PRECISION MEASUREMENT OF PION-PROTON ABSOLUTE DIFFERENTIAL CROSS SECTIONS AT ENERGIES SPANNING THE DELTA RESONANCE

By

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M. A. Sc. (Engineering Physics) University of British Columbia, 1990

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Doctor of Philosophy

in

THE FACULTY OF GRADUATE STUDIES

PHYSICS

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

May 1995

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Abstract

A measurement of the absolute differential cross section in the $\pi^p$ elastic reaction for incident pion laboratory kinetic energies of $T_\pi = 141.2, 168.8, 193.2, 218.1, 240.9,$ and $267.3$ MeV is described. Measurements where the scattered pion and recoil proton were detected in coincidence with the incident pion beam were taken at all energies. These measurements obtained data at six laboratory pion angles from $60^0$ to $155^0$. Single arm measurements, where only the scattered pions were detected at six near-forward angles from $20^0$ to $70^0$, were performed at the lowest four energies. Both the single arm and coincidence measurements employed a flat-walled supercooled liquid hydrogen target. The uncertainty in the proton density of this target was 0.5%. Solid CH$_2$ targets were also employed for coincidence measurements at the lowest four energies. The proton density uncertainties of these targets was 1%. Numerous measurements were performed to test the reproducibility of the cross sections under a variety of experimental conditions, and to elucidate potential sources of systematic uncertainty. Excellent consistency was found amongst all these data sets. In particular, the results obtained with the LH$_2$ and CH$_2$ targets were consistent at all energies. The final $\pi^+p$ data have typically 1-1.5% statistical and 1% normalization uncertainties, whereas these uncertainties are typically 1.5-2% and 1.5%, respectively, for the $\pi^-p$ data.

At incident pion energies above 190 MeV, there is reasonable agreement between the results from these measurements and those of Bussey et al. The agreement with the data of Sadler et al. at 263 MeV is excellent. Below 190 MeV, the data presented here are systematically below the data of Bussey et al., well outside the stated normalization uncertainties. The data are consistent within the stated uncertainties with
the Brack et al. results near 140 MeV, although the latter results are systematically a few percent lower.

At all energies and angles, the results from this work are in agreement within one standard deviation with the predictions of the Virginia Polytechnic Institute SP95 partial wave analysis (PWA) solution, although the data were not included in the analysis. Consequently, the work presented here adds support to this PWA which concludes that the pion nucleon coupling constant $\frac{g^2}{4\pi} = 13.7$, and that the mass of the $\Delta$ resonance is 1233.7 MeV, in disagreement with earlier results of partial wave analyses from other groups.
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Acknowledgement

It is 4:30 in the morning, and I am very tired, but it was worth it.

Thanks, Garth, for everything.

Good night.
Chapter 1

Motivation and Summary

Readers may well be asking themselves: why more πp cross-sections at energies dominated by the delta (Δ) resonance (i.e. ~100–350 MeV)? What more can be learned forty years after the pioneering pion-proton elastic scattering experiments of Anderson and Fermi et al.[1]? The answer is that even after all these years, there still exist significant discrepancies amongst various measurements of the total and differential cross sections at these energies. The disagreements are severe enough that many in the field have described the database as a “mess”. The inconsistencies have a number of serious implications not only for the πN system, but also for other related areas such as π-nucleus and NN scattering, and chiral perturbation theory. The obvious aim for experimentalists is to carry out new precise measurements to help resolve the problem as quickly as possible. The experiment described in this thesis represents our contribution to achieving this goal.

1.1 Historical Perspectives and the Current Controversy

The ongoing controversy over conflicting data sets has focused mainly on the 'low energy' region (up to an incident pion energy of about 70 MeV), and can be traced directly to the publication by Frank, et al. [2] in 1983 of π±p differential cross-sections from
30 to 90 MeV. These data differed significantly from the predictions of the Karlsruhe-Helsinki KH80 [3] partial-wave analysis solution 1. Partial wave analysis is a phenomenological method whereby the fundamental $\pi N$ scattering amplitudes can be parameterized by a set of "partial waves" determined by fits to the experimental data. The KH80 solution had been considered the "canonical" partial wave analysis for many years after its introduction. Consequently, these experimental results were subject to intense scrutiny and criticism. (An excellent historical account of these debates can be found in the $\pi N$ Newsletters 2 through 9 [4, 5, 6, 7, 8, 9, 10, 11]). As a result of the controversy involving the Frank data (see e.g. [12], [13]), a TRIUMF-Colorado–U.B.C group began a program to repeat and extend these measurements below 140 MeV. Expecting at first to re-confirm the KH80 predictions [14], the results of Brack et al. [15, 16, 17] instead essentially supported Frank, and furthermore indicated substantial discrepancies with the KH80 predictions at energies right up to 140 MeV. Subsequent experiments served only to cloud the picture further e.g. the partial-total cross-sections of Friedman et al. [18] supported the KH80 predictions up to 220 MeV, while the differential cross-sections of the Karlsruhe group up to 70 MeV [19, 20] tended to favour the results of the Frank and Brack data sets in most instances, but not in all. On the other hand, various low energy charge-exchange cross-section results at that time [21, 22, 23] tended to be in qualitative agreement with the KH80 solution. In addition to apparent contradictions between the data sets and the 'canonical' partial-wave solution, as well as between the data sets themselves, there have also been claims [13, 24] that the results from some experiments were 'internally inconsistent' i.e. the $\pi^+$ data could not be reconciled with the $\pi^-$ data in the context of a partial-wave analysis, which is based on well-founded conservation laws and the charge independence of nuclear forces. Even from this cursory overview, the nature of the confusion concerning this whole scenario is quite apparent.

1The basic ideas and methods of partial wave analyses are discussed in section 2.1
The importance of the low energy region of the $\pi N$ interaction relates to the determination of the $\pi N$ sigma term ($\Sigma$) from dispersion relation–constrained $\pi N$ partial–wave analyses (refer to 2.1 for more details). $\Sigma$ can be expressed as a function of low energy parameters describing the $\pi N$ interaction, parameters like scattering lengths and effective ranges, quantities which are particularly sensitive to the low energy data. For some time now there has been a substantial discrepancy between the 'experimentally' determined value $\Sigma$ (i.e. via PWA and dispersion relations, e.g. [25, 63]) and the 'theoretically' determined one $\sigma$ (via the baryon mass spectrum and chiral perturbation theory, ChPT [27]). In one interpretation, the discrepancy implies a large strange quark content in the nucleon contrary to conventional wisdom. The discrepancy could be real, simply the result of an incomplete theoretical calculation, or maybe due to an incorrect experimental value, which cannot be ruled out, given the confusion in the low energy database.

While the $\Sigma$ controversy was dominating the $\pi N$ scene, another no-less contentious issue arose which began a new thread of acrimonious debate which continues to this day. In 1987, the Nijmegen nucleon–nucleon ($N N$) PWA group published [29] a result for the $\pi^0 pp$ coupling constant, 0.072, substantially lower than the 'canonical' Karlsruhe-Helsinki result [3] for the charged–pion proton coupling constant, $f^2 = 0.079 \pm 0.001$, thus initiating a controversy over the apparent charge–independence breaking of the $\pi N$ interaction (see e.g. [30]). Later revisions by the Nijmegen group resulted [31] in a value 0.0745\pm0.0005, still a 5% discrepancy with the 'canonical' charged-pion result. Because the methodology of this group came under persistent attack (see e.g. [32]), their results have tended not to be embraced by the community.

The coupling constant controversy intensified when the Virginia Polytechnic Institute (VPI) group published in 1990 [33] their value for the charged–pion coupling constant, $0.0735 \pm 0.0015$, based on a $\pi N$ partial-wave analysis. This result prompted a

\footnote{For a good overview of this issue, see [28]}
new series of criticisms [34] and rebuttals [35].

The controversy has been heated, as a 5% change in \( f^2 \) would have numerous implications for model calculations (e.g. NN, \( \pi N \), \( \pi \)-nucleus), which include the \( \pi NN \) vertex. For example, Machleidt and Sammarruca showed [36] that the Bonn NN potential model could not accommodate such a change in the coupling constant in its description of the deuteron properties (e.g. quadrupole moment), which it describes rather successfully using the 'canonical' result 0.079. In the field of low energy QCD/ChPT, there is the well known discrepancy in the Goldberger–Treiman relation [37], which relates the product of \( f^2 \) and the pion decay constant \( F_{\pi} \) to the product of the proton mass and the proton axial-vector coupling constant \( g_A \) (see section 2.2). Using the 'canonical' value of \( f^2 \), there is a \( \sim 8\% \) discrepancy in this relation, which is somewhat larger than expected corrections can accommodate. The smaller coupling constant would remove the discrepancy.

### 1.1.1 What to do about the Controversy?

A crucial thread relates the \( \Sigma \) and \( f^2 \) controversies: the \( \pi N \) PWAs employed in the determinations of these constants are sensitive to the \( \pi^\pm p \) data in the \( \Delta \) resonance region, since the resonant P–wave amplitude dominates the full \( \pi N \) amplitude at energies up to about 350 MeV (refer to figure 2.4). This follows from the fact that all such analyses make use of dispersion relations to extrapolate the amplitudes from physical energies (accessible experimentally) to *unphysical* kinematical points: the so-called Cheng-Dashen point for the \( \Sigma \) (see 2.2), or the pion pole for \( f^2 \) (see 2.3.2). Many of the dispersion integrals used in the extrapolations are dominated by the \( \Delta \) resonance amplitudes. Thus even if there were no problems in the low-energy database, problems at energies around the \( \Delta \) could still render evaluations of \( \Sigma \) and \( f^2 \) uncertain.

\[ \pi^- p \rightarrow \pi^0 n \] charge-exchange differential cross-sections usually have substantially larger uncertainties than their charged counterparts, due to the experimental difficulties associated with gamma detection, and so contribute rather less to partial-wave analyses at low to intermediate energies (see e.g. [35].
Indeed, some significant discrepancies between data sets in the A region do exist. The total π⁺p cross-section results of Carter et al. [38] and Pedroni et al. [39] differ by up to several standard deviations, and both sets exhibit marked deviation from the KH80 predictions (illustrated in figure 1.1). The latter point is significant, since the KH80 solution has been the one historically used to determine the 'canonical' values of Σ and f². Furthermore, as has already been mentioned, the data of Brack et al. are consistently lower than the KH80 predictions, even up to 140 MeV, as shown in figure 1.2.
Figure 1.2: Ratios of the 140 MeV π⁺p differential cross-section data of Bussey et al. [40] (open circles) and Brack et al. [15] (solid squares) to the KH80 predictions. Only the Bussey data appeared in the KH80 database.
In the Δ region, the KH80 solution [3] depended exclusively on the results from a single group: the $\pi^\pm p$ total cross-sections of Carter et al. and the $\pi^\pm p$ differential cross-sections of Bussey et al. [40]. These data have very small uncertainties, and so are heavily "weighted" in partial wave analyses (see section 2.1.1). Both experiments were performed at the CERN SC in the late 1960's. Furthermore, throughout the aforementioned controversies (up to 1987), the Bussey data from 140 to 260 MeV were the only $\pi^\pm p$ differential cross-section data included in both the VPI and Karl-sruhe $\pi N$ data bases in that energy region. Since the Carter and Bussey data sets are consistent with each other [41], it follows from the Carter-Pedroni discrepancies that there could be a problem with the Bussey data as well.

A final concern should be mentioned here: many pion scattering experiments on nuclei other than hydrogen have been normalized to the $\pi p$ cross section data, by first measuring $\pi p$ (often on CH2 ) and using these data (or the phase-shift predictions based on the $\pi p$ data) as "known" input, hence establishing a normalization for the scattering on the nucleus under study 4. Clearly, a substantial change in the cross-sections and thus the "canonical" PWA solution could have significant repercussions as well for these types of measurements and their respective theoretical analyses.

Recently, several new $\pi^\pm p$ measurements have either been proposed or carried out to address (at least in part) these issues (e.g. $\pi^\pm p$ partial-total cross-sections from 40 to 500 MeV at LAMPF [44] and near-forward angle $\pi^\pm p$ differential cross-sections from 85 to 140 MeV [45]). However, none was designed to confront the differential cross-section results of Bussey et al. 5 and because of the significant discrepancies between the total cross sections of Carter et al. and Pedroni et al., and the differential cross sections of Brack et al. and Bussey et al. it is clear that the Bussey et al. data should

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4e.g. Gabathuler et al. [42] normalized their $\pi d$ elastic scattering cross-sections from 82–292 MeV to the $\pi^\pm p$ phase shift predictions, while more directly, Masterson et al. [43] normalized their $\pi d$ results at 143 MeV to the Bussey et al. data [40] shown in figure 1.2

5In fact, the TRIUMF proposal [46] of Brack et al. [15] states that data were to be measured at 140 MeV to compare with the "well determined" results of Bussey et al. [40].
no longer be considered immutable, but should be re-measured to help sort out these inconsistencies.

1.2 TRIUMF Experiment 645

Consequently, an experimental proposal (E645) was prepared and submitted to the TRIUMF Experiments Evaluation Committee, receiving a "high-priority" rating in July 1991. The experiment was completed in June 1992 after two months of beamtime on the M11 pion channel at TRIUMF.

1.2.1 Kinematical Region Measured

Absolute differential cross-sections for the $\pi^+p$ elastic scattering reactions were measured at incident pion laboratory kinetic energies of $T_r = 141.2, 154.6, 168.8, 193.2, 218.1, 240.9,$ and $267.3$ MeV. In addition to spanning the $\Delta$ resonance, the values of these energies were chosen:

a) to overlap the highest energy used by Brack et al. [15] (at $\sim 139$ MeV) and the lowest energy of Sadler et al. [47] (at $\sim 263$ MeV). Comparisons with these recent results provides a useful check of the consistency of our data with the modern database.

b) to coincide with the energies used for the measurements of the $\pi^+p$ analyzing powers of Sevior et al. [48] ($\sim 140, 167, 214,$ and $263$ MeV). Having differential cross-sections and analyzing powers at the same energy facilitates single-energy partial wave analyses (see section 2.1.1).

c) to provide a check of our normalization. Because of the dominance of the P33 partial wave near the peak of the $\Delta$ resonance, the $\pi^+p$ cross section at 193 MeV is constrained to be within roughly 2% of the partial-wave analysis predictions.

Both $\pi^+$ and $\pi^-$ data were obtained at all energies. The $\pi p$ coincidence measurements were taken at scattered pion laboratory angles of 60, 75, 95, 115, 135, and
Figure 1.3: Relationship of the $\pi^+ p$ kinematic ranges covered by this experiment (E645, solid points) to those of the other published results contained in the 1991 SAID [49] database. (see [50] for detailed references). Note that the $\pi^+ p$ data of Bussey et al. were the only data published since 1969 at energies between 140 and 267 MeV which existed in the KH80[3] database.
155 degrees ("set A") at all energies. In addition, \( \pi^+ \) data sets were obtained where these angles were shifted by \(-10\) degrees ("set B"). Single-arm results, where only the scattered pion was detected at lab angles of 20, 30, 40, 50, 60, and 70 degrees, were obtained at the lowest five energies. (This thesis will report on all these results apart from the single arm results at 154.6 MeV, and the set B results obtained with the liquid hydrogen target). Table 3.2 lists the geometrical configurations used in the measurements analyzed for this thesis, and figure 1.3 provides the relationship of the kinematical region spanned by these data compared to the VPI SAID [49] database in 1991.

1.2.2 Philosophy and General Description of the Experiment

Precision measurements of absolute differential cross-sections require much greater attention to experimental detail than needed in relative measurements such as asymmetry experiments. David Bugg, a physicist who has built a career based on these types of measurements, has provided "six requirements" which describe the essential elements of any measurement of absolute differential cross-sections:

*In a measurement of \( d\sigma/d\Omega \) the requirements are:*

1. to know beam intensity accurately,
2. to know beam composition accurately,
3. to know beam momentum accurately,
4. to know target length and density accurately,
5. to know solid angles accurately,
6. to avoid backgrounds.

(reprinted from D.V. Bugg [41])
The work of Brack et al. inspires another criterion which should be added to the above list:

*The confidence level of any claimed accuracy must be based on the extent to which the cross-section at a fixed kinematical point is independent of the experimental conditions.*

These Bugg "requirements" together with the above criterion provided the philosophical backbone to which all the experimental and analytical techniques of TRIUMF Experiment 645 were connected. The general features of how these items were addressed are outlined below, with the details deferred to later sections.

**To determine the beam intensity accurately**, the continuous-wave (CW) M11 pion beam available at TRIUMF coupled with coincidence beam detection of three thin, fast scintillation counters was used. The TRIUMF cyclotron (see section 3.1) provides one pulse, or "bucket" of protons on a fixed production target every 43.4 nanoseconds (corresponding to the 23.06 MHz frequency of the cyclotron radio-frequency acceleration system) in order to create a secondary beam of pions. Based on typical pion rates employed (1 MHz), only about one pion out of every 25 beam buckets causes a beam counter coincidence, implying ~1000 ns between consecutive pions, thereby avoiding the problem of signal pile-up characteristic of high intensity, low duty factor accelerators like the LAMPF linac. At these rates, using Poisson statistics, the probability of multiple pions in a single beam bucket was only about 3%, small enough to be readily handled in the analysis (see section 4.2.4). Consequently, beam intensities could be measured to much better than a percent accuracy at typical running rates.

**To determine the beam composition accurately**, the fact that the CW pion beams allow beam counting on an event-by-event basis was exploited, enabling pulse-height and timing information from the beam counters to be written to tape for every event. In addition, timing information obtained during special dedicated runs with nanosecond wide proton pulses ("phase restricted tune") enabled time separation of the pions,
muons, and electrons originating from the region near the production target at all energies (see section 4.2.1). During normal running, the broader timing distributions still allowed continuous pion/electron separation to be monitored up to ~200 MeV. The pulse height information was also used to identify and remove beam proton contamination. Monte Carlo simulations were used to accurately provide the corrections from pion decay both in the channel and downstream of the last channel magnet (see section 4.2.2). In this way, absolute beam compositions could be determined to better than 1% accuracy for $\pi^-$ and 0.5% for $\pi^+$.  

To determine the beam momentum accurately, pion–electron time-of-flight differences were measured both between scintillators contained within an evacuated beam pipe in the experimental area, and between the production target and a scintillator (see 3.5.4). Careful adjustment of constant-fraction discriminators (CFDs) reduced amplitude dependent time-walk to negligible levels, while accurate calibration using several time-to-digital (TDC) converters provided precise timing differences. Combined with well determined inter-scintillator and production target–scintillator separations, accuracies of ±0.25% in pion kinetic energy were achieved. The corresponding variations in the cross-sections are less than 1.4% for both $\pi^+$ and $\pi^-$.  

To determine the target length and density accurately, we employed both thin solid (CH$_2$) targets and a novel new flat-walled, supercooled liquid hydrogen (LH$_2$) target (sections 3.3.2 and 3.3.2 respectively). The linear thicknesses of the CH$_2$ targets were easily measured, while the composition was determined by chemical assay, yielding an overall proton density accuracy of about 1%. The supercooled LH$_2$ target was bubble-free, thus avoiding local density fluctuations. Prestressed mylar windows under zero differential pressure (i.e. the same on both sides) ensured a constant linear thickness with no bulging. Thermodynamic measurements determined the hydrogen density to high precision, resulting in an overall areal proton density uncertainty of
about 0.5%. This competes well with other modern target designs (e.g. [51]). An independent measurement using a technique whereby the energy loss of a beam of protons passing through the target was accurately measured, and the LH$_2$ thickness inferred from energy loss tables, yielded consistent results (see appendix A).

To determine the solid angles accurately, six pairs of scintillator telescope arrays were used to detect the scattered pions and recoil protons in coincidence (section 3.3.3). Use of thin transmission scintillators ensured virtually 100% detection efficiency for both $\pi^+$ and $\pi^-$ with negligible edge effects. The scintillator dimensions and distances to the target centre were measured precisely. The simple geometry facilitated straightforward Monte Carlo modeling of the pion decay corrections (section 4.1). Thin scintillators and targets, and minimal material surrounding the LH$_2$ target ensured low interaction losses for the scattered particles. Under such circumstances, solid angle uncertainties of $\leq 1\%$ were achieved.

To avoid backgrounds, $\pi p$ coincidence measurements (section 3.4.2) were carried out at central and near-backward angles using both the LH$_2$ and CH$_2$ targets. In addition, $\pi^+$ single arm measurements were undertaken (section 3.4.3) at near-forward angles using the LH$_2$ target. The tight ($\sim \pm 0.5^\circ$) geometry characterizing the $\pi p$ coincidence provided a stringent kinematical constraint which discriminated effectively against quasi-elastic scattering backgrounds when the pion-proton time-of-flight differences were measured. Carbon target and empty target runs effectively determined the small backgrounds from the CH$_2$ and LH$_2$ targets, respectively, to high accuracy. The material surrounding the LH$_2$ target was kept to a minimum, ensuring that backgrounds during $\pi^+ p$ single-arm running were kept to manageable levels. Straightforward background subtraction was carried out using data from an empty target which did not contain residual hydrogen gas, this having been replaced by helium from the surrounding pressure domes (section 3.3.2). Random “accidental” coincidences were non-existent at our low rate, event-by-event timing arrangement. In this way, total
background levels were typically 3% during $\pi^\pm p$ coincidence running, and 25% during $\pi^+ p$ single-arm measurements.

And, to obtain confidence in our quoted uncertainties, a very large number of test runs were performed (with the same statistics as the data runs) to probe possible sources of systematic error (section 3.5.3). In addition to acquiring $\pi^+ p$ and $\pi^- p$ data using both CH$_2$ and LH$_2$ targets at several energies, data were taken where the beam rates, target angles, CH$_2$ target thicknesses, counter geometry, and coincidence settings were all varied. The cross-sections measured from these runs provided a measure of our normalization and systematic uncertainties.

An extensive beam tuning program carried out before the experiment ensured that the incident pion beam properties were well understood (section 3.5.1), and provided valuable information for the Monte Carlo simulations required.

1.3 Scope and Layout of the Thesis

In Chapter 2, some theoretical aspects of the $\pi N$ system relevant to this work (e.g. partial-wave analyses and dispersion relations) are introduced. In addition, the importance of precision data around the $\Delta$ resonance as required for the determinations of the sigma term $\Sigma$ and coupling constant $f^2$ is elaborated upon. In Chapter 3, experimental details are discussed: the apparatus used, the data acquisition system employed, and the data-taking techniques utilized. The offline data analysis and Monte Carlo simulation methods are detailed in Chapter 4. The results are presented in Chapter 5, followed by an analysis of the quality, reliability, and impact of the data in Chapter 6.
Chapter 2

Theory

2.1 Phenomenology

This section outlines the phenomenology of the πN scattering amplitudes and their relation to partial wave analyses and dispersion relations. The application of the dispersion relations to the extraction of the πNN coupling constant is discussed in section 2.3.2. The notations used in these sections are based on those of references [52] and [53], to which the reader is referred for more information. Two-body kinematics in the centre-of-mass frame characterized by one pion and one nucleon (i.e. proton or neutron) in both the initial and final state of the reaction is assumed throughout.

2.1.1 Scattering Amplitudes

The scattering cross section $\sigma$ is an observable expressing the transition rate per unit particle flux (i.e. number of pions and nucleons crossing a unit area normal to the beam per unit time) of a particle in a free particle state $|i>$ going into another free particle state $|f>$ [53]. This can be written as

$$\sigma = \frac{4\pi^2}{W^2} \cdot \int d\Omega_f |T_{fi}|^2$$

(2.1)

where $W$ is the total energy of the interacting particles in the centre-of-mass system, $d\Omega_f$ is a solid angle element in the direction of the scattered pion, and $T_{fi}$ is the matrix
element of the transition operator connecting the initial and final states

\[ <f|T|i> = \delta^{(4)}(q_i - q_f) \cdot T_{fi} \]  

The \(\delta\)-function of the initial and final state 4–momenta ensures conservation of four–momentum.

The differential cross section is

\[ \frac{d\sigma}{d\Omega} = \frac{4\pi^2}{W^2} |T_{fi}(s,t)|^2 \]  

where the most general Lorentz invariant and parity conserving transition matrix (or “T–matrix”) is a function of two independent scattering variables, in this case two of the Lorentz invariant Mandelstam variables

\[ s = (p + q)^2 = W^2 \]
\[ t = (q - q')^2 \]
\[ u = (p' - q)^2 \]
\[ s + t + u = 2m^2 + 2\mu^2 \]

\( p (p') \) and \( q (q') \) are the 4–momenta of the initial (final) nucleons and pions, respectively, \( m \) and \( \mu \) are the nucleon and pion masses, \( s \) is the (total energy)\(^2\) in the centre–of–mass system, and \( t \) is the squared 4–momentum transfer, related to the (polar) scattering angle \( \theta \) between the initial and final state pions by

\[ \cos\theta = 1 + \frac{t}{2q^2} \]  

Here \( q \) is the magnitude of the centre–of–mass \( 3 \)–momentum.

The dependence of the cross section on the nucleon spins is introduced by defining another matrix operator \( M \) between the initial and final state nucleon Pauli spinors \( \chi \) in the rest frame

\[ \frac{2\pi}{q} T_{fi} = M_{fi} \]  

\[ \chi_f^\dagger M \chi_i \]  

\[ \chi_f^\dagger M \chi_i \]
where:

\[
M(s,t) = G(s,t) + iH(s,t)(\vec{\sigma} \cdot \hat{n})
\]  

(2.8)

and \(\vec{\sigma}\) is the vector of Pauli matrices, \(\hat{n}\) is the unit vector normal to the scattering plane; and \(H, G\) are called the spin flip, nonflip amplitudes respectively. With the spinors suitably normalized, the spin averaged differential cross section is written

\[
\frac{d\sigma}{d\Omega} = \frac{1}{2} \sum_{i,j} |M_{ij}|^2
\]

(2.9)

\[
= |G|^2 + |H|^2
\]

(2.10)

where

\[
G(s,z) = \sum_{l=0}^{\infty} [(l+1)f_{l+}(s) + lf_{l-}(s)]P_l(z)
\]

(2.11)

\[
H(s,z) = \sum_{l=1}^{\infty} [f_{l+}(s) - f_{l-}(s)]\sin\theta \frac{d}{dz}P_l(z)
\]

(2.12)

where \(z = \cos\theta\), and \(P_l(z)\) are the Legendre polynomials. The "partial wave amplitudes" for the state of total angular momentum \(J=L \pm 1/2\) (i.e. \(J=L \pm S\), where \(S=1/2\) (the nucleon spin) since pions are spinless) is

\[
f_{l\pm} = \frac{\eta_{l\pm} e^{2i\delta_{l\pm}} - 1}{2i}
\]

(2.13)

\[
\equiv \frac{T_{l\pm}}{q}
\]

(2.14)

where \(\delta_{l\pm}(s)\) are real parameters known as the "phase shifts" and \(0 \leq \eta_{l\pm} \leq 1\) are called "absorption coefficients". When only elastic scattering is energetically allowed, \(\eta_{l\pm} \equiv 1\), with \(1-\eta_{l\pm}\) parameterizing the amplitude describing transitions to inelastic channels (e.g. \(\pi p \rightarrow \pi\pi N\)).

The utility of this expression in describing strong interaction phenomena like \(\pi p\) scattering is due to the fact that the strong force has a finite range \(a (\sim 3\text{ fm})\), and so at a particular centre–of–mass momentum \(q\), only states with orbital angular momentum
L less than about $\sqrt{l(l-1)} \sim q^2 a/\hbar$ will contribute to the interaction. In practice, only the $l=0,1$ states contribute significantly to the $\pi p$ scattering amplitudes below about 250 MeV. Thus the parameter space to describe the $T$-matrix at each energy is reduced to a few partial waves from the infinite number that would have been required otherwise.

### 2.1.2 \( \pi N \) Isospin Structure

When considering the scattering of pions ($\pi^+, \pi^-, \pi^0$) from either the proton or the neutron, charge conservation leads to 8 different elastic scattering reactions, each of which can be, in principle, described by its own set of partial wave amplitudes. This picture is made more economical by introducing isospin symmetry, a symmetry which reflects the observation that the pions (nucleons) come in three (two) charge states with approximately equal masses ($m_{\pi^\pm}=139.57$ MeV, $m_{\pi^0}=134.96$ MeV) ($m_p=938.27$ MeV, $m_n=939.57$ MeV). The pions are considered as three states of the generic isospin 1 pion, and the proton and neutron are the two states of the isospin 1/2 nucleon $^2$.

Isospin symmetry was originally introduced to reflect the rather good symmetry describing the empirical observations of such things as the near equality of proton–proton and neutron–neutron cross sections (after correcting for electromagnetic effects) at low energies. A more modern interpretation arises from the quark model and the field theory of the strong interactions (quantum chromodynamics, or QCD). The QCD interaction does not distinguish among the quarks except through their masses [54]). If the masses of the up and down quarks $^3$ are equal ($m_u=m_d$), then isospin would be an exact symmetry. But in fact the quark masses are not equal, $m_d-m_u \sim 9-5=4$ MeV, and this difference plays the role of an isospin symmetry breaking parameter.

---

$^1$The eigenvalues of the orbital angular momentum operator $L$ are $L^2 \psi = l(l+1)\hbar^2 \psi$.  
$^2$It is for this reason that $\pi^+ p \rightarrow \pi^0 n$ is treated as just a special form of "elastic" pion–nucleon scattering.  
$^3$The up (u) and down (d) quarks are the elementary constituents of pions and nucleons.
which is small on the scale of the nucleon mass, and consequently provides an elegant explanation for the approximate isospin symmetry observed in nature. The realization that the u and d quark masses are different also solved a long standing puzzle regarding the neutron–proton mass difference, where calculations based only on the electromagnetic interactions between the constituent quarks found that the proton was heavier than the neutron because of the repulsive effects of the positive charge. However, recognition that the neutron has more d quarks than the proton solved the problem ([54]).

By analogy to the spin operators, the pions (nucleons) are identified by the total and third component of isospin, \(|I, I_3\rangle\), as are the two particle pion–nucleon states which combine the single particle states by the same Clebsch–Gordan coefficients used to combine the usual angular momentum states. Thus, with

\[
|p> = |1/2,1/2> \\
|n> = |1/2,-1/2> \\
|\pi^+> = |1,1> \\
|\pi^0> = |1,0> \\
|\pi^-> = |1,-1>
\]

then

\[
|\pi^+ p> = |3/2,3/2>
\]

\[
|\pi^- p> = \sqrt{1/3}|3/2,-1/2> - \sqrt{2/3}|1/2,-1/2>
\]

\[
|\pi^0 n> = \sqrt{2/3}|3/2,-1/2> + \sqrt{1/3}|1/2,-1/2>
\]

Assuming the scattering matrix does not depend on \(I_3\) (i.e. isospin invariance), then there are only two independent \(\pi N\) amplitudes (out of the original eight), and these depend only on the total isospin \(I\). These are related to the charge amplitudes by (e.g.
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\[ F_+ \equiv F_{\pi^+} \]

\[ F_1^I = \langle I=1/2 | S = I_1 = 1/2 \rangle \quad F_3^I = \langle I=3/2 | S = I_1 = 3/2 \rangle \]  
(2.15)

\[ F_+ = F_3^I \]  
(2.16)

\[ F_- = \frac{1}{3}(2F_1^I + F_3^I) \]  
(2.17)

\[ F_0 = -\frac{\sqrt{2}}{3}(F_1^I - F_3^I) \]  
(2.18)

The isospin "even" and "odd" combinations, which are related to the crossing symmetry of the amplitudes and used in dispersion relations (see section 2.1.4), are expressed as

\[ F_{ab} = F^+ \delta_{ab} + \frac{1}{2} F^- [\tau_a, \tau_b] \]  
(2.19)

\[ F^+ = \frac{1}{2}(F_- + F_+) \quad F^- = \frac{1}{2}(F_- - F_+) \]  
(2.20)

where the \( \tau_i \) are the initial and final state nucleon isospin Pauli matrices. Odd (even) expresses the symmetric (antisymmetric) behavior of the amplitudes under the interchange \( a \leftrightarrow b \).

2.1.3 Partial Wave Analysis

On including isospin, the partial wave amplitudes in equation 2.12 carry an additional index 1/2 or 3/2, corresponding to the vector sum of the isospin 1 pion and isospin 1/2 nucleon. The phase shifts are usually denoted with indices indicating twice the total angular momentum and isospin, \( \delta_{2J,2I} \) (e.g. \( \delta_{31} \)), but many other notations can be used (e.g. S31, \( \delta_{31} \), \( S_{31} \)).

The method of partial wave (or phase shift) analysis (PWA or PSA) attempts to find at each energy the set of partial waves, or equivalently, the phase shifts, which yield predictions for the cross sections (and other observables which depend on the polarization of the proton in the initial or final state, or both) that best fit the data \(^4\). For

\(^4\)Only the strong interaction part of the pion-nucleon reaction is considered here. In addition, there is also the non-trivial complication of the electromagnetic interaction (see e.g. [3]).
example, at the energies considered in this thesis, only S11, S31, P11, P13, P31, and P33 contribute significantly. The best fit to the data is obtained through a $\chi^2$ minimization where

$$
\chi^2 = \left( \frac{N-1}{\Delta N} \right)^2 + \sum_{\text{data}} \left( \frac{O_{\text{pwa}} - O_{\text{data}}}{\delta O} \right)^2
$$

where $O$ is an observable (like the cross section), with uncertainty $\delta O$ at each data point $(s,t)$, or $(q,\cos\theta)$, $\Delta N$ is an overall data normalization uncertainty, and $N$ is a free normalization parameter adjusted to give the best overall fit to the data.

Even at low energies with perfect data, the set of phase shifts which can describe the data is not unique [52]. These ambiguities can only be removed by other means e.g. by observing the interference of an observable with the well known electromagnetic amplitude, or by using constraints from dispersion relations. Although the partial wave amplitudes and the phase shifts are assumed to be smoothly varying functions of energy (except at S-wave resonance thresholds [52]), the $\chi^2$ minimization procedure is usually performed at discrete energies (hence these are called "single energy" PWAs). Vagaries in the data can thus yield phase shifts which do not vary smoothly with energy. A number of different procedures have been used to smooth the phase shifts as a function of energy (see e.g. [33] and [55] for two different methods), but it is generally held that dispersion relations provide the most theoretically well-founded approach [52]. The following section provides a brief introduction to dispersion relations and some of their applications to $\pi N$ scattering analysis.

### 2.1.4 $\pi N$ Dispersion Relations

Consider a function $g(t)$, where $g(t)=0$ for $t<0$. In scattering theory, this represents the principle of "causality", where the scattering amplitude is zero prior to the interaction taking place. The Fourier transform of $g(t)$, $f(v)$, is an analytic function in the upper

---

$^5$The uncertainties are standard deviations (68% confidence), if the probability distribution followed by the above expression is to be a true $\chi^2$ distribution.
half of the complex \( v \) plane. By Cauchy's theorem

\[
f(v) = \frac{1}{2\pi i} \oint_{\gamma} \frac{f(v')}{v' - (v \pm i\epsilon)}
\]

(2.22)

Substituting for the energy denominator

\[
\frac{1}{\sqrt{v'}} = P\frac{1}{\sqrt{v'} - v} \pm i\delta(v' - v)
\]

(2.23)

(where \( P \) denotes the principal value) and then taking the real and imaginary parts, one obtains the Hilbert transforms, or dispersion relations

\[
\Re f(v) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{3f(v')}{v' - v}
\]

(2.24)

\[
3f(v) = -\frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\Re f(v')}{v' - v}
\]

(2.25)

where the integrals are over the whole real line. These relations assume that \( f(v) \) vanishes sufficiently quickly that the expanding contour on the upper half plane does not contribute. More rapid convergence can be obtained by performing a "subtraction" i.e. subtracting the real (imaginary) part at some fixed energy from both sides of the above expressions e.g.

\[
\Re f(v_s) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{3f(v')}{v' - v_s}
\]

(2.26)

\[
\Rightarrow \Re f(v) - \Re f(v_s) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{3f(v')}{(v' - v)(v' - v_s)}
\]

(2.27)

Increased convergence is achieved at the expense of an extra parameter. Subtraction energies are usually chosen to correspond to some known or special energy e.g. in the \( \pi N \) case, the physical scattering threshold is often chosen since the subtraction constants are then related to the scattering lengths.

The earliest application of dispersion relations in scattering analysis was in the description of electromagnetic waves interacting with dispersive media, where the fact that the photon is massless simplified the determination of whether or not the amplitude had the required properties of causality and analyticity (see [56]). For the \( \pi N \)
system, these properties have been shown to apply only for the case of forward scattering [57] (i.e. $\theta \to 0$), but it is generally assumed to apply at all angles [52].

To obtain a manifestly Lorentz invariant form for the pion–nucleon amplitude, Dirac spinors must be used instead of the Pauli spinors in 2.7. The most general Lorentz invariant and parity conserving transition matrix element is given by

$$8\pi W_{fi} = \overline{u}_f(p') \left[ A(s,t) + \gamma^\mu \gamma^\nu \frac{Q^\mu Q^\nu}{m} B(s,t) \right] u_i(p)$$

(2.28)

where $(\gamma_\mu p^\mu - m)u(p) = 0$ is the free particle Dirac equation with $\overline{u}_f(p')u_i(p) = 2m\delta_{fi}$.

$Q^\mu = (q + q')/2$, $\chi^\mu = (q - q')/2$, and $v = (s-u)/4m$ is the so-called crossing variable.

The application of dispersion relations to $\pi N$ scattering follows from the "Mandelstam Hypothesis" [52], which states that the invariant amplitudes $A$ and $B$ are analytic functions of two complex variables, where the only singularities in the amplitudes occur on the real axis at single particle poles or from cuts due to the unitarity. Various dispersion relations of these amplitudes (or combinations of them e.g. $D = A + vB$) can be defined as functions of the variables $s, t, u$ (or combinations thereof) [52]. The simplest and most widely used are called "fixed-t" dispersion relations which are expressed in terms of the isospin odd and even amplitudes

$$\Re A^\pm(v, t) = \pi^{-1} p \int_{v_b}^\infty dv' 3A^\pm(v, t) \left[ (v' - v)^{-1} \pm (v' + v)^{-1} \right]$$

(2.29)

$$\Re B^\pm(v, t) = B_N^\pm(v, t) + \pi^{-1} p \int_{v_b}^\infty dv' 3B^\pm(v, t) \left[ (v' - v)^{-1} \pm (v - v')^{-1} \right]$$

(2.30)

where

$$B_N^\pm(v, t) = \frac{g^2}{2m} \left( (v_b - v)^{-1} \pm (v - v_b)^{-1} \right),$$

(2.31)

$v_b = (t - 2\mu^2)/4m$ is the location of the nucleon pole, $\mu$ is the pion mass, $v_{th} = t + t/4m$ is the threshold, $B_N$ is the nucleon pole (Born) term, and $\frac{g^2}{4\pi}$ is the $\pi NN$ coupling constant. The isospin crossing symmetry properties mentioned in equation 2.20 manifest
themselves here as

\[ A^\pm (v, t) = \pm A^\pm (v, t) \]
\[ B^\pm (v, t) = \mp B^\pm (v, t) \]

and these properties are used to get rid of the integrals at negative values of \( v \).

