THEORETICAL AND EXPERIMENTAL STUDIES

OF A HIGH DENSITY Z-PINCH

by

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ABSTRACT

A fast Z-Pinch in 1.22 Torr helium has been investigated electrically, photographically, and spectroscopically in order to determine the important parameters of the discharge. The dI/dt oscillogram has been used in conjunction with the circuit equations to find the current shell radius and velocity as functions of time. The dynamics have been modelled numerically using a modified snowplow model. The luminous zone of the plasma has been photographed end-on using a TRW image convertor camera. The radial distribution of the plasma measured on the end-on photographs is found to agree with both the electrical determination of the current shell radius and the radius given by the model.

The time-resolved electron temperature and electron density were measured spectroscopically using the He II 4686 Å emission line. The measurements show that the plasma has parameters of 10 eV and $10^{18}$ cm$^{-3}$ just before pinching and 37 eV and $8\times10^{18}$ cm$^{-3}$ during the 400 ns pinch phase. The results show that the plasma is well suited to the requirements of the future light mixing experiments.
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For help in interpreting the line profiles I would like to thank Dr. E. Källne.

Discussions with Dr. C. Tai, especially those concerning the electrical properties of the pinch are gratefully acknowledged.

While a detailed description of the low-inductance discharge bank has been included in this report, its construction was not part of the work described here. It was built and used for earlier experiments with a plasma focus by J. Burnett, under the supervision of Dr. J. Meyer.

I would like to express my thanks to glassblowers E. Williams and J. Lees for their excellent work.

Financial assistance of the National Research Council is gratefully acknowledged.
Chapter 1

INTRODUCTION

In this thesis the theoretical and experimental investigations of a fast Z-pinch are described. The project was carried out in preparation of various light mixing experiments which are to be conducted in the near future.

Detailed descriptions of the pinch, the energy storage bank, the circuit equations and the inductance of the circuit are given in Chapter 2. The measurement of \( \frac{dI}{dt} \) has provided a wealth of detailed information about the pinch. The various parameters which have been determined as functions of time using the \( \frac{dI}{dt} \) curve and the circuit equations are given in Chapter 3. The snowplow model, modified to include the effect of kinetic pressure is discussed in the following chapter. With three circuit equations included, the model consists of a closed set of equations which have been solved numerically. The predictions of the model are compared with the experimental data.

In Chapter 5 the optical and electrical arrangement which was used to photograph the plasma end-on is described. The photographs are presented in the following chapter. The plasma radius as function of time measured on the photographs is compared to the current shell radius calculated from the \( \frac{dI}{dt} \) curve and the radius given by the snowplow model.

The optical and electrical arrangement for spectroscopic
measurements is described in Chapter 7. The measured line profiles of the HeII 4686 Å line are presented in Chapter 8. The electron temperature and electron density which were deduced from the measured line profiles are compared to predictions of the modified snowplow model.
Chapter 2

DETAILS OF THE Z-PINCH AND
THE DISCHARGE CIRCUIT

2.1 The Z-pinch

A diagram of the Z-pinch used throughout this experiment is shown in Figure 2-1. The vessel is a 45.7 cm pyrex cylinder. The inside and outside diameters are 10.2 cm and 11.4 cm respectively. The electrode separation is 35.6 cm. The electrodes were made of copper and brass as shown in Figure 2-1. The constituent parts of the electrodes were soldered together using silver solder. Holes of 4.2 cm diameter were made into the electrodes to allow in future experiments convergent CO₂ laser radiation passage into the pinch.

Two holes of 1.9 cm diameter were drilled into the vessel wall allowing for ruby laser scattering experiments. Neither the CO₂ laser nor the ruby laser were used in the experiments described in this report. The holes in the vessel wall were simply sealed off with lucite plates during these experiments.

Helium gas was fed through continuously with a very low flow rate while the vessel was pumped on continuously. The pressure was measured at the vessel as indicated in the diagram, and was adjusted by varying the flow rate, using a needle valve on the gas inlet line.
Figure 2-1 The Z-Pinch.
In Table 2-1 the parameters of the pinch and the discharge circuit are listed. Unless stated otherwise, the filling pressure used for the experiments described in this report was 1.22 Torr. The initial number density of helium atoms in the table was calculated using the relation

\[ P_o = n_o kT_o. \]

The discharge circuit is shown in Figure 2-2. Although there are six 14 \( \mu \)F capacitors, each with its own four electrode spark gap, only one capacitor and one spark gap has been indicated for simplicity. Each spark gap is connected to the pinch with five 16 \( \Omega \) cables. Again, for simplicity, only one cable has been shown. There are thirty cables leading to the cable header. The spark gaps are triggered by pulses produced by firing one three electrode spark gap shown in the diagram.

2.2 Impedance of the Discharge Circuit and the Pinch

The discharge circuit inductance was determined to an accuracy of 12 % using various standard formulas to calculate the inductances of all the components and adding them up. The inductances of the 30 cables, the 6 coaxial spark gaps, the cable header, and the anode and cathode of the pinch, all of which were calculated using these formulas, are listed in Table 2-2. The inductance of the energy storage capacitors is also included in the table. The total inductance, not including the inductance of the pinch is thus

\[ L_c = 33 \text{ nH} \pm 4. \]
Table 2-1

Parameters of the Z-Pinch and Discharge Bank

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
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<tr>
<td>$P_o$</td>
<td>1.22 Torr</td>
<td>Helium filling pressure</td>
</tr>
<tr>
<td>$n_o$</td>
<td>$4.01 \times 10^{16}$ cm$^{-3}$</td>
<td>Initial number density of helium atoms at $T_o = 294 ; ^\circ$K</td>
</tr>
<tr>
<td>$\rho_o$</td>
<td>$2.68 \times 10^{-4}$ kg m$^{-3}$</td>
<td>Initial mass density</td>
</tr>
<tr>
<td>$r_o$</td>
<td>5.08 cm</td>
<td>Inner radius of vessel</td>
</tr>
<tr>
<td>$R$</td>
<td>5.72 cm</td>
<td>Outer radius of vessel</td>
</tr>
<tr>
<td>$\ell$</td>
<td>35.6 cm</td>
<td>Electrode separation</td>
</tr>
<tr>
<td>$C$</td>
<td>84 $\mu$F</td>
<td>Bank capacitance</td>
</tr>
<tr>
<td>$L_C$</td>
<td>32 nH</td>
<td>Bank inductance</td>
</tr>
<tr>
<td>$V_{Co}$</td>
<td>11.5 kV</td>
<td>Charging voltage</td>
</tr>
<tr>
<td>$\frac{1}{2} CV_C^2$</td>
<td>5.6 kJ</td>
<td>Bank energy</td>
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Figure 2-2 The Discharge Circuit.
Table 2-2

