Monte Carlo Simulation
and
Aspects of the Magnetostatic Design
of the TRIUMF
Second Arm Spectrometer
by
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B.A.Sc. University of British Columbia, 1985

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We accept this thesis as conforming
to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA
August 1988
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Department of Physics

The University of British Columbia
Vancouver, Canada

Date 6 OCT 1988
Abstract

The optical design of the TRIUMF Second Arm Spectrometer (SASP) has been completed and the engineering design started. The effects of the dipole shape and field clamps on the aperture fringe fields were studied. It was determined that a field clamp would be necessary to achieve the field specifications over the desired range of dipole excitations. A specification of the dipole pole edges and field clamps for the SASP is made.

A Monte Carlo simulator for the SASP was written. During the design this was used to study the profiles of rays passing through the SASP. These profiles were used in determining the positioning of the dipole vacuum boxes and the SASP detector arrays. The simulator is intended to assess experimental arrangements of the SASP.
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And of course I thank my parents Donald and Clarice Duncan who are so tolerant of a son who ran like hell from horticulture.
Chapter 1

Introduction

The TRIUMF Second Arm Spectrometer (SASP) is a high resolution, large solid angle, large acceptance, QQClam (Quadrupole, Quadrupole, Clamshell Dipole) type magnetic spectrometer. With construction about to start, the spectrometer should begin operation in mid 1990. Designed to complement the existing Medium Resolution Spectrometer (MRS) located in TRIUMF's Proton Hall at target 4BT2, the SASP will complete the Dual Arm Spectrometer System (DASS) allowing high resolution nuclear spectroscopy of three body reactions. This thesis describes aspects of the magnetostatic design of the SASP's dipole and presents a Monte Carlo simulation of the DASS/SASP system.

Part I of this thesis contains an examination of the SASP dipole, a description of the magnetic field profiles specified by the optical design, and a study of how such fields can be created with the magnet steel geometry. The primary tool for studying the dipole was a magnetostatic field simulation program called POISSON. The aspects of the dipole design this work considers are the internal field profile and the aperture fields of the magnet. The field profiles predicted by simulations of the magnet are compared with the fields specified by the optical design. The correct internal field was produced by adjusting the orientation of the dipole's pole pieces. The desired aperture fields were produced by adjusting the shape of the magnet's steel at the apertures and by the use
of a device called a field clamp which limits the extent of the fringe fields. Preliminary engineering specifications for the dipole apertures and field clamps are appended to this thesis.

Part II of this thesis presents a Monte Carlo simulation of the SASP which is part of a general simulation program called EASY. EASY is designed to study the performance of the complete Dual Arm Spectrometer System and to access experimental arrangements. The simulation of the spectrometer uses transport matrices to propagate a particle from the target to the SASP's focal surface, checking to see if the particle is blocked at various locations in the SASP. The simulation is capable of modelling, in detail, the spectrometer's focal surface detectors including the wire chambers and scintillators. The detectors can be modelled to any level of sophistication up to and including the effects of multiple scattering in the wire chamber windows and gasses. Aspects of the SASP investigated with the Monte Carlo simulation included the solid angle, the acceptance, the limiting apertures, and the downstream ray profiles. Studies of the limiting apertures were utilized in the design of the dipole, the vacuum system and the field clamps. Downstream ray profiles help position the spectrometer's detector array.

1.1 Why a Magnetic Spectrometer

An outline of the type of physics experiments proposed for the new DASS/SASP system is given in the proceedings of a two day workshop[1] which took place at TRIUMF in March 1986. Among the areas of interest are:

- Pion production, \((p,\pi)\), and associated pion production, \((p,\pi x)\).
- Nucleon knockout \((p,2p)\) and \((p,p'n)\)
- Charge exchange, \((n,p)\) and \((p,n)\)
Several experiments already approved for the DASS/SASP system are:

<table>
<thead>
<tr>
<th>EEC No.</th>
<th>Title</th>
<th>Spokesman</th>
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<td>416</td>
<td>Neutron knockout with SASP</td>
<td>C.A. Miller</td>
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<tr>
<td>417</td>
<td>Survey of the ((p, \pi^+)) reaction in the (\Delta) resonance region</td>
<td>P.L. Walden</td>
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<td>418</td>
<td>Nucleon effective polarization in Ca, Zr and Pb</td>
<td>W.J. McDonald</td>
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The SASP with its large solid angle and high resolution is a powerful instrument in itself, but it is the combination of the SASP with the MRS to make the Dual Arm Spectrometer System that is the most interesting aspect of the project.

While dual arm detector systems are not new (they are routinely made out of either a spectrometer and a detector telescope or two telescopes), a dual arm system consisting of two spectrometers operating together will achieve resolutions a factor of 10 to 100 times better than previously possible. Consider the following. The collision of a particle beam with a target can be classified by the number of particles produced in the reaction. A two body collision would be of the form

\[ m_B + m_T \rightarrow m_R + m_1 \]

and a three body collision

\[ m_B + m_T \rightarrow m_R + m_1 + m_2 \]

Where \( m_B \) is the incident particle, \( m_T \) the target nucleus, \( m_R \) the recoil particle, and \( m_1, m_2, \) etc. the scattered particles. Assuming the target nucleus does not break up in the collision, the scattered particles are usually simple entities (protons, electrons, pions, deuterons) of known masses, with the remains of the target nucleus making up the recoil particle. Often, the recoil nucleus is left in an excited state; the measurement
CHAPTER 1. INTRODUCTION

of the energy, cross section, and reaction dependence of these excited states is called nuclear spectroscopy. Measuring the mass of the recoil particle directly is not possible since it usually does not leave the target material, hence the term "missing mass". The alternative is to measure the energy and momentum of the scattered particles and then, knowing the incident beam energy and momentum,

\[
P_R = P_B - P_S
\]

\[
E_R = E_B + E_T - E_S
\]

where the target particle is assumed to be at rest. \( P_S \) is the total momentum of all the scattered particles. Then the missing mass is given by,

\[
m_R = \sqrt{E_R^2 - P_R^2}
\]

If the masses of the scattered particles are known, it is only necessary to measure their trajectories and momentum. This can be done with detector telescopes or with a magnetic spectrometer. A detector telescope can typically resolve a momentum to one part in \( 10^{-2} \) \( (10^{-3} \text{ at best}) \) while a magnetic spectrometer can do as well as one part in \( 10^{-4} \). As discussed in appendix A, the missing mass resolution of a two arm system is completely dominated by the poorer detector. Thus a dual arm experiment using only one spectrometer is not able to use that spectrometer's resolution efficiently. With the Dual Arm Spectrometer System however, TRIUMF will have a detector system capable of taking full advantage of the inherent resolution of both spectrometers.
1.2 The SASP Spectrometer

The proposed TRIUMF Second Arm Spectrometer (SASP) is a high resolution magnetic spectrometer designed to complement the existing Medium Resolution Spectrometer (MRS) at target 4BT2 in TRIUMF’s Proton Hall. The two spectrometer system is called DASS (Dual Arm Spectrometer System). The spectrometers can be operated independently in single arm modes or together in a dual arm mode which makes possible high resolution studies of three body reactions. The specifications of the SASP are given in table 1.1.

The SASP has a QQClam configuration, which consists of two quadrupoles followed by a 90° bend clamshell dipole. Unlike a regular dipole where the pole faces are set parallel to each other, a clamshell dipole has the pole faces set in a wedge. This adds additional focussing to the system. A particle scattered from the target in the 4BT2 target chamber enters the SASP horizontally where it is first focussed by the quadrupoles and then deflected 90° upwards by the dipole towards the detector array located at the spectrometer’s focal surface 1.5 metres above the dipole exit. The detector array consists of four sets of instruments: three Vertical Drift Chambers (VDCs), a set of trigger paddle scintillators, another scintillator further downstream and either a third scintillator or a Cerenkov counter. There is a front end chamber (FEC) which can be lowered into the beam between the target chamber and the first quadrupole. Although the SASP is capable of achieving high momentum resolution without it, this chamber is useful for calibrating the spectrometer and for gathering additional trajectory information.

The SASP can be made into a neutron detector using a CH target to convert the neutrons into protons which the spectrometer detects. At small angles, it is necessary to sweep the primary beam away from the target and in order to fit the sweeping magnet and conversion target before the SASP, Q1 must be removed and Q2 shifted
## SASP Specifications

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<th>Central Momentum ($P_0$)</th>
<th>Design</th>
<th>Maximum</th>
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<tr>
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<td>660 MeV/c</td>
<td>759 MeV/c</td>
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<th>Momentum Bite</th>
<th>-10% to +15% $\Delta P/P_0$</th>
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<th>Solid Angle</th>
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<tr>
<td>at -10% $\Delta P/P_0$</td>
<td>13.1 msr</td>
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<td>at -5% $\Delta P/P_0$</td>
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<td>at 0% $\Delta P/P_0$</td>
<td>15.4 msr</td>
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<td>at +5% $\Delta P/P_0$</td>
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<td>at +10% $\Delta P/P_0$</td>
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<td>at +15% $\Delta P/P_0$</td>
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<th>Resolution @660 MeV/c (with 2mr multiple scattering at focal plane)</th>
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<th>D/M</th>
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<td>at focal plane</td>
<td>bend plane</td>
<td>±102 mr</td>
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<td>at focal plane</td>
<td>non-bend plane</td>
<td>±42 mr</td>
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<th>Focal Plane Tilt</th>
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<th>Total Bend Angle</th>
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<th>Design</th>
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<tr>
<td>vertical</td>
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<td>horizontal</td>
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<th>40°</th>
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<tr>
<th>Minimum opening angle with the beam line</th>
<th>14°</th>
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Table 1.1: SASP Specifications
30cm back towards the dipole[2]. The vacuum box between Q2 and the dipole must be able to accommodate this.

The relationship between the SASP and the MRS is shown in figures 1.1 and 1.2. Both spectrometers rotate about the 4BT2 target chamber. The MRS pivots on a spherical bearing attached to the target chamber support pillar, while the SASP travels on two tracks using a raised guide rail between them to keep the spectrometer oriented on the target chamber. The only contact between the SASP and the target chamber is the vacuum pipe leading to the first quadrupole. This design, while technically more difficult than the MRS's pivot arrangement, allows the SASP to be installed without modifying the MRS support structure minimizing disruption of the MRS's operations. The vacuum pipe to the SASP will be attached to the target chamber with sliding seals that will allow it to rotate without breaking the vacuum. This is an improvement over the scattering chamber currently used for the MRS which uses a series of ports and bellows, making angel changes quite complicated. It is important that the smallest angle the two spectrometers can make with each other and with the beam line is as small as possible. Many experiments need to make measurements near 0° scattering (from the beam line) and others need to make measurements with a small angle between the scattered particles (i.e. a small angle between the spectrometers). The MRS is capable of approaching the beamline to less than 12° in its small angle configuration. While the SASP is a more compact device, it is also closer to the target chamber and so will have a closest angle of approach to the beamline of 14°. The closest angle of approach between the two spectrometers will be 40°.

1.3 Designing SASP

The process of designing the SASP started with an optical design that best met the physics requirements, which were the need for large solid angle, high resolution, and
Figure 1.1: Elevation view of the SASP and MRS. The target chamber and the SASP's detector arrays are only conceptual in this drawing. This figure reproduced with the permission of TRIUMF.
Figure 1.2: Plan View of the SASP and MRS. The SASP can come within 14° of the beamline and 40° of the MRS.
large momentum range. Optically the SASP consists of three elements, two quadrupoles and a dipole. The quadrupoles act as focusing elements while the dipole has both focusing and dispersive properties. The attributes of the quadrupoles that could be varied during the design process were their length, and the strengths of each harmonic field component (in fact the “quadrupoles” should really be called multipoles). The basic dimensions of the dipole were fixed early in the design but fine tuning and correcting aberrations in the optics could be accomplished by adjusting the dipole’s internal field profile and by altering the curved entrance and exit apertures. Working from a clamshell magnet design first suggested by H. A. Enge in conjunction with S. Yen, the optical design of the SASP was finalized by S. Yen [3].

The engineering design began after the optical design of the spectrometer was completed. The dipole design has three aspects: specification of the pole pieces such that they produce the desired field shape and are large enough to accept the particles passing through the spectrometer; design of the dipole side plates, return yokes, and coils used to produce the specified field strengths; and the concept of the dipole vacuum box used to allow particles to pass completely through the spectrometer in vacuum. The design of the SASP dipole is shown in figure 1.3. It is an H magnet with openings in the return yoke for the entrance and exit apertures. The dipole vacuum vessel consists of an aluminum ring (following the pole piece) shaped to form a vacuum seal between the pole pieces. Horns protrude from the dipole vacuum box at the entrance and exit apertures to connect it to the rest of the system. A coil encircles each pole piece outside the vacuum vessel. The basic structure of the dipole including the return yokes, coils, and vacuum box was designed by A. Otter[4], while the orientation of the pole pieces and the curvatures of the dipole apertures are specified in this work. Also specified here are field clamps — devices used to shape and limit the extent of the aperture fringe fields.
Figure 1.3: The layout of the SASP dipole. *This drawing reproduced with the permission of TRIUMF* Not shown are the entrance and exit vacuum horns.
Part I

Designing the SASP
Chapter 2

Spectrometers

2.1 Magneto-optical Systems

Consider a charged particle passing through a system of magnetic fields distributed symmetrically about a median plane (figure 2.1). In the case of the SASP spectrometer, this median plane bisects the dipole and quadrupole magnets. The $z - x$ planes of the entrance and exit coordinate systems lie on the median plane with $y_1$ parallel to $y_2$. It is conventional with bending magnets for $x$ to point in the direction of increasing radius of the magnet. Keeping this convention for the SASP, $x$ points vertically downward before the spectrometer with $y$ horizontal in the laboratory reference frame. After the spectrometer, both $x$ and $y$ are horizontal with $z$ pointing vertically up. The particle-field system is analogous to a light ray traversing an optical system. The trajectory of the particle is equivalent to the light ray [5]. The ray entering the system is described by

$$\vec{r}_1 = \begin{bmatrix} x_1 \\ \theta_1 \\ y_1 \\ \phi_1 \\ \delta \end{bmatrix}$$
Figure 2.1: A system of magnetic fields (in this case the SASP). A ray (the thick arrow) is transported from the $x_1, y_1$ plane (trajectory $\vec{r} = (x, \theta, y, \phi)$) to the $x_2, y_2$ plane.
and after traversing the system by

\[
\vec{r}_2 = \begin{bmatrix} x_2 \\ \theta_2 \\ y_2 \\ \phi_2 \\ \delta \end{bmatrix}
\]

where \( \delta \) is the percentage deviation of the charged particle momentum \( P \) from the central momentum of the system \( P_0 \).

\[
\delta = \frac{P - P_0}{P_0} \times 100\%
\]

As shown in figure 2.1, the components \( x \) and \( y \) give the position of the particle in the \( x - y \) plane. \( \theta \) is the angle between the projection of the ray onto the \( z - x \) plane and the \( z \) axis while \( \phi \) is the angle between the projection of the ray onto the \( z - y \) plane and the \( z \) axis. Note that there is no \( z \) component specified for the ray. This is because the trajectory of the particle is only examined at known planes along the "central trajectory" (the path followed by a ray with \( \delta = 0 \) entering the system along the \( z_1 \) axis). Distances \((x,y)\) are expressed in cm and angles \((\theta,\phi)\) in mrad. The central momentum and coordinate systems are defined such that if the ray enters the system along \( z_1 \) it will leave along \( z_2 \). This notation for describing rays was developed for TRANSPORT [6] — a popular magneto-optics program. The final trajectory, \( \vec{r}_2 \), is a function of the initial trajectory \( \vec{r}_1 \) and can be written as,

\[x_2 = x_2(x_1, \theta_1, y_1, \phi_1, \delta)\]  \hspace{1cm} (2.1)

\[\theta_2 = \theta_2(x_1, \theta_1, y_1, \phi_1, \delta)\]

\[y_2 = y_2(x_1, \theta_1, y_1, \phi_1, \delta)\]
\[ \phi_2 = \phi_2(x_1, \theta_1, y_1, \phi_1, \delta) \]

where \( \delta \) remains constant if the system contains no electric fields (ignoring radiative effects). These relationships can be written as Taylor expansions, for example,

\[
x_2 = \frac{\partial x_2}{\partial x_1} x_1 + \frac{\partial x_2}{\partial \theta_1} \theta_1 + \frac{\partial x_2}{\partial y_1} y_1 + \frac{\partial x_2}{\partial \phi_1} \phi_1 + \frac{\partial x_2}{\partial \delta} \delta + \frac{\partial^2 x_2}{2! \partial^2 x_1} x_1^2 + \frac{\partial^2 x_2}{2! \partial x_1 \partial \theta} x_1 \theta_1 + \ldots
\]

where the partial derivatives are evaluated at \( r_1^* = 0 \). Because the order of differentiation does not matter, many of the second and higher order partial derivatives are equal. For example, the derivatives with respect to \( x \) and \( \theta \):

\[
\frac{\partial^2 x_2}{2! \partial x_1 \partial \theta_1} = \frac{\partial^2 x_2}{2! \partial \theta_1 \partial x_1}
\]

The partial derivatives are written in a symbolic form that recognizes this property:

\[
(x/x) \equiv \frac{\partial x_2}{\partial x_1}
\]

\[
(x/x\theta) \equiv \frac{2\partial^2 x_2}{2! \partial x_1 \partial \theta_1}
\]

\[
\ldots
\]

where \((x/x\theta)\) combines both of the \(x, \theta\) derivatives making it unnecessary to have a \((x/\theta x)\) term — note that this notation does not use subscripts. The equation for \(x_2\) is rewritten as,

\[
x_2 = (x/x)x_1 + (x/\theta)\theta_1 + (x/y)y_1 + (x/\phi)\phi_1 + (x/\delta)\delta
\]

\[
+ (x/x^2)x_1^2 + (x/x\theta)x_1 \theta_1 + (x/xy)x_1 y_1 + (x/x\phi)x_1 \phi_1 + (x/x\delta)x_1 \delta
\]

(2.2)
\[ + \left( \frac{x}{\theta^2} \right)^2 \theta_1^2 + \left( \frac{x}{\theta y} \right) \theta_1 y_1 + \left( \frac{x}{\theta \phi} \right) \theta_1 \phi_1 + \ldots \]

These equations can be written in matrix notation,

\[ \vec{r}_2 = R^1 \vec{r}_1 + R^2 \vec{r}_1^2 + R^3 \vec{r}_1^3 + \ldots \]

where,

\[
R^1 = \begin{bmatrix}
\frac{x}{x} & \frac{x}{\theta} & \frac{x}{y} & \frac{x}{\phi} & \frac{x}{l} & \frac{x}{\delta} \\
\frac{\theta}{x} & \frac{\theta}{\theta} & \frac{\theta}{y} & \frac{\theta}{\phi} & \frac{\theta}{l} & \frac{\theta}{\delta} \\
\frac{y}{x} & \frac{y}{\theta} & \frac{y}{y} & \frac{y}{\phi} & \frac{y}{l} & \frac{y}{\delta} \\
\frac{\phi}{x} & \frac{\phi}{\theta} & \frac{\phi}{y} & \frac{\phi}{\phi} & \frac{\phi}{l} & \frac{\phi}{\delta} \\
\frac{l}{x} & \frac{l}{\theta} & \frac{l}{y} & \frac{l}{\phi} & \frac{l}{l} & \frac{l}{\delta} \\
\frac{\delta}{x} & \frac{\delta}{\theta} & \frac{\delta}{y} & \frac{\delta}{\phi} & \frac{\delta}{l} & \frac{\delta}{\delta}
\end{bmatrix}
\]

The definition of \( \vec{r} \) has been changed to conform to the convention used by TRANSPORT.

\[ \vec{r} = \begin{bmatrix}
x \\
y \\
\phi \\
l \\
\delta
\end{bmatrix} \]

where \( l \) is the difference between the distance travelled through the system by \( \vec{r} \) and the central ray (\( \vec{r}_1 = 0 \)). It has no effect on the other components and is not used in
this work. $R^2$ is a $6 \times 36$ element array and $r_1^{-2}$ is a 36 element column vector,

$$\begin{pmatrix}
  x_1^2 \\
  x_1 \theta_1 \\
  x_1 y_1 \\
  x_1 \phi_1 \\
  x_1 l_1 \\
  x_1 \delta_1 \\
  \theta_1 x_1 \\
  \theta_1^2 \\
  \vdots
\end{pmatrix}$$

Because the notation combines partial derivatives, half the terms in $R^2$ drop out making it an upper diagonal matrix.

Many of the terms in the matrices are in fact zero. For instance, because the momentum of the particle does not change, the only nonzero $\delta$ term is $(\delta/\delta) = 1$. More terms drop out because the magnetic fields are distributed symmetrically about the median plane. To see this, consider the $R^1$ matrix for a simple system consisting of a homogeneous magnetic field whose vector is parallel to $y$. A ray injected into the system will follow a helical path with an axis parallel to the $y$ axis. Assuming the field fills the entire region between the initial and final planes, a projection of the ray's trajectory onto the $x - z$ plane is a segment of a circular arc starting at $x_1$ and ending at $x_2$. The projection of the incident ray is tangent to the arc at $x_1$ making an angle of $\theta_1$ with respect to $z_1$. Similarly, the final ray is tangent to the circle at $x_2$ making an angle of $\theta_2$ with respect to $z_2$ (which is not necessarily parallel to $z_1$). The final position and angle of the projection, $x_2, \theta_2$ are dependent only on $x_1$ and $\theta_1$ and not on
CHAPTER 2. SPECTROMETERS

$y_1$ and $\phi_1$. Similarly $y_2$ and $\phi_2$ are only dependent on $y_1$ and $\phi_1$. $R^1$ then reduces to,

$$R^1 = \begin{pmatrix}
\frac{x}{x} & \frac{x}{\theta} & 0 & 0 & 0 & \frac{x}{\delta} \\
\frac{\theta}{x} & \frac{\theta}{\theta} & 0 & 0 & 0 & \frac{\theta}{\delta} \\
0 & 0 & \frac{y}{y} & \frac{y}{\phi} & 0 & 0 \\
0 & 0 & \frac{\phi}{y} & \frac{\phi}{\phi} & 0 & 0 \\
\frac{l}{x} & \frac{l}{\theta} & 0 & 0 & 1 & \frac{l}{\delta} \\
0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

(2.3)

2.2 Magnetic Spectrometers

A typical nuclear physics experiment has a particle beam colliding with a stationary target producing various scattered particles. The purpose of a magnetic spectrometer is to measure the momentum vector, $\vec{P}_s$ (magnitude and direction), of a charged scattered particle. The magnitude of the momentum is determined by passing the particle through a dispersive magnetic system which produces a correlation between one or more of the components of the particle’s final trajectory and the momentum magnitude. The momentum direction is determined either using position sensors before the spectrometer or by again producing correlations between the trajectory after the spectrometer and the initial trajectory before. The performance of the spectrometer is a function of the characteristics of the particle beam hitting the target, the kinematics of the reaction and the geometry of the apparatus.

Consider a spectrometer with optics described by the first order equation,

$$\vec{r}_2 = R^1 \vec{r}_1$$

where $\vec{r}_1$ is the ray at the target, $\vec{r}_2$ the ray at some plane after the spectrometer, and $R^1$ is given by equation 2.3. Determining the scattered particle momentum vector, $\vec{P}_s$ is therefore the same as measuring the components of the initial ray, $\vec{r}_1$. With appropriately placed detectors after the spectrometer, the trajectory $(x_2, \theta_2, y_2, \phi_2)$ of
the ray as it passes through the $z_2 = 0$ plane can be reconstructed. From equation 2.3, $x_2$ and $\theta_2$ are functions of $\delta$ (the momentum magnitude),

\[
x_2 = \left(\frac{x}{x}\right)x_1 + \left(\frac{x}{\theta}\right)\theta_1 + \left(\frac{x}{\delta}\right)\delta
\]

\[
\theta_2 = \left(\frac{\theta}{x}\right)x_1 + \left(\frac{\theta}{\theta}\right)\theta_1 + \left(\frac{\theta}{\delta}\right)\delta
\]

If $x_1$ and $\theta_1$ were known, $\delta$ could be determined directly with a position sensor measuring $x_2$ in the $z_2 = 0$ plane. This plane is called the spectrometer's focal plane.

In general, it is not possible to determine $\vec{r}_1$ completely using only the final trajectory. Equations 2.1 give the basic transfer functions through the spectrometer. There are four equations $(x_2(), \theta_2(), y_2(), \phi_2())$ and five unknowns $(x_1, \theta_1, y_1, \phi_1, \delta)$, making the system indeterminate. To get a solution, it is necessary to place constraints on the initial ray, forego some of the information, or use front end detectors to measure the initial trajectory of the ray. For many experiments the last option is undesirable for two reasons. First, these detectors alter the momentum and trajectory of the particle, decreasing the momentum magnitude resolution of the spectrometer. Second, being near the target and beam line, they receive a large flux of particles and are likely to saturate, limiting the rate at which the spectrometer can process data.

Constraining the ray entering the spectrometer can be done by various techniques. The "spot size" $(x_1, y_1)$ can be made arbitrarily small by decreasing the size of the particle beam hitting the target. Alternately, the angular range of the scattered particles can be constrained with a collimator (such as the entrance aperture of the spectrometer). Constraining $\vec{r}_1$ does not mean the values of its components are known exactly, only that they lie within a known range. The remaining uncertainty will produce an error in the reconstructed momentum vector of the scattered particle, $\vec{P}_s$. Whether this error is tolerable depends on the objectives of the experiment.
The last approach is to lose some of the information about $\vec{r}_1$. As with constraining $\vec{r}_1$, this will produce an error when reconstructing $\vec{P}_S$. The particle beam hitting the target is not perfectly monochromatic, meaning that while the average momentum of the beam will be some value $P_B$, a given particle will have a momentum deviating from the central momentum by a value, $\Delta P$. This deviation is represented as a percentage of the central momentum.

$$\delta_B = \frac{\Delta P}{P_B} \times 100\%$$

For the TRIUMF cyclotron, $\delta_B$ is normally distributed with a standard deviation of 0.1%. Ideally, the spectrometer should measure the momentum of the scattered particle, $P_S$, as a function of the beam particle momentum, $P_B$, but the smearing of the beam particle momentum will cause a smearing of the scattered particle momentum reducing the resolution of the spectrometer. By adjusting the beam and spectrometer optics using a technique called Dispersion Matching (see appendix B), the effects of $\delta_B$ can be eliminated, improving the spectrometer resolution. The trade off in dispersion matching is the loss of the ability to determine $x_1$ from the ray trajectory after the spectrometer (it can still be measured with detectors placed before the spectrometer).

Equation 2.4 shows $x_2$ as a function of $x_1$, $\theta_1$, and $\delta$. Calling $(x/x)$ the Magnification, $M$, of the spectrometer, and $(x/\delta)$ the Dispersion, $D$, equation 2.4 is rewritten as,

$$x_2 = Mx_1 + (x/\delta)\theta_1 + D\delta$$  \hspace{1cm} (2.5)

where $\delta$ is actually composed of two components (appendix B),

$$\delta = \delta_S + A\delta_B$$

Here, $\delta_S$ is momentum of the scattered particle for the monochromatic beam expressed
as a percentage deviation from the central momentum of the spectrometer, \( P_0 \),

\[
\delta_S = \frac{P_S - P_0}{P_0} \times 100\%
\]

and \( A \) is a constant that depends on the reaction kinematics and the experiment’s geometry. The beam is said to be dispersion matched when \( x_1 \) is made dependent on \( \delta_B \),

\[
x_1 = -\frac{D}{M} A \delta_B
\]

Note that at the target, the position of the beam particle, \( x_B \) is the same as the position of the scattered particle, \( x_1 \). Equation 2.5 reduces to,

\[
x_2 = (x/\theta) \theta_1 + D \delta_S
\]

To make \( x_2 \) a function of \( \delta_S \) only, the \((x/\theta)\) term is made as small as possible. This is called point to point focussing (since a position in the focal surface depends only on \( x_1 \) and \( \delta \), not \( \theta_1 \)).

Now consider the second and higher order terms in the Taylor expansion for \( x_2 \) (Equation 2.2). Since they are not wanted, any nonzero terms are called aberrations. The most important aberration is \((x/x\theta)\). This is the simplest in a series of aberrations which we can write in the form,

\[
(x/x\theta \delta_i)
\]

with \( i = 0, 1, 2, \ldots \). In the SASP optics design [3], aberrations of the form \((x/x\theta \delta^n)\), \( n = 0 \) to 4 are set to zero as well as other aberrations such as \((x/\phi^2)\), \((x/\theta^3)\), \((x/\theta \phi^2)\) and \((x/x\theta^3)\). The aberrations are zeroed by adjusting the shapes of the SASP’s quadrupoles (thus introducing multipole components) and by changing the curvatures of the dipole’s entrance and exit apertures. The non-chromatic aberrations (those without a \( \delta \) term)
are corrected using the two quadrupoles and the dipole entrance. The aberrations \((x/x\theta \delta^n)\) are corrected using the dipole exit curvature. There are an infinite number of aberrations in the spectrometer optics and only a few parameters that can be adjusted to eliminate them. While it is impossible to eliminate all the aberrations, in general they become less important as the order increases.

After as many aberrations as possible are accounted for with the spectrometer hardware, \textit{software corrections} can be applied. The particle momentum can be written as,

\[
\delta_S = \frac{1}{D^2} x_2 + f(x_2, \theta_2, y_2, \phi_2) + g(x_1, \theta_1, y_1, \phi_1)
\]

where \(f\) is a function of the ray trajectory after the spectrometer, and \(g\) is a function of the trajectory before. For the ideal spectrometer \(f\) and \(g\) are both zero. Assuming there are no front end detectors, \(g\) is indeterminate and will always produce an error in \(\delta_S\); \(f\) on the other hand can (at least in theory) be determined. \(f\) is written as a polynomial,

\[
f = ax_2 + b\theta_2 + cy_2 + d\phi_2 + ex_2^2 + fx_2\theta_2 + gx_2y_2 + h x_2\phi_2 + i\theta_2^2 + \ldots
\]

Combining this polynomial with the focal plane position to determine \(\delta_S\) is called a \textit{software correction}. To find the various coefficients in \(f\), rays of known momentum are sent through the spectrometer and a least squares or other parameter fitting routine used to find the best fit. For the SASP, aberrations are mostly corrected for central momentum rays using the hardware while software corrections become important for correcting the aberrations at the extreme ends of the momentum bite.
Chapter 3

SASP Dipole

3.1 RAYTRACE the Program

RAYTRACE is a general “ion-optics” computer code, written in FORTRAN, developed at MIT over the last 20 years[7]. The program user describes the various optical elements in the system to be simulated with standard devices supplied by RAYTRACE. These include multipole and dipole magnets, solenoids, and velocity selectors. In addition to tracing the trajectory of the charged particle (called a ray) through the system, RAYTRACE produces first and second order transport matrices for the system as well as some higher order terms, up to fifth order.

A particle with charge $q$ and velocity $\vec{v}$ passing through electric ($\vec{E}$) and magnetic ($\vec{B}$) fields is acted upon by the Lorentz force,

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

The fields the particle passes through are completely specified by RAYTRACE thus, if the velocity is known, the force acting on the particle anywhere in the system can be calculated. RAYTRACE solves the equations of motion using a fourth order Runge-Kutta stepwise integration technique [8]. A stepwise integration can be thought of as follows: suppose the particle is moving predominately parallel to the $z$ axis with
a known initial position and velocity. The force acting on the particle is determined and used to calculate the deflection of the particle from its original trajectory at a point further along the $z$ axis. The Runge-Kutta method expands on this approach by incorporating the force acting on the particle from several previous points. As the trajectory is extrapolated further from the starting point, the estimates become less accurate. This accumulated error can be made arbitrarily small by decreasing the increments of the steps — at the cost of increased calculation time. There are two other sources of error in this method: the inaccurate description of the magnetic fields; and the roundoff error carried through the calculation. The roundoff error is made negligible by using double precision arithmetic in the routines. The models of the magnetic fields used in RAYTRACE are idealized since the program does not account for the minor field variations of an actual system or the major effects of the saturation of a dipole magnet.

The various elements described in RAYTRACE are modelled with idealized analytic fields. For example, a quadrupole field extends an infinite distance from the optical axis and if a particle was to try to pass through the quadrupole element 10m from the axis it would still behave as if in an ideal quadrupole magnet. RAYTRACE does not consider the physical devices required to create such a field, making it a poor program to examine the effects of apertures. Other programs such as EASY (section 5.1) can be used for this. Because RAYTRACE is not designed to consider the feasibility of the devices it models, the user must be careful not to stake optical designs on devices that are impossible to build.

RAYTRACE does take into account the fringing fields at the apertures of magnets. The shape of the fringe field is specified with parameters input to RAYTRACE. To obtain the fringe field shape for a given problem, the user can either measure the field shape of an existing magnet or use a program like POISSON (appendix C) to
model the magnet. These RAYTRACE fringe fields do not accurately model the various effects found on real magnets. For example, the fringe field of a dipole will change at the edges of the aperture. The RAYTRACE model does not consider this effect.

Because of the large amount of computer time required for a calculation, RAYTRACE is not usually used to do the complete design of a magnetic system, but rather in conjunction with a faster (but less accurate) program such as TRANSPORT which would rough out the general layout of the system. RAYTRACE is then used to fine tune it. However, the clamshell dipole can not be modelled accurately with TRANSPORT so RAYTRACE was used for the entire optical design of the SASP.

### 3.1.1 Describing Dipoles in RAYTRACE

Figure 3.1 shows the major parameters used to describe a magnetic dipole. Originally RAYTRACE assumed that dipoles had homogeneous magnetic fields. This assumption has determined how the subsequent versions of the program have been developed. A particle travelling along the optical axis of the system with a momentum equal to the
central momentum of the dipole $P = P_0$ ($\delta = 0$) would travel in a circular path of radius $R$. This path (from $B$ to $C$ in figure 3.1) is called the central trajectory or central radius of the magnet. The particle traverses an arc $\phi$ along this path.

RAYTRACE uses four special coordinate systems to describe the dipole. All are right-handed with their origins on the optical axis.

A) **Incoming.** $z$ is parallel to the optical axis, pointing in the direction of motion. $x$ is parallel to the radius vector from the centre of curvature of the magnet to the point where the entrance field boundary passes through the central radius of the dipole.

B) **Entrance.** $z$ points out from the pole edge. $x$ is tangent to the pole edge, pointing in the direction of *decreasing* dipole radius.

C) **Exit.** $z$ points out from the pole edge. $x$ is tangent to the pole edge pointing in the direction of *increasing* dipole radius.

D) **Outgoing.** $z$ is parallel to the optical axis, pointing in the direction of motion. $x$ is parallel to the dipole radius vector from the centre of curvature of the magnet to the point where the exit field boundary passes through the central radius of the dipole.

The edges of a dipole magnet are called its Virtual Field Boundaries (VFB). The position of the VFB’s are such that a magnet with a Sharp Cutoff Fringe Field (SCOFF) would end exactly there. The VFB’s can be modified in several different ways (separately or in combination):

- The entrance/exit aperture can be tilted by an angle $\alpha/\beta$ with respect to the central trajectory.

- A VFB can be made part of the arc of a circle of arbitrary radius.
A VFB can be described by an eighth order polynomial.

The symmetry plane of the dipole is called the median plane. By symmetry arguments, it can be seen that the field lines always pass perpendicularly through this plane: that is, the field has only a $y$ component, $\mathbf{B} = (0, B_y, 0)$, on the median plane. RAYTRACE assumes that the field in an nonhomogeneous dipole varies radially from the center of curvature of the magnet (figure 3.1):

$$B_y(DR) = \frac{BF}{1 + NDX \frac{DR}{RB}} \quad (3.1)$$

where $RB$ is the central radius, $DR$ is the distance from that radius (positive $DR$ being further from the center of curvature) and $NDX$ is a dimensionless parameter determining the rate of drop off of the field. For a derivation of this formula, see section 3.3.

