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THE PRIMARY SPECIFIC IONIZATION IN GASES

of

POSITRONS AND ELECTRONS

by

Lorna Margaret Silver

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1.ABSTRACT

Theories of P.S.I.* produced by electrons passing through gases show that one would expect the number of primary ion pairs formed to vary with the electron density in the gas and with the mean ionization potential of the gas, to vary inversely as the square of the velocity of the electron at low energies reaching a minimum value approximately at a kinetic energy equal to the rest energy and to be independent of the sign of the beta particle producing the ionization.

It is consequently of interest to obtain measurements of the P.S.I. of beta particles at energies at which it is a minimum in various gases which are used for filling counters and ionization chambers including hydrogen, helium, neon, xenon, methane, chlorine and other quenching vapors such as methyl chloride which may become of practical concern for counters designed to operate over a wide temperature range. Present data on P.S.I. of electrons and mesons is limited to but few gases and the results are not in good accord. As mentioned, the P.S.I. is independent of the sign of the charge of the ionizing particle and, since all the indirect evidence and the principle of conservation of charge indicates the equality of the charge on the positron and negatron, it would be expected that the values of the P.S.I. would be identical for the two particles. However while it is known that the value of e/m for both particles is identical to within 2%, there is some possible theoretical indication that their masses might be slightly different.

* The primary specific ionization will hereafter be denoted by P.S.I.

Moreover the only direct estimation of the value of e^+ is still the measurement of P.S.I. from the original cloud chamber observation of a cosmic ray positron by Anderson¹ which established the charge equality of e^+ and e^- to within 20%. Thus it appeared of some fundamental importance to compare with precision the P.S.I. of both positive and negative electrons of similar velocities in a gas of high atomic number such as neon and a gas of low atomic number such as helium.

Finally it was hoped that the apparatus set up would be suitable for an investigation of the relation between the P.S.I. and velocity of the electron especially in the relativistic region in which the data is very meagre. This will need high energy beta sources (e.g. B^{12}) which will become available when the U.B.C. electrostatic generator is in operation.

Apparatus has been set up to determine the P.S.I. of electrons in various gases by determining the inefficiency of a Geiger counter filled with gas. To eliminate the difficulty of a variable path length of the particle through the ordinary cylindrical counter, rectangular and square envelope counters with thin windows have been constructed. The presence of a large thin window, even when a conducting surface, has been found to affect the spread of the discharge and results in the appearance of two size pulses analagous to the effect of an insulating bead on the centre of the wire. Thus, contrary to the usual theory of Geiger operation, it seems that the spreading of the discharge does involve a cathode mechanism in this design.

In order to select beta particles of homogeneous energy from the radioactive source, a small wedge shaped magnetic spectrograph has been built.

The counter inefficiency is determined by passing the beta particles through the inefficient counter into a 100% efficient counter operating at normal pressures, and recording the coincidence rate as a fraction of the "efficient counter" rate. Co^{56} was chosen as the most suitable positron emitter, and has been prepared in the Berkeley cyclotron by the $\text{Fe}^{54}(\text{d}, \text{n})\text{Co}^{56}$ reaction. RaE has been used as a negatron emitter.

II.

THEORY OF SPECIFIC IONIZATION

The P.S.I. is defined to be the number of primary ion pairs formed per cm. length of track in the gas, reduced to N.T.P. Two other similar quantities sometimes measured are the total specific ionization; i.e., the total numbers of ion pairs (primary, secondary, tertiary, etc.) formed per cm. at N.T.P., and the probable specific ionization; i.e., the number of primary ion pairs plus the number of secondary ion pairs having a specified upper limit to their energy which are formed per cm. at N.T.P.

J.J. Thomson²³ has calculated the energy transfer Q resulting from a collision of a particle of mass M , kinetic energy T , and charge ze , and an electron of mass m and charge e , when the electron suffers a deflection through an angle θ . His expression derived classically is

$$Q = \frac{4Mm}{(M+m)^2} T \cos^2 \theta$$

(See appendix 1.)

The closeness of collision can be specified by the impact parameter p :

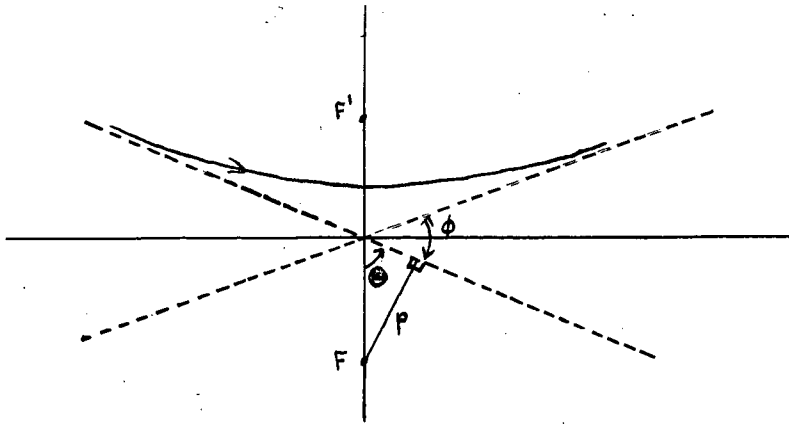


Fig.1. Impact Parameter in a Coulomb Field.

From conservation of momentum, it follows that for any p ,

$$\cos^2 \theta = \left(1 + \frac{mpv^2}{ze^2} \right)^{-2}$$

therefore
$$p^2 = \left(\frac{4T'}{Q} - 1 \right) \left(\frac{z^2 e^4}{4T'^2} \right)$$

(See appendix 2.)

where ze = charge on the bombarding particle

v = incident velocity

$T' = (m/M) T$.

An ionizing event requiring an ionization energy W will occur

whenever
$$p_0^2 = \left(\frac{4T'}{W} - 1 \right) \left(\frac{z^2 e^4}{T'^2} \right)$$

$$T' \gg W$$

and if

then
$$p_0^2 = \frac{z^2 e^4}{TW}$$

The number of electrons which will be removed from the atoms in 1 cm. of track (i.e., the P.S.I.) will equal the probability that the ionizing particle will come within radius p_0 of any electron, multiplied by the number of electrons per cc. in the gas; i.e.,

$$\text{P.S.I.} = NZ \left(\frac{\pi z^2 e^4}{T'W} \right) \dots\dots\dots(i)$$

Thomson's formula therefore stipulates that:

- (1) For a given incident velocity, P.S.I. is independent of M .
- (2) P.S.I. is proportional to v^{-1}
- (3) P.S.I. is proportional to z^2e^4
- (4) P.S.I. is independent of the sign of ze .
- (5) P.S.I. is proportional to $(M/H\rho)^2$ for ionizing particles of mass M selected by their $H\rho$ in a magnetic momentum analyser.

The equality of the magnitude of charge on positrons and negatrons was first established to a rough degree of approximation by Anderson¹ by a comparison of the specific ionizations of β^+ and β^- of relativistic energies such as occur in cosmic rays. All other evidence for this equality is essentially indirect and assumes the conservation of charge. The equality of e/m for β^+ and β^- has been established by Zahn and Spees²⁷ to an accuracy of 2%. Barnothy³ has suggested that theoretically the mass of the positron is 0.354% less than the mass of the electron due to the difference in sign of their mass defects, and a careful examination of the results of Zahn and Spees reveals that e^+/m is slightly greater than e^-/m . A precision measurement of e^+/m to this accuracy would, if this divergence in specific charge were established, be strong evidence of the exact equality of charge. Since the specific ionization is proportional to the square of the mass for a given velocity, or rather a given $H\rho$, a comparison of P.S.I. for positrons and negatrons will give direct evidence for the equality of mass and hence charge of the two particles.

Bethe has derived an expression for the P.S.I. by ascribing hydrogen-like wave functions to the atomic electrons, and

restricting the velocity of the bombarding electron to be much greater than the velocity of the atomic electrons in the Bohr orbits, and much less than the velocity of light. His formula is:

$$\text{P.S.I.} = \frac{(NZ) \pi e^4}{T' W} (0.285) \left(\ln \frac{42 m v^2}{W} \right) \dots\dots\dots (ii)$$

where $W = 13.5$ ev. for hydrogen

and $W(Z) = 13.5 Z$ for other gases.

The value for W , the average excitation potential of the whole atom, can be found more accurately empirically from stopping power data than theoretically. The theoretical expression given by Livingston¹⁷ is :

$$\text{Log } (W) = \left(1 - \frac{1.81}{Z} \right) \log I' + \left(\frac{1.81}{Z} \right) \log I_K$$

where I_K = average excitation potential of the K shell

$$= 1.103 Z^{\text{2eff.}} R_y$$

and R_y = ionization potential of the hydrogen atom.

$Z - 0.3$ = effective nuclear charge of the K shell.

I' = average excitation potential of the electrons outside the K shell.

$Z - 1.81$ = "effective" number of electrons.

To describe ionization by relativistic particles, Bethe⁴ applies an exact quantum mechanical treatment to the energy loss problem. He considers the distribution of Q into excitation and ionization energies, which varies with the atom bombarded, and he

shows that

$$-dT/dx = A/\beta^2 (\ln K + \ln \frac{\beta^2}{1-\beta^2} - \beta^2) \dots\dots\dots(iii)$$

where K depends inversely on the effective ionization potential of the gas and on whether it is primary or probable specific ionization which is being described. For P.S.I. in hydrogen,

$$K \doteq 10^5$$

$$A = 4z^2e^4NZ/mc^2$$

The ratio

$$(dT/dx)_{\text{pri}} \text{ P.S.I.} = V_0$$

= average energy expended per primary ion pair produced.

V_0 is a function of the nature of the gas only, and is independent of the energy and nature of the ionizing particle. Values for different gases are given in Table 1.

Table 1. V_0 for Different Gases.

Gas	Particle	Energy (Mev)	V_0 (ev.)	Reference
Air	Electron	8.3	32.0	11
Air	Proton	2.5-7.5	36.0	
Air	Alpha	7.8	35.1	
Air	"	5.3	35.6	
H ₂	"	"	36.0	20
He	"	"	31.0	
CO ₂	"	"	34.6	
Ne	"	"	27.8	18
A	"	"	24.9	
A	Electron	0.0174	26.9	20
Kr	Alpha	5.3	23.0	
Xe	"	1.3	21.4	

When changed to give P.S.I., equation (iii) becomes:

$$\text{P.S.I.} = \text{KNZ} \frac{(ze^2)^2}{mv^2} \left[\ln \left(\frac{\beta^2}{1-\beta^2} \right) (C) + b - \beta^2 \right] \dots\dots\dots(\text{iv})$$

where K is a constant which depends on the gas being ionized, and C depends on the ionization potential of the gas. A plot of (iv) is given in Fig.2

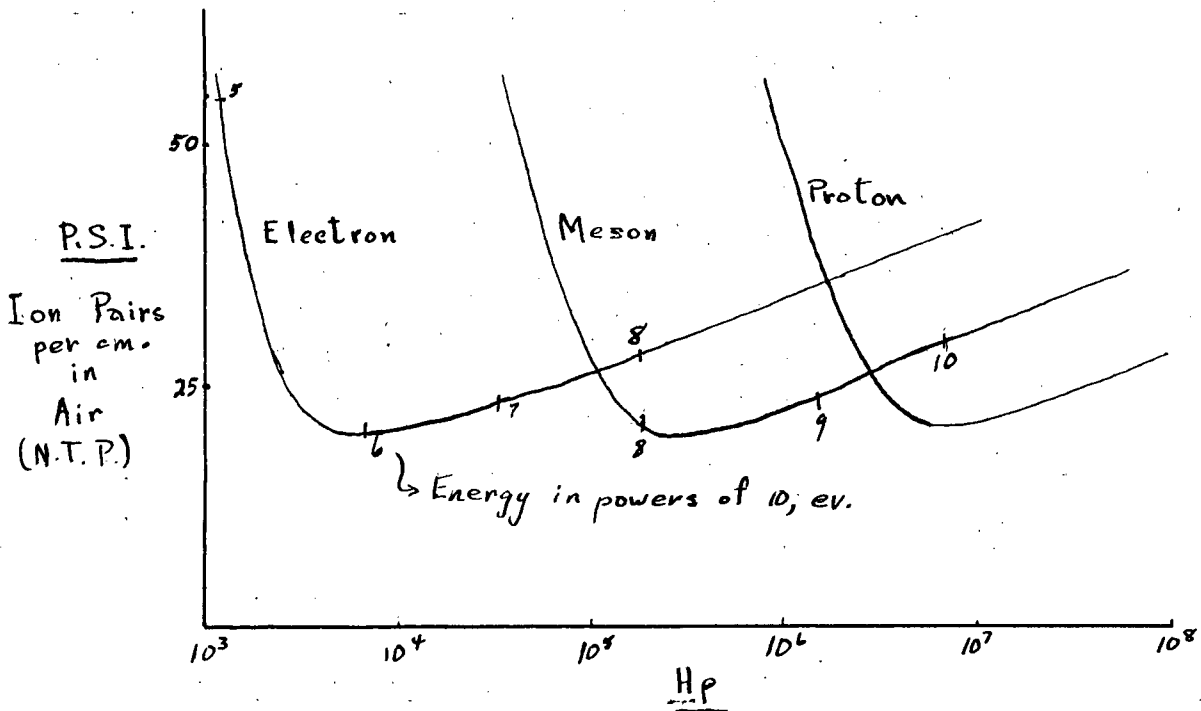


Fig.2. Ionization-Energy Curve.

The relativistic formulae differ significantly from the non-relativistic only when the energy is several times rest energy. The relativistic increase in the specific ionization is due to the increasing contraction of the electric field of the particle toward a plane perpendicular to its motion, thus increasing the impulse given to atomic electrons by particles passing at some distance from the atomic centre. An increase in $H\rho$ by a

factor of 5.10^4 only doubles the minimum value of the specific ionization.

III. Methods of Measurement of Specific Ionization and Available Results.

A) General Scheme.

The primary specific ionization of electrons, alpha particles, protons and mesons has been measured by two methods. In the first a cloud chamber is used in which the expansion is arranged to take place shortly before the passage of the ionizing particle through the chamber, so that condensation occurs before the ions diffuse an appreciable distance. The secondary electrons of low energies and correspondingly short ranges thus give rise to a cluster of closely spaced ions which appears as a blob about the primary ion pair. Counting the numbers of blobs per cm. gives the primary specific ionization. The energy of the particle is found from a measurement of the curvature of the track in a magnetic field. Inaccuracies arise from multiple scattering in the gas at low energies, and by the uncertainty in estimating the plane of the track. The amount of water vapor present is hard to estimate, and hence also introduces an error. Further, the blobs may overlap, be of various sizes, and be irregularly spaced, thus making accurate counting difficult.

