LOW ENERGY ELASTIC SCATTERING
AND THE PIONIC ATOM ANOMALY

By
MARK HANNA
B.Sc.(Hons), Concordia University, 1986

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE

in
THE FACULTY OF GRADUATE STUDIES

Department of Physics

We accept this thesis as conforming
to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

September 1988
© Mark Hanna, 1988
In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Physics
The University of British Columbia
Vancouver, Canada

Date October 11, 1988
Abstract

Differential cross sections for the elastic scattering of low energy (20 MeV) positive and negative pions from $^{12}$C and $^{40}$Ca have been measured. This measurement was performed at TRIUMF using the QQD low energy pion spectrometer at 11 angles between 45° and 125°. The $\pi^+$ $^{12}$C data are in good agreement with [OBG+83], as are the $\pi^-$ $^{12}$C data with [WBB+87], indicating that the overall normalization of the cross sections is good. However, the $\pi^\pm$ $^{40}$Ca data do not agree well with the previously published data of [WMR+88].

An optical potential model, whose parameters were determined from pionic atom data, was used to predict these differential cross sections. Two sets of parameters were used in the model. One set was determined from fits to “normal” pionic atom data, while another set was extracted from fits to “anomalous” data where level shifts and widths do not compare to theoretical values obtained when the normal set of parameters is used. In all the above cases, the experimental data best fits the optical model predictions of [FG80,Fri88b] when the normal set of parameters is used. The qualitative agreement of the data to the “normal” optical model predictions indicates that the pionic anomaly effects do not extend to positive pion energy values.
# Table of Contents

Abstract ii  
List of Tables v  
List of Figures vi  
Acknowledgements vii  

I Introduction 1  
I.1 The Pionic Atom ............................................. 1  
I.2 The Anomaly .................................................. 3  
I.3 Motivation for this Experiment ................................. 4  
I.4 Experiment 373 ................................................ 5  

II Theoretical Description 6  
II.1 Nuclear Scattering ............................................ 6  
II.2 The Optical Potential Model ................................... 7  
II.3 Outline of the $\pi$-Nucleus Potential ......................... 8  
II.4 Potential Model Calculations .................................. 10  

III The Experiment 15  
III.1 TRIUMF's M13 Low Energy $\pi - \mu$ Channel .................. 15  
III.2 QQD Spectrometer ............................................ 18  
III.3 Wire Chamber Design and Calibration .......................... 20  
III.4 Momentum Calibrations ....................................... 23  
III.4.1 Target Traceback Coefficients ................============ 24  
III.4.2 Magnet Transfer Coefficients .............................. 24
## List of Tables

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Fermi three parameter fit values for nuclear density distributions in $^{12}$C and $^{40}$Ca.</td>
<td>11</td>
</tr>
<tr>
<td>II</td>
<td>Optical potential parameters obtained from fits to normal and anomalous pionic atom data.</td>
<td>13</td>
</tr>
<tr>
<td>III</td>
<td>Optical potential parameters extrapolated to 20MeV.</td>
<td>14</td>
</tr>
<tr>
<td>IV</td>
<td>Specifications of targets used in Exp. 373.</td>
<td>28</td>
</tr>
<tr>
<td>V</td>
<td>Measured $\pi^+$ differential cross sections for $^{12}$C at 20 MeV.</td>
<td>42</td>
</tr>
<tr>
<td>VI</td>
<td>Measured $\pi^-$ differential cross sections for $^{12}$C at 20 MeV.</td>
<td>43</td>
</tr>
<tr>
<td>VII</td>
<td>Measured $\pi^+$ differential cross sections for $^{40}$Ca at 20 MeV.</td>
<td>44</td>
</tr>
<tr>
<td>VIII</td>
<td>Measured $\pi^-$ differential cross sections for $^{40}$Ca at 20 MeV.</td>
<td>45</td>
</tr>
<tr>
<td>IX</td>
<td>Optical potential model fit summary.</td>
<td>55</td>
</tr>
</tbody>
</table>
## List of Figures

1. The effect of the strong interaction on level shifts and widths .......................... 3
2. Schematic layout of the TRIUMF M13 pion channel ................................. 16
3. Schematic view of the QQD spectrometer ................................................. 19
4. Position information obtained from x and y TDC difference data ................. 22
5. The δ-difference (DDIF) cut ................................................................. 27
6. Target holder used for the Calcium target ............................................. 28
7. The experimental electronic logic ......................................................... 30
8. Typical time-of-flight spectrum for M13 and QQD .................................. 33
9. Difference in y position in WC4 and WC5 and box text ............................ 35
10. Beam sample spectrum for positive and negative polarity runs ................. 36
11. A comparison between [OBG+83] and TRIUMF π⁺ ¹²C data .................. 46
12. A comparison between [WBB+87] and TRIUMF π⁻ ¹²C data .................. 47
13. A comparison between [WMR+88] and TRIUMF π⁺ ⁴⁰Ca data ................. 48
14. A comparison between [WMR+88] and TRIUMF π⁻ ⁴⁰Ca data ................. 49
15. Normal and anomalous predictions compared to π⁺ ¹²C data ................. 51
16. Normal and anomalous predictions compared to π⁻ ¹²C data ................. 52
17. Normal and anomalous predictions compared to π⁺ ⁴⁰Ca data ................. 53
18. Normal and anomalous predictions compared to π⁻ ⁴⁰Ca data ................. 54
Acknowledgements

I would like to gratefully acknowledge Marty Rozon, who’s help throughout the production of this thesis has been greatly appreciated. His guidance and patience during what seemed like an eternal period of analysis, and his knowledge of experimental physics has proven invaluable. I wish to thank my supervisor, Dick Johnson, first and foremost for his persistence in getting me to complete this project, also for his ability to keep his unique sense of humor, sending me to Lake Louise, and making incredible culinary delicacies (not to mention his fearless handling of sharks). More thanks go to Eli Friedman and Oded Meirav for there assistance with the theoretical aspect of this document. Special thanks go to the “semi-cool dude”, Rinaldo Rui, for seeing the lighter side of a problem and keeping me laughing, and to “the baron”, Rigo Olsweski, (sorry about the van) whose computer wizardry and beer drinking is second to none.

Additional thanks go to Andrew (le hot racer!), John (la première étoile!), Chris (where’s the beef?), Reena, Jean, Vesna, Grant (don worry, be appy), Peter, Martin, Angela, Niall, Dave and to those killer ping pong paddle pushers Gio, Hon, and Richard. I couldn’t have gone through all those late nights without you, thanks a million for the fun and laughs. Marcello (le golfeur extraordinaire) also deserves special mention for indirectly turning me into an menacing biker.

Finally, I would like to sincerely thank my family for their patience and support throughout my academic years. I couldn’t have done it without you. I would also like to express my deep appreciation to Christy, who’s encouragement (and scolding for staying up late and eating junk food) kept me going during the final stages of this project.
Chapter I

Introduction

Man’s curiosity for understanding the basic forces in nature has always been an insatiable one. Only in the last century has man begun to unravel the mysteries that lie deep in the heart of matter. In 1935 Yukawa [Yuk35] proposed that the nuclear force, which tightly binds the atomic nucleus, is mediated by some massive particle (mass≈150 MeV), similar to the way that the electromagnetic interaction is mediated by massless photons. The pi-meson was first seen by Lattes et al. [LMPO47] in photographic emulsions exposed to cosmic rays on top of a mountain. These pions, identified with the Yukawa pion, were postulated to exist in three charge states. The pion had to exist in positive, negative, and neutral charge states to account for the known strong interactions. These states were later confirmed experimentally.

1.1 The Pionic Atom

Almost immediately after the discovery of the pion, the existence of mesic atoms were predicted by both Wheeler [Whe47] and Fermi and Teller [FT47]. A pionic atom is formed when a negatively charged pion is stopped in some material, then captured by an atom in a high atomic orbit. The pion, because of its large mass, orbits at levels much closer to the nucleus than that of a corresponding electron orbit. It then cascades down to lower energy orbits, first by Auger electron processes, and then by the emission of X-rays. Once the pion’s wavefunction
overlaps with the nuclear wavefunction, the cascade stops\(^1\). The pion then interacts strongly, and is absorbed by the nucleus.

