite to that required for sudden commencements; and (3) The permanent presence of the intrusion of charged particles whose intensity experiences a sudden decrease, as well as a sudden increase. Of these mechanisms, the first may be impossible since such an acceleration of charged particles in the ionosphere is hard to understand. The second and third may be possible, but they imply the presence of a complicated mechanism which has not so far been detected by any observations. Hence the magnetospheric boundary, rather than the ionosphere, seems to be more promising as the position of the source of the observed changes in the electromagnetic field.

* * * * *

The observed changes in the geomagnetic field are assumed to originate at the magnetospheric boundary. In this case, the field may be modulated during its propagation through the space between the source and the ground. To investigate the nature of its origin, therefore, it is necessary to eliminate the influence of the intermediate medium. It is particularly important to eliminate any modulation by the ionosphere - the lowest part of the upper atmosphere. For phenomena with a time scale of the order of world-wide changes (1 to 10^2 sec), the medium of the ionosphere behaves as a metallic conductor with high anisotropic conductivity, while the upper part of the atmosphere is the medium for magnetohydrodynamic wave propagation, as shown in Appendix II. Hence, by eliminating modulation by the ionosphere, that is, the effect of ionospheric screening, direct information about the origin of the observed changes in the geo-electromagnetic field can be obtained.

A general discussion of ionospheric screening is presented in Chapter III. The screening effect of the ionosphere has been discussed by various authors (Ashour and Price, 1948; Sugiura, 1949 and 1950; Watanabe, 1957; Francis and Karplus, 1960). They concluded that the screening effect is negligible for changes with a time scale longer than about 1 sec, and thus for most of the phenomena of interest in geomagnetism, ionospheric screening need not be taken into account. Hence the observed field has been regarded as equivalent to that incident on the ionosphere.

However, these results are based on questionable approximations. In their estimation of the ionospheric screening effect, the observed field was represented either by (1) an electromagnetic wave (e.g. by Francis and Karplus) or (2) a magnetostatic field (e.g. by Sugiura). Neither is an exact representation of the observed field, since (1) the observed field varies horizontally with a characteristic scale of the order of 10^8 cm, which is far smaller than the wavelength of an electromagnetic wave with a period longer than 1 sec, and (2) as far as the observed field is time dependent, the displacement current is not exactly zero, and the field in the neutral atmosphere is not exactly magnetostatic. Therefore certain errors must be introduced by adopting these approximate representations. However, the above authors did not estimate these errors and verify the applicability of the approximations.

Hence an attempt is made to relate the observed field with the field incident on the ionosphere avoiding the above approximations. The general solution of Maxwell's equations near the highly conductive earth is used to represent the observed field. It is found that the degree of ionospheric screening depends not only on the scale in time, but also on the scale in space and on the mode of the incident field. In the case when the time scale is about $10 - 10^2$ sec and the scale of the horizontal distribution on the ground is about 10^8 cm, as they are for world-wide changes, the results show that (1) for a certain mode screening may not be effective and the observed field is well approximated by the magnetostatic field, but (2) for another mode screening may have a significant effect. For this case a magnetostatic field is a poor approximation for the observed field although it may be impossible to detect the difference with the present accuracy of observations.

* * * * *

Using this result, the incident field corresponding to the main and preceding reverse changes is derived and a model of these changes is constructed in Chapter IV. It is found that for the main change ionospheric screening is not effective, and this part of the change may be explained by the compression (for a positive change) or by the expansion (for a negative change) of the magnetosphere due to a sudden change in the impact pressure of the solar corpuscular stream. But for the preceding reverse change, screening is significant and the incident field differs in direction by 90° from the observed field. This part of the change can be interpreted as a result of the deformation of the magnetosphere produced by the blowing of the lines of force towards the night side by the shear stress exerted by the solar corpuscular stream.

CHAPTER II

MORPHOLOGY OF WORLD-WIDE CHANGES

1. Data

Magnetograms were compared from several widely separated stations: Hartland (54.6°N, 79.0°) (geomagnetic coordinates are used throughout), Fredericksburg (49.6°N, 349.9°), Tbilisi (36.8°N, 122.0°), Honolulu (21.0°N, 266.4°), and Hermanus (33.3°S, 80.3°). Changes that were recorded almost simultaneously at all of them were selected. During the 3-month interval from April to June 1958, which is around sunspot maximum, at least 20 per cent of all 1-hour periods and at least 90 per cent of all days contain at least one such change. These are listed in Table 1.

For a study of the morphology of these changes, 24 of the larger ones were randomly selected. They are shown in Figure 1 by arrows on magnetograms from Honolulu. Magnetograms from 20 stations, most of which are uniformly distributed in middle- and low-latitude zones in the northern hemisphere as shown in Figure 2, were obtained from the IGY World Data Center. Although data were collected also from high-latitude stations, they could not be used in this analysis, because the phenomena studied were almost always obscured by local disturbances. The data confirm that

<u>Table 1. List of times of world-wide changes in the geomagnetic field from April to June 1958.</u> When more than one change was found during any one hour period, the largest has been tabulated. Negative changes are shown by writing the time in parenthesis.

Date	Time of Occurrence (GMT).
April 2	0230, 0540, 0750, (0820), 0910, (1010), (1250), 1320, (1430), 1510, 1650, 1740, 1810, (1930), 2030.
3	0330, (1820).
4	(1420), 2050, 2150.
5	(2140), 2220, 2310.
6	0000, 0520, 0820, 1100, (2320).
7	0540.
8	
9	1530, (1710),(2010), 2200, 2320.
10	0710, (0930).
11	0050, (0130), 0230, (0410), (0900), (1640), (1730), 1950, 2030, 2130.
12	(0400), (1010), 1440, 1720, 2040, (2300).
13	(0140), (0540), (0720), 1410, 1720, 1820.
14	0910, 1010, 1130, 1220, (1330), 1500, 1650, (1800), (2040), (2220), (2310).
15	
16	(1120), (1400).
17	
18	1910, 2210, (2330).
19	0500, 1540.

Table 1. (cont'd)

Date		Time of Occurrence (GMT)
April	20	
	21	
	22	1900, (2300).
	23	
	24	1120, 1830, 2300.
	25	0120, (0220), (0540), (1310), 2050.
	26	1250, 1350, (1440), 1530, (1610), 1710, 1820, 1910, 2110, 2230, 2340.
	27	1230, 1350, 1530.
	28	0030, (0740).
	29	0100
	30	
May	2	(1050), 1520, 1620.
	3	1730, 1840.
	4	
	5	
	6	0130, 0250.
	7	(0120), (0740), 1050.
	8	0140, (0120), 0220, 0500, 1250, 1350, 1800, (1930), 2240.
	9	0100, 0520, 0650, (0740), (0820), 1110, (1200), (1330), (1540).
	10	1340, 1650, 200, 2140.
	11	0100, 0200, 1510.
	12	(0140), 0430, (1520), 1700, (1750), 1810, 2010.

Table 1. (cont'd)

Date	Time of Occurrence (GMT)
May 13	0500, 2140, 2230, (2310).
14	(0020), 1720, 2130, (2220), 2300.
15	0220, (0930), 1010, 1320, 2050.
16	1300.
17	
18	1010.
19	
20	0030, 1100, 2310.
21	0100, 0320, (0410), 0500, 1730, 1850, (1930), 2200.
22	
23	0220.
24	
25	0310, (0650), 1040, (1530), 1620, (1730), (1830), 2000, 2300.
26	(0240), 0310, (0720), 1050, 1610, 2320.
27	0030, 1100, 1240, 1330, 1350, 1500, 1920, (2000), (2110).
28	
29	0420, (0810), (1040), (1350), 1520, 1720, (2330).
30	0110, 0820.
31	(1350), 1650.
June 2	0410, (0830), 1020, 1210, 1450, (1810).
3	0250.
4	

Table 1. (cont'd)

Date	Time of Occurrence (GMT)
June 5	0040, 0430, (0910), 2100.
6	0920, 1100, 1840, 1930, (2010).
7	0040, 1820, 2100.
8	1040, (1320), 1600, 1730, (1850), 1920, (2010), 2150, (2250), 2320.
9	0000, 0440, 0630, (1450), 1530, 1620, 1750, (1940), 2340.
10	0020, 1200, (1350), (2110).
11	0120, (1010), 1520, 1850.
12	0010, 1140, (1920), 2220.
13	0240, 1650.
14	1830, 2110.
15	0230, 0510, 0630, 0710, (0930), 1210, (1410), 1540.
16	0600, (1320).
17	2020, 2220.
18	0040, (0950), 1150, 1240, (1440), (1940), (2000), (2200), (2300), 2350.
19	0340, (0800), 0950, 1050, 1500, (1610), (1940), 2230.
20	(0240), (1130), (1320), 1820, 1900.
21	0210, (1130), (1200).
22	1010, 2020.
23	1040, 1630, 2300.
24	1050.
25	(1400), 1510, 2150.

Table 1. (cont'd)

Date	Time of Occurrence (GMT)
June 26	2130.
27	(2050).
28	0710, (0940), (1250), 1740, 1800, (1900), 2120.
29	

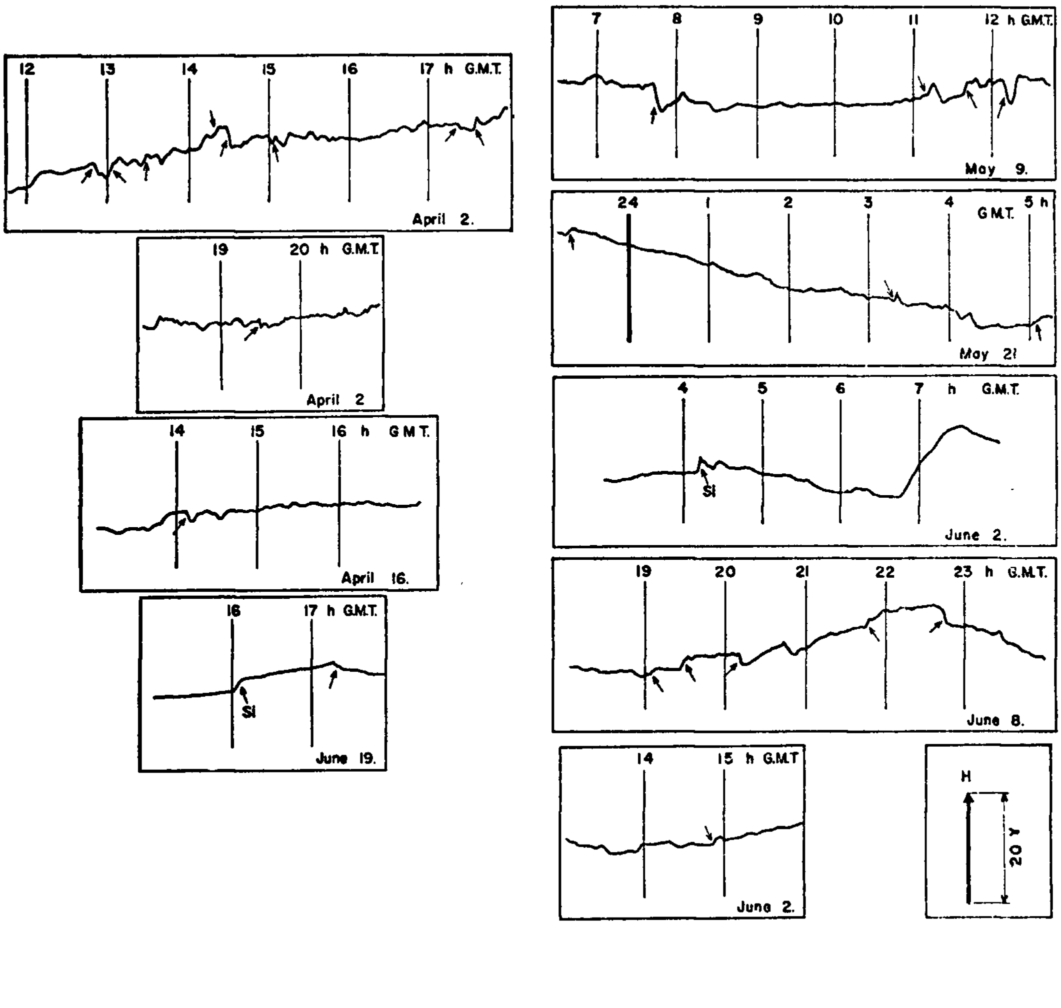


Figure 1. Examples of world-wide changes in the geomagnetic field (indicated by arrows) shown by the horizontal intensity magnetogram from Honolulu.

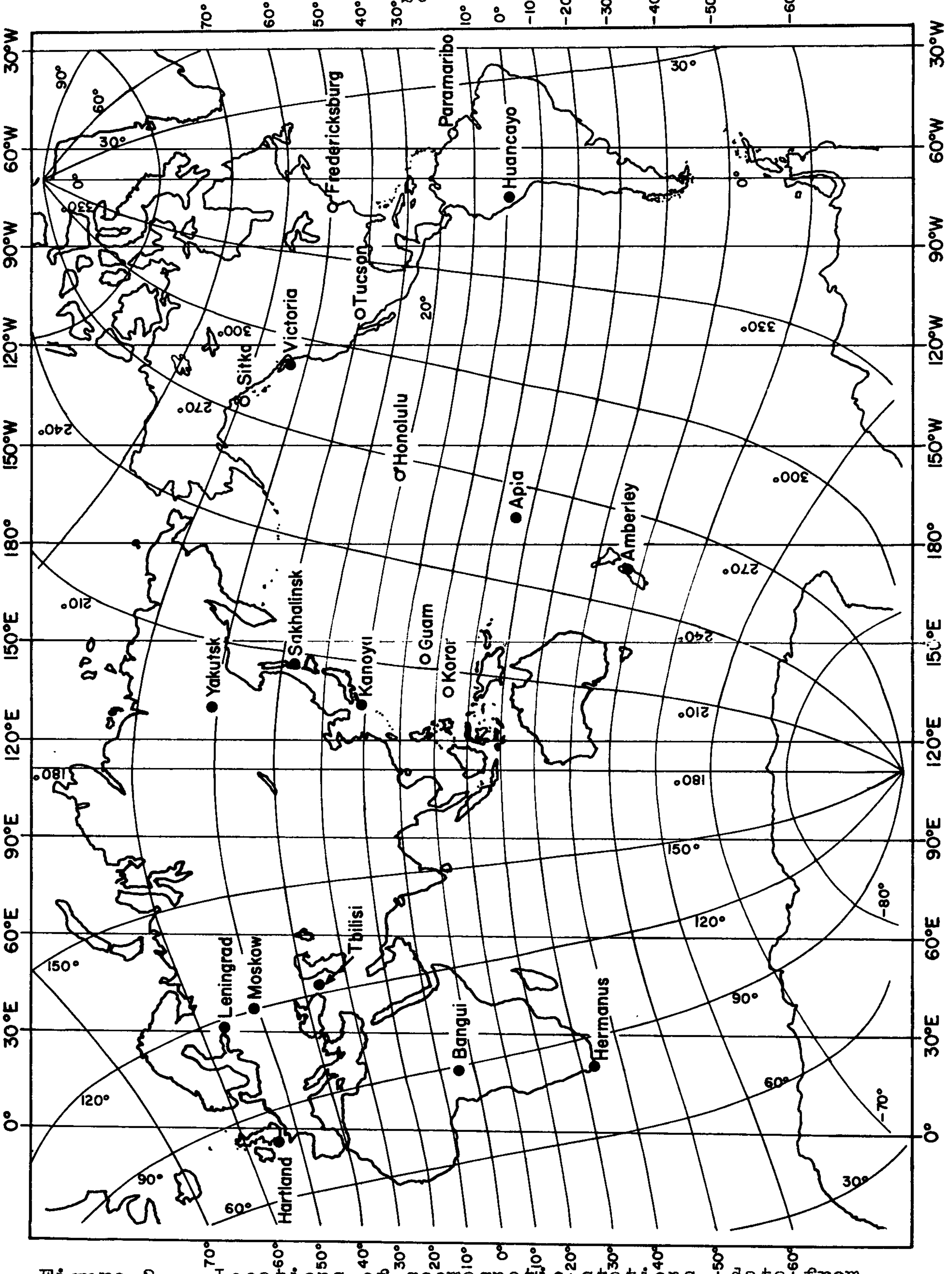


Figure 2. Locations of geomagnetic stations, data from which are used for analysis. From stations shown by closed dots, normal-run magnetograms are available; from those shown by open dots, normal- and rapid-run magnetograms are available.

these changes are of world-wide character, as can be seen by the examples in Figure 3. Among these changes are included those with a decrease as well as those with an increase in the horizontal intensity. The world-wide changes with a decrease in horizontal intensity are not the immediate recovery from an increase in it, like the recovery following a sudden impulse; although they mostly appear after an increase in the horizontal intensity, some tens of minutes usually pass between the two changes, and their magnitude frequently exceeds that of the preceding increase. Thus it seems proper to regard changes having a decrease in the horizontal component as independent phenomena. Changes having an increase in the horizontal intensity will be referred to as 'positive changes' and those with a decrease as 'negative changes'. Since the change in vertical intensity is far smaller than that in the horizontal intensity, these changes correspond to increases and decreases in the total force. Among the examples selected, 14 were positive and 10 were negative changes.

According to the Geomagnetic and Solar Data published in the Journal of Geophysical Research in 1958 and 1959, none of the selected cases were registered as a sudden commencement (abbreviated as sc) or a sudden impulse (abbreviated as si) by more than 5 stations, and, in fact, none of them preceded an interval of increased activity as an sc does, and only a few of them were impulse-shaped

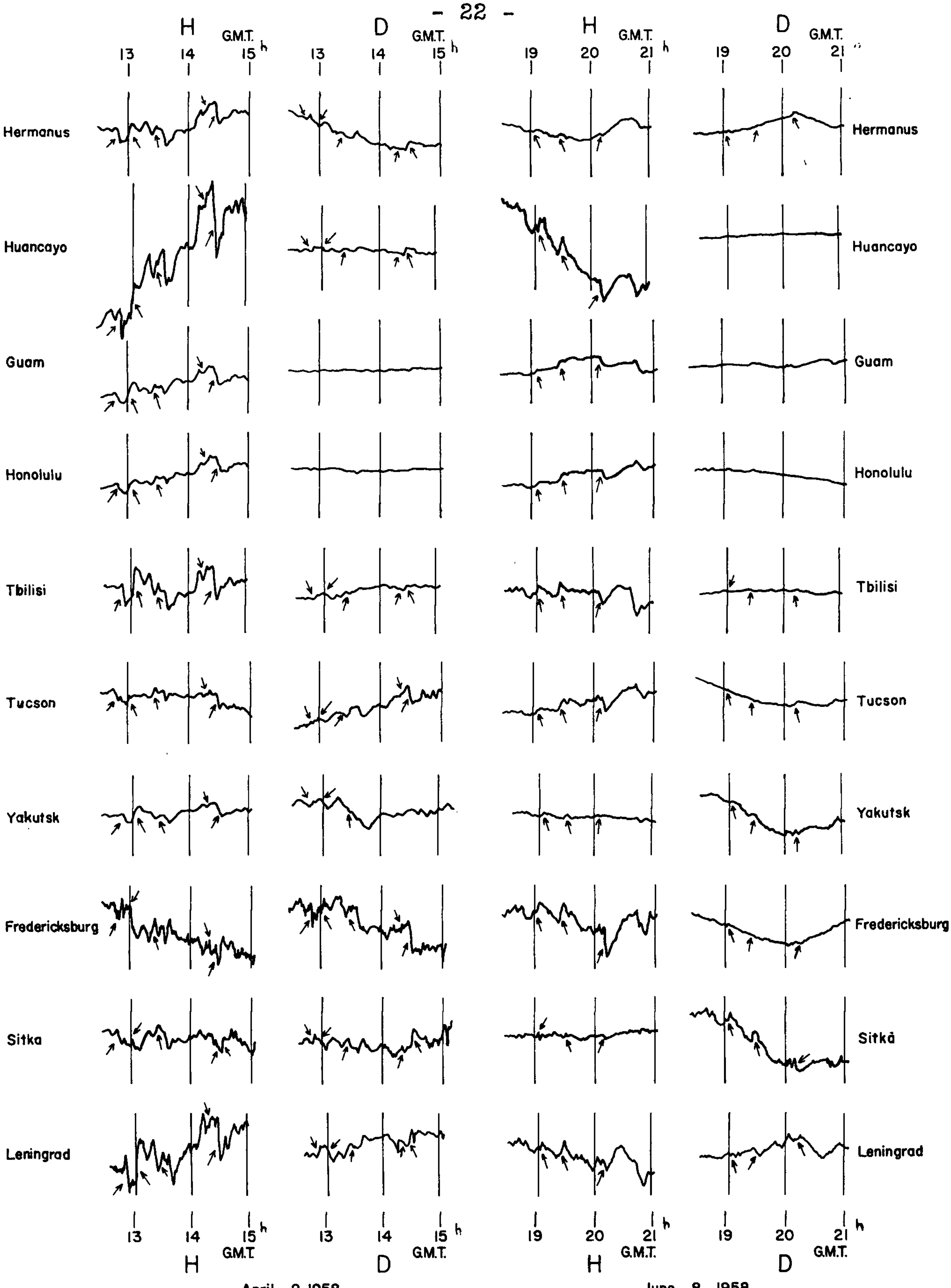


Figure 3. Comparison of magnetograms covering world-wide changes (indicated by arrows).

like an si. Only 3 of them occurred during the period of a magnetic storm that was reported by more than 3 stations. The average magnitude at 50°N of the change in the horizontal intensity of the selected cases is 16 γ . This is far smaller than changes appearing during a magnetic storm, and accordingly such an occurrence is seldom reflected in geomagnetic K indices.

For the sake of comparison, magnetograms were also obtained from the same stations at the time of 4 widely recognized sudden commencements during the same time interval; 1247 April 28, 1652 May 31, 0046 June 7, and 0713 June 28. Magnetograms were also obtained of 2 si's: 0408 June 2, and 1600 June 19 (only 3 si's were reported from more than 5 stations during this 3-month interval). Most of the data are normal-run magnetograms, but, from the 7 stations indicated by open dots instead of closed dots in Figure 2, rapid-run magnetograms were also available.

2. Form of the change

As can be seen in Figure 3, the traces of world-wide changes on the magnetograms of the horizontal intensity are similar in most parts of the world. At stations with geomagnetic latitude higher than about 40° at around 0800 LMT however (represented by Fredericksburg and Sitka in the record of April 2 and by Sitka in that of June 8), the form of the change differs from that at other stations.

Examination of rapid-run magnetograms shows that in this particular region the change is composed of two parts, as can be seen in Figure 4. First there appears a change having the same sign as that at all the stations, but immediately after this there is a change with opposite sign. Thus, in spite of the complexity in the trace of normal-run magnetograms in this part of the world, it is found that the horizontal intensity experiences a change in the same sense almost simultaneously once in any part of the world. Later, however, while this change continues in most parts of the world, another change, whose sign is opposite to that of the first, appears in a particular region.

The fact that the horizontal intensity changes once with the same sign on a world-wide scale distinguishes this phenomenon from other geomagnetic disturbances such as bays and solarflare effects, which are known to show changes of the same sign only in limited parts of the world. In the following description, the part of the change in which the horizontal intensity changes with the same sign all over the world will be called the 'main change'. The change that immediately follows the main change in some part of the world with its sign opposite to that of the main change will be called a 'following reverse change'. The over-all change is called 'positive' or 'negative' according to the sign of the main change. The regions where these following reverse changes are observed are found to be the same for both

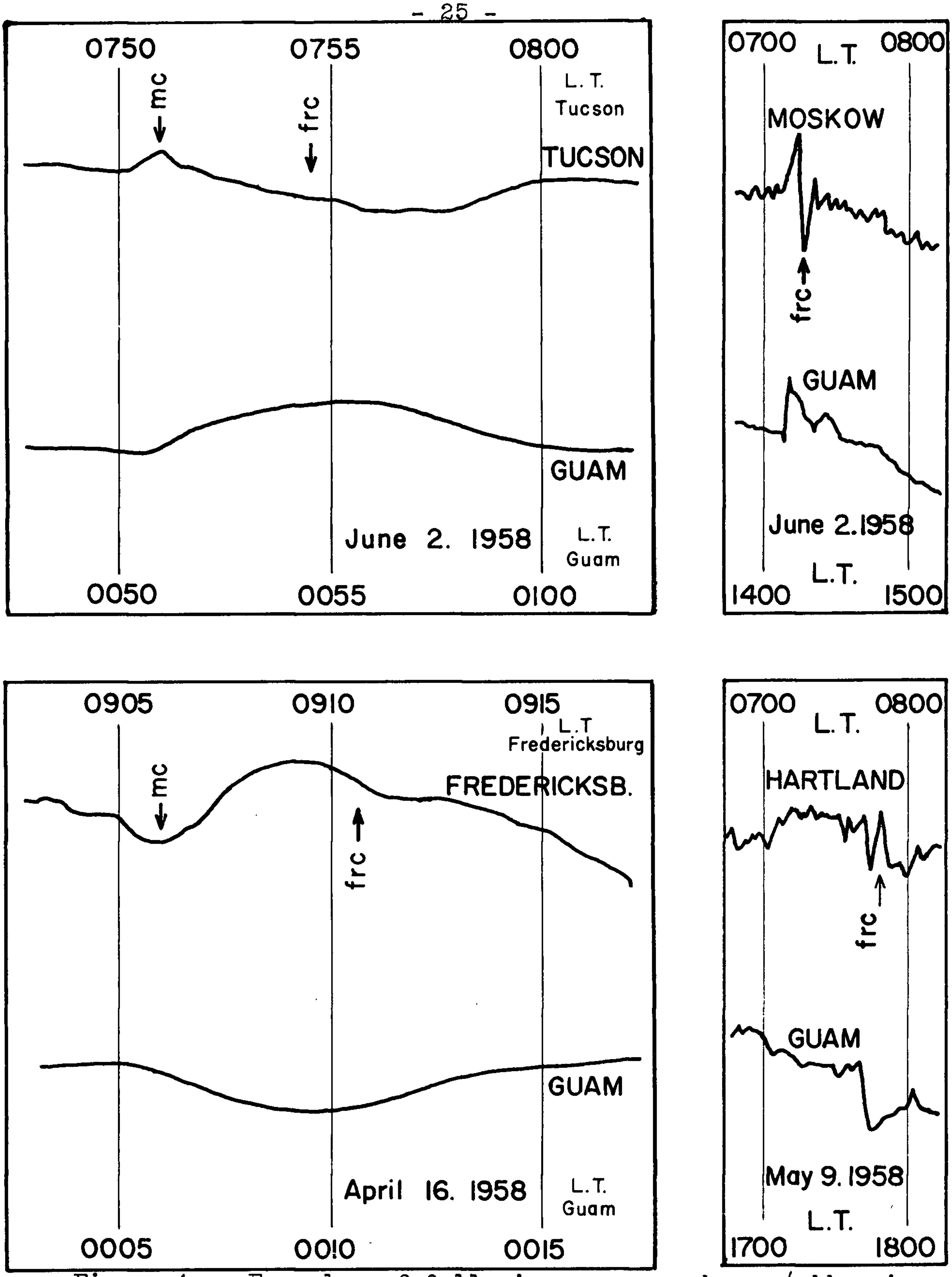


Figure 4. Examples of following reverse change (abbreviated to frc) recorded following main change (abbreviated to mc). Simultaneous records of Guam, where frc is never found, are attached as reference. Upper half,positive change; lower half, negative change.

positive and negative changes. They are a function of both local time and latitude as is shown in Figure 5. The duration of a main change when followed by a following reverse change ranges from a few to several minutes.

This type of reversal is known to occur in association with sc's (Oguti, 1956; Obayashi and Jacobs, 1957; and Matsushita, 1960). The region where this reverse change was recorded in 4 sample storms, which is also shown in Figure 5, is in good agreement with the results obtained by the authors mentioned above. For si's, Jackson (1952) gave some examples of this type of reverse change recorded in the same region, and the complexity of the form of the change, due to the existence of this reverse change, might explain the apparent minimum in the occurrence frequency of si's around 0800 LMT in middle and high latitudes, as observed by Ferraro, Parkinson, and Unthank (1951). A following reverse change is found in the same region also for 2 sample si's studied here. Figure 5 shows that the region of occurrence of the following reverse change for sc's is in good agreement with that for world-wide change analyzed here.

A reverse change of a different type is found to precede the main change on the afternoon side of the earth. As can be seen by the examples in Figure 6, this is a decrease in the horizontal intensity for a positive change and an increase in it for a negative change. It will be called here a 'preceding reverse change'. The preceding

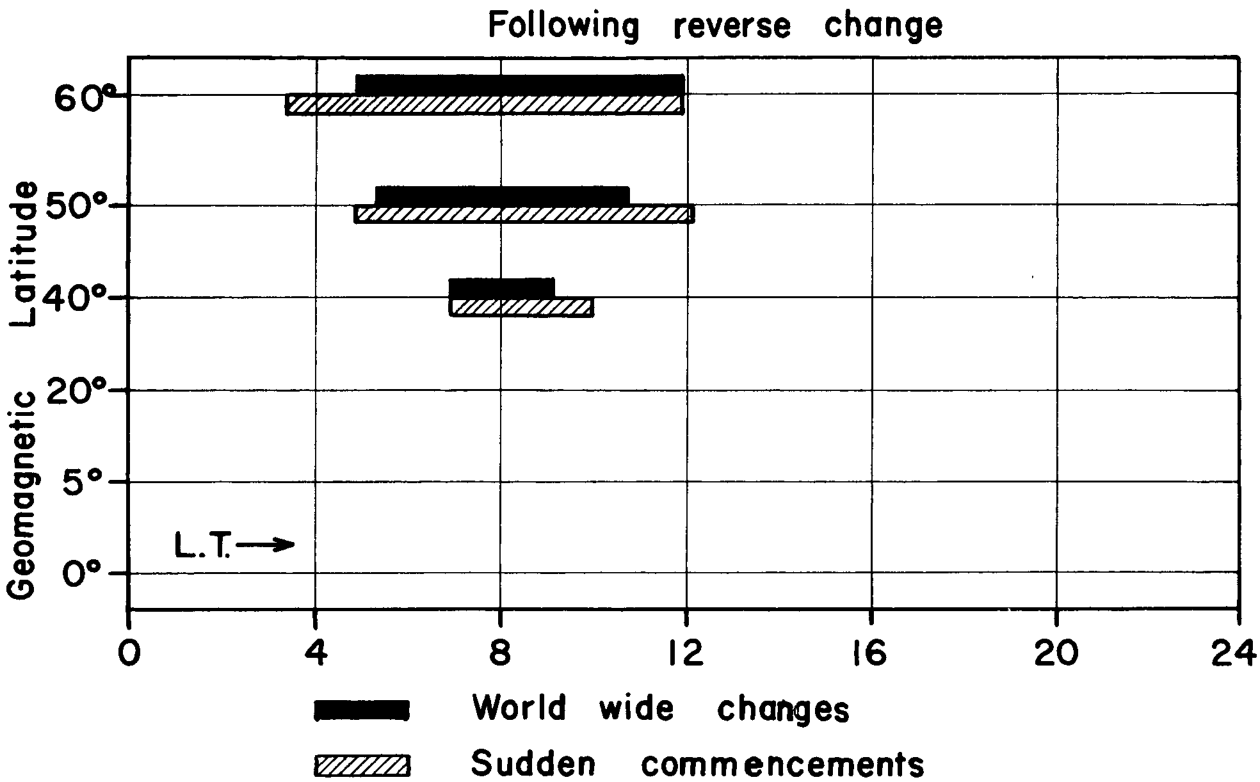


Figure 5. Region where following reverse change is found for world-wide changes and for sc's.

