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HIGH FIELD CURRENT FLUCTUATIONS IN N-TYPE GERMANIUM

ABSTRACT

The work reported here is an experimental and theoretical investigation of high-frequency electrical noise generated in extrinsic single-crystal n-type germanium at high electric fields. The electric field was pulsed so that the lattice temperature remained near 77°K. During the pulse, the electrons quickly reach a nonequilibrium steady-state due to their gaining energy from the electric field and brought to a steady-state by means of collisions with the lattice vibrations. Previous work has been concerned with noise measurements made at right angles to the electric field direction, where anisotropic behaviour was observed. The present measurements, made in the direction of the electric field, also show a high degree of anisotropy.

The electrical noise generated is described by the noise temperature, T_n , obtained by adapting the Nyquist formula to the non-equilibrium case. Measurement of T_n , performed at frequencies of 70 Mc/s and 30 Mc/s, indicated a uniform noise spectrum in this frequency range for all the samples used. The anisotropy of T_n suggested that T_n was explainable on the basis of the many-valley model of the conduction band of germanium, established by previous experimental investigations of the high-field mobility anisotropy.

A feature of the many-valley model is that electrons in different valleys of the conduction

band, will in general, exhibit different transport behaviour and as a result, transitions between these valleys will result in a noise phenomenon described as "intervalley noise". However, for measurements of T_n in the $\langle 100 \rangle$ direction, the "intervalley noise" will vanish, allowing a direct measure of the electron "heating" due to the electric field, the "hot electron noise". In the $\langle 111 \rangle$ and $\langle 110 \rangle$ directions, both intervalley and hot electron noise are expected. Both contributions to T_n are evaluated by means of Barrie's extension to the case of many-valley germanium of Stratton's high-field transport theory.

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by

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(Research Supervisor)

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CHAPTER 1 - INTRODUCTION

1.1 REVIEW OF PREVIOUS WORK

Until recently, investigations of noise in current-carrying single crystal semiconductors have been concerned with behaviour at relatively low frequencies and fields, where effects due to carrier population fluctuations are dominant. Recent experiments (Erlbach and Gunn, 1962) however, have revealed a source of noise in n-type germanium present when the electrons are subjected to high electric fields. This noise has been interpreted as an enhancement of the thermal or Nyquist noise, due to the electrons being "heated" by the field, a concept used earlier to explain the field dependence of the electron mobility.

An additional source of noise, predicted theoretically to be measurable under the high-current experimental conditions of this experiment, is the so-called "intervalley noise", which is due to conduction electron transitions between states of different effective mass.

We shall first review the noise work done on current-carrying semiconductor resistors, where the main concern is with carrier population fluctuations. Then, the high-field transport behaviour of n-type germanium will be discussed and applied to the calculation of both the "hot electron" and the "intervalley noise".

Electrical noise in any two-terminal network is the appearance across the terminals of a random fluctuating voltage and is described quantitatively by means of the

Thevenin and Norton equivalent circuits. The Thevenin description is as follows:

In a bandwidth Δf at frequency f the equivalent circuit is an impedance $Z(f)$ in series with a voltage generator $e(t)$, where $Z(f)$ is the circuit impedance at frequency f and the voltage generator $e(t)$ is, by convention, represented by its root-mean-square value, $\sqrt{\overline{e^2}}$, the bar indicating an average over a time interval much greater than $(\Delta f)^{-1}$. The fluctuating voltage $e(t)$ corresponds to what would be observed on a cathode ray oscilloscope placed across the terminals, if the oscilloscope bandwidth were Δf . The spectral density of $e(t)$ is defined as

$$S_e(f) \equiv \lim_{\Delta f \rightarrow 0} \left(\frac{\overline{e^2}}{\Delta f} \right)$$

The most basic two-terminal noise generating network to consider is that of a pure resistance R_0 in thermal equilibrium with its surroundings at an absolute temperature

T_0 . This ideal case was first considered by Nyquist from a theoretical point of view. He showed that $\overline{e^2}$, the mean-square value of the voltage generator in a frequency range Δf , is related to R_0 , T_0 , and f by Nyquist's theorem:

$$\overline{e^2} = 4KT_0 R_0 p(f) \Delta f$$

where K is Boltzmann's constant and $p(f)$ is the Planck factor

$$p(f) = \frac{hf}{KT_0 (e^{hf/KT_0} - 1)}$$

h being Planck's constant. We normally consider the

case $(hf)/kT_0$ much less than unity. Then Nyquist's theorem is

$$\overline{e^2} = 4kT_0 R_0 \Delta f$$

The proof of this classical limit of Nyquist's theorem is accomplished by purely thermodynamic arguments, using the second law of thermodynamics and the equipartition law, which gives it its general validity. However, for the case of a conductor in which the free electrons motion are described by a very simple model, Bakker and Heller(1939) were able to derive the Nyquist formula from an examination of the microscopic electron motions.

In addition to the thermodynamic derivation of Nyquist and the microscopic derivation of Bakker and Heller, the Nyquist formula has been verified experimentally by Johnson(1928). Also, Moullin determined Boltzmann's constant by means of thermal noise measurements and obtained agreement to within one per cent of values determined by other methods.

Another very basic noise source is the shot noise in the temperature-limited vacuum diode, which consists of a hot electron emitting cathode surrounded by an anode. The device is operated at high positive anode voltages in the current-saturated region of its current-voltage characteristic. In this condition the current is temperature-limited, because all electrons emitted by the cathode travel directly to the anode. Under these conditions, the diode current fluctuations at frequencies much less than the reciprocal of the transit-time takes on a simple form given by the Schottky theorem, if we assume that the emission process at the cathode is Poisson.

The Schottky theorem states,

$$S_I = 2qI_d$$

where S_I is the diode fluctuating current spectrum and I_d is the mean diode current. The equivalent circuit is then best described as a current generator given by the Schottky theorem in parallel with a frequency-dependent admittance which is normally very small in the radio frequency range. For practical noise diodes, the Schottky theorem does not explain the observed noise at all frequencies. At the lower audio frequencies, flicker effect enhances the observed noise and at frequencies in the microwave region transit-time effect lower the noise below that given by the Schottky theorem.

Another noise source, which is the main topic of this thesis, is the non-equilibrium case of a current-carrying single crystal semiconductor resistance. Its fluctuation emf. when in thermal equilibrium with its environment is given by the Nyquist theorem. However, the passage of direct current results in additional noise, which in general is a function of the biasing current, frequency, crystal orientation and the surface environment of the resistor. Most experimental and theoretical work has been concerned with the strongly frequency dependent low frequency noise, which may be orders of magnitude greater than the thermal noise even for relatively small values of current density.

We characterize the non-equilibrium noise of the semiconductor by a parameter, T_n , the noise temperature, obtained by using Nyquist's formula for the non-equilibrium

case and letting T_n substitute for the thermodynamic temperature. By definition of T_n , the mean-square voltage generator in the series representation is:

$$\overline{e^2} = 4KT_n R' \Delta f$$

where R' is the differential resistance; similarly in the current generator representation:

$$\overline{i^2} = 4KT_n G \Delta f$$

where G is the differential conductance. When no current flows, the noise temperature equals the thermodynamic temperature, from Nyquist's theorem.

Measurements performed over the frequency range of one Kc/s to six Mc/s by van der Ziel and associates (van der Ziel, 1954) on a crystal of near-intrinsic n-type germanium revealed the existence of two frequency-dependent components in the noise temperature, so that

$$\frac{T_n}{T_1} = 1 + \frac{A_1 I^2}{f} + \frac{A_2 I^2}{1 + (f/f_1)^2}$$

Here A_1 and A_2 are constants for the sample used, I is the direct current and f_1 is a constant independent of the direct current. The first term on the right-hand side is due to thermal noise. The last term, because of its frequency dependence, is interpreted as "generation-recombination noise". The remaining term is called "excess noise".

The generation-recombination noise is due to fluctuations in the electron and hole generation rates. Assuming that electron and hole population fluctuations occur due to hole-electron pair creation and hole-electron recombination, neglecting trapping and taking the transit-time much

greater than the lifetime, the noise temperature due to this process is given by

$$T_n^{gr} = \frac{q F^2 (\mu_n + \mu_p)^2 \sigma_p \tau_c}{K \mu_p (\sigma_n + \sigma_p) (1 + \omega^2 \tau_c^2)}$$

Here F is the electric field strength, μ_n, μ_p the electron and hole mobility, respectively and τ_c is the electron or hole lifetime. The conductivities σ_n and σ_p are given by

$$\sigma_n = n q \mu_n \quad \sigma_p = p q \mu_p$$

where p and n are the hole and electron densities, respectively.

Taking τ_c from the measured value of f_1 , $\frac{\sigma_p}{\sigma_n + \sigma_p}$ could then be determined from the experimental value of T_n^{gr} .

From $\frac{\sigma_p}{\sigma_n + \sigma_p}$ and the measured value of $\sigma_p + \sigma_n$, the resistivity of intrinsic germanium was calculated and found to be in good agreement with the accepted value.

The excess noise has been observed to have the following behaviour:

- 1) The frequency dependence is roughly f^{-1} over a range from the order of one Mc/s to 10^{-5} cycles per second.
- 2) Over a temperature range from 4°K to 240°K , it is relatively constant.
- 3) It is extremely sensitive to the surface condition and ambient environment.
- 4) Its dependence on the direct current is quadratic.

The $\frac{1}{f}$ spectrum may be obtained in a purely mathematical way if a distribution $g(\tau_c)$ of lifetimes is used such that $g(\tau_c)$ is proportional to τ_c^{-1} . The existence of both "slow trapping" and "fast trapping" results in a wide range of lifetimes necessary to obtain

the spectrum over the wide band of frequencies indicated above. The "slow trapping" has been associated with the "slow surface states" existing in the oxide layer on the semiconductor surface.

It is possible to explain the $\frac{1}{f}$ spectrum without assuming a distribution of lifetimes if we assume the conduction electrons reach the surface traps by tunnelling through a potential barrier of variable width. This also results in a wide temperature-independent lifetime distribution, as required by experiment.

So far, only low frequency, low field noise in current-carrying semiconductors has been discussed. The high-field, high frequency noise will be discussed later, after the rather complex transport properties of germanium have been examined.

1.2 TRANSPORT PROPERTIES OF A MANY-VALLEY SEMICONDUCTOR

Transport behaviour involves interactions between the electric field, the electrons, and the lattice. The electric field acts on the electrons to increase their energy and momentum and the electrons interact with the lattice vibrations to lose energy and momentum. The transport properties then describe a steady-state in which the electrons gain energy and momentum from the field at the same rate they lose it to the lattice vibrations.

The interaction between the field and a single electron depends on the effective mass of the electron for motion in the field direction. The effective mass is found from the solution of the Schroedinger equation for the electron

in the periodic field of the lattice. The energy eigenvalues $E(\vec{k})$ for the conduction band states are functions of the wave vector $\vec{k}$ which in turn are restricted to vectors having the values

$$2\pi \left(\frac{p_1}{S_1} \vec{b}_1 + \frac{p_2}{S_2} \vec{b}_2 + \frac{p_3}{S_3} \vec{b}_3 \right)$$

where p_1, p_2 , and p_3 are integers and $\vec{b}_1, \vec{b}_2$, and $\vec{b}_3$ are the primitive translation vectors of the reciprocal lattice. The crystal is assumed to be a rectangular parallelepiped having edges $S_1 \vec{a}_1, S_2 \vec{a}_2, S_3 \vec{a}_3$.

Let one minimum of the conduction band occur at $\vec{k}_0$ which is different from zero. If the x, y, z co-ordinate system is chosen so that the off-diagonal components of the effective mass tensor are zero, then the Taylor series expansion of $E(\vec{k})$ about the minimum energy state $\vec{k}_0$ is, for small values of $(\vec{k} - \vec{k}_0) = \Delta k_x, \Delta k_y, \Delta k_z$;

$$E(\vec{k}) = E(\vec{k}_0) + \frac{\hbar^2}{2} \left\{ \frac{(\Delta k_x)^2}{m_1} + \frac{(\Delta k_y)^2}{m_2} + \frac{(\Delta k_z)^2}{m_3} \right\}$$

where m_i , the effective electron mass for motion in the i -direction, ($i = x, y, z$) is given by

$$\frac{1}{m_i} = \frac{1}{\hbar^2} \left(\frac{\partial^2 E(\vec{k})}{\partial k_i^2} \right)_{\vec{k}_0}$$

For the lowest energy minimum of germanium, the values of

m_i are obtained from cyclotron resonance data at liquid helium temperatures. The most recent data are those of Levinger and Frankl (1961), quoted as:

$$m_1 = m_2 \equiv m_t = (0.08152 \pm 0.00008) m_0$$

$$m_3 \equiv m_l = (1.588 \pm 0.005) m_0$$

where m_0 is the free electron mass.

A complete picture of the band structure of germanium has been published by Herman (1955) as a result of his own theoretical work and data obtained from cyclotron resonance measurements. Considering only the conduction band, it is found that there are two classes of valley:

- 1) Four in the $\langle 111 \rangle$ direction of $\vec{k}$ space at the zone edge. The constant energy surfaces are ellipsoids of revolution given by equation 1.2.1 above, with $m_1 = m_2$, the z-direction being in the $\langle 111 \rangle$.
- 2) One minimum at $\vec{k} = 0$ about which the constant energy surfaces are spherical with effective mass $0.034 m_0$. The energy of this minimum is 0.896 e.v. above the valence band at 4°K so that only at high temperatures is there any degree of occupancy of this minimum.

