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THE STARK EFFECT OF HELIUM IN SOME B TYPE STARS.

by

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THE STARK EFFECT OF HELIUM IN SOME B TYPE STARS

Some years ago, a rather anomalous broadening of the HeI diffuse series triplet line $\lambda 4471$ became apparent, when attempts were made to establish the presence of Stark Effect in the spectra of dwarf B type stars by means of the broadened helium diffuse series lines. Several hypotheses have been put forward to explain this feature, but because little quantitative work on the intensities of helium lines in B type spectra has been done, it has not been possible to check these theories. The purpose of this investigation is to provide data for this purpose by careful intensity measurements of the profiles of helium lines on high dispersion plates. Six typical B type stars with strong helium lines have been selected for study.

Classification of the Stars:

Stars are classified according to the main features of their spectra. The spectral types as defined by the Harvard criteria are named O, B, A, F, G, K, M, R, N, S, and depend chiefly upon the various degrees of ionization in the stellar atmosphere. B type stars, with temperatures around 20,000°K, are amongst the hottest stars in the heavens. The chief absorption lines in their spectra are the HeI lines which reach a maximum intensity in this class, and the Balmer lines of hydrogen. There are a few relatively weak lines from the ionized atoms of several of the heavier elements, but on the

whole a B type spectrum is relatively free of all lines except those due to neutral helium and hydrogen.

The stars of a given spectral type are further classified according to their absolute magnitude, or intrinsic luminosity. Those of high luminosity are called giants, those of low luminosity dwarfs, while those of intermediate luminosity are called intermediates. As originally applied by Hertzsprung, these designations only referred to differences in absolute magnitude. However, subsequent work has shown that giant stars are, as the name implies, truly large stars with diameters up to 800x the sun, although their masses do not increase proportionately. Giant stars therefore have very low densities, and extremely tenuous atmospheres. Dwarf stars are small, dense stars, often smaller than the sun, with relatively dense atmospheres. Their surface gravity is on the average about 30 times the value of g on the earth.

The spectra of dwarf and giant stars of the same spectral class are somewhat different, the lines of hydrogen and helium particularly tending to have extensive wings in the dwarfs, but being sharp in the giants. This is in the absence of rotation of the star, for rotation broadens all spectral lines. The spectral type depends upon the average degree of ionization in the stellar atmosphere. Low density and high temperature favour ionization, hence dwarf stars of a given spectral type are at a higher temperature than giant stars of the same type. This higher temperature in dwarf atmospheres increases the Doppler broadening of the lines by the individual moving

atoms. Collisions between radiating atoms are also more frequent in the denser atmospheres of dwarf stars, hence collision is another factor tending to broaden the spectral lines. A third cause of broadened lines is Stark Effect. Owing to the relatively great density of electrons and ions in the atmosphere of a dwarf star, the absorbing and emitting atoms are in constantly changing electric fields. In these fields Stark components of a line appear, but since the fields are constantly changing the net result is a superposition of all possible components, and hence a broadening of the spectral line. Struve and Elvey¹ explained the greatly broadened wings of the hydrogen Balmer lines on this basis, and also some of the broadening of the HeI diffuse lines. Stark broadening is not noticeable in the lines from elements of higher atomic number since the displacements are too small. There is some evidence for the broadening of NI lines in stellar spectra, but on the whole Stark broadening is only appreciable in the hydrogen Balmer lines and the neutral helium diffuse series lines.

1. C. T. Elvey and O. Struve, Ap. J. 72, 277, 1930.

Facts of Stark Effect:

The main facts regarding Stark Effect in helium lines are as follows.^{2,3}

There is no simple relationship between displacement of a line from its zero-field position and the strength of the field. The relationship is linear only after the displacement becomes large with respect to the separation of the sub-levels characterized by different values of the quantum number l . Since the fine structure separations for helium are much larger than for hydrogen, the Stark displacements do not become linear until high fields are reached.

The sharp series and principal series of helium are displaced towards the red, the displacements increasing with serial number. The diffuse series lines are also shifted toward the red, and these displacements are much larger than for any of the other series.

Larger fields are not required for the production of electric combination lines, 200 volts/cm sufficing in some cases. The forbidden, or electric combination line, is usually displaced toward the violet although a red shift may occur at low fields and persist till moderate fields are reached. This may be seen in the behaviour of the 2p-5f line, $\lambda 4025$. Until the field reaches about 30 kv/cm the line moves to the red, above that it reverses direction and moves to the violet.

2. J.S. Foster, Proc. Roy. Soc. 117, 137, 1927.

3. J. S. Foster and A.V. Douglas, M.N. 99, 150, 1939.

The line 2p-4f, $\lambda 4470$, however, moves to the violet in all fields. The corresponding singlet lines behave in the same way as the triplet lines.

The intensities of Stark components do not always increase with increasing field; in some cases the line disappears, reappearing at higher fields. In emission the total intensity of the forbidden components and the permitted line equal the intensity of the permitted line at zero field.

In a stellar atmosphere a uniform field is not expected. However the motion and density of charged particles will create fluctuating fields which will have a permanent statistical effect. Fields of around 10^3 - 10^4 v/cm have been considered reasonable by Struve⁴ and others. The electron partial pressure p' is considered to vary with g , the surface gravity following the formula $p' \propto g^m$. Struve considers $m = \frac{1}{2}$ a likely value⁵. In any case it is evident that so long as m is positive, the greater g is, the greater the electron pressure, and the electric fields produced. Hence one would expect Stark effect to be present in dwarf stars and negligible in giants. Also increased pressures produce broadened energy levels.

