THE HYPERFINE STRUCTURE OF VERCTRY EXTRACTED FROM NEUTRON IRRADIATED GOID
by

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## Abstract.

The hyperfine structure of mercury extracted from neutron irradiated gold has been investigated with the did of a Fabry-Perot etalon. The theory and design of the interferometer are discussed in detail. The wavelengths of sixteen lines in the spectrum of $\mathrm{Hg}^{198}$, and the hyperfine structure of many lines in the spectrum of Hg199 have been evaluated and compared with former determinations. The ratio of $\mathrm{Hg}^{199}$ to $\mathrm{Hg}^{198}$ produced during the neutron bombardment of gold has been determined from intensity measurements of the hyperfine structure patterns. The result yields a value of (1.78士0.10) X $10^{4}$ barns for the neutron capture cross section of $A u^{198}$.

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## CHAPTER I: INTRODUCTION.

The transmutation of gold by neutron irradiation has been a source of growing interest during the past decade. In its natural state gold consists of but a single stable isotope, namely that of atomic weight 197. Early workers found that when Aul97 was subjected to a flux of slow neutrons the stable nucleus captured a neutron to form the radioactive Aul98 isotope. The latter then decayed by beta emission to $\mathrm{Hg}^{198}$. We may represent these reactions by the following equations:

$$
\left.\begin{array}{rl}
79 \mathrm{au}^{\prime 97} & +0 \mathrm{n}^{\prime} \rightarrow 79 \mathrm{au}^{198}+\gamma \\
& 79 \mathrm{au}^{\prime \prime 9} \rightarrow 80 \mathrm{Hg}^{188}+\mathrm{p}^{-}
\end{array}\right\} \quad \text { (1.1) }
$$

The half life of the associated beta decay has satisfactorily been determined as 2.69 days. The most recent and complete decay scherne for the $79^{\text {Au }}{ }^{198}$ has been proposed by Schiff and Metzger (1) and is depicted in Figure 1.

It now appears that the reactions of eqations (1.1) may allow a seemingly unrelated line of endeavour to be brought to a successful conclusion. For years physicists have been searching for a quantity to serve as the ultimate standard of length. The inadequacy of the International Prototype Metre was early recognized. In 1889 Michelson and Morley stated that the wavelength of the intense green line of mercury would probably be used as the ultimate standard of length. However, when they observed this radiation with their interferometer, they found it to be one of the most complex in nature. On the other hand the red light of cadmium was more nearly homogeneous. Consequently steps were taken to set up the latter as a standard.

Natural mercury consists of seven stable isotopes of mass


Figure 1: A decay scheme for $79^{A} u^{198}$.


Figure 2: A decay scheme for 7oAu199.
numbers $196(0.15 \%), 198(10.1 \%), 199(17.0 \%), 200(23.3 \%), 201$ (13.2\%), $202(29.6 \%), 204(6.7 \%)$. The resulting involved hyperfine structure patterns account for the complexity of the observed lines. W.F. Meggers (2) was among the first to suggest that if one of the even isotopes could be separated from the rest in sufficient quantities to make discharge tubes, the resulting green line would be an exceedingly monochromatic radiation of high intensity. He has further proposed that this green line should supersede the red line of cadmium as the ultimate standard of length. It would seem that the method of equations (I.I) is a simple and convenient means of arriving at this desired result.

The $\mathrm{Hg}^{198}$ initially prepared in this manner was found to be spectroscopically pure, but the yield was insufficient for the manufacture of lamps. Following World War II more effective neutron sources were available and lamps of several types were constructed. Later workers, using neutron fluxes as high as $10^{12}$ neutrons per cm. ${ }^{2}$ sec., discovered the presence of $\mathrm{Hg}^{199}$ in minute and varying quantities, and ascribed it to the presence of platinum as an impurity in the gold (neutron irradiated platinum resulting in $\mathrm{Hg}^{199}$ ). This was a false conjecture however. If Au ${ }^{198}$ were to possess an extremely high neutron capture cross section, then the following reactions would occur:
$\left.\begin{array}{rl}79 \mathrm{Au}^{198}+0 \mathrm{n}^{\prime} \rightarrow 79 \mathrm{Au}^{199}+8 \\ 79 \mathrm{Au}^{199} \rightarrow 80 \mathrm{Hg}^{199}+\mathrm{B}^{-}\end{array}\right\}(1.2)$

For large neutron fluxes the quantity of $\mathrm{Hg}^{199}$ produced in this manner would become appreciable.

Hence during the neutron irradiation a branching process takes place, the amount of $\mathrm{Hg}^{199}$ produced being dependent on the
flux and the Aul98 cross section. The half life of the beta radiation from Au199 is known to be 3.3 days. The decay scheme is shown in Figure 2.

We see then that an accurate determination of the neutron capture cross section of $A u^{198}$ is a problem of some importance. Hill and Mihelich (4) were among the first to correctly interpret the reactions involved in gold transmutation. By comparing the intensities of the $0.159 \mathrm{MeV} \gamma-r a y$ of Au 199 and the 411 KeV -ray of Aul98 they were able to estimate the ratio in which the two isotopes were produced. This in turn allowed an estimate of the cross section as $3.5 \times 10^{4}$ barns ( 1 barn $=10^{-24} \mathrm{~cm}{ }^{2}$ ). In a later paper (5) Hill mentions that this value is too high, and suggests $1.6 \times 10^{4}$ barns to be more appropriate.

It is proposed here to combine a determination of this cross section with the setting up of a Fabry-Perot etalon suitable for high resolution interferometry. It is well known that the intensity of a spectral line is directly proportional to the amount of the isotope giving the radiation. Hence if we resolve the hyperfine structure patterns due to the mercury extracted from neutron irradiated gold, and measure the relative intensities of the components due to each isotope, we obtain the ratio in Which the two isotopes were produced. This in turn allows the calculation of the desired cross section.

The succeeding chapters will deal fully with the theory and construction of the interferometer, and its application to the particular problem at hand.

CHAPTER II: THE THEORY OF THE FABRY-PEROT INTERFERONETER. The Fabry-Perot interferometer is one of the simplest and yet one of the most elegant of all optical instruments. It consists essentially of two plane-parallel partially reflecting flats separated by a narrow air space. Light incident on one of the plates suffers multiple reflections in passing through the instrument, and the resulting emergent beams interfere constructively and destructively to produce a series of circular fringes. The sharpness of the fringes increases rapidly with increasing reflectivity of the plates. Measurement of the fringe diameters allows the calculation of wavelengths. With a suitable choice of plate separation an extremely high resolving power is attainable.

The interferometer may be such that the width of the air gap is adjustable by motion of one or both plates, or nore commonly the plates are fixed and no change of separation is possible. The latter type is called the etalon. The most cominon metals used to gain high reflecting power are silver and aluminum, the latter especially in the uttra-violet region. The instrument is useful within the transmission range of quartz (approximately 2000 to 20,000 angstroms).

The theory of the instrument given here most nearly parallels that of K . W. Meissner (6).

Section 1: The Theory of the Interference Fringes.
Consider the two plates of the etalon with reflecting surfaces $S$, and $S_{2}$ a distance $t$ apart, and suppose a monochromatic plane wave is incident on $S$, as indicated in Figure 3.


Figure 3: The principle of the etalon.
Each time the ray strikes either $S_{\text {, or }} S_{2}$ it is partially reflected and partially transmitted. We will neglect here the rays returning to the first medium. The path difference between successive transmitted beams is the same. To evaluate this difference let the angle of incidence be $\theta$ and take the refractive index of air to be unity. Then IN is parallel to the original ray. The path difference between rays $I$ and 2 , $d$, is given by

$$
d=M N+N Q
$$

since the waves at $M$ and Q are in phase.
From the figure we see that

$$
\begin{align*}
& d=P Q \\
& d=2 t \cos \theta \tag{2.1}
\end{align*}
$$

If $\lambda$ be the wavelength of the incident light the phase difference between rays, denoted by $p$, is given by

$$
\begin{align*}
& p=\frac{2 \pi}{\lambda} d \\
& p=\frac{2 \pi}{\lambda} 2 t \cos \theta \tag{2.2}
\end{align*}
$$

Equation (2.2) does not account for the possibility of a phase change due to reflection at the metal films. This effect is generally small however (6) and will be neglected here.

By equation (2.2) the angle of incidence $\boldsymbol{\theta}$ determines the phase difference. The aggregate of all rays incident at 0 will form a circular ring of phase $p$ on leaving the etalon. Since
in an extended source all angles of incidence are present, there is a continuous change of phase across the resulting pattern. If $p$ is such as to give constructive interference we get bright circles; for destructive interference we get dark circles.

We will now develop a quantitative expression for the intensity distribution in the interference pattern. The incident wave, if taken to be of unit amplitude may be represented by the real part of $e^{i \omega \tau}$, where $\frac{\omega}{2 \pi}$ is the frequency of the light and $\tau$ is the time. If $R$ and $T$ are the fractions of the incident light reflected and transmitted respectively by each film, then the intensities of successive emergent trains are $T^{2}, R^{2} T^{2}, \ldots R^{2(k-1)} \mathbb{T}^{2}$ (for the list, and,...$k$ th trains). The corresponding amplitudes are then $T, R T, R^{2} T, \ldots$. Since between each of these there is an increase in phase angle of $p$, the emergent waves may be represented by the real parts of $T e^{i \omega \tau}$, $R T e^{i(\omega \tau-\beta)}, \ldots \mathrm{R}^{k-I_{T e}} i[\omega \tau-(k-1) \beta] \quad$. The sum of these partial vibrations will yield the resulting vibration. If there is an infinite number of these emergent trains, as is the case in practice for relatively large diameter plates and small separatimon, then the resultant wave of amplitude A is given by

$$
A e^{i \omega \tau}=T_{e}^{i \omega t} \sum_{i=1}^{p} R^{k-1} e^{-(k-1) \beta i}
$$

(Note, however, the discussion on page 7 .)
This becomes: $A e^{i \omega t}=T e^{i \omega t}\left(1+R e^{-\beta i}+R^{2} e^{-2 \beta i}+\cdots\right)$

$$
A_{e}^{i \omega \tau}=\frac{T_{e}^{i \omega \tau}}{1-R_{e}-\theta_{i}},
$$

and hence

$$
\begin{equation*}
A=\frac{T}{1-R_{e} R_{i}} \tag{2.3}
\end{equation*}
$$

The observable intensity $I$ is equal to $A A^{*}$, where $A^{*}$ is the complex conjugate of A. Therefore

$$
\begin{aligned}
& \text { A. Therefore } T^{2} \\
& I=\frac{T_{e}^{2}}{\left(1-R_{e}^{-p i}\right)\left(1-R e^{R i}\right)} \\
& 1+R^{2}-2 R \cos T^{2}
\end{aligned}
$$

and by substituting $\cos p=1-2 \sin ^{2} \frac{p}{2}$,

$$
\begin{equation*}
I=\frac{T^{2}}{(V-R)^{2}+4 R \sin ^{2} \frac{R}{2}} \tag{2.4}
\end{equation*}
$$

Equation (2.4) holds for an ideal case. Actually, for a beam incident at angle $\theta$, a shift of $\boldsymbol{Z} t \tan \theta$ across the plate occurs after each reflection and ultimately some of the beams are lost. This results in a broadening of the fringes and a decrease in intensity in the outer rings. To take the effect into account each term in the preceding summation should be multiplied by a correction factor, and the number of terns will now be finite. Such a calculation has been made for a square etalon by Geiger (15), and his results show that for small plate separations and reflectivity of $80 \%$ the effect is negligible.

