OPTIMIZATION OF MAGNETIC PROBE MEASUREMENTS IN TRANSIENT PLASMAS

by

JAROSLAV PACHNER, JR.
Dipl. Phys., Charles University, 1963

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

in the Department of PHYSICS

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

September, 1971
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Department of Physics
The University of British Columbia
Vancouver 8, Canada

Date Sep 22, 1971
ABSTRACT

A significant improvement in magnetic probe measurements has been achieved by developing a 3-coil magnetic probe which partially corrects for the boundary error, i.e. for the error caused by an exclusion of the plasma current from the space occupied by the probe. The spatial resolution $l$ of the three coil probe is roughly one-half of the probe radius, $a$. For current distributions which vary slowly with distance (scale length, $\lambda$) it is shown that the fractional error in the magnetic field is $0.2 \left( \frac{l}{\lambda} \right)^2$. For a conventional probe the error is at least four times as large.

Also it has been shown that spurious signals arising from poor probe geometry (and which often obscure the signals produced by the measured magnetic fields themselves) can be eliminated by making use of the symmetry of the discharge fields.

Measurements have been made on a Z-pinch discharge which confirm the claimed 3-coil probe performance by revealing a "fine" structure of the current distribution in
helium in the filling pressure range 0.5 to 4 Torr. The measurements are presented in the form of a catalogue of the spatial distribution of magnetic field and the current density for a Z-pinch discharge in He and Ar. Using the probe measurements a qualitative model of the collapse of the current sheet for 4 Torr in He is developed which differs from previous models.
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ACKNOWLEDGEMENTS

I wish to thank Dr. F. L. Curzon for suggesting this project and for his helpful guidance during the experimental work and the preparation of this thesis. Thanks are also due to Drs. B. A. Ahlborn, J. Meyer and R. Nodwell for improvements in the presentation of the thesis.

I am grateful to Mr. L. W. Funk whose measurements of the electron density helped in the interpretation of certain results. The discussions with Mr. J. D. Strachan are very much appreciated.

I am also indebted to Messrs. D. G. Sieberg and J. A. Zanganeh for the technical assistance, as well as to Mr. T. Matthews for the help with computations.
The major objective of this thesis was the improvement of magnetic probe measurements.

A conventional magnetic probe is a small coil of light-gauge wire placed at one end of a cylindrical insulating shield. The output voltage produced by the coil is proportional to the rate of change of magnetic flux passing through the plane of the coil. The time integrated signal is then proportional to the magnetic field. A typical arrangement for magnetic probing is shown in Fig. 1-1.

Magnetic probes can provide information on several important plasma parameters. From the magnetic field distribution in time and space we can deduce the plasma current density, electric field, electrical conductivity* and kinetic pressure (if the plasma density and velocity is known); see R. H. Lovberg in [2]. Magnetic probes are therefore widely

*Savic [1] and his co-workers have developed a novel magnetic probe which measures plasma conductivity directly. The technique depends on the effect of the plasma on the impedance of the coil immersed in the plasma. Since we are primarily concerned with magnetic field measurements this thesis contains no further discussion of conductivity probes.
Fig. 1-1 A typical magnetic probing arrangement. $R_0$-terminating resistor, $R_i$ and $C_i$-elements of integrator.
used in plasma physics. For example, they provide the principal diagnostic technique in pulsed plasma accelerators, mainly to reveal the current sheet position and the driving electromagnetic forces. They are also widely used in fusion devices, mainly to provide information on magnetic field configuration (R. H. Lovberg in [2]).

The conventional magnetic probes have, however, two serious limitations: the first shortcoming is the so-called boundary error caused by the exclusion of current in plasma from the space occupied by the probe. The second problem is the various spurious signals which often obscure the signals produced by the measured magnetic fields themselves.

In the past some spurious signals have been combatted with varying degrees of success. However, the techniques for their testing and elimination have not been reported in readily available literature.

With regard to the boundary error, several investigators attempted to account for it by correction procedures [3 to 6]. They studied how the measured probe signals could be used to calculate the true magnetic field and current density. However, Tam [6] showed that the application of correction procedures is impractical, since any small measuring error would be magnified by a factor of 20 in the correction procedure. He therefore concluded that normally the measured magnetic field is a better approximation to the unperturbed field than the corrected one.
Both of the above mentioned problems have been successfully eliminated by the techniques described in this thesis.

The origin and elimination of spurious signals associated with magnetic probe measurements in pulsed discharges is discussed in Chapter 2. The main contribution described in this chapter is in the proposed procedure for elimination of spurious signals arising from poor probe geometry by using the symmetry of discharge fields. The chapter also contains a summary of measuring techniques and methods of improving the probe design so as to reduce the spurious signals amplitude.

The major contribution of this thesis is presented in Chapter 3, where it is shown that a 3-coil probe can partially correct for the boundary error. The 3-coil probe gives $B_3$, a linear combination of 3 values of the perturbed field $B_p$, near the measuring point $r$. $B_3$ is a much better approximation to the undisturbed field $B_0$, than $B_p$, the signal obtained with a conventional probe.

Subsequent chapters are devoted to results obtained with the 3-coil probe in Z-pinch discharges in helium and argon. The objective was to demonstrate the capabilities of the 3-coil magnetic probe on a typical Z-pinch device and to provide a catalogue of spatial distributions of magnetic field and current density (Chapter 5).
The superior ability of the 3-coil probe has been demonstrated by observing a "fine" structure of the current sheet (i.e. the development of a "minor" current sheet inside of the main current sheet) in helium in the filling pressure range from .5 to 4 Torr. A complex structure of the current sheet has been observed before in helium [15 and 6] only in the high pressure limit where it is most pronounced.

The catalogue gives new information on the Z-pinch behaviour under different initial conditions. This will be useful in future experiments on Z-pinch discharges.

Finally in Chapter 6 we have compared our experimental results in helium with two types of the modified snowplow equation (the conventional piston-shock wave model in which the gas is trapped between the shock front and the current sheet which acts as a piston, and the York-Jahn model in which the current sheet moves through the cold gas, ionises it and the ions form a thin mass sheet outside the current sheet and are coupled to it by a radial electric field). For the limiting high pressure case (4 Torr in He), which is not described by either of the two models, we have presented a qualitative model which explains the main observed features.

The thesis is concluded with Chapter 7 which gives the summary of the original results and suggestions for
future work, centered on interesting features in the catalogue of magnetic field and current density distributions.
IMPROVEMENT OF MAGNETIC PROBE MEASUREMENTS

Chapter 2

SPURIOUS SIGNALS IN MAGNETIC PROBE MEASUREMENTS AND THEIR ELIMINATION

Ideally a magnetic probe should respond only to the magnetic field one wants to measure. However, even a very carefully built probe can pick up a number of undesirable signals, together with the wanted signal. The main trouble comes from the misalignment of the miniature probe coil with respect to its stem.

In this chapter we discuss various spurious electrostatic and magnetic signals obtained with magnetic probes in pulsed discharges. In section 2.1 we will show how the probe performance can be tested on a Z-pinch discharge and how distorted probe signals can be "corrected" to eliminate the influence of spurious signals of sufficiently small amplitude. Section 2.2 deals with elimination of spurious signals through the probe design and measurement techniques. Section 2.3 is a summary of the main results obtained in this chapter.
2.1 Test of the Probe Performance and Correction of Distorted Signals.

Let us consider a probe with the stem along the radius of axi-symmetric system with magnetic field components $B_r$, $B_\theta$ and $B_z$, Fig. 2-la. If the coil axis is not aligned exactly parallel to the direction of magnetic field we want to measure ($B_\theta$ say), the coil presents areas also to perpendicular directions, in this case $r$ and $z$. Therefore, we can generally write the signal measured by the probe at a given radial position in this form:

$$U_\theta(\theta, \alpha) = V_c + C_1(\theta, \alpha)B_r + C_2(\theta, \alpha)B_\theta + C_3(\alpha)B_z + C_4(\alpha)I$$  \hspace{1cm} (2.1)

$\theta$ gives the azimuthal position of the probe and $\alpha$ is the angle between the coil axis and the discharge axis, (Fig. 2-1b); it denotes the probe angular position when rotating it about its stem. In case of measuring $B_\theta$, $\alpha = 90^\circ$ or $270^\circ$. $V_c$ is the signal which results from the stray capacitance between the probe and the discharge electrodes. $C_1$, $C_2$ and $C_3$ are the sensitivities of the probe to $B_r$, $B_\theta$ and $B_z$ magnetic fields. $C_4I$ is a term representing the signal produced by stray magnetic flux from the total discharge current $I$; it is assumed that this flux is coupled to the probe through the leads outside the discharge vessel.
Fig. 2-1  (a) Axi-symmetric system for testing probe performance.  (b) Probe angular position when rotating it about its stem. Here the probe stem axis is perpendicular to the plane of the paper.
Consider that the probe coil is connected to two inner conductors of RG-22/U transmission line. Ideally the inner conductors are straight and parallel, Fig. 2-2(a). However, the cable may be deformed and the inner conductors twisted (Fig. 2-2(b)), thus presenting a loop to the magnetic flux.

![Diagram](image)

**Fig. 2-2** The external magnetic pickup loop

This loop may change its area when rotating the probe by 180°; therefore $0 < \beta \leq 1$ in the relation for $C_4$, (2.2). From axial symmetry it follows that:

$$
\begin{align*}
C_1(\theta, \alpha) &= -C_1(\theta + \pi, \alpha), \quad C_1(\theta, \alpha) = C_1(\theta, \alpha + \pi) \\
C_2(\theta, \alpha) &= -C_2(\theta + \pi, \alpha), \quad C_2(\theta, \alpha) = -C_2(\theta, \alpha + \pi) \\
C_3(\alpha) &= -C_3(\alpha + \pi), \\
C_4(\alpha) &= -\beta C_4(\alpha + \pi), \quad 0 < \beta \leq 1
\end{align*}
$$

(2.2)
These results are readily derived by considering what happens to the magnetic flux through the components of the probe area along the $r$, $\theta$ and $z$ directions, as $\alpha$ and $\Theta$ are varied. Therefore, by measuring on both sides of the discharge axis and also with the probe rotated by $180^\circ$ about its stem we get the following signals:

\[ U_{\theta 1} = U_\theta(\theta, \alpha) = C_1 B_r + C_2 B_\theta + C_3 B_z + V_c + C_4 I \]
\[ U_{\theta 2} = U_\theta(\theta, \alpha+\pi) = C_1 B_r - C_2 B_\theta - C_3 B_z + V_c - \beta C_4 I \]
\[ U_{\theta 3} = U_\theta(\theta+\pi, \alpha) = -C_1 B_r - C_2 B_\theta + C_3 B_z + V_c + C_4 I \]
\[ U_{\theta 4} = U_\theta(\theta, \alpha+\pi) = -C_1 B_r + C_2 B_\theta - C_3 B_z + V_c - \beta C_4 I \]

The above equations yield $C_1 B_r$ and $C_2 B_\theta$:

\[ C_1 B_r = \frac{1}{2} (U_{\theta 1} + U_{\theta 2} - A) = -\frac{1}{2} (U_{\theta 3} + U_{\theta 4} - A) \]
\[ C_2 B_\theta = \frac{1}{2} (U_{\theta 1} + U_{\theta 4} - A) = -\frac{1}{2} (U_{\theta 2} + U_{\theta 3} - A) \]

where

\[ A = \frac{1}{2} (U_{\theta 1} + U_{\theta 2} + U_{\theta 3} + U_{\theta 4}) = 2V_c + C_4 I (1-\beta) \]