4. O. Struve, Ap.J. 69, 173, 1929.

5. O. Struve, Ap.J. 69, 173, 1929.

Outline of the Problem:

In 1934 J. S. Foster and Miss A. V. Douglas⁶ analysed profiles of He lines in the spectra of some B type stars, and found very broad absorption lines near the normal positions of the diffuse series (singlets and triplets) of helium. The profiles are somewhat complex, and the absorption is greater on the violet half of the line. In Stark effect the diffuse lines are displaced to the red only, except the first member which is displaced very slightly towards the violet. The whole displacement increases with serial number, but does not vary directly with the electric field. Assuming the presence of Stark effect Foster and Douglas were able to explain qualitatively most of the absorption in the red half of the line. The presence of electric combination lines (2p-4f, 2p-5f, etc) explained part of the violet wing, but these lines did not have the right wavelengths or intensities to explain the total absorption, and a residual absorption was left at 1.2 - 1.7 angstroms from the diffuse lines $\lambda\lambda 4922$, 4471, 4388, 4026.

In a paper⁷ published in 1938 Struve studied the residual absorption near the line $\lambda 4471$ in the intermediate star α Herculis. He identified the violet wing as being chiefly $\lambda 4470$, the electric combination line 2p-4f. The ordinary Stark Effect shifts the 2p-4d line ($\lambda 4471$) towards the red;

6. J.S. Foster and A.V. Douglas, Nature 134, 417, 1934.

7. O. Struve, Observatory 61, 53, 1938.

the 2p-4f line (zero-field position $\lambda 4470$) is shifted towards the violet. Hence the lines should spread apart. If present, the separation of these lines could be detected on the high dispersion plates used by Struve. The profiles do not indicate any such effect. To explain this filling in between the lines, Struve considered the effect of collisional damping. By following a train of reasoning developed by Russel and Unsöld, Struve found that the results of this one line ($\lambda 4471$) could be explained, if in dwarf stars there is a damping constant for the He diffuse series which is approximately twenty times larger than the damping constant for other lines. He found that this damping constant depends upon the sensitivity of the given line to Stark Effect. He found also that this type of broadening is independent of the actual pattern of the Stark Effect, and will produce wings in relatively weak lines fully accounting for their symmetry. The symmetrical collision contour is superimposed over the unsymmetrical contour produced by the blended Stark effect, giving the net profile. Only the line $\lambda 4471$ is considered. In giants there should be no marked difference in the damping constant from its ordinary value owing to absence of electric fields.

In 1939 Foster and Miss Douglas⁸ studied helium line profiles in the stars α Herculis and γ Pegasi, and tried to account for the observed broadening of the HeI diffuse series lines both quantitatively and qualitatively by Stark Effect.

8. J.S. Foster and A. V. Douglas, M.N. 92, 150, 1939.

Near $\lambda 4922$ and $\lambda 4471$, they found areas of absorption which they could not explain by pure Stark effect, or by likely blends. These areas in each case totalled very nearly 50% of the line. To explain this absorption, Foster considered a possible cycle as indicated in fig. 1. and reasoned as follows:

If stellar conditions are such as to give diffuse lines 3 angstroms or more wide, and there is a reasonable departure from the Boltzman distribution, the broadening of the 4D level might be expected.

When radiation characteristic of the zero-field position of the electric combination line 2p-4f is considered, events in the direction indicated are more probable than in the reverse direction⁹. Thus even if there were no electric fields present to produce the f line (this could not be so, since conditions giving rise to the broadened levels would also give rise to electric fields), there would be relatively strong 2p-4d absorption in the region of the f line, since emission accompanying the absorption will be relatively weak here due to a leakage $4d \rightarrow 4f \rightarrow 3d \rightarrow 2p$. In varying fields it would be expected that the 2p-4f line be augmented at zero-field positions with its maximum intensity shifted toward the d line, and there should be no sharp distinction between the lines.

9. S. Rosseland, Ap.J. 63, 218, 1926.

This absorption is thus a secondary maximum in the d absorption and its existence is not proof that electric fields exist in stellar atmospheres. That such fields do exist however is evident from the greatly extended Balmer wings, and the broadened wings of the sharp series and diffuse series HeI lines.

As a direct consequence of a leakage of this sort, the lines 3d-4f and 2p-3d should be weakened by emission more than is the case in a normal star, i.e. any star in which conditions are not likely to make such a cycle possible. These conditions are most likely to be met in a dwarf or intermediate star, but not in a giant.

The 3d-4f line cannot be easily observed since it is in the infra red, 18693.4 angstroms for the singlets and 18684.2 angstroms for the triplets. However the 2p-3d line can be observed as it is 6678 angstroms for the singlets and 5876 for the triplets.

In a paper¹⁰ published in 1939 Goldberg also considers the problem of the abnormally large absorption near $\lambda 4470$. He shows by a consideration of transition probabilities, and the curve of growth effect, that fields necessary to make the electric combination line $\lambda 4470$ strong enough to take in all the observed absorption in α Herculis are not exceptionally large, but just what might be expected in the atmosphere of a dwarf star. He does not consider what displacement of $\lambda 4470$ would occur

10. L. Goldberg, Ap.J. 89 623, 1939.

because of these fields, but is content with saying that the observed absorption can be quantitatively explained by simple Stark absorption. He does not consider what might be the effect on any other of the diffuse lines.

The object of the present investigation is to examine spectral data, and obtain evidence in support of, or against, the Foster hypothesis. A decided weakening in absorption of the first member of the singlets and triplets will support Foster's hypothesis, and if there is no observable weakening of the first member, then it does not seem likely that Foster's hypothesis is correct.