Determination of the Inductance $L_C$

of the Discharge Circuit

<table>
<thead>
<tr>
<th>Number</th>
<th>Unit</th>
<th>Inductance per unit (nH)</th>
<th>Parallel Combination (nH)</th>
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<tr>
<td>30</td>
<td>16 Ω Coaxial cable, 3 m long</td>
<td>500 ± 75</td>
<td>16.6 ± 2.5</td>
</tr>
<tr>
<td>6</td>
<td>Coaxial spark gap</td>
<td>34 ± 6</td>
<td>5.7 ± 1</td>
</tr>
<tr>
<td>6</td>
<td>14 μF Capacitor</td>
<td>15</td>
<td>2.5</td>
</tr>
<tr>
<td>1</td>
<td>Cable header</td>
<td>5.4 ± 0.4</td>
<td>5.4 ± 0.4</td>
</tr>
<tr>
<td>1</td>
<td>Anode</td>
<td>1.0 ± 0.1</td>
<td>1.0 ± 0.1</td>
</tr>
<tr>
<td>1</td>
<td>Cathode</td>
<td>1.7 ± 0.1</td>
<td>1.7 ± 0.1</td>
</tr>
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</table>

Total: $L_C = 33 ± 4$ nH
As far as its electrical properties are concerned, the pinch may be considered to be a shorted coaxial transmission line of length \( \ell \), consisting of two concentric, conducting, hollow cylinders. The outer conductor represents the return conductor of radius \( R \), and the inner conductor represents the current flowing at the surface of the plasma, whose radius varies with time. The inductance of the line may be found using Ampère's law to be (Halliday and Resnick, 1967)

\[
L_p = \frac{\mu_0 \ell}{2\pi} \ln \frac{R}{r}.
\]

2.3 Electric Circuit and Circuit Equations

We assume that the resistance of the discharge circuit and of the pinch are both negligible. Once the bank has been triggered, the spark gaps become short circuits. The circuit is then simply a capacitor and a variable inductor in series as shown in Figure 2-3.

![Figure 2-3 Equivalent Circuit After t=0.](image)
In the diagram, C is the total bank capacitance of 84 μF and L is the total inductance in the circuit

\[ L = L_C + L_p, \]

\[ L = L_C + \frac{\mu_0 L}{2\pi} \ln \frac{R}{r}. \]  \hspace{1cm} (2-1)

The capacitor voltage is related to the current flowing out of the capacitor by

\[ \frac{dV_C}{dt} = -\frac{I}{C}. \]  \hspace{1cm} (2-2)

The inductor voltage is related to the magnetic flux \( \phi \) by Faraday's law of induction

\[ \frac{d\phi}{dt} = -V_L. \]

The inductor voltage is related to the capacitor voltage by

\[ V_L = -V_C, \]

so one may write

\[ \frac{d\phi}{dt} = V_C. \]  \hspace{1cm} (2-3)
The flux is the product of the inductance and current

\[ \phi = LI, \quad (2-4) \]

so equation 2-3 becomes

\[ \frac{d(LL)}{dt} = V_C. \quad (2-5) \]

These circuit equations will be made use of in the following two chapters.
3.1 Rogowski Coil

The rate of change $\frac{dI}{dt}$ of the discharge current was measured using a small Rogowski coil. This coil consists of 50 turns of enamelled wire wrapped around a 0.5 cm diameter cylinder. The coil was fastened to the end of a 50 $\Omega$ coaxial cable, with one end of the wire soldered to the center conductor, and the other end to the outer conductor. The coil was placed in a location in the cable header where the magnetic field due to the discharge current was high, with the coil axis parallel to $\vec{B}$ as shown in Figure 3-1.

![Figure 3-1 Rogowski coil](image-url)
The voltage $V_R$ in the cable is proportional to the first time derivative of the magnetic flux which passes through the coil, according to Faraday's law of induction:

$$V_R \propto \frac{d}{dt} \int_{\text{Coil}} \mathbf{B} \cdot d\mathbf{s}.$$

Since $\mathbf{B}$ at every point is proportional to the discharge current $I$ according to Ampère's law, the surface integral is also proportional to $I$. It follows that

$$\frac{dI}{dt} = AV_R,$$

where $A$ is a constant. Figure 3-2 shows the signal $V_R$ measured when the pinch is fired.

Figure 3-2 Rogowski Coil Signal; 100 V, 1 μs.

The simplest method which may be used to find the constant of proportionality $A$ involves integrating this curve twice. The first integration with respect to time gives the current, still in terms of $A$,
flowing out of the capacitor:

\[ I(t) = I(0) + \int_0^t \frac{dI}{dt} dt, \]

\[ I(t) = I(0) + A \int_0^t V_R dt. \]

The initial current is zero, so one may write

\[ I(t) = A \int_0^t V_R dt. \] (3-2)

The second integration gives the capacitor voltage:

\[ V_C(t) = V_C(0) - \frac{1}{C} \int_0^t I dt, \]

\[ V_C(t) = V_C(0) - \frac{A}{C} \int_0^t \int_0^t V_R dt' dt. \] (3-3)

Solving for A and setting \( t = \infty \) one obtains

\[ A = \frac{C[V_C(0) - V_C(\infty)]}{\int_0^\infty \int_0^t V_R dt' dt}. \] (3-4)

The charging voltage used throughout these experiments was \( V_C(0) = 11.5 \text{ kV} \) and the voltage remaining on the capacitors after the bank has been fired was \( V_C(\infty) = 0.75 \text{ kV} \). The signal \( V_R \) has negligible value after about \( t = 25 \mu s \). The signal \( V_R \) was digitized using the oscillogram shown in Figure 3-2 and others having different time scales from \( t = 0 \) until \( t = 25 \mu s \), and the numbers were typed onto computer
The first and second time integrals of $V_R$ were computed using the trapezoid rule and stored as functions of time in computer memory. The final value of the second integral was found in this way to be

$$
\int_0^t V_R \, dt' \, dt = 6.59 \text{ divisions } \mu s^2,
$$

so the constant $A$ according to equation 3-4 is

$$
A = 137 \text{ (kA/} \mu \text{s)/division.}
$$

Each division on the $dI/dt$ oscillogram thus represents 137 kA/$\mu$s.

