SOME EXPERIMENTAL CONSIDERATIONS ON THE
TRANSVERSE STERN-GERLACH EXPERIMENT FOR
FREE ELECTRONS

by

WILLIAM MARTIN VOGEL

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Department of Physics

The University of British Columbia,
Vancouver 8, Canada.

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ABSTRACT

Apparatus has been constructed to measure the g-factor of free electrons by means of a "transverse Stern-Gerlach experiment". The g-factor has not yet been measured, but preliminary experiments have been performed.

Some of the theoretical aspects of the experiment are discussed, and an order of magnitude for the effect calculated.

Construction and operation of the apparatus is described, and suggestions for improvements given.

Suggestions for a future experiment are given.

ACKNOWLEDGEMENTS

The writer gratefully acknowledges the assistance and encouragement of Dr. Myer Bloom, who suggested the experiment and supervised the writing of this thesis. Thanks are also due to Dr. Karl Erdman for suggestions regarding construction of the apparatus, and to Peter Haas who built the vacuum chamber and solenoid.

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CHAPTER I

INTRODUCTION

The problem of measuring the g-factor of the free electron has been investigated by a number of people, particularly in the last fifteen years. Previous to this the Dirac theory value for the g-factor seemed to fit all experimental evidence. In addition Bohr[1] had proven that a magnetometer experiment with free electrons was impossible, and that a conventional Stern-Gerlach experiment on free electrons could not be done. After the Second World War the availability of microwave equipment made possible experiments showing that the g-factor differed from 2 by a small but significant amount, and refinements in mathematical technique allowed the calculation of the first two radiative correction terms that account for this difference.

Most of the experimental work that has been done since then has been with bound electrons, and the free electron g-factor obtained by fairly laborious calculation. Thus, while the results have been in good agreement with the theory, it would be desirable to conduct an experiment with free electrons.

The large e/m ratio of the electron, however, makes such an experiment technically difficult, and only four experiments, including the present one, have been attempted with free electrons. Of these, only one has been successful to date.

Bloch[2] suggested using a magnetic field gradient and a very small electric field gradient to trap electrons that were in the lowest energy state ($n=0$, $s=-\frac{1}{2}$). Transitions would then be induced at either the cyclotron frequency or at the spin resonance frequency, raising the electrons to a higher energy state. The existence of the resonance would be monitored by the electrons leaving the trapping region. Preliminary experiments showed, however, that the small potential needed was too difficult to control.

Dehmelt[3] is attempting to measure the resonance frequency of the free electron in a mixture of argon and ionized sodium gases. The effect of the resonance will be a change in the rate of exchange collisions between the electrons and the oriented sodium atoms due to the resonant depolarization of the electron spins. The sodium atoms are oriented by optical pumping, and the change in exchange collision rate detected as a change in the optical density of the sodium vapor at the pumping frequency. The above effect has already been observed[3]. Obviously this experiment is not done in a vacuum, so that an experimental question remains as yet unanswered: Is it possible to extrapolate the results to zero density to obtain a value for the magnetic moment of the free electron?

The one successful experiment has been the work by

Crane et al[4,5,6] at the University of Michigan. They used relativistic electrons polarized by scattering. They then measured the polarization of the electrons after a known number of revolutions in a uniform magnetic field. Preliminary work in 1954[4] showed the feasibility of the idea, and a paper[5] in 1961 reported agreement with the first correction term. A recent paper[6] has reported refinements which have enabled them to show experimental agreement with the second correction term.

The work in progress here is an attempt to measure the resonant frequency of the free electron in a known field. The difficulty in doing this is that possible electron densities preclude normal resonance absorbtion techniques. It was suggested by Bloom and Erdman[7], however, that this difficulty might be overcome by inducing transitions using the interaction between the electron magnetic moment and an <u>inhomogeneous</u> oscillatory magnetic field (the "transverse Stern-Gerlach experiment"). This interaction would produce transitions involving changes in both the cyclotron and spin quantum numbers. The resonance would then be detectable as a resonant change in the dimensions of a beam of electrons. A brief description of some of the theoretical aspects of the transverse Stern-Gerlach experiment is given in Chapter 2.

A successful transverse Stern-Gerlach experiment has not yet been performed, but an apparatus has been designed and constructed for this purpose. Chapter 3 gives some of the considerations leading to the present design and gives a complete description of the apparatus as it exists now.

Chapter 4 gives suggestions which may be helpful in future work on this type of experiment.

CHAPTER II

THEORY

The energy level diagram for an electron in a uniform magnetic field B is shown in Figure 1. The energy levels are given in terms of a cyclotron quantum number n and spin quantum number m.

$$E_{nm} = (n + \tfrac{1}{2})\hbar\omega_c + m\hbar\omega_L + E_0 \qquad (1)$$

where $\omega_c = \dfrac{eB}{mc}$ (in Gaussian units) is the cyclotron frequency

$\omega_L = \dfrac{geB}{2mc}$ is the Larmor frequency and $g = 2(1 + a)$ is the g-factor of the free electron.

E_o is that part of the energy independent of n and m, namely the kinetic energy associated with motion parallel to the field.

The eigenfunctions are products of spatial wave functions (Landau wave functions[8]) and spin functions. These spin functions are eigenfunctions of S_z, the component of electron spin along $\overline{B}$, where $\overline{B}$ is assumed to be oriented in the z-direction.

One can now define three types of resonances, two of them well known, and the third corresponding to the transverse Stern-Gerlach experiment.

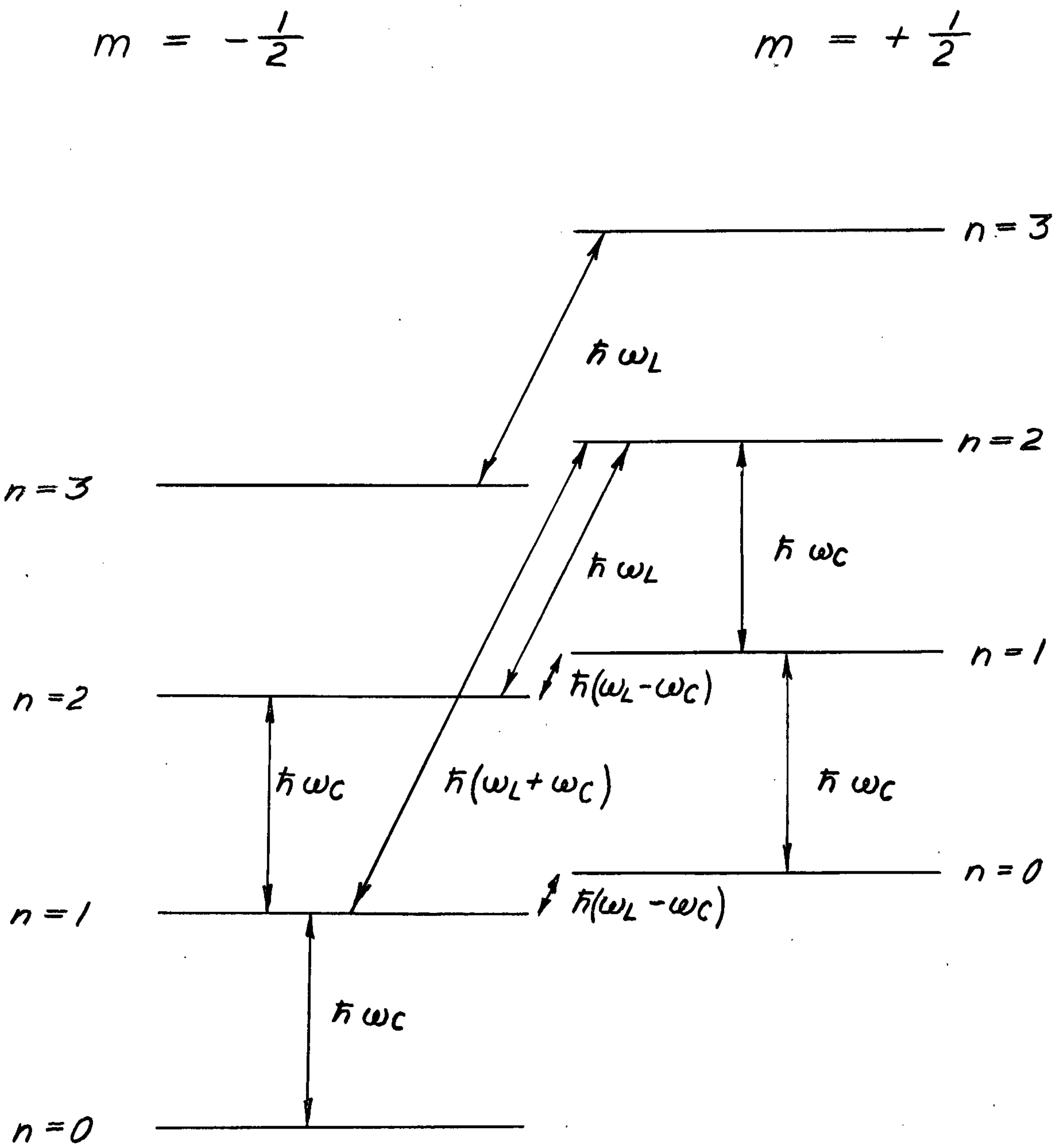


Fig. 1. Energy Levels of Electron in Magnetic Field

One resonance is produced by an oscillatory uniform electric field $E_x \cos \omega t$. For this the perturbation Hamiltonian is

$$\mathcal{H}' = e x E_x \cos \omega t \tag{2}$$

Since the Landau wave functions are similar to those for a linear harmonic oscillator*, there are non-vanishing matrix elements only for transitions where $\Delta n = \pm 1$, $\Delta m = 0$. These transitions correspond to the cyclotron resonance at $\omega = \omega_c$.

