AN EXPERIMENTAL INVESTIGATION OF THE MIXING
OF TWO OPTICAL FREQUENCY EM WAVES IN A PLASMA

by

LAWRENCE ALLAN GODFREY

B.Sc., University of British Columbia, 1971
M.Sc., University of British Columbia, 1973

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Department of PHYSICS

The University of British Columbia
2075 Wesbrook Place
Vancouver, Canada
V6T 1W5

Date S E P T. 1977
The effect of optical mixing of two tunable dye lasers at frequencies near the plasma frequency has been experimentally investigated in a helium plasma jet. It has been shown that the wave mixing produces longitudinal plasma oscillations at the frequency and wave vector of the mixing force. The driven waves were detected by scattering a third diagnostic light wave from their density fluctuation. The scattering signals increased to as much as seven times the signal observed when scattering from the thermal fluctuations alone. The spectrum of the spectral density function of the induced fluctuations has been measured, as well as the dependence of its amplitude on the power of the mixing light beams. These results agree well with theoretical calculations based on a simple model of the mixing effect of a single electron in the field of two electromagnetic waves. The response of the plasma to optical mixing at different frequencies has also been measured. This spectrum agrees in part with theoretical predictions, but has features not explained by the simple model.
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Chapter I

INTRODUCTION

Wave mixing was first proposed by Kroll, Ron and Rostocker in 1964 as a means of increasing the usually very small cross-section for scattering of laser light for plasma diagnostics. When two intense light sources are incident on a plasma, the electrons motion has a mixing effect which results a longitudinal driving force. The frequency of this driving force is equal to the frequency difference of the two incident EM waves. Matching this frequency difference to a plasma resonance allows one to drive well-defined plasma waves.

Besides being used as a diagnostic aid, optical wave mixing can be a possible tool to help fill the gap between theoretical and experimental studies of nonlinear wave-plasma interactions. Wave mixing can be classed as a nonlinear wave-plasma interaction in which different electromagnetic, electron, or ion wave motions cannot be considered as independent modes of the plasma. There has been a large increase in interest in nonlinear wave-plasma interactions during recent years, generated mainly by the quest for controlled nuclear fusion, but also by experiments to study and control the ionosphere with high power microwaves. Both these areas of research require detailed knowledge of plasma wave phenomena.
To date the experimental studies have been limited mostly to plasmas of density less than 10^{14} \text{ cm}^{-3} because of the necessity of using microwave power sources. Driving waves in a plasma generally requires that the frequencies of the driver be close to plasma normal mode resonance frequencies. Presently, the sources of high power EM radiation are unavailable between the range of a few GHz from microwaves and a few thousands of GHz from CO_{2} lasers. It has been impossible to do controlled studies of plasmas with densities of 10^{14} to 10^{19} \text{ cm}^{-3} until Stansfield, Nodwell and Meyer showed that plasma waves can be driven by mixing two intense, high frequency light sources.

Besides their limitation in densities, microwave experiments have the drawback that the smallest wavelength of the radiation is of the order of a millimeter. This is comparable to the sheath thickness of the plasma boundaries, in which the electron density goes to zero. The plasma boundary can not be considered sharp, a circumstance which complicates the interpretation of experimental results. It is also difficult to create plasmas which are uniform over the area of a microwave beam, a further complication to data interpretation. The perturbation caused by mechanical probes placed in the plasma vessels must also be considered.

Stable uniform plasmas with densities approaching of 10^{19} \text{ cm}^{-3} have only recently become available for plasma wave studies using CO_{2} lasers.
Wave studies at this density are still in their infancy.

Of course, irradiating DT pellets with Terawatt laser light beams can be considered as plasma waves studies, but the processes of these experiments are almost too complex to unravel.

Wave mixing then appears to be a very desirable diagnostic tool. With the beating of two laser beams, it should be possible to produce plasma waves of controlled amplitude, spectrum and wave vector distribution. The frequency of the waves can match the resonant modes of plasmas of any density, including the previously inaccessible $10^{14}$ to $10^{19}$ cm$^{-3}$ density range. The mixing experiments themselves test our interpretation of the physics of the interaction of the mixing light beams with the plasma, and the generated waves, with their well defined frequency and wave vectors, can be used for further wave-plasma studies. The mixing light beams are transparent to the plasma, and there are no mechanical probes to cause perturbations. The well defined interaction region is small enough that plasma density gradients usually need not be considered. Optical wave mixing, once its experimental feasibility and principles have been established, will allow a systematic study of wave-wave and wave-particle interactions in a wide variety of plasmas.

This thesis presents the experimental results of wave mixing of two tunable dye lasers near the electron resonance in a plasma with a density of $2 \times 10^{16}$ cm$^{-3}$. It
is both a verification and extension of Stansfield's work in which it was shown that optical wave mixing has a measurable effect on the plasma. The experiments presented here were chosen to test the theoretical results of Meyer which take into account the spatial and spectral content of the mixing beams.

Wave mixing has been reported by other authors, but mainly in the microwave region. Stern and Tzoar mixed two high frequency (\(\sim 30\) GHz) microwave beams in a 0.8 cm diameter mercury discharge and observed an increase in the naturally occurring radiation from the plasma at its resonance frequency (\(\sim 3\) GHz).

Kuhn, Leheny and Marshall did microwave mixing using a neon afterglow in a square X-band wave-guide. They obtained good quantitative agreement between their experimental results and theory modified to account for their finite geometry. The induced density fluctuations were observed by probes and scattering of microwaves.

Other authors have reported 'wave mixing', but their work is fundamentally different from the wave mixing reported here. These authors excited normal modes of the plasma with microwaves. It is these electron waves which interact, rather than the initial mixing beams.

Since this thesis is an experimental report, emphasis has been placed on the physics of the wave mixing as well as the techniques of the actual experiments. The main part of the thesis is therefore divided into four sections.
Chapter II presents a simple physical model of wave mixing in a plasma. The theory of Meyer is outlined. Experimental considerations are presented including checks on application of the theory to the experimental conditions.

Chapter III presents the experimental system. Included is a description of the plasma apparatus, the wave mixing light sources, and the optical and electrical arrangements.

The experiments themselves are presented in Chapter IV. The results are compared with theory. The method of data reduction has been placed in Appendix II to make this chapter more readable.

Finally a summary and conclusions are given in Chapter V. This chapter details the original contributions of the author. Suggestions for improvements of the present experimental arrangement, as well as for further possible studies are presented.
Chapter II

THEORY AND EXPERIMENTAL CONSIDERATIONS

In this chapter the physics of wave mixing is presented. The model of a test electron in the field of two high frequency electromagnetic waves is described. The results of this model are incorporated into the collisionless Vlasov equation so that the perturbations caused by the wave mixing can be evaluated. Two possible methods of detecting these perturbations are discussed. The scattering cross-section of an electromagnetic wave by the induced plasma waves is then evaluated, including the expected relative signal amplitudes for laser light scattering. Then the considerations in choosing the experimental arrangement and equipment used in the thesis are presented. Finally, the assumptions made in the theoretical derivation are evaluated for the actual experimental situation to check their validity.

A. Physical Model

An electron in the field of a single high frequency electromagnetic (EM) wave will not experience a detectable perturbation in its orbit. This assumes that the frequency of the light wave is so much higher than any
natural frequency of the plasma, that the electron, even though it will obtain a considerable velocity, does not have time to move from its orbit. When there are two high frequency EM waves present, this situation changes. The test particle 'mixes' the forces from the two waves, resulting in a low frequency driving force which can perturb the electron orbit. The mixing effect can be understood using the following description (Figure II-1).

The electron is accelerated by the electric field \( E_1 \) of the first EM wave. It then experiences a Lorentz force from the magnetic field \( B_2 \) of the second wave in the direction \( E_1 \times B_2 \). Simultaneously the electric field of the second wave imparts a velocity to the electron so that it also experiences a Lorentz force in the direction \( E_2 \times B_1 \). When these forces are added together and the sums performed, one contribution comes from a term whose frequency is the frequency difference of the two EM waves. This small acceleration, if the mixing frequencies are judiciously chosen to match a natural plasma resonance, can measurably perturb the electron from its natural orbit.

The magnitude and direction of the force at the frequency difference is calculated below, assuming plane monochromatic mixing waves. A more detailed calculation is outlined in section B.

The force applied to the electron by the two EM waves is
FIGURE II-1  Electron in the field of two EM waves.
\[
\mathbf{m}\ddot{\mathbf{v}} = e \sum_{i=1}^{2} \left( \mathbf{E}_i + \mathbf{v} \times \mathbf{B}_i \right)
\] ...(1)

where \(m, e\) and \(\mathbf{v}\) are the mass, charge and velocity of the electron, and \(\mathbf{E}_i\) and \(\mathbf{B}_i\) are the electric and magnetic fields of the \(i\)th EM wave with frequency \(\omega_i\) and wave vector \(\mathbf{k}_i\). We have for an EM wave at time \(t\) and position \(\mathbf{r}\):

\[
\mathbf{E}_i = \mathbf{E}_{i0} \cos(\theta_i)
\]

\[
\mathbf{B}_i = \frac{\mathbf{k}_i \times \mathbf{E}_i}{\omega_i}
\]

\[
\theta_i = \omega_i t - \mathbf{k}_i \cdot \mathbf{r} + \phi_i
\] ...(2)

Since the phase velocity of the mixing waves \(\frac{\omega_i}{k_i}\) is \(c\), the speed of light, to first order we may neglect the magnetic field term in (1). Integrating the first order acceleration \(\dot{\mathbf{V}}^{(1)}\):

\[
\dot{\mathbf{V}}^{(1)} = \frac{e}{m} \sum_{i=1}^{2} \mathbf{E}_i
\] ...(3)

we get the first order velocity \(\mathbf{V}^{(1)}\):

\[
\mathbf{V}^{(1)} = \frac{e}{m} \sum_{i=1}^{2} \frac{\mathbf{E}_{i0}}{\omega_i} \sin(\theta_i) + \mathbf{V}^{(o)}
\] ...(4)

We may arbitrarily choose the original velocity \(\mathbf{V}^{(o)}\) to be
zero for our test particle.

The electron has obtained a velocity given by (4). The maximum excursion $R_{\text{max}}$ that the electron will make from its position $R^{(0)}$ is found by integrating (4) again:

$$R = -\frac{e}{m} \sum_{i=1}^{2} \frac{E_i}{(\omega_i)^2} + R^{(0)}$$

$$R_{\text{max}} = -\frac{e}{m} \sum_{i=1}^{2} \frac{E_{i0}}{(\omega_i)^2}$$

...(5)

If the electric fields are strong enough that $V$ approaches $c$ (requiring $>90000$ MW focused to 100 microns), we still have for visible light that $R_{\text{max}}<10^{-7}$ m. This displacement is much too small to be measured by the techniques in this experiment. The velocity it does obtain, however, interacts with the magnetic field to produce a second order acceleration $\dot{V}^{(2)}$ of the test electron given by:

$$\dot{V}^{(2)} = -\frac{e}{m} V^{(1)} X \sum_{j=1}^{2} B_j$$

...(6)

Using (2) and (3) we have

$$\dot{V}^{(2)} = \frac{e^2}{m^2} \sum_{i,j=1}^{2} \frac{E_{i0} X[k_j X E_{j0}]}{\omega_i \omega_j} \sin(\theta_i) \sin(\theta_j)$$

...(7)

We now make the simplifying assumption that the mixing beams are polarized perpendicular to the plane defined by
(k_1, k_2). Then we have

\[ \dot{V}^{(2)} = \frac{e^2}{m^2} \sum_{i,j=1}^{2} \frac{E_i E_j}{\omega_i \omega_j} \left( \sin(\theta_i + \theta_j) + \sin(\theta_i - \theta_j) \right) k_j \]

\[ \ldots \text{(8)} \]

There are now a total of 8 terms for the second order velocity. The terms which oscillate at the sum frequencies can immediately be neglected, as we have shown above. We are left with

\[ \dot{V}^{(2)} = -\frac{e^2}{2m^2 c} \frac{E_{10} E_{20}}{\omega_1 \omega_2} \Delta \mathbf{k} \sin(\Delta \omega t - \Delta \mathbf{k} \cdot \mathbf{x} + \phi_1 - \phi_2) \]

\[ \ldots \text{(9)} \]

where

\[ \Delta \mathbf{k} = k_2 - k_1 \]

\[ \Delta \omega = \omega_2 - \omega_1 \]

\[ \ldots \text{(10)} \]

\( \Delta \mathbf{k} \) and \( \Delta \omega \) will be referred to as the mixing wave vector and frequency.

Note the important feature that the test electron experiences a longitudinal force since the direction of propagation is parallel to the wave vector \( \Delta \mathbf{k} \). The wave mixing should then drive longitudinal plasma electron waves at frequency \( \Delta \omega \) and wave vector \( \Delta \mathbf{k} \). When one considers the spatial and spectral structure of the mixing waves, then one would expect a spectrum of plasma waves with a spread in frequency and wave vector. The calculation of these spectra is outlined in the following section.
B. Theory

The effect of wave mixing on the plasma has been calculated by several authors. These include the original approximate calculation of the scattering cross section,¹ calculations for magnetic fields¹¹, for density gradients¹²,¹³,¹⁴, numerical simulations¹⁵,¹⁶, plasma heating¹⁷ and for the effect on the pump wave¹⁸,¹⁹. All of the above treatments assume plane monochromatic mixing waves, except for the latter which allows for finite beam diameters. The calculation which is most directly applicable to an experiment has been done by Meyer,⁷ who takes into account the spatial and spectral structure of the mixing waves. An outline of this derivation is given here.

Let us first anticipate some general results of this derivation. We have calculated in the above model the driving force on a free electron due to the mixing of two high frequency EM waves. Electrons in a plasma, however, are not free, but are influenced by the presence of other charged particles. Wave mixing can be thought of in terms of a variable frequency longitudinal driving force acting on the resonant system of ions and electrons that make up the plasma. The calculation of the effect of mixing EM waves on the plasma must first reflect the characteristics of the mixing forces. Their frequency and wave vector spectra will result in similar spectra for the driven density waves. The amplitude of the waves for different driving frequencies will depend on the resonant
response structure of the plasma system. This ability of the plasma to sustain the driven waves in turn will depend strongly on the wavelength of the driven waves compared to the plasma scale length. The correlation parameter, \( \alpha = \frac{k_d}{\Delta k} \), is a measure of this ratio, where \( k_d = e^2 N_0 / \varepsilon_0 \kappa T_e \) is the inverse Debye length.

The plasma responds to induced density fluctuations in such a manner that the fluctuations are reduced and charge neutrality is maintained. In practice, however, the charge density is not zero over scale lengths smaller than the Debye length: variations in electron density within a Debye sphere does not strongly influence the neighbouring plasma because of the shielding effect of the ions (Debye shielding). For small values of alpha, the wavelength of the driven wave is much smaller than the Debye length. Therefore, the plasma should respond to the driving force as if it were a system of independent electrons. It should be possible to drive plasma waves over a large frequency range.

The plasma is very efficient at maintaining charge neutrality on scale lengths larger than the Debye length. For large values of alpha, the wavelength of the induced wave is much larger than the Debye length, so that the driving force cannot produce plasma oscillations of appreciable amplitude. However, a uniform plasma does have natural oscillation resonances, one of which is near the plasma frequency. It should therefore be possible to drive large amplitude waves for large alpha, as long as
the driving frequency matches a plasma resonance.

The calculation of the effect of the mixing on the plasma should then reflect the sharp resonance for large alpha and the broad spectra for small alpha, as well as the frequency and wave vector structure of the driving force. Let us now proceed with the outline of the derivation.

The effect of wave mixing on the ions is neglected because of their large mass compared to the electrons. The second order acceleration of the electron, analogous to equation (9), is calculated assuming that the electromagnetic waves are generated by classical damped oscillators. The damping frequency $\gamma$ is much smaller than the central frequencies $(\omega_{10}, \omega_{20})$ of the two waves.

The second order acceleration is inserted into the collisionless Vlasov equation for the electrons. The usual Vlasov equation is used for the ions, in line with neglecting the wave mixing on the heavier particles.

Perturbation theory is then evoked to linearize the Vlasov equations. There is no external electric field, but a constant magnetic field is allowed. The equilibrium velocity distribution is assumed to be Maxwellian. Fourier transforms in space and Laplace transforms in time are performed.

Bernstein's$^{20}$ and Haqfors's$^{21}$ calculations aid in the evaluation of the density fluctuation. Realizing that the measurable quantity is not the density fluctuation $N_e(\Delta k, \Delta \omega)$, but is its spectral density
S(Δk, Δω) \propto \langle |N_e(Δk, Δω)|^2 \rangle, \hspace{1cm} \langle |N_e(Δk, Δω)|^2 \rangle is now calculated. To perform this derivation, it is assumed that the observation time T is much larger than the damping rate γ. Assumptions about the mixing beams must also be made. These are:

- the light waves are composed of a great number of modes during the time T,
- each light beam is considered to be a stationary random process with random starting times and phases for the modes,
- each light beam is assumed to be a plane Gaussian wave with effective diameter b.