Most of the pion's atomic life is spent at levels lower than the Bohr orbits of the electron. Due to the pion's large mass relative to the electron, it's orbiting levels around the nucleus are well within the lowest electron shell and thus can be treated like a hydrogen atom. The higher energy levels can be calculated directly from electromagnetic theory. However, when the pion gets close enough to the nucleus, it starts to feel the effects of the strong force, and that results in a shift and broadening of the energy levels. The lower the energy level, the stronger the nuclear interaction, and consequently only the lowest energy level is significantly affected before nuclear absorption occurs. It is therefore the last observable X-ray that is significantly shifted and broadened in energy from that of purely electromagnetic considerations. As well as this shift in energy, the lowest energy level is susceptible to "absorptive broadening" [Bri84]. This arises due to the shortened lifetime of the pion in an orbit where the strong force is encountered. The uncertainty principle states that \(\Delta E \cdot \Delta t \geq \hbar\), and hence the line width is broadened. Experimentally, these effects are viewed as a Lorentzian broadened X-ray line, whose centroid is shifted from a position expected from a purely electromagnetic interaction. This is shown in figure 1.

The theoretical description of the level shifts and widths in pionic atoms was initially deduced from low energy \(\pi^-\)–nucleon scattering data using first order perturbation theory. This proved to be inaccurate since the modification of the pion wavefunction due to the strong short range pion-nucleus interaction (leading to absorption), was not accounted for. This led to a semi-phenomenological

\(^1\)"Stops" is a rather strong word. If the negative pion reaches the 1s level (light atoms) obviously the cascade stops since it's at the ground state. Above \(^{20}\)Na the cascade doesn't exactly 'stop' when the wavefunctions overlap, rather nuclear absorption competes and rapidly dominates. In actuality, absorption may occur from different levels in the same atom. This is indicated experimentally by the small intensity of low transition X-rays in larger atoms.
Figure 1: The effect of the strong interaction on level shifts and widths.

approach using multiple scattering theory. The result was a nonlocal pion-nuclear potential derived by Ericson and Ericson [EE66]. The predictions of the potential model agreed satisfactorily with experimental level shift and width data in pionic atoms for elements throughout the periodic table. The optical potential model will be discussed in following chapter.

### 1.2 The Anomaly

The negative pion-nucleus optical potential has been very successful in reproducing experimental results on strong interaction level shifts and widths in pionic atoms [Fri88b]. In the past few years, improved experimental techniques have made it possible to study levels that were previously too broad and too weak to measure with any precision\(^2\). These levels, whose pion wavefunction overlap

\(^2\)The last observable X-ray (i.e. 2p → 1s) in medium to heavy nuclei (Na or Mg) is naturally weak since the cross section for absorption is already large at the 2p level. It is this transition that
more with the nuclear wavefunction than those of previously studied levels, have been measured using Compton suppression spectrometers [Bri84]. These types of spectrometers generally use a Sodium Iodide (NaI) or Bismuth Germanate (Bi$_4$Ge$_3$O$_{12}$) detector to identify Compton scattered electrons from the main solid state detector. The signal from this detector is then used as a veto for the Compton scattered event, thus creating a spectrum in which the total absorption peak for the X-rays is much more pronounced, distinct X-ray peak. Owing to this experimental advance, it was observed that a substantial fraction of levels in the 1s, 2p, and 3d range, have widths narrower than estimated by the optical model. These widths were seen to saturate$^3$ with increasing $Z$. Also, the shift of many of these levels were larger than predicted indicating a repulsive potential. New data may shed some light towards explaining these phenomena if they really exist.

1.3 Motivation for this Experiment

Optical potential models have successfully reproduced numerous pionic atom data throughout the periodic table. However, they continue to predict shifts and widths a factor of two or greater that experimentally observed for the transition to the last observable level in a particular atom. The anomalous shifts appear to be explainable for various reasons [BFG83,Sek82], which arise from a cancellation of effects from different parts of the optical potential [MS85]. These parts include repulsive s-wave and the attractive p-wave terms. The anomalous widths however, have remained without any reasonable explanation. The unexplained experimental existence of these widths indicates that there may be some important physical factor which was overlooked during the construction of the potential. If this is true, we have a very incomplete understanding of low energy pion physics. To

$^3$As $Z$ increases, the observed widths of the X-rays remains constant at a factor $\approx \frac{1}{2}$ of that predicted by theory. To date there is no explanation for this phenomenon.
determine whether or not the anomaly indeed exists at small positive pion energies, a low energy elastic scattering experiment was proposed to study elastic scattering from a nucleus that does not exhibit the anomaly and one that does.

1.4 Experiment 373

The pionic atom optical potentials can be successfully modified to model \( \pi \)-nucleus scattering at positive energies with some minor corrections for the energy increase. Therefore, we may predict differential cross sections for an angular distribution of a low energy elastic scattering experiment.

Experimentally there has been no recognition of anomalous effects for elastic \( \pi \)-scattering data in the range of 30 – 50 MeV [SMC79,SMOY80,SM83]. This indicates, assuming the anomaly exists, that either the energies are too high to show the effect unambiguously, or discrepancies in the analyses are too large. Thus the experiment should be performed at energies < 30 MeV.

An experiment employing the QQD spectrometer on TRIUMF’s M13 beam line was executed. To determine whether or not the effects of the anomaly are observable at small positive energies, positive and negative pions were scattered from targets made of \(^{40}\text{Ca}\) and \(^{12}\text{C}\) at 20 MeV. At this energy only a few partial waves contribute appreciably, namely the \( s \) and \( p \) waves. Calcium was chosen as an ideal target for two reasons; one being that due to its nuclear size, it should be large enough to exhibit the anomaly in the \( 1s \) state, the other being that since it is an \( N=Z \) nucleus, no complications due to isospin arise. Thus the anomalous effect is expected to appear more distinctly than \( \pi \)-scattering from heavier nuclei. Carbon is not expected to exhibit the anomaly but is useful for calibration and normalization purposes.
Chapter II

Theoretical Description

Experimental measurements of elastically scattered pions from nuclei yield data essential to the understanding of the strong interaction. This data takes the form of differential cross sections which are then used to construct theoretical models to explain experimental measurements. A brief description of the various processes involved in determining this link are discussed in the following sections.

II.1 Nuclear Scattering

The interaction between the pion and the target nucleus is represented by a potential which depends on the relative coordinate \( r \) relating the position of the pion to the centre-of-mass of the nucleus. This potential is then inserted into the Klein-Gordon equation. A mathematical description of the scattering of a relativistic spin zero particle (the pion) from a nucleus, can thus be expressed as [Fri83]

\[
\hbar^2 c^2 (\nabla^2 + k^2) \psi = [2E(V_n + V_c) - V_c^2] \psi
\]

where \( \hbar k \) is the c.m. momentum, \( E \) is the c.m. total energy of the pion, \( V_c \) is the Coulomb potential due to the finite charge distribution of the nucleus, and \( V_n \) is the nuclear potential. Quadratic terms involving \( V_n \) are usually omitted since their effects are small in comparison to the linear terms. The solution of this equation, \( \psi \), must be regular at the origin and have the asymptotic form of an incoming plane wave and an outgoing scattered spherical wave [Jac70], i.e.

\[
\psi \rightarrow \exp(ikz) + f(\theta) \frac{\exp(ikr)}{r}
\]
where $\theta$ is the scattering angle in the c.m. system and $f(\theta)$ is the scattering amplitude. The asymptotic form of $\psi$ can also be written in spherical coordinates as [BJ77]

$$\psi \rightarrow \frac{1}{2} \sum_i i^{i+1}(2i + 1)P_i(\cos \theta)[\phi_i(kr) - n_i \varphi_i(kr)]$$

where $\phi_i(kr)$ and $\varphi_i(kr)$ represent incoming and outgoing spherical waves, respectively, and $n_i$ represents the reflection coefficient\(^1\), usually expressed in terms of a complex phase shift, i.e.

$$n_i = \exp(2i\delta_i)$$

If $|n_i| = 1$, then the intensity of the incoming and outgoing waves are identical, thus the scattering is purely elastic. However, if inelastic and absorption processes take place then the intensity of the outgoing wave will be less than that of the incoming wave. Comparing equations 2, 3 and 4, we obtain for the scattering amplitude

$$f(\theta) = \frac{1}{2ik} \sum_i (2i + 1)(\exp(2i\delta_i) - 1)P_i(\cos \theta)$$

The differential cross section for elastic scattering can then be given by [Jac70]

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

II.2 The Optical Potential Model

An optical model is one that describes the effects of the strong interaction by means of an interaction potential. This potential is independent of the coordinates of the individual nucleons and depends only on the coordinate of the incident pion with respect to the nucleus as a whole [Kis55]. This semi-phenomenological

\(^1\)This coefficient is analogous to the index of refraction of some medium through which photons propagate. Here, the wavefunction of the pion is modified by its propagation through nuclear matter and is denoted by a complex index of refraction.
approach is based on the elementary pion-nucleon interactions

$$\pi + N \rightarrow \pi + N$$ \hspace{1cm} \text{(elastic scattering)} \hspace{1cm} (7)$$

and

$$\pi + N + N \rightarrow N + N$$ \hspace{1cm} \text{(absorption)}. \hspace{1cm} (8)$$

The model should successfully predict pion-nucleus scattering data in the energy region between $0 - 50$ MeV. Here, the effects of the $\Delta_{33}$ resonance do not cause too strong an absorption. Also, the optical parameters in the model, should vary uniformly with no sudden fluctuations over this energy region.