Using this value of $A$ and the stored integrals, the $dI/dt$, current, and capacitor voltage curves were found from equations 3-1, 3-2, and 3-3. These are plotted in Figure 3-3 from $t = 0$ to $t = 5 \mu$s. The high frequency of the $dI/dt$ curve is attenuated so much by the integration that it is barely visible on the current curve, and not visible at all on the voltage curve. The current has its first maximum at $t = 1.1 \mu$s. The value of the current at this time is

$$
I_{\text{max}} = 174 \text{ kA.}
$$

(3-5)

3.2 Passive Integrator Circuit

A good approximation to the current curve can be obtained by using a passive R-C circuit to "integrate" the Rogowski coil signal.
Figure 3-3  $\frac{dI}{dt}$, Current $I$ and Capacitor Voltage $V_c$. 
The circuit is shown in Figure 3-4. A 1 kΩ resistor and a 0.1 μF capacitor were used, so the time constant $RC$ should be 100 μs. The time constant was measured to be 88 μs.

![Passive Integrator Circuit](image)

Figure 3-4 Passive Integrator Circuit

The impulse response of the circuit is a decaying exponential:

$$f(t) = \frac{1}{RC} e^{-\frac{t}{RC}}.$$

The circuit output $V_I$ is a convolution between $f(t)$ and the input $V_R(t)$:

$$V_I(t) = \frac{1}{RC} \int_0^t e^{-\frac{t-t'}{RC}} V_R(t') \, dt'.$$

If $t << RC$, then $t' << RC$, since $t' \leq t$, and the exponential factor is approximately equal to unity, so one may write

$$V_I(t) = \frac{1}{RC} \int_0^t V_R(t) \, dt, \text{ if } t << RC.$$
If we are concerned with times much smaller than \( RC \), the signal \( V_I \) is a good approximation to the integral.

The choice of \( R \) and \( C \) depends on various considerations. The value of \( R \) should be much larger than 50 \( \Omega \) in order that the cable sees a 50 \( \Omega \) resistive impedance. The time constant \( RC \) should be much larger than the time over which the integral is sought. However, it should not be too large because the resulting signal decreases with \( RC \). Since the signal has to be amplified by the scope, the danger arises that any R.F. noise present at the scope input will be amplified as well and the signal-to-noise ratio ultimately decreases.

The measured signal \( V_I = \frac{I}{ARC} \) is shown in Figure 3-5. Comparison

![Figure 3-5 Discharge Current; 1 V, 500 ns.](image)

with the computed current curve of Figure 3-3 reveals that a high frequency component has been added to the integrated signal. If one ignores this 10 MHz frequency, the oscillogram agrees very well with the computed integral. On the oscillogram, one division represents 120 kA.
3.3 Measurement of the Capacitor Voltage

Using a high voltage potential probe which was constructed with 50 \( \Omega \) impedance characteristics, the voltage of the uppermost electrode of one of the capacitors was measured. Neglecting the voltage drop across the spark gap, this voltage is a good measure of the capacitor voltage. The voltage measured in this way is shown in Figure 3-6a, together with the current signal. A comparison with the voltage curve computed from the \( \frac{dI}{dt} \) curve (see Figure 3-3) shows that various high frequencies have been added in the measured signal, notably a 1.5 MHz signal and a small 10 MHz signal. If one again ignores these high frequencies, the oscillogram agrees very well with the computed curve. In particular, the two agree as to the time when the voltage crosses zero, about 6 \( \mu \)s after initiation of the current.

![Figure 3-6 Capacitor Voltage and Discharge Current With (a) B = 0, and (b) B = 120 G; Upper Trace: 20 V, Lower Trace: 1 V. Time: 1 \( \mu \)s.](image)

Using two magnetic field coils, each consisting of 1200 turns of wire carrying a current of 4 A, an axial magnetic field of 120 G was
applied. Figure 3-6b shows the voltage and current measured with the magnetic field turned on. The signals with the magnetic field turned on show considerably less 10 MHz noise.

3.4 The Changing Inductance

In first approximation, the current in a Z-Pinch may be considered to flow in an infinitely thin cylindrical shell of radius \( r \) (Uman, 1964). At time \( t = 0 \), the radius is \( r = r_o \). If the current shell were to remain at \( r = r_o \), the total inductance \( L \) would have the constant value

\[
L = L_o = L_C + \frac{\mu_0 L}{2\pi} \ln \frac{R}{r_o}
\]

\[
L_o = 32 \text{ nH} + \frac{(4\pi \times 10^{-7} \text{ H/m})(.36 \text{ m})}{2\pi} \ln \frac{.057}{.051}
\]

\[
L_o = 40 \text{ nH.} \quad (3-6)
\]

This could be realized experimentally by filling to a very high pressure. The circuit would be an ordinary L-C circuit whose current would be a sinusoid of frequency \( \omega_o = 1/\sqrt{L_o C} \) and amplitude \( I_o = V_{co}/(\omega_o L_o) \). This frequency corresponds to a quarter period of 2.9 \( \mu \)s, and with \( V_{co} = 11.5 \text{ kV} \), the amplitude is \( I_o = 525 \text{ kA} \). In Figure 3-7 this current is plotted together with the measured current.

The inductance \( L \) as function of time varies approximately as
the ratio of these two curves:

\[ L = \frac{L_0 I_0 \sin \omega_0 t}{I} , \]

where \( I \) is the measured current. From the diagram, it is evident that the measured current shows a strong dependence on the inductance. Consequently, the measured current is a very sensitive measure of the inductance.

3.5 Calculation of the Current Shell Radius from the \( \text{d}I/\text{d}t \) Curve

Assuming that the resistance of the circuit is negligible, at least during the time interval of interest \( 0 \leq t \leq 2.5 \mu s \), the
inductance may be found using only the dI/dt curve and the circuit equations. Once the inductance is known the current shell radius and velocity may be found.

The flux may be found by integrating equation 2-3. Since the current starts at zero, the flux starts at zero, so

$$\phi = \int_0^t V_C \, dt.$$  \hfill (3-7)

Using equation 2-4 the inductance is

$$L = \frac{\phi}{I}.$$ \hfill (3-8)

The inductance depends only on the radial distribution of the current density, the average radius of which may be found by solving for $r$ in equation 2-1:

$$r = R e^{\frac{2\pi}{\mu_0 c} (L_C - L)}.$$ \hfill (3-9)

We will refer to this radius as the current shell radius. The velocity of the current shell is defined using the finite difference formula

$$\dot{r} = \frac{\Delta r}{\Delta t}.$$ \hfill (3-10)

The flux, inductance, and current shell radius and velocity found using equations 3-7 through 3-10 are plotted in Figure 3-8. The radius has a minimum value of $r_{\text{min}} = 1.1 \text{ cm}$, 80 ns after the current
Figure 3-8 Total Inductance, Current Shell Radius and Velocity Calculated From the dI/dt Curve.
minimum at \( t = 1.8 \mu s \). The maximum speed reached by the current shell is 3.8 cm/\( \mu s \), at \( t = 1.7 \mu s \).

