Nonlinear response of two-dimensional waveguide-based photonic crystals and microstructured fibres

by

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ABSTRACT

The nonlinear response of two-dimensional waveguide-based photonic crystals and photonic crystal fibres is investigated in this thesis. First, the third order nonlinear response of planar waveguide-based photonic crystals is studied theoretically to estimate the influence of this nonlinear susceptibility on the specular reflectivity spectrum of these structures. It is shown that coupling of the incident field with leaky photonic modes introduces features in the specular reflectivity spectrum that are dependent on the input intensity. By changing the input intensity, the reflectivity of this structure at a desired wavelength can, theoretically, be switched from zero to one. This nonlinear property might have potential application in all-optical switching devices. Both degenerate (single beam) and nondegenerate (dual beam, pump/probe) geometries are considered.

In the second part of the thesis, the output spectrum of a microstructured fibre, which has a two-dimensional photonic crystal cladding is investigated as a function of power when ~100 fs laser pulses are launched into the fibre at an 80 MHz repetition rate. For launched average powers of ~20 mW to ~100 mW, the output spectrum is dramatically shifted and broadened compared to the input laser spectrum. This occurs through a combination
of nonlinear optical effects that are not currently understood. The detailed nature of the spectrum and the temporal properties of the light emitted from the photonic crystal fibre depend on the excitation wavelength and the coupling geometry. Under some conditions the spectrum consists of a series of discrete red-shifted components that shift monotonically further to the red at higher input powers. In this case the temporal shape of the output beam is found to be strongly asymmetric, with a rapid leading edge, and a slower decay. Under other conditions, the spectrum is more like a continuum, and the behaviour in the time domain is symmetric.
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CHAPTER 1

INTRODUCTION

The term "Photonics" describes a technology in which transmission and processing of data occurs partially or entirely by means of photons. A motivation for this technology is that by replacing relatively slow electrons with photons as the carriers of information, the bandwidth and speed of communication and computation systems can be dramatically increased. In telecommunication systems, optical fibres with their huge optical bandwidth already carry most of the information, but switching mostly relies on electronic or optoelectronic components. Even by using optoelectronic switches, optical signals have to be converted to electronic signals and then the switched signals have to drive an optical transmitter. However, to realize the full benefits of optical technology it will be necessary to build all-optical networks. It is self-defeating to convert the signal from optical to electronic forms every time it needs to be routed or switched.

Photonic crystals (PCs) offer a promising solution for this problem. PCs are microstructured materials in which the dielectric constant is periodically modulated on a length scale comparable to the desired optical wavelength of operation. They are usually viewed as the optical analogues of semiconductors: the periodic dielectric lattice modifies the propagation properties of the light in much
the same way as the microscopic atomic lattice of a semiconductor crystal modifies the propagation properties of electrons. Multiple interference between waves scattered from each unit cell of the structure may actually result in a "photonic bandgap", or a range of frequencies within which no propagating electromagnetic modes exist. Fig. (1.1) shows a diagram of one, two, and three-dimensional photonic crystals.

![Photonic Crystals Diagram](image)

Figure 1.1: Schematic diagram of one-dimensional (1D), two-dimensional (2D), and three-dimensional (3D) photonic crystals.

Although the bandgap property of photonic crystals facilitates the control and manipulation of light propagation, making optical switching elements from these structures requires the ability to dynamical tune their
properties. There are several ways to parametrically ad­
just the photonic crystal bandstructure. Many of these
methods involve incorporating foreign materials into the
photonic crystals, materials that can have their refrac­
tive index modified by varying some external parame­
ter. Examples include liquid Crystals [1], magnetic ma­
terial such as ferrite [2], and thermally-sensitive poly­
meric material such as Poly(3-alkylthiophene)[3]. The
alternative approach and the one that offers the ulti­
mate in switching speed, involves using light itself to
influence the propagation properties of photonic crystals
made from materials that respond nonlinearly to light
intensity[4, 5, 6, 7, 8, 9, 10, 11, 12].

Nonlinear optics of photonic crystals is therefore a
rapidly growing area of research. The strong optical con­
finement possible in these structures helps to reduce the
optical power levels needed to produce usable nonlinear
effects, as compared with bulk nonlinear optical materi­
als. Another advantage of PC systems for nonlinear opt­
ics is that the photon dispersion relation and the density
of electromagnetic states can be engineered over a wide
range of relevant parameter space, and these factors are
known to play important roles in nonlinear processes like
harmonic generation and frequency mixing. Huge non­
linear effects have already been reported [13, 14, 15, 16]
in photonic-crystal fibre excited by unamplified pulsed
laser systems. If research on the nonlinear-optical prop­
erties of PCs and PC fibres can result in a quantitative
understanding of these complex processes, then many
possible all-optical devices might be realized. Examples
include; limiters and switches, compact pulse compressors, and frequency converters \cite{17, 18, 19, 20}.

This thesis is concerned primarily with third-order nonlinear processes, one of which is the "optical Kerr effect" that effectively describes a material in which the refractive index is modified in proportion to the intensity of light in the material. That is \( n = n_0 + n_2 I \), where \( n_0 \) refers to the linear refractive index of the material, \( n_2 \) is the nonlinear Kerr coefficient, and \( I \) is the light intensity. In recent years the influence of the optical Kerr effect on the band structure of bulk photonic crystals has been considered theoretically \cite{21, 22, 23, 24} and experimentally observed in 1D silicon crystals using free-carrier effects \cite{25}. If free carriers are not involved, the Kerr effect phenomena can be used at very high speeds limited only by the optical pulse duration. The main challenge in this area is to develop a quantitative and efficient nonlinear numerical modelling tool, and use it to engineer structures that exhibit large nonlinear effects at moderate optical intensities that are compatible with optical communication systems (hundreds of milliwatts average power at GHz repetition rates).

Severe difficulties in making three-dimensional photonic crystals, and the existence of a mature technology for manufacturing electronic semiconductor circuits, make 2D planar photonic crystals an attractive solution for integrating various optical functions needed for processing information. Like bulk two-dimensional photonic crystals, planar photonic crystal semiconductor waveg-
uides, like in Fig.(1.1) (2D), can possess pseudo-photonic bandgaps that can be used to effectively localize photonic modes [26, 27, 28, 29, 30, 31, 32, 33]. Electromagnetic eigenmodes of two-dimensional planar photonic crystals (PPCs) can be classified as guided, leaky, or radiative. Both guided and leaky modes are substantially localized to the textured slab waveguide, while radiative modes are roughly uniformly distributed throughout all space. Guided modes are completely bound by total internal reflection to the waveguide core (they have an infinite lifetime) but leaky modes can couple to modes that propagate away from the waveguide, so they acquire a finite lifetime. Although the nonlinear properties of both guided and leaky eigenstates are of substantial interest, this thesis deals strictly with leaky modes. Leaky modes have been shown by this group [34] to strongly enhance the conversion efficiency for second harmonic generation from 2D photonic crystals. In that work, it was shown how the leaky modes of the 2DPPCs can really be thought of as microcavity resonances, similar to resonant photonic states in 1D Fabry-Perot cavities. Chapter 2 of this thesis extends this second harmonic study to consider the way in which the third order optical Kerr effect can be used to optically modify leaky mode properties in 2D PPCs.

Chapter 2 describes a Green’s function formalism developed and applied by the author to calculate the fields excited in the textured part of 2DPPCs by plane waves incident from the top half space. One of the advan-
tages of using a textured slab waveguide instead of a 1D Fabry-Perot cavity to enhance these nonlinear effects is the greater control the PPCs offer over the Q-values (mode linewidth or full-width at half maximum divided by the centre frequency) and dispersion of the leaky modes. The fully self-consistent calculation shows intensity dependent changes of the reflectivity spectrum associated with the optical renormalization of leaky photonic modes. The result of the calculation also shows that by changing the input intensity, the reflectivity of this structure at a desired wavelength can be switched from 0 to 1, which demonstrates the dynamic tunability of this structure with light intensity. Calculations also reveal that the magnitude of the optically induced frequency shift strongly depends on the bandwidth and spatial profiles of these leaky photonic eigenmodes. Both degenerate (a single strong excitation beam) and nondegenerate (one strong beam influencing the reflectivity of a weaker, independent probe beam) optical Kerr geometries are considered. Although preliminary experimental results related to these calculations have been obtained, they are not discussed in this thesis.

The effects predicted in chapter 2 will ultimately be measured using pulsed laser sources to excite the sample, and both time-resolved and time-integrated spectroscopy will be used to characterize the nonlinear reflection. In order to develop the relevant experimental techniques on a simpler, but related system, the author set up a time-resolved optical excitation and probe system to characterize the nonlinear transmission of $\sim$100 fs long laser
pulses through a commercially available photonic crystal fibre. Not only did this serve to help develop the apparatus needed to study the PPCs, it also revealed some novel results that might shed light on the nonlinear mechanisms at work in these fibres.

Chapter 3 describes the apparatus developed by the author to measure the time-integrated spectrum and the temporal properties of the broadband light emitted from these fibres when excited by more than 20 mW of mode-locked laser pulses at a repetition rate of 80 MHz. The spectra are recorded using a Fourier transform spectrometer. The time-domain investigations involved an optical cross-correlation experiment in which a nonlinear BBO (Beta-Barium Borate) crystal is used to produce a sum frequency signal from a portion of the 100 fs Ti:Sapp laser beam and the output beam from a photonic crystal fibre excited by the same 100 fs pulses. The experiment shows that the output spectrum of the fibre strongly depends on the input intensity and wavelength of the pump laser. At two different pump wavelengths, discrete shifts and supercontinuum generations are observed under different in-coupling conditions. The most interesting and novel result is that the temporal evolution of the up-converted signal for these two situations is markedly different. When the output spectrum consists of a series of discrete, broadened peaks, their temporal profile is highly asymmetric with a sharp leading edge, followed by a relatively long decay. In the supercontinuum case the temporal profile of the up-converted signal is symmetric. These observations may help identify the
nature of the complex nonlinear mechanisms responsible for these broadenings.
CHAPTER 2

THIRD-ORDER NONLINEAR RESPONSE OF 2D PLANAR PHOTONIC CRYSTALS

2.1 Nonlinear Polarization

One way to treat theoretically nonlinear optical effects is to expand the polarization of a medium under the influence of an applied electric field as a power series:

$$\bar{P}(t) = \bar{P}^{(0)}(t) + \bar{P}^{(1)}(t) + \bar{P}^{(2)}(t) + \bar{P}^{(3)}(t) + ... \quad (2.1)$$

in which $\bar{P}^{(n)}(t)$ is proportional to the nth power of the applied electric field. In Eq.(2.1), terms with $n \geq 2$ are responsible for nonlinear polarization. The third order nonlinear polarization, transformed to the frequency domain is:

$$\tilde{P}^{(3)}(\omega_\sigma) = \chi^{(3)}(-\omega_\sigma; \omega_1, \omega_2, \omega_3) \tilde{E}(\omega_1) \tilde{E}(\omega_2) \tilde{E}(\omega_3) \quad (2.2)$$

with $\omega_\sigma = \omega_1 + \omega_2 + \omega_3$ and where $\chi^{(3)}(-\omega_\sigma; \omega_1, \omega_2, \omega_3)$ represents the third-order susceptibility of the medium, which is in general dispersive. Table (2.1) summarizes the common third-order nonlinear phenomena [35], and associates names with the different physical responses. The distinguishing feature of each distinct process is the
Table 2.1: List of common third-order nonlinear phenomena.

