A CONVERSION ELECTRON AND
LOW ENERGY GAMMA-RAY SPECTROMETER

by

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ABSTRACT

A conversion electron and low energy gamma-ray spectrometer has been developed using a silicon lithium-drifted semiconductor detector. The spectrometer has a resolution of 2 Kev for 100 Kev electrons and photons under optimum conditions. The energies of these electrons and gamma-rays can be estimated to ±1 Kev, and their intensities to within ±6% with the standard sources available.

The electron capture decay of $^{153}\text{Gd} \rightarrow ^{153}\text{Eu}$ was investigated using this spectrometer. The branching capture ratios to the 172.9 Kev, 103.2 Kev, 97.4 Kev, and 0 Kev levels of $^{153}\text{Eu}$ were found to be 11%, 39%, 39% and 11% respectively. Possible $J^\pi$ values of $5/2^+$ or $3/2^+$ for the 172.9 Kev level, $5/2^+$ or $3/2^+$ for the 103.2 Kev level, and $3/2^-$, $5/2^-$, or $7/2^-$ for the 97.4 Kev level have been assigned. These values are in agreement with those found by other investigators.
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CHAPTER I
NUCLEI AND RADIOACTIVE DECAY

1. Introduction

Nuclei are known to be assemblies of fast-moving heavy particles called nucleons, which include both neutrons and protons. The total number of nucleons is called the mass number A, while the total number of protons, which determines the nuclear charge, is called the atomic number Z. Nuclei with the same Z are termed isotopes, and those with the same A, isobars.

The internal motion of the nucleons gives the nucleus such properties as energy, angular momentum $\mathcal{J}$ (or spin), parity $\pi$, and electric and magnetic moments, the values of which are measures of the way the nucleons are arranged in a nucleus; or in other words, of the nuclear structure.

Since a nucleus is a quantum mechanical system, it can only exist in certain discrete energy states, each corresponding to a particular internal nucleon configuration. Each energy state may be uniquely classified by its energy, spin, parity, and by its electric and magnetic moments. The latter two properties are difficult to measure in any state except the lowest or ground state, and so the first three quantities alone are normally used to define a state.

Radioactive decay is the process by which a nucleus can reach a state of lower total energy. It may do so by merely a rearrangement of its internal configuration with no change in the numbers or kinds of
nucleons present (excited state decay). In this case the excess energy is usually, though not always, carried off by a gamma-ray quantum. It may decay by the emission of a nucleon or a cluster of nucleons which results in a change in $A$ so that the residual nucleus belongs to a different nuclear species. Or finally, the radioactive decay may be of the type where the mass number $A$ remains constant, but a neutron converts to a proton, or a proton to a neutron. The emitted particles are positive or negative electrons and neutrinos, and this particular type of decay is called beta-decay. This is the decay process, followed by excited state decay, that is relevent to the work described in this thesis.

Of course, all such decays alter the nucleon configuration and hence alter the energy, angular momentum, parity, and moments of the nucleus. Measurements on the emitted radiation give information on the properties of the nuclear states involved. This is the field of nuclear spectroscopy.

All radioactive decay follows the usual statistical rule defined by the well-known equation

\[ \frac{dN}{dt} = -\lambda N \]

where $N$ is the number of parent nuclei present at time $t$ and $\lambda$ is the decay constant. Integration then gives

\[ N = N_0 e^{-\lambda t} \]

where $N_0$ is the number present at $t = 0$. This equation leads to the usual definition of the mean life time $\tau = \frac{1}{\lambda}$ and the half-life

\[ \tau_{1/2} = \frac{\ln 2}{\lambda} \]
2. Beta Decay

This process, mentioned briefly previously, may be described by the following three equations:

a) \( p \rightarrow n + e^+ + \nu \) \hspace{1cm} \text{positron decay (} \beta' \text{)}

b) \( n \rightarrow p + e^- + \bar{\nu} \) \hspace{1cm} \text{negatron decay (} \beta^- \text{)} \hspace{1cm} (I - 2)

c) \( p + e^- \rightarrow n + \nu \) \hspace{1cm} \text{orbital electron capture (E.C.)}

where \( n, p, e^+, e^-, \nu, \) and \( \bar{\nu} \) refer to the neutron, proton, positron, negatron, neutrino, and antineutrino respectively. Only process b) occurs for free particles, but all may occur inside the nucleus if the total energy of the system is lowered in the process. It is obvious that the emitted particles in beta decay are the positive and negative electrons and the neutrinos.

The energy requirements for the three beta decay processes are

\[
\text{a) } E_e = M(Z,A)c^2 - M(Z-1,A)c^2 - I - 2m_e c^2 > 0 \hspace{1cm} \beta^+
\]

\[
\text{b) } E_e = M(Z,A)c^2 - M(Z+1,A)c^2 - I' > 0 \hspace{1cm} \beta^-
\]

\[
\text{c) } E_e = M(Z,A)c^2 - M(Z-1,A)c^2 - B_e(Z) - I'' > 0 \hspace{1cm} \text{E.C.}
\]

In these equations

- \( E_e \) = total energy released in the decay
- \( M(Z,A) \) = atomic mass of the atom whose nucleus has atomic number \( Z \), and mass number \( A \)
- \( I \) = difference in ionization energy between parent and daughter atoms
- \( m_e \) = electron rest mass
- \( B_e(Z) \) = binding energy of the atomic electron before capture.
It can be seen that the recoil energy of the daughter atom has been ignored. This is a valid approximation except for the very light nuclei. In fact, the factor I is also customarily ignored except where $E_\nu$ is also small. In the context of these approximations then, the energy $E_\nu$ will be shared statistically between the beta particle and the neutrino in the first two processes, while in the third, the neutrino is emitted monoenergetically with total energy $E_\nu$.

**Positron and Negatron Decay**

The energy distribution of electrons in $\beta^+$ decay is given by \(^1,^2\)

$$P(E)\,dE = \frac{g^2}{2\,\pi^3} \, E(E^2 - m_c^2)^{1/2} \, (E - E_\nu)^2 \, F(E_\nu Z) \, |M|^2 \, dE \quad (I - 4)$$

where:

- $P(E)\,dE$ = probability of emission of a $\beta^+$ particle with total energy between $E$ and $E+dE$
- $g$ = coupling constant of the interaction
- $E_\nu$ = energy of the decay
- $F(E_\nu Z)$ = the correction to the distribution function arising from the interaction between the outgoing $\beta^+$ particle and the nuclear coulomb field
- $M$ = nuclear matrix element involved in the transition.

This equation is due originally to Fermi. It assumes that rest mass of the neutrino is zero, and the recoil of the daughter nucleus and the difference in ionization energies between the parent and daughter atoms are negligible.

Typical shapes of $\beta^+$ distributions are shown in Fig. 1.

The square of the nuclear matrix element, $|M|^2$, can be expanded in
FIG. 1.--
Typical Shapes of $\beta^\pm$ Distributions
a series of terms in $kR$, where $k = E/c\hbar$ and $R$ is the nuclear radius. Each term in the expansion represents the contribution from transitions involving particular angular momentum and parity changes. Selection rules governing these transitions have been deduced theoretically\(^{1,2,4}\), and will be summarized later.

The first and largest term in the expansion represents (according to Gamow-Teller selection rules\(^1\)) a transition in which $\Delta J = 0, 1$ and $\Delta \Pi = \text{No}$. This is called an allowed transition. If the transition is of this type the first term in $|M|^2$ is dominant (assuming $kR \ll 1$). If not, the first term vanishes.

The second term in the expansion of $|M|^2$ represents a transition in which $\Delta J = 0, ^\mp 1, ^\pm 2$ and $\Delta \Pi = \text{Yes}$. This is called a first forbidden transition. If the transition is of this type, the second term in $|M|^2$ is dominant. If not, the second term vanishes. In fact, the first non-zero term in the expansion always dominates, and determines the degree of forbiddenness of a transition.

The Gamow-Teller selection rules for the first few transition types are listed in Table I.

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<th>Ang. Mom. Change ($\Delta J$)</th>
<th>Approx. log ft value</th>
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<tr>
<td>Allowed</td>
<td>No</td>
<td>$0, ^\mp 1$</td>
<td>3 - 6</td>
</tr>
<tr>
<td>1st forbidden</td>
<td>Yes</td>
<td>$0, ^\mp 1, ^\pm 2$</td>
<td>6 - 10</td>
</tr>
<tr>
<td>2nd forbidden</td>
<td>No</td>
<td>$^\pm 2, ^\pm 3$</td>
<td>$&gt; 10$</td>
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The half-life of $\beta^+$ decay is given by
\[ t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{\int_{m_0 c^2}^{E_0} p(E) \, dE} \]  

and is strongly dependent on \( E_0 \) (\( t_{1/2} \) varies approximately as \( E_0^{-5} \)) and on \( |M|^2 \), which is included in the integral. If we define a function \( f \) as

\[ f = \int_{m_0 c^2}^{E_0} E \left( E^2 - m_0^2 c^4 \right)^{1/2} (E_0 - E)^2 F(E, \gamma Z) \, dE \]  

which is the integral over the \( \beta^+ \) spectrum independent of the nuclear state, then

\[ ft_{1/2} = \frac{\ln 2}{\frac{8}{2\pi^3} \int_{m_0 c^2}^{E_0} E \left( E^2 - m_0^2 c^4 \right)^{1/2} (E_0 - E)^2 F(E, \gamma Z) |M|^2 \, dE} \]

is a function called the comparative half-life, or \( ft \) value. The function \( f \) has been calculated and tabulated by Trigg and Feenburg.\(^3\)

The usefulness of calculating an \( ft \) value is that it is almost independent of \( E_0 \), but retains the strong dependence on \( |M|^2 \). It is therefore a measure of the forbiddenness of a transition. That is, since \( |M|^2 \) decreases rapidly with increasing degrees of forbiddenness, the \( ft \) value will increase rapidly also. Since \( ft \) values are large, it is more convenient to use their logarithms.

Typical log \( ft \) values for the different degrees of forbiddenness of \( \beta^+ \) transitions are included in Table I. The log \( ft \) value in itself, while useful in assessing the forbiddenness of a transition, is not completely definitive since overlaps of these values are fairly common.
Orbital Electron Capture

The only emitted particle in orbital electron capture is a neutrino, which interacts very weakly with matter, and is consequently almost impossible to detect. Therefore any information about this type of transition must come from secondary effects (x-rays, internal Bremsstrahlung, gamma-rays, etc.).

The probability for electron capture is given by

\[ P_i = \frac{2}{2\pi^3} \frac{\mathcal{E}}{Z_0(Z, Z)} |M|^2 \]  

where 

- \( P \) = the probability of an electron being captured from atomic orbital \( i \).
- \( M \) = is the same nuclear matrix element that is involved in \( \beta^+ \) decay and therefore orbital electron capture may be classified by the degrees of forbiddenness given in Table I.
- \( \mathcal{E}(E_0, Z) \) = a function that depends on the angular momentum, parity, and binding energy of the electron being captured. It also depends on the change in spin, parity, and energy between the parent and daughter nuclei.

Numerical values of \( \mathcal{E}(E_0, Z) \) have been tabulated by Trigg and Feenburg\(^2\) and by Zyryanova\(^21\).

The half-life for electron capture is given by

\[ t_{1/2} = \frac{\ln 2}{\lambda} \]

where

\[ \lambda = \sum_i \frac{\mathcal{E}}{2\pi^3} \frac{\mathcal{E}}{Z_0(Z, Z)} |M|^2 \]

the summation being over all orbital electron states.
In a similar manner to $\beta^-$ decay we can define a function $f$ as

$$f = \sum_i f_i (E_\gamma, Z)$$

then the $f_t$ value is

$$f_t = \frac{\ln 2}{\frac{2}{2\pi^3}} \sum_i f_i (E_\gamma, Z) \frac{M^2}{E_i}$$

which depends strongly on $|M|^2$ as in the $\beta^+$ case.

