A TIME CORRELATED STUDY OF THE
Z-PINCH DISCHARGE IN HELIUM

by

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A THESIS SUBMITTED IN PARTIAL FULFILMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in the department

of

PHYSICS

We accept this thesis as conforming to the
required standard

THE UNIVERSITY OF BRITISH COLUMBIA

May, 1968

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ABSTRACT

The structure in the collapse stage of a linear Z-pinch discharge in helium has been studied by optical methods. Observations with a framing camera, rotating mirror spectrograph, and monochromator have been correlated with magnetic field and current distributions determined by Tam (1967).

The luminous regions in a helium pinch are very faint. Therefore, up to twenty exposures have to be superimposed on the same framing camera or rotating mirror record. This requires a high degree of reproducibility in the initiation of the discharge.

At high initial pressures, a non-luminous shock wave at the inner edge of the collapsing current shell precedes the luminous plasma layer towards the centre of the discharge vessel. This shock front is followed by a region of predominantly HeI emission, while most of the HeII radiation occurs in the outer regions of the collapsing plasma shell. The separation into HeI and HeII radiating regions is consistent with spectroscopic measurements of temperature: higher temperatures occur at larger radii. Pressure and density in the non-radiating shock wave region are determined by calculations based on a simple model.

At low filling pressures, the HeI and HeII regions coincide. The position of maximum luminosity is observed to correspond with the position of maximum current density. The luminosity and current shells coincide with no shock wave preceding the luminous front.

Strong continuum radiation is emitted from the centre of the discharge tube as soon as the leading edge of the current shell reaches the axis. This leading edge is luminous at low initial pressures, but becomes a non-radiating shock front at higher filling pressures.

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ACKNOWLEDGEMENTS

I wish to thank my supervisor, Dr. F.L. Curzon, for his advice and help during the course of research. I am deeply indebted to Dr. B. Ahlborn for discussions and his help in the preparation of the thesis. Dr. J.H. Williamson's invaluable suggestions and Dr. R.A. Nodwell's patient counsel helped greatly in the completion of this work.

I am grateful to Messrs. W. Ratzlaff and J. Dooyeweerd for assembling the electronic systems. My thanks are also due to Mr. A. Fraser and Mr. J. Lees and their staffs, who provided excellent service in the construction of the apparatus. In particular, I owe much to Mr. A. Knop whose patience and skill made a viable switching device, and hence much of this research, possible.

Consultation with Dr. W.L. Lee and fellow students H.A. Campbell, H.G. James, S.S. Medley, and R.N. Morris helped clarify difficulties in dealing with experimental results. I am most deeply indebted to my wife whose understanding and encouragement far outweigh the assistance she provided in reducing data, drawing diagrams, and typing the manuscript.

Finally, I wish to acknowledge the generous financial assistance provided in the form of fellowships by the University of British Columbia and the National Research Council of Canada.

1.0 INTRODUCTION

Attempts to solve the problem of confining a very hot gas by means of a magnetic field have produced a variety of experimental configurations. The earliest and simplest arrangement is the Z-pinch discharge. Aside from its potential usefulness as a preionizing device, the Z-pinch provides a simple arrangement in which the formation and behaviour of a plasma can be studied.

In the Z-pinch discharge the plasma is sustained by an axial current I , (Figure 1.1), which is created by the discharge of energy from a capacitor bank. The axial current (I) produces an azimuthal magnetic field (B_θ), which in turn interacts with (I) to form a radial force (F_r). This force causes the plasma to constrict along the axis of the discharge tube. Such a constriction of the plasma is named the "pinch effect", while the period of maximum constriction is known as the "pinch stage" of the discharge. An advantage of this device is that the high current

Figure 1.1 Z-pinch discharge

R_0 = radius of discharge vessel

r = radius of plasma column

▨ = plasma

simultaneously generates and confines the plasma.

The early stage of constriction of the discharge is called the "collapse" or "pre-pinch" phase. During this period the plasma forms a hollow cylindrical shell of excited gas which collapses onto the axis of the discharge vessel. There is usually an abrupt, finite change in pressure and particle velocity between the inner surface of the hollow cylinder and the region of cold gas trapped within it. This zone of discontinuity is known as the "shock front".

The time interval between the initiation of discharge and the arrival on axis of the inner boundary belonging to the collapsing current shell is known as the "pinch" or collapse time. Its magnitude is characteristic of the particular discharge used. It spans the time taken to form the pinch and can be used to define the collapse and pinch stages of the discharge.

Many investigations have been made of the Z-pinch discharge in helium because of its relatively simple spectrum. Detailed studies of the luminosity front in the pre-pinch stage have been conducted by Böttcher (1961 to 1965) using photoelectric and photographic techniques. The action of the pinch in the axial region of the discharge has been investigated by H. Zwicker (1964a,b). Employing time-resolved spectroscopy, he has shown the occurrence of on-axis continua before and after the complete collapse of the luminosity front.

Both authors demonstrate agreement with a theoretical model of the fast-pinch discharge proposed by Jukes (1958) and Allen (1957), which in turn is based on a theory first presented by Guderley (1942).

Böttcher (1963) equated the motion of the luminosity front with that of a cylindrical shock wave and refined the Guderley theory to describe its action. Zwicker (1964a) invoked Jukes' version of the same theory to explain the appearance of on-axis continua.

Magnetic probe studies of the Z-pinch discharge have been conducted at this laboratory under the direction of F.L. Curzon. A "gradient probe" technique perfected by C.C. Daughney (1966) and S. Tam (1967) has yielded current and magnetic field distributions of the helium discharge in the pre-pinch stage. The experimental results agree with predictions based on a modified snowplow equation. However, this method does not yield information about the particle densities and temperatures within the collapsing current shell.

The research efforts mentioned above have concentrated on different aspects of the Z-pinch discharge because of limitations in the diagnostic devices employed. They provide detailed, but essentially uncoordinated information about the action of the pinch--from its inception to well after its formation in time. By extending the range of experimental observations over the complete life-history of the discharge, and by correlating them with known results, the following question can be clarified.

-What relation is there between the luminosity front and the collapsing current sheet which sustains the discharge?

This thesis attempts to resolve the problem above through the use of three different optical diagnostic methods. The advantage of investigating light radiated by the discharge is that the measuring devices leave the plasma undisturbed.

1) Photographic time-resolved spectroscopy with a fast medium-quartz spectrograph provides a survey of a large number of lines and their behaviour in time. Because of the calibration difficulties inherent in photographic work, only qualitative results are considered.

However, this method provides a guide to the identification, intensity, and time of formation for lines of interest. A qualitative estimate can be made of the purity of the discharge, as well as a choice of suitable interference filters for use in high speed photography.

2) A high speed framing camera can give information concerning the stability and gross structure of the discharge. By photographing the complete period of collapse and formation of the pinch with interference filters, any interesting features requiring further investigation can be correlated with respect to the time of their appearance.

3) The use of a grating monochromator serves to isolate lines determined by the above photographic procedures. A detailed study of line profiles can then be carried out to yield electron densities and temperatures within the plasma.

By using a Rogowski coil to monitor the discharge current, the above three methods can be correlated in time to give an overall picture of the Z-pinch discharge. The behaviour of the discharge in the collapse and pinch stages is then compared with results obtained from magnetic probe work.

Most of the techniques for time-resolution are standard and highly developed. However, the problem of adequate spatial resolution in photographic work for faintly luminous gases, such as helium, has been a serious obstacle until now. Unless the event can be reproducibly recorded, only diagnostic devices with a severely limited range of spatial or wavelength

resolution can be used. This in turn complicates the interpretation of observational results.

Two of the recording devices used in this work employ a rotating mirror. The discharge must be triggered by the motion of the mirror, since a recording depends on its proper placement and rotational speed. Because the light level of a single discharge in helium is far too low to produce a well exposed negative, a superposition technique must be introduced. This requires a high degree of accuracy in triggering the discharge. Fluctuations in the breakdown time duration of the discharge must also be eliminated so that an accurate superposition of many recordings can be achieved. Chapter 2 outlines solutions to the technical problems of accurate switching and timing of the discharge and diagnostic apparatus. The successful superposition of as many as 20 photographic records of the Z-pinch discharge in helium provides a valuable method for its study.

Chapter 3 shows the application of the time-resolution technique in spectroscopy. Low light levels in the collapse stage of the discharge make it impossible to obtain a viable record for even 20 superpositions off-axis. Because of this, only on-axis observations are executed systematically. These results are then correlated with monochromator recordings of the continuum and synchronized with the behaviour of the discharge current. The procedure gives an accurate determination of pinch times. It serves as a basis for comparison between the discharge system employed in this study and that used by Tam (1967) in magnetic probe work. Current traces and the appearance of secondary continua reveal the action of the discharge in the pinch and post-pinch regimes. This is discussed and contrasted with Zwicker's (1964a,b) findings.

Chapter 4 deals with framing camera results which are also based on a time-resolved photographic technique. Records are obtained of the collapse and pinch stages of the discharge. These are correlated with the observations of Chapter 3 to give a complete description of the discharge in the form of "collapse curves". By comparing them with Tam's (1967) results, the development of the pinch from the collapse stage is explained and the relation of luminosity fronts to the current sheet becomes clear. The conclusions arrived at by experimental method agree well with a qualitative description provided by the Jukes-Allen (1958) theory of fast-pinch discharges.

The structure of the collapsing plasma shell, revealed in Chapter 4, is now subjected to a quantitative analysis. Chapter 5 discusses in detail the monochromator-photomultiplier method of time-resolution. Electron temperatures and densities are determined in the luminous regions of the plasma. Calculations by Griem (1964) for relative intensities and relaxation times are compared with the experimental results. It is shown that an L.T.E. approximation can be assumed to be valid within the limits of experimental error.

The results of Chapter 4 indicate the presence of a non-radiating front which precedes the luminous shell for high filling pressures. Chapter 6 contains a proposed model for calculating the density in this non-luminous region, using electron densities and temperatures of Chapter 5. The results show that the non-luminous front can be regarded as a shock wave, in which very little kinetic energy of the gas passing through it is converted to ionization. Evidence in support of this conclusion is provided by the application of Guderley flow theory and a simple piston model for shock wave formation.

2.0 THE SUPERPOSITION OF PHOTOGRAPHIC RECORDS

It is often difficult to obtain good photographic records when the intensity of light emitted by a discharge is low. Such a situation occurs during the initial collapse stage of the Z-pinch. If the discharge has reproducible optical properties, it is possible to obtain usable records by superimposing several exposures of the same event on a single piece of film. This procedure is valuable in extending the limits of high speed photography and time-resolved spectroscopy. It assures the recording of sufficient light to exploit the advantages of spatial resolution and a wide field of view, which are characteristic of high speed framing camera pictures. It also allows the use of conventional spectrographs in place of large aperture optical systems, which are costly and generally have a lower wavelength resolution.

Figure 2.1 Photomultiplier trigger system for time-resolved photography

- Trigger beam, mirror in position A
- Recorded radiation, mirror in position B when discharge starts.

Figure 2.1 illustrates the time-resolution method for photographic records. Light from the discharge tube is reflected from the surface of a rotating mirror and imaged on the recording film. As the mirror rotates, the image of the discharge moves across the film, producing a record which has a time axis along the sweep direction of the mirror. (For a time-resolved spectrogram, the slit of the spectrograph replaces the film--Section 3.1. In the high speed framing camera, a series of lenses is interposed between the rotating mirror and recording film to form a sequence of images of the discharge--Section 4.1).

For faint discharges, several records must be superimposed to produce a useful photograph. This requires the discharge to be triggered at a certain mirror position so that each recorded event starts at the same point on the film. The discharge must also be triggered each time at the same frequency of rotation of the mirror, since this controls the scale of the time axis.

Both position and frequency selection are performed by a control circuit through the action of a photomultiplier trigger system (Figure 2.1). This arrangement automatically fires the discharge when the mirror reaches the preset speed and recording position. The trigger beam system is aligned in such a way that light from its source does not fall onto the film area swept out by the discharge image. The performance of the control circuit in conjunction with the trigger beam is outlined in Section 2.1.

It is necessary to assure commencement of the discharge when the rotating mirror is properly placed. This required the construction of a special discharge switch and a change in design of the conventional trigger pulse generator employed at this laboratory.

The rest of Chapter 2 may be omitted by a reader who is not interested in technical details. Apparatus and the solutions of associated technical difficulties, implied in the outline above, are dealt with in the following order:

Section 2.1 gives a brief sketch of the control circuit governing the time of the discharge of the Z-pinch. Jitter in the signal emanating from this system is found to be less than $0.1 \mu\text{sec}$.

Section 2.2 outlines the equipment and circuitry involved in producing the plasma. Before this thesis was undertaken, existing apparatus exhibited a considerable variation in the time of current breakdown over a long series of discharges. This posed a considerable problem in the execution of adequate time-resolved photography. A description is given of changes made to achieve an overall variation in breakdown of less than $\pm 0.2 \mu\text{sec}$.

Section 2.3 shows the method of measuring the discharge current. It is used for correlating the various techniques of time-resolution and is referred to frequently throughout the thesis. A comparison of parameters with those characteristic of the discharge studied by Tam (1967) is also given, since use will be made of his experimental results.

Descriptions of the time-resolved spectrograph, framing camera and monochromator arrangement are inserted in the remainder of the text to avert the monotony inherent in a large body of technical detail.

2.1 THE CONTROL CIRCUIT

The rotating mirror is the only moving part in the system. All events must be timed to happen when it is in the proper position, as indicated in Section 2.0. The timing action of the discharge is regulated by a control circuit designed by F.L. Curzon. It is described in detail by Neufeld (1966) and a synopsis of that account is provided in Appendix I for the sake of completeness.

Since the discharge must occur when the mirror is rotating at a given speed, the control system must contain a frequency gate. The trigger beam system causes the monitoring photomultiplier to emit a pulse each time the mirror swings through a certain position (mirror position A, Figure 2.1). The gate is tripped open only when the time interval (T) between pulses from the photomultiplier reaches a required value (waveforms e and f, Figure 2.2)

Figure 2.2

Control circuit and corresponding pulse trains

The output pulse from the gate is then delayed by a time T_d , allowing the mirror to swing into the proper recording position (mirror position B, Figure 2.1). The discharge is initiated by this delayed pulse.

