FINAL STATE INTERACTIONS IN THE REACTION

$$T(\text{He}^3, \text{He}^4)_{np}$$

by

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ABSTRACT

Triple correlation cross sections have been measured for the reaction $T(\text{He}^3, \text{He}^4)np$ in a complete experiment at a He^3 bombarding energy of 1.5 MeV. Three similar experimental geometries were used which allow the observation of low relative energies in the n-p system, and energies of 0.96 MeV in the He^4 -n system. Therefore the n-p singlet and $\text{He}^5(\text{g.s.})$ final state interactions were observed. Events from the two body reaction channel $T(\text{He}^3, d)\text{He}^4$ and overlapping kinematic contours were eliminated by particle identification. A least squares fit to the experimental triple correlation cross section for one geometry was made using two approximate theories for three body reactions. These were the Watson, and Phillips, Griffy and Biedenharn (P.G.B.) final state interaction theories. Both theories give the theoretical cross section to be proportional to a density of states (D.O.S.) function. The P.G.B. theory gives two forms for this function (P.G.B. 1. and P.G.B. 2.). The D.O.S. functions for the state of He^5 and Li^5 were calculated using only the P.G.B. 1 and Watson forms. The P.G.B. 1 form gives an inadequate description of both the n-p singlet and $\text{He}^5(\text{g.s.})$ final state enhancements. The $\text{He}^5(\text{g.s.})$ enhancement is well described by the Watson form of the D.O.S. function. The triple correlation cross section, for high proton energies, was dominated by a sequential breakup through the ground state of He^5 and by direct three body breakup. No evidence for contributions from the states of Li^5 or for any well defined contributions from the first excited state of He^5 were observed. The Watson and P.G.B. 2 forms of the singlet n-p D.O.S. function gave indistinguishable predictions of

the n-p singlet enhancement. The P.G.B. 2 form was used, for seven values of the n-p singlet scattering length, to fit the experimental data. The value of the singlet n-p scattering length extracted in the fitting procedure was

$$a_{np}^s = -21_{-4}^{+3} \text{ F.}$$

The large experimental errors assigned were caused by the sensitivity of the extracted value on the background terms included in each fit.

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CHAPTER I

INTRODUCTION

1. General

In recent years, reactions which produce more than two particles in the final state have been of considerable theoretical and experimental interest. Such reactions provide, at least in principle, a method of investigating the interaction between the final state particles; in many cases they provide the only means of investigating the interaction between pairs of short-lived particles. This is often the situation in high energy physics where most of the particles produced have short lifetimes. In fact, many of the "particles" of high energy physics are resonances in the interaction of other short-lived particles and can be observed only in multi-particle reactions. For example, the omega meson is a resonance in the three pion system. A similar situation in low energy nuclear physics is found in the neutron-neutron interaction. The 12 minute lifetime of the free neutron prevents the preparation of neutron targets, therefore the only direct means of measuring this interaction is by colliding beam experiments. Such experiments have not yet been performed, so to date, multi-particle systems are the only source of information on the neutron-neutron interaction.

In any reaction involving N particles in the final state, $3N - 4$ independent kinematic quantities require measurement in order to completely specify the reaction kinematics. Conservation of energy and momentum provide four constraining equations. The usual nuclear reaction produces two bound state particles, and only two kinematic quantities need be determined. Reactions which produce three bound state particles

require the determination of five kinematic quantities, which complicates the experimental determination of the reaction kinematics. This experimental complexity, which will be discussed later, in detail, has been largely overcome by the advent of on-line computers and multi-dimensional pulse height analysers. The measurement of a particular reaction in its entirety is, however, still a formidable and time consuming task. For this reason, all experiments to date have been limited to some particular kinematic region of the reaction studied.

The experimental complexity of multi-particle reactions is accompanied by the theoretical difficulty involved with the three or more body problem. This problem has not been solved exactly for even the simplest systems, making the analysis of multi-particle reactions difficult and always subject to theoretical uncertainties. Fortunately, many multi-particle nuclear reactions may be interpreted as a sequential process in which two of the final state particles are produced in a localized interacting state that is unaffected by the other final state particles. Two approximate theories^{1, 2} dealing with this particular reaction mechanism will be presented in Chapter V. The use of these theories provides a means of extracting information about the interaction of two final state particles from experimental data on a three particle reaction. However, until recently³ experimenters have had little success in detailed fits to experimental data and the analysis of multi-particle nuclear reactions has been qualitative.

A comprehensive review of publications on reactions involving more than two final state particles will not be attempted here. Work completed to 1964 is summarized in the "Proceedings of the Gatlinburg Conference on Correlations of Particles Emitted in Nuclear Reactions".⁴

Many of the experimental and theoretical problems in these proceedings are still pertinent. The measurements possible at low bombarding energies have been described by Holmgren⁵ and the theoretical treatment of scattering involving more than two bound state particles has been developed by several authors.^{1, 2, 6-9} Since the present work deals with a reaction in which two interacting nucleons are produced in the final state, the remainder of this chapter is restricted to a discussion of information available from this and similar reactions.

2. The Two Nucleon Interaction

The interaction of nucleons has been studied extensively, since this subject is of fundamental importance in nuclear physics. The properties and theoretical analysis of the low energy nucleon-nucleon interactions are presented in detail by Wilson¹⁰ and in standard nuclear physics texts.¹¹⁻¹³ Also, a comprehensive review of the isotopic spin dependence of the nuclear force is contained in "Isospin in Nuclear Reactions" edited by Wilkinson.¹⁴ Consequently, only a brief summary of the results relevant to the present experiment will be given here.

The possible low energy two nucleon systems can be classified according to the concepts of spin and isospin into an isospin singlet, spin triplet state and an isospin triplet, spin singlet state. The isospin singlet state, the well known spin triplet state of the deuteron, is the only known bound two nucleon system. The isospin triplet state includes the spin singlet interactions in the p-p, n-p and n-n systems. The fact that only the spin triplet state of the n-p system is bound, indicates the spin dependence of the nuclear force.

The low energy nucleon-nucleon interaction is insensitive to the

exact shape of the two nucleon potential. Only two parameters, the "effective range, r " and "scattering length, a " which describe the gross features of the potential can be extracted from the low energy experimental data. These parameters are contained in the shape independent effective range expansion of the s wave scattering phase shift ¹⁰⁻¹³

$$\begin{aligned} k \cot \delta_0 &= \left[k \cot \delta_0 \right]_{k=0} + \frac{1}{2} (rk^2) \\ &= -\frac{1}{a} + \frac{1}{2} (rk^2) \end{aligned} \quad (1.1)$$

which adequately describes the experimental phase shifts up to energies of 10 MeV. The effective range and scattering lengths for the n-p and p-p have been obtained from scattering experiments. ¹⁵⁻¹⁷ Henley ¹⁸ in a review of recent measurements of these parameters gives the values.

$$\begin{aligned} a_{pp}^s &= -7.817 \pm 0.007 \text{ F} & r_{pp}^s &= 2.810 \pm 0.018 \text{ F} \\ a_{np}^s &= -23.7146 \pm 0.0127 \text{ F} & r_{np}^s &= 2.76 \pm 0.07 \text{ F} \\ a_{np}^t &= 5.399 \pm 0.012 \text{ F} & r_{np}^t &= 1.726 \pm 0.014 \text{ F} \end{aligned} \quad (1.2)$$

As mentioned above, the only method at present of measuring the low energy n-n scattering parameters is through multi-particle reactions. The best measurement of the singlet scattering length is thought to come from the reaction $D(\pi^-, \gamma)2n$ ¹⁹⁻²¹ with the result ^{22,18}

$$a_{nn}^s = -18.42 \pm 1.53 \text{ F} \quad (1.3)$$

This is the simplest experiment to analyse theoretically since there are only two strongly interacting particles in the final state. However, Bander ²⁰ assigns a theoretical uncertainty, which is included in the value of a_{nn}^s above, of $\pm 1 \text{ F}$ caused by a slight sensitivity to the

effective range parameter. This parameter is difficult to determine from three particle reactions since the final state interaction theories^{1, 2} are valid only for very low relative energies. A recent independent measurement of the $D(\pi^-, \gamma)2n$ reaction²³ has given a singlet n-n scattering length of

$$a_{nn}^s = -11.2_{+1.9}^{-2.6}$$

and $a_{nn}^s = -13.1_{+2.5}^{-3.4}$ (1.4)

This discrepancy with the previous measurement²¹ indicates the need for further experimental and theoretical work on this reaction.

Charge symmetric reactions and energy levels in mirror nuclei suggest that the nuclear force is charge symmetric, i.e. that the n-n and p-p nuclear forces are the same in the same angular momentum and spin states. Also, an abundance of experimental evidence¹⁸ suggests that the stronger condition of charge independence, i.e. that the n-n, n-p and p-p nuclear forces are the same in the same angular momentum and spin state, may hold. The large negative values of the singlet scattering lengths indicate that the singlet states of the two nucleon systems are almost bound. This property makes the singlet scattering length very sensitive to the strength of the nucleon-nucleon potential. In fact, for a fixed radius of the potential the relation

$$\frac{\Delta a}{a} = C \frac{\Delta V_0}{V_0} \quad \text{where } C \sim 10 \quad (1.5)$$

can be obtained.¹⁸ The exact value of C is dependent on the potential shape. This amplification factor makes the measurement of the singlet scattering length one of the most sensitive and least ambiguous methods of checking the charge independence hypothesis.

To compare only nuclear forces the p-p scattering parameters must be corrected for electromagnetic effects. Corrections for coulomb effects and vacuum polarization have been done²⁴ to give

$$a_{pp} = -16.8 \text{ F to } -17.1 \text{ F}, \quad (1.6)$$

where the exact values are dependent on the shape of the two nucleon potential used. Henley¹⁸ shows that charge independence is broken by 2.1% and charge symmetry holds to within 0.8% if all direct electromagnetic effects are taken into account. The experimental test of charge symmetry relies, in part, upon the measurement of the n-n scattering length. An improved, or at least confirming measurement of this parameter in different physical situations has been the object of a number of recent investigations of three body nuclear reactions.

3. Two Nucleon Final State Reactions

A number of reactions, other than the $D(\pi^-, \gamma)2n$ reaction, produce two neutrons in the final state and are candidates for an investigation of the n-n singlet scattering length. Among these are the reactions $D(n,p)2n$, $T(n,d)2n$, $T(d, \text{He}^3)2n$ and $T(T, \text{He}^4)2n$. Measuring the singlet n-n scattering length by the investigation of these reactions has met with little success until recently, owing to the problem of fitting the experimental data with a theoretical model. For this reason it is useful to study experimentally a series of reactions in which the p-p, n-p and n-n interactions are evident in similar final states. The series of reactions which include the n-n final state reactions above are:

- (1) $D(n,p)2n$, $D(p,n)2p$
- (2) $T(n,d)2n$, $\text{He}^3(n,d)np$, $\text{He}^3(p,d)2p$ (1.7)

If a theoretical model is successful in extracting the low energy scattering parameters for the n-p and p-p systems (well known from elastic scattering experiments), then the n-n scattering parameters may be extracted from the appropriate reaction in a particular series with some confidence by applying the same model. This procedure, called the "comparison procedure" has been reviewed with a critical analysis of its application to incomplete experiments on the series of reactions (1) and (2) aforementioned, by van Oers and Slaus.²⁵ This paper contains a comprehensive list of experiments done on these reactions through 1966. The results are encouraging for the deuteron breakup reactions but the theoretical model used was incapable of explaining the trion breakup reactions. Trion spectra from the reactions $\text{He}^3(d, t)2p$ and $T(d, \text{He}^3)2n$ have been analysed by Haybron²⁶ and by Larson et al²⁷ in attempts to obtain the n-n scattering parameters. They also reach the conclusion that the final state interaction theories could not adequately explain the experimental data. Some doubt is therefore cast on the result

$a_{nn} = -16.1 \pm 1 \text{ F}$ obtain from these reactions by Baumgartner et al.²⁸

Recent work on the deuteron breakup reactions has given more promising results. Slobodrian et al²⁹ have made a comparative analysis of the deuteron breakup reactions at 20 MeV, obtaining a value

$a_{nn}^s = -16.7_{-3.0}^{+2.6} \text{ F}$. Grässler and Honecker³⁰ have analysed the neutron spectrum from the reaction $D(n, p)2n$ and obtained $a_{nn}^s = -16.2 \pm 2.2 \text{ F}$.

Niiler et al³ have analysed a complete experiment on the reaction

$D(p, \pi)2p$ with $6.5 \text{ MeV} \leq E_p \leq 13.0 \text{ MeV}$ and obtained a value of

$a_{np}^s = -23.9 \pm 0.8$ F, which agrees well with the value obtained from scattering experiments. Similar results from this reaction have been noted by Boyd, Donovan and Marsh³¹ and Jeremy and Grandy.³² These improved measurements give hope that the theoretical model may be used to analyse complete experiments, at least for the reaction $D(n,p)2n$ to extract the n - n scattering length with some confidence and accuracy.

Little work has been done on the trion-trion reactions leading to two nucleons and an alpha particle in the final state. The investigation of the nucleon-nucleon interactions in these reactions is difficult because of the strong interactions in the He^4 - n and He^4 - p systems. This is essentially the same problem found in the deuteron-trion reactions where resonances in the T - p and He^3 - n system can interfere with the nucleon-nucleon interaction measurement. However, the large Q values of the trion reactions make them accessible with He^3 and Triton beams from a low energy Van de Graaff accelerator.

The reaction $He^3(He^3, He^4)2p$ has been studied by a number of investigators³³⁻³⁸ since 1965. These investigations give evidence of a strong sequential decay through the ground state of Li^5 and of a singlet p - p final state interaction. No detailed analysis of the p - p interaction has been made. Blackmore,³³ in a series of complete experiments, has shown that the sequential decay theories predict spectral shapes which qualitatively agree with the experimental data for reasonable values of the p - p scattering length. Similar results were obtained by Slobodrian et al³⁷ in studies of the alpha particle spectra at forward angles. However, a value of the singlet p - p scattering length was not obtained from these measurements.

Alpha particle spectra from the reaction $T(T, He^4)2n$ have been

studied by Jarmie and Allen³⁹ and neutron spectra by Wong et al⁴⁰ for bombarding energies less than 2.0 MeV. Both experiments indicate contributions from the ground state of He⁵ and from a singlet n-n interaction. The n-n interaction, however, gave only a small contribution to these single particle spectra, therefore no quantitative analysis of this interaction was made. Gross et al⁴¹ have recently done a measurement of the alpha particle spectra at small forward angles for a bombarding energy of 22 MeV and have obtained $a_{nn}^s = -14 \pm 4 \text{ F}$.

The $T(\text{He}^3, \text{He}^4)np$ has been studied by a number of investigators.⁴²⁻⁴⁶ Most of the early work was concerned with the measurement of cross sections, branching ratios and angular distributions for bombarding energies below 1.5 MeV. In many cases not all the possible reaction channels were considered. The sequential decay through the ground states of He⁵ and Li⁵ was observed in most of these studies, although the contributions from these states was very dependent on the experiment performed. The singlet n-p interaction was first observed in this reaction by Smith et al⁴⁵ as a high energy knee in the alpha particle spectrum, and later by Blackmore³³ in a complete He⁴-p coincidence experiment. Blackmore showed that, as in the p-p interaction in the reaction $\text{He}^3(\text{He}^3, \text{He}^4)2p$, the sequential reaction theories predict the experimental spectral shape for reasonable values of a_{np}^s .

4. Present Work

In this work the reaction $T(\text{He}^3, \text{He}^4)np$ was studied in a complete He⁴-p coincidence experiment with two objectives: (1) to investigate the application of the final state interaction theories^{1, 2} to the states of He⁵ and the singlet n-p interaction observed in this reaction; and

(2) to analyse quantitatively the n-p singlet interaction and attempt to extract the n-p singlet scattering length. It is difficult to observe the states of Li^5 in a coincidence experiment using only charged particle detectors. Therefore, the He^4 -p final state interactions were not of particular interest in this work. Also, contributions to the three body coincidence spectra from the n-p triplet state are small relative to contributions from the singlet state for low relative energies in the n-p system.³ The n-p triplet interaction will, therefore, be ignored here.

