AN INVESTIGATION OF HIGH WAVE NUMBER TEMPERATURE
AND VELOCITY SPECTRA IN AIR

by

NOEL E. J. BOSTON

B.A. Sc., University of British Columbia, 1959

M.S., Texas A & M College, 1963

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Department of PHYSICS

The University of British Columbia
Vancouver 8, Canada

Date February 10, 1971

ABSTRACT

Turbulent velocity and temperature fluctuations in air were measured at a height of 4 meters over a tidal mud flat. Particular attention was focused on the high wave number, small scale region of the spectra of these fluctuations. The measurements of the velocity fluctuations were made with a constant temperature hot wire anemometer; the hot wire consisted of a platinum wire 5 μm in diameter and approximately 1 mm in length. Temperature fluctuations were measured with a platinum resistance thermometer which consisted of a platinum wire 0.25 μm in diameter and about 0.30 mm in length.

The velocity spectra results agree well with the classical results of Grant, Stewart and Moilliet (1962) and Pond, Stewart and Burling (1963). In addition, they extend the velocity spectrum in air to slightly higher wave numbers. Four "clean" spectra agree slightly better with the universal curve suggested by Nasmyth (1970) than with the classical results. This suggests that the one-dimensional Kolmogorov constant K' previously estimated may be too low. The mean value obtained with these data was 0.50.

The temperature spectra clearly show the shape of the one-dimensional temperature spectrum in air beyond the $-5/3$ region. These spectra show that in air there is no -1 region and that temperature and velocity spectra are very similar. The temperature spectrum falls off from the $-5/3$ slope at

slightly higher wave numbers than the velocity spectrum.

The value of the scalar constant K'_θ , which appears in the scalar $-5/3$ law, computed from these data was 0.81. Direct measurement was made of all parameters that enter into the calculation of it.

The coefficient of excess and skewness of derivatives of velocity and temperature signals were computed for sixteen cases. Large absolute values were obtained indicating the non-Gaussian character of high wave number, high Reynolds number turbulence. A wide range of values of both parameters was obtained. In general the coefficient of excess of the temperature derivative exceeds that of the velocity derivative. The effect of velocity sensitivity on the skewness of the temperature derivative signal was investigated and found to be non-negligible.

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I. INTRODUCTION

A. THE PROBLEM AND OBJECTIVES

During the past several years attempts have been made to measure the full one-dimensional spectrum of temperature fluctuations in the atmospheric boundary layer. Little difficulty was encountered in measuring low frequency (< 10 Hz), large scale (small wave number) temperature fluctuations but considerable difficulty was encountered in measuring high frequency, small scale (high wave number) temperature fluctuations. Neither the sensors nor the sensing systems that were used in the past were adequate. The first task of this research then was to construct a sensor with a sufficiently high frequency response and sufficiently small spatial resolution to measure high wave number temperature fluctuations in air and to incorporate it into a low noise electronic system of compatible response characteristics. The construction of platinum resistance wire sensors was embarked on even though most experimenters in the field believed that its required small diameter ($< 1 \mu\text{m}$) rendered the project impracticable.

The object was to determine the shape of the high wave number one-dimensional temperature spectrum and to evaluate the scalar constant, K'_0 , which appears in the expression for the universal $-5/3$ form of the temperature spectrum. During (and following) the development of the sensor and sensor

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system, experimental programs were initiated to measure high frequency temperature fluctuations in the atmospheric boundary layer. In order to evaluate K'_0 , information is required of the high wave number velocity spectrum which must be measured simultaneously. This opportunity was used to compare the high wave number velocity spectrum with previous measurements and recompute the one-dimensional Kolmogorov constant, K' , which appears in the well known $-5/3$ equilibrium form of the velocity spectrum.

Very few measurements of the statistics of high wave number velocity and temperature fluctuations have been made. Since such measurements yield insight into the physical nature of small scale turbulence, the coefficient of excess and kurtosis of derivatives of velocity and temperature signals were computed from these data.

In the course of this research several sets of measurements were made and analysed. Results from these previous measurements were used in designing the final experiment which culminated in obtaining data which provided the information necessary to meet the objectives of this research.

B. SIGNIFICANCE OF PROBLEM

The distribution of temperature in a turbulent field has a direct bearing on a number of problems in, and related to, geophysics since inhomogeneities in the atmosphere result in the scattering of sound waves and electromagnetic radiation. The important parameter in such scattering problems

is the refractive index, the variation of which is often related to the small scale structure of the temperature distribution. Further application of a knowledge of the small scale distribution of temperature is found in engineering disciplines such as chemical reactor systems, mixing of two fluids of equal density, and heat transfer.

The subject of heat transfer is especially significant to oceanography and meteorology. The direct estimate of sensible heat flux is given by the covariance (averaged product) of measured temperature fluctuations and vertical components of the simultaneous wind velocity fluctuations. The flux thus obtained can be compared with estimates from empirical relations which are based on profile measurements. The flux measurements usually attempt to cover the range of scales at which most of the flux occurs, which in practice requires frequency responses extending from about 0.001 Hz to 10 Hz (Miyake et al., 1970). Theoretical work on turbulence in shear flow on the other hand has been confined largely to considerations of high wave number turbulence and has not yet been entirely satisfactory in examining anisotropic scales. The objective in making high wave number turbulence measurements, as in the theory, is to achieve results that are characteristic of isotropic turbulence which are independent of how the turbulence is created.

II. PREVIOUS RESEARCH

A. THE VELOCITY SPECTRUM

The one-dimensional downstream velocity (energy) spectrum is defined so that

$$\bar{u}^2 = \int_0^{\infty} \phi(k) dk \quad (1)$$

where k is the radian wave number and u is the "downstream" component of velocity fluctuations. The bar indicates an ensemble average which is equivalent to a space or time average under certain assumptions (Lumley and Panofsky, 1964, p. 6 ff.) which are assumed to be satisfied provided the turbulence field is reasonably "stationary." The rate of energy dissipation is given by

$$\epsilon = 15\nu \int_0^{\infty} k^2 \phi(k) dk. \quad (2)$$

The function $k^2 \phi(k)$ is referred to as a dissipation spectrum and describes the distribution according to wave number of the rate of decay of turbulent energy to heat.

An important idea developed in modern turbulence theory is that of the energy cascade -- large scale motions are being constantly degraded into smaller and smaller scales. In the Kolmogorov (1941) theory, the idea of the energy cascade is applied seriously. The Kolmogorov theory should be applicable to all fields of turbulence provided the

Reynolds number (Re) is high enough (i.e. the ratio of inertial to viscous forces is large). A high Re guarantees that the wave number separation between the energy containing region of the spectrum and the viscous dissipation region is large. The small scale turbulence has lost its identity with the large scale generating forces and eventually becomes isotropic (Taylor, 1935).

The Kolmogorov ideas are contained in two hypotheses. The first states that for sufficiently high Re, statistical properties of locally isotropic scales are uniquely determined by the mean rate of dissipation of energy per unit mass, $\epsilon [(\text{cm/sec})^2 \cdot \text{sec}^{-1}]$, and kinematic viscosity $[\text{cm}^2 \text{ sec}^{-1}]$. Dimensional analysis then yields

$$\phi(k) = (\epsilon \nu^5)^{1/4} F(k/k_s) \quad (3)$$

for the form of the one-dimensional energy spectrum and

$$k^2 \phi(k) = k_s^2 (\epsilon \nu^5)^{1/4} (k/k_s)^2 F(k/k_s) \quad (4)$$

for the form of the energy dissipation spectrum. In these relations $k_s = (\epsilon/\nu^3)^{1/4}$ is the Kolmogorov wave-number, $1/k_s$ is known as the Kolmogorov microscale. Provided Re is high enough, $F(k/k_s)$ should be a universal function valid for all fields of turbulence. The results of Grant, Stewart and Moilliet (1962) from work in a tidal channel and Pond, Stewart and Burling (1963) working in the atmosphere bear this out. When their data are plotted on universal non-dimensional axes

the tidal channel measurements are indistinguishable from the atmospheric boundary layer measurements (Figure 1).

The second Kolmogorov hypothesis states that if Re is sufficiently large, there should exist a range of scales which are isotropic and in a local steady state but for which viscosity is not important. The statistical properties of the turbulence are determined by ϵ alone. Dimensional analysis then lead to the now well-known law of local isotropy

$$\phi(k) = K' \epsilon^{2/3} k^{-5/3} \quad (5)$$

where K' is an absolute constant. Again the results of Grant, Stewart and Moilliet, and Pond, Stewart and Burling (Figure 1) confirm this as their data exhibit a long region where the slope is $-5/3$ on the log-log plot. In fact the $-5/3$ region extends to anomalously low wave numbers where the turbulence could not possibly be isotropic. These and subsequent measurements by Weiler and Burling (1968) have led to a re-evaluation of ideas on local isotropy which is currently being carried out at the Institute of Oceanography, University of British Columbia (IOUBC).

According to Kolmogorov, the constant K' of Eq. (5) should be an absolute one, independent of the particular fluid or of the nature of the mean flow and the spectrum at low wave numbers. Pond et al. (1966) have summarized measured values of K' for four different flow fields (atmosphere [Pond], laboratory air jet [Gibson and Schwartz, 1963], ocean [Grant et al., 1962] and water tunnel [Gibson and Schwartz, 1963]).

Figure 1. Normalized One-Dimensional Energy (velocity) Spectra. From Pond (1965).

The mean value of K' was 0.48. However, Margolis and Lumley (1965), who conducted laboratory experiments in a curved turbulent mixing layer of rather unusual characteristics found that their values of K' varied significantly. They suggest this variation was due to differences in the ratio of production to dissipation of turbulent energy.

Since the Pond et al. (1966) paper other experimental results have been published. Kistler and Vrebalovich (1966) reported values of K' of 0.65 in very high Reynolds number grid turbulence; Shieh [see Gibson et al. (1970)] obtained a value of 0.59 in atmospheric measurements at high Reynolds number and Gibson et al. (1970) obtained a value of 0.69 from measurements made at a mean height of 2.25 m above the Atlantic Ocean. In addition two ~~theoretical values~~ are available. Kraichnan (1968) derived $K' = 0.58$ using the abridged Lagrangian history direct interaction approximation and Pao (1965) obtained a value of 0.55 by fitting his spectral cut-off function to measured viscous cut-off shapes with K' as the adjustable parameter. This list of K' values may be completed by adding the values of 0.53 obtained by Stewart, Wilson and Burling (1970) from measurements of the skewness of velocity derivatives and 0.56 obtained by Nasmyth (1970) from a new universal curve based on very clean tidal channel data.

These recent results cast doubt on the value of K' being exactly 0.48. The constant appeared to be better known in 1966 than four years later. These results clearly

demonstrated the need for further careful measurements of this important quantity.

An investigation of the one dimensional velocity spectrum was made by Nasmyth (1970). His new universal curve differed slightly from the generally accepted curve of Stewart and Grant (1962), being slightly sharper in the "knee" (break from $-5/3$ slope) and falling below it in the dissipation range of wave numbers. Although the difference is slight, the point was considered worth investigating.

B. THE TEMPERATURE SPECTRUM

1. General Considerations

The Kolmogorov ideas may be applied to turbulent fluctuations of passive scalars such as temperature fluctuations in the atmosphere and oceans, salt concentration fluctuations in the oceans or dye concentration fluctuations in laboratory experiments. A passive scalar is any quantity (including contaminants) that modifies and/or can be transported in a fluid but which does not introduce buoyancy effects. A scalar which did introduce buoyant type forces would be considered to be active. "Temperature" is active at large generating scales but not at the small scales discussed in this thesis. The symbol " θ " is used to indicate the "intensity" of a scalar, or as a subscript to indicate quantities which refer to scalar quantities.

If $\theta(C^0)$ is the deviation of the temperature from its "ensemble" mean, the one-dimensional temperature fluctuation spectrum is defined so that

$$\overline{\theta^2} = \int_0^{\infty} \phi_{\theta}(k) dk \quad (C^0)^2 \quad (6)$$

and the rate of scalar dissipation by

$$\epsilon_{\theta} = 6\kappa \int_0^{\infty} k^2 \phi_{\theta}(k) dk \quad (C^0)^2 \text{ sec}^{-1} \quad (7)$$

Here κ is the molecular diffusivity (conductivity) and $k^2 \phi_{\theta}(k)$ is a scalar dissipation spectrum and describes the distribution with wave-number of the rate of decay of the quantity labelled θ^2 . Since this may refer to temperature fluctuations, salt concentration fluctuations or dye fluctuations there is no convenient label such as "energy" that can be attached to it although it is often referred to now as "theta-squared-stuff," a name attributable to I. D. Howells. When θ is a temperature fluctuation the physical significance of $\overline{\theta^2}$ is that it is a measure of the inhomogeneity of the field and is closely related to entropy. Entropy change is given by

$$dS = \frac{dQ}{T + \theta} = \frac{C_p \theta}{T + \theta} \quad (8)$$

$$dS = \frac{C_p \theta}{T} \left(1 - \frac{\theta}{T}\right) \quad (9)$$

to the second order in θ/T , where T is the (ensemble) mean temperature (in degrees Kelvin) and Q is quantity of heat and C_p is the specific heat at constant pressure. Since $\overline{\theta} = 0$ then

$$dS = - \frac{C_p \overline{\theta^2}}{T^2} \quad (10)$$

which shows that $\overline{\theta^2}$ is a measure of the negative entropy (negentropy) of the fluid in motion. If $\overline{\theta^2}$ is dissipated in a turbulent fluid, positive entropy is produced (i.e., entropy increases).

