RAYLEIGH SCATTERING CROSS-SECTIONS OF NITROGEN AND ARGON

by

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ABSTRACT

Rayleigh Scattering from neutral nitrogen and argon at room temperature has been studied using a 12 megawatt Q-switched pulse ruby laser. The scattering angle was chosen to be 90 degrees from the incident beam. The relative differential scattering cross-section and the pressure dependence of the scattered signal of the scattering media were determined.

Measurements of the absolute differential cross-section of nitrogen and argon were also obtained. I found that the results agree very satisfactorily with the prediction of the theory within experimental error.

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CHAPTER I

INTRODUCTION

The motivation of this experiment has been two fold. Firstly; to confirm the agreement of the measured absolute differential scattering cross-section of neutral nitrogen and argon with the prediction from the theory. Secondly; to calibrate the scattered noise from the neutral particles of a laser-scattering experiment off a nitrogen or argon plasma.

Rayleigh Scattering has been previously done by numerous groups of Scientists[1]-[5]. It was first reported in 1965 by T.V. George, L. Goldstein, L. Slama and M. Yokoyama[1]-[2] that the absolute differential scattering cross-section of gases measured were approximately twice as large as predicted by the classical theory. Later in 1968, this experiment was redone by R. Rudder and D. Bach[3], who reported that their measured differential cross-sections were in good agreement with the theory. Rudder and Bach also reported that the laser monitoring technique used in George's experiment was not feasible and they had to select a different technique in this part which gave them very good results.

In this experiment, again a different laser monitoring device was used which showed a satisfactory operational result. The detail description of this device will be shown in Chapter III-B.

This thesis is presented in four major sections. The first section shows the general theory of Rayleigh Scattering and the determination of the absolute differential cross-section of gases by means of a tungsten ribbon lamp. Second section describes in much detail the apparatus used in this experiment. Third section shows the operation, the measurements obtained and a comparison of these experimental values with the theoretical values. Conclusion and Discussion are presented in the last section.

CHAPTER II

THEORY

A. General theory of Rayleigh Scattering is briefly reviewed here.

Consider a gas molecule located at the origin of a Cartesian Coordinate System. A monochromatic plane wave (laser beam) polarized in the z-direction is incident onto this particle, and its electric field is

$$\underline{E} = E_o \, e^{iwt} \, \hat{z}$$

Where $\hat{z}$ is the unit vector pointing towards the z-direction. (See Fig. 1)

As the particle (ideal gas particle) is isotropic, the induced dipole moment per molecule is

$$\underline{p}_j = E_o \alpha \, e^{iwt} \, \hat{z}$$

where α is the polarizibility of the Scattering medium. (Subscript "j" indicates j^{th} molecule).

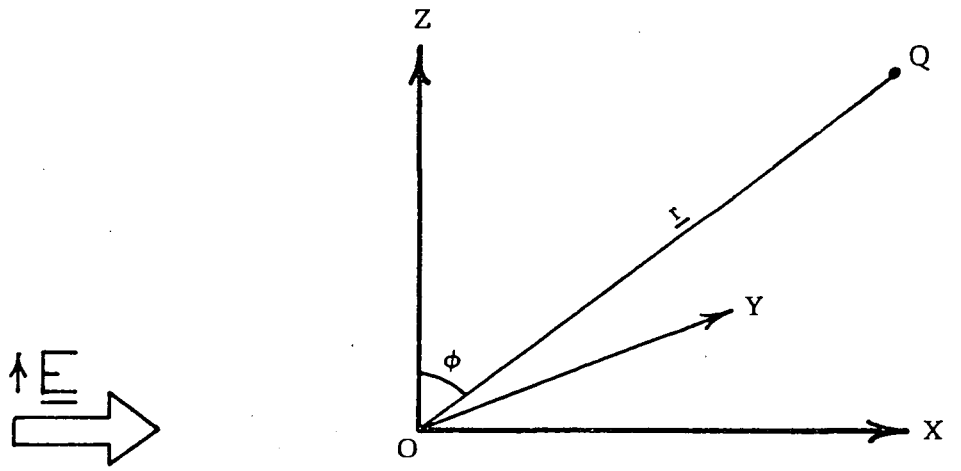


FIGURE 1

THE COORDINATE SYSTEM OF THE RAYLEIGH SCATTERING PROBLEM

If an observer Q is at a distance $\underline{r}$ from the origin and $\underline{r} \gg \lambda$ where λ is the wavelength of the incident monochromatic wave, then the electric field associated with this induced dipole moment is

$$\underline{E}_{sj} = -\frac{E_o k^2}{r} \hat{r} \times (\hat{r} \times \hat{z}) \alpha e^{i(wt - \underline{k}.\underline{r})}$$

where $\hat{r}$ is the unit vector along $\underline{r}$, and $\underline{k}$ is the wave vector.

Let Φ be the angle between $\hat{r}$ and $\hat{z}$, therefore,

$$\hat{r} \times (\hat{r} \times \hat{z}) = \sin\Phi \, \hat{u}$$

where $\hat{u}$ is a unit vector perpendicular to $\underline{r}$.

Therefore,

$$\underline{E}_{sj} = - \frac{E_o k^2}{r} \alpha \, \mathrm{Sin}\Phi \, e^{i(wt - \underline{k}.\underline{r})} \hat{u} \quad \text{---------} \quad (1)$$

Therefore, flux density received by Observer Q due to one molecure is

$$I_j = (\underline{E}_{sj} \cdot \underline{E}_{sj}^{*})$$

where $\underline{E}_s^{*}$ is the complex conjugate of $\underline{E}_s$.

Therefore,

$$I_j = \frac{E_o^2 k^4 \alpha^2}{r^2} \mathrm{Sin}^2\Phi$$

If the molecules do not interact, and if they are randomly distributed in space, the light scattered by a large number of molecules is expected to be completely incoherent, and the total flux density received at Q is the summation of I_j over all molecules inside the scattering volume.

Therefore,

$$I = \sum_{j=1}^{N} I_j = \frac{N E_o^2 k^4 \alpha^2}{r^2} \mathrm{Sin}^2\Phi$$

where N is the total number of molecules inside the Scattering volume.

If the scattering radiation is measured over an area of Δs, then the total flux measured will be

$$P_s = \Delta s \frac{N E_o^2 k^4 \alpha^2}{r^2} \sin^2\Phi$$

But $\frac{\Delta s}{r^2} = \Delta\Omega$, the solid angle subtended by the detecting system.

Therefore,

$$P_s = N E_o^2 k^4 \alpha^2 \sin^2\Phi \, \Delta\Omega \quad \text{----------(2)}$$

The intensity of the incident plane wave is

$$I_o \propto E_o^2 \quad \text{----------(3)}$$

By definition, the differential cross-section of the scattering medium per molecule is

$$\frac{d\sigma}{d\Omega} = \frac{\text{Total scatterd flux per molecule per solid angle}}{\text{Intensity of incident plane wave}}$$

$$= \left[\frac{P_s}{N\ \Delta\Omega}\right] \frac{1}{I_o} \qquad \text{----------(4)}$$

$$= \left[\frac{N\ E_o^2\ k^4\ a^2\ \sin^2\Phi\ \Delta\Omega}{N\quad \Delta\Omega}\right] \frac{1}{E_o^2}$$

Therefore,

$$\frac{d\sigma}{d\Omega} = k^4\ a^2\ \sin^2\Phi \qquad \text{----------(5)}$$

By the Clausius - Mossotti equation (See Schwartz[6]).

$$a = \frac{3}{N_v}\left(\frac{\epsilon - \epsilon_o}{\epsilon + \epsilon_o}\right)$$

where ϵ = permittivity of scattering medium

$= (1 + X_e)\,\epsilon_o$

ϵ_o = permittivity of free space

X_e = susceptibility of scattering medium

N_v = number of particles per unit volume

But the refractive index of the scattering medium in the optical region is

$$n = \left(\frac{\epsilon}{\epsilon_o}\right)^{1/2}$$

Therefore,

$$\alpha = \frac{3}{N_v}\left[\frac{n^2 - 1}{n^2 + 1}\right]$$

And,

$$\frac{d\sigma}{d\Omega} = \frac{9k^4}{N_v^2}\left[\frac{n^2 - 1}{n^2 + 1}\right]$$

As the gasses used obey Ideal Gas Law

$$N_v = \frac{p}{K\,T}$$

where p = pressure of gas chamber

K = Boltzmann constant

T = temperature of gas chamber

= room temperature

Therefore,

$$\frac{d\sigma}{d\Omega} = \frac{9k^4K^2T^2}{p^2}\left[\frac{n^2 - 1}{n^2 + 1}\right] \qquad \text{----------(6)}$$

From this equation, I can see that for a certain monochromatic incident wave at a certain temperature, the

scattering cross-section depends on the pressure as well as the index of refraction of the medium where the latter is a function of the former.

B. Pressure Dependence of Normalized Scattered Signals

Next I will derive the equation that shows the pressure dependence of the normalized scattered signal. Normalized scattered signal is defined as the scattered signal in volt measured across a 50 ohm load resistor from anode to ground of the photomultiplier divided by the peak value in volts of the photodiode laser pulse.

