**PHOTON ASYMMETRY
IN RADIATIVE MUON CAPTURE
ON CALCIUM**

By

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Abstract

The photon asymmetry (α_γ) and partial branching ratio ($R_{k>57}$), for radiative muon capture on ^{40}Ca , have been measured in order to determine the magnitude of the induced-pseudoscalar coupling constant, g_P . A large NaI(Tl) crystal (46cm $\phi \times$ 51cm) was used with an active NaI(Tl) converter (36cm $\times$ 30cm $\times$ 5cm) as the photon detector. The combined system had an energy resolution of 15% at 70 MeV; a factor of two improvement over previous similar experiments.

Simultaneous measurements of the photon asymmetry and the partial branching ratio have been performed twice in the past. From a theoretical stand-point the photon asymmetry measurement is of greater interest as it represents the least nuclear model-dependent way of extracting g_P . In the present experiment we have observed 3100 photons with energies greater than 57 MeV, after background subtraction. Of these, 2500 could be used in the determination of α_γ . A value of $\alpha_\gamma = 1.32^{+0.54}_{-0.47}$ is obtained from a fit to the photon time distribution.

For the first time in such measurements the photon asymmetry was visible in the time spectrum and an unconstrained fit is able to reproduce its known frequency and phase. The extracted asymmetry allows for the first time a meaningful limit to be placed on g_P which is free of the uncertainties associated with the extraction of g_P from the partial branching ratio. Our asymmetry result implies $g_P < 5.4g_P$, favouring a renormalization of the induced-pseudoscalar coupling constant. The partial branching ratio determined in this experiment is consistent with previous measurements. A model-dependent extraction of g_P from $R_{k>57}$ yields $g_P \sim (7 \pm 2)g_A$.

As a consequence of this work an improved experiment has recently been successfully performed at TRIUMF with increased statistics, improved energy resolution, and an improvement of approximately a factor of 15 in the signal-to-noise ratio.

We have also identified a potential systematic error of as large as 15% in the normalization of the photon asymmetry in past experiments. Such an error occurs from the use of the $V-A$ asymmetry distribution rather than a bound decay asymmetry distribution when performing Monte Carlo calculations of the decay electron asymmetry. This error affects the estimation of the residual muon polarization which is used to normalize the photon asymmetry.

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Chapter I

Introduction

I.1 The Muon

The muon was discovered in 1938 by Anderson and Neddermeyer [1] in cosmic rays. It immediately presented a puzzle as its mass of $\sim 200 m_e$ was close to that predicted for the carrier of the strong nuclear force by Yukawa [2]. Yet; if it was Yukawa's meson, it was surprising that it was not much more strongly absorbed in the atmosphere. A measurement of the lifetime of positive and negative muons, in carbon and in iron, by Conversi, Pancini and Piccioni in 1947 [3] established decisively that the muon interacted only weakly with nuclei. A major piece of the puzzle was solved with the discovery of the pion [4] in photographic emulsions exposed high in the Alps, and with the elucidation of the decay $\pi \rightarrow \mu + \nu$ [4,5]. Nature however had presented physics with its first "unnecessary" particle.

Since the days of the early cosmic ray studies the world of elementary particle physics has blossomed in an extraordinary fashion, as have the theoretical structures which attempt to explain this world. In the terminology of present day theories, the rather fundamental question "Why does the muon exist?" can be posed in many ways, but the answer remains unclear. This lack of understanding, at one level, has not prevented the study of muons from contributing to numerous fields of science including: elementary particle theory; quantum electrodynamics; atomic and molecular physics; nuclear structure; solid state physics and chemistry [6]. It is interesting to note that the process of nuclear muon capture [7,8], which played an important role in the initial identification of the muon as a weakly

interacting particle, is still of use today in studying the nature of the weak interaction. Before elaborating on the contribution that the study of radiative muon capture (RMC) on calcium can make to weak interaction theory, we present an overview of the theory.

I.2 Weak Interactions

I.2.1 The weak interaction Hamiltonian

For momentum transfers $q^2 \ll M_W^2$, where M_W is the mass of the W intermediate vector boson, weak interactions have been very successfully described [9] in terms of an effective theory of self-interacting vector currents. However, the theory fails when higher order weak processes are calculated, producing unrenormalizable infinities. The success of the “effective” theory is due in part to the very small contributions actually made by such higher order weak processes in a correct theory, which makes calculations performed in lowest order perturbation theory quite adequate in most cases. The complete renormalizable theory, in which the weak and electro-magnetic forces are unified, is the $SU(2) \times U(1)$ theory of Weinberg, Glashow and Salam [10,11,12], often referred to as the standard model. For the purposes of this thesis, the framework provided by the current-current interaction will be used. In this picture and within the previously stated limits of low momentum transfer, the existence of the intermediate vector bosons is ignored and the interaction between two incoming and two outgoing fermions takes place at a point. This four fermion interaction was first proposed by Fermi [13] and is also known as the universal Fermi interaction (UFI).

The general effective weak interaction Hamiltonian, $\mathcal{H}_w$, has the form

$$\mathcal{H}_w = -\frac{G}{2\sqrt{2}} [\mathcal{J}_\lambda \mathcal{J}_\lambda^* + \mathcal{J}_\lambda^* \mathcal{J}_\lambda] \quad (1)$$

where G is the Fermi coupling constant and $\mathcal{J}_\lambda$ is a vector current composed of a

hadronic and a leptonic part

$$\mathcal{J}_\lambda = \mathcal{J}_\lambda^H + l_\lambda . \quad (2)$$

The weak interactions do not conserve strangeness, so the hadronic current can be decomposed into strangeness conserving ($\Delta S = 0$) and strangeness changing ($\Delta S = \pm 1$) parts

$$\mathcal{J}_\lambda^H = \cos \theta \mathcal{J}_\lambda^{\Delta S=0} + \sin \theta \mathcal{J}_\lambda^{\Delta S=\pm 1} \quad (3)$$

where the relative strength of the two parts has been parametrized by θ , the Cabibbo angle [14]. This decomposition of the weak current forms a convenient basis for the classification of weak interactions. The various terms in the expansion of $\mathcal{J}_\lambda \mathcal{J}_\lambda^*$ are either: pure leptonic, giving rise to processes such as μ decay ($\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$); or semi-leptonic, giving rise to strangeness conserving processes such as μ^- capture ($\mu^- + p \rightarrow n + \nu_\mu$) and strangeness changing processes such as Λ decay ($\Lambda \rightarrow p + e^- + \bar{\nu}_e$); or non-leptonic, giving rise to strangeness changing processes such as $\Lambda \rightarrow p + \pi^-$. While strangeness conserving non-leptonic processes are possible, they compete with much stronger hadronic and electromagnetic processes and consequently are not easily seen.

I.2.2 The leptonic current

The leptonic current is found to have the explicit form

$$l_\lambda = -i\bar{\psi}_{\nu_e} \gamma_\lambda (1 - \gamma_5) \psi_e - i\bar{\psi}_{\nu_\mu} \gamma_\lambda (1 - \gamma_5) \psi_\mu \quad (4)$$

where ψ_l is a lepton field operator, γ_λ are the Dirac γ -matrices and $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$. In principle a more general current could be constructed from the five bilinear quantities in table I.

However, the vector minus axial vector (V-A) nature of the weak leptonic current is well established from muon decay and inverse muon decay experiments

Table I: The five bilinear quantities with definite properties under Lorentz transformations. Here $\sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$.

bilinear covariant	interaction	properties under space inversion (P)
$\bar{\psi}\psi$	scalar	+ under P
$\bar{\psi}\gamma_\mu\psi$	vector	space compts.: -under P
$\bar{\psi}\sigma_{\mu\nu}\psi$	tensor	
$\bar{\psi}\gamma_5\gamma_\mu\psi$	axial vector	space compts.: + under P
$\bar{\psi}\gamma_5\psi$	pseudoscalar	- under P

($\nu_\mu + e^- \rightarrow \mu^- + \nu_e$). Fetscher et al. [15] have summarized the constraints that experimental results now place on the coupling constants associated with each interaction term in the most general weak pure leptonic Hamiltonian. By allowing for complex coupling constants and for coupling to both right and left handed fermions, their most general Hamiltonian contains 20 real parameters minus one common arbitrary phase. Current experiments now put limits on all 19 free parameters that are consistent with a pure V-A weak leptonic current as given in equation 4. The explicit form of equation 4 is also a statement of two other experimental observations. One, is a statement of the similarity of the muon and the electron. There appears to be no justification for the introduction of separate coupling constants, hence the electron and the muon terms in equation 4 are given equal weight. This is often referred to as $\mu - e$ universality. The second observation is a statement of the difference between the electron and the muon. Equation 4 explicitly conserves the number of muons and electrons separately. This separation of the muon and electron families ($\mu^-, \mu^+, \nu_\mu, \bar{\nu}_\mu$ and $e^-, e^+, \nu_e, \bar{\nu}_e$) has been incorporated into the standard model. A violation of these conservation laws, if found, could help point the way to the next level of unification of the forces of nature, but as yet only very small upper limits have been placed on the

rates of such lepton number (muon and electron separately) changing processes.

A consequence of the exact V–A nature of the weak leptonic current is that the weak interaction couples only to left-handed particles (or right-handed anti-particles). This constitutes a violation of parity invariance. The $(1 - \gamma_5)$ in equation 4 has the effect of projecting out pure left-handed leptons. This absolute asymmetry in the way nature treats left and right-handed leptons is termed maximal parity violation.

I.2.3 The hadronic current

The hadronic current is not as clearly specified by theory and also not as well determined by experiment as is the leptonic current. Indeed, it is for the weak hadronic current that RMC in calcium has the potential of contributing to knowledge of the weak interactions.

At this point, discussion will be restricted to the experimentally accessible strangeness conserving processes. These are the semi-leptonic processes of $\beta^\pm$ decay, e^- -capture and μ^- -capture. The much less accessible process of neutrino scattering has made significant contributions to weak interaction physics but will not be considered further here.

Like the leptonic current, the hadronic current has both vector and axial vector parts.

$$\mathcal{J}_\lambda^H \Delta S=0 = \cos \theta (\mathcal{V}_\lambda + \mathcal{A}_\lambda) \quad (5)$$

The presence of the strong interaction considerably complicates the structure of these currents. However, they must be constructed from Lorentz covariants and this leads to convenient parameterizations. The only available covariants must be constructed from the γ -matrices and the four-momenta of the proton and neutron, p and n . For convenience, the momentum transfer is defined as $q = p - n$. Also

$q^2 = (p - n)^2$ is a Lorentz scalar. With these ingredients, the most general forms of the weak hadronic vector and axial vector currents are: [9]

$$\mathcal{V}_\lambda = i\bar{\psi}_p(f_V\gamma_\lambda + f_M\sigma_{\lambda\nu}q_\nu + if_Sq_\lambda)\psi_n \quad (6)$$

$$\mathcal{A}_\lambda = i\bar{\psi}_p(-f_A\gamma_\lambda\gamma_5 + if_P\gamma_5q_\lambda + f_T\sigma_{\lambda\nu}q_\nu\gamma_5)\psi_n \quad (7)$$

where ψ_p and ψ_n are nucleon field operators; and the f_α are form factors which are, in general, complex functions of q^2 . Assuming time-reversal invariance the f_α are required to be real functions. f_V and f_A are known as the main vector and main axial vector form factors. The other four form factors are said to be ‘induced’ by the strong interaction and occur in terms which are proportional to the momentum transfer. f_M is called the weak magnetism form factor in analogy with the anomalous magnetic moment form factor, F_2 , in the coupling between nucleons and the electromagnetic field.

$$\mathcal{J}_\lambda^{em} = i\bar{\psi}_{p'}(F_1\gamma_\lambda - F_2\sigma_{\lambda\nu}q_\nu)\psi_p \quad (8)$$

f_S is called the induced-scalar form factor. That the Lorentz vector if_Sq_λ gives rise to an induced-scalar interaction, is not immediately obvious. However, from energy-momentum conservation, it is easily shown [9] that if_Sq_λ can be replaced with $m_l f_S$ when a weak interaction matrix element involving $\mathcal{V}_\lambda$ is evaluated. Here m_l is the mass of the lepton participating in the weak process. Similarly, $if_P\gamma_5q_\lambda$ can be replaced by $m_l f_P\gamma_5$ and hence f_P is known as the induced-pseudoscalar form factor. Analogously, f_T is the induced-tensor form factor.

In the literature of nuclear muon capture and beta decay, ‘coupling constants’, g_α , as well as form factors are often quoted. They are related to one another as follows:

$$g_V(q^2) = f_V(q^2) \quad (9)$$

$$g_M(q^2) = 2M f_M(q^2) \quad (10)$$

$$g_S(q^2) = m_l f_S(q^2) \quad (11)$$

$$g_A(q^2) = f_A(q^2) \quad (12)$$

$$g_P(q^2) = m_l f_P(q^2) \quad (13)$$

$$g_T(q^2) = 2M f_T(q^2) \quad (14)$$

where M refers to the nucleon mass. The $g_\alpha(q^2)$ are in general only weak functions of q^2 , as will be shown, over the limited range of q^2 probed in beta decay and even in muon capture, hence it is not inappropriate to call them constants. $g_P(q^2)$ is a notable exception. The task of measuring these coupling constants is considerably simplified by several hypotheses: the conserved vector current hypothesis (CVC); the hypothesis of G invariance; the partially conserved axial current hypothesis (PCAC); and the $\mu - e$ universality already mentioned. The consequences of these hypotheses and some of the evidence for their validity is given below.

CVC [16] states that the divergence of the vector current is zero.

$$\partial_\lambda \mathcal{V}_\lambda = 0 \quad (15)$$

This gives rise to the conservation of the weak vector current and implies that the weak interactions of a bare hadron are the same as for a hadron plus its virtual pion cloud in the zero momentum transfer limit. Also, CVC directly predicts

$$g_S(q^2) = 0 . \quad (16)$$

The strongest statement of CVC, also called the isotriplet vector current hypothesis (IVC) [16], identifies the weak vector current as part of an isovector triplet together with the isovector part of the electromagnetic current. This allows $g_V(q^2)$ and $g_M(q^2)$ to be expressed in terms of electromagnetic form factors that can be obtained with precision from electron-proton and electron-deuteron

scattering [17]. Explicitly [18],

$$g_V(q^2) = \frac{G_D(q^2)}{(1 + q^2/4M^2)} \left[1 + \frac{q^2}{4M^2}(\mu_p + 1) \right] \quad (17)$$

$$g_M(q^2) = \frac{G_D(q^2)}{(1 + q^2/4M^2)} \left[\mu_p - \mu_n + \frac{q^2}{4M^2}\mu_n \right] \quad (18)$$

where

$$G_D(q^2) = \left(1 + \frac{q^2}{0.71 \text{GeV}^2} \right)^{-2} \quad (19)$$

and μ_p and μ_n are the anomalous magnetic moments of the proton and neutron respectively. At $q^2 = 0$ this reduces to

$$g_V(0) = 1 \quad (20)$$

$$g_M(0) = \mu_p - \mu_n = 3.706. \quad (21)$$

Many experimental tests of CVC exist. The branching ratio for π^+ beta decay of $R = (1.026 \pm 0.039) \times 10^{-8}$ [19] is found to be in good agreement with CVC predictions ($R = 1.07 \times 10^{-8}$), as is a measure of weak magnetism as observed in the beta decay of ^{12}B and ^{12}N to ^{12}C . This latter measurement [20] gives $g_M/g_V = 3.83 \pm 0.79$.

In 1958 Weinberg [21] gave the label ‘second-class’ currents to the g_T and g_S interaction terms and proposed several tests to search for the existence of such currents. The classification was made on the basis of G-parity (charge conjugation followed by 180° rotation in isospin space). The g_T and g_S terms were found to have G-parity opposite to the other axial vector and vector terms respectively. g_V , g_A , g_P and g_M are said to be first-class currents. Since the strong interaction conserves G-parity it is expected that a current induced by the strong interaction should have definite G-parity. On the basis of this and the success of current algebras, which would lose their validity if G-parity were not conserved [22], G invariance of the weak hadronic current is hypothesized. That is

$$g_T(q^2) = 0 \quad (22)$$

$$g_S(q^2) = 0. \quad (23)$$

The possibility of G-parity nonconservation has been raised [23]. An extensive least squares study of all available beta decay coupling constants yields [24]

$$g_T(0) = -0.37 \pm 0.28 \quad (24)$$

$$g_S(0) = -0.0005 \pm 0.0031 \quad (25)$$

which in the case of g_T is not a strong argument for G invariance. Note that CVC, which is firmly established, also predicts $g_S(0) = 0$; so a precision measurement of g_T is the sole test of G invariance. More recent work [25] gives $g_T(0) = -0.06 \pm 0.49$.

To this point it has been shown that $g_V(q^2)$ and $g_M(q^2)$ are well known and consistent with CVC; that $g_S \approx 0$; and that g_T , though poorly determined, is small and is not inconsistent with G invariance. PCAC places restrictions on the remaining two interaction terms, g_A and g_P .

Like CVC, PCAC [26,27] can be stated in terms of the divergence of the current,

$$\partial_\lambda \mathcal{A}_\lambda = f_\pi m_\pi^2 \phi_\pi \quad (26)$$

where f_π is the pion decay constant and ϕ_π is the pion field. In the limit $m_\pi \rightarrow 0$, the axial vector current would be conserved. It is partially conserved (that is, $g_A \approx g_V$) because the pion mass is small in comparison to the nucleon mass. The ratio $|g_A/g_V|$ is often called λ and in the case of beta decay and electron capture, where the low momentum transfer minimizes the effect of the induced couplings, the hadronic current is said to have a $V-\lambda A$ nature. For a free nucleon (the case of beta decay of the neutron) the main axial vector coupling constant is found to be $g_A = -1.262 \pm 0.005$ [28]. PCAC does not directly predict this value, but by

using experimental π -nucleon scattering cross-sections, related to weak axial current matrix elements by PCAC, good agreement has been obtained: Adler [29] calculated $g_A(0) = -1.24$ and Weisberger [30] arrived at $g_A(0) = -1.16$. Using dispersion-relation techniques and assuming that the divergence of the axial vector current is dominated by the pion at low q^2 [31] Goldberger and Treiman [32] arrived at

$$g_A(0) = \frac{g_{\pi NN} f_\pi}{\sqrt{2}M} = 1.32 \pm 0.02 \quad (27)$$

where $g_{\pi NN}$ is the pion-nucleon coupling constant. The numerical value is from the substitution of current estimates of the parameters. The 6% difference between this value and the experimental value from neutron decay is called the Goldberger-Treiman discrepancy. Using similar techniques, Wolfenstein [33] extended the work of Goldberger and Treiman to the pseudoscalar coupling constant finding

$$g_P(q^2) = \frac{\sqrt{2}f_\pi g_{\pi NN} m_l}{q^2 + m_\pi^2} - \frac{4M m_l g_A(0)}{m_A^2} - m_l I'(0) \quad (28)$$

where $m_\mu I'(0)$, in the case of muon capture, is conservatively estimated to be $< 0.1g_A(0)$ by assuming that this term is totally responsible for the Goldberger-Treiman discrepancy in g_A . The second term in equation 28 is proportional to the derivative with respect to q^2 of $g_A(q^2)$. Here a substitution has been made using the q^2 dependence of g_A experimentally determined from ν_μ and $\bar{\nu}_\mu$ scattering [18]

$$g_A(q^2) = g_A(0) \left(1 + \frac{q^2}{m_A^2}\right)^{-2} \quad (29)$$

where

$$m_A = 0.93 \pm 0.03 \text{ GeV} . \quad (30)$$

At the momentum transfer of ordinary muon capture (OMC) on hydrogen

$$\mu^- + p \rightarrow n + \nu_\mu \quad (31)$$

Table II: Values of g_P as deduced from OMC experiments on hydrogen.
The values presented are from the analysis of Bardin et al. [34].

Reference	Target	g_P
Bleser et al. 1962 [35]	liquid H ₂	6.0±8.0
Rothberg et al. 1963 [36]	liquid H ₂	11.0±4.3
Alberigi Quaranta et al. 1969 [37]	gas H ₂	10.3±3.9
Bystritskii et al. 1974 [38]	gas H ₂	7.9±5.9
Bardin et al. 1981 [39]	liquid H ₂	7.1±3.0
Average		8.7±1.9

$$q^2 = (E_\nu - p_\mu)^2 = 0.88m_\mu^2 \quad (32)$$

equation 28 gives

$$g_P = 8.4 \quad \text{or} \quad \frac{g_P}{g_A} = 6.7 . \quad (33)$$

This value of g_P is known as the Wolfenstein estimate. It is interesting to note that the second term in equation 28 represents only a 7% correction to g_P . The first term, due to the pion pole, dominates the induced-pseudoscalar form factor.

Mesonic effects are expected to modify both g_P and g_A when these interaction terms are studied in complex nuclei. g_P is expected to assume the Wolfenstein value only for free nucleons or very light nuclei. The only available data come from OMC on hydrogen and are summarized in table II. While the average of the OMC results, and indeed each individual result, is consistent with the Wolfenstein estimate, the errors are large and the measurements are subject to some uncertain systematic effects. Despite the level of agreement achieved, more precise measurements are still highly desirable.

I.3 Mesons in Nuclei

Much of nuclear physics has been built on the very successful picture of nuclei composed simply of protons and neutrons. It is well established that the nucleons themselves are constructed of quarks and that the basic nucleon-nucleon force at

ranges of about a Fermi or larger results from the exchange of mesons. If this is so then at some point the description of the nucleus in terms of protons and neutrons alone must break down, and the non-nucleonic degrees of freedom (mesons, quarks) become necessary for an the accurate description of the nucleus. A very active and interesting pursuit in nuclear physics at the moment is to search for manifestations of these non-nucleonic degrees of freedom. Mesons and quarks could manifest themselves in many ways. Here we shall discuss only what are termed meson exchange currents.

Three approaches are used in the calculation of meson exchange corrections. The first is to perform explicit calculations of the meson exchange diagrams. As meson exchange between nucleons is inherently a two-body process this involves the use of two-body wavefunctions. A second approach uses hadron form-factors calculated in some renormalization scheme but otherwise stays within the framework of the impulse approximation. Ohta and Wakamatsu [40] for instance consider π exchange between nucleons during muon capture but obtain an effective one-body correction by averaging their two-body Hamiltonian over one of the nucleon coordinates in infinite nuclear matter. The remainder of the calculation is then performed in the impulse approximation treating the nucleus as a collection of individual nucleons. Matrix elements of their one-body operators are summed over the individual nucleons assuming that all the many-body physics is contained in the wavefunctions used in the evaluation of the matrix elements. Self consistency (double counting) problems [41] can occur in meson exchange calculations as one is using wavefunctions derived from an inter-nucleon potential resulting itself from meson exchange. The third approach uses nuclear form-factors in the elementary particle model. Here CVC, PCAC and covariance arguments are employed to express the meson exchange corrections in terms of related physical processes for which data exist. In this way the detailed

calculations involving imprecisely known nuclear wavefunctions are avoided. The meson exchange corrections are contained in the form factors which are derived from experimental data. This method is limited by the availability of the specific experimental data required. Often additional assumptions must be made which are justified only by appealing back to the impulse approximation [18].

The second approach is the most widely used in calculating meson exchange corrections to muon capture. However, in very heavy nuclei its objective of retaining the impulse approximation framework is not completely met as additional terms appear in the effective Hamiltonian [42]. Calculations in medium and heavy nuclei are also open to difficulties associated with the accuracy of the nuclear wavefunctions used and their interpretation is often clouded by this difficulty. Few-body systems are preferable in this respect. The addition of meson exchange currents between nucleons has been found to be necessary to accurately account for several few-body experimental measurements of electromagnetic processes including the electrodisintegration of the deuteron [43] and the cross-section for thermal neutron capture on a proton [44].

Due to CVC the vector form factor at zero momentum transfer, i.e. the “vector charge”, is unchanged by the nuclear environment. However the nuclear medium introduces two effects which modify f_A and f_P . A virtual pion propagating in nuclear matter interacts by scattering from the other nucleons. The scattering can be described by a pion-nucleus optical potential and interpreted in terms of a modified pion field though the validity of the optical model for virtual pions is not firmly established. Even in the case of threshold scattering of the pion, the pion-nucleon interaction is dominated [45] by the p-wave resonance, the Δ (1232) isobar. The formation of Δ -hole states can be regarded as a polarization of the nuclear medium. The range of the pion is reduced, due to interactions with the surrounding nucleons, and can be

parameterized by an effective mass for the pion

$$\widetilde{m}_\pi^2 = \left(\frac{1 - \frac{1}{3}\alpha}{1 + \frac{2}{3}\alpha} \right) m_\pi^2 \quad (34)$$

where α is interpreted as a polarizability coefficient and the strength of the pion field is modified by $1/(1 + \alpha)$ in analogy with electrostatics.

Short-range correlations give rise to the second effect of the nuclear medium. Each of the nucleons (the sources of the pion fields) sits in a ‘correlation hole’ due to the strong repulsive nature of the short range nuclear forces. The electrostatics analogy can be developed further [41] and a detailed correspondence made between a pion source in a correlation hole and the electrostatics associated with an electric dipole in a hole in a dielectric medium. The Lorentz-Lorenz effect which modifies the strength of the electric dipole outside the hole also has an analog in nuclear matter and results, in infinite nuclear matter, in a reduction of the strength of the pion source (renormalization of the pionic vertex) by $\sim 23\%$ [46].

Following the notation of Ohta and Wakamatsu [47] the combination of these two effects on the weak hadronic form factors is given by

$$\widetilde{f}_A = f_A \left(1 + \frac{1}{3}\xi\alpha \right) \quad (35)$$

$$\widetilde{f}_P = f_P \left(1 + \frac{1}{3}\xi\alpha \right) \left(\frac{q^2 + m_\pi^2}{q^2 + \widetilde{m}_\pi^2} \right) \quad (36)$$

where the effect of short range correlations (and Pauli blocking) on the emitting nucleon are expressed in the screening parameter ξ . A value of $\xi = 1$ corresponds to the full Lorentz-Lorenz effect and $\xi < 1$ to a suppressed effect. There is recent evidence for the Lorentz-Lorenz effect in pion-nucleus scattering [48] where it appears as a modification to the p-wave scattering term in the optical model analysis. Rho [49] has calculated a value of 0.80 based on the work of Wilkinson [50] but in general the value of ξ is considered to be rather

uncertain [51]. The ratio

$$\frac{\tilde{f}_P}{f_P} = \left(\frac{q^2 + m_\pi^2}{q^2 + \tilde{m}_\pi^2} \right) \frac{\tilde{f}_A}{f_A} \quad (37)$$

is independent of ξ . Ericson [52] calculates a value of $\alpha = -0.75$ for infinite nuclear matter. This would lead (for $\xi \approx 1$) to substantial renormalizations of the form factors, namely,

$$\frac{\tilde{f}_A}{f_A} \rightarrow 0.75 \quad (38)$$

and

$$\frac{\tilde{f}_P}{f_P} \rightarrow 0.3 \frac{\tilde{f}_A}{f_A} \quad (39)$$

where equation 37 has been evaluated at $q^2 = -0.6m_\mu^2$ (appropriate for RMC).

However surface effects [46] are expected to diminish the renormalization effect for light and medium Z nuclei. Earlier Ohta and Wakamatsu [53] calculated an even larger effect resulting in $\tilde{f}_A/f_A = 0.65$. Such a large renormalization strongly affects the q^2 dependence of the pseudoscalar form factor. In the absence of renormalization f_P contains a pole at the pion mass. For RMC where “ $q^2 \rightarrow -m_\mu^2$ ” the enhancement due to the pion pole is large. The renormalization of the pion mass significantly alters the contribution of the pseudoscalar term to the RMC rate and may invalidate any comparison of experiment to theories in which the renormalized values of the form factors are not properly incorporated.

The experimental evidence for renormalization of f_A has been increasing in recent years. In 1974 Wilkinson [50] deduced a value of $\tilde{f}_A/f_A = 0.899 \pm 0.035$ from an analysis of Gamow-Teller beta decays between mirror nuclei for $A = 11 \rightarrow 21$. This analysis avoided many of the uncertainties due to the nuclear wavefunctions by using the mirror transitions. More recently studies of the (p, n) reaction [54] have revealed missing Gamow-Teller strength over a wide range of A that is interpreted in terms of a renormalization of the axial vector current. Consistent with the level of renormalization observed in the (p, n) studies is the

discovery by Buck and Perez [55] of a new and model independent way of analyzing beta decay and magnetic moment data of mirror nuclei. They find striking linear correlations in the data which are interpreted in terms of a ratio of $\tilde{f}_A/f_V = 1.00 \pm 0.02$. Rho [56] suggests that the nearness of this result to unity is tantalizing evidence that more than the optical model and the Lorentz-Lorenz effect is involved and that the axial current may well be conserved (like the vector current) in finite nuclear matter. This is speculation on Rho's part but clearly the question of the renormalization of the axial vector current has generated some interesting experimental and theoretical work. It is safe to say that the question as a whole is not yet settled. The evidence points at least toward a partial renormalization of f_A but the exact mechanisms are not established and may even contain new physics.

The remaining avenues for research span the low and intermediate energy nuclear physics domains. A precise measurement of the induced-pseudoscalar term of the axial vector current in radiative muon capture would be a valuable contribution to this field of study due to the sensitivity of the RMC rate to the features of the pion pole. Information from other probes such as beta decay and (p, n) reactions will contribute to the understanding of the axial vector form factor and so will complement rather than compete with any muon capture results.

I.4 Radiative Muon Capture in Calcium

Given that the measurement of g_P is of some interest, for the reasons given above, the question of how best to perform this measurement arises. Because $g_P \propto m_l$, the prospects for a measurement from beta-decay or electron capture are very small. This is well illustrated by the work of Bhalla and Rose [57] who obtained $|g_P/g_A| < 90$; and by Daniel et al. [58] who obtained $|g_P/g_A| < 5$ but later retracted their result [59]. The expected value is ≈ 0.05 for these electron

Table III: Values of g_P/g_A as deduced from OMC experiments on complex nuclei. Values of g_P/g_A quoted assume $g_T = 0$. Here P_{AV} and P_L refer to a measurement of the average polarization and longitudinal polarization respectively of the recoiling nucleus; Λ_μ and Λ_β refer to the μ^- capture rate and β decay rate to specific states in the nucleus; and $\gamma - \nu$ refers to the technique of $\gamma - \nu$ angular correlation.