Since the amplitudes \( A \) and \( B \) can be expressed in terms of the spin flip/nonflip amplitudes \( H \) and \( G \), as well as the partial wave amplitudes (see [52] for the relations), they are also suited for use in partial wave analyses. Their utility comes from the fact that the real part of the amplitude at some energy and momentum transfer (in this example) can be expressed in terms of an integral over the imaginary parts at all energies. This property is useful \( e.g. \) to determine the real part of the amplitude in energy regions where the data are sparse or inaccurate, or where data are nonexistent (\( e.g. \) at very low energies), and thus provides valuable additional information to partial wave analyses. The greatest utility of dispersion relations comes from their ability to determine an amplitude in kinematical regions outside the physical scattering domain (\( i.e. \ W < m+\mu \ and/or \ |\cos \theta| > 1 \)) in a theoretically well-founded way. The latter use of dispersion relations in the determinations of the \( zNN \) coupling constant and sigma term is discussed in sections 2.3.2 and 2.3.3.

### 2.2 Chiral Symmetry Breaking and the Sigma Term

It is generally accepted that quantum chromodynamics (QCD) is essentially the correct theory to describe the strong interactions between hadrons via the interaction of the constituent spin-1/2 quarks described by the Lagrangian density \(^6\)

\(^6\)Unless otherwise stated, refer to [60] for an excellent account of the principles outlined in this section.
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\[ L_{\text{QCD}} = \sum_q \bar{q}^c \left( \partial_\mu - i \alpha_s \frac{\lambda^a}{2} A^a_\mu \right) q - \frac{1}{2} \text{tr} G^a_{\mu \nu} G^{a \mu \nu} - \sum_q \frac{m_q}{2} \bar{q} q \] (2.32)

\[ = L_0 + L' \] (2.33)

In the QCD Lagrangian, q are the six quark fields ("up", "down", "strange", "charm", "bottom", "top"). A^a_\mu are the 8 gluon fields responsible for the force between the particles, the G_{\mu \nu} term is the gluon kinetic energy, and the last term represents the quark masses. Restricting our attention to only the three lightest quarks u, d, s, L_0 is invariant under interchanges amongst the quarks, as QCD only distinguishes the quarks through their masses. Thus theory is said to be SU(3) invariant. When restricting to the lightest two quarks (u, d), the theory is said to be SU(2) invariant.

Without quark masses, the theory is actually SU(3)_L \times SU(3)_R invariant, meaning that quarks with their spins aligned with their momentum vector ("right-handed") do not interact with those with the spins anti-aligned ("left-handed"), so the interaction is invariant under the interchange of these two groups of quarks separately. Thus, the theory is said to respect "chiral symmetry". This property leads to two conserved quantities ("currents")

\[ \partial^\mu V_\mu = \partial^\mu \bar{q} y^\mu q = 0 \] (2.34)

\[ \partial^\mu A_\mu = \partial^\mu \bar{q} y^\mu \gamma_5 q = 0 \] (2.35)

where V denotes the "vector" current (which has even parity), and A the "axial-vector" current (which has odd parity).

If the physical world of hadrons respected SU(3)_L \times SU(3)_R symmetry, the even parity neutron and proton would have e.g. odd parity partners with the same mass. However, this is not observed in nature. One explanation would be that this symmetry has been "spontaneously broken" i.e. the ground state of the theory has a more restricted symmetry than that of the Lagrangian of the theory. The spontaneous breaking of
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a continuous symmetry gives rise to massless states called "Goldstone bosons" [58] which couple to the current whose symmetry is broken, in this case $A_\mu$, yielding 8 such bosons for SU(3) (u,d,s quark) symmetry or 3 for SU(2) (u,d) quark symmetry. In the real world, these Goldstone bosons are identified as the pions, kaons, and $\eta$ in the case of SU(3), or just the pions for SU(2), as these bosons are light compared to the other mass scales (e.g. the nucleons). Albeit relatively small, these bosons do have some mass, which means that the Lagrangian is also "explicitly" broken by a mass term, since this term would couple the left and right-handed quark fields as well. The modern interpretation, then, is that the pions, kaons, and $\eta$ are quark–antiquark bound states and are the Goldstone bosons resulting from the spontaneous breakdown of the approximate SU(3) symmetry [54] of the QCD Lagrangian.

A number of important consequences follow from this formalism. The pions, kaons, and $\eta$ are known to decay, so the axial–vector operator whose symmetry was spontaneously broken must couple these particles to the vacuum i.e. (restricting our attention to the world of just u and d quarks),

$$<0|A^\mu|\pi> = iF_\pi p^\mu$$ \hspace{1cm} (2.36)

where $F_\pi$ is the pion decay constant, and the 4–momentum $p^\mu$ is the only 4–vector available to construct the matrix element. Taking the partial derivative $\partial_\mu = ip_\mu$ of both sides, and using $p^\mu p_\mu = m_\pi^2$, yields the "Partially Conserved Axial–Current" (PCAC)

$$\partial_\mu A^\mu = F_\pi m_\pi^2$$ \hspace{1cm} (2.37)

Since the pions are relatively light, the PCAC equation can be used in various expressions (e.g. matrix elements) in the limit $m_\pi^2 \rightarrow 0$, and then expanding these expressions in powers of the small pion mass to derive a number of important relations, two of which are considered here.

The "Goldberger–Treiman" relation [37] follows from the use of the PCAC equation
(2.37) above in the matrix element describing neutron $\beta$–decay. The result

$$F_{\pi}g_{\pi NN}^2 \approx m_p g_A(0)$$

which relates the strong interaction pion decay and $\pi NN$ coupling constants to the proton mass and the neutron weak decay axial–vector coupling constant, is confirmed experimentally to $\sim 8\%$ if one inserts the canonical values of $\frac{F_{\pi}^2}{4\pi} = 14.3$, $F_{\pi} = 93$ MeV, and $g_A(0) = 1.26$. However, the $8\%$ deviation is larger than corrections to this expression (which are reasonably well known) can accommodate. This discrepancy has been puzzled over in the literature for some time (for a recent account see [59]).

Another consequence of the PCAC hypothesis in the case of $\pi N$ scattering is the so-called “Adler Consistency Condition”. In the limit of the 4–momentum of one pion e.g. $q=0$ (i.e. “soft pion”)

$$T_{\pi N}^+(v, v_B; q^2, q'^2) \rightarrow T_{\pi N}^+(0, 0; 0, m_{\pi}^2) = 0 \tag{2.39}$$

(The equality is the same with $q$ and $q'$ interchanged). This states that the pion nucleon amplitude vanishes at the unphysical point $v = v_B = 0$.\footnote{\(v_B = -q \cdot q' / 2, \) and \(v = q' \cdot (p + p') / 2 = q \cdot (p + p') / 2, \) so both variables = 0 when either \(q'\) or \(q = 0\).} Assuming that the amplitude does not increase too sharply, it thus follows that the amplitude at physical energies near threshold (where the s–wave partial waves dominate) will be small, as indeed is observed.

It is important to note that these relations are not dependent on the quarks having mass, as they still hold in the limit of vanishing quark masses, and so they are only a measure of the spontaneous breaking of the symmetry.

The famous “sigma term” ($\sigma$) is a quantity which is a consequence of the explicit breaking of chiral symmetry, and so is related to the masses of the quarks. Since the derivation of the expression for $\sigma$ is somewhat involved\footnote{It is derived using the Ward identity of the double divergence of the time–ordered product of two axial–vector currents. This double–divergence can be shown to be equal to the sum of the $\pi N$ amplitude and two other terms, one of which involves the sigma term.}, only the result is presented
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here:

\[ T^+(0,0;0,0) = -\frac{\sigma(\mu^2)}{F_\pi^2} \]  

(2.40)

where

\[ \sigma(t) = \langle N(p')|\delta(x_0)\frac{\partial^2}{\partial x_0^2}N(p)|N(p)\rangle \]  

(2.41)

and \( t=(p-p')^2 \) is the 4-momentum transfer between nucleon states \(^9\).

The presence of the divergence of the axial-vector operator \( \partial^\mu A_\mu(0) \), means that the sigma term vanishes in the limit of zero quark masses, and so is a measure of explicit chiral symmetry breaking.

Expression 2.40 is not useful as written, since at the particular kinematic point \((0,0;0,0)\) involved, there are corrections due to the finite quark masses which are as large as \( \sigma \). However, if the amplitude is expanded in powers of \( m^2_\pi \) about this point to the point where the pions are “on-shell” (i.e. \( q^2=m^2_\pi \))

\[ T^+(0,0;m^2_\pi,m^2_\pi) = T^+(0,0;0,0) + m^2_\pi \frac{\partial T^+}{\partial q^2_1} + m^2_\pi \frac{\partial T^+}{\partial q^2_2} + O(m^4_\pi) \]  

(2.42)

then by using the Adler Consistency Condition (2.39) where

\[ T^+(0,0;m^2_\pi,0) = T^+(0,0;0,0) + m^2_\pi \frac{\partial T^+}{\partial q^2_1} + O(m^4_\pi) \]  

(2.43)

one finds that at the special kinematical point \((0,0;m^2_\pi,m^2_\pi)\)

\[ T^+(0,0;m^2_\pi,m^2_\pi) = -\frac{\sigma}{F_\pi^2} + O(m^4_\pi) \]  

(2.44)

and so the corrections are suppressed by four powers of the (already small) pion mass, and so can be handled (presumably) using known techniques. The special kinematical point \((0,0;m^2_\pi,m^2_\pi) = (v=0, t=2\mu^2)\) is called the “Cheng–Dashen” point, and it is to this point that the \( \pi N \) amplitude derived from partial wave analyses of the scattering data

\(^9\)The 4-momentum transfer \( t = 2\mu^2 \) when \( v_B=0 \).
is extrapolated in order to experimentally test the predictions of the theory (see section 2.3.3).

The controversy stems from the fact that the theoretical expectation does not agree with the experimental value (see e.g. [25, 52]). The theoretical calculation in 1988, which assumed that the nucleons did not contain strange quarks, yielded $\sigma \sim 40$ MeV, whereas the experimental results were near 60 MeV. The discrepancy could have been resolved if the strange quark content of the nucleon was about 20%, but such a large strange quark content is contrary to the conventional view of nucleons containing only up and down quarks. Recent improved theoretical calculations (without strange quarks) [90] have reduced the discrepancy to about 10%. Nonetheless, this discrepancy is still a puzzle which has yet to be resolved. New theoretical efforts have been delayed due to the discrepancies in the pion nucleon scattering database which have become apparent. Since it is not clear whether the discrepancy stems from the theoretical or the experimental determinations of $\sigma$, the necessity of having a consistent and accurate database is clear.

2.3 Influence of Cross Sections around the $\Delta$ Resonance

2.3.1 Phase Shift Analysis

Figure 2.4 shows the "partial-wave" total cross section for each of the s- and p-wave amplitudes for $\pi N$ scattering up to 400 MeV. The total cross sections for $\pi^+p$, $\pi^-p$, and charge exchange are simple sums of these contributions, weighted by the squares of the relevant factors listed in equation 2.18 (e.g. $\sigma^p_{\pi^p_{\text{TOT}}} = \frac{1}{6}(4\sigma_{s11} + 4\sigma_{p11} + 4\sigma_{p13} + \sigma_{s31} + \sigma_{p31} + \sigma_{p33})$)

This figure clearly demonstrates the dominance of the P33 amplitude in the $\pi N$ interaction at pion kinetic energies between about 75 and 350 MeV, especially for $\pi^+p$ scattering, where the only amplitudes which contribute are the isospin 3/2 (P33,
Figure 2.4: Contributions to total cross sections from each partial wave, which add incoherently to give the $\pi^+ p$, $\pi^- p$, or charge exchange cross sections. Note that only $P_{33}$, $P_{31}$, and $S_{31}$ contribute to the $\pi^+ p$ cross section. Note also that the energy of the peak of the $P_{33}$ cross section differs from the location of $\Delta$ resonance due to a kinematical factor.
P31, and S31) amplitudes. Because of the dominance of the P33 contribution, small changes in the P33 phase shift can have significant consequences on the phase shift analyses e.g. the discrepancy between the Brack et al. and Bussey et al. differential cross sections at 140 MeV can be explained by about a 2° change in the P33 phase shift (from about 55°). The importance of this phase shift in partial wave analysis is amplified further if dispersion relation constraints are employed in the analysis. The integrals in many of the commonly used dispersion relations receive significant contributions from, or are sometimes dominated by, the amplitudes in the energy region around the Δ. Since dispersion relations relate the real parts of an amplitude at a particular energy to the integral over the imaginary parts at all energies, the reach of the P33 partial wave is extended beyond the Δ region.

Consequently, since the experimental cross section data at energies around the Δ resonance define the P33 phase shift, any inaccuracies or erroneous normalizations in the data will have consequences over a broad energy range. Therefore it is crucial to have precisely measured and normalized data in this energy region.

2.3.2 The πNN Coupling Constant

The necessity of having an accurately known P33 partial wave is particularly relevant to determinations of the pion nucleon coupling constant \( \frac{f^2}{4\pi} \) (or equivalently \( f^2 \)). A number of approaches have been used to determine this constant from the πN scattering data, all of which are based on applications of dispersion relations (see [52]) which provide a theoretically well-founded technique to relate amplitudes at energies in the physical region of scattering data, to the unphysical region (i.e. \( W<\mu+m \)) where the nucleon pole is situated. The parameter \( f^2 \) reflects the strength of the interaction at a pion-two nucleon vertex i.e. \( \pi^- + p \rightarrow n \). Since \( m_n < m_p + m_\pi \), energy conservation is violated, and the process is said to be "virtual". However, the Mandelstam Hypothesis [52] states that since the πN amplitudes are analytic in the complex
(s,t) plane, dispersion relations can be applied at non-physical energies (within certain limits unimportant for this discussion, see [52]) as well as physical ones.

The nucleon pole (Born) term appears only in the dispersion relation for the invariant $B$ amplitude of equation 2.30, and it is from various combinations of this amplitude that the coupling constant can be determined. The $\pi^+ p$ forward $B$ amplitude (i.e. $B_+ = B^+ + B^-$ at $t=0=0$) is helpful in demonstrating the importance of the $P33$ partial wave in these determinations. The dispersion relation (2.30) is in this case

\[
\Re B_+(v) = \frac{8\pi m f^2}{v_B(v+v_B)} + \pi^{-1} P \int_{v_B}^{\infty} dv' \left[ \frac{3B_+(v)}{v-v'} - \frac{3B_-(v)}{v'+v} \right]
\]

where $v_B = -\mu^2/2m$, and $\Re$ and $\Im$ denote the real and imaginary parts of the amplitude respectively. This equation demonstrates explicitly the fact that the amplitude in the unphysical region at the nucleon pole, namely the first term on the right hand side, affects the $B$ amplitude at all energies. In practice, due to the $1/v$ dependence of this Born term, its effect becomes rather small at energies beyond the $\Delta$.

The $B$ amplitude can be expressed in term of partial waves as [53]

\[
B_+ = \frac{4\pi}{W+m} f_{S31} + \frac{4\pi}{W-m} f_{P31} + \frac{8\pi}{q^2} (m(2-W/m)) f_{P33} + \cdots
\]

The $P33$ amplitude dominates this expression, since the $S31$ amplitude is relatively small and is further suppressed by the $(W+m)$ denominator, and the $P31$ amplitude is much smaller than $P33$.

Since the determination of the $\pi$NN coupling constant is so dependent on the $P33$ amplitude, it is clear that precise data are required at energies spanning the $\Delta$ resonance.

One further comment about the application of the dispersion relations will be made here, since it is pertinent to the conclusions based on this work (see section 6.2.1). One way to determine the $\pi$NN coupling constant $f^2$ from equation 2.45, or its generalization to non-zero $t$ values, is to first perform a partial wave analysis of the $\pi$N
database, calculate the real and imaginary parts of the B amplitudes from the resulting partial wave amplitudes, determine $f^2$ at each kinematical point $(v,t)$, and then average $f^2$ over some kinematic region spanning the $\Delta$ resonance (see e.g. [53], [35], or [85]) where the data (hence the amplitudes) are believed to be most reliable. The averaging is necessary since variations in the data inevitably result in variations in $f^2$.

However, when dispersion relation constraints are applied simultaneously to the partial wave analysis, the situation is more tricky, since the dispersion relations depend on $f^2$ and thus $f^2$ must be specified a priori. This was the approach undertaken in the KH80 analysis, where it was believed that, after the analysis, a value of $f^2$ different from the input ($f^2 = 0.0790$, corresponding to $\frac{f^2}{4\pi} = 14.3$) could have resulted [61], although it was considered unlikely to do so [62].

An alternate approach has been developed recently by the V.P.I. group [81], where the dispersion relation constrained partial wave analysis was repeated for a range of $\frac{f^2}{4\pi}$ inputs, and then the coupling was extracted using a dispersion relation related to the one discussed above. When the input coupling constant was within about 0.7 of the optimal value (13.7), the extracted coupling was always the same as the input. (Beyond this range the extracted coupling would always be slightly closer towards the optimum than the input). It was found, however, that the $\chi^2$ significance of the partial wave fit varied smoothly with $\frac{f^2}{4\pi}$ and displayed a pronounced minimum. The value at this minimum was identified as the “true” value of $\frac{f^2}{4\pi}$, and its uncertainty was estimated from the width of the minimum and the consistency of the values similarly determined from $\chi^2$ fits for the $\pi^+p$, $\pi^-p$, and charge exchange data. This method, which appears to be the most obvious way of determining the optimal value of $\frac{f^2}{4\pi}$ from $\pi N$ scattering data, was only made possible recently by the greatly increased computing power available today compared to that of 1980.
2.3.3 The $\Sigma$ Term

The P33 partial wave amplitude also plays a role in determining the sigma term from $\pi N$ scattering data, again through the use of dispersion relations to relate the amplitudes in the physical region to the unphysical Cheng–Dashen point (see section 2.2). Many techniques have been devised for doing the extrapolations (e.g. see [52, 63]), but a recent analysis [25] demonstrates the link with the resonance energy data more directly. In this analysis, the results of a particular solution (in this case KH80) are assumed to provide "known" input above some energy (in this case 90 MeV). Forward dispersion relations for the $B^\pm$, $D^\pm=A^\pm+vB^\pm$, and $E^\pm=\frac{3}{2}D^\pm$ amplitudes are then used as constraints in a partial wave analysis of just the low energy (<90 MeV) data. This procedure fixes some subthreshold parameters of the D amplitude which in turn are related to the "experimental" sigma term via

$$\Sigma = F_\pi^2 D^+(v=0, t=2\mu^2)$$  \hspace{1cm} (2.47)

$$= F_\pi^2 (d_{00}^+ + 2\mu^2 d_{01}^+) + \Sigma_{corr}$$  \hspace{1cm} (2.48)

where the $d$'s are coefficients in the $(v,t)$ expansion of the D amplitude about the unphysical (0,0) point. The bar indicates that the Born term has been subtracted. The first term in the second expression is often denoted $\Sigma_d = F_\pi^2 (d_{00}^+ + 2\mu^2 d_{01}^+)$, and is the term that is expected to have the greatest sensitivity to changes in the physical $\pi N$ scattering amplitudes. The term $\Sigma_{corr}$ corrects for contributions to $D$ quadratic and higher in $t$ and can be estimated calculated using chiral perturbation theory [25] or by a dispersion relation for $D(v=0, t)$ [90].

The approach described places a large emphasis on the "known" amplitudes above 90 MeV. Such an emphasis is known to put severe constraints on the low energy data [64], due naturally to the P33 amplitude in the $\Delta$ region. So again, for a reliable determination of $\Sigma_d$ (hence $\Sigma$), it is essential that this amplitude, hence the data around

\footnote{At $v=0$, the D amplitude is equivalent to the $\pi N$ amplitude $\Sigma$ in equation 2.40}
the Δ resonance, be known accurately.
Chapter 3

Experimental Apparatus and Technique

3.1 TRIUMF Cyclotron

Experiment 645 was run at the TRIUMF meson factory in Vancouver, Canada, in May and June of 1992 (see figure 3.5). The TRIUMF cyclotron is an isochronous, sector-focused machine accelerating negative hydrogen ions ($\text{H}^-$) up to 520 MeV. $\text{H}^-$ cyclotrons have the advantage over proton machines that multiple proton beams of differing energies and currents can be extracted simultaneously with near 100% efficiency by placing carbon "stripping foils" at various radii on the path of the accelerated ions. After passing though the foils, the now fully stripped protons bend the opposite way in the cyclotron magnetic field and can thus be simply extracted down external beam lines. Two external beamlines are used for nuclear science at TRIUMF: beamline 1A (BL1A), which provides protons of beam currents up to about 140 $\mu$A to the "meson hall" for secondary particle production; and beamline 4 (BL4), which can accept up to 1 $\mu$A of proton to the "proton hall" for direct use in scattering experiments (along BL4B), or 10 $\mu$A for producing neutron beams and/or radioactive ions (along BL4A). Polarized proton beams can also be extracted into either line.

$\text{H}^-$ ions of 500 MeV kinetic energy at a cyclotron radius of 7.8 m have a revolution frequency of $\sim 4.6$ MHz. Acceleration is provided by four rows of quarter-wavelength

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1 More information can be found in [65]

2 A third extraction point (BL1C) has been used for isotope production, and is currently being transformed into a proton cancer therapy beamline
Figure 3.5: TRIUMF site plan.
radio-frequency (RF) resonant cavities, two above and two below the beam plane. To be able to fit into the cyclotron, these operate on the fifth-harmonic of this ion revolution frequency \textit{i.e.} 23.06 MHz. Consequently, a proton pulse emerges from each of the beamlines every \(\sim 43\) ns with a 100\% macroscopic duty factor. These pulses are nominally 3 ns wide, but can be adjusted to a maximum of 5 ns, and with some loss in flux, to a minimum of about 0.5 ns ("phase-restricted beam"). During normal running, the proton pulses in BL1A have a "double peak" time structure, resembling roughly two overlapping Gaussians with an overall 3 ns width, whereas during phase-restricted beam, pulses have a single narrow Gaussian structure. The narrow, single-peak structure of the phase-restricted beam was exploited during those dedicated runs in E645 designed to determine the pion/muon/electron composition of the beam via time–of–flight (TOF) down the M11 pion channel (see section 4.2.1).

Protons from BL1A impinge on two target stations (T1 and T2) for production of secondary pion and muon beams. The downstream T2 target is used primarily for production of muon beams, together with pion beams for the TRIUMF bio-medical channel. The upstream T1 target feeds the "high energy" M11 pion channel (used in E645), the low energy or stopped M13 pion channel, and the M15 surface muon channel. It contains several "thin" targets, typically an uncooled (2 mm), and a water-cooled (10 mm) pyrolytic graphite target, a 10 mm water target, and a water-cooled 12 mm beryllium target. Beryllium was used throughout E645 due to its superior \(\pi^-\) yield. Cooling power limits the proton beam spot to about 7 mm vertically and 2 mm horizontally at nominal 140 \(\mu\)A beam currents. Since the position of the proton beam on the production target affects the secondary beams as well (see sections 3.5.1 and 4.2.1), adjustments in position of proton beam spot transverse to the beam can be made with steering magnets in BL1A. Although the beam must normally be interrupted in order to alter the beam position, slight changes can be made without beam interruption. Profile monitors can be used at low currents to view the beam size and direction.
3.2 M11 Pion Channel

Experiment 645 was run on the TRIUMF M11 pion channel, a channel originally designed to produce pion beams up to 300 MeV kinetic energy. Recent modifications to the front-end septum magnet [66] have increased this limit to 350 MeV. With a nominal 140 µA proton beam current traversing a 12 mm Be target, maximum fluxes of $10^8 \pi^+$/sec (at 320 MeV/c) and $10^7 \pi^-$/sec (at 280 MeV/c) can be delivered to the experimental area [65]. Although there is no lower energy limit to M11, below ~ 70 MeV, it cannot compete in intensity and beam pion purity with the shorter M13 low-energy pion channel.

M11 consists essentially of three different sections: the front-end, the middle section, and the rear section [65]. Refer to figure 3.6 for a schematic layout of the channel. The optics of front-end part of the channel are such that pions produced at an angle of 2.89° to the primary proton beam are bent a further 4.11° by the dipole component of the BL1A Q9 (1AØ9) quadrupole situated downstream of the T1 production target. 1AØ9 also focuses vertically, thus increasing the vertical acceptance of the channel. After 1AØ9, pions are deflected 12 more degrees by the septum dipole, a magnet element which completes the separation of the pion and proton beams. After the septum, 15 cm thick tungsten ‘jaws’ (horizontal and vertical) control the geometrical acceptance into the channel, hence the pion rate, and to some extent the beam phase space parameters (see section 3.5.1).

The middle section consists primarily of the B1 dipole, and the quadrupoles Q1 and Q2. Together, these produce a dispersed double-focus at the channel midplane. B1 is also the primary momentum selector in the channel. Only particles with momenta within ~ ±2.5% of the central value are accepted after the 60° bend. (Section 3.5.4 provides details regarding the channel momentum calibration). Thick (2.5 cm) copper ‘slits’ situated just slightly upstream of the (18%/mm) momentum dispersed focus are used for selecting the desired momentum spread of the pion beam.
Figure 3.6: Schematic layout of the TRIUMF M11 pion channel [65]
Protons are a major contaminant of $\pi^+$ beams (roughly 10:1 protons to pions). Two sets of CH$_2$ absorber wheels at the midplane can be used to differentially degrade the momenta of the protons relative to the pions. The degraded protons will then be swept out of the pion beam by the downstream B2 dipole.

The downstream quadrupoles Q4, Q5, and Q6, and the dipole B2 are used to realize a doubly achromatic, double-focused beam spot at the experimental target positions. B2 sweeps out off-momentum particles, and in conjunction with Q4, achromatizes the beam. The quadrupoles can be adjusted to move the pion focus location somewhat from the nominal 'TINA' focus location, which is situated 230.1 cm from the Q6 midplane, and ~15.42 m from T1 (section 3.5.4). At this location, the horizontal focus is slightly upstream of the vertical focus and where the experimental targets are normally located (section 3.5.1).

Five sextupole magnets are used to correct second-order aberrations in the channel. SX1 and SX2 are used to correct the aberrations introduced by the B1 dipole fringe fields and similarly SX3, SX4 for B2. Situated near the dispersed focus, SX2.5 (also called SX6) corrects for chromatic aberrations. However, with two additional dipole elements, the septum and 1AQ9 not accompanied by correcting sextupoles, some remaining higher-order aberrations cannot be corrected. Section 3.5.1 provides a detailed analysis of the optical characteristics of the channel.

Of course, there will always be some contamination of the pion beam. As mentioned previously, the primary contaminant of $\pi^+$ beams comes from low energy protons from reactions in the T1 production target. Muons from pion decay, and electrons which arise from gamma-ray conversion in or near T1 following $\pi^0$ production, are the other dominant sources of beam contamination. Section 4.2.1 and appendix C provide a more detailed discussion and analysis of the beam composition.
3.3 Experiment 645 Apparatus

This section provides details of the E645 equipment situated in the M11 experimental area. For reference, a photograph of the setup taken just prior to dismantling is provided in figure 3.7.

3.3.1 Beam Monitoring

The incoming pion beam was detected by three ('beam-defining') scintillators operating in three-fold coincidence placed upstream of the target whose centre was at the TINA focus. All were 0.159 cm thick and constructed of NE110, wrapped by a single layer of 0.0025 cm aluminum foil and 0.0263 cm electrical tape. These counters were connected by short straight lucite lightguides to photomultiplier tubes with high-speed transistorized bases [67]. Counter alignment was done using a transit. The 2.54 cm wide by 10.2 cm high S1 counter was placed 90.3 cm upstream \(^3\) of the TINA focus, and 18.7 cm from the exit of the 20 cm diameter M11 beam pipe. It did not quite cover the full beam spot at that point. 1.27 cm wide by 4.45 cm high S2A and S2B counters were placed 41.0 and 40.5 cm, respectively, upstream of the TINA focus, and were mounted with the S2A phototube above the counter and the S2B phototube below. Muons from a pion decaying downstream of S1 can sometimes hit the light guide of a later counter causing Čerenkov light that can be detected by the phototube. The crossed lightguides of the almost-touching S2A and S2B counters ensured that such occurrences would not cause erroneous beam coincidences. This scintillation counter unit subtended a small solid angle at the target, projecting a cone with 0.7° horizontal by 3.3° vertical divergence onto the target. The exact spot sizes depended on the settings of the M11 jaws (see sections 3.2 and 3.5.1), but typically, the beam distributions at TINA were roughly \(1 \times 0.8 \text{ cm}^2\) and \(1^0 \times 4^0\) at half-maximum.

Two particle telescopes containing two scintillators in coincidence were used to

\(^3\)All distances between centres, unless otherwise stated
Figure 3.7: Experiment 645. The M11 beam pipe exit is seen in the lower right corner.
monitor beam intensity relative to the beam counters. One set just above the M11 beam pipe exit was mounted to detect the muon cone. The other set was mounted at beam height in the experimental area and placed to view backward pion scattering from the S2A,B counters.

Beam movement was detected by a four paddle hodoscope placed 248 cm downstream of the TINA focus. The paddles were arranged in the four corners of a square perpendicular to the beam, and centered at beam height. Relative rates of the up/down (left/right) summed pairs were continuously monitored to provide a record of beam centreing.

In calculations concerned with determining the fraction of beam bursts containing only one pion (see section 4.2.4), it was necessary to know the beam rate on target, a rate which was somewhat larger than the rate measured by the beam counters (since the beam counters did not intercept the whole beam). For this a 20.1 x 20.1 x 0.635 cm³ VETO paddle placed 123 cm downstream of the TINA focus was used. At this location it intercepted >95% of the incoming pion beam. It was viewed with a single standard TRIUMF phototube and base arrangement.

Finally, a capacitive pickup (TCAP) in the cyclotron was used to provide a signal to measure the relative TOF of particles down M11 to the beam counters. This was used for particle identification (see section 4.2.1).

### 3.3.2 Targets

**Liquid Hydrogen Target**

A key element in the experiment was the development of a thin, flat–walled supercooled liquid hydrogen target. This target provided protons at high–density within a cell of accurately known thickness, yet it was not thick enough to introduce large energy and interaction losses to the incoming or scattered beams, or large corrections to the effective solid angles due to finite source size effects (see section 4.1). Figures 3.8
Figure 3.8: **Top:** Plan view scale drawing of the E645 LH$_2$ target showing window and insulator thicknesses. **Bottom:** Inner flask assembly, showing target dimensions and operating conditions.
and 3.9 provide design drawings of the target including relevant physical parameters.

The LH$_2$ target was contained within the 14.99±0.03 mm thick, 152.4 mm aperture of a stainless steel ring, and two 0.115 mm pre-stressed mylar windows. The target LH$_2$ was cooled by a separate source of liquid hydrogen flowing inside the hollow stainless steel ring. This "cooling" hydrogen was liquefied once at the beginning of the experiment and then maintained at 15.6 to 16.0 psia (absolute pressure), where the pressure varied on a one minute sawtooth cycle. However, the target LH$_2$ was maintained at 18.05±0.05 psia, i.e. approx. 2.2psia overpressure (i.e."supercooled"). This overpressure prevented boiling and bubbling in the target, thus ensuring a uniform proton density with no local variations.

The entire target assembly was contained within a large 64.0 cm high, 48.4 cm outer diameter cylindrical stainless steel vacuum vessel with a 0.95 cm thick wall. An inner 0.03 mm copper heat shield at the target hydrogen temperature (20K) and
an outer shield at liquid nitrogen temperature (77K), both surrounded by aluminized-
mylar “superinsulation”, conducted away heat induced by infrared radiation to reduce
the cooling load on the target and to further ensure that no bubbles formed. Two
15.9cm high gaps in the vacuum vessel covered by 0.127mm kapton provided beam
access.

The prestressed mylar windows on the target cell were used to keep the linear
thickness of the target as uniform as possible. The deflection due to differential pres-
sures across the window was measured on a test bench at liquid nitrogen temper-
atures as 0.183cm/psid (psid=differential pounds per square inch) [68]. Although
quite small, this would still cause unacceptable bulging if the target cell were con-
tained in vacuum. Consequently, the cell was capped on both sides by 0.229mm thick
mylar domes containing gaseous helium at a pressure regulated to within 10 mpsid
(differential pressure) of the pressure in the cell, causing a maximum ±0.0356 mm
fluctuation in the cell width. A 14±1 cm liquid hydrogen column above the target
centre to the pressure regulation point produced a 14±1 mpsid hydrostatic head re-
sulting in a net 0.026±0.002mm outward window deflection. The helium pressure
and target–helium pressure differential was digitized and read-out online at regular
intervals by the data acquisition system (as shown in figure 3.10).

The linear thickness of the LH$_2$ target between the inside surfaces of the mylar
windows was 15.04±0.06 mm, comprised of 14.99 mm due to the machined depth of
the ring, a correction taking into account shrinkage when cooled to 20K, bulging of
the windows due to the hydrostatic head, and also a thin layer of epoxy bonding the
windows to the ring. The total uncertainty is the sum in quadrature of the individual
uncertainties.

Due to the failure of the target vapour bulb transducer, the target cell temperatures
could not be monitored continuously, but had to be determined by a different tech-
nique at four separate occasions throughout the experiment. The technique basically
Figure 3.10: **left:** Spectrum shows differential pressure (in arbitrary units) between LH$_2$ and the helium gas (in the domes) as a function of time. Each bin on the abscissa represents about a five minute interval. The window bulge fluctuations caused by these pressure deviations are included in the target thickness uncertainty; **right:** Similar spectrum for the absolute helium pressure (arbitrary units).
used the target cell itself as a vapour bulb. The target cell was filled with "freshly" condensed normal–hydrogen (i.e. 75% ortho, 25% para). Half–way through the target filling process (and for the last measurement, target emptying), pumping was discontinued and the target was allowed to reach equilibrium without any overpressure applied i.e. the LH₂ was at its boiling point. The vapour pressure at this point was provided by the helium pressure transducer together with the differential pressure transducer, both of which were regulating throughout this process. Using the manufacturers calibration curve, accurate to \( \leq 0.1 \text{ psia} \) [69], vapour pressures of 15.7, 15.5, 15.4, and 15.4 psia were measured 2, 94, 263, and 621 hours, respectively, after the first condenser fill, a time scale spanning the entire experiment. The resulting temperatures were inferred from vapour pressure tables [70]: 20.63, 20.58, 20.56, 20.55±0.02K, respectively, where the last measurement has a small correction applied reflecting the roughly 8% normal–to para–hydrogen conversion which occurred during the 16 hours after the target was filled. The observed temperature drop is consistent with normal–to para–hydrogen conversion in the cooling condenser fluid, which was kept at a constant average pressure throughout the run.

The target densities at each temperature were inferred from molar volume vs. temperature tables [70]. The average of the normal– and para–hydrogen densities was used since the normal/para ratio was unknown at any particular point in the experiment. Although for most runs the conversion from normal to para (about 0.5%/hour for the first 100 hours) would not have proceeded very far under normal conditions, unknown catalytic effects could have speed up the process. This introduces a 0.2% uncertainty to the target density, a value which completely dominates that arising from the temperature uncertainty of 0.02%/0.01K.

From the measured linear thickness together with the known average target density throughout the run, the target areal density was determined to be 106.2±0.5
mg/cm², or $63.43 \pm 0.32 \times 10^{-6}$ mb⁻¹, where the 0.5% uncertainty is considered to be a 1-standard deviation estimate.

The $0^0$ target angle and target centre location reference was set by sighting markers on the lower rim of the vacuum vessel using transits situated $90^0$ apart in the experimental area. These markers were set during target assembly and are believed to be accurate to about $0.25^0$ [68]. The vacuum vessel and support structure were mounted on an overhead platform from which the entire assembly could be rotated. The target angle could be set to $0.1^0$ according to markings at the outer edge of the rotating platform. Summing these two contributions in quadrature gives an overall target angle uncertainty estimate of $\pm 0.3^0$.

At one point during the experiment, a completely independent measurement of the target thickness was performed. This technique involved measurement of the energy of protons passing through the LH₂ target. The technique is described in more detail in Appendix A. Briefly, M11 beam protons were passed through a full target and stopped in a surface barrier detector array. The target was emptied, and layers of aluminum foil of precisely measured thicknesses were placed into the beam until the proton energy entering the detectors was the same as for the full target. With the knowledge of the relative stopping powers of protons in LH₂ and aluminum, an equivalent thickness of LH₂ could be inferred. The target was rotated to four different settings: $-40.0$, $0.0$, $38.5$, and $53.0^0$. A best fit to the thicknesses inferred at each angle resulted in:

$$X [\text{mg/cm}^2] = \frac{104.5 \pm 1.9}{\cos(\theta + (0.8^0 \pm 0.3^0))}$$

(not including an overall 1.8% normalization uncertainty, see figure A.52). The results for angles $\geq 0^0$ are within 1% of those obtained using the vapour bulb technique, but the measurement at $-40^0$ is outside the estimated uncertainty, signifying either a $\sim 2^0$ shift at this setting, or an overall $0.8 \pm 0.3^0$ angular offset. Considering as well

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4 $1 \text{mb}^{-1} \equiv 10^{-27}$ cm² is a useful unit to describe the proton number density when used in calculating cross sections.
the normalization uncertainties and the estimated accuracy of the vapour bulb result, the former interpretation seems likely, especially since there was some difficulty experienced at the time of the measurement moving the target to -40°. Most of the cross section measurements in this experiment were taken with a target angle of 53°, an angle for which the two techniques agreed. The forward angle and coincidence π⁺ cross section measurements overlapped at three angles, and these data do not support either a ~2° shift at the -40° setting, or a ~0.8° angular offset (see section 5.1.6).

Target empty running for background measurements was realized by evacuating the target cell of all the LH₂ and residual gas, and replenishing it with helium from the domes. The helium pressure was adjusted to maintain the same areal thickness as in target full operation: 15.8 psia at 53° and 16.1 psia at -40°.