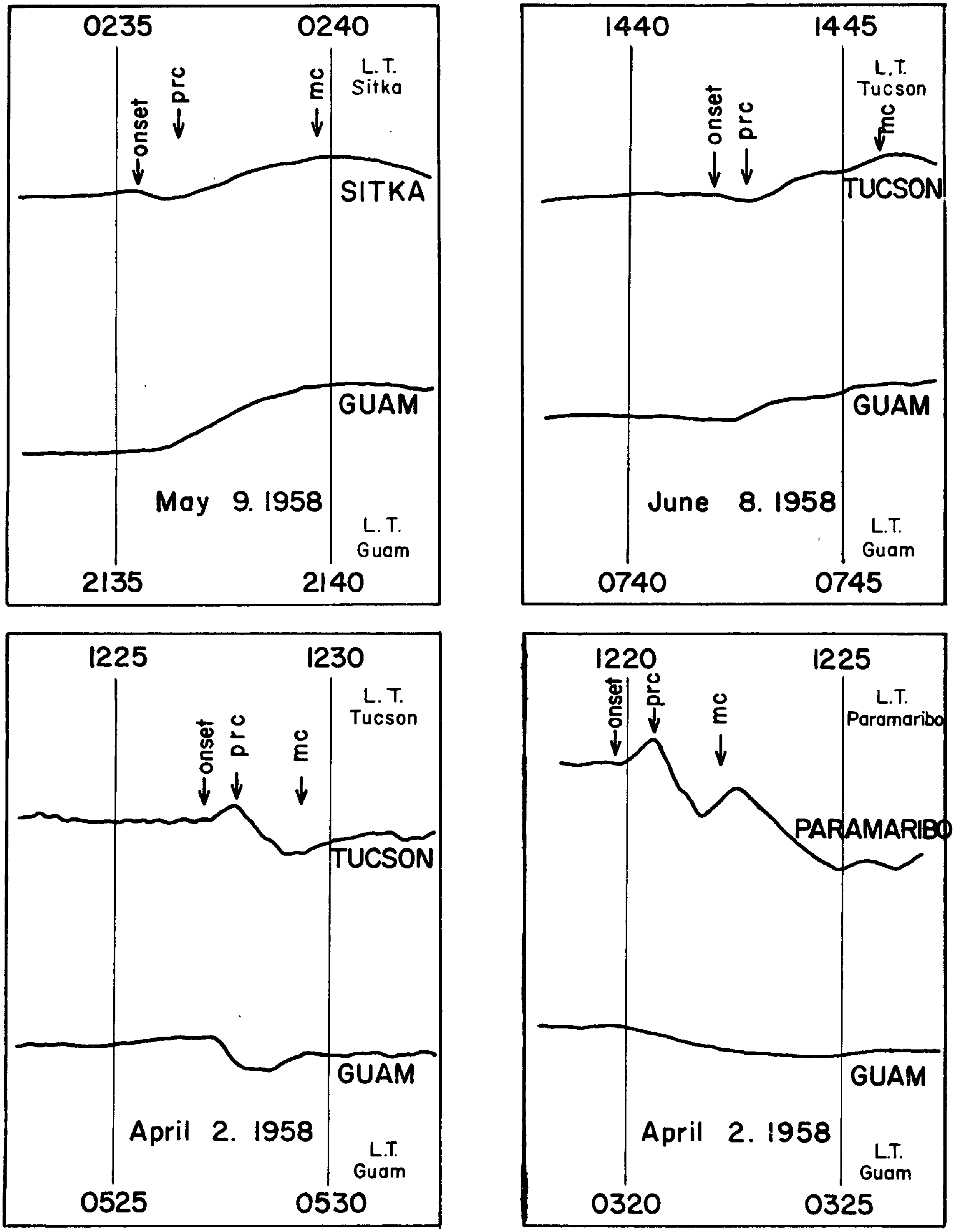


Figure 6. Examples of preceding reverse change (abbreviated to prc) recorded preceding main change (abbreviated to mc). Simultaneous records from Guam, where prc is not observed at this time, are attached as reference. Upper half, positive change; lower half, negative change.

reverse change can be distinguished from other variations recorded before the onset of the main change, first, by the closeness of the time of its onset to the time when the main change appeared at stations where no preceding reverse change was observed, and second, by the regularity of the region in which it appears. This reverse change usually lasts for about 1 minute, and can usually be detected only on rapid-run magnetograms.

This type of reversal is known to exist also in sudden commencements of magnetic storms, and is called a preceding reverse kick, the whole phenomenon being designated ssc*. It is found to occur also for si's in the same region. Figure 7 shows for both world-wide changes and sc's the regions where this preceding reverse change takes place. The region of appearance of ssc*'s obtained from the records of 4 sudden commencements is in good agreement with the results of studies by Nagata (1952) and Abe (1959). Although Matsushita (1960) obtained a different result, a note added to his paper states that the results may be changed if rapid-run magnetograms are used in the analysis. The ambiguity in the extent of the region is mainly due to the limitation in the number of rapid-run records that were available. It can be seen that the regions of occurrence of preceding reverse changes for sudden commencements and for world-wide changes correspond closely.

The forms of the changes in the horizontal component of the geomagnetic field described above are illustrated

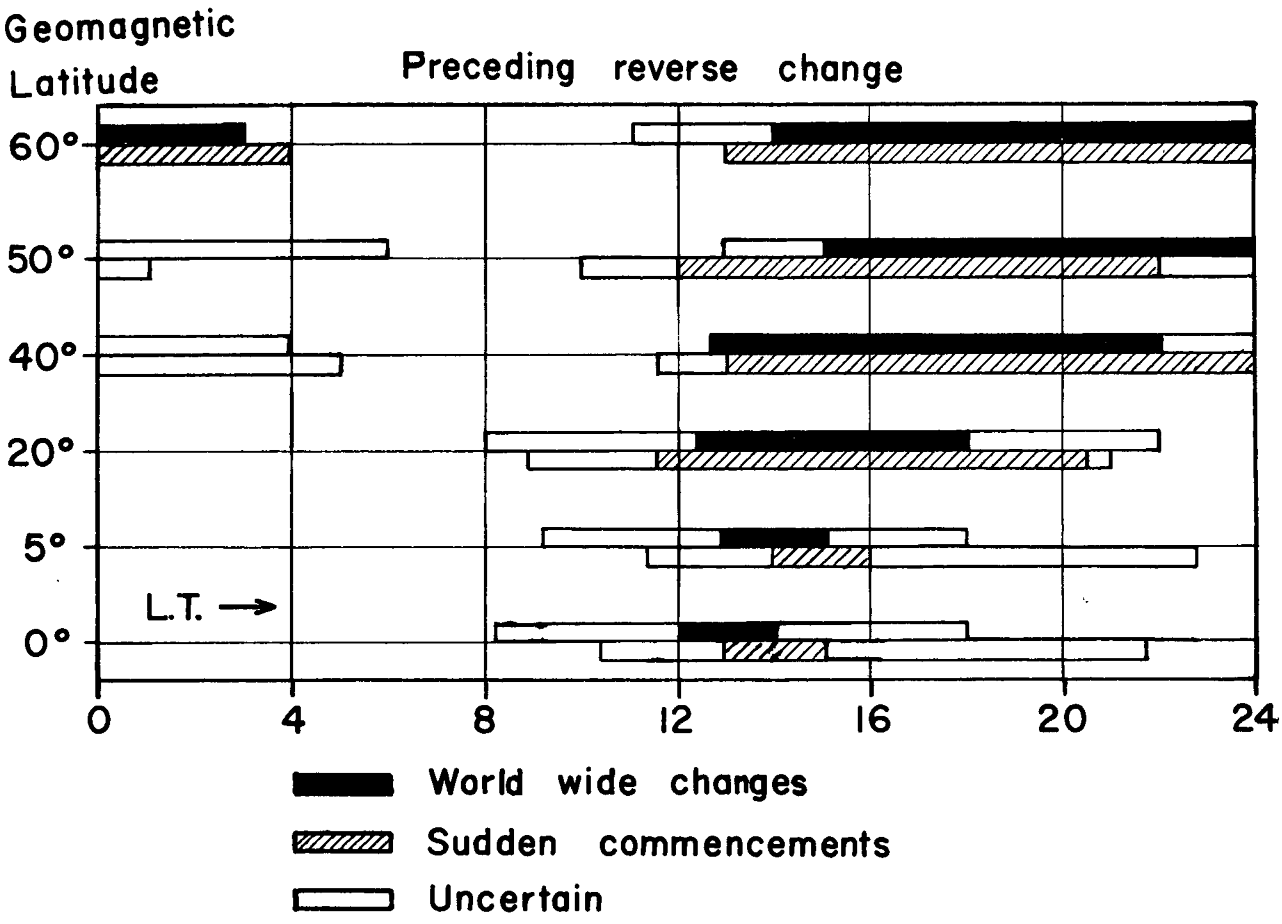


Figure 7. Region where preceding reverse change is found for world-wide changes and for sc's.

schematically in Figure 8. The only difference between a negative change and a positive change is in the sign of the whole change. All sc's, si's, and world-wide changes analyzed here correspond to one of the forms shown in this figure, the type depending on the latitude and local time. The difference between sc's, si's, and world-wide changes lies in their behaviour after the peak of the change is attained. For an sc, an interval of increased activity follows; for si, the change is supposed to be impulse-shaped. The world-wide changes analyzed here do not precede an interval of increased activity and are not necessarily impulse-shaped. An association with micropulsations cannot be inferred because of the low sensitivity of the rapid-run magnetograms used in the analysis.

3. Distribution of the time of onset

The way in which the changes spread successively all over the earth is of interest in a consideration of the physical mechanism of the changes. The time of onset is read on rapid-run magnetograms of the horizontal intensity because the change in this component is usually more conspicuous than that in other components. The error in reading is less than a few seconds. When a preceding reverse change is recorded, the time of onset of this part of the change is regarded as the time of onset of the phenomenon, because this time is closer than that of the onset of the main change to the time of onset of the phenomenon at other stations

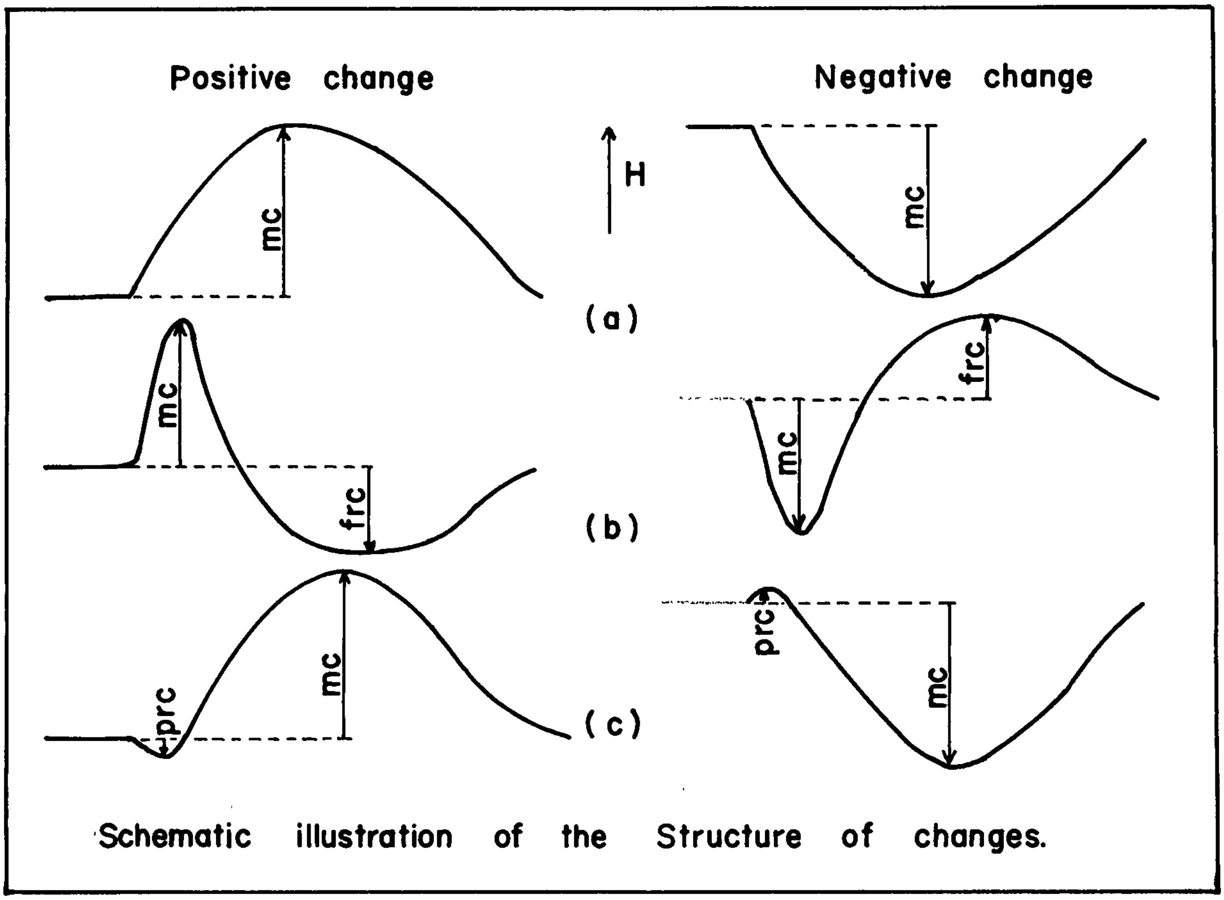


Figure 8. Schematic illustration of the three parts of change: (a) main change (mc), (b) following reverse change (frc), and (c) preceding reverse change (prc).

where a preceding reverse change is not observed. This has been noted for sc's by Gerard (1959) and by Williams (1960). A more systematic distribution of plots can be acquired by this method than by taking the time of onset of the main change everywhere. This strongly suggests that the preceding reverse change is an essential part of the phenomenon.

Local time dependence in low latitudes is studied by data from Guam (3.9°N, 212.8°), Honolulu (21.0°N, 266.4°), Tucson (40.4°N, 312.1°), and Paramaribo (17.0°N, 14.5°). As these stations cover only a 9-hour range in time, the local time dependence for a whole day is obtained by superimposing the local time dependence curves obtained for events that occurred at different GMT. This method is explained by the example shown in Figure 9, and Figure 10 is drawn by superimposing all the partial local time dependence curves obtained by means of this method. The effect of difference in the latitude of the stations can be eliminated statistically. Figure 10 shows that positive changes (including two cases of si), negative changes, and sc's spread in a similar manner from the noon side to the night side of the earth in low latitudes.

Latitude dependence is studied by a comparison of rapid-run magnetograms from two stations on approximately the same meridian, one in a low and the other in a middle-high latitude. Two pairs were chosen for this purpose:

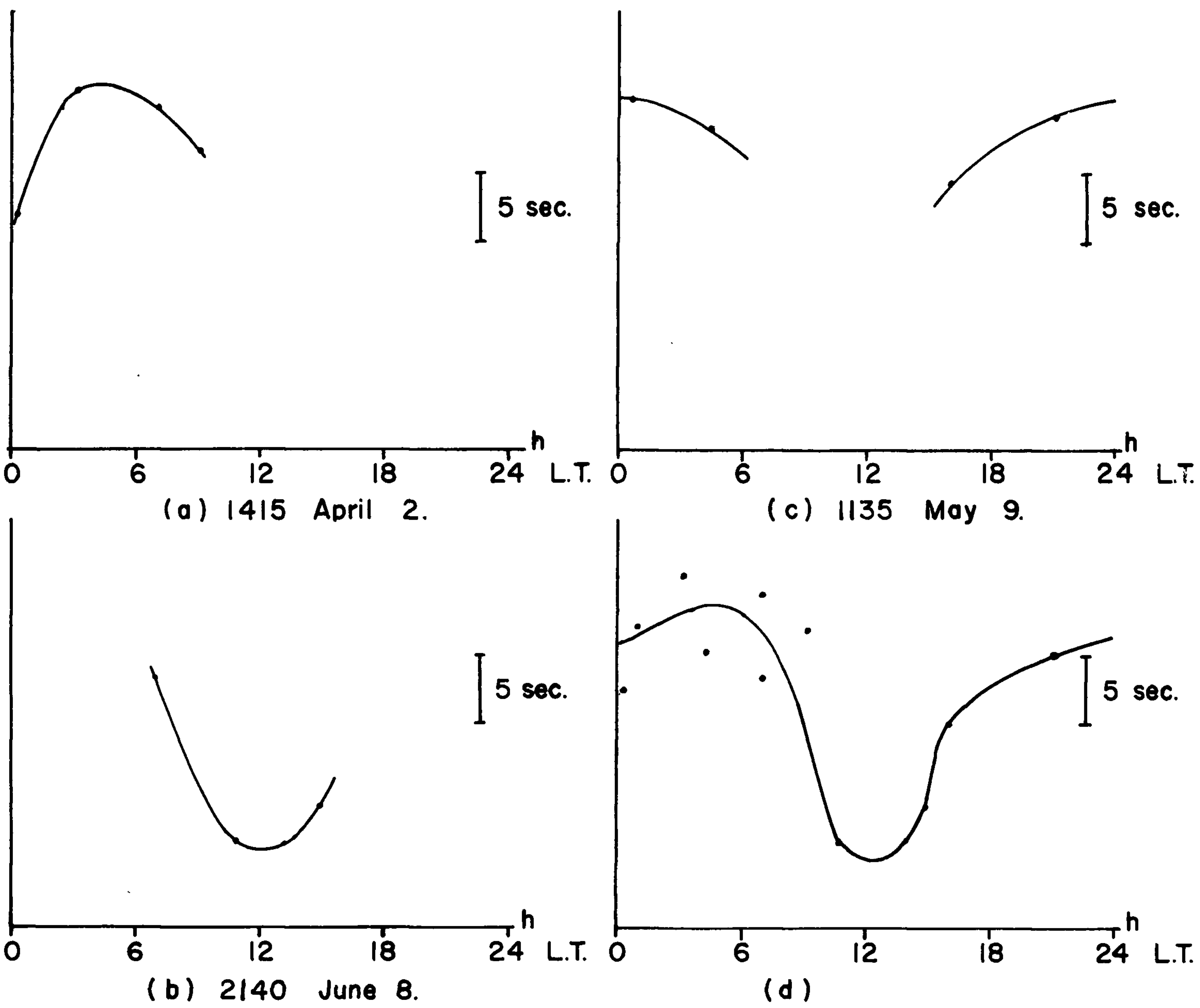


Figure 9. Illustration of the method used in obtaining the local time dependence curve of the time of onset in low latitudes. From curves (a), (b), and (c), each of which covers only a 9-hour range, curve (d) is derived, which shows the local time dependence for a whole day.

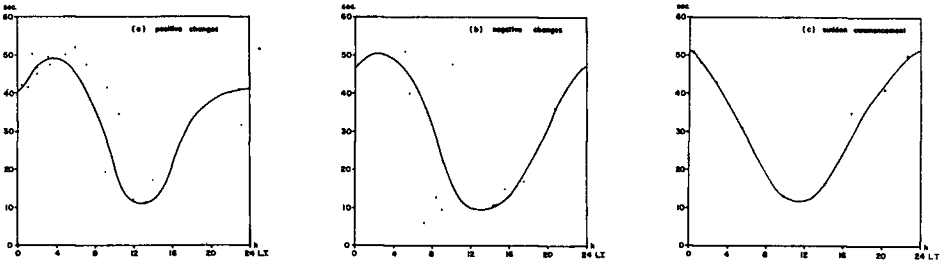


Figure 10. Local time dependence of the time of onset in low latitudes for (a) positive change, (b) negative change, and (c) sudden commencement.

Honolulu and Sitka (60.0°N, 275.4°) and Paramaribo and Fredericksburg (49.6°N, 349.9°). The amount of usable data is limited, because reliable reading of the time of onset becomes difficult in higher latitudes since the record is frequently obscured by local disturbances. Figure 11 shows the difference in time of onset between higher- and lower-latitude stations. It can be seen that the latitude dependence of the time of onset also seems to be similar for positive changes (including two cases of si's), negative changes, and sc's. The changes begin first in high latitudes at almost all local times.

Thus the distribution of the time of onset is the same for sc's and si's and world-wide changes both positive and negative. The overall similarity in the distribution of the time of onset between positive and negative changes supports the idea that a negative change should be regarded as an independent phenomenon, because, if a negative change is nothing more than a recovery following a positive change, it must first appear in the region shown in Figure 5 where the main change is interrupted by a following reverse change, while the main change still continues at other stations. From Figures 10 and 11 the world-wide distribution of the time of onset common to all these phenomena is obtained (see Figure 12). This is similar to the result obtained by Gerard (1959) and Williams (1960). Thus it seems that in all world-wide changes, sc's, and si's,

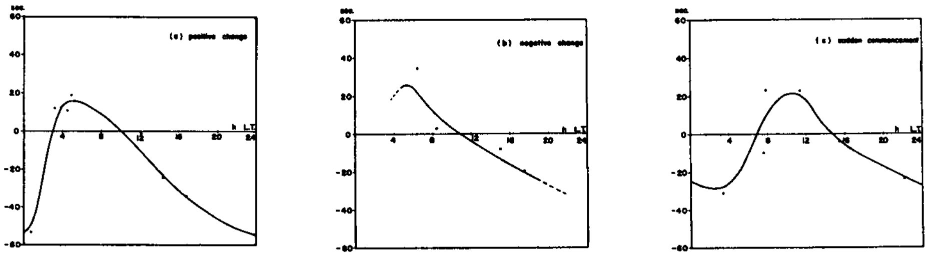


Figure 11. Difference in time of onset at middle-high latitude stations (Sitka or Fredericksburg) from that at low-latitude stations (Honolulu or Paramaribo): (a) positive change, (b) negative change, and (c) sudden commencement.

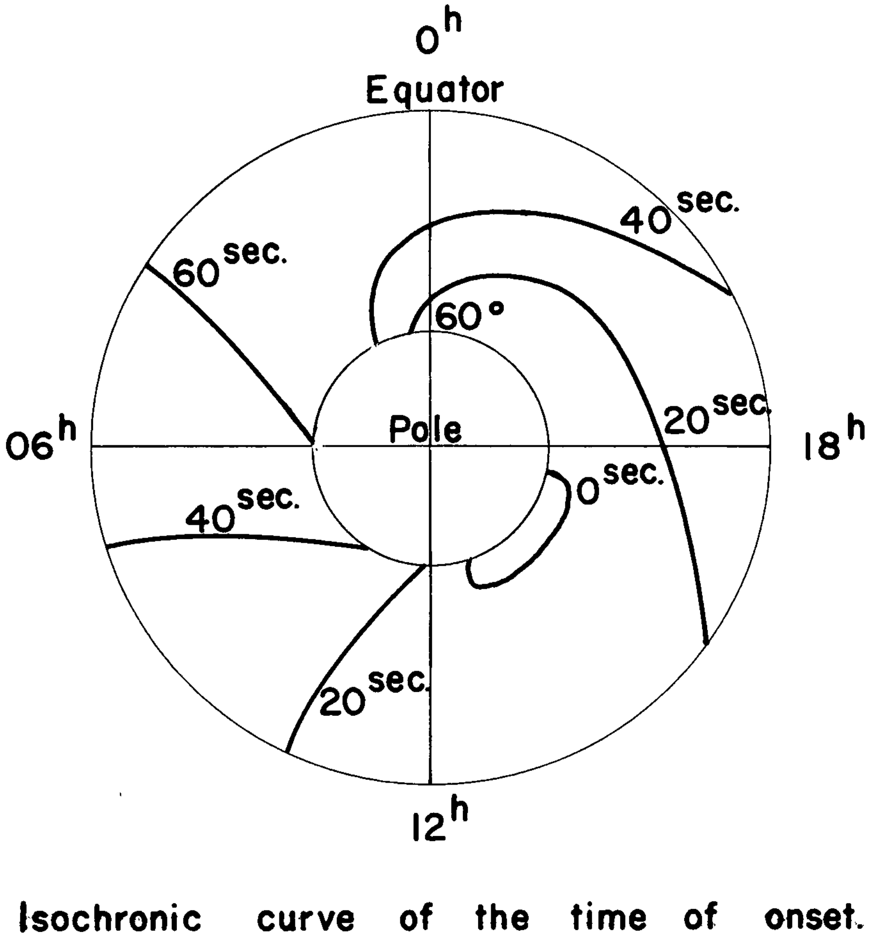


Figure 12. Isochronic curve of the time of onset of world-wide changes, si's, and sc's. Average value of the relative time of onset is given in seconds.

the preceding reverse change appears first in high latitudes on the afternoon side of the earth and then spreads toward the afternoon side in lower latitudes. Then, more than 20 seconds after the first appearance of the preceding reverse change, the main change appears outside the region where the preceding reverse change is recorded, and spreads from the noon to the midnight side. In the region where the preceding reverse change appears, the main change occurs about 1 minute after the onset.

In contrast to the present result, Troitskaya (1961) has reported from an examination of data from the USSR that the time of onset of sc is simultaneous everywhere. Since the USSR records sent to the IGY World Data Center have a slower speed of run (5.0 cm/hr) than those shown in her paper and those sent from other stations (20.0 cm/hr), it is impossible to check her result. Different methods of observing the phenomena are suggested as a possible cause of the discrepancy. Rapid-run tellurigrams are used by Troitskaya while rapid-run magnetograms are used here,and the frequency response of these two methods are not the same. But a comparison of the time of onset at Kanoya (tellurigram) and Guam (magnetogram) shows that the difference is small: -7 ± 12 sec for world-wide changes and 2 ± 3 sec for sc's. As these stations are nearly in the same meridian and differ in latitude by about 20°, this amount of difference in the time of onset is to be expected from the distribution shown in

Figure 12, and the difference in apparatus does not seem to affect the result significantly. Since tellurigram records shown in Troitskaya's paper are rich in high frequency oscillations, the determination of the time of onset may be more difficult than in the present analysis, and it is suspected that the time of onset taken by her might be erroneous.

4. Distribution of the magnitude of the change

The dependence of the magnitude of the change on latitude and local time is also studied. Since only normal-run magnetograms can be used for this analysis, it is difficult to read all three parts of the change. When a following reverse change appears, the magnitude of this part is read, because it is usually larger and lasts longer than the remaining parts of the change. In other cases, the magnitude of the main change is read. The magnitude of the change is measured from the level preceding the onset of the change, as shown in Figure 8. To combine the data from different cases, readings of the magnitude are normalized by the sum of the readings at 5 stations with latitude approximately 50°N: Hartland (54.6°N, 79.0°), Moscow (50.8°N, 120.5°) Yakutsk (51.0°N, 193.8°), Victoria (54.1°N, 293.0°) and Fredericksburg (49.6°N, 349.9°). The local time dependence of both the northward and the eastward components of the change at 60°N, 50°N, 40°N, 20°N, 5°N, and 0° obtained in this manner are shown in Figure 13, for positive changes,

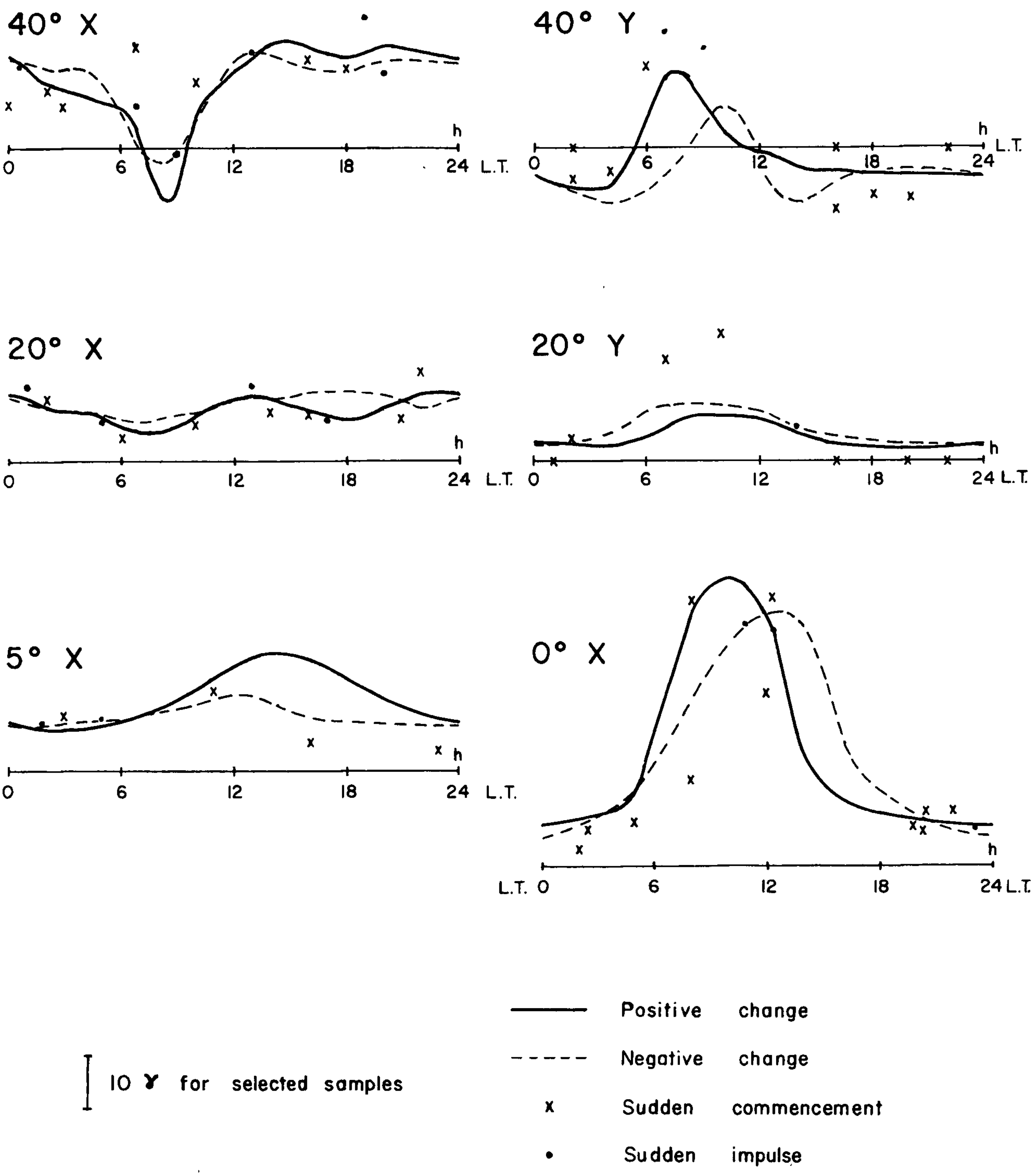

40° X
40° Y
20° X
20° Y
5° X
0° X
h
L.T.
0
6
12
18
24
Positive change
Negative change
Sudden commencement
Sudden impulse
10 γ for selected samples

Figure 13.

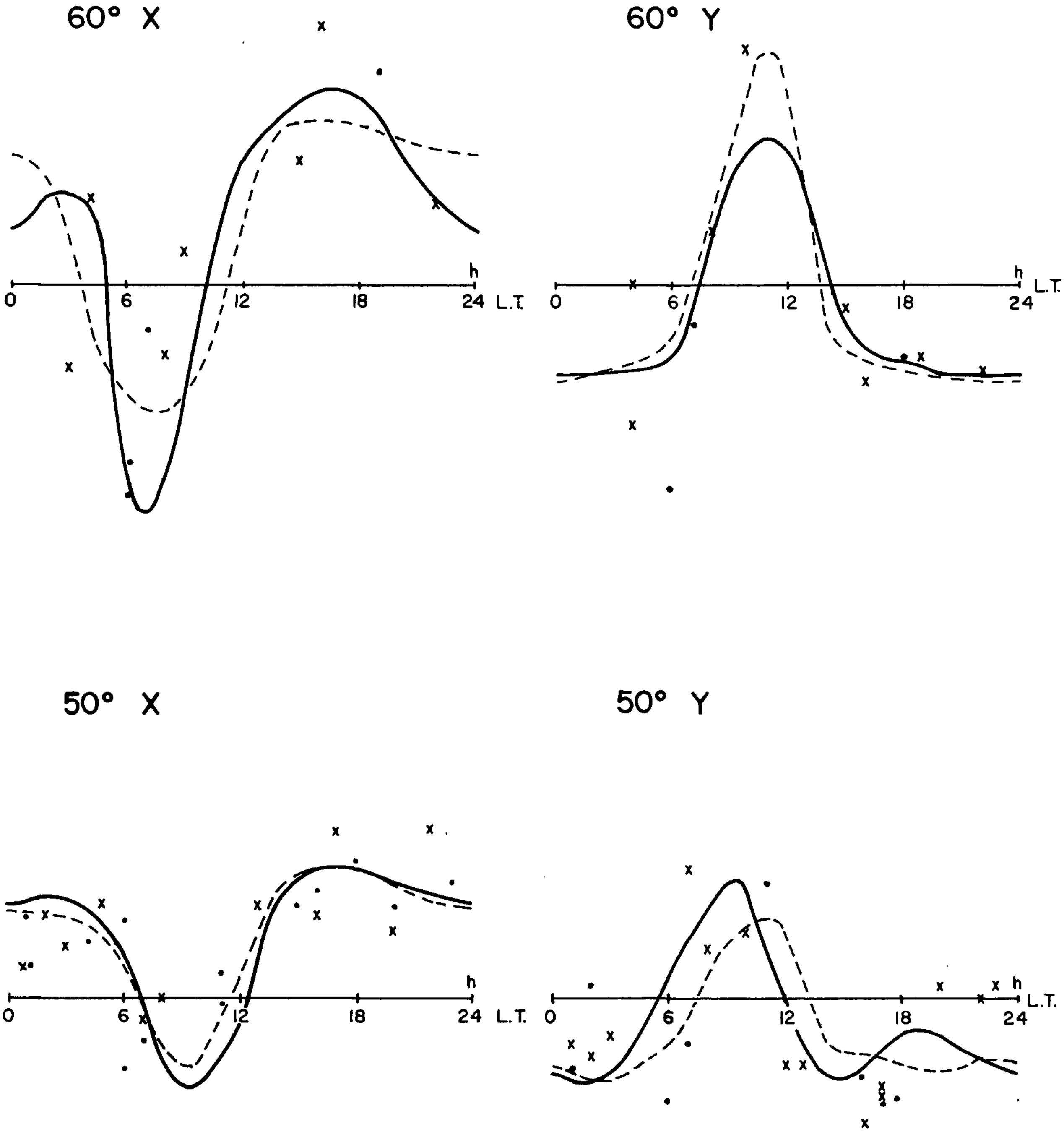


Figure 13. Distribution of magnitude of world-wide changes in the geomagnetic field as compared with that of si's and sc's: X, northward component; Y, eastward component. Scale corresponds to average of selected world-wide changes. Data are from Leningrad, Sitka (60°); Hartland, Moscow, Yakutsk, Victoria, Fredericksburg (50°); Tibilisi, Sakhalinsk, Tucson (40°); Kanoya, Honolulu, Paramaribo (20°); Bangui, Guam (5°); Koror and Huancayo (0°).

negative changes (with sign reversed), sc's, and si's. The change in the vertical component is less than one-third that in the horizontal component, and, since the sensitivity of this component is usually less than that of the other components, reliable readings are difficult and the plots fairly scattered. Therefore the distribution of the vertical component is not reproduced here. Near the equator, the change in declination becomes very small and the distribution of the eastward component at 5°N and 0° cannot be obtained.

