TEMPERATURE DEPENDENCE STUDIES

OF PERSISTENT CURRENTS IN SUPERCONDUCTORS

by

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B.Sc., University of British Columbia, 1963

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

in the Department

of

Physics

We accept this thesis as conforming to the

required standard

THE UNIVERSITY OF BRITISH COLUMBIA

April, 1966

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ABSTRACT

A study has been made of some energy and momentum properties associated with the electrons of superconducting indium. In the experiments, an electric current was induced in the indium sample and measurements were made to detect any effects of an increasing super-state electron density on the current. By lowering the sample's temperature appropriately, the electron density could be controlled.

The indium sample studied was in the form of a thin film, constructed by vacuum evaporation onto a glass substrate. Two indium wires connected the ends of the thin film to a copper wafer, forming a complete electric circuit. This circuit was electrically isolated and measurements of the current through the thin film were made with a search-coil coupled to the current's magnetic field.

A relationship between the current changes observed and some momentum properties of superstate electrons was then established. The assumption that all "virtual pairs" in a superconductor have a common momentum proved consistent with the experimental results.

Theoretical calculations are given which suggest that only those normal electrons with a preferred momentum can take part in increasing the super-conductive electron density. This implies that a superstate electron system may effect the transformation of internal energy into work. However, the experiments carried out have not been sufficiently sensitive to show this conclusively. The inference stated above is based on the experimental results in conjunction with the required agreement of theoretical calculations with accepted theory.

The indium-copper junction resistance had a resistance value which was markedly temperature dependent in the region below 3.4°K (the transition

temperature of pure indium). A variation of resistance between the extremes 9.4×10^{-10} Ω at 3.1°K and 8×10^{-10} Ω at 1.2°K was found. The cause of this large resistance change is ascribed to various effects but probably the most important one is the diffusion of copper impurity into the indium. Arguments in favor of this explanation are given.

A number of suggestions are also included which may be helpful to the design of experiments similar to the one reported in this thesis.

LIST OF ILLUSTRATIONS AND TABLES

ACKNOWLEDGEMENTS

A large number of people have been exceptionally helpful to me in giving their assistance and invaluable advice throughout the research work. I am particularly indebted to Dr. Brown who, as research supervisor, had originally suggested the study of Cooper-pair momentum properties. After taking a new post, private communications with him have proved very fruitful in ideas for this topic reported. In helping me with the experiments at all stages of the work, I am especially grateful to Dr. Critchlow. His contributions in suggestions, assistance and encouragement have been vital to the success of the research undertaken.

Also, Drs. Mathews and Crooks have kindly given much of their time in discussing the many technical aspects. I am grateful for their help and ready suggestions in the techniques of low temperature physics. In the construction of the cryostat, expert work, assistance and advice was given by Messrs. R. Weissbach, G. Brooks and J. Lees. At various other stages, the help obtained from Mrs. Thodos, Miss Kinne, Mr. Chapman and Mr. J. E. Tupper has been profitable.

Chapter 1

INTRODUCTION

1. Superconductivity and He II superfluidity

Superconductivity may be compared to the very similar phenomenon of superfluidity in liquid helium. Of their unusual and non-classicial properties, the most outstanding are zero viscosity in He II and zero resistivity in superconductors. A phase change to these quantum states occurs at a well defined temperature indicating that the low temperature phase is due to a collective behaviour of the particles. The superflow properties are caused by only a fraction of the available particles; helium atoms for superfluidity or electrons for superconductivity. With decreasing temperatures, this fraction increases.

2. Cooper-pairs

Despite the similarity of the superflow properties, they are exhibited by particles which have very dissimilar attributes, whereas helium atoms are neutral particles obeying the Bose-Einstein statistics, the electrons responsible for superconductivity are electrically charged and obey Fermi-Dirac statistics. This difference is significant if superfluidity is considered to be a boson condensation (Einstein, 1924; London, 1938).

However, these objections against the parallelism of the two phenomena can be overcome. Shafroth (1955) has shown that a condensation of electron-pair bosons might be the basis of superconductivity. The B.C.S. theory also ascribes the superconductive properties to electron pair interactions and experimental evidence gives support to this concept (Deaver and Fairbank, 1961; Doll and Nabauer, 1961).

3. Theories

At the present time, the description of superconductivity has progressed to a stage more detailed than thermodynamics and phenomenological equations. Many aspects of the phenomenon can now be better understood microscopically in terms of quantum mechanics.

The most successful microscopic theory is one proposed by Bardeen, Cooper and Schrieffer (1957). Experimental corroboration of B.C.S. theory has been extensive and it is shown to be essentially correct. The results of experiments reported here support the tenet of B.C.S. theory that the total momentum of all electron pairs is the same. However, other theories propose a common momentum feature also and outstanding examples are the boson condensation by Schafroth and London's phenomenological description. London's equations lead to the result that the mean local momentum is constant.

4. Common momentum experiments in He II and superconductors

In the He II, the feature of a common momentum for the superstate particles has gained experimental support from the results of Reppy and Depatie (1964). Their experimental procedure was as follows. At a temperature just below the lambda point, a quantity of helium in a beaker was set into rotation. After the beaker was clamped, normal fluid viscosity reduced the normal component's rotation to zero but allowed rotation of the superfluid at rates below a critical velocity. The superfluid density was then increased by decreasing the temperature of the helium.

At various lowered temperatures heat pulses destroyed the superfluidity and allowed the measurement of the total angular momentum possessed by both fluid components of the He II. It was found that the total angular momentum at the lower temperature was greater than the momentum of the superfluid at

the higher initial temperature in proportion to the increased superfluid density. From this result it can be concluded that the "normal" atoms with a thermal momentum equal to that of the "superfluid" atoms are preferred over others for entering the superstate.

This thesis is an account of experiments with superconductors which can be regarded as analogous to those of Reppy and Depatie. However, the super-conductivity case is considerably modified since we are dealing with charged particles instead of neutral atoms and the interaction of magnetic and induced electric fields with the electrons reduces the electrons' freedom of motion. Because of Lenz's law, the self-induced electric fields would certainly reduce, if not prevent, any effect analogous to the He II case. The feasibility of increasing an electric current in a superconductor by increasing the super-state electron density must certainly take into account the experimental geometry. Another problem encountered in the superconductivity case is shown by the application of thermodynamics. The detection of an effect is more difficult due to a much smaller internal energy change than in the He II case.

5. General topics

Topics helpful as a theoretical background will be discussed more fully in the next chapter. This will include a general description and a resume of some successful superconductivity theories. A section is also devoted to the specific application of theory to the experiments reported here and the derivation of equations to be used again later on.

The experiments have not exploited the potential range of investigation extensively. Consequently, a substantial amount of information about super-conductivity could become available through research in this direction. To facilitate the design of similar experiments, some suggestions are made which may prove useful.

In the calculations which follow, an attempt was made to keep the notation fairly consistent. The symbolism used is usually quite standard for low temperature physics but some of it is an individual style. A list of the symbolism is made in Appendix C which should clarify their definitions. With regard to units, the intention was to use MKS units throughout.

Chapter 2

THEORY

A. General Theory of Superconductivity

The most peculiar feature of superconductivity, discovered by Kamerlingh

Onnes in 1911, is the immeasurably small electrical resistivity. All attempts

at obtaining a value for it have yielded such a low maximum value that the

resistivity is now considered to be essentially zero. It is less than the

value for pure copper by a factor of 10^{-14} (Quinn and Ittner, 1962).

However, the low resistivity is only a single manifestation of an

unusual phenomenon and since its discovery numerous other properties have

been revealed. A list of some of these important experimental discoveries

is given on the following page.

At present, more than twenty metallic elements and numerous alloys and

compounds are known to exhibit a superconductive state. In most of these

superconductors the features associated with the superstate show a remarkable

similarity and this indicates that a specific consideration of the distinguish-

ing properties of the metals involved is not necessary. That is, an elementary

theory of superconductivity requires only a rudimentary amount of information

of the metals in order to successfully describe the features of the super-

state, and a test of the theories is the description of the properties,

listed on the following page, completely and fully. The experimental results

listed in this thesis should therefore be applicable to other superconductors

besides indium.

A theory of superconductivity, presented by Bardeen, Cooper and

Schrieffer (1957) gives results which are in excellent agreement with

experimental data. It is based on the Fermi model of a metal and assumes

that the superstate phase is caused by a small interaction between

Important properties of the superconductive phase

Infinite conductivity and critical magnetic field	Kamerlingh Onnes, 1911
Thermal conductivity change	de Haas and Bremmer, 1931
Discontinuity in specific heat	Keesom and Kok, 1932
Meissner effect	Meissner and Ochsenfeld, 1933
Isotope effect	Maxwell, Reynold et al, 1950
Energy gap for electrons	Goodman, 1953 Giaever, 1960
Flux quantization	London (theoretical) 1950 Fairbank and Deaver, 1961 Doll and Nabauer, 1961
Josephson tunneling effect	Josephson (theoretical) 1962 Anderson and Rowell, 1963
Modification of nuclear spin relaxation and ultrasonic attenuation	Bardeen, Cooper and Schrieffer, 1957

electrons and the lattice. This interaction is by means of phonons which, according to the theory, leads to a resultant attraction between electrons and this, in turn, lowers the internal energy to create a stable phase. A description of the B.C.S. theory is given in a concise summary by the authors as follows:

> " . . . the interaction between electrons resulting from virtual exchange of the phonons is attractive when the energy difference between the electrons' states involved is less than the phonon energy, $\hbar\omega$. It is favourable to form a super-conductivity phase when this interaction dominates the repulsive screened Coulomb interaction. The normal phase is described by the Bloch individual-particle model. The ground state of a superconductor, formed from a linear combination of normal state configurations in which electrons are virtually excited in pairs of opposite spin and momentum, is lower in energy by an amount proportional to an average $(\hbar\omega)^2$, consistent with the isotope effect . . . "

An important concept of the B.C.S. theory is the grouping of electrons into "virtual pairs" of equal total momentum. In the ground state the total momentum of the pairs is zero, whereas a current flowing in the metal will excite the common momentum to a non-zero value.

A similar result was obtained by London (1950) by means of a phenomeno-logical description of superconductivity. The two features of zero resistivity and perfect diamagnetism (Meissner effect) are described respectively by two electrodynamic equations:

$$e = \frac{\partial(\Lambda \dot{j})}{\partial t}$$

$$\vec{\nabla} \times (\Lambda \dot{j}) = -h$$

As is shown by London, a consequence of these two equations is that the mean local momentum is constant throughout a superconductor. The super-conductive state is therefore characterized by a long range order and a wide extension of the wave-function describing the superstate electrons.

Comparing the results of London and the B.C.S. theory, an agreement is obtained if the "particles" described by London are identified with the "virtual pairs" of the B.C.S. theory. Such a modification of London's equations amounts to doubling the charge of the particles and increasing the mass by some factor (not necessarily two).

The existence of electron pairs in the superconductive phase allows the description of the phenomenon in terms of a Bose-Einstein condensation. Properties of a charged boson-gas have been calculated by Schafroth (1955) and he shows the existence of a condensation and of a Meissner effect but whether such a model is valid for superconductivity remains to be determined. In the B.C.S. theory, although a pairing concept is used, the electron pairs are not bound. Hence, the Cooper-pairs are described as virtual-pairs and this concept differs slightly from the single particle bosons described by Schafroth. The electron-pair concept has been shown experimentally to be essentially correct.

London, using the empirical equations, has shown the conservation of the fluxoid, defined by the relation

$$\phi_c = \iint_S h \cdot dS + c \oint \Lambda j \cdot ds$$

The calculation is applicable to a multiply connected superconductor which encloses a magnetic field "h" and has a current density "j" through it.

By a quantum mechanical description of the superstate electrons, London shows that the fluxoid is quantized. Using the wave function $\psi = \psi_0 e^{iS}$ for the electrons, the requirement that this be a single valued function yields a quantization of the fluxoid in units of $\left(\frac{hc}{e}\right)$. The electric charge "e" of the superstate particles determines the value of the fluxoid.

Such a quantization has been demonstrated in the experiments of Doll and Näbauer (1961) and of Deaver and Fairbank (1961). Their results showed that the magnitude of the fluxoid unit is smaller by a factor of two than the value predicted by London. However, London had used the electron charge for the value $\left(\frac{hc}{e}\right)$ and the substitution of twice the electron charge (i.e. for Cooper-pairs) establishes an agreement between experiment and theory. A result of the B.C.S. theory, that electron-pairs are formed, is therefore confirmed by the experiments.