From equation (2.4) we get for the maximum intensity

$$
\begin{equation*}
I_{\max }=\frac{T^{2}}{(1-R)^{2}} \tag{2.5}
\end{equation*}
$$

which occurs for $\frac{\alpha}{2}=m \pi(m=0,1, \ldots)$. Using the value of $p$ of equation (2.2) this leads to $m \lambda=2 t \cos \theta$ (2.6) For values of $\theta$ satisfying (2.6) a bright fringe results. In is called the order of interference.

The minimum intensity is given by

$$
\begin{equation*}
I_{\text {min }}=\frac{T^{2}}{(V+R)^{2}} \tag{2.7}
\end{equation*}
$$

which occurs for $\not Z / 2=\left(m+\frac{1}{2}\right) \pi(m=0, \ldots)$. Therefore if $\theta$ satisfies

$$
\left(m+\frac{1}{2}\right) \lambda=2 t \cos \theta
$$

a dark fringe results. Along a diameter of the pattern the intensity varies between these two extremes according to equation (2.4).

Rewriting equation (2.4) thus:

$$
I=\frac{T^{2}}{(1-R)^{2}} \cdot \frac{1}{1+\frac{Y R}{(-R)^{2}} \sin ^{2} R / 2}
$$

and denoting by $F$ the quantity $4 R /(1-R)^{2}$, we get

$$
\begin{equation*}
I_{\text {min }}=I_{\max } / 1+F \tag{2.10}
\end{equation*}
$$

Note then that the sharpness of the fringes and the ratio: of maxima to minima are determined by $F$, the "coefficient of finesse". F in turn depends solely upon $R$, the reflection coefficient. The fringe sharpness increases rapidly with increasing $R$ indicated in Figure 4.

Figure 5 is a plot of $I_{\text {min }}$ as a function of $R$. The value of $I_{m i n}$ is given as a percentage of $I_{m a x}$.

In the ideal case $(R+T)=I$ and $I_{\max }$ is the same as the incident light intensity. $I_{\text {max }}$ is less in practice because of absorption by the metal film, ie., $R+T+A=I$ where $A$ is the absorption coefficient. The quantity A must always be considered in determining a feasible value of $R$ so that too low a transmission does not result. It is for this reason that silver is seldom used in the ultra violet since it has a large absorption coefficient throughout this region (7).

Section 2: The Fringe Falf-Width.
From equation (2.2), as $\theta$ increases, $\cos \theta$ decreases, and so p decreases. Hence the order of interference is greatest at the centre of the pattern.

We define the half width of a fringe as the width at half the maximum intensity. Suppose we consider the phase difference $I$ at this point, $i \cdot \theta ., \mathcal{L}$ is the value of $p$ when $I=\frac{I}{Z} I_{\max }$. We get from equations (2.5) and (2.9)

$$
\begin{aligned}
\frac{(1-R)^{2}}{(1-R)^{2}+4 R \sin ^{2 l / 2}} & =\frac{1}{2} \\
\sin \frac{l}{2} & =\frac{1-R}{2 \sqrt{R}} .
\end{aligned}
$$



Figure 4: Dependence of intensity distribution upon R. (7).


Figure 5: Dependence of $I_{\text {min }}$ on $R(7)$.

At the maximum $\quad p=2 \mathrm{~m} \pi$, and so. for $\boldsymbol{l}$ we may write

$$
\rho=2 \sin \pi+x
$$

Hence $\quad \sin \frac{l}{2}=\sin \left(m \pi+\frac{x}{2}\right)$
$\sin \frac{l}{2}= \pm \sin \frac{x}{2}$
giving $\quad \sin \frac{x}{2}=\frac{1-R}{2 \sqrt{R}}$.
Theoretically equation (2.12) allows the determination of $R$, the reflection coefficient of the plates. Remembering that the phase difference between maxima is $2 \pi$ we may measure $x$ and calculate $R$. In practice the method is likely to be unreliable because of fringe broadening due to temperature change, and other causes.

Section 3: The Resolving Power of the Etalon.
The resolving power of any spectrograph is defined by

$$
R_{0}=\frac{\lambda}{\Delta \lambda}=\frac{v}{\Delta v}
$$

where $\Delta \lambda(\Delta v)$ is the smallest change in wavelength (wave number) that can be resolved. From equation (2.6), since $\cos \theta$ is approximately unity for the etalon,

$$
M \lambda \doteqdot 2 t
$$

defines a variable order M. Differentiating,

$$
M \Delta \lambda \neq \lambda \Delta M=0 \quad \text { or } \quad \frac{\lambda}{\Delta \lambda}=\frac{M}{\Delta M}
$$

and since $m=1 / 2 \pi$
then $\Delta M=\Delta \mu / 2 \pi$.
Following Valasek. (8) it is concluded that two maxima are just resolvable when they cross at $I=0.405 I_{\max }$. If $(2 m \pi+y)$ is the phase difference at this point, equations (2.5) and (2.9) give

$$
1+\frac{4 R}{(1-R)^{2}} \sin ^{2}\left(1 \pi \pi+\frac{y}{2}\right)=\frac{1}{0.405}
$$

$$
\begin{aligned}
& \sin ^{2}(m \pi+y / 2)=0.368 \frac{10}{R} \frac{(1-R)^{2}}{R} \\
& \sin ^{2} \cdot y / 2=0.368 \frac{(1-R)^{2}}{R} \\
& y=2 \sin ^{-1}\left[\frac{0.368(1-R)^{2}}{R}\right]^{\frac{1}{2}}
\end{aligned}
$$

The change in phase difference between the two just resolved maxima is $2 y$, and so the resolving power becomes

$$
\begin{align*}
& R_{0}=2 \pi M / \Delta p \\
& R_{0}=2 \pi M / 2 y \\
& R_{0}=\pi M / 2 \sin ^{-1}\left[\frac{0.368(1-R)^{2}}{R}\right]^{\frac{1}{2}} \tag{2.13}
\end{align*}
$$

Since $M$ is proportional to t the resolving power increases with increasing separation of the plates.

We may note here that in practice an estimate of the resolveing power is given by

$$
R_{0}=M m
$$

where $n$ is the number of equivalent beams forming the interfereence pattern.

Section 4: The Interorder Separation.
Suppose two wavelengths $\lambda$, and $\lambda_{2}$ are present. Each will give rise to its own ring system. If $\lambda_{1}=\lambda_{2}$ the two systems coincide, and as $\left|\lambda_{2}-\lambda,\right|$ increases the systems diverge until the ring of order $m$ due to $\lambda$, coincides with the ring of order $(m+1)$ due to $\lambda_{2}$. This value of $\left|\lambda_{2}-\lambda\right| \equiv 0 \lambda$ is called the interorder separation.

If $m_{0}$ be the order of interference at the centre of the pattern where $\theta=00$ then

$$
m_{0} \lambda=2 t
$$

Differentiating,

$$
\begin{aligned}
& \Delta \lambda=\frac{\lambda \Delta m}{m_{0}} \\
& \Delta \lambda=\frac{\lambda^{2} \Delta m}{2 t} \quad \text { (neglecting the sign) }
\end{aligned}
$$

This last will be true at or near the centre of the system. For the interorder separation $\Delta m=1$ so that

$$
\begin{equation*}
\Delta \lambda=\frac{\lambda^{2}}{2 t} \tag{2.14}
\end{equation*}
$$

In terms of wave number $\quad \nu=\frac{1}{\lambda}$,

$$
\Delta v=\frac{1}{2 t}
$$

Section 5: Reduction of Observations.
We have seen that

$$
\begin{equation*}
m \lambda=2 t \cos \theta \tag{2.6}
\end{equation*}
$$

and at the centre of the pattern

$$
m_{0} \lambda=2 t
$$

These give the relation
$m=m_{0} \cos 0$,
and since $\theta$ is in general small we may approximate by

$$
\begin{equation*}
m=m n_{0}\left(1-\frac{\theta^{2}}{2}\right) \tag{2.16}
\end{equation*}
$$

The interference rings are formed at infinity and if brought to a focus by a lens of focal length $f$, the linear diameter of the $k$ th fringe is determined by

$$
D_{t} / 2=f \tan \theta .
$$

Again, for small $\theta$, this becomes

$$
D_{k / 2 f}=\theta
$$

$$
\text { and so } \quad m_{k}=m_{0}\left(1-\frac{D_{k}^{2}}{8 f^{2}}\right) \text {, }
$$

where $m_{k}$ is the order of the $k$ th ring.
Rearranging,

$$
\begin{equation*}
D_{k}^{2}=\frac{8 f^{2}}{m_{0}}\left(m_{0}-m_{k}\right) \tag{2.17}
\end{equation*}
$$

In general the order at the centre of the pattern is not integral, but is some positive fraction $\epsilon$. If $m$, is the order of the first fringe, then $m_{0}=m,+\epsilon$. The first ring exceeds the $k$ th by ( $k-1$ ) orders, and so the centre exceeds the $k$ th by ( $\epsilon+k-1$ ) orders, ie:

$$
m_{0}-m_{l}=\epsilon+k-1
$$

Substituting in equation (2.17),

$$
D_{k}^{2}=\frac{8 f^{2}}{m_{0}}(k-1+\epsilon)
$$

Similarly we could get

$$
D_{h+1}^{2}=\frac{8 f^{2}}{m_{0}}(A+\epsilon)
$$

Solving between these two equations for $\epsilon$,

$$
\begin{equation*}
\epsilon=\frac{D_{t+1}^{2}}{D_{k+1}^{2}-D_{t}^{2}}-k \tag{2.18}
\end{equation*}
$$

Note here that the differences between the squares of adjacent diameters is constant, and is in fact given by $\frac{8 f^{2}}{m_{0}}$ By averaging these differences we get a value of $\epsilon$ but this is not accurate since in actuality only the first and last fringes are utilized. Hence a least squares determination of $\mathcal{E}$ is necessary. Such a calculation has been made by Rolf and Darrell (9) and the results of their investigations will be briefly set down.

