SCATTERING OF LIGHT FROM
A PLASMA JET

by

Garth S.J.P. van der Kamp

B.Sc., University of British Columbia, 1966

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER IN SCIENCE

in the Department

of

PHYSICS

We accept this thesis as conforming to the

required standard

THE UNIVERSITY OF BRITISH COLUMBIA

May, 1968

In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and Study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the Head of my Department or by his representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of PHYSICS

The University of British Columbia
Vancouver 8, Canada

Date

ABSTRACT

The electron density in a plasma jet has been determined as a function of position in the jet by measurements of the coherent scattering of laser light. The electron density profiles thus obtained are compared with profiles obtained from measurements of spectral line intensities. The results of these two diagnostic methods are seen to agree within experimental error. For the experiment described here, the scattering is from correlated motion of the electrons, and the scattered light spectrum has distinct electron satellites. It is shown that the width of these satellites, observed in earlier work by Chan and Nodwell (1966), may be accounted for by the electron density gradients in the plasma. This thesis gives a short review of techniques and relevant theory, with emphasis on experimental problems. A brief discussion of perturbation of the plasma by the laser light is also included.

TABLE OF CONTENTS

		Page
	ABSTRACT	ii
	TABLE OF CONTENTS	iii
	LIST OF ILLUSTRATIONS	iv
	ACKNOWLEDGEMENTS	v
Chapter I	INTRODUCTION	1
Chapter II	THEORY	4
	A - Theory of Scattering	4
	B - Application to Diagnostics	8
	C - Apparatus and Technique	10
Chapter III	MEASUREMENTS OF EXCITATION TEMPERATURES	17
	A - Theory	17
	B - The Abel Transform	20
	C - Absolute Intensity Calibration	23
Chapter IV	RESULTS	28
	A - Perturbation of the Plasma	28
	B - Preliminary Scattering Results	30
	C - Plasma Diagnosis Results	31
	D - Satellite Widths	33
Chapter V	CONCLUSIONS	37
	A - Discussion of Results	37
	B - Suggestions for Future Work	37
	BIBLIOGRAPHY	39

LIST OF ILLUSTRATIONS

Figure		Page
1	Typical scattered light spectra	6
2	Schematic of Experimental Arrangement	11
3	Calibration of absorbtion filters	25
4	Perturbation of Plasma by Laser	28
5	Radial Electron Density Profile in Argon Jet	32
6	Radial Electron Density Profile in Helium Jet	32
7	Radial Electron Density Profiles in Argon Jet	34
8	Observed and Theoretical Satellite Profiles	35

ACKNOWLEDGEMENTS

My thanks go first of all to my supervisor, Dr. R.A. Nodwell, for his guidance during the experimental work and his patience during the preparation of this thesis. I would also like to thank Mr. M. Churchland for his aid and advice in the experimental work, and the members of the technical staff, in particular Mr. J. Dooyeweerd, for their frequent aid and useful suggestions.

CHAPTER I - INTRODUCTION

In recent years the scattering of laser light by a plasma has received much attention, both theoretically and experimentally. This interest is due at least in part to the advantages of using light scattering as a plasma diagnostic tool, the most important of these being the good resolution in space and time which can be obtained. Besides that there is the practical, and perhaps basic advantage that the reliability of this diagnostic method does not depend on intermediate theories and assumptions about the nature and properties of the plasma - the weak point in most other diagnostic methods.

The theory of the scattering of light has been derived and discussed by a number of authors such as Salpeter (1960), and Rostoker and Rosenbluth (1962). The case of the particular experiment described in this thesis is discussed by Williamson et al (1966), and by P.W. Chan (1966) When the plasma electrons act independently, the profile of the scattered light is essentially Gaussian, due to doppler broadening by the thermal motion of the electrons. When the electrons act coherently, the scattered light profile consists of a central peak, and of weak electron satellites, shifted to either side of the central line by approximately the plasma frequency. The satellite shift thus gives a measure of electron density. This latter case of coherent scattering holds for the experiment described here.

Among the experimental problems associated with scattering the low intensity of the scattered light is of overriding importance. For the experiment described here, the intensity of the observed scattered light is down by a factor of about 10^{-14} from the intensity of the incident light.

Several workers have reported the use of scattering as a diagnostic tool. Among these are Röhr (1967) who used the satellite shift to give a measure of electron density; Izawa et al (1966) who used scattering to obtain electron densities in a shock front; and, most important for the present purposes, Chan and Nodwell (1966) who first observed the satellites for scattering from a laboratory plasma, (plasma jet) and showed that the observed satellite shift was in good agreement with the expected values of the plasma parameters. The work described in this thesis is a continuation of the work reported by Chan and Nodwell.

One problem encountered by Chan and Nodwell was that the observed satellite width was much larger than that expected from the theory. This broadening was tentatively attributed to electron density gradients in the scattering volume. At first the purpose of the continued work was to verify this assumption. After some time it became apparent that to accomplish this purpose, electron density profiles of the plasma jet must be obtained. This consideration, together with the fact that satellite width could not be measured very reliably, changed the emphasis of the experiment towards plasma diagnostics, using the satellite shift to determine electron density.

In connection with this diagnostic approach, electron density profiles of the plasma jet were also obtained using spectral line intensity measurements, with a technique similar to that of Olsen (1963). These measurements allowed a comparison of the two diagnostic methods, both with regard to results and with regard to applicability and reliability. In addition of course such an independent measurement of plasma parameters makes it possible to verify that the satellite shift is indeed that

predicted by the theory. A detailed agreement of scattering observations with independently measured plasma parameters has as yet not been reported in the literature.

Finally, the electron density profiles obtained by the diagnostic methods mentioned above were used to determine what part of the observed satellite widths could be attributed to electron density gradients in the plasma.

This thesis reports and compares the results obtained by the two diagnostic methods one using scattering of light, and the other using spectral line intensities. Some results as to satellite widths, and perturbation of the plasma by the laser light are also given. Chapter II gives a summary and simple physical explanation of the relevant scattering theory, and also discusses the apparatus and experimental techniques used in scattering observations. This discussion is for a large part a continuation of the discussion given in P.W. Chan's thesis. Chapter III contains a short presentation of the theory and techniques used in the spectral line intensity measurements. Chapter IV reports and analyzes the experimental results on electron density gradients, satellite width, and perturbation of the plasma. Chapter V gives some conclusions based on these results, and also gives some suggestions as to future work.

The work reported in this thesis has been published recently, (Nodwell and van der Kamp, 1968).

CHAPTER II - SCATTERING OF LIGHT FROM A PLASMA

The theory for the scattering of light from a plasma has become well established in recent years. In fact, partly because of the difficulties inherent in the observation of scattering, the theory is at present well ahead of experiment, being easily capable of accounting for all experimental results obtained up till now. For that reason the discussion of scattering in this chapter will emphasize the practical problems involved in this particular experiment.

A - Theory of Scattering.

From a theoretical point of view the scattering of light by a plasma is a complex phenomenon. Some rather involved mathematics is required to describe it in full detail. The complete theory has been derived and discussed by a number of authors, including first of all Salpeter (1960), and Rostoker and Rosenbluth (1962). Williamson et al (1966) analyzes the particular case of the experiment described in this thesis, including a Fortran program to calculate the spectrum of the scattered light. A complete theoretical derivation is also given in the thesis by P.W. Chan (1966) who did the first work in this experiment.

The full theory will not be reproduced here, since the theoretical results relevant to the experiment can be simply summarized and qualitatively explained on the basis of plausible physical arguments. More detail can be found in the references given above.

The results of the full theory may be roughly summarized as follows. We consider a plasma with electron density N_e, electron temperature T_e, ion temperature T_i, and electron Debye length $L_d = \left(\frac{K_b T_e}{4\pi n_e e^2}\right)^{\frac{1}{2}}$ where K_b is Boltzmann constant and e the electronic charge. It turns out that the nature of the scattered light spectrum is characterized by

the parameter $\alpha = \frac{1}{|\Delta \underline{k}| L_d}$ where $\Delta \underline{k}$ is the difference between the wave vectors of the incident and the scattered light. As we will see, this parameter, α, has a simple explanation in terms of plasma waves.

Some typical scattered light spectra are shown in Fig. 1. The case $\alpha \ll 1$ corresponds roughly to the case of the wavelength of the incident light being much smaller than the Debye length. The electrons act individually and the scattered spectrum approaches that of Thomson scattering, doppler broadened by the thermal motion of the electrons. In the case $\alpha \gg 1$ the scattering is by collective motion of the electrons. The scattered spectrum has a narrow central peak, and weak electron "satellite" peaks separated from the central peak by approximately the plasma frequency. More precisely, the theory predicts that the satellite shift $\Delta \omega$ is given by

$$(\Delta \omega)^2 \approx \omega_p^2 + \frac{3 K_b T_e}{m_e} |\Delta \underline{k}|^2 = \omega_p^2 \left(1 + \frac{3}{\alpha^2}\right) \qquad (2-1, 2)$$

where ω_p is the plasma frequency and K_b the Boltzmann constant. The case $\alpha \approx 1$ has characteristics lying somewhere between the two extreme cases given above. We are particularly concerned with the case $\alpha > 1$.