We finally get

\[ \langle |N_e(Δk, Δω)|^2 \rangle = \langle |N_{th}(Δk, Δω)|^2 \rangle + \langle |N_{ind}(Δk, Δω)|^2 \rangle \]

... (11)

where

\[ \langle |N_{th}(Δk, Δω)|^2 \rangle = \frac{N V}{πΔω} \frac{\text{Im}(-F_e) |1 + Z\alpha^2 F_1|^2 + Z\alpha^4 \text{Im}(-F_1) F_2|^2}{|1 + \alpha^2 (F_e + ZF_1)|^2} \]

... (12)
$$N_{\text{ind}}(\Delta k, \Delta \omega)^2 = I_1 I_2 \cos^2 \frac{k_d}{(8\pi mc)^2} \left| \frac{F_e (1 + \alpha^2 F_1)}{1 + \alpha^2 (F_e + ZF_i)} \right|^2$$

$$x \frac{2\gamma}{\omega_1 \omega_2} \int_0^\infty P_1(\omega_1) P_2(\omega_2) d\omega_1 d\omega_2 \frac{4\gamma^2 + (\Delta \omega - \omega_1 - \omega_2)^2}{(4\gamma^2 + (\Delta \omega - \omega_1 - \omega_2)^2)^2}$$

$$x \frac{b^2}{\pi} \exp \left[ -b \left( \frac{(\Delta k)^2_x}{\sin^2(\beta \theta_{\text{mix}})} + (\Delta k)^2_y \right) \right]$$

...(13)

The variables in this expressions are:

- $Ze$ is the charge on the ions,
- $\alpha = k_d / |\Delta k|$ (correlation parameter),
- $k_d = e^2 N_0 / \epsilon_0 \kappa T$ (inverse Debye length),
- $\epsilon_0$ is the permittivity of free space,
- $\kappa$ is Boltzmann's constant,
- $F_e$, $F_i$, for no external magnetic field, reduce to:

$$F_e = F_e(x) = 1 - xe^{-x^2} \int_0^x e^{-t^2} dt - \frac{1}{2} \pi^2 xe^{-x^2}$$

$$x = \Delta \omega / (\Delta k)$$

$$a^2 = 2\kappa T_e / m$$

$$F_i = F_e \left[ \left( \frac{MT_e}{mT_i} \right)^{1/2} x \right]$$

...(14)

$T_e$, $T_i$ are the electron and ion temperatures,

$M$ is the mass of the ion,

$I_1, I_2$ are the mixing beam powers,
- $\beta$ is the angle between the electric field vectors of the mixing beams,
- $\gamma$ is the damping rate,
- $\omega_{i0}$ is the line centre of the spectral distributions $\{P_i(\omega_i)\}$ of the $i$th mixing beam. $P_i$ is normalized to unit area,
- $b$ is the radius of the effective size of the Gaussian plane waves,
- $\theta_{\text{mix}}$ is the mixing angle included between $k_1$ and $k_2$.
- the $z$-axis is taken as parallel to $\Delta k$,
- and the $y$-axis is perpendicular to the plane $(k_1, k_2)$.

In the above expression for the density fluctuations, $\langle |N_{\text{th}}(\Delta k, \Delta \omega)|^2 \rangle$ is immediately recognized as the contribution from the thermal fluctuations. These exist with or without wave mixing. The second term, $\langle |N_{\text{ind}}(\Delta k, \Delta \omega)|^2 \rangle$, is the result of the wave mixing. It can be rewritten in the form

$$\langle |N_{\text{ind}}(\Delta k, \Delta \omega)|^2 \rangle = \frac{1}{2} \int \frac{d^3}{(2\pi)^3} C(N_e, T_e) W(\Delta \omega) K(\Delta k) R(\Delta \omega/\Delta k; N_e, T_e)$$

... (15)

where

- $C$ is a constant for a particular plasma:

$$C = \frac{k^4_d}{(8\pi mc)^2}$$

... (16)
- $I_1$ and $I_2$ are the powers of the incident beams,
- $W$ is the effect of the finite frequency width of the mixing beams

$$W = \frac{2\gamma}{\omega_{10}^2 \omega_{20}^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{P_1(\omega_1) P_2(\omega_2) \, d\omega_1 \, d\omega_2}{4\gamma^2 + (\Delta\omega - \omega_1 + \omega_2)^2}$$

... (17)

This function is peaked at $\Delta\omega = \omega_{10} - \omega_{20}$.

- $K$ is the effect of the spatial structure of the mixing beams

$$K = \cos^2 \beta \frac{b^2}{2\pi} \exp \left[ -b^2 \left( \frac{(\Delta k)^2}{x} + (\Delta k)^2 \right) \right]$$

... (18)

This function, unlike the thermal spectrum, is sharply peaked in the direction of $\Delta k$.

- $R$ describes the response of the plasma to mixing at $\Delta k, \Delta \omega$

$$R = \left| \frac{F_e (1 + Z a^2 F_1)}{1 + a^2 (F_e + Z F_1)} \right|^2$$

... (19)

The experiments reported in this thesis were designed to test some of the predictions of this formula (15). First, the induced fluctuations were measured as a function of the mixing intensity to test the $I_1 x I_2$
dependence. Second, the spectrum of the induced wave was measured, keeping all parameters of the mixing constant. This tests the function \( W \).

Finally, the response function of the plasma was measured at different driving frequencies. This is the most important aspect of the thesis. Measurement of the intensity and frequency dependence \( I_1 \times I_2 \times W \) is necessary, but these are functions of the controllable mixing beams. A measurement of the response function tests our understanding of the physical processes involved in the interaction of the mixing light beams with the plasma.

Figure II-2 is a schematic of a possible spectrum with wave vector equal to \( \Delta k \), and scattering parameter alpha=1. The height of the spectrum due to the induced component is 'arbitrary' because of its dependence on the powers of the mixing waves. These powers have been chosen to give equal contribution from the thermal component and the induced component of the density fluctuation spectrum. The width of the induced spike is determined by the width of the spectral probability functions \( P_{ij}(\omega_i) \), as long as the mixing beams' spectral line profiles are narrower than the width of the response function \( R \). If the frequency difference between the two mixing waves is changed, then the position of the induced contribution will shift, and its peak height will change according to the value of the response function.

The response of the plasma for different values of alpha are illustrated in Figure II-3. The electron
FIGURE II-2  A typical induced and thermal fluctuation spectrum.
FIGURE II-3  Theoretical response of the plasma to different driving frequencies.
temperature and density have been fixed at \( N_0 = 2 \times 10^{16} \text{ cm}^{-3} \) and \( T_0 = 19000 \, ^\circ \text{K} \), so to change alpha, we must change the wave vector of the induced wave. The amplitude of the response functions have been normalized to unity. The actual values of the maximum response for alpha equal to 1, 2 and 4 are 0.35, 0.47 and 156, respectively. The curve for alpha equal to 10 is too narrow to resolve without special programming techniques. To illustrate a curve for very small alpha, parameters different than for our plasma must be used. The smallest possible alpha that is obtainable in this experiment is 0.97 for \( \Theta_{\text{mix}} = 180^\circ \). The broken line in Figure II-3 has been drawn for conditions exactly the same as for the alpha=1 case, except that the temperature is 190,000 \( ^\circ \text{K} \). This alpha=0.1 curve has a maximum value of \( \approx 0.98 \).

The resonance of the plasma, described by the response function \( R \), has the qualitative dependence on alpha as described in the introduction to this section. For small values of alpha, the wavelength of the wave being driven is much smaller than the Debye length. This means that the electrons tend to move independently of one another, and one would expect that the response of the plasma should have a broad spectrum.

For large values of alpha, the wavelength of the driven wave is much larger than the Debye length. Now collective effects are important, and the waves are strongly damped unless they satisfy the Bohm-Gross dispersion relation. The graphs show that the resonant
response does approach the plasma frequency from the high frequency side as alpha is increased.

The intermediate case is quite interesting, since the driven waves are governed both by the random motion of the particles and the longer range collective effects. We then have the broad resonances as for alpha=2.

In the next section, possible methods of observing the induced fluctuations are presented.

C. Methods Of Detection

Since Kroll, Ron, and Rostoker¹ first proposed wave mixing as a means of increasing the light scattering cross section of a plasma, scattering is an obvious possible choice to examine the effect of wave mixing. The scattering cross section is directly proportional to

\[ |\langle N_e(\Delta k, \Delta \omega) \rangle|^2 = |\langle N_{th}(\Delta k, \Delta \omega) \rangle|^2 + |\langle N_{ind}(\Delta k, \Delta \omega) \rangle|^2. \]

The scattering signal will be composed of two parts: the thermal fluctuation signal, and the induced fluctuation signal. If the mixing scattering signal as a function of frequency and wave vector is experimentally determined, then by subtracting the normal scattering signal component we are left with the induced scattering signal. This gives a relative measure of \( |\langle N_{ind}(\Delta k, \Delta \omega) \rangle| \) as a function of frequency and wave vector.

Another method of determining the effect of the mixing is to look at the intensity of one the of the
mixing beams, as suggested by Weibel\textsuperscript{18} in 1976. It has been shown by many authors\textsuperscript{22} that during the interaction of the three waves, energy is transferred from the higher frequency EM wave to the plasma wave and the lower frequency EM wave. As Weibel argues, if the frequency of one of the mixing beams is changed rapidly so that the mixing frequency is swept through a resonance of the plasma (a resonance of $R$), then this energy transfer will be modulated.

He suggests that, under favourable conditions, a power of the order of 10 MW in one of the mixing beams would be sufficient to produce a 1% change in the power of the other beam. He does not suggest a means of sweeping the frequency of one of the mixing beams on the time scale required for short pulse lasers ($\sim 10^6$ nm/sec). An experiment to test this concept would be to look at the intensity of the modulated beam with the mixing beams tuned to match a resonance of $R$. The intensity of the higher frequency beam should vary in the presence of the lower frequency beam. Measurements of the intensity of the higher frequency beam before and after the interaction region would indicate any change in intensity due to the mixing. Signal error considerations suggest that the minimum useful change in power would be of the order of 10%, requiring a 100 MW tunable light source. Such a high power requirement for a dye laser makes this method of observing the presence of the induced density fluctuation unattractive.
We are left with scattering diagnostics to detect the fluctuations. The experiments presented here, of course, were designed for scattering since Weibel's recent suggestions were not available when this research program was begun.

D. Signal Calculations

For the waves driven by the wave mixing to be detectable by laser scattering, the amplitude of the corresponding density fluctuation must be at least the order of magnitude of the thermal fluctuations at their maximum. Since thermal fluctuations are readily observable, the increase in scattering signal due to the wave mixing should also be detectable. The relative amplitude of the induced scattering signal as compared to the thermal scattering signal is calculated below.

The differential scattering cross section per unit frequency \(\frac{d\omega}{2\pi}\), per unit solid angle \(d\Omega\), per unit incident flux, for each electron is

\[
d^2\sigma = r_0^2 \left(1 - \sin^2 \theta \cos^2 \phi \right) S(\Delta k, \Delta \omega) \frac{d\omega \ d\Omega}{2\pi}
\]

...(20)

Here

- \(r_0\) is the classical electron radius,
- \(\theta\) is the scattering angle,
- \(\phi\) is the angle between the polarization vector of the incident light and the
scattering plane,

- $S$ is the spectral density function given by

$$S = \frac{2\pi}{N_0 V} \langle |N_e(\Delta k, \Delta \omega)|^2 \rangle$$

...(21)

$N_0$ is the average electron density,

$V$ is the scattering volume.

The amount of power scattered $I_{\text{scat}}$ is then

$$I_{\text{scat}} = \left( \frac{I_d}{A} \right) \left( \frac{2\pi}{N_0 V} \right) \frac{\langle |N_e|^2 \rangle}{N_0 V} \tau_o^2 \left( 1 - \sin^2 \theta \cos^2 \phi \right) \frac{d\omega}{2\pi} d\Omega$$

...(22)

where $I_d$ is the diagnostic power through the area $A$. One actually measures a range of $\omega$ and $k$, so that $I_{\text{obs}}$, the signal received is

$$I_{\text{obs}}(\Delta k_{\text{obs}}, \Delta \omega_{\text{obs}}) = \int d\Omega \int d\omega \ T_k(\Delta k - \Delta k_{\text{obs}}) \ T_\omega(\Delta \omega - \Delta \omega_{\text{obs}}) \ I_{\text{scat}}$$

...(23)

where $T_k$ and $T_\omega$ are the wave vector and frequency transmission functions of the detection system. The wave vector transmission function is centred at the wave vector $k_d - k_{\text{obs}}$, where $k_d$ and $k_{\text{obs}}$ are the wave vectors of the incident and scattered light, respectively. The frequency transmission function is centred at $\Delta \omega_{\text{obs}}$, the frequency shift of the detector from the diagnostic frequency.
These transmission functions should take into account the losses of the detection system, but for our purposes we may set them to unity at their maximums.

The solid angle integral can easily be approximated for the thermal and the induced fluctuation components to the scattered light. The thermal fluctuations do not have a strong angle dependence over the usually small solid angle of the detection system. The integral can then be replaced by the observation solid angle $d\Omega_{\text{obs}}$ times $\langle |N_{\text{th}}(\Delta k, \Delta \omega)|^2 \rangle$ calculated at $\Delta k = \Delta k_{\text{obs}}$.

The induced fluctuations are in the other extreme (see equations 13, 18). The response function of the plasma $R$ can be considered constant over the range of the observed $\Delta k$, but the wave vector function $K$ cannot. Since $b_k \gg 1$ in the exponential in $K$, the induced fluctuations scatter light in a very narrow cone in the direction $k_1 - k_2$. The integral over the solid angle of the observation system gives:

$$\int T(\Delta k - \Delta k_{\text{obs}}) R(\Delta \omega/\Delta k) K(\Delta k) \, d\Omega$$

$$= R(\Delta \omega/\Delta k_{\text{obs}}) \frac{\sin^2\theta_{\text{mix}}}{(\Delta k_{\text{obs}})^2 \sin^2\theta_{\text{scat}}} \quad \ldots (24)$$

and the observed signal is given by
\[ I_{\text{obs}} = \frac{I_d}{A} r_o^2 (1 - \sin^2 \theta \cos^2 \phi) \]

\[ \times \left[ d\omega_{\text{obs}} \int d\omega T_{\omega} |N_{\text{th}}(\Delta k_{\text{obs}}, \Delta \omega)|^2 \right. \]

\[ + \frac{I_1 I_2 C \sin^2 \theta \Theta_{\text{mix}}}{(\Delta k_{\text{obs}})^2 \sin^2 \theta_{\text{scat}}} \int d\omega_{\omega} R(\Delta \omega/\Delta k_{\text{obs}}) W(\Delta \omega) \right] \]

\[ \ldots (25) \]

This can be rewritten:

\[ I_{\text{obs}} = S_1 I_d + S_2 I_1 I_2 \]

\[ S_1(\Delta k_{\text{obs}}, \Delta \omega_{\text{obs}}) = r_o^2 (1 - \sin^2 \theta \cos^2 \phi) d\Omega \]

\[ \times \int d\omega_{\omega} < |N_{\text{th}}(\Delta k_{\text{obs}}, \Delta \omega)|^2 > \]

\[ S_2(\Delta k_{\text{obs}}, \Delta \omega_{\text{obs}}) = \varepsilon r_o^2 (1 - \sin^2 \theta \cos^2 \phi) C \]

\[ \times \frac{\sin^2 \theta \Theta_{\text{mix}}}{(\Delta k_{\text{obs}})^2 \sin^2 \theta_{\text{scat}}} \int d\omega_{\omega} R(\Delta \omega/\Delta k_{\text{obs}}) W \]

\[ \ldots (26) \]

The quantities \( I_d, I_1, I_2 \) and \( I_{\text{obs}} \) are measured in an actual experiment. The data then give values for \( S_1 \) and \( S_2 \), which are directly related to the spectral density functions for the thermal and the induced fluctuations. Note that to maximize the scattered light, all the lasers should be polarized perpendicular to the scattering plane.

The scattering volume \( V \) is determined by the the
intersections of the laser beams, and the viewing area of the detection system. The scattering volume for the thermal fluctuations are usually larger than those for the mixing, since the observation area is purposely made larger than the laser beam diameters to collect all the scattered light. The difference in volumes is reflected by the $\epsilon$ in equation (25). $\epsilon$ is the ratio of the mixing volume to the scattering volume.

Since the wave vector function $K$ is so peaked in the direction of the mixing wave vector, it is essential that the detection system be arranged so that its transmission is a maximum for $\Delta k$. This can easily be done as long as

$$\Delta k = k_1 - k_2 = k_d - k_{obs} \quad \ldots(27)$$

where $k_1, k_2$ are the wave vectors of the mixing beams, and $k_d, k_{obs}$ are the wave vectors of the incident diagnostic beam and the scattered light (Figure II-4). Since $k_1 \approx k_2$ and $k_d \approx k_{obs}$, the mixing and scattering angles are related by

$$|k_1| \sin^2 \theta_{mix} = |k_d| \sin^2 \theta_{scat} \quad \ldots(28)$$

If the spectral distributions $P_i(\omega_i)$ in $W(\Delta \omega)(10)$ are Lorentzian line profiles of full width half maximum $FW_{\omega_i}$:

$$P_i(\omega_i) = \frac{FW_{\omega_i}}{2\pi} \frac{1}{(\frac{1}{2}FW_{\omega_i})^2 + (\omega_i - \omega_{10})^2} \quad \ldots(29)$$
\[ \Delta k = k_1 - k_2 = k_d - k_{\text{obs}} \]

**FIGURE II-4**  Wave vector matching.
then $W$ reduces to

$$W(\Delta \omega) = \frac{F_\omega}{2\omega_1^2 \omega_2^2} \frac{1}{(h^2 F_\omega)^2 + (\Delta \omega - \omega_1 + \omega_2)^2}$$

...(30)

where $F_\omega = 4\gamma + F_{\omega_1} + F_{\omega_2}$. Generally, however, the distributions $P_i$ are not Lorentzians. The function $W$ can be calculated for experimentally determined line profiles as described in Appendix A.

