MEASUREMENT OF SMALL SHIFTS
OF WIDE LINES

by

MICHAEL U. POTTER

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Department of PHYSICS

The University of British Columbia
Vancouver 8, Canada

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ABSTRACT

A new technique has been devised to obtain time-resolved measurements of small spectral line shifts. Two linear neutral density wedges with opposite transmission gradients were placed over the top and bottom of a monochromator exit slit in such a way that a wavelength shift of a spectral line resulted in a change in light intensity transmitted through the wedges, which was monitored by two photomultipliers. To test this technique, an N II line, for which Griem has calculated the Stark shift as a function of electron density, was observed in the emission spectrum of a small theta-pinch using a monochromator of 10 Å/mm inverse dispersion. Space and time resolved line shift measurements were made and were associated with the axial velocity and the electron density fluctuations of the plasma. An uncertainty of 0.015 Å was estimated, which, for the 1.5 Å line used, corresponds to a shift to width ratio of 10^{-2}, believed to be lower than any previous shift measuring technique. Time of flight velocity measurements and conservation of mass considerations have confirmed the results as both reasonable and self-consistent.

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CHAPTER I

INTRODUCTION

(a) General Introduction

Plasma spectroscopy has become an extremely effective technique for measuring both atomic parameters and some macroscopic properties of the plasma. A great deal of information may be obtained by measuring the wavelength shift of a spectral line. Such shifts can be produced by mass motion (Doppler shifts), magnetic fields (Zeeman effect), and internal electric fields (Stark effect). In many typical laboratory plasmas the Stark effect is dominated by the impact of electrons with the emitting atoms or ions. Hence, velocities, magnetic fields and electron densities can, in principle, be determined from line shift measurements. The difficulty of these measurments is due to the very small shifts compared with the line width itself. For example, if a plasma has a particle velocity of 5 km/sec, then the Doppler shift of a spectral line of 5000 Å wavelength is $\lambda = \lambda \cdot v/c = 8 \times 10^{-2}$ Å. Stark shifts which, to a very good approximation, are linearly proportional to the electron number density, may by typically of the order of 10^{-1} Å for a plasma of $n_e \approx 10^{17}$ cm^{-3} and $T \approx 10^4$ °K. For a typical line width of 1 Å it is required to measure shift to width ratios of 10^{-2} to obtain 10% accuracy in line shift measurements of the above examples. The difficulty of such a measurement is enhanced by the fact that frequently only high inverse dispersion monochromators are available. For an inverse dispersion of 10 Å/mm the shift of these examples is of the order of 10^{-2}mm.

The purpose of this thesis is to describe a new technique for measuring such small line shifts of wide lines and to report some results obtained from a theta-pinch experiment applying this method to Stark and Doppler shifted lines. The important features of the technique are (1) no assumptions regarding the wavelength profile of the spectral line are necessary, (2) high time resolution is possible, and (3) high inverse dispersion (10 Å/mm) monochromators may be used.

(b) Discussion of Existing Methods

Several methods of measuring small line shifts have been previously reported. Photographic techniques are the most common and perhaps the easiest to apply. A spectrum is taken with a low inverse dispersion spectrograph of the plasma of interest, and, on the same plate, is placed a wavelength calibration provided by the spectrum of a low-pressure discharge, for which the line shifts are negligibly small. The main disadvantage of this technique is that, without further complication of the apparatus (rotating mirrors, drum cameras, etc), a time integrated spectrum is taken and no time resolution is possible.

In 1962 Drobowshevsky (ref. 1) reported that he had measured Doppler shifts of a few tenths of an Angstrom resulting from the azimuthal motion of a plasma in a homopolar device. His method consisted of observing a single line from a monochromator and splitting the wavelength profile with the intersection line of two prism faces perpendicular to each other, so that two photomultipliers were exposed to the light from the two halves of the line profile. He then rotated the prism assembly about an axis parallel to the direction of the light beam, so that it was set at some angle α, as shown in fig. 1. It is easily seen that a shift in wavelength of the

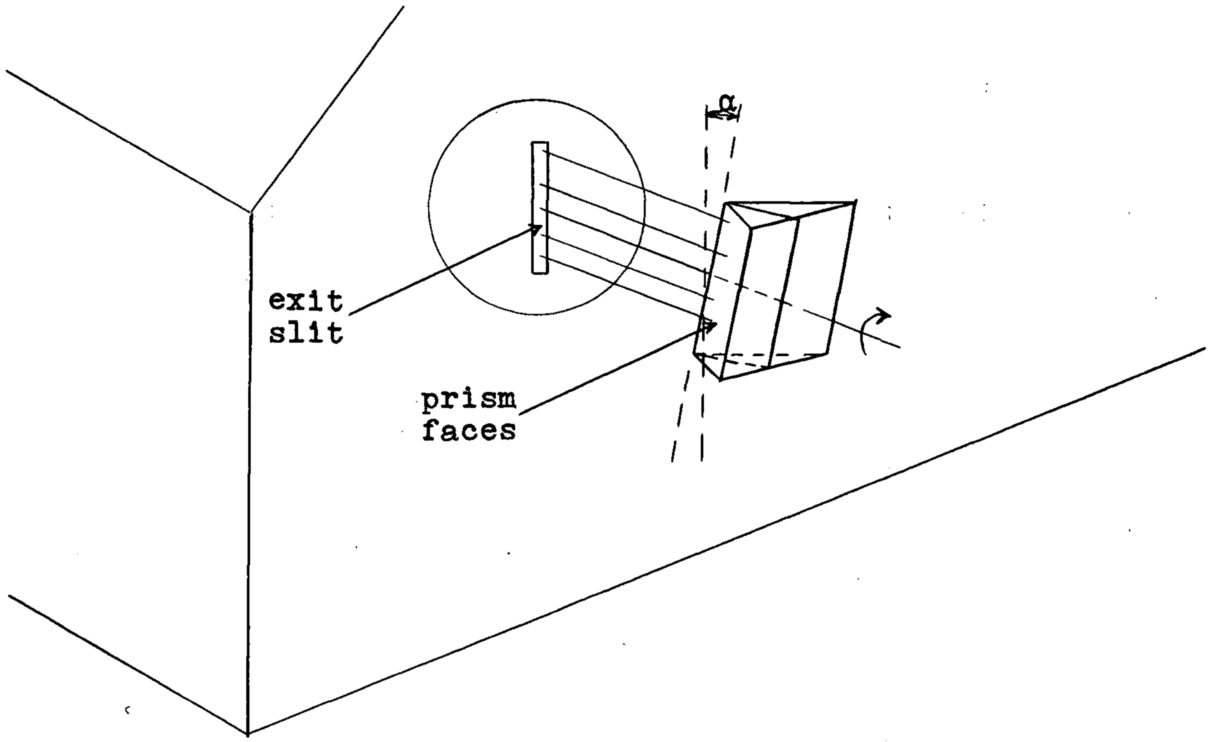


Fig. 1 Drobowshevsky's line shift measurement

line will result in an increased light intensity falling on one photomultiplier and a decreased intensity on the other, and from the variation in their voltage signals the shift can be calculated. The first difficulty associated with this technique is that it requires a knowledge of the line profile and the intensity distribution along the slit. These functions may be difficult to measure accurately and could very well vary during an experiment. The second difficulty arises if the line is narrow or the inverse dispersion of the monochromator is high, in which case the width of the prism intersection edge may no longer be small compared to the line width at the exit plane of the monochromator, so that irregularities in the edge will greatly influence the result.

Another line shift technique was applied by Keilhacker et al (ref.2). They used a Fabry-Perot interferometer and monitored the line profile photoelectrically with an array of eight fibre bundles arranged in a line. These eight fibres covered a spectral range of 2.54 Å. By monitoring the line profile in 0.32 Å steps it seems unlikely that shifts of less than 0.05 Å could be accurately measured. This sensitivity was sufficient to obtain useful information regarding the rotation of a plasma in a theta-pinch, since Doppler shifts of several tenths of an Angstrom were found. However, line shifts of interest in laboratory plasmas are not always so large. For example, it would be difficult to apply such a method to measure particle velocities in a membrane shock tube where shifts in the range 10^{-2} to 10^{-1} Å are to be expected.

Small line shifts have also been measured (ref. 3,4,5) by selecting two narrow segments on the wings of the line (fig. 2)

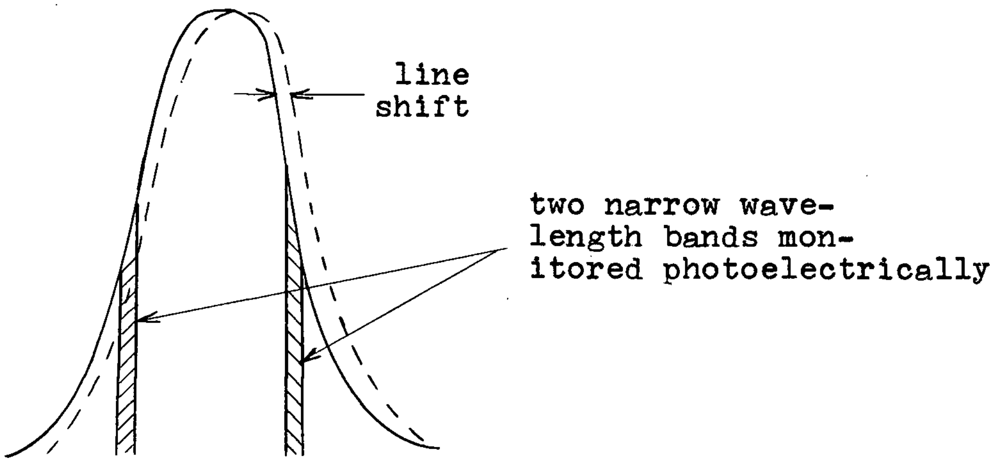


Fig. 2 Line shift measurement by selecting two narrow segments of the spectral line.

and recording the signal from each segment. As the line shifts, the relative magnitude of the signals will vary and the size of the shift can be calculated if the line profile is known. Generally, a small inverse dispersion instrument is required for this technique, except in Hirschberg's apparatus (ref. 3), which, by clever use of polarized light, separates two different wavelengths passing through a single exit slit. The major difficulty associated with this technique is that it is required to know the line profile before the shifts can be calculated. In addition, only a fraction of the total light intensity is used and an acceptable signal to noise ratio may be more difficult to obtain than if the entire line profile were used.

(c) Double Wedge Technique

The double wedge technique of measuring line shifts is designed to permit time resolved measurements of shifts down to 10^{-2} Å. The basic principle of this method, which has already been described (ref. 6), can be best understood by means of fig. 3. The monochromator is used to observe a single isolated line in the plasma radiation spectrum. At the exit slit are mounted two neutral density filters, W_1 and W_2. The transmission of these filters varies linearly in the x-direction and the transmission gradients are equal and opposite for the two filters. These filters are also aligned so they have equal transmission at the centre of the slit. All the light from the upper half of the exit slit passes through the filter W_1 and is monitored by photomultiplier P_1

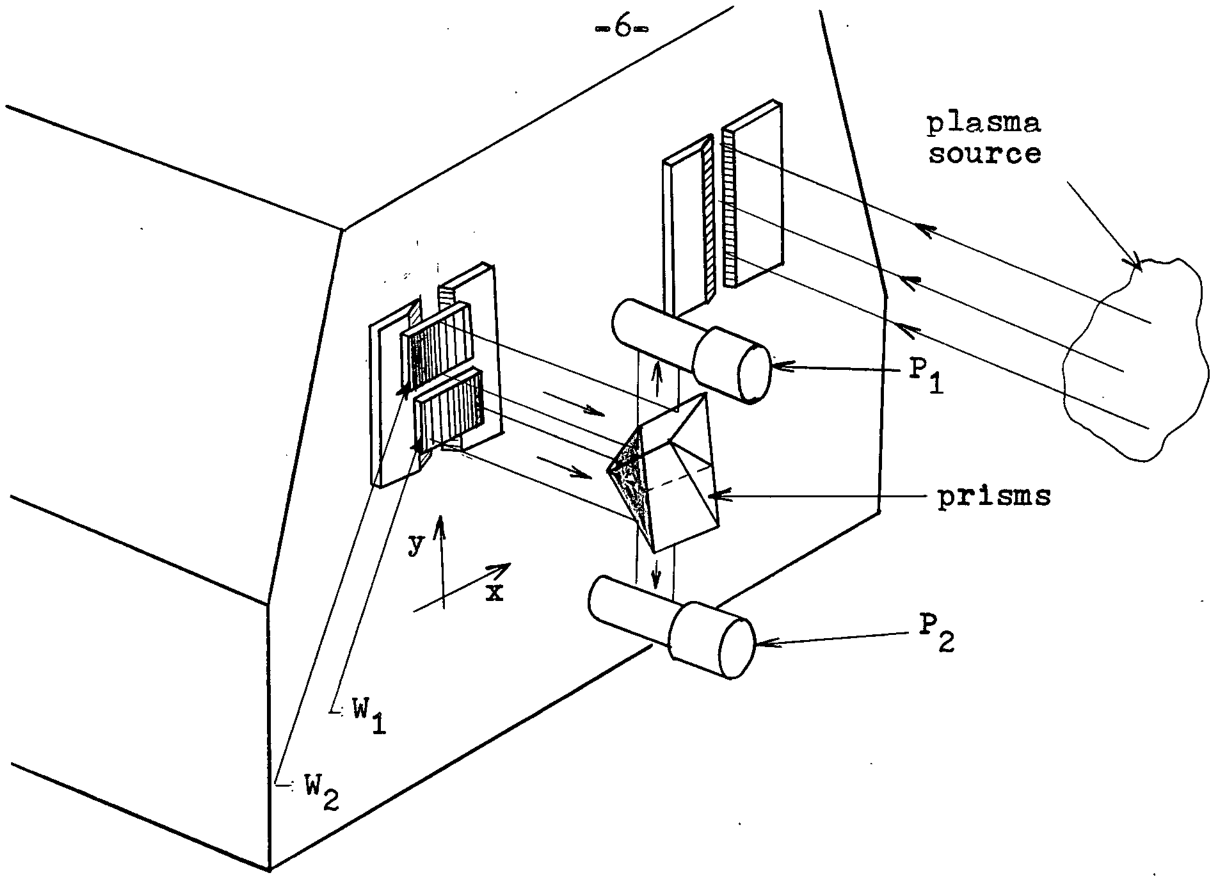


Fig. 3 Double wedge technique

and all the light from the lower half of the slit passes through W_2 and onto P_2. A shift in the wavelength of the spectral line will increase the intensity in one photomultiplier and decrease the intensity in the other. If the slope of the transmission gradients, the transmission at the centre of the slit, and the dispersion of the monochromator are all known, then the wavelength shift of the line can be calculated from measurement of the photomultiplier signals.

As will be proved in Chapter II, no assumptions need be made regarding the line profile or even the constancy of the line profile. This technique always measures the shift of the line centroid. By recording the photomultiplier signals on an oscilloscope a temporal history of the line shift can be obtained. Very good time resolution is possible and is limited only by the response of the photo-electronics which can be made as short as 3×10^{-8} sec.