The jitter for the delay system is smaller than $0.08 \mu\text{sec}$. The only other source of fluctuations would be from a change in speed of the rotating mirror. Section 3.1 deals with this problem in connection with the time-resolved spectrograph.

2.2 THE Z-PINCH DISCHARGE

It is essential that timing characteristics of the discharge be extremely reproducible to accomplish an accurate superposition of time-resolved records. Since pinch formation times are the order of 10 μ sec., the time between triggering the system and the breakdown of the discharge must be kept constant to within a fraction of a microsecond.

Such a condition imposes requirements not easily fulfilled by conventional equipment used until now in this laboratory. It was found necessary to modify the trigger pulse generator (Medley, 1965) and the discharge switch (Curzon, 1961). This decreased the variation in discharge breakdown to less than 0.2 μ sec. for as many as seventy consecutive firings of the discharge.

A sketch of the Z-pinch apparatus is provided in Figure 2.3.

Functions of the component parts are discussed in the following subsections:

Subsection 2.2a is a brief description of the gas vessel and adaptations made for observing radiation emitted by the discharge.

Subsection 2.2b outlines the action of the trigger generator and the change made in coupling it to the control circuit.

Subsection 2.2c gives the design and operation of the discharge switch. Table 2.i at the end of this chapter summarizes the main features of the equipment involved and compares them with the apparatus used by Tam (1967).

Figure 2.3 Discharge apparatus
(letters in brackets refer to Fig. 2.4)

2.2a DISCHARGE VESSEL AND CAPACITOR BANK

The horizontally-mounted vessel is a pyrex glass cylinder (dimensions given in Table 2-i, Section 2.3) within which brass electrodes are located at both ends. It is encased in a brass gauze (mesh interval 0.2 cm.) which serves as a return conductor and an efficient electrical screen. Provision is made for evacuating the tube and introducing the test gas at a desired pressure through perforations at the high tension end electrode (Figure 2.3).

To allow for photographic observations along the axis of the tube, the other electrode consists of a brass mesh beyond which an end plate is mounted, sealing the tube. This end plate is a large glass window through which the whole discharge cross-section can be photographed. In spectroscopic work, where only a small region of the plasma cross-sectional area is to be investigated, a small, easily replaceable quartz window is used instead. This is mounted on a one-half inch thick (1.27 cm.) brass plate (Figure 2.3), in which a quarter-inch (0.64 cm.) hole is drilled. The brass plate is large enough so that the window can be slid to any position across the diameter of the tube.

The capacitor bank (C_2 of Figure 2.4) is formed by ten N.R.G. low inductance storage capacitors, giving an overall capacity of 106 μF . These capacitors are charged to 10.5 K.V. for all experimental investigations.

2.2b DISCHARGE CIRCUIT AND TRIGGER GENERATOR

Figure 2.3 provides an idea of the placement and relative size of some of the component parts of the apparatus used in generating the Z-pinch. A schematic diagram of the whole system is given in Figure 2.4.

Figure 2.4

Triggering system and discharge circuit

- A--thyatron pulser, triggered by pulse from control circuit
- B--pulse transformer
- D--trigger generator circuit
- E--discharge vessel
- V--potential divider; R_F (180MΩ) total resistance of potential divider
- G--high-voltage charging unit

Trigger generator components:

- S_1 --three-electrode trigger spark gap
- C_1 --0.06 μF
- R_1 --50 Ω

Main discharge components:

- S_2 --three-electrode spark gap switch
- C_2 --(100 μF) capacitor bank

Figure 2.5

Trigger generator

- Brass
- Perspex
- Polyethylene

The control circuit (Section 2.1) sends out a pulse that trips the 2D21 thyratron pulse generator, A of Figure 2.4. Generator A forms a negative pulse (-300 volts, rise time 2 μ sec.) across the primary winding of pulse transformer B.* This in turn causes gap S_1 to break down, discharging capacitor C_1 and generating a pulse across resistor R_1 . This output pulse initiates breakdown of the three-electrode spark gap switch S_2 which discharges bank C_2 through vessel E.

The trigger circuit and generator D are modifications of those described by Medley (1965). That system used a pulsed light source, consisting of an open air spark gap enclosed by a quartz bulb, in place of the present pulse transformer B in Figure 2.4. Such an arrangement proved awkward to operate, since fogging of the quartz surface, changing atmospheric conditions, and damage to the electrodes in gap S_1 contribute to jitter in the overall breakdown time of the system over a large number of shots. Its replacement by the pulse transformer still serves, in conjunction with switch S_1 , to isolate the discharge circuit from the control electronics.

Figure 2.5 shows the trigger generator D in detail. It is constructed coaxially to suppress production of spurious electromagnetic signals. The 50 Ω output cable is terminated by the resistance R_1 (Figure 2.4) which consists of five chains of 22 Ω resistors arranged as a sheath around capacitor C_1 (Figure 2.5). The trigger capacitor C_1 is charged in parallel with C_2 through the potential divider F. The whole system can then be primed to discharge when switch S_1 is shorted by command from the control circuit.

* Pulse transformer Type U558, manufactured by Atkins, Robertson and Whiteford Ltd., Glasgow. Step-up ratio 3:100.

2.2c THE DISCHARGE SWITCH

The greatest source of jitter in the breakdown time of the system is caused by irregularities occurring in the main trigger switch S_2 (Figure 2.4). At the time this work was begun, a satisfactory discharge switch for 1-2 kilojoule condenser banks had been designed (Curzon, 1961). However, such a switch proved unsatisfactory for the use of a 5.8 kilojoule condenser bank and the rigid control of breakdown time over ranges of at least thirty shots. The salient features of design in the previous and final electrode systems are sketched in Figures 2.6a and 2.6b respectively. The improvements depicted by Figure 2.6b are incorporated into the discharge switch drawn in Figure 2.8.

In the electrode assembly of Figure 2.6a, it was found that the critical design parameter was the distance labelled B. This distance controls the delay between the breakdown of the trigger spark and the main spark gap channel: for $B \approx 3.2$ mm., the delay is about 2 μ sec.

When the system depicted by Figure 2.6a was used in conjunction with the 5.8 kilojoule bank, it was found that such a minimum delay of 2 μ sec. was indeed observable. However, when a series of ten shots was taken, a random fluctuation in occurrence of breakdown was noticed that varied from 4 to 10 μ sec. By advancing the trigger pin T to a position indicated in Figure 2.6b, the range of this fluctuation was cut down to approximately 4 μ sec. However, as the polyethylene insulator around tungsten tip T was eaten back, the fluctuations in the breakdown delay increased again (see Figure 2.7).

Figure 2.6

Electrode systems for spark gap switch
 a) Switch for 2 kilojoule capacitor banks
 b) Switch for 3 kilojoule capacitor banks

Figure 2.7

Oscillograms of discharge current
 (measured by Rogowski coil)
 Δt_1 —breakdown time ~ 2 microseconds
 Δt_2 —fluctuations in breakdown time ~ 6 microseconds

Since it was inconvenient to replace the polyethylene insulator at frequent intervals, a conventional spark plug was tried instead. The best insulation material available for our purposes was found to be the ceramic coating on the Lodge C₁ type spark plug. In order to eliminate problems due to wearing away of the conventional spark plug electrode tip, it was replaced by a tungsten rod 2.7 mm. in diameter. The rod projected 3.2 mm. beyond the surface of the ceramic insulator (Figure 2.8a). It was cemented into the ceramic with epoxy resin.

The hollow "ground" electrode, in which the trigger pin is lodged, was enlarged so that the specially prepared spark plug could be inserted without altering the geometry of the ceramic insulator. In order to prevent short circuiting of the main discharge to the walls, the sharp edge D of Figure 2.6a was rounded off to the geometry of Figure 2.6b.

A major problem in the design of the trigger switch is the insulation of the side walls of the housing that contains the electrode assembly (Figure 2.8b). Unless the walls are far enough away from the ground electrode and insulated adequately, the discharge can occur from wall to trigger electrode rather than across the gap C in Figure 2.8b. A thick coating of glyptal is applied to the inside of the brass container. The perspex bottom is corrugated with circular indentations (Figure 2.8b) to prevent accumulated debris from serving as a conductor from wall to electrode. Eventually, the glyptal coating on the walls will give way after about 200 shots. Before this can occur, the container is replaced by a spare one made up for this purpose. Both housings are used alternately on separate experimental runs after a fresh coating of insulator has been applied.

Figure 2.8a
Electrode assembly

Figure 2.8b
Discharge switch S₂

For optimum timing characteristics, the potential differences across gaps S_1 and S_2 (Figure 2.4) must be as close to the overvolting value as possible. The overvoltage is the potential difference across the gap which produces breakdown even though no trigger pulse is applied. The overvolting potential of gap S_1 is first adjusted to 11.5 kilovolts by charging the capacitor bank (C_2 , Figure 2.4) to 11.5 kilovolts and by increasing the length of the gap S_1 until it stops breaking down. Since each spark in S_1 produces a trigger spark in S_2 , the lid of S_2 is removed (Figures 2.4 and 2.8b). This prevents C_2 from discharging while the adjustment is made.

The top of the main discharge switch S_2 is then put in place and the electrode separation C (Figure 2.8b) is adjusted until the whole system overvolts at 10.75 kilovolts. The system is then ready for experimental work at an operational voltage of 10.5 kilovolts. It was found that this is the highest voltage at which the main discharge switch can be operated for an extended series of shots without serious damage to the electrodes. With this arrangement, the overall jitter is less than 0.2 μ sec. for a series of 50 to 100 shots.

2.3 CURRENT MEASUREMENTS

The measurement of the total discharge current employing a Rogowski coil is a standard experimental procedure. Current traces for initial pressures in helium of 500, 1,000, 2,000 and 4,000 μHg are illustrated in Figure 2.9. The period of the discharge current is approximately $3\frac{1}{2}$ to $3\frac{3}{4}$ μsec . Because a relatively short integration time was used, an accurate calibration of the total current can prove difficult. However, this is compensated for in that a superimposed high frequency structure becomes apparent during the first half-cycle of the discharge current (Figure 2.9). This effect would be smoothed out if a longer "integration time" were used.

This problem is dealt with in Appendix II, where it is shown that the observed current trace can give an underestimate of the "true" reading by a 20% error at worst. All current traces in the following sections are derived from the observed or "uncorrected" oscillograms.

As outlined in Appendix II, a calibration constant (k) can be estimated which relates the discharge current to the observed voltage output of the Rogowski coil circuit:

$$I = kv \quad \text{.....(1)}$$

$$\text{where } k = 9.4 \times 10^4 \text{ amps/volt} \pm 20\% \quad \text{.....(2)}$$

I = the discharge current

v = the output voltage of the integrator circuit

Equations (1) and (2) can be considered to hold true for the times of interest (0 to 10 μsec . after initiation of the discharge).

Figure 2.9 Output of Rogowski circuit vs. time.

In order to better understand the action of the discharge, it is necessary to correlate the results of optical investigations with magnetic field and current distributions observed by the magnetic probe method (Tam, 1967). The discharge system used in this study differed from Tam's only in the size of the condenser bank, which serves as the energy source for plasma production (Table 2-i). This in turn affects the discharge parameters of the system. A comparison of total discharge currents, observed by Tam and in the present Z-pinch system, is given in Figure 2.10. Although the magnitude of the discharge current differs, it will be shown that the action of the pinch in the collapse stage is identical for the two systems (Sections 3.3 and 4.6).

Figure 2.10 Comparison of discharge current in helium Z-pinch

— Tam's (1967) system
 --- 500 μHg filling pressure
 -.- 4,000 μHg filling pressure } Present system

Table 2-i Discharge system

A. COMPARISON OF DISCHARGE CIRCUIT PARAMETERS

	Tam (1967)	Present system
<u>Condenser Bank:</u>		
Total capacity	53 μ F	106 μ F
Total initial inductance	$0.12 \pm 0.01 \mu$ H	$0.09 \pm 10\% \mu$ H
Charging voltage	10.0 ± 0.2 kV	10.75 ± 0.05 kV
<u>Current Measurement</u>		
Maximum discharge current	200 kAmp	280 kAmp
Discharge current frequency	46 kHz	30 kHz
Rogowski coil sensitivity	$(1.86 \pm .11) \times 10^5$ amp/volt	$0.95 \times 10^5 \pm 10\%$ amp/volt
Integrator (RC passive, integration time constant)	24 msec.	47 μ sec.

B. APPARATUS COMMON TO BOTH SYSTEMS

<u>Discharge Tube</u> (pyrex)	
Inner diameter	15 cm.
Outer diameter	17 cm.
Electrodes	Brass
Electrode separation	59 cm.
<u>Vacuum System</u>	
Type 17 Balzers Oil Diffusion Pump	
Hyvac 14 Cenco Backing Pump	
Vacuum attainable	1 μ Hg
Leak rate	7 μ Hg/hr.
MacLeod Gauges	0-1 mm Hg, 0-10 mm Hg
Pyrani Gauge (Type GP-110 Pirani Vacuum Gauge)	
<u>Voltage Measurement</u>	
Convoy Microammeter in series with A.V.O. Multiplier	25 kV d.c.

3.0 PINCH AND POST-PINCH STAGES OF THE DISCHARGE

It is important to establish a correspondence between optical observations and the magnetic probe work conducted by Tam (1967). The correlation of results from these two different diagnostic methods can provide an otherwise unattainable insight into the action of the discharge.

A test for such a relation can be carried out by comparing "pinch times" measured by optical methods with those obtained through magnetic probe investigations. The pinch time is characteristic of the discharge circuit, geometry, and initial conditions of the Z-pinch apparatus. Identical values of this parameter would also prove the similarity of the discharge used by Tam with that employed here. The most accurate optical determination of the pinch time is achieved by synchronizing the discharge current with the output of a monochromator (Section 3.3). In order to determine a suitable wavelength for those measurements, several spectra were recorded by means of a time-resolved spectrograph. The time-resolved spectrum provides a history over a wide range of wavelength for visible radiation emanating from the discharge (Section 3.2).