The second method consists in measuring the efficiency of a Geiger counter; i.e., in finding the probability that at least

one ion pair is created in the gas by the passing particle.

The efficiency may be calculated as follows:

Assuming all path lengths in the counter are the same, and equal to l cms. then the average number of ion pairs produced in the tube is: $n = slp$.

where s = primary specific ionization

p = gas pressure in atmospheres.

Let $\omega(l)$ = probability that the particle goes a distance l without producing an ion pair.

Then $\omega(dl) = d\omega(l)$ = probability that the particle goes a distance dl without producing an ion pair.

Therefore $\omega(l) \cdot d\omega(l)$ = probability of it going a distance $(l+dl)$ without producing an ion pair

$$= \omega(l) (1 - slp)$$

Applying the boundary condition $\omega(0)=1$, and integrating,

we get $\omega(l) = e^{-slp}$

Therefore efficiency = $\varepsilon(l) = 1 - e^{-slp}$.

If the geometry is such that there is a continuous distribution of path lengths, it is possible and necessary to revise the expression for the efficiency as a function of (sp) .

The precision is limited in that the calculations assume that (1) the presence of one ion pair is sufficient to initiate a discharge (which is believed to be true in a properly operated Geiger counter)²⁴, and (2) that negligibly few entities capable of exciting a discharge are ejected by the particles from the inner surface of the counter cylinder. To check whether or not condition (2) is satisfied, an efficiency measurement could be made using a gas filling^{of} known P.S.I. and by checking the form of the functional relation by varying the filling pressure.

Ramsey¹⁹ has pointed out that a single segmented counter could be used to measure the P.S.I. By applying different voltages to different segments, one segment can be operated in the proportional region giving a current proportional to the number of ion pairs formed, while another segment is operated in the Geiger region giving a current which is proportional to the number of ionizing particles passing through it. The ratio of these two currents is proportional to the P.S.I. and is independent of the number of particles in the beam. To obtain the absolute value of P.S.I. for any gas, a calibration of the apparatus must be carried out using a gas of known P.S.I.

The results of several investigators are given in Table 2.

Table 2.

Available Data

1. Cloud Chamber.

Investigator	Gas	Particle	Momentum	P.S.I (N.T.P.)
Williams & Terroux ²⁶	H ₂	Electrons	$\sim 10^6$ ev/c	5.2
" "	O ₂	"	$\sim 10^6$	22
Kunze ¹⁶	Air	Mesons (?)	$\sim 10^9$	19
Conson & Brode ⁶	Air	" (?)	$\sim 10^9$	14-18
Hazen ¹³	He	Electrons	$10^6 - 10^7$	6.6
"	He	Mesons	$> 10^9$	6.5
Skramstad & Loughridge ²¹	N ₂	Electrons	600-2100 Kev.	19
" "	Ne	"	400-2100 Kev	12.6

2. Counter Efficiency

Danforth & Ramsey ¹⁰	Air	Mesons & Electrons	?	21
" "	H ₂	" "	?	6.2
Cosyns ⁷	A	" "	?	29.4
"	He	" "	?	5.9
"	H ₂	" "	?	6.0 ± 0.2
Curran & Reid ⁸	C ₂ H ₂ (OH)	Electrons	0.42 Mev.	79
" " ⁸	A	"	0.42 Mev	29.8
Hereford ¹⁵	H ₂	"	$E/mc^2 = 3$	4.5
" ¹⁵	H ₂	"	$E/mc^2 = 20$	5.7

Williams and Terroux claim an error of 2% in their measurement of $H\rho$, and an error of 5% due to the presence of water vapor and other gas impurities. They estimate that the error in judging the number of blobs per cm. produced by fast electrons is insignificant compared to the error of the first two causes. For slow electrons the method is very inaccurate because of the scattering. They find the variation of P.S.I. with velocity can be given by

$$\text{P.S.I.} = 5.2 \beta^{-1.5 \pm 0.2} \quad \text{for hydrogen.}$$

$$\text{P.S.I.} = 22 \beta^{-1.1 \pm 0.2} \quad \text{for oxygen.}$$

Their results show that equation (i) predicts the correct order of magnitude for P.S.I. but theoretical values are six times lower than experimental.

E.J. Williams²⁵ found that equation (ii) gave values within 10% of the experimental values for electrons having $\beta = 0.50, 0.75$ and 0.96.

Kunze did not find the expected increase of P.S.I. with velocity at energies greater than 2 Mev., nor did Anderson², who obtained a value of 31 ion pairs per cm. for energies greater than 10^7 ev. As Brode⁵ points out, this may be due to an error in the operation of the cloud chamber, giving a result which is too low by a factor of 2. This same mistake was made by Corson and Brode⁶ and is due to the fact that they were observing condensation on positive ions alone, instead of on ion pairs.

Hazen quotes a probable error of 1.6% for electrons and 1.7% for mesons. This covers an estimation of the percentages of alcohol and water vapor present in the chamber.

Skramstad and Loughridge estimate that the resolving power of the observer's eye limits the accuracy to 10-15% for N_2 , and 7-10% for Ne , depending on the velocity of the electron. They did not find an increase of P.S.I. with relativistic velocities. Their results are expressed as

$$P.S.I. = 19 \beta^{-1.15 \pm 0.15} \quad \text{for nitrogen.}$$

$$P.S.I. = 12.6 \beta^{-1.35 \pm 0.15} \quad \text{for neon.}$$

They also observed a few tracks in oxygen, two or three of which were definitely due to positrons and others to electrons. The results were indistinguishable in the two cases, and agreed with the values of Williams and Terroux.

The results of Danforth and Ramsey, Cosyns and Hereford, using the counter technique, all give a value for the P.S.I. of a particle of $E/mc^2 = 20$, where E is the total energy of the particle, which is an electron in the case of Hereford and a cosmic ray particle in the other cases.

For low energies, Hereford's results follow closely Bethe's theoretical curve, the minimum lies below the theoretical curve, and there are three experimental points to indicate that the P.S.I. does increase at relativistic velocities. Hereford's results appeared in the September 1948 issue of The Physical Review, sometime after the present experiment had begun and he used an experimental set up very similar to the one used by the author, but Hereford has used cylindrical glass envelope counters with graphite cathodes.

The majority of the results quoted in the literature are not in particular accord, which would suggest that the precision of the cloud chamber has been overestimated. The theory has been checked

and verified most closely in the region of the minimum of the ionization-energy curve.

IV. Experimental Arrangement.

A) General Scheme.

In this research the primary specific ionization of negatrons and positrons is obtained by measurement of the inefficiency of a Geiger counter operating at a low pressure of the gas under investigation. Essentially the method adopted is intended to give a direct comparison of the primary specific ionization of β^+ and β^- of various velocities. For this purpose a small magnetic analyser has been set up, the selection of either β^+ or β^- being made by reversal of the field direction while all other experimental conditions remain unchanged.

This method obviously can be applied to study the variation of P.S.I. with the energy of the bombarding particle and with the nature of the gas through which the particle passes. The gases it is proposed to study include argon, helium, chlorine, hydrogen, neon, and quenching vapors such as alcohol, methane and methyl chloride, while the energy range available for investigation is from 200 Kev. to 1.5 Mev., the lower limit being set by the energy spread introduced by the mica windows, and the upper limit by the sources at present obtainable.

B) Design of the β -ray Energy Analyser.

The magnet used is a 1/6 scale model of the deflection magnet designed for the U.B.C. Van de Graaf generator, but with different pole pieces to provide a 3/4 in. gap and wedge shaped field. It is capable of producing fields up to 14,000 gauss over the area of the wedge, i.e., 47 sq. cms. Details of the design and performance are given in Appendices 2 and 3. In addition to its simplicity, the main virtue of wedge refocussing is that it enables the velocities of the electrons to be analysed without the deflecting magnetic field straying over into the regions in which the electrons originate and where they are detected. Further, the yoke provides some shielding of the detector and the geometry and distances are such that excellent shielding from gamma rays and annihilation radiation from the source can be achieved by use of lead blocks, while still maintaining a reasonable solid angle and counting rate. The spread, or departure from perfect focus is given by

$$S = a\alpha^2 \sin \theta \quad (\text{See Appendix 4}).$$

where, for the wedge used,

$a = 30 \text{ cms.} = \text{distance from wedge apex to source,}$

$\theta = 21^\circ = \text{one half the wedge angle,}$

$\alpha = 5^\circ = \text{one half the collection angle.}$

whence, $S = 0.09 \text{ cms.}$

This means that a source 1 cm. wide will have an image of 1.09 cms. which is just less than the counter window width.

The dispersion, or the ability to separate two different velocities, as measured by the distance between the points at which

central rays corresponding to the two velocities would focus, is shown to be

$$D = 2a \sin \theta \frac{\Delta v}{v} \quad (\text{See Appendix 4}).$$

If we take $D = \pm 0.6 \text{ cms.} = \text{window width,}$

$$\text{then } \frac{\pm \Delta v}{v} = \pm D / (2a \sin \theta)$$

$$= \pm D / 21.48$$

$$\text{Now } \frac{\pm \Delta v}{v} = \pm \Delta p / p = \pm 1/2 (E/E) = \pm 0.6 / 21.48$$

$$\text{Therefore } \pm \Delta E/E = \pm 1.2 / 21.48 = \pm 6\%.$$

Thus, if the electrons were uniformly distributed in velocity up to the maximum velocity of electrons from the source, the magnet should select all those in a 6% band, i.e., 6 in 100 particles would enter the counter.

The vacuum box of the spectrometer is made of 3in. by 1 in. brass wave guide tubing which was cut and bent to a width of $1\frac{1}{4}$ in. in order that it fit between the pole pieces. The sources are placed on aluminium trays which fit snugly into a hole in one end plate. A $1\frac{1}{2}$ in. diameter mica window in the other end plate serves as exit slit for the electron beam. A lead baffle placed next to the exit flange serves to cut down the amount of scattered radiation in the beam arising from Compton and photo-electrons ejected from the walls of the vacuum box by gamma rays from the source and from annihilation radiation from positrons stopped in the walls. Lead blocks are set up to prevent direct gamma rays from the source entering the counters. The general layout of this apparatus is shown in Fig. 3 and Plate 1.

Fig 3 Layout of Apparatus.

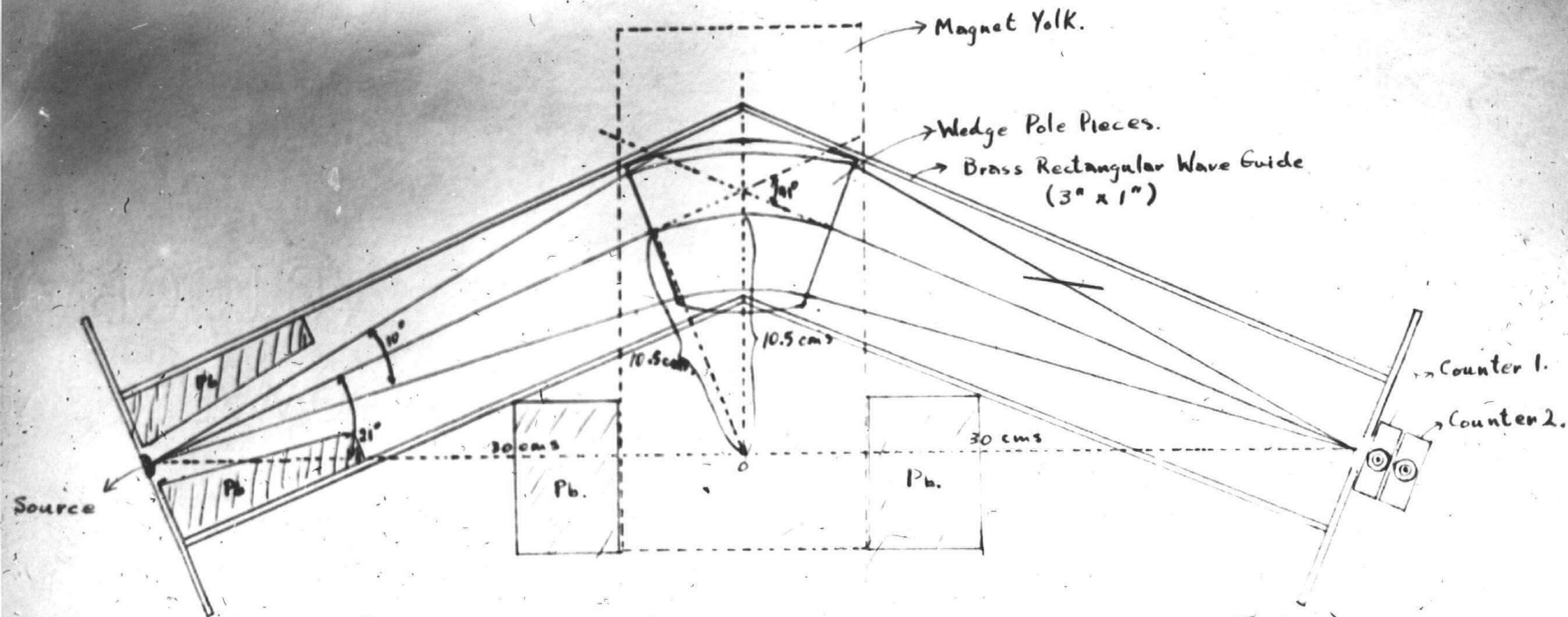
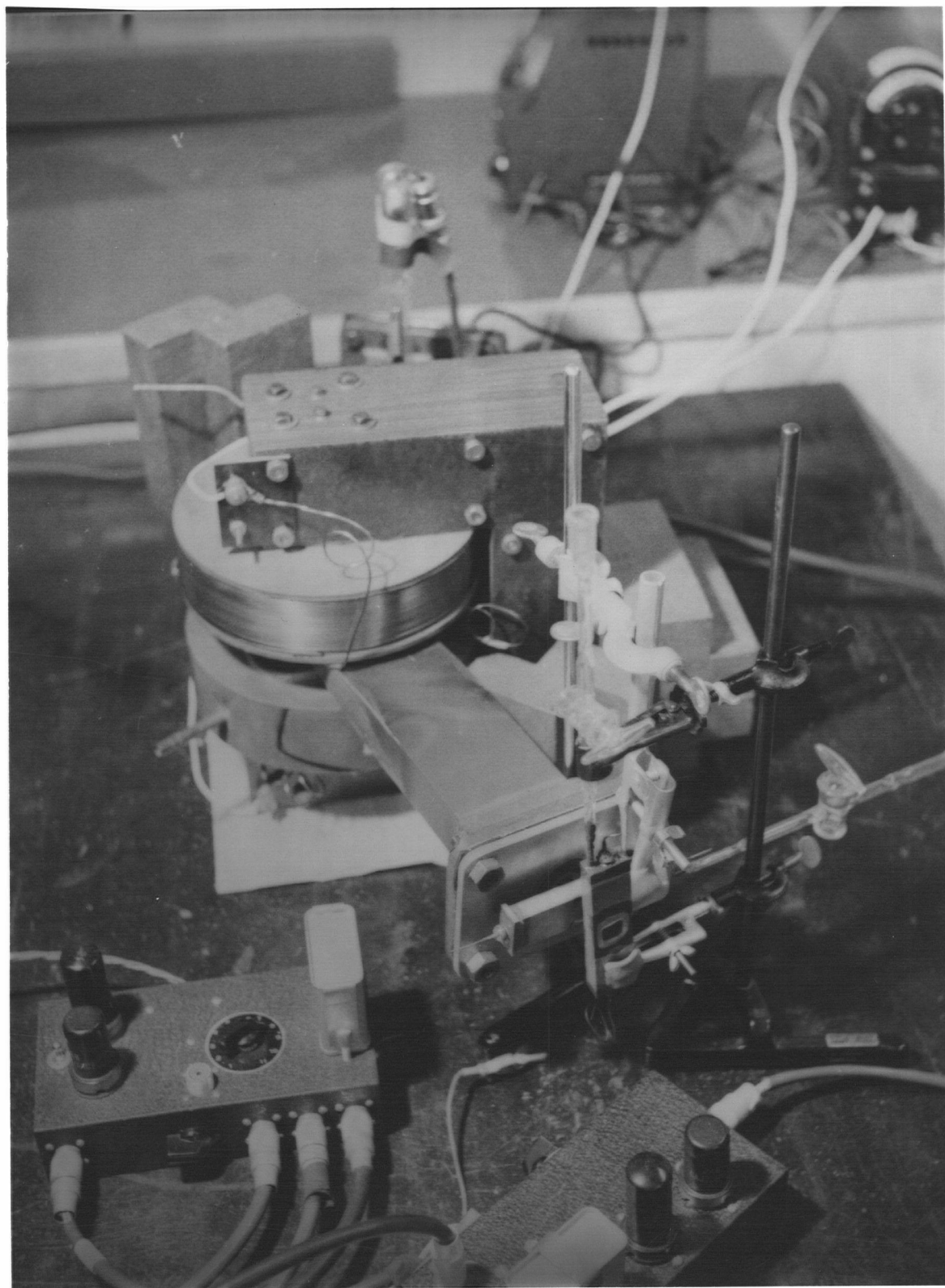