A plasma light source (theta-pinch) was set up to experimentally demonstrate the double wedge technique, and an N II line was observed for which the Stark shift has been calculated by Griem (ref. 7). It was expected that possible experimental error inherent in the apparatus was about 10^{-2} Å and the statistical uncertainty, due mainly to the non-reproducible nature of the plasma, was about 3×10^{-2} Å if nine repetitive measurments were taken. Checks have been made to show that the Doppler shift and Stark shift results obtained were both reasonable and self-consistent.

A major difficulty in the experimental procedure of this method is to balance two photomultipliers accuratel? for fast rising signals. Although response time is naturally limited by the rise time of the photo-electric system, a procedure was devised to eliminate the effect of small differences in response characteristics of the twin photomultiplier system. This is discussed in Chapter IV.

CHAPTER II

THEORY

(a) Fundamental Equation

In this section the fundamental equation will be derived whereby line shifts can be calculated from the double wedge technique.

Consider the intensity profile of an unshifted spectral line at the exit plane of a monochromator to be $I(x,y) = \Psi(x)\alpha(y)$. The function $\Psi(x)$ represents the line shape at the exit plane and is due to the natural line shape, the macroscopic properties of the plasma (such as rotational motion), the instrument profile and the slit width. The function $\alpha(y)$ represents the intensity distribution over the length of the exit slit. This is dependent upon the characteristics of the monochromator and upon the optics which focus the light from the plasma onto the entrance slit. The $\alpha(y)$ will also depend upon any non-uniformity of the plasma observed. During the experiment $\alpha(y)$ is considered to change only by a constant factor and the design of the apparatus (in particular, the entrance optics) must ensure that this assumption is valid.

The assumed form of $I(x,y)$ is justified if the monochromator slits are parallel and if the intensity of light at each point on the exit slit has equal contributions from all parts of the plasma observed so that there is no correlation between the position y and the plasma configuration. As described more fully in Chapter III both this assumption and the constancy of $\alpha(y)$ require astigmatic focussing of the plasma radiation on the entrance slit.

The point, $x = 0$, $y = 0$, is defined to be at the centre of the exit slit. The spectral line to be observed is positioned so that its centroid with respect to the x-coordinate lies at $x = 0$. Assume that the width w of the slit is very much greater than the width L of the line at the exit plane. Under these conditions the total light intensity passing through the upper half of the exit slit is given by

$$G_1 = \int_0^{d/2} \alpha(y)\,dy \int_{-w/2}^{w/2} \Psi(x)\,dx \tag{1a}$$

where d is the length of the slit. The light intensity passing through the lower half of the slit is

$$G_2 = \int_{-d/2}^{0} \alpha(y)\,dy \int_{-w/2}^{w/2} \Psi(x)\,dx \tag{1b}$$

Two photomultipliers monitor intensities G_1 and G_2. The signal recorded by each photomultiplier is proportional to the light intensity it receives, where the constant of proportionality may be varied by the photomultiplier power supply adjustment. These voltage signals can be written

$$S_1 = k_1 G_1 \tag{2a}$$

$$S_2 = k_2 G_2 \tag{2b}$$

The constants k_1 and k_2 are varied by adjusting the supply voltages to make $S_1 = S_2$.

In front of the upper half of the exit slit is placed a neutral density wedge whose transmission is given by

$$t_1(x) = t_\circ + bx \tag{3a}$$

where t_o and b are constants. In front of the lower half of the slit is placed a wedge with transmission

$$t_2(x) = t_o - bx \tag{3b}$$

This is shown graphically in fig. 4a. The light intensity of the upper half of the slit is now given by

$$g_1 = \int_0^{d/2} \alpha(y)dy \int_{-w/2}^{w/2} t_1(x)\Psi(x)dx$$

$$= t_o G_1 + b\int_0^{d/2} \alpha(y)dy \int_{-w/2}^{w/2} x\Psi(x)dx \tag{4a}$$

and the signal recorded by the upper photomultiplier is

$$s_1 = k_1 g_1$$

$$= k_1 t_o G_1 + k_1 b\int_0^{d/2} \alpha(y)dy \int_{-w/2}^{w/2} x\Psi(x)dx \tag{5a}$$

Similarly, for the lower half of the exit slit the photomultiplier signal is

$$s_2 = k_2 t_o G_2 - k_2 b\int_{-d/2}^{0} \alpha(y)dy \int_{-w/2}^{w/2} x\Psi(x)dx \tag{5b}$$

Since, for any value of y, the centroid of the intensity profile with respect to the x-coordinate lies at x = 0, then

$$\int_{-w/2}^{w/2} x\Psi(x)dx = 0 \tag{6}$$

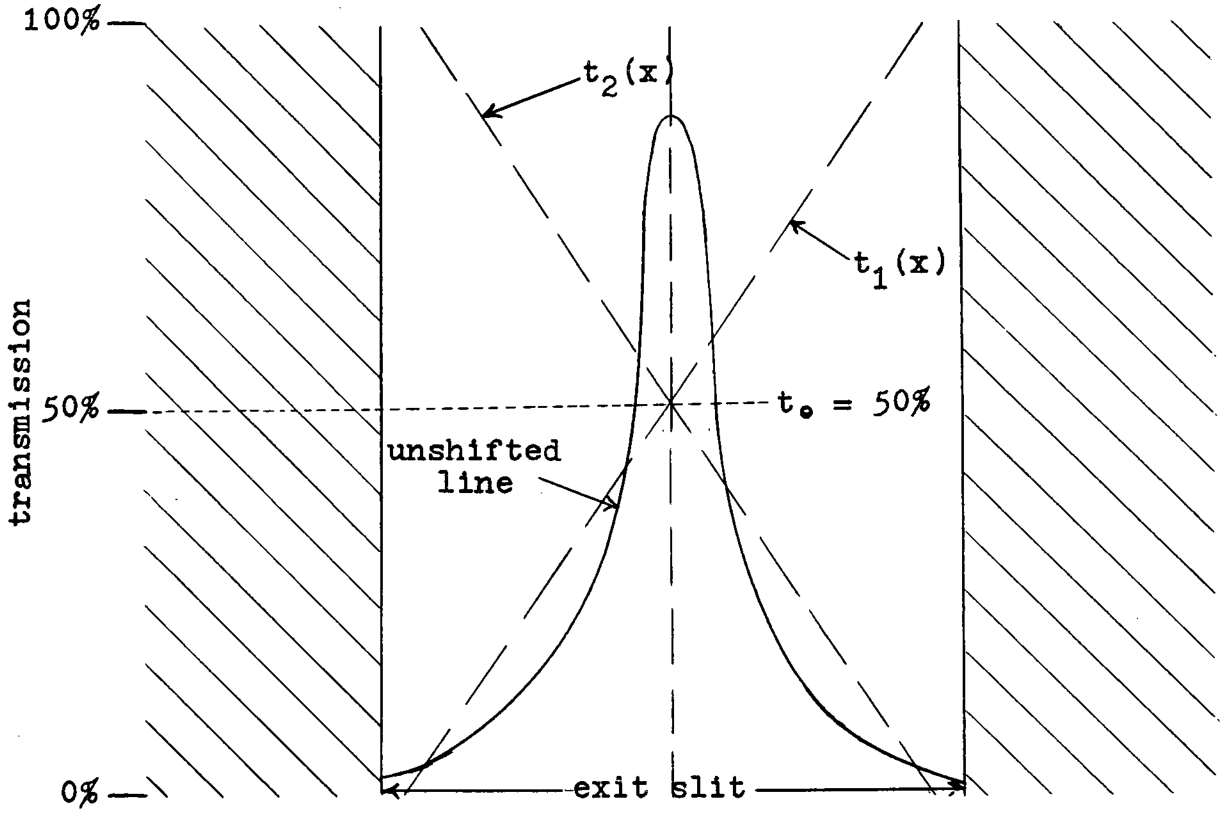


Fig. 4a Unshifted line at the exit plane

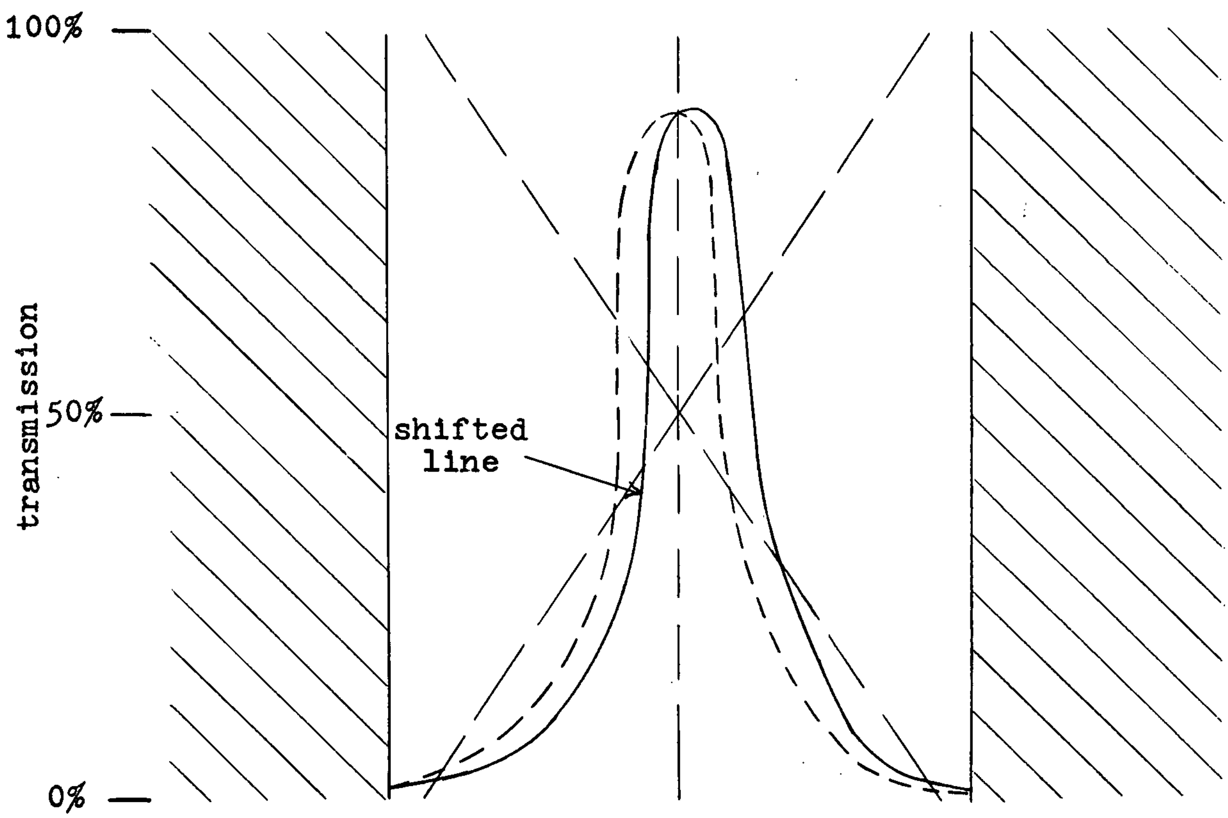


Fig. 4b Shifted line at the exit plane

and from equations (5a) and (5b)

$$s_1 = k_1 t_o G_1 \tag{7a}$$

$$s_2 = k_2 t_o G_2 \tag{7b}$$

Therefore,
$$s_1 = s_2 \tag{8}$$

Assume that the line centroid is shifted by an amount Δx and may change its shape in any manner (see fig. 4b). It is assumed that the optical setup does not change in any way so that the intensity distribution $\alpha(y)$ along the exit slit will change at most by a constant factor, C. One may write the intensity distribution of the shifted line as

$$I^*(x,y) = C\alpha(y)I^*(x - \Delta x) \tag{9}$$

No relationship is assumed between the functions I and I*. Now the signal recorded by the upper photomultiplier is

$$s_1^* = k_1 C\int_0^{d/2}\alpha(y)dy \int_{-w/2}^{w/2} t_1(x)\ I^*(x - \Delta x)dx \tag{10}$$

Let $x' = x - \Delta x$

$$s_1^* = k_1 C\int_0^{d/2}\alpha(y)dy \int_{-w/2 + \Delta x}^{w/2 + \Delta x} t_1(x' + \Delta x)I^*(x')dx' \tag{11}$$

Since small shifts, such that $x \ll w$ are considered, and the assuption, $w \gg L$, implies $I^*(x) \longrightarrow 0$ as $x \longrightarrow w/2$, then the limits of integration over x' can be changed to $\pm w/2$ with negligible error.

$$s_1^* = k_1 Ct_o \int_0^{d/2} \alpha(y)dy \int_{-w/2}^{w/2} I^*(x')dx'$$

$$+ k_1 Cb \int_0^{d/2} \alpha(y)dy \int_{-w/2}^{w/2} (x' + \triangle x) I^*(x')dx'$$

$$= k_1 Ct_o \int_0^{d/2} \alpha(y)dy \int_{-w/2}^{w/2} I^*(x')dx'$$

$$+ k_1 Cb \int_0^{d/2} \alpha(y)dy \int_{-w/2}^{w/2} I^*(x')x'dx'$$

$$+ \quad xk_1 Cb \int_0^{d/2} \alpha(y)dy \int_{-w/2}^{w/2} I^*(x')dx' \qquad (12)$$

Since the centroid of the shifted line lies at $x = \triangle x$ (ie. $x' = 0$) then the second term of equation (12) vanishes.

$$s_1^* = Ct_o S_1^* + \triangle x Cb S_1^* \qquad (13a)$$

Where S_1^* is the signal from the upper photomultiplier when observing the shifted line with no wedges. Similarily for the lower half of the exit slit

$$s_2^* = Ct_o S_2^* - \triangle x Cb S_2^* \qquad (13b)$$

From $S_1 = S_2$

$$k_1 \int_0^{d/2} \alpha(y)dy \int_{-w/2}^{w/2} I(x)dx = k_2 \int_{-d/2}^{0} \alpha(y)dy \int_{-w/2}^{w/2} I(x)dx$$

and it follows that

$$Ck_1\int_0^{d/2}\alpha(y)dy\int_{-w/2}^{w/2}I^*(x')dx' = Ck_2\int_{-d/2}^{0}\alpha(y)dy\int_{-w/2}^{w/2}I^*(x')dx' \tag{14}$$

Therefore, $$S_1^* = S_2^* = S^* \tag{15}$$

Equations (13a) and (13b) now become

$$s_1^* = CS^*(t_o + b\Delta x) \tag{16a}$$

$$s_2^* = CS^*(t_o - b\Delta x) \tag{16b}$$

Solve for Δx from equations (16a) and (16b)

$$\Delta x = \frac{s_1^* - s_2^*}{s_1^* + s_2^*}\frac{t_o}{b} \tag{17}$$

If the inverse dispersion, $d\lambda/dx$, of the monochromator is known then one can write

$$\Delta\lambda = \frac{s_1^* - s_2^*}{s_1^* + s_2^*}\frac{t_o}{b}\frac{d\lambda}{dx} = \frac{\Delta s}{\Sigma s}\frac{t_o}{b}\frac{d\lambda}{dx} \tag{18}$$

This is the fundamental equation used to calculate line shifts from the double wedge technique, where $\Delta\lambda$ is the wavelength change of the shifted line measured relative to the position where both wedges have the same transmission, t_o.