Let the constant difference

$$
D_{k+1}^{2}-D_{k}^{2} \equiv 8 f^{2} / m_{0}
$$

be denoted by No Then equation (2.18) gives

$$
\begin{aligned}
\epsilon & =N^{-1} D_{1}^{2} \\
1+\epsilon & =N^{-} D_{2}^{2}
\end{aligned}
$$

etc.
Let $\frac{\epsilon}{N^{-1}}=a, \quad \frac{1}{N}=\frac{1}{b}$.
Then $\quad \dot{D} \dot{,}^{2}=a$

$$
\begin{aligned}
& \dot{d}_{2}^{2}=a+b \\
& \vdots \\
& \dot{a}_{b}^{2}=a+(b-1) b
\end{aligned}
$$

Applying the method of least squares the normal equations in


$$
S=s a+\sigma b
$$

where

$$
\begin{array}{ll}
\Sigma=d_{1}^{2}+d_{2}^{2}+\cdots+d_{k}^{2} ; & a=1+2+3+\cdots+(k-1) \\
S=d_{2}^{2}+2 d_{3}^{2}+\cdots+(k-1) d_{k}^{2} ; & \sigma=1^{2}+2^{2}+\cdots+(k-1)^{2}
\end{array}
$$

When solved for a and bethe above equations lead to

$$
a=\frac{\sigma \Sigma+a S}{p \sigma-2^{2}} \quad, \quad b=\frac{h S-\infty \Sigma}{d \sigma-2^{2}},
$$

and finally we find for $\epsilon$

$$
\begin{align*}
& \epsilon=a / b \\
& \epsilon=\frac{\sigma \Sigma-a S}{\ell 5-a \Sigma} \tag{2.19}
\end{align*}
$$

Section 6: The Rectangular Array of Tolansky.
S. Tolansky (7) has devised an extremely useful method for the calculation of small wavelength differences (such as arise in hyperfine structure) from measurements of the fringe diameter e. Suppose two wavelengths $\lambda_{\text {, }}$ and $\lambda_{2}$ are present, $\lambda$, and $\lambda_{2}$ being components of a single line as in hyperfine structure. Then for $\lambda$, ,

$$
D_{k+1}^{2}-D_{k}^{2}=\frac{8 f^{2}}{m_{0}}=N,
$$

and similarly for $\lambda_{2 i n}$

$$
D_{l+1}^{\prime 2}-D_{l}^{\prime 2} \equiv \frac{8 f^{2}}{m_{0}^{\prime}}=N_{2}
$$

Since both m., and m, are of the order of 50,000 they differ by only about one part in 50,000 and so we may write $N_{1}=N_{2}=N$. If $\epsilon$, and $\epsilon_{2}$ are the fractional parts for $\lambda$, and $\lambda_{2}$ respectively then
and

$$
\epsilon_{1}=\frac{D_{k_{1}}{ }^{2}}{N_{1}=}-k
$$

Subtracting:

$$
\epsilon_{2}=\frac{D_{i+1}^{\prime}{ }^{2}}{N_{2}}-k
$$

$$
\begin{equation*}
\epsilon_{1}-\epsilon_{2}=\frac{D_{l+1}^{2}-D_{l+1}^{2}}{N} \tag{2.20}
\end{equation*}
$$

Since

$$
m_{0}=\frac{2 t}{\lambda}=2 t v
$$

then

$$
\begin{aligned}
& v_{1}-v_{2}=\left(m_{0}-m_{0}\right)^{\prime} / 2 t \\
& v_{1}-v_{2}=\left(m_{1}+\epsilon_{1}-m_{1}-\epsilon_{2}\right) / 2 t \\
& v_{1}-v_{2}=\frac{\epsilon_{1}-\epsilon_{2}}{2 t}
\end{aligned}
$$

$$
(2.21)
$$

From equations (2.20) and (2.21) it follows that $D_{k+1}^{2}-D_{k+1}^{\prime 2}=m$ is a constant.

From these properties we may construct the rectangular array. Let components of the pattern be a, b, c, .... and
the corresponding diameters be $\mathrm{D}_{\mathrm{ka}}, \mathrm{D}_{\mathrm{kb}}, \mathrm{D}_{\mathrm{kc}}$, ... The array is shown in Figure 6.


Figure 6: The rectangular array (7).
In the figure all values of $\mathbb{N}_{i j}$ should be the same and are averaged. All $m_{j k}$ in a horizontal row should be equal. The wave number separation between individual components is given by

$$
\begin{equation*}
v_{a}-v_{b}=\frac{n_{a b}}{2 t N} . \tag{2.22}
\end{equation*}
$$

Section 7: The Etalon Thickness.
The order of interference of the fringes in the pattern, and the thickness of the etalon, may be determined fairly readily by the method of exact fractions, as described by Candler (10). The two prerequisites are that the wavelength of the light be known, and that the thickness be known within the accuracy of a micrometer measurement. The method allows the determination of the width of the air gap to better than one part in $10^{8}$.

The fractional parts for three or four known lines of the spectrum are first calculated by the method of Section 5 . Suppose the etalon thickness by micrometer measurement is $t_{1}$. Then the approximate order of interference for the centre of the pattern due to $\lambda$, is

$$
m_{01}=\frac{2 t_{1}}{\lambda,}
$$

The exact fraction $E$, is known so that the precise order must be $\left(m_{0},+\epsilon_{1}\right) \pm x$, where $X$ is an integer. Since the ratios of the wavelengths are known, i.e. $\quad \lambda_{1} \lambda_{2}=\frac{m_{0_{2}}}{m_{01}}$, etc.,
then for each of the above possible orders the orders of the other lines may be calculated. For only one of these will the values of the fractional parts agree with the measured values, and so the precise order is obtained. The value of X above may be made small by appropriate choice of the lines.

The etalon thickness is then evaluated from the relation $m \cdot \lambda=2 t$. Using this calculated value of $t$, we may go back and improve the precision in the wavelength values to better than 0.0005 angstroms. Correspondingly the absolute wavelengths of any lines of the spectrum may be obtained from measurements of their iractional parts. In Chapter VI the method is used to evaluate some wavelengths in the spectrum of $\mathrm{Hg}^{198}$.

CHAPTER III: GENERAL THEORY
Section 1: Multiplet Structure
It has long since been established that the gross multiplet structure of spectral lines is caused by the interactions between the emission electrons. These electrons exert strong forces on one another due both to their electrostatic repulsion and the magnetic moments resulting from intrinsic spins and orbital angular momenta. Associated with each such electron is an orbital angular momentum vector $\vec{l}$ and a spin angular momentum vector $\vec{s}$, and for the case of one electron spectra these combine to form a resultant angular momentum vector $\vec{j}$. Because of quantization $\vec{l}$ and $\vec{\Delta}$ may not orient themselves in any arbitrary direction, but only in certain discrete directions, giving a set of discrete possible values for $\vec{j}, \quad j=(l+\infty),(l+-1), \cdots|l-\infty|$. Note that according to the results of quantum mechanics the values of the spin, orbital and total moments are respectively
 Atoms with the same $\vec{I}$ and $\vec{s}$ but different $\vec{j}$ have slightly different energy, and hence the splitting of the levels arises.

Conforming to standard spectroscopic nomenclature states for which $l=0,1,2,3,4, \ldots$ are called $s, p, d, f, g \ldots$ states respectively. (s meaning $I=0$ must not be confused with the spin quantum numbers.)

For the case of atoms with two or more valence electrons outside closed shells, an $\vec{l}$ and an $\vec{a}$ is associated with each, and the angular momentum of the atom as a whole is the resultant of the orbitals and spins of each electron. It is possible to form this resultant in many different ways. Since Russell-Saunders
or (IS) coupling presents some close analogies with the (JI) coupling of hyperfine structure, we shall discuss only it here. Under this scheme the coupling of the electron spins predominates and is too strong to be much influenced by the orbital angular momenta. Hen ce the $\vec{s}_{i}$ of the $i$ electrons may be combined to form a resultant $\vec{S}$, the spin of the atom. Similarly the $\vec{l}_{i}$ form a resultant $\vec{L}$ which in turn couples with $\vec{S}$ to produce $\vec{J}$ very similar to the preceding case of one electron. Again, values of $J$ are limited to

$$
J=(L+s),(L+s-1), \cdots(L-s)
$$

where now

$$
\begin{aligned}
& L=0,1,2, \cdots \\
& S=0, \frac{1}{2}, 1,3 / 2, \cdots .
\end{aligned}
$$

As before, $L=0,1,2, \ldots$ are termed S. P. D. ... states.
An electron in a particular state characterized by the values of $L, S, J$ may not release energy and pass over to any other state without exception. A number of selection rules govern the possible transitions. Only those are allowed for which:
(a) $\Delta J= \pm 1,0 \quad(J=0 \nrightarrow 0)$. This is quite general and is true for any type of coupling.
(b) $\Delta L= \pm 1,0 \quad(L=0 \rightarrow 0)$.

This is the Laporte rule for dipole radiation.
(c) $\Delta S=0 \quad$ is a special restriction in (LS) coupling. In general too only those transitions occur for which a single electron alters its value of 1. Rule (c) is sometimes violated, indicating that the coupling is not pure Russell-Saunders. An example of this is the mercury resonance line of wavelength 2536 angstroms, which is due to the transition $6 s^{2} S_{0}-6 \approx 6 \mu^{3} P^{0}$. Here $\Delta S=/$.

The value $(2 S+I)$ is called the multiplicity of the term and gives the number of possible values of $J$ provided $L>5$, in which case it also gives then the number of components in the term splitting.

Consider the energy $\left.W_{(L S}\right)$ of the coupling between $\vec{S}$ and
$\vec{L}$. This is given by

$$
W_{(L s)}=A^{\prime} \vec{L} \cdot \vec{S}
$$

where $A^{\prime}$ is called the interval factor. Using the vector model and the standard substitutions of quantum mechanics (replacing $s^{2}$ by $S(S+1)$, etc.), this becomes

$$
\begin{equation*}
W_{(L s)}=A^{\prime}\left[\frac{J(J+1)-<(<+1)-5(5+1)}{2}\right] . \tag{3.1}
\end{equation*}
$$

If now we consider states $L, S, J$ and $L, S, J-1$ we find immediately that

$$
\begin{equation*}
W_{(L s)}=A^{\prime} J \tag{3.2}
\end{equation*}
$$

This is the Lander interval rule, that the splitting between adjacent components is proportional to the higher $J$ value.

The statistical or quantum weight of a level is ( $2 \mathrm{~J}+\mathrm{l}$ ) and gives the number of independent eigen-functions describeing the state. The degeneracy is then ( $2 J+1$ ) and may be removed by applying a magnetic field (Zeeman effect).

Section 2: Hyperfine Structure.
Two types of hyperfine structure (hereafter abbreviated hes) occur in line spectra and we shall consider these in turn.
(a) Isotope Effect.