There are many forms of this potential, among which one of the most successful is the MSU potential [SMC79,SCH80,CMSB82] that follows the Ericson-Ericson potential for pionic atoms [EE66]. Both potentials are based on the Kisslinger potential [Kis55]. In general most potentials obtain good fits to data since they incorporate many similar features useful in describing $\pi$-nuclear interactions. In the next section we discuss the optical potential following the notation of Friedman and Gal [FG80].

II.3 Outline of the $\pi$-Nucleus Potential

The standard pion-nucleus optical potential which is inserted into the Klein-Gordon equation (eq. 1) has the form [EE66,KE69]

$$V_n(r) = \frac{1}{2\omega} [q(r) + \nabla \cdot \alpha(r) \nabla]$$ \hspace{1cm} (9)$$

where $\omega$ is the total pion centre-of-mass energy, $q(r)$ is the local momentum-independent part arising from pion-nucleon s-wave interaction. The local potential is written in terms of neutron ($\rho_n$) and proton ($\rho_p$) density distributions as

$$q(r) = -4\pi \left\{ \left( 1 + \frac{\omega}{m} \right) [b_0(\rho_n + \rho_p) + b_1(\rho_n - \rho_p)] + \left( 1 + \frac{\omega}{2m} \right) 4B_0 \rho_n \rho_p \right\}$$ \hspace{1cm} (10)$$
where \( m \) is the nucleon mass (\( \approx 931 \text{ MeV} \)). The constants \( b_0 \) and \( b_1 \) represent effective \( \pi \)-nucleon \( s \)-wave scattering lengths through isoscalar and isovector channels respectively. The constant \( B_0 \) is complex. Its imaginary part is related to the absorption of a pion on a pair of nucleons. Absorption on a single nucleon is improbable due to conservation of energy and momentum, and so the process is most likely to occur on proton-neutron pairs. The real part of \( B_0 \) describes dispersion effects.

The term \( \nabla \cdot \alpha(r) \nabla \) is the \( p \)-wave momentum-dependent part of the potential where \( \alpha(r) \) is decomposed as follows

\[
\alpha = \frac{\alpha_1}{1 + \frac{1}{3} \xi \alpha_1} + \alpha_2
\]  

(11)

with

\[
\alpha_1 = 4\pi \left( 1 + \frac{\omega}{m} \right)^{-1} \left[ c_0 (\rho_n + \rho_p) + c_1 (\rho_n - \rho_p) \right],
\]

(12)

and

\[
\alpha_2 = 4\pi \left( 1 + \frac{\omega}{2m} \right)^{-1} 4C_0 \rho_n \rho_p.
\]

(13)

Here the constants \( c_0 \) and \( c_1 \) represent effective \( p \)-wave scattering volumes through isoscalar and isovector channels respectively. The constant \( C_0 \) is a complex parameter in analogy with \( B_0 \). The modification of single nucleon terms due to nuclear correlations is represented by the so-called Lorentz-Lorenz (LL) effect\(^2\), and its strength depends on the value of \( \xi \). An extra term transforming the \( \pi \)-nucleon c.m. to \( \pi \)-nucleus c.m., by which \( p \)-wave \( \pi \)-nucleon interaction gives rise to terms of order \( \omega/m \), is included.

\(^2\)Nuclear pair correlations produce an effect similar to that caused by the scattering of electromagnetic waves in a dense, polarizable medium.
This local term has the form

\[ \Delta q_{\text{at}} = -4\pi \frac{\omega}{2m} \left\{ \left( 1 + \frac{\omega}{m} \right)^{-1} \nabla^2 [c_0(\rho_n + \rho_p) + c_1(\rho_n - \rho_p)] + \left( 1 + \frac{\omega}{2m} \right)^{-1} 2C_0 \nabla^2 (\rho_n \rho_p) \right\} \]  

(14)

The nucleon density distributions \( \rho_n \) and \( \rho_p \) appearing in the above equations are determined using a phenomenological form for the distribution. The proton and charge\(^3\) density distribution parameters were obtained from the fits of electron scattering data\(^4\) to a three parameter Fermi function\(^5\) \([\text{Jac70}]\)

\[ \rho_{PF}(r) = \left( 1 + \frac{r^2}{c^2} \right) \frac{\rho_0}{1 + \exp \left( \frac{r - c}{a} \right)} \]  

(15)

where \( \rho_0 \) is a normalization constant to a specified number of nucleons, \( c \) is the radius at half the nuclear density, and the diffuseness \( a \) is related to the nuclear skin thickness. The parameter \( w \) is referred to as a \textit{wine bottle} parameter which, depending on its value being either positive or negative, generates a hump or depression near the origin. The neutron density distribution parameters were obtained by similar fits from proton or \( \alpha \)-particle scattering data (or Hartree-Fock calculations). The values obtained from these fits are shown in table I.

II.4 Potential Model Calculations

The differential cross sections for elastically scattered low energy pions from nuclei are calculated using GLBKISS \([\text{Fri88a}]\), a computer program incorporating an optical model potential of the Ericson-Ericson MSU type. The program determines the solution to the Klein-Gordon equation with the potential in equation 9 and it is compared to the solution for only Coulomb scattering\(^6\) at a point well outside

---

\(^3\)Not shown above, but used in determining the Coulomb potential in target nuclei.

\(^4\)After correcting for the finite size of the proton charge.

\(^5\)Distribution forms such as this are not deduced from first principles but are phenomenological, which are found by experience to lead to agreement with a wide selection of data.

\(^6\)These are well known wavefunctions.
Table I: Fermi three parameter fit values for neutron, proton, and charge density distributions in $^{12}C$ and $^{40}Ca$ taken from Batty et al. [BFG83].
the nuclear radius where the effect of the strong force is sufficiently small. From the comparisons of these solutions, the phase shifts\(^7\) (eq. 4) are obtained and the differential cross section calculated (eq. 5,6). The optical potential parameters were initially determined from fits to strong interaction level shifts and widths in pionic atoms [EE66,KE69,BBF+79,FG80,BFG83]. This served as the basis for the determination of the parameters for normal states. Fits to anomalous states were also made [OBB+78,KPK+79,vEBD+84,TSS+84,TvEB+85,LTD+85,OFM+85] and the parameters determined. Several attempts to modify the Ericson-Ericson \(\pi\)-nucleus potential to account for these anomalous effects proved to be unsatisfactory [FG80,BFG83,Sek82,OTK84]. The normal and anomalous potential parameters, for a scattering energy of 20 MeV pions, used in GLBKISS were extrapolated from the pionic atom data parameters (see table II) and are shown in table III. These parameter sets were used for calculations to compare with our elastic scattering data as described in Chapter IV. The errors associated with these parameters are given in [Fri88b]. The large error in \(\xi\) is a result of the insensitivity\(^8\) of this parameter during the fit to pionic atom and elastic scattering data [MFA+88].

---

\(^{7}\)Outside the nucleus the wavefunction of the pion has a constant complex phase shift compared with the wavefunction of a purely Coulomb scattered pion. It is this shift that contains the information about the interaction which happened well inside the nucleus.

\(^{8}\)By changing the value of \(\xi\), it was required to adjust other parameters in the model to obtain similar fits, indicating that there are some correlations between the parameters. These fits did not substantially reduce the value of \(\chi^2\) per point.
Table II: Optical potential parameters obtained from fits to normal and anomalous pionic atom data [Fri88b].

<table>
<thead>
<tr>
<th>Optical Potential Parameters</th>
<th>Normal</th>
<th>Anomalous</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_0(m_\pi^{-1})$</td>
<td>-0.009</td>
<td>-0.009</td>
</tr>
<tr>
<td>$b_1(m_\pi^{-1})$</td>
<td>-0.094</td>
<td>-0.094</td>
</tr>
<tr>
<td>$B_0(m_\pi^{-4})$</td>
<td>-0.115+i0.055</td>
<td>-0.112+i0.030</td>
</tr>
<tr>
<td>$c_0(m_\pi^{-3})$</td>
<td>0.23</td>
<td>0.23</td>
</tr>
<tr>
<td>$c_1(m_\pi^{-3})$</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>$C_0(m_\pi^{-6})$</td>
<td>0.051+i0.053</td>
<td>-0.10+i0.074</td>
</tr>
<tr>
<td>$\xi$</td>
<td>1.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>
Table III: Optical potential parameters extrapolated to 20MeV from fits to elastic scattering data at higher energies. The values of $B_0$ and $C_0$ used in the program are multiplied by a factor of four. Note that only $b_0$ and $B_0$ vary significantly from their pionic atom values.
Chapter III
The Experiment

This low energy elastic scattering experiment was performed in the summer of 1986 at TRIUMF (TRI University Meson Facility). TRIUMF's cyclotron is capable of producing a primary 500 MeV unpolarized proton beam at a current of 140 μA. This high energy proton beam strikes a series of targets, usually beryllium or carbon, for the production of pions. These pions are then channeled to various secondary beam lines in the experimental areas. There are two such production targets in the Meson Hall. The first of these, called 1AT1, produced the pions for this experiment.