3.6 Initial Conditions

The radius \( r \) should start at the inner vessel radius with zero velocity:

\[
r(0) = r_o,
\]

(3-11)

\[
\dot{r}(0) = 0.
\]

In order to meet the second of these requirements it was necessary to adjust the time origin with respect to the \( \frac{dI}{dt} \) curve. The adjustment has the effect of displacing the flux curve vertically, but the current and voltage curves are hardly affected. The time origin was displaced to the right by 80 ns. The fact that the \( \frac{dI}{dt} \) oscillogram starts 80 ns early, according to the new time scale, is attributed to the interference caused by the switching of the high voltage. The \( \frac{dI}{dt} \) curve was set to zero before \( t = 0 \) as shown in Figure 3-8.

The minimum radius is very insensitive to the 80 ns time origin shift. The shift causes a change in \( r_{min} \) of only 10%. The changes in the radius and velocity curves around \( t = 0 \) are much more pronounced.

It may be seen from equation 3-9 that the radius curve depends in an exponential way on the external inductance \( L_c \). Varying \( L_c \) thus has the effect of scaling the entire radius curve. The other initial condition,
equation 3-11a was satisfied by varying $L_\text{C}$ until the initial radius coincided with the inner vessel radius $r_0$.

The purpose of varying $L_\text{C}$ in this way is two fold. First we obtain the properly scaled radius and velocity curves shown in Figure 3-8 and second we obtain another value for the inductance $L_\text{C}$. The value found as a result of this scaling procedure is

$$L_\text{C} = 32 \text{ nH} \pm 3,$$

in good agreement with the value calculated in the previous chapter using the dimensions of the components (see Table 2-2). The accuracy of the present method in determining $L_\text{C}$ is estimated to be better than $10\%$. 
4.1 Snowplow Equation

A dynamic model has been developed to describe the radial evolution of the plasma in a Z-Pinch discharge (Rosenbluth et. al., 1954). Descriptions are also given, for example in Uman (1964) and Jackson (1975). In this model the assumption is made that the plasma has zero resistance, so it cannot be penetrated by the magnetic field due to the current I. The current thus flows on the outside of the plasma, in the shape of a cylindrical shell of infinitesimal thickness.

The current shell is driven inward by the \( j \times B \) force, but is impeded by collisions with the molecules in its path. According to the model, the molecules are swept up and become part of the infinitesimal shell of radius \( r \). The momentum balance equation for such a system is

\[
\pi \rho_o \cdot \frac{d}{dt} \left( r_o^2 - r^2 \right) \frac{dr}{dt} = - \frac{\mu_0 I^2}{4\pi r}.
\]

The right side represents the \( j \times B \) force and the left side the time rate of change of momentum of the current/mass shell whose mass increases according to

\[
m = \pi \rho_o \cdot \left( r_o^2 - r^2 \right).
\]
4.2 Kinetic Pressure

After colliding with the current/mass shell, each particle is given approximately an energy of \( \frac{1}{2} m_p \left( \frac{dr}{dt} \right)^2 \). In a time \( dt \) the shell sweeps up \(-2\pi r \frac{dr}{dt} N \ dt\) particles. At time \( t \) the internal energy has increased to

\[
u = -\pi \lambda \int_0^t \rho r \left( \frac{dr}{dt} \right)^3 \ dt.
\]

Assuming total sweep-up, the volume is approximately

\[v = \pi r^2 \lambda,
\]

and the corresponding average mass density is

\[\rho = \rho_o \frac{r^2}{r^2}.
\]

The kinetic pressure is given by

\[P_K = \frac{2u}{3v},
\]

so the force \( F = 2\pi r \lambda P_K \) is

\[F = \frac{4\pi \rho_o^2 r^2}{3r} \int_0^t \frac{1}{r} \left( \frac{dr}{dt} \right)^3 \ dt.
\]
4.3 Snowplow Model with Kinetic Pressure Term

The snowplow equation is thus, dividing through by $\pi \rho_o \ell$

$$\frac{d}{dt} \left( r_o^2 - r^2 \right) \frac{dr}{dt} = - \frac{\nu_o I^2}{4\pi^2 \rho_o r} - \frac{4r_o^2}{3r} \int_0^t \frac{1}{r} \left( \frac{dr}{dt} \right)^3 dt.$$ (4-1)

In addition to this modified snowplow equation we use the circuit equations introduced in Chapter 2:

$$L = L_C + \frac{\nu_o \ell}{2\pi} \ln \frac{R}{r},$$ (4-2)

$$\frac{dV_C}{dt} = - \frac{I}{C},$$ (4-3)

$$\frac{d(I)}{dt} = V_C.$$ (4-4)

Equations 4-1 through 4-4 form a set of four equations in four unknowns, so a solution exists. There are two first order and one second order equations in the set, so in order to solve we need four initial conditions. These are

$$r(0) = r_o,$$

$$\dot{r}(0) = 0,$$ (4-5)

$$I(0) = 0,$$

$$V_C(0) = V_Co.$$
The numerical solution poses only one obstacle. Equation 4-1 must tell us what the first increment in radius should be for a given increment in time, but if we solve for $dr$ as a means for obtaining $\Delta r$ for a given $\Delta t$ we get

$$
\Delta r = \left\{ - \frac{\mu_0}{4\pi^2 \rho_0} \int_0^t \frac{I^2}{r} \, dt - \frac{4 \rho_0^2}{3} \int_0^t \frac{1}{r} \int_0^t \frac{1}{r} \left( \frac{dr}{dt} \right)^3 \, dt' \, dt \right\} \Delta t.
$$

At time zero, $r = r_0$, so both terms in the large brackets are of the form zero divided by zero. While equation 4-6 gives a valid increment for all succeeding steps, we clearly must use a different approach for the first step. It will be shown later in this chapter that the second term has a higher order time dependence around $t = 0$, so it is negligible at early times compared to the magnetic pressure term.