Notice that equation (18) does not indicate an absolute line position measurement. The double wedge technique is a sensitive line shift measuring device only insofar as relative line shifts are concerned. In this respect this technique has properties similar to a Faby-Perot interferometer, with which relative measurements have high resolution but absolute wavelengths are known only to the precision of the pre-selecting spectrograph. It will, however, become evident in section (c) of this chapter how relative line shift measurements can yield useful information about a plasma light source.

(b) Sensitivity of the Measurement

The critical measurement for the determination of a small line shift is the voltage difference, Δs. For a given line shift $\Delta\lambda$ this quantity will be maximum when the slope b of the wedges is greatest. The value of b is maximum when the wedges vary from 0% to 100% transmission over the full width L of the spectral line at the exit plane (and this is only possible for a symmetrical line)

$$\text{ie. } b = \frac{1}{L}\frac{d\lambda}{dx}$$

when transmission t_o will be 50%.
In this case equation (18) becomes

$$\Delta\lambda = \frac{\Delta s}{\Sigma s}\frac{L}{2} \qquad (19)$$

However, for a given experiment the quantity $\frac{L}{2(\Sigma s)}$ is a constant and the sensitivity of the voltage difference Δs to a given line shift $\Delta\lambda$ is maximized when $\frac{L}{2(\Sigma s)}$ is minimized. This occurs when the light intensity (which is proportional to Σs) is greatest. However the intensity must be maximized by adjustment of the entrance optics and increase of the plasma light intensity itself. If Σs is increased by opening the entrance slit then L is also increased by a proportional amount and the sensitivity of measurement will not be improved. In essence, equation (19) indicates that a bright, narrow line at the exit plane is required for best results of the line shift measurement.

However there are other considerations whereby one can optimize the entrance slit width for a given spectral line. It is a normally accepted result (ref. 8) that for a given number of photons incident on a photomultiplier the number of primary electrons emitted by the photo-cathode will be Poisson distributed about the mean. A unique feature of this distribution is that the variance is equal to the mean. By taking the variance of the shift Δx, as given by equation (17) while holding t_o, b, and Σs constant one has

$$\mathrm{var}(\Delta x) = [\mathrm{var}(s^*) + \mathrm{var}(s_2^*)]\left[\frac{t_o}{b(s^* + s_2^*)}\right]^2$$

but $\mathrm{var}(s_1^*) = \bar{s}_1$ (mean of s_1^*), and $\mathrm{var}(s_2^*) = \bar{s}_2$ (mean of s_2^*)

Therefore, $$\mathrm{var}(\Delta x) = \left[\frac{t_o}{b}\right]^2 \frac{1}{\Sigma s} \tag{20}$$

since $(\bar{s}_1 + \bar{s}_2) \approx \Sigma s$

However, Σs is proportional to the entrance slit width l, the intensity of the plasma M, and the centroid transmission t_o of the wedges. Therefore one can write $\Sigma s = Mlt_o$, and equation (20) becomes

$$\mathrm{var}(\Delta x) = \frac{t_o}{b^2 Ml} \tag{21}$$

The total line width L at the exit plane is approximately $l + l_o$, where l_o is the actual line width as determined by the natural line shape and the properties of the plasma. It is required that the wedge transmission vary from 100%

to 0% over the width $L = l + l_o$ (assuming the expected shift is small compared with L). Under these conditions the slope b is given by

$$b = \frac{1}{l_o + l}$$

which implies that

$$l = \frac{1}{b} - l_o \qquad (22)$$

so equation (21) becomes

$$\text{var}(\Delta x) = \frac{t_o}{b^2 M(\frac{1}{b} - l_o)} \qquad (23)$$

To find b for the minimum var(Δx), equate $\frac{\partial}{\partial b}[\text{var}(\Delta x)]$ to zero.

$$\frac{t_o}{M}(b - b^2 l_o)^{-2}(1 - 2bl_o) = 0$$

$$\text{ie. } 2l_o = \frac{1}{b} \qquad (24)$$

From equations (22) and (24) $l = l_o$. This result indicates that one should choose an entrance slit width equal to the actual line width. However, it should be kept in mind that this is required if one wishes to minimize the variance of the distribution of the voltage signal; but the possible errors introduced by other factors such as electromagnetic pick-up and noise may be of first consideration, especially when a bank discharge is involved.

(c) Application to Experiment

This method can be directly applied to measure shifts due to Doppler and Stark effects in a plasma with

with rotational symmetry and axial motion (such as a theta-pinch). To measure the Doppler shift due to the axial motion it is necessary to observe the plasma from two different directions with respect to the z-axis and , in effect, to compare the position of the line in each case. However, to measure the Stark shift one need only observe the plasma from one position, but the absolute shift can be evaluated only if a precisely known reference line can be observed. Unshifted standard lines may be obtained from low density discharges, such as that described by Minnhagen (ref. 9). However, for the experimental work reported in this thesis such a discharge was not available. Hence the Stark shifts were measured relative to an arbitrary wavelength (the wavelength where the transmission of both wedges is t_o), which yields electron density fluctuations; ie. <u>changes</u> in the electron density as a function of time.

The fundamental nature of the measurement described above can be better understood as follows. If one observes the plasma from some angle α from the axis of rotational symmetry then the line position observed can be expressed as

$$\Delta\lambda_1(t) = \Delta\lambda_s(t) + \Delta\lambda_d(t) + \lambda_o \qquad (24)$$

where: $\Delta\lambda_s$ is the absolute Stark shift

$\Delta\lambda_d$ is the Doppler shift due to mass motion at velocity V in the axial direction;

$$\text{ie. } \Delta\lambda_d = \lambda\frac{V\cos\alpha}{c}$$

λ_o is the position of the unshifted line, which is only known to the accuracy of the monochromator setting

The plasma is then observed from some other angle α' without changing the position of transmission t_0 (see fig. 5).

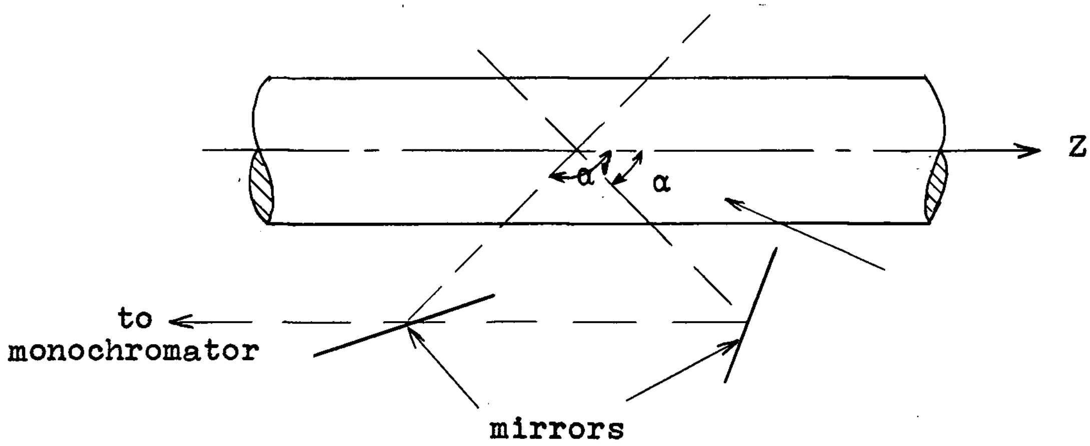


Fig. 5 Doppler shift measurement for a rotationally symmetric plasma

For this second case the line is observed at the wavelength

$$\lambda_2(t) = \Delta\lambda_s'(t) + \Delta\lambda_d'(t) + \lambda_0 \tag{25}$$

The values of $\Delta\lambda_s$ and $\Delta\lambda_s'$ will be equal if one can assume that essentially the same plasma configuration is observed from both direction α and α'. If $\alpha + \alpha' = 180°$ and one observes a rotationally symmetric, optically thin plasma with ideal optics this assumption is valid. Under these conditions $\Delta\lambda_d$ and $\Delta\lambda_d'$ are equal in magnitude and opposite in sign. By subtracting equation (25) from (24) the Doppler shift is given by

$$\Delta\lambda_d(t) = \frac{\lambda_1(t) - \lambda_2(t)}{2} \tag{26}$$

To obtain Stark shift data one observes the plasma from a direction perpendicular to the direction of mass motion, ie. $\alpha = 90°$, as shown in fig. 6.

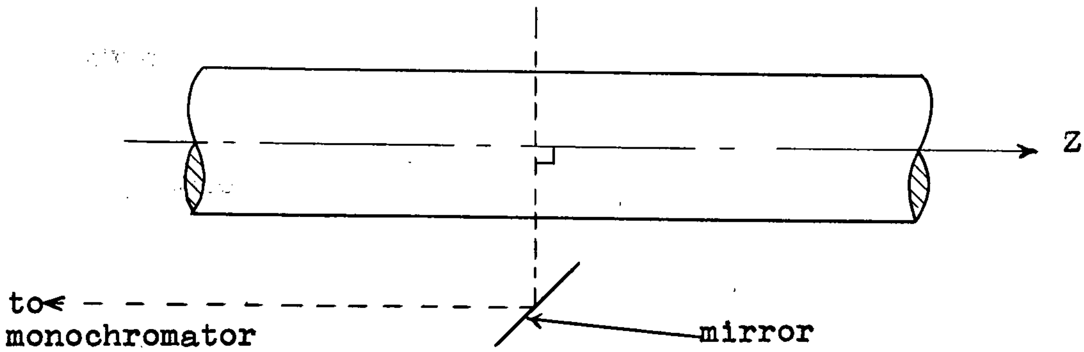


Fig. 6 Stark shift measurement of a rotationally symmetric plasm

Since $\Delta\lambda_d = \lambda \frac{V \cos\alpha}{c} = 0$ for $\alpha = 90°$ then equation (24) becomes

$$\lambda_1(t) = \Delta\lambda_s(t) + \lambda_o \tag{27}$$

and it follows that

$$\frac{\partial \lambda_1}{\partial t}\Delta t = \frac{\partial}{\partial t}(\Delta\lambda_s)\,\Delta t \tag{28}$$

and the fluctuations in time of the Stark shift can be calculated.

To apply and this theory in practice a plasma light source was required which was rotationally symmetric and whose spectral lines were expected to show Stark and Doppler shifts. It was expected that a small theta-pinch device with a conveniently long ringing period, not critically damped, should permit measurement of both shifts. Such a source was set up and is described in the following chapter.

CHAPTER III

APPARATUS

(a) General Description of Apparatus

A schematic diagram of the experimental setup is found in fig. 7. The plasma observed was created by a theta- pinch device. This consisted of a glass tube of 2.5 cm i.d. surrounded by a single turn copper coil 2 cm wide and of diameter 3.2 cm. A 15.6 μF capacitor bank charged to 15 kv was discharged through the coil. Breakdown of the gas in the tube was ensured by a pre-ionizing glow discharge. For these experiments the tube was filled with air at 350 μ Hg pressure.

The light emitted from a narrow cross-section ($\Delta Z \approx 0.5$ cm) of the plasma in the tube was focussed astigmatically on the entrance slit of a Spex 1700 II monochromator by means of a system of field stops and cylindrical lenses. The axes of these lenses were parallel to the monochromator slits since, with no focussing in this direction, the intensity distribution $\alpha(y)$ along the length of the slits was not sensitive to the optical alignment and could be expected to remain constant during the experiment. In addition, this ensured that a position on the entrance slit did not correspond to a particular region of the plasma cross-section and the wavelength profile of the line was expected to be the same (except for a constant factor) at all points on the slit. Thus, two of the important assumptions of Chapter II are validated. The optical system was arranged so that the light from the plasma could be observed from three different directions; perpendicular to the tube axis and at an angle of 45°

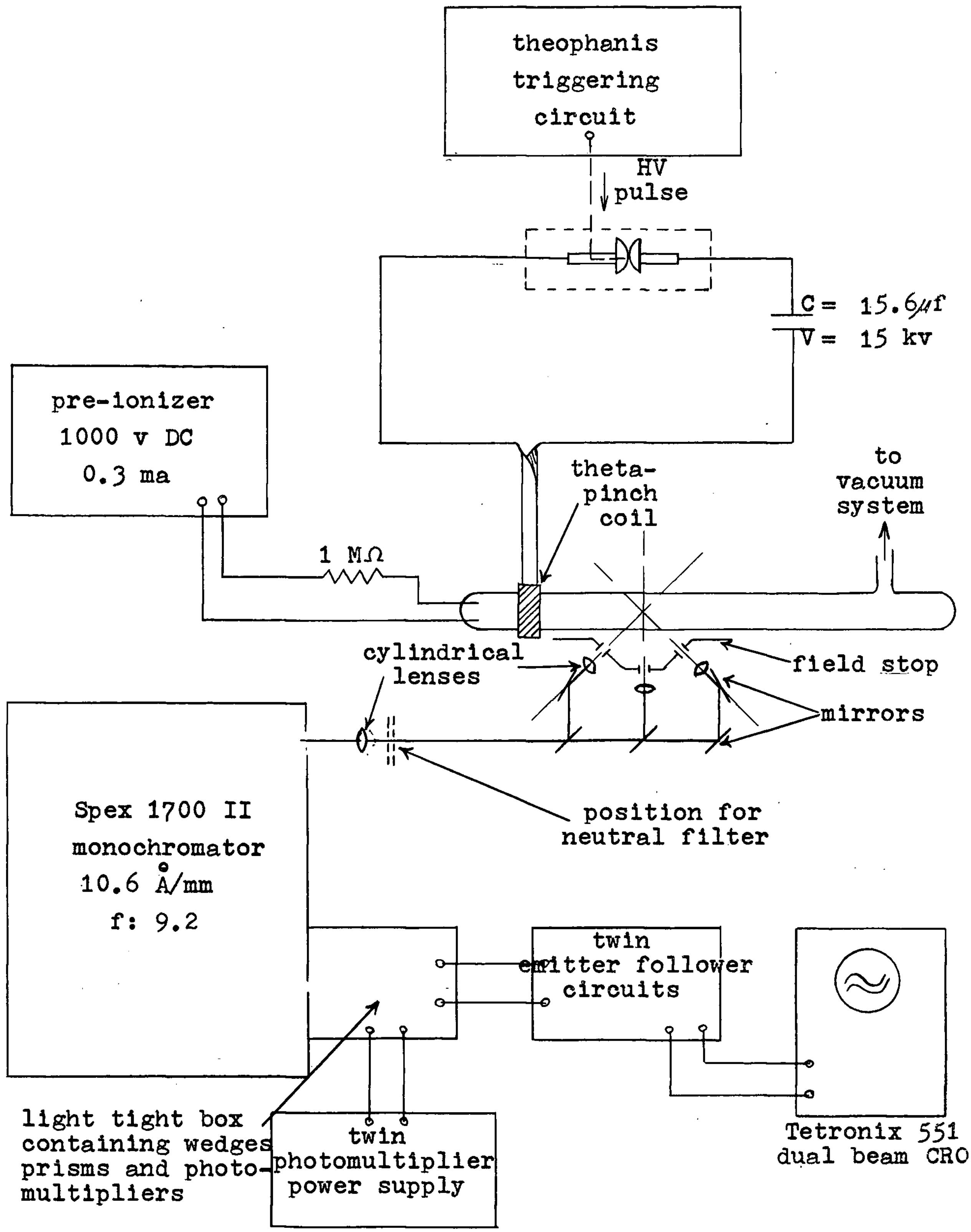


Fig. 7 Schematic diagram of apparatus

from the axis in both directions (see fig. 7).

