RADIATIVE MUON CAPTURE ON OXYGEN, ALUMINUM, SILICON, TITANIUM, ZIRCONIUM, AND SILVER

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Abstract

The photon spectra from radiative muon capture (RMC) on oxygen, aluminum, silicon, titanium, zirconium, and silver have been measured for photon energies greater than 57 MeV using a cylindrical pair spectrometer at TRIUMF. Ratios of RMC to ordinary muon capture (OMC) and values of the weak induced pseudoscalar coupling constant (specifically, $g_P/g_A$) have been calculated using the observed photon spectra in conjunction with theoretical nuclear RMC models. Present discrepancies between different theoretical calculations mean that experimental values of $g_P$ cannot yet be used to make meaningful tests of the partially conserved axial current hypothesis (PCAC). However, trends in RMC/OMC and $g_P/g_A$ with atomic number indicate an extreme sensitivity of RMC, and nuclear weak virtual pion currents in particular, to Pauli blocking. A future RMC experiment at TRIUMF with the nickel isotopes $^{58}$Ni, $^{60}$Ni, and $^{62}$Ni will help to clarify the present theoretical situation. Results from RMC on H and $^3$He are forthcoming shortly from TRIUMF, and will provide direct tests of PCAC.
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Chapter 1

Introduction

Physicists classify four fundamental forces in nature: strong, electromagnetic, weak, and gravitational. The gravitational and electromagnetic forces are observed on a macroscopic scale, but the strong and weak forces are observed only at the subatomic level. The four forces are listed in order of decreasing strength in table 1.1.

The electromagnetic and weak forces have been unified into a single description known as Weinberg-Salam-Glashow SU(2)×U(1) electroweak theory [2, 3, 4], or the Standard Model of electroweak interactions. This model has been very successful, most notably in its prediction and subsequent discovery of the $W$ and $Z$ intermediate vector bosons [5]. However, questions still exist concerning the form of the weak interaction at low energies, particularly when strongly-interacting particles (hadrons) are involved. The present study tests the form of the weak interaction in the presence of the strong interaction by investigating the semi-leptonic weak process of radiative muon capture on nuclei.

<table>
<thead>
<tr>
<th>force</th>
<th>coupling strength [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>strong</td>
<td>$\alpha_s(M_Z) = 0.116$</td>
</tr>
<tr>
<td>electromagnetic</td>
<td>$\alpha = 1/137$</td>
</tr>
<tr>
<td>weak</td>
<td>$G_F/\hbar c^3 = 1.16639 \times 10^{-5}\text{GeV}^{-2}$</td>
</tr>
<tr>
<td>gravitational</td>
<td>$G_N = 6.7071 \times 10^{-39}\hbar c(\text{GeV}/c^2)^{-2}$</td>
</tr>
</tbody>
</table>

Table 1.1: The four fundamental forces in nature.
1.1 Weak interactions

For weak processes at low energy, where the square of the momentum transfer is much less than the square of the mass of the mediating particle (i.e., \( q^2 \ll M_W^2 \) for a charged weak process such as muon capture), the \( W \) propagator \( \rightarrow \frac{1}{M_W^2} \) in the Feynman amplitude, and the interaction can be assumed to take place at a point. Furthermore, the strength of the weak interaction (see table 1.1) is such that it can be treated successfully by perturbation theory. The amplitude of a weak process is therefore calculated in perturbation theory from the “contact” interaction [6]

\[
\mathcal{H}_I(x) = \frac{G_F}{\sqrt{2}} J^\alpha(x) J^I_\alpha(x)
\]

where \( G_F \) is the Fermi weak interaction coupling constant, and \( J^\alpha \) is the weak 4-vector current. This contact interaction involving self-interacting weak currents was first proposed by Fermi in 1934 to describe the nuclear \( \beta \)-decay process [7, 8]. It allows a weak process to be completely determined once the currents are specified.

For a semi-leptonic process, the weak current \( J^\alpha \) is decomposed into leptonic and hadronic parts:

\[
J_\alpha(x) = J^L_\alpha(x) + J^H_\alpha(x).
\]

The experimental data on a wide range of leptonic and semi-leptonic processes indicate that the lepton fields enter the interaction in a purely “V—A” (vector — axial vector) form [6],

\[
J^L_\alpha(x) = \sum_i \bar{\psi}_i(x) \gamma_\alpha (1 - \gamma_5) \psi_i(x)
\]

\[
J^{L1}_\alpha(x) = \sum_i \bar{\psi}_\nu_i(x) \gamma_\alpha (1 - \gamma_5) \psi_i(x)
\]

where \( l \) labels the charged lepton fields \( e, \mu, \) and \( \tau \); \( \nu_i \) labels the corresponding neutrino fields; \( \psi_i(x) \) is a Dirac field linear in the absorption operators of the \( l^- \) leptons and the creation operators of the \( l^+ \) leptons; and the \( \gamma_\alpha \) are Dirac matrices with \( \gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 \).

The weak hadronic current is less well-known. However, retaining V—A structure, the weak hadronic current may be written as [9]

\[
J^H_\alpha(x) = [V_\alpha^{(0)}(x) - A_\alpha^{(0)}(x)] \cos \theta_c + [V_\alpha^{(\pm)}(x) - A_\alpha^{(\pm)}(x)] \sin \theta_c
\]
where $\theta_c$ is the Cabibbo angle which represents weak flavour mixing of the quarks, and the superscripts on $V_\alpha$ and $A_\alpha$ indicate strangeness-conserving ($\Delta S = 0$) or strangeness-changing ($|\Delta S| = 1$) currents. For a process such as muon capture, which involves only $u$ and $d$ quarks,

$$J^H_\alpha(x) = [V_\alpha(x) - A_\alpha(x)] \cos \theta_c$$

(1.6)

where the superscript (0) has been dropped from $V_\alpha$ and $A_\alpha$.

The $V_\alpha$ and $A_\alpha$ terms may be parameterized in order to retain Lorentz covariance of the weak current. The available covariants are the 4-momentum transfer $q_\alpha$ and the Dirac matrices $\gamma_\alpha$. For ordinary muon capture on the proton,

$$\mu^- + p \rightarrow n + \nu_\mu$$

(1.7)

the quantized fields available for the construction of the weak hadronic current are the proton and neutron fields $\psi_p(x)$ and $\psi_n(x)$. Therefore, the most general covariant expressions for $V_\alpha$ and $A_\alpha$ are [10] (see appendix B)

$$V_\alpha(x) = -i\bar{\psi}_p(x)[F_V(q^2)\gamma_\alpha - F_M(q^2)\sigma_\alpha \gamma^\lambda q^\lambda + iF_S(q^2)q_\alpha]\psi_n(x)$$

(1.8)

$$A_\alpha(x) = i\bar{\psi}_p(x)[F_A(q^2)\gamma_\alpha \gamma_5 + iF_P(q^2)\gamma_\alpha q_\alpha + F_T(q^2)\gamma_5 \sigma_\alpha \gamma^\lambda q^\lambda]\psi_n(x)$$

(1.9)

where $\sigma_\alpha \gamma_\lambda = \frac{i}{2}[\gamma_\alpha, \gamma_\lambda]$, and the $F_i$ are “form factors” which depend on $q^2$ and account for the composite structure of nucleons. $F_V$ and $F_A$ are the “bare” vector and axial vector form factors; $F_M$, $F_S$, $F_P$, and $F_T$ are the “induced” weak magnetic, scalar, pseudoscalar, and tensor form factors, whose associated currents are said to be induced by strong interactions inside the nucleon. The form factors are, in general, weak functions of $q^2$, and so a corresponding set of coupling constants is usually defined:

$$g_V(q^2) = F_V(q^2)$$

(1.10)

$$g_M(q^2) = 2MF_M(q^2)$$

(1.11)

$$g_S(q^2) = m_F S(q^2)$$

(1.12)

$$g_A(q^2) = F_A(q^2)$$

(1.13)
Chapter 1. Introduction

\[ g_P(q^2) = m_l F_P(q^2) \]  (1.14)
\[ g_T(q^2) = 2M F_T(q^2) \]  (1.15)

where \( M \) is the nucleon mass and \( m_l \) is the lepton mass (for muon capture, \( l = \mu^- \)).

Experimental and theoretical constraints on the weak form factors are as follows:

- Time reversal invariance – the assumption that the semileptonic weak interaction, Eq. (1.1), is time reversal invariant means that all weak form factors are real.

- \( G \) parity invariance – under \( G \) parity (a rotation in isospin space followed by charge conjugation: \( G = Ce^{i\pi/2} \)), the bare weak vector and axial vector currents transform as

\[ GV[A]G^t = V[-A]. \]  (1.16)

The induced weak magnetism and pseudoscalar currents also transform in this manner, and are called first-class currents. The induced scalar and tensor currents transform in the opposite manner,

\[ GV[A]G^t = -V[A] \]  (1.17)

and are called second-class currents. The strong vector and axial vector currents conserve \( G \) parity, so the weak currents induced by the strong interaction are expected to transform under \( G \) parity in the same manner as the bare weak vector and axial vector currents. That is, \( G \) parity invariance of the strong interaction requires that second-class currents vanish:

\[ F_s(q^2) = F_T(q^2) = 0. \]  (1.18)

- Conserved vector current hypothesis (CVC) – in electrodynamics, the equality of the observed (i.e. renormalized) charges of the positron (which is a lepton) and proton (a hadron) is attributed to the equality of the bare charges and the divergence-free nature of the electromagnetic (vector) current. Analogously, the observed near-equality of the muon (lepton) decay constant \( G_\mu \) and hadronic \( \beta \) decay vector coupling constants is
attributed to the (near) equality of the bare vector couplings and the divergence-free nature of \( V_\alpha(x) \) in Eq. (1.8):

\[
\partial^a V_\alpha(x) = 0. \tag{1.19}
\]

A suitable divergence-free vector current does exist, namely, the conserved isospin current of strong interactions, \( J_\alpha(x) \). The CVC hypothesis consists of identifying \( V_\alpha(x) \) and \( V^\dagger_\alpha(x) \) with the \((1 + i2)\) and \((1 - i2)\) components of \( J_\alpha \). Furthermore, the third \((I_3 = 0)\) component of \( J_\alpha \) is part of the electromagnetic current of hadrons according to the Gell-Mann-Nishijima-Nakano [12, 13] relation

\[
Q = I_3 + Y/2 \tag{1.20}
\]

where \( Q \) is the electromagnetic charge, \( Y \) is the hypercharge, and \( I_3 \) is the third component of isospin carried by a current. Therefore, there exist relationships between the semileptonic weak form factors and the (well-known from electron scattering experiments) hadronic electromagnetic form factors. In the \( q^2 \rightarrow 0 \) limit [10],

\[
F_V(q^2) = 1 \tag{1.21}
\]

\[
F_M(q^2) = \frac{\mu_p - \mu_n}{2M} \tag{1.22}
\]

\[
F_S(q^2) = 0 \tag{1.23}
\]

where \( M \) is the nucleon mass, and \( \mu_p \) and \( \mu_n \) are the anomalous magnetic moments of the proton and neutron respectively.

- \( \beta \) decay of the neutron – a value for \( F_A(q^2) \) at \( q^2 = 0 \) has been measured using neutron \( \beta \)-decay [1]:

\[
F_A(0) = 1.2573 \pm 0.0028 \tag{1.24}
\]

Therefore, we are left with the induced pseudoscalar form factor \( F_P(q^2) \) which is the focus of the present experimental study.

The axial current given in Eq. (1.9) is clearly not conserved; otherwise, \( \pi^- \rightarrow \mu^-\bar{\nu}_\mu \) decay could not occur. However, the axial current is made almost divergenceless by making the
partially conserved axial current hypothesis (PCAC), which relates the divergence of the axial current to the pion field [14, 15],

\[ \partial^\alpha A_\alpha(x) = f_\pi m_\pi^2 \phi_\pi(x) \]  

(1.25)

where \( f_\pi \) is the pion decay constant. Applying PCAC to Eq. (1.9) using Eq. (1.18), and using the Dirac equation gives

\[ 2MF_A(q^2) - q^2 F_P(q^2) = \frac{\sqrt{2} f_\pi m_\pi^2 G_{NN}}{q^2 + m_\pi^2} \]  

(1.26)

where \( G_{NN} \) is the strong coupling constant for the pion-nucleon vertex, \( M \) is the nucleon mass, and \( m_\pi \) is the pion mass. In the \( q^2 \to 0 \) limit, and assuming that the pion-nucleon coupling varies slowly with \( q^2 \) (i.e., \( G_{NN}(0) = G_{NN}(q^2) = G_{NN} \)), Eq. (1.26) reduces to the Goldberger-Treiman relation [16]

\[ F_A(0) = \frac{f_\pi G_{NN}}{\sqrt{2}M}. \]  

(1.27)

Assuming \( F_A(q^2) = F_A(0) \) and using Eq. (1.27) in Eq. (1.26) gives

\[ F_P(q^2) = \frac{2MF_A(0)}{q^2 + m_\pi^2} \]  

(1.28)

or, using Eqs. (1.13) and (1.14) for the coupling constants,

\[ g_P(q^2) = \frac{2Mm_\mu g_A(0)}{q^2 + m_\pi^2}. \]  

(1.29)

For ordinary muon capture on the proton, \( q^2 = 0.88m_\mu^2 \) giving the Goldberger-Treiman value for \( g_P \)

\[ \frac{g_P}{g_A} = 6.78 \]  

(1.30)

or, using Eqs. (1.13) and (1.24),

\[ g_P = 8.52. \]  

(1.31)

An objective of the present study is to test this value of \( g_P \) and hence the validity of PCAC by measuring the process of radiative muon capture on nuclei.
1.2 Radiative muon capture

In order to accurately measure the pseudoscalar contribution to the weak hadronic current, it is important to observe a weak process where this contribution is as large as possible. The form of Eq. (1.29) suggests a process which involves a heavy lepton. Also, the fact that the induced couplings enter into the semileptonic weak interaction proportional to $q^2$ (see Eqs. (1.1), (1.6), (1.8), and (1.9)) suggests a process with large 4-momentum transfers. Finally, the form of Eq. (1.29) suggests a process with momentum transfers near the pion pole. These criteria heavily favour tauon or muon capture processes over $\beta$ decay or electron capture processes. Muon capture processes are chosen for study due to the availability of very high quality muon beams.

The weak process of ordinary muon capture (hereafter referred to as OMC) on the proton,

$$\mu^- + p \rightarrow n + \nu_\mu$$

(1.32)

is one possible process for observing the induced pseudoscalar current. A low velocity muon becomes bound in an atomic orbital, and cascades in less than 1 ns to the $1s$ muonic orbital. This is followed by decay of the muon, or capture on the proton to produce a final state neutron and muon neutrino. Measurements of most $g_\rho$-dependent observables (e.g. polarizations, angular correlations) are limited by the detectability of these final state particles. A simple observable such as the muon capture rate can be measured, but in light nuclei where specific final states can be isolated, the measurement is difficult because the capture probability is small. In fact, only $10^{-3}$ of the muons which stop in a hydrogen target will undergo OMC; the rest will decay according to

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

(1.33)

which is called "Michel" decay. Measurements of $g_\rho$ as determined by measuring the OMC rate on hydrogen are shown in table 1.2. While all results agree with the Goldberger–Treiman value for $g_\rho$, it should be noted that the uncertainties are large. Measurements of the OMC rate in nuclei are made easier by the fact that the rate for muon capture goes as $Z_{eff}^4$, where
Table 1.2: Summary of \( g_P \) results from measurements of OMC on \( \text{H}_2 \), as presented by Bardin et al. [22].