For inhomogeneous field magnets such as the SASP dipole, the magnetic field is calculated analytically only on the median plane. Off the median plane, a Taylor expansion is used.

$$B_x = y \frac{\partial B_x}{\partial y} + \frac{y^3}{3!} \frac{\partial^3 B_x}{\partial y^3}$$

$$B_y = B_y + \frac{y^2}{2!} \frac{\partial B_y}{\partial y^2} + \frac{y^4}{4!} \frac{\partial^4 B_y}{\partial y^4}$$

$$B_z = y \frac{\partial B_z}{\partial y} + \frac{y^3}{3!} \frac{\partial^3 B_z}{\partial y^3}$$

The derivatives are evaluated in the median plane. The $y$ partial derivatives must be calculated without knowing $B_x$, $B_y$, or $B_z$. This can be done by looking at Maxwell’s
equations for a magnetostatic field,

\[ \nabla \times \vec{B} = \left( \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \right) \hat{i} + \left( \frac{\partial B_z}{\partial z} - \frac{\partial B_z}{\partial x} \right) \hat{j} + \left( \frac{\partial B_y}{\partial x} - \frac{\partial B_z}{\partial y} \right) \hat{k} = 0 \]  
(3.2)

\[ \nabla \cdot \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \]  
(3.3)

From equation 3.2,

\[ \frac{\partial B_z}{\partial y} = \frac{\partial B_y}{\partial z} \]  
(3.4)

\[ \frac{\partial B_x}{\partial z} = \frac{\partial B_z}{\partial x} \]  
(3.5)

\[ \frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} \]  
(3.6)

Any of the \( y \) derivatives can be calculated by converting it to a sum of partial derivatives in \( x \) and \( z \),

\[ \frac{\partial^n B_i}{\partial y^n} = \sum \frac{\partial^{m+p} B_y}{\partial x^m \partial z^p} \]  
(3.7)

where, \( n = m + p \) and \( i = x, y, z \). The derivatives of \( B_y \) in the median plane are then calculated numerically using a thirteen point grid. As an example of converting the \( y \) partial derivative to an \( x, z \) derivative, take \( \frac{\partial^3 B_x}{\partial y^3} \),

\[ \frac{\partial^3 B_x}{\partial y^3} = \frac{\partial^2}{\partial y^2} \frac{\partial B_x}{\partial y} \]  

Substituting in equation 3.6,

\[ \frac{\partial^2}{\partial y^2} \frac{\partial B_y}{\partial x} = \frac{\partial^2}{\partial x \partial y} \frac{\partial B_y}{\partial y} \]
since the order of differentiation does not matter and then using equation 3.3,

\[ \frac{\partial^2}{\partial x \partial y} \left[ -\frac{\partial B_x}{\partial x} - \frac{\partial B_z}{\partial z} \right] = -\frac{\partial^2}{\partial x^2} \frac{\partial B_x}{\partial y} - \frac{\partial^2}{\partial x \partial z} \frac{\partial B_z}{\partial y} \]

and using equations 3.4 and 3.6,

\[ \frac{\partial^3 B_x}{\partial y^3} = -\frac{\partial^3 B_y}{\partial x^3} - \frac{\partial^3 B_y}{\partial x \partial z^2} \]

The other derivatives can be handled similarly.

### 3.1.2 Fringe Fields in RAYTRACE

When calculating the fringe field of a magnet, RAYTRACE describes the field on the median plane as a function of the width of the magnet’s gap (called the gap width),

\[ B_y = B_i f(s) \]  

(3.8)

where \( s = z/D \), \( D \) is the gap width of the magnet, \( z \) the distance from the VFB (positive is out), and \( B_i \) is the field inside the magnet. The shape of the fringe field is specified with the coefficients in table 3.1. To calculate the field at a point in the aperture ("in" meaning the space just outside the magnet bounded by the curved pole edge), RAYTRACE calculates the distance \( s \) to the nearest point on the VFB in units of the dipole gap width. This distance is put into equation 3.8 along with \( B_i \). For a wedge magnet like the SASP dipole, the gap at the central radius is used. This is surprising since it would seem more logical to use the gap width at the point on the VFB used in the field calculation. The RAYTRACE code was modified to use the local gap width with almost no effect on the spectrometer performance. This is probably because the gap width changes only a small amount over the SASP dipole apertures. If
the change had been larger, the effect on the optics may have been significant. Fringe fields off the median plane are calculated with the same Taylor expansions used for the dipole interior.

The method used in RAYTRACE for calculating fringe fields should be considered approximate. Provided the aperture geometry is well behaved this approximation is adequate. "Well behaved" means the air gap changes slowly and the VFB has a gentle curvature. The effects of a curved aperture on the fringe field are considered in section 4.1.

3.2 Dipole Layout

The optics designed with RAYTRACE completely specify the magnetic fields everywhere in the spectrometer. It is necessary to translate these specifications into the physical hardware that will be built. RAYTRACE is designed so that the parameters used to specify the magnetic fields of a dipole can be readily translated into the major physical dimensions of the magnet's pole pieces.

The pole pieces are the two pieces of iron that shape the useful field region of a magnet. The smaller the gap between the pole pieces, the more intense the magnetic field. The SASP dipole's variable magnetic field is produced by flat pole pieces set at an angle to each other forming a wedge shaped gap (figure 3.2). The curvature of the pole edge determines the magnet's virtual field boundary. A bevel on the pole edge affects the shape and extent of the fringe field. Once the pole piece is specified, the dipole's return yokes, side plates and coils must be designed.

Referring to figure 3.1 the parameters describing the dipole are listed in table 3.1, producing the layout shown in figure 3.3. The short dashed line in figure 3.3 shows the approximate outline of the pole piece.
### Pole Piece

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>220,000.0 cm</td>
</tr>
<tr>
<td>( \phi )</td>
<td>( 2 \times (0.0298°) = 0.0596° )</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>(-75°)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>(-75°)</td>
</tr>
<tr>
<td>( 1/R_1 )</td>
<td>0</td>
</tr>
<tr>
<td>( 1/R_2 )</td>
<td>0</td>
</tr>
</tbody>
</table>

### Internal Field

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>( BF )</td>
<td>1.6073 T</td>
</tr>
<tr>
<td>( NDX )</td>
<td>1300.0</td>
</tr>
</tbody>
</table>

### Fringe Fields

<table>
<thead>
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</tr>
</thead>
<tbody>
<tr>
<td>( D )</td>
<td>10.0 cm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Entrance</th>
<th>Exit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_0 )</td>
<td>0.388258</td>
<td>0.388258</td>
</tr>
<tr>
<td>( c_1 )</td>
<td>1.563493</td>
<td>1.563493</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>-0.483961</td>
<td>-0.483961</td>
</tr>
<tr>
<td>( c_3 )</td>
<td>0.544297</td>
<td>0.544297</td>
</tr>
<tr>
<td>( c_4 )</td>
<td>-0.169069</td>
<td>-0.169069</td>
</tr>
<tr>
<td>( c_5 )</td>
<td>0.015043</td>
<td>0.015143</td>
</tr>
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</table>

### VFB Curvatures

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<thead>
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<th>Exit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_2 )</td>
<td>-684.58</td>
<td>-654.38</td>
</tr>
<tr>
<td>( S_3 )</td>
<td>(-5.2751 \times 10^5)</td>
<td>(-4.9721 \times 10^5)</td>
</tr>
<tr>
<td>( S_4 )</td>
<td>(-2.0709 \times 10^9)</td>
<td>(1.7044 \times 10^9)</td>
</tr>
<tr>
<td>( S_5 )</td>
<td>(-2.1053 \times 10^{13})</td>
<td>(-8.6203 \times 10^{12})</td>
</tr>
<tr>
<td>( S_6 )</td>
<td>0.0</td>
<td>(1.68105 \times 10^{16})</td>
</tr>
<tr>
<td>( S_7 )</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( S_8 )</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 3.1: RAYTRACE parameters for the SASP dipole.
Figure 3.2: A conceptual drawing of the SASP Clamshell dipole showing the pole pieces and wedge shaped air gap. Not shown are the dipole’s curved apertures and return yokes.

**Pole Piece** To describe the SASP dipole geometry in RAYTRACE, it is necessary to use a trick. As seen in figure 3.1, RAYTRACE assumes that magnets have a cylindrical symmetry with the central ray (travelling along the optical axis at the central momentum) making a circular trajectory of distance \( R \) from the centre of curvature. This trajectory is called the *central radius* or the *central arc* of the magnet. Although the field in the dipole can vary radially, the magnetic field along any circle centred on the centre of curvature is constant. The SASP dipole, with its flat pole faces, does not have cylindrical symmetry. As discussed in section 3.3, the field is constant along lines of constant gap. The trick is to model the SASP dipole in RAYTRACE, by increasing the "central" radius, \( R \), of the magnet and decreasing the angle spanning the dipole, \( \phi \), until the central arc looks approximately like a straight line segment. In the case of the SASP dipole, \( R = 220,000 \text{cm} \) and \( \phi = 0.0596^\circ \). This produces a 228.8cm long central arc that deviates from a straight line by less than 0.3mm (figure 3.3). The field is
Figure 3.3: SASP dipole pole piece geometry. The large value of $R$ makes the central radius appear as a straight line. The short dashed line shows the approximate outline of the pole piece.
CHAPTER 3. SASP DIPOLE

strongest at the small gap near the dipole apertures, decreasing with the increasing air gap. Unlike the normal RAYTRACE magnet where the central ray enters the dipole along the central radius, the central ray enters the clamshell dipole at a 45° angle to it. Inside the magnet, the central ray follows a trajectory that looks only approximately like a circle as it passes through regions of varying magnetic field.

The parameter $1/R_1$ ($1/R_2$) is used to give the entrance (exit) aperture a circular curvature. The sign of the parameters make the circular apertures convex or concave. The SASP dipole's pole edge curvature is described completely with the polynomial expression and these inverse radii are set to zero ($R_1(R_2)$ set to infinity).

Internal Field The internal field of the dipole is determined with the two parameters used in equation 3.1. $BF$ is the magnetic field at the central radius, and $NDX$ determines the rate at which the field drops off. The derivation of equation 3.1 and how the pole faces are set to produce the desired fields are described in section 3.3.

Fringe Fields The aperture fringe fields of the dipole are modelled in RAYTRACE using equation 4.1 where the distance from the pole edge is measured in terms of the dipole gap, $D$. The parameters $c_0, \ldots$ in table 3.1 describe the field shape. The SASP dipole uses the same field shape for both the entrance and exit apertures. The method used to determine the dipole fringe field is described in section 4.7.

VFB Curvatures The seven coefficients $S_2, \ldots$ are parameters in an eighth order polynomial (equation 4.3) which modifies the basic VFB's described with the $\alpha(\beta)$ and $1/\hat{R}_1(1/\hat{R}_2)$ parameters. Because the inverse radii are set to zero the basic boundary modified by the polynomial is a straight line. The shapes of the VFBs and the fringe fields for the two apertures are shown in section 4.2.
3.3 Interior Fields

3.3.1 Magnetic Field of a Clamshell Dipole

The magnetic boundary conditions between two materials of different permeability $\mu$ are,

$$B_{n2} = B_{n1}$$

$$B_{t2} = \frac{\mu_2}{\mu_1} B_{t1}$$

where $\mu = \mu_r \mu_0$ and $B_n(B_t)$ is normal(tangential) to the boundary. If material 1 is iron ($\mu_r \approx 500$), material 2 air ($\mu_r = 1$), and the B field in the iron is oriented approximately perpendicular to the interface, the tangential component in air goes to zero and the magnetic flux lines leave the iron perpendicular to the surface. (Actually, because of the large difference in permeabilities, the field lines will leave the iron perpendicular to the surface even if there is only a slight normal component to the field inside the iron).

A magnetic scalar potential $\Phi$ can be defined (see for example Hayt[10]),

$$\vec{B} = -\frac{1}{\mu} \nabla \Phi$$

That is, the B field is the gradient of the scalar potential. Since a surface of constant potential has the B field everywhere normal to it, the interface between the iron and air forms an equipotential surface.

The SASP dipole has flat pole faces set in a wedge shape (figure 3.4), forming two equipotential planes with cylindrical symmetry. The magnetic scalar potential between the pole faces can be calculated using Laplace's equation in cylindrical coordinates,

$$\nabla^2 \Phi = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \Phi}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 \Phi}{\partial \phi^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0$$  \hspace{1cm} (3.9)
Figure 3.4: The pole pieces of the SASP form two equipotential planes with cylindrical symmetry.

The equipotential planes are parallel to the $z$ axis and not dependent on either $\rho$ or $z$, therefore equation 3.9 reduces to,

$$\frac{\partial^2 \Phi}{\partial \phi^2} = 0$$

The solution for which is,

$$\Phi = A\phi + B$$

On the median plane, $\Phi = 0$ and on a pole face $\Phi = \Phi_0$ so,

$$\Phi = \frac{\Phi_0}{\alpha} \phi$$
were \( \alpha \) is the angle between the median plane and the pole face. Calculating \( \vec{B} \),

\[
\vec{B} = -\frac{1}{\mu_0} \nabla \Phi = -\frac{1}{\mu_0} \left( \frac{\partial \Phi}{\partial \rho} \hat{a}_\rho + \frac{1}{\rho} \frac{\partial \Phi}{\partial \phi} \hat{a}_\phi + \frac{\partial \Phi}{\partial z} \hat{a}_z \right)
\]

only the \( \partial/\partial \phi \) term survives leaving,

\[
\vec{B} = - \left( \frac{\Phi}{\mu_0 \alpha} \right) \frac{1}{\rho} \hat{a}_\phi
\]

Therefore, on the median plane of the clamshell dipole, the magnetic field is everywhere normal to the plane and inversely proportional to the distance from the apex of the wedge.

\[
B_y \propto \frac{1}{\rho}
\]  \hspace{1cm} (3.10)

### 3.3.2 The SASP Dipole's Internal Field

As discussed in section 3.2, RAYTRACE describes a magnet in terms of the central radius \( R \). For the SASP dipole \( R \) does not describe the trajectory along which a ray with the central momentum of the magnet, \( P_0 \), travels. Rather, the large value of \( R \) (220,000cm) is used to produce a magnet with planer geometry. In addition to the pole piece geometry, RAYTRACE describes the internal field of the magnet in terms of the central radius (equation 3.1). To determine the slope for the dipole pole faces that will produce the desired field profile, the internal field must be described in the same notation. Rearranging equation 3.10,

\[
B_y = \frac{\rho_0 B_0}{\rho_0 + x}
\]
where $B_0$ is the field at the central radius of the magnet ($R$), a distance $\rho_0$ from the apex of the wedge, and $x$ is the displacement from that central distance. In general, $\rho_0$ will not equal $R$ (the parameter used in RAYTRACE). For the SASP, these numbers differ by three orders of magnitude. Simplifying,

$$B_y = \frac{B_0}{1 + \frac{x}{\rho_0}}$$

Comparing this with the field used by RAYTRACE,

$$B_y = \frac{BF}{1 + NDX \frac{DR}{RB}}$$

where $RB$ is the central radius (also called $R$), $NDX$ is a field shape parameter, and,

$$\frac{NDX}{RB} = \frac{1}{\rho_0}$$

Assuming a 10cm gap at the central radius and using the SASP field parameters from table 3.1, we get a pole face slope of,

$$m = \frac{D_{0\frac{1}{2}}}{\rho_0} = D_{0\frac{1}{2}} \frac{NDX}{RB} = 5cm \frac{1,300}{220,000cm} = 0.02954545 \frac{cm}{cm}$$

where $D_{0\frac{1}{2}}$ is one half the dipole air gap at the central radius. The distance between a 10cm gap and a 15cm gap is

$$d = \frac{\Delta D_{\frac{1}{2}}}{m} = \frac{7.5 - 5.0}{0.02954545} = 84.61538cm$$
Extreme SASP Rays

<table>
<thead>
<tr>
<th>$x$ (cm)</th>
<th>$\theta$ (mrad)</th>
<th>$y$ (cm)</th>
<th>$\phi$ (mrad)</th>
<th>$l$ (cm)</th>
<th>$\delta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.0</td>
<td>-85.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-10.0</td>
</tr>
<tr>
<td>0.0</td>
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<td>0.0</td>
<td>0.0</td>
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</tr>
<tr>
<td>1.0</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>15.0</td>
</tr>
</tbody>
</table>

Table 3.2: Components used to generate the extreme rays sent through the SASP.

3.3.3 Dipole Design

The SASP dipole engineering design was produced by Allen Otter [4]. The size of the pole piece is determined by the momentum acceptance ("bite") of the spectrometer. The original design specified ±10%, meaning that the spectrometer could accept rays with $\delta$ ranging from $-10\%$ to $+10\%$. This specification was later changed to a momentum bite of $(-10\%,+15\%)$. RAYTRACE was used to draw the trajectories of "extreme" rays through the spectrometer (figure 3.5). The extreme rays deviate by the greatest distance from the central ray used in the optical design. In fact, even more diverse rays can get through the spectrometer but they will not necessarily pass through "good" magnetic field. The rays shown in figure 3.5 were generated by setting the components of the input rays to the values in table 3.2.

The dipole pole pieces were laid out around the extreme ray trajectories leaving a width of pole face approximately equal to a dipole gap width beyond the trajectory to ensure the ray doesn’t enter the dipole’s fringe field. It is this safety margin that increases the spectrometer acceptance beyond its design specification. Because the optics were not designed with these additional rays, aberrations may make them useless for high resolution spectroscopy.

The size of the dipole side plates and return yokes were estimated by modelling
Figure 3.5: RAYTRACE was used to draw the trajectories of extreme rays through the SASP. Shown is the central ray, the extreme $-10\%$ ray and the extreme $+10\%$ and $+15\%$ rays. This drawing reproduced with the permission of TRIUMF.
a cross section through the middle of the dipole (figure 3.6a) using the magnetostatic
modelling program POISSON (appendix C). Figure 3.6b shows the B field on the
median plane of the dipole predicted by POISSON compared to the analytic expression
used by RAYTRACE. The dipole is at approximately half the maximum excitation. In
the figure, $R$ is the distance from the central radius. The input files used by POISSON
to model the dipole cross section are listed in appendix E. The solid line is the $1/\rho$ field
profile used by RAYTRACE while the points show the field predicted by POISSON.
The spacing of the points is the grid size used to describe the problem in POISSON.
While the POISSON field at the central radius is not exactly equal to the required
RAYTRACE field, this can be corrected by adjusting the excitation of the dipole in
the model. For purposes of plotting, the field is scaled to be 1.0 at the central radius
($R = 0$). Note the field drops off at the sides of the pole piece.

Figure 3.7 shows a plot of the ratio of the POISSON field to the RAYTRACE
field for the cross section through the dipole. The fields are normalized at the central
radius ($R = 0$) and, ideally, the ratio should be exactly 1.0 across the dipole. Looking
at the low field (large $R$) side of the pole piece, the ratio and therefore the POISSON
field is too large. The cause of this difference between the POISSON model and the
analytic expression is not clear. The dipole is at about half the full excitation and
has not yet started to saturate. This difference (approximately 0.3%) might be due
to edge effects of the dipole or it might be due to the digitizing process in POISSON.
Regardless, a small difference like this in the dipole's field shifts the spectrometer's
focal surface but will not affect its performance.

Since RAYTRACE shows that the spectrometer design is tolerant to a small error
in the field profile, the POISSON result is accepted as correct and the slope of the dipole
pole faces are altered to match the POISSON field profile to the RAYTRACE profile
at the middle excitation. This changes the distance between the 10 and 15cm gaps
Figure 3.6: a) Cross-section of the SASP dipole modelled with POISSON. b) The interior field of the dipole. The solid line is the analytic expression used by RAYTRACE, the points are the field predicted by POISSON. R is the distance from the central radius of the dipole.
CHAPTER 3. SASP DIPOLE

Figure 3.7: The ratio of the POISSON field to the RAYTRACE field $B_P/B_{RT}$. The $B_P$ is scaled to exactly equal $B_{RT}$ at the central radius ($R=0$).

from 84.615cm to 83.777cm (figure 3.8). The new pole face slope is,

$$m = \frac{7.5 - 5.0}{83.777} = 0.02984113$$

The spectrometer design should be unaffected if POISSON turns out to be slightly off in its prediction of the dipole's internal field profile.

3.3.4 Dipole Saturation

Saturation is the term used to describe the nonlinear response of a magnet’s field to the exciting current in its coils. Below some critical value, the magnetic field will rise linearly with excitation. Above the critical current, the field stops rising linearly and levels off at a maximum value. When the excitation is above the critical value, the magnet is said to be saturating. Excitation is expressed as the current times the
number of turns of the coil — Amp-Turns (NA). The saturation properties of a magnetic material are characterized by its B-H curve — a plot of the magnetic field B produced by a given excitation H (expressed in Amp Turns or oersteds\(^1\)). Figure C.3 shows the B-H curves of some of the materials considered for the SASP dipole.

For a uniform gap dipole, the entire pole piece enters the saturation region at the same time so a series of plots of the field profile for different excitations would look the same — flat across the pole piece; dropping off at the edges. Only the magnitude of the field profiles would change with excitation. The SASP’s clamshell dipole behaves quite differently. Because the field varies across the pole piece, the small gap (high field) side will go into saturation before the large gap (low field) side. This changes the field profile across the magnet which is undesirable since the dipole field shape is an important part of the spectrometer optics. Figure 3.9 shows a series of field profiles for the SASP dipole at different excitations varying from 17000NA to 77000NA. The

\(^1\)one oersted equals \(\frac{4\pi}{10}\) Amp Turns
maximum excitation the dipole is intended to be operated at is 67000NA, although both the coils and the power supplies will be able to go 15% higher. When operating the SASP above the design excitation, saturation will degrade the magnetic field and, therefore, the spectrometer performance.

Figure 3.9 shows that at low excitation, the centre of the magnet is actually under excited. This is because the SASP dipole is an H magnet and for low excitations, the flux stays in the iron near the coils. Similar studies of an existing C style clamshell dipole, the LAMPF Low Energy Pion Spectrometer, or LEPS, (described in appendix D) did not show this effect for two reasons. First, the LEPS dipole is smaller so the flux does not have as far to go. Second, the return yoke is on the large gap side of the dipole. The large gap has a greater magnetic reluctance so flux prefers to travel to the lower reluctance, smaller gap side of the dipole. These two effects will tend to spread out the flux more uniformly than in the SASP dipole. As the excitation in the SASP dipole increases, the flux spreads out into the entire yoke producing the field at 37000NA. This was the excitation at which the pole face slope was adjusted to match the POISSON field profile to the RAYTRACE profile.

As the excitation is increased beyond 37000NA, saturation begins to alter the field profile and is quite noticeable (0.979% deviation from ideal) at 67000NA — the maximum design excitation. The POISSON field is scaled to be exactly equal to the RAYTRACE field at the central radius, which is on the high field, small gap side of the dipole. This makes the POISSON field appear high at the large gap side when in fact the field is low at the small gap side.

This saturation effect is potentially disastrous. Intuitively one would think that a 1% change in the dipole's field at 67000NA would destroy the spectrometer’s resolution. POISSON simulations using steels with better B-H curves were tried with only small improvements in the field profiles. The saturation effect is a fundamental problem
Figure 3.9: The SASP dipole internal field profile as a function of magnet excitation. Starting from the upper left, the excitations are: 17000NA, 27000NA, 37000NA, 47000NA, 57000NA, 67000NA, and 77000NA.
CHAPTER 3. SASP DIPOLE

with a clamshell dipole. The best that can be done is to adjust the pole face slope to minimize the distortion at one excitation which in turn limits the operating range of the spectrometer.

To determine the effect of the field sag on resolution, it is parameterized as a distortion to the ideal field (changing the notation from equation 3.1),

\[ B_y = \frac{B_0}{1 + n_x} \times CF(x) \]

where the correction factor \( CF \) is equal to 1 at \( x = 0 \),

\[ CF(x) = 1 + bx + cx^2 + dx^3 + ex^4 \]

This distorts the field uniformly over the length of the dipole and does not take into account the various effects due to the position of the return yokes. The 67000NA curves were used to study the distortion effects.

The only noticeable effect of the distortion on the spectrometer's characteristics was a shift of the focal surface a few centimeters further away from the dipole exit — the resolution was essentially unaffected. However, as discussed in section 2.2, part of the spectrometer's momentum determination is done using software corrections. The software corrections are very sensitive to the magnetic optics and the saturation effects will significantly alter them. This will have a major effect on the spectrometer's operating procedure. Because the optical properties and software corrections change with excitation, it will be necessary to have a new calibration for each setting of the spectrometer. Care is required to obtain these calibrations since the standard method of "stepping" the peak of some easily resolved reaction such as \( pp \to d\pi^+ \) across the focal surface by changing the spectrometer setting will change the optics and the focal surface being mapped.
As part of the study of the SASP dipole's internal field, it would have been useful to generate excitation (B-H) curves. However, attempts to replicate the measured excitation curve of the LEPS dipole (appendix D) failed. An excitation curve is a measure of both the local saturation of the dipole steel and of the flux "leaking" out of the yoke. The situation is analogous to a parallel resistance problem in an electric circuit. It becomes a question of whether the flux prefers to travel through the steel or the surrounding air. Both the SASP and LEPS dipoles are complicated three dimensional shapes with many possible paths for the flux. Trying to model these dipoles with two dimensional cross sections does not capture the nature of the problem. Simulations with the LEPS showed that one could replicate the true excitation curve of the magnet by enlarging the return yokes of the model but this arbitrary procedure cannot then be applied to an unknown magnet such as the SASP dipole.
Chapter 4
Dipole Apertures

In section 3.1 it was explained that the dipole aperture fields are used to correct various aberrations in the SASP optical design. If the aperture fields are not shaped correctly, the performance of the spectrometer will suffer. As with the interior of the dipole, the RAYTRACE optics design only produced a specification of how the aperture fields look — not the steel geometry. The specifications are given as sets of parameters in table 3.1 which describe the Virtual Field Boundary (VFB) and fringe field shapes for each aperture. These descriptions of the fringe fields are used to create the following engineering specifications:

1. The Pole Edge Boundary (PEB).
2. The Pole Edge Bevel.
3. The Field Clamp Boundary (FCB).

Although these items are not completely independent of one another, their effects on the aperture fringe fields can be roughly separated as follows. The pole edge boundary (PEB) is the curve the dipole pole piece follows. It determines the shape of the VFB which it is similar in shape to. The pole edge bevel is the rounding of the edge of the pole piece and shapes the upper part of the fringe field. The bevel also helps determine the behaviour of the fringe field with changing dipole excitation. The field
clamp boundary (FCB) describes the shape of the field clamp, which is a device placed near the aperture of the magnet to help shape the fringe field. While it can effect the field shape in several different ways, the most important function of a field clamp is to limit the extent of the fringe field; it "clamps" the field in. The main parameter for limiting the extent of the fringe field is the gap between the clamp and the PEB. Like the PEB, the FCB will depend on the VFB and on the PEB as well.

An engineering design has more requirements than meeting the RAYTRACE fringe field specification. The system must be feasible to build. For example, the best position for the field clamp conflicts with the dipole coils on one side and the dipole vacuum box on the other. The final relationship between the clamp, coil, and vacuum box is a compromise. The final specification for the dipole apertures and fringe fields may not be ideal, but it is the best practical design.

This work does not provide an engineering design for the SASP dipole apertures, but rather an engineering specification (appendix F). There are some aspects of the geometry that must be made exactly as specified (such as the three items listed above), there are other aspects that can be altered as necessary to develop a practical design. For example, constriction of the field clamp by the coil and vacuum box is resolved by having the clamp sandwiched between the coil and box with no clearance. This is impractical to build and either the coil and vacuum box will have to be moved slightly or the clamp made thinner.

4.1 Fringe Fields

Consider a dipole as a magnetic circuit. This is analogous to an electrical circuit: the magnet coils create a magnetomotive force (voltage) that induces flux (current) to flow through through the magnet's yoke and air gap which have a certain reluctance (resistance) that impedes the flow of flux (current). Like current in the electric circuit,
the flux is conserved around the magnetic circuit, but unlike the electric circuit, the flux can flow through the air surrounding the magnet as well as through the yoke. Also, unlike the resistance in the electric circuit, the reluctance of the steel is nonlinear. Generally the flux prefers to flow through the iron in the magnetic circuit but is forced to jump across the dipole's air gap. Crossing the air gap, the flux tends to bulge out producing a fringe field that drops off further from the magnet. This is the fringe field associated with a dipole aperture. The shape and extent of this field is determined by the shape of the pole edge, the size of the air gap, and the excitation of the magnet. Enge [9] produced a function that can describe the fringe field on the median plane where \( \vec{B} = (0, B_y, 0) \).

\[
B_y = \frac{B_0}{1 + e^{S(s)}}
\]

with,

\[
S(s) = c_0 + c_1 s + c_2 s^2 + c_3 s^3 + c_4 s^4 + c_5 s^5
\]

and \( s = z/D \), \( D \) is the width of the air gap, and \( z \) is the distance from the Virtual Field Boundary (VFB). For a fictitious magnet with a homogeneous internal field and no fringe field (it drops abruptly from the interior field value to zero at the VFB) the virtual field boundary lies at the field drop off. Such a magnet is said to have a Sharp Cut Off Fringe Field (SCOFF). An actual magnet has an Extended Fringe Field (EFF) and the VFB lies at the point where the integral of a SCOFF \( B \) field equals the total integral of the EFF \( B \) field,

\[
\int_{-\infty}^{\infty} B_{EFF}(s) ds = \int_{-\infty}^{VFB} B_{SCOFF} ds
\]

where \( \infty \) is far outside and \( -\infty \) far inside the magnet.

The angle a ray (particle) is deflected by a bending magnet is equal to its angular
Figure 4.1: The effect of a fringe field is to displace the trajectory and alter the bend angle of a ray.
velocity (caused by the $B$ field) integrated over time,

$$\theta = \int_0^t \omega dt$$

Converting from a time integral to a position integral,

$$\frac{dl}{dt} = v$$

and,

$$dt = \frac{dl}{v}$$

since the speed of the particle is constant in a purely magnetic field (i.e. no electric field). The angular velocity is given by the cyclotron frequency,

$$\omega = \frac{qB(l)}{m}$$

where $B(l)$ is perpendicular to the bend plane of the particle and varies with position. The angular deflection of the particle/ray then reduces to:

$$\theta = \int_0^l \frac{qB(l)}{mv} dl$$

or,

$$\theta = \frac{q}{p} \int_0^l B(l) dl \quad (4.2)$$

with $p$ being the momentum of the particle. Thus the angular deflection of the ray is proportional to the integral of $Bdl$. Suppose a bending magnet was designed assuming a sharp cut off fringe field (SCOFF), a homogeneous internal field and curved virtual field boundaries (figure 4.1). Tracing a particle (whose trajectory is called the SCOFF
ray) through the magnet, the path indicated with the solid line in figure 4.1 is made. Up to the VFB, the particle trajectory is straight. Entering the magnet, the particle travels along a circular trajectory of radius $R$ with centre $C_{\text{SCOFF}}$. Reaching the exit VFB, the particle resumes a straight trajectory. The total angle the particle (ray) is deflected through is

$$\theta_{\text{SCOFF}} = B_0 l_{\text{SCOFF}}$$

where $l_{\text{SCOFF}}$ is the length of the arc inside the magnet. Now suppose the magnet has an extended fringe field (EFF) extending a distance $\pm d$ from the VFB. The EFF ray enters the fringe field $d$ before the VFB and starts to deflect. The rate of deflection increases with increasing $B$ field until the particle reaches the homogeneous field where it has the same deflection (or radius of curvature) as the SCOFF case. Because the EFF ray started deflecting before the SCOFF ray, it is displaced from the SCOFF ray and will have a different centre of curvature, $C_{\text{EFF}}$. Reaching the exit, the EFF ray passes through the fringe field with gradually decreasing deflection and finally leaves the magnet displaced from the SCOFF ray. The curvatures of the magnet VFBs are such that the EFF ray has travelled a shorter distance through the magnet with a smaller $Bdl$ and therefore,

$$\theta_{\text{EFF}} < \theta_{\text{SCOFF}}$$

This result is dependent on the position and angle of the ray entering the magnet. An EFF ray entering the magnet on the opposite side of the aperture in figure 4.1 could leave with a greater deflection than the SCOFF ray. The result is the fringe field introduces aberrations to the optical system of the form,

$$(x/x'\theta^j)$$
where \(i, j\) are 0, 1, 2, \ldots. If the magneto optics are designed without consideration of these aberrations, the performance of the system will likely be adversely affected. In practice the fringe fields are included in the calculation; RAYTRACE describes fringe fields with parameters the user inputs to the program (see section 3.2). However, if the fringe field shape assumed in the optical design does not agree with real shape, aberrations will again appear. Some magnetic systems will be more sensitive to the fringe fields than others. For instance, the SASP’s resolution proved insensitive to small changes in the dipole fringe field shape. Such a tolerance to error will make the SASP easier to construct.

The RAYTRACE fringe field model assumes that the field at each point in the aperture is determined by the Enge function which is a function of the distance to the closest point on the aperture VFB. This model basically assumes that the field is unaffected by the aperture’s curvature and if this is not accurate, effects similar to the ones discussed in the SCOFF v.s. EFF magnet will be encountered. The fringe fields of a real magnet can differ from the model in one of two ways: as constructed, the magnet’s aperture fringe fields do not conform to the desired shape; or the fringe fields change shape with magnet excitation. Both effects can be present. The aperture field not conforming to the desired shape will affect the spectrometer performance regardless of the excitation it is being operated at while an excitation dependent fringe field will cause the spectrometer’s characteristics to change with excitation. It is already known that the SASP’s characteristics will have an excitation dependence caused by the saturation of the dipole's interior field (section 3.3.4), but the addition of aperture effects would aggravate the situation.

Both virtual field boundaries on the SASP dipole are concave and are thus susceptible to an effect called aperture filling. Consider a homogeneous field magnet with
an aperture whose VFB is a circular segment. Ideally the lines of constant field in the aperture should be circular and concentric with the VFB (and pole edge) as shown in figure 4.2a. For the circular pole edge however, flux moving straight out from the pole edge will interact with flux from other parts of the pole causing the fringe field to bulge (figure 4.2b). To understand this, take a straight VFB where, by symmetry, the lines of constant B field are parallel to the pole edge. If the straight VFB is bent into a circular arc, the length of the boundary and the total amount of fringe field flux associated with it remains constant but the space the flux has to pass through outside the magnet decreases. The flux “bunches up” which increases the local magnetic field. The contour lines will bulge out from the pole edge, the aperture appears to “fill in” with flux, and the VFB no longer follows the pole edge. The flattening of the contour lines decreases the effective curvature (i.e. increases the radius of curvature) of the VFB (figure 4.2b). The ratio of the extent of the fringe field and the radius of curvature of the aperture is given as a measure of this effect. If the ratio is small, the aperture filling is also negligible while if the ratio is large, the filling is significant. This is because the field at a point in the aperture (“in the aperture” meaning the space just outside the magnet) is effected only by the pole edge within about one fringe field extent of it. As the ratio of extent to radius decreases, a point in the aperture “sees” less of the pole edge and the fringe field becomes more like that of a straight boundary.

