**RADIATIVE MUON CAPTURE ON CARBON, OXYGEN AND
CALCIUM**

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Abstract

The partial branching ratio for photons of greater than 57 MeV ($G_{>57}$) has been measured for radiative muon capture (RMC) on ^{12}C , ^{16}O and ^{40}Ca in order to determine the magnitude of the induced pseudoscalar coupling constant g_p of the weak hadronic current. A time projection chamber (TPC) was used as a large solid-angle, medium-resolution pair spectrometer to detect the photons from RMC. The resulting spectra were essentially background-free, allowing a determination of the partial branching ratios to about 10% precision.

For ^{40}Ca , the partial branching ratio obtained is in good agreement with previous measurements, and yields the value $g_p/g_a = 5.7 \pm 0.8$ when compared with a phenomenological calculation of the nuclear response function and the value $g_p/g_a = 4.6 \pm 1.8$ when compared with a microscopic calculation of the nuclear response function. Both of these results suggest a slight (downward) renormalization of g_p/g_a in ^{40}Ca in comparison with the value expected for a free nucleon.

For ^{16}O , the partial branching ratio obtained in the present experiment is found to disagree strongly with one previous experiment but to agree with the second previous measurement. When compared with a phenomenological calculation of the nuclear response our result yields $g_p/g_a = 7.3 \pm 0.9$, consistent with an unrenormalized value of g_p . The same results compared to a microscopic calculation give the value $g_p/g_a = 13.6^{+1.6}_{-1.9}$ suggesting a large (upward) renormalization of g_p in the ^{16}O nucleus.

The first precise branching ratio has also been measured for RMC on ^{12}C . No calculation of the nuclear response specific to ^{12}C is available at present. However, when the present data are compared with a naive extrapolation of Fermi-gas model

calculations of the nuclear response for heavier nuclei, the branching ratio is found to be consistent with an unrenormalized value of g_p .

The present work has also provided valuable input, through the investigation of various systematic errors and potential backgrounds, into the design of a detection system for a forthcoming measurement of RMC on hydrogen.

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Glossary

Term	Defined
q^2	(square of momentum transfer) 3
g_p	(induced pseudoscalar coupling constant) 4
g_a	(axial vector coupling constant) 4
CVC	(Conserved Vector Current Hypothesis) 5
PCAC	(Partially Conserved Axial Vector Current Hypothesis) 6
OMC	(Ordinary Muon Capture) 7
RMC	(Radiative Muon Capture) 13
CEC	(Conservation of Electromagnetic Current) 18
EPM	(Elementary Particle Method) 21
E_{av}	(average nuclear excitation energy) 23
MEC	(Meson Exchange Currents) 28
MIA	(Modified Impulse Approximation) 28
TPC	(Time Projection Chamber) 30
IWC	(Inner Wire Chamber) 38
RFS	(Radio Frequency beam Separator) 43
k_{max}	(maximum photon energy) 145
PP	(Primakoff Polynomial) 145

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Chapter 1

Introduction

1.1 Weak Interactions

The weak and electromagnetic interactions are now believed to be well described by the extremely successful Weinberg-Salam-Glashow $SU(2) \times U(1)$ electroweak theory [1,2,3]. This is a renormalizable gauge theory with spontaneously broken symmetry which unifies the electromagnetic and weak interactions in one mathematical framework. The success of the theory was highlighted by the discovery of the W and Z intermediate vector bosons [4], which, along with the photon, are the gauge bosons that mediate the interaction.

Despite the success of the $SU(2) \times U(1)$ theory, there are still unsolved questions about weak interactions at low energies, specifically those involving strongly-interacting particles (hadrons). The presence of the strong interaction modifies the pure electroweak interaction to form an ‘effective’ hadronic weak interaction which can have an extra richness of structure. While there is also a renormalizable gauge theory of strong interactions, quantum chromodynamics (QCD), the calculations required for the application of QCD to the non-perturbative (i.e. low-energy) regime are largely intractable at present. Therefore the structure and couplings of the effective hadronic weak interaction are not calculable *ab initio*.

For processes at low energy, i.e. with momentum transfers $q^2 \ll M_W^2$ where M_W is the mass of the W boson, weak interactions have been successfully described in terms of an effective theory of self-interacting vector currents [5]. In this effective theory, introduced by Fermi [6,7], the interaction between two fermion currents is

assumed to take place at a point. The framework of this current-current interaction (which is a low-energy limit of the electroweak theory) will be used throughout this thesis.

Within this framework, a general effective weak interaction Lagrangian has the form

$$\mathcal{L}_{eff}(x) = -\frac{G_F}{\sqrt{2}} J_\lambda^\dagger(x) J^\lambda(x) + h.c. \quad (1.1)$$

where G_F is the Fermi coupling constant, and J^λ is a vector current which can be decomposed into hadronic and leptonic parts

$$J_\lambda(x) = J_\lambda^h + J_\lambda^l. \quad (1.2)$$

The various weak interaction processes can then be classified in terms of this decomposition, i.e. pure leptonic (e.g. muon decay $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$), semileptonic (e.g. muon capture $\mu^- + p \rightarrow n + \nu_\mu$) and non-leptonic (e.g. $K^+ \rightarrow \pi^+ \pi^0$).

1.1.1 The leptonic current

The leptonic current J_λ^l can be written directly in terms of the lepton fields

$$J_\lambda^l = \bar{\psi}_{\nu_e} \gamma_\lambda (1 - \gamma_5) \psi_e + \bar{\psi}_{\nu_\mu} \gamma_\lambda (1 - \gamma_5) \psi_\mu + \bar{\psi}_{\nu_\tau} \gamma_\lambda (1 - \gamma_5) \psi_\tau. \quad (1.3)$$

where γ_λ are the Dirac γ -matrices and $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$. For the present work, the third term involving the τ -lepton can be ignored, as the processes considered are at energies well below the threshold to produce τ 's.

The leptonic current given above is not the most general Lorentz covariant form that could be constructed from the available fields, but is of the specific V-A (vector minus axial vector) form postulated by Feynman and Gell-Mann [8]. The V-A structure is a consequence of the fact that the weak interaction only couples to left-handed particles (right-handed antiparticles). The V-A nature of the leptonic current is well established experimentally via neutrino scattering and muon-decay measurements.

1.1.2 The hadronic current

The hadronic current J_λ^h can be decomposed into parts having definite quark flavour SU(3) transformation properties. For a process involving only first-generation quarks (u, d) such as muon capture

$$J_\lambda^h = \cos \theta_c (V_\lambda - A_\lambda) \quad (1.4)$$

where θ_c is the Cabibbo angle, an element of the Kobayashi-Maskawa matrix [9] which represents the weak flavour mixing of the quarks. The structure of the terms V_λ and A_λ is not specified by theory. However retaining the (V-A) structure, and requiring Lorentz covariance, one can derive a convenient parameterization for these terms. The available quantities from which to construct these terms are the γ -matrices, the hadronic fields and the four-momentum transfer q_λ . In the specific case of the muon capture process

$$\mu^- + p \rightarrow n + \nu_\mu \quad (1.5)$$

the momentum transfer is $q = k_p - k_n$ (k_p and k_n are the proton and neutron four-momenta). The most general vector and axial vector terms are then

$$V_\lambda = i\bar{\psi}_p \left[f_v(q^2)\gamma_\lambda + f_m(q^2)\sigma_{\lambda\nu}q_\nu + if_s(q^2)q_\lambda \right] \psi_n \quad (1.6)$$

$$A_\lambda = -\bar{\psi}_p \left[f_a(q^2)\gamma_\lambda\gamma_5 + if_p(q^2)\gamma_5q_\lambda + if_t(q^2)\sigma_{\lambda\nu}q_\nu\gamma_5 \right] \psi_n \quad (1.7)$$

where ψ_n and ψ_p are the nucleon fields. The f_a are form factors which are, in general, complex functions of q^2 (a Lorentz scalar). The terms f_v and f_a are the vector and axial vector form factors. The form factors f_m ('weak magnetic'), f_s ('scalar'), f_p ('pseudoscalar') and f_t ('tensor') are 'induced' by the strong interaction. The induced form factors all appear in terms proportional to the momentum transfer q_λ . In general, the form factors themselves are only weak functions of q^2 . In processes such as β -decay and muon capture, which probe a limited range of q^2 , the form

factors are relatively constant, and so a related set of coupling ‘constants’ is often defined as follows

$$g_v(q^2) = f_v(q^2) \quad (1.8)$$

$$g_m(q^2) = 2M f_m(q^2) \quad (1.9)$$

$$g_s(q^2) = m_l f_s(q^2) \quad (1.10)$$

$$g_a(q^2) = f_a(q^2) \quad (1.11)$$

$$g_p(q^2) = m_l f_p(q^2) \quad (1.12)$$

$$g_t(q^2) = 2M f_t(q^2) \quad (1.13)$$

where M is the nucleon mass, m_l is the lepton mass, and the normalizations (the terms multiplying the form factors) are chosen for convenience.

The structure of these induced terms can be constrained by several general theoretical considerations. The assumption of time-reversal invariance dictates that all the form factors are real quantities. A second constraint comes from G-parity. The strong interactions are known to conserve G-parity (this is equivalent to a statement that the strong interactions are invariant under charge conjugation and rotations in isospin space). Since the strong interactions conserve G-parity, it is reasonable to expect that any currents induced by the strong interactions should have definite G-parity. Therefore we can expect that the induced form factors should undergo the same behaviour under a G-parity transformation as the corresponding bare vector or axial vector terms. Form factors having this property have been classified by Weinberg [10] as ‘first-class’ currents and those that do not as ‘second class’. The form factors f_s and f_t have opposite G-parity to the other vector and axial vector terms respectively and are therefore second-class currents; thus the expectation is

$$f_s(q^2) = 0 \quad (1.14)$$

and

$$f_t(q^2) = 0. \quad (1.15)$$

The possibility of second-class currents has been discussed, but there is no clear experimental evidence to date for their existence.

The conserved vector current hypothesis (CVC) puts additional constraints on the induced weak form factors. CVC states that the divergence of the vector current is zero

$$\partial_\lambda V_\lambda = 0 \quad (1.16)$$

i.e. that the vector coupling is unrenormalized by the strong interactions. CVC is a consequence of the isotriplet vector current hypothesis, which assumes that V_λ , $V_\lambda^\dagger$ and the isovector part of the electromagnetic current form an isospin triplet (this is a direct consequence of the $SU(2) \times U(1)$ structure of the electroweak theory). This allows the form factors $f_v(q^2)$ and $f_m(q^2)$ to be expressed in terms of electromagnetic form factors that can be precisely measured in electron scattering experiments. In the limit $q^2 \rightarrow 0$ these reduce to

$$f_v(0) = 1 \quad (1.17)$$

$$f_m(0) = 2M(\mu_p - \mu_n) \quad (1.18)$$

where M is the nucleon mass and μ_p and μ_n are the anomalous magnetic moments of the proton and neutron respectively. Applying Equation (1.16) directly to the vector current (1.6) also yields

$$f_s(q) = 0. \quad (1.19)$$

CVC is strongly supported by experimental evidence. For example, the experimental pion β -decay branching ratio agrees well with the CVC prediction [11], and measurements of β -decay of mirror nuclei agree with the predicted ratio of f_m/f_v [12].

In summary, the considerations discussed so far have shown $f_v(q^2)$ and $f_m(q^2)$ are well known (and consistent with CVC), f_t is expected to be zero because of G invariance and f_s is expected to be zero because of both CVC and G invariance. These expectations are all consistent with available experimental evidence. The remaining two terms, $f_a(q^2)$ and $f_p(q^2)$ require more discussion.

Unlike the weak vector current, the axial vector current A_λ is not exactly conserved (if it were, the pion would not decay). The non-conservation of the axial vector current is a consequence of chiral symmetry breaking in the QCD Lagrangian (i.e. the fact that the current-algebra quark masses are non-zero). The Goldstone boson of the symmetry breaking is the pion; in the limit of a massless pion (soft-pion limit), the symmetry would be unbroken and A_λ would be conserved. The symmetry is broken, however, and non-conservation of the axial current can be related to the pion field, ϕ_π ,

$$\partial_\lambda A_\lambda = f_\pi m_\pi^2 \phi_\pi \quad (1.20)$$

where f_π is the pion decay constant. This is known as the partially conserved axial vector current hypothesis (PCAC) [13,14]. Applying PCAC to the expression for A_λ (with $f_t(q^2)$ taken as zero), we get

$$2M f_a(q^2) - q^2 f_p(q^2) = \frac{\sqrt{2} f_\pi m_\pi^2 G_{\pi NN}}{q^2 + m_\pi^2} \quad (1.21)$$

where $G_{\pi NN}$ is the strong coupling constant for the pion-nucleon vertex, m_π is the pion mass and M is the nucleon mass. In the limit $q^2 \rightarrow 0$, this reduces to the Goldberger-Treiman relation [15]

$$f_a(0) = \frac{f_\pi G_{\pi NN}}{\sqrt{2}M} = 1.32 \pm 0.02 \quad (1.22)$$

where the assumption has been made that

$$G_{\pi NN}(0) = G_{\pi NN}(q^2) = G_{\pi NN} \quad (1.23)$$

that is, that the off-mass shell behaviour of the pion-nucleon coupling varies slowly with q^2 . The Goldberger-Treiman estimate for f_a is slightly different from the value measured using neutron β -decay, $f_a = 1.262 \pm 0.005$ [16]. The (5%) discrepancy between these values is related to the size of the chiral-symmetry breaking [17]. The Goldberger-Treiman relation can be used to relate f_a and f_p by substituting back into equation (1.21) and disregarding the q^2 dependence of f_a , giving

$$f_p(q^2) = \frac{2Mf_a(0)}{q^2 + m_\pi^2} \quad (1.24)$$

or, in terms of the coupling constants

$$g_p(q^2) = \frac{2Mm_l g_a(0)}{q^2 + m_\pi^2}. \quad (1.25)$$

Clearly, $g_p(q^2)$ is *not* slowly varying with q^2 , but rather has a pole at the pion mass. A simple interpretation is that the induced pseudoscalar contribution (g_p) is dominated by single pion exchange (the lightest available pseudoscalar particle). Despite the fact that g_p is a strong function of q^2 , its value at a specific q^2 is often used as a reference point. For ordinary muon capture (OMC) on the proton

$$\mu^- + p \rightarrow n + \nu_\mu \quad (1.26)$$

the kinematics give

$$q^2 = 0.88m_\mu^2. \quad (1.27)$$

Substituting this into equation (1.25) yields the Goldberger-Treiman value for g_p

$$\frac{g_p}{g_a} = 6.78 \quad (1.28)$$

or

$$g_p = 8.56 \quad (1.29)$$

The testing of this prediction for g_p , and thereby the testing of PCAC, was a goal of the work described in this thesis.

1.2 Mesons in Nuclei

Before discussing the experimental techniques available to measure g_p and the present state of knowledge of its magnitude, some discussion is needed of possible modifications of g_p (and the other axial weak couplings) in the nucleus. While CVC limits vector weak couplings to be the same in the nucleus as for a free nucleon, this need not be true for the axial current. Indeed, since the induced pseudoscalar coupling is thought to be brought about primarily via pion-exchange, it is natural to assume that modifications to the pionic field inside the nucleus would affect g_p . In general, some ‘renormalization’ of the axial couplings is allowed, and possibly even expected in the nucleus.

Any modification of the basic weak couplings of a nucleon when embedded inside a nucleus would be a manifestation of non-nucleonic degrees of freedom inside the nucleus. QCD describes the nucleons as being made up of quarks, and the nucleon-nucleon force (at large range) is well described in terms of meson-exchange. Therefore it would seem likely that at some level the description of the nucleus in terms of only protons and neutrons should fail, and meson or quark degrees of freedom would then be required. Despite this expectation, the ‘classical’ nuclear physics picture of a nucleus composed solely of protons and neutrons has proved remarkably successful. The search for evidence of non-nucleonic degrees of freedom in the nucleus is a topic of considerable interest at this time.

Various pieces of evidence point toward modifications of the form factors of a nucleon embedded inside the nucleus; these include the EMC effect [18], the ‘missing’ Gamow-Teller strength seen in (p, n) reactions [19] and systematics of β -decay in mirror nuclei [20]. These last two have been interpreted as indicating a strong renormalization of g_a in the nucleus, from its free nucleon value of 1.26 down to about 1.0. Discussion here will be limited to interpretations of such effects

in terms of mesonic degrees of freedom only (although it is conceivable that QCD (quark, gluon) degrees of freedom will ultimately be required; in fact QCD sum rules seem to be able to reproduce the observed quenching of g_a [21]).

The axial current can be modified in the nucleus in several ways. An emitted virtual pion, propagating through the nuclear medium can be scattered by other nucleons. This reduces the effective range of the pions and modifies the pion propagator; the m_π^2 is replaced by an effective mass [22]

$$\tilde{m}_\pi^2 = \left[\frac{1 - \frac{1}{3}\alpha}{1 + \frac{2}{3}\alpha} \right] m_\pi^2 \quad (1.30)$$

where α is interpreted as a measure of the nucleon polarizability. A second modification due to the nuclear medium is a modification of the pion-nucleon coupling due to the induced polarization of the medium around the emitting nucleon. The effective coupling is

$$\tilde{g}_{\pi NN} = (1 + \xi\alpha)g_{\pi NN} \quad (1.31)$$

where the screening parameter ξ takes into account the rather uncertain effect of Pauli blocking and short-range nucleon-nucleon correlations. The combination of these two effects on the axial form factors is given by [22]

$$\tilde{f}_a = f_a(1 + \xi\alpha) \quad (1.32)$$

$$\tilde{f}_p = f_p(1 + \xi\alpha) \left[\frac{q^2 + m_\pi^2}{q^2 + \tilde{m}_\pi^2} \right]. \quad (1.33)$$

This has the consequence that the ratio between the renormalized and unrenormalized values of f_p/f_a is independent of ξ and depends on α only through $\tilde{m}_\pi^2$

$$\frac{\tilde{f}_p}{\tilde{f}_a} = \left(\frac{q^2 + m_\pi^2}{q^2 + \tilde{m}_\pi^2} \right) \frac{f_p}{f_a}. \quad (1.34)$$

Ericson [23] has calculated that for infinite nuclear matter

$$\alpha = -0.75 \quad (1.35)$$

which would lead to substantial renormalizations of the form factors; at the momentum transfer appropriate for OMC ($0.88m_\mu^2$) this predicts

$$\frac{\tilde{f}_p}{\tilde{f}_a} = 0.33 \frac{f_p}{f_a}. \quad (1.36)$$

In finite nuclei surface effects are expected to reduce this renormalization. Nevertheless, large effects might still be expected.

1.3 The Induced Pseudoscalar Coupling

The interesting question of the value of g_p/g_a (or equivalently f_p/f_a) for the free nucleon and the possibility of its renormalization in the nucleus has motivated a fair amount of experimental effort. This section will provide a brief overview of this effort. It should be noted that most measurements of g_p are also sensitive to any possible induced tensor coupling (g_t), and in general the two effects are not easily distinguishable. In what follows the implicit assumption will always be made that $g_t = 0$.

As discussed above, the induced couplings appear in the weak currents in terms proportional to the momentum transfer q (see equations (1.6) and (1.7)). Therefore in β -decay or electron capture where the momentum transfer is relatively small, the effect of the weak couplings becomes small and difficult to observe. Due to the small electron mass, the expected value of g_p/g_a is ~ 0.05 ; a typical experimental result is that of Bhalla and Rose [24] who found $g_p/g_a < 90$. The prospects for learning something significant about g_p from these processes are clearly small.

1.3.1 Ordinary Muon Capture

A more promising process is that of ordinary muon capture (OMC) where the momentum transfer is much larger. While many possible observables in OMC are sensitive to g_p (e.g. polarizations, angular correlations), the nature of the final state

particles (n, ν) makes such measurements difficult. Unfortunately in complex nuclei the simplest possible observable, the capture probability, is much more sensitive to details of the nuclear structure than it is to g_p/g_a . This sensitivity can be reduced somewhat if the (more difficult) measurement is made of the capture rate to specific states of the final nucleus. Consequently it is only for hydrogen that measurements of the total muon capture rate have provided reliable information about g_p/g_a . A second fundamental experimental problem is the low probability of muon capture for light nuclei. The probability of muon capture in nuclei varies roughly as Z_{eff}^4 (where Z_{eff} is an effective nuclear charge). In hydrogen, only 10^{-3} of the muons stopping in the target will undergo nuclear capture; the remainder will simply decay via

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu. \quad (1.37)$$

Despite the experimental difficulties, a considerable body of data exists on the OMC rate in hydrogen. The results of all published measurements are summarized in Table 1.1. While the errors are large, the results are seen to be consistent with a value of g_p very close to the Goldberger-Treiman estimate. It should be noted that there is an additional difficulty in the interpretation of the rate in terms of g_p/g_a in liquid hydrogen, due to muonic-molecular effects. The transition rate from the ortho- to para-molecular state of the $p - \mu - p$ molecule has the most important effect. Only one measurement has been made (Bardin *et al.* [25]), and the results in Table 1.1 have been corrected using this measured transition rate. This correction modifies the extracted g_p/g_a by as much as 90%. However, a detailed analysis by Bakalov *et al.* [26] suggests that there are consistency problems with the measured ortho-para transition rate and the data for OMC on hydrogen. The data given in Table 1.1 represent the only information available to date on g_p for the free nucleon. Clearly more precise measurements would be welcome.

Table 1.1: Summary of values of g_p and g_p/g_a as determined from measurements of OMC on hydrogen. The values presented are from the analysis of Bardin *et al.* [25]

Reference	Target	g_p	g_p/g_a
Bleser <i>et al.</i> 1962 [27]	liquid H ₂	6.0 ± 8.0	4.8 ± 6.3
Rothberg <i>et al.</i> 1963 [28]	liquid H ₂	11.0 ± 4.3	8.7 ± 3.4
Alberigi Quaranta <i>et al.</i> 1969 [29]	gas H ₂	10.3 ± 3.9	8.2 ± 3.1
Bystritskii <i>et al.</i> 1974 [30]	gas H ₂	7.9 ± 5.9	6.3 ± 4.7
Bardin <i>et al.</i> 1981 [31]	liquid H ₂	7.1 ± 3.0	5.6 ± 2.4
Average		8.7 ± 1.9	6.9 ± 1.5

Table 1.2 provides a summary of the results of OMC measurements in complex nuclei that have been used to extract values of g_p . Measurements for ^{12}C include the average polarization of the recoiling nucleus along the muon-spin direction (P_{av}), the longitudinal polarization of the recoiling nucleus (P_L) and the capture rate to the ^{12}B ground state. The extracted values of g_p/g_a are seen to be quite dependent on the theoretical treatment of the nuclear structure. Nevertheless, the results suggest that g_p/g_a in ^{12}C is somewhat larger than the Goldberger-Treiman value. In ^{16}O the muon capture rate from the ^{16}O ground state to the first excited state in ^{16}N has been measured, and compared to the β -decay rate from this state back to the ^{16}O ground state. The ratio of these two rates is less sensitive to nuclear structure uncertainties. The results indicate a value of $g_p/g_a \sim 11-12$ in ^{16}O , again larger than the Goldberger-Treiman value. Finally, in ^{28}Si a measurement has been made of the angular correlation between the neutrino and nuclear de-excitation gamma-rays from a given excited state. Very different values for g_p/g_a have been extracted using two different calculations of the relevant nuclear matrix elements.

In summary, the existing OMC data on g_p/g_a in the nucleus is based on only three nuclei using three different techniques. The results appear to indicate values of g_p/g_a somewhat above the Goldberger-Treiman value, but more work is needed on both experiment and theory before definitive conclusions can be made about any

Table 1.2: Summary of values of g_p/g_a as determined from recent measurements of OMC in complex nuclei. P_{av} refers to a measurement of the polarization of the nucleus along the muon-spin direction, P_L refers to the longitudinal polarization of the recoiling nucleus and Λ_μ and Λ_β refer to OMC and β -decay to specific nuclear states.

Reference	Nucleus	Method	Theory	g_p/g_a
Possoz <i>et al.</i> [32,33]	^{12}C	P_{av}	[34]	7.1 ± 2.7
"	"	"	[35,36]	13.6 ± 2.1
"	"	"	[37]	15 ± 4
"	"	"	[38]	$10.3^{+2.1}_{-2.3}$
Kuno <i>et al.</i> [39]	^{12}C	P_{av}	[38]	$10.1^{+2.4}_{-2.6}$
Miller <i>et al.</i> [40]	^{12}C	Λ_μ	[37]	8.5 ± 2.5
Roesch <i>et al.</i> [41,42]	^{12}C	P_{av}/P_L	[43]	9.4 ± 1.7
"	"	"	[44]	7.2 ± 1.7
"	"	"	[38]	9.1 ± 1.8
Guichon <i>et al.</i> [45]; Hamel <i>et al.</i> [46]	^{16}O	$\Lambda_\mu/\Lambda_\beta$	[47]	11-12
Miller <i>et al.</i> [48]	^{28}Si	$\nu - \gamma$	[35,36]	12.9 ± 3.9
"	"	"	[49]	-1.9 ± 3.1

possible renormalization effects.

1.3.2 Radiative Muon Capture

A preferred process with which to measure g_p is radiative muon capture (RMC). In a small fraction of muon capture events, there is a high-energy photon in the final state, i.e.

$$\mu^- + p \rightarrow n + \nu_\mu + \gamma \quad (1.38)$$

This is essentially an internal bremsstrahlung process. The advantage of RMC over OMC in determining the induced weak coupling is that the momentum transfer q^2 is no longer fixed at $0.88m_\mu^2$. In fact if the photon is radiated by the proton, q^2 approaches $-m_\mu^2$ as $k \rightarrow k_{max}$. This is close to the pion pole in the induced pseudoscalar term, causing an enhancement in the effect of the pseudoscalar contribution of about a factor of 3. Therefore RMC has an increased sensitivity to g_p over OMC. An additional advantage of RMC is the fact that the photon in the final state

is relatively easy to detect (compared to the neutrons and neutrinos from OMC). There is a disadvantage, however: RMC is a relatively rare process. The branching ratio of RMC/OMC is of the order of 10^{-5} for high-energy photons (as will be discussed, only about the upper 1/3 of the spectrum is experimentally accessible, due to bremsstrahlung from muon-decay electrons).

Various observables in the RMC process are expected to be sensitive to g_p , including the rate (or branching ratio), the shape of the photon spectrum (to a smaller extent), the asymmetry of the photons with respect to the muon-spin axis (α_γ) and the photon circular polarization. Due to the small branching ratio of RMC, all experimental efforts to date have been inclusive measurements, where the final states of the nucleus are left unresolved. This has the consequence that the interpretation of the results becomes somewhat dependent on appropriate nuclear structure calculations. Exclusive measurements of RMC (i.e. to a specific final nuclear state) would have a much reduced sensitivity to the nuclear structure, but would be correspondingly more challenging experimentally.

Clearly a measurement of RMC on hydrogen would be of much interest, as it would avoid the difficulties of nuclear structure in the interpretation and would provide a value for g_p for the free nucleon. No such measurement has been made to date, largely due to the extreme rarity of the process: the combination of the OMC probability in hydrogen ($\sim 10^{-3}$) and the RMC/OMC branching ratio ($\sim 10^{-5}$) leads to a process that occurs only once for every 10^8 muons. Despite this, proposals exist to measure RMC on hydrogen [50,51].

Because of the formidable difficulty of measuring RMC on hydrogen, experimental (and theoretical) effort to date has concentrated on complex nuclei where the muon capture probability is higher, e.g. ^{40}Ca where 85% of muons stopping in the target will capture. Effort has also been concentrated on double closed-shell nuclei (e.g. ^{40}Ca , ^{16}O) where the nuclear structure calculational difficulties should be min-

imized. In addition, in complex nuclei the possibility of observing renormalization of the weak couplings adds additional interest.

A review of previous measurements of RMC will be given in Chapter 5, where they will be compared to the results of the present measurement. Chapter 2 will provide a review of the theory directly relevant to RMC on the nuclei studied in this work (^{12}C , ^{16}O , ^{40}Ca). Chapters 3 and 4 will discuss the experimental technique adopted and the methods used to analyze the data. Details of the extensive Monte Carlo modelling that was done for the experiment are given in an appendix. In Chapter 6 the results of this work are summarized and discussed in terms of their contribution to this general field of study.

Chapter 2

Theory

2.1 Radiative Muon Capture

The calculation of the various observables for radiative muon capture (e.g. the rate, energy spectrum, angular correlation between the photon direction and the muon spin) is a non-trivial task, and there have been many attempts, at various levels of complexity. An exhaustive review of the theory of RMC can be found in the article by Gmitro and Truöl [52]. The discussion here will concentrate on recent calculations of particular relevance to the targets under investigation. We will also concentrate primarily on calculations of the rate and the photon energy spectrum of RMC, as these were the quantities measured in this thesis.

The problem of calculating RMC on complex nuclei can be usually divided into two stages. The first stage is the determination of the amplitude for the ‘elementary’ process, that is, the amplitude for

$$\mu^- + p \rightarrow n + \gamma + \nu_\mu \quad (2.1)$$

The second stage is that of ‘embedding’ the elementary amplitude within the nucleus, that is, creating an effective Hamiltonian from the elementary amplitude and evaluating its matrix elements over all possible final states of the nucleus. This second step, dealing with the nuclear response function, is the more problematic, and will be discussed in a subsequent section.

2.2 The Elementary Amplitude

There have been three broad classes of approaches to the calculation of the elementary amplitude for RMC, and after some initial confusion, it appears [52,53] that all three now yield consistent results for all the important terms. They differ only in higher-order terms, which typically affect the RMC rate only at the level of a few percent.

The first of these approaches is perhaps the most intuitively appealing. This perturbative, or ‘diagrammatic’ technique was the first to be attempted historically (see, for example [54,55,56]). One considers a series of Feynman diagrams such as those of Figure 2.1, where a photon is emitted from every possible branch of the basic diagram for ordinary muon capture. Diagrams c) and d) describe radiation through the interaction of the anomalous magnetic moments of the neutron and proton. The additional terms e) to g) are needed to maintain the gauge invariance of the amplitude. Diagram e) is equivalent to the assumption of minimal electromagnetic coupling (i.e. the substitution $q_\lambda \rightarrow q_\lambda - eA_\lambda$) in the q -dependence of the weak magnetism form factor; diagram f) is from the analogous assumption for the pseudoscalar form factor, and diagram g) is due to the analogous assumption for the virtual pion propagator. Diagram g) can be understood as representing radiation from the virtual pion. The obvious question arises: have the correct counterterms all been added to the Born terms (the first four diagrams)? A naive way of expressing the same problem is to note that this method is largely an expansion of the amplitude in powers of k , the photon momentum, as motivated by soft-photon theorems [57]. Since there is another momentum appearing in the amplitude which is of comparable magnitude, q (the momentum transfer), an expansion in terms of both k and q seems more appropriate, and is more likely to avoid questions about convergence. It should also be noted that gauge invariance alone does not uniquely

determine the counterterms needed.