The theory also predicts that for a particular scattering angle and solid angle, the total scattered intensity is proportional to $\frac{1+\alpha^2}{1+2\alpha^2}$, while the integrated intensity of each satellite is proportional to $\frac{1}{2(1+\alpha^2)}$. The ratio of the intensity of each satellite to that of the central peak is thus $\frac{1+2\alpha^2}{2\alpha^4}$ which approaches $\frac{1}{\alpha^2}$ for large α.

These results can be qualitatively understood if we consider that the collective motions of the electrons are just plasma waves, so that in the case $\alpha \gg 1$, the scattered light spectrum is determined by the spectrum of the plasma waves. In particular, consider the diagram shown

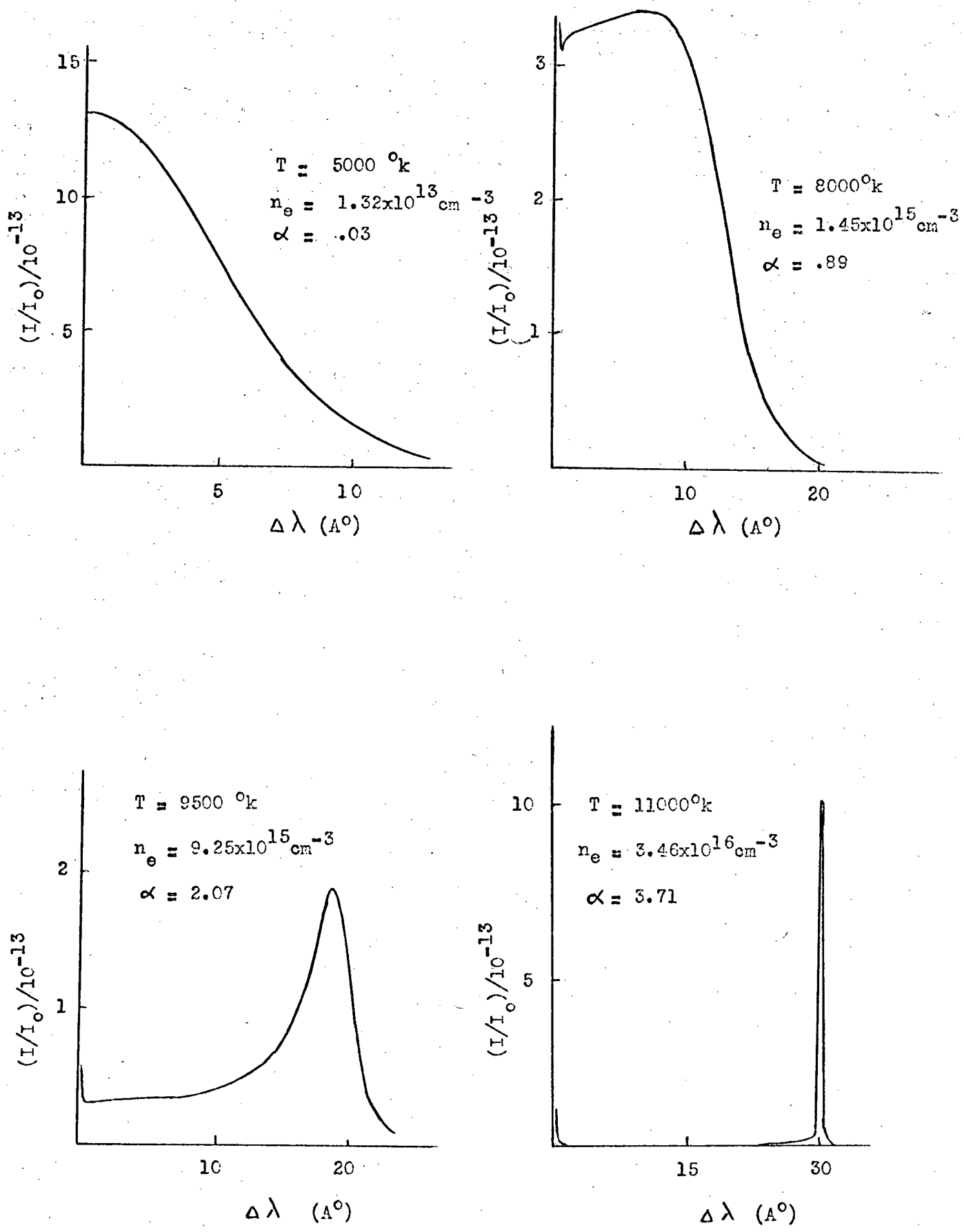
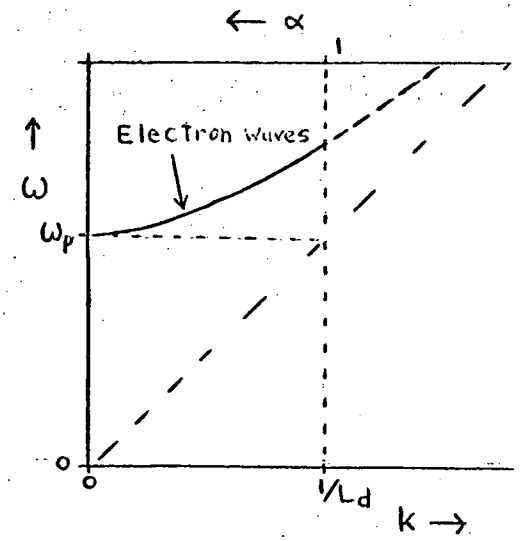


Figure 1 - Typical scattered light spectra for the argon plasma jet.



on the left (Holt and Haskell, 1965, p. 364). Only plasma waves with wave number $\underline{k} = \Delta \underline{k}$ contribute to the scattering. For $k \gg \frac{1}{L_d}$ $\left(\alpha = \frac{1}{kL_d} \ll 1\right)$ no plasma waves exist, since the wavelength is less than the Debye length L_d. For the corresponding values of α we would expect the spectrum of the scattered light to be characteristic of uncorrelated electron motion. For $k < \frac{1}{L_d}\left(\alpha = \frac{1}{kL_d} > 1\right)$ plasma waves can exist and produce satellites with frequency shift satisfying the dispersion equation of the plasma waves (Eq 2-1). The spectral width of the satellites is related to the Landau damping of the waves, which in turn depends on the derivative of the one-dimensional velocity distribution function at $\frac{\omega}{k}$ (the phase velocity of the plasma wave). Also, since the electrons most effected by the waves are the ones which travel with it at its phase velocity, the intensity of the satellites depends on the value of the velocity distribution at $\frac{\omega}{k}$. Thus both the width and integrated intensity of the satellites increase with decreasing α (increasing k).

Using this picture one can easily see how the intensity of the satellites can be enhanced through excitation of the plasma waves which do the scattering, that is by an increase of the number of super-thermal electrons travelling with the wave at its phase velocity.

The rest of the electrons are not correlated to each other, since their random thermal velocities are too high, but to the ions. Thus the

central peak exhibits a spectral distribution dependent on the ion motion, i.e. dependent on ion thermal velocities and ion waves.

B - Application to Diagnostics.

When scattering of light is used for plasma diagnostics, the problem is to determine the unknown plasma parameter from the observed scattered spectrum. The scattered signal usually is very weak, so that only some of the grosser parameters of the scattered spectrum such as satellite shift, integrated intensities, and halfwidths can be measured. Consequently only a few plasma parameters such as temperatures and densities can be determined.

In the experiment described here the scattering parameters that were measured were the satellite shift and halfwidth. For large α the satellite halfwidth which is strongly dependent on electron temperature, is too small to measure accurately. However, since the temperature dependence of the satellite shift is small, ($\frac{3}{\alpha^2} \ll 1$, see Eq. 1-2)a rough estimate of temperature, obtained from electron density through the Saha equation, is sufficient to allow an accurate evaluation of electron density from the observed satellite shift. In practice equation (1) for the satellite shift is found to be not sufficiently accurate for small α. The exact relation between electron density and satellite shift can be determined by direct computer calculation of the scattered light spectrum (Williamson et al 1966) with temperature dependent on electron density through the Saha equation. Table 1 gives plasma and scattering parameters for argon and helium at atmospheric pressure, and a scattering angle of 45°. Values intermediate between those given in the table were obtained by graphical methods, and by interpolation.

