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LINE INTENSITIES AND SPECTRAL TYPES IN B TYPE STARS

by

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ABSTRACT

Photometric measures of absorption line intensities in the spectra of forty-five B stars are given. The plates were taken and measurements made at the Dominion Astrophysical Observatory. Graphs have been prepared showing the intensities plotted against the Henry Draper subtype of the star for the stronger lines. Using these intensity measurements, those published by E. G. Williams, and unpublished measures by R. H. Petrie, criteria for determining spectral type from ratios of the intensities of the stronger lines are suggested. Also, from the variation of Balmer line intensity with luminosity, a method of determining absolute magnitudes is suggested. Spectral types and luminosities as determined from these criteria are given for 133 B type stars for which photometric measures of line intensity are available.

ACKNOWLEDGMENTS

I would like to acknowledge the kind help of Dr. J. A. Pearce, Dominion Astrophysicist, and the other members of the staff of the Dominion Astrophysical Observatory, including Dr. C. S. Beals, now Acting Dominion Astronomer, who all kindly contributed information and suggestions on the conduct of this investigation during my work at the Observatory. Especially I wish to thank Dr. R. M. Petrie, who supervised the work, for his many discussions of the problems involved, for the luminosity data he provided, and for permission to use his unpublished line intensity measurements.

LINE INTENSITIES AND SPECTRAL TYPES IN B TYPE STARS

The spectra of stars are classified according to their main absorption features. The spectral types are usually defined by the Harvard criteria and are named O, B, A, F, G, K, M, R, N, S, with subdivisions numbered from 0 to 9 in most types. These types depend chiefly on the degree of ionization in the stellar atmosphere and thus on its temperature. The degree of ionization is indicated by the variation in intensity of the absorption lines in the spectrum. As the temperature of the stellar atmosphere increases certain lines increase in intensity, reach a maximum, and then decrease. The temperature at which a line shows its maximum depends on the excitation potential of the lower level of the line and the ionization potential of the atom or ion. These factors determine the relative number of atoms able to absorb the line observed. This means that the atoms of high excitation and ionization potentials contribute most to the observed spectra of the hottest stars, while the lines of low excitation potential appear in the coolest stars.

In addition, in each type a further classification can be made on the basis of absolute magnitude or intrinsic luminosity into giants, intermediates and dwarfs. This difference in luminosity is only partially due to difference in the stellar mass, while the chief cause is that the giant stars have much more tenuous atmospheres than the dwarfs and thus a greater surface area. The pressure differences, which are equivalent to surface gravity differences, cause differences in the absorption spectra of the stars. In giants the absorption lines are sharper and the intensities of lines due to metallic ions are increased, while in dwarfs the lines are more diffuse and those due to hydrogen and helium are much broadened due to Stark effect. However, not all cases of diffuse lines in stellar spectra are due to luminosity effects, since high rotational velocities of stars tend to produce dish-shaped line profiles as a result of the integrated Doppler shift over the stellar disk.

An examination of a stellar spectrum as discussed above will give the temperature and surface gravity to classify the star in a two parameter system. However, there is also certain evidence to indicate that this is not sufficient definitely to characterize a stellar atmosphere. For example, among the hottest (Wolf-Rayet) and the coolest stars there appear differences that can hardly be explained except as variations in the abundance of various elements.

Among the hottest stars are those of spectral types O and B. They are comparatively rare, but owing to their great luminosity (a consequence of their high temperature) they can be seen for large distances in the galaxy. For this reason a knowledge of their motions proves useful in a study of the structure of the galaxy. For this to be of much value a knowledge of their distances is required. These distances can

be obtained if both the apparent and real magnitudes of the stars are known. To obtain criteria for determining spectral types and these absolute magnitudes was the object of the research upon which this thesis is based.

PREVIOUS INVESTIGATIONS

The Harvard classification is based on visual examination of low dispersion objective prism plates. Most of the revised classifications from various observatories such as Mount Wilson, Victoria, and the recent work of Morgan, Keenan, and Kellman¹ at Yerkes are also based on visual examination but of higher dispersion plates. Of these, the Victoria revised classification was made by J. A. Pearce largely on plates with a dispersion of 30 Angstroms/mm. at H γ , while that at Yerkes was made with dispersion 125 A/mm. at H γ . E. G. Williams,² using plates taken at Mount Wilson, obtained line intensities from spectrophotometric measures for O and B type stars. These measurements by Williams comprise about the only extensive list of measured line intensities in the B type stars. In addition, there are measurements of absorption line intensity for several stars made by Rudnick³ using a wide-slit photometer method.

The application of this method will be discussed briefly later. There

1. Morgan, Keenan and Kellman, An Outline of Spectral Classification.
 2. E. G. Williams, Ap. J., 83, 279, 1936.
 3. Paul Rudnick, Ap. J., 83, 439, 1936.
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have also been several lists of hydrogen line intensities published and a few special studies of special stars. These have not been referred to further in this discussion owing to lack of information about systematic differences in measurement between such values and those made at the Dominion Astrophysical Observatory.

Williams¹ used the line intensities obtained in his investigation to set up a classification scheme based on measured line intensities. However, as discussed by Petrie² in his investigation of the O type stars, Williams' criteria are based largely on lines of low intensity which are subject to large errors of measurement on low dispersion spectra. Since for a study of the fainter and more distant stars it is necessary to use low dispersion spectra and consequently spectra in which the weaker lines are obliterated, criteria based on the most intense lines in the spectra have to be determined. This is also required by the fact that many of these early type stars have shallow diffuse lines, increasing the difficulties of measurement. In B type stars by far the most intense lines are those of the Balmer series of hydrogen followed by the lines of the diffuse singlet and triplet series of helium and the line of ionized magnesium at $\lambda 4481$. However, it is well known that both hydrogen (chiefly in later subtypes) and helium (especially in early subtypes) are enhanced in intensity in dwarf and intermediate stars due to Stark effect caused by intermolecular electric fields, while the Mg II line is enhanced in the giants. These facts make it undesirable to use the absolute intensities of any of the measured lines for criteria. This is discussed by Williams¹, who comes to the conclusion that ratios of

1. E. G. Williams, Ap.J., 83, 305, 1936.

2. R. M. Petrie, J. R. A. S. C., 38, 337, 1944.

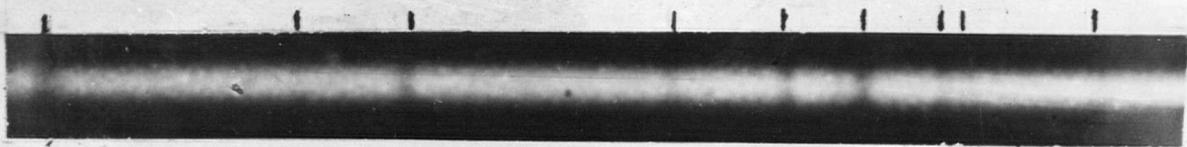
line intensities give the best guide to average characteristics.

OBSERVATIONAL MATERIAL

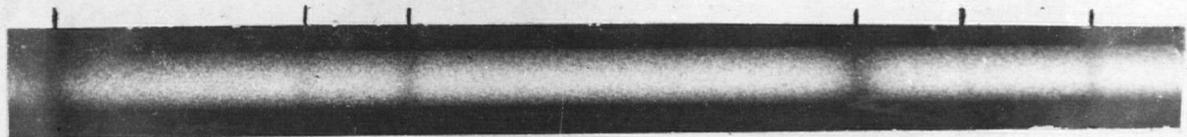
The observational material for this program consisted of 82 plates of 45 B type stars distributed among the Henry Draper catalogue sub-types as: B0, 3 stars, B1, 3, B2, 4, B3, 12, B5, 9,^{B8,9}/B9, 5. All these plates were taken with the 73 inch telescope of the Dominion Astrophysical Observatory by various members of the staff. The majority of the plates were taken using the LM spectrograph (i.e. the spectrograph was used in the one prism form and the medium focal length camera) with a dispersion of 30 Angstroms/mm. at H γ , although a few were taken with higher dispersion. Some typical spectra are shown in plate I.