If $A$ is exactly proportional to the total discharge current waveform, then $V_c = 0$. The departure of $A$ from the shape of $I$ provides information on the size of $V_c$. Using the expressions (2.4) we obtain radial profiles of $B_r$ and $B_\theta$ in arbitrary units. With the help of the Maxwell equation...
Curl \( \mathbf{B} = \mathbf{\mu}_0 \mathbf{J} \) we get the radial profile of the axial current density \( J_z \) in arbitrary units from \( B_\theta \) profile. Since the total discharge current \( I \) must be equal to the current carried by the plasma we can calculate sensitivity \( C_2 \) from equation

\[
I = 2\pi \int_0^R J_z(r) r \, dr
\]

where \( J_z = \frac{1}{\mu_0 r} \frac{\partial}{\partial r} (r B_\theta) \), and \( R \) is the inner radius of the discharge tube. Here \( I \) is measured by a Rogowski coil. The Rogowski coil is calibrated by equating the area under the current trace (in V. sec) to the total charge on the capacitor bank:

\[
\int I \, dt = k \int V \, dt = Q = CV_0
\]

For details of the calibration see Appendix I. By rotating the probe by 90° about its stem, we can now also measure \( B_z \) since

\[
C_3(\alpha + \pi/2) = C_2(\theta, \alpha) \quad C_3(\theta, \alpha + \pi/2) = - C_3(\alpha)
\]

We cannot find \( C_1 \) by the method used in determining \( C_2 \). By plotting \( C_1 B_r \) vs. \( r \) we can, however, get radial profiles of \( B_r \) in arbitrary units. We are not actually interested in measuring \( B_r \) with the probe we have just tested, however the information on \( C_1 B_r \) may simplify the measurements of \( B_\theta \).
If \( C_1 B_r = 0 \) or \( \ll C_2 B_\theta \) for all \( r \), we do not have to rotate the probe about its stem to get \( B_\theta \). We get \( B_\theta \) by measuring on both sides of the axis, which is easier:

\[
B_\theta = \frac{1}{2} \frac{1}{C_2} (U_{\theta 1} - U_{\theta 3}) = \frac{1}{2} \frac{1}{C_2} (U_{\theta 4} - U_{\theta 2}) \tag{2.6}
\]

In our case the plasma is produced by a conventional unstabilized linear Z-pinch discharge. Here \( B_r = B_z = 0 \) and therefore the equations (2.3) are much simplified. If \( V_c \) and \( C_4 \) differ from zero we use equation (2.6) for evaluating \( B_\theta \).

By the procedure described in this section we can determine the probe performance or its response to different pickup signals. The tested probe of reasonable performance can then be used for experimental measurements. The relations (2.4) (or (2.6)) and (2.5) enable the spurious signals to be eliminated.

The described procedure, of course, requires a reproducible plasma, e.g. plasma of a Z-pinch discharge at higher filling pressures (\( \geq 0.1 \) Torr for Argon, \( \geq 1 \) Torr for Helium). To estimate \( V_c \) and \( C_4 I \) tests have to be performed on the actual plasma.
2.2 Elimination of Spurious Signals Through the Probe Design and Measurement Techniques.

Generally we can say that the magnetic probe output is produced by an electromotive force which is induced in the probe circuit by external electromagnetic fields. Using the standard notation we can write for those fields

\[ \mathbf{E} = -\nabla \varphi - \frac{\partial \mathbf{A}}{\partial t} \]

\[ \mathbf{B} = \text{Curl} \ \mathbf{A} \]

We call the part of the probe signal which is produced by \( \varphi \) an electrostatic pickup. On the other hand a magnetic pickup is produced by the field components which are described by the vector potential \( \mathbf{A} \).

We will now discuss the various spurious pickup signals paying particular attention to their elimination through the probe design and measurement techniques.

2.2.1 The Electrostatic Pickup.

The electrostatic pickup enters the measuring circuit through capacitative coupling between the probe circuit and the discharge circuit, Fig. 2-3. If the stray capacities \( C_1 \) and \( C_2 \) are small compared with \( C_3 \) (voltage source far from the probe), then the induced voltages on the leads 1 and 2 are approximately the same, \( V_{c1} \approx V_{c2} \). They
Fig. 2-3 Capacitative coupling between the probe circuit and the discharge circuit.

Fig. 2-4 A balanced magnetic probe measuring circuit.
can be eliminated by using the balanced differential circuit with high common mode rejection ratio for frequencies of interest (Fig. 2-4). In case of large \( C_1 \) and \( C_2 \) compared with \( C_3 \) (voltage source close to the probe) it is likely that \( V_{c1} \neq V_{c2} \). The balanced differential circuit mentioned above will only reduce the electrostatic pickup, \(|V_c| = |V_{c1} - V_{c2}|\).

For a probe near the return conductor it is very likely that \( V_{c1} \neq V_{c2} \). Therefore we have to assume the presence of \( V_c \) in the probe signal as it was done in section 2.2 and check it.

In general the electrostatic pickup should not be a great problem. In 1965 Lovberg [2] reported that in his experience he did not see any electrostatic pickup on totally unshielded coils connected into 50 or 93\( \Omega \) transmission lines. From our experience we also do not recommend probe electrostatic shielding since it does not improve the signal.

2.2.2 The Magnetic Pickup

The magnetic pickup signals are induced in the measuring circuit by the rate of change of the magnetic flux passing through the various parts of the circuit. The only desirable component of this pickup is from the measured \( B \) which passes through the probe coil. We will now discuss all other undesirable components of the magnetic pickup.
Of primary concern is the component caused by the misalignment of the probe coil with respect to the stem (Fig. 2-5) and the first loop of the coil leads, which has to be considered as a part of the coil. To minimize the first loop, the leads should be brought together as shown in Fig. 2-6. If the coil form is removed after setting the coil, one lead is to be passed through the coil, (a). If the form remains with the coil, the double layer solenoid is the way to do it, (b). Otherwise, a slit would have to be made in the coil form to allow the lead to pass through. This can be somewhat difficult in the case of miniature coils.

If the coil leads are not tightly and uniformly wound together they may present a net area to the magnetic flux. It results in another undesirable signal component. To avoid this the tightness and uniformity of the loops should be checked visually using a magnifying glass. The electronic check can be performed by using the analogue of a Z-pinch described in the paragraph 3.3.5 (see Fig. 3-7a), with the cylindrical conductors close together. When sliding the probe stem radially through the cylinders, with the coil inside of the inner conductor (Fig. 2-7), the signal should be equal to zero. However, this component should be negligible if the probe is carefully made.

There is always the possibility of having the external magnetic pickup and therefore its presence should be checked
Fig. 2-5 Misalignment of the probe coil with respect to the stem.

Fig. 2-6 Two methods of bringing the magnetic probe coil leads together.
Finally we will discuss the high frequency component of the magnetic pickup. When the spark gap is initially closed the high frequency noise is radiated. Since magnetic pickup is proportional to $j\omega B$, it is picked up by all parts of measuring circuit and its magnitude depends on the geometry and the distance of the measuring circuit from the source. The high frequency pickup is severely damaging to the measurements at early times after the initiation of the discharge.

The following measures help to minimize the high frequency pickup:
(a) coaxialised triggering circuitry and spark gap switch as well as the measuring circuit,

(b) removing the oscilloscope further away from the discharge apparatus,

(c) shielding the transmission lines,

(d) putting the oscilloscope into a Faraday cage,

(e) another possibility is to delay the probe signal until the radiated noise has decayed by inserting an appropriate delay line into the measuring circuit close to the probe; if the major portion of the h. f. pickup enters the measuring circuit behind the delay line then we will see the clean trace on the oscilloscope.

2.3 Summary of Results on the Probe Misalignment

1. Everything except misalignment of the probe coil with respect to stem can be made negligible.

2. If the probe is calibrated in a "nice" system, then the orientation of the coil with respect to stem is known.

3. The probe can then be used in a "nasty" system by aligning the stem with some axis of the discharge.
Chapter 3

REDUCTION OF THE BOUNDARY ERROR BY THE PROBE
WITH THE SPATIALLY DISTRIBUTED
SENSITIVITY

3.1 Summary of Previous Work and Outline of Present Approach.

In Chapter 2 we have found the methods of eliminating the spurious signals associated with magnetic probe measurements. This chapter is devoted to the reduction of the most serious magnetic probe error, the boundary error, which is caused by the exclusion of the plasma electrical current from the space occupied by the probe. We attempt to solve this problem by designing the probe which itself accounts for the boundary error. Previously Malmberg [4] and Ecker, Kröll and Zoller [3] have calculated the effect of the boundary error on the accuracy of measured field profiles. Using these calculations Daughney [5] and Tam [6] attempted to develop correction procedures which would enable true field distributions to be calculated from measured fields. By splitting the plasma current into a number of current sheets and assuming that the magnetic field of a current sheet is distorted according to Malmberg's model
Daughney developed an integral equation for the correction of measured magnetic field, $B_p$:

$$B_o(r) = B_p(r) + \frac{1}{2} \int_0^R \frac{K(r-r')}{r'} \frac{d}{dr'} (r'B_o(r')) \, dr'$$

Since $B_p$ is only given as a set of measured values at different values of $r$ a numerical solution is necessary. Also the above integral equation has a singular kernel at $r=r'$; $K(r-r')$ resembles the curve shown in Fig. 3-8. Therefore the solutions are unstable to perturbations in $B_p$ which have a high spatial frequency. Tam studied in detail the effect of experimental errors on the corrected values of the current density. The results of his investigation show that the probe measurements have to be extremely accurate (within 1/4%) in order to obtain the fine structure of the true current since any small measuring errors will be magnified by a factor of 20 in the correction procedure. The perturbation on a thin current sheet is severe and the measured current density distribution is broadened and greatly reduced in amplitude. However, for smooth current density distribution, the measured current density is a better approximation to the true current density than the density corrected by Daughney's technique. As it has been pointed out an alternative solution to the problem of boundary
error is in the design of a probe which itself corrects for it. Our work is along this line.

For our calculations (section 3.2) we consider the effect of inserting a circular cylindrical probe guide along the symmetry axis (y) of a plane laminar current distribution (see Fig. 3-1). The probe moves inside the guide tube so that the disturbance of the current distribution does not depend on the position of the probe. We assume that the electrical conductivity of the plasma is a function of y alone, and that the undisturbed current, \( J_0 \), varies harmonically with time of an angular frequency, \( \omega \) \( \text{rad sec}^{-1} \). MKS units are used in the calculations.

Fig. 3-1 Geometry of plane current sheet.
The calculations show that the error in magnetic field can be reduced by using a multiple coil probe which is tested in the experiment described in section 3.3. of this chapter. Our planar model is valid for large radius current sheets, but as the results show it should give improvement even in a cylindrical system. As we are interested in the Z-pinch discharge the experiments are carried out on an analogue system with co-axial geometry. Section 3.4 is a discussion of the results and section 3.5, a summary of the main results of the investigations.