**Decay Schemes**

When a nucleus undergoes beta decay the decay sequence may be represented schematically as shown in Fig. 2. This particular example ($^{64}$Cu) was chosen as it decays by $\beta^+$, $\beta^-$ and E.C., all of which are in competition. It will be noted from spin and parity changes that all transitions are allowed. Included on the diagram are $f_t$ values and relative decay probabilities which have been deduced from experiments. These values are consistent with the allowed character of the decays.

**3. Excited State Decay (Gamma Decay and Internal Conversion)**

There are two main processes in excited state decay. They are the emission of electro-magnetic quanta (gamma rays), and the absorption of the decay energy by an atomic electron with its subsequent emission (internal conversion). The two processes, when they involve the same initial and final states, are always in competition.

The energy requirements for these processes are

$$E_\gamma = M^* (Z,A) c^2 - M(Z,A) c^2 > 0 \hspace{1cm} \text{gamma (\$)}$$

$$E_\gamma = M^* (Z,A) c^2 - M(Z,A) c^2 - B_e (Z) > 0 \hspace{1cm} \text{internal conversion (I.C.)}$$

(I - 9)
FIG. 2.---
Decay Scheme

$J^\pi = 1^+$

$\beta^-(38\%)$  
$E_\beta = 580$ Kev  
$\log ft = 5.3$

$J^\pi = 0^+$

$\beta^+(19\%)$  
$E_\beta = 657$ Kev  
$E.C.(43\%)$  
$E_\beta = 1677$ Kev  
$\log ft = 5.0$
In this equation the * refers to the higher energy state and Be (Z) is the binding energy of the electron before conversion. The other symbols are defined in Equation I - 4.

**Gamma Decay**

The transition probability for the emission of a gamma ray (to a first order approximation) is given by

\[
\omega_y = K \, k \left| \int \psi_f^* e^{-i \frac{k}{\hbar} R (E + m) \psi_i \, d \tau \right|^2 \quad (I - 10)
\]

In this equation

\[\omega_y = \text{the probability per unit time for emission of a gamma ray with energy } E = k \, c\]

\(K\) is a constant.
\(\psi_i\) and \(\psi_f\) = the initial and final wave functions of the nucleus.
\(R\) = is the radius of the nucleus.
\(E + m\) = represents the interaction causing the radiation.

\(E\) is electric in origin and has even parity.

\(m\) is magnetic in origin and has odd parity\(^4\).

This expression for \(\omega_y\) is often quite inaccurate but is useful for describing the method by which gamma radiation is classified.

The term \(e^{-i \frac{k}{\hbar} R}\) can be expanded in the usual form. That is:

\[e^{-i \frac{k}{\hbar} R} = \sum_{\ell=1}^{\infty} \frac{(-i \frac{k}{\hbar} R)^{\ell-1}}{(\ell-1)!}\]

Then the first term in

\[
\omega_y = K \, k \left| \int \psi_f^* \sum_{\ell=1}^{\infty} \frac{(-i \frac{k}{\hbar} R)^{\ell-1}}{(\ell-1)!} (E + m) \psi_i \, d \tau \right|^2 \quad (I - 11)
\]
represents the transition probability for dipole radiation, the second quadrapole radiation, etc.\(^4,5,6\).

Only terms of the lowest order in \(kR\) need be considered in the expansion unless the energy of the emitted gamma ray is large. For example:

- Taking \(R = 1.2 \times 10^{-13} \text{ cm} \ A^{1/3}\)
  \[ k = 5.05 \times 10^{10} \text{ E cm}^{-1} \]

where \(A\) is the mass number of the nucleus and \(E\) is the energy of the gamma ray in Mev.

Then \(kR \approx 6 \times 10^{-3} A^{1/3} E\).

Since \(E\) is usually less than 10 Mev, \(kR\) is usually much less than unity and only terms of the lowest order in \(kR\) need be retained. The following discussion is limited to this case.

The angular momentum of a gamma ray resulting from a \(2^\ell\)-pole transition is \(\ell\) with respect to the origin to which the multipole is referred\(^6\). If \(J_i\) and \(J_f\) are the initial and final angular momenta of the nucleus respectively, then the restrictions on \(\ell\) due to conservation of momentum are

\[ |J_f - J_i| \leq J_o \leq \ell \leq J_f + J_i \]

Because of the transverse nature of electro-magnetic radiation\(^4\), gamma rays must have angular momentum greater than or equal to one. Therefore, for \(J_o = 0\)

\[ 1 \leq \ell \leq J_f + J_i \]

and if \(J_f = J_i = 0\), no gamma radiation can occur. These restrictions set limits on the allowed values of \(\ell\) in the expansion of \(e^{-i kR}\).
Terms in the integral in equation I - 11 will vanish unless they have even parity. Because $\mathcal{E}$ and $\mathcal{M}$ have opposite parity, the second non-zero term will contain $\mathcal{E}$ if the first contains $\mathcal{M}$, and vice versa. The ratio of $\mathcal{M}$ to $\mathcal{E}$ is of the same order of magnitude as $kR$. That is

$$\frac{\mathcal{M}}{\mathcal{E}} \approx 5 \times 10^{-3} A^{-2/3}$$

Therefore, the first and second terms are of the same order of magnitude if the first contains $\mathcal{M}$. Only these two terms need be retained as contributions from all other terms are at least of order $(kR)^2$ smaller. Similarly, if the first term contains $\mathcal{E}$, all other terms may be ignored.

Transitions involving the emission of gamma rays are classified as electric (E) or magnetic (M) according to which are the dominant terms contributing to $\omega_\gamma$, and by the angular momentum (spin) of the emitted gamma ray. There are two distinct types of transitions. For the first type, the product of the initial and final wave functions of the nucleus has parity $(-1)^J$ ($-1 = \text{Yes, there is a parity change}, +1 = \text{No, there is not a parity change}$). Such transitions are called parity favored. The second type has a parity change of $-(-1)^J$. These are called parity unfavored transitions. Both cases are discussed below.

**Parity Favored Transitions**

The transition probability is given by

$$\omega_\gamma = \kappa \left| \sum_i \frac{U^* \mathcal{E}}{(J_i - 1)!} \frac{(i kR)^{J_i - 1}}{(j - 1)!} \mathcal{M} \right|^2$$

All other terms in equation I - 11 are zero or negligible compared to this term. Transitions of this type are classified as $EJ_\omega$.

If $J_\omega = 0$ the transition probability is given by
Transitions of this type are classified as $M_{1} + cE_{2}$, where $c$ represents the percentage of the transition that is $E_{2}$ in character.

Parity Unfavored Transitions

The transitions probability is given by

$$
\omega_{y} = K k \left| \sum \psi_{f}^{*} \left[ m + (-i k R) \varepsilon \right] \psi_{i} \right|^{2}
$$

Transitions of this type are classified as

$$
M J_{o} + c' E J_{o} + 1
$$

If $J_{o} = 0$ the transition probability is given by

$$
= K k \left| \sum \psi_{f}^{*} \varepsilon \psi_{i} \right|^{2}
$$

Transitions of this type are classified as $E1$.

The classification and relative transition probabilities of transitions involving the first few values of $J_{o}$ are given in Table II. The relative transition probabilities are only approximate as the error in equation I - 11 can be considerable.

An exact calculation of $\omega_{y}$ requires a nuclear model involving the rearrangement of the nucleons that will give the required changes in energy, spin, and parity of the nucleus. The experimental value of $\omega_{y}$, which can be obtained by measuring the number of gamma rays with energy $E_{o}$ emitted per unit time from a known radioactive source, is often used to check the adequacy of such a model.

Internal Conversion and Conversion Coefficients

Spin and parity changes of a transition can be deduced from the
### Table II

**Classification of Gamma Transitions**

#### Parity Favored Case

<table>
<thead>
<tr>
<th>( J_0 )</th>
<th>Parity Change</th>
<th>Class</th>
<th>Approx. Relative Value of ( \omega_\gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (0 ( \leftrightarrow ) 0)</td>
<td>No</td>
<td>M1 + (E2)</td>
<td>((k R)^2)</td>
</tr>
<tr>
<td>1</td>
<td>Yes</td>
<td>E1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>No</td>
<td>E2</td>
<td>((k R)^2)</td>
</tr>
<tr>
<td>3</td>
<td>Yes</td>
<td>E3</td>
<td>((k R)^4)</td>
</tr>
</tbody>
</table>

#### Parity Unfavored Case

<table>
<thead>
<tr>
<th>( J_0 )</th>
<th>Parity Change</th>
<th>Class</th>
<th>Approx. Relative Value of ( \omega_\gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (0 ( \leftrightarrow ) 0)</td>
<td>Yes</td>
<td>E1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>No</td>
<td>M1 + (E2)</td>
<td>((k R)^2)</td>
</tr>
<tr>
<td>2</td>
<td>Yes</td>
<td>M2 + (E3)</td>
<td>((k R)^4)</td>
</tr>
<tr>
<td>3</td>
<td>No</td>
<td>M3 + (E4)</td>
<td>((k R)^6)</td>
</tr>
</tbody>
</table>

internal conversion coefficients (I.C.C.). The I.C.C. of a transition is defined as

\[ \alpha = \frac{\omega_e}{\omega_\gamma} \]

where \( \omega_e \) is the transition probability for internal conversion. It should be noted that

\[ \frac{\omega_e}{\omega_\gamma} = \frac{N_{\text{c.e.}}(E_\gamma - B)}{N_\gamma(E_\gamma)} \]

where \( N_{\text{c.e.}}(E_\gamma - B) \) and \( N_\gamma(E_\gamma) \) refer to the number conversion electrons and gamma rays with energy \( E_\gamma - B \) and \( E_\gamma \) respectively, emitted per unit time; and that

\[ \omega_e = \omega_K + \omega_{L_1} + \omega_{L_II} + \ldots \ldots \]

where \( K, L_1, L_{II}, \ldots \) denote the different atomic orbitals.

The energy transfer between the nucleus and the atomic electron in internal conversion is a direct interaction between the bound electron and the same multipole field which otherwise would have resulted in the emission of a gamma ray. The transition probability for internal conversion therefore contains the same nuclear wave functions as the transition probability for gamma radiation.

It has been shown that, to a close approximation, the nuclear wave functions cancel in the calculation of internal conversion coefficients. The I.C.C.'s then depend only on the energy difference between the initial and final states, on the spin and parity change of the transition, and on the angular momentum, parity and binding energy of the electron being converted.

The internal coefficients can be written as
\[ \alpha = \alpha_K + \alpha_{L_I} + \alpha_{L_{II}} + \ldots \]

where

\[ \alpha_K = \frac{\omega_K}{\omega_Y} \]

\[ \alpha_{L_I} = \frac{\omega_{L_I}}{\omega_Y} \]

e tc.

These individual coefficients can be calculated and their values compared to those obtained experimentally to deduce the spin and parity change of a transition.

Theoretical internal conversion coefficients have been calculated by Sliv and Band\(^7\). They have calculated \( \alpha_K \) for nuclei with atomic number between 33 and 98 and \( \alpha_{L_I} \), \( \alpha_{L_{II}} \) and \( \alpha_{L_{III}} \) for nuclei with atomic numbers between 41 and 98. In both cases calculations were done for multipolarities up to \( J_\omega = 5 \) for both parity favored and parity unfavored transitions. Similar calculations have been done by Rose\(^8\).

Investigations of internal conversion coefficients therefore lead to knowledge of the spin and parity changes involved in the radioactive decay of a nucleus. Such investigations require a suitable spectrometer capable of measuring energies and intensities of electrons and gamma rays. A spectrometer with these features was constructed using a silicon lithium-drifted semiconductor detector. The development and characteristics of this spectrometer are the main topic of Chapters II, III and IV.
CHAPTER II

SEMICONDUCTOR DETECTORS

1. The P-N Junction

As is well known, electrical conduction in semiconductors is due to mobile carriers, such as electrons in the unfilled conduction band, or vacancies or "holes" in the valence band, which move under the influence of an external electric field. In a pure semiconductor crystal, the number of carrier electrons \(n\) and the number of carrier holes \(p\) are equal and depend only on the energy gap between the conduction and valence bands, and on the crystal temperature.