A suitable spectral line was centred at the exit slit. Immediately in front of the slit were mounted the two neutral density linear wedges. The light passing through each wedge was monitored by an RCA IP 21 photomultiplier and the voltage signal from each photomultiplier was passed through an emitter follower and displayed on a Tetronix 551 dual beam oscilloscope.

The details of the wedge mount are shown in fig. 8. This part of the apparatus had to be of precision construction since, for a shift of 10^{-1} Å, shot to shot stability of 10^{-3} mm in the wedge position was required if an accuracy of 10% was to be expected. The linear wedges, prisms and photomultipliers were mounted in a light tight box. The wedges were movable in the x-direction and were attached to a pair of micrometer drums in such a way that they could be adjusted either independently or together.

Photographs showing the important parts of the apparatus are shown in fig 9.

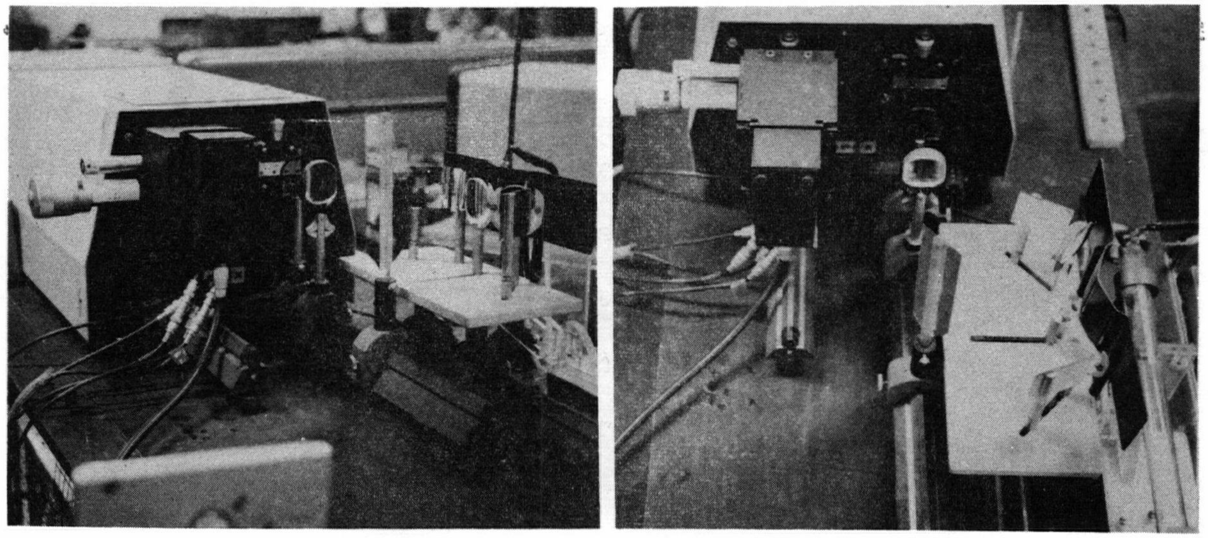
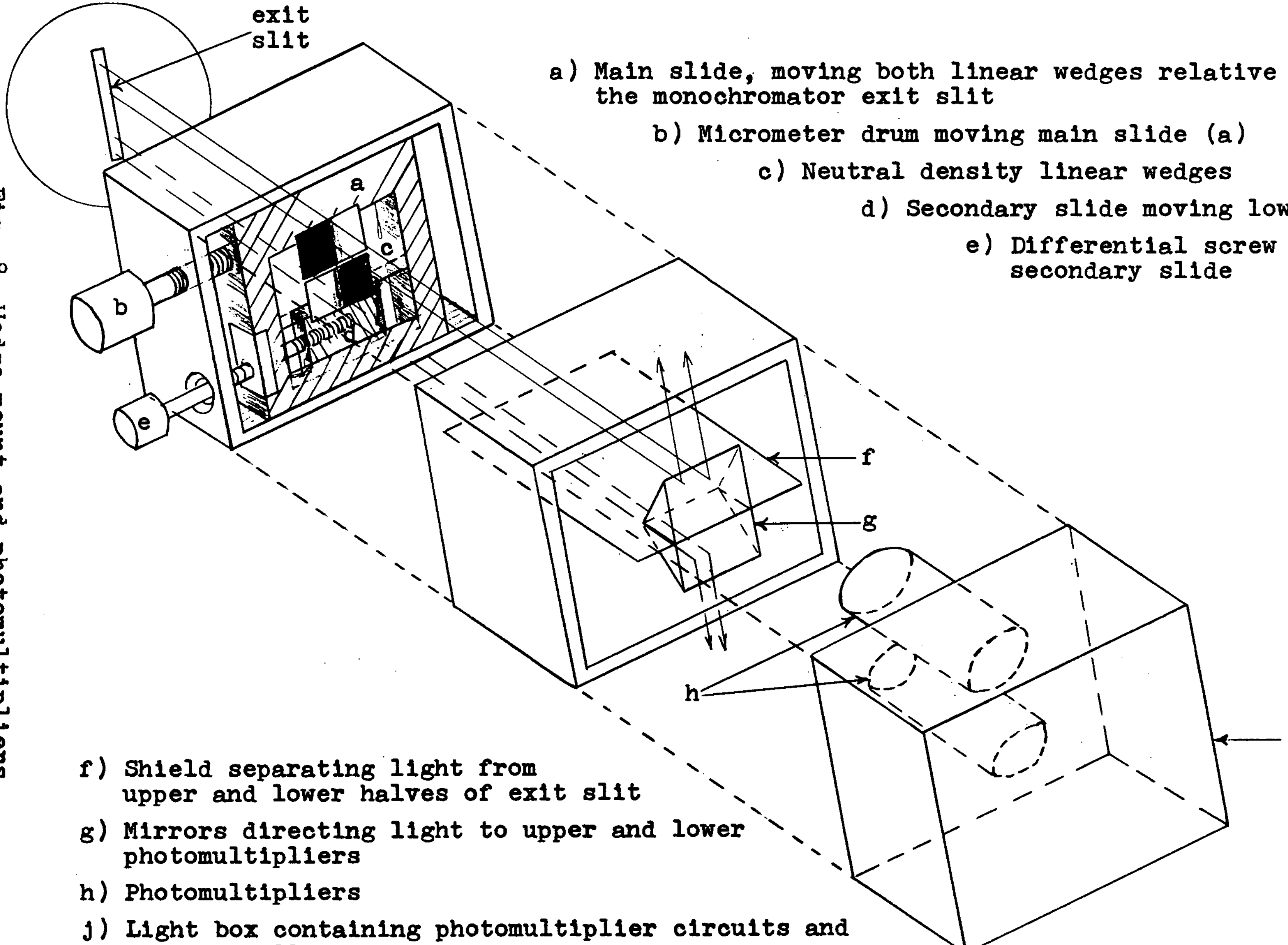


Fig. 9



a) Main slide, moving both linear wedges relative to the monochromator exit slit

b) Micrometer drum moving main slide (a)

c) Neutral density linear wedges

d) Secondary slide moving lower wedge

e) Differential screw moving secondary slide

f) Shield separating light from upper and lower halves of exit slit

g) Mirrors directing light to upper and lower photomultipliers

h) Photomultipliers

j) Light box containing photomultiplier circuits and emitter followers

Fig. 8 Wedge mount and photomultipliers

(b) Neutral Density Linear Wedges

To obtain a suitable neutral density linear wedge is of fundamental importance in applying this technique to measure small line shifts. For a total line width in the exit plane of 1 Å and a monochromator inverse dispersion of 10 Å/mm one requires a wedge to be linear over a distance of about 10^{-1} mm. A photographic technique for making such a wedge has been developed. It involves exposing a fine grained photographic plate to the half shadow region of a straight edge, as shown in fig. 10. A diffused light source (square with dimensions a x a) was placed at a height d above the plate. A razor

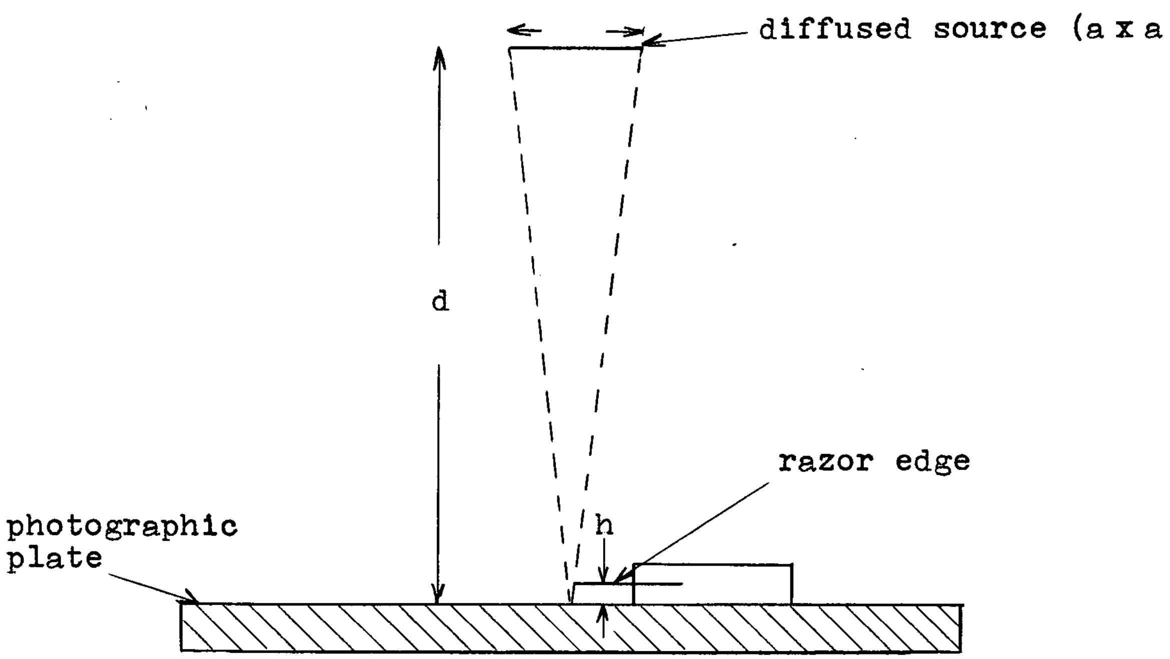


Fig. 10 Photographic technique of making the neutral density linear wedges

blade was held in a horizontal position at a height h with its edge directly beneath the centre of the source and parallel to one of its sides. Diffraction effects were negligible for the small distances between the razor edge and the plate and, therefore, the light intensity falling on the plate in the shadow region was a linear function of the distance along the plate and the width of the shadow was found from similar triangles.

As well as the simple dependence of the wedges on the geometry of the apparatus described above, their characteristics also depended on the H & D curve (density vs. log exposure) of the photographic plate. This effect is dealt with in Appendix I.

In order to understand the correspondence between the dimensions in fig.10 and the wedge characteristics, several wedges were made while varying a, d, and the exposure time. The transmission-distance relation of each was measured on a Kipp and Zonen microdensitometer with a Heath chart recorder at several points along the wedge. These results are summarized in table 1.

As seen in Chapter II the slope of the wedges is an important parameter in calculating line shifts. Although the microdensitometer, once its resolution was know, gave a good estimate of this quantity, a control line shift technique was developed giving a far more accurate average value for the slope. The Na 5890 line, emitted from a Gates sodium lamp was centred at the exit slit of the monochromator and the wedges placed in the wedge mount such that, for each, t_o lay at the line centroid. This was done by moving each wedge in the x-direction until the

Table 1

d = 50 cm, a = 3 cm, h = 0.64 cm

exposure	Δ x	Δ t	slope b
20 sec	0.264 mm	40%	157%/mm
25 "	0.224 "	48"	214 "
30 "	0.218 "	60"	275 "
35 "	0.106 "	50"	473 "
40 "	0.112 "	55"	491 "
45 "	0.092 "	56"	609 "
50 "	0.099 "	60"	607 "

d = 60 cm, a = 3 cm, h = 0.64 cm

exposure	Δ x	Δ t	slope b
20 sec	0.186 mm	28%	150%/mm
25 "	0.207 "	47"	227 "
30 "	0.165 "	53"	322 "
35 "	0.172 "	55"	320 "
40 "	0.152 "	55"	355 "
45 "	0.185 "	57"	309 "
50 "	0.106 "	56"	529 "
55 "	0.125 "	56"	448 "

d = 70 cm, a = 2 cm, h = 0.64 cm

exposure	Δ x	Δ t	slope b
30 sec	o.152 mm	33%	218%/mm
35 "	0.175 "	50"	303 "
40 "	0.172 "	45"	247 "
45 "	0.151 "	62"	375 "
50 "	0.150 "	58"	387 "
55 "	0.148 "	52"	351 "
60 "	0.145 "	41"	283 "
65 "	0.145 "	50"	345 "
70 "	0.132 "	48"	364 "
75 "	0.112 "	40"	357 "
80 "	0.106 "	47"	443 "
85 "	0.119 "	50"	420 "
90 "	0.098 "	47"	399 "

In the above table Δx and Δt are the distance and the change in transmission of the linear part of the wedge.

line intensity as seen by each photomultiplier was reduced by the factor t_o. The line was then shifted across the exit plane by means of a very small rotation of a glass plate suspended in the light path inside the monochromator. By means of careful measurement of the index of refraction and thickness of the glass, and the angle of rotation, one can calculate the magnitude of the shift of the line at the exit plane. If, at the same time, the voltage signals of the two photomultipliers are recorded then the equation (17) can be applied where the wedge slope b is the only unknown.

Such a measurment of the wedge slope was a valuable error check. There were three ways that the wedges could have introduced errors into the experiment: (1) they may not have been aligned parallel to the spectral line, (2) there were likely to be imperfections in the wedge due to non-uniformity and graininess of the photographic plate, and irregularities in the light source and razor blade used in making the wedge, (3) the shifted, or unshifted, line may have spread out to the non-linear region of the wedge.