**Solid Targets**

The solid targets used in this experiment consisted of 12.7 x 12.7 cm² square slabs of ρ=0.93 gm/cm³ CH₂, and a slab of 10 x 10 cm² square carbon graphite for background measurements. The targets used and their respective thicknesses are shown in table 3.1. The densities were obtained from measurements of the linear dimensions, together with weights measured using a Mettler balance. The linear thickness uniformity was measured with a machinists’ comparator, claimed to be accurate to accurate to 1x10⁻⁶ inch [71]. The hydrogen and carbon contents of the CH₂ targets were determined to 1% accuracy by chemical analysis provided by a commercial laboratory [71]. The stopping power for pions and protons in the graphite background target was midway between the CH₂ ‘D’ and ‘E’ targets, the solid targets which were most often used in the experiment.

The targets were held flat by thin aluminum frames gripping the targets around the edges and attached to an aluminum support bracket. The whole assembly was
### Table 3.1: Proton thicknesses for LH\textsubscript{2} and CH\textsubscript{2} targets, and carbon thickness for background graphite target. See text for descriptions and method used to determine proton densities.

<table>
<thead>
<tr>
<th>Target</th>
<th>Target Thicknesses (mg/cm\textsuperscript{2})</th>
<th>(10\textsuperscript{-6} mb\textsuperscript{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>LH\textsubscript{2}</td>
<td>106.2 ± 0.5</td>
<td>63.43 ± 0.32</td>
</tr>
<tr>
<td>CH\textsubscript{2} “A”</td>
<td>44.0 ± 0.1</td>
<td>3.78 ± 0.04</td>
</tr>
<tr>
<td>CH\textsubscript{2} “D”</td>
<td>185.8 ± 0.7</td>
<td>16.00 ± 0.16</td>
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<tr>
<td>CH\textsubscript{2} “E”</td>
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<td>25.30 ± 0.25</td>
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<td>graphite</td>
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<td></td>
</tr>
</tbody>
</table>

mounted onto a machinists’ rotating head providing accurate and reproducible angular adjustment. A transit mounted downstream of the target position was used to check the 90\textdegree position (‘edge on’) after every target change/angle adjustment. The 0\textdegree position was determined by placing a mirror on a target, shining a He-Ne laser through the transit viewpiece, and adjusting the vice angle and support apparatus adjustments (for vertical positioning) until the laser light was reflected back onto the laser exit aperture. With these techniques, the target angles were determined to ±0.2\textdegree (68% confidence).

### 3.3.3 Time-of-Flight Spectrometer

The time–of–flight (TOF) spectrometer consisted of the beam monitoring scintillators; six “arms” consisting of pairs of thin scintillation counters to detect the scattered pions; and six arms of thin scintillation counters to detect the recoil protons during coincidence running. Schematic diagrams of the spectrometer are illustrated in figure 3.11. A pion arm consisted of a two counter telescope viewing the target, with each telescope consisting of two 0.32 cm thick NE102 scintillators wrapped by a single layer of 0.0025 cm aluminum foil and 0.026 cm polyvinylchloride electrical tape. The scintillators were attached to ‘standard TRIUMF’ tubes and bases (XP2230 phototubes and resistor chain bases) via lucite lightguides bent at 90\textdegree. The telescopes were
Figure 3.11: **Top:** Plan view of TOF spectrometer in coincidence mode as modeled in GEANT, showing distances between centrelines of the counters and target, and pion arm angles for 'Set A'. Proton arm angles vary with energy (see text). **Bottom:** Elevated view of same setup, showing counter dimensions.
bolted onto a machined table, and both were positioned using a transit located at the target centre. The angles are estimated to be accurate to better than \( \pm 0.2^0 \). The rear-most '\( \pi 2 \)' counters were 4.00 cm wide by 10.00 cm high and were mounted 123.1\( \pm 0.3 \) cm from their centreline to target centre. The front '\( \pi 1 \)' counters were 4.9 cm by 16.5 cm, and situated 79.2 cm from the target centre. These dimensions and separations were chosen in order to define a telescope projecting to a \( \sim 6 \)cm horizontal, \( \sim 20 \) cm vertical spot at the target, large enough to cover the whole interaction region while not severely restricting the acceptance to muons from scattered pion decay (see section 4.1). Also, the placements were such that particles emanating from the target and passing through the \( \pi 1 \) counters would not strike the \( \pi 2 \) lightguide/phototube junction, which would otherwise cause erroneous 'Čerenkov' coincidences (see section 3.4.2).

For \( \pi p \) coincidence running, a set of six, 9.0 cm by 40.0 cm by 0.32 cm thick ('\( P 1 \)') scintillators mounted onto a machined table and transit-sighted into place were used as the recoil–proton detection arms. These scintillators were viewed at the top and bottom by standard TRIUMF phototubes coupled to lucite lightguides bent at 90\( ^0 \) so that the phototubes pointed radially. The scintillators were situated at a radius of 92.6\( \pm 0.3 \) cm. The base plates of the support apparatus for the counters were mounted onto a machined table which was sighted in using a transit. Each base plate was attached to the table with a bolt in a sliding groove at the rear of the table, and a c-clamp at the front. This arrangement was necessary since the proton counters needed to be moved after every energy change to the appropriate angles conjugate to the scattered pions. Angles determined using a transit at the target centre position were marked on the front edge of the table. These were used to match markers on the proton counters corresponding to the vertical centrelines of the scintillators. In this way, an accuracy of \( \sim \pm 0.1^0 \) was achieved.

The solid angle for the detection array was defined primarily by the \( \pi 2 \) counters.
Small corrections due to the geometric constraints imposed by the π1 and P1 counters were calculated by Monte Carlo (see section 4.1).

3.4 Measurement Technique and Electronics

This section outlines the general measurement techniques employed and the NIM-based electronics used to realize the system logic requirements. The electronic/logic diagram is shown in figure 3.18 and the timing diagram in figure 3.19: both will be referred to throughout this section. A discussion of the way the various signals were analyzed to obtain the cross-sections is presented in chapter 4.

3.4.1 Beam

The beam entering the TOF spectrometer was defined by the three beam counters detailed in section 3.3.1. Each counter signal was first fanned-out (LeCroy 428F) and then fed to a constant-fraction-discriminator (CFD) (Tennelec TC455) for threshold adjustment and precise timing. The counter voltages and CFD thresholds were set so that no logic pulses were generated by tube noise while still producing logic pulses for the smallest (essentially minimum ionizing) electron signals. A second S2B signal was fed to a discriminator (LeCroy 621BLZ) in which the threshold was set to keep only the large pulse height proton signals (S2BH=S2B-high), so that the inverting (complementary) output would indicate a "no-proton" event (S2BH). The combination of this signal and a differential absorber at the M11 midplane effectively removed all proton contamination from the beam definition (as shown in figure 3.12 and discussed in section 4.2.1).

An incident particle (π,μ, or e) was identified electronically by the four-fold coincidence (via a LeCroy 465 Coincidence Unit):

\[
\text{BEAM} \equiv S1 \cdot S2A \cdot S2B \cdot S2BH
\]
Figure 3.12: **left:** Pulse height distribution from the S2B beam scintillator at 218 MeV ($\pi^+$) without an M11 midplane absorber and without S2BH upper-level discrimination; **right:** Same spectrum, but with an absorber and S2BH discrimination. The absorber used here was thinner than the 0.251" one used for production runs.
The logic signal from the S2B counter defined the timing here and for the entire system (see figure 3.19). The tight angular definition of this telescope of beam counters ensured that all BEAM coincidences corresponded to a particle at the target, not counting calculable pion decay and hadronic interaction corrections (described in section 4.2.3 and appendix C).

The relative flight–times of the pions, muons, and electrons in the beam from the production target to the S2B counter were determined by measuring the time differences between pulses from the BEAM coincidence and the TCAP signal (a signal synchronous with the proton pulses in the cyclotron) supplied to the M11 counting room by the cyclotron control room. This information was used to determine the fraction of pions in the beam (as described in section 4.2.1).

If the beam counter information was read–out by the data acquisition system only when a πp scattering event was detected, it would not be possible to determine the electron or muon content of the beam since these particles do not generate πp events. So a special “beam samples” circuit was constructed to generate a data acquisition readout signal using only BEAM coincidences, when selected by a gate–generated pulse as a trigger (the electronics are shown in figure 3.18). This samples rate was adjusted by varying the widths and duty factor of a clock generator pulse stream in coincidence with BEAM. Since a BEAM pulse can come at any time, the coincidences between the BEAM signals and the gate generator were random in time. Since the huge majority of BEAM triggers do not cause πp events, the “beam samples” signals provided an unbiased sample of events striking the beam counters.

In order to assess the fraction of beam pulses containing more than one pion, the frequencies with which pions in adjacent time buckets were monitored. Particles arriving in a subsequent beam burst (2nd BEAM) were detected by AND–ing the BEAM signal with one delayed by 43 ns (i.e. period between bursts). Particles in three consecutive beam bursts (3rd BEAM) were identified by AND–ing the latter signal by a
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BEAM pulse delayed by 86 ns. In this way, BEAM, 2nd BEAM, and 3rd BEAM count rates provided information concerning the relative frequencies of at least one particle in one, two, and three consecutive beam buckets, respectively. With this information, the "Poisson parameter" \( \bar{\lambda} \) could be calculated. This parameter represents the average number of particles per beam bucket. Such information was necessary to correct BEAM for events with more than one pion in a bucket (as discussed in section 4.2.4).

For all \( \pi^+ \) measurements, the channel slit width was set to 18 mm, corresponding to a 1% FWHM \( \delta p/p \) momentum bite on target. Due to the lower available fluxes, the momentum bite was 2% for \( \pi^- \) runs, except for 267 MeV, where it was 2.5%. The front end jaws were adjusted at each energy to provide roughly 1 MHz BEAM rates, corresponding to typically 1.5 MHz and 2 MHz target rates for \( \pi^+ \) and \( \pi^- \), respectively. The target rate was essentially that of the VETO counter rate, since the VETO counter intercepted very nearly all the beam hitting the target. Some corrections (e.g. from pion decay) were required for an accurate estimate of the target rate, a rate which was required for the multiple-pion correction calculations (discussed further in section 4.2.4).

The four-paddle beam hodoscope was used to monitor variations in the beam position. The rate on each paddle was displayed by visual scalers for online monitoring, while the total counts were sent to CAMAC scalers for offline analysis. Sample offline spectra appear in figure 3.13. During any run, if the up/down rate ratio deviated substantially from unity (e.g. to \( \sim 0.5 \) or 2), the cyclotron control room would be notified and the proton beam steered on the production target to restore the pion beam position (discussed further in section 3.5.1). This criterion was derived from observations during the beam tuning phase prior to the experiment: when the proton beam was purposefully steered about 1 mm above (below) the center of the production target, the up/down hodoscope ratio changed to about 0.3 (3), and the full width of the M11 horizontal beam position distribution at the target increased to \( \sim 13 \) mm, from \( \sim 9 \) mm.
Figure 3.13: **left:** Ratio of beam counted in the left (looking downstream) two hodoscope scintillators versus the right two, as a function of time. Each bin represents about a minute; **right:** Spectrum for the up/down ratio. The beam movement at the target implied by this example is very small (<1 mm) and is considerably larger than for typical runs.
when the ratio was unity. The beam size was insensitive to horizontal movements of the proton beam on the production target.

**Pion Energy at Target Centre**

The pion momentum at the front of the channel is linearly related by equation 3.49 to the magnetic field strength of the B1 dipole, which is measured by an NMR probe set at the magnet midplane. The energy loss through the midplane absorber, the beam pipe exit window, and the in–beam counters (including tape), air, target windows, etc. (for the LH\textsubscript{2} target), and half the target material at the appropriate angle were determined using an energy loss program based on the full Bethe–Bloch equation [95]. This energy loss uncertainty is conservatively estimated as 10\% of the total loss (which was typically 2 MeV), and is added in quadrature to the ±0.25\% kinetic energy uncertainty from the calibration (equation 3.49) to give the total uncertainty.

Although the M11 pion beam energy was adjusted for every different target configuration to give the same energy at the target centre, the energies were not adjusted for the corresponding background runs, since primarily the backgrounds were small and the beam energy losses in the background targets were not much different from the foreground targets. Furthermore, the dominant background from quasi–elastic \(\pi p\) scattering from protons in carbon has a cross–section that is a much less sensitive function of energy than that of the \(\pi p\) interaction (because of Fermi momentum averaging), so the small energy changes between the foreground and background have a negligible effect on the measured background yields.

**3.4.2 \(\pi p\) Coincidence Detection**

Scattering events to the pion arms were identified by a \(\pi 1, \pi 2\) coincidence in any one of the six telescopes:

\[ \Pi_i \equiv \pi 1_i \cdot \pi 2_i \]
realized electronically in the same manner as for the beam counters. Figure 3.19 summarizes the timing protocol. Phototube voltages and discriminator thresholds were set to cut halfway into the electron signal, well below the smallest pion pulses, to ensure that no good pion events would be lost.

Pions and muons are identified by their TOF to the π2 counters relative to the BEAM signal. Neither the timing nor pulse height information from π2 could distinguish pions from those muons arising from pion decay between the target and π2. However, this was unnecessary, since this scattered pion decay was accurately modeled by Monte Carlo (discussed in section 4.1). As mentioned in section 3.3.3, careful placement of the π1 counters removed the possibility of Čerenkov events from particles hitting the π2 phototube/lightguide junction. The scintillation light arising from events hitting the scintillators have a longer pathlength to traverse before striking the phototube, so these events will be separated in time from the Čerenkov events. Figure 3.14 shows cases where the π counters were/were not removed from the EVENT coincidence. It is clear that the Čerenkov events are cleanly discriminated against.

Proton arm events were signaled by a coincidence between the "up" and "down" tubes of each of the P1 counters. Logic signals obtained by discriminating with CFDs were then fed to a LeCroy 624 meantimer to establish the timing gate. Prior to performing the actual experiment, each counter was placed in the beam, and the phototube voltages and discriminator thresholds were adjusted to cut half-way into the electron signal, ensuring that no protons would be lost.

Candidate πp scattering events were identified by the coincidence of BEAM with the coincidence output of signals from a pion arm and its conjugate proton arm:

$$\text{ARM}_i \equiv \text{BEAM} \cdot \Pi_i \cdot P_i$$

The timing was set such that only relatively fast particles in the pion arm and relatively slow particles in the proton arm would satisfy the ΠP coincidence.
Figure 3.14: **left:** $\pi 2$ Pulse height versus timing spectrum from a $\pi p$ coincidence run with the $\pi 1$ counter removed from the EVENT definition. Events from particles striking the $\pi 2$ phototube/lightguide junction ("Čerenkovs") are clearly separated; **right:** same, but with $\pi 1$ included in EVENT.
The πp scattering yield was derived from the spectra of pion-proton TOF differences. The tight geometry of the counter pairs greatly suppressed the dominant 3-body quasi-elastic \( \pi^\pm A \rightarrow \pi^\pm pX \) background, and, combined with the timing requirement, suppressed the quasi-free absorption \( \pi^+ A \rightarrow ppX \) background for \( \pi^+ \). These backgrounds from the coincidence measurements never exceeded about 7% of the foreground at any angle or energy. Figure 3.15 shows the worst case scenario.

The possibility of detection of a proton in the pion arm and a pion in the proton arm causing a coincidence was completely eliminated in all but a single case by the tight kinematical constraints imposed by the pion-proton counter pairs. The one case where such an event could occur was in the "Set B" configuration where both the pion and proton angles were \(~55^\circ\). Even in this case, only some pions were slow enough and some protons fast enough to satisfy the timing constraint. These events easily were separated from the true πp events by the TOF difference timing, and therefore did not present a problem in the analysis (see figure 3.16).

The targets were arranged so that the pion arms operated in reflection mode, and the proton arms in transmission mode i.e. the rear surface of the target (with respect to the incident beam) was oriented to face the proton counters. The target angles were chosen such that the lowest energy protons (i.e. at the largest angle) traveled through the thinnest part of the target. The requirement that these protons not suffer excessive energy loss and multiple scattering on the way to the P1 counter limited the proton angle to a maximum of about \(55^\circ\) at the lowest energy (141 MeV), corresponding to a minimum pion angle of \(~55^\circ\). For most runs ("set A"), the forward–most pion arm was set at \(60^\circ\), corresponding to a P1 counter angle of \(53^\circ\) at 141 MeV. The target angle was fixed at \(53^\circ\) for all energies. The backward–most pion angle was limited to \(155^\circ\) by the requirement that the corresponding proton counter angle at 267 MeV (8.6\(^0\)) be situated safely outside the cone of the incident beam.

Consistent with these constraints, one set of pion angles was chosen for most of the
Figure 3.15: πp TOF difference spectrum at 145° laboratory angle for 218.1 MeV π⁺ on a 3.2mm CH₂ target. This particular energy/angle/target combination has the largest background (~7% using a gate between channels -50 and 150) of all the coincidence runs measured in this experiment.
Figure 3.16: left: Proton versus pion timing for an arm where a proton in the pion arm and a pion in the proton arm ("reverse elastic") is kinematically allowed; right: same, but for an arm where reverse elastic events are geometrically forbidden.
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runs: Set A ∈ [60, 75, 95, 115, 135, 155] degrees, with θtgt = 53°. The proton angles were adjusted at every energy to the appropriate angles conjugate to the pion arms. These conjugate angles were calculated from relativistic kinematics assuming known beam energies. However, an M11 momentum recalibration after the run (see section 3.5.4) indicated that these beam energies were in fact somewhat different than assumed. The differences were very small, but nonetheless, any potential effect on the solid angles could easily be accounted for in the Monte Carlo simulation (see section 4.1).

3.4.3 Forward Angle Single Arm Pion Detection

For pion angles less than about 50°, the corresponding proton energies were not large enough for the protons to escape from the LH2 target. Consequently, a set of π⁺ runs at forward pion angles were undertaken at θπ ∈ [20, 30, 40, 50, 60, 70] degrees with the proton arms removed from the EVENT coincidence. This was realized by simply disengaging the proton input signal at each ΠP1 coincidence unit. Since the pion signal defined the ΠP timing, no other timing changes were necessary. An ARM event was then defined by the coincidence of the BEAM with any one of the pion arms.

Candidate np events were identified by the TOF to the π2 counter. Detecting only the scattered pions necessitated using only the LH2 target for these runs, since at forward angles, the elastic and quasi-elastic scattering backgrounds of pions on carbon would swamp the np signal if CH2 targets were used. Even with the LH2 target, the foreground to background ratios were considerably more severe in single arm mode than in coincidence mode, ranging (for π⁺) from about 1.5:1 at 20°, to about 7:1 at 70°. See figure 3.17 for a sample spectrum.

The reactions contributing the bulk of the π⁺p single arm background were: pion elastic scattering on the mylar windows and domes (carbon, oxygen, and hydrogen);
Figure 3.17: $\pi^+ p$ timing spectrum for 168.8 MeV $\pi^+$ at $20^0$ laboratory angle using the LH$_2$ target. The foreground/background ratio is the worst at this angle, dropping sharply as $\theta_\pi$ increases. The protons shown correspond to backward going pions. At the most forward angles, the protons were fast enough to satisfy the BEAM-II$_{\gamma}$ timing requirement.
pion quasi-elastic scattering on protons or neutrons bound in these nuclei (although mainly in carbon); and \( \pi^+ \) absorption on quasi-deuterons (proton-neutron pairs) in these same nuclei, producing two fast protons which can satisfy the \( \Pi \) timing gate (see figure 3.19). Despite sizable backgrounds, reliable background subtractions could be made nonetheless resulting in clean \( \pi^+p \) yields (see section 4.3.3).

However, this technique could not be used for \( \pi^-p \) scattering, in part due to the much lower signal-to-noise ratio, and in part due to the \( \pi^-p \rightarrow \pi^0n \) background. The \( \pi^-p \) cross section is about 9 times smaller than the \( \pi^+p \) cross section (see section 2.1), but elastic and quasi-elastic scattering on carbon and oxygen (the dominant background nuclei) are almost equal for \( \pi^+ \) and \( \pi^- \) (see e.g. [73]). Excessively long run times would be required to obtain statistical accuracies with \( \pi^- \) similar to \( \pi^+ \). Also, the \( \pi^0 \) from the competing \( \pi^-p \rightarrow \pi^0n \) reaction (which has roughly double the cross section) immediately decays into two gamma rays, which can pair produce an electron-positron pair in the \( \text{LH}_2 \) target vacuum vessel or in the \( \pi1 \) scintillator. These electrons could then cause a \( \pi1\pi2 \) coincidence. These events arrived at the \( \pi2 \) counter sooner than the elastically scattered \( \pi^- \) events (since the gammas and electrons/positrons travel at essentially the speed of light), but the timing resolution was not good enough to separate these out, so a detailed Monte Carlo simulation of these backgrounds would have been required to obtain useful results. Because of all these problems, only \( \pi^+ \) data were taken at the forward angles\(^6\).

The single arm runs were set up in transmission mode, with the target oriented such that the rear window faced the middle pion arm (-40°). The choice of pion angles was limited at the forward angle by the requirement that muons arising from decay of beam pions would not cause a pion arm coincidence. The other angles were chosen to fill in the angles not already covered by the two arm coincidence runs, but overlapping at one angle (60°) to provide a consistency check.

\(^6\) Actually, some \( \pi^-p \) single arm forward angle measurements were taken, where these effects were indeed observed.
3.4.4 Data Acquisition

The pulse height and timing signals from every scintillator in the TOF spectrometer were accumulated by CAMAC electronics and read-out by computer to 8mm video tape using the TRIUMF VDACS [72] data acquisition program (see figure 3.18). The pulse height signals from the linear fan-outs were fed directly to LeCroy 2249A integrating analogue-to-digital converters (ADCs), whereas the NIM logic timing signals from the CFDs were sent to LeCroy 2228 time-to-digital converters (TDCs), as well as to Kinetic Systems 3615 Scalers for accumulating the number of counter hits. Both the individual and the meantime signals from the proton counters were timed and scaled. As well, all the various counter coincidences were counted by the scalers, in particular BEAM, which was input into two different scalers as a consistency check (and safety precaution).

The scalers accumulated hit statistics continuously when the data acquisition was active, and were read-out by the CAMAC system in approximately one minute intervals during a run, after every pause in data acquisition, and at the end-of-run. On the other hand, the ADCs and TDCs were read-out only after a look-at-me (LAM) signal had been generated. The EVENT gate was the logical OR of all six pion-proton pair coincidences (or just pion arms for single arm runs) and the beam SAMPLE:

$$\text{EVENT} \equiv \sum_{i=1,6} \text{ARM}_i + \text{SAMPLE}$$

The LAM gate was essentially EVENT together with additional "inhibit" coincidences indicating whether or not the computer was busy processing data (BUSY), whether the beam was turned off (detected using a ratemeter), or whether another EVENT signal had immediately preceded the current one (detected using a fast inhibit). Due to this inhibiting, the number of LAMs and EVENTS were not equal, and so the number of $\pi p$ events recorded by the ADCs and TDCs (i.e. LAMs) were not equal to the number recorded by the scalers (EVENTs) for any given amount of BEAM recorded by the
Figure 3.18: Schematic diagram of nuclear instrumentation and logic circuits used in E645. Circuits for arms B through F are identical to arm A.
Figure 3.19: Schematic representation of relative timing and gate widths of the signals used to define a \( \pi \pi \) coincidence. Note that \( S2B \) defines the system timing.
scalers. Their ratio defines the live time (or duty factor) of the data acquisition system \( t.e. f_{lt} = \text{LAMs/EVENTs} \). The live time is an important factor entering into the BEAM normalization (see equation 4.53 and section 4.2).

The LAM gate was fanned-out and used as the TDC START pulse, the gate for the ADCs, and as the C212 input register STROBE signal (described below) which indicated to the CAMAC controller that data was ready to be read-out. The ADC gate widths were set at approximately 30ns, wide enough to include most of a signal, but smaller than the beam repetition period of 43ns to avoid pile-up and random coincidences. The TDCs operated in common start mode, with the LAM as the common START, and the counter signals as the STOP gates.

To reduce the number of ADC and TDC channels required to accumulate all the data from the six pion and proton arms, and therefore to conserve computer data tape, the data acquisition system employed a multiplexing scheme. In this scheme, only single \( \pi_1, \pi_2, \) or P1 ADC and TDC words were recorded by CAMAC for the arm which detected the \( \pi p \) EVENT. The accounting of which arm caused the EVENT was carried out by the C212 multiplexor pattern buffer. The \( \text{ARM}_i \) timing signals for each of the six arms, as well the beam SAMPLE signal, were fed to separate channels of the C212, where they were recorded as a binary "yes" or "no". In the offline analysis, the pattern buffer was checked to see which arm the recorded ADC and TDC signals were associated with, or whether instead the EVENT was a beam SAMPLE. One could also check whether more than one arm recorded a hit for the same EVENT, giving a measure of the rate of accidental coincidences. In practice, almost all of the runs did not register any simultaneous events in two or more arms, and the most observed for any run was two (out of many thousand events).
3.5 Measurement Program

3.5.1 Channel Tuning

A few months prior to the start of Experiment 645, the pole faces of the front-end septum magnet (see figure 3.6) were modified to achieve a higher maximum field with the given power supply [66]. Evidence from a brief beam tuning run carried out after this change\(^7\) suggested that the properties of the M11 beam were rather different from the time that the beamline was last studied in 1985 (see [65] and [74]). Since such a modification to a major beamline element could significantly alter the beam properties (including momentum calibration), a comprehensive beam tuning and calibration program was undertaken immediately prior the experiment presented in this work. Details of the setup are described in appendix B.

A number of beam properties that were investigated with this setup are summarized below:

- A doubly-achromatic double focusing tune producing an acceptable beam spot at the TINA location\(^8\) was established which differed from the previous tune mainly in the sextupole field settings. The dipoles (1AQ9, septum, B1, B2) and the other quadrupoles did not require any adjustment.

- The beam spot size and divergence were found to be strong functions of the aperture of the front-end rate restricting jaws. The beam size in the y-direction was sensitive only to the vertical aperture, as was the divergence, but much less so since it was primarily limited by the aperture of the final Q6 quadrupole. The horizontal divergence depended on the horizontal jaws primarily, but at large vertical jaw settings, a left-hand (looking along the pion beam) tail emerged. On

\(^7\)This run occurred a few months prior to E645 during low intensity operation of the cyclotron. The setup was similar to what will be described here.

\(^8\)The focus in the x-plane is slightly (~10cm) upstream of the target, but since the spot and divergence in this plane were small, it remains very nearly focussed at the target.
the other hand, the x-spot size was more-or-less equally sensitive to both horizontal and vertical aperture. The spectra with the jaws both wide open and closed down are shown in figure 3.20.

- The left hand tails observed in the horizontal spot size and divergence were found to be uncorrectable. A REVMOC/TRANSPORT simulation of the M11 channel showed that the 4 dipole elements (1AQ9, septum, B1, B2) introduce too many 2nd order aberrations to be correctable with the existing sextupoles. Appendix C discusses the M11 channel simulation in more detail.

- When correctly tuned, the beam travels exactly along the optical axis as it exits the channel. Small changes in the 1AQ9 or septum fields cause the beam to "porpoise" through the channel and emerge at an angle to the axis. Since the beam also followed the axis in the older tune, it appears that the modifications to the septum did not affect the beam steering. In particular, the central ray leaving the T1 production target entered the B1 dipole normally both now as well as previously. Since B1 is the primary momentum selector, this suggests that the momentum calibration with the new septum should be the same now as when previously calibrated (see section 3.5.4 and appendix D).

- The sensitivity of the beam parameters to the position of the proton beam on the production target was investigated. The beam is very insensitive to the horizontal position, but very sensitive to the vertical placement. A 1mm shift up (down) results in a much larger x-spot, with the beam centroid near the bottom (top) of the final quadrupole. This shift was easily detected by the hodoscope (see section 3.4.1), where the ratio of UP/DOWN rates in the hodoscope changed to about 10:1 (1:10) from unity. Hodoscope ratios of less than about 2:1 indicated no serious beam degradation.

During each run of E645, the values of the jaw apertures were recorded, so that
Figure 3.20: **Top:** Dependence of beam size at the target centre location on the aperture of the M11 rate restricting jaws. **Bottom:** Beam divergence dependence on jaw width. The observed left hand tails are uncorrectable with the available sextupoles (see text).
the beam size and divergence of each run was known. Consequently, the beam could be modeled accurately in all the Monte Carlo simulations. (see sections 4.1 and 4.2).

3.5.2 Foreground and Background Running

The pion and proton counter angular positions as well as the target angles used for the production runs (runs used to determine the cross-sections, as opposed to the systematic check runs) in E645 are listed in table 3.2.

An important element in measurements using background targets is the sequencing of the foreground (using \( \text{LH}_2 \) or \( \text{CH}_2 \) targets) and background runs. Ideally, a background target run immediately following a foreground run would have been desirable to limit any possible time-dependent systematic effects. However, several hours were required to fill or empty the liquid hydrogen target, making it impossible to conduct a target empty run immediately after a target full run was completed, or vice versa. Therefore, a series of target full (empty) runs was undertaken for each geometrical configuration, followed thereafter by all the respective target empty (full) runs. During target emptying (filling), the target would be moved out of the beam, and the time used to conduct other measurements e.g. data with solid targets. However, the existence of significant delays (up to a day) between target full and target empty runs was not expected to introduce any problems in this case since even large uncertainties in the relative foreground/background normalization (e.g. changes in solid angle from beam movement or size, or changes in target angle) would cause insignificant (two arm) or small (single arm) changes to the net yields due to the small background levels. Nonetheless, a small uncertainty from the relative foreground/background normalization (arising mainly from the uncertainty in the target angle) was included in the extracted net (foreground - background) yields (discussed in more detail in section 4.3).

For the case of \( \text{CH}_2 \) targets, data runs were followed immediately by background
<table>
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<th>$T_\pi$ [MeV]</th>
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<th>$\theta^\pi_{\text{lab}}$ [deg]</th>
<th>$\theta^\pi_{\text{p}}$ [deg]</th>
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<td>52.4 44.3 34.3</td>
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<tr>
<td></td>
<td>$\pi^-$</td>
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<td>$\pi^-$</td>
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</tr>
<tr>
<td>218.1</td>
<td>$\pi^+$</td>
<td>LH$_2$ -40 20 30 40 50 60 70</td>
<td>N/A</td>
<td>51.4 43.2 33.3</td>
</tr>
<tr>
<td></td>
<td>$\pi^-$</td>
<td>LH$_2$ 53 60 75 95 115 135 155</td>
<td>24.5 16.4 9.0</td>
<td>54.2 48.6 38.1</td>
</tr>
<tr>
<td></td>
<td>$\pi^+$</td>
<td>&quot;E&quot; 50 55 65 85 105 125 145</td>
<td>28.8 20.4 12.6</td>
<td>64.7 51.2 37.4</td>
</tr>
<tr>
<td>240.9</td>
<td>$\pi^-$</td>
<td>LH$_2$ 53 60 75 95 115 135 155</td>
<td>50.9 42.7 32.9</td>
<td>23.6 16.2 8.8</td>
</tr>
<tr>
<td>267.3</td>
<td>$\pi^-$</td>
<td>LH$_2$ 53 60 75 95 115 135 155</td>
<td>50.3 42.1 32.4</td>
<td>23.7 15.9 8.5</td>
</tr>
</tbody>
</table>

Counter distances: $\pi_1 = 79.2$cm; $\pi_2 = 123.1$cm; P1 = 92.6cm

Table 3.2: List of the geometrical configurations used in the production runs. The letters refer to the CH$_2$ targets (as explained in 3.1). The proton angles are not exactly kinematically conjugate to the pion angles, but are nonetheless only $<0.1^\circ$ off. (see section 3.5.4).
runs using the graphite target, except during those \( \pi^+ \) runs at 169 MeV designed to explore some systematic effects (see section 3.5.3). As only the two arm coincidence configuration was employed for the solid targets, the effect of a relative foreground/background normalization uncertainty on the cross sections was negligible (<0.1%) due to the very low background levels.

### 3.5.3 Systematic Checks

A main goal of Experiment 645 was to elucidate sources of systematic error by taking data under a wide variety of experimental conditions. These "systematic-check runs" were designed to probe each element of the differential cross-section: beam energy, target thickness, solid angle, yield extraction, and beam counting.

The \( \pi^+p \) coincidence measurements with both solid and liquid targets for pion energies of 141, 169, and 193 MeV, and for \( \pi^+ \) (only) at 218 MeV constituted a crucial systematic check of all the aforementioned elements (to varying amounts). Since the use of solid targets in \( \pi p \) elastic scattering experiments has been the subject of some recent criticism [41] these data were taken to provide the evidence necessary to resolve the dispute.

The \( \pi^+ \) near-forward, angle single arm data at 141, 169, and 218 MeV also provided a systematic check at the two angles of overlap with the coincidence runs, since the target angle (hence thickness), solid angles, and yield extraction are quite disparate in the two cases.

All of the other checks were carried out at the single energy 169 MeV \( \pi^+ \) in the two arm coincidence configuration. The large value of the \( \pi^+p \) cross-section at this energy permitted a wide variety of "systematics" data to be acquired reasonably quickly and to the same statistical accuracy (~1%) as the "normal" data. The checks that were performed at 169 MeV \( \pi^+ \) using either the liquid or solid targets are tabulated below, along with the elements of the differential cross-section they were designed to study.
Checks using Liquid Target

1. **Target Angle Varied**: Data were obtained for target angles of 45, 50, and 60 degrees, in addition to the 53° normally used for data. In addition to checking our knowledge of the target angles, these data check the solid angle determinations, since interaction losses and proton multiple scattering can change dramatically over that angular range, as well as beam interaction losses within the target.

2. **Beam Rate Varied**: Four pion beam rates at target from approximately 0.3 to 7 MHz were used to check the beam handling capabilities of our counters, the data acquisition live-time, and the multiple-pion correction schemes. The latter contributed the biggest effect, as the fraction of beam buckets with multiple pions varied from about 1% to 26% over that range (see section 4.2.4).

3. **Proton Counters Moved**: A run was taken where proton counters A, C, and E were positioned 6.5cm closer to the target, whereas the other counters were placed 10.0cm farther away. These data again check the solid angle calculations by varying the angular acceptance of the proton counters.

4. **Proton Counters Misaligned**: All the proton counters had their angles purposely shifted by +0.25° azimuthally from their nominal values to check the sensitivity of the solid angle calculations to misalignments on the order of the estimated angular uncertainty in the counter positions.

5. **π1 Counters removed from Coincidence**: With the π1 counters out of the event coincidence, a πp scattering event was defined by BEAM-π2-P1. As well as providing an additional check of the solid angle calculations, these data check whether events striking the π2 lightguides can cause spurious events, since the positioning of the π1 counters was arranged to remove this possibility when the π1 counters were included in the event coincidence (see section 3.3.3).
6. **Beam Momentum Spread Increased**: For one run, the beam momentum spread was increased to 3% $\delta p/p$ from the usual 1%. This data also checked the solid angle calculations, since the angular spread of the protons for a given pion angle increased under these conditions.

**Checks using Solid Targets**

1. **CH$_2$ Thickness Varied**: Runs were performed using CH$_2$ targets of thickness approximately 0.5, 2.0, 3.1, and 5.1mm. This set of runs checked the target thicknesses, the solid angles (primarily interaction losses and proton multiple scattering), beam interaction losses within the target material.

2. **Run with Scintillator Target**: Several runs were taken using an active, 15cm diameter (same as the LH$_2$ target) 0.33cm thick scintillator target. Several runs were performed to provide beam data to check predictions of the REVMOC beam Monte Carlo e.g. the determination of the pion target rate from the measured VETO rate (see section 4.2.4).

### 3.5.4 M11 Momentum Calibration

After channel tuning was completed, the channel momentum was recalibrated. The channel momentum is linearly proportional to the central B1 field, which is measured by an NMR probe. The recalibration technique was the same as that used in the original calibration [74], where a silicon surface barrier detector (SSBD) mounted in vacuum at the M11 exit was used to measure the kinetic energies of various light ions (e.g. $^2$H$^+$, $^3$He$^{++}$), from which the momentum of the channel could be determined. The results suggested a momentum roughly 1% lower at a given B1 field than the result in reference [74]. It was on the basis of this momentum calibration that the proton angles used in E645 were chosen (see table 3.2). However, subsequent to the completion of the experiment, it was discovered that this calibration technique was
not reliable, and that a second calibration would need to be undertaken. Appendix D provides the details of the surface barrier detector method and the reasons it was found to be unreliable for this application.

Because of the failure of the SSBD method, another momentum calibration program was undertaken in 1994. The new calibration method basically involved measurement of the TOF difference of pions and electrons in vacuum between two scintillators a known distance apart. The technique is straightforward, and was found to be reproducible over two calibration runs two months apart. Appendix D provides the details.

The resulting $M_{11}$ calibration was found to be linear in the $B_1$ field over the range 155 to 355 MeV/c:

$$P_{M_{11}}[\text{MeV}/c] = 0.03267 \cdot (B_1 - 17.1)[\text{Gauss}] \pm 0.15\%$$  \hspace{1cm} (3.49)

The uncertainty arises from the deviation of the calibration points over the above momentum range (see figure D.58), and corresponds to a $\pm 0.25\%$ uncertainty in the kinetic energy.

Since no dipole element or power supply in the beam line had been modified subsequent to E645 [66], this new calibration should also be applicable to the channel during the time the experiment was carried out. Substantiating evidence is provided by measurements of the $\pi$-e TOF difference from the T1 production target to the S2B scintillator during the phase restricted beam runs in the experiment. Using the $M_{11}$ channel length as calibrated by the later analyses (see appendix D), these results (141 to 267 MeV) are consistent with the new calibrations, albeit to less precision (roughly $\pm 0.5\%$).
Chapter 4

Data Analysis

This chapter presents the techniques used to extract from the TOF Spectrometer signals the differential cross section at laboratory kinetic energy $T_\pi$ and centre of momentum scattering angle $\theta_{cm}$:

$$\frac{d\sigma}{d\Omega}(T_\pi, \theta_{cm}) = \frac{Y(T_\pi, \theta_{cm}) \cdot \cos \theta_{tgt}}{N_\pi \cdot \Delta \Omega(T_\pi, \theta_{cm}) \cdot N_{prot} \cdot \varepsilon}$$  \hspace{0.5cm} (4.50)

where:

$Y = \text{Number of } \pi p \text{ events}$

$\theta_{tgt} = \text{Target angle}$

$N_\pi = \text{Number of incident pions that hit target}$

$\Delta \Omega_{eff} = \text{Effective solid angle for } \pi p \text{ detection}$

$N_{prot} = \text{Number of target protons/cm}^2$

$\varepsilon = \text{efficiencies (scintillators, computer)}$

The proton densities and target angles for each target used in this experiment and the methods used to determine them have already been detailed in section 3.3.2. The target proton densities can be found in table 3.1. Each of the other terms in expression 4.50 will be treated separately in the following sections. Details of the Monte Carlo simulation together with the techniques employed for analysis of the scintillator signals are presented here, whereas the results are deferred to chapter 5.
4.1 Solid Angle

The determination of the effective solid angle of the pion arm (for single arm operation), or pion and proton arm combination (for coincidence mode), was the most critical aspect of the analysis since it had to be done entirely by Monte Carlo simulation. Simulations can be fraught with problems if the approximations (which must necessarily be made) bias the results. It is for this reason that most of the systematic checks outlined in section 3.5.3 were performed. The ability of the simulation procedure outlined below to give consistent results for a variety of experimental conditions was a crucial test, the results of which provided some measure of the systematic errors introduced by this analysis.