III.1 TRIUMF's M13 Low Energy π − μ Channel

The facility's M13 beam line was used for this experiment. This low energy pion channel is capable of delivering pion beams with a momentum range of 20-150 MeV/c [OWMD81]. It views the 1AT1 production target at 135° to the primary proton beam. A series of dipoles, quadrupoles, and sextupoles achromatically focus the beam to the nuclear scattering target located 86 cm beyond the last quadrupole. This can be seen in figure 2. The first two quadrupoles (Q1 and Q2), and initial dipole (B1), are adjusted to create a dispersive focus at F1. The momentum of the beam is defined at F1 by suitable horizontal and vertical slit positions and widths. The momentum dispersion of the pion beam has been measured experimentally to be 1.22 cm/%Δp [OWMD81]. For a horizontal slit width of 30 mm (dispersion of 2.5% Δp) the π⁺ flux was ≈ 1.2 × 10⁶ particles/sec.
Figure 2: Schematic layout of the TRIUMF M13 pion channel and spectrometer. BL1A is the primary beam line and T1 is the pion production target; Q1-Q7, SX1-SX2, and B1-B2 are the channel quadrupole, sextupole and dipole magnets, respectively; F1 and F2 are the intermediate focus points; QT1-QT2 and BT are the spectrometer quadrupole and dipole magnets, respectively.
For the same setting, the $\pi^-$ flux was $\approx 3.2 \times 10^5$ particles/sec. Also situated at F1 is a CH$_2$ absorber used to stop heavier particles emerging from 1AT1 such as $\alpha$'s and protons. Following this, a set of quadrupoles (Q3,Q4,Q5), and sextupoles (SX1,SX2), are adjusted to produce a second dispersive focus at F2. Another set of momentum defining slits are located at this position. Finally a second dipole (B2), and two quadrupoles (Q6,Q7) bring the beam to a third and final achromatic focus at the scattering targets center. The beam line and spectrometer magnets were set and monitored throughout the experiment using PBX, a program to remotely select current values for each individual magnet. The actual magnetic fields were measured by NMR probes for the dipoles, and Hall probes for the quadrupoles and sextupoles.

Two in-beam counters consisting of NE110\(^1\) plastic scintillator were used. The first counter, B1, was situated preceding the scattering target whilst the second, B2, was placed behind the target. The coincidence of B1·B2 measured the absolute beam flux traversing the target. Two pairs of muon counters were located on either side of the M13 beam pipe at $\approx 15^\circ$ to the incident pion beam\(^2\). These counters were used to measure muons from the decay of pions in flight. The coincidence $\mu_1 \cdot \mu_2$ and $\mu_3 \cdot \mu_4$ are used as relative pion flux monitors. This is extremely useful since at very forward angles B2 cannot be placed behind the target as the spectrometer physically blocks the beam.

As mentioned earlier, the beam traveling down the M13 channel contains $\alpha$'s, $p$'s, $\pi$'s, $\mu$'s and $e$'s. The CH$_2$ absorber at F1 blocks the heavier $\alpha$'s and $p$'s while the lighter $\mu$'s and $e$'s are separated by their flight times through the channel. A capacitive probe (TCAP) located near the production target and the

---

\(^1\)These scintillators are fast signaling plastic detectors able to handle large flux and are manufactured by Nuclear Enterprises\(^TM\).

\(^2\)The optimum position of the muon counters is half the maximum muon cone angle since, at this position, the detectors are not too sensitive to fluctuations in beam position.
in-beam scintillator B1 supply the TDC (time-to-digital converter) stop and start, respectively, for this measurement. The result of this measurement is shown in figure 8 which indicates that a relatively clean pion beam was obtained.

III.2 QQD Spectrometer

The QQD (Quadrupole Quadrupole Dipole) low energy pion spectrometer was used to measure the elastically scattered pions for this experiment [SDB+84]. The two quadrupoles serve as a lens for enlarging the spectrometer’s solid angle to $\approx$16 msr. The dipole is used as a momentum analyser for the elastically scattered pions, as it deflects them $70^\circ$ to the left in the horizontal plane. These pions are brought to a dispersive focus beyond the final wire chamber at an angle of $72^\circ$ to the central momentum trajectory. The optical layout of the spectrometer can be seen in figure 3. Only the second quadrupole (QT2) which focuses in the vertical direction and dipole (BT) were used in this experiment. The first quadrupole (QT1), which focuses in the horizontal direction, was not used as it would increase the solid angle slightly ($\approx$5%) in exchange for decreasing the resolution in the target traceback.

Four delay-line multi-wire proportional chambers (DLMWPC) utilized by the QQD give position information for the elastically scattered pion. The first (WC1) and second (WC3) chamber positions are located ahead of QT1 and after QT2 respectively. There is room between the two quadrupoles for a second chamber but its use would not sufficiently improve first order position and angular resolution in the target traceback. The third (WC4) and fourth (WC5) chambers are positioned after BT. The momentum of the pion is determined by using the position information from the four chambers. Details of position and momentum calibration are given in sections III.3 and and III.4 respectively.

At the rear of the spectrometer there are three large plastic scintillator
Figure 3: Schematic view of the QCD spectrometer. The symbols WC1-WC5 are locations in which wire chambers are positioned; E1-E3, B1-B2, μ1-μ4 are the various scintillators associated with the spectrometer and beamline. The dashed line represents the central momentum trajectory of the scattered pion.
detectors, E1, E2, and E3 respectively. Only E1 and E2 were used for this experiment since a substantial fraction of pions would not make it beyond the first two scintillators at this low energy. A spectrometer event required the coincidence of B1-E1-E2 as this would most probably occur as a result of a particle traversing the QQD. Hardware coincidences and electronics setup are discussed with greater detail in section III.6.

III.3 Wire Chamber Design and Calibration

The DLMWPC's employed by the QQD were constructed at the University of Carleton Workshop [BPRW71,SDB+84]. Each chamber consists of 3 planes of equally spaced gold-tungsten wires. The anode plane at high positive potential, is sandwiched between two cathode planes at ground potential. The anode wires are oriented horizontally at with spacing of 2 mm to create a uniform electric field throughout the active area of the chamber. The cathode planes have a wire spacing of 1 mm and are oriented parallel and perpendicular to the anode wires. The horizontal cathode wires give position information in the y-direction while the vertical cathode wires give this information in the x-direction. The front 2 chambers (WC1,WC3) are filled with a helium-isobutane gas mixture to minimize multiple scattering\(^3\) while the back chambers (WC4,WC5) are filled with an argon-isobutane mixture. As the pion traverses the chamber it ionizes the gas inside producing electron-ion pairs. The electrons cascade towards and induce a signal on the closest anode which in turn capacitively couples to the neighbouring cathodes, producing signals in both x and y directions. One end of each cathode wire is connected to a printed circuit delay-line strip with a delay of 0.55 nsec between consecutive wires. The two ends of the delay-line are each connected to

\(^3\)Multiple scattering in the spectrometer due to mylar windows (for wire chambers) and gases (throughout QQD and wire chambers) limit the spectrometer resolution to \(\approx 1\) MeV.
amplifiers which are then fed to discriminators and time-to-digital converters (TDC’s).

Calibration of these chambers by converting TDC values to a position in millimeters is made convenient by the arrangement of these chambers. The conversion is defined by

\[ X = m \cdot t + b \]  

where \( X \) is the position in mm, \( m \) is the slope converting TDC values to mm, \( t \) is the TDC difference value, and \( b \) is the offset in mm to define the center of the chamber.