However, impurities in the crystal can cause an inequality between \(n\) and \(p\). When this is so we have an N-type or a P-type semiconductor, depending upon whether the electrons \(n\) or holes \(p\) are more numerous. These inequalities arise because, for example, the impurities may introduce levels high in the forbidden zone between the conduction and valence bands. An electron from such a level, when excited into the conduction band, becomes a carrier, while the hole left behind is trapped in position. Hence impurities of this type lead to N-type semiconductors. These impurities are called donor impurities as they "donate" electrons to the conduction band. Other impurities lead to levels lying low in the forbidden zone, near the top of the valence band. Valence electrons can easily be excited into these levels leaving mobile holes behind, while the electrons are trapped. These impurities are called acceptor impurities as they accept electrons from the valence band, leading to the formation of a P-type semiconductor. The energy level structures...
FIG. 3.--
Impurity Levels

Donor Impurity

Acceptor Impurity
for both types of impurities are shown in Fig. 3.

It is possible to convert P-type material to N-type material by diffusing a donor impurity into the crystal, and conversely, N-type material can be converted to P-type by acceptor diffusion. If the diffusion process extends only part way, the crystal will have two different regions. The junction between these regions is called a P-N junction. There will then be a diffusion of free electrons from the N-type region into the P-type region which will continue until the Coulomb repulsion from the charge buildup balances the diffusion forces. The effect is to create, at the junction, a region free of charge carriers, called the depletion layer. These layers are very narrow, being typically $10^{-6}$ cm.

However, the application of a reverse bias to the junction (negative voltage on the P-type side) causes a great increase in the width of the depletion layer, the increase depending on the voltage applied. Depletion layers $>3$ mm have been achieved. These depletion layers have extremely high resistivity. The only contribution to the conduction current, in the absence of external ionizing events, are those electron-hole pairs generated by thermal agitation, a process that can be reduced by cooling the crystal.

2. The P-I-N Crystal

A P-I-N crystal is made by diffusing a donor impurity into a P-type crystal, or an acceptor impurity into an N-type crystal. The impurity to be diffused into the crystal (say P-type) must be chosen so that the added donor electrons compensate for the acceptor levels which existed in the P-type material, leaving a region with no free charge carriers other
than those due to thermal agitation.

If proper diffusing techniques are used, only a thin layer of P-type material on one side of the crystal and a thin layer of N-type material on the other side will remain uncompensated. These layers have a low resistivity compared to the compensated region and act as electrical contacts for applying a reverse bias.

The compensated region has a high resistivity. In addition, the junctions between the P-type and compensated region, and the compensated and N-type regions, act like P-N junctions. Therefore a P-I-N crystal will act like a P-N junction under reverse bias, the only difference being that the compensated region of a P-I-N crystal can be made much larger (>2 cm) than the depleted region of a P-N junction. Also the width of the carrier free region of a P-I-N crystal is only slightly dependent on the applied reverse bias.

The compensated region of a P-I-N crystal has incorrectly been called intrinsic in the past. Hence the name P-I-N crystals.

3. Semiconductor Crystals as Particle Detectors

An energetic charged particle that penetrates a semiconductor crystal will interact with the crystal and lose energy in one of the following three ways.

a) The charged particle may impart sufficient energy to an electron to raise it to the conduction band, leaving a hole in the valence band. The energy of the primary particle is reduced by an amount equal to the width of the energy gap between the two bands. The remaining energy is shared randomly between the primary particle, the secondary electron, and
the secondary hole. Each of these will then lose their energies by processes a), b) or c).

b) The primary particle may lose energy by interacting with the crystal lattice itself. These interactions excite the lattice in optical and acoustical modes of vibration. The primary particle, after this loss of energy, will continue to lose energy by process a), b), or c).

c) When a particle has insufficient energy for either of the above processes, it must lose its remaining energy by thermal losses to the lattice material.

The overall energy loss process is shown schematically in Fig. 4.

When a reverse bias is applied to a crystal, the electrons and holes produced by process a) above cause the current across the crystal to increase. It is possible, by measuring this current increase, to deduce the energy lost by the primary particle. This will be described in more detail in later sections.

The number of electron-hole pairs that contribute to the current is given by

\[ n = \frac{E}{\xi} \]

where \( E \) is the energy lost in the carrier free region of the crystal and \( \xi \) is the average energy required to produce an electron-hole pair.

The value of \( \xi \) depends on the semiconductor material (usually germanium or silicon), on the type of primary particle, and on the temperature of the crystal.

The fluctuation in the number of electron-hole pairs is given by
FIG. 4.--
Energy Loss in Semiconductor Crystals

Primary Particle
Energy = $E$

Probability = $1 - r$

Excitation of electron to conduction band.

Energy Loss = $E_3$

Primary Part. Energy = $(E - E_3)(1 - p)$

Hole Energy = $p(E - E_3)/2$

Electron Energy = $p(E - E_3)/2$

These now become primaries for future generation if their energy is large enough.

$p$ is the fraction of energy lost by the primary particle.
\[ \langle \Delta n \rangle = \left( \frac{E}{\epsilon} \right)^{\frac{1}{2}} F \]

where \( \langle \Delta n \rangle \) is the RMS fluctuations in \( n \) and \( F \) is the so-called Fano factor; a statistical factor introduced by U. Fano\(^{10} \) in 1946. A value of \( F = 0 \) would imply that no energy was lost by processes b) or c) given previously, while a value of \( F = 1 \) would imply that the probability of producing an electron-hole pair approaches zero. Since the true picture is somewhere between these two extremes, the value of \( F \) must be between 0 and 1.
CHAPTER III

LITHIUM ION DRIFTED SILICON DETECTORS

1. Si(Li) Detectors for Photons and Electrons

A lithium ion drifted silicon detector (Si(Li)) is a P-I-N crystal operated under reverse bias. These crystals are made by diffusing lithium ions (a donor impurity) into a crystal of P-type silicon at a high temperature. The lithium compensates for the acceptor impurity in the P-type silicon as described earlier, creating a carrier free region. The N- and P-type regions are highly doped and supply good electrical contacts. Fig. 5 schematically shows some features of a Si(Li) detector.

The average energy required to produce an electron-hole pair in silicon by electrons depends only on the temperature of the crystal. This dependence is shown in Fig. 6. The number of electron-hole pairs produced in the carrier free region is linear with electron energy provided the electron loses all its energy in this region. This sets a minimum thickness for the compensated region and also requires a dead layer (window) thin enough that no appreciable amount of energy is lost as the electron passes through it. Fig. 7 shows the range of electrons in silicon as a function of energy. Fig. 8 shows the loss in electron energy in silicon per micron of dead layer as a function of electron energy.

An incident electron may be scattered out of the compensated region of the detector, as shown in Fig. 9. The "backscattered" electron will have lost only part of its energy in the detector. It has been shown that...
FIG. 5.-- Si(Li) Crystals

P-layer (grounded)

Incident particles

N layer (+ voltage applied)

Dead layer (window)

a) Physical

b) Concentration of carriers

c) Electric field due to applied bias
FIG. 6.--
$\varepsilon$ vs. Temperature

FIG. 7.--
Range of Electrons in Si
FIG. 8.--
Energy Loss in Dead Layer

Electron Energy (Kev/micron)

FIG. 9.--
Partial Energy Loss Processes

Backscattered Electron

Compensated region of detector

Electron too energetic to be stopped by this thickness of crystal

Incident Electrons

"Backscattered" Electron
that the fraction of electrons scattered out of a Si(Li) detector does not depend on electron energy. It depends only on the source-detector geometry used.

The detection of photons (gamma rays or x-rays) with a Si(Li) detector requires that the photon's energy be transferred to an electron (or electrons), and that the electron be detected. This transfer of energy can be accomplished by three processes; they are

a) **Photo-electric effect.**

The photon, with energy $E_\gamma$, is absorbed by an atomic electron with binding energy $E_B$. The electron is ejected from the atom with energy $E_\gamma - E_B$. The x-ray or auger electron emitted due to the vacancy in the atomic shell will interact also.

b) **Compton scattering.**

The photon suffers an inelastic collision with an atomic electron, losing energy $E_C$ to the electron. If $E_C$ is greater than the binding energy of the electron, it will be ejected from the atom. The photon is left with reduced energy and may undergo further interactions.

c) **Pair production.**

If the photon energy is greater than $2m_\text{e}c^2$, where $m_\text{e}$ is the electron rest mass, the photon may interact with the coulomb field of the nucleus, creating an electron-positron pair. The photon is completely absorbed and the excess energy is shared by the electron and positron. The excess energy is $E = E_\gamma - 2m_\text{e}c^2$.

The cross-sections as functions of photon energy for these three processes is shown in Fig. 10.

The measurement of the energy of a photon with a Si(Li) detector
FIG. 10.--
Photon Cross Section for Silicon

Absorption Cross Section in barns/atom

Photon Energy in Kev
requires that a known fraction of its energy be transferred to the
electrons, and that the electrons be completely stopped in the compensated
region.

In Compton scattering the photon is usually scattered out of the
detector after transferring an unknown fraction of its energy to the
electron. For this reason the Compton process is not a useful means of
measuring the photon's energy.

The photo-electric effect and pair production, on the other hand, transfer all of the photon's energy to electrons. In the case of the
photo-electric process, the full energy of the photon can be determined
by measuring the energy of the photo-electron produced, provided that it
is stopped in the compensated region. The probability that the x-ray
caused by the vacancy in the atomic shell will escape without undergoing
a photo-electric interaction in the compensated region is negligible as
the photo-electric cross-section for this energy (<2 Kev for silicon)
is very large.

The positron and electron produced in the pair process lose their
energy in the usual way. The positron, once it has lost its energy, will
interact with an atomic electron to form two photons, each with energy
\( m_e c^2 \). These photons may interact in the compensated region by either
the photo-electric or the Compton scattering processes, or they may
escape. A known fraction of the incident photon's energy will be
deposited in the detector provided neither of these two secondary photons
undergo Compton scattering. \( E\gamma \) is deposited if both photons produce
photo-electrons, \( E\gamma - m_e c^2 \) is deposited if one photon escapes and
the other produces a photo-electron, and \( E\gamma - 2m_e c^2 \) is deposited if
both escape.

The collection, amplification and analysis of these charge carriers produced by the energy loss in Si(Li) detectors may be used to produce an energy spectrum of the incident radiation.

2. Associated Electronics

The electronics required to collect, amplify and analyze the free charge carriers produced in the compensated region of a Si(Li) detector are shown diagrammatically in Fig. 11. The electronics described are those used in this report and are of "state-of-the-art" quality.

The preamplifier is a high gain, low noise, charge sensitive amplifier (Tennelec, model 135 M) that utilizes the inherent high input impedance and low noise of a field-effect transistor (FET). The equivalent circuit for a Si(Li) detector coupled to an FET preamplifier is shown in Fig. 12. $C_d$ and $R_d$ represent the crystal capacitance and resistance respectively. $R_s$ and $C_s$ are the resistance and capacitance in the connections between the crystal and the preamplifier. $R_L$ is the load resistance of the bias voltage supply (typically $10^9$ ohms). The parallel combination of $C_F$ and $R_F$ is the feedback impedance of the charge sensitive stage of the preamplifier.

If $R_s$ is small and $R_d$ and $R_L$ are large, the output voltage $V_o$ is given by

$$V_o = \frac{Q A}{C_d + C_s + C_F (A+1)}$$

where $A$ is the open loop gain of the preamplifier and $Q$ is the charge deposited on $C_d$; that is, $Q$ is the free charge carriers produced in the
FIG. 11.--

Schematic of Electronics

High voltage supply (0→±1000 V)

Crystal

Preamplifier

Pulse Shaper

Main Amp.

Baseline Restorer

Biased Amp.

Pulse Stretcher

To M.C.A.
FIG. 12.--
Equivalent Circuit for Detector-Preamp.

\[ A = R_{in}, \quad C_s = C_{FET}, \quad C_f, \quad R_f \]
compensated region of the detector.