A related type of aperture filling is caused by the pole piece’s finite size. Consider the fringe fields of a homogeneous magnet with a rectangular pole piece. If the magnet had no fringe fields, a contour plot of the magnetic field would parallel the pole edge, the field would jump from zero outside the magnet to the internal field value at the pole edge. However, an EFF magnet behaves differently. Near the centre of each face of the magnet, the field contours follow the pole edge, but at the corners of the magnet, the contour lines must make a 90° turn. The contour lines do not form a sharp right
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Figure 4.2: Aperture Filling. a) An ideal fringe field. b) Local aperture filling. c) Magnet edge effects. d) Edge effects on the aperture field.
angle, rather they make a gentle curve about the same size as the extent of the fringe field itself. Thus the corner of the magnet will perturb the fringe field along the pole edges. The larger the fringe field, the further from the corners the effect will be seen. If the fringe field extent is comparable in size to the sides of the magnet, the fringe field will appear to bulge without following the pole edge (figure 4.2c). In reality the lines of constant field are curving in at the corners rather than out at the centre of the pole edge. The effect of the finite dipole on a concave aperture is shown in figure 4.2d. This type of problem is always present to a degree since the aperture curvature cannot continue indefinitely. It is necessary to design the magnet apertures sufficiently wide to accept the particle beam. This keeps the edge of the aperture with its poor field clear of the particle. Both types of aperture filling can be lessened by minimizing the extent of the fringe field.

The problems discussed above are caused by the geometry of the magnet; the effects of magnet excitation are also important. Suppose the aperture has an acceptable fringe field at some low excitation for which none of the magnet steel is saturating. If the excitation is lowered, the field shape remains constant. But when the excitation is increased to put the magnet steel into saturation, the fringe field will change if the aperture is not correctly designed. If the magnet's pole edge bevel is improperly designed, the fringe field and the VFB will shift outwards as the magnet goes into saturation (see section 4.4). If the extent of the fringe field is a sizable fraction of the aperture radius, the aperture will fill in (figure 4.3a). Alternately, if the ratio of field extent to radius of curvature is small, the VFB will shift uniformly out from the aperture. This decreases the radius of curvature of the VFB (figure 4.3b). The edge effects of the finite sized dipole will compound the problem.
4.2 The SASP Dipole’s Apertures

Before specifying the steel geometry for the dipole apertures, the exact field shapes assumed by RAYTRACE must be known. RAYTRACE uses two attributes to describe the fringe field region of the dipole: the VFB curvature and the fringe field shape. The fringe field shape is described with the Enge function, 4.1, using the six coefficients found in table 3.1 (note both the entrance and exit apertures have the same fringe field shape). The VFB is described with an eighth order polynomial modifying the straight boundaries shown in figure 3.3 ($\frac{1}{R_1} = 0 = \frac{1}{R_2}$, i.e. the unmodified boundaries are straight):

$$ z = -R \sum_{n=2}^{8} S_n \left( \frac{x}{R} \right)^n $$

$$ R = 220,000.0cm $$
where $z$ and $x$ refer to the local coordinate systems at the entrance (B) and exit (C) shown in figure 3.3. The $S_n$ values used for the SASP dipole are given in table 3.1. Different VFB curvatures are used for the entrance and exit apertures. It is important to understand the regions where these polynomials are valid. Aberrations in the spectrometer optics are corrected for by using a "ray bundle" consisting of 198 particles injected randomly into the SASP's aperture. The extreme trajectories of the bundle in the dipole bend plane are shown in figure 3.5. The rays cross the dipole's entrance aperture between $x = -28\,\text{cm}$ and $x = +32\,\text{cm}$ using coordinate system B (figure 3.3). The rays cross the exit aperture in the region $x = -45\,\text{cm}$ to $x = +65\,\text{cm}$. The SASP's optics are thus only optimized for rays that enter and leave the dipole over these regions. Rays crossing the dipole VFBs outside the "good" regions will probably experience aberrations that will reduce the spectrometer's resolution. The term high resolution rays refers to rays that enter the SASP dipole over the good VFB regions. Rays that enter the dipole outside these regions are called low resolution rays. The polynomials describing the VFBs are only valid for the high resolution ray regions. For values of $x$ outside the high resolution regions, the polynomials become unrealistic and predict VFBs that are impossible to construct. This can be seen for the exit VFB shown in figure 4.4: the low end of the valid range is $x = -60\,\text{cm}$ and at $-100\,\text{cm}$ the $z_{VFB} = -40\,\text{cm}$ while at $x = -200\,\text{cm}$, it drops to $z_{VFB} = -3484.5\,\text{cm}$.

The concept of a virtual field boundary is obvious for a constant gap magnet with its homogeneous field (section 4.1) but not so obvious for the SASP's clamshell. Knowing the VFB definition, the Enge function (equation 4.1) is constructed so that $s = 0$ lies on the VFB of the homogeneous field magnet. To describe the fringe fields of a constant gap magnet, RAYTRACE calculates the position of the VFB and then uses the Enge function. RAYTRACE follows the same procedure for a nonhomogeneous field magnet. Ignoring aperture filling, it might be expected that the VFB is still the
Figure 4.4: Dipole Aperture Fringe Fields assumed by RAYTRACE. The lines are contours of constant B field while the points show the VFB. The fringe fields near the VFB give way to the straight internal field contours. a) Entrance b) Exit
point where the total area under a SCOFF curve equals the area under the EFF but this is not so. The Enge function can be interpreted as a factor,

\[ \frac{1}{1 + e^{s(s)}} \]

(varying from unity inside the magnet to zero outside) that modifies what the magnetic field would be at the point, \( s \), if the pole face extended indefinitely without interruption by the aperture,

\[ B_y(s) = \frac{B_0(s)}{1 + e^{s(s)}} \] (4.4)

where \( B_0(s) \) is a constant for a homogeneous field magnet. Figure 3.3 shows that approaching either of the SASP dipole apertures from inside the magnet along a direction perpendicular to the VFB, the air gap narrows causing the internal field to rise. If the aperture was not there, the field would continue to rise “outside” the VFB. Thus both the denominator and the numerator of equation 4.4 are increasing. The exponential in the denominator will eventually dominate but the fringe field of the sloped pole face magnet will drop off more slowly than the fringe field of the constant gap magnet. The integral of the resulting fringe field is greater than that of a SCOFF curve ending at the “VFB”. Thus the defined VFB lies further from the magnet than the specified VFB.

Another subtlety of the fringe fields of an nonhomogeneous magnet is that based on the assumption the Enge function is correct, the extent of the fringe field varies as a function of the dipole gap width. Equation 4.1 assumes that the fringe field extent is independent of the internal field, \( B_0 \), and only depends on gap width, \( D \). This means that for a homogeneous magnet, the width of the fringe field region is the same regardless of the excitation. This should hold true for a magnet operating below saturation. The nonhomogeneous field magnet has a changing air gap over the extent of the aperture and since the extent of the fringe field is determined by \( s = z/D \), the
fringe field region will be wider for the large gap (low field) side of the aperture. For the SASP, the gap varies from approximately 9cm to 11cm over the apertures and is 10cm where the optical axis crosses the VFB.

Figure 4.4 shows contour plots of the RAYTRACE aperture fields. The plots are in the local aperture coordinate systems $B$ and $C$ (figure 3.3). For the entrance plot, the increasing gap (decreasing field) is to the left; for the exit plot, the increasing gap is to the right. These plots show the bulging of the fringe field caused by the changing gap width. The straight contours at the bottom of each plot show the internal field whose contours are parallel to the lines of constant gap. Also marked as a set of points are the VFBs.

4.3 Pole Edge Boundary

Given the VFBs for the SASP, the Pole Edge Boundaries (PEBs) that produce them must be specified. Consider a beveled pole edge on a constant gap magnet. The flat pole face gives way to a bevel consisting of one or more angled cuts which give way to the usually vertical side of the pole piece. It is arbitrary which point is defined as the Pole Edge (PE) although it is convenient to use some easily recognized point such as the beginning of the bevel or the side of the pole piece. For this work the vertical side of the pole piece will be called the pole edge. Depending on the the dipole gap width and the shape of the bevel, the VFB can lie inside, outside, or at the pole edge. The distance from the VFB to the PE is called VFBPE; it is positive if PEB is outside VFB.

For a constant gap magnet, the fringe field, and therefore VFBPE, is constant for the entire aperture (ignoring any interactions between parts of the aperture discussed in section 4.1). Thus once the virtual field boundary shape and VFBPE are known, the pole edge boundary can be calculated. The procedure is to take points on the
VFB, draw perpendiculars to the tangent, move the distance VFBPE, and mark the points on the PEB. Repeating this, a line can be fitted through the PEB points and an analytic function produced for the PEB like the one used for the VFB (equation 4.3).

For the clamshell magnet, VFBPE is not constant over the aperture. This is because, even though the bevel is the same over the entire aperture, the air gap width and the apparent slope of the pole face along a perpendicular to the VFB changes with position on the aperture. The change in the apparent slope of the pole face can be understood by examining figure 4.4. The greatest apparent slope is where the line of travel is perpendicular to the gap lines — this is the true slope of dipole pole faces (m=0.0298 for the SASP). Alternately, if the line of travel is parallel to the lines of constant gap, the gap remains constant and the apparent slope is zero. As can be seen from figure 4.4, the angle the line of travel makes with the lines of constant gap is dependent on the where it crosses the VFB and thus the apparent pole face slope will change over the aperture.

The effect of the changing pole face slope is a changing VFBPE (the VFB – PE distance) over the extent of the aperture. To determine points on the pole edge boundary, VFBPE must be measured at several points on the aperture and parameterized with some function. This function can then be used like the constant VFBPE was for the homogeneous field magnet; points on the VFB are taken, normals to the VFB drawn, and the corresponding points on the PEB marked at the distance VFBPE along the normals. The PEB can be described with a polynomial similar to that used for the VFB.

VFBPE is calculated at a point on the VFB using a POISSON model of a cross section of the magnet aperture perpendicular to the VFB. Taking a point on the VFB, the gap width and the pole face slope along the perpendicular to the VFB is fixed by the dipole geometry. Keeping the pole edge bevel constant and ignoring the presence
of a field clamp, the only parameter in the system is VFBPE (figure 4.5a). The field profile looks approximately like the curves shown in figure 4.5b. The SASP geometry is such that the dipole gap decreases and the field rises as the apertures are approached from inside the magnet. Near the VFB the fringe field begins to dominate and the field reaches a peak value and then drops off outside the magnet. The shape of the peak and its position with respect to the VFB are determined by the pole edge bevel (discussed below). The peak is located between the pole edge and the VFB. VFBPE is used to match the internal fields and peaks of the POISSON and RAYTRACE fields. The field clamp does not affect this process because it only alters the fringe field profile.

The internal RAYTRACE field along the perpendicular to the VFB can be described by the expression,

\[ B_{RT} = \frac{1}{1 - cs} \]  

(4.5)

where the field is normalized to 1 at the VFB, \( s \) is the distance from the VFB (positive is out from the magnet), and \( c \) is a positive constant. This equation is a simplified version of equation 3.1 which describes the internal field of the dipole along a line perpendicular to the lines of constant gap. Based on the VFB - PE distance being displaced from the correct value, VFBPE, by a small amount, \( d \). The POISSON field is given by,

\[ B_p = \frac{1}{1 - c(t - [d + VFBPE])} \]

where \( t \) is the distance from the dipole pole edge. The POISSON field profile will only match the RAYTRACE profile if,

\[ t = s + VFBPE \]

or \( d = 0 \). If \( d > 0 \), the pole edge extends too far into the dipole aperture and the
Figure 4.5: Determining VFBPE and thus the Pole Edge Boundary (PEB). a) The VFBPE distance is changed so the POISSON model's internal field matches the RAY-TRACE internal field. b) Varying VFBPE changes the internal field profile (these curves are illustrative only).
POISSON field, $B_P(t)$, will drop off faster than the RAYTRACE field, $B_{RT}(s)$, as shown in figure 4.5 case 3. If $d < 0$, then the VFB – pole edge distance is too small and the POISSON field will drop off more slowly than the RAYTRACE field (figure 4.5 case 1). When $d = 0$, the POISSON and RAYTRACE field profiles match. Note that the magnitudes of the fields at the VFB are unimportant. Because the fields are described by an inverse relationship, the field profile is completely specified by one point on the curve which in this case is the virtual field boundary. The actual comparison of the POISSON and RAYTRACE curves is made by first rescaling the curves so that the peaks have field values of 1.0. Then the curves are shifted so that the peaks are superimposed and the internal fields compared.

### 4.4 Pole Edge Bevels

The flux in a magnetic circuit tends to concentrate at sharp points such as the corner of a square edged pole piece. This concentration increases the local magnetic field in the iron causing it to saturate before the rest of the pole piece. As the excitation increases, flux will "leak" out from the saturated corner causing the field to bulge out from the pole edge.

Braams [11] proposed a pole edge for dipoles that would prevent saturation effects. The so called Rogowski pole edge (suggested by Rogowski [12] for sparkless electrodes) is assumed to be an infinite cylinder which follows an equipotential surface of the magnetic scalar potential. For an air gap $D$ the shape of the pole is given by

$$z = \frac{D}{\pi} \phi$$

$$y = \pm \frac{D}{\pi} \left( \frac{\pi}{2} + e^\phi \right)$$
where $z$ is perpendicular to the pole edge in the median plane and $y$ is normal to the median plane. $\phi$ can be eliminated,

$$y = \pm D \left( \frac{1}{2} + \frac{1}{\pi} e^{\frac{z}{D}} \right)$$ \hspace{1cm} (4.6)

where the two solutions for $y$ represent the two pole pieces of the dipole. Inside the pole piece, the flux density (B field) is constant. When the pole goes into saturation, the entire tip will saturate at the same rate and the field profile will remain unchanged, although the magnitude of the field will no longer increase linearly with excitation. The Rogowski bevel produces a more extended fringe field than a square edged magnet, but when a magnet is to be operated over a range of excitations, it is more important to ensure a consistent field profile than a sharp cut off at low excitations. Figure 4.6 shows the ideal Rogowski bevel compared with the bevel that will be used on the SASP dipole. Since the SASP dipole's gap changes over the extent of the aperture, the bevel should change as well. To machine such a complex, smooth bevel would be quite difficult and expensive. In fact, it is adequate to approximate the Rogowski bevel with two simple cuts that remain constant over the extent of the aperture (the solid line in figure 4.6). The exact procedure for shaping the bevel is described in appendix F.1.
Figure 4.6: Rogowski pole edge bevel. Dashed line is the ideal bevel and the solid line is the actual bevel used on the SASP. The axes show the coordinate system used in equation 4.6.
4.5 Field Clamps

4.5.1 Modifying Fringe Fields

Often, the fringe field produced by the curvature and bevel of a dipole magnet is unsatisfactory. This could be due to the problems discussed in section 4.1 or perhaps the spectrometer optics behave differently than expected and the fringe fields must be modified to improve the spectrometer performance. To this end, there are two devices (having more or less opposite effects on the fringe field) that can be used to alter the aperture fields.

A snake [13] consists of two laminated soft iron bars suspended near and following the curvature of the pole edge (figure 4.7a). It has the effect of bulging the fringe field outward (figure 4.7b) which brings the VFB out as well. The snake does this by acting as a “stepping stone” for flux to cross the air gap. Flux can also flow along the snake from one part of the aperture to another because the surface of the snake forms an equipotential, linking together the entire aperture. The main use for snakes is to fine tune the shape of a virtual field boundary. The snake is mounted approximately in place and then distorted to produce the correct VFB. The laminations have the dual role of reducing eddy currents and hysteresis in the snake and making it flexible. This flexibility greatly simplifies fitting the snake to the aperture.

As discussed in section 4.1, concave apertures, such as the SASP’s, usually have the problem of fringe fields that are too extended. To correct for this, a device called a field clamp is employed. A field clamp (proposed in 1934 by Herzog [14]) consists of a rectangular iron tube which wraps around the magnet aperture (figure 4.8). The field clamp picks up flux that would otherwise extend into the aperture and then “short circuits” or shunts it around the edges of the aperture through the side plates, or return yokes, of the clamp. This decreases the extent of the fringe field and the effects
Figure 4.7: A snake is an iron strip suspended near the aperture of a magnet (a). It has the effect of "pulling" out the fringe field and the VFB (b).
Figure 4.8: A field clamp is an iron tube that wraps around the aperture of a magnet, shunting flux around the aperture through the sides of the clamp.

of saturation. The extent of the fringe field is approximately determined by the distance from the pole edge to the field clamp. By adjusting this distance over the extent of the aperture, the VFB shape can be altered. Unlike a snake however, the shape of the clamp can not easily be changed making it difficult to "fine tune" the fringe field.

An unclamped magnet can suffer from an effect called field reversal caused when the magnet coils are close to the aperture (such as on the SASP). The field reversal bends the particles in the opposite direction to that in the dipole and thus reduces the total $B_{dl}$. The effect this has on the SASP's resolution was not studied. Field reversal occurs as the fringe field caused by the pole piece drops off far from the pole edge and the coil field, which is opposite to the dipole field, dominates. This produces a reversed field that decays to zero further from the aperture. Simulations show that dependent on the geometry, this reversal can be 10% of the internal field and extend for several gap widths. Field reversal is completely eliminated by a field clamp which shunts both
the pole piece and coil flux around the aperture.

4.5.2 Modelling a Field Clamp

There are two basic methods of modelling a field clamp. The most accurate is to build a scale model of the clamp and the magnet whose field it is to modify. Hubner and Wollnik [15] performed a series of experiments with such a model studying the clamp characteristics. It would be difficult to build a useful model for the SASP dipole. The probe used to map the fields must be able to get into the air gap of the model. It would probably be necessary to build a $\frac{1}{4}$ or $\frac{1}{3}$ scale model which is almost large enough to be used as a separate spectrometer. The second method of studying a field clamp is with the aid of a computer simulation which was done for this work. The advantage of a computer simulation over constructing a scale model is that it is considerably less expensive and the geometry can be altered quickly and easily. The main disadvantage of using a computer simulation to study the dipole apertures is that only a two dimensional simulation program was available to model the three dimensional apertures, thus it is not feasible to study effects like aperture filling.

Ideally the field clamp should be modelled on the computer using a three dimensional magnetostatic code. No suitable program was available so the 2D code, POISSON (appendix C), with which the interior of the SASP dipole was studied (section 3.3), was used. A field clamp is a three dimensional object that fits into a curved asymmetric aperture, making it difficult to model using a two dimensional code like POISSON. The model is limited to slices of the clamp-dipole system which do not show how different parts of the aperture interact. Nor does it show how the flux moves through the return yokes of the clamp when investigating saturation.

A field clamp is modelled in two dimensions using a cross section with a "tail" on the end away from the dipole to connect the two halves of the clamp. This provides
the short circuit for the flux to cross the magnet air gap. The tail has to be situated far enough from the aperture to prevent it from influencing the magnetic field. An actual field clamp shaped like this would be useless since it would block the aperture. Figure 4.9 shows a typical configuration modelled in POISSON and the effect the clamp has on the fringe field. The dipole is modelled as a C magnet with the pole face sufficiently wide to ensure a homogeneous (flat) interior field. Modelling the clamp and dipole in this manner can be misleading. The presence of the clamp can appear to decrease the internal field of the dipole by 10% or more because the simulated field clamp provides a flux path which is a significant fraction of the size of the dipole return yoke. For an actual magnet, the clamp should not have a perceptible effect on the dipole’s internal field. Therefore this model will not quantitatively predict what excitation the clamp and dipole saturate at.

Figure 4.10 shows the relationship between the dipole, clamp and coil. The effect of the field clamp on the fringe field cannot be isolated from the effects of the dipole gap, the shape of the pole bevel, and the size and position of the coil. The main parameters in the system are:

- \( D \) The magnet gap width.
- \( D_F \) The field clamp gap width.
- \( S_F \) The field clamp – pole edge separation.
- \( S_C \) The coil – pole edge separation.
- \( S_{CF} \) The coil – field clamp separation.
- \( t_F \) The field clamp thickness.

Numerous POISSON simulations changing one or more parameters were run to determine their effects on the fringe fields. Most of the simulations used a square edged magnet rather than a beveled magnet to simplify the system.
Figure 4.9: Modelling a Field Clamp in POISSON. a) The POISSON geometry used to simulate a field clamp on a homogeneous field dipole — note the flux lines travelling through the field clamp rather than through the air gap. b) The magnetic field on the median plane of the dipole as calculated by POISSON without (dashed) and with (solid) the field clamp. The distance is measured in terms of gap widths from the pole edge.
4.5.2.1 Clamp Position ($S_F$ and $D_F$)

The width of the field clamp, $D_F$, tends to change the shape of the lower half of the fringe field curve. Decreasing $D_F$ raises the outer fringe field. This is because the clamp concentrates flux from the aperture at its tip. As the clamp is brought closer to the median plane, this flux increases the local B field. This effect is small and $D_F$ can be set arbitrarily. In the SASP a more important consideration than shaping the lower fringe field is that the clamp be above the plane of dipole pole face to allow rays to enter the dipole. A value for $D_F$ that meets this requirement is:

$$\frac{D_F}{D} = 1.4$$

$S_F$ is the main parameter for altering the field shape and varying the VFB position. For large values of $S_F$, the field clamp does not interact with the dipole. As $S_F$
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Figure 4.11: Fringe field for different values of $S_F$. There is a value of $S_F$ for which the field in maximally clamped (solid line). For smaller values of $S_F$ a step field builds up under the clamp (broken lines).

decreases, the clamp picks up flux from the air gap and the VFB shifts in. The field eventually reaches a point of “maximal clamping”. For smaller values of $S_F$, a residual “step” field builds up under the clamp (figure 4.11). For a square edged magnet with $D_F = 1.4D$ the maximal clamping is achieved with,

$$\frac{S_F}{D} \approx 0.6$$

Figure 4.12 shows the variation of the fringe field for a clamped magnet as a function of the dipole gap width, D, with the ratios $S_F/D$ and $D_F/D$ kept constant. The fringe field is adequately described by the Enge function (equation 4.1) which predicts that the field shape scales exactly as the dipole gap width, $D$. This indicates that the $D_F/D$ ratio must be kept constant when investigating the SASP dipole apertures with their changing gap widths.
Figure 4.12: The variation of the fringe field with D for constant ratios of $D_F/D$ and $S_F/D$.

4.5.2.2 Coil Position ($S_C$ and $S_{CF}$)

The coils for a magnet as large as the SASP dipole are difficult to construct according to design. The copper windings are wrapped against a form and encased with an epoxy which acts as a binding agent and insulator. Portions of the coil have a tendency to spring when released from the form. Movements of a few millimetres are to be expected. Thus it is important to know if the magnet’s fringe fields are significantly effected by the coil position and if so, how large an error in positioning can be tolerated. Simulations showed that for both the clamped and unclamped magnet, the VFB varied linearly with $S_C$. A 1cm shift of the coil away from the pole edge, shifted the VFB out 1.3mm for the unclamped magnet and 0.2mm for the clamped. For the vertical positioning of the coil, the field clamp coil separation, $S_{CF}$ does not significantly affect the fringe field. Shifting the coil from 1cm above the field clamp to 5cm above the field clamp, the VFB shifted 1.5mm outwards. Thus, coil positioning errors of the order of a few millimetres can be tolerated for the clamped magnet.
4.5.2.3 Clamp Thickness ($t_F$) and Return Yokes

The more flux the clamp is required to carry, the thicker it must be to prevent saturation. Figure 4.13 shows the fields produced by a 2cm thick clamp, a 5cm thick clamp and two clamps that change thickness from 2cm to 5cm. As expected, the 2cm clamp does not clamp the field as well as the 5cm one. The clamps with the 2cm "tongues" extending under the coil produce almost the same field as the straight 2cm clamp. This indicates that the important part of the clamp is the tip under the coil. The tongue has to carry both the flux from the dipole and that which is produced by the proximity of the coil. It is driven close to saturation by the coil and acts as a bottle neck limiting the amount of flux that can be drawn off the dipole. Therefore the clamp should be as thick as is practical.

Related to the clamp thickness is the size of the return yoke. This is difficult to model in two dimensions since unlike the POISSON models with their “tails”, the clamp yokes collect flux from over the entire aperture. Flux leaves the pole piece and enters the clamp perpendicular to the pole edge but then must turn to the sides and flow around the aperture through the yokes, back into the other half of the clamp and across the clamp – dipole gap into the other pole piece. The centre of the clamp has the smallest flux, but this accumulates towards the edges until the highest flux densities are reached at the return yoke. The return yoke must be thick enough to accept this flux without saturating. If it is important to keep the total amount of steel in the clamp to a minimum (cost or weight considerations say) but still maintain full clamping, the clamp should be made thin in the centre, thickening towards the edges. The final thickness of the clamp at the edge should be used for the return yoke as well. In practice the clamp weight is not a constraint and it is much simpler to make it all the same thickness.

A simplistic method of estimating the clamp thickness is to determine the B field
Figure 4.13: Several different clamp configurations. A 2cm thick clamp, a 5cm thick clamp, and two that are 5cm thick with a 2cm tongue under the coil. The points show the desired field contour and are the same for all four cases.
in the clamp model using POISSON and then multiply this field by the total width of
the aperture to determine the total flux that must pass through the two return yokes.
The return yokes are then made wide enough to accept the flux while keeping the flux
density (B field) below the saturation of the iron (approximately 15 kG).

If the return yokes are too small, the steel will saturate and flux will leak back
into the aperture. This prediction is confirmed from the studies of the LEPS dipole in
appendix D. There, a field clamp with only one return yoke produced a clamped field
with a bulge in it. The inadequate return path produced an effect somewhere between
a field clamp and a snake. If this problem is encountered on the SASP, more material
can be added to the clamp, so room to add additional material should be provided.

One might suppose that rather than building a field clamp that shunts flux around
the aperture, the clamp could be connected to the dipole return yoke (figure 4.14a).
This would still collect flux from the aperture with the advantage of a shorter route
to leave the clamp. Hence the clamp could be made thinner and occupy less space in
the dipole aperture. However, this is equivalent to creating a second pole piece on the
other side of the coil so that the flux flows in the opposite direction to that in the main
dipole air gap producing a large field reversal (figure 4.14b).

4.5.3 Comparing Computer and Scale Models

The POISSON results can be compared qualitatively to the scale model studies of
Hubner and Wollnik [15]. This comparison is qualitative because the scale model
studies placed the clamp closer to the median plane \((D_F \approx 0.625D\) compared to \(1.4D\)
for the computer studies) which seems to change the behavior of the clamp. Like the
POISSON studies, a residual field step was observed for small values of \(S_F\) although
the value was much smaller \((\leq 0.35D\) compared to \(\approx 0.6D\) for the POISSON studies).
Different from the POISSON studies, Hubner and Wollnik observed a build up of a
Figure 4.14: Shorting the clamp to the dipole return yoke (a) turns the clamp into a “negative” pole face producing the field reversal shown as a solid line in (b). The desired field is indicated by the dashed line.
residual field with increasing clamp thickness, $t_F$. They explained this as the clamp collecting more flux which then saturated the return yokes. The residual step was observed for $t_F \approx 0.67D$ and $S_F = 0.35D$. This geometry had the clamp much closer to the dipole and median plane than the POISSON studies and does not seem to disagree with simulations.

Also studied with the scale model were the clamp length and return yoke thickness — neither of which can be studied directly using the POISSON model. Hubner and Wollnik concluded that a clamp with length about $3D$ and return yoke thickness $2D$ would perform well. They produced a specification for an optimized clamp for a dipole with rounded edges (their approximation of a Rogowski bevel). The clamp dimensions were: $S_F = 0.83D$, $t_F = 0.5D$, $D_F = 0.83D$, length $3D$, and return yoke thickness $2D$. Note that the clamp is closer to the median plane than the dipole pole piece. Such a geometry is undesirable for the SASP as the clamp would greatly restrict the spectrometer's particle acceptance.

### 4.6 Clamp Design

From computer and scale model studies of the field clamp an ideal clamp for the SASP can be described. The clamp plates should be $0.5D$ thick, set parallel to the pole faces of the dipole at a height of about $0.7D$ above the median plane. The length of the clamp should be at least three gap widths and based on $S_F \approx 1.5D$ this makes the clamps project a total of $4.5D$ from the aperture. The exact clamp – pole edge separation, $S_F$, will be adjusted to make the fringe field follow the Enge function (i.e. scale with the gap width) over the aperture. The return yoke thickness will depend on the estimated flux through the field clamp which in turn depends on the width of the aperture and the excitation of the magnet. The clamp plates are set parallel to the pole face for two reasons. First, this keeps the geometry more or less constant over the
aperture. Second, it is simpler to construct a clamp made from flat plates.

Figure 4.15 contains conceptual drawings of the field clamp. Figure 4.15a shows a cross section taken through pole edge, coil, clamp, and vacuum box perpendicular to the pole edge while figure 4.15b shows a plan view of the aperture and field clamp. The clamp plate is parallel to the pole face but further from the median plane allowing room for the vacuum box. The vacuum box itself is displaced from the pole face plane to allow room for rays to pass from Q2 to the dipole. The dipole coil also lies parallel to the pole face. The coil is displaced 5cm from the pole face. 3cm accommodate the vacuum box and the ray bundle, and 2cm provide for the "tongue" of the field clamp to project under the coil. As discussed earlier, this impedes the effectiveness of the clamp. Beyond the coil, the clamp expands to its full 5cm thickness. The clamp continues parallel to the pole face until the "bend line" where it flares away from the median plane to allow passage of the ray bundle from Q2. The position of the bend line varies over the width of the aperture. This is the line where the clamp can start flaring without hitting the coil as shown in figure 4.15b. While the spectrometer's effective aperture would be increased by flaring the clamp immediately after the coil, this would necessitate a bulging, curved field clamp and the vacuum box would be difficult to construct. The width (in the bend plane) of the clamp is indeterminate. Monte Carlo studies (section 6) indicate that both apertures are fully illuminated by rays. Not all of these may be desirable so determination of the final width of the vacuum box and clamp will be deferred until the engineering design of the SASP is closer to completion.

This field clamp design coupled with the Rogowski pole edge bevel produced a fringe field different from the one used in the original RAYTRACE optical design (the original fringe field was also for a clamped magnet). However, when the new fringe field was parameterized and fed into RAYTRACE, it had no effect on the spectrometer performance. Therefore the desired field was defined to be that produced by the aper-
Figure 4.15: Conceptual field clamp design. Not to scale. a) Cross section perpendicular to the pole edge. b) Plan view of the aperture and field clamp.
ture design at $x_{\text{VFB}} = 0\text{cm}$ and $S_F = 14.0\text{cm}$. Once the PEB (section 4.3) was fitted to the internal RAYTRACE field using the parameter VFBPE, the field clamp parameter $S_F$ was adjusted to produce the desired fringe field over the aperture. $S_F$ was then described as a function of the VFB position. This function was used to produce points on the Field Clamp Boundary (FCB) which, like the PEB, was fitted with a polynomial curve.

4.7 Fitting the Clamp and Pole Edge

4.7.1 Fitting Procedure

As discussed earlier, most of the parameters of the aperture design were fixed (e.g. bevel shape, clamp thickness and displacement from the median plane) leaving only two unknowns: the VFB — pole edge distance (VFBPE); and the field clamp — pole edge separation ($S_F$). These parameters must be adjusted to produce the desired RAYTRACE fringe fields over the extent of the apertures. Since the exact widths of the field clamps have not been established, it is necessary to do this fitting process over a wider range than the field clamps are expected to extend. This results in the following ranges for $x_{\text{VFB}}$ (which are called $x_B$ (entrance) and $x_C$ (exit) in figure 3.3):

- Entrance: $(-50\text{cm},+50\text{cm})$
- Exit: $(-80\text{cm},+100\text{cm})$

The fitting process for each aperture is divided into four steps.

1. Fit VFBPE at several locations on the VFB.

2. Using the above VFBPE values, fit $S_F$ at the same locations.
3. Fit polynomial curves to the VFBPE and \( S_F \) data and use them to draw points on the Pole Edge Boundary (PEB) and the Field Clamp Boundary (FCB) in the same coordinate system as the VFB.

4. Fit polynomial curves to the PEB and the FCB.

The polynomial fits to the PEBs and FCBs can be utilized to layout and machine the dipole pole pieces and the field clamps.

POISSON was used to fit the parameters VFBPE and \( S_F \) by modelling cross sections taken through the field clamp — pole edge structure at various locations perpendicular to the VFB. Figure 4.16 shows the POISSON geometry used for the cross section at \( x_{VFB} = 40.0 \text{cm} \) for the entrance aperture. The POISSON model was run without the field clamp to fit VFBPE and then with the field clamp to fit \( S_F \).

4.7.1.1 Measuring VFBPE and \( S_F \)

Measuring VFBPE proceeds as follows: First, an initial estimate of the correct VFBPE distance is made, and the appropriate pole edge geometry described in an AUTOMESH (the POISSON input program — see appendix C) input file, appendix E contains an example. POISSON is run and the resulting field profile compared to the expected RAYTRACE profile. To compare the curves, it is necessary to normalize them by setting their peak values to 1.0 and superimposing the peaks. Since VFBPE is being fitted, only the interior RAYTRACE and POISSON fields are compared as discussed in section 4.3. The difference between the RAYTRACE and POISSON fringe fields is quantified by the area between the two curves, calculated using a numerical integration. The fitting process is repeated until the value for VFBPE is within \( \pm 0.05 \text{cm} \) of the value that produces a minimum area between the RAYTRACE and POISSON curves.

Once the VFBPE is fitted, a similar procedure is used to fit the field clamp parameter \( S_F \) using the optimal value for VFBPE. The difference between fitting VFBPE
Figure 4.16: a) POISSON model of the dipole aperture and field clamp. Position: Entrance, $z_{v/fb} = 40.0cm$ b) The POISSON field profile (points) compared to the desired RAYTRACE field profile.
and $S_F$ is that rather than only comparing the internal fields, the entire field profile is compared when fitting $S_F$. Use of only the fringe field is precluded because the internal and fringe fields are not completely decoupled. By fitting the entire field profile with $S_F$, perturbations to the internal field by the clamp are minimized. $S_F$, like VFBPE is fitted to an accuracy of ±0.05 cm.

Each iteration of fitting VFBPE and $S_F$ require new POISSON problem files with subtly different geometries. Furthermore, each location on the VFB has a different geometry since the clamp shape, pole face slope, and gap width change over the aperture (section 4.6). To create the POISSON problem files by hand would be very tedious and prone to error. Instead, the program MACAUTO (MACro processor for AUTOmesh) was written to act as a preprocessor for AUTOMESH (which in turn is a preprocessor for POISSON). Given as input $x_{VFB}$, VFBPE, and $S_F$, MACAUTO calculates the correct pole face slope, pole edge bevel, coil position, and field clamp cross section for either SASP dipole aperture.

Analysis of the POISSON predicted field profile was made by the program FRANA (FRinge field ANAlysis) which read the field shape from the POISSON output file, calculated the RAYTRACE field shape for the appropriate VFB location, normalized the fields, and calculated the area between them over the desired range. The algorithm FRANA uses to calculate the RAYTRACE fringe field is not exactly the same as that used by RAYTRACE. To calculate the field at a point in the fringe field region, RAYTRACE finds (approximately) the closest point on the VFB, determines the internal field at this point and then modifies this field by the Enge function (equation 4.1). The Enge function depends on the dipole gap at the VFB point and the distance of the desired point from the VFB in units of the gap width. This means that the field profile along a perpendicular to the VFB could be determined by various points on the VFB, depending on how convoluted the VFB is. Since the SASP VFBs are not convoluted,
the RAYTRACE algorithm was approximated by using the VFB point through which the perpendicular passes for all the field calculations. This should not have a significant effect on the results.

The fitting process was fully automated with the program FITTER which, given the range to try fitting the parameter over, would perform a binary search looking for the minimum of the area between the curves. Each iteration of the fit generated a new value of the parameter (VFBPE or $S_F$) which MACAUTO used to generate a new POISSON problem file. Next POISSON was run and the output analyzed with FRANA which fed its results back into FITTER. Each iteration took typically 5 to 8 CPU minutes on the VAX 8600 computer. Since on the average it takes eight iterations per fit, the process is lengthy and it is desirable to fit as few points as possible.