Plate 1 Layout of Apparatus



C) Choice of Sources.

RaE, with an E_{\max} of 1.17 Mev. and half life of 5.0 days, was chosen as negatron source owing to its availability. Both pure sources, prepared from RaD by electrochemical deposition on a nickel surface after removal of RaF by a similar electrochemical deposition on silver, and a source of RaD itself in the form of chloride evaporated to dryness on a mica sheet have been used. For purposes of this experiment the alpha emission from RaF and soft beta emission from RaD itself are not harmful. Reasonably thin sources were used, which were deposited on an area of about 1 cm. diameter, and thick backings.

The choice of a positron emitter for these experiments was a difficult one, since there are relatively few positron emitters with E_{\max} . greater than 1 Mev., still fewer of these have an adequate half life to be useful for these experiments in Vancouver. Further owing to the ready availability of sources from the Chalk River Pile it was preferable that the source should be producible by a neutron reaction in relatively high specific activity.

One of the positron emitters producible in a pile Cu^{64} from $\text{Cu}^{63}(n, \gamma)\text{Cu}^{64}$ reaction, has E_{\max} . of 0.66 Mev. and a half life of 12.8 hours. A considerable fraction of the active nuclei decay via β^- emission but the main objection to using this material as source was the cost of continued transport across Canada. Some Zn^{65} was prepared in the Chalk River Pile in the summer of 1948, while the author was there, by the $\text{Zn}^{64}(n, \gamma)\text{Zn}^{65}$ reaction, but on taking an absorption curve it was evident that the ratio of Zn^{65} nuclei which decayed by emitting a positron to those which

decayed by K capture and emitted a 1 Mev. γ ray was in the region of 1 in 200. This source only provides positrons of E_{\max} of 0.4 Mev., but the half life of 250 days would have been very suitable. Owing to the high ratio of gamma rays to positrons this source was not used.

Consideration was then given to sources which could be prepared with the aid of a cyclotron. Of these the following were considered:

Source	Preparation	E_{\max}	$T_{1/2}$	Comment
$^{52}_{25}\text{Mn}$	$^{52}_{24}\text{Cr}(d,2n)^{52}_{25}\text{Mn}$	0.77 Mev	6.5 days	Low E_{\max}
$^{56}_{28}\text{Co}$	$^{56}_{27}\text{Fe}(d,2n)^{56}_{28}\text{Co}$	1.5 Mev.	72 days	Good
$^{66}_{31}\text{Ga}$	$^{66}_{30}\text{Zn}(p,n)^{66}_{31}\text{Ga}$	3.1 Mev.	9.4 hrs.	$T_{1/2}$ too short
$^{22}_{11}\text{Na}$	$^{24}_{12}\text{Mg}(d,\alpha)^{22}_{11}\text{Na}$	0.58 Mev.	3.0 yrs.	Low E_{\max}
$^{90}_{41}\text{Nb}$	$^{90}_{40}\text{Zr}(d,2n)^{90}_{41}\text{Nb}$	1.0 Mev.	21. hrs	$T_{1/2}$ too short
$^{48}_{23}\text{V}$	$^{48}_{22}\text{Ti}(d,2n)^{48}_{23}\text{V}$	1.0 Mev.	16 days	E_{\max} slightly low.

Co^{56} was chosen as the positron emitter. Through the kindness of Dr. Hamilton, who is in charge of the 60 in. cyclotron used at Berkeley for trace preparation, a millicurie of Co^{56} has been prepared for use in this experiment. Since the range of the 25 Mev. deuterons from the Berkeley 60 in. cyclotron used for the bombardment of the iron is small, the Co^{56} will be concentrated in a thin surface layer of the target disc. It has been shown that 1/3 of the atoms decay by K-capture. The β^+ spectrum is believed to be simple, and therefore the momentum-number distribution can be

given by a Fermi plot:

i.e., the probability $\omega(\epsilon, \epsilon + d\epsilon) = K\epsilon(\epsilon^2 - 1)^{1/2}(\epsilon_0 - \epsilon)^2 d\epsilon$

where $\epsilon = E_{\text{Electron}}/m_0 c^2$, $\epsilon_0 = E_{\text{max}}/m_0 c^2$

From this, the average energy can be shown to be approximately 650 Kev.

D) Intensity Considerations.

The wedge magnet in use has the collection angle in one plane of $2/11$ radian. In the other plane the angle, as limited by the exit window which is $1/2$ in. at a distance of 28 in., is approximately $1/56$ radian. Thus the overall solid angle subtended by the counter at the source is

$$(1/4\pi) (1/56) (2/11) = 1/3872$$

Hence a millicurie source of Co^{56} can be expected to give

$$(3.7) (10^7) (1/3872) (6/100) (1/3) (1/3) = 50 \text{ counts/sec.},$$

where the factors of $1/3$ are allowances for K-capture and self-absorption in the source, while a millicurie source of RaE would be expected to give slightly less than 450 counts/sec., depending on the thickness. In fact with approximately $1/2$ millicurie of RaE the counting rate at the optimum value of H_p was found to be 170 counts/sec.

E) Counter Design.

In order to apply the equation for efficiency it is necessary to determine precisely the value of ' l ', the path length through the counter, or to evaluate theoretically the average value of ' l ' pertaining to the particular experimental arrangement. With the wedge analyser described, the direction of the electrons leaving the exit window are within 5° to the normal to the window. In order to define with some exactitude the path

length through the inefficient counter it was decided to use a rectangular envelope, and the whole counter design was based on use of 3 cm. wave guide brass tubing for this envelope. This has the obvious advantage over a cylindrical counter that not only is the distance between the windows known quite closely but that this distance can be made quite small - 1 cm. - so that relatively large values of P.S.I. can be measured with a not too low filling pressure, such as occur with gases like argon. Too low a pressure is rather undesirable as the stability of counter performance over a period of time is less certain, the amount of quenching vapor and hence life of the counter is low. Again it is relatively easy to insert windows of $1/2$ in. by 1 in. in mica of thickness as low as 2 milligrams/sq. cm. on a flat brass surface but almost impossible to do so over a cylindrical surface. Finally a closely packed coincidence arrangement is easily set up with such rectangular envelopes, and so enable a larger solid angle to be subtended at the source for the same counter volume than is possible with cylindrical geometry.

Curran and Reid⁹ have investigated some of the properties of rectangular counters. Using a collimated source they found that the effective counting volume of the counter coincided with its geometrical volume. Because of the reduced lengths of paths along which the positive ions produced in a discharge travel to the cathode, it was expected to observe shorter dead times than found in cylindrical counters of comparable cross sectional area, and this effect they verified.

There is a lower limit to the ratio of the length of the walls of a rectangular counter which is imposed by the fact that

the field must not be too high in the short direction to produce overshooting and yet the field in the long direction must be sufficient for an avalanche to occur. Curran and Reid point out that the ratio of maximum to minimum field strengths effective in a rectangular counter is given by

$$M = \frac{(4\ell^2/\pi^2) + \rho^2}{(4\ell^2/\pi^2) - \rho^2}$$

where ρ = 3 wire radius = distance at which multiplication process begins

ℓ = length of short wall.

For the counters used in the present experiment

$$\ell = 1 \text{ cm.}$$

$$\rho = 0.01905 \text{ cm.}$$

$$\text{hence } M = 1.002$$

One can therefore conclude that these counters should operate comparatively as well as cylindrical. The plateau lengths found were of the order of 60 Volts. This is more than adequate for accurate counting when the counter voltage can be derived from a well stabilized voltage supply. The power unit constructed for this purpose (See circuit diagram page 29) has a variation of 1/2% for a 10% variation in mains voltages, and after 15 minutes warming up period had less than 1 volt in 2000 drift over the next several hours.

Two square section brass counters, one with 3/8 in. square mica windows and one without, have also been built, partly for comparison with the rectangular shaped envelope, and show rather better plateau characteristics and less "window effect" (see paragraph "Results"). Ordinary bell type counters have been used for

the second 100% efficient counter.

Further, as an alternative scheme for determining P.S.I., a triple segmented counter (Fig. 4) has been constructed to function along the lines suggested by Ramsey. This has been operated successfully as three counters in one. In order to be used for actual measurement of P.S.I., it still needs to be supplied with an end window, and suffers from the rather small solid angle of electrons it will accept.

F) Counter Construction and Filling Techniques.

Scale diagrams of the counters used are given in Fig 5. The rectangular wave guide has the brass end plates silver soldered into place. The Kovar seals and filler tube are then soft soldered with acid core solder and a hand torch (gas and oxygen). Care is taken to keep the solder clean by wiping it with a wet rag before and after the seals are inserted. The central wire of 0.005 in. diameter tungsten, spot welded onto advance wire of 0.02 in. diameter, is then soft soldered in place, being pulled taut while the solder is hardening. The counter is now cleaned by placing it in a hot solution of 1 N. HNO_3 . Glyptal is first coated on top of the soft solder to prevent it being corroded by the acid. After the brass is nicely etched, the counter is quickly transferred to a water bath, where it is thoroughly rinsed. Finally the counter is rinsed with ethyl alcohol and allowed to dry in a vacuum dessicator. The next procedure is to seal mica windows onto the counter. Best Ruby mica is used which has been split under warm water by a very sharp tungsten needle. The thin sheets of mica are examined under polarized light to select those of uniform thickness. The seal used is Gelva V-7, a Vinyl Acetate Resin,

Fig. 4. Triple Counter

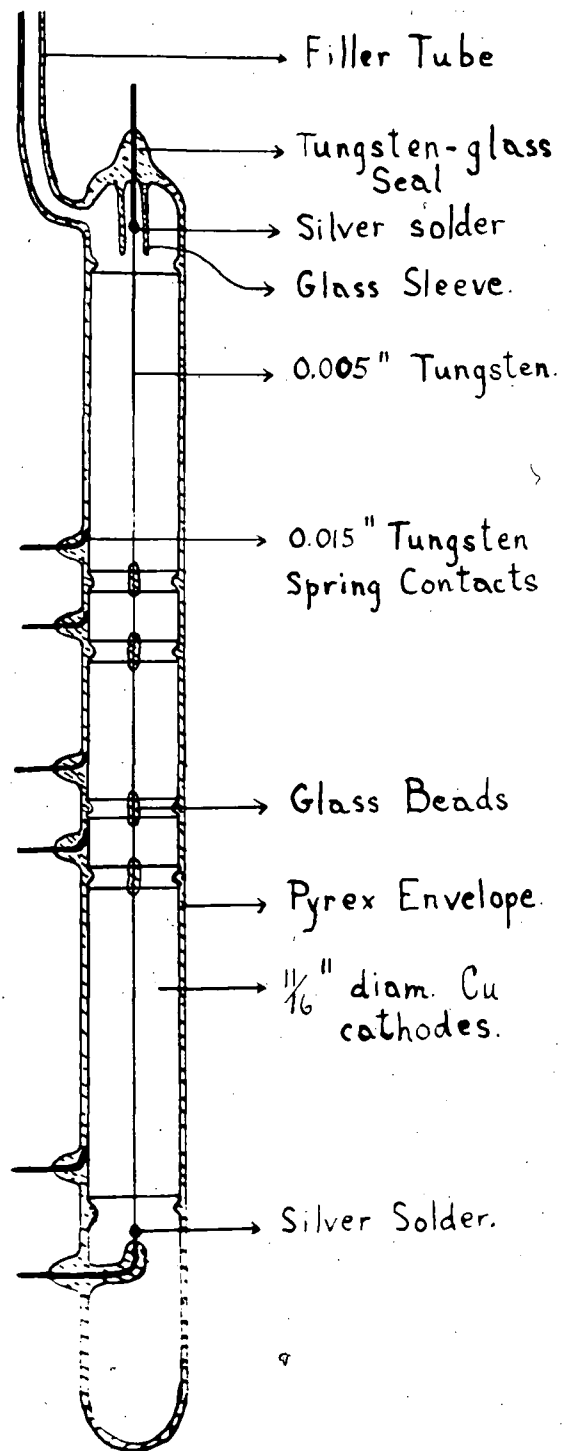


Fig 5. Rectangular and Square β -Counters.