2. Application of Kolmogorov-like Ideas

A first Kolmogorov-like hypothesis as applied to temperature fluctuations could be worded: at sufficiently high Re the statistical properties of the temperature fluctuations are determined by ϵ , ν , ϵ_θ and κ . Dimensional analysis [Corrsin (1951), for example] based on this hypothesis leads to

$$\phi_\theta(k) = \epsilon_\theta \epsilon^{-3/4} \nu^{5/4} H(\sigma, k/k_s) \quad (11)$$

for the one-dimensional temperature spectrum and

$$k^2 \phi_\theta(k) = k_s^2 \epsilon_\theta \epsilon^{-3/4} \nu^{5/4} (k/k_s)^2 H(\sigma, k/k_s) \quad (12)$$

for the scalar dissipation spectrum. In these relations σ is the Prandtl number ($\sigma = \nu/\kappa$), the ratio of kinematic viscosity to molecular diffusivity.

A second Kolmogorov-like hypothesis as applied to temperature fluctuations could be worded: if there exist a range of wave numbers which is isotropic but in which neither viscosity nor diffusivity is important then the only determining parameters are ϵ_θ and ϵ . Dimensional arguments then lead to

$$\phi_\theta(k) = K'_\theta \epsilon_\theta \epsilon^{-1/3} k^{-5/3} \quad (13)$$

where K_0' is an absolute constant. This result, when necessary, will be referred to as the "convective" subrange law. This result was predicted independently by Obukhov (1949) and Corrsin (1951). The shape has since been verified experimentally by Tsvang (1960); Gurvich and Kravchenko (1962) and Pond (1965). The spectrum of Pond is particularly striking as it exhibits almost three decades of $-5/3$ slope (on a log-log plot) but no significant drop from this slope at the highest wave-numbers because instrumental noise was encountered before the spectrum fell off.

The one-dimensional spectrum of temperature fluctuations in air at scales where neither viscosity nor conductivity are important may be considered to be well established to fall off as wave number to the $-5/3$ power however, as discussed later, the values of the constant or its existence has not previously been determined. The form at scales where viscosity and conductivity are important is not clear, as the next sections attempt to illustrate.

3. Shapes of Scalar Spectra in Dissipative Regions

a. Theories

Theoretical studies have enjoyed only qualified successes in predicting the shape of high wave number dissipation spectra of scalars.

Obukhov (1949) and Corrsin (1951) estimated the "cut-off" wave number (drop from $-5/3$) for the convective subrange as $k_c = (\epsilon/\kappa^3)^{1/4}$. Batchelor (1959) argued that this cut-off wave number is valid only when $\sigma \leq 1$. For $\sigma \gg 1$,

Batchelor suggested the existence of a viscous-convective subrange $[(\epsilon/\nu^3)^{1/4} \ll k \ll (\epsilon/\nu\kappa^2)^{1/4}]$ and a viscous-diffusive subrange $[(\epsilon/\nu\kappa^2)^{1/4} \ll k]$. He proposed a uniform straining model for these ranges and obtained the spectrum function

$$E_\theta(k) = -\epsilon_\theta \gamma^{-1} k^{-1} \exp[\kappa k^2/\gamma], \quad k \ll k_s \quad (14)$$

where $\gamma = -\frac{1}{2}(\epsilon/\nu)^{1/2}$. This form is illustrated in Figure 2.

For the case of strongly diffusive scalars ($\sigma \ll 1$), Batchelor, Howells and Townsend (1959) proposed the existence of an inertial diffusive subrange and obtained (Figure 3)

$$E_\theta(k) \approx 1/3 \epsilon_\theta \epsilon^{2/3} \kappa^{-3} k^{-17/3}, \quad k_c \ll k \ll k_s. \quad (15)$$

These two results [Equations 14 and 15] are perhaps the most significant theoretical results obtained to date. They predict definite forms for scalar spectra in dissipative regions for $\sigma \gg 1$ and $\sigma \ll 1$. For the case $\sigma \sim 1$, no clear definitive theory is available. Unfortunately this case is one of the most important since $\sigma \sim 0.7$ for air (under one atmosphere of pressure at 10°C). The $\sigma \sim 1$ problem has been considered by Howells (1960), Pao (1965), Kraichnan (1968) and Gibson (1968). For the most part these consist of modifications to the Batchelor (1959) and/or Batchelor, Townsend and Howells (1959) theories. The validity of the theories is difficult to comment on since sufficient spectral measurements over a wide range of Prandtl numbers are not available to confirm or disprove their predictions.

Figure 2. Scalar Spectrum for $\sigma \gg 1$ as Predicted by Batchelor (1958).

Figure 3. Scalar Spectrum for $\sigma \ll 1$ as Predicted by Batchelor, Howells and Townsend (1968).

b. Observations

The Batchelor (1959) form of the spectrum for weakly diffusive scalars has received some experimental support. Gibson and Schwarz (1963) provided water tunnel results of temperature and salinity (conductivity) fluctuations which were not inconsistent with Batchelor's theory. Nye and Brodkey (1966) measured dye concentration fluctuations in water and obtained $1\frac{1}{2}$ decades of a spectrum function nearly proportional to $1/k$ but no $-5/3$ region because the Re of the flow was too low. The most convincing results were provided by Grant, Hughes, Vogel and Moilliet (1968) who obtained temperature and velocity fluctuations in the open sea and in a tidal channel. Most of their temperature spectra (example, Figure 4) have well defined $-5/3$ and -1 regions but beyond this the shape of the spectra are not clear (due to noise and sensor frequency response limitations).

Few spectral measurements of strongly diffusive scalars are available. Rust and Sesonke (1966) found that for a fluid with $\sigma \approx 0.02$, the spectrum decreased approximately as wave number to the -3 power. Measurements of plasma fluctuations ($\sigma \approx 0.07$) by Granastein, Buchsbaum and Bugnals (1966) resulted in a spectrum that fell rapidly, the frequency to the $-5/3$ power dependence changing to a -7 power dependence in less than a decade.

For the case of near unity Prandtl number, only two high wave number spectral results are available. Lanza and Schwarz (1966) measured the one-dimensional spectrum of temperature fluctuations in air behind a heated grid in a

Figure 4. Temperature and Velocity Spectra from Run 2 of Grant et al. (1968).

wind tunnel. In order to obtain measurements over a wide range of non-dimensional wave numbers (k/k_s) they found it necessary to vary the grid Reynolds number and mesh length. In their method, several semi-empirical constants were required to calculate scalar spectral values. They also had to apply a finite-length correction of about 17% at $k/k_s = 1.0$. This does not mean their spectra are not correct, however their results are not as convincing as an experiment where less intermediate steps are used to arrive at final numerical values. Gibson et al. (1970) showed a single normalized spectrum of the derivative of temperature fluctuation signals obtained in air a few meters above the Atlantic (during the 1970 Barbados Oceanographic and Meteorological Experiment). The results of Lanza & Schwarz and of Gibson et al. are consistent only in that neither showed a -1 region. Not only did the spectra fall from a -5/3 region at different wave numbers, but also the shapes of the curves differed.

With only two cases of high wave number temperature spectra available, and these differing, the need for further results is very apparent. The temperature measurements presented in this thesis satisfy this need.

4. The One-dimensional Scalar Constant

The constant K'_0 of Eq. (13) is expected to be an absolute one, independent of the particular fluid or of the nature of the mean flow and spectrum at low wave numbers. Since most previously measured scalar spectra have been poorly determined at high wave numbers an accurate evaluation of

ϵ_0 from Eq. (7) has not been made. This in turn has prevented accurate evaluation of K'_0 from Eq. (13). To date, a variety of values has been reported. Swinbank (1955) and Taylor (1961), from measurements in the atmospheric boundary layer, estimated the scalar constant to be about 0.5 to 0.6. Similar values were obtained by Takeuchi (1962) and by Gibson and Schwarz (1963) in wind tunnel experiments. However, Gurvich and Zubkovsky (1966), from a survey of previous studies, quote a range of about 0.4 to 2. The study by Lanza and Schwarz (1968) led to a value of 0.78. Grant et al. (1968) obtained a value of 0.31 and Gibson et al. (1970) a value of 1.17. Panofsky (1969), basing estimates on observed spectra, obtained 0.7.

The range of values predicted by theoretical studies is no less impressive. Pao (1965) suggested 0.59 while Kraichnan (1966) predicted 0.208, based on his Lagrangian history direct interaction theory. Gibson (1968) argued for a value near 0.9.

Clearly, the range of values listed here indicates that the constant is simply not well known. In fact some question has been raised as to whether it is a constant at all. There have been suggestions that both K'_0 and K' can be expected to be functions of stability, or parameters related to it such as the Monin-Obukhov length. The measurements reported in this thesis were made when temperature was considered to be a passive scalar.

III. INSTRUMENTATION AND EXPERIMENTAL PROCEDURE

A. BACKGROUND

The experimental program for this thesis originally began with the temperature fluctuation system described by Pond (1965). Since his measurements became affected by noise before the dissipation region of the temperature spectrum was reached, improvements of his system were required. These included design of a smaller sensor, modification of the bridge for the chosen 0.25 μm diameter wire sensor, repackaging for optimum shielding of component parts, reorganization for easier operation, improved amplification systems and careful control of filtering. Modifications of the then existing electronic system allowed the temperature spectrum to be measured to just beyond the $-5/3$ (convection subrange) region but not far enough to unequivocally evaluate the area under the scalar dissipation spectrum. A different design was required; this is described in Section III-B-2.

Measurements made with the original system indicated that in order to obtain the desired spectra, improvement was required in the signal to noise ratio. The signal to noise ratio may be improved by increasing the temperature (overheat) at which the wire operates but this also deleteriously increases the sensitivity of the wire to velocity fluctuations. The alternative is to use a finer wire which has greater resistance per unit length. This increases the voltage drop across it thereby improving the signal to noise level ratio

for the same overheat. A sensor was constructed from platinum wire one-tenth the diameter of that used by Pond. At that time such a sensor had never been constructed and serious doubts existed regarding not only its construction but also its ability to withstand atmospheric field conditions.

B. TEMPERATURE FLUCTUATION SYSTEM

1. Sensor

The sensor of temperature fluctuations developed was a resistance thermometer consisting of a platinum wire $0.25\text{ }\mu\text{m}$ (0.00001 in.) in diameter and about 0.30 mm in length. This length to diameter ratio (in excess of 1000) is about five times that normally used and the diameter is ten times smaller than that previously considered "practical" (Hinze, 1959, p. 94).

a. Rationale for Dimensions of Sensor

As pointed out above, the primary reason for choosing such a small diameter wire was to improve the signal to noise ratio. However the wire time constant and wave number resolution also had to be considered. For measurements made as a time series at a fixed point these may be estimated from Taylor's (frozen turbulence) hypothesis (Taylor, 1938; or see Hinze, 1959, p. 40). For such measurements Taylor's hypothesis leads to the relation

$$k = \frac{2\pi f}{U} \quad (16)$$

where k is radian wave number (cm^{-1}), f is frequency (Hz) and U is mean wind speed (cm/sec). Eq. (16) implies that

turbulence of scales near $1/k$ are unchanged (frozen) during the time required for these scales to be swept past the sensor.

A series of measurements indicated that to obtain the desired spectra, temperature fluctuations in excess of 1 kHz would have to be measured. Assuming, to be safe, 2 kHz as the maximum frequency likely to be encountered at a mean wind speed of 6 m/sec, Eq. (16) indicated maximum wave numbers of 20 cm^{-1} would have to be resolved. This imposes the requirements that the sensor should have a length that does not exceed 0.5 mm and a frequency response curve that is flat to 2 kHz. The former point was satisfied; the latter required establishing the dynamic response of the wire.

The factors which affect the dynamic response of a thin platinum resistance wire thermometer are (i) lag due to the heat capacity of the wire, (ii) lag due to the heat capacity of the thin boundary layer of air next to the wire and (iii) heat transfer across the boundary layer. The first is simply the time constant of the wire. Expressions developed by Fabula (1962) and Krechmer (1954) lead to a time constant of about 10 microseconds for this wire. This exceeds by three orders of magnitude the anticipated requirements. The thermal inertia of the wire itself in no way distorts the turbulence at the maximum frequency of interest. The second factor, thermal capacity of the boundary layer of air about the wire, is small compared to the thermal capacity of the wire. In order for thermal capacities to be similar, the thickness

of the boundary layer would have to be approximately 22 times the diameter of the wire. As regards the third factor, Lighthill (1954) has made a detailed analysis of the phase lag in the heat transfer from a heated circular wire in a fluctuating stream in the range of Reynolds number for which a laminar boundary layer exists. The front quadrant of the wire is covered by a layer of nearly uniform thickness across which most of the fluctuating heat transfer is believed to occur. For the dimensions of the wire used here, the departure of the heat-transfer fluctuations from their quasi-steady form is completely negligible.