Observational Material:

Table 1. Stars Studied

Star	HD No.	m_v	Spectral Type	Luminosity Type
γ Pegasi	866	2.9	B2.5ss	intermediate
ρ Leonis	91316	2.9	B1.0s	giant
η Ursae Majoris	120315	1.9	B5.0nn	dwarf
ι Herculis	160762	3.8	B3.5	intermediate
67 Ophiuchi	164353	3.9	B5.0s	giant
55 Cygni	198478	4.9	B3.0e α	giant

Intensity measurements were made to verify the spectral classification. The results obtained by Dr. R. M. Petrie were in accordance with the spectral types assigned by Williams¹¹.

11. E.G. Williams, Ap.J. 83, 305, 1936.

Table 2. Instruments Used

Instrument	Dispersion at 5876 in A/mm	Dispersion at H γ in A/mm	Range covered
I M		30.1	4922-3820
II L		11.0	4471-4009
III M	30.3	10.5	5876
III R	30.3	10.5	6678-4922
III L		7.2	4922-4009
Litt II	29.4	8.7	6678-4009
Litt GI	14.6	14.7	6678-3820
Litt GIII		4.48	4388-3820
GI M	21.9		6678-5876
GI L	15.6		6678-5876
GII L	5.1		5876

The plates were taken at the Dominion Astrophysical Observatory by various members of the staff. Most of the plates were calibrated for intensity work by the rotating sector now in use, although a few of the plates taken some years ago were calibrated by the neutral tint wedge which was in use at that time.

Microphotometer tracings were made of the HeI diffuse series lines $\lambda\lambda 6678, 5876, 4922, 4471, 4388, 4144, 4026, 4009, 3820$. Line profiles were obtained using the intensitometer devised by Dr. C. S. Beals, and the equivalent widths of the lines derived. Plates I and II show typical profiles as obtained by the intensitometer. The two profiles taken from plates of γ Pegasi clearly show much complex structure which

PLATE I

PLATE II

is not present in the profiles of lines in 55 Cygni, a giant star. Note the resolution of the two helium lines $\lambda 4026$ (2^3P-5^3D) and $\lambda 4024$ (2^1P-7^1S) on the Litt GIII Plate of 55 Cygni. The dispersion here is 4.64 Å/mm. In intermediate and dwarf stars the region between these two lines is affected by absorption by the electric combination line $\lambda 4025$.

The mean equivalent width for each line is shown in the following tables. The separate determinations of equivalent width for each line together with the instrument used are shown in tables in the appendix.

The equivalent width of a line is a measure of its intensity. An absorption line in general does not completely absorb the continuous background, but only weakens it. The lines are in general V shaped, having more or less extensive wings, depending upon the conditions giving rise to the line. The equivalent width of a line is the energy absorbed by the line. This may be calculated if the line is considered as a rectangle with height equal to the continuous spectrum and area equal to that of the natural profile. The energy enclosed $W = \int_{\lambda_1}^{\lambda_2} E_{\lambda} d\lambda$ where E_{λ} is the Planck Radiation Law function $8\pi ch \lambda^{-5} (e^{\frac{hc}{\lambda kT}} - 1)^{-1}$. Over short ranges E_{λ} is constant hence $W = E_{\lambda} (\lambda_2 - \lambda_1)$. In practice when intensities are to be compared, but the actual energies absorbed are not required E_{λ} , the height of the continuous spectrum, is called unity. Hence the intensities of the lines are expressed in equivalent angstroms, i.e. $(\lambda_2 - \lambda_1)$.

Table 3. Mean Equivalent Widths (Giants)

Line	α Leonis	55 Cygni	67 Ophiuchi
6678	1.02	1.00	1.16
4922	.47	.64	.74
4388	.36	.49	.41
4144	.30	.43	.45
4009	.21	.30	.34
5876	1.19	.83 ¹²	.97
4471	.71	.74	.73
4026	.51	.62 ¹³	.64
3820	.33	.56	.65

12. 5876 is possibly weakened by emission in 55 Cygni and 67 Ophiuchi. 55 Cygni is classified as an emission line star.

13. The helium sharp series line $\lambda 4024$ has been subtracted from the measured profiles of $\lambda 4026$ in the giant stars 55 Cygni and 67 Ophiuchi.

Table 4. Mean Equivalent Widths (Dwarfs)

Line	γ Pegasi	ϵ Herculis	η Ursae Majoris
6678	.55	.47	.54
4922	.96	.91	.85
4388	.82	.64	.70
4144	.79	.62	.66
4009	.55	.44	.43
5876	.77	.61	.80
4471	1.28	1.07	1.01
4026	1.25	1.02	1.28
3820	1.14	.93	.60 ¹⁴

14. In η Ursae Majoris the wings of $H\gamma$ and $H\delta$ overlap owing to the great broadening of the profiles by the rapid rotation of the star and development of Stark wings. The height of the continuous spectrum probably was drawn too low for $\lambda 3820$ because of their overlapping.

fig 2.

fig 3.

$\times$ = SINGLETs
 $\odot$ = TRIPLETs

fig. 4.

x = SINGLETs
o = TRIPLETS

fig. 5

fig. 6.

X = SINGLETS
 ⊙ = TRIPLETS

fig. 7

SERIAL No. n

fig. 8

X = SINGLETS
O = TRIPLETS

SERIAL No. n

fig. 9.