The resulting fits to VFBPE and $S_F$ for both entrance and exit apertures are shown in figures 4.17 and 4.18 and tabulated in table 4.1. A total of seven points on the entrance VFB and nine points on the exit VFB were used for the fit. The behaviours of VFBPE and $S_F$ as functions of $x_{VFB}$ are quite different for the entrance and exit apertures. Part of this is because in the entrance coordinate system, dipole gap width decreases with increasing $x_{VFB}$ while in the exit coordinate system, gap width increases with increasing $x_{VFB}$. The sudden change in curvature of both VFBPE and $S_F$ for the exit at large negative $x_{VFB}$ is because of the change in curvature of the VFB which starts to decrease at about -60cm (compare with figure 4.4b).

### 4.7.1.2 Fitting PEB and FCB

The discrete points calculated for VFBPE and $S_F$ must be turned into polynomial descriptions of the Pole Edge Boundary (PEB) and the Field Clamp Boundary (FCB). These polynomials should look similar to the polynomials used to describe the VFB's in section 4.2 (but without the dependence on dipole central radius, $R$). The conversion
### Table 4.1: VFBPE and $S_F$ fits to various locations on the entrance and exit VFBs.

<table>
<thead>
<tr>
<th>Aperture</th>
<th>$x_{VFB}$ (cm)</th>
<th>VFBPE (cm)</th>
<th>$S_F$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entrance</td>
<td></td>
<td>±0.05</td>
<td>±0.05</td>
</tr>
<tr>
<td>-50.0</td>
<td>-0.05</td>
<td>15.70</td>
<td></td>
</tr>
<tr>
<td>-40.0</td>
<td>0.10</td>
<td>15.53</td>
<td></td>
</tr>
<tr>
<td>-20.0</td>
<td>0.15</td>
<td>15.13</td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>0.51</td>
<td>14.00</td>
<td></td>
</tr>
<tr>
<td>20.0</td>
<td>0.70</td>
<td>13.25</td>
<td></td>
</tr>
<tr>
<td>40.0</td>
<td>1.62</td>
<td>10.44</td>
<td></td>
</tr>
<tr>
<td>50.0</td>
<td>2.45</td>
<td>8.80</td>
<td></td>
</tr>
<tr>
<td>Exit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-80.0</td>
<td>0.10</td>
<td>14.70</td>
<td></td>
</tr>
<tr>
<td>-60.0</td>
<td>1.38</td>
<td>10.84</td>
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</tr>
<tr>
<td>-40.0</td>
<td>1.18</td>
<td>11.50</td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>0.51</td>
<td>14.00</td>
<td></td>
</tr>
<tr>
<td>20.0</td>
<td>0.10</td>
<td>15.00</td>
<td></td>
</tr>
<tr>
<td>40.0</td>
<td>0.15</td>
<td>15.20</td>
<td></td>
</tr>
<tr>
<td>60.0</td>
<td>0.25</td>
<td>15.44</td>
<td></td>
</tr>
<tr>
<td>80.0</td>
<td>0.30</td>
<td>14.70</td>
<td></td>
</tr>
<tr>
<td>100.0</td>
<td>0.35</td>
<td>14.13</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.17: VFBPE as a function of $x_{VFB}$ for a) Entrance and b) Exit apertures.
Figure 4.18: $S_F$ as a function of $x_{VF}$ for a) Entrance and b) Exit apertures.
CHAPTER 4. DIPOLE APERTURES

of the VFBPE and $S_F$ points into the PEB and FCB polynomials is accomplished in two steps. First polynomials are fitted to the data points creating analytic expressions for VFBPE and $S_F$ as functions of VFB location. These expressions are used along with the VFB curves to generate points on the PEB and FCB. Finally, polynomials can be fitted to the PEB and FCB points.

The choice of polynomial curves to fit to the VFBPE and $S_F$ data is limited. The highest order for the polynomial is the number of data points minus one. On the other hand because the data points do not vary smoothly, as high an order polynomial as possible is needed to fit the data reasonably. There is no such constraint when fitting the PEB and FCB curves however. The original VFB curves (which will affect the PEB and FCB curves) were of fifth order for the entrance aperture and sixth order for the exit. Thus, the PEB and FCB curves are expected to be at least this order. Because RAYTRACE uses eighth order polynomials in its calculations it was decided to do the same for the fits. This was done and the PEB's and FCB's generated. However, when investigating how dipole saturation (discussed later) shifts the VFB, a 20% loss in spectrometer resolution was apparent. Using the eighth order curves to fit the PEB and FCB introduced high order distortions to the fifth and sixth order VFBs.

As a remedy, the PEB and FCB data were refitted with the same order curves as the VFBs for each aperture. This smoothed out the high order distortions. To determine the effect this had on the VFB, the dipole–clamp geometry specified by the lower order curves was used with POISSON to calculate the VFBPE distance. These values for VFBPE are different from those calculated previously since the low order PEB and FCB curves do not follow the original data exactly. VFB curves of the same order as the original ones are then fitted to these data and fed back into RAYTRACE. This step prevents any of the high order distortions from showing up. The RAYTRACE studies showed that using the lower order PEB and FCB, the spectrometer resolution
is 10% less than the resolution the ideal optics produced — which is 10% better than
that produced by the high order polynomials.

As of the writing of this thesis, no further work has been done to decrease the
distortions caused by the pole edge and field clamp boundaries. The specifications
presented in this work produce a 10% loss in the spectrometer resolution. Further
work will be carried out later to attempt to improve the resolution. Specifically, the
ideal VFB and the POISSON produced VFB will be compared. A function relating
a shift in VFB to a shift in the PEB and FCB will be developed and used to apply
a correction to the PEB and FCB curves. Hopefully, this will produce an improved
specification for the dipole apertures. The polynomial curves and coefficients used to
describe the PEB and FCB are listed in appendix F along with the aperture and field
clamp specification.

4.7.2 PEB and FCB Tolerances

The loss in spectrometer resolution observed above helps estimate the effect of an
inaccurately machined dipole aperture or field clamp. On the basis that loss of spec­
trometer resolution is approximately a linear phenomena, the VFB curve with a 10%
loss is subtracted from the ideal VFB curve (figure 4.19), and the difference is said to
cause the 10% loss in resolution. Then, since the PEB and FCB are roughly the same
shape as the VFB, it can be said that a machining error the size of the VFB difference
will produce the 10% resolution loss. The assertion that the VFB shift is equal to the
corresponding shift in the PEB or FCB is a conservative estimate; the POISSON stud­
ies show that the VFB always shifts less than the steel geometry. Therefore a one to
one relationship overestimates the sensitivity to the machining. This method concludes
that a 1mm shift in the machined curve from the specified curve over a distance of 30cm
will produce a 10% loss in spectrometer resolution. The actual machining tolerances
Figure 4.19: Subtracting the 10% loss VFB from the ideal VFB curve a) Entrance. b) Exit.
are expected to be much smaller than this and, therefore, will not be a problem.

4.7.3 Saturation Effects

The resolution loss discussed above was observed when examining how the VFB shifts with magnet excitation. Ideally, the VFB should be unaffected by excitation. The aperture and field clamps were designed for a magnet operating at approximately half the maximum excitation — well into the linear range of the dipole. To see if there are significant saturation effects, the geometry specified by the PEB and FCB curves was used in a POISSON simulation using a magnet at the SASP's maximum operating excitation. The shift in the VFB was calculated and a new VFB curve constructed. This was then fed into RAYTRACE. Comparing the high and low excitation VFB curves visually, only a slight shift in the VFB is apparent. The RAYTRACE studies found no significant change in the spectrometer resolution caused by these small shifts (note that there is still the 10% resolution loss in both low and high excitation curves). Thus, the aperture design meets its goal of a fringe field unaffected by magnet setting.

4.7.4 Quality of the Calculation

The quality of the specifications presented are difficult to quantify. Ideally the results from the POISSON model should be compared with the field of an actual magnet of similar design to the SASP dipole. This was explored with the data from the LAMPF LEPS dipole (appendix D). Unfortunately, the field maps for the LEPS are not as extensive as desired and more importantly, the LEPS fringe fields differ in two major ways from the SASP fringe fields.

First, the pole edge bevel of the LEPS is a simple 60° slope, while the SASP will have a Rogowski bevel. The POISSON models indicate that these two types of bevel behave differently as a function of excitation. Second and more important, the LEPS is an unclamped magnet with a correspondingly more extended fringe field. It is precisely
Figure 4.20: A plot of the area between the RAYTRACE fringe field curve and the POISSON fringe field curve as a function of the distance of the field clamp from the pole edge ($S_F$) for two different POISSON convergence criterion $\varepsilon = 10^{-3}$ and $\varepsilon = 10^{-4}$. The units for area is fraction of the total RAYTRACE fringe field.

Figure 4.20: A plot of the area between the RAYTRACE fringe field curve and the POISSON fringe field curve as a function of the distance of the field clamp from the pole edge ($S_F$) for two different POISSON convergence criterion $\varepsilon = 10^{-3}$ and $\varepsilon = 10^{-4}$. The units for area is fraction of the total RAYTRACE fringe field.

an extended fringe field that POISSON is expected to model poorly.

Internal consistency in the POISSON model can be checked. Figure 4.20 shows a plot of the area between the RAYTRACE and POISSON fringe field curves called the residue as a function of $S_F$ for two values of $\varepsilon$, the POISSON convergence criterion. Plotted as a fraction of the total curve being fitted, $r$ is on the order of 1% and the change in $r$ with $S_F$ on the order of 0.1%. All the field clamp and pole edge calculations were done with $\varepsilon = 10^{-3}$. The oscillations of $\varepsilon$ are disturbing. $r$ was expected to vary smoothly, dropping with changing $S_F$, passing through a minimum, and then increasing again. The fact that the oscillation region decreases in width when the more stringent convergence criterion is used indicates that the oscillation is an artifact of POISSON and not the magnetic system being simulated. The oscillations were not anticipated; similar studies of the dipole were carried out with a simpler geometry (flat pole face,
coil and clamp parallel to the median plane) with no such oscillation observed. The over-relaxation process of fitting the differential equations may have become unstable and the solutions start to oscillate. These studies show that while \( S_F \) and VFBPE were fitted to \( \pm 0.05\text{cm} \), a more realistic estimate of the error might be ten times this. However, not all the points fitted experienced the oscillations and a look at the plots of VFBPE and \( S_F \) clearly show that the data is too smooth to have an error of \( \pm 0.5\text{cm} \).

Another problem encountered when fitting VFBPE and \( S_F \) was that for some positions on the pole edge, POISSON was incapable of creating the steel geometry from the input file. This happens when a complex geometry is described with too coarse a mesh. POISSON's mesh generator, AUTOMESH cannot fit all the features of the geometry with the mesh points available and gets stuck. A solution is to use a smaller mesh but this increases the execution time for the program and more important, there is a limit to the number of points that can be described in a problem. The current mesh size used for the SASP aperture simulations is approximately at this limit. The result of this reluctance of POISSON to fit some points on the VFB is that the VFBPE and \( S_F \) functions are not mapped out as thoroughly as is desired. For example, it would have been useful to have more points around the peak in VFBPE for the exit aperture.

These problems with POISSON indicate that the program has been stretched to its limit. POISSON was never intended for such detailed studies of small changes in the field. At this point it should be remembered that a three dimensional system is being approximated in two dimensions. Regardless of the accuracy of POISSON, the effects associated with a 3D aperture discussed in section 4.1 can not be modelled and may completely invalidate the results presented here. There is a distinct possibility that the aperture specifications given in this work will not produce a satisfactory field. Thus it is important to consider methods for altering the dipole fringe fields after construction.
of the dipole. Possible methods of doing this are, altering the pole edge boundary, and altering the field clamp. Altering the dipole pole edge would be a complicated task. The pole piece is made from a single piece of steel so it would be necessary to send both pole pieces out for machining. Alternately, the field clamp studies indicate that the VFB can be shifted by a centimetre or more by changing $S_F$. If the tongues of the field clamps are made detachable, modifying them or machining new ones could be readily accomplished. However, altering the dipole VFB's would be time consuming if it is necessary to remachine the field clamp tongues for each measurement. It would be useful to be able to do fast temporary modifications of the FCB. One way of doing this is by building up the FCB with thin iron strips attached to the field clamp with nonmagnetic fasteners. Once a satisfactory FCB was found with this method, the field clamp could then be sent out for machining.
Part II

Simulating the SASP
Chapter 5
Monte Carlo Method

There are several reasons for simulating the DASS/SASP system. As is discussed in chapter 6, simulations were used during the design of the SASP to calculate the profiles of the ray bundles passing through the spectrometer, helping to locate the vacuum vessels and detector arrays. Simulations can also predict the solid angle and data rates of the spectrometer allowing an estimate of the required run times of an experiment. If the kinematics of a reaction are included, a simulation can predict the distributions of the properties of the particles to be detected. This guides the experimenter in the design of detectors and the positioning of the experimental apparatus. By including the characteristics of the detectors in the simulation it is possible to study the performance and resolution of the detector systems. These studies can be used to track down problems in the real apparatus.

Any simulation of the DASS/SASP system must describe the properties of the incident particle beam, the target and the nuclear reactions within it, the scattering of the various reaction products towards the detectors, and the detectors themselves. The proton beam entering the target chamber at 4BT2 is created by accelerating negative hydrogen ions in the TRIUMF cyclotron, stripping them of electrons and injecting the remaining protons into beam line 4B. The energies of protons leaving the cyclotron are not identical, they have gaussian distributions. Typically a beam at 500MeV energy
will be made up of particles with energies varying 0.1% or more from this value. The beam enters a series of bending, focussing, and dispersive optics which correlate many of its properties. For instance a set of optical elements collectively called the *twister* is used to rotate the beam from a horizontally smeared spot to a vertical one and to dispersion match it (appendix B) with the result that the vertical position of the beam is linearly related to the proton momentum.

In the scattering chamber the proton beam hits the target which is usually a rectangular block oriented at an angle to the beam. Most of the protons will pass through the target without interaction and continue on "downstream". A small fraction of the beam particles do interact with the target atoms. The interaction can be a simple scattering of the beam particle off the target atom or a complex nuclear reaction where the beam and target nuclei are destroyed and new particles created. As the beam particle travels through the target, it may undergo multiple scattering and energy loss. Similarly, the reaction products may be deflected by the intervening material as they leave the target. The energies and trajectories of the reaction products are governed by the reaction kinematics which are in turn affected by the energy and trajectory of the incident proton. Of all the possible trajectories of the scattered particles, only those that encounter the detectors are of interest.

The detectors can be magnetic spectrometers such as the SASP and MRS, counter telescopes consisting of wire chambers and scintillators, or anything else that will register the passage of particles. All detectors have some common features however. They have a limited aperture through which the particles must pass, quantified as the detector's *solid angle*. The solid angle is the fraction of a sphere (4π steradians) surrounding the target that the detector occupies and is usually measured in millisteradians. All detectors have an efficiency less than 100% which varies depending on the type of detector and the number of sensing elements in the detector system. All detectors have an
inherent resolution meaning they do not measure particle properties exactly. Lastly, most detectors alter the properties (such as the energy and trajectory) of particles traversing them. This may be significant if a particle passes through several detectors in series.

Based on the purpose of the simulation, there are two basic simulation methods, analytic and Monte Carlo. An example of an analytic simulation is the program TRANSPORT [6] used to study beam optics. A particle beam injected into a system of optics is described by various parameters such as the centroid of the beam and its maximum extents. The optical system is described by mathematical formulas approximating how each optical element effects the beam. The effect of the entire optical system on the beam is determined by performing the mathematical transformation of each element in succession. This approach to modelling the system is adequate for studying beam lines where the behaviour of the particles in each part of the system is precisely known. An advantage of a program such as TRANSPORT is that it is fast and gives an accurate prediction of the general properties of optical systems. However, the analytic approach breaks down when investigating the kinematics of nuclear reactions in the target or the resolution of a detector. Random processes occur, for example where incident particles are injected into a target more than once with the same trajectory and energy and different reaction products are produced. Where reaction products are the same for each trial, they can have different trajectories or energies. To ascertain these effects with an analytic model, it is necessary to integrate the kinematics of the reaction over all possible beam profiles. Such an integration would have to be done numerically, and its limits and variables would be difficult to formulate.

The Monte Carlo type of simulation eliminates the need to describe difficult integrals. Monte Carlo is the name given to any technique that uses random numbers to solve a problem. For nuclear experiment, a Monte Carlo can be either a direct simu-
CHAPTER 5. MONTE CARLO METHOD

lation or an integration. Direct simulation consists of generating beam particles with properties following the beam's known distributions at the target and having these particles interact with the target particles. The random aspects of scattered particles are selected from random populations with the proper distributions. These reaction products are then transported through the detector system; setting off the various sensors and undergoing interactions in the detector materials. Where appropriate, numbers from a random distribution are generated and the effect calculated. After numerous runs, distributions of the states of the particles and the outputs of the detectors are developed and the average and extreme behaviours of the various parts of the system can be deduced. Where the momentum resolution of a spectrometer is being examined a Monte Carlo simulation of the spectrometer is run where the momenta of the particles injected are kept constant while the trajectory is varied. An ideal spectrometer with perfect detectors would produce a sharp momentum peak. However, the nonideal spectrometer and the imperfect detectors, will smear the momentum peak and the resolution of the spectrometer is then measured by the peak's width. To determine how much of the error in momentum is caused by the spectrometer optics and how much by the detector resolution, the simulation would be run with perfect detector resolution and the resultant resolution would be a function of the optics only.

While direct simulation of the system by a Monte Carlo shows general properties and trends, answers to specific questions such as the data rates for a specific experiment or the solid angle of a detector are often required. Such questions can be answered with a Monte Carlo integration. To see how a Monte Carlo can be used to evaluate an integral consider the following. Suppose the area within the region described by some known function \( f \) is to be found and that \( f \) is either impossible or very tedious to integrate. One familiar solution to this problem is to integrate the function with a numerical technique like Simpson's Rule [8]. A less familiar technique is to randomly generate
points uniformly distributed within a known region (call it $A_0$) that encompasses the region with the unknown area. The known region could be a simple box or it could be some elaborate shape. It is only necessary that it's area is known and that it completely engulfs the unknown region. For each randomly generated point, it is noted whether the point lies inside or outside the unknown area. For $N$ tries, say there are $n_*$ points that lie inside the area. Then as $N$ gets very large, the area enclosed by $f$ is,

$$A_f = \lim_{N \to \infty} \frac{n_*}{N} A_0$$

This is called the law of large numbers and can be generalized [16] to

$$\frac{1}{N} \sum_{i=1}^{N} f(u_i) \to \frac{1}{b-a} \int_{a}^{b} f(u)du$$

where $u_i$ is a uniform random number generated on the interval $(a, b)$ and $f(u_i)$ the function evaluated at that point. The technique can be extended to multi-dimensional integrals. This method of solving an integral is called a Monte Carlo.

The Monte Carlo approximates the correct solution to an integral. The error in the Monte Carlo summation is estimated as the standard error in the mean value of $f$.

$$\int_{a}^{b} f(u)du = (b-a)\bar{f} \pm (b-a)e$$

where $\bar{f}$ is the mean value of $f$,

$$\bar{f} = \frac{1}{N} \sum f(u_i)$$
and \( e = s/\sqrt{N} \). \( s \) is the sample deviation,

\[
s^2 = \frac{1}{N-1} \sum (f_i - \bar{f})^2 = \frac{1}{N-1} \left( \sum f_i^2 - N\bar{f}^2 \right)
\]

So the Monte Carlo evaluation of the integral is,

\[
\int_a^b f(u)du = \frac{b-a}{N} \sum_{i=1}^N f_i \pm (b-a) \sqrt{\frac{\sum f_i^2 - N\bar{f}^2}{N(N-1)}}
\]

As the number of points increase, the Monte Carlo approximation improves as approximately \( 1/\sqrt{N} \). Provided the function \( f \) is well behaved (always finite, piecewise continuous, finite number of discontinuities), a Monte Carlo integration will converge to the correct solution. However, it will not necessarily converge quickly. There is an art to setting up a Monte Carlo integral and selecting biased sampling populations to speed convergence [17].

For a particle beam experiment, the primary advantage of a Monte Carlo integration is that it is unnecessary to explicitly state the limits of integration. Applying Monte Carlo integration to the calculation of the solid angle of a detector, particles are generated at the experiment’s target and aimed into a cone centred on but larger than the detector. All the particles that penetrate the detector are recorded as hits and the solid angle of the detector is given as,

\[
\Omega_{\text{detector}} = \frac{n^*}{N} \Delta\Omega
\]

where \( \Delta\Omega \) is the solid angle of the cone. This is equivalent to integrating the differential solid angle of the detector as a function of the incident ray,

\[
\Omega_{\text{detector}} = \int \frac{d\Omega}{d\vec{r}_1} d\vec{r}_1
\]
where $r_1$ has four components, $x_1, \theta_1, y_1, \phi_1$, ($\delta$ is fixed). The limits of integration are complex functions dependent on the optics and the apertures of the detector system. If this integration was carried out with a numerical technique, these limits would have to be stated explicitly. The Monte Carlo implicitly considers the integration limits by simulating the behaviour of the particle traversing the detector.

Similarly, the data rate for an experiment could be calculated by injecting $N$ beam particles into the system for which $n^*$ products are detected. The data rate of the system is then,

$$R = \frac{n^*}{N} R_{\text{beam}}$$

where $R_{\text{beam}}$ is the rate at which the beam particles hit the target. It would be naive to just inject beam particles into the simulation and count the number of successful events in the detectors since only one in $10^6$ or fewer beam particles interact with the target — let alone pass through the detector system. Instead, the experimenter would force every beam particle to interact with the target and multiply the resultant data rate by the probability of the beam particle interacting with the target,

$$R = w_{\text{int}} \frac{n^*}{N} R_{\text{beam}}$$

where $w_{\text{int}}$ is the probability of an interaction and is called a weighting factor or just weighting. It is equivalent to changing the limits of the Monte Carlo integration. Weighting is also used when aiming a scattered particle towards a detector. Suppose a reaction scattered particles homogeneously in all directions. If the Monte Carlo faithfully simulated this, most of the particles generated would miss the detector which may have an acceptance of only a few millisteradians. Instead, the Monte Carlo would force the particles into a cone centred on and fully encompassing the detector’s aperture. Most of the reaction products would enter the detector but similar to the beam hitting
the target, the success rate would be improperly high. Another weighting factor,

\[ w_{\text{scat}} = \frac{\Omega_{\text{cone}}}{4\pi} \]

must be used in the calculation of the data rate.

\[ R = w_{\text{int}} w_{\text{scat}} \frac{n_s}{N} R_{\text{beam}} \]

If the Monte Carlo accurately models the geometry, kinematics, and probabilities of the experiment, no weighting factors are required. Where short cuts are taken to speed up the simulation, the probabilities are affected and a weighting factor must be used. Another way of looking at this is to imagine the particle starting the simulation with a weighting of unity. At each point in the simulation, a local weighting factor is multiplied to the particle's weighting factor. For most points, this local weighting factor is unity, but occasionally, such as for the scattering towards the target, the local weighting is a number less than one (the weighting factor must be in the range 0 - 1). If the particle spawns other particles during a reaction, then these daughter particles inherit the parent's weighting factor.

### 5.1 EASY, a Monte Carlo Simulator

There are several objectives for the design of the SASP's computer simulation. First it has to accurately model the transport of particles through the spectrometer to allow the examination of the effects of the various apertures in the system on the spectrometer's acceptance and solid angle. Using studies of the ray profiles at various locations in the spectrometer, the vacuum system could be designed to have a minimal effect on the solid angle. Many surfaces in a spectrometer can cause particles to scatter and still penetrate the spectrometer. Such particles will have their momentum and trajectories slightly
altered, and will decrease the resolution of the spectrometer. Sometimes these bad rays can be recognized by their trajectories through the detector array. Scattering from the pole face near the exit could be recognized by projecting the ray back from the detectors to the dipole exit. However, if the scattering occurs deeper in the spectrometer, it may be impossible to identify. A solution to this is to place "active collimators" (i.e. scintillators) at critical locations to identify bad rays passing through the system. The simulator can identify the optimum locations for the collimators and the event rates they will experience.

The second intent of the simulation is to study the performance of the SASP as a spectrometer. Specifically, such things as the position and shape of the focal surface, the momentum resolution of the spectrometer, and its ability to trace rays back to the target. The resolution and traceback properties would be studied in a variety of detector configurations such as with and without a front end chamber. The simulation must consider the effects of multiple scattering in the detectors and vacuum windows and position detector resolution. This information determines the optimal location for detectors and their necessary resolution. This is significant since the cost of wire chambers goes up with the increased resolution. It is pointless to build chambers ten times more accurate than the resolution of the spectrometer optics warrants. By studying the performance of the SASP with a computer model, a feel for the real system can be obtained which will speed up the commissioning process.

Finally, the computer simulation will be used to model experimental arrangements of the DASS/SASP system. A simulator permits data rates and the requirements for detector sizes and positions to be estimated. Any experimental arrangement is effected by the kinematics of the reaction being studied. For a simple case, such as in a two body reaction (section 5.3) the kinematics (but not the angular dependence of the cross section) can be determined analytically. For three body and more complex reactions,
approximate models have to be utilized. Even crude approximations to the desired reaction can provide much useful information. In addition to modelling the kinematics at the target, the simulator must reproduce the profile of the beam incident on the target and the resolution and efficiencies of the detectors.

The different aspects of an experiment can be thought of as a four level hierarchy:

1) Equipment
2) Acquisition Electronics
3) Analysis
4) Control

The *equipment* in an experiment includes all the hardware including vacuum systems, targets, and detectors. The *acquisition electronics* takes the raw data from the detectors, processes it, performs simple processing such as ensuring various detectors fire in coincidence, and then passes the data on to be analyzed and stored. *Analysis* consists of examining the data, determining which is valid and then reconstructing the desired information about the system. Analysis can be simple “on line” analysis to check that all the systems are performing properly, or the thorough “off line” analysis that examines the entire system. Finally, *control* represents the acts of the experimenter. The experimenter directly controls the particle beam, the position of a spectrometer about a target, detector settings and, beam characteristics.

**EASY (E)quipment and A)quisition S)ystem** attempts to mimic the experimental hierarchy. The first aspect of the experiment (equipment) is simulated by constructing the experimental apparatus out of standardized pieces of equipment (called *devices*). The effects of these devices on particles passing through them are simulated using a direct Monte Carlo simulation. The second aspect of the experiment (Acquisition Electronics) is modelled partly analytically and partly with a Monte Carlo. The analytic
aspect of the acquisition electronics is the description of the "logic" used in the experiment. Namely, which detectors must fire in combination to produce a valid signal. A Monte Carlo method is used to create the errors in the signals from the position sensors.

Analysis, the third aspect of an experiment, is completely analytic. Data taken from the simulation of the acquisition electronics can be analyzed by standard functions built into EASY or recorded for processing by other programs. The last aspect, Control of the experiment, is also completely analytic.

An experiment, or problem as it is called in EASY, is described in three parts using three separate files. First all the equipment (devices) used in the experiment are described in the problem's device file. Devices include targets and detectors, drift regions (air and vacuum) and the spectrometer optics. The simulations of some of devices are abstracted. For example, the beam optics simulated by the device beam. The beam optics describes what happens to the proton from its initial excitation at the ion source, its passage through the cyclotron and beamline optics up to the target chamber (for the DASS/SASP system the target is 4BT2). During its travels, the proton's characteristics (position, angular divergence, energy, etc.) form interdependences, correlations and limits. An example of a limit is that no beam particle will be found outside the beam pipe. The device beam simulates the beam optics by giving the particle the distribution of properties it should have just after leaving the cyclotron and then using a first order transport matrix to move the particle from the cyclotron to the target where it is checked to ensure it is still in the beam pipe. This simplification of the beam optics is not perfect. The proton could leave the beam pipe between the cyclotron and the target and re-enter before reaching the target without detection. This would produce an unrealistic beam profile on target. A more accurate model of the beam optics would calculate the particle's position at several locations in the beam line. Such a model requires massive computation and is not necessary for
simulation of the DASS/SASP system.

Devices like the optics of the SASP and MRS spectrometers are modelled with various transport matrices and a series of "aperture cuts" that block rays at strategic locations within the devices. As well as devices that mimic hardware in the experimental system, EASY has devices with no counterpart in the real world. For example the device backtrack drifts a ray backwards through space. This could be used to put a front end chamber before a spectrometer which expects the ray to start at the target. The ray would first be transported through the front end chamber within which it would undergo multiple scattering altering its trajectory. Then the ray would be back tracked to the target and passed into the spectrometer with its new trajectory.

Once the experimental hardware is described, EASY has to be told where data is to be collected and how to process it — analogous to the acquisition electronics and analysis components of an experiment. This is done in the data acquisition file (dac file for short) using EDAC (EASY Data ACquisition). EDAC is a language that allows the user to specify unique points in the experimental system and tell EASY to collect and process information from those points. If modelling a real experiment, the points data would be collected from are the outputs of the detectors. In the simulation however there is the freedom to collect data anywhere. This could be used to measure the trajectory of a ray with wire chambers and then compare the reconstruction with the ray's true trajectory. The data acquisition electronics in an experiment usually perform logical operations on the detector data, ensuring that the proper combination of detectors have/haven't fired before sending the information on to be processed further (this is called a hardware trigger). EDAC allows simple logical operations to be performed on the data collected from the system before further processing by EASY.

Basic processing by EASY consists of displaying data using histograms and scatter plots or storing the data "to tape" (actually to a file) for post processing by another
program. In addition to this, EDAC has constructs called *functions* that can perform calculations within EASY. Each function is based on a *generic function* which would perform a task, such as taking the output of two Vertical Drift Chambers and reconstructing the trajectory of a ray. The function would consist of a call to the generic function with *parameters* describing the geometry of the two VDC’s and with *arguments* which would be the outputs of the various wire planes on the chambers. The advantage of such a system is that frequently encountered problems can be described in a general way using a generic function and then the specific details easily added for each new experiment. Furthermore, different functions can be set up using the same generic function with different arguments and parameters. Thus both the SASP and MRS VDC’s could use the same trajectory reconstruction algorithm even though they have different geometries.

The final aspect of EASY is control. This takes the form of a series of *commands* found in the *command file*. Commands can mimic the control found in a real experiment such as changing the beam energy or setting a spectrometer’s central momentum. Commands can also turn off all the multiple scattering in the system or turn off the errors in position sensors. Some useful commands are *mapsolid* and *mapfocal* which map the solid angle and focal plane respectively of a device. Commands such as these can do the job of an entire elaborate experiment.

The hierarchy of EASY is as follows: first a system is described, then the data acquisition specified, and finally it is controlled. As in a real experiment, the control has no effect on the equipment and the electronics. The electronics and analysis can be altered without effecting the equipment. If the equipment is altered (detectors added, distances changed, etc.) the acquisition and analysis may have to be altered as well. A complete description of EASY is found in the manual *EASY, A Monte Carlo Simulator*[19].
5.2 Modeling the SASP Optics

As discussed in chapter 2, the SASP is modelled using the conventions of the program TRANSPORT, in which a projectile is represented as a six element ray propagating in the positive z direction.

\[
\begin{bmatrix}
x \\
\theta \\
y \\
\phi \\
l \\
\delta 
\end{bmatrix}
\]

where \((x, y)\) is the position of the ray in the plane perpendicular to the nominal direction of travel; \(\theta\) and \(\phi\) are the angles the ray makes with the \(z\) axis in the \(x-z\) and \(y-z\) planes (figure 2.1); \(l\) is the difference in path travelled by the ray and the central ray; and \(\delta\) is the ratio of the particle's momentum to the central momentum of the spectrometer, \(P/P_0\). The SASP is described with first and second order transfer matrices and many higher order terms.

\[
r_2^i = R^1 r_1^i + R^2 r_1^i + H(r_1^i)
\]

Individual terms are represented with the notation,

\[
r_2^i = \ldots + (r^i/r^j r^k \ldots) \times r_1^j \times r_1^k \times \ldots + \ldots
\]

For example,

\[(x/y\theta)\]

is the coefficient that modifies the product \(y_1 \theta_1\) when calculating \(x_2\).
5.2.1 Global Matrices

As part of the SASP’s optical design, RAYTRACE generated the sets of transfer coefficients to transport a ray from the target to the focal surface.

\[ R^1, R^2, H \]

where \( H \) is a set of eighteen high order terms. Each set of coefficients were for a specific ray momentum \((\delta = -10\%, -5\%, 0\%, +5\%, +10\%, +15\%)\). The first order matrix for 0% is,

\[
R_{0\%}^1 = \begin{bmatrix}
-0.59782 & 0 & 0 & 0 & 0 & 2.80849 \\
3.94159 & -1.67637 & 0 & 0 & 0 & 5.75814 \\
0 & 0 & -3.99876 & -0.20371 & 0 & 0 \\
0 & 0 & -12.65651 & -0.88448 & 0 & 0 \\
-1.45125 & 0.47013 & 0 & 0 & 1 & 1.04494 \\
0 & 0 & 0 & 0 & 0 & 1 
\end{bmatrix}
\] (5.1)

The coefficients were generated by tracing a standard bundle of 46 rays through the spectrometer and are only good for rays close (0.1%) to the momentum they were generated for. The quality of the transfer coefficients was tested by sending a bundle of 198 randomly generated rays through the spectrometer using both RAYTRACE and transfer coefficients. Taking the final trajectory of the ray calculated by RAYTRACE, \( r_2^{RAYTRACE} \), as correct, the accuracy of the coefficients is measured by subtracting components of the matrix final trajectory, \( r_2^{matrix} \) from the RAYTRACE components,

\[
r_i^{error} = r_i^{RAYTRACE} - r_i^{matrix}
\]

For example, the quantity,

\[
\theta_2^{error} = \theta_2^{RAYTRACE} - \theta_2^{matrix}
\]
is the difference between the angle the ray makes in the $x - z$ plane after the SASP as calculated by RAYTRACE and by the transfer coefficients. The dependency of these errors on the initial ray ($\vec{r}_1$) can be determined by plotting a component error against each of the initial ray components ($x_1, \theta_1, y_1, \phi_1, \delta$). An example of this is shown in figure 5.1a where $\theta_2^{\text{error}}$ is plotted against $\theta_1$. The errors in the other components showed similar dependences on the initial trajectory.

This presented a problem. It would be impractical for EASY to use the accurate but extremely slow RAYTRACE code to model the SASP. It has to be done with matrices to get a reasonable speed but the matrices generated by RAYTRACE are inaccurate. The situation is worse when considering how well a set of matrices model rays of differing momentums. The solution was to fit additional terms to one set of coefficients to produce a good fit to the true trajectories.