Reference	Method	Nucleus	g_P/g_A	Source of data
Possoz et al. 1974 [60]	P_{AV}	^{12}C	12 ± 5	own
Holstein 1976 [61]	P_{AV}	^{12}C	15 ± 4	[60]
Holstein 1976 [61]	Λ_μ	^{12}C	8.5 ± 2.5	[62]
Possoz et al. 1977 [63]	P_{AV}	^{12}C	7.1 ± 2.7	own
Parthasarathy and Sridhar 1979 [64]	P_{AV}	^{12}C	13.3 ± 1.8	[63]
Roesch et al. 1981 [65]	P_{AV}/P_L	^{12}C	9.4 ± 1.7	own
Kuno et al. 1986 [66,67]	P_{AV}	^{12}C	10.1 ± 2.4	own
Gagliardi et al. 1982 [68]	$\Lambda_\beta \Lambda_\mu$	^{16}O	10 ± 2.5	own
Heath and Garvey 1985 [69]	$\Lambda_\beta \Lambda_\mu$	^{16}O	≈ 12.5	own
Hamel et al. 1985 [70]	$\Lambda_\beta \Lambda_\mu$	^{16}O	11–12	own
Ciechanowicz 1976 [71]	$\gamma - \nu$	^{28}Si	-1.9 ± 3.1	[72]
Parthasarathy and Sridhar 1981 [73]	$\gamma - \nu$	^{28}Si	$13.5 \pm_{5.5}^{3.5}$	[72]

processes. In addition, at the very much higher momentum transfers of neutrino experiments g_P again becomes unimportant [33], leaving muon capture as the process most likely to reveal information on g_P .

Because the muons used in muon capture studies are typically polarized, there are many available observables that have some sensitivity to g_P . Unfortunately in OMC the final state particles (n, ν_μ) preclude the measurement of most of these observables. Table III summarizes the results of OMC experiments on complex nuclei which have been used to extract g_P . Together with table II it represents essentially all determinations of g_P from OMC experiments. It is seen immediately that the techniques used are not generally applicable to a wide range of nuclei and that in fact, only three nuclei have been studied, by three different techniques. With some exceptions, the results suggest g_P/g_A to be

somewhat higher than the Wolfenstein estimate of 6.7. However, g_P/g_A has been extracted with a variety of assumptions concerning meson exchange effects and the only general conclusion that can be drawn is that both theory and experiment must be improved before g_P/g_A can be reliably extracted.

The case of RMC is more hopeful. Assuming dominance of the pion pole term in the induced-pseudoscalar interaction gives a q^2 dependence of $(q^2 + m_\pi^2)^{-1}$ to g_P . When the photon is radiated by the proton, q^2 approaches $-m_\mu^2$ as k (the photon momentum) approaches m_μ . In comparison to OMC, where $q^2 = 0.88m_\mu^2$, this results in approximately a factor of 3 enhancement for high energy photons. This increased sensitivity — approximately one half of the high energy photon spectrum arises from the induced-pseudoscalar term [74] — comes with a penalty. The branching ratio of RMC/OMC is of order 10^{-5} for high energy photons. Despite this serious drawback, RMC has long been considered the method of choice for the measurement of g_P due to the increased sensitivity and the possibility of observing one of the final state particles, the photon. An assessment of the success of RMC in meeting this objective will be made when the results of this experiment are presented and compared to past efforts.

The strength of the induced-pseudoscalar interaction in RMC influences the RMC/OMC branching ratio; the shape of the photon energy spectrum (to a lesser extent); the photon muon-spin angular correlation (photon asymmetry); and the photon circular polarization. Because of the low branching ratio, all RMC experiments to date have been inclusive measurements, that is, the final states are unresolved and have all been summed together. For the extraction of g_P from the data, the theory must also be calculated for the inclusive process. Assumptions must be made to complete such calculations, and unfortunately, the results of the calculations are a function of these necessary assumptions. It turns out that the photon asymmetry and circular polarization are considerably less sensitive to the

details of the inclusive calculation as will be shown in chapter II. Unfortunately, the circular polarization of high energy photons is very difficult to measure effectively, eliminating this possibility.

To measure the fundamental radiative muon capture process, without the presence of possible renormalization effects, very light nuclei must be chosen. Proposals exist for RMC measurements on H [75,76] and on ^3He [77]. These proposals presently plan to measure only the radiative rate.

In the experiment reported in this thesis both the radiative rate and the photon asymmetry for radiative muon capture on calcium were measured. The emphasis was on improving the measurement of the photon asymmetry due to the decreased sensitivity of this observable to the details of the theoretical calculations. The fraction of muons captured (f_C) by a nucleus (as opposed to decaying in orbit) is proportional to Z_{eff}^4 [8] where Z_{eff} is the effective atomic number. For calcium, this implies that $f_C = 0.85$ as compared to 10^{-3} for hydrogen. This is one important reason for choosing calcium for this measurement. A second reason is that much of the theoretical effort has centered around calcium; partially in response to an early controversial measurement of the photon asymmetry in RMC on calcium [78].

Since g_P in calcium may be subject to modification by meson exchange currents, this experiment alone will neither determine the unrenormalized value of g_P , nor the extent of renormalization in calcium. The experiment is properly viewed as contributing to a much larger research effort whose goals include the testing of PCAC; the Wolfenstein estimate; the q^2 behavior of g_P (pion pole dominance of g_P); and the elucidation of meson exchange effects. Within this scope there is a need for improved RMC and OMC measurements from $Z=1$ to 82, as well as contributions from other areas of nuclear physics. With improved measurements, a corresponding theoretical effort would surely be generated.

An attempt has been made in this introduction to put the topic of radiative muon capture in calcium in perspective for the reader. In the final chapter, the results of the experiment will be discussed in terms of their contribution to this field, and suggestions for improvement will be made. In the next chapter, the theory directly relevant to the extraction of g_P from the data will be given. Chapter III deals with the description of the experiment, and chapter IV with the analysis of the data. Extensive Monte Carlo work was involved in the data analysis; details are given in an appendix. In chapter V, the results of the thesis are presented, discussed and compared with earlier work.

Chapter II

Theory

II.1 Muon Stopping History

The muons used in this experiment were derived from the decay of pions in flight.

The weak decay of the pion via

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \quad (40)$$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \quad (41)$$

occurs with a branching ratio of 0.99. The neutrinos are produced completely longitudinally polarized as a consequence of the V-A nature of the weak leptonic current. Since the pion has zero spin, the conservation of angular momentum forces the muons to also be completely longitudinally polarized (spin co-linear with momentum). This has been demonstrated experimentally to high precision [79].

The collection of muons from a decaying beam of pions, to form a muon beam for use in the experiment, is accomplished by the use of electromagnets. The details are described briefly when the muon beamline is discussed in chapter 3. The beam is formed from muons which, in the rest frame of the pion, had their momenta/spin vectors distributed over a conical region of phase space. Various detailed properties of the beamline, in addition to the magnitude of the Lorentz boost required to transform from the pion rest frame to the lab frame, determine the exact shape of this acceptance cone. The beam polarization, a quantity to be associated with the beam as an ensemble, is obtained by averaging the projection of the muon spin vectors onto the beam direction over this acceptance cone. This

results in what is called a kinematic depolarization of the beam. Typically, beam polarizations of $\sim 80\%$ are expected.

When a muon emerges from the evacuated beamline and encounters the target, it begins to lose energy via electromagnetic interactions with the target atoms. The process of slowing from velocities a significant fraction of the speed of light, c , to velocities of order αc (where α is the fine structure constant and αc is of order atomic electron velocities — ~ 2 keV for a muon) takes $\sim 10^{-9} - 10^{-10}$ seconds [80]. At this point the muon can be thought of as being thermalized. In the final stages of thermalization, a positive muon will capture and subsequently lose electrons, travelling some distance as the neutral bound system muonium ($\mu^+ e^-$). In metals however, Coulomb screening inhibits the existence of bound states, and the captured electrons are quickly lost to the conduction band [81]. The strong paramagnetic (dipole-dipole) interaction between the μ^+ and the e^- in muonium quickly depolarizes the muon. However, little or no depolarization is usually observed in good electronic conductors, presumably due to the effectiveness of the Coulomb screening [82]. Also, the rather homogenous interstitial environment of a metal, such as calcium, does not support the varying weak magnetic fields usually responsible for the relaxation of the polarization with time. So, for a μ^+ in a metal, little or no loss of polarization is expected as well as little relaxation of the polarization on the scale of the μ^+ lifetime.

A negative muon, once thermalized, undergoes electromagnetic interactions with the electrons in the immediate vicinity of the nucleus which finally captures the muon. The exact process is not well known. Capture into a mesoatomic orbital is assumed to take place in 10^{-13} to 10^{-14} seconds after thermalization [80]. The principal quantum number of the initial mesoatomic orbital is high, typically $n_0 > 14$, and the initial populations of the magnetic substates are approximately equal. From the capture orbital the muon cascades rapidly by electric-dipole

transitions to the 1S orbital. For high n , Auger transitions dominate; and at low n , radiative electric dipole transitions dominate the cascade. For a medium Z metal, such as calcium, the total time for the cascade is 10^{-15} to 10^{-14} seconds [83]. The μ^- will stay in the 1S orbital until it is captured by the nucleus or undergoes decay in orbit. The radius of the 1S orbital, for a μ^- captured by calcium, is approximately 13×10^{-15} m as compared to the nuclear radius of $\sim 5 \times 10^{-15}$ m. It is this significant overlap of the muon and nuclear wavefunctions which gives rise to the $\sim 85\%$ nuclear capture branching ratio.

A μ^- is also further depolarized during the cascade by spin-orbit effects. Explicit calculations for spin zero nuclei, such as ^{40}Ca , yield a spin-orbit depolarization of $\frac{5}{6}$ [84]. That is, the final polarization in the 1S state is $\frac{1}{6}$ of the initial polarization before the cascade. As there is no spin-orbit splitting of the 1S state, there is no further depolarization (or relaxation) after the 1S state is reached. These calculations assume that the initial magnetic substates are equally populated and apply in the limit of $n_o \rightarrow \infty$. In heavy nuclei a finite size effect slightly increases the depolarization. Experimental measurements confirm these estimates of the residual polarization [18].

In the case of ^{40}Ca the measured depolarization factor is found to be [85,86,87]

$$D_{\mu}^{Ca} = 0.137 \pm 0.010 . \quad (42)$$

This value should be compared with the expected value of $\frac{1}{6} = 0.167$. The ‘residual polarization’ of the negative muons in the 1S state, P_{res} , is the product of the beam polarization and the spin-orbit depolarization factor.

$$P_{res} = P_{beam} \cdot D_{\mu}^{Ca} \quad (43)$$

The residual polarization is an important factor in the interpretation of the photon asymmetry results. It is obtained in the experiment by measuring the

asymmetry of the electrons from μ^- decay in orbit. Some data were recorded with a μ^+ beam. This allows several calibration and consistency checks to be made on the data including a measurement of D_μ^{Ca} .

II.2 μ^+ Decay

A positive muon at rest in the target occupies interstitial or defect sites in the crystal lattice and decays via

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \quad (44)$$

with a mean lifetime of $2.19703 \pm 0.00004 \mu\text{sec}$. The energy-angle distribution of the positron for the most general local, non-derivative, parity-nonconserving Hamiltonian is

$$\begin{aligned} d^3\Lambda^{e^+}(x, \theta) = & \frac{d^3p}{(2\pi)^4} \frac{m_\mu E_o}{12} A \left\{ 6(1-x) + 4\rho \left[\frac{4}{3}x - 1 - \frac{1}{3} \frac{m_e^2}{E_o^2 x} \right] \right. \\ & + 6\eta \frac{m_e(1-x)}{E_o x} + \beta\xi \cos\theta \left[2(1-x) \right. \\ & \left. \left. + 4\delta \left(\frac{4}{3}x - 1 - \frac{1}{3} \frac{m_e^2}{m_\mu E_o} \right) \right] \right\} \quad (45) \end{aligned}$$

where θ is the angle between the μ^+ spin direction and the e^+ momentum; $E_o = (m_\mu^2 + m_e^2)/(2m_\mu)$ is the maximum e^+ energy; p and E are the momentum and energy of the e^+ ; $\beta = p/E$ is the velocity of the e^+ ; and $x = E/E_o$. The parameters A , ρ , η , ξ and δ are functions of the coupling constants and can be found in reference [88]. Table IV summarizes the status of some of these parameters. It is evident that agreement with the V-A theory is excellent. The parameter ξ is the most sensitive to the inclusion of the intermediate vector boson in the calculation of the energy-angle distribution of the e^+ . The modification of ξ by less than 2×10^{-6} [94], when the intermediate vector boson is included, demonstrates the adequacy of the local current-current description for μ decay.

Table IV: Status of the parameters in equation 45. The ‘V–A’ values are the values of the parameters corresponding to a pure V–A Hamiltonian.

Parameter(s)	V–A value	Measured value	Reference
ρ	0.75	0.7517 ± 0.0026	Peoples [89]
η	0	0.011 ± 0.085	Burkard et al. [90]
δ	0.75	0.7502 ± 0.0043	Stoker [91]
$\xi\delta/\rho$	1	> 0.9959	Carr et al. [92]
$\xi P_\mu\delta/\rho$	1	> 0.9975	Jodidio et al. [93]

When the V–A parameter values are substituted into equation 45 and radiative corrections are included to first order in α one obtains

$$d^2\Lambda^{e^+}(x, \theta) \propto \beta^3 x^2 \left\{ 3 - 2x - \frac{m_e^2}{E_o^2 x} + \frac{\alpha}{2\pi} f(x) + \beta \cos \theta \left[2x - 1 - \frac{m_e^2}{m_\mu E_o} - \frac{\alpha}{2\pi} g(x) \right] \right\} dx d\theta \quad (46)$$

where $f(x)$ and $g(x)$ are the radiative corrections [95] which include inner bremsstrahlung and virtual photon emission. The effect of the virtual photons is to modify the vertex function and the particle self-energies (renormalization of the charge and particle masses). The asymmetry of the decay positron is defined

$$A^{e^+}(x) = \frac{\frac{d\Lambda^{e^+}}{dx}(x, \theta = 0) - \frac{d\Lambda^{e^+}}{dx}(x, \theta = \pi)}{\frac{d\Lambda^{e^+}}{dx}(x, \theta = 0) + \frac{d\Lambda^{e^+}}{dx}(x, \theta = \pi)}. \quad (47)$$

With this definition the unnormalized decay positron angular distribution is given by

$$W^{e^+}(x, \theta) = 1 + A^{e^+}(x) \cos \theta. \quad (48)$$

In the analysis of data presented in this thesis, the following approximation was used for the energy-angle distribution of the decay positrons.

$$d^2\Lambda^{e^+}(x, \theta) \propto [(3x^2 - 2x^3) + (2x^3 - x^2) \cos \theta] dx d\theta \quad (49)$$

Figures 1 and 2 contrast the approximate distributions employed, to the

Figure 1: Energy distribution (arbitrary scale) of decay positrons from μ^+ decay at rest as a function of $x = E/E_0$. The solid curve corresponds to equation 46. The dashed curve corresponds to equation 49 which was the approximation used in the data analysis.

Figure 2: Asymmetry distribution of decay positrons from μ^+ decay at rest as a function of $x = E/E_0$. The solid and dashed curves have the same meaning as in figure 1.

ones described by equation 46. Significant discrepancies between the two formulas are seen for low x in the asymmetry distribution and for high x in the energy distribution. In terms of the analysis of the data, the discrepancy for $x > 0.3$ in the asymmetry distribution is the most important and here the difference is small. The discrepancy in the energy spectrum for high x is not as pronounced as shown in Figure 1 since many of the radiated photons accompany the electron and interact in the detector.

II.3 Bound μ^- Decay

It is assumed that no appreciable fraction of the negative muons stopping in the target decay before reaching the 1S orbital. This is consistent with the previously stated estimates of the times required for thermalization, atomic capture and the cascade.

When a muon decays in the field of a nucleus several effects alter the energy-angle distribution, even modifying the decay rate. The most important effect is that of the motion of the bound muon. This motion produces a Doppler smearing of the decay electron spectrum and also a relativistic dilation of the muon's lifetime (reducing the decay rate). A second major effect is similar to the Coulomb effect in ordinary beta decay. The Coulomb attraction of the electron to the nucleus increases the overlap of the muon and electron wavefunctions, thus tending to increase the decay rate and decrease the lifetime. Also the mass of the muon is effectively reduced by its binding energy to the nucleus. The phase space available for the decay varies as the fifth power of the effective mass, thereby inhibiting bound decay relative to free decay. The muon binding energy effect and the Coulomb effect nearly cancel each other in the computation of the bound decay rate. In addition to these main effects, there is a small nuclear recoil effect and small finite nuclear size effects. As is typical in nuclear physics, assumptions

and simplifications must be made to complete such calculations in a finite time.

While some calculations have been carried through to analytical results, the expressions are lengthy and will not be reproduced here. The more precise calculations employ numerical wavefunctions and hence the calculations are completed numerically and presented in tabular or graph form.

We introduce the following notation for the discussion of bound μ^- decay calculations. The energy-angle distribution for bound decay is given by

$$d^2\Lambda^{e^-}(x, \theta) = \frac{1}{2\pi}\beta^3 x^2 [\omega_R(x) + \omega_S(x) \cos \theta] dx d\theta \quad (50)$$

where $\omega_R(x)$ and $\omega_S(x)$ are the spectral functions associated with the decay rates R and S defined by

$$R = \int \beta^3 x^2 \omega_R(x) dx \quad (51)$$

$$S = \int \beta^3 x^2 \omega_S(x) dx . \quad (52)$$

Integration of equation 50 over θ gives the bound decay rate $\Lambda^{e^-} = R$. In analogy with equation 47 the asymmetry of the decay electron is given by

$$A^{e^-}(x) = \frac{\omega_S(x)}{\omega_R(x)} \quad (53)$$

and averaging the asymmetry, weighted by the energy spectrum ($\beta^3 x^2 \omega_R(x)$), gives

$$\langle A^{e^-} \rangle = \frac{\int \beta^3 x^2 \omega_R(x) \frac{\omega_S(x)}{\omega_R(x)} dx}{\int \beta^3 x^2 \omega_R(x) dx} = \frac{S}{R} . \quad (54)$$

This equation defines the physical significance of the quantity S . A similar integration using the approximate energy-angle distribution of equation 49 gives $\langle A^{e^+} \rangle = +\frac{1}{3}$, and for free μ^- decay $\langle A^{e^-} \rangle = -\frac{1}{3}$.

Several authors have calculated $\omega_R(x)$ for a variety of nuclei. The earliest calculation of note was by Gilinsky and Mathews [96]. They neglected the electron mass and also recoil effects by assuming infinite nuclear mass. Analytic

muon and electron wavefunctions were used which approximated the appropriate ones. The calculations were carried analytically to a point and then completed with numerical integration, the results being presented graphically for $Z = 26$. Johnson et al. [97], using approximations very similar to Gilinsky and Mathews, completed their calculation analytically by expanding in powers of αZ . Huff [98] employed improved numerical wavefunctions but still ignored the electron mass and nuclear recoil. His results were given in tables and figures for four nuclei.

Later Hänggi et al. [99] and Herzog and Alder [100] performed comprehensive numerical calculations which removed many of the objections to the earlier ones. They incorporated accurate electron and muon wavefunctions numerically generated by solution of the Dirac equation in a potential which included vacuum polarization and a realistic nuclear charge distribution. They also calculated changes to the phase space of the outgoing particles due to the recoiling nucleus and applied this as a correction to their results. Herzog and Alder further calculated the change to the electron energy spectrum due to internal bremsstrahlung. In ^{32}S (the only case to which this correction was applied) the energy spectrum in the vicinity of the most probable electron energy was attenuated approximately 3%. Other radiative corrections such as modifications to the weak decay vertex were not addressed.

All of the above authors are in agreement to better than 5% over the energy range $x = 0$ to 1.2 for the case of $Z = 26$. Note that in bound μ^- decay the recoiling nucleus participates in the conservation of energy and momentum. Thus the electron energy is not constrained to a maximum value of $E_e \approx \frac{m_\mu}{2}$ (equivalent to $x = 1$) as in the case of free muon decay. In the analysis of the data presented in this thesis the energy spectrum of Herzog and Alder [100] for ^{40}Ca has been employed. A cubic spline of the tabular data given by Herzog and Alder is presented in figures 3 and 4. The correction for internal bremsstrahlung has not

Figure 3: Energy distribution (arbitrary scale) of decay electrons from bound μ^- decay in ^{40}Ca as a function of $x = E/E_o$. The curve is a spline passing through the calculated points of Herzog and Alder [100].

Figure 4: Energy distribution of high energy decay electrons from bound μ^- decay in ^{40}Ca as a function of $x = E/E_o$. Note that the ordinate is logarithmic but that the normalization is as in figure 3.

Table V: Z dependence of the bound μ^- decay rate. R_V is the free (vacuum) rate and M is the mass of the nucleus.

$R(Z)/R_V$	Reference
$1 - (1 \text{ to } \frac{3}{2})(\alpha Z)^2$	Gilinsky and Mathews 1960 [96]
$1 - \frac{1}{2}(\alpha Z)^2 + O(\alpha Z)^4$	Johnson et al. 1961 [97]
$1 - \frac{1}{2}(\alpha Z)^2$	Huff 1961 [98]
$1 - \frac{1}{2}(\alpha Z)^2 - 0.06 \frac{m_\mu}{M}(\alpha Z)^2$	von Baeyer and Leiter 1979 [101]

been applied to the ^{40}Ca data and is not included in the figures nor in the analysis of the data presented in this thesis.

Several of the previous authors give expressions for the Z -dependence of the rate R . These are summarized in table V. Gilinsky and Mathews presented their calculations in the form of a figure and are at odds with the other authors. The final term in the expression of von Baeyer and Leiter is a nuclear recoil correction to R not considered by the other authors. It is small compared with $\frac{1}{2}(\alpha Z)^2$. Experimental data [102] support the majority of the authors. For ^{40}Ca one predicts $R(20) = R_V \cdot 0.989$. The nearest experimental measurement [102] is in ^{51}V with $R(23)^{exp} = R_V \cdot (1.00 \pm 0.04)$. For calcium this represents truly a small correction when calculating the fraction of muons captured by the nucleus, f_C . Nevertheless, the correction is made below. The lifetime associated with the disappearance rate of μ^- from the 1S orbital is given by

$$\tau^{40\text{Ca}} = \frac{1}{\Lambda_{decay}^{40\text{Ca}} + \Lambda_{capture}^{40\text{Ca}}}. \quad (55)$$

Experimental determinations are summarized in table VI. If one takes

$$\Lambda_{decay}^{40\text{Ca}} = (0.99 \pm 0.005)\Lambda_{decay}^{vacuum} \quad (56)$$

and defines

$$f_C = \frac{\Lambda_{capture}}{\Lambda_{decay} + \Lambda_{capture}} \quad (57)$$

Table VI: Experimental determinations of the bound μ^- lifetime in ^{40}Ca .
The indicated measurements were excluded from the average,
being $> 3\sigma$ from the most precise values.

$\tau^{40\text{Ca}}(\text{nsec})$	Reference
333 ± 7	Sens 1959 [104]
345 ± 3 *	Cramer et al. 1962 [105]
335.9 ± 0.9	diLella et al. 1971 [78]
365 ± 8 *	Hart et al. 1977 [106]
332.7 ± 1.5	Suzuki et al. 1986 [107]
335.0 ± 0.8	Average

then one finds [103] $f_C^{40\text{Ca}} = 0.849 \pm 0.005$.

A complete analysis of the data requires a knowledge of the asymmetry distribution, $A^{e^-}(x)$ for bound μ^- decay. Unfortunately only two calculations exist and several objections can be made to each. The first calculation, due to Gilinsky and Mathews [96] suffers from the previously stated simplifications and approximations. In table V Gilinsky and Mathews were shown to be in poor agreement with several other calculations of $R(Z)$, although at $Z = 20$ their value of $0.98R_V$ cannot be considered to be in poor agreement with $0.99R_V$ obtained by the other authors. Perhaps the largest error derives from the fact that their calculation is for $Z = 26$ and that $A^{e^-}(x)$ must be extracted, via equation 53, graphically from two small figures of $\beta^3 x^2 \omega_R(x)$ and $\beta^3 x^2 \omega_R(x)$, as tables or analytical expressions were not provided. The second calculation, by Johnson et al. [97], does provide analytical expressions (functions of Z). However, their expression for $\omega_S(x)$ could not be made to reproduce their figures. In addition to this, or possibly related, their technique of expanding in powers of αZ was rather strongly criticized by von Baeyer and Leiter [101] as being probably reliable for hydrogen, and being otherwise qualitative. Faced with these problems and objections, it was decided to use the asymmetry distribution derived from

Table VII: Results of Monte Carlo calculations of the average asymmetry for bound muon decay assuming different asymmetry distributions. The calculation of e^+ average asymmetry was performed as a check. H & A refers to Herzog and Alder [100], and G & M refers to Gilinsky and Mathews [96].

Energy distribution	Asymmetry distribution	$\langle A \rangle$ (± 0.0001)	fraction of $\pm 1/3$
$E^{e^-}({}^{40}\text{Ca})$ H & A	$A^{e^-}(Z = 26)$ G & M	-0.3312	0.994
$E^{e^-}({}^{40}\text{Ca})$ H & A	A^{e^+} (equation 49 and 48)	+0.2804	0.841
E^{e^+} (equation 49)	A^{e^+} (equation 49 and 48)	+0.3333	1.000

Gilinsky and Mathew's figures. This is shown in figure 5. Interpolation between figure 5 ($Z = 26$) and figure 2 ($Z = 0$; figure should be multiplied by -1 for free μ^- decay) to $Z = 20$ was considered but abandoned.

The importance of the asymmetry distribution was demonstrated in a Monte Carlo calculation of $\langle A \rangle$ as given by equation 54. The results are displayed in table VII. This average asymmetry directly affects our estimation, in the data analysis, of the residual polarization of the muons, prior to capture from the $1S$ state. The residual polarization is in turn our normalization for the photon asymmetry. A 15% difference is observed in the value of $\langle A \rangle$ between using $A^{e^-}(x)$ and $A^{e^+}(x)$. Only one calculation of S exists from which to calculate $\langle A \rangle$. Johnson et al. [97] give:

$$S = -\frac{1}{3}R_V(1 - \frac{5}{6}\alpha^2 Z^2) + O(\alpha^4 Z^4) \quad (58)$$

which yields $\langle A \rangle = -.331$ for $Z = 20$. In light of the criticisms of von Baeyer and Leiter, and the combination of a $Z = 20$ energy distribution with a $Z = 26$ asymmetry distribution, the agreement should be considered fortuitous. However, the $Z = 26$ asymmetry distribution is clearly a better choice than the free decay (A^{e^+}) one. It should be noted that von Baeyer and Leiter agree with Johnson et al. concerning the calculation of R (refer to table V).

Figure 5: Asymmetry distribution of decay electrons from bound μ^- decay as a function of $x = E/E_0$. The curve is derived from figures in reference [96] valid for $Z = 26$.

II.4 Ordinary Muon Capture in ^{40}Ca

OMC experiments in ^{40}Ca have been performed, but not with the objective of determining the weak coupling constants. Rather, the motivation has been to study the capture mechanism itself, and to probe the nuclear wavefunction and excited states. Information so gained may provide guidance for improvements in future RMC calculations. OMC in ^{40}Ca will be briefly described here because it is the dominant process occurring in the calcium target and it produces a significant background which partially dictates the design of the experiment.

The principal decay channel following nuclear muon capture is single neutron emission. The neutron energy spectrum can be viewed as the sum of three contributions: evaporation neutrons, giant-resonance neutrons, and direct neutrons. The contribution of evaporation neutrons is small and confined to very low energies. Excitation of collective states of the giant-resonance type is accomplished via doorway states of the one-particle-one-hole type which decay rapidly into more complicated configurations. De-excitation of these states is principally via particle emission. The high-energy end of the neutron spectrum is dominated by direct neutrons; the ones which were involved in the elementary process $\mu + p \rightarrow n + \nu_\mu$ occurring on one of the protons in the nucleus. These neutrons will exhibit an angular asymmetry with respect to the muon spin whereas the asymmetry is expected to be lost for giant-resonance neutrons in the formation of the compound nucleus.

The neutron energy spectrum following muon capture in ^{40}Ca has been measured several times, most recently by Kozlowski et al. [108], and earlier by Sundelin and Edelstein [109]. The spectrum is observed to decrease exponentially as

$$N(E_n) = N_o \exp(-E_n/T) \quad (59)$$

where $T \approx 9.5$ MeV [108]. Most theoretical work has concentrated on the calculation of the direct neutron process in attempt to reproduce the high energy end of the spectrum. Qualitative agreement is easily obtained above 20-30 MeV. Below 20-30 MeV, the observed excess of neutron intensity is presumed to be the signature of the giant-resonance neutrons. Bogan [110,111] calculated the direct neutron energy spectrum in the harmonic oscillator model for an effective Hamiltonian which was a sum of the elementary process over all the protons. He was able to obtain reasonable agreement with the experimental results of Sundelin and Edelstein but concluded that the neutron intensity was quite dependent on the nuclear model parameters, and thus OMC was an excellent probe of nuclear structure, particularly for the high momentum components of the proton wave functions. Bouyssy et al. [112,113] modified the calculations of Bogan, accounting for the final-state interaction between the residual nucleus and the neutron via an optical potential with surface and volume absorption. They were also able to obtain reasonable agreement with the experimental data and remarked on the sensitivity of the neutron energy spectrum to the real part of the neutron optical potential.

The earliest predictions of the asymmetry of the direct neutrons were due to Primakoff [114] who predicted $A^n = -0.4$. The earliest experimental results [115] appeared to confirm the sign of the asymmetry. Sundelin and Edelstein [109] obtained, however, a positive asymmetry which was convincingly confirmed by Kozłowski et al. [108]. Bogan was able to obtain a positive asymmetry, although not of the apparent amplitude observed, by the inclusion of momentum dependent terms in his effective Hamiltonian. The inclusion of these terms had a large effect on the asymmetry which was otherwise insensitive to the nuclear model parameters. Bouyssy et al. were able to obtain better agreement with the observed asymmetry by their inclusion of the final state interaction. They found

the asymmetry to be very sensitive to the behavior of the wave functions near the nuclear surface. The improved agreement was obtainable without destroying the agreement with the neutron energy spectrum.