Discharge current traces, in conjunction with time-resolved spectrographic and monochromator results, also yield information about the post-pinch stage of the discharge. The interpretation of these observations is contrasted with the work of Zwicker (Section 3.4). Section 3.1 outlines the optical arrangement and experimental procedure for obtaining time-resolved spectra of the Z-pinch discharge.

3.1 THE TIME-RESOLVED SPECTROGRAPH

A schematic diagram of the apparatus is presented in Figure 3.1. The instrumental arrangement is based on a design first proposed by A. Gabriel (1960). The rotating mirror assembly is a Barr and Stroud rotor type CP 5911, which is driven by compressed air. A brass housing with three replaceable quartz windows was constructed, and a light-tight box containing the trigger beam arrangement was attached to it. Two of the windows, at 90° to one another, were used as entrance and exit ports for the plasma light source. The third window was used for focussing the trigger beam onto the rotating mirror as shown in Figure 3.1.

The optical arrangement for the system is as follows. Light emanating from the discharge tube is focussed on slit S_1 . The slit S_1 is focussed via the concave mirror M_1 , the plane mirror M_2 , and the rotating mirror M_R on to the slit S_2 , which is the entrance slit of a Hilger medium quartz spectrograph. Slits S_1 and S_2 restrict the scanned cross-sectional area of plasma to less than 0.4 cm^2 at the centre of the tube. They are set at widths of 0.03 cm. and 20 microns respectively. The trigger beam system consists of a direct current lamp which reflects from the rotating mirror onto the photomultiplier P_1 , in such a way that it causes a sharp pulse output every half-revolution of the mirror.

In an experimental run, the condenser bank is first charged and set ready for firing. The rotating mirror is switched on and the pulses from the photomultiplier are fed into the control unit (Figure 3.1). When the mirror reaches the preset speed, the control unit automatically fires the discharge so that an image is cast on slit S_2 . The operator then switches off the mirror, re-charges the condenser bank, and repeats the process to superimpose a second image on the first.

Figure 3.1 Experimental arrangement for the time-resolved spectrograph

Oscilloscope #1 monitors the jitter in the time interval between the triggering of the discharge and the breakdown of the discharge current. This is done by simultaneously displaying the Rogowski coil output and the pulse emitted by the control circuit (i.e., the pulse of the delay unit, Figure 3.1). Oscilloscope #2 monitors jitter due to the irregular rotation of the mirror by exhibiting the first photomultiplier pulse which is produced after the discharge occurs. Typical oscillograms are displayed in Figure 3.2. Figure 3.2a shows a jitter in current breakdown of about $0.2 \mu\text{sec}$.

a)

b)

Figure 3.2 Four consecutive discharges in $1,500 \mu\text{Hg}$ helium

- a) Oscilloscope #1:
upper trace—signal from delay unit, 20 v/cm .
lower trace—Rogowski coil signal, 1 v/cm .
time scale = $2 \mu\text{sec./cm.}$, reading from right to left
- b) Oscilloscope #2:
output from amplifier, 5 v/cm .
time scale = $4 \mu\text{sec./cm.}$, reading from right to left

The jitter in the photomultiplier pulse (Figure 3.2b) is of the order 0.1 μ sec for a half revolution of the mirror over a period of 1 msec. This gives rise to an uncertainty of 0.01 μ sec in registering time-resolved spectra, since the recording of an event does not start until after the first 100 μ sec of each mirror sweep (i.e., the delay time between tripping and recording positions of the mirror).

The frequency gate can be preset to fire at the desired mirror speed by replacing the photomultiplier with a pulse generator and electronic counter. Optical alignment was made with the aid of a Spectraphysics Type 130C helium-neon laser. Table 3-i lists the component parts of the time-resolved spectrograph.

Table 3-i Time-resolved spectrograph

Rotating Mirror Assembly

Barr and Stroud Rotor Type CP5911, Ser. No. 1

Optical System

Quartz-water achromat		focal length = 32 cm. diameter = 4.5 cm.
Concave mirror	Optical Works Ltd., London, U.K., T 543	focal length = 6 feet diameter = 8 inches
Slit (S ₁ , Figure 3.1)	Hilger F. 1497	slit width = 0.03 cm.
Spectrograph	Hilger, medium quartz	f number $\approx$ 6 slit width $\approx$ 20 microns
Spectrographic plates	Kodak, Type I-O, I-F	

Frequency Calibration

Pulse generator	Hewlett Packard Model 212A
Electronic counter	Hewlett Packard Model 521C

Trigger Beam System

Photomultiplier	R.C.A. Type IP28
Power supply	Hewlett Packard, H.V. supply 0-3,000 volts Model 6516A 6 mA

Control Circuit

Frequency gate	Tektronix Type 162 waveform generator
	Tektronix Type 163 pulse generator
Delay unit	same as for frequency gate

Monitoring Equipment

Oscilloscope #1 (Figures 3.1, 3.2a)	Type 551, Dual-Beam, Tektronix
Oscilloscope #2 (Figures 3.1, 3.2b)	Type 545A, Single Beam, Tektronix
Rogowski coil (Table 2-i)	

3.2 TIME-RESOLVED SPECTRA OF THE PINCH

A wavelength must be chosen that is suitable for monochromator measurements of the pinch time. Hence, a survey of radiation behaviour in time must first be conducted over the visible spectrum. These experimental requirements are fulfilled by the time-resolved spectrograph.

The plasma column contracts to its minimum width, or pinch stage, in the axial region of the Z-pinch discharge. It is here that the greatest intensity of plasma radiation occurs. Even so, a superposition of at least ten discharges is found necessary to form a satisfactory time-resolved spectrographic plate. Therefore, only the "pinch" and "post-pinch" stages of the discharge can be analyzed by this method.

Figure 3.3 shows a series of time-resolved spectra taken end-on along the axis of the discharge vessel at different filling pressures. The beginning of the time scale coincides with the initial appearance of continuum at the bottom of each plate. For low filling pressures (250 to 1,000 μ Hg), a break (x) in the first continuum (I) is noticeable. In this same pressure region secondary continua (II and III) appear, which gradually decrease in intensity and finally disappear for the 2,000 μ Hg filling pressure. The persistence, self absorption, and broadening of the HeI lines (6678, 5876, 4470, and 3889 $\text{\AA}$) offer a striking contrast to the behaviour of the HeII 4686 $\text{\AA}$ line over the duration of the discharge and for increasing filling pressures. A decrease in the intensity and number of impurity lines is also evident as the filling pressure is increased.

Figure 3.3 Time-resolved spectra in helium

Figure 3.4 Time-integrated spectra in helium

- a) 250 μHg filling pressure in helium (I-O plate)
- b) 500 μHg filling pressure in helium (I-O plate)
- c) 750 μHg filling pressure in helium (I-O plate)
- d) 1,000 μHg filling pressure in helium (I-F plate)

(Calibrated against iron arc spectrum)

An estimate of the pinch time can be made from the geometry, mirror speed, and discharge current characteristics employed in recording the spectra of Figure 3.3. These measurements give values of 4 to 7 $\mu\text{sec.}$ for the 500 and 2,000 μHg cases respectively. A much more accurate method would be to use a photomultiplier with time resolution better than the limit of 0.25 $\mu\text{sec.}$ for the rotating mirror spectrograph (Figure 3.3g). Its output can be correlated easily with the discharge current, thus avoiding experimental errors in the determination of mirror speed and position (Section 3.3).

It is desirable to measure the intensity and time of occurrence for secondary continua, as well as the pinch time which can be characterized by the first appearance of radiation continuum at the axis. For this purpose a monochromator, set at a chosen continuum wavelength, is used in conjunction with the photomultiplier. A careful examination of the spectra in Figure 3.3 shows that a wavelength in the 4600 $\text{\AA}$ region is a good choice for monitoring the on-axis continua. The photomultiplier sensitivity is high and no impurity lines of consequence are encountered (Appendix III)

Spectral lines appearing in the pinch column can be identified by comparing the helium discharge spectrum with that of an iron arc. Figure 3.4 displays records obtained by the spectrograph without time resolution. They are referred to as "time-integrated" spectra. The value of time-resolved spectroscopy is amply demonstrated by comparing them with Figure 3.3. In the time-integrated spectra, the secondary continua are superimposed and there is no hint of self absorption in the HeI lines. Figures 3.4a to 3.4c were recorded on Kodak type I-O spectrographic plates, while a type I-F was used in Figure 3.4d. Spectral lines of wavelength

higher than HeII 4686 appear on the last plate only because of the different sensitivity for the two plate types. All spectra in Figure 3.3 were recorded on Kodak type I-F plates.

Figure 3.3g shows the discharge spectrum with the mirror in the static position. This gives an estimate of 0.25 μ sec. for the limit in time-resolution by the rotating mirror method, for no jitter in breakdown or recording.

3.3 PINCH TIMES

The "pinch time" is an easily measurable characteristic of the Z-pinch discharge. It is defined as the time interval between the initiation of the discharge and the on-axis arrival of the inner current shell boundary. This parameter can be observed by different diagnostic techniques and is therefore valuable in relating magnetic probe results with optical measurements.

The sudden appearance of continuum radiation at the axis of the discharge (Figure 3.3) indicates the presence of material which becomes highly excited in this region. This phenomenon can be used to measure the pinch time, since it signifies the beginning of the "pinch" stage of the discharge. It is found convenient to use the $4640 \text{ \AA}$ wavelength in monitoring continuum radiation from the axial region of the discharge (Appendix III). This allows the formation of subsequent continua to be timed as well.

A monochromator is focussed along the axis of the vessel and its output is displayed simultaneously with the discharge current on a double beam oscilloscope (Figure 3.5). The termination of spark gap noise from the trigger switch is considered as the time at which initiation of the discharge current occurs, since a backwards extrapolation of the discharge current trace gives the current $I = 0$ at this point (Figure 3.5). Such a situation is expected because the onset of current changes the impedance of the system and thus, suppresses the spark gap noise. Hence, the "formation" time of the discharge, as well as the occurrence of on-axis continuum, may be read to give the pinch time as defined above (T_p in Figure 3.5).

Figure 3.5 Determination of pinch time T_p helium discharge at 2,000 μHg filling pressure.

Pinch times observed by Tam (1967) are shown in column (2) of Table 3-ii below. They are taken from "current collapse shells" in Chapter 4 and represent the time of arrival on axis for the inner boundary of the current shell. The appearance times for on-axis continuum (T_p , Figure 3.5) are shown in column (3).

Table 3-ii Pinch times		
(1)	(2)	(3)
Initial pressure in helium (μHg)	"Collapse times" Tam (1967) (μsec)	On-axis continuum detectable at (μsec)
500	3.5	3.5
1,000	4.5	4.7
2,000	5.5--6	5.4
4,000	6.5--7	7

Tam's results are accurate to within 0.5 μsec . The optically determined pinch times in column (3) agree with Tam's measurements for the same initial filling pressures. The experimental error between the two methods is approximately 10%.

This agreement in pinch times forms the basis for relating Tam's magnetic probe measurements with a detailed optical study of the collapsing shell. A different diagnostic device must be used to study radiation behaviour for this preliminary phase of the discharge. However, spectrographic and monochromator records obtained thus far contain additional information about the pinch and post-pinch behaviour of the discharge column. For this reason, a further correlation of optical and magnetic probe results is deferred until Chapter 4.

3.4 BEHAVIOUR OF THE DISCHARGE AT PINCH AND POST-PINCH TIMES

Synchronization of time-resolved radiation records with the discharge current reveals the action of the discharge in the pinch and post-pinch regimes. In Figures 3.6 to 3.9, the discharge current is compared with composite radiation traces which are built up from time-resolved monochromator and spectrographic records (Appendix III).

A break in the first continuum is observable for pinches in the 250 to 1,000 μHg filling pressure region (Figures 3.3, 3.6, and 3.7). This break, or "dark line", occurs because the discharge axis does not correspond exactly with that of the vessel. A calculation verifying this conclusion is made in Section 4.5 on the basis of information gained from the collapse stage of the discharge. The appearance of the first continuum coincides with the on-axis arrival of the inner current shell boundary (Section 3.3). For this reason, it will be referred to as the "pinch continuum".

The occurrence of secondary continua in the 250 to 1,500 μHg filling pressure region (Figures 3.3a to 3.3e, and 3.6, 3.7) can be most readily explained by a second collapse of the current shell. Framing camera pictures in Chapter 4 show no evidence of additional collapse shells following the one responsible for the pinch continuum. The trapping probability of gas within the collapsing shell is very nearly 1 (Tam, 1967). This means that very little gas would be left free to form secondary collapse shells after the passage of the first current shell. Furthermore, the plasma column remains constricted in the axial region of the discharge for several microseconds after the first pinch (Chapter 4, Figures 4.12 to 4.15). It is during this stage that the secondary continua appear. Hence, these continua

can arise only from an interaction within the expanding gas column after the pinch.

The mechanism for the second collapse occurs in the following way. The kinetic energy of radial motion is thermalized during the first contraction responsible for the "pinch continuum". This causes an enormous pressure increase within the pinch column sufficient to overcome the confining magnetic pressure. The column therefore, expands reducing the brightness of the continuum. Because of the high density, the gas cools rapidly by radiation. This is borne out by the onset of self-absorption in the HeI lines and the formation of HeII lines in the latter half of the pinch continuum (Figure 3.3). Hence, the kinetic pressure is greatly reduced, but the electric current still exerts a large enough radial pressure to cause another collapse.

The disappearance of secondary continua as the filling pressure is increased can be understood by relating current discharge behaviour to the formation of the pinch. In Figures 3.6 to 3.9, the current always goes through a minimum when the pinch continuum appears and then rises again to a maximum. For low filling pressures, this maximum is the highest possible value the discharge current can reach (Figures 3.6, 3.7). At higher initial pressures, the pinch is not formed until the discharge current is already waning. Only a small, relative maximum can be attained afterwards by the discharge current (Figures 3.8, 3.9). Hence, magnetic pressure generated by the current is not sufficient to recompress the plasma column.