Therefore,

$$\frac{P_s}{P_o} = \text{constant} \times (\text{N.S.})$$

From equation (3) I know that the intensity of the incident beam is

$$I_o = E_o^2$$

Therefore, the power of the incident beam is

$$P_o = E_o^2 A_L$$

where A_L is the cross-section of the beam. From equation (4) I found that

$$\frac{d\sigma}{d\Omega} = \frac{P_s}{N \Delta\Omega} \frac{1}{I_o}$$

Therefore,

$$\frac{P_s}{I_o} = \frac{d\sigma}{d\Omega} N \Delta\Omega$$

Dividing both sides by A, I get

$$\frac{P_s}{I_o A} = \frac{P_s}{P_o} = \frac{d\sigma}{d\Omega} \frac{\Delta\Omega}{A} N$$

= constant x normalized scatterd signal.

From Ideal Gas Law

$$N = \frac{p V}{K T}$$

Therefore,

$$\frac{P_s}{P_o} = \frac{d\sigma}{d\Omega} \frac{\Delta\Omega \; V}{A_L \; K \; T} \; p \qquad \text{----------(7)}$$

For a certain scattering medium in a fixed scattering volume at constant temperature, the normalized scattered signal is a function of Pressure of the gas chamber only.

C. Absolute Calibration of Differential Scattering Cross-Section

To measure the absolute differential cross-section of the gases, I need to find the spectral response of the detecting system by using a tungsten ribbon lamp. Firstly; I have to calculate the flux density radiated from the tungsten lamp which is a grey body of emissivity of .432 at 6943Å and 2200° K. The intensity of a black body radiation (per unit solid angle per unit wavelength) at a particular wavelength is

$$I_{BB}(\lambda) = \frac{2hc^2}{\lambda^5} \left[e^{\frac{hc}{\lambda Kt}} - 1 \right]^{-1} \qquad \text{----------(8)}$$

To find the total intensity radiate to the solid angle subtended by the detecting system over the whole

instrument profile, I have to integrate $I_{BB}(\lambda)$ with respect to the wavelength over the instrument profile and multiply it by $\Delta\Omega$ the solid angle the detecting lens subtends. As the entrance slit and the exit slit of the monochromator is identical, the instrument profile is of triangular shape (See Fig. 12, Chapter IV-E).
Therefore,

$$\int I_{BB}(\lambda)\, d\lambda = I_{BB}(\lambda)\,\Delta\lambda$$

where $\Delta\lambda$ is the instrument width.
Total intensity radiated from the tungsten lamp to the detecting system is

$$I_T(\lambda) = eI_{BB}(\lambda)\,\Delta\lambda\,\Delta\Omega$$

where "e" is the emissivity of tungsten.
With this intensity, the photomultiplier would give corresponding signal in terms of anode current i_A which I measured by means of a known load resistor across the anode and ground. In my case a 10 Kilo-ohm resistor was used.

This information enabled me to obtain the spectral response of the detecting system.

$$S = \frac{i_A}{I_T(\lambda)}$$

$$= \frac{i_A}{e\ I_{BB}(\lambda)\ \Delta\lambda\ \Delta\Omega} \qquad \text{----------(9)}$$

Measurement of $\Delta\lambda$ will be shown in Chapter IV-E.

CHAPTER III

APPARATUS

The apparatus used in this experiment can be divided into four major parts: the ruby laser; the laser monitoring photodiode; the gas chamber and the light detecting system (Fig. 2).

A. The Ruby Laser

The ruby laser used is a TRG model 104A which has a Q-switching unit that can produce a 70 nanosecond pulse of up to a maximum energy of .8 joule. The output of the laser is reproducible to within 10% over an extended period of continuous operation at regular interval of one minute. The front mirror is a 30% reflecting sapphire resonator. The Q-switching unit consists of a rotating prism which performs at a speed of 30,000 r.p.m. The laser beam with diameter of 1 cm. was focused by a lens of 13.6 cm. focal length so as to maximize the detected scattered signal and minimize the stray light noise from the gas chamber. Further discussion on this matter will be seen under "Detecting System" in the same Chapter.

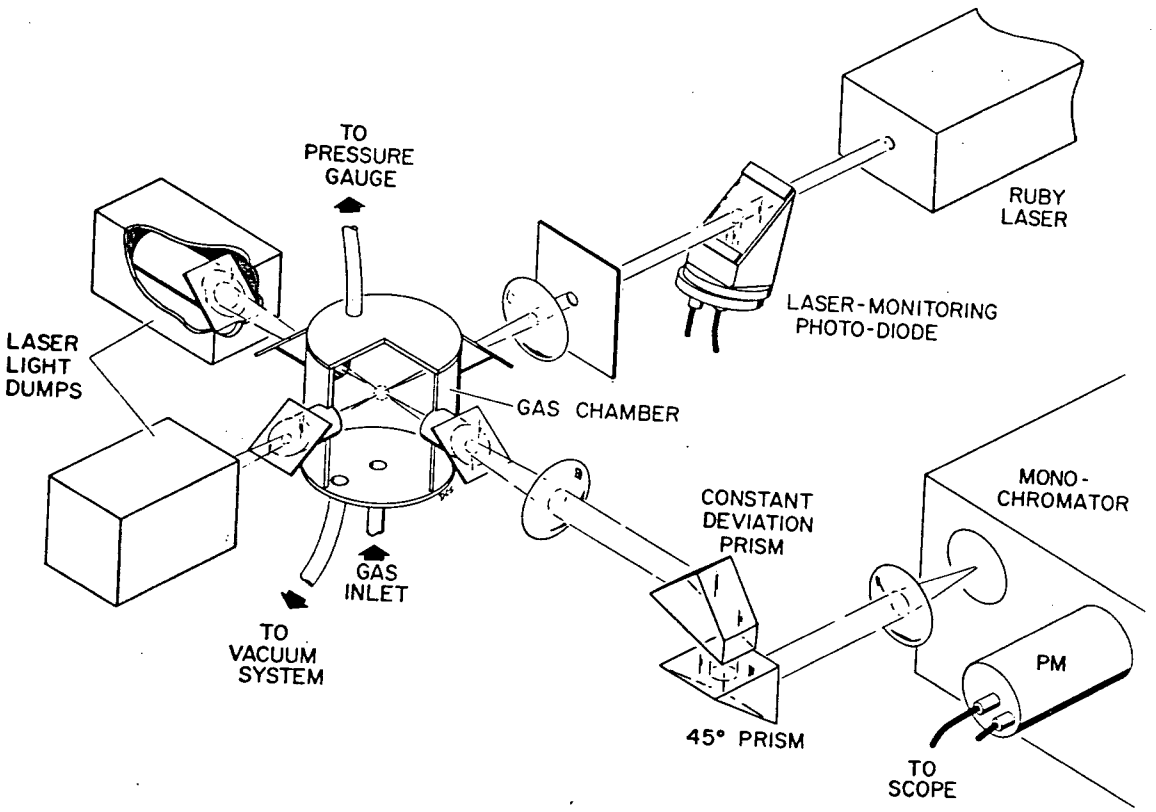


FIGURE 2 ILLUSTRATION OF THE EXPERIMENT

The laser light dump is a glass container with a Brewster window containing a strong copper sulphate solution. This glass container is placed in a black box with its Brewster window facing a small hole on the box big enough to allow the entrance of the laser beam.

The alignment of the laser plays the most important role in the laser output power. To do this, a good telescope is essential. The telescope, which was built by myself, has a set of very thin cross-hair at the common focal plane of the objective lens and the Gauss eye-piece. The Gauss eye-piece allows illumination of the cross-hair using a desk lamp.

The telescope is first aligned with the back mirror of the laser (which is the rotating prism) by the method of parallex of the cross-hair and its image reflected back from the prism. Now the three inch long ruby rod is inserted into the Laser head. By adjusting the position of the rod, the front surface reflection of the illuminated cross-hair is again aligned with the same method. Lastly, the front mirror of the laser is put in and it is also aligned. This method of alignment of the TRG ruby laser has proven to be most satisfactory and more efficient than using a He-Ne gas laser.

B. The Laser Monitoring Photodiode

This device (shown in Fig. 3) consists of a piece of plane glass placed at Brewster angle to the incident beam. It reflects about 4% of the beam towards a piece of ground glass and a neutral density filter of 1% transmission before the light hits the photodiode. This photodiode is a Hewlett Packard part number 5082 - 4220. It is biased by a 100 volt regulated power supply whose circuit diagram is shown in Fig. 4. The diode is operated with the 100 volt power supply, .01pf ceramic capacitor in parallel and a 50 ohm terminator (Fig. 5). It will give a linear signal of up to 3 volts across the 50 ohm terminator which matches the impedance of the transmitting cable and has a rise time of less than one nanosecond.

This monitoring device is calibrated against the energy measured by a TRG model 101 ballistic thermopile which is connected to a TRG model 102 energy meter of part 183 - 15. As the photodiode pulse shows a very consistent Gaussian line shape profile (Fig. 6), the power output of the laser will be the energy measured by the energy meter divided by the half width of the laser pulse. To measure the pulse width to a better accuracy, a fast Tektronix 454 Oscilloscope with rise time of 2.4 nanoseconds is used.