In Chapter II the reaction kinematics and experimental techniques for the study of three body reactions are given. The terms "complete" and "incomplete" experiments are explained. Chapter III describes the experiment and required experimental apparatus. Chapter IV describes the experimental procedure and gives a qualitative analysis of the results. Chapter V gives a review of the final state interaction theories. In Chapter VI these theories are applied to the triple correlation cross sections obtained, and a value of a_{np}^s extracted from the data. Chapter VII gives a summary of results and the conclusions. Details of three body reaction kinematics and the calculation of transformation jacobians are contained in Appendix A. Appendix B gives the evaluation of the He^5 and Li^5 phase shifts used in the calculation of the density of states functions. The experimental triple correlation cross sections are tabulated in Appendix C. Appendix D gives a brief description of the particle identification technique used.

CHAPTER II

REACTION KINEMATICS AND CONSIDERATIONS IN EXPERIMENTAL DESIGN

1. Reaction Kinematics

Three body reaction kinematics have been studied extensively by Bronson,⁴⁷ Blackmore,³³ Ohlsen⁴⁸ and many others.^{4, 49, 50} Therefore only a brief summary of the kinematics necessary for the understanding of this $T(\text{He}^3, \text{He}^4)np$ experiment will be presented. For completeness, Appendix A provides a more detailed discussion of the subject. Table I defines the notations to be used in this and later chapters. The velocity vector diagrams relating the coordinate systems relevant in three body reactions are shown in Figure 2.1.

Reactions involving three particles in the final state are of the general type:

$$\text{Projectile} + \text{Target} \rightarrow \text{Particle 1} + \text{Particle 2} + \text{Particle 3} \quad (2.1)$$

A schematic diagram of such a reaction is shown in Figure 2.2. To specify completely the reaction kinematics, nine kinematic variables must be determined. Conservation of energy and momentum provide four equations relating these variables, consequently only five are independent. Therefore, for complete kinematic determination, at least two detectors must be employed. These enable the simultaneous measurement of six kinematic variables, i.e. the energies and angles of two of the emitted particles. For notational convenience the detected particles will be labelled 1 and 2 (Figure 2.2). The reaction kinematics are over-specified if all six kinematic variables are measured. Therefore, the energy and momentum conservation equations yield a variable relationship

TABLE 1.

SYMBOLS USED IN DESCRIBING A THREE PARTICLE BREAKUP
 IN THE THREE RELEVANT COORDINATE SYSTEMS
 (Reference Figure 2.1)

<u>Coordinate System</u>		<u>Definition</u>
lab.	scm.	
T_i	t_i	kinetic energy of particle i.
V_i	v_i	velocity of particle i.
P_i	p_i	momentum of particle i.
Θ_i	θ_i	polar angle of velocity of particle i with respect to the beam direction.
Φ_i	ϕ_i	aximuthal angle of velocity of particle i with respect to the beam direction.
Δ_{ij}	δ_{ij}	angle between the two velocities i and j.
rcm. system for particle i emitted first		
$p_j^{(i)}$		momentum of particle j in system (i).
$\Theta_j^{(i)}$		polar angle of particle j with respect to the beam direction.
$\phi_j^{(i)}$		azimuthal angle of particle j with respect to the beam direction.
$\chi_j^{(i)}$		polar angle of particle j with respect to the recoil system.
V_{scm}		velocity of scm. system in lab. system.
$v_{rcm}^{(i)}$		velocity of rcm(i) system in scm. system.
Q		Q-value for overall reaction.
E_{jk}		internal or excitation energy of cluster (j+k).
m_i		mass of particle i.
M		total mass of system.

Useful relations:

$$\cos \Delta_{ij} = \cos \Theta_i \cos \Theta_j + \sin \Theta_i \sin \Theta_j \cos (\Phi_i - \Phi_j)$$

$$\cos \delta_{ij} = \cos \theta_i \cos \theta_j + \sin \theta_i \sin \theta_j \cos (\phi_i - \phi_j)$$

A. Relationship between Velocities in lab. and s.c.m. Coordinate Systems

B. Relationship between Velocities in s.c.m. and r.c.m.(i) Coordinate Systems

FIG. 2.1--Velocity Vector Diagrams Relating the Coordinate Systems Relevant in Three Body Reactions.

FIG. 2.2--Planar Schematic Diagram of a Three Body Reaction Showing the Polar Angles for a Detector Geometry with $\theta_A = 0^\circ$ and $\theta_B = 180^\circ$

of the form:

$$AP_2^2 - 2BP_2 + C = 0 \quad (2.2)$$

where

$$\begin{aligned} A &= (m_2 + m_3)/m_2 \\ B &= P_0 \cos \theta_{12} - P_1 \cos \Delta_{12} \\ C &= \left(1 + \frac{m_3}{m_1}\right)P_1^2 + \left(1 - \frac{m_3}{m_0}\right)P_0^2 - 2P_0P_1 \cos \theta_{11} - 2m_3Q \end{aligned} \quad (2.3)$$

For a given experimental geometry, the angles are fixed and the above equation becomes a quadratic expression for P_2 as a function of P_1 (or equivalently T_2 as a function of T_1). The simultaneous measurement of T_1 and T_2 therefore produces a kinematically allowed contour in the T_2 vs. T_1 plane. The contour shape is expressed by:

$$P_2^{(\pm)} = \frac{B(P_1) \pm \sqrt{B^2(P_1) - AC(P_1)}}{A} \quad (2.4)$$

There are six possible ways of labelling the three final state particles 1, 2 and 3. Therefore, if the particle masses are different, there will be six distinct contours of the form (2.4) for a given reaction and experimental geometry. As an example, a plot of the possible contours for the reaction $T(\text{He}^3, \text{He}^4)np$, which has a Q value of 12.095 MeV, is shown in Figure 2.3 for a particular geometry. There are only three contours plotted, each doubly degenerate since the neutron and proton masses are nearly equal.

The other quantities of kinematic interest are the internal or excitation energies in the three possible two particle system produced in the final state. These two body internal energies are given by the equations:

FIG. 2.3--Kinematic Contours for the Reaction
 $T(He^3, He^4)np$

$$\begin{aligned}
E_{23} &= Q + \frac{1}{(m_2 + m_3)} \left[P_0 P_1 \cos \theta_1 - T_0(m_0 - m_2 - m_3) - MT_1 \right] \\
E_{13} &= Q + \frac{1}{(m_1 + m_3)} \left[P_0 P_2 \cos \theta_2 - T_0(m_0 - m_1 - m_3) - MT_2 \right] \quad (2.5) \\
E_{12} &= \frac{1}{(m_1 + m_2)} \left[m_2 T_1 + m_1 T_2 - P_1 P_2 \cos \Delta_{12} \right]
\end{aligned}$$

It is usual to write these expressions as functions of only one of the particle energies, say T_1 , in which case E_{13} and E_{12} may be double valued due to the two possible values of T_2 . The two solutions are denoted by the superscripts (+) and (-) which indicate the solution for P_2 taken from Equation (2.4). There are therefore five possible internal energy functions associated with a particular kinematic contour, i.e. E_{23} , $E_{13}^{(+)}$, $E_{13}^{(-)}$, $E_{12}^{(+)}$ and $E_{12}^{(-)}$.

2. Experimental Techniques

Experiments which employ a single detector can measure at most, three of the five kinematic variables necessary for the determination of a three particle final state system. Such experiments are called "kinematically incomplete" or "incomplete" experiments and must measure an average over the undetermined variables. Such measurements are useful when the reaction is dominated by a sequential decay through a particular intermediate state and the first emitted particle is detected. The internal energy of the recoiling system is then defined and a relatively simple interpretation of the single particle spectra is possible. The sequential approximations has been found to be reasonably valid for a large number of three body reactions and therefore many such experiments have been successfully performed.^{38, 51-53} In fact, most of the quantitative measurements on three body reactions have been done

with these "incomplete" experiments²⁵⁻²⁸ because of the relative ease of experiment and analysis.

In two dimensional measurements, the energies of two of the emitted particles are measured simultaneously in two detectors, detector A and detector B. Such experiments determine six kinematic variables and are therefore called "kinematically complete" or "complete" experiments. The quantity of interest in these measurements is the distribution of events as a function of the energy and angles of one of the detected particles and the angles of the second detected particle. This distribution is called the triple correlation cross section and is denoted by:

$$\frac{d^3\sigma}{dT_1 d\Omega_1 d\Omega_2} = \sigma_t(T_1, \Theta_1, \Theta_2, \Phi_1, \Phi_2) \quad (2.6)$$

This cross section is a function of all five independent kinematic variables and is the quantity to which any theoretical model of three body reactions must conform.

In principle it is possible to measure a triple correlation cross section by measuring the energy distribution of particles detected in detector A under the condition that a particle was simultaneously detected in detector B. Without particle identification, such a measurement would represent a sum of six triple correlation cross sections as indicated in Equation (2.7)

$$\frac{d^3}{dT_A d\Omega_A d\Omega_B} = \sum_i \sum_{\substack{j \\ i \neq j}} \frac{d^3\sigma}{dT_i d\Omega_i d\Omega_j} \quad (2.7)$$

As was pointed out above, in a two dimensional energy measurement the kinematic overspecification of the system gives rise to six possible

kinematic contours, each corresponding to the detection of a specific pair of particles in the final state. Therefore, in the event that all these contours are distinct, this kinematic overdetermination provides a complete particle identification and the distribution of events along each contour represents a single term in the above summation. If the kinematic contours are not distinct this separation is inadequate and further particle identification or kinematic techniques (i.e. triple coincidence) must be employed if a single triple correlation cross section is to be measured.

The triple correlation cross section is found experimentally by summing the two dimensional coincidence events over a particular kinematic contour for each energy of one of the detected particles. In this way, the events in the two dimensional array are projected onto one of the energy axes to give the appropriate cross section. This projection can be interpreted as a single term in Equation (2.7) except under two exceptional kinematic conditions:

(1) When two or more contours overlap over some region, the summation in this region represents the sum of two triple correlation cross sections which are not experimentally separable. This situation often arises in two dimensional measurements. However, by careful choice of detector geometry it is usually possible to arrange that no such contour overlap occurs over a restricted region of interest. A single triple correlation cross section may be measured over this region. If the above is not the case, particle identification or triple coincidence must be used to eliminate all but one of the overlapping contours.

(2) Contaminant reactions give rise to events which fall on the kinematic contour. This is an unusual situation at low bombarding energies where few three body reactions are energetically possible and

the only contaminant reactions are two body processes. Two body reactions are usually eliminated by the coincidence condition set by the two detectors. If the experimental geometry does allow the coincidence condition to be satisfied, the energetics of the two body reaction usually yield a peak in the two dimensional spectrum, well removed from the three body kinematic contour. Most three body contaminant reactions have different kinematic contours and are usually separable by careful choice of detector geometry. If the experimental situation is such that events from a contaminant reaction in a kinematic region of interest cannot be avoided, these contaminant events must be removed by particle identification or triple coincidence techniques.

A third problem encountered in performing the projection of the two dimensional experimental data is the occurrence of a large enhancement when the kinematic contour becomes perpendicular to the energy axis. This kinematic peak often obscures structure in the measured cross section. Therefore, it is usual in this situation to project the data onto the other energy axis giving a different, but physically equivalent triple correlation cross section.

3. Cross Section Enhancements

One possible decay mode for three body reactions is a direct statistical breakup into the three final state particles. In this case the triple correlation cross section for a particular kinematic contour is dependent only on the phase space available to the final state particles. This cross section can be calculated exactly by the usual integration over energy and momentum conserving delta functions. Such an integration has been performed by many authors^{33, 47, 54} with the

result:

$$\frac{d^3\sigma}{dT_1 d\Omega_1 d\Omega_2} = \frac{m_1 m_3 P_1 P_2^2}{AP_2 + P_1 \cos \Delta_{12} - P_0 \cos \theta_2} \quad (2.8)$$

This statistical triple correlation cross section is referred to as the Phase Space Distribution (P.S.D.).

Deviations of the experimentally measured triple correlation cross section from the above P.S.D. must be caused by some correlation between the detected particles caused by final state particle interactions. Such correlations do exist⁴ and they can, in many cases, be associated with a range of relative energies between two of the final state particles. In particular, enhancements above the P.S.D. are found in triple correlation cross section measurements on the $T(\text{He}^3, \text{He}^4)_{np}$ reaction.³³ These enhancements can be associated with low relative energies between the neutron and proton and also with relative energies of 0.96 MeV between the neutron and alpha particle. The enhancements are attributed to the virtual n-p singlet state and the ground state⁵⁵ of He^5 respectively. In order to investigate such enhancements, it is necessary to choose a detector geometry in which the relative energies of interest are available to the system over some region of a kinematic contour and in which events in this region are not obscured by competing allowed events. The choice of such a geometry for the reaction $T(\text{He}^3, \text{He}^4)_{np}$ is considered in detail in the following chapter.

CHAPTER III

EXPERIMENTAL AND ELECTRONIC DESIGN

1. Introduction

This experiment studies the three body reaction, $T(\text{He}^3, \text{He}^4)np$.

The possible three body reaction channels are listed below.

Reaction	Q Value (MeV)
1. $\text{He}^3 + T \longrightarrow \text{He}^5(\text{g.s.}) + p \longrightarrow \text{He}^4 + n + p$	11.14
2. $\longrightarrow \text{He}^{5*} + p \longrightarrow \text{He}^4 + n + p$	
3. $\longrightarrow \text{Li}^5(\text{g.s.}) + n \longrightarrow \text{He}^4 + n + p$	10.13
4. $\longrightarrow \text{Li}^{5*} + n \longrightarrow \text{He}^4 + n + p$	
5. $\longrightarrow \text{He}^4 + (n,p) \longrightarrow \text{He}^4 + n + p$	
6. $\longrightarrow \text{He}^4 + n + p$	12.095

The main purpose of this experiment is to investigate the He^4 -n and singlet n-p final state interactions (reaction channels 1, 2, and 5) by measuring the triple correlation cross section for this reaction.

Chapter II described the necessity to determine a detector geometry which allows the observation of low relative energies in the n-p system and also relative energies in the He^4 -n system of 0.96 MeV over some kinematic regions in which no other kinematically allowed events are observable. Observation of low n-p relative energies is attained by fixing a He^4 detector, and placing a proton detector on the corresponding n-p

recoil axis. Such a geometry is given by $\theta_{\text{He}^4} = 90.0^\circ$ $\phi_{\text{He}^4} = 180.0^\circ$

$\theta_p = 61.4^\circ$ $\phi_p = 0.0^\circ$ (Figure 2.2). Since only charged particle detectors are used in this experiment, only two of the six possible kinematic contours will be observed. The four contours involving neutrons

are immediately eliminated. Figure 3.1 shows these two contours for the geometry given above. The internal energies of the three possible two body systems for the $\text{He}^4\text{-p}$ contour are also plotted on this diagram as functions of the proton energy T_1 . The regions on this contour corresponding to relative energies in the n-p system less than 500 keV and to a relative energy in the $\text{He}^4\text{-n}$ system of 0.96 MeV are marked on the diagram. The $\text{He}^5(\text{g.s.})$ enhancement should be easily resolved in this geometry. However, the n-p singlet enhancement will be obscured by events on the p-He^4 contour. Some method must therefore be used to eliminate events from the p-He^4 kinematic possibility. If this is done the above geometry will be excellent for observing the singlet n-p and $\text{He}^5(\text{g.s.})$ enhancements on the $\text{He}^4\text{-p}$ contour, provided no contaminant reactions interfere. Also, these enhancements will be well separated in proton energy.

2. Contaminant Reactions

The two body reaction channels (channels 7 and 8) and expected contaminant reactions (channels 9 and 10) are listed below.