As a final point, the extreme length to diameter ratio of the wire ensured that "end effects" and other effects were also negligible.

b. Construction of Sensor

The sensor was made from Wollaston wire consisting of a silver jacket about a platinum core. The outside diameter of the jacket is about $45\text{ }\mu\text{m}$ (0.0018 inches). The original wire (jacket and core) was prestressed into a V-shape and then soldered onto the end of a Flow Corporation Model HWP probe (Figure 5) in such a way that when the silver jacket was removed there was no undue stress on the thin platinum core. The silver jacket was electrochemically removed by placing the wire in a bubble of dilute nitric acid at the drawn tip of a U-tube (Figure 6) and applying 1.5 volts D.C. The jacket flakes off exposing the platinum core (Figure 7). By this method sensors of length of 0.25 mm to 0.50 mm were made.

Figure 5. "Wollaston" Wire Mounted on Probe.

Figure 6. The Etching Procedure.

(a) $0.25\mu\text{m}$ platinum wire core extending from silver jacket soldered to prongs.

(b) Magnification of (a) to show platinum core section. Distance between silver coated ends is approximately 0.4 mm.

Figure 7. Microscope photographs of platinum core used as the temperature fluctuation sensor. Photographs by Charles Woodhouse.

2. Circuitry

a. Electronics

The electronic system used to convert resistance changes of the wire to useable voltage fluctuations was derived from three previous systems developed during the course of this research. It was developed by National Electrolab Associates Limited of Vancouver, British Columbia in consultation with the Institute of Oceanography, University of British Columbia. It consists of an 80 kHz multivibrator, bridge, bridge amplifier, synchronous detector and D.C. amplifier (Figure 8). The multivibrator drives switching transistor 2N4126 to generate a square wave of constant amplitude which is applied to the bridge. The bridge (Figure 9) consists of two 2.2 k resistors, the probe resistance and balance adjustment resistors. Variable capacitors are required to balance out the capacitance of the 3 m cable to the probe. The bridge amplifier is an integrated circuit low noise amplifier with a fixed gain of 60 db. The rectifying 2N422 F.E.T. (Figure 8) is switched synchronously with the 80 kHz oscillator via the second 2N4126 transistor. An 80 kHz filter preceding the F.E.T. accepts only the "carrier" signal frequency band and the low pass filter following it partially smooths the carrier from the rectified output. The D.C. amplifier is an operational amplifier with fixed incremental gains of 3, 5, 10 and 20. Its frequency response is flat from D.C. to approximately 10 kHz. Complete circuit details and instructions in its use can be found in the operating

Figure 8. Block Diagram of Temperature Fluctuation Measurement System.

Figure 9. Bridge Network for Platinum Resistance Thermometer.

manual (National Electrolab Associates Ltd., 1130 East 7th Avenue, Vancouver 12, B. C. Project 116-01).

b. Bridge Design

The measurement of temperature fluctuations with a thin platinum wire takes advantage of the fact that a very slightly heated wire is a simple resistance thermometer. The wire is necessarily slightly heated by a small current in order to sense its change in resistance due to change in ambient temperature by measuring the change in voltage across it. If this heating is kept low, resistance changes due to cooling by the wind are negligible compared to resistance changes caused by temperature variations. The bridge design then consists essentially of calculating values of bridge resistors to ensure optimum current through the wire.

The rate at which heat is transferred from a cylinder (wire) to an air stream in steady flow has been found by King (1914) to be proportional to the square root of velocity. In this application his result reduces to the proportionality

$$H \propto L\Delta T\sqrt{Ud} + \text{constant} \quad (17)$$

where H is the rate of heat transfer to the air stream, L is the length of the wire, U is the mean wind speed and d is the diameter of the wire. ΔT is the overheat of the wire and is defined as

$$\Delta T = T_w - T_e \quad (18)$$

with T_w being the wire temperature and T_e the equilibrium temperature of the unheated wire. If the resistance of the

wire is R ohms and it carries a current of I amperes then the rate at which electrical energy is converted into heat in the wire is $I^2 R$ watts. In steady flow the rate of heat loss is equal to the rate at which heat is supplied to the wire, or

$$H = I^2 R \propto L \Delta T \sqrt{Ud} + \text{constant} \quad (19)$$

This expression, after suitable rearrangement, allows one to plot ΔT as a function of U for given values of I . The object is to select I such that the velocity induced change in ΔT is small compared with the effect of ambient temperature fluctuations for a typical range of velocities.

The optimum value of the current through the $0.25 \mu\text{m}$ wire chosen was determined experimentally. The probe was connected to a bridge (Figure 10) and the sensor placed in a small wind tunnel. A measured voltage was applied to the bridge and the output voltage recorded as a function of wind speed. The current through the wire was calculated and over-heat computed from an exact form of Eq. (19) [from Flow Corporation Bulletin #25, 1956]; this was plotted against $(\text{mean wind speed})^{1/2}$ in Figure 11. When the current was about 100 microamps through the wire, the velocity effect was larger than desirable (approximately equivalent to 0.030° per 1 m/sec change in wind speed at 4 to 5 m/sec). In order to be certain that no significant velocity dependence appeared in the temperature signal the $0.25 \mu\text{m}$ diameter wire was operated with a current of 50 microamps. A change in wind speed of 4 m/sec to 5 m/sec, for example, would be registered as a temperature change of

$$R_0 = 716\Omega \pm 560\Omega$$

Figure 10. Circuit Used to Test Response of Platinum Resistance Thermometer to Wind Speed.

Figure 11. Overheat (ΔT) as a Function of Wind Speed ($U^{1/2}$) for Various Currents through Platinum Wire Sensor.

only 0.005 C°. Further discussion of this problem may be found in Section V-C-2(b).

3. Calibration

A method of calibrating a platinum wire thermometer was outlined by Pond (1965). The method he proposed was adequate provided the sensor could be immersed in some low dielectric fluid. This is not possible with a 0.25 μm wire because surface tension of the fluid is sufficient to break the wire either during immersion or withdrawal. A static method was tried in which the sensor was capped and then placed in a temperature bath whose temperature could be raised or lowered and closely regulated at a given temperature. This method proved unsatisfactory for reasons that were not clear. Either equilibrium of the capped sensor and bath could not be achieved (even after several hours) or operating the sensor at zero wind speed (where the overheat vs. wind speed curve is steepest) caused instabilities in the electronics. A dynamic method which involved placing the probe in a small wind tunnel was tried but also found unsatisfactory because of the difficulty in controlling the temperature in the tunnel. Finally an indirect method was used.

Since a 2.5 μm diameter platinum wire could be placed in a low dielectric fluid, a calibration of such a wire was made and compared with a theoretical calibration based on the physical properties of platinum. The assumption was made that if there is good agreement between the two results, then

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a theoretical calibration of the 0.25 μm diameter wire should be adequate.

The resistivity and temperature coefficient of the wires were assumed to be those of bulk platinum. The Handbook of Chemistry and Physics (45th edition) lists these values as:

$$\begin{aligned} \text{resistivity } \rho_0 &= 11 \times 10^{-6} \text{ ohm-cm,} \\ \text{temperature coefficient } \alpha &= 3.7 \times 10^{-3} (\text{C}^\circ)^{-1}. \end{aligned}$$

The resistance of the core of the "Wollaston" wire used, as given by the manufacturer (Sigmund Cohn Corp.), is 6,000 ohms per foot (20 ohms/mm) for the coarse (2.5 μm diameter) wire and 600,000 ohms per foot (2,000 ohm/mm) for the fine (0.25 μm diameter) wire. The etched length is about 3.0 mm for the coarse wire and 0.30 mm for the fine wire which means the resistance of the coarse wire is about 60 ohms and that of the fine wire about 600 ohms. This agrees reasonably well with the resistance (R) as obtained from the resistivity (ρ_0) and the dimensions of the wire (length ℓ and area A):

$$R = \rho_0 \frac{\ell}{A} \quad (20)$$

The range of temperature encountered in these turbulence measurements is small so the change in resistivity can be assumed to be linearly proportional to change of temperature according to

$$\Delta\rho = \rho_0 \alpha \Delta T \quad (21)$$

From Eqs. (20) and (21)

$$\Delta R = \left(\frac{\rho_o \alpha l}{A} \right) \Delta T \quad (22)$$

The bracketed term has a value of 0.249 for the coarse wire and 2.49 for the fine wire. This is the change in resistance in ohms for each wire for a one C° temperature change.

A resistance box was substituted for the sensor in the bridge which then was balanced for typical sensor resistances (600 Ω to 900 Ω). At balance a change in the resistance box of one ohm with the D.C. amplifier of the platinum resistance thermometer electronics [Section III-B-2(a)] set at a gain of 20 resulted in a D.C. voltage level change of 0.4 volts, or $\Delta V / \Delta R = 0.4$ volts/ohm. Since, from the preceding paragraph, $\Delta R / \Delta T = 2.5$ ohms/C° then

$$\frac{\Delta V}{\Delta R} \cdot \frac{\Delta R}{\Delta T} = 1 \text{ volt/C}^\circ \quad (23)$$

at a gain of 20. All runs were made with the D.C. amplifier at a gain of 10 so that a voltage change of one volt corresponded to a temperature change of 0.5 C°.

The bridge, at balance, was found to be linear for over a 10 C° temperature change which is an order of magnitude greater than the rms temperature fluctuations encountered. The bridge was balanced before making each recording.

C. EXPERIMENTAL PROCEDURE

1. Location

The measurements were made at Boundary Bay, British Columbia (Figure 12). This location was the third of three

Figure 12. Location of Field Site.

sites used in the course of this research. The first, Spanish Banks, was not entirely satisfactory because temperature signals tended to be small in the stable air over the ocean. The second, Hanford, was not satisfactory because of the varied meteorological conditions encountered in air over a desert. The third, Boundary Bay, was a compromise between the first two. The air was slightly unstable as it blew across the beach off the ocean. The site is a tidal mud-flat, which is completely covered at high tide and is bare for about 2.5 km seaward at low tide. The site has been described in detail by Dobson (1969).

2. Equipment

a. Mast Arrangement

All instruments were supported from a portable mast at a height of 4 m. The signals from the sensors were carried by cables to a panel truck which served as the housing for the recording equipment. The truck was 30 m downwind from the mast.

b. Platinum Resistance Thermometer

The platinum resistance thermometer, which has already been described in detail, was mounted on the mast (at a height of 4 m) approximately 5 cm below the hot-wire probe. The platinum resistance thermometer electronics were placed at the foot of the mast.

c. Hot-wire Anemometer

A constant temperature hot-wire anemometer system was used (DISA Model 55D05). The unit was battery operated

and used in a 1:1 bridge ratio mode which requires a compensating cable in the balance arm whose impedance matches that of the probe cable. The hot-wire probe consisted of a platinum wire 5 μm in diameter and 1.0 mm in length. The hot-wire was mounted approximately 5 cm above the temperature sensor. Electronically this system can handle signals in the frequency range from DC to 100 kHz. Both the time response and wave-number limitation of the wire allow signals up to 1000 Hz to be measured without attenuation at winds speeds of 4 to 5 m/sec.

The system was calibrated in a wind tunnel of the Department of Mechanical Engineering at the University of British Columbia. A plot of output voltage squared versus the square root of the mean wind speed (Figure 13) was made to obtain the King's Law curve. The dynamic response of the system was obtained from the equation of the resulting straight line. If E represents voltage and U mean velocity then

$$E^2 = A + BU^{\frac{1}{2}} \quad (24)$$

where A and B are empirically determined constants. Differentiation of Eq. (24) yields

$$2EdE = B\left(\frac{1}{2}U^{-\frac{1}{2}}\right)dU \quad (25)$$

The differentials dE and dU represent voltage fluctuations (e) and velocity fluctuations (u) respectively so

$$e = \left[\frac{B \cdot \frac{1}{2}U^{-\frac{1}{2}}}{2E} \right] u \quad (26)$$

Figure 13. King's Law Curve for Hot-wire Anemometer.

is the equation relating these two parameters.

d. Cup Anemometer

The cup anemometer was manufactured by Thornthwaite Associates. It was mounted at 4 meters on the mast and separated horizontally by approximately 70 cm from the hot-wire and platinum resistance thermometer. The low inertia and internal friction of this instrument allow it to respond to wind paths of about 1 m, in practice equivalent to frequencies slightly greater than 2 Hz.