### 4.4 Taylor Series Expansions

In order to find the behaviour of $I$ and $r$ around $t = 0$, we may write a Taylor series expansion around $t = 0$:

$$
I(t) = I(0) + \dot{I}(0) t + \ddot{I}(0) \frac{t^2}{2} + \ldots
$$

$$
r(t) = r(0) + \dot{r}(0) t + \ddot{r}(0) \frac{t^2}{2} + \ldots
$$

Using the first three of the initial conditions 4-5 these become
\[ I(t) = I(0) t + \frac{\ddot{I}(0)}{2} t^2 + \ldots \quad (4-7) \]
\[ r(t) = r_0 + \frac{\ddot{r}(0)}{2} t^2 + \ldots \]

When the series 4-7 are substituted into the set of equations 4-1 through 4-4, one can solve for the constants \( I(0) \) and \( \ddot{r}(0) \):

\[ I(0) = \frac{V_C}{L_0}, \]
\[ \ddot{r}(0) = -\left( \frac{\mu_0}{3 \rho_0} \right)^{1/2} \frac{V_C}{2\pi L_0 r_0}, \quad (4-8) \]

where \( L_0 \) is the initial value of the inductance \( L \), given by equation 3-6. The Taylor series 4-7 become

\[ I(t) = \frac{V_C}{L_0} t + \ldots \quad (4-9) \]
\[ r(t) = r_0 - \left( \frac{\mu_0}{3 \rho_0} \right)^{1/2} \frac{V_C}{4\pi L_0 r_0} t^2 + \ldots \]

The desired initial increment in radius is thus

\[ \Delta r = -\left( \frac{\mu_0}{3 \rho_0} \right)^{1/2} \frac{V_C}{4\pi L_0 r_0} (\Delta t)^2 \quad (4-10) \]

The initial increment \( \Delta r \) is quadratic in the time increment \( \Delta t \). This of course explains why equation 4-6, which is linear in \( \Delta t \) leads to a result of zero divided by zero.
Using the initial conditions 4-5, equations 4-1 through 4-4 may be written in integral form, so the set becomes

\[ r = r_0 - \int_0^t \left( \frac{\mu_0}{4\pi^2 \rho_0} \int_0^t \frac{I^2}{r} \, dt + \frac{4r_o^2}{3} \int_0^t \frac{1}{r} \int_0^t \frac{1}{r} \left( \frac{dr}{dt} \right)^3 \, dt' \, dt \right) \frac{r}{r_o^2 - r^2} \, dt, \]

\[ L = L_C + \frac{\mu_0 \ell}{2\pi} \ln \frac{R}{r}, \]

\[ V_C = V_{C0} - \frac{1}{C} \int_0^t I \, dt, \]

\[ I = \frac{1}{L} \int_0^t V_C \, dt. \]

For the first step we must use 4-10 for \( \Delta r \), and for all succeeding steps 4-11a may be used. For early times it is best to use \( I(t) \) and \( r(t) \) given by equations 4-9 in the evaluation of the integrals occurring in equation 4-11a as follows:

\[ \frac{\mu_0}{4\pi^2 \rho_0} \int_0^t \frac{I^2}{r} \, dt = \frac{\mu_0 V_{C0}^2}{4\pi^2 \rho_o L_0^2 r_o} \int_0^t t^2 \, dt = \frac{\mu_0 V_{C0}^2}{12\pi^2 \rho_o L_0^2 r_o} t^3, \]

\[ \frac{4r_o^2}{3} \int_0^t \frac{1}{r} \int_0^t \frac{1}{r} \left( \frac{dr}{dt} \right)^3 \, dt' \, dt = \frac{4r_o^3}{3} \int_0^t \int_0^t t^3 \, dt' \, dt = \frac{\ddot{r}^3}{15} t^5. \]

The first of these has a \( t^3 \) dependence around \( t = 0 \), while the second has a \( t^5 \) dependence. The kinetic pressure term is therefore negligible compared to the magnetic pressure term at early times.
4.5 Numerical Integration

In order to perform the integration a computer program was written. The integrals were computed simply as sums. The integration step size used was $\Delta t = 10$ ns. The various constants which were used in the integration are listed in Table 4-1.

The results of the integration giving $I$, $V_C$, $L$, and $r$ as functions of time are plotted in Figure 4-1 and Figure 4-2. For comparison, the integration was done both with and without the kinetic pressure term. The derivatives $\dot{i}$ and $\dot{r}$ have been computed using the finite difference formulas

$$\dot{i} = \frac{\Delta I}{\Delta t},$$

$$\dot{r} = \frac{\Delta r}{\Delta t}.$$ 

By comparing Figure 4-1 and Figure 4-2 it may be seen that the kinetic pressure term has negligible effect until after about $t = 1.3$ $\mu$s.

4.6 Comparison With Experiment

The shape of the current curve given by the model, Figure 4-2 agrees quite well with the measured curve shown in Figure 3-3. The maximum current was 167 kA at $t = 1.07$ $\mu$s, in good agreement with the measured values (see equation 3-5).

The radius of the model also agrees closely with the current shell radius shown in Figure 3-8. These will be plotted on the same graph, together with the photographic measurements in Chapter 6.
Table 4-1
Constants Used in the Integration

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_0 )</td>
<td>5.08 cm</td>
<td>Initial radius of current/mass shell</td>
</tr>
<tr>
<td>( R )</td>
<td>5.72 cm</td>
<td>Radius of return conductor</td>
</tr>
<tr>
<td>( L_C )</td>
<td>32 nH</td>
<td>Inductance of external circuit</td>
</tr>
<tr>
<td>( \mu_0 )</td>
<td>( 4\pi \times 10^{-7} ) H m(^{-1} )</td>
<td>Permeability of free space</td>
</tr>
<tr>
<td>( \ell )</td>
<td>35.6 cm</td>
<td>Electrode separation</td>
</tr>
<tr>
<td>( V_{Co} )</td>
<td>11.5 kV</td>
<td>Charging voltage</td>
</tr>
<tr>
<td>( C )</td>
<td>84 ( \mu F )</td>
<td>Capacitance of bank</td>
</tr>
<tr>
<td>( P_0 )</td>
<td>1.22 Torr</td>
<td>Helium filling pressure</td>
</tr>
<tr>
<td>( T_0 )</td>
<td>294°K</td>
<td>Temperature of helium</td>
</tr>
<tr>
<td>( n_0 )</td>
<td>( 4.01 \times 10^{16} ) cm(^{-3} )</td>
<td>Number density from ( P_0 = n_0 kT_0 )</td>
</tr>
<tr>
<td>( \rho_0 )</td>
<td>( 2.68 \times 10^{-4} ) kg m(^{-3} )</td>
<td>Mass density from ( \rho_0 = m_{He} n_0 = 4m n_0 )</td>
</tr>
</tbody>
</table>
Figure 4-1  Snowplow Model Without Kinetic Pressure Term.
Figure 4-2 Snowplow Model With Kinetic Pressure Term.
Figure 4-3 shows three oscillograms which were obtained by firing the pinch at three different initial filling pressures. The program was run for these pressures and the results are plotted in Figure 4-4. There is good agreement between model and experiment.