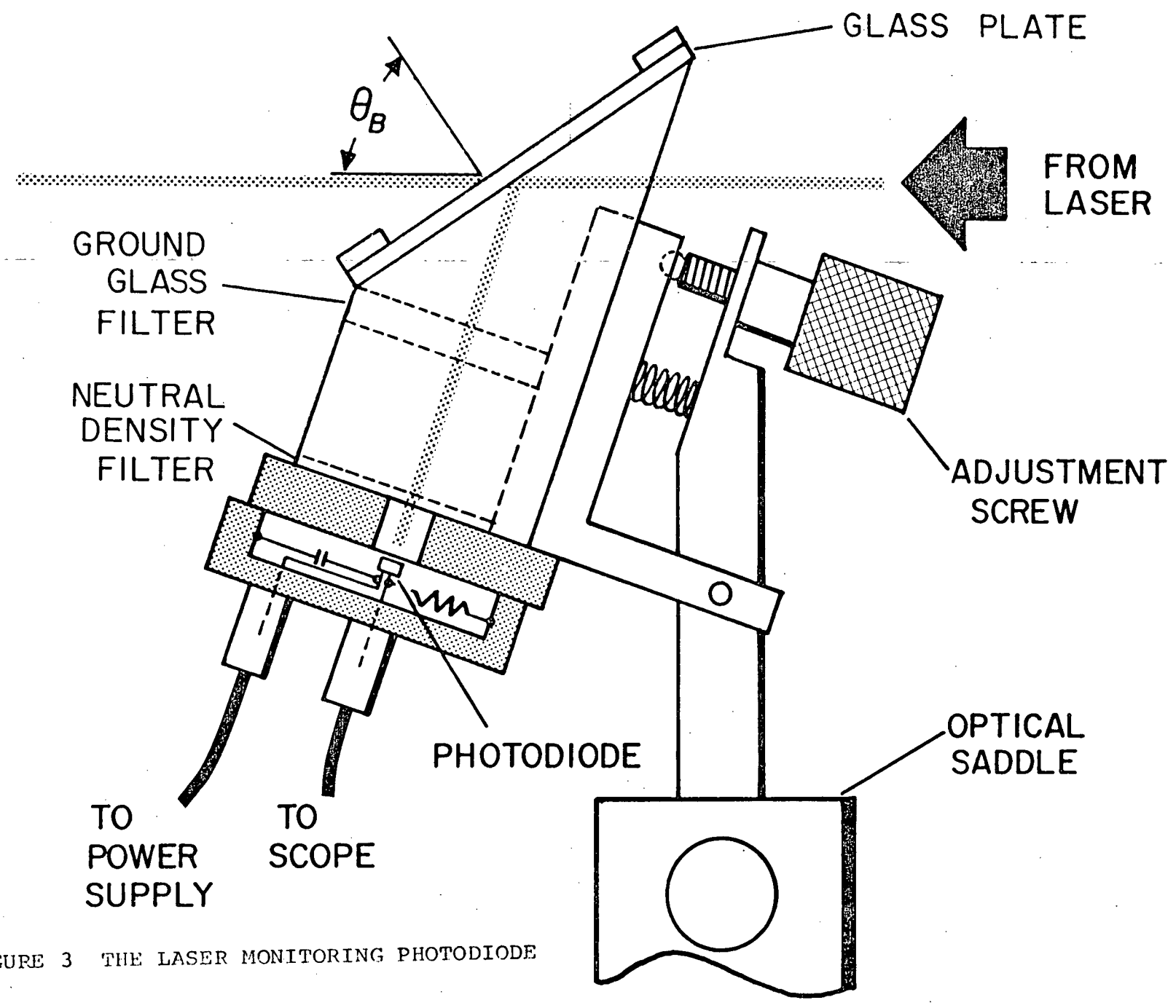


FIGURE 3 THE LASER MONITORING PHOTODIODE

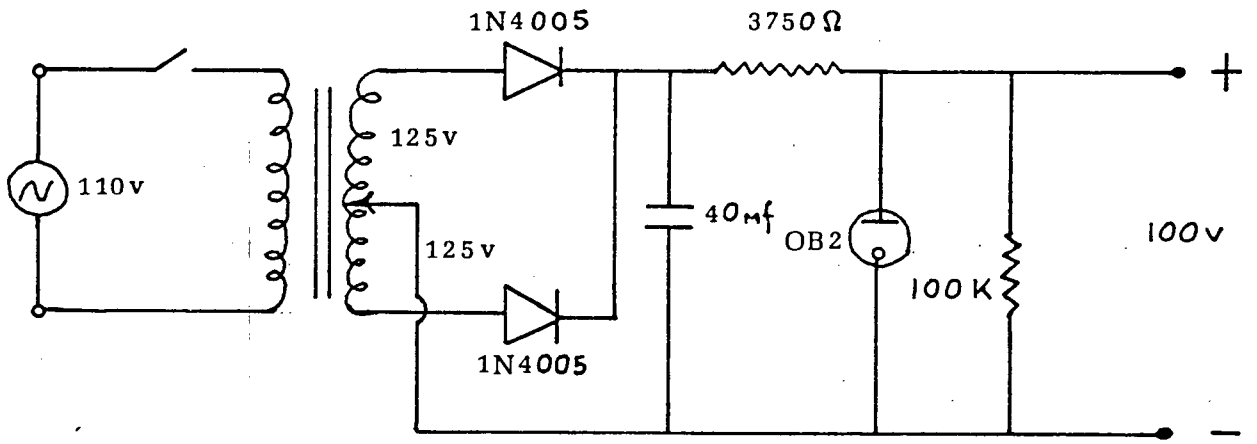


FIGURE 4

CIRCUIT DIAGRAM OF PHOTODIODE POWER SUPPLY

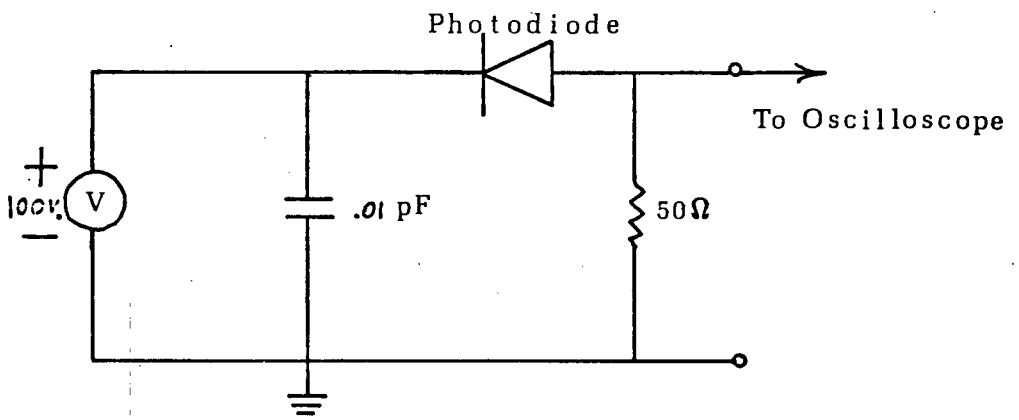


FIGURE 5

PHOTODIODE CIRCUIT

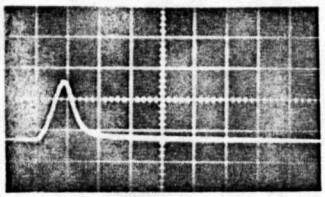


FIGURE 6

OSCILLOSCOPE TRACE OF LASER MONITORING PHOTODIODE PULSE

C. The Gas Chamber

The gas chamber is of four inches in diameter and five inches in height. The four Brewster windows protrude about three inches from the main cavity and are 90 degrees from each other. The chamber is built air tight such that the gases can be filled at controllable pressure. The top of the chamber is connected to a mercury pressure gauge which reads from vacuum to two atmosphere. Underneath the chamber are two outlets: one of which is connected to a vacuum pump and the other to the compressed gas tanks of the various types of gases.

This chamber, whose top can be removed, sits on a four-way adjustable base. This contributes to much easier optical alignment. Detail discussion on this part will be shown in Chapter IV-B.

The focusing lens of the laser and the detecting lens

sits in front of two adjacent windows of the chamber. The remaining two windows are occupied by the two light dumps which will help cut the stray light noise at the detecting system.

D. The Detecting System

This system consists of two lenses, one constant deviation prism, one 45 degree prism, a monochromator, a photomultiplier and an oscilloscope. The two lenses are used to collimate the scattered signal to the entrance slit of the monochromator, or I may say the image of the entrance slit of the monochromator is focused onto the scatttering volume which is situated in the middle of the gas chamber. The constant deviation prism is used to isolate the orders of the monochromator, and it is also used together with the 45 degree prism to rotate the collimated beam by 90 degrees. (This rotation of the beam is only for the convenience of the optical arrangement).

To minimize the stray light noise from the environment, it is best to confine the size of the entrance slit of the monochromator to a minimum without losing much of the

scattered signal, in order to be able to do an absolute calibration of the spectral response of the detecting system using a tungsten lamp, the height of the monochromator slit is limited to less than the width of the tungsten ribbon. Taking into consideration of all these factors, it was determined that the dimension of the slit to be a square pinhole of dimension .57 mm. x .57 mm.

The grating monochromator was built in our laboratory (H.W. Van Andel[7]). It is used in fourth order with a linear dispersion of 4.5Å per millimeter and a theoretical resolving power of 300,000. The effective aperture is limited by the grating and is such that the f-number is about 6.

The photomultiplier used to convert light signal into an electrical signal was a RCA type 7265. It is a fourteen stage tube with an S-20 spectral response which has a quantum efficiency of 3% at 6943Å. The phototube is biased with a total anode to cathode voltage of 1500 volts. The anode to ground load resistor is 50 ohms which matches the impedance of the transmitting cable. The potential distribution across the dynode chain is illustrated in Fig. 7.