On each plate a calibration was impressed in order to be able to determine the characteristic curve of the emulsion used. Most of the plates were calibrated by a calibrating spectrograph giving a series of spectra of differing intensity. These intensity differences were obtained by use of a rotating step sector in front of the slit of the calibrating spectrograph. The majority of the plates used had the ratio of the intensity of successive steps such that the common logarithm of the ratio was 0.2, but for a few of the older plates the logarithm of the ratio was 0.3 (i.e. intensity ratio 2:1). Both of these enable separate calibration curves to be drawn at different wavelengths so that each section of the spectrum is reduced by use of the correct calibration curve for that section. To determine wavelengths in the calibration a

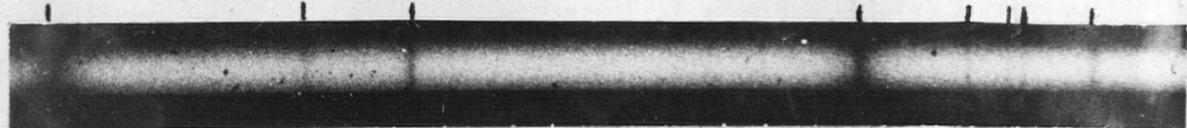
PLATE I
Typical Spectra



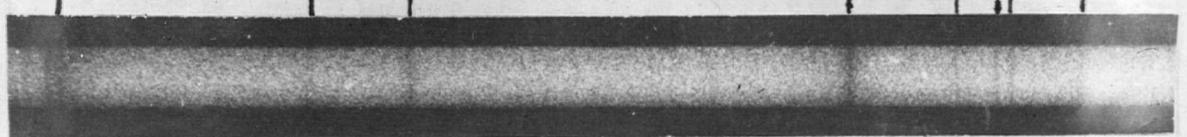
H.D. 37742 B0



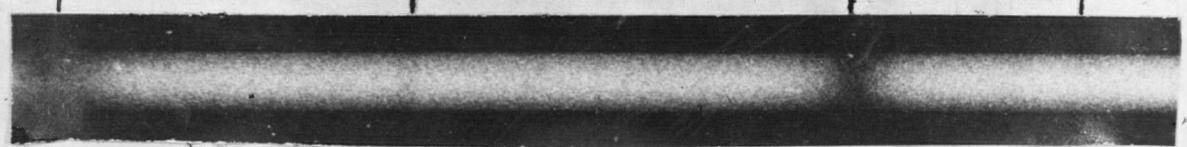
H.D. 221253 B2



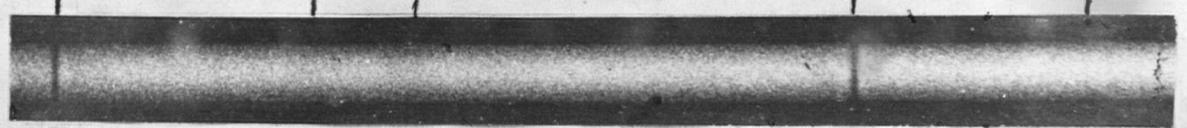
H.D. 11415 B3.5



H.D. 698 B5.5



H.D. 9531 B8



H.D. 10516

3970 He
4009 He I
4026 He I
4069 C III
4089 Si IV
4101 H δ
4116 Si IV
4121 He I
4128 Si II
4131 Si II
4144 He I

mercury arc is photographed with the calibrating spectrograph so that its spectrum appears next to the calibration. For most reductions in this work the calibration curve for $\lambda 4047$ was used for the portion of the spectrum between $\lambda 3933$ and $\lambda 4144$ and the calibration curve for $\lambda 4358$ for wavelengths between $\lambda 4267$ and $\lambda 4575$. A few of the oldest plates had been calibrated with a tube sensitometer illumened by white light with a filter. This impressed small spots of varying density on the plate. From this type of calibration only one mean calibration curve for all wavelengths on the plate could be obtained.

To obtain the equivalent widths of the lines measured, the usual practice of the observatory was used. This consists of first running the plates through the microphotometer described by Beals.¹ To transform the microphotometer record to a true intensity record, the semi-automatic intensitometer also described by Beals² was used. Most of the records were reduced in two stages, the galvanometer deflection being first reduced to a "log intensity" scale and on this the line of the continuous spectrum was drawn. The second stage consisted of transforming the "log intensity" to a true intensity curve, keeping the continuous spectrum at a height of 10 inches on the paper. The six Pleiades stars were measured later and reduced in only one stage directly to the true intensity, but with varying height of the continuous spectrum.

The intensity of the absorption lines was measured by drawing freehand a mean profile among the irregularities due to grain effect in the original plate as shown in Plate II. The area between this profile and the line of the continuous spectrum was measured with a planimeter. The planimeter readings were converted to the intensities of the lines

1. C. S. Beals, M. N., 96, 730, 1935.

2. C. S. Beals, J.R.A.S.C., 38, 44, 1944.

in equivalent Angstroms, i.e. the width of the line of rectangular profile and depth equal to the height of the continuous spectrum which would absorb the same total amount of energy from the continuous spectrum as the actual line does. For lines in the wing of the hydrogen lines the residual intensity of the wing at the position of the line being measured was taken as the level of the continuous spectrum for this purpose.

In addition to the intensity measures the apparent depth of the centre of the line profile was measured for the hydrogen and helium lines as a fraction of the height of the continuous spectrum.

ERRORS IN STELLAR SPECTROPHOTOMETRY

Stellar spectrophotometry is not of extreme accuracy. The reasons for this lie in the difficulties attendant upon the small light intensity received from the stars. In order to obtain spectra with a reasonably short exposure only a short slit can be used in the stellar spectrograph, since the stellar image, even with large telescopes, is very small, about 0.4 mm. in diameter. This allows only a small area of plate to be exposed and then be examined by the microphotometer. Thus the uncertainties introduced by the random effect of plate graininess are of fairly large magnitude. This effect causes uncertainties in the position of the continuous spectrum and of the mean profile of the lines as drawn on the tracing. In spite of the small length of slit, with good seeing the stellar image is too small to cover the whole slit length. In order to obtain even illumination of the slit this requires the guiding of the

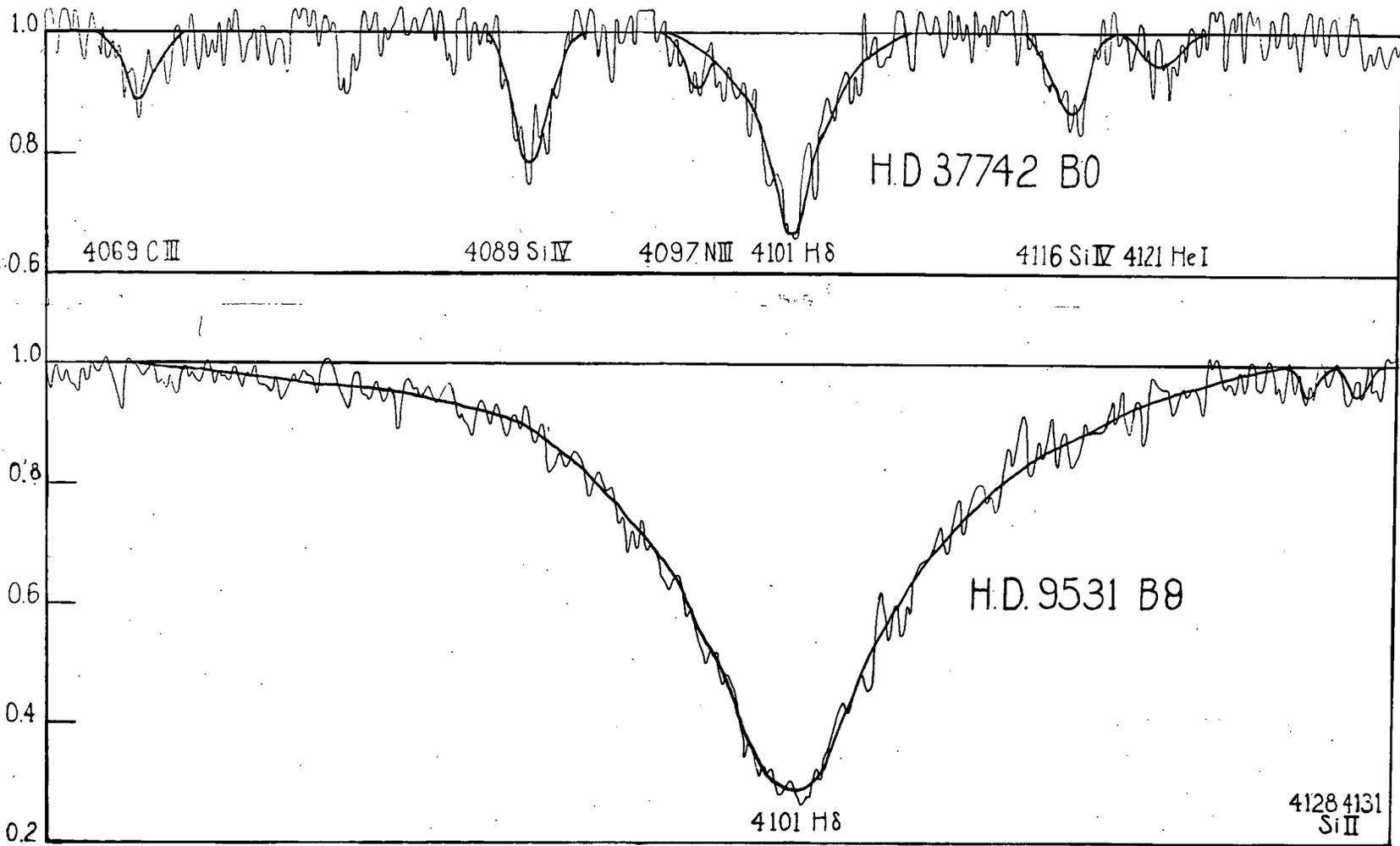


PLATE II

Rectified Line Profiles

telescope to be carried out so that the stellar image drifts along the length of the slit to expose each portion for the same time. If this has been imperfectly done, different portions along the slit of the spectrum will have been exposed to different positions of the characteristic curve of the plate used, while the microphotometer records an average blackening of the spectrum. If the characteristic curve were perfectly straight this average would not affect the result, but if the exposure of the weaker portion is such as to cause it to lie on the toe of the characteristic curve a considerable error occurs. Table I illustrates the differences in readings of equivalent width and depth obtained between an unevenly exposed plate and the mean of two properly exposed plates of the same star (ϵ Cassiopeiae).