The second resonance is produced by an oscillatory uniform magnetic field $H_x \cos \omega t$, for which the perturbation Hamiltonian is

$$\mathcal{H}' = \mu_x H_x \cos \omega t \tag{3}$$

This is independent of the space coordinates and thus affects only the spin dependent part of the wave function. Thus the only non-vanishing matrix elements involve transitions where $\Delta m = \pm 1$, $\Delta n = 0$. These transitions correspond to the

* Actually there are two quantum numbers, n and k, required to specify the Landau wave functions[8]. The energy depends only on n, but for each n, upon which, crudely speaking, the cyclotron radius depends, there may be very many possible values of k upon which, again crudely speaking, the position of the center of the cyclotron orbit depends.

electron spin resonance at $\omega = \omega_L$.

The third type of resonance is the one that this experiment is trying to use. It is produced by an oscillatory non-uniform magnetic field of the form

$$\bar{H}(\bar{r}) = \left[\bar{H}(0) + (\bar{r}\cdot\bar{\nabla})\bar{H}_1 + \text{(higher order terms)}\right]\cos\omega t \qquad (4)$$

and the perturbation Hamiltonian is

$$\mathcal{H}' = \bar{\mu}\cdot\bar{H}(\bar{r}) \qquad (5)$$

Since the perturbation is linear in $\bar{\mu}$, one can only obtain changes in the m quantum number of ± 1. Depending on the non-vanishing terms in the expansion of $\bar{H}(\bar{r})$, one can get any integral change in the n quantum number, i.e. the transverse Stern-Gerlach experiment corresponds to resonances at $\omega_L \pm p\omega_c$, where p = 0, ± 1, ± 2.. For a field of 100 gauss the resonant frequencies would be: $\frac{\omega_L}{2\pi}$ = 280.03 Mc., and $\frac{\omega_c}{2\pi}$ = 279.71 Mc.

It had been originally planned to induce transitions at $\omega_L - \omega_c$, as in this case the resonance would be a measure of the amount that g differs from 2 ($\omega_L - \omega_c$ = 320 kc. in a field of 100 gauss). However, calculations by Bloom[9], and by Rastall[10] have shown that to first order perturbation theory there is zero transition probability for a perturbing field of experimental interest. Consequently the experiment should be attempted at ω_L, $\omega_L + \omega_c$, etc. The accuracy to which such a frequency must be known must be much higher than for $\omega_L - \omega_c$ to achieve the same accuracy for the g-factor, but this can be largely overcome by simultaneous measurements of ω_L and ω_c, and by measuring the beat frequency $\omega_L - \omega_c$.

Calculation of the behavior of a beam of electrons travelling in a uniform magnetic field and subjected to an alternating field gradient appears very difficult. Attempts to calculate the quantum mechanical behavior of such a beam seems extremely complex when the beam radius is large compared with the radius of the lowest cyclotron orbit. It is plausible, however, that such a beam should behave classically, and the following discussion is confined to classical electrons.

The non-relativistic equations of motion for a particle of charge e, mass m and magnetic moment $\overline{\mu}$ in a magnetic field $\overline{B}_o + \overline{B}_1(\overline{r},t)$, where $\overline{B}_o$ and $\overline{B}_1(\overline{r},t)$ are time-independent uniform and time-dependent inhomogeneous magnetic fields respectively, are

$$m\ddot{\overline{r}} = \frac{e}{c}\dot{\overline{r}} \times (\overline{B}_o + \overline{B}_1) + (\overline{\mu}\cdot\overline{\nabla})\overline{B}_1 \tag{6}$$

$$\dot{\overline{\mu}} = \frac{ge}{2mc}\overline{\mu} \times (\overline{B}_o + \overline{B}_1) \tag{7}$$

In general $|B_1| \ll |B_o|$. A solution can be given if $\bar{B}_1$ is neglected in (7) and in the Lorentz force term in (6). The only justification that can be given for this approximation is that although the term $\left|\frac{e\dot{\bar{r}}}{c} \times B_1\right| \gg \left|(\bar{\mu}\cdot\bar{\nabla})\bar{B}_1\right|$, the latter term will produce resonance effects over a narrow frequency interval while the first term, although larger, will produce slowly varying effects as the frequency is changed in the vicinity of the resonance. The assumption that the nature of the resonance is not affected appreciably by the neglected term is made on intuitive grounds and not rigorously justified. It is potentially the weakest part of the present arguments.

With these approximations the solutions to (7) are

$$\mu_z(t) = \mu_z(0) = \mu\cos\theta$$

$$\mu_x(t) = \mu\sin\theta\cos(\omega_L t + \phi)$$

$$\mu_y(t) = \mu\sin\theta\sin(\omega_L t + \phi) \tag{8}$$

where θ, ϕ are the initial polar and azimuthal angles of $\bar{\mu}$ respectively, and $\bar{B}_o = B_o\bar{k}$, $\omega_L = \frac{geB_o}{2mc}$.

Equation (8) can now be substituted into the modified form of (6)

$$m\ddot{\bar{r}} \simeq \frac{e}{c}\dot{\bar{r}} \times \bar{B}_o + (\bar{\mu}(t)\cdot\bar{\nabla})\bar{B}_1(\bar{r},t) \tag{9}$$

The first term in (9) alone will produce cyclotron motion, i.e. motion in a circle with uniform circular velocity $\omega_c = \frac{eB_o}{mc}$.

The second term will produce small and unobservable effects except, it is hoped, near the "transverse Stern-Gerlach" resonances.

It is convenient at this point to work out these equations for two special cases. Case A is a two-dimensional quadrupole field, and Case B is a two-dimensional octupole field, with sinusoidal time dependence in each case.

Case A:

$$\bar{B}_1(\bar{r},t) = G_1 \cos\omega t \,(x\,\bar{i} + y\,\bar{j}) \tag{10}$$

A configuration of current elements producing such a field near the origin is shown in Figure 2.

The second term in the R.H.S. of (9) becomes

$$\bar{F} = F_x\,\bar{i} + F_y\,\bar{j} \tag{11}$$

where

$$F_x = \mu G_1 \sin\theta \cos(\omega_L t + \phi) \cos\omega t \tag{12}$$

$$F_y = \mu G_1 \sin\theta \sin(\omega t + \phi) \cos\omega t \tag{13}$$

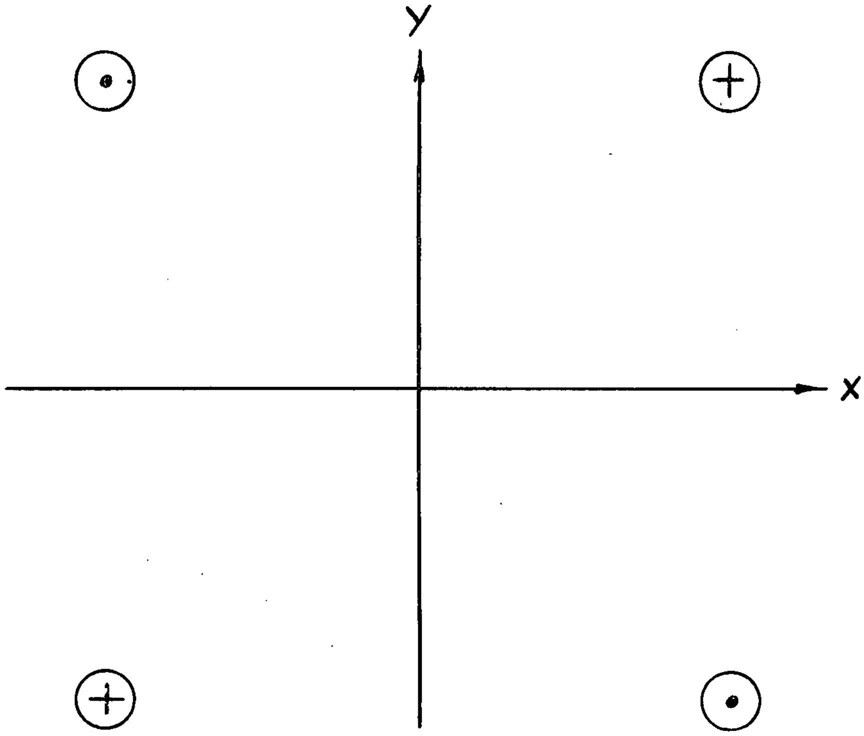


Fig. 2. Quadrupole Current Configuration

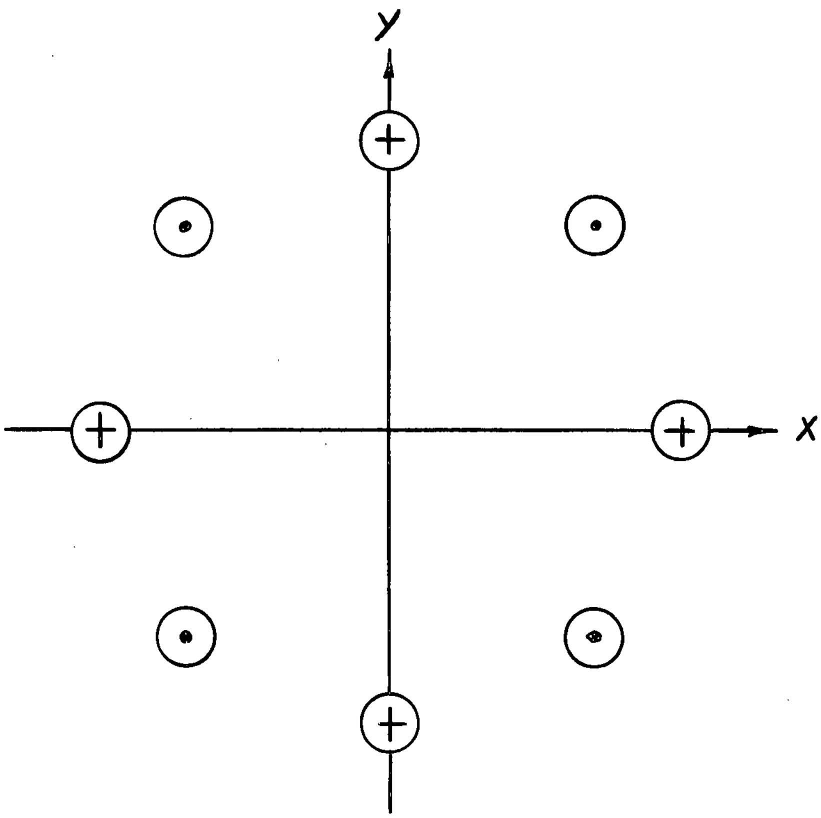


Fig. 3. Octupole Current Configuration

Clearly, the average value of these is non-zero only when $\omega = \omega_L$. If ϕ is zero the average value of the force is $\mu G_1 \sin\theta$ in the x direction. Solution of the equations give, at resonance, the cyclotron motion unchanged in a coordinate system moving with a uniform velocity

$$\dot{x} = 0 \, , \quad \dot{y} = \frac{\mu \sin\theta G_1}{m \omega_L} \tag{14}$$

Therefore, this resonance corresponds to a displacement of the cyclotron orbits but not in a change of radius. Quantum mechanically, this would correspond to a change in the k quantum number but not in n.