The cup system gave voltage "pips" as a function of cup revolution. These voltage counts were integrated in such a way to give a fluctuating D.C. voltage proportional to wind speed. The "pips" were superimposed on this signal. In this way a self check could be made of the system. The fluctuating D.C. voltage with "pips" was amplified and recorded on tape. These were later reproduced and manually converted to wind speed in the laboratory using the manufacturers calibration which was checked in a wind tunnel.

e. Quartz Thermometers

Two quartz thermometers (Hewlett-Packard Model 2801A) were used to get an indication of the vertical temperature gradient. The sensors were mounted at 1.5 m and 4 m on the mast. Their dimensions were 3/8 inch in diameter and 11/16 inch in length (H.P. type 2850A). These identical sensors were not shielded from radiation which means that absolute values of temperature are uncertain although relative values should be approximately correct.

3. Recording and Signal Conditioning

The signals from the platinum resistance thermometer, hot-wire and cup anemometer were recorded on separate FM (frequency modulated) channels of a magnetic tape recorder (Sangamo Model 3562) run at 60 inches per second (high frequency cut-off 20 kHz). The quartz thermometer digital meter reading was read onto the voice edge track. The recording system is outlined in Figure 14.

Since both the noise of the system and the maximum frequency of interest of the temperature signal were uncertain at the time of the experiment, recordings were made both without the low pass filter (marked L.P. FILTER) and with it at various settings to attenuate very high frequency noise.

The signal from the platinum resistance thermometer (PRT) electronics following L.P. FILTER and the signal from the hot-wire anemometer electronics (55D05) were treated similarly. Each was recorded in two different ways chosen to optimize the incompatible requirements of noise performance and frequency response. One channel was unfiltered (L.F. GAIN) and the other was differentiated. The purpose of the differentiation circuit was to improve the signal to noise ratio at high frequencies. L.F. GAIN provided gain not only for the direct signal but also for the signal prior to differentiation. The high frequency gain control (H.F. GAIN) determined the frequency at which there was unity gain in the differentiating circuit. H.F. GAIN was adjusted to provide optimum gain (prewhitening) of the frequencies of interest. The

Figure 14. Block Diagram of Recording System.

differentiating circuits were matched linear phase shift devices made (by Mr. F. E. Jerome) from Philbrick P85AU operational amplifiers. The circuit had a 6 db/octave gain up to 10 kHz (depending on H. F. GAIN setting) and then fell at 6 db/octave.

Recording at 60 ips with FM electronics ensured the best signal to noise ratio and provided sufficient frequency response (up to 20 kHz) to record faithfully the sharp rises and falls of the differentiated signal. It has the disadvantage of limiting the maximum length of a given run to 8 to 12 minutes, depending on the length of the tape.

IV. ANALYSIS

A. SELECTION OF DATA

1. Criterion

The primary criterion for selecting sections of data to be analysed was stationarity. Only those sections which exhibited reasonably uniform levels of turbulence, mean winds and temperature were selected for analysis. This is not to say that an analysis of the remaining sections is not important. However for this particular study these interesting meteorological situations have been avoided.

An example of selected data is given in Figure 15. Turbulence intensity of the smaller scales of both direct and differentiated signals remains reasonably uniform over the length of each section. The direct velocity signal shows that some low frequency oscillations are present but it is not difficult to see that the signal deviates very little from a mean wind speed of 3.0 m/sec. Low frequency oscillations are also present in the temperature signal but a mean is evident.

An example of the variety of temperature fluctuation signals encountered is shown in Figure 16. In this figure, part (a) shows the direct temperature signal shown in the previous figure. Part (b) shows a signal that is tending to be one-sided but the small scale fluctuations remain uniformly active throughout. Part (c) shows a signal exhibiting both quiet and active sections. It also shows a shape which is

Figure 15. An Example of Data Selected for Analysis.

Figure 16. Examples of Types of Temperature Fluctuation Signals Recorded.

often characteristic of temperature signals; a gradually rising leading edge and sharp trailing edge (saw tooth shape). Part (d) shows a section exhibiting markedly intermittent behavior. Of the records shown in Figure 16, parts (a) and (b) were selected for analysis, part (c), a borderline case, was also selected but part (d) was not.

2. Sections Analysed

Since the objectives of this thesis were concerned with examination of high wave number velocity temperature fluctuations, it was not necessary to examine long sections of data. Most sections analysed are one minute in length. Since the number of cycles in a given record decreases with decreasing frequency, spectral estimates become less reliable with decreasing frequency. Blackman and Tukey (1958; pp. 21-25) show that for a stationary Gaussian process estimates at each frequency are distributed as chi-square, with $d.f. = 2 \times (\text{length of record in seconds}) \times (\text{effective bandwidth in Hz})$ being the approximate number of degrees of freedom. The data analysed here is decidedly non-Gaussian so that such a criterion is no longer strictly applicable. However since the time series is broken into data "blocks," averages over consecutive blocks may be used to gain some information as to the stability of the spectral estimates over the interval of the signal. The digital analysis program (section IV-D) computes averages along blocks and across bands which are chosen by the investigator. Confidence levels may be computed by evaluating the variance of the

estimates of the spectral density in the band and dividing by the square root of the number of estimates. Observed 80% confidence levels of typical spectra are indicated by the vertical bars in Figure 17. The horizontal bars indicate the bandwidth over which the spectral value was estimated.

Data from five separate tapes were analysed. Sections were chosen, as discussed in the preceding section, from each of these tapes. A listing of tape number, section designation, mean wind speed and length of record is given in Table I. These data were collected on August 8, 1969 between noon and 8:00 p.m. (PDT). Table I is organized so that the first run made appears at the top of the table and the last at the bottom.

Changes in the stability of the lower atmosphere are to be expected when data are collected over such long periods. The only indication of stability was that given by the quartz thermometer. One (T1) was mounted about 1.5 m above the ground; the other (T2) was mounted at the top of the mast. For neutral conditions the difference $T1 - T2$ should be approximately $+ 0.04^{\circ}\text{C}$. In fact $T1 - T2$ ranged from $+ 1.80^{\circ}\text{C}$ at the beginning of the experiment to $+ 0.20^{\circ}\text{C}$ (and still becoming smaller) at the end of the experiment. However variations of 0.50°C were observed during runs. Occasionally, for example, the difference was zero. This means that in earlier runs buoyancy effects could be anticipated. Certainly the earlier temperature data are the most intermittent, frequently exhibiting the characteristic shape discussed in the previous section.

Figure 17. Typical Spectra Illustrating 80% Confidence Limits (vertical bars) and Band Widths (horizontal bars).

TABLE I
LISTING OF SECTIONS ANALYSED

Tape No.	Record Start Time PDT *	Section	Mean Wind (m/s) U	Reynolds Number** Re x 10 ⁶	Length of Record (Seconds)
202(2)	1330	A	3.43	0.9	35
		B2	3.39	0.9	30
		D	3.50	1.0	60
		F	3.42	0.9	23
203(1)	1427	A	3.35	0.9	60
		B	3.08	0.9	60
		C	3.00	0.8	60
		D	2.90	0.8	60
209(3)	1600	A	3.86	1.1	60
		B	4.03	1.1	60
		C	3.70	1.0	60
		D	3.68	1.0	60
RCA(1)	1629	A	4.30	1.2	60
		B	4.20	1.2	60
		E	3.50	1.0	60
		F	2.90	0.8	60
RCA 3(1)	1945	A	3.65	1.0	60
		B	3.93	1.1	60
		C	4.50	1.2	60

* Pacific Daylight Time

** Reynolds number = $Re = UZ/\nu$

Z = 4 m.

ν = kinematic viscosity

Since such large changes in $T_1 - T_2$ were observed with no apparent change in the spectra (though not the signals) one can conclude only that stability is probably not important in this discussion on high wave number turbulence.

For all runs analysed, with the exception of RCA 3(1), the wind was blowing from the south off the water and over the 1.5 km of bare tidal mudflat. During RCA 3(1), the wind was blowing from the east parallel to the shore.

B. INTERPRETATION OF DATA

Experimental turbulence measurements often are made as a time series of the phenomenon at a point fixed in space. Spectra are computed as functions of frequency. Theoretical work almost exclusively expresses results in terms of wave number. The relation between the two approaches is provided by Taylor's (frozen field) hypothesis [Eq. (16)]. In a boundary layer this is a good approximation only if $kz \gg 1$ where z is the height of measurement. For this research, $z = 400$ cm and wind speeds were typically about 400 cm/sec; Taylor's hypothesis should be valid for $f \gg 0.2$ Hz. This is always satisfied for the high frequency (high wave number) spectra considered here. Computations of parameters and constants were made with spectra in terms of frequency (see Figure 17 for example). Then all spectra were transformed to functions of wave number to allow comparisons with previous research. Since Eq. (16) is a linear transformation, the shape of a wave number spectrum will be the same as that of a frequency spectrum.

Frequency and wave number spectra are related by

$$f\phi(f) = k\phi(k). \quad (28)$$

The expression for the "inertial" subrange (Eq. (5)), in terms of f becomes

$$\phi(f) = \left(\frac{2\pi}{U}\right)^{-2/3} K' \epsilon^{2/3} f^{-5/3} \quad (29)$$

The expression for the rate of energy dissipation (Eq. (2)), becomes

$$\epsilon = 15\nu \left(\frac{2\pi}{U}\right)^2 \int_0^{\infty} f^2 \phi(f) df. \quad (30)$$

The corresponding expressions for the temperature spectrum are

$$\phi_{\theta}(f) = \left(\frac{2\pi}{U}\right)^{-2/3} K'_{\theta} \epsilon_{\theta}^{-1/3} f^{-5/3} \quad (31)$$

and

$$\epsilon_{\theta} = 6\kappa \left(\frac{2\pi}{U}\right)^2 \int_0^{\infty} f^2 \phi_{\theta}(f) df. \quad (32)$$

These were the basic expressions that were used in evaluating ϵ , K' , ϵ_{θ} and K'_{θ} from spectra such as shown in Figure 17. In addition, independent methods more suitably discussed in the "RESULTS" section were used to calculate ϵ .

C. ANALOG TO DIGITAL CONVERSION

1. General Description

Analysis of the data was accomplished through the use of a 10 bit analogue to digital converter (ADC) built at the Institute of Oceanography, University of British Columbia.

The ADC, based on Digital Equipment Corporation integrated circuit modules, is capable of digitizing (simultaneously) as many as ten channels of data. This instrument is interfaced (Figure 18b) with a small computer (Control Data 8092 Tele-programmer) which forms part of the computing facility of the U.B.C. Computing Centre. The 8092 was programmed to write digital tapes for subsequent processing on the IBM Model 360/67 computer at the Computing Centre.

2. Conversion Considerations

a. Analog Signal Level

To ensure good digitizing some amplification was necessary to provide an optimum signal input to the ADC. The signal level was always checked to ensure that maximum advantage was being taken of the full dynamic range of the ADC but that signal "peaks" did not overload the converter.

b. Aliasing

To prevent aliasing, the signal input to the ADC must contain no energy at frequencies greater than half the sampling rate; for a sampling rate F Hz, $F/2$ is called the folding or Nyquist frequency (Blackman and Tukey, 1951). This was done by using an analog low pass filter between the signal source and the converter input.

c. Sampling Frequency

The signal was sampled at least twice per cycle of the highest frequency with sufficient amplitude to produce an aliasing problem. An additional requirement in the case of the differentiated signals was that the sampling rate had

a) Digitization

b) Fourier coefficients computation

c) Spectra computation

Figure 18. Block Diagram of Analysis Procedure

to be rapid enough to define their sharp rises and falls.

Prior to this investigation the maximum frequencies of interest were unknown, so optimum filter settings and sampling rates had to be experimentally determined (see Section IV-D).

3. Procedure

The signal to be digitized went from the tape recorder to a D.C. amplifier (Preston Model 8300 XWB), was passed through a low pass filter (Krohn-Hite Model 3340) and then went to the ADC. The filter was set and the signal at the point just prior to the ADC was monitored on an oscilloscope. The gain of the D.C. amplifier was adjusted to take full advantage of the dynamic range of the ADC. When the sampling rate was set, the ADC was activated.

D. DIGITAL ANALYSIS PROGRAMS

Figure 18 (b and c) is a block diagram of the method used to perform the spectral analyses. The IOUBC Fast Fourier Transform package is a series of programs, some in FORTRAN IV and some in assembler language, which instruct the IBM 360/67 computer to read the tapes generated by the 8092 (Figure 18a), store the data in the computer memory and perform a Fast Fourier Transform (FFT) on them; the Fourier coefficients resulting from the FFT process are used to compute spectra. Routines for plotting and printing results are included in the package. In addition other specialized programs are included in the package but were not used in this investigation.

Each tape was checked for parity errors and sampling rate by the tape verify program (UBC program TVERIF) which also computes the first four moments of the distribution of digitized values. From these moments, the coefficients of excess and skewness were calculated. This program was run routinely on all digitized data. After verification the digital data tape was processed to yield another tape containing the Fourier coefficients of the digital time series (UBC program FTOR which is based on the discrete FFT technique). From this "coefficient tape" spectra were computed using UBC program SCOR. Use of these programs is described by Wilson et al. (1969). Dobson (1970) has described programs FTOR and SCOR in some detail.