The distribution shown in Figure 13 does not necessarily show simultaneous values of the change, but the figure probably represents the distribution of magnitude to a fair approximation. When the considerable dispersion of the plots, due possibly to the smallness of the values to be read from the records, is taken into account, Figure 13 can be regarded as showing close similarity between the world-wide changes analyzed here, sc's, and si's in the dependence of magnitude on latitude and local time. Latitude dependence of the average value of the change in a day can be obtained from Figure 13; it is shown in Figure 14. It can be seen that the amplitude of the change increases for both positive and negative changes near the equator.

The daytime enhancement, at the equator, in the magnitude of the change has been noticed by a number of authors (Vestine, 1953; Sugiura, 1953; Maeda and Yamamoto,

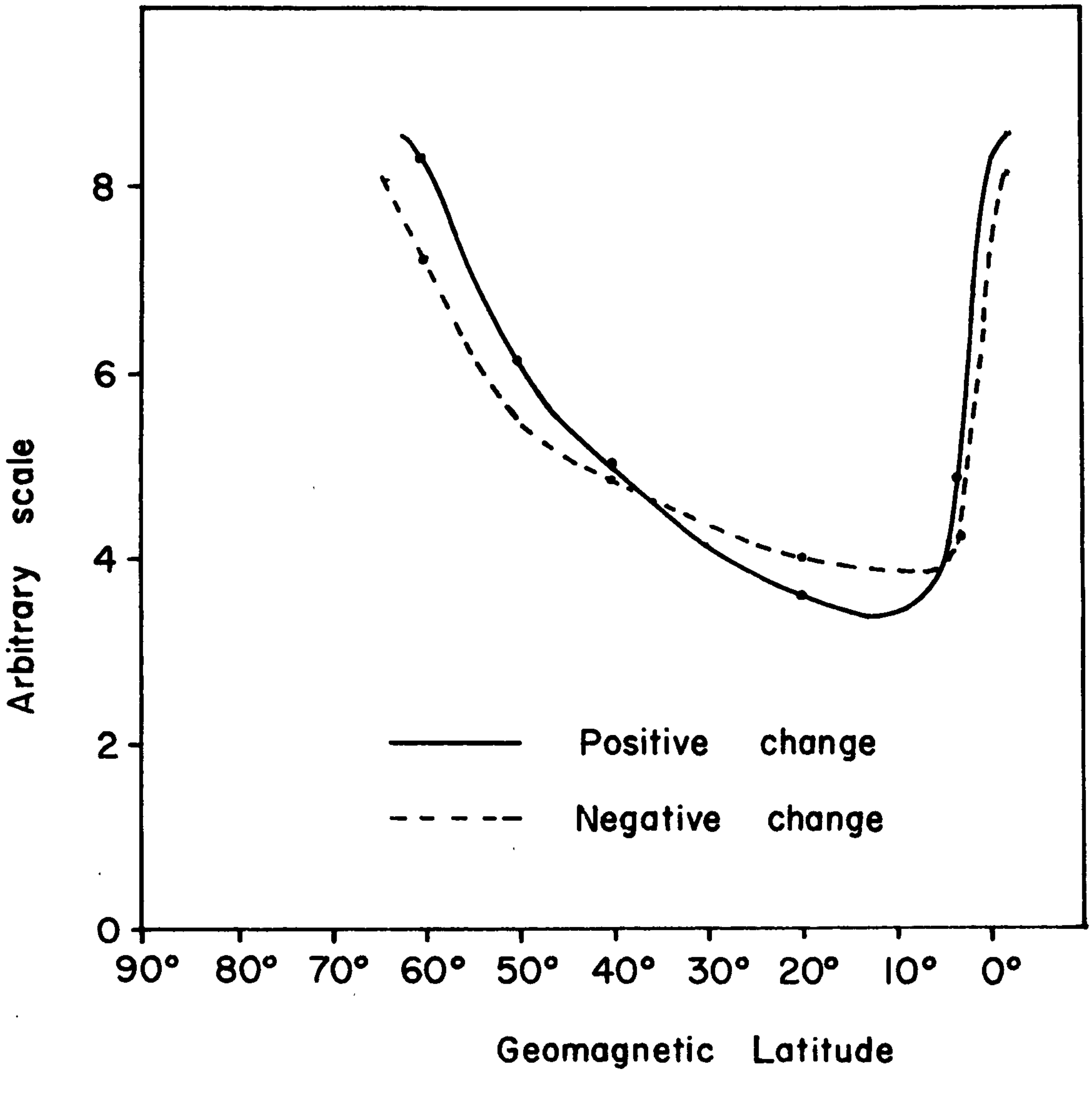


Figure 14. Latitude dependence of the magnitude of the horizontal component of world-wide changes in the geomagnetic field.

1960) for the main change of sc. But this type of enhancement is also found in the preceding reverse change. An example is shown in Figure 15 in records from Guam and Koror. These stations differ only by about one hour in local time, but have a difference in dip angle of about 12°, Koror being closer to the dip equator. It can be seen that the magnitude of the overall change at Koror is two or three times larger than that at Guam. The ratio of the magnitude of the change in the horizontal component at Koror to that at Guam is calculated for several cases recorded during the IGY when a preceding reverse change was present at both stations. This, and the ratio of the amplitude of Sq on a quiet day near the day of the sc are tabulated in Table 2. A close similarity is found between the ratios for the main change of sc and for Sq, in accordance with the earlier results of Forbush and Vestine (1955). The ratio for a preceding reverse change of sc is of the same order, but a few times larger than that for the main change or Sq.

Equatorial enhancement is found to occur in a similar manner for world-wide changes. Examples are given in Figure 16 (for a positive change) and Figure 17 (for a negative change). Magnetograms from several widely separated stations are presented to show the world-wide occurrence of the phenomenon. The magnitude of the overall change at Koror is a few times larger than that at Guam. (A preceding reverse change is distinguished from other small changes by

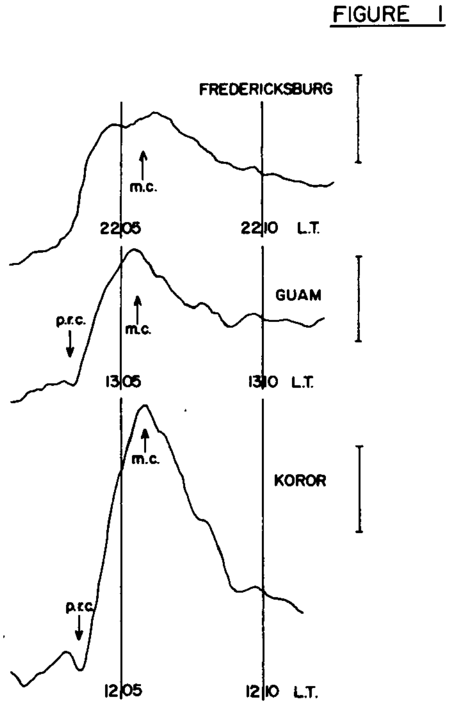


Figure 15. Equatorial enhancement of sc shown in a record from Koror.

Table 2. Ratio of the magnitude of the change in the horizontal component at Koror to that at Guam.

Date sc	LMT(Guam)	Preceding Reverse Change	Main Change	Sq
21 June 1958	1200	3.3	1.9	1.4
17 August 1958	1600	2.0	2.0	1.7
22 August 1958	1300	6.7	2.4	1.6
27 August 1958	1300	2.5	1.8	1.9
world-wide positive change				
21 May 1958	0900	3.0	1.8	1.6
02 June 1958	1400	3.9	2.1	2.0
world-wide negative change				
25 December 1957	1300	6.0	3.4	2.2

FIGURE 2

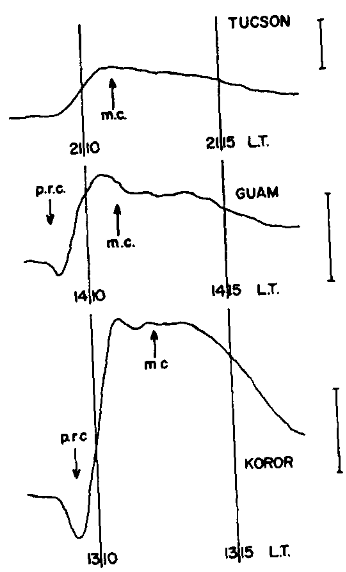


Figure 16. Equatorial enhancement of world-wide positive change, shown in a record from Koror. Records from other stations are presented to show the world-wide occurrence of the change.

FIGURE 3

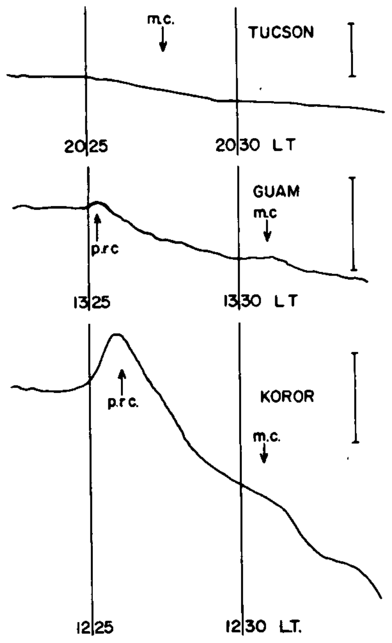


Figure 17. Equatorial enhancement of world-wide negative change shown in a record from Koror. Records from other stations are presented to show the world-wide occurrence of the change.

being confined to certain regions of the Earth and by the closeness of its time of onset to the time when the change is recorded in other parts of the world). Ratios of the magnitudes of changes are estimated in the same way as for sc's, and are given in Table 2. It can be seen that the ratios for the main change, preceding reverse change and Sq are related in a similar way to those for sc's, and moreover, the figures in the corresponding columns for sc and world-wide change are quite similar.

5. <u>Related phenomena</u>

Phenomena associated with the occurrence of world-wide changes in the geomagnetic field were looked for in the following records.

(1) Orbit of the satellite 1958γ(Explorer III).

The speed of orbiting of the artificial satellite is reported to vary during a magnetic storm (Jacchia, 1959), and explained by an increase in the atmospheric density. However, the orbit shown in the publication by NASA (technical report P-356) is a result of theoretical calculations on the basis of observations made every 48 hours. This time interval is too long to detect the effect of such small and frequent phenomena as world-wide changes, and no conclusions can be drawn.

(2) Vertical incidence ionograms.

Data were obtained from the stations tabulated in Table 3. Nothing particular can be found which may be

Table 3. Ionospheric stations.

Station	Geographic Latitude	Geographic Longitude
Akita	39.7°N	140.1°E
Baker Lake	64.3°N	96.0°W
Churchill	58.8°N	94.2°W
Fort Monmouth	40.3°N	74.1°W
Kokubunji	35.7°N	139.5°E
Meanook	54.6°N	113.3°W
Ottawa	45.4°N	75.9°W
Resolute Bay	74.7°N	94.9°W
St. Johns	47.6°N	52.7°W
Talara	64.6°S	81.3°W
Victoria	48.4°N	123.4°W
Wakkanai	45.4°N	141.7°E
Yamagawa	31.2°N	130.6°E

associated with the occurrence of world-wide changes. However, the time interval of 15 min between which these ionograms are taken may be too long to find out any effect related with world-wide changes which last only a few minutes.

(3) Ionospheric absorption.

Data were obtained from the stations tabulated in Table 4. No association is found with world-wide changes. Although an increase in absorption is reported to occur at the time of some sc's, more than half of the sc's studied do not show this feature (Ortner et al, 1962; Matsushita, 1961). Hence this does not seem to be an essential part of the phenomenon.

(4) Night airglow.

Data from two stations tabulated in Table 5 were used, but no relationship can be found. The hourly values used here may be quite unsuitable for the present purpose because of smallness of the duration of world-wide changes.

(5) Overall features of the geomagnetic field.

The daily number of occurrences of world-wide changes is compared with the K_p index and reproduced in Figure 18. It can be seen that the number of occurrences of world-wide changes is not related with K_p at all, and hence is independent of the degree of disturbance of the overall features of the geomagnetic field.

Table 4. Observatories recording ionospheric absorption.

Station	Geomagnetic latitude	Geomagnetic longitude
Barrow	71.5°N	156.4°W
Churchill	58.8°N	94.2°W
Colombo	06.6°N	80.0°E
Ft. Yukon	66.5°N	145.3°W
Thule	76.5°N	68.8°W
University Park	40.8°N	77.9°W

Table 5. Observatories recording night airglow.

Station	Geomagnetic latitude	Geomagnetic longitude
Haute Provence	43.9°N	5.7°E
l'Irsac a Lwiro	2.2°S	28.8°E

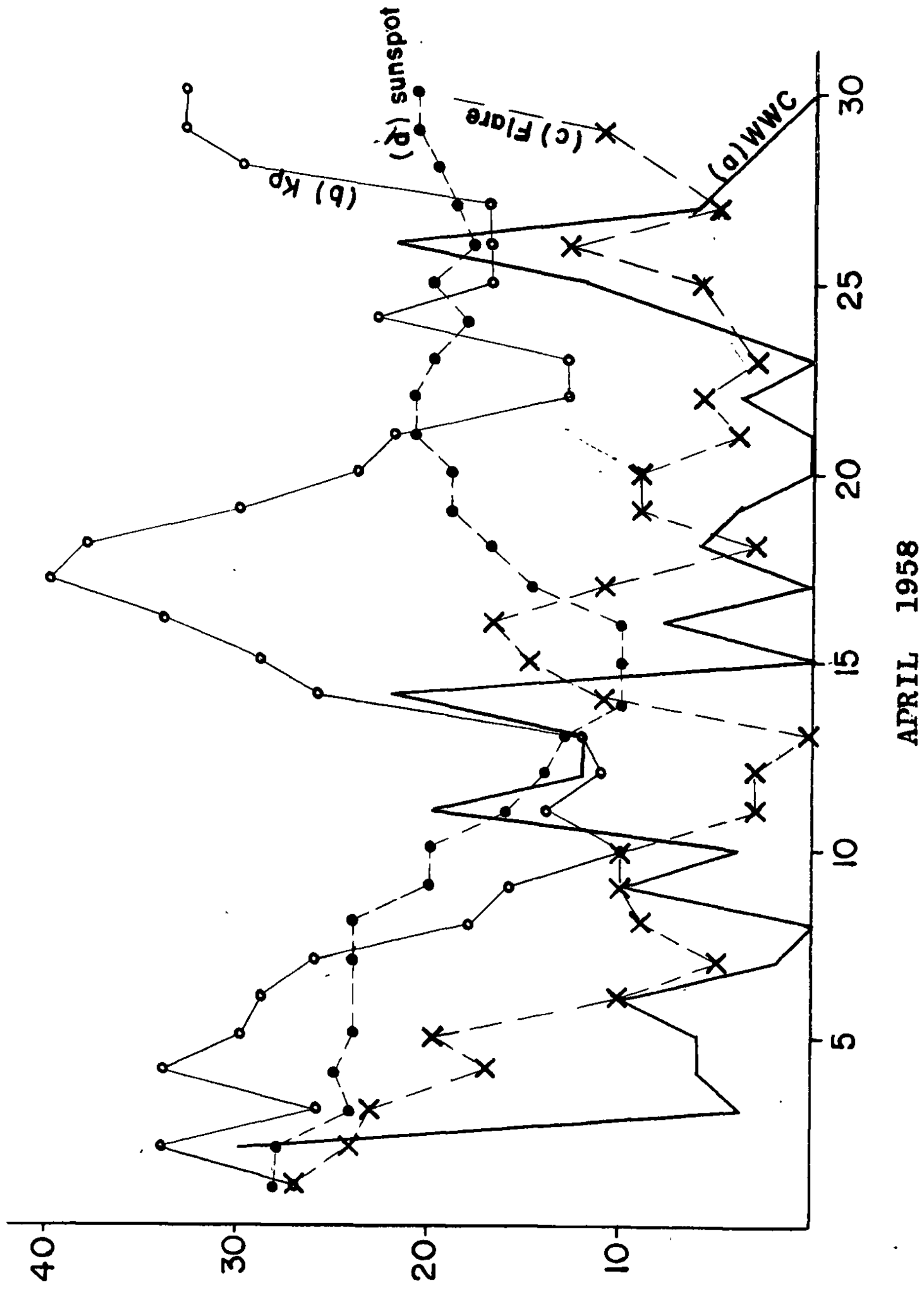


Figure 18. Variation in (a) the number of occurrences of world-wide changes (WWC) (multiplied by two), (b) the geomagnetic kp index, (c) the number of solar flares and (d) the sunspot number during April 1958.

(6) Solar phenomena.

The daily number of occurrences of solar flares (IGY solar activity report series No. 12, 1960) and the sunspot number are compared with the daily number of occurrences of world-wide changes. As can be seen from Figure 18, no correlation appears to exist between them. Bursts in solar radio noise (Solar-geophysical data, NBS, 1958) are also not related with world-wide changes. Since world-wide changes are small in magnitude and frequent in occurrence, it is not preferable to try and associate them with the limited number of particular events picked out in the published data. The original, continuously monitored record will have to be studied.

Thus so far, no phenomena can be detected which are related with world-wide changes in the geomagnetic field. But since most of the data used here is not suitable for the present purpose, a definite conclusion can not be drawn, and a detailed study will have to be made in the future.

6. Summary

World-wide changes are found to exist in the geomagnetic field. They are changes of magnitude less than about 20 γ in low and middle latitudes and are not registered as sc's or si's by most stations. However, they are quite similar to sc's or si's in most features such as form, manner of spreading over the earth, and distribution of magnitude. This conclusion is based on the analysis of randomly selected samples, but the other examples of changes selected at the beginning of this study but not subjected to detailed analysis are found to show the same characteristics. This fact, together with the broadness of the area covered by the 5 stations used to determine them, seems to justify the conclusion that all belong to the same class of phenomena. It follows that world-wide changes of the geomagnetic field are observed quite frequently; at least 20 per cent of every 1-hour period and at least 90 per cent of all days contained at least one of them during the 3-month period near sunspot maximum. The frequent occurrence of changes in the geomagnetic field on a world-wide scale is in accordance with the idea that world-wide features of the geomagnetic field are always related with the highly variable physical state of the solar atmosphere, although the relationship between individual events has not yet been found.

An sc is distinguished from a world-wide change by its association with an interval of increased activity. An si actually differs but little from a world-wide change. Si's are supposed to be impulse-shaped, but, as can be seen by the example of 1600 June 19 in Figure 1, some of those reported are not impulse-shaped, and, moreover, many of those that are really impulse-shaped are missed in the reports from the majority of stations. Thus si's seem to be nothing more than the world-wide changes that are widely recognized because of their large size.

The distribution of the horizontal component of the change in the geomagnetic field at the time of a world-wide (positive) change, sc, and si, is drawn schematically in Figure 19, making use of available rapid-run magnetograms and the results obtained in the previous sections of this paper. For a negative change, the direction of all the vectors is reversed. As only the distribution below 60° geomagnetic latitude is studied, the difference between geomagnetic and geographic latitude is ignored. Figure 19a shows the first stage of the change, in which only a preceding reverse change appeared in a limited region of the earth. Figure 19b, corresponding to the stage more than 1 minute after the stage shown in Figure 19a, represents the distribution of the main change. This stage continues from a few to several minutes, and then a following reverse change appears as shown in Figure 19c. In the region kept blank in Figure 19c, the field vectors are but little altered from those shown in Figure 19b.

The preceding reverse change and the following reverse change of sc are regarded by Oguti (1956) as similar phenomena differing only in the local time of the region of appearance. He proposed that the reverse change drifts westward from the afternoon side of the earth (where it appears as prc) to the morning side (where it appears as

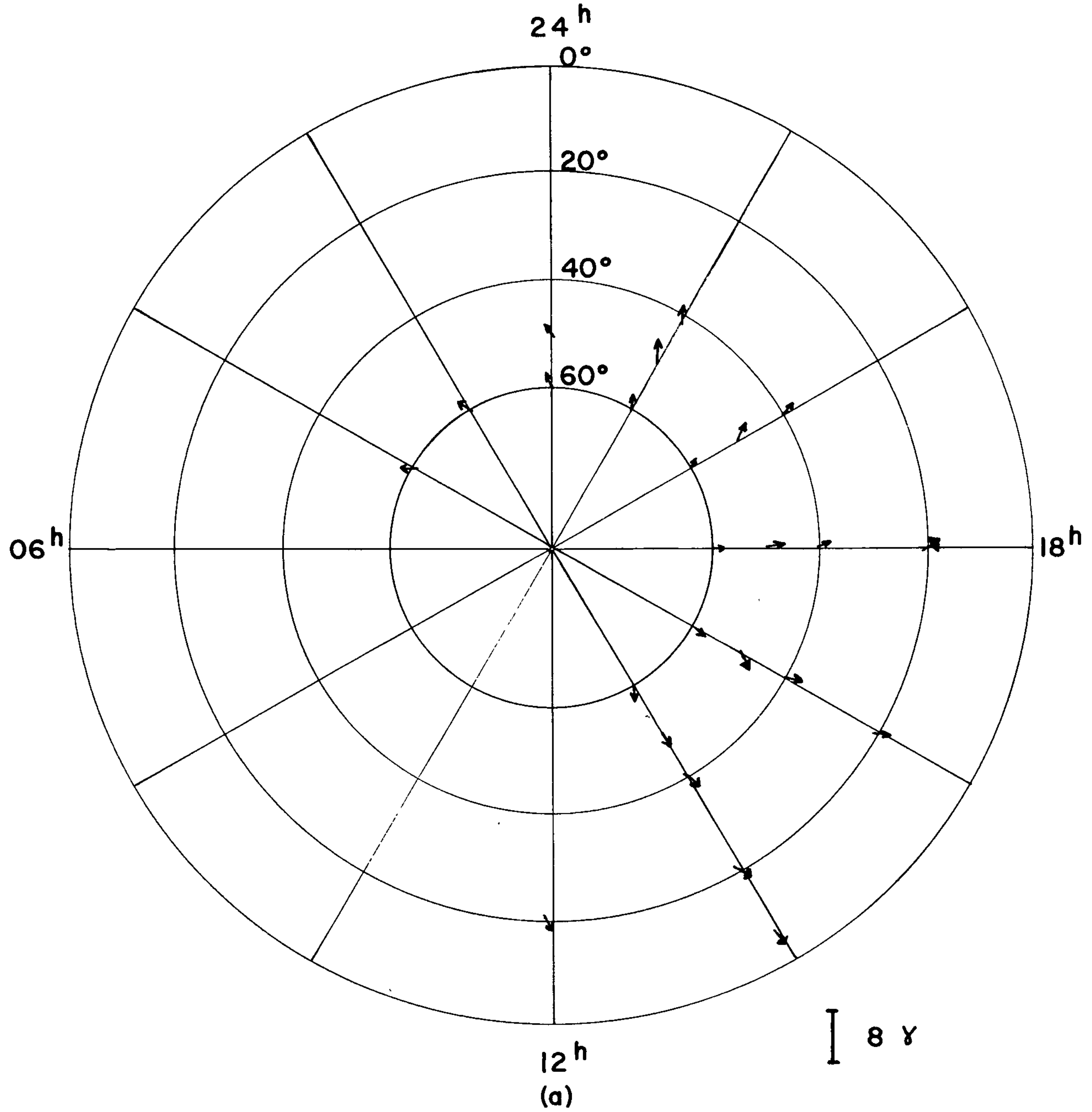


Figure 19a. Distribution of the horizontal component of a positive world-wide change in the geomagnetic field shown for three stages of the change. (For negative change, the direction of the vectors must be reversed). The scale given corresponds to the average of selected samples. (a) First stage, in which only preceding reverse change is observed in a limited region.

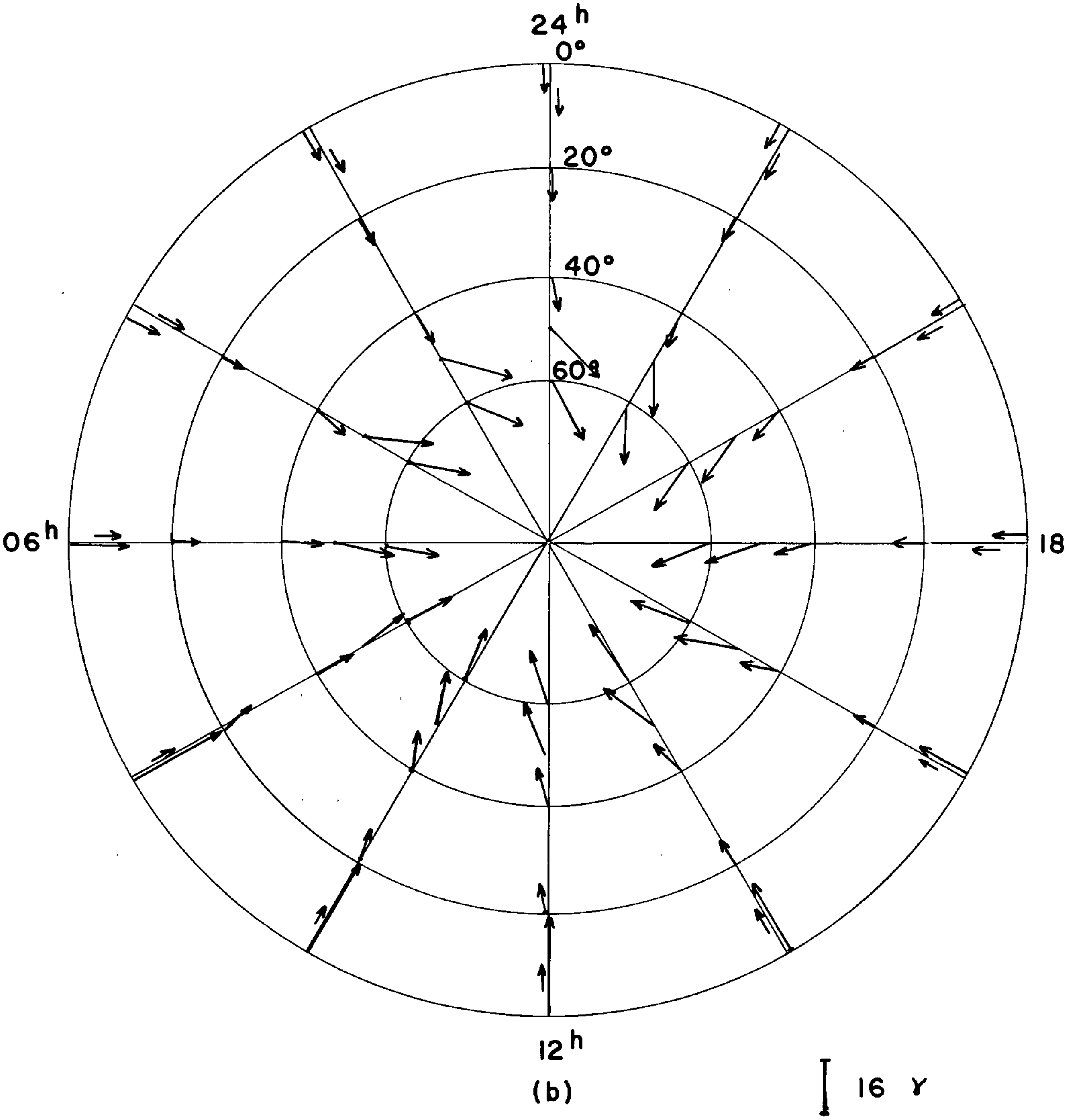


Figure 19b. (b) Second stage, in which main change is observed all over the world.

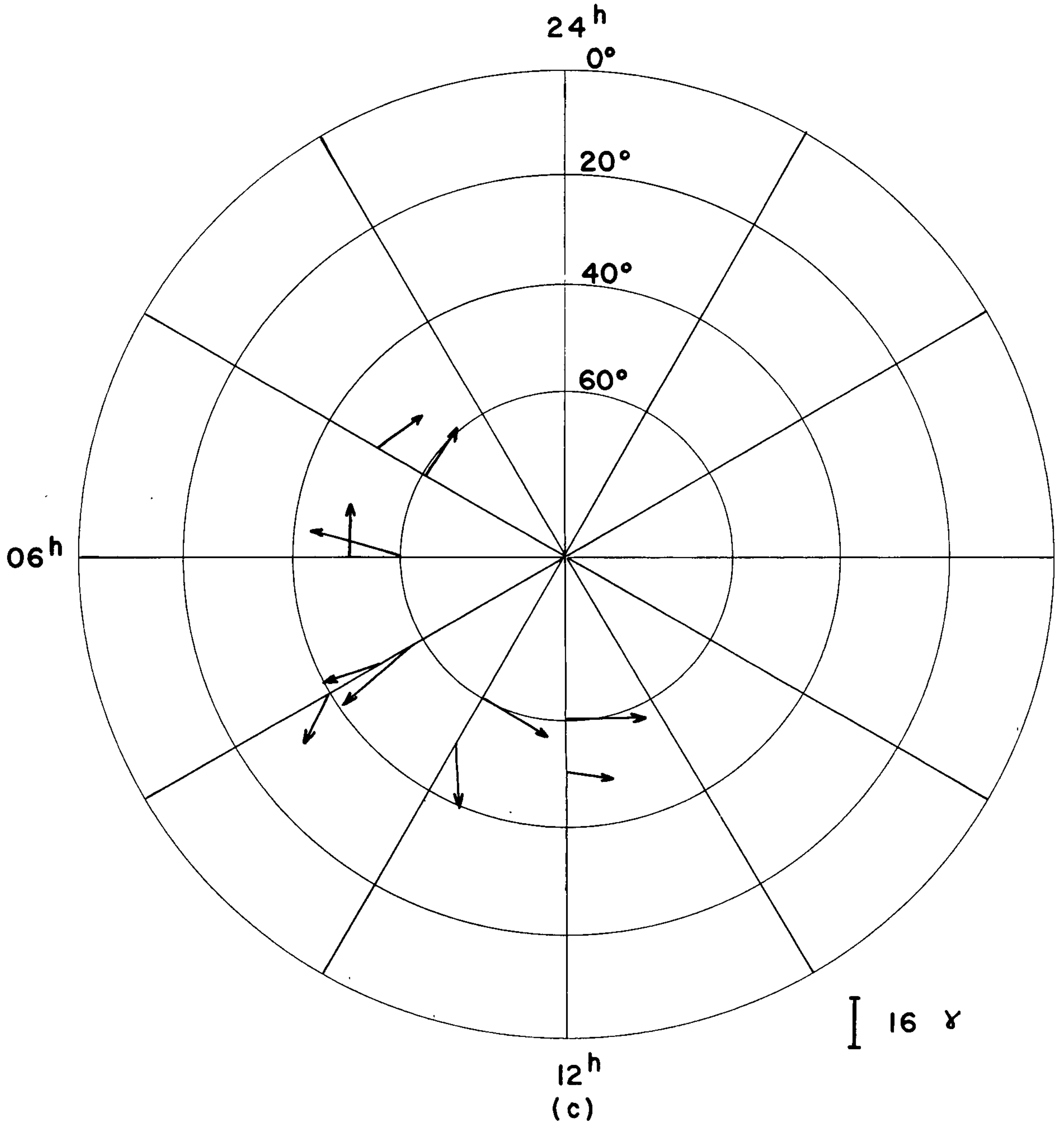
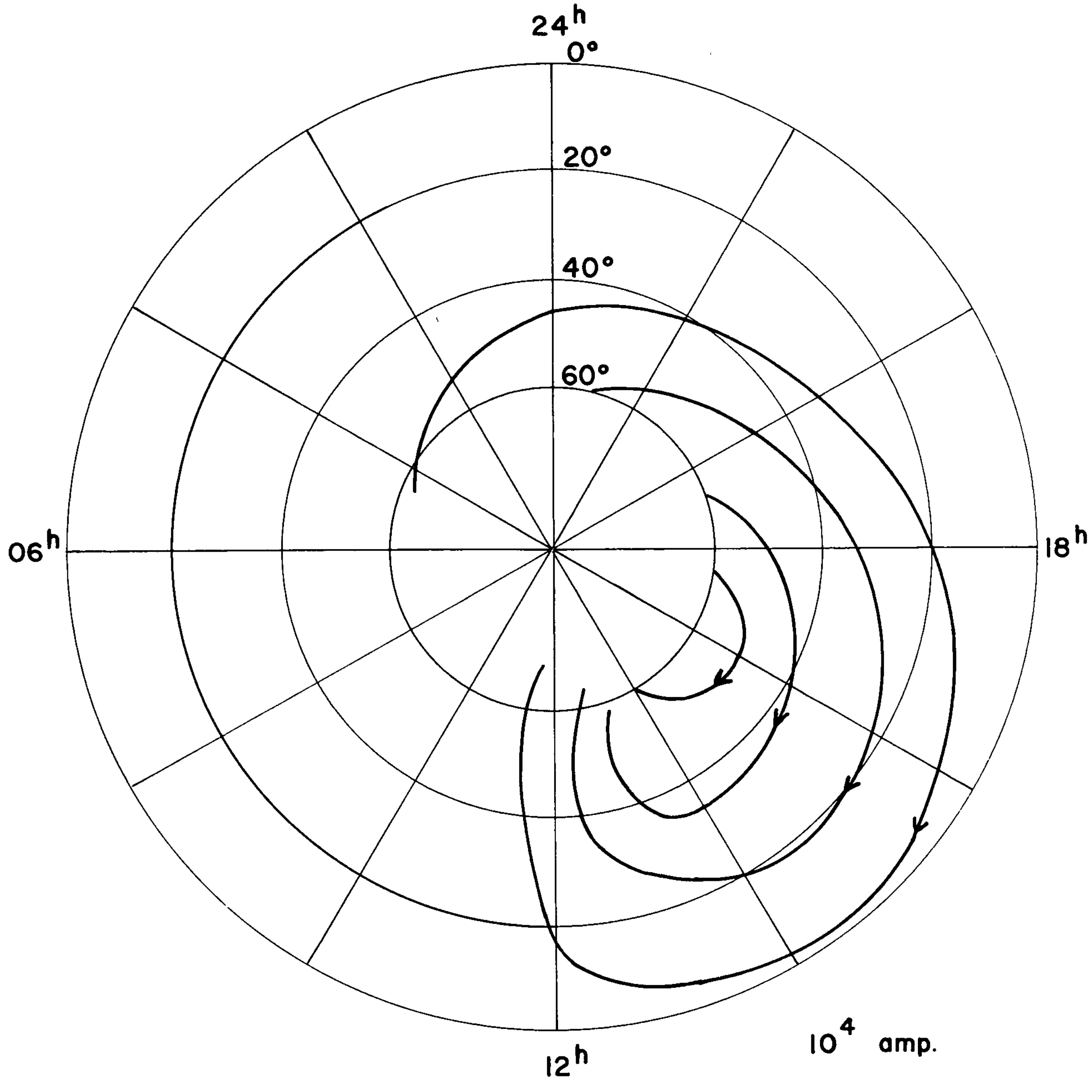


Figure 19c. (c) Third stage, in which following reverse change appears in the region shown.

frc). But the present result shows that prc and frc are quite different in the broadness of the region where they are observed, frc being more restricted to high latitudes. Hence it seems preferable to regard preceding reverse change and following reverse change as independent phenomena.