The effect of adding one or two neutrons to a nucleus is first considered. In our particular case we are dealing with $80 \mathrm{Hg}^{198}$ with an admixture of $80 \mathrm{Hg}^{199}$. The latter nucleus has a spin of $\frac{1}{2}$ unit and a magnetic moment of $\$ 0.504$ muclear magnetons, and therefore displays regular magnetic hrs. One measures an isotope shift by measuring the displace-
ment of the centre of gravity of this hfs. line pattern relative to the single line arising from ${ }_{80} \mathrm{Hg}^{198}$. The isotope shift due to the addition of two neutrons, such as is observed between the naturally occurring even-even mercury isotopes $\mathrm{Hg}^{198}, \mathrm{Hg}^{200}$, $\mathrm{Hg}^{202}$ results, as in general, in a regular spacing in the lines due to these isotopes. Also in general, one observes that the addition of but one neutron results in.less than a half the shift due to two neutrons. This effect is referred to as "odd-even staggering." In special cases the isotope shift increases irregularly, and this has been related to shell model structure.

In describing the observed isotope shifts one must emphasize that the two term systems are quite distinct, -further that it is a matter of definition whether one assigns (a) the same relative term values to the corresponding ground states, or whether (b) one sets the two ionization limits equal to zero (uses absolute term values). Obviously the two schemes (a) and (b) describe the same physical fact in different ways. It is apparently quíite pointless to say that the heavier isotope is shifted up unless one first makes clear which convention is being used. In the compilation due to Brix (used here in subsequent calculations) scheme (a) is used.

The isotope effect arises from a multiplicity of causes of which the chief known ones are (1) mass effect, (2) volume effect, (3) polarization effect. These are briefly considered in turn.

The correction, appropriate for the two body problem,
with nuclei of mass $M$ and $M+D \mathbb{N}$ is well known, and enlarges the term system of the heavier nucleus by the factor ( $1+\frac{\Delta M}{M}$ ). As well as this "wobbling nucleus" effect there is a further "specific effect" first calculated by Hughes and Eckart (16) for lithuim. A detailed perturbation method calculation is necessary.

The volume effect arises from the fact that the heavier isotope is generally larger. In the case of penetrating electrons this will increase the overlap between nuclear and electronic wave functions in the heavier nucleus, and so reduce the binding energy of the terms arising from penetrating electrons (17). This phenomenon has been exploted by Kopferman in confirming some predictions of the nuclear shell model (18).

The effect of polarization has also been treated by Breit (19), but the effects of nuclear mass and volume are so dependent on nuclear and electronic details that no verification has been well enough made to say whether it is measurable.
(b) Nuclear Spin.

Just as for the electrons we assume that the nucleus of an atom has an intrinsic angular momentum or spin with which is associated a magnetic moment. This nuclear magnetic monent interacts with the magnetic moment due to $\vec{J}$ to give a further splitting of the energy levels. Since the magnetic moment varies inversely as the mass the nuclear magnetic moment is much smaller than that due to the electrons, giving a much smaller, or hyperfine, splitting. An exception to
this general rule occurs in positronium where the splitting is of the same order as multiplet structures, since both moments are of the same magnitude (Bohr magnetons) (20).

The nuclear spin is denoted by $\vec{I}$. Just as before $\vec{L}$ and $\vec{S}$ combined to form $\vec{J}$, so now $\vec{J}$ and $\vec{I}$ combine to form a resultant $\vec{F}$, the total angular momentum of the atom (nucleus plus electrons). F may take the values (J+I), (J+I-I), --$\langle J-I|$, and only those transitions may occur for which $\Delta F{ }^{\prime} \neq 1,0$ ( $F=0 \rightarrow 0$ is forbidden). The energy of the coupling is given by $W_{(I, J)}=A \vec{I} \cdot \vec{J} \quad$ to a first approximation, and may be reduced to

$$
\begin{equation*}
W(I v)=A\left[\frac{F(F+1)-v(v+1)-I(I+1)}{2}\right], \tag{3.3}
\end{equation*}
$$ where $A$ is now the hes. interval factor. The splitting between components is proportional to the higher $F$ value since Russell-Saunders coupling holds quite rigorously in most cases.

The intensities of the various components of a hypermultiplet are governed by the Burger-Ornstein-Dorgelo sum rule which states that the sum of the intensities of all the lines of a hypermultiplet which belong to the same initial or final state is proportional to the statistical weight ( $2 \mathrm{~F}+1$ ) of the initial or final state respectively.

Section 3: The Spectrum of Mercury.
We shall now go on to investigate in detail the hes. of the more prominent lines of the mercury spectrum. We are interested here in only the two isotopes of atomic weights 198 and 199 respectively. In accordance with all nuclei of even mass number the former has a spin 0 . Since the possible values of $F$ are governed by $F=(J+I),---|J-I|$, then for $I=0, F$ may acquire only the one value $F=J$. Hence
there is no splitting of the Hg 198 terms. The second isotope has a nuclear spin of $\frac{1}{2}$ and so by a similar areasoning there are two possible values of $F$, namely $F=J \pm \frac{1}{2}$. Consequently the Hgl 199 terns with $J=0$ are single, and all other terms are doublets. For some of the lines there is a shift of the $\mathrm{Hg}^{198}$ component relative to the centre of gravity of the Hg199 structure because of the isotope effect.

Consider the specific example of the well-known green line of wavelength 5461 angstroms, which is due to a transition from a ${ }^{3} S_{1}$, to a ${ }^{3} P_{2}^{0}$ state. For the particular terms in question Brix and Kopferman (13) give as interval factors +718.5 and $+303.6 \mathrm{~cm} .^{-1}$ respectively for the $\mathrm{Hg}^{199}$ isotope. Furthermore the isotope shifts are $0.004 \mathrm{~cm} .^{-1}$ for the ${ }^{3} \mathrm{~S}$, , term and $O$ for the ${ }^{3} P_{2}{ }^{0}$ term. Using this data we may calculate the actual splitting to be observed, indicated in the energy level diagram of Figure 7. The transitions allowed by the selection rules are shown.

- When this line is viewed with a spectrograph its appear ance will be as depicted in Figure 8.

The structures of all other lines in the spectrum may be found in an analagous manner. This has been done for some 32 lines covering the wavelengti region from 2536 to 6716 angstroms, and these are tabulated in Appendix III. In all cases the interval factors and isotope shifts are those of Brix and Kopferman (13).

Section 4: The Disintegration Equations.
According to the well-knom theory of radioactive decay the number of atoms $N$ existing after time $t$ is given by

i
Figure 7: Inergy lovel diagram showing the hes of the Hg line 5461 A .

$\xi$
Figure 3: Schematic drawing of the hfs. of the line 5461 A.

$$
\begin{equation*}
N=N_{0} e^{-\lambda t} \tag{3.4}
\end{equation*}
$$

Where No is the number of atoms present at time to, and $\lambda$ is the decay constant or fraction of atoms decaying per unit time.

The half-life $\tau$ is defined as the time required for one -half of the active material present at any time to decay and is given by


For the case of transmutation by particle bombardment we require some quantity by means of which we may measure the amount of material transformed. The entity chosen for this purpose is called the "cross section." It gives the area of an imaginary circular disc associated with each nucleus such that if the bombarding particle passes through it the reaction takes place; otherwise it does not. Its value depends upon the reaction and upon the energy of the bombarding particle, and is measured in "barns" ( 1 barn $=10^{-24} \mathrm{~cm}^{2}$ ). In our particular case we are dealing with the "neutron capture cross section", which measures the probability that a target nucleus will capture a neutron from the incoming flux.

Consider then a quantity of $A u^{197}$ subjected to a flux of slow neutrons. We wish to calculate the amounts of the product elements, as represented by equations (1.1) and (1.2), present after a time t. The following notation will be adopted:

$$
\begin{aligned}
\sigma_{i} & =\text { neutron capture cross section } \\
\lambda_{i} & =\text { decay constant }
\end{aligned}
$$

$N_{2}=$ number of atoms present at time t where $i=1, \hat{2}, \overrightarrow{3}, 4, \ldots$
24.
for the elements $A u^{197}, \mathrm{Au}^{198}$, $\mathrm{Aul} 199, \mathrm{Hg} 198, \mathrm{Hg}^{199}$ respectively. Note that

$$
\begin{aligned}
& \lambda_{0}=\lambda_{3}=\lambda_{4}=0 \\
& \sigma_{2}=\sigma_{3}=\sigma_{4}=0 \quad \text { effectively }
\end{aligned}
$$

Let the neutron flux be $F$. Then we may write

$$
\begin{equation*}
\frac{d N_{0}}{d t}=-N_{0} \sigma_{0} F \tag{3.6}
\end{equation*}
$$

Equation (3.6) expresses the fact that the increase in the number of atoms of Au lg per unit time is the product of the probability an atom will capture a meutron, the number of neutrons, and the number of atoms. Similarly we may write

$$
\begin{align*}
& \frac{d N_{1}}{d t}=N_{0} \sigma_{0} F-\lambda_{1} N_{1}-N_{1} \sigma_{1} F  \tag{3.7}\\
& \frac{d N_{2}}{d t}=N_{1} \sigma_{1} F-\lambda_{2} N_{2}  \tag{3.8}\\
& \frac{d N_{3}}{d t}=\lambda_{1} N_{1}  \tag{3.9}\\
& \frac{d N_{4}}{d t}=\lambda_{2} N_{2}
\end{align*}
$$

(3.10)

This is a set of readily soluble linear differential equations. At time $t=0, \mathbb{N}_{0}=\mathbb{N}_{0}^{\circ}$ say, while $\mathbb{N}_{i}=0(i=1,2,3,4)$. Hence equation ( 3.6 ) gives immediately

$$
\begin{equation*}
N_{0}=N_{0}^{0} e^{-\sigma_{0} F t} \tag{3.11}
\end{equation*}
$$

Rewrite (3.7) thus:

$$
\left(\lambda,+\sigma_{1} F\right) N_{1}+\frac{d N_{1}}{d t}=\sigma_{0} F N_{0}^{0} e^{-\sigma_{0} F t}
$$

An integrating factor is $e^{+(\lambda,+\sigma, f) t}$. Multiplying by this factor and rearranging,

$$
\frac{d}{d t}\left(e^{+\left(\lambda,+\sigma_{1} f\right) t} N_{1}\right)=e^{\left[\lambda,+F\left(\sigma_{1}-\sigma_{0}\right)\right] t} \sigma_{0} F N_{0}^{0}
$$

which leads to

$$
N_{1}=\frac{\sigma_{0} F N_{0}^{0}}{\lambda,+F\left(\sigma_{1}-\sigma_{0}\right)}\left\{e^{-\sigma_{0} F t}-e^{-\left(\lambda,+\sigma_{1} F\right) t}\right\}(3.12)
$$