<table>
<thead>
<tr>
<th>Process</th>
<th>$\chi^{(3)}(-\omega_3; \omega_1, \omega_2, \omega_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.C. Kerr effect</td>
<td>$\chi^{(3)}(-\omega : 0, 0, \omega)$</td>
</tr>
<tr>
<td>(quadratic electro-optic effects)</td>
<td></td>
</tr>
<tr>
<td>D.C. Induced second harmonic generation</td>
<td>$\chi^{(3)}(-2\omega : 0, \omega, \omega)$</td>
</tr>
<tr>
<td>Third harmonic generation</td>
<td>$\chi^{(3)}(-3\omega : \omega, \omega, \omega)$</td>
</tr>
<tr>
<td>Four wave mixing</td>
<td>$\chi^{(3)}(-\omega_4 : \omega_1, \omega_2, \omega_3)$</td>
</tr>
<tr>
<td>Third-order sum and difference frequency mixing</td>
<td>$\chi^{(3)}(-\omega_3; \pm\omega_1, \omega_2, \omega_3)$</td>
</tr>
<tr>
<td>Optical Kerr effect</td>
<td>$\chi^{(3)}(-\omega_p; \omega_1, -\omega_p, \omega_p)$</td>
</tr>
<tr>
<td>(optically-induced birefringence)</td>
<td></td>
</tr>
<tr>
<td>Cross-phase modulation,</td>
<td></td>
</tr>
<tr>
<td>Stimulated Raman scattering</td>
<td></td>
</tr>
<tr>
<td>Stimulated Brillouin scattering</td>
<td></td>
</tr>
<tr>
<td>Optical Kerr effect</td>
<td>$\chi^{(3)}(-\omega : \omega, -\omega, \omega)$</td>
</tr>
<tr>
<td>(intensity-dependent refractive index)</td>
<td></td>
</tr>
<tr>
<td>Degenerate four wave mixing</td>
<td></td>
</tr>
<tr>
<td>Self and cross-phase modulation</td>
<td></td>
</tr>
<tr>
<td>Self-focusing</td>
<td></td>
</tr>
<tr>
<td>Two photon absorption, ionisation, and emission</td>
<td>$\chi^{(3)}(-\omega_1; -\omega_2, \omega_2, \omega_1)$</td>
</tr>
</tbody>
</table>

In the following section a Green’s function formalism is explained, which can be used to calculate the linear and nonlinear response of two-dimensional wave-guide
based photonic crystals. The nonlinearity considered in sections (2.3) and (2.4) is the optical Kerr effect in which the refraction index of the medium depends on the intensity of light \( I \),

\[ n = n_0 + n_2 I. \]

For the degenerate Kerr effect, \( n_2 \) in the above equation is related to the third-order susceptibility of the medium\[35\] as:

\[ n_2 = \frac{3 \text{Re} \chi^{(3)}(-\omega : \omega, -\omega, \omega)}{4 \epsilon_0 c n_0^2}. \]

### 2.2 Green's function formalism

The model developed by the author\(^1\) to treat the third order nonlinear response of planar waveguide based photonic crystals is an extension of the linear optical model \[33, 37, 38\] developed previously to describe the dispersion of photonic eigenstates in this geometry. This section therefore starts with a brief review of the general Green's function formalism used to efficiently solve the Maxwell's equations in this geometry. The following section extends the discussion to describe how the third-order nonlinearity was incorporated. The numerical model calculates the reflectivity spectrum of plane waves incident from the top half space for different incident field

\(^1\)The work described in this chapter was recently published in Ref\[36\]
strengths. The situation in which the 2DPC is excited only by one strong pump beam, and the other situation where it is excited by a strong pump and a weak signal beam at a different wavelength, are considered separately.

Fig. (2.1) shows a schematic diagram of the specific 2DPC which was used in the nonlinear simulations described next. It consists of an 80 nm thick GaAs core layer that is textured by a square lattice of air holes (with 164 nm radius), which penetrate the core layer completely on a pitch of 500 nm. Beneath this 2DPC was an 1800 nm thick alumina layer and a GaAs substrate. The fabrication process used to make samples like this consists of the following steps:

Molecular beam epitaxy (MBE) is used to grow single-crystal layers of Al$_{0.98}$Ga$_{0.02}$As and GaAs on the GaAs substrate. Etching the waveguide through a mask of PMMA (polymethyl-methacrylate) makes the holes that define the photonic crystal. The holes are formed in the mask using electron beam lithography system and then the etching is done with an electron cyclotron resonance (ECR) plasma etcher using chlorine. After etching, the structure is exposed to a moist environment at 425°C which converts the Al$_{0.98}$Ga$_{0.02}$As layer into an aluminium oxide (alumina) layer with a refraction index of $n \approx 1.6$ [39, 40] (the linear refraction index of GaAs is around $\sim 3.4$ for the range of wavelengths considered here). This oxidization results in stronger confinement of the light in the GaAs layer due to the higher (compared
to the Al$_{0.98}$Ga$_{0.02}$As layer) refractive index contrast between the GaAs waveguide layer and the layer beneath.

\[ \mathbf{E}_{\text{inc}}(\omega_p) \]
\[ \mathbf{E}_{\text{inc}}(\omega_S) \]

\[ \beta_{\text{inc}}^P, \beta_{\text{inc}}^S \]
\[ \beta_{\text{specular}}^P, \beta_{\text{specular}}^S \]

\[ \varphi \]
\[ \psi \]
\[ w \]
\[ d \]
\[ \Lambda \]

\[ \text{SUBSTRATE} \]

Figure 2.1: Two-dimensional waveguide-based photonic crystal with a square lattice of cylindrical holes.

We are seeking an integral solution of Maxwell's equations in the textured area of the two-dimensional photonic crystal etched with a square lattice of air holes. When this photonic crystal is excited by an incident field with electric field \( \mathbf{E}_{\text{inc}}(\mathbf{r}) \), the total field inside the grating will satisfy an inhomogeneous equation derived from the general Maxwell's equations [41],

\[ \nabla \times \nabla \times \mathbf{E}(\mathbf{r}) - \frac{\omega^2}{c^2} \mathbf{E}(\mathbf{r}) = 4\pi \frac{\omega^2}{c^2} \chi(\mathbf{r}) \mathbf{E}(\mathbf{r}) \quad (2.3) \]
where we assume a harmonic, $e^{-i\omega t}$ time dependence for all fields. $\chi(\vec{r})$ is a combination of the susceptibility of the untextured background and that which describes the scattering centres etched into the dielectric media:

$$\chi(\vec{r}) = \chi_B(\vec{r}) + \Delta \chi(\vec{r})$$  \hspace{1cm} (2.4)

$\chi_B(\vec{r})$ represents the linear susceptibility of the background and $\Delta \chi(\vec{r})$ represents the linear and nonlinear susceptibility of the textured core layer. Therefore Eq. (2.3) is expressed as:

$$\nabla \times \nabla \times \vec{E}(\vec{r}) - \frac{\omega^2}{c^2} (1 + 4\pi \chi_B(\vec{r}) \vec{E}(\vec{r})) = 4\pi \frac{\omega^2}{c^2} \Delta \chi(\vec{r}) \vec{E}(\vec{r})$$  \hspace{1cm} (2.5)

The Green's function solution of the above equation is [42]:

$$\vec{E}(\vec{r}) = \vec{E}^{\text{hom}}(\vec{r}) + \int_V dr' G_B(\vec{r}, \vec{r}') \Delta \chi(\vec{r}') \vec{E}(\vec{r}')$$  \hspace{1cm} (2.6)

where $\vec{E}^{\text{hom}}(\vec{r})$ is a solution of the homogeneous wave equation. These homogeneous solutions are the fields generated in the background, untextured multi-layer, when driven by an incident plane wave from the top half space. $G_B(\vec{r}, \vec{r}')$ is the solution of Eq. (2.5) for a point source in the textured region. If the medium is a slab waveguide structure and is textured only in the core layer with a periodic pattern, then (Appendix A) Eq. (2.6) Fourier transforms in the plane to yield:
\[ \mathbf{E}_e(\omega : \vec{\beta}_{\text{inc}} + \vec{\beta}_n, z) = \mathbf{E}_e^{\text{hom}}(\omega : \vec{\beta}_{\text{inc}} + \vec{\beta}_n, z) + \]
\[ \int_{-L/2}^{+L/2} dz' \mathcal{G}_e(\omega : \vec{\beta}_{\text{inc}} + \vec{\beta}_n, z, z'). \Delta \mathcal{F}(\omega : \vec{\beta}_{\text{inc}} + \vec{\beta}_n, z') \]

(2.7)

where \( \vec{\beta}_n \) is the reciprocal lattice vector of each Fourier component of the (in this case) square lattice,

\[ \vec{\beta}_n = \beta_g (ix + jy), \beta_g = \frac{2\pi}{\Lambda} \]