As suggested earlier, the degree of sensitivity, described above, of the neutron asymmetry and energy spectrum to the nuclear model employed, makes the extraction of information concerning g_A or g_P exceedingly difficult for OMC on ^{40}Ca . With improved data one might hope to put more stringent constraints on the nuclear model which may in turn be applied to improve RMC calculations.

The energy spectra and asymmetry of charged particles (p , d , t) emitted following μ^- capture on ^{40}Ca has also been measured [116]. The intensities are approximately an order of magnitude lower than the neutron intensities and the energy spectra exhibit a similar exponential behaviour. The asymmetries are consistent with zero. These particles are assumed to be associated with the de-excitation of the giant-resonance.

In the analysis of the data presented in this thesis, the high energy neutron spectrum of Bogan [111] will be used in an estimation of background to the experiment. The charged particles mentioned above are unlikely to escape our target and so are not expected to contribute to the background.

II.5 Radiative Muon Capture in ^{40}Ca

The calculation of the radiative muon capture energy and asymmetry spectra represents a significant challenge to theorists. The earliest attempts [114,117] are now nearly 30 years old. This long history has seen many variations of technique applied to the basic problem. In this brief review only the most general features of a few of the calculations will be given. Discussion will also be limited to calculations in “complex nuclei” such as ^{40}Ca . The situation with respect to calculations of RMC in nuclei such as ^3He (where an elementary-particle approach

is used) and even ^{16}O (where detailed microscopic calculations are done) is slightly different.

Essentially all of the calculations of RMC in complex nuclei have been performed in the impulse approximation where an effective Hamiltonian is constructed from the simple sum over all nucleons of the elementary process on a free nucleon. Several approaches have been taken to the calculation of the elementary amplitude. They will be discussed briefly later. Since RMC has only been observed as an inclusive process (without the identification of the final nuclear state) it is necessary to evaluate the matrix elements of the effective Hamiltonian by summing in some manner over a range of possible final states of the nucleus. Here again several approaches have been made to the computational problem of this summation. It was assumed until recently that much of the uncertainty introduced into the calculation by the various approximations (which allow this final summation over the final nuclear states) was largely cancelled if one also calculated the ordinary muon capture process in a way consistent with the RMC calculation and then normalized the RMC results with the OMC results. Christillin [118] has demonstrated the limitations of this statement by showing an $\sim 30\%$ variation in the partial branching ratio when two similar (but different) excitation spectra are assumed for the final nucleus. He concludes that unrealistic assumptions about the excitation spectrum can mask any renormalization effects. This sensitivity to the nuclear excitation spectrum limits the utility of the inclusive calculations and measurements. Despite these difficulties much progress has been made and there is currently reasonable consensus in the RMC calculations.

Rood and Tolhoek [119,120] pioneered the diagrammatic approach to the calculation of the elementary process ($\mu^- + p \rightarrow n + \gamma + \nu_\mu$). They explicitly considered the Feynman diagrams of figure 6. Diagrams e) to g) have to be included to restore gauge invariance. The induced pseudoscalar form factor is

Figure 6: Feynman diagrams contributing to radiative muon capture. Diagrams c) and d) show the photon coupled to the anomalous magnetic moments of the proton and neutron, μ_p and μ_n .

associated with diagram f). This approach naturally works well providing one has calculated all the relevant diagrams.

A second approach to the calculation of the elementary amplitude was developed by Adler and Dothan [121] who extended the use of the low energy theorem [122] to the non-zero current divergence of the axial vector interaction. Christillin and Servadio [123] later reformulated and confirmed this procedure. Christillin et al. [74] have also demonstrated the close correspondence between the results of this approach and the diagrammatic treatment of Rood and Tolhoek. This method is the starting point for most of the recent calculations.

The final approach to the elementary amplitude is known as the “elementary-particle” treatment. This was discussed briefly in chapter I in comparing various methods of calculating meson exchange corrections. Here as well, CVC and PCAC are used to link related experimental data to the calculation at hand. However it is necessary to invoke a “linearity hypothesis” which fixes the q^2 dependence of the radiative form factors. This approach was first proposed by Hwang and Primakoff [124] and later extended by Gmitro and Ovchinnikova [125]. Christillin [118] comments that in nuclei (such as ^{40}Ca) this approach does not appear to be useful.

The earliest solution to the problem of performing the summation over the final states of the nucleus was to invoke the closure approximation. This approximation assigns an average excitation energy to the final nucleus and evaluates the transition strength at that energy. The average energy is not fixed in any a priori way and is left as a free parameter of the theory. Thus in comparing experimental data to such theories a two parameter fit must be performed where the second parameter is of course g_P/g_A . The major objection to this approach from an experimentalist’s view point is that the value of g_P/g_A extracted from this fitting procedure is extremely sensitive to the average excitation energy. In fact

g_P/g_A is found to vary as k_{max}^6 , the maximum photon energy, where $k_{max} = m_\mu - \overline{E}_{excitation}$. From the theorist's point of view both the harmonic oscillator shell model wavefunctions usually employed and the obviously oversimplified excitation spectrum of the final nucleus raise serious doubts. The calculation of Rood and Tolhoek [119,120] is typical of these early calculations. All such calculations tend to produce absolute RMC and OMC rates, as well as relative RMC/OMC rates which are too high by factors of as much as two for nominal values of the coupling constants. For this reason fits of experimental data to these theories tend to produce negative values of g_P/g_A whereas positive values are expected. Rood, Yano and Yano [126] included a modification of the muon propagator due to the Coulomb field of a uniformly charged spherical nucleus in a calculation otherwise similar to that of Rood and Tolhoek. This produced a decrease of $\sim 24\%$ in the relative rate with respect to Rood and Tolhoek and this was sufficient to produce a positive value of g_P/g_A when compared to experimental data (although the sensitivity to k_{max} remains). In a later calculation with a more realistic excitation spectrum Christillin [127] observed only 5 – 10% effects when a bound muon propagator was introduced.

Some of the objections to the above theories were overcome by Fearing [128] who used the giant dipole resonance (GDR) model of Foldy and Walecka [129] to relate the dipole parts of the nuclear matrix elements relevant to RMC to integrals over the experimental photo-absorption cross-sections. The higher multipoles and the velocity dependent parts of the nuclear matrix elements were evaluated in the closure approximation using harmonic oscillator wavefunctions. The RMC spectrum is dominated however by the dipole parts and thus this incorporation of experimental data was a significant improvement. The calculation was improved by Sloboda and Fearing [130] when improved photo-absorption data became available. This second calculation included all terms to order $(1/M^2)$ where M is

the nucleon mass. All such terms were not consistently included in previous calculations. When fitted to the best data then available [106] a value of $g_P/g_A \simeq 0$ was found for ^{40}Ca . Due to the use of the closure approximation in the evaluation of the other multipoles, this calculation still has k_{max} as a free parameter and so is subject to some of the same practical difficulties when making a comparison to experimental spectra of limited statistics and moderate resolution. This work also investigated the effects of a 25% downward renormalization of g_A and found that while the relative (RMC/OMC) spectrum changed little the absolute RMC and OMC spectra were reduced by $\sim 60\%$ moving them toward agreement with the experimental spectra.

Christillin [131] has avoided the closure approximation by constructing a phenomenological nuclear response function. This resembles the GDR approach of Fearing to a point in that the nuclear response is divided into giant dipole and giant quadrupole parts. The difference however is that Christillin assumes simple Lorentzians for the two resonances rather than making use of experimental photo-absorption spectra. Above 75 MeV in the photon spectra only the GDR portion contributes while below 75 MeV both resonances contribute. In the case of OMC the contribution of the giant quadrupole resonance is approximately 30% and Christillin adjusts the strength of the quadrupole resonance in order to reproduce the observed OMC rate. Christillin admits that this nuclear response function is probably simplistic [118].

A final approach to the summation over the final nuclear states which also avoids the closure approximation is to perform a “microscopic” calculation where the summation is made over all partial transitions between the initial and final nucleus and each of these is evaluated at its correct energy. This has not been attempted with sophisticated wavefunctions in the case of ^{40}Ca but it has been done with simple shell-model wavefunctions (and with a limited model space) by

Gmitro et al. [132]. The result of these calculations was a partial branching ratio which differed from Christillin's [131] by roughly a factor of two and which produced large negative values of g_P/g_A when compared with experimental results. This discrepancy was largely remedied by modifying the calculation so that conservation of the nuclear electromagnetic current was enforced. This can be viewed as an implicit calculation of the meson exchange correction to the electromagnetic vertex. We refer to this calculation as the modified impulse approximation (MIA) of Gmitro et al. [133] and the previous one (without the conservation of electromagnetic current) as the impulse approximation (IA) of Gmitro et al. [132]. An unfortunate omission in this calculation (due to computational difficulties) is an explicit calculation of the contribution of the quadrupole region of the nuclear response function. As previously stated this part of the nuclear response function affects the OMC rate more than the RMC rate and so this omission can mostly be compensated for by adjusting the OMC rate. A 30% adjustment (in agreement with the quadrupole component required for Christillin to reproduce the OMC rate in his calculation) brings the MIA calculation into near agreement with Christillin's phenomenological calculation. When fitted to the most recent data [134] Gmitro et al. [133] obtain $g_P/g_A = 4.5 \pm 3$. However allowing a larger (still acceptable) quadrupole contribution would produce a value of $g_P/g_A = 11 \pm 4$. This sensitivity to the quadrupole contribution is a drawback in the use of these calculations.

Having summarized the various calculations available to which we must compare our data we now briefly give our justification for using only the calculations of Christillin [131] and Gmitro et al. (MIA) [133] in the data analysis to be presented in this thesis. We have measured the energy spectrum with only a moderate energy resolution of $\sim 15\%$. The sensitivity of the closure calculations (which includes the calculations by Fearing and Sloboda [128,130]) to the

parameter k_{max} precludes the extraction of reliable values of g_P/g_A from our data. Data taken with an intrinsically high resolution pair spectrometer exists [134] and these data are much better suited to the extraction of g_P/g_A in terms of the closure calculations. Aside from this reason we wish to be able to compare our results to the results of previous experiments. Here again this essentially restricts us to the two theories named above if we confine the comparisons to recent data which are free of the background problems associated with earlier data.

We present now the results of the calculations of Christillin [131] and Gmitro et al. [133] for ^{40}Ca . The energy spectra are given in figures 7 and 8. These energy spectra are relative ones having been normalized by the OMC rate. Note that the shapes of the spectra are different below ~ 75 MeV. This is due to the neglect of the quadrupole part of the nuclear excitation spectrum by Gmitro et al. [133] in their calculations. This is the region of the photon energy spectrum where it is energetically possible for the quadrupole resonance to be excited in the final nucleus.

Figure 9 shows the photon asymmetry as a function of photon energy for the calculation of Christillin [131]. In analogy with equations 47 and 48 the photon asymmetry is defined as

$$\Gamma(k) = \frac{\frac{d\Lambda_{RMC}}{dk}(k, \theta = 0) - \frac{d\Lambda_{RMC}}{dk}(k, \theta = \pi)}{\frac{d\Lambda_{RMC}}{dk}(k, \theta = 0) + \frac{d\Lambda_{RMC}}{dk}(k, \theta = \pi)} \quad (60)$$

and the RMC photon angular distribution is given by

$$W^\gamma(k, \theta) = 1 + \Gamma(k) \cos \theta . \quad (61)$$

It is seen to be nearly independent of energy and quite strongly affected by the value of g_P/g_A . The results of similar calculations in the theory of Gmitro et al. [133] are not yet finalized.

An important motivation for performing the measurement of the photon asymmetry described in this thesis was the relative insensitivity of α_γ to the

Figure 7: Radiative muon capture photon energy spectra calculated in the phenomenological model of Christillin [131,135] for several values of g_P/g_A .

Figure 8: Radiative muon capture photon energy spectra calculated in the modified impulse approximation of Gmitro et al. [133] for several values of g_P/g_A .

Figure 9: The photon asymmetry as a function of photon energy for several values of g_P/g_A calculated in the phenomenological model of Christillin [131].

Table VIII: The average photon asymmetry α_γ as calculated in several different models.

α_γ	Reference
0.75	Rood and Tolhoek [119,120]
0.78	Rood, Yano and Yano [126]
0.76	Sloboda and Fearing [130]
0.79	Christillin [131]
0.77	Gmitro et al. [132]

actual details of the calculations. Here we define α_γ to be the average photon asymmetry over the experimentally accessible region $k > 57$ MeV. That is

$$\alpha_\gamma = \frac{\int_{k>57} \Gamma(k) \cdot \Lambda_{RMC} dk}{\int_{k>57} \Lambda_{RMC} dk} \quad (62)$$

Experimentally it is only α_γ that has been measured to date due to the meager statistics available. We support the claim that α_γ is relatively insensitive to the assumptions of the calculation by presenting table VIII where α_γ is given for the Goldberger-Treiman value of g_P/g_A as calculated in several very different models. Fearing [136] has shown that the lowest order terms contributing to the deviation of α_γ from +1 (the pure V-A value) are of order $(1/M^2)$. That is there are no terms of order $(1/M)$. Previous calculations have not taken this into account and have not kept all terms of order $(1/M^2)$ assuming that they were small. Sloboda and Fearing [130] have calculated α_γ retaining all terms to order $(1/M^3)$ and find the insensitivity of α_γ to the details of the calculation indeed seems to be a valid assumption as table VIII shows. They find α_γ to be independent of the shell model parameters as well as the average excitation energy and the velocity terms in their calculations. They have also calculated the affect of a 25% downward renormalization of g_A and find no difference in $\Gamma(k)$ above ~ 60 MeV. This is the basis for our confidence in the purported *model-independence* of α_γ .

Figure 10 shows the dependence of α_γ on g_P/g_A and is the curve which is

actually employed in the extraction of our estimate of g_P/g_A from the experimental photon asymmetry.

The partial branching ratio, $R_{k>57}$, is defined in terms of the curves given in figures 7 and 8 as

$$R_{k>57} = \int_{k>57} \Lambda_{RMC}/\Lambda_{OMC}(k) dk . \quad (63)$$

Figure 11 shows the partial branching ratios as a function of g_P/g_A as extracted from integration of the curves in figures 7 and 8. These curves are also used in the analysis of the data presented in this thesis. We note that while the two curves in figure 11 do cross, the level of agreement of the two calculations is not striking. Inclusion of the omitted quadrupole resonance in the calculations of Gmitro et al. [133] would tend to raise $R_{k>57}$. However the normalization of the curve with respect to OMC would also have to be re-evaluated.

One final comment on the calculation of the RMC process in ^{40}Ca pertains to recent calculations [137] which attempt to “go beyond the impulse approximation” and include consistently meson exchange effects on RMC. These calculations show the effects of including these meson exchange corrections as compared to the standard impulse approximation. They find a suppression of the high energy end of the relative photon spectrum of about 10% and also a suppression of 10–20% in the absolute RMC rate. As the calculations were done for a single value of g_P/g_A it was not possible to use them to extract another estimation of the pseudoscalar coupling constant from the data. It is of course important to continue developing a consistent approach to RMC calculations which include meson exchange corrections.

Figure 10: Average photon asymmetry as a function of g_P/g_A . The curve due to Christillin [131] corresponds to figure 9 and is the one actually used in the analysis of our data. The curve due to Gmitro et al. [132] corresponds to their impulse approximation calculation without the imposed conservation of the electromagnetic current. The predictions of the photon energy spectra for these two calculations differ by a factor of two.

Figure 11: $R_{k>57}$ as a function of g_P/g_A for the calculations of Christillin [131] and Gmitro et al. [133].

Chapter III

Description of the Experiment

III.1 The M20A Backward Muon Channel at TRIUMF

The TRIUMF accelerator is a 6-sector isochronous cyclotron which accelerates H^- ions from 300 keV to a maximum energy of 520 MeV. For effective operation of the secondary pion and muon channels, a high current 12 keV unpolarized H^- ion source of the Ehlers type is used. The ion source beam is accelerated to 300 keV and injected axially into the cyclotron. Extraction is achieved by stripping the H^- ions of their two electrons in a thin carbon or aluminum foil. In this high current, unpolarized mode, the TRIUMF cyclotron routinely delivers a $140\mu A$ 500 MeV proton beam down primary beamline 1A (BL1A), to two meson production targets (1AT1 and 1AT2), in the meson hall. After 1AT2 the beam is stopped in the thermal neutron facility (TNF).

The primary proton beam is delivered with a 100% macroscopic duty cycle and a 23.055 MHz microscopic cycle. More detailed information about the TRIUMF cyclotron and primary beamlines can be found in the TRIUMF Users Handbook [138] and references therein.

The secondary beamline used to collect all the data described in this thesis is known as M20. The present M20 beamline was built in early 1983. Prior to 1983 an older version of M20 existed, whose description can be found in reference [138]. The design and operating characteristics of the present M20 [139,140] are in many ways superior to the old channel. It is likely that this experiment could not have been performed on the old M20 channel. For this reason, as well as to facilitate

later discussion of the sources of background to the experiment, a full description of the secondary channel will be given.

In figure 12 the layout of the current M20 channel is shown. Typically a water-cooled beryllium production target, whose dimension in the proton beam direction is 10cm, is used in the 1AT2 target station. Two quadrupole magnets (M20Q1 and M20Q2) form the injection section of the beam line and view the beryllium target at 55° to the proton beam with a 10 milliradian solid angle. The first bending magnet (M20B1) selects the charge and the momentum of particles which enter the decay section of the channel (M20Q4-5-6). The purpose of the decay section will be elaborated on after the complete description of the channel. The maximum magnetic field attainable in M20B1 limits the injection momentum into the decay section to a central value of 165.6 MeV/c with a 20% $\Delta p/p$ momentum spread. The quadrupole magnets of the decay section are powered by a single power supply and are connected so that the magnet polarities alternate. Other quadrupoles, M20Q3, M20Q7 and M20Q8, are used to match the optical characteristics of the decay section to the two bending magnets, the second of which, M20B2, is used to direct the beam into the experimental areas. Experimental area A, the one used in this experiment, has three additional magnet elements, M20AQ9, M20AQ10 and M20AQ11, which serve to focus the beam. The best focus is attained 1.15 m from the center of M20AQ11. Experimental area B is preceded by a $\mathbf{E} \times \mathbf{B}$ Wien velocity filter as well as focussing elements. To allow independent access to the two experimental areas, individual beam blockers can be positioned to intercept the beam entering either of the two final segments of the beamline. The beam blockers are physically blocks of copper (approximate dimensions 5 cm $\times$ 20 cm $\times$ 30 cm) and move on carriages located at the exit side of M20B2. Other beamline control elements of interest are the “jaws” and the “slits” (M20SL1) which allow some control over the number

Figure 12: The M20 channel.

and momentum spread of particles in the beamline.

Two types of channel tunes were used in the experiment. A ‘straight-through’ tune is one in which the magnetic field settings of M20B1 and M20B2 are such that particles momentum selected at M20B1 are directed into the experimental area by M20B2. A ‘decay’ tune is one in which muons, from the decay of pions in the decay section, are directed into the experimental area by M20B2. The Lorentz transformation, from the rest frame of the pions, where the muons are emitted isotropically to the laboratory frame, produces a distribution of muons that are highly forward peaked. Muons emitted parallel to the Lorentz boost are called forward muons and those emitted antiparallel are called backward muons. For the maximum pion injection momentum of 165.6 MeV/c, and from the kinematics of pion decay, forward muons are found to have a momentum of 173 MeV/c and backward muons, 86.5 MeV/c. In the laboratory frame, both forward and backward muon beams travel in the same direction. The design of M20A is optimized for backward muon beams although straight-through tunes are also possible. M20B on the other hand is optimized for forward muon beams as well as for straight through tunes. The small difference between the momenta of the undecayed pions and the forward muons is what necessitates the Wien filter in M20B. The large momentum gap between the undecayed pions and the backward muons is primarily responsible for the purity of backward decay beams. RMC experiments are very sensitive to pion contamination due to the $\sim 2\%$ branch of radiative π^- capture in calcium[141] as compared to the RMC rate of $\sim 2 \times 10^{-5}$.

In this experiment, a straight-through 100 MeV/c π^- tune was used to obtain much of the calibration data. The flux of particles far exceeded our needs, so the slits and the jaws were used to severely limit the rate, and simultaneously reduce the momentum spread to approximately 6% $\Delta p/p$ at the focus.

Both backward μ^+ and μ^- beam tunes were used in the experiment.

Table IX: Summary of backward μ^+ and μ^- tunes used in the experiment. Here μ_{stops} is the number of muons estimated to have stopped in the Ca target.

particle	Momentum (MeV/c)	Number of μ_{stops}	Fraction of data(%)
μ^-	86.5	4.46×10^9	2.6
	82.7	36.73×10^9	21.2
	76.0	131.81×10^9	76.2
		1.730×10^{11}	100.0
μ^+	76.0	1.04×10^9	100.0

Table IX summarizes the beam tunes used and the fraction of the data collected under each. For both tunes the channel was operated with the beam-defining elements (the jaws and the slits) retracted as far out of the beam path as possible. Experience with the channel has shown that scattering of pions in the edges of these elements can sufficiently degrade the pion momentum such that significant numbers of pions are directed by M20B2 into area A. For the backward μ^+ beam, significant numbers of pions in the beam produce an attenuation of the observed e^+ asymmetry due to the production of unpolarized μ^+ from π^+ decay at rest in the target. For the backward μ^- beam, the serious effects of pion contamination have already been mentioned.

For the backward μ^- tune, a stopping rate of approximately $3 \times 10^5 \text{ sec}^{-1}$ in the calcium target was achieved. For the backward μ^+ tune, the available flux was approximately a factor of 4.5 greater [140]. To reduce the flux to the level of the μ^- flux, we were advised [142] to defocus M20Q1 and M20Q2, in order to avoid using the slits and jaws and to avoid changes in magnetic elements of the beamline that determine the polarization of the μ^+ at the focus.

III.2 The Target

The target used in this experiment consisted of 2 blocks of natural calcium (96.97% ^{40}Ca) each measuring $10\text{cm} \times 10\text{cm} \times 1.9\text{cm}$. The target was prepared for our use by Oak Ridge National Laboratory. For ease of handling a thin coating of aluminium was evaporated onto all surfaces of the target. In between runs the target was stored in an evacuated desiccator to prevent surface oxidation.

In order to measure the photon asymmetry, or equivalently, the muon spin-photon angular correlation, the muon spin rotation technique (μSR) [143] was employed. In the μSR technique, muons stopping in the target precess in an externally applied magnetic field. For the asymmetric distribution of final state products following muon decay or capture, the probability of detection of one of these final state particles in a stationary detector varies periodically as a function of the time since the muon stop. The photon asymmetry can then be extracted from the time structure observed in a single photon detector.

To provide this precession field, a small H-magnet was used. The magnet had a gap of 16cm and the pole faces measured $23\text{cm} \times 30\text{cm}$. Field homogeneity over the volume of the target was measured to be better than 1%. In choosing the magnetic field, a decision was made not to use the stroboscopic method [81] of μSR in which the muon precession frequency (ν_{prec}) is matched to the cyclotron frequency

$$\nu_{\text{cyc}} = \nu_{\text{prec}} = \left| \frac{2\mu_{\mu}B}{\hbar} \right|. \quad (64)$$

Instead, a field of 0.040 T was used for which $\nu_{\text{prec}} = 5.46$ MHz as compared to $\nu_{\text{cyc}} = 23.055$ MHz. The reasons for this choice will be elaborated on later.

The calcium target was centered in all dimensions between the pole faces of the precession magnet, as shown in figure 13, and was positioned at the focus of the beamline. The two pieces of the target were placed one behind the other to

Figure 13: Schematic of the experimental geometry indicating the position of the magnet-target assembly with respect to the μ^- beam and the photon telescope. Detailed discussion of the geometry is found in the text.

Table X: Characteristics of backward μ decay beams as measured at the M20A focus.

Dimension	$FW_{\frac{1}{2}}M$ (in cm)	$FW_{\frac{1}{4}}M$ (in cm)	Divergence (in deg.)	
			of $FW_{\frac{1}{2}}M$	of $FW_{\frac{1}{4}}M$
horizontal	7.2	9.1	3.6	8.5
vertical	9.5	13.9	4.0	3.6

Table XI: Physical dimensions of the beam counters.

Counter	Dimensions (in cm)
$S0$	$25.0 \times 25.0 \times 0.32$
$S1$	$25.0 \times 25.0 \times 0.32$
$S2$	$15.0 \times 11.5 \times 0.16$
$S3$	$23.0 \times 11.5 \times 0.16$
$S4$	$23.0 \times 30.0 \times 0.32$
$S5$	$23.0 \times 30.0 \times 0.32$

produce a target of dimensions $10\text{cm} \times 10\text{cm} \times 3.8\text{cm}$ and the magnet and target assembly was oriented to bisect the 105° angle between the axis of the photon telescope and the μ^- beam.

The beam was collimated 50cm upstream of the target by a 10cm thick wall of lead with a rectangular aperture closely matched to the beam profile at that point. At the focus the beam had the properties given in table X.

The beam counters used to define an incoming particle and to define a μstop are labelled $S0$ to $S5$ in figure 13. $S4$ and $S5$ completely covered the pole faces of the magnet, and together with $S2$ and $S3$, formed a closed volume that contained the target. In the case of $S2$ and $S3$ part of the light guide attached to the plastic scintillator is also shown. Table XI gives further details on these beam counters.

Also used in the course of the experiment was a high pressure (100 atmosphere maximum) H_2 gas target, whose description and operation is detailed in reference [144]. When this target was in use, the calcium target and the scintillators $S2$ and $S3$ were removed from the precession magnet and the center of the gas target positioned accurately at the center of the magnet. Internal scintillators replaced the function of $S2$ and $S3$ in defining a stop in the gas.

III.3 The Photon Telescope

The photon telescope has two principle components: TINA, a large thallium doped crystal of sodium iodide ($NaI(Tl)$) and a smaller assembly of $NaI(Tl)$ detectors called the BARS (NaI Converter in figure 13).

The TINA crystal measures 46 cm diameter by 51 cm in length and is physically two equal cylindrical sections glued together. TINA, which stands for TRIUMF Iodide of Natrium, was manufactured by the Harshaw Chemical Company. The entire crystal is viewed by each of seven 12.5 cm RCA S83006E photomultiplier tubes. Many aspects of the performance of the crystal as an intermediate energy photon and electron spectrometer are found in reference [145].

The BARS is a package of seven rectangular $NaI(Tl)$ detectors, each measuring 5.08 cm $\times$ 4.35 cm $\times$ 35.56 cm, arranged such that the combined dimensions are 5.08 cm $\times$ 30.48 cm $\times$ 35.56 cm. Each bar is optically isolated from its neighbors by reflecting paper and is viewed by a 3.8 cm Hamamatsu R980 photomultiplier on each of its 5.08 cm $\times$ 4.35 cm faces. Thus the entire package has 14 photomultiplier tubes. Like TINA, the BARS were manufactured by the Harshaw Chemical Company. The BARS were designed to give position as well as energy information. Position resolution of 1-2 cm FWHM [146] has been obtained with a prototype bar by computing an energy asymmetry, $(E_L - E_R)/(E_L + E_R)$. Here E_L and E_R are proportional to the pulse-heights in the left and right-hand

photomultiplier tubes. The energy resolution of the bars at the center has been measured to be 3.2% FWHM at 40 MeV. The energy resolution over the whole bar is somewhat degraded due to uncertainties in the position variation of the pulse-heights. Further aspects of the performance of a scintillator array such as the BARS can be found in [146,147]. The function of the BARS is to be an efficient converter of photons into electron-positron pairs and that of TINA is to measure the bulk of the energy escaping from the BARS for a converted photon. By requiring energy deposition in both counters, a major source of background, arising from high-energy neutrons, is substantially reduced.

Two other important components of the photon telescope are a package of three plastic scintillators called the layered scintillator (LS 1,2,3 in figure 13) and a lucite Cerenkov counter ($\tilde{C}$ in figure 13). The layered scintillators each measured $38 \text{ cm} \times 38 \text{ cm} \times 0.32 \text{ cm}$, were optically isolated from each other and were viewed by separate photomultiplier tubes. Their function was to further reduce background in the experiment by confirming the passage of charged particles between the BARS and TINA, signifying an electromagnetic shower, and to provide an estimate of the number of particles in the shower from the deposited energy. The lucite used in the Cerenkov counter ($1.3 \text{ cm} \times 38 \text{ cm} \times 100 \text{ cm}$) is not a scintillant, and light is only produced by the Cerenkov process. For lucite, with an index of refraction of $n \sim 1.49$, the threshold for the production of Cerenkov radiation is

$$E_{th} > \frac{m_0 c^2}{\sqrt{1 - \frac{1}{n^2}}} = 1.35 m_0 c^2 \quad (65)$$

which gives thresholds of 0.69 MeV for $e^\pm$; 143 MeV for $\mu^\pm$ and 189 MeV for $\pi^\pm$. As the muons and pions in the beam have kinetic energies less than 30 MeV, a signal in the Cerenkov counter was taken to indicate the presence of $e^\pm$ from an electromagnetic shower. Although the counter was viewed by two photomultiplier

tubes, one at each end, the low photon yield [148] of ~ 350 per traversing $e^\pm$ (in the visible region, for 1.3 cm thickness) and the poor light collection geometry resulted in a detector of at best 75% efficiency, and no useful energy resolution.

Three other plastic scintillators formed the remainder of the photon telescope. A 48 cm $\times$ 58 cm $\times$ 0.32 cm counter was positioned in front of the BARS and served as a charged particle veto counter (V). A small 8.9 cm diameter $\times$ 0.32 cm counter (E) served to define a reduced solid angle for the collection of decay electron events. The third counter (H) measured 38 cm $\times$ 38 cm $\times$ 0.64 cm with a 23 cm diameter hole. Its function was to identify any charged particle leaving the target which could undergo bremsstrahlung in the collimator and mimic a neutral event.

All of these elements of the photon telescope were assembled in or on a heavy iron plate shielding box which had a 30 cm diameter hole on its front face. Two additional layers of large plastic scintillators were placed around the telescope. One group of six such counters were positioned over and to both sides of the TINA crystal inside the shielding box. These were called the “inner” counters. Eight counters mounted to the outside of the shielding box formed the “outer” counters. The purpose of these counters was to identify events related to the passage of cosmic rays through the apparatus.