1. Rectangular.

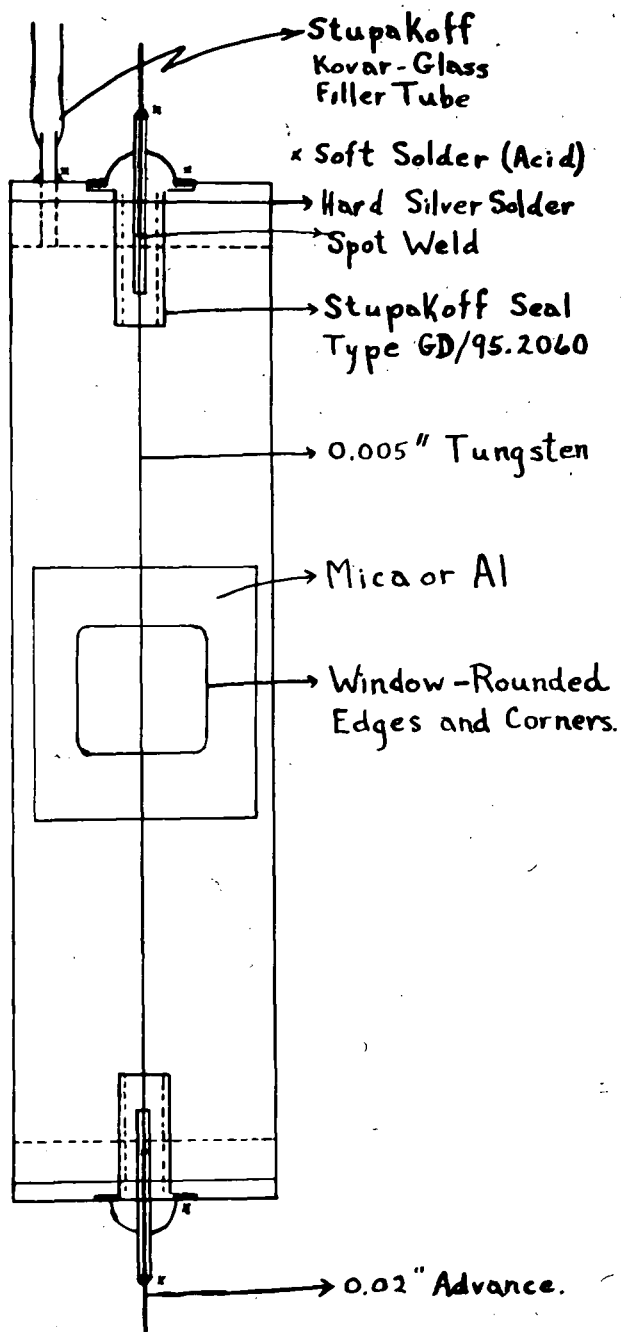
Envelopes: $1" \times \frac{1}{2}" \times 4"$ brass
wave guide tubing

Windows: $\frac{1}{2}" \times \frac{1}{2}"$

2. Square.

Envelopes: $\frac{7}{8}" \times \frac{7}{8}" \times 4"$ brass
tubing

Windows: $\frac{1}{2}" \times \frac{1}{2}"$



which is mixed in acetone. A thin layer is applied at a distance of $1/8$ in. from the edge of the hole to both mica and brass, and then freed from air bubbles by heating at 150° C. for $1/2$ hour. After this, the window is placed on the counter and weighted in position while the Gelva is baked to hardness at 150° C. for $3/4$ hour. Finally the mica is trimmed down and a coat of glyptal applied to the edges. The heating process causes the solder to soften and the wire to pull in and hence to require resoldering.

The cylindrical counters having Cu cylinders are cleaned by a chromic acid passivizing process. The mica window is then put in place and the glass bell (plus wire) is waxed onto the flange.

The filling and testing apparatus is shown in Plate 2. Taper joints are used to attach the counters to the system to facilitate the refilling process. The counters are thoroughly outgassed with the aid of the mercury diffusion pump, and the wires are glowd to burn off sharp points or dirt particles adhering to their surfaces. They are then filled in the usual way by allowing the organic vapor $1/2$ hour to diffuse before adding the main inorganic component. Several hours are left for diffusion and absorption by the counter surfaces to be complete before testing the counters electrically. This ageing process seems frequently to be necessary, counters often showing a much better plateau, etc., after this time than shortly after filling. The operating voltage does not noticeably alter during this time.

The counter operation is tested before it is removed from the filling system.

G) Electronic Equipment.

The electronic equipment used for testing, as shown in Plate 2, includes a head amplifier, stabilized power supply, a scaling unit and a pulse oscillograph which can be used with a triggered or self running sweep.

The head amplifier avoids the necessity of a long cable to the other apparatus with consequent large capacity across the counter and its resultant small output pulse size. It consists of a single 6AC7 amplifying stage with a low anode load to follow a rapid pulse rise, important in coincidence work, and also with simple negative feed back to extend the input voltage size over which the amplifier will work before saturating. The second stage is a 6AG7 cathode follower adequate to drive a long cable capacity so that the rate of rise of the pulse reaching the coincidence mixer is not appreciably reduced below the initial rate of fall of potential on the counter wire. Measurements confirm the expected amplifier voltage gain of 12 and maximum output pulse size of 28 volts, and also that the rise time with 6ft. of cable is less than $1/2$ microsec.

The 'Triggered' time base enables the start of the time base to be coincident with the arrival of the pulse, so that on the screen with a high counting rate the appearance is as shown in Plate 3: A delay time of $1/4$ microsec. in the signal lead enables the time base to be started ahead of the vertical displacement of the beam so the rate of rise of the pulse may be studied. This provides a very convenient means of measuring dead time and recovery time of a counter and of detecting multiple counts and pulses of different heights, and in fact of deducing most of the

Plate 2. Electronic Counter Testing Equipment

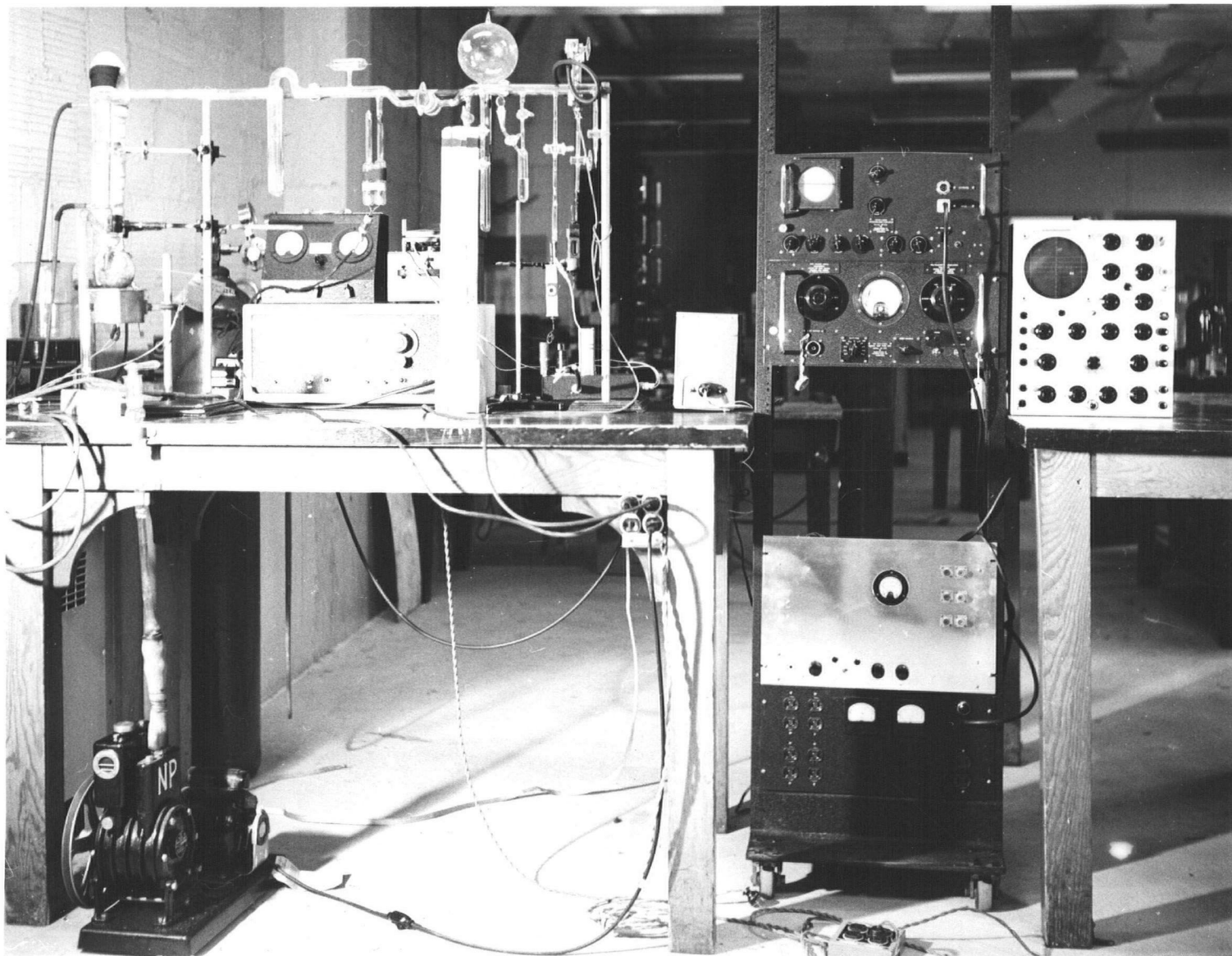
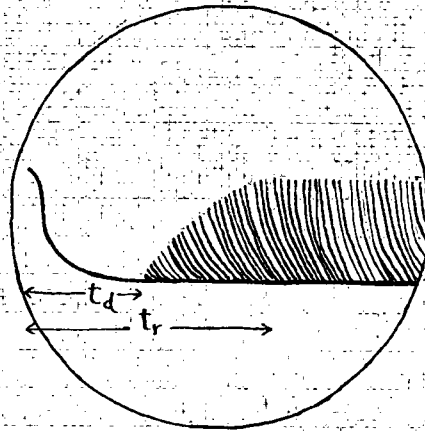


Plate 3. Dead Time Measurement.



$t_d = \text{Dead Time} = 50 \mu \text{ sec}$

$t_r = \text{Recovery Time} = 110 \mu \text{ sec}$

Tracing from negative of triggered oscilloscope pattern

troubles which arise in the counter.

The electronic equipment used in the main experiment is shown schematically in Fig. 6, and a photograph is shown in Plate 4. The head amplifier and stabilized power unit are of the same types as those mentioned previously. The coincidence mixer is a triple channel mixer but is used at present as a double channel mixer. Without special adjustment of the coincidence mixer, the resolving time was found to be 0.8 microsec., by feeding random pulses from two Geiger counters activated each by its own source into the mixer and counting the coincidences. From the usual formula $N_c = 2N_1N_2\tau$ the resolving time τ was deduced.

V.

Results.

Graphs of observed counting rates for typical specimen counters against voltage are given in Fig. 7. The bell type of β -counter and the rectangular and square gamma counters have useful plateaus, while the rectangular and square beta counters show very poor 'plateaus'.

One rectangular gamma counter was tested at reduced pressures and even with a total pressure of only 2 cms. had a plateau slope of 11% over a range of 100 volts. A square β -counter of 1.5 cms. total pressure showed a slope of 10% over 100 volts.

Although our value of M as previously calculated was 1.002 for the rectangular β -counters and hence, on the basis of Curran and Reid's report, should have operated successfully, it can be seen by the graph that this was not the case. The rectangular β -counters having windows of either Al or mica, when operated in

Fig 6. Schematic Electronic Arrangement.

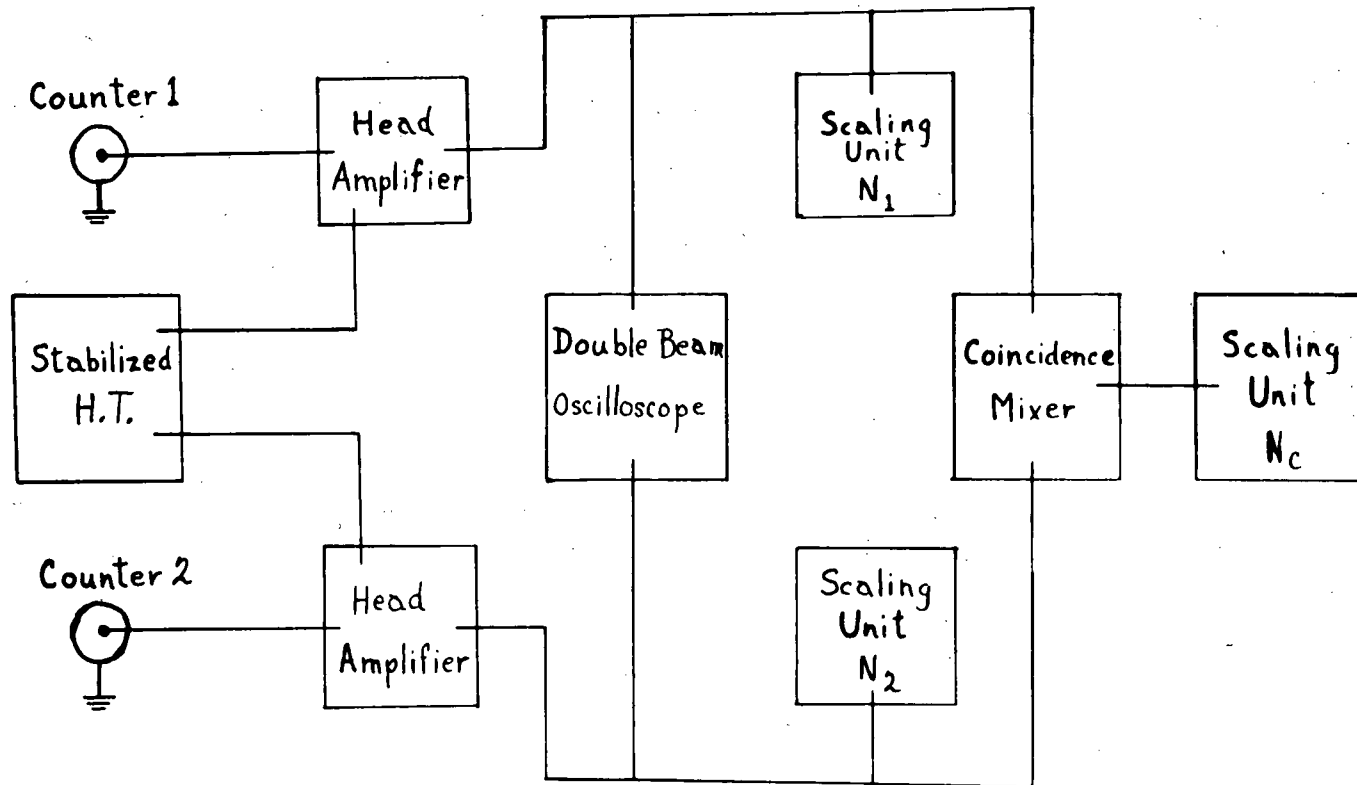


Plate 4. Spectrometer and Electronic Equipment.