Figure 3.6 Composite trace of on-axis continuum, 500 μHg

Figure 3.7.

Composite trace of on-axis continuum, 1,000 μHg .

Figure 3.8

Composite trace of on-axis continuum, 2,000 μHg

Figure 3.9 Composite trace of on-axis continuum, 4,000 μHg

Section 3.3 and supporting evidence in Chapter 4 imply that the first minimum in the discharge current appears very close to the time the contracting shell reaches its minimum diameter. It is well known that a sharp discontinuity occurs in the current waveform at the time of pinch formation (Zwicker, 1964a). The time interval for maximum contraction can be very brief. This would explain the sharpness in the current minima.

A second minimum in the discharge current trace occurs for the 500 and 1,000 μHg cases (Figures 3.6 and 3.7). The second minimum for the 1,000 μHg case is more in the nature of an "inflexion point", but it is distinguishable nevertheless. These second minima are reproducible from shot to shot under the same initial conditions. Furthermore, they occur just about the time of appearance for the second continuum in the 500 μHg case and for the third continuum in the 1,000 μHg initial pressure case. Figure 3.3 shows significant broadening in the helium line before and after the secondary continua (II, III). Indeed, the HeII 4686 line is widest during the third continuum. Since the line width increases with electron density, the existence of secondary continua can be ascribed definitely to a second contraction of the plasma cylinder.

Zwicker (1964 a,b) has conducted experiments similar to those described in this chapter. His observations were made at right angles to the axis of the Z-pinch discharge in helium. The range of initial pressures was comparable, although the discharge parameters were different from those used

here. He also observed secondary on-axis continua and related the first "pinch" continuum to the kink (or minimum) in his Rogowski coil trace.

Zwicker links the appearance of secondary continua to the onset of instabilities in the on-axis plasma column. The argument is that local instabilities occur along the axis of the discharge and, therefore, carry continuum across the diameter of the pinch column. As evidence of this, he cites the irreproducibility of the secondary continua from shot to shot for the same initial conditions.

In this thesis, observations of the pinch were taken end-on, along the axis of the discharge. Figure 3.3 shows distinct separations between the secondary continua. In Appendix III, Figures 1 and 2 show a possible jitter of $0.3 \mu\text{sec.}$ in the monochromator traces where secondary continua appear. Such a jitter seems too small for the secondary continua to be generated solely by instabilities. Although he does not state the magnitude of variation in time for the occurrence of secondary continua, Zwicker is quite emphatic that it is large from shot to shot for the same initial conditions.

A comparison of Zwicker's (1964a) and our discharge parameters shows that:

- i) Zwicker's current is much greater and its ringing period is much shorter than for this thesis: i.e.,

Maximum discharge current 380 kA vs. 280 kA observed here;
Discharge ringing period 14 $\mu\text{sec.}$ vs. 36 $\mu\text{sec.}$ observed here.

- ii) the change in amplitude of the discharge current, between its maximum value and the minimum attained at the first pinch, is greater in Zwicker's experiments than here: i.e.,

190 kA for 330 μHg vs. 19 kA for 500 μHg observed here;
95 kA for 1,000 μHg vs. 29 kA for 1,000 μHg observed here.

This suggests that changes in the conditions responsible for holding the plasma column together after the pinch vary much more violently in Zwicker's case than for the discharges studied in this thesis. Figure 3.3 shows further evidence of the comparative stability of the discharge studied here in contrast to that used by Zwicker. The highest filling pressure at which secondary continua occur is 1,500 μHg . Moreover, for the 10 μsec . interval covered by the plate, only one such additional continuum appears at this pressure. Zwicker shows two secondary continua for 2,200 μHg in helium, and at least one appears as high as 3,000 μHg (Zwicker, 1964a).

The occurrence of a second minimum in the discharge current traces, the broadening of helium lines and the small jitter-time for the secondary continua, point to a second collapse of the pinch column. This jitter of 0.3 μsec . can be caused by Rayleigh-Taylor instabilities as Zwicker suggests. Although instabilities are not primarily responsible for the formation of secondary continua, they can affect the reproducibility in their appearance on axis.

Zwicker's interpretation may be valid for his system, since it is much faster and therefore, more unstable. It is felt, however, that his secondary continua disappear with increasing filling pressure for the same reason as that given here--namely, the diminution of the discharge current at the pinch.

Qualitative agreement has been established with Zwicker's observations of the post-pinch regime for the Z-pinch discharge. On the other hand, the similarity of discharge systems demonstrated in Section 3.3 indicates that magnetic probe work done previously in this laboratory can be linked to our optical studies. Hence, further optical investigations in the discharge stages observed by Tam can provide a general picture of how magnetic and current distributions act to produce the Z-pinch.

4.0 KINEMATICS OF THE Z-PINCH DISCHARGE

"Kinematics" describes the motion of the plasma as a whole, without reference to the force or mass. Information can be obtained about the motion and macrostructure of the contracting plasma cylinder, from the time of its formation until its culmination in the pinch, by photographing successive stages of the discharge during the collapse regime. The high speed framing camera is preferable to a "smear" camera or image converter because of spatial resolution over a wider field of view, the larger number of film images available and the relative ease of interpreting the resulting photographs.

The action of the framing camera is based on the same principle used in time-resolved spectroscopy (Section 4.1). A similar superposition is followed in obtaining useable records (Section 4.2). Only a simple correction for perspective is needed to determine the true position and thickness of the hollow collapsing cylinder of plasma (Section 4.3).

Spatial distributions of HeI and HeII can be studied by taking framing camera photographs through suitable interference filters (Section 4.4). By synchronizing these results with other discharge parameters, the collapse stage can then be related to the post-pinch regime discussed in Chapter 3 (Section 4.5). A comparison with measurements performed by Tam (1967) can also be carried out to determine the connection between current distributions and radiation regions within the collapsing shell (Section 4.6). This provides an understanding of the action of the Z-pinch discharge which corresponds well with the Jukes-Allen theory for fast-pulsed discharges.

4.1 THE HIGH-SPEED FRAMING CAMERA

A Barr and Stroud Model CP5 framing camera is used, in conjunction with interference filters, to photograph the collapsing discharge column of the Z-pinch. In this camera the film remains stationary and a rotating mirror is used to pass light from the discharge through a series of fixed lenses (Figure 4.1).

The triggering process is identical to that of the time-resolved spectrograph (Section 3.1). A small light source (not shown in Figure 4.1) is mounted inside the camera and focussed on the rotating mirror. At a certain position, the reflection is recorded by the photomultiplier every time the mirror completes a full half-turn.

For a rotational speed of 5.5 kc/sec. the manufacturers give:

t_1 = time interval for image beam to travel from first to last lens.
= 7.5 μ sec.

t_2 = time period from trigger position of photomultiplier #1 to the first lens position.
= 3.4 μ sec.

For the pinch times observed in helium, it was found convenient to operate the framing camera at a speed of 2 kc/sec. This makes

t_1 = 20.6 μ sec.

t_2 = 9.4 μ sec.

For 60 lenses at 2,000 c.p.s., the time between images on the film =

$$\frac{20.6}{60} = 0.34 \mu\text{sec.}$$

A slight innovation was introduced to enable one person to operate the entire system. Contact switches were placed on the lever controlling the

compressed air input to the rotating mirror assembly. This allowed the operator to turn on the trigger beam, throw open the air valve and then open the camera shutter in one motion.

By recording the discharge current and the appropriate output signals from the control system, events recorded by the framing camera and the time-resolved spectrograph can be correlated through the use of a monochromator. The oscillograms monitoring the mechanical and electronic jitter are the same in every respect as those of Figures 3.2 a,b for the rotating mirror spectrograph.

Figure 4.1

Framing camera

4.2 DATA REDUCTION

The magnetic field and current distributions in a helium Z-pinch have been established by Tam (1967) for filling pressures of 500, 1000, 2000, and 4000 μ Hg. Since the system used in this thesis is identical to Tam's in its important features, it was decided to investigate the discharge at the same pressures. Spectroscopic observations in Chapter 3 indicate the plasma to be almost pure helium before the first pinch occurs, particularly for higher initial pressures. The spatial distributions of HeI and HeII can therefore be established by photographing the discharge end-on through suitable interference filters. This provides information which can be compared with Tam's data.

All photographs are made on Ilford HP3 film developed for 10 minutes in Ilford developer, number ID2. From its response curve (Figure 4.2) and an examination of helium lines dominant in the plasma (Figure 3.3), only the HeII 4686 Å and the HeI 4470, 5876 Å lines will register successfully on the film. Figure 4.3 shows the transmission curves for interference filters used in photographing the light emitted by these three helium lines.

Table 4-i gives the experimental conditions used in obtaining the photographs. The term "no filter" in the table means that no interference filters were employed. Such a record was made at the beginning and end of each experimental run as an additional check that no error in timing occurred.

Figure 4.4 shows a typical sequence of framing camera photographs. In order to correlate the photographs with current distributions measured by Tam (1967), the dimensions of the luminous shell must be measured. To determine the extent of the luminous regions in the plasma, half the field

Figure 4.2 Colour sensitivity of Ilford HP3 film

Figure 4.3 Transmission curves for interference filters

Filter	Transmission	Band pass $\Delta\lambda$
a) HeI 4470	64%	85 Å
b) HeII 4686	42%	27 Å
c) HeI 5876	49%	79 Å

Table 4-i Observational sequence and conditions for framing camera photographs

Order of observation	Type of filter	No. of shots per film strip in helium			
		500 μ Hg	1,000 μ Hg	2,000 μ Hg	4,000 μ Hg
1	No filter	4	1	1	1
2	HeII 4686	30	20	20	30
3	HeI 4472	15	10	10	15
4	HeI 5876	15	10	10	15
5	No filter	4	1	1	1

of view is masked by a "neutral density" filter of known transmission (Table 4-ii). For the purposes of this thesis, the boundaries of the luminous shell are then determined by the radii of the shell at which the intensity falls to about 35% of its peak value.

Figures 4.5 a,b show how the dimensions of the luminous shell are determined by measuring densities of the photographic image with a Jarrell-Ash microphotometer. The typical structure of the microphotometer records appears in Figure 4.5 c. The most important features are a luminous shell which collapses onto the discharge axis, and a luminous spot appearing at the centre of the discharge.

Table 4-ii Transmission of the "neutral density" filter used
 in determining boundaries of the collapsing shell

Wavelength	<u>Transmitted radiation</u> Incident radiation
HeI 4472	0.34
HeII 4686	0.37
HeI 5876	0.48

Figure 4.4

Segment of framing camera
film
Sequence of images on film
—20, 21, 22...etc.

For filling pressure of
2,000 μ Hg in helium,
no interference filter.

a) Analysis of image by microphotometer

b) Microphotometer trace

c) Spatially resolved parameters

- r_a - radius of axial spot
- r_i - inner radius of luminous plasma shell
- r_p - radius of peak intensity
- r_o - outer radius of luminous plasma shell
- R_o - radius of discharge vessel

Figure 4.5 Analysis of framing camera images.

4.3 CORRECTIONS FOR PERSPECTIVE

In order to ascertain the true dimensions of the plasma shell a correction must be applied to the microphotometer traces (Figure 4.5 c). Images recorded on film are affected by the length of the plasma column, as demonstrated in Figure 4.6.

The procedure is to treat the inner radius (r_i) of the film image as light coming from the back of the discharge tube, and the outer radius (r_o) as radiation from its front end. The radiation peak (r_p , Figure 4.5 c) is regarded as emanating from halfway down the tube. The later frames of the discharge in Figure 4.4 clearly show the high tension, or rear electrode of the discharge tube. Hence, the effect of perspective can be corrected by measuring the image and true cross-sections of the electrodes at both ends of the discharge.

If R_r = the radius of the rear electrode image (Figure 4.6),

R_f = the radius of the front electrode image (Figure 4.6) which corresponds to R_o of Figure 4.5 c,

and R_T = the true dimension of the electrode radius,

then the true r_i = image of r_i on film $\times \frac{R_T}{R_r}$

the true r_o = image of r_o on film $\times \frac{R_T}{R_f}$

The optical system of the framing camera experiment is such that a linear interpolation between the two electrode image sizes (R_f, R_r) is sufficient to place the position of peak intensity:

the true $r_p \approx$ image of r_p on film $\times \frac{2 R_T}{R_f + R_r}$

The soundness of the above assumptions for perspective is shown for the 500 μ Hg case where there is excellent correspondence with the current collapse shell in Figure 4.16. The close correlation between luminous and current shells is discussed in Section 4.6.

Figure 4.6

Effect of perspective on film image.

4.4 PEAK RADIATION AS A FUNCTION OF TIME

Before proceeding to a final synthesis of photographic data, the accuracy in recording the discharge can be checked by examining the behaviour of the peak radiation front within the collapsing shell. The radius of the radiation peak (r_p , Figure 4.5c) was measured in each image and plotted against the number of the frame. Since the frame interval is 0.34 μ sec., such graphs exhibit r_p as a function of time (Figure 4.7 to 4.10).

A striking feature appears in these graphs as the filling pressure is increased:

- i) At low pressures (500 and 1,000 μ Hg), the radiation peaks observed through HeI and HeII filters coincide with the peak observed when no filter is used at all.
- ii) At the higher pressures (2,000 and 4,000 μ Hg), the radiation peak observed through HeI filters coincides with the peak observed when no filter is used. However, the peak of the HeII 4686 $\text{\AA}$ radiation consistently follows behind the HeI radiation peak.

Only one discharge is required to obtain a usable photograph when no filter is used. At least ten or fifteen shots are needed to produce a viable photograph through the HeI filters (Table 4-i). The high degree of overlap for radiation peaks photographed with and without HeI filters demonstrates the smallness of errors in timing. The recording process is therefore accurate to 0.34 μ sec. at least (i.e., the duration of the interval between framing camera images).