Reaction			Q Value (MeV)
7.	$\text{T} + \text{He}^3 \longrightarrow \text{Li}^6 + \gamma$		15.79
8.	$\text{T} + \text{He}^3 \longrightarrow \text{He}^4 + \text{d}$		14.32
9.	$\text{He}^3 + \text{d} \longrightarrow \text{He}^4 + \text{p}$		18.352
10.	$\text{He}^3 + \text{He}^3 \longrightarrow \text{He}^4 + \text{p} + \text{p}$		12.859

The direct radiative capture reaction will not be observed in this experiment since only charged particle detectors are used. Its cross section at 2.73 MeV is $7.2 \mu\text{b/sr.}^{56}$ Consequently it is difficult to use this as a monitor reaction. This reaction channel may therefore be completely ignored.

FIG. 3.1-- He^4 -p and p- He^4 Kinematic Contour for the Reaction $\text{T(He}^3, \text{He}^4)\text{np}$ and Internal Energy Functions for the He^4 -p Contour

The reaction $\text{He}^3(\text{d},\text{p})\text{He}^4$ arises from the mass three HD^+ beam component striking the unavoidable He^3 target contamination. This reaction produces 16.7 MeV protons at a laboratory angle of 59° in coincidence with 3 MeV alpha particles at 90° . The coincidence condition will therefore be satisfied in the experimental geometry above due to the finite detector size. The full energy of the protons from this reaction was collected in the proton detector, therefore the contaminant peak will be removed from the $\text{T}(\text{He}^3, \text{He}^4)\text{np}$ kinematic contours and presents no experimental difficulties. The presence of a He^3 target contaminant also gives the three body contaminant $\text{He}^3(\text{He}^3, \text{He}^4)2\text{p}$ which, due to the almost equal masses of the neutron and proton and similar Q value has the same kinematic contours as the $\text{T}(\text{He}^3, \text{He}^4)\text{np}$ reaction. However, $\text{He}^3(\text{He}^3, \text{He}^4)2\text{p}$ and $\text{T}(\text{He}^3, \text{He}^4)\text{np}$ reaction cross sections are approximately equal, and the amount of He^3 contamination in the tritium target is small, therefore, few events are expected from this reaction. This can be verified by observing the p-p coincidence contour, since the corresponding n-p contour from the reaction $\text{T}(\text{He}^3, \text{He}^4)\text{np}$ is not observable in this experiment.

The two body reaction channel $\text{T}(\text{He}^3, \text{d})\text{He}^4$ produces 11.3 MeV deuterons at 63° in the laboratory in coincidence with 4.5 MeV alpha particles at 90° . This reaction will be observable with the chosen experimental geometry. The position of the d- He^4 peak in the two dimensional energy array is shown in Figure 3.1 and is removed from the three body He^4 -p contour. Slit scattering of the deuterons from the detector collimators however, gives rise to a significant number of event distributed across the top of the contour.³³ To eliminate such events, proton-deuteron discrimination or triple coincidence techniques are required.

It is possible to remove all the aforementioned contaminant reactions by performing a triple coincidence measurement, $\text{He}^4\text{-p-n}$, with the neutron detector subtending an angle of 40° between $\theta_3 = 70^\circ$ and $\theta_3 = 30^\circ$. This third detector would eliminate all two body reactions and the three body reaction $\text{He}^3(\text{He}^3, \text{He}^4)2\text{p}$. The p-He^4 contour would be observed under the triple coincidence condition up to $E_{\text{He}^4} = 3.0$ MeV, which is sufficiently low to stop any interference with the $\text{He}^4\text{-p}$ contour. However, events on the $\text{He}^4\text{-p}$ contour would not be observed, due to geometric losses from the third detector, above proton energies of 9.0 MeV. Therefore the n-p interaction would be observable without contaminant interference, but the $\text{He}^5(\text{g.s.})$ interaction would not be reliably observed. This triple coincidence technique has the added disadvantage of neutron detection efficiency which would reduce the coincidence counting rate by a factor of $1/2 - 1/3$. The experimental spectra would also be distorted by the variation of detection efficiency with neutron energy, which ranges from 0.5 to 11.0 MeV, and would have to be corrected for this effect.

The other method of eliminating the above contaminant reactions--charged particle identification--was used in this experiment. It provides the possibility of removing the deuterons and alpha particles from the 61.4° detector and therefore eliminates the p-He^4 contour and the $\text{T}(\text{He}^3, \text{d})\text{He}^4$ reaction. This method has three advantages; (1) high counting efficiency ($\sim 100\%$), (2) the $\text{He}^5(\text{g.s.})$ and n-p interactions are simultaneously observable, and (3) the two and three body reaction channels for the $\text{T} + \text{He}^3$ reactions are completely separable and independently measureable. It has the disadvantage that the $\text{He}^3(\text{He}^3, \text{He}^4)2\text{p}$ contaminant is not removed and the minimum proton energy detectable is

about 2.0 MeV.

3. Particle Identification Methods

The simplest method of particle identification for protons and alpha particles is placing a foil in front of the "proton" detector of such a thickness that the alpha energy will be greatly reduced without significantly affecting the proton energy. This method does not effectively distinguish between protons and deuterons since the energy loss difference between protons and deuterons in the foil is small. In proton-alpha identification, the proton energy is slightly reduced and the alpha particles which are not stopped give a low energy background.

Replacing the foil by a thin transmission detector (ΔE detector) which stops all alpha particles but transmits protons above a certain energy is a second method. The energy signal from this detector can be summed in coincidence with the energy signal from the thick stopping detector (E detector) to give the full proton energy without foil degradation. This method provides excellent discrimination between protons and alpha particles provided it is not necessary to detect low energy protons. However, no discrimination is made between protons and deuterons. This discrimination requires more sophisticated techniques of particle identification.

In this experiment an Ortec model 423 particle identifier was used to distinguish protons and deuterons. This identifier does an analogue computation on the energy signals from an E- ΔE detector telescope and produces a pulse which identifies the particle. The identification method used by the identifier was introduced by Goulding⁵⁷ and has subsequently been employed by many authors.^{58, 59} The character-

FIG. 3.2--Particle Identifier Characteristic Functions for Protons, Deuterons, Helium-3, Triton and Helium-4 for a 40μ ΔE Detector and a Function Exponent, ϵ , of 1.67.

istics of the computed particle identification function for a 40 micron ΔE detector and function exponent of 1.67 (Appendix D), is shown in Figure 3.2. This indicates the excellent particle discrimination obtained by the Goulding method. A short description of this identification method is contained in Appendix D.

4. Target and Scattering Chamber

A schematic diagram of the gas target cell and associated tritium handling system is shown in Figure 3.3. This system except for the pumping line E-F has been described previously by Blackmore.³³ The pumping line E-F allows the gas cell and manifold to be pumped into the activated charcoal trap without opening this system to the beam line vacuum chamber. The entire gas handling system was enclosed in a fume hood which was exhausted outside the building. Rubber gloves were sealed to this enclosure allowing manipulation of the manifold valves without subjecting the operator to possible direct exposure to tritium in the event of a gas leak. To protect against slow leaks in the tritium target cell which would not trip the ionization gauge sensing system the backing pump for the beam line pumping station was situated inside the fume hood and the backing pump on the main accelerator pumping system exhausted outside the building. All target cell windows were 25 μ in. grade C nickel foils^{*} and were attached to the gas cell with Delta Bond #152 conductive epoxy⁺. This adhesive has high heat conductivity and consequently improves the cooling of the beam entrance window.

Diagrams of the scattering chamber are shown in Figures 3.4 and 3.5. The entrance beam is defined by the collimator system A.B.C. and

* Chromium Corporation of America - Waterbury, Conn.

+ Wakefield Engineering - Wakefield, Mass.

FIG. 3.3--Schematic Diagram of the Gas Target Cell and Associated Tritium Handling System

TOP VIEW

TARGET CHAMBER

Scale :- $\frac{3}{4}" = 1"$

FIG. 3.4--Top View of the Scattering Chamber Showing the Gas Target Cell, Particle Detectors and Collimator Systems

SIDE VIEW

TARGET CHAMBER

Scale :- $\frac{3}{4}'' = 1''$

FIG. 3.5--Side View of the Scattering Chamber Showing the Gas Target Cell, Particle Detectors and Collimator Systems.

the reaction volume defined by the collimator systems E.F.G. and H.I.J. The collimator system E.F.G. was fixed to the scattering chamber at 90° to the beam axis and the system H.I.J. mounted on a rotatable arm attached to the front plate of the chamber. Details of the collimator systems are given in Table II. The size and shape of the target-defining collimators were chosen as a compromise between angular resolution, usable angles of rotation, coincidence count rate, kinematic broadening of the kinematic contour and alignment considerations. The angles of rotation allowed by the moveable detector were determined by the gas cell geometry³³ and were between 45° and 105° from the beam axis for the collimator system used.

The beam defining collimators and rotatable collimator system were aligned before the gas cell was put in place using a He-Ne gas laser. The 0° position of the angular scale indicator was set with this alignment to an accuracy of $\pm 0.25^\circ$. The gas cell entrance window was centered on the laser beam to complete the target geometry alignment.

5. Charged Particle Detectors

All particle detectors used in this experiment were silicon surface barrier detectors. The details of these are given in Table III. The 2 mm thickness E detector of the proton telescope stopped the 16.7 MeV protons from the reaction $\text{He}^3(d,p)\text{He}^4$ and thus removed this reaction contribution from the $\text{T}(\text{He}^3, \text{He}^4)\text{np}$ kinematic contour. The ΔE detector in the detector telescope was fully depleted and transmission mounted. The 50 micron depletion depth is sufficient to stop all alpha particles from any of the possible reactions but will transmit protons with an energy greater than 2.0 MeV. This places a lower energy limit on the protons which can be detected by the telescope. The limit is sufficiently

TABLE II
COLLIMATOR PARAMETERS

<u>Collimator</u>	<u>d (in.)</u>	<u>Size (in.)</u>	<u>Shape</u>	<u>Material</u>
A	$6.59 \pm .01$	$0.125 \pm .001$	Circular	S.S.
B	$4.86 \pm .01$	$0.187 \pm .001$	Circular	S.S.
C	$1.0 \pm .01$ to $3.25 \pm .01$	$0.175 \pm .001$	Circular tube	S.S.
E	$1.0 \pm .01$	$0.187 \pm .001$	Circular	Brass
F	$1.64 \pm .01$	$0.266 \pm .001$	Circular	Brass
G	$2.40 \pm .01$	$0.155 \pm .001$ x $0.375 \pm .001$	Rectangular	S.S.
H	$0.39 \pm .01$	$0.156 \pm .001$	Circular	S.S.
I	$1.75 \pm .01$	$0.281 \pm .001$	Circular	S.S.
J	$2.34 \pm .01$	$0.187 \pm .001$	Circular	S.S.

where d = distance from the centre of the scattering chamber
size = diameter, in the case of circular collimators
S.S. = stainless steel

TABLE III.

CHARGED PARTICLE DETECTORS

<u>Detector</u>	<u>Depletion Depth</u>	<u>Active Area</u>	<u>Resolution</u>	<u>Manufacturer</u>
E_1	2 mm	25 mm ²	< 30 keV	Ortec
E_1	50 μ	50 mm ²	< 50 keV	Nuclear Diodes
E_2	1 mm	50 mm ²	< 50 keV	Nuclear Diodes

$E_1 - \triangle E_1$ - form the proton detector telescope

E_2 - alpha particle detector at 90°

low that the main experimental results are unaffected.

6. Electronics

The electronics used in this experiment, select events defined by the simultaneous arrival of particles to two detectors. These events are labelled coincidence events and the two linear energy pulses produced by the detectors are amplified, shaped and presented to a Nuclear Data model 160 dual parameter pulse height analyser. The pulse height analyser digitizes the pulse heights and stores the result as an event in a 64 x 64 channel two dimensional array. Particle identification does not, in principle, alter this rather simple electronic design. It implies only one further constraint on the labelling of a coincidence event, i.e. the particle detected in a particular detector be identified as a proton. In practice, however, the electronics become much more complex.

The particle identifier electronics can be treated as a unit which produces three pulses; (1) a fast timing pulse, (2) a full particle energy pulse ($\bar{E} + \Delta E$) and (3) a particle characteristic pulse, the particle identifier output (P.I.O.). A block diagram of this unit is shown in Figure 3.6. The prompt bipolar pulses from the main shaping amplifiers are analysed by timing single channel analysers (T.S.C.A.) which are operated as lower level discriminators in the zero crossover timing mode. The thresholds of these are set as low as possible consistent with the elimination of amplifier noise, and each produces two fast logic pulses at the zero crossover point. The 150 nsec positive pulses are put into an overlap coincidence unit which gives a coincidence resolving time of 300 nsec. This relatively long resolving time was necessary due to time jitter on the low energy ΔE pulses caused by

FIG. 3.6--Block Diagram of the Particle Identifier Electronics (Signals Indicated by Arrows are Available to the Principle Electronics)

electronic noise. The random coincidence rate remains small since there is a slow E detector counting rate. The energy pulses from protons in the ΔE detector are small. These are amplified until most of the dynamic range of the shaping amplifier is used and later attenuated within the particle identifier. This is necessary to allow the 200 keV pulses from the high energy protons to pass the T.S.C.A. discriminator but unfortunately leads to the poor timing characteristics mentioned above. The fast negative logic pulses from the T.S.C.A. on the E channel provide the timing pulses available to the internal electronics. The pulse stretchers were modified to give an output pulse of $4\mu\text{sec}$ duration, the time required by the particle identifier to calculate the identifier function. When a coincidence between the E and ΔE detectors occurs, the delayed unipolar pulses from the shaping amplifiers are routed through the linear gates and pulse stretchers into the particle identifier which produces the $E + \Delta E$ and P.I.O. pulses.

The dual decade attenuator (D.D.A.) in conjunction with a precision pulse generator was used to calibrate and balance the E and ΔE channels of the particle identifier system. The balancing of the gains in the E and ΔE channels is very critical for the proper functioning of the particle identifier. The procedure followed will be described in detail in Chapter IV. The input pulses from the pulse generator were shaped to resemble particle pulses, enabling the approximate setup of the electronic timing with coincident E and ΔE pulses from the D.D.A.

A block diagram of the principle electronic system is shown in Figure 3.7. The signals available from the particle identifier section are indicated with arrows. The fast negative logic pulses from the

FIG. 3.7--Block Diagram of the Principle Electronics. (Signals Indicated by Arrows are Obtained from the Particle Identifier Electronics)

T.S.C.A. (1) are used as start pulses for the time-to-amplitude converter (T.A.C.). The stop signals for this conversion came from the fast timing pulses of the particle identifier system. These negative logic pulses are suitably delayed so that coincidence events between the two detectors appear as a peak in the time spectrum from the T.A.C. which experimentally has a width of 50 nsec. The window of T.S.C.A. (2) is set to accept pulses which fall within this peak. An output from T.S.C.A. (2) therefore indicates a coincidence event between the two detectors with a resolving time of 50 nsec. The window of T.S.C.A. (3) is set to accept only P.I.O. pulses which correspond to a proton identification. A coincidence event between T.S.C.A. (2) and T.S.C.A. (3) therefore implies that particles have arrived at the two detectors simultaneously and that one particle detected was a proton. These are the conditions required for the labelling of a valid coincidence event. The output from the overlap coincidence unit is used to gate open the N.D. 160, allowing the analysis of the two corresponding energy pulses.

Due to the time taken by the particle identifier to calculate the identification function ($4\mu\text{sec}$), it is necessary to delay both logic and linear pulses to establish the correct electronic timing. These delays are established by gate and delay generators (G.D.G.) and delay amplifiers. Scalers were used to monitor the two particle coincidence events and coincidence plus proton identification events. All electronic units with the exception of the nuvistor preamplifier⁶⁰ used in channel E_2 , are standard commercial modules which are listed in Table IV.

TABLE IV.