<table>
<thead>
<tr>
<th>Reference</th>
<th>Target</th>
<th>( g_P )</th>
<th>( g_P/g_A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bleser et al. 1962 [17]</td>
<td>liquid ( \text{H}_2 )</td>
<td>6.0 ± 8.0</td>
<td>4.8 ± 6.4</td>
</tr>
<tr>
<td>Rothberg et al. 1963 [18]</td>
<td>liquid ( \text{H}_2 )</td>
<td>11.0 ± 4.3</td>
<td>8.7 ± 3.4</td>
</tr>
<tr>
<td>Alberigi Quaranta et al. 1969 [19]</td>
<td>( \text{H}_2 ) gas</td>
<td>10.3 ± 3.9</td>
<td>8.2 ± 3.1</td>
</tr>
<tr>
<td>Bystritiskii et al. 1974 [20]</td>
<td>( \text{H}_2 ) gas</td>
<td>7.9 ± 5.9</td>
<td>6.3 ± 4.7</td>
</tr>
<tr>
<td>Bardin et al. 1981 [21]</td>
<td>liquid ( \text{H}_2 )</td>
<td>7.1 ± 3.0</td>
<td>5.6 ± 2.4</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>8.7 ± 1.9</td>
<td>6.9 ± 1.5</td>
</tr>
</tbody>
</table>

\( Z_{\text{eff}} \) is an effective nuclear charge. However, nuclear structure details dominate the capture rate in complex nuclei, thereby obscuring any dependence of the capture rate on \( g_P \). Other observables, such as the capture rate to a specific final nuclear state, are less sensitive to nuclear structure and have been used with carbon, oxygen, and silicon to extract values of \( g_P/g_A \). In the case of \( ^{12}\text{C} \), values of \( g_P/g_A \) have been extracted by measuring the average polarization of the recoiling nucleus along the muon-spin direction (\( P_{\text{av}} \)), the longitudinal polarization of the recoiling nucleus (\( P_{L} \)), and the capture rate to the \( ^{12}\text{B} \) ground state. For \( ^{16}\text{O} \), the OMC rate to the first excited state in \( ^{16}\text{N} \), relative to the \( \beta \)-decay rate back to the \( ^{16}\text{O} \) ground state, has been measured. Finally, for \( ^{28}\text{Si} \), observation of the angular correlation between the OMC neutrino and nuclear de-excitation gamma rays has provided values of \( g_P/g_A \). Unfortunately, the extracted values of \( g_P/g_A \) depend strongly on the theory used to describe the nuclear OMC process under investigation. Results are shown in table 1.3.

A complementary semileptonic weak process for measurement of the induced pseudoscalar current is radiative muon capture (hereafter referred to as RMC) on the proton,

\[
\mu^- + p \rightarrow n + \nu_\mu + \gamma. \tag{1.34}
\]

RMC has two distinct advantages over OMC. First, the photon in the final state is easy to detect so that an RMC branching ratio and an RMC photon energy spectrum can be measured. Both of these observables are directly related to \( g_P \). Second, the momentum transfer of RMC on the (stationary) proton is variable, ranging from \( q^2 = 0.88m_\mu^2 \) at 0 MeV photon energy to \( -m_\mu^2 \) at maximum photon energy. This is because there are three bodies
Table 1.3: Summary of $g_F/g_A$ results from measurements of OMC on nuclei. $P_{av}$ refers to measurement of the average polarization of the recoiling nucleus along the muon-spin direction; $P_L$ refers to measurement of the longitudinal polarization of the recoiling nucleus; $\Lambda_\mu$ and $\Lambda_\beta$ refer to measurement of the OMC and $\beta$-decay rates to specific nuclear states; and $\nu - \gamma$ refers to measurement of the angular correlation between the OMC neutrino and nuclear de-excitation gamma rays.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Nucleus</th>
<th>Method</th>
<th>Theory</th>
<th>$g_F/g_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Possoz et al. [23, 24]</td>
<td>$^{12}\text{C}$</td>
<td>$P_{av}$</td>
<td>[25]</td>
<td>7.1 ± 2.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[26, 27]</td>
<td>13.6 ± 2.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[28]</td>
<td>15 ± 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[29]</td>
<td>10.3 ± 2.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[30]</td>
<td>10.1 ± 2.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[26, 27]</td>
<td>2.6</td>
</tr>
<tr>
<td>Kuno et al. [30]</td>
<td>$^{12}\text{C}$</td>
<td>$P_{av}$</td>
<td>[29]</td>
<td>8.4 ± 2.5</td>
</tr>
<tr>
<td>Miller et al. [31]</td>
<td>$^{12}\text{C}$</td>
<td>$\Lambda_\mu$</td>
<td>[28]</td>
<td>8.4 ± 2.5</td>
</tr>
<tr>
<td>Roesch et al. [32, 33]</td>
<td>$^{12}\text{C}$</td>
<td>$P_{av}/P_L$</td>
<td>[34]</td>
<td>9.4 ± 1.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[35]</td>
<td>7.2 ± 1.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[29]</td>
<td>9.1 ± 1.8</td>
</tr>
<tr>
<td>Guichon et al. [36], Hamel et al. [37]</td>
<td>$^{16}\text{O}$</td>
<td>$\Lambda_\mu/\Lambda_\beta$</td>
<td>[38]</td>
<td>11 - 12</td>
</tr>
<tr>
<td>Miller et al. [39]</td>
<td>$^{28}\text{Si}$</td>
<td>$\nu - \gamma$</td>
<td>[26, 27]</td>
<td>12.9 ± 3.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[40]</td>
<td>-1.9 ± 3.1</td>
</tr>
</tbody>
</table>

in the final state, in contrast with OMC on the proton where the two-body final state fixes $q^2$ at $0.88m_\mu^2$. Experimentally, it is the high energy RMC photons that are observed (see section 4.1), so the observed momentum transfer in RMC is close to the pion pole, resulting in an enhancement of the pseudoscalar current by a factor of about 3 over OMC. However, RMC has the disadvantage of being a rare process. The RMC/OMC ratio for the high energy end of the RMC photon spectrum is on the order of $10^{-5}$. Therefore, measurement of RMC requires careful assessment of backgrounds. Also, due to the rarity of the process, nuclear RMC measurements are typically inclusive measurements, which means that theoretical nuclear structure calculations, used in the extraction of experimental results, are important when interpreting these results.

In the present experiment, RMC photons are observed via pair conversion in lead. Track reconstruction of the $e^+e^-$ pairs in a cylindrical drift chamber allows for measurement of the differential and integrated RMC photon energy spectra. In this study, RMC on six different nuclei is studied. Oxygen is chosen in order to compare with earlier oxygen results from PSI.
[41, 42], as well as a previous oxygen result from TRIUMF [43] which used a time projection chamber (as opposed to the drift chamber described in section 3.3.1) and a slightly different photon trigger ($2D$ instead of $\geq 1D$ – see section 3.4). Aluminum and silicon are chosen in order to verify an "isotope effect" (see section 6) seen in previous aluminum and silicon results [44]. Titanium is chosen for comparison with previous calcium results [41, 43, 45, 46, 47] to see if an "isotope effect" might also be present here. Zirconium and silver are chosen in order to investigate the decreasing trend in the RMC/OMC ratio with increasing atomic number (see section 6).

The next chapter will review the RMC theory which applies to the nuclei in the present study. In chapter 3 the experimental apparatus is discussed. Chapter 4 gives the detailed data analysis procedure, and all results are presented in chapter 5. Discussion and interpretation of results take place in chapter 6.
Calculations of RMC on the free proton and in nuclei are discussed below. While the elementary RMC process is well understood, the necessarily complex nuclear RMC process is more theory-dependent. A brief overview of elementary RMC theory is given in section 2.1. A more detailed discussion of the nuclear RMC theory with which the present study is concerned is given in section 2.2.2. An extensive review of RMC theory has been made by Gmitro and Truöl [48].

2.1 Elementary RMC

There have been three distinct approaches to the calculation of the amplitude for the elementary RMC process,

\[ \mu^- + p \rightarrow n + \nu_\mu + \gamma. \]  

(2.1)

All three yield consistent results, so the elementary RMC amplitude is believed to be well understood.

The first of these approaches is the evaluation of Feynman diagrams (i.e. perturbation theory). The standard diagrams [49, 50, 51] are shown in figure 2.1, and include radiation from the charge of the muon, the charge and magnetic moment of the proton, the magnetic moment of the neutron, and the charge of the exchanged virtual pion. The exchange diagram and the vertex-radiating diagram are needed to maintain gauge invariance of the amplitude. The relative strength of the pseudoscalar amplitude in the RMC process is largely determined
by the relative strength of the pion exchange diagram.

The second approach, followed by Adler and Dothan [52] and later by Christillin and Servadio [53], involves application of the soft photon theorem of Low [54] to the weak axial vector current. Both groups of authors show that the elementary RMC amplitude can be expressed in terms of the non-radiative amplitude (OMC) and the divergence of the axial vector current, and their results are in close agreement. Furthermore, the results of this second approach do not differ significantly [55] from the first diagrammatic approach discussed above.

The final approach, undertaken by Hwang and Primakoff [56] and extended by Gmitro and Ovchinnikova [57], is the construction of the elementary RMC amplitude from current conservation laws (conservation of electromagnetic current (CEC), CVC, and PCAC), the requirement of gauge invariance, and a “linearity hypothesis” which constrains the form of the weak form factors. Initial results were at odds with the diagrammatic approach (RMC rates were a factor of two lower for a given value of $g_P$); however, this discrepancy was attributed to problems with the linearity hypothesis, which were subsequently resolved [57].
2.2 Nuclear RMC

Once the elementary RMC process has been calculated, it is necessary to "embed" it in the nucleus in order to calculate the nuclear RMC process

$$\mu^{-}(Z,A) \rightarrow (Z-1,A)\nu\gamma.$$  \hspace{1cm} (2.2)

The simplest way to do this is to sum the elementary amplitude incoherently over all protons in the nucleus, i.e., to treat the interaction in impulse approximation (IA). The success of the IA requires that interactions involving two nucleons (via meson-exchange currents) are negligible. All of the nuclear RMC models discussed in section 2.2.2 use this approximation in conjunction with an appropriate nuclear response, except for Gmitro et al. [58] who use a "modified impulse approximation". Before discussing calculation of the nuclear response, it is important to consider effects of the nuclear medium on the weak pseudoscalar current which may result in deviations of $g_P$ from the Goldberger-Treiman value.

2.2.1 Quenching of $g_P$

As stated in section 2.1, the pseudoscalar amplitude in RMC is sensitive to the pion exchange diagram in figure 2.1. Therefore, nuclear effects which disturb the pion current, such as nucleon polarizability, short-range nucleon-nucleon correlations, and Pauli blocking of final-state neutrons, are expected to modify the value of $g_P$ in a nucleus. Scattering of an emitted virtual pion by other nucleons in the nucleus is taken into account by replacing $m_{\pi}^2$ in the pion propagator by an effective pion mass [59]

$$\tilde{m}_{\pi}^2 = \frac{m_{\pi}^2}{1 + \alpha}.$$  \hspace{1cm} (2.3)

where $\alpha$ is the nucleon polarizability. Also, the pion-nucleon coupling is modified by writing

$$\tilde{G}_{\pi NN} = (1 + \xi \alpha)G_{\pi NN}$$  \hspace{1cm} (2.4)

where $\xi$ accounts for short-range nucleon-nucleon correlations and Pauli blocking. Combination of the nuclear effects then results in a large "renormalization" of $g_A$ and $g_P$:

$$\tilde{g}_A = g_A(1 + \xi \alpha),$$  \hspace{1cm} (2.5)
The renormalized $g_P/g_A$ ratio does not depend on $\xi$, so with $\alpha = -0.75$ for infinite nuclear matter [60], and $q^2 = 0.88m_\mu^2$ for OMC,

$$\frac{\tilde{g}_P}{g_A} = 0.33 \frac{g_P}{g_A}. \quad (2.7)$$

Surface effects for finite nuclei are expected to reduce this "quenching" of $g_P$, but it is clear that the weak pseudoscalar current is significantly affected by the nuclear medium.

### 2.2.2 Nuclear response

In the IA, calculation of nuclear RMC requires only the elementary RMC amplitude and some method to account for initial and final state nuclear wavefunctions. The quantities of experimental interest that are calculated in a given theoretical model are the RMC/OMC ratio as a function of $g_P$, and the shape of the RMC photon spectrum. The RMC/OMC ratio is observed experimentally for photons with $k > 57$ MeV, and this partial RMC/OMC ratio is defined as $R_\gamma$. $R_\gamma$ is calculated (instead of pure RMC) in order to remove some dependency on nuclear effects which are common to both RMC and OMC. Also, the inclusive RMC branching ratio is calculated (rather than RMC to an exclusive final state) due to the experimental difficulty in measuring exclusive RMC branching ratios. The nuclear models discussed below are relevant to the nuclei in the present study.

- **Closure model:** this was the first model used to calculate nuclear RMC. The final state nucleus is assigned an average excitation energy $E_{av}$, and the nuclear RMC matrix element is evaluated at this transition energy. Fits to experimental data are accomplished by treating $E_{av}$ and $g_P$ as free parameters. Various authors [50, 61, 62, 63, 64, 65] have calculated RMC branching ratios and spectra in the closure approximation, but the extracted values of $g_P$ are extremely sensitive to $E_{av}$. Therefore, this model is not used in the present study to determine values of $g_P$. However, a closure RMC spectral shape, Eq. (5.1), is used here as a representative RMC photon spectrum in order to calculate...
values of $R_\gamma$. This polynomial shape was first derived in the closure approximation by Primakoff [66] by considering only the muon-radiating diagram in figure 2.1.

- Fermi gas model: this model avoids the closure approximation by using nuclear response functions calculated in the Fermi gas model. Christillin et al. [55] take nucleon-nucleon correlations into account by introducing an effective nucleon mass $M^*$ as a free parameter. The value of $M^*$ ($M^* = 0.5M$) is fixed by fitting model-predicted OMC rates to experiment; the model then reproduces OMC rates to better than 10% for a large number of nuclei with $Z \geq 42$. This model is used in the present study to determine values of $R_\gamma$ and $g_P$ for the heavy nuclei zirconium and silver.

- Semi-phenomenological “realistic” nuclear excitation model: this model avoids the closure approximation by using giant dipole (GDR) and giant quadrupole (GQR) resonances for the final state nuclei. Values of $R_\gamma$ for a given value of $g_P$ are $\sim 30\%$ lower than those predicted by the closure model, bringing theoretical results into closer agreement with experiment (assuming the Goldberger-Treiman value for $g_P$). Christillin’s [67] calculations of RMC on calcium are used to determine values of $R_\gamma$ and $g_P$ for titanium. Christillin and Gmitro’s [68] calculations of RMC on oxygen are used to determine the same quantities for the oxygen target.