III.4 Shielding and Sources of Background

Backgrounds in this experiment were a very important consideration. RMC experiments are prone to background contamination for three main reasons. The muon capture is not prompt with the muon’s arrival in the target but occurs with a lifetime of ~ 335 nsec (in Ca). To observe the capture with some reasonable probability, any interaction in the detector that occurred within 3 lifetimes (~ 1 μ sec) after a μ stop was accepted. In a prompt experiment of an otherwise similar

nature, this rather loose coincidence condition between the interaction in the detector and a particle stop, could be reduced to of order 10 nsec, thereby reducing the random background by a factor of 100. Secondly, if the particle to be detected were charged, then scintillators or wire chambers could be used to define the trajectory to ensure that it originated in the target. In this respect, the combination of the BARS and TINA offers little directional sensitivity, particularly for cosmic rays energetic enough to traverse the iron collimator. Finally, the small branching ratio, for RMC on ^{40}Ca , of $R_{k>57} \sim 2 \times 10^{-5}$ allows many improbable, moderate rate processes to compete with the true signal.

The backgrounds experienced in this experiment can be divided into three categories according to their origin: cosmic ray background; background from BL1A and M20; and background from processes in the target other than RMC.

The cosmic ray background has an energy spectrum for TINA-BARS coincidences that extends to several hundred MeV. Through the region of interest, between 50 and 100 MeV, the energy spectrum is nearly flat and is featureless. Near 200 MeV, which corresponds to a minimum ionizing particle traversing a maximum dimension of TINA, it rises slightly and at higher energies decreases exponentially. The time of arrival of a cosmic ray bears no relation to the time of a μstop , and hence the cosmic ray background constitutes a flat component in the time spectrum. The inner and outer cosmic ray veto counters already described are one measure against this background. A further measure was to build a substantial concrete roof over the experimental area from shielding blocks normally part of the BL1A shielding. The roof was formed of two $1.2 \text{ m} \times 1.8 \text{ m} \times 5.4 \text{ m}$ blocks supported on smaller blocks. The underside of the roof was 2.25 m above beam height. Directly over TINA the roof was 1 m above beam height and at that point was 2.4 m thick.

The most serious backgrounds were those related to the beam in BL1A and

M20. Figure 14 gives a more detailed view of the portion of the meson hall between the 1AT2 production target and the beam dump (TNF). In earlier attempts to perform this experiment at TRIUMF, backgrounds with energies ranging up to and in excess of 300 MeV were observed with the characteristic 23.055 MHz time structure that identified them as being cyclotron related. As seen in figure 14, the edge of the concrete shielding that surrounds BL1A intersects the M20A beamline in the vicinity of M20AQ9. Also indicated, is the position of the TINA crystal to the left of the M20A focus. Energies as high as those observed must be associated with BL1A (as opposed to M20); and to be detected at the position of TINA, they must have been neutral at least while traversing the substantial shielding of BL1A. The most likely source of high energy neutrons is the 1AT2 production target. Other possible sources include collimators (1AS1, 1AS2, 1AS3) and the TNF. To shield against these neutrons, the RMC collaboration requested that the shielding between 1AT2 and the experimental area be improved during the M20 rebuild. Accordingly, $\sim 8 \text{ m}^3$ of concrete shielding on the north side of M20Q4-5-6 was replaced with iron shielding blocks. In addition, the pillar to the south of TINA, which supported the overhead roof, was built of iron blocks providing an additional 90 cm of shielding directly beside TINA. Also, a 60 cm thick iron wall was constructed beside the M20B velocity separator.

Background from the beam in the M20 secondary channel ranged in energy up to $\sim 140 \text{ MeV}$, the π mass. Presumably, this background was related to the undecayed π^- 's interacting in the walls of the beamline's vacuum vessels. (It was also observed to have the characteristic 23.055 MHz time structure, and in addition, had a substructure indicating that the background was being produced at two or three locations, giving rise to slightly different time-of-flights to TINA.) The considerable iron shielding, as well as lead shielding at the end of the M20A

Figure 14: Detailed plan view of meson hall in the vicinity of the M20 secondary channel.

beamline, were intended to reduce this background as well.

Finally, there is background from interactions of the beam particles in and around the calcium target. For the purpose of discussion, this background will be sub-divided into that which is due to: μ^- stops in the calcium target; μ^- stops outside of the calcium target; and π^- contamination in the beam. Of the muons which stop in the calcium target, approximately 15% undergo decay in orbit and 85% undergo ordinary muon capture. Approximately 10^{-4} of the muon captures are accompanied by internal bremsstrahlung (RMC) with $\sim 2 \times 10^{-5}$ being in the energy region of > 57 MeV. High energy neutrons from OMC represent one unavoidable background. They can not be shielded against, hence the necessity of designing the photon telescope to have a high neutron rejection efficiency. High energy electrons, which undergo hard bremsstrahlung in the target, are further background which cannot be shielded against. To reduce their contribution to the RMC spectrum, the target was angled with respect to the telescope, to decrease the average pathlength in the target in the direction of the telescope. Also, the scintillators surrounding the target were inspected for energy deposition that could be associated with the continuing electron following bremsstrahlung. It should be noted that both of these backgrounds appear in the photon time spectrum with the same lifetime as that associated with the RMC events; the lifetime of a μ^- in Ca. Since different asymmetries are associated with these backgrounds than the RMC asymmetry, it is particularly important that the contributions of these processes be estimated. Detailed quantitative estimations can be found in the analysis of the data.

A less serious source of background is from μ stops outside of the target. Muons not stopping in the target are most likely to be in one of two environments: a plastic scintillator; or a 0.32 cm sheet of lead which was affixed to each pole face of the magnet before *S4* and *S5* were positioned. Negative muons in plastic

scintillators are associated with the carbon atoms and there they have a lifetime of $\sim 2.03 \mu\text{sec}$ [107], which implies a decay in orbit rate of 92%. In lead, only 3% of the muons decay in orbit and the μ^- lifetime is 75 nsec [107]. The difference between these lifetimes, and the μ^- lifetime in calcium, lends itself well to the fitting of several lifetime components in the time spectrum in order to extract estimates of the number of muons stopping outside the target. Conditions placed on the data in the data acquisition hardware, and in the later off-line computer analysis, helped to discriminate against such background events.

One final source of background is due to π^- contamination in the beam. However, the unique time signature of pion related events allows for effective discrimination against such events. The π^- interacts promptly in matter upon coming to rest, producing high-energy photons and/or neutrons. Events coincident in time with a particle traversing $S0$ and $S1$ are rejected. π^- contamination in the beam is reduced to a very small problem compared to backgrounds from π^- that avoid $S0$ and $S1$.

III.5 Data Acquisition

Approximately 50 analog signals were obtained from the various scintillation counters in the experimental area. These signals were transmitted, via low loss cables, to a "counting" room where the data acquisition electronics and computer were located. The signals were processed in an array of nuclear instrumentation modules (NIM) which was designed to identify potentially interesting "events" in the counters. For such events it was desired to record, on magnetic tape, 92 numbers which were proportional to signal amplitudes or time differences between the arrival of signals. To accomplish this task, electronic modules in a CAMAC (Computer Automated Measurement and Control) crate were activated by signals from the NIM modules and the information to be recorded was digitized by the

CAMAC modules. One of the CAMAC modules immediately generates a LAM (Look At Me) signal upon activation which generates a vectored interrupt in the PDP11/34 data acquisition computer. The 11/34 operates under Digital Equipment Corporation's RSX-11M multi tasking monitor. The two primary tasks are DA and MULTI [149]. DA manages the reading of the CAMAC crate, the buffering of events, and the taping of the assembled buffers. The task MULTI is a versatile histogramming package, with access to the DA buffers, that allow the experimenters to do on-line analysis of the data, and to monitor the experiment as the data is being recorded.

A brief summary of the data recorded for each event is given here. ADC refers to Analog to Digital Converter and TDC to Time to Digital Converter.

- individual Tina phototubes (7 ADC words)
- hardware sum of Tina phototubes (1ADC word, 2 TDC words)
- individual BARS phototubes (14 ADC words, 7 TDC words)
- hard sum of BARS phototubes (2 ADC words, 2 TDC words)
- beam and photon telescope plastic counters (11 ADC words and 9 TDC words)
- BARS pile-up information (2 ADC words)
- TINA pile-up information (12 ADC words)
- high pressure gas target scintillators (2 ADC words)
- pulse separator outputs (11 TDC words)
- cosmic ray veto counters (4 TDC words)

- cyclotron radiofrequency and capacitive probe pickup (4 TDC words)
- bit patterns (2 words)

In addition to this information, DA read 18 channels of scalers once per buffer. A scaler is a digital register which is incremented for every input logic pulse. The scalers were used to record the rates of various counters or various coincidences in the NIM electronics.

In more detail, figure 15 shows schematically the NIM electronics for the photon telescope. Signals from TINA's seven phototubes were passively split by a resistive delta circuit. One signal from each phototube was integrated by an RC circuit and then amplified by a factor of 10 before being passed to a peak-sensing ADC. The other signals were summed in a unity-gain summing amplifier with bridged direct-coupled current source outputs, which allowed for clipping of the signal at 260 nsec by reflection in a delay cable. This clipped signal was sent to various peak-sensing and charge-integrating ADC's and to a constant fraction discriminator (CFD), a module which gives a logic timing pulse that bears a constant relation to the timing of the input analog pulse independent (or nearly so) of the pulse-height. The TINA CFD signal was used wherever Tina timing was required in the logic.

Similarly, the analog signals from the BARS were passively split and one signal from each phototube summed and clipped. To operate the bases of the phototubes in a linear range, and to reduce the effect of noise on the signal cables, each of the BARS signals was amplified by a factor of 100 in the experimental area. The individual BARS signals were passed to peak-sensing ADCs and a TDC used on each bar. The BARS CFD timing signal was derived from the summed BARS signal. In addition, analog signals from the BARS and TINA were summed and passed to a third CFD which provided an E_{total} logic signal, with good timing

Figure 15: Schematic of the NIM electronics for the processing of the linear signals from the photon telescope. Here, as in figures 16 and 17, some details have been omitted for the sake of clarity. Where the function of a group of modules can be schematically shown by a single one, this has been done.

and an energy threshold of approximately 25 MeV.

Signals from the layered scintillator and the Cerenkov counter were not used in the hardware definition of an event. However, TDC and ADC (in the case of the layered scintillator only) information was recorded by CAMAC and was used in the on-line and off-line analysis of the data.

In figure 16 the trigger logic is given. The signals from many of the scintillators were measured in TDCs and/or ADCs. A μstop was defined as $(S0 \cdot S1 \cdot S2 \cdot (\overline{S3 + S4 + S5}))$ where: $\cdot$ denotes the logical AND operation; $+$ denotes the logical OR operation; and a bar above a symbol denotes the logical NOT operation. Each μstop generated a 1.1 μsec long μstop gate. A pile-up gate (PUG) was used to obtain an extending μstop gate.

A 'neutral event' was defined as $\text{TINA} \cdot \text{BARS} \cdot E_{\text{total}} \cdot (\overline{\text{neutral veto}})$ where the E_{total} condition was imposed to reduce the rate at which neutral events were written to magnetic tape and the neutral veto was defined as $S0 + S1 + H + VA + VB$. VA and VB refer to two photomultiplier tubes which viewed the veto counter (V). Each of the five logic signals in the neutral veto was set-up with a 50 nsec width.

A 'charged event' was defined as $\text{BARS} \cdot S3 \cdot E \cdot (VA + VB) \cdot (\overline{\text{charged veto}})$ where the charged veto was defined as $S0 + S1 + H + \overline{\text{pulser}}$. Here, the purpose of the pulser was to reduce the rate at which charged events were written to magnetic tape by requiring a random coincidence. The effect of the pulser was to reduce the charged rate by approximately a factor of 15.

A 'telescope event' was defined as neutral event + charged event, and a ' μstop event' was defined as $\mu\text{stop gate} \cdot \text{telescope event} \cdot \overline{\text{inhibit}}$. For each 'event' in the trigger logic, the CAMAC modules were activated by gate signals in the case of ADCs and bit pattern registers, and by start signals, in the case of TDCs. One of the CAMAC modules then produced a LAM, and the process of interrupting the computer and reading the CAMAC modules began. In the trigger logic this

Figure 16: Schematic of the NIM electronics for the trigger logic. Also shown are the various vetoes and the event inhibit logic.

final 'event' was defined as $\mu\text{stop event} + \text{pulser event}$, where 'pulser event' was defined as $S0 \cdot S1 \cdot S2 \cdot \text{pulser} \cdot \overline{\text{inhibit}}$. This pulser was independent of the one which reduced the charged event rate. The purpose was to obtain a small sample of events in which a beam particle had arrived at the target with no condition on signals in TINA or the BARS.

The inhibit signal is the logical OR of three signals. As soon as an 'event' has occurred, a 750 μsec long signal is generated. This signal provides an inhibit for a time greater than that required for the computer's interrupt handling routine to save the computer status and start reading the CAMAC crate. While the CAMAC crate is being read, a second signal from a module in the crate, provides the inhibit. Once the entire crate has been read and the modules cleared, the inhibit signal disappears and the system is ready for the next event. The third signal that can produce an inhibit is $(S0 + S1) \cdot (S4 + S5)$ which produces a 300 nsec long inhibit. This signal identifies beam particles which likely stopped in the lead sheet on the pole faces of the magnet. By not requiring an $S2$ signal, this inhibit is produced both for particles scattering out of the target and for those which missed the target completely. The effect of the 300 nsec inhibit is to greatly reduce the number of events from muons stopping in the lead on the pole face. Lead was used in order to keep this inhibit signal as short as possible.

Figure 17 shows the remaining NIM electronics, most of which are related to pile-up detection of various natures. A great simplification over typical pile-up detection circuits was achieved by the use of three non-standard electronics modules. These modules, called pulse separators [150], accept a train of up to four logic pulses at their inputs and then route the individual pulses into four separate outputs. The pulse pair resolution of the modules is better than 6 nsec. Each of the outputs is directed to a different TDC channel, resulting in a pulse separator-TDC system which behaves like a four hit TDC. Two of the pulse

Figure 17: Schematic of the NIM electronics for pile-up detection. Pile-up in the arrival of beam particles as well as in TINA and the BARS is searched for. The cosmic ray veto counter logic is also included.

separators were used to inspect for beam pile-up, their inputs being $S0 + S1$ and μstop signals. The third pulse separator was used to look for pile-up in the TINA CFD signal.

In addition to the TINA CFD pulse separator, pile-up in TINA was searched for by integrating the charge in the TINA analog pulse over 12 different positions along the pulse. This was achieved by delaying the pulse, by different amounts, into 12 channels of an ADC with a single gate. An improvement in this technique, where the gates rather than the analog signals were delayed and a separate-gate ADC was used, is reported in reference [145]. With this improvement, it was possible to see good neutron-gamma pulse shape discrimination (PSD) in TINA for energies greater than 30 MeV. However, no neutron-gamma PSD was observed in this experiment.

Because the volume of the BARS is so much smaller than TINA, pile-up, particularly from low energy neutrons, is much less of a concern. A very simple pulse-shape measurement was made on the summed BARS signal in which the charge in the pulse was integrated over a gate on the leading edge of the pulse and one on the trailing edge. The ratio of the charge in the two bins would be expected to be a constant or at least a slowly varying function of energy. Deviations from the expected ratio are an indication of pile-up. In both the BARS and TINA pulse-shape measurement, it was important to gate the ADC with BARS or TINA CFD timing respectively, rather than with event timing.

The cosmic veto counters were used to set bits in one of the bit pattern registers, indicating which of the counters fired. In addition, the logical OR of all the inner counters plus VA and VB (IN), and of all the outer counters plus the H counter (OUT), were made. An IN·OUT coincidence was also made. These signals were sent to TDC's arranged to look for IN and OUT signals for up to 9 μsec before the event time, and for IN·OUT signals for up to 16 μsec before event time.

Information from these TDC's was used in software for the rejection of probable cosmic ray related events.

III.6 Previous Runs

The RMC experiment in ^{40}Ca at TRIUMF has a history; principally one of fighting backgrounds. Some perspective on the experiment is gained from a brief summary of previous runs.

The RMC experiment described in this thesis received beam on four occasions, the first of which was in August of 1982. One week of beam time was granted on the old M20 beamline. The original version of the experimental geometry called for only TINA and the BARS, without counters in between, and without any cosmic ray veto counters. The beam was sporadic, but an overwhelming 23.055 MHz background signal was immediately seen in the neutral event time spectrum, whether the precession magnet was on or off. The intention had been to run in the stroboscopic μSR mode.

In November of 1982, the experiment returned to the old M20 beamline for one more week of beam. Previously, in August, the telescope had been oriented, without forethought, to look in the general direction of the 1AT2 production target. Now the telescope was rotated nearly 180° around the target and a 60 cm thick iron block placed between TINA and the 1AT2 target. The background remained overwhelming and it was impossible to see the precession frequency, this time set at ~ 6 MHz, due to the amplitude of the 23.055 MHz background. Some progress was made by placing a scintillator between TINA and the BARS. Following the experiment, some background measurements were made with and without the beam in the vicinity of the focus of the new M20A beamline. They compared very favourably to similar measurements in the old M20 area, a fact which was largely attributed to being a factor of two further from 1AT2.

In preparation for a one week test run in the new M20A area, in June of 1983, the layered scintillator and Cerenkov counters were designed and built; plans were made to construct the roof over the experiment; and to mount the inner and outer layers of cosmic ray veto counters. With the new experimental area came a factor of 6 increase in μ^- flux, but with the corresponding increase in π^- flux in the beamline also came an increase in M20 related background. In fact, the observed M20 related background rate scales as the product of the π^- flux and the μ^- flux since the livetime of the data acquisition electronics scales with the μ^- flux. During this run, the layered scintillator and the Cerenkov counter were each inserted separately between the BARS and TINA to determine their effectiveness in discriminating against the 23.055 MHz background. As a result of off-line analysis, it was concluded that at least a statistically significant excess of events had been seen in the 60-90 MeV region. However, off-line analysis was not conclusive about which of the layered scintillator and the Cerenkov counter performed better. A decision was made to use both counters together in a final three week long statistics collecting run in October and November of 1983.

In preparation for this final run, improvements were made to most aspects of the experimental geometry and to the shielding. During the run, the beam blocker for the other leg (M20B) mechanically failed and remained retracted (out of the beam) for the entire run. Over the course of the run, by manipulating the shielding and by adjusting the currents in magnets in both legs, the background was successfully reduced by almost a factor of 10. The data used for this thesis came exclusively from this final run and, although the quality of the data improved over the course of the run, all available data were summed in the analysis.

Chapter IV

Analysis of the Data

IV.1 Introduction

The analysis of the data was performed at TRIUMF using software packages supported for the most part by the TRIUMF computing services group. By linking user-written application-specific routines with these software packages, it was possible to read the MULTI-written magnetic tapes and construct histograms and scatterplots of functions of the data words of arbitrary sophistication. The preliminary reduction of the data consisted of making decisions concerning conditions to be placed on ‘good’ events and then selectively writing these events onto a second generation of magnetic tapes. This process of skimming the original data tapes was very convenient for the later analysis.

Common to all of the later analysis was the establishment of the energy calibration of TINA and the BARS, and the incorporation of the information from the layered scintillator and the Cerenkov counter into an estimate of the incident photon or electron energy. This estimate of the incident particle energy, the ‘reconstructed’ energy, was then used throughout the later analysis. Because energy from the electromagnetic shower, started in the BARS, was deposited in several active and non-active elements of the photon telescope, a Monte Carlo program (EGS3 [151]) was used to simulate the variations in the energy deposition patterns and to guide the design of an optimum energy reconstruction algorithm. Some details of the use of EGS3 for this and other related purposes will be given in this chapter. Other details are left to an appendix.

Aside from the establishment of the calibration, the remainder of the analysis can be divided into several sections with some degree of overlap. A principal division is between the analysis of the charged and the neutral data. By ‘charged’ data we refer to the class of events which satisfied the charged coincidence in the NIM electronics and which are presumably largely associated with electrons (or positrons) from muon decay in the target. Similarly ‘neutral’ events satisfied the neutral coincidence and are most likely associated with photons and neutrons. The charged data are analyzed primarily to determine the residual polarization of the negative muons prior to nuclear capture. The photon asymmetry (α_γ), energy spectrum, and partial branching ratio ($R_{k>57}$) are extracted from the neutral data. Knowledge of the efficiencies of the photon telescope, and of the data analysis cuts, for both background and good photon events are particularly important in the extraction of both the photon energy spectrum, and the partial branching ratio. This constitutes a significant part of the analysis and again involves the use of EGS3.

The analysis of the data reduces the experimental data to essentially six histograms, the {electron, positron and photon} energy and time spectra. Table XII summarizes the information extracted from each of these spectra and outlines some of the inter-relations between the data and the Monte Carlo calculations in the extraction of the final results.

This is a rather long and relatively detailed chapter. Although an attempt has been made to organize and structure the material presented here the reader may find it useful to occasionally refer back to table XII and to the table of contents where the structure and direction of the discussion in this chapter are more evident. Before discussing the energy calibration and the analysis of the charged and neutral data some comments will be made on the backgrounds encountered in the experiment.

Table XII: Summary of the information extracted from the principal histograms of the data analysis.

Histogram	Information Extracted and Comments
e^+ energy	Verification of both the energy calibration and the method of extracting P^{μ^+} via comparison with the Monte Carlo.
e^+ time	Extraction of the <i>beam</i> polarization (P^{μ^+}) via comparison to the Monte Carlo. Also input (with P^{μ^-}) to the measurement of the depolarization factor (D_{μ}^{Ca}) which is a check of the procedure.
e^- energy	Verification of both the energy calibration and the method of extracting P^{μ^-} via comparison with the Monte Carlo.
e^- time	Extraction of the <i>residual</i> polarization (P^{μ^-}) via comparison to the Monte Carlo.
γ energy	Extraction of the partial branching ratio ($R_{k>57}$) via comparison with theoretical calculations. Some of the experimental efficiencies involved in this calculation are calculated by Monte Carlo.
γ time	Extraction of the photon asymmetry (α_{γ}) via comparison with the residual polarization of the muon before capture (P^{μ^-}).

IV.2 Backgrounds

As has already been emphasized backgrounds of various natures constitute an important aspect of this experiment. In this section quantitative estimates of these backgrounds as well as some discussion of their impact on the data analysis are given.

IV.2.1 Cosmic ray background

Cosmic rays represented a significant background in this experiment. By passing through TINA and the BARS, but by avoiding the veto counter, a single cosmic ray can deposit sufficient energy in each to mimic a valid converted photon with a reconstructed energy in the RMC energy region. Since they arrive randomly with respect to the beam muons, they contribute a flat component to the photon time spectrum from which the asymmetry is extracted. While a flat component is more easily dealt with in the fitting of the time spectra than one with structure, the Poisson noise added to the time spectrum quickly begins to limit the fitting program's ability to find the weak asymmetry signal and also increases the errors associated with the fitted parameters.

Experience with the placement of a concrete roof over TINA in the past has shown that a reduction of $\sim 60\%$ in the cosmic ray rate observed in TINA can be achieved with a 1.2 m thick roof. In this experiment the roof subtended a larger solid angle than in previous experiments and had a minimum thickness of 1.2 m. Directly over TINA the roof was 2.4 m thick. Figure 18 displays the further reduction obtained by the use of the inner and outer veto counters and by other offline cuts on the data. The data were recorded during a cyclotron maintenance period in which no beam was produced. The gradual cutoff at low energies, displayed in the spectra, is an artifact of the ~ 25 MeV E_{total} hardware condition

Figure 18: Reduction of the cosmic ray background by various means. Curve a) is the reconstructed neutral energy spectrum for the cosmic rays penetrating the roof. Curve b) shows the effect of requiring no IN·OUT coincidence and curve c) the effect of additionally satisfying the layered scintillator dE/dx cut and other offline cuts.

on neutral data as well as the energy reconstruction process. It is seen that these measures reduce the cosmic ray background by more than a factor of 20. At this level they do not limit the precision attainable in the fitting of the photon asymmetry.

IV.2.2 Cyclotron related backgrounds

These are the backgrounds that appear with the cyclotron frequency in the neutral time spectra. As previously stated, a reduction of approximately a factor of 10 in this background was achieved over the course of the final data-taking run by rearranging and increasing the shielding and by tuning both M20A and M20B beamlines. While this improvement in the background was sufficient to allow the observation and the extraction of the photon asymmetry, the extent of the remaining background proved to limit the quality of the data. The extensive shielding appeared to have eliminated the very high energy background observed in previous runs and that which remained was consistent with a large component of π^- -induced photons and neutrons. The exact nature of this background and its source was not fully appreciated until after the conclusion of the run.

A decision was made after the run to analyze the data to completion but to also continue to try to understand the cyclotron related background in order to improve the quality of the data in future runs. The group examined several possibilities for moving the experiment further from the M20B channel (the presumed source of the background) or for transporting the pions further from the experiment. Neither alternative was very practical but some interesting calculations were performed [152]. The flux of π^- in the second bending magnet (M20B2) is approximately $1 \times 10^8 \text{ sec}^{-1}$ for nominal proton beam currents. This figure exceeded our naive expectations by about 2 orders of magnitude. Furthermore, beam studies showed that no more than 2/3 of the pions could be

made to exit M20B2 by the M20B port and that less than 10% of those could be transported by the M20B channel to the M20B focus. In short, essentially all of the substantial pion flux was being stopped in and around the M20B channel only 3 to 4 m from the experiment. As previously mentioned, the local beam blocker located in M20B2, which normally stops the beam from entering the M20B channel, had failed just prior to the run and was stuck in its retracted position. This had prevented us from experimenting with the effect of inserting this beam blocker on the background rate. During the run however, we believed that there was more to be gained by tuning the M20B leg, to transport the pions further away, than later calculations showed. With this clarified, it was evident that the background was best handled by inserting the M20B beam blocker in order to at least localize the source of background and then increase the shielding between this source and the detector. This philosophy was put to the test in October of 1984 when a three fold reduction of the background was demonstrated by simply inserting the blocker. Further reduction of the background was achieved by additionally increasing the shielding. It is very unfortunate that the option of inserting the blocker was not available during the collection of the data described in this thesis.

IV.2.3 OMC neutrons

Neutrons from ordinary muon capture are an unavoidable background which cannot be shielded against. In the neutral time spectrum they have the lifetime of a μ^- in calcium just as do the real RMC photons. This makes the estimation of their contribution to the observed rate and asymmetry difficult unless it can be demonstrated that the photon telescope effectively discriminates against them. This, of course, was the purpose of the NaI(Tl) active converter, the BARS.

Figure 19 shows the OMC neutron and RMC photon spectra for ^{40}Ca . The

Figure 19: The OMC neutron and RMC photon spectra for ^{40}Ca . Bog69 refers to Bogan 1969 [111] and Chr81 refers to Christillin 1981 [131]. Both curves are theoretical calculations, Christillin's is for $g_P/g_A = 8$. Koz85 refers to an experimental measurement of the OMC in Ca neutron spectrum by Kozłowski et al. [108]. The horizontal bar indicates a 60–90 MeV energy bin. The vertical error bar does not include normalization errors estimated at 12%.

partial branching ratio for high energy neutrons is seen to exceed that for high energy photons by approximately a factor of three. It is possible to measure the neutron discrimination afforded by the BARS with data recorded as part of the calibration runs. Interactions of negative pions in the high pressure hydrogen gas target produce both photons and neutrons. The calibration runs were recorded with a modified trigger which required only a signal in TINA coincident with a pion arrival. In this way both converted and unconverted events were written to magnetic tape. We define the time of flight (TOF) to be the difference in time between the pion stop and the event in TINA. Photons then have a constant TOF between the target and TINA while neutrons have a TOF that varies with energy and, for the relatively low momentum neutrons from the target, is several nanoseconds longer. Figures 20 and 21 show the discrimination against neutrons achieved when a minimum of 5 MeV is required to be deposited in the BARS. The curve at the bottom of the inset plot of figure 21 is a gaussian whose parameters were taken from a simultaneous fit of gaussians to the two peaks in figure 20. Its area is that which would correspond to a factor of 100 discrimination against the neutrons. That is

$$\left. \frac{N^n}{N^\gamma} \right|_{after\ BARS\ cut} = 0.01 \left. \frac{N^n}{N^\gamma} \right|_{before\ BARS\ cut} \quad (66)$$

From a comparison with the smooth line through the histogram in the inset plot it can be seen that the discrimination is of the order of a factor of 100 or more. Presumably the additional requirement of an energy signal > 0.15 MeV in the layered scintillator, consistent with the passage of electrons, further improves the neutron discrimination. We lack sufficient calibration data to measure this improvement in neutron discrimination.

Neutrons leave energy in a NaI(Tl) detector principally by elastic scattering. Because of the difference in mass between the neutron and the nuclei in the

Figure 20: Time of flight spectrum to TINA for photons and neutrons depositing 40 to 90 MeV in the photon telescope. There is no cut on energy in the BARS.

Figure 21: Effect of 5 MeV cut on BARS energy on the time of flight spectrum of figure 20. The inset plot has been scaled up by a factor of 20. The line through the inset histogram is only to guide the eye.

detector, only a very small energy is lost by the neutron to the recoiling nucleus in each collision. Even in a detector the size of TINA the neutron elastic scattering cross-section is such that the full energy of the neutron is rarely deposited in the detector. This effectively increases the neutron discrimination, by shifting the observed neutron energy spectrum to lower energies, and it is safe to conclude that the total contribution of OMC neutrons to the neutral energy spectrum is $\ll 1\%$. At this level the OMC neutrons can be ignored.

There is a flux of neutrons from the $1 \times 10^8 \pi^- \text{ sec}^{-1}$ which stop about 3-4 meters from the detector. At the NaI(Tl) detector, the flux is a function of the actual positions of the sources and of the shielding and is difficult to estimate. Whether the neutron discrimination achieved with the BARS and the layered scintillator is sufficient to eliminate these neutrons from consideration is a valid question. However neutral events induced by these neutrons do not exhibit the characteristic 335 nsec lifetime associated with good neutral events. It will be seen later that this allows their contribution to the neutral time spectrum to be subtracted out.