Analagously, equations (3.8), (3.9), and (3.10) give:

$$
\begin{gathered}
N_{2}=\frac{\sigma_{0} F^{2} N_{0}^{0} \sigma_{1}}{\lambda,+F\left(\sigma_{1}-\sigma_{0}\right)}\left\{\left(e^{-\sigma_{0} F t}-e^{-\left(\lambda_{2} t\right)}\right) \frac{F) \lambda}{\lambda_{2}-\sigma_{0} F}\right] \\
\left.-\frac{1}{\lambda_{2}-\left(\lambda,+\sigma_{1} F\right)}\left(e^{-\left(\lambda_{1}+\sigma_{1} F\right) t}-e^{-\lambda_{2} t}\right)\right\}
\end{gathered}
$$

$$
\begin{align*}
N_{3}= & \frac{F \lambda_{1} \sigma_{0} N_{0}^{0}}{\lambda,+F\left(\sigma_{1}-\sigma_{0}\right)}\left\{\frac{1}{\lambda,+\sigma_{1} F}\left(e^{-\left(\lambda_{1}+\sigma_{1} F\right) t}-1\right)-\frac{1}{\sigma_{0} F}\left(e^{-\sigma_{0} F t}-1\right)\right\} \\
N_{4}= & \frac{F^{2} \sigma_{1} \sigma_{0} \lambda_{2} N_{0}^{0}}{\lambda, f\left(\sigma_{1}-\sigma_{0}\right)}\left\{\frac{1}{\left[\lambda_{2}-\left(\lambda,+\sigma_{1} F\right)\right]\left(\lambda,+\sigma_{1} F\right)}\left(e^{-\left(\lambda, \sigma_{1} F\right) t}-1\right)\right. \\
& +\frac{1}{\lambda_{2}\left(\lambda_{2}-\sigma_{0} F\right)}\left(e^{-\lambda_{2} t}-1\right)-\frac{1}{\sigma_{0} F\left(\lambda_{2}-\sigma_{0} F\right)}\left(e^{-\sigma_{0} F t}-1\right) \\
& \left.-\frac{1}{\lambda_{2}\left[\lambda_{2}-\left(\lambda,+\sigma_{1} F\right)\right]}\left(e^{-\lambda_{2} t}-1\right)\right\}
\end{align*}
$$

The evaluation of $\sigma$, is to be made by measuring the ratio of the amounts of $\mathrm{Hg}^{198}$ and $\mathrm{Hg}^{199}$ produced. Immediately after activation for a time $t$ the ratio Au l99: Au l98, or the ratio $\mathrm{Hg}^{199}: \mathrm{Hg}^{198}$, may be found from the appropriate equations above. If these measurements are not made immedlately after extraction from the flux, the ratios will change due to the different decay rates of Au lg and Au ${ }^{199}$. The final result becomes very unwieldy.

Consider an alternative and much more convenient form. We are dealing essentially with a branching process which may be written symbolically as follows:

The two competing processes for Au ${ }^{198}$ transformation occur in the ratio $\frac{\sigma_{L} F}{\lambda,}$, the "branching ratio." If now the ratio $\mathrm{Hg}^{199}: \mathrm{Hg}^{198}$ be measured at such a time that all activity has ceased, then the result is precisely

$$
\begin{equation*}
\frac{\mathrm{H}^{199}}{\mathrm{H}^{198}}=\frac{\sigma_{1} F}{\lambda_{1}}, \tag{3.17}
\end{equation*}
$$

since all Au li formed decays to $\mathrm{Hg}^{199}$. The values of $\lambda$, and $F$ being accurately known, we may calculate $\sigma$, from the measured ratio of the mercury isotopes.

CHAPTER IV: THE APPARATUS.

Section 1: The Construction of the Etalon.
Fabry-Perot interferometers are generally designed with either a constant plate separation or a variable gap, the two types being equally common. The latter has the obvious advantage of ease in the adjustment of interorder separation and resolving power; the former has the advantage of stability -a prime requisite for long exposures. The etalon constructed here has a constant plate separation, but the design is such that the spacer may be removed and another of different length substituted with comparative ease. Hence the chief advantage of the second type is not completely lost.

The housing for the etalon is a thick-walled brass cylinder of inner diameter 7 cm . and length 12 cm . with three projecting studs placed symmetrically around one end. This casting is hinged to a thick brass plate at one end, the other resting on the point of a screw which turns into the same plate. The etalon tilt may then be readily adjusted. The 7 cm . diameter plates are made one of quartz and one of silica, giving the maximum possible range of transmission. They are 1 cm . in diameter and are made to fit securely into the housing. In order to avoid the production of false interference effects (7) their plane faces make angles of 15 minutes with each other, giving them a slightly prismatic shape. One face of either plate is optically flat.

Important factors to be considered in the design of an etalon are:

1. length and material of spacer.
2. means of adjusting parallelism of plates.
3. focussing lens
4. reflectivity of plates.

Considering the first of these, the lengths of the spacers were chosen after a study of the calculated hf's. separations to be expected in mercury was made, since the etalon thickness determines the interorder separation. It was decided to make three spacers of lengths $1,1.4$, and 3 cm . respectively. To guard against the expansion of the spacer due to changes of temperature and pressure during long exposures (this effect causing a broadening of the fringes) they were made of invar. Each separator consisted of a brass ring 0.8 cm . thick, 7 cm . in outer diameter, and 5 cm . in inner diameter. Three equally spaced projecting invar pins, equal in length within the accuracy of a microneter measurement, were mounted securely in the ring. In addition a groove parallel to the axis of the cylinder was cut in the bottom of the housing, and a pivot on the spacer ring fitted this exactly, thus guaranteeing reproducible positions. The pivot was so placed that the pins occupied the same relative positions as the studs on the casting (see Figure 9).

A fine adjustment for producing accurately parallel plates is provided by three ste日l springs mounted on the housing studs. For an etalon in frequent use the manipulation must be simple, rapid, and sensitive. The heads of the springs rest on a very thin brass ring placed against the

by each is controlled by a screw. The springs are so fastened in Brass forms that, to remove the plates, one need only loosen one screw in each form and swing them aside. The heads of the springs are small steel bits with spherical surfaces. The inner plate rests firmly against a cylindrical brass retainer which is held in place by the further end of the housing. This design gives effective plate diameters of 5 cm .

From Chapter II, Section 5, we know that the diameters of the fringes formed in the focal plane vary directly with the focal length of the objective. Since we desire large diameter rings a Hilger quartz fluorite achromat of 52 cm . focal length is mounted in the end of the housing opposite the plates. The lens has an aperture of 3.2 cm . and is separated by only a few centimetres from the 5 cm . aperture plates. It is hoped that by this arrangement each reflected beam will see the same effective aperture regardless of the angle of incidence $\theta$. The effect of the finite number of beams, mentioned in Chapter II, Section l, will thereby be minimized.

The plate reflectivity determines the fringe sharpness and the resolving power. Too high a reflection coefficient can not be used however because of the corresponding loss in intensity. Losses due to absorption in the metal film are important and must be kept to a minimum. Although silver is perhaps the most efficient material it has two serious disadvantages. It tarnishes quite rapidly on exposure to
the atmosphere, and it has a large absorption band in the near ultra-violet. Consequently, aluminum was chosen as more suitable. The deposition of the aluminum on the plates was performed by a standard evaporation procedure such as that described by Tolansky (7) or Strong (14). The reflecting coefficient was measured accurately as 0.805 by a multiple reflection procedure. The method of Chapter II, Section 2 was also applied.

Section 2: The Adjustment of the Etalon.
A method involving fringes of equal inclination (7) was used in order to gain accurate parallelism between the aluminized faces of the flats of the etalon. With the objective removed an extended mercury source was placed before the instrument, and the interference fringes viewed with the eye focussed for infinity by looking in a direction normal to the faces of the plates. By moving the eye across any diameter, the diameters of the rings were found to be continuously changing,i.e., the pattern appeared either to "open out" from or to "close in" to the centre. This indicated that a slight wedge angle existed between the plates, the pattern opening out towards the base of the wedge. The eye was then moved across diameters through each spacer pin in turn to determine which pins were too long. The front plate was removed, the spacer taken out, and the appropriate pegs carefully polished against a flat surface of crocus cloth. The etalon was reassembled and the observations repeated. This procedure was continued until the three pins were within a fraction of a wavelength of the same length, indicated by
less than one fringe opening across any diameter. The remaining fine adjustment was then easily carried out by changing the pressures exerted by the springs supplied for that purpose. It is to be noted that the procedure may become tedious and reqiires much patience -- the reward of great care in polishing being an early success. Other points worth mentioning are:
(a) In the later stages of polishing Bon Ami on glass was used in place of crocus cloth as being less abrasive.
(b) Exceeding care must be taken to ensure that the surfaces of the pins are kept flat so that optical contact between them and the metal film is obtained.
(c) The pins should be polished at both ends and not on just one.
(d) By placing the eye at some distance from the instrument to view the fringes the sensitivity of the method is increased since the centre of the pattern then fills the field of view. The centre is particularly sensitive to variations in thickness of the air gap.

Section 3: The Source.
Because of the extreme simplicity and convenience of the high frequency external electrode discharge, such a source was chosen for the high resolution measurements. High intensity and stability are further advantages of the tube. The electrodeless discharge lends itself readily to the cooling procedures necessary to reduce line breadth. In addition, the comparative ease of preparation of such a
preparation is given in Chapter V, Section 1 .
A detailed discussion of the action of the electrodeless discharge is given by Tolansky (7), and so only a brief survey will be given here. A tube containing gas or vapour at less than a few millimetres pressure is placed in close proximity to a coil carrying high frequency oscillations. An oscillating electric field is set up in the tube and produces great enough electron velocities to excite the gas atoms. The coupling between the high frequency energy and the tube is generally obtained by either inserting the tube within the coil, or clamping two electrodes to the ends of the tube and connecting these to the ends of the coil. The latter practice was adopted in this case, the electrodes consisting of fine copper coil enclosed in a brass or copper ring. The oscillator used to excite the discharge is basically the same as that described by G.P. Harnwell (24), and its circuit diagram is shown in Pigure 10. The oscillator is driven by a high voltage power supply.

The source itself was made of vycor and was 8 inches in length. Two inches at either end were $5 / 8$ inches in diameter to receive the electrodes; the middle 4 inches was $3 / 8$ inches in diameter. This narrow portion was enclosed by a quartz water jacket which was sealed to the bulbous ${ }^{\circ}$ ends with rubber sealing tape.

Meggers and Westfall (25) and Jacobsen and Farrison (21), have investigated thoroughly the action of similar mercury discharges and have found that the life of the tube is dependent on the frequency of the exciting oscillation,


Figure 10: Circuit diagram of refo oscillator.
increasing with increasing frequency. After a period of time, varying from 30 to 300 hours, the mercury is apparently driven into the walls of the tube. Providing no adsorbed gases are present the source may be rejuvenated by heating with a gas-air torch.

Section 4: The Spectrograph.
In the theory of the ring systems produced by the FabryPerot interferometer (Chapter II) we dealt wholly with monochromatic light waves. In the investigation of light composed of a great number of wavelengths each line gives its own fringe pattern and these are all superimposed. The resolution of this complex structure is performed by crossing the etalon with a spectrograph of smaller resolving power but sufficient dispersion to separate the various fringe systems. For this purpose a Hilger E-l quartz spectrograph was used. This instrument makes use of the Littrow mounting, and has a dispersion of approximately $12 \mathrm{~A} / \mathrm{mm}$. at 5000 A and $3 \mathrm{~A} / \mathrm{mm}$. at 2500 A .