The fitting process was done completely by hand. Different order terms and coefficients were added by trial and error to produce a reasonable agreement between RAYTRACE and the matrix predicted trajectories of the 198 ray bundles at each of the momentums -10%, -5%, 0%, +5%, +10%, and +15%. When finished, a total of 105 extra terms had been added to the original 0% matrices. The major additions were,

\[
\begin{align*}
(x/\theta^n) \\
(\theta/\theta^n) \\
(y/\phi^n) \\
(\phi/\phi^n) \\
(i/\delta^n)_{i=x,\theta,y,\phi}
\end{align*}
\]
Figure 5.1: $\theta_2^{error} = \theta_2^{RAYTRACE} - \theta_2^{matrix}$ plotted against $\theta_1$. The momentum of the rays are within 0.1% of $P_0$. a) The original RAYTRACE transfer matrices. b) After additional terms have been added to reduce the errors.
TABLE 5.1: The Standard deviations of the errors in the fits of the SASP matrices to the RAYTRACE trajectories.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\sigma_{x}(\text{cm})$</th>
<th>$\sigma_{y}(\text{mrad})$</th>
<th>$\sigma_{v}(\text{cm})$</th>
<th>$\sigma_{\phi}(\text{mrad})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10%</td>
<td>0.0840</td>
<td>0.7851</td>
<td>0.1293</td>
<td>0.6277</td>
</tr>
<tr>
<td>-5%</td>
<td>0.0496</td>
<td>0.4260</td>
<td>0.1098</td>
<td>0.2537</td>
</tr>
<tr>
<td>0%</td>
<td>0.0316</td>
<td>0.3982</td>
<td>0.0901</td>
<td>0.2328</td>
</tr>
<tr>
<td>+5%</td>
<td>0.0405</td>
<td>0.5472</td>
<td>0.0905</td>
<td>0.3647</td>
</tr>
<tr>
<td>+10%</td>
<td>0.0681</td>
<td>0.8241</td>
<td>0.1308</td>
<td>1.0840</td>
</tr>
<tr>
<td>+15%</td>
<td>0.1209</td>
<td>1.4579</td>
<td>0.1896</td>
<td>0.6769</td>
</tr>
</tbody>
</table>

where $n$ varied from one to seven. There were also some two and three variable terms such as

$$(x/\theta^4 \delta^2)$$

$$(y/\phi \theta \delta)$$

Some dependencies are left but are increasingly difficult to eliminate. The final fit for $\theta$ is shown in figure 5.1b; the standard deviation of the error in $\theta$ was reduced from 0.9mrad to 0.4mrad — a factor of 2.

These matrices are only good for rays in the range $|\theta| \leq 85\text{mrad}$, $|\phi| \leq 43\text{mrad}$, and $-10\% \leq \delta \leq +10\%$. Outside this range the high order terms blow up. A measure of the quality of the matrices is the standard deviation of the errors shown in table 5.1.

The position errors in table 5.1 are at the focal surface positions for each ray bundle. These position errors will increase at increased distances from the focal surface because of errors in the angles. A 1 mrad angle error will produce a 1 mm shift in the ray at a distance of 1 metre. For purposes of calculating aperture cuts and detector positions, this magnitude of error is insignificant. For momentum calculations and
track reconstruction, it is difficult to predict the effect the errors. The spot sizes of matrix bundles are approximately the same size as the RAYTRACE bundles. Thus the matrices get similar momentum resolutions to RAYTRACE. However, the error between the matrix and RAYTRACE rays is also about the same size as the spot size. Thus, while the spot sizes are similar, the matrix rays are basically uncorrelated to the RAYTRACE rays. This will effect the software corrections to the rays. Regardless of whether the matrices produce a better or worse resolution than RAYTRACE after software corrections, the matrices will require different corrections.

5.2.2 Aperture Cuts

An ideal simulation of the SASP would use a routine like RAYTRACE to trace each particle through the system examining its position at each step to see if it collided with a surface. Such a simulation, although slow, would give an accurate estimate of the spectrometer's solid angle and acceptance. As a shortcut, EASY only examines the position of the particle at nine key apertures in the system. This leads to an overestimate of the solid angle if a particle is travelling in a curved orbit that would strike a surface between apertures.

To ascertain the important apertures in the SASP, the focussing properties of the optical elements and the geometry of the vacuum vessels must be considered. A ray approaching the SASP must first travel the 70cm from the target to Q1. The first aperture it encounters is the entrance to the quadrupole, the aperture of which is assumed to be a circle of diameter 20cm. Inside Q1, the ray is focussed in the bend plane and defocused in the nonbend plane. Leaving Q1, it travels the through the vacuum pipe connecting Q1 to Q2. Inside Q2, the ray is defocused in the bend plane and focussed in the nonbend plane. The net effect of the two quadrupoles is a defocusing in the bend plane and a focussing in the nonbend plane. The ray's entrance
to the dipole is a slit varying in width from about 9 to 11 cm. Without focussing, part of the ray bundle would be blocked by the dipole's pole piece. Within the dipole, the rays are bent approximately 90° towards the vertical and in the process are momentum dispersed and focussed in the bend plane. Leaving the dipole the rays pass through the exit vacuum box and emerge from the vacuum system at the focal surface. The dipole does not focus in the nonbend plane. The ray bundle comes to a waist in the nonbend plane at or after the middle of the dipole depending on the ray momentum. The becomes about 30 cm wide when it reaches the focal surface. The trajectories of a ray bundle transported through the SASP by RAYTRACE are shown in figure 5.2. Figure 5.2a shows the bend plane of the spectrometer while figure 5.2b shows the nonbend plane. In the nonbend plane, the rays are projected onto a curved plane which follows the central trajectory through the spectrometer.

The nine apertures used in EASY's simulation of the SASP are shown in figure 5.3. They are in order from the target: the entrance and exit of each quadrupole; the constriction of the entrance vacuum box; the entrance of the dipole; the middle of the dipole; the exit of the dipole; and the constriction of the exit vacuum box. The method used to calculate the position of the ray depends on the aperture. For several of the apertures, RAYTRACE was used to generate transfer matrices from the target similar to those used for the complete SASP optics. For other apertures (such as aperture #5, the entrance vacuum box constriction) the ray is drifted from the previous aperture. These locations were chosen to find the limiting apertures of the spectrometer as is discussed in section 6.2. Only a few of the apertures are limiting and the balance can be removed to speed up calculations. The sizes and shapes of the apertures used by EASY are given in table 5.2. The meaning of the parameters given are explained in [19].
Figure 5.2: A $\delta = 0\%$ ray bundle traced through the SASP by RAYTRACE. a) Bend plane. Shows the quadrupoles, and the entrance and exit of the dipole. b) Nonbend plane. The rays are projected onto a curved surface that follows the central trajectory.
Figure 5.3: The SASP Apertures used by EASY
### Table 5.2: The SASP Aperture Cuts

<table>
<thead>
<tr>
<th>Aperture</th>
<th>Location</th>
<th>Type</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>Entrance Q1</td>
<td>quad</td>
<td>d</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>d2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>W</td>
</tr>
<tr>
<td></td>
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<td>#7</td>
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<td>#8</td>
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<td>slope</td>
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<td></td>
<td></td>
<td></td>
<td>intercept</td>
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</tbody>
</table>
5.2.2.1 Quadrupoles

A quadrupole focussing element has four pole pieces (spaced regularly about the optical axis) which run the length of the magnet and are shaped to produce the desired field. A sketch of a cross section of a quadrupole is shown in figure 5.4. While in a pure quadrupole magnet, the pole faces would have hyperbolic curvatures, the SASP "quadrupoles" have various higher order field terms which are created by slight distortions of the pole faces. As with a dipole, the magnetic field tends to bulge out from the ends of a quadrupole, increasing its effective length.

The quadrupoles were originally designed to hold a standard eight inch diameter (10.16cm radius) circular vacuum pipe. Subsequent studies with the Monte Carlo showed that the solid angle of the spectrometer could be increased if a "cruciform" shaped vacuum pipe was used (figure 5.4). A cruciform vacuum pipe increases the solid angle but many of the added rays are of poor quality. Only the region around the optical axis has a good quality field. The field between the poles is distorted by the coils and the limited extent of pole faces. However, the additional solid angle achieved
by the cruciform vacuum pipe will be useful for work where resolution is less important
than data rates. For high resolution work, the unwanted rays can be "vetoed" by active
collimators or ray tracing from the focal surface back through the spectrometer.

The entrance and exit of both Q1 and Q2 were used as apertures for the SASP
using the cruciform vacuum pipe. Apertures are placed at the effective edges of the
quadrupoles. The effective edges were used because transfer matrices to these locations
could be readily generated by RAYTRACE while transfer matrices to the pole edges
would be more difficult. This geometry should produce slightly low solid angles.

Straight line trajectories in the drift spaces imply that the acceptance of the
entrance apertures can be accurately determined. The exits of the quadrupoles differ
because the trajectories of the rays change. If a ray is diverging when it reaches the
aperture, it will be removed, even though the ray would have hit a surface before the
aperture. If the ray is converging an error can occur. In a quadrupole, the ray follows a
curved trajectory. In the focussing plane, a ray initially diverging will reach a maximum
displacement from the optical axis and then turn back. It is possible for the trajectory
to pass through the surface of the vacuum vessel, and still clear the exit aperture. Thus
rays that should be stopped will pass through the magnet.

For Q1, which focuses in the bend plane, aperture #2 provides a satisfactory
estimate of the aperture in the nonbend plane and a not so good estimate in the bend
plane. The opposite is true for the exit of Q2 (aperture #4). The size of the errors will
be small and not likely to effect the solid angle calculations.

5.2.2.2 Dipole

Three apertures are used to describe the dipole: the entrance (#6), the middle (#7),
and the exit (#8). Aperture #7 in the middle of the dipole has simple trapezoidal
cross section following the pole faces of the dipole. The width of the aperture in the
bend plane is set at the approximate extent of the "good field" region of the magnet. The entrance and exit apertures (#6 and #8) approximate the curvature of the dipole pole edges. The ray is transported to the \( z = 0 \) plane of the local aperture coordinate systems (figure 3.1). The distance from the \( z = 0 \) plane to the pole edge parallel to the \( z \) axis is calculated and the ray drifted this distance. The ray is then compared to a trapezoidal aperture which follows the projection of the pole faces into the local coordinate system. The widths of the apertures are set at approximately the true widths of the dipole apertures. This method is only exact if the ray is travelling parallel to the \( z \) axis. If the ray is at an angle, then it will not be drifted back to the exact crossing of the aperture. This procedure was taken to avoid the time consuming calculation of the intersection of two arbitrary lines and should not have a significant effect on the solid angle calculation. Lastly, the pole edge of the magnet is used for the aperture rather than the point where the bevel cuts the pole face. This will produce a slightly underestimated value for the solid angle.

Rays entering the dipole are diverging in the bend plane and converging in the nonbend plane. The dipole has little effect on the rays in the nonbend plane where they continue to converge. Low momentum rays reach a waist or focal point in the centre of the dipole after which they diverge. As the momentum increases, the focal point shifts downstream out of the dipole. Thus for low momentum rays, the bundle is diverging at the dipole exit while for high momentum rays, the bundle is converging. This property makes the dipole apertures reasonably accurate in the nonbend plane. High momentum rays leave the dipole still converging, any not blocked by the entrance will pass through the rest of the dipole. Low momentum rays hitting the pole face of the magnet will be blocked by the middle or exit apertures.

In the bend plane, rays of all momenta are focussed and leave the dipole converging. Rays can pass the entrance aperture, balloon out inside the dipole and then
converge passing the exit aperture. It is possible for extreme rays to leave the region of good field in the dipole without being detected. The middle dipole aperture (#7) lessens the chance of this by sampling the ray bundle at approximately its widest point.

5.2.2.3 Vacuum Boxes

The SASP dipole vacuum boxes are described in sections 4.7 and appendix F. Referring to figure 4.15, the vacuum box must extend outward from the pole edge beyond the coil after which it can flare upwards away from the ray bundle. The plate of the vacuum box is parallel to the dipole pole face but spaced further from the dipole's median plane. Because of the curvature of the dipole apertures, the vacuum box must extend some distance from the coil before flaring. The relation between the vacuum boxes, Q2, and the dipole is shown in figure 5.5. Ideally, the vacuum boxes should be far enough away from the median plane that they do not interfere with the ray bundle and form limiting apertures for the spectrometer. Since the coil and vacuum box thicknesses are fixed, raising the vacuum box up necessitates adding more steel to the dipole pole pieces and return yokes to maintain the same gap between the pole faces. A 1cm increase in the width of the box would require approximately an additional 800 kg of steel. This would also increase the minimum angle the SASP could make with the beamline and the MRS. The dimensions settled on are a compromise that result in the vacuum boxes blocking a small part of the high momentum ray bundles.

5.2.3 EASY's sasp_optics Device

The complete SASP optics (global matrices and aperture cuts) are simulated in EASY with the device sasp_optics. This device takes a ray at the target plane of 4BT2 and transports it to the exit of the SASP dipole. If the ray is blocked at any of the nine apertures in the spectrometer, it is lost. The exit coordinate system of sasp_optics has
Figure 5.5: The relationship between Q2, the dipole, and the vacuum boxes. The way the figure is drawn, rays appear to originate inside the Q2 vacuum pipe material. Because the rays are focusing in the nonbend plane, it is possible for rays that should be blocked by the quadrupole pole tips to pass unnoticed through the quadrupole. Similarly, rays could scatter off the pole tip without detection. The study of such scattering effects would best be studied with REVMOC or perhaps a combination of REVMOC and EASY.
z pointing vertically up and $x$ in the direction of increasing dipole radius (see section 8.1). While EASY’s `sasp_optics` device reasonably models the RAYTRACE predicted optics, because of the saturation effects of the SASP dipole, the optics on the real spectrometer will change. Therefore, EASY’s model will behave less and less like the SASP as it is run at higher central momenta.

5.3 Two Body Kinematics

A two body nuclear reaction is when two particles collide, interact, and two particles (not necessarily the incident ones) are emitted.

$$m_1 + m_2 \rightarrow m_3 + m_4$$

An example is the reaction $p(p, \pi^+)d$ where two protons collide and a pion and deuteron are emitted. The reaction is said to be isotropic if the reaction products are emitted with no preferred direction in the Centre of Mass (CM) Frame. The centre of mass frame is defined as the frame in which the total momentum of the incident and emitted particles is zero. Most reactions are not isotropic, rather the products have preferred directions determined by the reaction. This differs from the kinematics which specify the possible trajectories and energies of the reaction products and is dependent on the trajectories and energies of the incident particles. For example, the kinematics of the $p(d,\pi^+)d$ reaction with an incident proton energy of 500MeV, constrain the trajectory of the product deuteron to within a few degrees of the beam line. Most reactions are not isotropic, rather the reaction products have preferred directions determined by the reaction.

Apart from being the simplest reaction, a useful property of a two body reaction is that once the trajectory of one of the reaction products is determined, the momen-
turn and trajectory of both are fixed. To calculate the two body kinematics, consider two incident particles with known masses, energies, and momentums: \((m_1, E_1, \vec{P}_1)\) and \((m_2, E_2, \vec{P}_2)\). Also known are the masses of the reaction products: \(m_3\) and \(m_4\). First calculate the Lorentz transformation that takes one from the Lab Frame to the Centre of Mass frame. If the total momentum in the Lab Frame is along the \(z\) axis, then the transformation is described by the two quantities \(\gamma^*\) and \(\beta^*\).

\[
\begin{align*}
P^*_x &= P_x \\
P^*_y &= P_y \\
P^*_z &= \gamma^*(P_z - \beta^*E) \\
E^* &= \gamma^*(E - \beta^*P_z)
\end{align*}
\]

where the * denotes a CM value. Next the total energy available to the reaction in the CM frame, \(W\), is calculated. The total momentum of the reaction products in the CM frame is zero, giving the relations,

\[
E^*_3 + E^*_4 = W \\
\vec{P}^*_3 + \vec{P}^*_4 = \vec{0}
\]

Thus the reaction products travel in opposite directions in the CM frame. The method for calculating \(P^*_3\) and \(P^*_4\) is given in appendix H.1. For an isotropic reaction, a product can be emitted in any direction in the CM frame. The Monte Carlo simulates the reaction by randomly selecting a trajectory for one of the products. This fixes the trajectory of the other. In practice, only particles that hit the detectors are of interest. A
truly isotropic reaction would spray particles about and most would miss the detectors. This is avoided by scattering one of the reaction products into a cone centred on, and fully covering, the aperture of one of the detectors. This way, most of the particles generated at the target enter the detector, with only a few around the edges of the aperture missing. For example, in a \((p, \pi^+)\) reaction, where the pion is to be detected in the SASP spectrometer, a cone totally covering the aperture of the SASP in the centre of mass frame is specified. All the pions will be scattered isotropically into this cone. If the cone is chosen so that it is only slightly larger that the SASP, most of the particles will enter the spectrometer and the simulation will be efficient. By scattering the reaction product into a cone rather than a sphere, the symmetry of the reaction is destroyed. As discussed earlier, to make the reaction isotropic, a \textit{weighting factor} must be carried with each reaction product that states the probability of it being scattered into the cone. The weighting factor is given by,

\[ w = \frac{\Omega_D}{4\pi} \]

where \(\Omega_D\) is the solid angle of the cone in steradians. Thus if the cone is a full sphere, the weighting factor reduces to unity. This weighting factor is carried with the particle through the remainder of the simulation and is used in solid angle calculations and in plotting. To point the reaction product at the detector, it is necessary to calculate the angle it lies at in the CM frame. This angle is different than the lab frame angle due to Lorentz contraction. The method of calculating the CM detector angle \((\theta_D^*)\) is described in appendix H.2.

Once the CM trajectories of the reaction products are determined, the products are shifted to the lab frame,

\[ P_x = P_x^* \]
Figure 5.6: EASY's two body reaction. The scattered particle is pointed towards a detector.

\[ P_y = P^*_y \]

\[ P_z = \gamma^* (P^*_z + \beta^* E^*) \]

\[ E = \gamma^* (E^* + \beta^* P^*_z) \]

The reaction products are then handed to the rest of the simulation to be transported through the remainder of the system.

The reaction kinematics and the aiming of the products at a detector is handled in EASY by the devices `target` and `switchyard`. The incident beam interacts with a target particle somewhere in a rectangular target block described by `target`. The `target` routines calculate the kinematics and aim the reaction products at one of the detectors described in the `switchyard` device (figure 5.6). Currently, EASY can only handle isotropic two body kinematics but the routines are written to make modification relatively simple. For example, to make the reaction anisotropic, an angle dependent weighting factor is added to the reaction product. This would be multiplied with the scattering cone's weighting factor to produce the final weighting factor for the particle.
Chapter 6

Monte Carlo Studies of the SASP

Five basic Monte Carlo studies were completed for the SASP: the solid angle of the spectrometer as a function of momentum, the total acceptance of rays from the target plane, the limiting apertures of the spectrometer system, the ray profiles downstream of the dipole, and mapping the focal surface of the spectrometer. The EASY problem files used to generate the solid angle studies are listed in appendix I. All the solid angle studies shown used the spectrometer’s limiting apertures. Only blockage of the rays by the detectors and by vacuum vessels mentioned in section 5.2 were considered.

The SASP’s solid angle studies used several target spots. The target spot refers to the area in the target plane from which particles originate. For “high resolution” solid angles, a 2cm by 1cm target spot was used (the larger dimension is always x, the smaller y). For limiting apertures and the downstream ray bundles, a 10cm by 4cm target spot was used. The total acceptance of the SASP was checked with a 40cm by 20cm grid. The first two target spots had rays generated randomly over their areas while the third had rays generated at discrete points. All studies utilize the same method of pointing rays at the spectrometer. A ray generated at the target would be pointed randomly into a cone centred on the middle of Q1’s aperture. The cone was sufficiently large to fully illuminate the aperture.
6.1 High Resolution Solid Angle

The solid angle of a detector is a measure of the rate at which it detects particles. Suppose an imaginary sphere surrounds a point source radiating in all directions. The solid angle of a detector is the size of the cone whose base is the effective aperture of the device. If the detector accepted all the particles radiating from the source, it would have a solid angle of $4\pi$ steradians. A realistic solid angle for a spectrometer is on the order of a few millisteradians. The larger the solid angle of the detector, the more particles it accepts and the higher the data rates.

The effective aperture of a spectrometer is the cumulative effect of all the apertures the particles pass through. This can include the detectors if they do not accept all the particles that pass through the optics. Because there are dispersive elements in a spectrometer, the effective aperture and solid angle will be a function of momentum, usually having a peak near the central momentum. The momentum acceptance of a spectrometer is called its momentum bite and is usually measured as a percentage of the central momentum. The SASP was designed to have a momentum bite of (-10%,+15%).

A Monte Carlo simulation process is used to calculate the solid angle of a device by randomly sending rays into a cone that fully covers the device’s entrance aperture. The solid angle is then approximated by,

$$\Omega_{\text{device}} = \frac{n_*}{N} \Delta \Omega$$

where $N$ is the number of particles sent into a cone of size $\Delta \Omega$ and $n_*$ the number that get through. This is the Monte Carlo integration discussed in chapter 5. The error in
the calculated solid angle (from chapter 5) is:

$$\epsilon_\Omega = \sqrt{\frac{\sum f_i^2 - N\bar{f}^2}{N(N-1)}}$$

where \(f_i = 1\) if the particle makes it through the system and \(f_i = 0\) if it does not. Thus,

$$\bar{f} = \frac{1}{N} \sum_{i=1}^{N} f_i = \frac{n_*}{N}$$

and

$$\sum_{i=1}^{N} f_i^2 = n_*$$

since \(f_i^2\) is also either one or zero. The error is then,

$$\epsilon_\Omega = \frac{\Delta \Omega}{N} \sqrt{\frac{Nn_* - n_*^2}{N-1}}$$

and for \(N \gg 1\) and \(N \gg n_*\) this reduces to,

$$\epsilon_\Omega = \frac{\sqrt{n_*}}{N} \Delta \Omega$$

Typically, for \(N = 100,000\) and \(n_* = 10,000\), the error is about 1%. Rather than calculating the solid angle of the SASP for a point source, a distributed source that approximated the target spot size was used. This produces an averaged solid angle for that spot size and is a more reasonable measure of the data rates expected for the spectrometer. The spot size (based on a realistic beam spot at 4BT2) was 2cm vertical (\(x\)) and 1cm horizontal (\(y\)). This small size spot is called a “high resolution” spot because the aberrations that affect the momentum resolution of the spectrometer
increase with the target size. Larger target spots can be used where lower resolutions are acceptable.

Rays in the study were generated uniformly over the target spot. This is not realistic because most beam profiles are Gaussian with most of the particles at the centre of the spot. If the acceptance of the spectrometer drops off radially from the optical axis (and it does) a uniform spot size will underestimate the true solid angle.

Using the apertures described in section 5.2.2 with the high resolution spot size and 100,000 rays, the solid angles in table 6.1 were calculated. The EASY problem files utilized in this calculation are provided in appendix I. Table 6.1 contains solid angles for both the cruciform and circular vacuum pipes. For high resolution studies, the data for the circular vacuum pipe are more realistic since the cruciform pipe permits rays to pass through the poor field region of quadrupoles.

### 6.2 Limiting Apertures

The limiting apertures of the spectrometer determine if and where slit scattering may occur as well as determining the maximum solid angle of the system. Slit scattering occurs when particles graze a surface with a small change in momentum and trajectory.
Chapter 6. Monte Carlo Studies of the SASP

Table 6.2: Apertures hit by rays. The + indicates the top was hit and a - the bottom. A +- means that both top and bottom were hit. The apertures are shown in figure 5.3. Looking downstream, +x is vertically down and +y horizontally to the left.

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<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
<th>#9</th>
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</thead>
<tbody>
<tr>
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<td>x</td>
<td>y</td>
<td>x</td>
<td>y</td>
<td>x</td>
<td>y</td>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>-10%</td>
<td>+ + + + + + + +</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
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<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>+5%</td>
<td>+ + + + + + + +</td>
<td>+</td>
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<td>+</td>
<td>+</td>
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<td>+</td>
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<tr>
<td>+10%</td>
<td>+ + + + + + + +</td>
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<td>+15%</td>
<td>+ + + + + + + +</td>
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</tbody>
</table>

Such grazing degrades the momentum resolution of the spectrometer and the ability to trace the ray back to the target plane. Where sources of slit scattering are identified, passive and active collimators can be inserted to prevent it. A passive collimator is a thick aperture designed to prevent grazing. For high energy particles, such a collimator has to be quite thick. An active collimator is a scintillator that defines the aperture. When a particle passes through the scintillator, the unit fires and the event is vetoed by the data acquisition electronics. Too many particles hitting the active collimator saturates it and good events will be vetoed. While EASY is not designed to study the effect of particles grazing a surface (a program such as REVMOC [20] would have to be used), it can show the limiting apertures of the system and thus expose areas that may require further study.

The limiting apertures in the SASP were determined using the large 10cm by 4cm target spot although the high resolution spot (2cm by 1cm) produces similar results. Table 6.2 shows the apertures that rays hit. A (+) indicates that the top of the aperture was hit a (-) the bottom, and a (+-) both top and bottom. Aperture #1 is not included in the table since it is intentionally fully illuminated. All the apertures of the quadrupoles are illuminated and the constrictions of both vacuum boxes are
Table 6.3: Effective Apertures of SASP. Rays that do not pass through the approximately rectangular region (dimensions indicated in the this table) at the entrance to Q1 will not pass through the spectrometer.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$x_{\text{min}}$ (cm)</th>
<th>$x_{\text{max}}$ (cm)</th>
<th>$y_{\text{min}}$ (cm)</th>
<th>$y_{\text{max}}$ (cm)</th>
</tr>
</thead>
<tbody>
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<td>-12.19</td>
<td>12.45</td>
<td>-2.54</td>
<td>2.55</td>
</tr>
<tr>
<td>-5%</td>
<td>-11.94</td>
<td>12.45</td>
<td>-2.88</td>
<td>2.90</td>
</tr>
<tr>
<td>0%</td>
<td>-11.76</td>
<td>12.40</td>
<td>-3.44</td>
<td>3.45</td>
</tr>
<tr>
<td>+5%</td>
<td>-11.41</td>
<td>11.87</td>
<td>-3.81</td>
<td>3.79</td>
</tr>
<tr>
<td>+10%</td>
<td>-11.32</td>
<td>11.70</td>
<td>-3.93</td>
<td>3.99</td>
</tr>
<tr>
<td>+15%</td>
<td>-10.99</td>
<td>11.27</td>
<td>-3.73</td>
<td>3.72</td>
</tr>
</tbody>
</table>

illuminated in the nonbend high $x$ side of the dipole entrance. Although (#6) is illuminated, the number of rays that hit this aperture is insignificant. Essentially the limiting aperture in $x$ is the exit of Q2 (#4). Similarly the hits in apertures 7, 8, and 9 for the +15% rays are also very small. What is being observed is the inability of the dipole to deflect the extreme high momentum rays. As the momentum increases beyond +15%, these apertures dominate. In the nonbend plane the limiting aperture is the constriction of the exit vacuum box for all momenta. Figure 5.5 is a schematic representation of the vacuum system in the nonbend plane. The vacuum boxes intrude slightly on the ray bundle that would otherwise pass the dipole. The effect is on the order of a few percent and is unfortunate since ideally the dipole pole edges should form the aperture. These vacuum boxes are a potential source of slit scattering and active collimators will probably be installed at the entrance constriction. Scattering from the exit constriction can be detected by projecting a ray back from the focal surface detectors to the aperture making a collimator unnecessary.

The effective apertures at the entrance to Q1 for the different momenta are provided in table 6.3. The effective aperture tends to narrow in the $x$ direction and widen in the $y$ direction with increasing momentum. The aperture widens in the $y$ direction
because at higher momenta, the rays are deflected less in the y plane and fewer cross over the median plane and hit aperture #9, the exit vacuum box constriction. The aperture shrinks in the x direction because Q1 does not focus the rays as much and more hit Q2.

6.3 Target Plane Acceptance

The target plane acceptance describes how the solid angle of the spectrometer is a function of target plane position. For most experiments, which have a small beam spot on the target, the acceptance is not important. There are some experiments, however, which have large target spots and need a large acceptance. Two classes of these are (p, n) and (n, p) experiments. A (p, n) experiment hits the target with a proton beam and detects the emitted neutrons. An uncharged neutron cannot be detected directly by the spectrometer so it is converted into a proton in a hydrocarbon cell. The proton knocked out of the conversion cell by the neutron is detected in the spectrometer. A (n, p) experiment first converts the incident proton beam to a neutron beam with a conversion cell. The neutron beam hits the target and a proton is given off which is detected by the spectrometer. During either of these processes, the initially small beam will spread out becoming quite large. The larger the spectrometer acceptance, the greater the data rates for these experiments.

Maps of the SASP’s acceptance for different momenta are shown in figure 6.1 as contour plots (a two dimensional histogram of the 0% acceptance is shown in figure 6.2). The maps were created by generating 2000 rays at each point on a 2cm square grid of size 40cm (x) by 20cm (y) centred on the origin of the target plane. The solid angle at each point on the grid is shown on the contour plots as a function of the target plane position. Irregularities are caused by the poor statistics gathered from such small ray bundles. For all momenta, the SASP acceptance is approximately a rectangle with size
Figure 6.1: Target plane acceptance contour plots for -10%, -5%, 0%, +5%, +10%, and +15%. The target plane is located with its normal pointing along the spectrometer's optical axis. Positive x points vertically downward and positive y horizontally to the left (when looking downstream); the origin is at the optical axis. The irregularities are due to the low statistics used in the calculation.
$\pm 16\text{cm}$ in $x$ and $\pm 6\text{cm}$ in $y$. At low momentum, the acceptance is peaked at the origin (the optical axis) and falls off uniformly as distances increase. At high momentum, the acceptance becomes flatter and narrower (in $y$) with a small residual step around the edge.

Changes in acceptance can be explained by the apertures in the system and how focussing varies with momentum. The acceptance narrows with increasing momentum because the higher momentum rays are less focussed in the nonbend plane by the quadrupoles and tend to hit the dipole entrance pole edge. In the $x$ direction, the limiting aperture for all momenta is the exit of Q2. This aperture does not change significantly with momentum and the $x$ acceptance of the SASP remains approximately constant.

The acceptances shown here can be misleading. They were generated assuming only the SASP with the apertures described in section 6 were blocking the rays. As well, nothing is said about the quality of these rays. The SASP was optimized for rays from a small target spot and the matrices used to simulate the SASP were fitted to
this region. Rays from the edges of the acceptance will be of poor quality.

6.4 Downstream Ray Profiles

Design of the vacuum boxes and detectors of the spectrometer requires knowledge of the exact profiles of the ray bundle to be detected after the SASP. This is accomplished by drifting the ray bundles "downstream" (vertically upward in the laboratory) from the exit of the spectrometer. Figure 6.3 shows the -15%, -10%, +15% and +20% ray bundles drifted four metres downstream from the dipole exit in the bend and nonbend planes. Also shown on the diagram are the dipole exit and the focal surface (see section 6.5). These calculations used the large target spot (10cm by 4cm).

The SASP design specifies a momentum range of -10% to +15% but figure 6.3 shows that if the detectors are made just large enough to accommodate these ray bundles, a significant number of rays outside this range will pass through. The detectors will act as limiting apertures, reducing the effective solid angle for these extreme momenta.

6.5 Mapping the Focal Surface

For an ideal spectrometer the focal surface is the surface on which rays of constant momentum focus to a line (see figure 6.4a). One coordinate of the surface would have a one to one mapping with the momentum of the rays while the other coordinate would relate information about the ray's initial trajectory. Ideally, the focal surface should be a plane, then the momentum of the ray can be read off as the output of a wire chamber positioned on the plane. More likely, the focal surface will be curved but provided the rays still come to a sharp focus on the surface, the momentum can be determined simply by ascertaining the intersection of the ray with the surface.

It is not perfectly monochromatic rays that come to a focus. Consider a flat focal
Figure 6.3: Downstream Ray Profiles after the dipole exit for the large target spot. a) Bend Plane. b) Nonbend Plane. The bundles shown are: -15% (long dash), -10% (solid), +15% (dot-dash), and +20% (short dash). The focal plane is shown in the bend plane figure. Note in a) that the waists of the ray bundles lie above the focal surface. This is because the focal surface was calculated using the smaller high resolution spot size which produces ray bundles that focus closer to the dipole.
Figure 6.4: a) An ideal focal surface. One coordinate of the surface maps directly into the momentum of the ray while the other depends on the initial trajectory before the spectrometer in some way. b) A true spectrometer will not bring the rays to a sharp focus, instead it will be blurred.

surface (plane) with the x coordinate (call it $x_2$) directly proportional to the momentum of the ray. Then from appendix B, equation B.8,

$$x_2 = Mx_1 + D(\delta_S + A\delta_B)$$

where $M$ is the magnification of the system, $D$, the dispersion, $x_1$ the target plane position, $\delta_S$ the momentum of the particle being measured in the spectrometer, $\delta_B$ the momentum of the beam particle, and,

$$A = \frac{P_B \partial P_3}{P_0 \partial P_1}$$

$P_B$ and $P_0$ are the central momentums of the beam and scattered particles respectively while $P_1$ and $P_3$ are the absolute momentums of the beam and scattered particles respectively. Suppose that to map the focal plane, the beam is injected directly into
the spectrometer so,

\[ P_B = P_0 \]
\[ \frac{\partial P_3}{\partial P_1} = 1 \]
and \( A = 1 \). Thus,

\[ x_2 = Mx_1 + D(\delta_S + \delta_B) \]

Now suppose the beam was perfectly monochromatic \((\delta_B = 0)\). One still wants \( x_2 \) to be proportional to \( \delta_S \).

\[ x_2 = Mx_1 + D\delta_S \]

But there is still the \( x_2 \) dependence on \( x_1 \) due to the \( M \) term. This is resolved by having a dispersed beam meeting the condition,

\[ 0 = Mx_1 + D\delta_B \]

That is, once the \( x \) position in the target plane is chosen, an additional term,

\[ \delta_B = -\frac{M}{D}x_1 \]

is added to the original momentum, \( \delta_S \) that is being measured.

In an actual spectrometer, monochromatic rays will not come to a line focus, rather, they will be smeared out (figure 6.4b) making the focal point difficult to define. For purposes of mapping the SASP's focal surface, the focal point of a ray bundle is defined as the location where the bundle has a "waist" in \( x \). An additional complication in mapping the focal surface is that the nonmomentum coordinate of the surface \((y)\) is not uniquely determined by one component of the initial ray. That is, the first order
equation for $y$ is,

$$y_2 = R_{33}y_1 + R_{34}\phi_1$$

so $y$ is a function of both $y_1$ and $\phi_1$. To map the $y$ extent of the focal surface, $y_1$ and $\phi_1$ are incremented in discrete steps. Figure 6.5 shows the method for mapping the focal surface. Keeping $y_1$ fixed at zero (since $y$ is usually small), $\phi$ is shifted across the spectrometer aperture in several steps for each momentum. For the SASP, $D = -0.59892$ and $M = 2.81323$ giving $\frac{D}{M} = -4.697\text{cm/\%}$.

Figure 6.6a shows a plot of the positions of the waists of each ray bundle sent through the SASP in the $(x, y)$ plane. Figure 6.6b shows a perspective view of the focal surface generated by fitting a two dimensional polynomial to the points on the surface. The ripples are an artifact of the fitting routine. The edge of the focal surface is defined to be where the solid angle falls to approximately one third of the maximum. The method used here to define the focal surface differs from that used by RAYTRACE. For a given momentum, RAYTRACE defines the focal point as the intersection of two rays injected into the spectrometer: one pointed along the optical axis and the other pointed slightly off the axis. A comparison of these methods is shown in figure 6.7.

The solid line shows a cross section of the Monte Carlo defined focal surface in the median plane of spectrometer ($y = 0$). The different focal surfaces produced by the two methods are caused partly by the different ways of defining the surface and partly by the inaccuracies in the model of the optics used by EASY.