An 'equivalent' overhead current system is frequently employed as a convenient way to represent the distribution of changes in the geomagnetic field. This is an ionospheric current system drawn on the assumption that the observed magnetic field is entirely due to a closed electric current system flowing in the ionosphere. The equivalent current system constructed from the distribution of the magnetic field shown in Figure 19 is drawn in Figure 20. However, it should be noted that the distribution of a magnetic field that can be equivalently represented by a closed electric current system in the ionosphere is limited. Phenomena due to ionospheric currents which are divergent or convergent, or due to current systems which include flow along lines of force as an essential part, or due to hydromagnetic wave of some kind, can not be represented by an 'equivalent' current system. The equivalent current system is frequently employed nevertheless, because with the limited amount of data now available, the distribution can be approximated by that due to an ionospheric closed current flow. Thus there is no guarantee of the reality of an



(a)

Figure 20a. Equivalent overhead current system (assumed height 100 km) of a positive world-wide change in the geomagnetic field. The amount of electric current indicated in the figures flows between successive lines for the average of selected examples. (a) First stage.

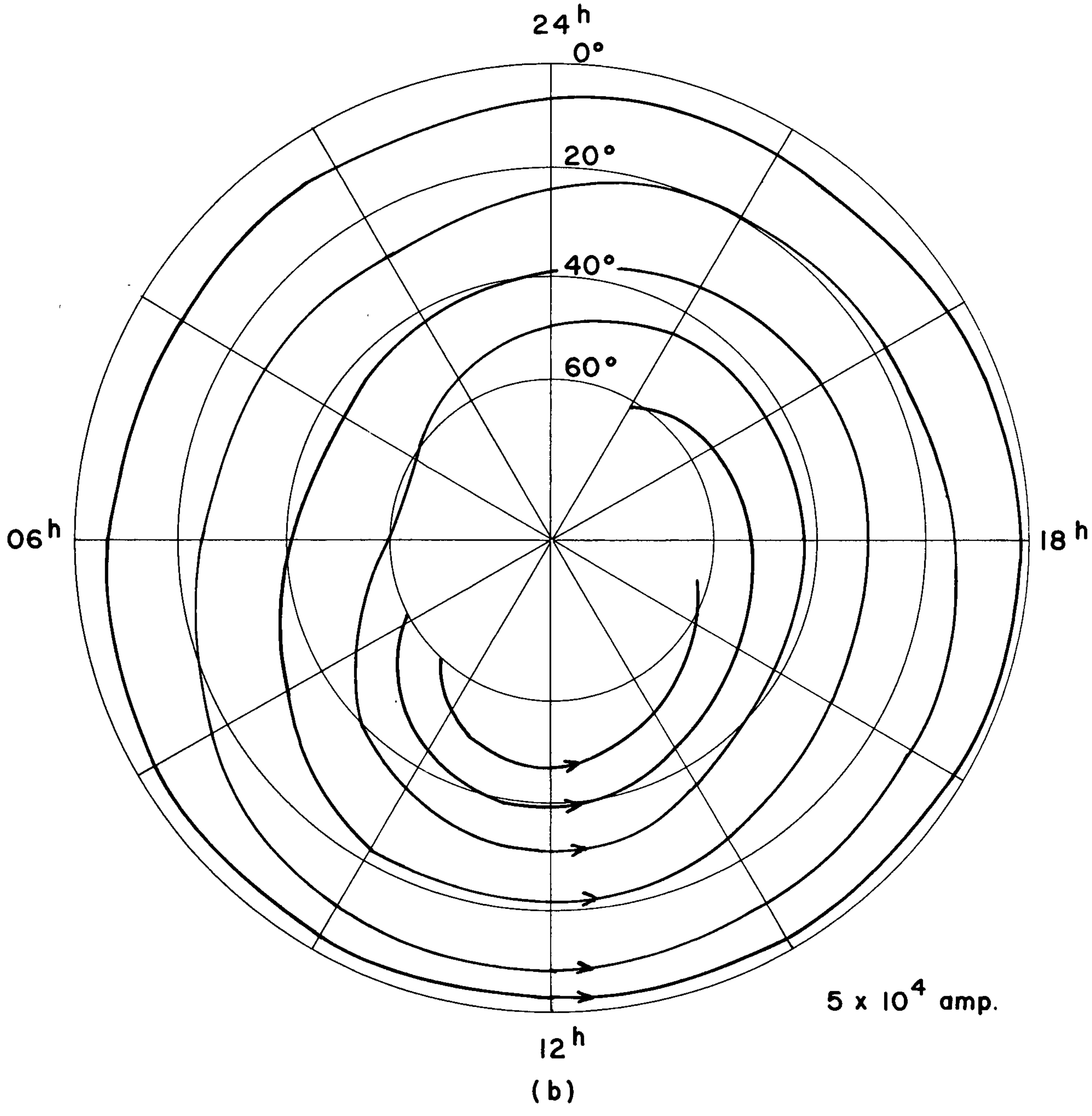


Figure 20b. (b) Second stage.

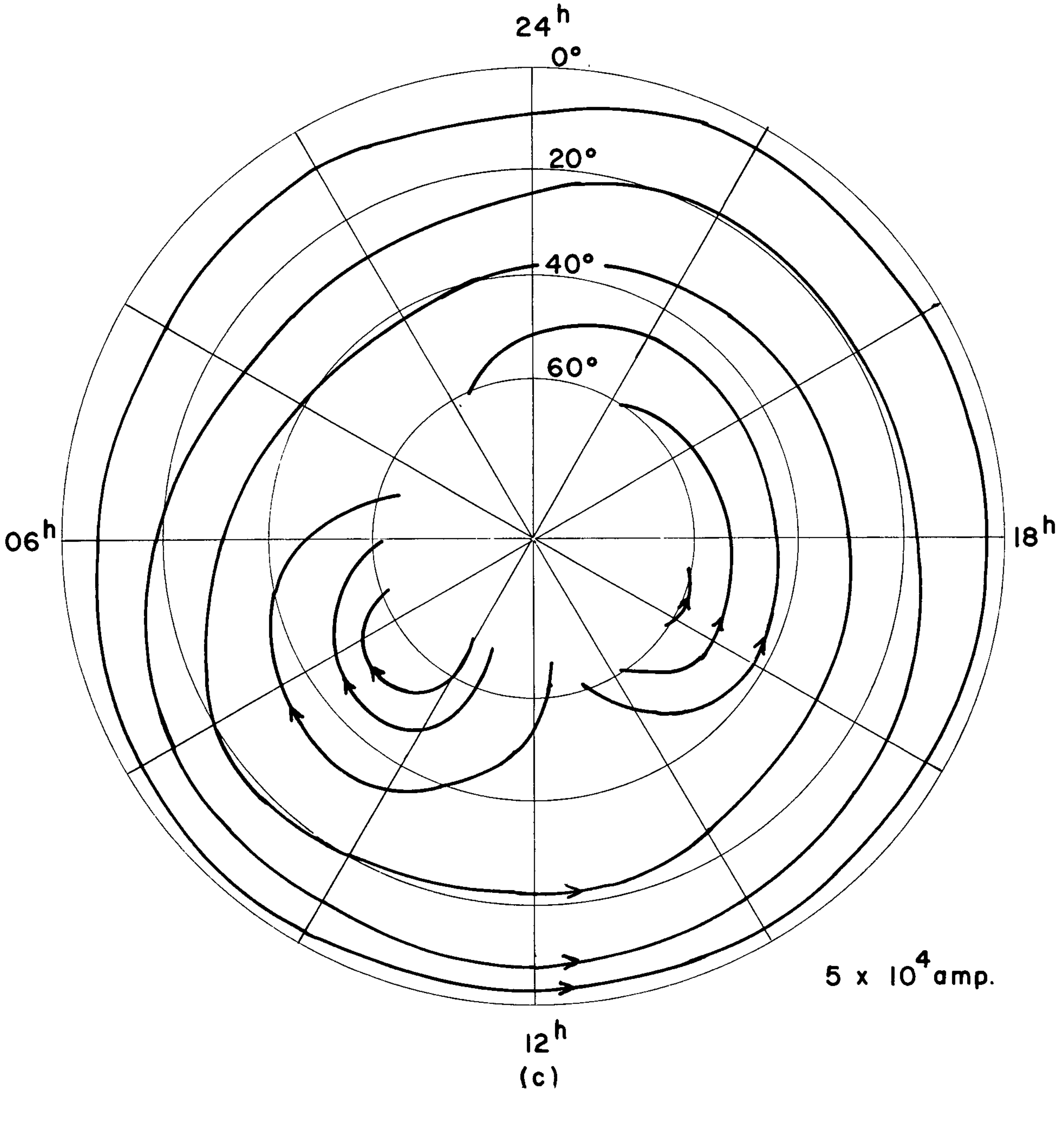


Figure 20c. (c) Third stage.

equivalent current system, which could also represent the distorted distribution of the magnetic field.

The equivalent current systems of preceding reverse change like the one shown in Figure 20a have been given by several authors. These show that the electric current flows northward around noon. If this is true, westward deflection of the magnetic field must be observed in middle and low latitudes around noon at the same time as prc in the afternoon side. Nevertheless, in no examples ever published (works already referred to and Sato, 1961, Sano, 1962), and in no cases studied by the author could such a change be found. Since the amount of rapid-run magnetograms is still limited, this finding can not be decisive but it seems to cast strong doubt to the validity of the equivalent current system for the representation of the observed magnetic field.

CHAPTER III

IONOSPHERIC SCREENING EFFECT

1. Electromagnetic field in the neutral atmosphere

From Maxwell's equations in free space:

$$\operatorname{curl} \underline{E} = -\frac{1}{c}\frac{\partial \underline{B}}{\partial t}, \tag{1}$$

$$\operatorname{curl} \underline{B} = \frac{1}{c}\frac{\partial \underline{E}}{\partial t}, \tag{2}$$

$$\operatorname{div} \underline{E} = 0, \tag{3}$$

$$\operatorname{div} \underline{B} = 0. \tag{4}$$

the electromagnetic field $\underline{E}$ and $\underline{B}$ in a neutral atmosphere can be written as

$$\underline{E} = \sum_{l,m,n,\omega} \underline{\mathcal{E}}^{l,m,n,\omega} e^{-i(lx+my+nz-\omega t)}, \tag{5}$$

$$\underline{B} = \sum_{l,m,n,\omega} \underline{\beta}^{l,m,n,\omega} e^{-i(lx+my+nz-\omega t)}, \tag{6}$$

where

$$l^2 + m^2 + n^2 = \frac{\omega^2}{c^2}. \tag{7}$$

The x, y and z axes are directed toward the south, east and vertically upwards respectively, and the ground is represented by z = 0. The horizontal components of $\underline{\mathcal{E}}^{l,m,n,\omega}$ and $\underline{\beta}^{l,m,n,\omega}$ are related by the equations

$$-\frac{i\omega}{c}\underline{\beta}_x^{l,m,n,\omega} = \frac{ilm}{n}\underline{\mathcal{E}}_x^{l,m,n,\omega} + \frac{i}{n}\left(-l^2+\frac{\omega^2}{c^2}\right)\underline{\mathcal{E}}_y^{l,m,n,\omega}, \quad (8)$$

$$-\frac{i\omega}{c}\underline{\beta}_y^{l,m,n,\omega} = \frac{i}{n}\left(m^2-\frac{\omega^2}{c^2}\right)\underline{\mathcal{E}}_x^{l,m,n,\omega} - \frac{ilm}{n}\underline{\mathcal{E}}_y^{l,m,n,\omega}. \quad (9)$$

The solid earth behaves almost as a perfect conductor for a change with a time scale of world-wide changes (Appendix A). Hence at z = 0, the horizontal component denoted by suffix h of the electric field is approximately zero, i.e.,

$$\underline{E}_h(0) = \sum_{l,m,n,\omega}\left(\underline{\mathcal{E}}_h^{l,m,n,\omega} + \underline{\mathcal{E}}_h^{l,m,-n,\omega}\right)e^{-i(lx+my-\omega t)} = 0.$$

Hence

$$\underline{\mathcal{E}}_h^{l,m,n,\omega} + \underline{\mathcal{E}}_h^{l,m,-n,\omega} = 0. \quad (10)$$

From equations (8), (9) and (10) it follows that:

$$\underline{\beta}_h^{l,m,n\omega} = \underline{\beta}_h^{l,m,-n,\omega}. \quad (11)$$

Using equations (10) and (11), equations (5) and (6) may be simplified,

$$\underline{E}_h = \sum_{l,m,\omega} \underline{\mathcal{E}}_h^{l,m,\omega}\left(e^{-inz} - e^{inz}\right) e^{-i(lx+my-\omega t)}$$

$$= -\sum_{l,m,\omega} 2inz\, \underline{\mathcal{E}}_h^{l,m,\omega}\, e^{-i(lx+my-\omega t)} \tag{12}$$

$$\underline{B}_h = \sum_{l,m,\omega} \underline{\beta}_h^{l,m,\omega}\left(e^{-inz} + e^{inz}\right) e^{-i(lx+my-\omega t)}$$

$$= \sum_{l,m,\omega} 2\, \underline{\beta}_h^{l,m,\omega}\, e^{-i(lx+my-\omega t)}, \tag{13}$$

where $$\underline{\mathcal{E}}_h^{l,m,\omega} = \underline{\mathcal{E}}_h^{l,m,n,\omega} = -\underline{\mathcal{E}}_h^{l,m,-n,\omega},$$

and $$\underline{\beta}_h^{l,m,\omega} = \underline{\beta}_h^{l,m,n,\omega} = \underline{\beta}_h^{l,m,-n,\omega}.$$

A summation over n is omitted since n^2 is uniquely determined when l,m and ω are given. $e^{\pm inz}$ is approximated by $1 \pm inz$ since $|nz| \ll 1$ in all cases to be discussed later.

The time scale of world-wide changes ranges from 10 to 10^2 sec, depending on the part of the change. Observation shows that their characteristic length of variation, measured along the surface of the earth, given by $1/\left|\frac{\nabla_h F}{F}\right|$ where F denotes the field strength and ∇_h is the horizontal gradient, is of the order of 10^8 cm in middle and low latitudes. Hence, in middle and low latitude regions, the electromagnetic field of world-wide changes may be represented by harmonics specified by

$$\sqrt{l^2+m^2} \sim 10^{-8} \quad cm^{-1},$$

$$\omega \sim 10^{-1} \text{ to } 10^{-2} \; sec^{-1}.$$

From the observed value of $\underline{\beta}_h^{l,m,\omega}$, $\underline{\varepsilon}_h^{l,m,\omega}$ can be derived using equations(8) and (9). The electromagnetic field $\underline{E}$(d) and $\underline{B}$(d) at the upper boundary of the neutral atmosphere z = d, i.e., at the lower boundary of the ionosphere, is obtained by substituting z = d into equations (12) and (13).

In the above representation of the electromagnetic field in the neutral atmosphere, the nature of the field is not specified; the solution of Maxwell's equations in the free space close to a perfect conductor is used in its general expression. This is in contrast to the discussions given by Ashour and Price (1948), Sugiura (1950) and Francis and Karplus (1960) on the ionospheric screening effect. These authors assumed, without any verification, that the observed field can be expressed either by an electromagnetic wave (Francis and Karplus) or by a magnetostatic field (Ashour and Price, Sugiura). Of these representations the former, the electromagnetic wave, seems inadequate. Observation shows that the characteristic length of variation in space is about 10^8 cm or less. Such a sharp variation can hardly be expected for an electromagnetic wave with a period longer than 10 sec, since the corresponding wavelength is longer than 3×10^{11} cm. This significant deviation of the observed field from an electromagnetic wave is physically reasonable, because the vertical scale of the neutral atmosphere is only 10^7 cm - less than about 10^{-4} of a wavelength. The source is too close for the changing electromagnetic field to have the properties of an electromagnetic wave. Hence an electromagnetic wave can not be regarded as a suitable approximation. With these given scale values in space and time, it follows from equation (7) that

$$n^2 = \frac{\omega^2}{c^2} - l^2 - m^2 \doteqdot -l^2 - m^2 \quad (14)$$

Since the horizontal distribution of the field may be represented by a Fourier series from observations on the ground, l and m are taken as real. It follows then from equation (14) that n is purely imaginary. The latter type of approximate representation, i.e., by a magnetostatic field, is characterized by the entire neglect of the displacement current. This neglect can not be taken for granted, as shown below.

The degree of the effect of ionospheric screening may be measured by the magnitude of $\underline{E}_h(d)$: since $\underline{E}_h$ is almost constant in the ionosphere as shown in the next section, $\underline{E}_h(d)$ is proportional to the magnitude of the ionospheric current which screens the incoming magnetic field. From equations (1) and (2), for dominant harmonic $\underline{\mathcal{E}}$ and $\underline{\beta}$, which compose $\underline{E}$ and $\underline{B}$, the following relationships hold:

$$|\operatorname{curl} \underline{\mathcal{E}}| \sim kL\mathcal{E} \sim \frac{\omega}{c}\beta , \tag{15}$$

$$|\operatorname{curl} \underline{\beta}| \sim k'L\beta \sim \frac{\omega}{c}\mathcal{E} , \tag{16}$$

where $\mathcal{E}$ and β denote orders of magnitudes of $\underline{\mathcal{E}}$ and $\underline{\beta}$, and $L \sim \sqrt{l^2+m^2} \sim n$ is the reciprocal of the characteristic length of variation of the field in space. Numerical factors k and k' satisfy

$$k, \quad k' \quad \lesssim \quad 1 . \tag{17}$$

(here every figure represents an order of magnitude). This means that the order of magnitude of curl $\underline{E}$ and curl $\underline{B}$ is obtained by multiplying the magnitude of the terms composing them by a factor which is smaller than unity. From equations (15) and (16) it follows that:

$$\frac{\varepsilon}{\beta} \sim \frac{\omega}{kLc} \sim \frac{k'Lc}{\omega}, \tag{18}$$

and

$$k k' \sim \left(\frac{\omega}{Lc}\right)^2 . \tag{19}$$

From equations (17) and (19) we have

$$\begin{aligned} 1 &\gtrsim k \gtrsim \left(\frac{\omega}{Lc}\right)^2, \\ 1 &\gtrsim k' \gtrsim \left(\frac{\omega}{Lc}\right)^2, \end{aligned} \tag{20}$$

where

$$\left(\frac{\omega}{Lc}\right)^2 \lesssim 1 \times 10^{-7}$$

for $\omega \lesssim 10^{-1}$ sec^{-1} and $L \sim 10^{-8}$ cm^{-1}.

It can be seen from equations (19) and (20) that except for certain special cases, both k and k' are smaller than unity. If the distribution only of the magnetic field or only of the electric field is under consideration, this will allow us to approximate the field by a magnetostatic or electrostatic field. However, in the case where relationships between the magnetic and electric fields are involved, this approximation by a static field is not permissible: since $\frac{\varepsilon}{\beta}$ is quite sensitive to the values of k or k' as can be seen from equation (18), k and k' must not be approximated by zero, small their values may be. A pair of k and k' values lies between two extreme cases. The first is when

$$k \sim 1 , \qquad k' \sim \left(\frac{\omega}{LC}\right)^2 \ll 1 . \tag{21}$$

The ratio of ε to β is then given by

$$\frac{\varepsilon}{\beta} = \frac{\omega}{LC} \lesssim 3 \times 10^{-4} . \tag{22}$$

This means that the electromagnetic field is, in the first approximation, a magnetostatic field with the non-static electric field of magnitude far smaller than that of the magnetic field. In the following, this case will be called a 'magnetostatic field'. It is the case adopted by Ashour and Price (1948) and by Sugiura (1950). (They represented the magnetic field in the neutral atmosphere by a potential field, i.e., k' is equated to zero. However, as k' cannot be set exactly to zero, as can be seen by equation (19), their assumption can be interpreted as assigning to k' its smallest possible value: $\left(\frac{\omega}{LC}\right)^2$). The opposite extreme case is when

$$k \sim \left(\frac{\omega}{LC}\right)^2 \ll 1 \, , \qquad k' \sim 1 \, . \tag{23}$$

In this case

$$\frac{\varepsilon}{\beta} = \frac{LC}{\omega} \gtrsim 3 \times 10^3 \tag{24}$$

This shows that the electromagnetic field can be described, to a first approximation, by an electrostatic field, with the magnetic field non-static and far smaller than the electric field. In the following, this case will be called an 'electrostatic field'. The real situation lies between these two extreme cases. In the case of an electromagnetic wave the ratio of ε to β is given by

$$\frac{\varepsilon}{\beta} \sim 1 \, . \tag{25}$$

Substituting equation (18) into equation (12), and noting that $L \sim n$ by equation (14), the order of magnitude of $\underline{E}_h(d)$ is given by

$$E_h(d) \sim n d \varepsilon \sim k' \frac{L^2 d C}{\omega} \beta \sim \frac{\omega d}{k C} \beta \, . \tag{26}$$

This shows that $E_h(d)$, and hence the degree of the effect of ionospheric screening, depends not only on ω - the time scale of the phenomenon, but also on L - the scale of the phenomenon in space, and on k - the degree of magnetostatic property of the observed field. Magnitudes of $\underline{E}_h(d)$ for various values of k are related by

$$k\left(E_h(d)\right)_k \sim \left(E_h(d)\right)_{magnetostatic} \sim \left(\frac{w}{Lc}\right)^2 \left(E_h(d)\right)_{electrostatic} . \tag{27}$$

Since $1 \gtrsim k$, $\left(E_h(d)\right)$ magnetostatic, which is the basis of estimation by Sugiura (1950), is the smallest value of $\left(E_h(d)\right)_k$ for various possible values of k. This means that by assuming the field in the neutral atmosphere to be magnetostatic, the ionospheric screening effect has been given the least possible significance. If the actual field is different from the magnetostatic field in the sense defined above, the ionospheric screening effect is more important than was formerly concluded. In the preceding discussion, no reason has been found why the field in the neutral atmosphere is a magnetostatic field. Therefore a general expression of the near source field given by equations (12) and (13) will be used in calculating the field in the upper atmosphere associated with the observed field. The validity of the magnetostatic field approximation will be checked afterwards by the results.

In the case of an electromagnetic wave, $E_h(d)$ is given by

$$\left(E_h(d)\right)_{wave} \sim n\,d\,\varepsilon \sim \frac{w}{c}\,d\,\beta . \tag{28}$$

since $n \sim \frac{\omega}{c}$ in this case. From equations (26) and (28) we have

$$(\underline{E}_h(d))_{wave} \sim (\underline{E}_h(d))_{magnetostatic} \tag{29}$$

Hence the degree of ionospheric screening is the same for an electromagnetic wave as for a magnetostatic field. This might explain why similar results are obtained by Sugiura (1950) and by Francis and Karplus (1960) in spite of the differences in the field representation in the neutral stmosphere - both of them concluding that the screening effect is insignificant for changes with a time scale longer than about a second.

2. Field in the ionosphere

Above the height of several tens of kilometers the following relationship holds:

$$4\pi\sigma > \omega, \tag{30}$$

where σ is the conductivity, and in this region the displacement current can be neglected in comparison with the conduction current. The electromagnetic field is then determined by,

$$\operatorname{curl} \underline{E} = -\frac{1}{c}\frac{\partial \underline{B}}{\partial t}, \tag{31}$$

$$\operatorname{curl} \underline{B} = \frac{4\pi}{c}\,\underline{j}, \tag{32}$$

$$\mathrm{div}\ \underline{E} = 4\pi q\ , \tag{33}$$

$$\mathrm{div}\ \underline{B} = 0\ , \tag{34}$$

and

$$\underline{j}_{\perp} = \sigma_1\ \underline{E}_{\perp} + \sigma_2\ \underline{E}_{\perp} \wedge \underline{b}_0\ , \tag{35}$$

$$\underline{j}_{\parallel} = \sigma_0\ \underline{E}_{\parallel}\ , \tag{36}$$

where the dielectric constant and the permeability are set equal to unity. $\underline{j}$ is the electric current density, q the electric charge density, and $\underline{b}_0$ the unit vector in the direction of the geomagnetic main field $\underline{B}_0$. The subscripts $\perp$ and $\parallel$ denote components perpendicular and parallel to $\underline{b}_0$ respectively, and σ_1 , σ_2 and σ_0 are conductivities in the three specified directions. An equation for $\underline{E}$ can be derived from these equations as

$$\mathrm{curl}\ \mathrm{curl}\ \underline{E} + \frac{4\pi}{c^2}\frac{\partial \underline{j}}{\partial t} = 0\ , \tag{37}$$

where $\underline{j} = \tilde{\sigma}\ \underline{E}$ and $\tilde{\sigma}$ the conductivity matrix. In the region where ω is smaller than ν_i: a half of the collision frequency of an ion with neutral particles (which is below 500 or 400 km for $\omega = 10^{-2}$ or 10^{-1} sec^{-1}), the medium behaves as a conductor with conductivity given by (Francis and Karplus, 1960, and Appendix II)

$$\sigma_0 = N_p e^2 \left(\frac{1}{m_e \nu_e} + \frac{1}{m_i \nu_i} \right),$$

$$\sigma_1 = N_p e^2 \left(\frac{\nu_e}{m_e(\nu_e^2 + \omega_e^2)} + \frac{\nu_i}{m_i(\nu_i^2 + \omega_i^2)} \right), \qquad (38)$$

$$\sigma_2 = N_p e^2 \left(- \frac{\omega_e}{m_e(\nu_e^2 + \omega_e^2)} + \frac{\omega_i}{m_i(\nu_i^2 + \omega_i^2)} \right),$$

where N_p is the number density of charged pairs, m_e and m_i are the masses of electrons and positive ions, e is the electron charge, ω_e and ω_i are the electron and ion cyclotron frequencies, and ν_e is the collision frequency of an electron. It is assumed in deriving these expressions for the conductivity that the static magnetic field $\underline{B}_o$ is uniform, the pressure and the gravitational force are negligible, and the ionosphere consists of electrons, one kind of positive ion and one kind of neutral molecule with the same mass as the co-existent positive ions.

The present discussion is concerned with middle and low latitude regions, and the ionosphere is assumed to be horizontally stratified, and the inclination ψ of the geomagnetic field is taken as constant. Equation (37) can be written in components as

$$\frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}\right)-\left(\frac{\partial^2 E_x}{\partial y^2}+\frac{\partial^2 E_x}{\partial z^2}\right)=-\frac{4\pi}{c^2}\frac{\partial j_x}{\partial t},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial E_x}{\partial x}+\frac{\partial E_z}{\partial z}\right)-\left(\frac{\partial^2 E_y}{\partial x^2}+\frac{\partial^2 E_y}{\partial z^2}\right)=-\frac{4\pi}{c^2}\frac{\partial j_y}{\partial t}, \qquad (39)$$

$$\frac{\partial}{\partial z}\left(\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}\right)-\left(\frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}\right)=-\frac{4\pi}{c^2}\frac{\partial j_z}{\partial t},$$

where $\underline{j}$ is given by

$$\underline{j}_x = \sigma_1(\underline{E}_x \sin\psi - \underline{E}_z\cos\psi)\sin\psi - \sigma_2\underline{E}_y\sin\psi + \sigma_0(\underline{E}_x\cos\psi + \underline{E}_z\sin\psi)\cos\psi,$$

$$\underline{j}_y = \sigma_1\underline{E}_y + \sigma_2(\underline{E}_x\sin\psi - \underline{E}_z\cos\psi), \qquad (40)$$

$$\underline{j}_z = -\sigma_1(E_x\sin\psi - E_z\cos\psi)\cos\psi + \sigma_2\underline{E}_y\cos\psi + \sigma_0(E_x\cos\psi + E_z\sin\psi)\sin\psi.$$

Equation (39) may be integrated upwards with respect to z to obtain $\underline{E}$ in the ionosphere from the given values of $\underline{E}$ and $\frac{\partial E}{\partial z}$ at the boundary. Since (σ) is assumed to depend only on z, the dependence of $\underline{E}$ on x and y does not vary with height: if the field is represented by a harmonic $e^{-i(lx+my)}$ in the neutral atmosphere it will also be proportional to $e^{-i(lx+my)}$ in the ionosphere. The following calculations will be made for a harmonic $e^{-i(lx+my-\omega t)}$. The total field is derived by summing this over l, m and ω.