The determination of the conversion factor \( m \), in the front chambers is achieved by observing the ‘picket fence’ structure obtained from the \( y \) TDC difference data. This is shown in figure 4(a). The \( y \)-direction cathode wires and anode wires are parallel. Thus, a signal induced in any one anode will produce a strong signal in the adjacent cathode wire, and a relatively weak signal in any neighbouring cathode. The \( x \)-direction cathode wires run perpendicular to the anode wires thus the anode signal will distribute a strong signal to one or more cathode wires resulting in a smooth spectrum. This can be seen in figure 4(b). Given that the physical wire spacing of the anodes is 2 mm, we may obtain the conversion factor from TDC differences to distance, i.e.

\[ 2 = m \cdot TDC_{\text{diff}} + b \]  

assuming \( b=0 \) initially, we have

\[ m = \frac{2}{TDC_{\text{diff}}} \]  

where \( TDC_{\text{diff}} \) is the difference between two consecutive pickets. The slope can be used for both \( x \) and \( y \) directions since each plane is identical except for their
Figure 4: (a) Picket fence structure obtained from $y$ TDC difference data used to determine wire chamber coefficients. (b) Position information obtained from $x$ TDC difference data. Note the smoother structure compared with $y$ TDC difference data.
orientation. Once $m$ is found, $b$ is adjusted to center the beam at the center of the chamber.

For the back chambers, the same method can be used for the $y$-direction information. However, these chambers are much larger than the front chambers and are segmented in $x$ to reduce dispersion and attenuation of signals in the delay-line. Wire chambers 4 and 5 each have 3 sections in the $x$-direction. Each section is 203 mm wide. The right-middle and middle-left edges overlap, so a particle passing through these points are likely to fire both sections. By examining the positions of double-hits in the central segment one can determine the conversion factor $m$, i.e.

$$203 = m \cdot TDC_{diff} + b$$

again assuming $b=0$ we have

$$m = \frac{203}{TDC_{diff}}$$

where $TDC_{diff}$ is the difference between the two double-hit peaks. Since all sections in any particular chamber are identical, the same conversion factor is assumed. The central offset is adjusted to position the double-hit peaks at $\pm 101.5$ mm. The left and right offsets are adjusted such that their respective edges with the central segment edges overlap.

### III.4 Momentum Calibrations

Two sets of coefficients are needed to completely define the trajectory and momentum of the pion through the spectrometer. These include the front end wire chamber coefficients for ray-tracing back to the target and a set of magnet transfer coefficients which are essential to calibrate the pion’s momentum through the spectrometer’s dipole BT.
III.4.1 Target Traceback Coefficients

A set of coefficients is needed to trace the pion's path through the two front end wire chambers back to the scattering target. These coefficients essentially define the pion's initial trajectory towards BT. The quadrupole magnet QT2, is positioned between WC1 and WC3, and thus the traceback is not simply a linear one. However, since QT2 is not a dipole, it is assumed the there will not be a large momentum dependence in the traceback. The coordinates required are the pion's position at the scattering target in the $x$ and $y$ ($X_0$ and $Y_0$). This results in

$$X_0 = a_1 \cdot X_1 + a_3 \cdot X_3$$  \hspace{1cm} (21)

and

$$Y_0 = b_1 \cdot Y_1 + b_3 \cdot Y_3 \ .$$  \hspace{1cm} (22)

Two special targets constructed of nichrome strips 3 mm. wide set about 10 mm. apart, one with horizontal strips, one with vertical strips, are placed at the scattering target position and spectrometer data is taken. From the analysis of this data the coefficients $a_1,a_3,b_1,$ and $b_3$ are adjusted to obtain the correct strip positions.

III.4.2 Magnet Transfer Coefficients

The passing of multi-energetic pions through the spectrometer's dipole results in the emergence of those pions at different positions in the back end wire chambers. The corresponding positions can then be converted to pion energies which can be expressed in terms of $\delta$, where

$$\delta = \frac{p - p_0}{p_0}$$  \hspace{1cm} (23)

Here, $p_0$ is the central momentum of the spectrometer and $p$ is the pion's momentum after scattering. The positions in the back chambers alone will not
clearly indicate the energy spectrum of the pions since the focal plane of the QQD lies at 72° to the central ray and is located beyond WC5 (see figure 3). We must therefore incorporate the information from the front end wire chambers to produce a good spectrum. The x and y positions in WC1 and WC3 can be employed directly to obtain the transfer coefficients in the software package QQDMP [Bar85]. It is these coefficients which relate the position information in the back chambers to a value δ (one for each chamber, δ4, δ5). The routine QQDMP assumes that the back-end wire chamber coordinates may each be written in terms of a polynomial of the front-end wire chamber coordinates and the momentum parameter δ,

\[ WC4X = A + B \cdot \delta_4 + C \cdot \delta_4^2 \]  

with

A being a polynomial of order m0 in front-end coordinates,
B being a polynomial of order m1 in front-end coordinates, and
C being a polynomial of order m2 in front-end coordinates.

The above expression may be meaningfully inverted to determine δ4 if the coefficients A, B, and C are known. Similarly δ5 can be obtained. The corresponding expression for WC4Y or WC5Y will have little or no δ-dependence and therefore may not be used for δ determination.

The data used to determine these transfer coefficients were obtained by elastically scattering 20 MeV pions from 12C at 0%, ±5%, and ±10% QQD central momentum settings. In the program QQDMP, one defines the elastic peak locations and attempts to minimize the peak widths. Once this is done, a multiple linear regression is performed to determine the coefficients A, B, and C. The values of δ4 and δ5 are then calculated and analysed further.
The analysis routine in QQDMP called QQDANA was used to optimize the data used for determining transfer coefficients. Cuts on the data were imposed to eliminate background events triggered in the spectrometer. The primary source of these events come from the decay of pions via $\pi \rightarrow \mu e$. Ideally, one would expect $\delta_4 = \delta_5$. However, if a pion decays into a muon, its path will deviate from the pion's trajectory by an angle limited by kinematics to $\pm 30^\circ$ [Gre], resulting in substantially different $\delta$'s. The muon will either hit some appendage within the spectrometer and stop or trigger an event. These events are eliminated by placing a cut on DDIF which is the difference between $\delta_4$ and $\delta_5$. If the difference was greater than 1.5%, the event was cut. Another useful cut is the ANGL cut. Using the value of $\delta_4$, the trajectory to WC5 is predicted, and a polar angle between the actual trajectory and the predicted one is calculated. If this value is too large, the event is cut. The DDIF and ANGL cuts overlap to a large extent, however it is useful to employ them both. A sample of the DDIF cut is shown in figure 5.

III.5 Targets

Two targets, one $^{12}$C, and one $^{40}$Ca, were used in this experiment. Both solid targets were held in plexi-glass target holders at beam height by a target ladder situated at the pivoting point of the QQD. The target holders were designed so that they do not interfere with the pion beam at any of the angles used in the experiment. A description of this holder is shown in figure 6.

The carbon target used was a 76 mm$^2$ graphite slate which should have been large enough to intercept the entire beam, however, this was not the case (see Appendix A). The calcium target was made of three self supporting plates of metallic natural calcium (97% $^{40}$Ca) of size 51 mm by 39 mm held in the target holder by a thin nylon thread. The nylon thread is thin enough that the
Figure 5: The $\delta$-difference (DDIF) cut. This represents the difference between $\delta_4$ and $\delta_5$ calculated from the $x$ position information in WC4 and WC5 respectively.

background it contributes is insignificant, thus eliminating the need for an "empty" target measurement. Specifications of these targets are shown in table IV.

III.6 Data Acquisition

This single arm elastic scattering experiment utilizing the QOD spectrometer was set up using the standard electronic arrangement described in previous theses from the PISCAT group [Roz85,Hes85,Bar85] without the use of the F2 counter$^4$.

The online data acquisition programs were run on a PDP11/34 computer using the STAR [Smi86] system to acquire experimental data and record it to magnetic tape. This data, in the form of ADC (analog-to-digital converter), TDC, scaler values and bit patterns, was produced by various modules in a CAMAC$^5$.

$^4$The F2 counter is a fast readout wire chamber placed at the second dispersive focus in the M13 channel. This chamber, when operating efficiently, gives additional momentum information of the incident pions.

$^5$The CAMAC system is an interface system which digitizes analog signals from the hardware
Figure 6: Target holder used for the Calcium target. The nylon line is shown as a dotted line threaded through holes in the target frame and anchored to the frame at the outermost holes. The solid Calcium target is interlaced through the nylon thread.