Case B:

$$\bar{B}_1(\bar{r},t) = G_2 \cos\omega t \left[(x^2 - y^2)\bar{i} - 2xy\bar{J} \right] \tag{15}$$

This can be produced by the current configuration shown in Figure 3. This case gives the Stern-Gerlach force in a similar manner to the above, but the forces depend on x and y.

If to a zeroth order approximation the particles are assumed to move in cyclotron orbits given by

$$x = x_0 + r\cos\omega_c t \tag{16}$$

$$y = y_0 + r\sin\omega_c t \tag{17}$$

the Stern-Gerlach force is given by (for $\phi = 0$)

$$F_x = \mu \sin\theta G_1 \begin{bmatrix} x_0[\cos(\omega_L+\omega)t + \cos(\omega_L-\omega)t] \\ -y_0[\sin(\omega_L+\omega)t + \sin(\omega_L-\omega)t] \\ +r[\cos(\omega_L+\omega_c+\omega)t + \cos(\omega_L+\omega_c-\omega)t] \end{bmatrix} \tag{18}$$

$$F_y = \mu \sin\theta G_1 \begin{bmatrix} y_0[\cos(\omega_L+\omega)t + \cos(\omega_L-\omega)t] \\ +x_0[\sin(\omega_L+\omega)t + \sin(\omega_L-\omega)t] \\ +r[\sin(\omega_L+\omega_c+\omega) + \sin(\omega_L+\omega_c-\omega)t] \end{bmatrix} \tag{19}$$

From this it can be seen that there will be a non-zero average value for the force only for $\omega = \omega_L$, and $\omega = (\omega_L + \omega_c)$.

For the quadrupole field an order of magnitude for the change in radius of the beam is $\frac{\mu G_1 \tau}{m \quad L}$, where τ is the time the electrons spend in the quadrupole field. This value is in agreement with that derived by Bloom[7] using energy considerations. For a field of 100 gauss, $\tau = 10^{-6}$ sec. and G = 10 gauss/cm.; the approximate change in radius is 10^{-7} cm.

CHAPTER III

APPARATUS

The small cyclotron radius of the electrons considered for this experiment (10^{-3} cm.) enabled the construction of a solenoid of large L/D while still being physically manageable. The diameter and length chosen were 2 inches and 1 meter respectively, giving an L/D of 20. Theoretically this gives a homogeneity of 1 part in a thousand over the 40 cm. center section chosen as the region where the inhomogeneous perturbing field would be set up. The choice of a small diameter test region necessitated that the apparatus producing the inhomogeneous field be slim, and with long leads. Also the electron source and the electron detector, which must be bigger than the solenoid diameter, had to be placed outside the solenoid. These considerations set the basic shape of the apparatus.

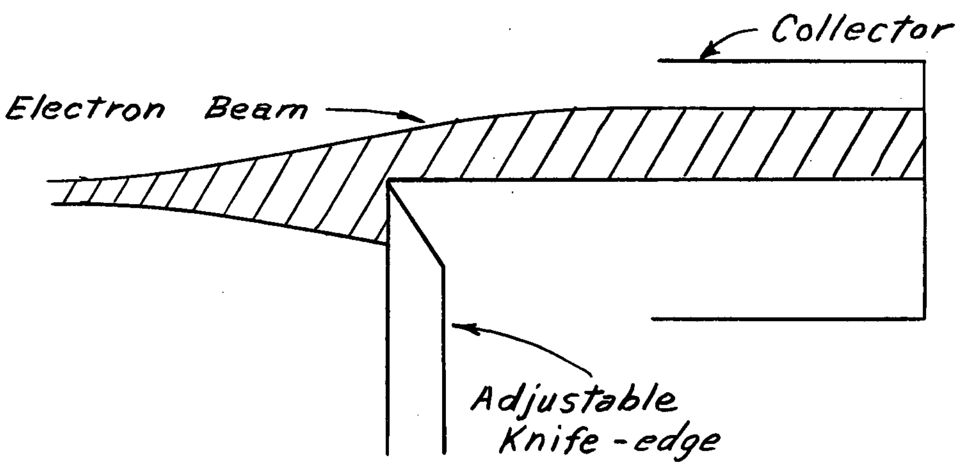
It was decided to use a quadrupole perturbing field produced by four wires arranged as in Figure 2, and 40 cm. in length.

All equipment in the source and detection chambers was designed to mount on the end plates for ease of construction.

As the end chambers are necessarily mounted outside the solenoid, they are in a weak field (essentially the earth's field). For this reason the apparatus was mounted parallel to the earth's field, and all possible magnetic material was excluded from this region of the apparatus. The electronics, with steel chassis and transformers, are kept at the other end of the aluminum equipment frame. The only exceptions to this are the micrometer screws used to adjust the source and collector.

Having the electron source and detector outside the solenoid, although dictated by design, has proven very useful as it produces an amplification in the size of the beam and any changes in it. The beam is approximately 100 times smaller in a 100 gauss field than in the earth's field. Thus a beam .5 cm. in diameter shrinks to 50 microns in the test region, and then expands to .5 cm. again in the detection chamber. Figure 4 shows how changes in radius of this expanded beam are to be detected with a knife edge cutting off part of the beam.

For the figures given at the end of the last chapter which predict a change in electron cyclotron radius of 10^{-7} cm., the fractional change in radius in the test region is of the order $\frac{10^{-7}}{5 \times 10^{-3}} \simeq \frac{1}{50,000}$. In the following chapter some suggestions for improvement of the present apparatus will be made, including a method for increasing the time spent in the test region, which would improve this ratio. The fractional change in radius could also be improved by using a smaller beam diameter.



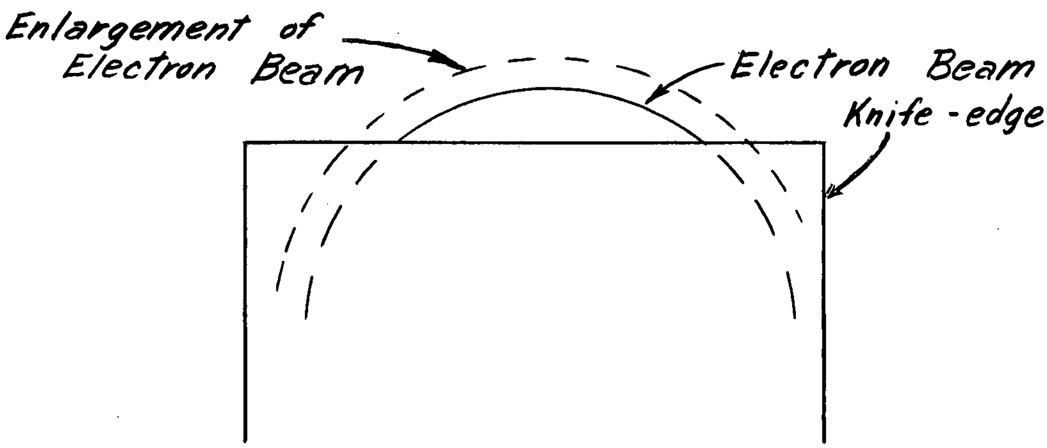


Fig. 4. Collector Knife-Edge

It was decided that it would be easier to do an A.C. experiment, rather than a D.C. one because the experiment consists basically of detecting small changes in a current. Also, the techniques used, such as phase sensitive detection, narrow banding, and high gain amplifiers are well developed and relatively easy to handle. The field gradient would be applied with the quadrupole lens at the cyclotron, or Larmor, frequency of the electrons, and F.M. modulated at some convenient audio frequency. (Alternatively, the longitudinal magnetic field could be modulated.) This should produce some change in current collected in the anode. The A.C. component of this current is fed to an amplifier, and then treated by standard techniques. The general shape of the apparatus is indicated in Figures 5, 7, and 8.

The vacuum chamber and system is outlined in Figure 6. As noted, the vacuum chamber is in two pieces, joined by two O-ring seals. This is to make it possible to separate the vacuum system from the solenoid that fits over the central pipe. The outside vacuum line is needed because the quadrupole lens fills most of the central pipe, and it would be difficult to achieve a good vacuum in one of the chambers without it. The water line supplying cooling for the diffusion pump, solenoid, and transistors is controlled by a solenoid valve and monitored by a pressure switch. In case of a failure of the water supply or bursting of a hose, the water and power supplies for the apparatus are shut off.