- Modified impulse approximation (MIA): Gmitro et al. [58] venture beyond the impulse approximation by considering nucleon-nucleon interactions via meson-exchange currents (MEC’s) at the electromagnetic vertex in figure 2.1. The MEC’s are accounted for using constraints which follow from continuity of the electromagnetic current. A microscopic model (shell model) is used for the initial and final state nuclei, and theoretical predictions are similar to those of the phenomenological model described above. The MIA was attempted because a purely microscopic model [58] for oxygen and calcium predicted values of $R_\gamma$ (for a given value of $g_P$) which were roughly double those predicted by the phenomenological model. Gmitro et al. [58] calculate RMC on oxygen and calcium, so the MIA is used here to determine values of $R_\gamma$ and $g_P$ for oxygen and
Chapter 2. Theory

titanium.

- Sum rule techniques: Roig and Navarro [69] use SU(4) symmetry, a Hartree-Foch scheme for the target ground state, and sum rule techniques (which are particularly adapted to the analysis of inclusive processes) to calculate RMC on carbon, oxygen, and calcium. Their values of \( R_\gamma \) are the lowest for a given value of \( g_P \), and are the closest to the experimental values (assuming the Goldberger-Treiman value for \( g_P \)). This model is used to determine values of \( g_P \) for oxygen and titanium.

To date, nuclear responses to RMC have not been calculated in a rigorous manner for aluminum, silicon, or titanium. However, as mentioned above, the nuclear response of calcium has been calculated, and is used in this study as an approximation to the response of titanium. Calculations for aluminum and silicon have been made in the Fermi gas model of Fearing and Welsh [70], but this Fermi gas model is only expected to be useful for \( Z > 20 \) nuclei. Fearing and Welsh utilize a relativistic mean field theory approach, where a relativistic Fermi gas model is used to describe medium-heavy nuclei, and a local density approximation along with realistic nuclear density distributions are used to relate the RMC process in infinite nuclear matter to finite nuclei. However, the aim of this model is to assess the reliability with which \( g_P \) can be extracted from experimental data, rather than make explicit predictions. In fact, implementing the realistic nuclear density distributions results in OMC rates that are significantly higher than the experimental values.

Moreover, a number of theoretical RMC calculations in nuclei have been done, and are used in the present study to extract experimental values of the partial RMC/OMC ratio, \( R_\gamma \), and the induced weak pseudoscalar coupling constant, \( g_P \). \( R_\gamma \) and \( g_P \) are sensitive to the pion exchange current in figure 2.1, so nucleon polarizability, short-range nucleon-nucleon correlations, and Pauli blocking in nuclei will affect the values of \( R_\gamma \) and \( g_P \) as extracted from nuclear RMC.
Chapter 3

Experiment

The experiment was carried out in the meson hall at the TRIUMF laboratory in Vancouver, British Columbia, Canada. Hardware requirements were: producing an intense, collimated $\mu^-$ beam; stopping this beam in targets of physical interest; detecting and high resolution tracking of RMC photons using pair conversion in lead; detecting beam, decay, and cosmic ray particles in scintillation counters; and writing data to tape for analysis.

3.1 TRIUMF muon and pion beams

Production of the primary proton beam at TRIUMF begins with injection of unpolarized $H^-$ ions in 5-ns-long pulses every 43.3 ns into the TRIUMF cyclotron. These ions are accelerated to 500 MeV at the cyclotron's outer edge, where thin carbon foils strip off the electrons to create $H^+$ (protons). The positively charged protons have opposite curvature (to $H^-$) in the cyclotron's magnetic field, which results in their extraction from the cyclotron into proton beam lines with very high efficiency. The primary 500 MeV proton beam has a macroscopic duty factor of 100%, and a typical current of between 100 and 140 $\mu$A.

Pions and their decay products (muons and electrons) are produced when the primary proton beam impinges upon the meson production target 1AT2, typically a beryllium strip extending 10 cm in the proton beam direction. The M9A beam channel views 1AT2 at 135° to the proton beam direction and has an acceptance solid angle of 25 msr. The RMC spectrometer (described in section 3.3) is mounted at the end of beam channel M9A. Additional information on the TRIUMF cyclotron, beam lines, and beam channels can be found in the
TRIUMF users handbook [71].

The muon beam used in this experiment is produced from pions decaying near the production target (called cloud muons). After transport by a series of dipole and quadrupole magnets to the RMC spectrometer, the cloud muon beam has a raw particle composition of \( \pi/\mu \sim 1 \) and \( e/\mu \sim 15 \). However, for RMC studies, an RF separator [72] utilizes the differences in particle time-of-flights to suppress the pion and electron contents of the beam to \( \pi/\mu \sim 10^{-3} \) and \( e/\mu \sim 5 \times 10^{-2} \). For radiative pion capture (hereafter referred to as RPC) studies, the separated \( \pi^- \) beam composition at the spectrometer is typically 96.3\% \( \pi^- \), 2.9\% \( e^- \), and 0.8\% \( \mu^- \). The muon beam has a momentum of 65 MeV/c, \( \delta p/p = 8\% \), a spot size of \( 5 \times 5 \text{ cm}^2 \) at the beam counters (see section 3.3.3), and a typical stopping rate in the targets of \( 5.0 \times 10^5 \text{ s}^{-1} \). The pion beam has a momentum of 81 MeV/c and a typical stopping rate in the targets of \( 5.6 \times 10^5 \text{ s}^{-1} \).

### 3.2 Targets

Six nuclear targets are used in the present RMC study: oxygen, aluminum, silicon, titanium, zirconium, and silver. The targets are composed of the naturally occurring elements, and their dimensions are listed in table 3.1.

The oxygen target consists of liquid D$_2$O in a polyethylene bag, held in the beam pipe by a lucite ring surrounded by a styrofoam holder. Liquid D$_2$O is chosen for its sufficient muon stopping power, and D$_2$O is used instead of H$_2$O because the RPC photon background arising from the small \( \pi^- \) content of the \( \mu^- \) beam (see section 3.1) and the charge exchange reaction, Eq. (4.1), is three orders of magnitude smaller for the deuteron than it is for the proton [73]. Furthermore, RMC on the deuteron is totally insignificant compared to that on oxygen.

The silicon target consists of granular silicon in a polypropylene container, held in the beam pipe by a styrofoam holder. The aluminum, zirconium, and silver targets consist of several metallic plates, and the titanium target of metallic shavings, all held in the beam pipe
<table>
<thead>
<tr>
<th>Target</th>
<th>Shape</th>
<th>Dimensions</th>
<th>No. of foils</th>
<th>Spacing of foils</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>disk</td>
<td>14.4 cm dia. × 3.0 cm thick</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>Al</td>
<td>disk</td>
<td>8.5 cm dia. × 0.15 cm thick</td>
<td>10</td>
<td>0.87 cm</td>
</tr>
<tr>
<td>Si</td>
<td>disk</td>
<td>8.6 cm dia. × 2.0 cm thick</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>Ti</td>
<td>disk</td>
<td>15.5 cm dia. × 20.0 cm thick</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>Zr</td>
<td>square</td>
<td>10.0 cm × 10.0 cm × 0.025 cm</td>
<td>15</td>
<td>0.69 cm</td>
</tr>
<tr>
<td>Ag</td>
<td>square</td>
<td>10.0 cm × 10.0 cm × 0.037 cm</td>
<td>7</td>
<td>0.96 cm</td>
</tr>
</tbody>
</table>

Table 3.1: Dimensions of RMC targets. All targets are mounted perpendicular to the beam axis.

by styrofoam holders. The motivation for using grains, plates, and shavings (as opposed to solid targets) is to reduce photon conversion in the target (increasingly significant for heavier targets).

A carbon target is used for calibration of the spectrometer (see section 4.2.3). It is composed of a single disk of graphite, 15.4 cm in diameter and 1.9 cm thick.

### 3.3 RMC spectrometer

The various components of the RMC spectrometer are shown in figures 3.1 and 3.2. The spectrometer has cylindrical symmetry, and an acceptance solid angle of $3\pi$. Starting in the target and extending radially outwards, a photon passes undetected through an $A$ counter, an $A'$ counter, and a $B$ counter. The photon then converts into an $e^+e^-$ pair in the lead converter which is sandwiched between the $B$ and $C$ scintillators. The $e^+e^-$ pair leaves a signal in one or more $C$ counters, and leaves two distinct circular patterns of cell hits in the drift chamber. One or more $D$ counters may be hit, depending on the topology and energy of the $e^+e^-$ pair. Conversely, a charged particle, such as a Michel electron, is detected in one of more of the $A$, $A'$, and $B$ counters (called "veto" counters); is detected in a $C$ counter; leaves a single circular track in the drift chamber; and may hit a $D$ counter. The components of the spectrometer are discussed in more detail below.
Chapter 3. Experiment

Figure 3.1: Global view of the RMC spectrometer, as configured for a hydrogen target. For the nuclear targets of the present study, the liquid protium refrigerator is replaced by a downstream veto scintillation counter.

Figure 3.2: Cross section of the RMC spectrometer, showing a two-photon event.
3.3.1 Drift chamber

The drift chamber is used for track reconstruction of $e^+e^-$ pairs arising from photon conversion in a 1.0 mm thick cylindrical lead converter. The thickness of the lead converter is chosen to provide efficient photon conversion and minimal momentum degradation of the $e^+e^-$ pair. The drift chamber, shown in cross section in figure 3.2, is composed of 4 cylindrical superlayers of 56, 64, 72, and 80 cells in a uniform magnetic field of $0.235 T \pm 0.5\%$. The cells range from 1.947 to 2.081 cm across at their radial midpoints, and contain 6 sense wires, each of which is instrumented with a discriminator and a LeCroy 1879 Fastbus pipelined TDC for multihit information in 2 ns bins. The sense wires are alternately staggered right and left of the cell midplane in order to resolve the left/right drift ambiguity. The wires in superlayers 1, 2, and 4 are strung axially, and provide $x$ and $y$ track coordinates. The wires of superlayer 3 are strung axially, followed by rotation of the end plates in opposite directions by 3.5°, so that the wires lie at a stereo angle of 7° with respect to the drift chamber axis. Superlayer 3 provides $z$ track coordinates. The drift chamber uses a gas mixture of 49.9% argon, 49.9% ethane, and 0.2% ethanol in order to achieve a drift velocity of 50 $\mu$m/ns at electric fields near 2 kV/cm. The hits in a cell are fit to a straight line, and the line segments in one axial layer are linked to segments in the other two axial layers to form circular $x$-$y$ tracks. The radius of a track and the magnetic field in the drift chamber give the $x$-$y$ component of the particle's momentum according to

\[ p(\text{eV/c}) = |q| \cdot B(\text{Tesla}) \cdot r(\text{cm}) \cdot 3 \times 10^6 \]  \hspace{1cm} (3.1)

where $|q|$ is the magnitude of the particle's charge in units of the proton charge $e$. More information on the design and performance of the drift chamber, as well as the track fitting technique, can be found elsewhere [74, 75].

3.3.2 Inner wire chamber

The inner wire chamber (IWC) is a dual-coordinate multiwire proportional chamber located inside the inner radius of the drift chamber. Its primary purpose is to provide a second
Chapter 3. Experiment

$z$ coordinate for charged particle tracks in the drift chamber, so that the $z$ component of a particle's momentum can be determined. The IWC consists of 768 axial anode wires arranged cylindrically and spaced by 2.21 mm, bounded on the inner and outer radii by cathode layers, each consisting of 384 spiral strips of aluminum supported by mylar. On the inner and outer cathode layers the aluminum strips are spaced by 3.06 mm and 3.19 mm, and lie at angles of $-45^\circ$ and $+45^\circ$ to the anode wires, respectively. The chamber uses a gas mixture of 77.6% argon, 22.2% isobutane, and 0.2% freon bubbled through methylal, with the cathode strips at ground and the anode wires at $+3.4$ kV. The $z$ coordinate of a track point is found from the intersection of the hit anode wire and the centroid of the induced pulses on a local strip cluster from at least one of the cathode planes. More information on the design and performance of the IWC can be found elsewhere [74, 76].

3.3.3 Beam, trigger, and cosmic ray counters

Four identical scintillation counters, of total thickness 0.635 cm, are located approximately 17 cm upstream of the target center. Coincident hits between these four counters are used to count beam particles, and the size of the signal in any one beam counter is used to discriminate between electrons, muons, and pions. A scintillation counter is also located downstream of the target, and serves as a veto in the definition of a particle stop.

Five cylindrical layers of azimuthally segmented scintillation counters surround the target. The first three layers, the $A$, $A'$, and $B$ counters, are located inside the lead converter; therefore, they act as photon vetos because a photon event should not leave a signal in any one of these counters. The $A$ and $A'$ rings are each segmented into 4 arc-shaped counters, and the inner $A$ ring is rotated with respect to the outer $A'$ ring by $45^\circ$ so that the joints do not overlap. The $B$ ring is segmented into 12 counters.

The $C$ ring is segmented into 12 counters, and is located just outside the lead converter; therefore, it is used in the identification of successful photon conversions. The $D$ ring is segmented into 16 counters, and is located outside the drift chamber. The $C$ and $D$ counters are used together to identify hit patterns characteristic of an $e^+e^-$ pair.
Chapter 3. Experiment

The top face of the spectrometer, and the upper parts of all four sides, are covered by a total of six scintillator-drift chamber units, each unit consisting of a rectangular scintillation counter and two rectangular drift chambers. This array of scintillators and drift chambers is used to reject photon events which arise due to cosmic ray showers. More information on the beam, trigger, and cosmic ray counters can be found elsewhere [74].

3.4 Data acquisition

TDC and ADC data from the component detectors of the RMC spectrometer is broken up into several YBOS data “banks” and written to tape when at least one of three trigger conditions is met:

- $\gamma$ trigger – this trigger selects $e^+e^-$ pairs arising from photons entering the lead converter. It is designed to maximize the acceptance of RMC and RPC photons, and minimize the acceptance of non-photon events, so that a CPU-manageable trigger rate is achieved. It also provides some rejection of single track events, such as Michel electrons that are missed by the $A$, $A'$, or $B$ veto counters and cosmic rays that are missed by the cosmic ray counters, through the definition

$$TRIG_\gamma = STROBE_\gamma \cdot EVENT \cdot AHC$$

where

- $STROBE_\gamma = \sum(A + A') \cdot \sum B \cdot \sum C \geq 1D$
- $EVENT$ = valid C-D pattern coincidence for an $e^+e^-$ pair
- $AHC$ = “analog hit counter”; valid pattern of drift chamber cell hits for an $e^+e^-$ pair.

The $\geq 1D$ condition, which requires that at least one the particles in an $e^+e^-$ pair reach the $D$ scintillators, rejects low energy photons such as X-rays resulting from cascade of a bound muon to the $1s$ muonic orbital.