The external parallel beam method of mounting the etalon relative to the spectrograph was used. The source is placed at the focus of a quartz lens and the interferometer placed in the parallel beam emerging therefrom. The fringe system is then focussed on the slit of the spectrograph by means of the quartz fluorite objective. The collimating lens need not be of high quality, but the objective must. The mounting requires the use of an extended source with uniform intensity distribution, but has the advantage that all the available light is employed in the useful production of fringes. Since the whole aperture of the etalon is used
the mirror coating must be extremely uniform, and the adjustment to parallelism requires especial care.

Note at this point that since the Littrow type spectrograph has unit magnification, the quartz objective determines the size of the ring system obtained on the photographic plate. For the photography of the interference patterns the slit of the spectrograph is opened to a width of about 1 mm .

## Section 5: The Step Sector.

For all photometric work employing the photographic plate, there must be provided on the plate a series of calibration marks of known relative intensity. The measured density of these marks then permits the construction of the Hurter and Driffield characteristic curve relating the density to the exposure. The calibration in this case was performed by the use of a rotating step sector placed immediately in front of the spectrograph slit. The sector consists of an opaque disc from which steps of angles $x, y, z, \ldots$ are removed. When it is rotated the total light intensity passing is reduced in the ratios $\frac{x}{360}, \frac{y}{360}, \frac{8}{360}, \ldots$ This particular sector contained six steps of angles 1.785, 3.685, 13.384, $38.484,149.685$, and 360.000 degrees respectively giving relative intensities of $1,2.064,7.498,21.560$, 83.857, 201.681. The radius of the largest step was 2.5 inches, and each step was cut to a depth of 2.5 mm . The sector was counterbalanced and mounted on the shaft of a 1/50 horsepower motor which gave a speed of 1725 revolutions per minute.
steps were made. In the first the sector was mounted on the table of a rotating prism spectrometer, with the prism removed. The radial cuts were viewed perpendicularly from above by means of a travelling microscope, and the botation of the table allowed the measurement of the angles. Secondly, the chords of each angle were measured with a Hilger comparator and the corresponding angles were calculated from the known radii of the steps. The results of the two methods agreod to better than $0.5 \%$.

The chief criticism of the use of the rotating sector is that the illumination is intermittent and may have a different effect from the same amount of continuous illumination. It has been shown (22) that there is a critical frequency above which the effects are the same, and for speeds of the magnitude used here accurate results are obtained.

Section 6: The Microphotometer.
The intensity measurements in the hfs. of the mercury lines was carried out with the aid of a Moll microphotometer. As originally designed this instrument uses a vacuum thermocouple to measure the variations in light intensity transmitted by the photographic plate. The plate is moved across the path of a fixed beam of lieht by means of an automatic driving mechanism. The currents from the thermoelement are carried to a galvanometer on which is mounted a mirror from which a beam of light is reflected to a photographic strip. The galvanometer deflections are therefore traced out on the kodabromide paper which is wound on a rotating drum.

As used here the recording mechanism was discarded in
favour of an automatic recording Brown potentiometer. The detector currents are transmitted to the recorder through ancn appropriate amplifying system. This arrangement has important advantages, not the least of which is convenience. The trace of the intensity pattern may be viewed as it is being reproduced, and the necessity of developing the paper is removed. The action of this type of recorder is in general more satisfactory.

A second variation, introduced initially through necessity, was the use of a lead sulphide infra-red detector in place of the vacuum thermo-element. Such a cell requires that the radiation falling on it be chopped. A motor driven chopper with 30 equally spaced holes rotated at 1800 revolutions per minute was placed in the path of the projecting lamp beam. In the later stages of experiment the proper thermocouple, originally unavailable, was installed.

Comparing the two detectors, the lead sulphide cell was found to be the more sensitive. While not as fast as the cell, the thermoelement was quite satisfactory. The chief drawback to the use of the cell was the introduction of noise, chiefly caused by the vibrations of the chopper. The great advantage on the other hand was the almost complete absence of "drift," i.e., the clear plate trace remajned constant over long periods of time. With the thermocouple considerable drift of the clear plate trace occurred, although prolonged examination showed it to be fairly linear. The reproducibility by both detectors was excellent, as was the reproducibility between the two.

A convenient method for focussing the microphotometer is outlined in Appendix II.

CHAPIER V: THE EXPERIPINTAL PROCEDURE.

Section 1: The Preparation of the Source.
The samples of gold containing the mercury were obtained from the Atomic Energy Commission at Chalk River, Ontario. They had been exposed to neutron bombardment in the atomic pile, a different value of the neutron flux being used for each sample. The gold is in the form of fine strips wound on thin aluminum wires, each sample consisting of a dozen or more of such pieces.

The nercury formed during the neutron irradiation is present as a volume distribution throughout the gold, and was distilled off by prolonged heating at a high temperature in vacuo. The distillation apparatus is shown in Figure 11. The long vycor tube, $A$, may be inserted in a furnace as desired. The vycor tube $C$ is the source described earlier (Chapter IV), Section 3). The column A is attached by means of a graded seal to the pyrex tubing and trap D. The system was thoroughly cleaned and the tube A inserted into an electrically heated furnace. A temperature of $450^{\circ} \mathrm{C}$ was maintained for a period of six hours, during which time the system was evacuated. The tubing outside the furnace was flamed with a torch to drive off all adsorbed gases. Helium was then introduced into the system and pumped out a few times, this flushing aiding in carrying out any trapped gas molecules. With the helium pressure maintained to prevent the entrance of air, the tip $B$ was broken off, the gold inserted into the vycor column, and the opening sealed off.


Figure 11: hercury distillation apparatus.

The trap $D$ and the source $C$ were immersed in liquid air, and pumping was continued until the helium pressure was reduced to a few microns. The gold was kept at a temperature of $450^{\circ} \mathrm{C}$ for several hours, and the escaping mercury vapour condensed on the cold walls of the source. The upper parts of the system were again flamed to drive all mercury into the source; any trapped water vapour was driven off; and the source was sealed off. It contained in addition to the mercury a slight pressure of helium to serve as a carrier. Very little trouble was encountered in producing with the source a stable discharge of high intensity.

Section 2: The Photography of the Interference Patterns. The source and etalon were mounted as described in Chapter IV, Section 4 , with a quartz lens of 5 cm . aperture and 20 cm . focal length being used as the collimator. In order to ensure that the apparatus was aligned correctly on the optical axis of the spectrograph the mercury source was replaced by an incandescent bulb, and the radiation from the slit viewed from the prism end. The correct adjustment was then easily made. The actual. focussing of the spectrograph itself was performed by a rapid convenient method described in Appendix I。

Suitable slit widths and exposure times were found using trial plates. The final plates taken for the photometric determination required exposure times varying from 2 to 10 minutes, depending on the line to be examined. A slit width of about lmm. was used. On each plate were an interference photograph of the mercury spectrum and a calibration exposure
with the step sector. For the latter the etalon was removed and the sector mounted immediately before the slit. Thus for each plate a characteristic curve could be obtained which was subject to the same conditions of development as the fringe pattern.

For every exposure care was tainen to ensure that the ring system was correctly centred on the slit, the slit running across a diameter. In some cases the centre of the pattern was placed at the centre of the slit; in others the centre of the pattern was set at one end of the slit so that only one half of eacb ring was visible. While either type was suitable for the measurement of intensity ratios, only the former served in wavelength determinations. The latter arrangement was used because more fringes could be obtained on each plate, and because of increasing sharpness of the maxima as one proceeds out from the centre.

All plates used were Eastman Type II - F (3), and were developed in Kodak formula D-19 and fixed in Kodak F-5.

Section 3: The Determination of Wavelengths.
On a number of suitable plates the fringe dianeters of several lines were measured for all components of the hfs. patterns. For most lines containing structure some of the $\mathrm{Hg}^{199}$ components were visible. The neasurements were made with a Hilger comparator, and the resulting values used to calculate the fractional parts by method of Chapter II, Section 5, and the wave number separations by the method of Chapter II, Section 6.

Section 4: The Measurement of the Mercury Ratio. The ratio $\mathrm{Hg}^{199}$ : Hg 198 in the sample was determined by measuring the relative intensities of the hes. components due to each isotope. The criteria determining which lines of the spectrum are most suitable for the purpose are:
(a) the line must be of high intensity.
(b) the Hgl99 must give rise to a simple hfs. With preferably no more than three components. .
(c) at least one component of the $\mathrm{Hg}^{199}$ structure must fall midway between the intense $\mathrm{Fg}^{198}$ orders in the fringe pattern.
(d) no other line may overlap the chosen line.

Condition (a) is necessary because of the low transmission of the etalon and the low intensity of the $\mathrm{Hg}^{199}$ components; conditions (b) and (c) considerably simplify the actual measurements; condition (d) prevents intemingling of two or more line patterns.

The etalon spacer of length $1 \mathrm{~cm} .(0.97720155 \mathrm{~cm}$.$) was$ used exclusively, and according to equation (2.15) it gives an interorder separation of $0.514 \mathrm{~cm} .^{-1}$. From the hfs. to be expected (Appendix III) the lines of wavelengths 5461, 4077, 3341, and 3125 angstroms were chosen as most appropriate for the intensity measurements. We might note that such apparently suitable structures as that of the line 3654 A must be ruled out by condition (d) above.

Ideally, the exposures would be taken so as to have the densities of these lines fall on the straight line portion of the characteristic curve. In practice the intensity ratio
between the $\mathrm{Hg}^{198}$ and $\mathrm{Hg}^{199}$ components was so great that the densities of poth could not fall on the straight line. With care however, this condition could be as nearly as possible attained. The patterns were traced with the microphotometer and the heights of the maxima measured to within $1 / 60$ inches.

In evaluating the true intensity ratio a number of factors must be considered. In the first place, the intensity of the maxima decreases with decreasing order of interference. This is inherent in the instrument itself and may be compensated for by the method suggested by Tolansky (7). A smooth curve is drawn through the peaks of successive maxima for each separate component. The construction simultaneously averages deviations due to local fluctuations in plate graininess and sensitivity. The intersections of any vertical line with these curves gives the relative heights of the peaks.

Secondy, the intensity between orders of the Hrér 198 pattern contributes to the density of the $\mathrm{Hg}^{199}$ component. The converse of this need not be considered since the Hg 198 components are so much more dense. To eliminate the effect, the lines were so chosen that the $\mathrm{Hg}^{199}$ maxima fell very nearly on the Hg ${ }^{198}$ minima. Using the measured value of the reflection coefficient and the measured height of the $\mathrm{Hg}^{198}$ peak, the intensity of $I_{m i n}$ of the $\mathrm{Hg}^{198}$ was calculated from equation (2.10). This value was then subtracted from the measured $\mathrm{Hg}^{199}$ intensity.