While basically flat in the nonbend ($y$) plane, the curvature of the SASP’s focal surface in the bend plane presents a problem. Ideally, the exit window of the dipole vacuum box would be laid directly on the focal surface. This minimizes the effect of multiple scattering in the exit window on track reconstruction since regardless of the change of ray’s trajectory by the window, it will still be traced back to the point where it crosses the focal surface. It is probably not possible to lay a thin exit window along the
Figure 6.5: Mapping the focal surface. A fan of rays with a constant momentum and that varies continuously in $x$ and $\theta$ (vertical dimensions) but constrained in $y$ and $\phi$ is "injected" into the spectrometer simulation. The ray bundle is focussed to a waist by the spectrometer optics and the location of this waist is defined as a point on the focal surface. By repeating this process for many different values of $y$ and $\phi$, a map of the focal surface is built up.
Figure 6.6: SASP Focal Surface. a) A projection of the focal surface onto the \((x,y)\) plane (horizontal in the lab with \(x\) in the spectrometer's bend plane). Each point is the focal point of a ray bundle (500 rays) with a different \(\delta_S\) and \(\phi_1\). \(\delta_S\) was stepped across the focal surface (\(x\) direction) in 2% steps from -20% to +24%. For each value of \(\delta_S\) 19 different values of \(\phi_1\) were used. b) A perspective view of the focal surface. The ripples are an artifact of the algorithm used to fit a polynomial surface to the points in (a).
Figure 6.7: The RAYTRACE focal surface (points) compared with the focal surface defined here (line). The RAYTRACE points are (from left to right): -10%, -5%, 0%, +5%, +10%, +15%.

curved focal surface since air pressure would cause it to buckle and kink. An acceptable compromise is struck by laying the window tangent to the focal surface at the point where the central ray crosses it. The exact position of this will be determined by field mapping of the completed spectrometer rather than with the Monte Carlo simulation.
Chapter 7

SASP Detectors

The SASP design is optimized for a momentum bite of (-10%,+15%) but these are not the extreme momenta that can pass through the system; higher and lower momentum rays will get through but with a reduced solid angle. After the dipole exit, these rays spread out over a larger range than the design (-10%,+15%). To detect all rays leaving the spectrometer would require an enormous detector array and costs would scale roughly linearly with size. The SASP's detector system will capture the rays within the designed momentum bite. Outside this range, the detector array will become one of the spectrometer's limiting apertures. Figure 7.1 shows the proposed SASP detector array in relation to the focal surface and the dipole exit. The detector stack consists of three Vertical Drift Chambers (VDCs) set at an angle of 46° to the horizontal followed by a six segment scintillator hodoscope parallel to the VDCs acting as the main trigger for the stack. Mounted further downstream is a horizontal scintillator followed by either a Cerenkov counter or second horizontal scintillator. The detectors and electronics are described more fully in [21]. Not shown in figure 7.1 is the SASP's Front End Chamber (FEC) located between the target and the first quadrupole.

The three identical VDCs are stacked tightly against each other and are set right against the exit window of the vacuum system. The exact exit window position will be determined after the finished spectrometer has its field mapped and the position of the
Figure 7.1: The SASP detector array shown with respect to the dipole exit, the focal surface, and the Proton Hall roof
focal surface determined. The intent is to have the exit window of the vacuum tank lie on the focal surface. This will minimize the effect of multiple scattering in the window on tracing the rays back to the focal surface. In practice it is difficult to curve the exit window to conform to the curvature of the SASP’s focal surface (figure 7.1) so a flat window will be laid tangent to the intersection of the central ray with the focal surface.

### 7.1 Vertical Drift Chambers

A Vertical Drift Chamber [22] is a type of wire chamber that detects both the position and angle of a particle track. Set at approximately 45° to the nominal particle track, the VDC wire plane is composed of sense and guard wires sandwiched between cathode planes (figure 7.2) producing *drift cells* having uniform fields except in the immediate vicinity of the sense wires. When a particle passes through three adjacent cells, it leaves a trail of electrons that drift to the sense wires at a constant velocity (because of the
uniform fields). The position and angle of the particle track through the wire plane can then be reconstructed from the drift times. Because the particle must pass through at least three cells, there is a minimum track angle to the normal that the VDC can accept.

\[
\theta_{\text{min}} = \tan^{-1} \frac{w}{h}
\]

where \( w \) is the width of a cell and \( h \) is the height. The VDC cell geometry and tilt angle (\( \theta_{VDC} \)) are selected so that all the desired rays can be detected, for the SASP, the minimum angle was determined with the Monte Carlo simulation. The worst case is for -10% rays, and \( \theta = 28.38^\circ \). The VDCs were thus designed for a minimum angle of 28° and the chamber tilt (\( \theta_{VDC} \)) increased from the nominal 45° to 46°. This gives an error margin of approximately a degree. The SASP VDC's are almost identical in design to the ones currently employed on the MRS spectrometer [23]. The differences being that the SASP VDCs are larger to cover the SASP's bigger focal plane and the drift cells are taller to allow a smaller \( \theta_{\text{min}} \).

### 7.1.1 VDC Construction

Each of the SASP's three VDC's has two wire planes. The \( x \) plane has wires running in the \( y \) direction and gives a position in the chamber's \( x \) direction. The \( u \) plane wires are tilted at an angle of 30° to the \( x \) plane wires. Using the \( u \) and \( x \) coordinates the \( y \) coordinate of the ray can be reconstructed.

Figure 7.3 shows a cross section of a VDC. It consists of a mylar gas window followed by an aluminized mylar cathode plane, the \( x \) wire plane, a cathode plane, the \( u \) wire plane, a cathode plane, and another gas window. The windows and cathode planes are made out of 0.001" (0.00254cm) mylar. Between each plane is a 0.625" (1.5875cm) gas layer. The gas is under a slight pressure, causing the windows to bulge out approximately 0.5" (1.27cm) making the outer gas layers 1.125" (2.8575cm). The
gas mixture used is 50% isobutane \((C_4H_{10})\) 50% argon. The position resolution of the VDC's is 0.15mm and the angular resolution is 0.5mrad. The minimum detectable angle to the normal is 28°.

### 7.1.2 VDC Modelling

Each layer of the VDC is modelled in EASY as a drift region with an appropriate radiation length which determines the extent of multiple scattering of the ray. The simulation only scatters the ray and does not consider energy loss. The wire planes are modelled with the `vdc` option of EASY's `pos_sensor` device. This option outputs both a position and angle for the ray passing through the wire plane. If the angle of the ray to the normal through the VDC is less than the \(\theta_{\text{min}}\) parameter, the `pos_sensor` considers it a bad event and does not fire. An actual wire chamber would output wire numbers and timing information from which the position and angle of the track is reconstructed. The `pos_sensor` device skips all this and directly outputs a position
in cm and an angle in mrad. The position and angle reported by EASY will not be the exact values for the track. Rather, there will be random errors following normal distributions added to the detector outputs. The sizes of the position and angle errors are given as parameters for the pos_sensor. The position errors can be "turned off" using the command @poserror. For the level of simulation that EASY is intended for, these outputs are adequate. (It is possible that a more advanced simulation would produce signals closer to the true outputs of the detectors. These could then be fed into a simulation of the data acquisition electronics.) The ray is completely unaffected by the pos_sensor unlike a true wire plane where there is a small chance that the ray can hit the wire and be deflected. Finally, the user can specify the efficiency of the wire chamber which determines the probability that it will fire for each event.

7.2 Front End Chamber

The SASP has a Front End Chamber (FEC) between the target chamber and the first quadrupole. Having a FEC improves the trace back of the trajectory to the target and makes calibration of the system easier. However, an FEC degrades the momentum resolution of the spectrometer and can not take a high flux of particles. This severely limits the data rate for the system. Unlike the MRS, the SASP can avoid this problem by retracting the FEC and the operating without it. The SASP optics allow good operation with or without an FEC.

7.2.1 FEC Construction

The SASP FEC will be similar to the MRS FEC and consists of a sandwich of mylar, gas, and wire planes. A mylar window is followed by a gas layer followed by an aluminized mylar cathode, another gas layer, an $x$ wire plane, gas, a cathode, gas, an $x'$ plane, gas, a cathode, gas, a $y$ plane, gas, a cathode, gas, a $y'$ plane, gas, a cathode, gas,
and an exit window. Each layer of gas is 0.5" (0.127cm) thick but the windows bulge outward under pressure adding an extra 0.5". The windows and cathodes are made out of 0.001" (0.00254cm) mylar. The gas used is isobutane at one third atmosphere. The position resolution of the wire planes is something less than 0.5mm. The chamber is inserted into the vacuum pipe between the target and the quadrupole (hence the windows bulge outward). The MRS chamber can handle data rates of some one to two million events per second before saturating.

7.2.2 FEC Modelling

The FEC is modelled similarly to the VDCs, each region represented by a drift space with the appropriate radiation length. The wire planes are simulated with the pc option of pos_sensor which outputs the position, but not the angle of the ray.

7.3 Track Reconstruction

The trajectory of a ray after the dipole can be reconstructed using the $x$ and $u$ positions from any pair of the three VDCs in the detector stack. Three VDCs are used for redundancy and to provide information on detector efficiencies. EASY has built into it a generic function called vdc2 which will automatically calculate the trajectory of the ray at an arbitrary reference plane called the canonical plane. This calculation uses the outputs of the detectors and so includes the position errors when reconstructing the trajectory. The algorithm for vdc2 is given in appendix J.
Chapter 8

Spectrometer Models

This chapter (along with appendices K and L) describes how the SASP and MRS spectrometers are modelled using EASY. For a full description of EASY and an explanation of the various constructs found in this chapter and the appendices see EASY, A Monte Carlo Simulator [19]. The models of the spectrometers presented are intended to be run as "black boxes", where details such as the exact dimensions and compositions of detectors are preset. The SASP and MRS definitions are stored in files that are referenced by the simulation using the models. To update the models only one set of files must be changed rather than every EASY problem using them. The models of the SASP and MRS are complex and consequently slow to execute. Many problems do not need as thorough a simulation of the spectrometers as provided here and would benefit from simplification.

The SASP and MRS models contain the different regions that the particles pass through, but not necessarily with the correct geometry. While the model may describe the separation between wire chambers as, say, 5.7cm, the actual value may be different. The SASP model is not truly representative because the spectrometer has not been built and various features of the design may change. For the MRS, several dimensions were estimated. Final measurements of the spectrometer will be made in the future. While the models are known to be inaccurate, this does not make them useless. For
example, provided the geometry of the models are used, rays can still be traced back to the focal surfaces of the SASP and MRS. Inaccurate geometry will to a small degree adversely affect results when studying the effects of multiple scattering and detector errors on ray reconstruction.

### 8.1 SASP Model

EASY's SASP model is based upon the device `sasp_optics`, described in section 5.2, which transports a particle from the target plane to the exit of the spectrometer. The other components of a complete SASP model are the FEC and the focal plane detectors (also called the detector stack). The FEC is a small wire chamber located between the target chamber and the first quadrupole. The chamber contains four wire planes named x0, x0p, y0, and y0p ('p' is for 'prime'). The detector stack consists of three VDCs placed directly against the exit window of the dipole vacuum system which is located tangent to the focal surface. Adjacent to the wire chambers is a six segment set of "trigger paddle" scintillators and located horizontally above (downstream of the dipole) are two other scintillators, a Cerenkov counter is not used in the model. The VDCs are labelled in order of distance from the vacuum exit window, VDC1, VDC2, and VDC3. The VDCs have x and u wire planes called x1, u1, x2, u2, x3, and u3. EASY's SASP model is contained in the two files `SASP_DEF.DEV` and `SASP_DEF.DAC` which are listed in appendix K.

As indicated in section 5.2, while `sasp_optics` provides a reasonable approximation of the SASP's RAYTRACE optics design, it will differ significantly from the true SASP optics when the spectrometer is run at high magnet settings. Due to saturation distortions of the dipole's magnetic field, the actual spectrometer will have a focal surface that shifts position with excitation. Thus EASY's model of the SASP should be thought of as a reasonable approximation that is much better behaved than the real
CHAPTER 8. SPECTROMETER MODELS

spectrometer.

8.1.1 SASP Device Definitions

In EASY, complex pieces of equipment (called devices) can be built up out of simpler devices. Commencing with basic devices such as \texttt{drift}, \texttt{scint}, and \texttt{pos\_sensor}, elaborate detector arrays can be constructed. For example, the device \texttt{sasp} is defined as,

\begin{verbatim}
define sasp{
    sasp\_fec
    sasp\_optics
    sasp\_stack
}
\end{verbatim}

This definition of the SASP is placed in the user's device file. The three devices used in the definition of the SASP (\texttt{sasp\_fec}, \texttt{sasp\_optics}, and \texttt{sasp\_stack}) are pre defined in the file \texttt{SASP\_DEF.DEV} (see appendix K for a listing), and are included in the user's problem description using the \#\texttt{include} command. Of the three components of \texttt{sasp}, only \texttt{sasp\_optics} is a basic device built into EASY. The device \texttt{sasp\_stack} describes the spectrometer system from the exit of the dipole through the vacuum window, the three wire chambers, and the three scintillators. Each wire chamber is a device called \texttt{sasp\_vdc} made out of simpler devices; and the six segment trigger paddles are approximated as a single large scintillator. A series of complex rotations and translations (using the "devices" \texttt{rotate} and \texttt{translate}) are used to change the coordinate systems to produce the correct orientation of the wire chambers and scintillators.

Modelling the front end chamber may seem difficult at first glance since it must come between the target and the spectrometer, and \texttt{sasp\_optics} requires the particle start at the target plane for the transfer matrices to work. This problem is solved by first drifting the particle to and through the FEC where it undergoes various multiple
scattering and then projecting the new trajectory back to the target plane using the device backtrack. Because sasp.fec is modular, the simulation of the SASP can also be run without a FEC by deleting it from the SASP description.

### 8.1.2 SASP EDAC Definitions

In EASY, data is gathered and processed in an experiment using EDAC (EASY Data ACquisition system) instructions in the problem’s data acquisition file. EDAC instructions are of two types, definitions and actions. Definitions can be of three types, addresses, booleans, and functions. Addresses tell EASY which locations in the system to gather data from. Booleans are logical expressions that determine if a specific combination of events happen in the system. For example, a boolean could be defined to be true only if all the scintillators in the SASP detector stack fire. Functions process data from the system after each particle has passed through it. For example, a function can be defined to take the data from two VDCs in the detector stack and calculate the trajectory of the ray at the focal plane.

Actions are used to display and store data collected from the experiment; there are four basic types. The actions hist and scat will histogram and scatter plot data from any point in the system. The location to gather data can be specified with either a device path or with an address. The address is really just a short hand for naming points in the system to collect data from. The action tape records data to a file where it can be examined by different programs after the simulation is completed. The last type of action is called a conditional action. This is like the if then else construct found in most programming languages. It checks to see if a condition is true and if so, processes the specified actions. The condition can be made up of booleans defined prior to the conditional action or of references to points in the system. Like the address, the boolean is a short hand for long expressions. Conditional actions allow the user
to set up simple logic in his data acquisition system. For example, he can instruct EDAC to only histogram the x readout of VDC1 if both the FEC and the downstream scintillators fire. This way the user can study the efficiencies and performances of compound systems such as the SASP detector stack.

To simplify the user’s data acquisition files, some commonly used addresses, booleans, and functions have been defined in the file SASP.DEF.DAC (listed in appendix K). By including this file in his dac file (using the #include command), the user can use these definitions. There are ten position sensors and three scintillators that signals can be taken from in the SASP system (although there is nothing stopping you from taking data from anywhere else as well). The front end chamber has four wire planes; two x and two y. Each of the three VDC’s has an x and a u plane. The three scintillators consist of the trigger paddles (TP) next to the VDC’s and the two downstream scintillators S1 and S2. Table 8.1 lists the addresses of the various detectors in the SASP system. To create a scatter plot of the x readout in VDC1 versus the u readout in VDC3 the following statement would be placed in the dac file.

Table 8.1: SASP Detector Addresses in EASY
Table 8.2: SASP detector booleans in EASY.

\[
\begin{align*}
sasp_{vdc1} & \text{_fired} \\
sasp_{vdc2} & \text{_fired} \\
sasp_{vdc3} & \text{_fired} \\
sasp_{fec} & \text{_fired} \\
sasp_{tp} & \text{_fired} \\
sasp_{scints} & \text{_fired} \\
sasp_{vdc5} & \text{_fired} \\
sasp_{trigger} &
\end{align*}
\]

The $s$ following an address tell’s EDAC the location is a sensor; the ranges of the scatter plot axes are specified with the \texttt{min} and \texttt{max} statements.

Because the detectors are not one hundred percent efficient, a particle traversing the SASP’s detector array will not necessarily trigger all the detectors. In a real experiment, the acquisition electronics would be wired to look for certain combinations of detectors firing before processing the data. This can be simulated in EASY using booleans and logical expressions. Using \textit{conditional actions} in EDAC, an experimenter can insist on any desired combination of detectors firing before processing the data. Listed in table 8.2 are the standard boolean expressions defined in SASP.DEF.DAC. The booleans \texttt{sasp\_vdc1\_fired}... are true if both the $x$ and the $u$ planes of each VDC fire. The boolean \texttt{sasp\_fec\_fired} is true if at least one $x$ plane and one $y$ plane in the FEC fire. The boolean \texttt{sasp\_trigger} is used if the processing is to wait for all the detectors to fire; it is true if all the VDCs, the FEC, and all three scintillators have fired.
Table 8.3: Reconstructing the SASP canonical ray using the generic function, vdc2. The components of the canonical ray, \( \vec{r}_c \) are calculated by vdc2 and made available to the user as outputs 0 — 3 of the function.

8.1.3 SASP Track Reconstruction

As discussed in section 7.3, EASY has a built in generic function called vdc2 which takes wire plane readouts from two vertical drift chambers and reconstructs the trajectory of a ray at a canonical plane. The SASP dac definition file, SASPJDEF.DAC has defined two functions using vdc2. The function sasp_vdcl2 takes the outputs of VDC1 and VDC2 and reconstructs the trajectory of the canonical ray, \( \vec{r}_c \) at the horizontal plane at the exit of the SASP dipole (\( z_c \) points vertically upwards along the optical axis). Similarly the function sasp_vdcl3 reconstructs the canonical ray at the same plane using the outputs of VDC1 and VDC3. Table 8.3 describes the outputs of sasp_vdcl2; the outputs of sasp_vdcl3 are similar.

8.2 MRS Model

EASY’s MRS model is basically the same as the SASP model except that the spectrometer’s geometry is different. The MRS is a Quadrupole - Dipole (QD) spectrometer bending the rays nominally 60° upwards from the horizontal into a detector array located approximately 4.2m from the dipole exit. The MRS is shown in figure 1.1. The MRS’s focal surface is approximately a plane oriented at 45° to the the central ray. The focal plane detectors consist of two VDCs and a set of trigger paddles located immediately after the vacuum box window which is situated at and parallel to the focal
surface. Downstream of the VDCs and trigger paddles can be either a focal plane polarimeter or a pair of scintillators. Like the SASP, the MRS has a front end chamber located between the target and the quadrupole, although unlike the SASP, the MRS cannot run without the device.

The MRS optics are modelled in EASY with the device, mrs_optics and is simulated with first and second order transfer matrices generated by the program TRANSPORT (the MRS's optics are simpler than the SASP's so second order transfer matrices are adequate). There are five apertures checked in the MRS optics: the entrance and exit of the quadrupole, the entrance and exit of the dipole, and the exit window of the vacuum box at the focal plane. The MRS's FEC is the same design as the SASP's but the VDCs differ from the SASP VDCs. Specifically, the layouts of the chambers are the same but the MRS VDCs have narrower gas spaces than the SASP, are oriented at 45° to the optical axis, and are much smaller than the SASP VDCs (100cm by 35cm compared to the SASP's 200cm by 40cm).

The EASY model of the MRS is structured much like that of the SASP. For simplicity, the MRS model uses two downstream scintillators rather than the focal plane polarimeter. The MRS device definitions are in the file MRS.DEF.DEV while the EDAC definitions are in MRS.DEF.DAC (both files are listed in appendix L). The MRS detector addresses and detector booleans are listed in tables 8.4 and 8.5 respectively. The function mrs_vdc12 calculates the canonical ray at a plane perpendicular to the optical axis at the exit of the MRS dipole ($z_c$ is oriented at 60° to the horizontal). The outputs of mrs_vdc12 are listed in table 8.6.
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<table>
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<th>mrs.x0</th>
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<td>mrs.x0p</td>
</tr>
<tr>
<td></td>
<td>mrs.y0</td>
</tr>
<tr>
<td></td>
<td>mrs.y0p</td>
</tr>
<tr>
<td>VDC #1:</td>
<td>mrs.x1</td>
</tr>
<tr>
<td></td>
<td>mrs.u1</td>
</tr>
<tr>
<td>VDC #2:</td>
<td>mrs.x2</td>
</tr>
<tr>
<td></td>
<td>mrs.u2</td>
</tr>
<tr>
<td>TP:</td>
<td>mrs.tp</td>
</tr>
<tr>
<td>S1:</td>
<td>mrs.s1</td>
</tr>
<tr>
<td>S2:</td>
<td>mrs.s2</td>
</tr>
</tbody>
</table>

Table 8.4: MRS Detector Addresses in EASY

| mrs.vdc1.fired |
| mrs.vdc2.fired |
| mrs.fec.fired  |
| mrs.tp.fired   |
| mrs.scints.fired|
| mrs.vdc1s.fired|
| mrs.trigger    |

Table 8.5: MRS detector booleans in EASY.

<table>
<thead>
<tr>
<th>$x_c$</th>
<th>&lt;mrs.vdc12(0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_c$</td>
<td>&lt;mrs.vdc12(1)</td>
</tr>
<tr>
<td>$y_c$</td>
<td>&lt;mrs.vdc12(2)</td>
</tr>
<tr>
<td>$\phi_c$</td>
<td>&lt;mrs.vdc12(3)</td>
</tr>
</tbody>
</table>

Table 8.6: Reconstructing the MRS canonical ray using the generic function, vdc2.
Part III

Summary and Appendices
Chapter 9

Summary

Part I of this thesis investigated various aspects of the magnetostatic design of the SASP clamshell dipole. Using the magnetostatic code, POISSON, the excitation behaviour of the dipole's internal field was studied and its effects on the spectrometer performance determined. The dipole's aperture fringe fields were characterized and a specification for the aperture steel shape and field clamps that produces the fringing fields specified in the optical design was presented. The accuracy and applicability of POISSON to the SASP dipole, was assessed by limited studies of the LAMPF Low Energy Pion Spectrometer.

The wedge shaped air gap of a clamshell dipole produces a magnetic field that varies as the inverse of the distance from the apex of the wedge and as consequence the dipole does not enter saturation uniformly over the pole piece causing the field to sag on the high field side of the dipole at high excitations. This phenomena produces a 0.979% distortion of the SASP dipole's field. While its primary effect on the spectrometer's behaviour is to shift the focal surface, the changing field profile necessitates recalibration of the spectrometer for each momentum setting.

The dipole apertures are used to correct for aberrations in the spectrometer optics with virtual field boundary shapes described by fifth and sixth order polynomials. Although the SASP's optical design allows for the effects of fringe field in the dipole's
apertures, it did not address the steel geometries required to produce the required VFB and fringe field shapes. Characteristics of the fringe fields were studied by modelling cross sections of the dipole apertures were modelled with the two dimensional magnetostatic program POISSON. To produce the desired field shapes, a device called a field clamp was employed to remove excess flux from the aperture and limit the extent of the fringe field. Aided by a Rogowski pole edge bevel, the field clamp also prevents the aperture fringe fields from changing shape with magnet excitation. The POISSON simulations were used to fit the dipole steel and field clamp geometries to the RAYTRACE field specifications. The curves formed by the dipole pole edge and the leading edge of the field clamp (called the Pole Edge Boundary and Field Clamp Boundary) were fitted with polynomials similar to those used to describe the virtual field boundaries. These curves will be used to layout the dipole pole pieces and field clamps for machining. The fringe fields produced by the geometry described by the PEB and FCB do not produce VFBs identical to the ones specified in the SASP optics design. The aperture geometry specified in this thesis produces a 10% loss in the spectrometer's resolution. Before dipole construction commences further investigation to reduce the resolution loss due to the apertures is required.

Studies of the dipole utilized a two dimensional magnet simulation program to model a three dimensional object. Simulating cross sections through the dipole cannot capture such three dimensional behaviour in the clamshell as the distribution of flux in the return yokes or the way flux is distributed in the concave dipole apertures. POISSON's performance was assessed by studying the clamshell dipole used for LAMPF's Low Energy Pion Spectrometer. The LEPS dipole differed from the SASP and had limited maps of the magnetic field so quantitative comparison with POISSON was difficult. The LEPS data showed that POISSON reproduces the general behaviour of a clamshell's fringe fields, but did not indicate whether POISSON is quantitatively pre-
cise. POISSON had a tendency to produce erratic results when small perturbations were made in the magnet geometry. POISSON has been extended to its limit fitting the SASP dipole aperture geometries. Study of subtle effects would probably produce meaningless results. Design of the SASP field clamps must allow for alterations to permit the fringe fields to be fine tuned. A procedure for using maps of the dipole field must be developed to facilitate modification of the clamps.

In Part II of this thesis a Monte Carlo simulation of the Second Arm Spectrometer was developed. The simulation program, called EASY (Equipment and Acquisition SYstem simulator), was used to study the extent and profiles of rays transiting the spectrometer to help facilitate positioning vacuum boxes and detectors. After the commissioning of the spectrometer, EASY will be utilized to assess experimental arrangements of both the SASP and the DASS (Dual Arm Spectrometer System). Monte Carlo studies performed in this work included: solid angle of the spectrometer as a function of momentum, limiting apertures in the spectrometer system, target plane acceptance, downstream ray profiles, and mapping of the focal surface.

Rays passing through the spectrometer are alternately focussed and defocused by the quadrupoles. The ray bundle leaves Q2 diverge in the dipole bend plane and converge in the nonbend plane. After passing through the dipole, the rays are deflected and focussed in the bend plane and come to a focus at the spectrometer’s focal surface. In the nonbend plane, the rays — only marginally effected by the dipole — cross over the median plane and diverge after the dipole. The point of cross over (which can be thought of as a focus in the nonbend plane) varies with momentum. The high momentum rays are deflected least and cross the median plane near the dipole exit while the low momentum rays cross near the entrance. The limiting aperture of the spectrometer in the bend plane is the exit of Q2 which restricts the amount of divergence the ray bundle has after leaving the quadrupole. In the nonbend plane, the
constriction of the dipole's entrance vacuum box (where it projects under the dipole coil) is the limiting aperture for the high momentum rays while the exit vacuum box constricts low momentum rays. Ideally, the limiting apertures in the nonbend plane should be the entrance and exit of the dipole but space constrictions have forced the vacuum boxes to protrude slightly into the ray bundle. The solid angle caused by the vacuum boxes is only a few percent smaller than what it would be if the dipole pole piece formed the limiting apertures.

The solid angle of the spectrometer could be increased 20% by switching from the conventional circular vacuum pipe through the quadrupoles to a cruciform shaped pipe which follows the quadrupole's pole tips. The SASP's solid angle using the cruciform vacuum box varies from 15.4 msr at the central momentum to about 13 msr at the extreme design momentums ($\delta = -10\%$ and $\delta = +15\%$). These numbers should be taken with caution since some of the rays traversing the edges of the cruciform vacuum box will pass through regions of poor magnetic field which leads to a degradation in the spectrometer's resolution. A more realistic solid angle for "high resolution" rays is 12.5 msr at the central momentum and about 10 msr at the extreme momenta.

The target plane acceptance of the spectrometer is the region in the target plane from which rays will pass through the spectrometer. For the SASP, this is a rectangle approximately 8cm horizontally and 16cm vertically, peaked at the centre and falling to zero at the edges. In this enormous spot, the centre is approximately flat and accommodates reasonably well the largest spot size (4cm by 10cm) likely to be used in the system. Spectrometer optics were optimized for a smaller spot size so the SASP will not achieve its full resolution using such a spot.

While the MRS has a flat focal surface (plane), the SASP's focal surface is akin to a paraboloid. This makes the process of momentum calculation for the SASP more complicated than for the MRS. As well, it is not possible for the spectrometer's vacuum
window to follow the focal surface (desirable to minimize the effects of multiple scattering on the spectrometer resolution). The focal surface encompasses a much larger momentum range than the SASP optics were designed for — approximately -20% to +25%. Rays leaving the dipole exit illuminate much of the region above the spectrometer. It is not practical to construct a detector array that detects all these rays, so the array is designed to accept only the full ray bundle from the design momentum bite. Even this requires wire chambers 2m long and 40cm wide.

EASY’s model of the SASP was based on first and second order transport matrices produced by RAYTRACE. These matrices proved inaccurate and many higher order terms were added. The final matrix model behaves similarly but not identically to the RAYTRACE optical design. A monochromatic ray bundle transported through EASY’s SASP model would focus to a position on the model’s focal surface with a spot size similar to that of the RAYTRACE model. Beam spots on the focal surfaces of the EASY model and the RAYTRACE model look similar, but are only loosely correlated. A ray passing through the left side of the RAYTRACE focal surface spot could pass through the centre or right side of the EASY spot. The software corrections used to improve the resolution of the spectrometer will be different for the two cases. EASY will be able to predict the qualitative behaviour of the SASP, but a procedure to optimize the momentum resolution developed with EASY can not be transferred directly to the real spectrometer — although it may indicate the method to be used. Because the SASP dipole’s properties change with excitation, the EASY model of the SASP optics will only match the real optics at low magnet settings.

The last chapter of this thesis presents complete EASY models of SASP and MRS, simulating all the detectors, drift regions, vacuum windows, and such items found on the spectrometers. These models are maintained in special files which can be accessed by any simulation of the spectrometers. Neither model is numerically accurate, meaning
that while all the objects found in the spectrometer systems are described in the models, the orientation of these objects with respect to one another is not correct. In the case of the MRS, this is due to an incomplete knowledge of the detector positions and will be corrected in the near future. For the SASP, the detector arrays do not exist yet and are subject to change. While these models are not exact replicas of the spectrometers, they are self consistent. For example, the SASP model can still be used to calculate the reconstruction of a ray’s trajectory after the spectrometer. The discrepancies in the models would become important when considering the effects of multiple scattering in windows and such. The SASP and MRS models will be maintained and updated to make them as accurate as possible.

While they are thorough, these models are also very slow to execute. They are intended to be prototypes for anyone running simulations of the DASS/SASP system. A user completely unfamiliar with the spectrometer system can use these models as a black box from which data is collected. As the user becomes more concerned with the details of the spectrometers, he can look into the particulars of the simulation. An added advantage of keeping the prototypes of the spectrometer models is that any changes to the system only have to be made once rather than in every EASY simulation that uses the models.
Appendix A

Resolving Missing Mass

Consider an experiment where the mass of a "recoil" particle is to be measured,

\[ m_B + m_T \rightarrow m_R + m_1 + m_2 + \ldots \]

where \( m_B \) and \( m_T \) are the beam and target particle of known mass and \( m_R \) is the recoil particle. Then scattered products \( m_1, m_2 \) etc. are particles of known mass which have their momentums measured in various "detector arms" of the experimental apparatus. This appendix determines how the calculated resolution of the recoil (or "missing") mass depends on the measured momentums \( m_1, m_2, \) etc.

Where the target particle is stationary, the momentum of the recoil particle can be calculated by,

\[ \vec{P}_R = \vec{P}_B - \vec{P}_S \]  \hspace{1cm} (A.1)

and the energy,

\[ E_R = E_B + E_T - E_S \]  \hspace{1cm} (A.2)

where \( \vec{P}_S \) and \( E_S \) are the total momentum and energy of all the scattered particles \( (m_1, m_2, \) etc.). The missing mass can then be calculated from,

\[ m_R = \sqrt{E_R^2 - P_R^2} \]  \hspace{1cm} (A.3)

where \( P_R = |\vec{P}_R| \). The error in a missing mass, \( \Delta m_R \) is given by,

\[ \Delta m_R = \sqrt{\left( \frac{E_R \Delta E_R}{m_R} \right)^2 + \left( \frac{P_R \Delta P_R}{m_R} \right)^2} \]  \hspace{1cm} (A.4)

Often, the \( P_R \) is small compared to the mass of the recoil particle and \( \Delta m_R \) reduces to
APPENDIX A. RESOLVING MISSING MASS

(since $E_R \approx m_R$),

$$\Delta m_R = \Delta E_R$$  \hspace{1cm} (A.5)

where

$$\Delta^2 E_R = \Delta^2 E_B + \Delta^2 E_T + \Delta^2 E_S$$

The target particle is at rest with a known mass so $\Delta E_T = 0$ while the error in the beam energy is due to its momentum spread. The error in $E_S$ is given by,

$$\Delta^2 E_S = \Delta^2 E_1 + \Delta^2 E_2 + \ldots$$

Suppose only the momentum of each scattered particle is measured by the detectors. Then, since the masses of the scattered particles are known exactly, the error in the energy of each particle is,

$$\Delta E_i = \frac{P^2_i}{E_i} f_{P_i}$$  \hspace{1cm} (A.6)

where,

$$f_{P_i} = \frac{\Delta P_i}{P_i}$$

is the *fractional error* in the momentum. This is the usual way of rating the resolution of a spectrometer. Then,

$$\Delta m_R = \sqrt{\left(\frac{P^2_B}{E_B} f_{P_B}\right)^2 + \left(\frac{P^2_1}{E_1} f_{P_1}\right)^2 + \left(\frac{P^2_2}{E_2} f_{P_2}\right)^2 + \ldots}$$  \hspace{1cm} (A.7)

In a single arm experiment (only one scattered particle, $m_1$) and where the error in the beam momentum is negligible, the missing mass resolution is given by,

$$\Delta m_R = \frac{P^2_1}{E_1} f_{P_1}$$  \hspace{1cm} (A.8)

while for a two body experiment the resolution is given by,

$$\Delta m_R = \sqrt{\left(\frac{P^2_1}{E_1} f_{P_1}\right)^2 + \left(\frac{P^2_2}{E_2} f_{P_2}\right)^2}$$  \hspace{1cm} (A.9)

Consider the reaction, $A(p, 2p)B$ and assume that the scattered protons each have approximately the same momentum, $P$ (and hence the same energy, $E$). Then, $\Delta m_R$ is
APPENDIX A. RESOLVING MISSING MASS

given by,
\[
\Delta_{m_R} = \frac{P}{\sqrt{E}} \sqrt{f_{P_1}^2 + f_{P_2}^2} \tag{A.10}
\]

where \(f_{P_1}\) and \(f_{P_2}\) are the momentum resolutions of the two detector arms. Suppose one detector is a magnetic spectrometer with a resolution of \(f_P = 10^{-4}\); the other a counter telescope with a resolution of \(f_P = 10^{-3}\). Then the missing mass resolution would be,
\[
\Delta_{m_R} = \frac{P}{\sqrt{E}} \sqrt{(10^{-4})^2 + (10^{-3})^2} = 1.005 \times 10^{-3} \frac{P}{\sqrt{E}}
\]

The resolution of the system is completely dominated by the poorer detector. Basically the same holds true if the scattered particles have different masses and momentums although some improvement can be made by using the detector with the higher resolution to measure the larger momentum. In conclusion, to achieve a high resolution measurement of the missing mass in a two arm experiment, it is necessary for both arms to have high resolution detectors.
Appendix B

Dispersion Matching

In a particle beam – magnetic spectrometer system the momentum resolution of the spectrometer is limited by the spectrometer optics, the spot size of the particle beam on the target, and the energy spread of the particle beam. By use of Dispersion Matching, the effects of the spot size and incident beam energy spread can be reduced or eliminated.