From the last of equation (39) it follows that

$$\left\{ (l^2+m^2) + \frac{4\pi i \omega}{c^2}(\sigma_1 \cos^2\psi + \sigma_0 \sin^2\psi) \right\} \underline{E}_z$$

$$= i\left(l \frac{\partial \underline{E}_x}{\partial z} + m \frac{\partial \underline{E}_y}{\partial z} \right) \tag{41}$$

$$- \frac{4\pi i \omega}{c^2}\left\{ (-\sigma_1 + \sigma_0)\sin\psi \cos\psi \, \underline{E}_x + \sigma_2 \cos\psi \, \underline{E}_y \right\} .$$

Substituting equation (41) into the first two of equation (39) we have

$$\left\{ -m^2 - \frac{4\pi i \omega}{c^2}(\sigma_1 \cos^2\psi + \sigma_0 \sin^2\psi) \right\} \frac{\partial^2 \underline{E}_x}{\partial z^2} + lm \frac{\partial^2 \underline{E}_y}{\partial z^2}$$

$$= -\frac{4\pi i \omega}{c^2} 2il(-\sigma_1 + \sigma_0)\cos\psi \sin\psi \frac{\partial \underline{E}_x}{\partial z}$$

$$- \frac{4\pi i \omega}{c^2}\left\{ il\sigma_2 \cos\psi + im(-\sigma_1 + \sigma_0)\cos\psi \sin\psi \right\} \frac{\partial \underline{E}_y}{\partial z}$$

$$- \left\{ m^2(l^2+m^2) + \frac{4\pi i \omega}{c^2}(l^2+2m^2)(\sigma_1 \sin^2\psi + \sigma_0 \cos^2\psi) \right.$$

$$\left. + \left(\frac{4\pi i \omega}{c^2}\right)^2 \sigma_0 \sigma_1 \right\} \underline{E}_x \tag{42}$$

$$+ \left\{ lm(l^2+m^2) + \frac{4\pi i \omega}{c^2} lm(\sigma_1 \cos^2\psi + \sigma_0 \sin^2\psi) \right.$$

$$\left. + \frac{4\pi i \omega}{c^2}(l^2+m^2)\sigma_2 \sin\psi + \left(\frac{4\pi i \omega}{c^2}\right)^2 \sigma_0 \sigma_2 \sin\psi \right\} \underline{E}_y$$

and

$$
\begin{aligned}
& lm\,\frac{\partial^2 \underline{E}_x}{\partial z^2} + \left\{ -l^2 - \frac{4\pi i\omega}{c^2}\left(\sigma_1 \cos^2\psi + \sigma_0 \sin^2\psi\right) \right\} \frac{\partial^2 \underline{E}_y}{\partial z^2} \\
&= -\frac{4\pi i\omega}{c^2} \left\{ im\left(-\sigma_1 + \sigma_0\right)\cos\psi \sin\psi - il\sigma_2 \cos\psi \right\} \frac{\partial \underline{E}_x}{\partial z} \\
&\quad - \Big\{ -lm\left(l^2 + m^2\right) - \frac{4\pi i\omega}{c^2}\, lm\left(\sigma_1 \cos^2\psi + \sigma_0 \sin^2\psi\right) \\
&\qquad + \frac{4\pi i\omega}{c^2}\left(l^2 + m^2\right)\sigma_2 \sin\psi + \left(\frac{4\pi i\omega}{c^2}\right)^2 \sigma_0 \sigma_2 \sin\psi \Big\} \underline{E}_x \\
&\quad - \Big\{ l^2\left(l^2 + m^2\right) + \frac{4\pi i\omega}{c^2}\, l^2\left(\sigma_1 \cos^2\psi + \sigma_0 \sin^2\psi\right) \\
&\qquad + \frac{4\pi i\omega}{c^2}\left(l^2 + m^2\right)\sigma_1 \\
&\qquad + \left(\frac{4\pi i\omega}{c^2}\right)^2 \left(\sigma_1^2 \cos^2\psi + \sigma_1 \sigma_0 \sin^2\psi + \sigma_2 \cos^2\psi\right) \Big\} \underline{E}_y .
\end{aligned}
\tag{42}
$$

These are ordinary differential equations giving $\underline{E}_x$ and $\underline{E}_y$ as functions of z. The magnetic field is obtained from $\underline{E}$ by equations (31) and (41) as

$$
\begin{aligned}
-\frac{i\omega}{c}\underline{B}_x &= \left[\left(l^2 + m^2\right) + \frac{4\pi i\omega}{c^2}\left(\sigma_1 \cos^2\psi + \sigma_0 \sin^2\psi\right) \right]^{-1} \\
&\quad \Big[lm\,\frac{\partial \underline{E}_x}{\partial z} - \left\{ l^2 + \frac{4\pi i\omega}{c^2}\left(\sigma_1 \cos^2\psi + \sigma_0 \sin^2\psi\right) \right\} \frac{\partial \underline{E}_y}{\partial z} \\
&\quad + \frac{4\pi i\omega}{c^2}\, im \left\{ \left(-\sigma_1 + \sigma_0\right)\cos\psi \sin\psi\, \underline{E}_x + \sigma_2 \cos\psi\, \underline{E}_y \right\} \Big] ,
\end{aligned}
\tag{43}
$$

and

$$-\frac{i\omega}{c}\underline{B}_y = \left[(l^2+m^2)+\frac{4\pi i\omega}{c^2}(\sigma_1\cos^2\psi+\sigma_0\sin^2\psi)\right]^{-1}$$

$$\left[\left\{m^2+\frac{4\pi i\omega}{c^2}(\sigma_1\cos^2\psi+\sigma_0\sin^2\psi)\right\}\frac{\partial \underline{E}_x}{\partial z} \right. \tag{43}$$

$$-lm\frac{\partial \underline{E}_y}{\partial z}$$

$$\left. -\frac{4\pi i\omega}{c^2}il\left\{(-\sigma_1+\sigma_0)\cos\psi\sin\psi\,\underline{E}_x+\sigma_2\cos\psi\,\underline{E}_y\right\}\right].$$

Since the following condition holds in most parts of the ionosphere,

$$\sigma_0 \gg \sigma_1, \ |\sigma_2| \quad , \tag{44}$$

equation (42) can be approximated as

$$-\sin^2\psi\frac{\partial^2 \underline{E}_x}{\partial z^2} = -2il\cos\psi\sin\psi\frac{\partial \underline{E}_x}{\partial z} - im\cos\psi\sin\psi\frac{\partial \underline{E}_y}{\partial z}$$

$$-\left\{(l^2+2m^2)\cos^2\psi+\frac{4\pi i\omega}{c^2}\sigma_1\right\}\underline{E}_x$$

$$+\left\{lm\sin^2\psi+\frac{4\pi i\omega}{c^2}\sigma_2\sin\psi\right\}\underline{E}_y \quad , \tag{45}$$

and

$$-\sin^2\psi\frac{\partial^2 \underline{E}_y}{\partial z^2} = -im\cos\psi\sin\psi\frac{\partial \underline{E}_x}{\partial z}$$

$$-\left\{-lm\sin^2\psi+\frac{4\pi i\omega}{c^2}\sigma_2\sin\psi\right\}\underline{E}_x$$

$$-\left\{l^2\sin^2\psi+\frac{4\pi i\omega}{c^2}\sigma_1\sin^2\psi\right\}\underline{E}_y \, .$$

And equation (43) as

$$-\frac{i\omega}{c}\underline{B}_x = -\frac{\partial \underline{E}_y}{\partial z} + im \cot\psi \, \underline{E}_x ,$$

and

$$-\frac{i\omega}{c}\underline{B}_y = \frac{\partial \underline{E}_x}{\partial z} - il \cot\psi \, \underline{E}_y . \tag{46}$$

$\underline{E}_h$ at the boundary is given by equation (12). $\frac{\partial E_h}{\partial z}$ at the boundary may be obtained from $\underline{B}_h(d)$ and $\underline{E}_h(d)$ by means of equation (46). It was shown in the preceding section that for given E_h, B_h is largest in the 'magneto-static' case and is then given by $O\left(\frac{\sqrt{l^2+m^2}\, c}{\omega} E_h\right)$. Hence the order of magnitude of $\frac{\partial E_h}{\partial z}$ at the boundary is given by

$$\left(\frac{\partial E_h}{\partial z}\right)_d \gtrsim \sqrt{l^2+m^2}\; E_h(d) . \tag{47}$$

The variation of $\underline{E}_h$ with height can be evaluated by separating it into two parts. The first can be estimated from the gradient of $\underline{E}_h$ at the boundary given by equation (47). This will result in the variation $\Delta_1 E_h$ across the ionosphere:

$$\Delta_1 E_h \gtrsim \left(\frac{\partial E_h}{\partial z}\right)_d h \sim \sqrt{l^2+m^2}\; h \; E_h(d) , \tag{48}$$

where h is the thickness of the ionosphere which is of the order of 10^7 cm. With $\sqrt{l^2+m^2} \sim 10^{-8}$ cm^{-1}, $\Delta_1 E_h$ does not exceed $E_h(d)$ and the variation coming from this source can be neglected. The second part is due to the variation of $\frac{\partial E_h}{\partial z}$ inside the ionosphere given by equation (45). Here terms proportional to $\frac{\partial E_h}{\partial z}$ are negligible, since the variation in $\frac{\partial E_h}{\partial z}$ due to these terms is of the order of $\sqrt{\ell^2+m^2}\, h\, \frac{\partial E_h}{\partial z}$, which is smaller than $\frac{\partial E_h}{\partial z}$ by an order of 10. The largest value of $\Delta\frac{\partial E_h}{\partial z}$ is then expected at the height of maximum conductivity. Approximating this region by a layer of thickness 100 km with $\sigma \sim 10^{6}$ cgs, $\Delta\frac{\partial E_h}{\partial z}$ across this region is estimated as $10^{-7}\omega E_h$. The variation $\Delta_2 E_h$ in E_h resulting from this amount of change in $\frac{\partial E_h}{\partial z}$ is

$$\Delta_2 E_h \sim (10^{-7}\omega E_h)\, h \sim \omega E_h \,.$$

When ω is smaller than 10^{-1} sec^{-1}, or when the time scale of the change is longer than about 10 sec, $\Delta_2 E_h$ does not exceed E_h. The contribution to $\Delta\frac{\partial E_h}{\partial z}$ from other parts of the ionosphere is also negligible. Thus the variation of $\underline{E}_h$ with height can be neglected inside the ionosphere for cases of world-wide changes where a time scale is longer than about 10 sec. In the range of ω where the variation of $\underline{E}_h$ across the ionosphere is appreciable, the ionospheric screening effect may be more significant than estimated here.

Hence in the relevant range of ω, the electric field $\underline{E}_h$ inside the ionosphere and $\underline{E}_h(D)$ at the upper boundary of the ionosphere $z = D$ are equivalent and can be written as

$$\underline{E}_h = \underline{E}_h(D) = \underline{E}_h(d), \tag{49}$$

and $\dfrac{\partial \underline{E}_h}{\partial z}$ at z = D is given from equation (45) by

$$\begin{aligned}
&\left(\frac{\partial \underline{E}_x}{\partial z}\right)_D - \left(\frac{\partial \underline{E}_x}{\partial z}\right)_d \\
&= \frac{E_x}{\sin^2 I}\int_d^D \left\{(l^2+m^2)\cos^2 I + \frac{4\pi i\omega}{c^2}\sigma_1\right\} dz \\
&- \frac{E_y}{\sin^2 I}\int_d^D \left\{ lm\sin^2 I + \frac{4\pi i\omega}{c^2}\sigma_2 \sin I\right\} dz, \\
&\left(\frac{\partial \underline{E}_y}{\partial z}\right)_D - \left(\frac{\partial \underline{E}_y}{\partial z}\right)_d \\
&= \frac{E_x}{\sin^2 I}\int_d^D \left\{-l\cdot m\sin^2 I + \frac{4\pi i\omega}{c^2}\sigma_2 \sin I\right\} dz \\
&+ \frac{E_y}{\sin^2 I}\int_d^D \left\{ l^2\sin^2 I + \frac{4\pi i\omega}{c^2}\sigma_1 \sin^2 I\right\} dz .
\end{aligned} \tag{50}$$

Substituting equations (49) and (50) into equation (46) and neglecting terms of small order, we have

$$\underline{B}_x(D) - \underline{B}_x(d) = \frac{4\pi}{c}\,\frac{\Sigma_2}{\sin I}E_x + \frac{4\pi}{c}\Sigma_1 E_y$$

and

$$\underline{B}_y(D) - \underline{B}_y(d) = -\frac{4\pi}{c}\,\frac{\Sigma_1}{\sin^2 I}E_x + \frac{4\pi}{c}\,\frac{\Sigma_2}{\sin I}E_y, \tag{51}$$

where Σ_1 and Σ_2 are integrated conductivities given by

$$\Sigma_1 = \int_d^D \sigma_1 \, dz \sim 10^{13} \quad cgs ,$$

and

$$\Sigma_2 = \int_d^D \sigma_2 \, dz \sim -10^{13} \quad cgs .$$

Thus the ionosphere can be approximated by a metallic shell with anisotropic conductivity.

3. Field in the magnetosphere

Above a certain height, ranging from about 400 km for $\omega \sim 10^{-1}$ Sec^{-1} to about 500 km for $\omega \sim 10^{-2}$ sec^{-1}, ω is larger than ν_i. In this region, the electrical conductivity is given by(Francis and Karplus, 1960: and Appendix II.)

$$\sigma_0 = \frac{N_p e^2}{m_e \nu_{en}} ,$$

$$\sigma_1 = \frac{N_p m_i c^2}{B_0^2} i\omega , \qquad (52)$$

$$\sigma_2 = \frac{N_p m_i^2 c^3}{e B_0^3} \omega^2 .$$

In this region the following relationships hold between the magnitudes of σ_i.

$$\sigma_0 \gg |\sigma_1| \gg \sigma_2 . \qquad (53)$$

In contrast to the ionospheric conductivity given by equation (38), the conductivity given above depends on ω. This part of the upper atmosphere behaves as a magnetohydrodynamic medium. In terms of the Alfven wave velocity:

$V = \sqrt{\dfrac{B_0^2}{4\pi N_p m_i}}$, σ_1 can be written as

$$\sigma_1 = \frac{1}{4\pi}\left(\frac{c}{V}\right)^2 (i\omega)$$

The Alfven wave velocity V varies significantly with height: it increases from 2×10^7 cm $\sec^{-1}$ at a height of 500 km to 5×10^8 cm $\sec^{-1}$ at a height of 3000 km and then decreases towards the outer boundary of the magnetosphere (Dessler et al, 1960). In the following discussion, however, V will be approximated by 1×10^8 cm $\sec^{-1}$. Then for $\sqrt{l^2+m^2} \sim 10^{-8}$ cm^{-1} and for $\omega \lesssim 10^{-1}$ sec $^{-1}$, the following relationships hold.

$$\frac{4\pi\omega\sigma_0}{c^2} > l^2 + m^2,$$

$$\left|\frac{4\pi\omega\sigma_1}{c^2}\right| = \left(\frac{\omega}{V}\right)^2 < l^2 + m^2 . \tag{54}$$

Using equations (53) and (54), equations (42) can be approximated by

$$-(-iN)^2 \sin^2\psi\, \underline{E_x} = -2il\cos\psi\sin\psi(-iN)\underline{E_x} - im\cos\psi\sin\psi(-iN)\underline{E_y}$$
$$- \left\{(l^2+2m^2)\cos^2\psi + \frac{(i\omega)^2}{V^2}\right\}\underline{E_x} + lm\sin^2\psi\,\underline{E_y}, \tag{55}$$

and

$$-(-iN)^2 \sin^2\psi \, \underline{E}_y = -im \cos\psi \sin\psi \, (-iN) \underline{E}_x + lm \sin^2\psi \, \underline{E}_x - \left\{ l^2 \sin^2\psi + \frac{\sin^2\psi (iw)^2}{V^2} \right\} \underline{E}_y ,$$

where $\underline{E}_x$ and $\underline{E}_y$ are assumed to vary as e^{-iNz} with height. From these equations, a dispersion equation may be obtained.

$$X^4 + \left(-m^2 + Y^2 - \frac{2}{V^2}\right) X^2 - \frac{Y^2 - m^2}{V^2} + \frac{1}{V^4} = 0 ,$$

where (56)

$$X = -iN \sin\psi - il \cos\psi ,$$

$$Y = -iN \cos\psi + il \sin\psi ,$$

and $\underline{E}_x$ and $\underline{E}_y$ are related by

$$\frac{\underline{E}_y}{\underline{E}_x} = \frac{im Y}{\sin\psi \left(X^2 + Y^2 + \frac{w^2}{V^2} \right)} . \tag{57}$$

Equation (56) can be solved to give

$$-iN = \pm \sqrt{l^2 + m^2} \quad \text{or} \quad il \cot\psi \pm \frac{iw}{V \sin\psi} . \tag{58}$$

For all four modes, $|N|$ is of the same order as l or m, and hence equation (41) can be approximated as

$$\frac{\underline{E}_z}{\underline{E}_x} = - \cot\psi . \tag{59}$$

This shows that the electric field in this region is approximately perpendicular to the direction of the magnetic lines of force as a result of the high conductivity in that direction.

$\underline{E}$ in the magnetosphere can now be written as

$$\underline{E} = \sum_{l,m,w} \Big[E_1^{l,m,w} \left(\frac{-im \sin\psi}{\sqrt{l^2+m^2}\cos\psi + il\sin\psi}, \; 1, \; \frac{im\cos\psi}{\sqrt{l^2+m^2}\cos\psi + il\sin\psi} \right) \cdot e^{-i(lx+my-wt)+\sqrt{l^2+m^2}(z-D)}$$

$$+ E_2^{l,m,w} \left(\frac{-im\sin\psi}{-\sqrt{l^2+m^2}\cos\psi + il\sin\psi}, \; 1, \; \frac{im\cos\psi}{-\sqrt{l^2+m^2}\cos\psi + il\sin\psi} \right) \cdot e^{-i(lx+my-wt)-\sqrt{l^2+m^2}(z-D)} \tag{60}$$

$$+ E_3^{l,m,w} \left(\sin\psi, \; \frac{im\sin\psi}{il + \frac{iw}{V}\cos\psi}, \; -\cos\psi \right) \cdot e^{-i\left\{lx+my+\left(-l\cot\psi - \frac{w}{V\sin\psi}\right)(z-D)-wt\right\}}$$

$$+ E_4^{l,m,w} \left(\sin\psi, \; \frac{im\sin\psi}{il - \frac{iw}{V}\cos\psi}, \; -\cos\psi \right) \cdot e^{-i\left\{lx+my+\left(-l\cot\psi + \frac{w}{V\sin\psi}\right)(z-D)-wt\right\}} \Big].$$

In the first two modes, the electric field varies exponentially with height. Of these two modes, mode 1, which diverges as $z \to \infty$, represents the field incident from above, and mode 2, which converges to zero as $z \to \infty$, represents the one reflected from below. The second two modes represent wave propagations in $\pm$ z directions with an amplitude determined as a function of the x, y and z coordinates. In the northern hemisphere where $\psi > 0$, mode 3 corresponds to an incident wave propagating downwards, and mode 4 to the reflected wave propagating upwards. In the southern hemisphere where $\psi < 0$, the roles of modes 3 and 4 are interchanged. The magnetic field $\underline{B}$ can be derived by substituting equation (60) into equation (43) under conditions (53) and (54)

$$\begin{aligned}
\underline{B} = -\frac{c}{i\omega}\sum_{l,m,\omega}\Bigg[& E_1^{l,m,\omega}\left(l,\ m,\ i\sqrt{l^2+m^2}\right)\frac{-im^2\cos\psi\sin\psi - l\sqrt{l^2+m^2}}{l^2+m^2\cos^2\psi} \\
& \cdot e^{-i(lx+my-\omega t)+\sqrt{l^2+m^2}(z-D)} \\
& + E_2^{l,m,\omega}\left(l,\ m,\ -i\sqrt{l^2+m^2}\right)\frac{-im^2\cos\psi\sin\psi + l\sqrt{l^2+m^2}}{l^2+m^2\cos^2\psi} \\
& \cdot e^{-i(lx+my-\omega t)-\sqrt{l^2+m^2}(z-D)} \\
& + E_3^{l,m,\omega}\left(\frac{-im\sin^2\psi\,\frac{i\omega}{V}}{il+\frac{i\omega}{V}\cos\psi},\ \frac{i\omega}{V},\ \frac{im\sin\psi\cos\psi\,\frac{i\omega}{V}}{il+\frac{i\omega}{V}\cos\psi}\right) \\
& \cdot e^{-i\left\{lx+my+\left(-l\cot\psi-\frac{\omega}{V\sin\psi}\right)(z-D)-\omega t\right\}} \\
& + E_4^{l,m,\omega}\left(\frac{im\sin^2\psi\,\frac{i\omega}{V}}{il-\frac{i\omega}{V}\cos\psi},\ -\frac{i\omega}{V},\ \frac{-im\sin\psi\cos\psi\,\frac{i\omega}{V}}{il-\frac{i\omega}{V}\cos\psi}\right) \\
& \cdot e^{-i\left\{lx+my+\left(-l\cot\psi+\frac{\omega}{V\sin\psi}\right)(z-D)-\omega t\right\}}\Bigg].
\end{aligned} \tag{61}$$

The present solution is different from the 'magnetohydrodynamic wave' usually adopted. This comes from the second of the relationships (54) which, in terms of the magnetohydrodynamic wave length $\lambda = \frac{2\pi V}{\omega}$, can be written as

$$\frac{2\pi}{\sqrt{\ell^2+m^2}} < \lambda ,$$

showing that the observed field varies in space with a scale shorter than the wavelength. This is the situation analogous to what existed in the neutral atmosphere.

The magnetospheric field given by equations (60) and (61) is related to the observed field in the neutral atmosphere through equations (49) and (51), where $\underline{E}_h(D)$ and $\underline{B}_h(D)$ are given by substituting z = D into equations (60) and (61). The effect of the ionospheric screening is considered for the case where $|\ell| \sim |m|$, i.e. where the scale of the variation of the observed field is similar for both north-south and east-west directions. The orders of magnitudes of the incident field in the northern hemisphere are then given by

$$\begin{aligned} E_1 &\sim \frac{4\pi\omega}{\ell c^2} I_x + \frac{4\pi\omega}{\ell^2 c^2}(m I_y + \ell I_x) + \frac{\omega}{\ell c}\beta_y \\ &\quad + \frac{\omega}{\ell^2 c}(m\beta_x - \ell\beta_y) + d\,(m\varepsilon_x - \ell\varepsilon_y) , \\ E_3 &\sim \frac{4\pi\omega}{\ell c^2} I_x + \frac{1}{\ell - \frac{\omega}{V}}\frac{4\pi\omega}{c^2}(m I_y + \ell I_x) + \frac{\omega}{\ell c}\beta_y \\ &\quad + \frac{1}{\ell - \frac{\omega}{V}}\frac{\omega}{c}(m\beta_x - \ell\beta_y) + d\,\ell\,\varepsilon_x + d\,\ell\,\varepsilon_y , \end{aligned} \tag{62}$$

where terms which always appear in a certain combination are written in brackets, and I_x, I_y, β_x and β_y are related to $\mathcal{E}_x$ and $\mathcal{E}_y$ by

$$I_x = \frac{-\Sigma_1}{\sin^2\psi} z\,i\,n\,d\,\mathcal{E}_x + \frac{\Sigma_2}{\sin\psi} z\,i\,n\,d\,\mathcal{E}_y \sim \Sigma\, l\, d\, \mathcal{E},$$

$$I_y = \frac{-\Sigma_2}{\sin\psi} z\,i\,n\,d\,\mathcal{E}_x - \Sigma_1\, z\,i\,n\,d\,\mathcal{E}_y \sim \Sigma\, l\, d\, \mathcal{E},$$

$$-\frac{i\omega}{c}\beta_x \sim k\,l\,\mathcal{E},$$

$$-\frac{i\omega}{c}\beta_y \sim k\,l\,\mathcal{E},$$

and $$m\beta_x - l\beta_y \sim |\operatorname{curl} \underset{\sim}{\beta}| \sim \frac{\omega}{c}\mathcal{E}.$$

Details of the derivation are presented in Appendix B. With $l, m, n \sim 10^{-8}\ cm^{-1}$, $\Sigma \sim 10^{13}\ cgs$, $d \sim 10^{7}\ cm$ and $V \sim 10^{8}\ cm\ sec^{-1}$, the orders of magnitudes of the terms in the first of equation (62) are estimated as follows:

$$\frac{4\pi\omega}{l c^2} I_x \sim \omega\mathcal{E},\quad \frac{4\pi\omega}{l^2 c^2}(m I_y + l I_x) \sim \omega\mathcal{E},\quad \frac{\omega}{l c}\beta_y \sim k\mathcal{E},$$

$$\frac{\omega}{l^2 c}(m\beta_x - l\beta_y) \sim 10^{-5}\omega^2\mathcal{E},\quad d\,(m\mathcal{E}_x - l\mathcal{E}_y) \sim 10^{-1} k\mathcal{E}.$$

Similarly, the magnitudes of the terms in the second of equation (62) are

$$\frac{4\pi\omega}{l c^2} I_x \sim \omega\mathcal{E},\quad \frac{1}{l\frac{\omega}{V}}\frac{4\pi\omega}{c^2}(m I_y + l I_x) \sim \mathcal{E},\quad \frac{\omega}{l c}\beta_y \sim k\mathcal{E},$$

$$\frac{1}{l\frac{\omega}{V}}\frac{\omega}{c}(m\beta_x - l\beta_y) \sim 10^{-5}\omega\mathcal{E},\quad d\,l\,\mathcal{E}_x \sim 10^{-1}\mathcal{E},\quad d\,l\,\mathcal{E}_y \sim 10^{-1}\mathcal{E}.$$

Since $10^{-1} \gtrsim \omega \gtrsim 10^{-2}\,\mathrm{sec}^{-1}$ and $1 \gtrsim k$, equation (62) can then be approximated by,

$$\begin{aligned} E_1 &\sim \underset{(\omega\varepsilon)}{\frac{4\pi\omega}{\ell c^2} I} + \underset{(k\varepsilon)}{\frac{\omega}{\ell c}\beta}, \\ E_3 &\sim \underset{(\varepsilon)}{\frac{4\pi V}{c^2} I} + \underset{(k\varepsilon)}{\frac{\omega}{\ell c}\beta}. \end{aligned} \tag{63}$$

Written below each term is its order of magnitude.

The observed magnetic field $\underline{\beta}$ consists of the incident field and the field $\underline{b}$ due to the ionospheric current, where b is given by

$$b \sim \frac{4\pi}{c} I \tag{64}$$

If b is smaller than the observed field, the ionospheric screening effect can be neglected since the observed field must approximately be the same as the incident field. But if on the other hand b is comparable with, or larger than the observed field, $\underline{b}$ must have cancelled the significant part of the incident field, i.e. the screening is effective. Hence a comparison with unity of the ratio of b to β can be used as a criterion of the significance of the ionospheric screening effect.

Now let us consider the case where only the field of mode 1 is incident on the ionosphere. Then from equations (63) and (64) it follows that

$$\begin{aligned} E_1 &\sim \frac{\omega}{\ell c} b + \frac{\omega}{\ell c}\beta, \\ 0 &\sim \frac{V}{c} b + \frac{\omega}{\ell c}\beta. \end{aligned} \tag{65}$$

Hence we have

$$b \sim \frac{\omega}{\ell v} \beta \sim \omega \beta \quad . \tag{66}$$

Since $\omega \lesssim 10^{-1} sec^{-1}$, it follows then that

$$\frac{b}{\beta} < 1 \quad ,$$

that is, the field due to the ionospheric current is smaller than the observed field. Thus for the incidence of the field of mode 1, the ionospheric screening effect will not be significant. The order of magnitude of k in this case can be seen from equations (63) and (65) to be

$$k \sim 1 \quad . \tag{67}$$

This shows that the field in the neutral atmosphere is 'magnetostatic', and the approximation adopted by earlier work is correct in this case.

But when the field of mode 3 is incident on the ionosphere, from equations (63) and (64) we have

$$\begin{aligned} 0 &\sim \frac{\omega}{\ell c} b + \frac{\omega}{\ell c} \beta \quad , \\ E_3 &\sim \frac{v}{c} b + \frac{\omega}{\ell c} \beta \quad . \end{aligned} \tag{68}$$

It follows then

$$b \sim \beta \tag{69}$$

for any ω ranging from 10^{-1} sec^{-1} to 10^{-2} sec^{-1}. In this case, the ionospheric screening effect cannot be ignored.

Due to the anisotropy of the inospheric conductivity, this amount of b can produce a significant difference in the direction, if not in the magnitude, between the incident and the observed magnetic field. The order of magnitude of k is then

$$k \sim \omega \qquad \text{i.e.} \qquad 10^{-2} \lesssim k \lesssim 10^{-1}, \tag{70}$$

as can be seen from equations (63) and (68). This shows that the observed field is not 'magnetostatic'. The present result is in accordance with the discussion given in section 1 that ionospheric screening may be effective when the observed field is sufficiently different from the 'magnetostatic field'.

The above deviation of the observed field from the magnetostatic field is, however, practically impossible to detect by observation: from equations (19) and (70) it follows that $10^{-6} \lesssim k' \lesssim 10^{-5}$ and accordingly five to six figure accuracy is required in the determination of $|\,\mathrm{curl}\ \underline{\beta}\,|$, and hence in the observation of $\underline{B}$ to detect this difference. The situation in the southern hemisphere can be discussed in a similar way by solving equations (49) and (51) for E_1 and E_4, instead of E_1 and E_3.

The present result is obtained for the special case where $|l| \sim |m|$, but it will be seen in the next section that a similar conclusion may be derived for the case where $|l| \gg |m|$. Hence it may generally be concluded that ionospheric screening can be effective for changes in the geomagnetic field with a time scale of 10^{-1} - 10^{-2} sec^{-1}. The degree of screening depends not only on the time scale, but also on the scale in space and on the mode of the incident field, and will have to be estimated in each individual case. It is also found that the observed field can not always be represented by a magnetostatic field, and an underestimation of the screening effect can sometimes result if the observed field is always approximated by a magnetostatic field or an electromagnetic wave.

After completion of the present calculations, it became known that Dr. J. W. Dungey had estimated the magnitude of the ionospheric current based on similar ideas to those presented above (private communication). But his calculations are carried out for the extreme case which is called 'electrostatic field' in this paper, for which the magnitude of the ionospheric current is the largest as can be seen from equation (27). Since the actual field is quite different from the electrostatic field as shown above (k': $10^{-6} \sim 10^{-5}$ while $k' \sim 1$ for the case of an electrostatic field), his results will lead to an overestimation of the screening effect.

APPENDIX A

For changes with a time scale of 10 - 10^2 sec, the earth may be approximated as a perfect conductor for the following reason.

Due to the finiteness of the conductivity of the earth, the horizontal component of the electric field at the ground does not vanish completely, and strictly speaking equation (10) must be replaced by

$$\underline{\varepsilon}_h^{\,l,m,n,\omega} + \underline{\varepsilon}_h^{\,l,m,-n,\omega} = \Delta \underline{\varepsilon}_h^{\,l,m,\omega} \tag{A1}$$

Accordingly the electric field in the neutral atmosphere is given, instead of by equation (12), by

$$\underline{E}_h = \sum_{l,m,\omega} \left\{ 2inz\, \underline{\varepsilon}_h^{\,l,m,\omega} + \Delta \underline{\varepsilon}_h^{\,l,m,\omega} \right\} e^{-i(lx+my-\omega t)} \tag{A2}$$

$\Delta \underline{\varepsilon}_h^{\,l,m,\omega}$ may be neglected, and the earth may be approximated by a perfect conductor, if

$$2nd\, \varepsilon_h^{\,l,m,\omega} \gg \Delta \varepsilon_h^{\,l,m,\omega} \tag{A3}$$

since under this condition the field in the upper atmosphere may approximately be determined independently of $\Delta \underline{\varepsilon}_h^{\,l,m,\omega}$.

ε is related to β by equation (18) as

$$\varepsilon \sim \frac{\omega}{kLc}\,\beta \tag{A4}$$

As discussed in the latter part of section 1, the ratio $\frac{\varepsilon}{\beta}$ is smallest when the field is magnetostatic, i.e. when k = 1. Hence the lower limit of ε is given by

$$\varepsilon_{min} \sim \frac{\omega}{Lc}\beta \quad . \tag{A5}$$

Then the left-hand side of (A3) is

$$2nd\,\varepsilon_h^{l,m,\omega} \gtrsim 10^{-3}\,\omega\beta \tag{A6}$$

with $L, n \sim 10^{-8}\,cm^{-1}$ and $d \sim 10^{7}\,cm$. Observations show that $\Delta\varepsilon_h/\beta$ is of the order of 1 mV/γ , i.e. 10^{-6} cgs in the relevant range of ω, i.e.

$$\Delta\varepsilon_h^{l,m,\omega} \sim 10^{-6}\,\beta \tag{A7}$$

(A6) and (A7) show that the the condition (A3) is satisfied for $10^{-2} \lesssim \omega \lesssim 10^{-1}\,sec^{-1}$, and the earth may be approximated by a perfect conductor.

APPENDIX B

Derivation of equation (62)

From equations (49) and (51) it follows that

$$-\frac{im\sin\psi}{\sqrt{l^2+m^2}\cos\psi+il\sin\psi}E_1-\frac{im\sin\psi}{-\sqrt{l^2+m^2}\cos\psi+il\sin\psi}E_2$$

$$+\sin\psi\,E_3+\sin\psi\,E_4=-2ind\,\mathcal{E}_x, \tag{B1}$$

$$E_1+E_2+\frac{im\sin\psi}{il+\frac{iw}{v}\cos\psi}E_3+\frac{im\sin\psi}{il-\frac{iw}{v}\cos\psi}E_4$$

$$=-2ind\,\mathcal{E}_y, \tag{B2}$$

$$-\frac{l(-im^2\cos\psi\sin\psi-l\sqrt{l^2+m^2})}{l^2+m^2\cos^2\psi}E_1-\frac{l(-im^2\cos\psi\sin\psi+l\sqrt{l^2+m^2})}{l^2+m^2\cos^2\psi}E_2$$

$$+im\frac{\sin^2\psi\,\frac{iw}{v}}{il+\frac{iw}{v}\cos\psi}E_3-im\frac{\sin^2\psi\,\frac{iw}{v}}{il-\frac{iw}{v}\cos\psi}E_4$$

$$=\frac{4\pi iw}{c^2}I_y+\frac{2iw}{c}\beta_x, \tag{B3}$$

$$-\frac{m(-im^2\cos\psi\sin\psi-l\sqrt{l^2+m^2})}{l^2+m^2\cos^2\psi}E_1-\frac{m(-im^2\cos\psi\sin\psi+l\sqrt{l^2+m^2})}{l^2+m^2\cos^2\psi}E_2$$

$$-\frac{iw}{v}E_3+\frac{iw}{v}E_4$$

$$=\frac{-4\pi iw}{c^2}I_x+\frac{2iw}{c}\beta_y, \tag{B4}$$

where $$I_x = -\frac{\Sigma_1}{\sin^2\psi}\, 2ind\, \varepsilon_x + \frac{\Sigma_2}{\sin\psi}\, 2ind\, \varepsilon_y, \tag{B5}$$

$$I_y = -\frac{\Sigma_2}{\sin\psi}\, 2ind\, \varepsilon_x - \Sigma_1 \cdot 2ind\, \varepsilon_y, \tag{B6}$$

and $$-\frac{i\omega}{c}\beta_x = \frac{i\ell m}{n}\varepsilon_x + \frac{i}{n}\left(-\ell^2 + \frac{\omega^2}{c^2}\right)\varepsilon_y, \tag{B7}$$

$$-\frac{i\omega}{c}\beta_y = \frac{i}{n}\left(m^2 - \frac{\omega^2}{c^2}\right)\varepsilon_x - \frac{i\ell m}{n}\varepsilon_y. \tag{B8}$$

The summation over ℓ, m and ω are dropped, and in the following calculation, terms of the order of $\frac{\omega}{V}/\ell, m$ will be ignored.