<table>
<thead>
<tr>
<th>Target</th>
<th>Nucleus</th>
<th>Mass Density</th>
<th>$N_{tgt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>graphite</td>
<td>$^{12}$C</td>
<td>378</td>
<td>$1.90 \times 10^{22}$</td>
</tr>
<tr>
<td>calcium</td>
<td>$^{40}$Ca</td>
<td>414</td>
<td>$6.23 \times 10^{21}$</td>
</tr>
</tbody>
</table>

Table IV: Target mass densities and scattering center densities of targets used in Exp. 373
crate. A specific CAMAC module, the C212 unit, generated LAM (look-at-me) signals enabling the PDP11/34 to read data created by a spectrometer hardware event. In this experiment, a hardware event was defined as the coincidence \textbf{B1}·\textbf{E1}·\textbf{E2} with the start signal given by \textbf{B1} and the stop given by \textbf{E1}. The back scintillators \textbf{E1} and \textbf{E2} each have two phototubes whose output was measured in a meantiming circuit to make the event timing position independent. A schematic diagram depicting the electronic logic used is shown in figure 7.
Figure 7: The experimental electronic logic.
Chapter IV
Data Analysis

The off-line analysis utilizing the REPQQD [OH87] software package is discussed in this chapter along with the calculation of absolute differential cross sections and their errors. The constraints and normalization of the data are also presented. Finally, the results are fitted using the optical potential model discussed in Chapter II.

IV.1 Off-Line Analysis Program

Tapes containing experimental data were analysed off-line using REPQQD, a software package similar to the on-line STAR system but specifically fabricated to analyse QQD spectrometer data. This analysis routine allows the user to read events and scalers from tape, create and calculate new parameters, apply software constraints for the removal of background and 'bad' events, and construct one or two dimensional histograms of these parameters. The histograms can then be saved for later use such as peak fitting.

IV.2 Event Definitions

There are two types of events written to tape; spectrometer events and beam sample events. The definition of a good spectrometer event is an M13 pion scattering from a specific target location, firing all spectrometer wire chambers and scintillators without decaying in flight. We require the electronic coincidence B1·E1·E2. Beam sample events are defined by the coincidence B1·B2. These
events are selected at random throughout the run and used for determining the pion fraction of the incoming beam. Both types of events are essential in the determination of the differential cross sections. They are discussed below, along with their various constraints.

IV.2.1 Spectrometer Events

There are several cuts and tests to be passed before a pion can be considered a true event. For example, the event must initially satisfy the spectrometer hardware event defined above. Following this, the particle must lie within timing limits for pions imposed in the beam time-of-flight spectrum. This spectrum is generated from the occurrence of spectrometer events and is the time it takes the particle to travel from TCAP to B1 (see figure 8 (a)). Another similar constraint is obtained from the time-of-flight spectrum through the QQD (E1T). This cut is useful for the removal of large amounts of background events caused by pions decaying to muons within the spectrometer. This can be seen in figure 8(b).

Moreover, all 4 wire chamber anodes are required to have fired and reasonable TDC values obtained. These values are checked for validity by requiring the sum of the TDC values for both ends of the delay line of each section of each chamber to lie within specified limits. The back chambers have an additional requirement that both right and left sections in $x$ not fire simultaneously. These tests are useful when the chambers experience noise. Cuts are also placed on the traceback to the target. This is explained in detail in Appendix A. At 20 MeV, approximately 50% of the pions decay during their flight through the spectrometer. To eliminate these events, two separate constraints are placed on the position information given by the rear wire chambers. These are the DDIF (discussed in section III.4) and the DY45 cuts. The latter is made by imposing limits on $y$ position data in WC4 and WC5. If the pion decays to a muon, its trajectory deviates from the initial path of
Figure 8: (a) Typical time-of-flight spectrum for particles travelling from 1AT1 production target to B1 scintillator. (b) Time-of-flight of particles traversing the QQD spectrometer. Most of the background is easily removed by placing more stringent constraints.
the pion by an angle limited by kinematics to < 30°. If the \( y \)-difference was greater than 3 cm, the event was cut as shown in figure 9(a). Finally a box test was applied to determine which events which were clearly true pions. This was possible by examining a two dimensional scatterplot of the time-of-flight through the spectrometer (E1T) vs. \( \delta_4 \) (or \( \delta_5 \)) and eliminating inelastically scattered pion and background events. An example of this is shown in figure 9(b).

The result is a relatively clean elastic scattering peak used in determining the differential cross sections.

IV.2.2 Beam Sample Events

As mentioned previously, the beam sample events are selected at random throughout each run. Defined by the coincidence \( \text{B1-B2} \), these events are the measure of the flight time of particles traveling from the 1AT1 production target to the inbeam scintillator \( \text{B1} \). From these spectra we may determine the actual pion fraction delivered to the scattering target. For positive polarity runs (see figure 10(a), 82% of the beam were \( \pi^+ \), whereas for negative polarity runs (see figure 10(b)) only 49% of the beam contained \( \pi^- \) particles. At a specific distance restricted by kinematics and the size of \( \text{B1} \), a muon from a decaying pion may still intercept \( \text{B1} \) and be considered a pion. A small correction factor for this contamination was taken into account in determining the true pion flux.

IV.3 Cross Sections

The absolute differential cross section including normalization factors is defined as

\[
\frac{d\sigma}{d\Omega_{CM}} = \frac{N_{scat}}{\varepsilon_{decQQD} \cdot \varepsilon_W \cdot \varepsilon_d} \cdot \frac{1}{N_{inc} \cdot \varepsilon_\pi \cdot \varepsilon_{decB1}} \cdot \frac{1}{\frac{\rho \cdot N_{At}}{A \cdot \cos \theta_{pt}}} \cdot \frac{1}{\Delta\Omega_{lab}} \cdot J_{lab\rightarrow CM}
\]  

(25)

where

\( N_{scat} \) is the number of scattered pions traversing the \( QQD \)
Figure 9: (a) Difference in $y$ position in WC4 and WC5. Scale is in mm $\times 10$. (b) Scatterplot of $\delta_5$ vs E1T. Here the muons are easily separated from true pion events.
Figure 10: (a) Beam sample spectrum for positive polarity runs. (b) Beam sample spectrum for negative polarity runs. Note the increase in the number of electrons.
\( \varepsilon_{decQQD} \) is the correction for \( \pi \)-decay through the spectrometer

\( \varepsilon_{WC} \) is the normalization due to wire chamber efficiencies

\( \varepsilon_{dt} \) is the correction for computer dead time

\( N_{inc} \) is the number of incident pions

\( \varepsilon_\pi \) is the pion fraction of the incoming beam

\( \varepsilon_{decB1} \) is the correction for \( \pi \)-decay before B1

\( \rho \cdot t \) is the target thickness in g/cm\(^2\)

\( N_{Av} \) is Avagadro’s number (6.022 \( \times \) 10\(^{23} \) atoms/g-mole)

\( A \) is the atomic weight of the scattering target

\( \theta_{tgt} \) is the target angle in the lab frame

\( \Delta \Omega_{\text{lab}} \) is the spectrometer solid angle

and

\( J_{\text{lab-CM}} \) is the Jacobian for the lab to C.M. transformation

The number of scattered pions (\( N_{\text{scat}} \)) was determined by fitting the sum of two Gaussian functions, one with large width and one narrow, to the elastically scattered pion peak. Fits to the data were performed using OPDATA [BK87]. This data manipulation program was employed once all background was effectively removed. At this energy about 50% of the pions decay before reaching the back scintillators. Therefore, the values obtained by the fit have still to be corrected for pion decay through the 2.38 m long spectrometer. This factor is given by

\[
\varepsilon_{decQQD} = \exp \left( \frac{m_\pi \cdot l_{QQD}}{p \cdot \tau \cdot c} \right)
\]  

(26)
where $m_\pi$, $p$, $\tau$, are the pion mass, momentum, and mean-life, and $\ell_{QGD}$ is the spectrometer decay length. Other corrections such as wire chamber efficiencies and computer dead time must be taken into account with this particular setup. With the initial wire chamber constraints discussed in section IV.2.1, we may define the efficiency of any particular chamber (WC3 for example) as

$$Eff_3 = \frac{WC1 \cdot WC3 \cdot WC4 \cdot WC5}{WC1 \cdot WC4 \cdot WC5}$$

where WC$m$ indicates a valid firing in both $x$ and $y$ sections of the $m^{th}$ chamber. The total efficiency is thus defined as

$$\varepsilon_{WC} = Eff_{tot} = Eff_1 \cdot Eff_3 \cdot Eff_4 \cdot Eff_5$$

and was typically $\sim 80\%$. During the processing of an event, the computer is busy and cannot recognize any forthcoming events until it is finished with the event at hand. A correction for this dead time is defined as

$$\varepsilon_{dt} = \frac{N_{LAM}}{N_{SPECT} + N_{BS}}$$

where $N_{LAM}$ is the number of times the computer witnesses an event and $N_{SPECT}$ and $N_{BS}$ are the number of spectrometer and beam sample events respectively.