Because timing errors are so small, the spatial separation of the HeI and HeII peaks at higher pressures must be regarded as significant. It can be explained by noting that gas in the outer boundaries of the luminous shell has been heated for a longer time than the gas on the inner edge. Furthermore, Joule heating is more efficient in the neighbourhood of the HeII peak since it has been established that the current density increases towards the outer edge of the collapsing shell (Tam, 1967).

Figure 4.7 Radius of radiation peaks vs. time, He 500 μHg

Figure 4.8

Figure 4.9

Radius of radiation peaks corrected for perspective vs. time, He 2,000 μHg

--○-- HeII 4686

—△— combination of no filter, HeI 5876, HeI 4470

Figure 4.10 Radius of radiations peak corrected for perspective vs. time, He 4,000 μHg

4.5 SYNCHRONIZATION OF FRAMING CAMERA PHOTOGRAPHS WITH OTHER DISCHARGE PARAMETERS

The reproducibility in creating and recording the Z-pinch discharge is now established. In order to make maximum use of information provided by the framing camera records, it is desirable to correlate them with other discharge parameters. By synchronizing each set of film images with the total discharge current, a connection can be made between the collapse phase and the pinch stage explored in Chapter 3.

A direct relation between the discharge current and film image can be determined through the use of a monochromator. This instrument is placed behind a small window in the end electrode of the discharge tube. The distance of the window from the axis of the discharge is set at 1 inch or 2 inches. (In reality these distances are 3.18 cm. and 5.08 cm. respectively since the axis of the discharge does not coincide exactly with that of the vessel.) The monochromator is set on the HeII 4686 Å line. Its output is fed into a double beam oscilloscope and displayed simultaneously with the discharge current (Figure 4.11). Synchronization is achieved by noting that the framing camera image showing HeII peak radiation at a radius of 1 inch (or 2 inches) must occur when the maximum in the first monochromator pulse registers on the oscillogram. The position of this point in time can then be read directly from the oscillogram with respect to the initiation of the discharge.

Figure 4.11 Synchronization of framing camera images with the total discharge current

The graphs in Figures 4.12 to 4.15 were plotted in this manner. They will be referred to henceforward as "luminosity graphs" to distinguish them from the current collapse graphs obtained by the magnetic probe method (Tam, 1967). They are composite images built up from the film strips described in Table 4-i (Section 4.2). The inner radius is that of the "no-filter" and "HeI" film strips. The outer radius of the luminosity shells is that of the "no-filter" and "HeII" film strips. The peak radiation position is that belonging to the peak radiation radii of the "HeII" film strips. Because of low light levels in the 500 μ Hg case, only the "no-filter" film strip was used. This is the reason no bars representing the spread in observation points are shown on the inner and outer boundaries of the luminosity curve of Figure 4.12.

The on-axis continua discussed in Section 3.4 are included in the luminosity graphs as well. The photomultiplier voltage is displayed against time. It can be seen that intense continuum radiation appears in the axial region of the discharge only when the collapsing luminous shell approaches its minimum diameter. Just one collapse shell is evident before the appearance of on-axis continuum. This and the persistently small cross-section of the plasma column during the post-pinch regime support the conclusion that secondary continua occur because of an additional collapse within the pinch column (Section 3.4).

The sequence of framing camera images in Figure 4.4 clearly shows an "axial spot" which appears in association with appreciable radiation before the luminous shell is truly formed. The outer radius of this radiant spot is drawn in the luminosity graphs (Figures 4.12 to 4.15). The luminosity curves indicate what seems to be a partial collapse before the main trigger switch is completely conducting. These effects are not evident on Tam's collapse curves (Figures 4.16 to 4.19) since his observations are triggered by the rise in discharge current after the initial transient has died away.

The monochromator response to "axial spot" radiation can be estimated from framing camera pictures. Assuming its radiation to be continuum-like for the 2,000 and 4,000 μHg initial pressure cases, its photomultiplier signal would be about 1/30 of the signal produced by the pinch continuum. Since the monitoring oscilloscopes were adjusted to measure pinch continuum intensities, there would be insufficient gain for the detection of the axial spot.

Further information is necessary to determine the nature of axial spot radiation. Observations might be conducted in a direction perpendicular to the discharge axis and close to the high-tension electrode. It may then be possible to decide whether the axial spot can be regarded simply as an "electrode burn", or if it is evidence of at least a partial column of luminous plasma appearing before the main pinch takes place.

The accuracy in relating the collapse stage observations to the pinch and post-pinch regime can be demonstrated by considering the "dark line" in the first continuum for low initial pressure discharges (Figures 3.3, 3.6, and 3.7). This break in the pinch continuum can be explained by the fact that the discharge axis is slightly eccentric with respect to the axial viewing hole. For the 500 μ Hg filling pressure case in Figure 4.12, the velocity of the outer boundary of the luminous shell can be evaluated as it hits the axis:

$$= 3.33 \text{ cm}/\mu\text{sec.}$$

The pinch continuum first appears at 7.43 μ sec. on the luminosity graph and the "dark line" occurs 0.85 μ sec. later. If the break in the first continuum is truly caused by eccentricity in the viewing hole, then all of the luminous shell will have passed this observation point in 0.85 μ sec., i.e., the predicted thickness of the luminous shell at 7.43 μ sec. $= 0.85 \times 3.33 \text{ cm.} = 2.83 \text{ cm.}$

Since continuum appears at 7.43 μ sec. in Figure 4.12, the inner radius of the total current shell is zero. Therefore, the thickness of the shell at 7.43 μ sec. can be read from the luminosity graph as the outer radius of the luminous region. The value obtained is 2.83 cm., which is identical to that deduced above.

This calculation shows that the "axial" viewing port is not located on the axis of the discharge. In fact, a burn on the electrode caused by the pinch is found 0.25 inches away from the geometrical centre of the high-tension electrode. The agreement between predicted and observed values of shell thickness attests the accuracy in timing for the observational methods used.

The break in the first continuum caused by the eccentric viewing port shows that the diameter of the pinch must be less than 0.5 inches in the 500 μ Hg case. In higher initial pressure plasmas (2,000 to 4,000 μ Hg), this "dark line" is not observed since more material is pushed onto the axis, making the pinch column larger in diameter.

Figure 4.12 Luminosity curve, 500 μHg

Figure 4.13 Luminosity curve, 1,000 μHg

Figure 4.14 Luminosity curve, 2,000 μHg

Figure 4.15 Luminosity curve, 4,000 μHg

4.6 STRUCTURE OF THE Z-PINCH DISCHARGE

The similarity in pinch times, measured by magnetic probe and optical methods, forms the basis for relating current density distributions obtained by Tam (1967) to the luminosity regions observed in the collapse of the plasma shell. Figures 4.16 to 4.19 display the current shells measured by Tam for 500, 1,000, 2,000 and 4,000 μHg filling pressures in helium. The boundaries of the current shell are determined by the radii at which the current density falls to 50% of its maximum value.

Tam shows that most of the mass is contained within these boundaries by employing a modified snowplow equation:

$$\frac{d}{dt} \left[\alpha (R^2 - r^2) \frac{dr}{dt} \right] = - \frac{1}{\rho_o \rho_o} (r'^2 B'^2 - r B^2)$$

It is assumed that the inner surface of the shell moves with the same velocity as that of the center of mass across the layer,

where, r (r') = radius of the inner (outer) surface of the current shell

R = radius of the inner wall of the discharge vessel

B (B') = measured magnetic flux density at inner (outer) surface

ρ_o = initial density of gas within the current shell at the beginning of the discharge

α = trapping factor showing the fraction of gas retained by the collapsing shell.

Numerical solutions of the above equation were obtained for the collapse curve of the inner current shell boundary and compared with those measured by the gradient probe. Tam determines the best fit between theory and experiment to be:

$\alpha = 0.9$, for helium at 1, 2, 4 mmHg. initial pressure
 $\alpha = 0.75$, for helium at 500 μ Hg. initial pressure.

Therefore, the boundaries of the current shell in Figures 4.16 to 4.19 contain almost all the material swept up by the collapsing current.

The luminosity shells of Figures 4.12 to 4.15 are represented by the cross-hatched areas in the current shell collapse graphs of Figures 4.16 to 4.19. An important result of correlating the luminosity and current collapse shells is that the HeII peak radiation region coincides with Tam's observation points for peak current density. This holds true for all cases. The luminosity shell can therefore be identified with the main current-bearing region of the collapsing plasma column. Although the boundaries of the luminosity shell are defined by a drop to 35% of the peak intensity, its width corresponds well with the dimensions of the current shell in the 500 and 1,000 μ Hg. initial pressure cases (Figures 4.16 and 4.17). From the considerations of the previous paragraph this means that almost all the moving gas is visible in the framing camera pictures.

For higher filling pressures (2,000 and 4,000 μ Hg.) such a situation no longer holds true. Figures 4.18 and 4.19 show that a non-luminous, conducting body of gas precedes the radiating region of the current shell. It is the on-axis arrival of this relatively cold gas which is responsible for the appearance of continuum in the axial region.

In the 4,000 μ Hg. case, an examination of Tam's current density profiles shows the formation of a subsidiary current shell in front of the radiating region. Its peak value is initially less than 50% of the maximum current density in the luminous region of the collapsing plasma column. Its inner

Figure 4.18 Current shell collapse, magnetic probe method
He 2,000 μ Hg

boundary is sketched in Figure 4.19 and represents the radius at which the current density in this "minor" current shell falls to half its peak value. Hence, the apparent "splitting" of the inner current shell boundary in the 4,000 μHg . filling pressure case.

In Figures 4.18 and 4.19 the luminosity shell containing most of the discharge current is slowed down. It is brought to rest before it reaches the axis for the 4,000 μHg . filling pressure case. This is due to pressure exerted by material carried onto the discharge axis in front of the luminous region. Tam's (1967) current density profiles for 4,000 μHg filling pressure show the beginning of a reflection for the "minor" current shell after it reaches the axis. This explains the relatively poor agreement in pinch times (t_p) when he used the snowplow model ($t_p \approx 8 \mu\text{sec.}$) to fit his observed current collapse ($t_p \approx 7 \mu\text{sec.}$) for this case. The model assumes negligible kinetic pressure outside the main current shell.

The slowing down of the current bearing layer can be explained qualitatively by the "Jukes-Allen" model for a fast-pinch discharge. The theory for this model (Jukes, 1958) is outlined in Section 6.2 in connection with Guderley flow. The action of the collapsing plasma column can best be understood by assuming that most of the current resides within a narrow shell around the radiation (current) peak. A cold shock front precedes the current layer. The kinetic energy of this shock is thermalized upon hitting the discharge axis. Material carried by it is heated and expands, exerting pressure on the incoming current layer. This is known as the "throttling process": the reason the current layer can never reach the axis (Figure 4.20). This phenomenon is well illustrated in the correlated luminosity and current shells of Figures 4.18 and 4.19.

Figure 4.20

Schematic diagram of Jukes's model for Z-pinch contraction

The Jukes-Allen theory is valuable in that it predicts the behaviour of the forerunning shock front independently of the current layer. That it does not hold true for the low initial pressure cases (500 and 1,000 μHg) will be demonstrated more fully in Section 6.2. However, Figures 4.16 and 4.18 show that the luminous region coincides almost exactly with the current shell and that, therefore, a firm demarcation between shock and current layers is not possible. For low initial pressures (500 to 2,000 μHg) the snowplow model seems adequate. For higher initial pressures (2,000 μHg and above) the Jukes-Allen theory must be invoked to account for significant kinetic pressure outside the main collapsing shell.

The above discussion of discharge action helps to explain qualitatively the occurrence of the "dark line" in the pinch continuum for low initial pressures (Figures 4.12, 4.13). If the viewing port is eccentric with respect to the discharge axis, then the continuum appearing before the "dark line" can be identified with material rushing onto the axis. All continuum after

this break arises from excited material expanding outwards and away from the axis of the discharge.

The following collapse stage characteristics of the Z-pinch discharge emerge from observations discussed in this and previous sections:

- i) Appreciable luminosity appears before the current shell is fully formed. This is caused by an initial transient current from the triggering apparatus. It is also possibly responsible for the formation of an inner "axial spot".
- ii) The time of appearance for continuum in the axial region of the discharge coincides with the arrival time on the discharge axis of the inner current shell boundary. These "pinch" or "collapse" times can be predicted by the Snowplow model of the Z-pinch discharge at low initial filling pressures (500 to 2,000 μHg).
- iii) The peak HeII radiation of the collapsing luminosity shell coincides with the peak current region of the corresponding current shell. The luminosity shell can therefore be identified with the main current-carrying region of the collapsing plasma column.
- iv) At lower filling pressures (500 to 1,000 μHg in helium), both current and luminosity shells almost completely coincide.
- v) At higher filling pressures (2,000 to 4,000 μHg in helium), the current shell is slowed down and, in the 4,000 μHg case, brought to rest before it reaches the discharge axis. There is a discernable separation between current and luminosity fronts. In addition, a difference in spatial distributions of HeI and HeII becomes distinguishable within the luminosity shell. This behaviour can be described qualitatively by the Jukes-Allen model.

From the observations above, the collapsing shell is composed of two regions:

- 1) an inner, relatively cold shock layer followed by
- 2) a hotter, luminous shell.

In helium pinches generated with low initial pressures (500 to 1,000 μHg) the "shock" and "luminous" layers almost coincide. Progressing through higher initial pressures (2,000 μHg and above), the distinction between these two regions becomes more apparent.

5.0 DETERMINATION OF DENSITIES AND TEMPERATURES WITHIN THE LUMINOUS PLASMA

Measurements made until now have furnished insight only into the spatial distribution of helium and front velocities in the collapsing current shell. In order to understand the plasma more completely it is necessary to know its temperature and density. Having evaluated these parameters within the excited gas during several stages of its formation, the consistency of a given dynamical theory can then be checked. This in turn would help to explain the basic mechanisms responsible for the action of the Z-pinch.