(2.8)

where \( \Lambda \) is the pitch of real space lattice. In all of these equations \( \vec{\beta}_{\text{inc}} \) is the in-plane wave vector of the incident field. A detailed derivation of the Green's function that propagates the perturbing polarization into the same textured slab area can be found elsewhere \([33, 37, 38]\), and the result is:

\[ \mathcal{G}_e(\omega : \vec{\beta}, z, z') = \frac{2\pi i \bar{\omega}^2}{\omega_c(\beta)} \times \]

\[ \left\{ [\theta(z - z') e^{i\omega_c(\beta)(z-z')} + \theta(z' - z) e^{-i\omega_c(\beta)(z-z')} + \right. \]

\[ + [r_{s_{\text{up}}} e^{-i\omega_c(\beta)(z+z'-L)} + r_{s_{\text{down}}} e^{i\omega_c(\beta)(z+z'+L)} + \right. \]

\[ + r_{s_{\text{up}}} r_{s_{\text{down}}} (e^{i\omega_c(\beta)(z-z'+2L)} + e^{-i\omega_c(\beta)(z-z'-2L)}) \frac{1}{D_s} ] \mathcal{S}(\vec{\beta}) \mathcal{S}(\vec{\beta}) + \]

\[ [\theta(z - z') e^{i\omega_c(\beta)(z-z')} + \frac{r_{p_{\text{up}}} r_{p_{\text{down}}} e^{i\omega_c(\beta)(z-z'+2L)}}{D_p} ] \mathbf{P}_c(\vec{\beta}) \mathbf{P}_c(\vec{\beta}) + \]
\[ \theta(z' - z) e^{i\omega_c(\beta)(z - z')} + \frac{r_{pup} r_{pdown} e^{-i\omega_c(\beta)(z - z' - 2L)}}{D_p} \hat{p}_{c-}(\vec{\beta}) \hat{p}_{c-}(\vec{\beta}) + \]

\[ \frac{1}{D_p} [r_{pup} e^{-i\omega_c(\beta)(z + z' - L)} \hat{p}_{c-}(\vec{\beta}) \hat{p}_{c+}(\vec{\beta}) + \]

\[ r_{pdown} e^{-i\omega_c(\beta)(z + z' + L)} \hat{p}_{c+}(\vec{\beta}) \hat{p}_{c-}(\vec{\beta})] \]

\[ - \frac{4\pi}{\epsilon_c} \delta(z - z') \hat{z} \hat{z} \]  

(2.9)

The various reflection coefficients in this Green’s function, \( r_{s_{up}} \), \( r_{s_{down}} \), \( r_{p_{up}} \), and \( r_{p_{down}} \) are the net reflection coefficients of the s and p polarized fields inside the textured layer as they try to propagate upwards and downwards respectively. They include all multiple reflections from the untextured background. The other parameters appearing in the Green’s function include

\[ D_s = 1 - r_{s_{up}} r_{s_{down}} e^{i\omega_c(\beta)2L} \]

\[ D_p = 1 - r_{p_{up}} r_{p_{down}} e^{i\omega_c(\beta)2L} \]  

(2.10)

where \( L \) is thickness of the slab. The unit vectors \( \hat{s}(\vec{\beta}) \), \( \hat{p}_{c+}(\vec{\beta}) \), \( \hat{p}_{c-}(\vec{\beta}) \), and \( \hat{z}(\vec{\beta}) \) are basis vectors specific to each Fourier component of the electric field. They are defined as:

\[ \hat{s}(\vec{\beta}) = \hat{k}(\vec{\beta}) \times \hat{z}(\vec{\beta}) \]

(2.11)
and

\[ \hat{p}_{c\pm}(\beta) = \frac{k \hat{z} \mp \omega_c(\beta)\hat{k}}{\tilde{\omega}\sqrt{\epsilon_c}} \]  \hspace{1cm} (2.12) 

and finally \( \omega_c(\beta) = \sqrt{\omega^2\epsilon_c - \beta^2} \), \( \tilde{\omega} = \omega/c \), and \( \hat{k} \) is the unit vector in the direction of the incident in-plane wave vector.

In Eq. (2.7) \( \Delta P \) is the total polarization inside the textured layer resulting from the incident field. In the linear situation this polarization is written as:

\[ \Delta P(\omega : \beta_{inc} + \beta_n) = \Delta \chi^{(1)} (\omega : \beta_{inc} + \beta_n) \]

where summation over the repeated \( \beta_k \) is implied. If the thickness of the grating is very small relative to the wavelength of the incident field, the electric field, polarization, and susceptibility can be considered constant in the \( z \) direction, and after doing the integration, the solution for the field components in the grating layer is:

\[ \bar{E}(\omega : \beta_{inc} + \beta_n) = \bar{E}_{\text{hom}}(\omega : \beta_{inc} + \beta_n) + \]

\[ \bar{G} (\omega : \beta_{inc} + \beta_n) \Delta \chi^{(1)} (\omega : \beta_{inc} + \beta_n) \]

\[ \]  \hspace{1cm} (2.14)
Dropping the frequency and wave vector arguments, this can be expressed more simply as:

\[ \vec{E} = \vec{E}_{\text{hom}} + \hat{M} \vec{E} \]  \hspace{1cm} (2.15)

with \( \hat{M} = G \Delta \chi^{(1)} \). By considering a finite numbers of Fourier components, the above equation transforms to a coupled linear system of equations, that has a self-consistent solution.

Usually we do use this calculation to model the specular reflectivity spectrum of two-dimensional photonic crystals as a function of the incident frequency and in-plane wavevector (equivalent to the incident angle). This is because such spectra are relatively easy to obtain experimentally, and they, together with the simulations, provide a means of determining the dispersion of leaky modes of the photonic crystal. To get the specular reflectivity from Eq.(2.14), which provides only the fields inside the textured layer, we need another Green’s function that propagates the polarization associated with the fields in the grating layer to fields propagating in the upper half space of photonic crystal. This propagator is given by [33, 37, 38]:

\[
\tilde{g}_{\text{UHS}} (\omega : \vec{\beta}, z, z') = \frac{2\pi \tilde{\omega}^2}{\omega_c(\vec{\beta})} e^{i\omega_c(\vec{\beta}) z} e^{i\omega_c(\vec{\beta}) (L/2 - z')} \times \\
\left[ \frac{\tilde{t}_{\text{sup}}}{D_s} (1 + r_{\text{down}} e^{i\omega_c(\vec{\beta}) (L/2 - z')}) \tilde{s}(\vec{\beta}) \tilde{s}(\vec{\beta}) + \right]
\]
\[
\frac{t_{\text{up}}}{D_p} (\hat{\mathbf{p}}_{c+}(\vec{\beta}) \hat{\mathbf{p}}_{c+}(\vec{\beta}) + \hat{\mathbf{p}}_{c+}(\vec{\beta}) \hat{\mathbf{p}}_{c-}(\vec{\beta}) r_{\text{down}} e^{i\omega_c(\beta)(L/2+z')})
\]

(2.16)

and finally the specular field of interest is:

\[
\vec{E}_{\text{UHS}}(\omega : \vec{\beta}_{\text{inc}} + \vec{\beta}_n; z) = \vec{E}_{\text{UHS}}^{\text{hom}}(\omega : \vec{\beta}_{\text{inc}} + \vec{\beta}_n; z) + \\
\int_{-L/2}^{+L/2} dz' \hat{g}_{\text{UHS}}(\omega : \vec{\beta}_{\text{inc}} + \vec{\beta}_n, z, z'). \Delta \vec{P}(\omega : \vec{\beta}_{\text{inc}} + \vec{\beta}_n; z')
\]

(2.17)

The specular reflectivity spectrum of the structure described in Fig.(2.1) in the near infrared is shown in Fig.(2.2). For this calculation the in-plane wave vector of the incident field was oriented along the X-direction of reciprocal lattice Brillouin zone with \( \vec{\beta}_{\text{inc}} = 0.01 \beta_g \hat{x} \). There are three s-polarized and one p-polarized modes with unit reflection in this range of frequencies. These modes represent resonance coupling of the incident field to leaky modes of this two-dimensional photonic crystal. In general these modes have Fano-like line shapes, but for certain filling factors \( (\pi r^2/\Lambda^2) \) and in-plane wave vectors the modes can have Lorentzian line shapes like the highest energy mode, S3, in Fig.(2.2).

Fig.(2.3) shows the dispersion diagram of these leaky modes in the X-direction of the first Brillouin zone close to the zone centre. These are the four lowest energy bands at zone centre, and the corresponding eigenstates
are principally composed of different linear combinations of fields with wavevectors close to the four smallest reciprocal lattice vectors that describe the square lattice [37].

Figs. (2.4) through (2.7) show real space diagrams of the electric field magnitude within one unit cell associated with the four modes represented in the specular reflectivity plot of Fig. (2.2). The real space electric fields were obtained by summing all 81 Fourier field components, which were used in the linear simulation. The high-energy S3 mode is largely concentrated in the air
holes, the low energy mode S1 is largely localized in the
dielectric background, and the two intermediate modes
S2, and P1 are more uniformly distributed within the
unit cell. The white circle shows the perimeter of the air
hole in each unit cell. This is a general feature of the
field configuration in dielectric media. To minimize the
electromagnetic energy, the lowest frequency mode,S1,
concentrates its displacement field in regions of high di-
electric constant. Although the higher-order modes tend
to concentrate their displacement fields in regions of high
dielectric constant, they should also remain orthogonal
to the lower-order mode and each other, therefore their
field distributions cannot be understood as simply as those of the lowest-order mode[44].

In the following section we describe how the inclusion of a third order nonlinear response can modify these reflectivity spectra. The basic effect responsible for these altered reflectivity spectra is the effective modification of the refraction index of the core material at high intensities (the Kerr effect). With this in mind, the effects should be greatest when strong fields are created inside the textured slab area. From the real space plots one would therefore expect that the S1 mode should exhibit a stronger nonlinear sensitivity than the other modes that are not so concentrated in the material. However, the net nonlinear response depends not just on the fraction
Figure 2.5: Real space plot for S2-mode at $\beta_x/\beta_g = 0.01$

Figure 2.6: Real space plot for P1-mode at $\beta_x/\beta_g = 0.01$
of the field inside the material part of the textured core, but also on the total field strength associated with the leaky mode.

The internal field for each mode also depends on its linewidth. Modes with smaller linewidth (or high Q-value) support higher internal fields. Fig.(2.8) shows the variation of inverse $Q (Q = \delta \omega / \omega_0$, or full-width at half maximum divided by the centre frequency) of the four bands shown in Fig.(2.2) above. This figure shows that the S1 and S3 modes have very high Q-values, especially near zone centre. They are therefore expected to yield a much larger nonlinear responses, than the relatively broad S2 and P1 modes. The infinite Q values suggested for modes S1 and S3 at zone centre in Fig.(2.8) are
an artifact of the calculation that assumes infinite plane wave excitation. Finite-size crystals excited by finite-size beams will have large, but non-infinite lifetimes at zone centre.