3.2 Calculation

3.2.1 Current Flow Around the Probe Guide

The undisturbed current flow \( J_0 \) satisfies the equation

\[
J_0 = J_0(y) e^{j\omega t} \hat{z}
\]  

(3.1)

where \( t \) is time and \( \hat{z} \) is a unit vector along the z-axis of a cartesian co-ordinate system. \( J_0(y) \) is determined by using Maxwell's equations, assuming that the displacement current is negligible and that the conductor is ohmic, i.e.

\[
J_0 = G(y) E(y) e^{j\omega t} \hat{z}
\]  

(3.2)
We have

\[ \text{Curl Curl } E = -\frac{\partial}{\partial t} \left( \text{Curl } B \right) \]

which can be rewritten as

\[ \text{Curl Curl} \left( \frac{J_0}{\sigma(y)} \right) = -\frac{\partial}{\partial t} \left( \mu_0 J_0 \right) \]

Since \( \text{Curl Curl} = \text{grad div} - \nabla^2 \)
we get in our case

\[ \text{Curl Curl} \left( \frac{J_0}{\sigma(y)} \right) = \frac{\partial^2 \left( \frac{J_0}{\sigma(y)} \right)}{\partial y^2} \]

Therefore

\[ \frac{\partial^2 \left( \frac{J_0(y)}{\sigma(y)} \right)}{\partial y^2} = j\mu_0 w J_0(y) \quad (3.3) \]

(\( \sigma(y) \) is the electrical conductivity, \( E(y) \) the electric field strength, \( j = (-1)^{1/2} \) and \( \mu_0 = 4\pi \times 10^{-7} \) Farads \( \text{m}^{-1} \)).

We next assume that the probe guide changes \( J_0 \) by an amount \( \Delta J \), where

\[ \Delta J = \Delta J_\rho + \Delta J_\theta \quad (3.4) \]

(The suffices \( \rho \) and \( \theta \) denote the radial and azimuthal components of \( \Delta J \), Fig. 3-1). Further, we assume the following: \( \Delta J \cdot y = 0 \), \( \Delta J \) varies harmonically with time at a
frequency \( w \), and that there is no accumulation of charges
in the current sheet, i.e. \( \text{div } \mathbf{J} = 0 \). By again applying
the Maxwell's equations in cylindrical polars we find
similarly that

\[
\left( \frac{\partial^2}{\partial \phi^2} + \frac{3}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial y^2} \right) \Delta \phi = j \mu_0 w \Delta \phi \quad (3.5a)
\]

\[
\frac{\partial}{\partial \phi} \left( \int \phi \Delta \phi \right) + \frac{\partial \Delta \phi}{\partial \theta} = 0 \quad (3.5b)
\]

The equations are solved subject to the boundary conditions

\[
\Delta \phi \to 0 \quad \text{as} \quad \phi \to \infty \quad (3.6a)
\]

\[
\Delta \phi_{\rho} = -\left( \phi_{\rho} \right)_{\rho} = -\phi_{\rho} \cos \theta \quad \text{at } \rho = a \quad (3.6b)
\]

(no current flow to the boundary of the hole made by the
probe guide; the radius of this hole is \( a \)). The solution
of equations (3.5) is
\[ \Delta J_\rho = -J_0(y) \frac{a^2}{\rho^2} \cos \theta \, e^{j\omega t} \quad (3.7a) \]
\[ \Delta J_\theta = -J_0(y) \frac{a^2}{\rho^2} \sin \theta \, e^{j\omega t} \quad (3.7b) \]

For \( \rho < a \), there is no current at all so that
\[ \Delta J = -J_0(y) \, e^{j\omega t} \frac{x}{\rho} \quad (3.8) \]

The most important feature of the above result is that the disturbance to \( J_0 \) at any plane \( y = \text{constant} \) can be obtained by multiplying \( J_0 \) by functions of \( \rho, \theta \) and \( a \) which are frequency independent.

### 3.2.2 \( \Delta B \), the Boundary Error

We now derive an expression for the change in magnetic field \( d(\Delta B) \) produced by the hole in a current sheet located between \( y \) and \( y + dy \) (\( \hat{x} \) is the direction of the probe coil axis).

From the Biot-Savart law it follows that
\[ d(\Delta B) = dy \int_0^{2\pi} \int_0^\infty \frac{\mu_0 \Delta J \times \hat{R} \, \rho \, d\theta \, d\rho}{4\pi |\hat{R}|^3} \quad (3.9) \]
where $R$ is the vector displacement of the point $Y$ where $d(\Delta B)$ is measured, from the point $(\phi, \Theta)$ in the current sheet at which $\Delta J$ is measured (Fig. 3-1); $dy$ is the thickness of the current shell. Since $d(\Delta B(Y))$ is measured on the axis of the probe,

$$R = \hat{y} (Y - y) - \sigma$$  \hspace{1cm} (3.10)

where $\hat{y}$ is a unit vector along the probe axis. Eliminating $\Delta J$ and $R$ from equation (3.9) by means of equations (3.7), (3.8) and (3.10) leads to the result

$$d(\Delta B(Y)) = \frac{\mu_0}{4\pi} \int_{0}^\infty dy \int_{0}^{2\pi} d\Theta \left[ \int_{0}^{a} \int_{0}^{\infty} I_1 dy + \int_{0}^{\infty} I_2 dy \right] \hspace{1cm} (3.11)$$

where

$$I_1 = \frac{Y - y}{\left[ (Y - y)^2 + \sigma^2 \right]^{3/2}} \hspace{1cm} (3.12)$$

$$I_2 = I_1 \left( \frac{a}{\sigma} \right)^2 \cos(2\Theta)$$

The integral $I_1$ is due to the fact that current is excluded from regions occupied by the probe guide, the second integral arises from the distorted current in the plasma itself. By
symmetry, the integrals containing $I_2$ vanish, so that distortions in the magnetic field arise solely because the probe guide excludes current from the region of the plasma which it occupies. Evaluating the integrals in equation (3.11) leads to the result

$$d(\Delta B(Y)) = \frac{\mu_0 J_0(y)dy}{2} \left[ \text{sgn} \zeta - \frac{\zeta}{(\zeta^2 + a^2)^{1/2}} \right]$$ \hspace{1cm} (3.13)

where,

$$\zeta = \gamma - y, \text{ and } \text{sgn} \zeta = 1 \text{ for } \zeta > 0$$

$$\text{sgn} \zeta = -1 \text{ for } \zeta < 0$$ \hspace{1cm} (3.14)

For a nest of current sheets, we integrate equation (3.13) over all values of $y$ to obtain

$$\Delta B(Y) = \frac{\mu_0}{2} \int_{-\infty}^{+\infty} J_0(y)dy \left[ \text{sgn} \zeta - \frac{\zeta}{(\zeta^2 + a^2)^{1/2}} \right]$$

$$= \frac{\mu_0}{2} \int_{-\infty}^{+\infty} J_0(y)dy K(\zeta)$$ \hspace{1cm} (3.15)

$K(\zeta)$ resembles the curve shown in Fig. (3-8) and it is apparent that the error, $\Delta B$, is primarily determined by the form of the current density near the measuring point $Y$. 
3.2.3 Measured Field \( B_p(Y) \) and Current \( J_p(Y) \)

The measured magnetic field \( B_p(Y) \) produced by a set of current sheets is obtained by adding \( B_o(Y) \), the undisturbed magnetic field, to equation (3.15).

\[
B_p(Y) = B_o(Y) + \Delta B(Y) \tag{3.16}
\]

where

\[
B_o(Y) = -\frac{\mu_0}{2} \int_{-\infty}^{+\infty} J_o(y) \text{sgn} \frac{\xi}{\sqrt{\xi^2 + \alpha^2}} \ dy \tag{3.17}
\]

By manipulating the above equations the following expression for \( B_p \) is obtained.

\[
B_p(Y) = -\frac{\mu_0}{2} \int_{-\infty}^{+\infty} \frac{\xi}{\sqrt{\xi^2 + \alpha^2}} \ J_o(y) \ dy \tag{3.18}
\]

Substituting \( B_p(Y) \) into Maxwell's equation, \( \text{curl} \ B_p = \mu_0 J_p \) yields an equation for the apparent current density \( J_p(Y) \), namely,

\[
J_p(Y) = -\frac{1}{2} \int_{-\infty}^{+\infty} J_o(y) \ dy \left[ \frac{\partial}{\partial Y} \left( \frac{\xi}{\sqrt{\xi^2 + \alpha^2}} \right) \right] \tag{3.19}
\]
3.2.4 Reduction of Boundary Error

Equation (3.18) is derived assuming that $B_p$ is sampled by a single coil probe located at $Y$. Suppose however that the probe is spread out so that its sensitivity at a point $Y'$ near $Y$ is $T(Y-Y')dY'$. (We are interested only in measuring the azimuthal magnetic field $B_\theta$. Therefore we deal only with the probe sensitivity to $B_\theta$; here $T \equiv C_2$ of Chapter 2.) For such a probe, the value of the perturbed field, $B_p'$ is

$$B_p'(Y) = -\frac{\mu_0}{2} \int_{-\infty}^{+\infty} J_0(y) dy \int_{-\infty}^{+\infty} T(Y-Y') \frac{Y'-y}{[(Y'-y)^2 + a^2]^{1/2}} dY' \tag{3.20}$$

Hence if,

$$\int_{-\infty}^{+\infty} T(Y-Y') \frac{Y'-y}{[(Y'-y)^2 + a^2]^{1/2}} dY' = \text{sgn} (Y-y) \tag{3.21}$$

$B_p'$ becomes identical with $B_0$, the unperturbed magnetic field (see equation (3.17)).

By changing the signs of $Y$, $Y'$ and $y$ in the above equation it is easily verified that

$$T(Y-Y') = T(Y'-Y) \tag{3.22}$$
i.e. that the probe is symmetric about the measuring point, Y.

It is not feasible to make a probe with complicated spatial sensitivity. Nevertheless, the spatial resolution can be improved with a probe consisting of several coils, distributed along the y-axis. Here again a large number of coils is not practical because it is difficult to adjust their sensitivities and spacing sufficiently accurately to gain any real advantages over a single coil probe. We have therefore considered the problem of optimizing the performance of a three coil probe.

3.3 The Three Coil Probe

3.3.1 The Basic Idea

The basic idea of the three coil probe can be best explained by considering the magnetic field near a hole of a radius a in a single plane current sheet (Fig. 3-1). Fig. 3-2(a) shows the unperturbed field $B_0$ as well as the perturbed field $B_p$; Fig. 3.2(b) depicts $\partial^2 B_p(y) / \partial y^2$. It is clear that a linear superposition of $B_p$ and $\partial^2 B_p / \partial y^2$ closely resembles $B_0$. The three coil probe we have devised provides such a superposition. The centre coil (sensitivity $T_2$) is situated halfway between the two outer coils (each of sensitivity $T_1$).
The axes of the three coils are along the x-direction, and are spaced by a distance d.

The output of the probe, $B_3$, is

$$B_3(Y) = T_2 B_p(Y) + T_1 [B_p(Y+d) + B_p(Y-d)]$$

which on expanding in a Taylor-series reduces to

$$B_3(Y) = (T_2 + 2T_1) B_p(Y) + T_1 \frac{\partial^2 B_p}{\partial Y^2}$$

i.e. linear superposition of $B_p$ and $\frac{\partial^2 B_p}{\partial Y^2}$.