IV.2.4 Bremsstrahlung

The contribution of bremsstrahlung from the decay electrons in the target is easily estimated. Figure 22 shows the energy spectrum of the decay electrons as curve a). This energy spectrum is given by

$$N^{e^-}(E) = N_{\mu} \cdot (1 - f_C) \cdot d\Lambda^{e^-}/dE \quad (67)$$

where N_{μ} is the number of μ^- stops in the calcium target and $d\Lambda^{e^-}/dE$ is the bound decay electron spectrum of Herzog and Alder [100] normalized to unit area. Curve b) is the bremsstrahlung spectrum due to the electron spectrum of curve a) for 0.2 radiation lengths of calcium. This is the average distance out of the target

Figure 22: Contribution of bremsstrahlung from decay electrons to the neutral energy spectrum. Curve a) is the energy spectrum of the decay electrons. Curve b) is the bremsstrahlung spectrum calculated from curve a). Curve c) is the 'observed' energy spectrum in the photon telescope after the detector solid angle, efficiencies and lineshape have been taken into account. Further details are found in the text.

for electrons headed in the direction of the detector. The spectrum is calculated as

$$N^\gamma(k) = \int \frac{t}{k} F(t, k, E) \cdot N^{e^-}(E) dE \quad (68)$$

where t is the target thickness in radiation lengths and $F(t, k, E)$ is similar to the bremsstrahlung functions given by Tsai and Whitis [153] which are valid for much higher energies. EGS3 was used for guidance in modifying $F(t, k, E)$. At 1 GeV EGS3 was found to be in excellent agreement with $F(t, k, E)$ as given by Tsai and Whitis and it was assumed that the discrepancies observed at lower energies were due to the limitations of the formulas as stated by these authors. Curve c) is the spectrum of photons which would be detected by the photon telescope. Here the solid angle, the efficiencies of the hardware and software cuts, and the detector response function have been taken into account. These effects will be considered in more detail when the acceptance of the detector is discussed in the calculation of the partial branching ratio for RMC.

Because bremsstrahlung overwhelms the low energy portion of the photon spectrum, RMC measurements have typically been restricted to photon energies in excess of 57 MeV. In the estimation presented here 33 counts due to bremsstrahlung of decay electrons in the target are seen to fall above 57 MeV (curve c)). This represents $\sim 1\%$ of the number of observed photons after the background subtraction has been performed.

IV.2.5 Muon stops outside of the target

The fraction of muons stopping in environments other than calcium can be estimated by fitting the charged time spectra with a μ SR function which has more than one lifetime component. A general two component μ SR function has the form

$$N(t) = N_1^{t=0} e^{-t/\tau_1} [1 + A_1 \cos(\omega t + \phi)] + N_2^{t=0} e^{-t/\tau_2} [1 + A_2 \cos(\omega t + \phi)] + B \quad (69)$$

where the $N_i^{t=0}$ are normalization constants, τ_i are the mean lifetimes, A_i are the asymmetries, $\omega = 2\pi\nu_{prec}$ is the precession frequency, ϕ is an initial phase, and B is a flat (random) background term. Due to variation in the magnetic field over the volume of the target, there is a finite width which can be associated with the precession frequency. Over several precession periods muons, initially precessing in phase, gradually become de-phased and the asymmetry signal is attenuated. This attenuation of A_i with time is reasonably modelled as an exponential relaxation and A_i in equation 69 can be replaced by $A_i e^{-\lambda_i t}$. The necessity for including such relaxation factors was tested early in the data analysis and values of λ consistent with zero were always found. Hence a relaxation parameter was not included in later analysis.

By fixing τ_C at 2.03 μsec , the lifetime for a μ^- in carbon [107], parameter values for $N_{Ca}^{t=0}$ and $N_C^{t=0}$ were extracted from the charged time spectra. These values are related to the number of μstops by

$$\frac{\mu\text{stops in C}}{\mu\text{stops in Ca}} = \frac{N_C^{t=0}}{N_{Ca}^{t=0}} \times \frac{\tau_C}{\tau_{Ca}} \times \frac{f_C^{Ca}}{f_C^C} \times \frac{\Omega_{Ca}}{\Omega_C} \quad (70)$$

where Ω_{Ca}/Ω_C is the ratio of solid angles for μstops in calcium to μstops in carbon. This ratio was estimated to be 1.5 ± 0.5 since decay electrons from μstops in $S4$ and $S5$ will have a smaller effective solid angle than stops in the target or in $S2$ or $S3$. With this assumption the fraction of the total μstops estimated to have been in a carbon-like environment (the scintillators and their wrappings) was found to be $1.1 \pm 0.7\%$.

The μ^- lifetime in calcium found in the above two component fit was 352 ± 4 nsec. This is to be compared to the value of 335.0 ± 0.8 of table VI. The reason for the discrepancy is unclear. The TDC was calibrated at the end of the run and the 23.055 MHz structure present in the neutral time spectra confirmed the calibration. Systematic errors in the calibration are certainly less than 1 %. If

the lifetime of the carbon component was left as a free parameter in the fit then the best fit corresponded to a reduced intensity for the carbon component and a longer lifetime of 358 ± 4 nsec for the calcium component. The presence of a shorter lifetime component, such as one corresponding to the lifetime of a μ^- in iron or lead, would tend to cause a shorter lifetime to be fitted for the calcium component in a two component fit. No such component could be found in the charged data. A small part of the data was recorded with a carbon target replacing the calcium one. The targets were of the same dimensions and of comparable densities so the scattering of muons out of the two targets should be quite similar. Because the lifetime of the μ^- in carbon is several times longer than the calcium lifetime, a short lifetime component would stand out even more for the carbon target run. Figure 23 shows the fit to the carbon target data. Within the limitations of the low statistics no short lifetime component is seen. It can be concluded that the contribution to the spectrum from μ stops in the poletips of the precession magnet is very small. The 300 nsec $(S0 + S1) \cdot (S4 + S5)$ veto in combination with the lead sheet on the poletips appears to work quite well at reducing this background. The reason for the discrepancy in the μ^- lifetime remains unresolved. The effect of pre and post muon pile-up on the μ SR signal has been calculated by Garner [154] in detail. Both tend to distort the time spectrum in a way which leads to an underestimation of the muon lifetime when the spectrum is fitted. It should be pointed out that the experiment was not designed with a precision measurement of the lifetime as an objective. Such experiments usually employ lower stopping rates and record the time spectrum over several μ seconds as opposed to the 1 μ sec window used in this experiment. A similar large value for $\tau_{\mu^-}^{Ca}$ of 365 ± 8 nsec was observed in the earlier calcium RMC experiment of Hart et al. [106].

Figure 23: Fit to μ^- in carbon time spectrum to inspect for short lifetime components. None are evident.

IV.2.6 Beam contamination

Both pions and electrons are present in the muon beam to some degree. They are only of concern if they interfere with the counting of μ stops or if they produce a background in the detector. Since the two bending magnets, M20B1 and M20B2, select different momenta (those corresponding to backward muons from the decay of a 165 MeV/c pion beam), any electrons or pions which enter the experimental area were likely scattered in the beamline. The flux of electrons in the beam was not constantly monitored but it was measured earlier to be $\sim 2\%$ in normal TOF. Electrons, momentum-selected at the second bending magnet, enter the area with a kinetic energy of approximately 80 MeV. These electrons will not stop in the target (in general) and therefore they do not interfere with the counting of muon stops. Some fraction of the electrons however will undergo hard bremsstrahlung in the target and mimic a valid μ stop. Due to conservation of momentum the bremsstrahlung photon distribution is highly forward peaked. It is also in prompt coincidence with the particle stop and so will not contribute to the observed energy spectrum. From figure 22 it is seen that hard bremsstrahlung occurs about one in a thousand times making the error in the counting of the μ stops negligible.

Pions will stop in the target and although they interact promptly they have the potential, as previously mentioned, to contribute to the observed neutral energy spectrum. To estimate the fraction of pions in the muon beam some data were taken with the prompt ($S0 + S1$) veto disabled. When this was done a sharp peak could be observed in the neutral time spectrum from the 2% branch for photon production. The area of this peak was determined as well as the area under the decay electron time spectrum. The fraction of pions in the beam was determined from these two areas by correcting for the differences in charged and

neutral event solid angle, livetime, and branching ratio.

$$\frac{\pi^-}{\mu^-} = \frac{\text{prompt neutral events}}{\text{charged events}} \times \frac{\Omega^c}{\Omega^n} \times \frac{L^c}{L^n} \times \frac{BR^c}{BR^n} \quad (71)$$

This calculation yielded $\pi^-/\mu^- = 6 \pm 1 \times 10^{-5}$. At this level the inclusion of pion stops with muon stops does not significantly affect the estimation of the total number of muon stops (used as a normalization in the calculation of the partial RMC branching ratio). Also with the prompt veto in place it is expected that pions make a negligible contribution to the neutral energy spectrum.

Some data were also taken with a negative pion beam stopping in the calcium target with the prompt veto disabled in the NIM electronics. The observed neutral energy spectrum is shown in figure 24. As will become evident when the neutral energy spectrum for the RMC data is shown, there is certainly a component present in the spectrum which resembles figure 24. Actually, negative pions stopping in any medium Z material produce similar spectra [155].

If the pion contamination in the beam is as small as 6×10^{-5} then one is forced to conclude that the observed contribution to the RMC neutral energy spectrum is due to pions which did not pass through *S0* and *S1* nor stop in the target. Rather, it appears more likely that the observed contamination of the neutral energy spectrum was due to pions stopping in the vicinity of the M20B leg of the secondary channel.

IV.3 Energy Calibration

IV.3.1 The π^-p reaction at rest

Most of the data useful in the energy calibration of the photon telescope was recorded with the high pressure gas target [144] replacing the calcium target. This target was filled with 100 atmospheres of hydrogen gas and the pion beam momentum was selected to maximize the number of pion stops in the gas itself.

Figure 24: Neutral energy spectrum for negative pions stopping in calcium.

Internal scintillators provided signals useful in identifying stops in the gas.

Stopped negative pions also undergo atomic capture and then cascade towards the 1S orbital much as negative muons do. There are some differences which are well described by Koch [83] but they are not of consequence here. The π^- and the proton interact strongly via the reactions

$$\pi^- + p \rightarrow \begin{cases} \gamma & + & n & (39.6\%) \\ \pi^0 & + & n & (60.4\%) \end{cases} \quad (72)$$

and the π^0 decays immediately into two photons. The photon from the 39.6% radiative capture branch is mono-energetic with an energy of 129.4 MeV. The photons from the decay of the π^0 are Doppler shifted by the kinetic energy of the π^0 . The energy spectrum of a single observed photon from the decay of the π^0 is a flat rectangular distribution which begins at 54.9 MeV and ends at 83.0 MeV. This corresponds very conveniently to the energy region of interest for the RMC measurement.

IV.3.2 Role of EGS3

The program EGS3, as modified for the present experiment by Dorothy Sample, writes events to magnetic tape in a format very similar to MULTI. These events can therefore be studied using the standard analysis programs employed in the analysis of the real data. In the case of the EGS3 events, the information written to magnetic tape was the energy (in MeV) deposited in the various elements (active and non-active) of the photon telescope as well as other information not available in the real data set, such as the initial coordinates, direction cosines, and energy of the photon within the target.

EGS3 makes no assumptions about the energy resolution of the detector elements and simply gives the energy deposited. In the NaI(Tl) detectors the observed response to mono-energetic photons is an asymmetric distribution of

finite width with a low energy tail. The low energy tail and part of the finite width are due to the inefficiency of the detector system to capture or contain all of the electromagnetic shower. This contribution to the detector response is well modelled by EGS3. Variations in the concentration of the thallium dopant in the NaI(Tl) crystals are responsible for most of the remaining width of the response function, with photon statistics also making a small contribution. It has been found [145] that the observed response function can be obtained by convoluting a gaussian with the EGS3 lineshape. The width of this gaussian is a function of energy ($\sigma = k \cdot E^{1/2}$ adequately describes this energy dependence).

The calibration procedure, using data generated by EGS3, is an iterative one in which the goal is to find the parameters of the calibration and the values of k which produce the best agreement between the experimental spectrum and the Monte Carlo data. It is assumed that three values of k are required; one for each of TINA, the BARS, and the layered scintillator. Each of the individual BARS and layered scintillators is calibrated separately with its own unique calibration parameters.

IV.3.3 TINA and the BARS calibration

It is common practice at TRIUMF to make some attempt, in setting up an experiment with TINA, to ‘balance’ TINA’s photomultiplier tubes by individually varying the high voltages so as to obtain the same gain in each tube. Conventional wisdom is that equal gains produce the best resolution, although the resolution has never been observed to be a strong function of the gains. Typically, the gains are estimated by observing the position in an ADC spectrum of the 129.4 MeV photon, from the π^-p reaction at rest, when all but one of the tubes are switched out at a preamplifier.

However, a year prior to the final run, TINA was severely fractured in

shipping. An ADC spectrum of a single tube now shows distortion of the expected spectrum, with some tubes exhibiting multiple '129.4 MeV' peaks. It is rather amazing that, when all seven tubes are summed, the energy resolution achieved is essentially the same as before the shipping incident. Nevertheless, the fracturing has complicated the balancing procedure by making the estimation of the tube gains very difficult. This appears to be particularly true for the nearly uncollimated geometry in which TINA was used in this experiment. The energy resolution for TINA (all seven tubes) initially observed in the calibration data was 9.5% at 129.4 MeV. This can be compared to resolutions of better than 5% [145] obtained in collimated geometries. In the off-line analysis of the data a software balance was sought which would minimize the width of the 129.4 MeV peak. The general function minimization routine MINUIT [156] was used for this purpose and the resolution was improved to 7.1%.

The energy deposited in TINA is

$$E_{TINA} = K \sum_i G_i (TINA_i - ped_i) \quad (73)$$

where $TINA_i$ and ped_i are the ADC word and the ADC pedestal for the i th tube, G_i is a relative calibration constant, K is an overall calibration constant, and the sum is over all seven tubes. The G_i were the free parameters of the MINUIT minimization and were initially set to 1, which appeared to be within 10% of the 'equal gains' solution. However, the fitted values were found to be (in order G_1 to G_7) 1.137, 0.704, 0.871, 0.626, 1.906, 1.039 and 0.716 which exhibit deviations in excess of 10%.

One expects the best resolution to be obtained when the gains of the individual tubes are truly equal. It has always been assumed that approximately the same number of photons enter each photomultiplier tube after a large electromagnetic shower and that therefore equal pulse-heights in the seven tubes

signifies equal gains. A possible interpretation of the above results, in which the best resolution was obtained with relative calibration constants ranging from 0.626 to 1.906, is that the fractures in TINA greatly distort the assumed uniform distribution of photons over the collecting surface. In fact, tube 5 appears to collect three times as many photons as tube 4. Whether or not this interpretation is correct is of some interest to future users of TINA. It would also be of interest to learn whether the resolution can be improved, in the case of a collimated geometry, by similar techniques.

The software balance described above was used throughout the analysis because of the improved resolution. The data useful in determining the calibration of TINA consisted of runs 60-62 where the BARS, the Cerenkov counter and the layered scintillator were not yet installed, and runs 63-69 and 162-167 where all counters were installed. Since the trigger for a neutral event (for the calibration) did not include the BARS, both converted and unconverted events were recorded.

It was found necessary in the analysis of the μ^- in Ca data to impose a cut on the neutral data which required that all three of the layered scintillators had fired. This was called the LST cut and it will be discussed in more detail later. Figures 25 and 26 show the TINA energy spectra for converted photons with and without the LST cut applied. The same calibration for TINA is used in both figures and also in a comparison of the unconverted photon spectrum to the Monte Carlo. The calibration used represents a compromise between these three cases. This comparison of experimental data to Monte Carlo was also made for runs 162-167 and there slightly different calibration constants were found. In particular, the sum of the seven pedestals had drifted up 0.9 MeV and the gain had also increased 1%. The drift of the pedestals could be followed using the pulser events. It was seen to be a smooth function of run number. The overall gain of TINA could not be tracked with sufficient precision and it was assumed that

Figure 25: TINA energy spectrum and the smoothed Monte Carlo results for converted photons *with* the LST cut applied. The data is from runs 63-69.

Figure 26: TINA energy spectrum and the smoothed Monte Carlo results for converted photons *without* the LST cut applied. The data is from runs 63-69.

the 1% change in gain occurred gradually. Calibration parameters, which were the simple average of these early and late calibration runs, were used in the analysis of the data. The differences in the calibration parameters between the early and the late runs were assigned as systematic uncertainties in these parameters.

The only adjustable parameters in the EGS data were the width of the additional gaussian smearing and thresholds in TINA and the BARS. The threshold in the BARS comes into play in the definition of a converted event. These thresholds were set to correspond to the experimental data, and the width parameter and its associated systematic error were found by fitting the width of the 129.4 MeV peak, in the EGS data, to the experimental peaks, for both the early and late calibration runs.

The calibration of the BARS proceeded in a different manner but still quite dependent on EGS3. Zych et al. [147] have shown that if the pulse-height varies exponentially as a function of distance from the photomultiplier tube, then the energy deposited in a bar is given by

$$E_i = C_i \sqrt{L_i \cdot R_i} \quad (74)$$

where L_i and R_i are the pulse-heights (with the pedestals subtracted) in the left and right-hand end of the i th bar. C_i is a calibration constant. This relation was employed but it was found necessary to apply a small parabolic correction

$$E_i^{corr} = E_i \cdot (1 + AP_i^2) \quad (75)$$

where A is a constant for all bars and P_i is the position in the i th bar given by

$$P_i = M_i(L_i - R_i)/(L_i + R_i) + B_i \quad (76)$$

where the B_i have been chosen to place $P_i = 0$ at the centre of the bar.

From symmetry one expects that the energy spectrum in bar 1 should be similar to that in bar 7. Similarly, bar 2 can be paired with bar 6 and bar 3 with

bar 5. The intercalibration of the BARS was achieved by comparing the energy spectra of bars 1 and 7 with the sum of the EGS3 energy spectra for bars 1 and 7. This was necessary, as the shape of the energy spectrum for converted photons was observed (both in the data and in EGS3) to be a function of the distance from the detector axis. This is at least partially due to the changing solid angle of TINA for the electromagnetic shower.

When each of the individual bars was intercalibrated in the above fashion, the energy in the BARS as a whole

$$E_{BARS} = \sum_i E_i^{corr} \quad (77)$$

was histogrammed for the early and late calibration runs. A comparison of the two histograms showed that any change in calibration was less than 1%. A systematic error of $\pm 1\%$ in the calibration was assumed as this was sufficient to generate easily discernable differences between the EGS3 and the experimental E_{BARS} spectra. Figures 27 and 28 show the BARS energy spectra for converted photons with and without the LST cut applied. Without a peak in the spectra it was not possible to fix the width of the gaussian smearing function as was done for TINA. The FWHM of the BARS energy resolution has been measured separately to be $\sim 18.5\%$ at 662 keV and $\sim 3.2\%$ at 40 MeV. These values are fit by $\sigma_{BARS} = 0.065E^{0.572}$. The choice of $\sigma_{BARS} = 0.065(\pm 0.065)\sqrt{E}$ for the width of the smearing function and its associated systematic error certainly covers the actual values and has a surprisingly small effect in the calculation of later systematic errors.

IV.3.4 Energy reconstruction

It is not sufficient to simply add the energy deposited in the BARS and TINA together and associate this sum with the photon energy since the charged particles

Figure 27: BARS energy spectrum and the smoothed Monte Carlo results for converted photons *with* the LST cut applied. The data is from runs 63-69.

Figure 28: BARS energy spectrum and the smoothed Monte Carlo results for converted photons *without* the LST cut applied. The data is from runs 63-69.

in the electromagnetic shower can deposit significant energy in the material between the BARS and TINA. The energy deposited in the layered scintillator was measured, albeit with poor resolution, and it can be added to the BARS plus TINA energy. The energy deposited in the back face of the BARS package, the Cerenkov counter, and the front face of TINA can not be measured since these are not scintillating materials. The energy in these non-active elements of the photon telescope is primarily a function of the actual number of charged particles which pass between the BARS and TINA in the shower. The total thickness of the non-active elements is approximately 2.70 g/cm^2 . An inspection of the energy deposited in the layered scintillator can provide an estimate of the number of charged particles in the intervening shower and hence a correction can be made for the energy deposited in the non-active elements. The reconstructed energy is then the sum of the BARS, TINA, and layered scintillator energies plus this estimate of the energy in the non-active elements.

The energy loss of a fast particle traversing a thin layer of material is given by the Landau distribution [157]. The distribution is asymmetric with a large high energy tail. EGS3 does a reasonable job of modelling the Landau distribution observed in each of the three elements of the layered scintillator. The modelling improves as the energy thresholds in EGS3 are lowered, but the computation time increases dramatically and a compromise must be made. Some insight into the consequences of this compromise is given by Rogers [158,159]. By using three elements in the layered scintillator, the Landau distribution is sampled three times. Scatterplots of the energy in one element against the energy in another are consistent with the distributions expected for independent samples. If a histogram is made of the lowest energy of the three independent samples, then the high energy tail of the Landau distribution is much less pronounced and the resulting histogram has nearly symmetric peaks. Figures 29 and 30 show this effect. The

discrepancy in these figures between the Monte Carlo results and the experimental data can be partially attributed to the compromise mentioned above. Neither figure has an LST cut on it. The exponential low energy background seen in figure 29 is due to Compton scattering or photoelectric absorption of low energy photons in single elements of the layered scintillator. This disappears when the LST cut is applied and also in figure 30 when all three elements are sampled. It is surprising that so few of the electron-positron pairs appear to escape the BARS and traverse the layered scintillator. It is perhaps even more surprising that the electromagnetic shower between the BARS and TINA is completely neutral for 55% of the converted photons. The experimental data and EGS3 both agree on this number. This is an unexpected consequence of having a 2 radiation length thick converter. The implication is that the layered scintillator cut is only 45% efficient for good converted photon events.

EGS3 was used to look at the energy deposited in the non-active elements and the correlation with the minimum layered scintillator energy of figure 30. Figures 31 and 32 display the results where no additional gaussian smearing has been applied to the energy in the non-active elements. It is seen that a single electron deposits ~ 5 MeV in the non-active elements and a pair deposits ~ 10 MeV. Table XIII summarizes the energy reconstruction for charged and neutral events.

The analysis of the data is not particularly sensitive to the choices of $E_{MISSING}$ or the specification of regions of minimum layered scintillator energy given in table XIII. However, it is important that the same energy reconstruction algorithm be used in both the experimental data and the Monte Carlo data. Due to the LST cut, neutral events with < 0.15 MeV as a minimum layered scintillator energy are discarded. It was also found that discarding events with a minimum layered scintillator energy of > 1.50 MeV was quite effective in eliminating a

Figure 29: Layered scintillator energy spectrum and the smoothed Monte Carlo results. The peak at 1.8 MeV corresponds to a single electron and that at 3.6 MeV to a pair. The energy spectrum is the sum of the energies in the three elements of the layered scintillator.

Figure 30: Smallest energy deposited in a single element of the layered scintillator and the smoothed Monte Carlo results. Note the improved separation of the single electron peak (0.6 MeV) and the pair peak (1.2 MeV).

Figure 31: Relief plot of minimum layered scintillator energy in a single element versus energy in the non-active elements from EGS3. The one and two electron peaks are indicated. Figures 30 and 32 are the projections on the axes.

Figure 32: Energy in the non-active elements of the photon telescope from EGS3. The one and two electron peaks are evident at 5 and 10 MeV.

Table XIII: Energy reconstruction for charged and neutral events. $E_{MISSING}$ is the energy added to the BARS plus TINA plus layered scintillator energy to compensate for energy deposited in non-active elements. In the case of charged events, 10.2 MeV corresponds to the average energy deposited in the calcium target by the decay electron as determined by EGS3.

Minimum LS energy (MeV)	$E_{MISSING}$ (MeV)	
	neutral	charged
0-0.15	—	10.2
0.15-0.90	5.0	15.2
0.90-1.50	10.0	20.2
>1.50	—	20.2

portion of the neutral background while retaining most of the good photon events.

Figures 33 and 34 show the reconstructed energy spectra with and without the LST cut applied. The accuracy of the energy calibration is of interest particularly in the region of the RMC data (from 57 to 90 MeV). Also for the calculation of the partial branching ratio, a systematic error in the calibration at 57 MeV must be assigned. This systematic error is taken to be ± 1 MeV although figures 33 and 34 suggest it may be less. The degree of disagreement near 80 MeV has prompted this choice. A somewhat independent check of the energy calibration near 50 MeV is provided by the decay electron and positron energy spectra of figures 35 and 36. The agreement obtained here between the experimental data and EGS3 supports the energy calibration and the assignment of the systematic error.

IV.3.5 Photon response functions

EGS3 Monte Carlo calculations were performed for mono-energetic photons of 50 to 100 MeV in steps of 10 MeV. The simulations were performed with photons whose initial coordinates were chosen from a realistic muon stopping distribution

Figure 33: Reconstructed energy spectrum and the smoothed Monte Carlo results *with* the LST cut applied. The experimental data are from runs 63-69 for photons from the π^-p reaction at rest. The attenuation of the π^0 photons by the LST cut, relative to the radiative capture photons, is seen when this figure is compared to figure 34. The Monte Carlo spectrum was normalized to the area in the experimental spectrum.

Figure 34: Reconstructed energy spectrum and the smoothed Monte Carlo results *without* the LST cut applied. The experimental data are from runs 63-69 for photons from the π^-p reaction at rest.

Figure 35: Decay electron reconstructed energy spectrum from bound μ^- decay and the smoothed Monte Carlo results.

Figure 36: Decay positron reconstructed energy spectrum from μ^+ decay and the smoothed Monte Carlo results.

in the calcium target. The reconstructed energy spectra were formed using the values for TINA, BARS, and layered scintillator smearing functions determined in the energy calibration and reconstruction. Appropriate values for the BARS and TINA thresholds were also used. The Monte Carlo spectra obtained were fit to the function

$$N(E_{obs}) = Ae^{(E_{obs}-B)/D} \cdot \left[1 - \operatorname{erf} \left(\frac{E_{obs} - B}{C} \right) \right] \quad (78)$$

where E_{obs} is the observed energy, and A, B, C and D are parameters which can be expressed as function of E_γ , the incident photon energy. For a response function of unit area

$$A = \frac{1}{2D} e^{C^2/4D^2} \quad (79)$$

The results of these Monte Carlo calculations are shown in figures 37 and 38.

IV.4 Charged Data Analysis

IV.4.1 Role of EGS3

The observed positron or electron asymmetry is a function of the $\mu^\pm$ polarization. For a given interval in observed (reconstructed) energy, the observed asymmetry is also a function of energy loss in the target, multiple scattering in the target, finite target and solid angle effects, and the energy reconstruction algorithm itself. EGS3 was used to simultaneously model all these effects and to predict the asymmetry that would be observed, for any interval in reconstructed energy, assuming a muon polarization of 100%. The actual muon polarization was then expected to be given by the ratio of the experimental asymmetry to the EGS3 asymmetry.

In performing these Monte Carlo calculations a realistic muon stopping distribution in the calcium target was employed. It was obtained from a separate Monte Carlo energy loss program into which the relevant experimental geometry

Figure 37: Monte Carlo response functions for 50, 60, 70, 80, 90, and 100 MeV photons. For clarity only the Monte Carlo data for 100 MeV are superimposed on the fit. The variation in area under the response function reflects the energy dependence of the experimental acceptance.

Figure 38: Fits to the energy dependence of the parameters of the photon response function.

and the known beam characteristics of table X were inserted. Also, in the EGS3 Monte Carlo calculations, the appropriate energy and asymmetry distributions (specified in chapter II) for μ^+ and bound μ^- decay were used. Additional details on the extraction of the asymmetry from the Monte Carlo results are given in the appendix.

Differential asymmetry distributions were formed by sub-dividing the reconstructed energy spectra into the following 10 MeV wide intervals: 15-25 MeV, 25-35 MeV, 35-45 MeV, and 45-55 MeV. Integral asymmetry distributions were formed from the following intervals in reconstructed energy: 15-55 MeV, 25-55 MeV, 35-55 MeV, and 45-55 MeV. Both integral and differential asymmetry distributions were employed in the extraction of the $\mu^\pm$ polarizations from the experimental and EGS3 data. Slightly different systematic errors can be associated with the use of one or the other of these distributions. In particular, it should be noted that the two distributions are not independent of one another, as they are constructed from the same initial data.

IV.4.2 Treatment of the experimental data

The experimental definition of a charged event was quite restrictive and consequently the charged data contain no significant backgrounds. Also, since the absolute acceptance for charged events does not enter into the RMC rate calculations, the detailed nature of the cuts that were applied to the data are of little interest. These cuts were essentially pile-up rejection cuts. Only one cut, the 2nd μ_{stop} cut, need be described.

The 2nd μ_{stop} cut was implemented in software using the TDC information from the μ_{stop} pulse separator. Events which had more than one μ_{stop} , within a time window that extended from 939 nsec before the event to 171 nsec after the event, were discarded. This represented $\sim 20\%$ of the recorded events. Events

surviving this cut, for which the μ_{stop} occurred between 939 and 67 nsec before the event, were termed foreground events. Those for which the μ_{stop} occurred 25 to 171 nsec after the event were termed background events. Events with a μ_{stop} between 67 nsec before the event to 25 nsec after the event were discarded by hardware and software prompt timing cuts.

Without the 2nd μ_{stop} cut, the time distribution is distorted in a fashion which is a function of the μ_{stop} rate. With the possibility of more than one muon in the target at a given time, the apparent lifetime of the muon is decreased. In addition, the random background under the muon related events, which is flat when the 2nd μ_{stop} cut is used, is described by an exponential when the cut is removed. Muons which arrived more than 939 nsec before the event were not observed in the μ_{stop} pulse separator TDCs because they fell outside the TDC range. A small fraction of these ‘old’ muons will not have decayed and they will be present in the target from time to time. The contribution of old muons to the foreground and background time windows was calculated by Monte Carlo and found to be 0.75% and 0.085% respectively, for $\tau_{\mu^-}^{Ca} = 335$ nsec and for the average μ_{stop} rate employed in the present experiment ($\sim 250\text{K/sec}$). Using the observed lifetime of $\tau = 352$ nsec does not significantly alter the Monte Carlo results. It should be added that since the effect of old muons is to shorten the observed lifetime, their presence cannot explain the discrepancy between our fitted lifetime and the world average of 335 nsec.

The experimental data was sorted to build the charged event time spectra corresponding to the various differential and integral energy bins. The fitting routine MINUIT [156] was used to fit a μSR function to the time data in order to extract the experimental asymmetries. As previously stated, the asymmetry relaxation factor was ignored as well as the carbon lifetime component and any

possible iron or lead components. The μ SR function employed was thus reduced to

$$N(t) = N^{t=0} e^{-t/\tau} [1 + A \cos(\omega t + \phi)] + B . \quad (80)$$

The MINOS command of MINUIT finds the true confidence intervals (errors) on the parameters of the fit by analyzing the exact behaviour of the χ^2 function. In all analysis presented here, 1σ errors (68% confidence limit) have been used.