Results:

From a consideration of figures 2-7 which are a plot of serial number against equivalent width for lines of the diffuse series of helium, a difference between giant stars, intermediates, and dwarfs is at once apparent. In the giants, ρ Leonis, 67 Ophiuchi, and 55 Cygni, the first member of both the singlet and triplet series is the strongest line of the series, and the intensities of the lines decrease with serial number much as the f values calculated by Goldberg¹⁵ (see fig. 9). The first members are not relatively so strong as the first f values, but some incipient emission in these lines might be expected as the stars are of early B type.

In the intermediate and dwarf stars, γ Pegasi, ϵ Herculis, and η Ursae Majoris, the first member of each series is one of the weakest lines measured, the second member being much stronger. Stark effect broadens the second and succeeding members but does not affect the first member. The first member should therefore have relatively the same strength as in a giant star of the same spectral class or one of slightly earlier class. That this is not the case is evident.

When the various hypotheses put forward to explain the relatively large absorption near the second member of the singlets ($\lambda 4922$) and of the triplets ($\lambda 4471$) are considered, it is seen that a weakening of $\lambda 6678$ and $\lambda 5876$ is a direct consequence of the cycle proposed by Foster and Douglas.

The intensity of a spectral line is a function of the

15. Goldberg, Leo, Ap.J. 20, 414, 1939.

number of atoms absorbing a given wavelength and emitting it. If more atoms are absorbing than emitting, then the line is an absorption line. If the equilibrium is shifted so that more atoms are emitting energy of the given wavelength than absorbing it, the line will be an emission line. The equilibrium between absorption and emission, and hence the strength of the line, is governed by the temperature and density of the star's atmosphere. If some other effect causes an emission or absorption of energy apart from the general equilibrium, then the line will be strengthened or weakened accordingly. This is what occurs in the leakage cycle proposed by Foster and Douglas. Energy is absorbed at $\lambda 4470$ owing to level broadening, but it is not re-emitted there in proportion to the equilibrium rate in force, but is emitted at a higher wavelength, $\lambda 5876$. This emission at $\lambda 5876$ is not balanced by any absorption at this wavelength, hence this line i.e. $\lambda 5876$ is weakened in absorption.

According to the explanations of the extra absorption at $\lambda 4470$ by Struve and Goldberg, the leading members of the series should be unaffected, and hence have the same relative strength as in a giant of the same or neighbouring classes. The observed weakening of $\lambda 6678$ and $\lambda 5876$ could not possibly be due to emission as ordinarily considered in emission line stars, for then it should be more noticeable in the giant than the intermediate or dwarf stars. It is therefore concluded that the observed weakening of the leading members, relative to the other lines of the diffuse series, is strong evidence in favour of the cyclic transitions suggested by Foster and Douglas.

A similar plot of data secured by Unsöld¹⁶ on γ Scorpii, a dwarf B0 star, shows the same weakening (fig. 8). It is also interesting to note that H.H. Plaskett¹⁷ qualitatively noticed this weakening of the leading member of the series, when he studied the spectrum of 10 Lacertae, an O9 star classified by Williams as an intermediate. This weakening of $\lambda 6678$ and $\lambda 5876$ is also reported by E.G. Williams¹⁸ for the dwarf stars γ Ursae Majoris and α Leonis.

A consideration of figs. 2-7 shows that in most cases the equivalent widths for $\lambda 4144$ and $\lambda 4026$ are above the smooth curve. The deviation in $\lambda 4144$ is larger in the later spectral types e.g. B5 (67 Ophiuchi), than the earlier ones e.g. B1 (α Leonis). It seems likely that the extra absorption is due to a blend with sulphur lines as the profile of $\lambda 4144$ in 67 Ophiuchi has broad shallow wings, although this star is a sharp lined giant. Of the elements mentioned by Struve¹⁹ as likely to be represented in a B type spectrum, only oxygen and sulphur have lines in this region. Moreover the oxygen multiplet has very low intensity, whereas the sulphur multiplet lines have the following wavelengths and intensities.²⁰

16. A. Unsöld, Zeit. f. Astrophys. 21, 22, 1941

17. H.H. Plaskett, Pub. D.A.O., 1, 325, 1922

18. E.G. Williams, Ap. J. 83, 279, 1936

19. O. Struve, Ap. J. 74, 225, 1931

20. C.E. Moore, A Multiplet Table of Astrophysical Interest.

Table 5 Sulphur Multiplet

4142.42	8	4168.37	5
4145.05	8	4189.68	4
4153.05	10	4217.16	4
4162.64	10		

In table 6 are given the measured intensities of the only two lines of the sulphur multiplet which are free from blends and which are clearly visible on moderate dispersion plates of the stars studied. The assumption that the relative intensities of the lines are the same in stellar spectra as those given by C. Moore,²¹ gives an estimate of the combined strength of the chief lines causing the blend. When this amount is subtracted from the total equivalent width for $\lambda 4144$, the point lies much closer to the curve.

Table 6 Measured Equivalent Widths

	γ Peg.	55Cyg.	α Herc.	67Ophi.
line	B2.5	B3.0	B3.5	B5.0
4162	.03		.01	.05
4153	.07	.05	.04	.06
According to multiplet table $4142+4145 = .8(4153+4162)$ giving				
4142+4145	.08	.04	.04	.09

The most likely line blending with $\lambda 4026$ is the 2^1P-7^1S helium I line $\lambda 4024$. The preceding members in this series can be seen on all the plates taken. On high dispersion plates of the giant star 67 Ophiuchi $\lambda 4024$ is seen to be distinctly

21. C.E. Moore, A Multiplet Table of Astrophysical Interest.

separated from $\lambda 4026$. In intermediate or dwarf stars, however, the Stark components cause a blend over this region. The measured equivalent widths of the two preceding lines in the sharp series of HeI are given in table 7, and the equivalent width of $\lambda 4024$ is estimated as above from the intensities given by C. Moore. When this amount is subtracted from the total measured equivalent width of $\lambda 4026$, the line is closer to the smooth curve. There is no immediately obvious line of any other elements likely to be present which could cause such a blend.