Figure 4-3 Capacitor Voltage and Discharge Current for Three Filling Pressures:
(a) 0.1 Torr, 
(b) 1.0 Torr, 
(c) 1.5 Torr.
Upper trace: 20 V, Lower trace: 1 V. 
Time: 1 μs.
4.7 Estimates of Electron Density and Temperature

Assuming total sweep-up, all the helium atoms are within a cylinder of radius \( r \). The average density of electrons assuming total ionization is

\[
n_e = 2n_0 \frac{r_0^2}{r^2}.
\]  

(4-13)

Using this density, the total density of ions and electrons is

\[
n = n_i + n_e = \frac{3}{2} n_e,
\]

so we may estimate the temperature using this density and the kinetic
pressure given by the model, as follows:

\[ P_K = n_0 kT_0 + n_1 kT_1 + n_e kT_e , \]

\[ P_K = nk\bar{T} , \]

\[ \bar{T} = \frac{P_K}{nk} . \] (4-14)

The electron temperature and density have been computed using the values of \( r \) and \( P_K \) given by the program, and will be compared in Chapter 8 with the spectroscopic measurements.
Chapter 5

EXPERIMENTAL ARRANGEMENT USED FOR
TAKING END-ON PHOTOGRAPHS

5.1 The Experimental Set-up

In order to photograph the plasma a TRW image converter camera with a fast framing plug-in unit was employed. The set-up used to photograph the plasma end-on is shown in Figure 5-1. The object to be photographed is a long luminous cylinder. It was possible to photograph the luminous cylinder only after about \( t = 1.5 \) \( \mu \text{s} \) since before this time it is hidden behind the electrodes. The radius of the holes in the electrodes through which the photographs were taken is 2.1 cm.

The optical set-up is a modified pinhole camera. It accepts only that light which is emitted parallel to the Z-axis, to within a tolerance defined by the diameter of the pinhole. The luminous cylinder projects onto a circle on the photocathode, provided that the pinhole is small.

The lens \( L_2 \) was included only because it is part of the TRW camera package. Its inclusion has the effect of decreasing the size of the image on the photocathode. The aperture behind lens \( L_2 \) (not shown) was kept wide open. In any pinhole camera the aperture stop must be the pinhole. Any other apertures can easily ruin the image.
Figure 5-1 Optical and Electrical arrangement Used for Taking End-on Photographs (Schematic).
5.2 Sequence of events

In order to take a picture the shutter is activated. It remains open for .01 seconds. It is within this time interval that the events described below occur. The Polaroid photographs containing three images is subsequently removed from the rear of the TRW camera.

There exist a pair of electrical contacts in the TRW camera which close at the time the shutter is opened. The trigger unit for the capacitor bank is activated by this switch. When the bank fires, current begins to flow in the vessel. Light produced by this current flow is transported into the shielded room via the light fiber, where it is converted into a negative electrical signal by means of a photomultiplier. A positive pulse is created at the start of this signal, which is fed into the TRW delay unit. The delay unit waits for the time $t_{\text{delay}}$ which has been dialed in, and then produces a 300 V pulse which is fed into the trigger input of the camera. The camera then opens and closes its gating grid three times and simultaneously deflects the electron beam so that it is focussed at three positions on the photoanode.

By means of a resistor divider network within the TRW camera, three 8 V pulses are produced, corresponding to the three high voltage gating pulses, to serve as a monitor. These pulses are fed back into the shielded room, attenuated using a 10x attenuator, and added to the discharge current signal using the "Add" feature of the oscilloscope. This composite signal constitutes the upper oscilloscope trace. A typical oscillogram is shown in Figure 5-2. The lower trace is the photomultiplier signal, which has been made to appear positive using the "Invert" feature.
of the oscilloscope. The three exposure pulses are shown in Figure 5-3 with a smaller time scale.

Figure 5-2  Discharge Current With Three Exposure Pulses Added. Upper Trace: 1 V, Lower Trace: 2 V. Time: 500 ns.

Figure 5-3  Image Converter Camera Exposure Pulses, With Exposure Times of (a) 5 ns, and (b) 20 ns. 1 V, 20 ns.
5.3 Alignment of the Optical System

In order to align the pinhole of the pinhole camera, the TRW camera was put aside, leaving only the pinhole, the lens $L_3$, and the pinch. First the lens was aligned using the Helium-Neon laser beam which had previously been adjusted to go through the center of each electrode. The lens was fixed in the position where it was found that the laser beam was not deflected by the lens. The laser was then turned off and the pinhole was introduced and aligned in the $x$, $y$, and $z$ directions using the method illustrated in Figure 5-4.

![Figure 5-4 Alignment of the Pinhole Camera.](image)

The eye is placed very close to the pinhole, and the pinhole is moved in the three directions until the electrode apertures appear superimposed. If the pinhole is at the wrong $x$ or $y$ position, the electrodes appear displaced with respect to each other; if it is at the wrong $z$ position the electrodes appear to have different sizes.
Using this method the pinhole was aligned to an accuracy of ±0.5 mm in each of the three directions.

Once the pinhole was aligned, the TRW camera was put into place as near as possible to the pinhole as shown in Figure 5-1.
6.1 End-on Photographs

The luminous zone of the plasma was photographed using the arrangement which was described in detail in the previous chapter. The optical set-up is a pinhole camera "focussed at infinity". There are no perspective effects to be corrected for. The photos show intensity as function of radius and angle directly. The z-dependence has been integrated out by the optical set-up. The plasma is optically thin at visible wavelengths, so the intensity on the photographs is, assuming the pinhole is small:

\[ I(r, \theta) = \frac{1}{\lambda} \int_{-\lambda/2}^{\lambda/2} I(r, \theta, z) \, dz, \]

where \( I(r, \theta, z) \) is the intensity of light emission in the plasma. The form of this integral shows that the signal-to-noise ratio is very high. It represents an averaging process in the z-direction, averaging over a length \( \lambda = 36 \text{ cm} \) of plasma.

The photographs are shown in Figures 6-2 through 6-6. The vessel is shown in each figure together with a scale representing radial
distance. The pinhole diameter $d$, the magnification factor $M$, and the exposure time are stated for each. For the series of photographs shown in Figure 6-2 the focal length of lens $L_3$ (see Figure 5-1) was 23 cm. For all the rest, the focal length of lens $L_3$ was 12 cm.

The horizontal striations which appear on some of the photographs are not real. They are caused by the gating grid wires, within the TRW camera.

A finite pinhole size has the effect of blurring the image by an amount approximately equal to the pinhole diameter. The pinhole was accordingly kept very small. Fortunately there was enough plasma light to make this possible.

In Figure 6-1 a totally over-exposed photograph is shown in order to illustrate the frame size of the photographs shown in Figures 6-2 through 6-6.