If $A$ is large, the output voltage reduces to

$$V_o = \frac{Q}{C_F}$$

The pulse shaper is the integrating differentiating network shown in Fig. 13. Its characteristics are given in Table III. The pulse from the preamplifier is shaped to give the best signal to noise gain in the main amplifier.

The main amplifier is a Nuclear Chicago, Model 27001. Its gain is continuously variable from 6 to 400.

The baseline restorer is an Ortec, Model 438. Its function is to insure that a pulse is not superimposed on the tail of a preceding pulse; that is, it prevents pile-up of the individual signals.

The biased amplifier (Ortec, Model 408) discriminates against pulses below a given voltage. This discriminator voltage is adjustable. All pulses above this voltage are amplified linearly and their baseline restored to zero.

The pulse stretcher (Ortec, Model 411) insures that the pulses are compatible with the input to the multi-channel analyser (MCA), a Nuclear Data, Model 110.

The MCA consists of an analog-to-digital converter, and a 128 word memory (128 channels). Its readout is visual (Nuclear Data 410 Display), or by a teletype printer and paper tape puncher.

A typical output of this system is shown in Fig. 14. The radioactive source used was $^{57}$Co, which decays by electron capture to $^{57}$Fe.
FIG. 13.--
Pulse Shaper Network

$T_1$, $T_3$, $T_4$ are 2N4124

$T_2$, $T_5$ are 2N4126
Table III

Characteristics of the Pulse Shaper

<table>
<thead>
<tr>
<th>Integration</th>
<th>Differentiation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Const. ((\mu) sec.)</td>
<td>(C_1) (pf)</td>
</tr>
<tr>
<td>0.2</td>
<td>20</td>
</tr>
<tr>
<td>0.47</td>
<td>47</td>
</tr>
<tr>
<td>1.0</td>
<td>100</td>
</tr>
<tr>
<td>2.0</td>
<td>200</td>
</tr>
<tr>
<td>4.7</td>
<td>47</td>
</tr>
<tr>
<td>10.0</td>
<td>1000</td>
</tr>
</tbody>
</table>

Amplification

<table>
<thead>
<tr>
<th>(R_3) (ohms)</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>2</td>
</tr>
<tr>
<td>47</td>
<td>4</td>
</tr>
<tr>
<td>100</td>
<td>8</td>
</tr>
</tbody>
</table>
Typical Low Energy Spectrum

Source: $^{57}$Co
Detector: $S_1$
Kev/chan: .378
The four peaks are due to the gamma rays and conversion electrons from the 136.3 and 121.9 Kev transitions in $^{57}$Fe. The first peak is due to the K conversion electrons from the 121.9 Kev transition. The second is due to gamma rays from this transition, as well as the L+M+... conversion electrons whose energy is only slightly lower than the gamma ray's energy. The third and fourth are due to the K conversion electrons and gamma rays plus L+M+... conversion electrons from the 136.3 Kev transition.

3. Inherent Resolution of the Detector-Analyser System

The peaks in Fig. 14 represent total energy loss by mono-energetic particles in the compensated region of the detector. The width of these peaks at half maximum (FWHM - full-width-half maximum) is called the resolution $\omega$, and is due to fluctuations in the response of the detector-analyser system to these mono-energetic particles.

The fluctuations in the response have two independent causes. They are

a) Statistical fluctuations in the number of electron-hole pairs produced in the detector. The relationship between the FWHM and the fluctuations in the number of pairs $\langle \Delta n \rangle$ is

$$\omega_c = 2.35 \varepsilon \langle \Delta n \rangle$$

assuming that the peak is gaussian shaped. From equation II - 1 the FWHM becomes

$$\omega_c = 2.35 (E \varepsilon)^{1/2} F$$

b) Fluctuations due to noise generated in the detector and associated electronics. A detailed analysis of noise has been done by Goulding.
He shows that there are three main sources of noise, which may be summarized as follows. The first, shot noise, results from electron-hole pairs generated by thermal excitation in the detector. Its contribution to $\omega$ is given by

$$\omega_s = \left( K_s C_T^2 T R_{eq} S_s \right)^{\frac{1}{2}}$$

The second, flicker noise, is generated by fluctuations in the current flowing into the input stage of the preamplifier, and its contribution is

$$\omega_f = \left( K_f C_T^2 S_f \right)^{\frac{1}{2}}$$

Finally, leakage noise is caused by currents over the surface of the detector and its contribution is

$$\omega_L = \left( K_L i S_L \right)^{\frac{1}{2}}$$

In the above equations, the $K$'s are constants, $C_T$ is the total capacitance of crystal and input stage, $T$ is the crystal temperature (in °K), $R_{eq}$ is the equivalent noise resistance of the preamplifier input, the $S$'s are factors that depend upon the shape of the pulses, and $i$ is the leakage current.

Since these noise sources are independent of each other, and independent of the number of electron-hole pairs, the total resolution is given by

$$\omega_T = \left( K_s C_T^2 T R_{eq} S_s + K_f C_T^2 S_f + K_L i S_L + \varepsilon E \left( 2.35 \varphi^2 \right)^{\frac{1}{2}} \right)$$

For good resolution, each term in this equation must be made as small as possible.

$R_{eq}$ is set by the type of input to the preamplifier used. It cannot
be changed without redesigning the preamplifier.

The factors $S$ can be lowered by the proper choice of time constants in the pulse shaper. The usual procedure for finding the best time constants is to minimize the resolution by changing $C_1$ and $C_2$ in the pulse shaper after all other parameters have been fixed.

$C_T$ can be lowered by using low capacitance connectors between the crystal and preamplifier, and by the proper choice of detector. The capacitance of a Si(Li) detector, assuming it can be treated as a parallel plate capacitor, is

$$C_d = \varepsilon A/W$$

where $\varepsilon$ is the permittivity of silicon. Therefore

$$C_d = 1.05 \text{ A/W picofarads}$$

Here $A$ is the area of the detector in square centimeters and $W$ is the thickness of the compensated region in centimeters.

The relationship between noise and total input capacitance for the 135 M preamplifier is \(^{16}\)

$$\omega = 1.5 \text{ Kev + .018 Kev/pf (in silicon)}$$

The Fano factor ($F$) in the equation for $\omega_c$ depends on the crystal material. It is very difficult to measure accurately, but most reports give values for silicon ranging from .2 to .4 \(^{9,17}\).

The temperature of the crystal effects three terms, in the equation for $\omega_T$. Decreasing the temperature decreases the contribution to the resolution due to shot noise by decreasing the number of thermally produced electron-hole pairs. It also decreases the contribution due to
leakage current by decreasing $i$, but increases the contribution due to
fluctuations in the number of electron-hole pairs produced by increasing
$\epsilon$ (see Fig. 6).

The leakage current also varies with the reverse bias applied to
the crystal. The bias voltage must be high enough to accelerate the
electrons and holes away from each other before they are able to reunite.
Unless great care is taken in the preparation of the crystal and in keep­
ing the surface of the crystal free from contaminates, the term involv­
ing the leakage current will dominate over all the others and will effect­
villy set the minimum resolution obtainable.

Since the crystal is usually cooled much below room temperature,
it must be placed in a vacuum chamber to insure that no condensation
occurs on the crystal surface, as this will add to the surface leakage
current. Fig. 15 shows the vacuum chamber used for cooling the crystal.

In summary, once the detector-analyser system has been chosen, there
are only three adjustments that can be made to improve the resolution.
They are

i Temperature of the crystal

ii Bias voltage

iii Time constants of the pulse shaper.

In order to determine the best resolution obtainable with the
detector-analyser system described in Section III - 2, the crystal
temperature was varied by changing the length of the cold-finger. At
each setting, the temperature was measured by copper-constantin thermo­
couples, and the resolution optimized by adjusting the bias voltage and
the time constants. The results of this procedure for the two crystals
FIG: 15.--

Crystal Chamber

to vacuum pump

Liquid Nitrogen Chamber

Cold Finger

Crystal

Removable Absorber
Source
Thin A window
FIG. 16. --

Plot of Temperature vs. Optimized Resolution for Two Detectors

Resolution (FWHM)

Temp. in °C

S_1 Detector

K_1 Detector

-190 -180 -170 -160 -150 -140 -130 -120 -110 -100 -90

-2.0 -3.0
used are shown in Fig. 16. The characteristics of these crystals are given in Table IV. The optimum resolution for the $S_1$ detector was reached at $T = 190^\circ C$, $V = 400$ volts, and time constants of $2 \mu\text{secs}$. The optimum resolution for the $K_1$ detector was reached at $T = -100^\circ C$, $V = 350$ volts and time constants of $2 \mu\text{secs}$. The peak used was the 121.9 Kev gamma peak of $^{57}\text{Co}$.

4. Other Contributions to the Resolution

Peaks due to monoenergetic particles may be broadened by effects other than the fluctuations in the response of the detector analyser system. These effects are source charging, source absorption, scattering from surrounding materials, and dead layer absorption.

Sources that emit electrons will quickly charge themselves to high (and sometimes fluctuating) potentials, thus altering the energy of the emitted electrons. The source must therefore be well grounded.

The effect of too thick a source is to degrade the energy of electrons coming from below the surface by collision losses (source absorption). This adds a low energy component, or tail, to the peak, resulting in a greater width. To keep this peak broadening to a minimum, the source must be kept thin, the maximum thickness depending on the source material and the electron energy. An example of peak broadening due to source absorption is shown in Fig. 17. The peak is the 114 Kev K conversion peak of $^{57}\text{Co}$. The $S_1$ detector was used to obtain both spectra.

Thin sources may be made in a number of different ways\(^1\), but those used in this experiment were prepared by subliming the source material onto a thin aluminum backing, as explained below. The sublimation chamber is shown in Fig. 18. A drop of a solution of the source material
FIG. 17.--
Effect of Source Thickness on Resolution

Peak is at 114.8 Kev

Thick Source

Thin Source

3.3 Kev

2.5 Kev
### Table IV

Characteristics of the Two Detectors

<table>
<thead>
<tr>
<th>Detector</th>
<th>Simtec type KQ2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obtained from Simtec Ltd. in October, 1968</td>
<td></td>
</tr>
<tr>
<td>Recommended reverse bias</td>
<td>400-800 volts</td>
</tr>
<tr>
<td>Thickness (W)</td>
<td>2 mm</td>
</tr>
<tr>
<td>Area (A)</td>
<td>50 mm²</td>
</tr>
<tr>
<td>Window (Dead Layer)</td>
<td>.2 microns</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Detector</th>
<th>Kevex type A80-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obtained from Kevex Corporation in February, 1968</td>
<td></td>
</tr>
<tr>
<td>Recommended reverse bias</td>
<td>300-700 volts</td>
</tr>
<tr>
<td>Thickness (W)</td>
<td>5 mm</td>
</tr>
<tr>
<td>Area (A)</td>
<td>80 mm²</td>
</tr>
<tr>
<td>Window (Dead Layer)</td>
<td>5 microns</td>
</tr>
</tbody>
</table>
FIG. 18.--

Sublimation Chamber

Glass viewer

Source backing

Source Material

Tungsten ribbon

Copper

To vacuum pump

Ceramic feed through

To pulser

Tungsten Ribbon

Source material
FIG. 19.--

Pulser Circuit

To Variac
(0 - 60 VAC)

To Tungsten Ribbon
is deposited on the tungsten ribbon. The material is then dried with an infra-red lamp and the chamber evacuated. A large current (≈10 Amps) is pulsed through the ribbon, and as a result a local hot-spot develops in the region of the droplet where the ribbon width has been reduced. The source material is sublimed off the ribbon and onto the aluminum backing. The thickness of the source can be varied by the number of current pulses used, the length of the pulses, or by the amount of source material deposited on the ribbon. The current pulse generator is shown in Fig. 19.