In both the two arm and single arm configurations, the \( \pi 2 \) counter defined the "geometric" solid angle (\( \Delta \Omega_{\text{geom}} \)) for detecting scattered pions. This solid angle was 2.646±0.012 milli-steradians (msr), corresponding to a ±0.93° horizontal and a ±2.33° vertical angular size at \( r(\pi 2) = 123.1 ± 0.3 \text{cm} \) (see section 3.4.2). However, due to a number of effects which are discussed below, not all scattered pions (recoil protons) emanating from the \( \pi p \) reaction vertex that "would have" hit a \( \pi 2 \) (P1) counter did so, and some that "would have" missed, actually hit. Consequently, since the number of \( \pi p \) events detected at a particular pion angle \( Y \) in equation 4.50 is different from the "actual" number \( \pi p \) events produced in the target, an "effective" solid angle \( \Delta \Omega_{\text{eff}} \) was required to compensate for these effects. This effective solid angle differed from the geometric solid angle due to the effects illustrated in the following paragraphs:

**Interaction Losses:** A pion (proton) could suffer a hadronic elastic or inelastic interaction on the way to the \( \pi 2 \) (P1) counter, thus escaping detection due to the narrow angular size of the counters and the fact that these reactions "spread" out the reaction products over a comparably much larger angular range. To compensate for this effect, the effective solid angle \( \Delta \Omega_{\text{eff}} < \Delta \Omega_{\text{geom}} \). This correction was substantial and ranged between about 2 and 5% depending on target and pion scattering angles (see
Finite Source Size: A finite incident beam spot implies a finite volume in the target from which a scattered pion could originate, making the distance to the π2 counter (hence the solid angle) different for each pion. \( \Delta \Omega_{\text{eff}} \) is then a weighted average over all the possible source vertices. In the radial direction, the solid angle "seen" by a pion originating farther away than the centre of the target (hence farther away from the π2 counter) averages out with one originating closer than the target centre. On the other hand, all pions originating from vertices displaced transversely from the centre have longer pathlengths to the π2 counter than the central vertex, hence see smaller solid angles. In this experiment, this effect was very small (<0.2%).

Interfering Structures: A scattered particle that would have otherwise missed the π2 (P1) counter could have hit some experimental structure, multiply scattered and subsequently hit the counter in question, thereby artificially increasing \( \Delta \Omega_{\text{eff}} \). This effect occurred when using the LH₂ target, and was non-negligible only for the most forward pion arm in the two arm setup. Here the pions that would have missed the π2 counter could have scattered off the target ring and subsequently hit the counter. The effective solid angle of the π2 counter was increased by about 10% from this effect. However, the additional geometrical constraints imposed by the π1 and P1 counters prevented this effect from affecting \( \Delta \Omega_{\text{eff}} \) (see figure 4.23).

Pion Decay: In the absence of interaction losses, interfering structures, and finite source size effects, the effective solid angle of the π2 counter would still equal the geometric, even in the presence of pion decay, for isotropic pion angular distributions. Pions that would have hit the counter but which subsequently decayed with the daughter muon missing the counter are balanced by "off-angle" pions that would have missed the counter but subsequently decayed with the muon hitting the counter ("in/out cancellation" effect). In actuality, the pion angular distribution is not isotropic.
but only approximately so over the typical angular range of pion scattering angles detectable by the \( \pi 2 \) counters (\( \sim \pm 10^0 \)), and so the in/out cancellation was not quite exact. In any case, the intermediate \( \pi 1 \) counters destroyed the “open” geometry and constrained the number muons from off-angle pion decay that could have been detected. Therefore some pions that would have hit were not compensated for, so \( \Delta \Omega_{\text{eff}} \) was decreased for this effect too, typically 2–4\% (see e.g. figure 4.22).

**Proton Counter Constraint:** For a monochromatic incident pion beam, in the absence of multiple scattering, pion decay, finite source size, or interaction losses, the TOF spectrometer was designed such that every proton conjugate to the pions that hit the \( \pi 2 \) counter, hit the \( P1 \) counter. However, an incident beam of finite size and momentum spread kinematically spreads out the recoil proton for a given pion angle, as does proton multiple scattering en route to \( P1 \). Pion decay also spread out the pion angles that could produce a \( \pi 2 \) hit, thus also spreading out the conjugate proton beam. The net result was that the proton counters did constrain the overall solid angle somewhat, contributing also to a decrease in \( \Delta \Omega_{\text{eff}} \) (typically about 7\%) (see figure 4.22).

**Counter Edge Effect:** The phototube voltages and discriminator thresholds for all the TOF spectrometer scintillation counters were set using beam pions which passed through the front face and exited the rear face of the counters (see section 3.3.3). However, it was in principle possible for a pion (muon, proton) to enter near the edge of the scintillator and exit “out the side”, thus reducing the path length in the scintillator and consequently the light output, possibly below the threshold level. This effect reduced the “effective” surface area of the counter and hence reduced \( \Delta \Omega_{\text{eff}} \). In this experiment, assuming a point source and straight line trajectories, the area around the edge of the \( \pi 2 \) counter front face where a pion or muon enters, and then exits the “side” face, comprises only 0.5\% of the total frontal area. Given that most of these events would yield detectable light outputs due to the low thresholds used, the edge effect
inefficiency was actually considerably smaller than 0.5%. The same was true for the more heavily ionizing protons through the edges of the P1 counters. Consequently, this is considered a negligible effect.

**Monte Carlo Simulation**

Although each of the above effects could be accurately calculated analytically, the combined effect could not, due to the significant correlations among them. The only way to accurately estimate $\Delta \Omega_{\text{eff}}$ was through a Monte Carlo simulation of the $\pi p$ scattering process in the TOF spectrometer.

The basic principle behind the Monte Carlo determination of $\Delta \Omega_{\text{eff}}$ was: if a region of known area $A$ was uniformly covered by $N$ randomly generated points, the area $A_r$ of an arbitrarily shaped region entirely within $A$ was estimated by $A_r = (N_r/N) \cdot A$, where $N_r$ was the number of points contained within the interior region. Similarly, with all the relevant physical processes associated with $\pi p$ scattering (multiple scattering, pion decay, etc.) and the details of the experimental configuration (scintillators, target, air, etc.) included, then by generating $N_{mc}$ $\pi p$ scattering events, where the scattered pion was randomly and uniformly distributed within the solid angle $\Delta \Omega_{mc}$ chosen to be much larger than the $\pi 2$ counter solid angle and large enough to accommodate all events which could possibly result in a $\pi 2$ "hit", the effective solid angle $\Delta \Omega_{\text{eff}}$ for a $\pi 2$ counter wholly contained in $\Delta \Omega_{mc}$ was estimated as:

$$\Delta \Omega_{\text{eff}} = \frac{N_H}{N_{mc}} \cdot \Delta \Omega_{mc} \left( \pm \frac{100}{\sqrt{N_H}} \% \right)$$

where $N_H$ was the number of $\pi 2$ counter hits. The uncertainty follows from the Poisson limit to the applicable binomial statistics, since the number of counter hits $N_H$ was large ($\sim 10,000$) for most simulations, and the probability that the counter would be hit was small (e.g. $\Delta \Omega_{\text{eff}} / \Delta \Omega_{mc} \sim 2.5/600 = 0.004$).

The precision of the simulation can be increased arbitrarily, subject to available computing power, but the main goal of the simulation was the accuracy to which $\Delta \Omega_{\text{eff}}$
could be estimated. The latter required the relevant physical processes and the experimental conditions to be modeled as faithfully as possible. For this, the GEANT [75] detector description and Monte Carlo particle tracking program was used to simulate the $\pi p$ scattering reaction in the TOF spectrometer.

4.1.1 GEANT description of TOF Spectrometer

The GEANT program contains sophisticated detector description routines which permitted accurate modeling of all the elements of the TOF spectrometer. The compositions, dimensions, and positions of all in-beam, pion arm, and proton arm counters (including wrapping tape) were modeled exactly (see figure 3.11), as was the CH$_2$ target. The LH$_2$ target was also faithfully modeled, including all windows, dome, heat shield and super-insulation layers, stainless steel vacuum vessel, target cooling ring, and liquid hydrogen coolant in the ring (refer to figures 3.8 and 3.9, and the figures in appendix E). Only the plumbing and central target support were not modeled, as they were well removed from the "active" region. The accurate modeling ensured that the geometry dependent effects discussed previously were properly treated. In particular, since the LH$_2$ target rings interfered somewhat with the smallest angle pion arm in the two arm configurations, an accurate rendition of the cooling ring assembly was essential.

The particle tracking routines provided in GEANT include practically all the known physical processes involving elementary particles, and the routines provide options for each, including turning each one "on" or "off". In the TOF spectrometer simulation, the most relevant processes were: multiple scattering using the Molière prescription, energy loss with Landau fluctuations, particle decay, and direct atomic electron knock-out (i.e. delta rays). The hadronic interaction routine, GHEISHA, was turned "off" since it was found to be unreliable in the low energy regime considered here.
These interaction losses were introduced afterwards "by hand" to obtain the final results (see section 4.1.3). The precise modeling of the apparatus and these physical processes, as well as the fine step-by-step tracking of each particle through all the media, provided all the necessary ingredients for a faithful simulation of the πp scattering process in the experimental system employed for this experiment.

**Simulation Details**

The initial conditions for the incident pion beam were constructed by first randomly choosing a position, angle, and momentum from the known (see below) Gaussian distributions at the centre of the scattering target, and then projecting the trajectories back towards the Q6 magnet midplane location, taking into account the different x and y plane foci, and the constraint imposed by the Q6 aperture. The size, divergence, and momentum spread of the incident pion beam at the target centre location were known for all the various scattering arrangements from the beam phase space measurements, obtained for a variety of channel settings during the beam tuning program prior to the start of the experiment (see section 3.5.1). Since the beam size, divergence, and momentum spread were different for each measurement, a separate simulation was necessary for every configuration (energy, polarity, target angle, etc.) used during the experiment in order to ensure faithful modeling of the target source size and πp scattering kinematics.

The πp interaction vertex in the target was generated randomly with uniform probability along the incident beam path between the points where the beam entered and exited the target. Since the targets used were thin and the energy loss was small, this was an appropriate approximation.

The initial trajectories of the scattered pions were chosen randomly within a solid angle window in the laboratory frame (i.e. $\Delta \Omega_{\text{mc}}$ in equation 4.51) large enough (typically 600 to 800 msr) to accommodate all trajectories that could result in decay muons
hitting the π2 counter. Following two body kinematics, the energy of the scattered pion was calculated using the polar angle between the scattered and incident pions, and the energy of the incident pion at the target vertex. Since the minimum solid angle window size required varied with pion energy, hence polar angle, the average number of generated πp vertices required to achieve a certain statistical accuracy in the solid angle also varied (see equation 4.51). Consequently, a single window encompassing all the arms would have had to be large enough to accommodate the most backward angle pion, where the decay muon Jacobi angle was largest, thus "wasting" window size on the more forward angles where the Jacobi angles were smaller. Thus to optimize computing time, the solid angle determinations for a particular experimental configuration were conducted for one pion (or pion–proton) arm at a time.

The scattered pion trajectories were chosen to uniformly illuminate the Monte Carlo window. Although in reality the polar distribution of the scattered pions are a function of polar angle, the distribution is essentially constant over the small angular size (±1.8°) of the π1 counters. Thus for those ~95% of the π2 counter hits that were pions at π1 (e.g. see sample output in appendix F.3), the cross section was very nearly constant. The other 5% corresponded to hits involving a muon at π1, most of which arose from pions produced within about 10° of the central angle, so for these events as well, the uniform illumination approximation was reasonable.

For the two arm configuration simulations, the initial energy and trajectory of the recoil protons were such that two body kinematics involving both the incident and scattered pions was satisfied.

The basic tracking sequence for each event in the Monte Carlo simulation can be
summarized as follows: A pion was created at the position of the Q6 quadrupole mid-plane (as outlined above), and tracked towards the target. If the pion missed the target, or decayed before reaching it, the pion or muon was tracked until it left the experimental area. If the pion had not decayed, but had traversed all the in–beam counters, and passed through the target, a πp scattering vertex was randomly chosen at some point along the trajectory within the target. A polar and azimuthal angle within a predefined window was randomly chosen for the scattered pion, and its momentum calculated as a function of the incident pion energy at the vertex following two–body kinematics. For two arm simulations, a proton was also generated with a trajectory and momentum satisfying two–body kinematics. The scattered particles were then tracked until they left the experimental area. A new incident pion was then created, and the process repeated until the end of the simulation.

**Solid Angle Extraction**

The TOF (time–of–flight) spectrometer identifies πp scattering events by the TOF difference between the π2 and P1 counters for the two arm runs, or by simply the TOF to the π2 counter for single arm runs (see sections 4.3.2 and 4.3.3). The pion arm cannot distinguish between scattered pions and their decay muons. So the net πp yield spectra (i.e. after appropriate background subtraction) contain only these events involving a pion or a decay muon in the pion arm, and, for the two arm runs, a proton in the proton arm.

The GEANT Monte Carlo simulated only the πp reaction, since the solid angles ΔΩ_{eff} derived from the simulation were to be used with the net πp yields Y (in equation 4.50) discussed above. Therefore it was sufficient in the simulation to determine whether or not the pion arm was hit by a pion, muon, or electron, and whether the proton arm was hit by a proton. Since the background reactions were not simulated, it was not necessary to generate TOF spectra in order to select πp events.
At the end of an event, a record of the counters that were hit, the particles that hit them, and various coincidences between the counters was written. A table of accumulated hit statistics was generated at the end of the simulation. From this table, the solid angles of the following arm combinations: π2 hit only, π1·π2 hit, and π1·π2·P1 hit were determined according to equation 4.51. These solid angles included all the effects discussed previously (finite source size, pion decay, multiple scattering, etc.) except hadronic interaction losses. These hadronic loss corrections were applied to the solid angles “by hand” afterwards using another program.

Appendix F provides an example of the final output for one arm.
Figure 4.21: The results of a high statistics (0.3%) GEANT simulation of the solid angles for a 137.8 MeV run using a CH$_2$ target at 53°. No hadronic interaction losses are included. The $\pi$2 solid angle equals the geometric solid angle as expected.

### 4.1.2 Solid Angle Simulation Results

#### CH$_2$ Targets

When all interactions were "turned off" and a point source (narrow pencil beam on a thin target) was used, the solid angle subtended by the $\pi$2 counter ($\Delta\Omega_{\text{eff}}(\pi2)$) equaled the geometric solid angle as expected. In this case, the $\pi$1 and P1 counters posed no constraints, and the solid angles defined by the $\pi2\cdot\text{P1}$, $\pi2\cdot\pi1$, $\pi2\cdot\pi1\cdot\text{P1}$ counter combinations ($\Delta\Omega_{\text{eff}}(\pi2\cdot\text{P1})$, $\Delta\Omega_{\text{eff}}(\pi2\cdot\pi1)$, and $\Delta\Omega_{\text{eff}}(\pi2\cdot\pi1\cdot\text{P1})$ respectively) also equaled the geometric solid angle $\Delta\Omega_{\text{geom}}$. This was still true after introducing a realistic CH$_2$ target thickness and beam size (i.e. finite source), while keeping pion decay and multiple scattering "turned off".

Since the source size effect was negligible, the $\pi$ solid angle ($\Delta\Omega_{\text{eff}}(\pi2)$) should have still equaled the geometric ($\Delta\Omega_{\text{geom}}$) even in the presence of pion decay and multiple
scattering (neglecting hadronic interaction losses), due to the "in/out cancellation" effect discussed previously. Figure 4.21 shows the result for a simulation of all six arms in a two arm configuration using a 2mm CH₂ target at 53°. The effective solid angles of all π2 counters were within 0.3% of geometric, and the average was within <0.15%. The fact that ΔΩ_{eff}(π2) did equal ΔΩ_{geom} as expected constituted an important check of the simulation.

The effective solid angle for the π2-π1 combination ΔΩ_{eff}(π2-π1) decreases smoothly with increasing angle. The reason is that the pion energy decreases with increasing angle, and thus the number of decays increases as well as the size of the muon decay cone, making the chance of a π2-π1 coincidence increasingly less likely. On the other hand, ΔΩ_{eff}(π2-P1) and ΔΩ_{eff}(π2-π1-P1) are reasonably constant with angle as a result of two competing effects: with increasing pion angle, the recoil proton energy increases, decreasing the spread in the recoil proton "beam" at P1 (due to decreased multiple scattering), and thus tends to increase ΔΩ_{eff}. However, with increasing pion angle,
the angle of the pion decay muon cone increases, thus increasing the angular range of pions leaving the target that can result in a pion arm coincidence. This translates into a broader recoil proton angular range, the limits of which could be outside the P1 arm acceptance. This tends to decrease $\Delta \Omega_{\text{eff}}$. The drop at the most forward angle is due to the fact that the associated recoil proton has a small recoil kinetic energy ($\sim 25$ MeV) resulting in significant multiple scattering before reaching P1. These effects were generally reproduced in all the CH$_2$ two arm simulations.

These simulations were run to achieve (typically) 0.7% statistical uncertainties in the solid angles. From the discussion above, it was demonstrated that $\Delta \Omega_{\text{eff}(\pi 2)}$ equaled $\Delta \Omega_{\text{geom}}$ when the simulation was run to high statistical accuracy. It follows that at lower statistical accuracy, deviations in the $\Delta \Omega_{\text{eff}(\pi 2)}$ solid angle from $\Delta \Omega_{\text{geom}}$ are due simply to statistical fluctuations. Figure 4.22 shows the result of an identical simulation to that of figure 4.21, but at lower statistical accuracy. The plot at right in figure 4.22 shows that the ratios of the $\Delta \Omega_{\text{eff}(\pi 2-P1)}, \Delta \Omega_{\text{eff}(\pi 2-P1)}$, and $\Delta \Omega_{\text{eff}(\pi 2-P1)}$ solid angles to the $\Delta \Omega_{\text{eff}(\pi 2)}$ solid angle are smooth functions of angle, and almost independent of the statistical accuracy. Therefore the simulations can be carried out to a lower statistical accuracy (for a given amount of computing time) without compromising the precision, by "normalizing" the results to $\Delta \Omega_{\text{geom}}$ or to the average $\Delta \Omega_{\text{eff}(\pi 2)}$, thereby decreasing the individual uncertainties by a factor of $\frac{1}{\sqrt{6}}$. This latter approach was used for all solid angle determinations carried out in this experiment.

Separate simulations using the background graphite target were not performed, primarily since the CH$_2$ targets were used only in two arm mode, where the backgrounds were exceedingly small (see section 4.3.2). Also, since the graphite and CH$_2$ targets were of similar thickness (see table 3.1), then in any case the differences in source size and multiple scattering would also have been small. So it was assumed that apart from hadronic interaction loss corrections, the solid angles for detecting a pion and proton in coincidence were the same for both the CH$_2$ and the graphite
Figure 4.23: **left:** Solid angles for a LH$_2$ target run. The large $\pi$2 and $\pi$2-$\pi$1 values at 60° are not statistical fluctuations, but reflect the fact that pions are multiply scattered in the target cooling ring into these acceptances. **right:** Ratio to $\Delta\Omega_{\text{eff}}(\pi2)$ for the solid angles shown at left. The smooth angular dependencies for the largest five angles are similar to that seen in figure 4.21 for a CH$_2$ target. Apart from the solid angles at 60°, for use in the cross section calculations, these solid angles were smoothed by normalizing to the average $\Delta\Omega_{\text{eff}}(\pi2)$ solid angle.

targets.

**LH$_2$ Target**

The LH$_2$ target simulations differed somewhat from those outlined above in that the source size was larger, and the target ring and vacuum vessel had to be included in addition to the target cell since they could influence the scattered particles. Neither the ring nor the cryostat presented an obstacle for recoil protons, but the pion arms at all angles projected back to the target position a solid angle telescope which intercepted the vacuum vessel. The most forward arm in two arm mode (arm "A", 60° with a 53° target angle) projected back to the target position a solid angle which also intercepted the target ring. However, simulations with a pion scattering window large enough to encompass the vacuum vessel showed that the vessel had a negligible effect on the solid angles, since it did not intercept any of the pions which could have
produced muons that cause a $\pi 2 \cdot \pi 1$ coincidence. Pions would have had to multiple scatter by $\geq 20^\circ$ to enter the $\pi 2 \cdot \pi 1$ acceptance, which was highly unlikely. Adding the proton counter constraint in the two arm mode suppressed the effect to unobservable levels. This implied that $\Delta \Omega_{\text{eff}}(\pi 2)$ should have been very nearly equal to $\Delta \Omega_{\text{geom}}$ for the simulations, just as, in fact, was observed.

However, the target ring did influence $\Delta \Omega_{\text{eff}}(\pi 2)$ and $\Delta \Omega_{\text{eff}}(\pi 2 \cdot \pi 1)$ for the angle mentioned above. Because of the multiple scattering in the ring, pions that would have otherwise missed the pion arm counters were thereby able to scatter into the acceptance. This implied that $\Delta \Omega_{\text{eff}}(\pi 2)$ would be larger than $\Delta \Omega_{\text{geom}}$, as was observed (see figure 4.23). For $\Delta \Omega_{\text{eff}}(\pi 2 \cdot P1)$ and $\Delta \Omega_{\text{eff}}(\pi 2 \cdot \pi 1 \cdot P1)$, inclusion of the proton arm removed this ring effect, since the pion trajectories that would intercept the ring were not associated with protons that would hit the P1 counters.

For none of the other angles or energies did the ring or the vacuum vessel produce constraints of this kind. Thus, the average of these $\pi 2$ solid angles equaled the geometric value (see 4.23). Following the CH$_2$ target example, the single arm $\Delta \Omega_{\text{eff}}(\pi 2 \cdot \pi 1)$ results and the two arm $\Delta \Omega_{\text{eff}}(\pi 2 \cdot \pi 1 \cdot P1)$ results (except arm "A") were normalized to the average of the $\pi 2$ solid angles. Appendix F.3 illustrates just such an averaging.

Simulations using an empty target showed that the decreased pion multiple scattering had no effect on $\Delta \Omega_{\text{eff}}(\pi 2 \cdot \pi 1)$, however, in two arm mode, the decreased proton multiple scattering resulted in a significantly increased $\Delta \Omega_{\text{eff}}(\pi 2 \cdot P1)$ and $\Delta \Omega_{\text{eff}}(\pi 2 \cdot \pi 1 \cdot P1)$, especially for the most forward pion angle (i.e. lowest energy recoil protons). Backgrounds were very small in two arm mode (as discussed in detail in section 4.3.2), and so use of an empty target solid angle had a negligible effect on the background normalization factor $\kappa$ (see equation 4.62), hence on the cross sections. On the other hand, such was not the case for the single arm mode, where the backgrounds were

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1The vacuum vessel window gap shown in figure 3.9 was designed such that all pion trajectories that could result in a muon that could cause a $\pi 2 \cdot \pi 1$ coincidence would pass through the window and not the vessel wall.
much larger (see section 4.3.3). Here, although $\Delta \Omega_{\text{eff}}(\pi_2-\pi_1)$ did not differ between the full and empty target simulations, the different empty target solid angles due to the different hadronic interaction losses did have a significant effect on $\kappa$, hence the cross sections. Consequently empty target solid angles appropriately corrected for hadronic interaction losses were employed in the analysis. These hadronic losses are discussed in the next section (4.1.3).

Finally, it should be noted that the solid angle results from these simulations show that inclusion of the $\pi_1$ and/or P1 counters affected the geometric solid angle by no more than $\sim 9\%$ in the worst case, and more typically by only $\sim 5\%$. Thus the systematic uncertainty expected for these simulations should be smaller than about $1\%$ (i.e. $10\%$ uncertainty on the corrections). Additionally, the results of the systematic checks which are outlined in section 5.2.1 provide even tighter constraints on systematic errors involved in the experiment.

### 4.1.3 Hadronic Interaction Corrections

The simulations in the preceding sections included all the effects which could modify the geometric solid angle, except the cases where pions or protons that would have hit their respective counters interacted with a nucleus and produced products that were not detected. GEANT was found to be unreliable in simulating these pion and proton hadronic interaction losses accurately in the energy region encountered in the experiment. Consequently, a separate program was developed to determine these losses, which then could be applied to the GEANT results.

Pions and protons can interact with nuclei either elastically (i.e. two body reaction where the particle "bounces" off), inelastically (i.e. two body reaction where the nucleus is left in an excited state), or non-elastically (i.e. three or more body reaction

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2GEANT was designed as a simulation tool for high energy physics, consequently many of the approximations used in the hadronic interaction routines are not appropriate at low energies.
where the particle can scatter from a proton or neutron within a nucleus, or be absorbed and create other reaction products). The elastic cross sections are denoted $\sigma_{el}$, while the sum of the inelastic and non-elastic cross sections are denoted $\sigma_{\text{reactions}}$ (for "reaction"). The sum $\sigma_{el} + \sigma_{\text{reactions}}$ is called the total cross section, $\sigma_{\text{tot}}$. To correct for hadronic interaction losses, such corrections require not only the probability that a pion or proton would interact, but also whether the resulting reaction products could be registered in the counter arms in lieu of the incident particles (and satisfy the timing gate).

The target (and surrounding material in the case of LH$_2$) and the $\pi 1$ counter were the two regions where the large majority of hadronic interactions would occur. For such interactions, the $\pi 2$ and P1 counters severely restricted the solid angle into which a hadronic interaction product could go and still be detected. It was assumed that if a pion or proton interacted in- or non- elastically with a nucleus, then it would be considered "lost", as it was very unlikely the daughter products (which are spread over a broad angular range) would strike the appropriate counter. Although the probability that a pion or proton would scatter forward is considerably larger for elastic scattering (e.g. $\pi$-C, $\pi p$, pp, or p-C), it was still rather unlikely that they would be detected. However, for the case where the particles interacted in material near the $\pi 2$ or P1 counters, then the total loss would be somewhere between that calculated using the total cross sections for all the material between the interaction point and the counter under consideration, and that using the reaction cross sections only.

The probability of a particle interacting in material of number areal density $z$ (where the interaction cross section is $\sigma$ millibarn $^3$) is

$$\text{Interaction Probability} = 1 - e^{-z\sigma}$$

(4.52)

The total and reaction hadronic interaction losses for pions and protons was determined using a program which applied equation 4.52 starting at the centre of a target

$^3$1 mb $\equiv 10^{-27}$ cm$^2$. 
and traveling straight line paths to their respective counters. The total and reaction cross sections for pion and proton scattering on hydrogen and carbon were obtained from published results [76] and listed into tables for interpolation. Pion and proton cross sections on other nuclei (with atomic number $A$, e.g. oxygen $A=16$) were extrapolated from the carbon results using the rule-of-thumb: $\sigma(A)=\sigma(^{12}\text{C})\cdot\left(\frac{A}{12}\right)^{3}$. Most losses occurred on hydrogen and carbon, so this approximation was adequate. Even though the cross sections are energy dependent, the energy losses were not large in this experiment, so the interaction losses were calculated disregarding such energy losses.

The different path lengths through the targets for each arm were taken into account. Also, a correction was made for the fact that $\sim 5\%$ of the $\pi 2 \cdot \pi 1$ coincidences were due to muons, which do not suffer hadronic interactions at these energies.

The total and inelastic interaction losses were calculated for each different material encountered between the target centre and the $\pi 2$ and $P 1$ counters, summed separately, and then applied to the solid angles obtained from the GEANT simulations. Since the chance that a pion and its recoil proton both interacted hadronically was very small, the summed pion losses were simply added to the summed proton losses in the two arm case. For this case, the final solid angle was estimated to be: $\Delta \Omega_{2\text{arm}} = \Delta \Omega_{\text{tot}} - (\Delta \Omega_{\text{tot}} - \Delta \Omega_{\text{inel}})/4$, where $\Delta \Omega_{\text{tot}}$, $\Delta \Omega_{\text{inel}}$ was the solid angle calculated with total, inelastic cross section losses, respectively. This "educated guess" was a way of accounting for those elastically scattered pions and protons (and even fewer inelastically scattered ones) which would in fact be detected by the counters. In practice, the uncertainty introduced by this estimate was small. Typical final losses were 2–3%. The largest losses were for the forward–most pion angle, where the pion traverses the largest path length through the target, and where the proton cross sections were largest. Nonetheless, these losses were never more than $\sim 7\%$. An example output from this program can be found in appendix F.4.

For single arm runs, the expression for the final solid angle reads: $\Delta \Omega_{1\text{arm}} = \Delta \Omega_{\text{tot}}$.
(ΔΩ_{tot} - ΔΩ_{inel})/3$, where the difference with respect to the two arm calculation reflects the fact that there was some in/out cancellation of scattered pions suffering an additional elastic scattering in the target hydrogen. *i.e.* a scattered pion rescattering out of the pion arm acceptance could be compensated by a pion original traveling outside the acceptance, and then rescattering in. Again, this estimate introduced only a small additional uncertainty. Typical final losses were 2–3%.

### 4.2 Beam Intensity Determination

The main goal of this experiment was to provide data with precise normalization and reliably estimated normalization uncertainty. In effect, this meant a precise knowledge of the target proton densities and angles ($N_{prot}, \cos \theta_{tgt}$), and the number of pions which hit the target ($N_{\pi}$) and resulted in the measured πp yield $Y$. The target proton densities and angles were discussed in section 3.3.2; the latter term is detailed here.

The determination of $N_{\pi}$ follows from a product of six terms:

$$ N_{\pi} = B \cdot f_{LT} \cdot f_{\pi} \cdot f_{D} \cdot f_{L} \cdot f_{S} \quad (4.53) $$

where:

- $B =$ **BEAM** coincidences recorded by hardware scalers
- $f_{LT} =$ Data acquisition live time fraction (efficiency)
- $f_{\pi} =$ Pion fraction to Q6 (measured e.g. during phase restricted tune)
- $f_{D} =$ Pion survival fraction after decay from Q6 to target centre
- $f_{L} =$ Pion survival fraction after interaction losses to target centre
- $f_{S} =$ Multiple pions in target correction factor

**BEAM** (*i.e.* S1\cdot S2A\cdot S2B\cdot S2BH) was counted by two separate channels of the hardware scalers, and checked by an online visual scaler, and consequently was considered to be virtually error–free (see section 3.3.3). As mentioned in section 3.4.4, the
live time $f_{LT}$ was measured as the ratio of the hardware scalers LAM/EVENT \textit{i.e.} $\pi p$ events registered by the data acquisition system (and written to tape) versus those detected by the TOF Spectrometer electronics. These scalers were checked by online scalers, and so the uncertainty on this number was also expected to be negligible. The determinations of the other factors in equation 4.53 are very important and rather involved, and so each will be discussed separately in the following sections.

4.2.1 Beam Composition

The number of beam counts $B$ in equation 4.53 which actually resulted in a pion in the target was calculated in five steps. The first involved ensuring that there was no proton contamination in the $\pi^+$ beams; the second involved determination of the muon and electron contamination from pions decaying near the T1 production target; the third and fourth steps involved determination of the latter contamination in the channel from the region after the production target to the last quadrupole, and after the last quadrupole, respectively. The final step involved calculating pion interaction losses through the in-beam counters, air, and target. The third and fourth steps were necessary since the muon contamination originating within the channel or downstream of the last quadrupole could not be distinguished from pions. These two steps were performed with Monte Carlo simulations, and are discussed in detail in section 4.2.2. The procedures for dealing with the hadronic losses are presented in section 4.2.3. Proton removal will be covered in the following section, with the details of the second step (contamination of muons and electrons originating near T1) presented in the subsequent section.

Residual Proton Contamination

As mentioned in section 3.4.1, protons were removed from the BEAM definition using the midplane absorber and the S2BH upper level discriminator. However, at the
Identifying Residual Protons in Beam

worst case: 267.2 MeV $\pi^+$

Figure 4.24: Spectra used to check for proton contamination in the beam counts. At 267 and 241 MeV, some (higher energy) protons were not rejected by the midplane absorber/S2BH combination. The cuts shown define the proton contamination fraction used to correct $f_p$.

highest two energies, some protons managed to “leak” into the BEAM definition. The protons were easily identified by the TCAP SAMPLE spectrum and large pulse heights in all the in-beam counters (see figure 4.24). The corrections were never large (<0.5%), and so introduced only a small additional uncertainty to $f_p$ (<0.1%).

Muon and Electron Contamination from T1: Phase Restricted Beam

The muon and electron contamination of the M11 beam from pion decay near the region of the T1 production target T1 could be determined using the particle TOF to the S2B counter with respect to the cyclotron TCAP signal. At low energies, the length of the channel allowed all three components to be easily resolved during normal operation [15], but the normal 3ns width of the proton beam limits the clean separation of particles to pion energies of about 110 MeV. At the energies considered here, this timing width limitation meant that the muon (and at the highest energies even the
Chapter 4. Data Analysis

Figure 4.25: **left:** Normal time structure measured with respect to the TCAP probe exhibited during a 141 MeV $\pi^-$ run. The electrons are clearly separated, but the muons are totally obscured under the pion and electron peaks; **right:** TCAP time spectrum, but during a dedicated run using phase restricted primary proton beam on the pion production target. In this case the pions, muons, and electrons are clearly identified.
electron) contribution was indistinguishable from the dominant pion peak. Consequently, a series of runs with a reduced proton pulse width were dedicated to measuring the beam composition. Coupled with a fine-tuning of the in-beam scintillator CFD pole-zero timing adjustments, an ~1 nanosecond resolution (with a clean Gaussian distribution) was achieved in this "phase-restricted" tune operation, much narrower than the ~3ns double-Gaussian time structure seen during normal runs (refer to figure 4.25). This phase-restricted beam operation was performed twice during the experiment.

Runs were taken at all the energies explored in this experiment, using the same T1 production target and midplane absorbers employed in the data production runs. The midplane slits were nominally set to 9mm in the first series of measurements, and 36mm in the second, whereas the jaws were 90mm x 90mm for both series. Tests were run where the slit widths were varied between 9mm and 45mm. The electron contribution originated from pair production in the target of the gammas from π⁰ decay, which depended on the proton beam location on the target. Consequently, runs were also undertaken where the proton beam was purposefully steered on the T1 production target. Since only beam counter information was required, all pion and proton arms were removed from the EVENT definition.

The π:μ:e ratios were determined from Gaussian fits to the TCAP timing spectra. Except at the highest two energies, the three peaks could be clearly identified. At these highest energies, the muon peak was obscured by the tail of the pion distribution. However, by fixing the muon peak position using the expected π-μ and μ-e TOF difference at those momenta, robust fits were found. The electron component could be clearly separated at all energies. See figure 4.26 for some example spectra and fits.

The pion fraction results (fπ) from the two series are summarized in figure 4.27.

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4The beam current was reduced a factor of 100 during these runs, effectively turning off most other experiments taking place at TRIUMF, which was why these were undertaken only twice and for short periods.
Figure 4.26: Examples of Gaussian fits to TCAP-S2B TOF spectra taken during phase restricted beam operation designed to determine the pion fraction of the M11 beam. The location of the muon peaks at the highest energies (e.g. 267 MeV at right) were fixed using the expected $\pi-\mu$, $\mu-e$ TOF separations.
Immediately obvious is the difference in the $\pi^-$ results between the two series, at 140 MeV roughly 2% to 0.5%, and the constancy of the $\pi^+$ results between the series, differing by less than 0.2% at all energies. As explained in appendix C, REVMOC [98] pion and decay muon transport simulations of the M11 channel showed that the $\pi/\pi+\mu$ fraction varied with midplane slit setting, since the muons originated from a more distributed source at T1, and so were not focussed in the same way as the pions at the midplane: consequently, narrow slit settings favour the pions over the muons. This effect was expected to be larger for the electrons, since their source was even more distributed, and since they were a much larger fraction of the $\pi^-$ beam than are the muons. The results here appear to bear this out. There were many fewer muons and positrons in the $\pi^+$ beam, and so the effect there was much less apparent.

The REVMOC simulations also showed that the fractions depended on the geometry of the beam counters, since larger (smaller) counters would intercept a larger (smaller) fraction of the muons and electrons which form a "halo" around the pion beam. Because of this, these results cannot be compared directly to those of reference [99] where the phase restricted beam technique was also used. Nonetheless, the trends of the results shown in figure 4.27 are consistent with those of reference [99].

Tests where the proton beam was purposefully moved on the production target showed that these results were insensitive to all but the most severe maladjustments. The result of steering the M11 beam was monitored by the beam hodoscope. The largest effect was seen for an extreme left/right steering during $\pi^-$ where the pion fraction increased by 1%. Such a proton beam maladjustment would normally be corrected immediately by the cyclotron control room during higher current (normal) operation since the beam would trip radiation monitors downstream of T1.
Figure 4.27: Percentage of pions, muons, and electrons in the M11 beam defined by the in-beam scintillators as measured using phase restricted beam. The differences between the series are due in part to changing electron contamination from different midplane slit settings (see text). The relative $\pi/(\pi+\mu)$ fraction is constant ($\leq 0.2\%$ difference at 141 MeV) between the series. These fractions are insensitive to typical drifts of the proton beam location on the production target.
Muon and Electron Contamination from T1: Normal Beam

The $\pi^+p$ results in figure 4.27 show that the pion fraction is near 1 at all energies, and the results of the two series were consistent to $\sim \pm 0.1\%$. However, the $\pi^-p$ results show that while the pion fraction is still large ($>\sim 0.9$), there is a discrepancy between the two series (2% at 140 MeV decreasing to about 0.5% at 270 MeV). Consequently, multiple Gaussian "fits" were performed on the TCAP spectra during normal operation in order to decide which of the two series (if either) was correct, and to check whether large variations in the pion fraction occurred between runs.