Fig. 3-2(c) shows the improvement in $B_p$ which results from the 3-coil probe.
3.3.2 Optimization of the Probe Design

For the above described probe we can write its sensitivity as

\[ T(Y-Y') = T_1 \left[ \delta(Y-Y' + d) + \delta(Y-Y' - d) \right] + T_2 \delta(Y-Y') \]  \hspace{1cm} (3.23)

and the integral in equation (3.21) is some function \( F(Y-y) \).

The requirement that \( F(Y-y) = \text{sgn}(Y-y) \) as \( Y-y \to \infty \) imposes the following condition on \( T_1 \) and \( T_2 \),

\[ 2T_1 + T_2 = 1 \]  \hspace{1cm} (3.24)

By using the equation (3.23) and assuming \( y=0 \) (current sheet at the origin) we can write \( F(\propto 3\text{-coil probe signal}) \) as

\[ F(Y) = T_1 \left( \frac{Y+d}{[(Y+d)^2 + \alpha^2]^{1/2}} + \frac{Y-d}{[(Y-d)^2 + \alpha^2]^{1/2}} \right) \]  \hspace{1cm} (3.25)

\[ + T_2 \frac{Y}{(Y^2 + \alpha^2)^{1/2}} \]

To optimize the probe design we examine the behaviour of \( F(Y) \) as the parameters \( T_2/T_1 \) and \( d/a \) are varied. For given \( T_2/T_1 \) we find \( T_1 \) and \( T_2 \) by using the equation (3.24). The typical \( F(Y) \) curve is shown in the insert of Fig. 3-3. As the value of \( d/a \) is increased for a given \( T_2/T_1 \), the distance \( Q \) at which \( F(Q) = 1 \) is reduced. However, at the same time the peak value of \( F(Y) \), denoted \( P \),
is increasing. We therefore place an upper limit for the peak value of \( F(Y) \), \( P \leq 0.05 \) (i.e. 5% error limit in \( B_0 \) measurement for \( Y > Q \)), and try to minimize \( Q \) by varying \( T_2/T_1 \) and \( d/a \).

It is found that if \( d/a \) is reduced then the corresponding best value of \( T_2/T_1 \) also is reduced. Furthermore, the fit of \( F \) to \( \text{sgn} \) improves. However, the lowest value of \( d/a \) is limited by the difficulty of making probes with \( d < 2 \) mm. In order to minimize the perturbation of plasma by the probe, we would like to keep \( a \) as small as possible. Therefore for a given probe radius \( a \), the probe will have a better performance with the smallest possible coil spacing \( d \).

Table 3-1 and Fig. 3-3 show that the change in optimum \( F(Y) \) for \( 0.5 \leq d/a \leq 0.8 \) is not important (i.e. a probe with \( d/a = 0.5 \) (smallest coil spacing) is only marginally better than a probe with \( d/a = 0.8 \) (largest coil spacing)).
<table>
<thead>
<tr>
<th>$T_2/T_1$</th>
<th>-2.4</th>
<th>-2.52</th>
<th>-2.7</th>
<th>-3.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>d/a</td>
<td>.50</td>
<td>.57</td>
<td>.65</td>
<td>.80</td>
</tr>
<tr>
<td>example: d(mm) corresponding to $a = 3$ mm</td>
<td>1.5</td>
<td>1.7</td>
<td>2</td>
<td>2.4</td>
</tr>
</tbody>
</table>

**Table 3-1.** The optimum values of $T_2/T_1$ and d/a.

The effect on the probe performance of changing $T_2/T_1$ and d/a from their optimum values is shown in Fig. 3-4. Fig. 3-4(a) and (b) show the effect of mistuning $T_2/T_1$ by ±5%. It is apparent that there is less deterioration in the fit of $F(Y)$ to $\text{sgn}(Y)$ if $T_2/T_1$ exceeds the optimum value, than if $T_2/T_1$ is less than its optimum value. Therefore the coil sensitivities should, if anything, be adjusted so that the value of $T_2/T_1$ is larger than the optimum. Fig. 3-4(c) and (d) show the effect of incorrect d/a on the probe performance. The curve 0 is the $F(Y)$ for optimum values of $T_2/T_1$ and d/a. Curves 1, 2 and 3 correspond to an increase of 10, 20 and 30% in the effective probe radius which would for example be produced if the probe strongly cooled the plasma. Curve 4 depicts $F(Y)$ for d/a increased by 10%. (Such a change could occur if the probe was incorrectly constructed.) It is clear that in the event of misconstruction it is best to have d/a less than the optimum value.
Fig. 3-4  Computed probe performance with parameters $T_2/T_1$ and $d/a$ slightly changed from their optimum values.

(a) & (b): $T_2/T_1$ mistuned by $+5\%$ (curve $\oplus$), or by $-5\%$ (curve $\ominus$).

(c) & (d): $d/a$ incorrect; curves 1, 2, 3 correspond to 10, 20, 30\% increase of $a$ (plasma cooling), curve 4 to 10\% increase of $d/a$.

Curves 0 correspond to the optimum combinations of $(T_2/T_1, d/a) = (-2.4, .5)$ for (a) and (c), $(-2.7, .65)$ for (b) and (d).
Using the results obtained in this paragraph (planar geometry) as a guide we want to minimise the boundary error in a Z-pinch discharge (coaxial cylindrical geometry).

3.3.3 Probe Construction

The coils are single layer solenoids consisting of twenty turns of 37 A.S.W.G. copper wire. They were wound on a 0.122 cm diameter wire, which was removed after setting the coils in nail polish. To eliminate emf.s produced by spurious magnetic flux, the leads to the coils were tightly twisted together.

The coils are held in a lucite holder (Fig. 3-5a) with a spacing of $d = 2.1$ mm, which is equal to $0.57a$ (see Table 3-1 above) where $a = 3.7$ mm is the radius of the probe guide in the Z-pinch.

The probe electrical circuit is shown in Figure 3.5b. $L_1$ and $L_2$ denote the outside coils, $L_3$ the centre one. $P_1$, $P_2$ and $P_3$ are potentiometers (0-100 $\Omega$ Bourns E-Z Trim), which enable us to adjust the sensitivities of the coils. The potentiometers are enclosed in a metal casing.

3.3.4 Adjustment of Coil Sensitivities

The adjustment of the coil sensitivities is made with a circuit consisting of three coil probe, twin-feeder
cable and "dual integrator" (Fig. 3-7b) since this is the system to be used in the actual Z-pinch system. Its output is fed into Tektronix 1A1 plug-in in a type 551 oscilloscope.

By feeding a common signal (Tektronix 190B sine wave generator) into the "dual integrator" (Fig. 3-7b) and connecting the output to a differential amplifier (Tektronix 1A1 plug-in in a type 551 oscilloscope) the potentiometers can be adjusted until the sensitivities of the two branches are identical. For identical signals at CD (Fig. 3-7b) zero output is observed in the oscilloscope.

The above theory requires that the sensitivities of L1 and L2 (Fig. 3-5b) be equal and that $T_2/T_1 = -2.52$. 

**Fig. 3-5** 3-coil magnetic probe:

a) holder of coils with channels for coil leads, distances in mm,

b) electrical circuit.
To equalize the sensitivities of L1 and L2 (Fig. 3-5b) we put the probe in a spatially uniform magnetic field generated by the simple equipment shown in Fig. 3-6. By short circuiting L3 and adjusting P1 and P2, the outputs from L1 and L2 can be equalized (indicated by zero signal on the oscilloscope with the inputs to both channels of the plug-in added together). Using the differential input the sum of the outputs from the two coils (V1 + V2) can be measured. The short circuit is then removed from L3 and P3 adjusted until the complete probe output is 0.26 times the signal V1 + V2. In all cases the amplitude of the probe signal is divided by the amplitude of the Rogowski coil signal so as to eliminate small changes in magnetic field caused by variations in the charging potential of the capacitor bank and the heating of the damping resistor (Fig. 3-6).

3.3.5 Magnetic Field in Co-axial System

Having adjusted the probe for optimum performance in a planar system, we tested and optimized its performance in a co-axial system analogue to a Z-pinch discharge.

The cylindrical conductors consist of aluminum foil on cardboard tubes, and the end plates are made of copper sheet (Fig. 3-7a). The concentric conductors form
Fig. 3-6 System for adjustment of coil sensitivities, previously used by Tam [6]. (Current leads width \( b = 20 \text{ cm} \), length \( = 20 \text{ cm} \). separation \( h = 1 \text{ cm} \).)

Fig. 3-7 a) Co-axial system for testing probe performance, previously used by Daughney [5].

b) Measuring circuit of the probe.
part of the inductance of parallel resonant circuit which is fed by a sinusoidal generator of variable frequency (anode supply 2 kV, maximum power output 100 W). The current is monitored by a Rogowski coil.

The magnetic probe moves in a guide tube 7.8 mm in diameter which is placed diametrically across the concentric cylinders. The position of the probe is monitored by a travelling microscope. For these measurements, the dual integrators were replaced by dual resonators (Fig. 3-7b) in order to improve the voltage gain of the probe. The resonators are tuned approximately to the resonant frequency of the analogue system. By slightly mistuning, it is possible to adjust the trimmer capacitors so that the two branches have the same gain. The gains are balanced by the same technique used for the dual integrator (see paragraph 3.3.4). The gains are equal to an accuracy of about 1%. The output of the resonators is fed into a differential amplifier (Tektronix W plug-in) which greatly reduces common mode signals produced by stray capacitance between the probe and the concentric conductors. (The elimination of these signals requires that the two branches of the probe circuit be accurately balanced, since capacitatively coupled signals are largest in the very region where the magnetic field strength is lowest). As a precaution, the magnetic field at a given
radial position \( r \) was measured as a function of \( \alpha \) where \( \alpha \) is the angle between the coil axes and the z axis of the coaxial system. The stray capacity between the probe and the concentric conductors should not depend on \( \alpha \), hence the probe signal \( U(r, \alpha) \) is given by the expression

\[
U(r, \alpha) = V_c + V_B(r) \sin \alpha
\]

where \( V_B \) is the signal produced by the magnetic field, and \( V_c \) is the capacitative signal. \( V_B(r) \) can be computed from values of \( U(r, \alpha) \) for \( \alpha = \pi/2 \) and \( \alpha = 3\pi/2 \) radians, using the following expression

\[
V_B(r) = \frac{|U(r_1 \pi/2)| - |U(r_1 3\pi/2)|}{2} \quad (V_B < V_C)
\]

\[
V_B(r) = \frac{|U(r_1 \pi/2)| + |U(r_1 3\pi/2)|}{2} \quad (V_B > V_C)
\]

(If \( V_B > V_C \), the probe signal will go through zero as the probe is rotated about its stem; in the converse situation, no zero in the probe signal occurs.)