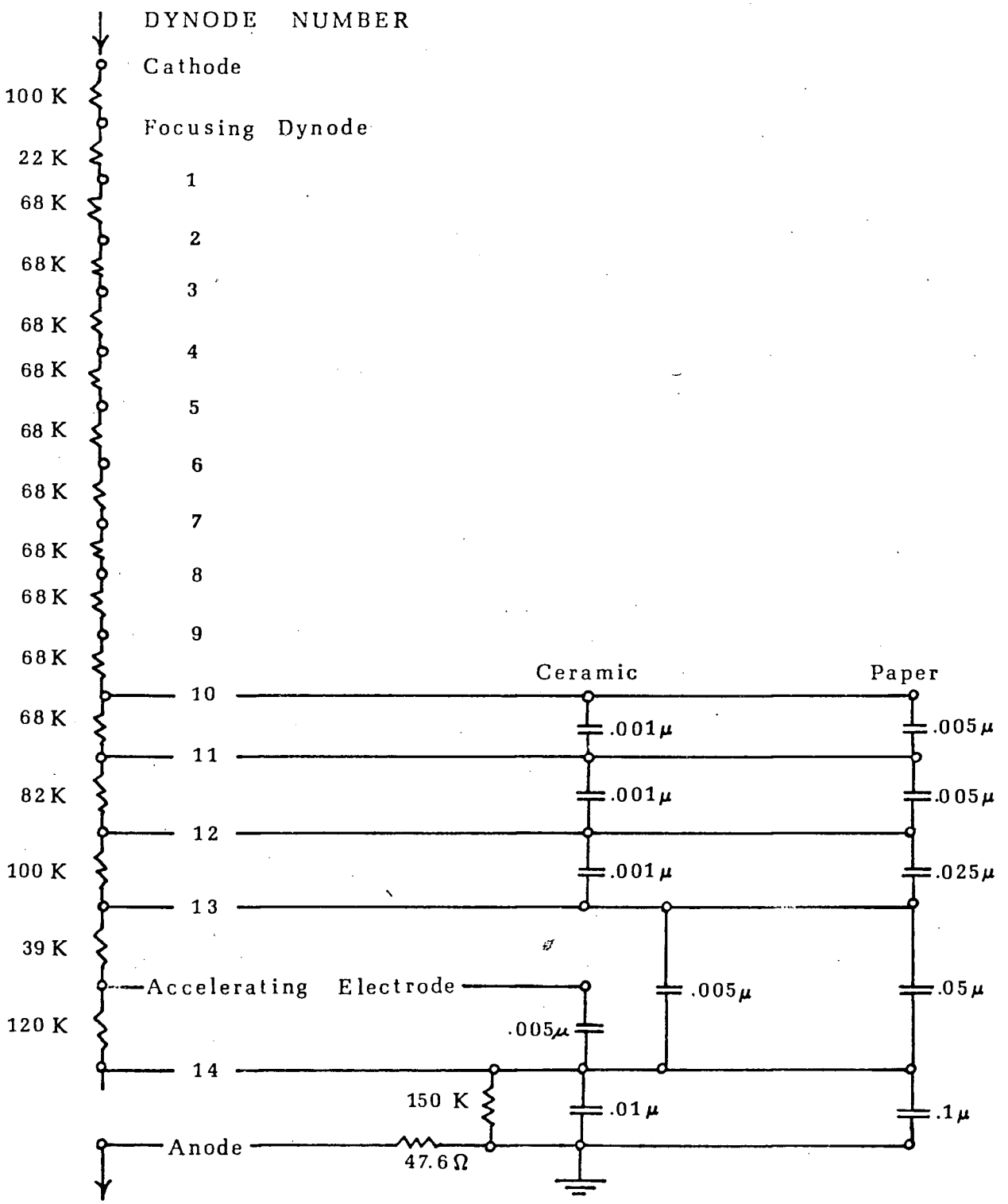


FIGURE 7

POTENTIAL DISTRIBUTION OF THE PHOTOMULTIPLIER DYNODE CHAIN

A Tektronix 551 dual beam oscilloscope together with plug-in units Type L and Type K was used. The photodiode laser pulse is shown on the lower trace of the screen with the Type K plug-in unit and the photomultiplier pulse on the upper trace with the Type L plug-in unit. This oscilloscope together with plug-in Type L has a rise time of 16 nanoseconds while with Type K has a rise time of only 14 nanoseconds. Signals were recorded on Polaroid films of type 410 at a sweeping speed of 100 nanoseconds per division.

CHAPTER IV

PROCEDURE AND RESULTS

A. General Features of the Ruby Laser

The ruby laser can be operated in two modes: namely the normal mode and the Q-switching mode. The normal mode gives maximum energy of 1.5 joules, and the Q-switching mode gives a maximum energy of .8 joule. The Q-switching laser pulse half-width was measured to be 35 nanoseconds which gives a peak power of 20 megawatts.

The laser output power very much depended on the alignment of the laser and the cooling system of the laser head. This cooling system consists of a car radiator with cold water running through it to cool the air which is being pumped by a suction fan to the laser head. The shortest duration between firing the laser is one minute which gives an average power output of 12 megawatts. If the laser was fired at four minute intervals, the output can reach its peak power of 20 megawatts. Because of the time consumed in waiting for the laser head to cool off, a sacrafice on the laser power was necessary. Therefore, it was fired

every one minute.

The Q-switching unit of the laser has a delay time adjustment which synchronizes the pumping action of the flash tube and the position of the rotating prism. This also contributes slightly to the lasing power.

B. Alignment of the Optical System

The alignment of the whole system was done by using a He-Ne gas laser by shining the beam through the exit slit of the monochromator with a diffuser immediately in front of the slit. The light coming out from the entrance slit, after going through the lenses and the prisms, was focused onto the centre portion of the gas chamber where a piece of transparent film is placed to mark the scattering volume. (The removable top of the gas chamber allows the placement of this film). On the other hand, the laser beam is also being focused onto the same scattering region by adjusting the focusing lens.

The next step is to search for the source of stray light. By trial and error, numerous buffers were installed

to cut down this light to its minimum. It was found that the remaining stray light came from the inside wall of the gas chamber which had already been painted dull black.

C. Observation of the Pressure Dependence of the Normalized Scattered Signal

As shown in Chapter II-B equation (7)

$$\frac{P_s}{P_o} = \frac{d\sigma}{d\Omega} \frac{\Delta\Omega \, V}{A_L \, K \, T} \, p$$

where P_s/P_o is proportional to the normalized scattered signal. In a certain set of fixed conditions of the scattering medium, that is for constant $\frac{d\sigma}{d\Omega}$, $\Delta\Omega$, V, A and T, the normalized scattered signal is linearly dependent on the pressure.

To obtain this linear relation, the normalized scattered signals were recorded at five different pressures. Fig. 8 shows the theoretical points and the experimental points with error bars. Each experimental point is an average