The scattering of electrons between states in the same valley (intravalley scattering) and between states of different valleys (intervalley scattering) occurs in accordance with the selection rules:

$$\vec{k}' = \vec{k} \pm \vec{q}$$

where $\vec{k}'$ is the final and $\vec{k}$ the initial electron wave vector and $\vec{q}$ is the wave vector of the phonon involved in the transition (+ for absorption and - for emission).

Conservation of energy of the system requires that

$$E(\vec{k}') = E(\vec{k}) \pm \hbar u |\vec{q}|$$

with the + and - meaning the same as above and u is the velocity of sound in the crystal, which may be a function of $\vec{q}$. The probability P_{ij} of a transition

in unit time from a state in valley i to a state in valley j is

$$P_{ij} \propto |M_{ij}|^2 f(E')$$

where $f(E')$ is the density of states at the state to which the electron is scattered. Here it is assumed that $\hbar u |\vec{q}|$ varies little along the E' surface.

The matrix element $M_{ij} = [n(\vec{q})]^{1/2} D_{ij}$ for absorption

$$= [n(\vec{q}) + 1]^{1/2} D_{ij} \quad \text{for emission}$$

where $n(\vec{q})$ is the number of phonons present initially in the mode involved, and D_{ij} is independent of the occupation of the phonon states.

For intervalley scattering D_{ij} is nearly independent of the locations of the initial and final states in their respective valleys. Hence, for absorption, the transition probability for a transition to a state of energy $\Delta E + \hbar \omega$ in valley j is

$$W_a = W \frac{(\Delta E + \hbar \omega)^{1/2}}{e^{\hbar \omega / kT_L} - 1}$$

W being constant.

For emission,

$$W_e = W \frac{(\Delta E - \hbar \omega)^{1/2}}{1 - e^{-\hbar \omega / kT_L}} \quad \text{for } \Delta E > \hbar \omega$$

$$= 0 \quad \text{for } \Delta E \leq \hbar \omega$$

The density of states in both cases is proportional to the square-root of the final state energy.

Fig.1.1

Ellipsoidal constant-energy surfaces of conduction band electrons of germanium in $\vec{k}$ -space. Only two of the four minima are shown.

BC and AC represent, respectively, intravalley and intervalley transitions.

1.3 HIGH FIELD TRANSPORT THEORY

Many papers have been published on the theory of high field transport in n-type germanium, the most recent of which have dealt with the anisotropy of the mobility and the changes in valley population with field. Other important considerations include the form of the velocity distribution at non-zero fields and the effect of electron-electron collisions on the form of the distribution. Any theory used to explain the observed current fluctuations in the present experiment must determine the variance of the field component of electron velocity and the drift velocities of the various valleys as a function of the electric field.

The ideal approach to the problem is the determination of the velocity distribution function by solving the Boltzmann transport equation for high fields, assuming electron-electron collisions and impurity, acoustical, optical and intervalley scattering. The nearest approach to this ideal is the work of Reik and Risken(1962), who solved the Boltzmann equation for a many-valley semiconductor under the restriction of high electron energy only, which would apply only for very high electric fields. They found the energy distribution to be Maxwellian, even ignoring electron-electron collisions.

Stratton(1957) has carried out an analysis of the role of electron-electron collisions in determining the velocity and energy distributions. His order of magnitude estimates of the average energy exchanged in electron-electron and

acoustical mode scattering lead him to assert that a critical electron density, n_a , given by

$$n_a \approx 2 \times 10^{14} \left(\frac{T_L}{293} \right)^2 \left(\frac{E}{k T_L} \right)^2 \text{ cm}^{-3}$$

exists, where T_L is the lattice temperature and E the electron energy. For densities greater than n_a , the energy distribution will tend towards a Maxwellian for energies neighboring on E . In order that the momentum distribution be Maxwellian, displaced in the case of high fields, the value of n must be greater than

$$10^{15} \left(\frac{E}{k T_L} \right)^2$$

at a lattice temperature of 77°K.

In the present experiment, the carrier density was about $8 \times 10^{14} \text{ cm}^{-3}$. According to Stratton's analysis then, the assumption of a displaced Maxwellian momentum distribution for this case is not valid, for electron energies much greater than $k T_L$. What form the distribution takes then is not known. However, the role of electron-electron collisions in determining the distribution may be found by comparing the results of Stratton's analysis with those of Yamashita and Watanabe(1954), who found the form of the velocity distribution at high fields by ignoring the effects of electron-electron collisions. Their solution, the Pisarenko distribution, differs from the Maxwellian except at zero field. Although electron-electron collisions may influence the shape of the velocity distribution, as far as the interpretation of the present experiment is concerned, it is necessary to know only the extent to

which such collisions affect the mean value and the mean-square value of the velocity distribution. This was done by calculating the mean and mean-square velocity at an arbitrary field of 87 volts per cm. for both the displaced Maxwellian and the Pisarenko distributions. In these calculations, acoustical mode scattering and a scalar effective mass were used. The results of the calculations were that the mean values differ by 5% and the mean-square by 11%. Hence it was concluded that the displaced Maxwellian distribution provides a reasonable value of the first and second moments, even for the case of a tensor effective mass as is used in Barrie's extension of Stratton's theory.

The theory of Barrie was used to provide all first and second moments of the various valley velocity distributions needed in the theoretical expressions for the current fluctuations. As well, it provided the valley electron temperatures, which were used to calculate the high field valley populations. The Barrie theory has been tested experimentally as far as the dependence of drift velocity on field (Barrie and Burgess, 1962). The theory predicts correctly the occurrence of mobility anisotropy at 77°K, but the actual values of drift velocity are in only fair agreement with the theory.

Another approach in interpreting high field transport behaviour is that of Paige (1960). He assumed a Maxwellian energy distribution in each valley and by employing scalar effective mass theory in his expressions for mobility was able to deduce the valley electron temperatures and

valley mobilities for the current direction close to the $\langle 111 \rangle$ direction. Published data on the angle between current and field was used in his analysis. For the principal directions this method of analysis is of no value since the current and field directions are the same.

1.4 BARRIE'S EXTENSION OF STRATTON'S HIGH FIELD THEORY

In this theory the band scheme of germanium discussed in Section 1.2 is assumed. Also, the mean electron energy is assumed low so that only the four $\langle 111 \rangle$ minima are occupied and contribute to the current. Other assumptions and approximations used are the following:

- 1) The distribution $f(\vec{k})$ of electrons in one valley in $\vec{k}$ -space is Maxwellian at all fields and is displaced about a field-dependent part $\vec{k}_0$, that is;

$$f(\vec{k}) \propto e^{-E(\vec{k}-\vec{k}_0)/kT}$$

Here the origin in $\vec{k}$ -space is taken at the zone edge in the $\langle 111 \rangle$ direction.

- 2) The steady-state equations from which the parameters $\vec{k}_0$ and T are found contain the factors $\left(\frac{\partial f(\vec{k})}{\partial t}\right)_{ac}$ and $\left(\frac{\partial f(\vec{k})}{\partial t}\right)_{op}$, the rate of change of the distribution function due to acoustical and optical mode scattering, respectively. The above factors may be expressed as sums involving the transition probabilities $\rho_a(\vec{k}, \vec{k}+\vec{q})$ and $\rho_e(\vec{k}+\vec{q}, \vec{k})$, which represent, respectively, transitions by absorption of a phonon $\vec{q}$ to a state $\vec{k}+\vec{q}$ and transitions by emission of a phonon $\vec{q}$ to a state $\vec{k}$.

Ordinary perturbation theory is used to calculate $\rho_a(\vec{k}, \vec{k} + \vec{q})$ and $\rho_e(\vec{k} + \vec{q}, \vec{k})$. This implies use of the Born approximation and the resultant expression for the transition probability,

$$\rho = \frac{2\pi}{\hbar} B(\vec{q}) n(\vec{q}) \delta\{E' - E + \hbar \omega(\vec{q})\}$$

where E' and E are the final and initial energies, respectively.

$B(\vec{q})$ is the square of the absolute value of the matrix component of the perturbing potential, V , connecting the two states, and is defined by

$$B(\vec{q}) = \int \phi_{\vec{k}'}^* V \phi_{\vec{k}} dx dy dz$$

$\phi_{\vec{k}'}$ and $\phi_{\vec{k}}$ being the wave functions of the initial and final states.

$n(\vec{q})$ is the average number of phonons in the $\vec{q}$ -mode, and is given by:

$$n(\vec{q}) = [e^{\epsilon(\vec{q})/kT_L} - 1]^{-1}$$

3) $B(\vec{q})$ is assumed to be isotropic in $\vec{q}$ -space.

Since intervalley scattering is ignored in the determination of the valley mobilities, we need consider only the intravalley optical and acoustical scattering.

According to Herring(1955), in the case only of intravalley acoustical scattering may $B(\vec{q})$ be anisotropic due to the possibility of shear strains producing deformation potentials.

The Barrie equations of Section 5.1 reduce to the original equations of Stratton(1958) if m_t and m_l are

equal. Since the original Stratton theory predicts an isotropic mobility at all fields, it is quite inappropriate for the interpretation of the present experiment.

1.5 PREVIOUS DISCUSSIONS OF HIGH FIELD NOISE

P.J.Price(1959) first discussed the extension of the Nyquist theorem to the case of a conductor biased at strong electric fields such that the relation between the current density and electric field is no longer linear. He considered the fluctuation of velocity of a single electron, neglecting any electron-electron interaction and was able to relate the spectral density of the field component of electron velocity fluctuation to a component of the differential diffusion dyadic. He applies his result to two high field cases: one in which the constant energy surfaces in $\vec{k}$ -space are spherical and scattering only by acoustical mode interactions; the other, applicable only in the case of very high fields, where only scattering by emission of optical phonons is important. In both cases the noise temperature is of the same order as $\frac{\langle E \rangle}{K}$, where $\langle E \rangle$ is the mean electron energy. The frequency dependence of T_n is uniform up to frequencies the order of the reciprocal of the mean-free-time, corresponding to about 10^{10} cycles per second.

Gurevich(1962) discussed the current fluctuations in the non-equilibrium state of a semiconductor at high field. He assumed spherical energy surfaces in $\vec{k}$ -space, acoustical mode scattering and no electron-electron interaction. His expression for $S_I(\omega)$ in the direction of the

applied field, where $S_I(\omega)$ is the spectral density of current fluctuations due to the electron thermal motions, contains a parameter τ_s , which results in dispersion in $S_I(\omega)$ for frequencies of the order τ_s^{-1} , where τ_s is of the order 10^{-10} seconds. For frequencies much less than τ_s^{-1} , his theory predicts the current spectrum to be frequency independent and to be proportional to the square-root of the applied field.

Another source of noise also predicted to extend out to high frequencies is the intervalley noise, first discussed by Price(1960). This source of current fluctuation is due to the random rate of electron transitions between the various valleys of the conduction band and is present only when the valleys are "quasi-isolated", in the sense that an electron localized in one valley tends to execute many collisions within one valley before making a transition to another valley. A necessary condition for the existence of this effect is the variation of electron mobility between the valleys.

Using the experimental values of the intervalley transition rate obtained by Weinreich, Sanders, and White(1959) at 77°K, Price predicted that the noise measured in the $\langle 100 \rangle$ direction with the direct current in the $\langle 110 \rangle$ direction should be detectable with a joule heating due to the applied field of the order of 100 watts per cubic centimeter and that the spectral density should drop off at microwave frequencies, since the mean lifetime of an electron in a valley is of the order of 10^{-9} seconds.

CHAPTER 2 - THEORY OF CURRENT FLUCTUATIONS IN A MANY-VALLEY SEMICONDUCTOR

Let us suppose the semiconductor sample to be in the form of a homogeneous single-crystal bar of length L and of uniform cross-section with current leads attached to its two ends. Electrons are assumed to be the majority carrier and the hole population taken to be negligible. We assume a uniform electric field F exists in the sample resulting in a stationary fluctuating current, $I(t)$, whose spectral density, $S_I(\omega)$, we wish to calculate.

The current $I(t)$ is due to the motion of the free electrons of the sample and is given by

$$I(t) = \frac{q}{L} \sum_{j=1}^{N(t)} u_j(t)$$

where $N(t)$ is the free electron population at time t and $u_j(t)$ is the velocity component of the j th electron in the direction of the longitudinal sample axis.

Let us now assume that the total population $N(t)$ is composed of two semi-isolated groups of population $N_1(t)$ and $N_2(t)$ with each group having drift velocities in the field which are, in general, different. By semi-isolated we mean that electron transitions occur between the groups but not at a rate sufficient to affect the individual group energy or velocity distributions. $N_1(t)$ and $N_2(t)$ will fluctuate due to random transitions occurring between the groups.

$N(t)$ will also fluctuate, but its variance is sufficiently

small (Appendix 1) that $N(t)$ has been taken constant.

The current in the sample at time t is then:

$$I(t) = \frac{q}{L} \sum_{j=1}^{N_1(t)} v_j(t) + \frac{q}{L} \sum_{k=1}^{N_2(t)} w_k(t)$$

where $v_j(t)$ and $w_k(t)$ are, respectively, the instantaneous axial components of velocity of the j th electron in group 1 and of the k th electron in group 2.

The current may be expressed in terms of the variables

$N_1(t)$, $N_2(t)$, $\Delta v_a(t)$, and $\Delta w_a(t)$, the latter two being the arithmetic-mean velocity fluctuations, defined by

$$\Delta v_a(t) = \frac{1}{N_1(t)} \sum_{j=1}^{N_1(t)} \Delta v_j(t) \quad \Delta w_a(t) = \frac{1}{N_2(t)} \sum_{k=1}^{N_2(t)} \Delta w_k(t)$$

where

and

$$\Delta v_j(t) = v_j(t) - \bar{v}$$

$$\Delta w_k(t) = w_k(t) - \bar{w}$$

Then

$$I(t) = \frac{q}{L} \left[N_1(t) (\bar{v} + \Delta v_a(t)) + N_2(t) (\bar{w} + \Delta w_a(t)) \right]$$

We assume that $\text{cov}[v_a(t), w_a(t+\tau)]$ equals zero. This means that there is assumed no appreciable velocity "carry-over" from one valley to another and seems reasonable since the matrix element for intervalley scattering is nearly independent of the locations of the initial and final states in their respective valleys (Herring, 1955).