Table 7 Measured Equivalent Widths

	γ Peg.	55Cyg.	μ Herc.	670phi.
line	B2.5	B3.0	B3.5	B5.0
4437	.14	.11	.14	.08
4168	.10	.06	.07	.05
4024		.04		.13
According to multiplet table 4024 = .5(4168+4437) giving				
4024	.12	.08	.10	.07
Average	.12	.06	.10	.10

However after this correction $\lambda 4026$ is still the most intense helium line in the spectra of the two dwarf stars γ Ursae Majoris and γ Scorpii. Apparently some effect peculiar to dwarf stars is strengthening this line. When curves of growth (curves showing the relative increase of line strength with increasing number of absorbing atoms) are plotted from the data for each star, the scatter around the curve for individual observations is small except for the lines $\lambda\lambda 4026, 5876, 6678$. The levels associated with the singlet line $\lambda 4388$, and the corresponding

triplet line $\lambda 4026$ are very nearly of the same energy, so that one would expect any extra absorption in $\lambda 4026$ due to level broadening would also be found in $\lambda 4388$. Moreover, the curves of growth show that if anything $\lambda 4388$ is on a steeper part of the curve than $\lambda 4026$. Thus any increase in the number of absorbing atoms should have greater effect on the strength of $\lambda 4388$ than $\lambda 4026$. Actually it is $\lambda 4026$ which is alone strengthened to a degree more than seems likely for pure Stark broadening. Hence we may conclude that this is some effect peculiar to the $\lambda 4026$ line, and the observed discrepancies are not a curve of growth effect. It is unlikely a Foster leakage would occur only in the triplets, when the triplet and singlet levels are so close together.

Further data on dwarf stars covering the whole range of B stars from B0 to B8 must be obtained before any definite conclusion can be reached on this apparently excessive strengthening of $\lambda 4026$.

Foster and Douglas in their analysis of the HeI diffuse series lines in μ Herculis and γ Pegasi, claim that the leakage at $\lambda 4922$ and $\lambda 4471$ amounts to well over 40% of the line. Since some of this energy is emitted again at $\lambda 6678$ and $\lambda 5876$ we can estimate the change produced in the equivalent width of these lines. If the continuous background of the spectrum is considered to behave as black body radiation, it is possible to calculate what the change in equivalent width is at $\lambda 6678$ for a given change at $\lambda 4922$.

If there are N transitions causing the leakage absorption

at wavelenth λ_1 , then the energy absorbed is,

$$N h \nu_1 = N h c \lambda_1^{-1} = W_1 8 \pi c h \lambda_1^{-5} (e^{\frac{h c}{\lambda_1 k T} - 1})^{-1} \quad (1)$$

$$\text{giving } N = W_1 8 \pi \lambda_1^{-4} (e^{\frac{b}{\lambda_1} - 1})^{-1} \quad (2)$$

where $b = \frac{h c}{k T}$ since for a given star T is constant.

There are also N transitions emitting energy at λ_2 , hence

$$N h \nu_2 = N h c \lambda_2^{-1} = W_2 8 \pi c h \lambda_2^{-5} (e^{\frac{h c}{\lambda_2 k T} - 1})^{-1} \quad (3)$$

$$\text{giving } N = W_2 8 \pi \lambda_2^{-4} (e^{\frac{h c}{\lambda_2 k T} - 1})^{-1} \quad (4)$$

equating (2) and (4) and transposing terms

$$W_2 = W_1 \left(\frac{\lambda_2}{\lambda_1} \right)^4 \frac{(e^{\frac{b}{\lambda_2} - 1})}{(e^{\frac{b}{\lambda_1} - 1})} \quad (5)$$

To evaluate the constant b, T is assumed to be 20,000°.

Tables 8-10 show the results obtained first when it is assumed that W_1 equals 40% of the 2P-4D line, secondly when W_1 equals 30% of this line. All the other corrections that have been suggested are shown and applied. Figures 10-15 are plots of the net equivalent widths against serial number.

Comparing the corrected values with the measured equivalent widths in giant stars, we see that a correction of 40% makes the first member of each series in the dwarf and intermediate stars much more intense than the corresponding line in a giant star of about the same spectral class. It also makes the second member of the series less in the case of the dwarf and intermediate stars than it is in giant stars. The remaining lines are in general stronger in the dwarf and intermediate stars than in the giants. We would expect that in dwarf and intermediate stars the first member of both the singlets and triplets should have much the same intensity as in giant stars, whereas the higher members should be somewhat

Table 8.

γ Pegasi

	W	correction	net W
6678	.55	plus 40% Foster leakage +.76	1.31
		plus 30% Foster leakage +.58	1.13
4922	.96	less 40% Foster leakage -.38	.58
		less 30% Foster leakage -.29	.67
4388	.82	none	.82
4144	.79	less sulphur blend -.08	.71
4009	.55	none	.55
5876	.77	plus 40% Foster leakage +.90	1.67
		plus 30% Foster leakage +.67	1.44
4471	1.28	less 40% Foster leakage -.51	.77
		less 30% Foster leakage -.38	.90
4026	1.25	less 4024 blend -.12	1.13
3820	1.14	none	1.14

Table 9.