Scattering of electrons or photons from the chamber walls, source backing, source holder, etc. does not normally broaden a peak. Compton scattering through small angles can, on occasion, produce a peak that looks like a full energy peak. In most cases, however, the energy lost in the scattering processes leaves a residue that appears at considerably lower energies than the undistorted peak and is usually smeared over the lower energy background as a continuum. The exception to this is the case of low-energy photons, which may be scattered at large angles with only a slight loss in energy. From the Compton process one can deduce that

$$E'_Y = \frac{E_Y}{1 + \frac{E}{m c^2} (1 - \cos \Theta)}$$

where $E'_Y$ and $E_Y$ are the energies of the scattered and incident gamma rays respectively and $\Theta$ is the scattering angle. The cross-section for this process is very large for $E_Y < 100$ Kev and $20^\circ < \Theta < 100^\circ$. Hence large mass concentrations should be kept as far away from the source and detector as possible. Fig. 20(a) shows two types of source holders and Fig. 20(b) shows their effect on the 41 Kev x-ray peak of europium.
FIG. 20.--

Two Types of Source Holders and Their Effect on Low Energy Photon Resolution

Ring holder

a) Thin Al with source sublimed onto it

Al or lucite ring

Electrical contact

Thin Al

Cardboard

Cardboard holder

b)

Effects of Backscattering from Source Holder on 41 KeV X-Rays
FIG. 21 a).

Effects of Dead Layer on Low Energy Electrons

Counts

Channel No.

Electron Peak

$\gamma$-97.4

$\gamma$-103.2

K$_1$ Detector

Counts

Electron Peaks

$\gamma$-97.4

Electron peak

$\gamma$-103.2

S$_1$ Detector

Channel No.
Fig. 8 showed the energy loss of electrons passing through the dead layer of a Si(Li) detector. Figs. 21 shows the effect of the dead layers of the K\textsubscript{1} and S\textsubscript{1} detectors on a portion of the \textsuperscript{153}Gd electron spectrum. The spectrum taken with the K\textsubscript{1} detector (21(a)) shows the two gamma peaks at 97.4 and 103.2 Kev and a degraded electron peak at lower energy. The spectrum taken with the S\textsubscript{1} detector (21(b)) has three electron peaks, two of which are superimposed on the gamma peaks. The gamma spectrum by itself, taken with the electrons absorbed by a thin aluminium absorber, is included for comparison of the two spectra.
CHAPTER IV

ANALYSIS OF SPECTRA

1. Computer Analyses

A computer program was written to simplify the analysis of spectra obtained with the detector-analyser system described above. It is explained in detail in Appendix A. As stated there, a value of $\chi^2/(n-m) = 1$ is indicative of a good fit, although it was found experimentally that fits with values of $\chi^2/(n-m)$ between 0 and 3 still fit the function $\chi^2$ to the data very well. If $\chi^2/(n-m)$ was greater than 3, the fit was rerun using different portions of the total spectrum, as the goodness of fit is very sensitive to any non-linearity in the background. It was found that all the spectra analysed with this program could be fitted with a goodness of fit parameter, $\chi^2/(n-m)$, less than 3.

The program is capable of separating peaks that overlap; that is, peaks whose energy difference is less than the resolution of the system. Fig. 22 shows two such peaks. They are the $K_\alpha$ and $K_\beta$ x-ray peaks of silver that result from the electron capture decay of $^{109}$Cd. Their energies are 22.1 and 25.0 Kev respectively. The solid curve is a plot of the function fitted to this data. Figs. 23(a) and 23(b) are the components of this composite peak given by the computer program.

To check the accuracy of the separation of these peaks, the ratio of their intensities was found and compared with the accepted value of 4.64 given in reference 18. This ratio is

$$\frac{K_\alpha}{K_\beta} = (5.5 \pm .3)R$$
FIG. 22.--

- Experimental data
- Fitted function

Channel No.
FIG. 23.--
Components of Peak in Fig. 22

a) \( K_\alpha \) component

b) \( K_\beta \) component

Counts

Channel No.
R is the correction factor for the difference in efficiency of the \( S_1 \) detector for the two different energies. This correction factor is \( 0.9 \pm 0.05 \) (see Fig. 26). The ratio of their intensities is therefore \( 4.9 \pm 0.5 \), which is in agreement with the value quoted above.

Different values of gain of the amplifier system were used to determine the effect of gain on the uncertainty in the position of the peaks and on the reliability of the intensity estimates given by the fitting routine. The peak used was the 114.9 Kev K-conversion electron peak resulting from the electron capture decay of \( ^{57} \text{Co} \). The spectra of this peak were taken for the same length of time for each gain setting so the intensities would be the same. The results for the error in position and intensity are given in Table V.

<table>
<thead>
<tr>
<th>Gain (Kev/Channel)</th>
<th>Peak Uncertainty (Channel)</th>
<th>Peak Uncertainty (Kev)</th>
<th>Intensity Error (Percent.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>±.3</td>
<td>±.6</td>
<td>±5.6</td>
</tr>
<tr>
<td>1.0</td>
<td>±.2</td>
<td>±.2</td>
<td>±4.0</td>
</tr>
<tr>
<td>0.5</td>
<td>±.1</td>
<td>±.05</td>
<td>±3.0</td>
</tr>
<tr>
<td>0.3</td>
<td>±.1</td>
<td>±.03</td>
<td>±2.0</td>
</tr>
<tr>
<td>0.1</td>
<td>±.3</td>
<td>±.03</td>
<td>±2.0</td>
</tr>
</tbody>
</table>

From Table V it can be seen that a gain setting of 0.3 Kev/Channel will minimize the error in the intensity and position of a peak. Since the MCA has 128 channels the best results are obtained if the biased amplifier is set so that a range of 35 \( \rightarrow \) 45 Kev is covered by the MCA.
2. Energy Calibration

This work is mainly concerned with low energy electrons and photons; that is, with energies up to approximately 120 Kev. Three different biased amplifier settings are needed to cover this range of energies if the minimum error possible in peak position and intensity is to be obtained.

Each of these three settings was calibrated using as many photon sources of known energy as were available. The energy range of each setting, and the sources along with the energies of their photon peaks are listed in Table VI. The results of these calibrations are plotted in Figs. 24(a), (b), and (c).

A least square fit using a function of the form \( y = a + bx + cx^2 \) was done on the calibration data. The values of a and b found for each setting are given on the calibration curves (Figs. 24). The value of c was found to be at least a factor of \( 10^4 \) smaller than the value of b for each setting. Since the maximum error in ignoring the \( x^2 \) term is then approximately .04 Kev (which is less than the error in the energies of most calibration sources) the calibration curves were assumed linear. It should be noted that the errors associated with the values of a and b given on the calibration curves are only the statistical errors of the fitting procedure, and do not contain the possible errors in the quoted calibration energies.

The energy of conversion electron peaks can be obtained from these calibration curves if the energy lost by an electron in the dead layer of the detector is known. Simtec quotes a value for the dead layer of the \( S_1 \) detector of .2 microns. This value was checked using the conversion
<table>
<thead>
<tr>
<th>Source</th>
<th>Energy (Kev)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Setting 1</strong></td>
<td>Energy Range $6 \rightarrow 50$ Kev</td>
<td></td>
</tr>
<tr>
<td>$^{57}$Co</td>
<td>$14.35 \pm .05$</td>
<td>18</td>
</tr>
<tr>
<td>$^{109}$Cd ($K_\alpha$)</td>
<td>$22.1 \pm .1$</td>
<td>Ag x-rays</td>
</tr>
<tr>
<td>($K_\beta$)</td>
<td>$25.0 \pm .1$</td>
<td></td>
</tr>
<tr>
<td>$^{133}$Ba ($K_\alpha$)</td>
<td>$30.9 \pm .1$</td>
<td>Cs x-rays</td>
</tr>
<tr>
<td>($K_\beta$)</td>
<td>$35.1 \pm .1$</td>
<td></td>
</tr>
<tr>
<td>$^{153}$Gd ($K_\alpha$)</td>
<td>$41.3 \pm .1$</td>
<td>Eu x-rays</td>
</tr>
<tr>
<td>($K_\beta$)</td>
<td>$47.3 \pm .1$</td>
<td></td>
</tr>
<tr>
<td><strong>Setting 2</strong></td>
<td>Energy Range $38 \rightarrow 85$ Kev</td>
<td></td>
</tr>
<tr>
<td>$^{153}$Gd ($K_\alpha$)</td>
<td>$41.3 \pm .1$</td>
<td>Eu x-rays</td>
</tr>
<tr>
<td>($K_\beta$)</td>
<td>$47.3 \pm .1$</td>
<td></td>
</tr>
<tr>
<td>$^{241}$Am</td>
<td>$59.54 \pm .02$</td>
<td>18</td>
</tr>
<tr>
<td>$^{133}$Ba</td>
<td>$81.0 \pm .2$</td>
<td>22</td>
</tr>
<tr>
<td><strong>Setting 3</strong></td>
<td>Energy Range $75 \rightarrow 128$ Kev</td>
<td></td>
</tr>
<tr>
<td>$^{133}$Ba</td>
<td>$81.0 \pm .2$</td>
<td>22</td>
</tr>
<tr>
<td>$^{154}$Eu</td>
<td>$86.9 \pm .2$</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>$105.3 \pm .2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$122.9 \pm .2$</td>
<td></td>
</tr>
<tr>
<td>$^{57}$Co</td>
<td>$121.91 \pm .05$</td>
<td>18</td>
</tr>
</tbody>
</table>
FIG. 24 a).

Setting #1

\[ a = 6.09 \text{ Kev} \pm 0.01 \]
\[ b = 0.3756 \text{ Kev/Ch.} \pm 0.0005 \]

Energy (Kev) = 0.3756 (Channel No.) + 6.09
FIG. 24 b).

Setting #2

Energy (Kev) = 0.3631 (Channel No.) + 38.33

a = 38.33 Kev $\pm$ 0.01
b = 0.3631 Kev/Ch. $\pm$ 0.0005

153 Gd $\gamma$
241 Am
133 Ba

Energy (Kev)

Channel No.
FIG. 24 c)---

Setting #3

$E_{\text{Kev}} = 0.4063 \times \text{Channel No.} + 75.22$

$a = 75.22 \pm 0.01$

$b = 0.4063 \pm 0.0005$
electron standards listed in Table VII. The apparent energies of their peaks were found from Figs. 24, and the difference between these energies and those given by Reference 18 was used to deduce the thickness of the dead layer (see Fig. 8).

Table VII

<table>
<thead>
<tr>
<th>Source</th>
<th>Energy (Kev) from Ref. 18</th>
<th>Energy (Kev) from Fig. 24</th>
<th>E (Kev)</th>
<th>Dead Layer (microns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{109}\text{Cd}$ ($K_{87.7}$)</td>
<td>$62.2 \pm 0.2$</td>
<td>$61.9 \pm 0.1$</td>
<td>$0.3 \pm 0.3$</td>
<td>$0.3 \pm 0.3$</td>
</tr>
<tr>
<td>$^{109}\text{Cd}$ ($L_{87.7}$)</td>
<td>$84.2 \pm 0.2$</td>
<td>$84.1 \pm 0.1$</td>
<td>$0.1 \pm 0.3$</td>
<td>$0.1 \pm 0.3$</td>
</tr>
<tr>
<td>$^{57}\text{Co}$ ($K_{122.0}$)</td>
<td>$114.9 \pm 0.2$</td>
<td>$114.6 \pm 0.1$</td>
<td>$0.3 \pm 0.3$</td>
<td>$0.3 \pm 0.3$</td>
</tr>
<tr>
<td>$^{57}\text{Co}$ ($K_{136.3}$)</td>
<td>$129.2 \pm 0.2$</td>
<td>$129.2 \pm 0.1$</td>
<td>$0.0 \pm 0.3$</td>
<td>$0.0 \pm 0.3$</td>
</tr>
</tbody>
</table>

Averaging these values gives a dead layer of $0.175 \pm 0.3$ microns, which is in agreement with the value of $0.2$ microns quoted by Simtec. Since better electron standards were not available, it was impossible to find the thickness more accurately. The value of $0.2$ microns was therefore used to find the correction to the electron energies.

It was found that the position of the calibration peaks could change by as much as one channel due to changes in electronic gain. To guard against this happening while a spectrum was being taken, calibration runs were performed before and after each spectrum containing peaks of unknown energies was taken. If the shift in the calibration peaks was greater than $0.2$ channels, the data was discarded and the spectrum taken again. For each spectrum used, new values of a and b were calculated for the
calibration curves.