This leads to the following expression:

$$\begin{aligned} \frac{1}{q^2} \text{cov}[I(t), I(t+\tau)] &= (\bar{v})^2 \text{cov}[N_1(t), N_1(t+\tau)] + (\bar{w})^2 \text{cov}[N_2(t), N_2(t+\tau)] + \\ &+ \bar{v} \bar{w} \left\{ \text{cov}[N_1(t), N_2(t+\tau)] + \text{cov}[N_2(t), N_1(t+\tau)] \right\} + \\ &+ \overline{N_1(t) N_1(t+\tau)} \text{cov}[v_a(t), v_a(t+\tau)] + \overline{N_2(t) N_2(t+\tau)} \text{cov}[w_a(t), w_a(t+\tau)] \end{aligned}$$

The current spectral density, $S_I(\omega)$, is given by the Weiner-Khintchine theorem,

$$\begin{aligned} \frac{L^2}{q^2} S_I(\omega) = & (\bar{v})^2 S_{N_1}(\omega) + (\bar{w})^2 S_{N_2}(\omega) + 2\bar{v}\bar{w} S_{N_1, N_2}(\omega) + \\ & + 4 \int_0^\infty \overline{N_1(t) N_1(t+\tau)} \text{cov}[v_a(t), v_a(t+\tau)] \cos \omega \tau d\tau + \\ & + 4 \int_0^\infty \overline{N_2(t) N_2(t+\tau)} \text{cov}[w_a(t), w_a(t+\tau)] \cos \omega \tau d\tau \end{aligned} \quad (2.1)$$

The group population spectra are found in Appendix 2.

The field and crystal-orientation dependent drift velocities, $\bar{v}$ and $\bar{w}$, are determined from some appropriate transport theory. There remains only the calculation of $\text{cov}[v_a(t), v_a(t+\tau)]$ and $\text{cov}[w_a(t), w_a(t+\tau)]$. These may be obtained directly from their definitions, as follows, for group 1:

$$N_1(t) \Delta v_a(t) = \sum_{j=1}^{N_1(t)} \Delta v_j(t)$$

$$N_1(t+\tau) \Delta v_a(t+\tau) = \sum_{k=1}^{N_1(t+\tau)} \Delta v_k(t+\tau)$$

Multiplying these two equations together and averaging;

$$\overline{N_1(t) N_1(t+\tau)} \text{cov}[v_a(t), v_a(t+\tau)] = \bar{N}_1 f_1(\tau) \text{cov}[v(t), v(t+\tau)] \quad (2.2)$$

where $f_1(\tau)$ is the fraction of electrons present in group 1 at time t that remain in group 1 until time $t+\tau$.

$v(t)$ is the axial velocity component of any group 1 electron.

The departure of electrons from group 1 to group 2 is governed by the equation (Appendix 2),

$$\frac{dN_1(t)}{dt} = -p_{12} N_1(t)$$

where $p_{12}N_1$ is the probability per unit time of a transition from group 1 to group 2.

Hence $f_1(\tau)$ has the form $f(\tau) = e^{-p_{12}\tau}$.

$\text{cov}[v(t), v(t+\tau)]$ may be found if we make the assumption that intravalley scattering involves only elastic collisions. Then, if η is the angle of deflection as a result of a single scattering event,

$$\text{cov}[v(t), v(t+\tau)] = \text{var } v e^{-\tau/\tau_1}$$

where the relaxation time τ_1 is given by (Wannier):

$$\frac{1}{\tau_1} = \left\langle \frac{1 - \cos \eta}{S_1} \right\rangle$$

S_1 is the time between collisions for electrons in group 1. The average indicated by the angular brackets is performed over all possible intravalley transitions. Similar expressions may also be written for group 2 electrons, in which case τ_2 and S_2 replace τ_1 and S_1 respectively.

Assuming that $\frac{1}{\tau_1}$ is much greater than p_{12} , the time dependence of the right-hand-side of equation 2.2 is determined by $\text{cov}[v(t), v(t+\tau)]$ so that equation 2.2 takes the form

$$\overline{N_1(t)N_1(t+\tau)} \text{cov}[v_a(t), v_a(t+\tau)] = \overline{N_1} \text{var } v e^{-\tau/\tau_1}$$

Similarly if $\frac{1}{\tau_2}$ is much greater than p_{21} ,

$$\overline{N_2(t)N_2(t+\tau)} \text{cov}[w_a(t), w_a(t+\tau)] = \overline{N_2} \text{var } w e^{-\tau/\tau_2}$$

The current spectral density, as given by equation 2.1, with the above expressions along with the population

spectra taken from Appendix 2, now takes the form:

$$\begin{aligned} \frac{L^2}{q^2} S_I(\omega) = & \frac{4(\bar{v}-\bar{w})^2 p_{12} p_{21} N}{(p_{12}+p_{21})[\omega^2 + (p_{12}+p_{21})^2]} + 4\bar{N}_1 \text{var} v \int_0^\infty e^{-z/\tau_1} \cos \omega z dz \\ & + 4\bar{N}_2 \text{var} w \int_0^\infty e^{-z/\tau_2} \cos \omega z dz \end{aligned}$$

Carrying out the integrations gives

$$\begin{aligned} S_I(\omega) = & \frac{4q^2}{L^2} \left\{ \frac{(\bar{v}-\bar{w})^2 p_{12} p_{21} N}{(p_{12}+p_{21})[\omega^2 + (p_{12}+p_{21})^2]} + \right. \\ & \left. + \frac{\bar{N}_1 \text{var} v \tau_1}{1 + (\omega \tau_1)^2} + \frac{\bar{N}_2 \text{var} w \tau_2}{1 + (\omega \tau_2)^2} \right\} \end{aligned}$$

The differential conductance, $G_i(\omega)$, due to the electrons of the i th group is, (Wannier)

$$G_i(\omega) = \frac{q^2 \bar{N}_i \tau_i}{L^2 m_i (1 + \omega^2 \tau_i^2)}$$

where m_i is the effective mass of the i th electron in the field direction and τ_i is the intravalley velocity relaxation time.

We may check these formulas by considering the equilibrium zero field case. Then $\bar{v}$ and $\bar{w}$ are zero and the intravalley velocity relaxation times are equal to τ_0 , the same for all valleys.

If we assume the total conductance is equal to the sum of the separate group conductances, ignoring the contribution of inter-group transitions to $G(\omega)$, then, in equilibrium

$$G(\omega) \equiv G^0(\omega) = \frac{q^2 \tau_0}{L^2 (1 + \omega^2 \tau_0^2)} \left[\frac{\bar{N}_1}{m_1} + \frac{\bar{N}_2}{m_2} \right]$$

The current spectral density is, at equilibrium,

$$S_I(\omega) = 4 \frac{q^2}{L^2} \left[\bar{N}_1 \text{var} v + \bar{N}_2 \text{var} w \right] \frac{\tau_0}{1 + (\omega \tau_0)^2}$$

In the above expression, p_{12} and p_{21} are taken as zero, which is consistent with the earlier assumption that $G^\circ(\omega) = G_1^\circ(\omega) + G_2^\circ(\omega)$

In the equilibrium case, $\text{var} v$ and $\text{var} w$ obey the simple relations given in Appendix 3,

$$m_1 \text{var} v = K T_L \quad m_2 \text{var} w = K T_L$$

The current spectral density then correctly assumes the form given by the Nyquist formula,

$$S_I(\omega) = \frac{4 q^2 \tau_0}{L^2 (1 + \omega^2 \tau_0^2)} \left[\frac{\bar{N}_1}{m_1} + \frac{\bar{N}_2}{m_2} \right] K T_L = 4 G^\circ(\omega) K T_L$$

Returning to the non-equilibrium case, we make the following inequalities in accordance with the experimental conditions:

$$\omega \tau_j \ll 1, \quad p_{ij} \gg \omega$$

The current spectrum then assumes its low-frequency form,

$$S_I = \frac{4 q^2 N (\bar{v} - \bar{w})^2 p_{12} p_{21}}{L^2 (p_{12} + p_{21})^3} + 4 \frac{q^2}{L^2} \left[\bar{N}_1 \tau_1 \text{var} v + \bar{N}_2 \tau_2 \text{var} w \right]$$

The noise temperature, T_n , of the sample under non-equilibrium conditions is a convenient way of describing its behaviour. It is found using the Nyquist formula

$$S_I = 4 K T_n G$$

where G is the sample differential conductance, given by the sum of the contributions from each group of the electron population.

Thus
$$G = G_1 + G_2 = \frac{\partial(I_1 + I_2)}{\partial V}$$

where $\bar{I}_i$ is the mean current carried by group i electrons and V is the applied voltage.

From the field dependence of $\bar{v}$ and $\bar{w}$, G_1 and G_2 may be found. The total differential conductance is then

$$G = \frac{q}{L^2} \left[\bar{N}_1 \frac{d\bar{v}}{dF} + \bar{N}_2 \frac{d\bar{w}}{dF} \right]$$

and the noise temperature

$$T_n = \frac{q^2 N (\bar{v} - \bar{w})^2 p_{12} p_{21}}{L^2 G (p_{12} + p_{21})^3 K} + \frac{G_1 m_1 \text{var } v + G_2 m_2 \text{var } w}{K G}$$

Thus we find that the noise temperature contains a contribution due to the difference in drift velocity of the different valleys. This contribution, termed intervalley noise by Price(1965) is, due to the $(\bar{v} - \bar{w})^2$ factor, strongly dependent on the sample orientation.

The second contribution, the "hot electron" contribution, represents the increase in sample noise due to the increase in the variance of the velocity distribution in each valley.

CHAPTER 3 - EXPERIMENTAL APPARATUS AND TECHNIQUE

A series of determinations of noise temperature, T_n , and differential conductance, G , were made as functions of applied electric field on rectangular, single crystal n-type germanium samples whose lengths were in the $\langle 111 \rangle$, $\langle 110 \rangle$, and $\langle 100 \rangle$ crystal directions. The samples in each case were cooled to liquid nitrogen temperatures and measurements were performed at frequencies of 30 Mc/s and 70 Mc/s. The same measurement technique was used for all samples.

Basically, the apparatus consisted of:

- 1) A low output impedance voltage pulse generator for supplying $10\mu\text{sec.}$ pulses to samples under study. The sample resistances were of the order of one hundred ohms.
- 2) A variable frequency multivibrator for triggering the voltage generator and the delay multivibrator (see 4 below).
- 3) A low-noise gated 30 Mc/s amplifier, along with 30 Mc/s and 70 Mc/s preamplifiers and output meter for measuring the sample noise only during a $5\mu\text{sec}$ interval centered on the sample voltage pulse.
- 4) One delay multivibrator and square-wave voltage generator for supplying the gating pulse to the amplifier.
- 5) A temperature-limited noise diode which served as a noise reference source.
- 6) A set of deposited carbon resistors whose conductance

at the two measuring frequencies were known.

- 7) A sample mount consisting of appropriate filters and designed for four-terminal sample noise measurements. The mount had to have provision for holding liquid nitrogen and yet be compact so as to minimize the lead inductance between the sample and the input terminals of the preamplifier.

3.1 APPARATUS DESIGN CONSIDERATIONS

The apparatus served to determine the noise equivalent circuit of the sample under high field conditions but with minimal sample heating. For the desired impurity range, of the order of 10^{15} cm^{-3} used in the experiment, practical sample sizes results in resistances in liquid nitrogen of the order of 100 ohms. Since electric fields of at least several hundred volts per centimeter are required to produce a measurable amount of noise, joule heating is extreme unless very short pulses are used. Hence, the average power dissipated by the sample must be very small. With these requirements in mind a pulse duration of $10 \mu\text{sec.}$ was chosen, with a repetition rate of from nine pulses per second to 210 per second.

The next consideration is the measurement of the sample noise. Since the noise must be measured only under pulsed field conditions, the measurement frequency should be much greater than the reciprocal of the pulse duration so that a number of cycles of the noise should be visible. This indicates a value of 10 Mc/s to 100 Mc/s. A choice of frequency in this range is also desirable since the

$1/f$ and the excess noise terms would be expected to be small, as discussed in Chapter 1. As a check that the frequency-dependent noise sources are negligible, measurement of noise temperature at two widely different frequencies in this range are desirable. The lowest of the two frequencies was chosen at 30 Mc/s, because of the availability of commercial amplifiers at this frequency. A higher frequency of 70Mc/s was chosen arbitrarily.

The 30 Mc/s amplifier consisted of a low noise cascode preamplifier followed by five synchronously tuned 30 Mc/s pentode stages. Vacuum diode detection was used. The detector was followed by a two stage video amplifier with its last stage a cathode follower.

The 70 Mc/s amplifier consisted of the same apparatus as used at 30 Mc/s except the 30 Mc/s preamplifier was replaced by a 70 Mc/s cascode and a mixer stage.

In order to achieve maximum sensitivity of the output meter to the pulse sample noise, the amplifier gain was gated so that only during the "pulse on" time was the gain non-zero. This was achieved by replacing the last radio-frequency stage of the amplifier, normally a 6AK5 pentode, by a 6AS6, which is similar to the 6AK5 except that its suppressor grid is not internally connected to the cathode. A gating pulse sufficient to "cut-off" the 6AS6 was then applied to its suppressor grid. The duration of the gating pulse was set at about one-half that of the sample pulse so as to avoid any transient voltages from entering the amplifier due to the switch-on and switch-off of the sample current.