![Figure 2.8: The inverse Q of the same bands shown in Fig.(2.3).](image)

**2.3 Degenerate Kerr effect**

Starting with the degenerate optical Kerr effect in which a single strong pump field with frequency $\omega$, and in-plane wave vector $\beta_{\text{inc}}$, illuminates the two-dimensional photonic crystal, the total polarization in the thin grating region can be expressed as:
\[
\Delta \vec{P}(\omega : \vec{\beta}_{inc} + \vec{\beta}_n) = \Delta \chi^{(1)}(\omega : \vec{\beta}_n - \vec{\beta}_k) \vec{E}(\omega : \vec{\beta}_{inc} + \vec{\beta}_k) + \\
\chi^{(3)}(-\omega; \omega, -\omega, \omega : \vec{\beta}_n - \vec{\beta}_m + \vec{\beta}_l - \vec{\beta}_k) \times \\
\vec{E}(\omega : \vec{\beta}_{inc} + \vec{\beta}_m) \vec{E}^*(\omega : -\vec{\beta}_{inc} - \vec{\beta}_l) \vec{E}(\omega : \vec{\beta}_{inc} + \vec{\beta}_k)
\]

(2.18)

where again summation is implied over repeated indices in product terms. By dropping the frequency and wave vector arguments, this can be written as:

\[
\Delta P_\mu = \Delta \chi^{(1)}_{\mu\gamma} E_\gamma + \chi^{(3)}_{\mu\alpha\delta\gamma} E_\alpha E_\delta^* E_\gamma,
\]

(2.19)

The third-order polarization can be written as:

\[
\Delta P^{(3)}_\mu = \theta^{(3)}_{\mu\gamma} E_\gamma,
\]

(2.20)

where explicitly the \(\theta^{(3)}\) tensor is:

\[
\theta^{(3)}_{\mu\gamma}(\omega : \vec{\beta}_n - \vec{\beta}_k) = \chi^{(3)}_{\mu\alpha\delta\gamma}(-\omega; \omega, -\omega, \omega : \vec{\beta}_n - \vec{\beta}_m + \vec{\beta}_l - \vec{\beta}_k) \times \\
E_\alpha(\omega : \vec{\beta}_{inc} + \vec{\beta}_m) E_\delta^*(\omega : -\vec{\beta}_{inc} - \vec{\beta}_l),
\]

(2.21)

and therefore the total polarization is:

\[
\Delta P_\mu = (\Delta \chi^{(1)}_{\mu\gamma} + \theta^{(3)}_{\mu\gamma}) E_\gamma = \chi^{eff}_{\mu\gamma} E_\gamma,
\]

(2.22)
where $\chi^{\text{eff}}$ is the total effective susceptibility in the textured area, which explicitly depends on the intensity of the incident light.

Here we assume that the matrix elements of the $\chi^{(3)}$ tensor are determined by the symmetry of the bulk material that makes up the photonic crystal (GaAs in this case). For $43m$ (GaAs) and $m3m$ (Si, Ge) group symmetry, the only nonzero elements of the $\chi^{(3)}$ tensor can be categorized in four groups [43]:

\[
\begin{align*}
\chi_{xxxx}^{(3)} &= \chi_{yyyy}^{(3)} = \chi_{zzzz}^{(3)}, \\
\chi_{xxyy}^{(3)} &= \chi_{yyxx}^{(3)} = \chi_{yyzz}^{(3)} = \chi_{zzxx}^{(3)} = \chi_{zzyy}^{(3)}, \\
\chi_{xyzy}^{(3)} &= \chi_{yyyy}^{(3)} = \chi_{yzyy}^{(3)} = \chi_{zyzz}^{(3)} = \chi_{zzxx}^{(3)}, \\
\chi_{xyyx}^{(3)} &= \chi_{yyyy}^{(3)} = \chi_{yzyx}^{(3)} = \chi_{zzyz}^{(3)} = \chi_{zzxx}^{(3)}.
\end{align*}
\]

(2.23)

where the subscripts refer to the principal atomic crystal axes. In this case the $\theta^{(3)}$ matrix components can be written as:

\[
\begin{align*}
\theta_{xx}^{(3)} &= \chi_{xxxx}^{(3)} E_x E_x^* + \chi_{xyyx}^{(3)} E_y E_y^* + \chi_{zuzz}^{(3)} E_z E_z^* \\
\theta_{xy}^{(3)} &= \chi_{xxyy}^{(3)} E_x E_y^* + \chi_{xzyy}^{(3)} E_y E_x^* \\
\theta_{xz}^{(3)} &= \chi_{yyyy}^{(3)} E_x E_z^* + \chi_{zuzz}^{(3)} E_z E_x^*.
\end{align*}
\]
\[
\begin{align*}
\theta^{(3)}_{yx} &= \chi^{(3)}_{yxyx} E_x E_y^* + \chi^{(3)}_{yxyx} E_x^* E_y \\
\theta^{(3)}_{yy} &= \chi^{(3)}_{yyxy} E_x E_y^* + \chi^{(3)}_{yyyy} E_y E_x^* + \chi^{(3)}_{yyzy} E_z E_z^* \\
\theta^{(3)}_{yz} &= \chi^{(3)}_{yyzz} E_x E_y^* + \chi^{(3)}_{yzyz} E_z E_y \\
\theta^{(3)}_{zx} &= \chi^{(3)}_{zzxx} E_x E_x^* + \chi^{(3)}_{zzzx} E_z E_x^* \\
\theta^{(3)}_{zy} &= \chi^{(3)}_{zzzy} E_z E_y + \chi^{(3)}_{zyzy} E_y E_z \\
\theta^{(3)}_{zz} &= \chi^{(3)}_{zzzz} E_x E_x^* + \chi^{(3)}_{zyzz} E_y E_y^* + \chi^{(3)}_{zzzz} E_z E_z^* 
\end{align*}
\]

(2.24)

The Green's function tensor in this formalism, Eq. (2.9), is expressed in a coordinate system with \((\hat{k}, \hat{s}, \hat{z})\) basis vectors associated with each Fourier component of the field, but the \(\theta^{(3)}\) tensor is naturally expressed in the fixed \((\hat{x}, \hat{y}, \hat{z})\) coordinate system. Therefore we need a transformation from one coordinate system to another. Suppose we calculate the third-order polarization in the \(\{\hat{x}, \hat{y}, \hat{z}\}\) coordinate system.

\[
\Delta P^{(3)}(\vec{\beta}_n) = \theta^{(3)} (\vec{\beta}_n - \vec{\beta}_k) E(\vec{\beta}_k),
\]

(2.25)
The transformation of this vector to the new coordinate system proceeds as follows:

\[
\Delta \vec{P}^{(3)}(\vec{\beta}_n) = \langle \vec{\beta}_n \rangle \Delta \vec{P}^{(3)}(\vec{\beta}_n),
\]

or

\[
\Delta \vec{P}^{(3)}(\vec{\beta}_n) = \langle \vec{\beta}_n \rangle \theta^{(3)} (\vec{\beta}_n - \vec{\beta}_k) \tilde{E}(\vec{\beta}_k),
\]

(2.26)

but this vector in the new coordinate system can be calculated from:

\[
\Delta \vec{P}^{(3)}(\vec{\beta}_n) = \langle \vec{\beta}_n \rangle \theta^{(3)} (\vec{\beta}_n - \vec{\beta}_k) \tilde{E}'(\vec{\beta}_k),
\]

(2.27)

therefore the transformation of the \( \theta^{(3)} \) tensor can be calculated from the following equation:

\[
\theta^{(3)} (\vec{\beta}_n - \vec{\beta}_k) = T(\vec{\beta}_n) \theta (\vec{\beta}_n - \vec{\beta}_k) T^{-1}(\vec{\beta}_k)
\]

(2.29)
in which:

\[
\theta' (\vec{\beta}_n - \vec{\beta}_k) = \begin{pmatrix}
\theta_{kk}(\vec{\beta}_n - \vec{\beta}_k) & \theta_{ks}(\vec{\beta}_n - \vec{\beta}_k) & \theta_{kz}(\vec{\beta}_n - \vec{\beta}_k) \\
\theta_{sk}(\vec{\beta}_n - \vec{\beta}_k) & \theta_{ss}(\vec{\beta}_n - \vec{\beta}_k) & \theta_{sz}(\vec{\beta}_n - \vec{\beta}_k) \\
\theta_{zk}(\vec{\beta}_n - \vec{\beta}_k) & \theta_{zs}(\vec{\beta}_n - \vec{\beta}_k) & \theta_{zz}(\vec{\beta}_n - \vec{\beta}_k)
\end{pmatrix}
\]

\[
T(\vec{\beta}_n) = \begin{pmatrix}
\hat{k}(\vec{\beta}_n).\hat{x} & \hat{k}(\vec{\beta}_n).\hat{y} & \hat{k}(\vec{\beta}_n).\hat{z} \\
\hat{s}(\vec{\beta}_n).\hat{x} & \hat{s}(\vec{\beta}_n).\hat{y} & \hat{s}(\vec{\beta}_n).\hat{z} \\
\hat{z}(\vec{\beta}_n).\hat{x} & \hat{z}(\vec{\beta}_n).\hat{y} & \hat{z}(\vec{\beta}_n).\hat{z}
\end{pmatrix}
\]

\[
\theta (\vec{\beta}_n - \vec{\beta}_k) = \begin{pmatrix}
\theta_{xx}(\vec{\beta}_n - \vec{\beta}_k) & \theta_{xy}(\vec{\beta}_n - \vec{\beta}_k) & \theta_{xz}(\vec{\beta}_n - \vec{\beta}_k) \\
\theta_{yx}(\vec{\beta}_n - \vec{\beta}_k) & \theta_{yy}(\vec{\beta}_n - \vec{\beta}_k) & \theta_{yz}(\vec{\beta}_n - \vec{\beta}_k) \\
\theta_{zx}(\vec{\beta}_n - \vec{\beta}_k) & \theta_{zy}(\vec{\beta}_n - \vec{\beta}_k) & \theta_{zz}(\vec{\beta}_n - \vec{\beta}_k)
\end{pmatrix}
\]