![Figure 6-1 Frame Size of the Photographs Taken With the TRW Camera.](image)

Plots of radius against time are shown in Figure 6-7. The lengths of the vertical lines represent the thickness of the luminous region as measured on the end-on photographs of Figures 6-2 through 6-6.
Figure 6-2  End-on Photographs, Showing the Luminous Region of the Z-Pinch Plasma.
Figure 6-3  End-on Photographs, as in Figure 6-2, But With a Higher Magnification, and a Smaller Pinhole.
Figure 6-4  End-on Photographs, as in Figure 6-3, But Using a He II 4686 Å Filter, and a Longer Exposure Time.
Figure 6-5  End-on Photographs, as in Figure 6-3, But With an Applied Axial Magnetic Field of 120 Gauss.
Figure 6-6  End-on Photographs, as in Figure 6-5, But Using a He II 4686 Å Filter, and a Longer Exposure Time.
Figure 6-7 Radius vs. Time as Measured on the End-on Photographs: (a) From Fig. 6-2, (b) From Fig. 6-3, (c) From Fig. 6-4, (d) From Fig. 6-5, (e) From Fig. 6-6.
The plasma appears very different when photographed using only its emitted 4686 Å light than it does when photographed using all wavelengths. The plasma emits primarily continuum radiation, which is proportional to the square of the electron density. The photographs which were taken without the He II filter thus show the regions of high electron density. The photographs which were taken with the He II filter show preferentially the hot \( T_e \geq 7 \text{ eV} \) region of the plasma.

6.2 Radius vs. Time Graph

In Figure 6-8 the measurements of all the end-on photographs which were taken with \( B = 0 \) are plotted, as in Figures 6-7a, 6-7b, and 6-7c. For comparison, the current shell radius from Chapter 3 (see Figure 3-8), and the radius given by the snowplow model (see Figures 4-1 and 4-2) are also plotted in Figure 6-8. In order that the diagram remain as uncluttered as possible, the uncertainties of the measurements in the temporal direction have not been shown in Figure 6-8. The uncertainties are indicated in Figures 6-2 through 6-6.
Figure 6-8 Radius vs. Time Graph Comparing the Photographs With the Current Shell Radius From Chapter 3 and the Snowplow Model From Chapter 4, Both With and Without Kinetic Pressure $P_k$. 
Chapter 7

EXPERIMENTAL ARRANGEMENT USED FOR
SPECTROSCOPIC MEASUREMENTS

7.1 The Experimental Set-up

In order to obtain time-resolved line profiles of the singly ionized helium line He II 4686 Å a commercial optical multichannel analyzer (O. M. A.) was employed. It was used in gated mode with a gating pulse width of 50 ns. An oscillogram of the gating pulse is shown in Figure 7-1. This pulse is produced by a circuit which discharges a length of high voltage cable charged to 1300 V through a 50 Ω terminating resistor.

Figure 7-1 650 V Gating Pulse;
1 V, 20 ns.

The arrangement used is shown in Figure 7-2. The pinch discharge is imaged onto the entrance slit of a monochromator, and
Figure 7-2  Optical and Electrical Arrangement Used for Measuring Time-Resolved Line Profiles of the He II 4686 Å Line (Schematic).
the O. M. A. head is mounted in place of the exit slit.

7.2 Sequence of Events

The activation of the push button alerts the "O. M. A. Sync" unit to output a pulse at the time it receives the next clock pulse from the O. M. A. console. The pulse is fed into the trigger unit of the capacitor bank, causing the pinch to fire. Light from the pinch is sent into the shielded room via the light fiber, and converted into a negative electrical signal by the photomultiplier. An inverted pulse generator creates a 2 V pulse at the start of the light signal, and this pulse is fed into the TRW delay unit. After a time $t_{\text{delay}}$ has passed this unit puts out a 25 V pulse which triggers the square pulse generator. The 650 V square pulse is sent to the O. M. A. head gate input, and by means of a "T" connector is returned into the shielded room. After attenuating the pulse by a factor of 1/880, the 0.7 V signal is added to the current signal using the "Add" feature of the oscilloscope. This constitutes the upper oscilloscope trace. A typical oscillogram is shown in Figure 7-3.

Figure 7-3 Discharge Current With Gating Pulse Added;
Upper Trace: 1 V, Lower Trace: 2 V. Time: 500 ns.
The lower trace is the photomultiplier signal, which has been made to appear positive using the "Invert" feature of the oscilloscope.

The light which fell onto the 500 channels of the O. M. A. head during the 50 ns square pulse is displayed on a second oscilloscope, and plotted by the console on the X-Y plotter.
Chapter 8

THE HE II 4686 Å PROFILES AND TIME-RESOLVED ELECTRON DENSITY AND TEMPERATURE

8.1 Response of the O. M. A.

The line profiles of the He II 4686 Å line were measured using the arrangement which was described in detail in the previous chapter. While the profiles were being collected during the course of the experiment it became apparent that the response of the O. M. A. was not at all uniform across the 500 channels, and that there was considerable blurring (cross-talk) occurring between the various channels. The electron optics within the O. M. A. head was producing a very poor image. The gating pulse height which had been used until that time was 1000 V.

In order to improve the situation, the response of the O. M. A. to an input consisting of a series of "spikes" was monitored while varying, from shot to shot the gating voltage. In this way it was possible to monitor both the overall shape of the response curve and the cross-talk while varying the gating voltage.

To calibrate the O. M. A. it would have been undesirable for a number of reasons to dismount the head from the monochromator. It was accomplished without dismounting the head using the following method.
8.2 Calibration of the O. M. A.

First the monochromator was turned to the \( \lambda = 0 \) setting. This setting places the zero-order light which is reflected from the grating onto the O. M. A. head. Next the entrance slit of the monochromator was dismounted and remounted with the slit horizontal, 90° relative to its normal orientation. The slit was masked along its length using the mask shown in Figure 8-1. The masked slit is thus imaged onto the O. M. A. head via two curved mirrors and the grating acting as a plane mirror, in a way which is not wavelength dependent. The slit could still be adjusted in order to vary the amount of light.

The lens \( L_2 \) of Figure 7-2 was removed, and the mirror \( M_2 \) was masked with white paper. The purpose of removing the lens and covering the mirror with white paper was to ensure that whatever fraction of pinch light which entered the monochromator would pass through each opening of the masked entrance slit with equal intensity. The size of the aperture \( A_2 \) was reduced somewhat in order to ensure that light from any part of the white paper which passed through the masked entrance slit is accepted by the first monochromator mirror. If any light is lost at this mirror aperture, the image would again be non-uniform.
Figure 8-2  Response of the O. M. A. in (a) D.C. Mode and (b) Gated Mode.
The resulting O. M. A. signal using D.C. mode, illuminating the white paper with a 100 Watt lamp, is shown in Figure 8-2a. In Figure 8-2b are shown the resulting signals using gated mode, with various values of the gating voltage. Fortunately, the pinch at $t = 1.8 \mu$s gave enough light within 50 ns to make this possible. On the basis of this plot, it was decided that the He II profiles which had already been collected at a gating voltage of 1000 V should be discarded, and that a new set of data should be taken at a gating voltage of 650 V.