Such transients, if present, would be visible on the monitoring oscilloscope. Their possible presence was also checked by placing a metal resistor across the amplifier input terminals in place of the germanium sample and pulsing it with the same voltage and current magnitudes. Since no increase in the amplifier output meter reading occurred, it was concluded that the system was free from transient effects.

Due to the low repetition rate and the short duration of the noise pulse from the amplifier, the measurement of the noise power of the pulsed sample was out of the question. Instead, a "peak-reading" voltmeter was used for the output meter. Since the measuring procedure called for a comparison of two noise sources, the peak-reading voltmeter was used to indicate the equivalence of two noise sources.

3.2. SAMPLE HOLDER DESIGN

The first experimental sample holder tried was a styrofoam bottle with stainless steel connections to the two current leads of the sample. The current leads served also as the noise sensing electrodes. The sample was cut in the form of a rectangular cross-section bar with gold wire leads alloyed to the two ends, so that the pulse current and the sample noise were both conducted by the same electrodes. This arrangement is perfectly satisfactory if no additional noise is introduced at the junctions between the semiconductor and the wire lead. However, since precautions against junction noise were taken by van der Ziel and associates, (van der Ziel, 1954) in their low frequency

measurements referred to in Chapter 1, junction noise may also be present at the higher frequencies as well. Preliminary two-terminal noise temperature measurements in $\langle 111 \rangle$ samples showed consistency between samples, independent also of the direction of pulse current. In the $\langle 100 \rangle$ samples, however, noise much in excess of the $\langle 111 \rangle$ sample noise and dependent on the direction of current flow through the sample was observed. This behaviour was interpreted as evidence of junction noise and the two-terminal measurements were discontinued in favour of four-terminal determinations.

The four-terminal measurements were made on samples cut as shown in Fig.3.1. The pulse current leads are attached to the two ends and the noise measurements made between the two side contacts. In this manner, any noise generated by the passage of current through a metal-semiconductor junction does not appear directly across the input terminals of the amplifier. However, part of the noise generated at the current electrodes may still enter the amplifier unless efforts are made to isolate the current electrodes from ground potential. This was done by means of filters, as described in Section 3.3, tuned to the center of the amplifier pass band. It is then necessary to check that any junction noise entering the amplifier is small compared to the noise appearing across the side contacts of the sample. This was done by using two values of impedance for each filter. All filters had band-widths much less than the amplifier pass band and hence the

absolute value of impedance, $|Z|$, of each filter over the pass band was determined by its capacitance, C , and inductance, L , values. The relation for Z is

$$|Z| \approx \frac{\omega}{C_0|\omega - \omega_0|(\omega + \omega_0)}$$

where $\frac{\omega}{2\pi}$ is the frequency and $\frac{\omega_0}{2\pi}$ is the resonant frequency of the filter. The filter capacitance, C_0 , is composed of an external variable part and the distributed coil capacitance. Filters having different values of C_0 were used to vary the amount of attenuation of any junction noise present at the current electrodes. Since no dependence of noise temperature on filter capacitance was found, it was concluded that all filters were successful in attenuating the junction noise well below the side electrode noise.

3.3 MEASUREMENT OF FILTER CONSTANTS

It was decided to mount the filters along with the sample in the liquid nitrogen, since cooling the coils increased the value of the parallel resistance, R_1 , and hence decreased the filter bandwidth. We assume each filter to consist of a parallel combination of the inductance L_0 , distributed capacitance C_d , and external capacitance C_1 , with shunt resistance R_1 .

L_0 and C_d for each filter in liquid nitrogen were found by adding known values of fixed mica condensers for C_1 and determining the resonant frequency for each value of C_1 . The resonant frequency was found using a

grid-dip meter whose frequency was compared against a Marconi Q-Meter as standard. L_0 was then found from the slope of a plot of ω_0^{-2} vs. C_1 , and C_d was obtained by subtracting C_1 from the total capacitance $\omega_0^{-2} L_0^{-1}$, for each value of C_1 . The final value of C_d was found by averaging each of these values.

The shunt resistance, R_1 , was found in the following way. The variable mica condensers were now placed in parallel with each coil, the combination now constituting the form of the filters used in the later noise temperature measurements. Both coil losses and dielectric losses in the variable condenser will contribute to R_1 .

The filter was mounted in a styrofoam container holding liquid nitrogen. Electrical connections consisting of $\frac{1}{8}$ inch by $\frac{3}{8}$ inch brass strips were sealed into the bath bottom so that the filter, cooled to liquid nitrogen temperatures could be connected directly across the Q-Meter terminals. The connections being short and of low inductance contributed little to the coil impedance. The measurement procedure consisted of resonating the Q-Meter with a high Q coil at the filter resonant frequency. The filter was then connected across the condenser terminals of the Q-Meter and the Q-Meter capacitor changed slightly, if necessary, to bring it into resonance. Then the decrease of the Q-Meter reading, due to the shunt resistance of the filter, allows the value of the shunt resistance to be found.

Fig.3.1 - Typical four probe sample -Scale 17:1

Fig.3.2. Sample holder and nitrogen bath - Scale 1.66:1

Fig.3.3.3

The diagram shows a control system with the following components and connections:

- Block 1** is connected to **Block 2** and **Block 17**.
- Block 2** is connected to **Block 3**.
- Block 3** is connected to **Block 4**.
- Block 4** is connected to **Block 5**.
- Block 5** is connected to **Block 6**.
- Block 6** is connected to **Block 7** (inductor with a switch) and **Block 8** (inductor with a switch).
- Block 7** and **Block 8** are connected to **Block 9** (a horizontal bar with a central tap).
- Block 9** is connected to **Block 10** (a vertical line with a capacitor).
- Block 10** is connected to **Block 11** (a circle with a vertical line and a switch).
- Block 11** is connected to **Block 12**.
- Block 12** is connected to **Block 13** (dashed box) and **Block 15**.
- Block 13** is connected to **Block 12** by a dashed line.
- Block 15** is connected to **Block 14** and **Block 16**.
- Block 16** is connected to **Block 17**.
- Block 17** is connected to **Block 1**.

Apparatus used in the determination of sample noise temperature and differential conductance. Numbers and corresponding unit titles follow in Fig.3.3a.

Apparatus Unit Titles (refer to Fig.3.3)

<u>Number</u>	<u>Description</u>
1	Variable frequency multi-vibrator
2	10 μ sec. rectangular pulse generator
3	Pulse amplifier
4	Fixed resistor
5	Pulse attenuator
6	Oscilloscope
7 & 8	Filters
9	Germanium sample
10	Liquid nitrogen bath
11	5722 Noise diode
12	30 Mc/s or 70 Mc/s pre-amplifier
13	100 Mc/s oscillator
14	Peak reading voltmeter
15	30 Mc/s gated amplifier
16	4 μ sec. rectangular pulse generator
17	3 μ sec. delaying multi-vibrator

3.4 NOISE TEMPERATURE MEASUREMENT PROCEDURE

The first step in the determination of the sample noise temperature consisted in determining the differential sample conductance under the pulsed electric field. The following procedure accomplished this.

Set the noise diode current near its maximum rated value and across the input terminals of the amplifier place a known conductance, G . Adjust the amplifier gain to give a reading X on the amplifier output meter, which we designate by OM . Then, replace G by the sample, pulsed by the electric field. Noting the reading of OM , we increase the pulsed field applied to the sample until OM reads X . If the noise diode current has been chosen high enough to "swamp out" all other noise sources in the sample or the known conductance G , then the sample conductance will now equal the known conductance. To check that all other noise sources have been overwhelmed by the noise diode, it is only necessary to repeat the above procedure for a smaller value of the noise diode current. The same field should be obtained in both cases.

Once the sample differential conductance has been found, the measurement of the sample noise temperature proceeds in either of two ways, denoted by cases A and B.

Case A (Used for noise temperature greater than room temperature)

Considering the sample's Norton noise equivalent circuit, the mean-square noise current generator in the

frequency range Δf is $\overline{i_s^2}$ and is given by

$$\overline{i_s^2} = 4KT_n G \Delta f$$

It is in parallel with the conductance G .

With the sample across the amplifier input the output meter reading is some value, say X . The known conductance along with its noise current generator, denoted by $\overline{i_r^2}$, is then substituted for the sample and the noise diode current varied until OM reads X once again. Then, the two current generators are the same, since the measured noise is the same in both cases and both generators are in parallel with the same value of conductance. Since the noise diode current generator and the sample noise generator are independent, we write

$$\overline{i_s^2} = \overline{i_r^2} + \overline{i_n^2}$$

where $\overline{i_n^2}$ is the noise diode current generator.

Substituting the formulas for the above current generators, leads to

$$4KT_n G \Delta f = 4KT_o G \Delta f + 2qI_d \Delta f$$

where I_d is the mean noise diode current.

The noise temperature is then

$$T_n = T_o + \frac{qI_d}{2KG}$$

Case B (Used for noise temperatures less than room temperature)

In this case, the known conductance is first connected across the amplifier input and the gain adjusted so that OM reads some convenient value X . The sample under

field is then connected to amplifier input and the noise diode current adjusted until OM reads X . Equating current generators gives,

$$\overline{I_r^2} = \overline{I_s^2} + \overline{I_n^2}$$

and the noise temperature is

$$T_n = T_o - \frac{q I_d}{2KG}$$

Trial and error alone determine whether case A or B is appropriate. For example, if T_n is greater than T_o and case B is tried, then OM will read greater than X when the sample is applied to amplifier input and it is obvious that the noise temperature of the sample is greater than room temperature, and case A procedure must be used to determine the noise temperature.

3.5 MEASUREMENT OF PULSE VOLTAGES

For each determination of noise temperature, the sample current I and the voltage V applied across its current terminals were also found. I was found from the voltage drop across a fixed 118 ohm wire resistor in series with the sample but at room temperature. All voltage measurements were obtained by applying the unknown voltage pulses to a 6.15 to 1 pulse attenuator to which a variable D.C. voltage, $V_{d.c.}$, was applied at an internal point, through a switch. Using the oscilloscope as a detector, the switch was opened and closed, while $V_{d.c.}$ was varied until it equalled the attenuated pulse voltage.

The resistors used in the compensated pulse attenuator were of deposited carbon construction and of low

voltage coefficient. The attenuation was checked at a 280 volt pulse amplitude and found to be within one per cent of the low voltage value.

3.6 DETERMINATION OF SAMPLE ELECTRIC FIELD

Experimentally it was found that for the sample geometry used, a side electrode and the center line of the side-arm extended through the sample form an equipotential surface. The electric field F in the "active" part of the sample, that is, the part between the side probes was then taken as

$$F = \frac{V_2 - V_3}{L}$$

where V_2 and V_3 are the side electrode voltages and L is the center line separation of the side-arms. Empirically, it was found that

$$V_1 = C_2 (V_2 - V_3)$$

where V_1 is the voltage across the current electrodes, and C_2 is constant for each sample. This means that the electric field may be determined from a measurement of V_1 alone. That is,

$$F = \frac{V_2 - V_3}{L} = \frac{V_1}{C_2 L}$$

The length $C_2 L$ is found to be less than the sample current electrode separation; the difference is likely due to penetration of the current electrodes into the sample interior and the consequent lowering of the current electrode separation.

3.7 SAMPLE PREPARATION

The single crystal from which the samples were prepared was grown from the melt by the Czochralski technique. The axis of the crystal was in the $\langle 110 \rangle$ direction. A value of impurity density was chosen so as to produce a resistivity of about five ohm-cm at the center of the crystal. This was the value of resistivity for which the most recent experimental work on anisotropy of hot electron conductivity had been done and so for the sake of continuity in the experiments this seemed the most reasonable choice. The grown crystal exhibited six faces along the direction of growth. Laue x-ray analysis established four of these faces as $\langle 111 \rangle$ and two as $\langle 100 \rangle$. The $\langle 110 \rangle$ direction was found as well.

Next, slices were cut at right angles to the growth axis of the crystal by means of a 0.003 inch diameter tungsten wire, driven in a reciprocating wire saw. Carborundum was used as the cutting abrasive. The slices were then lapped to a thickness of about 0.4 millimeter with a variation in thickness over each slice of about 5%. Four terminal samples (Fig.3.1) were cut from the slices using an ultrasonic cutting tool and carborundum as the cutting abrasive. The samples were then etched in medium CP4 solution for about 45 seconds to clean and prepare the surface for the attachment of the four gold wire leads. These leads consisted of 0.005 inch diameter gold, doped with 0.6% antimony, according to the Sigmund Cohn Co., the manufacturer. When brought into contact with

the etched germanium in a nitrogen gas atmosphere at a temperature above the gold-germanium eutectic temperature, a liquid phase consisting of germanium, antimony, and gold is formed. The temperature is then lowered and the liquid phase solidifies to form an alloy region between the gold wire and the sample. Upon solidification, a germanium regrowth layer, highly doped with antimony forms on the sample surface. This junction has been termed a n^+-n junction, the n^+ referring to the highly doped regrowth layer, and the n to the sample conductivity. Electrically, these junctions perform in such a manner that they are highly impervious to hole current while allowing electron current passage. Expressions have been derived for the hole current but depend in part on the difficult to determine properties of the highly doped regrowth layer of the junction. In this experiment, the possibility is always assumed that hole injection takes place and indirect means adopted to assess its importance.

As a final step, the samples were etched for five seconds in a medium CP4 solution.