We can now calculate the relative scattering signals using values of the plasma parameters and experimental conditions for this experiment, and assuming Lorentzian line profiles for the mixing beams:

$$\omega_2 = 2.30 \times 10^{15} \text{ sec}^{-1} \ (820.0 \text{ nm}),$$
$$\omega_1 = 2.31 \times 10^{15} \text{ sec}^{-1} \ (816.1 \text{ nm}),$$
$$\omega_1 - \omega_2 = 1.09 \times 10^{13} \text{ sec}^{-1} \ (31.0 \text{ nm}),$$
$$F_{\omega_2} = 3.7 \times 10^{12} \text{ sec}^{-1} \ (1.3 \text{ nm}),$$
$$F_{\omega_1} = 2.3 \times 10^{12} \text{ sec}^{-1} \ (2.3 \text{ nm}),$$
$$\tau_0 = 2.82 \times 10^{-15} \text{ m},$$
$$V_{\text{norm}} = 2.5 \times 10^{-12} \text{ m}^3,$$
$$V_{\text{mix}} = (2/3) V_{\text{norm}},$$
$$N_0 = 2.0 \times 10^{16} \text{ cm}^{-3},$$
$$T_e = 19000 \ \text{K},$$
$$\theta_{\text{obs}} = 110^0,$$
$$\theta_{\text{mix}} = 150.7^0,$$
$$\alpha = 1.0,$$
$$I_1 = I_2 = 7 \text{ MW},$$
$$d\Omega_{\text{obs}} = 5.0 \times 10^{-3} \text{ Sr} \ (f/12.5).$$
T₂ is a symmetric function, unit height, 0.2 nm and 0.6 nm wide at the top and the base, respectively.

We get for the thermal term:

\[
\frac{I_{th}}{I_d} = 5 \times 10^{-15}
\]

and for the mixing term:

\[
\frac{I_{mix}}{I_d} = 20 \times 10^{-15}
\]

The total signal we observe during the mixing should be 5 times the signal observed from the thermal fluctuations alone. The actual power varies between 5 and 8 megawatts, so the expected mixing signal will be between 2 and 4 times the thermal signal. The fact that the experimental line profiles are not true Lorentzians will change the value for the induced signal only a small amount, since the frequency transmission function is much wider than the line profiles (i.e. the shape is not important, just the total integrated intensity).

E. Experimental Design

The calculations of the scattering signals in section C indicate that high power light sources are necessary to study wave mixing successfully. The mixing lasers must have similar frequencies, and at least one should be tunable to allow tuning of their frequency difference to a plasma resonance. The diagnostic laser should produce at least 10 MW at a frequency which matches photomultiplier responses in order to give reasonable size
scattering signals with our experimental conditions. The diagnostic laser should also have a frequency which is significantly different from the frequencies of the mixing lasers. If the wavelengths are too close, then to satisfy the wave vector matching conditions, the detection system will be colinear with one of the mixing beams. Stray light would then be a severe problem.

Table II-1 illustrates the most reasonable choices of laser systems which satisfy the above conditions.

System 2 has the advantage that one of the mixing beams can be a very high power solid state laser. The added cost (a second solid state laser) and complexity (frequency doubling) are large disadvantages.

System 3 also has the advantage of mixing with a high power laser. The diagnostic dye laser can be chosen to lase at frequency which matches the maximum response of photomultipliers. This wavelength is much different than the mixing wavelengths. Unfortunately, commercial flashtube pumped dye lasers have only recently been able to produce the megawatts of power required for the diagnostics, and even then an oscillator amplifier system is required.

The first system then was chosen for this experiment. If two tunable dye lasers are used for the wave mixing, then they are automatically in the same spectral region. The dye lasers can either be flashtube pumped or laser pumped. Laser pumping has the advantage
TABLE II-1  Possible laser systems.

<table>
<thead>
<tr>
<th>SYSTEM 1</th>
<th>MIXING BEAM</th>
<th>MIXING BEAM</th>
<th>DIAGNOSTIC BEAM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dye Laser</td>
<td>Dye Laser</td>
<td>Ruby Laser</td>
</tr>
<tr>
<td>SYSTEM 2</td>
<td>Dye Laser</td>
<td>Ruby Laser</td>
<td>Frequency Doubled Glass Laser</td>
</tr>
<tr>
<td>SYSTEM 3</td>
<td>Dye Laser</td>
<td>Ruby Laser</td>
<td>Dye Laser</td>
</tr>
</tbody>
</table>
that the laser required for scattering diagnostics could also supply the pump energy. Laser pumped dye lasers have higher efficiencies than flashtube units. The operation frequency of a laser pumped dye laser is always lower than the pump (diagnostic) beam. Thus the mixing and diagnostic lasers are spectrally separated, and the inverse frequency dependence of the mixing can be exploited. Also the timing of the lasers is automatic since a laser pumped dye laser lases almost coincident with the pump.

The choice of a plasma for a new diagnostic experiment is governed by several criteria. It should be reliable, easily available, preferably well studied, and satisfy the conditions of the theory to be tested. The open air D.C. plasma jet with which our laboratory is familiar is a good candidate. Electrode design improvements now allow its operation at several hundred amperes indefinitely. It has been studied by several authors with spectroscopic\textsuperscript{23} and laser diagnostics\textsuperscript{24,25,26}. However, the plasma produced by the plasma jet is not in local thermodynamic equilibrium,\textsuperscript{27} and is not collisionless. These aspects of the jet are considered in section E.

The actual experimental arrangement must facilitate examining the induced density fluctuations as a function of frequency and wave vector. The range of the driving frequencies which produce a measurable response in the plasma, a function of the mixing wave vector, must not
be too narrow to resolve. Also the ever present thermal fluctuations must be considered. For these reasons, a diagnostic scattering angle of 110° with corresponding mixing angle of 150.7° were chosen to give a thermal scattering spectrum which is essentially flat out to 3 nm from the ruby line (alpha=1.0). The angular separation of the laser beams is adequate to allow a co-planar system, unlike in Stansfield's work.

The frequency dependence of the scattered light can be observed by tuning a dye laser and/or the detector to different frequencies. Observing the wave vector dependence is much more difficult, since a choice of scattering and mixing angles fixes the observation wave vector. The entire optical assembly must be reorganized to change \( \Delta k_{\text{obs}} \). Because of this difficulty, the present experiment was performed as a single observation wave vector.

F. Validity Considerations

In deriving the expressions (11-13) for the density fluctuations, many assumptions were made, both about the wave mixing light sources and the plasma. These assumptions will now be examined for validity, beginning with the mixing beams.

The light sources are considered as being generated by classical damped oscillators. The number of modes present is large enough, and their phases and
starting times uncorrelated so that each beam can be considered as a stationary random process. We can only say that this is a plausible description of a laser system. Assumptions about the damping rate $\gamma$, however, are well satisfied. It is assumed that

$$\frac{1}{T} \ll \gamma \ll \omega_1, \omega_2$$

...(31)

where the damping rate can approximated by the time for a single pass through the oscillator ruby rod. The damping rate is then $1 \times 10^8$ sec$^{-1}$, much smaller than the central frequency of the dye lasers, $2.6 \times 10^{15}$ sec$^{-1}$. The inverse observation time $(1/T)$ is $4 \times 10^7$ sec$^{-1}$, completing the inequality. The estimate of the damping rate is very crude, since it depends on the unknown details of the laser system.

The laser outputs are assumed to be plane Gaussian beams. This assumption is not likely to be satisfied for our lasers. Observation of the near field output using foot-print paper indicates that it is 'spotty'. That is, the smooth variation of burn expected for a Gaussian beam is not present. This uneven output will have several consequences. First, one must now question the form of the function $K(\Delta k)$ and the mixing intensity dependence of the induced fluctuations as given in (15). If the spiking is random both in space and time, then the average value of the integrals in $k$ space required to obtain (15) would be expected to still give the $I \times I$ dependence, but on a longer time scale than
before. Since some structure can be seen in the burn patterns of the lasers, the time scale for the spiking is not sufficiently short to average out the beam intensity during one laser pulse. This suggests that only on the average of several shots will we get the $I_1 x I_2$ dependence.

This structure can also be considered as a source of error for the mixing experiment. The laser monitors measure an average power of the dye lasers, and do not truly represent the product of the electric fields in the mixing region.

$K(\Delta k)$ should still be sharply peaked around $\Delta k$, but may be reduced in amplitude. Since the detection system essentially integrates over all the wave vectors present in the induced fluctuations, its exact form is unimportant for our considerations.

There are three main assumptions about the plasma. First, the plasma is supposed to be collisionless. Since the waves in a plasma are Landau damped, this criterion really states that the waves must be damped before collisions disrupt the wave motion.

The Landau damping rate for waves with phase velocity near the thermal velocity of the electrons cannot be calculated using the approximate solution for small wave vectors. Ecker,28 using a more exact calculation, gives a value for the Landau damping rate $\omega_L$ roughly equal to electron plasma frequency $\omega_p$:

$$\omega_L = 0.85 \omega_p$$
The plasma frequency is $8 \times 10^{12}$ rad/sec.

The collisional damping rates are dependent on the choice of the velocity to be used for the electrons. An electron in the mixing region experiences the induced oscillation, but still retains its thermal velocity. Since its thermal velocity is so much higher than the velocity which it obtains through the mixing process, the average thermal velocity should be used. The collision rates $\nu_{ea}$ between electrons (e) and species (a) can then be taken directly from Appendix C:

$$\omega_{en} = 0.05 \omega_\ell$$

$$\omega_{ei} = 0.005 \omega_\ell$$

$$\omega_{ee} = 0.07 \omega_\ell$$

The subscripts n and i refer to the neutrals and the ions.

These collision rates are at least ten times smaller than the Landau damping rate. Since the waves are damped 10 e-foldings before being interrupted, collisions can be expected to play only a small role in the damping. The collisionless requirement is approximately satisfied.

The fact that collisions have little effect on the spectrum of the density fluctuations for this choice of $k$ is backed by the excellent agreement between scattering theory for collisionless plasmas and experimental spectra for this plasma source. Collisions
will have a lesser effect on the very wide spectrum for \( \alpha = 1 \) in this report, so one should consider the fits for spectrums with higher \( \alpha \) (smaller \( \Delta k \)), such as in \(^{(26)}\).

It is also assumed that the plasma is in local thermodynamic equilibrium (LTE). That is, the ions and electrons should have Maxwellian velocity distributions characterized by the same temperature. The plasma from the plasma jet is not in LTE,\(^{(27)}\) since the ion temperature is lower than the electron's. However, laser scattering experiments show that at least the electron velocity distribution is Maxwellian\(^{(26)}\).

Different temperatures for the ions and electrons will have no effect on the results of this experiment, since there is no change to the thermal fluctuations in the spectral region of interest. This is because the phase velocity of the waves being examined are so much larger than the thermal speed of the ions that there are virtually no ions with sufficient velocity to interact with the wave. It is only when \( T_i / T_e = (M/m)^{1/2} \) that the ion distribution makes a contribution to the regions examined. This reasoning also applies to the high phase velocity induced waves, with the added feature that the ions are considered stationary in the fields of the mixing beams because of their higher mass.

The Vlasov equations were linearized when the density fluctuations were calculated. This requires that the fluctuations be much smaller than the average density.
The smallness of the perturbation can be verified in two ways. First, we know that the thermal fluctuations are much smaller than the average density. The induced fluctuation spectrum measured in these experiments are only a few times the thermal level, so they too can be expected to be much smaller than the thermal level. Secondly, Stansfield*, using plane monochromatic wave approximations, has shown that the amplitude of the induced wave is \( fN \), where \( N \) is the mean electron density, and \( f \) is given by the expression

\[
f \approx \frac{e^2}{m^2} \frac{E_{10}E_{20}}{\omega_{10}\omega_{20}} \frac{1 - \epsilon_{e}}{\epsilon_{e}} v_{ph}^{-2}
\]

...(32)

Here \( \epsilon_e \) is the electronic dielectric function and \( V_{ph} \) is the phase velocity of the wave. Using the value of the field of \( 3 \times 10^6 \) \( V/m \) (10 MW focused to 0.2 mm diameter spot) and a maximum value of \( (1 - \epsilon_e)/\epsilon_e = 5 \) at \( V_{ph} \) equal to the mean thermal velocity, we get \( f = 5 \times 10^{-7} \). Thus, the amplitude of the driven waves are much smaller than the average density.

One great advantage that laser scattering claims as a diagnostic tool is that the plasma is not perturbed by the diagnostic beam. This is only true if the energy absorbed from the diagnostic beam is small compared to the thermal energy.

Kunze\(^{29}\) gives for the energy absorbed by the electrons through inverse bremsstrahlung as:
\[
\frac{\Delta T_e}{T_e} = 5.32 \times 10^{-7} \frac{N_o}{(\kappa T_e)^3/2} \left[ 1 - \exp\left( -\frac{\hbar \nu}{\kappa T_e} \right) \right] \frac{I_d}{A} \Delta T
\]

... (33)

Unfortunately, this expression for the amount of energy absorbed is maximum for low temperature, high density plasmas such as ours. The units for this equation are cm\(^{-3}\) for density, ev for \(kT\), cm for wavelength, watts cm\(^{-2}\) for laser intensity, and seconds for the pulse length \(\Delta T\). Using 40 MW focused to 150 micron diameter gives \(I_d/A = 2.3 \times 10^{11} \text{MW cm}^{-2}\) for the 20 nanosecond pulse. This formulae then gives the energy absorbed as \(\frac{\Delta T_e}{T_e} = 2\).

This assumes that the relaxation time amongst the electrons is much smaller than the length of the laser pulse, and that the energy stays with the electrons. The first requirement is satisfied since the energy exchange time between electrons is approximately \(3 \times 10^{-12}\) sec (Appendix C). The assumption that the energy remains entirely with the electrons is not valid. We are not interested in the temperature relaxation time between the electrons and the ions and neutrals, which is long because of the reduced energy transfer as temperatures approach their LTE values. We are interested in the energy exchange rate at the present temperature difference. Spitzer\(^{30}\) gives this energy loss rate as:
\[
\frac{dT_e}{dt} = \frac{T_i - T_e}{t_{eq}}
\]

\[
t_{eq} \approx \frac{27}{N_0 A n A} (T_e)^{3/2} \text{ sec}
\]

where \( A \) is 9 times the number of electrons in the Debye sphere, \( T_e \) and \( T_i \) are in °K, and \( N_0 \) in cm\(^{-3}\). The value for the rate of change of temperature for the temperature difference in the interaction region is:

\[
d(T_e)/dt = 1 \text{ ev/(4x10}^{-12} \text{ sec}).
\]

Thus, the energy deposited to the electrons will quickly be carried to the ions. As discussed in Appendix C, the neutrals and ions are close to equilibrium between themselves, so that the neutrals will also absorb some of the energy. The neutrals, because of the increasing population of higher atomic energy levels and radiation losses, can absorb energy without raising their kinetic energy an equivalent amount. Therefore, the energy deposited to the electrons will result in a raised neutral and ion temperature, and possibly a slightly increased electron density.

There is some evidence that the diagnostic laser (the most intense of the three lasers) is not perturbing the plasma an appreciable amount. If the diagnostic power is increased, one would expect the perturbation to increase. The change in plasma parameters would change
the scattering cross section, which would show up as a change in slope of a scattered intensity versus incident intensity graph. Such a graph (Figure B-1) shows no significant deviation from a straight line, indicating that the calculation of the energy absorbed is an over-estimate.

The calculation of the induced fluctuations is 'quasi-static' since it is assumed that the mixing beams are present for all time. Actual changes in the laser intensity will be followed closely by the induced fluctuations so long as the time scale of the changes are longer than the response time of the plasma. The dye lasers used sometimes show a tendency to mode-lock, but the modulations are a few nanoseconds wide. The plasma can respond to changes in mixing intensity on the order of the inverse plasma frequency ($\approx 10^{-12} \text{ sec}$). We can therefore assume that the mixing light waves produce plasma waves with no 'phase lag'.

In summary then, the assumptions used in the derivation of the spectrum of the induced density fluctuations hold for our experiment, except for plane Gaussian mixing beams. The effect of non-Gaussian beams is expected to give some departure from the simple dependence $I_1 \times I_2 \times K(\Delta k)$, but only in the form of possibly reduced signals with more shot to shot variation.
Chapter III

EXPERIMENTAL APPARATUS

This chapter is devoted to a description of the experimental apparatus and electronics. The optical system is illustrated in Figure III-1. Its main components include a ruby laser which provides the power for the diagnostic scattering as well as the pumping of the dye laser oscillator-amplifiers. The diagnostic beam and dye laser mixing beams are directed by mirrors and prisms toward the plasma jet where they are monitored and focused into the centre of the plasma. The scattered light is collected by another set of lenses and analyzed using a monochromator and photomultiplier. The laser monitors and photomultiplier signals are delayed with respect to each other and recorded on oscillograms. The spectra of the dye lasers are also monitored by a second monochromator, and an optical multichannel analyzer after they pass through the interaction region. An electronic logic system controls the firing of the ruby laser, since it must lase only when the remaining diagnostic equipment is 'ready'. A more detailed description of the experimental system follows.
FIGURE III-1  The complete optical system.

Legend:  D, Brewster angle dump; F, glass funnel and optical fibre; GP, thin glass plate; H, Rayleigh horn light dump; I, iris; L, lens; M, mirror; MON, Brewster angle laser monitor (shown rotated 90° about the laser optical axis).
A. The Plasma Jet

A plasma jet produces a steady state, open air plasma by passing a D.C. current through a gas which flows between two electrodes. Morris has shown that in the radial direction, the electron temperature and density of this plasma is constant to 5% within a range of 1 mm. Design improvements of the plasma jets used by Chan and others have made this plasma source reliable for extended periods of operation. These characteristics make this simple device an excellent experimental plasma source: relatively inexpensive, reproducible and reasonably uniform, long lived, and free of surrounding windows which can restrict the diagnostic geometry and increase stray light in scattering experiments.

Figure III-2 is a schematic of the plasma jet. Helium gas flows between the tungsten cathode and the copper anode where it is partially ionized, and then out of the anode orifice, creating a free standing plasma flame. The design improvements mainly consist of increased water cooling of the anode by directing the water symmetrically over the inside of the copper surface, with higher flow rates in the regions of highest heat loading. Improvements were also made to the tungsten cathode design. Now copper is first melted onto tungsten with a nickel interface and then machined and soldered to a brass tube. The tungsten-nickel-copper-brass design gives a stronger joint and better heat conduction compared to soldering. Also the cathode water flow was redesigned
FIGURE III-2  The plasma jet.
to eliminate stagnation points.