Table 1 - Plasma and scattering Parameters for Argon and Helium Jets

TEMP (^{o}k)	EL DEN (cm^{-3})	NEUTRALS DEN (cm^{-3})	PARTITION FN	ALPHA	SAT SHIFT (Å)
ARGON					
8000	.145E16	.914E18	1.000	0.89	----
9000	.535E16	.805E18	1.000	1.61	15.5
10000	.151E17	.704E18	1.000	2.57	22.3
11000	.346E17	.599E18	1.001	3.71	30.2
12000	.667E17	.480E18	1.003	4.93	39.7
13000	.109E18	.349E18	1.006	6.06	49.7
14000	.152E18	.224E18	1.014	6.91	58.1
15000	.184E18	.125E18	1.030	7.34	63.7
16000	.199E18	.643E17	1.063	7.39	66.3
17000	.202E18	.325E17	1.124	7.21	66.8
18000	.197E18	.172E17	1.231	6.93	66.2
19000	.190E18	.987E16	1.409	6.63	65.2
HELIUM					
12000	.621E15	.610E18	1.000	.48	----
13000	.159E16	.561E18	1.000	.73	----
14000	.355E16	.517E18	1.000	1.05	----
15000	.713E16	.475E18	1.000	1.44	18.2
16000	.130E17	.433E18	1.000	1.89	23.0
17000	.220E17	.388E18	1.000	2.38	27.7
18000	.344E17	.339E18	1.000	2.89	32.7
19000	.503E17	.286E18	1.000	3.40	36.9
20000	.686E17	.230E18	1.001	3.88	41.9
21000	.873E17	.175E18	1.002	4.27	46.3
22000	.104E18	.125E18	1.003	4.56	50.0
23000	.117E18	.843E17	1.005	4.73	52.7

Note: These data are for pressure of 1 atm. Argon composition data are from Popenoe and Schumaker (1965), Helium data from Drawin and Felenbok (1965).

C - Apparatus and Technique.

A diagram of the experimental arrangement for the observation of scattering is given in Fig. 2. As has been mentioned before, this experiment is a continuation of the work done by P.W. Chan, and the apparatus and technique are essentially those which Chan used. Thus the following discussion will be largely confirmed to various improvements which have been made, and the reader is asked to refer to Chan's thesis for comparison or additional information.

(a) Laser. The laser used was a TRG Model 104A Q-switched ruby laser, with a Daly-Sims attachment. The Daly-Sims accessory shortens the time during which lasing can take place, and thus increases the power of the laser pulse, while eliminating the possibility of multiple pulses. The combined units are rated for a pulse-width of 30 nanoseconds, beam divergence of 10 milliradians, and peak power of 30 megawatts. The TRG Model 104 laser used by Chan was rated for a peak power of 10 megawatts. Thus the model 104A represents an important improvement in laser power, and in the resultant intensity of scattered light.

(b) Plasma Jet. The plasma jet is similar to the one used by Chan. It gives a reproducible plasma with an electron density between 10^{16} and 10^{17} cm^{-3} and a temperature of 1 to 2 ev. The jet is operated at atmospheric pressure with a current of 150 to 300 amperes and a gas (argon or helium) flow rate of 75 to 150 millilitres per second. As the flow rate through the jet is increased to about 100 ml/sec there is a marked transition from laminar to turbulent flow. Most of the experimental work was done in the laminar flow regime because it gives a much steadier and more reproducible plasma. For more detail on the construction, operation,

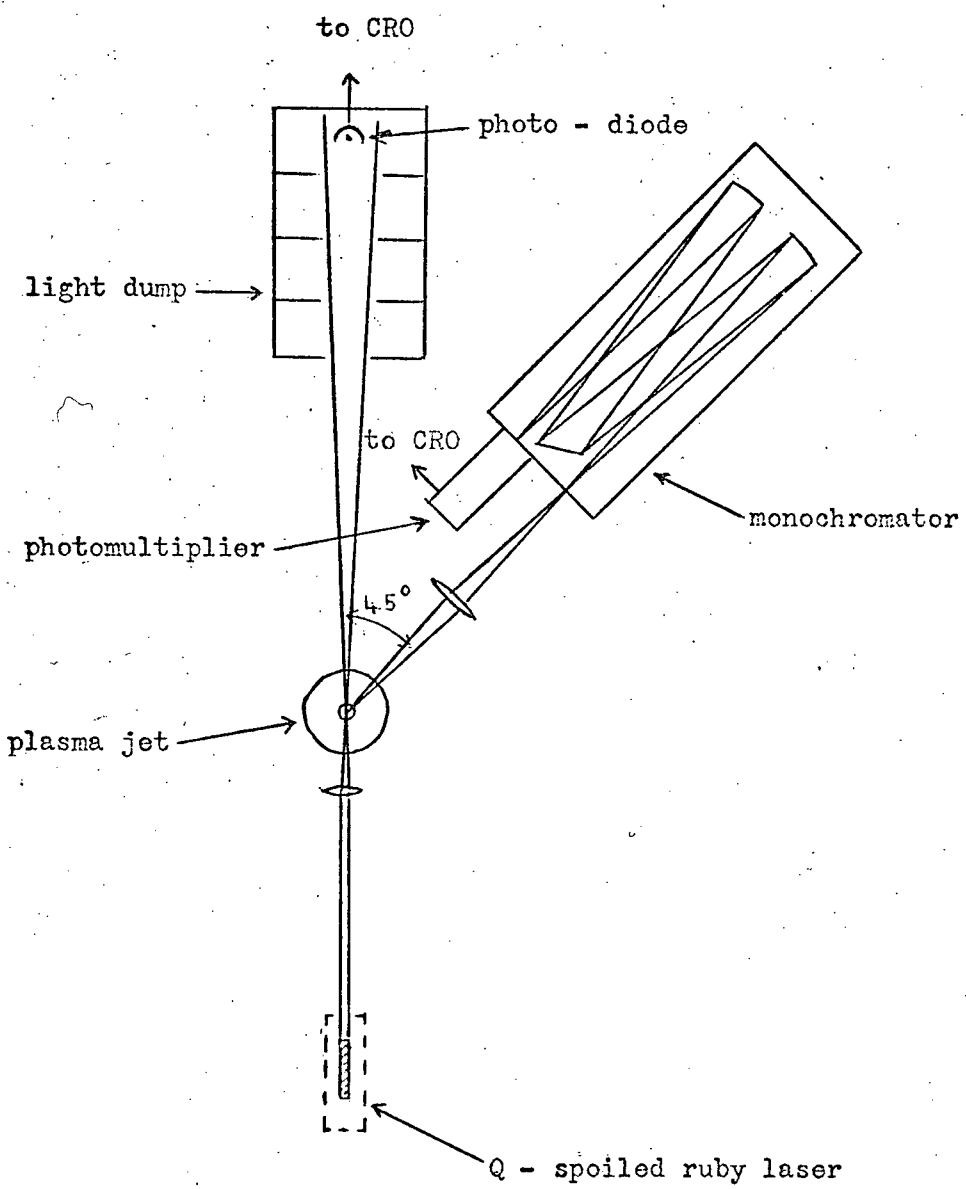


Figure 2 - Schematic of the Experimental Arrangement

and properties of the jet, the reader is referred to the thesis by P.W. Chan (1966) and R.M. Morris (1968). The jet was mounted on a base which allowed controlled motion of the jet in any direction, so that, by moving the jet, the laser focus could be positioned at any pre-selected point in the jet to within .02 millimetres.

(c) Monochromator. The monochromator, a Jarrell-Ash 82-010, was the same as the one used by Chan. It has an f number of 10, and a reciprocal dispersion of 16 Angstroms per millimetre at 6700 Angstroms. The monochromator is blazed at 7500 A°, so that ultraviolet light entering the monochromator appears mostly in the second order, that is in the red and infrared. Thus for observation of light signals above about 6500 A°, a red transmitting filter was usually needed to absorb the ultraviolet light.

(d) Photomultiplier. The photomultiplier used in all the work was an EM I 9558B. It is linear for a dc anode current up to 1 ma, and for up to five times greater values of current under pulsed conditions. The photomultiplier has a peak quantum efficiency of 30% at 5500 A°, and an efficiency of about 3% at the laser wavelength.

(e) Optical System and Reduction of Stray Light. Not surprisingly, it was found that the simpliest optical system was most satisfactory. The laser beam was focussed on the plasma through a simple convex lens with 10 cm focal length. (Doublet lenses could not be used in the direct laser beam because the glue at the interface was soon burned through absorption of the beam). After passing through the plasma the laser light was absorbed in a large box, painted dull black inside, and suitably baffled. A fast-rise photo diode, inside the box, monitored

the laser pulse, and provided a triggering pulse for the CRO. This pulse also gave a useful check on the laser intensity, so long as the diode was operated in the linear region of small signals (less than 1 volt).

The plasma was focussed on the monochromator entrance slit through a pair of achromat doublets, with approximately one-to-one magnification.

Baffles to reduce stray light seemed to be more harmful (probably through reflection and diffraction) than beneficial, and since they were rather a hindrance during alignment, they were finally dispensed with altogether. In any case, by far the most serious source of stray light signals was found to be at the junction between the photomultiplier and the exit slit of the monochromator. The only other important concerns in the reduction of stray light were that the laser beam should not strike the top of the jet, and should not reflect off the front surface of the absorbing box. And even when this occurred, most of the stray light entered the photomultiplier directly through the gap between it and the exit slit, and not through the monochromator. The reason for this is that the satellite frequency is relatively far removed from the laser line. Any stray light entering through the monochromator entrance slit shows up mainly as part of the central peak. Partly for this reason, and partly through careful shielding, stray light could be reduced to negligible proportions.