The maximum possible error in the energy of unknown peaks was estimated by summing the different errors involved. The error in peak position from the fitting procedure ranged from .03 keV for intense peaks to .2 keV for weak peaks. The error in the calibration curves was estimated to be a maximum of .2 keV (.15 keV for the error due to energy uncertainty of the calibration peaks and .05 keV for their position uncertainty). The maximum error in electron energy due to an error in the value used for the thickness of the dead layer of the detector was .24 keV for 100 keV electrons and .5 keV for 20 keV electrons (assuming a maximum error in the thickness of .3 microns). The maximum error in the energies of unknown peaks is therefore .4 keV for photons and .6 and .9 keV for 100 and 20 keV electrons respectively. The actual error in measured energies is probably much less than these values for all but very low intensity peaks.

3. Separation of Electron and Photon Spectra

Quite often electron and photon peaks overlap in a spectrum. It is convenient therefore to be able to separate the electron and photon spectra. The procedure used was as follows: First a total spectrum containing both electron and photon peaks was taken. An aluminum absorber thick enough to stop all electrons\(^1\) was then placed between the source and detector and a spectrum containing only photon peaks was taken. The intensities of the photon peaks was corrected for the absorber and subtracted from the total spectrum, leaving only peaks due to electrons.

The photon peaks were corrected for the absorber by an empirical equation of the form
\[ I = I_0 \exp \left( A / (E - E_t)^B \right) \]

Here \( A, B, \) and \( E_t \) are constants that are varied to fit experimental absorption data, \( I \) and \( I_0 \) are the reduced and initial intensities respectively, and \( E \) is the photon energy. Fig. 25 is a plot of this function along with the experimental absorption data. The values \( A = 5, \) \( E_t = 7, \) and \( B = 1.5 \) give a satisfactory fit to the experimental data.

It should be noted that this correction function cannot be used to correct the total photon spectrum for the absorber. That is, one cannot multiply the counts found in each channel by the value of \( I/I_0 \) at the energy corresponding to that channel and get the same spectrum as one would find without the absorber. The reason is that the monoenergetic photons that pass through the absorber without losing any energy (fraction \( I/I_0 \)) will have a spread in energy due to resolution and a low energy continuum resulting from Compton scattering. Therefore if one applied this correction continuously the peaks would be distorted as the correction on the lower energy side would be too large and the correction on the higher energy side would be too small. The lower energy continuum would also be much too large. Therefore the value of \( I/I_0 \) corresponding to the energy of each peak was found and this value was used to correct the data over that peak region after the background from higher energy photons was subtracted. Only the peaks without background were subtracted from the total spectrum. The result is that the electron spectrum contains the background from photons as well as the background from electrons.

4. Efficiency Calibrations

The efficiency of the Si detector for photons as a function of photon energy was found using standard sources obtained from I.A.E.A.,
FIG. 25.--

Absorption Curve for Photons in Aluminum Absorber

Calculated from $\frac{I}{I_0} = \exp\left(5/(E-7)^{1.5}\right)$

* - experimental points
Vienna. The relevant characteristics of these sources are listed in Table IX. The sources are heat sealed between two thin plastic discs and cold-welded between two thin aluminum discs. The fraction of the intensity of the photon peaks absorbed by this encapsulation was found by measuring the decrease in intensity when one side of a dummy (no source) capsule was placed between the source and the detector. These fractions were used to correct the number of photons radiated into the solid angle of the detector \((8.2 \times 10^{-3} \text{ Std})\) per unit time for each calibration peak. These intensities are included in Table IX.

The ratio of the actual intensity per unit time of a given photon peak as measured with the \(S_1\) detector to the calculated intensity for that peak is the intrinsic efficiency of this detector for that photon peak's energy. This ratio is plotted in Fig. 26 for the five energies listed in Table IX. The low energy portion of this curve \((<30 \text{ Kev})\) is extrapolated to match curves given in reference 12.

The efficiency of the \(S_1\) detector for electrons was assumed constant (see Chapter III - 1). It has not been measured for this detector. A value of \(80 \pm 5\%\) is assumed, which is in agreement with measurements made by various groups\(^{23,24,25}\). Low energy electron standards have only recently become available to this laboratory and this efficiency will be measured in the near future.

The error in the photon efficiency curve is estimated to be \(\pm 5\%\). This value is the sum of the errors in the source strength, in the fraction of total radiation represented by a peak, and in the measured intensity of that peak. The total error may be larger than 5% as any error in the calculated solid angle subtended by the detector has not been included.
Table VIII
Photon Efficiency Calibration Data

<table>
<thead>
<tr>
<th>Source</th>
<th>Strength* ((\text{mc}))</th>
<th>(t_{\frac{1}{2}})</th>
<th>Energy of Cal. peak (Kev)</th>
<th>Percent of disintegration</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{137}\text{Cs})</td>
<td>10.4 ± .2</td>
<td>30.5 years</td>
<td>32.1 ± .1</td>
<td>5.7 ± .2</td>
</tr>
<tr>
<td>(^{241}\text{Am})</td>
<td>10.4 ± .1</td>
<td>432.9 years</td>
<td>59.54 ± .02</td>
<td>35.9 ± .6</td>
</tr>
<tr>
<td>(^{203}\text{Hg})</td>
<td>20.3 ± .2</td>
<td>46.8 days</td>
<td>72.873 ± .001</td>
<td>9.7 ± .5</td>
</tr>
<tr>
<td>(^{57}\text{Co})</td>
<td>11.4 ± .1</td>
<td>271.6 days</td>
<td>121.97 ± .03</td>
<td>85.0 ± 1.7</td>
</tr>
</tbody>
</table>

*at January 1, 1970; 00:00 Universal time

<table>
<thead>
<tr>
<th>Source</th>
<th>Energy (Kev)</th>
<th>Absorption (%) by capsule</th>
<th>Intensity** (photons/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{137}\text{Cs})</td>
<td>32.1 ± .1</td>
<td>6.5</td>
<td>1300</td>
</tr>
<tr>
<td>(^{241}\text{Am})</td>
<td>59.54 ± .02</td>
<td>3.8</td>
<td>8974</td>
</tr>
<tr>
<td>(^{203}\text{Hg})</td>
<td>72.873 ± .001</td>
<td>2.8</td>
<td>28188</td>
</tr>
<tr>
<td>(^{57}\text{Co})</td>
<td>121.97 ± .03</td>
<td>2.1</td>
<td>23918</td>
</tr>
<tr>
<td>(^{203}\text{Hg})</td>
<td>279.19 ± .008</td>
<td>1.4</td>
<td>21059</td>
</tr>
</tbody>
</table>

**Intensity calculated to be radiated into the solid angle subtended by the detector, after corrections have been made for the life-time of the source and absorption by the capsule.
FIG. 26.

Photon Efficiency of $S_1$ Detector

% vs. Kev

- $^{137}\text{Cs}$
- $^{241}\text{Am}$
- $^{203}\text{Hg}$
- $^{57}\text{Co}$
- $^{203}\text{Hg}$
CHAPTER V

DECAY OF $^{153}$GADOLINIUM

1. Introduction

The electron capture decay of $^{153}$Gd to $^{153}$Eu has been investigated previously by a variety of techniques $^{26-30}$. The decay scheme deduced by Lederer et al. $^{18}$ from these measurements is given in Fig. 27. There is a disagreement between the values given by different authors for the capture branching ratios, especially to the ground state. It was therefore decided that a re-investigation of this decay using the Si(Li) spectrometer described in the previous chapters would be informative.

2. Source Preparation

The $^{153}$Gd source material was obtained from Union Carbide Corp. Its chemical form was GdCl$_3$ dissolved in 1 N. HCl. The specific activity of $^{153}$Gd in the material was 5.46 millicuries/milligram.

Source material was sublimed onto a thin aluminum backing (0.8 mg/cm$^2$) in the manner described earlier. The maximum thickness of source material that would not degrade the low energy electron peaks by source absorption was found experimentally by preparing successively thinner sources until no improvement in peak shape was noted. This thickness was estimated to be $\approx 10$ $\mu$gm/cm$^2$ from the specific activity (5.46 m Ci/mgm), the source strength ($\approx 50$ $\mu$Ci), and the source area ($\approx 1.5$ cm$^2$). The aluminum backing was mounted on cardboard (see Fig. 20) to reduce backscatter.

Since GdCl$_3$ is hygroscopic, sources must be kept in a vacuum, in a desiccator, or covered with collodion $^{22}$ so that they do not absorb
FIG. 27.--
Main Transitions in the Decay of $^{153}\text{Gd}$

$^{5/2^+}$

$J^\pi$ Kev

$5/2^-$, 97.4

$7/2^+$

$3/2^+$

$153\text{Gd}$

$14\% ft = 5.8 E_0 = 70 \text{ Kev}$

$41\% ft = 6.5 E_0 = 139.8 \text{ Kev}$

$36\% ft = 6.6 E_0 = 145.6 \text{ Kev}$

$0.6\% ft = 8.4 E_0 = 159.6 \text{ Kev}$

$9\% ft = 7.8 E_0 = 243 \text{ Kev}$
moisture from the atmosphere and therefore become thicker. The sources
could not be covered with collodion as this would absorb energy as the
electrons passed through it. They were therefore stored in a desiccator
when not in the crystal chamber.

3. $^{153}$Gd Spectra

The total spectrum and the photon spectrum of $^{153}$Gd for each of the
three biased amplifier settings, along with the energy calibration data,
were taken with the Si$_1$ detector. The intensity of the peaks in the photon
spectra were corrected for the absorber (see Chapter IV) and subtracted
from the total spectra to get the three electron spectra. These three
spectra were normalized to the same time (the less intense regions were
taken over longer periods of time), matched in energy using the overlap
between the settings, and then plotted as one composite spectrum in Fig.
28. The same matching procedure was used to obtain the composite photon
spectrum (Fig. 29).

The energies of the peaks in these two spectra were found using the
energy calibration data and the channel position of the peaks as given by
the computer fitting program. The energies of the electron peaks were
corrected for the absorption in the dead layer of the Si$_1$ detector. These
energies are included in Figs. 28 and 29.

The origins of the electron peaks were deduced by subtracting the
binding energies of the K-, L-, or M+N- shell atomic electrons of europium
from the energies of the gamma peaks and comparing these values with the
energies of the electron peaks. The origins of the gamma peaks were
deduced by comparing their energies with the transition energies given on
the decay scheme (Fig. 27). Both the electron and photon peaks, along
FIG. 28.—

Conversion Electron Spectrum of $^{153}$Gd

Setting 1

Setting 2

Setting 3

Counts ($10^3$)

Channel No.

21.1
36.5
48.9
54.6
61.9
68.1
89.5
95.4
101.5

2 4 6 8 10 12 14 16 18 20 22 24 26

20 40 60 80 100 120 140 160 180 200 220 240 260
FIG. 29.