The most highly developed method of determining temperature and density is through the analysis of plasma radiation (Griem, 1964; Cooper, 1966). This chapter is therefore concerned with measuring these parameters only in the highly luminous regions of the plasma. A model for calculating plasma densities in the faintly luminous shock zone of the collapse will be outlined in Chapter 6.

The difficulties of radiation calibration inherent in photographic work has been avoided by the use of a monochromator-photomultiplier arrangement (Section 5.1). Electron densities and temperatures in the collapse stage of the discharge are determined by analyzing helium line profiles. Absolute intensity measurements are used to evaluate these parameters (Section 5.2). This entails the assumption of local thermal equilibrium (L.T.E.). Plasma temperatures and particle densities are then computed at two separate stages of the discharge collapse.

Using relative intensity measurements, values of T_e are compared for L.T.E. and a semi-coronal model developed by Griem (Section 5.3). It is shown that an error of 20% in the estimate of T_e can occur because of the different de-excitation mechanisms involved.

The effect of self absorption on the shape of measured line profiles is examined (Section 5.4). Calculations demonstrate that errors arising from this phenomenon are less than, or of the order of, those due to corrections made to the monochromator traces. The effect of self absorption can also be used to estimate the accuracy of this correction procedure.

An evaluation of electron-particle collisional relaxation times shows that an approximation to L.T.E. conditions may be assumed. This allows the equating of particle temperatures with deduced electron temperatures (Section 5.5). Thus, estimates of particle densities can be made within the collapse stage of the discharge and later used to verify the shock structure of the plasma in Chapter 6.

5.1 THE MONOCHROMATOR-PHOTOMULTIPLIER METHOD OF TIME RESOLUTION

A Spex Model #1700 II monochromator was used in conjunction with an EMI photomultiplier tube Type 9558B to observe plasma radiation (Figure 5.1). A dual beam oscilloscope displayed the discharge current simultaneously with the photomultiplier output at the wavelength investigated. This serves as a useful tie-in with results from the time-resolved spectrograph and the framing camera.

Figure 5.1 Experimental arrangement for monochromator

By using a monochromator, the radiation intensity at only one given wavelength can be determined each time the discharge is fired. For a study of continuum radiation only one such oscillogram is necessary to determine the electron density or temperature, depending on the optical depth of the plasma (Section 5.3). However, in the prepinch stage of the discharge, it is light emitted from the bound levels of the helium atoms and ions which predominates. In this case, electron densities can be determined by observing the broadening of these helium lines which is caused by the

perturbing electrons. It is therefore necessary to record line profiles: i.e., plots of intensity versus wavelength for a spectral region occupied by a helium line at a specific instant of time during the discharge. This may be done by changing the wavelength monitored by the monochromator each time the discharge is fired. Since the discharge characteristics are reproducible, the resulting sequence of oscillograms can be used to construct the required line profile (Figure 5.2). Each profile required at least 30 shots. They are therefore determined only when the peak intensity reaches points at 3.18 cm. or 5.08 cm. from the discharge axis. Only the collapse stage of the 2,000 and 4,000 μ Hg cases was thoroughly explored in this manner because of considerable impurity radiation in the low initial pressure discharges (Figure 3.3). Radiation profiles of the HeI 3889 $\text{\AA}$ and HeI 5876 $\text{\AA}$ lines were used to evaluate electron densities.

Electron temperatures for the collapse stage were determined by measuring the absolute intensity of the HeII 4686 $\text{\AA}$ line. This was done by evaluating the area of the profile (Figure 5.2) and then calculating the total power radiated by the line calibrated against black body emission. The monochromator was calibrated for this purpose by using a carbon arc.

The reciprocal dispersion and instrumental broadening for the first order spectrum of the monochromator were found to be practically constant for the region of interest (3,800 to 6,000 $\text{\AA}$). For a slit width of 8 microns the instrumental broadening is 0.2 $\text{\AA}$.

Figure 5.2

HeII 4686 Å line profile
 (observed at 3.18 cm. off-axis for 2,000 μ Hg filling pressure)
 $T_e \approx 39.2 \times 10^3$ °K, $n_e \approx 0.89 \times 10^{17} \text{ cm}^{-3}$

A description of the measurements for these instrumental functions and the absolute intensity calibrations is provided in Appendix IV.

Optical alignment of the system was made with the aid of a Spectra-physics Type 130C helium-neon laser. The same quartz-water achromat was used as for the time-resolved spectrograph (Section 3.1). The f number for the achromat = 7.0, and matches that of the monochromator with an f number = 6.8. It was decided to use the same control unit described in Section 2.2 in order to duplicate the timing of events as much as possible. The signals issued by the triggering photomultiplier (Figure 2.1) are replaced by the output of a pulse generator in Figure 5.1.

5.2 ABSOLUTE LINE INTENSITY MEASUREMENTS

Electron temperatures (T_e) and densities (n_e) can be evaluated from line profiles of the type displayed in Figure 5.2. The method is as follows.

By choosing an appropriate electron temperature (T_e) as a starting point, Griem's (1964) table of half-widths can be used to obtain the corresponding electron density (n_e) from the observed half-width of a HeI line (HeI 5876 Å or HeI 3889 Å).

If one assumes that double ionization is negligible, ($n_{\text{He}^{++}} \ll n_{\text{He}^+}$) then: $n_{\text{He}^+} = n_e$
Knowing the total power radiated per unit solid angle by the HeII 4686 line $\left[\sum_{\text{He}^+} (\lambda = 4686) \right]$ from absolute intensity measurements, the following formula can be used:

$$\sum_{\text{He}^+}(\lambda) = \frac{1}{4\pi} A_n^m n_{\text{He}^+} \frac{g_m}{Z_{\text{He}^+}(T_e)} e^{-E_m/kT_e} \frac{hc\ell}{\lambda} \dots\dots(1)$$

where A_n^m = Einstein emission coefficient from upper state m to lower state n
 g_m = the statistical weight factor for state m
 $Z_{\text{He}^+}(T_e)$ = partition function (Appendix VI)
 ℓ = length of the discharge tube = 59 cm.

Solving for T_e , using HeII 4686, equation (1) becomes:

$$T_e = \frac{5.8972 \times 10^5}{2.3026 \log_{10} \frac{0.11003 \times n_{\text{He}^+}}{Z_{\text{He}^+}(T_e) \sum_{\text{He}^+}(4686)}} \dots\dots(2)$$

with values $\lambda = 4686 \times 10^{-8}$ cm.
 $A_n^m = 1.732 \times 10^8 \text{ sec}^{-1}$ (Griem, 1964)
 $g_m = 32$
 $E_m = 50.80 \text{ e.v.} = 81.38 \text{ ergs.}$ (Moore, 1959)

For a given T_e , the following parameters are known:

$n_{\text{He}^+}(T_e)$ ---from Griem (1964)

$Z_{\text{He}^+}(T_e)$ ---calculated by computer (Appendix VI)

$\sum_{\text{He}^+}(4686)$ ---determined by observation and remains fixed
(Table 5-i)

T_e can be evaluated from equation (2). Starting with 35,000° K, the right hand side of equation (2) is iterated until two successive values of T_e agree to the second decimal place. Usually not more than three iterations, using a desk calculator, are necessary. This procedure also yields a final value for the electron density n_e .

For HeI 5876, equation (1) becomes:

$$n_{\text{He}^0} = 59.7833 \sum_{\text{He}^0}(5876) Z_{\text{He}^0}(T_e) \exp\left(\frac{2.6652 \times 10^5}{T_e}\right) \dots\dots(3)$$

for

$$\lambda = 5876 \times 10^{-8} \text{ cm.}$$

$$A_n^m = 0.706 \times 10^8 \text{ sec}^{-1}, g_m = 15 \text{ (Wiese, 1966)}$$

$$E_m = 22.97 \text{ e.v.} = 36.78 \text{ ergs (Moore, 1959)}$$

$Z_{\text{He}^0}(T_e)$ = a complicated partition function given by Wiese
(Appendix VI)

The density of neutral atoms n_{He^0} can be determined by equation (3) since the following parameters are known:

T_e ---from the iteration procedure of equation (2)

$Z_{\text{He}^0}(T_e)$ ---since T_e is now fixed (Appendix VI)

$\sum_{\text{He}^0}(5876)$ ---determined by absolute intensity measurements
(Table 5-i)

It should be noted that equation (1) and consequently equations (2) and (3) depend on the existence of thermal equilibrium in the He^+ (He^0) excited states right down to the ground state. This is implied by assuming

$$n_m = n_{\text{He}^+} \frac{g_m}{Z_{\text{He}^+}(T_e)} e^{-E_m/kT_e}$$

Such an assumption may not be valid. Indeed, Mewe (1967) shows that for a helium plasma, of $n_e \sim 10^{17}$ and $35,000^\circ \text{ K} < T_e < 40,000^\circ \text{ K}$, the excited levels of the He^+ species can be considered in thermal equilibrium with the continuum for $m \geq 2$. The ground state population ($m = 1$) of He^+ can depart quite significantly from an L.T.E. distribution. If the plasma is optically thin with respect to Lyman radiation of He^+ , the ground state population can be 30 times greater than that predicted by L.T.E. However, for a plasma which is optically thick to radiation from the ground He^+ state, the departure from equilibrium is greater by only a factor of 1.1 to 2.0. The expected error in T_e is approximately 20% if n_{He^+} is within a factor 10 of the value predicted from thermal equilibrium. This is due to the logarithmic dependence of T_e on n_{He^+} in equation (2), where $n_e \approx 10^{17} \text{ cm}^{-3}$ and $T_e \approx 40,000^\circ \text{ K}$.

For the neutral helium species in the same plasma, L.T.E. can be assumed valid only for $m \geq 3$. No estimates are readily available for the departure from L.T.E. values in cases of $m < 3$. At most, a "departure factor" of the same order quoted for the helium ions can exist, since the ground state to first level excitation energy is less for neutral helium (20 e.v. for He^0 , versus 40 e.v. for He^+). Even with the ground state population of neutral helium raised by a factor 30 above its expected L.T.E. value, neutral helium would still be negligible compared with ionized helium. The range of T_e determined here falls in the 2 to 11 e.v. temperature region, where it is known that rapid "burning out" of neutral helium takes place (Mewe, 1967).

Table 5-i contains the measured radiated power and half-widths of the lines necessary for the iterative procedure outlined above. The mono-

chromator response to radiation was calibrated by means of a carbon arc (Appendix IV). Plasma line intensities were calculated by measuring the area under line profiles, such as the one in Figure 5.2. These values were then used to determine the power radiated by the HeI 5876 Å and HeII 4686 Å lines. The width of the observed plasma lines is assumed to be a combination of Lorentzian broadening due to electron collisions and a Doppler type of instrumental broadening. The instrumental broadening is quite small (0.2 Å in Appendix IV) compared with the observed full line half-widths. Nevertheless, a correction was applied of the form

$$\omega = \sqrt{\omega_1^2 - (0.2)^2}$$

where ω = full half-width used in calculations
 ω_1 = observed full half-width

to better approximate the natural line width.

The HeI and HeII regions referred to in Table 5-i are a consequence of the different spatial distributions for these two species (Section 4.4). The "HeI region" is more densely populated and at a slightly lower temperature than the "HeII region"---hence, the terminology, since more HeI would be in the first region.

The temperature T_e was determined by equation (2) for the bracketed values of n_e in Table 5-i. The choice was based on the quality and degree of change in the profile from the "HeI" to the "HeII" region. The other n_e for the corresponding region was calculated using the deduced T_e . This served as a check on the accuracy of the results. The apparent discrepancy of 20% is due to the variation in optical depth of the plasma for light of different frequencies. This effect is discussed in Section 5.4.

Equation (2) shows that the calculated temperatures have a maximum error of less than 10%, assuming L.T.E. holds true down to the ground state.

Table 5-i

Absolute intensity evaluation of n_e , T_e

A. Initial pressure 2,000 μ Hg in helium

Off-axis observ'n point (cm.)	Region invest- igated.	Helium line ($\text{\AA}$)	Radiated power $\mathcal{E}(\lambda)$ erg/sec.sr.*	Full half-width ($\text{\AA}$)	$T_e \times 10^3$ ($^{\circ}$ K)	$n_e = n_{He^+}$ $\times 10^{17}$ (cm) $^{-3}$	% diff'ce in n_e	Neutral helium $n_{He^0} \times 10^{13}$ (cm) $^{-3}$
3.18	HeI	HeII 4686	1.295×10^9	2.65	37.66	(1.486) 1.295	15	15.6
		HeI 5876	1.466×10^9	5.421				
		HeI 3889		3.106				
	HeII	HeII 4686	1.454×10^9	2.65	39.23	(0.891) 1.159	20	8.93
		HeI 5876	1.008×10^9	3.247				
		HeI 3889		4.270				
5.08	HeI	HeII 4686	0.598×10^8	0.925	33.40	0.532 (0.516)	3	6.24
		HeI 5876	2.958×10^8	1.952				
		HeI 3889		1.3				
	HeII	HeII 4686	0.716×10^8	0.925	34.21	0.371 (0.384)	3	3.51
		HeI 5876	1.965×10^8	1.36				
		HeI 3889		0.967				

B. Initial pressure 4,000 μ Hg in helium

3.18	HeI	HeII 4686	3.292×10^9	4.90	39.44	1.627 (1.863)	12	20.7
		HeI 5876	2.329×10^9	5.922				
		HeI 3889		4.661				
	HeII	HeII 4686	3.861×10^9	5.8	40.55	1.394 (1.710)	18	7.86
		HeI 5876	0.987×10^9	5.071				
		HeI 3889		4.270				
5.08	HeI	HeII 4686	2.795×10^8	2.05	35.32	0.808 0.996	23	12.0
		HeI 5876	7.934×10^8	2.956				
		HeI 3889		2.505				
	HeII	HeII 4686	3.273×10^8	2.15	35.68	0.89 0.902	13	8.4
		HeI 5876	6.023×10^8	3.256				
		HeI 3889		2.266				

* sr. = unit steradian

Since the parameters n_{He^+} , Z_{He^+} , $\Sigma_{\text{He}^+}(4686)$ enter into the logarithmic part of equation (2), this small error appears even if there is:

- an error of 20% in estimating n_e
- an additional 20% margin for determining the radiated power $\Sigma(\lambda)$, and
- a possible 20% error for the oscillator strength (and hence, Einstein emission coefficient).