Below the transition temperature the entropy of the superconductive phase is lower than in the normal phase with a difference depending in the temperature. This increased order of the superstate can be entirely associated with a finite fraction of the metal's free electrons and the formation of this finite fraction is actually the condensation responsible for superconductivity. Although artificial, the division of the free electrons into a "normal fluid" and a "superfluid" leads to "two-fluid" models which give a good qualitative description of superconductive properties. In the phenomenological theory of Gorter and Casimir (1934), the temperature dependence of the superstate electron density has the form,

$$n(T) = n(0)\left(1 - \frac{T^4}{T_c^4}\right)$$

Complete order is associated with the electron-pair density "n" and the temperature dependence is in fair agreement with experiment.

The B.C.S. theory also gives a temperature variation of the density close to the form above. However, the London equations only contain the density as a parameter which has to be evaluated by other methods. At absolute zero, the electron-pair density is of the order of 10^{27} m^{-3} in most ideal bulk superconductors.

B. Theory of Experiment

Those electrons in a superconductor which are not in the superstate belong to the one-particle Bloch functions (Cooper, 1960). But even though one can make the separation between independent-states for normal electrons and correlated-states for superstate electrons, the electrons themselves are not restricted to a single one of the two phases. A continuous interchange of electrons between the normal-states and the superstate occurs. Statistical fluctuations of the superstate electron density are therefore expected but the time-average of the density is determined by the temperature of the super-conductor.

The flow of an electron current through a superconductor excites the superstate electrons above the ground-state energy level. But even with the exchange of electrons between the normal and superstate, no energy loss by the current can be detected for currents below a critical value. With a current flowing, the superstate electrons are therefore in a metastable state.

It can be concluded that, at a constant temperature, the momentum distribution of the electrons leaving the superstate must be identical with the distribution of the normal Bloch electrons entering the superstate to maintain equilibrium conditions. Such a process is possible if the superstate electron-pairs are selective about the momentum of electrons entering the superstate. Selectivity occurs if a normal electron can enter the superstate only with a momentum suitable for a vacancy created in the superfluid. The requirement that newly-formed electron-pairs must have a momentum equal to that of the superstate-pairs originally present would produce such a selectivity. This is consistent with accepted theories of superconductivity.

When the temperature is decreasing, a similar concept for the selection of normal electrons can be used. However, instead of a constant electron density, the superstate density increases by the creation of more pairs. And if these newly-formed electron-pairs have a common momentum equal to that of the other electron-pairs, effects are produced which can be tested experimentally.

On account of the theory above, a measurable effect is expected when the temperature of a current-carrying superconductor is decreased. The current density in the superconductor should increase if all newly-formed Cooper-pairs have the expected non-zero momentum.

A theoretical calculation of the effect on a current for a specific super-conductor circuit is given below. The circuit is a model of the actual experimental design and the result of these calculations will also be compared to the experimental results.

C. Theoretical Calculations

A schematic diagram of the sample coil circuit is shown in Figure 1 below. Other details of the sample coil are given in Appendix A. The thickness "d" of the superconductor is of the order 10^{-5} cm. which permits the approximation of a uniform current density. Other details of the circuit are listed beside the diagram. It will also be assumed that the electric field is a constant since the vector magnetic potential is roughly constant within the superconductor. This is only true for an infinitely long solenoid and the flux leakage through the ends of the coil are neglected to simplify the calculations. In the calculation, the electron-pair density will be considered as a time dependent parameter.

i

indium
coil

$8 \times 10^{-10} \, \Omega$

Search-coil

Galvanometer

G

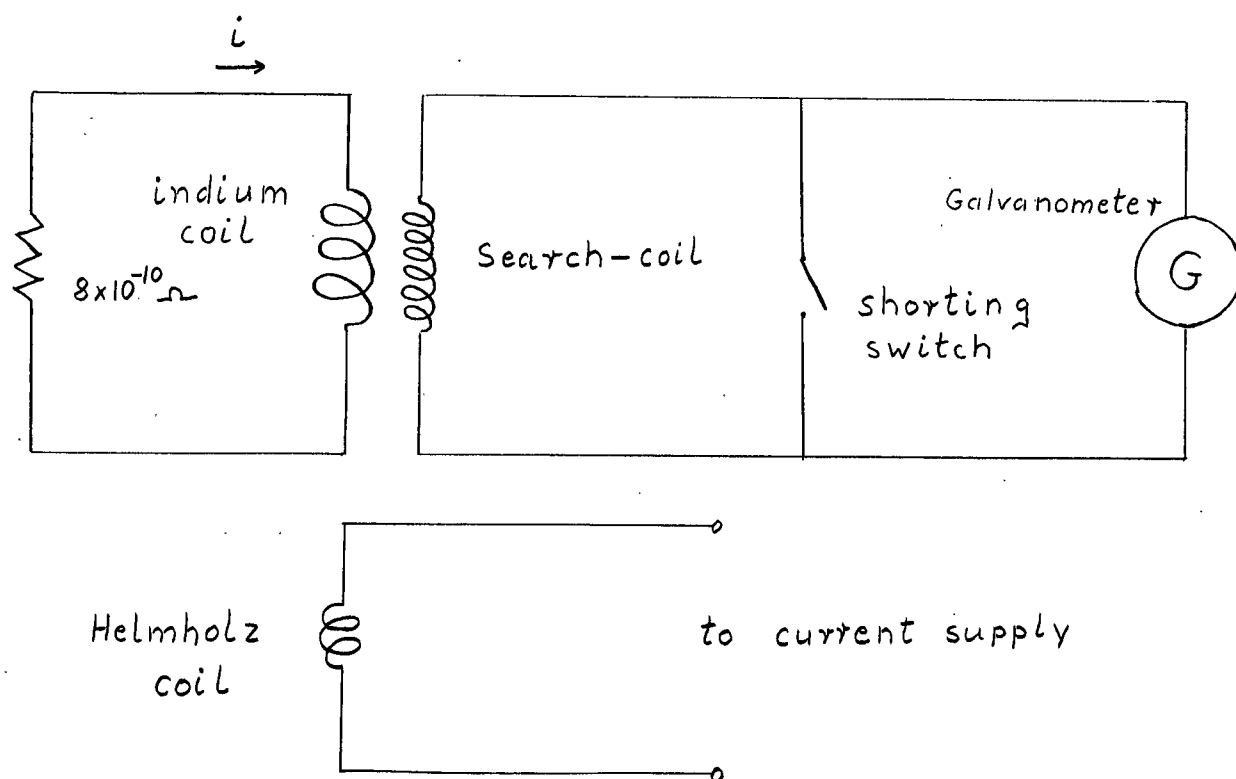shorting
switch

Helmholz
coil

to current supply

Figure 1: The search-coil and indium coil circuit

1. Using London's equation

   The current "i" in Figure 1 is described by Faraday's law as,

   $$L \frac{di}{dt} = -1e - iR \qquad (2.1)$$

The dimension l, d, and w of the sample are given in Appendix A. Substitution of London's equation $e = \frac{\partial(\Lambda j)}{\partial t}$ for the electric field makes the equation soluble for the current. But since the electron-pair density is time dependent, the parameter Λ is also time dependent.

   Therefore, $\quad L \frac{di}{dt} = -\ell \frac{d}{dt}(\Lambda j) - iR \qquad$ where $\Lambda = \frac{m}{q^2 n}$

   Hence, $\quad \left(L + \alpha \Lambda\right) \frac{di}{dt} = -\left(R + \alpha \frac{d\Lambda}{dt}\right) i \qquad \alpha = \frac{\ell}{d \cdot w}$

   and $\quad \frac{di}{dt} = -\frac{\left(R + \alpha \frac{d\Lambda}{dt}\right)}{\left(L + \alpha \Lambda\right)} i \qquad (2.2)$

2. Using an energy method

   For the initial conditions, a current is assumed to flow in the circuit. The total energy $\mathcal{E}_T$ consists of the potential energy $\frac{1}{2} Li^2$ and the kinetic energy of the super-current.

   Therefore, $\quad \mathcal{E}_T = \frac{1}{2} Li^2 + (nV) \frac{1}{2} m v^2$

   or $\quad \mathcal{E}_T = \frac{1}{2}\left(L + \alpha \Lambda\right) i^2 \qquad (2.3)$

Dissipation of this energy, associated with the super-current, occurs at the rate $i^2 R$. There are two special cases to consider for the increase of the electron-pair density:

a) The increase in density is produced by electrons of selected momentum. With a current flowing, the total momentum is non-zero and therefore some energy is contributed to the electron system. The energy gain occurs at the rate,

$$\frac{d\varepsilon}{dt} = \frac{1}{2} m v^2 \frac{d}{dt} (n V)$$

or

$$\frac{d\varepsilon}{dt} = -\frac{1}{2} \frac{d\alpha \Lambda}{dt} i^2 \qquad (2.4)$$

Therefore,

$$\frac{d\varepsilon_T}{dt} = -i^2 R + \frac{d\varepsilon}{dt}$$

and hence,

$$\frac{di}{dt} = -\frac{\left(R + \alpha \frac{d\Lambda}{dt}\right)}{(L + \alpha \Lambda)} i \qquad (2.6)$$

b) With the assumption that the newly-formed electron-pairs have a total momentum of zero, irrespective of the other pairs, the energy change associated with the current will be zero. This corresponds to a process with no selection of normal electrons.

Therefore,

$$\frac{d\varepsilon_T}{dt} = -i^2 R$$

and using (2.3)

$$\frac{di}{dt} = -\frac{\left(R + \frac{1}{2} \alpha \frac{d\Lambda}{dt}\right)}{(L + \alpha \Lambda)} \qquad (2.8)$$

In this case there is no selection of the normal electron momentum for the transition. The normal electrons therefore contribute no energy to the current. But the electron-pairs are still considered to attain a common momentum. This is only a hypothetical case and no explanation can be given of the approach to equilibrium.

3. Using Quantum Mechanics

   With a ψ-function description of the superstate electrons, the behaviour of a current in a closed superconductor loop has been calculated by Keller and Zumino (1961). The resultant behaviour of the current was found to be;

$$i = \frac{n'(\hbar c/e)}{(L + \alpha \Lambda)} \qquad (2.9)$$

At a constant temperature, the quantization of the current and also the fluxoid is apparent. However, the temperature dependence of Λ indicates a consequent temperature dependence of the persistent current. The temperature variation of "i" and its quantization are not mutually exclusive events since it is only the unit of quantization that is changing.

Chapter 3

EXPERIMENTAL DETAILS

A. Design of apparatus

The cryostat is designed to permit a variation of temperature in order to study the behaviour of electric currents in a superconductor. A liquid helium bath for the sample, contained in a silvered glass dewar of 3-liter capacity, provides the necessary low temperatures for the experiments. Variations of the temperature are brought about by pumping off the helium vapour above the bath with a high-speed pump. In general, the design is quite conventional.

The pumping rate of the helium vapour is controlled by a system of valves. A 2-inch valve and a smaller by-pass valve permit rapid cooling of the bath and particular openings of them determine a consequent temperature range. Variations of the temperature were easy to obtain by this method.

Long-term temperature regulation is made with a "cartesian diver" type manostat (Gilmont ; 1946). No drift of temperature was noticeable with the manostat in operation.

To establish a thermal equilibrium between the superconductor and helium, the sample is actually immersed in the bath and, since it is in the form of a thin film, excellent thermal contact is made.

Because the temperature of the bath changed often, a problem with non-uniform temperatures was anticipated. To counteract such conditions two corrective measures are employed. Firstly, a heater at the bottom of the dewar creates convection currents and turbulence in the bath. The stirring action prevents pockets of denser helium from collecting in the dewar bottom. Secondly, a copper jacket surrounds the sample to reduce any vertical temperature gradients in the vicinity. Large openings in the jacket's top cover still allowed

evaporation of helium.

Temperatures in the range 1.2°K to 4.2°K are available with the cryostat. However, all experiments were done below the indium transition temperature of 3.4°K.

A copper tube connects the inside of the dewar to a mercury manometer. Measurements of the dewar pressure determined the bath temperature on the basis of the 1958 conversion (Physica 24, 1958). Since measurements by such a method correspond to the surface temperature of the bath, a carbon resistor was also placed into the helium close to the sample. When the temperature changed rapidly, readings of the resistor were used.