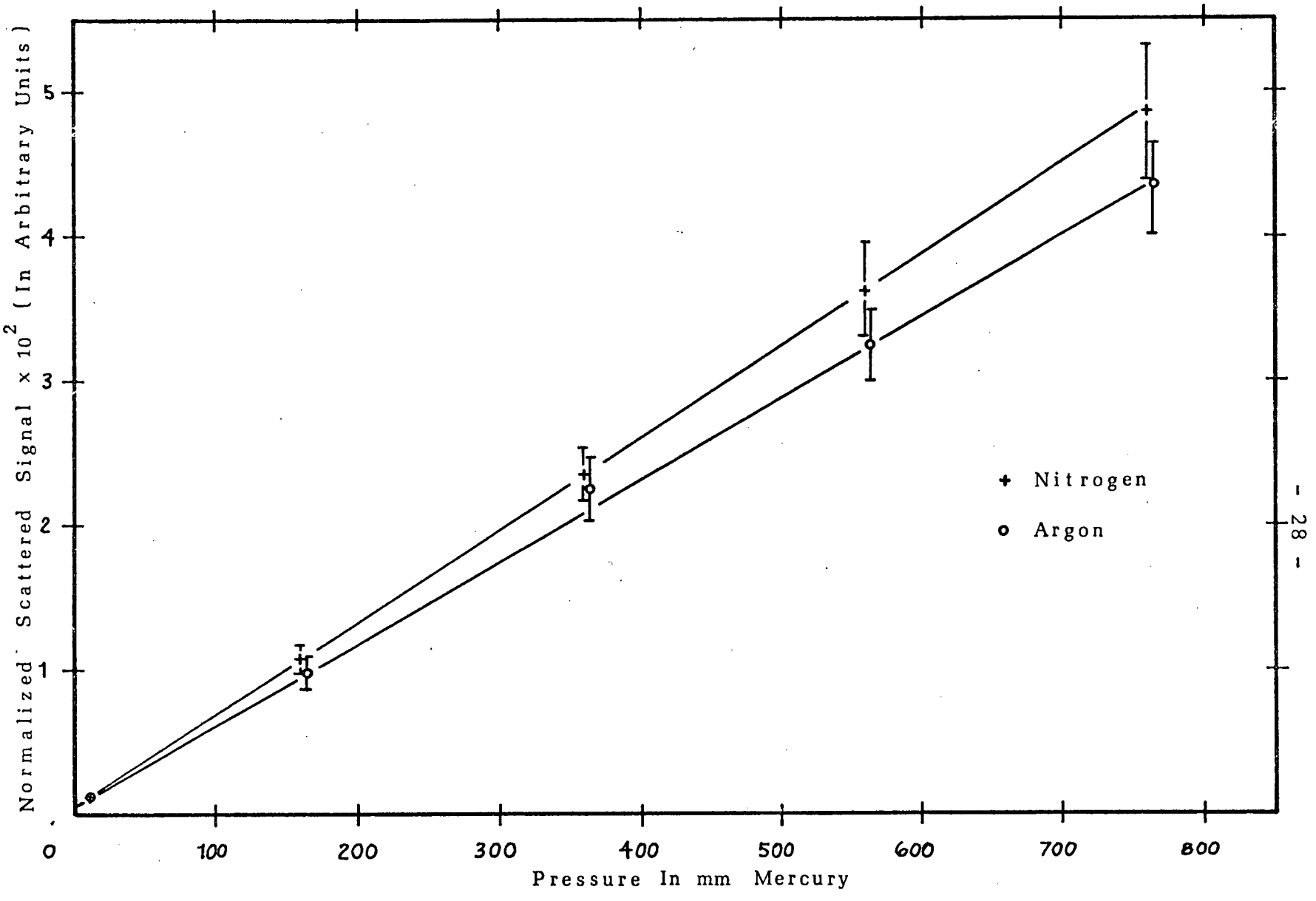


FIGURE 8 PRESSURE DEPENDENCE OF SCATTERED SIGNAL OF N_2 AND Ar

of twelve recordings. The error bar is the root mean square value of the deviation from the mean. The two plots show good agreement. The signal to noise ratio was calculated to be the of the order of 100.

D. Measurement of Relative Differential Scattering Cross-sections

From equation (5) of Chapter II-A, I found that

$$\frac{d\sigma}{d\Omega} = k^4 \alpha^2 \sin^2 \Phi$$

where k is the wave number = $\frac{2\pi}{\lambda}$ (λ = 6943Å , wavelength of the ruby laser); α is the polarizibility of the scattering medium; and Φ is 90 degrees, the angle between the incident electric field and the axis of observation.

By the Clausius - Mossotte equation

$$\alpha = \frac{3}{N_V} \left[\frac{\epsilon - \epsilon_o}{\epsilon + \epsilon_o} \right]$$

where,

$\epsilon = (1 + X_e)\,\epsilon_o$

$= $ permittivity of scattering medium

ϵ_o = permittivity of free space

X_e = susceptibility of scattering medium

N_v = number of scattering particles per unit volume

$= \dfrac{p}{K\,T}$

p = pressure of gas chamber

T = temperature of gas chamber

K = Boltzmann constant

At room temperature and one atmosphere, substitute all the appropriate values in the equation, I get

$$\left(\frac{d\sigma}{d\Omega}\right)_{N_2} = k^4\ a^2_{N_2}$$

$$= 2.11 \times 10^{-32}\ m^2$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{Ar} = k^4\ a^2_{Ar}$$

$$= 1.87 \times 10^{-32} m^2$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{N_2} \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{Ar} = \frac{2.11 \times 10^{-32}}{1.87 \times 10^{-32}}$$

$$= 1.13$$

I have shown that

$$\frac{P_s}{P_o} = \frac{p\ V\ \Delta\Omega}{A_L\ K\ T}\ \frac{d\sigma}{d\Omega}$$

$$= \text{constant} \times (\text{N.S.})$$

where (N.S.) = normalized scattered signal.

Therefore at constant temperature and pressure, and the same detecting system, the normalized scattered signal is linearly proportional to the differential cross-section.

Therefore,

$$\left(\frac{d\sigma}{d\Omega}\right)_{N_2} \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{Ar} = \frac{(\text{N.S.})_{N_2}}{(\text{N.S.})_{Ar}}$$

From experiment:

(N.S.) = measured normalized scattered signal - stray light noise (in arbitrary units)

From Fig. 8

$$(N.S.)_{N_2} = (4.857 - .05) \times 10^{-2} \pm 9.5\%$$

$$= 4.807 \times 10^{-2} \pm 9.5\%$$

$$(N.S.)_{Ar} = (4.332 - .05) \times 10^{-2} \pm 7.4\%$$

$$= 4.282 \times 10^{-2} \pm 7.4\%$$

Therefore,

$$\frac{(N.S.)_{N_2}}{(N.S.)_{Ar}} = \frac{4.807 \times 10^{-2}}{4.282 \times 10^{-2}}$$

$$= 1.12$$

This value shows good agreement with theory's prediction.

E. Measurement of the Absolute Differential Cross-section of Nitrogen and Argon

Before the cross-section can be obtained absolutely I need the following parameters: (i) room temperature, (ii) the load resistance across anode and ground of the photomultiplier, (iii) the size of the entrance slit of the monochromator, (iv) the scattering volume and total

scattered flux, (v) the half-width of the laser pulse, (vi) calibration of the laser power versus the photodiode pulse, (vii) the instrument profile.

(i) Room temperature: this can be easily measured by a thermometer. $T = 296.3^{\circ} K$

(ii) The load resistance across anode and ground of the photomultiplier is $R = 47.6$ ohms.

(iii) The size of the entrance slit of the monochromator was measured by a traveling microscope. $A = .57 \times .57\ mm^2$.

(iv) The scattering volume and the total scattered flux.

Let the laser beam be focused to the origin of a Cartesian coordinate system (Fig. 9). Let "f" be the focal length of the focusing lens, "R" be the radius of the laser beam and "W" be the height of the entrance slit of the monochromator (the beam was rotated 90 degrees).

First we take a small strip of the scattering volume of thickness "dx" and distance "x" from the origin (as shown in diagram). Let "r" be its radius.

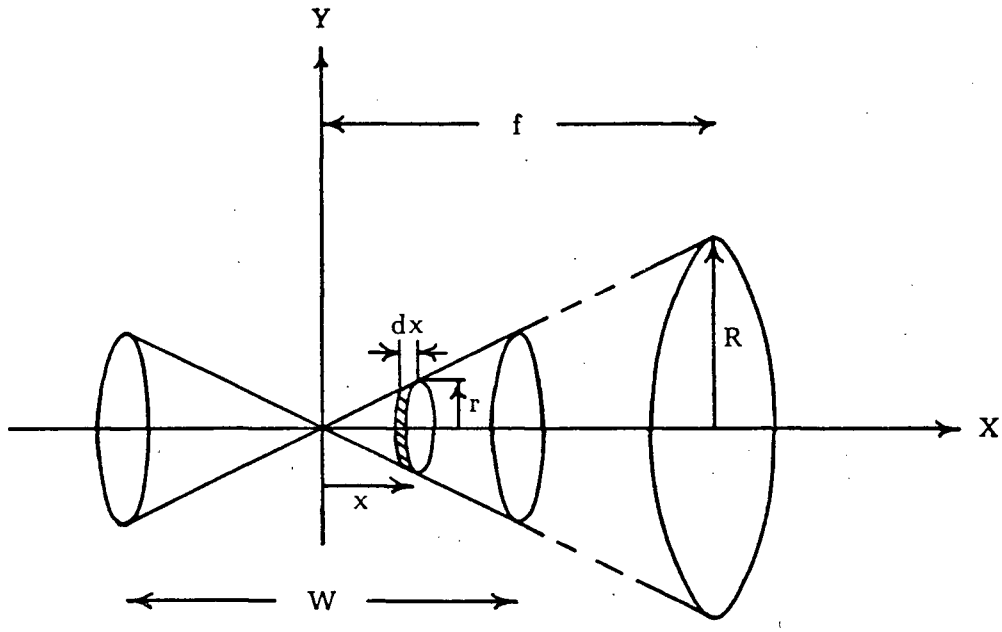


FIGURE 9

ILLUSTRATION OF SCATTERING VOLUME
(Much Exaggeration)

If surface area of strip = A_s

Therefore, volume of strip $dV = A_s dx$

As the gases used obey Ideal gas law, therefore number of scattering particles in this strip is

$$dN = \frac{p\ dV}{K\ T} = \frac{p\ A_s\ dx}{K\ T}$$

From equation (4) of Chapter II-A (For the strip)

$$\frac{d\sigma}{d\Omega} = \left[\frac{dP_s}{dN\ \Delta\Omega}\right] \frac{1}{I_o}$$

where dP_s = total flux radiated from the strip measured by the detecting sytem

Therefore,

$$\frac{d\sigma}{d\Omega} = \frac{dP_s \; K \; T}{p \; A_s \; dx \; \Delta\Omega} \quad \frac{A_s}{P_o}$$

$$dP_s = \frac{p \; P_o \; \Delta\Omega}{K \; T} \quad \frac{d\sigma}{d\Omega} \; dx$$

Therefore the total flux measured over all the scattering volume is

$$P_s = \int_{-\frac{W}{2}}^{\frac{W}{2}} \frac{p \; P_o \; \Delta\Omega}{K \; T} \quad \frac{d\sigma}{d\Omega} \quad dx$$

$$= \frac{p \; P_o \; \Delta\Omega}{K \; T} \quad \frac{d\sigma}{d\Omega} \quad W \quad \text{---------- (10)}$$

From this equation I can see that the scattered flux only depends on the slit width, not the size of the laser beam provided the slit height is not too great.

(v) Half width of photodiode laser pulse.