α Herculis

	W	correction	net W
6678	.47	plus 40% Foster leakage +.72	1.19
		plus 30% Foster leakage +.54	1.01
4922	.91	less 40% Foster leakage -.36	.55
		less 30% Foster leakage -.27	.64
4388	.64	none	.64
4144	.62	less sulphur blend -.04	.58
4009	.44	none	.44

Table 9. (cont)

	W	correction	net W
5876	.61	plus 40% Foster leakage +.76	1.37
		plus 30% Foster leakage +.56	1.17
4471	1.07	less 40% Foster leakage -.43	.64
		less 30% Foster leakage -.32	.75
4026	1.02	less 4024 blend -.10	.92
3820	.93	none	.93

Table 10.

7 Ursae Majoris

	W	correction	net W
6678	.54	plus 40% Foster leakage +.76	1.37
		plus 30% Foster leakage +.50	1.04
4922	.85	less 40% Foster leakage -.34	.51
		less 30% Foster leakage -.25	.60
4388	.70	none	.70
4144	.66	less sulphur blend -.09	.57
4009	.43	none	.43
5876	.80	plus 40% Foster leakage +.70	1.50
		plus 30% Foster leakage .53	1.33
4471	1.01	less 40% Foster leakage -.40	.61
		less 30% Foster leakage -.30	.71
4026	1.28	less 4024 blend -.10	1.18
3820	.60	none	.60

fig. 10.

X = SINGLETS
O = TRIPLETS

fig. 11.

fig. 12.

fig. 13.

X = SINGLETS
O = TRIPLETS

fig. 14.

X = SINGLETS
O = TRIPLETS

fig. 15.

more intense in dwarfs and intermediates than in giants, since the effective Stark broadening increases with serial number.

It thus seems that a 40% correction raises the first member and lowers the second member of each series too far above and below a pure Stark curve. If the correction is 30% of the 2P-4D line, then the results are much closer to the expected curve. In each case for the dwarf and intermediate stars, $\lambda 6678$ and $\lambda 5876$ have about the same intensity as in the giant stars, while $\lambda 4922$ and $\lambda 4471$ are about the same, or even lower than in giants.

Actually the leakage need not be very great to have an appreciable effect on the intensities of $\lambda 6678$ and $\lambda 5876$.

Recalling that

$$W_2 = W_1 \left(\frac{\lambda_2}{\lambda_1} \right)^4 \frac{(e^{\frac{b}{\lambda_2}} - 1)}{(e^{\frac{b}{\lambda_1}} - 1)}$$

and substituting in values for b with $T = 20,000^\circ$, and for

$\lambda_2 = 6678$, $\lambda_1 = 4922$, it is seen that $W_2 = 2.00W_1$ for the singlets, and for the triplets $W_2 = 1.76W_1$. In other words a leakage of energy corresponding to an equivalent width of .1 angstroms at $\lambda 4922$ results in an emission of energy of .2 equivalent angstroms at $\lambda 6678$, thus effectively weakening this line. A leakage of .1 equivalent angstroms at $\lambda 4471$ results in an emission of .176 equivalent angstroms at $\lambda 5876$.

Foster and Douglas estimated the leakage absorption to be over 40% of the 2P-4D line. However it seems more reasonable upon the above evidence to estimate this leakage as forming only about 30% of the line. The method used by Foster and Douglas in estimating the relative absorption of Stark components in the

lines under consideration is by no means exact. Hence their results may be subject to considerable uncertainty.

Work done subsequent to writing this thesis has shown that the results given in the paper by Foster and Douglas (M.N., 99, 150, 1939) have been misinterpreted. When the area marked X, in the profiles of $\lambda 4471$ for γ Pegasi and μ Herculis shown in this paper, and the total area, Z, of the line are compared it is found that X equals about 30% of Z, in complete agreement with the results announced above. A careful reading of the paper by Foster and Douglas reveals that they claim $\frac{X}{Z-X} \doteq 45\%$ not $\frac{X}{Z} \doteq 45\%$ as was assumed in the above development. The solution of the equation $\frac{X}{Z-X} \doteq 45\%$ for $\frac{X}{Z}$ yields the result that $\frac{X}{Z}$ is about 30%, a value found to be consistent with the reconstruction of the expected Stark Intensity curve.

Summary of Results:

Data have been presented which support the hypothesis of a leakage cycle as proposed by Foster and Douglas to explain the strong absorption near the 2p-4d lines of HeI in the spectra of dwarf B type stars. However it seems likely that this absorption constitutes about 30% of the line rather than over 40% as proposed by Foster and Douglas. The hypotheses suggested by Struve and Goldberg to explain the absorption near $\lambda 4471$ are not satisfactory, since both hypotheses indicate relatively no change in the strengths of the leading members of the diffuse series in giant and dwarf stars of the same spectral type, whereas a weakening of these lines has been observed.

An excessive absorption near $\lambda 4026$ has been found in dwarf stars. From the data examined no explanation of this effect can be offered. A further analysis of the spectra of class B dwarf stars is required before any statement on this point can be given.

I would like to acknowledge the kind assistance of all members of the staff of the Dominion Astrophysical Observatory, Victoria, B.C., who most generously took plates of the stars studied, and contributed much to the progress of this investigation by many stimulating discussions of the problem. In particular I would like to thank Dr. R. M. Petrie, upon whose suggestion this investigation was undertaken, for his many helpful suggestions concerning the problem, and for his guidance in selecting data. The analysis of the data and

writing of the paper was concluded at the University of British Columbia under the direction of Dr. Wm. Petrie.

APPENDIX

The separate determinations of equivalent width for each line, together with the instrument used, are shown in the following tables 11-16. The weighted mean equivalent width for each line with the probable error is given at the bottom of each column. The assigned weights are indicated first in each column, then the equivalent width of the line in angstroms. The weights are relative, and depend on the dispersion and actual plate features.