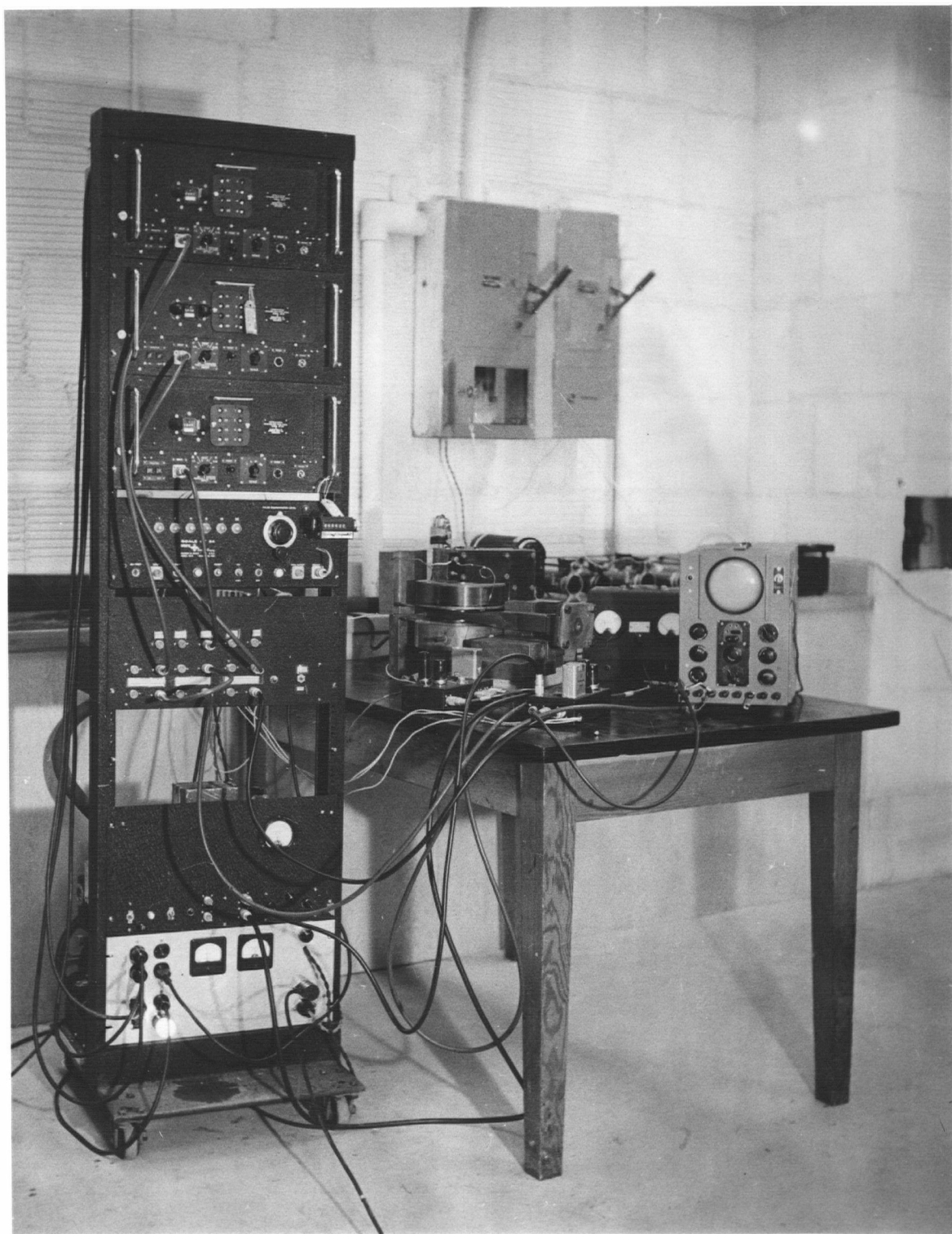


Fig. 7. Characteristic Curves of Various Counters.

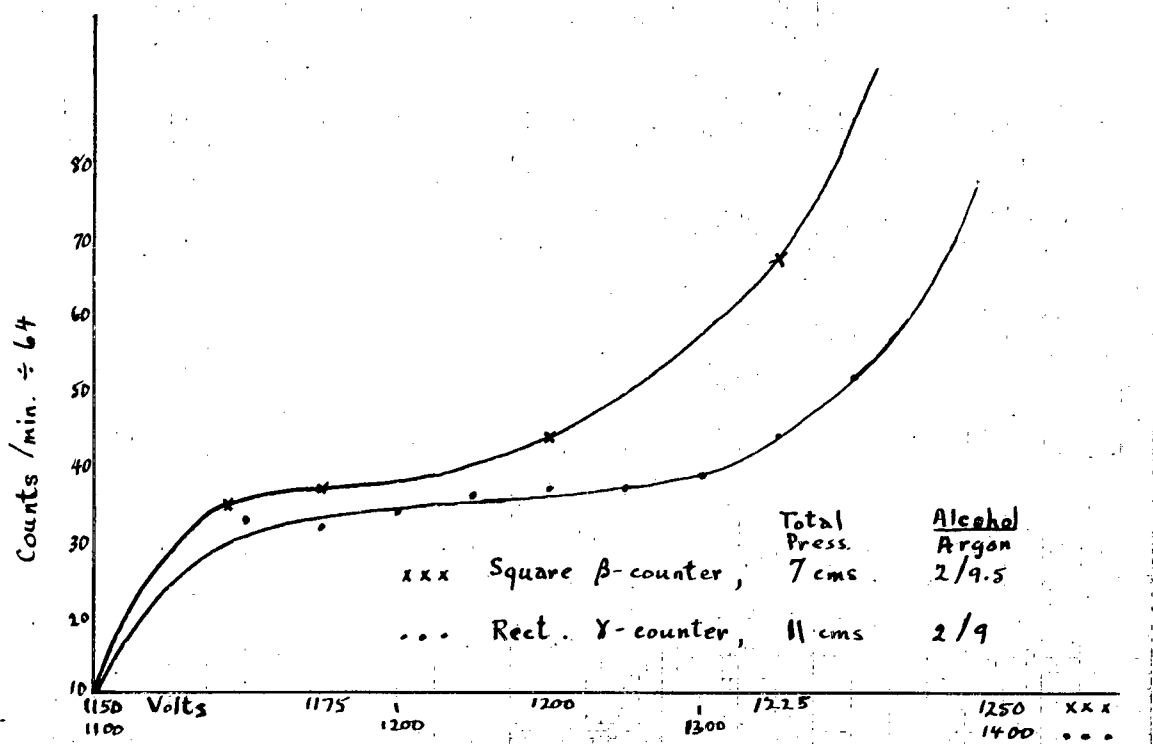
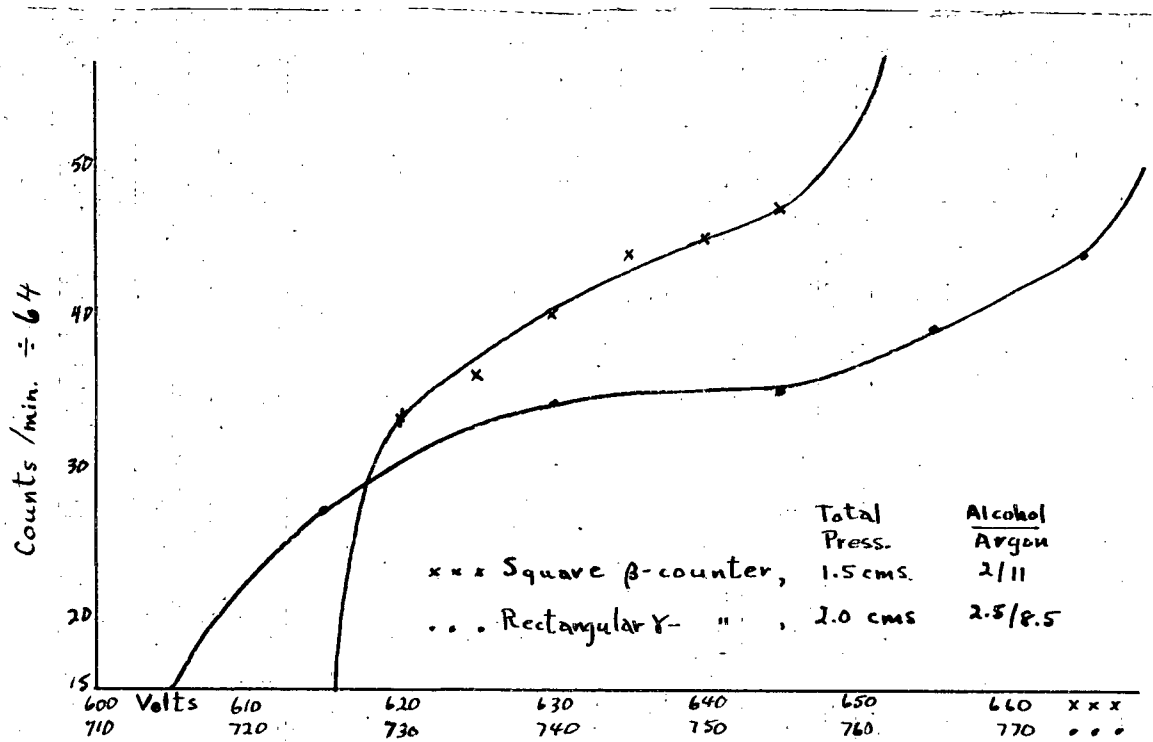
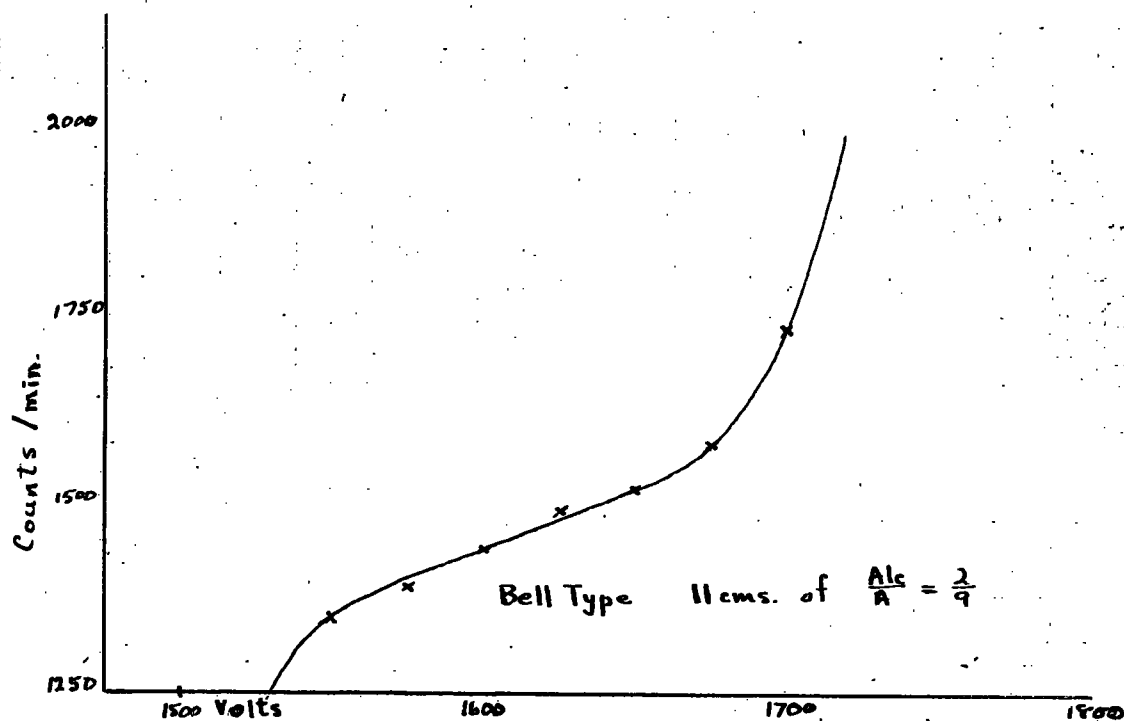
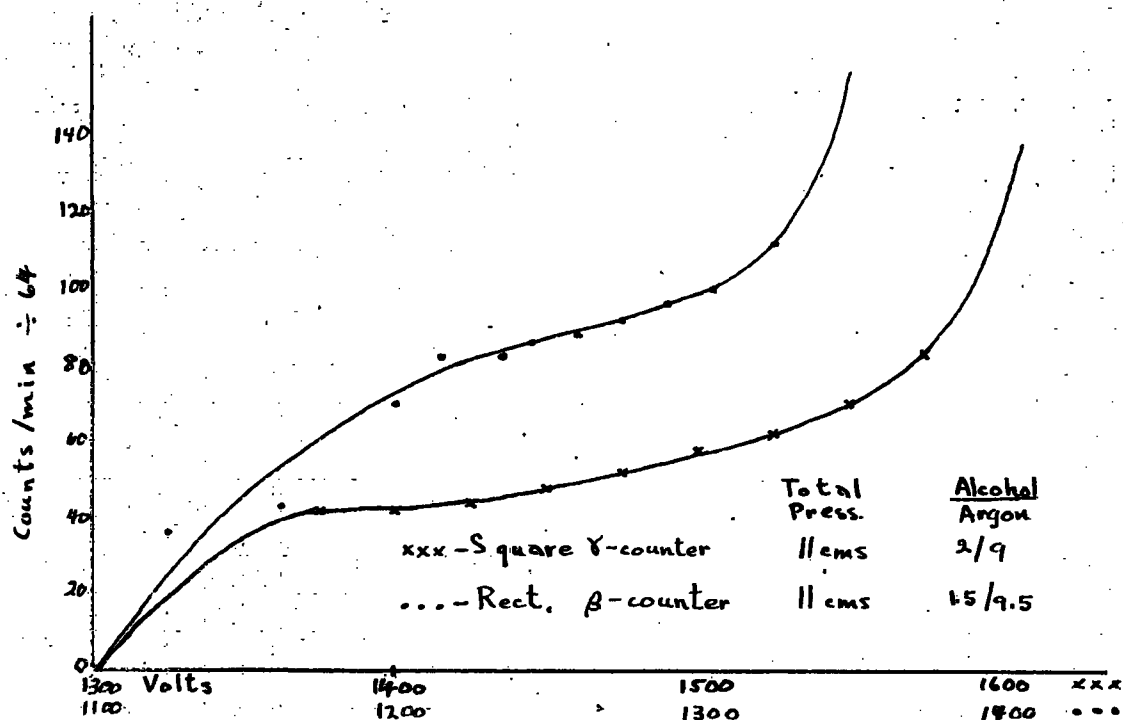


Fig. 7 (cont)



their 'Geiger' region, gave pulses of two distinct heights analogous to the effect of a bead at the centre of the wire, as was checked by plotting counting rate against scalar discriminator setting. This effect was only partially improved when the mica was made conducting on the outside by coating it with graphite, which seems to indicate, contrary to the usual theory, that a cathode mechanism such as the ejection of electrons from the cathode by quanta emitted in the initial avalanche, is involved in the spread of the discharge. To check this last hypothesis a point β source was placed against the window centre and the counting rate noted, then the source was moved to the window's edge and the counting rate again recorded. The value of the ratio

Efficiency at Edge was 1.34 for a graphite covered mica window,
Efficiency at Centre

and was 1.91 for the same window without graphite. The efficiency ratios were found to be the same when counting only large pulses and when counting both large and small pulses and also for a gamma source. Thus it would appear that the efficiency of the counter in the central region where the windows are is very low, even when the field distribution in this region is made uniform.