Subtracting (B2) from (B1) we have

$$\begin{aligned} &-\frac{m}{\ell}\frac{im\sqrt{\ell^2+m^2}\cos\psi\sin\psi}{\ell^2+m^2\cos^2\psi}(E_1 - E_2) + \left\{\frac{m}{\ell}\frac{im\cdot i\ell\sin\psi\cos\psi}{\ell^2+m^2\cos^2\psi} - 1\right\}\cdot \\ &\cdot(E_1+E_2) + \frac{m}{\ell}\sin\psi\frac{\omega\cos\psi}{V\ell}(E_3 - E_4) \\ &= -2ind\left(\frac{m}{\ell}\varepsilon_x - \varepsilon_y\right). \end{aligned} \tag{B9}$$

Equation (B4) can be written as

$$\begin{aligned} &\frac{im^3\cos\psi\sin\psi}{\ell^2+m^2\cos^2\psi}(E_1+E_2) + \frac{\ell m\sqrt{\ell^2+m^2}}{\ell^2+m^2\cos^2\psi}(E_1 - E_2) \\ &-\frac{i\omega}{V}(E_3 - E_4) = -\frac{4\pi i\omega}{c^2}I_x + 2\frac{i\omega}{c}\beta_y. \end{aligned} \tag{B10}$$

Subtracting (B4) from (B3), we have

$$\left(\frac{m^2}{l}\sin^2\psi\,\frac{i\omega}{v}+\frac{i\omega}{v}\,l\right)(E_3-E_4)-i\frac{m^2}{l^2}\left(\frac{\omega}{v}\right)^2\sin^2\psi\cos\psi\cdot$$

$$\cdot(E_3+E_4)=\frac{4\pi i\omega}{c^2}(m I_y+l I_x)+2\frac{i\omega}{c}(m\beta_x-l\beta_y), \qquad \text{(B 11)}$$

From (B2) and (B11), it follows that

$$(E_1+E_2)+\frac{l^2+m^2\sin^2\psi}{m\frac{\omega}{v}\sin\psi\cos\psi}(E_3-E_4)$$

$$=-2ind\varepsilon_y+\frac{l}{im\left(\frac{\omega}{v}\right)^2\sin\psi\cos\psi}\left\{\frac{4\pi i\omega}{c^2}(m I_y+l I_x)+\frac{2i\omega}{c}(m\beta_x-l\beta_y)\right\}. \qquad \text{(B 12)}$$

Eliminating (E_3 - E_4) in (B9) by (B12), we have

$$\frac{(l^2+m^2)(l^2+m^2\sin^2\psi)}{(l^2+m^2\cos^2\psi)\,m\frac{\omega}{v}\sin\psi\cos\psi}(E_1+E_2)+\frac{m\sqrt{l^2+m^2}}{l\frac{\omega}{v}}(E_1-E_2)$$

$$=\frac{1}{l\frac{i\omega}{v}}\left\{\frac{4\pi i\omega}{c^2}(m I_y+l I_x)+2\frac{i\omega}{c}(m\beta_x-l\beta_y)\right\}$$

$$+\frac{l^2+m^2\sin^2\psi}{m\frac{\omega}{v}\sin\psi\cos\psi}\,2ind\left(\frac{m}{l}\varepsilon_x-\varepsilon_y\right). \qquad \text{(B 13)}$$

From (B9) and (B10) it follows that

$$
\begin{aligned}
&\frac{i\left(m^4\sin^2\psi\cos^2\psi - l^2(l^2+m^2)\right)}{l^2\left(l^2+m^2\cos^2\psi\right)}\left(E_1+E_2\right) \\
&+ \frac{2m^2\sqrt{l^2+m^2}}{l\left(l^2+m^2\cos^2\psi\right)}\sin\psi\cos\psi\left(E_1-E_2\right) \\
&= \frac{m}{l^2}\sin\psi\cos\psi\left(-\frac{4\pi i\omega}{c^2}I_x + 2\frac{i\omega}{c}\beta_y\right) \\
&\quad + 2nd\left(\frac{m}{l}\varepsilon_x - \varepsilon_y\right).
\end{aligned}
\tag{B14}
$$

Then (E_1+E_2) and (E_1-E_2) can be given as

$$
\begin{aligned}
&D\left(E_1+E_2\right) \\
&= \left\{\frac{m}{l^2}\sin\psi\cos\psi\left(-\frac{4\pi i\omega}{c^2}I_x + 2\frac{i\omega}{c}\beta_y\right) + 2nd\left(\frac{m}{l}\varepsilon_x - \varepsilon_y\right)\right\}\frac{im\sqrt{l^2+m^2}}{l\frac{\omega}{v}} \\
&- \left[\frac{1}{l\frac{i\omega}{v}}\left\{\frac{4\pi i\omega}{c^2}\left(mI_y + lI_x\right) + 2\frac{i\omega}{c}\left(m\beta_x - l\beta_y\right)\right\}\right. \\
&\left. + \frac{l^2+m^2\sin^2\psi}{m\frac{\omega}{v}\sin\psi\cos\psi}\, 2ind\left(\frac{m}{l}\varepsilon_x - \varepsilon_y\right)\right]\frac{2m^2\sqrt{l^2+m^2}}{l\left(l^2+m^2\cos^2\psi\right)}\sin\psi\cos\psi ,
\end{aligned}
\tag{B15}
$$

where

$$
\begin{aligned}
D = &\frac{i\left(m^4\sin^2\psi\cos^2\psi - l^2(l^2+m^2)\right)}{l^2\left(l^2+m^2\cos^2\psi\right)}\,\frac{im\sqrt{l^2+m^2}}{l\frac{\omega}{v}} \\
&- \frac{2(l^2+m^2)(l^2+m^2\sin^2\psi)\,m^2\sqrt{l^2+m^2}}{l\left(l^2+m^2\cos^2\psi\right)^2 m\frac{\omega}{v}\sin\psi\cos\psi}\sin\psi\cos\psi .
\end{aligned}
$$

and

$$
\begin{aligned}
&D(E_1 - E_2) \\
&= \left[\frac{1}{l\,\frac{i\omega}{v}} \left\{ \frac{4\pi i\omega}{c^2}(m I_y + l I_x) + \frac{2i\omega}{c}(m\beta_x - l\beta_y) \right\} \right. \\
&\qquad \left. + \frac{l^2 + m^2\sin^2\psi}{m\frac{\omega}{v}\sin\psi\cos\psi}\, 2ind\left(\frac{m}{l}\varepsilon_x - \varepsilon_y\right) \right] \frac{i\left(m^4\sin^2\psi\cos^2\psi - l^2(l^2+m^2)\right)}{l^2(l^2+m^2\cos^2\psi)} \\
&\quad - \left\{ \frac{m}{l^2}\sin\psi\cos\psi\left(-\frac{4\pi i\omega}{c^2} I_x + 2\frac{i\omega}{c}\beta_y\right) + 2ind\left(\frac{m}{l}\varepsilon_x - \varepsilon_y\right) \right\} \\
&\quad \cdot \frac{(l^2+m^2)(l^2+m^2\sin^2\psi)}{(l^2+m^2\cos^2\psi)\, m\frac{\omega}{v}\sin\psi\cos\psi} .
\end{aligned}
\tag{B16}
$$

Hence under the condition that $|l| \sim |m|$, orders of magnitude E_1 and E_2 are given as,

$$
\begin{aligned}
E_1,\ E_2 \sim\ & \frac{4\pi\omega}{l c^2} I_x + \frac{4\pi\omega}{l^2 c^2}(m I_y + l I_x) + \frac{\omega}{lc}\beta_y \\
& + \frac{\omega}{l^2 c}(m\beta_x - l\beta_y) + 2d(m\varepsilon_x - l\varepsilon_y) .
\end{aligned}
\tag{B17}
$$

Now, from (B1) and (B2), we have

$$\left(-\frac{im\sin\psi}{\sqrt{l^2+m^2}\cos\psi+il\sin\psi}+\frac{im\sin\psi}{-\sqrt{l^2+m^2}\cos\psi+il\sin\psi}\right)E_1$$
$$+\left(\sin\psi+\frac{im\sin\psi}{il+\frac{iw}{v}\cos\psi}\;\frac{im\sin\psi}{-\sqrt{l^2+m^2}\cos\psi+il\sin\psi}\right)E_3$$
$$+\left(\sin\psi+\frac{im\sin\psi}{il-\frac{iw}{v}\cos\psi}\;\frac{im\sin\psi}{-\sqrt{l^2+m^2}\cos\psi+il\sin\psi}\right)E_4 \tag{B18}$$
$$=-2imd\left(\varepsilon_x+\frac{im\sin\psi}{-\sqrt{l^2+m^2}\cos\psi+il\sin\psi}\varepsilon_y\right),$$

and (B11) leads to

$$E_4=\frac{1}{-\frac{im^2\frac{iw}{v}\sin^2\psi}{il-\frac{iw}{v}\cos\psi}-\frac{iw}{v}l}\left\{-\left(\frac{im^2\frac{iw}{v}\sin^2\psi}{il+\frac{iw}{v}\cos\psi}+\frac{iw}{v}l\right)E_3\right.$$
$$\left.+\frac{4\pi iw}{c^2}(mI_y+lI_x)+2\frac{iw}{c}(m\beta_x-l\beta_y)\right\}. \tag{B19}$$

From (B17), (B18) and (B19), orders of magnitude of E_3 and E_4 are given as,

$$E_3,\,E_4\sim\frac{4\pi w}{lc^2}I_x+\frac{1}{l\frac{w}{v}}(mI_y+lI_x)+\frac{w}{lc}\beta_y$$
$$+\frac{1}{l\frac{w}{v}}\frac{w}{c}(m\beta_x-l\beta_y)+2dl\varepsilon_x+2dl\varepsilon_y. \tag{B20}$$

CHAPTER IV

THEORY OF WORLD-WIDE CHANGES

It follows from the discussion in Chapter III that the screening effect of the ionosphere may be appreciable for world-wide changes. Hence the observed field which is analyzed in Chapter II must be corrected for the modulation by the ionosphere before we can infer the cause of the phenomenon which is supposed to operate at the magnetospheric boundary.

The main change of world-wide changes has a time scale of a few minutes. As illustrated in Figure 19b, its magnitude changes but little with local time, but varies with latitude, in middle and low latitudes, with a characteristic scale of the order of 10^8 cm. Hence for this part of the change we have,

$$\omega \sim 10^{-2}\ \mathrm{sec}^{-1},$$
$$l \sim 10^{-8}\ \mathrm{cm}^{-1},$$
$$|m| \ll |l| .$$

On the other hand, the preceding reverse change, which has a time scale of several tens of seconds, is observed only in a limited range of local time as shown by Figure 19a. In middle and low latitudes, the characteristic distance of variation of its magnitude is of the order of 10^8 cm both latitudinally and longitudinally. However, to avoid the mathematical complexity encountered in the preceding chapter (see Appendix B), the variation with local time is neglected, and for the preceding reverse change we take ω, ℓ and m as

$$\omega \sim 10^{-1} \ sec^{-1},$$
$$\ell \sim 10^{-8} \ cm^{-1},$$
$$|m| \ll |\ell| \ .$$

The change in the horizontal component of the electromagnetic field at the upper boundary of the neutral atmosphere (z = d) is given by

$$\underline{B}_h(d) = 2 \sum_{\ell,\omega} \underline{\beta}_\ell^{\ell,0,\omega} e^{-i(\ell x - \omega t)},$$

$$\underline{E}_h(d) = 2|\ell| d \sum_{\ell,\omega} \underline{\mathcal{E}}_\ell^{\ell,0,\omega} e^{-i(\ell x - \omega t)},$$

where $\underline{B}_h(d) \doteqdot \underline{B}_h(o)$: change observed on the ground,

and

$$\frac{i\omega}{c} \beta_x^{\ell,0,\omega} = |\ell| \ \mathcal{E}_y^{\ell,0,\omega},$$
$$\frac{i\omega}{c} \beta_y^{\ell,0,\omega} = \frac{\omega^2}{|\ell| \ c^2} \ \mathcal{E}_x^{\ell,0,\omega} . \qquad (71)$$

From the condition that $|m| \ll |\ell|$, the expressions (60) and (61) for the electromagnetic field in the magnetosphere can be simplified to give

$$
\begin{aligned}
\underline{E} = \sum_{l,\omega} \Big[& E_1^{l,0,\omega}(0, 1, 0)\, e^{-i(lx-\omega t)+|l|(z-D)} \\
& + E_2^{l,0,\omega}(0, 1, 0)\, e^{-i(lx-\omega t)-|l|(z-D)} \\
& + E_3^{l,0,\omega}(\sin\psi, 0, -\cos\psi)\, e^{-i\{lx+(-l\cot\psi-\frac{\omega}{V\sin\psi})(z-D)-\omega t\}} \\
& + E_4^{l,0,\omega}(\sin\psi, 0, -\cos\psi)\, e^{-i\{lx+(-l\cot\psi+\frac{\omega}{V\sin\psi})(z-D)-\omega t\}} \Big],
\end{aligned}
\tag{72}
$$

$$
\begin{aligned}
\underline{B} = -\frac{c}{i\omega}\sum_{l,\omega} \Big(& E_1^{l,0,\omega}(-|l|, 0, -il)\, e^{-i(lx-\omega t)+|l|(z-D)} \\
& + E_2^{l,0,\omega}(|l|, 0, -il)\, e^{-i(lx-\omega t)-|l|(z-D)} \\
& + E_3^{l,0,\omega}\left(0, \frac{i\omega}{V}, 0\right) e^{-i\{lx+(-l\cot\psi-\frac{\omega}{V\sin\psi})(z-D)-\omega t\}} \\
& + E_4^{l,0,\omega}\left(0, -\frac{i\omega}{V}, 0\right) e^{-i\{lx+(-l\cot\psi+\frac{\omega}{V\sin\psi})(z-D)-\omega t\}}
\end{aligned}
\tag{73}
$$

For modes 1 and 2, the electric field is perpendicular to the meridian, and for modes 3 and 4 it is in the meridional plane. In the magnetosphere where the conductivity is infinite, the following relationship holds:

$$\underline{E} = -\frac{1}{c}\, \underline{u} \wedge \underline{B}_0 ,$$

where $\underline{u}$ is the velocity of the plasma and $\underline{B}_0$ is the main geomagnetic field which is in the meridional plane. This shows that modes 1 and 2 are associated with a plasma motion in the meridional plane (poloidal), while modes 3 and 4 are accompanied by a plasma motion perpendicular to the meridian (toroidal). The incident field is composed of modes 1 and 3 in the northern hemisphere, and of modes 1 and 4 in the southern hemisphere.

The electromagnetic field in the neutral atmosphere and that in the magnetosphere are connected by equations (49) and (51)

$$
\begin{aligned}
E_1 + E_2 &= 2|\ell| d\, \mathcal{E}_y, \\
\sin\psi\,(E_3 + E_4) &= 2|\ell| d\, \mathcal{E}_x, \\
|\ell|\,(E_1 - E_2) &= \frac{4\pi i\omega}{c^2} I_y + 2\frac{i\omega}{c}\beta_x, \\
-\frac{i\omega}{V}(E_3 - E_4) &= -\frac{4\pi i\omega}{c^2} I_x + 2\frac{i\omega}{c}\beta_y .
\end{aligned}
\tag{74}
$$

where prefixes are omitted. Hence the amplitudes E_1, E_3 and E_4 of the incident field can be related to $\mathcal{E}_x$ and $\mathcal{E}_y$ by

$$
\begin{aligned}
2E_1 &= 2|\ell| d\, \mathcal{E}_y + \frac{4\pi i\omega}{|\ell| c^2} I_y + 2\mathcal{E}_y, \\
2E_3 &= \frac{2|\ell| d}{\sin\psi}\mathcal{E}_x + \frac{4\pi V}{c^2} I_x + \frac{2i\omega V}{|\ell| c^2}\mathcal{E}_x, \\
2E_4 &= \frac{2|\ell| d}{\sin\psi}\mathcal{E}_x - \frac{4\pi V}{c^2} I_x - \frac{2i\omega V}{|\ell| c^2}\mathcal{E}_x ,
\end{aligned}
\tag{75}
$$

where

$$I_x = \frac{\Sigma_1}{\sin^2\psi} 2i|\ell| d\, \mathcal{E}_x - \frac{\Sigma_2}{\sin\psi} 2i|\ell| d\, \mathcal{E}_y ,$$

$$I_y = \frac{\Sigma_2}{\sin\psi} 2i|\ell| d\, \mathcal{E}_x + \Sigma_1 \cdot 2i|\ell| d\, \mathcal{E}_y .$$

From these equations the change in the field at ground level resulting from the incidence of each mode may be obtained. When only the field of mode 1 is incident, it follows from equation (75) that

$$2E_3 = \frac{2i|\ell| d}{\sin\psi} \mathcal{E}_x + \frac{4\pi V}{c^2} I_x + \frac{2i\omega V}{|\ell| c^2} \mathcal{E}_x = 0 ,$$

in the northern hemisphere,

and $$2E_4 = \frac{2i|\ell| d}{\sin\psi} \mathcal{E}_x - \frac{4\pi V}{c^2} I_x - \frac{2i\omega V}{|\ell| c^2} \mathcal{E}_x = 0 , \qquad (76)$$

in the southern hemisphere.

With $\ell \sim 10^{-8}\, cm^{-1}$, $d \sim 10^{7}\, cm$, $10^{-1} \gtrsim \omega \gtrsim 10^{-2}\, sec^{-1}$, $V \sim 10^{8}\, cm\, sec^{-1}$, and $|\sin\psi| \sim 1$, orders of magnitude of the terms in equation (76) may be estimated

first term $\sim 10^{-1}\, \mathcal{E}_x$,

second term $\sim \mathcal{E}_x,\ \mathcal{E}_y$,

third term $\sim 10^{-7}\, \mathcal{E}_x$.

Hence it follows that

$$I_x = 0$$

i.e.

$$\frac{\Sigma_1}{\sin\psi}\,\varepsilon_x = \Sigma_2\,\varepsilon_y\,, \tag{77}$$

in both hemispheres. This shows that ε_x and ε_y are of the same order of magnitude. Denoting this by ε, the magnitude of the observed magnetic field may be written from equation (71) as

$$\beta \sim 2\,\frac{c}{\omega}\,\ell\,\varepsilon\,.$$

The order of magnitude of the magnetic field due to the ionospheric current is given by

$$b \sim \frac{8\pi}{c}\,\Sigma\,\ell d\varepsilon\,.$$

It follows from these estimates that

$$\frac{b}{\beta} \sim \frac{4\pi\omega\,\Sigma\,d}{c^2} \sim \omega\,. \tag{78}$$

Since $10^{-1} \gtrsim \omega \gtrsim 10^{-7}\,\mathrm{sec}^{-1}$ for the present case, equation (78) shows that $b < \beta$: the contribution from the ionospheric current to the observed change in the magnetic field is negligible. Thus for the incidence of the field of mode 1, ionospheric screening is not effective.

Substituting equation (77) into equation (71), we have

$$\begin{aligned} \beta_x &= \frac{|\ell|\,\Sigma_1\,c}{i\omega\,\Sigma_2\sin\psi}\,\varepsilon_x\,, \\ \beta_y &= -\left(\frac{i\omega}{|\ell|\,c}\right)^2 \frac{\Sigma_2}{\Sigma_1}\sin\psi\;\beta_x\,, \end{aligned} \tag{79}$$

for both hemispheres. With $\omega \lesssim 10^{-1} sec^{-1}$ and $|\Sigma_1| \sim |\Sigma_2|$, it follows that

$$|\beta_x| \gg |\beta_y| ,$$

ie, the observed direction of the change in the magnetic field is approximately in the meridional plane. Substituting equations (77) and (79) into the first of equation (75), we have

$$\begin{aligned} 2E_1 &= 4\pi \left(\frac{i\omega}{|\ell| c}\right)^2 \frac{\Sigma_2^2}{\Sigma_1} \frac{2|\ell| d}{c} \beta_x \\ &\quad + 2\left(|\ell| d + \frac{8\pi i\omega \Sigma_1 d}{c^2} + 1 \right) \frac{i\omega}{c|\ell|} \beta_x \\ &\doteqdot 2 \frac{i\omega}{c|\ell|} \beta_x . \end{aligned} \tag{80}$$

This shows that to an observed increase in the horizontal component ($Re(i\omega\beta_x) = Re\left(\frac{d\beta_x}{dt}\right) < 0$) corresponds an incidence of the field with $E_1 < 0$ in both hemispheres. The incident electric field is then directed westward, and the associated motion of the magnetospheric plasma is compressional. A decrease in the horizontal component, on the other hand, implies $E_1 > 0$; the incident electric field is directed eastward and the associated plasma motion is expanding. Vectors of the incident and the observed field for the case of incidence of mode 1 are illustrated in Figure 21.

The screening current in middle and low latitudes is given by equations (76) and (79) as,

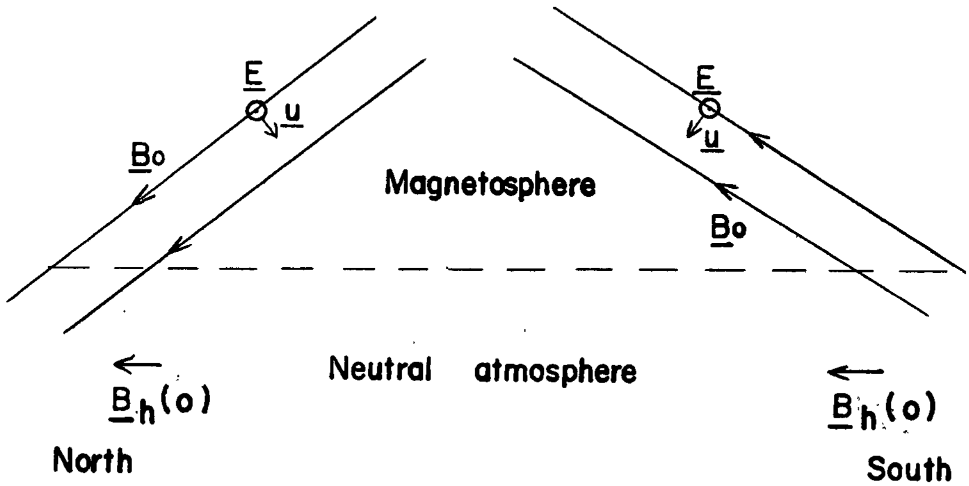


Figure 21. Vectors of the observed and incident fields when the field of mode 1 is incident. $\underline{E}$: incident electric field, $\underline{u}$: velocity of the associated plasma motion, $\underline{B}_h(o)$: horizontal component of the observed field, and $\underline{B}o$: geomagnetic main field.

$$I_x = \mp \frac{c^2}{4\pi V} \frac{2|\ell| d}{\sin\psi} \varepsilon_x \tag{81}$$

$$= - \frac{i\omega c d \Sigma_2}{2\pi V \Sigma_1} \beta_x \, ,$$

in the northern hemisphere,

$$= \frac{i\omega c d \Sigma_2}{2\pi V \Sigma_1} \beta_x \, ,$$

in the southern hemisphere,

$$I_y = 2\left(\Sigma_2 + \frac{\Sigma_1^2}{\Sigma_2}\right) \frac{i\omega d \Sigma_2}{|\ell| \Sigma_1} \beta_x \, .$$

When the fields of mode 3 (in the northern hemisphere) and mode 4 (in the southern hemisphere) are incident, it follows from equation (75) that

$$2E_1 = 2|\ell| d \varepsilon_y + \frac{4\pi i \omega}{|\ell| c^2} I_y + 2\varepsilon_y = 0 \, , \tag{82}$$

in both hemispheres. The orders of magnitude of each term are given by

first term $\sim$ $10^{-1} \varepsilon_y$,

second term $\sim$ $\omega \varepsilon_x$, $\omega \varepsilon_y$,

third term $\sim$ ε_y .

Hence for equation (82) to hold, ε_x must be of the order of $\omega^{-1} \varepsilon_y$ as given by

$$\frac{4\pi i \omega}{|\ell| c^2} \frac{\Sigma_2}{\sin\psi} 2|\ell| d \varepsilon_x + 2\varepsilon_y = 0 \, . \tag{83}$$

Since $\omega \lesssim 10^{-1} sec^{-1}$, it follows that $|\mathcal{E}_y| \ll |\mathcal{E}_x|$, and $\mathcal{E}_x$ gives the order of magnitude of the incident field $\mathcal{E}$. From equations (71) and (83) we have

$$\beta_x = - \frac{8\pi \Sigma_2 |l| d}{c \sin \psi} \mathcal{E}_x ,$$

$$\beta_y = - \frac{i\omega}{|l| c} \mathcal{E}_x , \tag{84}$$

in both hemispheres. With $\Sigma_2 \sim -10^{13}$ cgs, $l \sim 10^{-8}\ cm^{-1}$, $d \sim 10^7$ cm and $\omega \lesssim 10^{-1} sec^{-1}$, it can be seen that

$$|\beta_x| \gg |\beta_y|$$

and the order of magnitude of the observed change in the magnetic field is given by

$$\beta \sim \frac{8\pi \Sigma l d}{c} \mathcal{E} .$$

The order of magnitude of the magnetic field produced by the ionospheric current is given by

$$b \sim \frac{8\pi \Sigma l d}{c} \mathcal{E}$$

where

$$\Sigma : |\Sigma_1| \sim |\Sigma_2| .$$

Hence it follows that

$$\frac{b}{\beta} \sim 1 . \tag{85}$$

This shows that the contribution from the ionospheric current to the observed change in the magnetic field is not negligible: the ionospheric screening effect has to be taken into account in this case.

Substituting equation (84) into equation (75), it may be seen that the incident field in the northern hemisphere is characterized by a factor E_3 given by

$$2E_3 = -\frac{V\Sigma_1}{c\Sigma_2 \sin\psi}\beta_x , \tag{86}$$

and in the southern hemisphere by

$$2E_4 = \frac{V\Sigma_1}{c\Sigma_2 \sin\psi}\beta_x .$$

The increase in the horizontal component ($\beta_x < 0$) thus corresponds to $E_3 < 0$ and $E_4 < 0$. The horizontal component of the incident electric field is directed northwards in the northern hemisphere and southwards in the southern hemisphere. The associated motion of the magnetospheric plasma is toroidal and is directed westwards in both hemispheres, as illustrated in Figure 22.

In both hemispheres, the incident magnetic field is directed perpendicular to the meridian; eastward in the northern and westward in the southern hemisphere, while the observed change in the geomagnetic field is approximately in the meridional plane. Thus in the case of incidence of modes 3 and 4, the direction of the change in the magnetic

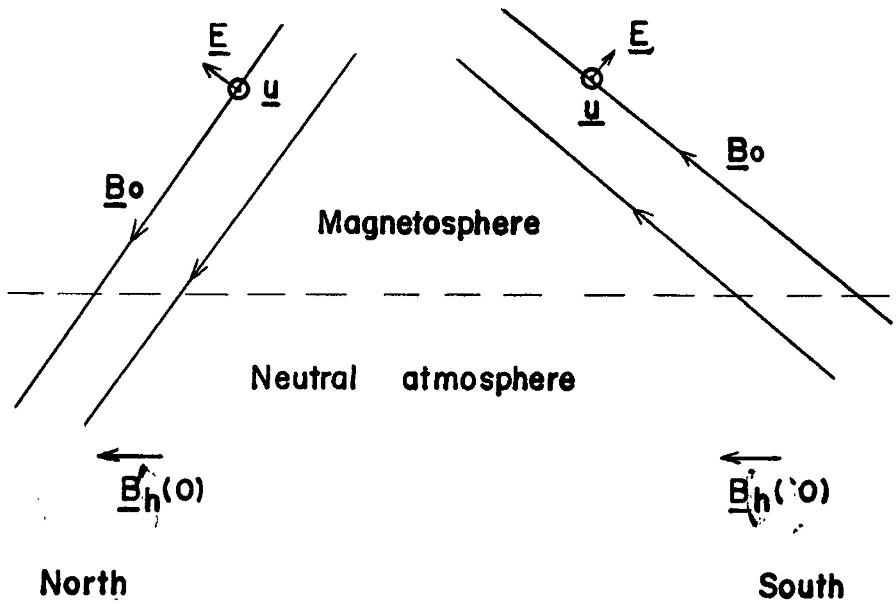


Figure 22. Vectors of observed and incident fields when the field of mode 3 (in the northern hemisphere) and of mode 4 (in the southern hemisphere) is incident. Symbols are the same as in Figure 21.

field is rotated through 90° by the screening effect of the ionosphere. The screening current in middle and low latitudes is given from equations (75) and (84) by

$$I_x = -\frac{c\Sigma_1}{4\pi\Sigma_2 \sin\psi}\,\beta x \,, \qquad I_y = -\frac{c}{4\pi}\,\beta x \,, \tag{87}$$

in both hemispheres.

Thus the incident field of both modes is found to give rise to a change in the meridional component of the geomagnetic field. The change in the E-W component is found to be small compared to that in the meridional component. Since this result is closely related to the condition that $|m| \ll |\ell|$, the direction of the change in the magnetic field actually observed may not necessarily be in this way if $|m|$ is not negligible compared with $|\ell|$, as it is for the preceding reverse change. Hence in relating the observed field to the incident field in the magnetosphere using the present results, it may be reasonable to take note only of the meridional component of the magnetic field.

The main change is an increase (for a positive change) or a decrease (for a negative change) in the horizontal component of the geomagnetic field all over the world. If this part of the change is assumed to result from the incidence of the field of mode 1, it follows that the magnetospheric plasma is contracting (for a positive change) or expanding (for a negative change) everywhere at the time of a main change. This is consistent with the generally accepted idea that the main change of sudden commencements (which has the same characteristics as positive changes) is due to the compression of the magnetosphere resulting from the sudden increase in the impact pressure of the solar corpuscular stream. Negative changes may then be related with a sudden decrease in it. If, on the other hand, we assume that the main change is related to the incidence of modes 3 and 4, it follows that the magnetospheric plasma is rotating westwards (for a positive change) and eastwards (for a negative change) everywhere at the time of the main change. The incident electric field is then directed outwards (for a positive change) or inwards (for a negative change) with respect to the magnetospheric boundary. This seems also to indicate a sudden change in the impact pressure of the solar corpuscular stream. Due to their larger inertia, protons in the stream may penetrate farther into the magnetosphere than electrons, and this causes a polarization directed outwards at the magnetospheric boundary. When the physical state of the corpuscular stream is kept stationary, the electric field due to this polarization may be neutralized by the displacement of the magnetospheric plasma. But when the impact pressure of the stream suddenly changes, the polarization produced by the stream plasma is also suddenly changed, and until it is neutralized by the resulting displacement of the magnetospheric plasma it will cause an electric field with the sense indicated above. It seems therefore that the main change of world-wide changes is the result of a change in the size of the magnetosphere resulting from a change in the impact pressure of the solar corpuscular stream.

The preceding reverse change is a decrease (for a positive change) or an increase (for a negative change) in the horizontal component observed immediately preceding the main change in most parts of the afternoon side of the Earth. If interpreted as a result of the incidence of mode 1, the appearance of a preceding reverse change indicates the occurrence of an expansion (for a positive change) or of a compression (for a negative change) of the afternoon side of the magnetosphere immediately preceding its world-wide compression (for a positive change) or expansion (for a negative change). This model is, however, hard to accept since the afternoon side of the magnetosphere is the very region where the influence of the stream is felt in the first place, as can be seen by the distribution of the time of onset illustrated in Figure 12. The change conveyed to the ground on the afternoon side of the Earth by mode 1 must be in the same sense as that transmitted to the remaining parts of the Earth.

If, on the other hand, we relate this part of the change to the incidence of modes 3 and 4, the associated flow of the magnetospheric plasma will be directed eastwards (for a positive change) or westwards (for a negative change) on the afternoon side. Such a flow seems probable, since as the corpuscular stream flows by the magnetosphere, a shear stress is exerted on it, bending the lines of force towards the night side. An increase in the impact pressure may be associated with an increase in the shear stress, and lines of force, together with the magnetospheric plasma, will be brought further towards the night side. When the impact pressure is weakened, the shear stress will also be weakened, and the lines of force, together with the magnetospheric plasma, will return to the sunward side. Hence positive changes are associated with eastward flow and negative changes with westward flow of the magnetospheric plasma on the afternoon side. The absence of preceding reverse changes on the morning side is consistent with this idea, because the direction of flow on this side is opposite to that on the afternoon side, and the resulting change of the geomagnetic field on the ground has the same sense as that of the main change.