The superconductor sample is a 4-turn coil which has a very low resistance connected across its ends to form an L-R circuit. Details of its dimensions and construction are given in Appendix A. Currents in the sample were induced by establishing a flux through the coil and then switching off the external magnetic field. A Helmholz coil of inside diameter sufficient to fit over the dewars produced the external field. The current supply to the Helmholz coil consists of a remote controlled motor-generator and fields up to 500 gauss were attainable. Induced currents in the sample were less than 1 ampere.

The inductance of the coil could not be measured at room temperature because of its large resistance. To get the value of it, two other methods were employed.

From the dimensions of the coil a standard formula valid for wire coils was used to give an approximate value. Also, by constructing a replica of the thin film coil with copper wire of the same width, a direct measurement could be made. Both of these methods gave similar values and this was used in the calculations for the actual inductance. The value is $L = 3 \times 10^{-7}$ henry

STAINLESS-STEEL TUBE

THIN FILM COIL

SEARCH-COIL

3000-turn COIL

PYREX TUBE

COPPER JACKET

CARBON THERMOMETER
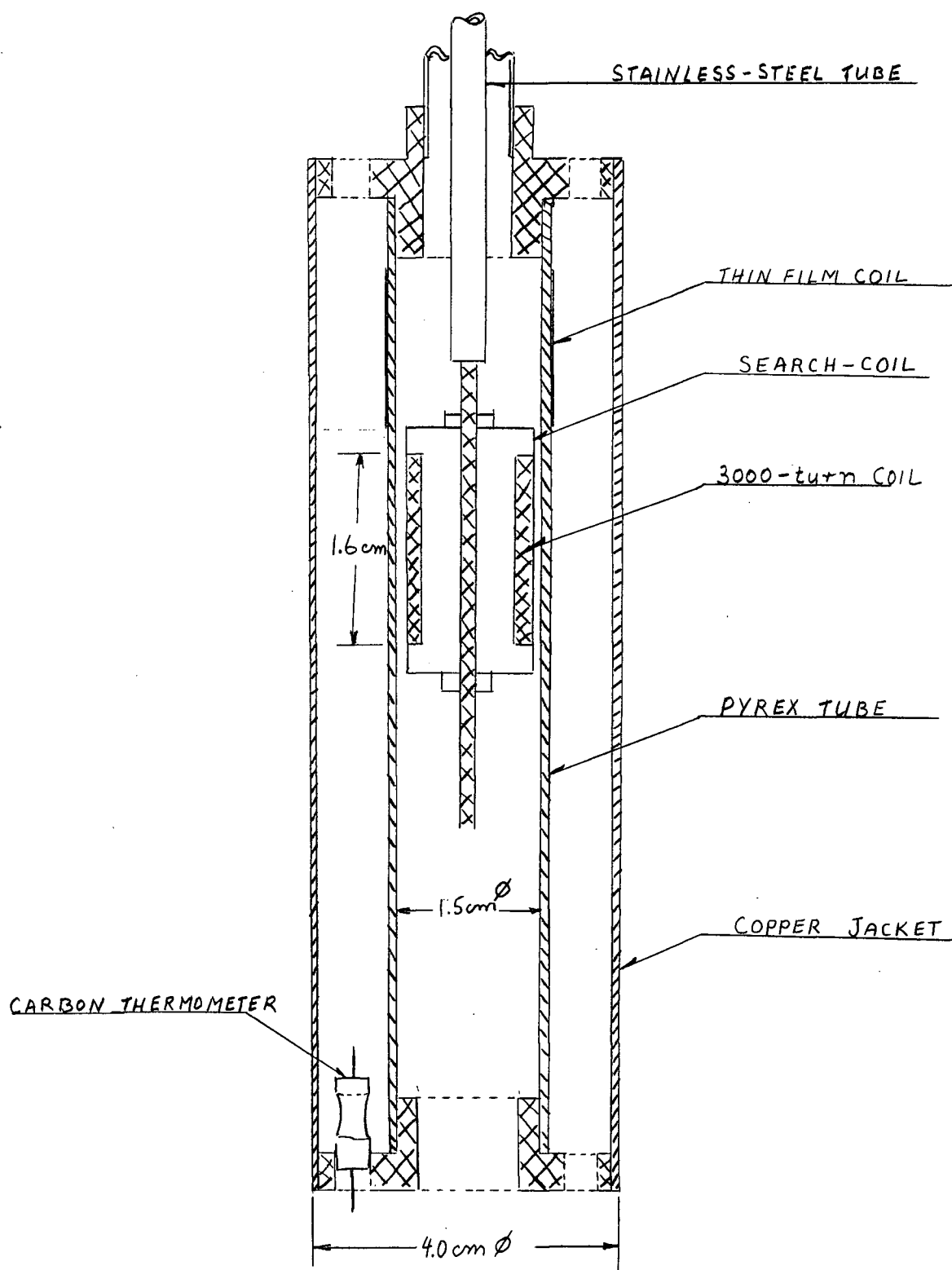
1.6 cm

1.5 cm $\emptyset$

4.0 cm $\emptyset$

Figure 2: MOUNTING OF SAMPLE AND SEARCH-COIL

Since the magnetic flux produced by the superconductor coil is proportional
to the current through it, a search-coil magnetometer is calibrated to give
current values. Calibration of the magnetometer was made by passing known
currents through a replica of the sample coil.

Details of the 3000 turn search-coil wound with #37 wire are shown in
Figure 2. It connects to a galvanometer and is movable through a fixed
distance within the sample coil. Deflections of the galvanometer are
proportional to the flux change in the search-coil. A stainless steel tube,
passing through an O-ring seal in the dewar cap, moves the search coil through
a fixed distance within the sample coil. The illustration in Figure 2 shows
the experimental arrangement. In operation, the search-coil is displaced by
means of the tube leading through the dewar cap. This is done by hand.

Readings of the galvanometer deflection were always made on the downward
stroke of the search-coil, with the galvanometer close to its equilibrium
position before each deflection. During the switching of the Helmholz coil
current, a short circuit protected the galvanometer and also kept it in an
equilibrium position. Figure 1 shows a schematic diagram of the search-coil
and sample coil circuits.

The magnetometer ratings are as follows:

Number of turns on the search-coil . . . . 3000 turns

Magnetometer sensitivity to sample current . . . . 5 x 10^{-3} Amp/mm

Galvanometer sensitivity . . . . 2 x 10^{-8} Amp/mm

Magnetometer sensitivity to magnetic field . . . . .01 gauss/mm

In order to reduce possible thermal emf's in the wires leading to the
galvanometer, they are made continuous except for the connection to the

search coil. Both leads of gauge #27 are soldered into hollow tube Kovar seals in the dewar cap.

B. Experimental procedure

The procedure consisted of inducing a current in the indium sample and then, as the temperature changed, observing its behaviour. Observations were also made within the temperature-held constant.

Measurements of the current, in terms of galvanometer deflections, were made at regular 15-second intervals over a period of about 5 minutes. The length of each stroke of the search-coil had been standardized during the magnetometer's calibration.

The current induced in the sample was proportional to the flux through the sample coil initially. By using a trial and error method the current was brought close to its critical value, which is a temperature-dependent quantity. This was done by increasing the magnetic field of the Helmholz coil for each successive trial until measurements showed a maximum value.

The experiment consists of two separate parts. In one part the current behaviour was observed with the superconductor at constant temperatures while in the other the temperature was decreasing during the measurements of the current.

1. Constant temperature part

With the temperature held constant by the manostat the current in the L-R circuit of the sample was observed for a period of about 5 minutes. This procedure was then repeated at various other convenient temperatures in the range 1.2°K to 3.2°K. By plotting the observed current values against time on a semi-log graph, the behaviour of the sample current could be

analyzed easily. A typical result of these measurements is shown in

Figure 3.

2. Variable temperature part

At a temperature of $2.9^{O}K$, which is close to the indium transition

temperature yet still low enough to permit a fairly large current to flow in

the sample, a current was induced by means of the Helmholz coil. Measurements

of the current were then repeated every 15 seconds as before. However, the

temperature remained constant only for about one minute. After observation

of the current for such a period the temperature of the helium was rapidly

reduced. Repetitions of this procedure at various pumping rates and with the

current flowing in a reverse direction completed the investigation of a

possible temperature effect on a supercurrent.

The observed current values were plotted against time on a semi-log graph.

Typical results of this procedure are shown in Figures 6, 7 and 8.

Since the temperature was rapidly decreasing during the pumping procedure

it was difficult to obtain accurate temperature readings. A list of elapsed

pumping time and approximate temperature is shown in Table I.

Chapter 4

RESULTS AND ANALYSIS

A. Introduction

By considering the inductance of the indium sample's L-R circuit and also the momentum of the superstate electrons, a description of the current is given by equations 2.2 (2.6) or 2.8.

With an energy gain,

$$\frac{di}{dt} = -\frac{\left(R + \alpha \frac{d\Lambda}{dt}\right)}{(L + \alpha \Lambda)} i \qquad (2.2) \text{ and } (2.6).$$

With no energy gain,

$$\frac{di}{dt} = -\frac{\left(R + \frac{1}{2}\alpha \frac{d\Lambda}{dt}\right)}{(L + \alpha \Lambda)} i \qquad (2.8)$$

The parameters "α" and "Λ" are constant for the sample coil and are given by the physical dimensions. Their values are calculated below. Expansion coefficients at the temperature below 3.3°K are assumed to have a negligible effect. From the dimensions of the indium superconductor (see Appendix A), the value of α is

$$\alpha = 1.4 \times 10^9 \text{ m}^{-1} \qquad \text{where } \alpha = \frac{1}{w.d}$$

An estimate of "Λ" can also be made on theoretical grounds. Since the superstate electrons carry current as virtual pairs, their charge is twice the electronic charge and the particle mass is approximately twice the electronic mass. The correct value of the latter quantity is the effective mass but this is unknown at the present time. Using the pair density "n" as a parameter, the value of "Λ" and its derivative are then given by,

$$\Lambda = \frac{4 \times 10^7}{n} \quad \text{henry-meters}$$

$$\frac{d\Lambda}{dt} = \frac{4 \times 10^7}{n^2} \frac{dn}{dt} \frac{\text{henry-meters}}{\text{second}}$$

There is considerable uncertainty in the electron-pair density of the sample. Using the magnetic field penetration depth $\lambda(o) = 6.4 \times 10^{-6}$ cm for a bulk sample of indium (Lock, 1951), one gets the density $\bar{n}(o) = 3 \times 10^{21}$ cm^{-3} at 0°k.

Where $\quad \lambda^2(T) = \frac{c^2}{4\pi} \frac{m}{q^2 n(T)} \quad cm^2 \qquad (4.1)$

However, a correction must be made for a thin film due to the reduction of the mean free path (Pippard, 1953). Assuming that the mean free path of the indium sample is the thickness "d" of it, the correction required for the density is the factor 1/2 (Waldram, 1961).