It was found that fixing the value of the background term (B) to the average value per nsec observed in the background time window, and fixing the angular frequency (ω) to the fitted value of the energy interval with the largest asymmetry, saved computer time and did not artificially lower the MINOS errors of the asymmetry parameter (A). Examples of fits to the electron and positron time spectra are given in figures 39 and 40. The raw charged time spectra show a sharp distortion (width ~ 5 nsec) at intervals of 49 nsec. In figure 39 the raw data have been rebinned for clarity by a factor of 10 (bin width 10.10 nsec) but the effect of this periodic distortion is still seen as a deviation from the fitted curve approximately every fifth bin. This problem was first noted in the electron time spectrum due to the ample statistics, but was later also detected in the positron time spectrum and the neutral time spectrum. Further work showed it to be present in runs where the precession magnet was off, in runs where a carbon target was used in place of the calcium one, and in runs where the prompt hardwired 50 nsec $S0 + S1$ veto was disabled. It was also found to be independent of all software cuts. All this leads to the conclusion that the 49 nsec distortion of the time spectrum is not related to any process in the target. One must also conclude that it cannot be related to any cyclotron-related background as the cyclotron period is 43.37 nsec and the time calibration of the TDC is known to 1%. The final conclusion was that it was related to the electronics in some way. The pulse separators were tested after the experiment and no problems were found. The

Figure 39: Example of a fit to the electron μ SR time spectrum from bound μ^- decay. The energy interval is 15-55 MeV. The errors on the data are smaller than the points.

Figure 40: Example of a fit to the positron μ SR time spectrum from μ^+ decay. The energy interval is 15-55 MeV. Note the larger asymmetry and the longer lifetime.

source of the periodic distortion of the time spectra remains unresolved.

The effect of the periodic distortion is to greatly increase the χ^2 of the fits to the electron time spectra where the statistics were high. As it was of some concern what the effect of these distortions would be on the fitted asymmetry and its MINOS error, a Monte Carlo study was performed. Data files were generated that were in all respects similar to the experimental data but with varying intensities of this 49 nsec distortion. Each data point in the file was selected from a Poisson distribution whose mean was given by the μ SR function plus a distortion term. Fits to these generated data files showed no perceptible change to the fitted value of the asymmetry or its MINOS error until the intensity of the 49 nsec distortion was approximately twice that observed in the experimental data. Based on this Monte Carlo study, it was concluded that the 49 nsec distortions could be safely ignored in the extraction of the asymmetry from the experimental data. Some efforts were made to include the distortion in the fit by modifying the μ SR function to include several Fourier components of ~ 20 MHz. These fits took an inordinate amount of computer time and did not result in fits with a χ^2_ν ($= \chi^2/\text{degrees of freedom}$) of 1.

IV.4.3 Extraction of $\mu^\pm$ polarizations

Systematic errors in the extraction of the $\mu^\pm$ polarizations were estimated by varying the parameters of the calibration by reasonable estimates of their errors. For the experimental data, the BARS calibration and the BARS threshold were varied. The calibration of TINA contributes a very small error, as most of the energy for a charged event is dissipated before TINA is reached and so was not included in the estimation of systematic errors. Also, the TINA threshold does not contribute an error as TINA is not required in the definition of a charged event. For the Monte Carlo EGS3 data, the BARS threshold, and the BARS and

TINA smearing widths, were varied to estimate systematic errors. The systematic and statistical errors calculated in this fashion are not independent of each other and so were not added in quadrature.

Tables XIV, XV, XVI, and XVII summarize the asymmetries obtained from the experimental and Monte Carlo data, for both electrons and positrons, and show the contributions of the statistical and systematic errors to the total error for each energy interval. The $\mu^\pm$ polarizations were extracted from these data by performing one parameter fits of the EGS3 asymmetries to the experimental asymmetries. The parameter was a multiplicative scaling factor applied to the EGS3 results. Since the decay electron/positron distributions in the Monte Carlo corresponded to 100% polarized muons, the scaling factor is our best estimation of the polarization of the muons in the experimental data. Figures 41, 42, 43, and 44 show the fits of EGS3 results to the differential and integral asymmetry spectra for the decay electrons and positrons.

In the fitting procedure, the error associated with each point was taken to be the EGS3 error and the experimental error added in quadrature, as these are independent of each other. Errors on the scaling factors were found by increasing and decreasing the scaling factor until the value of χ_ν^2 increased by 1

$$\chi_\nu^2 = \chi_\nu^2|_{min} + 1 \quad (81)$$

and for the one case where χ_ν^2 exceeded 1

$$\chi_\nu^2 = 2 \chi_\nu^2|_{min} \quad (82)$$

was used. The results of these fits are summarized in table XVIII. As previously stated, the integral and differential asymmetries are subject to slightly different systematic errors. The final results for the $\mu^\pm$ polarizations shown in table XVIII are the simple averages of the differential and integral asymmetries, as are the quoted errors.

Table XIV: Asymmetries obtained from the *experimental electron* data from bound μ^- decay. For each differential and integral energy interval the contributions of statistical and systematic errors to the total error are shown.

Energy Interval	Asymm.	Stat. Error	Systematic Errors		Stat. + Sys. Error
			BARS thres.	BARS cal.	
15-25	-0.0162	0.0021	0.0001	0.0002	0.0024
25-35	-0.0295	0.0020	0.0002	0.0004	0.0026
35-45	-0.0639	0.0021	0.0001	0.0003	0.0025
45-55	-0.0862	0.0038	0.0003	0.0012	0.0053
15-55	-0.0400	0.0012	0.0005	0.0001	0.0018
25-55	-0.0496	0.0014	0.0002	0.0002	0.0018
35-55	-0.0682	0.0019	0.0001	0.0003	0.0023
45-55	-0.0862	0.0038	0.0003	0.0012	0.0053

Table XV: Asymmetries obtained from the *Monte Carlo electron* data from bound μ^- decay. For each differential and integral energy interval the contributions of statistical and systematic errors to the total error are shown.

Energy Interval	Asymm.	Stat. Error	Systematic Errors			Stat. + Sys. Error
			BARS σ	BARS thres.	TINA σ	
15-25	-0.1236	0.0183	0.0061	0.0074	0.0007	0.0325
25-35	-0.3376	0.0172	0.0036	0.0010	0.0010	0.0228
35-45	-0.5581	0.0169	0.0006	0.0037	0.0000	0.0213
45-55	-0.6958	0.0254	0.0052	0.0157	0.0011	0.0474
15-55	-0.3584	0.0098	0.0010	0.0057	0.0000	0.0165
25-55	-0.4698	0.0112	0.0008	0.0002	0.0004	0.0126
35-55	-0.5924	0.0142	0.0018	0.0009	0.0003	0.0172
45-55	-0.6958	0.0254	0.0052	0.0157	0.0011	0.0474

Table XVI: Asymmetries obtained from the *experimental positron* data from μ^+ decay. For each differential and integral energy interval the contributions of statistical and systematic errors to the total error are shown.

Energy Interval	Asymm.	Stat. Error	Systematic Errors		Stat. + Sys. Error
			BARS thres.	BARS cal.	
15-25	0.088	0.023	0.0076	0.0020	0.033
25-35	0.213	0.022	0.0010	0.0060	0.029
35-45	0.388	0.022	0.0017	0.0005	0.024
45-55	0.638	0.037	0.0040	0.0220	0.063
15-55	0.266	0.012	0.0042	0.0006	0.017
25-55	0.334	0.014	0.0009	0.0005	0.016
35-55	0.442	0.024	0.0015	0.0034	0.024
45-55	0.638	0.037	0.0040	0.0220	0.063

Table XVII: Asymmetries obtained from the *Monte Carlo positron* data from μ^+ decay. For each differential and integral energy interval the contributions of statistical and systematic errors to the total error are shown.

Energy Interval	Asymm.	Stat. Error	Systematic Errors			Stat. + Sys. Error
			BARS σ	BARS thres.	TINA σ	
15-25	0.1921	0.0189	0.0036	0.0009	0.0007	0.0241
25-35	0.2797	0.0171	0.0030	0.0046	0.0009	0.0256
35-45	0.5270	0.0156	0.0039	0.0034	0.0006	0.0234
45-55	0.7969	0.0168	0.0038	0.0029	0.0008	0.0243
15-55	0.3952	0.0091	0.0006	0.0022	0.0001	0.0120
25-55	0.4687	0.0102	0.0004	0.0007	0.0005	0.0119
35-55	0.6085	0.0121	0.0009	0.0014	0.0003	0.0147
45-55	0.7969	0.0168	0.0038	0.0029	0.0008	0.0243

Figure 41: Fit of EGS3 asymmetries to the experimental differential electron asymmetry spectrum. The line is only to guide the eye.

Figure 42: Fit of EGS3 asymmetries to the experimental integral electron asymmetry spectrum. The line is only to guide the eye.

Figure 43: Fit of EGS3 asymmetries to the experimental differential positron asymmetry spectrum. The line is only to guide the eye.

Figure 44: Fit of EGS3 asymmetries to the experimental integral positron asymmetry spectrum. The line is only to guide the eye.

Table XVIII: Results of fits of EGS3 asymmetries to experimental asymmetries for the extraction of the muon polarization.

Data	Fit	EGS3 Scaling Factor	χ^2_ν	P^μ
μ^-	differential	0.112 ± 0.013	2.29	0.111 ± 0.010
	integral	0.111 ± 0.006	0.65	
μ^+	differential	0.745 ± 0.074	0.88	0.730 ± 0.058
	integral	0.715 ± 0.041	0.65	

The depolarization factor, D_μ^{Ca} , is given by the ratio of P^{μ^-}/P^{μ^+} . Using the polarizations of table XVIII, D_μ^{Ca} is found to be

$$D_\mu^{Ca} = 0.152 \pm 0.018 \quad (83)$$

which is in good agreement with the value of 0.137 ± 0.010 derived from previous measurements and quoted in chapter II.

IV.5 Neutral Data Analysis

IV.5.1 Treatment of the experimental data

Under this heading the various off-line cuts which were applied in the analysis of the neutral data will be discussed and their effects on the data quantified. One cut, the LST cut, which required that all three layered scintillator fired (exceeded an energy threshold) was implemented very early in the data analysis. The skimmed neutral data contained ~ 148 K events with energies $> \sim 50$ MeV. When the LST cut was applied to these events, only 18.7 K survived. At these energies the efficiency of the LST cut for photons has been shown to be $\sim 45\%$; this implies

that ~ 40 K of the 148 K events were photons. Presumably, the remainder are related to neutron interactions in the telescope. It has already been shown that the BARS alone provide sufficient neutron discrimination to deal with the OMC neutrons generated in the target. One must conclude that the layered scintillator is important in further increasing the neutron discrimination to deal with the copious π^- -induced neutrons from the M20B beamline.

The Cerenkov counter was not utilized in the energy reconstruction algorithm and it was in fact also not utilized in any cut to further reduce the background. The principal reasons for this were its low efficiency and the fact that it triggered on Compton scattered photons and on stopping electrons. The latter reason meant that it could not reliably be incorporated into the energy reconstruction algorithm. Its low efficiency ($\sim 75\%$) meant that when it was used in combination with other cuts, it lowered the overall acceptance and did so in a way which was not easily measured nor calculated by Monte Carlo. In retrospect, it did not fulfill its purpose and, in fact, degraded the overall telescope energy resolution somewhat.

The 2nd μ stop cut has already been described. It was used in the extraction of the electron and photon asymmetries since only one muon was desired in the target at one time. Since our precession frequency was non-stroboscopic, a second muon arriving at the target is out of phase with the first. Without the 2nd μ stop cut, the observed asymmetry is attenuated by these out of phase muons even if the distortions of the lifetime and background are corrected for. In the determination of the photon energy spectrum and partial branching ratio, the 2nd μ stop cut was not used in order to increase the acceptance.

The information provided by the S3 TDC and ADC was used in an S3 cut which discarded events where energy was deposited in the S3 scintillator. This cut can be viewed as a software charged event veto cut supplementing the hardware

veto provided by the V counter. By referring back to figure 22 it can be seen that an efficiency of $>99.9\%$ is required of the combination of *S3* and V to limit the contribution of high energy electrons from bound μ^- decay to $<1\%$ of the RMC spectrum. Individual scintillators tested in an electron beam prior to the start of the run were all shown to have efficiencies in excess of 99%, so the combination of *S3* and V should effectively eliminate any measurable contribution from the high energy electrons.

Another possible consequence of the *S3* cut is that some neutral events observed in the telescope, arising from bremsstrahlung of the decay electrons in the target, will be discarded if the continuing electron following bremsstrahlung reaches *S3*. The calculated contribution of high-energy electron bremsstrahlung to the neutral energy spectrum has been calculated and shown to be $\sim 1\%$ of the RMC photon rate. This calculation was for bremsstrahlung in the 0.2 radiation length path out of the calcium target. Because the high-energy electron and hence bremsstrahlung spectrum falls off so sharply with energy, the photons observed in energy region II are more likely to be due to very hard bremsstrahlung, where the electron is left with little energy, than to bremsstrahlung with sufficient energy left to the electron such that it reaches *S3*. Consequently, the *S3* cut does not discard many events of this type in energy region II. An EGS3 Monte Carlo confirmed this when only 3 of the 33 high-energy electron bremsstrahlung cases were seen to deposit energy in *S3*.

The other cuts applied in the final analysis of the data are placed in five groups for the purpose of discussion. The first group is labelled the cosmic (COS) cut. It was formed from information derived from the cosmic veto counters and is intended to suppress the contribution of cosmic rays to the spectra. The second group is labelled the pile-up (PU) cut. It consisted of a BARS pile-up cut, utilizing the charge-integrating ADCs on the rising and falling edge of the BARS

signal; a TINA-BARS timing cut; a 2nd TINA cut, utilizing the TINA pulse separator; and a BARS-BARS timing cut which requires a time coincidence between all BARS which fired within a time window encompassing the event. The next group is labelled the prompt (PRO) cut. This cut checked the *S2* TDC and the *S0 + S1* pulse separator TDCs to discard events in prompt time coincidence with the arrival of a beam particle into the target or the area. The next group is labelled the magnet pole-tip (*S4/S5*) cut. This cut inspected the *S4* and *S5* TDCs and ADCs for evidence of interactions in the pole-tips or in the scintillators themselves. The last group is labelled the layered scintillator dE/dx (DEDX) cut. This cut discards events in which the energy deposited in each of the three elements of the layered scintillator exceeds 1.5 MeV.

The neutral energy spectrum can be divided into three regions as in figure 45. Region I is dominated by bremsstrahlung from the decay electrons. Region II contains the RMC spectrum as well as some background and Region III should contain only background. It is necessary to know the efficiency of these cuts for accepting good photon events and it is desirable to be able to estimate what discrimination is achieved against the background. If we define the ‘signal’ as those events falling in the exponential distribution in the time spectrum and the ‘noise’ as those events making up the flat component in the time spectrum, then it is possible to measure the fraction of the signal and of the noise affected by each of these cuts. In this context ‘signal’ includes the RMC photons, any OMC neutrons, and the decay electron bremsstrahlung, while ‘noise’ includes the cosmic ray background and any π^- -induced photons and neutrons. It has already been argued that the OMC neutrons are negligible and that the bremsstrahlung is $\sim 1\%$ of the RMC spectrum above 57 MeV, so the signal should be overwhelmingly RMC photons. Table XIX summarizes the effects of these five cuts in each of the three energy regions. It is also possible to look at the 23.055 MHz (43 nsec)

Figure 45: Neutral energy spectrum divided into regions: I (<57 MeV), II (57-90 MeV) and III (>90 MeV). Curve A) is the energy spectrum before cuts and curve B) is the energy spectrum of the events discarded by one or more cuts.

Table XIX: Effects of the offline cuts on the neutral energy spectrum. % signal or noise discarded means the fraction of signal or noise that would be discarded due to a particular cut if no other cuts were used. S/N refers to the signal to noise ratio. The errors quoted are derived from the Poisson statistics on the time spectra.

Cut	Energy Region	% 'signal' Discarded	% 'noise' Discarded	S/N After Cut
NONE	I	0 ± 0	0 ± 0	3.12 ± 0.08
	II	0 ± 0	0 ± 0	0.34 ± 0.03
	III	0 ± 0	0 ± 0	-0.004 ± 0.028
COS	I	0.57 ± 0.16	3.9 ± 0.5	3.23 ± 0.09
	II	-2.6 ± 2.8	10.6 ± 0.9	0.39 ± 0.04
	III	1 ± 220	11.5 ± 1.0	—
PU	I	3.7 ± 0.2	5.1 ± 0.5	3.16 ± 0.09
	II	4.8 ± 2.1	5.2 ± 0.6	0.34 ± 0.03
	III	20 ± 209	6.4 ± 0.7	—
PRO	I	0.8 ± 0.1	2.4 ± 0.4	3.17 ± 0.08
	II	1.3 ± 0.8	0.7 ± 0.2	0.33 ± 0.03
	III	-13 ± 109	1.2 ± 0.3	—
S4/S5	I	2.9 ± 0.4	25.6 ± 1.3	4.07 ± 0.15
	II	4.4 ± 0.7	0.2 ± 0.1	0.32 ± 0.03
	III	-46 ± 302	0.6 ± 0.2	—
DEDX	I	0.9 ± 0.2	5.7 ± 0.6	3.28 ± 0.09
	II	13.7 ± 5.2	33.8 ± 1.8	0.44 ± 0.06
	III	231 ± 1542	37.5 ± 1.9	—
ALL 5	I	8.5 ± 0.5	39.1 ± 1.7	4.69 ± 0.22
	II	19.2 ± 6.2	45.6 ± 2.2	0.50 ± 0.07
	III	222 ± 1505	51.0 ± 2.3	—

structure in the time spectra to observe how this is affected by the cuts. This was done by constructing histograms of the foreground time window modulo 43 nsec. Figures 47, 49, and 51 are such histograms for each of the three energy regions and for all cuts. Figures 46, 48 and 50 are the time spectra for each of the energy regions for all cuts. The last three lines of table XIX were computed from the areas under the foreground and background regions of these time spectra. Information such as that presented in these figures and in table XIX was used in the final specification of these cuts. In some instances cuts or parts of cuts were found to have no selectivity, discarding the same fraction of the signal as of the noise. These cuts were modified or abandoned.

Some general observations can be made from table XIX. In region III there is no signal and this is reflected in the % signal discarded always being consistent with zero. The COS cut is highly selective, discarding only noise. The PU cut shows no selectivity as would be expected for random pile-up. The PRO cut affects very few events as most prompt events were eliminated in hardware. When the time spectrum modulo 43 nsec is constructed for the events discarded by each of these five cuts, it is observed in energy region II that the COS and the $S4/S5$ cuts discard no 43 nsec noise as would be expected. The PU cut discards only a representative amount of 43 nsec noise showing no selectivity, and the PRO cut and the DEDX cut are seen to discard relatively large amounts of 43 nsec noise. The $S4/S5$ cut and the DEDX cut deserve more detailed consideration.

The $S4/S5$ cut is observed to discard a large fraction of the noise in region I. μ stops in the $S4$ and $S5$ counters have a long lifetime and most would survive the $300 \text{ nsec } (S0 + S1) \cdot (S4 + S5)$ veto intended for μ stops in the lead on the pole-tips of the magnet. These muons bear no relation in time to the next or the previous μ stop and so contribute a flat component to the time spectrum and are thus classified as 'noise'. Furthermore, the hole counter which was designed to identify

Figure 46: Neutral time spectrum for energy region I. Curve A) is for the case of no cuts and curve B) are the events which would be discarded by the combination of all five cuts. The spectrum has been binned by 43 nsec so as to average out the 43 nsec time structure.

Figure 47: Neutral foreground time spectrum modulo 43 nsec for energy region I. Curve A) is for the case of no cuts and curve B) are the events which would be discarded by the combination of all five cuts.

Figure 48: Neutral time spectrum for energy region II. Curve A) is for the case of no cuts and curve B) are the events which would be discarded by the combination of all five cuts. The spectrum has been binned by 43 nsec so as to average out the 43 nsec time structure.

Figure 49: Neutral foreground time spectrum modulo 43 nsec for energy region II. Curve A) is for the case of no cuts and curve B) are the events which would be discarded by the combination of all five cuts.

Figure 50: Neutral time spectrum for energy region III. Curve A) is for the case of no cuts and curve B) are the events which would be discarded by the combination of all five cuts. The spectrum has been binned by 43 nsec so as to average out the 43 nsec time structure.

Figure 51: Neutral foreground time spectrum modulo 43 nsec for energy region III. Curve A) is for the case of no cuts and curve B) are the events which would be discarded by the combination of all five cuts.

electrons originating from the target which might undergo bremsstrahlung in the collimator, offers incomplete protection for electrons originating from *S4* or *S5*. In this way, a large number of neutral events could be generated due to scattered muons decaying in *S4* or *S5* and subsequent bremsstrahlung in the collimator.

The $4.4 \pm 0.7\%$ discarded signal due to the *S4/S5* cut in region II also requires explanation. It is difficult to argue that this cut can discard this fraction of the RMC photon events. Pair production occurs in the target for $\sim 15\%$ of the RMC photons but would also leave a signal in *S3* and hence be discarded by the *S3* cut unless followed by hard bremsstrahlung. The probability of undergoing bremsstrahlung to produce a photon in energy region II however makes the possible contribution of this process small. It must be assumed that the 4.4% discarded signal is actually associated with the high energy electrons from bound muon decay which somehow escape the other cuts.

The DEDX cut, which discards events in which greater than 1.5 MeV was deposited in each of the three layered scintillators, is seen to discard approximately one third of the noise in energy region II. This cut would be expected to be particularly effective at discarding cosmic ray events due to the long pathlength in the layered scintillator for single cosmic rays which traverse the edges of both TINA and the BARS. The DEDX cut is the principal cut responsible for reducing the cosmic ray background from that shown in curve b) of figure 18 to that of curve c). As mentioned previously the DEDX cut also discards large amounts of 43 nsec noise. A possible explanation for this could be elastic neutron scattering off the protons in the plastic scintillators after elastic scattering in the BARS. The DEDX cut also is observed to discard approximately 14% of the signal in energy region II. This is to be expected as figure 30 shows. However it should be mentioned that figure 30 corresponds to the calibration data and not to the RMC data. With a different energy spectrum the ratio of one to two electron

events would be altered somewhat.

The overall cut efficiency, $\mathcal{E}_{CUTS}$, for the acceptance of real RMC photon events is of interest in the calculation of the detector acceptance for the measurement of the partial branching ratio. As will be clarified later the effect of the LST cut and the 2nd μ stop cut are accounted for separately and are not included in $\mathcal{E}_{CUTS}$. Also not included are the $S3$ and $S4/S5$ cuts as these have been argued not to discard real RMC photon events. The random rejection of RMC events by noise in these three counters is estimated to be less than 0.1%. Additionally, it is assumed that $\mathcal{E}_{CUTS}$ is not a function of energy over energy region II. The percent signal discarded in energy region II by the four cuts (excluding the $S4/S5$ cut) is found to be 16.2 ± 6.1 . $\mathcal{E}_{CUTS}$ is taken to be the complement of this yielding 0.838 ± 0.061 .

IV.5.2 Extraction of the photon asymmetry

In the analysis of the charged data it was necessary to perform Monte Carlo calculations to extract the muon polarization from the observed electron asymmetry. Since, in the case of photons, there is no loss of energy nor any multiple scattering in the target it was not necessary to run similar Monte Carlo calculations for the neutral data. For both the charged and neutral data the observed asymmetry is slightly attenuated by finite detector and target size effects due to the averaging of the angular distribution over the angular acceptance. For the charged data this effect is included in the Monte Carlo calculations. The appendix describes this effect in some detail. For the neutral data the size of this effect can be estimated from

$$r = \frac{2 \sin(\theta/2)}{\theta} \quad (84)$$

where r is the attenuation factor and θ is the angle over which the angular distribution has been averaged. This equation is strictly valid for angular averaging over a single dimension. In the case of a two dimensional detector it provides an upper limit to the size of the effect. Neglecting the finite target size gives a value for θ of 20° for the experimental geometry used resulting in $r = 0.995$. Including the finite target size increases θ very little. The total effect is therefore estimated to be less than 0.5%; much smaller than the fitting errors associated with extracting the asymmetry from the experimental data. In contrast to the electron and positron data there were insufficient neutral data to construct meaningful integral and differential photon asymmetry distributions although these would have been of some interest. Consequently only a single fit to the neutral data representing all of the available neutral data is presented here.

Figure 52 illustrates some of the problems in fitting the neutral time spectrum to extract the photon asymmetry. The data is displayed with 1 nsec binning and with this time dispersion, the 43 nsec cyclotron-related background as well as the poor signal-to-noise ratio are evident. Two questions arise... How best to deal with the 43 nsec background in the fitting procedure to extract the asymmetry? and... Is there any sensitivity of the fitted asymmetry to the time and energy windows placed on the data fitted? This second question will be dealt with first.

The signal-to-noise ratio deteriorates as the energy increases so it is potentially worthwhile to consider lowering the upper limit of the energy window used in the fitting. The lower limit is fixed at 57 MeV by the criterion of requiring not more than a 1% contribution to the neutral spectrum from decay electron bremsstrahlung in the target. Table XX shows the effect of varying the upper limit of the accepted energy region. The table shows that essentially no signal is gained by using an upper energy limit of greater than 80 MeV. For this reason the

Figure 52: Neutral time spectrum with 1 nsec binning. Both the background time and the foreground time windows are shown for an energy window of 57–80 MeV. The 43 nsec cyclotron-related background is clearly evident.

Table XX: Number of events in the background-subtracted neutral time spectrum ($67 \leq t \leq 883$ nsec) as a function of the upper limit of the accepted energy region.

Upper Energy Limit (in MeV)	Remaining Events ($\pm \sim 200$)
75	2324
80	2517
85	2512
90	2520

energy window used was 57-80 MeV. It was also observed that the MINOS errors of the extracted asymmetry increased in size when upper energy limits greater than 80 MeV were employed. This is attributable to an increase in the included background.

Similarly the upper limit of the time window was lowered to see if discarding the region of the time spectrum which contains little signal improved the MINOS errors on the extracted asymmetry. Only when more than 200 nsec of the time spectrum was excluded from the fit was any effect seen and then the MINOS errors were observed to increase rather than decrease. It was decided to include as much of the long time neutral data as possible to retain as much of the signal as possible. Varying the lower limit of the accepted region of the time spectrum had a significant effect not only on the MINOS errors but also on the extracted asymmetry. These results are summarized in table XXI. Immediately evident is the wide variation in the fitted asymmetry with the lower time limit, t_1 . This is to be expected for the size of the error in the fitted asymmetry. However it must be noted that the various asymmetries in table XXI do not represent independent measurements as they are based on the substantially the same data. It is also observed that the MINOS errors slowly increase as t_1 is raised. This is due to the exclusion of increasing amounts of the signal from the fitting region. The value of

Table XXI: Effect on the extracted asymmetry and its MINOS error of varying the lower limit of the accepted time region.

Lower Time Limit (in nsec)	Asymmetry	χ^2_ν
45	0.117 ± 0.041	27.9/16
47	0.110 ± 0.042	28.5/16
48	0.106 ± 0.042	29.8/16
49	0.099 ± 0.041	28.2/16
53	0.090 ± 0.043	26.5/16
57	0.092 ± 0.044	23.6/16
58	0.098 ± 0.044	24.7/16
61	0.131 ± 0.045	20.7/16
65	0.146 ± 0.046	17.4/16
67	0.128 ± 0.046	15.7/16
69	0.122 ± 0.046	14.6/16
71	0.124 ± 0.047	13.7/16
73	0.117 ± 0.047	12.4/16
75	0.136 ± 0.048	12.0/16
78	0.123 ± 0.048	10.7/16
80	0.150 ± 0.048	11.6/16
85	0.178 ± 0.049	20.4/16
88	0.203 ± 0.050	15.4/15
98	0.150 ± 0.048	20.3/15
108	0.162 ± 0.048	15.4/15

χ^2_ν is seen to be poor for $t_1 \leq 58$ nsec presumably due to some distortion of the time spectrum which is confined to the region below 58 nsec. If the MINOS errors are adjusted, to compensate for this, by multiplying them by $\sqrt{\chi^2_\nu}$ when $\chi^2_\nu > 1$, then the smallest MINOS errors occur in the vicinity of $t_1 = 67$ nsec. Since this is also the point where χ^2_ν first reaches a value of 1, the value of $t_1 = 67$ nsec was chosen as the value to be used in the extraction of the photon asymmetry. This is an important decision in the analysis of the data as it greatly affects the value of the asymmetry determined from the data. The choice of $t_1 = 67$ nsec is justified as it includes in the fitting region the maximum amount of the signal possible while avoiding the region of the time spectrum suspected of distortion. $t_1 = 67$ nsec was also used in the determination of the energy spectrum and the partial branching ratio to avoid any effects attributable to suspected distortion.

The second question concerning the fitting of the neutral time spectrum was how best to deal with the 43 nsec background present in the data. The period of the precession signal buried in figure 52 is 183 nsec. It is not easily discernable in the presence of the background. To add to the difficulties the short background time window limits our knowledge of the background. Two essentially different methods (background subtraction and rebinning) were used in dealing with the 43 nsec background. In the first method a best estimate of the shape of the 43 nsec time structure was obtained by ‘signal averaging’ over the three 43 nsec periods in the background time window. This estimate of the structure of the background was then replicated periodically across the entire foreground time window and a background subtraction was then performed. The background subtracted data were then rebinned before fitting with MINUIT. The errors associated with the individual time bins in the spectrum were calculated assuming Poisson statistics on the original data and taking the signal averaging and background subtraction operations into account. The second method involved rebinning the original data

Table XXII: Summary of fits to the neutral time spectrum to extract the photon asymmetry. The expected values listed are derived from fits to the electron time spectra except for the expected value of B which is from the background time window and is further explained in the text. An overall normalization was also a free parameter of both fits but is not shown.