The transformations of the two sets of basis vectors for each Fourier component are:

\[
\hat{s}(\vec{\beta}_n) = \frac{(\vec{\beta}_n)_y}{|\vec{\beta}_n|} \hat{x} - \frac{(\vec{\beta}_n)_x}{|\vec{\beta}_n|} \hat{y}
\]

\[
\hat{k}(\vec{\beta}_n) = \frac{(\vec{\beta}_n)_x}{|\vec{\beta}_n|} \hat{x} + \frac{(\vec{\beta}_n)_y}{|\vec{\beta}_n|} \hat{y}
\]

\[
\hat{z}(\vec{\beta}_n) = \hat{z}
\]

(2.30)
The explicit form of each component of the $\theta^{(3)}$ tensor in
the new coordinate system is thus:

\begin{align*}
\theta_{kk}(\tilde{\beta}_n - \tilde{\beta}_k) &= \frac{(\tilde{\beta}_k)_x}{|\tilde{\beta}_k|} \left( \frac{(\tilde{\beta}_n)_x}{|\tilde{\beta}_n|} \theta_{xx}(\tilde{\beta}_n - \tilde{\beta}_k) + \frac{(\tilde{\beta}_n)_y}{|\tilde{\beta}_n|} \right) \\
\theta_{yx}(\tilde{\beta}_n - \tilde{\beta}_k) &= \frac{(\tilde{\beta}_k)_y}{|\tilde{\beta}_k|} \left[ \frac{(\tilde{\beta}_n)_x}{|\tilde{\beta}_n|} \theta_{xy}(\tilde{\beta}_n - \tilde{\beta}_k) + \frac{(\tilde{\beta}_n)_y}{|\tilde{\beta}_n|} \theta_{yy}(\tilde{\beta}_n - \tilde{\beta}_k) \right] \\
\theta_{ks}(\tilde{\beta}_n - \tilde{\beta}_k) &= \frac{(\tilde{\beta}_k)_y}{|\tilde{\beta}_k|} \left[ \frac{(\tilde{\beta}_n)_x}{|\tilde{\beta}_n|} \theta_{xx}(\tilde{\beta}_n - \tilde{\beta}_k) + \frac{(\tilde{\beta}_n)_y}{|\tilde{\beta}_n|} \theta_{yx}(\tilde{\beta}_n - \tilde{\beta}_k) \right] - \\
\theta_{xk}(\tilde{\beta}_n - \tilde{\beta}_k) &= \frac{(\tilde{\beta}_k)_x}{|\tilde{\beta}_k|} \left[ \frac{(\tilde{\beta}_n)_x}{|\tilde{\beta}_n|} \theta_{xy}(\tilde{\beta}_n - \tilde{\beta}_k) + \frac{(\tilde{\beta}_n)_y}{|\tilde{\beta}_n|} \theta_{yy}(\tilde{\beta}_n - \tilde{\beta}_k) \right] \\
\theta_{sk}(\tilde{\beta}_n - \tilde{\beta}_k) &= \frac{(\tilde{\beta}_k)_x}{|\tilde{\beta}_k|} \left( \frac{(\tilde{\beta}_n)_y}{|\tilde{\beta}_n|} \theta_{xx}(\tilde{\beta}_n - \tilde{\beta}_k) - \frac{(\tilde{\beta}_n)_x}{|\tilde{\beta}_n|} \theta_{yx}(\tilde{\beta}_n - \tilde{\beta}_k) \right) - \\
\theta_{ss}(\tilde{\beta}_n - \tilde{\beta}_k) &= \frac{(\tilde{\beta}_k)_y}{|\tilde{\beta}_k|} \left[ \frac{(\tilde{\beta}_n)_y}{|\tilde{\beta}_n|} \theta_{xy}(\tilde{\beta}_n - \tilde{\beta}_k) + \frac{(\tilde{\beta}_n)_x}{|\tilde{\beta}_n|} \theta_{yy}(\tilde{\beta}_n - \tilde{\beta}_k) \right]
\end{align*}
\[
\theta_{sz}(\vec{\beta}_n - \vec{\beta}_k) = \frac{(\vec{\beta}_n)_y}{|\vec{\beta}_n|} \theta_{xz}(\vec{\beta}_n - \vec{\beta}_k) - \frac{(\vec{\beta}_n)_x}{|\vec{\beta}_n|} \theta_{yz}(\vec{\beta}_n - \vec{\beta}_k)
\]

\[
\theta_{zk}(\vec{\beta}_n - \vec{\beta}_k) = \frac{(\vec{\beta}_k)_x}{|\vec{\beta}_k|} \theta_{zx}(\vec{\beta}_n - \vec{\beta}_k) + \frac{(\vec{\beta}_k)_y}{|\vec{\beta}_k|} \theta_{zy}(\vec{\beta}_n - \vec{\beta}_k)
\]

\[
\theta_{zs}(\vec{\beta}_n - \vec{\beta}_k) = \frac{(\vec{\beta}_k)_y}{|\vec{\beta}_k|} \theta_{zx}(\vec{\beta}_n - \vec{\beta}_k) - \frac{(\vec{\beta}_k)_x}{|\vec{\beta}_k|} \theta_{zy}(\vec{\beta}_n - \vec{\beta}_k)
\]

\[
\theta_{zz}(\vec{\beta}_n - \vec{\beta}_k) = \theta_{zz}(\vec{\beta}_n - \vec{\beta}_k)
\]

Finally, the self-consistent solution of the field that incorporates this third-order nonlinearity is calculated using a similar equation to that which was used in the linear situation (Eq. (2.15)):

\[
E_{\alpha} = E_{\alpha}^{\text{hom}} + G_{\alpha\mu}(\chi^{(1)}_{\mu\gamma} + \theta^{(3)}_{\mu\gamma})E_{\gamma}
\]

(2.32)

or

\[
\vec{E} = [\mathbb{1} - \mathcal{G}(\mathbf{i}^{(1)} + \mathbf{i}^{(3)})]^{-1} \vec{E}^{\text{hom}}
\]

(2.33)

where \(\mathbf{i}^{(3)}\) tensor in Eq.(2.33) depends on the field solution \(\vec{E}\), through Eq.(2.21). Therefore we can only solve
Eq. (2.33) iteratively. To start the iteration loop we calculate the $\tilde{\Theta}^{(3)}$ tensor by using $\tilde{E}^{\text{hom}}$ as the field solution. Then new values of the field are found by solving Eq. (2.33). This solution then is used to find new values of $\tilde{\Theta}^{(3)}$ tensor. We repeat this process until we reach a self-consistent solution. For having a fast convergence, the average of the last two field values are used to guess the solution in each iteration step.

Numerical values of the diagonal elements of the GaAs third order susceptibility are taken to be $-9.74 \times 10^{-11} \text{ esu}$ [45]. This value was the result of a susceptibility measurement at 1064nm or 9398cm$^{-1}$ wavenumber (which is very close to the range of resonance modes considered in this simulation). Unfortunately we are unaware of any report of the measured values for the off-diagonal elements of the third-order GaAs susceptibility. We therefore assume that the off-diagonal elements are equal to the diagonal elements for most of the calculations.

Fig. (2.9) shows the reflectivity spectrum for the S3 mode at different incident intensities. By increasing the input intensity, the peak of the mode shifts to higher energies (blue shifts). This is consistent with the negative value of third-order susceptibility in this range which means that the light reduces the effective refractive index of the material. The S3 mode has a $Q \approx 3000$, and it responds fairly strongly to the field even though most of the S3 mode is localized in the air hole. As the graph shows, the reflectivity at 10744.5cm$^{-1}$ increases
from $\approx 0.25$ at low powers to around 1.0 at input power $\approx 3800\text{kHz/cm}^2$.

![Graph showing specular reflectivity of the S3 mode for different incident intensities.](image)

Figure 2.9: Specular reflectivity of the S3 mode for different incident intensities; the dashed line shows the linear reflectivity and the solid lines are for intensities of 765, 2342, and 3871 kHz/cm$^2$ for increasing blue shifts respectively.

Fig.(2.10) shows the degenerate response of the S1 mode at different incident intensities. This mode has a relatively narrow, Fano-like line shape and a $Q$-value of $\approx 12000$, significantly higher than the S3 mode. It also has its field localized primarily in the GaAs part of the textured layer. All of these factors result in the S1 spectrum shifting the same amount, measured in units of linewidths, as the S3 spectrum, at much lower abso-
lute field strengths. It only requires an incident intensity of $155kW/cm^2$ to change the reflectivity from 0 to 0.8 at $9679cm^{-1}$. In the case of the S3 mode, incident intensities of $\sim 3871kW/cm^2$ are required to change the reflectivity from 1 to 0.4.

![Figure 2.10: Specular reflectivity for the S1 mode for different incident intensities; The dashed line shows the linear reflectivity and the solid lines are for intensities of 30, 48, 107, and 155 $kW/cm^2$ for increasing blue shifts respectively.](image)

The P1 and S2 modes also blue shift at high incident powers, but a very high input power is needed to induce a given percentage (with respect to the linewidth) shift because they have a relatively small Q value ($\approx 50$).
Fig. (2.9) shows that the line shape of the S3 mode becomes multi-valued for incident intensities higher than \( \sim 4000\text{ kW/cm}^2 \). This suggests the possibility of observing bistable behaviour at a fixed frequency as a function of the optical power. Fig.(2.11) illustrates such bistable behaviour for the S3 mode at 1074.5\( cm^{-1} \). A clear hysteresis loop is evident in the range of 4700 to 6000\( kW/cm^2 \). The bistability behaviour simply shows the dynamic response of the reflectivity to the applied incident field. In other words, the reflectivity can be switched from one value to another (all-optical switching) simply by changing the applied incident power.
As mentioned above, the values of the off-diagonal elements of the third-order susceptibility of GaAs are not known. The simulations in Figs. (2.9-2.11) were obtained by setting the off-diagonal elements equal to the diagonal elements. To indicate the significance of this assumption, Fig. (2.12) shows the simulated reflectivity for the S3 mode obtained by setting the diagonal elements equal to zero (dash-dot line) and when they are equal to the diagonal elements (solid line).

Figure 2.12: Reflectivity in the vicinity of the S3 mode for nonzero and zero off-diagonal elements of the \( \chi^{(3)} \) tensor.