Therefore, $\quad n(o) = 1.5 \times 10^{21} \quad$ cm^{-3}

The temperature dependence of the density is given by the equation (Lynton, 1962): $\quad n(T) = n(o) \left(1 - \frac{T^4}{T_c^4} \right) \qquad (4.2)$

Even though equations 4.1 and 4.2 are idealized relations, good agreement with experiments is found. Applying equation 4.2, the density at 2.9°K is $n(2.9) = 6 \times 10^{20}$ cm^{-3}

Table I

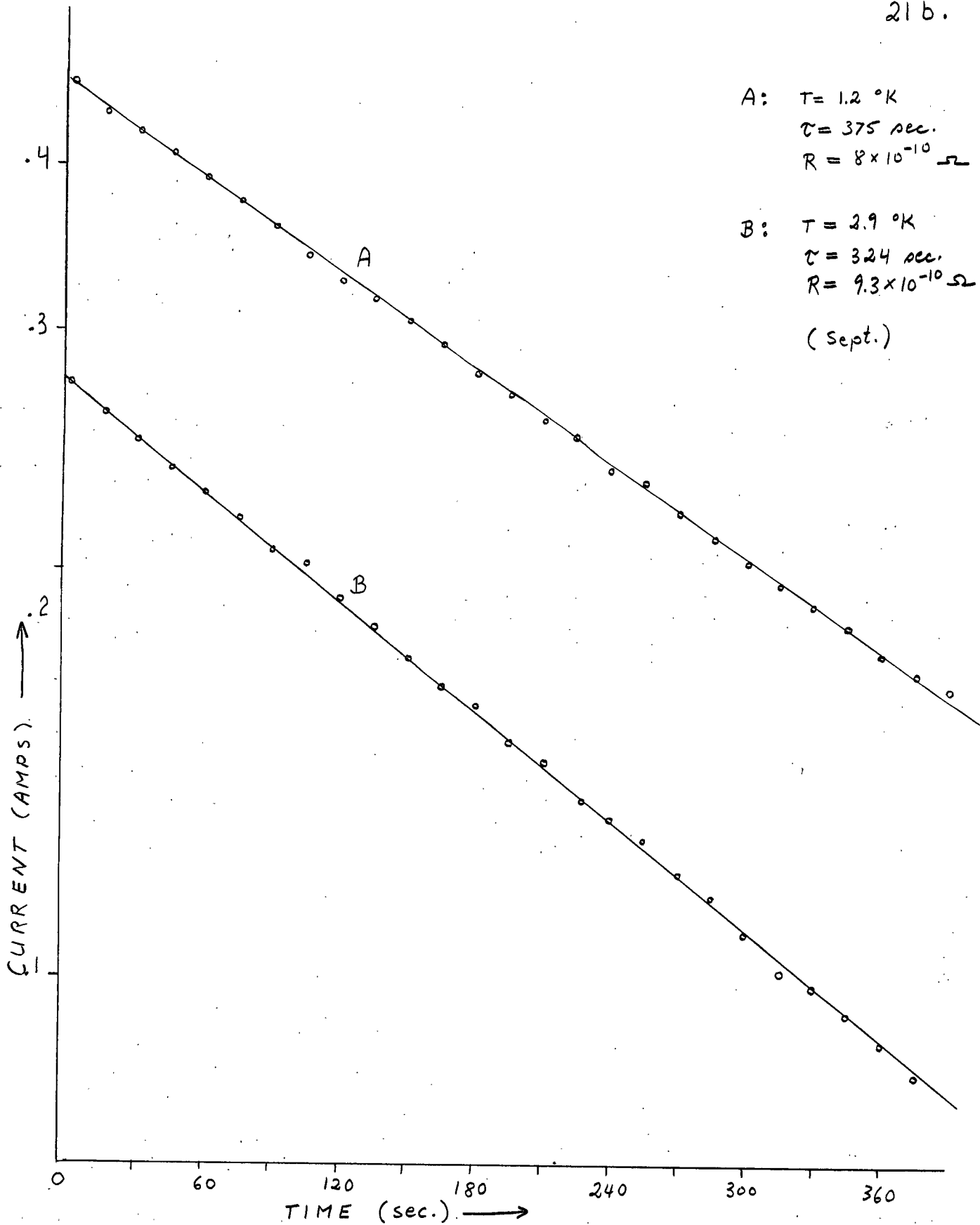| Pumping time (sec.)	Temperature ($^\circ$K)
0	3.0
30	2.1
120	1.5

FIGURE 3: Log plot of current at constant temperature

An estimate of the density can also be made by a second method. According to the Fermi model of a metal and the B.C.S. theory, the fraction "r" of electrons involved in the superconducting condensation are those within a distance kT_c of the Fermi surface E_F (Rosenberg, 1965).

Therefore,
$$r = \frac{kT_c}{E_F} = 10^{-4} \tag{4.25}$$

Since the density of free electrons in indium is 10^{22} cm^{-3}, the Cooper-pair density is $n = 10^{+18}$ cm^{-3}. This differs by two orders of magnitude from the value obtained with equation 4.1.

B. Constant temperature results

At a constant temperature, Λ is a constant also and equation 2.2 can be simplified to

$$\frac{di}{dt} = -\frac{R}{L + \alpha\Lambda} i$$

If R is independent of the current, the solution of the equation above is an exponential decay with a time constant τ given by:

$$\tau = \frac{L + \alpha\Lambda}{R}$$

Where
$$i = i_0 e^{-\frac{R}{L + \alpha\Lambda} t} \tag{4.3}$$

The observed current values also showed an exponential decay, thus establishing agreement with equation 4.3. A typical result for the current values is shown in Figure 3 for temperatures of 1.2°K and 2.9°K. The straight line fitted to the experimental points is always within their uncertainty.

The time-constant of the circuit was found to be temperature dependent. It increased with decreasing temperatures as shown in Figure 4. This figure also suggests that the transition temperature is about 3.3°K. Since "Λ"

22b.

$$\tau = \frac{t}{\log i_0 - \log i}$$

(Dec.)

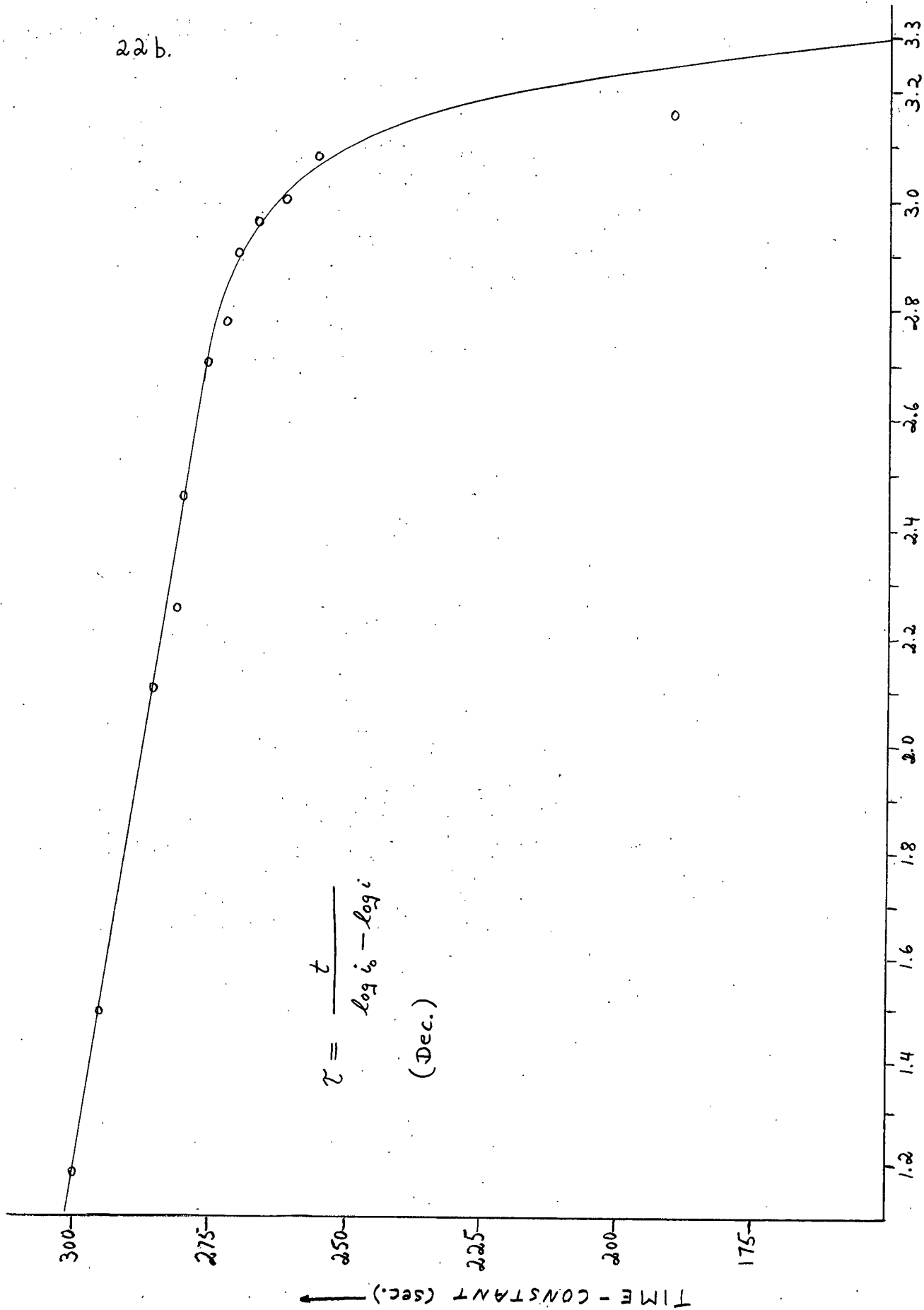TEMPERATURE (°K) $\longrightarrow$

TIME-CONSTANT (sec.) $\longleftarrow$

Figure 4: TEMPERATE DEPENDENCE OF TIME-CONSTANT

decreases with decreasing temperatures due to the electron-pair density variation, it cannot cause the temperature dependence of the time-constant. Therefore the resistance must be strongly temperature-dependent to overshadow any effect by Λ. Thus R must decrease with decreasing temperatures.

An estimate of Λ shows that in comparison to L, $\alpha\Lambda$ can be neglected for a first order approximation. Using the electron density calculated above from equation 4.25 for the indium thin film at 2.9°K, the density is

$$n\ (2.9)\ =\ 6 \times 10^{25}\ m^{-3}$$

Therefore, $\quad \alpha\Lambda\ =\ 8 \times 10^{-10}\ \text{henry}$

This value, which is a maximum for $\alpha\Lambda$, is much smaller than the inductance L where $\quad L = 3 \times 10^{-7}\ \text{henry}$

In comparison to this, the value of $\alpha\Lambda$ can be neglected. This permits the calculation of R from the time-constant and the results are plotted against temperature in Figure 5. A repetition of the experiments, made after an elapsed time of 3 months, showed that the junction resistance had increased (Dec.). The increase was by a factor of 1.2, constant almost for all temperatures.

The variation of resistance with temperature may be caused by a layer of indium, in contact with the copper wafer, remaining normal even below the indium transition point. The depth of such a layer may then depend on temperature to produce the resistance variation. Using the resistivity of indium and the resistance change from 2.9°K to 1.2°K, an estimate of the depth δ is then

$$\frac{\delta \cdot \rho}{A}\ =\ \Delta R$$

where A = surface area

$\rho = 2.4 \times 10^{-7}\ \Omega\ \text{-cm}$
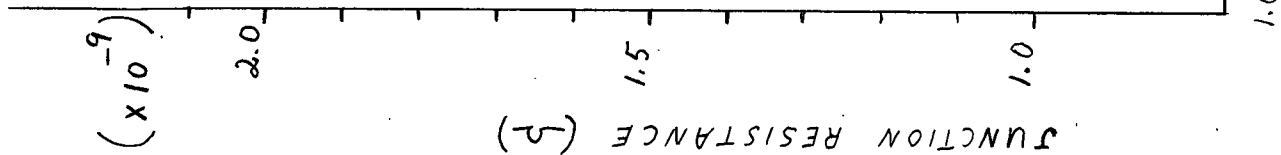
$\Delta R = 3 \times 10^{-10}\ \Omega$

Figure 5: TEMPERATURE DEPENDENCE OF RESISTANCE

23b.

This gives $\delta \approx 10^{-3}$ cm

This is much larger than the coherence length of 10^{-5} cm for the superstate electrons. However, copper impurities in the indium close to the copper wafer may lower the transition point of indium to such a depth. This is especially probable since the indium was fused to the copper by heating them. If so, the increase in resistance over a period of 3 months may have been due to greater diffusion of the copper into the indium.

Although the resistance was temperature-dependent, at a constant temperature in the range covered the resistance was ohmic. This can be seen in Figure 3 for two temperatures. Other temperatures gave similar results. The resistance R was found to be independent of current in the range from 0.3 amperes to 0.06 amperes.

With the current in a reverse direction, the junction resistance at 1.9°K and 2.9°K was found to be lower by a factor of 0.9 than the value in the forward direction. The resistance in the reverse direction was not measured at other temperatures. However, the reverse resistance was found to be ohmic at the two temperatures.

C. Variable temperature results

The temperature dependence of a supercurrent was studied dynamically in the range 1.2°K to 2.9°K. The method consists of decreasing the temperatures rapidly and observing its effect on the current.