In case (1), if the wedges were parallel to each other, then the slope b' as it appears to the spectral line would be constant (provided the line lay within the linear region of the wedge at all positions along the exit slit) and would be related to the true slope of the wedge by $b' = b \cos \alpha$, where α is the angle between the wedges and the line. This effective slope b' would be directly measured by the control line shift technique and was, in fact, the value to be used to calculate the line shifts of the plasma spectral line. If, however, the upper and lower wedges were not parallel to each other then they would have different effective slopes. This would be evident from the control line shift experiment since, under these circumstances, the sum of the voltage signals from the upper and lower photomultipliers would not remain constant.

In cases (2) and (3) the slope b would not be constant in the region of the wedges where the light passes through. However, these sources of error would also be evident from the control line shift experiment since the voltage signal difference Δs would vary non-linearly with the line shift.

It can be concluded that this technique gives an accurate value for the wedge slope and also represents a complete check on the wedges and their positioning in the wedge mount.

CHAPTER IV

EXPERIMENT AND RESULTS

(a) Preparations for Line Shift Measurements

To perform the experiment it was first necessary to select a strong isolated line. In order to survey the available lines of the theta-pinch at 350 μ Hg pressure of air the axis of the tube was imaged onto the entrance slit of a Hilger E 1 spectrograph and a time integrated spectrum (superposition of 10 shots) was taken. Fig. 11 shows a section of this spectrum and indicates the correspondence between the position on each spectral line with the position on the tube axis.

The N II 3994 line was selected for observation since it fulfilled all three of the important requirements for making the line shift measurement: (1) it was one of the strongest lines of the spectrum, (2) it was isolated, having no observable lines within a few Angstroms, (3) its wavelength was close to that of the peak of the photomultiplier response curve. In addition, Stark effect calculations have been made by Griem (ref. 7) for this line so that line shifts can be related to electron densities.

The line was scanned by the monochromator with the slits at a very narrow setting and the natural line width plus instrument profile was thereby estimated as 0.5 Å. A few tests showed that in order for the light intensity to record a strong signal on the photomultipliers for a wide range of positions along the theta-pinch tube it was necessary to open the entrance slit up to 0.1 mm. The width of the line

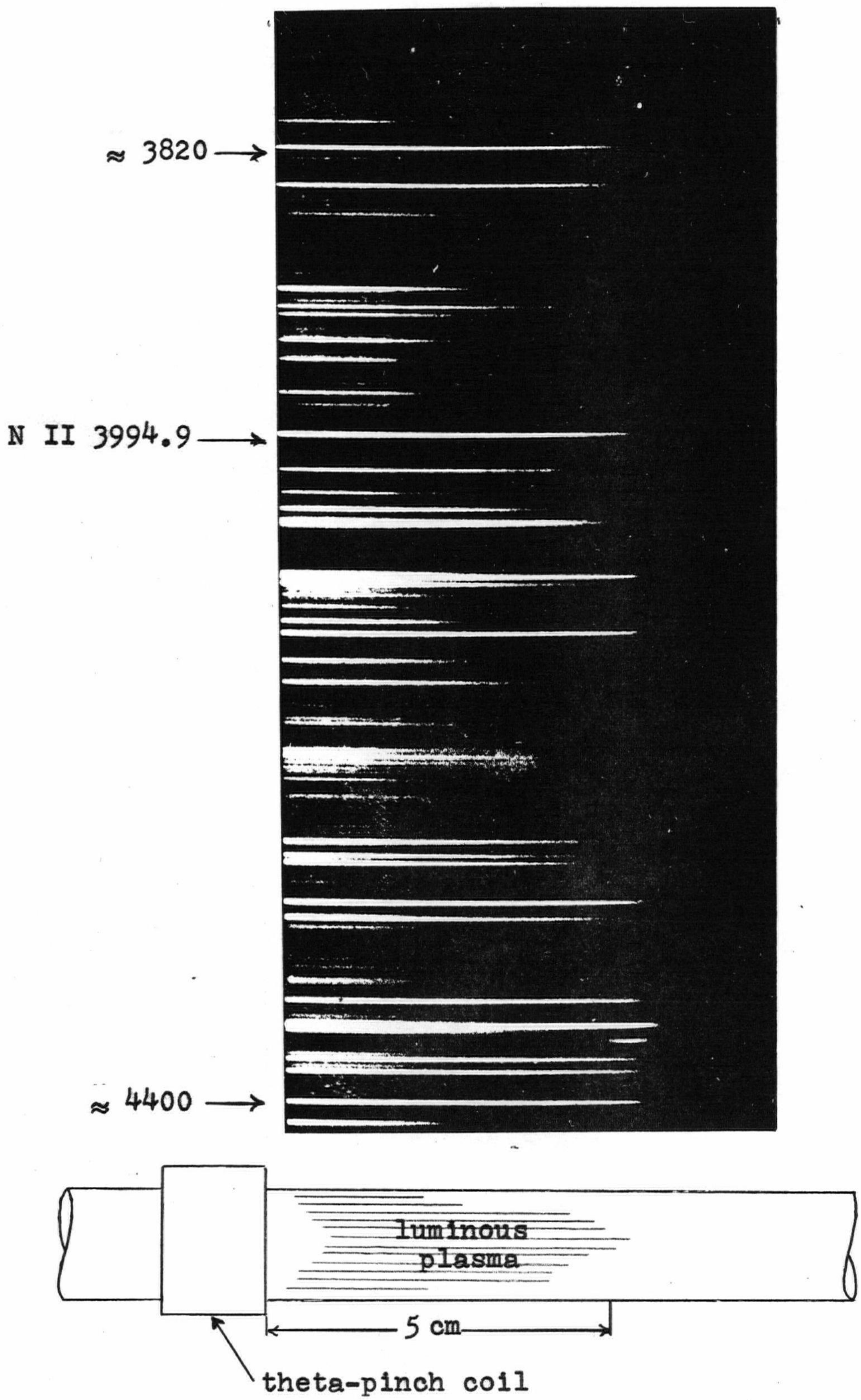


Fig. 11 Spectrum of theta-pinch

at the exit plane is approximately the sum of the line width estimated above, and the entrance slit width; i.e. about 1.5 Å, since the monochromator inverse dispersion is 10 Å/mm. This indicates that a wedge is required which is linear over the range 0.15 mm plus the expected shift.

From the investigation of the wedge properties in Chapter III (see table 1) it was seen that a wedge made with $h = 70$ cm, $\alpha = 0.6$ mm and exposure time 45 sec would be suitable for the line to be observed. A microdensitometer trace of this wedge is shown in fig.12. The slope of the wedge was measured to be 375 %/mm and its range of linearity $\approx$ 0.15mm. Since the resolution of the instrument was 0.015 mm (estimated by scanning a razor blade edge) the actual rangeof linearity was likely to be about 0.165 mm.This trace shows fluctuations in the linear region. These were due to graininess, irregularities in the film and dust, which were all random effects. They would be evident when a narrow strip of the wedge is scanned, as in the microdensitometer, but not when the integrated effect of all positions along the wedge is considered, as in the line shift apparatus.

This wedge was cut in two and each half placed in the wedge mount such that their transmission gradient lay in opposite directions. They were aligned under a travelling microscope before being fixed to the exit slit assembly. The control line shift experiment was then used to accurately measure the slope and test the linearity of the wedges. The results of this check are discussed more fully in a later section of this chapter.

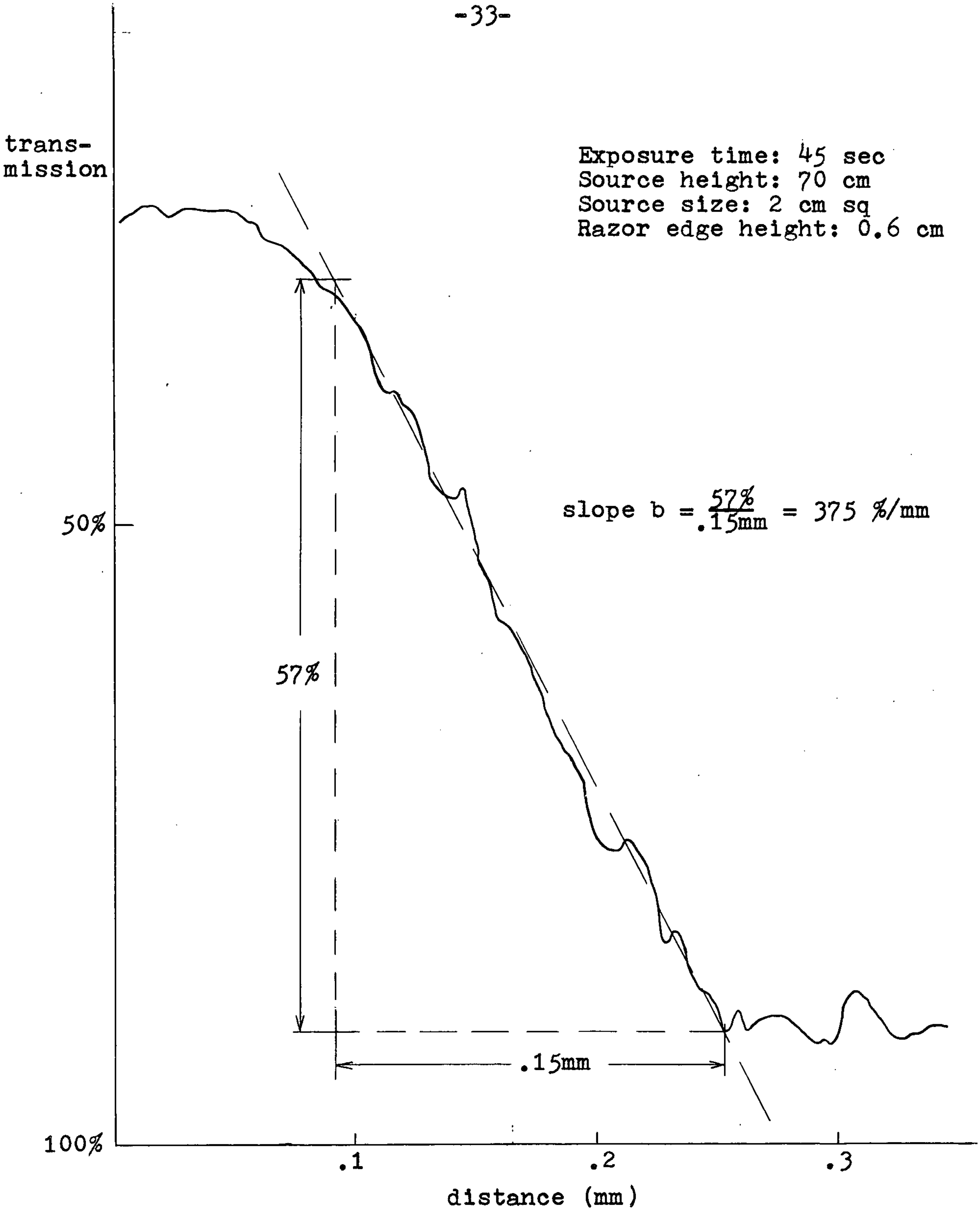


Fig. 12 Microdensitometer trace of linear wedge

(b) Measurement of Small Line Shifts

A time resolved picture of the N II 3994 line intensity emitted by the plasma at a particular position in the theta-pinch tube is shown in fig. 13.

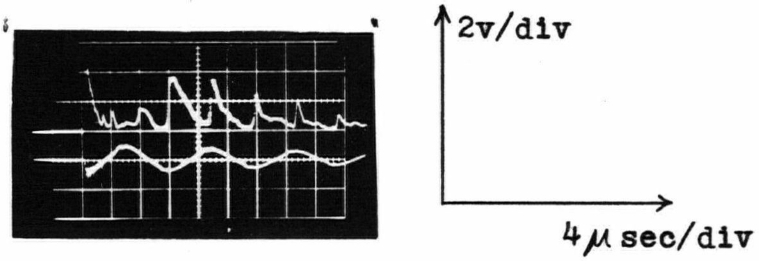


Fig. 13 N II 3994 line intensity (upper trace) and di/dt in the drive loop

The upper trace shows the photomultiplier signal and the lower trace is the voltage signal from a sensor coil near to the drive loop, and represents di/dt in the loop. The entrance optics focussed the cross-section of the discharge tube at Z = 0.5 cm observed from a direction perpendicular to the Z-axis onto the monochromator entrance slit. As expected the light intensity fluctuates as a series of decaying pulses with a frequency twice the bank ringing frequency. It was decided to measure the line shifts in the first four main pulses of light that appear in fig. 13.

As indicated in Chapter II, the first step in making the measurement was to balance the voltage signals from the two photomultipliers with no wedges in front of the exit slit. Fig. 14 shows the sum (upper trace) and difference (lower trace) of these signals for the best photomultiplier balance.

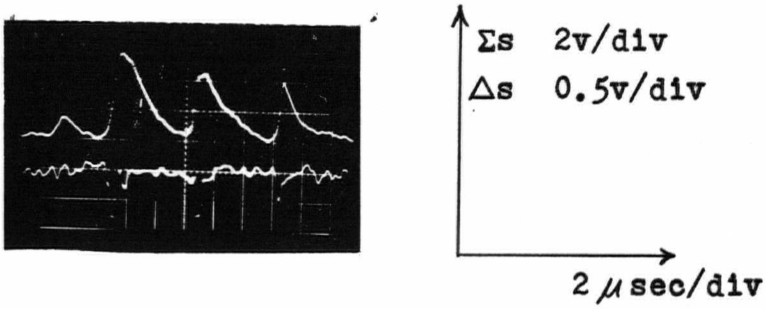


Fig. 14 Signal sum (upper trace) and signal difference (lower trace) for best photomultiplier balance

It was found that, in general, the completely balanced condition (zero difference signal for all time) could not be obtained. This was due to several effects of both a systematic and a random nature, which are listed below.

A. Systematic Effects

1. Distortion of the signal by non-ideal electronic apparatus (emitter followers, transmission lines, oscilloscope).
2. Different frequency characteristics of the two emitter followers.
3. Electromagnetic pick-up due to the bank discharge.
4. Systematic variation of the distribution function $\alpha(y)$ with time.

B. Random Effects

1. Electronic and photoelectric noise of all kinds.
2. Random variation of $\alpha(y)$ due to non-reproducibility of the plasma.