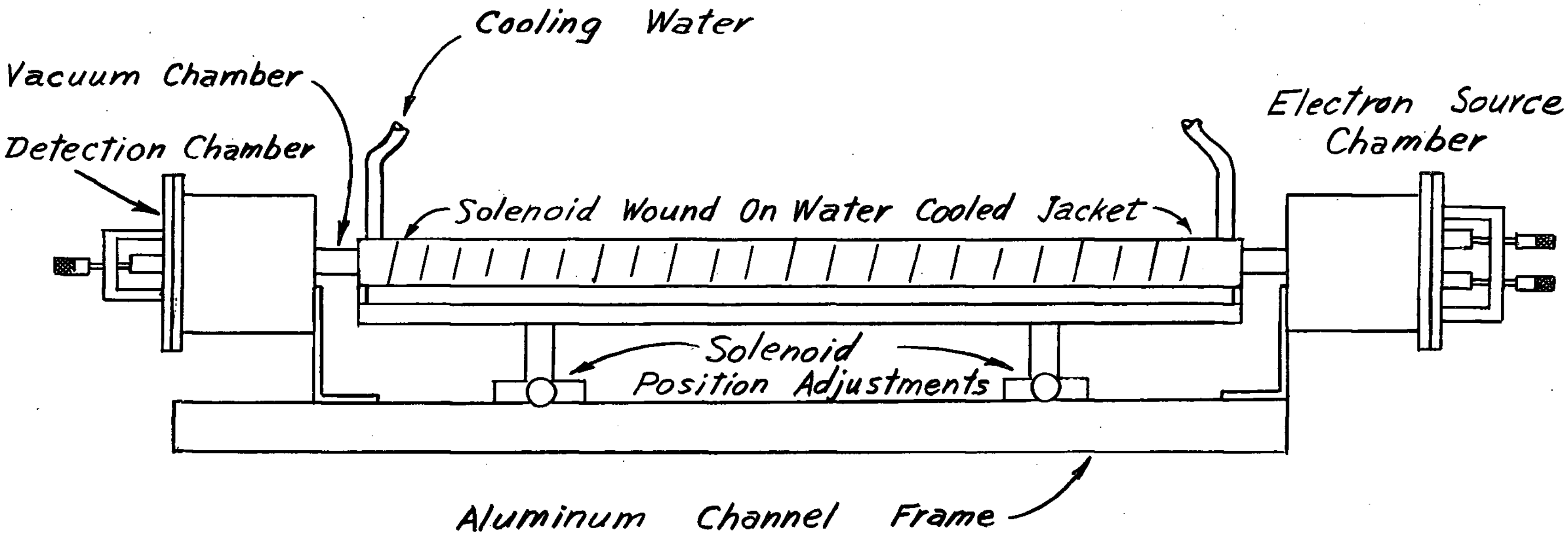


Fig. 5. General View of Apparatus

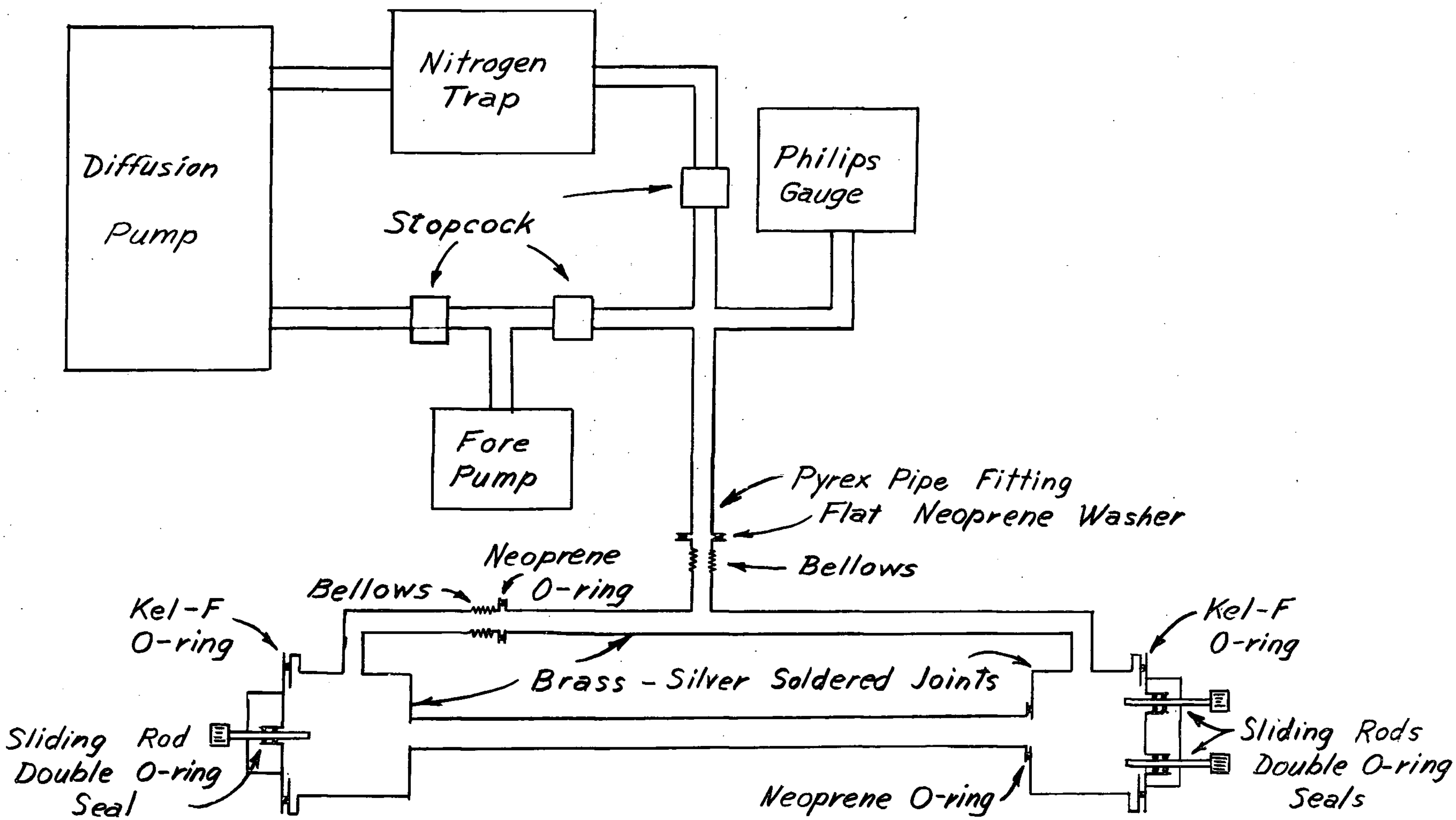


Fig. 6. Vacuum System

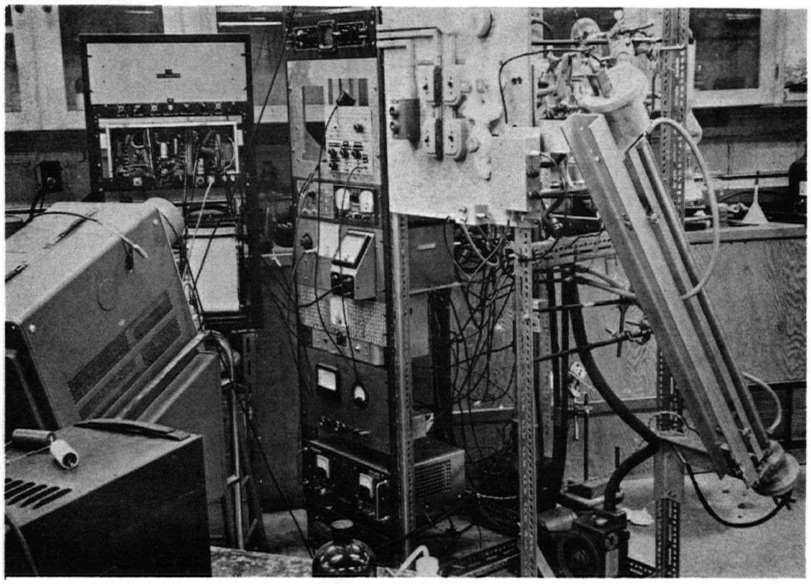


Fig. 7. General View of Equipment Rack

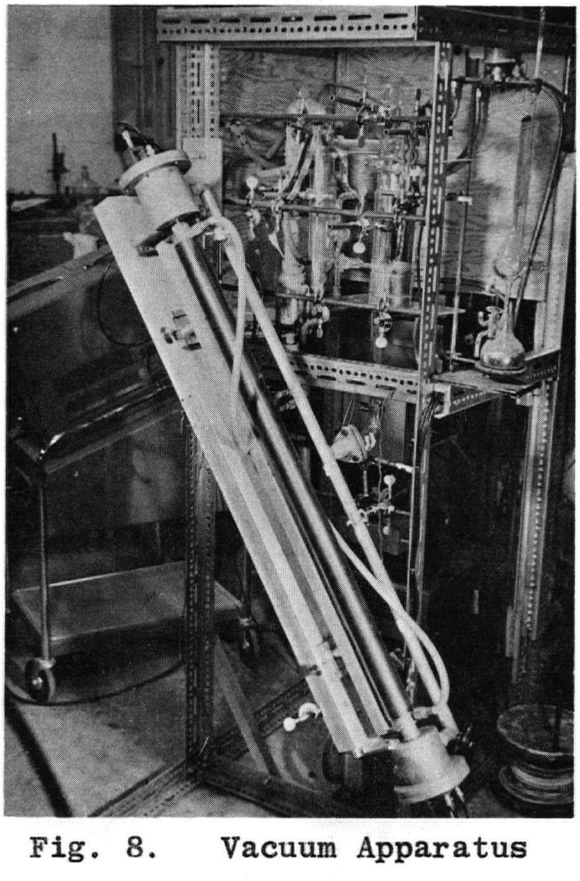


Fig. 8. Vacuum Apparatus

To avoid excessively long pumping times, it is necessary to bake out the system whenever any new equipment is placed into the vacuum chamber. In all cases, baking decreases the pumping time. The two end chambers were heated by two industrial heat lamps and aluminum reflectors. However, the limited space between the central tube and the solenoid precluded the use of a resistance wire coil for heating. Consequently a surplus 2 K.V.A. pole transformer was rewound to produce 1400 amps at 1.2 volts with 130 volts, 20 amps input. This was used to resistance heat the central tube, using the aluminum channel frame for a return line. With 70 volts on the primary from a Variac, the tube maintains a temperature of 170°C. This is as high as can be safely used, as the melting point of the solder used on some parts of the quadrupole lens is about 180°C. The heat lamps maintain the end caps at about 125°C. The temperature is monitored at five points on the apparatus by copper-constantan thermocouples.