The jet is idled at low current, about 70 amperes, with a welding supply (Figure III-3). This reduces the duty cycle of the battery as well as increases the lifetime of the electrodes. The DC welding supply has maximum ratings of 500 amperes, 60 volts and 10 KVA. When the lasers are ready to be fired, the current source is switched by relays to a 48 volt, 240 ampere-hour battery, and the current raised in six steps to 230 amperes by combining 1 ohm, 1000 watt resistors in parallel with the ballast resistor. The large number of steps reduces the erosion of the anode, thereby increasing the reproducibility of the jet. The current is then trimmed to 230 amperes using a current regulated power supply. This power supply, connected in parallel with the jet and isolated from the battery by a diode, acts as a variable current source of up to 30 amperes, regulated to better than 1%. The jet is allowed a few seconds to come to equilibrium before an experimental shot is made.

The jet current is monitored by measuring the voltage drop across a 10⁻⁴ ohm shunt with a digital voltmeter which reads to the nearest tenth of a millivolt. This gives a direct reading of the current: 0.1 mv = 1 A. The shunt is accurate to 1% and the voltmeter to 0.1 mv, so that the absolute current accuracy is ±3 A, and the current reproducibility is ±1 A.

The axis of the jet is vertical, and perpendicular to the monochromator and laser axis. The
FIGURE III-3  The plasma jet power supply circuit.
helium gas flow rate is $8 \times 10^{-3}$ l sec$^{-1}$ (25 standard cubic feet per hour).

In order to maintain the charge on the battery during an experiment, a charging unit is connected to the battery. When the jet is using the welder current source, a relay switches power to the charger.

B. The Ruby Laser

A ruby laser system capable of high output power is required because the ruby laser provides both the pumping power for the dye lasers and the incident radiation for the scattering diagnostics. A Q-switched oscillator-amplifier combination was constructed for this experiment, similar to that designed by Albach$^{32}$.

Both the oscillator and amplifier employ 6 inch long by 1/2 inch diameter Brewster cut ruby rods. The optical pumping of each rod is provided by two linear flashlamps in double elliptical cavities. Up to 4 kilojoules of energy in a 1 millisecond pulse is provided by a separate capacitor bank for each flashlamp. The ruby rods and flashlamps are enclosed in glass jackets to permit water cooling. Refer to Albach and Churchland$^{33}$ for construction details.

The oscillator laser cavity has a Laser Optics 'ekalon' Fabry-perot etalon of 66% reflectivity ($R$) as the output mirror, and a dielectric coated, >99% $R$ rear reflector. Q-switching is done using a Glan-air prism
polarizer and Pockels cell placed between the ruby rod and rear reflector. The etalon aids in longitudinal mode control, and 8 mm diameter irises in the cavity help to control transverse modes. The prism polarizes the ruby light perpendicular to the scattering plane.

The laser system, with fresh optical components, is capable of delivering over 500 megawatts peak power in a 20 nanosecond pulse, but it is regularly operated at less than 200 megawatts to preserve components. The spectral line width of this laser system can be expected to be less than 0.07 nanometers as indicated by measurements on lasers of similar construction. To try to maintain reproducibility, the ruby laser was fired no more than three times every ten minutes. However, during the course of the hundreds of shots required for an experiment, the pulse characteristics from shot to shot vary more than 10%, and the output power could drop 50% by the end of a run.

C. The Tunable Dye Lasers

Two tunable, oscillator-amplifier dye lasers, illustrated by Figure III-4, were constructed for this experiment. Several different dye laser configurations were examined in the search for high power, narrow spectral line output characteristics. It is not within the scope of this thesis to describe these studies, so only information on the actual system used is presented.
FIGURE III-4  Dye laser system.
The dye lasers are capable of producing 5 to 10 megawatts of power in a 15 nanosecond full width half maximum (FWHM) pulse. The spectral line shape is variable from less than 0.1 to more than 0.3 nm FWHM by varying the diameter of the irises. Figure IV-5 compares typical output profiles as measured by an optical multichannel analyzer. The dye lasers can be tuned through more than 15 nm centred at 810 nm.

The oscillator output reflector is a 30% R plane mirror. The plane grating rear reflector provides tuning. This 1200 lines per mm grating, manufactured by Bausch and Lomb, is blazed for 750 nm in first order. The 32 mm wide ruled area has lines 30 mm long. It is Littrow mounted with the lines perpendicular to the scattering plane. The grating mount has two opposing, pre-stressed angle bearings for smooth rotational adjustment. This tuning is done by a large drum micrometer on a lever arm approximately 10 cm long. The lever arm can be adjusted so that a 0.01 mm micrometer movements gives 0.1 nm tuning (1 micron = 0.01 nm). The maximum of the output can be tuned to a desired wavelength to within ±0.02 nm, and is stable to the same figure.

The dye cell windows, flat to λ/4, are mounted at the Brewster angle. The optical path length through the dye is approximately 10 mm. A flow system with a 300 ml reservoir is used to prolong the operating time for the dye lasers. The dye used is 3,3′-diethyl-2,2′-
FIGURE III-5  Dye laser beam spectral line profiles.
thiatricarbocyanine iodide (DTTC iodide) dissolved in dimethyl sulfoxide (DMSO) at a concentration of $2 \times 10^{-5}$ Molar for the oscillator and $4 \times 10^{-5}$ for the amplifier. The dye solution is replaced during an experiment whenever the output power begins to drop, usually after about 75 laser firings. The Brewster angle dye cells and the natural tendency for dye lasers to lase with the same polarization as the pumping laser ensure that the dye laser outputs are polarized perpendicular to the scattering plane.

Irises and a Galilean telescope placed in the cavity help to control the spectral line shape. The 1 cm maximum diameter irises reduce the angular divergence of the output, and therefore reduce the spread in angle of incidence at the grating. The telescope, with antireflection coated lenses of -4 cm and 16 cm focal lengths, expand the dye laser beam so that more lines of the grating are illuminated. This improves the resolving power of the grating and also has the extremely favourable effect of reducing the power flux on the grating. The longer lifetime of the grating with the telescope present makes the beam expander almost mandatory. With the telescope, the grating lasts indefinitely, but can be damaged in a single shot without it.

Each oscillator is pumped with 15 megawatts of the ruby laser output, and each amplifier with about 65 megawatts. The gain of the amplifier is 25, which means, for 10 megawatts output power, only 4% of the pump energy
is converted to oscillator output energy. This is much below the 20% reported possible for this dye. The 20% figure, however, refers to output with little wavelength selection, which can account for a factor of two or more in the difference in efficiencies. Care was not taken to maintain high purity of the dye and solvent. This, combined with the use of only medium quality optics, makes the 4% efficiency not unreasonable, although below expectations.

The spectral line shape of the output is very sensitive to the position of the oscillator pumping beam. If the pump is off-centre, then the output profile is skewed. A 1/4 mm displacement of the 10 mm diameter pumping beam produces changes in the spectral output which are readily seen using the OMA monitor.

D. The Complete Experimental System And Diagnostics

Figure III-1 on page 46 is a schematic of the optical system. The output from the ruby laser is divided into three beams by dielectric coated mirrors (M1,2) of selected reflectivity. Forty percent of the power is directed to pump one of the dye laser systems, an equal amount is directed to the second dye laser, and the remaining 20% is transmitted through the mirrors for laser scattering diagnostics. The reflectivities quoted for all the dielectric mirrors are the suppliers’ figures for the angles of incidence in the experiment.
The diagnostic beam goes through an iris which makes the focused spot more symmetric. It is then directed to the plasma by two mirrors of >99% R (M3,4) and focused by a 10 cm focal length lens (L1). After passing through the plasma, the diagnostic beam is trapped in a glass Rayleigh horn.

The dye laser pumping beams are themselves divided into two parts. The glass plates (GP1,2) reflect approximately 20% into the dye cells of the oscillators. The remaining pumping beams are redirected by prisms (P1,2) into the amplifier cells. After being mostly absorbed by the dye solutions, the pumping beams then enter Brewster angle dumps.

The dye laser mixing beams are directed towards the plasma by prisms (P3,4), and then focused into the interaction region by 10 cm focal length lenses (L2,3). After passing though the plasma, the dye laser beams are collected by glass funnels. Optical fibres at the base of the funnels direct some of the collected light to the entrance slit of a monochromator-optical multichannel analyzer combination (henceforth referred to as the OMA).

The Princeton Applied Research Model 1205 OMA is a vidicon device with an image intensifier first stage. It is placed at the exit port of a monochromator so that the image plane of the image intensifier is coincident with the exit focal plane of the monochromator. This produces an image of the exit plane of the monochromator on the vidicon surface. Light on the vidicon surface
produces a charge proportional to the light intensity. The charge is measured by an electron beam which scans across the vidicon surface in 500 line segments, or channels. The signal from each channel is then displayed digitally and on an oscilloscope. A single count represents two photons incident on the vidicon at maximum gain for the image intensifier at its best spectral response. The OMA is oriented so that the channels are parallel to the entrance slit of the monochromator. Thus each narrow vertical channel plays the role of a monochromator exit slit and a photomultiplier. A complete spectrum of 500 points can be obtained on a single shot basis. As any experimenter who has painfully processed photographic plates or point by point spectra can well imagine, the 500 channel OMA is an extremely valuable tool.

A frosted glass plate and neutral density filters placed between the entrance slit and the ends of the glass fibres serve to control the intensity of light reaching the OMA. The monochromator has a dispersion of 10 Å/mm. This, combined with the 2.5 microns channel spacing, gives a combined dispersion of 0.025 nm per channel. Due to the cross talk between channels, the resolution of the OMA is 1.5 channels FWHM. For the 10 micron entrance slit used, the system resolution is 0.04 nm, quite sufficient to resolve the 0.1 to 0.3 nm FWHM dye laser spectral lines. Since the wavelength spread of the 500 channels of the OMA is greater than the
wavelength difference of the dye lasers, both lasers can be displayed simultaneously. For every firing of the lasers, the spectral lines are plotted by an x-y chart recorder. Also the total integrated intensity, and the maximum signal and its channel position is recorded using digital information supplied by the OMA. Since the OMA has a free running read and display cycle which cannot be controlled externally, a timing system is required. This system is described in section F.

A lens combination (L4,5) collects scattered light and focuses it onto the entrance slit of the second monochromator. The baffles and irises in the collection system, set for f/12.5, are very important for stray light reduction. These aids, combined with the laser and Rayleigh horn viewing dump, make the stray light levels negligible in the regions of the scattering spectrum of interest. The irises are indicated in Figure III-1, but the baffles, since their position is a unique function of the exact optical arrangement, are not.

The entrance slit of the monochromator is masked in the vertical direction and has its width set so that it forms a 300 micron square viewing hole. This helps to define the scattering volume in the plasma, since any light which does not go through the image of the entrance slit in the plasma will not go through the entrance slit itself. An iris placed inside the monochromator restricts the f-number from its full f/6.3 to f/12.5. This is to ensure good stray light rejection, but at the same time
maintain sufficient normal scattering signals. The mixing signals, because of their small solid angle, will be unaffected by this change in f-number.

The inverse dispersion of the monochromator is 1 nm/mm, with a maximum resolution of 0.01 nm. The exit slit is set to 300 microns or wider, depending on the experiment being performed, to give a 0.3 nm FWHM or wider transmission function. The scattered light is detected by a room temperature, GaAs photocathode photomultiplier (RCA-C31034B) with the standard dynode resistor chain suggested by RCA. The photomultiplier (PM) is restricted to very low light levels indicated by a maximum 30 second average anode current rating of 10 microamps at 1500 volts. To reduce the average current to this level when the PM is exposed to the continuum radiation of the plasma jet, it is necessary to insert a chopping wheel in the detection system.

To allow a maximum reduction in the plasma light levels, the chopping wheel is placed as close to the entrance slit as possible, with an aperture just sufficient to let the converging light through. The 3 mm wide aperture is at 9.8 cm radius on a 25 cm diameter aluminium disk. The free running chopping wheel, rotated by a 3600 RPM synchronous motor, must also be considered in the timing sequence. The chopping wheel reduces the average light level by 1/100, giving a PM anode current of 1 microamp at 1500 volts, well below the stated maximum.

Besides monitoring the spectral output of the
dye lasers, the diagnostic and mixing laser beams are monitored by pin photodiodes. Brewster angle glass plates (GP 3, 4, 5) placed just before the focusing lenses reflect some of the laser light onto frosted glass plates. Some of the dispersed light reaches the photodiodes through neutral density filters which reduce the intensity to the linear operation level for the diodes. The signals from these diodes, as well as the photomultiplier signal, are transmitted by 50 ohm coaxial cables to delay cables and a Tektronix 7904 oscilloscope. The cables are terminated with 50 ohms at the oscilloscope, but not at the diodes or the PM. The delay cables combined with two dual plug-in units allow all four signals to be displayed on the oscilloscope simultaneously. The ruby laser signal is displayed immediately, the dye lasers with 100 and 200 nsec delay, and the PM with 400 nsec delay. The delay cables are lengths of very low loss 50 ohm cable. The rise time for the dye laser monitor system is about 1 nsec (0.8 nsec for the oscilloscope and less than 1.0 nsec for the diodes). The ruby laser monitor system rise time is restricted to 1.5 nsec by the slower plug-in unit at the oscilloscope, and the PM system to 3 nsec by the slow response of the PM.

The signal-to-noise for the displayed signals is 20 to 1. This large figure is possible because of the very low D.C. PM current from the low continuum radiation of a Helium plasma. The noise pickup from the fast falling high voltage Pockels cell driver is reduced to a
negligible level by sheathing the signal cables in extra grounding braid, and housing the PM assembly in a brass case.

E. Alignment Procedure

There are over 50 optical components in this experiment, so that careful alignment and reliable mounts are necessary. The table on which most of the components are mounted consists of 1/2 inch thick aluminium plates, tapped with 1/4-20 holes on 2 inch centres. This homemade optical table is far superior to triangular optical rails, but does not perform as well as commercial units would be expected to.

Alignment is accomplished with the aid of four separate HeNe CW lasers, one for each optical axis. These lasers require additional steering mirrors and pin holes, but greatly reduce the set-up time for a run. One HeNe beam enters the ruby laser through the rear reflector. This is possible since this mirror has greater than 99\% R at 694 nm, but less than 50\% R at 633 nm. The other beams cross above the centre of the plasma jet toward the dye lasers and the monochromator. The beams are co-planar and parallel to the table top.

The anode of the plasma jet is replaced by an alignment template which has four pairs of pin holes, each pair 35 cm apart. These pin holes have been made to define accurately the angles for the four optical axis
above the plasma jet. The HeNe beams are then directed by mirrors (M1,2 for the ruby axis, extra mirrors for the others) through the pin holes. The jet mount allows height and tilt adjustments to make the template coplanar with the alignment beams. The consistency of the angles is better than $10^{-3}$ radians (0.3 mm in 30 cm), as indicated by observing the beam positions at several meters distance after different alignments. The accuracy is estimated at the same value.

The anode is replaced and the optical components for the dye lasers are centred, starting at the plasma jet. The focusing lenses L2,3 are inserted at a later time. The standard technique of observing the reflections from the laser mirrors is used to align the laser cavities. The grating must then be rotated to reflect 820 nm rather than 633.

Because the operating wavelength for the dye lasers is outside the visible region, focusing the telescopes by conventional means is difficult. However, the extremely high gain of the dye lasers can be used to an advantage, since the oscillators will lase even if the telescopes are poorly aligned. By monitoring the spectral outputs with the OMA while the telescopes are adjusted, focusing is easily and accurately done.

The scattered light collection system is aligned next. Mirror M5, lenses L4,5, and the chopping wheel are on a separate optical bench with vernier distance scales on the optical mounts. The chopping wheel aperture was
cut to be in the scattering plane when it reaches its highest position. Mirror M5 is adjusted so that the alignment beam is reflected parallel to the axis of the bench. The monochromator is centred so that the HeNe beam enters the entrance square and is centred on the first mirror. Lenses L4,5 are centred.

To aid in fixing the distance between the plasma jet and the focusing and collection lens, a pin mounted in a plug which fits into the anode aperture is used. The pin defines the centre of the vertical axis of the jet, and its tip positions the interaction region.

The light between the two collecting lens must be as parallel as possible in order to keep f-numbers matched and to reduce light loses to a minimum. A telescope, with cross-hairs in its focal plane and focused on infinity for visible light, is used to do the initial adjustment: when the lenses are one focal length from the pin or entrance square, the image is in the plane of the cross hairs. To overcome possible error in alignment of the telescope, an iterative process is used to make fine adjustments. The entrance square is illuminated from inside the monochromator, and the image is checked by parallax for coincidence with the pin. One lens is adjusted to make the image co-planar and the distance moved is recorded. The lens is then moved one-half the distance back and the second lens moved the same distance to compensate. The image is checked for parallax again, and the procedure repeated. Usually only two iterations
are required. This procedure fixes the position of the good quality achromat collecting lenses to better than 0.5 mm. Note that a step filter which transmits above 620 nm is used when viewing the pin and image.

The distance between the plasma and the laser focusing lenses is determined by trial and error, since this distance depends on the unknown divergence of the laser beams. The lasers are fired with an unexposed, developed piece of negative film in the region of the focal volume. When the lenses are the correct distance from the plasma, the smallest damage spot on the film is centred above the jet. The depth of focus of the laser beam-lens system is about 1 mm, so that this adjustment can easily be done.

The actual centering of all beam spots and collection volume in the interaction region must be performed just before the experiment itself. The image of the entrance square, with the step filter in the light path, is centred on the pin. The focused spot of the ruby laser diagnostic beam is checked to be coincident with the focused spot of its HeNe alignment beam, and centred at the tip of the pin. The pin is removed and the film inserted so that the entrance square image and the ruby alignment beam spot are centred at the same point on the film. The lasers are then fired individually, and the focusing lenses adjusted so that the laser damage spots are in the centre of the image of the square. The 100-200 micron diameter laser spots are usually within one radius
of the final position before the fine adjustments, which are accurate to about 25 microns.