Finally, to obtain the desired ratio, recall that the Hgl99 maxima measured represent the contribution of only
one component of the $\mathrm{Hg}^{199}$ structure. The intensity ratios among these various components are calculated on the basis of the intensity sum rule of Chapter III, Section 2 assuming strict (JI) coupling. Hence the measured $\mathrm{Hg}^{199}$ intensity must be increased by the appropriate factor. These three corrections having been applied we obtain the true intensity ratio of the hfs. components, which is in fact the ratio $\mathrm{Hg}^{199}: \mathrm{Hg}^{198}$. Several determinations of this quantity were made from each of the four lines on a number of plates with slightly varying exposures. The mean of these then determined the neutron capture cross section of Aul98 according to equation (3.17).

CHAPTER:VI: THE EXPERTMENTAL RESULTS.
Section 1: The Calculation of Absolute Wavelengths. For as many of the $\mathrm{Hg}^{198}$ lines as possible the correct values of the fractional parts were calculated by a least squares application as outlined in Chapter II, Section 5. Using these values and Meggers' wavelengths (17), the order of interference and etalon thickness were determined. The line 3341 A was chosen to calculate the approximate order, and the exact orders were found for 15 other lines by the method of exact fractions. The wavelengths of these 15 lines were then calculated relative to Meggers' value of 3341. 4814 A . The results of these investigations are set down in Tables 1 and 2 .

The unambiguity of the method of exact fractions for determing the order of interference is depicted in Table 1. The measured fractions for each line are shown in column 2, and the calculated orders in column 3. The coincidence of observed and calculated fractions is obvious.

Table 1: Calculated order of interference.

| Wavelength (A) | Observed <br> fraction | Order of Interference |
| :---: | :---: | :---: |
| 3341.4814 | 0.121 | $58,489.121$ |
| 3125.6700 | 0.486 | $62,527.493$ |
| 3131.5510 | 0.068 | $62,410.068$ |
| 3131.8420 | 0.261 | $62,404.269$ |
| 3650.1567 | 0.997 | $53,369.991$ |
| 3654.8393 | 0.399 | $53,474.392$ |
| 3663.2808 | 0.173 | $53,351.168$ |
| 4046.5715 | 0.752 | $47,297.753$ |
| 4077.8379 | 0.433 | $44,84.435$ |
| 4358.3376 | 0.855 | $35,789.957$ |
| 5460.7532 | 0.999 | $33,874.162$ |
| 5769.5984 | 0.169 |  |
| 5790.6626 | 0.941 |  |

Using the value of $m$. for the line 3341 A the equation $m_{0} \lambda=2 t \quad$ gives for $t$ the value $t=0.97720155 \mathrm{~cm}$.

We get here a striking example of the accuracy obtainable in measurement of length when the measurement is performed using a monochromatic wavelength of light as standard. The wavelengths of 15 lines were calculated, using the measured fractions and the value of $t$, from the equation $m_{0} \lambda=2 t$. These are found in Table 2, where for comparison the values obtained by Meggers and Kessler (28), and Burns and Adains (27) are also listed.

Table 2: Wavelengths of $\mathrm{Hg}^{198}$ (angstroms)

| Bedford | Neggers and Kessler | Burns and Adams |
| :---: | :---: | :---: |
| 5790.6625 | 5790.6626 | 5790.6626 |
| 5769.5972 | 5769.5984 | 5769.5984 |
| 5460.7519 | 5460.7532 | 5460.7532 |
| 4358.3377 | 4358.3376 | 4358.3372 |
| 4077.8380 | 4077.8379 | 4077.8379 |
| 4046.5716 | 4046.4715 | 4046.5712 |
| 3663.2808 | 3663.2808 | 3663.2808 |
| 3654.8393 | 3654.8393 | 3654.8392 |
| 3650.1566 | 3650.1567 | 3650.1564 |
| 3341.4814 | 3341.4814 | 3341.4814 |
| 3131.8424 | 3131.8420 | 3131.8423 |
| 3131.5510 | 3131.5510 | 3131.5513 |
| 3125.6706 | 3125.6700 | 3125.6698 |
| 3021.4997 | 3021.4997 | 3021.4996 |
| 2967.2842 | 2967.2833 | 2967.2832 |
| 2536.5068 | 2536.5064 | 2536.5063 |

We must note that the wavelengths in columns 2 and 3 of Table 2 are relative to the green line 5460.7532 angstroms. In the calculations here it was not thought advisable to use this line as standard because only three fringes could be measured for it, reducing the accuracy in the fractional part. Consequently Meggers' value of 3341.4814 was taiken as standard because 7 or 8 fringes could be measured for it.

Section 2: The Hyperfine Structure Separations.
For those lines in which the $H g 199$ structure was readily visible, the fringe diameters were measured and the wave number separations calculated by the method of Tolansky (Chapter II, Section6). The measurements were made with a Hilger comparator, and in some cases also from the photometric trace. A typical example of the rectangular array is shown in Table 3, the data being obtained from the diameters of the fringes of the line

Table 3: Rectangular array for $\lambda=4077$ A․


In Table 4 are listed the measured wave number separations, the separations calculated from the data of Brix and Kopferman (Appendix III), and the separations found by Schuler and Keyston (23).

Table 4: Hyperfine Structure of $\mathrm{Hg}^{199}$.

| WRVELENGTH | WAVE | NUMBER | SEPARATIONS | (CM. ${ }^{-1}$ ) |
| :---: | :---: | :---: | :---: | :---: |
| (A) | $\begin{aligned} & \text { BEPFORO } \\ & \text { (COMPARATOR) } \\ & \hline \end{aligned}$ | BEDFORA (PNOTOMETER) | BRIX KOPFERMAN | SCHULER KEYSTON |
| 5461 | -0.257 | -0.259 | -0.2591 | -0.278 * |
| 4358 | -0.212; +0.113 |  | -0.2216; +0.1168 | $-0.240 ;+0.106^{*}$ |
| 4077 | -0.243 | -0.233 | -0.2425 | -0.249 |
| 4046 | +0. 352 |  | +0.3633 | +0.36 2 |
| 3341 | -0.255 | -0.224 | -0.2346 |  |
| 3125 | +0.775 | to. 768 | +0.7749 |  |
| 2536 | $+0.232$ |  | +0.2214 | $+0.233$ |

Except for the values of Schuler and Keystone, the separations are relative to the $\mathrm{Hg}^{198}$ component as zero in each case. Schuler and Keyston's data is relative to $\mathrm{Hg}^{200}$ as zero in some cases (\%) since they do not give their values of the isotope shift between $\mathrm{Hg}^{198}$ and $\mathrm{Hg}^{200}$ for all the lines.

The comparator value for the line 3341A is the result of only one measurement and so may be subject to error. The values of the other lines are the means of six or seven determinations from different plates. The evaluations from the photometric traces are not as accurate.

Section 3: The Neutron Capture Cross Section of Au ${ }^{198}$. Twenty-one independent determinations of the ratio Hg ${ }^{199}$ : $\mathrm{Hg}^{198}$ were made from the four lines on six different plates. The mean of these gives for the ratio:

$$
\frac{\mathrm{Hg}^{199}}{\mathrm{Hg}^{198}}=0.274 \pm 0.015
$$

This mercury was distilled from a gold sample irradiated with a neutron flux of $4.6 \times 10^{13}$ neutrons per $\mathrm{cm} .^{2}$ sec. Since the half-life of Aul98 is 2.69 days equation (3.5) gives for the decay constant

$$
\begin{aligned}
& \lambda_{1}=\frac{\ln 2}{2.69 \mathrm{~d}} \\
& \lambda_{1}=2.98 \times 10^{-6} \mathrm{sec} .^{-1}
\end{aligned}
$$

Using equation (3.17) we then get for the neutron capture cross section of Aul98

$$
\begin{aligned}
& \sigma_{1}=\frac{\lambda_{1}}{F} \mathrm{Hg}^{199} / \mathrm{Hg}^{198} \\
& \sigma_{1}=(1.78 \pm 0.10) \times 10^{-20}
\end{aligned}
$$

Or in the common units, $\sigma_{1}=1.78 \times 10^{4}$ barns.

It is felt that an accuracy of the order of $5 \%$ was attained in the determination of the mercury ratio. The
chief reason why the error could not be reduced was that in almost every case the $\operatorname{Hg}^{198}$ maxima produced a density approach:ing the shoulder of the characteristic curve. If the exposure were shortened to prevent this, then the $H g 199$ maxima receded on to the toe of the curve. The curve itself was determined by the six steps of the sector, and it is possible that with more steps a corresponding increase in accuracy might be gained.

The second important factor affecting the determinations was the drift resulting from the thermocouple, or on the other hand the noise from the lead sulphide cell. These were particularly troublesome in measuring the $H_{g}^{199}$ maxima. Variations in clear plate trace between the interference pattern and the calibration marks may also have affected the result.

The value of $1.78 \times 10^{4}$ barns for the cross section is in good agroement with Hill's value of $1.6 \times 10^{4}$ barns (5), and is in support of his assumption that his first estimate of $3.5 \times 10^{4}$ barns is too high (4).

In order that the green line of Hg 198 be used as the standard of length it should be free from all structure. If the $\mathrm{Hg}^{198}$ is to be prepared by the neutron irradiation of gold, then less than $1 \%$ of $\mathrm{Hg}^{199}$ should be present to ensure the freedom from his. This in turn gives as a maximum value of the neutron flux to be used $1.7 \mathrm{X} 10^{12}$ neutrons per $\mathrm{cm} .^{2} \mathrm{sec}$.

Appendix I: Method of Focussing a Prism Spectrograph.
Assuming that a focus in the visible region may readily be obtained by direct observation (although the considerations of Section 3 to follow may be applied as a final adjustment even in the visible region), an approximate focus for the ultra-violet regions may be found by simple calculations. The final adjustments may then be made with a minimum of difficulty. In most spectrographs regulation of prism rotation and translation and plate tilt may be made, and we shall consider these in turn.