ELECTRONIC MODULES

1. Particle Identifier Electronics

<u>Unit</u>	<u>Manufacturer</u>	<u>Model</u>
Preamplifier	Ortec	109A
Shaping Amplifier	C.I.	1410
Overlap Coincidence	Ortec	418
Linear Gate	Ortec	426
Pulse Stretcher	Ortec	411
Particle Identifier	Ortec	423
D.D.A.	Ortec	422
Pulser	B.N.C.	PB-2
T.S.C.A.	C.I.	1435

2. Principle Electronics

Shaping Amplifier	Ortec	440A
Delay Amplifier	Ortec	427
T.S.C.A. (1)	C.I.	1435
T.S.C.A. (2)	C.I.	1435
T.S.C.A. (3)	Ortec	420A
T.A.C.	Ortec	437A
Overlap Coincidence	R.I.D.L.	27351
Scalers	C.I.	1470

Ortec - Oak Ridge Technical Enterprises Corporation
Oak Ridge, Tennessee

C.I. - Canberra Industries, Sturup Nuclear Division
Middletown, Connecticut

R.I.D.L. - Nuclear Chicago, Melrose Pk, Ill.

B.N.C. - Berkeley Nucleonics Company
Berkeley, California

CHAPTER IV

EXPERIMENTAL PROCEDURE AND RESULTS

1. Introduction

The beam, target, and energy calibration are discussed initially in this chapter. The subsidiary spectra taken to check the performance of individual elements within the experimental design are next described. As an example, the two dimensional energy spectrum taken for the geometry $\theta_2 = 90^\circ$, $\theta_1 = 62^\circ$ is presented. Finally, a qualitative analysis of the triple correlation cross sections is provided.

2. Beam and Target

The experiment was performed using a 1.88 MeV He^{3+} beam from the University of British Columbia Van de Graaff accelerator. The beam energy is reduced by 0.38 MeV due to energy losses in the 25 μ in. gas cell entrance foil, giving a 1.5 MeV beam in the target gas. Some spreading of the beam profile is expected because of the target entrance foil. However, this effect should not be of major importance in this experiment. For a 1/8" diameter beam spot, energy loss in the entrance foil placed an upper limit of 0.5 μ A on the beam current. Beam heating ruptures the foil at higher currents.

The gas target cell was filled to a pressure of 75 Torr with 99% pure tritium gas from the Oak Ridge National Laboratory. This pressure corresponds to 5 curies of tritium in the 20 cm^3 volume of the gas cell pressure gauge system. The gas pressure dropped steadily after the beam had been on target for approximately 24 hours, indicating that beam heat-

ing effects had produced a leak in the entrance foil. Fortunately, the leak was slow, so safe tritium levels were maintained in the laboratory.

3. Energy Calibration

The precision pulser, dual decade attenuator (D.D.A.) and associated channel matched charge terminators were calibrated in energy using the 5.47 MeV alpha particles from an Am^{241} source situated in a vacuum chamber separate from the target chamber. Each channel of the D.D.A. has three switches which accurately define the input pulse attenuation. In the calibration, these were set at 5.47 and the pulser output amplitude adjusted until the charge pulses from the charge terminators were equivalent to the charge deposited in a detector by the 5.47 MeV alpha particles. The pulser amplitude was fixed at this calibration point. Two simultaneous pulses, calibrated in energy over a range of 0 - 9.99 MeV, and independently variable in steps of 0.01 MeV were then available from the D.D.A. charge terminators. The energy calibration was checked by measuring the positions of the He^5 (g.s.) in the single particle proton spectra (Figure 4.3) at angles of 50° - 100° in 10 degree steps. All measured peak positions agreed with the value calculated from kinematics, corrected for energy loss in foils, to an accuracy of ± 30 keV.

The N.D. 160 dual parameter pulse height analyser was calibrated in energy by simulating a coincidence event with the energy calibrated pulses from the D.D.A. The energy calibration curves for the proton and alpha particle channels were found to be linear and gave the following calibration over the 64 channel spectrum.

Particle Channel	Energy Slope	Slope Intercept
proton	$187 \pm 1 \text{ keV/ch}$	$200 \pm 50 \text{ keV}$
alpha	$218 \pm 1 \text{ keV/ch}$	$0 \pm 50 \text{ keV}$

The errors are assigned assuming an absolute energy determination of ± 50 keV. The maximum observable proton energy was 12 MeV. Higher energy pulses saturate the proton linear amplifier and are rejected by T.S.C.A. (1) (Figure 3.7). The 5.47 MeV pulser calibration point was checked before and after all calibration runs to test the pulser stability.

The energy calibrated pulser-charge terminator system described above was also used to check the particle identifier operation and to balance the E and ΔE amplifier gains. The E and ΔE gains were balanced by gating identical charge pulses through first the E channel of the identifier electronics (Figure 3.6) and then through the ΔE channel. In each case the E + ΔE signal from the particle identifier, observed through a biased amplifier with an overall gain of 20, was stored in a pulse height analyser. The amplifier gains were adjusted until the same pulse height was obtained from both energy channels. This procedure was repeated periodically throughout the experiment. The amplifier gains were sufficiently stable that no significant adjustment was required after the initial balancing.

4. Subsidiary Spectra

The energy pulses from each detector and from the E - ΔE combination were analysed individually for unexpected contaminant reactions. Typical examples of these spectra are shown in Figures 4.1 and 4.2. Figure 4.3 shows a typical spectrum of the full energy pulses from the E - ΔE detector telescope with the pulse height analyser gated open by a proton identification. The contributions to these spectra from the expected reactions and decay modes are identified by the notation:

(particle)
(reaction number),

FIG. 4.1--Single Particle Spectra for the Reaction $T(\text{He}^3, \text{He}^4)np$ from Detectors E_1 and $E_1 + \Delta E_1$

FIG. 4.2--Single Particle Spectra for the Reaction $T(\text{He}^3, \text{He}^4)np$ from Detectors E_2 and ΔE_1

FIG. 4.3--Single Particle Proton Spectrum at 60° from the Reaction $T(\text{He}^3, \text{He}^4)np$

where the reaction number refers to the two lists of reactions in Chapter III.

The increased width of the $(\alpha)_8$ peak in Figure 4.2(b) over the width of the same peak in Figure 4.2(a) is caused by; (1) straggling in a 100 μ in. nickel foil placed in front of the ΔE detector to stop He^3 particles elastically scattered from the target gas and entrance foil and, (2) by the larger gain of the ΔE amplification system. The proton spectrum (Figure 4.3) indicates an unstructured background typical of three body reactions and a high energy peak due to protons from the sequential breakup through the ground state of He^5 . This proton peak was observed in Figures 4.1 and 4.2(a) only as a shoulder on the $(d)_8$ peak. No evidence of a significant contribution from any reaction other than those listed in Chapter III is seen in the spectra taken. The high energy protons from the reaction $\text{He}^3(d,p)\text{He}^4$ are not observable in these spectra. The alpha particles from this reaction gave no significant contribution to the spectra in Figure 4.2. Only a small fraction of the alpha particles from the $T(\text{He}^3, \text{He}^4)np$ reaction observed in the single particle spectra will contribute to the two parameter spectra. However, alpha particles observed from the reaction $\text{He}^3(d,p)\text{He}^4$ will always contribute. Therefore, this contaminant reaction cannot be ignored in the two parameter measurement on the basis of these single particle spectra.

5. Particle Identifier Spectra

The particle identifier system was checked by a two dimensional analysis of the particle identifier output (P.I.O.) and total energy ($E + \Delta E$) pulses using the N.D. 160 two parameter pulse height analyser. Two such two dimensional spectra are shown in Figures 4.4 and 4.5. The

FIG. 4.4--Dual Parameter Particle Identifier Spectrum of P.I.O. vs. Energy from the Reaction $T(He^3, He^4)np$
 Showing Channels with Greater than 50 Events.

FIG. 4.5--Dual Parameter Particle Identifier Spectrum of P.I.O. vs. Energy from the Reaction $T(\text{He}^3, \text{He}^4)np$ Showing Channels Greater than 10 Events

ordinate in these spectra represents the P.I.O. in arbitrary units, and the abscissa, the detected particle energy in MeV. The two spectra are identical except that the minimum number of events displayed in the two dimensional array is lower in Figure 4.5. Figure 4.4 shows that the proton band from the three body reaction $T(\text{He}^3, \text{He}^4)np$ and the deuteron peak from the two body reaction $T(\text{He}^3, d)\text{He}^4$ are well separated. Therefore most of the particles detected in the $E - \Delta E$ detector telescope are correctly identified. The inclusion of channels containing between 10 and 50 events, in Figure 4.5, shows the effects of slit scattering on the identifier system. Deuterons scattered from the front collimator of the detector telescope are identified as deuterons but are degraded in energy. They form a deuteron band in the identifier spectrum, well separated from the proton band. Deuterons and protons scattered from the collimator between the E and ΔE detectors no longer satisfy the range energy relationship used by the particle identifier and are the cause of the diagonal bands in Figure 4.5. Some deuterons will be identified as protons because of this slit scattering in the second collimator. This problem could be eliminated by placing the E and ΔE detectors close together without an intervening collimator. Unfortunately, this was not possible with the detectors used here. The number of incorrect deuteron identifications is small compared to the total number of particles detected. However, the experimental geometry is such that these events will always satisfy the coincidence condition between the proton and alpha particle detectors. Therefore, these incorrect identifications will appear as a more significant contribution to the two dimensional energy spectra.

6. Two Dimensional Energy Spectra.

Two dimensional, T_2 vs T_1 , spectra for the reaction $T(\text{He}^3, \text{He}^4)np$

were taken with the proton detector telescope positioned at 59° , 62° and 65° from the beam axis. Low relative energies in the n-p system are observable at all these angles. The coincidence counting rates were about 6 - 8 counts per minute for a beam current of $0.5 \mu\text{A}$ and a gas target pressure of 75 Torr. Approximately 10,000 coincidence events were collected at each angle, involving a total running time of over 24 hours per angle.

The two dimensional spectrum for $\Theta_1 = 62^\circ$ is shown in Figure 4.6. The only distinct features in this spectrum are the $\text{He}^4\text{-p}$ kinematic contour, a contribution due to incorrectly identified deuterons and the low proton energy cut-off caused by the ΔE detector. Unfortunately, the deuteron contribution occurs at energies corresponding to low relative energy in the n-p system. The main effect is removed from the three body kinematic contour, however, and does not interfere with the measurement of the triple correlation cross section. The region in which events caused by slit-scattered protons from the contaminant reaction $\text{He}^3(\text{d,p})\text{He}^4$ are expected is marked on the spectrum as the band "s". A number of channels in this region, especially in the 59° spectrum where the effect is expected to be largest, contain 5 - 10 events. This contaminant reaction is therefore present. However, the contribution to the triple correlation cross section should be small and will be neglected. In the measured spectra, there are no channels along the p-p coincidence contour for the reaction $\text{He}^3(\text{He}^3, \text{He}^4)2\text{p}$ containing more than 4 events. Therefore, contributions from this reaction on the $\text{He}^4\text{-p}$ contour are small and can be neglected.

The solid line in Figure 4.6 represents the calculated $\text{He}^4\text{-p}$ contour for the reaction $\text{T}(\text{He}^3, \text{He}^4)\text{np}$. A geometric broadening of this

FIG. 4.6--Two Dimensional He^4 -p Coincidence Spectrum from the Reaction $T(\text{He}^3, \text{He}^4)np$

contour is due to the finite size of the detectors. This broadening is particularly noticeable at high proton energies where the reaction kinematics are sensitive to the polar angles of the detected particles. The geometric broadening observed does not present a serious problem. The two dimensional spectra are very well defined and indicate that the triple correlation cross section should be extracted with little interference from background reactions.

7. Triple Correlation Cross Sections

The experimental triple correlation cross sections were extracted from the two dimensional energy spectra by summing over the $\text{He}^4\text{-p}$ kinematic contours as described in Chapter II. Projections onto the proton energy axis for the proton angles $\theta_1 = 59^\circ$, 62° and 65° are shown in Figures 4.7 - 4.9. The projected triple correlation spectra are normalized to 5×10^6 counts in the deuteron peak from the reaction $\text{T}(\text{He}^3, \text{d})\text{He}^4$ observed in the fixed 90° detector (Figure 4.2(a)). These triple correlation cross sections all show, (1) a strong peak at high proton energies due to the sequential breakup through the ground state of He^5 , (2) an enhancement in the region of low relative energies in the n-p system ascribed to the n-p singlet interaction, and (3) a structureless background. The He^5 ground state peak occurs at energies of 0.96 MeV in the $\text{He}^4\text{-n}$ system as expected.⁵⁵ The contribution from this reaction increases with increasing centre of mass angle, consistent with the angular distribution results of Kuhn and Schlenk.⁴⁶ The shape of the n-p singlet enhancement varies in the measured spectra. However, this variation could be purely statistical, and no definite conclusions can be drawn. The data at 62° displays the most symmetric n-p singlet peak

FIG. 4.7--Triple Correlation Cross Section for the Reaction $T(\text{He}^3, \text{He}^4)np$ for $\theta_1 = 59^\circ$

FIG. 4.8--Triple Correlation Cross Section for the Reaction $T(\text{He}^3, \text{He}^4)np$ for $\theta_1 = 62^\circ$

FIG. 4.9--Triple Correlation Cross Section for the Reaction $T(\text{He}^3, \text{He}^4)np$ for $\theta_1 = 65^\circ$

and was, therefore, chosen for the detailed analysis in Chapter VI. No evidence of significant contributions from the Li^5 states, the first excited state of He^5 or contaminant reactions is seen in the experimental triple correlation cross sections.

CHAPTER V

THEORY OF FINAL STATE INTERACTIONS

1. Introduction

Nuclear scattering experiments on complex nuclei, in principle, must be compared to a many body analysis. Fortunately many reactions allow a two body approximate model analysis, (e.g. Projectile + Target

Reaction Product + Residual Nucleous). Reactions which result in three particles, such as this $T(\text{He}^3, \text{He}^4)np$ reaction, must involve three body analysis. Consequently, the approximations made in predicting results from such reactions are different than those for two body systems. In addition, three body analysis is complicated and hence, not as well understood as two body analysis.

The mathematical formalism for the exact solution for the non-relativistic scattering of a three body system was first put in an elegant form by Faddeev⁴⁷ in 1960. This formalism assumes no uniquely three body interactions and the resulting equations require for their solution the exact interaction for all pairs of particles. Approximate forms of the two body potential have been used by Lovelace,⁴⁸ and by Aaron, Amado and Yam,^{49, 50} which make the solutions of Faddeev's equation more amenable to calculation. Aaron and Amado⁵¹ have calculated the proton spectrum for the reaction $D(n,p)2n$ using their exact theory (ref. 49, 50) and separable s wave spin dependent two body forces. They reproduce the major experimental features; however, the results are not sufficiently sensitive to resolve the n-n scattering length. This failure is attributed to the simplicity of the two nucleon interaction used. A more com-

plete form of the two nucleon interaction, however, makes the calculation possible only through the use of a very large, high speed computer. The work of Aaron and Amado is an encouraging application of approximations to the exact three body theory of Faddeev. However, more approximate theories are generally used in the analysis of experiments on three body reactions and these will be presented and applied in this paper.

2. Watson Theory

Watson¹ introduced the term "final state interaction" to describe any interaction between final state products of a nuclear reaction which can influence the properties of the reaction cross section. These interactions are pictured as playing no part in the "primary interaction" which produces the given final state, but once the primary interaction has taken place, interactions in the final state can appreciably affect the angular and energy distributions of the emitted particles. Watson imposes the following restrictions on the system he describes;

(1) The primary reaction must be short ranged, i.e. that it may be confined to a volume a^3 , where a is the range of the interaction.

(2) The relative energies of the particles interacting in the final state must be low.

(3) The final state interaction must be strong and attractive.

Watson decomposes the interaction, $\mathcal{V}$, which causes the transition from the initial state a to the final state B , into a primary interaction, V , and a final state interaction, v , such that

$$\mathcal{V} = V + v, \quad (5.1)$$

and derives the approximate transition amplitude

$$T_{B,a} = (\phi_B^{(-)}, v \psi_a^{0(+)}), \quad (5.2)$$

where $\phi_B^{(-)}$ and $\psi_a^{0(+)}$ satisfy the Lippman Schwinger equations

$$\begin{aligned} \phi_B^{(-)} &= \chi_B + \frac{1}{E_a - i - H_0} v \phi_B^{(-)} \\ \psi_a^{0(+)} &= \chi_a + \frac{1}{E_a - i - H_0} v \psi_a^{0(+)} \end{aligned} \quad (5.3)$$

$$\text{where } H_0 \chi_a = E_a \chi_a$$

$$\text{and } H_0 \chi_B = E_B \chi_B$$

Equations 5.3 indicate that all effects of the final state interaction, v , are carried in the function $\phi_B^{(-)}$.