Clearly there is a considerable difference between the two approximations. It is obviously important in this case to get values for the off diagonal elements, and per-
haps quantitative comparisons with these types of reflectivity experiments might be a way of measuring them.

### 2.4 Nondegenerate Kerr effect

We now consider the nondegenerate optical Kerr effect in which a strong pump beam with frequency $\omega_p$ and in-plane wavevector $\vec{\beta}_{pump}$ is used to modify the effective bandstructure as probed by a weak signal beam that can in general have a frequency $\omega_s$ and in-plane wavevector $\vec{\beta}_{probe}$. For simplicity in the following discussion, it was assumed that the two beams have the same in-plane wave vector $(0.01\beta_g \hat{x})$.

The nondegenerate simulation proceeds as follows. The fields generated by the pump beam alone are first solved for in the textured layer using the algorithm described in the preceding section. The effective susceptibility in the presence of the strong pump beam, including the Kerr effect is then used to calculate the reflectivity spectrum of the weak probe beam using:

$$E_\alpha(\omega_s, \vec{\beta}_{probe} + \vec{\beta}_n) = E_{\alpha}^{hom}(\omega_s, \vec{\beta}_{probe} + \vec{\beta}_n) + G_{\alpha\mu}(\omega_s, \vec{\beta}_{probe} + \vec{\beta}_n) \times \Delta \chi^{(1)}_{\mu \nu \gamma}(\omega_s; \omega_s: \vec{\beta}_n - \vec{\beta}_k) E_{\gamma}(\omega_s: \vec{\beta}_{probe} + \vec{\beta}_k) + \chi^{(3)}_{\mu \nu \sigma \gamma}(\omega_s; \omega_p, -\omega_p, \omega_s: \vec{\beta}_m - \vec{\beta}_m + \vec{\beta}_l - \vec{\beta}_k) \times E_\alpha(\omega_p: \vec{\beta}_{pump} + \vec{\beta}_m) E^*_\delta(\omega_p: -\vec{\beta}_{pump} - \vec{\beta}_l) \times$$
\[ E_{\gamma}(\omega_\text{s} : \vec{\beta}_\text{probe} + \vec{\beta}_k) \]
two bands.

The result of the degenerate and nondegenerate simulations presented in this chapter, and published in Ref[36] suggests the potential for using 2DPCs as optical switching and optical limiting components. Simply by changing the power of an incident field, the reflectivity of the structure can be parametrically controlled without relying on electronic controls. The major problem in using this particular phenomenon in applications is the need for a rel-
atively high incident power. This can be traced to the relatively small off-resonant, intrinsic third order susceptibility of GaAs assumed in these simulations. The required power levels could be reduced by either introducing free-carriers (excited by the incident field) or by tuning the pump and/or probe beams near a direct bandgap in the host semiconductor heterostructure[46]. Both of these approaches would increase the effective third-order susceptibility over that assumed here, but at the expense of causing some absorption of the light. This may or may not be tolerable depending on the specific application.
CHAPTER 3

PHOTONIC CRYSTAL FIBRES

The concept of photonic crystals was incorporated in optical fibre technology first by Russell et al., and subsequently by a large number of research groups around the world [47, 48, 49, 50, 51]. Fig.(3.1) shows a scanning electron micrograph of the cross section of a silica-based "photonic crystal fibre" (PCF) used in the work described below.

Figure 3.1: A scanning electron micrograph, in cross section of the photonic crystal fibre used for the studies described in this chapter

In conventional optical fibres, a relatively high index made by Crystal Fibre Company in Denmark
core is surrounded by a lower index cladding glass in order to support bound optical modes that propagate only in the core, due to total internal reflection from the core-cladding interface. By surrounding the core of the fibre by a periodic array of air holes, as in Fig. (3.1), it is also possible to support bound modes that propagate only in the core, but the group velocity dispersion (GVD), $\frac{d}{d\lambda}(1/v_g)$, of PCFs can differ markedly from conventional fibres. The GVD parameter indicates how fast a short pulse will spread as it propagates through the fibre (this is a big problem in fibre optic communication systems, and is the main reason why long-haul fibre systems operate at the zero dispersion wavelength of typical silica single mode fibres at 1.55 microns). In conventional fibres, the GVD is "normal" (negative) for wavelengths shorter than 1.3 $\mu$m and "anomalous" (positive) for longer wavelengths. By engineering the air hole size and pitch in photonic crystal fibres, one can create fibres with anomalous group velocity dispersion in the visible and near infrared part of the spectrum.

The GVD spectrum of a fibre is important not only in optical communication systems, but also in the context of the nonlinear propagation of light through fibres. Much work has been done over the past 20 years concerning the influence of nonlinear response on the propagation of light through single mode fibres (SMFs). In fact, a viable solution to the negative effects of pulse broadening actually makes use of the third order nonlinear response of silica. When a fibre is operated in the anomalous dispersion regime, it is possible to excite optical solitons that
propagate without distortion by cancelling the effect of group velocity dispersion through self-phase modulation [13, 14, 52].

PCFs offer a particularly attractive medium in which to study guided wave nonlinear optics for two reasons. First, the high index contrast between the \(~1 \, \mu m\) diameter silica core and the surrounding photonic crystal lattice is relatively large compared to conventional SMF, thus reducing the effective mode diameter from \(~10 \, \mu m\) to \(1.8 \, \mu m\). The smaller mode volume means that the electric field strength in the silica is proportionately higher for a given guided power level. Second, by being able to tune the dispersion, it is possible to make fibres in which the anomalous dispersion region and the near-zero dispersion point are in the near infrared part of the spectrum, near 800 nm. This means that relatively convenient short-pulse laser systems can be used to study the propagation of light through such fibres.

Already, several groups have reported that the output spectrum from PCFs like that shown in Fig. (3.1) can assume a broad, continuum shape when pumped with laser pulses ranging in duration from 40 fs to 0.8 ns, at average power levels of a few \(mW\) [15, 16]. The shape of the nonlinearly shifted output spectrum varies from fibre to fibre, and is strongly dependent on the pump wavelength and power. Although it is clear that several nonlinear phenomena are likely operating in concert to produce such extraordinary spectra, an exact explanation for the effect has yet to be determined. In the present work,
two distinctly different forms of broad output spectra were observed, and the temporal nature of the output fields was measured and correlated with the spectra.

3.1 Experiment setup

Fig. (3.2) shows the fibre's linear dispersion properties. Table (3.1) summarizes the important properties of the photonic crystal fibre shown in Fig. (3.1).

![Dispersion of the photonic crystal fibre described in Table(3.1) provided by the fibre manufacturer](image)

Figure 3.2: Dispersion of the photonic crystal fibre described in Table(3.1) provided by the fibre manufacturer

The objective of the experiments described below was to characterize the spectral and temporal properties of the light emanating from the PCF when \(\sim 100\) fs pulses
CHAPTER 3. PHOTONIC CRYSTAL FIBRES

<table>
<thead>
<tr>
<th>Material</th>
<th>Pure silica</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cladding diameter</td>
<td>$80 \pm 5\mu m$</td>
</tr>
<tr>
<td>Coating diameter</td>
<td>$195 \pm 5\mu m$</td>
</tr>
<tr>
<td>Coating material</td>
<td>Acrylate</td>
</tr>
<tr>
<td>Core diameter</td>
<td>$1.0 \pm 0.2\mu m$</td>
</tr>
<tr>
<td>Pitch (distance between air holes)</td>
<td>$1.6 \pm 0.2\mu m$</td>
</tr>
<tr>
<td>Ratio between air hole diameter and pitch</td>
<td>$&gt;0.85$</td>
</tr>
<tr>
<td>Numerical aperture</td>
<td>$&gt;0.5@633nm$</td>
</tr>
<tr>
<td>Zero dispersion wavelength</td>
<td>$600 \pm 100nm and 1150 \pm 150nm$</td>
</tr>
<tr>
<td>Attenuation</td>
<td>$&lt;0.5dB/m@635nm$</td>
</tr>
<tr>
<td></td>
<td>$&lt;1dB/m@944nm$</td>
</tr>
<tr>
<td></td>
<td>$\approx 1dB/m@1250nm$</td>
</tr>
<tr>
<td></td>
<td>$&lt;10dB/m@1390nm$</td>
</tr>
</tbody>
</table>

Table 3.1: Photonic crystal fibre specifications.

ranging in wavelength from 760nm to 850nm are launched into it using a 40X microscope objective.

Fig. (3.3) shows the setup of the experiments and table (3.2) describes all of the optical elements. The Ti-sapphire laser is a Tsunami model 3960-S1S that emits 100fs pulses at a mode-locking repetition rate of 82MHz. The centre wavelength of the laser can be varied from 750 to 830nm with a maximum average power of $\sim 800mW$ at 800nm.
Table 3.2: Optical elements of the cross-correlation experiment

By using a beam splitter (BS), this laser pulse is divided into two parts. The part that is directed towards the retro-reflector $RF_1$ on the translation stage is referred to as the gate beam. It is focused by $L_3$ onto the surface of the nonlinear crystal (NLC) used to up-convert the output in order to measure its temporal behaviour.