- $Q$ rate trigger – this trigger selects a fraction of the charged particles leaving signals in the $A$, $A'$, or $B$ veto counters. It is designed to accept some of the charged particles
arising from muon decay in the target and cosmic rays, and is used for background and normalization studies of the RMC spectrometer. It is defined by

$$TRIG_Q = Q1 + Q2$$

(3.3)

where

$$Q1 = \sum B \cdot \sum C \cdot \sum D \cdot Q$$

$$Q2 = > 1(A + A') \cdot \sum C \cdot \sum D \cdot Q$$

and $Q$ is a variable width gate, usually set so that the $Q$ trigger rate is several percent of the $\gamma$ trigger rate.

- **R rate trigger** – this trigger selects a random sample of beam particles. If no charged particle trigger occurs during the $Q$ gate, then another gate, called the $R$ gate, is generated. If at least two out of four beam counters fire in coincidence with this gate, an $R$ rate trigger occurs. The width of the $R$ gate is usually adjusted so that the $R$ trigger rate is about 10% of the $\gamma$ trigger rate. R-rate data is used to monitor the pion, muon, and electron contents of the beam, beam counter efficiency, and drift chamber behaviour. It is also used directly in the calculation of RMC rates, since it provides information on the accuracy of the muon STOP scaler, defined by

$$STOP_\mu^- = 1 \cdot 2 \cdot 3 \cdot 4 \cdot RF_\mu \cdot V + A + A' \cdot MV$$

(3.4)

where

$1 \cdot 2 \cdot 3 \cdot 4 = 4$-fold beam counter coincidence

$RF_\mu$ = correct RF separator timing for a muon

$V$ = downstream veto counter

$MV$ = Master Veto – due to equipment faults, computer busy, or low beam rate.

More information on RMC data acquisition and monitoring can be found elsewhere [74].

RMC data for the oxygen, aluminum, and silicon targets was taken in January-February 1992. Data for the titanium, zirconium, and silver targets was taken in May-June 1994. The 1992 data had a faulty discriminator on the $1 \cdot 2 \cdot 3 \cdot 4 \cdot RF_\mu$ Fastbus TDC, but TDC
data was still available due to the acquisition of both CAMAC and Fastbus TDC data for all spectrometer components. Data on tape is unpacked, decoded, and analyzed using the RMCOFIA (Radiative Muon Capture OFfline Interactive Analysis) analysis package developed at TRIUMF. The raw data on tape is first passed through a “skim” analysis in order to reject $\gamma$ events that are unworthy of full analysis due to insufficient or poor drift chamber data. The remaining $\gamma$ events, and the $R$ and $Q$ rate events, are written to three separate tapes for further analysis. The $\gamma$ and $R$ rate analyses are pertinent to the calculation of nuclear RMC, and are discussed in sections 4.2 and 4.3.
Chapter 4

Analysis

4.1 Backgrounds

The goal of the RMC data analysis is to extract, for a given nucleus, an RMC photon spectrum, and to use this spectrum to determine \( R_v \) (the RMC/OMC ratio for \( k > 57 \) MeV) which, when compared to theoretical predictions, gives a value for \( g_P \). In order to extract the RMC photon spectrum, a number of backgrounds must be completely removed by hardware and software cuts, while maintaining good RMC photon acceptance. These backgrounds are

- radiative pion capture (RPC) on the nuclear target, as well as other materials in the spectrometer. Pion capture on the proton at rest has two major branches:

\[
\begin{align*}
\pi^- + p &\rightarrow \pi^0 + n \quad (60.4\%) \\
\pi^0 &\rightarrow \gamma + \gamma \quad (98.8\%) \quad (4.1)
\end{align*}
\]

\[
\begin{align*}
\pi^- + p &\rightarrow \gamma + n \quad (39.6\%) \quad (4.2)
\end{align*}
\]

RPC on nuclei has a branching ratio \( \sim 2 \times 10^{-2} \) per captured pion, whereas RMC has a branching ratio \( \sim 10^{-5} \) per captured muon. The RPC photon spectrum peaks around 115 MeV, but extends down into the observable RMC photon spectrum, so it must be removed without compromising the number of RMC photons. This is achieved by minimizing the RPC background by suppressing the pion content of the beam as outlined in section 3.1, and rejecting photon events that occur in prompt coincidence.
with a beam particle, because RPC is a strong process whereas RMC is weak. A typical nuclear RPC photon spectrum is shown later in this chapter in figure 4.5.

- bremsstrahlung of muon-decay (Michel) electrons, and radiative muon decay (RMD): 
  $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \gamma$. For free muons, both of these processes have a kinematic endpoint of 53 MeV, so this unavoidable background is removed by rejecting photons with energies less than 57 MeV (to allow for spectrometer energy resolution). The observable RMC photon spectrum is then limited to energies greater than 57 MeV. However, muon binding effects and any high-energy tail in the spectrometer response may produce muon decay photons with energies greater than 57 MeV. By simulating the complete muon decay photon spectrum (i.e. radiative muon decay and bremsstrahlung for bound and free muons) in the spectrometer Monte Carlo (see section 4.2.3), the number of muon decay photons with energies greater than 57 MeV is found to be $\sim$0.8% of the observed RMC spectrum for the oxygen target, and $<0.2\%$ of the observed RMC spectra for all other targets considered here. This is consistent with the number of (non-RMC) photons observed above 100 MeV after all cuts and extrapolated back to 57 MeV for each of the targets. Therefore, the Monte Carlo accurately simulates this background, which is significant only for the oxygen target. However, in the case of the oxygen and silicon targets, this background is removed along with RMC photons from non-target materials by running a $\mu^-$ beam on the “empty” targets (see next item and section 5.2). Furthermore, by running a $\mu^+$ beam on a liquid hydrogen target so that all capture processes are “turned off” leaving only muon-decay processes, the contribution to the energy spectrum above 57 MeV due to the high-energy tail in the spectrometer response has been found to be negligible [44]. Moreover, the background from muon decay photons in the observable RMC energy region (>57 MeV) is only significant for the oxygen target, and in this case it is successfully removed.

- RMC on non-target materials, most importantly the target container if the nuclear material under investigation is in liquid or granular form. This background is estimated
by running the $\mu^-$ beam on the empty target container, and observing the number of RMC photons per incident muon.

• cosmic rays. The vast majority of all photons arising due to electromagnetic showers from cosmic ray particles are rejected by software cuts (see section 4.2.1). The remaining cosmic ray background after these cuts has been measured to be $2.3 \pm 0.4$ photons/day in the energy range of interest (57-100 MeV) [44]. Based on the livetimes of the RMC data collections for the targets considered here, this background is 0.23% of the RMC signal for O, and $\leq 0.10\%$ for all other targets. Therefore, the after-cuts cosmic ray background is negligible. Furthermore, in the case of the oxygen and silicon targets, any cosmic ray background is removed along with RMC photons from non-target materials by running a $\mu^-$ beam on the "empty" targets.

4.2 Counting RMC photons — $\gamma$ rate analysis

In order to determine $R_\gamma$, the absolute number of RMC photons and the absolute number of muon stops must be known. The absolute number of RMC photons is found by applying software cuts to the $\gamma$ rate data; by determining the efficiencies of these software cuts (also using $\gamma$ rate data); by finding the absolute photon acceptance of the spectrometer (using Monte Carlo data), and by calibrating this acceptance to a well-known radiative capture process, namely, RPC on carbon.

4.2.1 Software cuts

The software cuts are described in the order in which they are applied to the data:

1. veto cut — reject photons with hits in $A$, $A'$, or $B$, in a 60 ns time window around $STROBE_\gamma$. This cut is a repeat of the hardware veto, Eq. (3.2), and is intended to remove a large number of bremsstrahlung photons arising from Michel electrons.
2. **cstrobe cut** — reject photons that do not fire a $C$ counter. This is necessary in order to remove a large fraction of non-photon events. The photon spectrum from a $\mu^-$ beam incident upon the silicon target, after the veto and cstrobe cuts, is shown in figure 4.1.

3. **tracking cut** — reject photons based on $e^+$ and $e^-$ track fitting parameters. These parameters include the number of points used in the fit, the $\chi^2$ of the fit, the proximity of the track to IWC hits, and the distance from the center of the spectrometer to the apex of the track. This cut improves the photon energy resolution of the spectrometer, and removes many bremsstrahlung and cosmic ray photons.

4. **photon cut** — reject photons based on physical parameters of the observed photon. These parameters include the relative geometries of the Pb converter and the tracks of a $e^+e^-$ pair; the $z$ component (i.e. the component along the beam axis) of the closest distance between the path of the extrapolated photon and the center of the spectrometer; and the difference in the $z$ components of the $e^+$ and $e^-$ momenta. This cut removes photons which have geometrical sources other than the nuclear target. The photon spectrum from a $\mu^-$ beam incident upon the silicon target, after all cuts up to and including the photon cut, is shown in figure 4.1.

5. **random cut** — reject photons constructed from mismatched $e^+$ and $e^-$ tracks. The drift chamber has a "memory" time of 250 ns; hence it is possible for a false photon to be constructed from the $e^+$ track of an asymmetric photon conversion in the Pb converter and the $e^-$ track of a Michel decay or photon conversion in the target, where the $e^-$ track passes through the drift chamber outside the veto cut time window, but inside a window up to 250 ns before and up to 250 ns after $STROBE_\gamma$. Other possibilities for false photon construction involve the $e^+$ track of a photon conversion in the target, and the $e^-$ track from a Michel decay or photon conversion in the Pb converter. Unlike the veto cut, the identities of the $e^+$ and $e^-$ particles are constrained here by the $A$, $A'$, $B$ hit pattern.
Figure 4.1: Photon spectrum from a $\mu^-$ beam incident upon the silicon target, at various levels of software cuts: (a) after all cuts up to and including the strobe (unshaded) and photon cuts (shaded); (b) after all cuts up to and including the cosmic (unshaded) and prompt cuts (shaded). There are no photons below $\sim 30$ MeV because the spectrometer photon acceptance goes to zero here. This is a result of the $\gamma$ trigger condition that at least one of the tracks of a conversion $e^+e^-$ pair reach the $D$ scinillators (see Eq. (3.2)).

6. **cosmic cut** — reject photons induced by cosmic ray particles. These events are primarily identified by hit patterns in the cosmic ray drift chambers, and further identified by hit patterns in the $C$ and $D$ counters (a high energy cosmic ray particle will leave a straight track through the spectrometer, so that opposite $C$ and $D$ counters are likely to fire). The photon spectrum from a $\mu^-$ beam incident upon the silicon target, after all cuts up to and including the cosmic cut, is shown in figure 4.1.

7. **prompt cut** — reject photons from RPC on the target. RPC is a strong process (as opposed to RMC which is weak), so that RPC photons are identified by prompt $\gamma$-beam particle coincidences. The photon spectrum from a $\mu^-$ beam incident upon the silicon target, after all cuts up to and including the prompt cut, is shown in figure 4.1.

8. **energy cut** — after all previous cuts, there is still a large peak centered at 53 MeV in the photon energy spectrum, which is due to bremsstrahlung of Michel electrons,
and radiative muon decay photons. Therefore, all photons with energy less than 57 MeV are rejected (to allow for finite energy resolution), and only the partial RMC photon spectrum above 57 MeV is retained for calculation of results and comparison with theory.

The resulting photon spectrum, shown in figure 4.2, is clearly a pure RMC photon spectrum, although there are a few non-RMC photons at high energy (>100 MeV). These photons are $\mu^-$ decay photons and, as stated in section 4.1, occur in a number which is significant only for the oxygen target in which case they are successfully removed.

### 4.2.2 Cut efficiencies

The software cuts described in section 4.2.1 are designed to completely remove non-RMC photons. In so doing however, they also remove some RMC photons. These RMC photons are accounted for by calculating the following cut efficiencies.
Figure 4.3: Time spectrum of hits in one $A$ counter for events passing all cuts up to the random cut in the RMC on Si data, where 0 ns is the time of $STROBE_\gamma$. Off-time regions $S$ and $T$ are used in the combined $A$, $A'$, and $B$ time spectrum of events passing all cuts up to and including the prompt cut to calculate the efficiency of the veto cut, $\epsilon_v$.

$\epsilon_v$: corrects for RMC photons cut by the hardware and software veto cuts, by looking in two off-time windows on opposite sides of $STROBE_\gamma$ (regions $S$ and $T$ in figure 4.3), each with the same width as the veto cut ($60$ ns), to see if there is a hit in $A$, $A'$, or $B$ (due to Michel electrons, beam particles, or noise) for events passing all cuts up to and including the prompt cut.

$$\epsilon_v = 1 - \frac{S + T}{2 \cdot (# \text{ events})}$$

(4.3)

where $S,T =$ number of events with a hit in $A$, $A'$, or $B$ in the $S,T$ off-time window.

$\epsilon_{cs}$: corrects for good photons cut by the csstrobe cut, because a $C$ counter was inefficient.

$$\epsilon_{cs} = C \text{ counter efficiency} = 0.976 \pm 0.005.$$  

(4.4)

$\epsilon_r$: corrects for RMC photons cut by the random cut by finding the number of prompt photons (which are ultimately cut by the prompt cut) in the energy range $50 - 80 \text{ MeV}$
that fail the random cut. The prompt photons in this energy range are representative of RMC photons because they have energies, or topologies, similar to reconstructed RMC photons.

\[ \epsilon_r = 1 - \left( \frac{\text{# prompt } \gamma, \text{50 - 80 MeV, that are cut}}{\text{# prompt } \gamma, \text{50 - 80 MeV}} \right) \]  

(4.5)

\( \epsilon_c \): corrects for RMC photons cut by the cosmic cut by finding the number of prompt photons (which are ultimately cut by the prompt cut) in the energy range 100 - 140 MeV that fail the cosmic cut. The prompt photons in this energy range consist solely of photons from a single-photon reaction; RPC photons in the 50 - 80 MeV range may come from a two-photon reaction, Eq. (4.1), so even if only one photon is reconstructed in this range, it is likely that opposite \( D \) counters will have fired, to which the cosmic cut is sensitive. Therefore, only the prompt photons in the energy range 100 - 140 MeV are representative of how many RMC photons are cut by the cosmic cut.

\[ \epsilon_c = 1 - \left( \frac{\text{# prompt } \gamma, \text{100 - 140 MeV, that are cut}}{\text{# prompt } \gamma, \text{100 - 140 MeV}} \right) \]

(4.6)

\( \epsilon_p \): corrects for RMC photons cut by the prompt cut. This correction factor is found by fitting the spectrum of photons cut by the prompt cut to the function \( a \cdot RMC + b \cdot RPC \), where \( RMC \) is the final RMC spectrum (i.e., after the energy cut), \( RPC \) is the pure RPC spectrum for the target under investigation, and \( a \) and \( b \) are variable parameters.

\[ \epsilon_p = \left( 1 + a \right)^{-1} \]

(4.7)

If the RMC:RPC and RMC:bremsstrahlung photon ratios in the prompt cut photon spectrum are very small (true for nuclei with \( Z \leq 6 \)), \( \epsilon_p \) is found by doing a Poisson calculation, given a beam rate and the bremsstrahlung photon production rate (i.e., the free muon decay rate). This is how \( \epsilon_p \) is calculated when analyzing RMC runs on the “empty” targets (i.e., the styrofoam and plastic containers).