Watson then assumes that only two particles in the final state are produced with low relative momentum, and factors out of $\phi_B^{(-)}$ that part $g_k^{(-)}(r)$ which describes the relative motion of this pair. The transition amplitude may then be written as

$$T_{B,a} = (g_k^{(-)}, \bar{R}) \quad (5.4)$$

where $\bar{R}$ is a transition operator which does not explicitly depend on the relative momentum k . If k is small enough that only s waves need be considered, and coulomb interactions are neglected then

$$g_k^{(-)} = \frac{e^{-i\delta}}{kr} \sin(kr + \delta) \quad (5.5)$$

which for $kr \ll 1$ gives

$$g_k^{(-)} = \frac{e^{-i\delta}}{k} \sin \delta f(r) \quad (5.6)$$

and

$$T_{B,a} = \frac{e^{-i\delta}}{k} \sin \delta (f(r), \bar{R}) \quad (5.7)$$

This transition amplitude gives the explicit dependence of the cross section on the relative momentum of the two interacting final state particles to be

$$\nabla(E_k \mathcal{R}_k) = W \frac{\sin^2 \delta(k)}{k} \quad (5.8)$$

where W is independent of k and $\delta(k)$ is the s wave phase shift for the scattering of the pair of low relative energy particles. Migdal⁵² derived a similar expression for the case when coulomb interactions were allowed and therefore this theory is usually referred to as the Watson-Migdal theory.

Phillips⁶⁵ has shown an extension of the Watson calculations to include coulomb interactions and assuming a boundary radius a for the primary interaction uses the exact outside wave functions to obtain

$$\nabla(E_k \mathcal{R}_k) = W \frac{\sin^2(\delta_\ell + \phi_\ell)}{p_\ell} \quad (5.9)$$

where $p_\ell = ka/A_\ell^2$

$$A_\ell^2 = F_\ell^2 + G_\ell^2 \quad (5.10)$$

$$\phi_\ell = \tan^{-1} \left[\frac{F_\ell(ka)}{G_\ell(ka)} \right]$$

Here, F_ℓ and G_ℓ are the regular and irregular solutions to the coulomb wave equation. The formulae are extended to include arbitrary ℓ values.

3. Phillips, Griffy and Biedenharn (P.G.B.) Theory

Phillips, Griffy and Biedenharn² consider the case of a distinctly sequential decay such that we may consider the three body breakup as a two step process

The cross section for the first reaction is given according to perturbation theory by

$$\sigma_D^*(\Theta, \phi, k_b) = \frac{\mu_a \mu_b k_a}{4\pi \hbar^4 k_b} \left| \langle B + b, E_b | H' | A + a, E_a \rangle \right|^2 \quad (5.11)$$

Discrete states in the system B greatly affect the energy dependence of this cross section. This energy dependence may be factored out of the matrix element by introducing a density of states function (E_B) and writing (5.11) in the form

$$\sigma_D^*(E_B) = \frac{\mu_a \mu_b k_a}{4\pi \hbar^4 k_b} \left| \langle B + b, E_B | H' | A + a, E_a \rangle \right|^2 \rho(E_B) \quad (5.12)$$

For a system B which has a set of discrete energy states E_n , the density of states function is given by

$$\rho(E_B) = \sum_n \delta(E_B - E_n) \quad (5.13)$$

The vectors $|B + b, E_B\rangle$ may therefore be defined for all E_B in the formulation (5.12).

P.G.B. generalize this formalism to include the possibility of the system B being produced in a localized decaying state, and calculate a "generalized density of states function" (D.O.S. function) for this case by two methods.

(1) The method of enumeration of states employed for discrete energy states is extended by treating the continuum states as discrete. In this, the generalized D.O.S. function is defined by

$$\rho_a(E_b) = \lim_{R \rightarrow \infty} [\rho_R(E_B) - \rho_{aR}(E_B)] \quad (5.14)$$

where $\rho_R(E_B)$ is the number of states per unit energy interval when the system B is confined to be within a sphere of radius R and $\rho_{aR}(E_B)$ is the number of states per unit energy interval when B is confined within this sphere of radius R but excluded from a concentric sphere of radius a (the primary interaction radius). The results of this calculation gives

$$\rho(E_B) = \frac{1}{\pi} \frac{d}{dE_B} (\mathcal{J}(E_B) + \mathcal{P}_L(E_B, a)) \quad (5.15)$$

where $\mathcal{J}(E_B)$ is the scattering phase shift for the system C + c and $\mathcal{P}_L(E_B, a)$ is the hard sphere phase shift defined in Equations (5.10).

(2) The energy variation of the matrix element associated with the final state continuum nature is extracted by renormalizing the final state wave function, which amounts to calculating the probability that the continuum state has c + C within the volume of interaction. The result obtained by this method is

$$\begin{aligned} \rho_L(E_B) = \frac{\mu}{\hbar^2 k} \left[\frac{d}{dk} (\mathcal{J}_L + \mathcal{P}_L) - \frac{1}{2} \left(\frac{1}{k} - 2 \frac{\partial A}{\partial k} \right) \sin 2(\mathcal{J}_L + \mathcal{P}_L) \right. \\ \left. - \frac{1}{k} \left(A \frac{\partial^2 A}{\partial r \partial k} - \frac{\partial A}{\partial r} \frac{\partial A}{\partial k} \right) \sin^2 (\mathcal{J}_L + \mathcal{P}_L) \right] \end{aligned} \quad (5.16)$$

where A is defined in Equations (5.10). For the special case when the radial part of the wave function describing the system B can be taken as independent of energy for $r \leq a$ this renormalization procedure gives

$$\rho_L(E_B) = \frac{2\mu a}{\pi \hbar^2 k^2} \frac{\sin^2(\mathcal{J}_L + \mathcal{P}_L)}{p_L} \quad (5.17)$$

which is exactly the spectral shape obtained from the Watson theory (Equation 5.9).

From the above we see that the Watson-Migdal and P.G.B. final

state interaction theories may be combined and the results summarized by writing the cross section for the three body reaction in the form

$$\nabla_D^*(E_B) = M^2 \rho_e(E_B) \quad (5.18)$$

where M is some energy independent function and $\rho_e(E_B)$ is a generalized density of states function which takes one of the following forms

$$(1) \quad \frac{1}{\pi} \frac{d}{dE_B} [\delta_e(E_B) + \phi_e(E_B, a)]$$

$$(2) \quad \frac{1}{k} \left[\frac{d}{dk} (\delta_e + \phi_e) - \frac{1}{2} \left(\frac{1}{k} - 2 \frac{\partial A}{\partial k} \right) \sin 2(\delta_e + \phi_e) - \frac{1}{k} \left(A \frac{\partial^2 A}{\partial k \partial r} - \frac{\partial A}{\partial r} \frac{\partial A}{\partial k} \right) \sin^2 (\delta_e + \phi_e) \right]$$

$$(3) \quad \frac{\sin^2(\delta_e + \phi_e)}{p_e}$$

For convenience these forms of the D.O.S. function will be labelled the P.G.B. 1, P.G. B. 2 and Watson forms respectively.

There is no a priori means of choosing the form of D.O.S. function to be used in any particular experimental situation. Blackmore³³ has shown that for an isolated resonance with a Breit Wigner shape the functions (1) and (3) give the same results if the energy shift function

Δ_e is linear in energy and the level width Γ_e is independent of energy. This is not the case if the phase shifts do not exhibit a resonance or for energies near threshold, and in these cases the two functions will produce different results. The function (2) presents computational difficulties for the general coulomb case and is usually ignored for this reason. For the simple case of s wave n-p scattering however, this function reduces to

$$(2') \quad \frac{1}{k} \left[\frac{d}{dE} (\delta_c + \delta_l) - \frac{1}{2k} \sin 2(\delta_c + \delta_l) \right]$$

which is easily calculated and gives results similar to the Watson form.

The generalized density of states functions for the ground and first excited states of Li^5 and He^5 have been calculated for the P.G.B. 1 and Watson forms and are shown in Figures 5.1 and 5.2. These calculations have been done previously by Blackmore; however, for completeness, the details will be presented in Appendix B. All three forms of the D.O.S. function have been used in the calculations for the n-p singlet state and the results are plotted in Figure 5.3. The phase shifts required for the n-p scattering were calculated using effective range theory with the parameters given on the diagram. A complete discussion of all D.O.S. functions as applied to the particular reaction $T(\text{He}^3, \text{He}^4)_{np}$ will be given in the following chapter.

FIG. 5.1--Theoretical Density of States Functions (Calculated from the Watson and PGB 1 Forms) for the Ground and First Excited States of He^5

FIG. 5.2--Theoretical Density of States Functions (Calculated from the Watson and PGB1 Forms) for the Ground and First Excited States of Li^5

FIG. 5.3--Theoretical Density of States Functions, Calculated from the Watson, PGB1 and PGB 2 Forms, for the n-p Single State

CHAPTER VI

ANALYSIS OF EXPERIMENTAL DATA

1. Introduction

The final state interaction theories in Chapter V describe the cross section, σ_{D^*} , for the production of a state D^* which decays according to the diagram below, in which time development is indicated by arrows.

All particles are labelled by the P.B.G. convention (Equation (5.11)).

The final state particles are labelled 1, 2 and 3 in agreement with the notation of Table I. The conventions that particle 1 is detected in detector 1, having a solid angle $d\Omega_1$ and particle 2 is detected in detector 2 having a solid angle $d\Omega_2$ will be used.

The probability, σ_{seq}^c , that the above sequential breakup will be observed in a complete experiment, assuming a finite detector geometry and energy resolution is given by the product of three terms. These are:

- (1) the probability of the formation of the state D^* , given by

$$\int_{E_B} \sigma_{D^*}(E_B) dE_B, \quad (6.1)$$

(2) the probability of detecting the particle 1, given by

$$\int_{\Omega_1(s.c.m.)} f(\theta_1, \varphi_1) d\Omega_1(s.c.m.) \quad (6.2)$$

where $f(\theta_1, \varphi_1)$ is the total system centre of mass (s.c.m.) angular distribution for the breakup of D^* , and

(3) the probability of detecting particle 2 given that particle 1 was detected. This condition defines the recoil centre of mass frame (r.c.m.). This probability is:

$$\int_{\Omega_2(r.c.m.)} g(\chi_2^{(1)}, \varphi_2^{(1)}) d\Omega_2(r.c.m.) \quad (6.3)$$

where $g(\chi_2^{(1)}, \varphi_2^{(1)})$ is the r.c.m. angular distribution for the breakup of the system B. The expression for ∇_{seq}^c is therefore:

$$\nabla_{seq}^c = \int_{E_B} \int_{\Omega_1(s.c.m.)} \int_{\Omega_2(r.c.m.)} \nabla_{D^*} f(\theta_1, \varphi_1) g(\chi_2^{(1)}, \varphi_2^{(1)}) d\Omega_1(s.c.m.) d\Omega_2(r.c.m.) dE_B \quad (6.4)$$

Equation (6.4) represents the final state interaction theory result for a two dimensional measurement.

It is necessary to transform the integrals of Equation (6.4) into integrals over the kinematic variables in the laboratory frame, since the limits of these variables in the r.c.m. and s.c.m. frames are not known directly. The formal transformation is well known from elementary calculus and gives:

$$\nabla_{seq}^c = \int_{T_1(lab)} \int_{\Omega_1(lab)} \int_{\Omega_2(lab)} J_2 \nabla_{D^*} f(\theta_1, \varphi_1) g(\chi_2^{(1)}, \varphi_2^{(1)}) d\Omega_1(lab) d\Omega_2(lab) dT_1 \quad (6.5)$$

where J_2 is the jacobian for the transformation of the r.c.m. and s.c.m. variables into the laboratory system. Equation (6.5) gives the theoretical triple correlation cross section in the laboratory frame as:-

$$\frac{d^3 \overline{\nabla}}{dT_1 d\mathcal{N}_1 d\mathcal{N}_2} = J_2 \overline{\nabla}_D^* f(\theta_1, \phi_1) g(\mathcal{N}_2^{(1)}, \phi_2^{(1)}) \quad (6.6)$$

The intermediate breakup processes are assumed isotropic in their respective centre of mass systems for simplicity. With this customary assumption, the angular distribution functions are constant. Equation (6.6) then reduces to:

$$\frac{d^3 \overline{\nabla}}{dT_1 d\mathcal{N}_1 d\mathcal{N}_2} = \text{const. } J_2 \overline{\nabla}_D^* \quad (6.7)$$

The theoretical triple correlation cross section to be used in this analysis will take this form.

2. The Reaction $T(\text{He}^3, \text{He}^4)np$

There are five possible sequential decay schemes for the reaction $T(\text{He}^3, \text{He}^4)np$. These are:

A triple correlation cross section of the same general form as Equation (6.7) may be written for each of these decay schemes. The total triple correlation cross section, (assuming no interference between the reaction channels) is a linear sum of terms representing each reaction channel plus a statistical three body breakup term. Labelling the proton, alpha

particle and neutron in the final state 1, 2, and 3 respectively, this becomes:

$$\frac{d^3 \sigma_{\text{tot}}}{dT_1 d\Omega_1 d\Omega_2} = a_1 J_{1L} \rho(E_{\text{He}5}) + a_2 J_{1L} \rho(E_{\text{He}5*}) + a_3 J_{3L} \rho(E_{\text{Li}5}) \\ + a_4 J_{3L} \rho(E_{\text{Li}5*}) + a_5 J_{2L} \rho(E_{\text{np}}) + a_6 [\text{P.S.D.}] \quad (6.8)$$

Equation (6.8) will be used to fit the experimental triple correlation cross sections. The a_i are arbitrary constants and the statistical breakup term (P.S.D.) is given by Equation (2.8).

Structure can occur in the theoretical triple correlation cross section from enhancements in either the jacobians J_{iL} or the density of states functions $\rho(E_B)$. Structure arising from the jacobians represents a purely kinematic effect and must be carefully considered before physical information can be extracted from the experimental data. The discussion in Appendix A demonstrates that the transformation jacobians have the form:

$$J_{iL} = \frac{M}{m_2 p_i p_j} \times \frac{p_1 p_2^2}{AP_2 + p_1 U_{12} - p_0 U_2} \quad j \neq i \quad (6.9)$$

Written in terms of the phase space distribution the jacobians become:

$$J_{iL} = \frac{M(\text{P.S.D.})}{m_1 m_2 m_3 p_i p_j} \quad j \neq i \quad (6.10)$$

The transformation jacobians for the reaction $T(\text{He}^3 \text{He}^4)_{\text{np}}$ and for the geometry used in this experiment are shown in Figure 6.1 as functions of the proton energy. The effects of these kinematic transformations on the final state interaction theory enhancements will now be considered in detail for each term in the theoretical triple correlation cross

FIG. 6.1--Kinematic Transformation Jacobians from the r.c.m. and s.c.m. to the Laboratory Frame for the Reaction $T(\text{He}^3, \text{He}^4)np$

section (Equation 6.8).

The phase space distribution, contained in all terms of Equation (6.8), is monotonically increasing with proton energy. The denominator, $AP_2 + P_1 U_{12} - P_0 U_2$, represents the derivative of the equation for the kinematic contour (Equation (2.2)) with respect to the momentum of the alpha particle, P_2 . Thus the denominator is zero at the maximum proton energy, giving a singularity at that point. This singularity is mathematical and, as shown later, will produce a high proton energy enhancement when integrations to include finite geometry are performed. This result applies to all singularities in the following discussion.