3.8 SAMPLE TEMPERATURE RISE DUE TO JOULE HEATING

With no voltage pulse applied to the sample, it will remain in thermal equilibrium with the liquid nitrogen bath surrounding it at the ambient temperature, θ_a . We now consider the sample temperature when a succession of voltage pulses of duration α and frequency $(\alpha+\beta)^{-1}$, where β is the off-time between pulses. After a sufficiently long time the sample will be in a steady-state

when the heat generated by a single pulse is removed from the sample during the off time. This heat removal can be by conduction and convection to the liquid nitrogen and by conduction through the sample lead wires. We assume that the thermal conductivity of the sample is high enough so that the radial temperature gradient is small and that the heat conducted by the leads is small compared to the losses from the surfaces to the liquid nitrogen. The sample then may be characterized by a uniform temperature, $\theta(t)$ which, in the steady-state is cyclic in time t .

In the steady-state, $\theta = \theta_i$ at the start of the voltage pulse, rises to θ_f at the end of the pulse, then decays back to θ_i at the start of the next pulse. Assuming Newton's law of cooling, we get, for t in the range from zero to α ,

$$\frac{d\theta}{dt} = \frac{P}{M_0 C_p} - \frac{(\theta - \theta_a)}{t_0}$$

where P is the pulse power dissipated during the pulse,

M_0 is the sample mass, C_p is its specific heat, and t_0 is its thermal time constant.

In the interval $\alpha < t < \alpha + \beta$,

$$\frac{d\theta}{dt} = - \frac{(\theta - \theta_a)}{t_0}$$

By experiment, t_0 was found to be about 0.05 seconds.

For all measurements, α is constant and equal to 10^{-5} sec.

Then, for α/t_0 much less than unity,

$$\theta_i \approx \theta_a + \frac{P \alpha}{M_0 C_p (e^{P/t_0} - 1)}$$

Also,

$$\theta_f \approx \theta_a + \frac{\propto P}{M_o C_p} \frac{e^{P/t_o}}{(e^{P/t_o} - 1)}$$

Putting in extreme values of $P = 200$ watt

$$M_o = 4.4 \times 10^{-3} \text{ gm}$$

$$C_p = 0.153 \frac{\text{joule}}{\text{gm degree}}$$

Then $\theta_i - \theta_a \approx 0.4^\circ$

$$\theta_f - \theta_a \approx 3^\circ$$

So, the maximum "background" temperature rise is less than 0.5 degrees, and the rise during the pulse of the order of 3 degrees.

Taking the case of low electric field and high pulse repetition rate results in the following values:

$$P = 9.0 \text{ watt}$$

$$\beta = 4.8 \times 10^{-3} \text{ sec.}$$

Then $\theta_i - \theta_a \approx 1.4^\circ$

and $\theta_f - \theta_a \approx 1.5^\circ$

3.9 INFLUENCE OF SKIN EFFECT ON SAMPLE DIFFERENTIAL CONDUCTANCE

The skin depth, δ , is given by

$$\delta = \left(\frac{2}{\omega \sigma \mu_o} \right)^{1/2}$$

where $\frac{\omega}{2\pi}$ is the frequency, σ the conductivity, and μ_o is the permeability of the medium.

For a cylindrical sample of radius r_o , the ratio of the resistance at frequency f to the zero-frequency resistance, R_o , is approximated as

$$\frac{R_f}{R_o} \simeq 1 + \frac{1}{48} \left(\frac{r_o}{\delta} \right)^4$$

The above approximation holds only for skin depth much greater than the radius.

At 77°K, sample resistivity was about 0.3 ohm-cm. and the mean cross-sectional area of the order 0.1 mm².

Then,

$$\frac{R_f}{R_o} \simeq 1 + 3.25 \times 10^{-7}$$

and skin effect is completely negligible.

In the case of the fine gold wire leads, we find skin effect is appreciable, but can be neglected anyway. The resistivity of the gold wire was about 10⁻⁶ ohm-cm. For a diameter of 0.005 inches, the ratio R_f/R_o is about five. For a typical length of 3 mm,, the effective resistance is about 0.01 ohms, which is small compared to the sample resistance and can therefore be neglected.

3.10 INVESTIGATIONS OF SPURIOUS SOURCES OF NOISE

Hundred volt pulses resulting in currents of the order of one ampere were applied to the sample. The possible existence of transient voltages as well as noise generated in the soldered junctions and pressure metal-to-metal contacts entering the amplifier must be investigated.

Positioning the "on" pulse to the amplifier at the center of the voltage pulse supplied to the sample minimizes the transients. Any large transients picked up by the amplifier should be visible on the oscilloscope monitoring the output noise of the amplifier. None was observed in the course of the experiments. Small transients as well as

spurious noise generated at the soldered joints and pressure contacts were investigated in the following way.

A metal resistor in the form of a fine wire was substituted for the sample in the bath of liquid nitrogen. The impedance of the wire was resistive of the same order as a typical germanium sample and had as well a small inductive component in series.

No additional noise was observed from the amplifier when the wire sample was pulsed. Hence, it was concluded that the system was free from transient effects from the sample pulse as well as spurious electrode noise external to the sample.

Another possible source of noise is due to optical generation of hole-electron pairs in the sample due to the room illumination. This was checked and found to be non-observable since the pulsed sample noise did not vary when the illumination was changed from the normal amount to zero, when all lights were turned off and the sample bath covered with a black cloth.

3.11 SAMPLE NOISE DUE TO HOLE GENERATION AT THE CURRENT ELECTRODES

The possible existence of additional noise due to hole injection at the positive sample current electrode can only be investigated indirectly in the case of the four terminal samples. For the two terminal noise measurements a more direct check was possible.

The first noise temperature measurements, performed

on rectangular bars in the $\langle 111 \rangle$ direction were about the same as obtained later with the four terminal samples. To eliminate possible hole injection noise in the two terminal measurements, a special sample consisting of a rectangular filament in the $\langle 111 \rangle$ direction with a large block end was cut. The usual gold wire leads were alloyed to one corner of the block and to the end of the filament. The block electrode dimensions were chosen so that any holes injected into the block electrode would drift on the average only one-half the block electrode length during the pulse duration. Hence, if the block electrode were positive, during the voltage pulse the high resistance part of the sample would be relatively free of injected holes. Measurements of noise temperature on this sample showed no dependence on pulse polarity and were the same as obtained with the previous two terminal samples. Hence it was concluded that the noise measured in the $\langle 111 \rangle$ direction was not due to hole injection.

In the case of the four terminal samples, further information on hole injection was obtained with noise temperature measurements on the same sample at both 70 Mc/s and 30 Mc/s. Any noise due to hole injection should exhibit a strong frequency dependence over this range, since the hole current spectral density should be of the form

$$S_{I_h}(\omega) = 4q I_p \frac{(1 - \cos \omega t_p)}{\omega^2 t_p^2}$$

where I_p is the hole current and t_p , the hole transit

time is of the order of 10^{-7} seconds.

Here it is assumed that hole recombination may be neglected due to their brief transit time. The lifetime of holes was determined for a typical sample and found to be about 10^{-4} seconds, which is several orders of magnitude greater than the transit time.

The absence of frequency dependent noise in the samples used then indicates that hole injection noise could be considered small.

CHAPTER 4 - EXPERIMENTAL RESULTS

The experimental results consist of noise temperature, T_n , and differential conductance, G , measured at both 30 Mc/s and 70 Mc/s, as a function of electric field, F , for the sample axis in the $\langle 111 \rangle$, $\langle 110 \rangle$, and $\langle 100 \rangle$ directions. The sample material, of n-type germanium was of one value of impurity density. The lattice temperature was within the range of 77°K to 80°K .

The most prominent features of the noise temperature are its apparent lack of frequency dependence over the range 30 Mc/s to 70 Mc/s as well as its high degree of anisotropy.

The lack of frequency dependence suggests that the events contributing to the current fluctuations have a mean duration τ , such that $\omega\tau$ is much less than unity. At 70 Mc/s then, τ is less than 5×10^{-10} seconds. Certainly the carrier mean free time is less than τ so that thermal noise would be expected to be frequency independent in the range of measurements. Also, since the average time an electron spends in one valley of the conduction band is of the order of 2×10^{-10} seconds, (Appendix 5) any modulation of the current arising from such valley-valley transitions would also be independent of frequency in the measured range.

Erlbach and Gunn(1962), by measuring the current fluctuations at right angles to the electric field also found anisotropic behaviour at a lattice temperature of

77°K. However, the dependence of the noise temperature, T_n' , representing the current fluctuations perpendicular to the field, on field was found to be linear with electric field over most of the range investigated by them, which was of the region up to one thousand volts per centimeter. For our experimental results, both $\log T_n$ vs. F and $\log T_n$ vs. $\log F$ plots were made to see if any simple relationship existed between T_n and F but none was found.

Most of the experimental error in both T_n and G was due to fluctuations in the meter monitoring the output noise of the amplifier, the maximum estimated error in a single reading being about 10% for the noise temperature and about 7% for the differential conductance. An error of about 4% was present in the values of field.

Fig. 4.1

Fig. 4.2

Fig. 4.7

Fig. 4.4

Fig. 4.6

CHAPTER 5 - COMPARISON OF EXPERIMENTAL RESULTS WITH THEORY

In this chapter, the results of Chapter 4 are compared with the theoretical values of T_n and G obtained from the formulas of Chapter 2 after the appropriate field-dependent and orientation-dependent parameters are found.

The final expression for the sample noise temperature, as derived in Chapter 2, shows two independent terms: the intervalley and the hot electron contributions.

To evaluate the intervalley contribution, the following is required:

- 1) The drift velocity of electrons in any valley, for the electric field in any one of the three principal crystal orientations. Barrie's extension of Stratton's theory (Barrie and Burgess, 1962) as discussed later, supplies the required drift velocities.
- 2) The absolute values of the intergroup transition probabilities, P_{ij} , at any field strength in the three principal crystal directions. For the case of zero field the P_{ij} may be taken from the data of Weinreich et al, (Appendix 3). For non zero fields, the different valley electron temperatures, as found from the Barrie theory, must be used to deduce the steady-state valley populations and the intervalley transition probabilities. This is done in Appendix 2.
- 3) The sample differential conductance, calculated from the values of drift velocity given by the Barrie theory. From the drift velocities the mobilities are found;

the quantities finally used for the differential conductance are $\frac{d}{dF}(\mu_1 F)$ and $\frac{d}{dF}(\mu_2 F)$.

The noise temperature contribution due to "hot electrons" requires the differential conductance of each of the valley groups; this is obtained from the mobility results, as in 3) above. As well as the differential conductance, the valley electron temperatures are needed; these are given by the Barrie theory.

5.1 DETERMINATION OF GROUP DRIFT VELOCITY FROM THE BARRIE THEORY

Barrie's extension of Stratton's high field theory (Barrie and Burgess, 1962) consists of two equations relating the drift velocity, $\vec{v}^{**}$ of electrons in a single valley to the electric field, $\vec{F}^*$, where the star denotes values in the transformed $\vec{k}$ -space.

The equations are:

$$\begin{aligned} \vec{F}^* \cdot \vec{v}^{**} = & \frac{32}{9\pi} \sqrt{\frac{m_t^2 m_g}{m_o^3}} \cdot \frac{3}{4q} (2\pi m_o K T_L)^{1/2} \left[\frac{4 m_o^2 K T_L}{3\pi \hbar^4} \left(\frac{C^2}{MN} \right) \left(\frac{r_o}{r} \right)^{3/2} \left(1 - \frac{r}{r_o} \right) \left(\frac{2 m_t + m_g}{3 m_o} \right) + \right. \\ & \left. + \frac{3 m_o K \theta_o}{4\pi \hbar^4} (B_o V') \left(\frac{r_o}{r} \right)^{1/2} \left\{ (n_o + 1) e^{-r} - n_o \right\} \frac{r}{2} e^{1/2} K_1 \left(\frac{r}{2} \right) \right] \end{aligned}$$

$$\begin{aligned}
F^* = \vec{J}^{**} \left(\frac{32}{9\pi} \right) \sqrt{\frac{m_e^2 m_e}{m_o^3}} \left(\frac{3}{4q} \right) (2\pi m_o K T_L)^{1/2} \left(\frac{r_o}{r} \right)^{1/2} & \left[\frac{4m_o^2 K T_L}{9\pi \hbar^4 u^2} \left(\frac{C^2}{MN} \right) + \right. \\
+ \frac{m_o^2}{2\pi \hbar^4} B_o V' \left\{ \left([n_o + 1] e^{-r} + n_o \right) \frac{r^2}{4} e^{1/2} K_2 \left(\frac{r}{2} \right) + \right. & \\
+ \left. \left([n_o + 1] e^{-r} - n_o \right) \frac{r^2}{4} e^{1/2} K_1 \left(\frac{r}{2} \right) \right\} & \left. \right]
\end{aligned}$$

where $n_o = \frac{1}{e^{\theta_o/T_L} - 1}$ $r = \frac{\theta_o}{T}$ $r_o = \frac{\theta_o}{T_L}$

C and B_o are constants and V' is the crystal volume.

$$K_n(z) = \frac{\Gamma(\frac{1}{2}) (\frac{1}{2} z)^n}{\Gamma(n + \frac{1}{2})} \int_1^\infty e^{-zt} (t^2 - 1)^{n-1/2} dt$$

Hence, if $\vec{J}^{**}$ is known, the drift velocity, $\vec{U}$, of the valley electrons may be found from the relations:

$$\vec{J}^{**} = \frac{\hbar}{m_o} \vec{J}_{k_o}^* \quad \text{and} \quad \vec{U} = \sqrt{\frac{m_o}{m_t}} U_x^{**}, \sqrt{\frac{m_o}{m_t}} U_y^{**}, \sqrt{\frac{m_o}{m_z}} U_z^{**}$$

It should be noted that the Barrie equations are rather easy to apply if the direction of the electric field is known, as well as its magnitude. This is the situation in the present experiment, in which the samples were orientated along the principal crystal directions. Then, because of the symmetrical arrangement of the valleys about

the longitudinal sample axis, the field was known to be in the same direction as the longitudinal axis.