Although the muons in the TCAP spectra were totally obscured by the pion peak during normal running, the electron peaks could be easily identified up to about 218 MeV. For these spectra, multiple Gaussian line shapes (usually two, but sometimes three were required) were fit to the pion peak, then the muon and pion peaks were predicted using the phase restricted beam results for the peak separations, peak width ratios, and relative abundances. The resulting "fit" was overlayed on the TCAP beam SAMPLE spectrum (see figure 4.28). An additional complication entered here: during normal beam operation, the timing spectra were not simply sums of Gaussian spectra, but instead had right hand "tails". These tails could be seen in the TCAP spectrum for single pion $\pi p$ events, which can be identified by a IIPI\text{VETO EVENT} cut, since only pions could yield a pion arm-proton arm coincidence. This spectrum was normalized to the SAMPLE spectrum to elucidate the magnitude of the pion tail under the muon and electron peaks (see figure 4.28). The electron peak predictions from the "quasi-fits" and the pion tail were used to estimate the actual electron fraction. Since the $\pi/($p+\mu$)$ ratio was found to vary less than 0.2% from the phase restricted beam results, $f_\pi$ could then be estimated. In all cases, the $\pi^-$ results were within the results of the two phase restricted beam series, whereas the $\pi^+$ results were within 0.2% of the phase restricted beam results. At the lower energies where the electron component could be reliably estimated, the result for $f_\pi$ extracted in the above manner was used
in the cross section calculations; at the higher energies, the phase restricted beam result was used. Since the $\pi^-$ results were always between the results of the two phase restricted beam series, the estimates at the higher energies were not expected to vary more than $\sim 0.5\%$ from those shown in figure 4.27. For $\pi^+ p$, the phase restricted beam results were used at all energies.

The precise nature of $f_\pi$ should be emphasized here. As shown in appendix C, muons from pion decay after the last quadrupole could not be distinguished from pions through TOF, so the "pion" fraction in the previous discussion actually contains the contribution from these decay muons. The fraction $f_\pi$ represents pion fraction at the $Q6$ quadrupole for the beam whose pions, muons, and electrons would eventually cause a BEAM coincidence. To get the fraction of BEAM coincidences that were pions in the target, $f_\pi$ must be corrected for pion decay in the channel and downstream of $Q6$ ($f_D$) and for hadronic interaction losses ($f_L$). Details of these corrections are presented next.

4.2.2 Pion Decay

Pion Decay downstream of $Q6$

Pion decay downstream of the last quadrupole could be calculated straightforwardly by Monte Carlo simulation, since the decay process is well understood and can be modeled exactly. The GEANT program was used to determine the factor $f_D$ which (neglecting interaction losses) represented the fraction of pions starting at the $Q6$ magnet midplane which caused a BEAM coincidence and subsequently hit the target. Since this factor was dependent on the beam counter geometry and beam phase space, the
Figure 4.28: Example of a beam sample TCAP timing spectrum taken during normal running. Two Gaussians are fit to the pion peaks, and then the muon and electron contributions are inferred using parameters obtained from the phase restricted beam runs. The pion "tail" is inferred from a TCAP spectrum subject to a $\pi$P-VETO (i.e. single pion) EVENT. These fits are used to determine $f_\pi$. 
simulations were repeated for every run using the beam phase space parameters measured during the channel tuning phase undertaken prior to the experiment (see section 3.5.1). Appendix F.2 presents an example GEANT output for such a beam simulation run.

The results for $f_D$ varied between about 0.961 at 141 MeV to 0.973 at 267 MeV, and were rather insensitive to the beam size. Considering the decay correction only up to the S2B counter, the pion survival fraction was 99.2%, virtually independent of energy and beam size. Consequently, $f_R$ actually contained about 0.8% muons. These muons were corrected for using $f_D$. The GEANT results for the pion decay to the S2B counter and to the target were checked using the REVMOC beam transport program, and the two simulations agreed to better than 0.1% at all energies (see appendix C).

**In-channel Pion Decay**

One source of muons not yet considered were those originating from pion decay within the channel. The muons in the "muon" peak shown in figure 4.26 had the maximum TOF difference with respect to the pions, and so were identified as those originating near the production target. The $\pi - \mu$ TOF difference for muons originating after Q6 was small compared to the timing resolution, and consequently those muons would appear under the "pion" peak. This implies that muons originating between T1 and Q6 would have a timing distribution spread between these two extremes. The results from the lower energy phase restricted beam fits (e.g. 169 MeV in figure 4.26) show that this contribution should have been rather smaller than the "muon from T1" contribution, since a "gap" existed between the pion and muon peaks. Intuitively, this contribution was expected to be small, since most muons emerged at an angle to the incident pion direction (the "Jacobi angle", varying from $\sim 9^0$ at 140 MeV to $6^0$ at 270)

\[5\text{The same phase space parameters were used for the simulations determining the effective solid angles (see section 4.1).}\]
with a momentum much smaller than the pion, so that these muons would either immediately strike the beam pipe walls or be bent away eventually by the magnets. Only that small fraction of muons which originated at a small angle to the incident pion direction would have a chance to survive a BEAM coincidence, since these muons would have nearly the same momentum as the pions and would start near enough to the beam axis that they could conceivably be transported through the channel and strike the BEAM counters. Because of these considerations, the contribution to the beam contamination from these muons was expected to be very small.

Nonetheless, a Monte Carlo simulation of the entire M11 beamline was undertaken using the REVMOC beam transport program, where pions and their decay muons are transported from the T1 production target through to the in-beam counters and scattering target. The details of this simulation are presented in appendix C. The results of these simulations confirmed that the contamination from muons originating between T1 and Q6 was small, with only about a 0.2±0.1% contribution not already accounted for by either $f_R$ or $f_D$. Therefore the correction $f_R \rightarrow 0.998f_R$ was included in the beam normalization factor for all runs.

### 4.2.3 Pion Beam Interaction Losses

The simulations discussed above neglected pion loss through hadronic interaction since the REVMOC and GEANT simulations of these interactions were not sufficiently reliable in this application. However, the factor $f_R$ already included hadronic loss effects to the S2B counter, since it was measured empirically e.g. in the phase restricted beam runs. Therefore, only the hadronic losses of pions in the target (and shields, windows, etc. in case of the LH2 target), in the air, and in the S2B counter (after a hit had been registered by the electronics) were required to be included “by hand” to the beam normalization factor.

The hadronic loss calculations were done using the same program used to correct
the solid angles for hadronic losses (see section 4.2.3 and appendix F.4). Since the targets were thin, the 'average' interaction loss in the target corresponded to the loss calculated to the target centre. All the pions which interacted in the air and in the various shields and windows surrounding the LH$_2$ target were assumed lost, since it was very unlikely that the pion would continue forward to the target. For the S2B counter, pion losses were considered for the tape on the downstream face of the counter, and for the final third of the scintillator. The latter estimate followed from the fact that pions which interacted beforehand either would not have generated a scintillation pulse large enough to satisfy the CFD threshold requirement, or the interaction products (e.g. protons) would have resulted in large pulse height signals that would have been vetoed via the S2BH upper-level discriminator.

The maximum total hadronic loss correction was $\sim$2.0%, for the case 168.8 MeV $\pi^+$ LH$_2$ target oriented at 60°. Consequently, the uncertainty introduced by the various assumptions and hadronic cross section estimates were small, even considering an uncertainty estimated as about 15% of the total correction (10% from the pion–nucleus cross section estimates and 10% from the assumptions made about the interaction losses).

4.2.4 Multiple Pion Correction

A BEAM coincidence registered by the beam scintillators did not signify that a single particle passed through all the counters, but instead signified that at least one particle did so. More than one pion with the correct kinematics to reach the TINA target location could be produced by a single (~3ns wide) proton pulse on the T1 production target. Consequently, more than one pion could be inside the target during the TOF Spectrometer electronics EVENT gate, thus multiplying the probability that a $\pi p$ interaction could occur. However, for such events, only a single BEAM hit is registered. Therefore the total beam counts $B$ had to be corrected for these multiple pion events,
or, these events had to be rejected outright from the analysis. These two approaches to calculate the single-pion beam correction $f_s$ are presented in the following sections.

**Poisson correction of Multiple Pions**

The first technique involved accepting all $\pi p$ scattering events, including those arising from multiple pions in the target, and then correcting the beam counts $B$ appropriately to compensate. Since the distribution of the number of pions in a single beam bucket followed the Poisson distribution, this was called the "Poisson" correction scheme.

The rate of at least one pion on target ($R_{tgt}$) during each beam burst was limited by the proton beam bucket rate on the $T1$ production target, namely $v=23.058 \text{ MHz}$. Note that $R_{tgt}^\pi$ was what would be measured by in-beam scintillators large enough to intercept the entire beam. If the distribution of the number of pions in a single beam bucket followed the Poisson distribution, then it follows that the probability of at least one pion per bucket is equal to (1 - probability of no pions) i.e. $P(1) = 1 - P(0) = 1 - e^{-\lambda}$, so that

$$R_{tgt}^\pi = v \cdot (1 - e^{-\lambda})$$

The total number of pions per second ($R_{tgt}^{tot}$) impinging on the target (including multiples) was

$$R_{tgt}^{tot} = v \cdot (1 \cdot P(1) + 2 \cdot P(2) + \cdots + n \cdot P(n) + \cdots)$$

$$= v \cdot \sum_{n=1}^{\infty} \left( n \cdot \frac{\lambda^n e^{-\lambda}}{n!} \right)$$

$$= v \cdot \lambda$$

so the Poisson parameter $\lambda = \ln(v/(v - R_{tgt}^\pi))$ represented the average number of pions in a beam bucket.
In words, the multiple pion correction factor is described as

$$ f_S^p = \frac{\text{Rate (or number) of pions hitting target AND at least one hitting BEAM}}{\text{Rate (or number) of pion BEAM coincidences}} $$

(4.54)

where the numerator follows from the requirement that at least one pion in the beam must hit the in-beam counters to generate an BEAM signal. If the beam counters intercepted all the pions that hit the target, then it follows that $f_S^p = \frac{R_{tgt}^{tot}}{R_{tgt}} = \lambda/(1-e^{-\lambda})$.

For an M11 beam rate of ~2 MHz, $f_S$ was ~1.04 i.e. a 4% correction. However, if the in-beam counters did not intercept the entire beam, as is the case in this experiment, then this expression for $f_S$ requires modification. The following heuristic argument demonstrates why this is so:

Consider 100 pion buckets that hit the target, 4 of which contained two pions i.e. a 4% multiple pion fraction, (96+4.2)/100 = 1.04. If the in-beam counters intercepted only 1/2 the beam hitting the target, then 48 of the one pion events would have registered BEAM coincidences, but 3 of the two pion events would have been registered, since the chance that at least one of the two pions would be registered is 3/4. Therefore, using expression 4.54, the actual correction is (48+3.2)/(48+3) = 1.059, a substantial increase over 1.04.

An expression including this effect and correct to all orders (i.e. 2, 3, ... pion events) was derived analytically [77]. Consider different Poisson parameters for that fraction of the M11 beam striking the target $\lambda_T$, and that fraction causing a BEAM coincidence $\lambda_B$. Define $f = (\lambda_B/\lambda_T)$, and $g = 1-f$, where $f$ is the probability that a pion in the full beam intercepted the in-beam counters, and $g$ is the probability that it did not. Then for the numerator in equation 4.54, consider $n$ pions in the full beam on target. The probability that at least one hit the BEAM counters is: $(1 - \text{all miss})$ i.e.

$$ P_n(\text{at least 1 in BEAM}) = (1 - g^n) \frac{\lambda_T^n}{n!} \cdot e^{-\lambda_T} $$

Therefore the rate was: $v \cdot \sum_n n \cdot P_n$, which, after a little bit of algebra, simplifies to
\[ \lambda_T(1 - ge^{-\lambda_B}). \] The denominator in equation 4.54 is clearly: \( v \cdot (1 - e^{-\lambda_B}) \), making the final result:

\[ f_S^p = \frac{\lambda_T(1 - ge^{-\lambda_B})}{1 - e^{-\lambda_B}} \tag{4.55} \]

where:

\[ \lambda_T = \ln \left( \frac{v}{v - R_{\text{tgt}}^\pi} \right) \]
\[ \lambda_B = \ln \left( \frac{v}{v - R_{\text{beam}}^\pi} \right) \]

\( R_{\text{tgt}}^\pi \) = Pion rate on target

\( R_{\text{beam}}^\pi \) = BEAM pion rate hitting target

\[ g = 1 - \frac{\lambda_B}{\lambda_T} \]

Using the results from section 4.2.1, the rate of pions hitting both the BEAM counters and the target \( R_{\text{beam}}^\pi \) was simply \( R_{\text{beam}}^{\text{hit}} \cdot f_x \cdot f_D \cdot f_L \). However, determination of the target pion rate was somewhat more involved, since it had to be inferred by correcting "backward" from the observed VETO counter rate.

The VETO counter was constructed and positioned to intercept the entire M11 beam (in the idealized case of no decay and interaction losses). To obtain the target pion rate from the observed VETO hit rate, an upward correction for pions interacting after the target, or decaying and the muon missing the VETO, is expected, as is a downward correction for VETO hits which were not pions in the target (e.g. electrons). A Monte Carlo simulation was required to: **a.** calculate these decay corrections, and **b.** determine the pion fraction for the full beam striking the target. The target pion fraction was not equal to \( f_x \cdot f_D \cdot f_L \) as might be expected since the target area was much larger than that of the beam counters, so it intercepted a larger fraction of the muon and electron "halo" surrounding the pions than did the in-beam counters (refer to appendix C). Consequently the pion fraction and decay corrections were different from
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$f_\pi$ and $f_D$ respectively. This was confirmed by a REVMOC simulation comparing the pion fraction at the target with and without including the in-beam counters as beam constraints (see figure C.56).

These simulations showed that for $\pi^+$, where the electron contamination was very low, $R^\pi_{tgt} \approx (0.97) \cdot R_{\text{VETO}}$ for all energies to a good approximation ($<0.3\%$). These simulation results were confirmed by runs in Experiment 645 using a 15cm diameter (i.e. same as the LH$_2$ target) scintillator target for two different beam intensities, $R_{\text{beam}} = 1$ and 2 MHz.

For $\pi^-$, the electron contamination was much larger than for $\pi_\nu$, and so a modification of the REVMOC code was required. In this case, electron transport through the M11 channel was modeled in separate runs, where the relative intensities of the electrons with respect to the pions and muons were adjusted to reproduce the known electron fraction measured during the phase restricted beam runs (see figure 4.27). Results for the pion rate on target varied from about $R^\pi_{tgt} \approx (0.85) \cdot R_{\text{VETO}}$ at 140 MeV to about $R^\pi_{tgt} \approx (0.95) \cdot R_{\text{VETO}}$ at 267 MeV.

Fortunately, the multiple pion correction $f^\pi_G$ was rather insensitive to these approximations at the beam intensities used for the data runs (i.e. $R_{\text{beam}} \approx 1$ MHz, $R_{tgt} \approx 1.5-2.0$ MHz). At these intensities, a $\pm 5\%$ variation in $R^\pi_{tgt}$ corresponded to $\delta f^\pi_G \approx \pm 0.4\%$, which was small compared to the typical $0.8\%$ pion fraction uncertainties associated with $\pi^-$ beams.

The assumption that the number of pions in a single beam bucket followed a Poisson distribution was confirmed experimentally. Circuits were constructed to detect BEAM hits in 2 and 3 consecutive beam buckets after an initial BEAM event. Using Poisson statistics, the probability of at least one hit in $m$ consecutive beam buckets after an initial BEAM event is $(1 - e^{-\lambda})^m$, so that $\lambda_{\text{exp}} = -\ln(1 - \frac{N_n}{N_{n-1}})$, where $N_n$ is the number of hits in the $n^{th}$ consecutive beam bucket. If the number of pions per bucket did indeed follow the Poisson distribution, then $\frac{N_n}{N_i} = \frac{N_i}{N_j}$, and $\lambda_{\text{exp}} = \lambda_{\text{beam}}$. In practice, these relationships
Comparison of Experimentally Determined vs. Calculated Poisson Parameter $\lambda$

$\lambda = -\ln(1 - \frac{N_2}{N_1}) = 0.042$

$\lambda = \ln\left(\frac{\nu}{\nu - R_{beam}}\right) = 0.039$

$N_1(1-e^{-\lambda})^n$

Figure 4.29: Number of BEAM coincidences registered by the scalers, and the number of BEAM coincidences registered for 2 and 3 consecutive beam bursts (43ns apart). The solid line shows the Poisson parameter $\lambda$ inferred from ratios between these scalers, whereas the dashed line is the prediction using the measured BEAM coincidence rate (see text). In practice, the latter value was used to determine the single pion fraction of the beam.
were found to be always true in this experiment. Figure 4.29 shows an example from the run outlined in appendix F.

**VETO correction of Multiple Pions**

The chance of two pions in a beam burst *both* interacting in the target is exceedingly small, so if one pion caused a πp event, then the other would through the target towards the VETO counter. Therefore, instead of correcting the accumulated BEAM hits for the multiple pion events, these multiple pion events could be *rejected* by detecting the extra (second, etc.) pion with the VETO counter at the time when a πp event was detected by the TOF spectrometer. In this analysis, these events were eliminated from both the πp yields (see sections 4.3.2 and 4.3.3) and the accumulated BEAM hits B. VETO hits were easily identified by a plot of its pulse height versus timing spectra, subject to the constraint of a ππ EVENT, as shown in figure 4.30.

With the multiple pion events removed from the measured yields, the number of BEAM hits had be corrected to compensate. Using some of the notation from the analysis in the previous section, the multiple pion "veto" correction factor $f'_S$ was:

$$f'_S = \frac{\text{BEAM hits with only 1 pion in bucket}}{\text{measured BEAM}}$$

(4.56)

$$= \frac{f \cdot (\lambda e^{-\lambda})}{1 - e^{-\lambda}}$$

(4.57)

where as before $f$ is the fraction of the total M11 beam on the target intercepted by the in-beam scintillators, and $\lambda$ is the Poisson parameter appropriate to the full beam on target.

The fraction $f$ and parameter $\lambda$ must be expressed in terms of the VETO counter hits and misses. As for the Poisson correction, the fact that the BEAM rate was different than the target rate complicated the calculation of the VETO beam correction factor. The details of the derivation for $f'_S$ are deferred to appendix G, while only the
Figure 4.30: VETO pulse height vs. timing dot plot subject to a ΠΠ coincidence event in any arm of the TOF spectrometer. The box identifies VETO hits, signifying events with more than one pion in a beam bucket. The VETO hits at the lower right occurred one beam burst (43ns) after the ΠΠ EVENT, and were badly out of time with the ADC gate, so only a small portion of the scintillator pulse was measured.
result is quoted here:

\[ f_S' = K_V \cdot \left( \frac{1}{1 + K_V (1 - e^{\lambda_V})} \right) \cdot \lambda_V \]  

(4.58)

where:

\[ K_V = \frac{V_M}{V_H} \]

\[ \lambda_V = \ln\left( \left(1 + K_V^{-1}\right) \left(\frac{f}{1 - ge^{-\lambda_V}}\right) \right) \]

\[ V_M = \text{VETO misses subject to PIP EVENT} \]

\[ V_H = \text{VETO hits subject to PIP EVENT} \]

\[ f = 1 - g = \frac{\lambda_B}{\lambda_V} \]

\[ \lambda_B = \ln\left( \frac{v}{v - R_{\text{beam}}} \right) \]

In this expression, \( \lambda_V \) is defined recursively. Consequently, in practice, the approximation using \( g=0 \), \( \lambda_0 = \ln(1 + K_V^{-1}) \) was put into the expression for \( \lambda_V \), and then the expression was iterated until the series converged. The final value was then input into equation 4.58, thus ensuring that the equation was self-consistent. One could also have used \( \lambda_T \) from equation 4.55 as initial input into the iteration. In practice, \( f_S' \) was insensitive to (reasonable) initial values of \( \lambda_0 = \), and in fact differed little from the expression using \( \lambda_T \) as input without iteration, since \( \lambda_T \) should have equaled \( \lambda_V \) if the Poisson and VETO multiple pion correction schemes were to give consistent cross-sections (see results in section 5.1).

When the target pion rate equals the beam pion rate, \( R_B^p = R_T^p \) (\( i.e. f=1, g=0 \)), the above expression simplifies to:

\[ f_S'(g = 0) = K_V \cdot \ln\left(1 + K_V^{-1}\right) \]
where for $K^{-1}_v$ small:

$$
\approx K_v \left( K^{-1}_v - \frac{1}{2} (K^{-1}_v)^2 \right)
$$

$$
= 1 - \frac{1}{2} \left( \frac{V_H}{V_M} \right)
$$

$$
\approx \frac{1}{1 + \frac{1}{2} \left( \frac{V_H}{V_M} \right)}
$$

which is an expression that has been used in the past on the pion channels at TRIUMF for VETO doubles correction (see e.g. [71]). The latter expression is appropriate in the limit of low beam rates and large in-beam counters. Nonetheless, it turns out that this approximation is a good one at the rates typical to this experiment. For the example shown in appendix F.5, with a 1.4 MHz pion rate on target, the VETO correction was 0.954, whereas it would be 0.953 using the above approximation.

The discussion above ignored the effect of pion decay after the pion passed through the target, resulting in a muon which does not strike the VETO, so that the extra pion is inadvertently not rejected in the analysis. It turned out that this effect could be ignored, since the effect contributes to two effects which canceled to a good approximation. See section 4.3.4 for a discussion of this effect.

### 4.3 Yield Extraction

The $\pi p$ scattering yield at a particular pion scattering angle (i.e. for a particular TOF spectrometer arm) $Y(T, \theta_{cm})$ corresponding to a given number of incident pions in the target $N_x$ was identified by the $\pi 2-P1$ TOF difference in the two arm coincidence case, or by the $\pi 2$ TOF in the single arm case. Both cases are presented in detail
in the following sections. The pulse height and timing signals of the TOF spectrometer were multiplexed into single π1, π2, P1 ADC or TDC data words, respectively, by the data acquisition system (see section 3.4.4). The C212 pattern buffer recorded which pion-proton arm pair (or just the pion arm for single arm runs) detected the πp event. A logical AND of the appropriate C212 bit pattern with the scintillator signals de-multiplexed the ADC and TDC signals of the six arms for the off line data analysis. The logical OR of the C212 channels identified a πp event in any of the TOF spectrometer arms.

This scheme was also used to gain a measure of the rate of random accidental coincidences i.e. ΠP1 (two arm) or Π1 (single arm) coincidences not caused by a beam pion. The logical exclusive or (XOR) of the six C212 buffer patterns would not equal the logical or (OR) if more than one arm registered a coincidence during a single BEAM event identified by the in-beam scintillators. Such events could be caused by background radiation in the experimental area, or possibly by another particle from a three-body reaction on non-hydrogen nuclei in the target. The fact that there was almost never any "multiple arm" coincidences (<0.05%) indicated that the accidental background was totally negligible, and consequently this source of background is not considered in the discussion that follows.

4.3.1 Foreground/Background Normalization

The "pure" or net πp scattering yield \( Y(T,\rho) \) in equation 4.50 was defined as the yield resulting from the "foreground" target (CH\(_2\) or LH\(_2\)) minus an appropriately normalized yield from the "background" target (carbon or empty LH\(_2\) target cell). The foreground yield contained contributions from both πp scattering on the hydrogen target, and all other pion scattering reactions on the surrounding material which managed to satisfy the kinematical and geometrical constraints of the TOF spectrometer. The yields from

\[ \text{\footnotesize\cite{footnote}} \]

6The chance of two (or more) \( \pi p \) events occurring in a single beam burst was extremely unlikely, even for high beam rates with a large multiple-pion fraction.
the foreground and background runs are expressed as:

\[ Y_{fg} = Y + Y_{back}^{fg} \]

\[ = K_{fg} \cdot (N_{\text{prot}} \cdot \sigma_{\pi p} + N_{\text{back}} \cdot \sigma_{\text{back}}) \]  

(4.59)

and:

\[ Y_{back}^{bg} = K_{bg} \cdot N_{\text{back}} \cdot \sigma_{\text{back}} \]  

(4.60)

Here \( K_{fg, bg} = \frac{N_b - \Delta \Omega}{\cos \theta_{fg}} \), \( \sigma_{\text{back}}^{fg, bg} \) are the “generic” cross section for all the background processes, \( Y_{back}^{fg, bg} \) are the background yields, and \( N_{fg, back} \) are the background nuclei densities for the foreground and background runs, respectively.

In this experiment, \( \sigma_{back}^{fg, bg} = \sigma_{back}^{bg} \) to an excellent approximation since the background processes (all three–or–more body) which contributed to the background vary much more slowly with incident pion energy than \( \pi p \) elastic scattering, and the incident pion kinetic energies in the foreground and background runs were nearly equal (<0.5 MeV difference). Solving equation 4.60 for \( \sigma_{back}^{bg} \) and substituting into equation 4.59, one obtains (after some algebra):

\[ Y_{fg} = Y + \left( \frac{K_{fg}}{K_{bg}} \cdot \frac{N_{fg, back}}{N_{back}} \right)Y_{back}^{bg} \]  

(4.61)

consequently, substituting for \( K \), the net yield is:

\[ Y = Y_{fg} - \left( \frac{N_{fg, back}}{N_{fg}} \cdot \frac{\Delta \Omega_{fg}}{\Delta \Omega_{bg}} \cdot \frac{\cos \theta_{fg, tgt}}{\cos \theta_{back, tgt}} \cdot \frac{N_{back}^{fg}}{N_{back}^{bg}} \right)Y_{back}^{bg} \]  

(4.62)

\[ = Y_{fg} - \kappa \cdot Y_{back}^{bg} \]

Apart from the obvious target and beam dependencies, the foreground/background normalization factor \( \kappa(T_\pi, \theta_\pi) \) in general depended on the pion angle. This dependence was via the effective solid angle, \( \Delta \Omega_{eff} \), which differed between the foreground and background targets due mostly to different proton multiple scattering and/or pion and proton hadronic interactions in the target cell (see section 4.1). The differences
in the foreground and background solid angles for the two arm coincidence setups could be reliably determined by the Monte Carlo simulation; however, this ratio had to be determined empirically from the observed proton background in the single arm cases (see sections 4.3.3 and 5.1.1). Also, a different value of $\kappa$ was used in the cross section calculations depending on which multiple pion correction scheme (Poisson or VETO) was used to correct the beam (see sections 4.2.4 and 4.3.2).

4.3.2 Two Arm Coincidence

As outlined in section 3.4.2, candidate $\pi p$ scattering events using the two arm coincidence technique were identified by the $\pi 2$-P1 (pion–proton) TOF difference. The proton TOF to the P1 counters was measured in two ways: via the meantimer signal, or by taking the average of the signals from the top and bottom phototubes. In practice, both resulted in identical yields, and so only the meantimer signal was used. The foreground TOF difference peaks were narrow Gaussians with $\sigma \sim 300$ps. Most of the observed background under the foreground peak stemmed from pion quasi-elastic scattering off protons bound in heavier nuclei (mostly carbon) in the target region. This background was almost negligible at the smaller angles at 141 MeV, and reached the maximum level of about 7% at the backward angles for $218$ MeV $\pi^+ p$ using a CH$_2$ target (illustrated in figure 4.31).

The $\pi p$ scattering yield is derived in software from wide (several $\sigma$) gates placed around the net (after background subtraction) spectra (e.g. figure 4.31). Foreground $\pi 2$ and P1 counter pulse height and timing signals are shown in figure 4.32 for the same pion–proton arm combination as in 4.31, as well as their relationship to the TOF difference. These plots demonstrate that no gates other than the TOF difference gate were required to identify $\pi p$ scattering unambiguously, due to the very low background levels.
Figure 4.31: Foreground, normalized background, and subtracted $\pi p$ TOF difference spectra for the pion-proton arm pair with the worst signal-to-noise ratio for all the two arm runs. A typical software yield-defining gate is shown by the vertical dashed lines.
Figure 4.32: **top 4:** Typical foreground π2 and P1 counter timing and pulse height spectra. These are for the same configuration as figure 4.31. **bottom 4:** Relationship of the above spectra to the π2–P1 TOF difference. Virtually all the events outside the TOF gate shown are removed by the background subtraction.
4.3.3 Single Arm

For the single arm runs, only $\pi^+ p$ data were analyzed, for the reasons outlined in section 3.4.3. Candidate $\pi^+ p$ scattering events were identified by the TOF to the $\pi 2$ counter with respect to the BEAM coincidence. The background materials in the beam, mainly carbon, were a more serious problem in this case. Without the proton arm coincidences to discriminate against $\pi^+ A \rightarrow \pi^+ p + X$ quasi two-body events, pion elastic scattering and absorption on carbon contributed much larger background levels than in the two arm runs. The timing resolution was good enough to separate pions from all but the fastest absorption protons, but $\pi^+ p$ and the elastically and quasi-elastically scattered pions on carbon had very similar velocities and so could not be separated except via a background spectrum subtraction. Refer to figure 4.33 for an example of a $\pi 2$ timing spectrum at the most forward angle (20°).

As for the two arm case, the foreground/background normalization factor can be determined using equation 4.62. However, in the single arm setup, the pions scattering in the full LH$_2$ target can rescatter in the surrounding target material (e.g. vacuum vessel, target ring) and the resulting final state pions or protons can be detected by the pion arms$.^7$ This serves to enhance the pion arm acceptance when the target is full relative to when it is empty. In principle, the added acceptance due to pion rescattering can be determined via Monte Carlo simulation. However, as mentioned in section 4.1.3, GEANT was found to be unreliable in simulating pion hadronic interactions at the energies found in this experiment. The normalization technique adopted for the single arm runs was to count the proton background with boxes on the $\pi 2$ pulse height versus TOF spectra when the target was full and when it was empty (see figure 5.34) and using the ratio as the normalization factor. This is the same technique that was used by Brack, et al., in reference [16]. The uncertainty in the net yield arising from

$^7$In the two arm setup, the proton arms constrain the scattered pions to trajectories exiting the vacuum vessel within the exit window, so such pion rescattering does not occur in this case.
Figure 4.33: **top:** Example π2 timing spectrum and software gate used to extract πp yield from single arm runs. The background level was maximal here, decreasing with larger angle. A small 1.5 bin shift was applied to the background spectrum before subtraction. **bottom:** π2 foreground pulse height vs. timing spectrum, showing clear separation of pions and protons. The πp yield was defined using the gate shown.
the uncertainty in the normalization factor (from the counting statistics) was added in quadrature to the other statistical uncertainties. There is the question of whether the "extra" pion contribution from rescattering equals the "extra" proton contribution. It was found from a dedicated run measuring single arm and coincidence yields simultaneously that in fact these two contributions were equal at the level of the statistical accuracy in the measurement. This measurement is discussed in more detail in section 5.1.1.

Despite the sizable backgrounds, the foreground–background subtraction resulted in a "clean" $\pi^+p$ yield peak on a negligible residual background. Due to the smaller energy loss of the pions in the background (emptied of LH$_2$) target, and timing drifts in the electronics, the background peaks shifted slightly (1 or 2 bins, corresponding to 50 to 100ps) with respect to the foreground peak. The background peaks were therefore shifted before the subtraction was performed. In practice, the shift made no statistical difference to the yields. The extracted net yields were defined by a software gate placed around the $\pi^+p$ peak. Slightly wider/narrower gates were used to check the sensitivity to the cut placement, and any differences were included in the overall statistical uncertainty (refer to section 5.2.1).

### 4.3.4 Yields and Multiple Pion Correction

In extracting the yields, the case had to be considered where there were two or more pions were present in the target during the same BEAM event gate, thereby doubling (tripling, etc.) the likelihood of a $\pi p$ scattering event. The two methods (Poisson and VETO) employed to correct the incident BEAM counts ($N_i$) were detailed in section 4.2.4.

In the Poisson correction scheme, the number of incident pions detected by the in–beam counters was increased by the factor in equation 4.55 to reflect the fact that some fraction of the time (following Poisson statistics) there will be more than one pion
in the target during a single beam burst. In this scheme, the $\pi_2$-P1 TOF difference spectra includes contributions from both single and multiple pion BEAM events, so no additional constraints were applied to the yield spectra. The “extra” pions were dealt with in both the correction to $N_\pi$, and the fact that such events were included in the yields, resulting in a cross-section 4.50 that was properly normalized.

In the VETO correction scheme, a multiple pion event was identified by a particle hitting the VETO paddle in the same beam burst that a $\pi\Pi$ coincidence was detected in one of the TOF spectrometer arms. These events were easily identified by plotting the VETO pulse height versus the timing signal subject to a $\pi\Pi$ EVENT as shown in figure 4.30, and identifying the VETO hits by a software box cut. Events which satisfied this cut were then removed from the $\pi_2$-P1 TOF difference spectra, resulting in a yield corresponding to only single-pion BEAM events. Correcting the resulting incident beam $N_\pi$ for these rejected events using equation 4.58 resulted in a cross-section (equation 4.50) corresponding to only single-pion beam events.

Two special cases need to be considered in the VETO correction scheme: the case where an event was VETO-ed but shouldn't have been, and vice versa.

In the first case, the “extra” particle(s) in the beam bursts could have been muons or electrons, which passed through the target and hit the VETO counter, while a pion in the same burst caused a $\pi\Pi$ event. These $\pi\pi$ events would have been rejected using the VETO cut, but since the incident BEAM event was also corrected for such events, the effect cancels, the net result being simply a loss of statistics.

In the second case, there could have been two (or more) pions in the target in a beam burst, where one caused a $\pi\Pi$ event, and the other continued en route to the VETO paddle, but interacted (e.g. decayed) prior to reaching counter with interaction products (e.g. decay muon) escaping detection by the VETO. Here, the events should have been rejected but were not. However, since these events were included in both
$N_x$ and the yields, the effect again cancels. Results from GEANT simulations (see section 4.2.4) showed only $\sim 6\%$ of the beam pions in the target did not cause a VETO hit by either the pion or its decay muon, so in any case the effect was quite small. Consequently, neither of the above scenarios present any problems to the extraction of correctly normalized $\pi p$ yields when using the VETO multiple pion correction scheme.
Chapter 5

Results

The results of analyzing the data using the techniques outlined in the preceding two chapters are presented here in three steps. First are the results of the runs designed to elucidate sources of systematic error, second is the uncertainty analysis, which takes into account the results of the previous section, and finally the tables and plots of the final results. Conclusions based on these results are presented in chapter 6.

5.1 Systematic Checks

5.1.1 Single Arm versus Coincidence

For one run at 169 MeV $\pi^+$ using the normal two arm LH$_2$ setup, the proton arms were removed from the EVENT coincidence, putting the system in single arm pion detection mode, but the proton arm information was still written to tape. This enabled both single arm yields and two arm yields to be obtained simultaneously. Comparison of the single arm and two arm cross sections showed that there was a significant pion rescattering effect when the target was full (section 5.1.1). The bottom graph in figure 5.34 shows the ratio of the single arm to the two arm cross sections when the foreground/background normalization factor for the single arm results was calculated using equation 4.62 (not including the effects of pion rescattering in the solid angle), and from the ratio of the counts in the proton background boxes shown at the top of the figure. The two arm results were normalized using equation 4.62 in both
cases. The error bars in the empirically normalized results reflect the uncertainty in the normalization factor arising from the counting statistics in the boxes \(^1\). Clearly, normalizing the results empirically using the proton background boxes adequately accounts for the pion rescattering effect in the full target run, even for effects as large as 15–20\% (±5\%).

Consequently, the foreground/background normalization was determined empirically from the measured proton background boxes for all single arm forward angle runs. Corrections to the naive "beam" normalizations (i.e. equation 4.62) were typically 5–10\%, depending on angle. The corresponding changes to the cross sections varied from about 0.5–2.5\%, depending also on the amount of background.

It is likely that the number of "extra" pions is not exactly equal to the number of "extra" protons from the pion rescattering. The results in figure 5.34 imply that the difference was less than roughly 10–30\%. For all but the most forward angle, factoring in an additional 20\% uncertainty in the relative "extra" pion/proton abundances increased the uncertainty in the cross sections by amounts less than the statistical uncertainties in all cases. However, due to the very large background fraction in the most forward (20°) arms (roughly 50\%), the cross sections were very sensitive to the normalization factor, and the systematic uncertainty in the relative "extra" pion and proton abundances introduced uncertainties that were large compared to the other statistical uncertainties. Therefore, for this angle only, the approximation to the "true" normalization from the empirical normalization method outlined above was deemed invalid, and the data at this angle (at each energy) were removed from the final data set.

It should be emphasized that the pion rescattering effect on the foreground solid angle could in principle be determined by Monte Carlo simulation, so that the foreground/background normalization in the single arm runs could be calculated as for

\(^1\)Since the single arm and two arm results shown were obtained from the same data set, all other uncertainties (statistical and systematic) cancel to a substantial extent.
Figure 5.34: top: π2 pulse height versus TOF spectra with the LH$_2$ target full and empty for a single arm run where pion–proton coincidence data was obtained simultaneously. The boxes identify the proton background, where the ratio of the counts in the full and empty target boxes determines the foreground/background normalization factor. bottom: The ratios of the single arm to the two arm cross sections for the above run. The solid circles are the results using the “box” normalization, whereas the open boxes show the results using “beam” normalization (equation 4.62) where the effect of pion rescattering in the vacuum vessel, etc., was not included in the solid angles. The error bars reflect the ±5% uncertainty in the “box” normalization, whereas the underlined percentages show the differences in the two sets of normalization factors.
the two arm runs using equation 4.62. With the GEANT Monte Carlo program continually being updated, the possibility of calculating these corrections should be explored if the program's treatment of pion hadronic interactions is found to be satisfactory.

5.1.2 Beam Rate Test

Data were obtained for the 168.8 MeV π⁺p coincidence setup using the LH₂ target at five pion beam rates on target: 0.34, 0.87, 1.4, 3.1, and 6.7 MHz i.e. two lower and two higher than the ~1.4 MHz rate used in most of the π⁺p production runs. One empty target run with a 1.5 MHz rate was used for background subtraction in all cases. Since the beam rate was adjusted using the front end jaws of the M11 channel, the pion beam size on target also changed. However, GEANT simulations (discussed in section 4.1) showed that the effect on the solid angle was less than 0.2%. Consequently, only the solid angles corresponding to the "normal" beam size were used in all cases.

Corrections for multiple pions in the target during a single EVENT gate were made using both the VETO and Poisson correction schemes outlined in section 4.2.4. The cross section results are displayed in figure 5.35. The beam correction factors varied from 1% at 0.34 MHz to 4% at 1.4 MHz and to 26% at 6.7 MHz. The cross sections corrected by the VETO and Poisson schemes differed by on average 0.3% at 6.7 MHz, and smaller at the lower rates. As is evident from this figure, there is no monotonic systematic variation within the statistical uncertainties (~1.3%) in the cross sections over this range. For all production π⁺p runs in coincidence mode, the cross sections corrected by the Poisson and VETO schemes never differed by more than 0.3%.