The monitoring laser pulse was recorded with a sweeping speed of 50 nanoseconds per division using a fast Tektronix 454 oscilloscope which has a rise time of 2.4 nano-

seconds. The laser pulse is of Gaussian line profile. A typical trace is shown in Fig. 6 (Chapter III-B). The pulse showed very good reproducibility when the laser was fired regularly every minute. Therefore the peak value of the pulse is proportional to both the laser power and the laser energy where the former is equal to the latter divided by the half-width of the pulse.

Averaging over ten traces, the half pulse width was determined to be Δt = 35.75 nanoseconds.

(vi) Calibration of the laser power versus the photodiode pulse.

As shown in part (v) that the peak value of the laser pulse is proportional to both the laser energy and the laser power, therefore I could calibrate the linear relation between the laser power and the peak pulse value. The former was measured by a TRG model 101 ballistic thermopile with the measured values divided by the half laser pulse width Δt ; and the latter was recorded by a Tektronix 551 oscilloscope. A plot of the laser energy versus the peak pulse value is shown in Fig. 10.

Therefore E = linear relation between the laser power and the peak laser pulse value

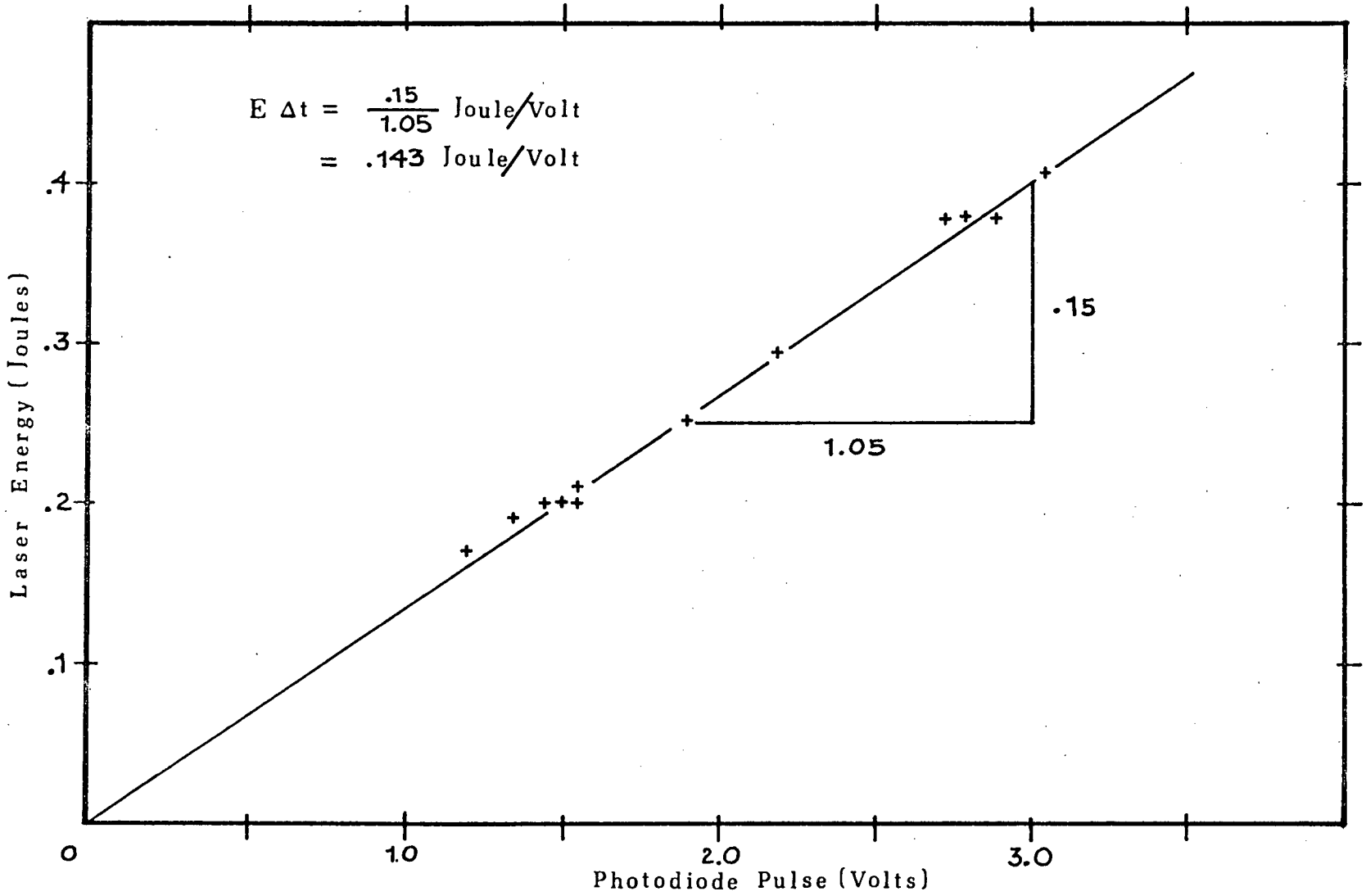


FIGURE 10

CALIBRATION OF LASER ENERGY WITH LASER MONITORING PHOTODIODE PULSE

$$= \frac{.143}{\Delta t} \text{ watts per volt}$$

(vii) Spectral Response of the Detecting System

To determine the exact total scattered flux received at the detector, a calibration of the spectral response of the detecting system is essential. This was done by placing a tungsten ribbon lamp at the centre of the gas chamber with the tungsten filament situated exactly where the scattering took place. Without altering any part of the detecting system the lamp was turned to a certain brightness. I measured the temperature of the lamp with a pyrometer and the anode current of the photomultiplier responded to the flux at 6943Å. To get a better accuracy in the latter measurement, a Fluke model 8100A digital voltmeter was used to measure the anode current across a 10 kilo-ohm resistor.

Temperature of lamp

$$t = 2200^{\circ} \text{ K}$$

Anode current

$$i_A = \frac{30.9}{10 \times 10^{-3}} \times 10^{-3} \text{ amp.}$$

$$= 3.09 \times 10^{-6} \text{ amp.}$$

From equation (9) Chapter II-C

$$S = \frac{i_A}{I_T(\lambda)}$$

$$= \frac{i_A}{e\, I_{BBB}(\lambda)\, \Delta\lambda\, \Delta\Omega}$$

From equation (8) Chpater II-C

$$I_{BB}(\lambda) = \frac{2hc^2}{\lambda^5}\left[e^{\frac{hc}{\lambda kt}} - 1\right]^{-1}$$

At $t = 2200^{\circ}$ K

And $\lambda = 6943\text{Å}$

$$I_{BB}(\lambda) = 6.00 \times 10^{10} \frac{\text{joules}}{m^3 \text{ sec steradian}}$$

e = emissivity of Tungsten at 2200°K and 6943Å

$= .432$

Therefore,

$$S = \frac{3.09 \times 10^{-6}}{.432 \times 6.00 \times 10^{10}\, \Delta\lambda\, \Delta\Omega}$$

$$= \frac{1.19 \times 10^{-16}}{\Delta\lambda\ \Delta\Omega} \text{ m}^2 \text{ amp sec joule}^{-1}$$

(viii) The Instrument Profile

To determine the Spectral Response specifically, I have to know $\Delta\lambda$, the instrument width. (Later I will show that $\Delta\Omega$ will cancel out when finding the absolute cross-section).

This part was done by using a Neon Geissler tube. As both of the slits of the monochromator had identical width, the measurements had shown a very good triangular shape. A typical instrument profile of 6929.5Å is shown in Fig. 11. Several lines in the red region of the neon spectrum were used, namely 6929.5Å , 7032Å , 7174Å and 7245Å. The average half width of the instrument profile was determined to be

$$\Delta\lambda = 4.22\text{Å}$$

A comparison of this instrument profile with the normalized scattering profile of argon at one atmosphere was done. They appeared to be in good agreement (Fig. 12).

With the above parameters available, I am now able to calculate the absolute differential cross-sections of nitrogen and argon.