To address the above questions, another approach to the calculation of the elementary amplitude was taken by Adler and Dothan [58]. They extended the soft-photon theorem of Low [57] for the electromagnetic current, to the weak axial current, and showed that the radiative amplitude could be expressed in terms of the non-radiative amplitude (i.e. that for OMC) and the divergence of the axial current. This was demonstrated to be exact for orders k^{-1} and k^0 , which enabled the amplitude to be determined up to, and including, terms linear in q and k , thus determining the leading order non-Born terms. Through the use of PCAC, the divergence of the axial current was related to physically measurable quantities such as the pion photoproduction amplitude.

A similar, but independent method is that of Christillin and Servadio [59], who also used soft-photon theorems to relate the higher-order terms to the divergence of the axial current. This technique differed from that of Adler and Dothan primarily in that here a simultaneous expansion was done in powers of q and k , rather than separate expansions for each variable. They obtained results in almost exact agreement with the results of Adler and Dothan; neither set of results was significantly different from those of the ‘diagrammatic’ approach (the differences have been discussed by Christillin *et al.* [60]).

An even more ambitious approach was that of Hwang and Primakoff [61,62] who attempted to reconstruct the RMC amplitude from current conservation laws: conservation of electromagnetic current (CEC), CVC and PCAC, as well as gauge invariance. They required an additional dynamical assumption, the ‘linearity hypothesis’ which constrained the functional form of the radiative weak form factors. The results of this calculation were quite different from the previously described techniques, predicting RMC rates a factor of two smaller, for the same values of g_p . This result was surprising, in that the other techniques themselves satisfy CEC,

Figure 2.1: Feynman diagrams contributing to RMC

CVC, and PCAC, and are gauge invariant. This discrepancy has been attributed [63,64,65] to problems with the linearity hypothesis; in particular the q^2 -dependence of the pseudoscalar form factor was ignored for the hadron-radiating diagram, which is just where the largest enhancement to the rate is expected from g_p . In addition, the pion-radiating term was not considered. Nevertheless, the basic idea of this method is appealing, as it provides a scheme for going to higher orders in q and k .

Gmitro and Ovchinnikova [65] have extended the Hwang and Primakoff approach, while avoiding the problems with the linearity hypothesis; the q^2 -dependence of the pseudoscalar form factor was included correctly, although the other weak form factors were assumed constant (this is reasonable since they vary by less than 5% for $q^2 \leq |m_\mu^2|$). The results were quite consistent with both the ‘diagrammatic’ method and the soft-photon technique. It should be noted that the use of additional dynamical assumptions (such as vector-meson dominance) could be incorporated naturally into this theory to determine higher-order terms in the RMC amplitude.

In summary, the calculation of the elementary amplitude appears to be under control, at least to the level of some small (few percent) higher order terms. A few more comments on the elementary amplitude are appropriate, however. Most calculations of physical observables based on one of the amplitudes mentioned above have been made using a non-relativistic approximation. This is usually done by performing a Foldy-Wouthuysen transformation [66] to produce an effective Hamiltonian. This is effectively an expansion in powers of $1/M$ where M is the nucleon mass. Fearing [63] has done a fully relativistic calculation for RMC on the proton in the ‘diagrammatic’ approach, and found that differences from the non-relativistic expansion are small ($\leq 10\%$); Sloboda and Fearing [67] have investigated the effect of terms of order $1/M^2$ on RMC in ^{40}Ca and found them to be small, except for those obtained from the squaring of $1/M$ terms in the Hamiltonian (which are already included in the non-relativistic approach).

The possibility of the effect of intermediate excitations of the $\Delta(1232)$ on RMC has been examined by Beder and Fearing [68] in a fully relativistic calculation. They find a 7-8% enhancement of the rate at the high-energy end of the photon spectrum for RMC on the proton; they comment that the effect for RMC on nuclei, however, might be quite different.

2.3 Elementary-Particle Approach to Nuclear RMC

Once a technique has been adopted to determine the amplitude for RMC on the proton, a calculation of RMC in a nucleus must deal with the structure of the nucleus in question. A natural temptation, when confronted with the problem of choosing the correct nuclear wave functions and calculating the matrix elements, is to avoid the problem altogether, and treat the nucleus as a structureless object (i.e. an ‘elementary particle’). In the Elementary Particle Model (EPM), [65,63,69,61] the initial and final states of the nucleus are treated as structureless objects, characterized by their spin, parity and isospin. The nuclear structure is absorbed into the definitions of the weak form factors, which are obtained by comparison with data from other weak and electromagnetic reactions on the nucleus in question (e.g. OMC, β -decay, pion photoproduction). Of course, one must still calculate transitions to each individual final state separately, and sum the results. The EPM has been applied to calculations of RMC on ^3He by several authors [69,63,61,70], and for specific exclusive transitions in ^{12}C by Hwang and Primakoff [71], and by Gmitro and Ovchinnikova [65].

The application of the EPM to RMC in nuclei has been examined in detail by Klieb and Rood [70,72], who have criticised it on several grounds. One difficulty is that it is necessary to make definite assumptions about the forms of the nuclear weak form factors, which unlike the ones for the nucleon, are all strong functions of q^2 . Because of these strong dependences on q^2 , derivatives of the form factors

and other higher-order non-Born terms become numerically much more important than in nucleon case. Unfortunately, because these nuclear form factors do not correspond to the exchange of any stable particles, there is very little guidance to the functional form of their q^2 -dependence. A second difficulty with applying the EPM to a two-vertex process such as RMC, is that intermediate excited states of the nucleus are ignored (the analogous case for the nucleon is the excitation of virtual $\Delta(1232)$ states, which are far away in energy, and therefore is expected to be small [68]). Due to these problems, and also due to the fact that the EPM has not been applied to inclusive RMC in any of the nuclei studied in this thesis, the EPM will not be considered further in this thesis.

2.4 Nuclear Response in RMC

Having resigned ourselves to dealing with the complexities of nuclear structure, i.e. to explicitly summing our matrix elements over all final nuclear states, a new question arises: can we use the elementary amplitude for RMC on the proton, and simply sum incoherently over all the nucleons? This approach, the impulse approach, implicitly ignores diagrams in which two nucleons participate (which can come about through meson-exchange). As well, the relation between the small and large components of the free nucleon spinors is assumed to hold true in the nucleus as well. The majority of the calculations of nuclear RMC have been made within the framework of the impulse approximation, with the exception of recent work by Gmitro *et al.* [73] and that of Achmedov *et al.* [74], which will be discussed later.

A few general features of the results of calculations of RMC on nuclei should be mentioned here. The first is that the *shape* of the photon energy spectrum is usually found to be quite insensitive to g_p ; instead it is the total rate of RMC (or equivalently, the rate above some lower photon energy limit) that is most sensitive to g_p . However, this absolute rate of RMC is also found to be quite sensitive to the

details of the nuclear structure model used in the calculation. This sensitivity can usually be considerably reduced if, instead, the branching ratio of radiative capture relative to the ordinary capture process is considered (i.e. RMC/OMC). This is not unexpected, as the same nuclear wave functions are used in the calculation of both OMC and RMC, and uncertainties should largely cancel. The idea is to calculate OMC using the same approximations or assumptions that are used in the calculation of RMC, and then to normalize the results to the well-determined experimental OMC rate. However, if unrealistic nuclear excitation spectra are used, the ratio of RMC/OMC can still show considerable sensitivity to the details of the model (e.g. [75]).

In the interests of brevity, this discussion will be limited to calculations of inclusive RMC on the nuclei considered in this thesis (^{40}Ca , ^{16}O , ^{12}C). In fact, however, it is just these nuclei on which most of the theoretical effort has concentrated.

The earliest method used to calculate the nuclear matrix elements for RMC was the closure approximation. In the closure approximation, the assumption is made that one can assign an average excitation energy (E_{av}) to the final nucleus. The transition strength is then evaluated at that excitation energy. Usually E_{av} is considered as a free parameter, and so any comparison to experimental data involves a simultaneous fit to both g_p and E_{av} . This poses a difficulty for data taken with limited energy resolution (as is the case for most RMC data). The extracted value of g_p is extremely sensitive to the value of E_{av} . Early examples of closure model calculations are those of Rood and Tolhoek [56] who used harmonic oscillator shell-model wave functions in a calculation of RMC on ^{40}Ca . Rood, Yano and Yano [76] extended the same calculation by considering modifications to the free muon propagator in the nucleus. Both of these calculations predicted values of RMC/OMC too large (by up to a factor of two) when the canonical values of the weak couplings were used.

Fearing [77] improved on the closure model for ^{40}Ca by using measured photoabsorption strength functions for evaluation of the electric dipole matrix elements. This is sensible since it is known [52] that for typical medium-energy probes (radiative pion capture, OMC, inelastic electron scattering etc.), the final nucleus is left in collective excited states (predominantly dipole) such as those seen in photonuclear experiments, e.g. the giant dipole resonance (GDR). The other multipoles were calculated within the closure approximation using harmonic oscillator wave functions. The principal result was a lowering of the RMC rate in the high-energy end of the photon spectrum (where the excitations are mainly electric dipole), to bring the predictions closer in line with experiment. The calculation was further improved by Sloboda and Fearing [67] using more recent photoabsorption data and Hartree-Fock wave functions; they also considered $1/M^2$ order terms in the non-relativistic reduction of the amplitude. The same authors have also investigated ways of reducing the dependence of the RMC rate on E_{av} through the use of sum rules [78].

The closure model was also used by Borchini and De Gennaro [79] in a calculation of RMC in ^{40}Ca . They applied Migdal's quasiparticle method [80] to account for the residual nucleon-nucleon interaction. A large dependence of the extracted rate for RMC on the value chosen for E_{av} was exhibited in this calculation as well.

The closure approximation has been criticised by Christillin [81]. The essential argument is that, in assuming that all the transition strength is concentrated at a single energy, one is implicitly including states of the nuclear excitation spectrum which are energetically forbidden to contribute to the high-energy part of the spectrum. Therefore the closure approximation is at its worst in the experimentally accessible portion of the spectrum.

One approach used to avoid the closure model, is that adopted by Christillin *et al.* [60], where the nuclear response function was calculated in the Fermi gas model, and applied to medium-heavy nuclei ($Z \geq 42$). A free parameter used in

this model is an effective nucleon mass M^* , which serves to account for nucleon-nucleon correlations. The value of M^* is fixed by fitting to experimental results for OMC; the model was able to reproduce measured OMC rates to better than 10% for most nuclei considered. This was one of the few calculations to consider systematics of RMC across the table of nuclides. Despite the serious reservations one should have about applying the Fermi gas model to light nuclei (especially closed shell nuclei), this is one model to which we will compare our data, in general terms at least.

A second attempt was made by Christillin [81,75] to calculate RMC without using closure; a phenomenological nuclear excitation spectrum was used for ^{40}Ca . This was similar to the giant dipole resonance model of Fearing, except that rather than using photoabsorption data, simple Lorentzian lineshapes were used, for both the GDR and the giant quadrupole components, and closure was not imposed. The relative strengths of the dipole and quadrupole terms were adjusted empirically to reproduce the observed OMC rate in ^{40}Ca . This model predicted values for RMC/OMC as much as $\sim 30\%$ smaller than typical closure model results for the same value of g_p (bringing the theory closer to existing experimental data, if the Goldberger-Treiman value for g_p is assumed). This clearly demonstrates the fact that the sensitivity to the nuclear structure model is not entirely eliminated by considering the ratio of RMC to OMC. We will compare our results for ^{40}Ca to this model, among others. The predicted dependence on g_p/g_a of the partial branching ratio for photons above 57 MeV ($G_{>57}$) is shown in Figure 2.2.

A similar phenomenological approach has been utilized by Christillin and Gmitro [82] for RMC on ^{16}O . Again, the GDR and GQR were considered as simple Lorentzian strength distributions, and detailed account was taken of low-lying individual states in the final ^{16}N nucleus. In this case, much less sensitivity to the

Figure 2.2: The partial branching ratio $G_{>57}$ as a function of g_p/g_a for ^{40}Ca , from the calculations of Christillin ('Chr') [81] and Gmitro *et al.* [73] ('GOT')

details of the nuclear structure model was found for RMC/OMC than was the case for ^{40}Ca . The predicted dependence of the partial branching ratio on g_p/g_a is shown in Figure 2.3.

Of course the 'ideal' calculation would be a truly 'microscopic' one, where the summation is made over all the individual partial transitions between initial and final states, evaluated at the correct excitation energies, and where realistic nuclear wave functions are used. However, such a method poses large calculational difficulties, especially for reasonably heavy nuclei (such as ^{40}Ca). The closest thing to a complete microscopic calculation of RMC has been the work of Gmitro *et al.* [83]. For ^{16}O n-particle, n-hole (where $n=0,1,2$) shell-model wave functions were used for the initial state; the most important 1^- $T=1$ and 2^- $T=1$ levels were included for the ^{16}N final state. The wave functions used were tested by comparison between predictions for electron scattering and radiative pion capture

Figure 2.3: The partial branching ratio $G_{>57}$ as a function of g_p/g_a for ^{16}O , from the calculations of Christillin and Gmitro [82] ('CG') and Gmitro *et al.* [73] ('GOT')

on ^{16}O and existing data. In addition, the model was able to reproduce the measured OMC rate. The results show that the inclusion of 2-particle, 2-hole correlations are extremely important for both RMC and OMC. The authors also calculated RMC for ^{40}Ca ; however, due to computational difficulties, only simple 1-particle, 1-hole wave functions were used (implicitly ignoring quadrupole excitations). Both RMC and OMC rates were thereby likely overestimated.

The results of these microscopic calculations were somewhat distressing; for both ^{16}O and ^{40}Ca the RMC/OMC predictions were approximately a factor of two larger than the earlier phenomenological calculations [82,81] for the same value of g_p . In addition, the microscopic results indicated a much greater dependence of RMC on g_p than obtained in the phenomenological approach. This was especially disturbing for the ^{16}O case, where the microscopic calculation would seem to be quite complete.

One possible explanation for the difficulty with the microscopic calculations is in the neglect of meson-exchange corrections (MEC), implicit in the impulse approximation. An early discussion of meson-exchange effects in RMC was given by Ohta [84], who estimated that a reduction in the RMC yield as large as 40% could be expected from MEC. This result, however, has been severely criticised for inconsistencies in the model used [52]; for example, the kinematics relevant to OMC rather than RMC were assumed. A more consistent treatment of MEC to the weak interaction vertex was provided by Achmedov *et al.* [85], who found a 10-12% reduction in RMC/OMC for ^{40}Ca ; however, these calculations were only performed for one value of g_p , so these results cannot be used to extract g_p from the present ^{40}Ca data.

A second possibility is that the problem lies in meson-exchange corrections connected, not with the weak vertex, but rather with the electromagnetic vertex. This has been investigated by Gmitro *et al.* [73]. They found that MEC effects at the electromagnetic vertex could (largely) be accounted for through constraints determined by imposing continuity of the electromagnetic current. This was done by an extension of the Siegert theorem [86] which relates transverse electric current multipoles directly to the time derivatives of electric charge multipoles (in the soft-photon limit). Gmitro *et al.* found that these MEC corrections were rather large for RMC; explicit calculations of RMC/OMC for both ^{16}O and ^{40}Ca were performed in this ‘modified impulse approximation’ (MIA). The results were much smaller than their earlier microscopic calculations, and were more consistent with the predictions of the phenomenological models for the case of ^{40}Ca . Again, only simple 1-particle, 1-hole wave functions were used for ^{40}Ca ; an empirical adjustment was made to compensate for the missing quadrupole component. The results for RMC/OMC, however, were quite sensitive to this quadrupole component, and values of g_p extracted by comparison to recent data [87] were found to more than

double if a larger, but still reasonable, quadrupole contribution was assumed. The ^{16}O calculation does not suffer from this drawback. The predicted dependence of the partial branching ratio on g_p/g_a for ^{40}Ca and for ^{16}O are shown in Figures 2.2 and Figure 2.3 respectively.

Finally, a very different approach has been investigated recently by Fearing and Walker [88]. They performed a calculation of RMC using relativistic mean field theory to investigate the effects of the renormalization of the nucleon mass due to the nuclear medium. The nuclear medium was considered as a relativistic Fermi gas. The most important effect was found to be Fermi motion, which reduces the RMC/OMC branching ratio by factors of up to three. Therefore such effects are clearly important for extracting g_p/g_a from a measurement of RMC. Fearing and Walker note that Fermi motion effects will already have been incorporated to some degree in typical non-relativistic calculations through terms of order p/M .

In summary, while there has been considerable theoretical effort on the calculation on RMC, there are relatively few calculations available for comparison with the present data. We choose not to consider any of the closure model calculations. This is due both to the theoretical objections discussed above, and to the sensitivity of the predictions to E_{av} , which, because of the moderate energy resolution of the present experiment, is not readily extractable from the data. This leaves only a few calculations useful for comparison: the Fermi gas model of Christillin *et al.* [60], which unfortunately was used only for medium-heavy nuclei, and so affords no direct predictions for the nuclei considered in this thesis; the phenomenological models of Christillin [81] for ^{40}Ca and of Christillin and Gmitro [82] for ^{16}O ; and the MIA ‘microscopic’ calculation of Gmitro *et al.* for ^{40}Ca and ^{16}O . At present there exists, to our knowledge, no calculation for inclusive RMC on ^{12}C .

Chapter 3

Description of the Experiment

The measurement of RMC requires a detection system that is capable of detecting photons in the energy range 50 MeV to 100 MeV with good efficiency and at least moderate resolution; it should be insensitive to the large backgrounds of neutrons from muon capture and electrons from muon decay. Also desirable is the ability to reject both photons from radiative pion capture (due to pions in the muon beam) as well as cosmic-ray induced events, and tracking capability to ensure that the photons actually originate in the target. The detection system chosen for this experiment fulfills these requirements and is described in this chapter. It was based on the TRIUMF time projection chamber (TPC), which was used to detect and track the electron and positron pair produced by the conversion of RMC photons in a Pb-scintillator converter package surrounding the target. Figure 3.1 shows a cut-away view of the detection system. Scintillation counters were used to count the number of beam particles stopping in the target, and along with multiwire proportional chambers and other scintillation counters provided the photon trigger. The TPC and the trigger counters were located in a region of uniform magnetic field, so that the energy of the e^+e^- pair, and hence the photon energy, could be determined from the curvature of the tracks. The magnet was surrounded by an external set of scintillators and drift chambers to identify cosmic-ray induced events.

Figure 3.1: Perspective view of the TRIUMF TPC. The numbered elements are (1) magnet iron, (2) magnet coil, (3) exterior trigger scintillators a) W counters, b) E counters, (4) EWC (exterior wire chambers), (5) TPC endcap support frame, (6) central electric field cage wires, (7) central high voltage plane, (8) outer electric field cage wires, (9) converter package, (10) IWC (interior wire chamber) (11) TPC endcap proportional wire modules.

Table 3.1: Characteristics of the TRIUMF TPC.

Drift Volume	1.15 m^3
Drift Field	25 kV/m
Electron Drift Velocity	$7.0 \text{ cm}/\mu\text{s}$
Maximum Drift Length	0.343 m
Chamber Gas	80% Ar 20% CH ₄
# of Sectors	12
# of Anode Wires per Sector	12
# of Segmented Cathode Pads per Sector	636
Anode Wire Diameter	$20 \mu\text{m}$
Anode Wire Voltage	1750 V
Wire Gain	$\sim 5 \times 10^4$
Minimum $r\phi$ resolution	$\sim 200 \mu\text{m } \sigma$
z resolution	$1.5 \text{ mm } \sigma$

3.1 The Time Projection Chamber

The principal component of the detection system used in the present experiment was the TRIUMF time projection chamber (TPC). The TRIUMF TPC has been described in detail elsewhere [89,90,91]. Table 3.1 details some of the relevant characteristics. The TPC is a large volume, hexagonal drift chamber, located in parallel and uniform (axial) electric and magnetic fields. The ionization electrons created by charged particles that traverse the detector volume drift under the influence of the electric field towards a set of proportional wire chambers at the end caps of the detector. The drift field of $E = 250 \text{ V/cm}$ was provided by a high voltage grid located at the midplane of the TPC, and by two concentric columns of field shaping wires around the drift volume and one column of wires along the central tube.

Each of the 6 sectors of each of the end caps of the TPC had 12 anode wires (144 in all); located under each anode wire was a plane of segmented pads centered every 6 mm (636 pads in each sector; 7632 in all). As the drifting ionization electrons reached the end caps a segment of the track would pass through a slot into these

proportional wire chambers. Coordinates of the track in the x - y (transverse) plane were determined from the anode wire position and from the distribution of induced charge on the cathode pads. The final (z) coordinate of the track was obtained from the time of the electron's drift (hence the term 'time projection chamber'). Thus up to 12 sets of (x,y,z) position coordinates were determined for each charged particle track in the TPC, allowing a reconstruction of the helical trajectory of the tracks. The momentum of the track was determined from the radius of curvature, and the origin of the track in (x,y,z) was also found. In addition, the cathode pad pulse heights provided up to 12 independent samples of the particle's energy loss (dE/dx) in the chamber gas, which enabled electrons and positrons to be distinguished from heavier particles.

The cost of fully instrumenting the 7632 cathode pads would be prohibitive, and so the pads were multiplexed; each individual pad on a given sector was connected in parallel with the corresponding pad on every other sector. Since the original TPC experiment (TRIUMF Expt. # 104, a search for muon-electron conversion [92]) involved the detection of single tracks, the possibility of confusion in the track reconstruction due to the multiplexing was small. Possible ambiguities were reduced by the fact that the anode signals were not multiplexed, and so the assignment of the cathode information to a given sector could be made on the basis of which anode wires had fired. However, in the present experiment, a good event involved at least two tracks in the TPC (the e^+e^- pair), and thus ambiguities became somewhat more troublesome, requiring some extra effort in the data analysis.

The cathode pad signals were amplified externally to the TPC by LeCroy TRA510 amplifiers; the anode signals were amplified by on-board hybrid preamplifiers.

The chamber gas mixture used was 80% argon, 20% methane at atmospheric

pressure. This provided a drift velocity of $V_d = 7.0 \text{ cm} \cdot \mu\text{s}^{-1}$ with the drift field used (250 V/cm) so the total drift time was therefore up to 5 μs . One of the advantages with the TPC type of drift chamber is that the ionization electrons tend to spiral along the parallel electric and magnetic field lines, which has the effect of strongly reducing their transverse diffusion (longitudinal diffusion is unaffected).

At high rates, the positive ions generated by the avalanche at the anode wires, if allowed to stream back into the drift volume, would seriously distort the electric fields in the drift region. To minimize this, a set of grid wires was installed at the entrance to each anode wire region [91]. These wires were normally biased at a potential that served to prevent both the ionization electrons from entering the proportional region and also the positive ions from leaving the proportional region into the drift volume of the TPC. The grid potential was then pulsed to the ‘open’ condition, which allowed the passage of ionization electrons only when a valid trigger was determined by the trigger electronics (described below). This substantially reduced the rate of firing of the anode wires, and therefore also reduced ageing effects in the chamber.

The various effects that determine the spatial resolution of the TPC have been studied previously and are well understood [90] ; the resolution was found to depend on the track crossing angle, the diffusion of the drift electrons, the $\mathbf{E} \times \mathbf{B}$ effects near the anode wire and the discrete nature of the ionization. The best observed resolution was $\sigma = 200 \text{ } \mu\text{m}$ in the (x, y) plane; the z -resolution was about $\sigma = 2 \text{ mm}$. As will be discussed later, the photon energy resolution was not limited by the spatial resolution of the TPC, but rather by other effects. More information on the principles of operation of the TPC, and details about other TPC’s can be found elsewhere [93].

The magnetic field was provided by a large volume magnet which was obtained from the University of Chicago. The useable volume was $1.5 \text{ m} \times 1.2 \text{ m} \times 0.9 \text{ m}$;

the TPC, trigger counters and beam counters were mounted within this volume. The main coil of 22 water-cooled aluminum pancakes of 16 turns each, along with two end coils, each of 4 pancakes, provided an axial field of 0.25 T with a measured uniformity of $\delta B/B < 0.3\%$. The magnet was capable of providing a field of up to 1.0 T, and various field strengths were used during early tests. A 20 cm diameter hole in each pole face allowed access for the beam, beam counters, targets, converter scintillator package, various light guides and cables.

3.2 Beam and Trigger Counters

The scintillators used to count the incident beam particles and to define a stop in the target are shown in Figure 3.2. They consisted of four beam counters B1-B4, each 104 mm (horiz.) by 158 mm (vert.). The first three scintillators were 6.3 mm thick. The last scintillator (B4) was 0.8 mm thick, to minimize the counting of false stops due to those low-energy particles which stopped in the last counter. Each beam counter was viewed by two photomultiplier tubes to maximize its light collection efficiency. These counters acted as live degraders to ensure that the incident muon beam stopped in the target, and also served to aid in the rejection of background events due to pions in the beam (the majority of pions were stopped in these counters, due to their shorter range). In addition, the counters enabled the identification of events in prompt coincidence with an incoming beam particle; this allowed further rejection of pion-induced background events. Also included upstream of the beam counters was a Pb collimator which was lined with scintillators (BV1-BV4) to minimize and reject the scattered beam. The entire beam counter assembly was recessed into the magnet so that final beam counter was 35.0 cm upstream of the center of the TPC, where the target was placed. Timing and pulse height information were recorded for each of the individual beam counters.

Figure 3.2: Schematic cross section of the detector and the beam and trigger counters.

Table 3.2: Dimensions of the Trigger counters.

Counter	length (cm)	width (cm)	distance from center (cm)	Thickness (mm)
IA	49.5	2.8	15.9	3.0
IB	30.5	3.0	17.1	3.0
IC	27.9	3.3	18.4	3.0
W	80.0	61.5	61.5	12.7
E	80.0	66.7	67.0	6.4

A disk-shaped veto scintillator (V) was located 14.5 cm downstream of the target to reject any particles which passed through the target. This counter was connected to a single photomultiplier tube via an air and plastic light guide extending out the back of the magnet.

Surrounding the target was a package with three layers of scintillators. Each layer consisted of 18 individual counters 3.0 mm thick, together forming 18-sided polygons in cross section. Each counter was a trapezoid in cross-section, the edges cut at an 80° angle to minimize gaps between the counters. The package was oriented so that every three adjacent counters corresponded to one of the sectors of the hexagonal TPC. The inner two layers (IA1-IA18, IB1-IB18) served a dual purpose, both to complete the definition of an incident muon stop in the target, and to veto charged particles coming from the target at the time of the event. All the counters were connected via light guides to photomultiplier tubes outside the downstream end of the magnet. The physical dimensions of these counters are summarized in Table 3.2. The IA layer was 15.9 cm in inner radius and 49.5 cm in length and each counter was read out by a Hamamatsu R1450 photomultiplier tube. The IB layer was 17.1 cm in inner radius and 30.5 cm in length; groups of three adjacent counters were connected to a single Hamamatsu R1398 photomultiplier tube (6 in total). A cylindrical sheet of Pb of 0.6 mm in thickness (later replaced by a 1.0 mm thick sheet) and 25.1 cm in length was placed between the second (IB)

and third (IC) layers of counters; this served to convert photons from the target into electron-positron pairs for detection in the TPC. The final layer of counters (IC1-IC18) with an inner radius of 18.4 cm and a length of 27.9 cm was used to signal the presence of the charged leptons from converted photons. Each of these counters was read out by a Hamamatsu R1450 photomultiplier tube. The segmentation of these counters allowed the rejection of backgrounds through the ability to look for a valid spatial trigger pattern, and also kept the singles rate in any given counter low.

The converter package was surrounded by a cylindrical wire chamber (IWC) of inner radius 99 mm, outer radius 120 mm and length 470 mm. The chamber had 192 anode wires strung axially at a spacing of 3.58 mm, and two layers of 128 helical cathode ribbons, inside and outside the anodes. The cathodes were of opposite pitch, each completing a rotation of 90° along the length of the chamber; the combination of the anode and cathode information thus gave both the axial and azimuthal coordinates of a charged particle passing through the IWC.

Just outside the TPC were the outer trigger counters, which were made up of 6 sets of planar counters, each covering a sector of the TPC. Each set had 3 counters: a wire chamber (EWC1-EWC6) and two scintillators, W1-W6 and E1-E6. Closest to the TPC were the counters W1-W6 which were 80 cm in the axial direction, 61.5 cm wide and 1.3 cm thick. Each W-counter was read out by a BBQ wave shifter bar at the downstream end of the scintillator; these were connected to light guides that exited the magnet between the main and the end coils of the magnet. The EWC wire chambers had 64 anode wires in the azimuthal direction and 16 cathode wires in the axial direction. Due to problems with the efficiency of these chambers, they were not used in the present experiment, and will not be discussed further. The final layer of scintillators E1-E6, were approximately 80 cm long, 67 cm wide and 0.64 cm thick. Each of these were connected to two adiabatic light guides which

Table 3.3: Dimensions of the Targets. In each case, the first dimension listed is the thickness in the beam direction.

Target	Dimensions
^{40}Ca	$1.9\text{cm} \times 10\text{cm} \times 10\text{cm}$
D_2O	$\sim 3.0\text{cm} \times 15.2\text{cm}\phi$
^{12}C	$1.9\text{cm} \times 15.2\text{cm}\phi$
polyethylene (CH_2)	$2.0\text{cm} \times 12.5\text{cm}\phi$

exited the magnet between the main and end coils, attached to two photomultiplier tubes. Two of the scintillators (E2 and E5) were actually split in two to correspond to the two light guides, but the signals from the tubes were summed electronically, and the segmentation was ignored in the subsequent data analysis.

Surrounding the magnet was a series of counters to veto cosmic-ray induced events (see Figure 3.3). These consisted of 12 large area drift chambers of various sizes (the CRDC's) and 6 groups of large scintillators. The scintillator groups were made of 23 different sheets of scintillator read out by 18 phototubes, coupled to the scintillators via BBQ wave shifter bars. The top and all four sides of the magnet were covered by a double layer of drift chambers and a single layer of scintillator.

3.3 Targets

Four different targets were used during the course of the experiment: calcium, oxygen, carbon and polyethylene (CH_2). The physical dimensions of these targets are summarized in Table 3.3. The calcium target consisted of a $10\text{ cm} \times 10\text{ cm} \times 1.9\text{ cm}$ block of natural calcium (96.97% ^{40}Ca), which had a thin layer of aluminum evaporated on all surfaces for ease of handling. Between runs the target was stored in a desiccator filled with nitrogen to reduce surface oxidation.