Using this method of measuring \( V_B \), we determined the magnetic field as a function of the distance from the axis of the concentric conductors. The measurements were carried out for the following conditions: radius of outer conductor \( r_o = 10.16 \) cm; radius of inner conductor \( r_i = 5.71, 2.54 \) or \( 0.635 \) cm.
3.3.6 Spatial Resolution

The unperturbed magnetic field \((B_0)\) varies as \(1/r\) between the concentric conductors; elsewhere it is zero. The perturbed magnetic field \((B_p)\) was measured with a single coil probe and plots of \(\Delta B\) versus \(u\) were constructed where

\[
\Delta B(u) = \frac{B_p(u) - B_o(u)}{B_o(0^+)}
\]

and \(u = r_i - r\) or \(u = r - r_o\) is the distance from the nearest concentric conductor (of the radius \(r_i\) or \(r_o\)) to the measuring point \(r\). These curves are plotted with crosses in Fig. 3-8.

Similar measurements were made with the three coil probe. However the sensitivity of L3 had to be adjusted slightly to account for the cylindrical geometry of the current sheet. This adjustment was made at \(r_i = 5.71\) cm; successive plots of \(\Delta B\) versus \(u\) were made, altering \(P_3\) for each plot until the best one was obtained. \(T_2/T_1\) was measured by the technique outlined in the paragraph 3.3.4 and was found to have the value -2.8 (planar theory requires \(T_2/T_1 = -2.52\)). Plots of \(\Delta B\) for all the conductors were then obtained with the modified probe. These are marked with circles in Fig. 3-8. As a final check the magnetic field produced by a solid rod of radius 0.635 cm was measured (Fig. 3-9). The solid circles are the calculated values,
Fig. 3-8 Improved spatial resolution of the 3-coil probe. Plots of $\Delta B$ vs. $u$; $\Delta B$ = normalised error of the measured magnetic field, $u$ = distance from the current sheet, x-single coil probe, o-three coil probe.

Fig. 3-9 Unperturbed magnetic field of high curvature measured with 3-coil probe - o, calculated field values - .
and the open circles the measured field. Even for distances as close to the axis as 1 cm, there is no discernible difference between the two sets of results. Hence the three coil probe is no worse than a single coil probe for unperturbed fields in an axi-symmetric system. Furthermore as Fig. 3-8 shows the three coil probe markedly improves the spatial resolution of magnetic field measurements.

In dynamic problems, obtaining accurate values for the large fields is important. Hence we define the spatial resolution for \( u < 0 \). The resolution distance \( \ell \) is taken to be the distance for which \( \Delta B(u) \geq 5\% \). With this definition for the single coil probe \( \ell \approx 5 \text{ mm} \) (i.e. \( 1.5a \)), whereas for the three coil probe \( \ell \approx 2 \text{ mm} \) (i.e. \( a/2 \)).

3.3.7 Boundary Error for Smooth Current Distributions

The plots in Fig. 3-8 show that if the current density changes rapidly over the distance \( \ell \), the errors in \( B \) can be very large. However in most practical systems, the scale length over which \( J \) changes will exceed \( \ell \). It is therefore instructive to consider the error in \( B \) for "smooth" current distributions. By analogy with equation (3.15), in an axi-symmetric system the error in the magnetic field (\( \Delta B(r) \)), is

\[
\Delta B(r) = \int_0^r \mu_0 J_0(s) \Delta B(r,s) ds
\]
where $\Delta B(r,s)$ is the error at $r$ produced by a current shell of unit strength and radius $s$. Expanding $J_0$ in a Taylor series about $r$, and noting that $\Delta B$ vanishes outside the range $\pm l$, leads to the result,

$$\Delta B(r) = \mu_0 \int_{r-l}^{r+l} \left( J_0(r) - (r-s)\left(\frac{\partial J_0}{\partial s}\right)_r \right) \Delta B(r,s) ds$$

Fig. 3-8 shows that $\Delta B(r,s)$ is a function of $u = r-s$ only (i.e. the shape of the curves does not vary very much with $r_1$), and that $\Delta B(r-s) \approx -\Delta B(s-r)$. Using these assumptions enables $\Delta B(r)$ to be written in the form

$$\Delta B(r) = -2\mu_0 l^2 \left(\frac{\partial J_0}{\partial r}\right) \int_0^1 g \Delta B(g) dg \quad (3.27)$$

where $g = (r-s)/l$. Since $\Delta B(g) \leq 0.5 (1-g+0.1)$ (see Fig. 3-8) an upper estimate of $\Delta B(r)$ for a smooth current distribution is

$$\Delta B(r) < -0.217 \mu_0 l^2 \left(\frac{\partial J_0}{\partial r}\right) \quad (3.28)$$

If the characteristic scaling length of $J_0$ is $\lambda$ it follows from Maxwell's equation $\text{Curl } B = \mu_0 J_0$, that

$$\mu_0 \left| \frac{\partial J_0}{\partial r} \right| \approx \left| \frac{1}{\lambda^2} B_0(r) \right|$$
hence

\[ \left| \frac{\Delta B(r)}{B_0(r)} \right| < 0.217 \left( \frac{\ell}{\lambda} \right)^2 \]  \hspace{1cm} (3.29)

For a single coil probe \( \ell \) is more than a factor two larger than it is for a three coil probe, so that the three coil probe improves the measuring accuracy of \( B_0(r) \) by at least a factor of four.

3.4 Discussion

The above model for "boundary" errors, although less specialized than earlier ones (Ecker, Kröll and Zöller [3] and Malmberg [4]) is still restrictive because no account is taken of time dependent conductivities, nor of the dependence of the conductivity on the electric field strength. Nevertheless as the simple planar model shows, the main error in \( B \) arises because the electric current is eliminated from the region occupied by the probe guide. In the current shell itself, the current density is reduced above and below the probe guide (Fig. 3-1), and increased on both sides of it. The perturbations in magnetic field produced by these distortions tend to cancel each other. Even for more sophisticated models a partial cancellation should still occur so that distortions of the current flow in the
plasma would only make minor corrections to $B_p$ calculated on the basis on the planar model. It therefore appears that a three coil probe will produce significant improvements in the accuracy of magnetic field measurements, even in systems which are more complicated than the simple models considered above. For this expectation to occur the system should not depart greatly from the planar model. In the Z-pinch discharge for example "a" should be small compared to the radius at which $B$ is measured, as well as to the thickness of the current shell ($\lambda$).

3.5 Advantages of the Three Coil Probe

The calculations and measurements show that a three coil probe can be constructed which partially corrects for the changes in magnetic field caused by the presence of the probe. For optimum performance, the coils are spaced by $0.57a$, where $a$ is the radius of the probe guide, and the central coil has a gain - 2.8 greater than the two outer coils. The spatial resolution $\ell$ of the probe is $\sim a/2$ and the measuring error ($\Delta B/B_0$) for magnetic fields which have a slow spatial variations is $\sim 0.2(\ell/\lambda)^2$, where $\lambda$ is the scale length for magnetic field variations. If the effective probe radius is larger (large cooling of plasma) than the radius $a$, according to which the coil
spacing was calculated, then $\frac{\partial^2 B_p}{\partial r^2}$ is reduced. The probe correction of $B_p$ is therefore smaller than it should be. However, even in this case $B^3_p$ is still a better approximation of $B_0$ than $B_p$ (see Fig. 3-4 (c) and (d)).
APPLICATION OF 3-COIL PROBE TO A Z-PINCH DISCHARGE

Chapter 4

THE EXPERIMENTAL APPARATUS

The superior ability of the 3-coil probe, described in Chapter 3, is demonstrated by its application to a Z-pinch discharge (in He and Ar) previously used by Daughney [5] and Tam [6]. The objective was to test the 3-coil probe in a real plasma and compare our results with the results of previous magnetic probe measurements of Daughney and Tam. Furthermore, we provide a catalogue of spatial distributions of magnetic field and current density and use these data to elucidate the acceleration mechanism of the current sheet at various filling pressures.

This chapter is divided into three sections. Our experiments were conducted on the Z-pinch device which is described in section 4.1. The discharge current was measured by the standard techniques which are described in Appendix I. The last two sections deal with magnetic probe measurement (section 4.2) and calibration (section 4.3).
4.1 The Z-pinch

The main components of the Z-pinch discharge system are the discharge vessel, the capacitor bank, the vacuum system and the triggering system. The basic specifications of the discharge apparatus are listed in Table 4-1.

The discharge vessel is a Pyrex tube of 17 cm O.D., 15 cm I.D. and 75 cm length with plane brass perforated electrodes. The detailed design of the discharge tube is shown in Fig. 4-1. The main difference from the usual design is the presence of the guide tube (for magnetic probe) mounted diametrically across the discharge tube. The usual practice is to introduce the probe into the discharge vessel through an o-ring seal in the wall. The guide tube is a quartz tube of .74 cm O.D. sealed at the discharge vessel with Araldite epoxy resin. The vessel can be evacuated and then filled to the required pressure with the working gas (Ar or He in our experiments).

The circuit diagram of the discharge apparatus is shown in Fig. 4-2. The capacitor bank (53 μF total capacity) is formed by five N.R.G., low inductance, storage capacitors connected in parallel. These capacitors are charged to 12 kV for all experiments. We have therefore 3.8 kJ of stored energy. The triggering circuit, conventional in design, is described in detail by Medley [14].
TABLE 4-1

DISCHARGE APPARATUS SPECIFICATIONS

CAPACITOR BANK AND LEADS

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total capacity (N.R.G. Type 203 low inductance)</td>
<td>53 µF</td>
</tr>
<tr>
<td>Inductance of capacitor bank and current leads</td>
<td>12 ± 0.1 µH</td>
</tr>
<tr>
<td>Width of leads</td>
<td>15 cm</td>
</tr>
<tr>
<td>Length of leads</td>
<td>1 m</td>
</tr>
<tr>
<td>Separation of leads (polyethylene)</td>
<td>2 mm</td>
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</table>

DISCHARGE VESSEL

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge vessel</td>
<td>Pyrex</td>
</tr>
<tr>
<td>Electrodes</td>
<td>Brass</td>
</tr>
<tr>
<td>Electrode separation</td>
<td>59 cm</td>
</tr>
<tr>
<td>Inside tube diameter</td>
<td>15 cm</td>
</tr>
<tr>
<td>Outside tube diameter</td>
<td>17 cm</td>
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</table>

VACUUM SYSTEM

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 170 Balzers oil diffusion pump</td>
<td></td>
</tr>
<tr>
<td>Hyvac 14 Cenco backing pump</td>
<td></td>
</tr>
<tr>
<td>Base Pressure</td>
<td>1 mTorr</td>
</tr>
<tr>
<td>Leak rate</td>
<td>1 mTorr/hr</td>
</tr>
</tbody>
</table>

CURRENT MEASUREMENT

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>'Rogowski' coil</td>
<td></td>
</tr>
<tr>
<td>Integration time constant</td>
<td>340 µsec.</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>450 kA/V</td>
</tr>
<tr>
<td>Maximum discharge current</td>
<td>200 kA</td>
</tr>
<tr>
<td>Discharge current frequency</td>
<td>50 kHz</td>
</tr>
</tbody>
</table>

VOLTAGE MEASUREMENT

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conway microammeter in series with</td>
<td></td>
</tr>
<tr>
<td>A.V.O. multiplier</td>
<td>25 kV D.C., 500 M Ω</td>
</tr>
<tr>
<td>Charging voltage</td>
<td>12.0 ± .1 kV</td>
</tr>
</tbody>
</table>
TABLE 4-1 (Cont'd)

PRESSURE MEASUREMENT

<table>
<thead>
<tr>
<th>Type</th>
<th>Measurement Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>GP-110 Pirani vacuum</td>
<td>1 to 2000 mTorr</td>
</tr>
<tr>
<td>gauge</td>
<td></td>
</tr>
<tr>
<td>Vacustat absolute</td>
<td>1 to 1000 mTorr</td>
</tr>
<tr>
<td>gauges</td>
<td>.01 to 10 Torr</td>
</tr>
</tbody>
</table>
Fig. 4-1 Cross-section of discharge tube.
Fig. 4-2  Z-pincho discharge circuit.