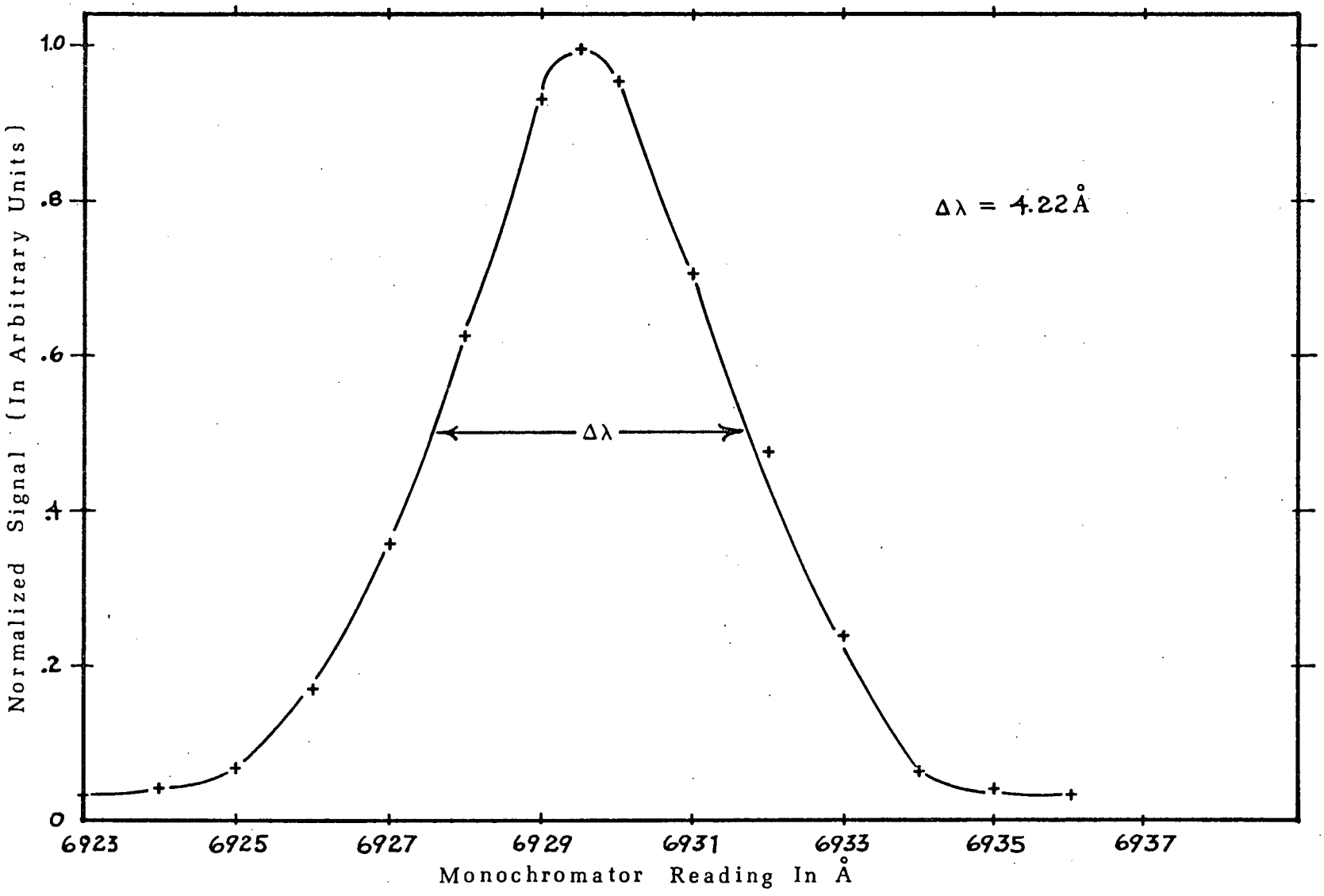


FIGURE 11 INSTRUMENT PROFILE OF NEON LINE 6929.5 Å

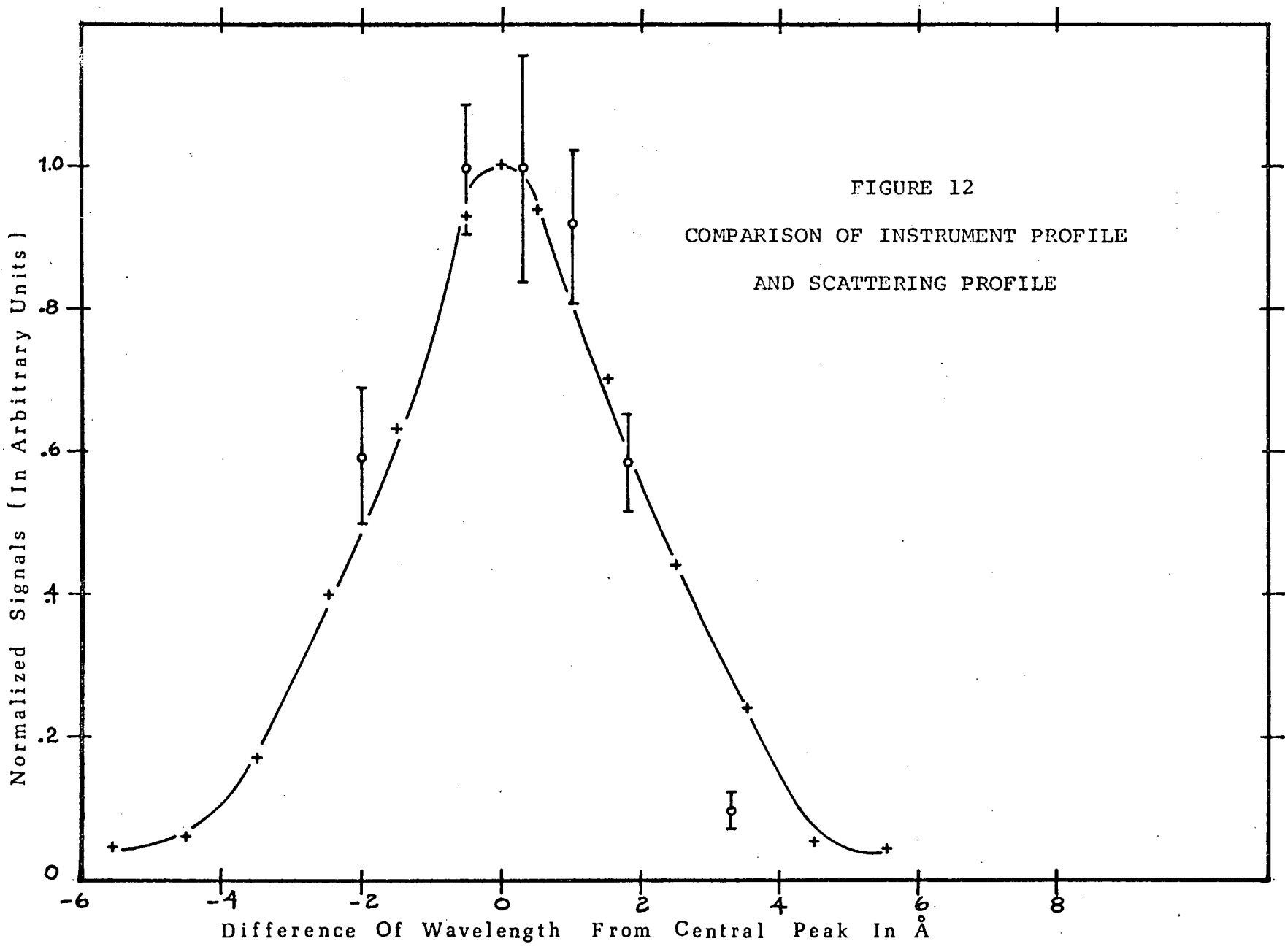


FIGURE 12

COMPARISON OF INSTRUMENT PROFILE

AND SCATTERING PROFILE

From equation (10) Chapter IV-E.

$$P_s = \frac{p\, P_o\, \Delta\Omega}{K\, T} \frac{d\sigma}{d\Omega} W$$

Therefore,

$$\frac{d\sigma}{d\Omega} = \frac{K\, T}{W\, \Delta\Omega\, P} \frac{P_s}{P_o} \qquad \text{----------(11)}$$

The total scattered flux observed at the detecting system is

$$P_s = \frac{\text{scattered signal in volt}}{\text{load resistor from anode to ground of photomultiplier}} \times \frac{\text{area of slit}}{\text{spectral response}}$$

$$= \frac{(S.S.)}{R} \times \frac{A}{S}$$

The total input power of the ruby laser

P_o = the photodiode laser pulse peak value in volt

x linear relation of laser power versus laser pulse

= (L.S.) x E

Therefore,

$$\frac{P_s}{P_o} = \frac{(S.S.)}{(L.S.)} \times \frac{A}{E\ S\ R}$$

But,

$$\frac{S.S.}{L.S.} = \text{Normalized Scattered signal (defined in Chapter II-B)}$$

$$= (N.S.)$$

Substitute this into equation (11), I get

$$\frac{d\sigma}{d\Omega} = \frac{K\ T\ A}{\Delta\Omega\ p\ E\ R\ SW} \times (N.S.) \qquad \text{----------(12)}$$

and substitute for S, the spectral response

$$\frac{d\sigma}{d\Omega} = \frac{K\ T\ \Delta\lambda\ A}{W\ E\ R\ 1.19 \times 10^{-16}} \times \frac{(N.S.)}{p}$$

K = Boltzmann constant in joules per degree Kelvin

T = Temperature of gas chamber in degrees Kelvin

W = Height of monochromator slit in meter

A = Area of slit in square meter = W^2

p = Pressure of gas chamber in newton per square meter

E = Linear relation between laser power and laser pulse in watts per volt

R = Load resistance from anode to ground of photomultiplier in ohms

(N.S.) = Normalized scattered signal (unitless)

Substitute the constant values, I get

$$\frac{d\sigma}{d\Omega} = 4.32 \times 10^{-26} \times \frac{(N.S.)}{p} \ m^2$$

At one atmosphere and room temperature

$$(N.S.)_{N_2} = 4.807 \times 10^{-2} \pm 9.5\%$$

$$(N.S.)_{Ar} = 4.282 \times 10^{-2} \pm 7.4\%$$

$$p = 1.007 \times 10^{-5} \text{ newton } m^2$$

Therefore,

$$\left(\frac{d\sigma}{d\Omega}\right)_{N_2} = 4.32 \times 10^{-26} \times \frac{4.807 \times 10^{-2}}{1.007 \times 10^{5}} \pm 9.5\% \ m^2$$

$$= 2.07 \times 10^{-32} \pm 9.5\% \ m^2$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{Ar} = 4.34 \times 10^{-26} \times \frac{4.282 \times 10^{-2}}{1.007 \times 10^{5}} \pm 7.4\% \ m^2$$

$$= 1.86 \times 10^{-32} \pm 7.4\% \ m^2$$

To compare these values with the theoretical values, I can calculate from equation (5) Chapter II-A

$$\frac{d\sigma}{d\Omega} = k^4 \alpha^2$$

k = Wave number

$$\alpha = \frac{3}{N_v} \frac{\epsilon - \epsilon_o}{\epsilon + \epsilon_o} = \text{Plarizibility of scattering medium}$$

N_v = number of scattering particles per unit volume at one atmosphere and room temperature

$$\alpha_{N_2} = 1.78 \times 10^{-30} \ m^3$$

$$\alpha_{Ar} = 1.68 \times 10^{-30} \ m^3$$

Therefore,

$$\left(\frac{d\sigma}{d\Omega}\right)_{N_2} = 2.11 \times 10^{-32} \ m^2$$

And

$$\left(\frac{d\sigma}{d\Omega}\right)_{Ar} = 1.87 \times 10^{-32} \text{ m}^2$$

The theoretical values and the experimental values show agreement well within experimental error.