A further check was made by moving a point gamma source along the narrow edge of the counter and again the efficiency was found to drop in the central region. Tests made with collimated beta sources did not show any significant change in large to small pulse size as the region into which the betas were fired through the window was changed.

These results force one to believe that the cathode mechanism and not merely the field distortion prevents the central region

from having the same Geiger operating voltage as the rest of the counter. Naturally this results in very poor plateaus for these beta counters so far tested.

An attempt was made to measure the real efficiency of such a counter by plotting the value of N_c , the coincident counting rate, against voltage on the rectangular counter, the bell counter voltage being adjusted to be at the start of its slope. The spurious counts which arise in the rectangular counter as the voltage is raised should not affect the coincidence rate significantly until the state of almost continuous discharge is reached, for

$$N_c = 2 N_1 N_2 \tau,$$

with $N_2 = 50$ counts per sec. and hence N_c should also be about 50 counts per sec., and with $\tau = (1/2)(10)^{-6}$ sec., N_1 can be

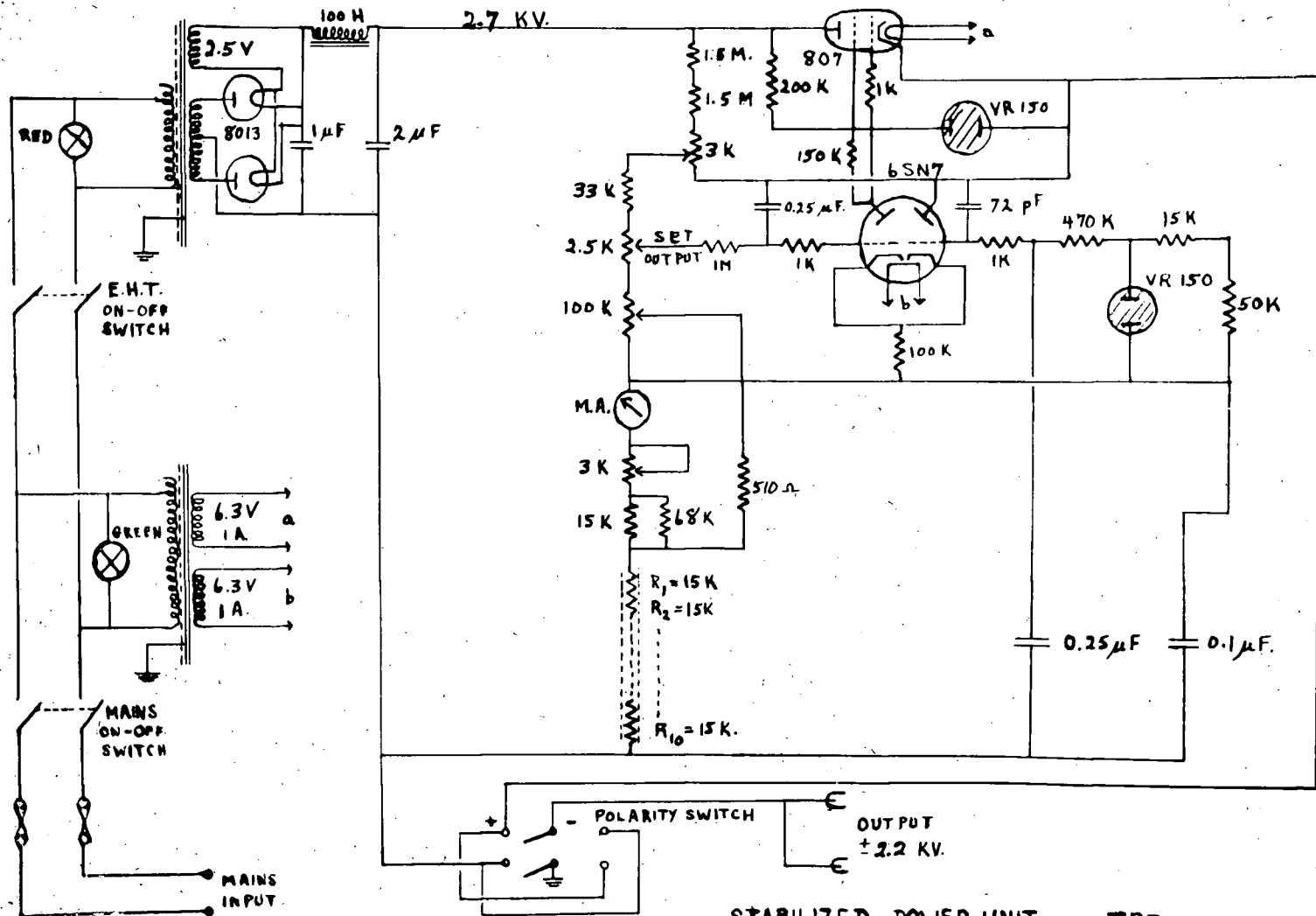
$$\frac{5}{(2)(50)(1/2)(10)^{-6}} = 10^5 \text{ counts per sec. before the chance}$$

'spurious' coincidence rate amounts to 10% of the real rate.

The results tabulated below bear this out and indicate that even at the highest voltages reached before discharge sets in the counter efficiency was only 96% instead of the computed figure of 100% for the filling.

Background N_2	118 counts in 2 min., constant value.						
Background N_c	20	"	" " "	"	"	"	"
Current (Amps)	0.2	0.1	0.05	0.15	0.25	0.35	0.45
N_c counts/2 min.	2268	1284	574	1822	2149	1388	395
N_2 Counts /2 min	2494	1441	702	2024	2428	1551	499
Efficiency= $(N_c - 20) / (N_2 - 118)$	95.2%	95.5%	94.8	94.8	92.5	95.3	98.5

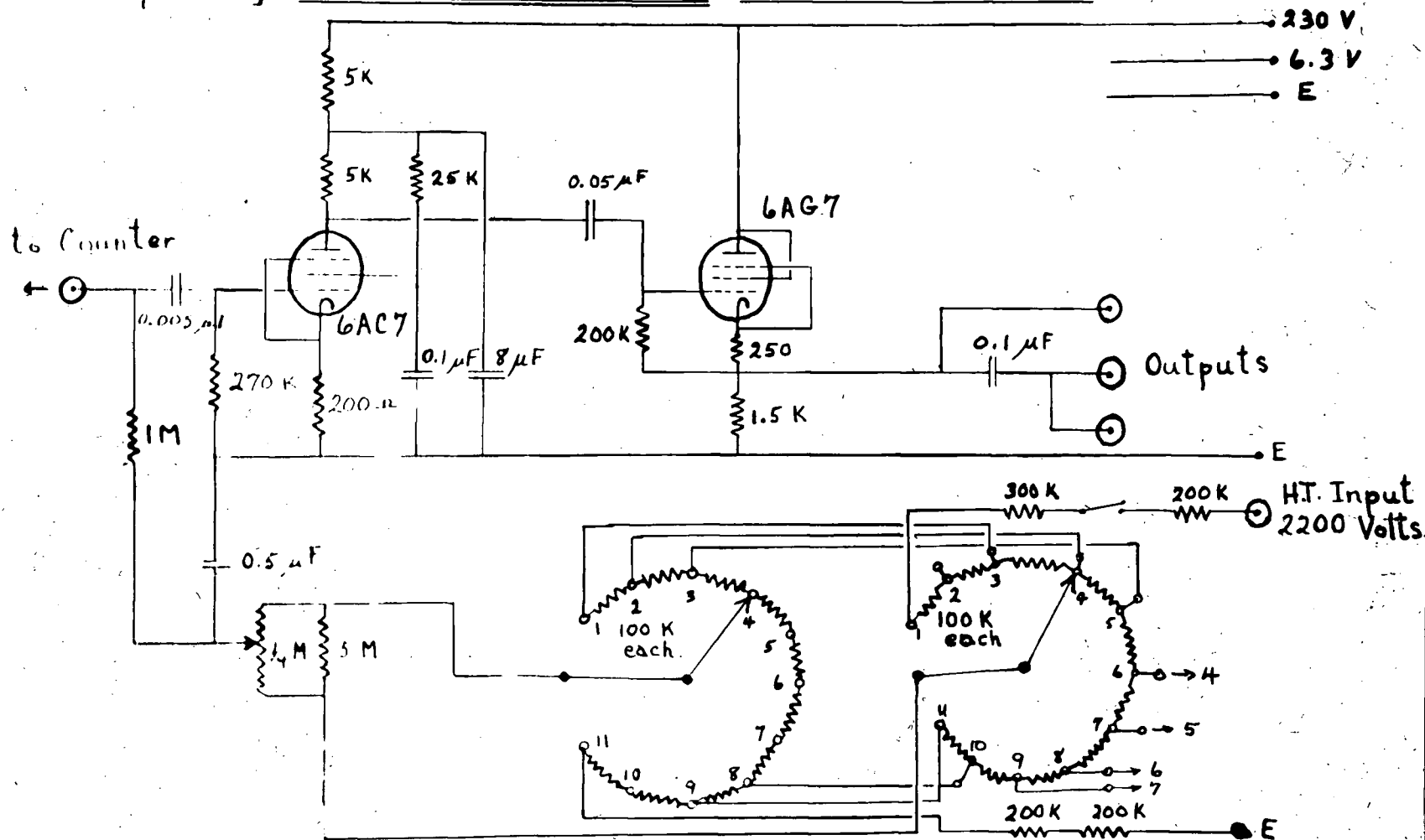
It was obvious at this stage that further work was needed to improve the behaviour of the rectangular beta counters. Firstly it is intended to sputter copper onto the mica in an attempt to equalize the work functions of the cathode materials and hence make the counter 100% efficient, and secondly to use a "quench unit" to reduce voltage on counter by 250 volts once discharge has occurred and keep it at this 'below Geiger threshold' level for 300 microsec. to eliminate multiple pulses and so flatten the plateau and so be sure that both counters will fire once only when an ionizing particle passes through their sensitive volumes. A unit has been made up to the circuit shown and appears to work satisfactorily.

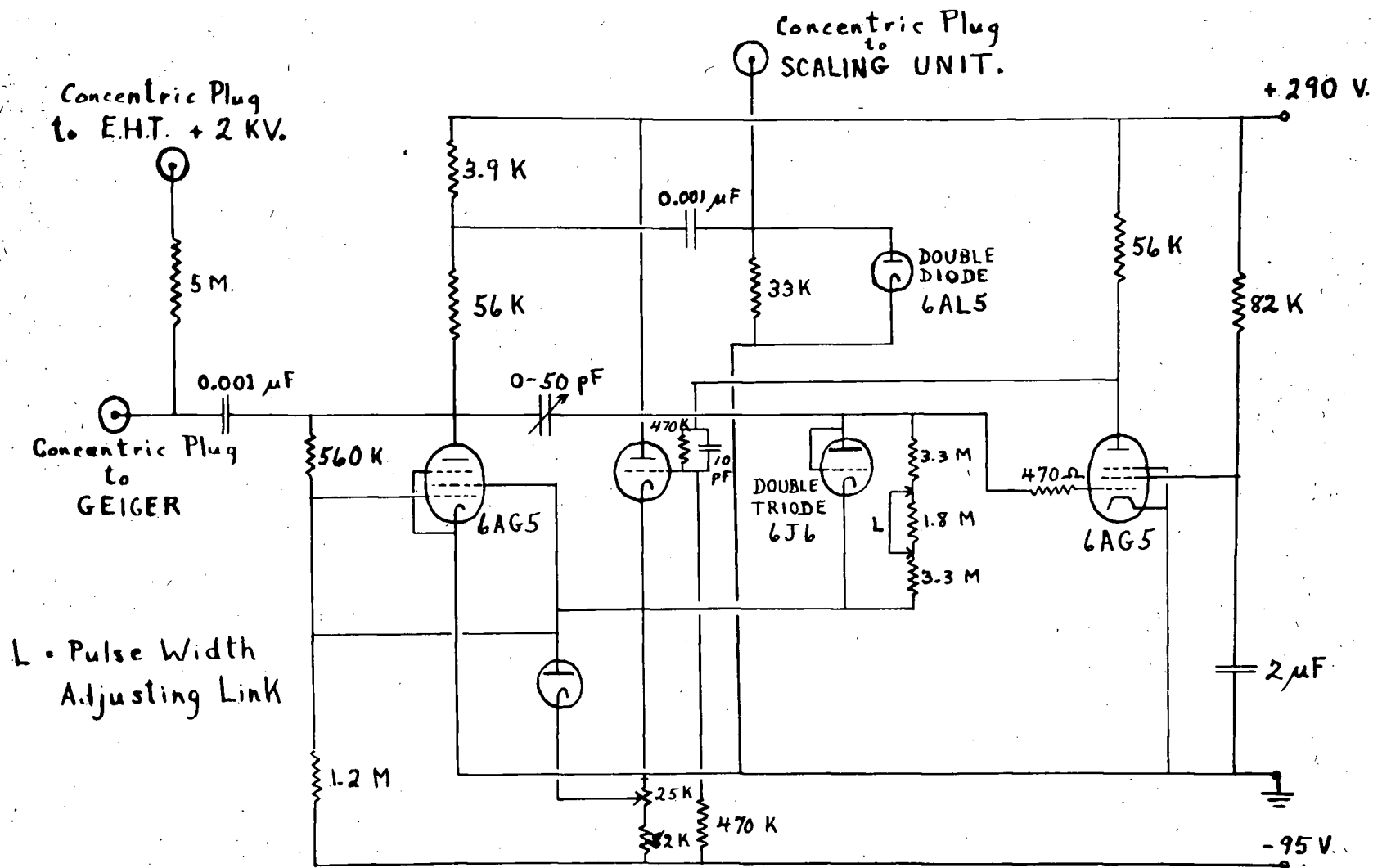


STABILIZED POWER UNIT - T.R.E. DESIGN.