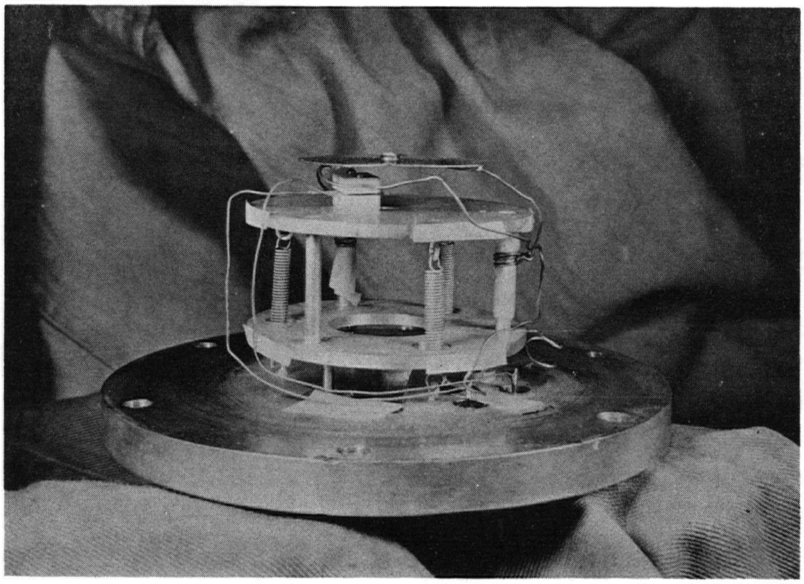
When new equipment is to be outgassed it takes about 4-5 days of baking to achieve a vacuum of 10^{-5} mm.Hg. After the original baking, 24 hours usually suffices.

All surfaces in the vacuum chamber with the exception of the accelerating anode, the collecting anode, and the quadrupole wires are coated with 'dag' dispersion No. 154, manufactured by the Acheson Colloid Co., a dispersion of graphite in alcohols. This material has been found to be very effective in reducing surface charge effects in ion beam apparatus. As well as being coated, all movable parts in the chamber are connected to ground with coated wires to reduce the possibility of charges building up.

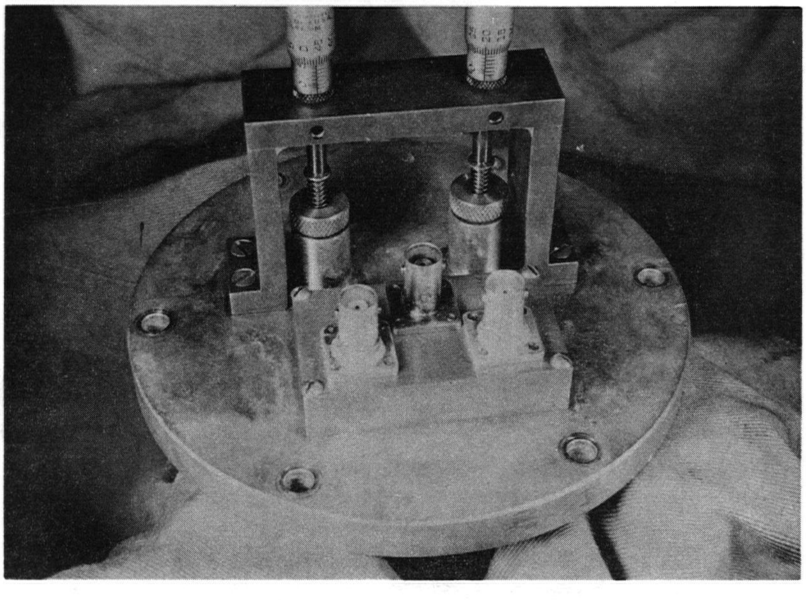
The beam is produced by a tungsten hairpin filament, .003 inches in diameter, heated by D.C., and which can be made negative with respect to ground. To reduce space charge effects near the filament and to shape the beam, an accelerating anode is placed in front of the filament. The filament and anode can be moved at right angles to the axis of the apparatus by two micrometer screws working through vacuum seals. In this way the beam can be adjusted to go down the center of the quadrupole lens. The electrical leads for the filament, anode, collector, and quadrupole lens come in through kovar seals, three in each end.

In the detector end, there is a knife edge, adjustable by means of a micrometer screw, which cuts off part of the beam and keeps it from reaching the collector cup. Also in this end are the two lead-ins for the quadrupole lens. See Figures 9 and 10 for pictures of the end plates.

The quadrupole lens is made of brass and copper rods, with spacers of Lava, a machinable ceramic. The quadrupole wires are separated from the beam by a dag-coated glass tube. This coating must meet two opposing requirements. It must be much thinner than the skin depth at the operating frequency, and its resistance from one end of the test

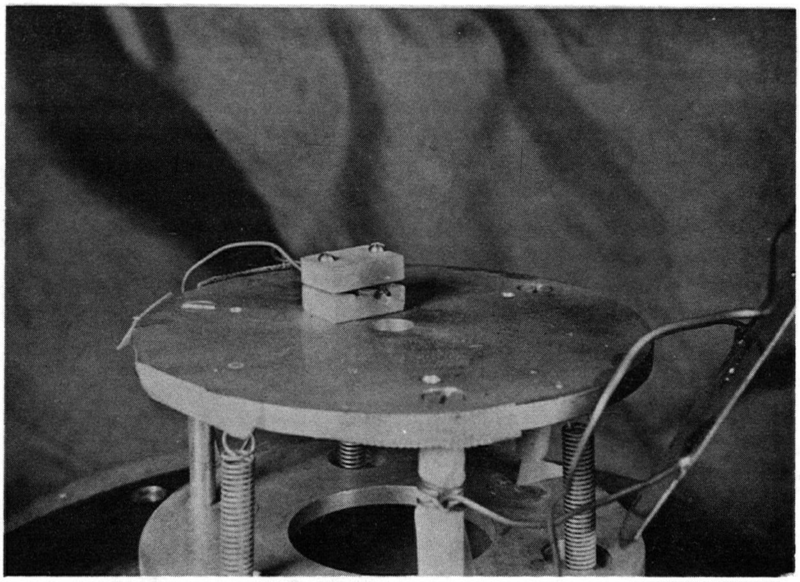


Top Side of Plate



Bottom Side of Plate

Fig. 9. Source End Plate



Close up of Filament at Source End

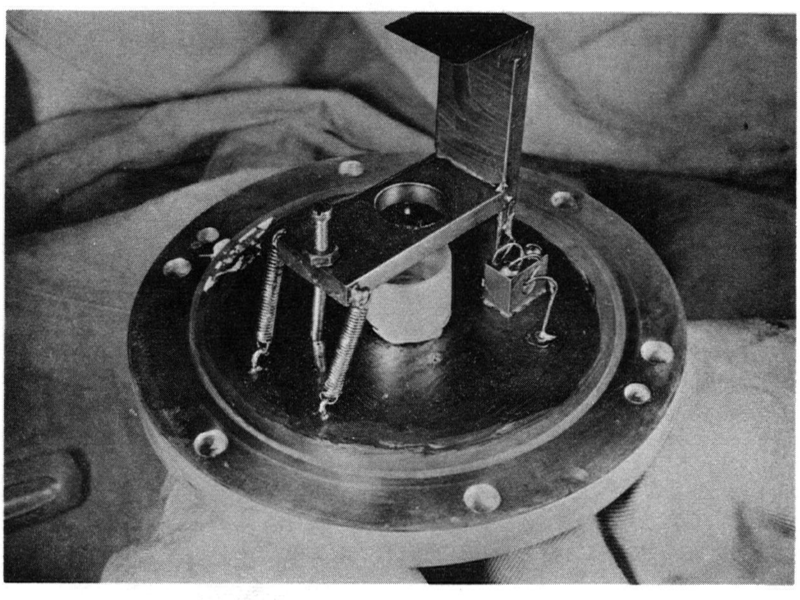


Fig. 10. Collector End Plate

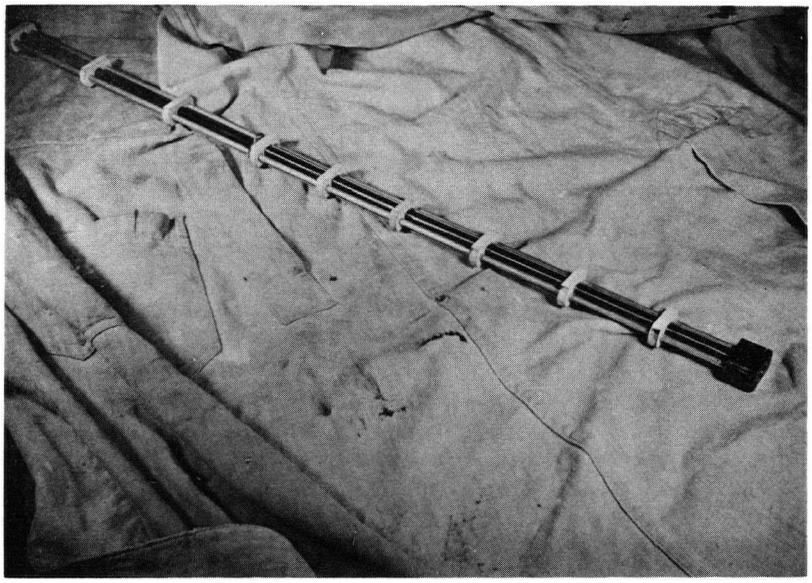
chamber to ground multiplied by the capacitance to ground must be less than the period of the R.F. applied. The first requirement is to let in the R.F. magnetic field, and the second is to screen out the effects of the potential between the quadrupole wires as a result of the current flow, which may affect the electrons directly through their electric charge. Figure 11 is a picture of the lens, and Figure 12 is an enlarged drawing of a section of the lens.

The electonics are in three sections. One section supplies currents and voltages for the electron source. Another supplies the regulated current for the solenoid, and the third section detects, amplifies, and displays the resulting changes in the beam. See block diagram in Figure 13.

The electron source electronics are fairly straightforward. Filament current should be set to approximately 1.1 amps. It may be possible to use higher currents than this but no higher has been used. It might be advisable to burn out a filament to see how much current it will take.