Table I

Line	H δ		H γ		Helium I						MgII
	W	D	W	D	4009		4026		4471		4481
					W	D	W	D	W	D	W
Poor Plate	7.0	0.75	8.5	0.75	0.44	0.22	1.40	0.46	2.14	0.57	0.71
Good Plates	5.8	0.62	5.8	0.68	0.42	0.16	1.08	0.38	0.90	0.33	0.33

This example is an extreme one, since the plate in error is very noticeably poorly exposed, but a small amount of the same effect must remain in the spectra measured although in all other cases apparently evenly exposed plates were used.

On account of these almost irremediable errors in the spectrographic process a variation of 10% in the measurements obtained is often

not considered extreme. In the measurement of the B type stars even larger errors are not surprising. One reason for this is the fact that many of the lines are extremely diffuse and hence their measured intensities will be very sensitive to errors in drawing both the level of the continuous spectrum and the mean profile. In later type stars, where the lines are deep and narrow, a small error in height on the tracing will produce only a small error in the measured area, while a small error in the case of a shallow diffuse line will cause a large error in the measured area.

MEASUREMENT OF LINE INTENSITIES

Table II gives the measured line intensities in equivalent Angstroms. For the Hydrogen lines and stronger Helium I lines the intensity is given in the column headed W, while that headed D gives the relative depth of the centre of the line. The intensities and depths as given are mean values in the case where two plates were measured. The columns of the table give first the number the star is given in the Henry Draper Memorial Catalogue, followed by the constellation name. Then comes the visual apparent magnitude, while the next three columns give the spectral type, first as assigned at Harvard by Miss Cannon and listed in the Henry Draper Catalogue, then the Victoria Revised type, as given by J. A. Pearce from visual estimates,¹ and lastly, types as determined from these measured intensities. The next two columns give the absolute

1. Pub. D.A.O., vol. 5, No. 2, 1931.

magnitudes as determined from the measures and the number of plates of the star on which the measures are based, while the remaining columns give the actual measures. In the measures a dash (-) indicates that the line was not noticeable on the tracing, a question mark that the presence of a slight absorption is doubtful, while a blank means, in general, that no detailed examination has been made. In the case of line depths, the blank means that, although an intensity has been measured, the line is so diffuse and shallow that the depth observed is unreliable. The note 2sp indicates that the star is a spectroscopic binary with two similar spectra visible. In this case, unless the two spectra coincide, the depth measurement has no meaning.

In Table III are given line intensities of β Orionis, B8, α Lyrae, A0, and α Cygni, A2, as determined by measuring with the planimeter the profiles published by Williams and Hiltner.¹ The two A type stars were measured to see that the criteria obtained would not be inconsistent with a smooth junction with the A type stars.

In addition to the stars measured in this program, there have also been used in the discussion of results, measurements for 27 B stars made by R. M. Petrie, partly for this program and partly in his work on binary stars. These measurements are mainly unpublished. The measurements given by Williams² for 64 stars have also been used, since a comparison of measures by the different measurers, as plotted in Plate III, fig. 1, indicate no serious systematic differences.

The values of line intensities in equivalent Angstroms from these three sources have been plotted against the Henry Draper subtype for H, HeI, MgII, and CII in Plate III, figs. 2, 3, 5, 6. For hydrogen in fig. 2 the mean of H γ and H δ has been plotted, for neutral helium

1. W. A. Hiltner and R.C. Williams, Photometric Atlas of Stellar Spectra.

2. E. G. Williams, Ap. J., 83, 279, 1936.

the mean of the four lines $\lambda 4026$, $\lambda 4471$ (diffuse triplets), $\lambda 4388$, and $\lambda 4144$ (diffuse singlets), for ionized magnesium the line $\lambda 4481$, and for ionized carbon the line $\lambda 4267$ has been plotted. In fig. 4 the ratio of the sum of the intensities of the two singlet lines to the sum of the intensities of the two triplet lines has been shown. Subtypes 4, 6, and 7 are missing from all graphs because they are not used in the Henry Draper classification.

The very large scatter shown in all graphs is partly due to poor classification. This portion should be removed by the use of a classification scheme based on measured line intensities. This will not remove all the scatter apparent on the diagrams, since there are variations in intensity in each subtype, due to luminosity effects, variations due to errors in measurement, especially of weak lines, and probably also variations due to other natural differences between the individual stars.

TABLE II - STELLAR DATA

Henry Draper Number	Name	Appt. Mag.	Spectral Type			Meas'd Abs. Mag.	Plates
			HD	Vict. Revd.	Meas'd		
9531	Boss 340	5.77	B9		B8	+2.3	2
10516	ϕ Persei	4.19	B0p	B0ne	-	-	2
11415	ϵ Cassiopeiae	3.44	B3	B5s	B3.5	-1.3	2
17573	41 Arietis	3.68	B8	-	B7.5	+1.1	2
20315	30 Persei	5.38	B5	B7n	B6	+1.6	2
20319		6.02	B9	-	B7	+1.4	1
20365	29 Persei	5.30	B3	B3	B3	-0.1	1
20418	31 Persei	5.08	B3	B3	B4	-1.2	1
20809	Boss 767	5.30	B3	B3	B4	-1.3	1
21362	Boss 783	5.64	B5	B8nn	B5	-1.8	1
21428	34 Persei	4.67	B5	B5	B3.5	-0.3	2
23180	\circ Persei	3.94	B1	B2k	B0.5	-2.8	2
23324	Boss 855	5.63	B8	-	B7	+0.5	2
23432	Boss 861	5.85	B8	-	B7	+1.7	2
23441	+24 ^o 556	6.46	B9	-	B9	+2.2	2
23753	Boss 872	5.51	B8	-	B7.5	+0.3	2
23850	Boss 877	3.80	B8	-	B7	-0.7	2
23950		5.92	B9	-	B9.5	+1.1	2
25940	48 Persei	4.03	B3p	B3e	B3	-2.2	1
29763	τ Tauri	4.33	B5	B5n	B3.5	+0.9	1
30836	π 4 Orionis	3.78	B3	B2s	B2	-3.0	2
31237	π 5 Orionis	3.87	B3	B2s	B1	-2.8	2
33328	λ Eridani	4.34	B2	B3nk	B1	-2.1	2
35039	22 Orionis	4.65	B3	B3ns	B1.5	-1.5	2
35411	η Orionis A	3.44	B1	B0k	B1	-1.8	2
35468	γ Orionis	1.70	B2	B2s	B2	-2.0	2
35715	ψ Orionis	4.66	B2	B2	B1	-1.7	2
36267	A Orionis	4.32	B3	B4n	B5	+0.1	2
36822	ϕ ' Orionis	4.53	B0	B0ssk	B0.5	-2.6	2
37742	ζ Orionis br	2.05	B0	B0nk	B0	-4.7	2
37756	Boss 1399	5.00	B3	B3	B1	-1.6	1
42560	ξ Orionis	4.35	B3	B3nn	B3	-0.5	2
45542	ν Geminorum	4.06	B5	B5ne	B5	-1.1	2
58715	β Canis Min.	3.09	B8	-	B8	-0.4	2
109387	κ Draconis	3.88	B5p	B5e	B5	-2.0	2
116658	α Virginis	1.21	B2	B2	B2	-2.9	2
144217	β ' Scorpii	2.90	B1	B2k	B2.5	-2.0	2
149212	15 Draconis	4.98	B8	-	B9.5	+2.9	2
155763	γ Draconis	3.22	B5	B8s	B7.5	-2.3	2
177756	λ Aquilae	3.55	B9	-	B9.5	+1.9	2
180554	1 Vulpeculae	4.60	B5	B5n	B3.5	-1.5	2
196867	α Delphini	3.86	B8	-	B8	+1.3	2
198667	5 Aquarii	5.50	B8	-	B9.5	+0.1	2
202904	ν Cygni	4.42	B3p	B3nek	B2	-1.5	2
212120	2 Lacertae	4.66	B5	B5	B4.5	-0.8	2

TABLE II (Cont'd)