The oxygen target was approximately 500 ml of 99.96% pure D_2O , held in a polyethylene bag with walls 0.15 mm in thickness. The bag was roughly cylindrical

Figure 3.3: Schematic of cosmic-ray veto counters.

with a radius of 7.6 cm and a length of about 3 cm. Water was used to provide sufficient muon stopping power, and D₂O was used rather than H₂O to reduce the background due to photons from the decay of π^0 's produced by any π^- 's in the beam via the reaction $\pi^-p \rightarrow \pi^0n$. The branching ratio for pion charge exchange on the deuteron is more than three orders of magnitude smaller than that on the proton [94]. The fraction of muons that capture on the deuteron in the D₂O rather than on the oxygen is completely negligible. A small fraction of the data was taken with an identical target of H₂O.

The carbon target was a solid cylinder of graphite of radius 7.6 cm and length 1.9 cm. The CH₂ target, required for the calibration runs using radiative pion capture, was a solid cylinder of polyethylene of radius 6.25 cm and length 2.0 cm.

The target thicknesses were chosen to ensure that the muon beam was stopped near the center of each target. Each target, when used, was placed in the geometrical center of the TPC.

3.4 Muon and Pion beams

The M9 stopped- π/μ channel at TRIUMF [95] provided the muon and pion beams used in this experiment. The TPC and the rest of the detection system was mounted at the end of a 10 m long extension to M9 (see Figure 3.4). The beams are produced when the primary proton beam from the TRIUMF cyclotron impinges on the meson production target 1AT2, typically a strip of beryllium 10 cm long in the beam direction. M9 views the 1AT2 target at 135° with respect to the incident beam, with a solid angle acceptance of 25 msr. The pions thereby produced, along with their decay products (muons and electrons) are transported to the detection system via a series of dipole and quadrupole magnets. The primary proton beam of 500 MeV kinetic energy had a microstructure of 5 ns-long pulses every 43.3 ns and a macroscopic duty factor of 100%. The current was typically between 100

and 140 μA . More information on the TRIUMF cyclotron and the other primary and secondary beamlines can be found in the TRIUMF users handbook [96] and references therein.

The muon beam used in this investigation was produced by pion decays near the production target; these are known as cloud muons. However, the beam is not entirely composed of muons; not all the pions decay by the time the beam reaches the detector; and there are also electrons, primarily due to conversion of photons produced by π^0 decay in the production target. For the 73 MeV/c negative muon beam, after 19 m of drift (i.e. at the TPC) the raw beam composition is $\pi/\mu = 1$ and $e/\mu = 10$. These contaminations would cause difficulties for two reasons: the large electron rate would be too large for the beam counters to handle and it would also cause a high rate of low-energy particles in the detector; secondly (and more seriously), the pions would cause a huge background due to radiative pion capture in the target. As will be discussed later, the branching ratio for radiative pion capture is typically 3 orders of magnitude larger than that for radiative muon capture, and so it is essential in a measurement of RMC to reduce the pion contamination of the beam.

To suppress both the electron and pion content in the muon beam, a radio frequency particle separator (RFS) [97] was used in the beam line. This made use of the differences in the time-of-flight of the three components of the beam. Figure 3.5 shows the time structure of the beam at the position of the RFS for a 73 MeV/c beam tune. The pions and electrons arrive at about the same time relative to the cyclotron RF structure (the electrons are from the subsequent beam-burst) and the muons arrive approximately 1/2 an RF period earlier. The RFS consists of an RF electric field (20 kV/cm), transverse to the beam direction, driven at the 23 MHz cyclotron frequency. The field is produced by 1.0 m long plates with a 15 cm gap, with power supplied by a 120 kW RF amplifier. The phase of the RF voltage

Figure 3.4: The M9 stopped π/μ channel.

Figure 3.5: Time structure of the beam at the RF separator.

relative to the cyclotron RF structure is then tuned so that the $q\mathbf{v} \times \mathbf{B}$ deflection of the muons in a .01 T horizontal DC magnetic field is just cancelled by the force due to the electric field; the muons thus pass through the RFS undeflected. By the time the pions and electrons arrive, the sign of the electric field has reversed, and the magnetic and electric fields reinforce each other to deflect these particles out of the beam. In practice, it is not possible to have a complete 180° phase separation between both the pions and muons and the electrons and the muons. The phase of the RFS was typically tuned to minimize the pion flux relative to the muon flux. After the separator the beam composition was measured to be $\pi/\mu = 10^{-3}$ and $e/\mu = 10^{-2}$.

The M9 channel is capable of providing $1.3 \times 10^6 \mu^- \text{ s}^{-1}$ at the TPC (for 140 μA of protons on 1AT2) [92]. However, the beam has a relatively large spot size ($6 \times 10 \text{ cm}^2$). For this experiment, a new tune for M9 was developed to minimize the spot size and also the divergence of the beam, at the expense of muon flux.

This tune was undertaken to minimize both the false trigger rate due to muons that scatter into the IC counters and subsequently decay, and also to reduce the background from radiative muon capture of these scattered muons in the trigger counters or the converter. This new tune provided a muon flux of $5.0 \times 10^5 \mu^- \text{ s}^{-1}$ with $\delta P/P = 10\%$ incident on the beam counters with a spot size of $5 \times 5 \text{ cm}^2$ and a reduced divergence. This yielded typical stopping rates of $4.0 \times 10^5 \mu^- \text{ s}^{-1}$ in the C and D₂O targets and $3.4 \times 10^5 \mu^- \text{ s}^{-1}$ in the Ca target (due to the smaller area of the Ca target). The rates could be adjusted using a set of slits and jaws in the beamline.

A positive muon beam was also used for background studies; a 73 MeV/c μ^+ beam was produced by simply reversing the polarity of the magnets in the beam line. There is typically 5 times more flux available for the positive beam than for the negative beam, and so the slits and jaws were used to reduce the beam intensity, in order to produce μ^+ stopping rates in the target similar to the rates used for the negative muon beam.

To perform the calibration of the detection system, the radiative pion capture reaction was measured on several targets. Because the pions have a shorter range in the beam counters than the muons, the momentum of the beam line was tuned to 89 MeV/c for these runs to ensure that the pions stopped in the targets, and the phase of the RFS was tuned to transmit pions. At this momentum the time structure of the beam at the RFS allows only a partial separation of the beam, and the beam composition was typically $\mu/\pi = 0.5$, $e/\pi = 1.7$. At this momentum, the three particle types all arrive at different times at the beam counters, and so the pion stops in the target could be cleanly separated from muon or electron stops by a timing cut in the electronics. The negative pion flux was varied with the use of the slits and jaws to provide stopping rates in the targets between $1.0 \times 10^3 \pi^- \text{ s}^{-1}$ and $4.0 \times 10^5 \pi^- \text{ s}^{-1}$ to characterize the rate dependent properties of the system.

3.5 Trigger Electronics

In an experiment such as this, where several hundred words of data must be written to tape and subsequently analyzed for each candidate event, and where the flux of charged particles through the detector is on the order of 10^6 s^{-1} , it is essential that the data be recorded only for those events that stand a good chance of being a valid converted photon. This is ensured by the fast trigger electronics. Through setting various logical requirements on the signals from the beam and trigger counters, the trigger system was able to reduce the event rate from $5 \times 10^5 \text{ s}^{-1}$ to a trigger rate of $< 10 \text{ s}^{-1}$. Most of the trigger electronics were conventional NIM electronics.

The electronics used in this experiment was quite complicated, comprising over 240 NIM modules, most with multiple independent channels, in 32 NIM bins; 126 CAMAC modules, also mostly with multiple independent channels, in 7 CAMAC crates; several assorted FASTBUS and custom built modules, as well as many delay units. Clearly, a detailed description of all the electronics is impractical. Instead, a simplified and schematic description of the essential parts of the trigger logic will be given in this section. Linear signals from the trigger counters were summed where appropriate (i.e. where several photomultiplier tubes viewed a single counter) and were converted to NIM logic signals via conventional discriminators; this first stage of the logic will not be shown in the diagrams or discussed further. Where several modules can be represented by one module in the schematic diagrams, this has been done. The following section (3.6) will include a summary of the data digitized by CAMAC, but will not give details on exactly how the signals got there, as this is relatively straightforward.

The photon trigger can be divided into four stages, each successive level further reducing the false event rate. The first stage is shown schematically in Figure 3.6 ,

Figure 3.6: Schematic of first stage of the the trigger electronics.

and is concerned with defining a valid μ -stop in the target (or π -stop for calibration running). A coincidence of all four beam counters B1-B4 was made; if the timing of this relative to the cyclotron RF was appropriate for the time-of-flight to the TPC of the muon (or pion, for calibration runs), the *BEAM* condition was met. The *STOP* was defined as $STOP \equiv BEAM \cdot \overline{V + \Sigma BV}$ and $STOP\bar{I} \equiv STOP \cdot \overline{\Sigma IA + \Sigma IB}$. This $STOP\bar{I}$ eliminates particles scattered into the converter package and thereby completes the definition of a particle stopping in the target. Finally, the livetime (*LT*) is defined by $LT \equiv STOP\bar{I} \cdot \overline{BLANK}$ which defines the deadtime-corrected number of μ -stops (or π -stops). *BLANK* is made up of a logical OR of all the inhibiting signals, including computer-busy, time-to-process-events from higher levels in the trigger, RFS out of phase-lock, magnetic field and various chamber high voltage faults, as well as inhibits generated by the PDP program MONIT (described later).

The *LT* signal initiated a time gate, during which photon events due to RMC

were accepted. For the calcium target, this was provided by an updating discriminator, and lasted for 950 ns; for the oxygen and carbon targets, the gate was produced by a pileup gate and was 4.0 μ s in length. These gates were at least twice the muon lifetime in the respective targets, thereby ensuring that the majority of the RMC events would occur during the gate.

For calibration runs using radiative pion capture, the gate was 30 ns long, since pion capture occurs in prompt coincidence with the pion stopping in the target. For studies of background and efficiency using cosmic-rays during beam-off periods this first stage of the trigger was bypassed.

The second level trigger was concerned with the signals from the converter package and the outer trigger counters, and was designed to ensure that a photon conversion had taken place in the Pb converter, and that the two tracks of the pair passed through the TPC volume to reach the outer scintillators (W1-W6 and E1-E6). The condition *CONV* was defined by $CONV \equiv \Sigma IC \cdot (\overline{\Sigma IA + \Sigma IB}) \cdot \Sigma(E+W)$. This condition therefore corresponded to a valid conversion and at least one of the outer trigger counters having been hit.

The individual IA, IB and IC signals that corresponded to an individual TPC sector were each sent to one of the Northern Electronics MB10/6 programmable logic units. These modules made up an important part of several stages of the trigger; each module accepted up to 10 inputs and provided up to 6 independent outputs, each one of which could be programmed to be present for any given logical combination of the 10 inputs. These modules aided in rapid changes of the trigger during initial test runs, and also allowed changes to other trigger configurations (such as efficiency tests with cosmic rays and studies of systematics using muon-decay electrons) with a minimum of changes to the hardware. At this stage, the MB10/6 produced an output ($= IABC$) for the condition that one of the 3 IC's in

Figure 3.7: Schematic of second stage of the trigger electronics.

that sector had fired and none of the IA 's or IB 's had. This condition is redundant with the $CONV$ condition, but was found to be useful for reducing the trigger rate from events that 'leak' through the $CONV$ condition through jitter in the timing signals.

The outputs from these first MB10/6's were sent to one of another set of 6 MB10/6's, along with individual signals from relevant E and W counters. Again, each of the MB10/6's corresponded to one sector of the TPC, and the inputs were the $IABC$ condition from that sector, along with the E and W counter signals from that sector, as well as those from the two adjacent sectors. These units gave an output (IEW) for the condition $IEW_i \equiv I_i \cdot (W_j + E_j) \cdot (W_k + E_k)$ where $j \neq k$ and $j, k = i-1, i$ or $i+1$. Here $1 \leq i, j, k \leq 6$ (cyclic) are the TPC sector numbers. Thus this condition required that at least one of the outer scintillators had fired in at least two of the three outer sectors corresponding to the $IABC$ signal. This ensured that there were at least two tracks in the TPC, and that each was sufficiently energetic

Figure 3.8: Schematic of the third and fourth stages of the trigger electronics.

to reach the outer counters. The rate of ΣIEW was typically $20\text{--}30\text{ s}^{-1}$ during RMC data-taking, most of which was from muon-decay electrons that failed to fire the IA and IB counters and fired two outer counters via a delta ray or low-energy bremsstrahlung photon.

The IEW_i signals fed into several other stages in the logic, the first of these being the *EVENT* coincidence which was the first part of the third level trigger (see Figure 3.8). The *EVENT* was defined by $EVENT \equiv LT \cdot \Sigma IWC \cdot \Sigma IEW \cdot CONV$ where ΣIWC was a logical OR of the signals from the IWC anodes. The rate of *EVENT* was typically $8\text{--}15\text{ s}^{-1}$ during RMC data-taking.

The *EVENT* signal was used to provide gates to all the ADC's and registers, to start the time-to-process events inhibit, to provide starts for the TDC's and to open a $6.0\text{ }\mu\text{s}$ long gate (LG). The length of the LG was determined by the maximum drift time in the TPC of about $5\text{ }\mu\text{s}$. A coincidence was made between this gate and

the individual IEW_i to provide the sector gates SG_i , i.e. $SG_i \equiv LG \cdot IEW_i$. The sector gates were then used to pulse the TPC grids to the 'open' condition, to allow passage of the ionization electrons to the anode wires in the particular sectors that triggered. If an anode wire at a given radial position fired during the time of the SG, a $2.0 \mu\text{s}$ gate was then applied to the 2280 ADC's for the cathodes corresponding to that wire. In addition, blanking gates were applied to the inner anode wires, so that the early portion of the SG was vetoed for these wires. Geometry dictates that tracks originating at the center of the TPC necessarily have long drift times, and so these blanking gates ensured that the anode wires fire only for tracks coming from the target; this helped to reduce the number of spurious signals from other tracks in the chamber at the time of the event. The end of the LG also provided the *CLEAR* and *RESET* signals to the ADC's, TDC's and registers if the computer-busy signal was not present, that is if the final stage of the trigger had not been satisfied.

The fourth level trigger used information from the TPC itself, along with the IWC, and is shown schematically in Figure 3.8. This final stage of the trigger was necessarily delayed by the requirement of waiting for the maximum drift time of the TPC. Another set of MB10/6's checked that the IWC anodes had fired in the TPC sector corresponding to the IEW_i signal; this condition produced an output $TRIG \equiv IWC_i \cdot IEW_i$. The *TRIG* signal was fed (after a $6.0 \mu\text{s}$ delay) into a final MB10/6 unit, along with the 12 gated outputs from the TPC anode wires (multiplexed by sector here). If the *TRIG* signal was present and if ≥ 6 anode wires had fired, a *LAM* (look at me) signal was generated, i.e. $LAM \equiv TRIG \cdot \geq 6$ TPC anodes, and the event was written to tape. If no *LAM* was generated, a *CLEAR* was sent to all the CAMAC modules and the *BLANK* turned off. The *LAM* rate for RMC data-taking was typically $4\text{-}10 \text{ s}^{-1}$.

The trigger logic described above was designed for the selection of photon events; during various tests, background studies and efficiency studies other trig-

gers were used. In general, the trigger could be changed simply by loading another set of files to the MB10/6 programmable logic units, with a minimum of changes to the hardware. Runs were taken with beam off to look at cosmic-ray induced photon background using the photon trigger with the LT condition removed from the *EVENT* (as there are no μ -stops expected). However, there was also another mode of running with cosmic-rays, one used to monitor the efficiency of the TPC, the trigger counters and the cosmic ray counters. During these runs, the magnetic field was set to zero, so that tracks in the TPC would be straight lines, the IA, and IB counters were removed from the trigger, and the trigger requirement was essentially any combination of an IC and a W or E-counter. This trigger provided straight cosmic-ray tracks that passed through the chamber. By extrapolating the reconstructed track through the various trigger and cosmic-ray counters, the efficiency of each counter could be determined by measuring how often it actually fired when the track passed through it. This mode of running was used repeatedly before, after and during data taking periods.

Also, during some studies, single tracks from muon-decay electrons were examined. For these runs, the IA and IB counter vetoes were removed, as well as the two-sector requirement, and the trigger used was essentially that used in the muon-electron conversion experiment [92].

3.6 Data Acquisition

After a valid trigger (LAM) was generated by the trigger electronics, a process began whereby relevant information from the detection system was digitized and put onto magnetic tape, and a subset of the data was analyzed immediately for on-line monitoring of the experiment. The data acquisition system has been described elsewhere [98] and consists of a multi-crate CAMAC acquisition system (7 crates in all), external memory, and PDP 11/34 and VAX 11/750 computers. The PDP

computer handled the tasks of reading the data for each event from the CAMAC modules, transferring it to tape, updated various histograms of the raw data and also performed several basic monitoring and control processes. It also passed a fraction of the events to the VAX computer via DECNET. The VAX then analyzed this subset of the events, with the identical software used in the off-line data analysis.

For each event, typically 700 words of data were written to tape. The most important component of the data was the information from the TPC itself. The analogue pulse height on each of the 144 anode wires and the 636 12-fold multiplexed cathode pads was digitized using the LeCroy 2280 CAMAC ADC system. This consisted of 20 modules, each with up to 48 channels of 12 bit ADC, and a single auxiliary processor module that had pedestal memory for each of the ADC channels. The processor was operated in a pedestal subtraction mode, in which the processor individually subtracted previously measured pedestals for each ADC channel, and only channels with data over a preset digital threshold were read out, along with a fixed number of channels on either side. Thus, rather than reading out each ADC channel for every event, a compact data set was read out; this contained all the information on the wires and pads that actually had signals. To ensure that the ADC's were gated on at the correct time, the ADC system was modified so that each module with cathode pad inputs from the same anode wire was gated only if that anode wire had fired.

During the time that the 2280 processor was performing the digitization and compacting of the TPC ADC data, the readout of all other CAMAC modules was taking place. After the 2280 processor completed its conversion of the data a second *LAM* was generated and the PDP began to read out the 2280 ADC data. A description of the data in the other CAMAC modules follows below.

The logic signals from all 144 TPC anodes were also sent to bit registers to provide the pattern of hits in the chamber. The time information from the anode

wires was multiplexed by sector, and digitized in 12 TDC channels. Therefore, if two tracks fired a given anode wire, only the arrival time of the earliest ionization to reach the anode wire was digitized. For a subset of the data, a FASTBUS pipeline TDC system (LeCroy 1879 modules) was used, with each of the 144 anode wires going to a separate TDC channel, and with each of the TDC channels having multi-hit capability, thus effectively de-multiplexing the time information. Unfortunately, due to both hardware and software problems, only a relatively small fraction of the data set had this information available, and so the FASTBUS TDC information was not used in the present analysis.

As well as the information from the TPC itself, information from the beam, trigger and cosmic-ray counters was also digitized by CAMAC and written to tape for each event. These included:

- the ADC values for the beam counters B1-B4, V, BV1-B4
- the ADC values for the trigger counters IC1-IC18, W1-W6, E1-E6
- the ADC value for the IWC proportional chamber
- the Bit Register patterns for all the trigger and beam counters, including the pattern of hits in the IWC and EWC's
- the TDC values (relative to the *EVENT* time) of all cosmic ray counters
- the TDC values (relative to the *EVENT* time) for the time of the μ -stop, and various combinations of trigger counters, mainly to look at pileup effects
- the CAMAC scaler values for the TPC anode wires and 34 of the different logical combinations in the trigger electronics

At the beginning of each buffer of data, a record header was written to tape; this included information such as the time, the magnetic field, the main and end coil

currents in the TPC magnet, the phase of the RFS, the TPC anode high voltage and the currents and voltages of the IWC and EWC chambers.

At the beginning of each run, a run header was written to tape, which included the settings of the MB10/6 logic units, the high voltages of all the scintillators and wire chambers, along with various operator-entered comments such as the target, beam conditions etc. A hardcopy of these run headers, along with a sample of the scaler rates, was automatically printed.

The PDP computer provided the capability of a final level trigger, the 'pre-taping cuts', whereby events that failed to meet certain simple criteria determined by the PDP would not be written to tape and a reset command sent to the electronics. In this experiment two pre-taping cuts were used: events were cut if the number of words of data from the 2280 ADC's was too small to be a valid two-track event, and also if the number of hits on the cathode pads was too small to be a valid event. These cuts were able to reduce the rate of events written to tape by another factor of 50-60% , with essentially no loss of good data ($< 2\%$ typically).

The PDP computer ran a monitor program (MONIT) that periodically displayed scaler rates, high voltage levels, magnetic field, and statistics on the fraction of events cut by the pre-taping cuts. Data acquisition could then be suspended if any these parameters fell out of specified bounds. The PDP also ran tasks to measure and load pedestal values to the 2280 ADC system processor, to load the memories of the MB10/6 programmable logic units, to control the high voltages of the scintillation counters and to display histograms of the raw data.

The data acquisition system was able to handle the maximum trigger rate during the RMC data taking of $\leq 10 \text{ s}^{-1}$ with a dead time of $\leq 20\%$ when the PDP was not asked to histogram the raw data; the dead time increased to about 30% when the histogramming was turned on.

3.7 Run Summary

Data were recorded for the experiment described in this thesis in several running periods. Initial development of the new tune for the M9 channel took place during a one week period in June 1985. Tests of part of the converter package, development of the trigger logic, optimization of the magnetic field strength and initial photon acceptance measurements took place during two weeks in August 1985. The full converter package with 0.6 mm of Pb was installed and tested during a two week period in November-December 1985. The final trigger logic was established during this period and initial studies of RMC on the calcium target were made.

During a month-long run in May 1986, high-statistics data were taken on RMC on both calcium and carbon with the 0.6 mm Pb converter, as well as detailed studies of the rate-dependences of the photon acceptance, and measurements of backgrounds due to radiative pion capture as well as bremsstrahlung from muon-decay electrons. The Pb converter was switched to a 1.0 mm thick one for a five-week long run in July-August of 1986, during which time RMC data on calcium and carbon were taken, along with continued investigation of rate dependences in the data and high-energy tails in the photon response function.

Measurements of RMC on calcium and oxygen with the 1.0 mm thick Pb converter were made during four weeks in November-December 1986, along with more data on radiative pion capture backgrounds and rate dependences in the acceptance.

Finally, beam was allocated for a week in January 1987 and for two weeks in May of 1987 primarily to examine various contributions to the systematic error; smaller data sets were obtained for RMC on calcium and oxygen during these periods.

Between running periods, extensive measurements were made of the efficiencies of the various components of the detection system using cosmic-rays, and, in

addition, long runs were taken to measure the cosmic-ray background rate.

Chapter 4

Data Analysis

4.1 Introduction

The analysis of the data obtained in this work was performed on the TRIUMF Data Analysis Centre VAX cluster of computers (two VAX 8650's and a VAX 780), as well as on the TPC750 VAX 750, various μ -VAX computers, and a few other computers (i.e. wherever the author could scrounge available CPU time). Various computer programs were used for different aspects of the data analysis; besides SOFIA (described in the following section), several TRIUMF general-purpose routines such as OPDATA, REPLAY, FLOWA, PLOTDATA, EDGR etc. were very useful for manipulation, fitting and plotting of data.

As this chapter is relatively long and involved, a brief overview is presented here. The first three sections describe, respectively, the algorithms used to reconstruct and fit the tracks in the TPC, the Monte Carlo simulation of the detector, and the software cuts used to select good events. A discussion of the determination of the detector response function, and its analytical parameterization follows. The next section details the various backgrounds to the measurement, their relative significance and the techniques used to deal with them. A significant rate dependence was observed in the photon acceptance, and the next section examines the cause of this rate effect, and the means used to correct for it. Finally, the last two sections discuss the final normalization of the detector acceptance, and various small corrections required to extract the experimental RMC branching ratios.

Data were taken with two different Pb converters, one 0.6 mm thick and one

1.0 mm thick. The larger data set (and the only one in the case of ^{16}O) was with the 1.0 mm thickness. Aside from the increased acceptance (and somewhat worsened energy resolution), the characteristics of the data, and the methods of analysis were very similar to the data with the 0.6 mm converter. In this chapter, in the interest of brevity, data shown will be from the 1.0 mm converter, unless significantly different for the 0.6 mm data, whereupon both will be shown.

4.2 Track Reconstruction

The first stage of the data analysis was the reconstruction of the charged particle tracks from the TPC. This was done using the program SOFIA (Sophisticated OFFline Interactive Analysis) [99]. SOFIA was developed at TRIUMF specifically for analysis of data from the TPC; it performs event-by-event analysis in either batch or interactive mode. In addition to track fitting, SOFIA was used for other aspects of the data analysis. These include the generation of a graphical representation of selected events (called a 'PHOTO'), histogramming and the application of user-defined cuts. These other functions will be discussed later in this chapter. The track reconstruction routines used in this experiment were somewhat different from those which have been used previously for analysis of TPC data [92], and consequently they will be discussed in some detail here.

The first step in searching for tracks in the TPC was the decoding of the TPC cathode ADC data. The program searched for 'clumps'; these were clusters of adjacent cathodes with non-zero ADC values. Each clump was fitted by a median technique to find the center of gravity of the induced signal on the cathodes; this then represented the x -coordinate of a segment of a track. More involved fits to the shape of the distribution of charge in a clump were found to be time-consuming, and to have little effect on the results.

Since the cathode pads for all the TPC sectors were multiplexed together, the

cathode clumps needed to be assigned to the correct sector. This was accomplished through the use of the anode wires (which were not multiplexed) and the trigger information. Clumps were assigned to a sector only if the corresponding anode wire had valid ADC and TDC information, and if the trigger pattern indicated that the sector should have data. With the two-track data from converted photons, there were often ambiguities; in these cases the clumps were assigned to all the sectors (usually only two) that satisfied the above criteria. These remaining ambiguities were then (usually) sorted out later in the track reconstruction. For each clump, the y -coordinate was just the y -coordinate of the corresponding anode wire; the z -coordinate was determined from the TDC value of that anode. Thus a pattern of (x, y, z) coordinates of track segments in the TPC was produced.

SOFIA then began attempting to reconstruct tracks from these (x, y, z) coordinates. For each candidate track, the fitting routines proceeded in two steps: first, an attempt was made to find the circular projection of the helical track on the (x, y) plane; when this was found, a fit was then made to the helix using the z -coordinates as well.

The circle fitting proceeded as follows: a track starting point was chosen from the points on the innermost wire with data; points that had previously been tried as a starting point, or that were included in a previously found track were not used as starting points. After a starting point was found, the point closest to it in the next wire (in the radial direction) was chosen; this continued until four points were found.

The four points were then fitted analytically to a circle. The radius (R) and center (X, Y) were found by minimizing

$$\chi_c^2 = \sum_i \left[(x_i - X)^2 + (y_i - Y)^2 - R^2 \right]^2. \quad (4.1)$$

If χ_c^2 was less than a user-defined limit χ_{xy}^2 , the points were accepted as a trial circle. If χ_c^2 was greater than χ_{xy}^2 , the point that contributed the most to χ_c^2 was rejected and a new point was searched for on the next wire with data.

After a trial circle was found, all the remaining points (that had not already been used in any other track) were tested for inclusion. For each trial circle, the point closest to the circle was added to the fit. If the new value of χ_c^2 was acceptable (i.e. $< \chi_{xy}^2$), the point was retained in the fit, and the values of R and (X, Y) were updated. This continued until all the points had been tested for inclusion. The trial circle was retained even if only four points had been found.

The (x, y) points in the circle were then tested for inclusion on a helix, by fitting their respective z -coordinates to a linear function of the anode wire number. This simple procedure identified the (x, y, z) -coordinates that were likely candidates to be on the helix. A minimum of three such candidate points was required; otherwise the track was rejected. The three candidate points were then fit analytically to a helix by minimizing

$$\chi_h^2 = \sum_i \left[m \cdot \tan^{-1}((y_i - Y)/(x_i - X)) + b - z_i \right]^2 \quad (4.2)$$

where m was the slope and b the intercept. Again, if χ_h^2 was greater than a limit χ_z^2 , the point contributing the most to χ_h^2 was rejected and another point on the trial circle was tested for inclusion on the helix. After a trial helix was found, the remaining points on the trial circle were tested, and each was added to the fit if χ_h^2 was not increased beyond χ_z^2 . If a helix was not found, the trial circle was rejected. If a helix was found, the points on it were stored in an array to prevent their being used on other tracks.

The procedure was then repeated; new starting points were tried until there were too few unused points remaining to form a valid track. At this point, the

program attempted to search for a candidate 'pair' of tracks. If no tracks had been found, the program went on to look at the next event. If two or more tracks had been found they were tested as a 'pair' as described next; the case of only one track having been found is a special one, and will be discussed later.

The criteria for a valid 'pair' were that the two tracks were of opposite curvature (i.e. opposite charge), that they crossed the converter within 10 cm of each other, and that their opening angle was less than 45° . The opening angle was defined as the angle between the tangents to the two circles at their point of closest approach, or at their intersection(s) if they intersected. This angle is small ($< 20^\circ$) for e^+e^- pairs from a converted photon for the photon energies of interest here. If the event had several tracks, all possible combinations of two tracks were tried until an acceptable 'pair' was found. If a valid 'pair' was found the track parameters were stored and the analysis continued.

If an acceptable pair was not found, but there were at least two tracks in the event, a new stage in the track searching was initiated. Analysis of preliminary data had shown that real e^+e^- often had not been identified by this point in the track fitting. This was often due to one, or both, of the tracks crossing between two TPC sectors; there are gaps between the sectors which are not instrumented, and consequently tracks crossing these gaps often had relatively few points. To partly recover this lost efficiency in the track reconstruction, the following procedure was developed.

In a good e^+e^- pair, the two tracks should project back to a single common point at the converter; this fact was used to guide the track fitting routines. The track with the largest number of points (i.e. presumably the best determined track) was projected to the converter; the (x,y,z) -coordinate of the intersection of the track with the converter (the 'CP') was then used as an additional point on each of the other tracks in the event. The circle fitting was then repeated with the CP in the fit;

if χ_c^2 was too large, the point contributing the most to χ_c^2 (except for the CP) was then removed from the fit. The fitting procedure continued much as before, except that the CP was forced to remain on the track; however a track was only accepted if the inclusion of the converter point did not increase the radius of curvature of the circle by more than 20% . The CP was tried on each of the other tracks until a good ‘pair’ was found. This procedure was found to increase the reconstruction efficiency without affecting the position or energy resolution significantly.

If this procedure failed to find an acceptable ‘pair’, or if only one track had been found originally, a second related procedure was adopted. Again the converter crossing point (CP) of the track with the most points (or the only track in the case of a single track) was determined. This was then used as a starting point for the circle (and helix) fitting to search for new tracks, which proceeded as before. To ensure that any ‘pairs’ found were in fact likely to be real, more restrictive values were used for χ_{xy}^2 , and for the opening angle of the ‘pair’. Again, this technique was found to increase the efficiency for identification of good photons without worsening the energy or position resolution.

If a valid pair was not found by this time, the analysis then proceeded to the next event. The entire track reconstruction process was ‘tuned’ using real photons from capture calibration data, as well as Monte Carlo simulated data.

All the data obtained were analyzed in a first pass, which simply selected all events that qualified as ‘pairs’, and wrote these events to a second generation of magnetic tapes, thus reducing the data set to a manageable size for subsequent analysis.