A. Trigger spark gap switch with isolated ultra-violet trigger pulse Q
B. Main spark gap trigger generator
C. Trigger generator charging pick-off
D. Main spark gap switch
E. Discharge vessel
F. Z-pincho capacitor bank
G. High voltage supply
This circuit has two important features. First, the switch A triggered by a U.V. light source (Fig. 4-2) isolates the high voltage discharge circuit from the initial trigger generator. Secondly, this circuit is constructed coaxially in order to minimize radiation of the high frequency noise.

4.2 Magnetic Probe Measurement

For our investigation we use the three coil probe described in section 3.3. The probe parameters are:

\[ \frac{d}{a} = 0.57 \] and \[ \frac{T_2}{T_1} = -2.8. \] This probe, in comparison with a single coil probe, improves the spatial resolution of magnetic field measurements by at least a factor of two.

The magnetic probe measuring circuit is a balanced differential circuit identical to the one shown in Fig. 2-4 and discussed in paragraph 2.3.2. It is connected to a W plug-in of a Tektronix 551 dual beam oscilloscope. The oscilloscope is placed in the Faraday cage. Throughout the experimental measurements, we have triggered the time base of the oscilloscope by using the internal triggering mode with a signal supplied from the Rogowski coil, connected to the lower beam (type G plug-in). The recorded current waveform serves also as a time reference for the correlation of magnetic field observed at different radial positions.

To eliminate spurious pickup signals we took the precautions described in section 2.2. Then we tested the
probe performance by the procedure described in section 2.1. The signal of the magnetic probe used during our experiments was found free of all spurious pickup signals except for some high frequency pickup after the initiation of the discharge (we did not use any delay). The typical oscillogram is shown in Fig. 4-3.

The probe moves inside the quartz guide tube which runs diametrically across the discharge vessel midway between the electrodes. The radial position of the probe is measured with respect to the discharge tube axis, which is determined first by centering the center probe coil on the axis. It is found that the axis of the discharge vessel coincides with the Z-pinch discharge axis. \(B_\theta(0) = 0\) up to the pinch time for all discharges. \(B_\theta(0)\) also remains zero at post-pinch times for the discharges with higher filling pressures, \(>1\) Torr of He. Such plasmas are slower as well as more stable.

The experimental procedure is to record the magnetic field \(B_\theta\) as a function of time \(t\) at 5 mm intervals for \(80\) mm \(r\) \(\geq 0\). During one traverse we take alternate points and pick up the odd points on a return traverse. This is a simple and sensitive test for the possible drift of discharge characteristics during the run (R. H. Lovberg [2]). Any significant drift then would show up as a tendency of the points to zig-zag.
Fig. 4-3 A typical oscillogram. $B_0$ - upper trace, $I$ - lower trace (Ar, .5 Torr, $r = 3.5$ cm, $2 \mu\text{sec/cm}$)

From this set of results we construct the radial profiles of the magnetic field for the selected times. Using the Maxwell equation $\text{Curl } B = \mu_0 J$ we then obtain the axial current density radial profiles. The shot-to-shot reproducibility demanded by this procedure was satisfactorily obtained for all filling pressures with the exception of 0.5 Torr of He which is irreproducible after the pinch time $T \approx 4.5 \mu\text{sec}$; see Fig. 4-4. The calibration of the magnetic probe is given in the following section.

The method of the data reduction is described in Appendix II and the experimental results are presented in the next chapter.
Fig. 4-4 Demonstration of the discharge reproducibility. An overlay of two traces of $B_\theta$, (a) and (b), of three traces (c).
4.3 Magnetic Probe Calibration

Our calibration procedure which makes use of the experimental apparatus has the advantage that it ensures the calibration of the complete magnetic probe measuring circuit including the oscilloscope. It is based on the fact that the total discharge current $I$ (measured by the calibrated Rogowski coil) must at all times be equal to the current carried by the plasma.

$$I = 2\pi \int_0^R J_z r \, dr$$  \hspace{1cm} (4.1)

where $J_z$ is the axial current density and $R$ is the inner radius of the discharge tube. Using the procedure outlined in previous section we first obtain from the measurements with the uncalibrated probe the radial profiles of $B_\theta$ in volts and $J_z$ in arbitrary units.

$$B_\theta [\text{Wb m}^{-2}] = B_\theta [\text{Volt}] \cdot k [\text{Wb m}^{-2}/\text{Volt}]$$  \hspace{1cm} (4.2)

$$J_z [\text{Amp m}^{-2}] = J_z [\text{a.u.}] \cdot C [\text{Amp m}^{-2}/\text{a.u.}]$$  \hspace{1cm} (4.3)

where $k$ is the magnetic probe sensitivity, $C$ is the current density calibration constant, and $[\text{a. u.}] = \text{arbitrary units}$. Using then the equations (4.1) and (4.3) we determine the current density calibration constant $C$. Since the axial
current density $J_z$ is now known absolutely, we can finally determine the magnetic probe sensitivity $k$. The $z$-component of the Maxwell equation $\mu_0 \nabla \times B = \text{Curl } B$ (in cylindrical coordinates) is

$$\mu_0 J_z = \frac{1}{r} \frac{\partial}{\partial r} (r B_\theta) \quad (4.4)$$

Using the equations (4.2) - (4.4) we have for the magnetic probe sensitivity

$$k \left[ \text{Wb m}^{-2}/\text{V} \right] = \mu_0 \frac{C}{r} \frac{J_z}{\left( \frac{B_\theta}{r} + \frac{\partial B_\theta}{\partial r} \right)} \quad (4.5)$$

For a selected time and radial position we can find the values of $J_z$ [a.u.], $B_\theta$ [Volt] and $\partial B_\theta / \partial r$ [Volt/m] (from the radial profiles of $J_z$ and $B_\theta$); the current density calibration constant $C$ [Amp m$^{-2}$/a.u.] is already known. Substituting these parameters in equation (4.5) gives $k$.

For the actual calibration we have constructed twelve $B_\theta$ radial profiles (for different selected times and filling pressures). With the help of a digitizer and a computer we have then generated the corresponding $J_z$ radial profiles and computed the integral which appears on the r.h.s. of the equation (4.1). (The detailed description of the data reduction is given in Appendix II.) In this way we have determined the current density calibration constant $C$ and
therefore the axial current density $J_z$ with the accuracy of 4%.

The consequent calculation of the magnetic probe sensitivity was done for ten selected combinations of time and radial position. In this way it was established that a 1 Volt deflection on the oscilloscope is equivalent to a magnetic field of 16.7 Weber/meter$^2$,

$$ k = 16.7 \text{ Wb m}^{-2}/\text{V} $$

The error associated with the calibration procedure is about 4%. The largest source of the error is in the computing of the integral of the equation (4.1).
Chapter 5

THE CATALOGUE OF THE MAGNETIC FIELD AND CURRENT DENSITY DISTRIBUTIONS

This section is a summary of our experimental results. We will first present our results in form of graphs. This will be followed by a brief discussion, where we will point out the significant features of the results and choose the problems which will be treated in greater detail in the next chapter.

On the following pages we present graphs of the azimuthal magnetic field and the axial current density radial profiles for the discharges in argon at 0.1, 0.25, 0.5, 1.0 and 2.0 Torr initial pressures and in helium at 0.5, 1.0, 2.0 and 4.0 Torr initial pressures respectively (Fig. 5-1). Each graph is identified with the gas chemical symbol and initial pressure. Individual curves are labelled with numbers which give the time in microseconds after the initiation of the discharge. To allow for the correlation of some other measurements on a Z-pinch of the same parameters, we also give graphs of the discharge current waveforms (Fig. 5-2).
Fig. 5-1  Radial profiles of azimuthal magnetic field and axial current density.

(a) He, .5 Torr,  (e) Ar, .1 Torr
(b) He, 1 Torr,   (f) Ar, .25 Torr
(c) He, 2 Torr,   (g) Ar, .5 Torr
(d) He, 4 Torr,   (h) Ar, 1 Torr
    (i) Ar, 2 Torr

Each graph is identified with the chemical symbol of the working gas and its initial pressure. Individual curves are labelled with numbers, which give the time in μsec after the initiation of the discharge.
(i)
Fig. 5-2  Total discharge current waveforms.

(a)  He, .5 Torr,   (e)  Ar, .1 Torr
(b)  He, 1 Torr,    (f)  Ar, .25 Torr
(c)  He, 2 Torr,    (g)  Ar, .5 Torr
(d)  He, 4 Torr,    (h)  Ar, 1 Torr
(i)  Ar, 2 Torr
The presented results can be divided into two groups according to the filling gas, argon or helium. The "argon results" do not reveal any fine structure of the current sheet. On the other hand the "helium results" look much more interesting.

In argon we have the same kind of profiles as the ones obtained by Daughney [5] with a single coil probe, only our accuracy is better. However, there is one interesting feature which should be noted. At the time of the total discharge current reversal (t ~ 10 sec), i.e. when $I = 0$, there is an appreciable current flowing in the plasma. The current at the discharge tube center maintains the direction of the first halfperiod and slowly decays while the new current sheet carrying the current in opposite direction is being formed at the tube wall. It can be seen best at 1 Torr of initial pressure. There must be, therefore, a closed current loop in the plasma and when $I = 0$ the plasma current at the center must be equal to the plasma current at the walls.

In helium we see the development of a current sheet inside the main (= original) current sheet in the whole region of the investigated initial pressures. Similar effects have been observed by Kogelshatz et. al. [15] and by Tam [6] for high initial pressures, where this effect is
most pronounced. For the low initial pressures (0.5 and 1.0 Torr) the collapse is fast and strong. Under these conditions the main current sheet collapses onto the axis, absorbing the preceding inner sheet and forming a single narrow channel of high current density. At the high initial pressure (4 Torr) the collapse is considerably slower. Here the main current sheet never reaches the axis. It is stopped at \( r \approx 2.5 \text{ cm} \) and eventually we have a current density distribution with a central peak and steady coaxial current shell. At 2 Torr of the filling pressure we have an intermediate case with a peculiar current structure.
Chapter 6

DISCUSSION OF RESULTS

This chapter is divided into two sections. In section 6.1 we compare our experimental results in helium with two types of the modified snowplow equation, the conventional piston-shock wave model and the York-Jahn model in which ions form a thin mass sheet outside the current sheet. In section 6.2 we present a qualitative model for the limiting high pressure case (4 Torr in He) which cannot be described by either of two models.

6.1 Comparison of Experimental Results with Theory

6.1.1 Outline of Snowplow Model

In this section we want to determine whether the moving current sheet of our Z-pinch in helium behaves as an approximation to a "snowplow."