INTENSITY MEASURES

H.D. No.	Hydrogen				Helium I							
	γ (4340)		δ (4101)		4471		4026		4388		4144	
	W	D	W	D	W	D	W	D	W	D	W	D
9531	12.00	.71	11.28	.72	0.52	.10	0.38	.12	0.34	.06	0.12	
10516	em		em		0.75		0.74		-		0.84	
11415	5.84	.68	5.80	.62	0.90	.33	1.08	.38	0.66	.24	0.50	.23
17573	10.44	.65	10.06	.68	0.41	.10	0.49	.12	0.22	.07	0.10	
20315	9.29	.62	9.15	.64	0.66	.10	0.60	.12	0.48		0.18	
20319	9.80	.72	11.20	.70	0.50	.11	0.48	.10	0.23		0.25	.08
20365	6.44	.59	5.88	.60	1.15	.26	1.14	.31	0.74	.20	0.48	.15
20418	6.19	.50	6.64	.51	1.02	.17	1.24	.21	0.64	.10	0.70	.13
20809	6.52	.54	6.08	.56	0.92	.18	0.97	.26	0.46	.09	0.35	.09
21362	7.02	.51	6.62	.53	0.69	.13	0.58	.11	0.41	.08	0.42	.08
21428	6.89	.60	6.60	.61	1.04	.24	1.20	.32	0.68	.14	0.60	.18
23180	3.49	.52	2.73	.50	1.07	.36	1.03	.41	0.66	.30	0.48	.24
23324	9.58	.62	9.78	.64	0.41	.09	0.50	.11	0.28	.06	0.17	.05
23432	11.38	.70	10.44	.71	0.57	.10	0.59	.15	0.42	.08	0.15	.06
23441	11.52	.66	11.69	.71	0.21	.05	0.27	.07	0.14	.04	0.06	
23753	9.46	.59	8.98	.61	0.36	.06	0.42	.10	0.32	.07	0.09	.04
23850	8.25	.62	7.85	.62	0.34	.07	0.53	.10	0.33	.05	0.14	.04
23950	10.62	.75	9.70	.72	0.21	.08	0.34	.10	-?		?	
25940	4.77	em	5.16	em	0.86	.19	1.00	.24	0.75	.18	0.44	.12
29763	8.40	.61	7.80	.63	1.64	.30	1.33	.32	0.72	.19	0.64	.17
30836	3.92	.54	3.59	.56	1.13	.37	1.00	.42	0.61	.30	0.53	.27
31237	3.96	.52	3.19	.48	1.18	.36	0.94	.34	0.63	.29	0.63	.24
33328	4.34	.37	3.83	.38	1.28	.18	1.19	.20	0.81	.12	0.55	.11
35039	4.92	.64	4.66	.62	1.50	.48	1.25	.50	0.86	.42	0.98	.41
35411	4.75	2sp	3.85		1.42		1.18		0.74		0.72	
35468	4.88	.52	4.38	.52	1.18	.36	1.21	.43	0.75	.30	0.80	.33
35715	5.03	2sp	4.06		1.45		1.18		0.97		0.49	
36267	7.66	.60	7.72	.60	0.89	.18	0.86	.22	0.37	.09	0.28	.08
36822	3.75	.50	2.77	.46	1.29	.40	1.02	.38	0.71	.28	0.32	.21
37742	1.74	.38	1.49	.34	0.88	.26	0.57	.21	0.30	.11	0.20	.07
37756	4.50	.51	4.51	.51	1.43	.35	1.31	.39	0.76	.26	0.89	.29
42560	6.64	.56	6.29	.60	1.28	.29	1.24	.30	0.70	.18	0.67	.18
45542	7.12	.55	6.26	.58	0.82	.14	0.61	.16	0.45	.07	0.23	.08
58715	9.20	.55	9.56	.66	0.43	.08	0.33	.06	0.26		0.10	
109387	6.02	.44	4.80	.48	0.70	.18	0.41	.12	0.46	.13	0.19	.06
116658	4.28	2sp	3.52		1.19		0.88		0.68		0.48	
144217	4.35	2sp	5.37		1.12		1.02		0.76		0.59	
149212	11.69	.75	13.20	.78	0.29	.08	0.24	.08	-		0.07	
155763	5.68	.62	6.42	.70	0.33	.17	0.42	.21	0.14	.09	0.18	.09
177756	10.79	.66	11.36	.67	0.30	.06	0.21	.06	-		?	
180554	5.76	.56	5.67	.58	0.84	.18	0.96	.26	0.58	.12	0.60	.15
196867	10.46	.68	10.28	.73	0.36	.07	0.30	.07	0.27	.05	0.04	
198667	9.32	.74	8.96	.75	0.23	.13	0.20	.11	0.12		0.08	
202904	5.05	.38	5.02	.47	1.25	.26	1.25	.28	0.75	.19	0.79	.19
212120	6.74	2sp	6.80		0.77		0.86		0.39		0.47	

TABLE II (Cont)

INTENSITY MEASURES

H.D. No.	He I		Mg II	C II	CaII	Si II		Si IV	O II		
	4009	4121	4481	4267	3934 (K)	4128	4131	4089	4070	4072	4076
9531	-	-	0.43		0.22	0.13	0.08				
10516	--	-	-		-						
11415	0.42	0.24	0.33	0.20	0.26	0.09	0.10				
17573	-	-	0.29		0.11	0.04	0.08				
20315	-	0.09	0.58		0.14	0.18	0.13				
20319	-	-	0.40		0.49	0.18	0.13				
20365	0.44	0.14	0.31	0.22	0.24						
20418	0.21	0.25	0.31	0.19	0.14	0.18	0.18				
20809	?	?	0.21	0.14	?						
21362	?	0.12	flaw	?	0.04						
21428	0.48	0.29	0.31	0.18	0.22						
23180	0.45	0.41	0.18		0.23	0.10	0.04	0.11	.19	.13	.23
23324	-	-	0.34		0.18	0.17	0.14				
23432	-	-	0.57		0.31	0.12	0.14				
23441	-	-	0.26		0.15	0.07	0.10				
23753	-	-	0.31		0.04	0.05	0.07				
23850	?	-	0.29		0.10	0.09	0.10				
23950	-	-	0.36		0.29	0.19	0.14				
25940	0.35	0.14	0.39	0.18	0.12						
29763	0.43	0.28	0.28	0.25	0.25	0.09	0.12				
30836	0.60	0.33	0.31		0.24				.15	.10	.18
31237	0.47	0.40	0.29	0.34	0.19	0.04	0.06	0.06	.06	.07	.12
33328	0.48	0.30	0.22	0.13	0.13				.07	.08	.13
35039	0.57	0.47	0.32	0.29	0.19				.11	.08	.10
35411	0.48	0.35	0.22	0.24	0.13			0.17		.43	
35468	0.57	0.40	0.32	0.28	0.11				.07	.09	.15
35715	0.53	0.37	0.15		0.08			0.18		.47	
36267	0.18	0.22	0.26	0.13	0.12	0.14	0.11				
36822	0.28	0.27	0.20	0.12	0.24			0.37	.25	.16	.18
37742	0.07	0.17	-	0.06	0.13			0.55			
37756	0.73	0.40	0.39		0.14			0.08	.07	.06	.12
42560	0.49	0.24	0.33	0.24	0.20						
45542	0.20	0.05	0.44	?	0.26	0.02	0.07				
58715	-	-	0.42	-	0.14	0.14	0.21				
109387	0.16	0.08	0.29		0.05	0.04	0.03				
116658	0.40	0.32	0.26		0.03			0.04		.39	
144217	0.36	0.42	0.28	0.25	?			0.10	.07	.06	.07
149212	-	-	0.54		0.58	0.14	0.22				
155763	0.21	0.09	0.25	0.09	0.14	0.10	0.09				
177756	-	-	0.39		0.16	0.09	0.08				
180554	0.44	0.12	0.38	0.18	0.21	0.04	0.06				
196867	-	-	0.41		0.42	0.12	0.11				
198667	-	-	0.39		0.32	0.18	0.12				
202904	0.55	0.24	0.29	0.20	0.07	0.02	0.09				
212120	0.22	0.16	0.35	0.20	0.25	0.11	0.14				

TABLE II (Cont.)