E. EFFECTS OF SAMPLING RATE AND FILTERING

In order to determine optimum sampling rates and filter settings, a 35 second sample [202(2) A] of a differentiated temperature signal was digitized several times. At a sampling rate of 6000 Hz it was digitized at three different filter settings (no filter, 2 kHz and 1 kHz). This was repeated at sampling rates of 4000 Hz and 2000 Hz. The differentiated temperature signal was chosen because it is the most difficult signal to analyze. It has a large dynamic range, extremely rapid rise and fall times and its spectrum was "guessed" prior to the observations to extend significantly to perhaps 1 or 2 kHz. A differentiated temperature signal from tape 202(2) was chosen because it was recorded without being low

pass filtered; therefore all of the signal and noise is available on this tape.

The dependence on sampling rate of the differentiated temperature spectrum computed from the unfiltered signal is illustrated in Figure 19 (a, b and c). This figure shows that the use of a 2000 Hz sampling rate aliases noise from frequencies greater than the Nyquist frequency (1000 Hz) into frequencies between 100 and 1000 Hz. The digitizations at 4000 Hz and 6000 Hz produce similar spectra to about 550 Hz. Aliased noise is encountered by the 4000 Hz digitization at this frequency and at 630 Hz by the 6000 Hz digitization. Another aspect very evident in these spectra is 60 Hz noise which appears at its fundamental and 180 Hz.

Low pass filtering with -3 db cut-off at 2 kHz before digitizing greatly reduces the noise at 60 Hz and 180 Hz (Figure 19b) probably because of better isolation in the grounding system of the filter. The slowest digitization rate is again unsatisfactory. The two higher digitization rates produces identical spectra out to about 1500 Hz; these begin to drop off beyond this frequency because of filter attenuation of the signal.

The effect of a 1 kHz cut-off to the low pass filter is very noticeable (Figure 19c); in this case all three digitization rates produced identical spectra to about 700 Hz. Beyond this frequency the slowly sampled (2000 Hz) spectrum begins to deviate and the two at higher rates follow the filter cut-off curve.

Figure 19. Spectrum Dependence on Sampling Rate.

Figure 20 shows the dependence of the observed spectrum on filter cut-off frequency; a constant high sampling rate (6000 Hz) was used in all three cases. Both of the filtered spectra show the filter cut-off well. The unfiltered spectrum follows the 2 kHz spectrum right up to where the filter becomes effective. The spectra from the filtered signals, as has been pointed out, show less noise than the spectra from the unfiltered signal.

The points of the unfiltered spectrum consistently fall below the points of the two filtered spectra. This is because the two filtered signals were amplified (by a small unknown amount) before digitizing but the unfiltered signal was not.

Clearly a sampling rate of 2 kHz is insufficient to allow adequate analysis of the differentiated temperature signal unless the signal is low-pass filtered at 1 kHz prior to digitizing. There is however a possibility of losing some high-frequency information if this is done. If the signal is filtered at 2 kHz both 4000 Hz and 6000 Hz sampling rates appear to be sufficient. Only with unfiltered data would the 4000 Hz rate be insufficient. In order to be safe, all of the differentiated temperature signals were filtered at 2 kHz and sampled at 6 kHz.

The other signals that were analysed were the undifferentiated temperature signal, the undifferentiated velocity signal and the differentiated velocity signal. The differentiated velocity signal had less noise at high frequencies and did not have as sudden step rises as the temperature

Figure 20. Spectrum Dependence on Low Pass Filter Setting. Sampling Rate = 6 kHz.

signal. It extended to rather lower frequencies so it was digitized at 2 kHz after being low pass filtered at 1 kHz cut-off. The same treatment was used with the two direct signals.

F. NOISE ANALYSIS

The experimental success of this investigation depended on constructing a very small, rapid response temperature sensor and in separating the high frequency temperature signals from electrical noise. Of the two, the latter proved to be by far the more challenging and frustrating.

Initial noise records were made by placing a cap over the sensor while attempting to leave all other parts of the system unchanged. With the ambient temperature signal thus eliminated, the remaining signal would be due to the noise of the system. The difficulty with this approach is that the operating point of the wire sensor is changed. The wire which was operating in a linear part of the overheat curve (Figure 11) is forced to operate where this curve is most non-linear (ΔT increasing rapidly). This results in a large change in the D.C. level which required either a change in the amplification or a rebalance of the bridge. The noise record obtained in this way was, then, atypical of the system and in fact turned out to be larger than some observed high frequency signals.

At this point use was made of the intermittent nature of the temperature signal itself to provide an evaluation of the noise spectrum. The temperature signal, as was previously

mentioned, at times was very patchy, exhibiting bursts of activity separated by quiescent periods (Figure 16c and d). A close examination of these quiet periods revealed that in many cases there was no sign of the "one-sided" activity typical of temperature on the direct signals and no activity on the differentiated signals. These quiet periods were often accompanied by dropping (slowing) wind (Figure 21). Since there was no apparent temperature signal, these sections might indicate the electrical noise of the system. Examination of several quiet period spectra showed little variation from case to case, quite unlike that associated with varying temperature levels; the resulting spectra then are assumed to represent the system "noise."

An example of the excellent agreement of three "noise" spectra taken from differentiated temperature signals is shown in Figure 22. Two of the spectra 202(2) C and 202(2)G, from the same tape, are almost superimposed for frequencies in excess of 40 Hz. The third spectrum is from a different tape. Since it is the noise at high frequencies that is of such importance to this investigation this agreement is particularly gratifying. The noise spectra also clearly indicate the 60 Hz contamination and its harmonic at 180 Hz which was so evident in Figures 19 and 20. All spectra clearly define the attenuation due to the 2 kHz filter through which these signals were passed prior to digitization.

The noise defined in this manner was subtracted from each of the differentiated temperature spectra. Two spectra,

Figure 21. An Active Temperature Signal Followed by a Quiescent Period.

Figure 22. Spectra of "Noise" as Determined from Quiescent Sections of Differentiated Temperature Signals.

before and after noise subtraction, are shown in Figure 23. In almost all cases the noise and signal were the same magnitude at frequencies of 1 kHz and above, and in some cases at slightly lower frequencies.

Attempts were not made to subtract noise from the three other signals (undifferentiated temperature, undifferentiated velocity and differentiated velocity). In the case of the velocity signals no such corresponding "quiet" sections appear so no "noise" record is available. The undifferentiated temperature signal was used primarily to provide a check on the differentiated signals; in particular on the level at the lower frequency end of the temperature spectrum where noise is not a problem.

Figure 23. Spectra of Differentiated Temperature Signals without and with "Noise" Subtraction.

V. RESULTS

A. VELOCITY SPECTRUM RESULTS

1. Computation of ϵ

a. Indirect Method

An indirect estimate of ϵ can be made from kinematic stress and logarithmic wind profile relations. In conditions of neutral stability the kinematic stress is given by

$$U_*^2 = kz \frac{dU}{dz} \quad (33)$$

where $k = 0.4$ is the von Karman constant (Lumley and Panofsky, 1964, p. 103). Further assume the rate of mechanical production of turbulent energy, $U_*^2 \frac{dU}{dz}$, equals the dissipation rate so that

$$\epsilon = U_*^2 \frac{dU}{dz} . \quad (34)$$

The kinematic stress can also be related to the drag coefficient, C_D , by

$$C_D = \frac{U_*^2}{U^2} \quad (35)$$

where U is the mean wind speed adjusted to a fixed height (Priestley, 1959, pp. 21-11). From Equations (33), (34) and (35),

$$\epsilon = \frac{C_D^{3/2} U^3}{KZ} . \quad (36)$$

A value of C_D for the site and wind conditions has been measured by McBean (1970); this is $3(\pm 0.4) \times 10^{-3}$.

This method of estimating ϵ was considered to be the least reliable of those used because the conditions under which it was derived may not always hold. Further, Eq. (34) may not be strictly valid since the two regions of scale sizes in which production and dissipation occur are probably not distinct but may overlap. Nevertheless, the computation was made to provide an approximate check on values obtained by other methods.

b. "Inertial" Range Method

When the spectrum $\phi(f)$ has been measured, ϵ can be estimated by choosing values of $\phi(f)$ from the best straight line in the frequency range in which the observed form is $-5/3$ and substituting in Eq. (29). A value of 0.48 (see page 8) was assumed for the constant K' .

c. Direct Method

The energy dissipation rate was estimated directly from a summation of scales contributing to viscous dissipation according to Eq. (30).

Table II summarizes the results obtained from the three methods. Best agreement was obtained between the direct [ϵ (area)] and the "inertial" range [$\epsilon(K')$] methods (Figure 24). This is consistent with results obtained by Weiler and Burling (1967) in the sense that they found more consistency between C_D values computed from direct stress and "inertial" range methods than between those estimated from profiles and

TABLE II
COMPARISON OF ϵ AS COMPUTED BY THREE METHODS

Tape No.	Section	U(m/sec)	$\epsilon(C_D)$	$\epsilon(K')$	$\epsilon(\text{area})$
202(2)	A	3.43	42	*	64
	B2	3.39	39	73	69
	D	3.50	43	56	46
	F	3.42	40	99	103
203(1)	A	3.35	38	36	45
	B	3.08	30	33	44
	C	3.00	28	65	82
	D	2.90	24	32	48
209(3)	A	3.86	59	63	62
	B	4.03	67	42	33
	C	3.70	52	137	91
	D	3.68	51	80	57
RCA(1)	A	4.30	80	62	63
	B	4.20	74	104	95
	E	3.50	43	125	100
	F	2.90	24	172	125
RCA3(1)	A	3.65	50	214	184
	B	3.93	63	193	130
	C	4.50	94	195	135

* No velocity spectrum was obtained for 202(2)A due to computer malfunction which prevented computation of $\epsilon(K')$.

Figure 24. Comparison of (a) $\epsilon(\text{area})$ and $\epsilon(C_D)$ and (b) $\epsilon(\text{area})$ and $\epsilon(K')$.

other methods. Whereas there is some scatter in the values, the general agreement is good (i.e., all are of the same order of magnitude).

2. Computation of K'

The value of K' was estimated assuming Eq. (29) was valid and using the "direct" value of ϵ obtained from Eq. (30). Based on 17 estimates (Table III) a value of 0.50 with a standard deviation of ± 0.086 (standard error of the mean = 0.02) was obtained.

This agrees well with the value of 0.49 with standard deviation of 0.040 (standard error of the mean = 0.01) obtained by Pond (1963) for atmospheric boundary layer results. The mean of all values reported previously to 1967 is 0.48 with standard deviation of 0.055 (standard error of the mean = 0.009). The only values that appear anomalous are the values of 0.65 reported by Kistler and Vrebalovich (1966); 0.59 reported by Sheih (1969) and 0.69 reported by Gibson et al. (1970). This last result will be discussed in a later section.

3. Normalization of Spectral Results

a. Velocity Spectra

Velocity spectra were normalized according to Eq. (3). The advantage of normalizing results is that it allows comparison not only between different trials of the same experiment but also between previous researches. Agreement of results indicates that the data were properly recorded and analysed as well as providing a self consistent body of results on which to test theories.

TABLE III
THE ONE-DIMENSIONAL KOLMOGOROV CONSTANT

Tape No.	Section	$\phi(10 \text{ Hz})$	$\epsilon^{2/3}$	K'
202(2)	A	--	16.0	--
	B2	2.58	16.8	0.499
	D	2.21	12.8	0.419
	F	3.17	22.0	0.465
203(1)	A	1.59	12.7	0.41
	B	1.43	12.5	0.397
	C	2.20	18.9	0.410
	D	1.34	13.2	0.366
209(3)	A	2.55	15.7	0.484
	B	1.99	10.3	0.599
	C	----	----	-----
	D	2.82	14.8	0.586
RCA1(1)	A	2.71	15.8	0.476
	B	3.77	20.8	0.510
	E	3.78	21.5	0.560
	F	4.24	25.0	0.611
RCA3(1)	A	5.57	32.4	0.532
	B	5.44	25.7	0.624
	C	6.41	26.3	0.656
Average				0.503
Standard deviation				± 0.086
Standard error of mean				0.021

Velocity spectra of the sections for a specific tape were normalized and plotted on a single diagram (Figure 25 a, b, c, d and e). These groups of spectra were compared by superimposing them on a single plot (Figure 26).

These figures show that the spectra very nearly overlap with the break from the $-5/3$ region occurring near $\log k/k_s = -1$. Since the $-5/3$ region was clearly defined, no attempt was made to extend the spectra to lower wave numbers.