Fit	Parameter	Free or Fixed	Value ($\pm$ MINOS errors)	Expected Value	χ^2_ν
1	A	free	0.128 ± 0.046	—	15.7/16
	τ	free	0.389 ± 0.026	0.352 ± 0.004	
	ϕ	fixed	-88	-88 ± 2	
	ν	fixed	5.45	5.45 ± 0.02	
	B	fixed	1.00	1.00 ± 0.043	
2	A	free	$0.133^{+0.053}_{-0.046}$	—	14.3/13
	τ	free	$0.415^{+0.113}_{-0.077}$	0.352 ± 0.004	
	ϕ	free	-81^{+41}_{-38}	-88 ± 2	
	ν	free	$5.24^{+0.31}_{-0.42}$	5.45 ± 0.02	
	B	free	$0.96^{+0.13}_{-0.18}$	1.00 ± 0.043	

(without background subtraction) by 43 TDC channels. At 1.01 nsec/channel this corresponded to a new bin width of 43.43 nsec which was conveniently close to the cyclotron period of 43.37 nsec. In this way any structure in the time spectrum periodic with the cyclotron is averaged out and the background can be treated as a flat component in the fitting procedure. A side effect of rebinning to this degree is that the asymmetry is attenuated by a factor of 0.911 as per equation 84. Equation 84 is strictly valid in this (one dimensional) case. Here $\theta = 2\pi(43/183)$. Both methods give the same results with the same MINOS errors and show the same behaviour of χ^2_ν with respect to variation of t_1 . The second method is preferred for simplicity and it is the one employed here although the first method is more flexible in that it is not restricted to rebinning by exactly 43 channels .

Table XXII shows the results of two fits to the neutral time spectrum where the data was rebinned by 43 nsec and a flat background was included in the fit. In

the table the flat background B is given as a multiplicative factor. A value of 1.00 for B corresponds to the background per bin observed in the background time window. The 4.3% error in the expected value of the fitted background term is due to limited statistics in the background time window. As a further check on the level of the background observed in the background time window a calculation was done of the number of counts in this window which would be required in order for the background subtraction to yield zero integral counts for energies greater than 100 MeV (where there should be no RMC signal). The required number of counts was 527 ± 29 which compares well with the observed number of counts of 537 ± 23 .

Fit 1 in table XXII is the fit corresponding to $t_1 = 67$ nsec in table XXI. Fit 2 is the fit from which our final value of the asymmetry will be taken. It is noteworthy that in both fits the fitted parameters are in good agreement with each other and with the expected values. Applying the correction for the attenuation of the asymmetry due to the rebinning of the data, the value of

$$A_\gamma = 0.146^{+0.058}_{-0.050} \quad (85)$$

is obtained. Fit 2 is also displayed in figures 53 and 54.

IV.5.3 Photon energy spectrum

To increase the statistics the 2^{nd} μ_{stop} cut was removed for this part of the neutral data analysis. The same time window was used as in the extraction of the photon asymmetry for consistency. With the 2^{nd} μ_{stop} cut removed it is not possible to deduce the scaling factor, which must be applied to the background before background subtraction, from the ratio of the foreground to background time windows as was done when the 2^{nd} μ_{stop} cut was applied. However the necessary scaling factor can be deduced from the requirement that the integral number of counts above 100 MeV be zero after background subtraction. This

Figure 53: Fit to the neutral time spectrum for the extraction of the photon asymmetry. The time spectrum has been rebinned by 43 bins and a flat background term has been included in the fitting function.

Figure 54: Fit to the neutral time spectrum for the extraction of the photon asymmetry. The time spectrum has been rebinned by 43 bins and a flat background term has been included in the fitting function as in figure 53. Here the background and the exponential decay have been removed from the plot to show the asymmetry term alone.

approach was tested when the 2nd μ stop cut was applied and was found to work well. The scaling factor found in this way was 8.78. That is, the background energy spectrum corresponding to the background time window was multiplied by 8.78 before being subtracted from the energy spectrum corresponding to the foreground time window. This large scaling factor is unfortunate in that it amplifies the Poisson scatter on the measured background energy spectrum. As the background is believed to be smoothly varying and structureless it was considered appropriate to smooth the background numerically before doing the background subtraction to partially remove this disadvantage. Figures 55, 56 and 57 show the details of this background smoothing and subtraction.

If F_i is the foreground energy spectrum and $\overline{B}_i$ is the scaled up and smoothed background energy spectrum, then the background subtracted energy spectrum (S_i) is given by

$$S_i = F_i - \overline{B}_i \quad (86)$$

and errors on individual bins are given by

$$\sigma_{S_i} = \sqrt{F_i + 8.78 \cdot \overline{B}_i} . \quad (87)$$

The integral area of the background subtracted energy spectrum above 57 MeV observed energy is found to be

$$A_{>57} \equiv \sum_{i \geq 57} S_i = 3133 \pm 261 \quad (88)$$

independent of whether the smoothed background ($\overline{B}_i$) or the unsmoothed background (B_i) is used.

It is desirable in general to deconvolute the observed energy spectrum from the experimental response function to produce a true energy spectrum which is more easily compared with theoretical calculations. This was attempted in this experiment but the results were found to be unsatisfactory and they have not

Figure 55: Neutral energy foreground and background spectra before background subtraction. The background has been scaled up by a factor of 8.78 so that the integral area above 100 MeV is zero after background subtraction.

Figure 56: Neutral energy foreground and background spectra before background subtraction in the RMC energy region. The line through the background spectrum is the smoothed background spectrum which is used in the background subtraction.

Figure 57: Background subtracted neutral energy spectrum. The error bars shown take account of the factor of 8.78 in the scaling up of the background to account for the different foreground and background time windows.

been included here. Several deconvolution algorithms were used to attempt solutions to the matrix equation

$$\mathbf{T}_{ij}\mathbf{E}_j = \mathbf{S}_i \quad (89)$$

where $\mathbf{E}_j$ is the true energy spectrum sought and $\mathbf{T}_{ij}$ is the matrix formed from the response functions whose ij^{th} element is the probability of the observed energy falling in the i^{th} bin of the observed energy spectrum for a photon whose energy falls in the j^{th} bin of the true energy spectrum. The normalization of the columns of $\mathbf{T}_{ij}$ is such that

$$\sum_i \mathbf{T}_{ij} = 1 \quad (90)$$

when the range of i is sufficient to encompass the complete response function. This definition of the normalization of $\mathbf{T}_{ij}$ ignores for the moment any energy dependent efficiencies of the photon telescope as well as the energy independent efficiencies.

The exact solution to equation 89 can be found by inverting $\mathbf{T}_{ij}$. This however produces a vector $\mathbf{E}_j$ which oscillates rapidly between positive and negative values and which requires severe smoothing in order to produce a sensible result. An alternative to smoothing after the deconvolution is to smooth $\mathbf{S}_i$ before the matrix multiplication. Again it is found that rather severe smoothing is required to produce a sensible result. Iterative techniques to solve equation 89 without matrix inversion, but which tend to drive toward the exact solution produce similar results.

The deconvolution algorithm used by Hart [160] is another iterative approach which avoids matrix inversion. Rather than driving $\mathbf{E}_j$ toward the exact solution, the algorithm converges to a 'statistically acceptable' solution. In the

present context a statistically acceptable solution is one for which

$$\sum_i \left\{ \left| \mathbf{S}_i - \sum_j \mathbf{T}_{ij} \mathbf{E}_j \right| - \sigma_{S_i} \right\} \Rightarrow 0 \quad (91)$$

or one for which χ^2_ν tends to unity as in

$$\sum_{i=1}^N \left[\frac{\left| \mathbf{S}_i - \sum_j \mathbf{T}_{ij} \mathbf{E}_j \right|}{\sigma_{S_i}} \right]^2 \Rightarrow N . \quad (92)$$

This algorithm was found to be unstable on our data set and many widely varying statistically acceptable solutions could be found which were strongly influenced by the initial guess for $\mathbf{E}_j$ supplied to the program. This difficulty may have been associated with the large errors associated with the energy spectrum due to the unfavourable statistics of the background subtraction or it may have also been related to the limited usefulness of the convergence criterion (equation 91) for a small number of bins. Rather than prejudice the resulting true energy spectrum by arbitrarily choosing one of the statistically acceptable solutions it was decided not to deconvolute our energy spectrum but instead to convolute the theoretical energy distributions with our response functions in order to make the comparison of experiment to theory. The results of this are presented in the next chapter.

IV.6 The Photon Acceptance Equation

In order to make the comparison of theory to experiment it is necessary to include all of the energy dependent and energy independent efficiencies in the convolution of the theory with the response functions. All such efficiencies are incorporated into the acceptance equation

$$\mathbf{N}_i^\gamma = N_\mu \cdot f_C \cdot P_T \cdot \mathcal{E}_{El} \cdot \mathcal{E}_{CUTS} \cdot \sum_j \mathbf{T}_{ij} \cdot \mathcal{E}_j^{MC} \cdot \mathbf{B}_j \quad (93)$$

where $\mathbf{N}_i^\gamma$ is the photon energy spectrum to be compared with the experimental one, N_μ is the number of muons estimated to have stopped in the calcium target,

f_C is the fraction of muons which undergo nuclear capture in calcium, P_T is the probability per muon stop that the capture or decay occurs within the foreground time window used in the data analysis, $\mathcal{E}_{El}$ is the efficiency for a valid photon event to survive the various hardware (electronic) vetoes, $\mathcal{E}_{CUTS}$ is the efficiency for a valid photon event to survive the various offline data analysis cuts not explicitly accounted for elsewhere in the acceptance equation, $\mathbf{T}_{ij}$ is the matrix of response functions, $\mathcal{E}_j^{MC}$ is a vector of the energy dependent efficiencies calculated by Monte Carlo, and $\mathbf{B}_j$ is the theoretical energy distribution.

The vector $\mathbf{N}_i^\gamma$, derived from the theoretical energy distributions, is analogous to the experimental quantity $\mathbf{S}_i$. To complete the analogy we define $A_{>57}^{th}$ to be

$$A_{>57}^{th} \equiv \sum_{i \geq 57} \mathbf{N}_i^\gamma \quad (94)$$

To further clarify the notation we also note that

$$\mathbf{B}_j \equiv (\Lambda_{RMC}/\Lambda_{OMC})_j \quad (95)$$

where the units are photons/ μ^- capture/MeV. Hence the partial branching ratio associated with a given $\mathbf{B}_j$ is

$$R_{k>57}^{th} \equiv \sum_{j \geq 57} \mathbf{B}_j \quad (96)$$

in units of photons/ μ^- capture. With these definitions $A_{>57}^{th}$ can be viewed as a function of g_P/g_A or $R_{k>57}^{th}$. In the next chapter the results of the experiment are extracted from the data by comparing $A_{>57}$ to $A_{>57}^{th}$.

The values of the constant terms in equation 93 are summarized in table XXIII. f_C and $\mathcal{E}_{CUTS}$ have been discussed previously, this chapter concludes with brief discussions of N_μ , P_T , $\mathcal{E}_{El}$, and $\mathcal{E}_j^{MC}$.

Table XXIII: Values of the constant terms in the acceptance equation. Π refers to the product of the constant terms. The errors have been added in quadrature.

Term	Value
N_μ	$(1.741 \pm 0.012) \times 10^{11}$
f_C	0.849 ± 0.005
P_T	0.739 ± 0.006
$\mathcal{E}_{El}$	0.974 ± 0.003
$\mathcal{E}_{CUTS}$	0.838 ± 0.061
Π	$(8.92 \pm 0.81) \times 10^{10}$

IV.6.1 N_μ

The number of μ stops was counted in one of the CAMAC scalers. The value obtained from the scaler for the sum of all the RMC data was 1.730×10^{11} . Several small corrections to this value were made in arriving at the value and the error quoted in table XXIII. These corrections are summarized in table XXIV.

The effect of any inefficiencies in the $S0$, $S1$, and $S2$ beam counters is to cause an underestimation of the number of actual muon stops in the target. However if the μ stop logical condition is not fulfilled then the electronics are not activated and such muons only disrupt the calculation of the experimental acceptance by increasing the number of ‘old’ muons in the target. This effect has been calculated to be $\ll 1\%$.

The effect of any inefficiencies in the $S3$, $S4$, and $S5$ beam counters is to cause an overestimation of the number of actual muon stops in the target. The misidentified muons presumably stop in either a carbon, iron, or lead environment rather than a calcium one. To the extent to which such events occur long or short lifetime components should be observed in the time spectra. No short lifetime components have been seen and the correction for the long lifetime carbon component has already been as applied as shown in table XXIV. The moderate

Table XXIV: Corrections to the CAMAC scaler value for N_μ .

Correction	Comments
$\downarrow 1.1 \pm 0.7\%$	stops in carbon
$\uparrow 1.73\%$	random vetoing of μ stops (at scaler only) by 300 nsec $(S0 + S1) \cdot (S4 + S5)$ inhibit
$\uparrow 0.0003 \times 10^{11}$	two muons from the same proton beam burst arriving simultaneously and being counted as one
$\uparrow \ll 0.001 \times 10^{11}$	two muons in adjacent proton beam bursts being counted as one because beam counters didn't recover fast enough. (beam counters were clipped)
$\downarrow < 0.0001 \times 10^{11}$	pions counted as muons
$\downarrow \sim 0.00007 \times 10^{11}$	electrons counted as muons

beam intensity and the good beam optics of the M20A muon channel along with the 43 nsec microstructure of the TRIUMF cyclotron have allowed the extraction of N_μ with relative ease.

IV.6.2 P_T

In the limit of zero μ_{stop} rate the probability per μ_{stop} of a muon capturing or decaying within the foreground time window $[t_1, t_2]$ is given by

$$P_T = \frac{1}{\tau} \int_{t_1}^{t_2} e^{-t/\tau} dt \quad (97)$$

where τ is the muon lifetime in the target. As the rate increases from zero several possibilities arise which complicate the calculation of P_T . A 2nd muon arriving before t_2 has the effect of extending the foreground time window for the first muon as well as chopping a hole in the lifetime curve of the first muon due to the prompt (hardware and software) vetoes which encompass its arrival time. Also as the μ_{stop} rate increases the number of 'old' muons which decay in the background time window increases. When the background subtraction is performed on the experimental data the background in the foreground time window is oversubtracted due to these 'old' muons. This effect can be corrected for in the calculation of P_T . Due to the calculational difficulties presented by these rate dependent effects the calculation of P_T was performed by Monte Carlo. The result presented in table XXIII is for an assumed muon lifetime of 335 nsec. Using the experimentally observed muon lifetime of 352 nsec resulted in a change of 0.001 ± 0.006 to P_T . The error associated with P_T was calculated by allowing the rate to vary by 10% and the values of t_1 and t_2 to vary by ± 2 nsec. 80% of the error is attributable to the variation of t_1 .

Table XXV: Contributions to the 'electronic' or hardware efficiency for the survival of good photon events.

Veto	Length	% Events Lost
$(S0 + S1) \cdot (S4 + S5)$	300 nsec	1.73%
$S0 + S1$	50 nsec	0.81%
VA+VB+H noise	50 nsec	0.02%

IV.6.3 $\mathcal{E}_{El}$

$\mathcal{E}_{El}$ is less than 1.00 due to the random vetoing of good events by the various inhibits and vetoes in the electronics. It is not a measure of the CAMAC deadtime which does not enter into the acceptance equation due to the inhibiting of the scalers during the reading of the CAMAC crate. Table XXV gives the size of the contributions to $\mathcal{E}_{El}$.

To compute these accidental veto probabilities a weighted average was performed over all of the RMC data to account for deviations from the average μ_{stop} rate of ~ 250 K/sec. The veto rates required in the calculation were obtained from the CAMAC scalers (which included a clock signal) on a run by run basis. The probabilities given in table XXV were calculated using

$$\mathcal{E}_{El}^{veto_i} = \frac{\sum_{runs} veto_i \text{ rate} \cdot veto_i \text{ length} \cdot \mu_{stops}/run}{\sum_{runs} \mu_{stops}/run} . \quad (98)$$

In the case of the $S0 + S1$ veto probability the $S0 + S1$ rate was replaced in equation 98 by the $(S0 + S1 - \mu_{stop} - (S0 + S1) \cdot (S4 + S5))$ rate to avoid double counting problems.

A 10% error in the calculation of the veto probabilities of table XXV was assumed in the value of $\mathcal{E}_{El}$ given in table XXIII.

IV.6.4 $\mathcal{E}_j^{MC}$

The energy dependent efficiencies which were calculated by Monte Carlo in this experiment can be separated into three parts. The first is the efficiency for the photon to escape the target without undergoing pair production or Compton scattering. In the energy region of interest the pair production cross-section is rising and the Compton scattering cross-section is falling. The sum of the two is nearly energy independent over the accessible RMC spectrum. Pair production is the dominant process and this efficiency is labelled $(1 - f_{pp})$ to reflect this fact.

The second part of $\mathcal{E}_j^{MC}$ is a combination of the detector fractional solid angle, the BARS photon conversion efficiency (which is defined by the BARS energy threshold), and TINA's energy threshold for the definition of a neutral event. This combination is somewhat inter-related and is not further separable in a meaningful way.

The final part of $\mathcal{E}_j^{MC}$ is the efficiency for the electromagnetic shower originating in the BARS to produce a charged particle which traverses all three of the layered scintillators. This efficiency is surprisingly low, a fact which is attributed to having a photon converter which was 2 radiation lengths thick. Over the region of interest this efficiency is observed to rise slowly as would be expected. Of the three parts of $\mathcal{E}_j^{MC}$ this is the only one which can be compared with experiment and even then it is only the efficiency averaged over an energy region which can be compared to the Monte Carlo and not the energy dependence of the efficiency explicitly. Reasonable agreement was obtained when this check was performed.

These three parts of the energy dependent efficiency as well as their product are shown in figures 58, 59, 60, and 61. The Monte Carlo calculations of figures 58, 59, and 60 cover the energy region of the π^0 photons from the π^-p

Figure 58: The efficiency for a photon to escape the calcium target without interaction.

Figure 59: The solid angle and conversion efficiency of the photon telescope. Also included is the efficiency associated with the TINA and BARS energy thresholds in the definition of a neutral event.

Figure 60: The efficiency for the production of at least one charged particle in the electromagnetic shower between the BARS and TINA with its subsequent detection in all three of the layered scintillators. Elsewhere in the text this has been called the layered scintillator efficiency.

Figure 61: The energy dependent efficiency, $\mathcal{E}_j^{MC}$, calculated by Monte Carlo. This is the product of the curves in figures 58, 59, and 60.

reaction at rest. Estimates of the errors associated with the parameterizations of the energy dependence of these efficiencies were made by varying the parameters of the energy reconstruction of the Monte Carlo data within their previously stated estimates of errors. The error on the product of these efficiencies was then calculated by adding the errors on the three parts in quadrature bin by bin. This produced an error on the product which varied slightly with energy as shown in figure 61.

The estimation of this efficiency relies heavily on the Monte Carlo program. It predicts a energy dependence which varies by approximately a factor of two over the accessible region of the RMC energy spectrum or the π^0 photon spectrum. The validity and accuracy of this approach is difficult to quantitatively assess but is best displayed by refering back to figures 33 and 34. These are figures of the reconstructed energy spectrum for the calibration data. The fit of the Monte Carlo to the experimental data over the region of the π^0 box has not been adjusted in any way and implicitly contains some parts of this efficiency.

Chapter V

Results and Discussion

V.1 The Photon Asymmetry and g_P/g_A

The true photon asymmetry is obtained by normalizing the experimentally observed photon asymmetry (equation 85) by the residual muon polarization prior to capture. This yields

$$\alpha_\gamma = \frac{A_\gamma}{P^{\mu^-}} = \frac{0.146^{+0.058}_{-0.050}}{0.111 \pm 0.010} = 1.32^{+0.54}_{-0.47} \quad (99)$$

where the errors have been added in quadrature. Physically one should have $|\alpha_\gamma| \leq 1$ since the true photon asymmetry is bounded by the magnitude of the residual polarization. The result presented here although statistically compatible with the physically meaningful range of α_γ requires a critical examination.

The extracted value of α_γ could be mistakenly large for two possible reasons; either the experimentally observed photon asymmetry is too large or the deduced residual muon polarization is too small. Both of these possibilities will be commented on in turn.

All experimental effects such as interactions of the photons within the target and finite target and detector sizes tend to attenuate rather than enhance the observed asymmetry. The size of these effects in any case is so small for our experimental geometry that it was unnecessary to explicitly correct for them in the data analysis. (Recall that the finite target and detector size effects for the neutral data were argued in chapter IV to be less than 0.5%.)

It is difficult to argue that any mechanism exists in the data collection or

analysis which can affect the asymmetry information in the time spectra. This leaves the possibility of an error in the fitting of the time spectra to extract the asymmetry. First it should be recalled that an acceptable χ^2_ν of 14.3/13 was obtained for the fit to the neutral data. Referring back to table XXI and recalling the important role of the choice of the lower time limit of the fitted region of the time spectra it is seen that few other choices of the value of t_1 give an asymmetry (the asymmetries shown in the table were uncorrected for the attenuation due to the rebinning of the data by 43 nsec) significantly lower than that corresponding to the chosen value of $t_1 = 67$ nsec. The lowest acceptable asymmetry in the table is only 10% lower. While we have stated in chapter IV the basis of our choice of t_1 there is no justification in selecting an alternate value on the basis of the magnitude of the asymmetry associated with it. Another possibility would be an over-estimation of the level of the background underlying the time spectra. This would have the effect of causing an artificial increase in the fitted value of the asymmetry but would also be accompanied by an artificial decrease in the fitted lifetime. There is no evidence in the fits displayed in table XXII which suggest that this occurred. In particular, for fit 2 of table XXII all parameters were free and any correlations between the parameters of the fit are properly included in the errors as calculated by the MINOS command of MINUIT. Lastly, evidence has been presented that the fraction of muons stopping in carbon-like environments is small. An explicit carbon component was not included in the fitting of the data due to the increase in computation time associated with the additional parameters of the fit. Even an under-estimation of the carbon component is unlikely to account for an artificially high asymmetry. Using the data of references [161,162,163] the ratio D_μ^{scint}/D_μ^{Ca} is found to be 0.47. That is, muons stopping in plastic scintillator will be depolarized more completely than those stopping in calcium and so this effect would not raise the average observed

asymmetry.

Turning our attention to the possibility of an under-estimation of the residual polarization we begin by admitting that a higher polarization of approximately 80–85% was anticipated prior to the detailed analysis of the data. The actual polarization of the M20A backward muon beam had never been measured and so we performed a series of measurements after the ‘low’ polarization value was extracted from our charged data. Varying target thicknesses of carbon and calcium were placed in backward μ^+ and μ^- beams as well as in surface muon beams (intrinsically 100% polarized). When the asymmetries observed in a simple stack of plastic scintillators were normalized against the surface muon runs, and target thickness effects compensated for, the result was an inferred beam polarization of $73 \pm 5\%$ where the error was largely associated with some inconsistencies observed in the surface muon data. This is in good agreement with the measured beam polarization of $73.0 \pm 5.8\%$ from the data presented in this thesis. Both 76 and 86.5 MeV/c muon beam momentums were used in the collection of the experimental data. Results of this later measurement of the beam polarization gave $P_{76}^\mu = (1.02 \pm 0.02)P_{86.5}^\mu$, showing little variation of the beam polarization over this momentum range. These measurements of the beam polarization in varying configurations provided an independent check of several other aspects of the charged data analysis. The detailed analysis of these tests involved the Monte Carlo calculation of the stopping distribution of 76 and 86.5 MeV/c muon beams in carbon and calcium targets of several different thicknesses (a total of 10 beam/target/thickness combinations were used) and the use of EGS3 in calculating the energy loss and multiple scattering of the electrons in the target and scintillator stack. When all corrections were applied the agreement between the 10 configurations was at the $\pm 1\%$ level providing confidence in the technique used and in the Monte Carlo codes. Without these corrections the

disagreements between the different configurations were approximately at the 10% level. With this essential confirmation of our method of extracting the muon polarizations we have confidence that no major effect was excluded from the data analysis technique and that the residual polarization and its stated error as extracted from the charged data are reasonable and not artificially low.

Figure 62 displays the photon asymmetry obtained in this experiment against the theoretical curve of α_γ versus g_P/g_A used in the extraction of a value of the induced-pseudoscalar coupling constant from the asymmetry measurement. The curve due to Christillin [131] is the one for which we quote values. The results of a simple graphical extraction of g_P/g_A are

$$\frac{g_P}{g_A} = < 5.4 \text{ (68\% c.l.)} \quad (100)$$

Clearly our result does not significantly constrain the allowed values of g_P/g_A . It can be said that our result favours a renormalized value of g_P/g_A over the unrenormalized Goldberger-Treiman value of 6.7 (Wolfenstein estimate) but caution should be exercised in reading more into the numbers than is truly justified. For example, it is valid to question the graphical extraction of the 68% confidence interval when the most probable value falls outside the bounded physical region. Alternative approaches exist [164] but none are considered wholly satisfactory or universally acceptable. A very conservative approach involves a renormalization of the probability distribution over the physical region alone. This bounded approach yields $g_P/g_A < 11.3$ (68% c.l.) when applied to our asymmetry measurement.

Given that our value of the photon asymmetry by itself does not place serious constraints on g_P/g_A it is of interest to compare it to previous measurements and to combine it with these. Table XXVI summarizes all previous measurements of the photon asymmetry in calcium.

Figure 62: Extraction of g_P/g_A from the experimental photon asymmetry by comparison with the theoretical curves of Christillin [131] and Gmitro et al. [132]. The hatched area indicates the experimental asymmetry value $\pm$ one standard deviation (the upper limit is at 1.86).

Table XXVI: Summary of all previous experimental determinations of the photon asymmetry in calcium. The individual entries are discussed in some detail in the text.

α_γ	Reference
$\leq -0.32 \pm 0.48$	DiLella et al. 1971 [78]
0.90 ± 0.50	Hart et al. 1977 [106]
0.82 ± 0.76	Frischknecht et al. 1980 [165]
0.99 ± 0.26	Hasinoff et al. 1984 [166]
0.92 ± 0.43	Döbeli et al. 1986 [167]
$1.32^{+0.54}_{-0.47}$	present measurement

Three of the entries in table XXVI require only brief comments, the results of Hart et al. [106] and Döbeli et al. [167] will be discussed more fully. The first measurement of the photon asymmetry, performed by DiLella et al. [78], employed only a bare NaI(Tl) detector without any converter for the photons. The authors estimate a background of 45% due to the inclusion of neutrons from OMC. Their raw asymmetry of -0.32 ± 0.48 is made even more negative when a correction is applied for the neutron background which has a positive asymmetry [108,109], hence the reason for the inequality in table XXVI. This explanation does not help to explain the differences between the measurement of DiLella et al. and the others of table XXVI. One must assume that additional backgrounds or other problems contributed to the observed discrepancy. All other measurements since this first one have employed essentially the same technique using a converter (active or non-active) to discriminate against the neutron background followed by a large NaI(Tl) detector.

The measurement due to Frischknecht et al. [165] is a report of a feasibility study for the experiment eventually reported by Döbeli et al. [167]. A 2 cm NaI(Tl) active converter was used with a large NaI(Tl) detector and the stroboscopic method of μ SR was employed. The reported errors include some

estimate of systematic errors in the residual polarization but the final error is dominated by statistical errors on the sample of ~ 1900 photon events.

The entry in table XXVI by Hasinoff et al. [166] is a preliminary report of the results of the data presented in this thesis. The published experimental photon asymmetry of $\alpha_\gamma \cdot P^{\mu^-} = 0.110 \pm 0.027$ has been normalized with our residual polarization of 0.111 ± 0.010 to obtain the value of α_γ given in the table. Our present value, from the completed analysis, of $1.32^{+0.054}_{-0.047}$ supercedes the previously published value and should be used in the future when referring to the results of this experiment. The small discrepancy between the two values is likely attributable to the inclusion (in the previously published value) of some decay-electron bremsstrahlung (which has an opposite asymmetry) in the energy region fitted. The increased errors now being reported are primarily associated with larger MINOS errors. These fitting errors are strongly coupled to the number of events in the time spectrum which has decreased with the exclusion of some of the bremsstrahlung events. A more thorough analysis of the systematic uncertainties also contributes to an increase in the final value for the error for the asymmetry in the experiment.

The two remaining entries in table XXVI are the ones to which our result should be compared. The agreement between the measurements is statistically acceptable. Despite this several comments should be made. Concerning the results of Hart et al. [106,160], where a 3 mm lead converter was used with a large NaI(Tl) detector, some assumptions which were made in the extraction of their residual polarization from the electron data deserve re-examination in light of the experience gained in the analysis of the present experiment. Hart et al. assumed that the asymmetry of the electrons from bound μ^- decay averaged over all energies was within 1% of the V-A value of $-1/3$, based on the calculations of Johnson et al. [97] which have since been strongly criticized by von Baeyer and

Leiter [101]. Irrespective of this later criticism Hart et al. used a bound-decay energy spectrum with the V–A asymmetry distribution (as opposed to a bound decay asymmetry distribution) in their Monte Carlo calculations. As we have shown in table VII this combination of energy/asymmetry distributions results in an average asymmetry which falls short of $-1/3$ by 15%. Also in the analysis of our data we have found systematic errors of a similar magnitude (but opposite direction) if the full response function of the photon telescope is not included in the Monte Carlo results before the comparison of the Monte Carlo results are made to the experimental data. Hart [160] does not give sufficient details in his thesis to be certain whether the full response function was incorporated into the extraction of the residual polarization or not. This is potentially a larger correction for Hart's data than it was for ours as his response function has a width of 29% FWHM (to be compared with 15% for the present experiment). In the case of Hart's data these two effects may nearly cancel and in view of the magnitude of his errors on α_γ these criticisms should not be construed as necessitating an adjustment of his published result. The errors on his fitted asymmetry are comparable to the size of our errors, although we have twice as many events, due to the lower background experienced in his experiment. The asymmetry in Hart's time spectrum is not evident to the eye nor to the fitting program unless several parameters of the fit are constrained (frequency, phase, lifetime, and background). By performing a constrained fit on his data Hart ignores additional contributions to the final error due to uncertainty in the values of the fixed parameters.

It is appropriate at this point to remind the reader that a calculation of the electron asymmetry distribution for bound muon decay on calcium does not exist and that in this experiment we have actually employed a distribution valid for ^{56}Fe . There is an unaccounted for systematic error therefore associated with our choice of asymmetry distribution. We estimate this error to be significantly

smaller than the error associated with using the V–A asymmetry distribution, that is $\ll 15\%$. As for errors associated with the effect of using a possibly imprecise response function in the extraction of the residual muon polarization, we believe that our rather detailed attention to the comparison of Monte Carlo and experimental data, in general, renders these errors small and that they are sensibly included in the estimations of our systematic errors.