1-\( \epsilon_i \) is the fraction of RMC photons cut by the \( i^{th} \) cut. Furthermore, \( N_\gamma \) (after all cuts) = \( N_\gamma^{\text{exp}} \) is a number of RMC photons (all non-RMC photons have been removed). Therefore,
application of only one correction factor (e.g., $\epsilon_r$) to the experimental number of RMC photons gives

$$N_{\gamma}^{\text{exp}} = N_{\gamma}^{\text{true}} - N_{\gamma}^{\text{true}}(1 - \epsilon_r) = N_{\gamma}^{\text{true}} \cdot \epsilon_r$$  \hspace{1cm} (4.8)

so

$$N_{\gamma}^{\text{true}} = \frac{N_{\gamma}^{\text{exp}}}{\epsilon_r}. \hspace{1cm} (4.9)$$

After application of $\epsilon_r$, we are still left with a pure number of RMC photons. Application of all the correction factors then yields

$$N_{\gamma}^{\text{true}} = \frac{N_{\gamma}^{\text{exp}}}{\epsilon_v \cdot \epsilon_{cs} \cdot \epsilon_r \cdot \epsilon_c \cdot \epsilon_p}. \hspace{1cm} (4.10)$$

**Note:** when analyzing RPC data (e.g., for the purposes outlined in section 4.2.3), the random, cosmic, prompt, and energy cuts are not used, because the number of random, cosmic, and bremsstrahlung photons in the RPC spectrum is negligible (this is what allows the use of RPC photons in the calculation of $\epsilon_r$ and $\epsilon_c$ above). Furthermore, RPC photons are, by definition, prompt. Therefore, in the RPC case,

$$N_{\gamma, RPC}^{\text{true}} = \frac{N_{\gamma, RPC}^{\text{exp}}}{\epsilon_v \cdot \epsilon_{cs}}. \hspace{1cm} (4.11)$$

### 4.2.3 Absolute photon acceptance

The intrinsic photon acceptance of the RMC spectrometer is found by means of a Monte Carlo simulation, which uses the framework of the CERN software package GEANT [77]. The Monte Carlo simulation is broken down into three stages. First, a specially tailored Monte Carlo program developed at TRIUMF for dealing with beam optics, called REVMOC [78], is used to simulate the muon beam entering the spectrometer. A second Monte Carlo simulation, the target Monte Carlo, uses this beam to determine the muon stopping distribution in the target. The accuracy of REVMOC and the target Monte Carlo is verified by comparing Monte Carlo predicted stopping fractions to experiment. Finally, in the spectrometer Monte Carlo, a theoretical RMC photon spectrum is generated isotropically from the muon stopping distribution in the target. All important aspects of the spectrometer and target,
including geometry, chemical and physical properties of materials, and trigger conditions, are input into the Monte Carlo code. The Monte Carlo generated data is written to disk in YBOS format, and analyzed with RMCOFIA using the identical tracking and photon cuts as were used in the analysis of the experimental RMC data (the Monte Carlo generated data contains only RMC photons, and does not take detector inefficiency into account, so the veto, cstrobe, random, cosmic, and prompt cuts are redundant). The number of Monte Carlo photons after analysis, compared to the original number of photons generated in the target, gives the absolute photon acceptance of the spectrometer, for a given theoretical input RMC photon spectrum.

The calibration of the spectrometer (i.e., the normalization of the absolute photon acceptance, as determined by Monte Carlo, to a well-known radiative process) is achieved using the radiative pion capture photon spectrum from carbon [79]. At the beginning and/or end of each experimental running period for a target, the beam is switched from $\mu^-$ to $\pi^-$, and RPC on C data is taken. This RPC data is analyzed using exactly the same cuts as were used in the corresponding RMC analysis. However, due to the differing geometries of the carbon and RMC targets, the photon cut parameter $z_{close}$ (the z component of the closest approach of the path of the extrapolated photon to the center of the spectrometer) differs between the two analyses. A typical $z_{close}$ distribution, that for RMC (and RPC, due to the small $\pi^-$ content of the $\mu^-$ beam) on Si, is shown in figure 4.4. Also shown in figure 4.4 is the $z_{close}$ distribution with the prompt photons and the “wraparound” photons (one of the tracks of a $e^+e^-$ pair did not reach the radius of the D scintillators) removed. In both the RMC analysis for a given target, and its corresponding RPC on C analysis, the peak in the $z_{close}$ distribution corresponding respectively to RMC or RPC photons originating in the target is fit to a gaussian function, and the acceptable $z_{close}$ window is typically set to the mean $\pm 3\sigma$ of this gaussian (in the case of the titanium target and its corresponding RPC on C analysis, this window is $\pm 2\sigma$). After analysis, the agreement between the shapes of the Monte Carlo and experimental RPC on C photon spectra is excellent. This agreement is shown in figure 4.5 for the RPC on C data taken during the RMC on Si data acquisition.
Figure 4.4: The $z_{\text{close}}$ distribution for RMC (and RPC, due to the small $\pi^-$ content of the $\mu^-$ beam) on silicon. $z_{\text{close}}$ is the $z$ component of the distance of closest approach of the path of the extrapolated photon to the center of the spectrometer. The shaded area excludes prompt photons and "wraparound" photons (one of the tracks of a $e^+e^-$ pair did not reach the radius of the $D$ scintillators).

Figure 4.5: Comparison of the known RPC on C photon spectrum [79] after convolution with the spectrometer Monte Carlo and analysis by RMCOFIA (error bars), to the experimental RPC on C photon spectrum (stars). The spectra have been normalized so that each has the same integral number of counts, and scaled so that the size of the error bars (Poisson statistics) reflects the accuracy of the spectrometer Monte Carlo in reproducing experimental data.
period. The experimental RPC on C branching ratios, calculated for each nuclear RMC target, are in reasonable agreement with the weighted average of three mutually consistent measurements [79, 80]: (1.83 ± 0.06)%. The ratios (listed as “F” in table A.1) of the experimental RPC on C branching ratio values to this given value are applied to the experimental RMC branching ratios in order to calibrate the spectrometer, i.e., in order to account for the small inaccuracy in the absolute photon acceptance of the spectrometer as determined by Monte Carlo.

4.3 Counting $\mu^-$ stops — $R$ rate analysis

The absolute number of muon stops is found from the STOP scaler and associated corrections as determined from $R$ rate data and Monte Carlo simulations of muon and pion beams.

4.3.1 STOP scaler

The muon STOP scaler is constructed from a coincidence between the four beam counters 1·2·3·4 and the RF separator time window for a muon $RF_\mu$, in turn put into coincidence with $MV$ and $V + A + A'$, where $V$ is the downstream veto counter. Therefore, in order to determine the absolute number of muon stops, it is imperative to know the muon tagging accuracy of $RF_\mu$ when presented with the raw pion beam ($\mu^-:\pi^-:e^- \sim 1:1:10$); the muon detection accuracy of 1·2·3·4 when presented with the muon beam ($\mu^-:\pi^-:e^- \sim 1000:1:50$); and the detection efficiencies of $V$, $A$, and $A'$. The determination and application of the relevant correction factors are discussed in section 4.3.2.

4.3.2 Correction factors

$C_{bm}$: corrects for missed stops due to inefficiencies of the beam counters. A term could also be added here to account for the efficiency of the downstream veto counter, but because this efficiency has been measured to be $\sim 97\%$ and the vast majority of muons stop in the target, any correction to the number of stops due to the efficiency of the
downstream veto counter may be neglected.

\[ C_{bm} = \frac{n4 + n3}{n4 + n1} \]  \hspace{1cm} (4.12)

where \( n4 = \# \) of 4-fold beam counter coincidences

\( n3 = \# \) of 3-fold beam counter coincidences

\( n1 = \# \) of instances where a 1-fold beam counter coincidence occurs in the dead time window previous to a 3-fold coincidence. When this happens, the STOP scaler can still see the 4-fold coincidence, (if it occurs), but the 1 • 2 • 3 • 4 • \( RF^\mu \) Fastbus TDC, used in \( R \) rate analysis, cannot.

\( C_{REV MOC} \): corrects for \( \mu^- \) stops which occur in the target container or beam counter 4, and not in the target material itself, as determined by a Monte Carlo simulation of the \( \mu^- \) beam incident upon the entire target (container and all).

\[ C_{REV MOC} = \left( 1 + \frac{N_{cont} + N_{bm4}}{N_{targ}} \right)^{-1} \]  \hspace{1cm} (4.13)

where \( N_{cont} \) = number of stops (per incident beam particle) in the target container

\( N_{bm4} \) = number of stops (per incident beam particle) in beam counter 4

\( N_{targ} \) = number of stops (per incident beam particle) in the target material.

\( C_e \): corrects for over-counting of \( \mu^- \) stops, due to \( e^- \) in the beam. \( e^- \) are identified by \( dE/dx \) in the beam counters, and counted per Fastbus 1 • 2 • 3 • 4 • \( RF^\mu \) TDC entry. This correction involves the relative probabilities for \( e^- \) and \( \mu^- \), of the same momentum, to make a valid STOP. However, these probabilities are equal in first order because, for \( \mu^- \) running, the vast majority of \( \mu^- \) stop in the target, thereby making a valid STOP, and the vast majority of \( e^- \) pass through the target and downstream veto counter, leaving a below-threshold signal in the veto counter, making a valid STOP (Note: for \( \pi^- \) running, the similar masses of the \( \pi^- \) and \( \mu^- \) mean that they also have roughly the
same probability for making a valid STOP).

\[ C_o = \left(1 + \frac{\# e^-}{\# \mu^-}\right)^{-1}. \]  

\( C_u \): corrects for under-counting of \( \mu^- \) stops, due to \( \mu^- \) missing an RF tag. These RF-muons are identified by \( dE/dx \) in the beam counters, and counted per Fastbus 1·2·3·4·RF\(_\mu\) TDC entry.

\[ C_u = \left(1 - \frac{\# \mu^- \text{ missed by RF}}{\# \mu^-}\right)^{-1}. \]  

\( C_m \): corrects for multiple particles per beam burst, i.e., one or more muons are missed due to the presence of another muon. By counting the number of additional Fastbus 1·2·3·4·RF\(_\mu\) entries in 14 subsequent beam buckets, \( n_{\text{mult}} \), per original Fastbus 1·2·3·4·RF\(_\mu\) entry (assuming that all of the additional hits are muons – the beam counter ADC’s are read only once per event, so beam particle identity is unknown for additional beam particles in an R-rate event), the number of additional muons in a beam bucket in which a muon has already been detected is \( n_{\text{mult}}/14 \).

\[ C_m = \left(1 - \frac{\# \text{ additional unseen } \mu^-}{\# \mu^-}\right)^{-1}. \]  

Equivalently, with \( r = \text{beam rate} \), and \( \Delta t = 43 \text{ ns} \) (beam bucket width – see section 3.1),

\[ C_m = \frac{r\Delta t}{1 - e^{-r\Delta t}}. \]  

\( C_{md} \): corrects the number of \( \mu^- \) stops, due to a \( \mu^- \) stop being vetoed by a Michel \( e^- \) hitting the downstream veto counter \( (V) \), or a \( \mu^- \) stop being added by a Michel \( e^- \) returning to fire all four beam counters in coincidence with RF\(_\mu\). This correction is found by doing a Monte Carlo simulation of Michel decays from the \( \mu^- \) stopping distribution in the target (also determined by Monte Carlo).

\[ C_{md} = \left(1 + \frac{(\# \ e^- \text{ enter bm } 4,3,2 \rightarrow 1) - (\# \ e^- \text{ enter } V)}{\# \ e^- \text{ enter A}}\cdot (0.16)\cdot (1 - \epsilon_r)\right)^{-1} \]

where \( V \) is the downstream veto counter, and \( 1 - \epsilon_r \) is the fraction of RMC photons that have a Michel \( e^- \) track pointing to the C counter that fired as a result of the RMC
photon. The Michel $e^-$ is detected if there is a hit in any one of three possible $A$ and $A'$ counters (mapped back from the $C$ counter that fired) in a 500 ns window around $STROBE_{\gamma}$. The factor of 0.16 in Eq. (4.18) arises from the fact that there are a total of 8 $A$ and $A'$ counters, and the fact that $1 \cdot 2 \cdot 3 \cdot 4 \cdot RF_\mu$ and $V + A + A'$ must fall within 30 ns of each other in order to form a STOP $(\frac{8}{3} \cdot \frac{30}{500} = 0.16)$.

Unlike the experimental number of RMC photons after cuts, $N^{exp}_\gamma$, the experimental number of muon stops contains "bad" stops (i.e., non-muon stops which must be removed first).

$$N^{exp}_{\text{stops}} = N_{\mu^-} + N_{e^-} = N_{\mu^-} \left(1 + \frac{N_{e^-}}{N_{\mu^-}}\right)$$

(4.19)

so

$$N_{\mu^-} = \frac{N^{exp}_{\text{stops}}}{1 + N_{e^-}/N_{\mu^-}} = N^{exp}_{\text{stops}} \cdot C_\circ.$$ (4.20)

But

$$N_{\mu^-} = N_{\text{good}} + N_{\text{decays}} = N_{\text{good}} \left(1 + \frac{N_{\text{decays}}}{N_{\text{good}}}\right)$$

(4.21)

so

$$N_{\text{good}} = \frac{N_{\mu^-}}{1 + N_{\text{decays}}/N_{\text{good}}} = N_{\mu^-} \cdot C_{\text{md}}.$$ (4.22)

Then, because $C_{\text{bm}}$, $C_{\text{m}}$, and $C_{\text{u}}$ are defined such that $1-C_{\text{bm}}^{-1}$, $1-C_{\text{m}}^{-1}$, and $1-C_{\text{u}}^{-1}$ represent fractions of good muon stops lost,

$$N^{\text{true}}_{\text{stops,all}} = N_{\text{good}} \cdot C_{\text{bm}} \cdot C_{\text{m}} \cdot C_{\text{u}}.$$ (4.23)

But

$$N^{\text{true}}_{\text{stops,all}} = N_{\text{targ}} + N_{\text{cont}} + N_{\text{bm4}} = N_{\text{targ}} \left(1 + \frac{N_{\text{cont}} + N_{\text{bm4}}}{N_{\text{targ}}}\right)$$

(4.24)

so

$$N_{\text{targ}} = N^{\text{true}}_{\text{stops}} = \frac{N^{\text{true}}_{\text{stops,all}}}{1 + (N_{\text{cont}} + N_{\text{bm4}})/N_{\text{targ}}} = N^{\text{true}}_{\text{stops,all}} \cdot C_{\text{REVMOC}}.$$ (4.25)

Combining Eqs. (4.20), (4.22), (4.23), and (4.25),

$$N^{\text{true}}_{\text{stops}} = N^{\text{exp}}_{\text{stops}} \cdot C_{\text{bm}} \cdot C_{\text{REVMOC}} \cdot C_\circ \cdot C_{\text{u}} \cdot C_{\text{m}} \cdot C_{\text{md}}.$$ (4.26)
Chapter 4. Analysis

Note: in the analysis of RPC data, where the number of pion stops is used, the correction factors to the pion STOP scaler data are analogous to those above for the muon STOP scaler data, except that $RF_\mu$ is replaced by $RF_\pi$, and $C_{md}$ is replaced by $C_{pd}$.