I also attempted to measure the scattering cross-section of Helium. Detailed discussion is shown in the Appendix.

F. Error Analysis

From the results I obtained, I can say the experimental values are in good agreement with the prediction from the theory. It was unfortunate that the standard deviation of the values obtained were 9.5% for nitrogen and 7.4% for argon. I suspect that the error was mainly due to the impurities in the gases. Before each sampling of data, the gas chamber was flushed at least six times with the high purity gas (purity of 99.999%) to avoid mixture of impurities. Even with this precaution, it was not possible to get 100% pure

gas. There was further indication showing the big deviation of scattered signal from gas with impurities. A general grade nitrogen which has 99.9% purity was used. The normalized scattered signal measured was at least four times bigger than expected.

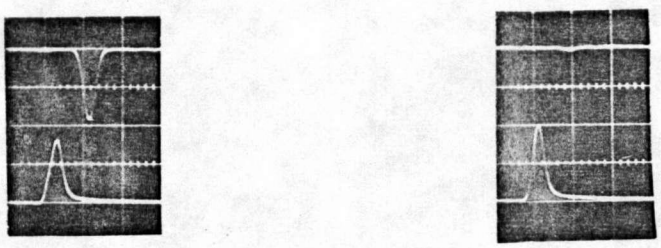
Also from equation (11) of Chapter IV-E I have shown that

$$\frac{P_s}{P_o} = \frac{\Delta\Omega \; W \; p}{K \; T} \; \frac{d\sigma}{d\Omega} \propto (N.S.)$$

Therefore, for any gas at a certain temperature and pressure, the size of the Normalized Scattered signal will increase with the height of the monochromator entrance slit provided the laser beam is narrower than the width of the slit. By focusing down the laser beam, I am able to close down the width of the monochromator to .57 mm. and this helped enormously in eliminating the stray light noise. Unfortunately the height of the slit was limited by the size of the tungsten ribbon lamp filament, therefore the slit was chosen to have both the height and the width equal. To illustrate this part, the detecting system was unfocused from the scattering volume. A comparison of the scattered signals of both focusing case and unfocusing case is shown in

Fig. 13. The lower trace is the monitoring laser pulse and the upper trace is the scattered signal.

Source of stray light was found to be coming from inside the gas chamber. This was tested by means of buffers placed at different regions of the system. The inside wall of the gas chamber was painted dull-black to cut down reflection from the surface. The diameter of this chamber was limited by the focal length of the lenses available at the time it was built. I believe if the chamber size was bigger, the stray light noise would be further decreased.



FOCUSED UNFOCUSED

FIGURE 13

ILLUSTRATION OF SCATTERED SIGNAL

WITH FOCUSED AND UNFOCUSED DETECTING SYSTEM

CHAPTER V

CONCLUSION AND DISCUSSION

A. Conclusion

The measurement of the absolute scattering cross-sections of argon and nitrogen have shown close agreement with the theory's prediction within experimental error.

The main objective of this experiment was to calibrate the scattered signal from neutral particles in a laser-plasma interaction experiment using a nitrogen or argon plasma. The results also confirm the work done by Rudder and Bach.

B. Discussion

This technique could be applied to the measurement of scattering cross-sections of gas mixtures at constant pressure, and study the turbulence and perturbation of particle density of gas flow.

For further work to be done in Rayleigh Scattering, one should consider using a CO_2 laser. There are numerous reasons to choose the CO_2 laser, but the most important reason is its operating power can reach the gigawatt range, and its repetition rate can be much more frequent than the ruby laser. To further improve the experiment, one should also be cautious with the choice of the type of photomultiplier. In this experiment, a RCA 7265 phototube with S-20 response was used. With the new tubes now available in the market, one could easily choose a tube with much better sensitivity and quantum efficiency in the red spectrum.

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APPENDIX

Helium was also used as a scattering medium, but its scattering cross-section is of a factor of 70 times smaller than those of nitrogen. Therefore, the measured values of the cross-section of helium was not satisfactory. Regardless , the pressure dependence of helium scattered signal showed fairly good agreement with the theory (Fig. 14).

The theoretical value of absolute differential scattering cross-section of helium is

$$\left(\frac{d\sigma}{d\Omega}\right)_{He} = .0291 \times 10^{-32} \ m^2$$

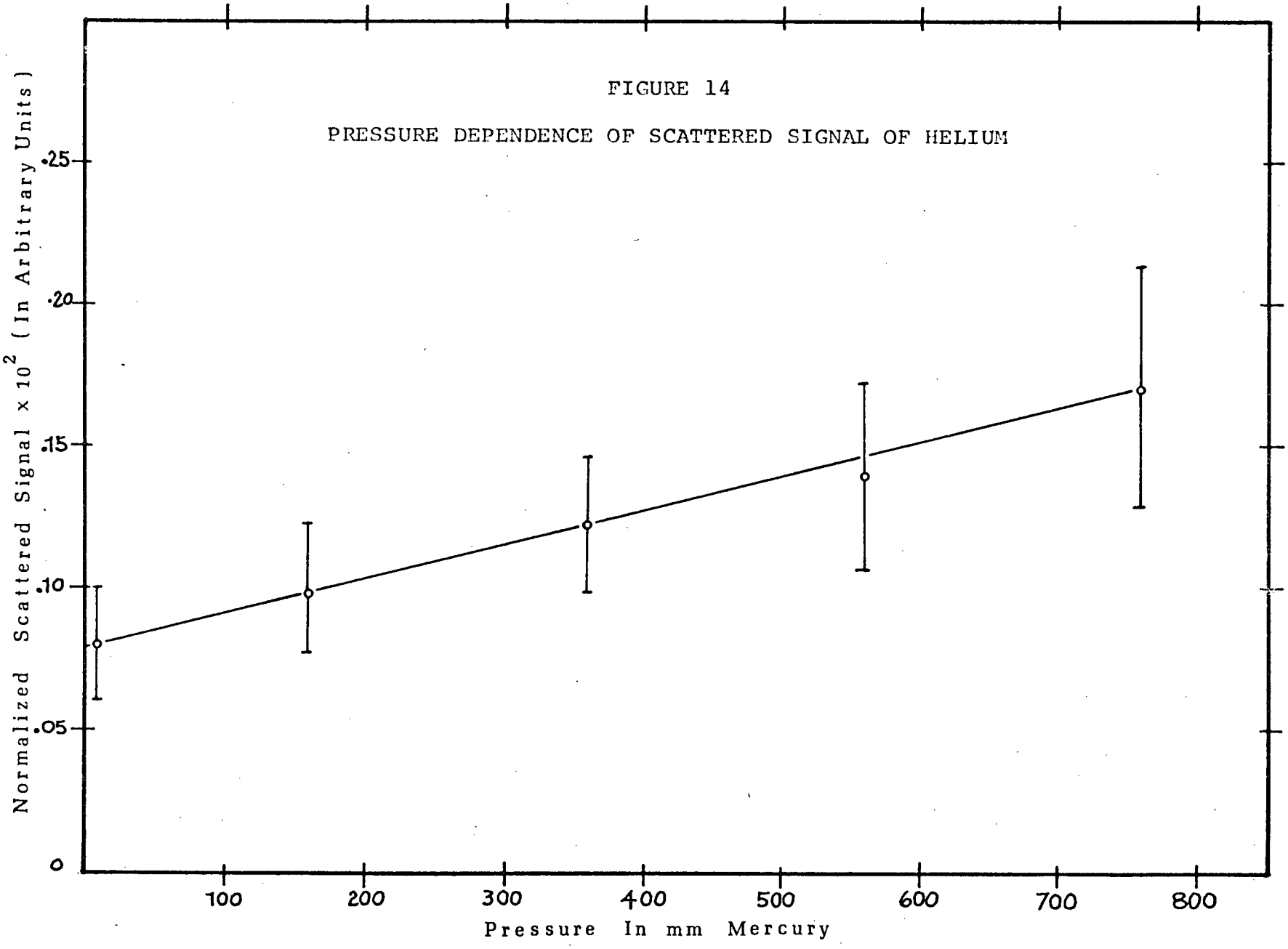
The measured value was

$$\left(\frac{d\sigma}{d\Omega}\right)_{He} = .036 \times 10^{-32} \ m^2$$

$$\text{Error Percentage} = \frac{.036 - .0291}{.0291} \times 100\%$$

FIGURE 14

PRESSURE DEPENDENCE OF SCATTERED SIGNAL OF HELIUM



= 24%

The signal to noise ratio in this part was approximately equal to 1 in comparison to 100 for argon and nitrogen.