Following the derivation of Blomqvist [24], consider the two body reaction,

\[ m_1 + m_2 \rightarrow m_3 + m_4 \]  \hspace{1cm} (B.1)

where \( m_1 \) is a beam particle incident on the target particle \( m_2 \) with the reaction product \( m_3 \) being detected in the spectrometer while \( m_4 \) is the undetected recoil particle. The incident (beam) particle, \( m_1 \) travels with a momentum \( P_1 \) which deviates by a small amount \( \Delta P_1 \) from the central momentum of the beam, \( P_B \). The scattered and recoil particles have momentums \( P_3 \) and \( P_4 \) respectively while the target particle, \( m_2 \) is at rest. For convenience, all these momenta are assumed to be positive while the small deviation of \( P_1 \) from \( P_B \) can be either positive or negative. The momentum of the detected particle, \( P_3 \) will depend on the kinematics of the reaction including the momentum of the beam particle, \( P_1 \).

\[ P_3 = P_3(P_1) \]  \hspace{1cm} (B.2)

Since \( P_1 \) deviates from \( P_B \) by only a small amount, can be equation B.2 rewritten Taylor expansion,

\[ P_3 = P_S + \frac{\partial P_3}{\partial P_1} \Delta P_1 + \frac{1}{2} \frac{\partial^2 P_3}{\partial P_1^2} (\Delta P_1)^2 + \ldots \]  \hspace{1cm} (B.3)

where \( P_S \) is the momentum of the scattered particle if the incident beam was monochro-
matic. While $P_3$ is the momentum of the particle that enters the spectrometer, it is $P_S$ that one ultimately wishes to measure. Now expressing $P_3$ in terms of the central momentum of the spectrometer, $P_0$,

$$\delta = \frac{P_3 - P_0}{P_0} \times 100\%$$

For a monochromatic beam,

$$\delta = \frac{P_S - P_0}{P_0} \times 100\%$$

and the relative momentum of the incident beam,

$$\delta_B = \frac{\Delta P_1}{P_B} \times 100\%$$

Equation B.3 can now be written as,

$$\delta = \delta + A\delta_B + B\delta_B^2 + \ldots$$

(B.4)

where,

$$A = \frac{P_B \partial P_3}{P_0 \partial P_1}$$

and,

$$B = \frac{1}{2} \frac{P_B^2 \partial^2 P_3}{P_0 \partial P_1^2}$$

Blomqvist evaluates $A$ as,

$$A = \frac{P_B \beta_B \left(1 - \frac{E_S}{m_2}\right) + \frac{P_S}{m_2} \cos \theta}{P_0 \beta_S \left(1 + \frac{E_B}{m_2}\right) - \frac{P_B}{m_2} \cos \theta}$$

(B.5)

where,

$$\beta_i = \frac{P_i}{E_i}$$

and $\theta$ is the angle between the incident and scattered particles ($P_1$ and $P_3$). $E_B/E_S$ is the energy of the incident/scattered particle for the monochromatic beam. Blomqvist derives a similar expression for $B$. 


To first order, the equation describing the focusing in the x direction is:

\[ x_2 = R_{11} x_1 + R_{12} \theta_1 + R_{16}\delta \]  
(B.6)

Assuming a point to point focus \((R_{12} = 0)\) and calling \(R_{11}\) the Magnification of the system, \(M\), and \(R_{16}\) the Dispersion, \(D\),

\[ x_2 = M x_1 + D\delta \]  
(B.7)

Where \(x_1\) is the position of the particle in the target plane and \(\delta\) is given by equation B.4. Ignoring the second and higher order terms in \(\delta_B\),

\[ x_2 = M x_1 + D(\delta_S + A\delta_B) \]  
(B.8)

In order to dispersion match the beam, the \(x_1\) and \(\delta_B\) terms are made to cancel each other out,

\[ 0 = M x_1 + D A\delta_B \]

or substituting for \(A\),

\[ x_1 = \frac{D P_B \partial P_3}{M P_0 \partial P_1} \]  
(B.9)

Thus by making \(x_1\) a function of \(\delta_B\), the first order focusing of the spectrometer is a simple function of \(\delta_S\), the quantity to be measured,

\[ x_2 = D\delta_S \]

Since the dependence on the unknown quantities \(x_1\) and \(\delta_B\) has been eliminated, the resolution of the spectrometer is improved.

This calculation assumes that the spectrometer has no aberrations — i.e. there are no terms higher than first order for \(x_2\). In practice this is not true and if these terms or if the second and higher order terms in \(\delta_B\) in equation B.4 can not be ignored, the dispersion matching condition (equation B.9) becomes increasingly complicated and harder to achieve. The advantage of the dispersion matching method is that it eliminates the effects of the target spot size and the beam dispersion simultaneously. The disadvantage of the method, is that it can produce large extended spots on the target requiring large targets and spectrometers with large vertical acceptances.
Appendix C

Magnetostatic Field Modelling

To design the field clamps and study excitation effects on the SASP and LEPS, their magnetostatic fields had to be modelled. Ideally this should be done in three dimensions since both the SASP and LEPS are decidedly 3D objects. Unfortunately the 3D program now in use, GFUN [25], is not suited for studying the fringe field regions on a magnet. A more likely candidate, TOSCA [26], has only recently become available to TRIUMF and is not ready for use. Both GFUN and TOSCA are limited in the number of points used to describe an object to be modelled, which is a problem with a complex geometry such as the SASP's. So instead of modelling the dipoles in 3D, cross sections of the magnets were modelled in two dimensions. The computer code used to model these cross sections was POISSON — a program in common use at TRIUMF.

C.1 Solving Magnetostatic Field Equations

In two dimensions, the magnetic vector potential has only a component out of the plane of the problem. It can be shown[27] that solving for the vector potential in this case reduces to solving the two dimensional Poisson's equation,

\[ \nabla^2 \phi(x,y) = \rho(x,y) \]  \hspace{1cm} (C.1)

where \( \phi(x,y) \) is some scalar function proportional to the vector potential and \( \rho(x,y) \) is the local current density. In two dimensions, the current \( \vec{J} \) is either into or out of the plane. The Poisson equation can in turn be solved by a technique called point successive relaxation [28].

Consider a rectangular grid (figure C.1). Solving Poisson's equation over a con-
Figure C.1: A rectangular grid over which the discrete Poisson’s equation is solved

Continuous region is equivalent to solving for every point on the grid,

\[ \phi(i, j) = \frac{\phi(i - 1, j) + \phi(i + 1, j) + \phi(i, j - 1) + \phi(i, j + 1)}{4} + \rho(i, j) \]  

(C.2)

That is at each point, \( \phi \) is equal to the average of its neighbours. If every point on the grid meets this condition, the region satisfies the discrete form of Poisson’s equation. Note that as the grid size becomes arbitrarily small, equation C.2 converges to equation C.1.

The input to a discrete Poisson problem is similar to the continuous case; regions at constant potential, \( \phi \), are specified as points of fixed \( \phi \) on the grid. If a region has current, \( \rho(x, y) \), this is specified at each grid point. Boundary conditions must also be specified; the two types possible are Neumann and Dirichlet. A Neumann boundary condition is a constant potential on the boundary while the Dirichlet condition specifies that the first derivative of \( \phi \) is zero,

\[ \frac{\partial \phi}{\partial \hat{n}} = 0 \]

where \( \hat{n} \) is the unit vector normal to the boundary.

There are two main techniques for solving the discrete form of Poisson’s equation for the region. Finite element analysis sets up a system of equations for all the points
on the grid and then solves them simultaneously using matrix algebra. If the equations are "well behaved" they can be solved exactly. If the equations are not, then an approximation to the correct solution can be made to any accuracy. The other method, called *successive relaxation* looks at a point on the grid, sets it to the average of its neighbours and then goes to the next point and repeats the process (using the new value of the last point in the calculation). If this is repeated over the entire grid many times, the values of $\phi$ converge to the correct solution (indicated when values of $\phi$ do not change from one iteration to the next). The more iterations used, the better the accuracy of the final value. While the finite element method appears to be more rigorous than the relaxation method, under many conditions they require similar amounts of computing time. In addition, the successive relaxation method has the considerable advantage of being simple to implement.

A slight modification to successive relaxation, is *successive over-relaxation* which speeds convergence by "over correcting" the potential at each point. First the difference between the old potential at the point and the average of its neighbours is calculated,

$$
\Delta \phi = \left[ \frac{\phi_1 + \phi_2 + \phi_3 + \phi_4}{4} + \rho \right] - \phi_{old}
$$

(C.3)

where $\phi_1 \ldots$ are the four neighbours. In successive relaxation, $\Delta \phi$ would be added to $\phi_{old}$ to produce $\phi_{new}$. In over-relaxation, $\Delta \phi$ is first multiplied by a constant greater than 1,

$$
\phi_{new} = \phi_{old} + r\Delta \phi
$$

causes the potential to overshoot. As the values approach the solution, the relaxation parameter, $r$, is reduced to unity. This process can reduce the number of iterations required by a factor of two or better.

A problem that does not appear with the continuous Poisson's problem is having the boundaries of the different regions conform to the grid. If a boundary is curved, the regular array of grid points can not follow it. The smooth curved boundary will be transformed into a "stepped" boundary that follows the grid. This changes the behaviour of the field and the effects of the boundary will not be observed. A solution is to distort the grid to conform to the boundary. It is difficult for a rectangular grid to be distorted to a curved boundary without introducing kinks and jumps. A triangular grid (or "mesh") does not have this problem and is more commonly used. the number of neighbours is changed from four to six. Because the grid is not regular, equation C.3 is no longer a simple average of the neighbouring points and weighting factors are
used to account for the relative distances of the neighbours,

\[
\Delta \phi = \left[ \frac{w_1 \phi_1 + w_2 \phi_2 + w_3 \phi_3 + w_4 \phi_4 + w_5 \phi_5 + w_6 \phi_6}{6} + \rho_0 \right] - \phi_{old}
\]

### C.2 POISSON the Program

POISSON is actually a group of programs designed to solve for the magnetic fields and forces in two dimensional arrangements of magnetic materials, conductors, and free space [29]. It does this by converting the continuous problem into a discrete problem on a triangular “logical mesh” and then solving for the magnetic vector potential on this mesh using successive over-relaxation [27]. POISSON originated as TRIM in the late 1960’s. TRIM required the user to manually translate the problem to the logical mesh, POISSON now does that automatically.

To solve a problem with POISSON, the shapes of the various regions (steel, air, current) to be modelled are described in a file which a program called AUTOMESH translates into the logical mesh. This mesh is an array of triangles contorted to follow the outlines of the different regions; the outlines of the problem’s regions are assigned nodes on the mesh. Next a program called LATTICE distorts the logical mesh to conform to the problem’s geometry. Now the program POISSON solves Poisson’s equation over the mesh using successive over-relaxation. Boundary conditions at the edges of the modelling region (which is rectangular) have to be specified. The boundary conditions can be either Neumann \((\vec{B} = B_n \hat{n})\) or Dirichlet \((B_n = 0)\). Dirichlet conditions do not allow the lines of magnetic flux to pass through the boundary while Neumann conditions cause the flux lines be perpendicular to the boundary. In POISSON, the boundary conditions can be set by the user but the default values are for the top and sides of the “world box” (the modelling region) to be Dirichlet and the bottom Neumann. Flux can only enter or leave the box through the bottom boundary. The other boundaries confine the flux. This has the effect of mirroring the magnet geometry about the bottom of the box. Thus for symmetric problems, only half the geometry need be described.

These boundary conditions make it difficult to model the extended fringe field of a magnet. The Dirichlet and Neumann conditions force all the magnetic flux in the system to be contained within the world box. Most magnets have leakage, that is, the flux does not stay within the magnetic material or the air gaps. Instead it moves through free space outside the magnet. When the magnet is modelled with POISSON,
a finite sized box must be used with all the flux contained within this region. Since
B is flux per unit area, this has the effect of increasing the local B field. Thus, when
modelling the fringe fields of a magnet, it is important to make the free space region
outside the magnet large to minimize the effect on the field. This in turn requires a
larger mesh for the problem and more computing time to find a solution.

Figure C.2 shows an H magnet modelled with POISSON. The default boundary
conditions are used which has the effect of mirroring the magnet about the bottom of
the box. Figure C.2c,d show the flux lines in the magnet and the B field along the x
axis (centre of the magnet’s air gap) calculated by POISSON.

C.3 Magnetic Materials

Many of the regions modelled in a POISSON problem are, of course, magnetic and
respond nonlinearly to the applied magnetic field. This property is described with a
material’s B-H curve. POISSON allows the B-H curve of the material being modelled
to be specified as a table of values. POISSON interpolates the points on the table to
calculate the magnetic field produced in a material for a given excitation.

When modelling the SASP and LEPS dipoles, several different steels with differing
magnetic properties were examined. The B-H curves are shown in figure C.3. Apart
from the effects of composition, the magnetic properties of the steels can also change
quite dramatically due to different ways of production.
Figure C.2: A simple example of a H magnet modelled with POISSON. (a) shows the original problem and (b) the logical mesh describing it. c) The flux line in the magnet. d) the B field (in Gauss) along the median plane of the air gap.
Figure C.3: Several B-H curves were used when modelling the SASP. Curve 2 is POISSON material #2 (1006 steel), curve 3 is material #3 (1010 steel) and curve 4 is material #4 (1020 steel).
Appendix D

The LEPS

Study of the SASP dipole with POISSON had no precedent. There was no estimate of the accuracy of the calculations. To facilitate understanding the modelling, the clamshell magnet used for the LAMPF Low Energy Pion Spectrometer (LEPS) was modelled and the POISSON predicted results compared against actual field measurements. The LEPS consists of a vertical bend clamshell dipole magnet separated from the target by about 30cm. As the only magnet in the spectrometer, the dipole acts as both the bending and the focusing element. The dipole (figure D.1) is a clamshell dipole with a single return yoke around the large gap (low field) side, this is quite different from the SASP which has two return yokes. Other differences are that the LEPS is about half the size of the SASP (the SASP dipole is about 110cm wide while the LEPS dipole is about 55cm), and has a steeper slope for the pole face (figure D.1b). Like the SASP, both the entrance and exit apertures are concave but neither have field clamps. The central field at the 8.5cm gap is 17.647 kG compared with the SASP’s 16.073 kG at its 10.0cm gap. The RAYTRACE parameters for the LEPS’s optics are given in table D.1 (refer to figure 3.1). For comparison with the SASP studies, two aspects of the LEPS design were investigated: the aperture fringe fields, and the saturation effects in the main body of the dipole.

The pole edge curves of the LEPS do not follow the VFB curves used in its RAYTRACE design (table D.1), instead they are approximated by circles. The pole edges are beveled with a simple 60° cut as opposed to the Rogowski bevel the SASP will have (see section 4.4).
Figure D.1: LEPS dipole a) Layout showing the pole piece, coil return yoke and extreme rays. b) Cross section showing wedge shaped air gap.
Table D.1: LEPS dipole RAYTRACE parameters. Compare with the SASP parameters in table 3.1.
APPENDIX D. THE LEPS

Table D.2: LEPS track positions. $x$ is shown in figure D.2.

<table>
<thead>
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<th>$x$ (in.)</th>
<th>$x$ (cm)</th>
</tr>
</thead>
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<td>$-4.1275$</td>
</tr>
<tr>
<td>2</td>
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<td>$0$</td>
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<td>$2$</td>
<td>$5.0800$</td>
</tr>
<tr>
<td>6</td>
<td>$3$</td>
<td>$7.6200$</td>
</tr>
</tbody>
</table>

Table D.2: LEPS track positions. $x$ is shown in figure D.2.

D.1 Field Mapping

The mapping of the LEPS's magnetic field was limited to a set of measurements taken at the entrance aperture (under different field settings and geometries) and an excitation curve taken with a retractable probe positioned at one point inside the dipole. This probe is normally used to set the dipole field during an experiment. It is unfortunate that the data are limited since a clamshell magnet is expected to behave quite differently from an ordinary dipole and a thorough mapping is required to get a proper understanding of its characteristics. The entrance aperture was mapped with a probe assembly that measured the field along the six tracks shown in figure D.2; the $x$ positions of the tracks are listed in table D.2. Six different combinations of configuration and excitation were mapped.

1. no clamp, mid excitation
2. no clamp, high excitation
3. clamp, no return yoke, mid excitation
4. clamp, no return yoke, 0.5 inches closer to dipole, mid excitation
5. clamp, no return yoke, 0.5 inches closer to dipole, high excitation
6. clamp, with return yoke, 0.5 inches closer to dipole, mid excitation

The terms used (based on information provided from LAMPF) are qualitative. High excitation presumably means the maximum design excitation (for which $BF = 17.647kG$) and mid excitation approximately $2/3$ the maximum. The field clamp position is shown on figure D.2; the “0.5 inches closer” means along the line AB in figure D.2. Since the
Figure D.2: The six tracks along which the LEPS field was mapped. The pole edge boundary (PEB) is described by a circular arc centred at A. This is quite different from the SASP PEBs which are described by eighth order polynomials.
results from the LEPS data are only qualitative, the exact excitations used and the exact positions of the field clamp are not critical. Note that the tracks for the field profiles are not perpendicular to the pole edge. This presented a problem when trying to model them in POISSON which can only model the field in a plane perpendicular to an infinitely long straight boundary.

Figure D.3 shows field profiles along the six tracks for three different mappings (1,4,6). The profiles for field map 1 (mid excitation, no clamp) are the solid lines. The dipole is approached from the left, with z as defined in figure D.2. These maps approximate those generated by POISSON in section 4.5. To the left of the peak is the fringe field region, to the right is the $1/\rho$ internal field.

The term "field clamp" used in the field map descriptions is misleading. The LEPS is close to the target scattering chamber, connected to it by an extension of the dipole vacuum box. A field clamp must fit inside this extension. A proper field clamp, with its two return yokes, would block the dipole aperture so the LAMPF group used only the top and bottom plates without the return yokes, therefore this is not a true field clamp — it is a snake (section 4.5). This geometry should produce a bulge in the field which is the opposite of what is required. The field profiles for the clamp with no side plates (map 4) are shown as the short dashed lines in figure D.3. The expected bulge is seen.

Field map 6 was of a clamp with one side plate (attached at on the side of the clamp furthest from track 1). This configuration was not considered seriously since it blocked the dipole aperture. It is difficult to predict the effect of a single return yoke. The field profiles are shown in figure D.3 as the dot-dash line. If the clamp plates and the single return yoke are sufficiently large, the flux will be properly shunted across the gap and the field clamped. The field profiles along track #6 shown in figure D.3 look like a snake with some of the flux removed. This indicates that the return yoke is shunting some of the flux around the gap while the rest leaks across. This is equivalent to a regular clamp with saturating return yokes. The geometry of the clamp is such that the gap between the clamp and pole edge is largest for the low numbered tracks. Therefore, the effect of the clamp on the fringe field should decrease with decreasing track number; this is seen clearly in figure D.3.

Another way of examining the field maps is by drawing contour plots. In a contour plot, evidence of the aperture filling in and a change in the field shape with excitation as discussed in section 4.1 would be investigated. Figure D.4 shows contour plots of map 1 (mid excitation) and map 2 (high excitation). The diamonds show the pole
Figure D.3: Magnetic field profiles along the six tracks for the mid excited dipole. **Solid:** Map #1 no clamp. **Dash:** Map #4 clamp with no yoke (snake). **Dot-dash:** Map #6 clamp with one yoke.
Figure D.4: Contour plots of the LEPS entrance aperture fields. The diamonds show the pole edge. The $x$ coordinate is plotted in reverse with $x$ as defined in figure D.2 positive down. a) Low excitation. b) High excitation.
edge. The $z$ coordinate is plotted in reverse with positive $x$ down as defined in figure D.2. If the two contour plots were superimposed they would appear almost identical except for magnitude. The only difference between them is the field begins to bulge out slightly more at the bottom (low field side) of the high excitation plot than the mid excitation plot. Thus there are no significant saturation effects. On the other hand, both fringe fields change from concave at the pole edge to convex further out. This is more likely due to the proximity of the sides of the dipole than to aperture filling effect discussed in section 4.1.

Another effect observed (more apparent on the low excitation plot) is a contour line crossing from outside the pole edge at the high field end of the aperture to inside at the low field end. This is qualitatively what is expected if the fringe field function (equation 4.1) is dependent on gap width. As the gap increases, the fringe field spreads out over a larger region with the result that the lines of constant $B$ move towards the interior of the dipole.

In addition to the field mapping of the entrance completed before final assemble of the spectrometer, data is available from a retractable probe that is inserted into the air gap to the position shown in figure D.1. This probe is used to set the field of the magnet. It produces the excitation curve shown as the solid line in figure D.8.

### D.2 Modelling the LEPS Fringe Fields

Modelling the LEPS fringe fields proved to be difficult. For comparing the true fields with the POISSON predicted fields, the most convenient tracks for the field maps would have been along perpendiculars to the pole edge. This was not the case so it was necessary to project the POISSON field profile onto the field map track. For this to be a valid procedure, the assumption is made that the fringe field along a perpendicular to the pole edge is essentially equal to that from a straight boundary (the bulged field in the contour plots show this is not strictly true). This projection is equivalent to a rotation about some axis or reference point. This is the point where the internal field, dominated by the pole face slope, gives way to the external field dominated by the pole edge geometry. The location of this point is not obvious. A reasonable first guess is the point where the bevel intersects the pole face. If the correct reference point is selected, the peaks of the field map profile and the POISSON projection should coincide. If not, they will be shifted. By shifting the peaks about, the correct reference point can be located. An example of the POISSON problem file used to model the LEPS aperture
is listed in appendix E. An alternate approach would be to use the field maps to interpolate the field along the line that POISSON models, but it would still suffer from the bulging fringe field. As well, there could be a problem deciding how to interpolate between the tracks.

In the POISSON model, the pole face slope was set equal to that seen by the field mapping probe along its track as opposed to the slope along the perpendicular to the pole edge. The projection is correct only outside the dipole. Inside, a different projection is required with a different reference point. This introduces a new parameter to be fitted to the field maps and adds no information. The method should be acceptable if the interior and fringe fields are decoupled as expected. By using the pole slopes along the profile tracks, direct agreement between POISSON and the field maps for the dipole’s internal field near the pole edge can be observed.

This process for fitting the POISSON data to the mapped fields is less than ideal as it involves several assumptions, a magic number (the reference point), and ignores the observed field bulge. It kills the hope of getting quantitative numbers out of the comparison. The best that can be hoped for is a qualitative idea of how well POISSON agrees with the true fields (i.e. whether there are systematic discrepancies).

Figure D.5 shows the comparison of the POISSON calculation, the RAYTRACE expected field and the actual field maps for the mid excited dipole without a clamp (map 1). The RAYTRACE field is not valid to the left of the peak. Like the POISSON field, it is calculated along the perpendicular to the pole edge and is projected onto the field map track. As mentioned above, this projection is only valid for the external field and distorts the internal RAYTRACE field to the left of the peak.

The internal fields predicted by POISSON agree well with the observed fields except for tracks 5 and 6 which are out near the edge of the pole where the field is expected to drop. Seeing the effects of the dipole sides on the internal field supports the premise that the bulge observed in the contour plots is caused by the dipole side. The POISSON fringe fields agree approximately with the actual fringe fields having about the same extent and slope. The peaks agree well which indicates the original assumption that the fringe field starts to dominate at the start of the bevel is reasonable. This indicates that the actual curvature of the bevel has to be carefully controlled to produce the correct VFB curvature. The fact that the actual fringe field is not consistantly above the POISSON fringe field indicates that the there is little aperture filling (see section 4.1) by the flux. That is, the different parts of the aperture do not interact with each other. These calculations were repeated with the high excitation field maps (2) with
Figure D.5: The POISSON predicted fringe fields (points) compared with the RAYTRACE expected field (dashed line) and the measured fringe field (solid line) for the mid excited dipole without a clamp (map 1). Because of transformations done to the POISSON and RAYTRACE data, the RAYTRACE field is not valid to the left of the peak.
almost the same results as expected from the contour plots.

D.3 Modelling the LEPS Interior Fields

The study of the LEPS's internal field was prompted by the saturation effects predicted by POISSON in section 3.3. Figure D.6 shows a cross section through the LEPS modelled with POISSON (the POISSON problem file is listed in appendix E). The top plate and return yoke are much larger than the actual ones on the LEPS. This is because the 2D POISSON model would predict much more leakage flux from the iron than is the case on the real LEPS with its extensive return yoke (figure D.1). The leakage flux lowers the field in the dipole air gap. This was remedied by increasing the size of the return yoke and top plate until the field at the central gap of the dipole agreed roughly with that actually observed in the dipole. The dimensions shown in figure D.1 are the result.

Figure D.7 shows an excitation spectrum of the LEPS internal field predicted by POISSON similar to the one produced for the SASP (figure 3.9). Unlike the SASP, the LEPS does not suffer centre field sagging at low excitation. This is because the
Figure D.7: The profile of the LEPS internal field for various dipole excitations. Starting from the upper left, the excitations are 14000NA, 34000NA, 60000NA, and 64000NA.
Figure D.8: The measured LEPS excitation curve (solid) and several different attempts to model it with POISSON (points).

LEPS is a C magnet and the flux tends to spread more uniformly into the center of the pole piece. At low excitations, the field is uniformly 0.362\% high over most of the pole. Which means that the fringe field region extends slightly past the central radius gap of 8.5cm. At maximum excitation, the LEPS has a saturation distortion of (subtracting the base level),

$$1.737\% - 0.362\% = 1.375\%$$

which is approximately a third larger than the SASP’s distortion at maximum excitation. One would expect the LEPS dipole to be worse than the SASP dipole because it only has one return yoke making it easier for flux to leak out. However this could be off set by the LEPS’s smaller size and the greater saturation of the LEPS is probably not significant.

Figure D.8 shows the measured excitation curve (at the LEPS's field probe position) and various attempts to model it with POISSON. The curves are normalized so that the linear portions at low excitation agree with each other. The upper POISSON curves use the geometry shown in figure D.6 based on two different types of steel excitation curves. Again, the rational behind this geometry was to match the field in the POISSON model with that observed in the real system. Thus it is not surprizing that
the POISSON curves are close to the observed curve. This is, of course, a cheat since by changing the geometry around, the LEPS curve could be matched exactly.

The lower curve was thought to be a more honest approach. Here, the ratio of the widths of the pole piece and return yoke in the POISSON model were set equal to the ratio of the areas of the pole piece and return yoke for the real magnet,

\[
\frac{w_{pp}^{\text{POISSON}}}{w_{ry}^{\text{POISSON}}} = \frac{A_{pp}^{\text{real}}}{A_{ry}^{\text{real}}}
\]

while the top plate in the model was made the same thickness as that on the real magnet. The idea is that since POISSON models the cross section through an infinitely long dipole, setting the ratio of width's in the model to the ratio of areas in the real magnet should produce the same reluctance. The top plate was kept the same, since it projects onto the two dimensional plane as it would on the real magnet. On reflection, this was a mistake. If the objective is to make the reluctance per unit length of each part of the model the same as the total reluctance of the real magnet, then,

\[
\frac{t_{tp}^{\text{POISSON}}}{w_{pp}^{\text{POISSON}}} = \frac{A_{tp}^{\text{real}}}{A_{pp}^{\text{real}}}
\]

where \(A_{tp}^{\text{real}}\) is the cross sectional area of the top plate — which is the same relation as for the return yoke. What should be defined as the cross section of the top plate is not obvious since the return yoke wraps around it (figure D.1). What could be done is to make the top plate wedge shaped, with the thickness of the wedge corresponding to the cross section along a circle centered on the pole piece. This would make the top plate small over the pole piece, and large over the return yoke. It turned out that the pole piece and return yoke lengths used in figure D.6 were very close to the proper ratio. Decreasing the thickness of the top plate to the real thickness (which is not a valid model) produced the lower curve in figure D.8.

While this model is not the best guess, it does illustrate the sensitivity of the excitation curve to changes in the yoke and top plate. What it also says is that trying to model the excitation spectrum of the LEPS or the SASP with POISSON is a risky business. Both magnets are extreme examples of a three dimensional parallel path problem. That is, flux from the pole piece can travel by many different routes through the yoke and this can not be simulated in two dimensions. By fiddling with the geometry, the POISSON excitation curve could be matched to the observed curve but once done, the same procedure could not be applied to the SASP since it has a
much different geometry. This problem requires the application of a three dimensional magnetostatic code such as TOSCA.

D.4 Summary of the LEPS Studies

The nature of the LEPS data is such that qualitative agreement could be made with the POISSON predictions but not quantitative. It is a successfully passed null test, meaning that while the LEPS data do not disagree with the POISSON calculations, they do not necessarily agree either. The main points to be learned from this experience, are:

1. The unclamped LEPS fringe field bulges — more likely due to the proximity of the side of the pole piece than any effects of the aperture itself.
2. The side plateless field clamp behaves like a snake as expected.
3. The importance of a large enough return yoke on a field clamp is illustrated by the clamp with one yoke.
4. Predicting B-H curves from a POISSON model is difficult.
5. The POISSON model produced an internal field profile of the LEPS that agreed with the observed internal field along the probe tracks.
Appendix E

POISSON Input Files

This appendix contains sample problem files for the four types of POISSON simulation done for the SASP and LEPS dipoles. A POISSON problem file contains the description of the regions to be simulated. The problem file is converted into a logical mesh file from which the grid POISSON preforms the over-relaxation on is generated. The last section has the POISSON input file that describes the B-H curves used for the SASP dipole pole piece. A POISSON input file is used to set various options in the program and define new B-H curves. The options include the tolerance POISSON is to carry the calculation to.

E.1 SASP Field Clamp

This is a sample problem file for the SASP field clamp. The cross section is through the \( x = 0 \) position on the VFB with the end of the clamp 14.1546cm from the pole edge (located at 153.0cm). The dipole is set for an excitation of 67000NA. This file produces figure 4.16. There are five regions specified in the file. The first is an air box that outlines the entire problem region. The second region defined is the dipole itself which is assumed to be made from 1020 steel. The third and fourth regions are the coils and the fifth region is the field clamp.