After inducing a current in the sample, measurements of it were made at regular intervals of 15 seconds. During the first minute of the measurements the temperature was kept constant to check the behaviour of the current. Then, after these initial observations, the temperature was decreased rapidly.
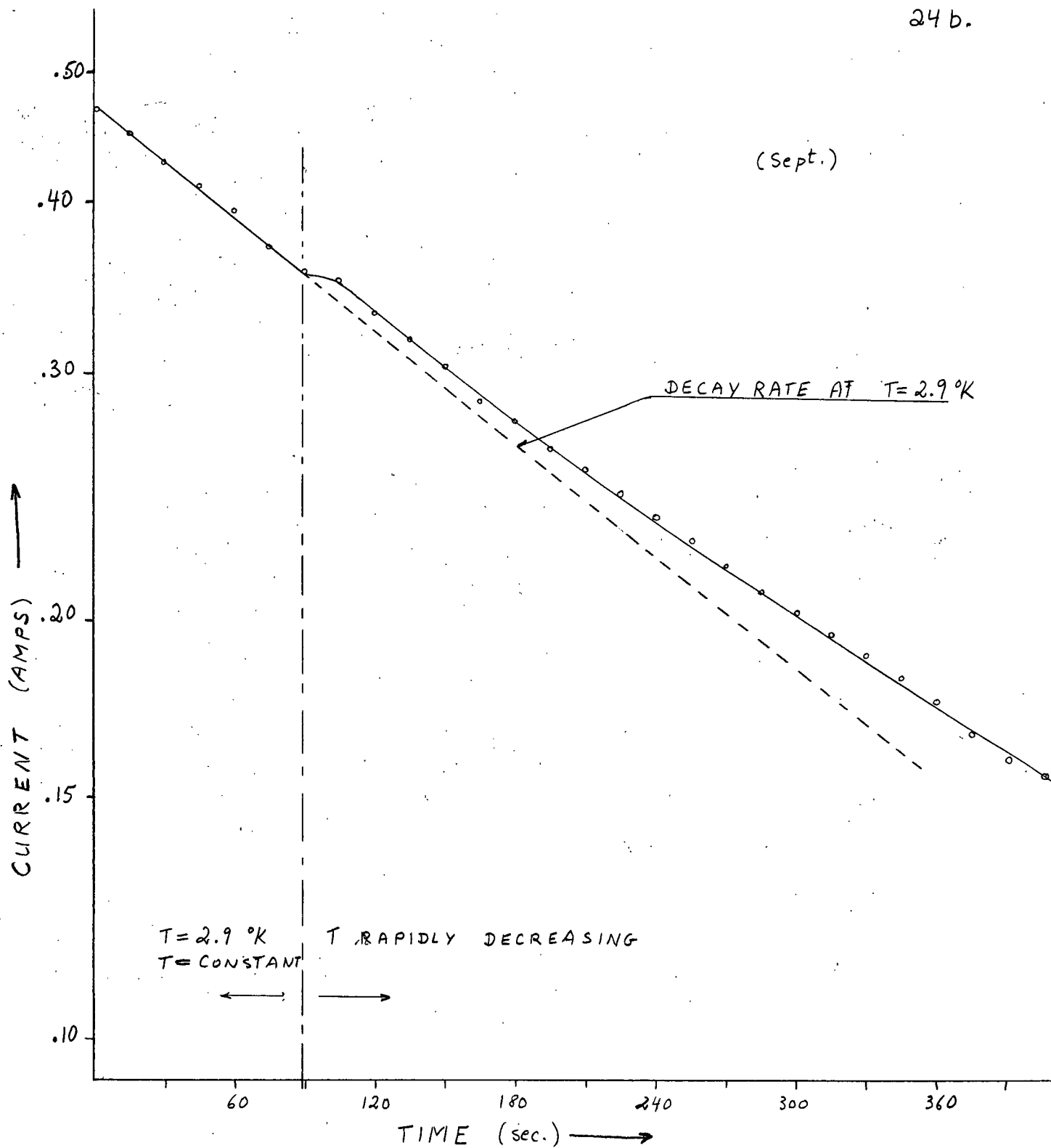
Figure 6: LOG PLOT OF CURRENT DECAY

The maximum rate of decrease is given in Table I. A graphical plot of the current values, using the maximum pumping rate, is shown in Figure 6.

The rate of decay during the constant temperature phase is known from the previous results above. From those results a straight line of the correct slope is drawn through the constant temperature current values. The line is also extended, by means of dashes, past the constant temperature region. Using this as a reference line, the behaviour of the current can be determined when the temperature decreases. An example of this construction can be seen in Figure 6.

From the graph of Figure 6 it can be seen that the current in the sample decayed at a slower rate in the region of the decreasing temperature than in the region of constant temperature. This is indicated by the departure of the observed values from the dashed reference line.

The current behaviour in the decreasing temperature region has 3 characteristic features:

a) A slow rate of decay occurs in the first 15 seconds of temperature decrease. This appears as a rapid departure of the current values from the dashed line.

b) The rate of decay then increases again approaching the rate at the constant temperature.

c) Finally, the rate of decay appears as a considerably smaller value than the constant temperature rate at the beginning.

Results similar to those illustrated in Figure 6 have been obtained in all repetitions of the procedure where the temperature decreased rapidly. The currents had been induced to flow in the same direction except for one case. Those results, with the current in a reverse direction, are shown in
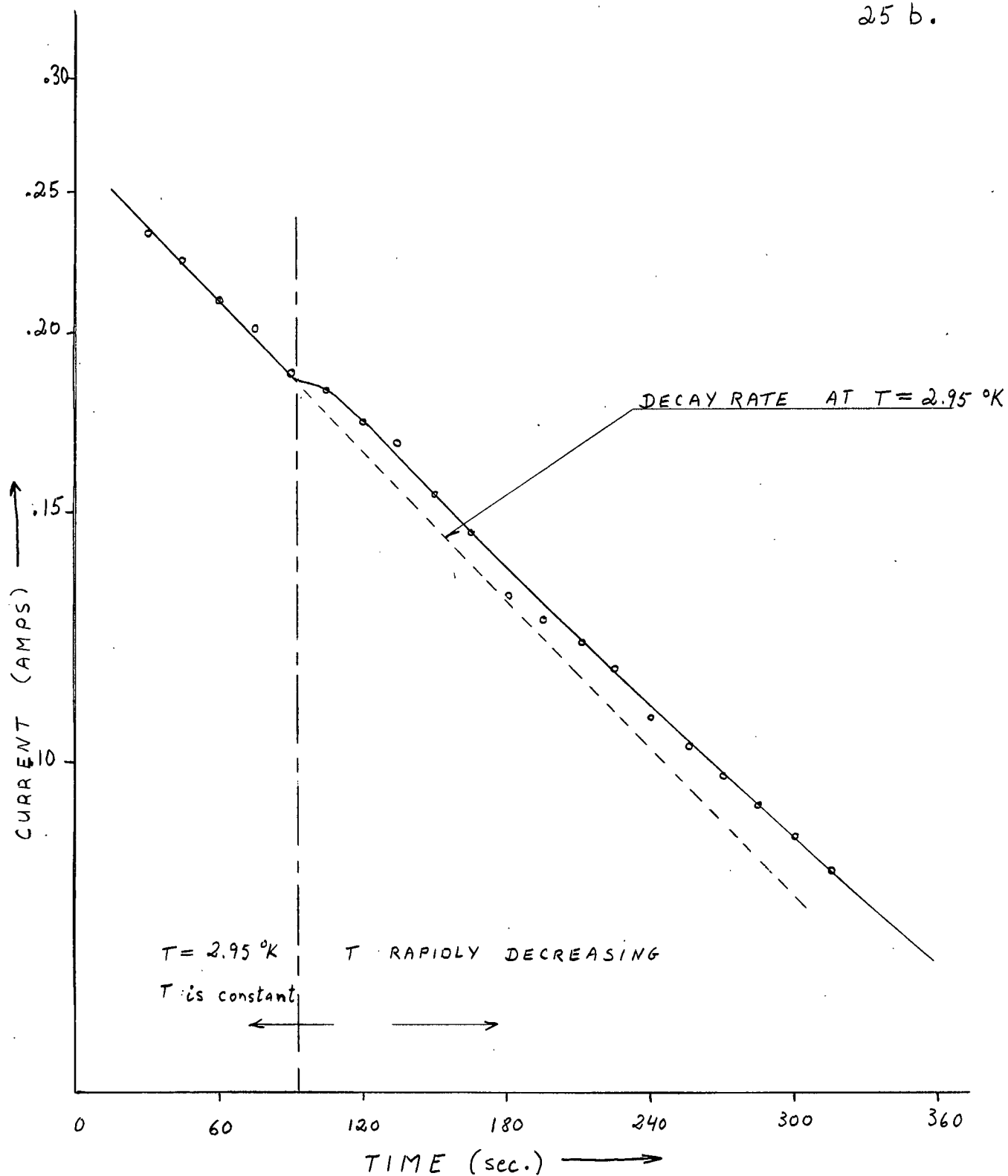
Figure 7: LOG PLOT OF CURRENT (Dec.)

Figure 9. Similar results are also found for that case.

The data has been obtained during two periods. Whereas the initial results are from September, later results are from the period of December. These two periods are shown on the graphs. In Figure 8 (Dec.), two points are not within experimental accuracy with respect to the smooth line approximation. This remains unexplained and Figure 9, which was obtained later on, does not show this apparent error again.

Parts b) and c) of the three characteristic features above can be explained by the resistance's temperature dependence. However, part a) cannot be explained on that basis and the feature will be referred to as an "anomaly". An enlargement of the anomaly in Figure 6 is shown in Figure 10.

The anomaly appears as a region where the current's rate of decay is very small. A simple argument will show that the temperature dependence given in Figure 5 cannot be responsible for it.

The assumption that the resistance's temperature dependence is causing the rate of decay to change implies that the rate of decay of the current "i" will be monotonically decreasing with time. Therefore the function F(t) will also be monotonically decreasing.

Where $F(t) = \log i$

Since the resistance is monotonically decreasing with temperature, which is also decreasing with time, the derivative $F'(t)$ must be monotonically increasing since:

$$F'(t) = -\frac{R}{L}$$

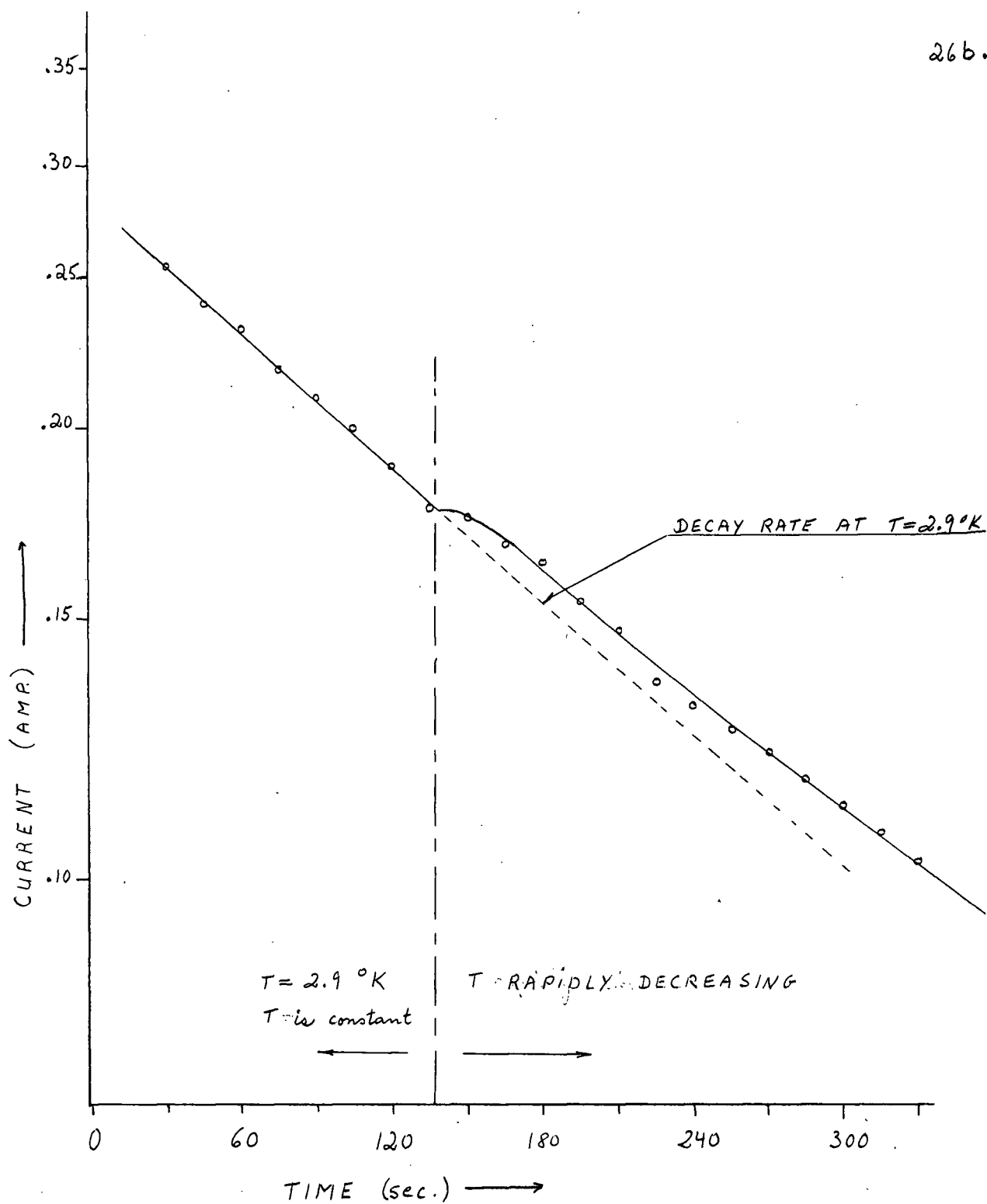A quantity $\mathcal{T}$ is defined below, where t_1 is the time at which pumping is initiated and $t > t_1$.