The irises and baffles are placed when the optics for that section have been aligned. The laser dumps and funnel collectors are finally inserted, stray light checked, and the experiment performed.

F. Electrical System And Timing

Figure III-6 is a block diagram of the electrical system. The logic unit is necessary since the ruby laser must be fired only during the 10 microseconds that the chopping wheel window is in place, and the 760 microseconds that the OMA is ready to accept light.

The logic unit receives pulses from the OMA and chopping wheel which occur at known times before these units are 'ready'. The OMA prepulse is a standard part of the unit. The chopping wheel prepulse is obtained when light from a light emitting diode is reflected from a silvered section of the rotating shaft onto a matched photodiode. Rotation of the pickup on the shaft housing gives coarse adjustment of the prepulse delay time. The horizontal movement of the entire assembly provided by its optical mount gives fine adjustment. This pulse is processed by a discriminator and pulse shaping network before being sent to the logic unit.

When it is known that the OMA and chopping wheel window will entirely overlap, a pulse with variable delay
FIGURE III-6 Electrical system.
is sent to various components. This pulse goes directly to the high voltage trigger unit for the oscillator flashlamps, through a delay generator to the trigger unit for the amplifier flashlamps, and through another delay generator to trigger the Pockels cell driver. A coil of wire around the coaxial cable to the Pockels cell picks up enough signal to trigger the oscilloscope when the Pockels cell voltage is lowered.

Figure III-7 gives more detail of the logic sequence and timing. A represents the chopping wheel window, and B, its prepulse. This prepulse triggers two pulses of variable width: C is a timing window, and D is the delayed out timing. If the leading edge of the prepulse E from the OMA is within the timing window C, then the acceptance time for the OMA and the chopping wheel will overlap, and the lasers can be fired. When E and C overlap, a pulse is sent to the OMA memory enable G to indicate that the OMA is to store the signals from its vidicon during the next read cycles. For pulsed light signals it is best to scan the vidicon and add the information into memory for several read cycles. To enable the OMA console controls to determine the number of read cycles, the memory enable pulse is more than one second long. Only when memory enable G is high and the delay timing pulse D is lowered does a trigger pulse H go to the other electronics.

The widths and delays of the various pulses were determined in the following manner. To ensure a minimal
FIGURE III-7 Logic sequence and timing pulses.
wait until the two units are synchronized, the timing window $T_3$ should be as wide as possible. It is limited in width by the width of the ready time $T_6$ of the OMA. The OMA prepulse delay $T_5$ is fixed by the its electronics, and the chopping wheel window $T_1$ is fixed by the speed of the motor. $T_7$, the time between the firing of the oscillator flashlamps and the laser itself, are fixed by the requirements of maximum output power and the capacitor bank discharge time. Similarly, the delays for the oscillator flashlamps and Pockels cell triggering, $T_8$ and $T_9$, are fixed. So the following times are determined by the requirements of the experiment:

\[
T_1 = 18
\]
\[
T_3 = T_6 = 760
\]
\[
T_5 = 832
\]
\[
T_7 = T_9 = 800
\]
\[
T_8 = 600.
\]

All times are in microseconds.

If the OMA prepulse occurs at the beginning of the timing window, then the end of the OMA window is required to overlap the chopping wheel window. This insures that as the prepulse $E$ overlaps the timing window $C$ at later times, the machine windows will still overlap, though at earlier times within the OMA window:

\[
T_5 + T_6 = T_2 + T_1 / 2
\]
\[
T_2 = 1591.
\]

The delay $T_4$ must be adjusted so that the oscillator flashlamps capacitor bank has enough time to discharge
before the laser is fired:
\[ T_4 \cdot T_9 = T_2 \cdot T_1 / 2 \]
\[ T_4 = 800. \]

Now all the pulse widths and delays \( T_1-T_9 \) are defined.
In this chapter the experiments are described and the results discussed. First a description of the plasma as determined by Thomson scattering diagnostics is given. Then the general experimental conditions and procedures are presented, and the method of data reduction is outlined. Finally the details of the experiments are presented as well as a discussion of the results.

A. Thomson Scattering Measurements of Electron Temperature and Density

A ruby laser light scattering experiment was performed to determine the electron density and temperature of the plasma to be used in the main experiments. The experimental arrangement was exactly as described in Chapter III, except that the mirrors for diverting ruby laser light for pumping the dye lasers were not present. Further details of this standard diagnostic method are not given. Figure IV-1 shows a typical scattering spectrum of the normalized scattered light signal as detected by the photomultiplier as a function of wavelength shift from the ruby laser light wavelength.
FIGURE IV-1 Thomson scattering spectrum to determine electron temperature and density.
The error bars are the standard deviations of the mean of usually 9, but at least 6 measurements. The solid curve is a least squares fit of the experimental data to the theoretical scattering spectrum as derived by Salpeter, assuming a Maxwellian velocity distribution for the electrons. The theoretical spectrum was convoluted with the monochromator transmission function. The parameters that were varied for the fit were relative amplitude, electron temperature, and the correlation parameter alpha. The electron density was then inferred using the fitted values for temperature and alpha.

As can be seen from the quality of the fit, the plasma is very well described by a Maxwellian electron velocity distribution with a temperature of 19,500 °K, and a density of $2.16 \times 10^{16} \text{ cm}^{-3}$. No unusual features can be seen on this spectrum, nor on other Thomson scattering spectra obtained at this time.

The electron temperature and density of the plasma is in the range of values as measured on similar plasma jets using laser scattering and spectroscopic techniques. The fitted values are identical to those obtained by the author in a previous laser scattering experiment with a similar plasma jet.

B. General Experimental Conditions And Procedure

Table IV-1 lists the experimental conditions which were held constant during all experiments. In this
1. Ruby Laser Characteristics

- **wavelength**: 694.27 nm
- **linewidth**: 0.07 nm or less
- **total output energy**: 3.6 J
- **pulse FWHM**: 20 nsec
- **total average output power**: 180 MW
- **diagnostic beam power**: 40 MW
- **beam diameter**: 8 mm
- **focused beam diameter**: 0.15 mm
- **focused intensity**: 2x10^11 W/cm²
- **focused beam f-number**: f/12.5

2. Dye Laser Characteristics

- **wavelength**: 810-820 nm
- **linewidth**: 0.1 to 0.3 nm
- **total output energy**: 100 mJ
- **pulse FWHM**: 15 nsec
- **average output power**: 8 MW
- **beam diameter**: 6 mm
- **focused beam diameter**: 0.2 mm
- **focused intensity**: 2x10^10 W/cm²
- **focused beam f-number**: f/17

3. Other Parameters

- **diagnostic scattering angle**: 110 degrees
- **mixing angle**: 150.7 degrees
- **light collection f-number**: f/12.5
- **He flow rate**: 8x10^-3 l sec⁻¹
- **anode-cathode distance**: 9.5 mm
- **anode-mixing region distance**: 1.8 mm

**TABLE IV-1** Parameters of the experiment held constant.
table, the effective size of the focal spots are assumed equal to the area $A$ of the damage spot on the negative film used in the alignment. The power densities $I$ are calculated using the time width $T$ of the pulses, the total energy $E$, and the area:

$$I = \frac{E}{TxA}$$

The same general procedure is used for all the mixing experiments. The spectral lineshapes of the dye lasers are monitored on every shot, and the ruby laser pumping beams adjusted to maintain reasonably symmetric lineshapes. The wavelength and intensity at the line centre, and the total integrated intensity are recorded, using the digital information from the OMA. The ruby laser and dye laser output power is monitored by photodiodes on every shot, and recorded by photographing the oscilloscope trace.

The mixing experiments require that the scattered light signals be determined with and without the dye laser beams in the interaction region. To accomplish these measurements, a pair of shots are made with and without the mixing beams. The parameter being investigated (for instance the mixing frequency) is changed to a new value, and another pair of shots obtained. The entire range of the parameter is covered in this stepping manner. The parameter is then reset to its original value, and its range covered several more times. This procedure ensures that all values of the parameter are covered evenly, and that any changes in experimental
conditions will be easily noted. Each pair of shots, including their associated recording of OMA data and laser cooling time, require an average of 7 minutes.

The exit slit of the monochromator was set between 300 and 600 microns wide, depending on the spectral resolution desired. The narrower slit gives a triangular instrument profile with FWHM of 0.3 nm. The wider slit gives a trapezoidal instrument profile with 0.6 nm FWHM.

The oscillograms are analyzed as described in the following section. Typical oscillograms for normal and mixing shots are illustrated in Figure IV-2. These oscillograms correspond to the experimental conditions for Figure IV, at a wavelength shift of -3.05 nm.

C. Data Reduction

An outline of the measurement methods for the oscillograms and the evaluation of the numbers obtained is presented in this section. A detailed description of the methods and their justification are given in Appendix B.

The data acquisition rate for the mixing experiments presented in this thesis is unfortunately rather low. For this reason it is imperative that as much and as accurate information as possible be extracted from the oscillograms. Different analysis techniques were tried using the data from the experiment described in Section D of this chapter. It was concluded that the best
FIGURE IV-2  Typical Oscillograms.
analysis method is to digitize the oscillograms; that is, to measure the height of the signals as a function of time (distance) along the oscillogram. Numerical methods can then be used to evaluate the parameters of the mixing experiments, employing integration of the signals over time.

The photomultiplier signal at time t \( (I_{\text{obs}}(t)) \) is related to the ruby laser power \( I_d(t) \) and the dye laser powers \( I_1(t) \) and \( I_2(t) \) through the parameters \( S_1 \) and \( S_2 \) by the formula (see equation (26)):

\[
I_{\text{obs}}(t) = S_1 I_d(t) + S_2 I_d(t) I_1(t) I_2(t)
\]

\( \ldots (35) \)

\( S_1 \) and \( S_2 \) can change as some parameter of the experiment is changed. (Recall that \( S_1 \) is the normal scattering cross section and \( I_1 I_2 S_2 \) is the mixing scattering cross section.) Estimates for the parameters are made using the formula:

\[
S_1 = \frac{\sum_{i=1}^{N} \frac{\int I_{\text{scat}}^i \, dt}{\int I_d^i \, dt}}{\sum_{i=1}^{N} \left( \int I_d^i \, dt \right)^2}
\]

\( \ldots (36) \)

\[
S_2 = \frac{\sum_{j=1}^{M} \frac{\int I_{\text{scat}}^j \, dt - S_1 I_d^j \, dt}{\int I_d^j I_1^j \, dt}}{\sum_{j=1}^{M} \left( \int I_d^j \, dt \right)^2}
\]

\( \ldots (37) \)
where there are $N$ normal scattering shots and $M$ mixing shots, and the superscripts refer to the shot number. These formulae are least squares estimates of the parameters $S_1$ and $S_2$ using equal weighting for all points, as described in Appendix B.

To evaluate the integral of the triple product $I_d x I_1 x I_2$, it is necessary to determine the position on the oscillograms that correspond to the same time for the 3 different signals. The method used to determine these relative time origins is described in Appendix B.

The standard deviation of the individual estimates of the parameter $S_1$ is reduced by 2 as compared to the more usual method of measuring the heights of the signals alone. This means that it requires 4 times fewer shots to obtain the same accuracy for the mean value of $S_1$ if one digitizes the oscillograms rather than measure the peaks. We therefore have compensated for the fact that we are only able to collect an average of 8 shots for the mixing and normal scattering combined at each parameter setting.

D. Dependence Of Mixing Signal On Mixing Power

It is obvious that the amount of light scattered due to light mixing in the plasma will depend on the diagnostic beam intensity as well as the intensity of the dye laser mixing beams. This experiment was performed to verify the relationship between the laser intensities and
the light detected by the photomultiplier predicted by theory.

Determining the scattered signal dependence on power is in fact essential if we are to proceed further. When other parameters of the experiment are investigated, we are required to compensate for the variations in the detected signal due to changes in scattering and mixing powers. If the lasers gave more consistent output power, this correction would not be necessary.

The width of the exit slit of the monochromator was adjusted to 600 microns. The centre of its transmission function was set to correspond to a wavelength shift of $-2.87 \text{ nm}$ from the ruby laser light wavelength. The dye laser whose optical axis corresponds most closely to the ruby laser axis was set to 820 nm. To set the mixing frequency to the same value as the observation frequency, the second dye laser was tuned to a wavelength of 816.1 nm.

An absorption cell was placed in the path of the pumping beam for the oscillator of the second dye laser. The cell contained cryptocyanine dissolved in distilled water. To vary the output power of this second dye laser, three different concentrations of cryptocyanine solution were used. Also the iris's of this laser were changes from 8 to 6 mm diameter. This varies the area of the output beam, and therefore the total output power. Changing the iris apertures also changed the linewidth of this dye laser, as is recorded using the OMA monitor. A
series of oscillograms were taken, alternating one normal scattering shot with two mixing shots.

The normal scattering cross section, measured by $S_1$, is first evaluated using equation (36). The least squares estimate of $S_1$ is $1.53\pm0.03$. The standard deviation of the points themselves is approximately 10%.

The mixing signals can now be evaluated. The PM signal is a sum of the normal scattering signal and the mixing signal:

$$I_{\text{obs}} = S_1 I_d + S_2 I_1 I_2$$

(38)

To verify this formula we first must subtract the normal signal from the total PM signal and compare this with the triple product. This gives a value for $S_2$ of $1.01\pm0.09$.

The data is represented on two separate graphs. The first graph, Figure IV-3, illustrates the total PM signal for both the normal scattering and the mixing shots as a function of the calculated signal using equation (38) above and the fitted values for $S_1$ and $S_2$. The data should fit the solid curve whose slope is unity and intercept is the origin.

This graph has several important features. The first of course is that the data fit the equation above quite well. No trend away from the straight line can be seen, although the fluctuations of the data are quite large.

Another feature is that the fluctuations in the
FIGURE IV-3  Total PM signal compared to the fitted values.
total PM signal for the mixing case follows the trend of the normal scattering signals. The dotted lines encompass all the data points which correspond to normal scattering alone. The majority of the points for the mixing shots also fall within these lines.

The last feature is that the increase in signal due to mixing is quite significant. The largest PM signals for mixing are more than twice as large as the signal for normal scattering alone, with approximately the same diagnostic beam intensity. This corresponds very well with the calculation of the relative magnitude of the mixing and normal scattering signals in Chapter II. The calculations there indicate a ratio of mixing signal to normal signal of 4:1. Making a correction for the wider instrument profile used here gives a ratio of 3:1, as compared to the actual value of 1:1. The slightly lower experimental value may be due partly to the non-Gaussian laser beams, as discussed in Chapter II. However, anticipating the results of the measurement of the response of the plasma to different driving frequencies, the main reason for the low experimental ratio is the choice of mixing frequency. The signal at slightly different mixing frequencies can be expected to be a factor of 3 to 4 higher than at the frequency for this experiment. This would make the ratio of mixing signal to normal signal approximately 4:1. Considering the possible error in the mixing and scattering volumes used in the calculation of the expected signal of 40% or more, this
agreement is excellent.

A second method of illustrating the data is to plot the PM signal due to mixing alone as a function of the triple product, i.e. \( I_{\text{obs}} - S_1 x I_d \) versus \( I_d x I_1 x I_2 \). This graph, Figure IV-4, should stand on its own as a good fit to a straight line through the origin. The origin intercept can only be assumed if the value of \( S_1 \) is accurately known, as is the case. However, unlike a graph of normal scattering signal versus ruby laser signal, some data points can (and do) become negative.

The solid curve is a least squares fit through the origin, and the dashed curve is a least squares fit for a straight line with an intercept. These two fitted lines are virtually identical.

This diagram graphically illustrates how the variations in the signal due to mixing alone are compounded by the normal scattering signal variations. The standard deviation of the estimate for \( S_2 \) is just under 9%, but the standard deviation of the points themselves is approximately 50%.

The validity discussion of Chapter II indicated that the effect of the non-Gaussian character of the mixing laser beams would only result in possible reduction of mixing effect and increase in signal error. The relative signal sizes indicate very little reduction, and the increase in the scatter of points for the mixing is small. Apparently the different intensity structure of the beams has had little effect.
FIGURE IV-4 Photomultiplier signal due to light mixing alone as a function of the product of the laser powers.
E. Spectrum Of The Induced Fluctuations

As predicted by equation (13), the spectrum of the density fluctuations driven by the light mixing should be of the form

\[ <|N_{\text{ind}}(\Delta k, \Delta \omega)|^2> \propto R(\Delta \omega/\Delta k)W(\Delta \omega) \]

where \( W(\Delta \omega) \) is a double convolution of the normalized frequency distribution of the light sources driving the waves with a lorentzian of \( \text{FWHM}=4\gamma \), and \( R(\Delta \omega/\Delta k) \) is the response function of the plasma. This experiment was performed to verify that the mixing spectrum is determined by the dye laser spectra in this manner. The dye lasers were fixed at frequencies similar to those in the previous experiment, corresponding to 820.75 and 816.15 nm. The expected maximum in the spectrum of the induced spectral density function is at a wavelength shift of -2.81 nm from the ruby laser line. The scattering light signal was observed at different monochromator settings about this frequency shift (\( \Delta \lambda=-2.79 \text{ nm} \)). The scattered light signal due to mixing (\( I_{\text{mix}} \)) then gives the spectrum of the induced wave, convoluted with the instrument transmission function \( T_\omega \), for these dye laser spectra (from equation (26)):

\[ I_{\text{mix}}(\Delta \omega_{\text{obs}}) \propto \int T_\omega(\Delta \omega - \Delta \omega_{\text{obs}}) R(\Delta \omega/\Delta k) W(\Delta \omega)d\omega \]
To obtain some resolution of the mixing spectrum, the width of the instrument transmission function should not be wider than the spectrum being observed. In general, the smaller the width is, the higher the resolution and the smaller the signal. In our case, since the width of the entrance slit of the monochromator is fixed to define the scattering volume, we can optimize resolution and signal strength by setting the exit slit to the same width. The transmission function then is 0.36 nm wide, about equal to the expected width of the induced spectrum.