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$REG NREG=5 MAT=1 DX=2.0 DY=1.5 XMIN=0.0 XMAX=250.0 YMAX=150.0 NPOINT=5$
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$P0 X=250.0 Y=0.0$
$P0 X=250.0 Y=150.0$
$P0 X=0.0 Y=150.0$
```

204
APPENDIX E. POISSON INPUT FILES

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$PO X=153.0 Y=30.0$
$PO X=173.0 Y=30.0$
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$PO X=86.0 Y=9.0$
$PO X=86.0 Y=25.0$
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$PO X=70.0 Y=9.0$
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$PO X=167.2114 Y=9.5676$
$PO X=171.4942 Y=9.4459$
$PO X=171.5793 Y=12.4447$
$PO X=172.0195 Y=27.9384$
$PO X=157.5254 Y=28.3502$
$PO X=157.0000 Y=9.8577$
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$PO X=240.0000 Y=16.2067$
$PO X=230.2098 Y=16.2067$
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\section*{APPENDIX E. POISSON INPUT FILES}

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$PO X=245.0000 Y=0.0000 $
$PO X=240.0000 Y=0.0000 $

\textbf{E.2 SASP Dipole Interior Field}

This is the cross section through the centre of the SASP dipole. The pole piece is made out of a 1020 steel (the default POISSON magnetic material — type 2), while the pole piece in made from 1010 steel (material type 4 defined in section E.5). This file produces figure 3.6.

\begin{verbatim}
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\end{verbatim}
\end{verbatim}
E.3 LEPS Dipole Fringe Field

The POISSON problem file used to model the LEPS fringe field along track #3 (see figure D.2). The pole face slope, points (88.0,7.3802) - (149.0488,4.5192) is set at the angle seen by the probe while the bevel, (149.0488,4.5192) - (153.0,11.3629) is taken from a plane perpendicular to the pole edge. This file produces a geometry similar to D.6.

LEPS33 50000.0

$REG NREG=5 MAT=1 DX=2.0 DY=1.5 XMIN=0.0 XMAX=250.0 YMAX=150.0 NPOINT=5$

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APPENDIX E. POISSON INPUT FILES

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$PO X=65.0 Y=26.0$
$PO X=88.0 Y=26.0$
$PO X=88.0 Y=7.3802$
$PO X=149.0488 Y=4.5192$
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$PO X=158.0 Y=8.0$
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$PO X=171.0 Y=7.0$
APPENDIX E. POISSON INPUT FILES

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$PO \ X=163.0 \ Y=4.0$

E.4 LEPS Dipole Interior Field

The POISSON problem file used to model the internal field of the LEPS dipole. This file produces the geometry shown in figure D.6.

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$PO \ X=75.0 \ Y=0.0$
$PO \ X=75.0 \ Y=20.0$
$PO \ X=98.0 \ Y=20.0$
$PO \ X=98.0 \ Y=6.65$
$PO \ X=101.0 \ Y=6.5$
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$PO \ X=152.5 \ Y=3.925$
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$PO \ X=90.0 \ Y=8.0$
$PO \ X=90.0 \ Y=20.0$
$PO \ X=87.0 \ Y=20.0$
E.5 POISSON Input File

The POISSON input file that defines the B-H curves of two new steel types, #3 and #4. #4 is a good 1010 steel that is used for the SASP dipole pole piece.

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### APPENDIX E. POISSON INPUT FILES

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-1 DUMP
Appendix F

Aperture Specification

This appendix contains specifications of the pole edge, field clamp, and vacuum box geometry for the SASP dipole apertures. It is a first order design and will be modified before a final engineering design is achieved. For example, the exact widths of the dipole apertures and the relative positions of the return yokes are not yet known. Specified here are:

- Pole edge bevel shape.
- Dipole entrance and exit aperture curvatures.
- Field clamp design.
- Dipole entrance and exit vacuum box designs.

The basic dipole geometry and the coordinate systems used are shown in figure 3.3. Positions along a VFB ($x_{VFB}$) is referred to by the $x$ coordinate of the appropriate coordinate system, B or C. Note that $z$ is positive out for both. Some assumptions made about the dipole geometry are:

- The coil cross section is 14.5 cm horizontal by 18.5 cm vertical
- The coils lie in planes parallel to the dipole pole faces.
- Coil — pole edge separation is 2.186559in (5.538599cm) for the entrance aperture and 2.317238in (5.8857845cm) for the exit aperture.
- The lower extent of the coil is 5.0 cm above the plane of the pole face.
- The vacuum box is made from material 2.0 cm thick.
- Both the dipole and field clamp are made of 1020 hot rolled steel as specified by POISSON material #4 (the B-H curve for which is shown in appendix C).
F.1 Pole Edge Bevel

A Rogowski bevel is approximated for the SASP dipole's apertures by taking 75° and 30° cuts perpendicular to the pole edge as shown in figure F.1. If the pole pieces were flat, a cross section through the bevel would follow the three points A, C, and D shown in figure F.1b. C would be 5cm and A 11cm from the pole edge (which due to the SASP dipole's geometry is perpendicular to the median plane). However, the dipole's pole pieces are sloped making the 30° cut intersect the pole face at B rather than A. Because the apparent slope of the pole face varies depending on where the perpendicular to the PEB is taken, the distance from the pole edge to B would vary over the aperture. The resulting curves made by points B and C are shown in figure F.1a. The effect of the sloped pole face on the distance from B to the pole edge is exaggerated.

One possible method for cutting the bevel is shown in figure F.2. Point E, the corner of the uncut pole piece is taken as a reference point. Note that because the dipole gap varies over the extent of the aperture, the distance of E from the median plane varies as well. First a 75° cut (with respect to the median plane) is made such that the distance from E to D is 22.1244cm (figure F.2b). Then a 30° cut is made such that the height of C above E is 3.4641cm (figure F.2c). This method makes it unnecessary to calculate the distance from B to the pole edge.

F.2 Vacuum Box Design

To simplify the construction of the vacuum boxes, it is desirable to construct them from flat plates. As shown in figure 4.15, each box consists of a flat plate that sits under the coil and field clamp parallel to the dipole pole face. The inner side of this plate becomes a flange that seats against the side plates of the dipole. The end of the plate that protrudes from the clamp butts against a sloped plate that flares to accomadate the spreading ray bundle (figure F.3).

F.2.1 Entrance

The entrance vacuum box must connect the dipole to the exit of the second quadrupole. The distance between the quadrupole and the dipole along the optical axis is approximately 120cm normally. The exact distance will depend on the final design of the quadrupole. However, the SASP will be run in some configurations where the quadrupole is 30cm closer to the dipole. The beam pipe leaving the quadrupole is the
Figure F.1: SASP dipole bevel. a) The cuts are made perpendicular to each point on the PEB. b) The bevel is comprised of a $75^\circ$ and a $30^\circ$ cut. The intersection of the $30^\circ$ cut and the pole face varies over the aperture.
Figure F.2: Cutting the bevel. a) The uncut pole piece. b) First the 75° cut is made. c) Followed by the 30° cut.

Figure F.3: Simplified drawing of an aperture vacuum box. Does not show the changing gap width or curvature of the aperture. The field clamp is a tube symmetric about the median plane that wraps tightly about the vacuum box.
28cm wide cruciform shaped pipe discussed in section 5.2.2. The rays fill most of the region between the quad and the dipole. In the nonbend plane, the quad has a 28 cm wide aperture compared to the dipole's approximately 10 cm aperture. The vacuum box will have to be flared beyond the coils to connect to the quad. This flare must accommodate both the 120cm and 90cm separations between the quadrupole and dipole. In the bend plane, the rays leave the quadrupole over its entire 28cm aperture and enter the dipole aperture over a range of approximately -47.5 cm to 44.0 cm in coordinate system C (figure 3.3). Thus while the vacuum box must increase in width from the dipole to quad in the nonbend plane, it must decrease in the bend plane.

The vacuum box has to fit under both the coil and the field clamp (figure 4.15). The coil–pole face separation is 5.0 cm; the tongue of the field clamp that projects under the coil is 2.0 cm thick; the vacuum box is 2.0 cm thick. This leaves a gap of 1.0 cm between the vacuum box and the pole face. Without this gap, the solid angle of the spectrometer would be reduced by approximately 10%. Figure F.4 shows a side and bottom view of the entrance vacuum box. As shown, it consists of one section (probably cast as part of the main dipole vacuum box) extending back from the dipole to the quad where it couples with the quad's vacuum pipe. This does not consider the need to move Q2 back towards the dipole. An improved design would have the flange closer to the dipole with an intermediate box between the dipole box and quadrupole.

F.2.2 Exit

Figure 6.3 shows the cross sections of ray bundles after the SASP for $\delta = -10\%, 0\%$, and $+15\%$. Also shown are the maximum widths of the ray bundles at each cross section (in the nonbend plane). These bundles are for the largest target spot that would be used with the spectrometer, $\pm 5\text{cm} (x)$ by $\pm 2\text{cm} (y)$. The vacuum box will have to flare slightly in the nonbend plane to clear the ray bundle. The exit window for the vacuum box should lie on the focal surface of the spectrometer. However, the studies of the SASP dipole indicate that the exact location of the focal surface will not be known until the spectrometer is built and its field mapped. Thus the vacuum box will be built in two sections. The first will be attached to the dipole vacuum box with a flange located after the field clamp. The second section will be added later to extend the vacuum to the focal surface. Figure F.5 shows a side view of the exit vacuum box.
Figure F.4: Entrance Vacuum box design, side and bottom view.
Figure F.5: Exit Vacuum box, side view. Also shown are the extreme rays for -10% and +15% as well as points on the focal surface.
F.3 Field Clamp Design

As shown in figure F.3, the clamps consist of tubes fitted tightly around the vacuum boxes and butting against the coils with tongues projecting under the coils. The thickness of the tongue is 2.0 cm while the balance of the tube is 5.0 cm thick. The clamp is symmetric about the median plane. The tubes must protrude beyond the end of the fringe field which extends approximately 30 cm out from the pole edge (figure 4.16b). A reasonable value for the extent of the tube is 50 cm from the pole edge. POISSON studies indicate that 5cm thick return yokes is adequate to carry the flux from both apertures. However, because the POISSON calculations are uncertain, it is important to leave room to add more steel to the return yokes if necessary. The clamps are shown in figures F.6 and F.7.

The 2 cm tongue under the coil carries most of the flux in the clamp. Making the clamp fit tightly to the coil relieves this bottleneck and helps prevent saturation problems. This means that there are two complex curves to be fashioned for each clamp: the fit to the coil, and the leading edge of the tongue. It would be desirable to fashion the clamp such that the tongue detaches to make modifications to it if necessary after construction of the dipole.

F.4 PEB and FCB Specification

As stated in the design procedure, polynomial descriptions of the pole edge (PEB) and field clamp edge (FCB) were generated. These descriptions are in the local coordinate systems shown in figure 3.3 (the same as the original VFB descriptions).

\[ z = \sum_{i=0}^{8} a_i x^i \]

where \( z \) and \( x \) are in cm. This polynomial is good for the range \( x = \pm 50.0 \text{cm} \) for the entrance and \( x = -80.0 \text{cm} \) to \( x = 100.0 \text{cm} \) for the exit. Outside this range the polynomials give unreliable results. The coefficients are listed in table F.1 and points on the PEB and FCB are listed in tables F.2 and F.3. Also listed are points on the inner and outer boundaries of the coils. The curves produced by the coefficients in table F.1 are shown in figures F.8 and F.9. Also shown in the figures, are the coil positions.
Figure F.6: Entrance field clamp, side view.
Figure F.7: Exit field clamp, side view.
Figure F.8: Entrance pole edge, coil, and field clamp edge.

Figure F.9: Exit pole edge, coil, and field clamp edge.
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Table F.1: PEB and FCB coefficients
# APPENDIX F. APERTURE SPECIFICATION

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Table F.2: Entrance PEB, FCB, and coil curves.
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Table F.3: Exit PEB, FCB, and coil curves.
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### APPENDIX F. APERTURE SPECIFICATION

**Dipole Exit**

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Appendix G

Random Numbers

EASY, being a Monte Carlo simulator, makes extensive use of random numbers, particularly uniformly and normally distributed ones. Algorithms for uniform random number generators are common [18]; EASY generates uniform random numbers between 0 and 1 using the standard routines built into the C programming language. This random number generator is initialized at the start of the program, meaning if a simulation is run twice exactly the same results will be produced each time. This is advantageous when debugging a simulation, each run will be identical making it easier to track down problems. On the other hand one often wants to duplicate a Monte Carlo using different numbers to see if an observed effect is due to chance statistics or if it is a real phenomena. The only way to do this as EASY is currently written is to increase the number of events simulated. Ideally, the seed number of the random number generator should be set in the program to either duplicate a random number sequence or to generate a new one. The usual random number generator produces a sequence of numbers that appear random but in fact will eventually repeat after a large number of iterations — this is independent of the seed number used to start the generator. But a Monte Carlo uses a large number of iterations and there is a (albeit small) chance that the simulation will start to repeat itself. The user thinks he is getting good statistics from a run but in fact they are almost useless. One method of avoiding this is for subsequent runs of the Monte Carlo to use different algorithms for the random number generators.

There are many algorithms available for calculating random numbers with a Gaussian distribution (see [16]). Most of these methods are concerned with using approximations to make the calculation as fast as possible. In EASY calculating a random number is only a small part of the total simulation so speed is not a factor. Thus a slow but exact method of calculating Gaussian random numbers is used.

Starting with two uniform random numbers between 0 and 1, \( u \) and \( v \), the trans-
formation

\[ x = \sqrt{-2 \ln u \cos(2\pi v)} \]
\[ y = \sqrt{-2 \ln u \sin(2\pi v)} \]

is used. This produces two normally distributed random numbers with mean 0 and standard deviation 1. To convert \( x \) and \( y \) to numbers with mean \( m \) and standard deviation \( \sigma \),

\[ x' = m + (\sigma \times x) \]
\[ y' = m + (\sigma \times y) \]

A nice feature of this method is that it takes only a little more time to calculate two numbers than one.
Appendix H

2 Body Kinematics

H.1 Calculating CM Energies and Momentums

Suppose that one of the incident particles \( m_2 \) is stationary (as in a target) \( \text{Lab Frame} \) and the other \( m_1 \) is moving with an kinetic energy \( T_1 \) in a beam.

\[
E_1 = m_1 + T_1
\]
\[
E_2 = m_2
\]

The total CM energy available to the reaction \( W \) is given by,

\[
W^2 = 2T_1m_2 + (m_1 + m_2)^2
\]

then calculating \( \gamma^* \) and \( \beta^* \),

\[
\gamma^* = \frac{E^{\text{LAB}}}{W} = \frac{T_1 + m_1 + m_2}{W}
\]
\[
\beta^* = \sqrt{1 - \frac{1}{\gamma^{*2}}}
\]

The magnitude of the momentums of the reaction products \( P_3^* = P_4^* \) is calculated by,

\[
P_{3,4}^* = \sqrt{\frac{(W^2 - (m_3 + m_4)^2)(W^2 - (m_3 - m_4)^2)}{4W^2}}
\]
H.2 Aiming at a Detector

At what angle $\theta^*$ does one point a scattered particle with momentum $P^*$ in the centre of mass frame, to hit a detector lying at an angle of $\theta$ in the lab frame? Start with the relation,

$$E^* = \gamma(E - \beta P \cos \theta)$$

and defining $A = \left(\frac{E^*}{\gamma}\right)^2$ and $B = \beta \cos \theta$,

$$A = (E - BP)^2 = E^2 - 2BEP + B^2P^2$$

substituting $E^2 = P^2 + m^2$

$$A = P^2 + m^2 - 2B\sqrt{P^2 + m^2}P + B^2P^2$$

rearranging and squaring both sides, one gets a quadratic in $P^2$,

$$[(1 + B^2)^2 - 4B^2]P^4 - [2(1 + B^2)(A - m^2) + 4m^2B^2]P^2 + (A - m^2)^2 = 0$$

for which there are two solutions,

$$P^2 = \frac{(1 + B^2)(A - m^2) + 2m^2B^2}{(1 - B^2)^2} \pm \frac{2B\sqrt{m^2(1 + B^2)(A - m^2) + m^4B^2 + (A - m^2)^2}}{(1 - B^2)^2}$$

What do the two solutions for $P$ mean? For a given $E^*$ and $\theta$, there are two possible values of $P$. From $E^* = \gamma^*(E - \beta^*P \cos \theta)$ there are two cases,

$$\begin{align*}
\theta < \frac{\pi}{2} & \quad E^* < E \\
\theta > \frac{\pi}{2} & \quad E^* > E
\end{align*}$$

So when choosing $P$ from the two possible solutions,

$$\begin{align*}
0 \leq \theta \leq \frac{\pi}{2} & \quad \text{choose larger } P \\
\frac{\pi}{2} \leq \theta \leq \pi & \quad \text{choose smaller } P
\end{align*}$$
Note that if $P^*$ is less than $\gamma^* \beta^* m$, there is a maximum scattering angle. And then using

$$P^* \sin \theta^* = P \sin \theta$$

one can solve for $\theta^*$

$$\theta^* = \sin^{-1} \left( \frac{P}{P^* \sin \theta} \right)$$
Appendix I

EASY Input Files

This appendix lists the files used to generate the various Monte Carlo studies of the SASP using EASY. To describe a problem to EASY, the user must create three files: the device file; the data acquisition file and the command file. Each of these files has the same name as the problem (in this case it will called solid) and a distinctive suffix; .dev, .dac, and .cmd respectively. As well, the problem must be identified to EASY in the problem file called EASY.PRB. This is done with the line:

@problem 1 solid

where solid is the name of our problem. All these examples are for a ray bundle of +10% (accept for the focal plane mapping) assuming the central momentum of the SASP is 660.0 MeV/c. To change to a different momentum, the variable ray momentum in the @mapsolid command is changed to the appropriate value.

1.1 Solid Angle

The SASP’s “high resolution” solid angle was calculated by generating rays smeared out over a rectangle 2cm by 1cm in the target plane. The system consists only of the sasp_optics. The function marker tells the command @mapsolid that the ray made it successfully past this point (in this case the end of the SASP optics). EASY automatically calculates the solid angle of the device and prints it out to the problem’s output file, solid.out. The command @setsasp sets the central momentum of the SASP and the command @end tells EASY that the simulation is over.

Device File (solid.dev)

    define main{
        sasp_optics
I.2 Limiting Aperture

This problem is almost the same as for the high resolution solid angle except that a larger spot size is used and the command @saspcuts prints the SASP aperture cuts scatter plots to the output file solid.out. Three sets of scatter plots are generated for each aperture for a total of 27. The PASSED plots show all the rays that pass the aperture cut. The MISSED plots show all the rays that hit the aperture. The THRU plots show the rays that made it through the entire system as they appear at each aperture.

Device File (solid.dev)

define main{
    sasp_optics
}

Data Acquisition File (solid.dac)

#address saspend=[sasp_optics]
#function mymark=marker(@saspend/x;)

Command File (solid.cmd)
I.3 Target Plane Acceptance

While the device and data acquisition files are the same as for the above studies, the command @mapsolid is changed to produce 2000 rays at each point on a grid of size 40cm by 20cm with a 2cm spacing. The solid angle for each point on the grid is printed to a file called (in this case) solid.sol along with the coordinates of the grid point. This data can then be changed into a matrix with 21 rows and 11 columns by the program VECTOMAT.C for plotting by a program such as PLOTDATA (a standard TRIUMF plotting package).

Device File (solid.dev)

    define main{
        sasp_optics
    }

Data Acquisition File (solid.dac)

    #address saspend=[sasp_optics]
    #function mymark=marker(saspend/x;)

Command File (solid.cmd)

    @setsasp 660.0 \set Po of SASP to 660.0 MeV/c
    @mapsolid p \smeared source
        -20.0 20.0 2.0 \xmin, xmax, xstep
        -10.0 10.0 2.0 \ymin, ymax, ystep
        726.0 70.0 \ray momentum, dist to aperture
        0.100 \scattering cone size (in str)
    @saspcuts \print out SASP aperture cuts
    @end
I.4 Downstream Ray Profiles

The downstream ray profile calculation uses the same target spot size as the limiting aperture calculation but in addition to transporting the rays through the SASP, they are drifted downstream in 50cm steps. The data acquisition file sets up scatter plots of the ray bundle at each step and the command @analize prints them out.

**Device File (solid.dev)**

```plaintext
define drifting{
    drift
    aperture entrance none
    length 50.0 \ 50.0 cm
    radlength -1.0 \ vacuum
    aperture exit none
}

define main{
    sasp_optics
    drifting \ 50cm downstream
    drifting \ 100cm
    drifting \ 150cm
    drifting \ 200cm
    drifting \ 250cm
    drifting \ 300cm
    drifting \ 350cm
    drifting \ 400cm
}
```

**Data Acquisition File (solid.dac)**

```plaintext
#address saspend=[sasp_optics]
#address d1=[drifting,1]
#address d2=[drifting,2]
#address d3=[drifting,3]
```
#address d4=[drifting,4]
#address d5=[drifting,5]
#address d6=[drifting,6]
#address d7=[drifting,7]
#address d8=[drifting,8]

#function mymark=marker(@saspend/x;)

scat(sasp exit) saspend/y(y cm) min=-100.0 max=100.0:
       saspend/x(x cm) min=-100.0 max=100.0;
scat( 50cm downstream) @d1/y(y cm) min=-100.0 max=100.0:
       @d1/x(x cm) min=-100.0 max=100.0;
scat(100cm downstream) @d2/y(y cm) min=-100.0 max=100.0:
       @d2/x(x cm) min=-100.0 max=100.0;
scat(150cm downstream) @d3/y(y cm) min=-100.0 max=100.0:
       @d3/x(x cm) min=-100.0 max=100.0;
scat(200cm downstream) @d4/y(y cm) min=-100.0 max=100.0:
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scat(250cm downstream) @d5/y(y cm) min=-100.0 max=100.0:
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scat(300cm downstream) @d6/y(y cm) min=-100.0 max=100.0:
       @d6/x(x cm) min=-100.0 max=100.0;
scat(350cm downstream) @d7/y(y cm) min=-100.0 max=100.0:
       @d7/x(x cm) min=-100.0 max=100.0;
scat(400cm downstream) @d8/y(y cm) min=-100.0 max=100.0:
       @d8/x(x cm) min=-100.0 max=100.0;

Command File (solid.cmd)

@ssetsasp 660.0 \set Po of SASP to 660.0 MeV/c
@mapsolid s \smeared source
   -1.0 1.0 2.0 \xmin, xmax, xstep
   -0.5 0.5 2.0 \ymin, ymax, ystep
726.0 70.0 \ray momentum, dist to aperture
0.100 \scattering cone size (in str)
100000 \number of rays
@analize \print out downstream cross sections
I.5 Focal Surface

Device File (solid.dev)

```plaintext
define main{
    sasp_optics
}
```

Data Acquisition File (solid.dac)

```plaintext
#address saspend=[sasp_optics]
#function mymark=marker(@saspend/x;)
```

Command File (solid.cmd)

```plaintext
@setsasp 660.0 \set Po of SASP to 660.0 MeV/c
@focus -1.0 1.0 -4.70 \x: min max D\M
   -85.0 85.0 \theta: min max
   10.0 24.0 0.5 \delta: min max step
   -0.05 0.05 0.005 \y: min max step
   -50.0 50.0 5.0 \phi: min max step
   500 \number
@end
```
Appendix J

Track Reconstruction Using VDCs

The EASY generic function \texttt{vdc2} reconstructs the trajectory of a ray passing through two vertical drift chambers at some arbitrary plane (called the canonical plane) using the position outputs of the VDCs. This plane is not necessarily parallel to the VDCs and is usually the focal plane. The geometry using two identical chambers is shown in figure J.1. Each chamber has an $x$ plane and a $u$ plane readout where the $u$ axis is at some angle $\theta_u$ to the $x$ axis. The chambers are parallel and set at an angle $\theta_{VDC}$ to the optical axis. The lower chamber is called VDC1 and the upper, VDC2. The $x$ origin of each chamber lies on the optical axis while the $u = 0$ position lies on the perpendicular running through $x = 0$. The spacing between $x$ planes of the two chambers is $D$ and the separation between the $x_1$ plane and the canonical plane (which is set at some angle $\theta_{rp}$ to the optical axis) is $d_{rp}$.

1. Convert the individual chamber readouts to the $(x_1,u_1)$ coordinate system.

   \[
   x'_1 = x_1 \\
   u'_1 = u_1 \\
   x'_2 = x_2 + O_{21} \\
   u'_2 = u_2 + O_{21} \cos \theta_u
   \]

   where $O_{21}$ is the distance between the origins of $x_1$ and $x_2$.

2. Calculate the ratios

   \[
   \frac{v_x}{v_z} = \frac{(x'_2 - x'_1)}{D} \\
   \frac{v_u}{v_z} = \frac{(u'_2 - u'_1)}{D}
   \]

   240
Figure J.1: Reconstructing a track using two VDC's
where \( v_x, v_u, v_z \) are the \( x, u, \) and \( z \) velocities of the particle.

3. The \( u_1 \) plane is above (or downstream) of the \( x_1 \) plane. Calculate the \( u \) position of the ray in the \( x \) plane.

\[
x_r = x_1'
\]
\[
u_r = u_1' - d \frac{v_u}{v_z}
\]

4. Calculate the \( y \) position and velocity in the \( x_1 \) plane.

\[
y_r = \frac{u_r}{\sin \theta_u} - \frac{x_r}{\tan \theta_u}
\]
\[
\frac{v_y}{v_z} = \frac{v_x}{v_z} \frac{\sin \theta_u}{\tan \theta_u} - \frac{v_x}{v_z}
\]

5. Rotate the velocity to a plane parallel to the canonical plane.

\[
\frac{v_{xc}}{v_{zc}} = \frac{v_x \cos \theta_r + \sin \theta_r}{-\frac{v_x}{v_z} \sin \theta_r + \cos \theta_r}
\]
\[
\frac{v_{yc}}{v_{zc}} = \frac{v_y}{v_z} \frac{\sin \theta_r + \cos \theta_r}{\sin \theta_r + \cos \theta_r}
\]

where \( \theta_r = \theta_{rp} - \theta_{VDC} \) is the angle between the canonical plane and the VDC plane.

6. Calculate the distance to the canonical plane and the displacement of the canonical plane's origin.

\[
d_{rpp} = d_r + x_r \sin \theta_r
\]
if \( d_{rp} < 0 \) (the canonical plane is below the VDC1)

\[
O_p = O_{r1} + x_r \cos \theta_r
\]
else

\[
O_p = x_r \cos \theta_r - O_{r1}
\]

and the position on the canonical plane,

\[
x_c = d_{rpp} \frac{v_{xc}}{v_{zc}} + O_p
\]
7. Calculate the angles the canonical ray makes with the $z_c$ axis,

\[
\theta_c = \tan^{-1} \left( \frac{v_{zc}}{v_{zc}} \right)
\]

\[
\phi = \tan^{-1} \left( \frac{v_{zc}}{v_{zc}} \right)
\]
Appendix K

EASY's SASP Model

The complete EASY definition of the SASP spectrometer is kept in the two files SASP.DEF.DEV and SASP.DEF.DAC.

K.1 Device File

`EASY's SASP Model

The complete EASY definition of the SASP spectrometer is kept in the two files SASP.DEF.DEV and SASP.DEF.DAC.

K.1 Device File

```bash
#***************************************************************
#
# SASP Definition File
#
# 26 July 1988
#
# The geometry used for this model is approximate only.
#
# To use this SASP definition, define the following device in
# your DEV file,
#
# define sasp{
#     sasp_fec
#     sasp_optics
#     sasp_stack
# }
#
# To run the SASP without a front end chamber, delete the SASP_FEC
# from the definition.
#
244`
define sasp_cathode{
    drift \mylar
    aperture entrance none
    length 0.00254
    radlength 28.7
    aperture exit none
}

define sasp_vdc_gas{
    drift \gas
    aperture entrance none
    length 1.5875
    radlength 34
    aperture exit none
}
\ Estimated rad length for 50% Ar and 50% isobutane at STP

define sasp_vdc_gas_bulged{
    drift \gas
    aperture entrance none
    length 2.8575
    radlength 34 \estimate
    aperture exit none
}

define sasp_fec_gas{
    drift \gas
    aperture entrance none
    length 1.27
    radlength 16930 \isobutane 0 STP (should to be 1/3 atm)
    aperture exit none
}
define sasp_fec_gas_bulged{
   drift \gas
   aperture entrance none
   length 2.54
   radlength 16930 \isobutane @ STP (supposed to be 1/3 atm)
   aperture exit none
}

define sasp_fec_window{
   drift \mylar window
   aperture entrance rect -10.0 10.0 -10.0 10.0
   length 0.00254
   radlength 28.7
   aperture exit none
}

\ The SASP's FEC consists of four wire planes separated by
\ gas and cathodes. The cell is pressurized at 1/3 atm and
\ the windows bulge out 0.5". While the FEC is really a
\ proportional counter, the VDC pos sensor is used because the
\ PC option is not yet implemented --- just ignore the angle
\ information.
define sasp_fec_cell{
   sasp_fec_window
   sasp_fec_gas_bulged
   sasp_cathode
   sasp_fec_gas
   pos_sensor -10.0 10.0 0.80 0.0 \x0 plane
   vdc 0.05 0.5 0.0
   sasp_fec_gas
   sasp_cathode
sasp_fec_gas
pos_sensor -10.0 10.0 0.80 0.0 \ x0' plane
   vdc 0.05 0.5 0.0
sasp_fec_gas
sasp_cathode
sasp_fec_gas
pos_sensor -10.0 10.0 0.80 90.0 \ y0 plane
   vdc 0.05 0.5 0.0
sasp_fec_gas
sasp_cathode
sasp_fec_gas
pos_sensor -10.0 10.0 0.80 90.0 \ y0' plane
   vdc 0.05 0.5 0.0
sasp_fec_gas
sasp_cathode
sasp_fec_gas_bulged
sasp_fec_window
}

\ The SASP's FEC consists of a drift from the target
\ plane to the FEC, transport through the FEC, and then
\ a bactrack to the target plane. This positions the
\ ray ready for transportation through the SASP_OPTICS
\ device.
define sasp_fec{
   drift \ vacuum
       aperture entrance none
       length 50.0
       radlength -1
       aperture exit none
   sasp_fec_cell
   backtrack 65.25778
}
APPENDIX K. EASY'S SASP MODEL

\define sasp_vdc_window{ 
  \drift \mylar window 
  \aperture entrance rect -100.0 100.0 -20.0 20.0 
  \length 0.00254 
  \radlength 28.7 
  \aperture exit none 
}

\The SASP's VDCs consist of a window followed by gas, a cathode plane, gas, wire plane (x), gas, cathode, wire plane (u), gas, cathode, gas, window. The gas mixture is 50\% Ar and 50\% isobutane.
define sasp_vdc{ 
  sasp_vdc_window 
  sasp_vdc_gas_bulged 
  sasp_cathode 
  sasp_vdc_gas 
  pos_sensor -90.0 90.0 0.80 0.0 \x plane 
    \vdc 0.015 0.5 28.0 
  sasp_vdc_gas 
  sasp_cathode 
  sasp_vdc_gas 
  pos_sensor -100.0 100.0 0.80 30.0 \u plane 
    \vdc 0.015 0.5 28.0 
  sasp_vdc_gas 
  sasp_cathode 
  sasp_vdc_gas_bulged 
  sasp_vdc_window 
}

\The SASP_STACK transports the ray starting at the exit of the dipole through the vacuum to the exit window of the SASP vacuum, to and through the 3 VDCs
the trigger paddles, and the 2 downstream scintillators.
The ray is rotated to a frame where the VDCs appear horizontal
are drifted through them, and then rotated back to the lab frame
(z points vertically up) to be drifted through the downstream
scintillators.

define sasp_stack{
  drift \vacuum
    aperture entrance none
    length 158.0
    radlength -1
    aperture exit none
  rotate y -46.0
  drift \mylar vacuum window
    aperture entrance none \ change this
    length 0.00254
    radlength 28.7
    aperture exit none \ change this
  drift \air
    aperture entrance none
    length 1.3867767
    radlength 30420.0
    aperture exit none
  translate 6.0468723 0.0 0.0

  sasp_vdc
  drift \air
    aperture entrance none
    length 5.7023
    radlength 30420.0
    aperture exit none
  translate 18.411728 0.0 0.0
  sasp_vdc
  drift \air
    aperture entrance none
    length 5.7023
APPENDIX K. EASY'S SASP MODEL

radlength 30420.0
aperture exit none
translate 18.411728 0.0 0.0
sasp_vdc
drift \air
    aperture entrance none
    length 25.78
    radlength 30420.0
    aperture exit none
translate 22.087778 0.0 0.0

scint
    aperture entrance rect -105.0 105.0 -21.0 21.0
    length 0.5
    radlength 42.4
    aperture exit rect -105.0 105.0 -21.0 21.0
translate 0.51776515 0.0 0.0

rotate y 46.0
drift \air
    aperture entrance none
    length 110.97782
    radlength 30420
    aperture exit none
scint
    aperture entrance rect -90.0 90.0 -22.0 22.0
    length 0.5
    radlength 42.4
    aperture exit rect -90.0 90.0 -22.0 22.0
drift \air
    aperture entrance none
    length 59.5
    radlength 30420
    aperture exit none
scint
appendix entrance rect -100.0 100.0 -25.0 25.0
length 0.5
radlength 42.4
aperture exit rect -100.0 100.0 -25.0 25.0
}

K.2 Data Acquisition File

Standard EDAC definitions for SASP.DEF.DEV

25 Jan 1988

#address sasp_x1=[sasp;sasp_vdc,1;pos_sensor,1]
#address sasp_u1=[sasp;sasp_vdc,1;pos_sensor,2]
#address sasp_x2=[sasp;sasp_vdc,2;pos_sensor,1]
#address sasp_u2=[sasp;sasp_vdc,2;pos_sensor,2]
#address sasp_x3=[sasp;sasp_vdc,3;pos_sensor,1]
#address sasp_u3=[sasp;sasp_vdc,3;pos_sensor,2]

#address sasp_tp=[sasp;scint,1]
#address sasp_s1=[sasp;scint,2]
#address sasp_s2=[sasp;scint,3]

#address sasp_x0=[sasp_fec;pos_sensor,1]
#address sasp_x0p=[sasp_fec;pos_sensor,2]
#address sasp_y0=[sasp_fec;pos_sensor,3]
#address sasp_y0p=[sasp_fec;pos_sensor,4]
#boolean sasp_vdc1_fired=(sasp_x1 && sasp_u1)
#boolean sasp_vdc2_fired=(sasp_x2 && sasp_u2)
#boolean sasp_vdc3_fired=(sasp_x3 && sasp_u3)
#boolean sasp_fec_fired=((sasp_x0 || sasp_x0p)&&(sasp_y0 || sasp_y0p))

#boolean sasp_tp_fired=(sasp_tp)
#boolean sasp_scints_fired=(sasp_s1 && sasp_s2)
#boolean sasp_vdcs_fired=(sasp_vdc1_fired && sasp_vdc2_fired && sasp_vdc3_fired)

#boolean sasp_trigger=(sasp_vdcs_fired && sasp_tp_fired && sasp_scints_fired && sasp_fec_fired)

#function sasp_vdc12=vdc2(sasp_x1,sasp_u1,sasp_x2,sasp_u2; 17.78,3.17754,30,46,160,0)
#function sasp_vdc13=vdc2(sasp_x1,sasp_u1,sasp_x3,sasp_u3; 35.56,3.17754,30,46,160,0)
Appendix L

EASY's MRS Model

The complete EASY definition of the MRS spectrometer is kept in the two files:

MRS.DEF.DEV
MRS.DEF.DAC

L.1 Device File

(MRS Device Definition File)

26 July 1988

The geometry used in this file is approximate only.

To use this MRS definition, define the following device in your DEV file:

define mrs{
    mrs_fec
    mrs_optics
    mrs_stack
}
define mrs_gas_bulged{
    drift
        aperture entrance none
        length 2.54
        radlength 34
        aperture exit none
}
\estimate of 50\% Ar and 50\% isobutane mix at STP radlength

define mrs_gas{
    drift
        aperture entrance none
        length 1.27
        radlength 34 \estimate
        aperture exit none
}

define mrs_fec_gas{
    drift
        aperture entrance none
        length 1.27
        radlength 16930
        aperture exit none
}
\isobutane at STP radlength (NOTE that correct pressure \is 1/3 atm).

define mrs_fec_gas_bulged{
    drift
        aperture entrance none
        length 2.54

APPENDIX L. EASY'S MRS MODEL

radlength 16930
aperture exit none

define mrs_cathode{
  drift
  aperture entrance none
  length 0.00254
  radlength 28.7
  aperture exit none
}

define mrs_vdc_window{
  drift
  aperture entrance rect -52.5 52.5 -17.5 17.5
  length 0.00254
  radlength 28.7
  aperture exit none
}

\ MRS VDC consists of 2 wire planes sandwiched between gas and
\ cathode layers.
define mrs_vdc{
  mrs_vdc_window
  mrs_gas_bulged
  mrs_cathode
  mrs_gas
  pos_sensor -50.0 50.0 0.80 0 \x plane
    vdc 0.015 0.5 28.0
  mrs_gas
  mrs_cathode
  mrs_gas
  pos_sensor -50.0 50.0 0.80 30 \u plane
    vdc 0.015 0.5 28.0
define mrs_fec_window{
    drift
        aperture entrance rect -4.0 4.0 -4.0 4.0
        length 0.00254
        radlength 28.7
        aperture exit none
}

\ The MRS FEC consists of 4 wire planes (2 x and 2 y)
\ with gas at 1/3 atm. Using VDC pos sensor type instead
\ of PC because PC not implimented yet. Just ignore the
\ angle information.
define mrs_fec_cell{
    mrs_fec_window
    mrs_fec_gas_bulged
    mrs_cathode
    mrs_fec_gas
    pos_sensor -4.0 4.0 0.80 0.0 \x0 plane
        vdc 0.015 0.5 28
    mrs_fec_gas
    mrs_cathode
    mrs_fec_gas
    pos_sensor -4.0 4.0 0.80 0.0 \x0' plane
        vdc 0.015 0.5 28
    mrs_fec_gas
    mrs_cathode
    mrs_fec_gas
    pos_sensor -4.0 4.0 0.80 90.0 \y0 plane
        vdc 0.015 0.5 28
The complete MRS FEC device. Drifts the ray through vacuum to the FEC, through the FEC and then back to the target plane so it is ready for transport through the MRS OPTICS.

```
define mrs_fec{
    drift \vacuum
    aperture entrance none
    length 56.0
    radlength -1
    aperture exit none
    mrs_fec_cell
    backtrack 71.25778
}
```

The complete MRS detector stack. Drifts the ray from the exit of the MRS dipole through vacuum, out the vacuum window, through the 2 VDC's, the trigger paddles (approximated as a single scintillator) and through the downstream scintillators. Note the Focal Plane Polarimeter is not implemented.

```
define mrs_stack{
    drift \vacuum
    aperture entrance none
    length 419.6
    radlength -1
```
aperture exit none
rotate y -45.0
drift \vacuum window
    aperture entrance none
    length 0.00254
    radlength 28.7 \mylar
    aperture exit none
drift \air
    aperture entrance none
    length 1.4124175
    radlength 30420 \air
    aperture exit none
translate 5.2292135 0.0 0.0
mrs_vdc \VDC1
    drift \air
    aperture entrance none
    length 23.952272
    radlength 30420 \air
    aperture exit none
translate 34.124973 0.0 0.0
mrs_vdc \VDC2
    drift \air
    aperture entrance none
    length 10.0
    radlength 30420 \air
    aperture exit none
translate 16.3577 0.0 0.0
scint \TP
    aperture entrance rect -60.0 60.0 -30.0 30.0 \garbo
    length 0.5
    radlength 42.4 \polystyrene scintillator
    aperture exit none
translate 0.5 0.0 0.0
rotate y 45.0
drift \air
APPENDIX L. EASY'S MRS MODEL

aperture entrance none
length 36.074388
radlength 30420
aperture exit none

scint \S1
aperture entrance rect -60.0 60.0 -30.0 30.0 \garbo
length 0.5
radlength 42.4 \polystyrene
aperture exit none

drift \air
aperture entrance none
length 56.65
radlength 30420
aperture exit none

scint \S2
aperture entrance rect -60.0 60.0 -30.0 30.0 \garbo
length 0.5
radlength 42.4 \polystyrene
aperture exit none

}

L.2 Data Acquisition File

\ Standard EDAC definitions for MRS_DEF.DEV
\ 25 Jan 1988
\ 
#address mrs_x1=[mrs;mrs_vdc,1;pos_sensor,1]
#address mrs_u1=[mrs;mrs_vdc,1;pos_sensor,2]
#address mrs_x2=[mrs; mrs_vdc,2; pos_sensor,1]
#address mrs_u2=[mrs; mrs_vdc,2; pos_sensor,2]

#address mrs_tp=[mrs; mrs_stack; scint,1]
#address mrs_s1=[mrs; mrs_stack; scint,2]
#address mrs_s2=[mrs; mrs_stack; scint,3]

#address mrs_x0=[mrs_fec; pos_sensor,1]
#address mrs_x0p=[mrs_fec; pos_sensor,2]
#address mrs_y0=[mrs_fec; pos_sensor,3]
#address mrs_y0p=[mrs_fec; pos_sensor,4]

#boolean mrs_vdc1_fired=(mrs_x1 && mrs_u1)
#boolean mrs_vdc2_fired=(mrs_x2 && mrs_u2)
#boolean mrs_fec_fired=((mrs_x0 || mrs_x0p) && (mrs_y0 || mrs_y0p))

#boolean mrs_tp_fired=(mrs_tp)
#boolean mrs_scints_fired=(mrs_s1 && mrs_s2)
#boolean mrs_vdcs_fired=(mrs_vdc1_fired && mrs_vdc2_fired)

#boolean mrs_trigger=(mrs_vdcs_fired && mrs_tp_fired
                           && mrs_scints_fired
                           && mrs_fec_fired)

#function mrs_vdc2=vdc2(mrs_x1,mrs_u1,mrs_x2,mrs_u2;
                         34.124973, 2.54254, 30, 45, 424.9942859, 0)
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