## 1. Prism Rotation:

By Snell's law the angles of incidence, i, and refraction, $r$, and the refractive index of the prism, $n$, are related thus:

$$
\begin{equation*}
n=\frac{\sin i}{\sin r} \tag{7.1}
\end{equation*}
$$

In most spectrographs the prism is at the position of minimum deviation so that $\Omega=\frac{A}{2}$, where $A$ is the angle of the prism. Equation (7.1) becomes

$$
\begin{equation*}
n=\frac{\sin i}{\sin A / 2} \tag{7.2}
\end{equation*}
$$

Suppose now that the instrument has been focussed in the visible region, and that the wave-lengths at extreme ends of the plate are respectively $\lambda_{1}$ and $\lambda_{2}\left(\lambda_{1}>\lambda_{2}\right)$. For a focus in the near ultra-violet the prism must be rotated so that $\lambda$, is moved to the opposite end of the plate. From tables of the variation of $n$ with $\lambda$ we may calculate $i$ for any $\lambda$ from equation (7.2), and in particular we may calculate $i$, and $i_{2}$
for $\lambda_{1}$, and $\lambda_{2}$. To move $\lambda_{2}$ to the desired position for the new focus we must rotate the prism through an angle
$\left(i_{2}-i, i_{1}\right)$. In general the amount of rotation will be somewhat less than this to allow for overlap in the two regions. The procedure may be repeated for the far ultra-violet focus.
2. Prism Translation:

The new region is now centred on the plate. In order to focus the spectrum we must consider the variation of the focal length, $f$, of the camera lens with $m$. We have

$$
\begin{equation*}
\frac{1}{f}=(x-1) C \tag{7.3}
\end{equation*}
$$

where $C$ is the sum of the curvatures of the lens surfaces. From the visible focus we may measure for $\lambda$, ( $f$,say), and hence calculate $C$ from equation (7.3) since $f_{1}, \lambda$, , and $n$, are known. C is a constant of the lens. From the tables again we find $m_{2}$ for $\lambda_{2}$ and using the value of $C$ calculate $f_{2}$. Since for the near ultra-violet focus $\lambda_{2}$ occupies the position of $\lambda$, , in the visible focus, we must translate the prism through a distance $\left(f_{1}-f_{2}\right)$.

## 3. Plate Tilt:

Suppose that for a given plate tilt $\lambda_{1}$ is focussed at prism translation $\chi_{1}$ and $\lambda_{2}$ at prism translation $\chi_{2}$. We wish to know by how much we must increase the plate tilt to give a focus across the whole plate. We shall consider the general case where the axis of rotation of the plate holder is not in the plane of the plate.

In Figure 12, $O$ is the axis of rotation of the plate
AP. The angle of incidence $i$ is to be increased by $d \theta$. Let the difference $\left(x_{2}-x_{1}\right)$ be denoted by $\Delta x$.


Figure 12: Regulation of plate tilt.
Note that the angle $\alpha$ is constant and hence the angle at Q is also $d \theta$. From the figure

$$
\begin{align*}
& \alpha=\pi-\varphi-\frac{\pi}{2}-\psi \\
& \alpha=\frac{\pi}{2}-(\varphi+\psi) \tag{7.4}
\end{align*}
$$

For small $d \theta$ the arc $A B$ is essentially equal to the chord $A B$ so that in triangle $Q A B$ by the law of sines

$$
\begin{equation*}
\frac{n d \theta}{\sin d \theta}=\frac{l}{\sin \left[\frac{\pi}{2}+\alpha\right]} \tag{7.5}
\end{equation*}
$$

By equation (7.4) this becomes

$$
\begin{equation*}
\frac{n d \theta}{\sin d \theta}=\frac{l}{\sin [\pi-(\varphi+\varphi)]} \tag{7.6}
\end{equation*}
$$

Again for small $d \theta, \sin d \theta \sim d \theta$, and since
$|\sin [\pi-(\varphi+\psi)]|=\operatorname{frin}(\varphi+\psi))$ equation (7.6) reduces to

$$
\begin{equation*}
n=\frac{l}{\sin (\phi+\psi)} \tag{7.7}
\end{equation*}
$$

Now

$$
\begin{equation*}
d s=(d+l) d \theta \tag{7.8}
\end{equation*}
$$

or $d s=[d+1$ fin $(\rho+\psi)] d \theta$,
and also $d_{0}=\Delta x \operatorname{cas}\left(\frac{\pi}{2}-\phi\right)$

$$
\begin{equation*}
d s=\Delta x \sin \varphi \tag{7.9}
\end{equation*}
$$

Equations (7.8) and (7.9) give for $d \theta$

$$
\begin{equation*}
d \theta=\frac{\Delta x \sin \phi}{d+\Omega \sin (\varphi+4)} \tag{7.10}
\end{equation*}
$$

To give a focus over the whole plate we must increase the plate tilt by an amount $d \theta$ given by equation (7.10).
4. Example of Method:

The procedure outlined was used to focus the Hilger E-I quartz spectrograph. Table 5 gives the variation of $n$ with $\lambda$ (28).

Table 5: Variation of $n$ with $\lambda$ (28).

| $\lambda(\dot{A})$ | $m$ | $\lambda(\dot{A})$ | $m$ |
| :---: | :---: | :---: | :---: |
| 7065 | 1.5405 | 2749 | 1.5875 |
| 5893 | 1.5442 | 2573 | 1.5962 |
| 4861 | 1.5497 | 2313 | 1.6140 |
| 4341 | 1.5540 | 2195 | 1.6250 |
| 3404 | 1.5674 | 1990 | 1.6509 |

Cauchy's dispersion formula may be written

$$
m=A+\frac{B}{\lambda^{2}}
$$

where $A, B$ are constants. If we substitute in turn $n_{1}, \lambda_{1}$ and $n_{2}, \lambda_{2}$ and subtract:

$$
\begin{aligned}
\dot{n}_{1}-n_{2} & =B\left(\frac{1}{\lambda_{1}^{2}}-\frac{1}{\lambda_{2}^{2}}\right) \\
\Delta m_{1} & =B \Delta\left(\frac{1}{\lambda^{2}}\right)
\end{aligned}
$$

$$
(7.11)
$$

Table 6 gives the values of $\Delta m, \Delta\left(\frac{1}{\lambda^{2}}\right)$, and $B$ for .the wavelength intervals of Table 5.

Table 6: Evaluation of $B$.

| $\Delta\left(\frac{1}{\lambda^{2}}\right)\left(A^{-2}\right)$ | $\Delta m$ | $B\left(\dot{A}^{2}\right)$ |
| :--- | :--- | :--- |
| $0.089 \times 10$ | 0.0037 | $4.17 \times 10$ |
| 0.133 | 0.0055 | 4.13 |
| 0.108 | 0.0043 | 3.98 |
| 0.331 | 0.0134 | 4.05 |
| 0.451 | 0.0201 | 4.46 |
| 0.202 | 0.0087 | 4.31 |
| 0.357 | 0.0178 | 5.00 |
| 0.206 | 0.0110 | 5.36 |
| 0.449 | 0.0259 | 5.79 |

Note that B is not actually constant but increases with decreasing wavelength.

For the Littrow mount. $A=60^{\circ}$, sothat by equation (7.2) $\sin i=\frac{m}{2}$. In the visible region $\lambda, \sim 7000 A$ and $\lambda_{2} \sim 3 / 25 \dot{A}$. Using a value of $B$ in the neighbourhood of $3125 \dot{A}$ and Cauchy's dispersion formula we may extrapolate from Table 5 to find $m$ for 3125A. The result is $n=1.5749$, and hence $i_{2}=51^{\circ} 27^{\prime}, \quad i=50^{\circ} 22^{\prime}$. $\therefore \quad\left(i_{2}-i_{1}\right)=95^{\prime}$.
Since 1 prism rotation division equals 15.17 seconds we must rotate the prism 393 divisions, or to give overlap say 370 divisions.

In the visible, $f,=69$ inches and hence equation (7.3) gives $C=37.2$ inches ${ }^{-1}$, and for $\lambda_{2}$ we get $f_{2}=64.8$ inches. Hence we must translate the prism through 4.2 inches.

For the regulation of plate tilt note that the theory of Section 3 is a second order correction. The plate is first set for an estimated focus, and then the correct tilt calculated. For the spectrograph in question

$$
\text { On } 22^{\circ} ; \quad 4 \sim 22^{\circ} 36^{\prime} ; \quad \Omega=2.56 \text { inches; } d \sim 9 \text { inches. }
$$

Hence equation (7.10) reduces to

$$
\begin{equation*}
d \theta=0.0348 \Delta x . \tag{7.12}
\end{equation*}
$$

$\Delta x$ is found from the first plate and $d \theta$ calculated. Note that a slight compensation in prism translation may be neeessary since in increasing the plate tilt both ends of the plate move and not only the outer end. The necessary adjustmont may be calculated from equation (7.12)

Table 7 shows the agreement between calculated and actual focus.

Table 7: Agreement of calculated focus with actual focus.

| near U.V. |  |  | far U.V. |  |
| :---: | :---: | :---: | :---: | :---: |
|  | calculated | actual | calculated | actual |
| prism rotation | 1330 | 1330 | 920 | 920 |
| $\begin{aligned} & \text { prism } \\ & \text { trans- } \\ & \text { lation } \end{aligned}$ | 30 | 33 | 15 | 15 |
| plate tilt | 8.0 | 8.0 | 15.2 | 15.4 |

Appendix II: Method of Focussing a Microphotometer.
In the Moll microphotometer a light beam is focussed in the plane of the plate holder by a small objective. An exactly similar lens collimates this light, which is then focussed upon the thermocouple. With the beam focussed in the plane of the holder we wish to calculate by how much the lenses must be moved when plates of varying thickness are inserted. Two cases arise, depending on whether the emulsion is placed against the holder or away from the holder.
(a) Emulsion against holder:

If the objective and collimator be respectively $L$, and $L_{2}$, then $L$, remains fixed. For a plate of thickness $d$, $I_{2}$ must be moved away from the holder a distance $d$.
(b) Emulsion away from holder:

The focus must always be in the plane of the emulsion. In this case, if the plate has refractive index $n$, then $L$, must be moved a distance $\frac{d}{m}$ and $I_{2}$ a distance $d$. If further a plate of thickness $(d \pm x)$ replaces that of thickness d, 1 , must be moved a distance $\frac{x}{m}$ and $I_{2}$ a distance $x$. The two lenses are always moved in the same direction.

Case (a) has the advantage of a simpler adjustment since only $I_{2}$ need be moved. Case (b) has the advantage that the emulsion will not be scratched by resting against the plate.

## Appendix III: The Hyperfine Structure of $\mathrm{Hg}^{199}$.

The hfs. of 30 lines in the spectrum of $\mathrm{Hg}^{199}$ has been calculated by the procedure of Chapter III, Section 3. In all cases the data used is that of Brix and Kopferman (13). The relative intensities of the components have been evaluated by means of the intensity sum rule of Burger, Dorgelo, and Ornstein. The results of the investigation are listed in Table 8. Note that the wave number assigned to each component gives the separation of that component from the single $\mathrm{Hg}^{198}$ Iine.


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## PLATE I:

Fabry-Perot pattern of the line 5461
angstroms showing the hfs. due to the
two components, $\mathrm{Hg}^{198}$ and $\mathrm{Hg}^{199}$.

#  <br> <br> |ㅔㅔ||IIIIIIII 

 <br> <br> |ㅔㅔ||IIIIIIII}

PIATE II:
At the right, the fringe system
due to the line 4077 angstroms.
Note that the line 4046 angstroms
at the left also exhibits hfs.

## - $111111|1| 1||||||||||||\mid$

PIATE III:
Interference pattern of the line
3341 angstroms showing the two ring systems due to the two Hg components.

PLATE IV:
Off-centre Fabry-Perot pattern
of the line 3125 angstroms. At
the right is the doublet, 3131.8
and 3131.6 angstroms.