The monochromator wavelength setting was changed in 0.2 nm increments from 690.77 to 692.27 nm. Between 4 and 8 shots, with an average of 6, were made at each setting for mixing, with similar statistics for normal scattering. The oscillograms were analyzed as previously described with the resulting spectrum shown in Figure IV-5. Since the normal scattering spectrum changes by less than 2% over this wavelength region, the normal scattering cross section \( S_1 \) was assumed to be constant. An average value of \( S_1 \) was used when determining the mixing signal. The scattering signals at different wavelengths had a standard deviation of ±7%, indicating that any real change in \( S_1 \) of a few percent is negligible. The standard deviation of the mean of \( S_1 \) is 2%.

As usual the spectra of the dye lasers were recorded on every mixing shot. This allows a direct comparison of the obtained spectrum with the spectrum
FIGURE IV-5  Spectrum of the induced wave.
predicted by equation (40). The FWHM of the spectral lines, for a single shot, were 0.19 to 0.22 and 0.10 to 0.12 nm for the two dye lasers. Single shot spectra are shown in Figure III-5. The centre maximum and the width of the dye laser profiles change from shot to shot by ±1 OMA channels from their average values. This has the effect of increasing the effective width of the profiles for the entire run. Six representative spectra, approximately every 10th shot, were chosen as typical profiles for the run. These spectral lines, with 0.13 and 0.23 nm FWHM are illustrated in Figure IV-6. The Lorentzian and Gaussian line profile comparisons in this figure, least squares fits for amplitude and width, show that neither adequately describe the assymetric experimental profiles. Using the results of appendix A and the measured instrument transmission function, equation (40) was evaluated numerically to give the solid curve in Figure IV-5. The amplitude of the theoretical curve relative to the experimental points was determined using a least squares fit with weights equal to the inverse of the variance of the experimental points. The instrument width is also indicated on the diagram.

We again have excellent agreement between theory and experiment. Our instrument width is not sufficiently narrow to make out fine structure of the induced spectrum, but the widths and general shape match very well.
FIGURE IV-6  Spectral line profiles (Average of several shots).
The most important aspect of the derivation of the induced fluctuation is the function \( R(\Delta k, \Delta \omega) \), the response of the plasma to different driving frequencies and wave vectors. The experiment described here is designed to evaluate the response function.

Recalling equation (26), and using the form for \( \omega \) which assumes Lorentzian profile mixing beams, we have that the mixing signal received is

\[
I_{\text{mix}}(\Delta \omega_{\text{obs}}) \propto \int \frac{T(\Delta \omega - \Delta \omega_{\text{obs}}) \cdot R(\Delta \omega/\Delta k) \cdot d\omega}{(\Delta \omega)^2 + (\Delta \omega - \omega_{10} + \omega_{20})^2}
\]

The detection system frequency transmission function \( T_\omega \) is peaked at \( \Delta \omega = \Delta \omega_{\text{obs}} \), and the Lorentzian style denominator is peaked at \( \Delta \omega = \omega_{10} - \omega_{20} \). Thus to get a reasonable size signal due to mixing, the detector must be tuned to \( \Delta \omega_{\text{obs}} = \omega_{10} - \omega_{20} \). The expected width of the response function of the plasma is much larger than the width of the transmission function and the Lorentzian denominator. In this case, the mixing signal at \( \Delta \omega_{\text{obs}} = \omega_{10} - \omega_{20} \) is a measure of the response of the plasma to the driving frequency \( \Delta \omega_{\text{obs}} \). If a dye laser is tuned to different frequencies, and is 'followed' by the detection system, keeping \( \Delta \omega_{\text{obs}} = \omega_{10} - \omega_{20} \), we have a direct measure of the response function for different driving frequencies. The expected response function is more than 4 nm wide, so we have good resolution using a 0.3 nm wide transmission function.
The exit slit of the monochromator was set for the triangular instrument profile with 0.3 nm FWHM. The frequency difference between the monochromator setting and the ruby line, and between the dye lasers, were changed in increments of $1.17 \times 10^{12}$ sec$^{-1}$, from $2.74 \times 10^{12}$ to $2.52 \times 10^{13}$ sec$^{-1}$. This corresponds to 0.3 nm steps from 693.55 to 687.85 nm.

Between 3 and 7, with an average of 3.9, and between 3 and 8, with an average of 3.6 shots were recorded for the mixing and normal scattering, respectively. The spectra of the dye lasers were monitored on every mixing shot to ensure consistent lineshape and frequency. The dye and ruby laser output powers were recorded for normalization purposes as usual. The oscillograms were analyzed as described earlier, except that the fitted values of the thermal signals rather than the experimental values were used when calculating the mixing signals.

The normal shots constitute a laser scattering diagnostic experiment by themselves. That is, the spectrum of the scattering from the thermal fluctuations as a function of frequency gives the electron temperature and density for this particular run. The normal scattering data are presented in Figure IV-7. The solid line is the theoretical least squares fit using the method described in section A.

There are two important features of this spectrum. First, the least squares values for the
FIGURE IV-7  Thomson scattering spectrum for mixing experiment.  
Dye laser beams blocked off.
temperature (19200 °K) and density \((2.13 \times 10^{22} \text{ m}^{-3})\) are exactly the same for the earlier diagnostic run in section A. This again illustrates the reproducibility of the plasma. The second feature is the few points that are more than one standard deviation away from the theoretical curve. On a purely statistical basis one expects approximately \(1/3\) of the data points to lie one standard deviation from the expected curve. However, the variations in signal may be real. This is indicated by the smooth change from above the fitted curve to below it. Also, the frequencies of the sharp transitions at \(\Delta \lambda = -3.0\) and \(-1.5 \text{ nm}\) are harmonics of each other. The plasma frequency wavelength shift has been indicated on the graph.

When analyzing the data to obtain the response function of the plasma, it is necessary to subtract the thermal scattering signal from the total signal. The estimate for the thermal signal \((S_t)\) for a particular frequency can be found from the few shots at that one frequency, but a better estimate is to choose the value of the theoretical fit for that frequency. In this way the data from the entire run is used, resulting in a more accurate estimate of the mean value of the signal at any one frequency. The fitted values for \(S_t\) were therefore used.

The resulting spectrum of the response function of the plasma as a function of frequency is displayed in Figure IV-8. This graph obviously departs violently from...
FIGURE IV-8  Measured response of the plasma to different driving frequencies.
the smooth spectrum described in Chapter II. The broken line is drawn as a (suggestive) aid to the eye. The previous two experiments were done with the mixing frequency corresponding to the region of reduced signal around $\Delta \lambda = -2.8 \text{ nm}$.

There are certain trends to this spectrum. The peak at $\Delta \lambda = -3.15 \text{ nm}$ has a signal approximately four times the maximum normal scattering signal. This corresponds closely to the expected signal due to the induced fluctuations, so one would expect that the valleys in the spectrum are the anomalies rather than the peaks. This idea is also backed by the fact that the theoretical response function, calculated using the experimental values for the temperature and density, and normalized to the last six data points, fits the envelope formed by the maximum values very well. This is indicated by the solid line in Figure IV-8.

A second feature has to do with the position of the valleys. There are definite valleys at $\Delta \lambda = -2.85$ and $-4.35 \text{ nm}$, but the reduced signals at $\Delta \lambda = -1.65$ and $-1.05 \text{ nm}$ may not be statistically significant. If we assume all four are valleys, then their positions may be related in a systematic way. Because of the speculative nature of these observations, further discussion is restricted to Appendix D.

Another trend in the spectrum of the response function is the correlation of the discontinuity around $\Delta \lambda = -3.0 \text{ nm}$ to the similar one at the same position in the
thermal scattering spectrum. This may indicate that the anomalous scattering signal is enhanced by the effect of the wave mixing. The similar discontinuity for the normal scattering spectrum at 1/2 the frequency shift does not have such a correlation.

It might be suggested that the large variations in the spectrum are at least partly due to using the fitted thermal signals rather than the actual experimental signals. If the actual signals are subtracted from the total PM signal to the give the mixing signal, then we obtain a spectrum with the same features as before. The drop in signal at $\Delta \lambda = -1.65$ nm is increased, but the others remain approximately the same. The small change is to be expected, since the mixing signal is more than 30 times larger than the variations of the thermal signals from the fitted curve.

This spectrum is not unique. For instance, an experimental run with a smaller anode-cathode distance produced a plasma with fitted temperature and density of 24000 °K and $1.1 \times 10^{16}$ cm$^{-3}$. The response spectrum of the plasma is again sharply modulated. However, there is no readily seen relationship between the positions of the valleys as is suggested in Appendix D.

There are two general reasons for the unexpected modulations in the response function of the plasma. First, our physical model may be wrong or incomplete. Second, there is possibly a phenomenon not related directly to the wave mixing which does not allow the waves
to develop to the expected amplitude. Before it may be concluded that the model is incorrect, all possibilities which come under the second category must be exhausted.

There are two possible methods through which the energy from the mixing beams will not produce the proper amplitude waves. First, the mixing beams may be attenuated at frequencies which correspond to the valleys. Second, the mixing beams may produce the expected mixing waves, but they are damped by some process in the plasma.

If the mixing beams are attenuated at specific frequencies, the driving force would be reduced to below the expected value. Then the calculated integrals of the triple product of the laser powers would be larger than their actual value. This lowers the estimate of $S_2$, creating a 'valley'. However, data is available which excludes the possibility of the dye lasers being attenuated at the valleys and not at the peaks.

The dye laser photodiode monitors placed before the laser beams are focused into the plasma measure the dye laser power as a function of time, integrated over all frequencies. The OMA monitor measures the dye laser powers as a function of frequency, integrated over time. The OMA light collecting funnels are placed after the dye lasers pass through the interaction region. We therefore have a measure of the dye laser powers before and after the interaction region simply by integrating the OMA spectra with respect to frequency, and the photodiode signals with respect to time. The integral of the line
profiles were recorded during the actual run using built-in OMA functions. The time integrals are done using the digitized oscillograms and the computer programs written for integrating normal scattering signals.

The data is presented in Figure IV-9 and IV-10, where the signal after the interaction region is plotted as a function of the signal before. Figure IV-9 compares the data for the fixed frequency dye laser for the three data points around $\Delta \lambda = -2.55$ nm to the data for the adjacent peaks ($-19.5, -3.15,$ and $-3.45$ nm). Figure IV-10 illustrates the same information for the variable frequency dye laser. The error bars are the estimated error in measuring the area under the oscillogram traces, determined in Appendix B. The solid curves are least square fits through the origin for the signals at the valleys, and the broken line for the peaks.

These graphs show that there is no statistically significant absorption of the dye laser beams as their frequency difference is tuned from the peak to the valley. We can exclude frequency dependent attenuation of the beams as a cause of the modulations.

Possible damping of the driven waves cannot be so easily ruled out. The validity discussions in Chapter II indicate that Landau damping is the dominant damping mechanism, but there are other possible mechanisms not yet considered. An example is strong coupling of the mixing waves to other natural modes of the plasma. If sufficient energy is transferred to other waves, then the scattered
FIGURE IV-9  Dye laser power before and after the mixing region: the fixed frequency laser.
FIGURE IV-10  Dye laser power before and after the mixing region: the variable frequency laser.
light signal could be reduced. However, the following considerations suggest that nonlinear mode coupling is not the explanation.

For the mixing waves with frequencies and wave vector \((\Delta \omega, \Delta \mathbf{k})\) to be coupled to other waves \((\Delta \omega_1, \Delta \mathbf{k}_1)\), a third wave \((\Delta \omega_0, \Delta \mathbf{k}_0)\) must already be present to satisfy the momentum and energy conservation requirements:

\[
\Delta \omega_1 = \Delta \omega + \Delta \omega_0 \\
\Delta \mathbf{k}_1 = \Delta \mathbf{k} + \Delta \mathbf{k}_0
\]

...(42)

(See for instance Sagdeev and Galeev38). The existing waves \((\Delta \omega_0, \Delta \mathbf{k}_0)\) must be normal modes of the plasma, as must the waves produced \((\Delta \omega_1, \Delta \mathbf{k}_1)\). Because the driven waves have a large wave vector, either \(\Delta \mathbf{k}_1\) or \(\Delta \mathbf{k}_0\) must also be large. Density waves with large wave vectors are not usually normal modes of the plasma, so the requirement that both the existing and coupled waves be normal modes is not satisfied.

If there is only one wave \((\Delta \omega_0, \Delta \mathbf{k}_0)\) already existing, then all the produced waves will have different frequencies, but identical wave vectors. A class of waves with such a dispersion relation is improbable, so that one must conclude that there is a set of waves present for the driven waves to interact with.

The amplitude dependent energy transfer must be from high frequency waves to lower frequencies. Therefore, for the energy transfer to be efficient, the frequency of the existing and produced waves must be lower
than the mixing waves, and the amplitude of the existing wave must be considerable. Laser scattering experiments performed at at least five different wave vectors by different authors have not observed large amplitude waves present.

It is concluded that neither attenuation of the mixing beams, nor nonlinear coupling of the driven waves to other waves can explain the modulations of the response function. However, we cannot choose between damping and an incomplete model as the cause of the unexpected modulations. Other damping mechanisms must be considered, and further theoretical work is required before we make a definite conclusion.

In the above analysis, only possible anomalous behavior resulting from the interaction of the mixing beams and the electrons and ions has been considered. It is also possible, however, for the laser beams to interact with the neutrals in the plasma. This interaction would be at distinct frequencies because of the atomic energy level structure of the atoms. The small cross-sections for these interactions may be compensated by the very high density of the neutral atoms ($\sim 10^{18}$ cm$^{-3}$).

Such interactions cannot be predicted by the simple model which has been presented. However, one would expect that such interactions would produce modulations in the scattered light signal which are not symmetric with respect to the diagnostic laser frequency. A measurement of the symmetry of the response function will indicate if
laser-neutral interactions is a possible cause of the response function modulations.
Chapter V

CONCLUSIONS AND SUGGESTIONS

The effect of optical mixing of two tunable dye lasers at frequencies near the plasma frequency has been experimentally investigated. It has been shown that the wave mixing produces longitudinal plasma oscillations at the frequency and wave vector of the mixing force. The driven waves were detected by scattering a third diagnostic light wave from their density fluctuation.

The wave mixing experiments presented in this thesis require a very reliable plasma source because of the long operation times. With the improvements which I have made to the electrode design, as well as my addition of the current regulated power supply and battery charging unit, the plasma jet can now be used for the length of time required.

I have been able to produce an increase in scattering signal due to the optical mixing an order of magnitude more than previously reported. The scattering signals increased to as much as seven times the signal observed when scattering from the thermal fluctuations alone.

I have made the first measurements of the dependence of the amplitude of the induced spectral density on the power of the optical mixing beams. This
dependence agrees well with theory, even though the dye lasers were not plane Gaussian waves as assumed in the derivation. The power dependence was used to normalize the signals in the subsequent experiments.

The frequency spread of the mixing beams produces a spectrum of induced waves. The accuracy of my measurement of the spectrum of the induced spectral density function is a factor of two better than previous measurements. The experimental spectrum agrees very well with the theoretical profile convoluted with the monochromator frequency transmission function.

I have made the first measurements of the response of the plasma to different mixing frequencies. This spectrum departs radically from the theoretical predictions. The spectrum is modulated, with changes in mixing signal by as much as a factor of four at adjacent measurement points. The envelope formed by the peaks of the spectrum fit the theoretical curve well. This good agreement, combined with the absolute value of the maximum signal being close to the expected value, indicates that the valleys in the spectrum are anomalies rather than the peaks. I have shown that nonlinear mode coupling and frequency dependent attenuation of the mixing laser beams are probably not the cause of the modulations in the response spectrum. They are perhaps due to an unknown damping mechanism, or perhaps to an incomplete physical model. No definite conclusion can be made at this time.

As happens frequently, these experiments have
raised questions while answering others. It has been shown that the wave mixing produces plasma waves of appreciable size and known spectral content. This method of producing waves can be used to study wave-wave and wave-particle interactions, and perhaps wave mixing will become a valuable tool for plasma physicists. However, further work must be done to understand completely the response of the plasma to different driving frequencies.

There are some improvements to this experiment which I strongly recommend. The first is to improve the data acquisition rate. This can be accomplished by using higher repetition rate laser systems, and an 'automatic' multi-channel detection system. The techniques of using optical multi-channel detectors with analogue to digital conversion and recording of data directly onto magnetic tape are becoming quite standard, and would be invaluable to this experiment.

The second improvement would be to change the method of detection of the induced waves. Presently the amplitude of the waves is small, so that the scattering cross section is only a few times the thermal value. The signals can be difficult to detect and evaluate, and alignment is critical. A possible different detection method is to look at the coupling between allowed and forbidden components of spectral lines. The electric fields associated with the driven electron waves should couple these spectral lines through the A.C. Stark effect. Baranger and Mozer have shown that plasma waves
should produce satellites around forbidden components, and Hicks that harmonics are also possible. Ringler detected such satellites caused by waves which appear naturally in the plasma. The electric field strength of these waves were the same order of magnitude as waves driven here.

If Thomson scattering is still used, then recent developments in commercially available dye lasers could be exploited. Flashtube pumped dye laser oscillator-amplifier systems are now available with ten's of megawatts output power, 0.1 nm linewidth, 500 nsec FWHM pulses. The wave mixing could be done using a ruby laser and a dye laser, the diagnostics performed with a second dye laser (System 3 in Table II-1). Such a system has many advantages. The quantum efficiency of the detectors at the wavelength of the diagnostic laser are high. The ruby laser could improve the mixing by orders of magnitude over the presently attained level. The normalization of the signal to the laser powers can be done easier, since the integral of the product of the laser powers is just the product of the dye laser powers times the integral of the ruby laser pulse. (The much longer dye laser pulses can be considered constant as compared to the diagnostic pulse.) This last integral and the multiplication can be done electronically.