HEAD AMPLIFIER

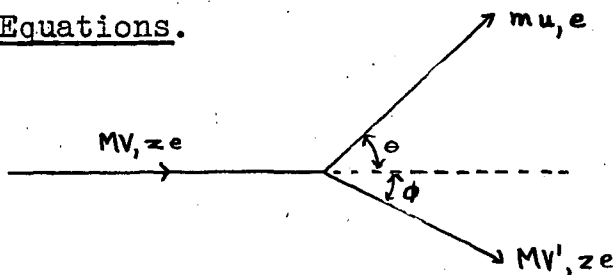
incorporating VOLTAGE CONTROL UNIT for T.R.E. Power Unit.





QUENCH UNIT FOR SELF QUENCH GEIGER.

APPENDIX 1.

THE ENERGY TRANSFER RELATIONS.1) Collision Equations.

Conservation of momentum requires that

$$a) \quad M^2 V'^2 \cos^2 \phi = m^2 u^2 \cos^2 \theta - 2MVmu \cos \theta + M^2 V^2$$

$$b) \quad M^2 V'^2 \sin^2 \phi = m^2 u^2 \sin^2 \theta$$

$$\text{hence c) } M^2 V'^2 = m^2 u^2 - 2MVmu \cos \theta + M^2 V^2$$

Conservation of Energy requires that

$$d) \quad 1/2 M V'^2 = 1/2 M V^2 - 1/2 m u^2$$

Combining C) and d) we get

$$e) \quad M^2 V'^2 = M^2 V^2 - M m u^2$$

$$f) \quad (Mm + m^2) u^2 = 2MVmu \cos \theta$$

$$\text{therefore } u = \frac{2MmV}{(M+m)m} \cos \theta$$

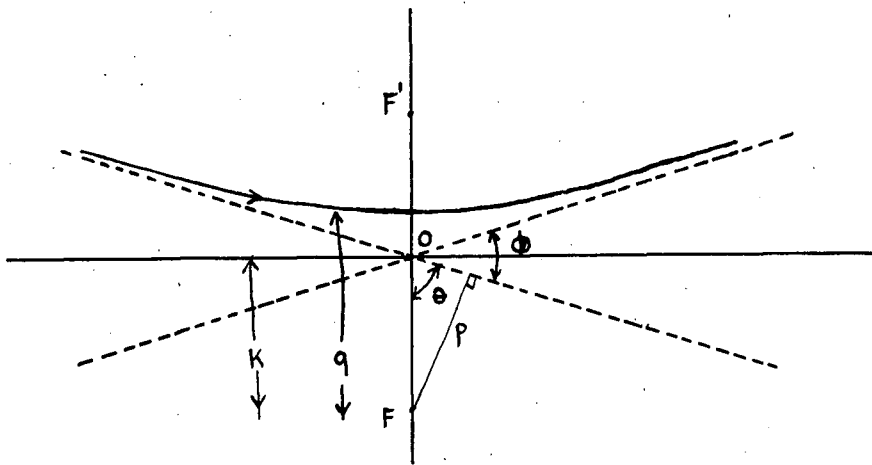
or $Q = \text{energy transferred}$

$$= 1/2 m u^2 = \frac{1}{2} \frac{m^4 M^2 V^2}{(m+M)^2} \cos^2 \theta$$

$$= \frac{(4Mm)}{(M+m)^2} 1/2 M V^2 \cos^2 \theta$$

2) Impact Parameter in Coulomb Field.

The particle will be attracted towards the electrons with a force of ze^2/r^2 and will describe a hyperbola with respect to the second particle, as shown below:



In this sketch

q = closest distance of approach.

p = impact parameter = closest distance of approach
if particle were not deflected.

k = distance of focus of hyperbola from origin.

ϕ = angle of deflection of incident particle.

θ = angle of deflection of electron with respect to
the incident direction.

The energy relationship at the closest distance of approach is

$$\frac{1}{2} mv^2 = \frac{1}{2} mv_0^2 + ze^2/q$$

where v = velocity of particle at an infinite distance

and v_0 = velocity of particle at origin.

Let $K = ze^2/mv^2$

hence $v_0^2/v^2 = 1 - 2K/q$

From angular momentum considerations we have

$$mvp = mv_0q$$

therefore $v_0^2/v^2 = p^2/q^2$

whence $p^2/q^2 = 1 - 2K/q$ (A)

From hyperbola geometry

$$q = e(1 + \cos \theta)$$

and $e = p/\sin \theta$

therefore $q = p(1 + \cos \theta)/(\sin \theta)$

and $p^2/q^2 = (1 - \cos^2 \theta) / (1 + \cos \theta)^2 = (1 - \cos \theta)(1 + \cos \theta)$

By substituting in (A) and multiplying by $(1 + \cos \theta)$ we get

$$(1 - \cos \theta) = 1 + \cos \theta - (2k/p)(\sin \theta)$$

hence $2k/p = 2 \cos \theta / \sin \theta$

and $p/k = \tan \theta$

now $\tan^2 \theta = \sin^2 \theta / \cos^2 \theta = (1 - \cos^2 \theta) / \cos^2 \theta$

therefore $1 = \cos^2 \theta (1 + p^2/k^2)$

therefore $\cos^2 \theta = 1 / (1 + \frac{m^2 p^2 v^4}{z^2 e^4})$

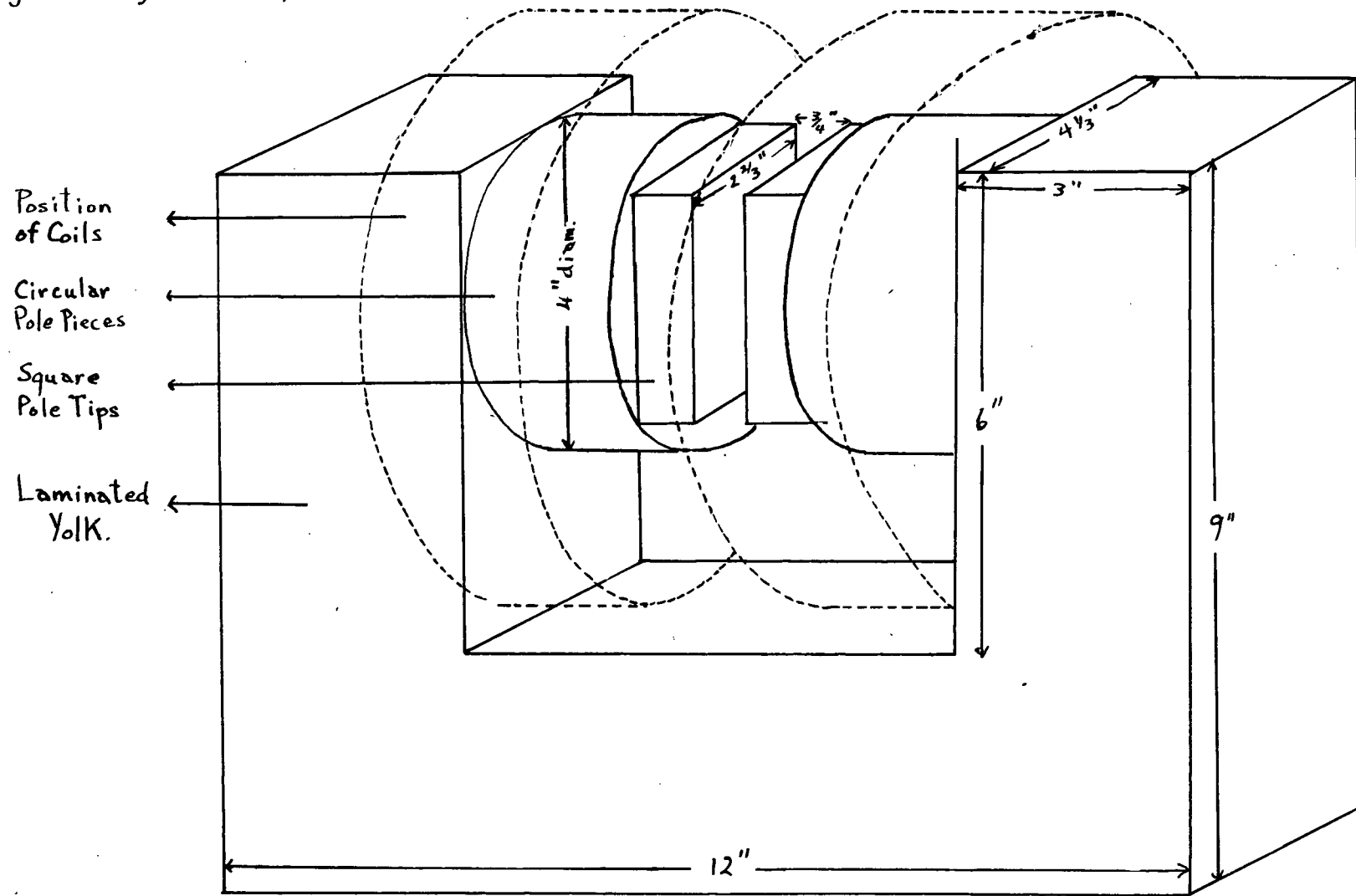
3) Evaluation of the Parameter p.

$$Q = 4mM / (M - m)^2 \quad T \cos^2 \theta = 4mM / (M - m)^2 \quad T / (1 + \frac{m^2 p^2 v^4}{z^2 e^4})$$

hence $m^2 p^2 v^4 / z^2 e^4 = ((4mMT / Q(M - m)^2) - 1)$

and $p^2 = (4T' / Q - 1)^2 e^4 / 4T'$

Fig. 6 Magnet for Spectrometer



APPENDIX 2.MAGNET DESIGN.

The 1/6 model of the magnet designed for the U.B.C. Van de Graaff generator has been modified to have an air gap of 3/4 in. Not shown in the diagram Fig. 8) are the coils which have been wound with 1100 turns of No. 13 gauge formex copper wire and provided with a water cooling layer such that a maximum current of 16 amps. can be passed through without serious overheating.

Hence $NI_{\max} = 17,600$ amp. turns

Therefore $M.M.F._{\max} = 4\pi NI/10 = 22,106$ gilberts
 $= H_{\text{air}} l_{\text{air}} + H_{\text{iron}} l_{\text{iron}}$

Assuming a maximum value for H_{iron} of 40 Oersteds to correspond to 10,000 lines/sq.cm. in the iron yolk used which is made of laminated low carbon steel, then

$$M.M.F._{\max} = 22,106 = H_{\text{air}} \left(\frac{12}{16}\right)(2.54) + (40)\left(\frac{172}{6}\right)(2.54)$$

Therefore $H_{\text{air}} = 11$ Kilogauss

With the model as used, saturation would obviously occur in the poles which are not tapered to allow for leakage flux. However as it is a 'model' with enlarged air gap, clearly the limitation lies not in iron saturation but in the current which can be passed through the coils.

APPENDIX 3.MAGNET PERFORMANCE.

This magnet's performance is given in the following graphs, as found by a ballistic galvanometer and search coil. On the next diagram for comparison is shown the performance of the magnet when modified to have wedge shaped pole pieces of approximately the same total area as the square tips.

Fig 9. Magnet Calibration at High Fields.