Table 11

♂ Leonis

Diffuse Singlets					
	2 ¹ P-3 ¹ D	2 ¹ P-4 ¹ D	2 ¹ P-5 ¹ D	2 ¹ P-6 ¹ D	2 ¹ P-7 ¹ D
	6678.149	4921.929	4387.928	4143.759	4009.270
III M	1.02	1			
I M		1 .46	1 .43	1 .37	1 .17
		1 .44	1 .30	1 .35	1 .20
		1 .52	1 .35	1 .19	1 .25
	1.02	.47±.01	.36±.03	.30±.04	.21±.01

Table 11 (cont). Diffuse Triplets				
	$2^3\text{P}-3^3\text{D}$	$2^3\text{P}-4^3\text{D}$	$2^3\text{P}-5^3\text{D}$	$2^3\text{P}-6^3\text{D}$
	5875.618	4471.477	4026.189	3819.606
GII L	3 1.29			
III M	1 1.10			
	1 1.00			
I M		1 .86	1 .46	1 .23
		1 .63	1 .59	1 .45
		1 .63	1 .48	1 .32
	$1.19 \pm .03$	$.71 \pm .05$	$.51 \pm .03$	$.33 \pm .05$

Table 12 55 Cygni

Diffuse Singlets					
	$2^1\text{P}-3^1\text{D}$	$2^1\text{P}-4^1\text{D}$	$2^1\text{P}-5^1\text{D}$	$2^1\text{P}-6^1\text{D}$	$2^1\text{P}-7^1\text{D}$
	6678.149	4921.929	4387.928	4143.759	4009.270
GI L	2 .83				
GI M	3 1.14				
III M	1 .93				
II L			3 .52	2 .39	2 .26
			2 .39		
III L		4 .65			
I M		1 .61	1 .51	1 .51	1 .36
Litt-			4 .52		5 .30
GIII					5 .30
	$1.00 \pm .05$	$.64 \pm .02$	$.49 \pm .01$	$.43 \pm .02$	$.30 \pm .00$

Table 12 (cont.)		Diffuse Triplets			
	$2^3\text{P}-3^3\text{D}$	$2^3\text{P}-4^3\text{D}$	$2^3\text{P}-5^3\text{D}$	$2^3\text{P}-6^3\text{D}$	
	5875.618	4471.477	4026.189	3819.606	
GI L	3 .76				
	3 .89				
III M	1 .89				
IIIL		3 .81	2 .51		
		2 .76			
I M		1 .52	1 .62	1 .56	
			1 .66		
Litt-			3 .64	4 .56	
GIII			3 .67	4 .57	
	.83±.02	.74±.03	.62±.02	.56±.00	

Table 13

67 Ophiuchi

		Diffuse Singlets				
	$2^1\text{P}-3^1\text{D}$	$2^1\text{P}-4^1\text{D}$	$2^1\text{P}-5^1\text{D}$	$2^1\text{P}-6^1\text{D}$	$2^1\text{P}-7^1\text{D}$	
	6678.149	4921.929	4387.928	4143.759	4009.270	
III R	1 1.19	4 .81				
	1 1.12	4 .66				
II L		4	4 .44	4 .51	4 .36	
			4 .39	4 .44	4 .39	
				4 .43	4 .28	
I M		1 .81	1 .42	1 .44	1 .37	
		1 .70	1 .49	1 .44	1 .31	
		1 .79				
Litt-II			1 .32	1 .35	1 .29	
	1.16±.03	.74±.02	.41±.01	.45±.01	.34±.01	

Table 13 (cont.) Diffuse Triplets				
	$2^3\text{P}-3^3\text{D}$	$2^3\text{P}-4^3\text{D}$	$2^3\text{P}-5^3\text{D}$	$2^3\text{P}-6^3\text{D}$
	5975.618	4471.477	4026.189	3819.606
III R	1 1.03			
	1 .91			
II L		4 .67	4 .55	
		4 .70	4 .68	
		4 .78	4 .63	
I M		1 .70	1 .73	3 .66
		1 .87	1 .73	2 .78
				2 .51
LittIII		3 .73	2 .64	
	.97±.04	.73±.01	.64±.01	.65±.03

Table 14

 γ Pegasi

Diffuse Singlets					
	2^1P-3^1D	2^1P-4^1D	2^1P-5^1D	2^1P-6^1D	2^1P-7^1D
	6678.149	4921.929	4387.928	4143.759	4009.270
Litt-	1 .26	4 .92	2 .84	2 .76	2 .40
GI	3 .65		2 .75	2 .88	2 .36
		4 .93	4 .81	3 .67	3 .45
III L		4 .83	4 .90	3 .83	3 .57
II L			3 .74	3 .93	4 .62
			3 .76	3 .64	4 .52
I M		1 1.26	1 .91	1 .70	1 1.04
		1 1.44	1 .93	1 1.10	1 .75
	$.55 \pm .07$	$.96 \pm .03$	$.82 \pm .01$	$.79 \pm .02$	$.55 \pm .02$
Diffuse Triplets					
	2^3P-3^3D	2^3P-4^3D	2^3P-5^3D	2^3P-6^3D	
	5875.618	4471.477	4026.189	3819.606	
Litt-	2 .59	2 1.28	2 1.24	3 .90	
GI	3 .89	2 1.40	2 1.10		
III L		4 1.17	3 1.08		
		4 1.29	3 1.24		
II L		3 1.30	3 1.42		
		3 1.19	3 1.20		
I M		1 1.40	1 1.51	1 1.56	
		1 1.46	1 1.57	2 1.30	
	$.77 \pm .03$	$1.28 \pm .01$	$1.25 \pm .02$	$1.14 \pm .08$	

Table 15.