While 650 V is clearly the best voltage to use, there still remains some cross-talk between the channels, primarily near the edges. Also, the response at 650 V is still not uniform. The overall shape of the response curve is quite similar for voltages of 500 V through 1200 V. A gating voltage of 650 V is a significant improvement over 1000 V, but the improvement is mainly a reduction of cross-talk.

8.3 The He II 4686 Å Profiles

After returning to the configuration shown in Figure 7-2, many shots were made at various monochromator settings, at various times relative to the pinch current initiation, using a gating voltage of 650 V and a pulse duration of 50 ns as shown in Figure 7-1. As stated earlier, the O. M. A. response was not uniform even at 650 V. The response curve is essentially as shown in Figure 8-3a and 8-3b. Figure 8-3a shows the O. M. A. signal at three wavelength settings far enough removed from the 4686 Å line to show only continuum. If the O. M. A. response had been uniform all these curves would have been essentially flat.
Figure 8-3 O.M.A. Response. These Shots Were Made at $t = 1.8 \mu s$ at Wavelengths Far Enough Removed From the He II 4686 Å Line to Show Primarily Continuum.
Figure 8-3b shows the O. M. A. signal at 4900 Å with and without a neutral density filter of .3 density. The curve labelled "4" was used as the response curve and all the data were divided by this curve.

The line profiles which were found in this way are presented in Figure 8-5. These profiles are the result of piecing together a mosaic of curves, after dividing each one by the curve labelled "4" in Figure 8-3b. The times are in microseconds relative to the initiation of the pinch current. This is the same time scale as was used in previous chapters.

Figure 8-4 shows typical oscillograms which were used while taking the data in order to correlate the gating pulse with the pinch discharge. The times correspond to those in Figure 8-5 and 8-6.

8.4 **Electron Density and Electron Temperature**

Figure 8-6 shows an "eyeball fit" to the data of Figure 8-5. These profiles were used to determine the temperature and density of the electrons. The electron density was found using the line full-widths at half maximum ($\Delta \lambda_{\text{FWHM}} = 2 \Delta \lambda_{\text{HWHM}}$), and the relationship (Griem, 1964):

$$N_e = C(N_e, T_e) \left( \frac{\Delta \lambda_{\text{FWHM}}}{\Delta \lambda_{\text{HWHM}}} \right)^{3/2}.$$  

The values of $C(N_e, T_e)$ were calculated using tabulated Stark profiles (Griem, 1974).

The electron temperature was found using the line to continuum ratio. To do this the area of 100 Å of continuum and the area under the
Figure 8-4 Monitor Oscillograms, as in Figure 7-3, Showing the 50 ns Gating Pulse Added to the Current Signal, and the Photomultiplier Signal. The Times Are Those Used for the He II Line Profiles of Figures 8-5 and 8-6. Upper Trace: 1 V, Lower Trace: 2 V. Time: 500 ns.
Figure 8-5 He II 4686 Å Profiles. These are the O. M. A. Results, After Having Been Divided by the Instrument Sensitivity Curve Shown in Figure 8-3b. The Curves Have Been Normalized to Maximum Intensity.
Figure 8-6  He II 4686 Å Profiles: an "Eyeball Fit" to the Data of Figure 8-5.
line after subtracting the continuum were measured from the line profiles of Figure 8-6. Using the ratio of these areas the temperature may be found (Griem, 1964).

The line to continuum ratio, the half-widths at half maximum intensity $\Delta \lambda_{\text{HWM}}$ and the corresponding electron density and temperature are listed as function of time in Table 8-1. The electron density and temperature are plotted in Figure 8-7. The error bars represent the scatter of the data in Figure 8-5 which were used to find the density and temperature. The error bars in the temporal direction indicate the duration of the gating pulse over which the spectra were integrated by the O. M. A.

The solid curves in Figure 8-7 are the density and temperature found using equations 4-13 and 4-14, in Section 4.7. While these estimates agree favorably with the measurements before $t = 1.7$ $\mu$s, which according to the photographic measurements is about 50 ns after the plasma has reached the Z-axis, the measurements show that these parameters are 3-4 times higher than the simple analysis would indicate during the pinch phase.
Table 8-1
Determination of Electron Density and Temperature

<table>
<thead>
<tr>
<th>Time (μs)</th>
<th>Line to Continuum Ratio</th>
<th>$\Delta \lambda_{\text{HWHM}}$ (Å)</th>
<th>$T_e$ (eV)</th>
<th>$N_e$ (10^{18} cm^{-3})</th>
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Figure 8-7 Electron Density and Electron Temperature.
DISCUSSION AND CONCLUSIONS

The equivalent circuit (see Figure 2-4) and the circuit equations are based on the assumption that the circuit resistance is negligible. That this is a good assumption at least during the time interval of interest $0 \leq t \leq 2.5$ μs has been verified indirectly in two ways. First, the current shell radius found from the $dI/dt$ curve agrees with the end-on photographs. Second, the radius and current given by the modified snowplow model in Chapter 4 are in good agreement with the photographs and the measured current respectively. Had the resistance been significant, inconsistencies between each of these analyses and the photographs and the measured current would have resulted.

The current shell radius found from the $dI/dt$ curve shows clearly the expected piston-like behaviour (see Figure 6-8). The outer radius of the luminous zone stays approximately 0.5 cm ahead of the current shell throughout the pinch. That it should stay ahead by this distance makes sense since the current shell has a finite thickness. A detailed analysis of the current density distribution which was done using a Z-Pinch of somewhat different parameters (Pachner, 1971) shows that while there is some current flowing at all radii at all times, most of the current flows within about 0.5 cm of the current density maximum.

The parameters resulting from the numerical integration of the modified snowplow model (see Figure 4-1) are in good agreement with those
found from the $dI/dt$ oscillogram. In particular, the agreement between the current of the model and the measured current during the first two microseconds shows that the kinetic pressure term gives a reasonable estimate of the pressure. Using the pressure and radius of the model the electron density and temperature have been predicted.

The end-on photographs show that the plasma shell first reaches the Z-axis at $t = 1.65\ \mu s$. Up until this time the temperature and density found from the model agree favourably with the spectroscopic observations (see Figure 8-7). After the shell has reached axis, however the measured parameters are 3-4 times higher than those found from the model. The discrepancy is attributed to shock heating and shock compression which were not included in the model.
REFERENCES