4.3 Monte Carlo

As is common in experiments of this kind, a computer program to simulate the response of the detection system was required (these programs are known as Monte

Carlo routines). This was needed for several reasons. The acceptance and energy resolution of the detector for photons are strong functions of the photon energy, and so extraction of a branching ratio, as well as determining the shape of the spectrum for RMC, requires a knowledge of this acceptance and its photon energy dependence (the ‘response function’). These would be possible to determine experimentally if a source of monoenergetic photons, variable over the relevant range of energies, was available. This was not the case and so a Monte Carlo simulation was required.

A second motivation for the Monte Carlo program was to aid in the optimization of the various software cuts used, as well as in the tuning of the track reconstruction algorithms. In addition, the Monte Carlo was used in the estimation of the importance of various backgrounds. A final motivation was the use of the Monte Carlo to understand the origin of, and to correct for, the observed rate dependence of the photon acceptance.

The Monte Carlo program was written using the framework of the CERN program known as GEANT (version 3.10) [100]. GEANT provides routines to track various particles through a user-specified geometry, simulating an assortment of relevant physical processes, such as energy loss, multiple scattering and bremsstrahlung (for charged particles), pair-production and Compton scattering (for photons) etc. GEANT allows the user to record relevant quantities and to store them in a user-defined format. In the present case, routines were written to digitize and store the results of each simulated ‘event’ in exactly the same format as for the actual experimental data written to tape. Thus, Monte Carlo ‘data’ could be analyzed with the same track reconstruction routines (i.e. SOFIA) and identical cuts as were used to analyze the experimental data. Therefore direct comparisons of all the resulting distributions could be made. Results of the Monte Carlo simulation and comparisons with the experimental data will be presented throughout this chapter; a detailed discussion of the Monte Carlo, and also some further comparisons with

measured data, is left to an appendix.

4.4 Cuts

The events that passed the track reconstruction algorithms as valid ‘pairs’ were not, in general, all from real photons. Also, of those that were real photon events, some fraction were poorly fit and hence the reconstructed photon energy was relatively far from the true energy. Finally, of those events that were real photons and were well fit (i.e. with good energy resolution), some fraction were not genuine RMC photons originating in the target, but were a result of one of several background processes. The track searching and reconstruction algorithms already described were optimized for maximum acceptance, that is, to recognize as many candidate photons as possible; consequently the percentage of false or poorly-resolved events recognized as ‘pairs’ was relatively large. Consequently, reasonably restrictive cuts had to be placed on the events to select out well-resolved, genuine RMC photons. These cuts were based upon the characteristics of the fitted tracks in the TPC plus information from the trigger counters and other components of the detection system.

The cuts were devised and tuned in several ways. In the calibration data (radiative pion capture), almost all of the triggers were indeed valid photons, so this data could be used to identify the characteristics of good events; Monte Carlo data was used in this way also; since the ‘true’ energy of each event was known, effects leading to poor energy resolution could be studied. Especially valuable was the data taken with a μ^+ beam. In this case, since no physical muon-induced process could result in a photon of greater than 53 MeV (the endpoint of the free muon decay spectrum), any event above this energy was either not a real photon or a poorly resolved one, or some sort of background event (e.g. cosmic-ray induced). Those events above 53 MeV could be studied in detail, and cuts developed or adjusted to

reject them.

The decision on what cuts to use, and how restrictive to make them, was to some extent arbitrary. Once the cuts are stringent enough to select only real photon events, more restrictive cuts represent a compromise between energy resolution and photon acceptance since, in general, each cut will reject a certain number of good, well-resolved photons. The cuts chosen were optimized such that the error on the final results due to the backgrounds and energy resolution was comparable with the final statistical error on the branching ratio. In the case of ^{40}Ca , due to the much larger signal from RMC, the cuts could have been loosened to gain somewhat in acceptance without a significant cost in increased background. This was not done, however, both for the sake of simplicity, and also to maintain compatibility in the comparison of the results from each target.

The determination of the efficiency of each cut was made using both the pion calibration data, and GEANT Monte Carlo simulated data. With only one exception, identical cuts were used in the analysis of RMC data, pion calibration data and GEANT data, so that the acceptance determined from the calibration data represented the actual acceptance for RMC data. The one exception was the PROMPT cut, which served to reject pion-induced background, and so was inappropriate for the pion calibration data. The PROMPT cut, along with the cosmic-ray background cut (COSMIC), will be discussed in later sections, as they were both cuts designed to eliminate specific backgrounds.

It should be noted that there are strong correlations between many of the cuts; a poorly fit event was likely to manifest itself in several of the quantities cut upon. Also, some of the cuts (such as those on the location of the origin of the photon) serve several purposes: to eliminate real background photons (i.e. events not due to RMC in the target), spurious 'pairs' and also poorly-resolved RMC events (as position and energy resolution are obviously correlated).

Before discussing the individual cuts, it is necessary to discuss the nature of the spurious ‘pairs’ that need to be eliminated, and to explain the reasons that some real events had poor energy resolution. As will be discussed in more detail later, the amplitude of the cathode pad signals was very low during this experiment. This meant that some of the cathode clumps were barely over threshold, which caused a large error in determining the centroid of the clump; in fact the amplitude was often too low for any clump on the pad to be over threshold, and so the (x,y) point was not found at all. Combined with this, was the fact that any tracks crossing between TPC sectors had no points in the dead regions between the sectors. Thus, the reconstructed tracks often had as few as four points used for the fit; the curvature (and therefore the momentum) of the track was then poorly determined - a single point could throw the fit off drastically. This contributed greatly to the degradation of the energy resolution, especially to the ‘tails’ of the resolution function. The source of the spurious ‘pairs’ is then also easily understood. The activity in the TPC during the measurement of RMC was dominated by electrons from muon decay; at the muon stopping rates used, there were often several decay electrons in the chamber within the time window defined by the TPC drift time. Consequently, when a trigger occurred, there were often several tracks present in the chamber. A ‘pair’ could then be produced by one electron track along with either another, poorly fit electron track (where the fit gave the wrong sign for the curvature), or with the re-entrant portion of a track that wraps-around. The problem is exacerbated by spurious points due to electronic noise or other activity in the TPC.

While the probability of such an occurrence producing a spurious ‘pair’ was clearly very low, it was not negligible in comparison with the very rare RMC process. Fortunately, the vast majority of such events are easily identified and rejected by the cuts. Confidence that no such events remain after the cuts was gained by comparison of all the characteristics (distributions in time, space, quality of the fit

etc.) of the remaining events with those of genuine photons (obtained from both pion calibration data and GEANT).

The cuts can be roughly ordered into a few groups. The first group is concerned with the quality of the track reconstruction. CHISQ was a cut on the χ^2_c of the circle fit of each track in the pair; it was only slightly more restrictive than the upper limit χ^2_{xy} used in the track reconstruction routines. PHICONV and ZCONV are restrictions on the maximum difference, in azimuthal angle (ϕ) and z-location respectively, between the converter crossing points of the two tracks in the pair. The two tracks from a valid converted photon should intersect at a unique point in (x, y, z) at the converter (equivalently, at a unique (ϕ, z)). Large deviations are indicative of poorly-fitted tracks, or a spurious pair. THETA and THETAXY are upper limits on the opening angle of the pair, and the projection of the opening angle on the (x, y) -plane (see Figure 4.1). The opening angle of the pair is defined to be the angle between the tangents to the two helices, at their point of closest approach. Photons of the relevant energy in this experiment have small opening angles ($< 5^\circ$); the observed opening angle was usually somewhat larger due to multiple scattering in the Pb, and finite position resolution. Again, large opening angles indicate poor track reconstruction or false pairs. The THETA cut was somewhat less restrictive than the THETAXY to allow for the poorer resolution of the z-component of the fit.

Occasionally, SOFIA would attempt to fit different tracks to the same physical track, usually when the track had several points with large deviations from the physical track; DISTINCT was a cut that required a certain number of points in each track to be larger than 2.0 cm away from any other fitted track, to eliminate these events.

There were two remaining cuts that can be grouped with the ‘fitting’ cuts; the MISSING and FALSE cuts. These cuts were concerned with how many points

Figure 4.1: Histograms of various parameters used to select good events. The histograms are π^- -calibration data before and after (hatched region) all cuts were applied. The arrows show the locations of the software cuts. The parameters are a) THETA the opening angle, b) the opening angle in the (x, y) plane, c) and d) track mismatch between the TPC and IWC in z and ϕ , e) the closest radial approach to the target center and f) the z -position of the converter-crossing point.

were used in the fitted tracks. For each track in the pair, the program added up the number of anode wires that were crossed by the fitted track, but on which there was no point found in the fit (the dead regions between sectors do not contribute to this sum). If the sum exceeded the value set by MISSING in either of the tracks, the event was rejected. This cut was very valuable in removing events that were poorly reconstructed. The program also found the number of anode wires that had fired in the TPC, but which were not used in either fitted track; if this exceeded the value set by FALSE, the event was cut. This cut rejected events that were badly fitted, as well as events where there was a third distinct track in the chamber (a third track in the chamber may lead to some ambiguity as to which tracks were from the converted photon).

The next group of cuts utilized information from the trigger counters. IAIB rejected any event where an IA or IB counter had its bit set in the register (to veto charged particles); in principle, this is redundant with the hardware trigger, but it was found to be required, due to inefficiency in the electronics (likely from events with marginal timing). ICIWC rejects events where both an IC counter and a hit in the IWC were found in each of two sectors of the TPC (this usually indicated a third track in the TPC).

The WE cut rejected events where the outer portion of the trigger consisted solely of one W counter and one E counter in the adjacent sector. This trigger pattern could have been caused by a single track crossing between sectors at the outer counters; therefore the two-track nature of the event was less certain. This restriction could have been imposed at the hardware level; however its usefulness in rejecting spurious events was not discovered until after data-taking was completed.

WCROSS and ICCROSS were quite similar cuts. WCROSS required that if either fitted track in the pair projected to a W counter, that counter must have fired. ICCROSS put a similar restriction on the IC counter. In both cases, allowance was

made for a small deviation of the track from the relevant counter, to allow for the finite position resolution in the TPC.

ZIWC and PHIWC (see Figure 4.1) put restrictions on the maximum deviation between the fitted tracks and the position of the nearest hit in the IWC, in both the z -coordinate and azimuthal angle (ϕ) respectively.

There were two cuts which were concerned with the location of the origin of the photon (see Figure 4.1). RTARGET was a cut on the distance from the TPC center, of the intersection with the plane $z=0$ of the projected direction of the photon (the targets were located at $z=0$). If this distance was larger than the value set by RTARGET (approximately the target radius), then the event was rejected. This helped to eliminate events originating in the IA and IB trigger counters. ZTARGET required that the location in z where each track in the pair crossed the converter be within a specified range; this rejected events coming from the beam counters or the veto counter (V) downstream of the target.

Finally, the DEDX cut rejected events where one of the tracks in the pair was too heavily ionizing to be an electron (or positron). The sum of the pulse heights in each cathode clump was calculated, for all the points in each fitted track. These were corrected for the track crossing angle and the pitch angle (because the size of the track segment that intersects the entrance slot to the proportional region depends on these angles); the largest two and the smallest two values in each track were dropped (to reduce the effect of Landau fluctuations on the average), and the remainder were averaged. This provided a relative estimate of the ionization rate (i.e. dE/dx) for the particle; if this was larger than a specified value, the event was rejected. This eliminated the (extremely few) events where one of the tracks was a heavily ionizing particle (e.g. μ, π, p, α etc.).

4.5 Response Function

The detailed response of the detector (that is, the efficiency, energy calibration, resolution and lineshape as a function of incident photon energy) was determined through the use of the GEANT Monte Carlo simulation. Monoenergetic photons were generated at 10 MeV intervals between 40 and 150 MeV; typically 4×10^5 photons were generated at each energy. The initial locations of the photons were sampled from a realistic stopping distribution in the target. The resulting simulated events were then analyzed by SOFIA, as per actual data. The reconstructed photon energy spectra were then fitted to a response function of the following form:

$$R(E_\gamma, E) = A \exp \left[-\frac{1}{2\sigma_0^2}(E - E_0)^2 \right] \quad \text{for } E_1 < E < E_2 \quad (4.3)$$

$$R(E_\gamma, E) = B \exp \left[-\frac{1}{2\sigma_1^2}(E_1 - E)^2 \right] \quad \text{for } E < E_1 \quad (4.4)$$

$$R(E_\gamma, E) = C \exp \left[-\frac{1}{2\sigma_2^2}(E - E_2)^2 \right] \quad \text{for } E > E_2 \quad (4.5)$$

where

$$E_1 = E_0 - \frac{\sigma_0^2}{\sigma_1} \quad (4.6)$$

$$E_2 = E_0 + \frac{\sigma_0^2}{\sigma_2} \quad (4.7)$$

and

$$B = A \exp \left[-\frac{\sigma_0^2}{2\sigma_1^2} \right] \quad (4.8)$$

$$C = A \exp \left[-\frac{\sigma_0^2}{2\sigma_2^2} \right] \quad (4.9)$$

and A , E_0 , σ_0 , σ_1 , and σ_2 , are free parameters. E_γ is the actual photon energy and E is the reconstructed energy. This functional form is essentially a Gaussian with high- and low-energy tails, and was chosen simply because it gave a reasonable

Table 4.1: Coefficients of polynomial fits to the energy dependence of the photon response function parameters.

Converter	Parameter	P_0 (MeV)	P_1	P_2 (MeV) ⁻¹	P_3 (MeV) ⁻²
0.6 mm	E_0	-7.548	1.1403	-1.028×10^{-3}	—
	σ_0	-3.365	0.2026	-2.216×10^{-3}	1.188×10^{-5}
	σ_1	6.402	-0.2560	3.697×10^{-3}	-1.389×10^{-5}
	σ_2	1.983	-0.0641	9.606×10^{-4}	1.880×10^{-7}
1.0 mm	E_0	-9.823	1.1381	-1.013×10^{-3}	—
	σ_0	5.186	0.1900	3.240×10^{-3}	-9.608×10^{-3}
	σ_1	-11.786	0.5410	-7.128×10^{-3}	-3.715×10^{-5}
	σ_2	-1.700	0.1312	-1.604×10^{-3}	1.001×10^{-6}

representation of the lineshape (for clarity, the function is not given in normalized form). The parameters E_0 , σ_0 , σ_1 , and σ_2 were then fitted to polynomial functions of E_γ , e.g.

$$\sigma_0 = P_0 + P_1 E_\gamma + P_2 E_\gamma^2 + P_3 E_\gamma^3 \quad (4.10)$$

The results of these fits are shown in Figures 4.3, 4.4, 4.5 and 4.6, and the values of the coefficients P_0 , P_1 , P_2 and P_3 for each fit are given in Table 4.1. Figure 4.2 shows the energy spectrum for 70 MeV photons with the fit superimposed. The chosen functional form clearly parameterizes the GEANT results well, except for the details of the extreme high-energy ‘tail’ of the lineshape. This tail will be seen to be rather important, and will be discussed in more detail later. With the detector response function parameterized as above, the Monte Carlo predictions for the observed spectrum for any given spectrum are easily produced analytically, thereby avoiding the necessity of running lengthy Monte Carlo simulations for each input spectrum.

The final parameter, A , is related to the photon acceptance. Some of the factors contributing to the energy dependence of the acceptance can be directly extracted from the Monte Carlo. Figures 4.7 and 4.9 show the pair-production

Figure 4.2: Photon energy spectrum for 70 MeV photons from GEANT and parameterization of detector response function, 1.0 mm converter.

Figure 4.3: Photon response function parameters E_0 and σ_0^2 for 0.6 mm converter.

Figure 4.4: Photon response function parameters σ_1 and σ_2 for 0.6 mm converter.

Figure 4.5: Photon response function parameters E_0 and σ_0^2 for 1.0 mm converter.

Figure 4.6: Photon response function parameters σ_1 and σ_2 for 1.0 mm converter.

probability in the Pb converter as a function of photon energy; it rises slightly with photon energy. Figures 4.8 and 4.10 show the trigger efficiency versus photon energy. This falls rapidly for lower photon energies, due both to the falling pair-production probability, and to the trigger requirement that two sectors of the outer counters fire. As the photon energy falls, the probability becomes increasingly smaller for both the e^+ and the e^- to have sufficient energy to reach the outer counters, and so fewer events satisfy the trigger. In fact, at low photon energies (≤ 50 MeV) the majority of the triggers are due to pairs with very asymmetric energy-sharing: the higher energy lepton reached one of the outer counters, and the trigger was satisfied either by that particle crossing over to hit an outer counter in an adjacent sector, or by a low-energy bremsstrahlung photon firing an outer counter in another sector. Of course, this behaviour is strongly dependent on the chosen magnetic field. Figures 4.11 and 4.12 show the photon acceptance as a function of photon energy for the two converter thicknesses. The absolute normalization of

Table 4.2: Coefficients of fit to the energy dependence of the photon acceptance.

Converter	A_0	A_1 (MeV)	A_2 (MeV)	A_3
0.6 mm	2.044×10^{-3}	77.88	28.32	2.026×10^{-3}
1.0 mm	1.583×10^{-3}	80.11	25.16	2.792×10^{-3}

the acceptance shown is a ‘zero-rate’ value; the absolute normalization and the rate dependence of the acceptance will be discussed later. The fact that the acceptance falls to zero for low photon energies was an advantage in reducing the background from the bremsstrahlung of electrons from muon decay, but it does mean that the acceptance was not at its maximum over the energy range of the RMC spectrum. The energy dependence of the acceptance was parameterized by a function of the form

$$\epsilon\Omega = A_0 \left\{ \operatorname{erf} \left[\frac{(E_\gamma - A_1)}{A_2} \right] - \operatorname{erf} \left[\frac{(A_1 - E_\gamma)}{A_2} \right] \right\} + A_3 \quad (4.11)$$

where the fitted values of the coefficients A_0 , A_1 , A_2 and A_3 are given in Table 4.2. The functional form was chosen simply for convenience, and represents the GEANT results well.

Comparison of the parameters σ_0 , σ_1 , and σ_2 for the 0.6 mm and the 1.0 mm converter cases shows the loss in resolution that accompanied the gain in acceptance in using a thicker converter. The comparison of final results obtained with the two different converter thicknesses provides a valuable check on the data analysis procedure.

The detector response function produced by GEANT can be checked by comparing its predictions to data taken during the pion calibration runs. For both radiative pion capture in ^{12}C , and the radiative pion capture and pion charge exchange reactions on the proton, high quality data with good energy resolution already exist from other experiments. These spectra can then be folded with the detector response function as parameterized above, and compared with the data.

Figure 4.7: Photon conversion efficiency for 0.6 mm converter, calculated in GEANT.

Figure 4.8: Photon trigger efficiency for 0.6 mm converter, calculated in GEANT.

Figure 4.9: Photon conversion efficiency for 1.0 mm converter, calculated in GEANT.

Figure 4.10: Photon trigger efficiency for 1.0 mm converter, calculated in GEANT.

Figure 4.11: Photon acceptance for 0.6 mm converter, calculated in GEANT.

Figure 4.12: Photon acceptance for 1.0 mm converter, calculated in GEANT.

Figure 4.13: Radiative pion capture on ^{12}C compared to GEANT prediction, 1.0 mm converter.

Figure 4.13 shows this comparison for radiative pion capture on ^{12}C , using the data of Perroud *et al.* [101] for the input spectrum; the agreement is impressive.

The data for the π^-p reaction were obtained by subtracting a suitably normalized spectrum of π^-C from π^-CH_2 . The π^-p reaction at rest has two branches:

$$\pi^-p \rightarrow \pi^0 n \quad (60.4\%) \quad (4.12)$$

$$\pi^-p \rightarrow \gamma n \quad (39.6\%) \quad (4.13)$$

where the radiative capture gamma-ray has an energy of 129.4 MeV, and the π^0 nearly always decays into two photons (98.8 % branch). The ratio between these

Figure 4.14: Photon spectrum from π^-p at rest, compared to GEANT prediction, 1.0 mm converter.

two processes is called the Panofsky ratio, and is well known [102]. The π^0 has a kinetic energy of 2.9 MeV, and so the photons from the π^0 decay are Doppler-shifted in the lab frame; the energy spectrum of a single observed photon is a rectangular distribution between 54.9 MeV and 83.0 MeV. Fortuitously, this is just the energy range of the useable portion of the RMC spectrum. Figure 4.14 shows the π^-p spectrum compared with the Monte Carlo prediction. A few small effects had to be considered. Firstly, there was a finite probability that, given an otherwise valid π^0 -decay photon event, the second photon might have interacted in the target (Compton scattering, pair production) to produce charged particles that could then

hit the IA counters and veto the event. This would not occur with the radiative capture gamma, because there is only one of these photons at a time. This vetoing modifies the observed Panofsky ratio, and its effect was calculated using GEANT. A second effect is due to the different photon interaction probabilities in the C and CH₂ targets. Because of the differing densities and radiation lengths, a photon was more likely to interact in the C target than the CH₂ target. If one normalizes the C and CH₂ data simply by the number of pion stops (LT 's), this effect is neglected, and too few π^- C events are subtracted from the π^- CH₂ spectrum. This effect was also calculated using GEANT. Both of these (small) effects have been compensated for in figure 4.14. The agreement here is somewhat less impressive, especially at the upper end of the π^0 portion of the spectrum. Part of this may be related to the necessary subtraction of the π^- C spectrum, but it is more likely attributable to inaccuracies in the simulation of the low-energy tail of the response for the 129 MeV photon. The detector resolution was quite poor for photons with energies as large as 129 MeV; at these energies, the tracks in the TPC have very large radii of curvature (i.e. are quite straight), and so very small changes to the fitted track cause large changes in the momentum calculated. Thus it is perhaps not too surprising that the Monte Carlo does not reproduce the exact details of the resolution at these energies. It should be noted that the RMC spectrum has an endpoint below these energies. At the low-energy end of the π^0 spectrum where one has more confidence in the accuracy of the Monte Carlo, the data are well-reproduced by GEANT.

Because only the high-energy portion of the RMC photon spectrum is used in the calculation of the partial branching ratio, a systematic error due to the energy calibration at 57 MeV must be assigned. This was estimated from comparisons between GEANT and the data for the $^{12}\text{C}(\pi^-, \gamma)$ spectrum, the low-energy end of the π^0 box and the spectrum of bremsstrahlung photons from positrons from μ^+ decay (this last spectrum will be discussed in the next section). The energy

calibrations of the GEANT predictions for each of these spectra were varied and compared to the data. The difference between the predicted calibration and the calibration that yielded the best fit to the data (i.e. with the smallest χ^2) was taken as the systematic error in the calibration. In all three cases the error was less than 400 keV, and so an overall energy calibration error of 400 keV was assumed. The high-energy end of the π^0 box and the 129 MeV peak were not used in the determination of this uncertainty, due to the resolution problem mentioned above. If they were used, the energy calibration error would be increased to about 1.1 MeV. Note that these are less relevant, as it is the error in the energy calibration at 57 MeV that contributes to the error in the partial branching ratio.

4.6 Backgrounds

The measurement of a relatively rare process such as RMC requires that considerable attention be paid to all possible sources of backgrounds. This section describes the methods used to reduce or eliminate potential sources of backgrounds (i.e. real photons that are not due to RMC on the target) and provides quantitative estimates of their effects.

4.6.1 Radiative Pion Capture

One background that has typically plagued previous measurements of RMC is radiative pion capture. Since muons are produced from pion decay, there is usually some pion content in the muon beam. Table 4.3 shows the branching ratios for radiative pion capture for the three targets investigated in this work. Clearly, the branching ratios are several orders of magnitude larger than those for RMC, and so even a very small pion contamination in the beam can cause a photon background larger than the RMC signal. Figure 4.15 shows measured radiative pion capture spectra from these three targets, which were obtained by tuning the

Table 4.3: Radiative pion capture branching ratios.

Target	(π^-, γ) Branching Ratio	Reference
^{12}C	$(1.83 \pm 0.06) \%$	[101,103]
^{16}O	$(2.27 \pm 0.24) \%$	[104]
^{40}Ca	$(1.82 \pm 0.15) \%$	[103]

channel for pions. The spectra are all quite similar, and while they peak at photon energies somewhat above the endpoint for RMC (~ 90 MeV), with the present energy resolution a substantial fraction of the spectrum lies in the energy region of the RMC spectrum.

This potential background was reduced in several ways. The first stage was the RF separator, after which the π/μ ratio in the beam was approximately 10^{-3} . Secondly, due to the shorter range of the pions, the majority of pions remaining downstream of the RF separator stopped in the beam counters rather than passing through to the target. The final stage of reduction was achieved through the exploitation of the fact that radiative pion capture, being a strong interaction process, occurs in prompt coincidence with the pion stopping in the target. A bit register was set if any two of the beam counters fired in coincidence with a photon trigger, and so events with this ‘prompt bit’ set could be rejected, as they were likely pion-induced. This was the PROMPT cut. Figures 4.16, 4.17 and 4.18 show the spectrum of all such prompt events for the three targets; in the oxygen and carbon cases, one can clearly see a radiative pion capture spectrum. There are also some events that are in fact RMC and bremsstrahlung events that were randomly in coincidence with an incident beam particle (the correction for the loss of these events due to the PROMPT cut will be discussed later). From the observed number of radiative pion capture events in the prompt spectrum, the pion content of the beam reaching the target was determined to be typically 1.5×10^{-5} in reasonable agreement with the

Figure 4.15: Measured radiative pion capture photon spectra, 1.0 mm converter.

Figure 4.16: Prompt photon events from ^{40}Ca , 1.0 mm converter.

value measured in the $\mu - e$ conversion experiment of $\sim 2.0 \times 10^{-5}$ [92]. The larger radiative pion capture signal seen in the carbon data (see Figure 4.18) is due to a subset of the data (about 20% of the total) where the pion content of the beam was an order of magnitude larger than normal, due to the phase of the RF separator being improperly set.

The pion-rejection efficiency of the PROMPT cut was measured separately by tuning to a pion beam, and determining the fraction of radiative pion capture events that did not have the prompt bit set. This was found to be $(0.61 \pm 0.10)\%$. The number of pion-induced events remaining in the RMC spectrum after the prompt cut could then be determined by applying this fraction to the number of radiative pion capture events seen in the prompt spectrum; the results are given in Table 4.4. The background was clearly at an acceptable level in all cases. The overall suppression factor of the pion-induced background was typically 10^7 . It should be noted that the suppression efficiency could have been further improved by placing cuts on the

Figure 4.17: Prompt photon events from ^{16}O , 1.0 mm converter.

Figure 4.18: Prompt photon events from ^{12}C , 1.0 mm converter.

Table 4.4: Background from Radiative Pion Capture.

Target	Converter	(π^-, γ) events before PROMPT cut	(π^-, γ) events after PROMPT cut	% of RMC signal
^{12}C	0.6 mm	~ 7	0.04 ± 0.01	0.04 ± 0.01
	1.0 mm	~ 700	4.3 ± 0.7	0.90 ± 0.14
^{16}O	1.0 mm	~ 40	0.24 ± 0.04	0.06 ± 0.01
^{40}Ca	0.6 mm	~ 7	0.04 ± 0.01	< 0.01
	1.0 mm	~ 10	0.06 ± 0.01	< 0.01

pulse heights of individual beam counters at the time of the event as was done in the $\mu - e$ conversion experiment. In that experiment an overall pion-suppression factor of about 10^{10} was achieved, using the same beam counters. These additional cuts were not required here, as the background was already at an acceptably low level. A comparison of the non-prompt RMC spectra with the radiative pion capture spectra clearly shows the almost complete absence of pion-induced photons after the PROMPT cut.

In principle, there is an additional possible pion-induced background: photons from pions that did not pass through the beam counters (and therefore which would survive the prompt cut). These could be due to pion interactions upstream in the beamline, or even from other beamlines. This has been investigated thoroughly by Virtue *et al.* [105] in their measurement of the photon asymmetry in RMC on ^{40}Ca , where it was found to be a severe problem for their NaI-based measurement. In the present measurement this background was not a problem, as the trigger and geometry of the detector selected photons originating in the target. Photons entering the detector from elsewhere in the experimental hall have a negligible probability of being detected. This was explicitly verified in the cosmic-ray background runs; a relatively large fraction of this data was taken when beam was being delivered to the neighbouring beamlines, but no difference was observed in the photon background rate from that observed when the cyclotron was off.

4.6.2 Cosmic rays

Photons from electromagnetic showers induced by cosmic rays were another possible source of background. More than 750 hours of cosmic-ray background data were obtained, between and after various data taking runs. A relatively large beam-off trigger rate of about $.018 \text{ s}^{-1}$ was observed. These were overwhelmingly composed of single charged particles traversing the chamber, and skimming the IC-counters (thereby missing the IA and IB veto scintillators), and firing the outer trigger counters in two sectors (i.e. one in the top half of the TPC and one in the bottom half). None of these single-track events survived the cuts to select photon events.

A small rate of real photons was observed ($3 \times 10^{-4} \text{ s}^{-1}$) in the cosmic ray background data. In most of these, one or more of the cosmic-ray drift chambers or scintillators had fired in prompt coincidence with the event, presumably due to other components of the electromagnetic shower. Also, for many of these events, a larger multiplicity of trigger counters (IC, IWC, E and W counters) was observed than was usual for a single photon event in the calibration data. Consequently, a set of cosmic-ray cuts was developed, based upon various combinations of cosmic-ray counters and trigger counters having fired. After these cuts were applied, only a few events remained, corresponding to a rate of (1.6 ± 0.2) events/day. The energy spectrum of these events is shown in figure 4.19. The fact that the spectrum appears to peak between 50 and 120 MeV is likely just a reflection of the fact that the detector acceptance was optimized for this energy range. The rate for cosmic-ray photons in the energy range of the RMC spectrum (57 to 100 MeV) was (0.8 ± 0.1) events/day. The number of cosmic-ray photons in the RMC spectrum can be estimated from this rate, using the total time of each measurement, and correcting for the fraction of real time that the *LT* gate was open (since a cosmic-

Figure 4.19: Measured cosmic ray background photon spectrum, 1.0 mm converter.

Table 4.5: Cosmic ray background.

Target	Converter	Total Live Time (10^5 s)	Cosmic Ray Events in RMC region	% of RMC signal
^{12}C	0.6 mm	2.444	1.6 ± 0.2	1.6 ± 0.2
	1.0 mm	7.992	7.4 ± 0.9	1.5 ± 0.2
^{16}O	1.0 mm	4.536	4.2 ± 0.5	1.2 ± 0.1
^{40}Ca	0.6 mm	2.442	1.6 ± 0.2	0.20 ± 0.03
	1.0 mm	3.223	3.7 ± 0.4	0.18 ± 0.02

ray photon would have to fall within this gate to satisfy the trigger). Table 4.5 shows the results for the three targets (as almost all the cosmic-ray background data were obtained with the 1.0 mm converter, the background rate for the data taken with the 0.6 mm converter was estimated by scaling by the relative photon acceptances of the two converter thicknesses). In each case the cosmic-ray background was $\leq 2\%$ of the RMC signal.