The five groups of velocity spectra presented are very similar. Figure 26 shows that the 202(2) spectra drop off slightly more quickly at high wave numbers. The "universal" curve for velocity spectra determined by Grant et al. (1962) from oceanic turbulence is found to fit very closely to these groups of spectra. The group 202(2) fall below the Grant et al. curve at high wave numbers. Recently Nasmyth (1970) has produced a new approximation to Kolmogorov's Universal function based on very "clean" spectra of oceanic turbulence. The new curve lies very near the old in the low and mid-range of wave numbers, has a slightly sharper "knee" and then falls below the old curve (Figure 27). His spectra were compared with the 202(2) spectra and the Grant et al. spectra (Figure 28). Since no "noise" was subtracted from the 202(2) spectra (or any of the other velocity spectra) and the signal to noise level for the 202(2) runs was the best of the data presented here, agreement over the viscous dissipation region can be taken as partial support for the universal curve proposed by Nasmyth. Those spectra which were not

Figure 25. Normalized Velocity Spectra: (a) 202(2);
(b) 203(1); (c) 209(3); (d) RCA(1); (e) RCA3(1).

Figure 26, Composite of All Normalized Velocity Spectra.

Figure 27. Comparison of Stewart and Grant (1962) Universal Spectrum Function (dashed) with Nasmyth (1970) New Universal Curve (solid). Stewart and Grant Curve has been displaced upward towards right for easier comparison.

Figure 28. Comparison of 202(2) Spectra with Nasmyth (1970) and Grant et al. (1962) Spectra.

as "clean" agree better with the old curve of Grant et al. (1962).

b. Energy Dissipation Spectra

Energy dissipation spectra were normalized according to Eq. (4). Since the log-log plot is notoriously insensitive to differences and discrepancies, the normalized energy dissipation spectra have been plotted on a linear scale. The same sequence has been followed (Figure 29a, b, c, d and e) as with the velocity spectra results. The composite of the energy dissipation spectra is shown in Figure 30.

These results compare very well with those of Pond (1965) and Grant et al. (1962). The dotted lines on Figure 30 are the envelope of the points from Grant et al. (1962). Although there are several points which fall outside the envelope, approximately 70% of them are attributable to the 202(2) group of spectra which was previously noted to differ slightly from the other four groups. Figure 30 is remarkably similar to the results of Pond et al. (1966). Their spectra should be more reliable (statistically) since they were computed from records ten times longer than these records.

Dissipation spectrum 202(2)A was compared with the dissipation spectrum presented by Gibson et al., (1970). Figure 31 (plotted log-log) shows that the level of 202(2)A is lower in the "inertial" range but does not fall as quickly in the dissipation region. Since the 202(2) group

Figure 29. Normalized Energy Dissipation Spectra:
 (a) 202(2); (b) 203(1); (c) 209(3);
 (d) RCA1(1); (e) RCA3(1).

Figure 30. Composite of All Normalized Energy Dissipation Spectra.

Figure 31. Comparison of 202(2)A Energy Dissipation Spectra with Gibson et al. (1970) Energy Dissipation Spectra.

was "cleanest" of the five groups of spectra and shown to agree closely with the Nasmyth "universal" curve then the Gibson et al. curve appears to fall off too quickly at high wave numbers. The energy dissipation rate ϵ as estimated from the dissipation spectra by Eq. (30) is about the same for the two spectra (of Figure 31) since the additional area of the Gibson et al. spectrum in the "inertial" range approximately cancels its area deficit in the "dissipation" range [relative to 202(2)A]. If $\phi(k)$ is too large for the non-normalized Gibson et al. spectrum, an evaluation of K' from Eq. (29) could be too large by the ratio $\phi(k)_{\text{Gibson}}/\phi(k)_{202(2)A}$. Application of this correction results in a value of K' of approximately 0.55 for the Gibson et al. data. Although this is still larger than most previously reported results, it agrees closely with the value of 0.56 estimated by Nasmyth (1970) from his new "universal" curve and is in line with the value of 0.58 predicted by Kraichnan (1968) from his abridged Lagrangian history direct interaction approximation. The value obtained using the 202(2)A data was 0.50.

The reasons for the discrepancies of the two dissipation spectra of Figure 31 are not immediately clear although differences in recording may be responsible. Gibson et al. first band-pass filtered the hot wire anemometer signal between 2 Hz and 2 kHz with active filters (attenuation rate 24 db/octave). The signal was then recorded on an FM tape recorder operated at 7 1/2 inches/sec. At this speed the

frequency response of the recorder extends to 2.5 kHz and the signal to noise ratio is 35 db. The hot wire anemometer data used in this thesis were unfiltered (before recording) and recorded on an FM tape recorder at 60 in/sec at which speed the frequency response of the tape recorder extends to 20 kHz and the signal to noise ratio is 46 db.

High pass filtering at 2 Hz removes the D.C. level and hence information on the mean wind speed at which the anemometer was operating. This can lead to calibration errors which would cause errors in the energy level of the spectra. This could account for the difference in level in the "inertial" range.

The steeper slope found by Gibson et al. at high wave numbers could result from excessive low pass filtering during the recording stage. The anemometer signal was low pass filtered twice; at 2 kHz and 2.5 kHz. Some attenuation can be expected to occur at lower frequencies (since these are the -3 db points) where analyses were made. Further comment on this point would require knowledge of the characteristics of the filter used. This information was not available at the time of this writing.

Other causes for different spectral shapes would involve the anemometer system used, size of sensor and analysis. The DISA model 55D05 used in this work is, in fact, in direct competition with the Thermo-Systems Model 1054A used by Gibson et al. However a significant difference is that they used a linearizer with their system. In general, the

use of linearizers is not recommended for measuring levels of turbulence at low wind speeds. The sensors were only slightly different in dimensions. The analyses were almost identical as regards sampling rate and the low pass filtering used to prevent aliasing. Gibson et al. do not indicate if a noise correction was made to their spectrum. Too much high frequency noise subtracted from their spectrum would cause it to fall too steeply. No noise was subtracted from the velocity spectra used in this study.

B. TEMPERATURE SPECTRUM RESULTS

1. Computation of ϵ_θ

The value of ϵ_θ was estimated from Eq. (32). Results are included in Table IV.

2. Computation of K'_θ

The value of K'_θ was determined from Eq. (31). Values of $\phi_\theta(f)$ were chosen from the best straight line in the frequency range in which the observed form is of $-5/3$. The value of ϵ was that estimated from Eq. (30) [i.e. $\epsilon(\text{area})$]. Errors in estimating ϵ do not seriously affect K'_θ since it enters into Eq. (31) to the $-1/3$ power. An additional point of interest is that K'_θ is estimated from Eq. (31) without involving the absolute value of the temperature spectrum. Since $\phi_\theta(f)$ appears to the first power on the left side of Eq. (31) and ϵ_θ on the right side involves $\phi_\theta(f)$ linearly as an integrand, calibration factors converting volts to $^\circ\text{C}$ cancel out.

Based on 16 estimates (Table IV) a value of K'_θ of 0.81 86 with a standard deviation of 0.079 (standard error of mean = 0.02) was obtained. Since there have been no generally well established values of K'_θ (see Section II-B-4) a comparison with earlier results is difficult. The closest of previously reported values is the 0.78 of Lanza and Schwarz (1966). Private discussion with G. McBean and S. Pond revealed they had obtained values between 0.8 and 0.9 although considerably more variation was encountered by them and their values are not determined by the direct method. The results of Lanza and Schwarz and of Gibson et al. (1970) will be examined more closely in later sections.

3. Normalization of Spectral Results

a. Temperature Spectra

Temperature spectra were normalized according to Eq.(11). Since the universal argument $H(\sigma, k/k_s)$ is a function of two parameters, a single universal plot cannot be made of scalar spectra for all Prandtl numbers. The spectra would lie on a universal curve in the "convection" ($-5/3$ region) and visco-convective (-1 region) subrange but not in the diffusive dissipation region if normalization were carried out according to Eq.(11)(Figure 32). In order to be universal in the visco-convective and diffusive dissipation regions, dimensional analysis shows a second plot of $\phi_\theta(k)k_s^3\nu\sigma^{\frac{1}{2}}\epsilon_\theta^{-1}$ versus $k(k_s\sigma^{\frac{1}{2}})^{-1}$ is necessary (Figure 33). These derivations are discussed by Gibson (1962).

Universal plots in support of these normalization procedures were given by Gibson and Schwarz (1963) for cases of $\sigma = 7$ (temperature fluctuations in water) and $\sigma = 700$ (salt concentration fluctuations in water). Scalar spectra

TABLE IV
THE ONE-DIMENSIONAL SCALAR CONSTANT, K'_θ

Tape No.	Section	ϕ_θ (10 Hz)	ϵ_θ	K'_θ
202(2)	A	6.40(-7)	0.119	0.833
	B2	1.10(-6)	0.0582	0.745
	D	2.50(-8)	0.0491	0.699
	F	1.80(-6)	0.0362	0.903
203(1)	A	8.80(-7)	0.0425	0.723
	B	9.20(-7)	0.0468	0.723
	C	2.15(-6)	0.0479	0.823
	D	9.00(-7)	0.0197	0.715
209(3)	A	1.30(-6)	0.0233	0.791
	B	3.40(-7)	0.0486	0.771
	C	1.70(-6)	0.0331	0.851
	D	5.30(-7)	0.0914	0.826
RCA1(1)	A	1.10(-6)	0.0175	0.861
	B	7.80(-7)	0.0141	0.856
	C	6.60(-7)	0.0133	0.993
	F	6.60(-7)	0.0160	0.895
Average				0.813
Standard deviation				0.079
Standard error of mean				0.021

Figure 32. Normalization of Scalar Spectra for $\sigma \gg 1$ According to Eq. (14). After Gibson (1962).

Figure 33. Alternate Normalization of Scalar Spectra for $\sigma \gg 1$. After Gibson (1962).

measured by several investigators for Prandtl numbers of 0.7 (temperature fluctuations in air), 7 and 700 have been gathered in a universal plot of the second type by Lanza and Schwarz (1966).

Since this investigation deals only with one fluid, normalization of the first type only has been carried out.

Temperature spectra of the sections from each tape were normalized and plotted on a single diagram (Figure 34 a, b, c and d). These groups of spectra were compared by superimposing them on a single plot (Figure 35).

The spectra normalize very well in the "dissipation" region and not quite as well in the " $-5/3$ region." The break from the $-5/3$ region occurs just beyond $\log k/k_s = -1$. Since the $-5/3$ region was clearly defined, no attempt was made to extend the spectra to lower wave numbers.

These spectra demonstrate clearly the shape of the one-dimensional temperature spectrum in air beyond the " $-5/3$ " region, a fundamental objective of this thesis. There is no " -1 " region and the differences between temperature and velocity spectra (Figure 36) are slight. The " $-5/3$ " region of the temperature spectrum extends to slightly higher wave numbers than the velocity spectrum but the fall-off rate is approximately the same. The curves defining the "dissipation" regions appear to parallel each other.

There are few previous measurements with which to compare these data. The results of Gibson and Schwarz, (1963) for example, compare favorably in the " $-5/3$ " region but the

Figure 34. Normalized temperature Spectra: (a) 202(2); (b) 203(1); (c) 209(3); (d) RCA1(1).

Figure 35. Composite of All Normalized Temperature Spectra.

Figure 36. Comparison of Normalized Temperature and Velocity Spectra.

agreement becomes poorer at higher wave numbers. The opposite agreement is provided by Lanza and Schwarz (1966) [Figure 37]. Their spectrum agrees well [compared with 203(1) spectra] at high wave numbers in the "dissipative" region but their data fall below the 203(1) spectra in the " $-5/3$ " region. The similarity at high wave numbers is surprisingly good when one considers the extremely different conditions under which these observations were made [see Section II-A-3(b)]. The fact that their value of K' (0.78) is close to the value obtained here (0.81) is probably fortuitous.

b. Scalar Dissipation Spectra

Temperature dissipation spectra were normalized according to Eq. (12). They are plotted linearly in the same sequence as for the temperature spectra (Figures 38a, b, c and d). The composite of these (Figures 39) illustrates not only the similarity of shape but the moderately narrow envelope in which the points lay.

The temperature dissipation spectra peak lies very nearly at the same wave number ($k/k_s \approx 0.08$) as the velocity spectra peak. However the temperature dissipation spectra are broader. This conclusion is based on the sum total of the results rather than comparing any two individual spectra.

Temperature dissipation spectrum 203(1)D was compared with the temperature dissipation spectrum presented by Gibson et al. (1970). The log-log plot (Figure 40) shows discrepancies of the two spectra which are similar to the discrepancies noted in the velocity dissipation spectra

Figure 37. Comparison of Lanza and Schwarz (1966) Temperature Spectrum with 203(1)C Temperature Spectrum.

Figure 38. Normalized Scalar (temperature) Dissipation Spectra: (a) 202(2); (b) 203(1); (c) 209(3); (d) RCA1(1).

Figure 39. Composite of All Normalized Scalar (Temperature) Dissipation Spectra.