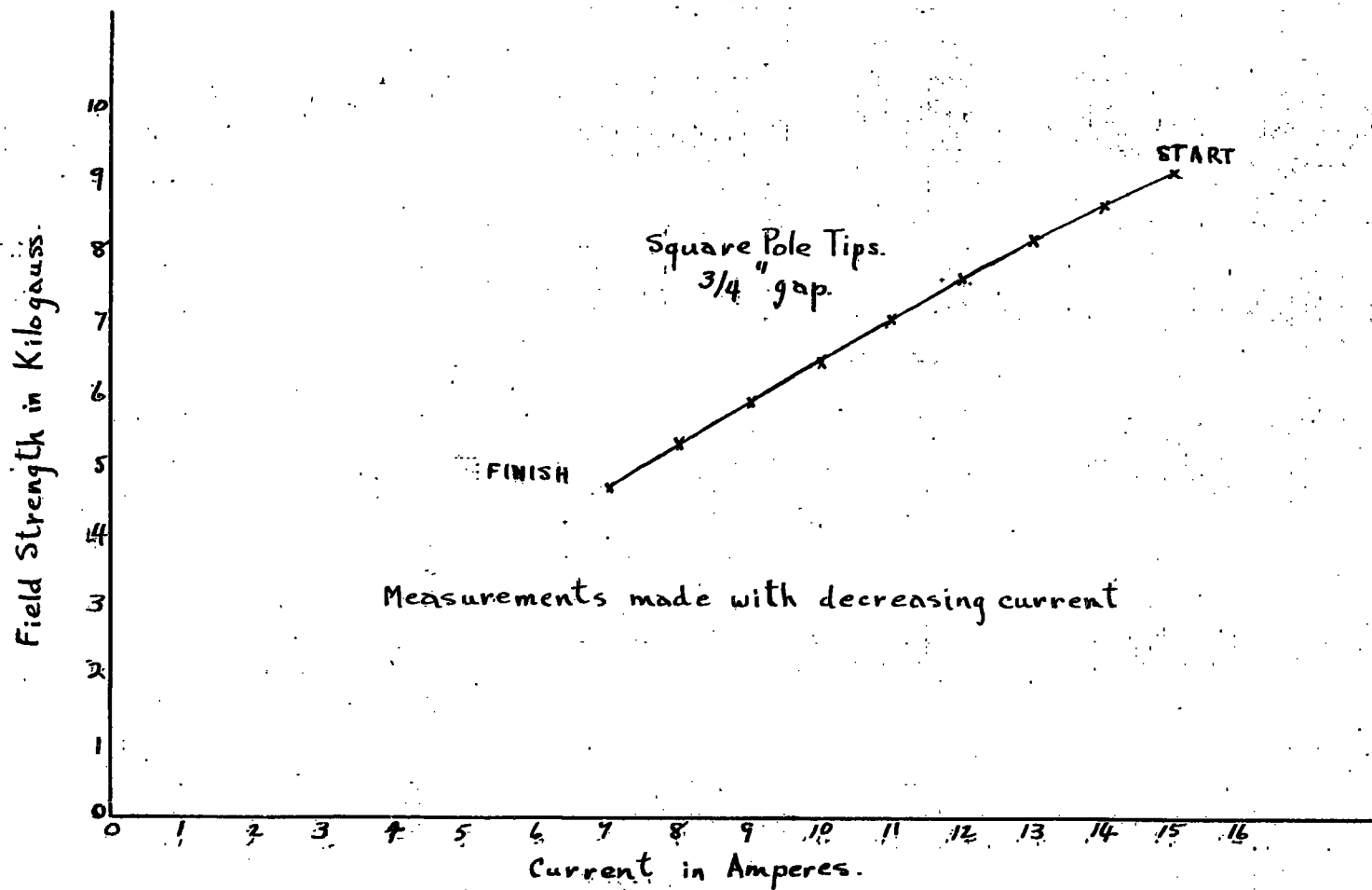
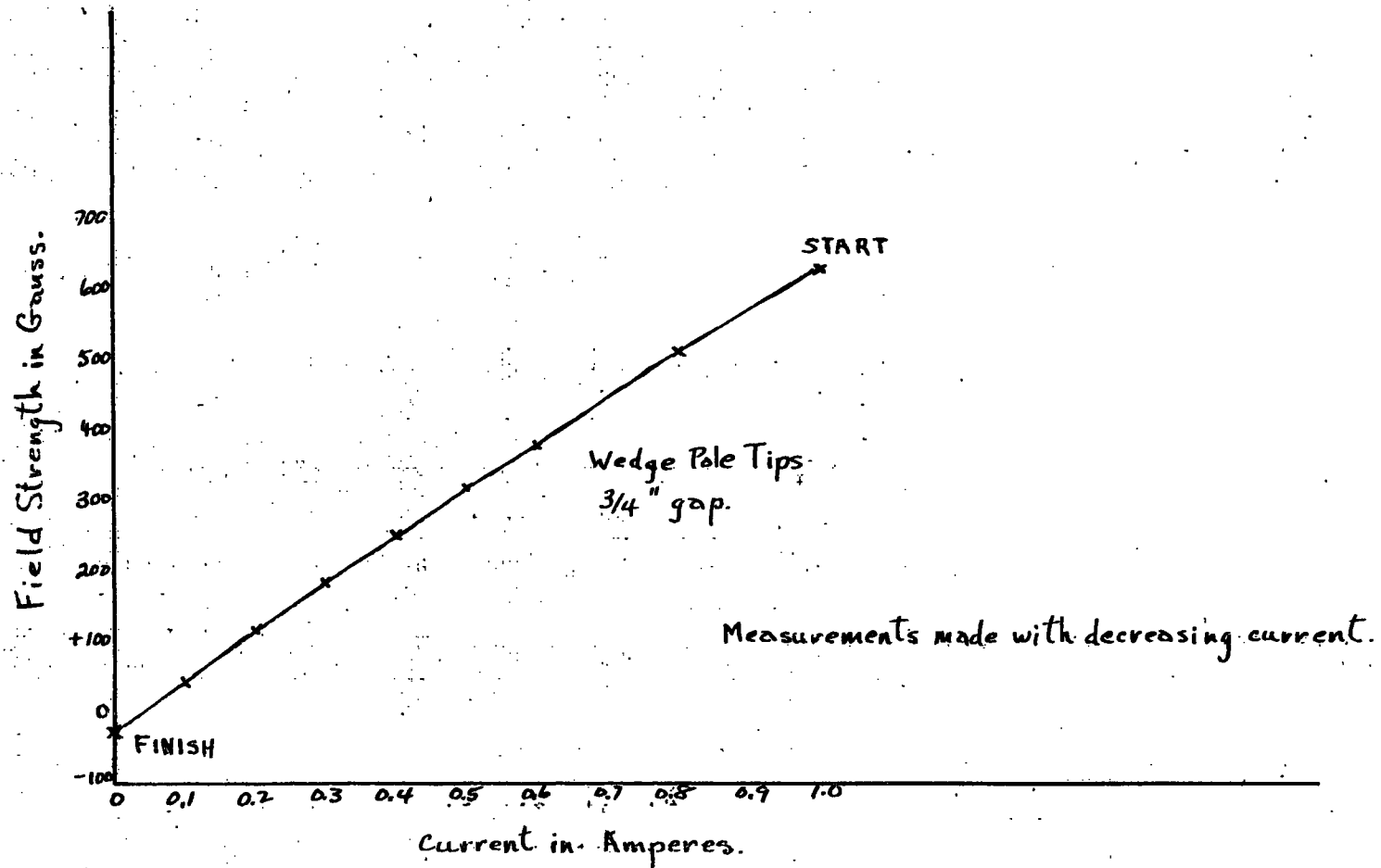
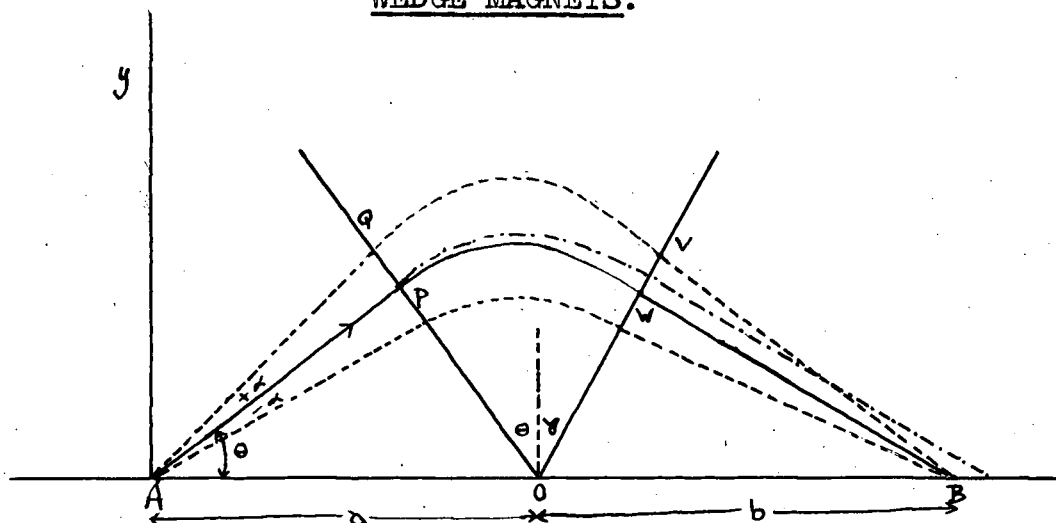


Fig.10 Magnet Calibration at Low Fields.



APPENDIX 4.MAGNETIC REFOCUSING OF ELECTRON PATHS.WEDGE MAGNETS.¹¹

The electron paths in the wedge are arcs of circles which are tangent at the edges of the field to the entrance and exit directions.

Consider a homogeneous beam of electrons of velocity v entering the entrance slit A making the angle θ with the base line and entering the field at P perpendicular to OPQ. If the field H is set to turn the beam into an arc of radius R where

$$R = a \sin \theta = OP = OW$$

where

$$HR = mv/e$$

then the centre of curvature of the arc will be O, and the exit beam will leave the wedge face at W perpendicular to OWV and enter the collector at B on the axis. All other beams e.g. AQV, with similar velocity and with the angle $\pm \alpha$ to AP will be refocussed so as to cross very close to B. That is, the best refocussing for a divergent pencil from A occurs at B, where

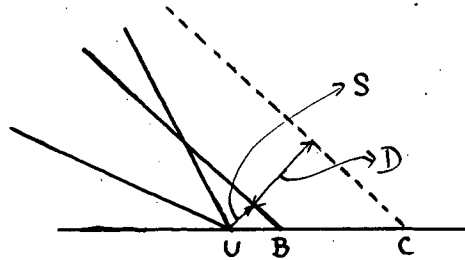
$$b = a \sin \theta / \sin \gamma$$

The departure from perfect focus - the Spread -

$$S = UB \sin \gamma, \text{ where } UB \text{ is the spread along the}$$

base line due to beams making angles of $\pm \alpha$ with the central path APWB.

If one considers a beam of velocity $v + \Delta v$ starting along AP, its radius of curvature will be larger and it will intersect the base line at C. The ability to separate two different velocities is called the dispersion - D -



$$D = BC \sin \gamma$$

D and S are defined as lengths perpendicular to the ray path as the slit of the collector will normally be placed perpendicular to the path. The ratio D/S gives a measure of the theoretical resolving power.

Derivation of Equation for S:

Let AOB be the x-axis and AY the y-axis. Consider the path AQVU of the beam at $+\alpha^\circ$ to the normal beam.

Coordinates of Q:

$$x_1 = a \cos \theta \cos (\theta + \alpha) / \cos \alpha$$

$$y_1 = a \cos \theta \sin (\theta + \alpha) / \cos \alpha$$

Coordinates of T:

$$x_2 = a ((\cos \theta \cos (\theta + \alpha) / \cos \alpha + \sin \theta \sin (\theta + \alpha) \cot \gamma))$$

$$y_2 = a ((\cos \theta \sin (\theta + \alpha) / \cos \alpha - \sin \theta \cos (\theta + \alpha) \cot \gamma))$$

Coordinates of V:

$$x_3 = \frac{+2(x_2 + y_2 \cot \gamma + a \cot^2 \gamma) + 4(x_2 + y_2 \cot \gamma + a \cot^2 \gamma)^2 - 4 \operatorname{cosec}^2 \gamma (x_2^2 + y_2^2 + 2ay_2 \cot \gamma + a^2 \cot^2 \gamma - a^2 \sin^2 \theta)}{2 \operatorname{cosec}^2 \gamma}$$

$$y_3 = (x_3 - a) \cot \gamma$$

obtained from intersection of a circle about T:

$$x^2 + y^2 + 2ax + 2by + c = 0$$

and second edge of wedge: $y = (\cot \gamma)x - a \cot \gamma$

Coordinates of U: (where intersects AO)

$$x_4 = x_3 + y_3 (y_3 - y_2) / (x_3 - x_2)$$

$$y_4 = 0$$

$$\text{Hence } S = 's' \sin \gamma = (a + b - x_4) \sin \gamma$$

$$= a(\sin \gamma + \sin \theta) - \sin \gamma ((x_3 + y_3(y_3 - y_2) / (x_3 - x_2)))$$

On expanding in powers of α and dropping terms in $\alpha^{3,4}$, etc.

$$S = a \alpha^2 / 2 ((\sin^2 \theta / \sin \gamma + \sin^2 \gamma / \sin \theta))$$

$$\text{When } \theta = \gamma, S = a \alpha^2 \sin \theta$$

$$\text{When } \theta = \gamma = 90^\circ, S = a \alpha^2 \text{ the usual } 180^\circ \text{ focussing case.}$$

Derivation of Equation for D:

Increase of Δv in v increases R by ΔR where $\Delta R / R = \Delta v / v$ if H is kept fixed.

If this is drawn in, one can find a new position and new angle at which the beam leaves the field and its intercept on ZOB. (Same procedure as before). Expanding gives:

$$D = a \sin \theta / \sin \gamma \cdot \Delta v / v (\sin \theta + \sin \gamma)$$

$$\text{When } \theta = \gamma, D = 2a \sin \theta \cdot \Delta v / v$$

$$\text{When } \theta = \gamma = 90^\circ, D = 2a \Delta v / v.$$

Resolving Power:

D/S is plotted as a function of θ and γ in units of $(2/\alpha^2)(\Delta v/v)$. The maximum value occurs for values such that

$2 \sin \gamma = \sin \theta$, giving the ratio $D/S = 1.1/3$ times that for 180° case of $\theta = 45^\circ, \gamma = 45^\circ$ case.

BIBLIOGRAPHY.

- (1) Anderson, C. D., Phys. Rev. 44, 406, 1933.
- (2) Anderson, C. D., Phys. Rev. 50, 263, 1936.
- (3) Barnothy, Phys. Rev. 74, 844, 1948.
- (4) Bethe H., Handbuch der Physik, Vol. 24, 1, p 523, 1933.
- (5) Brode, R. D., Rev. Mod. Phys. 11, 222, 1939.
- (6) Corson, D. R. and Brode, R. D., Phys. Rev. 53, 733, 1938.
- (7) Cosyns, M., Bull. Tech. Ass. Ing. Brux., 173-265, 1936.
- (8) Curran, S. C. and Reid, J. M., Nature 160, 866, 1947.
- (9) Curran, S. C. and Reid, J. M., Rev. Sc. Inst. 19, 67, 1948.
- (10) Danforth, W. E., and Ramsey, W. E., Phys. Rev. 49, 854, 1936.
- (11) Gray, D. H., Proc. Camb. Phil. Soc. 40, 72, 1944.
- (12) Gurney, R. W., Proc. Roy. Soc. A 107, 332, 1925.
- (13) Hazen, W. E., Phys. Rev. 63, 107, 1943.
- (14) Hazen, W. E., Phys. Rev. 65, 259, 1944.
- (15) Hereford, F. L., Phys. Rev. 74, 574, 1948.
- (16) Kunze, P., Zeits. f. Physik, 83, 1, 1933.
- (17) Livingston, Rev. Mod. Phys., July, 1937. p.265
- (18) Nicodemus, D. B., PhD. Thesis, Stanford, 1946.
- (19) Ramsey, W. E., Phys. Rev., 61, 97, 1942.
- (20) Schnieder, K., Ann. der Physik, 35, 445, 1939.
- (21) Skramstaß, H. K. and Loughridge, D. H., Phys. Rev. 50, 677, 1936.
- (22) Stephens, W. E., Phys. Rev. 45, 513, 1934.
- (23) Thomson, J. J., Phil. Mag. 23, 449, 1912.
- (24) Wilkinson, D. H., Phys. Rev.
- (25) Williams, E. J., Proc. Roy. Soc. A 135, 108, 1931.
- (26) Williams, E. J., and Terroux, F.R., Proc. Roy. Soc. A 126, 289, 1929
- (27) Zahn, C.T. and Spees, A.H., Phys. Rev., 58, 861, 1940.