(f) Statistical methods. The one problem which characterizes the whole experiment is the weakness of the scattered light signal. For a typical experimental situation, the intensity of scattered signal in one satellite is about 10^{-12} of the incident light, and the number of photons actually observed is of the order of 100. Taking into consideration the signal fluctuations due to the cascade in the photomultiplier, plus

the noise due to stray light and background light from the jet, we see that the expected signal-to-noise ratio may be little better than unity. Statistical methods can be applied involving many shots at each wavelength setting, but the number of shots is limited because the electrodes of the plasma jet begin to deteriorate after a few hours. Thus satellite observations must be done quickly, with as few laser shots as possible.

If statistical methods have limited usefulness, the next step is to maximize the scattered signal, and minimize stray light and background light from the jet. We will enumerate the various possibilities point by point.

(g) Intensity of scattered signal. We have for the integrated intensity of each satellite

$$n = \left(\frac{U}{h\nu}\right) L\, r_o^2 \sin^2\gamma \frac{n_e}{2(1+\alpha^2)}\; q(\Delta\Omega)$$

n is the number of scattered photons observed.

U is the total energy of the incident light and

$h\nu$ is the energy of each incident photon.

U is limited by the available power, and by perturbation of the plasma by the laser.

L is the length of scattering volume.

It is determined by the width of the entrance slit and the magnification of the optical system. Increasing L decreases the spatial resolution, and increases the broadening of the observed satellites due to density gradients in the plasma.

$r_o = 2.82 \times 10^{-13}$ cm is the radius of the electron.

γ is the angle between the plane of polarization of the incident light and the scattered plane. In this case $\gamma = \frac{\pi}{2}$ and $\sin^2 \gamma = 1$.

n_e is the electron density. For large α the term $\frac{n_e}{1+\alpha^2}$ is practically constant, independent of n_e, since α^2 is proportional to n_e.

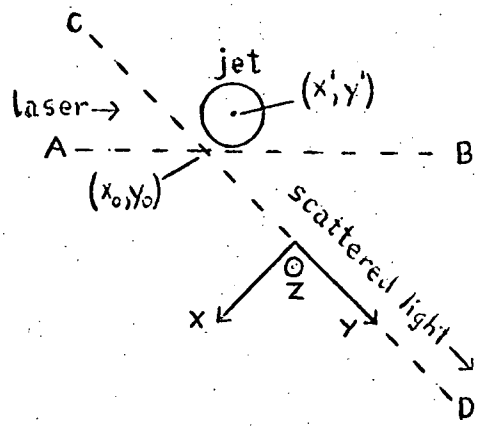
q is the quantum efficiency of the photomultiplier, approximately .03. for the EMI 9558 B .

$\Delta\Omega$ is the solid angle into which the light is scattered. In principle $\Delta\Omega$ could be maximized by placing a large powerful collimating lens very close to the plasma. Angular resolution would be decreased, but the effect of that on the scattered spectrum is small, and moreover spatial resolution could be improved

(i) Reduction of background radiation from the plasma. Plasma luminosity is due to spectral and continuum radiation. Spectral radiation can be avoided by not working at wavelengths near spectral lines, and by choozing a gas with few spectral lines near the laser wavelength. The continuum due to recombination radiation and bremstrahlung is dependent mainly on the temperature and density of the electron. It can not be avoided, but its effect can be greatly reduced by working at low electron densities (satellite intensity is independent of n_e for large α) and by restricting observation to regions where the plasma column, equal to plasma emissivity integrated along the line of sight.

(j) Positioning of Scattering Volume in Jet. One important purpose of the experiment was to measure the satellite shift as a

function of the position of the scattering volume in the jet, the scattering volume being defined as that part of the focussed laser beam from which scattered light passes through the monochromator. Thus it was important to have accurate measurements of the distance of the scattering volume from the central axis of the jet. This was done in the following manner. Consider the diagram. The jet moves, while the scattering volume, lying at the intersection of AB and CD, remains fixed in the coordinate system xyz (The y axis lies in the direction of the scattered light. The z axis is parallel to the central axis of the jet).



The height z of the scattering volume above the cathode tip of the jet is easily measured with vernier calipers. If the position of the jet axis is given by the coordinates (x' , y'), we want the values, x_o and y_o , of x' and y' when the scattering volume lies on the jet axis. Moving the jet along the x-axis gives a profile of the dc light from the jet as a function of x', with a clear maximum at x_o. To determine y_o we use the fact that in argon the Ar II lines can be strongly excited by the laser light. The resultant large signal, measured as a function of y' has a maximum at y_o . For any position (x' , y') of the jet, the separation of the scattering volume from the axis is just $\left((x_o - x')^2 - (y_o - y')^2\right)^{\frac{1}{2}}$.

The whole procedure must of course be repeated from time to time while the jet is being run, so that allowance can be made for changes in position of the jet due to the changes in the electrodes.

CAPTER III - MEASUREMENTS OF EXCITATION TEMPERATURES.

In conjunction with use of scattering for plasma diagnostics, another method was used, namely that of measuring excitation temperature through plasma emissivity. The purpose was to compare the results obtained by scattering with those obtained by a more established method, and also to compare the potential usefulness of the two methods. Since the primary concern of the experiment was scattering, and not excitation temperature measurements, the latter will be treated rather quickly here, only a short mention of the difficulties involved. The method may be outlined as follows. The photomultiplier is calibrated for absolute intensity measurements with a standard source. Intensity profiles of the jet are taken using the photomultiplier, and are unfolded by the Abel transform to give radial emission profiles. The emission of the plasma is in turn related to its temperature. The method is very similar to that used by Olsen (1963), who applied it to the mesurement of atomic transition probabilities.

A - Theory.

(a) Emissivity as Function of Temperature. For the emissivity of a plasma due to one atomic spectral line we have

$$\epsilon = \frac{1}{4\pi}(E_m - E_n) A_{mn} n_m \qquad (3-1)$$

ϵ is the emissivity per unit angle.

E_m and E_n are the energies of the upper and lower states for this transition;

A_{mn} is the atomic transition probability for the transition, and

n_m is the number density of atoms in the upper energy state.

(Transition probabilities for helium were taken from tables of atomic transition probabilities published by the U.S. Department of Commerce

(Wiese et al - 1966). These are theoretically calculated values with an estimated possible error of about 10%. For the transition probability of the Argon I 6965 line the value measured by Popenoe and Schumaker (1965) was used. The possible error quoted for it is 11%.)

If we assume that collisional processes predominate over spontaneous radiational transfer in excitation of the upper state, then n_m is determined by the energy distribution of the colliding particles.

Criteria given by Griem (1966) show that for typical values of n_e and T in the plasma jet, this assumption holds for all energy levels with principal quantum greater than 1. In this case n_m is given by

$$n_m = n_o \frac{g_m}{Z(T)} e^{-E_m/k_bT} \qquad (3-2)$$

n_o is the density of atoms in the ground state

g_m is the degeneracy of the upper state

Z(T) is the partition function of the atom

T is the excitation temperature of the upper state;

T is the same for all states for which the above assumption holds, and should also be equal to T_e, the electron temperature, since electron collisions predominate in the process of excitation. The density n_o of particles in the ground state, effectively equal to the density n_a of neutrals, is dependent on the temperature of the various species in the plasma. The value of n_a is governed by the following equations;

neutrality

$$n_e + n_i = 0 \qquad (3-3)$$

gas law

$$k_b(n_e T_e + n_i T_i + n_a T_a) = P = 1 \text{ atm} \qquad (3-4)$$

Saha

$$\frac{n_e n_i}{n_a} = 2 \frac{Z_i(T)}{Z_a(T)} \left(\frac{m k_b T}{2\pi \hbar^2}\right)^{3/2} e^{-E_a/k_bT} \qquad (3-5)$$

Z_i and Z_a are the partitions of the singly ionized and neutral states.

E_a is the ionization of the neutral atom, corrected for plasma interactions.

T in the Saha equation is essentially electron temperature T_e, since ionization is mainly by collision with electrons. In order to solve these equations, we must assume complete thermal equilibrium, that is we assume that the energy distributions of all various species are characterized by the same temperature T. Using eq. 3-3 eq. 3-4 then becomes

$$(2n_e + n_a) k_b T = P \qquad (3-6)$$

Equations 3-5 and 3-6 are two equations in three unknowns n_e , n_a and T, (P = 1 atmosphere) which can be solved for n_a as a function of T.