There are many possible future experiments. These include the ones already mentioned, namely the investigation of the response function of the plasma and
the Baranger-Hozer effect. Other experiments include a direct measurement of the Landau damping rate. If the mixing is done at sufficiently small angles, the damping rate of the driven waves will be low enough that they can propagate an appreciable distance from the interaction region. The amplitude of the wave as a function of distance would give the damping rate. This experiment would require a plasma with much lower collision rates than the plasma jet.

Of course, the present experiment could (and should) be performed using changed geometry to study the waves of different wave vector. Using the present equipment, it is possible to reduce the mixing wave vector by more than a factor of two without changing from a coplanar system.
Bibliography


Appendix A

DOUBLE CONVOLUTION OF A LORENTZIAN
WITH TWO ARBITRARY LINE PROFILES

We have, in the theoretical derivation of the spectral density \( \langle |N_{ind}(\Delta k, \Delta \omega)|^2 \rangle \) (equation (13)), the double integral I:

\[
I(\omega) = 2\gamma \int_{-\infty}^{\infty} \frac{d\omega_1 d\omega_2 P_1(\omega_1) P_2(\omega_2)}{4\gamma^2 + (\omega - \omega_1 + \omega_2)^2}
\]

\[\text{(A1)}\]

\( \gamma \) is a constant and \( P_i(\omega_i) \) is the spectral distribution of the \( i \)th mixing light source normalized to unity:

\[
\int_{-\infty}^{\infty} d\omega_1 P_i(\omega_1) = 1
\]

\[\text{(A2)}\]

If the profiles can be described by Lorentzian with FWHM \( F_{\omega_i} \) and line centres \( \omega_{i0} \):

\[
P_i(\omega_1) = \frac{F_{\omega_i}}{2\pi} \frac{1}{(b_2 F_{\omega_i})^2 + (\omega_1 - \omega_{i0})^2}
\]

\[\text{(A3)}\]

then the integral \( I(\omega) \) simply reduces to
\[ I(\omega) = \frac{P_W}{2} \frac{1}{(\frac{1}{2}P_W)^2 + (\omega - \omega_{10} + \omega_{20})^2} \]

...(A4)

where \( P_W = 47 \cdot P_{W1} + P_{W2} \). In general, however, we do not have Lorentzian line profiles.

If the line profiles are measured as a function of frequency, then by linearly interpolation between the measured points we have a numerical description of the line profiles:

\[ P_i(\omega_i) = 0 \quad \omega_i < c_{i,1} \]
\[ P_i(\omega_i) = a_{ik} \omega_i + b_{ik} \quad c_{i,k} \leq \omega_i \leq c_{i,k+1} \]
\[ P_i(\omega_i) = 0 \quad \omega_i > c_{i,n} \]

...(A5)

The \( a_{ik} \) and \( b_{ik} \) are determined by the measured values \( P_i(\omega_i) \) at \( \omega_i = c_{ik} \), \( k=1,n \):

\[ a_{ik} = \frac{P_i(\omega = c_{i,k+1}) - P_i(\omega = c_{i,k})}{c_{i,k+1} - c_{i,k}} \]
\[ b_{i,k} = P_i(\omega = c_{i,k}) - a_{i,k} c_{i,k} \]

...(A6)

Using this linear analytical expression for the \( P_i(\omega_i) \), it is possible to integrate \( I(\omega) \) numerically. However, we must eventually calculate the observed quantity which is proportional to the convolution of this double integral with the monochromator transmission function and the response function of the plasma:
Performing this triple integral numerically with standard integration routines available to users of the IBM 360-67 computer at U.B.C. is prohibitively expensive. Thus, the integral $I(\omega)$ should be evaluated analytically. This is done below.

Substituting the linear expressions for the line profiles (eq. 5) into $I(\omega)$ gives:

$$I(\omega) = 2\gamma \sum_{i=1}^{n-1} \sum_{k=1}^{m-1} \int_{c_{2,i}}^{c_{2,k+1}} \int_{c_{1,i}}^{c_{1,k+1}} \frac{(a_{2,i} \omega_2 + b_{2,i})(a_{1,i} \omega + b_{1,i})}{4\gamma^2 + (\omega - \omega_1 + \omega_2)^2} \, d\omega_1 \, d\omega_2 \, du$$

where there are $m$ and $n$ points describing the profiles $P_1$ and $P_2$ respectively. Using the substitutions

$$x_i = \frac{\omega_i}{2\gamma} \quad \quad \quad \quad p_{i,k} = a_{2,i}a_{1,k}$$

$$u = \frac{\omega}{2\gamma} \quad \quad \quad \quad q_{i,k} = a_{2,i}b_{1,k}$$

$$a_{i,j} = 2\gamma a_{i,j} \quad \quad \quad \quad r_{i,k} = a_{1,k}b_{2,i}$$

$$b_{i,j} = b_{i,j} \quad \quad \quad \quad s_{i,k} = b_{2,i}b_{1,k}$$

$$\delta_{i,j} = \frac{c_{i,j}}{2\gamma}$$

we have

$$I_{\text{mix}}(\Delta \omega_{\text{obs}}) \propto \int_{-\infty}^{\infty} T(\Delta \omega - \Delta \omega_{\text{obs}}) R(\Delta \omega) I(\omega) \, d\omega$$

\ldots (A7)
\[ I(\omega) = 2 \sum_{i=1}^{n-1} \sum_{k=1}^{m-1} I_{1,k}(\omega) \]

\[ I_{1,k}(\omega) = \int_{\delta_{2,i}}^{\delta_{2,i+1}} dx_2 \int_{\delta_{1,k}}^{\delta_{1,k+1}} dx_1 \left[ \frac{p_{1,k}x_1x_2 + q_{1,k}x_2 + r_{1,k}x_1 + s_{1,k}}{1 + (u - x_1 + x_2)^2} \right] \]

... (A10)

Also define

\[ I_{i,k}^P = \iint \frac{dx_1 dx_2 x_1 x_2}{1 + (u - x_1 + x_2)^2} \]

\[ I_{i,k}^Q = \iint \frac{dx_1 dx_2 x_2}{1 + (u - x_1 + x_2)^2} \]

\[ I_{i,k}^R = \iint \frac{dx_1 dx_2 x_1}{1 + (u - x_1 + x_2)^2} \]

\[ I_{i,k}^S = \iint \frac{dx_1 dx_2}{1 + (u - x_1 + x_2)^2} \]

... (A11)

Then

\[ I_{1,k} = p_{1,k} I_{1,k}^P + q_{1,k} I_{1,k}^Q + r_{1,k} I_{1,k}^R + s_{1,k} I_{1,k}^S \]

... (A12)

The integral with respect to \( x_1 \) is done first, using the transformation \( x_1 = -z + (u + x_2) \). The \( x_1 \) integral range is now \( z_{k,0} \) to \( z_{k,1} \) where

\[ z_{k,\ell} = z_{k,\ell}(x_2) = u + x_2 - \delta_{1,k+\ell} \]

... (A13)

The integration with respect to \( x_2 \) is then performed using the transformation \( x_2 = z - u + \delta_{i,k+\ell} \). The integral range now
is $z_{k,l}^{i,0}$ to $z_{k,l}^{i,1}$ where

$$z_{k,l}^{i,m} = u - \delta_{1,k+l} + \delta_{2,i+m}$$

... (A14)

The details of these straightforward integrals are not given.

Define

$$A_{i,k,l}^{m} = \int_{Z_{k,l}^{i,0}}^{Z_{k,l}^{i,1}} (Z_{k,l}^{i})^{n} \arctg Z_{k,l}^{i} \, dz_{k,l}$$

$$L_{i,k,l}^{n} = \int_{Z_{k,l}^{i,0}}^{Z_{k,l}^{i,1}} (Z_{k,l}^{i})^{n} \ln[1 + (Z_{k,l}^{i})^{2}] \, dz_{k,l}$$

... (A15)

Then

$$I_{i,k}^{S} = (-1) \sum_{k=0}^{1} (-1)^{k+1} A_{i,k,l}^{0}$$

$$I_{i,k}^{R} = \sum_{k=0}^{1} (-1)^{k+1} b_{1,k,l}^{0} - A_{i,k,l}^{1} - \delta_{1,k+l} A_{1,k,l}^{0}$$

$$I_{i,k}^{Q} = (-1) \sum_{k=0}^{1} (-1)^{k+1} A_{i,k,l}^{1} + (\delta_{1,k+l} - u) A_{i,k,l}^{0}$$

$$I_{i,k}^{P} = \sum_{k=0}^{1} (-1)^{k+1} b_{1,k,l}^{1} + 2(\delta_{1,k+l} - u) L_{1,k,l}^{0} - A_{1,k,l}^{2} - (2\delta_{1,k+l} - u) A_{1,k,l}^{1} - \delta_{1,k+l}(\delta_{1,k+l} - u) A_{1,k,l}^{0}$$

... (A16)

The standard integrals $A_{i,k,l}^{n}$ and $L_{i,k,l}^{n}$ are:
\[ L_{i,k,\ell}^{1} = \frac{1}{2} \sum_{m=0}^{1} (-1)^{m+1} [\ln(1 + Z^2) + Z^2 \ln(1 + Z^2) - Z^2] \]
\[ L_{i,k,\ell}^{0} = \sum_{m=0}^{1} (-1)^{m+1} [Z \ln(1 + Z^2) - 2Z + 2 \arctg(Z)] \]
\[ A_{i,k,\ell}^{2} = \frac{1}{3} \sum_{m=0}^{1} (-1)^{m+1} [Z^3 \arctg(z) - \frac{1}{2}Z^2 + \frac{1}{2} \ln(1 + Z^2)] \]
\[ A_{i,k,\ell}^{1} = \frac{1}{2} \sum_{m=0}^{1} (-1)^{m+1} [\arctg(Z) + Z^2 \arctg(Z) - Z] \]
\[ A_{i,k,\ell}^{0} = \sum_{m=0}^{1} (-1)^{m+1} [Z \arctg(Z) - \frac{1}{2} \ln(1 + Z^2)] \]

\[ \ldots (A17) \]

where the sub- and super-scripts of Z have been dropped.

Combining terms, we finally obtain:
where $Z = Z_{k,l}^{i,m}$ as above.

This analytical solution can be checked by using the $P_i(\omega_i)$ which describe Lorentzian line profiles as in equation A3. The results of equation A18 can then be compared directly to equation A4. Excellent agreement (1 part in $10^4$) is found between the two with

$$F_{W_1} = 5.66 \times 10^{12} \text{ sec}^{-1},$$
$$F_{W_2} = 7.01 \times 10^{12} \text{ sec}^{-1},$$
$$\gamma = 10^9 \text{ sec}^{-1}.$$

These corresponds to 0.20 and 0.25 nm FWHM line for the
narrow and wider profile respectively, centred at 820 and 816.1 nm. The P's were described by points every 0.02 nm, over a range that was 8 and 10 times the FWHM for the profiles. The large ranges are necessary because of the contribution to the double integral from the intense wings.
Appendix B

DATA REDUCTION

In this appendix a detailed description of the data reduction is given. First, two different methods of measuring the signals on the oscillograms are evaluated. Then the methods actually used for digitizing the oscillograms and reducing the data are presented. The errors in the measurement are analysed. Finally, the different methods of evaluating the scattering parameters are discussed and the actual method used is justified.

A. Oscillogram Analysis

As indicated in Chapter IV, the data acquisition rate for the mixing experiments described in this thesis is unfortunately very low. For instance, the spectrum in section F of Chapter IV require approximately 20 hours continuous work, including the "last minute" alignment. However, there are only an average of four shots for each wavelength position for the mixing, and four shots for the normal scattering. Therefore, it is imperative that as much and as accurate information as possible be extracted from the oscillograms as possible.

Consider a normal scattering experiment where
the photomultiplier signal $I_{\text{norm}}$ is equal to a constant $S_1$ times the ruby monitor signal $I_d$: $I_{\text{norm}} = S_1 x I_d$. The constant $S_1$ will change for different scattering angles and f-numbers, wavelengths, optical transmission efficiency of the lenses and gratings, and plasma conditions. It is necessary to obtain an estimate of $S_1$ from a series of measurements of $I_{\text{norm}}$ and $I_d$ while attempting to keep these parameters constant. The most simple method for estimating $S_1$ is to measure the maximum height of the ruby laser monitor and photomultiplier signals using a ruler or magnified scale. An appropriately weighted average of the ratio of the measured heights $I_{\text{norm}}/I_d$ now gives $S_1$.

This method makes two basic assumptions. It is first assumed that the peaks in the signals occur at the same instant in time. Secondly, it is assumed that the peak photomultiplier signal is a linear function of the peak scattering light, even though the photomultiplier integrates the signal because of its slow risetime. These two assumptions are reasonable, and spectra obtained in this manner can fit the theory very well. However, this method has a drawback in that not all the information from oscillograms is being used: the photomultiplier signal is related to the laser signal at every instant in time, not just at the maximum values. If we could determine the heights of the signals as a function of time then we could do an average of the ratio of heights for the two signals at the same instant in time. Alternatively, we could
calculate the ratio of the area's of the signals. This latter method is convenient since one does not have to determine accurately the relative time origins of the two signals. Also the ratio of the areas is probably more correct, for statistical reasons as indicated in section B, and because the PM signal is already slightly integrated.

The latter method of determining the constant S has the disadvantage that the measurement of the height of the signals as a function of time is an extremely tedious task. Therefore, a compromise is usually made in which the peak heights of the signals are measured, but a larger number of observations are taken. This is not possible for the mixing experiments described in this thesis.

The improvement in data reduction made by using the heights of the signals as a function of time as compared to measuring the peaks heights only is illustrated by analyzing the normal scattering signals of the experiment in Chapter IV, Section D. When the 21 normal scattering oscillograms are analysed by measuring the peak heights of the signals alone, one obtains a value of 20% for the standard deviation of the estimates of S about the mean of $S_1$. When the the areas of the signals are determined, this standard deviation is reduced to 10%. Figure A-1 illustrates this improvement. The different standard deviations indicate that, to obtain equally accurate estimates of the mean value of $S_1$, four times as many observations must be made when the peak heights of
FIGURE B-1 Thomson scattering photomultiplier signal as a function of incident ruby power.
the signals are used instead of the areas. Obviously, for this experiment, the method of measuring the heights of the signals as a function of time is necessary. By using this analysis method, the four shots at each value of experimental parameter is equal to more than 10 shots using the simpler method.

It is appropriate at this point to detail the method of determining the signal heights as a function of time. A commercial digitizer is used (Instronics Limited, 'Gradicon'), which gives the position of a curser in the form of $x,y$ co-ordinates punched on regular computer data cards. The coordinates are given in thousandths of an inch. The curser consists of a set of fine cross-hairs with a dot at their intersection, etched in clear plastic. The diameter of the dot is approximately 100 microns. The cross-hairs and oscillograms are viewed through a plano-convex lens temporarily mounted on the curser. The lens, with a magnification of over 10, makes positioning the centre of the curser on the 400 microns wide image of the oscilloscope trace on the oscillograms relatively easy.

The first oscillogram is taped to the digitizer table so that the horizontal graticule lines are parallel to the $x$-axis of the table. This is checked by moving the curser along the graticule line and observing changes in the $y$-coordinate. A single computer card is taped to the table so that the remaining oscillograms can be mounted parallel to the $x$-axis by butting them against this stop. To avoid parallax error, a small red dot was placed at the
centre of the lens. The faint image of this dot is maintained above the curser centre and the point of measurement.

The actual digitizing is performed in the following manner. The curser is set to the intersection of the main graticule lines at the centre of the oscillogram. The \((x,y)\) coordinates are set to \((0,0)\). The coordinates of the 4 intersection points of the graticule lines 3 divisions up and down and 3 divisions left and right of the origin are measured. These coordinates can be used to determine the relative magnification of this set of oscillograms as compared to the calibration oscillograms described later. Next, the curser is set so that its horizontal line is centred on the baseline of the oscillogram trace, and its vertical line bisects the ruby laser monitor pulse. The position of the curser is recorded so that the position of the ruby pulse on the oscillogram is known. The \((x,y)\) coordinates are reset to \((0,0)\). Now all measurements of height are relative to the baseline, and distance along the oscillogram are relative to the approximate centre of the ruby pulse. The curser is finally moved along the oscilloscope trace, and its position punched on cards. Approximately 25 \((x,y)\) coordinates are punched for each signal.

Now the integration of the signals can be done numerically, linearly interpolating between the coordinates of the signals. The integration ranges are determined in the following manner. First, the time
origins of the oscillograms is shifted so that half the area of the ruby pulse is located on each side of the new origin. The small shift in origin is only a few percent of the total integration range of approximately 1 cm, and in fact makes less than 1/4 % difference in the value of the integral. The heights of the oscillograms as a function of distance are added together. This gives the heights as a function of distance for an "average oscillogram", and the points at which the signals go to zero can easily be determined. Equal lengths of integration ranges is used for both the photomultiplier and the ruby signals.