The possible errors arising from random effects could be reduced by taking several identical measurements and calculating the mean. However, in the case of the systematic effects it was necessary to ensure that they were held constant during the entire measurement so that the deviation of the Δs trace from zero without the wedges was identical to the Δs signal obtained with wedges, in the absence of any shift. Since these possible systematic errors may depend non-linearly on the magnitude of the photomultiplier signals, it was required that the signals without wedges be reduced to approximately those with the wedges. This was done by inserting a homogeneous neutral density filter in front of the entrance slit, with transmission $t_n \approx t_o$. In this way the intensity with and without the wedges was kept approximately equal and any systematic effects could be expected to remain equal. Under these conditions, the signal difference Δs_f, recorded with the filter in position, multiplied by the factor t_o/t_n represents the amount of the signal difference Δs_w, recorded with the wedges in position, which is due to systematic, reproducible effects other than line shifts. Equation (18) is now modified as

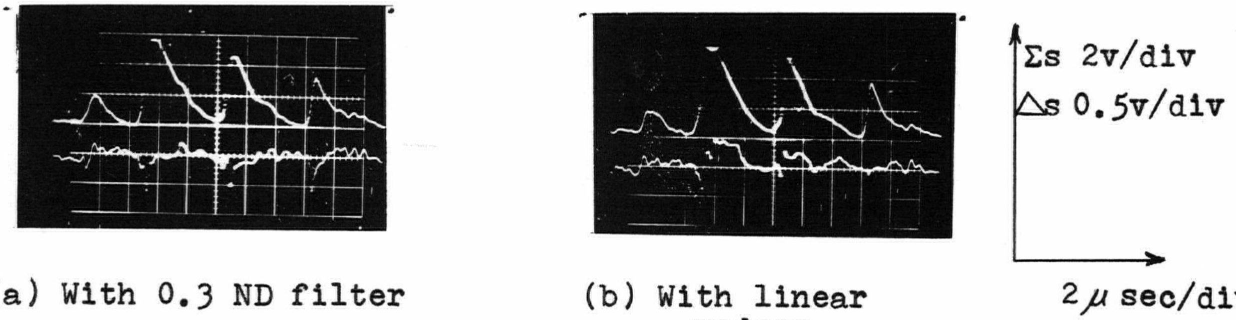
$$\Delta\lambda = \frac{\Delta s_w - \Delta s_f \frac{t_n}{t_o}}{\Sigma s_w} \frac{t_o}{b} \frac{d\lambda}{dx} \tag{29}$$

(c) Doppler Shift Measurement

A series of line shift measurements were taken by observing the N II 3994 line from two directions, $\alpha = 45°, 135°$ and from equations (26) and (29) the Doppler shift was given by

$$\Delta\lambda_d = \frac{1}{2}\left\{\left[\frac{\Delta s_w - \frac{t_o}{t_n}\Delta s_f}{\Sigma s_w} \frac{t_o}{b} \frac{d\lambda}{dx}\right]_{45°} - \left[\frac{\Delta s_w - \frac{t_o}{t_n}\Delta s_f}{\Sigma s_w} \frac{t_o}{b} \frac{d\lambda}{dx}\right]_{135°}\right\}$$

A typical set of traces, from which the calculations of Doppler shifts were made, is shown in fig. 15.

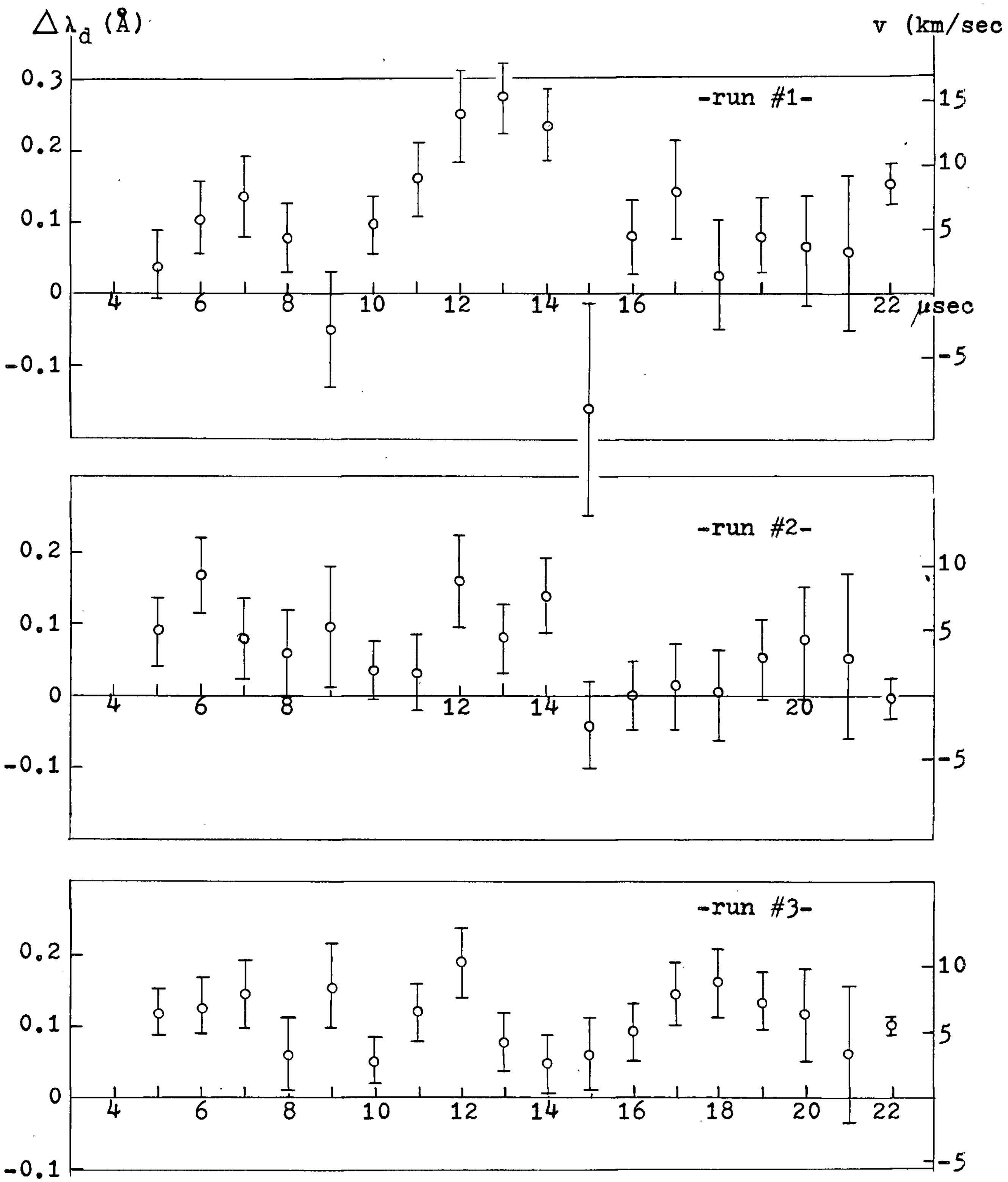


(a) With 0.3 ND filter

(b) With linear wedges

Fig. 15 Σs and Δs traces from which Doppler shifts are calculated

The two observed cross-sections of the theta-pinch intersected at Z = 2.5 cm. Three separate runs under identical conditions were taken to obtain an accurate time resolved axial flow velocity at this position, and to check the reproducibility of the final results. For each run all the measurements required were taken independently nine times, and for each point in time the mean values of the voltage signals were calculated. By means of such repetitive measurement it was possible to reduce the uncertainty due to random fluctuations in the signal. Fig. 16 shows the axial velocity as a function of time for each of the three runs. The error bars represent the standard error of the mean, calculated from the nine measurements. Fig. 17 shows the mean of the three runs plotted along with the intensity of the light signal and di/dt as measured by the sensor coil. It can be seen that there is a correlation



NB. time t = 0 is arbitrary but is the same for all measurements

Fig. 16 Doppler shifts due to axial motion as a function of time (Z = 2.5)

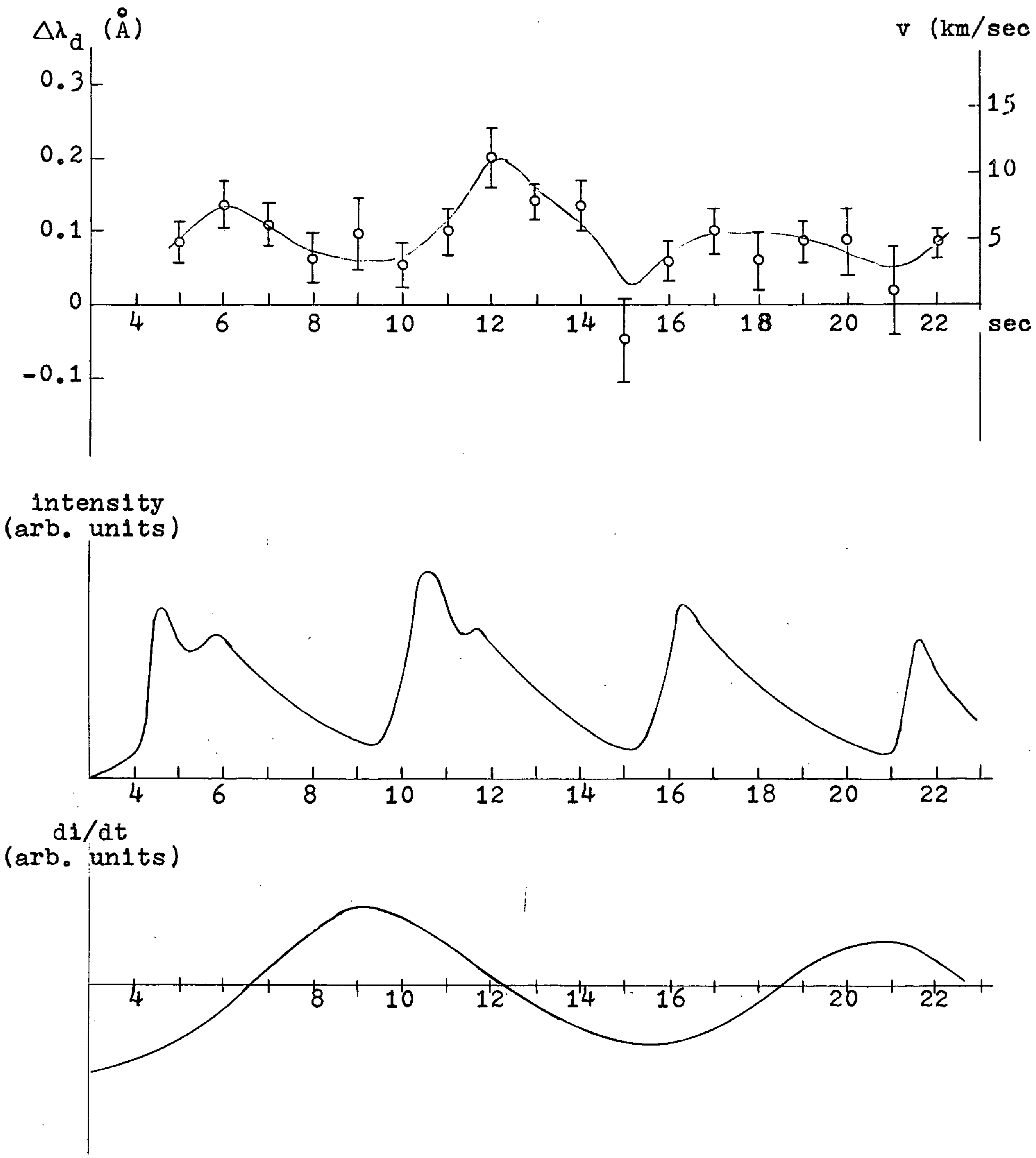


Fig. 17 Mean of Doppler shift measurements, N II 3994 line intensity, and di/dt of drive loop

between the fluctuations of all three curves, the light intensity and velocity fluctuate with double the bank frequency. The intensity peak is delayed about 1.5 sec from the peak di/dt, which is the order of magnitude of the time expected for the particles to travel from the place of generation to the region being observed.

As a verification of these results a series of time of flight measurements was made. The absolute intensity of the N II 3994 line was observed from direction $\alpha = 90°$ for several Z-positions down the theta-pinch tube. These traces were all similar to the sum (Σs) curve of fig.15 except that the intensity pulses gradually change their shape while proceeding down the tube. However, certain features could be detected in all the oscillograms and it was assumed that these corresponded to luminous fronts travelling down the tube. As expected these fronts were recorded at a later time for positions further away from the drive loop. The complete set of oscillograms is reproduced in Appendix II. The displacement curve of the first intensity pulse is plotted in fig.18. The straight line in this figure represents the slope of the displacement curve at $Z = 2.5$ cm. This indicates a front velocity at this position of 6 ± 1 km/sec averaged over the time $t = 6\mu$sec to 8μsec. This front is likely to be a shock front and a Mach number of 20 would be expected. The particle velocity should then be slightly smaller than the front speed, and, indeed, the Doppler shift measurments (see fig.17) yield 4.8 km/sec for this time interval. It is interesting to note that Simkinson (ref. 10), in his studies of a similar theta-pinch

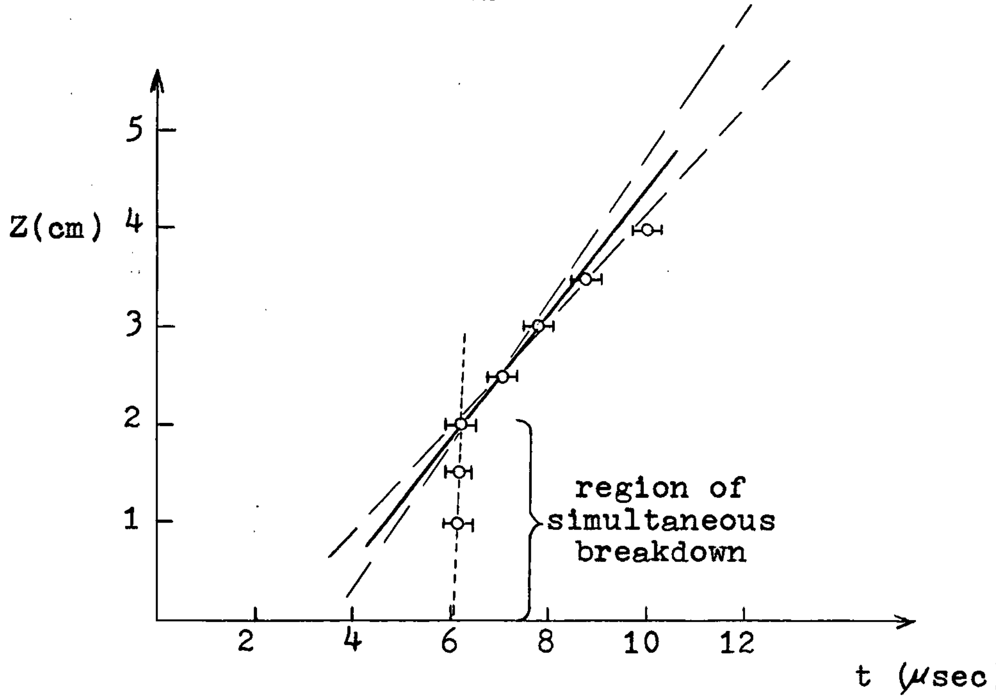


Fig. 18 Time of flight measurements

device, observed shock fronts and measured a front velocity of 6 km/sec.