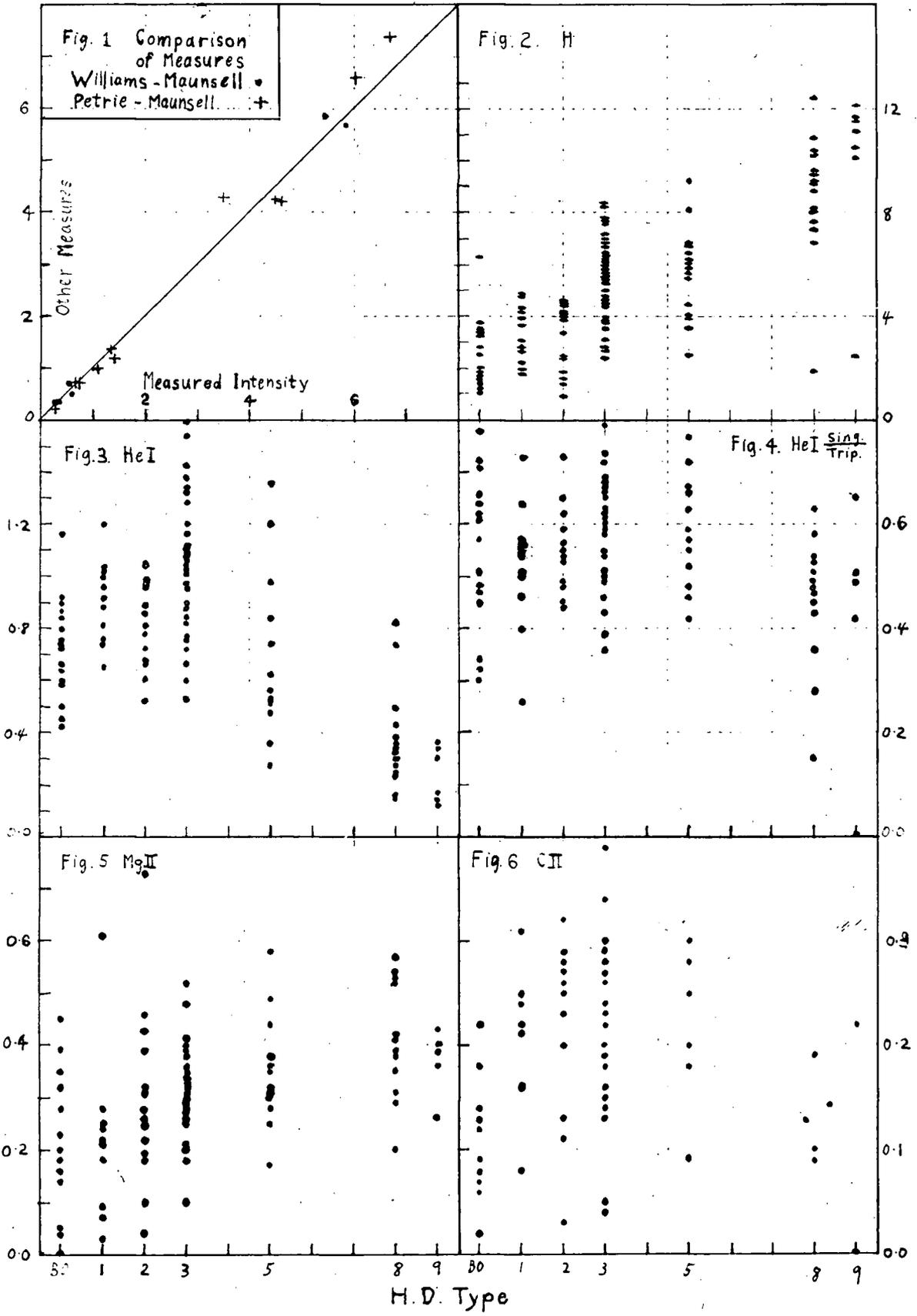
H.D.No.	Si III			SiIV	NI I	NI II	FeII	CII	CIII	HeI	HeII	
	4553	4568	4575	4116	3995	4097	4233	3920	4069	3927	4200	4542
9531							0.12					
10516							em					
20319							0.12					
23180				0.03								
30836								0.35		0.35		
31237	0.43	0.37	0.22		0.06			0.20		0.45		
33328	0.08	0.07	0.04									
35039	0.20	0.12	-		0.08							
35411	0.18	0.07	-	0.13	0.11							
35468	0.24	0.18	?		0.12							
35715				0.04								
36822	0.28	0.25	0.16	0.22	0.08	0.21						
37742				0.28	?	0.10			0.22		0.17	0.12
37756								0.20		0.46		
144217	0.21	0.08	0.17	0.05	?							
149212							0.16					
177756							0.18					
196867							0.08					

TABLE III - MEASUREMENTS FROM TRACINGS
IN ATLAS

H.D. No.	Name	H.D. Type	Appt. Mag.	Equivalent Widths				
				4340 H γ	4101 H δ	4471 HeI	4026 HeI	4388 HeI
34085	β Orionis	B8p	0.3	1.86	2.02	0.50	0.57	0.25
172167	α Lyrae	A0	0.1	19.5	14.7	0.09	0.13	-
197345	α Cygni	A2p	1.3	2.86	3.04	0.18	0.12	0.04

H.D. No.	4144 HeI	4009 HeI	4121 HeI	4481 MgII	3933 CaII	4233 FeII	4267 CII	4128 SiII	4131 SiII
34085	0.37	0.22	0.17	0.53	0.63	0.16	0.23	0.33	0.32
172167	-	0.03	-	0.26	0.78	0.14	-	0.06	0.05
197345	-	0.03	0.04	0.90		0.60	-	0.35	0.34

PLATE III



SPECTRAL CLASSIFICATION

With the exception of the work of Williams¹, practically all classifications so far published of the B type stars are based on visual examination of plates of the stellar spectrum. In this category, for example, fall the determinations of Miss Cannon for the Henry Draper catalogue which are based on the criteria given in the introduction to the catalogue², the Victoria Revised classification, by J. A. Pearce, using visual estimates of line intensity and based on criteria as yet unpublished, a scheme given by E. G. Williams and D. L. Edwards,³ and that given by Morgan, Keenan, and Kellman.⁴ These visual estimation methods enable an experienced person to classify spectra rapidly and fairly consistently. However, such methods do not guarantee absolute agreement between different observers, even when using the same plates as Tables I and II given by Williams and Edwards show.³ A more serious difficulty lies in the fact that an inexperienced observer cannot take the classification criteria given and expect to produce results agreeing with those obtained by those who laid down the criteria. Other objections to the visual method are that it is almost impossible to use the same visual criteria for plates of all dispersions, hence introducing inconsistencies, and that errors are introduced by the usual diffuse lines (often caused by rotational effects), by poor exposure, and by luminosity effects, since they cannot be visually estimated with precision.

1. E. G. Williams, Ap. J., 83, 305, 1936.

2. Harvard Annals, 91, 6, 1918.

3. E. G. Williams and D. L. Edwards, M. N., 98, 467, 1938.

4. Morgan, Keenan, Kellman, An Outline of Stellar Classification.

A classification scheme based on measured line intensities should be free from these disadvantages since it is based on objective measures of the line intensities, which should be the same for all observers, rather than on subjective estimation which will certainly vary from observer to observer and probably from time to time for the same observer.

The criteria advanced by Williams¹ provide an objective scheme of this type, but, as discussed above, it is chiefly based on measurements of weak lines which are difficult to measure accurately on low dispersion spectra, especially if the lines are at all diffuse in character.

Theoretically a classification scheme based on ratios of the intensities of lines due to the same element in different stages of ionization would give a good indication of temperature since it should be independent of abundance effect. For example, since in B type spectra lines due to SiIII, SiIII, and SiIV appear, it should be possible to base a classification on the ratios SiIII/SiIII and SiIII/SiIV. However, this does not appear very practical since the lines are of low intensity, and hence are unsuitable because of the effect of accidental errors in measurement.

Another possible theoretical classification could be based on an extension of the argument suggested by Goldberg² for the ratio of the diffuse singlets to the diffuse triplets of HeI. However, as shown in Plate III, figure 4, the measured results show considerable variation in this ratio within one spectral subtype; together with a flat maximum for the mean values in the early subtypes. Further consideration also shows that even on a theoretical basis curve of growth effects and the line

1. E. G. Williams, Ap.J., 83, 310, 1936.

2. Leo Goldberg, Ap. J., 89, 623, 1939.

broadening due to Stark effect would complicate the ratio and prevent its practical use.

The outstanding variation in B type spectra is the increase in intensity of the lines of the Balmer series of hydrogen with decreasing temperature. The hydrogen lines also increase so much in intensity with decreasing luminosity that they can only be used for type criteria if the absolute magnitude is already reasonably well known.

Hence it appears that a practical scheme will have to be based on ratios of the lines of highest intensity, i.e. those of hydrogen, helium, and ionized magnesium. The greatest change in ratios between subtypes occurs in the case where one member of the ratio is increasing in intensity with advancing type, while the second is decreasing. Since the hydrogen and magnesium lines increase in intensity with decreasing temperature, while the helium lines first increase in intensity, then reach a maximum and decrease, the obvious ratios to try are those of hydrogen to helium and magnesium to helium. However, since both hydrogen and helium lines are enhanced in dwarfs, although hydrogen to a greater extent, while the magnesium line is enhanced in the giants, it is obvious that one ratio will not be suitable for all luminosity groups. Hence a final classification cannot be assigned until the luminosity has been determined. Similarly to obtain the correct luminosity a knowledge of the spectral type is required (see next section). However, the criteria are such that a successive approximation method will work to give both spectral type and luminosity. Hence an assumption of a preliminary type or luminosity (by visual examination of the spectrum or otherwise) enables a final determination to be reached fairly rapidly.