However, this is not the case in the single arm π⁺p runs, where a much larger variation in the cross sections calculated with the two correction schemes was observed. The Poisson correction factors for these runs were the same as those from the coincidence setups at the same beam rate and energy, whereas the VETO correction factors were considerably larger, especially in the target empty runs. e.g. at
Figure 5.35: 168.8 MeV $\pi^+$ laboratory cross sections for the coincidence configuration using the LH$_2$ target for a range of pion rates on target. The uncertainties shown are purely statistical. The correction used to get the fraction of BEAM events with only a single pion ($f_s$, equation 4.53) varied from 1% at 0.3 MHz to 26% at 7 MHz. The results using the “VETO” and “Poisson” correction schemes differed by at most 0.3% at the highest rate, and progressively smaller at lower rates. The solid double arrow signifies the rate set during production runs.
168.8 MeV $\pi^+ p$, with a target pion rate of 1.53 MHz, the Poisson correction factor was $f^P = 1.047$, while the VETO correction was $f^V = 0.928$. For a coincidence run at the same rate, the VETO correction was 0.950. For the empty target run with a 1.85 MHz target pion rate, $f^P = 1.059$, and $f^V = 0.876$, a much larger correction than expected in terms of the modest increase in beam rate over the foreground run. The average discrepancy between the Poisson and VETO corrected cross sections is about 0.7% in all the single arm runs (141.1, 168.8, 218.1 MeV), but a systematic dependence with angle was observed, with the VETO-corrected cross sections much smaller than the Poisson corrected results at $20^0$ (4% at 218 MeV, 2% for the others), equal or slightly smaller at $30^0$, and then larger by about the same amount at the larger angles (2.5% at 218 MeV, 1.5%-2% for the others).

For a given beam rate, a reason why the multiple pion fraction would change would be if the target was smaller than the beam, in which case different targets/angles would yield a different target pion rate. However, the LH$_2$ (and CH$_2$) targets used here were many times larger than the beam size, and so there should have been no effect on the multiple pion fraction. Since the Poisson scheme relied solely on the measured beam rates, the resulting correction should not change with the target (for a given rate) as indeed is observed. This points to the VETO scheme as the source of the discrepancy.

One potential source of spurious VETO hits arises from energetic electrons ("deltas") which were knocked forward out of the target or preceding material by a pion which subsequently caused a detected $\pi p$ event. However, the maximum number of $\delta$ electrons which could reach the VETO counter is around 1% of the scattered pion yield in the worst case $^2$ (i.e. 218 MeV) and independent with angle. The fraction decreases depending on how much of the electron pulse-height spectrum was rejected by the VETO discriminator threshold, and how many miss the VETO acceptance. Since in

$^2$This estimation uses the formula for $\delta$ electron production found in the Particle Data Booklet [84] and also takes into account the electron energy loss to the VETO counter.
all coincidence runs the VETO and Poisson corrected results agreed to better than 0.3%, it appears that this effect was indeed small.

The most likely explanation is associated with pion reactions on the background nuclei (mostly carbon) which result in more than one charged particle in the final state, with one of them a pion detected by the TOF Spectrometer, and one of the others a particle detected by the VETO counter. The fact that the most forward angles are associated with largest background is consistent with the most forward angle yielding the largest discrepancy. This also explains the abnormally large correction in the empty target runs. In section 4.3.4, it was argued that it doesn’t matter that an event is vetoed/not vetoed "by accident", since the correction applied to the beam is canceled by the fact that the event was/wasn’t rejected from the scattering yield. So even under these circumstances, the Poisson and VETO corrected cross sections agree. However, the multiple pion correction was applied as a beam normalization i.e. a correction averaged over all the angles. Consequently, if all the arms were not equally likely to be vetoed, those angles inducing a large correction to the beam would get a smaller correction than they should, whereas the other angles would receive a larger correction. The fact that the average deviations of the Poisson and VETO results were reasonably similar and that the observed systematic angular dependence of the deviations with angle supports this explanation as well. Consequently, only the Poisson correction scheme was used with the single arm results.

Note that this effect was not a concern for the coincidence experiments, since the backgrounds were very much lower, and the pion arm–proton arm coincidence requirement severely restricted the phase space available for background reactions to yield another charged particle that goes forward and hits the VETO counter.
5.1.3 CH$_2$ Thickness Test

Data using the coincidence setup at 168.8 MeV $\pi^+$ were accumulated for CH$_2$ target thicknesses of $\sim$0.5mm ("A"), 2.0mm ("D"), 3.2mm ("E"), and 5.2mm ("D"+$\pi$) $^3$. These data provided a check for the target proton density uncertainty (1%), the target angle uncertainty (estimated as 0.2°, corresponding to $\pm$0.5% in $\cos\theta_{tg}$), the incident beam normalization via hadronic interaction losses in the target, and the solid angle determination, via the varying scattered particle multiple scattering and hadronic interaction losses. These data were obtained at various stages during the experiment, separated by as much as two weeks.

The cross sections for each of the targets are shown in figure 5.36. The error bars for the cross sections shown (typically $\pm$1.5%) are purely statistical. Also shown are the calculated solid angles for each target, plotted as a ratio to the $\pi^2$ solid angles determined from the GEANT simulations. In this ratio, the statistical error of the simulation virtually cancels since the number of $\pi^2$ counter hits are almost perfectly correlated with the number of $\pi^2\cdot P1$ coincidence hits. The error bars on the solid angle ratios in figure 5.36 reflect only the uncertainties arising from the hadronic loss corrections. The beam pion losses vary from 1.2% for target A to 2.4% for target E+D, while the solid angles between these targets differ between 6.5% and 1.5% at the extremal angles. The results are completely consistent within the relative normalization uncertainties: e.g. $\sim$1.7% amongst targets A, D, and E, and $\sim$2.1% between A and E+D. The solid and dashed horizontal lines are the weighted averages of the cross section using the three thinnest targets and are included to better visualize the results. Considering the order of magnitude change in target thickness from target A to E+D, these results provide good verification that the solid angles and the various uncertainties are being properly estimated.

$^3$Refer to table 3.1 for the table of proton densities for these targets.
Figure 5.36: **top:** 168.8 MeV \(\pi^+\) laboratory cross sections for the coincidence setup using various CH\(_2\) target thicknesses. The uncertainties shown are purely statistical. Each thickness has a normalization uncertainty dominated by target proton density (1%) and the \(\pm 0.2^\circ\) target angle error (0.5%). **bottom:** The solid angles used for the above cross sections, plotted as a ratio to the \(\pi^+\) solid angle generated during the same GEANT solid angle simulation. The counting statistics virtually cancel in this ratio, leaving only the uncertainties due to hadronic interaction losses shown.
5.1.4 Configuration Change Tests

Key elements of this experiment were: a) the use of the both liquid LH$_2$ and solid CH$_2$ targets, and b) the use of both coincidence $\pi p$ and single arm $\pi$ detection techniques with the LH$_2$ target. A very important test of the experimental and data analytical method was whether or not these configurations yielded the same cross sections at a given energy and polarity.

Data at 141.2, 168.8, and 218.1 MeV $\pi^+ p$ were obtained using all the above techniques at each energy. The results are shown in figure 5.37, plotted as ratios to the

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**Figure 5.37**: $\pi^+$ cross section ratios to the KH80 PWA predictions for the three energies with LH$_2$ single arm, and LH$_2$ and CH$_2$ coincidence data. The VPI SP95 PWA predictions are shown for comparison. $\theta_{gt} = -40^\circ$ for the single arm runs, and $53^\circ$ for the coincidence runs, except for $50^\circ$ at 218.1 with the CH$_2$ target. Uncertainties shown (~1.3-2.0%) are statistical only. These data are consistent at the level of the estimated ~1% normalization uncertainties.
KH80 PWA predictions for clarity. The VPI SP95 predictions are also overlayed as another basis for comparison. The uncertainties shown (1.3-2.0%) are statistical only. There is good agreement amongst the data at all energies, with consistency well within the ascribed relative normalization uncertainties: ~1.5% in all cases (see table 5.3). The agreement is especially impressive between the 168.8 LH2 and CH2 coincidence results. Here, the LH2 data are the average over the three lowest rate runs shown in figure 5.35, while the CH2 data are the average of the results using targets “A”, “D”, and “E” shown in figure 5.36. These data have 0.8% and 0.7% normalization and statistical uncertainties, respectively.

In addition, π⁻p coincidence data were obtained at 141.2, 168.8, and 193.2 MeV with the LH2 and CH2 “D” targets. Results are shown in figure 5.38. There is very good agreement between the LH2 and CH2 sets, completely consistent with the relative ~2.0% normalization uncertainties.

The LH2 and CH2 data at a given energy shown in figures 5.37 and 5.38 were run at different times during the four week long experiment, so these data also demonstrate the excellent reproducibility of the beam pion energy. However, there was one run at 193.2 MeV π⁺p where the CH2 cross sections were ~3.5% larger than the LH2 results (see figure 5.39). This was the only occasion where there was not excellent agreement between runs at a given energy, polarity, and configuration. Moreover, these results cannot be reconciled with the constraints imposed by the partial wave analyses at this energy near the Δ resonance (discussed further in section 5.1.5 and illustrated in figure 5.41). Although the reason for the anomalously large CH2 result is not certain, the most likely explanation is that the M11 channel was not set to the proper energy. A ~2 MeV increase in the beam energy for the CH2 run would account for the difference, and this energy offset is within the range possible if the position of the M11 momentum selecting midplane slits was improperly set. As a result, in the data sets summarized in section 5.2, this CH2 run was the only one excluded.
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Comparison of $\pi^-$ Coincidence Results with LH$_2$ (●) and CH$_2$ (★) Targets

Figure 5.38: $\pi^-p$ coincidence cross section ratios to the KH80 PWA predictions at the three energies (141.1, 168.8, and 193.1 MeV) where data with both LH$_2$ and CH$_2$ targets were obtained. The uncertainties shown (~1.5-2%) are statistical only. The data are completely consistent with the ~1.4% normalization uncertainty of each data set.
Figure 5.39: Data from the only occasion in the experiment where there was not excellent agreement between CH$_2$ and LH$_2$ target results at a given energy and polarity. The most likely explanation for the discrepancy is that the M11 channel was set at the wrong energy in the CH$_2$ run (see text).

Data at 168.8 MeV $\pi^+p$ were taken with several other changes to the standard coincidence configuration. These changes were designed to further test the GEANT solid angle determinations. Results were obtained where the proton P1 counters were purposefully misaligned by $+0.25^0$, where the P1 counter radii were changed to 85.5cm or 102cm (from 92cm), where the $\pi$1 counter was removed from the EVENT coincidence, and where the incident beam momentum spread was increased to 3% $\Delta p/p$ (from 1%). The cross section results, and the solid angles determined for each configuration, are shown in figure 5.40.

The relative normalization uncertainties between the data sets vary. The target thickness uncertainty, and essentially the pion decay, interaction loss, and multiple pion fraction uncertainties are common to all, whereas the target angle uncertainties are different and independent amongst the normal, "momentum spread", and the other three runs taken as a group (implying a relative 1% normalization uncertainty), but common to this group (implying essentially no relative normalization error amongst them). Again, the data show good consistency within the statistical, or
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168.8 MeV $\pi^+\text{LH}_2$ Target @ 53°

Cross Sections (top) and Solid Angles (bottom) under Various Configurations

Figure 5.40: top: 168.8 MeV $\pi^+p$ coincidence cross sections using the LH$_2$ target, measured under various experimental configurations and times during E645 (A...F refer to the counter arms, A being 60°). For clarity, the cross sections are plotted as a ratio to the KH80 PWA prediction and slightly offset from each other. The uncertainties shown are statistical only. bottom: Solid angles generated for the above cross sections, plotted as ratios to the $\pi$2 solid angle $\Omega(\pi^2)$ obtained from the same GEANT simulation run.
statistical and relative normalization, uncertainties, with all results in agreement, on average, at about the 1% level.

The results of figures 5.37, 5.38, and 5.40 are a very good indication that the solid angles and beam normalizations have been reliably determined using the procedures outlined in chapter 4.

5.1.5 Energy and Overall Normalization Test

A useful check of the beam energy and normalization of the LH2 data was done by measuring the $\pi^+p$ cross section at an energy (193.2 MeV) near the $\Lambda$ resonance ($\sim$189-193 MeV, see section 6.2). The data are plotted as ratios to the SP95 predictions in figure 5.41. At this energy, the P33 phase shift is $\delta_{P33} \approx 92^0$ and the other phase shifts are small: $\delta_{S31} \approx 15^0$, $\delta_{P31} \approx 5^0$ (see figure 6.50). SP95, other various VPI solutions (including FA93), KH80, and the trends of the VPI single energy solutions at lower and higher energies [79] show that $\delta_{S31}$ and $\delta_{P33}$ are certainly within $\pm 1^0$ and $\pm 2^0$ of $15^0$ and $92^0$, respectively, at this energy.

The effects on the cross sections of $\pm 1^0$ and $\pm 2^0$ changes in the SP95 phase shifts are demonstrated in figure 5.41. Both have about a maximum 1% effect on the prediction over the angular range of the data. Large fractional changes in $\delta_{P31}$ have negligible effect here. The size of these effects on this ratio is almost constant over roughly 190±5 MeV, due to the dominance of the P33 partial wave and the smallness of the others. Therefore, the $\pi^+p$ cross section data should lie within about $\sim$2% of the PWA solutions over this range. As is apparent in the figure, the data are well within this range. This is a fairly good test of both the normalization and the beam energy, since the cross sections vary with energy here at $\sim$0.5-1.0%/MeV, depending on angle, and so the results are a good indication that both are well understood.

\footnote{At this energy, the KH80 and SP95 PWA solutions differ by less than 1.5% for 193.2 MeV $\pi^+p$.}
Figure 5.41: 193.2 ± 0.6 MeV $\pi^+ p$ (with the LH$_2$ target) coincidence cross sections as a ratio to the SP95 PWA prediction. Shown are the effects on the SP95 prediction of large ±1$^\circ$ and ±2$^\circ$ changes in the $\delta_{S31}$ and $\delta_{P33}$ phase shifts, respectively. Due to the dominance of the P33 partial wave (i.e. $\delta_{P33} \sim 92^\circ$ at this energy), the cross sections should lie within about 2% of the PWA solutions. The changes in the cross section ratios due to the energy uncertainty are smaller than the statistical uncertainties shown. The data at this energy provide a check of the beam energy and normalization, and it is clear that these data are consistent with the constraints.
5.1.6 LH₂ Target Angle Test

A check of the uncertainty in the LH₂ target angle was undertaken by taking π⁺p coincidence data at 168.8 MeV with target angles of 45°, 53° ("normal"), and 60°. The target angle uncertainty is estimated to be ±0.3° based on the mechanical measurements performed during assembly and alignment of the target prior to the experiment (see section 3.3.2). This angular uncertainty corresponds to an uncertainty in \( \cos \theta_{\text{tgt}} \) of 0.5%, 0.7%, and 0.9% , respectively, for the angles listed above.

The results are shown in figure 5.42 along with the solid angles input for each of the runs ⁵. Overlayed on the plot are bands corresponding to the normalization uncertainty arising from the estimated ±0.3° target angle uncertainty, within which all the points for a given target angle can simultaneously be shifted up or down. The data are consistent within the normalization bands. These data are also consistent with a systematic angular offset of about 0.9°, but the evidence is weak as it depends almost entirely on the rather high point at \( \theta_\pi=115 \) degrees. Nonetheless, it may or may not be coincidental that this offset agrees with the one suggested by the independent target thickness measurement outlined in Appendix A. The effects of such an offset are significant.

Referring to figure 5.37, a ~0.9° offset would normalize the single arm LH₂ results upwards by 1.2%, and coincidence results downwards by 1.9%. Such a renormalization destroys the otherwise perfect agreement between the CH₂ and LH₂ coincidence results at 168.8 MeV (which have much smaller statistical and normalization uncertainties than the other data sets). The π⁻p data shown in figure 5.38 are also inconsistent with a 1.9% drop in the LH₂ data. Such a renormalization would clearly place the LH₂ data systematically 2–3% lower than the CH₂ data.

⁵The \( \theta_\pi=60° \) data point at the 45° target angle is flagged and removed from consideration since, unlike the other angles, an uncertain fraction of straight line pion trajectories from the target to the π₂ counters pass through the target ring, and so the hadronic interaction losses could not be reliably determined. No contribution for such loss corrections, which would raise the cross section, is included in the data point shown.
Figure 5.42: top: 168.8 MeV $\pi^+$ coincidence cross sections for various LH$_2$ target angles. The uncertainties (~1.3%) are statistical only, whereas the lines show the normalization effect of a ±0.3° change in target angle. The starred point corresponds to a setting where some scattered pions following trajectories straight to the $\pi$ counter pass through the target cooling ring (see text). bottom: Solid angles used in the above cross sections, plotted as a ratio to $\pi_2$ solid angle. Only uncertainties from hadronic losses are shown. The starred point does not include a correction for hadronic loss through the target ring.
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Comparing to the KH80 solution in figure 5.37, the agreement between the LH$_2$ single arm and the coincidence results would appear to improve at 169 MeV from a relative 3.1\% shift in the cross sections, but comparing instead to the SP95 solution and taking into account the statistical and normalization uncertainties, the evidence is not compelling. The effect of the hypothesized angular shift would neither improve nor deteriorate the agreement between the single and two arm results at 141 and 218 MeV, but instead would imply that the backward hemisphere (coincidence) cross sections were systematically lower with respect to KH80 than the forward hemisphere cross sections, although the SP95 solution, which predicts all of the $\pi^-p$ data in this work rather well, predicts the opposite behavior.

The 193.3 MeV $\pi p$ results shown in figure 5.41 cannot be reconciled with a 1.9\% drop in normalization. A 1.9\% drop in the cross sections could be compensated by a $\sim$2 MeV increase (\textit{i.e.} $\sim$1\%) in the beam energy, but this possibility is completely inconsistent with the results of the M11 channel calibration shown in figure D.58.

Finally, the target thickness results of the independent proton energy loss measurement summarized in Appendix A were in agreement with the vapour bulb measurements for target angles $\geq 0^\circ$. The application of an $0.9^\circ$ angular offset hypothesized here would remove this consistency as well.

Therefore, considering all the evidence presented above, there is little likelihood of the existence of a $\sim 0.9^\circ$ offset in the LH$_2$ target angle suggested (albeit weakly) by figure 5.42 and the target thickness measurement shown in figure A.52. In fact, such an offset would generate more inconsistencies than it removes. It is important to emphasize that the evidence from figure 5.42 for the large angle offset outside the estimated $\pm 0.3^\circ$ uncertainty arises largely on the basis of a single point, and nonetheless the data are consistent within the quoted statistical and normalization uncertainties. Furthermore, the data from the target thickness measurement illustrated in figure A.52 are consistent with a $1.5^\circ$ error in the target angle at the -40$^\circ$ orientation, and no
target offset, since the data points at target angles ≥0° are consistent with the vapour bulb result (see Appendix A). Although the data are consistent with zero target angle offset, the uncertainties suggest that the error in the target angle should be increased somewhat from the originally estimated ±0.3° (corresponding to a ±0.7% normalization uncertainty for a target angle of 53°). On this basis, the value for the angular offset of the LH₂ target was estimated to be 0°±0.4° (corresponding to a ±1.0% normalization uncertainty for a target angle of 53°).

5.2 The Absolute Differential Cross Sections

The final results for the π⁺p and π⁻p elastic absolute differential cross sections in the centre–of–mass system are listed in tables 5.4, 5.5, 5.6, 5.7, 5.8, and 5.6 at the end of this section. The uncertainties quoted are at the 1 standard deviation (i.e. 68% confidence) level. Effects from the energy and normalization uncertainties are not included in the errors associated with each data point. The data are also plotted as a function of the centre–of–mass angle in figure 5.43, along with the predictions of the KH80 [3] and SP95 [78] partial–wave analyses.

The data were obtained from runs taken with a fixed experimental configuration (i.e. fixed beam rate, target angle, etc.), except for the 168.8 MeV π⁺p LH₂ and CH₂ coincidence results, which are weighted averages of runs with three different rates, and three different target thicknesses, respectively. The averaging of the LH₂ results is justified by figure 5.35, where the differences in the data at the three lowest rates (the lowest two taken ~1 week after the other) are randomly distributed within the statistical uncertainties. The averaging of the CH₂ “A”, “D”, and “E” target results is justified by figure 5.36, where the data from these CH₂ targets are consistent with no dependence on target thickness.
5.2.1 Uncertainty Analysis

Uncertainties in the Absolute Normalization

The normalization uncertainties quoted in the tables are based on the following considerations:

- **Target Angle, \( \cos \theta_{\text{tgt}} \):** \( \pm 0.4^0 \) for the LH\(_2\) target, corresponding to a \( \pm 1.0\% \) uncertainty in \( \cos \theta_{\text{tgt}} \) at \( 53^0 \) (for the coincidence setup) and \( \pm 0.6\% \) at \( -40^0 \) (for the single arm setup), and \( \pm 0.2^0 \) for the CH\(_2\) targets, corresponding to \( \delta \cos \theta_{\text{tgt}} = 0.5\% \). These estimates are based on the mechanical measurements done during the experiment and the results discussed in section 5.1.6. The uncertainty for the 168.8 MeV \( \pi^+ p \) CH\(_2\) results is reduced by a factor of \( 1/\sqrt{3} \), since the data are averaged over three targets which were setup and aligned independently. (It is assumed that the angular uncertainty introduced during each alignment is random and larger than any systematic angular misalignment common to all runs).

The uncertainty for the 168.8 MeV \( \pi^+ p \) LH\(_2\) coincidence data was reduced by \( 1/\sqrt{2} \), since one of three runs in the average (at the 1.5 MHz target pion rate) was obtained more than a week prior to the other two, during which period the target was removed and repositioned several times.

- **Multiple Pion Correction, \( f_5 \):** \( \pm 10\% \) of the value of \( f_5 \) determined for each run. This is justified by the excellent agreement exhibited by the results discussed in section 5.1.2 and shown in figure 5.35. For the 168.8 MeV \( \pi^+ p \) data, which was an average over runs with \( \sim 1, 2, \) and 4\% corrections, 2\% was chosen for the weighted mean.

- **Pion Fraction, \( f_4 \):** The uncertainties range from \( \pm 0.3\% \) at 141.1 MeV to \( \pm 0.1\% \) at 267.2 MeV for the \( \pi^+ p \) data. This follows from the direct measurements obtained with the phase restricted beam discussed in section 4.2.1 and shown in
figure 4.27, and checked at 141.1 and 168.8 MeV by following the fitting procedure to the TCAP timing spectra outlined in the same section. For the $\pi^- p$ data, the uncertainties range from 0.9% at 141.1 MeV to 0.3% at 267.2 MeV. Up to 193 MeV, the uncertainties follow from the fits to the TCAP spectra using the results from figure 4.27 as a guide. At the higher energies, these fits could not be performed reliably and so only the results from figure 4.27 were employed. The results at the lower energies were always within the results of the two series of measurements shown in figure 4.27, and so this is justified.

- **Pion Decay, $f_D$:** $\pm 0.2\%$ in all cases, since the results of the GEANT and REVMOCC simulations used to generate these corrections agreed to $<0.1\%$ (refer to section 4.2.2). Another $0.1\%$ was added for the uncertainty in the contribution from pion decay within the channel (see appendix C). The statistical accuracies of the simulations were negligible compared to these estimates.

- **Hadronic Interaction Loss, $f_L$:** This uncertainty was estimated to be 15% of the calculated loss to the centre of the target. The uncertainty was from about 0.3% at 168.8 MeV $\pi^+ p$ on the LH$_2$ targets, to 0.1% at 193.2 MeV $\pi^- p$ on the 2mm CH$_2$ target. For the case of the 168.8 MeV $\pi^+ p$ where results were averaged over 0.5, 2, and 3.2mm CH$_2$ targets, the uncertainty from the 2mm target (0.2%) was applied to the average.

- **Target Proton Density, $N_{prot}$** The uncertainty in the proton density of the LH$_2$ target is estimated to be $\pm 0.5\%$ from the vapour bulb measurements taken during the experiment (see section 3.3.2) and confirmed by results obtained by measuring proton energy loss through the target (see appendix A). The proton density uncertainty of the CH$_2$ targets is $\pm 1\%$, as measured by chemical analysis by a commercial laboratory [71] (see section 3.3.2). For the 168.8 MeV $\pi^+ p$ data,
this uncertainty was reduced by $1/\sqrt{3}$ since it is assumed that the thickness uncertainties of these targets are uncorrelated.

- **Beam and Computer Live Time, B and $f_{LT}$**: The uncertainties in both these quantities are considered negligible. The BEAM counting was done with three different scaler modules: two of which were recorded on tape, and a third visual scaler online. Their agreement was to $<<0.1\%$ in all cases. $f_{LT}$ was measured by the ratio of scalers registering the EVENTS recorded to tape to the EVENTS detected by the TOF Spectrometer. Again, the results from the scalers written to tape and the visual scalers were in perfect agreement. In the data presented in the tables, $f_{LT}$ was 0.98 or better, and it is highly unlikely that the uncertainty in this number would be larger than 0.1%.

All of the normalization uncertainties outlined above were combined in quadrature to yield the values quoted in the tables. As an example, table 5.3 shows the uncertainties and their sum for all the 168.8 MeV data.

**Angle Dependent Uncertainties**

The experimental uncertainties which vary with the pion scattering angle come from the counting statistics in the foreground and background runs, the statistics in the Monte Carlo determinations of the solid angles, the uncertainties in the hadronic loss corrections of the scattered pions and recoil protons, and the uncertainty in the distance from the $\pi2$ counter to the target centre ($\pm0.5\%$ corresponding to $\pm0.3$ cm). The hadronic loss uncertainty is included with the other statistical errors of each data point since the hadronic losses vary with particle energy and the amount of material the particles traverse as they exit the target, both of which depend on the scattering angle.
1σ Uncertainties at 168.8 MeV

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<td>(f_{LT})</td>
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**“Statistical”**

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Table 5.3: Breakdown of absolute normalization (i.e. scattering angle independent) and average angle dependent uncertainties for the cross sections at 168.8 MeV. Uncertainty estimates for hadronic interaction losses of the scattered pions and protons are included with the statistical errors since the target material traversed by the pions and protons depends on the scattering angle. The quantities in parentheses reflect the fact that the final \( \pi^+p \), \( \text{LH}_2 \), and \( \text{CH}_2 \) results are averages over several runs (see text). The uncertainties at other energies are similar to those shown here. The normalization factors are defined in equations 4.50 and 4.53.
Referring to section 4.3.1 and equation 4.62 (reproduced here), the net yield is calculated from

\[ Y = Y_{fg} - \kappa \cdot Y_{bg}^{\text{back}} \]

where \( Y_{fg} \) is the yield measured in the foreground runs, \( Y_{bg}^{\text{back}} \) is the yield measured in the background runs, and \( \kappa \) is the foreground/background normalization factor. The uncertainty in \( Y \) is then taken to be:

\[ \Delta Y = \sqrt{ (\delta Y_{fg})^2 + \left( \frac{\delta \kappa}{\kappa} \right)^2 + \left( \frac{\delta Y_{bg}^{\text{back}}}{Y_{bg}^{\text{back}}} \right)^2 } \]  

(5.63)

where the uncertainties in the foreground and background yields are Poisson distributed (i.e. \( \delta Y_{fg} = \sqrt{Y_{fg}} \)) and the foreground/background normalization uncertainty \( \delta \kappa \) comes mainly from the target angle uncertainties in the foreground and background runs. In practice, the second term involving \( \kappa \) is negligible in the coincidence measurements where the background is very small, but it does have a non-negligible effect on the single arm runs, where backgrounds are typically 25% (but up to 50% at 20°) of the foreground yields. Also, for the single arm runs, there is some uncertainty in the yields arising from the placement of the software cuts used (see section 4.3.3 and figure 4.33). Gates 20 channels wider and narrower than the chosen gate were defined, and the variation in the resulting yields were added in quadrature to the other uncertainties. In practice, these variations were never larger than half of the statistical uncertainties. Refer to appendix F.5 for an example output where these errors are computed.

In section 4.1 it was shown that the solid angles determined from the GEANT simulations could be smoothed, and therefore the statistical errors improved, by normalizing the solid angles to the average solid angle of \( \pi \) counters determined during the same simulation run. Refer to appendix F.3 for an example calculation. The hadronic loss uncertainty (at 68% confidence) is estimated to be 10% of the loss suffered by the
pions and protons (see section 4.1.3). One problem arises from the number of particles which suffer a hadronic reaction but the reaction products are still detected as a \( \pi p \) event. The uncertainty in this estimate is taken to be a quarter the difference between the losses computed using the inelastic (reaction) cross sections (where all the particles suffering such interactions are assumed lost) and the losses computed using the total cross sections (where some of the elastically scattered particles could go forward and be detected). Refer to section 4.1.3 and an example calculation in appendix F.4.

The final "statistical" uncertainties are the quadrature sums of all these uncertainties. Refer to appendix F.4 for an example calculation. In practice, the uncertainties are dominated by the statistical uncertainty from the simulation, and the uncertainty in the \( \pi 2 \) counter radius. As an example, a list of uncertainties for the 168.8 MeV data is shown in table 5.3.

### 5.2.2 Radiative Corrections

At all energies there is the possibility of pion bremsstrahlung occurring (i.e. \( \pi p \rightarrow \pi p \gamma \)), but it is known to have a very small cross section (\(<0.1\) mb/sr) [80]. In this thesis, radiative corrections to the differential cross sections have not been considered. However, they are expected to be small with respect to the experimental uncertainties for the following reasons: In the two arm coincidence setup, if the energy carried away by the radiative gamma ray was sizable, the kinematic correlations of the scattered pion and recoil proton would change, and an event which would otherwise cause a \( \pi p \) coincidence event would be lost. However, this effect does not occur in single arm mode. In this case, radiative energy losses would cause some pions that would have hit the pion arm counters to miss them, and some that would have missed the counters to hit them, thus canceling the effect. Figure 5.37 shows that the single arm and coincidence results are consistent with each other, suggesting that radiative corrections, if
present, are not large with respect to the uncertainties. Consequently no corrections for this process has been applied to the data.
Chapter 5. Results

Absolute Differential Cross Section Results

Figure 5.43: The absolute differential cross sections in the centre-of-mass system for all energies and configurations reported in this work. The uncertainties shown are statistical only. The curves are the predictions of the KH80 (solid) and SP95 (dashed) partial wave analyses.
Table 5.4: Absolute centre-of-mass differential cross sections at 141.2 MeV. All the stated uncertainties are at the 1σ level. The statistical uncertainties are the quadrature sum of the solid angle uncertainties (including those from hadronic interaction loss) and counting statistics (including those from background subtraction). The normalization uncertainty (∆N) is the quadrature sum of the individual contributions (see text).
Table 5.5: Centre-of-mass differential cross sections at 168.8 MeV. The LI\textsubscript{2} coincidence results are the weighted average of three runs with the lowest beam rates shown in figure 5.35. The CH\textsubscript{2} results are the weighted average over targets "A", "D", and "E" shown in figure 5.36.

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
& \theta^0_{\text{c.m.s}} & \frac{d^2 \sigma}{dt d\Omega} (\pi^+ p) [\text{mb sr}] & \pm \delta(\text{stat}) & \frac{d^2 \sigma}{dt d\Omega} (\pi^- p) [\text{mb sr}] & \pm \delta(\text{stat}) \\
\hline
\text{LH}_2 \text{ Target, Single Arm} & \pm 0.1^0 & \Delta N = 1.1\% & & \pm 0.1^0 & \Delta N = 1.3\% \\
\hline
38.4 & 21.38 & 0.38 & x & 38.4 & 21.38 & 0.38 & x \\
50.6 & 16.66 & 0.24 & x & 50.6 & 16.66 & 0.24 & x \\
62.5 & 12.95 & 0.18 & x & 62.5 & 12.95 & 0.18 & x \\
73.9 & 10.02 & 0.16 & x & 73.9 & 10.02 & 0.16 & x \\
84.9 & 8.27 & 0.15 & x & 84.9 & 8.27 & 0.15 & x \\
\hline
\text{LH}_2 \text{ Target, } \pi p \text{ Coincidence} & \pm 0.1^0 & \Delta N = 0.9\% & \Delta N = 1.3\% & \pm 0.1^0 & \Delta N = 0.8\% & \Delta N = 1.5\% \\
\hline
73.9 & 10.35 & 0.12 & 1.198 & 0.019 & 73.9 & 10.35 & 0.12 & 1.198 & 0.019 \\
90.2 & 8.58 & 0.08 & 0.859 & 0.014 & 90.2 & 8.58 & 0.08 & 0.859 & 0.014 \\
110.1 & 11.33 & 0.10 & 0.974 & 0.015 & 110.1 & 11.33 & 0.10 & 0.974 & 0.015 \\
128.4 & 17.51 & 0.15 & 1.496 & 0.021 & 128.4 & 17.51 & 0.15 & 1.496 & 0.021 \\
145.2 & 24.26 & 0.19 & 2.101 & 0.028 & 145.2 & 24.26 & 0.19 & 2.101 & 0.028 \\
161.0 & 29.25 & 0.23 & 2.466 & 0.033 & 161.0 & 29.25 & 0.23 & 2.466 & 0.033 \\
\hline
\text{CH}_2 \text{ Target, } \pi p \text{ Coincidence} & \pm 0.1^0 & \Delta N = 0.8\% & \Delta N = 1.5\% & \pm 0.1^0 & \Delta N = 0.8\% & \Delta N = 1.5\% \\
\hline
73.9 & 10.32 & 0.08 & 1.163 & 0.020 & 73.9 & 10.32 & 0.08 & 1.163 & 0.020 \\
90.2 & 8.63 & 0.08 & 0.884 & 0.018 & 90.2 & 8.63 & 0.08 & 0.884 & 0.018 \\
110.1 & 11.35 & 0.10 & 0.999 & 0.021 & 110.1 & 11.35 & 0.10 & 0.999 & 0.021 \\
128.4 & 17.77 & 0.14 & 1.468 & 0.029 & 128.4 & 17.77 & 0.14 & 1.468 & 0.029 \\
145.2 & 23.90 & 0.18 & 2.091 & 0.037 & 145.2 & 23.90 & 0.18 & 2.091 & 0.037 \\
161.0 & 29.27 & 0.21 & 2.516 & 0.043 & 161.0 & 29.27 & 0.21 & 2.516 & 0.043 \\
\hline
\end{array}
\]
\[ T_x = 193.2 \pm 0.6 \text{ MeV} \]

<table>
<thead>
<tr>
<th>( \theta^0_{\text{cms}} )</th>
<th>( \frac{d\sigma}{d\Omega}(\pi^+ p) [\text{mb/sr}] )</th>
<th>( \pm \delta(\text{stat}) )</th>
<th>( \frac{d\sigma}{d\Omega}(\pi^- p) [\text{mb/sr}] )</th>
<th>( \pm \delta(\text{stat}) )</th>
</tr>
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<td>( \pm 0.1^0 )</td>
<td>( \Delta N = 1.2% )</td>
<td>( \Delta N = 1.4% )</td>
<td>( \Delta N = 1.1% )</td>
<td></td>
</tr>
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<tr>
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<td>0.836</td>
<td>0.014</td>
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<tr>
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<td>1.000</td>
<td>0.018</td>
</tr>
<tr>
<td>129.0</td>
<td>15.03</td>
<td>0.16</td>
<td>1.603</td>
<td>0.025</td>
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<td>20.72</td>
<td>0.21</td>
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<td>25.39</td>
<td>0.25</td>
<td>2.767</td>
<td>0.040</td>
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</table>

Table 5.6: Centre-of-mass differential cross sections at 193.2 MeV.
Table 5.7: Centre–of–mass differential cross sections at 218.1 MeV.
### Chapter 5. Results

#### $T_x = 240.9 \pm 0.8 \text{ MeV}$

<table>
<thead>
<tr>
<th>$\theta^0_{\text{cms}}$</th>
<th>$\frac{d\sigma}{d\Omega}(\pi^+ p) \ [\text{mb/}\text{sr}]$</th>
<th>$\pm \delta(\text{stat})$</th>
<th>$\frac{d\sigma}{d\Omega}(\pi^- p) \ [\text{mb/}\text{sr}]$</th>
<th>$\pm \delta(\text{stat})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pm 0.1^0$</td>
<td>$\Delta N = 1.2%$</td>
<td>$\Delta N = 1.3%$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>76.3</td>
<td>6.40</td>
<td>0.08</td>
<td>0.830</td>
<td>0.013</td>
</tr>
<tr>
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<td>4.14</td>
<td>0.05</td>
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<td>0.010</td>
</tr>
<tr>
<td>112.5</td>
<td>4.74</td>
<td>0.06</td>
<td>0.685</td>
<td>0.012</td>
</tr>
<tr>
<td>130.3</td>
<td>x</td>
<td>x</td>
<td>1.143</td>
<td>0.018</td>
</tr>
<tr>
<td>146.6</td>
<td>10.80</td>
<td>0.11</td>
<td>1.657</td>
<td>0.024</td>
</tr>
<tr>
<td>161.8</td>
<td>13.11</td>
<td>0.13</td>
<td>2.074</td>
<td>0.029</td>
</tr>
</tbody>
</table>

Table 5.8: Centre-of-mass differential cross sections at 240.9 MeV. The $\pi^+$ point at 130.3$^0$ has been removed since the corresponding proton counter for this run was found to be seriously misaligned.

#### $T_x = 267.3 \pm 0.9 \text{ MeV}$

<table>
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<tr>
<th>$\theta^0_{\text{cms}}$</th>
<th>$\frac{d\sigma}{d\Omega}(\pi^+ p) \ [\text{mb/}\text{sr}]$</th>
<th>$\pm \delta(\text{stat})$</th>
<th>$\frac{d\sigma}{d\Omega}(\pi^- p) \ [\text{mb/}\text{sr}]$</th>
<th>$\pm \delta(\text{stat})$</th>
</tr>
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<tr>
<td>$\pm 0.1^0$</td>
<td>$\Delta N = 1.1%$</td>
<td>$\Delta N = 1.1%$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>77.2</td>
<td>4.75</td>
<td>0.06</td>
<td>0.727</td>
<td>0.011</td>
</tr>
<tr>
<td>93.5</td>
<td>2.83</td>
<td>0.04</td>
<td>0.515</td>
<td>0.010</td>
</tr>
<tr>
<td>113.3</td>
<td>3.07</td>
<td>0.04</td>
<td>0.583</td>
<td>0.012</td>
</tr>
<tr>
<td>131.0</td>
<td>4.88</td>
<td>0.06</td>
<td>0.847</td>
<td>0.016</td>
</tr>
<tr>
<td>147.1</td>
<td>7.06</td>
<td>0.08</td>
<td>1.280</td>
<td>0.023</td>
</tr>
<tr>
<td>162.1</td>
<td>8.65</td>
<td>0.10</td>
<td>1.564</td>
<td>0.032</td>
</tr>
</tbody>
</table>

Table 5.9: Centre-of-mass differential cross sections at 267.3 MeV.
Chapter 6

Discussion and Conclusions

6.1 Comparison with Previous Work

As mentioned in chapter 1, prior to the work presented in this thesis, the only comprehensive set\(^1\) of cross section data for energies spanning the \(\Delta\) resonance were those of the Queen Mary College experiment of Bussey \textit{et al.} [40] performed at CERN, and it is primarily with these data that the work described in this thesis will be compared. At \(\sim 140\) MeV, there are the results of two experiments at TRIUMF by Brack \textit{et al.} [15] (also mentioned in chapter 1) and the more recent [17, 45]. Near 263 MeV, there are relatively recent results (1987) by the LAMPF group of Sadler, \textit{et al.} [47]. The results from these two groups (Brack and Sadler) serve as low and high energy "benchmarks" for the results reported in this work. Of particular interest are the Brack results of 1986 [15], since the techniques employed in that experiment (including the TOF Spectrometer) were similar to those applied here, except that the 1986 results were restricted to the use of \(\text{CH}_2\) targets.