<table>
<thead>
<tr>
<th>Optical elements</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1 - M_{10}$</td>
<td>Dielectric mirrors (IR)</td>
</tr>
<tr>
<td></td>
<td>25.4 mm Diameter, 1/5 wave, 0.5 – 18 μm</td>
</tr>
<tr>
<td>$M_{11}, M_{12}$</td>
<td>Dielectric mirrors (UV)</td>
</tr>
<tr>
<td></td>
<td>25.4 mm Diameter, 1/5 wave, 250 – 600 nm</td>
</tr>
<tr>
<td>HWP</td>
<td>Half-wave plate</td>
</tr>
<tr>
<td>PL</td>
<td>Polarizer</td>
</tr>
<tr>
<td>$L_1$</td>
<td>Objective lens</td>
</tr>
<tr>
<td></td>
<td>40X, Newport F-L40B</td>
</tr>
<tr>
<td>$L_2, L_3$</td>
<td>Lens with respectively 10 and 15 cm focal length</td>
</tr>
<tr>
<td>$L_4, L_5$</td>
<td>UV lens</td>
</tr>
<tr>
<td></td>
<td>Focal length 7.5 cm, AR coated, 250-430 nm</td>
</tr>
<tr>
<td>NLC</td>
<td>Beta-Barium Borate (BBO) crystal</td>
</tr>
<tr>
<td></td>
<td>$5 \times 5 \times 0.5$ mm, cut at $38^\circ$</td>
</tr>
<tr>
<td>$F_1$</td>
<td>BG40 filter</td>
</tr>
<tr>
<td>PM</td>
<td>Parabolic mirror</td>
</tr>
<tr>
<td></td>
<td>$90^\circ$ off axis, 28.75 mm focal length</td>
</tr>
<tr>
<td>BS</td>
<td>50/50 Beam splitter</td>
</tr>
<tr>
<td>$RF_1, RF_2$</td>
<td>Newport Gold-coated retro-reflector</td>
</tr>
<tr>
<td>DL</td>
<td>Delay line translation stage</td>
</tr>
</tbody>
</table>
Another part of the laser beam passes through a half-wave plate (HWP) and a linear polarizer (PL) and is coupled by $L_1$ onto the cleaved input facet of the photonic crystal fibre. Ideally, the laser should be optically isolated from the fibre by using a Faraday rotator, which
prevents reflections from the fibre facet going back to the laser. However, an isolator was not available at the time of this experiment, and back reflections were not found to be a problem. A parabolic mirror (PM) collimates the output of the fibre and lens $L_2$ focuses this beam (referred to as the fibre beam) onto the nonlinear crystal at the same point as the gate beam is focused. A CCD camera positioned to display the surface of the nonlinear crystal helps to overlap the two beams (fibre and the gate) at the same point. The nonlinear crystal is BBO\footnote{Beta-Barium Borate}, and it rests on a motorized rotation stage that has an accuracy of 0.01°.

If the gate and fibre signals overlap in time and space inside the BBO crystal, a pulse at the sum frequency can be generated. By varying the relative arrival time of the gate and fibre signals on the BBO crystal, it is possible to map out the temporal profile of the fibre signal, which is generally much longer than the gate pulses. Lens $L_4$ collimates the up-converted signal and mirrors $M_{11}, M_{12}$, and lens $L_5$ images it on the entrance of a monochromator (Digikrom DK242 from CVI Laser Corporation). Filter $F_1$ helps to block any IR beam passing through the monochromator. A photomultiplier tube (Model 9784A made by Thorn EMI Electron Tubes Ltd.) at the exit of the monochromator is used to monitor the sum generation signal in a photon counting mode (Hamamatsu model C3866).
A computer running Labview software controls the rotation stage, monochromator, and photon counting system. The Labview codes used in this experiment are summarized in Appendix B.

### 3.2 Phase Matching

Efficient up-conversion can only occur if the BBO crystal is oriented at an angle that allows phase matching of the nonlinear conversion process [35, 53]. The BBO crystal is a uniaxial crystal characterized by ordinary ($n_o$) and extraordinary ($n_e$) refractive indexes, both of which exhibit considerable dispersion across the visible and near infrared part of the spectrum. This dispersion is described by Sellmeier's equation[54]:

\[
\begin{align*}
n_o^2 &= 2.7359 + \frac{0.01878}{\lambda^2 - 0.01822} - 0.01354\lambda^2 \\
n_e^2 &= 2.3753 + \frac{0.01224}{\lambda^2 - 0.01667} - 0.01516\lambda^2
\end{align*}
\]

where $\lambda$ is in units of $\mu$m. BBO is transparent between 0.19 - 3.5$\mu$m. Fig.(3.4) shows the geometry of the interaction of gate and fibre beams within the nonlinear crystal.

The phase matching condition depends on the polarization of the interacting beams. The "Type I" configuration used in this experiment is such that the gate and fibre beams are polarized along the ordinary axis, and
the sum frequency signal is generated in the extraordinary polarization. The phase matching condition for sum frequency generation in this situation is then given by:

$$\sin^2(\theta_m) = \frac{(1/n_s^2) - (1/n_{o,s}^2)}{(1/n_{e,s}^2) - (1/n_{o,s}^2)}$$  \hspace{1cm} (3.2)$$

where $\theta_m$ is the phase matching angle defined in Fig. (3.4) and $n_{o,s}^2$ and $n_{e,s}^2$ are the squared refractive indices of the ordinary and extraordinary beams at the sum generation wavelength. Also $n_s$ is the refractive index of the upconverted beam which depends on the angle and
wavelengths of the interacted beams.

There is a unique phase matching angle for each combination of gate and fibre wavelengths and you can up-convert different fibre wavelengths by tuning the BBO crystal. There is a Maple code described in Refs. [55, 56] which calculates the phase matching angle, $\theta_m$, for the sum generation situation. By entering the gate and fibre wavelengths and also the angle between these two beams, the Maple code gives you the desired rotation angle of the nonlinear crystal. In this experiment the angle between the gate and fibre beams was set to $14^\circ$.

### 3.3 Experiment procedure

Excellent alignment of all optics is required to overlap the gate and fibre pulses temporally and physically inside the BBO crystal. Under optimum conditions the typical up-converted count rate from the PMT is $\sim 50,000$ units for average fibre output powers of 2.0 mW. The following briefly outlines the alignment procedure used to detect these weak signals:

1-For upconversion of each portion at output spectrum of the fibre, changing the delay between gate and fibre beam is necessary. This is because of group velocity dispersion of the fibre. The gate beam must be incident on the translatable retro-reflector $RF_1$ so that changing the delay does not alter the spatial overlap of the two beams on the BBO crystal surface. This requires the gate beam and the retro-reflector's axis both to be parallel to the
axis of translation stage. By installing an iris before mirror $M_7$, we make sure that the gate beam passes through the aperture for all positions of the translation stage. The beam splitter and mirror $M_2$ are used to align the translation axis of the moving stage.

2-As explained above, the BBO crystal has to be rotated over a range of angles ($\sim 10$ degrees) in order to measure the temporal evolution of the full spectrum of the fibre emission. The two beams must obviously not be allowed to walk off each other under this rotation. Therefore, the two beams must strike the crystal precisely on the axis of the rotation state it is mounted on. To fulfill this requirement a thin piece of Teflon tape was attached to the surface of the BBO crystal. At first by moving the crystal on its translation stage and looking at the location of the gate beam on the monitor during the rotation, the axis of rotation can be found easily. Then by aligning mirror $M_{10}$, the fibre beam can be focused at the same point.

The next step is to adjust the optical paths of two beams. First we set the input power to the fibre at a low level, $\sim 10$ mW, in order that the output spectrum of the fibre exhibits minimal broadening. Under this condition the gate and fibre beams are at the same wavelength (we typically align the system at 800 nm). Measuring the separate paths with a ruler is not sufficiently accurate to ensure that the two pulses will overlap in the BBO crystal. By inserting a fast Silicon PIN (P-type, Intrinsic, N-type) detector (Newport, model 818-BB-21) after the
filter $F_1$ to intercept both beams, and by adjusting the delay line while observing the 50 ohm terminated signal on an oscilloscope (Tektronix TDS 350) it is possible to get the relative delay to within $\sim 100$ ps, which means the uncertainty in the zero delay between the beams is $\sim \pm 1.5 \text{cm}$.

To assist in fine-tuning the delay stage, a chopper was installed after the mirror $M_8$ to modulate the fibre output at $270 \text{Hz}$. This facilitated using a lock-in amplifier to monitor the up-converted signal as measured on a sensitive silicon detector (UDT-455 UV) placed directly after the filter $F_1$. Previous researchers using the same basic setup\cite{55, 56}, employed a mini-shaker with a second retro-reflector to periodically sweep the delay over a 20 ps interval while searching for the overlap condition. In the present work, the chopper/lock-in technique was found to be easier to work with.

Experience showed that by moving the delay line a few mm from the position in which the gate and fibre beams had nominally equal lengths as measured by the fast PIN detector, the upconverted signal can be detected on the lock-in amplifier. After the zero delay position is determined in this way, the chopper is turned off and mirrors $M_7$ and $M_{10}$ are tweaked, and the BBO crystal is rotated in $0.01^\circ$ steps to maximize the DC upconverted signal at the optimal pulse overlap.

When the power into the fibre is increased, and the output spectrum shifts, the crystal has to be rotated and
a new zero delay must be determined using the same procedure outlined above. The alignment of collection optics after the filter $F_1$ is similar to that described in Refs.[55, 56].

3.4 Results

Figs.(3.5) and (3.6) show the output spectra from the Ti-sapp laser when it was tuned to 803nm and 829nm with $\sim 13.5\, \text{nm}$ and $\sim 8.5\, \text{nm}$ bandwidths respectively.

Before doing the cross-correlation experiment the time-integrated output spectrum of the photonic crystal fibre was measured using a Fourier transform spectrometer (BOMEM DA8). When the fibre was pumped at 803nm, the output spectrum consists of a series of well-defined peaks, each much broader than the input spectrum, and all shifted to the red. The higher the pump power, the larger the maximum shift, and the larger number of satellite peaks. Fig.(3.7) shows the output spectrum of the fibre when it was pumped at 803nm with 20mW input power. In this case the output power of the fibre was $\sim 1.0\, \text{mW}$. The second peak in this figure was centred at 885nm with a $\sim 25\, \text{nm}$ bandwidth. Also Fig.(3.8) shows the spectrum with 100mW input power and $\sim 2.2\, \text{mW}$ output power. There are two sharp peaks in this figure around 942 and 1128nm with respectively 31 and 38nm bandwidths.
In contrast, when the fibre pumped at 829nm, the output is more uniformly shifted to the red, but again, the maximum shift increases with increasing launch power. Fig.(3.9) shows the output spectrum of the fibre when it was pumped at 829nm and the input and output powers were 100 and 2.0mW respectively. At the highest powers used in the experiment, 100mW, the output spectrum extended as far as 1300nm, and this maximum shift was approximately the same for the two different wavelengths.
Figure 3.6: Output of the Ti-sapp laser at 829 nm.

Figure 3.7: Spectrum of the fibre with 20 mW input at 803 nm.
Figure 3.8: Spectrum of the fibre with 100 mW input at 803 nm.