The original and also simplest snowplow model [12] assumes that the Z-pinch discharge consists of a thin collapsing cylinder of infinite conductivity and infinitesimal thickness, and that the sheet sweeps all the particles on
its way. Under these assumptions the snowplow equation can be written as

$$\frac{d}{dt} \left[ \rho_0 \bar{r} \left( r_o^2 - r^2 \right) \frac{dr}{dt} \right] = -F(t)$$

where \( r(t) \) is the radius of the collapsing sheet, \( r_o \) its initial value, \( \rho_0 \) the initial gas density, and \(-F(t)\) the Lorentz force acting on a unit length of the cylinder.

The discrepancy of the original snowplow model usually arises from the fact that the real current sheet does not have an infinite conductivity and is by no means thin.

More detailed snowplow models recognise that the current sheet has a finite thickness with an interior structure. The conventional models assume that the current sheet is a piston driving a shock wave ahead of it. In the model designed recently by York and Jahn [11] the current sheet moves through cold gas and ionizes (or heats) it. The ions form a thin mass sheet outside the current sheet and are coupled to it by a radial electric field.

### 6.1.2 Common Features of Refined Snowplow Models

From our measurements we know that the current sheet has an initial thickness \( d; d = R - r_o \), where \( R \) is the inner discharge vessel radius and \( r_o \) is the initial radius
of the inner surface of the sheet. For both models we will assume that only a fraction \( \nu_0 \) of the mass, originally occupying the volume between \( R \) and \( r_0 \), will take part in the collapse. The mass of this gas \( (m_0) \) is given by

\[
m_0 = \nu_0 \rho_0 \pi \left( R^2 - r_0^2 \right)
\]

where \( \rho_0 \) is the initial gas density.

The dynamic equation is derived from the rate of change of momentum of the mass (associated with the moving current sheet) due to the forces acting on it. In the snowplow models this force is equal to the Lorentz force provided that the difference in kinetic pressure across the moving mass sheet is negligible. The Lorentz force density is given here by the product \( J_z \times B_\theta \). We can therefore write for the total Lorentz force between radii \( r_1 \) and \( r_2 \)

\[
F = 2\pi \int_{r_1}^{r_2} J_z B_\theta r \, dr
\]

By expressing \( J_z \) in terms of \( B_\theta \) (using again the Maxwell equation \( \mu_0 \mathbf{J} = \nabla \times \mathbf{B} \)) we get

\[
F = \frac{\pi}{\mu_0} \left( \int_{r_1}^{r_2} B_\theta^2 \, dr + r_2 B_\theta^2(r_2) - r_1 B_\theta^2(r_1) \right)
\]  

(6.1)
This expression is used later in both models for calculating the Lorentz force.

6.1.3 Piston-shock Wave Model

It is assumed that the collapsing current sheet acts as a piston. A shock front is thus produced which precedes it. In this case the gas inside the current sheet is undisturbed until it is encountered by the shock front and the particles which are then swept up are assumed to form a thin mass layer enclosed by the current sheet.

We assume that the piston radius \( r_p \) is equal to the radius of the maximum Lorentz force acting on element of the current sheet; see Fig. 6-1. We denote the shock front radius \( r_s \) and we further assume that the mass moves with the piston speed. For the mass associated with the current sheet we can write therefore, assuming 100% trapping:

\[
M = m_0 + \rho_0 \pi (r_0^2 - r_s^2) \quad (6.2)
\]

From our measurements we do not know the position of the shock front. However, \( r_s \) can be calculated for given \( r_p \), and the density ratio across the shock front \( \alpha = \rho_1 / \rho_0 \):

\[
r_s^2 = \frac{\nu_0 (R^2 - r_0^2) - \alpha r_p^2 + r_0^2}{1 - \alpha} \quad (6.3)
\]
Fig. 6-1 Typical radial profiles of axial current density $J_z$, Lorentz force $F$, and the assumed kinetic pressure $p$.

Here $r_s$ has physical meaning only for $r_s^2 > 0$.

The modified snowplow equation can be written now as

$$M \ddot{r}_p + \dot{M} \dot{r}_p = -F \quad (6.4)$$
where $M$ is given by (6.2) and $F$ by (6.1).

The acting force $F$ is evaluated for the limits $r_s$ and $r_a$, where $r_a$ is the outside radius ($r_a > r_s$) for which $\mathbf{J}_z \times \mathbf{B}_\theta$ is zero. The typical radial profiles of the Lorentz force, the axial current density and the assumed kinetic pressure, showing the approximate positions of $r_s$, $r_p$ and $r_a$, are given in Fig. 6-1. The kinetic pressure outside the limits $r_s$ and $r_a$ is assumed to be negligible in the absence of better knowledge, and therefore $\int \text{grad} \ p$ can be neglected when calculating $F$ over these limits.

It is also quite possible that the kinetic pressure falls to a negligible value already at $r_p$; $F$ is therefore computed also for the upper limit $r_p$ and the results are compared.

The piston velocity $\dot{r}_p$ and its acceleration $\ddot{r}_p$ are calculated from the collapse curves shown in Fig. 6-2.

To be able to calculate the shock front position $r_s$ (using the equation (6.3)) we need to know the density ratio across the shock front, $\alpha = \rho_1 / \rho_0$. We have calculated $\alpha$ for given Mach numbers (estimated from the collapse curves) using the iterative procedure suggested by Ahlborn and Salvat [16]. It is valid for a strong shock (Mach no. $> 5$) and thermal equilibrium. The enthalpy of helium as a function of temperature and pressure necessary for the procedure can be found in Ref. [17]. We have plotted $\alpha$ as a function of Mach number in Fig. 6-3.
Fig. 6-2  Collapse curves: (a) He, .5 Torr, (b) He, 1 Torr  
(c) He, 2 Torr, (d) He, 4 Torr  

○...position of the main current sheet  
△...position of the Lorentz force peak  
○...position of the main electron density peak  

taken from Funk's data [18], available only for 4 Torr.
Fig. 6-3 Density ratio across the shock front in helium as function of Mach number; initial pressure $p_0 \approx 10^{-3}$ atm, $T_0 = 290^\circ$K, $c = .965$ km/sec.
6.1.4 York-Jahn Model

According to York and Jahn [11] the neutral gas acceleration by a propagating current sheet occurs in three distinct phases (see Fig. 6-4): a leading region of intense electron current conduction (the current sheet), producing full gas ionization with subsequent radial and axial ion acceleration in radial electric and azimuthal magnetic fields; a following narrow region (≈ 1 cm thick) of intense mass concentration (the radial electric field in the current sheet is the mechanism for transferring the J x B force density from electrons to the ions); and a "wake" region of induced ionized-gas flow, following the current and mass sheets with fractional radial velocity. The fraction of the gas trapped by the sweeping current sheet approaches 100% and the spatial separation between the position of maximum current density (or Lorentz force peak) and the mass sheet remains constant during the collapse.

For our calculation we assume that the mass sheet is situated at \( \text{r}_m \), where \( \text{r}_m = \text{r}_p + \delta \) and \( \delta \) is a small constant. The optimum value of \( \delta \) is found for the best agreement of our results with the model. Analogously with (6.2) we can write for the mass of the sheet

\[
M = m_0 + \rho \pi (r_o^2 - r_m^2)
\]  

(6.5)
Fig. 6-4 York-Jahn model radial profiles of axial current density $J_z$ and mass density $\rho$.
According to the model the acting force $F$ is then calculated with the upper limit of $r_m$ (mass sheet is very thin) and for the simplicity we put the lower limit on the discharge axis. Since the kinetic pressure on the discharge axis (before the pinch) and outside $r_m$ is negligible, $\text{grad } p$ integrated over these limits can be neglected. The equation of motion is then identical with (6.4).

6.1.5 Confirmation of the Validity of the York-Jahn Model at Low Pressures.

Using our experimental data we can calculate the rate of change of momentum ($\dot{P}$) and the driving Lorentz force ($F$) of the snowplow equation for the two models described above; note that $\dot{P} = \text{l.h.s. of the equation (6.4)}$. If one of the models gives $\dot{P} \approx -F$, then we assume that the model provides a good description of the dynamics of the collapse. We have varied the initial trapping coefficient $\gamma_0$, and in the case of the York-Jahn model also the spatial separation $\delta$ of the mass sheet from the Lorentz force peak to get the best agreement. From the initial conditions we know $\gamma_0$, and from the collapse curves $r_p$ and its time derivatives, $\alpha$ and $r_s$ are computed as described above. The digitised $B_\theta$ radial profiles enable us to compute the Lorentz force for given limits. We have done these
computations for several times after the formation of the current sheet, and only for large current sheet radii. The results are presented in Table 6-1.

The agreement with the piston-shock wave model is poor in comparison with the York-Jahn model. The York-Jahn model provides very good agreement for the case of .5 Torr initial pressure. We have also reasonably good agreement in the case of 1 Torr initial pressure for the times from 2 to 3.5 μsec after the discharge initiation. Here we can clearly recognize the decrease of the ratio of \( \dot{P} \) to \(-F\) (Table 6-1). Since \( \dot{P} \) cannot be larger than the value calculated, \( F \) must be reduced. Hence other effects, such as ion-neutral drag and kinetic pressure gradient have to be included when refining the models. (This cannot be done until more diagnostic information is available.) These effects obviously become more important at increased pressures. Therefore the 4 Torr case is not described satisfactorily. We discuss qualitatively this limiting high pressure case in the next section.

6.2 Qualitative Description of the Z-pinch in Helium at High Initial Pressures.

We have seen in the previous section that neither of the two models presented there can describe the collapse at high initial pressures in helium. We do not have enough
Table 6-1. Comparison of experimental results with the models of Z-pinch collapse. Agreement is reached if \( \dot{P} \), the rate of change of momentum \( \approx -F \), the driving Lorentz force. (r = current sheet radius; for both models \( v_o \approx 0.5 \) (for 0.5 and 1 Torr), \( \approx 0.8 \) (for 2 and 4 Torr), for Y-J model \( \sigma = 1 \) mm (for 0.5 and 1 Torr), = 3 mm (2 Torr), = 4 mm (4 Torr))

<table>
<thead>
<tr>
<th>P</th>
<th>t (μsec)</th>
<th>r (cm)</th>
<th>York-Jahn</th>
<th>piston-shock wave</th>
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information to build a detailed model which would be satisfactory. Lovberg [8] has shown on a theta pinch the importance of knowing the electric field, magnetic field, particle density and temperature profiles in the plasma, before attempting to apply detailed models. We will therefore limit ourselves to a qualitative model describing the main observed features at 4 Torr of initial pressure. Here we have to point out that our model is just a hypothesis. It is up to the further investigation to prove whether our ideas are correct or wrong.

We have at our disposition the measurements of the azimuthal magnetic field and axial current density presented in this thesis, and the measurements of electron density presented in L. W. Funk's Ph.D. thesis [18]. Funk's laser interferometer measurements of the electron density were performed on a Z-pinch apparatus of the same dimensions and discharge circuit parameters. A Rogowski coil measurements of the total discharge current yield identical current waveforms. We therefore conclude that our independently obtained results can be correlated.

Examination of the experimental results brings to the attention the following features:

1. The main electron density peak is outside the current sheet during the collapse (Fig. 6-2). Consequently
the electron density in the current sheet is relatively low.