$C_{pd}$: corrects the number of $\pi^-$ stops, where the $\pi^-$ decays before stopping in the target, but the STOP definition is still satisfied ($\tau_{\pi^-} = 26$ ns, whereas $\tau_{\mu^-} = 2.2$ $\mu$s, so this correction is insignificant for $\mu^-$ beam). This correction factor is determined by means of a Monte Carlo simulation of the $\pi^-$ beam incident upon the target.

$$C_{pd} = \left(1 + \frac{\text{# $\mu^-$ stopping in target or $bm4$}}{\text{# $\pi^-$ stopping in target}}\right)^{-1}. \quad (4.27)$$

The derivation of the true number of $\pi^-$ stops in RPC data is similar to the above for $\mu^-$ stops in RMC data, and is given by

$$N_{\pi^-\text{stops}}^{\text{true}} = N_{\pi^-\text{stops}}^{\text{exp}} \cdot C_{bm} \cdot C_{REVMOG} \cdot C_0 \cdot C_u \cdot C_m \cdot C_{pd}. \quad (4.28)$$

4.4 Addition of uncertainties

The $\epsilon_i$ efficiencies are all statistically independent; therefore, their uncertainties are added in quadrature when calculating $R_\gamma$. The $\epsilon_i$ involve quantities of the form $X = n_1/N$, where $n_1$ is a subset of $N$, with $n_1 + n_2 = N$. The uncertainty in $X$, $dX$, is easily calculated to be

$$dX = X \left(\frac{1}{n_1} - \frac{1}{N}\right)^{1/2}. \quad (4.29)$$

$C_{REVMOG}$ and $C_{pd}$ are calculated by independent Monte Carlo simulations, and are statistically independent of all other correction factors. $C_{md}$ is found by doing a Monte Carlo simulation and by using the value of $\epsilon_r$; hence the uncertainty to be added in quadrature is

$$(d[\epsilon_r \cdot C_{md}])^2 = (C_{md} \cdot d\epsilon_r + \epsilon_r \cdot dC_{md})^2. \quad (4.30)$$

d$C_{pd}$ and $dC_{md}$ are found by varying the $z$ location of the target in their respective Monte Carlo simulations. $dC_{REVMOG}$ is found by comparing Monte Carlo data to experimental data for runs involving empty target containers, and determining the accuracy with which
the Monte Carlo predicts the detection efficiency of the downstream veto counter (which is independent of target). $C_0$, $C_u$, and $C_m$ are not statistically independent of one another (unless, as in the case of the O, Al, and Si data, $C_0$ and $C_u$ are calculated using CAMAC TDC data, but $C_m$ is still calculated using Fastbus TDC data, due to a faulty discriminator on the $1 \cdot 2 \cdot 3 \cdot 4 \cdot RF_\mu$ Fastbus TDC – see section 3.4. In this case, $C_m$ is independent of all other correction factors, but $C_0$ and $C_u$ are not independent of each other). The uncertainty that is added in quadrature to the other uncertainties involved in calculating $R_\gamma$ is

\[ (d[C_0 \cdot C_u \cdot C_m])^2 = (C_u \cdot C_m \cdot dC_0 + C_0 \cdot C_m \cdot dC_u + C_0 \cdot C_u \cdot dC_m)^2. \] (4.31)

$C_u$ involves a quantity of the form $Y = n_1/n_2$, with $n_1 + n_2 = N$. In this case,

\[ dY = Y \left( \frac{1}{n_1} + \frac{1}{n_2} + \frac{2N}{n_2^2} \right)^{1/2}. \] (4.32)

The uncertainty in the absolute photon acceptance depends on the form of the RMC photon spectrum input in the Monte Carlo. If the spectrum is taken bin by bin from theory, then the uncertainty is found by varying the Monte Carlo determined muon stopping distribution in the target. The two extremes of a point stopping distribution in the middle of the target, and a uniform stopping distribution throughout the target, are used to generate photons in the spectrometer Monte Carlo. This was done explicitly for the Al and Si targets (and C for RPC), and the resulting uncertainty due to the accuracy of the Monte Carlo determination of the muon stopping distribution was found to be $\leq 2\%$. An uncertainty of 2% was then applied to the absolute photon acceptance of the other targets. Because these two extremes in the muon stopping distribution are unrealistic, the uncertainty in the absolute photon acceptance is over- rather than underestimated.

If the RMC photon spectrum is in analytic form with a variable parameter (specifically, $k_{\max}$ in the Primakoff polynomial, given in section 5.1), then the uncertainty in the absolute photon acceptance has a second contribution which is estimated by determining the values of the absolute photon acceptance when the high and low limits on $k_{\max}$ are used for the RMC spectrum input in the Monte Carlo. The determination of the limits on $k_{\max}$ is given in section 5.1.
Chapter 5

Results

5.1 Calculation of $R_\gamma$ and $g_P/g_A$

Two methods are used for the extraction of $R_\gamma$ and $g_P/g_A$ from the experimental RMC data: the integral method and the shape method. The integral method is used when the nuclear RMC response has been calculated for the nucleus under investigation, and gives experimental values for both $R_\gamma$ and $g_P/g_A$. This is the case for O, Ti, Zr, and Ag. The shape method is used in all cases, and only gives values for $R_\gamma$.

The integral method involves passing the theoretical RMC spectrum through the Monte Carlo, normalizing the number of generated photons (i.e., the number of photons input into the Monte Carlo) to the experimental number of captured muons (i.e., the corrected number of muon stops multiplied by the OMC capture fraction – see table 5.1), analyzing the resulting spectrum with RMCOFIA, and obtaining a predicted number of observed RMC photons above 57 MeV as a function of both the theoretically calculated $R_\gamma$ and $g_P/g_A$. These functions are fit to simple polynomials. The intersection of the polynomial functions with the experimental number of RMC photons above 57 MeV gives experimental values for $R_\gamma$ and $g_P/g_A$.

The shape method involves passing a representative theoretical RMC photon spectral shape through the Monte Carlo in order to determine the photon acceptance of the spectrometer and thus a value for $R_\gamma$. The spectral shape used is the closure or Primakoff [66].
Table 5.1: OMC capture fractions and mean muon capture lifetimes in nuclear targets [81].

<table>
<thead>
<tr>
<th>Element</th>
<th>$f_{\text{capture}}$</th>
<th>$\tau$ (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.0777 ± 0.0007</td>
<td>2026.3 ± 1.5</td>
</tr>
<tr>
<td>O</td>
<td>0.1844 ± 0.0009</td>
<td>1795.4 ± 2.0</td>
</tr>
<tr>
<td>Al</td>
<td>0.6095 ± 0.0005</td>
<td>864.0 ± 1.0</td>
</tr>
<tr>
<td>Si</td>
<td>0.6587 ± 0.0005</td>
<td>756.0 ± 1.0</td>
</tr>
<tr>
<td>Ti</td>
<td>0.8530 ± 0.0006</td>
<td>329.3 ± 1.3</td>
</tr>
<tr>
<td>Zr</td>
<td>0.9529 ± 0.0004</td>
<td>110.0 ± 1.0</td>
</tr>
<tr>
<td>Ag</td>
<td>0.9634 ± 0.0006</td>
<td>87.0 ± 1.5</td>
</tr>
</tbody>
</table>

The polynomial (see section 2.2.2), given by

\[
\frac{dG(x)}{dk} \sim (1 - 2x + 2x^2)x(1 - x^2) \tag{5.1}
\]

where $\frac{dG(x)}{dk}$ is the differential RMC photon spectrum; and $x = k/k_{\text{max}}$, where $k$ is the RMC photon energy and $k_{\text{max}}$ is the maximum RMC photon energy. $k_{\text{max}}$ is directly related to $E_{av}$, discussed in section 2.2.2. The value of $k_{\text{max}}$ chosen for a given nucleus is that for which the Primakoff spectrum, after Monte Carlo convolution and RMCOFIA analysis, gives the best fit to the experimental RMC spectrum. The uncertainty in $k_{\text{max}}$, which contributes to the uncertainty in the absolute photon acceptance (see section 4.4), is found by normalizing each Monte Carlo spectrum to the experimental spectrum (i.e., multiplying each Monte Carlo spectrum by a factor such that the Monte Carlo and experimental spectra have the same integrated number of photons); fitting each normalized Monte Carlo spectrum to the experimental spectrum and obtaining a value for the $\chi^2$ goodness-of-fit statistic; plotting $\chi^2$ vs. $k_{\text{max}}$ and fitting this to a simple polynomial; and extracting the uncertainty in $k_{\text{max}}$ using the fact that for a single free parameter, the 68.3% confidence interval is in the region $\Delta\chi^2 = 1$ [82]. The $\chi^2$ vs. $k_{\text{max}}$ curve for the oxygen target is shown in figure 5.1. After RMCOFIA analysis, the number of RMC photons above 57 MeV, compared to the original number of Primakoff photons above 57 MeV generated in the Monte Carlo, gives the absolute RMC photon acceptance of the spectrometer. Writing the photon acceptance as $\text{Acc}$, we
Chapter 5. Results

Figure 5.1: $\chi^2$ vs. $k_{max}$ for fits of Primakoff spectra generated by Monte Carlo to the experimental RMC on O spectrum. The solid curve is a quadratic fit to the data points.

have

$$\text{Acc} \cdot \sum_{j>57\text{MeV}} G_j = \sum_{i>57\text{MeV}} \sum_j A(i,j) \cdot G_j$$  \hspace{1cm} (5.2)

where $G_j$ is the physical RMC photon energy spectrum in units of photons/MeV/capture, and $A(i,j)$ is the absolute probability of a photon of energy $E_j$ being reconstructed at an energy $E_i$. The experimentally observed “true” photon energy spectrum from RMC, $N_{\gamma,i}^{\text{true}}$, is related to the physical photon energy spectrum, $G_j$, by

$$N_{\gamma,i}^{\text{true}} = N_{\text{stops}}^{\text{capture}} \cdot \sum_j A(i,j) \cdot G_j$$  \hspace{1cm} (5.3)

where $f_{\text{capture}}$ is the fraction of muons that undergo OMC in the target (see table 5.1). Summing both sides of Eq. (5.3) over $i > 57$ MeV, and using Eq. (5.2), gives

$$R_{\gamma} = \sum_{j>57\text{MeV}} G_j = \frac{N_{\gamma>57}^{\text{true}}}{f_{\text{capture}} \cdot \text{Acc} \cdot N_{\text{stops}}^{\text{true}}}$$  \hspace{1cm} (5.4)

where $R_{\gamma}$ is the $k > 57$ MeV RMC/OMC ratio. In terms of experimentally observed numbers of photons and stops, this is

$$R_{\gamma} = \frac{N_{\gamma>57}^{\text{exp}}}{f_{\text{capture}} \cdot \text{Acc} \cdot N_{\text{tries}}}$$  \hspace{1cm} (5.5)
where, using Eqs. (4.10) and (4.26),

$$N_{\text{tries}} = N_{\text{stops}}^{\exp} \cdot \epsilon_v \cdot \epsilon_{cs} \cdot \epsilon_r \cdot \epsilon_c \cdot C_{bm} \cdot C_{REVMOOC} \cdot C_o \cdot C_u \cdot C_m \cdot C_{pd}.$$  

(5.6)

$N_{\text{tries}}$ is defined such that $N_{\text{tries}} \cdot f_{\text{capture}}$ is the number of Monte Carlo photon “tries” that reduces to the observed experimental number of photons after Monte Carlo convolution and RMCOFIA analysis. The calibration of the spectrometer is then taken into account by multiplying $R_\gamma$ by $F$ (see section 4.2.3 and appendix A). **Note:** in RPC analysis, the RPC spectrum is observed at all energies. Therefore, the RPC branching ratio is given by

$$R_{RPC} = \frac{N_{\gamma,RPC}^{\exp}}{Acc \cdot N_{\pi-\text{tries}}}$$  

(5.7)

where, using Eqs. (4.11) and (4.28),

$$N_{\pi-\text{tries}} = N_{\pi-\text{stops}}^{\exp} \cdot \epsilon_v \cdot \epsilon_{cs} \cdot C_{bm} \cdot C_{REVMOOC} \cdot C_o \cdot C_u \cdot C_m \cdot C_{pd}.$$  

(5.8)

## 5.2 O, Al, Si, and Ti

No theoretical RMC photon spectra, with associated values of $R_\gamma$ and $g_P/g_A$, are currently available for aluminum or silicon nuclei (the theory of Fearing and Welsh [70] is only realistic for heavy nuclei). Therefore, the shape method is used to determine $R_\gamma$, and values for $g_P/g_A$ are not obtained for Al and Si. However, three calculations of RMC on O [58, 68, 69] are available for direct comparison with experiment. Similarly, three calculations of RMC on Ca [58, 67, 69] are available for comparison with RMC on Ti data (assuming that the nuclear response of Ti is similar to that of Ca). Therefore, the integral method is used to obtain values of $g_P/g_A$, and both the integral and shape methods are used to obtain values of $R_\gamma$, for O and Ti. In the special case of Roig and Navarro [69], RMC photon spectra are not available, so a modified integral method is used to determine values of $g_P$: theoretical values of $R_\gamma$ are plotted vs. $g_P$ and fit to a simple polynomial; an experimental value for $g_P$ is found from the intersection of this polynomial function with the experimental value of $R_\gamma$ as determined by the shape method. The nuclear RMC models used for O and Ca are discussed in section 2.2.2.
Chapter 5. Results

All quantities involved in the calculation of $R_\gamma$ and $g_P/g_A$ for O, Al, Si, and Ti are shown in appendix A, table A.1. As stated in section 3.2, the Si target is granular, and held in place by a polypropylene, $(C_3H_6)_n$, and mylar, $(C_5H_4O_2)_n$, container. The oxygen target is composed of liquid D$_2$O, held in place by a polyethylene bag, $(CH_2)_n$, and a lucite ring, $(C_5H_8O_2)_n$. Because the muon capture lifetimes in C, O, and Si are similar (see table 5.1), RMC data was taken using the empty D$_2$O and Si targets, to determine the fraction of RMC photons coming from the container in each case. The number of RMC photons from the empty target is normalized to the number of $1 \cdot 2 \cdot 3 \cdot 4 \cdot RF_\mu \cdot MV$ hits for the full and empty targets, and corrected (via Monte Carlo) for the number of muons which stop in the container when the container is full as opposed to empty. The resulting fraction of RMC photons from the target container is shown in table A.1 as $empfrac$. The aluminum target is composed of aluminum plates, and the titanium target of metallic Ti shavings, both held in place by styrofoam target holders. Because the styrofoam is of very low density, its RMC photon contribution is negligible.

The photon acceptance, given as $Acc$ in table A.1, is that found from the shape method. Comparisons of the Primakoff RMC spectra (after Monte Carlo convolution and RMCOFIA analysis) to the experimental RMC spectra for Si, Al, O, and Ti are shown in figures 5.2 and 5.3. Values of $R_\gamma$ and $g_P/g_A$, obtained for O and Ti using the integral method, are read directly off figure 5.4. Values of $g_P$ for O and Ti, obtained using $R_\gamma$ as determined by the shape method, are read directly off figure 5.5. Comparisons of the theoretical RMC on O and Ca spectra (after Monte Carlo convolution and RMCOFIA analysis) to the experimental RMC on O and Ti photon spectra are shown in figures 5.6 and 5.7. Final numerical results are shown in table 5.2.
Figure 5.2: Comparison of the experimental RMC on Si and Al photon spectra with the closure spectral shapes given by Eq. (5.1) after these shapes have been convoluted by the spectrometer Monte Carlo, and analyzed by RMCOFIA. The solid line in each figure is the spectral shape for the best fit value of $k_{\text{max}}$. 