The density of states functions for the ground state and first excited state of He^5 in Equation (6.9) are multiplied by the jacobian J_{1L} . Figure 6.1 indicates that this jacobian has no structure except a singularity at the maximum proton energy. This singularity is due to two effects: (1) the energy in the He^5 recoil system approaches zero at high proton energies, and (2) the phase space distribution singularity. The density of states functions for the states of He^5 are plotted as functions of the proton energy in Figure 6.2. The units of the abscissa are arbitrary and the two forms of the D.O.S. functions have been normalized to give approximately the same height to the ground state peak. Also, the plotting routine places the zero for the dotted curves 1/8 in. higher than the solid curves. This convention will be followed throughout the following discussion. Both forms of the theoretical functions indicate a strong peak at high proton energies due to the He^5 ground state and a broad peak due to the first excited state. The peak positions and shape predicted by the two forms of D.O.S. function are slightly different.

FIG. 6.2--Density of States Functions for the States of He⁵ as Functions of the Proton Energy for the Reaction
 $T(\text{He}^3, \text{He}^4)np$

FIG. 6.3--Theoretical Triple Correlation Cross Sections for the States of He⁵ for the Reaction T(He³, He⁴)np

The products of the D.O.S. functions with the jacobian J_{1L} , representing the first and second terms in Equation (6.8) are plotted as functions of the proton energy in Figure 6.3. The kinematic effects on the theoretical He^5 ground state peak are; (1) to enhance contributions for high energy protons, shifting the peak positions slightly, (2) to decrease the tail at low proton energies, and (3) to produce a second sharp peak at the maximum proton energy. The kinematic effects on the theoretical He^5 first excited state enhancement are more pronounced. The broad peak predicted by the Watson form of the D.O.S. function is sharpened and enhanced relative to the ground state peak. The peak predicted by the P.G.B.I. form is completely eliminated. A sharp peak at high proton energies, due to the singularity in the transformation jacobian, occurs for both forms of the He^{5*} D.O.S. function.

The D.O.S. functions for the ground state and first excited state of Li^5 in Equation (6.8) are multiplied by the jacobian J_{3L} . Figure 6.1 indicates that this jacobian exhibits a peak for proton energies of about 11 MeV in addition to the phase space singularity. The 11 MeV peak appears when the neutron energy in the s.c.m. frame goes to zero ($p_3 \rightarrow 0$) giving a second singularity in the transformation. The density of states functions for the Li^5 states are plotted as functions of the proton energy in Figure 6.4. Both forms of the theoretical functions indicate a strong peak due to the ground state of Li^5 at low proton energies and a broad peak due to the first excited state. The products of the D.O.S. functions with the jacobian J_{3L} , representing the third and fourth terms in Equation (6.8), are plotted as functions of the proton energy in Figure 6.5. The kinematic effect on the theoretical Li^5 state enhancements is the reduction of the final state interaction effects.

FIG. 6.4--Density of States Functions for the States of Li⁵ as Functions of the Proton Energy, for the Reaction T(He³, He⁴)np

FIG. 6.5--Theoretical Triple Correlation Cross Sections for the States of Li^5 , for the Reaction $\text{T}(\text{He}^3, \text{He}^4)\text{np}$

Consequently, the theoretical Li^5 predictions possess only a kinematic peak corresponding to the transformation J_{3L} at high proton energies. The lowest proton energy measureable in this experiment was 2.0 MeV, so the kinematic enhancement will be the only observable contribution from the Li^5 states. The two terms in Equation (6.8) can therefore be replaced by one term of the form J_{3L} . This single term requires only that some interaction is present, i.e. $\rho(\text{Li}^5) \neq 0$, and does not depend on the specific shape of the theoretical D.O.S. function.

The D.O.S. function for the singlet n-p interaction in Equation (6.8) is multiplied by the jacobian J_{2L} , Figure 6.1. This jacobian displays the phase space singularity and a strong narrow peak at proton energies corresponding to low relative energies in the n-p system. The D.O.S. functions for the n-p singlet interaction are plotted as functions of the proton energy in Figure 6.6. Two of the theoretical forms, the P.G.B. 2 and Watson forms, exhibit a double peaked behaviour with a minimum at proton energies corresponding to zero relative energy in the n-p system. The other, P.G.B. 1 form, exhibits a single peak with a maximum at these proton energies. The products of the D.O.S. functions with the jacobian J_{2L} are plotted as functions of the proton energy in Figure 6.7. The kinematic effect on the double peaked functions is to remove the minimum and replace the two peaks with a single broad peak. The kinematic effect on the single peaked function is to greatly reduce the predicted peak width. Information in the triple correlation cross section on the singlet n-p interaction will, therefore, be contained in the shape and width of the n-p singlet enhancement, not in its position. In fact, it is obvious from the experimental data, Figures 4.7 - 4.9, that the measured enhancement is not extremely narrow, and the P.G.B. 1

FIG. 6.6--Density of States Functions for the n-p Singlet States as Functions of Proton Energy, for the Reaction $T(\text{He}^3, \text{He}^4)np$

FIG. 6.7--Theoretical Triple Correlation Cross Section for the n-p Singlet State, for the Reaction $T(\text{He}^3, \text{He}^4)np$

form of the D.O.S. function may be eliminated as a possible description of the singlet n-p final state interaction.

3. Integrations

The integration of the above theoretical functions over finite target and detector geometry is important for two dimensional measurement analysis since there are singularities in the coordinate transformations

J_L . In principle the multiple integral

$$\begin{aligned} \nabla_{\text{seq}} = & \int_{r_b} \int_{\theta_b} \int_{\phi_1} \int_{\phi_2} \int_{z_b} \int_{\theta_1} \int_{\theta_2} \int_{T_1} \frac{d^3 \nabla_{\text{tot}}}{dT_1 dR_1 dR_2} \sin \theta_1 \sin \theta_2 d\theta_1 d\theta_2 d\phi_1 d\phi_2 dT_1 d\theta_b dr_b dz_b \\ & (6.11) \end{aligned}$$

must be evaluated. Integrations over r_b , θ_b and z_b are included to account for finite beam diameter and length over the gas target. To perform the integration Equation (6.11) would take a large amount of computer time. Therefore, a one-dimensional line beam approximation to the experimental geometry was used. Equation (6.11) in this approximation becomes

$$\begin{aligned} \nabla_{\text{seq}} = & \int_{T_1} \int_{z_b} \int_{\theta_1} \int_{\theta_2} \frac{d^3 \nabla_{\text{tot}}}{dT_1 d\theta_1 d\theta_2} \sin \theta_1 \sin \theta_2 d\theta_1 d\theta_2 dz_b dT_1 \quad (6.12) \end{aligned}$$

This approximation should include all important geometric effects since the reaction kinematics are not strongly azimuth dependent for the geometry used in this experiment.

Integrations over beam length and polar angles were carried out numerically for each term of Equation (6.8) for 500 values of the proton energy, E_1 , to give values of the differential cross section

$$\frac{d\overline{V}_B}{dT_1} = \int_{z_b} \int_{\theta_1} \int_{\theta_2} \frac{d^3\overline{V}_B}{dT_1 d\Omega_1 d\Omega_2} \sin\theta_1 \sin\theta_2 d\theta_1 d\theta_2 dz_b \quad (6.13)$$

The differential cross section for the He^5 states were calculated using both the P.G.B. 1 and Watson forms of the D.O.S. functions. The n-p singlet differential cross sections were calculated using a number of values of the singlet scattering length. These were: (1) $a_{np}^s = -23.7 \text{ F}$ using the Watson form of the D.O.S. function, and (2) $a_{np}^s = -16.0 \text{ F}$, -18.0 F , -20.0 F , -22.0 F , -23.7 F , -26.0 F , and -28.0 F using the P.G.B. 2 form. Differential cross section values for intermediary proton energies were calculated from the 500 tabulated values by Aitken interpolation. To illustrate the effect of integration on the theoretical triple correlation cross sections, the differential cross section, Equation (6.13), for the phase space distribution is shown in Figure 6.8 as a function of the proton energy. The singularity at the maximum proton energy is removed and becomes a broad enhancement at high proton energies.

4. Fitting Procedure and Results

A linear least square fit to the experimental triple correlation data, for $\theta_1 = 62^\circ$, was done using the equation

$$\begin{aligned} \overline{V}_{\text{seq}}(T_i) = & a_1 \int_{T_i - \Delta T}^{T_i + \Delta T} \frac{d_{np}}{dT_1} dT_1 + a_2 \int_{T_i - \Delta T}^{T_i + \Delta T} \frac{d\overline{V}_{\text{He}^5}}{dT_1} dT_1 \\ & + a_3 \int_{T_i - \Delta T}^{T_i + \Delta T} \frac{d\overline{V}_{\text{P.S.D.}}}{dT_1} dT_1 + a_4 \int_{T_i - \Delta T}^{T_i + \Delta T} \frac{d\overline{V}_{\text{He}^5}^*}{dT_1} dT_1 + a_5 \int_{T_i - \Delta T}^{T_i + \Delta T} \frac{d\overline{V}_{\text{JL}}}{dT_1} dT_1 \end{aligned} \quad (6.14)$$

FIG. 6.8--The Integrated Phase Space Distribution for the Reaction $T(\text{He}^3, \text{He}^4)np$

where T_i = central energy of the i^{th} channel

ΔT = half the channel width

The channel width, $2 \Delta T$, was taken as constant over the entire spectrum and energy losses in the exit foils folded in only in the calculation of T_i . Each data point was weighted by counting statistics. The least squares results are shown as a solid line in Figures 6.9 - 6.14. The data points are displayed by vertical lines whose length represents counting statistics uncertainty. The values of the parameters a_i (normalized to the $\text{He}^5(\text{g.s.})$ contribution) obtained from the least squares fit, the D.O.S. function form used for the He^5 states and singlet n-p state, and the value of the singlet scattering length, a_{np}^s , are indicated on the diagrams. Omitted values of a_i have not been used in the least squares fit and are zero.

5. States of Li^5 and He^5

All terms in Equation (6.14), with the exception of the n-p singlet term, give their most significant contribution to the triple correlation cross section at high proton energies, Figures 6.3, 6.5 and 6.8. Therefore, fits to this portion of the triple correlation data (channels 47 - 60) were made for a number of possible combinations of terms to determine the contribution from each term. Also, fits were made for different energy calibrations near the calibration determined experimentally. An energy calibration of 0.1875 MeV/ch. with a slope intercept of 0.20 MeV gave the best results. The χ_n^2 values for the fits obtained with this energy calibration and the Watson form of the $\text{He}^5(\text{g.s.})$ D.O.S. function are listed in Table (V). This table shows that:

- (1) Low values of χ_n^2 , which indicate a good fit to the experi-

TABLE V

χ_n^2 VALUES FOR FITS TO THE HIGH ENERGY REGION OF THE
TRIPLE CORRELATION CROSS SECTION

Terms Included	χ_n^2	Coefficients			
		a_2	a_3	a_4	a_5
$\text{He}^5(\text{g.s.})$	75	1.01			
$\text{He}^5(\text{g.s.}) + \text{He}^{5*}$	7.5	0.92		$.138 \times 10^{-1}$	
$\text{He}^5(\text{g.s.}) + J_{3L}$	71	1.0			.005
$\text{He}^5(\text{g.s.}) + J_{3L} + \text{He}^{5*}$	7.2	.92		$.137 \times 10^{-1}$	.005
$\text{He}^5(\text{g.s.}) + \text{P.S.D.}$	0.78	.72	.23		
$\text{He}^5(\text{g.s.}) + \text{P.S.D.} + J_{3L}$	0.62	.70	.23		-.005
$\text{He}^5(\text{g.s.}) + \text{P.S.D.} + \text{He}^{5*}$	0.76	.738	.21	$.20 \times 10^{-1}$	

mental data, are obtained only if both the $\text{He}^5(\text{g.s.})$ and P.S.D. contributions are included.

(2) The contribution due to the states of Li^5 , J_{3L} , is small and of either sign.

(3) The inclusion of the He^{5*} contribution gives only a slightly improved fit.

It can be concluded from these results that:

(1) The high energy end of the triple correlation cross section is dominated by a sequential breakup through the ground state of He^5 and direct three body breakup.

(2) There is no evidence of effects due to the states of Li^5 . Therefore, the term J_{3L} will be omitted in further calculations.

(3) The effects of the first excited state of He^5 are not well defined. Fits to the entire triple correlation spectrum, as will be shown later, are improved by the inclusion of this term for some values of the singlet n-p scattering length. However, little information on this state can be obtained from the experimental data.

A typical fit to the triple correlation data at $\theta_1 = 62^\circ$ obtained using the first four terms of Equation (6.14) and the Watson form of the $\text{He}^5(\text{g.s.})$ D.O.S. function is shown in Figure 6.9. The experimental data is well reproduced. A similar fit using the P.G.B. 1 form of the $\text{He}^5(\text{g.s.})$ D.O.S. function, shown in Figure 6.10, indicates that this form incorrectly predicts the $\text{He}^5(\text{g.s.})$ peak position. The predicted peak position is dependent on the energy calibration. Therefore, fits to the high energy and of the triple correlation data were done for energy calibrations which correctly positioned the P.G.B. 1 $\text{He}^5(\text{g.s.})$ peak. The best results were obtained for an energy calibration of 0.190 MeV/ch. with a slope

FIG. 6.9--Example Least Squares Fit to the $T(\text{He}^3, \text{He}^4)np$ Triple Correlation Cross Section Using the Watson Form of the $\text{He}^5(\text{g.s.})$ D.O.S. Function

FIG. 6.10--Example Least Squares Fit to the $T(\text{He}^3, \text{He}^4)np$ Triple Correlation Cross Section Using the PGB1 Form of the $\text{He}^5(\text{g.s.})$ D.O.S. Function

intercept of 0.20 MeV. The χ_n^2 results for two of these fits were:

<u>Terms Included</u>	χ_n^2
$\text{He}^5(\text{g.s.}) + \text{P.S.D.}$	3.7
$\text{He}^5(\text{g.s.}) + \text{P.S.D.} + \text{He}^{5*}$	3.5

The energy calibration required to correctly position the ground state peak is outside the range of the expected error in the experimental energy calibration (Chapter IV). Also, the χ_n^2 values obtained for fits to the high energy portion of the triple correlation data were significantly higher than those obtained for similar fits using the Watson form of the $\text{He}^5(\text{g.s.})$ D.O.S. function. It must be concluded, therefore, that the P.G.B. 1 form does not adequately represent the effects of the $\text{He}^5(\text{g.s.})$ final state interactions. Since this form also failed to predict the effects of the n-p final state interaction, it will not be used in further calculations.

6. The n-p Singlet Interaction

Fits to the experimental data were made using the P.G.B. 2 and Watson forms of the singlet n-p D.O.S. function with $a_{np}^s = -23.7$ F. The quality and character of the fits obtained differed insignificantly. Niiler et al.³ found the P.G.B. 2 form gave slightly better results in fitting data on the reaction $D(p,n)2p$. Therefore, this form of the D.O.S. function was chosen for the investigation of the n-p singlet interaction.

Fits to the experimental data were made for $a_{np}^s = -16$ F, -18 F, -20 F, -22 F, -23.7 F, -26 F, and -28 F. The fits for $a_{np}^s = -16$ F, -20 F, -23.7 F and -28 F are shown in Figures 6.11 - 6.14. In each fit, the singlet n-p, $\text{He}^5(\text{g.s.})$, He^{5*} and P.S.D. amplitudes are determined by the least squares procedure. χ_n^2 was calculated over the entire spectrum to

FIG. 6.11--Least Squares Fit to the $T(\text{He}^3, \text{He}^4)_{np}$ Triple Correlation Cross Section for $a_{np}^s = -16 \text{ F}$

FIG. 6.12--Least Squares Fit to the $T(\text{He}^3, \text{He}^4)np$ Triple Correlation Cross Section for $a_{np}^s = -20 \text{ F}$

FIG. 6.13--Least Squares Fit to the $T(\text{He}^3, \text{He}^4)\text{np}$ Triple Correlation Cross Section for $a_{np}^s = -23.7 \text{ F}$

FIG. 6.14--Least Squares Fit to the $T(\text{He}^3, \text{He}^4)\text{np}$ Triple Correlation Cross Section for $a_{\text{np}}^s = -28 \text{ F}$

give a measure of the quality of each fit. Also, to illustrate the character of the fits obtained over the n-p singlet peak, values of χ_n^2 , assuming nine degrees of freedom, were calculated for eleven data points on the low and high energy sides of the peak. These χ_n^2 values are denoted χ_T^2 , χ_L^2 and χ_H^2 respectively, and are plotted as functions of the singlet scattering length in Figure 6.15. These χ_n^2 plots indicate that large values of the singlet scattering length give good fits to the low energy side of the n-p peak and low values give good fits to the high energy side. The χ_T^2 curve reflects these characteristics of the fit by giving a shallow minimum at $a_{np}^s = -21$ F.