The detailed calculation has been done (Popenoe and Schumaker (1965), Drawin and Felenbok (1965)) and the results are tabulated in Table I. The condition that $T_e = T_i = T_a$ is a stringent one, since the equilibriation time for electrons and ions or neutrals is relatively long, due mainly to the difference in mass. There are indications that this condition does not hold for the helium plasma jet (Morris 1968).

Finally combining the equations given above, we obtain for the emissivity :

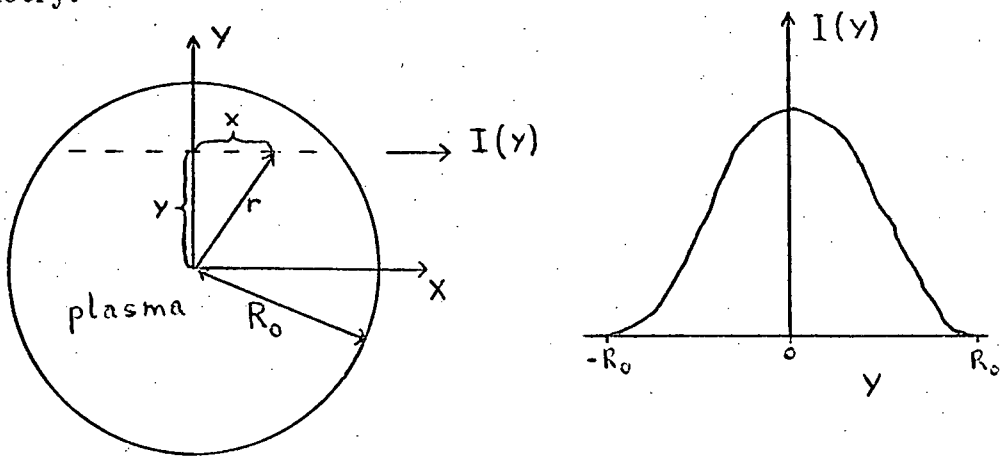
$$\epsilon = \frac{1}{4\pi}(E_m - E_n) A_{mn} \frac{n_a(T)}{Z(T)} g_m e^{-E_m/k_b T} \qquad (3-7)$$

the validity of this equation being subject to the harsh condition of complete thermal equilibrium. The equation can be readily inverted by graphical techniques, to give T, and therefore also n_e , as a function of ϵ .

II The Abel Transform.

We have seen in the previous section how emission is related to temperature if the plasma is in complete thermal equilibrium. Thus we may determine the plasma temperature by measuring emission. But temperature and therefore emission are functions of position in the plasma, and in actual fact we are limited to observing total emission, i.e. intensity, from a region with varying emission. From such data emission as a function of position, can be obtained by means of the Abel transform.

In our particular case we may assume a plasma with cylindiscal symmetry.



For the experimental arrangement shown in the diagram we have :

$$I(y) = 2\int_0^{(R_o^2 - y^2)^{1/2}} \epsilon(r)\, dx \qquad (3-8)$$

$$= 2\int_y^{R_o} \frac{r\,\epsilon(r)}{(r^2 - y^2)^{1/2}}\, dr$$

$I(y)$ is the observed intensity as a function of y

$\epsilon(r)$ is the emission coefficient as a function of r

Equation 3-8 holds only if the plasma is optically thin. Using the estimates given by Griem (1965) we find that the assumption of optical thinness is valid for the case of the plasma jet. The spectral profiles of some of the more intense lines do however show some signs of self-absorption, amounting in some cases to about 5 to 10 per cent of the total intensity of the line.

The inversion of equation 3-8 to find $\epsilon(r)$ is accomplished by means of the Abel transform :

$$\epsilon(r) = \frac{1}{\pi} \int_{R_0}^{r} \frac{(dI(y)/dy)}{(y^2 - r^2)^{1/2}} \, dy$$

Direct numerical integration methods for determining $\epsilon(r)$ are not very satisfactory because of the derivative in the integrand. The observed function $I(y)$ has various errors, random and otherwise, imposed on it, which cause the derivative $dI(y)/dy$ to be poorly defined. A more satisfactory method is to approximate $I(y)$ by suitable least-square-fit polynomials and perform the integration analytically .

In general we write

$$I(y) = \sum_{i=0}^{N} a_i F_i(y)$$

$F_i(y)$ are the expansion polynomials.

a_i are coefficiente to be determined by fitting.

The Abel transform $\tilde{F}_i(r)$ of the $F_i(y)$ can be found analytically.

$$\tilde{F}_i(r) = \frac{1}{\pi} \int_{R_0}^{r} \frac{F_i'(y)}{(y^2 - r^2)^{1/2}} \, dy \qquad (3\text{-}9)$$

and then

$$\epsilon(r) = \sum_{i=0}^{N} a_i \tilde{F}_i(r)$$

which is the required solution.

The choice of the $F_i(y)$ depends on the nature of the experiment. One tries to select them such that they will not give unphysical maxima, minima, or inflection points. For the "bell-shape" intensity profile of the plasma jet Freeman and Katz (1960) used

$$F_i(y) = (R_0^2 - y^2)^i \qquad (3\text{-}10)$$

where R_0 is the distance from the center of the jet to the point where the intensity drops to zero. The index i varies from zero to about six. These are the polynomials used in the solution of the Abel transform for this experiment.

A routine integration (Eq. 3-9) yields for the transform $\tilde{F}_i(r)$;

$$\tilde{F}_i(r) = \frac{2^{2i}(i!)^2}{(2i)!}(R_0^2 - r^2)^{2i-1}$$

The problem now remaining is to find the coefficients a_i by means of a least squares fit. The quantity to be minimized is

$$\sum_j \left[O(R_j) - \sum_i a_i F_i(R_j)\right]^2$$

O(Rj) is the observed intensity at the point

Rj , and $F_i(Rj)$ is given by Eq 3-10.

The algebraic manipulations involved are routine and need not be reproduced here.

The actual computation was done by computer, using a Fortran program written for the purpose. It was found that the best fit was obtained for a fitting polynomial of the form

$$F(y) = \sum_{i=2}^{4,5} a_i (R_0^2 - y^2)^i$$

Starting the summation at $i = 2$ forces F and its first derivative to zero at the end point $y = R_0$. This condition corresponds to the expected physical situation. If F is of higher order $i > 5$, it begins to fit the noise on the intensity profile, and a physically untrustworthy emission profile results. The transform was applied to each half of the intensity profile separately, since the jet is not likely to be completely symmetric. With this procedure small physically impossible discontinuities will occur at the centre of the emission profile, which can however be largely eliminated by a judicious choice for the centre of the intensity profile.

III - Absolute Intensity Calibration.

The final problem in determining excitation temperature from integrated line intensity, is to relate the electrical signal from the photomultiplier to the absolute intensity of the radiation emitted by the source. To do this, we use a standard source, the carbon arc, which may be assumed to be a blackbody radiator with a temperature of 3800° k. The carbon arc is placed in the position of the jet, so that light from either source is collected through the same optical system.

For the intensity of light per unit solid angle from the carbon arc we have :

$$I_c(\lambda) = \int_{\lambda_1}^{\lambda_2} S(\lambda)\, d\lambda$$

$$\approx (\lambda_2 - \lambda_1)\, S\left(\frac{\lambda_2 + \lambda_1}{2}\right) \qquad (3-11)$$

$\lambda_2 - \lambda_1$ is the width of the exit slit in angstroms

$S(\lambda)$ is the energy flux from a blackbody per unit solid angle

given by

$$S(\lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_b T} - 1} \qquad 3\text{-}12$$

For the integrated intensity of the line radiation we have:

$$I_l = \int_{-\infty}^{\infty} i(\lambda)\, d\lambda$$
$$\approx \int_{\lambda-\delta}^{\lambda+\delta} i(\lambda)\, d\lambda$$

The exit slit width equals 2δ, and should be made large enough to envelop most of the line, $i(\lambda)$. (This condition may be difficult to satisfy - if the line profile is Lorentzian, a slit with a width of 10 full line half widths would still miss 6% of the total line radiation).

If the photomultiplier is linear, the voltage output signal is proportional to the intensity of incoming radiation. We have for the voltage due to the carbon arc continuum radiation

$$V_c(\lambda) = k(\lambda)\, I_c(\lambda)\, \Delta\Omega$$

$k(\lambda)$ is a calibration factor

$\Delta\Omega$ is the solid angle of light accepted by the optical system

Similarly for I_l we have

$$I_l = \frac{V_l}{k(\lambda)\Delta\Omega}$$
$$= I_c(\lambda)\, V_l / V_c(\lambda) \qquad (3\text{-}13)$$

We measure V_l and $V_c(\lambda)$, and compute $I_c(\lambda)$ using equations 3-11 and 3-12. Thus the integrated line intensity I_l is determined. I_l is measured as a function of the position y of the jet. This intensity profile $I_l(y)$ is converted to a radial emission profile $\epsilon_l(r)$ using Abel transform. $\epsilon_l(r)$ in turn gives teperature $T(r)$ through equation 3-10.

B - Apparatus and Technique.