Table 5-ii summarizes electron temperature (T_e) and density (n_e) determinations, together with the resulting kinetic pressures:

$$P_{\text{kin}} = (n_e + n_i + n_o) kT$$

and densities $\rho = n_i m_i + n_o m_o + n_e m_e$

It is assumed that $n_e = n_i$, and $T_e \sim T_i \sim T_o$

where subscript e = electron mass and density
 i = helium ion mass and density
 o = neutral helium mass and density

The table also contains estimates of the magnetic pressure $P_m = \frac{B^2}{2\mu_0}$ which were calculated from magnetic field distributions measured by Tam (1967). These values are for the maximum pressure exerted on the outer boundary of the current shells in Figures 4.18 and 4.19.

The comparison of P_{kinetic} with P_m provides a check for consistency in the determination of n_e and T_e . At the 5.18 cm. radial position, the gas pressure cannot exceed the magnetic pressure, since the shell is still accelerating. Since the velocity changes slowly this would mean $P_{\text{kin}} \leq P_m$.

It should be noted that this condition is not fulfilled close to the axis for the higher initial pressure (Table 5-ii). However, this agrees with an earlier observation that the current layer never reaches the axis in the 4,000 μHg filling pressure case (Section 4.6).

Table 5-ii

Absolute Temperature Measurements

Initial pressure	Off-axis position	Region investigated	$T_e \times 10^3 K$	$N_e \times 10^{17}/cm^3$	Densities		Pressures	
					$gm/cm^3 \times 10^{-7}$	Relative to initial density	$dyne/cm^2 \times 10^6$	
2,000 μHg	3.18 cm	HeI	37.7	1.49	9.96	2.28	1.55	1.54
		HeII	39.2	0.89	5.98	1.37	0.97	
	5.08 cm	HeI	33.4	0.52	3.46	0.79	0.48	1.09
		HeII	34.2	0.38	2.58	0.59	0.36	
4,000 μHg	3.18 cm	HeI	39.4	1.86	12.49	1.43	2.03	0.29
		HeII	40.6	1.71	11.46	1.31	1.91	
	5.08 cm	HeI	35.3	0.90	6.05	0.69	0.88	0.99
		HeII	35.7	0.90	6.01	0.69	0.88	

5.3 ELECTRON TEMPERATURE FROM THE SEMI-CORONAL MODEL

The temperature determinations reported in the previous section are based on the assumption that the plasma is in local thermal equilibrium. For this to occur, collisional excitation and de-excitation rates of the atomic energy levels must greatly exceed population rates determined by radiative processes. This may not occur for low-lying energy levels, in which case collisional excitation can be balanced by radiative decay. This is the coronal or Elwert steady-state model. In such a condition, the population of the ground states of HeI and HeII can exceed the value expected from thermal equilibrium calculations, as shown in Section 5.2. For this model, Griem (1964, pp. 273-4) shows that the electron temperature can be determined from the intensity ratio of the HeII 4686 Å and HeI 5876 Å spectral lines.

Table 5-iii is constructed from Griem's calculations and the intensity ratios in Table 5-i. The spread in electron densities calculated from HeI 3889 and HeI 5876 half-widths ranges from 2% to 30% and is included in the table below. The magnetic pressure calculations of Table 5-ii have again been included to compare with the estimated kinetic pressures. It is seen that $P_{kin} > P_m$ in almost all cases, so that the assumption $T_e = T(ions) = T(atoms)$ seems doubtful if this model is to be employed.

Table 5-iii
Griem's semi-coronal model

Initial pressure	Off-axis position	Region investigated	$T_e \times 10^3 K$	$N_e \times 10^{17}/cm^3$	% difference in n_e	Densities		Pressures	
						gm/cm ³ x 10 ⁻⁷	Relative to initial density	P_{kin}	P_m
2,000 μHg	3.18 cm	HeI	48.4	1.51	20	10.09	2.31	2.01	1.54
		HeII	50.0	0.91	31	6.06	1.38	1.25	
	5.08 cm	HeI	43.9	0.52	2	3.51	0.80	0.63	1.09
		HeII	45.7	0.39	5	2.62	0.60	0.49	
4,000 μHg	3.18 cm	HeI	50.3	1.90	21	12.73	1.45	2.64	0.29
		HeII	57.5	1.76	24	11.76	1.34	2.64	
	5.08 cm	HeI	45.5	0.92	23	6.13	0.70	1.15	0.99
		HeII	46.9	0.91	2	6.10	0.70	1.18	

Electron temperatures ranging from 35,000° K to 44,000° K are obtained when the L.T.E. model is applied to the observed HeI 5876 to HeII 4686 intensity ratios (Griem, 1964; Mewe, 1967). Although experimental error in determining line intensities may affect the actual value of T_e , the application of the same intensity ratios to the L.T.E. and coronal models should show the effect of departures from equilibrium. As stated in Section 5.2, a 20% error in T_e would be caused by a factor of 10 in the HeII ground state population.

Such an error is seen to occur on comparing the ranges of T_e :

$$44,000^\circ \text{ K} \leq T_e (\text{coronal}) \leq 50,000^\circ \text{ K}$$

$$35,000^\circ \text{ K} \leq T_e (\text{L.T.E.}) \leq 44,000^\circ \text{ K}$$

By assuming a lower starting temperature for the iteration procedure in Section 5.2, we are assured of a better estimate of the total HeI population (equation (3), Section 5.2). This, and the consistency criterion involving magnetic pressures, justifies the use of Table 5-ii for further investigations of the Z-pinch in Chapter 6.

5.4 THE EFFECT OF SELF ABSORPTION ON LINE PROFILES

For the same plasma conditions, a consistent difference of about 20% appears between electron densities calculated alternatively from HeI 3889 and HeI 5876 line profiles (Tables 5-i and 5-iii). The reason is that the plasma is not uniformly transparent to radiation from all wavelengths in the visible spectrum. For an absorbing, homogeneous, L.T.E. plasma, the ratio of observed line intensity $I(\lambda)$ to that emitted in the ideal (or optically thin) case, $E(\lambda)$, is

$$\frac{I(\lambda)}{E(\lambda)l} = \frac{1 - e^{-\tau(\lambda)}}{\tau(\lambda)} \quad \dots\dots(1)$$

where $\tau(\lambda)$ is the optical depth at wavelength

l is the geometrical depth of plasma along the line of sight

$E(\lambda)$ is the intensity per unit volume in the optically thin case

The magnitude of $\tau(\lambda)$ is greatest at the centre of the line profile. This causes a depression in the maximum intensity region of the profile, thus distorting its shape and causing observed half-widths to appear greater than their true value. An expression for the effect of self absorption on

the observed line half-width can be derived for Lorentzian profiles assuming that emission and absorption occur as in a black body cavity. The ratio of the observed half-width (δ) of a spectral line to its true half-width (ω) is found to be

$$\frac{\delta}{\omega} = \left\{ \frac{\tau_o}{\ln \left[\frac{2}{1 + \exp(-\tau_o)} \right]} - 1 \right\}^{\frac{1}{2}} \dots\dots(2)$$

where τ_o is the optical depth at the line centre o .

Self-absorption causes the observed total intensity (I_t) of the line to be smaller than its true value (E_t). For Lorentzian profiles, the relation is

$$\frac{I_t}{E_t} = \left(1 - \frac{\tau_o}{4} + \frac{3}{16}\tau_o^2 - \frac{5}{384}\tau_o^3 \dots\dots \right) \dots(3)$$

Derivations of equations (2) and (3) are provided in Appendix VI along with an expression for τ_o and a graph showing its influence on observed half-widths.

Table 5-iv displays values of τ_o calculated from observed line half-widths, particle densities and temperatures given in Table 5-i. The expected underestimate of total line radiation is calculated from equation (3) for the HeI 5876 and HeII 4686 lines. The table shows that observed intensities can be 20% less than their true value. However, a correction procedure must be made to the monochromator traces before the helium line profile can be constructed (Appendix V). This results in roughly a 20% overestimate of the intensity at each monitored wavelength in the profile.

Hence, the correction procedure would tend to compensate errors in observed intensities due to self-absorption. Therefore, an error in line intensities of the order of 10% can be expected.

The consistency of the correction procedure in Appendix V can be demonstrated by comparing the effects of line broadening due to self-absorption with the observed line half-widths. The spread in half-width due to optical thickness is worked out for each line. The percentage difference in broadening due to self-absorption between HeI 5876 and HeI 3889 is then shown. Since the electron densities (n_e) of Sections 5.2 and 5.3 were calculated using the half-widths of these lines, the effect of self-absorption should be reflected in the percentage difference of n_e . These values are recorded in Table 5-iv, for the absolute intensity measurements and the Griem semi-coronal model. The last three columns of the table show a discrepancy between calculated and "observed" percentage difference of at most 10%. This justifies the assumption of the 10% error in line intensities stated above.

It should be noted that self-absorption effects for the HeI 3889 line are negligible. Therefore, electron densities calculated from the half-width of these line profiles would be closest to their true value. These quantities were used whenever possible. Since the slope of the line profile in its half-height region varies slowly, a 20% error in evaluating its shape would cause an error much smaller than this in determining its observed half-width. However, a maximum error of 20% is assigned to the calculation of electron densities.

Table 5-iv Influence of self absorption on line intensities and half-widths

Initial pressure ($\mu\text{Hg.}$)	Off-axis observ'n point (cm.)	Region investigated	Helium line	Half of half-width $\Delta\lambda$ $\times 10^{-8}\text{cm.}$	$T_e \times 10^3$ ($^\circ\text{K}$)	τ_0	% error in total line intensity	Calculated spread in $\Delta\lambda$ for self absorption (%)	Calculated % diff'ce for self-absorption between He I lines	% diff'ce in N_e from observed He I half-widths	
										Absolute intensities Table 5-i	Semi coronal Table 5-iii
2,000	3.18	HeI	HeII 4686	1.325	37.66	0.745	16	21	25	15	20
			HeI 5876	2.711		0.933	19	26.5			
			HeI 3889	1.553		0.047		1			
		HeII	HeII 4686	1.325	39.23	0.789	16	22	31	20	31
			HeI 5876	1.624		1.007	20	31			
			HeI 3889	2.135		0.022		< 1			
	5.08	HeI	HeII 4686	0.463	33.40	0.120	3	3	7	3	2
			HeI 5876	0.976		0.627	14	17.5			
			HeI 3889	0.650		0.028		< 1			
		HeII	HeII 4686	0.463	34.21	0.131	3	3.5	16	3	5
			HeI 5876	0.680		0.57	12	16			
			HeI 3889	0.484		0.024		< 1			
4,000	3.18	HeI	HeII 4686	2.45	39.44	0.956	18	27	36	12	20
			HeI 5876	2.961		1.237	24	36.5			
			HeI 3889	2.33		0.047		1			
		HeII	HeII 4686	2.9	40.55	1.074	21	31	17	18	24
			HeI 5876	2.536		0.612	13	17			
			HeI 3889	2.135		0.021		< 1			
	5.08	HeI	HeII 4686	1.025	35.32	0.228	5	6	28	23	23
			HeI 5876	1.478		1.022	20	29			
			HeI 3889	1.252		0.044		1			
		HeII	HeII 4686	1.075	35.68	0.251	6	7	19	13	2
			HeI 5876	1.628		0.688	15	19			
			HeI 3889	1.133		0.029		< 1			

The relatively small values of τ_0 indicate an average error of 20% in the observed intensities and calculated n_e due to the neglect of self absorption effects. This is of the order of the estimated experimental error in constructing the line profiles. The values in Tables 5-i and 5-ii must therefore be regarded as estimates only. However, as demonstrated in Section 5.2, they are adequate in determining electron temperatures and yield kinetic pressures consistent with Tam's magnetic probe results.

It should be noted that self absorption can be quite significant for higher particle densities occurring near the inner boundary of the collapsing shell. Indeed, Böttcher (1964) has found that the optical depth in this zone can be so great as to render the method of Section 5.2 useless. However, knowing the extent of the current layer and the much lower particle densities in its luminous regions, one can construct a model to give estimates of the density at the shock front. This will be done in Chapter 6.

Extreme cases of self absorption are noticeable in the axial radiation spectra of Figure 3.3. Calculations (Appendix VIIc) show that for expected plasma values of $n_i \approx 10^{18} \text{ cm}^{-3}$, $T_e > 40,000^\circ \text{ K}$, the optical depth of the plasma is of the order $\tau(\lambda = 4640 \text{ Å}) \geq 4$. For this reason, and because of possible non-homogeneity in the heating of the axial plasma column along its length, the monochromator voltages in Figures 3.6 to 3.9 (Section 3.4) have not been used to calculate electron densities or temperatures.

5.5 RELAXATION TIMES

Two methods of measuring the electron temperature have been used in this thesis. The first method, using absolute intensity measurements, assumes that excited states of the atom are brought into local thermal equilibrium (L.T.E.) with the free electrons. The free electrons have a Maxwellian velocity distribution characterized by a temperature T_e . In the second method, where Griem's (1964) calculations for the semi-coronal model are employed, it is assumed that the upper excited states are in L.T.E. with the free electrons. However, the relative populations of HeI and HeII are controlled by a balance between collisional ionization and radiative recombination. It is essential to establish whether population densities can relax to their steady state values much faster than a time characterizing changes in macroscopic plasma properties. In most cases, the pinch time ($\sim 10 \mu\text{sec.}$) determines the rate of change in the plasma temperature and is therefore used as a standard of comparison for relaxation times.

The observed plasma conditions can be represented by:

$$n_e = 10^{17}/\text{cm}^3, \quad T_e = 40,000^\circ \text{K} = 3.4 \text{ e.v.}, \quad \frac{n_{\text{He}^+}}{n_{\text{He}^0}} \sim 10^3 \dots\dots(1)$$

These are averages of values found in Tables 5-i to iii.