The singlet n-p scattering length could not be included as a parameter in the least squares fitting procedure due to the computer time required. In the fitting procedure adopted above, fits obtained for different values of a_{np}^s were dependent on the values of the four parameters, $a_1 - a_4$, in the Equation (6.14). The parameter values for each fit obtained are listed in Table VI. The interaction between these parameter values and the singlet scattering length in determining the fit to the experimental data is not known. Therefore, no quantitative estimate of the error in determining the scattering length can be made directly from the least squares fitting procedure.

The most significant parameter affecting the fit obtained for a given scattering length is expected to be the contribution from the first excited state of He^5 , as the coefficient, a_4 , varies considerably in the fits obtained. To illustrate this effect, fits to the experimental data were made for four constant values of a_4 ; $a_4 = 0.0, .445 \times 10^{-2}, .89 \times 10^{-2}$ and $.134 \times 10^{-1}$. The parameters a_2 and a_3 were also fixed at, $a_2 = 0.738$ and $a_3 = 0.194$. The values of χ_n^2 for each of these values of a_4 are

TABLE VI

LEAST SQUARES FIT AMPLITUDES

Scattering Length	a_1	a_2	a_3	a_4
-16	.242	.758	.167	0.0
-18	.247	.749	.179	0.0
-20	.252	.741	.190	0.0
-22	.256	.738	.194	$.410 \times 10^{-2}$
-23.7	.259	.737	.195	$.795 \times 10^{-2}$
-26	.264	.735	.196	$.125 \times 10^{-1}$
-28	.268	.734	.197	$.159 \times 10^{-1}$

plotted as functions of the singlet n-p scattering length in Figure 6.16. The minimum value of χ_n^2 varies from $a_{np}^5 = -21 \text{ F}$ for $a_4 = 0.0$ to $a_{np}^5 = -24 \text{ F}$ for $a_4 = 0.134 \times 10^{-1}$. Unfortunately, the contribution of the first excited state of He^5 to this reaction is not known. Therefore, there is no a priori way of choosing a value of a_4 . The minimum value of χ_n^2 for each value of a_4 is below that expected from the statistical accuracy of the experimental data. Therefore, it is not possible to choose between the values of a_{np}^s determined from these curves.

The discussion above indicates the sensitivity of the value of a_{np}^s extracted from the experimental data to the background subtraction terms. This sensitivity is reflected in the broad minimum in χ_T^2 . The error in a_{np}^s was therefore determined from values of χ_T^2 representing a departure of the theoretical fit from the experimental data of one standard deviation. This gives the value of the singlet scattering length to be

$$a_{np}^s = -21_{-4}^{+3} \text{ F}$$

It is felt that the large errors assigned in this way, realistically reflect the uncertainty in the determined value of a_{np}^s caused by background subtractions.

A similar analysis procedure could be carried out for the experimental data at $\theta_1 = 59^\circ$ and $\theta_1 = 65^\circ$ to give three independent measurements of the singlet scattering length. The extracted value of a_{np}^s would be somewhat improved by the combination of the three independent determinations. However, the quality of the fits to this data cannot be expected to be significantly better than that obtained here for the $\theta_1 = 62^\circ$ data and large errors would have to be associated with each.

FIG. 6.16-- χ_n^2 Values vs. Singlet n-p Scattering Length for Fixed a_2 and a_3 and Four Values of a_4

value of a_{np}^s due to uncertainties in background contributions. Therefore it is unlikely that more significant information would be obtained by carrying out this analysis.

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CHAPTER VII

SUMMARY, RESULTS AND CONCLUSIONS

The previous sections described an investigation of the three body reaction $T(\text{He}^3, \text{He}^4)np$ in a complete experiment, at a bombarding energy of 1.5 MeV. The purpose of this, and similar experiments, is to investigate the application of approximate three body reaction theories to a reaction which produces two nucleons in the final state, with a view to extracting the p-p, n-p or n-n singlet scattering lengths from three body reaction data. The p-p and n-p scattering lengths are known accurately from elastic scattering data. Unfortunately, the n-n scattering length cannot be measured directly. Three body reactions which produce two neutrons in the final state provide a means of investigating this parameter. The accurate determination of the n-n scattering length is of fundamental importance in nuclear physics, as it provides a sensitive test of charge symmetry of the nuclear force. Confidence in its determination by the application of a theoretical model to the analysis of a three body reaction can be attained only if the same model can be applied to similar reactions to give accurate values of the n-p or p-p scattering lengths. Therefore, the extraction of the n-p scattering length from this $T(\text{He}^3, \text{He}^4)np$ experiment is of contemporary interest.

The approximate three body reaction theories used in this work were introduced by Watson¹ and by Phillips, Griffy and Biedenharn.² These theories consider the three body reaction to proceed by a sequential process in which one particle is emitted, leaving the remaining final state particles in a two body scattering state. These theories give the

Reaction cross section to be proportional to a density of states (D.O.S.) function which is entirely dependent on the properties of the two body state. The form of the D.O.S. function is different in the two theories. In fact, Phillips, Griffy and Biedenharn give two functional forms, (P.G.B. 1, and P.G.B. 2).

It has been shown⁶⁸ that the Watson and P.G.B. 1 forms of the D.O.S. function give the same results if the width Γ is independent of energy and the energy shift Δ_L is a linear function of energy. This is not the case for energies near threshold or for non-resonant phase shifts. The calculated D.O.S. functions show that these two forms give results which differ slightly for the $\text{He}^5(\text{g.s.})$ enhancements and drastically for the n-p singlet enhancement. The P.G.B. 1 form predicts an extremely narrow n-p singlet enhancement which is not observed experimentally. Therefore, this form of the D.O.S. function provides a completely inadequate description of the n-p final state interaction.

The Watson form of the $\text{He}^5(\text{g.s.})$ D.O.S. function predicts the resonance peak to occur at an n- He^4 internal energy of 0.91 MeV, which is close to the value expected from scattering experiments.⁵⁵ This form, with a statistical breakup contribution, provides an excellent fit to the $\text{He}^5(\text{g.s.})$ enhancement for the experimentally determined energy calibration. The P.G.B. 1 form predicts the $\text{He}^5(\text{g.s.})$ peak to occur at an internal energy of 0.81 MeV, and therefore, predicts this peak to appear at proton energies 100 keV above the position determined experimentally and predicted by the Watson form. This deviation of the peak position is only slightly beyond that explainable by the expected errors in the energy calibration. However, fits to the experimental $\text{He}^5(\text{g.s.})$ enhancement, for an energy calibration which correctly positions the P.G.B. 1 $\text{He}^5(\text{g.s.})$ peak, give

significantly larger χ_n^2 values than similar fits using the Watson form. These results suggest that the P.G.B. 1 form of the D.O.S. function does not give an adequate description of any final state interactions observed in this $T(\text{He}^3, \text{He}^4)np$ experiment.

The P.G.B. 2 form of the D.O.S. function was not calculated for the states of He^5 or Li^5 because of the computer time required for coulomb function evaluation. This form was calculated for the n-p singlet interaction as the absence of coulomb effects simplifies the computation. The P.G.B. 2 and Watson forms of the n-p singlet D.O.S. function were found to be very similar. In fact, fits to the experimental triple correlation cross sections using these two forms were indistinguishable. It is expected from these singlet n-p results, that the P.G.B. 2 form, if calculated, would give similar results as predicted by the Watson form for the states of Li^5 and He^5 .

The most significant, and perhaps surprising, result obtained from the analysis of this $T(\text{He}^3, \text{He}^4)np$ experiment is the excellent quality of the least squares fits, obtained using the Watson and P.G.B. 2 forms of the D.O.S. functions, to the experimental triple correlation data. These indicate that considering the three body breakup as a distinctly separable two-step process is valid, to a close approximation, even in the case of the broad singlet n-p interaction. The least squares results show that, in the kinematic region observed, the $T(\text{He}^3, \text{He}^4)np$ reaction proceeds predominantly through three reaction channels; (1) a sequential decay through the ground state of He^5 , (2) a direct three body breakup, and (3) a sequential decay through a virtual singlet n-p state. These three mechanisms accurately account for the major features of the measured triple correlation cross section. No evidence was found for contributions

from the states of Li^5 or for significant contributions from the first excited state of He^5 . The observation of the Li^5 states is not favoured kinematically, in the geometry used; therefore, their absence here does not reflect on the importance of these states to the total reaction cross section.

The value of the singlet n-p scattering length obtained from least squares fits to the triple correlation cross section for a number of values of a_{np}^s was

$$a_{np}^s = -21_{-4}^{+3} \text{ F},$$

where the quoted errors represent a departure of the fit from the experimental data of one standard deviation. The determination of this scattering length was found to be very dependent on contributions from the $\text{He}^5(\text{g.s.})$, He^{5*} and statistical breakup terms included in the least squares fits.

Unfortunately, the measurement of branching ratios are difficult, especially for the He^{5*} and statistical breakup channels, therefore these contributions are not known a priori. Also, contributions from the states of Li^5 , which were ignored here, may give a structureless background in the n-p peak region. The shapes of the contributions from the sequential decay channels were taken as described by the approximate three body theories even for internal energies well off resonance. It is not obvious that this is a valid assumption, as the sequential approximation certainly breaks down at these energies. There is no distinct contribution from the first excited state of He^5 , therefore it was not possible to determine if the shape of this contribution is described by the final state interaction theories. Since the Watson form of the D.O.S. function described the $\text{He}^5(\text{g.s.})$ and singlet n-p contributions, it was assumed that this form

also describes the He^{5*} contributions. The conservative estimation of the error in the determination of a_{np}^s used here, reflects the uncertainties in these background subtractions.

As mentioned in Chapter VI, the triple correlation data obtained here for $\Theta_1 = 59^\circ$ and $\Theta_1 = 65^\circ$ could be analysed to give two more independent measurements of a_{np}^s . This analysis would give an improved value of the singlet scattering length, due essentially to an increase in counting statistics. The uncertainties in background subtractions would be present in all determinations. To obtain a significant improvement over the present measurement, it is necessary to determine the contributions from the He^5 and Li^5 states and direct three body breakup more accurately. This would require the investigation of the $T(\text{He}^3, \text{He}^4)np$ reaction in a number of different kinematic regions, chosen by careful consideration of the three body reaction kinematics. Since the measurement and analysis of each triple correlation cross section is involved and lengthy, such a program would be quite formidable.

The reaction $T(T, \text{He}^4)2n$ could be studied in a similar experiment to this $T(\text{He}^3, \text{He}^4)np$ experiment to extract a value of the n-n scattering length. All charged particle contaminant reactions and all kinematic contours except the He^4 -n contour would be eliminated by replacing the proton counter used here by a neutron counter. However, the experimental difficulties involved with neutron detection and energy determination would be encountered. The states of Li^5 would not be observed but would be replaced by similar contributions from the He^5 system. The background subtractions in such an experiment would, therefore, be very similar to those encountered in this $T(\text{He}^3, \text{He}^4)np$ experiment. It is expected, then, that errors in a_{nn}^s of ± 4 F would have to be assigned due to uncertainties

in these background terms. Therefore, since much cleaner reactions which produce two neutrons in the final state are available, i.e.

$D(n,p)2n$ and $D(\pi^-, \gamma)2n$, this reaction will not appear attractive for the determination of a_{nn}^s until more detailed studies of the contributions from other reaction channels have been done.

APPENDIX A

1. Three Body Reaction Kinematics

The kinematics for three body reactions have been discussed by many authors.^{33, 47, 48} Because kinematic considerations are especially important for this experiment, this section will comprise a brief review for completeness. All symbols to be used are defined in Table I. The velocity vector diagrams of the relevant coordinate systems are shown in Figure 2.1.

Conservation of energy and momentum in the laboratory frame give the equations

$$\underline{P}_0 = \underline{P}_1 + \underline{P}_2 + \underline{P}_3 \quad (\text{A1-1})$$

$$T_0 + Q = T_1 + T_2 + T_3 \quad (\text{A1-2})$$

Using the nonrelativistic relation between momentum and energy

$$P_i^2 = 2MiT_i, \quad (\text{A1-3})$$

and eliminating P_3 from Equations (A1-1) and (A1-2) gives the quadratic equation,

$$A P_2^2 - 2B P_2 + C = 0 \quad (\text{A1-4})$$

where

$$A = 1 + m_3/m_2$$

$$B = P_0 \cos \theta_2 - P_1 \cos \Delta_{12} \quad (\text{A1-5})$$

$$C = (1 + m_3/m_1)P_1^2 + (1 - m_3/m_0)P_0^2 - 2P_0P_1 \cos \theta_1 - 2m_3Q$$

The solutions of this equation give the allowed kinematic contour in the

P_2 vs P_1 plane

$$P_2^{\pm}(P_1) = (B(P_1) \pm \sqrt{B^2(P_1) - A C(P_1)})/A \quad (A1-6)$$

The solutions are of course physically restricted to those which satisfy the conditioning equations

$$B^2 - AC \geq 0$$

$$P_2^{\pm}(P_1) \geq 0 \quad (A1-7)$$

Using these equations it is straightforward to calculate the corresponding contour in the T_2 vs T_1 plane

$$T_2^{\pm}(T_1) = 2m_2 P_2^{\pm 2}(P_1) \quad (A1-8)$$

Equations (A1-1) and (A1-2) may be solved again in terms of energy to give the more explicit equivalent equation,

$$(m_1+m_3)T_1 + (m_2+m_3)T_2 - P_0 P_2 \cos \theta_2 - P_0 P_1 \cos \theta_1 + P_1 P_2 \cos \Delta_{12}$$

$$= m_3 Q + T_3 (m_3 - m_0) \quad (A1-9)$$

The position and energy of the third (undetected) particle are the only remaining kinematic variables to determine the laboratory frame. These are easily obtained from Equations (A1-1) and (A1-2), and are,

$$T_3 = T_0 + Q - T_1 - T_2,$$

$$\cos \theta_3 = (P_0 - P_1 \cos \theta_1 - P_2 \cos \theta_2)/P_3, \text{ and} \quad (A1-10)$$

$$\tan \Phi_3 = \frac{P_1 \sin \theta_1 \sin \Phi_1 + P_2 \sin \theta_2 \sin \Phi_2}{P_1 \sin \theta_1 \cos \Phi_1 + P_2 \sin \theta_2 \cos \Phi_2}$$

The system centre of mass (s.c.m.) parameters are obtained from the laboratory parameters by the transformation implied by the defining

vector relations

$$\begin{aligned}\vec{v}_i &= \vec{V}_i - \vec{V}_{scm} \\ V_{scm} &= \frac{m_0 V_0}{M}\end{aligned}\quad (A1-11)$$

The corresponding velocity vector diagram is shown in Figure 2.1(a). From this diagram or Equation (A1-11) the relations,

$$\begin{aligned}v_i^2 &= V_i^2 + V_{scm}^2 - 2V_i V_{scm} \cos \Theta_i, \\ \phi_i &= \Phi_i, \text{ and}\end{aligned}\quad (A1-12)$$

$$v_i \cos \Theta_i = V_i \cos \Theta_i - V_{scm},$$

between the laboratory and s.c.m. variables are easily obtained.