When adjusting the voltages for maximum beam current, considerable interaction was found between anode voltage and filament bias. Generally anode voltages of about 50 volts produced the maximum current. Higher voltages usually lowered the beam current and also produced local heating of the anode, which was detrimental to the vacuum.

The solenoid current regulator is similar to that by Patlach[11]. However, some changes have been made to the



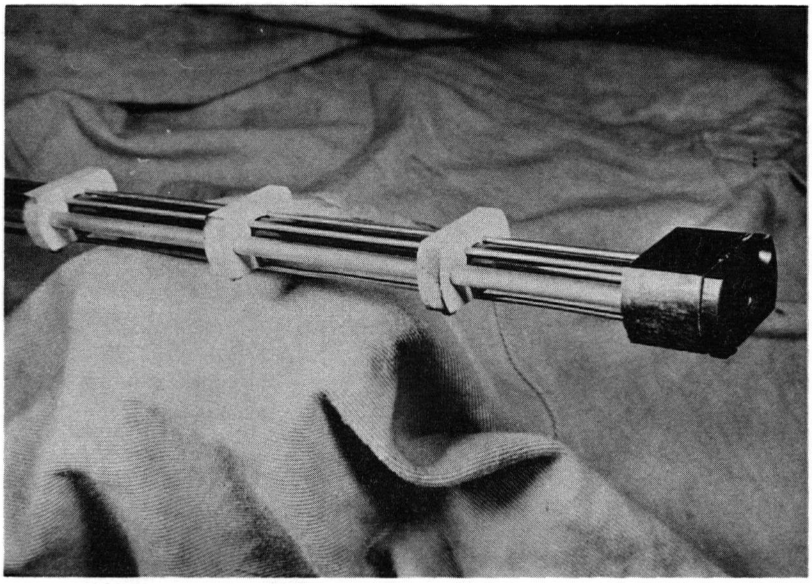


Fig. 11. Views of Quadrupole Lens

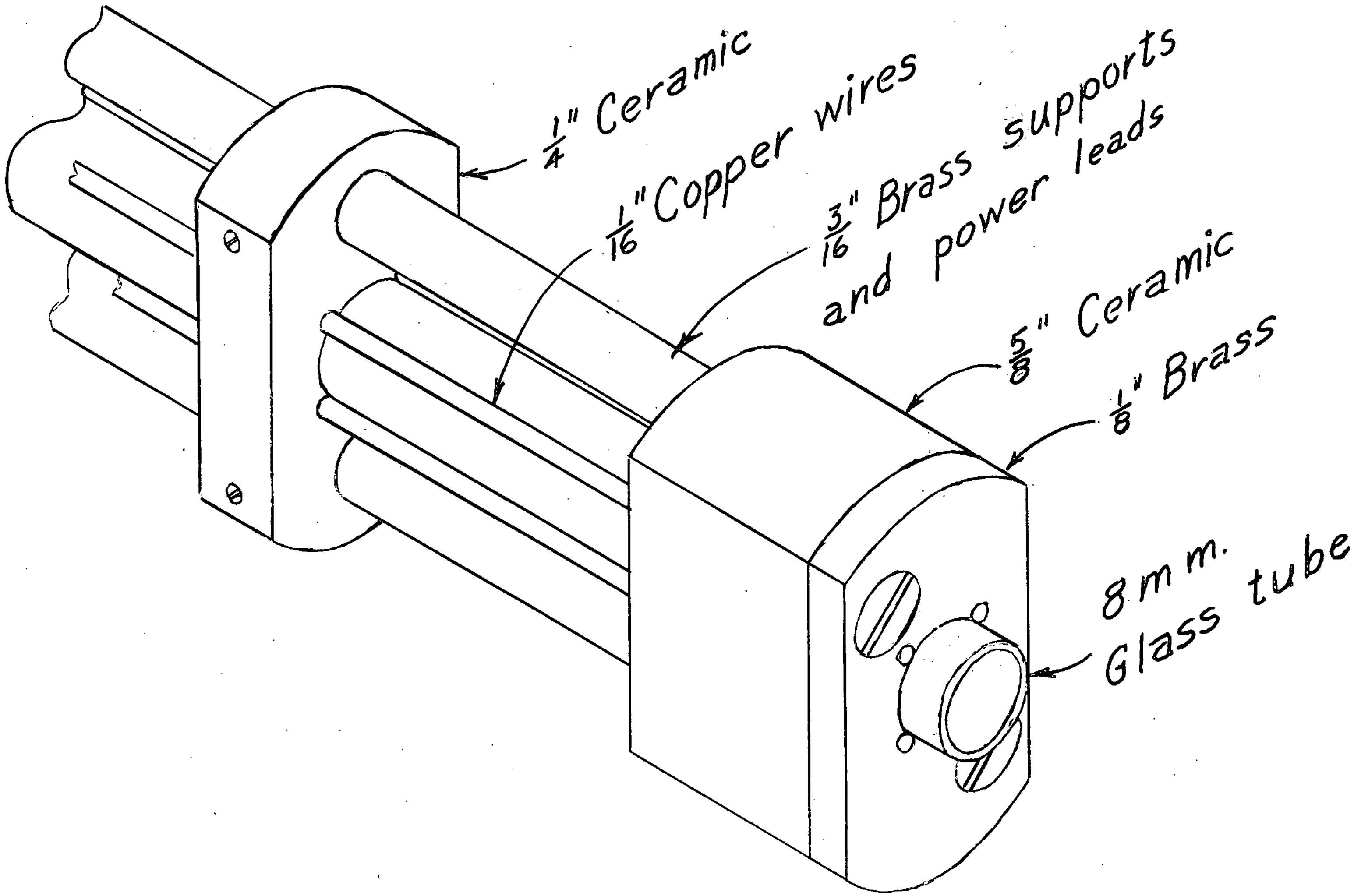


Fig. 12. One Section of Lens

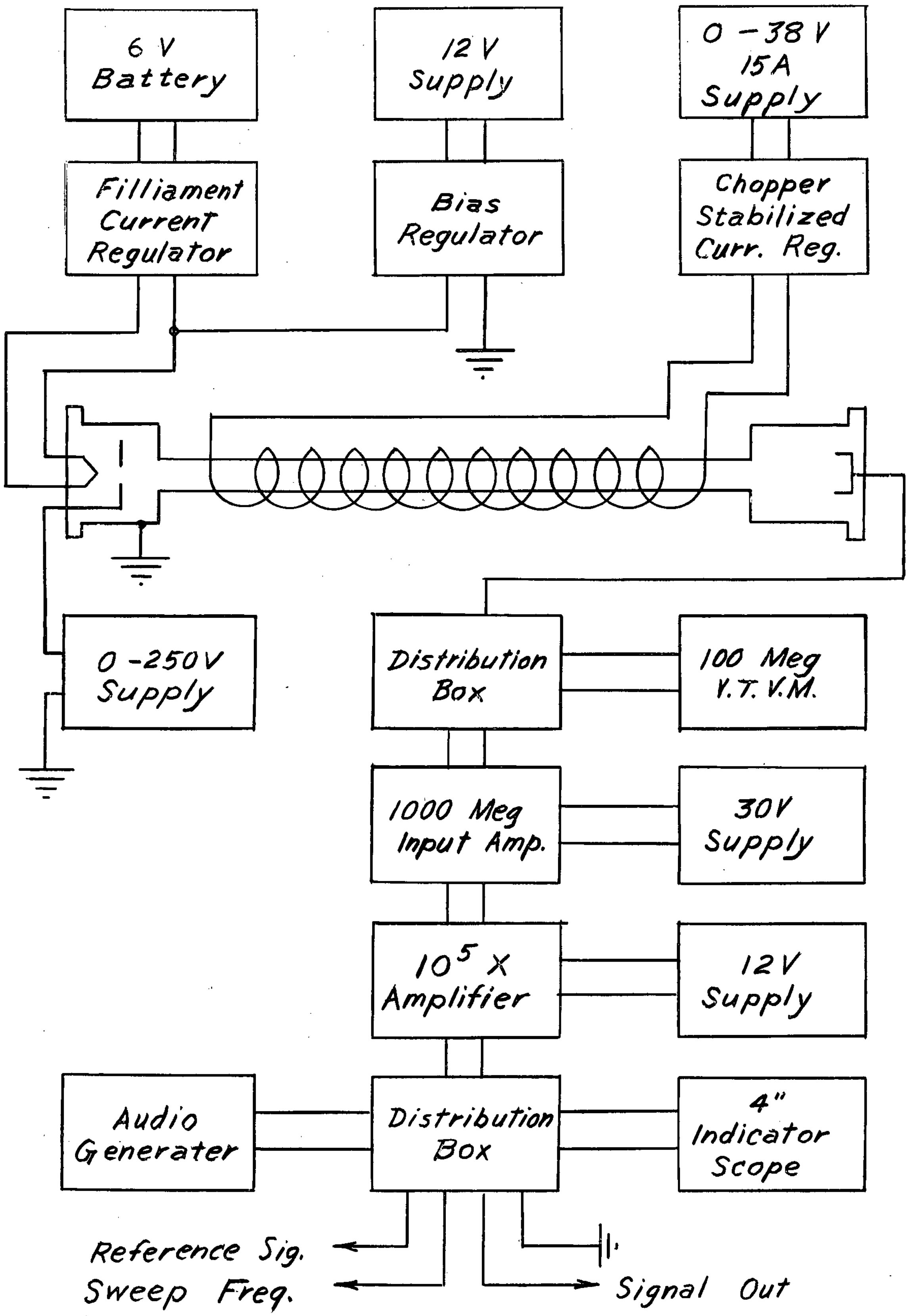


Fig. 13. Block Diagram of Electronics

Brown null indicator, and the A.C. amplifier is of a different design (see Figure 14). At present there is no meter indicating voltage across the pass transistors. This should be added as the transistors tend to overheat when voltages of over 5 volts are put across them at high currents. This is caused by poor heat sink design and could be corrected by some structural modifications. At present the reference supply is a mercury cell. However, the plug-in unit could be easily replaced by an A.C. powered model if complete freedom from batteries was desired.