Figure 40. Comparison of 203(1)D Scalar Dissipation Spectrum with Gibson et al. (1970) Scalar Dissipation Spectrum.

comparison [Section V-A-3(b)]. The Gibson et al. "inertial" range level is too high and the fall-off is too steep compared to the 203(1)D spectrum. The result is that the areas under the two spectra (when plotted linearly) are about the same (hence same scalar dissipation rate) but the $\phi_0(k)$ value used to compute K'_θ from Eq. (13) will be too large. Taking this factor into account lowers the Gibson et al. value of K'_θ to approximately 0.79 which is in keeping with the results obtained here.

The reasons for the discrepancies of these temperature spectra are probably the same as the reasons for the discrepancies of the velocity spectra which were discussed in Section V-A-3(b). In addition the dimensions of the two temperature sensors are somewhat different. Gibson et al. used a platinum wire with a diameter of 0.6 μm and length 2 mm. The platinum wire used in this study was 0.25 μm in diameter and about 0.40 mm long. Whereas the frequency response of both wires is more than adequate there may be some wave number limitation encountered by the Gibson et al. wire at the highest frequencies. Noise was subtracted from the temperature spectra presented here. It is not known if a noise correction was made to the Gibson et al. spectrum. As was pointed out for the velocity spectrum case, excessive noise corrections will cause the spectrum to fall too steeply.

C. COEFFICIENT OF EXCESS AND SKEWNESS OF SIGNALS

The coefficient of excess and skewness of derivatives of velocity and temperature were computed for 16 cases. These

same indices were computed also for four cases of "noise."

Coefficient of excess as used here is defined as

$$\text{c.e.} \equiv \left[\frac{\overline{e^4}}{(\overline{e^2})^2} \right] - 3. \quad (37)$$

The term "flatness factor" will be reserved for

$$F \equiv \overline{e^4} / (\overline{e^2})^2. \quad (38)$$

The term "kurtosis" will be avoided since it has at least three definitions in reputable literature. Skewness is defined as

$$S \equiv \overline{e^3} / (\overline{e^2})^{3/2}. \quad (39)$$

1. Velocity derivative

a. Coefficient of Excess

The values of the coefficient of excess (c.e.) for velocity derivatives are listed in Table V.

The c.e. of a random (Gaussian) signal is zero ($F = 3$). The usual values of c.e. quoted for velocity (as measured in relatively low Reynolds number turbulence) are not very much different from zero (Batchelor, 1953). This has led to models of turbulence (Golitsyn, 1962, for example) that depend on the Millionshchikov hypothesis that second and fourth moments of the velocity distribution are related in the same way as those for a Gaussian random variable.

The situation for high frequency components of velocity appears to be markedly different. Pond (1965) found a value of 17 for the c.e. for the velocity derivative of data

TABLE V

COEFFICIENT OF EXCESS AND SKEWNESS OF DERIVATIVES
OF VELOCITY AND TEMPERATURE SIGNALS

Tape No.	Section	Vel. Derivative		Temp. Derivative		Record Length (sec)
		c.e.	S	c.e.	S	
202(2)	A	10.2	-0.743	20.3	-.0132	35
	B2	12.5	-1.22	17.1	+.195	30
	D	18.5	-0.487	21.4	+.140	60
	F	21.2	-2.22	22.6	+.00206	23
203(1)	A	4.74	- .401	13.6	-.0819	60
	B	4.85	- .399	22.8	+.596	60
	C	5.03	- .377	12.2	+.203	60
	D	7.98	- .548	13.4	+.307	60
209(3)	A	21.2	-1.44	32.0	+.315	60
	B	5.25	- .394	13.9	+.171	60
	C	8.59	- .660	30.7	+.673	60
	D	13.6	- .841	15.5	+.0390	60
RCA1(1)	A	30.8	-3.14	18.1	+.286	60
	B	12.5	- .693	11.0	+.251	60
	E	14.2	- .678	12.3	+.257	60
	F	11.3	- .734	6.01	+.0511	60

obtained with a hot wire anemometer. Stewart et al. (1970) reported a range of values of 9.3 to 17.9 for six cases of differentiated velocity. Gibson et al. (1970) found values ranging from 10 to 23 for four cases of velocity derivatives measured over the Atlantic. The range of values obtained here is 4.7 to 30.8 which encompasses all previous values. Evidently a large range of values is possible although values between 10 and 20 seem most frequent. The c.e. is a measure of the "peakedness" of a signal which is clearly related to "intermittency." The scatter of values encountered may be due to a number of effects; it may be dependent on the Reynolds number of the flow, the height (nearness to boundary) at which measurements are made, the stability of the atmosphere or the length of the data record analysed. Unfortunately the data presented here are not sufficient to determine any dependence on such parameters. What these results do, is confirm that the c.e. for high Reynolds number turbulence is greater than zero--usually F [Eq. (38)] is much greater than one--which illustrates that high wave number turbulence is decidedly non-Gaussian in character. Predictions by Novikov and Stewart (1964) for the spectrum of fluctuations of the dissipation rate require that the flatness factor be large compared to 3. These results indicate that this requirement is satisfied for very large Reynolds number turbulence. These results also show that the c.e. of velocity derivatives has a larger range of values than has been previously measured.

b. Skewness

The values of the skewness for velocity derivatives are given in Table V.

Odd moments, such as skewness (normalized third moment), describe the symmetry or lack of symmetry of a signal. A Gaussian signal or any symmetrical signal has zero skewness; these are statistically symmetric above and below their averages. A positively skewed signal has positive excursions that are much larger, but less frequent, than negative ones.

Kolmogorov similarity theory calls for the skewness of the velocity derivative to be an absolute constant, but there is no theory predicting its value. The best value at present may be expected to be obtained from high Reynolds number turbulence data.

There is a direct relation connecting the skewness of the velocity difference at two points separated by a distance r with the Kolmogorov constant K' of the $-5/3$ region of a one-dimensional downstream velocity spectrum. The relation is

$$S(r) = 0.100 (k)^{-3/2} \quad (40)$$

The condition that this relation holds is that the separation r is small enough that the turbulence is locally isotropic and large enough that viscosity is unimportant (Pond et al., 1963). Stewart et al. (1970) found empirically that for such a band of scales $S(r)$ is close to 0.26. This compares with values of 0.31 calculated from K' by Pond et al. (1966) and 0.39 and 0.36 reported by Stewart (1963) as a correction to

some values measured by Gurvich (1960). The value of 0.26 corresponds to a Kolmogorov constant K' of 0.53.

The range of values of $S(0)$ obtained by Stewart et al. (1970) is -0.56 to -0.76 (for six cases). A slightly wider range (-0.44 to -0.85) was obtained by Gibson et al. (1970) for four cases over the Atlantic. They obtained one slightly larger value (-0.89) in a measurement over the Pacific. Nasmyth (1970) computed values that ranged from -0.089 to -0.487 with a mean of -0.237. Whereas the lowest (absolute) value obtained with the present data (-0.37) is similar to the above results, the highest (absolute) values seem excessive. If all values greater than one in absolute value are ignored, the skewness ranges from -0.37 to -0.84 which is in keeping with these previous results. A visual examination of the analog records failed to reveal any obvious differences that could account for the four large values. However three of the four spectra were poorly defined at frequencies less than about 10 Hz. The direct, undifferentiated, temperature signals for these same four cases had a clearly defined "-5/3" region to frequencies as low as 1 Hz which indicates the differentiated signals at some stage became contaminated with low frequency noise. Because of this, these large values probably should be viewed with suspicion for the present. Such a feature however has no influence on the results of earlier sections, since the contribution of such scales to dissipation is negligible.

There is a tendency for large values of c.e. to accompany large (absolute) values of skewness (Figure 41). However such plots can be misleading since the eye tends to place too much weight on the two suspicious upper left point. If these points are ignored, then there appears to be a tendency for the c.e. to be independent of skewness, having a constant value of about -0.7. Clearly the values of skewness and c.e. of these data neither negate nor confirm a particular relation for skewness.

2. Temperature Derivative

a. Coefficient of Excess

The values of the coefficient of excess for temperature derivatives are given in Table V.

The temperature derivative, if anything, is expected to have larger values for the c.e. than the velocity derivative. Gurvich (1967) obtained values of 15 and 1400! Gibson et al. (1970) from four cases, obtained a range of values from 22 to 39. The range of values of c.e. for these data is 6.0 to 32. In the first twelve cases, the values of the c.e. of the temperature derivative exceed those of the velocity derivative. In the last four cases the opposite is true.

A random variable with a very large coefficient of excess is characterized by a few very large positive (or negative) spikes occupying a small fraction of the time. This suggests that in the case of temperature signals, the coefficient of excess is going to depend on both the stability of the atmosphere and the height at which the measurement is made. This

Figure 41. Comparison of Coefficient of Excess and Skewness of Differentiated Velocity Signals.

is because the temperature signal exhibits increasing intermittency (1) with increasing height (at least during unstable conditions) and (2) with increasing instability. Intermittency as used here means the periods of quiescence become longer relative to the active periods. This being the case, the coefficient of excess of temperature signals should (1) increase with height and (2) increase with instability. The results of Gibson et al. (1970) support the former. As regards the latter point, the results presented here do not discredit it. The lowest values of c.e. were obtained in the last four runs RCA(1) which were the least unstable of the four groups. Because of the large changes of T1 - T2 observed (see Section IV-A-2) further inferences on the stability dependence are unwise.

A signal with a large c.e. also becomes difficult to analyse. Gibson et al. (1970) point out that in order to obtain the value of 1400 reported by Gurvich, there must be only about 7 samples of extreme temperature amplitude in 10,000 samples. Clearly it is necessary to take large sample sizes in order to measure a large coefficient of excess with any statistical significance.

These data show that a large range of values may be obtained for the coefficient of excess of temperature derivatives and that this value is in general larger than that for the velocity derivative. The results are similar to, but not as consistently large as, those of Gibson et al. (1970).

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b. Skewness

The values of the skewness for temperature derivatives are given in Table V.

There have been very few measurements made of this quantity. Nasmyth's (1970) values ranged between ± 0.1 with a mean very close to zero. Gibson et al. (1970) obtained a range of values of -0.040 to -0.72 . The range obtained for these data is much larger. The positive values run from approximately zero (random signal) to $+0.67$. There are also two negative values which are very nearly zero. The significance of the difference in sign between these and the Gibson et al. values is not clear since details of their computation were not immediately available.

There is at most only a loose correlation between the skewness and the coefficient of excess (Figure 42). In general the absolute values of these results are somewhat lower than the Gibson et al. (1970) results. No conclusions are drawn from them.

Temperature signals should, according to theory, exhibit zero skewness. These data indicate that the skewness of the temperature derivative is almost certainly not zero. During the final drafting of this thesis a paper was received from Wyngaard (1970) in which he showed that although the velocity sensitivity of a resistance wire thermometer is completely negligible for most purposes, it causes the measured skewness of the temperature derivative to be large and positive. He suggested that the values given by Stewart (1969) and

Figure 42. Comparison of Coefficient of Excess and Skewness of Differentiated Temperature Signals.

Gibson et al. (1970) for skewness measured with fine resistance wires in the atmosphere could have been the result of contamination from velocity sensitivity in the measurement of a locally isotropic temperature field. Since Stewart used data obtained with the temperature measuring system described in this thesis an evaluation of the significance of Wyngaard's comments is of interest.

Basing estimates on typical values of experimental parameters from the Gibson et al. (1970) study he found the measured skewness due to velocity sensitivity alone to be about 0.6. Since the values reported by Stewart (1969) and Gibson et al. (1970) were in the range 0.4 to 1.0 the implication of Wyngaard's calculation is readily apparent. It is of interest to repeat Wyngaard's calculations using the parameters applicable to this study. Following Wyngaard's notation, the superscript "m" will be used to indicate quantities determined from experimental measurements. The skewness may then be written

$$S^m = \frac{\overline{(\partial\theta/\partial x)^3}^m}{\left[\overline{(\partial\theta/\partial x)^2}^m\right]^{3/2}} \quad (41)$$

The numerator (3rd moment) was shown by Wyngaard to involve non-linear interactions between temperature and velocity derivatives, namely

$$\overline{\left(\frac{\partial\theta}{\partial x}\right)^3}^m = -3c \left[\frac{\partial\theta}{\partial x} \frac{\partial\theta}{\partial x} \frac{\partial u}{\partial x} \right] - c^3 \overline{\left(\frac{\partial u}{\partial x}\right)^3} \quad (42)$$

where c is the velocity sensitivity in $^{\circ}\text{C} \cdot \text{sec} \cdot \text{cm}^{-1}$ of the resistance wire thermometer. Since c is small the second term on the right of Eq. (41) may be neglected so that

$$\overline{\left(\frac{\partial \theta}{\partial x}\right)^3}^m = -3c \overline{\frac{\partial \theta}{\partial x} \cdot \frac{\partial \theta}{\partial x} \cdot \frac{\partial u}{\partial x}} \quad (43)$$

Using a form of the temperature spectrum proposed by Pao (1965), the triple correlation term may be expressed as

$$\overline{\frac{\partial \theta}{\partial x} \cdot \frac{\partial \theta}{\partial x} \cdot \frac{\partial u}{\partial x}} = -0.48 \epsilon_{\theta} \epsilon^{1/2} \kappa^{-3/2} \quad (44)$$

where for ease of calculation the three dimensional Kolmogorov constant has been taken as one. All terms on the right of Eq. (44) are available so that the triple correlation can be easily computed. Since c is known experimentally to be $5 \times 10^{-5} \text{ } ^{\circ}\text{C} \cdot \text{sec}/\text{cm}$ (page 34) then the third moment of the temperature derivative can be estimated from Eq. (44).