The final coordinate system of interest in three body reactions is the centre of mass system of the particle recoiling from the initial break-up. This is called the recoil centre of mass system (r.c.m.) and is defined by the vector relations

$$\begin{aligned}\vec{v}_j^{(i)} &= \vec{v}_j - \vec{v}_{rcm}^{(i)} \\ \vec{v}_{rcm}^{(i)} &= \frac{m_i}{m_j + m_k} \vec{v}_i\end{aligned}\quad (A1-13)$$

The corresponding velocity vector diagram is shown in Figure 2.1(b). The superscript (i) in this notation indicates that the particle (i) was the first emitted particle and therefore particles j and k form the recoiling system. The equations,

$$\begin{aligned}v_j^{(i)2} &= v_j^2 + v_{rcm}^{(i)2} + 2 v_{rcm}^{(i)} v_j \cos \zeta_{ij}, \\ v_j^{(i)} \cos \Theta_j^{(i)} &= v_j \cos \Theta_j + v_{rcm}^{(i)} \cos \Theta_i, \text{ and} \\ \tan \phi_j^{(i)} &= \frac{v_j \sin \Theta_j \sin \phi_j + v_{rcm}^{(i)} \sin \Theta_i \sin \phi_i}{v_j \sin \Theta_j \cos \phi_j + v_{rcm}^{(i)} \sin \Theta_i \cos \phi_i}\end{aligned}\quad (A1-14)$$

relating the s.c.m. variables to the r.c.m. variables are easily obtained from Figure 2.1(b) or Equations (A1-13).

The final quantity of kinematic interest is the energy available in the r.c.m. system. This energy is of primary importance in the final state interaction theories and is given by

$$\begin{aligned} T_{jk} &= \frac{1}{2} m_j v_j^{(i)^2} + \frac{1}{2} m_k v_k^{(i)^2} \\ &= \frac{1}{2} m_j v_j^{(i)^2} (1 + m_j/m_k). \end{aligned} \quad (\text{A1-15})$$

With the equation for the allowed kinematic contour in the s.c.m. frame,

$$\mathcal{E}_{\text{tot}} = \frac{m_i + m_k}{m_k} \mathcal{E}_i + \frac{m_j + m_k}{m_k} \mathcal{E}_{ji} + \frac{p_i p_j}{m_k} \cos \theta_{ij}, \quad (\text{A1-16})$$

we may write internal energy in terms of the s.c.m. energy of the first emitted particle.

$$T_{jk} = \mathcal{E}_{\text{tot}} - \frac{M}{m_j + m_k} \mathcal{E}_i \quad (\text{A1-17})$$

From this equation the laboratory representation of the internal energy can be found as

$$T_{jk} = Q + \frac{1}{m_j + m_k} \left[p_i p_0 \cos \theta_i - T_0(m_0 - m_j - m_k) - MT_i \right] \quad (\text{A1-18})$$

Writing these out explicitly, and noting that from Equation (A1-9)

$$\begin{aligned} (m_1 + m_2)Q + (p_0 p_3 \cos \theta_3 - T_0(m_0 - m_1 - m_2) - MT_3) \\ = m_1 T_2 + m_2 T_1 - p_1 p_2 \cos \Delta_{12} \end{aligned} \quad (\text{A1-19})$$

the three internal energy functions become

$$\begin{aligned} T_{12} &= \frac{1}{(m_1 + m_2)} \left[m_2 T_1 + m_1 T_2 - p_1 p_2 \cos \Delta_{12} \right], \\ T_{13} &= Q + \frac{1}{(m_1 + m_3)} \left[p_0 p_2 \cos \theta_2 - T_0(m_0 - m_1 - m_3) - MT_2 \right], \text{ and} \end{aligned} \quad (\text{A-20})$$

$$T_{23} = Q + \frac{1}{(m_2+m_3)} \left[P_0 P_1 \cos \theta_1 - T_0(m_0-m_2-m_3) - MT_1 \right] .$$

2. Jacobians

There are four transformation jacobians required to transform all the theoretical triple correlation cross sections into the laboratory frame. The first of these is the transformation from the s.c.m. to the laboratory frame which is given notationally by

$$J_L = \frac{\partial (\varepsilon_i \mu_i \phi_i \mu_j \phi_j)}{\partial (T_i \mu_i \bar{\phi}_i \mu_j \bar{\phi}_j)} \quad (A2-1)$$

$$\text{where } \mu_i = \cos \theta_i$$

There are also three possible jacobians for the transformation from the r.c.m. frame to the s.c.m. frame, which are dependent on the experimental configuration. In the first case the first emitted particle is detected and the cross section is projected as a function of the energy of this particle. In this case the jacobian is given notationally by,

$$J_1 = \frac{\partial (T_{jk} \mu_j^{(i)} \phi_j^{(i)} \mu_i \phi_i)}{\partial (\varepsilon_i \mu_j \phi_j \mu_i \phi_i)} \quad (A2-2)$$

In the second case the first emitted particle is detected but the cross section is projected as a function of the energy of the detected breakup particle in which case,

$$J_2 = \frac{\partial (T_{jk} \mu_j^{(i)} \phi_j^{(i)} \mu_i \phi_i)}{\partial (\varepsilon_j \mu_j \phi_j \mu_i \phi_i)} \quad (A2-3)$$

In the final case both detected particles come from the breakup of the recoiling system and,

$$J_3 = \frac{\partial(\tau_{jk} \mu_j^{(i)} \phi_j^{(i)} \mu_i \phi_i)}{\partial(\varepsilon_j \mu_j \phi_j \mu_k \phi_k)} \quad (A2-4)$$

These jacobians have been evaluated by Bronson⁴⁷ by calculating the appropriate partial derivatives using the equations of section A1. This is a straightforward but laborious task. Blackmore³³ has derived the same results in a more elegant form by calculating the phase space factor in the three reference frames. The results of these calculations are

$$J_L = \frac{P_1 P_2^2 [A P_2 + P_1 \mu_{12}]}{P_1 P_2^2 [A P_2 + P_1 \mu_{12} - P_0 \mu_2]}, \quad (A2-5)$$

$$J_1 = \frac{M P_2^2}{m_2 P_2^{(1)} [A P_2 + P_1 \mu_{12}]},$$

$$J_2 = \frac{M P_1 P_2}{m_2 P_1^{(2)}} \frac{1}{[A P_2 + P_1 \mu_{12}]}, \quad \text{and}$$

$$J_3 = \frac{M P_1 P_2}{m_2 P_1^{(3)} P_3} \frac{1}{[A P_2 + P_1 \mu_{12}]}$$

The total transformation is a product of the transformation J_L times one of the transformations J_χ and may be written in the form,

$$J_{\chi L} = J_\chi J_L = \frac{M}{m_2 P_i P_j^{(\ell)}} \frac{P_1 P_2^2}{[A P_2 + P_1 \mu_{12} - P_0 \mu_2]}, \quad j \neq \ell. \quad (A2-6)$$

With the transformations (A2-6) all the integrations required for the analysis of the complete experiment results may be performed.

APPENDIX B

THE Li^5 & He^5 DENSITY OF STATES FUNCTIONS

The calculation of the density of states functions (expressions 5.19) for the states of He^5 and Li^5 require the determination of the phase shifts $\mathcal{J}_\ell$ and $\mathcal{P}_\ell$. These phase shifts are known quite well from the elastic scattering of nucleons from He^4 . The p wave phase shifts $\mathcal{J}_1^{(+)}$ and $\mathcal{J}_1^{(-)}$ which correspond to the first two states of Li^5 and He^5 are fitted quite well by single level dispersion theory. These phase shifts may therefore be written in the standard form

$$\mathcal{J}_\ell = B_\ell - \mathcal{P}_\ell$$

where $B_\ell = \tan^{-1} \left[\frac{\frac{1}{2} \Gamma_\ell}{T_\ell + \Delta_\ell - T} \right]$ (B-1)

and $\mathcal{P}_\ell = \tan^{-1} \left[\frac{F_\ell}{G_\ell} \right] \Big|_{\rho = kb}$

Here F_ℓ and G_ℓ are the regular and irregular coulomb wave functions. The state width Γ_ℓ is written in terms of a reduced width γ_ℓ and a penetration factor P_ℓ so that

$$\frac{1}{2} \Gamma_\ell = P_\ell \gamma_\ell^2 \quad \text{(B-2)}$$

where $P_\ell = \rho / A_\ell^2$ (B-3)

$$A_\ell^2 = F_\ell^2 + G_\ell^2$$

The resonant energy T_ℓ is defined with the energy shift function Δ_ℓ where

$$\Delta_\ell = -\gamma_\ell^2 (\ell + g_\ell) / \rho = kb$$

$$g_\ell = kR F^{-1} \left[\frac{\partial F_\ell}{\partial \rho} - G_\ell / A_\ell^2 \right] \quad (\text{B-4})$$

and is therefore not the same as the observed energy of the state.

The values of the parameters b , T and 2 for the first two states of the Li^5 and He^5 systems are given in Table 1B. The parameters for the Li^5 system are from Barnard et al.⁶⁶ and those for the He^5 system from Hoop and Barschall.⁶⁷ These parameters, together with the single level dispersion relations above, allow the evaluation of the D.O.S. functions for the mass five systems.

TABLE 1B

Parameters Obtained for States of Li^5 and He^5 from Single Level Dispersion Theory

State J	Phase Shift	b (fm.)	T (MeV)	² (MeV)
$\text{Li}^5 \ 3/2^-$	δ^+	3.0	4.79	8.23
$\text{Li}^5 \ 1/2^-$	δ^-	3.0	19.79	15.28
$\text{He}^5 \ 3/2^-$	σ^+	2.9	3.10	8.5
$\text{He}^5 \ 1/2^-$	σ^-	2.9	16.0	12.5

APPENDIX C

Tabulated Triple Correlation Cross Section for the Reaction $T(\text{He}^3, \text{He}^4)np$

In each of the following tables the proton energy and unmodified experimental data are given.

I. Geometry $\theta_1 = 62^\circ$ $\theta_2 = 90^\circ$
 $\phi_1 = 0^\circ$ $\phi_2 = 180^\circ$

Incident Energy = 1.5 MeV

<u>Energy (MeV)</u>	$\frac{d^3\sigma}{dT_1 d\Omega_1 d\Omega_2}$ (arb.units)	<u>Energy (MeV)</u>	$\frac{d^3\sigma}{dT_1 d\Omega_1 d\Omega_2}$
Particle Stopped	0	5.20	235
Particle Stopped	0	5.38	249
1.13	0	5.57	211
1.29	0	5.75	198
1.45	0	5.94	185
1.61	0	6.12	173
1.78	0	6.31	164
1.95	0	6.49	142
2.12	0	6.68	150
2.30	0	6.86	146
2.48	0	7.05	107
2.65	41	7.23	108
2.83	49	7.42	120
3.01	50	7.61	121
3.19	59	7.79	110
3.37	74	7.98	111
3.55	96	8.16	126
3.73	103	8.35	103
3.92	124	8.53	111
4.10	154	8.72	112
4.28	173	8.90	135
4.46	205	9.09	125
4.65	229	9.28	141

(cont'd.)

II. Geometry (cont'd.)

<u>Energy (MeV)</u>	$\frac{d^3\sigma}{dT_1 d\Omega_1 d\Omega_2}$ (arb.units)	<u>Energy(MeV)</u>	$\frac{d^3\sigma}{dT_1 d\Omega_1 d\Omega_2}$
3.37	60	7.98	128
3.55	77	8.16	132
3.73	107	8.35	124
3.92	128	8.53	133
4.10	150	8.72	135
4.28	118	8.90	233
4.46	153	9.09	298
4.65	118	9.28	241
4.83	136	9.47	511
5.02	110	9.65	707
9.84	147	11.15	685
10.03	158	11.33	429
10.21	161	11.51	220
10.40	152	11.70	110
10.58	167	11.89	17
10.77	200	12.08	2
10.96	218	12.26	0

III. Geometry $\Theta_1 = 59^\circ$ $\Theta_2 = 90^\circ$
 $\Phi_1 = 0^\circ$ $\Phi_2 = 180^\circ$

Incident Energy = 1.5 MeV

<u>Energy (MeV)</u>	$\frac{d^3\sigma}{dT_1 d\Omega_1 d\Omega_2}$ (arb.units)	<u>Energy(MeV)</u>	$\frac{d^3\sigma}{dT_1 d\Omega_1 d\Omega_2}$
Particle Stopped	0	5.20	74
Particle Stopped	0	5.38	103
1.13	0	5.57	108
1.29	0	5.75	144
1.45	0	5.94	153
1.61	0	6.12	201
1.78	0	6.31	233

(cont'd.)

I. Geometry (cont'd.)

<u>Energy (MeV)</u>	$\frac{d^3\sigma}{dT_1 d\Omega_1 d\Omega_2}$ (arb.units)	<u>Energy(MeV)</u>	$\frac{d^3\sigma}{dT_1 d\Omega_1 d\Omega_2}$
4.83	267	9.47	135
5.02	269	9.65	164
9.84	163	11.15	644
10.03	201	11.33	386
10.21	250	11.51	155
10.40	279	11.70	44
10.58	432	11.89	0
10.77	590	12.08	0
10.96	738	12.26	0

II. Geometry $\Theta_1 = 59^\circ$ $\Theta_2 = 90^\circ$
 $\Phi_1 = 0^\circ$ $\Phi_2 = 180^\circ$

Incident Energy = 1.5 MeV

<u>Energy (MeV)</u>	$\frac{d^3\sigma}{dT_1 d\Omega_1 d\Omega_2}$ (arb.units)	<u>Energy(MeV)</u>	$\frac{d^3\sigma}{dT_1 d\Omega_1 d\Omega_2}$
Particle Stopped	0	5.20	195
Particle Stopped	0	5.38	202
1.13	0	5.57	229
1.29	0	5.75	217
1.45	0	5.94	209
1.61	0	6.12	241
1.78	0	6.31	236
1.95	0	6.49	218
2.12	0	6.68	206
2.30	0	6.86	189
2.48	1	7.05	166
2.65	41	7.23	172
2.83	49	7.42	159
3.01	59	7.61	150
3.19	52	7.79	136

(cont'd.)

III. Geometry (cont'd.)

<u>Energy (MeV)</u>	$\frac{d^3 \nu^-}{dT_1 d\mathcal{R}_1 d\mathcal{R}_2}$ (arb.units)	<u>Energy(MeV)</u>	$\frac{d^3 \nu^-}{dT_1 d\mathcal{R}_1 d\mathcal{R}_2}$
1.95	0	6.49	252
2.12	0	6.68	257
2.30	0	6.86	290
2.48	0	7.05	247
2.65	36	7.23	246
2.83	46	7.42	228
3.01	63	7.61	236
3.19	47	7.79	144
3.37	76	7.98	152
3.55	151	8.16	166
3.73	126	8.35	204
3.92	129	8.53	213
4.10	118	8.72	233
4.28	124	8.90	296
4.46	125	9.09	413
4.65	97	9.28	436
4.83	118	9.47	904
5.02	112	9.65	856
9.84	98	11.15	563
10.03	110	11.33	243
10.21	111	11.51	0
10.40	124	11.70	0
10.58	140	11.89	0
10.77	122	12.08	0
10.96	167	12.26	0

APPENDIX D

PARTICLE IDENTIFICATION

The Ortec model 423 particle identifier used in this experiment employs the empirical relationship between the range, R , and incident particle energy E_T ,

$$R = a E_T^\xi \quad (D-1)$$

where a is a constant dependent on the type of particle stopped and is almost independent of energy. The exponent, ξ , for energies above 10 MeV is 1.73. For lower energies slightly lower values of this exponent give improved identification.

For a totally depleted transmission detector of constant thickness T and a thick stopping detector, as diagrammed above, the following relationships can be derived from Equation (D-1).

$$\begin{aligned} T + R_2 &= a E_T^\xi = a (E_2 + \Delta E)^\xi, \\ \text{and } R_2 &= a E_2^\xi, \\ \text{therefore, } T/a &= (E_2 + \Delta E)^\xi - E_2^\xi \end{aligned} \quad (D-2)$$

The quantity T/a is a constant which is dependent on the particle type. This quantity is computed by the particle identifier from the energy signals from an $E - \Delta E$ detector combination, and a proportional pulse generated at the particle identifier output (P.I.O.). The identification characteristics are shown in Figure 3.2.

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