Since all of the above data sets were obtained at similar, but not identical, energies, the large energy dependence of the cross sections makes direct comparisons of the different data virtually meaningless. As a result, comparisons are best accomplished by comparing each data set with the results of an energy-dependent partial wave analysis at the relevant energy. Since in practice all the modern PWAs (\textit{e.g.} KH80 [3], FA93

\(^{1}\)There are some \(\pi^+p\) data at two energies each, from Otterman \textit{et al.} [50] and Troka \textit{et al.} [50], but these have considerably larger uncertainties and do not contribute significantly to the database.
Chapter 6. Discussion and Conclusions

[81], and SP95 [78]) display a similar energy dependence for the cross sections, any
of these analyses can be used for this purpose. The approach used in this thesis is
to examine ratios of the data to PWA prediction, since in this way differences among
the data sets become more apparent. Such differences would not be visible on a plot
of cross section versus angle like that shown in figure 5.43.

In this thesis the KH80 [3] PWA analysis is used as the reference PWA prediction
since it has long been considered the "canonical" solution from which many conse-
quences of πp scattering have been derived (e.g. the pion nucleon coupling constant
$\frac{g^2}{4\pi}$). Also shown are recent PWA results of the VPI group, FA93 [81] and SP95 [78]. (All
these predictions are evaluated at the same energies as investigated in this work).

The most recent VPI analysis, SP95, is similar to FA93 in that each solution em-
loys dispersion relation constraints applied in the same fashion. In addition, the
πNN coupling constant $\frac{g^2}{4\pi}$ was varied to provide the best fit to the data. They differ
in that SP95 includes some new data (e.g. the new Brack et al. results [45]), and the
normalization uncertainties of the Bussey et al. data were increased to 5% (from <1%
[82]) to resolve a conflict between the single energy and energy dependent solutions
[83].

Cross section ratios of the results from this experiment together with the predic-
tions of SP95 and FA93 to the PWA values of KH80 are shown in figures 6.44, 6.45,
and 6.46.

Consider first the comparison of this work with the other data sets. In the following
discussion, the ~1% normalization uncertainties ascribed to the Bussey data will be
assumed [82] and not the larger 5% value applied in the SP95 analysis. One sees
that this work and that of Bussey et al. are consistent within the uncertainties at all
energies above the resonance, although the statistical uncertainties from this work
are smaller. The two data sets differ systematically at 169 MeV, but only at the 1

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2SP95 also employed a new technique to search for resonances in the partial waves, but this only
affected energies much higher than those of our experiment.
Figure 6.44: Comparison of 141.1 and 168.8 MeV data with results of previous work, plotted as ratios to the KH80 PWA [3] predictions at their respective energies. Only statistical errors are shown. The dashed (FA93 [81]) and double lines (SP95 [78]) are recent PWA predictions from the VPI group. The Bussey et al. data have claimed normalization uncertainties of <1\% (although this was increased to 5\% in SP95), while the uncertainty in the Brack data sets are \sim 2.5\%. Only the Bussey data appeared in the KH80 solution. There is excellent agreement between our E645 results and SP95, although though our results were not included in SP95.
Figure 6.45: Comparison of 193 and 218 MeV data with the results of Bussey et al., and the recent VPI FA93 and SP95 PWA solutions. Only the Bussey et al. data appeared in the databases of the KH80 and VPI solutions. The agreement of the current results with the SP95 solution is again excellent.
Figure 6.46: Results at 241 and 267 MeV compared to results from previous work. The Bussey et al. data appear in all the PWAs, whereas the more recent (1987) Sadler et al. data at 263 MeV [47] appear only in the VPI database. The Sadler et al. data have normalization uncertainties of 3% and 5% for the \( \pi^+p \) and \( \pi^-p \) data respectively, whereas the Bussey et al. uncertainties were claimed to be about 1% (but increased to 5% in SP95). The agreement of our results with those of Sadler et al. data and the SP95 predictions is excellent.
Figure 6.47: Ratio of $\pi^+ p$ to $\pi^- p$ cross sections at 141 MeV compared to the KH80 and SP95 PWAs and the two data sets of Brack et al. near this energy. The data are plotted as ratios to the KH80 prediction. The excellent agreement among these data sets suggests that the differences in the $\pi^+ p$ and $\pi^- p$ cross sections evident in figure 6.44 arise from a systematic effect common to both pion polarities (e.g. effective solid angle determination).

standard deviation level. The results are in clear disagreement at 141 MeV.

Both of the Brack et al. data sets at 141 MeV are consistently $\sim$2-3% lower than this work, but nonetheless consistent at the edge of the relative $\sim$3% normalization uncertainty. Figure 6.47 shows the ratio of the $\pi^+ p$ and $\pi^- p$ cross sections for the results of this work and those of Brack et al. at this energy\(^3\). The excellent agreement amongst the data sets further emphasizes the consistency between the results of this work and those of Brack.

Our E645 results and those of Sadler et al. at $\sim$267 MeV are perfectly consistent in both the normalization and the shape of the cross sections. Note that our data are

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\(^3\)The $\pi^+ p$ and $\pi^- p$ Bussey et al. data near 141 MeV were not performed at the same energy or pion scattering angles, and so a comparison with the results of this work and those of Brack et al. is not possible.
measured to a greater statistical precision than any of the other data sets.

Our results are in clear disagreement with the KH80 PWA analysis at energies below the resonance (i.e. at 141 MeV and 169 MeV). However, the analysis is in general agreement above the resonance with the present data.

The consistency of our work with the world's data excluding the CERN data sets is demonstrated more forcibly by the most striking feature of these figures: even without appearing in the database, there is excellent agreement at all energies between the results of our work and the predictions of the SP95 solution. Although the agreement with the FA93 solution is also very good, it is not as good as with the SP95 solution. This is significant, since the difference is probably due to the fact that the Bussey et al. normalization uncertainties were relaxed to 5% in SP95 from <1% in FA93. This change greatly reduced the effect of the Bussey et al. data on the fits, giving the other data sets in the database more "weight" in defining the partial waves. The dominance of the P33 partial wave implies that the difference between the FA93 and SP95 solutions which are apparent around the resonance will have implications for all the data from about 100 to 350 MeV.

The excellent agreement between our results and the predictions of the SP95 solution can be used to test the consistency of our data with the total cross section data of Pedroni et al. [39] and Carter et al. [38]. Figure 6.48 shows these total cross sections plotted as ratios to the SP95 solution. The SP95 solution clearly favours the Pedroni et al. results over their whole energy range, and so by extension, implies good agreement between Pedroni and the differential cross sections of our experiment.

In should be noted, however, that the Carter et al. data quote much smaller uncertainties than those of Pedroni et al., especially at the lower (<150 MeV) energies (typically ~<0.5% versus ~2-3% respectively), and so as long as they remain in the

\[^{4}\text{Note that the uncertainties of total cross sections of Carter et al. were not subject to the increase given to the Bussey et al. data.}\]
Figure 6.48: The total cross sections of Pedroni et al., and Carter et al., plotted as ratios to the SP95 predictions. Comparison of this plot with the previous figures shows that the SP95 solution is consistent with the Pedroni data and the differential cross section data from our work, implying that our data and the Pedroni et al. data are consistent with each other.
database with their claimed uncertainties, they will dominate the partial wave analysis fits.

Since our work is to some extent an extension of the earlier work of Brack et al., it is interesting to observe how well the lower energy data of Brack are fit by the SP95 solution. Figure 6.49 shows that at 87 and 117 MeV, the Brack et al. data, which have 2-3% normalization uncertainties, are systematically ~3% lower than the SP95 predictions, just as is the case at 140 MeV. However, as noted above, the Carter et al. total cross sections also have an influence on the partial wave solutions in this energy region, and so it is quite conceivable that with a relaxation of the uncertainties on the Carter data (or even removal from the database), a future partial wave analysis could favour the Brack data more than SP95 currently does. On the other hand, the systematic ~2-5% difference between the Brack et al. data, and the SP95 predictions (and our data at 141 MeV), might indicate a normalization problem in the Brack data. Nevertheless, the trend appears clear: the SP95 solution is more consistent with the TRIUMF differential cross sections (Brack, our data) and the Pedroni et al. total cross sections than with the CERN results (Bussey, Carter).

6.2 Influence on Partial Waves

The SP95 solution can be used as an indication of the impact of our data on the scattering phase shifts. The s- and p-wave phase shifts for the SP95 and KH80 solutions are shown in figure 6.50. The most obvious differences between the solutions occur in the P31 and (especially) S11 phase shifts, where SP95 is consistently smaller in magnitude than KH80. However, the most significant difference between these two phase shift solutions is not apparent in the figure.

Figure 6.51 shows a close-up view of the P33 phase shift in the region where it crosses 90° i.e. near the Δ resonance. The SP95 solution has in fact two P33 phase
Figure 6.49: Ratios of previous work to the KH80 predictions plotted near 87 and 117 MeV. Also shown is the VPI SP95 solution results for comparison. Only the Bussey et al., data appears in the KH80 database, whereas all of the data shown are included in the VPI database. The Frank et al. data [2] shown have large (~ 20%) normalization uncertainties.
Comparison of Phase Shifts from the SP95 and KH80 Partial Wave Analyses

Figure 6.50: Comparison of the hadronic phase shifts from the KH80 and SP95 phase shift solutions. Since the SP95 solution already predicts the data from this work rather well, the differences shown are indicative of the changes expected once our data are included in the database.
Figure 6.51: The P33 phase shift near Δ resonance energies from the KH80 [3] and SP95 [78] PWA solutions. The SP95 solution allows for a difference in the Δ⁰ and Δ⁺⁺ resonance masses by including different $\delta_{P33}$ phases for $\pi^- p$ and $\pi^+ p$, respectively, whereas the KH80 solution does not. The arrows show the "average" Δ masses quoted by the Particle Data Group (PDG) [84], and the SP95 analysis (obtained by Breit-Wigner fits to the P33 partial-wave).
shifts, one for $\pi^+p$ scattering, which forms the $\Delta^{++}$ (uuu quark combination) resonance state, and one for $\pi^-p$ elastic and $\pi^-p \rightarrow \pi^0n$ scattering, which forms the $\Delta^0$ state (udd quark combination). This provides for a mass splitting between the resonant states, a splitting which is expected on the basis of the u and d quark mass difference (see section 2.1 for a discussion). The KH80 solution makes no such provision.

The $\Delta$ resonance mass is different from the energy at which $P_{33}$ crosses $90^0$, since other $\pi p$ scattering processes besides $\Delta$ formation contribute to this partial wave. Consequently, the $\Delta$ mass and width parameters are derived from fits to the $P_{33}$ phase shift using a Breit–Wigner resonance form plus a background (see e.g. [3]), a background which is assumed small and slowly varying with energy, much like the $S_{11}$, $S_{31}$, $P_{31}$, and $P_{13}$ partial waves. Nonetheless, the differences in the $90^0$ crossing angles persist in the differences in fitted masses due to the smallness of the background.

Figure 6.51 also indicates that the SP95 solution suggests a $\Delta$ resonance mass about 1.5 MeV heavier than that of the KH80 solution, based on the shift in the $90^0$ crossing energies. The decrease in the value of the $P_{33}$ phase shift below the resonance caused by the energy shift is almost wholly responsible for the decrease in magnitude and shape differences observed between the cross section predictions of the two PWA solutions there. Table 6.10 shows the mass and widths determined from a number of sources. The CBC [85] results are from a partial wave analysis using only the Carter et al. and Bussey et al. data sets around the resonance. The average quoted by the Particle Data Group [84] is based primarily on this and the KH80 analysis. The SP95 solution also suggests a $\Delta$ mass 1.5 MeV heavier than these other determinations. Note that all these analyses used the same forms for the Breit–Wigner line shape and background contribution.

Since the SP95 analysis came to the above conclusion on the $\Delta$ mass without using

---

\(5\)The $P_{11}$ partial wave has a rapid rise after \(\sim 150\) MeV due to the $P_{11}$ "Roper" resonance at \(\sim 485\) MeV pion kinetic energy.
Table 6.10: Δ resonance mass and width parameters obtained from various partial wave analyses. The KH80 Δ mass is an estimate based on the P33 90° crossing energy shown in figure 6.51. The CBC result [85] is obtained from fits to the Bussey et al. differential and Carter et al. total cross sections. The SP95 database includes both the Pedroni et al. and Carter et al. total cross section sets, although the solution clearly favours Pedroni (see figure 6.48). Results of recent partial wave analyses of NN scattering data favour the $\frac{\Delta^2}{4\pi}$ result from the SP95 solution (see text).

<table>
<thead>
<tr>
<th>Source</th>
<th>$M^\text{avg}_{\Delta}$ [MeV]</th>
<th>$2\Gamma$ [MeV]</th>
<th>$M_{\Delta^0} - M_{\Delta^{++}}$ [MeV]</th>
<th>$\frac{\Delta^2}{4\pi}$</th>
</tr>
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<tbody>
<tr>
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<td>115.6</td>
<td>$\sim$0.5</td>
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<tr>
<td>KH80</td>
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<td>-</td>
<td>14.3 ± 0.4</td>
</tr>
<tr>
<td>CBC</td>
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In any case, the increased precision of our data compared to that of Bussey et al., Brack et al., and Sadler et al. is expected to lead to a reduction in the uncertainties in the phase shifts and resonance parameters once they are included in a new analysis.

6.2.1 Consequences for the πNN Coupling Constant

Table 6.10 also shows the results for the πNN coupling constant derived from the SP95, KH80, and CBC analyses. The 4% difference in the SP95 and KH80/CBC values can be attributed, in part, to the shift upwards in the Δ mass and the decrease in the resonance width, since both serve to decrease the strength of the dominant P33 partial wave component in the dispersion integrals used to determine $\frac{\Delta^2}{4\pi}$ (see section 2.3.2). The SP95 value for $\frac{\Delta^2}{4\pi}$ was obtained using the same technique described in [81], from which the value 13.75 was found (see also section 2.3.2).

At this time (1995), the controversy over the value of this coupling constant still persists, but the fact that our cross section data described here are well described by the SP95 solution certainly strengthens the case for a lower value of the coupling
constant. If it were decided to increase the uncertainties of (or remove) the Carter et al. total cross sections below the resonance energy, it is conceivable that the value of $\frac{G^2}{4\pi}$ might drop even further below 13.7 once our differential cross section data are included in a new analysis, since the Carter total cross sections almost certainly have the effect of "propping up" the P33 partial wave at energies below the resonance.

In fact, a value of $\frac{G^2}{4\pi}$ near 13.6 would be consistent with recent results of partial wave analyses of NN scattering data [91]. Furthermore, new results suggest that such a value of the coupling constant can be reconciled with the Bonn NN potential model. It was mentioned in chapter 1 that Machleidt and Sammarruca argued [36] that the Bonn model could not accommodate the smaller coupling constant when used in its description of the deuteron. However, Machleidt and Li recently showed [86] that the smaller coupling is the only single way to fix a problem that all meson-exchange models have with describing the NN $^3P_0$ partial wave. With the lower coupling, however, an additional source of tensor attraction is required to reconcile the models with the deuteron properties. Additional NN tensor attraction of the required amount has been found by the Jülich group [87] in the form of correlated $\pi-\rho$ exchange, a process which is missing in the Bonn model. Also, recent improvements (over that of the Bonn model) in the description of correlated two pion exchange [88] suggest that the description of the higher NN partial waves is improved with the smaller coupling constant than with the larger one. These results effectively mute one of the severest criticisms of the lower coupling constant, namely, that in the context of the otherwise very successful Bonn NN model, it could not be reconciled with the well established properties of the deuteron.

As mentioned in chapter 1, the observed discrepancy in the Goldberger–Treiman relation when using the larger value of the coupling constant is eliminated when the smaller value of the coupling constant is used. In fact it is stated in reference [59] that "it has been hoped for a long time that" $\frac{G^2}{4\pi}$ would decrease, thus lowering the amount
of the discrepancy. Our results together with the recent SP95 PWA results suggest, as outlined above, that perhaps their hopes have been fulfilled.

6.2.2 Consequences for the Sigma Term

The consequences of our data on evaluations of the sigma term ($\Sigma$ or $\Sigma_d$) will only become apparent after a careful re-analysis of the full database. Nonetheless, something can be learned from a comparison of our data with the FA93 analysis shown in figures 6.44, 6.45, and 6.46. Sainio reported [89] a determination of $\Sigma$ using the FA93 solution as input into the program previously used to obtain $\Sigma$ from the KH80 solution [90]. It was found that $\Sigma \approx 65$ MeV, larger than the ~60 MeV result previously obtained with the KH80 solution, and the previous 64 MeV result of Koch [63], and consequently further from the theoretical expectation of about 50 MeV. Since the evaluation of the sigma term based on this particular application of dispersion relations has a significant connection to the resonance energy data (see also section 2.2), it was expected that significant changes to the P33 partial wave might have significant observable consequences for $\Sigma$. In fact, as pointed out by Sainio [89], the program could accommodate the data of Brack et al. [15, 16, 17] when using the FA93 solution as input at higher energies, whereas previously using the KH80 solution as input it could not [90]. However, this result suggests that the effect is fairly small. The fact that differences between SP95 and FA93 are smaller than the differences between FA93 and KH80 makes it difficult to assess whether or not our data will have much effect on future determinations of $\Sigma_d$. On the basis of this result at least, it appears that the solution of the discrepancy must be found elsewhere.
Chapter 6. Discussion and Conclusions

6.3 Concluding Remarks

The primary motivating factor behind the experiment described in this thesis was to provide precise new absolute differential cross section data at energies spanning the \( \Delta \) resonance in the hope that the results would shed light on the serious discrepancies observed between the Pedroni \textit{et al.} and Carter \textit{et al.} total cross sections, and the Bussey \textit{et al.} and Brack \textit{et al.} differential cross sections. In order to do so, it was necessary to build as many checks as possible into the experiment to ensure the highest possible confidence in the quoted normalization uncertainties, since the discrepancies mentioned, are only at the few percent level. There is every indication that this work has met both objectives.

With these results, which span the low energy \( \pi N \) region (containing recent data of Brack \textit{et al.}) and the region above the \( \Delta \) resonance (with recent data by Sadler \textit{et al.}), it should now be possible to evaluate more clearly the reliability of the quoted uncertainties characterizing the various data sets used in the phase shift analyses. The increased precision of our results compared to previous work will also improve the accuracy of the phase shifts. This will in turn reduce the experimental uncertainties in the determinations of the \( \pi NN \) coupling constant, the sigma term, and in the parameters of the \( \Delta \) resonance, and thereby establish new standards in these fundamental constants.
Bibliography


[12] D.V. Bugg, in [4], p15 ; J.S. Frank, in [4], p21
[14] G. Smith, private communication
[34] G. Höhler in [5], p66


[41] D.V. Bugg, in [4], p15


[49] Scattering Analysis Interactive DialIn program, Virginia Polytechnic and State University, Blacksburg, Virginia; contact Prof. R.A. Arndt for more information


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[61] G. Höhler et al. Handbook of Pion Nucleon Scattering, Physik Daten Nr.12-1, Universität Karlsruhe (1979) Karlsruhe (1979); also G. Höhler, private communications

[62] E. Pietarinen, private communication


[64] M.E. Sainio, private communication


[66] D.F. Ottewell, TRIUMF M11 Beamline Coordinator, private communication

[67] The transistorized tubes and bases are proprietary designs of the University of Regina, Saskatchewan nuclear physics group. Contact Dr. George Lolos for more information.

[68] W. Kelner, TRIUMF Targets Group, private communication

[69] D. Healey, TRIUMF Targets Group, private communication


[76] $\sigma_{tot}(\eta p, pp)$: from SAID program [49]
\[ \sigma_{tot}(\pi C): \text{from [73]} \]
\[ \sigma_{tot}(pC), \sigma_{el}(pC): \text{from a Dirac Optical Model program ("RUNT") by Dr. E.D. Cooper,} \]
\[ \text{Fraser Valley College, B.C., Canada.} \]

[77] The original derivation is due to Dr. G. Jones, Physics Dept., University of British Columbia, private communication


[79] This observation follows by inspection of the phase shifts from single energy analyses of the database. These single energy fits can be viewed in the SAID program [49]


[82] R.A. Arndt, private communication. The original paper ([40]) quotes no normalization uncertainties. The uncertainties were updated recently to $\sim 0.5$-1% in correspondence from D.V. Bugg to R.A. Arndt

[83] R.A. Arndt, private communication


[93] R. Ristinen, University of Colorado, Boulder, private communication


[95] LOSSPROG, An Energy Loss Program. L.G. Greeniaus, University of Alberta

[96] The assistance of my brother, Roberto Pavan, in the setting up and debugging of the REVMOC transport codes is gratefully acknowledged.

[97] G. Stinson, TRIUMF /University of Alberta, private communication. His assistance with the TRANSPORT and REVMOC codes is gratefully acknowledged.


[101] C. Joram, Universität Karlsruhe, private communication. He noted that his group had noticed a similar effect and pointed out the references which explained the discrepancy.


[106] Semiconductor Detectors, eds. G. Bertollini, A. Coche, North Holland, Amsterdam (1968)

Appendix A

LH₂ Target Thickness Measurement

An independent measurement of the LH₂ target thickness was undertaken during one of the periods of phase restricted operation undertaken in Experiment 645. The technique was based on measuring the energy lost by M11 beam protons (available when a midplane absorber is not used) through the target, and then using energy loss tables to infer the LH₂ thickness. The method will only be outlined here: a more detailed description is to be published [92].

The proton energy loss in just the LH₂ target material was determined by measuring the energy loss difference between full and empty targets. The proton energies were measured by a downstream array of 0.4, 2.2, and 2.2mm silicon detectors. The first two detectors were used in a ΔE-E arrangement to distinguish the heavily ionizing protons from passing pions and electrons. The energy of the proton beam was arranged so that the protons would stop in the second detector when the target was full, and in the third detector when the target was empty. With the target then emptied, layers of aluminum foil were added to the beam until the energy deposited in the detectors was the same as when the target was full. The thicknesses of these aluminum absorbers are known to 1% [93]. The ratio of the proton stopping power in hydrogen to aluminum multiplied by this aluminum thickness gives the thickness of hydrogen via:

\[ X = \int_{E_{\text{in}}}^{E_{\text{out}}} \left( \frac{dE}{dX} \right)^{-1} dE \]

The proton stopping powers were taken from the accurate Janni tables [94].
This formula requires knowledge of the proton stopping power in the region of the LH₂ target cell, which was significantly different from than at the region of the aluminum foils (at the energies used here ~31 MeV), since the protons lost energy in all the downstream target material (windows, heat shields, air) before reaching the absorbers. An energy loss program [95] was used to determine the proton energies at the entrance and exit of the LH₂ target cell. The proton energy at the exit of the M11 beam pipe was inferred from the channel momentum calibration, and checked by measuring the proton energy deposited in the silicon detectors and working backwards to the beam pipe. For the latter method, the detectors were first calibrated using an Americium α-particle source together with a precision voltage pulser. The two approaches yielded consistent results.

This technique was carried out at four target angles: -40.0, 0.0, 38.5, and 53° each with a ±0.25° uncertainty. The results are shown in figure A.52. Except at 0°, the error bars were dominated by the angular uncertainty and the stopping power uncertainty at the LH₂ target cell arising from the uncertainty in the proton energy. At 0°, protons "leaked" through the second silicon counter, making it more difficult to assess accurately the total proton energy loss. A 1% uncertainty in the aluminum foil thicknesses and a 1.6% uncertainty of the proton stopping power in hydrogen as given in the Janni tables combine for an overall 1.8% normalization uncertainty.

Fitting the data (by least-squares) to the expression $X = X_0/\cos(\theta + \theta_0)$ yielded the result: $X_0 = 104.5 \pm 0.5$ mg/cm², $\theta_0 = 0.8^0 \pm 0.3^0$. Adding the normalization error gives $X_0 = 104.5 \pm 1.9$ mg/cm². The data point at -40° is the only one not to agree with the vapour bulb result ($X_0 = 106.2 \pm 0.5$ mg/cm², outlined in section 3.3.2), and is solely responsible for the lower value of $X_0$ and the offset. The energy lost by the protons in the target should have been the same when the target was rotated to -40° and 40°, however, the proton energy lost for the target oriented at -40° was significantly less that
Figure A.52: Results from the two different \( \text{LH}_2 \) target thickness measurement methods. The vapour bulb result is described in section 3.3.2. The uncertainties in the proton energy loss results include the effect of a \( \pm 0.25^\circ \) angular uncertainty. The normalization error band does not include the contribution from the statistical errors, although the normalization error is included in the best fit thickness error estimate.
for $40^\circ$. Consequently, the positive angle measurement angle was repeated with excellent consistency (this time at $38.5^\circ$ since the proton energy lost in the $-40^\circ$ orientation was consistent with that expected for $-38.5^\circ$), whereas the $-40^\circ$ data was not repeated. There was some difficulty experienced in moving the target to the $-40^\circ$ orientation for this measurement, since it was the first time that rotation to the $-40^\circ$ orientation was attempted, and the apparatus was "strained" and had to be "coaxed" into place. So it is possible that instead of a $0.8^\circ$ offset implied by these measurements, a $1.5^\circ$ error was made when endeavoring to set the target to the $-40^\circ$ orientation. Section 3.3.2 contains a discussion of the implications of this discrepancy.

\footnote{After the initial attempt, no more difficulties were experienced when moving the target to the $-40^\circ$ target angle position.}
Appendix B

Channel Tuning Setup

The setup used to measure the beam size and divergence of the M11 beam during the pre-experiment tuning run is shown schematically in figure B.53. The 20cm square wire chambers (WC) were of the proportional delay–line type, with roughly a 2mm resolution in the \(\hat{y}\)–coordinate, and 1mm in the \(\hat{x}\)–coordinate. A wire chamber installed in the diagnostic box at the M11 midplane was used to measure the beam size at the dispersed–focus. This wire chamber could be inserted into or removed from the beam remotely. The wire chambers in the experimental area were sighted into to the nominal central beam trajectory with a transit, and the position calibrations were done using a collimated Ruthenium \(\beta\) particle source. The front wire chamber was placed at the TINA focus location, and the beam triggering scintillators were placed behind the last chamber in order to minimize sources of multiple scattering prior to the final focus. The 30cm M11 vacuum beam pipe had an additional 20cm inner diameter pipe connected to it to extend to within 48cm of the TINA focus. This was also done to reduce multiple scattering. The four–paddle hodoscope was installed at the same position it had during E645, so that it could be calibrated to monitor beam movements during the experiment.

The beam tuning was carried out for 140 MeV \(\pi^-\) without a midplane absorber in the channel, although the midplane wire chamber did act as a thin absorber. A gate was placed on the beam time–of–flight to the scintillators (referenced to the cyclotron
Appendix B. Channel Tuning Setup

Figure B.53: Schematic view of setup used during beam tuning. The M11 midplane wire chamber was located in the diagnostics box at the midplane (see figure 3.6).

TCAP signal in order to remove electrons and muons, since, being secondary particles, they had substantially different beam properties than did the pions.
Appendix C

Simulation of M11 Beamline with REVMOC

A Monte Carlo simulation of the entire M11 beamline was undertaken primarily to determine the number of muons from in-channel pion decay (i.e. pion decay between the septum and the final quadrupole) that would cause a BEAM coincidence (see section 3.4.1), and whether these could be distinguished from the beam pions during phase restricted beam operation (see section 4.2.1) [96]. The TRIUMF REVMOC [98] Monte Carlo beam transport package was selected for its ease of use, and its ability to use directly those magnet parameters derived with the TRANSPORT [100] beamline design program.

The TRANSPORT code originally used to design the M11 beamline [97] was able to find a unique solution of magnet settings that would provide a momentum-dispersed double-focus at the M11 midplane, and a doubly-achromatic double-focus at the TINA position with no higher order aberrations. It correctly predicts the observed 18% $\delta p / p$ midplane momentum dispersion and maximum 2.5% FWHM momentum bite after the B2 dipole (refer to figure 3.6). However, the magnet settings (especially the sextupoles) from this solution are rather different than the experimentally determined tune. When the magnet parameters from this solution (e.g. field strengths, fringe fields) are input into REVMOC, Gaussian beam shapes are predicted, at variance with observation (see figure 3.20).
Measured vs. Simulated

M11 Beam Parameters

Figure C.54: Comparison of measured M11 beam parameters with front-end jaws opened wide (H:140cm, V:120cm) and narrow (H:50cm, V:30mm) versus REVMOC simulation predictions.
The above solution assumes that the 1AQ9 quadrupole is aligned along the central ray of the beamline, when in reality it is aligned with the BL1A proton line, and the M11 optics uses the off-axis dipole component of the quadrupole to bend the pion beam onto the septum magnet (see section 3.2). Consequently, a new solution with a more accurate 1AQ9 simulation was found where REVMOC now predicts beam properties much closer to those observed.

1AQ9 was approximated by a quadrupole (of half the original strength) along the BL1A axis, followed by a dipole of appropriate strength, followed by another quadrupole along the M11 optical axis. A TRANSPORT refit was required after this change. The new solution has a dispersed double-focus at the midplane, and is doubly-achromatic and focuses in the y-plane at TINA, but the focus in the x-plane is ~5cm upstream of TINA, as observed experimentally (see section 3.5.1).

With the new 1AQ9 definition, all second order aberrations in M11 cannot be corrected with the available sextupoles, with one aberration remaining (free to be chosen) leading to the skewed x-size and horizontal divergence distributions at TINA. An unremovable aberration causing left hand tails is exactly what was observed during the pre-experiment channel tuning period (section 3.5.1). Moreover, simulating the M11 front-end rate restricting jaws in REVMOC reproduces the observed dependence of the beam phase space at TINA on the jaw aperture with reasonable accuracy (see figure C.54). Some improvements to the simulation are certainly possible, but the accuracy is considered good enough to warrant confidence in the results of the pion decay simulations which now follow. These TRANSPORT and REVMOC input files can be found at the end of this appendix.

C.1 In-channel Pion Decay

The sources of muons causing BEAM coincidences can be divided into roughly three regions: near the T1 production target, in the M11 channel between the septum and
the last magnet, and downstream of the last magnet. Muons from near T1 originate from the decay of pions produced with a broad spectrum of energies and angles, but only those muons with momenta within the momentum bite of the channel survive to exit the beam pipe. The TOF difference between these muons and the pions is maximal. The muons originating in the channel are doubly constrained: there are far fewer of them, since the pion rate is far lower than near T1, and only a small fraction of pions decay to muons having a momentum near the channel momentum; and these muons are emitted within a few degrees (in the lab) of the incident pion direction, and so could be lost by geometrical constraints. The time distribution of these muons with respect to the pions has a broader distribution. Muons originating downstream of the last magnet (Q6) are constrained geometrically by the beam counters, but have no momentum constraints.

The contribution to the E645 BEAM coincidences from muons originating downstream of Q6 has to be determined by Monte Carlo since the muons cannot be distinguished from pions via TOF or energy loss. Fortunately, it is rather straightforward to model, since there are no magnets. This was done using both the REVMOC and GEANT (see section 4.2.2) for all the E645 energies. The two simulations gave essentially identical results (within \( \leq 0.08\% \)). The fraction of BEAM coincidences caused by muons that were pions at Q6 is virtually constant at \( 0.82 \pm 0.02\% \) at all E645 energies (not including pion interaction losses).

Muons from pion decay near T1 can be identified experimentally by TOF to the S2B counter (with respect to the TCAP cyclotron signal) when the proton beam pulse time spread is narrowed to \( \sim 1\) ns width during phase restricted beam operation (see section 4.2.1 and e.g. figure 4.25). This contribution was modeled at 276 MeV/c by REVMOC. The time spread in the simulation was put in manually. The angular distribution of pions from T1 was chosen to just fill the Q6 aperture to save computing
time, whereas realistically the pions emerge from all angles. The momentum distribution was chosen to be similar to that observed at the exit of the channel (see [65]). The angular and momentum distributions of the pions leaving T1 are uncorrelated in REVMOC, whereas in reality the situation is much more complicated, since for E645, the pions originate from inclusive production by protons on beryllium.

Despite the rather crude approximation of the source, figure C.55 shows that the simulation does a reasonable job of modeling reality. However, the muon component in the right hand peak is underestimated by a factor of 2. The simulation shows that roughly 10% of the muons in the right hand peak originate from pions with momenta larger than the channel momentum. A more realistic description of the initial pion momentum and angular distribution would increase the muon fraction and improve the agreement. Nonetheless the modeling is good enough to elucidate a number of effects:
Figure C.56: REVMOC simulation showing effect of beam counters on target $\mu$ fraction. The pion distribution hardly changes in the two cases, whereas the muon fraction increases substantially. Note that the in-beam counters severely restrict the contribution from in-channel pion decay (see figure C.57).
The beam scintillators significantly affect the muon fraction. Since the muons originate from a distributed source, the muon beam size is somewhat larger than the pions'. Consequently, the pion fraction results shown in section 4.2.1 do not apply to the M11 channel in general. In particular, the muon fraction at the target constrained by the beam counters is significantly different than the unconstrained case (see figure C.56). Since the electrons, which cannot be modeled by REVMOC, originate from an even more diffuse source ($\pi^0$ production followed by decay into gammas followed by $\gamma$ conversion into electrons), one would expect this effect to be even larger in that case. This has implications for the multiple pion correction outlined in section 4.2.4.

The slits at the dispersed focus have a small effect on the muon fraction. This can be understood from the larger muon beam phase space throughout the channel: muons of the central momentum are not necessarily on axis like the pions. Again, one would then predict a larger effect on the electron fraction. This explains the observed variation in electron fraction between the two phase restricted tune runs (see section 4.2.1 and figure 4.27), one of which used 9mm slits, the other 36mm. The observed variation in the muon:pion ratio is considerably smaller, as predicted.

The front-end jaw apertures do not affect the pion fraction appreciably. Both the pion and muon beams fill the beam pipe at that point, and so adjusting the jaws cuts into both beams more or less equally.

The muon contribution from pions decaying between $Q6$ and after the immediate vicinity of the T1 source was determined by starting the pion beam at several different points along the channel, and observing the resultant time distributions. The first order transfer matrix from T1 to these locations was calculated using TRANSPORT, and input into the REVMOC code in lieu of all the magnetic elements these matrices
REVMOC $\pi - \mu$ ΔTime-of-Flight from several M11 Magnets

Figure C.57: REVMOC simulations of $\pi-\mu$ time-of-flight differences at 274 MeV/c starting at different points along the M11 channel. Note that right-hand muon peak originates from the region near the production target, and that there is very little contribution between the Septum and the Q6 quadrupole i.e. the last M11 magnet. After Q6, the muon contamination is easily determined by REVMOC or GEANT.
represent using the "MAT" card (refer to [98] for the program description). In the simulation, the pion phase space parameters are chosen at T1, multiplied by the matrix representing transport to a particular beamline location, and then the pion is tracked and decay allowed. The resulting $\pi, \mu$ time distributions for the pion beam starting at the SEPTUM and sextupole SX1 entrances, and the Q6 exit with 274 MeV/c central momentum are shown in figure C.57.

The figure confirms graphically that muons originating after Q6 cannot be separated from the pions, and that the large majority of muons in the right hand peak originate near T1. Only a relatively small number of muons originating between the septum and Q6 survive to cause a BEAM coincidence. Note that most of these muons are under the right hand peak, and so will be counted with the muons from T1 when fitting the peaks obtained from the phase restricted beam runs (see e.g. figure C.55).

Therefore only a very small fraction of muons will be unaccounted for following the procedure outlined in section 4.2.1. This otherwise unaccounted-for contribution is estimated to be $0.2\% \pm 0.1\%$ at all energies. Consequently, it is clear that in-channel pion decay does not pose a problem to the beam normalization analysis in this experiment.

Again this conclusion applies only for this experiment, since as observed in figure C.56, the in-channel contribution increases when a larger fraction of the total beam is intercepted. This is expected, since muons (and electrons) from all sources form a "halo" encompassing the pion beam, and the larger the pion beam counters are, the more muons and electrons they will accept.

---

\[1\] At higher momenta, there will be fewer pions decaying, but the resulting muons will be traveling more in the forward direction, thus compensating in a manner similar to what was observed in the case of decay downstream of Q6 through the beam counters.
C.2 M11 TRANSPORT Code

The following TRANSPORT file simulates the M11 beamline with the 1AQ9 quadrupole approximated as quadrupole–dipole–quadrupole combination. The file does not simulate the front-end channel jaws or the midplane slits. Refer to TRANSPORT manual [100] for program details.