The complete procedure in solving the final equations of the Barrie theory then is as follows. This solution will give the electron temperature, T_i , for each valley as well as the drift velocities for any value of applied field, provided the field and sample current are in the principal crystal directions.

- 1) Solve the equations for v^{**} and F^* by inserting values of electron temperature, T , in the equations, starting with T equal to the lattice temperature and proceeding upwards.
- 2) Knowing the applied field $\vec{F}$, allows $\vec{F}^*$ to be found. From the dependence of $\vec{v}^{**}$ on $\vec{F}^*$, and knowing $\vec{v}^{**}$ and $\vec{F}^*$ are parallel, gives $\vec{v}^{**}$. Knowing F^* gives T , and hence the valley populations may be found. From $\vec{v}^{**}$ the drift velocity $\vec{v}$ of each valley is obtained. Then, taking components of $\vec{v}$ in the field $\vec{F}$ direction gives the group drift velocities, $\bar{v}$ and $\bar{w}$.

The relationship between some of these quantities is shown in a table on the following page.

The following table summarizes the information on the group transport properties:

Sample Axis Direction	ψ_1	ψ_2	$\frac{F_1^*}{F}$	$\frac{F_2^*}{F}$	$\frac{\bar{v}}{v^{**}}$	$\frac{\bar{w}}{v^{**}}$
100	54°8	---	β_1	---	β_1	---
110 Cold Valleys	35°2	---	β_4	---	β_4	---
Hot Valleys	--	90°	---	β_2	---	β_2
111 Cold Valley	---	0°	---	β_3	---	β_3
Hot Valleys	70°5	---	β_5	---	β_5	---

ψ_i is the angle between the major axis of a constant energy ellipsoid and the field direction.

$$\beta_1 = \left(\frac{2m_o}{3m_t} + \frac{m_o}{3m_e} \right)^{1/2}$$

$$\beta_4 = \left(\frac{2m_o}{3m_e} + \frac{m_o}{3m_t} \right)^{1/2}$$

$$\beta_2 = \left(\frac{m_o}{m_t} \right)^{1/2}$$

$$\beta_5 = \left(\frac{8m_o}{9m_t} + \frac{m_o}{9m_e} \right)^{1/2}$$

$$\beta_3 = \left(\frac{m_o}{m_e} \right)^{1/2}$$

5.2 CALCULATION OF DIFFERENTIAL CONDUCTANCE

The low frequency value of the differential conductance for one valley is given by

$$G_i = \frac{\bar{N}_i q^2 \tau_i}{m_i L^2}$$

This is also equal to $\frac{\partial \bar{I}_i}{\partial V} = q \frac{\bar{N}_i}{L^2} \frac{d(\mu_i F)}{dF}$

where μ_i is the group i mobility, $\bar{I}_i$ is the mean current of group i electrons, and $F = V/L$ is the applied field.

Values of $\mu_i F$ were obtained from Barrie's theory. At 50 volt per cm. intervals, the function $\mu_i F$ was numerically differentiated, using a five-point differentiation formula. The error involved in the differentiation was estimated in the following way. The differentiation was repeated using a three-point formula, and the results compared with the five-point formula values. No difference greater than 2.5% was found between them.

The total conductance G was found by adding the contributions from both groups,

5.3 DETERMINATION OF CARRIER DENSITY AND ELECTRON POPULATION

In the $\langle 110 \rangle$ direction sample, the resistivity was found to be 2.46 ohm-cm at 23°C, with an estimated error of 6% due to thickness and width variations along the sample length. Taking the mobility as being due to the lattice contribution of scattering only with the imp-

urity component small, results in a value of $6.5 \times 10^{20} \text{ m}^{-3}$ for the carrier density. To check that the impurity scattering is small at room temperature, the mobility as given by the Debye-Conwell theory, which includes the effect of both impurity and lattice scattering, was calculated and found to be in agreement with the lattice mobility alone to within less than one per cent.

In the $\langle 100 \rangle$ and $\langle 111 \rangle$ directions, the measured resistivities were 2.03 and 2.00 ohm-cm. respectively, with maximum errors of about 6%. The carrier densities are essentially the same at $8.0 \times 10^{20} \text{ m}^{-3}$.

In calculating the total sample electron population, the noise generating part of the sample of interest in the present investigation was considered to consist of the main body of the sample (Fig. 3.1) between the center lines of the side probes and excluding the side probes themselves, which would be largely low field regions.

The values of the total population obtained were:

$$\langle 110 \rangle \quad \bar{N} = 1.66 \times 10^{11}$$

$$\langle 100 \rangle \quad \bar{N} = 3.85 \times 10^{11}$$

$$\langle 111 \rangle \quad \bar{N} = 5.17 \times 10^{11}$$

6.4 THE HOT ELECTRON CONTRIBUTION TO THE NOISE TEMPERATURE

As given in Chapter 2, this contribution is

$$T_n^{\text{ho}} = \frac{\bar{N}_1 m_1 \text{var } v \frac{d(\mu_1 F)}{dF} + \bar{N}_2 m_2 \text{var } w \frac{d(\mu_2 F)}{dF}}{K \bar{N} \frac{d}{dF} (\mu_1 F + \mu_2 F)}$$

In Appendix 3, it is shown that for the distribution in $\vec{k}$ -space assumed by Barrie, the velocity variances are given by

$$\text{var } v = \frac{k T_1}{m_1} \quad \text{var } w = \frac{k T_2}{m_2}$$

where m_1 and m_2 are, respectively, the group 1 and group 2 effective masses in the field direction and T_i is the electron temperature of the i th group.

The final expression for the hot electron contribution is then

$$T_n^{\text{h.e.}} = \frac{\bar{N}_1 T_1 \frac{d(\mu_1 F)}{dF} + \bar{N}_2 T_2 \frac{d(\mu_2 F)}{dF}}{\bar{N} \frac{d(\mu_1 F + \mu_2 F)}{dF}}$$

The values of $\bar{N}_1/\bar{N}$ and $\bar{N}_2/\bar{N}$ are found from the valley population ratio, n_1/n_2 . For the sample axis in the $\langle 110 \rangle$ direction,

$$\frac{\bar{N}_1}{\bar{N}} = \frac{\frac{n_1}{n_2}}{1 + \frac{n_1}{n_2}}$$

For the $\langle 111 \rangle$ direction,

$$\frac{\bar{N}_1}{\bar{N}} = \frac{\frac{n_1}{n_2}}{3 + \frac{n_1}{n_2}}$$

Values of $\frac{n_1}{n_2}$ are found using relation A2.1 of Appendix 2, and require only the value of the group electron temperatures, T_1 and T_2 .

CHAPTER 6 - CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

The work described in the preceding chapters is the first experimental investigation of high current, high frequency noise in the field direction in an extrinsic semiconductor.

Two independent sources of noise have been used to interpret the results, intervalley noise and hot electron noise. An additional source, due to fluctuations in the total electron population, has been shown theoretically to be negligible.

In two crystal orientations, $\langle 111 \rangle$ and $\langle 110 \rangle$, the intervalley noise is calculated to dominate the hot electron noise, with the resulting $\langle 111 \rangle$ noise temperature greater than that of the $\langle 110 \rangle$. The calculated values of noise temperature agree well with the experimental results for the $\langle 111 \rangle$ direction and rather poorly for the $\langle 110 \rangle$.

In the $\langle 100 \rangle$ direction, the intervalley noise is predicted to vanish and only the hot electron contribution appearing in the noise temperature. The calculated hot electron noise is much smaller than the intervalley noise in the other two crystal orientations. This agrees roughly with the experimental results.

One possible source of error in the experimental values of noise temperature is due to slight misalignment of the sample axis with respect to the principal crystal direction. This would be particularly undesirable in the $\langle 100 \rangle$ samples,

where the noise temperature is so much smaller than for the other orientations. Any such misalignment would result in different transport properties of the different valleys and consequently, intervalley noise. For this reason, it is somewhat speculative to draw any conclusions from the discrepancies between theory and experiment for measurements in this direction.

Discrepancies between actual and predicted noise temperatures may be due to several simplifications contained in the theory. These are:

1) In the $\langle 111 \rangle$ and $\langle 110 \rangle$ current directions, the intervalley noise calculations include transitions only between the four $\langle 111 \rangle$ minima of the conduction band. However, at high field strengths when the electron temperatures of these valleys are high, transitions to the $\langle 000 \rangle$ minimum take place. Because of the low effective mass of 0.034 electron masses, electrons in this minimum should possess high mobility and would contribute to the intervalley noise.

2) The intervalley transition rate under equilibrium conditions was found from rather scanty data of Weinreich et al. Only five values of sample impurity were investigated and interpolating their data was rather uncertain.

In conclusion, it may be added that the experimental values of the differential conductance show quite good agreement with the predictions of the Barrie theory at the higher values of field. The poor agreement at low fields may be attributed to the neglect of impurity scattering in the theory.

From the results of this investigation it has not been proven that the anisotropy of noise temperature is due to the intervalley noise component alone.

Measurements made over a much greater frequency range than reported here and extending into the microwave region would be expected to show a frequency dependence if the major source of noise were due to intervalley transitions whereas the hot electron noise should be uniform beyond the microwave spectrum, dropping off with frequency at the order of 10^{12} cycles per second.

Some additional information might also be obtained from noise temperature determinations made at different lattice temperatures. Increasing the lattice temperature from 77°K decreases the electron mobility and increases the intervalley transition rate, resulting in a reduction in the intervalley noise.

Other semiconductors possessing less complex band structure might also be investigated. In particular, indium antimonide, which is believed to have spherical constant energy surfaces in $\vec{k}$ -space would show no intervalley noise.

Noise analogous to intervalley noise, and due to hole transitions between the degenerate valence bands of p-type germanium might be observable with the present experimental arrangement. Since the constant energy surfaces of the holes are spherical, the noise temperature should be isotropic.

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APPENDIX 1 - FLUCTUATIONS IN ELECTRON POPULATION OF SAMPLE

We wish ultimately to calculate the fluctuations in the total electron population of a typical sample under high electric field conditions. To do this we must first consider the equilibrium population fluctuations.

The system to be considered is a homogeneous impurity semiconductor crystal containing a fixed number N_0 of stationary positive ions and a fluctuating number $N(t)$ of electrons whose mean number $\bar{N}$ equals N_0 so as to preserve space-charge neutrality. Such a system constitutes a plasma in which the positive ions instead of being free are rigidly bound to fixed locations. The characteristics of such a system are determined by two interactions; the coulomb interaction which tends to create a uniform distribution of electrons amongst the fixed ions and the effects of thermal agitation of the electrons which tends to disrupt the uniform distribution. These two competing processes result in electrostatic shielding of both ions and electrons. In the case where the mean electron-electron coulomb interaction is small compared to the mean electron kinetic energy, the Debye-Hückel theory is applicable, and the effect of shielding results in the formula

$$V(\vec{r}) = \frac{q}{4\pi\epsilon r} e^{-r/D}$$

for the average electrostatic potential $V(\vec{r})$ at a distance $\vec{r}$ from the charge q . The length D , the Debye length, is given by

$$D = \left(\frac{kT\epsilon}{n_0 q^2} \right)^{1/2}$$

where ϵ is the permittivity and n_0 the mean electron density. Thus, enclosing every fixed or mobile charge in the medium is a sphere of charge neutrality whose radius is of the order of the Debye length.

Since the electrons are in thermal equilibrium with the lattice at a temperature T , they possess random thermal motions appropriate to T but constrained by the coulomb interaction. Such random motions and their associated current density fluctuations are described by the fluctuation-dissipation theorem, which is a generalization of the Nyquist theorem of current and voltage fluctuations in a dissipative network.

By applying the fluctuation-dissipation theorem to the longitudinal modes of propagation, Burgess(1965) found for the spectrum of charge density fluctuations:

$$\langle \rho_{\vec{r}}^2 \rangle = \frac{kT\epsilon \Gamma^2}{1 + D^2 \Gamma^2}$$

The Fourier component of wave vector $\vec{\Gamma}$ has the form

$$\rho_{\vec{r}} e^{i(\Gamma_x x + \Gamma_y y + \Gamma_z z + \phi)}$$

which, when integrated over the volume of the sample yields an expression for $(\Delta N)_{\vec{r}}$, the population fluctuation due to the $\vec{\Gamma}$ Fourier component.