Phase shift somewhere in the A.C. amplifier produced oscillations at about 14 kc., so the amplfier open loop gain had to be brought below one by the time it got to that frequency. As a consequence there is considerable high frequency noise voltage on the solenoid, and due to the low inductance of the solenoid (200 microhenries), it appears as an appreciable noise current. It has not yet been determined if this is going to cause any difficulty. If it does, a high current choke of about 1 henry in series with the solenoid should clear it up.

The beam current, if allowed to impinge on a collector that is at a slight positive potential, should be a reasonable approximation of a constant current source. Thus if the current is forced by a bias battery to flow through a high resistance, any desired voltage can be obtained across this resistance. However, the use of very high load resistances

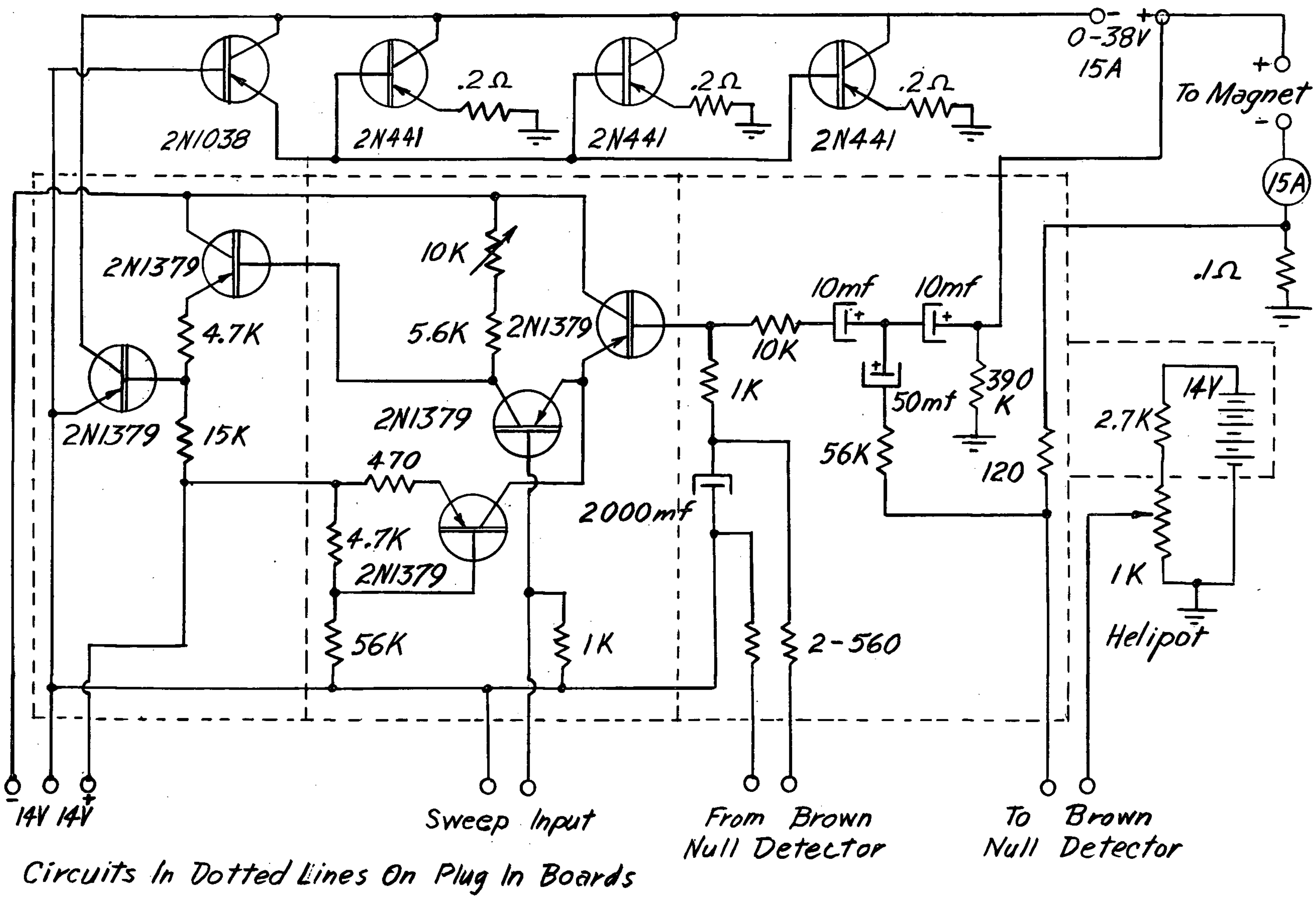


Fig. 14. Magnet Current Regulator

produces leakage problems, and, more important, the problem of A.C. and D.C. voltage monitors with high input impedances. Thus 100 megohms has been chosen as the maximum resistance. Also, to keep the collector voltage reasonable it has been decided to vary the load resistance up to 100 megohms so as to keep the total voltage to about 1 volt.

Some simple calculations show that this is a reasonable choice with respect to thermal noise in the resistor and shot noise in the electron beam. The resistance noise is given by the Nyquist relation

$$E_1 = [4kTR\,(\text{Band Width})]^{1/2} \tag{20}$$

$$= [1{\cdot}6 \times 10^{-20} R\,(B.W.)]^{1/2}\,\text{Volts at } 300°\text{K},$$

while that due to shot noise in the beam is[12]

$$E_2 = [2Ie\,(B.W.)]^{1/2} R \tag{21}$$

where e is the electronic charge and I is the beam current. At RI = 1,

$$\frac{E_2}{E_1} = \frac{[2e\,(B.W)]^{1/2} I^{1/2}}{I^{1/2}[4kT(B.W)]^{1/2}} = \left(\frac{2e}{4kT}\right)^{1/2} \simeq 4{\cdot}5 \tag{22}$$

Thus shot noise voltage is large enough so that the resistance noise may be ignored. It is of no value to increase the value of RI beyond this, as the shot noise increases linearly with resistance.

A calculation of noise voltage for a bandwidth of 1000 cycles/sec. and RI = 1 gives approximate values

$$\begin{aligned} &3 \times 10^{-5} \text{ Volts at } I = 10^{-6} \text{ amps} \\ &3 \times 10^{-4} \text{ Volts at } I = 10^{-8} \text{ amps} \end{aligned} \tag{23}$$

This means that for a 10 nanoamp current the noise current will be about 1/1000 of the total current. However, narrowing the passband of the system can improve this considerably.

The load resistance and bias is provided by the distribution box. A filter with a time constant of 5 seconds keeps noise from the VTVM from getting into the input amplifier. When all the resistance is switched out the 100 megohm filter resistance provides the A.C. load, and this resistance plus the 100 megohm resistance of the VTVM make up the D.C. load.

The A.C. signal is fed from the distribution box to a high impedance unity gain amplifier[13]. This device has an input impedance in excess of 1000 megohms, and could be adjusted higher if needed. From here the signal is fed into a transistor amplifier with a gain of about a million.

This in turn drives the vertical plates of the oscilloscope, and could be fed to a phase sensitive detector if needed (see Figures 15 and 16).

The oscilloscope horizontal sweep is supplied by an audio signal generator. Also, two phase shifted outputs are provided, one for sweeping the magnetic field, and the other as the reference voltage for a phase sensitive detector.

As a preliminary experiment an attempt was made to observe the cyclotron resonance of a beam of electrons. As would seem reasonable, it was found that for H_o near 75 gauss, the beam current was directly proportional to the magnetic field.

The quadrupole lens used (see Figures 11 and 12) was designed for use at $= (\omega_L - \omega_c)$, and its performance at 200 Mc. is not known. The spacing of the wires producing the quadrupole field is 1 cm., and length is 40 cm.

As higher power equipment was not available, use was made of a low power signal generator inductively coupled to the quadrupole lens. With the coupler used and the arbitrary length of cable connecting the signal generator and the lens, it was found that a dip in the output voltage of the signal generator occurred every 25 Mc. between 200 and 400 Mc., indicating that maximum power was being fed into the lens at these frequencies.

The cyclotron resonance was observed at 222 Mc. by sweeping the solenoid magnetic field at 200 cycles and