The number of incoming particles was determined by the scaler monitoring the coincidence $B1 \cdot B2$. For runs where the $QQD$ was at 50° or less, the incident flux was given by $B1$ and then normalized to $B1 \cdot B2$ using values obtained for runs where $B2$ was used$^1$. The beam sample spectrums mentioned in section IV.2.2 were used to determine what fraction of these particles were pions. This correction factor is given by

$$\varepsilon_\pi = \frac{N_\pi}{N_\pi + N_\mu + N_e}$$

$^1$Normalizing to $B1 \cdot B2$ could also be done using scaler values from the coincidence $\mu_1 \cdot \mu_2$ and $\mu_3 \cdot \mu_4$. 

38
where \( N_\pi, N_\mu, N_e \), are the number of pions, muons and electrons\(^2\) respectively. As discussed earlier, pions decaying 43 mm, before \( \text{B1} \)\(^3\) may still trigger the scintillator and be considered a pion. Although the likelihood of triggering \( \text{B1-B2} \) is small, the possibility still exists and must be corrected for. This is especially true for runs where \( \text{B2} \) cannot be put in coincidence. Similar to equation 26, this correction is given by

\[
\varepsilon_{\text{decB1}} = \exp \left( -\left( \frac{m_\pi \cdot (\ell_{B1} + \ell_{TGT})}{p \cdot \tau \cdot c} \right) \right) \tag{28}
\]

where \( \ell_{B1} = 43 \text{ mm} \) upstream of \( \text{B1} \) and \( \ell_{TGT} \) is the distance from \( \text{B1} \) to the scattering target. The value of \( \varepsilon_{\text{decB1}} \) was typically \( \sim 95\% \).

The target thickness depends on the target’s angle with respect to the QQD. To minimize straggling due to electromagnetic scattering, the target angle was chosen to be

\[
\theta_{\text{tgt}} = \frac{1}{2} \theta_{\text{QQD}}. \tag{29}
\]

where \( \theta_{\text{tgt}} \) is measured from the beam axis to the normal to the target. At this angle, all pions scattered into the spectrometer travel the same distance through the target, no matter where the scattering occurs. The effective thickness the pion encounters is then

\[
t_{\text{eff}} = \frac{t_{\text{tgt}}}{\cos \theta_{\text{tgt}}} \tag{30}
\]

where \( t_{\text{tgt}} \) is the actual target thickness in cm. The number of scatterers per unit area in the target is calculated from

\[
N_{\text{tgt}} = \frac{\rho \cdot t_{\text{eff}} \cdot N_{\text{Av}}}{A}. \tag{31}
\]

The values of \( N_{\text{tgt}} \) for both carbon and calcium targets are given in table IV.

\(^2\)Discriminator thresholds reject many of the electrons.

\(^3\)At this distance upstream from \( \text{B1} \) a muon from a decayed pion may still trigger \( \text{B1} \).
The spectrometer solid angle used was $\Delta \Omega = 0.016$ steradians. This value is based on previous group work [Bar85, Hes85]. Since $\Delta \Omega$ is constant throughout the cross section calculations, any associated error can be absorbed in the overall normalization of the data.

IV.4 Cross Section Errors

The cross section error may be determined by summing the individual fractional errors of the contributing terms in quadrature. These terms do not include errors that effect all points equally, since they are absorbed in the normalization. Hence the error is written as

$$
\left( \frac{\Delta \left( \frac{d\sigma}{d\Omega_{CM}} \right)}{\frac{d\sigma}{d\Omega_{CM}}} \right)^2 = \left( \frac{1}{N_{\text{scat}}} \right)^2 + \left( \frac{\Delta WC}{WC} \right)^2 + \left( \frac{\Delta X_{0\text{traceback}}}{X_{0\text{traceback}}} \right)^2 + \left( \frac{\Delta \text{flux}}{\text{flux}} \right)^2 + \left( \frac{\Delta \pi_{\text{decay}}}{\pi_{\text{decay}}} \right)^2.
$$

(32)

The wire chamber error has been estimated by Rozon [Roz85]

$$\frac{\Delta \text{Eff}}{\text{Eff}} = \left( \frac{1 - \text{Eff}}{N} \right)^{\frac{1}{2}}$$

(33)

where Eff is the fraction of events that pass the cut and N is the number of events in the cut region. This error is typically 1% or less. The target traceback error is the error estimated in the fitting procedure discussed in Appendix A. The contribution here ranges from a maximum of 8% or less for both positive and negative polarity runs. The error in beam flux is brought about from the normalization of $B1 \cdot B2$ for runs where $B2$ could not be used. It is an estimate taken to be 1%, the stability limit of the muon counters. A major contribution to background events is from the decay of pions. The error associated with this correction is estimated to be no more than 4%. This is because the majority of muons decay at large angles with respect to the initial pion. Those that do decay

$^4$The spectrometer solid angle remains reasonably constant for the range of pion momentum measured in this experiment (see [SDB+84]).
at very forward angles (within 5°) either have too little or to great a momentum to be detected within the limits of the elastic scattering peak in the back wire chambers of the QD.

IV.5 Results and Comparisons

The differential cross sections and their errors for $^{12}$C and $^{40}$Ca are shown in tables V, VI, VII, and VIII. The beam energy is given as 20 MeV. This is the average pion energy at the target's center after allowing for energy losses in the mylar windows, scintillator, and target. The errors shown are statistical.

The $\pi^+$ $^{12}$C results agree well with the data of [OBG+83]. At backward angles, it would appear that the TRIUMF data is consistently lower but well within the statistical limits, except for the 125° data point which disagrees substantially. Figure 11 shows the comparison of these results. Similarly the $\pi^- 12$C data agree nicely with [WBB+87]. This can be seen in figure 12. A comparison of the $\pi^+$ $^{40}$Ca and $\pi^- 40$Ca data are shown in figures 13 and 14, respectively. Here, the disagreement beyond 60° is obvious, especially at very back angles. The current TRIUMF data lies below the the data of [WMR+88]. An observation of the $\pi^- 40$Ca data shows that the Coulomb-nuclear interference minimum lies at 65°. It is the location of this minimum which is critical for the determination of the anomaly. These two sets of data for $^{12}$C and $^{40}$Ca constitute all available 20 MeV experimental data to date. The fact that there were two experiments ([WBB+87], [WMR+88]) to collect low energy negative pion data could be a factor in the discrepancy between the published data and the TRIUMF data. Since the TRIUMF cross sections were calculated from data collected in one experiment, the positive and negative scattering comparison may be more reliable.
<table>
<thead>
<tr>
<th>Polarity</th>
<th>$\theta_{CM}$ (deg)</th>
<th>$\frac{d\sigma}{d\Omega_{CM}}$ (mb/sr)</th>
<th>$\Delta \frac{d\sigma}{d\Omega_{CM}}$ (mb/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+$</td>
<td>45.6</td>
<td>7.77</td>
<td>$\pm$ 0.13</td>
</tr>
<tr>
<td></td>
<td>50.6</td>
<td>5.01</td>
<td>$\pm$ 0.24</td>
</tr>
<tr>
<td></td>
<td>55.7</td>
<td>4.09</td>
<td>$\pm$ 0.18</td>
</tr>
<tr>
<td></td>
<td>60.7</td>
<td>3.75</td>
<td>$\pm$ 0.14</td>
</tr>
<tr>
<td></td>
<td>65.7</td>
<td>3.48</td>
<td>$\pm$ 0.20</td>
</tr>
<tr>
<td></td>
<td>69.8</td>
<td>3.63</td>
<td>$\pm$ 0.23</td>
</tr>
<tr>
<td></td>
<td>75.8</td>
<td>3.30</td>
<td>$\pm$ 0.26</td>
</tr>
<tr>
<td></td>
<td>82.9</td>
<td>3.78</td>
<td>$\pm$ 0.25</td>
</tr>
<tr>
<td></td>
<td>90.8</td>
<td>3.99</td>
<td>$\pm$ 0.28</td>
</tr>
<tr>
<td></td>
<td>105.8</td>
<td>4.56</td>
<td>$\pm$ 0.33</td>
</tr>
<tr>
<td></td>
<td>125.8</td>
<td>4.78</td>
<td>$\pm$ 0.33</td>
</tr>
</tbody>
</table>

Table V: Measured $\pi^+$ differential cross sections for $^{12}$C at 20 MeV.
<table>
<thead>
<tr>
<th>Polarity</th>
<th>$\theta_{CM}$ (deg)</th>
<th>$\frac{d\sigma}{d\Omega_{CM}}$ (mb/sr)</th>
<th>$\Delta \frac{d\sigma}{d\Omega_{CM}}$ (mb/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^-$</td>
<td>45.6</td>
<td>8.56</td>
<td>± 0.32</td>
</tr>
<tr>
<td></td>
<td>50.6</td>
<td>4.65</td>
<td>± 0.18</td>
</tr>
<tr>
<td></td>
<td>55.7</td>
<td>2.20</td>
<td>± 0.08</td>
</tr>
<tr>
<td></td>
<td>60.7</td>
<td>1.31</td>
<td>± 0.11</td>
</tr>
<tr>
<td></td>
<td>65.7</td>
<td>0.77</td>
<td>± 0.07</td>
</tr>
<tr>
<td></td>
<td>69.8</td>
<td>0.75</td>
<td>± 0.12</td>
</tr>
<tr>
<td></td>
<td>75.8</td>
<td>0.85</td>
<td>± 0.07</td>
</tr>
<tr>
<td></td>
<td>82.9</td>
<td>1.42</td>
<td>± 0.12</td>
</tr>
<tr>
<td></td>
<td>90.8</td>
<td>2.46</td>
<td>± 0.19</td>
</tr>
<tr>
<td></td>
<td>105.8</td>
<td>4.33</td>
<td>± 0.31</td>
</tr>
<tr>
<td></td>
<td>125.7</td>
<td>5.93</td>
<td>± 0.40</td>
</tr>
</tbody>
</table>

Table VI: Measured $\pi^-$ differential cross sections for $^{12}$C at 20 MeV.
Table VII: Measured $\pi^+$ differential cross sections for $^{40}$Ca at 20 MeV.