Thus the preceding reverse change seems to result from a sudden change in the shear stress of the solar corpuscular stream exerted on the magnetosphere. A similar idea was presented by Parker (Wilson and Sugiura, 1961) to explain the regularity in the sense of rotation of the initial part of sudden commencements. But his idea, which ignores the inospheric screening effect, fails to explain the appearance of the preceding reverse change in the horizontal component only on the afternoon side of the Earth. According to his model, the direction of the initial variation of the magnetic vector of sudden commencements should be longitudinal everywhere, in contradiction with observations. The regularity in the sense of rotation is not discussed here since (1) In the present model which neglects the variation of the magnitude of the change with local time, the longitudinal component of the change in the geomagnetic field can not be reliable and (2) A strong objection is presented by Matsushita (1962) to the results of analysis given by Wilson and Sugiura.

Since the characteristic distance in which the Alfven wave velocity varies is shorter than the wave length corresponding to an angular frequency of $10^{-1} \gtrsim \omega \gtrsim 10^{-2}$ sec^{-1} (see Appendix II), the estimation of the time of propagation of the change from the magnetospheric boundary to the ground on the basis of ray theory is misleading. Hence the distribution of the times of onset given by Figure 12 can not readily be explained. The preceding reverse change precedes the main change possibly because the deformation of the magnetosphere by a shear stress may be achieved in an interval of time short compared with that required for a change of its size by the impact pressure.

CHAPTER V

CONCLUSIONS

Summarizing the results of this thesis, the influence of the solar corpuscular stream on the geomagnetic field can be described as follows.

The upper atmosphere of the Earth is composed of plasma with the geomagnetic field frozen into it. The impact pressure of the corpuscular stream which is always flowing out from the sun confines the geomagnetic field, together with the Earth's atmosphere, in a space of finite dimensions called the magnetosphere. At the same time, the solar corpuscular stream exerts a shear stress on the magnetosphere as it passes by, and geomagnetic lines of force are bent towards the night side. Thus in a stationary state the configuration of the geomagnetic lines of force may be illustrated in Figure 23.

The intensity of the solar corpuscular stream is as variable as the physical state of the solar atmosphere. A sudden change in its intensity gives rise to a sudden change in the impact pressure and the shear stress which it exerts on the magnetosphere, and this will result in a sudden change in the configuration of the geomagnetic lines of force. The result is observed at ground level as a sudden, world-wide change in the geomagnetic field. Such changes are observed quite frequently: almost every day around sunspot maximum. A few of these changes are followed by an interval of increased activity and are known as sudden commencements of a magnetic storm. But most of these changes are not associated with the beginning of a magnetic storm. This difference may result from the difference in the energy spectrum of the relevant flow of the solar corpuscular stream.

If the intensity of the corpuscular stream is suddenly strengthened, the impact pressure and the shear stress are suddenly increased, and the geomagnetic lines of force will be further compressed, and further dragged towards the night side of the Earth from the stationary configuration as shown in Figure 23. After being modulated by the screening effect of the ionosphere, the effect of this process is observed in the horizontal component of the geomagnetic field as a decrease immediately followed by an increase (in a region covering most of the afternoon side of the Earth) or as an increase (in the remaining regions). If the intensity of the stream is suddenly weakened, on the other hand, the resulting sudden decrease in the impact pressure and the shear stress changes the stationary configuration of the magnetosphere to the form illustrated in Figure 23. The effect of this process on the horizontal component of the geomagnetic field at ground level produces a sudden increase immediately followed by a

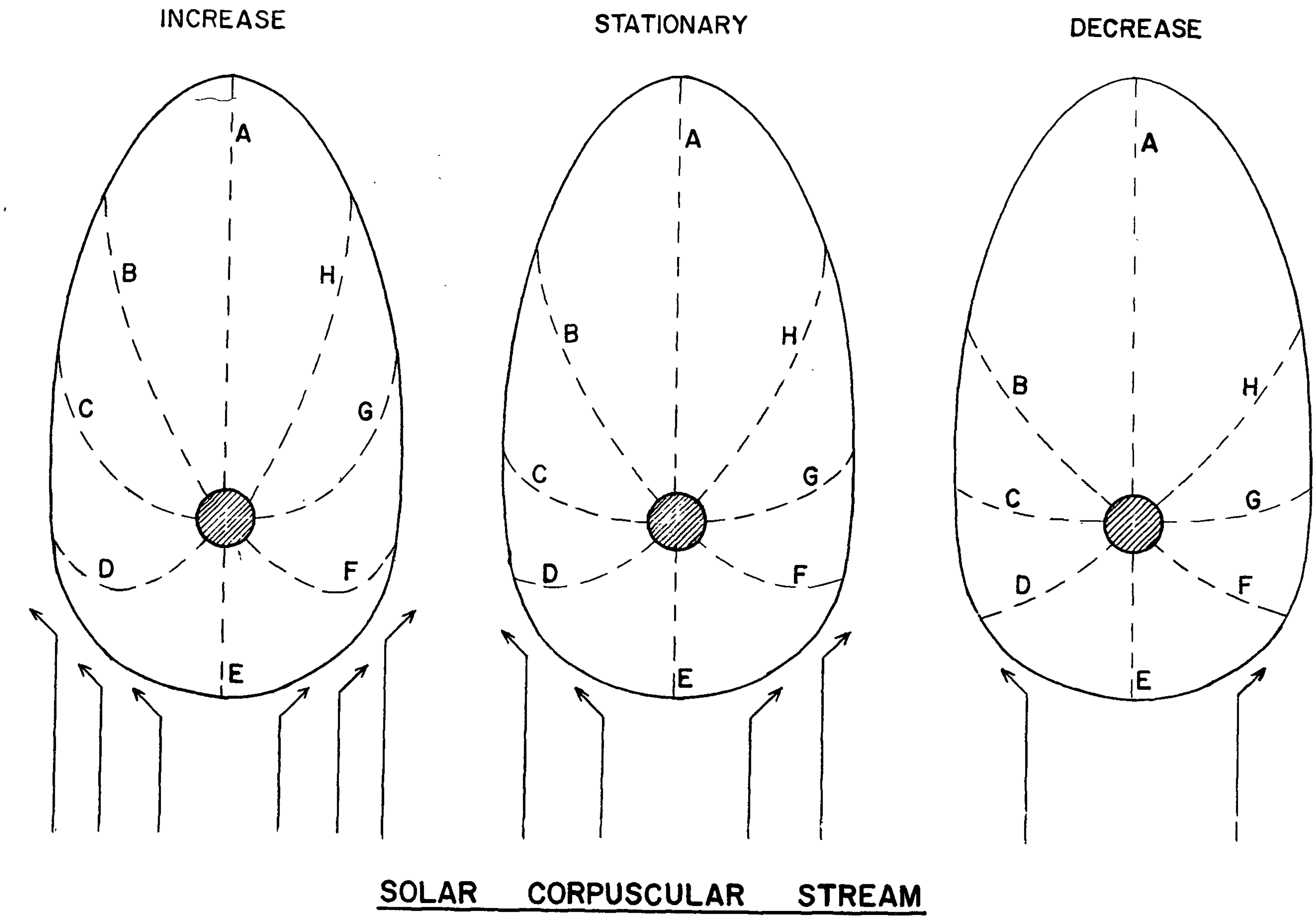


Figure 23. Schematic illustration of the configuration of lines of force. Dotted curves represent the projection on the equatorial plane of representative lines of force. Center: stationary configuration. Left: after an increase in intensity of the corpuscular stream. Right: after a decrease in it.

decrease (in a region covering most of the afternoon side of the Earth) or a sudden decrease (in the remaining regions).

The present model seems to give a reasonable explanation for such observed features as (1) the frequent occurrence of world-wide changes in the geomagnetic field, (2) the existence of positive and negative changes which are morphologically identical, (3) the world-wide occurrence of the main change and (4) the occurrence of the preceding reverse change in a limited region covering most of the afternoon side of the Earth. But in the present treatment, (1) the horizontal uniformity of the physical state of the ionosphere and the geomagnetic main field is assumed, and given physical parameters are representative of the state in middle and low latitudes, and (2) variation in the magnitude of the preceding reverse change with local time is neglected. Further considerations without making these assumptions are necessary to explain (1) the regularity in the sense of rotation of the magnetic vector and (2) the equatorial enhancement of the main and preceding reverse change. The cause of the following reverse change also remains to be explained.

APPENDIX I

Classification of sudden impulses in the geomagnetic field

Committee No. 10 (on rapid variations and Earth currents) of the International Association of Geomagnetism and Aeronomy (I.A.G.A.) issued a special report at the 1957 meeting of the International Union of Geodesy and Geophysics (I.U.G.G.) in Toronto. This report contained the resolutions of an earlier meeting held that year in Copenhagen. Three kinds of sudden impulses in the geomagnetic field were defined.

ssc A sudden impulse followed by an increase in activity lasting at least one hour. The more intense activity of the storm may appear immediately or it may be delayed a few hours.

ssc* This is similar to an ssc, except that the sudden impulse is immediately preceded, on at least one component, by one or more small reverse oscillations. In case the reverse movement has approximately the same amplitude as the principal movement, it will be reported as ssc (not ssc*).

si If the observer sees an important sudden impulse during a storm, but doubts that it represents the beginning of a new storm, he should report it as si.

As knowledge of the world-wide morphology of these changes is accumulated, the incompleteness of the present classification has become more apparent. The main defect is the lack of regard of world-wide features of the phenomenon. This is perhaps natural, since the above specifications were given as indications for the classification of phenomena at an individual station. Hence phenomena are classified only by characteristics observed at a single station regardless of world-wide features of the phenomena. This makes the terminology used in the present classification inadequate for the description of the overall features of the phenomenon. For example, the present definition of ssc* could apply to four different types of changes. Since these four types of changes are entirely different in character, confusion is unavoidable if the same terminology ssc* is used for their description.

According to the present definition of si, sudden impulses which do not occur during a magnetic storm are not counted as si, although both in reports from stations and in published papers such occurrences are usually called si's. Actually si's are observed quite frequently, and since they show the same morphology as sc's, they must be classified in the same way as sc's. (To make the revision in terminology as small as possible, the terminology si is retained, although in the main part of this thesis these are called 'world-wide changes' since they are not always impulse-shaped.)

These indicate that we are now in a position to try and revise, on the basis of our accumulated knowledge, the present classification of sudden impulses in the geomagnetic field.

* * * * *

The observed features of sudden impulses in the geomagnetic field are summarized below. Descriptions always refer to the horizontal component, since considerable clarity can be attained by noting this component only.

1. There are sudden changes in the geomagnetic field which are observed all over the world with certain features in common within a time interval of about one minute. The sense of the change is either an increase or a decrease. The part of the phenomenon which is observed on a world-wide scale will be called the main impulse. The overall phenomenon is called positive or negative according to the sign of the main impulse. The overall morphology of the phenomenon is independent of the sign of the main impulse.

2. A sudden impulse may be classified into two categories - those which mark the beginning of a magnetic storm and those which do not. Their overall morphology is identical in both cases.

3. The main impulse may be accompanied, in some regions, by a reverse impulse. These are of two types. The first, which precedes the main impulse, will be called a preceding reverse impulse.

4. The second, which follows the main impulse, will be called a following reverse impulse.

The possible confusion inherent in the present classification is shown in the following example. Consider the changes shown in Figure 24a and 24b. The change illustrated in Figure 24a may be either

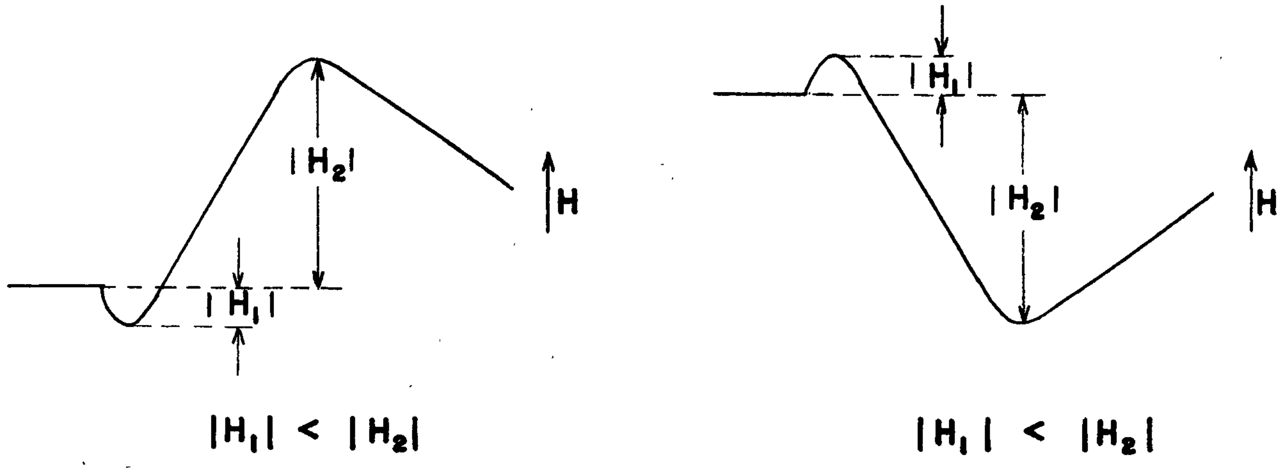
(1) A positive main impulse preceded by a preceding reverse impulse, or

(2) A negative main impulse followed by a following reverse impulse.

The change illustrated in Figure 24b may be either

(1) A positive main impulse followed by a following reverse impulse, or

(2) A negative main impulse preceded by a preceding reverse impulse.



(a) (b)

Figure 24. Examples of sudden impulses.

All these four cases are apparently different. But all of them are sudden impulses which are accompanied, on at least one component, by one or more small reverse oscillations. Hence if they are followed by an interval of 'an increase in activity lasting at least one hour', then all of them must be called ssc*, thus losing any distinction between them. The same difficulty is encountered if we try to classify sudden impulses in the same way as sudden commencements into si and si*. Moreover, all of them must be called ssc if $|H_1|$ is comparable with or larger than $|H_2|$ (it is not rare that following reverse change is smaller than main change), since then the 'reverse movement has approximately the same amplitude as the principal movement'. This introduces further confusion. It is impossible to identify the nature of the phenomenon using only the two terms - ssc and ssc*.

* * * * *

Hence an attempt is made to improve the present classification. The terminology used by Matsushita (1957, 1960, 1962) is adopted as a basis. This is preferred to the terminology used by Akasofu and Chapman (1960), since this can be extended to the case of negative change in a simpler way. This is extended in the following respects.

1. A clear distinction is made between the terminology for the phenomenon as a whole and that for the phenomenon as seen in various parts of the world.

2. Since their morphologies are the same, similar classification and symbols are used for the four following cases - a positive impulse preceding a period of increased activity, for a negative impulse preceding a period of increased activity, for a positive impulse not preceding a period of increased activity, and a negative impulse not preceding a period of increased activity.

Sudden impulses will first be divided into two categories: those which are observed within about one minute all over the world, although the shape of the impulse may not be exactly the same, and those which are not. Only the first of these two categories are studied in this thesis. World-wide sudden impulses are then classified according to their relationship with the interval of increased activity and the sign of the main impulse, as follows:

1. sc (positive sudden commencement). A phenomenon which is observed as a sudden increase in the horizontal component all over the world, and which precedes the increase in activity lasting at least one hour all over the world. In some regions the sudden increase in the horizontal component is accompanied by a decrease.

2. (sc) (negative sudden commencement). A phenomenon similar to an sc but with the sign of the overall impulse reversed. (This class of phenomena has not been clearly identified in records.)

3. si (positive sudden impulse). A phenomenon which is similar to an sc but which does not precede the increase in activity in the manner specified in 1.

4. (si) (negative sudden impulse). A phenomenon which is similar to an si but with the sign of the overall impulse reversed.

The above are the classifications of the phenomena as a whole. Each of these will appear in various shapes in different parts of the world, and will be denoted in the following way.

1.1 SC At a place where only the main impulse is observed at the time of the sc, the phenomenon will be called SC.

1.2 ¯SC At a place where the main impulse and a preceding reverse impulse are observed at the time of the sc, the phenomenon will be called ¯SC.

1.3 SC¯ At a place where the main impulse and a following reverse impulse are observed at the time of the sc, the phenomenon will be called SC¯.

1.4 ¯SC¯ At a place where the main impulse, preceding reverse impulse and following reverse impulse are observed, the phenomenon will be called ¯SC¯.

2.1 (SC) Replace sc in 1.1 by (sc)

2.2 (¯SC) Replace sc in 1.2 by (sc)

2.3 (SC¯) Replace sc in 1.3 by (sc)

2.4 (¯SC¯) Replace sc in 1.4 by (sc)

3.1 SI Replace sc in 1.1 by si

3.2 ¯SI Replace sc in 1.2 by si

3.3 SI¯ Replace sc in 1.3 by si

3.4 $^{-}$SI^{-} Replace sc in 1.4 by si

4.1 (SI) Replace sc in 1.1 by (si)

4.2 ($^{-}$SI) Replace sc in 1.2 by (si)

4.3 (SI^{-}) Replace sc in 1.3 by (si)

4.4 ($^{-}$SI^{-}) Replace sc in 1.4 by (si)

Since the above detailed classification is impossible using data from only one station, the present official terminology ssc and ssc* might have to be retained for reporting. But in an investigation of sudden impulses, the terminology suggested above is preferable.

APPENDIX II

THE OHMIC LAW IN THE UPPER ATMOSPHERE

1. Introduction

In electromagnetic theory, the property of the medium is expressed by the following three equations: (1) an equation representing the relationship between the magnetic induction $\underline{B}$ and the magnetic field intensity $\underline{H}$, (2) an equation representing the relationship between the electric displacement $\underline{D}$ and the electric field intensity $\underline{E}$ and (3) an equation relating the electric current density $\underline{j}$ to the electric field intensity $\underline{E}$. In the upper atmosphere of the Earth the first two of these relationships are given by $\underline{B} = \underline{H}$ and $\underline{D} = \underline{E}$ to a good order of approximation. The third, which may be called the Ohmic law in a generalized sense of the word, is derived from the equation of motion, the equation of continuity and the equation of state of the gas, and is expressed either as,

$$\underline{j} = \tilde{\sigma}\ \underline{E}$$

or as

$$\underline{j} = \tilde{\sigma}'\ (\underline{E} + \frac{1}{c}\underline{v} \wedge \underline{B}_0),$$

where $\underline{v}$ is the velocity of the center of mass of the gas, and $\underline{B}_0$ is the geomagnetic field. $\tilde{\sigma}$ and $\tilde{\sigma}'$ are conductivity matrices consisting generally of operators.

The difference between these two representations lies in the frame of reference in which $\underline{j}$ is derived. In the first, $\underline{j}$ is derived in a frame in which $\underline{E}$ is observed. In the second $\underline{j}$ is derived in a frame which is moving with velocity $\underline{v}$ with respect to the frame in which $\underline{E}$ is observed, and since the relativistic variation in $\underline{j}$ is small for the values of $\underline{v}$ to be discussed later, $\underline{j}$ is approximately the same as $\underline{j}$ observed in the same frame as $\underline{E}$. From a mathematical point of view, in the first representation, the variable $\underline{v}$ is already eliminated, and the property of the medium is sufficiently expressed by a conductivity matrix $\tilde{\sigma}$. In the second representation, on the other hand, $\underline{v}$ is still included, and hence to express the property of the gaseous medium, equations relating $\underline{v}$ to the electromagnetic field must also be used together with the Ohmic law.

In spite of the greater simplicity of the first expression of the Ohmic law, the second representation is usually adopted by most workers, because it has been found that in most cases studied in geomagnetism and aeronomy, the behaviour of a gas under an electric field is expressed as a drift of the center of mass in a direction perpendicular both to the geomagnetic main field and to the direction of the electric field, with a velocity given by,

$$\underline{v} = \frac{c}{B_0^2}(\underline{E} \wedge \underline{B}_0)$$

and the second representation of the Ohmic law is then simplified as,

$$\underline{E} + \underline{v} \wedge \underline{B}_0 = 0$$

(Alfven, 1950; Cowling, 1957; Dungey, 1958; and others). But, in the ionosphere, this simple expression does not always hold. Particularly for angular frequencies ω ranging from 10^{-1} to 10^{-3} sec^{-1}, which correspond to a time scale of 6 to 6000 seconds and include much of the phenomena treated in geomagnetism and aeronomy, this expression can not be used (Watanabe, 1962). In this situation, the second expression seems to lose its advantage, and might only add to the complexity of the problem by including the variable $\underline{v}$ explicitly and thus increasing the number of basic equations. Hence for a study of world-wide changes, sc's and micropulsations in the geomagnetic field, which have a time scale of one second to several minutes, it seems preferable to use the first formulation of the Ohmic law without including $\underline{v}$ explicitly.

Ionospheric data are tabulated in Table 6.

2. Fundamental equations

The equations of motion for the three constituents (electron, singly ionized positive ion and neutral particle) of the model ionosphere are given by

Table 1a. Ionospheric data

Altitude (km)	N_n	N_e	T	κ_p	ν_{en}	ν_i	ν_{ie}
80	$2{\cdot}84^{14}$	$1{\cdot}00^{3}$	168	$3{\cdot}53^{-12}$	$1{\cdot}98^{6}$	$6{\cdot}87^{4}$	$2{\cdot}56^{1}$
90	$3{\cdot}90^{13}$	$3{\cdot}80^{4}$	176	$9{\cdot}75^{-10}$	$2{\cdot}78^{5}$	$9{\cdot}47^{3}$	$6{\cdot}80^{2}$
100	$6{\cdot}00^{12}$	$1{\cdot}80^{5}$	208	$3{\cdot}00^{-8}$	$4{\cdot}48^{4}$	$1{\cdot}48^{3}$	$2{\cdot}48^{3}$
120	$6{\cdot}30^{11}$	$2{\cdot}00^{5}$	390	$3{\cdot}18^{-7}$	$9{\cdot}78^{3}$	$1{\cdot}60^{2}$	$1{\cdot}18^{3}$
140	$1{\cdot}07^{11}$	$2{\cdot}40^{5}$	662	$2{\cdot}24^{-6}$	$1{\cdot}95^{3}$	$2{\cdot}82^{1}$	$8{\cdot}85^{2}$
160	$4{\cdot}00^{10}$	$3{\cdot}00^{5}$	926	$7{\cdot}50^{-6}$	$4{\cdot}43^{2}$	$1{\cdot}01^{1}$	$4{\cdot}98^{2}$
180	$2{\cdot}00^{10}$	$4{\cdot}00^{5}$	1115	$2{\cdot}00^{-5}$	$2{\cdot}22^{2}$	$5{\cdot}46^{0}$	$5{\cdot}05^{2}$
200	$1{\cdot}07^{10}$	$6{\cdot}00^{5}$	1230	$5{\cdot}60^{-5}$	$1{\cdot}11^{2}$	$2{\cdot}97^{0}$	$6{\cdot}77^{2}$
220	$6{\cdot}60^{9}$	$8{\cdot}60^{5}$	1305	$1{\cdot}30^{-4}$	$6{\cdot}39^{1}$	$1{\cdot}87^{0}$	$8{\cdot}83^{2}$
240	$4{\cdot}60^{9}$	$1{\cdot}30^{6}$	1356	$2{\cdot}84^{-4}$	$3{\cdot}94^{1}$	$1{\cdot}32^{0}$	$9{\cdot}55^{2}$
260	$3{\cdot}30^{9}$	$1{\cdot}80^{6}$	1400	$5{\cdot}45^{-4}$	$2{\cdot}58^{1}$	$9{\cdot}65^{-1}$	$1{\cdot}63^{3}$
280	$2{\cdot}35^{9}$	$2{\cdot}20^{6}$	1430	$9{\cdot}37^{-4}$	$1{\cdot}75^{1}$	$6{\cdot}92^{-1}$	$1{\cdot}92^{3}$
300	$1{\cdot}82^{9}$	$2{\cdot}60^{6}$	1455	$1{\cdot}43^{-3}$	$1{\cdot}14^{1}$	$5{\cdot}42^{-1}$	$2{\cdot}21^{3}$
350	$8{\cdot}30^{8}$	$2{\cdot}14^{6}$	1488	$2{\cdot}58^{-3}$	$4{\cdot}77^{0}$	$2{\cdot}52^{-1}$	$1{\cdot}75^{3}$
400	$4{\cdot}70^{8}$	$1{\cdot}53^{6}$	1500	$3{\cdot}26^{-3}$	$1{\cdot}66^{0}$	$1{\cdot}46^{-1}$	$1{\cdot}57^{3}$
450	$2{\cdot}50^{8}$	$1{\cdot}33^{6}$	1500	$5{\cdot}33^{-3}$	$7{\cdot}25^{-1}$	$7{\cdot}90^{-2}$	$1{\cdot}11^{3}$
500	$1{\cdot}43^{8}$	$9{\cdot}52^{5}$	1500	$6{\cdot}65^{-3}$	$3{\cdot}12^{-1}$	$4{\cdot}61^{-2}$	$8{\cdot}20^{2}$
600	$4{\cdot}73^{7}$	$7{\cdot}39^{5}$	1500	$1{\cdot}56^{-2}$	$6{\cdot}35^{-2}$	$1{\cdot}55^{-2}$	$6{\cdot}30^{2}$
700	$1{\cdot}64^{7}$	$6{\cdot}17^{5}$	1500	$3{\cdot}76^{-2}$	$1{\cdot}95^{-2}$	$5{\cdot}52^{-3}$	$5{\cdot}30^{2}$
800	$5{\cdot}81^{6}$	$4{\cdot}91^{5}$	1500	$8{\cdot}46^{-2}$	$5{\cdot}75^{-3}$	$2{\cdot}05^{-3}$	$4{\cdot}25^{2}$

* T means the ionospheric temperature.

<u>Table 6.</u> <u>Ionospheric Data</u> (after Watanabe, 1962). N_n: the number density of neutral particles, N_e: the number density of electrons, $\kappa_p = \frac{N_p}{N_n}$, T: the ionospheric temperature, and $\nu_e = \nu_{ei} + \nu_{en}$: the electron collision frequency, and ν_i : the collision frequency of an ion with neutral particles.

$$N_e m_e \frac{d\underline{u}_e}{dt} + N_e \nu_{ei} \frac{m_i m_e}{m_i + m_e} (\underline{u}_e - \underline{u}_i) + N_e \nu_{en} \frac{m_e m_n}{m_e + m_n} (\underline{u}_e - \underline{u}_n)$$
$$= - \nabla P_e - e N_e \left(\underline{E} + \frac{1}{c} \underline{u}_e \wedge \underline{B} \right), \tag{1}$$

$$N_i m_i \frac{d\underline{u}_i}{dt} + N_i \nu_{ie} \frac{m_i m_e}{m_i + m_e} (\underline{u}_i - \underline{u}_e) + N_i \nu_{in} \frac{m_i m_n}{m_i + m_n} (\underline{u}_i - \underline{u}_n)$$
$$= - \nabla P_i + e N_i \left(\underline{E} + \frac{1}{c} \underline{u}_i \wedge B \right), \tag{2}$$

$$N_n m_n \frac{d\underline{u}_n}{dt} + N_n \nu_{ni} \frac{m_n m_i}{m_n + m_i} (\underline{u}_n - \underline{u}_i) + N_n \nu_{ne} \frac{m_n m_e}{m_e + m_n} (\underline{u}_n - \underline{u}_e)$$
$$= - \nabla P_n , \tag{3}$$

where m_e, m_i and m_n are the masses, $\underline{u}_e$, $\underline{u}_i$ and $\underline{u}_n$ the average velocities, N_e, N_i and N_n the number densities, and P_e, P_i and P_n the partial pressures, of an electron, an ion and a neutral particle respectively. The second and third terms on the left hand side of each equation represent the exchange of momentum due to collisions between different kinds of particles, ν_{ie} , ν_{en} , and ν_{in} represent collision frequencies of an ion with electrons, an electron with neutral particles and an ion with neutral particles respectively. Three other collision frequencies are related to them by the equations

$$N_e \nu_{ei} = N_i \nu_{ie}, \qquad N_i \nu_{in} = N_n \nu_{ni}, \quad N_e \nu_{en} = N_n \nu_{ne}, \tag{4}$$

(In deriving these collision frequencies, it is assumed that the direction of relative velocity between colliding particles becomes completely random after each collision.)

The equations of continuity relating the $\underline{u}$'s and N's are

$$\frac{\partial N_e}{\partial t} + \mathrm{div}\,(N_e \underline{u}_e) = 0, \tag{5}$$

$$\frac{\partial N_i}{\partial t} + \mathrm{div}\,(N_i \underline{u}_i) = 0, \tag{6}$$

$$\frac{\partial N_n}{\partial t} + \mathrm{div}\,(N_n \underline{u}_n) = 0. \tag{7}$$

Let us assume that when a changing electromagnetic field is not present, the velocity of each constitutent is zero and that the density and partial pressure are constant and homogeneous. Pressure P_s and number density N_s (s = e, i and n) are replaced by $P_s + p_s$ and $N_s + n_s$, where capital letters denote the value corresponding to the state before the changing electromagnetic field is introduced, and small letters denote the perturbation associated with the changing electromagnetic field. As the gas should be electrically neutral in the stationary state, $N_e = N_i$, and will be denoted by N_p. Magnitudes of perturbing quantities are assumed to be small enough to allow the linearization of the above equations. Neglecting also terms of the order m_e/m_i or m_e/m_n (in comparison to the remaining terms), equations (1) - (7) become

$$N_p m_e \frac{\partial \underline{u}_e}{\partial t} + N_p \nu_{ie} m_e (\underline{u}_e - \underline{u}_i) + N_p \nu_{en} m_e (\underline{u}_e - \underline{u}_n)$$

$$= - \nabla p_e - e N_p \left(\underline{E} + \frac{1}{c} \underline{u}_e \wedge \underline{B}_0 \right) , \tag{1'}$$

$$N_p m_i \frac{\partial \underline{u}_i}{\partial t} + N_p \nu_{ie} m_e (\underline{u}_i - \underline{u}_e) + N_p \nu_{in}' m_i (\underline{u}_i - \underline{u}_n)$$

$$= - \nabla p_i + e N_p \left(\underline{E} + \frac{1}{c} \underline{u}_i \wedge \underline{B}_0 \right) , \tag{2'}$$

$$N_n m_n \frac{\partial \underline{u}_n}{\partial t} + N_p \nu'_{in} m_i (\underline{u}_n - \underline{u}_i) + N_p \nu_{en} m_e (\underline{u}_n - \underline{u}_e) \tag{3'}$$

$$= - \nabla p_n ,$$

where $\nu'_{in} = \frac{1}{2} \nu_{in}$ (assuming $m_i = m_n$),

$$\frac{\partial n_e}{\partial t} + N_p \operatorname{div} \underline{u}_e = 0, \tag{5'}$$

$$\frac{\partial n_i}{\partial t} + N_p \operatorname{div} \underline{u}_i = 0, \tag{6'}$$

$$\frac{\partial n_n}{\partial t} + N_n \operatorname{div} \underline{u}_n = 0, \tag{7'}$$

where $\underline{B}_o$ denotes the main geomagnetic field which is far stronger than the changing magnetic field. Assuming that the change in state takes place adiabatically, p_s and n_s ($s = e$, i and n) are related by the equations

$$p_e = a_e^2 m_e n_e, \qquad a_e = \sqrt{\frac{\gamma K T}{m_e}}, \tag{8}$$

$$p_i = a_i^2 m_i n_i, \qquad a_i = \sqrt{\frac{\gamma K T}{m_i}}, \tag{9}$$

$$p_n = a_n^2 m_n n_n, \qquad a_n = \sqrt{\frac{\gamma K T}{m_n}}, \tag{10}$$

(Note: K = Boltzman constant; k = magnitude of wave vector.)

The above equations can be solved in the form,

$$\underline{j} = \tilde{\sigma}\ \underline{E} \tag{11}$$

where

$$\underline{j} = N_p e\,(\underline{u}_i - \underline{u}_e), \tag{12}$$

and $\tilde{\sigma}$ is a conductivity matrix.

From Maxwell's equation in a medium where ε and μ can be approximated by unity:

$$\mathrm{curl}\ \underline{E} = -\frac{1}{c}\frac{\partial \underline{B}}{\partial t}, \tag{13}$$

$$\mathrm{curl}\ \underline{B} = \frac{4\pi}{c}\underline{j} + \frac{1}{c}\frac{\partial \underline{E}}{\partial t}, \tag{14}$$

$$\mathrm{div}\ \underline{E} = 4\pi q, \tag{15}$$

$$\mathrm{div}\ \underline{B} = 0, \tag{16}$$

where

$$q = e\,(n_i - n_e), \tag{17}$$

it follows that

$$\nabla^2 \underline{E} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \text{grad} \cdot \text{div}\, \underline{E} + \frac{4\pi}{c^2}\frac{\partial \underline{j}}{\partial t} . \tag{18}$$

The right hand side of equation (18) includes the effects of the presence of a charged medium on the propagation of electromagnetic waves. The first term represents the effect due to the inhomogeneity produced in the charged medium, and the second term represents the effect due to the presence of electric currents.