(a) Carbon Arc. The carbon arc used in the absolute intensity calibration was a standard piece of apparatus, and has been adequately described elsewhere (see Null and Lozier (1962) and Maclatchy (1966)). Under the proper conditions of electrode positions and arc current, as described in the literature, the arc will operate very quietly and steadily, and the intensity of light coming from the anode surface is then well reproducible and characteristic of a blackbody with emissivity of one and a temperature of 3800°k. The arc was placed in the position of the plasma jet, as the best and simplest assurance that the same optical system was used for light coming from the jet and from the arc. The position of the arc was adjusted so that the image of the monochromator entrance slit was focussed on the centre of the anode surface of the arc.

(b) Calibration of neutral density filters. The voltage signal due to the carbon arc continuum was measured over a range of wavelengths from 4000 A° to 8000 A° in small steps of wavelength. For most of this range, the intensity of the carbon arc light was enough to saturate the photomultiplier, and absorbing filters were needed. The filters used were gelatine Wratten ND filters. Their density is wavelength dependent, and thus it was also necessary to measure this density as a function of wavelength. It turns out that if we represent the density of a filter by

$$d(\lambda) = m(\lambda)\, d_0$$

d_0 is the density at some particular wavelength, (5000 A° in our case) then $m(\lambda)$ is the same for all filters, independent of density.

The results for $m(\lambda)$ are given in Fig. 3 . This same calibration

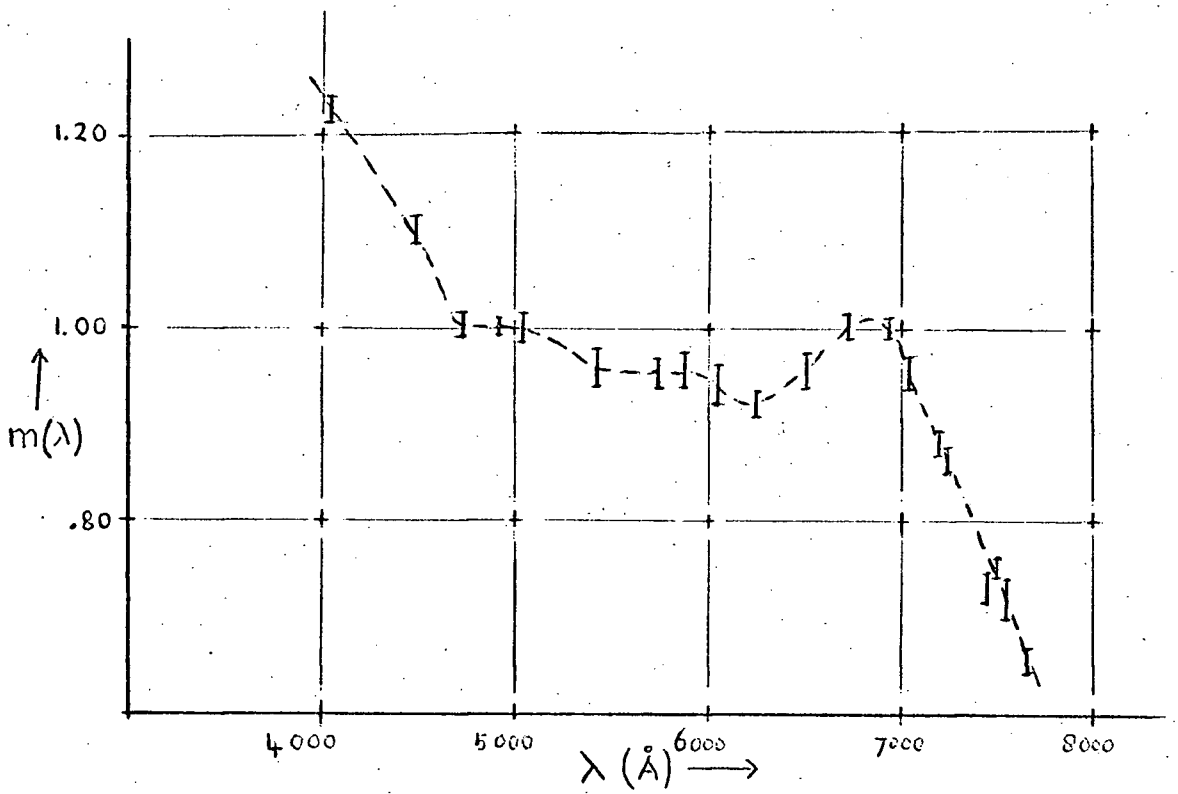


Figure 3 - Calibration of absorbtion filters $\left(d(\lambda) = m(\lambda)\, d_{\lambda=5000}\right)$

is useful for the measurement of intensity of light coming from the plasma, since, especially for the spectral lines, this intensity is in many cases enough to saturate the photomultiplier. The actual voltage V_c or V_l used in equation 3-13, is then calculated from the observed voltage V' by

$$V_{l,c} = V'_{l,c}\, 10^{-m(\lambda)\, d_o}$$

d_o is the total density (at 5000 A°) of the absorption filters used in reducing the light intensity.

<u>(c) Integrated line intensity as function of position.</u> In order to do the Abel unfolding, integrated line intensities have to be measured as a function of position y across the jet. A potentiometer was mechanically connected to the mechanism for moving the jet, in such a manner that

horizontal position of the jet was linearly related to change in voltage at the potentiometer tap. With this voltage fed to the horizontal input of an oscilloscope, and the signal from the photomultiplier simultaneously fed to the vertical input, continuous intensity profiles could be very easily obtained.

In order to allow for the background continuum of the jet, intensity profiles were taken at wavelength of the centre of each line, and at some wavelength near the line but well away from all spectral lines. The intensity of the background continuum was much less than the intensity of the spectral lines, so that errors in measurements of the background were not very important. Subtracting the background profile from the observed line intensity profile, gives the true line intensity profile. During most of the final experimental work observations of line intensities were done between observations of scattering. This procedure gave repeated quick checks on fluctuations in position and temperature of the jet, so that the effect of fluctuations could be allowed for and minimized.

IV - RESULTS.

The important results of this experiment came from the final series of experimental runs, which involved a combination of scattering observations and excitation temperature measurments. During earlier work some interesting data and observations on satellite halfwidths and perturbation of the plasma were obtained, and these are reported in this chapter along with the principal results.

Perturbation of the Plasma. It is to be expected that the extreme intensities of light in the focussed laser beam will have some perturbing effect on the plasma, and such effects were indeed observed.

Heating of the plasma due to absorption of laser light is evidenced by a momentary increase in the continuum radiation when the laser is fir ed, and by the appearance of spectral radiation from species of higher ionization. Under conditions of high laser power and large electron densities such heating signals were observed. Some rough graphs of their dependence on laser power and electron density are given in Fig. 4.

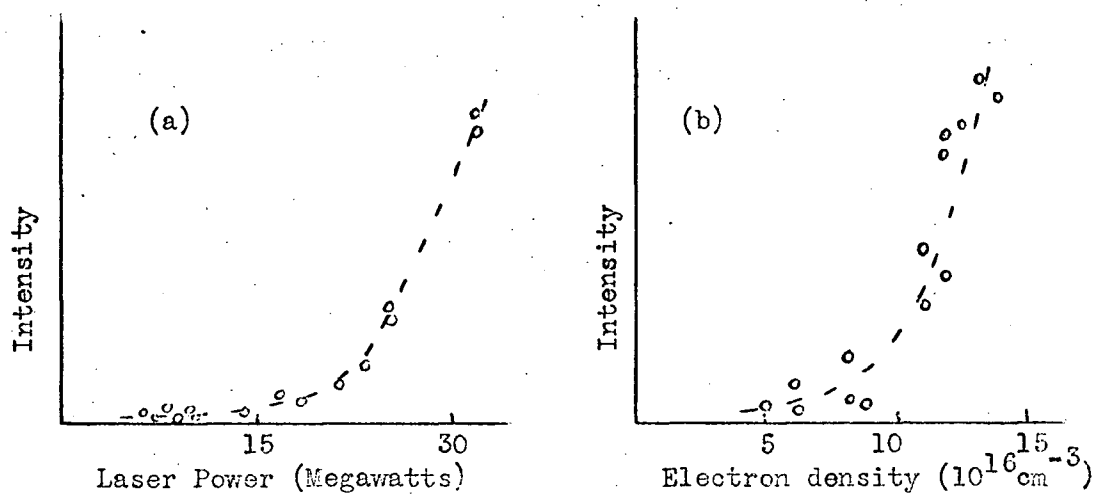


Figure 4 - Perturbation of plasma by laser. Circles are experimental points.
(a) shows the dependence of the increase of continuum signal on laser power.
(b) shows the variation of the intensity of Ar II radiation with electron density at constant power of laser.