The ratios suggested as suitable for type criteria are given

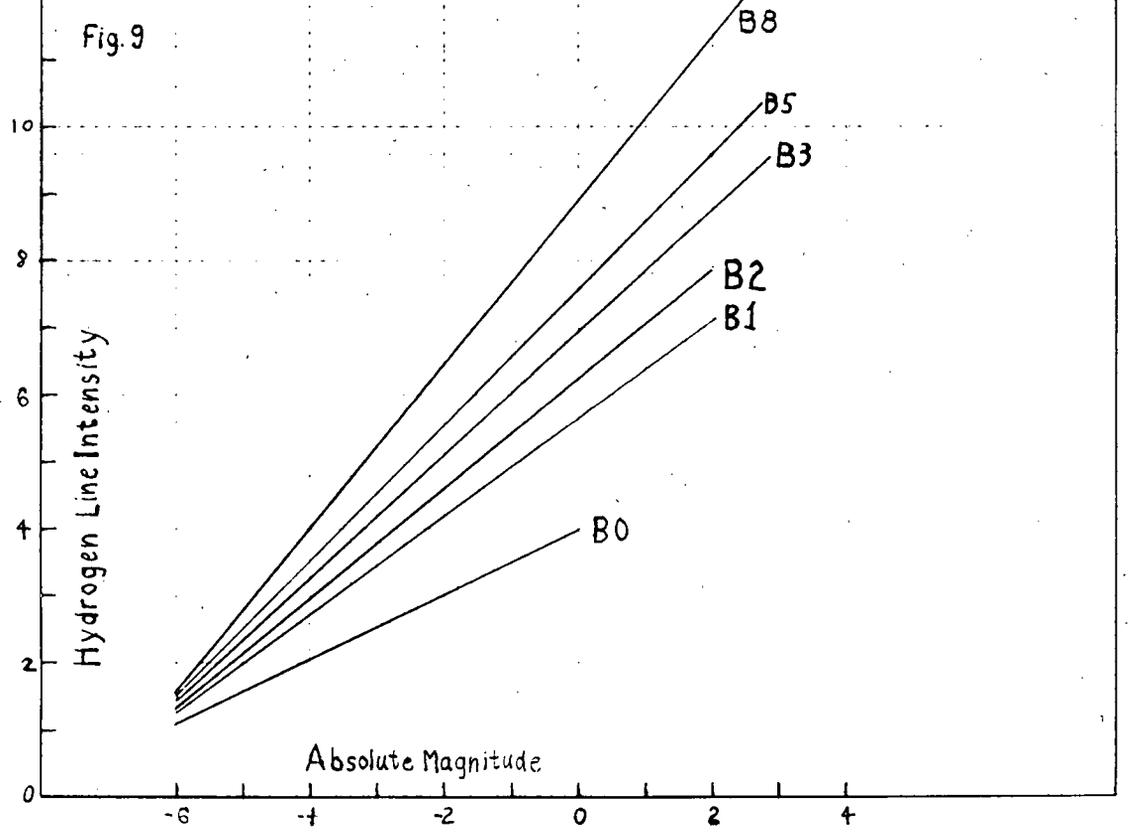
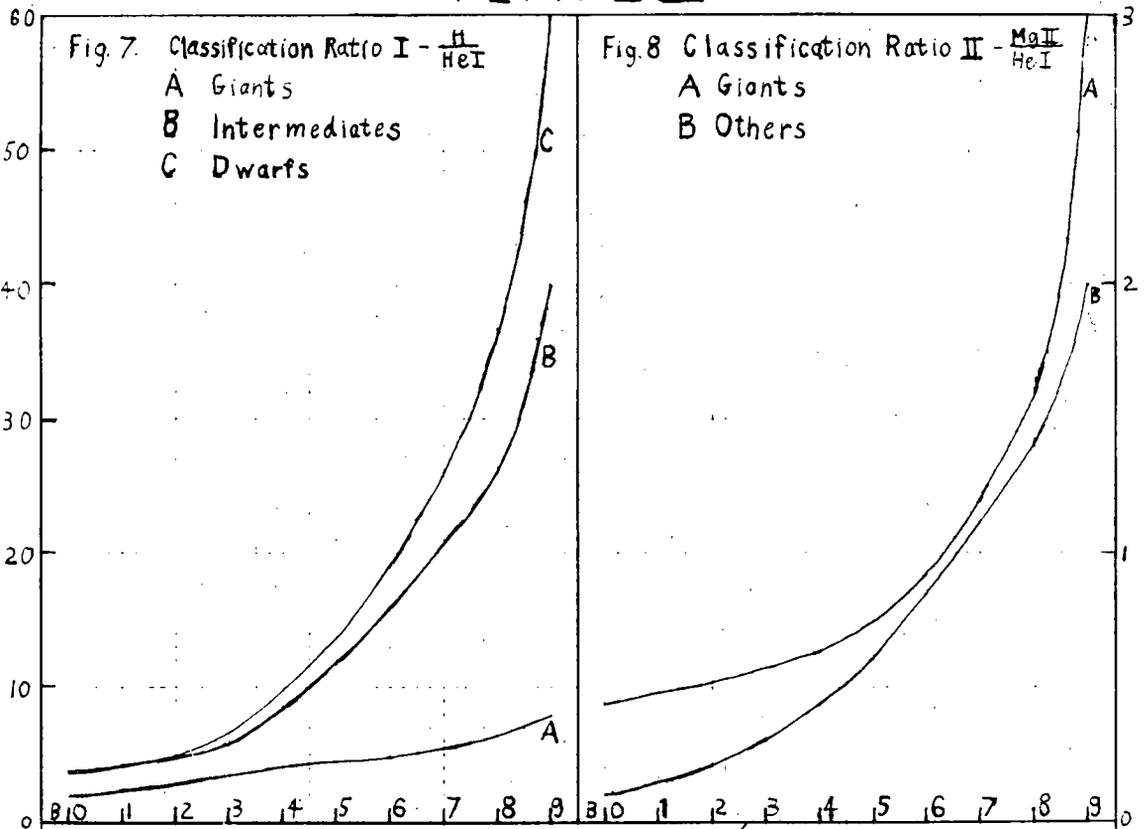
in Table IV and plotted in Plate IV, figures 7 and 8. The ratio H/HeI is the ratio of the mean intensity in equivalent Angstroms of H γ and H δ to the mean intensity of the HeI lines $\lambda 4471$, $\lambda 4026$, $\lambda 4388$, and $\lambda 4144$. The ratio MgII/HeI is the ratio of the MgII line $\lambda 4481$ to the mean of the four HeI lines. The H/HeI ratio was given greater weight in the early subtypes where the MgII line is weak.

The line depths measured along with the intensities have not been used for classification purposes because of the fact that the recorded effect is not due wholly to the true line depth but is considerably modified by the spectrograph in redistributing the light in the spectrum by diffraction effects. This varies with the spectrograph used so that criteria including such material could not be used on spectra taken with a variety of spectrographs with different dispersions.

TABLE IV - SUGGESTED TYPE CRITERIA

Type	Ratio H/HeI			Ratio MgII/HeI	
	Giants	Intermediates	Dwarfs	Giants	Intermediates and Dwarfs
B0	2.0	3.6	3.6	0.44	0.10
B1	2.4	4.4	4.4	0.48	0.16
B2	2.8	5.0	5.0	0.52	0.22
B3	3.4	6.0	6.6	0.58	0.30
B4	4.0	8.6	9.8	0.64	0.44
B5	4.4	12.0	13.6	0.74	0.62
B6	4.8	16	19	0.92	0.86
B7	5.6	21	26	1.2	1.12
B8	6.4	26	36	1.6	1.4
B9	8.0	40	60	3.0	2.0

PLATE IV



LUMINOSITY CRITERIA

The main effect of luminosity apparent in a B type stellar spectrum is the very large increase in the strength of the hydrogen lines due to Stark effect. To enable luminosities to be determined from the measurements of this program it was necessary that the stars originally used have known absolute magnitudes. As the results obtained are to be used to study galactic structure, it is desirable to use distance criteria obtained independently of consideration of galactic structure. The absolute magnitude criteria used by Williams¹ were based on distances determined by measurement of the intensity of the K lines due to interstellar calcium, which in turn he calibrated against distances determined from galactic motions. The stars chosen for this program were largely those whose distances were known from non-spectroscopic criteria. These included stars with known trigonometric parallaxes of greater than 0".010 and other stars whose parallax could be computed from group or cluster motions. For most of the stars used Dr. R. M. Petrie computed absolute magnitudes from this data and in a few cases gave the magnitude as determined from the K line intensity by using the relationship given by Evans.² For the six Pleiades stars the absolute magnitudes were kindly calculated by Dr. J. A. Pearce, using the cluster parallax as calculated by him³. The material used is naturally rather weak in the giant stars, since they are rarer and on the average are at a much greater distance and hence there are very few in the region close to the sun to which the

1. E.G. Williams, Ap. J., 83, 326, 1936.

2. J. W. Evans, Ap. J., 93, 275, 1941.

3. J.A. Pearce, JRASC, 40, 143, 1946.

choice of stars with relatively large trigonometric parallax restricts the distance.