Analysis of the cosmic-ray background events indicated that the rate could be

reduced by another factor of about three, by making some of the other cuts more restrictive, as well as by tightening the cosmic-ray cuts further. For example, since cosmic-ray photons do not originate in the target, the location of their conversion point along the z -axis (ZTARGET) was distributed uniformly along the converter, and so a large fraction of them could be rejected by making the cut on ZTARGET tighter, at some cost in acceptance, however. This was not considered to be necessary, since the cosmic-ray background rate with the present cuts was already at an acceptable level.

There was some probability for a valid RMC photon to be rejected by the cosmic-ray cuts, if it was randomly in coincidence with a cosmic-ray shower. This was rather unlikely, and since the probability would be the same for this to happen to a photon in the calibration data, the measured acceptance was implicitly corrected for this effect.

4.6.3 Bremsstrahlung

Some fraction of the muons stopping in the target will not undergo nuclear capture, but will decay instead. These muon decay events can produce photons in one of two ways: either via bremsstrahlung of the decay electrons in the target or surroundings (external bremsstrahlung), or via radiative muon decay (internal bremsstrahlung).

For the decay of the free muon, neither of these processes can produce photons of greater than 53 MeV (the end point of the electron spectrum from muon decay). Below this energy, however, any RMC experiment has to deal with a copious background of photons from these processes, especially for light nuclei, where the majority of muons in the target decay rather than capture. This background therefore limits measurements of RMC to the portion of the spectrum above the 53 MeV threshold. Typically a lower limit of 57 MeV has been used, to allow for finite

detector resolution. This represents only about the top third of the RMC photon spectrum.

In fact, the situation is slightly more complicated than described above, because the decay electron spectrum from muons bound in orbit around a nucleus is not the same as the spectrum for free muons. The orbital motion of the muon produces a Doppler smearing of the energy spectrum, and the nuclear recoil effects can also distort the spectrum, producing electrons of energy > 53 MeV. The bound muon decay spectrum has been calculated by several authors (although not always for the specific nuclei considered in this thesis); here, the work of Hangii *et al.* [106] is used. They derived an analytical expression for the energy spectrum, as a function of the Z of the nucleus, using a plane-wave Born approximation for the final electron wave function, and ignoring nuclear recoil effects. They found that for $Z \leq 20$ this expression provided a very good approximation to the more complete calculation which included nuclear recoil; therefore it is applicable to the nuclei under investigation here. Figure 4.20 shows the resulting electron energy spectrum from this analytic expression (equation (13) of [106], corrected for several misprints, as per Herzog and Alder [107]) for ^{16}O . As one goes to nuclei with smaller Z , the spectrum becomes closer to the free muon decay spectrum. Figure 4.21 shows the high-energy portion of this spectrum, demonstrating that there is a (very small) contribution to the spectrum from electrons of energy up to the muon mass, 105.7 MeV.

To determine the effect of this on our data, a special version of GEANT was used. In this version, electrons were generated in each of the targets, with the energies sampled from the bound muon decay spectrum for the respective target, and with initial positions sampled from a realistic muon stopping distribution in the target. In each case, more than 10^6 electrons were generated. Each electron was tracked until it either stopped, or hit one of the veto scintillators (thereby vetoing

Figure 4.20: Electron energy spectrum from bound muon decay in ^{16}O .

Figure 4.21: Electron energy spectrum from bound muon decay in ^{16}O , high energy region.

Figure 4.22: Bremsstrahlung photon energy spectrum from bound muon decay in ^{12}C .

the event), and the energies of all bremsstrahlung photons produced were recorded. Figure 4.22 shows the resulting photon spectrum for the ^{12}C case. For each target, the spectrum for all photons that were not vetoed was then folded with the detector response function, to produce the expected experimental spectrum. Figure 4.23 shows the resulting spectrum for ^{12}C .

As mentioned above, however, external bremsstrahlung is not the only contributing process here. The radiative muon decay (internal bremsstrahlung) process also produces photons from muon decay. For the targets used here, especially ^{12}C , the decaying electrons typically traverse only a small fraction of a radiation length of the target, and consequently have a relatively small probability to undergo external bremsstrahlung. Radiative muon decay then becomes an important contribution to the photon spectrum. The branching ratio and photon spectrum for radiative decay of *free* muons has been calculated by Behrends *et al.* [108]; however no cal-

Figure 4.23: Bremsstrahlung photon energy spectrum from bound muon decay in ^{12}C , folded with detector response function.

ulation exists for radiative decay of *bound* muons. Thus, in this analysis, the free radiative decay spectrum has been used (which ends at 53 MeV), and the implicit assumption has been made that any distortions to the photon energy spectrum due to the muon's being in a nuclear orbit are relatively small.

The calculated sum of the external and internal bremsstrahlung spectra for each target, folded by the detector response function, can then be compared to the observed spectra. Of course, the contribution from RMC must be subtracted out. Figure 4.24 shows this comparison for ^{12}C , where a simple polynomial ansatz has been used for the RMC spectrum. The ^{16}O case is similar. For ^{40}Ca the RMC signal overwhelms the bremsstrahlung background, and no such comparison was attempted. While the agreement in the shape of the spectrum is reasonable, the absolute magnitude of the predicted bremsstrahlung spectrum had to be normalized *up* by 12% in the ^{16}O case and *down* by 8% for the ^{12}C case. There are several reasons

Table 4.6: Fraction of bound decay bremsstrahlung spectrum above 57 MeV. The effect of this on the RMC signal depends on the relative ratio of RMC to bremsstrahlung for the given target (see text).

Target	% of observed spectrum > 57 MeV
^{40}Ca	1.29 ± 0.21
^{16}O	0.23 ± 0.05
^{12}C	0.18 ± 0.03
free decay	0.20 ± 0.03

that this agreement should not be expected to be much better. The magnitude of the contribution from external bremsstrahlung is very dependent on the details of the stopping distribution in the target, which is not well known (note that the RMC signal is much less sensitive to this stopping distribution, as it is only the few percent of photons that are lost due to interactions in the target that are affected). Also, there may be contributions from electrons that undergo bremsstrahlung elsewhere than the target; their contribution is dependent on the small inefficiency of the veto scintillators. Finally, the bremsstrahlung events are at an energy where the detector acceptance is exceedingly low (see Figures 4.11 and 4.12), and therefore not as well determined by the Monte Carlo. The observed bremsstrahlung spectrum is a convolution of a photon spectrum that falls rapidly with energy with an acceptance that rises steeply with energy, and so its magnitude is very sensitive to small errors in either. In light of these considerations an agreement of better than about 20% is reasonable.

With the bremsstrahlung spectra produced for each target, the fraction of the observed spectrum appearing above 57 MeV can be predicted. These are given in Table 4.6 and are compared with the prediction for bremsstrahlung from free muon decay. The results clearly show that for ^{12}C and ^{16}O , the fraction of the bremsstrahlung spectrum appearing above 57 MeV is consistent with that from free

Figure 4.24: Bremsstrahlung photon energy spectrum from bound muon decay in ^{12}C , folded with detector response function, compared to data. The histogram is the ^{12}C μ^- data with the RMC signal subtracted, using a simple polynomial ansatz for the theoretical RMC spectrum. The smooth curve is the predicted bremsstrahlung spectrum (from Figure 4.23).

decay bremsstrahlung, i.e. that the muon binding effects are small. Only in the ^{40}Ca case is there a significant difference. In principle, the normalized bremsstrahlung spectra could be used to predict directly the background underneath the RMC signal due to bremsstrahlung. This was not done; the reason is presented below.

As discussed earlier, the parameterization of the detector response function does not accurately represent the extreme ‘tails’ of the detector response. It also could be argued that GEANT itself may not accurately model these tails in the response, as they are due to rather rare occurrences in the details of the track reconstruction. Thus, there is likely an additional contribution from photons which were less than 57 MeV in energy, but were reconstructed as being above 57 MeV. Rather than rely on the Monte Carlo to reproduce the contribution from this high-energy tail in the detector response, this background was explicitly measured using the μ^+ beam.

When a μ^+ stops in the target, it does not become bound to a nucleus, and therefore it cannot undergo nuclear capture. Instead, all the incident muons will decay with the free muon lifetime. Thus the RMC process is ‘turned off’ but the bremsstrahlung processes remain. Any event observed above 53 MeV is therefore due to the high-energy tail of the detector resolution (or due to a cosmic ray). This allows an experimental determination of the high-energy tail. The details of the μ^+ bremsstrahlung spectrum are slightly different from those for the μ^- case, due to the effects discussed above. However, the differences are small, and the contribution from the high-energy tail can be considered independently from the bound decay effects. A second difference between the μ^- and μ^+ spectra is due to the additional process of positron annihilation in-flight. This causes an enhancement of the photon spectrum, but it does not significantly affect the shape of the spectrum near the endpoint.

Data were taken with the μ^+ beam on both ^{12}C and ^{16}O targets, and, as

expected, the results were very similar. Figure 4.25 shows the sum of the two spectra. A small high-energy tail in the response function is clearly visible (the expected number of cosmic-ray events in this spectrum is ~ 3). The offending events were examined in detail, and they each appeared to be genuine photons. In several cases the cause of the poor resolution was apparent, but each case was different. As discussed earlier, there is a compromise between resolution and acceptance, and several of these events could have been rejected by making some of the cuts more restrictive, at considerable expense in acceptance, however. On the other hand, a loosening of the cuts would entail an increase in the high-energy tail. As a general comment, the high-energy tail would have been smaller if there had been more points found per track in the TPC; this will be discussed in more detail when the rate dependence is considered.

The observed high-energy tail, defined as the fraction of the bremsstrahlung events observed above 57 MeV, was $(2.1 \pm 0.3)\%$. The error is purely statistical. By normalizing to the number of bremsstrahlung events below 57 MeV in the μ^- data (after subtracting the contribution from RMC below 57 MeV), the contribution of the high-energy tail to the observed spectrum above 57 MeV can be estimated. Table 4.7 gives these estimates. For the ^{12}C and ^{16}O cases, the tail calculated from the μ^+ spectrum alone has been used; as discussed above, the differences between the *free* decay bremsstrahlung spectrum (the μ^+ spectrum) and the *bound* muon decay spectra for these targets are negligible (see Table 4.6). For ^{40}Ca , the contribution estimated from the μ^+ data has been added to the *difference* between the contributions from free decay and bound decay in ^{40}Ca (given in Table 4.6), to account for the muon-binding effects. This gives an *effective* high-energy tail of $2.1\% + 1.1\% = 3.2 \pm 0.4\%$ for ^{40}Ca .

The tail contribution was quite significant, especially for ^{12}C , and this is in

Figure 4.25: Photon energy spectrum from μ^+ stopping in ^{16}O and ^{12}C . The events above 53 MeV are due to the high-energy tail in the detector response function.

Table 4.7: Background from high-energy tail of the detector response function.

Target	Converter	# of events > 57 MeV	% of RMC signal
^{12}C	0.6 mm	22 ± 3	17 ± 2
	1.0 mm	82 ± 12	14 ± 2
^{16}O	1.0 mm	24 ± 4	6.2 ± 0.9
^{40}Ca	0.6 mm	7 ± 1	0.50 ± 0.07
	1.0 mm	6 ± 1	0.30 ± 0.04

fact the single largest background in this measurement. Since μ^+ data were taken only using the 1.0 mm converter, the assumption has been made that the high-energy tail was the same for the 0.6 mm converter. This is reasonable, since the high-energy tail was not (primarily) due to energy losses in the Pb, but rather to the track-fitting.

For the final analysis, the normalized μ^+ data were simply subtracted from the μ^- spectrum to yield the final RMC spectrum; in principle, this would allow the RMC data below 57 MeV to be used in the extraction of the branching ratios, however, this was not done, simply because of the added uncertainty due to the subtraction of a large background, and also due to the minor differences between the shapes of the μ^+ and μ^- bremsstrahlung spectra.

4.6.4 Muon stops outside the target

Some fraction of the incident beam did not stop in the target, but instead was scattered into the target surroundings, which was primarily composed of scintillator material (the IA and IB counters and the veto counter). Since the scintillator plastic is a compound of carbon and hydrogen, these muons provide a potential source of background due to the very small fraction of them that undergo RMC (mainly from the carbon). This was potentially a serious problem, especially for the ^{16}O and ^{12}C targets, where the RMC rate from the target material is also small. A

second potential problem was due to the fraction of these scattered muons which passed through the IA and IB counters and stopped in the Pb converter. Due to the strong Z-dependence of the OMC rate, almost all muons stopping in Pb would capture, and so the probability of RMC would be correspondingly large.

Some reduction in this background was afforded via the cuts on the origin of the photon (ZTARGET and RTARGET), as well as by the (small) reduction of solid angle for photons originating from the target surroundings. This reduction factor was estimated via GEANT, and was found to be $\eta = (0.51 \pm 0.15)$, where the error reflects the uncertainty in the knowledge of the spatial distribution of the muon stops outside the target. Because a muon stopping in the IA or IB counters (or the Pb) would fire the IA counter, it would not produce a $STOP\bar{I}$, and consequently would not produce an LT gate. Therefore, a photon from RMC in the target surroundings would be uncorrelated in time with the LT gate, and so the background was also reduced by the fraction of time that the LT gate was not open.

The fraction of the muon beam that scattered into the scintillators surrounding the target was measured by CAMAC scalers (i.e. by looking at $STOP \cdot I$ rather than $STOP\bar{I}$). The contribution to the RMC spectrum from this background can then be estimated if one makes an assumption about the RMC branching ratio for the surroundings (i.e. for carbon) relative to that for the target. At this stage we will approximate this by the ratio of the OMC rates, that is, we will assume that RMC/OMC is the same for ^{40}Ca , ^{16}O and ^{12}C ; the calculation can then be corrected using the final results of this thesis (we shall see that the approximation is indeed reasonable). Table 4.8 gives the fraction of scattered muons, the ratio of OMC rates, the reduction due to the LT gate, and the resulting background/signal ratio; the error is dominated by the error in the reduction factor η estimated via GEANT (the fraction of muons scattered outside the target was larger for the Ca target

Table 4.8: RMC from muon stops in the scintillators, estimated from the muon stopping rate in the scintillators. The relative OMC rate is the rate of OMC in the carbon of the scintillators relative to the OMC rate in the target. The LT factor is the reduction factor in this background due to the finite LT gate.

Target	(scattered μ 's)/(μ -stops)	relative OMC rate	LT factor	% of RMC signal
^{12}C	$0.15 \pm .01$	1.00	0.80	6.1 ± 1.8
^{16}O	$0.15 \pm .01$	0.417	0.78	2.5 ± 0.8
^{40}Ca	$0.35 \pm .02$	0.090	0.33	0.5 ± 0.2

than for the others, due to its somewhat smaller size). This calculated correction does not include possible contributions due to muon stops in the Pb converter, as the fraction of the scattered muons that reached the Pb is unknown. This fraction is expected to be quite small; the scattered muons were already at relatively low energy due to energy loss in the beam counters, and so most should have stopped in the IA counter. A GEANT simulation of the muon's stopping distribution suggests that $< 1\%$ of the scattered muons end up in the Pb, but this estimate was very sensitive to the (poorly known) characteristics of the fringes of the muon beam.

This calculation can be checked by examining the time distribution of the photon events. As mentioned, photons from RMC in the target surroundings would be uncorrelated in time with the LT gate, and so would provide, in principle, a time-independent background to the time distribution; conversely, they should show a correlation with the signal $STOP \cdot I$. Therefore, the background contribution can be extracted by fitting the time distribution of the photon events (relative to the time of the LT gate) and determining the component that does not exhibit the correct muon lifetime in the given target. Pileup effects, however, make this procedure slightly more complicated than stated above.

At the beam rates used in this investigation, several muons were likely to be in the target at any given time. Consequently, any given photon event could be

related to one of several previous muon stops. Thus the time difference between the last muon stop (i.e. the *LT* gate) and the photon event produces an apparently shorter lifetime than the actual muon lifetime in the target. In addition, a time-independent background produces an apparent lifetime due to this pileup effect. These distortions can be calculated exactly, given the incident muon rate and the muon lifetime in the target. A small Monte Carlo routine (LTCORR) was written to produce the distorted lifetime distribution; this routine also calculated the fraction of good photons events rejected by random coincidence with another incoming muon (via the PROMPT cut) and fraction of events lost because they occurred after the end of the *LT* gate.

The time distribution of the good photon events (relative to the *LT* gate) was fitted to a sum of two exponentials, one with the apparent lifetime appropriate for the target, and one with the apparent lifetime of a time-independent background at the relevant muon stopping rate. The amplitudes of the two exponentials were free parameters in the fit. Fits were performed both for all photon energies (i.e. including the bremsstrahlung background) and also for those photons in the RMC region ($E_\gamma > 57$ MeV); the bremsstrahlung background should exhibit the same time dependence as the RMC signal, and this adds considerably to the statistics of the time distribution. Figure 4.26 shows the fit for the 1.0 mm converter ^{16}O data for all photon energies. Table 4.9 gives the results of these fits. Note that the estimated background here includes all photons that are uncorrelated with the *LT* signal, that is, it includes RMC from the scintillators and the Pb converter as well as cosmic ray background. The results are, in all cases, consistent with the values calculated from the stopping rates. However, these time distribution results have larger uncertainties, due to the difficulties in fitting the low-statistics data.

An alternate way of considering this background was to assume the background is as calculated using the muon stopping rate in the scintillators (Table 4.8).

Table 4.9: Background due to RMC in material surrounding the target, determined from fits to the time distribution of photon events.

Target	Converter	All photons % of signal	$E_\gamma > 57$ MeV % of signal	Calculated % of signal (from Table 4.8)
^{12}C	0.6 mm	$3.1^{+4.4}_{-3.1}$	$4.2^{+9.1}_{-4.2}$	6.1 ± 1.8
	1.0 mm	8.2 ± 3.6	< 7.3	"
^{16}O	1.0 mm	< 2.5	< 7.5	2.5 ± 0.8
^{40}Ca	0.6 mm	< 3.1	< 2.2	0.5 ± 0.2
	1.0 mm	< 1.3	2.3 ± 1.8	"

Table 4.10: Muon lifetimes, determined from fits to the time distribution of photon events, compared with best existing data [109].

Target	τ_μ (ns)	Accepted τ_μ (ns) [109]
^{40}Ca	322 ± 15	332.7 ± 1.5
^{16}O	1690 ± 67	1795.4 ± 2.0
^{12}C	2110 ± 90	2026.3 ± 1.5

The time distribution of the photon events were then fitted to the same two exponentials as before, with the relative amplitude now fixed, and the effective muon lifetime in the target as the free parameter. This effective muon lifetime could then be corrected to a ‘true’ lifetime (τ_μ) by interpolating the results of the pileup correction simulation (LTCORR). The results are given in Table 4.10 and compared with the best existing data for τ_μ in the three targets. The level of agreement is impressive, especially considering that the present experiment was not designed to measure the muon lifetime. This adds confidence that the level of background due to muons that stop elsewhere than in the target was small and well understood.

For the data taken with the ^{16}O target only, an independent means of checking this background was available. For these data, a channel of multi-hit FASTBUS TDC was available with the input being $STOP \cdot IA$, i.e. a particle scattered into the IA scintillator, and the TDC stop being the time of the photon event. Therefore,

Figure 4.26: Time distribution of photon events from ^{16}O target, relative to the last LT gate. All photon energies are included. The fitted curve is an exponential with the effective muon lifetime in ^{16}O (i.e. with account taken of pileup).

Figure 4.27: Time distribution of photon events from ^{16}O target, relative to muons stops in the scintillators; $E_\gamma > 57$ MeV only. The fitted curve is an exponential with the muon lifetime in ^{12}C plus a constant.

the RMC signal should be uncorrelated with the time of the hits in this TDC, but any background from RMC in the scintillator or the Pb should show the appropriate lifetime. Figure 4.27 shows this time distribution for $E_\gamma > 57$ MeV, along with a fit to a flat component (the signal from the target) and an exponential component with the muon lifetime in carbon (the background). The results $((4.5^{+5.0}_{-4.5})\%$ of the RMC signal using $E_\gamma > 57$ MeV; $(3.2 \pm 3.0)\%$ of the RMC signal using all photon energies) are again consistent with the calculated background. Note that any background from Pb would appear entirely in the prompt bin (the muon lifetime in Pb is 75 ns), where there was in fact a ‘hole’; the hole was at least partially due to the PROMPT cut.

Due to the rather large uncertainties in the estimates of this background obtained from the fits to the time distributions, we will use the estimate calculated

from the measured stopping rates to account for this background.

4.6.5 Other Backgrounds

In addition to the background processes discussed above, a few other backgrounds to a measurement of RMC are conceivable. One of these is the copious flux of neutrons from ordinary muon capture. This has posed serious troubles for most previous RMC measurements, as the NaI detectors typically used are in fact very efficient neutron detectors. The branching ratio for high-energy neutrons is larger than that for RMC photons by about a factor of three, and so very efficient neutron discrimination is required. This background was not a problem in the present experiment, as the TPC is completely insensitive to neutrons. Similarly, muons scattered into the TPC, or protons from μ^- capture in the target or (n, p) reactions in the target area, would be easily rejected by their large energy loss in the TPC (i.e. by the *DEDX* cut). As single track events they would also be extremely unlikely to mimic a valid e^+e^- pair.

A final possible background is due to the electrons that comprise $\sim 1\%$ of the muon beam. It is possible that an electron could undergo hard bremsstrahlung in the target and produce a high-energy photon. Due to conservation of momentum, the high-energy photon spectrum from bremsstrahlung is very forward-peaked, and so these photons would have only a very small probability of converting in the Pb converter and triggering the detector. Of course, such events would be in prompt coincidence with the electron firing the beam counters, and so they would be rejected by the cut on the prompt bit. This background was therefore negligible.

4.7 Rate Dependence

Early in the course of this experiment, a troubling aspect of the performance of the detector was found: there was a significant rate dependence of the observed photon acceptance. This was seen most clearly in the pion calibration runs, where it was observed that the number of good photon events per stopped pion (i.e. photons per LT) decreased as the pion stopping rate was increased. Obviously, this is a potential problem in making an absolute measurement of the RMC rate. A considerable amount of effort was devoted, both during the experimental runs and in offline data analysis, to finding the cause of this rate effect, and to finding a means to correct for it. Only a brief account of this effort is given here.

Figure 4.28 shows the observed rate effect from the pion calibration runs, for both π^-CH_2 and π^-C , and illustrates several features of the effect. The photon acceptance falls approximately exponentially, when plotted as a function of the rate of firing of one of the sets of anode wires of the TPC ('Wire # 1 rate' refers to the summed rate of all 12 of the innermost set of wires of the TPC). The rate of firing of the TPC anode wires is, of course, proportional to the charged particle activity in the chamber. A similar exponential dependence was found if the acceptance is plotted versus other rates that are also related to the activity in the TPC, e.g. the pion stopping rate, incident beam rate, or the singles rates in the trigger counters. One also sees clearly that the effect is, at least for the pion calibration runs, independent of the target used. The physical radiative pion capture probability obviously cannot be dependent on the rate that pions are stopped in the target. There are two possibilities to explain the rate effect: either the number of pion stops was being overestimated at high rates, or the detector acceptance for photons was decreasing at high rates. As will be seen, the latter was the case.

Rate dependences in detectors are, of course, not unexpected. Several possi-

Figure 4.28: Rate dependence of photon acceptance, 1.0 mm converter.

ble explanations were initially considered, but these were unable to account for the observed effect. One possibility was the effect of dead-time in the electronics and data acquisition system. Dead-time was correctly compensated for through a series of 'blanks' in the electronics, which disabled the counting of the incident beam particles when the electronics or computer was busy, and thereby unable to accept an event. Several tests were used to confirm that dead-time was indeed being correctly accounted for. These included artificial changes in the trigger, and in ancillary processes running on the PDP computer, thereby changing the dead-time, while maintaining the beam rate constant. The observed acceptance remained constant, thereby demonstrating that the rate effect was not caused by dead-time.

A second obvious possibility was a systematic miscounting of the number of incident pions, due either to pileup or efficiency changes in the beam counters. Pileup occurs when two or more incident particles arrive in the same beam-bucket, and therefore within the time-resolution of the electronics, and are counted as one single particle. Even at the maximum rates used ($\sim 4.0 \times 10^5 \text{ s}^{-1}$), this effect was negligible ($\sim 1\%$), and in fact would act in the direction opposite to the observed effect. Similarly, the efficiency of the beam counters would, if anything, likely *decrease* with increasing rate, and so could not account for the rate dependence. The efficiency of the individual beam counters was measured as a function of incident rate, and no variation was found.

The incident beam rate was typically varied through the use of the slits and jaws in the beamline. Changes in the positions of the slits and jaws also affect other characteristics of the beam, including the profile and momentum dispersion. These changes could conceivably affect the stopping distribution of the incident pions (or muons), thereby affecting the observed acceptance (either through solid-angle effects, or problems with misidentifying particles that stop in the veto scintillators as stops in the target). This possible explanation was eliminated empirically, by

two sets of systematic studies. In one study, the rate was varied solely by using the vertical slit (which primarily changes the beam profile) and then the rate was varied only by using the horizontal slit (which affects both the profile and momentum dispersion). The observed rate dependence was identical in the two cases. A second study was done, in which the incident beam flux was varied by lowering the cyclotron ion-source current, leaving the slits and jaws untouched. Again, the identical rate dependence was observed. Clearly, the rate dependence was unrelated to the manner in which the flux was varied.

There is, in fact, one factor that was expected to contribute to a rate dependence in the counting of the incident beam. There was a finite probability that a particle stopping in the target will be accepted as a valid LT , but the trigger from a resulting photon was rejected because an IA or IB scintillator had fired. The singles rates in the IA and IB scintillators were due not only to beam particles scattered into them, but also due to bremsstrahlung photons and delta-rays from beam particles, x -rays from the RF separator and electrons from muon decay in the target (recall that the pion calibration beam was composed of similar fractions of π 's, μ 's and e 's). In principle, this should not cause a problem for the IA scintillators, as the IA scintillators were required *not* to have fired for a $STOP\bar{I}$ (and thus a LT) to be generated. However, small differences in the timing in the electronics could cause the signal in the IA not to veto the $STOP\bar{I}$, but to veto the trigger at the $CONV$ coincidence or in the MB10/6 units. As the IB's were not used in the definition of $STOP\bar{I}$, a signal from one of these would also prevent the event from triggering. Because the singles rates in the IA and IB counters were beam-rate dependent, the probability of the event being rejected through a random coincidence between the particle stop and a hit in the IA and/or IB counters would increase with beam flux, thereby producing a decrease in the observed acceptance.

The random vetoing effect described above was observed in the data, and it

Figure 4.29: Rate dependence of the trigger rate for $^{12}\text{C}(\pi^-, \gamma)$. The line is a guide to the eye.

contributed to the rate effect. However, this was a relatively minor effect, and could explain only a small fraction of the observed rate dependence. The random vetoing could be directly monitored from the trigger rate (i.e. the LAM/LT ratio); see figure 4.29. Note that the variation in this ratio in fact represents *all* contributions to the rate-dependence from the trigger electronics.

To ensure that the variation in the trigger rate was indeed dominated by the random vetoing effect, several studies were performed. In one study, the rate dependence was measured with the RF separator turned completely off; this increased the relative numbers of μ 's and e 's in the pion beam, thereby changing the singles rates in the IA and IB counters for the same pion stopping rate. The trigger rate was shown to vary with the IA and IB singles rates in the same manner as with the RF separator on. Also, a series of runs were taken in which the phase of the RF separator was varied, with the incident flux maintained constant. By changing

the phase, the relative proportions of particles in the beam could be continuously varied. Again, the trigger rate was found to vary in the same manner with the IA and IB singles rates. In all these cases, if the observed photon acceptance was corrected for this random-vetoing effect (using either the IA and IB singles rates, or the observed trigger rates to correct the acceptance), then the observed (residual) rate effect was found to be the same, independent of the composition of the beam. Therefore, this small component of the rate effect was well understood and easily corrected for. In the following discussion of the rate dependence it will be assumed that this random vetoing effect has already been corrected for.

Since only a small variation in the trigger rate (i.e. LAM/LT) was attributable to the random vetoing effect, the rate effect could not be attributed to rate dependent variations in the efficiencies of any of the trigger counters. As a final assurance that the rate dependent acceptance was independent of the trigger electronics, data were obtained with a single-track trigger, to detect electrons from muon decay. Here, the trigger was quite different from the photon trigger; in particular, there was no vetoing based on the IA and IB counters. A very similar rate dependence of the single track acceptance was observed (see Figure 4.30). Similar behaviour was observed in measurements of pion elastic scattering and the decay $\pi^+ \rightarrow e^+ \nu$ (these were made during other experiments performed with the TPC at about the same time as the present work). Clearly, the observed rate dependence was not an artifact of the trigger (or the particular track-fitting algorithms and cuts, as these were different for each of the different reactions). Consequently, the cause of the rate dependence must be a loss of efficiency of the TPC itself.

Various rate dependent effects are possible in TPC-like detectors, and several have been discussed by Sauli [110,111]. In the present case, the rate dependent acceptance was directly attributable to a reduction in amplitude of the cathode pad

Figure 4.30: Rate dependence of acceptance for muon decay electrons, for the ^{12}C target. The line is a fit to an exponential dependence on the wire rate .

signals. The physical origin for this reduction in amplitude will be discussed later; here a phenomenological description is given.

Analysis of pion calibration data taken at different rates uncovered several quantities that varied systematically with rate. The number of points found per helical track (N_{xyz}) decreased monotonically with increasing rate, as did the average amplitude of both the cathode pad and anode wire signals. As the rate increased, a larger fraction of events failed the track reconstruction routines, and a larger fraction of ‘pairs’ found were rejected by the cuts. In addition, as the rate increased, the innermost wires of the TPC were used relatively less often in the helix fits. Figures 4.31, 4.32 and 4.33 show, respectively, the rate dependence of the number of points per helix (N_{xyz}), the dE/dx and the relative use of the inner TPC wires in the helix fit. Since the physical dE/dx of the e^+ and e^- is not a function of rate, the observed dE/dx is a measure of the cathode pad pulse height. The anode wire

Figure 4.31: Rate dependence of number of points in helix fit for each track.
The line is a guide to the eye.

amplitudes also show a similar rate dependence (this is expected, as the cathode signal is just an image of the anode signal).

The variation of all these quantities with rate can be understood as being due to a decrease in the amplitude of the cathode pad signals. Because of the stochastic nature of the ionization process, for a given gain in the TPC proportional wire region, a broad distribution of pulse-heights will be observed on the cathode pad (and anode wire) signals. The gain of the system was such that, at low rates, the majority of these signals were large enough to exceed the hardware and software thresholds. However, as the effective gain was decreased, more and more signals are lost as they fall below threshold, and consequently fewer points per track are found in the event. Consequently the probability of correctly reconstructing the tracks is reduced, and so the photon detection efficiency decreases. The behaviour of the relative usage of the wires in the helix fit is also readily understood. Figure 4.34

Figure 4.32: Rate dependence of cathode pad amplitude. The line is a guide to the eye.