**Silicon**

<table>
<thead>
<tr>
<th>$k_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a 85 MeV</td>
</tr>
<tr>
<td>b 88 MeV</td>
</tr>
<tr>
<td>c 95 MeV</td>
</tr>
</tbody>
</table>

**Aluminum**

<table>
<thead>
<tr>
<th>$k_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a 87 MeV</td>
</tr>
<tr>
<td>b 88 MeV</td>
</tr>
<tr>
<td>c 89 MeV</td>
</tr>
</tbody>
</table>
Figure 5.3: Comparison of the experimental RMC on O and Ti photon spectra with the closure spectral shapes given by Eq. (5.1) after these shapes have been convoluted by the spectrometer Monte Carlo, and analyzed by RMCOFIA. The solid line in each figure is the spectral shape for the best fit value of $k_{\text{max}}$. 
Figure 5.4: Number of RMC photons above 57 MeV, $N_\gamma$, vs. $R_\gamma$ and $g_P/g_A$ for the RMC on oxygen theories of Gmitro et al. (GOT) [58] and Christillin and Gmitro (CG) [68], and the RMC on Ca theories of Gmitro et al. (GOT) [58] and Christillin (C) [67]. The hatched regions are the experimental number of RMC photons above 57 MeV for oxygen and Ti.
Figure 5.5: $R_x$ vs. $g_P/g_A$ for the RMC on O and Ca theory of Roig and Navarro (RN) [69]. The hatched regions are the experimental values of $R_x$ for O and Ti as determined by the shape method.
Figure 5.6: Comparison of the experimental RMC on O photon spectrum with the theories of Gmitro et al. (GOT) [58] and Christillin and Gmitro (CG) [68]. The theoretical spectra have been convoluted by the spectrometer Monte Carlo, analyzed by RMCOFIA, and normalized to reflect the results in figure 5.4.
Figure 5.7: Comparison of the experimental RMC on Ti photon spectrum with the RMC on Ca theories of Gmitro et al. (GOT) [58] and Christillin (C) [67]. The theoretical spectra have been convoluted by the spectrometer Monte Carlo, analyzed by RMCOFIA, and normalized to reflect the results in figure 5.4.
5.3 Zr and Ag

Calculations of RMC on Mo and Sn [55] and calculations of RMC on Zr and Ag [70] are available for comparison with experimental RMC on Zr and Ag data (assuming that the nuclear responses of Mo and Sn [55] are similar to those of Zr and Ag). Either theory may be used to determine experimental values of $R_\gamma$ and $g_P/g_A$ for Zr and Ag using the integral method, but only the Mo and Sn theory of Christillin et al. [55] is used, for the reasons discussed in section 2.2.2 and in order to compare with previous results [44].

All quantities involved in the calculation of $R_\gamma$ and $g_P/g_A$ for Zr and Ag are shown in appendix A, table A.1. The photon acceptance, given as $Acc$ in table A.1, is that found from the shape method. Comparisons of the Primakoff RMC spectra (after Monte Carlo convolution and RMCOFIA analysis) to the experimental RMC spectra for Zr and Ag are shown in figure 5.8. Values of $R_\gamma$ and $g_P/g_A$, obtained for Zr and Ag using the integral method, are read directly off figure 5.9. Comparisons of the theoretical RMC on Mo and Sn spectra (after Monte Carlo convolution and RMCOFIA analysis) to the experimental RMC on Zr and Ag photon spectra are shown in figure 5.10. Final numerical results are shown in table 5.3.
Figure 5.8: Comparison of the experimental RMC on Zr and Ag photon spectra with the closure spectral shapes given by Eq. (5.1) after these shapes have been convoluted by the spectrometer Monte Carlo, and analyzed by RMCOFIA. The solid line in each figure is the spectral shape for the best fit value of $k_{\text{max}}$. 
Figure 5.9: Number of RMC photons above 57 MeV, $N_\gamma$, vs. $R_\gamma$ and $g_P/g_A$ for the RMC on Mo and Sn theory of Christillin et al. (CRS) [55]. The hatched regions are the experimental number of RMC photons above 57 MeV for Zr and Ag.
Figure 5.10: Comparison of the experimental RMC on Zr and Ag photon spectra with the RMC on Mo and Sn theory of Christillin et al. (CRS) [55]. The theoretical spectra have been convoluted by the spectrometer Monte Carlo, analyzed by RMCOFIA, and normalized to reflect the results in figure 5.9.
Table 5.2: Experimental values of $R_t$, $g_P/g_A$, and $k_{max}$ for O, Al, Si, and Ti.

<table>
<thead>
<tr>
<th>Z</th>
<th>$R_t$ ($10^{-5}$)</th>
<th>$g_P/g_A$</th>
<th>$k_{max}$ (MeV)</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1.47 ± 0.05</td>
<td>5.1$^{+0.9}_{-1.2}$</td>
<td>—</td>
<td>Gmitro et al. [58]</td>
</tr>
<tr>
<td></td>
<td>1.37 ± 0.06</td>
<td>4.1 ± 0.2</td>
<td>—</td>
<td>Christillin and Gmitro [68]</td>
</tr>
<tr>
<td></td>
<td>1.61 ± 0.16</td>
<td>—</td>
<td>87.1 ± 1.9</td>
<td>Primakoff [66]</td>
</tr>
<tr>
<td></td>
<td>1.61 ± 0.16</td>
<td>6.0$^{+1.0}_{-1.1}$</td>
<td>—</td>
<td>Roig and Navarro [69]</td>
</tr>
<tr>
<td>Al</td>
<td>13</td>
<td>1.36 ± 0.12</td>
<td>—</td>
<td>Primakoff [66]</td>
</tr>
<tr>
<td>Si</td>
<td>14</td>
<td>1.96 ± 0.20</td>
<td>—</td>
<td>Primakoff [66]</td>
</tr>
<tr>
<td>Ti</td>
<td>22</td>
<td>1.42 ± 0.04</td>
<td>&lt; 0</td>
<td>Gmitro et al. [58]</td>
</tr>
<tr>
<td></td>
<td>1.19 ± 0.07</td>
<td>&lt; 0</td>
<td>—</td>
<td>Christillin [67]</td>
</tr>
<tr>
<td></td>
<td>1.28 ± 0.10</td>
<td>—</td>
<td>88.1 ± 2.0</td>
<td>Primakoff [66]</td>
</tr>
<tr>
<td></td>
<td>1.28 ± 0.10</td>
<td>2.9$^{+0.8}_{-0.9}$</td>
<td>—</td>
<td>Roig and Navarro [69]</td>
</tr>
</tbody>
</table>

Table 5.3: Experimental values of $R_t$, $g_P/g_A$, and $k_{max}$ for Zr and Ag.

<table>
<thead>
<tr>
<th>Z</th>
<th>$R_t$ ($10^{-5}$)</th>
<th>$g_P/g_A$</th>
<th>$k_{max}$ (MeV)</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zr</td>
<td>40</td>
<td>1.28 ± 0.07</td>
<td>—</td>
<td>Christillin et al. [55]</td>
</tr>
<tr>
<td></td>
<td>1.28 ± 0.07</td>
<td>—</td>
<td>87.1 ± 2.0</td>
<td>Prima koff [66]</td>
</tr>
<tr>
<td></td>
<td>1.26 ± 0.11</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Ag</td>
<td>47</td>
<td>1.21 ± 0.06</td>
<td>2.1$^{+0.8}_{-0.7}$</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>1.16 ± 0.10</td>
<td>—</td>
<td>87.3 ± 1.9</td>
<td>Christillin et al. [55]</td>
</tr>
<tr>
<td></td>
<td>1.16 ± 0.10</td>
<td>—</td>
<td>—</td>
<td>Prima koff [66]</td>
</tr>
</tbody>
</table>
Due to the model dependency of nuclear RMC (see table 5.2), the final $R_\gamma$ results of the present study are those found using the closure or Primakoff polynomial (i.e., the shape method described in section 5.1). These results, along with previous results also obtained in the closure model, are presented in table 6.1. The widely different values of $R_\gamma$ for Si and Al, as observed by Armstrong et al. [44], are confirmed by the present results. Although Armstrong et al. and the present experiment used the same experimental apparatus, this agreement is a validation of the experimental and analytical procedures of each, because Armstrong et al. used a $2D$ photon trigger (see Eq. (3.2)), whereas the present experiment used a $\geq 1D$ trigger. Furthermore, Armstrong et al. used an analytic form for their detector response and acceptance, whereas in the present experiment, the detector acceptance was modelled independently for each nuclear target in the Monte Carlo. The $\sim 3\sigma$ difference between the values of $R_\gamma$ for Al and Si is qualitatively explained by Pauli blocking of the final state neutron, which reduces the available phase space for both RMC and OMC. RMC, with its 3-body final state, is suppressed more than OMC; therefore, a decrease in $R_\gamma$ (= RMC/OMC) is expected for nuclei such as Al which have neutron excesses. This "isotope effect" was first noted by Primakoff [66]. Ti also has a neutron excess (whereas Ca does not), so the $\sim 4\sigma$ difference between the present $R_\gamma$ result for Ti, measured here for the first time, and the previous result for Ca (see table 6.1) is a further example of this isotope effect.
Phase space arguments also account for the decreasing trend of $R_\gamma$ with $Z$, shown in figure 6.1. As $Z$ increases, the neutron Fermi momentum becomes significantly larger than that of the proton, resulting in less available phase space for RMC. The present $R_\gamma$ results for Zr and Ag, measured here for the first time, are consistent with previous results for other high $Z$ nuclei on the $R_\gamma$ vs $Z$ plot.

The present $R_\gamma$ result for O is $\sim 25\%$ lower than the previous results of Armstrong et al. [43] (see tables 6.1 and 6.2) and Döbeli et al. [41] (see table 6.2), and is $\sim 60\%$ lower than the previous result of Frischknecht et al. [42] (see table 6.2). Although Armstrong et al. also did their work at TRIUMF, their C, O, and Ca experiment utilized a time projection chamber (TPC) in place of the present drift chamber (section 3.3.1), and they found a beam rate-dependency in their photon acceptance (an exponential decrease in acceptance with particle flux in the TPC). They were only able to observe the functionality of this rate dependence for RPC on C and RMC on Ca, because the small RMC rates for C and O make observation of the beam rate-dependence of the photon acceptance impractical. However, they were able to simulate the observed rate-dependence of the RPC on C and RMC on Ca acceptances in their Monte Carlo. Therefore, their Monte Carlo was used to correct the photon acceptance in the C and O cases, and their $R_\gamma$ result for O agrees with the previous result of Döbeli et al. [41]. Nevertheless, it may be possible that they overestimated the effect of the beam rate on the photon acceptance in the C and O cases, arriving at acceptance values that are too low and corresponding $R_\gamma$ values that are too high.

Experimental values of $R_\gamma$ are plotted vs. neutron excess, $\alpha = (N - Z)/A$, in figure 6.2. The decrease in $R_\gamma$ with $\alpha$ is explained above by phase space arguments which result from Pauli blocking of the final state neutron. The low value of $R_\gamma$ for O compared to that of Si or Ca (i.e., the other $Z = N$ ($\alpha = 0$) nuclei) is qualitatively explained by the larger energy gap between the filled proton shell and the empty neutron shell for O compared to that of Si or Ca. This energy gap reduces the available phase space for muon capture.

Theoretical predictions of $R_\gamma$ vs. $Z$ are compared to experimental results in figure 6.3. In figure 6.3(a), the theory of Christillin et al. [55] agrees well with the experimental $R_\gamma$ vs.
Z plot for $Z \geq 40$ and $g_P/g_A = 0$. The theories of Christillin [67] and Christillin and Gmitro [68] are included for comparison with the low $Z$ results, and there is an intriguing agreement between experiment and theory for both the $g_P/g_A = 0$ and $g_P/g_A = 6.8$ curves, even though these two theoretical curves have distinctly different shapes at low $Z$. In figure 6.3(b), the theory of Fearing and Welsh [70] reproduces the shape of the experimental $R_\gamma$ vs. $Z$ plot for $Z \geq 40$, but only after the theoretical values of $R_\gamma$ are scaled by 0.38. Comparison between theory and experiment for $Z < 20$ is included in the figure, but the model of Fearing and Welsh is not expected to be applicable for these nuclei [70].

<table>
<thead>
<tr>
<th>$Z$</th>
<th>$\alpha$</th>
<th>$R_\gamma \times 10^{-5}$</th>
<th>$k_{max}$ (MeV)</th>
<th>$\chi^2$/d.o.f.</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>0</td>
<td>1.61 ± 0.16</td>
<td>87.1 ± 1.9</td>
<td>1.46</td>
<td>present work</td>
</tr>
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<td>Al</td>
<td>0.0364</td>
<td>1.36 ± 0.12</td>
<td>88.5 ± 1.2</td>
<td>1.58</td>
<td>present work</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.43 ± 0.13</td>
<td>90 ± 2</td>
<td>1.12</td>
<td>Armstrong et al. [44]</td>
</tr>
<tr>
<td>Si</td>
<td>0.00304</td>
<td>1.96 ± 0.20</td>
<td>87.8 ± 1.7</td>
<td>1.91</td>
<td>present work</td>
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<tr>
<td></td>
<td></td>
<td>1.93 ± 0.18</td>
<td>92 ± 2</td>
<td>1.73</td>
<td>Armstrong et al. [44]</td>
</tr>
<tr>
<td>Ca</td>
<td>0.00195</td>
<td>2.09 ± 0.19</td>
<td>93 ± 2</td>
<td>1.56</td>
<td>Armstrong et al. [44]</td>
</tr>
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<td>Ti</td>
<td>0.0810</td>
<td>1.28 ± 0.10</td>
<td>88.1 ± 2.0</td>
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<td>present work</td>
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<td>0.123</td>
<td>1.26 ± 0.11</td>
<td>87.1 ± 2.0</td>
<td>1.31</td>
<td>present work</td>
</tr>
<tr>
<td>Mo</td>
<td>0.124</td>
<td>1.11 ± 0.11</td>
<td>90 ± 2</td>
<td>0.82</td>
<td>Armstrong et al. [44]</td>
</tr>
<tr>
<td>Ag</td>
<td>0.129</td>
<td>1.16 ± 0.10</td>
<td>87.3 ± 1.9</td>
<td>1.01</td>
<td>present work</td>
</tr>
<tr>
<td>Sn</td>
<td>0.158</td>
<td>0.98 ± 0.09</td>
<td>87 ± 2</td>
<td>1.14</td>
<td>Armstrong et al. [44]</td>
</tr>
<tr>
<td>Pb</td>
<td>0.208</td>
<td>0.60 ± 0.07</td>
<td>84 ± 3</td>
<td>0.85</td>
<td>Armstrong et al. [44]</td>
</tr>
</tbody>
</table>

Table 6.1: Values of $R_\gamma$, $k_{max}$, and the corresponding $\chi^2$ of fit as determined by the shape method (closure model). $\alpha = (N - Z)/A$ is the neutron excess, where $A$ is the atomic mass of the natural element [1], and $N = A - Z$. 
Chapter 6. Discussion

Figure 6.1: $R_\gamma$ vs. $Z$. Plotted values are from table 6.1, where the solid circles are the present closure model results, and the solid squares are previous closure model results [44].