By using the absolute magnitudes as obtained above a plot (Plate IV, figure 9) of total absorption of the hydrogen lines against absolute magnitude was made. This was used to evaluate the "measured" luminosities given.

CLASSIFICATION OF STARS

The system of classification as developed above has been applied to the stars measured, the results being included in Table II, and to the stars measured by Williams and by Petrie, for which the results are given in Table V. This table is arranged similarly to the first columns of Table II. In the last column of Table V is indicated by a W or a P whether the intensities used were measured by Williams¹ or by Petrie.²

The correlation between the measured type and the H.D. type and that between the measured type and the Victoria Revised type are shown in Plate V. The general tendency seems to be for the measured type to be somewhat earlier than the visual estimates. This is reasonable, since the frequent diffuseness of the lines, tending to obliterate the weak lines, makes the spectrum appear later on visual examination.

1. E. G. Williams, Ap. J., 83, 279, 1936.

2. R. M. Petrie, unpublished.

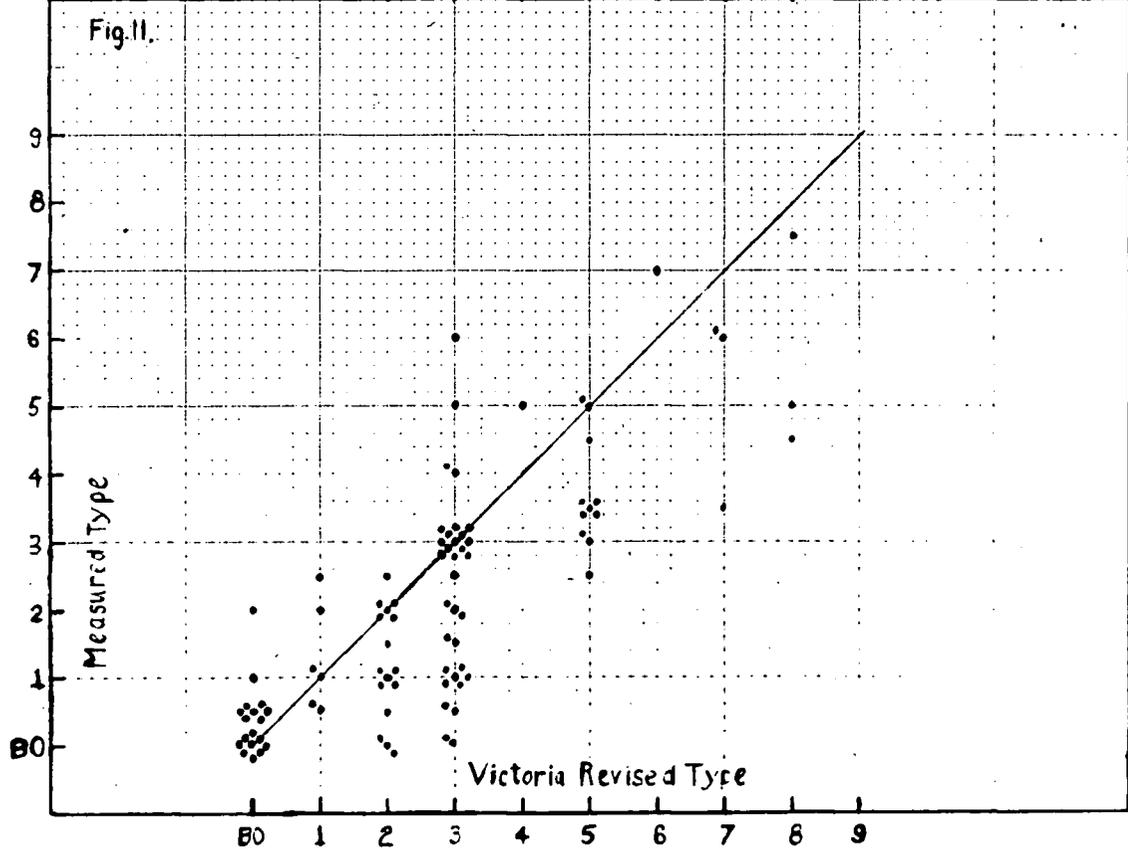
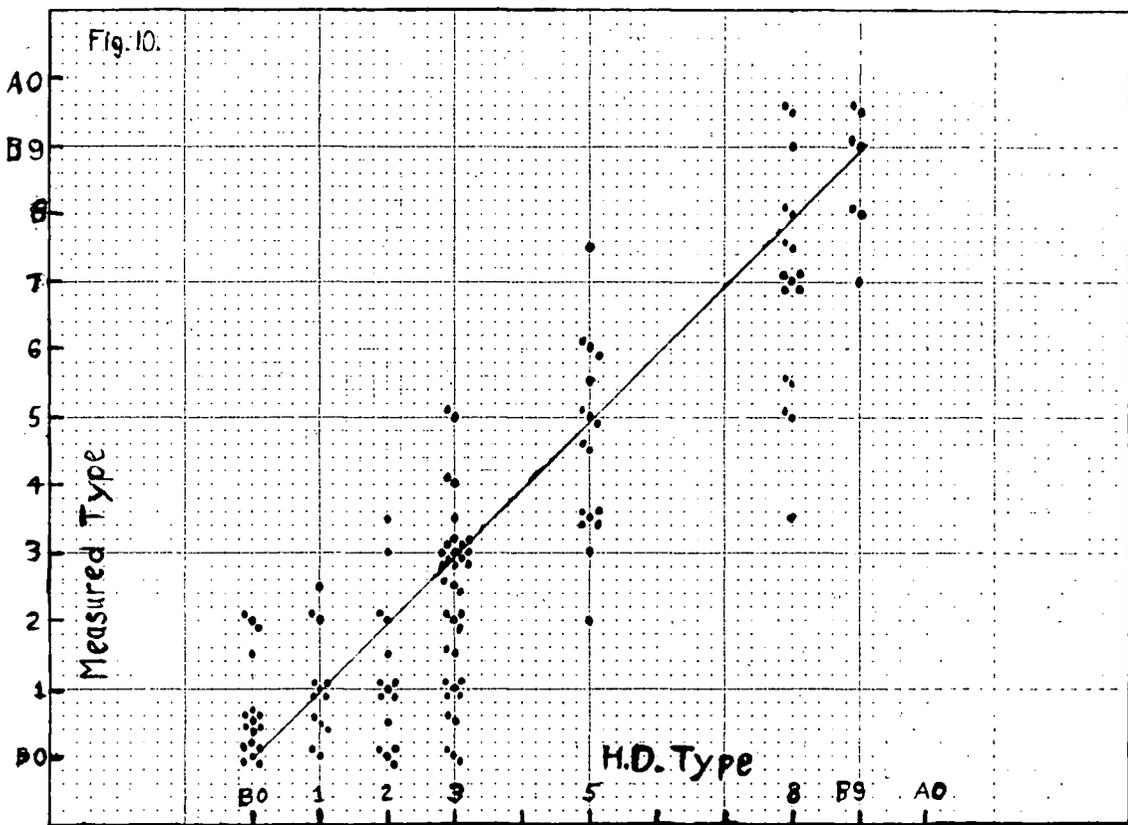
TABLE V - TYPES FOR STARS MEASURED BY OTHERS