Figure 4.33: Rate dependence of relative usage of the inner 4 wires of the TPC in the helix fits. The line is a guide to the eye.

Figure 4.34: Distribution of signal amplitudes and singles rates across the TPC wires. Wire # 1 is the innermost set of wires and wire # 12 is the outermost. The lines are guides to the eye.

shows the variation of both the pad amplitude and rate of firing of the TPC wires as one moves from the innermost to the outermost wires. Where the singles rate was high, the relative amplitude was small and the wire was used relatively less often in the track fitting. This again shows the empirical relationship between the rate and the signal amplitude.

The data clearly show the loss in amplitude of the cathode pad signals. The question arises as to whether or not this loss in amplitude can explain the magnitude of the observed rate dependence. This was confirmed in detail using the GEANT Monte Carlo. However, a simpler indication that this was true is discussed first. A series of $^{12}\text{C}(\pi^-, \gamma)$ runs were taken where the incident pion flux was maintained constant, but the high voltage on the TPC anode wires was reduced. Consequently, the amplitude on the cathode pad signals was reduced, fewer points were found per track, and the photon acceptance decreased. Figure 4.35 shows the relative

photon acceptance versus dE/dx for these runs, as well as for $^{12}\text{C}(\pi^-, \gamma)$ taken with a constant anode voltage but with the rate varied. A similar dependence of the acceptance on the pad amplitude was observed in the two cases, although they differ somewhat in detail (note that the rate dependence runs have been corrected for the random vetoing effect). The difference can be explained in terms of the differing distributions of amplitudes across the TPC wires in the two cases. As the anode voltage was varied, the relative distribution of amplitudes on the TPC wires did not change; only the overall normalization of the amplitudes changed. In contrast, when the rate was varied, the relative distribution of amplitudes was found to change. For an individual wire, the signal amplitude depended approximately quadratically on the singles rate on that wire. Due to these differing distribution of amplitudes, it is reasonable to expect some difference in the dependence of the acceptance on dE/dx , which is effectively an average amplitude over all the wires. The general agreement seen in Figure 4.35 adds support to the idea that the observed rate dependence was attributable to a loss of amplitude with rate.

Confirmation that the observed rate effect was due to the reduction in the cathode pad amplitude was provided using GEANT. As discussed in detail in the appendix, the relative amplitude of the signals on the TPC cathode pads and anode wires were parameters input at the digitization stage. These parameters were adjusted so that the relative amplitudes agreed with the relative amplitudes observed empirically, both in the distribution across the TPC (i.e. moving from inner wires to outer wires), and in the way this distribution varied when an overall amplitude factor was varied. This single overall amplitude factor could then be adjusted at execution time to reproduce the empirical effect on the amplitudes observed in the data at different rates. The dependence of the Monte Carlo acceptance on the amplitudes could then be studied.

Figure 4.35: Acceptance versus pad amplitude for data taken with TPC anode high voltage varied and with rate varied, for $^{12}\text{C}(\pi^-, \gamma)$. The closed circles are data taken at the same rate but with the anode voltage varied and the open circles are data taken at the same anode voltage but with the rate varied. The lines are guides to the eye.

With the amplitudes varied in the above manner, a decrease in the photon acceptance was found in GEANT, similar to that observed experimentally. The GEANT results showed that the majority of the effect was due to the reduction in the cathode pad signals; the reduction in the (larger) anode signals was not enough to reduce a significant fraction of these signals to below threshold. The loss in acceptance was due to either the signal on the pads falling completely below threshold, and hence the (x, y, z) point being lost completely from the helix fit, or to the centroid of the cathode pad clump being incorrect due to the signals on the pads being only slightly above the noise level. The latter case worsens the quality of the fit, thereby increasing the probability of the event failing the cuts.

The sensitivity of the acceptance to other digitization parameters chosen in the Monte Carlo (besides the overall amplitude factor) was studied. The acceptance was also found to be rather sensitive to the value chosen for the noise on the cathode pads. As discussed in the appendix, the noise could be determined with reasonable accuracy from the data. However, significant variations in acceptance (for the same amplitude factor) could be produced by varying the cathode pad noise within a range consistent with the data. Acceptances differing by as much as $\pm 20\%$ could be obtained with the same amplitude factor, i.e. for a given dE/dx . Consequently, a variable other than dE/dx was required to determine the photon acceptance uniquely. Various studies with GEANT showed that N_{xyz} , the number of points found per helix, was such a variable. When the photon acceptance was plotted against N_{xyz} , all the GEANT results, independent of the digitization parameters chosen, lay on the same smooth curve. This is shown in Figure 4.36, where the measured $^{12}\text{C}(\pi^-, \gamma)$ data are superimposed. The Monte Carlo and the data are in almost perfect agreement (the absolute normalization of the acceptance will be discussed in the next section). This demonstrates that the rate dependence is phenomenologically understood, as it is reproduced in the Monte Carlo, and that

the photon acceptance for a given set of data can be determined simply by examining N_{xyz} . Since the loss in acceptance was due to points being lost from the fit, it is reasonable that the acceptance is determined by the number of points found per track. The agreement seen in Figure 4.36 was found to survive over wide ranges of variation in the values chosen for the cathode pad noise, anode wire noise, cathode clump width etc. in GEANT. In fact, small variations in the consistency of pion calibration data taken during different beam periods were also virtually eliminated by plotting the acceptance against N_{xyz} (these variations were apparently due to changes in the physical cathode pad noise).

A small complication to this should be noted. The functional dependence of the photon acceptance on N_{xyz} depends somewhat upon the photon energy. This is largely a geometrical effect. Because of their differing curvatures, the arc lengths of tracks from different photon energies are different, and so the number of points in the track fit will also differ. There are also some other effects which cause the variation of the acceptance with N_{xyz} to depend on photon energy. Consequently, when correcting for the rate dependence of the RMC data using N_{xyz} , GEANT runs with an RMC photon energy spectrum were used.

The error in the acceptance found in this manner was simply due to the precision with which N_{xyz} could be determined from the data. N_{xyz} was estimated from the data by finding the centroid of the distribution. With a limited data set for RMC on any given target, the error in N_{xyz} (and therefore in the acceptance) was dominated by the statistics in the sample.

A second method was also used to account for the rate dependence. This was based on the empirical exponential dependence of the acceptance on the rate of firing of the TPC wires as observed in the pion calibration data (see Figure 4.28). This method assumed that the same exponential dependence existed for the RMC

Figure 4.36: Acceptance versus N_{xyz} for $^{12}\text{C}(\pi^-, \gamma)$ from GEANT compared to data. The closed circles are the data, the open circles are GEANT results. The curve is a spline fit to the GEANT results.

Figure 4.37: Rate dependence of RMC events from ^{40}Ca . The line is the rate dependence found in the pion calibration runs.

data, and extrapolated the RMC data to a zero-rate value using the measured wire rates. This suffers from the drawback of the assumption that the wire rate uniquely determines the acceptance under the somewhat different conditions of the pion calibration and RMC runs. Despite this, the results obtained using this method were in good agreement with those found with the first method. For the ^{40}Ca target, data were taken at a few different rates. Figure 4.37 shows the results obtained, compared with the rate dependence found from pion calibration data; the rate dependences are clearly consistent. For ^{16}O and ^{12}C the RMC rate was so small that measurements at different rates were impractical. Despite the good agreement between the two methods used to account for the rate dependence, the final results given here are based on the first method only, to avoid the assumption mentioned above.

Finally, the reason for the rate dependence of the amplitude of the TPC signals

must be addressed. However, it should be noted that a detailed understanding of the cause of this reduction in amplitude is not needed to correct for its effect, as was discussed above. The most likely cause of rate-dependent effects is that of space-charge buildup due to positive ions. Due to the low mobility of ions (typically 10^3 times slower than electrons), the positive ions produced in the avalanche remain for relatively long times ($\sim$ several ms) before being neutralized on a cathode. A relatively large space charge can therefore build up at high rates.

As discussed by Sauli [111], space charge effects can be divided into several categories, the two important ones in this case being space charge in the drift volume and space charge in the proportional region.

The problem of space charge in the drift volume of a TPC has been known for some time. The primary consequence is a distortion of the drift electric field, both in magnitude and direction. These distortions modify the drift velocity and change the trajectories of the drifting electrons. This can have serious effects on the position resolution of the detector. A partial solution to this problem is to use a gated-grid system such as the one used in this experiment (see Section 3.1). Except when the trigger condition was satisfied, drift electrons were prevented from entering the proportional region, and positive ions were prevented from entering the drift volume. Especially at the low trigger-rates of this experiment, the number of positive ions in the drift region should therefore be relatively small. Examination of the characteristics of the fitted tracks as a function of drift distance showed only a small dependence of the amplitude and resolution. This was consistent with electron diffusion effects measured in cosmic-ray runs (i.e. at ‘zero’-rate). A test was also done in which the trigger rate was artificially reduced by pre-scaling; this would then reduce the positive ion buildup in the drift volume, as the grids were gated on less frequently. No change in pulse amplitude (or the rate dependence) was observed under these conditions. Therefore there was no evidence that the dramatic

amplitude changes observed were related to space charge in the drift volume.

The problem of space charge in the proportional region is a different one. The major effect is a reduction in the electric field close to the anode wire, and therefore in the effective gain of the chamber. The gated grid system reduces the ion production rate due to drift electrons in the chamber, but does not affect those produced from ionization due to primary particles traversing the proportional region. In the present case the rate of firing of the anode wires was typically 10^3 times higher than the trigger rate, indicating that the rate was mainly due to ionization from these primary particles (although there was probably some fraction due to inefficiencies in the blocking by the gated grids). A typical critical rate for gain-reduction due to space charge is 10^9 electrons $\text{s}^{-1}\text{mm}^{-1}$, dependent somewhat on geometry etc. [111]. A minimum-ionizing particle crossing the proportional region will generate ~ 300 electron-ion pairs. At the gain of the TPC (5×10^4), for the innermost TPC anode wire this corresponds to a rate of $7 \times 10^3 \text{ s}^{-1}$, which is of the same order as typical rates used in the present experiment. Thus space charge buildup in the proportional region is a reasonable explanation for the amplitude reduction.

Rate dependent amplitudes (and therefore acceptance) had been observed in earlier experiments performed with the TRIUMF TPC, although the effects were much smaller than found here (typically a loss of 20% in acceptance at the highest beam rates used) [92]. There are two reasons that the effects were more dramatic in the present case. The first is due to ageing effects in the TPC. In the $\mu - e$ conversion experiment, the cathode pad amplitudes were observed to decrease slowly as a function of time, presumably due to carbonization on the anode wires and cathode pads. Consequently, the average amplitudes became closer to threshold and so the positive-ion space charge effect became more important. By the end of the present experiment, the accumulated charge on the inner wires was approximately 10^{16} electrons/mm, which is of the same order that was found to exhibit ageing

effects in a previous version of the TPC [91]. In principle, this loss of amplitude could be compensated for by increasing the TPC anode voltage (at the expense of accelerating the ageing process); however, the chamber would not operate much above the 1750 V used without breakdown.

The second reason for the larger rate effects seen in the present work was the much lower magnetic field used (2.5 kG vs. 9.0 kG). Because of the lower magnetic field, a much larger fraction of the muon-decay electrons and other primary particles were able to get out to the outer wires of the TPC. Consequently, the rates of firing of the TPC wires were much higher for similar beam rates, and so the space charge effect was relatively larger. This was demonstrated in a series of $^{12}\text{C}(\pi^-, \gamma)$ runs taken at a higher magnetic field (5.0 kG), where the rate dependence was observed to be considerably reduced. A higher magnetic field, however, would have entailed a loss of acceptance (at the trigger level) for photons in the RMC energy range.

4.8 Photon Acceptance

There are two possible methods to determine the absolute normalization of the photon acceptance ($\epsilon\Omega$). The first is to rely upon the Monte Carlo simulation to estimate the acceptance. This has the disadvantage that the results become extremely dependent upon the accuracy of the simulation; also, estimation of the uncertainty in the acceptance due to the reliability of the Monte Carlo is difficult. The second approach, adopted here, was to normalize the absolute acceptance using a process that has a well-known branching ratio, namely radiative pion capture. This has the disadvantage that the knowledge of the acceptance is limited by the error in the branching ratio used in the normalization. Added confidence is given to the procedure if the Monte Carlo prediction of the absolute acceptance is close to the normalized acceptance, as was the case here.

The ideal target to use in a normalization of intermediate-energy photon ac-

ceptance is hydrogen. The branching ratio between the radiative capture and pion charge-exchange process is well known, and the photons produced are of convenient energies (see Section 4.5). Unfortunately, a suitable hydrogen target was unavailable for this experiment. The compromise of using a hydrogen-containing compound, and subtracting the non-hydrogenous contribution using a second target (i.e. subtracting measured (π^-, γ) spectra from CH_2 and C) has attendant difficulties. The statistical error is increased, and one must correctly account for the differing photon attenuation factors for the two targets. Also, one needs to know the fraction of pions that capture on the hydrogen as opposed to the other element. For CH_2 , there has been some controversy in the literature about the correct value for this capture fraction, $W(\text{CH}_2)$. A weighted average of five measurements [112,113,114,115,103] gives $W(\text{CH}_2) = (1.26 \pm 0.12)\%$ (the mutual agreement of these data is poor, and so the error in $W(\text{CH}_2)$ has been scaled up by the square root of the reduced χ^2 to reflect this). Due to the relatively large error of $W(\text{CH}_2)$, and the difficulties mentioned above, the CH_2 data was not used in the normalization (however, see Table 4.12 below).

Of the other targets for which radiative pion capture spectra were obtained in this work, the branching ratio for ^{12}C has the smallest uncertainty. The weighted average of three, mutually consistent measurements [101,103] for this branching ratio is $(1.83 \pm 0.06)\%$. In this experiment, high statistics data was obtained for $^{12}\text{C}(\pi^-, \gamma)$ over a wide range of beam rates. The normalization of the absolute photon acceptance was therefore made using this reaction.

There are a few small effects that need to be accounted for, before the measured photon rate per LT can be compared to the GEANT prediction. The first of these corrections was for the number of incident pions that decay in flight. An incident π^- that decayed after passing through the beam counters, but before stopping in the target would be counted as a valid LT , but could not produce a

radiative capture photon. The size of this correction was determined using a special version of GEANT. The fraction of incident pions that decayed to muons and which subsequently stopped in the target was found to be $(4.4 \pm 0.6)\%$, where the error reflects the range of variation observed with trials using different reasonable beam parameters.

A second small correction was due to pileup in counting the incident pions (i.e. two π^- 's in the same beam bucket); this was 1.2% at the highest rate used. The third correction was for π^- that stop in the IA counters surrounding the target, but that are counted as a *LT* due to the inefficiency of the IA's. The average IA counter efficiency was measured to be $>95\%$ using cosmic-rays and 18% of the incident π^- beam stopped in the IA counters. Since the radiative pion capture rate from the scintillator is similar to that from the carbon target, this correction is less than 1%. Finally, the effect of vetoing the event due to the IA and IB counters firing randomly in coincidence with the photon event has been discussed and corrected for in the previous section.

The acceptance determined using the $^{12}\text{C}(\pi^-, \gamma)$ data was 7% *larger* than that predicted by the GEANT Monte Carlo. While this is a relatively small discrepancy, its direction is puzzling. Usually, a Monte Carlo simulation would be expected to *overestimate* an experimental acceptance, due to inefficiencies in the detector components that are unaccounted for in the simulation. In the present case, however, the neglect of inefficiencies in the veto counters IA and IB could cause the Monte Carlo to underestimate the observed acceptance. Photon events that interacted in the IA or IB counters, or ones in which a delta-ray or low-energy bremsstrahlung photon from the e^+e^- pair interacted in the IA or IB counters, could pass the electronics trigger due to veto inefficiency, but would fail to trigger the Monte Carlo. An indication that this may be the correct explanation is that the trigger rates predicted by GEANT were typically 5% lower than those actually observed (however,

Table 4.11: Contributions to the error in the absolute normalization of the photon acceptance ($\epsilon\Omega$). The errors have been added in quadrature to give the total error.

(π^-, γ) Branching Ratio Error	3.4 %
Pion Decay in Flight	0.6 %
Photon Absorption in Target	2.0 %
Rate Effect	3.6 %
Statistics	0.7 %
Total Error	5.4 %

it should also be noted that the trigger rate predicted by GEANT was found to be rather sensitive to the low-energy cutoffs used in the simulation; see appendix).

The factors contributing to the error in the absolute normalization are listed in Table 4.11. One of these has not been discussed above, the effect of photon absorption in the target. The probability of a photon interacting in the target (by pair-production or Compton scattering) is dependent on the average path length of the photon in the target. Consequently, the photon interaction probability predicted by GEANT depends somewhat on the distribution assumed for the origins of the photons. The distributions assumed were realistic ones based on GEANT calculations of the pion (and muon) stopping distributions. The widths of the distributions were adjusted slightly so that the predicted histograms of RTARGET and ZTARGET agreed well with those observed in the data. To assign an error, the Monte Carlo was run with stopping distributions varied between the extremes of all photons originating at the center of the target and of photon origins distributed uniformly throughout the target, and the variation in the photon absorption was determined. Because these extremes produced predictions for RTARGET and ZTARGET that were in obvious disagreement with the data, the extracted uncertainty is likely to be an overestimate.

From the acceptance determined as above, the measured (π^-, γ) data on other targets can then be used to extract experimental branching ratios for these targets.

Table 4.12: Experimental radiative pion capture branching ratios. For CH₂ the branching ratio listed is W(CH₂), the fraction of π^- 's that capture on hydrogen.

Target	Branching Ratio (%)	Reference
¹⁶ O	2.24 ± 0.48	[103]
	2.20 ± 0.33	[103]
	2.27 ± 0.24	[104]
	2.17 ± 0.20	present work
⁴⁰ Ca	1.82 ± 0.15	[103]
	1.94 ± 0.35	[116]
	1.78 ± 0.15	present work
CH ₂	1.26 ± 0.12	[112,113,114,115,103]
	1.49 ± 0.14	present work

The results are summarized in table 4.12. The errors on the branching ratios include the error of the normalization of the detector acceptance, as well as additional contributions from photon absorption in each target, the rate effect, and statistics. The statistics were poorer for these data and consequently the statistical error and the rate dependence error are somewhat larger than for ¹²C. For the ¹⁶O case, there was an additional complication due to radiative pion capture on the deuteron, which contributes about 4.5% to the observed spectrum. This has been accounted for in the results given in Table 4.12. The good agreement between the present work and previous results for these branching ratios adds confidence in our knowledge of the detector acceptance.

4.9 Branching Ratio Calculation

The observed photon energy spectrum (N_i^γ) from RMC is related to the physical photon energy spectrum, G_j , by

$$N_i^\gamma = N_\mu \cdot f_{capture} \cdot \sum_j R_{ij} \cdot \epsilon \Omega_j \cdot G_j \quad (4.14)$$

where N_μ is the number of (dead-time corrected) muon stops in the target,

$f_{capture}$ is the fraction of muons that undergo nuclear muon capture, R_{ij} is the detector response function (i.e. the relative probability of a photon of energy E_j being reconstructed at an energy E_i) and $\epsilon\Omega_j$ is the absolute photon detection efficiency for photons of energy E_j . $\epsilon\Omega_j$ is the efficiency with a given set of gains on the cathode pads, and so implicitly accounts for the rate effect. The observable of interest, $\mathbf{G}_j$, in units of photons/MeV/capture, is therefore implicitly defined in terms of known and measured quantities. $\mathbf{G}_j$ can then be determined by convoluting it with the detector response function and acceptance and comparing with the observed spectrum. Due to the experimental difficulties discussed earlier, only a partial branching ratio (e.g. $E_\gamma > 57$ MeV) is accessible. The number of photon events above an observed energy of 57 MeV

$$N_{>57}^\gamma = \sum_{i>57\text{MeV}} \mathbf{N}_i^\gamma \quad (4.15)$$

can be considered as a function of the theoretical partial branching ratio

$$G_{>57} = \sum_{j>57\text{MeV}} \mathbf{G}_j \quad (4.16)$$

which is given in units of photons/capture. Alternatively, $N_{>57}^\gamma$ can be considered as a function of g_p . Thus g_p can be determined by comparing the number of photons predicted in the experimental spectrum (above 57 MeV) for a given g_p to $N_{>57}^\gamma$.

The factors R_{ij} and $\epsilon\Omega_j$ and their uncertainties have been discussed earlier in this chapter. The factor $f_{capture}$, the fraction of muons that undergo OMC in the target, is taken from the literature, and the values used are listed in table 4.13. The remainder of this section describes the determination of N_μ and $N_{>57}^\gamma$.

4.9.1 N_μ

N_μ was determined from the number of LT (i.e. the number of dead-time corrected $STOP\bar{I}$) after correcting for a few small effects. The first correction (f_{pileup})

Table 4.13: The fraction of muons that undergo OMC in the target [109].

Target	$f_{capture}$ (%)
^{12}C	7.67 ± 0.06
^{16}O	18.38 ± 0.11
^{40}Ca	85.0 ± 0.2

was to account for the possibility of two μ^- arriving in the same beam-bucket, and therefore being counted as only a single particle. This was easily calculated from the incident flux and was a 1.3–1.7% correction, depending on the rate. The individual beam counter efficiencies were measured to be $>99\%$, so that the probability that an incident μ^- did not fire the beam counters was $<1\%$; thus no correction was required for this. Due to the relatively long ($2.2\ \mu\text{s}$) lifetime of the muon, the probability of a μ^- decaying in flight between the beam counters and the target was also negligible.

The probability of an incident beam e^- being incorrectly counted as a muon stop was also small. In the case of the 73 MeV/c μ^- beam, the electrons in the beam arrive at the beam counters at the same time as muons from the subsequent beam-burst, and hence they could not be rejected by timing cuts in the electronics. The e^- 's have a much longer range than μ^- 's of the same momentum and in general they passed completely through the target and veto counter. The probability of an incident electron producing a stop was measured to be 23% (likely due to inefficiency in the veto counter). Since only 1% of the incident beam were electrons, the fraction of false LT due to electrons was $f_e = 0.23\%$. The π^- content of the beam ($\sim 10^{-5}$) was small enough to have a negligible effect on the counting of the μ^- beam.

Finally, due to inefficiencies in the IA counters, an incident μ^- that scattered into these counters may have been misidentified as a muon stop in the target (the background of RMC photons from these scattered muons has already been dis-

Table 4.14: The values of N_μ and $N_{>57}^\gamma$.

Target	Converter	N_μ (10^{10})	$N_{>57}^\gamma$
^{12}C	0.6 mm	12.182 ± 0.038	116 ± 16
	1.0 mm	30.851 ± 0.092	497 ± 43
^{16}O	1.0 mm	20.358 ± 0.061	361 ± 34
^{40}Ca	0.6 mm	7.407 ± 0.022	1299 ± 88
	1.0 mm	9.635 ± 0.029	2159 ± 134

cussed). This over-counting of muon stops in the target can be determined from the average efficiency of the IA counters ($\sim 96\%$) and the measured fraction of the beam scattered into these counters. This correction (f_{IA}) was between 0.8 and 1.4%, depending on the target.

From the above corrections, the number of genuine dead-time corrected muon stops in the target was then calculated from

$$N_\mu = LT \cdot (1 - f_{IA}) \cdot (1 - f_e) \cdot (1 + f_{pileup}) \quad (4.17)$$

The error in N_μ due to these corrections was 0.3 %. The values of N_μ for each target and converter thickness are given in Table 4.14.

4.9.2 $N_{>57}^\gamma$

The number of photons observed above 57 MeV for each target ($N_{>57}^\gamma$) must be corrected for each of the various backgrounds, as discussed earlier. $N_{>57}^\gamma$ must also be corrected for the number of otherwise valid photons that were rejected by the trigger electronics. Photons could be rejected due to one of three causes: random vetoing from the IA and IB counters, vetoing from the PROMPT cut and finally the finite length of the LT gate. The first of these effects has been discussed, and was easily calculated from the singles rates in the IA and IB counters. The other two effects were calculated using the small Monte Carlo routine LTCORR, mentioned

earlier. The PROMPT cut will reject an otherwise valid photon event if another μ^- arrives in random coincidence with the event. This probability therefore depends on the incident beam rate and the effective width of the PROMPT veto (30 ns). Of course, a μ^- may generate a valid RMC photon less than 30 ns after it stops in the target, and such an event would also be rejected by the PROMPT cut. Finally, if the RMC photon was produced after the LT gate, and if a subsequent incident μ^- did not arrive to update the LT gate, then the photon would not satisfy the *EVENT* coincidence and so no trigger would be produced. These two corrections were relatively small (7-10% in total) at the rates used. The uncertainty on these corrections was estimated by varying the parameters used in the Monte Carlo (e.g. the beam rate, muon lifetime, LT gate width, width of the PROMPT veto) by reasonable amounts, and was found to be $\sim 1\%$. A check on this calculation was made by examining the energy spectrum of photons rejected by the PROMPT cut. The fraction of RMC photons rejected by the prompt cut could be estimated by making an approximate subtraction of the radiative pion capture contribution. This remaining fraction was, in all cases, in good agreement with the fraction predicted using LTCORR.

The values of $N_{>57}^\gamma$ for each target and converter thickness are given in Table 4.14. The error in $N_{>57}^\gamma$ for each case includes the errors due to the above corrections, as well as errors from three other sources. These are the rate dependence of the acceptance, the energy calibration and the effect of photon absorption in the target. The rate dependence error was dominated by the uncertainty on the centroid of the N_{xyz} distribution for each data set, as discussed earlier. The uncertainty in the energy calibration at 57 MeV (± 400 keV) contributes to the error in $N_{>57}^\gamma$. The error was taken as the number of events within 400 keV of the nominal 57 MeV cut-off, and was typically 2-4%, depending on the target. Finally,

Table 4.15: Summary of the contributions to the error in $N_{>57}^\gamma$. All errors are listed in % of $N_{>57}^\gamma$. The total error was obtained by adding the contributing errors in quadrature.

Source of error	^{40}Ca		^{16}O	^{12}C	
	1.0 mm	0.6 mm	1.0 mm	1.0 mm	0.6 mm
Statistics	2.2	2.8	5.5	4.5	9.3
Rate effect	4.7	5.2	5.8	5.5	7.6
Radiative pion capture	~ 0	~ 0	0.01	0.14	0.01
Cosmic Rays	0.02	0.03	0.14	0.2	0.2
Bremsstrahlung	0.04	0.07	1.2	3.0	4.9
μ stops outside target	0.2	0.2	0.8	1.8	1.8
Energy calibration	2.6	2.3	3.4	2.7	2.4
Photon absorption in target	2.0	2.0	2.3	2.1	2.1
PROMPT and finite- LT	0.9	0.9	1.2	1.1	1.1
total	6.2	6.8	9.4	8.6	13.5

the error due to photon absorption in the target was estimated by GEANT in the same manner as for the pion calibration data (see section 4.2) and was typically $\sim 2\%$, depending on the target.

Chapter 5

Results and Discussion

5.1 ^{40}Ca

The photon energy spectrum from ^{40}Ca obtained with the 1.0 mm converter is compared with the theoretical energy spectra from Christillin [81] and that from Gmitro *et al.* [73] in Figures 5.1 and 5.2 respectively. The theoretical spectra have been convoluted with the detector acceptance and response function. The errors shown on the data are the statistical errors only. The sensitivity of the RMC rate to g_p is evident from these figures.

Two somewhat different methods were used to extract the partial branching ratio ($G_{>57}$) and g_p/g_a from the data. The first method was to compare the predicted integral number of counts above 57 MeV with that observed ('integral method'). The second method was to perform a χ^2 -fit of the theoretical spectra to the data ('spectrum fitting method'). The integral method is discussed first.

Typically, existing theoretical calculations provide spectra and branching ratios for only a few values of g_p/g_a . In the integral method, each of these spectra was convoluted with the detector acceptance and response function and then multiplied by N_μ to give the predicted number of counts above 57 MeV (see equation 4.14). This predicted $N_{>57}^\gamma$ can be considered as being a function either of g_p/g_a or of the theoretical branching ratio, $G_{>57}$. Simple polynomial fits were made to the predicted $N_{>57}^\gamma$ as a function of $G_{>57}$ (Figure 5.3) and as a function of g_p/g_a (Figure 5.4). Also shown in these figures is a hatched region which represents the observed $N_{>57}^\gamma$, which corresponds to the area under the experimental spectrum above 57 MeV. The ex-

Figure 5.1: Photon energy spectrum from ^{40}Ca , 1.0 mm converter, compared to theory of Christillin [81]. The theoretical spectra have been convoluted with the detector response function.

Figure 5.2: Photon energy spectrum from ^{40}Ca , 1.0 mm converter, compared to theory of Gmitro *et al.* [73]. The theoretical spectra have been convoluted with the detector response function.

Figure 5.3: $N_{>57}^{\gamma}$ from ^{40}Ca , 1.0 mm converter, as a function of $G_{>57}$ for the theories of Christillin [81] ('Chr') and Gmitro *et al.* [73] ('GOT').

perimental error indicated by the hatched region includes the errors on $N_{>57}^{\gamma}$, N_{μ} and the normalization of the acceptance, added in quadrature. Thus this includes *all* the contributions to the experimental error.

The extraction of $G_{>57}$ and g_p/g_a along with their uncertainties from Figures 5.3 and 5.4 is straightforward. The reason that the same $N_{>57}^{\gamma}$ gives slightly different branching ratios $G_{>57}$ for the two theories is the energy dependence of the photon acceptance. With an energy-dependent acceptance, theoretical energy spectra with different shapes yield, for the same branching ratio, different numbers of observed events. The near linearity of the curves in Figure 5.3 reflects the fact that the theoretical energy spectra do not change much in shape as g_p/g_a is varied, but mainly change in normalization. The similarity between the curves of Christillin [81] and those of Gmitro *et al.* [73] seen in Figure 5.3 reflects the similarity of the spectral shapes predicted for the two theories. The resulting values of $G_{>57}$ and

Figure 5.4: $N_{>57}^{\gamma}$ from ^{40}Ca , 1.0 mm converter, as a function of g_p/g_a for the theories of Christillin [81] ('Chr') and Gmitro *et al.* [73] ('GOT').

g_p/g_a are given in Table 5.1.

The data set for ^{40}Ca taken using the 0.6 mm converter has somewhat fewer events but better energy resolution than the 1.0 mm converter data. Figures 5.5 and 5.6 show these data with same two theories superimposed. The theoretical spectra have again been convoluted with the appropriate detector efficiency and response function. Figures 5.7 and 5.8 show $N_{>57}^{\gamma}$ as a function of $G_{>57}$ and of g_p/g_a respectively for this data. The resulting values of $G_{>57}$ and g_p/g_a are given in Table 5.1.