The estimate \( S_1 \) is now made in using the following two formulae:

\[
S_1 = \frac{\sum_{i=1}^{N} \left( \int I_{\text{norm}}^i \, dt \right) \left( \int I_d^i \, dt \right)}{\left( \sum_{i=1}^{N} \left( \int I_d^i \, dt \right)^2 \right)}
\]

\[
S_1 = \frac{\sum_{i=1}^{N} \left( \int \left( I_{\text{norm}}^i \right) \, dt \right)}{\sum_{i=1}^{N} \left( \int I_d^i \, dt \right)}
\]

\[\ldots (B1)\]

where \( I_{\text{norm}}^i \) and \( I_d^i \) are the signals for the \( i \)th experimental shot of a total of \( N \) shots. These estimates are in fact least squares estimates using different weighting methods, as discussed in Section B.
Evaluating the mixing signals is not as straightforward. The signals on the mixing oscillograms are theoretically related in the following manner:

\[ I_{\text{obs}} = S_1 I_d + S_2 I_1 I_2 \]  

...(B2)

where \( I \) and \( I \) are the photomultiplier and ruby laser monitor signals, and \( I \), and \( I \) are the dye laser monitor signals. The PM signal due to the mixing alone is

\[ I_{\text{mix}} = I_{\text{obs}} - S_1 I_d = S_2 I_1 I_2 \]  

...(B3)

As this equation indicates, to make an estimate of the mixing constant \( S_2 \) for this particular parameter setting, we must subtract the expected signal due to normal scattering. To work as accurately as possible we must again do the integral of these signals with respect to time.

The evaluation of the triple product integral by numerical methods is a simple task, once the signals are digitized. However, the relative time origins are now very important. The dye lasers begin to lase after the ruby laser, so that their lasing peaks do not coincide. It is not sufficient to do the integration starting at the rising edge of the signals, nor centred on their peaks.

The true relative time origins were determined by connecting the cables which carry the signals from the photodiode monitors to a pulse generator. A 20 nsec wide pulse is sent down all three cables and oscillograms made.
Measurements of the shifts of the delayed signals relative to the direct signal correspond to the shifts of the origins of the dye laser pulses to the ruby laser pulse. There is no time shift caused by the monitors, since they are identical in construction and equidistant from the scattering volume. This technique gives an absolute measurement of the origin shifts, independent of oscilloscope and camera linearity, cable lengths and lasing times.

Now that the time origins are determined, the integral of the triple product can be evaluated. This integration is performed using the trapezoid rule with .001 inch increments. The ruby laser signal time origin is redefined, and the heights of the signals are determined by linear interpolation, as is done in the normal scattering case. The same integration range for the ruby pulse is used. The value of the integral is then given by:

\[
\int_{x_1}^{x_n} I \, dI_1 \, dI_2 \, dt = \frac{1}{2} \sum_{i=1}^{n-1} [x_{i+1} - x_i]
\]

\[
x \left[ H(x_{i+1}) H(x_{i+1} + d_1) H(x_{i+1} + d_1) + H(x_i) H(x_i + d_1) H(x_i + d_1) \right]
\]

... (B4)

where \( H(x) \) is the height of the oscillogram trace at the distance \( x \), \( d_1 \) and \( d_2 \) are the shifts of the time origins, \( x_{i+1} - x_i = .001 \) inches, and the ruby laser is in the region \( x_1 \) to \( x_n \).
This experiment is not being done on an absolute scale. That is, the scattering intensities are being determined relative to each other. This allows us to multiply the calculated signals by constants to give convenient numbers to handle. The constant $S_1$ is of the order of unity without any adjustment, but the triple product integral is multiplied by $10^3$.

This digitizing technique, and especially the Textronix 7900 series oscilloscope, proved to be surprisingly linear. In the analysis we have made the assumption that the oscillograms are linear in time, that the constant relating distance along the oscillogram to real time does not change significantly along the trace. The specifications for this oscilloscope indicate that this constant should not change more than 3% across the entire face, and not more than 1% in the middle half. Measurements of the linearity using a 50 MHz pulse generator indicate that this oscilloscope is well within specifications. It is linear to within less than 1% over any 150 nsec (3 cm) distance. Thus no correction for nonlinearity of the time scales is necessary.

Using the origin shift calibration oscillograms, the shift in the signals varies only .01 inches when the first pulse starts at the graticule 1 cm in from the edge of the face as compared to starting 4 cm in from the the edge (about 1/4 %). The changes in origin shift due to different starting positions of the ruby pulse are approximately ±.002 inches, which also can be neglected.
(This is a measure of the average linearity over 3 cm rather than the linearity within the 3 cm.)

The oscillograms for the calibration pulses can be used to indicate the errors involved in determining the areas and the triple product integral. The measured area of a calibration pulse whose width is equal to that of the ruby laser pulse has a standard deviation of 1%. The standard deviation of the triple product integral is 2%. Another estimate of the measurement errors can be found by going through the digitizing sequence several times using the same experimental oscillogram. This gives a standard deviation of 4% for the area, and 5% for the triple product integral. This larger error can be attributed to the fact that the rectangular calibration pulses are more regular and easier to see than the laser monitor pulses. The standard deviations of $S_1$ and $S_2$ are much larger than 5%, so that the measurement accuracy is quite sufficient for our purposes.

The measurement of the peak heights of the signals are surely also accurate to 5%. Why, therefore, is the standard deviation of the estimates for $S_1$ twice as large? Obviously there must be sources of error other than measurement alone which are handled better by the integration method. These sources include innacuracy in determining the baseline, and the variations in the width of the ruby laser pulse. Because of the fast risetime of the ruby monitor, a change in total energy with the same relative change in width can maintain the same peak
height. However, due to the integrating effect of the PM, its signal peak height will change. This can readily be seen if one considers the exaggerated case of a square pulse for the ruby laser, and an integration time constant for the PM of the order of the width of the ruby pulse. The PM pulse would then be approximately twice the width of the ruby pulse, and its height proportional to the ruby pulse width.

The fact that the scattering signal is composed of mixing signal and normal Thomson scattering signal is a disadvantage in the data reduction. The signal due to mixing alone has its own large fluctuations which are compounded by those of the normal scattering.

B. Evaluation Of Relative Scattering Cross Sections

We now have numbers which represent the incident ruby diagnostic beam $I_d$, the mixing product $I_d x I_1 x I_2$ and the scattered light $I_{obs}$. These numbers must be used to evaluate $S_1$ and $S_2$ for one particular set of experimental parameters, in the the formula

$$\int_{I_{obs}}^{I_d} dt = S_1 \int_{I_d}^{I_1} dt + S_2 \int_{I_d}^{I_1 x I_2} dt$$

... (B5)

$S_1$ and $S_2 x I_1 x I_2$ are proportional to the normal and mixing scattering cross sections, respectively, and the superscripts i refer to the ith scattering measurement.
The integral signs will be dropped for simplicity.

Let us restrict ourselves to the normal scattering experiments first. Three of many possible estimates for $S_1$ using the $n$ measurements are:

\[ S_{1a} = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{I_{1i}^{\text{obs}}}{I_{1i}^{\text{d}}} \right) \]

\[ S_{1b} = \sum_{i=1}^{N} \left( I_{1i}^{\text{obs}} \right) \frac{1}{\sum_{i=1}^{N} \left( I_{1i}^{\text{d}} \right)} \]

\[ S_{1c} = \frac{\sum_{i=1}^{N} \left( I_{1i}^{\text{obs}} I_{1i}^{\text{d}} \right)}{\sum_{i=1}^{N} \left( I_{1i}^{\text{d}} \right)^2} \]

It is shown below that these three estimators are in fact just least squares estimates using different weighting methods.*2, *3

Consider the situation of a random variable $Y$ which is a known function of $X$:

\[ Y = F(X ; \alpha_1, \alpha_2, \ldots ) + \varepsilon \]

... (B7)

The $\alpha_i$ are unknown parameters of the function $F$, and $\varepsilon$ is a random variable which represents the errors involved in a real experiment. A series of measurements $(X_i, Y_i)$ are made from which the "best" estimates $\hat{\alpha}_i$ for the parameters $\alpha_i$ are to be made. The accepted definition of
"best" is usually estimators which are linear in the \( Y_i \), have an average value which is equal to the actual parameters, and have the smallest variance. If the random variable \( \epsilon \) has zero expectation value and variance \( \sigma^2 \), then the best estimators are found using weighted least squares method. That is, the choice of parameters which minimizes the sum

\[
S = \sum_{i=1}^{N} W_i \left[ Y_i - F(X_i; a_1, a_2, \ldots) \right]^2
\]

\[
W_i = [\sigma(\epsilon_i)]^{-2}
\]

are the best estimates of the true parameters. Setting \( \partial S / \partial a_i = 0 \) for the different \( a_i \) gives a set of equations which can be solved for the parameters in terms of the measured quantities \( (X_i, Y_i) \).

A simple example is a normal scattering experiment with a CW laser for the light source\(^*\). Now \( Y = \alpha X + \epsilon \), where \( Y \) is the number of scattered photons, \( X \) is the length of time to obtain the counts, and \( \alpha \) is proportional to the scattering cross section. One would expect that the error \( \epsilon \) would have zero average value and some variance \( \sigma^2 \). The dependence of \( \sigma \) on \( X \) will be discussed presently. This experiment then qualifies for the least squares method. Setting

\[
S = \sum_{i=1}^{N} W_i \left[ Y_i - a_1 X_i \right]
\]

\[\ldots (B9)\]
we have
\[ \hat{\alpha}_1 = \frac{\sum_{i=1}^{N} W_i Y_i X_i}{\sum_{i=1}^{N} W_i X_i} \]

... (B10)

Now consider different weighting schemes. For the simple case above let us assume that \( \epsilon \) is governed by Poisson statistics only. In this case, the variance of \( \epsilon \) is proportional to the true number of counts in the time \( X \), which in turn is directly proportional to \( X \). Therefore, \( W \propto X^{-1} \), and \( \hat{\alpha}_1 \) is
\[ \hat{\alpha}_1 = \frac{\sum_{i=1}^{N} Y_i}{\sum_{i=1}^{N} X_i} \]

... (B11)

This, of course, corresponds to the \( S_{10} \) estimator in (B6).

Consider next the case for constant percent error. This would correspond to our simple CW scattering experiment if, whenever the counting rate is reduced, a longer period of time is used to obtain more counts and constant standard deviation. Now
\[
\sigma / \mu = c \\
W_1 = c \chi_1^{-2} \\
\hat{a}_1 = \frac{\sum_{i=1}^{N} Y_i}{\sum_{i=1}^{N} \chi_i} \\
c = \text{constant}
\]

This corresponds to the $S_{1a}$ estimator in equation (B6).

Finally we have the case where $\epsilon$ is constant, independent of $X$. This corresponds to the signal being buried in noise, so that the fluctuations in signal are due to the background alone. This gives

\[
W_1 = c \\
\hat{a}_1 = \frac{\sum_{i=1}^{N} \chi_i Y_i}{\sum_{i=1}^{N} (\chi_i)^2}
\]

which corresponds to the $S_{1c}$ in (B6).

The weighting scheme which is chosen is dictated by the conditions of the experiment, and is never as clear cut as in the above simple examples. Some general observations can be made however.

First, in an actual experiment, the values of $X$ can never be absolutely known. If the variance of $X$ cannot be neglected, the analysis is still valid, but now
will also take into account the variations in $X$.

Second, if one is able to maintain the experimental variable $X$ nearly constant, then all estimators will be exactly equivalent, independent of the form of the dependence of $\epsilon$ on $X$. Keeping the $X$'s constant may be convenient from the point of view of not having to determine the weighting scheme, but it does not always give good estimates of the parameters. Consider the case for $Y = mX + b + \epsilon$. Keeping $X$ constant gives no estimate of $m$ or $b$. It will, however, give an estimate of the distribution of $\epsilon$.

Third, if a large number of measurements are made, then the three estimators are essentially equivalent. This follows directly from the fact that the expectation values of all the estimators are equal to the true values in the limit of a large number of observations. For example, consider the case of the normal scattering signals in Chapter V, Section D. The standard deviation of the estimate of $S_1$, for 20 measurements, is 3%, but the three estimators of $S_1$ differ by only 2/3%.

We thus see that the weighting scheme is only important when one must change $X$, or when a small number of measurements are made. The present work has few measurements and large variations in laser powers, so that the weights must be chosen.

The weighting scheme valid for this experiment is between the weighting for Poisson statistics and for
constant error \( (S_{1b}, S_{1c}, \text{ equation (B6)}) \). The photomultiplier signal is proportional to the number of photons scattered into the collection system. Since there are only of the order of 100's of photons in the scattered light detected by the PM, statistical fluctuations are an important source of error. The measurement inaccuracy, which is at least partly independent of the area, contributes a similar amount to the total error. We must choose between the two weighting schemes, but fortunately the estimates for \( S_1 \) and \( S_2 \) using weighting schemes (b) and (c) give essentially the same results. Scheme (a), which assumes constant percent error, is radically different. As an example, the estimate for \( S_2 \) for the data of Chapter V, Section D, using 38 points, has a standard deviation of the estimator of 10%. The three estimators differ by less than 4%. However, if the centre 5 data points are analyzed separately, schemes (b) and (c) give a 10% smaller value for \( S_2 \), but are still within 2% of each other. Weighting scheme (a) gives a value that is 47% low.

For these experiments, the values for the parameters presented are those calculated using the weighting for constant error. The values using scheme (b) were calculated to look for large discrepancies, but are not presented. In all cases, schemes (b) and (c) were found to be equivalent.

\( S_1 \) is determined from the normal scattering alone, as outlined above. \( S_2 \) will be determined from the
mixing shots where the photomultiplier signal will be the sum of the normal scattering signal and the mixing signal:

\[ I_{\text{mix}} = I_{\text{obs}} - S_1 I_d + \epsilon' \]

... (B14)

We also have from the theory that

\[ I_{\text{mix}} = S_2 I_d I_1 I_2 (+ \epsilon'') \]

... (B15)

\( \epsilon' \) represents the errors involved in the measurement of the observed and diagnostic laser signals, and the calculation of \( S_1 \). \( \epsilon'' \) represents the error involved in the calculation of the triple product integral. This gives

\[ S_2 I_d I_1 I_2 = I_{\text{obs}} - S_1 I_d + \epsilon' - \epsilon'' \]

... (B16)

We now have the same situation as described above with

\[ Y = I_{\text{obs}} - S_1 I_d \]

\[ X = I_d I_1 I_2 \]

\[ a_1 = S_2 \]

\[ \epsilon''' = \epsilon'' - \epsilon' \]

... (B17)

Assuming that the random variable \( \epsilon''\) still has average value zero and a new variance \((\sigma')^2\), we can again use the least squares analysis. The weighting scheme should lie between Poisson statistics and constant error. The
estimates for $S_2$ were calculated using weighting schemes (b) and (c). Since both schemes proved to be equivalent, only the results of the first are presented.
Appendix C

PLASMA JET COLLISION RATES

The theoretical calculation of the induced density fluctuation in Chapter II requires that the dominant damping mechanism is Landau damping. The collision frequencies must be calculated to ensure that collisional damping is not important. This appendix collects information from different sources on the collision frequencies between the electrons and other species present in the plasma.

Ecker\textsuperscript{28} has calculated the collision rates and energy exchange times for test electrons moving in a field of Maxwellian electrons or protons. The average collision rate between electrons for the temperature and density of our plasma is:

\[ \nu_{ee} = 1.25 \nu^{(0)} \]

where \( \nu^{(0)} = 4 \times 10^{11} \) rad/sec. (Spitzer's\textsuperscript{30} formula give a value lower by a factor 2.)

The energy exchange rate between electrons is required for the calculation of the perturbation of the plasma caused by the laser beams. Ecker gives a value the energy exchange rate \( T_{ee} \) of:
\[
\tau_{ee} = 1.3/\nu(0).
\]

Ecker also gives a value for the electron-ion collision rate \(\nu_{e-i}\). This collision rate is, correcting for the lower temperature of the ions:

\[
\nu_{e-i} = 0.09 \nu(0).
\]

Calculation of collision phenomena between charged particles and neutrals depends strongly on the choice of the interaction potential. It is thus appropriate to use measured values for the electron-neutral collision rates. Using the experimental results of Brode, the electron-neutral collision rates \(\nu_{e-n}\) are:

\[
\nu_{en} = 0.8 \nu(0).
\]

Ecker gives a value for the Landau damping rate for a plasma with electron temperature and density of the plasma jet of:

\[
\omega_1 = 17 \nu(0)
\]

Thus we see that the collision frequencies are an order of magnitude smaller than the damping rate.
Appendix D

DISCUSSION OF THE LOCATION OF THE DIPS IN THE MEASURED RESPONSE FUNCTION

It has been indicated in Chapter IV, Section F, that there is a possible relationship between the positions of the modulations in the measured response function of the plasma. There are definite valleys at $\Delta \lambda = -2.85$ and $-4.35$ nm, and possible valleys at $-1.65$ and $-1.05$ nm. If we assume that all four are valleys, and plot the positions $\Delta \lambda_n$ of the nth valley as a function of n, we get the excellent straight line shown in Figure IV-9 (A). In this graph, the starting integer for the first 'valley' was chosen as $n=1$ since it produced a straight line plot. If $\Delta \lambda_n$ is plotted for different starting $n$, the curves deviate radically from straight lines. However, note that if one plots $(\Delta \lambda_n)^2$ rather than $\Delta \lambda_n$ versus $n^2$, starting now at $n=0$, a second straight line fit is obtained, with curves for different choices of starting $n>0$ (Figure D-1 (B)). This dual dependence of $\Delta \lambda_n$ on n is related to the slight arbitrary nature on the initial choice of n, and also to the mathematics of small numbers. If

$$(\Delta \lambda_n)^2 = (\lambda_0)^2 + (\lambda_1)^2 n^2$$

... (D1)
FIGURE A-1  Position of the nth valley as a function of n.
Then

\[ \Delta \lambda_n = \lambda_0 \left[ 1 + \frac{1}{2} \left( \frac{\lambda_1}{\lambda_0} \right)^2 n^2 \right] \]

...(D2)

for \( \lambda_1 / \lambda_0 \ll 1 \), where and are constants. For the case \( \lambda_1 / \lambda_0 \gg 1 \), it is possible to imagine that changing the index by one could again produce a reasonable straight line fit. Thus, if there is a definite relationship amongst the valleys, we do not know if it is of the form (D1) or (D2). The error bars in Figure D-1 (B) represent \( \pm 1/2 \) FWHM of the monochromator transmission function, \( T_\omega \). The same error bars (±1.5 nm) are slightly larger than the circles in (A).