(d) Stark Shift Measurements

Time resolved Stark shift measurements were made by observing the N II 3994 line perpendicular to the theta-pinch tube axis at three positions, Z = 1, 2, 3 cm. Fluctuations in the Stark shift were calculated from equation (18) but since no zero shift reference source was available absolute shifts were not measured. Fig. 19 shows the results. Note that the three runs were performed so that they all had the same arbitrary zero shift position.

The equations of conservation of mass and the ideal gas law permit an order of magnitude check on the Stark shift results. One can compare the total number of particles inside the theta-pinch volume with an estimate of the number of charged

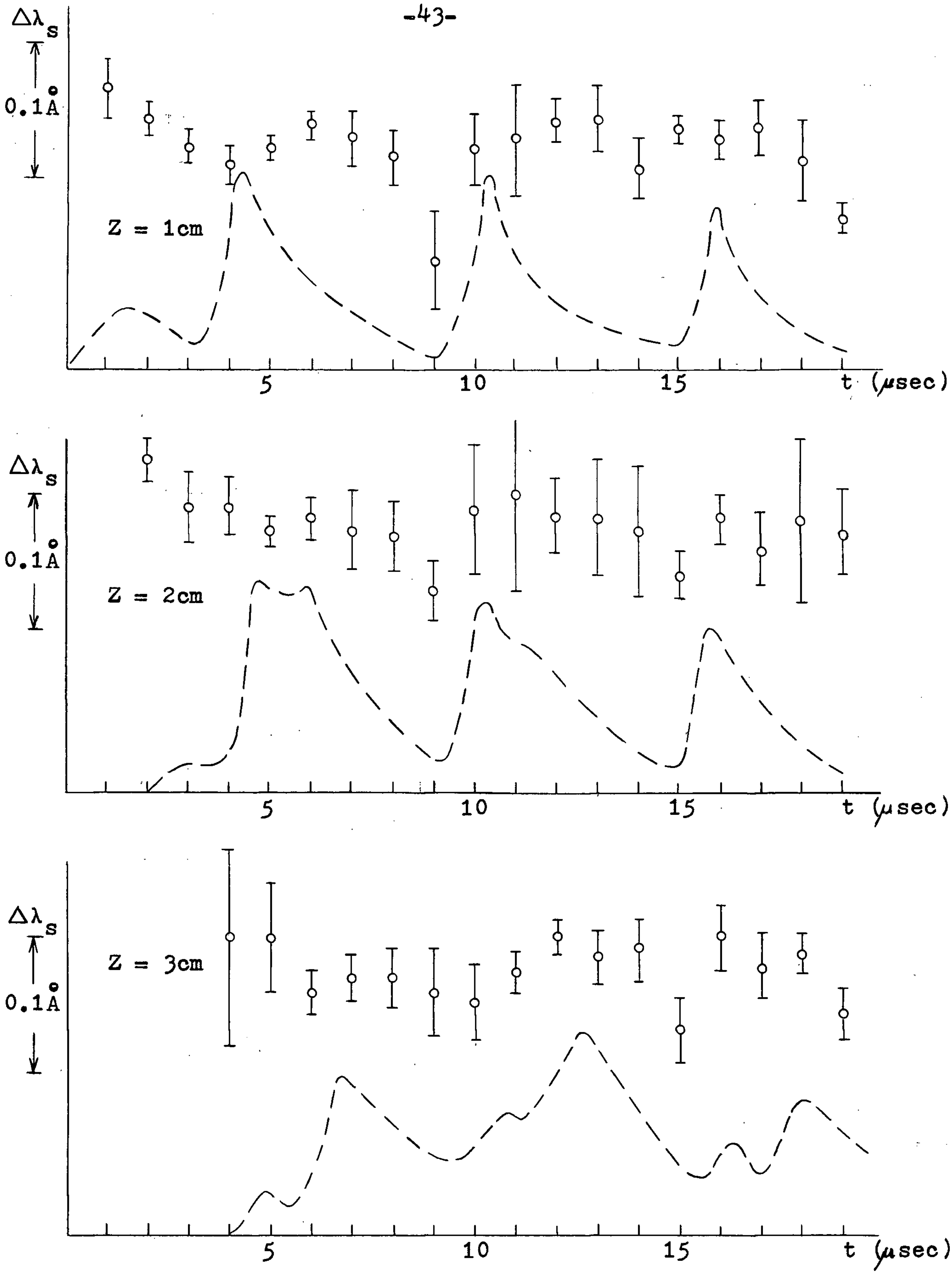


Fig. 19 Stark shift measurements (Z = 1,2,3 cm) as function of time; plotted with line intensity---

particles observed to travel through a cross-section at $Z = 2.5$ cm. From application of the ideal gas law one finds that prior to discharge the theta-pinch tube contains 1.7×10^{17} molecules/cm^3.

Assume a mixture of N_2 and O_2 molecules so that upon dissociation one has 3.4×10^{17} atoms/cm^3, and if one has complete (single) ionization then the electron density, $n_e = 3.4 \times 10^{17}$ electrons/cm^3. In the oscillograms taken in the time of flight velocity measurement (see Appendix II) it appears that the section of the theta-pinch tube up to $Z = 2$ cm breaks down almost simultaneously. If one assumes that this includes the entire region of the tube from the mid-plane of the loop to 2 cm in front of the loop then, since the loop is 2 cm wide, the total breakdown region has a volume of $\pi r^2 \times 3$ cm, where $r = 1.25$ cm, the inside radius of the tube. If the gas is completely (singly) ionized then a maximum total number of electrons of $3.4 \times 10^{17} \times 3 \times \pi \times r^2 = 5 \times 10^{18}$ electrons would be produced.

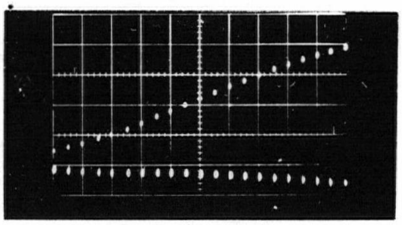
This figure may be compared to the minimum number of electrons observed experimentally to cross a given cross-section of the tube at $Z > 2$ cm. A minimum value for the Stark shift can be estimated as equal to the magnitude of the fluctuations observed. From fig. 19, $Z = 2$ and 3 cm, this was about 0.1 Å. This is equivalent to assuming that the zero Stark shift is represented by the smallest $\Delta\lambda_s$ observed (at $Z = 2$ cm, $t = 9.0$ sec), which puts a lower limit on all measured shift since, for this particular line the Stark shift is always toward the red. The graph in fig. 18 represents the 16 μsec period after the gas breaks down. From fig. 16 it is observed that the mean velocity over this time range at Z = 2.5 cm was 5.3 km/sec. The electron density calculated by Griem (ref.7) which corresponds to a Stark shift of 0.1 Å is $2 \times 10^{17} cm^{-3}$. From this data, if one assumes that the electron density is equal to the ion density, then one can calculate the total number of ions crossing the cross-section at Z = 2.5 cm.

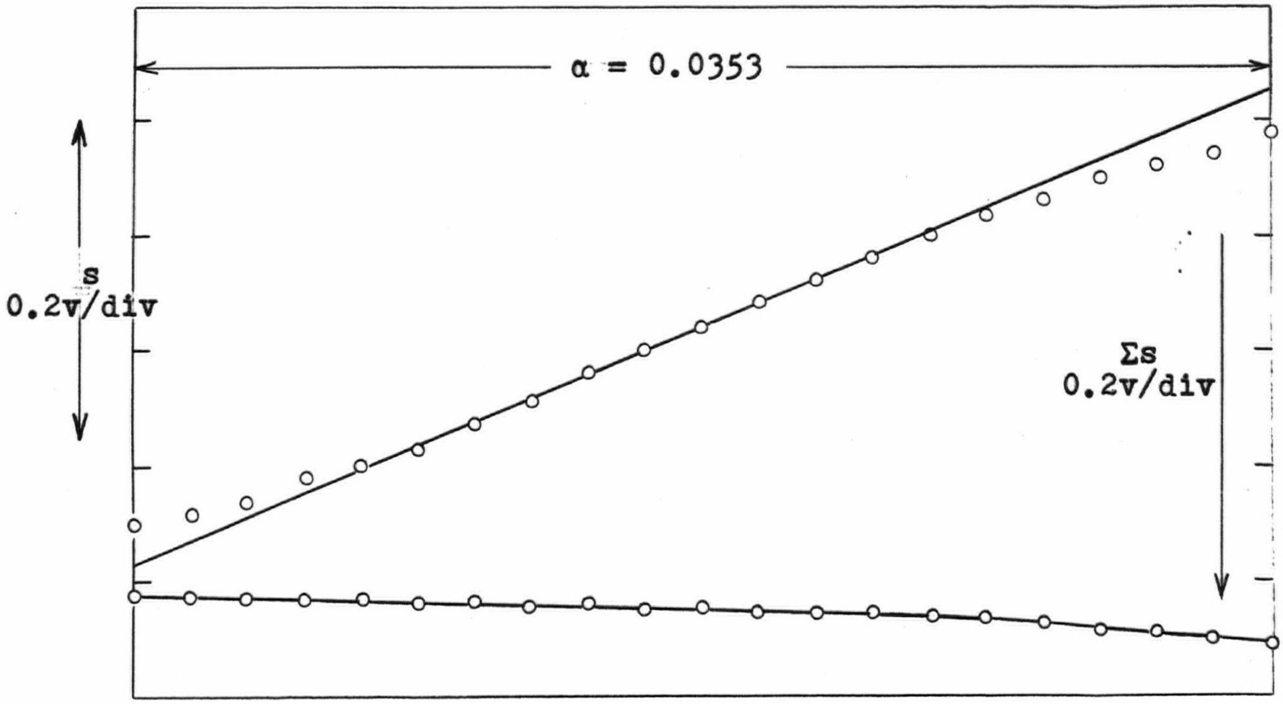
$$n_i A v_i t \approx 2 \times 10^{18}$$

where $A = 1\ cm^2$ is the estimated area on the axis through which the electrons and ions escape the magnetic confinement in the theta-pinch. This estimated number of 2×10^{18} ions can be accounted for by the initial available total number of 5×10^{18}. Therefore, this check can verify that the Stark and Doppler shift results as reasonable and self-consistent.

(e) Systematic Errors in the Measurement

As indicated in section (b) of this chapter a good check on the possible errors introduced by the wedges is the control line shift experiment. The results of this test, using the Na 5890 line and shifting the image by rotating a piece of glass 12.6 mm thick of index of refraction 1.5177, is shown in fig. 20. The glass was rotated in steps and each step is represented by a dot on the oscillogram. Care was taken to ensure that the total shift represented by this oscillogram exceeded the maximum width of the wedges that would be used to measure the N II line shift. The entrance slit width for this experiment was 0.05 mm so that the line at the exit plane





thickness of glass, d = 1.26 cm
index of refraction, n = 1.5177
angle of rotation, α = 0.0353 rad.

$$x = d\alpha(1 - \frac{1}{n}) = 0.1517$$

$$b = \frac{s}{\Sigma s}\frac{t_e}{x} = \frac{.80}{.66}\frac{0.5}{.1517} = 400\%/\text{mm}$$

Fig. 20 Results from control line shift experiment had at least that width. It was calculated that the total line shift seen in fig.20 was 0.1517 mm, which is approximately the width of the wedges expected to be used in the line shift measurement. It can be seen that in this range the signal difference Δs departs from the linear by a maximum of 0.06v and the signal sum, Σs is constant to within 0.04v. Therefore, $\Delta s/\Sigma s$ has a possible error of $\frac{\delta(\Delta s)}{\Delta s} + \frac{\delta(\Sigma s)}{\Sigma s} = \pm 7.5\% \pm 6.0\% = \pm 13.5\%$ However, this is an unduly pessemistic upper limit for the possible error since, allowing for the line width used (0.05 mm), fig. 20 represented a total wedge range of 0.2 mm. It was assumed that as long as the N II line was properly centred on the wedge and the shifts were small (of the order of 10^{-1} Å) then the error due to non-linearity and positioning of the wedges was < 10%. This error could, in principle, be further reduced by using a small section of a wider wedge.

From fig. 20 the central line shift experiment gave a value for the wedge slope b of 400%/mm. This is slightly more than the wedge slope measured by the microdensitometer, but the value given by the control line shift technique is expected to be more reliable.

CHAPTER V

CONCLUSIONS

(a) General Conclusions

It was concluded that the double wedge technique is an accurate method by which line shifts of a theta-pinch plasma can be measured within an uncertainty of about 0.015 Å for a 1.5 Å wide line. From this result it is believed that this method is more accurate than any time resolved line shift measurement that has been reported in the literature. This experiment showed that, with the N II 3994 line observed, Doppler shifts and Stark shifts were measured with an uncertainty corresponding to velocities of ± 1 km/sec and electron density fluctuations of $\pm 3 \times 10^{16}$ cm^{-3}. With such resolution one could easily measure the ratio of Stark shifts for different lines to compare the predictions of the theory. With improved reproducibility of the source or with phase sensitive detection techniques for slowly varying sources, shift to width ratios of 10^{-3} should be measurable. The Doppler and Stark shift measurements were in good agreement with time of flight measurements and did not violate the conservation of mass. They were also in good agreement with Simpkinson's results (ref. 10) and indicated that a shock front was generated by the discharge.

The value of this technique is obvious. It is possible to make time and space resolved measurements of both mass velocity and electron density without disturbing the plasma. The knowledge of these parameters alone yield important information regarding the behaviour of any laboratory plasma.

(b) Future Research

In future work an accurate check of the double wedge technique should be made by observation of a shock tube plasma, for which the velocity can be calculated.

A further improvement of this technique would be to construct a standard low density source of sufficiently strong, well-defined lines which have negligibly small Stark shift. The low pressure AC discharge described by Minnhagen (ref. 9) would be appropriate. Accurate absolute Stark shift measurements could then be made for any line whose wavelength is close to a suitable line in the standard source (such that they both lay within the linear region of the wedges). This would mean that absolute electron densities could be calculated.

A further application of this technique could be Zeeman shift measurements. A slight modification of the apparatus would be necessary so only one of the components would be observed.

APPENDIX I

THE PHOTOGRAPHIC PROCESS AND THE LINEAR WEDGE

In order to make a linear neutral density wedge in the manner described in Chapter III one must show that if a photographic plate is exposed to a linear shadow then the developed plate will have a linear transmission gradient.