Several aspects are apparent from an examination of the results given in Table 5.1. The results from the two different converter thicknesses are in excellent agreement. These data had different acceptances and resolutions, and were taken during different running periods at different beam rates. The good agreement adds confidence in the data analysis method and the error estimates. A weighted av-

Figure 5.5: Photon energy spectrum from ^{40}Ca , 0.6 mm converter, compared to the theory of Christillin [81]. The theoretical spectra have been convoluted with the detector response function.

Figure 5.6: Photon energy spectrum from ^{40}Ca , 0.6 mm converter, compared to the theory of Gmitro *et al.* [73]. The theoretical spectra have been convoluted with the detector response function.

Table 5.1: Summary of the results for the partial branching ratio $G_{>57}$ and g_p/g_a for ^{40}Ca from the theories of Christillin [81] and Gmitro *et al.* [73] using the ‘**integral method**’.

Theory	Converter	$G_{>57} (10^{-5})$	g_p/g_a
Christillin [81]	1.0 mm	2.24 ± 0.17	5.9 ± 0.8
	0.6 mm	2.13 ± 0.17	5.4 ± 0.8
	average	2.18 ± 0.16	5.7 ± 0.8
Gmitro <i>et al.</i> [73]	1.0 mm	2.08 ± 0.15	$5.1^{+1.8}_{-2.0}$
	0.6 mm	2.00 ± 0.14	$4.1^{+1.8}_{-2.0}$
	average	2.04 ± 0.14	$4.6^{+1.7}_{-1.8}$

Table 5.2: Summary of the results for the partial branching ratio $G_{>57}$ and g_p/g_a for ^{40}Ca from the theories of Christillin [81] and Gmitro *et al.* [73] using the ‘**spectrum fitting method**’. ‘PP’ refers to the Primakoff polynomial closure approximation (see text).

Theory	Converter	$G_{>57} (10^{-5})$	k_{max} (MeV)	g_p/g_a	χ^2_{dof}
Christillin [81]	1.0 mm	2.19 ± 0.18	—	5.7 ± 0.8	1.39
	0.6 mm	2.07 ± 0.18	—	5.2 ± 0.8	0.80
	average	2.13 ± 0.16	—	5.4 ± 0.8	—
Gmitro <i>et al.</i> [73]	1.0 mm	2.02 ± 0.16	—	4.4 ± 2.0	1.93
	0.6 mm	1.90 ± 0.15	—	2.7 ± 1.9	1.46
	average	1.96 ± 0.15	—	3.6 ± 1.9	—
PP (Primakoff polynomial)	1.0 mm	$2.16^{+0.27}_{-0.30}$	91.6 ± 3.0	—	1.52
	0.6 mm	$2.14^{+0.26}_{-0.32}$	91.9 ± 3.6	—	0.72
	average	$2.16^{+0.22}_{-0.28}$	91.7 ± 2.3	—	—

average of the results from the two converters is also given in Table 5.1. The error on the weighted average has been calculated taking into account the fact that the two measurements are not independent, but have several correlated errors, e.g. the errors on $\epsilon\Omega$, $f_{capture}$ etc. .

The results for both the partial branching ratio ($G_{>57}$) and for g_p/g_a are in agreement for the two theories, although a slightly smaller branching ratio and g_p/g_a is obtained using the theory of Gmitro *et al.* [73]. It is interesting to note that the reasonable agreement between the values for g_p/g_a for the two theories is

Figure 5.7: $N_{>57}^{\gamma}$ from ^{40}Ca , 0.6 mm converter, as a function of $G_{>57}$ for the theories of Christillin [81] ('Chr') and Gmitro *et al.* [73] ('GOT').

somewhat fortuitous, as the two calculations 'cross' near the experimental value of $N_{>57}^{\gamma}$. In fact, for values of g_p/g_a much different from about 7.0, the two theories predict quite different branching ratios and therefore different values for $N_{>57}^{\gamma}$ (see Figures 5.4 and 5.8).

The second method used to extract $G_{>57}$ and g_p/g_a from the data was the spectrum fitting method. In this method, the theoretical spectrum (convoluted by the detector response) which was closest to the data was chosen. A χ^2 -fit of this spectrum to the data was made, with the normalization of the theoretical spectrum as the free parameter. The theoretical branching ratio for the particular spectrum was then multiplied by this normalization factor to give the experimental $G_{>57}$. In the cases where the χ_{dof}^2 (χ^2 per degree of freedom) of the fit was significantly larger than 1, the error on the normalization factor from the fit was scaled up by $\sqrt{\chi_{dof}^2}$. Since the errors on the individual channels in the energy spectrum do not include

Figure 5.8: $N_{>57}^{\gamma}$ from ^{40}Ca , 0.6 mm converter, as a function of g_p/g_a for the theories of Christillin [81] ('Chr') and Gmitro *et al.* [73] ('GOT').

the overall errors (due to the normalization of the acceptance, the rate dependence, N_{μ} etc.), the error from the χ^2 -fit was added in quadrature to these other errors to obtain the error on $G_{>57}$. The value of g_p/g_a was then obtained from the partial branching ratio using a polynomial fit of the theoretical branching ratio as a function of g_p/g_a . The resulting fits are shown in Figures 5.9 and 5.10 for the 1.0 mm and 0.6 mm converter data respectively. The values for $G_{>57}$ and g_p/g_a obtained are given in Table 5.2.

The third entry in Table 5.2 labelled 'Primakoff polynomial' requires further explanation. In the closure approximation, if only the muon-radiating diagram is considered, the RMC photon spectrum can be expressed in a simple analytic form [117,81]:

$$\frac{dG(x)}{dk} \sim (1 - 2x + 2x^2)x(1 - x)^2 \quad (5.1)$$

with

$$x = \frac{k}{k_{max}} \quad (5.2)$$

where k is the photon energy and k_{max} is the maximum photon energy in the closure approximation, which is directly related to E_{av} , the average excitation energy of the nucleus. This expression was first derived by Primakoff [117] and herein is labelled the ‘Primakoff polynomial’ (‘PP’). This, in fact, can provide a reasonable first-order representation of more complete calculations of the RMC spectrum, if a suitable value is chosen for k_{max} . The problem of the sensitivity of closure model calculations to E_{av} (or equivalently, to k_{max}) has been discussed. Nevertheless, to facilitate comparisons to other experiments that used this, or other closure model expressions, results are presented for the branching ratio and k_{max} determined from a two-parameter fit to our data. The larger error on $G_{>57}$ found using the Primakoff polynomial simply reflects the sensitivity to k_{max} . The best fit spectrum is also shown in Figures 5.9 and 5.10. The present results, when compared to the closure model calculations of Sloboda and Fearing [67] would yield negative values for g_p/g_a ; similar results would be obtained for earlier closure model calculations of the nuclear response function [79,56].

The results for both $G_{>57}$ and g_p/g_a obtained using the spectrum fitting method (Table 5.2) are quite consistent with those obtained from the integral method (Table 5.1). The consistency between the two data sets taken using the two different converters is again apparent. It is interesting to note that in both cases the fit to the theory of Christillin [81] has a somewhat smaller χ^2 than that of Gmitro *et al.* [73].

The spectrum fitting method has the disadvantage that is based on the assumption that the shape of the RMC spectrum does not vary significantly as g_p/g_a is varied. As has been stated, this appears to be generally true. In addition, the

Figure 5.9: Photon energy spectrum from ^{40}Ca , 1.0 mm converter, compared to the best fits from the spectrum fitting method. ‘Chr’ refers to the theory of Christillin [82], ‘GOT’ to that of Gmitro *et al.* [73] and ‘PP’ to the Primakoff polynomial.

spectrum fitting method is more sensitive to inaccuracies in the detector response function. Because of these drawbacks, we consider the results obtained from the integral method to be the final results of this experiment.

Table 5.3 compares the present results to other recent measurements of RMC on ^{40}Ca . The two earliest experiments are not listed in the table, those of Conversi *et al.* [118] and Rosenstein and Hammerman [119]. Both of these experiments used NaI(Tl) counters without photon converters; consequently they were very sensitive to neutron-induced backgrounds. The results of these measurements are not directly comparable to the more recent ones, as they quoted total rather than partial branching ratios (using available theories to extrapolate the energy spectrum below 57 MeV).

It is apparent from Table 5.3 that there is good agreement among all the recent

Figure 5.10: Photon energy spectrum from ^{40}Ca , 0.6 mm converter, compared to the best fits from the spectrum fitting method. 'Chr' refers to the theory of Christillin [82], 'GOT' to that of Gmitro *et al.* [73] and 'PP' to the Primakoff polynomial.

measurements for $G_{>57}$ and the present results. Despite this agreement, there is some variation in the extracted values of g_p/g_a . The individual experiments have used somewhat different techniques, and they will be discussed in some detail below.

The measurement of Hart *et al.* [120,121] was made using a NaI(Tl) detector with a 3.0 mm Pb photon converter. They obtained a relatively background-free measurement, with, however, rather poor energy resolution ($\sim 30\%$ FWHM). They normalized their results to their measured electron spectrum from muon decay, rather than to the number of muon stops. This has the disadvantage that the resulting RMC branching ratio is quite sensitive to the observed μ^- lifetime in ^{40}Ca . They observed a lifetime of $\tau_\mu = 365 \pm 8$ ns which is somewhat larger than the accepted value of 333 ns [109]. Their result for $G_{>57}$ would be $\sim 9\%$ lower if the accepted value for τ_μ were used. The partial branching ratio was obtained

Table 5.3: Summary of RMC results for ^{40}Ca from recent measurements. The partial branching ratio $G_{>57}$ is given in units of 10^{-5} . For theories using the closure approximation, the value of k_{max} found is given. ‘RYY’ refers to the theory of Rood, Yano and Yano [76], ‘Chr’ refers to the theory of Christillin [81], ‘GOT’ refers to the theory of Gmitro *et al.* [73] and ‘PP’ refers to a fit to the Primakoff polynomial (see text).

$G_{>57}$ (10^{-5})	Theory	k_{max} (MeV)	# events	g_p/g_a	reference
2.11 ± 0.14	RYY	86.5 ± 1.9	1229	6.5 ± 1.6	Hart <i>et al.</i> [120,121]
2.07 ± 0.20	—	—	2450	—	Frischknecht <i>et al.</i> [87]
1.92 ± 0.20	RYY	90.8 ± 0.9	”	3.5 ± 1.3	”
1.96 ± 0.20	Chr	—	”	4.6 ± 0.9	”
$2.35^{+0.32}_{-0.30}$	Chr	—	3133	7.6 ± 1.6	Virtue <i>et al.</i> [105]
$2.15^{+0.27}_{-0.26}$	GOT	—	”	$6.0^{+2.8}_{-2.9}$	”
2.30 ± 0.21	Chr	—	3234	$6.3^{+1.0}_{-1.5}$	Döbeli <i>et al.</i> [122,123]
2.26 ± 0.21	PP	92.5 ± 0.7	”	—	”
2.18 ± 0.16	Chr	—	3458	5.7 ± 0.8	present work
2.04 ± 0.14	GOT	—	”	$4.6^{+1.7}_{-1.8}$	”
$2.15^{+0.19}_{-0.22}$	PP	91.7 ± 2.3	”	—	”

by assuming that the RMC spectrum was a power series, convoluting it with the detector response function and fitting it to the data. The error quoted on $G_{>57}$ is quite small, but it does not include any contribution from uncertainty in the energy calibration at 57 MeV, despite the poor energy resolution of the detector. The data were fitted to the closure model calculation of Rood, Yano and Yano [76] to yield $g_p/g_a = 6.5 \pm 1.6$. As is common to closure model calculations, this result is quite sensitive to the value found for k_{max} .

The next entry in Table 5.3 is that of Frischknecht *et al.* [87], which was performed using a high resolution (1.2% FWHM), small solid-angle ($\epsilon\Omega \sim 3 \times 10^{-5}$) pair spectrometer. To provide a reasonable signal with such a small solid-angle required the use of a very high ($5 \times 10^6 \text{ s}^{-1}$) muon beam rate. This caused difficulties in estimating the number of muon stops, as the rate was too high to count the muons individually. The muon stopping fraction in the target was measured at low

beam intensities, and scaled up for higher rates. The experiment also suffered from a relatively large pion-induced background, in which 40% of the raw photon spectrum was pion-induced. The prompt rejection efficiency was rather poor (71%), so even after the removal of prompt events, an additional subtraction of pion-induced background was required. This subtraction was normalized to the photon spectrum above the RMC endpoint. Because of the good energy resolution of the pair spectrometer, the deconvolution of the detector response function from the RMC data was simple, and their energy spectrum is probably the most reliable of the existing data.

The first entry for $G_{>57}$ from Frischknecht *et al.*'s data given in Table 5.3 is a model-independent result obtained from summing the events in their deconvoluted spectra. The other two entries are from fitting the theories of Rood, Yano and Yano [76] and Christillin [81] to their deconvoluted spectrum. The branching ratios found from fitting to the spectrum are somewhat smaller than the model-independent result. This occurs because the data were observed to be slightly above the fitted theory at lower photon energies (~ 57 to 62 MeV). This might indicate either a deficiency in both of the calculations, or that the bremsstrahlung background extended slightly higher in energy than expected.

It is interesting to compare the value for g_p/g_a extracted using the calculation of Rood, Yano and Yano to the value found by Hart *et al.* using the same theory. Despite the good agreement seen in the branching ratios, the two experiments give somewhat different values of g_p/g_a using the same theory. This is due to the different values found for the other free parameter in a closure model fit, k_{max} . This exhibits the well-known sensitivity of closure model calculations to k_{max} ; the RMC rate varies roughly as k_{max}^6 . Fits of the data of Frischknecht *et al.* to other closure model calculations [77,78,67,79] gave negative values for g_p/g_a . It should be noted that three different photon converters were used in their experiment, and while they

each gave consistent results for $G_{>57}$, the values extracted for g_p/g_a were not quite as consistent. For example, the values obtained by fitting to the Rood, Yano and Yano [76] theory were 1.1 ± 1.1 , 4.8 ± 3.4 and 6.6 ± 2.6 , compared to a value of 3.5 ± 1.3 when fitting to the sum of all the data. Therefore there is some room for doubt about the magnitude of the quoted error .

The value for g_p/g_a of 4.6 ± 0.9 obtained for Christillin's theory [81] from the data of Frischknecht *et al.* [87] was obtained from a fit to their photon spectrum. They state that simply using the model-independent branching ratio would give a somewhat higher value for g_p/g_a (which they do not give). However, it is apparent from figures given in their paper [87] that these authors have misinterpreted the work of Christillin somewhat. In reference [81] Christillin presents spectra and branching ratios for different values of g_p in terms of a parameter ' Δ ', given via

$$g_p = \Delta \left[\frac{2m_N m_\mu}{(m_\pi^2 + q^2)} \right] g_a \quad (5.3)$$

where q^2 is evaluated at the kinematics of OMC, i.e. $q^2 = 0.88m_\mu^2$. Therefore $\Delta = 1$ gives the Goldberger-Treiman value for g_p/g_a of 6.8 (or $g_p = 8$), and hence deviations from $\Delta = 1$ indicate deviations from the Goldberger-Treiman value. Frischknecht *et al.* have apparently interpreted $\Delta = 1$ to refer to $g_p/g_a = 8.0$, not 6.8. Unfortunately, this mistake appears to have propagated through the literature as it appears to be present in the results of Virtue *et al.* [105] and most notably in the review article of Gmitro and Truöl [52] (Döbeli *et al.* [122,123] appear to have used $g_p/g_a = 7.0$ for $\Delta = 1$) . As will be discussed later, the same misinterpretation has been made by various authors in interpreting the calculation of Christillin and Gmitro [82] for ^{16}O . With the correct interpretation of Δ , a value of $g_p/g_a = 5.1 \pm 1.2$ was obtained by this author by comparing the model-independent branching ratio to the theory of Christillin [81].

Frischknecht *et al.* also fit their data to the theory of Gmitro *et al.* [73] and

obtained $g_p/g_a = 3.0 \pm 0.6$ from a fit to their spectrum and $g_p/g_a = 5.8 \pm 2.4$ using the model-independent branching ratio. It should be noted that Gmitro *et al.* [73] did not present calculations for any value of g_p/g_a less than 4.5, so that the extrapolation to $g_p/g_a = 3.0$ is ill-defined.

Preliminary results are given in Table 5.3 for the experiment of Virtue *et al.* [105]. In this experiment, a NaI(Tl) was used with an active, segmented photon converter also made of NaI(Tl). Better energy resolution (15% FWHM at 70 MeV) was achieved than that of Hart *et al.* [120,121]. The beam rate was low enough to allow individual counting of muon stops in the target. They were troubled, however, by large cyclotron-related backgrounds in their RMC signal; this led to an unfavourable signal/noise ratio of about 0.5 and limited the accuracy of their result. It should be noted that the experiment was designed primarily to measure the photon-muon spin asymmetry (α_γ) rather than the branching ratio. As mentioned, the value for g_p/g_a extracted from $G_{>57}$ using the theory of Christillin *et al.* [81] apparently suffered from the misinterpretation of Δ discussed above. Their branching ratio would indicate a value of $g_p/g_a = 6.5 \pm 1.4$ using the correct interpretation, which is somewhat lower than the value quoted in Virtue's thesis [105].

The results of Döbeli *et al.* [122,123] given in Table 5.3 represent one of several targets investigated by this group. Two large NaI(Tl) detectors were used; the larger was a modular array of 64 elements with a 4.0 mm Pb photon converter and an energy resolution of 18% FWHM at 70 MeV. The smaller detector was a single NaI(Tl) crystal with an active NaI(Tl) photon converter and a resolution of 22% FWHM at 70 MeV. High muon beam rates were used (up to $3.0 \times 10^6 \text{ s}^{-1}$) and this caused similar problems to those encountered by Frischknecht *et al.* [87]. The stopping fraction in the target was determined at low rates and scaled up to the high rates used in the experiment. A rather large (48%) correction had to be made to the scaling factor due to the influence of the slits (used to reduce the

beam rate) on the beam profile (and therefore on the stopping fraction). Only 18% of the incident beam stopped in the target. Relatively large (20%) pion-induced backgrounds had to be subtracted (as the prompt rejection efficiency was poor: 77-85%), as well as empty-target and neutron-induced backgrounds. Despite these large corrections, the agreement between the results from the two detectors was good. The result given in Table 5.3 supersedes the somewhat different preliminary result of $G_{>57} = (1.73 \pm 0.36) \times 10^{-5}$ which was given in reference [124] and which was quoted in the review article by Gmitro and Truöl [52]. It should be noted that the error on the result given by Döbeli *et al.* [122,123] for $G_{>57}$ was derived by those authors by considering the results from their two detectors as being totally independent. However, they are not independent since the large (7%) error on the number of muon stops was common to both detectors. A proper combination of the errors would increase the error on $G_{>57}$ from ± 0.21 to ± 0.24 , and would therefore increase the error on the extracted value of g_p/g_a somewhat.

The value found for k_{max} in the present work (using the Primakoff polynomial) agrees well the values for k_{max} found by Frischknecht *et al.* [87] and Döbeli *et al.* [122,123], but is somewhat higher than that found by Hart *et al.* . As noted earlier, the extracted value of g_p/g_a from Hart *et al.* using theory of Rood, Yano and Yano [76] would be reduced if a larger k_{max} were used, which would bring the result into agreement with that of Frischknecht *et al.* for the same theory.

The present results for $G_{>57}$ are in good agreement with all the existing data, and are more precise than all except the results of Hart *et al.* . The present measurement, with superior energy resolution to all previous experiments except that of Frischknecht *et al.* , is essentially background-free, and the results using two different photon converters are in good agreement.

Averaging the results of the five experiments listed in Table 5.3 gives the value

$$G_{>57} = (2.15 \pm 0.08) \times 10^{-5} \quad (5.4)$$

Comparing this result to the phenomenological calculation of Christillin [81] gives the value $g_p/g_a = 5.5 \pm 0.5$ or $(81 \pm 7)\%$ of the Goldberger-Treiman value. Comparing the branching ratio to the MIA calculation of Gmitro *et al.* gives $g_p/g_a = 6.0 \pm 1.0$ or $(89 \pm 15)\%$ of the Goldberger-Treiman value. The values for g_p/g_a found using closure model calculations tend to be much lower, often leading to (improbable) negative values of g_p/g_a . In both the present results and those of Frischknecht *et al.*, the theory of Gmitro *et al.* yields a lower value of g_p/g_a if the spectrum fitting method is used rather than the integral method. If the three values of g_p/g_a extracted in this way from Frischknecht *et al.*, Virtue *et al.* and the present work are averaged, they give $g_p/g_a = 3.1 \pm 0.6$. However, the χ^2_{dof} of the fits tend to be poor. The poorer χ^2_{dof} and lower value of g_p/g_a appear to be due to the calculation underestimating the rate at lower photon energies. Frischknecht *et al.* [87] have speculated that this may be due to the fact that quadrupole excitations were omitted in the calculation. Therefore the lower value of g_p/g_a extracted by fitting the theory of Gmitro *et al.* to the shape of the spectrum should be viewed with caution.

Measurements of another observable in RMC, the photon-muon spin asymmetry (α_γ) have also been made for ^{40}Ca . ^{40}Ca is in fact the only nucleus for which data on α_γ exist. The advantage of this observable over a measurement of the branching ratio is that α_γ appears to be less sensitive than $G_{>57}$ to the details of the nuclear structure model used, making the extraction of g_p/g_a from the data more reliable. The disadvantage is that the asymmetry signal is small and rather difficult to measure. The existing data on α_γ are summarized in Table 5.4. Only the results of Hart *et al.* [120,121] exist in final form; the other data are still preliminary. A

detailed discussion of all these data is given in the thesis of Virtue [105]. The error claimed by Schaad *et al.* [125,124,126] is significantly smaller than that of the other two measurements, but must be treated with some suspicion due to the improbably large χ^2_{dof} of ~ 5.3 found for their fit to the asymmetry signal. Taking the errors at face value, however, gives a value for g_p/g_a of $2.1 \pm_{7.0}^{4.4}$ using the MIA theory of Gmitro *et al.* [127]. This is consistent with both $g_p = 0$ and the Goldberger-Treiman value, as well as with the results of the branching ratio measurements. The error on the experimental value of α_γ would have to be decreased significantly before the precision on the extracted g_p/g_a would be competitive with that determined from the branching ratio data. It should also be noted that the prediction for the dependence of α_γ on g_p/g_a predicted by Gmitro *et al.* [127] in the MIA calculation is significantly different from that found in earlier calculations [67,81,83], and this casts doubt on the purported ‘model-independence’ of the calculation of α_γ .

In summary, the branching ratio results in ^{40}Ca , when compared to the two most recent calculations, indicate a value of g_p/g_a somewhat less than the Goldberger-Treiman value, and the much less precise asymmetry results are consistent with this as well. This could indicate a ‘quenching’ of g_p in the ^{40}Ca nucleus. However, the difference from the Goldberger-Treiman value is of the same order as the difference between the values extracted from the two theories. Thus a definitive statement about any renormalization of g_p in ^{40}Ca awaits more consistency in the theoretical treatment of the nuclear response.

5.2 ^{16}O

The photon energy spectrum from ^{16}O is compared to the theoretical energy spectra from Gmitro *et al.* [73] and from Christillin and Gmitro [82] in Figures 5.11 and 5.12 respectively. The theoretical spectra have been convoluted with

Table 5.4: Summary of existing results for the photon-muon spin angular correlation (α_γ) for ^{40}Ca . A value of unity for (α_γ) indicates $g_p = 0$.

α_γ	reference
0.90 ± 0.50	Hart <i>et al.</i> [120,121]
0.94 ± 0.14	Schaad <i>et al.</i> [125,124,126]
$1.32^{+0.54}_{-0.47}$	Virtue <i>et al.</i> [105]
1.00 ± 0.23	Pouladdej <i>et al.</i> [128]

the detector acceptance and response function. The experimental energy spectrum has had the normalized μ^+ bremsstrahlung spectrum subtracted away, as discussed in Section 4.6.3. The errors shown include the effect of this background subtraction. Due to the low statistics, the spectrum has been displayed with 2 MeV energy binning.

In the same way as was done for ^{40}Ca , both the integral method and the spectrum fitting method were used to extract $G_{>57}$ and g_p/g_a from these data. Figures 5.13 and 5.14 show $N_{>57}^\gamma$ as a function of $G_{>57}$ and g_p/g_a respectively. Again, the hatched area corresponds to the experimental result with all contributions to the experimental error included. The results are given in Table 5.5.

The extracted values for $G_{>57}$ using the two theories are in excellent agreement, and yet the values for g_p/g_a are very different. The result of $g_p/g_a = 7.3 \pm 0.9$ using the phenomenological calculation of Christillin and Gmitro [82] is in agreement with the Goldberger-Treiman estimate, but the result $g_p/g_a = 13.6^{+1.6}_{-1.9}$ from the modified impulse approximation (MIA) calculation of Gmitro *et al.* [73] indicates a large upwards renormalization of g_p in the ^{16}O nucleus. The two calculations are clearly in conflict. This suggests that the ‘agreement’ between the MIA calculation [73] and the phenomenological calculation [81] for ^{40}Ca seen earlier is probably fortuitous. It is also interesting to note that the present results for the ^{16}O branching ratio compared to the earlier IA calculation of Gmitro *et al.* [83] (without the

Table 5.5: Summary of the results for the partial branching ratio $G_{>57}$ and g_p/g_a for ^{16}O from the theories of Gmitro *et al.* [73] and Christillin and Gmitro [82] using the ‘integral method’.

Theory	$G_{>57} (10^{-5})$	g_p/g_a
Gmitro <i>et al.</i> [73]	2.18 ± 0.21	$13.6^{+1.6}_{-1.9}$
Christillin and Gmitro [82]	2.22 ± 0.23	7.3 ± 0.9

Table 5.6: Summary of the results for the partial branching ratio $G_{>57}$ and g_p/g_a for ^{16}O from the theories of Gmitro *et al.* [73] and Christillin and Gmitro [82] using the ‘spectrum fitting method’. ‘PP’ refers to a fit to the Primakoff polynomial (closure approximation).

Theory	$G_{>57} (10^{-5})$	k_{max} (MeV)	g_p/g_a	χ^2_{dof}
Gmitro <i>et al.</i> [73]	2.13 ± 0.24	—	$13.1^{+1.8}_{-2.0}$	1.15
Christillin and Gmitro [82]	2.16 ± 0.24	—	7.1 ± 0.9	1.35
PP	2.29 ± 0.39	87.8 ± 4.2	—	1.04

continuity equation constraint) would yield $g_p/g_a < 4.0$. As an historical note, the present branching ratio would lead to a negative value for g_p/g_a if compared to the closure model calculation for ^{16}O of Rood and Tolhoek [56].

The results of the spectrum fitting method for ^{16}O are given in Table 5.6 and the resulting fits are displayed in Figure 5.15. The theory of Gmitro *et al.* has a slightly smaller χ^2_{dof} than that of Christillin and Gmitro; the fit to the Primakoff polynomial with $k_{max} = (87.8 \pm 4.2)$ MeV gives a slightly better representation of the data. The results of the spectrum fitting method are in excellent agreement with the results of the integral method. The integral method results will be considered as the final results for the same reasons discussed in the ^{40}Ca case. The much larger error on the value of $G_{>57}$ extracted using the Primakoff polynomial is due to the uncertainty in k_{max} .

The present results are compared to the other existing data for RMC on

Figure 5.11: Photon energy spectrum from ^{16}O compared to the theory of Gmitro *et al.* [73]. The theoretical spectra have been convoluted with the detector response function.

Figure 5.12: Photon energy spectrum from ^{16}O compared to the theory of Christillin and Gmitro [82]. The theoretical spectra have been convoluted with the detector response function.

Figure 5.13: $N_{>57}^{\gamma}$ from ^{16}O as a function of $G_{>57}$ for the theories of Christillin and Gmitro [82] ('CG') and Gmitro *et al.* [73] ('GOT').

Figure 5.14: $N_{>57}^{\gamma}$ from ^{16}O as a function of g_p/g_a for the theories of Christillin and Gmitro [82] ('CG') and Gmitro *et al.* [73] ('GOT').

Figure 5.15: Photon energy spectrum from ^{16}O compared to the best fits from the spectrum fitting method. ‘CG’ refers to the theory of Christillin and Gmitro [82], ‘GOT’ to that of Gmitro *et al.* [73] and ‘PP’ to the Primakoff polynomial.

^{16}O in Table 5.7. Only two previous measurements exist. The first is that of Frischknecht *et al.* [129] from the same group that provided the results for ^{40}Ca given in reference [87]. The same high-resolution pair-spectrometer was used as for their ^{40}Ca measurement. The ^{16}O results are still in preliminary form and only a few details of the experimental technique are available. A D_2O target was used with two Au photon converters. High muon stopping rates were used ($\sim 4 \times 10^6 \text{ s}^{-1}$). Presumably, similar problems in normalizing to the number of muon stops were encountered as in the ^{40}Ca case. Their raw photon spectrum showed a very large pion-induced background, several times that of the RMC signal. Despite the preliminary nature of these data, the results have been compared to theory in several papers [82,73,52]. The measured branching ratio indicates a value of $g_p/g_a > 16$ when compared to the calculation of Christillin and Gmitro [82] and

Table 5.7: Summary of the results for the partial branching ratio $G_{>57}$ and g_p/g_a for ^{16}O from all existing measurements. ‘GOT’ refers to the theory of Gmitro *et al.* [73], ‘CG’ refers to the theory of Christillin and Gmitro [82] and ‘CRS’ refers to a comparison to an extrapolation of the Fermi-gas model calculations of Christillin *et al.* [60].

$G_{>57}$ (10^{-5})	Theory	# events	k_{max}	g_p/g_a (MeV)	reference
6.2 ± 0.8	CG	800	—	$>> 16$	Frischknecht <i>et al.</i> [129]
2.44 ± 0.47	CG	325	—	8.4 ± 1.9	Döbeli <i>et al.</i> [122,123]
2.39 ± 0.46	CRS	”	89.9 ± 5.0	$6.0^{+2.0}_{-3.0}$	”
2.22 ± 0.23	CG	361	—	7.3 ± 0.9	present work
2.18 ± 0.21	GOT	”	—	$13.6^{+1.6}_{-1.9}$	”
2.29 ± 0.39	CRS	”	87.8 ± 4.2	$4.9^{+2.2}_{-2.4}$	”

$g_p/g_a \gg 16$ when compared to the MIA calculation of Gmitro *et al.* [73]. If compared to the earlier IA calculation of Gmitro *et al.* [83] the branching ratio indicates $g_p/g_a = 14 \pm 1$. In all cases, the data of Frischknecht *et al.* [129] indicated a large upward renormalization of g_p in the ^{16}O nucleus. However, their branching ratio disagrees strongly with the one measured in the present work (by more than four standard deviations).