The mechanism of this heating effect is not well understood. Spitzer (1962) gives a formula for the absorption of photons by free electrons in an inverse bremstrahlung process. It has the form :

$$k \approx 3 \times 10^{-36} n_e^2 T^{1/2} \quad cm^{-1}$$

k is the absorption coefficient. With a maximum heating effect, Ar III radiation does not appear, while Ar II radiation is momentarily strongly in evidence. From this fact we can deduce an approximate figure for the absorption of laser energy by the electrons, and the result agrees within an order of magnitude with that computed from the above equation. Only a small fraction, about 10^{-4} , of the incident light is absorbed, so that the amount of absorption is difficult to measure directly. The curve in Fig. 4-(b) could possibly be in agreement with the n_e^2 dependence given in the above equation, but the results in figure 4-(a) seem to indicate a non-linearity with respect to laser power, though there is no indication of such a non-linearity in the equation.

The heating signal itself presents a problem because it tends to obscure signal due to scattered light. A more basic problem is of course that diagnostic results obtained by laser scattering cannot be immediately trusted since the scattering volume is inevitably perturbed to some degree by the laser. In the experiment described here these perturbation effects were avoided as much as possible by use of laser powers well below the bend in the curve given in Fig. 4-(a). The heating signal has one advantage, in that it allows an accurate determination of the position of the scattering volume in the jet. (see chapter II (j)).

One other perturbation effect is the momentary decrease in the intensity of some Ar I spectral lines due to passage of laser light through the plasma. This decrease is of the order of 5 to 10 per cent of the total line intensity, but since most of the observed line radiation comes from plasma regions outside the scattering volume, the small decrease or "negative emission" probably represents a very large decrease in the intensity of line radiation emitted from the scattering volume. The "negative emission" effect may well be related to the heating signals described above through a general increase of electron and excitation temperatures in the scattering volume.

B - Preliminary Scattering Results.

As was mentioned in the introduction, the original purpose of this experiment was to determine what part of the observed satellite width could be attributed to electron density gradients in the scattering volume. At first it was attempted to achieve this purpose by varying the size of the scattering volume, while keeping its position in the jet fixed. The results, it was hoped, would show a decrease of satellite width with decrease in length of scattering volume, tending to the theoretical satellite width for a uniform plasma at zero scattering volume length. The actual results were not conclusive, though they were compatible with the expected result. A decrease of satellite width with decrease in the length of the scattering volume was observed, but the expected results were obscured by poor signal-to-noise ratios in the satellite observations, and by fluctuations in the plasma jet, both during and between runs. The method was

abandoned as unsatisfactory, and the original object of the experiment was obtained by the determination of electron density profiles in the plasma jet.

C - Plasma Diagnosis Results.

During the final experimental runs observations of scattering were combined with measurements of spectral line intensities. The same optical system was used for both methods, except that a wider exit slit was used for the intensity measurements. The two methods are described in Chapters II and III.

Figure 5 shows the variation of electron density with radial position at a fixed height (12.5 mm) above the cathode of an argon jet operated at a current of 280 amperes. The electron densities as determined by scattering are indicated by the experimental points with error bars indicated. The electron densities as determined by three separate measurements of the Ar I 6965 line are also shown. These profiles were taken in between scattering observations, and the differences between the three are an indication of the magnitude of the fluctuations in the jet. The discontinuity at the centre of these profiles is due to fitting and Abel inversion as described in Chapter III(b).

Figure 6 shows similar curves for electron density in the helium jet at a fixed height of 6.6 mm and a current of 220 amperes. No error bars are given for the position of the scattering observations because the position of the scattering volume could not be completely determined due to lack of heating signal in the helium jet. The plasma was quite reproducible in this case, as shown by the two

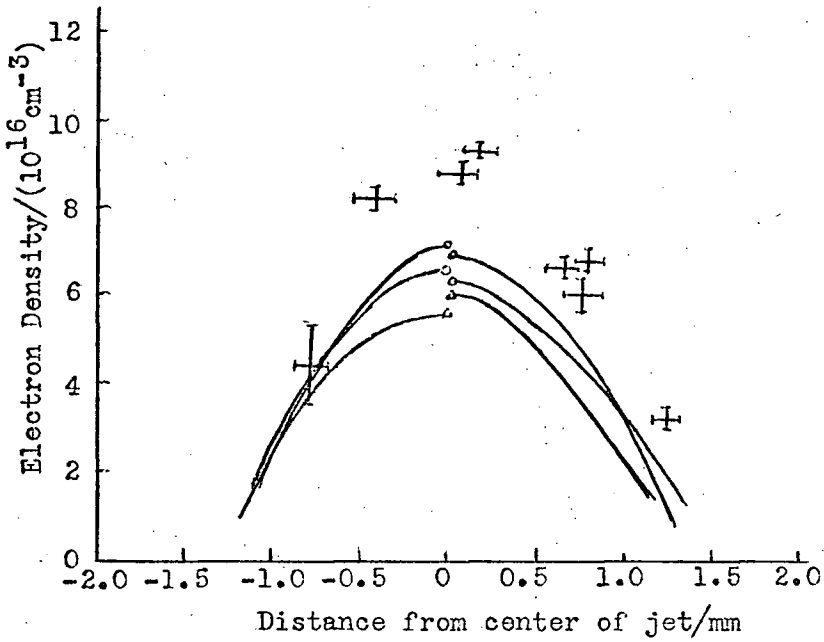


Figure 5. Electron densities in the argon plasma jet. Error bars indicate estimated errors for points determined by scattering. Continuous curves represent line intensity measurements.

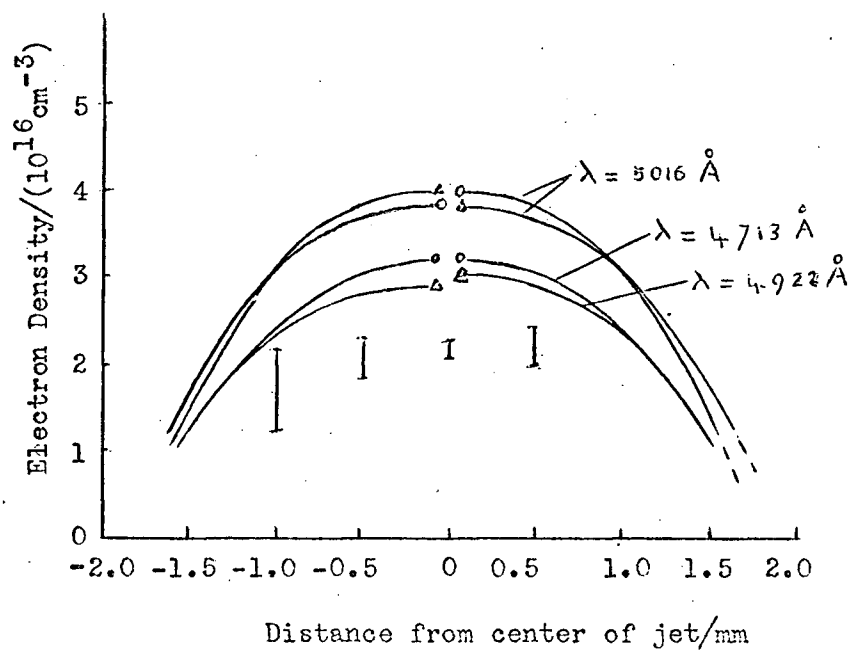


Figure 6. Electron densities in the helium plasma jet. Error bars indicate estimated errors for points determined by scattering. Continuous curves represent line intensity measurements.

typical profiles obtained from the intensity of the He I 5016 line, but different spectral lines give different electron densities.

Comparing the results of the two methods, we see that there is agreement as to the shape of the profile, much flatter for helium than for argon, but there is also a systematic discrepancy, the cause of which is difficult to determine. The densities as determined by scattering are measured directly with small possible errors, and are dependent only slightly on the assumption that electron temperature and density are related through the Saha equation. These scattering results are consequently much more reliable than the results obtained by the line intensity method which involves absolute calibration of the detector system (± 10%), integrated line intensities (± 5%), approximate transition probabilities (± 10%), Abel unfolding (± 5%), and the assumptions of complete local thermal equilibrium and an optically thin plasma. A thorough discussion of the errors involved in the spectroscopic temperature measurement can be found in the papers by Olsen (1963) and Popenoe and Schumaker (1965). The total possible error may be as high as 30% or more, sufficient to account for the discrepancies noted above.

Figure 7 shows a more complete set of electron density profiles for the argon jet operated at 280 amperes. The curves shown are a combination of results from scattering and line intensity measurements.

D - Satellite Widths.