$\langle (\Delta N)_{\vec{r}}^2 \rangle$ is then found, after averaging over an ensemble,

$$\langle (\Delta N)^2 \rangle_{\vec{r}} = \frac{8KTE\Gamma^2}{q^2(1+D^2\Gamma^2)} \frac{(1-\cos\Gamma_x L)}{\Gamma_x^2} \frac{(1-\cos\Gamma_y A)}{\Gamma_y^2} \frac{(1-\cos\Gamma_z B)}{\Gamma_z^2}$$

the phase factor ψ having disappeared. L , A , and B are the dimensions of the parallelepiped sample in respectively the x , y , and z directions. The expression for $\text{var}N$ is obtained by integrating over all modes of $\vec{\Gamma}$, the mode density being $(8\pi^3)^{-1}$ per unit volume of $\vec{\Gamma}$ space, to yield:

$$\text{var}N = \frac{KTE}{\pi^3 q^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\Gamma^2}{(1+D^2\Gamma^2)} \frac{(1-\cos\Gamma_x L)}{\Gamma_x^2} \frac{(1-\cos\Gamma_y A)}{\Gamma_y^2} \frac{(1-\cos\Gamma_z B)}{\Gamma_z^2} d\Gamma_x d\Gamma_y d\Gamma_z$$

An upper limit to $\text{var}N$ is found immediately if we ignore the one term in the denominator of the integrand. This corresponds to the case of an infinite Debye length, or the absence of coulomb interaction. This limit is:

$$\text{var}N \leq \bar{N}$$

Returning to the general case, the integration over one of the components of $\vec{\Gamma}$ say Γ_x is performed using the following relation, which is established easily by contour integration:

$$\int_{-\infty}^{\infty} \frac{(x^2 + \lambda_1^2)(1 - \cos \lambda_2 x) dx}{(x^2 D^2 + \lambda_3^2) x^2} = \frac{\pi (\lambda_3^2 - D^2 \lambda_1^2)}{D \lambda_3^3} + \frac{\pi \lambda_2 \lambda_1^2}{\lambda_3^2}$$

The expression for $\text{var}N$ then becomes:

$$\text{var}N = \frac{KTE}{\pi^2 q^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \frac{1}{D[1 + D^2(\Gamma_y^2 + \Gamma_z^2)]^{3/2}} + \frac{L(\Gamma_y^2 + \Gamma_z^2)}{1 + D^2(\Gamma_y^2 + \Gamma_z^2)} \right\} \frac{(1 - \cos\Gamma_y A)}{\Gamma_y^2} \frac{(1 - \cos\Gamma_z B)}{\Gamma_z^2} d\Gamma_y d\Gamma_z$$

For integration over Γ_y we require the solution of the following integral:

$$I_1 = \int_{-\infty}^{\infty} \frac{(1 - \cos \Gamma_y A) d\Gamma_y}{[1 + D^2(\Gamma_y^2 + \Gamma_z^2)]^{3/2} \Gamma_y^2}$$

Letting $\Gamma_y A = \lambda$ and $\alpha^2 = D^2 \Gamma_z^2$ results in

$$I_1 = \frac{2A}{\alpha^3} \int_0^{\infty} \frac{(1 - \cos \lambda) d\lambda}{[1 + (\frac{D\lambda}{\alpha A})^2]^{3/2} \lambda^2}$$

In the limit of D/A much less than unity, I_1 approaches an upper limit,

$$I_1 \leq \frac{2A}{\alpha^3} \int_0^{\infty} \frac{(1 - \cos \lambda) d\lambda}{\lambda^2} = \frac{A\pi}{\alpha^3}$$

Returning to the expression for varN and integrating over Γ_y leads to an expression for the upper limit of varN

$$\begin{aligned} \text{varN} \leq \frac{KTE}{\pi^3 q^2} \int_{-\infty}^{\infty} \left\{ \frac{A\pi^2}{D[1 + D^2 \Gamma_z^2]^{3/2}} + \frac{L\pi^2}{D[1 + D^2 \Gamma_z^2]^{3/2}} + \right. \\ \left. + \frac{\pi^2 AL \Gamma_z^2}{1 + D^2 \Gamma_z^2} \right\} (1 - \cos \Gamma_z B) d\Gamma_z \end{aligned}$$

The integration over Γ_z is similar to that for Γ_y , except the parameter α is now equal to unity. Finding an upper limit to the Γ_z integration leads to the final expression for the upper limit of varN , with the assumption that all sample dimensions are much greater than the Debye length:

$$\text{varN} \leq \bar{N} D \left(\frac{1}{L} + \frac{1}{A} + \frac{1}{B} \right)$$

Putting in the following values, which are appropriate for a typical sample in the present experiment under equilibrium conditions:

$$n_0 = 8 \times 10^{20} \text{ m}^{-3}$$

$$\xi = 16 \xi_0$$

$$A \approx 3 \times 10^{-4} \text{ m}$$

$$T = 77^\circ \text{ K}$$

$$D = 8.6 \times 10^{-8} \text{ m}$$

$$B \approx 4 \times 10^{-4} \text{ m}$$

and using the result for $\text{var}N$ results in

$$\frac{\text{var}N}{N} \leq 4 \times 10^{-4}$$

We see from this the importance of the Debye length in determining the population fluctuations in a volume V of the plasma. Of particular importance is the minimum dimension of V in cases of samples shaped like thin slabs or long rectangular bars, as in the present experiment.

In applying these results to an equilibrium germanium sample, we are overestimating the fluctuations, since we have assumed all surfaces of V to be permeable to the passage of longitudinal density fluctuations, whereas only the ohmic end contacts allow unimpeded electron flow to and from the sample.

In order to extend this analysis to the non-equilibrium high field case, we must first evaluate the effect of electron drift, due to the applied field, on the population fluctuations. This may be done by considering the population fluctuations in a volume identical to the sample volume, which is moving at the drift velocity u in, for convenience, the x -direction of a set of axes at

rest with respect to the equilibrium plasma. As in the equilibrium case, we need $(\overline{\rho_F^2})'$, the $\vec{F}$ -spectrum in the moving coordinate system.

A plane wave of circular frequency ω and wave vector $\vec{F}$ as observed in the fixed coordinates will appear in the moving coordinates to have the same wave vector $\vec{F}$ but circular frequency $\omega' = \omega - u\Gamma_x$. In the stationary coordinate system, Burgess(1965) showed that the $\vec{F}$ -spectrum of the longitudinal space-charge density modes is given by

$$\overline{\rho_F^2} = \frac{2KT\epsilon_0^2\Gamma^2}{\pi\omega} \Im m\left(\frac{1}{\mathcal{E}(\Gamma,\omega)}\right)$$

Since in the moving system only a frequency shift occurs,

$$\overline{\rho_F^2} = (\overline{\rho_F^2})'$$

where $\omega' = \omega - u\Gamma_x$.

Integrating $(\overline{\rho_F^2})'$ over ω' gives $(\overline{\rho_F^2})'$ for the moving system.

$$\begin{aligned} (\overline{\rho_F^2})' &= \int_0^\infty (\overline{\rho_F^2})' d\omega' = \frac{2KT\epsilon_0^2\Gamma^2}{\pi} \int_0^\infty \frac{1}{(\omega' + u\Gamma_x)} \Im m\left[\frac{1}{\mathcal{E}(\Gamma,\omega' + u\Gamma_x)}\right] d\omega' \\ &= \frac{2KT\epsilon_0^2\Gamma^2}{\pi} \int_{u\Gamma_x}^\infty \frac{d\omega}{\omega} \Im m\left[\frac{1}{\mathcal{E}(\Gamma,\omega)}\right] = \frac{2KT\epsilon_0^2\Gamma^2}{\pi} \left\{ \int_0^\infty \frac{d\omega}{\omega} \Im m\left[\frac{1}{\mathcal{E}(\Gamma,\omega)}\right] - \int_0^{u\Gamma_x} \frac{d\omega}{\omega} \Im m\left[\frac{1}{\mathcal{E}(\Gamma,\omega)}\right] \right\} \end{aligned}$$

The first term is identical to the equilibrium value, and the second, due to drift, we now consider.

Since $\frac{1}{\omega} \Im m\left[\frac{1}{\mathcal{E}(\Gamma,\omega)}\right]$ is an even function of ω , the

drift term is odd in $\vec{r}_x$; hence, including the drift term in the $\vec{r}$ -space integral in the evaluation of $\text{var}N$ results in no change from the equilibrium value.

A second effect of the high field on fluctuations in the total population is in the possible enhancement of the thermal velocities of the electrons as distinct from the drift just considered. This may be taken into account if we adopt the concept of an electron temperature to describe the non-equilibrium velocity distribution, as in the Barrie theory, and assume a non-equilibrium Debye length exists which is proportional to the square-root of the electron temperature.

APPENDIX 2 - CALCULATION OF GROUP POPULATION SPECTRAL DENSITIES

Let us consider the j th group, still restricting ourselves to the case of two groups in all. Then, for small fluctuations in the group population we may write the following two Langevin equations for the population fluctuations, ΔN_j :

$$\frac{d\Delta N_j(t)}{dt} = p_{kj} \Delta N_k(t) - p_{jk} \Delta N_j(t) + \Psi_j(t) \quad (A2.1)$$

where $p_{jk} N_j(t)$ is the probability per unit time of a transition from group j to group k .

$\Psi_j(t)$ is a stochastic term representing the fluctuations in $\frac{d}{dt} \Delta N_j(t)$.

Let us denote the Fourier transform of any suitable function $x(t)$ of time by

$$\tilde{x}(\omega) \equiv \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt$$

Then, Fourier transforming equation A2.1 yields the two equations:

$$\tilde{\Delta N}_j(\omega) (p_{jk} + i\omega) - p_{kj} \tilde{\Delta N}_k(\omega) = \tilde{\Psi}_j$$

with the solution

$$\tilde{\Delta N}_j(\omega) = a_j(\omega) \tilde{\Psi}_j(\omega) + b_k(\omega) \tilde{\Psi}_k(\omega)$$

where

$$a_j(\omega) = \frac{i\omega + p_{kj}}{A(\omega)}$$

$$b_k(\omega) = \frac{p_{kj}}{A(\omega)}$$

$$A(\omega) = i\omega (p_{12} + p_{21}) - \omega^2$$

The group population spectral densities are defined by

$$S_{N_j, N_k} = \lim_{T \rightarrow \infty} \frac{2}{T} \tilde{N}_j(\omega) \tilde{N}_k^*(\omega)$$

and

$$S_{N_j} = \lim_{T \rightarrow \infty} \frac{2}{T} \tilde{N}_j(\omega) \tilde{N}_j^*(\omega)$$

From the solution of equation A2.1 we arrive at the following expression for $S_{N_j}(\omega)$:

$$S_{N_j}(\omega) = |a_j(\omega)|^2 S_{\psi_j} + |b_k(\omega)|^2 S_{\psi_k} + [a_j(\omega) b_k^*(\omega) + a_j^*(\omega) b_k(\omega)] S_{\psi_j, \psi_k} \quad (A2.2)$$

The spectrum of ψ_j is related to the events constituting ψ_j by:

$$S_{\psi_j} = 2\nu |\tilde{r}(t)|^2$$

if the events are mutually independent. Here ν is the mean rate of occurrence of events, a typical event having the form $r(t)$, and commencing at time t equals zero. Two processes contribute to ν : electron transitions between groups, occurring at the rate $p_{12} \bar{N}_1 + p_{21} \bar{N}_2$ and entry and exit from the sample boundaries due to the passage of current, occurring at the rate

$$\frac{\bar{I}_j}{q} = \frac{\bar{N}_j}{t_j}$$

where t_j is the transit-time.

Since $\frac{1}{t_j}$ is much smaller than p_{ij} , the major contributor to ν is the intergroup transition rate.

$r(t)$ is a delta function, $\pm \delta(t-t_0)$ where t_0 is the time of transition, with the positive sign for entry and

the negative sign for exit. The spectrum S_{ψ_j} is then:

$$S_{\psi_j} = 2(p_{12}\bar{N}_1 + p_{21}\bar{N}_2) = \frac{4p_{12}p_{21}\bar{N}}{p_{12} + p_{21}}$$

S_{ψ_j, ψ_k} is found by considering $\psi_1(t) + \psi_2(t)$. If we ignore fluctuations in the total electron population, as discussed in Appendix 1,

$$\frac{d}{dt} \Delta N_1(t) + \frac{d}{dt} \Delta N_2(t) = 0$$

Then, adding equations A2.1 gives

$$\psi_1(t) + \psi_2(t) = 0$$

The spectrum of $\psi_1 + \psi_2$ is related to S_{ψ_1} , S_{ψ_2} and S_{ψ_1, ψ_2} by:

$$S_{\psi_1 + \psi_2} = S_{\psi_1} + S_{\psi_2} + 2S_{\psi_1, \psi_2}$$

Since $S_{\psi_1 + \psi_2}$ is zero,

$$S_{\psi_1, \psi_2} = -2(p_{12}\bar{N}_1 + p_{21}\bar{N}_2) = -\frac{4p_{12}p_{21}\bar{N}}{p_{12} + p_{21}}$$

The coefficients in equation A2.2 have the values:

$$|a_j(\omega)|^2 = (p_{kj}^2 + \omega^2) / [\omega^4 + (p_{12} + p_{21})^2 \omega^2]$$

$$|b_k(\omega)|^2 = p_{kj}^2 / [\omega^4 + (p_{12} + p_{21})^2 \omega^2]$$

$$a_j(\omega) b_k^*(\omega) + a_j^*(\omega) b_k(\omega) = 2p_{kj}^2 / [\omega^4 + (p_{12} + p_{21})^2 \omega^2]$$

Equation A2.2 then yields for the population spectrum of the j th group:

$$S_{N_j}(\omega) = \frac{4p_{12}p_{21}\bar{N}}{(p_{12} + p_{21})[\omega^2 + (p_{12} + p_{21})^2]}$$

For frequencies $\omega \ll p_{jk}$, $S_{N_j}(\omega)$ approaches S_{N_j} ,

where

$$S_{N_j} = \frac{4 p_{12} p_{21} N}{(p_{12} + p_{21})^3}$$

Since

$$\tilde{\Delta N}_j + \tilde{\Delta N}_k = 0$$

$$\tilde{\Delta N}_j \tilde{\Delta N}_k^* + \tilde{\Delta N}_k \tilde{\Delta N}_j^* = 0$$

and

$$S_{N_j, N_k} = -S_{N_k}$$

Hence

$$S_{N_j, N_k} = -\frac{4 p_{12} p_{21} N}{(p_{12} + p_{21})^3}$$

APPENDIX 3 - CALCULATION OF VALLEY POPULATIONS AND THE INTERVALLEY TRANSITION PROBABILITIES

As stated in Chapter 1, the probability of an electron transition between valleys due to absorption of a lattice phonon of energy $\hbar\omega$ is:

$$W_a = W \frac{(\Delta E + \hbar\omega)^{1/2}}{e^{\hbar\omega/kT} - 1}$$

where ΔE is the initial energy of the electron above the valley minimum.