Figure 3.9: Spectrum of the fibre with 100 mW input at 829 nm.
Silica is a “Kerr medium”, which means that the local refractive index depends on the local intensity of light, just as in the 2DPCs discussed in Chapter 2. While in Chapter 2 we only considered harmonic (constant amplitude) excitation, here we are explicitly dealing with a pulsed excitation. In a Kerr medium a spatial and temporal variation of the intensity will result in a time and space dependent refraction index. When a short pulse passes through a length $L$ of this medium it experiences an intensity-dependent phase shift described by [35, 53]:

$$\phi(t) = -n_2 I(t) \omega_0 L/c$$ \hspace{1cm} (3.3)

where $\omega_0$ is the frequency of the optical carrier. The pulse experiences a corresponding spectral broadening equal to:

$$\delta\omega(t) = \frac{d}{dt} \phi(t)$$ \hspace{1cm} (3.4)

Assuming the pulse has a hyperbolic secant profile (as is the case for the Ti-Sapp laser: Tsunami user’s manual) with width $\tau_0$:

$$I(t) = I_0 sech^2(t/\tau_0)$$ \hspace{1cm} (3.5)

then the maximum frequency shift due to the “Kerr effect” is:

$$\delta\omega_{max} = \frac{n_2 \omega_0 I_0 L}{c\tau_0}$$ \hspace{1cm} (3.6)
In the present case $\tau_0 \sim 100 fs$, and in silica $n_2 = 3 \times 10^{-20} \text{m}^2/\text{W}$. At an average power of $2mW$ inside the $\sim 1\mu m$ core of the fibre, the maximum wavelength shift attributable to the conventional Kerr effect is $\delta \lambda_{\text{max}} = 79nm$. This shows that Kerr-related (third order) self-phase modulation cannot explain the $\sim 500nm$ shift observed in both the discrete and continuum broadening regimes.

To reveal the temporal behaviour of the output of the photonic crystal fibre, the cross-correlation measurement is necessary (a common experiment to characterize short, non-transform limited pulses). The setup of this experiment was shown in Fig.(3.3). It can be used both to obtain spectra from the monochromator at a fixed gate-signal delay, and to obtain time-delay data at a fixed setting of the monochromator. Fig.(3.10) shows the spectrum of the upconverted signal at the satellite peaks located near 885 nm (see Fig.(3.7)), 939 nm, and 1128nm (see Fig.(3.8)) respectively. The spectra in Fig.(3.10) were obtained with the delay (between the gate and fibre beams) fixed at 0ps, the monochromator bandwidth (resolution) set to 1 nm, and the nonlinear crystal phase matching angle set to optimize the upconversion at the quoted peak wavelengths in the time-integrated spectra. The time of counting for each wavelength for these spectra was 3 seconds.
Figure 3.10: Intensity versus wavelength at the peaks of the discrete spectra in Figs. (3.7) and (3.8). From left to right these spectra were obtained with nonlinear crystal phase matching angles and delays set to (21.43°, 19.8 ps), (22.93°, 26.4 ps), and (26.87°, 59.4 ps).

Fig. (3.11) shows the temporal delay curves obtained by fixing the monochromator (with a 1 nm bandpass) at the peak wavelengths of Fig. (3.10), and varying the delay between the gate and fibre beams. Each point in this time-delay data was obtained by averaging for 10 seconds. Fig. (3.12) superimposes the time-delay and spectral data on a single plot. The corresponding set of plots obtained when the gate beam was at 829 nm and the BBO phase matching angle and the delay were optimized for upconversion at 990 nm, 1060 nm, and 1140 nm (of the spectrum shown in Fig. (3.9)) are shown in Figs. (3.13-3.15).
Figure 3.11: Intensity versus delay between the gate and fibre beams at the peak wavelengths of Fig. (3.10). The left spectrum corresponds to the left spectrum of Fig. (3.10), and so on.

Figure 3.12: Three dimensional plot of intensity versus delay and wavelength for the discrete spectra (composite of Figs. (3.10) and (3.11)).
Figure 3.13: Intensity versus wavelength at three different wavelengths in the continuum spectrum in Fig.(3.9). From left to right these spectra were obtained with nonlinear crystal phase matching angles and delays set to \((24.08^\circ, 29.7 \text{ ps}), (25.58^\circ, 39.6 \text{ ps}),\) and \((27.09^\circ, 62.7 \text{ ps})\).

The delay values needed to optimize the signal strength, with the BBO crystal set to upconvert different wavelengths, can be plotted as a function of wavelength to estimate the group velocity delay characteristics of the fibre. This information is shown in Fig.(3.16), which shows the delay values as a function of the peak in the detected wavelengths for each of the BBO angle settings. Also shown in Fig.(3.16) is the delay that one would expect assuming all of the light propagates in the lowest order bound mode of the fibre (dashed line). This latter curve was obtained by graphically measuring the slope of the dispersion curve of the lowest bound mode in Fig.(3.2).
Figure 3.14: Intensity versus delay between the gate and fibre beams at the peak wavelengths of Fig.(3.13). The left spectrum corresponds to the left spectrum of Fig.(3.13), and so on.

Figure 3.15: Three dimensional plot of intensity versus delay and wavelength for the continuum spectrum (composite of Figs.(3.13) and (3.14)).
Figure 3.16: Delay versus peak up-converted wavelengths for discrete (squares) and continuum (circles) spectra. The dashed line is the delay expected from Fig. (3.2) assuming all of the light propagates in the lowest order mode of the fibre.

The close overlap of these two curves shows that all of the light is propagating in the lowest order fibre mode, at least in the range of $800 - 1300nm$.

Although the exact mechanism responsible for these spectra is yet unclear, the correlation of the asymmetric behaviour of time delay results for the discrete spectra, Fig. (3.11), versus the symmetric time-delay behaviour for the continuous spectrum, Fig. (3.14), provides new and potentially useful information that might help to determine the nature of the underlying mechanism. A simple calculation shows that the asymmetric time-delay data might be evidence of a very rapidly varying phase, with respect to frequency shift, in the case of the dis-
crete nonlinear spectra. The symmetric time-delay data in the continuum case is consistent with a slowly varying phase, again with respect to frequency, in the continuum spectrum.

To reveal the exact mechanism behind these results, additional experiments that measure the time-delay spectra at all frequencies would help.
3.5 Conclusion

A Green's function formalism was used to describe the third-order nonlinear response of the planar photonic crystal waveguides, which contain Kerr materials. An iterative numerical solution for degenerate Kerr effect showed the dependence of reflectivity spectrum in the vicinity of leaky photonic crystal modes with the incident intensity of a single pump beam. It was shown that the Q value and line shape of these modes (which can be engineered by changing the thickness of the oxide cladding, the air filling-fraction and the in-plane wavevector of the incident field) have a great influence on reflectivity spectrum of this structure at higher powers. The nondegenerate Kerr effect was also considered to study the reflectivity spectrum of a relatively weak signal beam when a strong pump beam changed the effective susceptibility of the medium. In this case the overlapping between the field distributions of the pump and signal beams in the texture area had a great influence. Observed bistability behaviour and the shifting of reflectivity at high powers may be useful in all-optical applications such as power-limiting and optical switching.

The temporal and spectral response of a photonic crystal fibre, pumped by a 100fs Ti-Sapp laser, was also observed by using time-integrated spectroscopy and an optical cross-correlation measurement. The output spectrum of the fibre measured by a Fourier transform spectrometer showed discrete (pump wavelength at 803 nm) and continuous spectral broadening (pump wavelength
at 829 nm) that depend on pump wavelength and pump intensity. A simple calculation proved that the self-phase modulation cannot be the only nonlinear mechanism which caused $\sim 500\text{nm}$ spectral broadening when the fibre pumped with 100 mW input power. The time-resolved spectroscopy of the up-conversion of different portions of the output spectrum of the fibre showed asymmetric temporal behaviour for the discrete spectrum and symmetric behaviour for the continuous spectrum. More experiments would be required to gain a full microscopic understanding of the observed strong nonlinear effects.
APPENDIX A

FOURIER COEFFICIENTS CALCULATION

The susceptibility of a two-dimensional photonic crystal with a square lattice in real space is:

\[
\chi(\vec{r}) = \chi_b + (\chi_g - \chi_b)\theta(R - |\vec{r}|)
\]

where \(\chi_b\) and \(\chi_g\) are the susceptibilities of the background and the grating respectively and \(R\) is the radius of the holes. The Fourier coefficient of susceptibility can be calculated from:

\[
\chi(\vec{G}) = \chi_b \delta_{G0} + \frac{\chi_g - \chi_b}{V_{cell}} \int d\vec{r} e^{i\vec{G}.\vec{r}} \theta(R - |\vec{r}|)
\]

where \(\vec{G}\) are the reciprocal lattice vectors and \(V_{cell}\) is the volume of the unit cell, which in this case will be area of the unit cell. The above integration can be written as:

\[
\int d\vec{r} e^{i\vec{G}.\vec{r}} \theta(R - |\vec{r}|) = \int_0^\infty r \theta(R - r) dr \int_0^{2\pi} d\phi e^{iG.r\cos\phi}
\]

but the angular part of the integration can be written in terms of a Bessel function:

\[
J_0(x) = \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{ix\cos\phi}
\]
therefore

\[ \int d\vec{r} e^{i \vec{G} \cdot \vec{r}} \theta(R - |\vec{r}|) = 2\pi \int_0^R r J_0(Gr) dr \]

By using the following recurrence relation for Bessel functions:

\[ \frac{d}{dx} [x J_1(x)] = x J_0(x) \]

finally the susceptibility in Fourier space can be written as:

\[ \chi(\vec{G}) = \chi_0 \delta_G + \frac{2\pi R(\chi_g - \chi_b)}{GV_{cell}} J_1(GR) \]
APPENDIX B
LABVIEW CODES

All necessary LabView codes, which were used in the cross-correlation experiment, are summarized in the following table.

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>rotate.vi</td>
<td>Rotate nonlinear crystal with 0.01° accuracy</td>
</tr>
<tr>
<td>Read.vi and write.vi</td>
<td>Read and write specific parameters for monochromator. They are used for changing slit size and position of grating. In this experiment widths of entrance slit and middle slit of the monochromator were set at 2mm</td>
</tr>
<tr>
<td>scanandsave.vi</td>
<td>Control the monochromator and detector where in this situation is photomultiplier. It also collects data of the number of photons versus wavelength in a text file</td>
</tr>
</tbody>
</table>

Table B.1: LabView codes.
BIBLIOGRAPHY


[52] I. F. Mollenhauer, R. H. Stolen, and J. P. Gordon, “Experimental observation of picosecond pulse nar-