From equations (5') and (6') it follows that

$$\frac{\partial q}{\partial t} + \text{div}\, \underline{j} = 0 .$$

Hence using equation (15) we have,

$$\text{div} \frac{\partial E}{\partial t} + 4\pi\, \text{div}\, \underline{j} = 0 . \tag{18'}$$

This is not an independent equation, however. It follows directly from equation (18) by taking the divergence of both sides, since $\underline{E}$ and $\underline{j}$ are time-dependent. The electric field $\underline{E}$ can be derived from equations (11) and (18). The associated magnetic field $\underline{B}$ can be derived from equations (13), (14) and (16).

3. The Ohmic law

Equations (1) - (10) will be solved to derive $\tilde{\sigma}$. Let us assume that every variable in the preceding equations has the form: $e^{-i(\underline{k}\cdot\underline{r}-\omega t)}$. Then equations

(1') - (3') can be written as,

$$N_p m_e (i\omega) \underline{u}_e + N_p \nu_{ie} m_e (\underline{u}_e - \underline{u}_i) + N_p \nu_{en} m_e (\underline{u}_e - \underline{u}_n)$$

$$= i\underline{k}\, p_e - e N_p \left(\underline{E} + \frac{1}{c} \underline{u}_e \wedge \underline{B}_0 \right), \tag{1''}$$

$$N_p m_i (i\omega) \underline{u}_i + N_p \nu_{ie} m_e (\underline{u}_i - \underline{u}_e) + N_p \nu'_{in} m_i (\underline{u}_i - \underline{u}_n)$$

$$= i\underline{k}\, p_i + e N_p \left(\underline{E} + \frac{1}{c} \underline{u}_i \wedge \underline{B}_0 \right), \tag{2''}$$

$$N_n m_n (i\omega) \underline{u}_n + N_p \nu'_{in} m_i (\underline{u}_n - \underline{u}_i) + N_p \nu_{en} m_e (\underline{u}_n - \underline{u}_e)$$

$$= i\underline{k}\, p_n, \tag{3''}$$

and using equations (5') - (7'), (8) - (10) become,

$$p_e = a_e^2 m_e N_p \frac{1}{\omega} (\underline{k} \cdot \underline{u}_e), \tag{8'}$$

$$p_i = a_i^2 m_i N_p \frac{1}{\omega} (\underline{k} \cdot \underline{u}_i), \tag{9'}$$

$$p_n = a_n^2 m_n N_n \frac{1}{\omega} (\underline{k} \cdot \underline{u}_n). \tag{10'}$$

It can be shown that the terms representing the pressure gradient in equations (1'') - (3''), viz. $i\mathbb{k}p_e$, $i\mathbb{k}p_i$ and $i\mathbb{k}p_n$ may be neglected in comparison with other terms under certain conditions which are fulfilled in the present problem. These conditions are,

$$
\begin{aligned}
&|N_p m_e \omega \underline{u}_e|, \quad |N_p \nu_{ie} m_e \underline{u}_e|, \quad |N_p \nu_{en} m_e \underline{u}_e|, \\
&|N_p \nu_{ie} m_e \underline{u}_i|, \quad |N_p \nu_{en} m_e \underline{u}_n|, \quad \left|\frac{e}{c} N_p \underline{u}_e \wedge \underline{B}_0\right| \\
&\gg \frac{a_e^2 m_e N_p}{\omega} |\underline{k}(\underline{k}\cdot\underline{u}_e)|,
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
&|N_p m_i \omega \underline{u}_i|, \quad |N_p \nu_{ie} m_e \underline{u}_i|, \quad |N_p \nu'_{in} m_i \underline{u}_i|, \\
&|N_p \nu_{ie} m_e \underline{u}_e|, \quad |N_p \nu'_{in} m_i \underline{u}_n|, \quad \left|\frac{e}{c} N_i \underline{u}_i \wedge \underline{B}_0\right| \\
&\gg \frac{a_i^2 m_i N_p}{\omega} |\underline{k}(\underline{k}\cdot\underline{u}_i)|,
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
&|N_n m_n \omega \underline{u}_n|, \quad |N_p \nu'_{in} m_i \underline{u}_n|, \quad |N_p \nu_{en} m_e \underline{u}_n|, \\
&|N_p \nu'_{in} m_i \underline{u}_i|, \quad |N_p \nu_{en} m_e \underline{u}_e| \quad \gg \frac{a_n^2 m_n N_n}{\omega} |\underline{k}(\underline{k}\cdot\underline{u}_n)|.
\end{aligned}
\tag{21}
$$

These may be summarized as,

$$\left(\frac{\omega}{k}\right)^2 \gg \frac{\gamma KT}{m_e},\quad \frac{\omega}{\nu_{ie}'}\frac{\gamma KT}{m_e},\quad \frac{\omega}{\nu_{en}}\frac{\gamma KT}{m_e},\quad \frac{\gamma KT}{m_i},$$
$$\frac{\omega}{\nu_{ie}}\frac{\gamma KT}{m_i},\quad \frac{\omega}{\nu_{in}'}\frac{\gamma KT}{m_i},\quad \frac{\gamma KT}{m_n},\quad \frac{\omega}{\nu_{in}'}\frac{N_n}{N_p}\frac{\gamma KT}{m_i},$$
$$\frac{\omega}{\nu_{en}}\frac{N_n}{N_p}\frac{\gamma KT}{m_e},\quad \frac{\omega}{\nu_{ie}}\frac{\gamma KT}{m_e},\quad \frac{\omega}{\nu_{in}'}\sqrt{\frac{N_n}{N_p}}\frac{\gamma KT}{m_i},\tag{22}$$
$$\frac{\omega}{\nu_{en}}\sqrt{\frac{N_n}{N_p}}\frac{\gamma KT}{m_e}.$$

From the ionospheric data shown in table 6, it can be seen that on the right hand side of the inequality (22), the term $\frac{\omega}{\nu_{en}}\frac{N_n}{N_p}\frac{\gamma KT}{m_e}$ is the largest at any height. Hence the condition (22) may be represented by

$$\frac{1}{k^2} \gg \frac{\gamma KT}{m_e}\frac{1}{\omega \nu_{en}}\frac{N_n}{N_p}. \tag{22'}$$

It can be seen that the condition (22) can be satisfied for $\omega = 10^{-2}\ \mathrm{sec}^{-1}$ only if

$$\frac{1}{k} \gg 3 \times 10^9 \tag{23}$$

This means that there should be little difference in the characteristics of the phenomenon over distances of about 30,000 km (300° along a meridian). This is a rather severe condition that does not seem to be fulfilled in the case of world-wide changes. But it can be seen that all terms appearing in the condition (22) other than the largest are smaller than $\left(\frac{\omega}{k}\right)^2$ if

$$\frac{1}{k^2} \gg \frac{\gamma KT}{m_i}\frac{1}{\omega \nu_{in}'}\frac{N_n}{N_p}. \tag{22''}$$

This can be satisfied when ($w = 10^{-2}$ sec^{-1})

$$\frac{1}{k} \gg 1 \times 10^{8} , \tag{23'}$$

i.e., when features of the phenomena are similar over distances of 1000 km (10^{o} along a meridian), which seems to be true in the present case. Hence it may be said that when the condition (23') is fulfilled, all the terms in equations (1') - (3') are larger than the pressure gradient terms with the exception of one term - that which is related to the largest term on the right hand side of (22). This is $N_p \nu_{en} m_e \underline{u}_n$ on the left hand side of equation (3'), which is the smallest among the non-pressure gradient terms. Hence equations (1') - (3') may be approximated consistently by condition (23') by dropping, in addition to the pressure gradient terms, the term $N_p \nu_{en} m_e \underline{u}_n$ in equation (3'). The upper limit of the error introduced by this approximation is given by the ratio of the largest of the dropped terms to the smallest of the retained terms, i.e. by

$$\mathrm{Max}\left[\frac{\gamma KT}{m_i} \frac{k^2}{w \nu_{in}} \frac{N_n}{N_p} , \quad \frac{m_i \nu_{in}'}{m_e \nu_{en}} \right] , \tag{23''}$$

which is small enough - ranging from 10^{-3} at 80 km to 10^{-1} at 500 km. This estimation of the upper limit of the error is quite strict, and the real order of magnitude of the error could be considerably smaller than that given here.

Equations (1'') - (3'') are solved under this approximation. The components of $\underline{u}_e$ and $\underline{u}_i$ perpendicular to the main magnetic field $\underline{B}_o$ are denoted by $\underline{u}_e^{\perp}$ and $\underline{u}_i^{\perp}$, and written as,

$$\underline{u}_e^{\perp} = \frac{1}{\Delta}\left(\Delta_e \, e \, \underline{E}^{\perp} + \Delta_e' \frac{e}{c} \underline{u}_e^{\perp} \wedge \underline{B}_o + \Delta_e'' \frac{e}{c} \underline{u}_i^{\perp} \wedge \underline{B}_o \right), \qquad (24)$$

$$\underline{u}_i^{\perp} = \frac{1}{\Delta}\left(\Delta_i \, e \, \underline{E}^{\perp} + \Delta_i' \frac{e}{c} \underline{u}_i^{\perp} \wedge \underline{B}_o + \Delta_i'' \frac{e}{c} \underline{u}_i^{\perp} \wedge \underline{B}_o \right), \qquad (25)$$

where

$$\Delta = \alpha (i\omega)^3 + \beta (i\omega)^2 + \gamma (i\omega) + \delta,$$

$$\Delta_s = \alpha_s (i\omega)^2 + \beta_s (i\omega) + \gamma_s,$$

$$\Delta_s' = \alpha_s' (i\omega)^2 + \beta_s' (i\omega) + \gamma_s',$$

$$\Delta_s'' = \alpha_s'' (i\omega)^2 + \beta_s'' (i\omega) + \gamma_s'',$$

where $s = e$ or i,

and

$$\alpha = N_p^2 N_n m_e m_i m_n,$$

$$\beta = N_p^2 N_n m_e m_i m_n \nu_{en},$$

$$\gamma = N_p^2 N_n m_e m_i m_n \nu_{en} \nu'_{in} ,$$

$$\delta = 2 N_p^3 m_e^2 m_i \nu_{en} \nu'_{in} \nu_{ie} ,$$

$$\alpha_e = - N_p^2 N_n m_i m_n ,$$

$$\beta_e = - N_p^2 N_n m_i m_n \nu'_{in} ,$$

$$\gamma_e = 0 ,$$

$$\alpha_e' = - N_p^2 N_n m_i m_n ,$$

$$\beta_e' = - N_p^2 N_n m_i m_n \nu'_{in} ,$$

$$\gamma_e' = - N_p^3 m_e m_i \nu_{en} \nu'_{in} ,$$

$$\alpha_e'' = 0 ,$$

$$\beta_e'' = N_p^2 N_n m_e m_n \nu_{ie} ,$$

$$\gamma_e'' = N_p^3 m_e m_i \nu_{en} \nu_{in}',$$

$$\alpha_i = N_p^2 N_n m_e m_n,$$

$$\beta_i = N_p^2 N_n m_e m_n \nu_{en},$$

$$\gamma_i = 0,$$

$$\alpha_i' = 0,$$

$$\beta_i' = - N_p^2 N_n m_e m_n \nu_{ie},$$

$$\gamma_i' = - N_p^3 m_e m_i \nu_{en} \nu_{in}',$$

$$\alpha_i'' = N_p^2 N_n m_e m_n,$$

$$\beta_i'' = N_p^2 N_n m_e m_n \nu_{en},$$

$$\gamma_i'' = N_p^3 m_e m_i \nu_{en} \nu_{in}',$$

From equations (24) and (25) it follows that

$$\left\{ 1 + \frac{\Delta_e' \Delta_e'}{\Delta^2} \frac{e^2 B_0^2}{c^2} + \frac{\Delta_e'' \Delta_i'}{\Delta^2} \frac{e^2 B_0^2}{c^2} \right\} \underline{u}_e^{\perp}$$

$$+ \left\{ \frac{\Delta_e' \Delta_e''}{\Delta^2} \frac{e^2 B_0^2}{c^2} + \frac{\Delta_e'' \Delta_i''}{\Delta^2} \frac{e^2 B_0^2}{c^2} \right\} \underline{u}_i^{\perp}$$

$$= \frac{\Delta_e}{\Delta} e \underline{E}^{\perp} + \frac{1}{\Delta^2} (\Delta_e' \Delta_e + \Delta_e'' \Delta_i) \frac{e^2}{c} \underline{E}^{\perp} \wedge \underline{B}_0 , \tag{26}$$

$$\left\{ \frac{\Delta_i' \Delta_e'}{\Delta^2} \frac{e^2 B_0^2}{c^2} + \frac{\Delta_i' \Delta_i''}{\Delta^2} \frac{e^2 B_0^2}{c^2} \right\} \underline{u}_e^{\perp}$$

$$+ \left\{ 1 + \frac{\Delta_i' \Delta_e''}{\Delta^2} \frac{e^2 B_0^2}{c^2} + \frac{\Delta_i'' \Delta_i''}{\Delta^2} \frac{e^2 B_0^2}{c^2} \right\} \underline{u}_i^{\perp} \tag{27}$$

$$= \frac{\Delta_i}{\Delta} e \underline{E}^{\perp} + \frac{1}{\Delta^2} (\Delta_i' \Delta_e + \Delta_i'' \Delta_i) \frac{e^2}{c} \underline{E}^{\perp} \wedge \underline{B}_0 .$$

The components of $\underline{u}_e$ and $\underline{u}_i$ parallel to $\underline{B}_0$, denoted by $\underline{u}''_e$ and $\underline{u}''_i$ are given by

$$\underline{u}_e'' = \frac{\Delta_e}{\Delta} e \underline{E}'' , \tag{28}$$

$$\underline{u}_i'' = \frac{\Delta_i}{\Delta} e \underline{E}'' . \tag{29}$$

The solution of equations (26) - (29) can, in general, be written in the form,

$$\underline{u}_e^{\perp}, \underline{u}_i^{\perp} \sim f \frac{(i\omega)^{11} + \cdots\cdots}{(i\omega)^{12} + \cdots\cdots} e \underline{E}^{\perp} + g \frac{(i\omega)^{11} + \cdot}{(i\omega)^{12} + \cdots} \underline{E}^{\perp} \wedge \underline{B}_0 \tag{30}$$

$$\underline{u}_e'', \underline{u}_i'' \sim h \frac{(iw)^2 + \cdots}{(iw)^3 + \cdots} e \underline{E}'', \tag{31}$$

where $(iw)^s + \cdots$ represents a power series in (iw) of order s, and f, g, h are non-zero constants. The complete expression must be derived if we wish to find out the characteristics over the entire frequency range. But in the present problem, we are interested in a limited range of frequency, and further approximation is possible.

The following condition holds between 80 and 500 km for ω ranging from 10^{-1} to 10^{-2} sec^{-1}.

$$\mathrm{Min}\,(\nu_{en}, \nu_{in}', \nu_{ie}) \gg \omega$$
$$\gg \mathrm{Max}\left(\frac{N_p m_e \nu_{en}}{N_n m_n}, \frac{N_p m_i \nu_{en} \nu_{in}'}{N_n m_n \nu_{ie}}, \frac{N_p m_i \nu_{in}'}{N_n m_n} \right). \tag{32}$$

This is equivalent to

$$\nu_{in}' \gg \omega \gg \frac{N_p m_i \nu_{en} \nu_{in}'}{N_n m_n \nu_{ie}}.$$

Then Δ, Δ_s, Δ_s' and Δ_s'' may be approximated as,

$$\Delta = N_p^2 N_n m_i m_e m_n \nu_{en} \nu'_{in} (i\omega)\left(1 + \frac{m_e \nu_{ie}}{m_i \nu'_{in}} + \frac{\nu_{ie}}{\nu_{en}}\right),$$

$$\Delta_e = - N_p^2 N_n m_i m_n \nu'_{in} (i\omega),$$

$$\Delta_e' = - N_p^2 N_n m_i m_n \nu'_{in} (i\omega)\left(1 + \frac{m_e \nu_{ie}}{m_i \nu'_{in}}\right),$$

$$\Delta_e'' = N_p^2 N_n m_n m_e \nu_{ie} (i\omega), \tag{33}$$

$$\Delta_i = N_p^2 N_n m_e m_n \nu_{en} (i\omega),$$

$$\Delta_i' = - N_p^2 N_n m_n m_e \nu_{ie} (i\omega),$$

$$\Delta_i'' = N_p^2 N_n m_e m_n \nu_{en} (i\omega)\left(1 + \frac{\nu_{ie}}{\nu_{en}}\right).$$

Above the 120 km level, where $\omega \gg \nu'_{in}$, equations (26) - (29) can be solved to give,

$$\underline{j}^{\perp} = \frac{N_p e^2 \nu'_{in}}{m_i \omega_i^2} \underline{E}^{\perp} - \frac{N_p e^2}{m_i \omega_i} \frac{\nu'^2_{in}}{\omega_i^2} \underline{E}^{\perp} \wedge \underline{b}_0 , \tag{34}$$

$$\underline{j}^{\parallel} = \frac{N_p e^2}{m_e \nu_{en}} \underline{E}^{\parallel}, \tag{35}$$

where $\underline{b}_0$ denotes a unit vector in the direction of $\underline{B}_0$. Between 80 and 120 km, where $\omega \ll \nu'_{in}$, the solution is,

$$\underline{j}^{\perp} = \frac{N_p e^2}{m_i \nu'_{in}}\left(1 + \frac{\nu_{en}\nu'_{in}}{W_i W_e}\right) e\underline{E}^{\perp} - \frac{N_p e^2}{m_i W_i} \underline{E}^{\perp}_{\wedge} \underline{b}_0 , \tag{36}$$

$$\underline{j}^{\parallel} = \frac{N_p e^2}{m_e \nu_{en}} \underline{E}^{\parallel} . \tag{37}$$

Thus the conductivity below the 500 km level is real and independent of ω. The ionosphere behaves like a conductor with anisotropic conductivity given by

$$\tilde{\sigma} = \begin{pmatrix} \sigma_1 & \sigma_2 & 0 \\ -\sigma_2 & \sigma_1 & 0 \\ 0 & 0 & \sigma_0 \end{pmatrix}, \tag{38}$$

where the z-axis is taken in the direction of $\underline{B}_0$, and from 80 to 120 km,

$$\sigma_1 = \frac{N_p e^2}{m_i \nu'_{in}}\left(1 + \frac{\nu_{en}\nu'_{in}}{W_i W_e}\right),$$

$$\sigma_2 = -\frac{N_p e^2}{m_i W_i} ,$$

$$\sigma_0 = \frac{N_p e^2}{m_e \nu_{en}} ,$$

and from 120 to 500 km,

$$\sigma_1 = \frac{N_p e^2 \nu'_{in}}{m_i W_i^2}$$

$$\sigma_2 = -\frac{N_p e^2 \nu_{in}'^2}{m_i w_i^3},$$

$$\sigma_0 = \frac{N_p e^2}{m_e \nu_{en}}.$$

These are tabulated in table 7, and illustrated in figure 25. Above the 500 km level, either of the following conditions is satisfied:

$$\nu_{in}' \ll \omega \ll \nu_{en}, \quad \nu_{ie}, \tag{39}$$

or

$$\nu_{in}', \nu_{en} \ll \omega \ll \nu_{ie} \tag{40}$$

ω taken to lie between 10^{-1} and 10^{-2} sec^{-1} does not exceed ν_e anywhere, because ν_{ie} is of the order of 10^2 sec^{-1} with a magnetospheric temperature of 10^5 °K and a plasma density of 10^2 cm^{-3}. Under the condition (39), Δ, Δ_S, Δ_s', Δ_s'' may be written as

$$\Delta = N_p^2 N_n m_e m_i m_n (\nu_{en} + \nu_{ie})(i\omega)^2,$$

$$\Delta_e = -N_p^2 N_n m_i m_n (i\omega)^2,$$

$$\Delta_e' = -N_p^2 N_n m_i m_n (i\omega)^2,$$

$$\Delta_e'' = N_p^2 N_n m_n m_e \nu_{ie} (i\omega), \tag{41}$$

$$\Delta_i = N_p^2 N_n m_n m_e \nu_{en} (i\omega),$$

$$\Delta_i' = -N_p^2 N_n m_n m_e \nu_{ie} (i\omega),$$

$$\Delta_i'' = N_p^2 N_n m_n m_e (\nu_{en} + \nu_{ie})(i\omega).$$

Table 7. Electrical conductivity in the ionosphere.*

Altitude (km)	σ_0	σ_1	$\lvert\sigma_2\rvert$
80	3.10^{4}	1.10^{4}	3.10^{4}
90	6.10^{6}	5.10^{4}	8.10^{5}
100	1.10^{8}	2.10^{5}	3.10^{6}
120	3.10^{9}	4.10^{6}	5.10^{6}
140	7.10^{9}	7.10^{5}	1.10^{5}
160	3.10^{10}	5.10^{5}	4.10^{4}
180	3.10^{10}	2.10^{5}	6.10^{3}
200	5.10^{10}	9.10^{4}	2.10^{3}
250	1.10^{11}	4.10^{4}	2.10^{2}
300	1.10^{11}	2.10^{4}	4.10^{1}
350	1.10^{11}	8.10^{3}	8.10^{0}
400	1.10^{11}	4.10^{3}	2.10^{0}
450	1.10^{11}	2.10^{3}	6.10^{-1}
500	1.10^{11}	1.10^{3}	2.10^{-1}

*Cgs Gauss units are used. These may be transformed into other systems of units by,

$$\sigma_{esu} = \sigma_{Gauss},$$

$$\sigma_{emu} = \frac{1}{c^2}\sigma_{Gauss},$$

$$\sigma_{MKS} = \frac{10^{11}}{c^2}\sigma_{Gauss}.$$

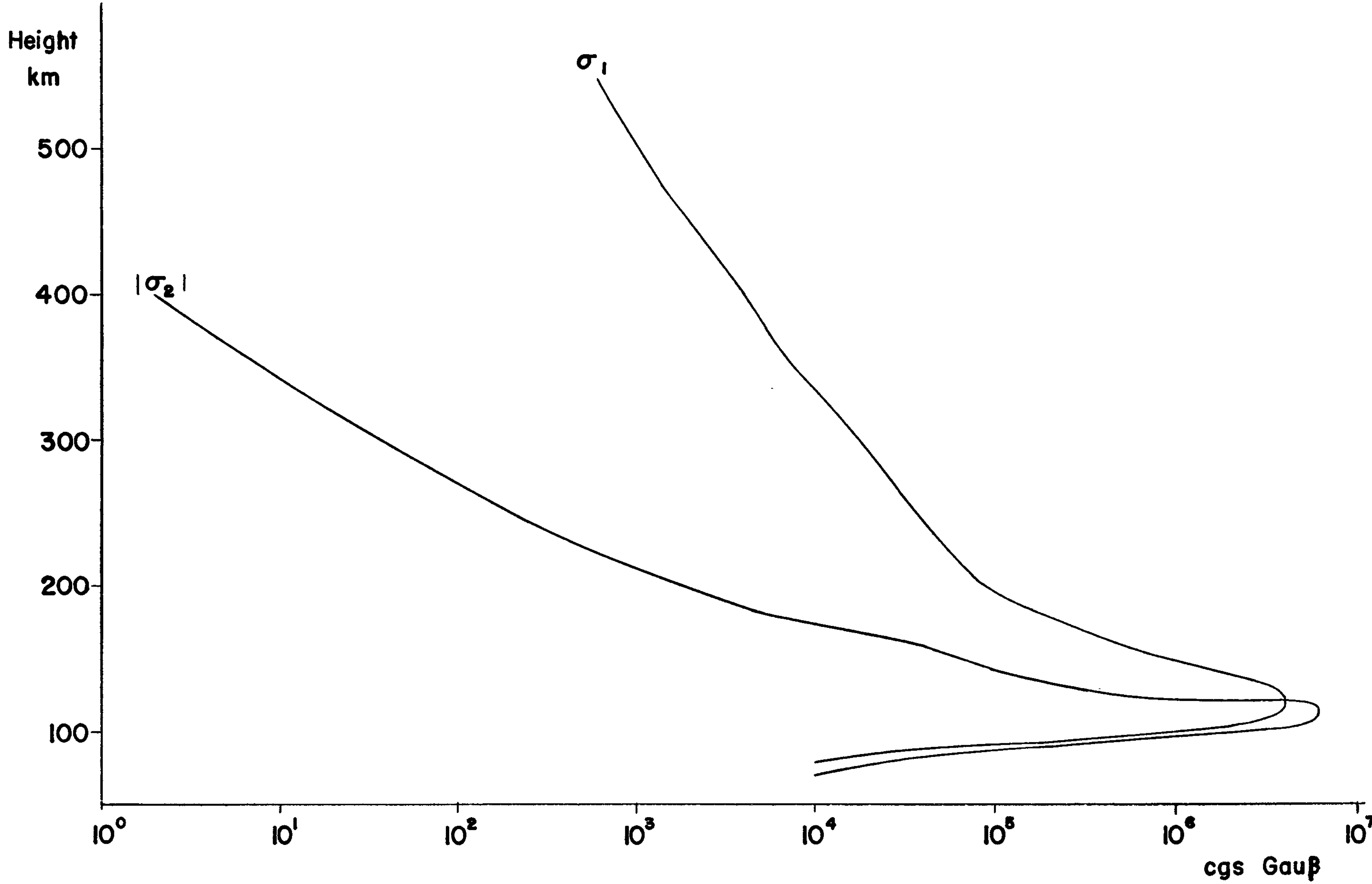


Figure 25. Distribution of the electrical conductivity with height.

Substituting these into equations (26) - (29) it follows that,

$$\underline{j}^{\perp} = \frac{N_p m_i c^2}{B_0^2}(i\omega)\underline{E}^{\perp} - \frac{N_p c e}{B_0}\frac{(i\omega)^2}{\omega_i^2}\underline{E}^{\perp}\wedge \underline{b}_0, \tag{42}$$

$$\underline{j}^{\parallel} = \frac{N_p e^2}{m_e(\nu_{en}+\nu_{ie})}\underline{E}^{\parallel}, \tag{43}$$

Under condition (40) we have,

$$\begin{aligned}
\Delta &= N_p^2 N_n m_e m_i m_n \nu_{ie}(i\omega)^2,\\
\Delta_e &= -N_p^2 N_n m_i m_n (i\omega)^2,\\
\Delta_e' &= -N_p^2 N_n m_i m_n (i\omega)^2,\\
\Delta_e'' &= N_p^2 N_n m_n m_e \nu_{ie}(i\omega),\\
\Delta_i &= N_p^2 N_n m_n m_e (i\omega)^2,\\
\Delta_i' &= -N_p^2 N_n m_n m_e \nu_{ie}(i\omega),\\
\Delta_i'' &= N_p^2 N_n m_n m_e \nu_{ie}(i\omega).
\end{aligned} \tag{44}$$

It follows then that,

$$\underline{j}^{\perp} = \frac{N_p m_i c^2}{B_0^2}(i\omega)\underline{E}^{\perp} - \frac{N_p c e}{B_0}\frac{(i\omega)^2}{\omega_i^2}\underline{E}^{\perp}\wedge \underline{b}_0, \tag{45}$$

$$\underline{j}^{\parallel} = \frac{N_p e^2}{m_e \nu_{ie}}\underline{E}^{\parallel}. \tag{46}$$

Hence above 500 km the conductivity is given by

$$\sigma_1 = \frac{N_p m_i c^2}{B_0^2} (i\omega) ,$$

$$\sigma_2 = - \frac{N_p c e}{B_0} \frac{(i\omega)^2}{\omega_i^2} , \tag{47}$$

$$\sigma_0 = \frac{N_p e^2}{m_e(\nu_{en}+\nu_{ie})} .$$

The main difference between the conductivity matrix given by (38) and that given by (47) lies in the dependence of σ_1 on ω. Below about 500 km, σ_1 is a constant independent of ω, and the medium behaves as an anisotropic conductor. Above the 500 km level, on the other hand, σ_1 is proportional to $i\omega$. Substituting this into the Maxwell equation (13);

$$(\operatorname{curl} \underline{B})^{\perp} = \frac{4\pi}{c} \underline{j}^{\perp} + \frac{1}{c} \frac{\partial \underline{E}^{\perp}}{\partial t} \doteqdot \frac{1}{c} \left(\frac{4\pi N_p m_i c^2}{B_0^2} + 1 \right) i\omega \underline{E}^{\perp} , \tag{48}$$

it can be seen that this has an effect equivalent to alternating the dielectric constant. Hence the phase velocity of the propagation of the electromagnetic field becomes entirely different from c. This mode of propagation of the electromagnetic field is called a magnetohydrodynamic wave.

Equation (48) has been solved in the case of transverse propagation by Spitzer (1956). But it is more customary to rewrite $\underline{j}$ as,

$$\underline{j} = \tilde{\sigma}' \left(\underline{E} + \frac{1}{c} \underline{V} \wedge \underline{B}_0 \right) , \tag{49}$$

$\underline{v}$ is the velocity of the center of mass of the gas and $\hat{\sigma}'$ is the conductivity matrix as determined in a frame which is moving with the velocity $\underline{v}$, $\underline{v}$, defined by,

$$\underline{V} = \frac{N_p(m_i \underline{u}_i + m_e \underline{u}_e) + N_n m_n \underline{u}_n}{N_p(m_i + m_e) + N_n m_n}, \tag{50}$$

is given from equations (1) - (3) by

$$\rho \frac{d\underline{V}}{dt} = \frac{1}{c} \underline{j} \wedge \underline{B}_0, \tag{51}$$

where

$$\rho = N_p (m_i + m_e) + N_n m_n .$$

Substituting equations (42) or (45) into equation (1) and noting that $N_p \gg N_n$ in the magnetosphere, we have,

$$N_p m_i (i\omega) \underline{V}^{\perp} = \frac{B_0}{c}\left(\frac{N_p m_i c^2}{B_0^2} (i\omega) \underline{E}^{\perp} \wedge \underline{b}_0 + \frac{N_p c e}{B_0} \frac{(i\omega)^2}{\omega_i^2} \underline{E}^{\perp} \right) \tag{52}$$

It follows from equations (49) and (52) that,

$$\underline{E}^{\perp} + \frac{1}{c} \underline{V} \wedge \underline{B}_0 = \frac{i\omega}{\omega_i} \underline{E} \wedge \underline{b}_0,$$

$$\text{and} \qquad 4\pi \underline{j}^{\perp} = \frac{\omega_p'^2\, i\omega}{\omega_i^2} \underline{E}^{\perp} - \frac{\omega_p'^2 (i\omega)^2}{\omega_i^3} \underline{E}^{\perp} \wedge \underline{b}_0 \tag{53}$$

where ω_p' is the proton plasma frequency defined by

$$\omega_p' = \left(\frac{4\pi N_p e^2}{m_i} \right)^{\frac{1}{2}}$$

Hence

$$\sigma_1' = -\frac{\omega_p'^2 (i\omega)}{4\pi \omega_i^2},$$

$$\sigma_2' = -\frac{\omega_p'^2}{4\pi \omega_i}, \tag{54}$$

$$\sigma_0' = \sigma_0 .$$

As $\omega \ll \omega_i$ inside about ten earth radii for ω : $10^{-1} \sim 10^{-2}\ sec^{-1}$, $|\sigma_2'| \ll |\sigma_1'|$ and it follows that,

$$\left| \underline{E}^{\perp} + \frac{1}{c} \underline{V} \wedge \underline{B}_0 \right| \doteqdot \left| \frac{i\omega}{\omega_i} \underline{E}^{\perp}_{\wedge} \underline{b}_0 \right| \ll |\underline{E}^{\perp}| . \tag{55}$$

Hence

$$\underline{E}^{\perp} + \frac{1}{c} \underline{V} \wedge \underline{B}_0 \doteqdot 0 . \tag{56}$$

The theory of magnetohydrodynamic wave propagation is constructed from this equation which implies that the magnetic lines of force are virtually frozen into the conductive medium (Alfven, 1950: Cowling, 1957; Dungey , 1958).

But it must be noted that the applicability of the theoretical ray treatment adopted by most previous workers is quite limited for problems in geomagnetism and aeronomy. Ray theory is valid only when the medium is sufficiently uniform within a distance of a wavelength, i.e., it is appropriate only for phenomena whose characteristic period of variation T satisfies $\frac{VT}{L} \ll 1$, where V is the Alfven velocity and L a characteristic distance of variation in V . Table 8 shows that this condition requires $T \ll 1\ sec$ in most parts of the magnetosphere. Hence for most phenomena studied in geomagnetism and aeronomy, theoretical ray treatment is inadequate.

Table 8. Check of the uniformity of the magnetosphere.

Altitude (km)	V(cm/sec)	L(cm)	V/L(sec^{-1})
7.10^3	3.10^8	5.10^8	6.10^{-1}
3.10^3	5.10^8	2.10^8	3.10^0
1.10^3	5.10^7	4.10^7	1.10^0
5.10^2	5.10^7	3.10^7	2.10^0

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