<table>
<thead>
<tr>
<th>Polarity</th>
<th>$\theta_{CM}$ (deg)</th>
<th>$\frac{d\sigma}{d\Omega_{CM}}$ (mb/sr)</th>
<th>$\Delta\frac{d\sigma}{d\Omega_{CM}}$ (mb/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+$</td>
<td>45.2</td>
<td>82.80</td>
<td>$\pm$ 3.21</td>
</tr>
<tr>
<td></td>
<td>50.2</td>
<td>45.21</td>
<td>$\pm$ 2.52</td>
</tr>
<tr>
<td></td>
<td>55.2</td>
<td>38.43</td>
<td>$\pm$ 2.03</td>
</tr>
<tr>
<td></td>
<td>60.2</td>
<td>34.45</td>
<td>$\pm$ 1.85</td>
</tr>
<tr>
<td></td>
<td>65.2</td>
<td>27.91</td>
<td>$\pm$ 1.53</td>
</tr>
<tr>
<td></td>
<td>69.5</td>
<td>24.57</td>
<td>$\pm$ 1.35</td>
</tr>
<tr>
<td></td>
<td>75.2</td>
<td>20.15</td>
<td>$\pm$ 1.61</td>
</tr>
<tr>
<td></td>
<td>82.7</td>
<td>19.16</td>
<td>$\pm$ 1.21</td>
</tr>
<tr>
<td></td>
<td>90.3</td>
<td>16.97</td>
<td>$\pm$ 0.93</td>
</tr>
<tr>
<td></td>
<td>105.2</td>
<td>14.86</td>
<td>$\pm$ 0.79</td>
</tr>
<tr>
<td></td>
<td>125.2</td>
<td>10.91</td>
<td>$\pm$ 0.60</td>
</tr>
</tbody>
</table>
Table VIII: Measured $\pi^-$ differential cross sections for $^{40}$Ca at 20 MeV.
Figure 11: A comparison between [OBG+83] and TRIUMF $\pi^+^{12}$C data. The stars are the TRIUMF data.
Figure 12: A comparison between [WBB+87] and TRIUMF $\pi^-{^{12}}C$ data. The stars are the TRIUMF data.
Figure 13: A comparison between [WMR⁺88] and TRIUMF $\pi^+{^{40}\text{Ca}}$ data. The stars are the TRIUMF data.
Figure 14: A comparison between [WMR+88] and TRIUMF $\pi^{-}$ $^{40}$Ca data. The stars are the TRIUMF data.
IV.6 Optical Potential Model Comparisons

The optical potential discussed in Chapter II was used to produce theoretical curves for the elastic scattering of pion from $^{12}$C and $^{40}$Ca at 20 MeV. Both normal and anomalous parameters were used and compared to experimental data. These comparisons are shown in figures 15, 16, 17, and 18.

The anomaly was not expected to be overly noticeable for the carbon data. Even though both normal and anomalous curves differ minimally, it is clear that the normal fit is superior. It is again noted that at large back angles, theory and experiment disagree. The position of the Coulomb-nuclear interference minimum in the $\pi^-\ ^{12}$C data lies at 70°, which according to theory indicates no anomaly. The $\pi^+\ ^{12}$C data shows little if any variation in this position. Qualitatively, both curves for the $\pi^+\ ^{40}$Ca data fit equally well. This is a direct indication that the anomalous effect is suppressed for positive pion data, as expected. The accentuation of this effect is clearly visible for the elastic scattering of negative pions from larger nuclei, as in the $\pi^-\ ^{40}$Ca data. Here, the normal curve generally fits better at angles $\leq 90^\circ$, especially in the region around the minimum ($\approx 65^\circ$). At back angles however, the anomalous curve fits quite well as the data is lower than theoretically predicted. Overall, the indication of the data shows little or no support for the existence of the anomaly at positive pion energy values. Qualitatively this is seen in table IX.

IV.7 Conclusion

From the figures shown, it is obvious that there is no quantitative agreement between the anomalous curves generated by GLBKISS and the experimental data. However, the normal optical potential parameters [Fri88b] give rise to curves which are a closer fit to experimental data. Statistically, the normal optical
Figure 15: Normal and anomalous predictions compared to $\pi^+ {^{12}}C$ data. The solid line represents the prediction using normal optical model parameters. The dashed line represents the anomalous case.
Figure 16: Normal and anomalous predictions compared to $\pi^-{^{12}}C$ data. The solid line represents the prediction using normal optical model parameters. The dashed line represents the anomalous case.
Figure 17: Normal and anomalous predictions compared to $\pi^+ {^{40}}\text{Ca}$ data. The solid line represents the prediction using normal optical model parameters. The dashed line represents the anomalous case.
Figure 18: Normal and anomalous predictions compared to $\pi^- \, ^{40}\text{Ca}$ data. The solid line represents the prediction using normal optical model parameters. The dashed line represents the anomalous case.
Table IX: The optical potential model fit summary for $^{12}$C and $^{40}$Ca are shown for both normal and anomalous curves.

Potential model predictions are favoured, as shown in table IX. However, the ambiguities observed do not clearly rule out the effect of the anomaly. Also, it was found that the parameter $\xi$ in the optical model is too insensitive to definitely discard the existence of the anomaly.

The difficulty lies in the gathering of high quality experimental data to test the anomaly. The QQD spectrometer was used for the first time at 20 MeV. At this energy, the resolution of the spectrometer decreases rapidly\textsuperscript{5}. A much shorter spectrometer to reduce the number of decaying pions is required. The problem of the target ladder alignment should have been avoided. Extreme care must be taken when setting up an experiment to obtain data of such low energy.

Using the knowledge and experience gained from this experiment, it is possible to perform an experiment to obtain better data at this low energy. More data points should be taken around the interference minimum region to clearly determine its location. Once this is done, a more rigorous comparison can be made to determine the existence of the anomaly, however, better data at this time may not be justifiable due to theoretical uncertainties.

\textsuperscript{5}The optimum resolution is obtained at energies of the order of 50 MeV.
Bibliography


56


Appendix A
Traceback Fitting

The scattering location in the target is determined by projecting the pion's path back to the target from position information given by WC1 and WC3 as discussed in section III.4.1 of Chapter II. During the analysis of the data it was found that the target chamber was out of alignment with respect to the beam. This was observed in the target traceback histograms. Since the target was in transmission mode (see equation 29), the traceback for 2 identical spectrometer angles on either side of the beam\(^1\) should be similar. However, this was not the case. Target traceback histograms from data taken at Q\(_{\text{QD}}\) angles to the right of the beam displayed the target edges, as those on the left did not. The loss in scattering data resulted in substantially lower cross sections.

This was compensated for by fitting a Gaussian to the beam profile (which remained constant throughout the experiment) as seen in the \(x\) direction by the traceback histograms. The traceback data in the \(y\) direction remained virtually unaffected. During the fitting procedure, the width (\(\sigma\)) of the Gaussian remained fixed while the area was varied. The ratio of the fitted area to the actual area was the factor used in modifying the cross section. The error of the fit was determined by varying \(\sigma\) and the area under the function in order to obtain maximum and minimum values for which the fit remained reasonable. This method yielded errors typically \(\sim 8\%\).

The assumption that the beam profile remained constant is based on the fact\(^1\) The physical size of the M13 experimental area prevents data to be taken at Q\(_{\text{QD}}\) angles \(\sim 70^\circ\) to the left as measured downstream from the M13 channel.
that the beamline characteristics such as magnet currents and slit positions and widths remained unchanged.