2. The velocity of the collapse is constant in the time interval from 4 to 6 μsec (Fig. 6-2). Maximum magnetic pressure is also constant in this time interval (Fig. 5-1(d)).

3. The current sheet forms a narrow layer of about 1/2 cm halfwidth at 6 μsec (r = 4 cm) and preserves its shape and peak current density up to 8 μsec (r = 2.5 cm) when it is brought to rest (Figs. 5-1(d) and 6-2). The main electron density peak stops at r ≈ 3 cm.

4. Meanwhile a "minor" current sheet appears in the current density distribution ahead of the main current sheet. It arrives on the axis at 7 μsec and reaches maximum current density at 8 μsec, when the main current sheet is brought to rest (Fig. 5-1(d)).

5. The degree of ionization is low. Nowhere in the outer region (for r > 2.5 cm) does the electron number density exceed 8 x 10^16 cm^-3 (i.e. 60% of the initial number density).

Our model qualitatively explains the main observed features, namely the spatial separation between the current sheet and the main electron density peak, the mechanism which brings the current sheet to rest before reaching the axis, and the occurrence of the axial current spike.
The model consists of a current sheet driven shock wave. The current sheet is a region of low density and high temperature plasma. After its formation and initial acceleration it travels with a constant velocity of almost 10 km/sec for about 2 μsec before it starts decelerating. Since the sound speed in room temperature helium is about 1 km/sec we estimate the shock wave Mach number to be 10. For the Mach number 10 the density ratio across the shock front \( \frac{s_1}{s_0} = 4 \); see Fig. 6-3. We have therefore a strong non-ionising shock wave driven by a leaky piston (= current sheet).

The sequence of events during the state of dynamic equilibrium (4 - 6 μsec) is as follows:

The shock front passes through the cold gas raising its initial density \( s_0 \) to \( s_1 = 4s_0 \) and heating it; the velocity of these heated neutral particles is \( v_1 < v_{\text{shock}} \).

As these neutrals are passing through the Joule heated current sheet they are heated again, which raises their specific volume. They cannot expand inward and are therefore expelled outward (as in the jet engine). The Lorentz force is balanced by the friction of these neutrals on ions and by the gradient of the kinetic pressure. In this time interval the current sheet is therefore moving with the constant velocity due to its inertia.
The excited expelled neutrals relax in some characteristic time, equivalent to a characteristic distance behind the current sheet, forming thus the region of high electron density observed by Funk.

The Lorentz force is controlled by the external discharge circuit. The observed state of dynamic equilibrium (4 to 6 μsec) is then disturbed when the discharge current and consequently the Lorentz force goes down (see Figs. 5-2(d) and 6-5). The kinetic pressure gradient now exceeds the Lorentz force and the resulting force is directed outward. This force is increasing and gradually brings the current sheet to rest (through the increased viscous drag arising from ion-neutral collisions).

After the current sheet starts decelerating, the shock front is driven to the discharge axis by its own momentum. Due to the focusing effect of this cylindrically converging shock wave (analogous to imploding detonations [19]) the gas temperature (and also pressure and density) is tremendously increased. This temperature increase is responsible for the corresponding raise of conductivity and for the occurrence of the axial current spike.
Fig. 6-5 Maximum Lorentz force on element of current sheet at 4 Torr of He vs. time after the discharge initiation.
Chapter 7

CONCLUSIONS AND PROPOSALS FOR FUTURE WORK

7.1 Conclusions

The main contributions of this thesis are to the improvement of magnetic probe measurements.

1. Spurious signals arising from poor probe geometry can be eliminated by using the symmetry of discharge fields. This is done by taking linear combination of probe signals at different locations in the discharge and at different orientations of the probe coil(s) with respect to the discharge axis.

2. The 3-coil probe which has been devised, substantially reduces the boundary error caused by the presence of the probe in the plasma.

(a) It has been verified that the 3-coil probe improves the spatial resolution $\ell$ of the magnetic field by at least a factor of 2 compared to a conventional probe. For the 3-coil probe $\ell \approx a/2$, where $a$ is the probe radius.
(b) For smooth current distributions the magnetic field measuring error \((\Delta B/B_0)\) is \(\sim 0.2 \left( \ell / \lambda \right)^2\), where \(\lambda\) is the scale length for spatial variations of the undisturbed current density \(J_0\). The 3-coil probe therefore improves the measuring accuracy of the undisturbed field \(B_0\) by at least a factor of four.

(c) The computer simulation of the probe misconstruction (incorrect coil sensitivity or spacing) and the increase of the effective probe radius (because of plasma cooling) show that even in this case the 3-coil probe gives better approximation to \(B_0\) than conventional probe.

3. Measurements of magnetic field \(B_\theta\) and current density \(J_z\) in Z-pinch discharges in helium at initial pressures between .5 and 4 Torr confirm the superior ability of the 3-coil probe. A "fine" structure - the development of a "minor" current sheet inside of the main current sheet has been observed. (A complex structure of the current sheet in He has been observed before [14 and 5] only in the high pressure limit where it is most pronounced.)

A catalogue of \(B_\theta\) and \(J_z\) radial profiles has been compiled for He and Ar and gives information on the Z-pinch behaviour and driving electromagnetic forces under
different initial conditions. It suggests regions of interest for further investigation of the acceleration process and the current sheet structure (using other diagnostic techniques).

7.2 Proposals for Future Work

The details of the gas acceleration process and of the current conduction mechanism in the plasma sheet (such as in a Z-pinch) are not yet completely understood. In the past several years the research in this field has been therefore directed towards the investigation of the sheet structure [6-10].

The results of our measurements in the Z-pinch discharge ($B_\theta$ and $U_z$ radial profiles) contain a great deal of information which can contribute to the better understanding of the above mentioned problems. The comparison of our experimental results in He with two types of modified snowplow equation (the conventional piston-shock wave model and the York-Jahn model in which ions form a thin mass sheet outside the current sheet) gives approximate agreement with the York-Jahn model at lower filling pressures. A qualitative model of the limiting high pressure case of 4 Torr in He, which cannot be described by either of two models, has been presented.
It is clear that additional information (namely the distribution of the kinetic pressure and electric field in the discharge) are necessary for the explanation of the gas acceleration process. It can be obtained with the special pressure probe described by York and Jahn [11], and with the coaxial differential probe described by Burkhardt and Lovberg [7]. The present discharge vessel should therefore be replaced with the vessel of the same size, but suitable for the suggested probing. Results obtained from this probing under the same experimental conditions could be correlated with our results using the current waveforms (Fig. 5-2).

The results at 4 Torr in helium show a steady state configuration (from 8 to 10 sec), with decaying discharge current in the large volume \( r \approx 2.5 \text{ cm} \) relatively quiescent plasma of approximately constant density. This suggests the use of a Z-pinch as a spectroscopic source suitable for line-broadening and continuum studies, as well as for laser scattering experiments. Further investigation should be conducted to determine whether the variation of discharge conditions (initial pressure and charging voltage) would provide a similar stable plasma over a wide range of temperatures and densities.
BIBLIOGRAPHY


APPENDIX I

MEASUREMENT OF THE DISCHARGE CURRENT

The discharge current $I$ is measured with a Rogowski coil (S. L. Leonard in [1]) constructed from a 40 cm length of RG 65A/U delay cable with the outer conductor removed. The discharge current carrying lead is passed through the torus opening. To reduce the capacitative coupling between the coil and the discharge circuit the Rogowski coil is wrapped with a 0.001 inch thick piece of brass shim stock, which does not close on itself electrically to permit penetration of the magnetic field. This shield is electrically grounded.

The Rogowski coil output signal is integrated to obtain a signal proportional to the discharge current. The balanced RC-type integrator is similar to the one shown in Fig. 2-4. The integration time constant is $340 \mu$sec now.

The enlarged oscillograms of the current waveforms were used to calibrate the current measuring system. The areas bounded by the waveform and the zero-current baseline were measured with a planimeter. The graticule area
was used to calibrate the planimeter readings in $\mu$sec-volts. It is known that the area under the current waveform must be equivalent to the total charge on the capacitor bank

$$\int I \, dt = Q = CV$$

where
- $I$ is the discharge current
- $Q$ is the charge on the capacitor
- $C$ is the capacitance
- $V$ is the charging voltage.

In this way it was established that a 1 V deflection on the oscilloscope is equivalent to a discharge current of 450 k A. The error associated with the calibration procedure is about 3%. The largest source of the error is in the measurement of the area under the current trace.

To check the accuracy of the calibration we have constructed another simple Rogowski coil and calibrated it using the above outlined procedure. The simultaneous measurements of the discharge current by both Rogowski coils agrees to 2%.
APPENDIX II

DATA REDUCTION

In this appendix we explain in detail how we obtain from the recorded data the radial profiles of the azimuthal magnetic field $B_\theta$ and the axial current density $J_z$.

Throughout the experimental measurement we have simultaneously displayed on the CRT both the magnetic field (upper beam) and the discharge current (lower beam) as a function of time. This was done for the probe radial positions from 0 to 80 at 5 mm intervals. The oscilloscope traces have been recorded on polaroid film (see Fig. 4-3).

We have then measured $B_\theta$ with the help of a grid and a magnifying glass for the selected times for which we wished the radial profiles to be constructed. The current waveform served as a time reference for the correlation of $B_\theta$ observed at different radial positions. The grid was ruled on lucite to fit the photographed graticule of the oscilloscope. It divides the large division of the graticule into ten small divisions. The reading error is approximately
2 of the small division. The $B_\theta$ values obtained in this way have been tabulated and then plotted to give the desired $B_\theta$ profiles.

By applying the $z$-component of the Maxwell equation $\mu_0 J_z = \text{Curl } B$ in cylindrical co-ordinates, the desired radial profiles of the axial current density $J_z$ can be obtained. This is a very tedious procedure. However, with the help of a digitizer and a computer it can be made easier as well as more accurate.

In our case we have been using the digitizer of the UBC Geophysics Department. It is a drum type digitizer (with automatic variable feed) of their own construction. It records only $y$ co-ordinate ($B_\theta$) with 0.1 mm resolution and $\pm 0.2$ mm accuracy. The $x$ co-ordinate ($r$) is determined to within 1% accuracy by the length of the digitizing interval $\Delta$ (variable) and the ordinal number of the $y$ co-ordinate. We used $\Delta = 0.5$ mm, however, since the radius was twice expanded on the graphs we had the effective $\Delta = 0.25$ mm. Generally we can say that the instrument error can be
neglected in comparison with the possible operator error. After some practice we have been able to follow the $B_\theta$ curve with the digitizing cursor within the thickness of the curve. However, since the calculation of $J_z$, eq. (II-1), involves differentiation it is very sensitive to inaccuracies in $B_\theta$. This problem was solved on the one hand by computing the derivative over a longer interval of $4\Delta$ (= 1 mm in real), and on the other hand by plotting large numbers of points (20 per cm). Since $B_\theta$ profile is a smooth curve, $J_z$ profile should be also smooth. The scatter of the points then indicates the inaccuracy of the digitizing.

Using the digitized $B_\theta$ radial profiles we have generated and plotted the $J_z$ radial profiles with the help of an IBM 360/67 computer and a Calcomp plotter.