For emission of a phonon of energy $\hbar\omega$, the transition probability is

$$W_e = \frac{W(\Delta E - \hbar\omega)^{1/2}}{1 - e^{-\hbar\omega/kT}} \quad \text{for } \Delta E \gg \hbar\omega$$

$$= 0 \quad \Delta E < \hbar\omega$$

Assuming that the energy distribution of the electrons in a single valley is Maxwellian at an electron temperature T , the energy distribution is given by

$$f(\Delta E) = a(T) (\Delta E)^{1/2} e^{-\Delta E/kT}$$

$a(T)$ is related to the single valley population density, $n_1(T)$ by

$$n_1(T) = a(T) \int_0^{\infty} (\Delta E)^{1/2} e^{-\Delta E/kT} d(\Delta E) = a(T) \Gamma(3/2) (kT)^{3/2}$$

Considering now the transitions from an energy shell $d(\Delta E)$ in one valley, by absorption, we find that the transition probability per unit time is proportional to

$$\frac{f(\Delta E) (\Delta E + \hbar\omega)^{1/2}}{e^{\hbar\omega/kT_L} - 1}$$

The corresponding expression for emission is

$$\frac{f(\Delta E) (\Delta E - \hbar\omega)^{1/2}}{1 - e^{-\hbar\omega/kT_L}}$$

Hence, the total transition probability, $\theta_1(T)$, from a valley at electron temperature T to another valley of arbitrary electron temperature, is proportional to

$$\begin{aligned} & \frac{a(T)}{e^{\hbar\omega/kT_L} - 1} \int_0^\infty (\Delta E)^{1/2} (\Delta E + \hbar\omega)^{1/2} e^{-\Delta E/kT} d(\Delta E) + \\ & + \frac{a(T)}{1 - e^{-\hbar\omega/kT_L}} \int_{\hbar\omega}^\infty (\Delta E)^{1/2} (\Delta E - \hbar\omega)^{1/2} e^{-\Delta E/kT} d(\Delta E) \end{aligned}$$

The above integrals are readily evaluated in terms of the Bessel function

$$K_1(z) = \frac{\Gamma(\frac{1}{2}) (\frac{1}{2} z)}{\Gamma(\frac{3}{2})} \int_1^\infty e^{-zt} (t^2 - 1)^{1/2} dt$$

Then,

$$\theta_1(T) \propto a(T) T K_1\left(\frac{\hbar\omega}{2kT}\right) \left[\frac{e^{\hbar\omega/2kT}}{e^{\hbar\omega/kT_L} - 1} + \frac{e^{-\hbar\omega/2kT}}{1 - e^{-\hbar\omega/kT_L}} \right]$$

Expressing $a(T)$ in terms of $n_1(T)$,

$$\theta_1(T) \propto n_1(T) T^{-1/2} K_1\left(\frac{\hbar\omega}{2kT}\right) \left[\frac{e^{\hbar\omega/2kT}}{e^{\hbar\omega/kT_L} - 1} + \frac{e^{-\hbar\omega/2kT}}{1 - e^{-\hbar\omega/kT_L}} \right]$$

Then the transition probability per electron, denoted by

$$\phi_1(T) \text{ is } \frac{\theta_1(T)}{n_1(T)}.$$

Suppose two valleys having electron temperatures T_1 and T_2 are in a steady-state of population and that all four valleys are characterized by either T_1 or T_2 . Then the steady-state condition $\theta_1(T_1) = \theta_1(T_2)$ allows the valley population ratio $\frac{n_1}{n_2}$ to be found.

The relation is

$$\frac{n_1}{n_2} = \left(\frac{T_1}{T_2}\right)^{1/2} \frac{g(T_2)}{g(T_1)} \frac{K_1\left(\frac{\hbar\omega}{2kT_2}\right)}{K_1\left(\frac{\hbar\omega}{2kT_1}\right)}$$

where

$$g(T) = \frac{e^{\hbar\omega/2kT}}{e^{\hbar\omega/kT_L} - 1} + \frac{e^{-\hbar\omega/2kT}}{1 - e^{-\hbar\omega/kT_L}}$$

and has the bounds:

$$\text{for } T \text{ equal } T_L, \quad g(T_L) = \operatorname{csch}\left(\frac{\hbar\omega}{2kT_L}\right)$$

$$\text{and for } T \text{ much greater than } \hbar\omega, \quad g \approx \coth\left(\frac{\hbar\omega}{2kT_L}\right)$$

Since the valley population at zero field, when T equals T_L , is one-quarter the total electron population, the absolute values of n_1 and n_2 may now be found.

The intergroup transition probabilities p_{12} and p_{21} introduced in Chapter 2 are found from the values of $\phi_1(T)$ for the valley concerned, together with the value of $\phi_1(77^\circ\text{K})$

and the absolute value of p_{12} and p_{21} at 77°K, which are evaluated in Appendix 5.

For example, for the case of one valley at electron temperature T_1 and the other three at electron temperatures T_2 , we have, denoting the single valley by subscript 1:

$$p_{12} = p_{12}^{o'} \frac{\phi_1(T_1)}{\phi_1(77^\circ)}$$

$$p_{21} = p_{21}^{o'} \frac{\phi_1(T_2)}{\phi_1(77^\circ)}$$

where $p_{12}^{o'}$ and $p_{21}^{o'}$ are the transition probabilities per electron at zero field for the special case of one valley at electron temperature T_1 and the other three valleys at electron temperature T_2 . As shown in Appendix 5, $p_{12}^{o'}$ equals $3 p_{21}^{o'}$.

APPENDIX 4 - CALCULATION OF VELOCITY VARIANCE IN ARBITRARY DIRECTION OF DISPLACED MAXWELLIAN DISTRIBUTION

Let k_x, k_y, k_z be the components of a vector in $\vec{k}$ -space, taking the origin of the coordinate system at the center of one of the constant energy ellipsoids. The k_z axis is directed along the major axis of the ellipsoid.

Taking an arbitrary direction α with direction cosines $\alpha_1, \alpha_2, \alpha_3$ we require the variance of the displaced Maxwellian distribution in this direction.

An electron in state $\vec{k}$ has velocity components v_x, v_y, v_z , related to $\vec{k}$ by :

$$v_x = \frac{\hbar}{m_t} k_x \quad v_y = \frac{\hbar}{m_t} k_y \quad v_z = \frac{\hbar}{m_e} k_z$$

Denoting its component of velocity in the direction α by v_α we have

$$v_\alpha = \alpha_1 v_x + \alpha_2 v_y + \alpha_3 v_z$$

Averaging over all electrons in the valley, denoting the average by $\overline{v_\alpha}$:

$$\overline{v_\alpha} = \frac{\int_{\vec{v}} v_\alpha g(\vec{v}) d\vec{v}}{\int_{\vec{v}} g(\vec{v}) d\vec{v}}$$

The mean-square value is

$$\overline{v_\alpha^2} = \frac{\int_{\vec{v}} v_\alpha^2 g(\vec{v}) d\vec{v}}{\int_{\vec{v}} g(\vec{v}) d\vec{v}}$$

where $g(\vec{v})$ is the electron velocity distribution.

Since $g(\vec{v}) d\vec{v} = \frac{\hbar^3}{m_t^2 m_e} f(\vec{k}) d\vec{k}$

$$\overline{v_{\alpha}^2} = \int \vec{v}_{\alpha}^2 f(\vec{k}) d\vec{k} / \int f(\vec{k}) d\vec{k}$$

Using the expression for $f(\vec{k})$ of the displaced Maxwellian distribution,

$$f(\vec{k}) = a(T) e^{-\frac{E(\vec{k}-\vec{k}_0)}{KT}}$$

with
$$E(\vec{k}) = \frac{\hbar^2}{2m_0} \left\{ \frac{m_0}{m_t} k_x^2 + \frac{m_0}{m_t} k_y^2 + \frac{m_0}{m_l} k_z^2 \right\}$$

and
$$E(\vec{k}-\vec{k}_0) = \frac{\hbar^2}{2m_0} \left\{ \frac{m_0}{m_t} (k_x - k_{0x})^2 + \frac{m_0}{m_t} (k_y - k_{0y})^2 + \frac{m_0}{m_l} (k_z - k_{0z})^2 \right\}$$

Then, by direct calculation,

$$\begin{aligned} \text{Var } v_{\alpha} &= \left(\frac{\alpha_1^2}{m_t} + \frac{\alpha_2^2}{m_t} + \frac{\alpha_3^2}{m_l} \right) KT \\ &= \left(\frac{\sin^2 \theta_{\alpha}}{m_t} + \frac{\cos^2 \theta_{\alpha}}{m_l} \right) KT = \frac{KT}{m} \end{aligned}$$

where θ_{α} is the angle between the direction α and the v_z axis, and m is the inertial effective mass for acceleration in the α direction, given by (Wilson)

$$m = \frac{m_t m_l}{m_l \sin^2 \theta_{\alpha} + m_t \cos^2 \theta_{\alpha}}$$

APPENDIX 5 - INTERVALLEY TRANSITION RATE AND THE ACOUSTOELECTRIC EFFECT

The introduction of an acoustical wave into a lattice of n-type germanium atoms will shift the four minima or valleys of the conduction band, whereas in the undeformed lattice the minima have the same energy. This means that the probability of occupation of states in the valleys will change and as a consequence the valley populations will redistribute due to intervalley transitions.

If the acoustical wave frequency is high enough, the redistribution will not be complete, and if it is much greater than the intervalley rate, little or no redistribution will occur. Hence, measurements of the redistribution as a function of frequency afford a direct means of gaining data about the intervalley transition rate.

The acoustoelectric effect (Weinreich, G. et al. 1958), allows information to be determined about the redistribution. The effect is the appearance of a d.c. voltage in the direction of propagation of a travelling acoustical wave.

As far as intervalley transitions are concerned, the relevant quantity determined from the acoustoelectric effect is the intervalley transition time, τ_{iv} , which the authors define from the equation,

$$R_{12} = \frac{2}{3\tau_{iv}} (n_1 - n_2)$$

where R_{12} is the net rate of entry of electrons into group 2 from group 1. Here, each of the two groups consists

of two valleys, and n_i is the number of electrons per unit volume in group i .

The above relation allows the determination of p_{ij}° , the probability per unit time of an electron making a transition from group i to group j , where groups i and j each consist of two valleys, for the equilibrium case when $p_{12}^{\circ} = p_{21}^{\circ}$.

In terms of p_{ij}° ,

$$R_{12} = n_1 p_{12}^{\circ} - n_2 p_{21}^{\circ}.$$

Since, in equilibrium, $n_1 = n_2$ then $p_{12}^{\circ} = p_{21}^{\circ}$.

Hence $R_{12} = p_{12}^{\circ} (n_1 - n_2)$

Equating R_{12} to Weinreich's expression gives

$$p_{12}^{\circ} = \frac{2}{3\tau_{iv}}$$

For the case of one valley in one group and three in the other, we may also determine the p_{ij}° 's, denoted now by $p_{ij}^{\circ'}$ for this case to avoid confusion with the previous case of two valleys in each group.

In this case, the gross rate of entry of electrons to the second group of three valleys from the one-valley first group is, at equilibrium

$$\frac{n}{4} p_{12}^{\circ'}$$

where n is the total free electron concentration.

Equating this to the gross rate from group 2 to group 1:

$$p_{12}^{\circ'} = 3 p_{21}^{\circ'}$$

Considering a single valley in the case of two groups, each containing two valleys, at equilibrium the gross rate from

the one valley to all the other valleys is

$$\left(\frac{n}{4} p_{12}^0 \right) \times \frac{3}{2}$$

since $\frac{n}{4} p_{12}^0$ is the rate to the two valleys of the second group.

Hence, substituting for the expressions obtained for the gross rate in terms of $p_{12}^{0'}$, gives,

$$\frac{n}{4} p_{12}^0 \times \frac{3}{2} = \frac{n}{4} p_{12}^{0'}$$

and, in terms of τ_{iv} ,

$$p_{12}^{0'} = \frac{1}{\tau_{iv}}$$

$$p_{21}^{0'} = \frac{1}{3\tau_{iv}}$$

Measured values of the intervalley transition rate, R , equal to $\frac{1}{\tau_{iv}}$, for the case of antimony as the impurity have been obtained by Weinreich and Tell, and reported by Price and Hartman(1964). Their value of the donor contribution, R_{donor} to R is:

$$R_{donor} = 0.4 \times 10^{-6} N_i \left(\frac{100^\circ}{T_L} \right)^{1/2} \text{ cm}^3 \text{ sec}^{-1}$$

for the lattice temperature, T_L , in the range from 20°K to 100°K . N_i is the donor density, which in the present experiment was the order of 10^{15} cm^{-3} . Hence, for this experiment

$$R_{donor} = 4.5 \times 10^8 \text{ sec}^{-1}$$

From Weinreich et al(1959), the phonon contribution at 77°K , read from his graphs is

$$R_{phonon} = 5.0 \times 10^9 \text{ sec}^{-1}.$$

Since the donor contribution is small and is expected to decrease at high fields when the valleys are at electron temperatures higher than the lattice temperature, the donor contribution was neglected altogether. The zero field value of the intervalley rate then is taken as the phonon contribution alone.