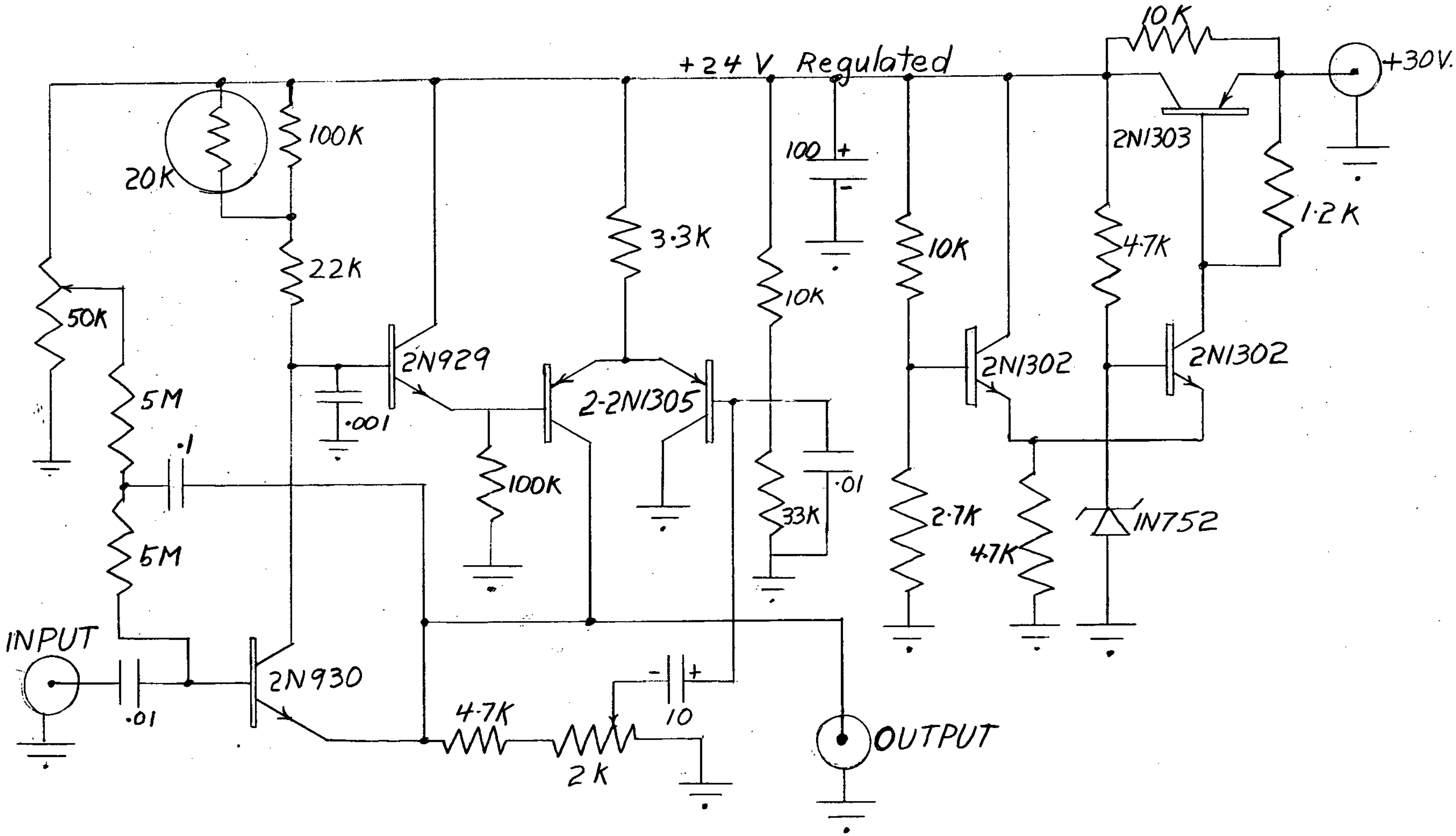


Fig. 15. High Input Impedance Amplifier

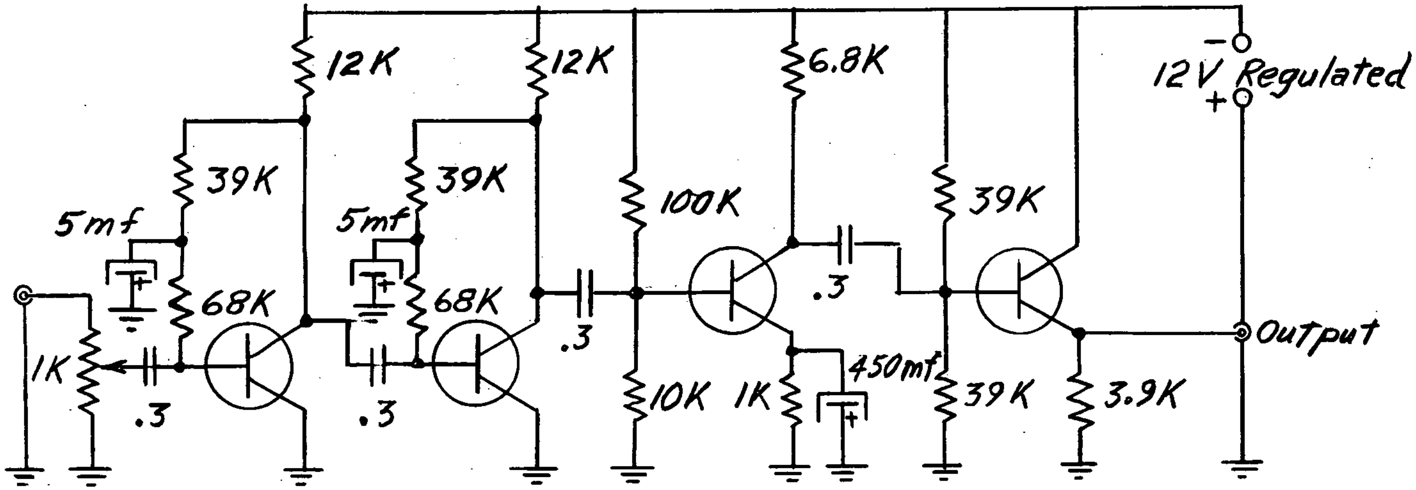


Fig. 16. High Gain Amplifier

observing the A.C. component of signal on an oscilloscope. The change in the total beam current at resonance was in the order of 5%, and the width of the resonance was about 4 Mc.

The average value of the electron energy was 10 e.v., corresponding to a velocity of 1.4×10^8 cm./sec. For this velocity the transit time through the 40 cm. test region is .3 μsec. This is in good agreement with the .25 μsec. predicted from the line width.

Since the Larmor frequency and the cyclotron frequency differ by only 254 Kc. at this magnetic field, it is obvious that the 4 Mc. line width will make it impossible to detect any resonance effect at the Larmor frequency. Some suggestions for improving the resolution are given in Chapter 4.

CHAPTER IV

POSSIBLE IMPROVEMENTS IN THIS APPARATUS

Increasing the resolution of the resonances is the most important problem. For the experiment to give useful results the number of Larmor periods spent in the test section must be increased from the present 60 to something like 10^4 periods.

The only practical way to achieve this seems to be to trap the electrons in the test region by some sort of electric field gate. The central glass tube could be made in two sections with a metal ring at each end of the section that extends through the test region. Negative potentials of 10-20 volts on these rings should trap the electrons within this region and hold them for a millisecond or so. This should not only increase the resolution but also greatly increase the change in the beam at resonance.

The major stumbling block in using a trapping method is that the electrons should not undergo many collisions with residual gas in the vacuum, and this means that a vacuum of 10^{-7} mm.Hg. or better will be needed. Two fast oil diffusion pumps should be used instead of the present small mercury pump, and they should be mounted one at each end of the apparatus with short, straight connections. This should not only make possible the necessary vacuum, but should also shorten pumping times considerably.

Some empirical testing with small bias coils and permanent magnets has proven fruitful in increasing beam current, and could be used to keep the beam diameter small where it leaves the 8 mm. glass tube on the collector end of the quadrupole lens.

CHAPTER V

SUGGESTIONS FOR FUTURE EXPERIMENTS

One idea that could be incorporated in a new experiment is the use of the modified solenoid developed by Garret[14]. This would enable the production of a homogeneous field using a larger diameter solenoid. This would have the obvious advantage of enabling the vacuum chamber to be much larger. This in turn would simplify the vacuum considerations (only one pump would be needed), and would simplify the construction of the field gradient lens. Also, the apparatus could be much shorter because the source and collector could be close to the ends of the field gradient lens. The fact that the solenoid is notched means that electrical leads could be brought in to the center of the apparatus, and the vacuum pump line could be taken out. Being able to bring in the R.F. signal to the center of the lens would be very useful, especially at higher frequencies.

It has been suggested by Fairbanks[15] that a magnetic mirror and time of flight approach could be used to produce an electron beam in which all the electrons have $n = 0$ and $m = -\frac{1}{2}$, and Adler et al[16,17,18] have shown that it is possible to cool the cyclotron orbits of an electron beam by coupling the beam to a cold load tuned to the cyclotron frequency. It appears possible that electrons could be cooled by this method very nearly to liquid helium temperatures. Assuming that the cyclotron orbits of the beam are in thermal equilibrium, then the probability of them being in quantum state n is given by[19]

$$W(n) = \frac{e^{-\frac{n\hbar\omega}{kT}}}{\sum_{n=0}^{\infty} e^{-\frac{n\hbar\omega}{kT}}} = e^{-\frac{n\hbar\omega}{kT}}\left(1 - e^{-\frac{\hbar\omega}{kT}}\right) \qquad (24)$$

and for $T = T°K$, $\omega = 6 \times 10^9$/sec., and $n = 0$

$$W(0) \simeq [1-(1-1/100)] = 1/100$$

Thus approximately 1% of the electrons would be in the lowest cyclotron orbit, and half of these should have $m = -\frac{1}{2}$.

If this "cooled" beam of electrons now passes to a region where there is a larger magnetic field and this is done adiabatically so that no transitions occur, the magnetic energy of the electron, $(n + \frac{1}{2} + \frac{mg}{2})\frac{e\hbar}{mc}$, is larger. This increased energy is subtracted from the translational energy of the electron. Thus, if the translational energy of the electron is less than the increase in magnetic energy, the electron cannot enter the higher field, and it is reflected. This is known as a magnetic mirror. The electrons that get through such a high field region must have translational energies greater than $h\omega_c$, where ω_c is the cyclotron angular velocity in the mirror field, or have $n = 0$, $m = -\frac{1}{2}$, or both. The ones in the first category can be eliminated by pulsing the beam and considering only those that arrive after a specified time.

The fraction of electrons with energies between E and E + ΔE is, for a one-dimensional Maxwell-Boltzman distribution[20]

$$\frac{\Delta N}{N} = \frac{e^{-\frac{E}{kT}}\Delta E}{(4\pi kTE)^{1/2}} \tag{25}$$

and when T = 2000°K, ω = 1.7 x 10^{11}, E = $\frac{\hbar\omega}{2}$, and ΔE = $\hbar\omega$,

$$\frac{\Delta N}{N} = e^{-\frac{\hbar\omega}{2kT}}\sqrt{\frac{\hbar\omega}{2\pi kT}} \simeq \sqrt{\frac{\hbar\omega}{2\pi kT}} \simeq 1/100$$

Thus the beam of polarized electrons will be 1/10,000 of the starting beam, and should be a feasible current to work with.

After passing through the magnetic mirror, the beam could go to the quadrupole lens, or its equivalent, and then through another magnetic mirror. This would reject all those electrons that had undergone transitions between the two mirrors. Thus the experiment would entail the measurement of an actual current change, rather than a change in beam dimensions. Probably more important, it would enable working with sizeable beam diameters, which would mean larger currents with lower electron densities.

Changes could also possibly be made in the method of producing transitions. If a large magnetic field was used, wave guide techniques might be used, with the electron beam travelling down the center of a waveguide or resonant cavity.

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