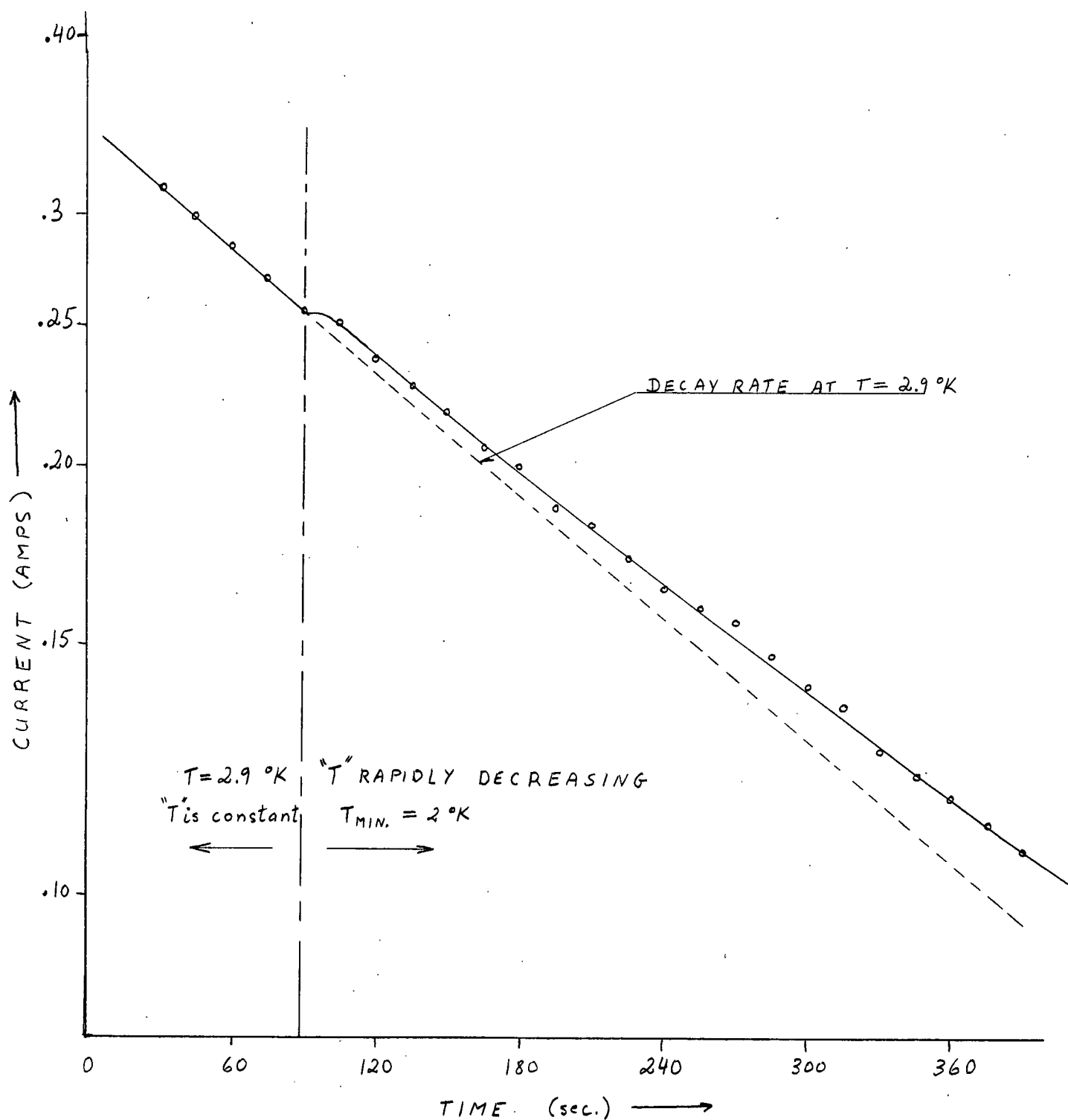Figure 8: LOG PLOT OF CURRENT IN REVERSE DIRECTION (Dec.)

Figure 9: LOG PLOT OF CURRENT vs. TIME (Dec.)

$$\bar{\tau}^{-1} = -\frac{F(t) - F(t_1)}{(t - t_1)}$$

or

$$\bar{\tau}^{-1} = \frac{-1}{(t - t_1)} \int_{t_1}^{t} F'(t') \, dt'$$

Therefore,

$$\frac{d\bar{\tau}^{-1}}{dt} = \frac{1}{(t - t_1)^2} \int_{t_1}^{t} F'(t') \, dt' - \frac{F'(t)}{(t - t_1)}$$

or

$$\frac{d\bar{\tau}^{-1}}{dt} = \frac{-1}{(t - t_1)^2} \int_{t_1}^{t} F''(t') \, t' \, dt'$$

Since $F'(t)$ is a monotonically increasing function, its derivative $F''(t)$ must therefore be positive everywhere. Therefore,

$$\frac{d\bar{\tau}^{-1}}{dt} < 0$$

and hence,

$$\frac{d\bar{\tau}}{dt} > 0 \quad everywhere$$

From the data of Figure 6, values of $\bar{\tau}$ are calculated and shown in Figure 11. It can be seen that $\bar{\tau}$ does not have the property derived above. Therefore, the assumption made above that R is responsible for the anomaly is not valid and some other explanation is required.

An explanation of the anomaly is given by the theoretical calculations of the experiment in Chapter 2, part C. It is based on the common momentum feature of the Cooper-pairs.

Since the exact current behaviour in the anomaly is not known, an approximation will be made, as shown in Figure 10. The piecewise linear approximation of the anomaly consists of an initial constant current (for roughly 6 seconds) followed by an exponential decay. Theoretical calculations in Chapter 2 are in good agreement with the experimental results approximated in the above fashion if $R + \alpha \frac{dI}{dt} = 0(10^{-10})$. Substitution of
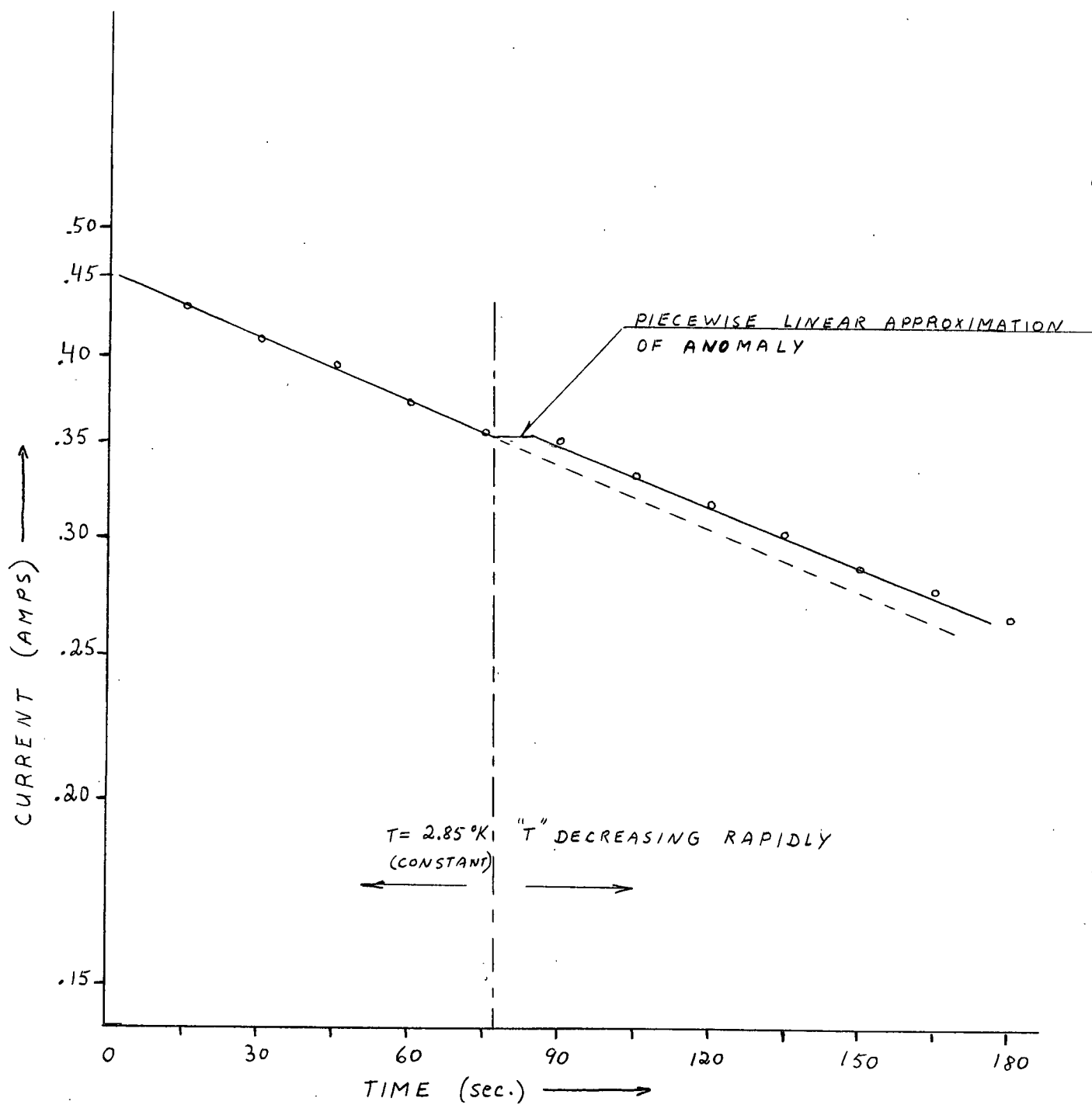
Figure 10: ENLARGEMENT OF ANOMALY IN FIGURE 6.
(Sept.)

values from the sample coil dimensions and other constants of the indium superconductor into equations 2.2 or 2.6 is shown below. Calculation shows that the value of α, Λ and n are of magnitude such that the requirement of the current can be satisfied.

From Table I, the pumping rate in the first 30 seconds produces the average "decreasing-temperature" rate:

$$\frac{\Delta T}{\Delta t} = 3 \times 10^{-2} \frac{{}^\circ K}{\sec}$$

Hence, over a period of $\Delta t = 6$ seconds, $\Delta T = 0.2{}^\circ K$

Now, since Λ has the temperature dependence $\dfrac{\Lambda(T_1)}{\Lambda(T_2)} = \left(\dfrac{1 - T_2{}^4/T_c{}^4}{1 - T_1{}^4/T_c{}^4} \right)$

therefore, $\quad \Lambda(2.9) = 1.5 \, \Lambda(2.7)$

and, $\quad \Delta \Lambda = -.5 \, \Lambda(2.9)$

To satisfy the requirement $R + \alpha \dfrac{d\Lambda}{dt} \approx 0$, the value of $\Lambda(2.9)$ is now calculated.

Using the values,

$R(2.8) = 9 \times 10^{-10} \, \Omega$
$\alpha = 1.4 \times 10^9 \, m^{-1}$

we get $\quad \Delta t = 6 \, sec.$

$\Lambda(2.9) = 7.7 \times 10^{-18} \, \Omega-m-sec.$

and, $\quad n(2.9) = 5 \times 10^{24} \, m^{-3}$

This value of "n" is smaller than the value obtained from penetration depths. However, the density is of the order calculated on the basis of equation 4.25.