Photon Spectrum of $^{153}$Gd

Counts $(10^3)$

<table>
<thead>
<tr>
<th>Channel No.</th>
<th>Setting 1</th>
<th>Setting 2</th>
<th>Setting 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>41.3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>47.3</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>64.8</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td>97.4</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td>103.2</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>160</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>180</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>220</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>240</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak Energy (Kev)</td>
<td>Peak Origin</td>
<td>Intensity (Counts)</td>
<td>S1 Detector Eff. (%)</td>
</tr>
<tr>
<td>------------------</td>
<td>-------------</td>
<td>--------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>21.1</td>
<td>K\textsubscript{69.7}</td>
<td>13107 ± 7.3%</td>
<td>80 ± 5</td>
</tr>
<tr>
<td>36.5</td>
<td>Auger\textsuperscript{1}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>48.9</td>
<td>K\textsubscript{97.4}</td>
<td>10516 ± 5.3%</td>
<td>80 ± 5</td>
</tr>
<tr>
<td>54.6</td>
<td>K\textsubscript{103.2}</td>
<td>37702 ± 2.3%</td>
<td>80 ± 5</td>
</tr>
<tr>
<td>61.9</td>
<td>L\textsubscript{69.7}</td>
<td>1866 ± 12.3%</td>
<td>80 ± 5</td>
</tr>
<tr>
<td>68.1</td>
<td>M+N\textsubscript{69.7}</td>
<td>200 ± 40%</td>
<td>80 ± 5</td>
</tr>
<tr>
<td>89.5</td>
<td>L\textsubscript{97.4}</td>
<td>1894 ± 4.8%</td>
<td>80 ± 5</td>
</tr>
<tr>
<td>95.4</td>
<td>L\textsubscript{103.2} + M+N\textsubscript{97.4}</td>
<td>7789 ± 3.1%</td>
<td>80 ± 5</td>
</tr>
<tr>
<td>101.5</td>
<td>M+N\textsubscript{103.2}</td>
<td>1886 ± 4.8%</td>
<td>80 ± 5</td>
</tr>
<tr>
<td>41.3</td>
<td>K x-rays</td>
<td>51230 ± 1.5%</td>
<td>30.6± 1.5</td>
</tr>
<tr>
<td>47.3</td>
<td>K x-rays</td>
<td>7943 ± 3%</td>
<td>18.6± .9</td>
</tr>
<tr>
<td>69.8</td>
<td>172.9 103.2</td>
<td>298 ± 15%</td>
<td>5.8± .3</td>
</tr>
<tr>
<td>97.4</td>
<td>97.4 0</td>
<td>1810 ± 3.6%</td>
<td>2.0± .1</td>
</tr>
<tr>
<td>103.2</td>
<td>103.2 0</td>
<td>1056 ± 4.1%</td>
<td>1.75± .09</td>
</tr>
</tbody>
</table>

*after the M\textsubscript{97.4} conversion electrons were subtracted.
with their deduced origins, are listed in Table X.

The peak intensities found by the fitting program, and their intensities corrected for the detector efficiency, are included in Table X. The contribution to the intensity of the 95.4 Kev electron peak due to the M conversion electrons from the 97.4 Kev transition was calculated using the M conversion coefficients of Rose and assuming that the transition was E1. This contribution was subtracted to give the intensity of L\textsubscript{103.2} conversion electrons. Intensities of peaks resulting from other transitions were too weak to be observed above the background.

4. Conversion Coefficients

The K-conversion coefficients and the L-conversion coefficients (Table XI) were calculated and compared to the theoretical coefficients given by Sliv and Band. The multipolarities of the transitions can be assigned from these comparisons. Comparison of the K-conversion coefficients require that the 69.7 Kev transition be a mixture of M\textsubscript{1} and E\textsubscript{2}, the 97.4 Kev transition be E\textsubscript{1}, and the 103.2 Kev transition be a mixture of M\textsubscript{1} and E\textsubscript{2}. The L-conversion coefficients are in agreement with these assignments.

The K- and L-conversion coefficients contain the rather large uncertainties in the efficiency of the S\textsubscript{1} detector for photons and electrons. The ratio of these coefficients should be more exact as these efficiencies cancel out. That is, the ratio $\alpha_K/\alpha_L$ is equal to the number of K-conversion electrons from a transition divided by the number of L-conversion electrons from that transition, as measured by the detector. These ratios are given in Table XII along with the theoretical values of $\alpha_K/\alpha_L$. The percentage $E2$ admixture in the 69.4 Kev and 103.2 Kev transitions was calculated from these ratios to be 0% $\rightarrow$ 20% and 16% $\rightarrow$ 32%.
### Table X

**K-Conversion Coefficients** ($\alpha_K$)

<table>
<thead>
<tr>
<th>Transition Energy (Kev)</th>
<th>Measured Values</th>
<th>El</th>
<th>Theoretical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>4.6</td>
<td>2.85</td>
</tr>
<tr>
<td>69.7</td>
<td>4.2 ± 1.3</td>
<td>4.6</td>
<td>2.85</td>
</tr>
<tr>
<td>97.4</td>
<td>0.2 ± 0.04</td>
<td>1.6</td>
<td>1.2</td>
</tr>
<tr>
<td>103.2</td>
<td>1.02 ± 0.17</td>
<td>1.43</td>
<td>1.07</td>
</tr>
</tbody>
</table>

**L-Conversion Coefficients** ($\alpha_L$)

<table>
<thead>
<tr>
<th>Transition Energy (Kev)</th>
<th>Measured Values</th>
<th>El</th>
<th>Theoretical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>6.67</td>
<td>5.34</td>
</tr>
<tr>
<td>69.7</td>
<td>0.60 ± 0.22</td>
<td>0.09</td>
<td>5.34</td>
</tr>
<tr>
<td>97.4</td>
<td>0.035 ± 0.007</td>
<td>0.25</td>
<td>0.83</td>
</tr>
<tr>
<td>103.2</td>
<td>0.18 ± 0.03</td>
<td>0.209</td>
<td>0.86</td>
</tr>
</tbody>
</table>

### Table XI

**$\alpha_K / \alpha_L$**

<table>
<thead>
<tr>
<th>Transition Energy (Kev)</th>
<th>Measured Values</th>
<th>El</th>
<th>Theoretical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>-</td>
<td>7.07</td>
</tr>
<tr>
<td>69.7</td>
<td>7.0 ± 1.2</td>
<td>-</td>
<td>7.07</td>
</tr>
<tr>
<td>97.4</td>
<td>5.6 ± 1.1</td>
<td>6.67</td>
<td>-</td>
</tr>
<tr>
<td>103.2</td>
<td>5.3 ± 0.6</td>
<td>-</td>
<td>6.84</td>
</tr>
</tbody>
</table>

respectively. The admixture of E2 in the 103.2 Kev transition is consider-
ably higher than the value of 1.7% given by reference 18. The 97.4 Kev
transition is assumed to be pure E1 as the next allowed multipole has a
transition probability that is reduced by a factor of \((kR)^4\) (see Table II).

5. Capture Branching Ratios

The fraction of total capture transitions that go to a given level in
the daughter nuclei in electron capture decay is called the capture branch-
ing ratio to that level.

Defining the terms

\[ N_E = \text{Number of capture transitions to the excited state at energy} \]
\[ E \text{ above the ground state in } ^{153}\text{Eu} \]

\[ I_e(E_\nu) = \text{Intensity of the } e = K, L, M+N+ \text{ conversion electrons from} \]
\[ \text{the } E_\nu \text{ transition in } ^{153}\text{Eu} \]

\[ I_\gamma(E_\nu) = \text{Intensity of the gammas from the } E_\nu \text{ transition in } ^{153}\text{Eu}. \]

Then from the intensities given in Table X the number of capture
transitions to the excited levels of \(^{153}\text{Eu}\) are

\[ N_{172.9} = I_\gamma (69.7) + I_K (69.7) + I_L (69.7) + I_{M+N+} (69.7) \]
\[ = (2.33 \pm .32) \times 10^4 \]

\[ N_{103.2} = I_\gamma (103.2) + I_K (103.2) + I_L (103.2) + I_{M+N+} (103.2) \]
\[ - N_{172.9} = (8.1 \pm 1.2) \times 10^4 \]

\[ N_{97.4} = I_\gamma (97.4) + I_K (97.4) + I_L (97.4) + I_{M+N+} (97.4) \]
\[ = (8.15 \pm .71) \times 10^4 \]

\(N_{172.9}\) was subtracted from the total radiation from the 103.2 Kev
transition to get \(N_{103.2}\) as the 172.9 Kev level feeds the 103.2 level via
the 69.4 Kev transition.

The number of capture transitions to the ground state of $^{153}$Eu can be deduced from the intensity of the K x-rays. This calculation requires that the fraction of K-capture to $L+M+N-$capture transitions is known, that the number of K x-rays resulting from transitions other than K-capture to the ground state of $^{153}$Eu is known, and that the K-fluorescent yield ($W_K$) of europium is known.

The K-fluorescent yield is the fraction of vacancies in the K-atomic shell that result in K x-rays (the rest give rise to Auger electrons). The K-fluorescent yield of europium is $0.92^{18}$.

The intensity of K x-rays resulting from transitions other than K-capture to the ground state is the sum of the K-conversion electrons from all transitions and the K-capture transitions to all other levels, multiplied by the K-fluorescent yield. If all the capture transitions are assumed to be allowed or first forbidden non-unique ($\Delta J = 0, \pm 1$), then the fraction of the total capture transitions that are K-captures can be calculated from probability ratios given by Brysk and Rose$^{33}$ and by Zyryanova$^{21}$. These ratios are

$$\frac{\omega_{L_I}}{\omega_K} = 0.128 \frac{Q - E - B_{L_I}}{Q - E - B_K}^2$$

$$\frac{\omega_{L_{II}}}{\omega_{L_I}} = 0.041$$ (ignoring the difference in the L shell binding energy)

$$\frac{\omega_{M+N}}{\omega_{L_I} + \omega_{L_{II}}} = 0.229 \frac{(Q - E - B_{M+N})^2}{Q - E - B_L}$$

$L_{III}$ capture is not allowed.
In these equations $\omega_e$ is the probability of an electron in the $e$ atomic shell being captured, $Q$ is the energy difference between the ground states of $^{153}\text{Gd}$ and $^{153}\text{Eu} (= 243 \text{ Kev})$, $E$ is the energy of the excited state the capture transitions is going to, and $B_e$ is the binding energy of the $e$ atomic shell in Eu.

The number of K-capture transitions to any level in $^{153}\text{Eu}$ is

$$N_e(K) = N_E / \left(1 + \frac{\omega_{L\text{I}}}{\omega_K} (1 + \frac{\omega_{L\text{II}}}{\omega_{L\text{I}}})(1 + \frac{\omega_{M+N}}{\omega_L})\right)$$

The number of K x-rays that do not result from the K-capture to the ground state is

$$N(K \text{x-ray}) = (N_{172.9}(K) + N_{103.2}(K) + N_{97.4}(K) + I_K(69.7) + I_K(97.4) + I_K(103.2))W_K$$

$$= (19.2 \pm 2.1) \times 10^4$$

The number of K x-rays resulting from the K-capture transitions to the ground state is

$$N_{K\text{x-ray}} + N_{K\beta} - N(K \text{x-ray}) = (1.6 \pm 1.6) \times 10^4$$

The number of K-capture transitions to the ground state is

$$N_0(K) = \frac{16200}{W_K} = (1.8 \pm 1.8) \times 10^4$$

and the number of capture transitions to the ground state is

$$N_0 = N_0(K) \left(1 + \frac{\omega_{L\text{I}}}{\omega_K} (1 + \frac{\omega_{L\text{II}}}{\omega_{L\text{I}}})(1 + \frac{\omega_{M+N}}{\omega_L})\right)$$

$$= (2.3 \pm 2.3) \times 10^4$$

The total number of capture transitions to all levels in $^{153}\text{Eu}$ is therefore...
\[ N = N_{172.9} + N_{103.2} + N_{97.4} + N_0 \]

\[ = (2.09 \pm 0.25) \times 10^5 \]

The capture branching ratios are therefore given by \( \varphi_E = \frac{N_E}{N} \).

These values, expressed as percentages, are given in Table XIII, along with those found by other investigators.

Table XII

<table>
<thead>
<tr>
<th>( \varphi )</th>
<th>Measured ( \log ft ) values in brackets</th>
<th>Ref. 26</th>
<th>Leutz (1960) from Ref. 26</th>
<th>Ref. 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>172.9</td>
<td>11 \pm 3 (5.8)</td>
<td>17</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>103.2</td>
<td>39 \pm 10 (6.6)</td>
<td>32</td>
<td>50</td>
<td>32</td>
</tr>
<tr>
<td>97.4</td>
<td>39 \pm 8 (6.6)</td>
<td>40</td>
<td>35</td>
<td>30</td>
</tr>
<tr>
<td>0</td>
<td>11 \pm 11 (7.3)</td>
<td>9</td>
<td>4</td>
<td>24</td>
</tr>
</tbody>
</table>

The reduced lifetimes (ft values) of the capture transitions can be calculated using these capture branching ratios and the electron capture probabilities given by Zyryanova. They are

\[
ft = (1.683 \, (Q - E - B_K)^2 + 0.2196 \, (Q - E - B_{LI})^2 + 0.0092 \, (Q - E - B_{LII})^2) \, \frac{\ln \frac{2}{\lambda_E}}{\lambda_E}
\]

where the energies are in units of \( m_0c^2 \),

\[ \lambda_E = N_E \lambda / N \] is in sec\(^{-1}\), and

\( \lambda \) is defined by \( t_\frac{1}{2} = \frac{\ln 2}{\lambda} = 242 \) days. The logarithm of these values are given in Table XIII. These log ft values indicate that the
electron capture transitions are allowed or first forbidden.