The only other previous measurement of RMC in ^{16}O is that of Döbeli *et al.* [122,123]. Details of their experiment have been given above in the discussion of their ^{40}Ca data, but there are some factors unique to the ^{16}O case that should be mentioned. An H_2O target was used, and combined with the poor prompt-rejection of their method this lead to a very large pion-induced background (50% of the RMC signal *after* the elimination of ‘prompt’ events). Relatively large empty target (18%), neutron-induced (10%) and cosmic-ray induced (8%) background subtractions were also required. Only the data from their modular NaI(Tl) array were available for this measurement. The data from the single NaI(Tl) were swamped by pileup events, in which a bremsstrahlung photon and a subsequent muon-decay electron were summed in energy to yield an apparent RMC photon. These pileup events

were not rejected properly in the electronics (the effect was small in the ^{40}Ca case, and so the data from the single NaI(Tl) were still useable for that target).

Unfortunately, this pileup effect was not discovered until the publication of preliminary results at two different conferences: a partial branching ratio of $(6.5 \pm 1.3) \times 10^{-5}$ was given in reference [124] and an even higher value of $(9.3 \pm 1.9) \times 10^{-5}$ was given in reference [126]. These preliminary results, in apparent agreement with the pair-spectrometer results of Frischknecht *et al.* [129], were quoted in the review article of Gmitro and Truöl [52], and led to the conclusion by those authors that g_p/g_a in ^{16}O was significantly higher than the Goldberger-Treiman estimate.

The final result of Döbeli *et al.* [122,123], however, disagrees with their preliminary result and with the result of Frischknecht *et al.* [129], and instead agrees well with the present result (see Table 5.7). A final comment is necessary on the size of the error quoted by Döbeli *et al.* [122,123]. They normalized the acceptance of their modular NaI(Tl) detector using π^- capture on the proton obtained from a subtraction of $\pi^- \text{CH}_2$ and $\pi^- \text{C}$ spectra. This requires a knowledge of the probability of π^- capture on the proton in CH_2 ($W(\text{CH}_2)$). They used the value $W(\text{CH}_2) = 1.26 \pm 0.06\%$. As discussed earlier, the lack of internal consistency of the data for $W(\text{CH}_2)$ indicates a value with a larger uncertainty, $W(\text{CH}_2) = 1.29 \pm 0.18\%$, which would lead to a larger error on their value of $G_{>57}$ for ^{16}O .

As mentioned, there has been some confusion in the literature about the meaning of the parameter ' Δ ' used by Christillin to represent different values of g_p/g_a ; in particular, the figures given by Gmitro and Truöl [52] referring to the phenomenological calculation for ^{16}O [82] are incorrect. Döbeli *et al.* apparently used $g_p/g_a = 7.0$ as the Goldberger-Treiman value ($\Delta = 1$), slightly different from the canonical $g_p/g_a = 6.78$ used here. Use of the canonical value would decrease the value extracted by Döbeli *et al.* to $g_p/g_a = 8.1 \pm 1.8$.

Finally, it should be noted that Döbeli *et al.* [122,123] used a value for f_{capture}

(the fraction of muons stopping in ^{16}O that undergo OMC) of $(17.5 \pm 0.5)\%$, based on the muon lifetime results summarized by Eckhause *et al.* [130]. As discussed earlier, more recent data are available (see reference [109]) which yield $f_{\text{capture}} = (18.38 \pm 0.11)\%$, the value used in this work. Applying this value to the results of Döbeli *et al.* [122,123] would further reduce their branching ratios by another 5%, which would improve their agreement with the present results.

Döbeli *et al.* [122,123] do not compare their results to the MIA calculation of Gmitro *et al.* [73], stating in their paper that ‘Shell model (MIA) and phenomenological response lead to consistent descriptions for ^{16}O and ^{40}Ca ’. However, this is *not* true for ^{16}O , as has been discussed earlier (it is also not true for either nucleus if one considers the observable α_γ). A comparison of their results with the MIA calculation would lead to the value $g_p/g_a \sim 15$.

Döbeli *et al.* also compare their results to the Fermi-gas model calculation of Christillin *et al.* [60]. This calculation was performed for selected medium-heavy nuclei ($Z > 42$). To apply this to ^{16}O , Döbeli *et al.* used the following technique: for each value of g_p/g_a given in reference [60] they fit the calculated values of $G_{>57}$ to a linear function of Z (the nuclear charge). The fits were constrained to pass through the $G_{>57}$ calculated by Christillin [81] for ^{40}Ca for the appropriate value of g_p/g_a . Note that this latter calculation for ^{40}Ca used a phenomenological Lorentzian GDR and GQR for the nuclear response, a very different approach from the Fermi-gas model. These linear fits were then extrapolated to low- Z nuclei, e.g. ^{16}O . Branching ratios were then extracted by comparing the data to the Primakoff polynomial (a closure model result), since this technique provides no description of the shape of the RMC spectrum. We consider this procedure dubious, at best. For the purposes of comparison, the value of g_p/g_a extracted using this procedure is also given in Table 5.7. The much larger error on g_p/g_a than those determined using the other two models is due to the uncertainty in the experimental value of k_{max} . Döbeli *et*

al. [123,122] apparently have not included the effect of the (large) error on k_{max} on their extracted $G_{>57}$ and g_p/g_a . The value of g_p/g_a extracted from the present data using this extrapolation of the Fermi-gas model agrees with that found from the phenomenological calculation of Christillin and Gmitro [82]. We do not ascribe much significance to this, however, due to the somewhat questionable technique of extrapolating a calculation for medium-heavy ($A > 92$) nuclei to ^{16}O .

The present results for $G_{>57}$ agree with the results of Döbeli *et al.* [123,122] and the agreement of these two experiments casts doubt on the very different results of Frischknecht *et al.* [129]. The interpretation of the present branching ratio in terms of g_p/g_a gives two very different values when compared to the available calculations of the nuclear response for ^{16}O . A firm statement about possible renormalization of g_p/g_a in ^{16}O away from the Goldberger-Treiman value awaits a clarification of the theoretical situation. Unfortunately, it does not appear possible to select between the two calculations by a measurement of α_γ in ^{16}O . Not only would this pose formidable experimental challenges, but the ratio $G_{>57}/\alpha_\gamma$ is not significantly different for the two calculations [127,52].

It is suggestive, however, that the value of g_p/g_a found using the MIA theory of Gmitro *et al.* [73] is consistent with the values of g_p/g_a ($\sim 11 - 12$) determined from OMC and β -decay measurements in ^{16}O [131,132,46]. It is perhaps relevant that the explicit effects of meson-exchange corrections have been included (to some degree) in both the MIA calculations and the calculations used to extract g_p/g_a from the OMC and β -decay data.

5.3 ^{12}C

The extraction of a partial branching ratio and value for g_p/g_a is somewhat more problematic for the ^{12}C data. This is true because there is not, at present, any theoretical calculation available for inclusive RMC on ^{12}C . One approach, used

Figure 5.16: Photon energy spectrum from ^{12}C , 1.0 mm converter, compared to the theory of Christillin and Gmitro *et al.* [82] for ^{16}O . The theoretical spectra have been convoluted with the detector response function.

by Döbeli *et al.* [122,123], is to apply the available calculations for ^{16}O [73,82] to the ^{12}C case. This is obviously a dubious technique, as both calculations are based on quite specific details of the nuclear structure of ^{16}O , and should not be expected to be applicable to a much different nucleus. Nevertheless, for the purposes of comparison with the results of Döbeli *et al.* [122,123], this approach was also used here.

Figures 5.16 and 5.17 show the comparison of the photon spectrum for ^{12}C obtained with the 1.0 mm converter, compared to the theoretical spectra for ^{16}O from Christillin and Gmitro [82] and Gmitro *et al.* [73] respectively. As before, the errors on the data include the effect of the subtraction of high-energy tail events from the bremsstrahlung background. The results for the data taken with the 0.6 mm converter are similar, but have much poorer statistics. The results of the integral

Figure 5.17: Photon energy spectrum from ^{12}C , 1.0 mm converter, compared to the theory of Gmitro *et al.* [73] for ^{16}O . The theoretical spectra have been convoluted with the detector response function.

method are listed in Table 5.8.

The spectrum fitting method was also used for the ^{12}C data. The results are given in Table 5.9, and the fits obtained are shown in Figures 5.18 and 5.19 for the data taken with the 1.0 mm and 0.6 mm converters respectively. For the 0.6 mm converter data the statistics were too poor to provide a stable two-parameter fit to the Primakoff polynomial.

There is good agreement between the extracted branching ratios from the two data sets (1.0 mm converter and 0.6 mm converter), for both the integral and spectrum fitting methods. However, the spectrum fitting method gives lower branching ratios than does the integral method. The results of the spectrum fitting method for the 1.0 mm converter data show that neither of the two ^{16}O calculations provide a good representation of the observed ^{12}C spectrum (the statistics of the 0.6

Figure 5.18: Photon energy spectrum from ^{12}C , 1.0 mm converter, compared to the best fits from the spectrum fitting method. ‘CG’ refers to the theory of Christillin and Gmitro [82] and ‘GOT’ to the theory of Gmitro *et al.* [73] for ^{16}O . ‘PP’ refers to the Primakoff polynomial.

mm converter data are too poor to confirm this). This is not too surprising, as the calculations are being applied to a different nucleus. However, it could be argued that the bad fits are at least partially due to an underestimate of the effect of the bremsstrahlung spectrum, which would increase the yield at lower photon energies. The Primakoff polynomial gives a reasonable fit with a k_{max} of about 85 MeV. The branching ratio extracted using this fit is somewhat higher than those determined using the ^{16}O calculations, because more of the spectrum is concentrated at lower photon energies where the acceptance is smaller.

The only other data on RMC on ^{12}C is that of Döbeli *et al.* [122,123]. Just as for their ^{16}O data, only the modular NaI(Tl) detector gave useable data, since the single crystal NaI(Tl) detector was swamped by pileup events. The preliminary results for the partial branching ratio of $(7.4 \pm 1.8) \times 10^{-5}$ presented in reference [124]

Table 5.8: Summary of the results for the partial branching ratio $G_{>57}$ and g_p/g_a for ^{12}C from the theories of Gmitro *et al.* [73] and Christillin and Gmitro [82] for ^{16}O , using the ‘integral method’.

Theory	Converter	$G_{>57} (10^{-5})$	g_p/g_a
Gmitro <i>et al.</i> [73]	1.0 mm	2.21 ± 0.18	$13.8^{+1.5}_{-1.7}$
	0.6 mm	2.47 ± 0.34	$15.9^{+2.3}_{-2.9}$
	average	2.27 ± 0.18	$14.3^{+1.5}_{-1.6}$
Christillin and Gmitro [82]	1.0 mm	2.25 ± 0.22	7.4 ± 0.9
	0.6 mm	2.54 ± 0.34	8.6 ± 1.4
	average	2.34 ± 0.23	7.8 ± 0.9

Table 5.9: Summary of the results for the partial branching ratio $G_{>57}$ and g_p/g_a for ^{12}C from the theories of Gmitro *et al.* [73] and Christillin and Gmitro [82] for ^{16}O using the ‘spectrum fitting method’. ‘PP’ refers to a fit to the Primakoff polynomial (closure approximation).

Theory	Converter	$G_{>57} (10^{-5})$	k_{max} (MeV)	g_p/g_a	χ^2_{dof}
Gmitro <i>et al.</i> [73]	1.0 mm	1.86 ± 0.24	—	$10.5^{+2.4}_{-2.8}$	1.79
	0.6 mm	2.42 ± 0.57	—	$15.5^{+3.8}_{-5.1}$	1.05
	average	1.94 ± 0.22	—	$11.3^{+1.9}_{-2.4}$	—
Christillin and Gmitro [82]	1.0 mm	1.90 ± 0.22	—	6.1 ± 0.8	1.69
	0.6 mm	2.38 ± 0.43	—	7.9 ± 1.7	1.03
	average	2.00 ± 0.20	—	6.5 ± 0.7	—
PP	1.0 mm	2.64 ± 0.63	85.0 ± 4.9	—	1.04

Figure 5.19: Photon energy spectrum from ^{12}C , 0.6 mm converter, compared to the best fits from the spectrum fitting method. Labels as for 5.18

and the later value of $(10.6 \pm 3.0) \times 10^{-5}$ [126] were based on data from the single NaI(Tl) and are therefore erroneous. Again, these preliminary results were quoted in the review by Gmitro and Truöl [52] and were considered by those authors to add evidence of a systematic renormalization (upwards) of g_p in light nuclei (even without a theoretical calculation specific to ^{12}C , it would be difficult to reconcile branching ratios this large with the Goldberger-Treiman value of g_p/g_a).

The branching ratio listed in Table 5.10 for Döbeli *et al.* [122,123] is their final result, based only on the modular NaI(Tl) detector, and supersedes the preliminary results given above. A 6.0 mm Pb photon converter was used, which gave much poorer energy resolution. Only 75 ± 46 events were observed, and the pion-induced background was again large (46% of the RMC spectrum *after* the prompt cut). No estimate was provided for the neutron-induced background for this target, but the empty-target background was very large (52% of the RMC signal). Finally, due to

Table 5.10: Summary of the results for the partial branching ratio $G_{>57}$ and g_p/g_a for ^{12}C from all existing measurements. ‘GOT’ refers to the theory of Gmitro *et al.* [73] and ‘CG’ refers to the theory of Christillin and Gmitro [82] for ^{16}O . ‘CRS’ refers to a comparison with an extrapolation of the Fermi-gas model calculation of Christillin *et al.* [60]. For the present data, there are fewer events in the ‘CRS’ since there only the 1.0 mm converter data could be used, as discussed in the text.

$G_{>57}$ (10^{-5})	Theory	# events	k_{max}	g_p/g_a (MeV)	reference
2.7 ± 1.8	CG	75	—	9.5 ± 7.2	Döbeli <i>et al.</i> [122,123]
”	CRS	”	—	$7.2^{+5.6}_{-11.8}$	”
2.34 ± 0.23	CG	613	—	7.8 ± 0.9	present work
2.27 ± 0.18	GOT	”	—	$14.3^{+1.5}_{-1.6}$	”
2.64 ± 0.63	CRS	497	85.0 ± 4.9	$6.7^{+2.8}_{-3.6}$	”

an unknown efficiency of one of the beam counters, this data had to be normalized to ^{40}Ca data taken under the same conditions. All of these effects lead to the very large error on $G_{>57}$. Their $G_{>57}$ agrees with the much more precise results of the present work, but puts no significant constraints on g_p/g_a on its own.

Without a calculation of RMC specific to ^{12}C , the interpretation of the present results in terms of g_p/g_a must be considered somewhat tentative. Just as in the case of ^{16}O , the phenomenological calculation of Christillin and Gmitro yields a value for g_p/g_a not significantly different from the Goldberger-Treiman value, while the MIA calculation of Gmitro *et al.* indicates a value substantially in excess of the Goldberger-Treiman estimate. The extrapolation of the Fermi-gas model of Christillin *et al.* [60] is also consistent with the Goldberger-Treiman value. On the naive assumption that the specifics of the nuclear structure do not affect the RMC branching ratio very strongly, then the approximate agreement of $G_{>57}$ for ^{12}C and ^{16}O would suggest that g_p/g_a is not very different in the two nuclei. The group of Gmitro *et al.* is intending [133] to perform an MIA calculation for ^{12}C , which would allow a more definitive interpretation of the present results in the near future.

Chapter 6

Conclusions

We have measured the branching ratio and photon energy spectrum for radiative muon capture on ^{40}Ca , ^{16}O and ^{12}C . These constituted the first measurements of RMC using a large solid-angle pair-spectrometer. The large solid angle of the detector allowed us to obtain good statistics while avoiding the very high muon beam fluxes that have caused difficulties for some previous experiments. Essentially background-free spectra were obtained which lead to a precision of $\approx 10\%$ or better on the measured branching ratios. For two of the targets (^{40}Ca and ^{12}C) measurements were made using two different photon converters, providing a valuable consistency check on the data.

For ^{40}Ca , good agreement was found for the partial branching ratio $G_{>57}$ with other recent measurements. Comparing to the two most recent calculations of the nuclear response, a value for g_p/g_a of 5.7 ± 0.8 was obtained using the phenomenological calculation of Christillin [81] and a value of 4.6 ± 1.8 using the microscopic calculation of Gmitro *et al.* [73]. In common with another recent measurement (Frischknecht *et al.* [87]) a slight preference was found for the spectral shape predicted by Christillin. The values for g_p/g_a extracted using the two theories indicate a small downward renormalization of g_p/g_a from the unrenormalized Goldberger-Treiman prediction of $g_p/g_a = 6.7$, although the effect is only about 1σ in each case. The ‘renormalization’ is about the same size as the difference between the values extracted from the two calculations, and so should be regarded with some caution. Clearly, more work is required on the theoretical side before a definitive statement

can be made about the presence (or absence) of a small renormalization of g_p in ^{40}Ca .

For ^{16}O , the extracted partial branching ratio was found to disagree completely with one previous measurement (Frischknecht *et al.* [129]) and to agree with the other previous measurement (Döbeli *et al.* [122]). The experimental error was reduced from 19% to 10% . A reasonable fit to the spectral shape was found for both the phenomenological calculation of Christillin and Gmitro [82] and the microscopic calculation of Gmitro *et al.* [73]. However, very different values of g_p/g_a were extracted using these two different theories. A value of $g_p/g_a = 7.3 \pm 0.9$ was obtained using the phenomenological calculation, which is in good agreement with the Goldberger-Treiman value. With the microscopic calculation a value of $g_p/g_a = 13.6^{+1.6}_{-1.9}$ was found, in strong disagreement with the Goldberger-Treiman value and indicating a strong upwards renormalization of g_p/g_a in ^{16}O (or a failure of PCAC). It is apparent that some theoretical effort is required to resolve the contradictions between the two calculations before it can be stated that g_p is renormalized in ^{16}O .

For ^{12}C the first reliable data on RMC were obtained. In the absence of a calculation of the nuclear response specific to ^{12}C , the data were compared to the two recent calculations for ^{16}O ; neither was found to reproduce the observed shape of the photon energy spectrum. Comparison of the integrated rate above 57 MeV to these two calculations gave very similar values to those found for these calculations in the ^{16}O case. A fit to the closure-approximation ‘Primakoff polynomial’ yielded a reasonable fit with $k_{max} = 85 \pm 5$ MeV and a partial branching ratio of $(2.6 \pm 0.6) \times 10^{-5}$. Comparing this with a naive extrapolation of the Fermi-gas model calculations for medium-heavy nuclei by Christillin *et al.* [60] gave a value of $g_p/g_a = 6.7^{+2.8}_{-3.6}$ in agreement with the Goldberger-Treiman value. The present results have prompted a theoretical effort to calculate the nuclear response for RMC in ^{12}C [133].

Based on the data available at that time, Gmitro and Truöl [52] concluded that the RMC results on ^{12}C and ^{16}O were incompatible with the Goldberger-Treiman value for g_p/g_a . The present results, along with those of Döbeli *et al.* [123,122] contradict this conclusion, at least using phenomenological and Fermi-gas calculations of the nuclear response.

To put the present results into perspective, Figure 6.1 presents a summary of the existing data for the RMC branching ratio ($G_{>57}$) as a function of the nuclear charge Z . For ^{40}Ca , the average of the five most recent measurements (including the present experiment) is plotted. The present results indicate that for nuclei with $Z \leq 20$, the RMC branching ratio appears to be rather constant at about 2.2×10^{-5} ; however the data of Döbeli *et al.* [122] for higher- Z nuclei (specifically ^{165}Ho and ^{209}Bi) indicate that this does not seem to hold for very heavy nuclei.

In summary, the present results when compared to phenomenological and Fermi-gas models [81,82,60] yield values of g_p/g_a consistent with the Goldberger-Treiman estimate (except in the case of ^{40}Ca where a slight ‘quenching’ is indicated at the 1σ level). However a comparison to the microscopic modified impulse approximation calculation [73], yields a highly renormalized value of g_p/g_a for ^{16}O while giving a slightly ‘quenched’ value for ^{40}Ca . Until a resolution of the differences between the existing theoretical treatments for ^{16}O (and to a lesser degree for ^{40}Ca) is arrived at the question of the renormalization of g_p in light nuclei is still open.

6.1 Suggestions for Further Work

The present work demonstrates that a technique has been developed to measure RMC branching ratios reliably, even for relatively light nuclei. In fact, one of the primary motivations for the present work was to uncover the various systematic effects and backgrounds that limit a measurement of RMC on light nuclei, with

Figure 6.1: RMC branching ratio ($G_{>57}$) versus the nuclear charge Z . 'TPC' refers to the present experiment, 'FRI 82' to the results of Frischknecht *et al.* [129] and 'DOB 88' refers to the results of Döbeli *et al.* [122]. For the case of ^{40}Ca , the average of the five most recent measurements (including the present one) is plotted ('world average'). For clarity, in the ^{12}C and ^{16}O cases the different data have been slightly displaced along the Z axis.

the objective of designing a measurement of RMC on hydrogen. This objective has been met, and the present work has provided very valuable input to the design of a detection system for TRIUMF experiment 452 [50], a measurement of RMC on hydrogen. This experiment is expected to begin data-taking early in 1989.

The primary limitation of the present experiment was the rate dependence of the photon acceptance. This reduced the photon acceptance (and therefore the statistics acquired) and also added uncertainty to the normalization of the acceptance. In addition, because of the loss of points in the reconstructed tracks, the energy resolution was degraded. This loss contributed to the major background encountered, namely bremsstrahlung events in the high-energy tail of the detector response. These considerations dictated the design of the detector for experiment 452. The detector designed (and constructed) is a conventional large volume drift chamber [50]. This detector should not exhibit the strong rate-dependent acceptance observed in the (ageing) TPC. Extensive Monte Carlo studies were done with the emphasis on minimizing the high-energy tail in the acceptance. Other design considerations of the drift chamber, electronics, trigger counters and beam-line benefitted greatly from the experience gained from the present experiment.

It is expected that, in addition to the hydrogen measurement, the new detector will be used to perform some nuclear RMC measurements. It would be interesting to turn to heavier nuclei to verify the results of Döbeli *et al.* [122] that appear to indicate a systematic ‘quenching’ of g_p/g_a for high- Z targets. A measurement of RMC on ^3He would also be desirable, as the interpretation in terms of g_p/g_a would be relatively unambiguous. Finally, the possibility exists of attempting exclusive RMC measurements on selected light nuclei, for which the nuclear structure uncertainties would be minimized.

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Appendix A

GEANT Monte Carlo

The Monte Carlo simulation of the experiment was performed using the framework of the GEANT program (version 3.10) [100]. This appendix will primarily be concerned with the user-written routines that were specific to the present application.

Each GEANT ‘event’ began with the generation of an initial particle. For most of the present applications, this was a photon, with energy sampled from a relevant energy spectrum (e.g. RMC spectrum, $^{12}\text{C}(\pi^-, \gamma)$ spectrum, etc.). The initial location of the particle (usually in the target) was sampled from a realistic muon (or pion) stopping distribution. The initial direction was sampled from a random azimuthal (ϕ) direction; the polar angle was sampled by choosing from a flat distribution in $\cos\theta$ between $\sqrt{2}/2$ and $-\sqrt{2}/2$, i.e. between 45° and 135° to the beam axis. The restricted range in θ was implemented simply to reduce the running time of the program, as it was found that initial photons outside this range in θ had negligible probability of triggering the detector. For all results extracted from the Monte Carlo, the normalization was corrected for this reduced solid angle.

All the relevant geometry of the detector was included in the Monte Carlo. The geometry ‘ended’ at the coils of the magnet surrounding the detector, as any particle entering the magnet would be likely to be absorbed in the coils or the magnet steel. There were a few approximations made in the geometry, either to reduce execution time or to simplify coding. The IA and IB scintillators were cylindrical in the Monte Carlo rather than 18-sided polygons. This had negligible

effect on the results (recall that these were veto counters, and should not fire for a valid event anyway). The walls of the EWC and IWC wire chambers were ignored (these were quite thin, and should have little effect). The central high-voltage grid of the TPC was not included; this was mostly empty space and should also have a negligible effect. Finally, the light guides for all scintillators were empty space (air). The detector geometry was such that any particle passing through these light guides would be unlikely to enter the TPC active volume, and so they should be ignorable. Aside from the above approximations all components of the detector were accurately represented in GEANT.

GEANT provides routines to simulate all important physical processes undergone by the relevant particles in the simulation (primarily photons, electrons and positrons). These processes included

- Pair production
- Compton scattering
- Photoelectric effect
- Bremsstrahlung
- Positron Annihilation
- Multiple scattering of charged particles (using Molière theory)
- Energy loss of charged particles (using Landau distribution or discrete production of δ -rays)

Some tests were done of the effect of using the discrete production of δ -rays to simulate energy loss. The only significant difference from sampling using a Landau distribution was that the ‘trigger-rate’ increased by about 10% due to events where

a δ -ray from an electron or positron passing through the converter traversed the TPC volume and stopped in an outer trigger scintillator (E or W counter).

All secondary particles produced by the initial particle were also tracked and allowed to interact, unless they were below a user-defined threshold energy (100 keV for most of the Monte Carlo run). In the case where the initial particle was a photon (i.e. the majority of the simulations), the simulation of a given Monte Carlo event was aborted early under either of two possible conditions. The first condition was if over 100 keV in energy was deposited in any IA or IB veto scintillator. Since this event would be vetoed by the trigger condition that there be no signals from any IA or IB counter, tracking of the event was ended immediately. The second condition was if the initial photon had entered the active volume of the TPC (i.e. had passed through the converter package and the inner walls of the TPC) without interacting (i.e. producing any secondary particles). Early tests with GEANT had shown that any such photon had a negligible probability of producing a valid trigger, so the tracking of that event could be safely halted. These two measures served to considerably reduce the CPU time required per generated photon without sacrificing any accuracy in the simulation.

For any Monte Carlo ‘event’ that satisfied the same logical requirements as those demanded by the hardware trigger electronics, the information from the event was stored for subsequent analysis. Routines were added to the Monte Carlo in order to provide the results of each event in the same format as the actual experimental data written to tape. This allowed the same version of the analysis routines (SOFIA) to be used to analyze the GEANT ‘data’ and actual data. The routines written to digitize the information provided by GEANT are discussed below.

The digitization of the information from the trigger counters (scintillators and wire chambers) was relatively straightforward. In the case of the scintillators, the relevant register bits were set if energy was deposited in a scintillator; because the

pulse-height information from the scintillators was not used in the data analysis it did not need to be encoded by the Monte Carlo. In the case of the IWC, the position of each charged particle crossing the chamber was ‘smeared’ by the measured resolution of the chamber and stored as bit register information. Information from the EWC’s (external wire chambers) was not digitized, because they were not used in the data analysis.

The information from the TPC required more work, however. GEANT provided a set of (x, y, z) coordinates for the charged particle tracks that traversed the TPC active volume; this had to be translated into a series of ADC and TDC values for the TPC cathode pads and anode wires. The digitization parameters described below were determined from several sources: fits to cosmic-ray data, fits to pion calibration data, and beam-off measurements of electronic noise. These empirical parameters determine the spatial resolution (and therefore largely determine the energy resolution) of the TPC; the dependence of these parameters on drift length, track crossing angle, magnetic field etc. were as given in reference [90].

The original (x, y, z) coordinates that were digitized were the locations where the given track crossed the projection along the z -axis of the (x, y) location of one of the anode wires. The z -coordinate was converted to a time value using the measured electron drift velocity, smeared by a resolution factor and converted to a TDC value using the known TDC calibration. Only the ‘earliest’ hit on a given TDC was encoded, to reproduce the single-hit capability of the CAMAC TDC’s.

For the cathode pad ADC data, the (x, y) coordinate was smeared by a resolution factor and converted to a position that represented the centroid of the cathode pad ‘clump’. A clump amplitude was chosen from a Landau distribution, modified according to the wire crossing angle, drift length (to account for diffusion) and pitch angle of the track. The clump width was also chosen, based again on track crossing angle etc. The pulse heights on each cathode pad were determined

from the resulting (Gaussian) distribution. Subsequent hits on the same pad were simply summed if they were within the time determined by ADC gate width and ignored otherwise. The anode ADC data was determined in a similar fashion. An overall 8% inefficiency for the anode wires was added to simulate the effect of bad and marginal wires in the TPC; the 8% was determined from measured efficiencies using cosmic-rays.

The cathode and anode ADC data were then multiplied by two amplitude factors. The first depended on the anode wire, and increased as one moved outward from the TPC center, i.e. wire #2 had a higher gain than wire #1 etc. . This variation was determined from fitting to pion calibration data, and represented both the differential effect of ageing on the TPC wires, as well as the higher rates on the inner wires. The second factor was a common, overall amplitude normalization. This represented the effect of changing the rate through the chamber (assuming that the distribution of charged particles in the TPC did not alter significantly as their rate was varied). This factor was varied at execution time to simulate different rates. The anode and cathode ADC data were also smeared using the measured electronic noise factors, and converted to the same format used by the LRS 2280 ADC's for actual data, including the effects of pedestal subtraction and compactification.

The dependence of the GEANT results on the various parameters used in the digitization was studied by varying them within reasonable ranges. Aside from the overall amplitude factor (i.e. the 'rate'), the only critical parameter was the cathode pad noise factor. The predicted photon acceptance and energy resolution were found to be quite sensitive to this noise factor. The reason is that an increase in the noise can push the individual cathode pads above or below threshold, thereby distorting the cathode pad clump or even losing it entirely from the fit. It could be argued that the electronic noise measured with the beam off could be different from

the actual noise with beam on, and so the beam-off noise might be inappropriate to use in the Monte Carlo. However, the dependence on the cathode pad noise of the various observed quantities in the data was studied using GEANT. It was found that several of these (e.g. χ^2 of the fit, observed width of the clumps) could be used as ‘indicators’ of the pad noise, these were mutually consistent with the observed data only when the noise was chosen very near that measured with the beam off.

Finally, the accuracy of the Monte Carlo simulation using GEANT was checked by comparing the results of the analysis of GEANT data with that for actual data, for a host of observed quantities. A sample of these comparisons for the $^{12}\text{C}(\pi^-, \gamma)$ reaction is given in the following figures. The agreement is seen to be very reasonable in all cases.

Figure A.1: Comparisons between $^{12}\text{C}(\pi^-, \gamma)$ data (dots) and GEANT predictions (histogram) for various quantities. The quantities are a) ϕ , the photon azimuthal angle, b) θ , the photon polar angle, c) the perpendicular deviation of the clump centroid from the fitted track, d) the χ^2 of the circle fit, e) the cathode pad pulse amplitude, and f) the width of the cathode pad clumps.

Figure A.2: Comparisons between $^{12}\text{C}(\pi^-, \gamma)$ data (dots) and GEANT predictions (histogram) for various quantities. The quantities are a) the photon opening angle, b) the photon opening angle in the (x, y) -plane, c) and d) the track mismatch between the TPC and the IWC in z and ϕ , e) the number of points in the helix fit (N_{xyz}) and f) the dE/dx of the tracks.