With a current of I = .3 Amps flowing through the indium coil at 2.9${}^\circ$K, the density $n = 10^{24} \, m^{-3}$ is large enough to keep the kinetic energy of the Cooper-pairs below that of the energy gap $2 \mathcal{E}(T)$.

28 b.

TO 1500 AT $(t-t_1) = 15$

$$\bar{\tau} = \frac{(t-t_1)}{\log(i_i/i)}$$

(FROM DATA SHOWN IN FIG. 6)

COOLING IS INITIATED AT TIME "t_1".

$T = 2.85 \,°K$ (CONSTANT)

PARAMETER $\bar{\tau}$ (sec.)

600 — 550 — 500 — 450 — 400 — 350 —

TIME INTERVAL $(t-t_1)$ ⟶ (sec.)

0   30   60   90   120   150   180   210   240
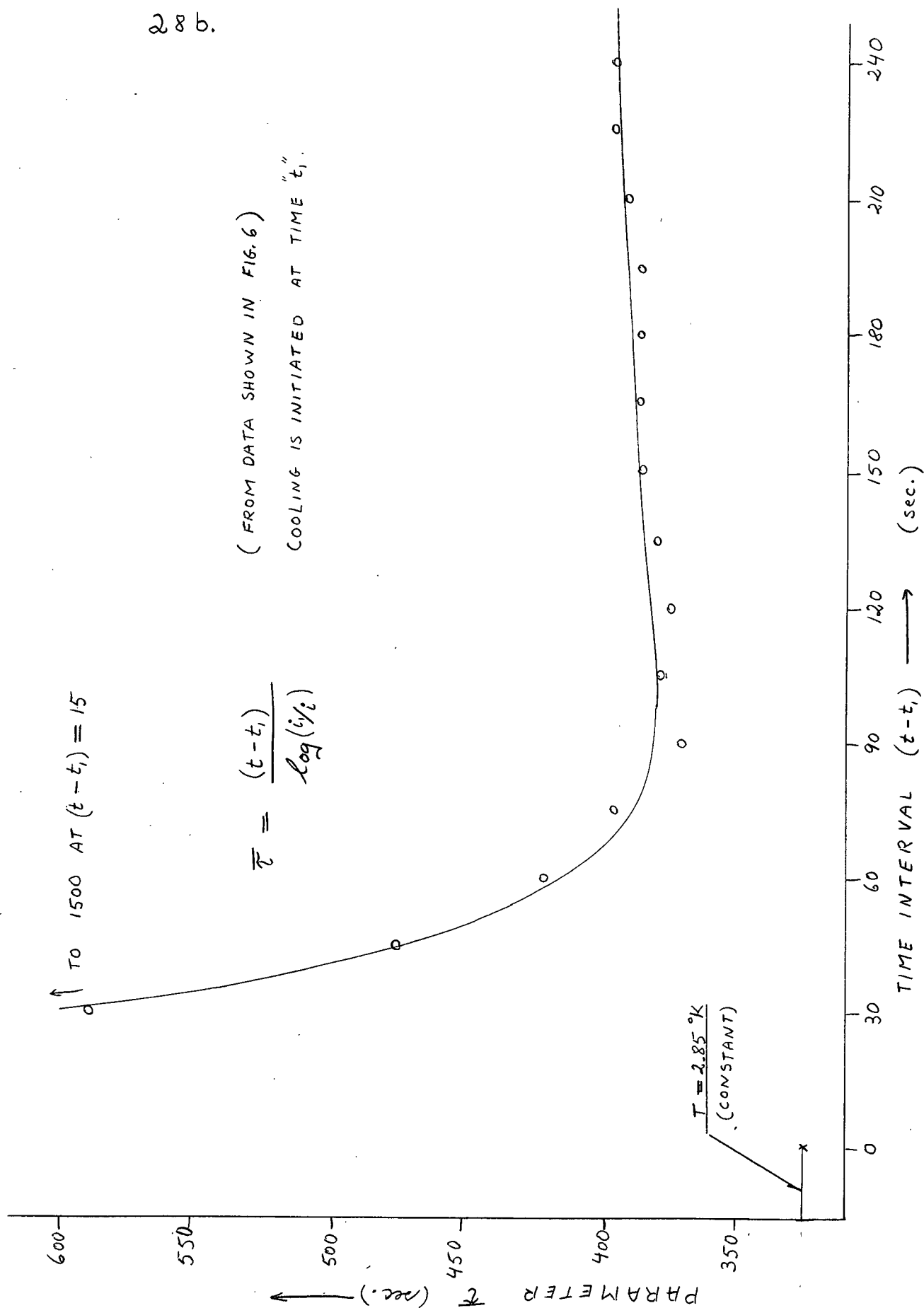
Figure 11: $\bar{\tau}$ AT VARIOUS TIME INTERVALS

That is $\quad \frac{1}{2} m v^2 < 2 \mathcal{E}(T)$

or $\quad \frac{1}{2} \left(\frac{I \alpha}{\ell}\right)^2 \frac{\Lambda}{n} < 2 \mathcal{E}(T) \qquad$ where $\quad I = \frac{\ell}{\alpha} n q v$

Hence $\quad \frac{1}{2} m v^2 = 2 \times 10^{-24}$ Joules

and since $\frac{\mathcal{E}(T)}{\mathcal{E}(0)} = .2$ at $2.9^\circ K$, therefore $2 \mathcal{E}(T) = k_B T_c = 4 \times 10^{-23}$ Joules

The current is therefore less than critical (as required) for the density
used above.

The calculations using equation 2.6 and the evaluated parameters α,
Λ and n are therefore not inconsistent with the experimental results of
Figure 10. The quantitative calculations above also apply to the experimental
results obtained in repetitions of the decreasing temperature procedure.
These results are all very similar including the results for the reverse
current.

Equations 2.2 and 2.6 are identical even though different methods were
employed for their derivation. They were shown to describe the experimental
results to within the experimental accuracy. However, equation 2.8 differs
from equations 2.2 and 2.6, due to different assumptions for its derivation,
but it also gives a good description of the experimental results.

Equation 2.8 has, $\qquad \frac{di}{dt} = - \frac{\left(R + \frac{1}{2} \alpha \frac{d\Lambda}{dt}\right)}{\left(L + \alpha \Lambda\right)} i$

This equation was obtained with the assumption of a common momentum of the
electron-pairs, but it was also assumed that no selection of the normal
electrons occurs and this distinguishes the result from equation 2.6 .

There is a difference between the equations by a factor of 2 in one term. Since only orders of magnitude are significant, the experimental results obtained for this thesis are not sufficiently accurate to decide the correctness of the two distinct equations. However, theoretical calculations based on generally accepted theory indicates that equations 2.2 and 2.6 are correct. This is shown in part E below. It is also shown in part E that an experimental method can be used to verify the accuracy of equations 2.2 and 2.6 even though electron densities and effective masses of Cooper-pairs are not accurately known at present.

D. Discussion of Errors

The anomaly in the current behaviour, caused by decreasing the sample temperature, has been explained on the basis of the common momentum feature. In addition to this feature, the principle of selection of normal electrons has also been used. To examine the validity of the explanation, thermodynamic principles are applied. The assumptions which had been made for the calculations are also discussed.

1. Thermodynamic requirements

Equation 2.6 is derived by assuming that normal electrons entering the superstate are selected according to their momentum. The newly-formed pairs must have a momentum equal to that of the other pairs originally present. This produces an increase of the kinetic energy associated with the current.

Essentially, the process is a transformation of internal energy into available work, and since the efficiency of this transformation must be less than 100%, the experimental results can be tested on this requirement. The calculations are given below.

Using the specific heat of indium at 2.8°K as $c = 12 \times 10^{-3}$ J/$^{\circ}$K mole the sample, of volume 5×10^{-11} m^3, has the heat capacity:

$$C_s = 3 \times 10^{-8} \text{ joules/deg.}$$

Therefore, $\quad \Delta U = C_s \Delta T \quad = 6 \times 10^{-9}$ joules $\quad$ where $\Delta T = 0.2^{\circ}$K

From Figure 10 the energy gain in the anomaly over a period of 6 seconds is,

$$i^2 R \, \Delta t \quad = 2.4 \times 10^{-10} \text{ joules} \quad \text{where } i = .3 A$$

The observed energy gain of the current system is therefore less than the change in internal energy as required by the second law of thermodynamics.

2. Correction factors

The electron density $n = 5 \times 10^{24}$ m^{-3} at 2.9°K, calculated by using equation 2.6 and the data of Figure 10, was considerably less than the value calculated from the penetration depth ($n = 6 \times 10^{26}$). However, the data gave a density which is still consistent with theory. The assumptions made in deriving equations 2.2 and 2.6 may require modifications to give a better description of the experiment.

It is assumed that the current in the sample is uniformly distributed across the thin film. However, since the cross-section of the indium film is a flat rectangle, edge effects are likely to create a non-uniform current distribution. This appears as a reduced cross-sectional area which consequently increases α . Therefore a correction for the edge effect increases the calculated electron-pair density.

A similar problem is associated with an incomplete transition of the sample into the superstate. This is especially prominent close to the transition temperature. Should it be the case that the current is restricted

to superconducting channels within the thin film (Pippard, 1951), the cross-sectional area is again decreased. The region outside of the channel may then become superconductive at lower temperatures. As well as decreasing the cross-sectional area of the thin film, the effect is to enhance the increase in the total number of electrons involved when cooling the sample. It is the total increase of electrons which appears as an energy gain (equation 2.4). Therefore, the temperature-dependent increase of the cross-sectional area increases the calculated electron-pair density.

Since the energy gap of a superconductor is modified by a current, the electron-pair density is also affected (B.C.S. theory; Parmenter, 1959). Therefore the penetration depth measured by Lock (1951) may not give an accurate value of the density in the sample with a current, comparable to the critical value, flowing through it.

The theoretical calculations were applied to an exaggeration of the anomaly. Instead of a constant current during the initial cooling period, the current actually decreased, but at a reduced rate. Therefore, an over-adequate condition was established for the calculations.

An additional inaccuracy may be introduced by using $m = 2\,m_e$ for the mass of the Cooper-pairs. Their effective mass may actually be higher than the value used.

Although the minimum value of the density ($n = 10^{24}$) was required to give a constant current as calculated from equation 2.6, consideration of all the factors mentioned above would improve the value substantially. Each correction discussed tends to increase the calculated density. The density has been made a test for the agreement of the calculations with the data.

3. Thermoelectric effects

It has been shown above that the change in the junction resistance is inadequate in explaining the observed anomaly. However, the copper in the junction may produce a thermal emf even though superconductors are not known to give measurable voltages.

The effect of a thermal emf on the current, which may be responsible for the anomaly, is examined below. It is shown that thermoelectric effects are not adequate for explaining two aspects of the anomaly.

(a) Since the sign of a thermoelectric voltage depends on the thermal gradient, a reversal of the current direction in the sample is not expected to reverse any thermal voltage in the sample circuit or in the search-coil circuit. However, the anomaly observed with the current in a reverse direction was found to be similar to that with a forward current. This result is contrary to the expected anomaly with a thermoelectric effect.

(b) The anomaly in the observed current corresponds to an energy gain (over a period of 6 seconds) of 10^{-10} joules. The voltage V required to generate this energy is given by,

$$\frac{V^2}{R} = 10^{-10} \text{ joules}$$

or, $$V = 10^{-10} \text{ volts}$$

This voltage is an unreasonably large thermoelectric effect for a copper wafer immersed in liquid helium at about 3°K. The thermoelectric power of pure copper is less than 10^{-6} volts/ K (MacDonald,1955). Therefore, it is unlikely that a thermal effect produces the observed anomaly.

4. Experimental accuracy

The experimental uncertainty in the current measurements is $\pm$ .5 mm as observed on the meterstick. This corresponds to uncertainties in the current of 5 x 10^{-3} amperes. As can be seen in Figure 6, the anomaly is within experimental accuracy. However, in Figure 7 (Dec.) two data points are not within the uncertainty. These occur shortly after the anomaly and their departure from a smooth line approximation cannot be explained. Since only incremental deflections of the galvanometer were measured, slow drifting of it was not effective in causing errors. In fact, no such drifting of the galvanometer was observed.