The electron density profiles obtained from the satellite shifts were used to predict the spectral shape of the observed satellites. In particular, the observed profile of a satellite will be a convolution

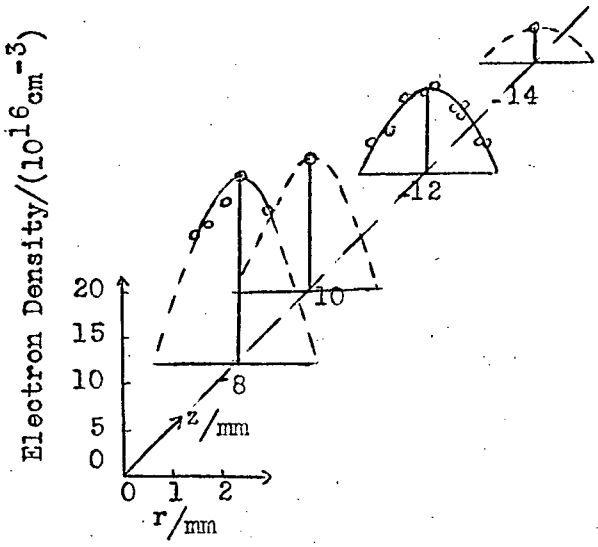
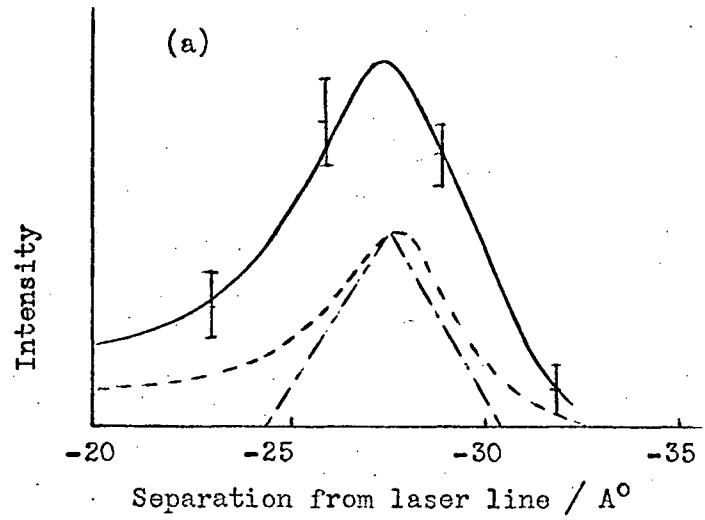


Figure 7. Electron densities in the argon plasma jet. Current is 280 A. z is the distance along the jet axis measured from the cathode tip. r is the separation from the center of the jet. Circles represent scattering measurements. Dashed curves are profiles from spectroscopic measurements normalized to fit the electron density as determined from scattering.

of the instrument profile, the satellite profile for a uniform plasma, and the broadening due to gradients in the scattering volume.

Figure 8(a) shows an observed satellite for the helium jet. Here the electron density is practically constant over the scattering volume, and no correction for variation in electron density need be applied. The theoretical curve is thus calculated by convolution of the monochromator response profile with the satellite profile for a uniform plasma. The experimental points fit the predicted curve very well. In fact the width of the observed satellite fixes the electron temperature at 17000 $\pm$ 3000 ^{o}k, in good agreement with the value of 17000°k calculated from the electron density through the Saha equation.

Figure 8(b) shows an observed satellite for the argon jet. Here the satellite profile for a uniform plasma is quite narrow (about .4 A^{o}



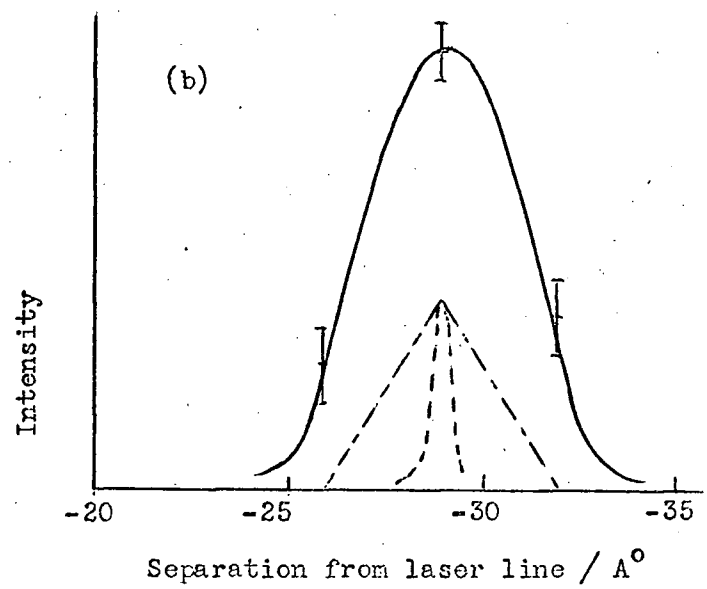


Figure 8. Comparison of observed and predicted satellite profiles. Error bars represent standard deviations for experimental points; -.-. instrument profile; - - - scattered light profile for uniform plasma; —— convolution of instrument profile, scattered light profile, and broadening due to electron-density gradients.

halfwidth), and the broadening is primarily due to electron density gradients in the scattering volume. Again the experimental points are in good agreement with the theoretically expected results. Thus it is verified that the observed broadening of the satellites can be accounted for by the spatial inhomogeneity of the plasma.

CHAPTER V - CONCLUSIONS.

Discussion of Results.

We have seen in the previous chapter that the width of the satellites, first observed by Chan and Nodwell (1966) can indeed be accounted for by the variations in electron density within the scattering volume. Because of poor signal-to-noise ratios encountered in the satellite observations, these results are not entirely conclusive, but they make any large discrepancy from the theory highly unlikely.

It has also been shown that the values for plasma parameters obtained from scattering and from line intensities agree within the possible experimental error of about 30%. Thus it is confirmed that the observed value of the satellite shift agrees with the theoretical value within this range of accuracy.

On the whole, the scattering method of plasma diagnosis is more direct and reliable, but the line intensity method is faster and easier, and thus allows more complete data gathering. The intensity method however depends on the assumption of complete local thermal equilibrium in the plasma. The results given here, and those obtained by Morris (1968), indicate that this condition does not strictly hold for the helium jet. It is difficult to determine how valid the results of the line intensity method are, but in any case we have here a point in favour of the scattering method of plasma diagnosis, which does not depend on questionable assumptions about the nature of the plasma.

Suggestions for future work.

The method of using light scattering to obtain electron densities described in this thesis could perhaps be improved by

a measurement of the integrated intensity of the scattered light because this intensity is roughly proportional to the electron density. If the detector system is calibrated by Rayleigh scattering, or by means of a standard source, one or two shots of the laser would be sufficient to determine electron density. The signal would be much larger than for satellite observations, and noise problems would therefore be considerably less. Stray light of course would be more important. Electron density profiles could be very easily obtained by this method, and it would in any case be useful in determining the position of the scattering volume in the jet by the method described in chapter II (j).

Another possible method of observing scattered light would be by phase-sensitive detection techniques. The problem of the low intensity of scattered light could probably be overcome, but fluctuations in the plasma could also give difficulties. An interesting possibility is the use of an image converter camera to take pictures of the entire scattered light spectrum. If such an approach were successful it could yield a great deal of information about the plasma under investigation.

BIBLIOGRAPHY

Chan, P.W., 1966, Ph D Thesis, University of B.C.

Chan, P.W., and R.A. Nodwell, 1966, Phys. Rev. Letters, 16, 267

Drawin, H.W., and P. Felenbok, 1965, "Data for Plasmas in Local Thermodynamic Equilibrium", Gauthier - Villars - Paris.

Freeman, M.P., and S. Katz, 1960, J. Opt. Soc. Am., 50, 826.

Griem, H.R., 1964, "Plasma Spectroscopy", McGraw - Hill.

Holt, E.H., and R.E. Haskell, 1965, "Foundations of Plasma Dynamics", The MacMillan Company, New York.

Izawa, Y., H. Kabuto, M. Yokoyama, and C. Yamanaka, 1966, Tech. Dept. Osaka Univ., 16, 515.

MacLatchy, C., 1966, M Sc Thesis, University of B.C.

Morris, R.N., 1968, M Sc Thesis, University of B.C.

Nodwell, R.A., and G.S.J.P. van der Kamp, 1968, Can. J. Phys., 46, 833.

Noll, M.R., and W.W.Lozier, 1962, J. Opt. Soc. Am 52, 1156.

Olsen, H.N., 1963, J. Quant. Spectry. Rad. Transfer, 3, 59.

Popenoe, C.H., and J. B. Schumaker Jr., 1965, J. Res. Natl. Brr. Stds., 6, 495.

Röhr, H., 1967, Phys. Letters, 25, 167.

Rosenbluth, M.N., and N. Rostoker, 1962, Phys. Fluids, 5, 776.

Salpeter, E.E., 1960, Phys. Rev., 120, 1528.

Spitzer, L. Jr., 1965, "Physics of Fully Ionized Gasses", Interscience Publishers.

Wiese, W.L., M.W. Smith, and B.M. Glennon, 1966, "Atomic Transition Probabilities H through Ne", U.S. Dept. of Commerce, NSRDS - NBS 4, Vol. 1.

Williamson, J.H., R.A. Nodwell, and A.J. Barnard, 1966, J. Quant. Spectry. Rad. Transfer, 6, 895.