6. Spin and Parity Assignments

The ground states of $^{153}\text{Eu}$ and $^{153}\text{Gd}$ have been assigned the values of $J^\pi = 5/2^+$ and $3/2^+$ respectively by atomic beam experiments. The 103.2 and the 69.7 Kev transitions in $^{153}\text{Eu}$, as they are $M1^+ (E2)$ in character, require that $\Delta J = 0, \pm 1; \Delta \pi = 1$. The 97.4 Kev transition, being $E1$ in character requires that $\Delta J = 0, \pm 1; \Delta \pi = -1$. These requirements give the parity of the 103.2 and 172.9 Kev levels of $^{153}\text{Eu}$ as positive and the parity of the 97.4 Kev level as negative.

Therefore there is no parity change between the ground state of $^{153}\text{Gd}$ and the 172.9, 103.2 and 0 Kev levels of $^{153}\text{Eu}$. The capture transitions to these levels is therefore assumed to be allowed ($\Delta J = 0, \pm 1; \Delta \pi = 1$) as the next type of transitions that does not have a parity change is second forbidden with a log $ft > 10$. The capture transition to the 97.4 Kev level of $^{153}\text{Eu}$ involves a parity change and is therefore assumed to be first forbidden ($\Delta J = 0, \pm 1, \pm 2; \Delta \pi = -1$), which is in agreement with the log $ft$ value of 6.6.

The possible $J^\pi$ values of the excited states of $^{153}\text{Eu}$ are therefore

\begin{align*}
97.4 \text{ Kev} & \quad J^\pi = 3/2^-, 5/2^-, 7/2^- \\
103.2 \text{ Kev} & \quad J^\pi = 3/2^+, 5/2^+ \\
172.9 \text{ Kev} & \quad J^\pi = 3/2^+, 5/2^+
\end{align*}

These values of $J^\pi$ are in agreement with those given on Fig. 27.
CHAPTER VI

CONCLUSIONS

The spectrometer described in previous chapters has proved successful in investigating low energy transitions resulting from electron capture decays. Although its resolution does not compare favourably with that obtained by large magnetic spectrometers for electrons\(^2\), or bent crystal spectrometers for low energy gamma rays\(^{28}\), it is sufficient to separate the K-, L-, and M+N conversion electron peaks and the gamma ray peaks resulting from the excited state transitions associated with these decays. It has, however, three advantages over these spectrometers. Namely, that large portions of the energy spectrum can be taken at one time, that the electron and gamma ray spectra can be taken using the same source and the same geometry, and that the efficiency is considerably greater.

It should be noted that much better resolution (<0.4 Kev compared to 2 Kev) for low energy x-rays has been obtained using Si(Li) detectors\(^{34}\). To achieve this resolution the input stage of the preamplifier must be cooled with liquid nitrogen to reduce the electronic noise of the input stage. The crystal must have a small surface area and a large compensated region to reduce the input capacitance of the preamplifier. And the chamber must be evacuated to at least 10\(^-7\) Torr. to reduce condensation on the surfaces of the crystal to a minimum. It is felt that the problems involved with this type of system when sources or absorbers are changed (since they must be placed inside the chamber) would nullify the advantages of achieving better resolution.

The resolution of the type of spectrometer used can be improved by using germanium crystals instead of silicon as the average energy required
to produce an electron-hole pair in germanium (2.9 eV) is less than in silicon (3.7 eV). Germanium is not used, however, as these crystals must be kept at a temperature less than -30°C or the lithium ions will drift out of the compensated region. It is therefore very difficult to keep these crystals free of surface condensation when sources or absorbers are being changed.

The uncertainty in the intensity of photon peaks can be reduced by a better calibration of the efficiency of the detector. This has not been done yet as the calibration sources were not received until December 1969, and the electronic equipment has been in almost constant use in other experiments since that time.

The uncertainty in the intensity of electron peaks can be reduced by measuring the efficiency of the detector rather than using a value deduced from backscatter experiments done by other workers. The efficiency can be measured by detecting the K-conversion electrons that are in coincidence with the K x-rays of an electron capture decay that only involves one excited state. The electron spectrum will contain a peak due to full energy loss K-conversion electrons and a lower energy continuum due to the backscattered K-conversion electrons. The ratio of the intensity of the peak to the total intensity is the efficiency of the detector.

Once the backscatter fraction of the Si(Li) detector is known, it will be possible to obtain β spectra with this spectrometer and to correct these spectra for backscattering. It will also be possible to separate the different β groups involved in a decay by detecting those β particles that are in coincidence with the radiation from transitions that de-excite different levels of the daughter nucleus.
A new crystal chamber has been built with facilities for mounting two Si(li) detectors. This chamber will be used to measure the backscatter fraction of electrons using the coincidence method described above. It will also be used to determine the sequence of transitions involved in a given decay by measuring coincidences between radiation resulting from the different transitions.

This chamber has a liquid nitrogen capacity of 15 litres, and can keep the Si(Li) detectors cooled for approximately three days (compared to ≈3 hours for the chamber described in Chapter III) without refilling. This will allow longer runs to be taken without the constant attention needed previously. The chamber also has facilities for inserting absorbers between the source and detectors without opening the chamber, which would require warming the crystals to room temperature. It will therefore be much more suitable for investigating radioactive nuclei than the chamber described in this thesis.
APPENDIX A

Fitting Routine

The purpose of a least square fit is to find values of the parameters \( P_k \), which minimize the function

\[
R^2 = \sum_i a_i (y_i - \bar{y}_i (P_1, P_2, \ldots, P_m))^2 \quad (A - 1)
\]

In this equation the summation is over all data points. The \( y_i \)'s are the experimental data, the \( a_i \)'s are the weights associated with the \( y_i \)'s, and the \( \bar{y}_i \)'s are the values of the function that is used to represent the data.

A necessary and sufficient condition for \( R^2 \) to be a minimum, as a function of the \( P_k \)'s, is

\[
\frac{d R^2}{d P_k} = 0 \quad \text{for all} \quad k.
\]

These equations can be solved exactly if \( \bar{y} (P_1, P_2, \ldots, P_m) \) is a linear function of the \( P_k \)'s. If not, no explicit solution exists. In this case, a method of linearization can be used. One such method is that of Gause\(^{31}\).

This method consists of linearizing the function \( \bar{y} \) with respect to a set of parameters \( \delta P_k \) by using a truncated Taylor's series. That is, if

\[
P_k = P^o_k + \delta P_k, \quad \text{where} \quad P^o_k \quad \text{is the initial estimate of the parameter} \quad P_k,
\]

then

\[
\bar{y} = \bar{y}^o + \sum_{k=1}^{m} \left( \frac{\partial \bar{y}}{\partial P_k} \right) \delta P_k + \text{higher derivatives}
\]

Here \( \bar{y} = \bar{y} (P_1, P_2, \ldots, P_m) \)

\[
\bar{y}^o = \bar{y}^o (P^o_1, P^o_2, \ldots, P^o_m)
\]
and the derivatives are evaluated at the initial estimates of the parameters. The function
\[ y = y^* + \sum_k \left( \frac{\partial y}{\partial P_k} \right)_0 \delta P_k \]
is a linear function of the \( \delta P_k \)'s, and
\[ R' = \sum_i a_i (y_i - y^*) - \sum_k \left( \frac{\partial y}{\partial P_k} \right)_0 \delta P_k \]
is a minimum when
\[ \frac{dR'^2}{d(\delta P_k)} = 0 \quad \text{for all } k. \quad (A - 3) \]
These equations can be solved for \( \delta P_k \) exactly and a new estimate for \( P_k (P_k^1 = P^*_k + \delta P_k) \) used in equation (A - 2). This iteration process can be continued until \( \delta P_k \leq \Delta_k \) for all \( k \) simultaneously. \( \Delta_k \) is usually chosen so that
\[ \bar{y}_i (P^*_1, P^*_2, \ldots, P^*_m) \approx y_i (P^*_1 + \Delta_1, P^*_2 + \Delta_2, P^*_m + \Delta_m) \]
for all \( i \).

Equations A - 3 can be written as a matrix equation.

Writing \( B = (B_1, B_2, \ldots, B_m) \)
\[ B_\ell = \sum_i a_i (y_i - y^*_i) \left( \frac{\partial y_i}{\partial P_\ell} \right)_0 \]
\[ C = \text{n x n matrix} \]
\[ C_{\ell k} = \sum_i w_i \left( \frac{\partial y_i}{\partial P_\ell} \right)_0 \left( \frac{y_i}{P_k} \right)_0 \]
\[ \delta P = (\delta P_1, \delta P_2, \ldots, \delta P_m) \]
Then equations A - 3 become

$$C \delta P = B$$

which has solutions

$$P = A B$$

where $A$ is the inverse of the matrix $C$.

The "sample standard deviation" in the final estimate of the parameter $P_k$ is

$$\sigma_k = \left| \frac{\chi^2}{n-m} A_{kk} \right|^{\frac{1}{2}}$$

where $\chi^2$ is the value of $R^2$ for the final parameters, $n$ is the number of data points, $m$ is the number of parameters, and $A_{kk}$ is the diagonal element of the matrix $A$.

The "goodness of fit" can be estimated by the quantity $\chi^2$. If $(a_i)^{-\frac{1}{2}}$ is a good estimate of the standard deviation in $y_i$, and the experimental points are actually represented by the function $\bar{y}$, $\chi^2$ will have an approximately chi-squared distribution with $n-m$ degrees of freedom and $\chi^2/n-m$ should then be approximately unity. Then if one puts $a_i^{-\frac{1}{2}} = (y_i)^{\frac{1}{2}}$ (the standard deviation in $y_i$), $\chi^2/n-m$ will be approximately unity if $\bar{y}_i \to y_i$ for all $i$.

The function used to represent portions of spectra containing peaks was the sum of Gaussian peaks superimposed on a linear background. That is

$$\bar{y}_i = \sum_{j=1}^{J} H_j \exp \left( 41n 2 \left( \frac{i - \omega_j}{\omega_j} \right)^2 \right) + a + ib$$

In this equation

$$J = \text{number of peaks in the portion of spectrum}$$
\( H_j \) = height of the \( j^{th} \) peak

\( i \) = channel number

\( \mu_j \) = position of the \( j^{th} \) peak in channels

\( \omega_j \) = FWHM of the \( j^{th} \) peak in channels

\( a + ib \) = linear background.

A computer program was written to fit this function to the experimental data using the method described above. The input requirements of the program are the number of channels, the counts in each channel, the number of peaks, the convergence criteria, and the initial estimates of \( \mu_j \) and \( \omega_j \). The program estimates initial values for \( H_j \), \( a \) and \( b \).

The output of the program contains the final estimates of the parameters \( \mu_j \), \( \omega_j \), \( H_j \), \( a \) and \( b \), along with their standard deviations, \( \sigma_k \). It also contains the value of \( \bar{y}_i \) at each channel, the value of \( \bar{y}_{ij} = H_j \exp(4 \ln 2 \left(\frac{i - \mu_j}{\omega_j}\right)^2) \) (the fitted peak minus background) for each channel, the intensity of each peak

\[
(I_j = (\frac{\pi}{4 \ln 2})^{\frac{1}{2}} H_j \omega_j)
\]

with its standard deviation, and the goodness of fit parameter, \( \chi^2/n-m \).
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