Figure 6.2: $R_\gamma$ vs. $\alpha$ (neutron excess). Plotted values are from table 6.1, where the solid circles are the present closure model results, and the solid squares are previous closure model results [44].
Figure 6.3: Experimental $R_\gamma$ vs. $Z$ compared with (a) the theories of Christillin et al. [55] ($Z \geq 40$), Christillin [67] ($Z = 20$), and Christillin and Gmitro [68] ($Z = 8$); and (b) the theory of Fearing and Welsh [70], scaled by 0.38. Theoretical values of $R_\gamma$ for $Z < 40$ are shown in (b) even though the model of Fearing and Welsh is not expected to be applicable for such nuclei. Experimental values of $R_\gamma$ are shown by the solid circles (present closure model results) and solid squares (previous closure model results [44]), and are taken from table 6.1.
Chapter 6. Discussion

6.2 \( g_P \)

The \( g_P/g_A \) results of this and other RMC experiments are shown in table 6.2 and plotted vs. atomic number in figure 6.4. The model dependence of nuclear RMC is clear in the cases of O and Ca, and indicates that nuclear RMC must have a better theoretical understanding before experiment can make any tests of PCAC. The present Ti, Zr, and Ag results indicate that \( g_P \) may be quenched to a large degree in nuclei, although the \( g_P/g_A \) values for Ti cannot be taken too seriously, as nuclear responses for Ca were adopted in this case, and the assumption that the nuclear responses of Ca and Ti are similar, at least with respect to RMC, appears to be invalid when one considers the large difference in their \( R_y \) values (see section 6.1). The present \( g_P/g_A \) values for O are marginally lower than the Goldberger-Treiman value, and only display a slight model dependence. Previous \( g_P/g_A \) values from RMC on O are significantly higher, and, in the case of Armstrong et al. [43], are strongly model-dependent. The discrepancy between the present and previous [43] results as extracted from RMC on O is discussed in section 6.1.

In figure 6.4, the decrease in \( g_P/g_A \) with Z for \( Z \geq 40 \) is model-dependent, and merely reflects the Z-dependent quenching of \( g_P \) present in the Fermi gas model of Christillin et al. [55] (this model was used to extract values of \( g_P/g_A \) for all of the \( Z \geq 40 \) results in figure 6.4). However, in the model of Christillin et al., the values of \( g_P/g_A \) that are required to reproduce the experimental values of \( R_y \) (i.e., the values of \( g_P/g_A \) in table 6.2 for \( Z \geq 40 \)) do not agree with the values of \( g_P/g_A \) required to reproduce the well-known OMC rates [81]. Also, although both Christillin et al. and Fearing and Welsh [70] are able to reproduce the shape of the \( R_y \) vs. \( Z \) plot for \( Z \geq 40 \) (see figure 6.3), Fearing and Welsh are able to do so without the requirement of a Z-dependent quenching of \( g_P \) (note, however, that Fearing and Welsh's absolute values of \( R_y \) are roughly 2.5 times larger than experimental values, assuming the Goldberger-Treiman value of \( g_P \)). Fearing and Welsh also find that \( R_y \) is quite sensitive to various inputs in the Fermi gas model, which suggests that it is difficult to extract values of \( g_P \). Clearly then, the present theoretical situation for nuclear RMC is not such that experiment can make conclusive statements about \( g_P \) and PCAC.
Table 6.2: Values of $g_P/g_A$ and $R_y$ as determined by the integral method. “GOKS” refers to Gmitro et al. [83]; “CG” refers to Christillin and Gmitro [68]; “GOT” refers to Gmitro et al. [58]; “RN” refers to Roig and Navarro [69]; “C” refers to Christillin [67]; and “CRS” refers to Christillin et al. [55].
Figure 6.4: $g_p/g_A$ as a function of atomic number. Plotted values are from table 6.2 (integral method results). The acronyms in the symbol legend correspond to the different theoretical models used to extract $g_p/g_A$. They are given in full in the caption to table 6.2.
Chapter 7

Conclusions

A test of the PCAC prediction of the induced weak pseudoscalar coupling constant, \( g_P \), has been made by measuring the photon spectrum from RMC in nuclei. Values of \( g_P \) and the \( g_P \)-dependent \( R_\gamma \) (the RMC/OMC ratio for \( k > 57 \text{ MeV} \)) are extracted by comparing experimental results with theoretical predictions. Due to inconsistencies between different theoretical calculations, a meaningful test of PCAC using nuclear RMC is not yet possible.

\( R_\gamma \) has been measured in six nuclei: oxygen, aluminum, silicon, titanium, zirconium, and silver. The present results for aluminum and silicon agree with previous results [44], and indicate a sensitivity of \( R_\gamma \) to Pauli blocking. The present \( R_\gamma \) result for Ti, when compared to previous results for Ca, further shows a sensitivity of \( R_\gamma \) to Pauli blocking. The present \( R_\gamma \) result for O is \( \sim 25\% \) lower than previous results, but one previous result [43] may have an overestimated beam rate-dependent correction.

The present results for zirconium and silver are consistent with previous high Z results, and indicate either a Z dependent quenching of \( g_P \) to zero in heavy nuclei (Christillin et al. [55]), or Z independent effects which do not alter the form of the Goldberger-Treiman relation given in Eq. (1.29) (Fearing and Welsh [70]). This discrepancy will be resolved by the proposed RMC-Nickel experiment at TRIUMF, where RMC will be measured on three Ni isotopes which do not cross a neutron shell closure: \(^{58}\text{Ni}, ^{60}\text{Ni}, \) and \(^{62}\text{Ni} \). Further theoretical nuclear RMC work will then be needed in order to determine meaningful values of \( g_P \). Furthermore, nuclear RMC models for Al, Si, and Ti have not yet been attempted. In light of the present \( g_P \) result for O as extracted from Roig and Navarro [69], sum rule
techniques look promising for calculation of RMC on nuclei in addition to C, O, and Ca.

Measurements of RMC on H and $^3$He are forthcoming shortly from TRIUMF, and will provide direct tests of the PCAC value of $g_p$. 
Bibliography

Bibliography


Appendix A

Quantities used in the calculations of $R_\gamma$ and $g_P/g_A$
Appendix A. Quantities used in the calculations of $R_\gamma$ and $g_p/g_A$

<table>
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<tr>
<th></th>
<th>O</th>
<th>Al</th>
<th>Si</th>
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<tr>
<td>$N_{\gamma&gt;57}^{exp}$</td>
<td>2365 ± 49</td>
<td>2787 ± 53</td>
<td>3028 ± 55</td>
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<tr>
<td>$N_{\text{stop}}^{exp}$</td>
<td>1.29881 x 10^{11}</td>
<td>5.41003 x 10^{10}</td>
<td>4.01114 x 10^{10}</td>
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<tr>
<td>$\epsilon_v$</td>
<td>0.9613 ± 0.0006</td>
<td>0.9705 ± 0.0010</td>
<td>0.9730 ± 0.0010</td>
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<tr>
<td>$\epsilon_{cs}$</td>
<td>0.976 ± 0.005</td>
<td>0.976 ± 0.005</td>
<td>0.976 ± 0.005</td>
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<tr>
<td>$\epsilon_r$</td>
<td>0.943 ± 0.004</td>
<td>0.968 ± 0.005</td>
<td>0.968 ± 0.005</td>
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<tr>
<td>$\epsilon_c$</td>
<td>0.957 ± 0.002</td>
<td>0.948 ± 0.004</td>
<td>0.940 ± 0.006</td>
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<tr>
<td>$\epsilon_p$</td>
<td>0.940 ± 0.013</td>
<td>0.927 ± 0.009</td>
<td>0.910 ± 0.007</td>
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<tr>
<td>$C_{bm}$</td>
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<td>1.019 ± 0.003</td>
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<tr>
<td>$C_{REVMOC}$</td>
<td>0.99558 ± 0.00004</td>
<td>0.99788 ± 0.00002</td>
<td>0.9771 ± 0.0002</td>
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<tr>
<td>$C_o$</td>
<td>0.987 ± 0.003</td>
<td>0.991 ± 0.002</td>
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<tr>
<td>$C_u$</td>
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<td>1.022 ± 0.004</td>
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<td>$C_{md}$</td>
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<table>
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<tr>
<th></th>
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<tr>
<td>$N_{\gamma&gt;57}^{exp}$</td>
<td>1644 ± 41</td>
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<td>743 ± 27</td>
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<td>$N_{\text{stop}}^{exp}$</td>
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<td>1.49912 x 10^{10}</td>
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<td>$\epsilon_v$</td>
<td>0.908 ± 0.003</td>
<td>0.912 ± 0.004</td>
<td>0.913 ± 0.005</td>
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<tr>
<td>$\epsilon_{cs}$</td>
<td>0.976 ± 0.005</td>
<td>0.976 ± 0.005</td>
<td>0.976 ± 0.005</td>
</tr>
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<td>$\epsilon_r$</td>
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<td>0.983 ± 0.002</td>
<td>0.985 ± 0.004</td>
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<tr>
<td>$\epsilon_c$</td>
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<td>0.933 ± 0.003</td>
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<td>$\epsilon_p$</td>
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<td>$C_{bm}$</td>
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<td>1.0208 ± 0.0014</td>
<td>1.15 ± 0.05</td>
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<tr>
<td>$C_{REVMOC}$</td>
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<td>$C_o$</td>
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<td>0.9887 ± 0.0010</td>
<td>0.983 ± 0.011</td>
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<tr>
<td>$C_u$</td>
<td>1.017 ± 0.005</td>
<td>1.0117 ± 0.0011</td>
<td>1.008 ± 0.009</td>
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<tr>
<td>$C_m$</td>
<td>1.030 ± 0.010</td>
<td>1.0300 ± 0.0017</td>
<td>1.017 ± 0.013</td>
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<tr>
<td>$C_{md}$</td>
<td>1.0003 ± 0.0002</td>
<td>1.0004 ± 0.0002</td>
<td>1.0003 ± 0.0002</td>
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<tr>
<td>$empfrac$</td>
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<tr>
<td>$Acc$</td>
<td>0.0071 ± 0.0003</td>
<td>0.0070 ± 0.0004</td>
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<tr>
<td>$F$</td>
<td>0.95 ± 0.06</td>
<td>0.93 ± 0.04</td>
<td>1.03 ± 0.04</td>
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</table>

Table A.1: Quantities used in the calculations of $R_\gamma$ and $g_p/g_A$ for O, Al, Si, Ti, Zr, and Ag. $N_{\gamma>57}^{exp}$ is the observed number of photons (> 57 MeV) after all software cuts, and $N_{\text{stop}}^{exp}$ is the raw (uncorrected) number of beam particle stops in the target. See sections 4.2.2, 4.2.3, 4.3.2, 4.4, and 5.2 for details on all quantities.
Appendix B

Derivation of equations (1.8) and (1.9)

Comparing Eq. (2.213a) of Marshak, Riazuddin, and Ryan (MRR) [10] with Eq. (1.3) in the present thesis, and using Eq. (2.228) of MRR makes it clear that the $J_\lambda(x)$ in Eq. (2.229) of MRR is $J_{\lambda}^{H_i}(x)$ in the notation of the present thesis. Therefore, $V_\lambda(0)$ and $A_{\lambda}(0)$ in Eqs. (2.231a) and (2.231b) of MRR are actually $V_{\lambda}^i(0)$ and $-A_{\lambda}^i(0)$ in the notation of the present thesis (compare Eq. (2.229) in MRR with Eq. (1.6) of this thesis). Hence, from Eqs. (2.231a) and (2.231b) of MRR with an obvious change in notation for the form factors, we have

\begin{align*}
V_\alpha^i(x) &= i\bar{\psi}_n(x)[F_V(q^2)\gamma_\alpha - F_M(q^2)\sigma_\alpha q^\lambda - iF_S(q^2)q_\alpha]\psi_p(x) \\
A_\alpha^i(x) &= -i\bar{\psi}_n(x)[F_A(q^2)\gamma_\alpha\gamma_5 + iF_P(q^2)\gamma_5 q_\alpha - F_T(q^2)\gamma_5 \sigma_\alpha q^\lambda]\psi_p(x).
\end{align*}

From Mandl and Shaw [6] we have

\begin{align*}
\gamma_\alpha^i &= \gamma_0 \gamma_\alpha \gamma_0, \quad \gamma_5^i = \gamma_5, \quad \gamma_\alpha^2 = \gamma_5^2 = 1, \\
\{\gamma_\alpha, \gamma_5\} &= 0, \quad \sigma_\alpha = \frac{i}{2}[\gamma_\alpha, \gamma_5], \\
\bar{\psi} &= \psi^\dagger \gamma_0,
\end{align*}

and using the reality of the form factors (see section 1.1) we derive Eqs. (1.8) and (1.9):

\begin{align*}
V_\alpha(x) &= -i\bar{\psi}_p(x)[F_V(q^2)\gamma_\alpha^i - F_M(q^2)\sigma_\alpha \gamma_\alpha q^\lambda + iF_S(q^2)q_\alpha]\gamma_0^\dagger \psi_n(x) \\
&= -i\bar{\psi}_p(x)[F_V(q^2)\gamma_\alpha^i \gamma_0 \gamma_\alpha \gamma_0 - F_M(q^2)\gamma_0 \sigma_\alpha \gamma_0 q^\lambda \\
&\quad + iF_S(q^2)\gamma_0 \gamma_\alpha q_\alpha] \gamma_0^\dagger \psi_n(x) \\
&= -i\bar{\psi}_p(x)[F_V(q^2)\gamma_\alpha - F_M(q^2)\sigma_\alpha q^\lambda + iF_S(q^2)q_\alpha] \psi_n(x).
\end{align*}
Derivation of equations (1.8) and (1.9)

\[ A_\alpha(x) = i\psi_\alpha(x)[F_A(q^2)\gamma_5\gamma_\alpha - iF_F(q^2)\gamma_5 q_\alpha - F_T(q^2)\sigma_\alpha\gamma_5 q_\lambda]\gamma_0\psi_n(x) \]  

\[ = i\psi_\alpha(x)[F_A(q^2)\gamma_5\gamma_0\gamma_\alpha - iF_F(q^2)\gamma_5\gamma_0 q_\alpha \] 

\[- F_T(q^2)\gamma_0\sigma_\alpha\gamma_0\gamma_5 q_\lambda]\gamma_0\psi_n(x) \]  

\[ = \bar{\psi}_\alpha(x)[F_A(q^2)\gamma_\alpha\gamma_5 + iF_F(q^2)\gamma_5 q_\alpha + F_T(q^2)\gamma_5\sigma_\alpha q_\lambda]\psi_n(x) \]