H.D. No.	Name	Appt. Mag.	Spectral Type			Meas'd Abs. Mag.	Meas- urer
			H.D.	Vict. Revd.	Meas'd		
358	α Andromedae	2.15	A0p	-	B8	-1.1	P
698	+57°28	7.08	B5	-	B5.5	-3.5	P
886	γ Pegasi	2.87	B2	B2ss	B1	-2.0	W
1486	TV Cassiopeiae	var	B9	-	B8	+2.7	P
3360	ζ Cassiopeiae	3.72	B3	B2sk	B2	-2.0	W
4180	\circ Cassiopeiae	4.70	B2	B5n	B3	-2.6	W
4727	\rightarrow Andromedae	4.42	B3	B5s	B4	0.0	P
5394	γ Cassiopeiae	2.25	B0p	B0nne	B0.5	-	W
13854	Boss 507	6.42	B1p	B0sk	B0	-4.0	W
14134	Boss 519	6.66	B0	B2sk	B2	-5.5	W
21291	Boss 781	4.42	B9p	-	B9	-5.3	W
22928	δ Persei	3.10	B5	B8n	B4.5	-1.8	W
23480	Boss 865	4.25	B5	B7n	B6	-0.6	W
24398	ζ Persei	2.91	B1	B1s	B2	-4.9	W
24534	X Persei	var	B0p	B0nne	B0	-	W
24760f	ϵ Persei f	8.1	-	-	B9	+6	W
25833	+33°785	6.61	B3	B3k	B3	-0.4	P
28446b	l Cam b	5.86	B1	B2nk	B0	-2.7	W
28446f	l Cam f	6.61	B1	B0sk	B0.5	-1.8	W
29376	+7°676	6.89	B5	B5k	B3	-1.1	P
32343	Boss 1195	5.31	B3p	B3e	B3	-1.8	W
32630	η Aurigae	3.28	B3	B3	B2.5	-2.7	W
34085	β Orionis	0.34	B8p	-	B7	-5.7	W
34333			B1	-	B2	-2.7	P
35497	β Tauri	1.78	B8	-	B5.5	-1.1	W
36371	χ Aurigae	4.88	B1	B3ss	B2	-5.4	W
36862	λ ¹ Orionis f	5.56	Oe5	B1sk	B0	-0.1	W
37128	ϵ Orionis	1.75	B0	B0k	B0.5	-5.2	W
37202	ζ Tauri	3.00	B3p	B3e	B3	+1.6	W
38771	κ Orionis	2.20	B0	B0k	B0.5	-5.2	W
39698	Boss 1464	5.89	B2	B3	B0.5	-1.6	W
40111	Boss 1475	4.90	B2	B0	B0	-3.1	W
41117	χ ² Orionis	4.71	B2p	B2ssk	B0	-6.3	W
41534	Boss 1517	5.64	B3	B3	B1	+0.1	W
44701	-3° 1413	6.58	B5	B5n	B3.5	-0.8	P
44743	β Canis Ma	1.99	B1	B1ss	B1	-3.8	W
58050	+15° 1564	6.37	B3	B3e	B3	-1.7	W
74280	η Hydrae	4.32	B3	B5n	B1.5	-1.5	W
87737	η Leonis	3.58	A0p	-	B4	-3.1	W
87901	α Leonis	1.34	B8	B6n	B7	-0.7	W
89688	Boss 2748	6.53	B3	B3	B1.5	-1.5	W
91316	ρ Leonis	3.85	B0p	B0sk	B0.5	-4.9	W
93521	+38° 2179	6.89	B3	B3nn	B0	-2.4	W
100600b	90 Leonis b	6.0	B3	B3	B3	0.0	W
100600f	90 Leonis f	7.3	B3	B5	B5	+0.1	W

TABLE V (Cont.) TYPES FOR STARS MEASURED BY OTHERS

H.D. No.	Name	Appt. Mag.	Spectral Type			Meas'd Abs. Mag.	Meas- urer
			H.D.	Vict. Revd.	Meas'd		
120315	η Ursae Ma	1.91	B3	B3n	B3	+0.7	W
136175	U Coronae Bor	var	B8	B3	B5	+1.6	P
139892	β GC7352 b	5.07	B8	B8n	B5.5	+2.7	P
147394	ζ Herculis	3.91	B5	B7s	B3.5	+0.7	W
148184	χ Ophiuchi	4.85	B3	B3e	B0	-2.6	W
148479	α Scorpii f	6.5	A3	-	B2.5	+1	W
149438	ζ Scorpii	2.91	B0	B1s	B0.5	-2.2	W
149757	γ Ophiuchi	2.70	B0	B0nnk	B2	-4.2	W
149881	+14°3086	6.59	B2	B2k	B0	-3.2	W
156247	U Ophiuchi	var	B8	B5nk	B3.5	-0.5	P
156633	μ Herculis	var	B3	B3	B3	-0.2	P
160762	ι Herculis	3.79	B3	B3s	B3	-3.6	W
162732	Boss 4514	6.43	B8	-	B9	0.0	W
164353	Boss 4548	3.92	B5p	B8s	B6	-5.0	W
169454		6.8	B0	-	B2	-5.4	W
175227	DI Herculis	var	A	B5	B4	-0.9	P
180939	RS Vulpeculae	var	B8	-	B5	0.0	P
185507	ϵ Aquilae	5.17	B3	B3	B1	-0.1	P
187811	Boss 5068	4.91	B3	B5ne	B2.5	-1.1	W
190603	Boss 5150	5.69	B0	B0ssk	B0	-5.8	W
190967		7.92	B3	-	B0	-3.3	P
192422	+38°3956	7.10	B2	B0sk	B0	-4.7	W
193536	+45°3139	6.28	B1	B2k	B1	-1.2	P
194279	+40°4150	7.05	B0	B0sk	B0	-5.5	W
197911		7.9	B5	-	B2.5	-3.0	P
198478	Boss 5361	4.89	B2	B2sk	B1	-5.9	W
198846	Y Cygni	var	B2	B0nnk	B1	-3.2	P
199081	Boss 5375	4.68	B3	B3k	B3	+1.3	P
199140	+27°3909	6.44	B3	B1s	B2.5	-4.0	P
200120	Boss 5410	4.86	B0p	B3nne	B3	-0.4	W
204172	Boss 5512	5.84	B0	B0k	B0	-4.0	W
205021	β Cephei	3.32	B1	B1	B0.5	-1.6	W
206165	Boss 5563	4.87	B2p	B2sk	B1.5	-5.3	W
208185	+62°1992	7.7	B3	-	B1	-0.2	W
208392	+61°2216	7.10	B3	B3nnk	B1	-1.5	W
208947	+65°1691	6.28	B3	B3k	B1	+0.8	W
212455	+55°2756	8.4	B2	-	B3.5	-5.0	W
212571	π Aquarii	4.64	B1p	B1nnek	B1	-3.9	W
213420	Boss 5810	4.54	B3	B3k	B0.5	-1.6	W
216014	+64°1717	6.83	B3	B0k	B0.5	-1.1	P
218066			B5	-	B2	-2.1	P
221253	Boss 6046	4.89	B3	B3k	B2	0.0	P
224151	Boss 6142	6.05	B0	B0k	B0.5	-3.8	W
228911			B3	-	B0	+0.5	P

PLATE V



CONCLUDING REMARKS

The progress of this research has shown that satisfactory criteria for determination of spectral types and luminosity from photometrically measured line intensities can be obtained. The results above give suggested numerical values for these criteria and record some types and luminosities determined by their use. Since the reduction and measurements for thirty-nine of the forty-five stars for which line intensities are given took the greatest part of a four-month summer assistantship at the Dominion Astrophysical Observatory, it is obvious that, unless a more rapid method of obtaining intensities can be found, an application of this method to any large body of material would prove impracticable because of the time involved. However, a considerable increase in speed can be obtained by restricting the spectral regions measured to the lines being used for classification. A further saving of time can be made by reducing the microphotometer tracings to true intensity records by only one stage instead of the two stages used for most of this material. Owing to the relatively large probability of error in the measurements from a single plate it is not advisable to measure only one plate of a star, but a system by which only four lines in each stellar spectrum are measured should prove about as satisfactory as the system here suggested. The four lines suitable for this purpose are H γ at 4340, the HeI diffuse singlet at λ 4388, the HeI diffuse triplet at λ 4471, and the MgII line at λ 4481. These have the advantage that they all lie in a short region of

the spectrum, and hence one calibration curve will prove sufficient to reduce the microphotometer record for all the lines. Use of this scheme would make very little change in the intensity ratios for spectral type since in general the two hydrogen lines are of approximately equal intensity and similarly for the two helium singlets and the two triplets.

Since the process of reduction of intensity records is so time-consuming it is natural to consider the possibility of taking readings of line intensities directly from the plate as was done by Rudnick.¹ This method consists of using a photometer with a wide slit so that the complete width of the line being examined falls inside the slit. The galvanometer deflection caused by the amount of light falling on the sensitive element of the photometer can be read visually and by the use of suitable calibration the equivalent widths can be determined directly. The difficulties in this method lie in the large variation in width of the spectral lines being measured, the relatively small depths of many of them, and the fact that the reading obtained is not directly proportional to the equivalent width and behaves in such a manner that the difference from proportionality varies with the shape of the line. It is obvious that the relative effect produced by an absorption line of given equivalent width is approximately inversely proportional to the effective width of the analyzing slit of the photometer. Thus in a spectrum with sharp lines, a comparatively narrow slit can be used and the light transmitted through the photographic plate will be significantly more than that through a similar length of the continuous spectrum. Hence an accurate reading can easily be obtained. However, because of the large number of diffuse line stars, the minimum width of the analyzing slit will be that which

1. Paul Rudnick, Ap. J., 83, 439, 1936.

will include a 10 Angstrom length of the spectrum. If the maximum depth of the center of the line is only about 5% of the continuous spectrum as in the case of many of the lines measured, the change between the absorption by the portion of the plate including the line and that of a portion including only continuous spectrum will be very small, of the order of 2 or 3%. Since the errors inherent in the method (plate grain, etc.) will be related to the total amount of light transmitted by the plate while the reading desired will be the difference between two such quantities, the result will have a considerable probability of error. This does not exhaust the difficulties of this method, since to include the total width of the hydrogen lines (including the wings due to Stark effect) would require about 50 or 60 Angstroms of spectrum to be examined at once.

For these reasons it seems that the most practical system for obtaining line intensities for classification purposes is that described above, using only four lines and making reductions in only one step.

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