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16.00 'K1' 7.000000 0.000000;
16.00 'K2' 8.000000 0.000000;
3.0 'DT09' 0.40820;
3.0 ' ' -0.00500;
5.00 '1AQ9' 0.24630 -3.51131 10.48000;
20.0 ' ' 180.00000;
2.0 ' ' 0.00000;
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2.0 ' ' 0.00000;
20.0 ' ' -180.00000;
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3.0 'SEPI' 0.51990;
3.0 ' ' -0.00500;
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2.0 ' ' 0.00000;
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5.00 '11Q1' 0.49050 1.53687 10.48000;
3.0 'DQ12' 0.30010;
5.00 '11Q2' 0.49050 -1.13326 10.48000;
3.0 'DQ31' 0.21740;
18.00 'SEX1' 0.23060 -0.25227 10.16000;
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### Appendix C. Simulation of M11 Beamline with REVMOC

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SENTINEL

SENTINEL
C.3 Sample REVMOC M11 File

The following REVMOC input file simulates the pion beam and its decay muons through the M11 channel at 274 MeV/c. The magnet settings were scaled from the TRANSPORT solution outlined above. In this example, the vertical jaws are set to a 100mm aperture, the horizontal jaws to 30mm, and the midplane horizontal slits to 18mm (corresponding to 1% δp/p FWHM). There is no midplane absorber. For program details, refer to the REVMOC manual [98].

```
2 ### 274 MeV/c T1-LH2 100x30x18 VJxHJxSL ###
! STARTING POINT
  D START 0.0001 10.48
! Drift to 1AQ9 (-0.5cm to compensate 1AQ9 "dipole")
  D DT09 0.40310 10.48
** 1st 1/2 1AQ9
  Q 1AQ9 0.24630 -4.00875 10.48
! Fringe field card for 1AQ9 "dipole"
! G/2 K1 K2
  FRIN P1AQ9 10.48 0.0 0.0
! Rotation for left-hand bend
  R 180.0
! Bender Pole Edge ON ENTRANCE (angle, 1/r)
  ENTR 1/R3 0.0 0.0
** SIMULATING 1AQ9 BENDING by putting a thin (say 1cm)
** dipole 1/2 way through. Bends pions 4.11 deg. Field
** is high due to short length. Not a concern here
** since its only a simulation device
! 1AQ9 bending: Leff[m], KG, grad, 2nd grad, P0
!  B 1AQ9B 0.0100 65.56161 0.0 0.0 0.274
!
! Bending Magnet Pole Edge ON EXIT
  EXIT 1/R4 0.0 0.0
! Rotation (back to right-hand)
  R -180.0
** 2nd 1/2 1AQ9
  Q 1AQ9 0.24630 -4.00875 10.48
! FRINGE FIELD parameters for subsequent bending magnets
! G/2 K1 K2
  FRIN S1B1B2 10.000 0.3500 4.400
! thin drift as Q9 exit checkpoint
  D Q9OUT 0.0001 10.48
! Drift to Septum (-0.5cm to compensate 1AQ9 "dipole")
  D SEP1 0.5139 20.0
```
Appendix C. Simulation of M11 Beamline with REVMOC

! thin drift as septum entrance checkpoint
D INSEPT 0.0009 10.0
! Rotation
R  180.0
! Bending Magnet Pole Edge ON ENTRANCE
ENTR 1/R3  0.0 0.0
! Septum: Leff, KG, grad, 2nd grad, central momentum
B 11S1  0.58780 3.29736 0.0 0.0 0.274
! Bending Magnet Pole Edge ON EXIT
EXIT 1/R4  0.0 0.0
! Rotation
R  -180.0
! Drift to JAWS
D DS1J  0.16085 10.48
! ** Simulating Jaws with a drift space of RESTRICTED dimensions
! ** This is done because decay product (muon) suffers NO dE/dX in REVMOC
! ** Space is 15cm (6") long to match thickness of H+V Tungsten jaws
! ** First the Horiz. gap, then Vert. (wide open is ~ : 8.0 -8.0 8.0 -8.0)
! D JAWS  0.15  1.5 -1.5  5.0 -5.0
! ! Drift to first quad
D DJQ1  0.16085 10.48
! First Quadrupole
Q 11Q1  0.49050 1.75459 10.48
! Drift to Second Quadrupole
D DQ12  0.30010 10.48
! Second Quadrupole
Q 11Q2  0.49050 -1.29381 10.48
! Drift Space to Sextupole
D DQS1  0.21740 10.48
! First Sextupole
SEX SEX1  0.23060 -0.28801 10.16
! Drift Space to B1
D S1B1  0.2380 10.16
! Rotation
R B1IN  180.0
! Bending Magnet Pole Edge ON ENTRANCE
ENTR 1/R1  7.7 0.00000
! First bending magnet: Leff, KG, grad, 2nd grad, central momentum
B 11B1  1.19620 7.96197 0.0 0.0 0.274
! Bending Magnet Pole Edge ON EXIT
EXIT 1/R2  7.7 0.00000
! Rotation
R B1EX  -180.0
! Drift space after B1
D DBS2  0.33450 10.16
! Second sextupole
Appendix C.  Simulation of M11 Beamline with REVMOC

\[\text{SEXX SEX2 0.2360 1.05834 10.16}\]

! Drift space MID after second sextupole
  D MID 0.5170 10.16
  ! MIDPLANE absorber. CH2 thickness here must be subtracted from
  ! the MID drift space.
  D ABS 0.0001 10.16
    1. 0.935
    0.8571 6. 0.1429 1.

! ** Simulating slits at mid-plane by drift of RESTRICTED dimensions
! ** 2x2.5cm (1") long to represent Copper slits
! ** card gives aperture (wide open usually : 5. -5. 5. -5.)
! 
  D SLITS 0.05 0.9 -0.9 5.00 -5.00
!
! Remaining drift space to Q3
  D DMQ3 0.41890 10.16
  ! 3rd Quadrupole
  Q 11Q3 0.50040 -3.60887 10.48
  ! Drift to Q4
  D DQ34 0.32140 10.48
  ! 4th Quadrupole
  Q 11Q4 0.40310 5.55214 15.24
  ! Drift to SEX6
  D DQ44 0.34076 15.24
  ! Sixth Sextupole
  SEX SEX6 0.23060 -0.24249 15.24
  ! Drift to Q5
  D DQ45 0.34076 15.24
  ! 5th Quadrupole
  Q 11Q5 0.50040 -3.61875 10.48
  ! Drift space after Q5
  D DSQ3 0.28110 10.48
  ! 3rd Sextupole
  SEX SEX3 0.2360 -1.00128 10.16
  ! Drift space after sex3
  D B2IN 0.31780 10.16
  ! Rotation
  R 180.0
  ! Bending Magnet Pole Edge ON ENTRANCE
  ENTR 1/R3 0.0 1.48570
  ! Second bending magnet: Leff, KG, grad, 2nd grad, central momentum
  B 11B2 1.19060 7.91148 0. 0. 0.2740
  ! Bending Magnet Pole Edge ON EXIT
  EXIT 1/R4 0.0 1.48570
  ! Rotation
  R -180.0
  ! ** Steer beam by SHIFTING dx [mr] (i.e. like a B2 scan)
  SH STEER 0.0 0.75 0.0 0.0
! B2 EXIT CHECKPOINT
  D B2EXIT 0.00100 15.2
! Drift Space after B2
  D DBS4  0.48355  15.2
! Fourth Sextupole
  SEX SEX4  0.2360 -0.54517  15.24
! Drift Space to Q6
  D SQ6   0.21030  15.2
! Sixth Quadrupole
  Q 11Q6  0.40310 -3.13182  15.24
** c/I Q6--> tina=230.6cm Q6 length=40.31cm ==> Q6 exit-->tina=210.45
  D Q6EXIT  0.001  15.0
! First (wide) section of vacuum pipe after end of Q6
  D PIPE1  0.9081  15.0
!
! ### FOLLOWING IS SPECIFIC TO EXPT 645 ###
! Second (narrower) section of pipe to exit
  D PIPE2  0.1051  10.0
! Thin MYLAR drift as channel exit
  D EXIT   * 0.00025  10.
     1.  1.39
       0.625  6.  0.042  1.  0.333  8.
! air between M11 exit and S1
  D M11-S1  * 0.1862  10.
     1.  0.001205
       0.22222  8.  0.77778  7.
! 1st scintillator
  D S1    * 0.0016  1.27 -1.27  5.1  -5.1
     1.  1.032
       0.92308  6.  0.07692  1.
! air between S1 and S2a
  D S1-S2A  * 0.4914  10.
     1.  0.001205
       0.22222  8.  0.77778  7.
! 2ND scintillator
  D S2A    * 0.0016  0.635 -0.635  2.225  -2.225
     1.  1.032
       0.92308  6.  0.07692  1.
! AIR SPACE
  D S2A-B  * 0.0034  10.
     1.  0.001205
       0.22222  8.  0.77778  7.
! 3rd scintillator LOCATION (JUST AIR IN THIS CASE)
  D S2B    * 0.0016  0.635 -0.635  2.225  -2.225
     1.  1.032
       0.92308  6.  0.07692  1.
! Drift space of air to Target
  D S2B-TGT  * 0.4015  10.
     1.  0.001205
Appendix C. Simulation of M11 Beamline with REVMOC

0.22222 8. 0.77778 7.
! 1/2 LH2 @ 53 deg @ 'Tina' location: Looks elliptical to the beam
D TINA * 0.0075 4.5 7.5
  1. 0.0708
  1. 1.

!
INIT RANDN 99876
X 0.25 100.0 0. 0. 0. 5.
Y 0.70 325.0 0. 0. 0. 5.
P MOMEN 0.274 0.008 0.2740 0. 2.0 -0.65 0. 1.
G STATS 1. 4132.
M Pion * 0.1395679 0.0 0.0 26.03 1.0 0. 0. 180.
  1.0 0.105658 0.0
OUTPUT 1 TINA
NSPAC 6. -1.
-1. -1. -1. -1. -1. -1. -1. -1. -1. -1.
P START 120. 0.240 0.320 -4.
P TINA 100. 0.020 0.274
X TINA 100. 10.0 0.0
DX TINA 100. 100.0 0.0
Y TINA 100. 10.0 0.0
DY TINA 100. 200.0 0.0
END
INIT RANDN 109153
P MOMEN 0.266 0.008 0.2740 0. 2.0 -0.65 0. 1.
G STATS 1. 3600.
END
INIT RANDN 1054
P MOMEN 0.282 0.008 0.2740 0. 2.0 -0.65 0. 1.
G STATS 1. 4700.
END
INIT RANDN 155
P MOMEN 0.290 0.008 0.2740 0. 2.0 -0.65 0. 1.
G STATS 1. 4850.
END
INIT RANDN 1090856
P MOMEN 0.298 0.008 0.2740 0. 2.0 -0.65 0. 1.
G STATS 1. 4900.
END
INIT RANDN 987657
P MOMEN 0.306 0.008 0.2740 0. 2.0 -0.65 0. 1.
G STATS 1. 5044.
END
INIT RANDN 689258
P MOMEN 0.314 0.008 0.2740 0. 2.0 -0.65 0. 1.
G STATS 1. 4980.
END
INIT RANDN 10959
P MOMEN 0.322 0.008 0.2740 0. 2.0 -0.65 0. 1.
Appendix C. Simulation of M11 Beamline with REVMOC

G STATS 1.4876.
END
INIT RANDN 111150
PMOMEN 0.330 0.008 0.2740 0. 2.0 -0.65 0. 1.
G STATS 1.4636.
END
INIT RANDN 8656
PMOMEN 0.338 0.008 0.2740 0. 2.0 -0.65 0. 1.
G STATS 1.4500.
FINIS
Appendix D

M11 Momentum Calibration Details

D.1 $\pi$-e TOF Difference

A momentum calibration of the M11 channel was undertaken in March and May 1994 using a technique of measuring the TOF difference between pions and electrons between two scintillators in vacuum. Using pions and electrons has the advantage that for our energies of interest, they have very similar energy losses in scintillator, thus minimizing discriminator walk-effects, and the electrons travel essentially with the velocity of light. Results were obtained using a few different variations of this straightforward technique.

In one configuration, the M11 beam pipe was extended by $\sim$6m, and two scintillators (SA,SB) of precisely measured thicknesses were positioned in the beam pipes through vacuum–sealed ports. The distance between upstream faces of these counters was 570.6±0.3cm. The scintillator signals were discriminated with ORTEC 462 CFDs, and the scintillator voltages and CFD pole-zero adjustments were carefully setup to reduce the CFD walk <10ps. The timing resolutions were about 0.9ns FWHM. The CFD signals were fed to LeCroy 2228 TDCs (of nominally 50 ps/channel gain) and then read–out by computer to be copied to tape for offline data analysis. The exact gain of each TDC channel used was calibrated using an ORTEC 462 time calibrator. The gains were checked by two other 462 calibrators, and by noting the TDC channel difference between pion peaks one cyclotron RF period apart ($1/23.058$MHz = 43.369ns).
Figure D.58: Figure shows the % deviations of each π-e TOF M11 pion kinetic energy measurement point from the best fit (i.e. calibration) line. Also shown is the calibration from the 1985 analysis [74], which used a different technique (see section D.2).

An accuracy of ±0.06 ps/channel was achieved. Each timing signal was fanned-out and sent to two different TDCs as a further check. The pion and electron timing peak centroids in the offline analysis were determined by Gaussian peak fitting programs. The M11 midplane slits were set to accept a 0.3% δp/p momentum bite in order to reduce time spread, although results with broader settings were obtained as a check.

Using the above configuration, the pion velocity, and therefore the M11 channel momentum, follows simply from the knowledge of the π-e TOF difference between the two scintillators, the scintillator separation, and the pion energy loss in the upstream scintillator. This was repeated at ~158, 184, 245, and 282 MeV/c.

A second technique uses essentially the above configuration. Data are first obtained with the two scintillators touching at the downstream port position. A single
\(\pi\)-e TOF difference peak is observed, and this establishes a reference time \(t_0\). By measuring the differences of the pion and electron timing peaks with respect to \(t_0\) with one scintillator moved back to the upstream position, the pion velocity follows from the ratio of those two differences, and the result is independent of the TDC gain and scintillator separation. Results at 158 and 184 MeV/c agreed with the above first method well within the uncertainties.

The pathlength and the 0.9ns timing resolution limited the above two methods to at most 290 MeV/c. To go to higher momenta, the much longer flight path from the T1 production target to the downstream scintillator was employed. The scintillator times were referenced to the cyclotron TCAP signal, and phase restricted beam was used to minimize the proton time spread and to provide clean Gaussian-shaped timing distributions (see e.g. 4.25). The M11 channel length was calibrated by insisting on consistency between the momentum inferred from the \(\pi\)-e TOF difference between the two scintillators (as before), and the one inferred from the difference between the upstream scintillator and TCAP. From the weighted average of these results at 158, 184, and 245 MeV/c, the channel length from T1 to the TINA focus was determined to be 1542.1\(\pm\)1.5 cm\(^1\) An uncertainty of \(\pm\)2 cm was used in the calculations. Measurements at \(\sim\)158, 245, 302, 336, and 358 MeV/c (i.e. about 70 to 245 MeV) were conducted using this technique.

The results from these measurements are shown in figure D.58. The data from the second technique essentially overlap the first technique results, and are not included here for clarity. Results obtained using a broader momentum spread also agree within uncertainties and are also not shown. Data using the first method were obtained in both running periods (March and May), while the third method was used only in May. The calibration is made with respect to the B1 dipole field, which is measured by an

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\(^1\)This is 12.2 cm longer than quoted in the TRIUMF Users' Manual [65]. That determination was done by a direct measurement during the channel construction, with an estimated uncertainty of about \(\pm\)10 cm [66].
NMR probe positioned at the dipole midplane. B1 is chosen over B2 since it is the primary momentum selector in the channel. As a function of the B1 field (in Gauss) the new momentum calibration reads:

\[ P_{M11}[\text{MeV/c}] = 0.03267 \cdot (B1 - 17.1) \pm 0.15\% \]

which corresponds to about a \( \pm 0.25\% \) uncertainty at 68% confidence level in pion kinetic energy in this energy region. The uncertainty is estimated from the spread in the calibration points from the best fit line.

**D.2 Surface Barrier Detector Method**

A momentum calibration was attempted during the pre–experiment tuning phase, and again during March 1994, using the technique of stopping light ions produced at T1 in a silicon counter in vacuum at the beam pipe exit. Protons (\( ^{+}H_1 \)), deuterons (\( ^{+}H_2 \)), tritons (\( ^{+}H_3 \)), alphas (\( ^{++}He_4 \)), and helium–3 nuclei (\( ^{++}He_3 \)) are stopped in the counters and their pulse–heights measured by a multi–channel–analyzer. The absolute energy is calibrated by an americium–241 \( \alpha \) source and a precision research pulser. With the knowledge of the mass and absolute energy deposition of each ion, the channel momentum can be inferred.

It was on the basis of the pre–experiment analysis using the above method that the proton angles conjugate to the pion arms were chosen for E645 (see section 3.4.2). In fact, this technique was used originally in 1985 [74], and the calibration from that analysis had been used by TRIUMF experimenters since that time.

In March 1994, the technique was repeated with more detail and at more energies than the pre–E645 run. These results were consistently \( \sim 0.3\% \) in momentum below the \( \pi-e \) TOF difference results conducted at the same time. It was discovered [101] that the assumption that each ion of the above ions is equally efficient at creating electron–hole pairs in the silicon is not valid [102, 103, 104, 105]: alpha particles are
most efficient, with all others less so in varying degrees. Our results from the March 1994 run confirmed the relative 0.6% defect between alphas and protons between 10 and 20 MeV seen by Martini [102] and Kemper [103], and the 0.1% defect between alphas and doubly charged He\(_3\) ions described by Haynes [105]. Martini speculates that the different rate of K\(\alpha\) x-ray excitation by the incident nuclei is the source of the energy defect, since these x-rays will leave the detector volume without depositing their full energy. Other pulse-height defect effects include Rutherford scattering energy losses [106] (i.e. the recoiling silicon nuclei do not produce e-hole pairs) and other nuclear reaction losses [107]. Although inclusion of the alpha-(proton,helium-3) defect improved the agreement somewhat, it is difficult, and perhaps not even possible with present knowledge, to make all the corrections in a complete and consistent way. Therefore it was concluded that this technique is not reliable for this application, and only the \(\pi\)-e TOF results would be used for the final calibration.

The old 1985 calibration [74] using the SSBD method is also shown in figure D.58. The \(~0.25\)% discrepancy in momentum (0.35% in pion kinetic energy) between this and the new calibration is consistent with the size of the effects outlined above. This suggests that perhaps the calibration presented here is the “true” M11 calibration that has been in effect at least since 1985. However, this is true only to the extent that the NMR probe monitoring the B1 field registers the same field value now as it did 10 years ago, which might not be the case if the probes have been repositioned in the magnet. Considering this, it is difficult to make a firm conclusion about the actual calibration in 1985.\(^2\)

---

\(^2\)An uncertainty in this calibration of about \(\pm0.5\) MeV has been quoted by Brack \textit{et al.} (see \textit{e.g.} [15]) for their differential cross section results to 140 MeV. The old calibration is indeed within 0.5 MeV of the new calibration at 140 MeV, less at lower energies, and so would be within their quoted uncertainties.
Appendix E

GEANT LH$_2$ Target Specifications

Geometry and specifications of key LH$_2$ target components as defined in the GEANT Monte Carlo program used to determine the solid angles are shown in the figures on the following pages. These figures were generated by the interactive version of the program. The actual design specifications of this target can be found in figures 3.8 and 3.9.
Figure E.59: LH₂ cryostat and target modeled in GEANT.
Figure E.60: LH$_2$ target cell and ring modeled in GEANT.
Appendix F

Solid Angle Calculation and Cross Section Outputs

Typical outputs from the routines used to determine the effective solid angles and the differential cross sections for each arm in the TOF Spectrometer are presented in the following sections. This example shows the calculation for 168.8 MeV $\pi^+$ on a LH$_2$ target at 53$^0$ for a two arm coincidence run. The first section shows the GEANT simulation output for arm D (i.e. $\theta_x=115.0^0$, $\theta_p=25.3^0$), the second shows the pion decay correction output, the third displays the LOTUS 123 output for normalizing the two arm solid angles with respect to the average "$\pi$2 only" solid angle, and the fourth section shows the hadronic loss calculations for all the arms. The final section shows the Lotus 123 summary pages which combine the solid angle data with the yield data, target data, and the various beam normalization factors for both the full and empty targets to automatically calculate the differential cross sections and their uncertainty. Details of all of the above are found in chapter 4.
Appendix F. Solid Angle Calculation and Cross Section Outputs

F.1 Sample GEANT Output

******************************
** FINAL STATISTICS **
******************************

TOT : 4634899.
S1 : HIT 4205161. | PI2 : HIT 17151.
   PI 4138311. | PI 15149.
   MU 66845.  | MU 2002.
   EL 5.  | EL 0.
   P 0.  | P 0.
   PI 4008462. | PI 38.
   MU 35687.  | MU 17442.
   EL 3.  | EL 26.
   P 0.  | P 16562.
S2B : HIT 4043631. | VRTX : PI 3898381.
   PI 4007158. | BEAM HIT * S2B_BAR 4043219.
   MU 36469.  | * PI 4007114.
   EL 4.  | * PI * TGT PI 3904747.
   P 0.  | * PI * VRTX PI #GOOD3898381.
TGT : HIT 4035411. | TGT PI * VETO HIT 1761.
   PI 3904747. | * HIT * VETO HIT 33274.
   MU 130652. | GOOD PI * PI1 HIT 83901.
   EL 12. | * PI * PI2 HIT 17151.
   P 0.  | * PI * PI PROT 16562.
VETO: HIT 55197. | PI1 HIT * PI2 HIT 16652.
   PI 285.  | PI2 HIT * PI HIT 16526.
   MU 54855. | * * PI PROT 16526.
   EL 57.  | PI1,2 H * P1 HIT 16134.
   P 0.  | * * * P1 PROT 16134.
PI1 : HIT 83905. | GOOD PI * PI1,2 HIT 16652.
   PI 77540. | * * PI2 H * P1 P 16526.
   MU 6357. | * * TWO-ARM 16134.
   EL 8.  | GOOD PI * PI1, PI2 MU 1007.
   P 0.  | YIELD * VETO HIT 0.

********** Effective Solid Angles **********
#
#   PI2 = 2.646 +/- 0.76 % #
#   P1 * PI2 = 2.550 +/- 0.78 % #
#   PION ARM = 2.569 +/- 0.77 % #
#   TWO ARMS = 2.489 +/- 0.79 % #
#
******************************************************************************
(beam hit & tgt pi) / beam hit = Fd = 0.9658
(good pi & pi1,2 mu) / (good pi & pi1,2 hit) = 6.0473 %
The initial pion kinetic energy is \( = 171.300 \text{ MeV} \)

The input beam parameters are:
- \( \text{Sigma-X} = 0.4000 \text{ cm} \)
- \( \text{Sigma-Y} = 0.2300 \text{ cm} \)
- \( \text{Sigma-Theta} = 0.3400 \text{ deg} \)
- \( \text{Sigma-Phi} = 1.8500 \text{ deg} \)
- \( \frac{dP}{P} = 1.0005 \% \text{ FWHM} \)

The Monte Carlo Window is:
- \( \text{Theta}(l) = 92.00 \text{ deg} \)
- \( \text{Theta}(u) = 138.00 \text{ deg} \)
- \( \text{Beta} (l) = -22.00 \text{ deg} \)
- \( \text{Beta} (u) = 22.00 \text{ deg} \)

\[ \Rightarrow \ D_{\text{OMEGA}} = 601.51 \text{ msr} \]

The Target parameters are:
- \( \text{Thickness} = 1.50 \text{ cm} \)
- \( \text{Angle} = 53.00 \text{ deg} \)

The Pion Arm considered is:
- \( \text{ARM #} = 4 \)
- \( \text{at THETA} = 115.00 \text{ deg} \)
F.2 GEANT Pion Decay Calculation Output

***************
** FINAL STATISTICS **
***************

TOT:
2000000.

S1 : HIT
PI 1723045. | PI2 : HIT 2.
P 1694403. | PI 0.
MU 28640. | MU 0.
EL 2. | EL 2.
P 0. | P 0.

S2A : HIT
PI 1361067. | PI 33.
P 1348534. | PI 1.
MU 12533. | MU 23.
EL 0. | EL 9.
P 0. | P 0.

S2B : HIT
PI 1365969. | VRTX: PI 0.
P 1353442. | BEAM HIT 1349558.
MU 12527. | * PI 1337545.
EL 0. | * PI * TGT PI 1303650.
P 0. | * PI * VRTX PI #GOOD 0.

TGT : HIT
PI 1801195. | TGT PI * VETO HIT 1683188.
P 1725399. | * HIT * VETO HIT 1705858.
MU 75759. | GOOD PI * PI1 HIT 0.
EL 37. | PI * PI2 HIT 0.
P 0. | * PI * PI1 PROT 0.

VETO: HIT
PI 1713212. | PI1 HIT * PI2 HIT 0.
P 1584300. | PI2 HIT * PI HIT 0.
MU 128875. | * * PI1 PROT 0.
EL 37. | PI1,2 H * PI HIT 0.
P 0. | * * PI PROT 0.

PI1 : HIT
5. | GOOD PI * PI1,2 HIT 0.
PI 0. | * * PI2 H * PI P 0.
MU 2. | * * TWO-ARM 0.
EL 3. | GOOD PI * PI1, PI2 MU 0.
P 0. | YIELD * VETO HIT 0.

******************************************************************************
# (BEAM pi & TGT pi) / BEAM hit = Fd = 0.9660 #
******************************************************************************
### Appendix F. Solid Angle Calculation and Cross Section Outputs

#### F.3 Sample Lotus 123 Output File

**GRANT RUN SUMMARY - NO HADRONIC LOSSES**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SET : A</td>
<td>POLARITY : PLUS</td>
<td>ANGLE : 53</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ARM</th>
<th>ANG</th>
<th>p12</th>
<th>sld ang</th>
<th># &lt;P12&gt; normlzd Avg'd</th>
<th>SLD ANG [msr]</th>
<th>ERR [%]</th>
<th>MU CORR</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>60</td>
<td>2.688</td>
<td>2.474</td>
<td>0.84%</td>
<td>2.474</td>
<td>0.84%</td>
<td>5.10%</td>
</tr>
<tr>
<td>B</td>
<td>75</td>
<td>2.635</td>
<td>2.482</td>
<td>0.78%</td>
<td>2.492</td>
<td>0.40%</td>
<td>5.30%</td>
</tr>
<tr>
<td>C</td>
<td>95</td>
<td>2.640</td>
<td>2.496</td>
<td>0.84%</td>
<td>2.502</td>
<td>0.40%</td>
<td>5.60%</td>
</tr>
<tr>
<td>D</td>
<td>115</td>
<td>2.646</td>
<td>2.489</td>
<td>0.79%</td>
<td>2.489</td>
<td>0.40%</td>
<td>6.00%</td>
</tr>
<tr>
<td>E</td>
<td>135</td>
<td>2.667</td>
<td>2.507</td>
<td>0.80%</td>
<td>2.487</td>
<td>0.40%</td>
<td>5.90%</td>
</tr>
<tr>
<td>F</td>
<td>155</td>
<td>2.643</td>
<td>2.485</td>
<td>0.80%</td>
<td>2.488</td>
<td>0.40%</td>
<td>6.20%</td>
</tr>
</tbody>
</table>

**AVG P12 = 2.646 (geom = 2.646)**

% err = 0.009

(%) = 0.36%

**LOSSLESS CORRECTIONS**

- **S1**: 1723045
- **S2A**: 1361067, # F_DECAY = 0.9660
- **S2B**: 1365969
- **BEAM**: 1349558
- **VETO**: 1713212, \( K_{\text{veto}} = 1.0251 \)
- **BPI*TPI**: 1303650, \( K_t = 0.9867 \)
- **TPI**: 1725399
- **TPI*VH**: 1681188
- **TH*VH**: 1705858

--- **COMMENTS** ---

A) pi2 averaged over Arms B..F only ; Arm A affected by target ring
### F.4 Hadronic Loss Calculation Output

168.8 MeV PI+ kinetic energy @ target center
1.5 cm LH2 target @ 53.0 degrees

| THET_$\pi$ | 60. | 75. | 95. | 115. | 135. | 155. |
| T_$\pi$ | 134.4 | 120.9 | 104.4 | 91.0 | 81.4 | 75.5 |

| SLD ANG | 2.474 | 2.492 | 2.502 | 2.489 | 2.487 | 2.488 |
| % stats | 0.84 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 |

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<th>% LOSS</th>
<th>TOT RXN</th>
<th>TOT RXN</th>
<th>TOT RXN</th>
<th>TOT RXN</th>
<th>TOT RXN</th>
<th>TOT RXN</th>
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<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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<td>0.1</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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<td>0.1</td>
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<td>1.1</td>
<td>0.8</td>
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<td>0.7</td>
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<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>AIR</td>
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<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

| P SUM | 3.1 | 2.5 | 2.4 | 1.8 | 1.9 | 1.4 | 1.6 | 1.1 | 1.4 | 1.0 | 1.4 | 0.9 |

| SLD ANG | 2.399 | 2.413 | 2.446 | 2.460 | 2.463 | 2.477 | 2.461 | 2.473 | 2.463 | 2.475 | 2.466 | 2.478 |
| max%err | 0.30 | 0.29 | 0.28 | 0.26 | 0.26 | 0.24 | 0.26 | 0.24 | 0.23 | 0.23 | 0.23 | 0.23 |

| THET_$\pi$ | 52.5 | 44.3 | 34.3 | 25.1 | 16.8 | 9.1 |
| T_$\pi$ | 34.4 | 47.9 | 64.4 | 77.8 | 87.4 | 93.3 |

<table>
<thead>
<tr>
<th>% LOSS</th>
<th>TOT RXN</th>
<th>TOT RXN</th>
<th>TOT RXN</th>
<th>TOT RXN</th>
<th>TOT RXN</th>
<th>TOT RXN</th>
</tr>
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<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
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<tr>
<td>MYLAR</td>
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<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
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<tr>
<td>HE GAS</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
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<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>KAPTON</td>
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<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
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<td>0.4</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

| P SUM | 1.2 | 0.6 | 0.9 | 0.5 | 0.8 | 0.4 | 0.6 | 0.4 | 0.6 | 0.4 | 0.6 | 0.4 |

| TOT SUM | 4.3 | 3.1 | 3.3 | 2.3 | 2.7 | 1.8 | 2.3 | 1.5 | 2.0 | 1.3 | 1.9 | 1.2 |

| SLD ANG | 2.370 | 2.398 | 2.423 | 2.448 | 2.445 | 2.467 | 2.445 | 2.464 | 2.449 | 2.466 | 2.452 | 2.469 |
| max%err | 0.58 | 0.51 | 0.45 | 0.40 | 0.36 | 0.34 |

| FINAL | 2.377 | 2.416 | 2.442 | 2.438 | 2.441 | 2.444 |
| SLD ANG | +1.07% | +0.72% | +0.69% | +0.67% | +0.65% | +0.65% |
### Appendix F. Solid Angle Calculation and Cross Section Outputs

% Beam Hadronic Interactions

<table>
<thead>
<tr>
<th></th>
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<th>SHIELDS</th>
<th>MYLAR</th>
<th>1/2 TGT</th>
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<tr>
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<td>0.10</td>
<td>0.07</td>
<td>0.08</td>
<td>0.18</td>
<td>1.09</td>
</tr>
</tbody>
</table>

Uncorrected Pion Decay/Veto Factors:

- $f_d = 0.9660$
- $k_v = 0.97$

(from GRANT)

CORRECTED BEAM LOSS/VETO FACTORS:

- $F_D = 0.9492$
- $k_v = 0.96$

(MT target)

- $(= 0.9596)$
- $(= 0.97)$

**** COMMENTS ****

a) Arms B..F averaged to $\pi^2 = 2.654 \pm 0.010$ (.40%)
b) Arm A influenced by target ring, not included in average
c) Final SLD ANG = TOT - (TOT-RXN)/4
d) Uncertainty [%] = $\sqrt{\text{stat}^2 + (0.1*\text{loss})^2 + (0.5*\text{max%err})^2 + 0.45^2}$
   from $-0.3\text{cm}$ to $\pi^2$ from tgt------
e) 1/3 S2B losses considered since $f_{\pi}$ already includes losses to S2B, so only losses _after_ BEAM registered contribute
f) Pion rate on target = $k_v * $ Veto hit rate
### Sample Cross Section Summary Output

**Experiment Summary**

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<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
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<td>6.5454E+09</td>
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<tr>
<td>Beam 2</td>
<td>4.2443E+09</td>
</tr>
<tr>
<td>Beam 3</td>
<td>4.2704E+09</td>
</tr>
<tr>
<td>Beam 4</td>
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<tr>
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<td>Beam 34</td>
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<td>Beam 35</td>
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</tr>
<tr>
<td>Beam 36</td>
<td>17132</td>
</tr>
</tbody>
</table>

**Beam Factors**

- $\Phi_{\pi} = 0.9880$
- $\Phi_{\text{prot}} = 1.0000$
- $\Phi_{\text{MC}} = 0.9473$
- $\Phi_{\text{it}} = 0.9817$
- $\Phi_{\text{p}} = 1.0435$
- $\Phi_{\text{veto}} = 0.9539$
- $\Phi_{\text{in-chan}} = 0.9980$

**Effective Beam**

- $\Phi_{\text{beam}} = \Phi_{\pi} \Phi_{\text{it}} \Phi_{\text{decay}} \Phi_{\text{sin-gles}} (1 - \Phi_{\text{prot}})$

**Poisson Parameters**

- $\lambda' = 0.0422$
- $\lambda = 0.0655$
- $\lambda_{\text{cut}} = 0.0369$

**Yields**

- Poisson: 327402428326637343614228646656
- Veto: 327372439326775343784223246445

**Effective Cross Section**

- $\sigma_{\text{eff}} = \Phi_{\text{beam}} \lambda' \lambda_{\text{cut}}$

**Normalization**

- No external triggers found.
EXPERIMENT 645 DATASUMMARY

BACKGROUND

RUN t: 99 T pi [MeV]: 168.80 POLARITY: PLUS

DATE: TARGET: MTT: TOT: ANGLE: 53

SET: A C, C', C.

LOG PAGE: 133 - LOG NOTES

LM2 MONOK? YES B2 SCAM NO CMAR LOG

SETTINGS

VJ = MID S Bl [G) NOD: D: 1.07 Nd/p: p= ABSORBER: MOOD U/D: 0.92

AVG RATES - BEAM MONITORS


EXPERIMENT: 1.61 1.14 1.13 1.08 1.62 1.25 27 1.39

GRANT: 1.61 1.27 1.28 1.26 1.60

EXP = VIS? SAMPLES [Mz]: [2.1 SCALARS

BEAM: 2.64839E+09 EVENTS: 7370 BUSY_BAR: 7367 CLOCK [sec]: 2755.9

2nd BUCKET: 1.398090E+08 LAMS: 7367 SAMPLES: 5786

3rd BUCKET: 6.590600E+06 SUSI: 7367 S2B_BAR: 2.455680E+07 EXPVIS?

BEAM FACTORS

P_p: 0.98801 — P_prot: 1.0000 F_d [MC): 0.9577 P_l: 0.9996

P_a) poiss): 1.0506 F_s) veto): 0.9515 P_d [chan]: 0.9980

POISSON PARAMETERS

BEAM2/BEAM1 = 0.0472 —> LAMBDA' = 0.0483 Tgt_p: veto = 0.9700 LAMBDA = 0.0730

BEAM3/BEAM2 = 0.0471 BEAM/R = 0.0478 R_tgt — pi = 1.5517 LAMBDA_CUT = 0.0451

BEAM*P_p*P_l*P_decay*P_singles: 0.98800.99960.95771.04721.0000

VETO*91*91*

YIELDS

ABCDEP

BEAM_eff*

POISSON: 4354014046258961562

VETO: 4504123946419001552
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Appendix G

Derivation of "Veto" Multiple Pion Correction

Consider the total rate of pions incident on the target (including multiples), where the probability of n pions in a beam bucket follows a Poisson distribution:

\[ R_{tot} = v \left( 1 \cdot P1 + 2 \cdot P2 + \cdots \right) \]

\[ = v \left( e^{-\lambda} + 2 \cdot \frac{\lambda^2}{2!} e^{-\lambda} + \cdots \right) = v\lambda \]  \hspace{1cm} (G.64)

where \( \lambda \) is seen to represent the average number of pions per beam bucket, and \( v \) is the cyclotron frequency 23.058 MHz. As outlined in section 4.2.4, the in-beam counters cannot distinguish multiple hits, and so the BEAM rate corresponds to "at least one pion" in the beam, or \( B_{meas} = v \cdot (1 - e^{-\lambda}) \). For the case of the "veto" correction scheme, we want only the fraction of beam bursts with only one pion, \( B_{\text{in}} = B_{\text{meas}} \cdot f_S^\nu \). Clearly, the single-pion rate is the first term in G.64, modified by the fraction \( f \) of the total beam covered by the in-beam counters, so that

\[ f_S^\nu = \frac{f \cdot \lambda e^{-\lambda}}{1 - e^{-\lambda}} \]  \hspace{1cm} (G.65)

The VETO counter is used to reject events from the \( \pi p \) scattering yields when a hit is registered in the VETO counter at the same time that a HII EVENT is registered by the TOF Spectrometer. Therefore, the "miss veto", or \( V_M \), rate corresponds to the case where the only pion in the beam caused an EVENT \( i.e. \)

\[ R(V_M) = \sigma \cdot v(f \cdot \lambda e^{-\lambda}) \]

where \( \sigma \) represents heuristically the differential cross section, solid angle, etc., from equation 4.50 necessary to calculate the scattered pion rate from the beam intensities
i.e.

\[
\frac{\text{Single pion EVENT rate}}{\text{Single pion BEAM rate}} = \frac{\sigma \cdot \nu(f \cdot \lambda e^{-\lambda})}{\nu(f \cdot \lambda e^{-\lambda})} = \sigma
\]

Now consider more than one pion in a beam bucket. The probability of \textit{at least} one pion hitting the in–beam counters is \(1 - \text{Prob(\text{all missed})}\), \textit{i.e.} \(1 - g^n\). where \(g = 1 - f\). So the rate of \(n\) pions where at least one hit the in–beam counters and target (causing an EVENT) is

\[
R(n \pi^i; 1 \text{ caused EVENT}) = \sigma \nu \left(1 - g^n\right) \frac{\lambda^n}{n!}
\]

The rate of VETO counter hits subject to EVENT ("hitveto" or \(V_H\)) is

\[
R(V_H) = R(V_H) = \sum_{n=2}^{\infty} R(n \pi^i; 1 \text{ caused EVENT}) = \sigma \nu \lambda e^{-\lambda} \left(e^\lambda - 1 - g(e^\lambda - 1)\right)
\]

Therefore, defining \(K_v = \frac{V_H}{V_M}\), using equations G.65 and G.66, one can show after a little algebra that

\[
K_v = \frac{1 - e^{-\lambda}}{f e^{-\lambda}} + (e^\lambda - 1)
\]

so that

\[
\frac{f \cdot e^{-\lambda}}{1 - e^{-\lambda}} = K_v \left(\frac{1}{1 + K_v(e^\lambda - 1)}\right)
\]

One now requires an expression for \(\lambda\) in terms of \(K_v\) to complete the derivation. Again using equations G.65 and G.66, one can show that

\[
1 + K_v^{-1} = \frac{V_M + V_H}{V_M} = e^\lambda \frac{1 - ge^{-\lambda}}{f}
\]

so that

\[
\lambda_v = \ln\left(\frac{f}{1 - ge^{-\lambda}}\right)
\]

So in this approach, \(\lambda\) is defined recursively. One method to do this is to start with the approximation \(g=0\), \(\lambda_0 = \ln(1 + K_v^{-1})\) as the first step in the iteration procedure.

By substituting equation G.67 and G.68 into equation G.65, one arrives at the final expression for \(f_S^v\):

\[
f_S^v = K_V \left(\frac{1}{1 + K_v(1 - e^\lambda)}\right) \cdot \lambda_v
\]