An analysis of the experimental results suggests that the explanations based on the common momentum feature and the principle of selection is valid. Other explanations, based on effects in the junction resistance, were found to be inadequate.

E. General discussion

The calculations based on equation 2.6, which assumes a selection of the normal electrons, are identical to calculations based on accepted theory. It has been shown above that, using London's equation $e = \frac{\partial(\Lambda \gamma)}{\partial t}$, the description of the current behaviour in an L-R circuit is given by equation 2.2. This equation is identical to equation 2.6.

For an inductive circuit with no resistance, the current behaviour has been calculated by Keller and Zumino (1961). Using quantum mechanical methods their resultant equation (in terms of the notation used in this thesis) is,

$$i = \frac{n'\left(h c / 4\right)}{\left(L + \alpha \Lambda\right)}$$

$$(4.4)$$

Using equation 2.6 with the simplification R = 0, we get,

$$\frac{di}{dt} = -\frac{\alpha \frac{d\Lambda}{dt}}{L + \alpha \Lambda} \, i$$

This equation can be integrated directly to obtain,

$$i = \frac{Constant}{\left(L + \alpha \Lambda\right)} \tag{4.5}$$

Therefore, the temperature dependence of the current in this equation is identical to that of equation 4.4. However, the latter equation shows the additional propery of flux quantization. This is purely a quantum mechanical feature.

Upon making the simplification R = 0 for equation 2.8, the result is,

$$\frac{di}{dt} = -\frac{\frac{1}{2} \alpha \frac{d\Lambda}{dt}}{L + \alpha \Lambda} \, i$$

This equation can also be integrated directly to obtain,

$$i = \frac{Constant}{\sqrt[2]{L + \alpha \Lambda}} \tag{4.6}$$

But this result is unique and its validity is contradicted by accepted theory. Because of the difference in the temperature of the current between equations 4.5 and 4.6, experimental measurements with no resistance in the circuit can be used to determine the accuracy of the two equations.

Discussions and calculations in this thesis have been entirely restricted to thin film superconductors. By means of a modification of the superconductor thickness, application of the equations can be made to bulk samples. Since the currents in a superconductor are effectively restricted to flow in a depth given by the penetration depth λ , substitution of λ for the thickness d of a thin film gives a simplified description of the current in a bulk sample. Therefore α becomes temperature-dependent and is now defined to be,

$$\alpha = \frac{\ell}{\lambda w}$$

This value can be substituted into all the equations above to obtain their application to bulk samples.

Chapter 5

CONCLUSIONS

It has been shown in the previous chapters that a study of the temperature

dependence of supercurrents provides a method for obtaining information of basic

processes in a superconductor. The feature of a common momentum by the electron-

pairs in indium is shown to be in agreement with the experimental results.

Not only is this feature sufficient to produce the results observed, it was

found to be necessary for providing an explanation. Other conjectured causes

were found to be inadequate.

The assumption that only those normal electrons with a preferred momentum

are selected by the superconductor to increase the superstate density is also

consistent with the experimental results. The preferred momentum requires that

newly-formed Cooper-pairs have a total momentum equal to that of other Cooper-

pairs. However, this selection principle is not necessary for an explanation

of the observed results. The experimental accuracy has not been sufficient

to resolve any expected effects caused by the selection of normal electrons.

It should, however, be possible to detect this principle by an improved

experimental design. In order to obtain agreement between theoretical calcula-

tions, (assuming that Cooper-pairs have a common momentum) and calculations

derived from generally accepted theory, it is necessary to assume a selection

principle.

The assumption of this principle also offers an explanation of the meta-

stable state of a supercurrent. It provides a mechanism for a supercurrent

to have fluctuations in the electron-pair density without dissipating energy.

The dissipation of energy due to transitions of electrons between the excited

current-carrying state and the lowest ground state is compensated for by the

selection of electrons which again provide an energy gain to the current.

The results obtained by studying the electric current flowing in an indium thin film and in an indium-copper-indium junction are as follows:

(1) At a constant temperature, the behaviour of a persistent current in a superconducting indium coil connected across a low resistance is an exponential decay. The rate of decay is given by the time-constant of the circuit. Starting at 2.9°K, a rapidly decreasing temperature reduces the rate of decay to a value considerably below that observed at a constant temperature. The decrease in the rate of decay occurs in the interval 2.9°K to 2.7°K. Subsequently, the rate of decay increases again to a larger value.

(2) The indium-copper junction has a temperature-dependent resistance in the range 1.2°K to 3.2°K. Shortly after constructing the junction the resistance had the values 8×10^{-10} _Ω_ at 1.2°K and 9.4×10^{-10} _Ω_ at 2.9°K. Three months after constructing it these values had increased to 1×10^{-9} _Ω_ at 1.2°K, 1.05×10^{-9} _Ω_ at 1.9°K and 1.1×10^{-9} _Ω_ at 2.9°K. With the current flowing in a reversed direction, the resistance at 1.9°K is 9.5×10^{-10} .

The temperature dependence is a monotonically increasing function with temperature. Close to the indium transition temperature of the indium the rate of increase is faster than at lower temperatures. At constant temperatures the resistance is ohmic over a current range from .3 to .06 amperes.

Although useful results were obtained, considerable improvement of the experimental design can be made. In performing the experiments, practical guidelines were obtained for improved experimental methods. A list of some suggestions is given below.

(i) To extend the experimental range closer to the transition temperatures and also to increase the accuracy of the experiments, a magnetometer of greater sensitivity is desired.

(ii) To obtain a more detailed observation of the sample current, an automatic recording method should be used.

(iii) Increasing the ratio of length to cross-sectional area for the sample would enhance the momentum effects to be studied.

(iv) A simplification in the analysis of the data is made by keeping the junction resistance at a constant temperature.

(v) An experimental study of a superconductor with no resistance in the circuit is suggested. To improve the accuracy of such a design, the inductance should be reduced to minimum value.

Appendix A

CONSTRUCTION OF THE SAMPLE COIL

Construction of the indium thin film was by vacuum evaporation onto a glass tube substrate. To obtain a simultaneous deposit around the tube, the bulk sample to be evaporated completely surrounded the substrate. This was achieved by using a heater in the form of a circular loop which enclosed the tube symmetrically.

At a pressure of 10^{-6} mm Hg before evaporation, the value rose to 10^{-4} mm Hg during the evaporation procedure.

To shape the film into a coil, the substrate was fitted to a lathe and carved into the desired geometry. By using the threading mechanism of the lathe and a blunt wooden tool, the 4-turn coil took its shape with no difficulties. A diagram of the coil is shown in Figure 12. The dimensions of the coil are as follows:

Length l	=	.25 m
Width w	=	9×10^{-4} m
Thickness d	=	2000 Å
Inductance L	=	3×10^{-7} H (4-turns)

Connections to the ends of the coil consisted of indium wires which were simply soldered to the thin film and substrate. By measuring the resistance of the coil at room temperature and its dimensions, the calculated thickness was found to be 2000 Å. However, measurements of the inductance could not be made directly at room temperature due to the high resistance of the film, but measurement of the inductance of a copper wire replica served as a value to be used in the calculation. The inductance is 3×10^{-7}H.
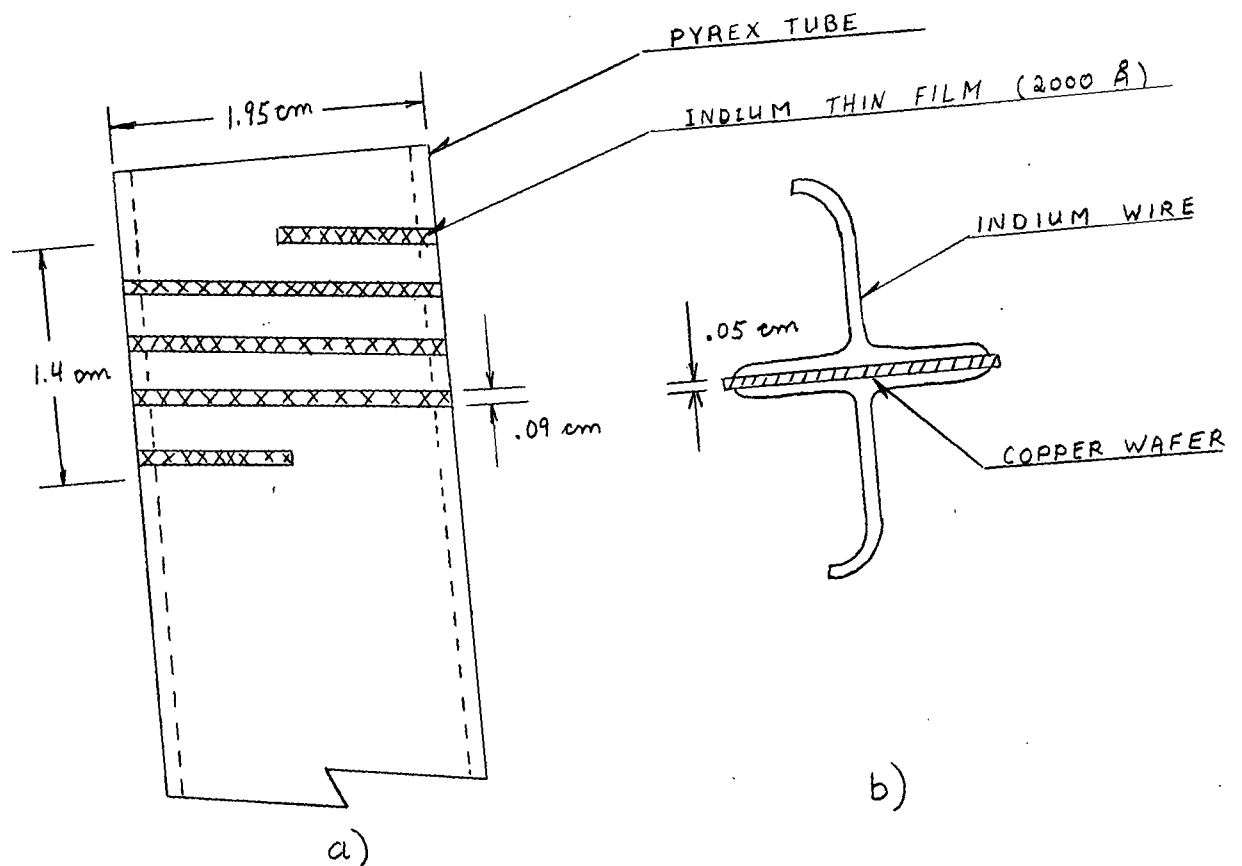
Figure 12:   a) THIN FILM COIL

b) JUNCTION RESISTANCE

Appendix B

CONSTRUCTION OF THE JUNCTION RESISTANCE

The junction resistance consists of a thin copper wafer which has indium contacts soldered to both large surfaces. A contact area of 1 cm^2 is made by each lead. In Figure 12 a section view of the junction can be seen.

After cleaning the surfaces of the copper wafer, the indium contacts were fused to the wafer by heating the parts to the melting temperature of indium. It was necessary to use acid flux in order to get the indium to "wet" the copper properly. Calculations of the junction resistance at $4^\circ K$ gave the value $5 \times 10^{-10} \Omega$. The measured value is $9 \times 10^{-10} \Omega$ at $3^\circ K$. The dimensions of the copper wafer are:

$$Width \quad = \quad 1.3 \ cm$$

$$Length \quad = \quad 1.4 \ cm$$

$$Depth \quad = \quad .05 \ cm$$

Appendix C

LIST OF SYMBOLS

A list of the symbols used in the calculations is given below. There is some overlap in these definitions but such cases are usually redefined or else obvious.

d   –   thickness of thin film

l   –   length of the indium film

w   –   width of the indium coil

V   –   volume of the indium coil

Λ –   parameter defined by $\Lambda = \dfrac{m}{q^2 n}$

α –   parameter defined by $\alpha = \dfrac{\ell}{d \cdot w}$

τ –   time constant of the L-R circuit

λ –   penetration depth into a superconductor

δ –   an effective depth

m   –   mass of the Cooper-pairs

q   –   charge of the Cooper-pairs

n   –   density of the Cooper-pairs

v   –   drift velocity of Cooper-pairs

e   –   electric field

j   –   current density

i   –   total current

T   –   temperature of the sample

T_c –   transition temperature of the sample

L   –   inductance of the sample circuit

R   –   resistance of the junction

k   –   Boltzman constant

E_F –   Fermi energy

c   –   speed of light

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