From the relation

$$\epsilon_{\theta} = 6\kappa \overline{\left(\frac{\partial \theta}{\partial x}\right)^2} \quad (45)$$

the final term necessary to compute the velocity induced skewness from Eq. (41) can be found. These results are presented in Table VI.

Clearly, the velocity sensitivity has an important effect on temperature derivative statistics. In the results presented here it is of the same magnitude in almost one-third of the cases and small, but non-negligible, in the remaining cases.

TABLE VI

VELOCITY INDUCED SKEWNESS OF TEMPERATURE DERIVATIVE SIGNALS

Tape No.	Section	ϵ_θ	$\epsilon^{1/2}$	$\overline{\left(\frac{\partial \theta}{\partial x}\right)^3}^m$	$\left[\overline{\left(\frac{\partial \theta}{\partial x}\right)^2}^m\right]^{3/2}$	Velocity Induced Skewness
202(2)	A	1.2 (-1)	8.0	8.3 (-4)	3.4 (-2)	2.5 (-2)
	B2	5.8 (-2)	8.3	4.2 (-4)	1.2 (-2)	3.6 (-2)
	D	4.9 (-2)	6.8	2.9 (-4)	8.9 (-3)	3.2 (-2)
	F	3.6 (-2)	10	3.2 (-4)	5.7 (-3)	5.6 (-2)
203(1)	A	4.3 (-2)	6.7	2.5 (-4)	7.2 (-3)	3.4 (-2)
	B	4.7 (-2)	6.6	2.7 (-4)	8.3 (-3)	3.2 (-2)
	C	4.8 (-2)	9.1	3.8 (-4)	8.6 (-3)	4.4 (-2)
	D	2.0 (-2)	6.9	1.2 (-4)	2.3 (-3)	5.2 (-2)
209(3)	A	2.3 (-2)	7.9	1.6 (-4)	2.9 (-3)	5.4 (-2)
	B	4.9 (-2)	5.7	2.3 (-4)	8.8 (-3)	2.7 (-2)
	C	3.3 (-2)	9.5	2.7 (-4)	4.9 (-3)	5.6 (-2)
	D	9.1 (-2)	7.6	6.0 (-4)	2.3 (-2)	2.7 (-2)
RCAI(1)	A	1.8 (-2)	7.9	1.2 (-4)	1.9 (-3)	6.4 (-2)
	B	1.4 (-2)	9.8	1.2 (-4)	1.4 (-3)	8.6 (-2)
	E	1.3 (-2)	10	1.2 (-4)	1.3 (-3)	9.1 (-2)
	F	1.6 (-2)	11	1.6 (-4)	1.7 (-3)	9.4 (-2)

Only weak inferences may be drawn about the temperature derivative signal in this light. Apparently it may have a rather large skewness but not always. The scatter in the results suggests that perhaps the sample is too small. Since higher derivatives take longer to converge (longer in the ensemble sense), it may be necessary to analyse longer sections of data or average over a large number of records before one can be more definite about the skewness of the temperature derivative. Because of Wyngaard's criticism, previous values of skewness of temperature derivatives should be viewed with scepticism until the velocity sensitivity effect has been accounted for.

3. Noise Derivative

The temperature derivative "noise" signal (Section IV-F) should be expected to have values of c.e. and skewness indicative of a random signal. Both should be very small or zero. These indices were calculated for the temperature derivative "noise" for one section of each of the four tapes. The very small values obtained (Table VII) show that these signals have much different statistics than the turbulent signals and may be justifiably treated as noise.

TABLE VII
COEFFICIENT OF EXCESS AND SKEWNESS OF
TEMPERATURE DERIVATIVE "NOISE" SIGNAL

Tape No.	Section	c.e.	S	Record Length (sec)
202(2)	G	+0.511	-0.0187	9
203(1)	D'	+0.249	+0.0448	7
209(3)	B'	+0.383	+0.0357	16
RCA1(1)	H	-0.0992	+0.00545	5

VI. SUMMARY AND CONCLUSIONS

A. REVIEW OF OBJECTIVES

The objectives of this investigation were to provide additional information on the high wave number velocity (energy) spectrum and to provide new information on the high wave number temperature spectrum. The former involved (1) computing normalized energy spectra from velocity data and comparing them with previously measured energy spectra and (2) providing additional evaluations of the one-dimensional Kolmogorov constant K' .

The new investigations of the temperature spectrum involved the development of a sensor and equipment to measure high wave number temperature fluctuations and analysis of the observed data (1) to determine the shape of the one-dimensional temperature spectrum and (2) to evaluate the scalar constant K'_θ .

B. TEMPERATURE FLUCTUATION MEASUREMENTS

The measurement of temperature fluctuations presents no conceptual difficulties and there are several ways of measuring them. However, to measure temperature fluctuations at frequencies much greater than 10 Hz in the atmosphere presents special practical difficulties. The signal is often small which means the electronic noise of the detection system becomes of paramount importance. The sensor must be small (and, as a result, very fragile) to explore the

frequencies of interest yet it must operate in exposed atmospheric conditions. These were the primary difficulties that were overcome to make the temperature fluctuation measurements.

1. The Sensor

To my knowledge a wire with a diameter as small as $0.25\text{ }\mu\text{m}$ has not been used to measure temperature fluctuations in the atmosphere elsewhere. Such a thin wire had been considered to be impractical. Hinze (1959, p. 94) states that $2.5\text{ }\mu\text{m}$ is considered a practical lower limit for the diameter of platinum wires. There was some doubt that even if the sensor could be constructed, the wire could not withstand atmospheric wind conditions. Extensive experience now has been obtained with wires of this size in a variety of conditions. One wire was used over a desert (near Richland, Washington) for 6 hours in winds ranging from 4 to 7 m/sec. At this same site, on a separate occasion, another wire remained intact for over an hour in a sand storm with winds of 12 m/sec. These wires also have been used successfully to measure temperature fluctuations over the ocean at the Spanish Banks field site operated by I.O.U.B.C. At this location a single wire has lasted for several days. At Boundary Bay no breakages were encountered once the probe was mounted on the mast. It appears then, that a properly constructed wire of these dimensions may be operated under the same conditions that a $2.5\text{ }\mu\text{m}$ hot wire would be expected to operate. However, this wire compares unfavorably to hot

wires in that it is very susceptible to breakage by jarring or knocking. Extreme caution must be exerted not only in mounting such probes but also in ensuring that no jarring of the support occurs after mounting.

2. Electronics

The system used to convert resistance changes of the wire to useable voltage fluctuations was derived from three previous systems two of which I developed during the course of this research. Each of these operated with sensors of the same size to detect temperature fluctuations. However their noise levels, each lower than the preceding design, were too large to allow unambiguous definition of the shape of the temperature spectrum. Only the system described in this thesis met with unqualified success; this was developed in cooperation with National Electrolab Associates who have since built similar systems which are used (McBean, 1970) and which may be purchased commercially. Further research on an electronic detection system could undoubtedly lead to designs with even lower noise levels. This should probably be done if further statistical properties of temperature signals are to be investigated. Ideally one would like to recover the "universal" temperature spectrum presented in this work without the necessity of subtracting noise.

3. Characteristics of Temperature Signals

A cursory visual examination reveals that temperature signals differ significantly from velocity signals. The temperature signal tends to be one-sided with active warm

periods separated by cool quiescent periods. These features appear to be functions of both stability and height. The active periods could be associated with the passage of plumes. Often, though not always, a quiet section of the temperature signal corresponds to a gradual slowing of the mean velocity accompanied by a slight drop in the down stream turbulence level. Qualitative inspection of temperature records in convection by Webb (page 206, Lumley and Panofsky, 1964) revealed a similar phenomenon; in rising air the traces of both temperature and vertical velocity are active, in sinking motion the traces of vertical velocity are still arrive but the temperature signal is quiet. Clearly both horizontal and vertical velocity fluctuations contribute to fluctuations of temperature. A detailed examination of these correlations would be worthwhile.

Data collected during the last runs, when the atmosphere was approaching stable conditions, showed less of the intermittency associated with the earlier runs. The velocity and temperature records, though by no means identical, became more similar as the stability increased.

The intermittent nature of temperature signals warrants special investigations. An intermittency factor could be introduced and examined as a function of stability and height. In addition, active and quiet periods could be examined separately both for their spectra and statistics. Finally, correlations with other flow parameters, such as vertical and horizontal velocity just mentioned, could be made.

C. UNIVERSAL RESULTS

1. Velocity

The velocity spectra results agree well with the classical results of Grant, Stewart and Moilliet (1962) and Pond, Stewart and Burling (1963). In addition, they extend the velocity spectrum for measurements in air to slightly higher wave numbers. However of more interest is the fact that the best spectra agree more closely with the universal curve suggested by Nasmyth (1970) than with the classical results. Since the Nasmyth curve is steeper in the dissipation region than the previous best universal curve, then the energy dissipation rate may have been overestimated in the past. If this is the case, the value of the one-dimensional Kolmogorov constant K' previously estimated may be too low. The mean value obtained with these data was 0.50. A slightly higher value, as suggested by Nasmyth, would not be surprising.

In order to determine the value of K' more accurately, a very carefully controlled low noise experiment is required. Furthermore the experiment should be repeated under a variety of stability conditions in order to resolve questions concerning the "universality" of the "constant" K' .

2. Temperature

The temperature spectra results are considered to be the most important aspect of this research. They show the previously unknown shape of the one-dimensional temperature spectrum in air at wave numbers beyond the $-5/3$ region. These spectra show that in air there is no -1 region and that

temperature and velocity spectra are very similar. The temperature spectrum falls off from the $-5/3$ slope at slightly higher wave numbers than the velocity spectrum. This conflicts with Corrsin (1951) and Obukhov (1949) who believed that the ratio of temperature microscale $\eta_\theta = (\kappa^3/\epsilon)^{1/4}$ to Kolmogorov microscale $\eta = (\nu^3/\epsilon)^{1/4}$ should be given by $\eta/\eta_\theta = \sigma^{3/4}$. Since for air $\sigma < 1$, then η should be less than η_θ which implies temperature spectra should not extend as far as velocity spectra. The shape at high wave numbers of the spectra obtained with these data is similar to the shape at high wave numbers of the spectrum obtained by Lanza and Schwarz (1966).

With the exception of the Gibson et al. (1970) spectrum no previous data are available for comparison with the temperature dissipation spectra. Since there were several questions regarding the methods used in recording their data, lack of agreement with those results is not considered to be serious. However it would be wise to be cautious until one sees how each stands up to results of future measurements.

The value of K'_θ computed from these data was 0.81. This value is probably the most accurate of those evaluated up to this time. It is the first evaluation in which direct measurement was made of all parameters that enter into the calculation of it. This constant has been sought for some time and its evaluation is considered a major contribution of this research.

Again a cautionary note is in order. This value should be regarded as valid for air in slightly unstable conditions. Whether or not it remains a constant under more varied meteorological conditions remains to be seen. This would depend strongly on how the temperature spectrum was affected by non-neutral conditions. If the spectrum was altered at frequencies below 10 Hz, which seems likely, then the value of K'_0 should remain essentially unaltered. If non-neutral conditions made their effect felt at higher frequencies where the major contributions to ϵ_0 occur then the value could alter appreciably.

There has been a wide range of values of K'_0 reported in the literature. For the most part experiments have been performed by competent workers and flaws in their methods are not in evidence. Perhaps greater emphasis should be given to the role of the Prandtl number in these experiments, the suggestion being that K'_0 could have a Prandtl number dependence which is not evident from dimensional arguments.

D. STATISTICS OF VELOCITY AND TEMPERATURE DERIVATIVES

Only general conclusions can be drawn from the computed values of coefficient of excess and skewness of the velocity and temperature derivatives. The large (absolute) values obtained serve once again to illustrate the highly non-Gaussian character of high wave number, high Reynolds number turbulence. No constant values were obtained; on the contrary a wide range of values was indicated. In general the

coefficient of excess of the temperature derivative exceeds (but not always) that of the velocity derivative.

These studies point out that the statistics of high wave number turbulence deserve a rather careful, thorough investigation. The investigation should take into account not only dependences on Reynolds number, nearness to boundary and fluid stability but also look into the effects, if any, of analysis procedures (sampling rate, filtering) and sensor system response (velocity sensitivity, phase change and amplitude attenuation of signals) on values of coefficient of excess and skewness.

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