The final data analysis of the data set for which preliminary results were reported by Döbeli et al. [167] is still in progress. Fewer details are available on the actual extraction of the asymmetry from their data but an experimental time spectrum was published along with the preliminary results. My comments are confined to this time spectrum. The published spectrum contains approximately 7000 events; a large number compared with previous experiments and with this present one for which we have ~ 2500 events. The sinusoidal variation (the asymmetry signal) in the time spectrum should be better defined in this larger data set than in ours but this is not the case. Indeed if one calculates the reduced chi-squared of their fit to the their time spectrum, which can be simply and accurately done from their figure, the result is $\chi^2_\nu = 2.7$. The probability of such a high value is $< 10^{-3}$. This indicates a rather severe problem with their data at least at the time of the publication. There is one very significant difference between their experiment and the one described in this thesis. We chose in this experiment to precess the muons at a frequency different from the cyclotron frequency so that the cyclotron-related background and the asymmetry signal could be separated from one another. In their experiment the stroboscopic method was used and hence both the background and the signal had the same frequency. The phase of the background however may have been different from the signal depending on the length of the flight-path(s) from the background source(s) to the detector. Their quoted error on the asymmetry was actually ± 0.26 not

± 0.43 as appears in the table. In light of the obvious problems with their data I have taken the liberty of scaling up their errors by $\sqrt{\chi^2_\nu}$ as is standard practice when faced with improbable values of χ^2_ν . The resulting error of ± 0.43 is still smaller than the errors associated with the asymmetry in this experiment.

A more recent report from Döbeli et al. [168] gives an asymmetry of 0.80 ± 0.17 for a data set containing ~ 27000 events. However, the severe background problems with the experiment are only accentuated by the increased statistics. The reduced chi-squared of their fit is $\chi^2_\nu = 5.3$, a highly improbable value.

Despite the problems associated with a large background in our experiment the asymmetry is visible in our time spectrum and unconstrained fits to the time spectrum are able to find the signal and produce fitted parameters which are within error limits of their known values. Indeed our quoted errors are obtained from a completely unconstrained fit and in this way all correlations between the parameters of the fit are included in our errors. This is a statement of the quality of our data, despite its shortcomings, which cannot be made about the data of Döbeli et al. and Hart et al. and it is our contention that the differences in the quality of the data are not adequately reflected in the differences observed in the quoted errors. This may be partly attributed to the comprehensive study of the systematic errors, associated with the extraction of α_γ , performed in the course of the analysis of the data being presented in this thesis.

Combining the results of the present experiment with those of Hart et al., Frishknecht et al., and Döbeli et al. one arrives at a weighted average of $\alpha_\gamma = 1.02 \pm 0.25$. Extracting g_P/g_A from figure 62 as before, we obtain

$$\frac{g_P}{g_A} = < 8.0 \text{ (68\% c.l.)} \quad (101)$$

Here as well, the interpretation of the asymmetry result is that a renormalized

value of g_P/g_A is favoured over an unrenormalized one but that the Golberger-Treiman value is not excluded.

V.2 The Partial Branching Ratio and g_P/g_A

Figures 63 and 64 show the results of convoluting the theoretical energy spectra of Christillin [131] and Gmitro et al. [133] with our acceptance equation. Explicitly

$$N_i^\gamma = 8.92 \times 10^{10} \cdot \sum_j T_{ij} \cdot \mathcal{E}_j^{MC} \cdot B_j \quad (102)$$

where B_j are the theoretical energy spectra in units of photons/capture/MeV and N_i^γ are the convoluted spectra plotted in figures 63 and 64 in units of counts/MeV. We have defined $A_{>57}^{th}$ as the area under these curves. $A_{>57}^{th}$ can be viewed, as is shown in figures 65 and 66, as being a function of either g_P/g_A or of $R_{k>57}^{th}$, the theoretical partial branching ratio. Also shown on these figures is a hatched region which corresponds to the area under the background-subtracted neutral energy spectrum. The experimental errors indicated by this hatched region include a conservative error of ± 200 counts due to an estimated uncertainty of ± 1 MeV in the determination of the position of 57 MeV. Previously $A_{>57}$ was quoted as 3133 ± 261 counts which, with the addition of this final error in quadrature, becomes

$$A_{>57} = 3133 \pm 329 . \quad (103)$$

In the extraction of the final results of this experiment from figures 65 and 66 there are errors due to the uncertainty in the overall normalization of equation 102 as well as due to the uncertainty in the experimental quantity $A_{>57}$. As given in table XXIII this uncertainty in the normalization of the theoretical curves is $\pm 9.1\%$. Table XXVII summarizes the final values extracted for the partial branching ratio and g_P/g_A from figures 65 and 66. It is seen that the contributions to the final error limits from the variation of the normalization of

Figure 63: Energy spectra of Christillin [131] shown convoluted with our acceptance function and overlaid on the experimental background-subtracted energy spectrum.

Figure 64: Energy spectra of Gmitro et al. [133] shown convoluted with our acceptance function and overlaid on the experimental background-subtracted energy spectrum.

Figure 65: $A_{>57}^{th}$ as a function of g_P/g_A for the theories of Christillin (Chr81) [131] and Gmitro et al. (Gmi86) [133]. The hatched region is the experimental value of $A_{>57}$. Note the suppressed zero on the vertical scale.

Figure 66: $A_{>57}^{th}$ as a function of $R_{k>57}^{th}$ for the theories of Christillin (Chr81) [131] and Gmitro et al. (Gmi86) [133]. The hatched region is the experimental value of $A_{>57}$.

Table XXVII: Summary of the extraction of $R_{k>57}$ and g_P/g_A from the experimental data. The errors associated with the normalization of the convoluted theoretical curves and the experimental quantity are shown separately and have been added in quadrature to obtain the errors on the final results. $R_{k>57}$ is given in units of 10^{-6} photons/capture.

Quantity	Theory	$\delta A_{>57}$	δNorm	Final Results
$R_{k>57}$	Gmi86	+1.97 -1.93	+1.90 -1.56	$21.5^{+2.7}_{-2.5}$
	Chr81	+2.27 -2.31	+2.25 -1.87	$23.5^{+3.2}_{-3.0}$
g_P/g_A	Gmi86	+2.01 -2.26	+1.92 -1.81	$6.0^{+2.8}_{-2.9}$
	Chr81	+1.15 -1.22	+1.10 -1.01	$7.6^{+1.6}_{-1.6}$

the theoretical curves and from the uncertainty in $A_{>57}$ are equal. A large part of both of these uncertainties is directly attributable to the meager statistics and the unfavourable signal to noise ratio. (Table XXIII) shows that δNorm is dominated by the uncertainty in $\mathcal{E}_{CUTS}$ which is estimated from the data and is limited by the statistics of the large background subtraction.) We estimate that the uncertainty in the extracted rate could be reduced if greater statistics were available by at least a factor of two before the systematic errors of our technique would begin to limit the ultimate precision.

We shall first compare our extracted partial branching ratio with those obtained in previous experiments before commenting on g_P/g_A . Table XXVIII summarizes the values obtained in recent determinations of $R_{k>57}$. Two older measurements of the partial branching ratio exist due to Conversi et al. [170] and Rosenstein and Hammerman [171]. Both of these experiments used bare NaI(Tl) detectors without a photon converter and were thus susceptible to high neutron backgrounds. While they measured the energy spectrum above 60 MeV they used available theories to extrapolate the energy spectrum to lower energies and quoted total rather than partial branching ratios. Thus their measurements are not directly comparable to the others in table XXVIII and are now of historical

Table XXVIII: Summary of the partial branching ratios $R_{k>57}$ obtained in recent measurements. The number of photons in the background subtracted energy spectrum for energies greater than 57 MeV is also given along with the resolution of the detector used. It is interesting to note that although the number of photons per measurement has generally increased with time, the quoted errors have also tended to increase rather than to decrease.

$R_{k>57} (10^{-6})$	# Photons	$\Delta E/E$	Reference
21.1 ± 1.4	1229 ± 44	29%	Hart et al. 1977 [106,160]
20.7 ± 2.0	2450	1.5%	Frischknecht et al. 1985 [134,169]
17.1 ± 3.6	≥ 7000	25%	Döbeli et al. 1986 [167]
22.5 ± 2.9	3133 ± 329	15%	present measurement

interest only.

The four determinations of $R_{k>57}$ in table XXVIII are seen to be in essential agreement. The value quoted for the present measurement is the simple average of the two values given in table XXVII. Despite the level of agreement achieved it is of interest to comment on each of the other determinations and to point out the differences in technique used as well as their strengths and weaknesses with respect to the present experiment.

Hart et al. normalized their photon spectrum against their electron spectrum rather than against the number of stopped muons. This has the advantage of cancelling several solid angle and detector efficiency terms from their expression for the partial branching ratio. However a 25% correction had to be estimated and applied to the data to adjust for electrons which did not escape their target or which failed to exceed the energy threshold requirements of their detector. They observed a lifetime for the μ^- in calcium of 365 ± 8 nsec which was incorporated into their normalization. If the established value of 335 nsec is used instead their extracted rate would fall $\sim 8.6\%$. As the reason for the high lifetime is not known it is not clear whether such a correction should be applied. $R_{k>57}$

was obtained from their deconvoluted energy spectrum by fitting a power series to it. The errors which they quote for $R_{k>57}$ are directly coupled to their quoted errors in the number of photons observed which does not include a contribution due to any uncertainty in the position of 57 MeV in their spectrum despite their limited energy resolution. The background in their energy spectrum was quite manageable permitting them to produce for the first time reliable values for both the partial branching ratio and the photon asymmetry although their quoted errors appear not to include some systematic contributions.

The determination of $R_{k>57}$ due to Frischknecht et al. [134,169] differs from all the others in that it was performed with a high resolution pair spectrometer. The use of this intrinsically small solid angle detector was possible due to a 5 MHz muon stopping rate attained during their run. Most of the uncertainty in their final value is attributable to difficulties inherent with estimating the number of μ stops at a high stopping rate. They measured the stopping fraction of the beam at low beam intensities but had to remove the veto counter from behind the target when using the full intensity and depend on the measurement made at low intensity. A correction for μ stops in a lead collimator also had to be applied. This could be estimated by projecting the photon direction (deduced from reconstructing the electron-positron pair momenta in the spectrometer) back to the plane of the target. With such excellent resolution the deconvolution of the photon spectrum was trivial and their energy spectra represent the most reliable to date. Their data were recorded with three different photon converters and when analyzed in three pieces presented the opportunity for making some consistency checks. In the case of the partial branching ratio the checks confirmed the consistency in their treatment of the data. The value of $R_{k>57}$ quoted is quite independent of theory and relatively free of experimental uncertainties.

Few details are available concerning the measurements of Döbeli et al.

reported in preliminary form in reference [167]. They employed two large NaI(Tl) detectors. The largest ($\sim 80\%$ of the combined solid-angle) used a lead converter and the smaller one had an active NaI(Tl) converter. They state that the quoted errors “arise mainly from the uncertainty of the energy calibration and the detector acceptance”. The number of photons in their energy spectrum is not given. It can be assumed to be ≥ 7000 , the number of events in their time spectrum from which their photon asymmetry was measured.

We turn now to a discussion of the values of g_P/g_A as extracted from our data and from previous experiments. g_P/g_A can be extracted from the data in two ways. So far only the first way, in which g_P/g_A was obtained from a comparison of the area under the experimental energy spectrum to the areas under the theoretical energy spectra (essentially from $R_{k>57}$), has been presented. In our case this comparison was done by convoluting the theoretical spectra with our acceptance function rather than deconvoluting the experimental data. The comparison is just as valid in either case but we have chosen not to present deconvoluted data as we believe this introduces an increased systematic error and an arbitrariness to the data which we associate with the unfavourable statistics of our background subtraction. The second way of extracting g_P/g_A is by fitting the energy spectra to the theories with g_P/g_A as the free parameter. As the theoretical spectra are not available as analytic functions of g_P/g_A we estimated the value of g_P/g_A , for which the χ^2 of the fit would be a minimum, by calculating χ^2 for the theories of Christillin [131] and Gmitro et al. [133] for the values of g_P/g_A shown in figures 63 and 64 and then assuming χ^2 varied quadratically in the region of the minimum. Due to the large errors associated with the experimental data (which properly reflect the uncertainty in the area under the background subtracted spectrum but which have a large correlated contribution from the background subtraction) the minimum values of χ^2 are significantly less than one per point.

Table XXIX: Summary of the determinations of g_P/g_A for recent measurements. The values derived from both the partial branching ratio and from fitting the energy spectra are shown. The partial branching ratio is also given for convenience. RYY74 refers to the theory of Rood, Yano and Yano [126]. The entries in the table are discussed at length in the text.

Reference	Theory Used	Fitting to Shape of Energy Spectra		From $R_{k>57}$
		g_P/g_A	χ^2_ν	g_P/g_A
Hart et al. [106,160] ($R_{k>57} = 21.1 \pm 1.4$)	RYY74	6.5 ± 1.6	24.8/23	—
	Chr81	—	—	6.7 ± 0.8
Frischknecht et al. [134,169] ($R_{k>57} = 20.7 \pm 2.0$)	RYY74	3.5 ± 1.3	60/36	—
	Chr81	4.6 ± 0.9	49/36	6.5 ± 1.2
	Gmi86	3.0 ± 0.6	poor	5.1 ± 2.5
present measurement ($R_{k>57} = 22.5 \pm 2.9$)	Chr81	7.3 ± 1.6	3.9/27	7.6 ± 1.6
	Gmi86	6.3 ± 3.5	7.8/27	6.0 ± 2.8

The errors which we quote on the values of g_P/g_A extracted in this way correspond to an increase of χ^2 by χ^2_{min} rather than by n , the number of degrees of freedom (= number of points -1). The results of this extraction of g_P/g_A are summarized in table XXIX along with the results of previous determinations of g_P/g_A .

Two comments on table XXIX will be made before the detailed discussion of the entries. There is not an entry for the data of Döbeli et al. [167] as they have not yet produced more than a preliminary branching ratio. All the entries in the final column, where g_P/g_A is given as calculated from the rate, are due to this author. Frischknecht et al. [134] give a value of 5.8 ± 2.4 for g_P/g_A as the value corresponding to their experimental rate for the theory of Gmitro et al. [133]. This is in slight disagreement with calculations presented in reference [133].

The results of Hart et al. for the theory of Rood, Yano and Yano [126] can be compared to the fit of Frischknecht et al. to this same theory. It is noticed that while the partial branching ratios could hardly agree better there is not agreement on the extracted value of g_P/g_A when fits are made to the theoretical energy

spectra. The calculations of Rood, Yano and Yano were done in the closure approximation and the reader is reminded of the difficulties associated with the dependence of g_P/g_A on the parameter k_{max} which must be fit simultaneously with g_P/g_A . The k_{max}^6 dependence of g_P/g_A is well known and this is precisely the reason that a fit to the theory of Rood, Yano and Yano was not performed for our data. Hart et al. found $k_{max} = 86 \pm 1.8$ MeV and Frischknecht et al. found $k_{max} = 90.8 \pm 0.9$ MeV. Concerning the value corresponding to the extraction of g_P/g_A from Hart's rate for the theory of Christillin (which has the smallest associated error of any entry in that column), it should be emphasized again that although the statistics in these experiments have increased with time so have the associated errors due to an increasing appreciation of the systematic errors associated with the experiment. In this regard Hart's errors should be considered optimistic.

In comparing Frischknecht's extracted values (from fitting to the shape of the energy spectra) to the values extracted by this author (from their observed rate) an inconsistency is noticed. A similar inconsistency is not found for the results of this experiment. The quoted errors on the values extracted from the energy spectra also seem optimistic in Frischknecht's case, particularly for the theory of Gmitro et al. It is not clear actually how this particular value was obtained as the calculation of Gmitro et al. does not extend below $g_P/g_A = 4.5$ and the extrapolation necessary to fit the data is not well defined. As previously mentioned Frischknecht et al. were able to perform some consistency checks since their data were collected in three different configurations (different converters in the pair spectrometer). For the determination of g_P/g_A from fitting to the theory of Christillin, Frischknecht et al. found values [134] of 3.3 ± 0.3 , 6.7 ± 0.8 , and 5.3 ± 0.6 from their consistency checks. When the data were summed and fitted together the value of $g_P/g_A = 4.5 \pm 0.9$ which appears in table XXIX was found.

It seems that their consistency checks leave room for some doubt about the magnitude of their quoted errors.

Frischknecht et al. also observed that the fit below ~ 75 MeV to the theory of Gmitro et al. was rather poor. A value of χ^2 was not given but the data fall consistently far above the theoretical energy spectrum. This was explained in terms of the omission of the quadrupole resonance region of nuclear excitation in the calculations of Gmitro et al. [133]. Such an omission would be expected to lower the predicted spectrum in this energy region. In contrast we do not find such an effect when fitting to the theory of Gmitro et al. Our value for χ^2 is larger however for the fit to Gmitro's calculation than for the fit to Christillin's. It may be that our statistical errors partially mask a similar effect but the magnitude of the deviation below 75 MeV must still be smaller in our case than it was for Frischknecht et al.

Chapter VI

Conclusions

We have measured the photon asymmetry and energy spectra for radiative muon capture in ^{40}Ca . A thorough investigation of the response function of our detector has enabled us to assign realistic systematic errors to the quantities measured. We have obtained a lifetime spectrum for the RMC photons in which the asymmetry signal is clearly visible and to which unconstrained multiparameter fits can be made. For the first time such fits are able to find the signal and also reproduce (within error limits) the other parameters of the fit which are known from the decay electron data.

The measurement of the photon asymmetry represents the least nuclear model-dependent approach to the extraction of a value of g_P/g_A from the data and is the principal result of this thesis. We find a value of $1.32^{+0.54}_{-0.47}$ for the photon asymmetry which corresponds to a value of $g_P/g_A < 5.4$, thus favouring a renormalized value of g_P/g_A . For the first time it has been possible to place a meaningful limit on the induced-pseudoscalar coupling constant from a photon asymmetry measurement.

In the course of the analysis of this experiment a potential systematic error of as much as 15% in the normalization of the photon asymmetry was identified. Such an error can occur from the use of the V–A asymmetry distribution rather than a bound-decay electron asymmetry distribution when performing Monte Carlo calculations of the decay electron asymmetry. Previous experiments have used the V–A asymmetry distribution but sufficient details are not available

concerning their normalization procedure to state with certainty that errors have resulted.

The analysis of the RMC energy spectrum was performed using two methods and we have obtained an internal consistency between these two methods not found in the analysis of the experimental data of Frischknecht et al. [134]. Our experimental resolution of 15% represents an improvement of a factor of approximately two over that of both earlier experiments which have simultaneously measured the energy spectrum and the photon asymmetry. Part of the significance of this experiment is that it was the forerunner to a much improved experiment which has already been successfully performed at TRIUMF and is presently in the data analysis stage.

From the measurement of the partial branching ratio, $R_{k>57}$, we have extracted values of g_P/g_A of 7.6 ± 1.6 and 6.0 ± 2.8 corresponding to analysis in terms of the theories of Christillin [131] and Gmitro et al. [133] respectively. When a fit is performed to the shape of the energy spectrum very good agreement with these values is obtained. However we consider the values extracted via the partial branching ratio to be our final numbers. These values are in good agreement with the unrenormalized Goldberger-Treiman prediction of $g_P/g_A = 6.7$. In this respect we are in agreement with the work of Hart et al. [106] but are in conflict with the recent work of Frischknecht et al. [134] who obtained $g_P/g_A = 4.5 \pm 0.9$ from fits to their energy spectrum. However, as pointed out earlier, these data are subject to internal consistency problems. We are also in slight conflict with our own value of g_P/g_A as derived from the photon asymmetry. However, as the uncertainties associated with the theories used to extract g_P/g_A from the partial branching ratio are likely comparable to the experimental uncertainties this does not represent a serious internal inconsistency in our measurements.

The success of radiative muon capture in determining a value of g_P/g_A has

thus far not been inspiring. While the experimental determination of the rate is presently certain to about 10%, the uncertainty in the theories currently limits further progress in inclusive RMC measurements. The less model-dependent extraction of g_P/g_A through a measurement of the photon asymmetry awaits improved data. Improved data, in turn, is very much linked to a higher flux of clean negative muons. This point deserves further comment. The severe problems evident in the preliminary asymmetry results of Döbeli et al. [167,168] may indicate that the stroboscopic technique is not useful for RMC asymmetry measurements in the presence of significant backgrounds. The inability to measure the background spectrum and the fact that the background events arrive with exactly the same frequency but with a different phase than the RMC photon events makes the interpretation of such stroboscopic experiments difficult. However improved beam intensities and target-detector technologies may allow for the measurement of radiative muon capture to identified states in the final nucleus which would greatly simplify the calculations of the energy spectrum and the radiative rate and thereby allow further progress in this direction. The implicit coincidence condition involved in such exclusive measurements may well alleviate the background problems and enable the full use of future high-intensity muon beams.

This experiment has suffered from a large beam-related background which has limited the quality of the data. Significant progress was made prior to and over the course of the experiment in the reduction of this background. In the improved experiment recently conducted at TRIUMF the background problem was further reduced to the point where the quality of the data is now limited by statistics. The signal-to-noise ratio in the new experiment was approximately 7 which is to be compared with the ratio of 0.5 observed in this experiment.

It is typical at this point in a thesis to offer suggestions for the improvement

of the experiment. Such suggestions arising from the experience gained in the analysis of the data of this experiment have already been successfully applied to the second TRIUMF asymmetry experiment (E364). We recommended the removal of the Cerenkov counter from the photon telescope and the positioning of TINA closer to the BARS. This improved the detector resolution and helped to reduce our dependence on the Monte Carlo calculations. We recommended the construction of a new precession magnet to allow the positioning of the photon telescope closer to the target. The new magnet has increased the detector acceptance by a factor of three. Decreasing the angle of the target with respect to the beam allowed the use of a thinner target with a shorter path length in the target for the decay electrons which again tends to reduce our dependence on the Monte Carlo calculations. We have also increased the background time window in the experiment which allowed the collection of improved statistics on the background thereby permitting a more favourable background subtraction in the energy spectrum. Finally, the identification and resolution of the background problem has permitted the collection of a much improved data set with a greatly improved signal-to-noise ratio.

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Appendix A

Monte Carlo Calculations

The EGS3 Monte Carlo program [151] was used throughout the analysis of the data presented in this thesis. The basic program provides the framework for modelling the electromagnetic shower due to an incident electron, positron or photon on an experimental apparatus of arbitrary geometry. All relevant physical processes for the interaction of these particles with matter are included explicitly in the program. All particles generated in the shower are followed individually by the program until their energy falls below a user-specified threshold. At this threshold energy the particles are considered to have deposited their remaining energy locally and so the detailed tracking of the particle is halted.

To this basic program the user must add routines which specify the dimensions, orientation and composition of the elements of the experimental apparatus. In the present experiment the geometry of the photon telescope was broken down into a series of rectangular and cylindrical elements each of which was composed of a single "medium", such as iron, calcium, sodium iodide, etc. The basic program provided for the generation of media database files which contained the medium-dependent information required by EGS3. The representation of the experimental geometry attained in this fashion was actually quite accurate and involved approximately 40 geometrical elements and 7 different media. The output of the program was a file written in MULTI format so as to be readable by the standard data analysis packages available at TRIUMF. The file consisted of an event-by-event record of the energies deposited in each of the

elements of the experimental geometry as well as the initial coordinates, direction cosines and energies of the incident particles.

Four sets (I–IV) of Monte Carlo data were generated for the experiment. Set I was used in the calibration of the active elements of the detector and in the estimation of the energies deposited in the non-active elements. Initial photon energies were chosen from the π^-p photon energy spectrum; the initial direction cosines were chosen from an isotropic distribution; and the initial coordinates were chosen from a distribution which approximated the pion stopping distribution in the high-pressure hydrogen gas target. This set of Monte Carlo data was also useful in the optimization of the energy reconstruction algorithm and in estimating various detector efficiencies. Set II consisted of a series of shorter Monte Carlo calculations with mono-energetic photons. These data were used to determine the photon response function lineshape as a function of energy and to obtain analytical expressions for the lineshapes.

Monte Carlo data sets III and IV were simulations of the response of the photon telescope to the decay electrons and positrons from polarized negative and positive muons in the calcium target. These data were used to establish the asymmetries that would be expected for any given interval in reconstructed energy and for 100% muon polarization. These asymmetries were then used to determine the $\mu^\pm$ polarizations in the experiment. The initial coordinates for the electrons and positrons were chosen from a realistic stopping distribution for muons in our calcium target. The beam momentum distribution (p_x, p_y, p_z) , and profile (x, y) at the target face were included in the Monte Carlo calculation of the stopping distribution. The initial energies of the particles were chosen from the V–A decay positron energy distribution or from the bound decay electron energy distribution of Herzog and Alder [100] as appropriate. The direction cosines were then chosen from an angular distribution whose asymmetry was given by the V–A decay

positron asymmetry distribution or the bound decay electron asymmetry distribution of Gilinsky and Mathews [96].

In previous experiments the residual polarization of the μ^- has been deduced by assuming an integrated asymmetry (over all electron energies) of $-1/3$ would correspond to 100% polarization. Typically a small correction is applied for electrons not escaping the target and for those not exceeding the energy threshold of the detector. Although it certainly depends on the details of the experimental geometry and on the way in which these corrections are calculated, it is our experience that this approach represents an oversimplification of the problem of determining the residual polarization. We have already emphasized that one makes an immediate 15% error if the combination of a bound decay energy spectrum and the V-A asymmetry distribution is used. In addition the correction due to energy loss in the target and detector threshold, which increases the magnitude of the expected asymmetry (for 100% polarization), is not necessarily well modelled by a sharp cut-off as is usually presumed. For our target the most probable energy loss was approximately 10 MeV. However the energy loss distribution observed in the Monte Carlo data was asymmetric (with a high energy tail) and had a FWHM of ~ 7 MeV. Simply assuming the most probable value certainly introduces an error in this case. We find as well that multiple scattering of the electrons in the target can produce as much as a 5% attenuation of the expected asymmetry. A potentially large effect can be associated with the proper inclusion of the response function or detector lineshape for these low energy electrons. Part of the lineshape is given by the Monte Carlo program but part must be added in an ad hoc fashion. In our Monte Carlo calculations additional Gaussian smearing was added to the energies deposited in the various elements to simulate instrumental effects not otherwise included. The effect of this additional blurring of the instrumental resolution can decrease the expected asymmetry by as

much as 10% at the high energy end of the decay electron energy spectrum.

So, with the Monte Carlo data of sets III and IV the problem is how to calculate the expected asymmetry (for 100% polarization) for any given interval in reconstructed energy. It is conceptually simpler to ignore, for the moment, any effects due to the presence of the target. For the case of 100% polarization one can picture, at the location of the target, an ensemble of muons with their spins aligned and precessing in the horizontal plane due to the applied external magnetic field. Now the maximum and minimum probability of detecting a decay electron (or positron), P_{max} and P_{min} , will occur when the muon spin (the ensemble polarization vector) is either aligned or anti-aligned with the axis of symmetry of the detector. The asymmetry then can be defined in terms of these two probabilities. Actually these probabilities and the asymmetry (Monte Carlo) are most naturally expressed as functions of the electron/positron energy, x ,

$$|A_{MC}(x)| = \frac{P_{max}(x) - P_{min}(x)}{P_{max}(x) + P_{min}(x)}. \quad (104)$$

However this is not really the asymmetry in which we are interested. We need the asymmetry expressed as a function of the observed (or reconstructed) energy. It is useful though to continue with the asymmetry given by the above equation a little further. For a given value of x the probability of detecting a decay electron/positron is simply the fraction of the particles ‘thrown’ in the Monte Carlo which hit the various counters and deposit sufficient energy to satisfy the definition of a charged event in the experiment. The particles must be thrown (i.e. have their direction cosines chosen) from an angular distribution which corresponds to the energy x . If the geometry of the detector and the physics of polarized muon decay are included in a realistic fashion then $A_{MC}(x)$ is found by running the Monte Carlo for two cases: one where the polarization vector is aligned with the detector axis and one where it is anti-aligned. EGS3 is relied

upon to follow the particle and its electromagnetic shower in order to decide whether the definition of a charged event has been satisfied. This is a simple prescription and could be repeated for many values of x in order to map out the energy dependence of A_{MC} .

In reality a wide range of x may contribute to a given interval in reconstructed energy. The transformation from $A_{MC}(x)$ to $A_{MC}(E)$ where E is the reconstructed energy is accomplished by selecting x from the appropriate energy distribution and building histograms of the reconstructed energy for each of the two cases. Then $A_{MC}(E)$ is computed from the fraction of the thrown events which fall within the given interval in reconstructed energy for each of the two cases. If we now label the two cases as 0° and 180° then

$$A_{MC}(E) = \frac{P_0(E) - P_{180}(E)}{P_0(E) + P_{180}(E)} . \quad (105)$$

This is now the quantity of interest. It includes modifications of the observed asymmetry arising from geometrical effects (both finite target and finite detector), as well as energy loss, energy threshold, and energy reconstruction (instrumental resolution) effects. The finite target effects are of course only included if the initial coordinates of the particles have been chosen from a realistic (3-dimensional) stopping distribution.

If the target is now included in the Monte Carlo calculation, that is, if the geometrical element representing the target is assigned a non-vacuum medium, then the effects of energy loss in the target and multiple scattering in the target are automatically included as EGS3 tracks the particles through the target material. For a target which is positioned symmetrically with respect to the detector axis (for instance at 90° to the axis) the multiple scattering and energy loss occur symmetrically and the maximum and minimum probabilities still occur when the polarization vector is at 0° and 180° . However if the target is placed

asymmetrically, such as for our actual experimental geometry, then energy loss and multiple scattering occur to a greater extent on one side of the target and the directions of the polarization vector which correspond to the maximum and minimum observed intensities in the detector are rotated away from 0° and 180° although they remain separated by 180° . The size of this effect was found by Monte Carlo calculation to be only 3 ± 0.5 degrees for our stopping distribution in the calcium target. The error made by performing the Monte Carlo calculations for 0° and 180° can be corrected by dividing the asymmetry obtained from these two cases by the cosine of 3° but this is a completely negligible correction.

To summarize, the expected observed asymmetry of the decay electrons (or positrons) from 100% polarized muons was found by running two Monte Carlo calculations in which the polarization vector was either aligned or anti-aligned with the axis of symmetry of the detector. Histograms of the reconstructed energy were constructed in each case. The asymmetry for a given reconstructed energy interval was calculated from the number of Monte Carlo events which fell within the interval in the two cases. These results were used to determine the muon beam polarization (μ^+ case) and the residual μ^- polarization by comparing the Monte Carlo results with fitted experimental asymmetries for the same interval in reconstructed energy.