Herculis

Diffuse Singlets					
	$2^1\text{P}-3^1\text{D}$ 6678.149	$2^1\text{P}-4^1\text{D}$ 4921.929	$2^1\text{P}-5^1\text{D}$ 4387.928	$2^1\text{P}-6^1\text{D}$ 4143.759	$2^1\text{P}-7^1\text{D}$ 4009.270
II L			3 .74	3 .70	3 .44
			3 .67	3 .65	3 .51
			3 .62	3 .47	3 .45
			3 .65	3 .60	3 .35
			3 .64		
			3 .69		
			3 .55		
			3 .58		
			3 .68		
			3 .64		
			3 .69		
			3 .58		
			3 .57		
			3 .76		
			3 .62		
			3 .60		
			3 .60		
			3 .58		
			3 .66		
			2 .64		
I M		1 .70	1 .88	1 .67	1 .57
		1 1.15	1 .58	1 .64	1 .41

Table 15 (cont)

	$2^1\text{P}-3^1\text{D}$	$2^1\text{P}-4^1\text{D}$	$2^1\text{P}-5^1\text{D}$	$2^1\text{P}-6^1\text{D}$	$2^1\text{P}-7^1\text{D}$
I M		1 1.16 1 11088 1 .80	1 .85	1 .54	1 .45
LittGI	3 .47	3 .85	2 .96	2 .70	2 .43
III R	1 .60	3 .86			
III M	1 .34 1 .59				
	$.47 \pm .03$	$.91 \pm .03$	$.64 \pm .01$	$.62 \pm .01$	$.44 \pm .01$
Diffuse Triplets					
	$2^3\text{P}-3^3\text{D}$	$2^3\text{P}-4^3\text{D}$	$2^3\text{P}-5^3\text{D}$	$2^3\text{P}-6^3\text{D}$	
	5875.618	4471.477	4026.189	3919.606	
II L		3 1.07 3 1.13 3 1.00 3 .83 3 1.06 3 1.09 3 1.38 3 1.05 3 1.30 3 1.01 3 .90 3 .88 3 .88 3 1.10	6 1.07 6 .92 6 .90		

Table 15 (cont)

	$2^3\text{P}-3^3\text{D}$	$2^3\text{P}-4^3\text{D}$	$2^3\text{P}-5^3\text{D}$	$2^3\text{P}-6^3\text{D}$
II L		3 1.06 3 1.52 3 1.03 3 1.16 3 .97 3 .91 2 .95		
I M		1 1.23 1 1.19 1 1.30	2 1.15 2 1.17 1 1.13	1 .96 1 1.12 1 .93 1 .93
LittGI	3 .58	2 1.20	3 1.10	1 .73
III R	1 .76			
III M	1 .55 1 .61			
	$.61 \pm .02$	$1.07 \pm .01$	$1.02 \pm .01$	$.93 \pm .04$

Table 16

7 Ursae Majoris

Diffuse Singlets					
	2^1P-3^1D	2^1P-4^1D	2^1P-5^1D	2^1P-6^1D	2^1P-7^1D
	6678.149	4921.929	4387.928	4143.759	4009.270
III R	3 .39	3 .65			
	3 .66	3 1.01			
	2 .56	2 .94			
II L			3 .73	3 .57	3 .41
			3 .78	3 .81	3 .40
			2 .75	2 .54	2 .46
I M		1 .78	1 .38	1 .73	1 .27
			1 .62		1 .70
	$.54 \pm .03$	$.85 \pm .04$	$.70 \pm .03$	$.66 \pm .01$	$.43 \pm .01$
Diffuse Triplets					
	2^3P-3^3D	2^3P-4^3D	2^3P-5^3D	2^3P-6^3D	
	5875.618	4471.477	4026.189	3819.606	
III R	3 .68				
	3 .91				
	2 .80				
II L		3 .96	3 1.35		
		3 1.17	3 1.41		
		2 1.02	2 1.14		
I M		1 .67	1 1.26	1 .47	
		1 1.03	1 .94	1 .74	
				1 .60	
	$.80 \pm .01$	$1.01 \pm .03$	$1.28 \pm .03$	$.60 \pm .05$	

BIBLIOGRAPHY

Elvey, C.T., Struve, O., Ap. J. 72, 277, 1930.

Foster, J.S., Proc. Roy. Soc. 117, 137, 1927

Foster, J.S., Douglas, A.V., Nature 134, 417, 1934.

M.N. 99, 150, 1939.

Goldberg, Leo, Ap. J. 89, 623, 1939.

Ap. J. 90, 414, 1939.

Merrill, P.W., Adams, W.S., Ap. J. 97, 98, 1943.

Moore, C.E., A Multiplet Table of Astrophysical Interest.
Princeton, N.J., Princeton University
Observatory, 1933.

Plaskett, H.H., Pub. D.A.O. 1, 325, 1922.

Rosseland, S., Ap. J. 63, 218, 1926.

Struve, O., Ap. J. 69, 173, 1929.

Ap. J. 70, 85, 1929.

Ap. J. 70, 237, 1929.

Ap. J. 74, 225, 1931.

Observatory 61, 53, 1938.

Unsöld, A., Zeit. f. Astrophys. 21, 22, 1941.

Williams, E.G., Ap. J. 83, 279, 1936.

Ap. J. 83, 305, 1936.

Woolley, R.R., Observatory 60, 235, 1937.