The most important characteristic of a photographic plate or film is the density vs. log exposure relation, usually known as the "H & D" curve (ref.11). A typical curve is shown in fig. 21. The linear portion of the H & D curve, which has

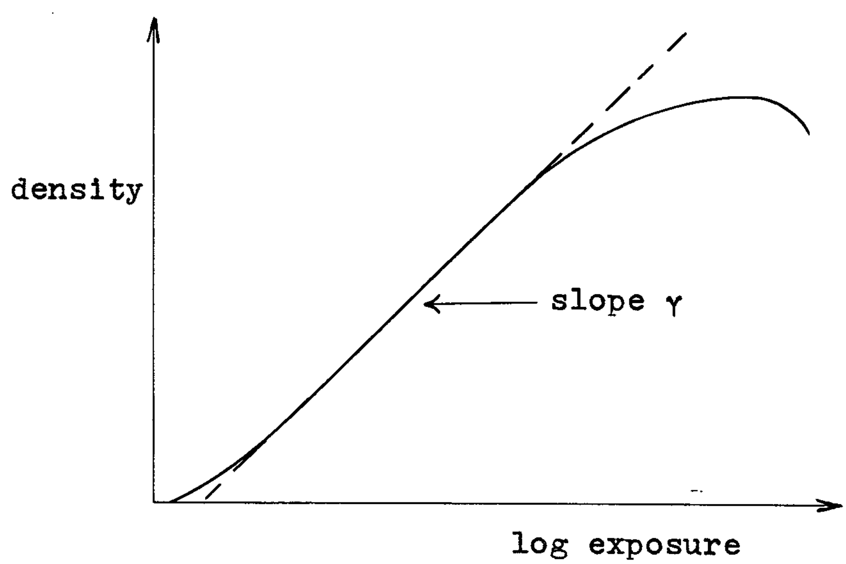


Fig. 21 Typical H & D curve

constant slope γ, covers most of the useful range of exposures for ordinary films. The value of γ and, indeed, the shape of the curve in general, depends on the development, i.e. type of developer and length of development.

In the region of constant γ the H & D curve can be expressed by the equation

$$D = k + \gamma \log E \tag{31}$$

$$= \log (E^{\gamma} k_1) \tag{32}$$

where: $k = \log k_1$ -- the density axis intercept of the extended linear portion of the H & D curve.

E is the exposure -- $E = It$, where I is the light intensity and t is the length of time film is exposed.

D is the density of the exposed film.

Now, the transmission of the exposed film is given by

$$D = -\log T \tag{33}$$

$$\text{therefore,} \; -\log T = \log (E^{\gamma} k_1) \tag{34}$$

and it follows from equations (33) and (34) that

$$T = (E^{\gamma} k_1)^{-1} \tag{35}$$

Since for a linear shadow $E \propto x$ it is seen that the transmission of the wedge can never be linear, but will always vary as $E^{-\gamma}$. For certain films, however, the transmission-distance curve will be linear to a very close approximation.

In order to make the wedges used in this experiment Ilford Special Lantern Contrasty plates were used. This plate is described by Ilford as a "fast, non-colour-sensitive, fine grain plate." Its H & D curve is shown in fig.22,(ref. 12). From this figure equation (35) becomes

$$T = (1.8 \times 10^6) E^{-2.5}$$

This equation is plotted in fig.23 for transmissions from 10% to 90%. It can be seen that this plot is linear from about 25% to 80% transmission. This is in good agreement with the range of linearity measured in fig. 12 for the wedge that was used in the apparatus.

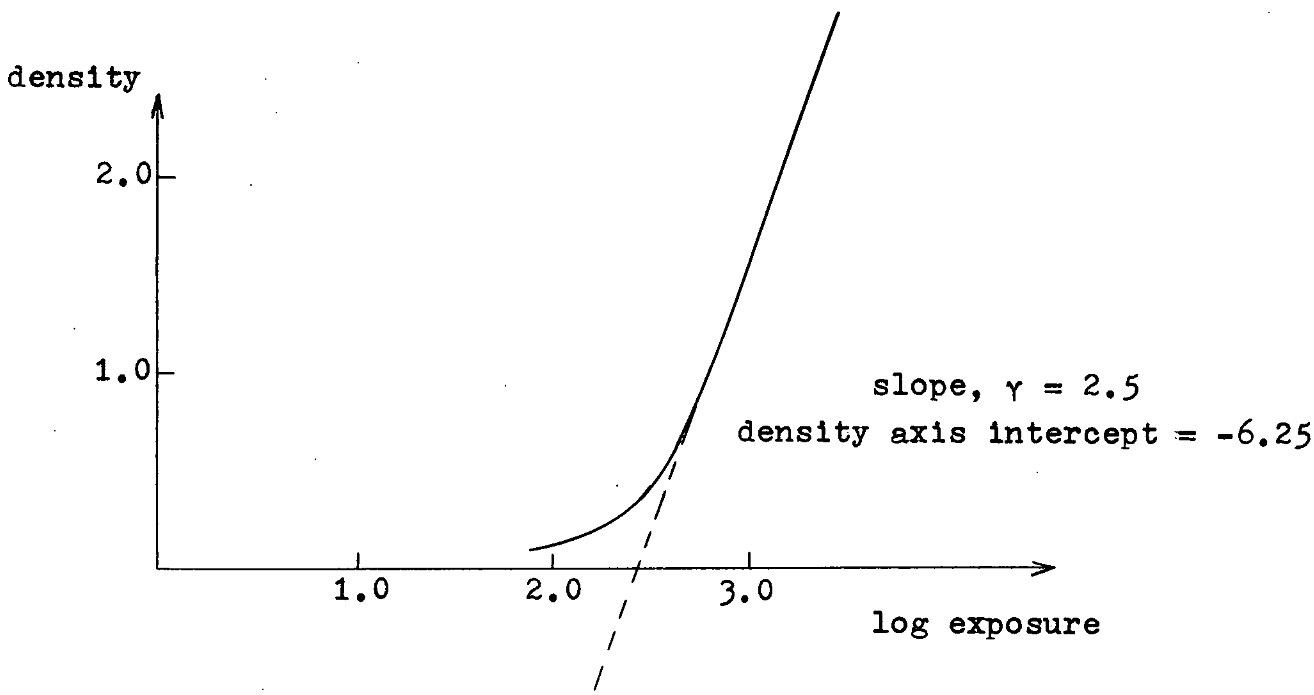


Fig. 22 H & D curve for Ilford Special Lantern Contrasty

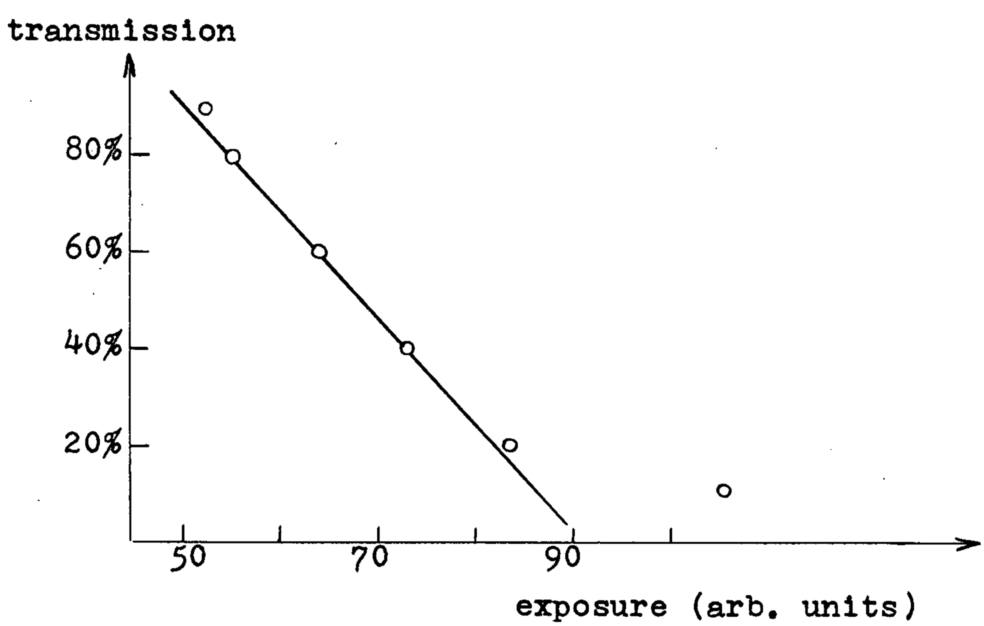


Fig. 23 Calculated transmission-exposure curve for Ilford Special Lantern Contrasty

APPENDIX II

TIME OF FLIGHT VELOCITY MEASUREMENTS

As a check on the velocities recorded by the line shift measurements a series of time of flight measurements were made and the velocity of the luminous front was calculated as a function of its position in the theta-pinch tube. To do this the tube was observed from a direction perpendicular to its axis and the oscillogram was recorded representing the time resolved N II 3994 line intensity at various distances, Z, down the axis from the drive loop. Care was taken to stabilize the oscilloscope trigger and horizontal setting so that different oscillograms had the same time zero. Fig. 24 shows the traces taken. Since the second peak is a clearly marked feature of all these oscillograms, its position in time as a function of Z was measured and plotted in fig. 25. The slope of the curve

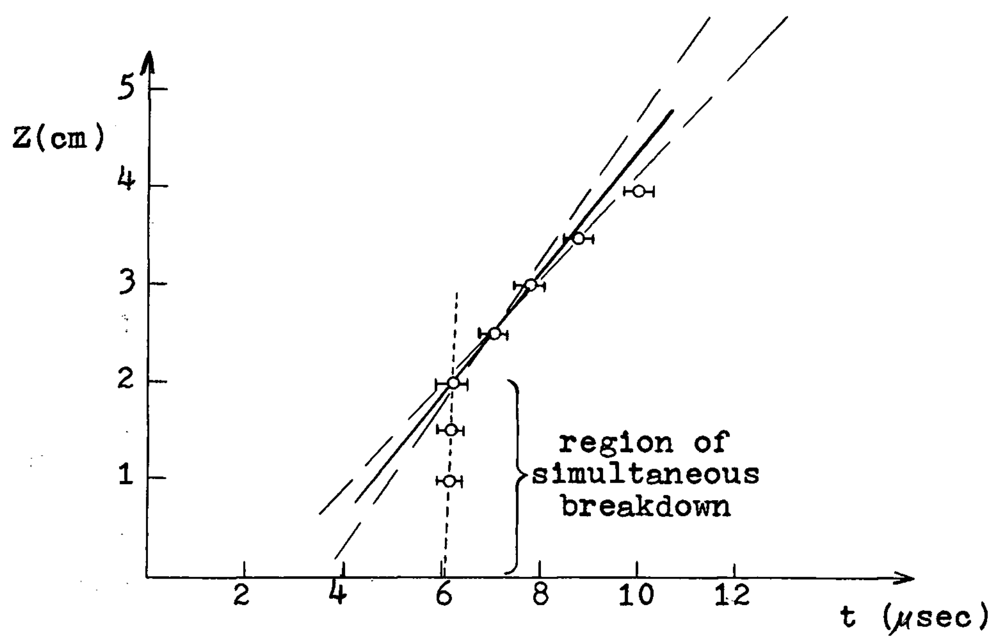


Fig. 25 Time of flight measurements

represents the velocity of the luminous front which is represented by the second intensity pulse. The slope of the dotted line is

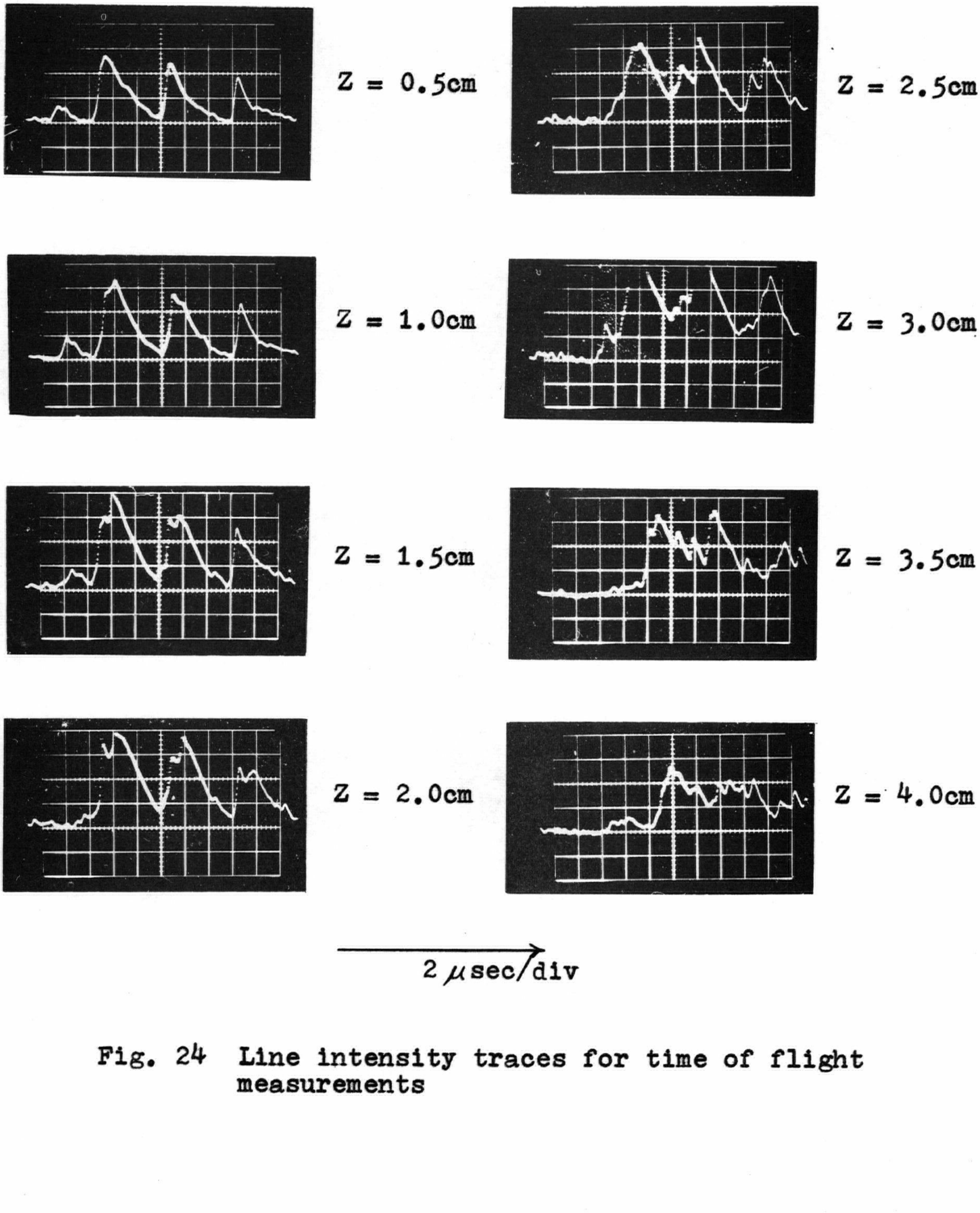


Fig. 24 Line intensity traces for time of flight measurements

about 40 km/sec, which is an unexpectedly high axial velocity for a theta-pinch. It is quite likely, however, that the breakdown of the gas occurs almost simultaneously for regions of the tube very close to the drive loop ($Z < 2$cm.). Of special interest is the velocity at the point $Z = 2.5$ cm since this is where the line shift velocity measurements were taken. The solid line represents this velocity, averaged over $2 \leq Z \leq 3$ and $6 \leq t \leq 8$ and shows a slope of 6 ± 1 km/sec.

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