Let us first consider electron-electron and electron-particle (HeI, HeII) relaxation times. Spitzer (1956) gives the electron-electron relaxation time as

$$\tau_{ee} \approx \frac{0.266 T_e^{3/2}}{n_e \ln \Lambda} \dots\dots(2)$$

where T_e is in $^\circ\text{K}$ and $\ln \Lambda$ is a slowly varying function of n_e and T_e , usually of the order of 10.

From statement (1), it is seen that

$$\tau_{ee} \approx 5 \times 10^{-12} \text{ sec.} \quad \dots\dots(3)$$

and therefore, the electron velocity distribution is Maxwellian.

Griem (1964; p. 155, equ'ns 6--69a,b) derives electron-particle relaxation times in a homogeneous transient plasma. For a single ionization and $v \approx 10^8 \cdot Z \cdot \left[\frac{kT}{Z^2 E_H} \right]^{\frac{1}{2}}$ cm/sec., the electron-neutral equilibration time

is:

$$\tau_{K^0} \approx \left[3 \times 10^{-7} \frac{E_H}{kT} \frac{3/2}{N_e} \right]^{-1} \frac{N_a}{N_a^1} \frac{M}{m} \text{ sec.} \quad \dots\dots(4)$$

and the electron-ion relaxation time is:

$$\tau_{K^Z} \approx \left[3 \times 10^{-7} \frac{Z^2 E_H}{kT} \frac{3/2}{N_e} \right]^{-1} Z \frac{M}{m} \text{ sec.} \quad \dots\dots(5)$$

where,

v = velocity of the electrons
 $Z = 0$ for neutrals, $Z = 1, 2, \dots$ for ions
 M = atom or ion mass
 m = electron mass
 N_a^Z = ion density
 N_a = total density

Griem gives values of: $\tau_{K^0} = 10^{-7}$ sec. (electron-neutral)

$\tau_{K^1} = 10^{-8}$ sec. (electron-ion)

for a hydrogen plasma of $N_e \approx 10^{16}/\text{cm}^3$, $kT = 1$ e.v., and 10% ionization.

Substituting values of statement (1) for a helium plasma in equations (4) and (5) gives:

$$\tau_{K^0} \approx 2.5 \times 10^{-8} \text{ sec. (electron-neutral) } \dots\dots(6a)$$

$$\tau_{K^1} \approx 2.5 \times 10^{-8} \text{ sec. (electron-ion) } \dots\dots(6b)$$

Statements (3) and (6a,b) show the electron-electron and electron-particle relaxation times are all less than 10^{-7} seconds. Hence, it can be assumed that the velocity distributions of electrons, HeI and HeII particles are all controlled by the identical temperature T_e .

However, for L.T.E. to hold, the relaxation time is determined by the rate of collisional de-excitation of the first excited state. For hydrogenic systems this can be expressed as (Griem, 1964; p. 153, equ'n 6-65):

$$\tau_1^{Z-1,a} \approx \frac{1.1 \times 10^7 Z^3}{f_{21} N_e} \frac{N_a^Z}{(N_a^Z + N_a^{Z-1})} \frac{E_2^{Z-1,a}}{Z^2 E_H} \left(\frac{kT}{Z^2 E_H} \right)^{\frac{1}{2}} \exp \left[\frac{E_2^{Z-1,a}}{kT} \right] \text{sec.} \quad \dots\dots(7)$$

where, $E_2^{Z-1,a}$ = excitation energy of the resonance line for HeII (40e.v.)

$\frac{N_a^Z}{N_a^Z + N_a^{Z-1}}$ = accounts for the fact that not all ground-state atoms or ions need be excited or ionized

E_H = ionization energy of hydrogen = 13.6 e.v.

f_{21} = absorption oscillator strength from ground to resonance state ≈ 1 .

According to Griem, equation (2) yields a value of the collisional excitation time for ionized helium:

$$\tau_1^{1,2} \approx 0.3 \mu\text{sec.}$$

where his helium plasma has $T = 4$ e.v., $n_e = 10^{18}/\text{cm}^3$

Using the average values in statement (1) yields a relaxation time

$$\tau_1^{1,2} \approx 9.0 \mu\text{sec.} \quad \dots\dots(8)$$

This is comparable to the pinch time of $10 \mu\text{sec.}$

The relaxation times for the higher excited levels of the HeII ions are much less than the pinch time. For helium plasma conditions of statement (1) and principal quantum number m , an excited state of the HeII ions

($m > 1$) has the following equilibration time (Griem, 1964; p. 154, equ'n 6-67):

$$\tau_m^{Z-1} \approx \frac{8.9 \times 10^{-10}}{m^4} \times 10^{\frac{14.1}{m^3}} \dots\dots(9)$$

so that, for $m = 2$, $\tau_2^{Z-1} \approx 3.2 \times 10^{-3} \mu\text{sec.}$ \dots\dots(10)

Thus, the relaxation times for states of $m > 1$ are so short that partial L.T.E. between excited states is established almost instantaneously.

Although the relaxation time for the lowest state of HeII is comparable to the pinch time, its population is probably different by less than a factor of 10 from the value assumed in L.T.E. Since the electron temperature depends logarithmically on the population density, the maximum error in T_e would still be only 15% (Section 5.2, equation (2)).

For the coronal model, McWhirter (1965) gives a relaxation time of $\frac{10^{12}}{n_e}$ seconds. Hence, for the electron densities observed ($n_e \approx 10^{17} \text{ cm}^{-3}$), the relaxation time is $10 \mu\text{sec.}$, which is again comparable to the pinch time. Temperatures calculated from the coronal model still depend logarithmically on the intensity ratios of the HeII 4686 and HeI 5876 lines. As Griem's results show, at $40,000^\circ \text{ K}$, an error of a factor of ten in the intensity ratio causes an error in T_e of roughly 14%.

It is evident from the above discussion that a true estimate of T_e can only be obtained by solving in detail the rate equations for the excited levels in the two atomic species (HeI and HeII). Nevertheless, the fact that T_e depends logarithmically on the population densities lends confidence to the estimate of T_e as $40,000^\circ \text{ K} \pm 15\%$. Since the temperatures deduced from the L.T.E. model and the semi-coronal model are practically the same

when relative intensity ratios are used (Section 5.3), no conclusions can be drawn regarding which model best describes the plasma in the Z-pinch. This question cannot be resolved by comparing relaxation times, which turn out to be almost identical.

6.0 DYNAMICS OF THE PINCH

The motion of the collapsing luminous shell has been analyzed and related to that of the current shell. Time and spatially resolved measurements of temperature and density within the luminous plasma have been conducted at successive stages of the collapse. A synthesis of this information can now be made in an effort to understand the dynamics of the Z-pinch for high initial pressure discharges (2-4 Torr).

Framing camera pictures show that a non-luminous front precedes the luminosity shell. Its existence is disclosed by the occurrence of axial continuum before the luminous shell can reach the axis of the discharge. An attempt is made to determine the density of material in this leading (shock) edge of the collapsing current shell. It is difficult to measure this density directly because of the faintly luminous and optically thick region in which it occurs. However, a simple model can be constructed to relate it to densities observable in the luminous region of the current shell (Section 6.1). From evaluations of this density, it is found that shock heating occurs in the non-luminous region, but that very little kinetic energy is used in ionizing the gas as it enters the collapsing current shell. These findings are confirmed by a more rigorous theory which takes the cylindrical geometry of the discharge and ionization processes into account (Section 6.2).

The pressure at the shock front can be calculated from particle velocities observed in the luminous region behind it. A simple model is used of a shock produced by the uniform motion of a piston through a gas at rest. This yields a gas pressure in the shock front which approximately equals the spectroscopically determined pressure further back in the collapsing shell. Both are

in turn, consistent with the driving magnetic pressure, so that the inward moving shell is a dynamically stable structure.

6.1 DENSITIES WITHIN THE SHOCK LAYER

The determination of densities in the shock layer can elucidate the processes responsible for heating the gas as it is trapped by the collapsing current shell. This can be demonstrated by considering the following simple model.

Since the current peak converges towards the discharge axis at supersonic speeds, it will drive a shock wave ahead of it (Figure 6.1). Also, as the radius versus time curves of Sections 4.5 and 4.6 show, the collapse velocity can be regarded as approximately constant. Provided the shock front is far enough from the discharge axis for curvature effects to be negligible, one would expect the condition of the shock heated gas to be fairly well described by the standard Rankine-Hugoniot theory (Thompson, 1962). In a frame of reference at rest with respect to the shock front the following conservation equations hold:

$$\text{mass:} \quad \rho_1 u_1 = \rho_2 u_2 \quad \dots\dots(1)$$

$$\text{momentum:} \quad p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \quad \dots\dots(2)$$

$$\text{energy:} \quad \frac{1}{2} u_1^2 + \frac{p_1}{(\gamma_1 - 1) \rho_1} = \frac{1}{2} u_2^2 + \frac{p_2}{(\gamma_2 - 1) \rho_2} + W \quad \dots(3)$$

with p , ρ and u representing the gas pressure, density and velocity respectively. Subscript 1 refers to the cold gas ahead of the shock front and 2 denotes the gas behind it (Figure 6.1). W is the energy used up per unit mass in ionizing the cold gas as it enters the shock front. The parameter γ is the ratio of enthalpy to internal energy and is represented by $\gamma = 5/3$

for a monatomic gas in which no ionization process occurs (Lunkin, 1959; Ahlborn, 1967).

Figure 6.1 Model of shock formation

If W is zero (i.e., no kinetic energy is used up in ionizing the particles) then it can be easily shown that the density behind the shock front (ρ_2) is four times that of the cold gas entering the shock (ρ_1):

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{g_2 + 1}{g_2 - 1} = \frac{\gamma + 1}{\gamma - 1} = \frac{5/3 + 1}{5/3 - 1} \approx 4$$

If the kinetic energy of cold gas entering the shock front is partially used up in ionizing it, then u_2 will be reduced below the value observed when $W = 0$ in equation (3). It follows from equation (1) that ρ_2 will increase correspondingly. Böttcher (1963) has shown that for ionization of helium by shock heating

$$\frac{\rho_2}{\rho_1} \approx 10$$

For helium ($T \approx 30,000^\circ \text{ K}$ and $p \approx 1$ atmospheres), g_2 is about 1.2 (Ahlborn, 1967). Hence, a measurement of the particle densities behind the shock front can provide decisive evidence on whether the incoming gas is ionized by shock heating.

Densities have been measured within the "luminosity" shell at 5.03 cm. and 3.18 cm. off-axis for the 2,000 and 4,000 μHg cases. As stated in

Section 4.6, there is a definite separation of the collapsing current shell into shock and luminosity regions. Since the cold shock front does not register on framing camera film before it hits the axis, Tam's collapse curves must be used to fix its approximate position (Figures 4.18 and 4.19). Furthermore, Tam's work can provide an estimate for the overall average density of the total material within the collapsing shell.

Since the densities shown in Table 5-ii are local, and not average, it is necessary to assume an overall structure for the collapsing shell before information can be extracted concerning the shock layer density.

This structure is outlined in Figure 6.2. The outer boundaries of the current shell have been established by current density measurements (Tam, 1967). It has been shown that at least 90% of the gas is swept up into the current shell that passes through it and resides in the region bounded by the positions at which the current density falls to half its maximum value.

Figure 6.2 Model of the collapsing current shell

Let:

R_o = radius of vessel wall

r_{im} = inner }
 r_{om} = outer } radii of Tam's "magnetic probe" shell

r_{il} = inner radius of the luminosity shell

r_{p1} = radius of the "HeI peak"

r_{p2} = radius of the "HeII peak"

Particle densities at the HeI (ρ_1) and HeII (ρ_2) peak radiation positions have been determined spectroscopically. It is assumed that the particle density (ρ_x) in the region between the inner edges of the current and luminous shells varies slowly. Since relatively little light and current is detectable in this zone, ρ_x represents, at worst, an average value if it is treated as a constant.

The evaluation of the particle density (ρ_x) in the non-luminous region is carried out in the following manner. The overall average density of the current shell (ρ_{TM}) is determined first. Then observed local densities ρ_1 and ρ_2 are used to readjust the density distribution within the shell according to Figure 6.2.

i) If ρ_{TM} = the average density within the total "magnetic shell" as measured from Tam's collapse curves (relative to the initial density ρ_o) then,

$$\int_{r_{im}}^{r_{om}} \rho_{TM} r dr = \alpha \int_{r_{im}}^{R_o} r dr$$

so that,

$$\rho_{TM} = \alpha \left[\frac{R_o^2 - r_{im}^2}{r_{om}^2 - r_{im}^2} \right] \dots\dots(4)$$

where α takes into account the fact that not all particles are swept up by the collapsing current layer. Tam (1967) shows $\alpha \approx 0.9$ for 2,000 and 4,000 μHg .

ii) Integrating over the various layers of the collapsing shell gives:

$$\int_{r_{im}}^{r_{om}} \rho_{TM} r dr = \int_{r_{im}}^{r_{il}} \rho_x r dr + \int_{r_{il}}^{r_{p1}} \rho_I r dr + \int_{r_{p1}}^{r_{p2}} \rho_{II} r dr + \int_{r_{p2}}^{r_{om}} \rho_{III} r dr \dots\dots(5)$$

where:

- ρ_x = the unknown density of the shock layer (relative to the initial density ρ_0); it is assumed a constant and therefore, an average quantity;
- ρ_I = the density of the shell bounded by the inner radius of the luminosity layer and the radius of the "HeI peak", (relative to ρ_0);
- ρ_{II} = the density of the shell bounded by the "HeI peak" and the "HeII peak", (relative to ρ_0);
- ρ_{III} = the density of the shell bounded by the "HeII peak" and the outside radius of Tam's "magnetic probe" shell, (relative to ρ_0).

For the three areas with relative density ρ_s , $s = I, II, III$, a linear interpolation is used between known densities, so that:

$$\rho_s = K_s r + b_s \dots\dots(6a)$$

where, $K_s = \frac{\rho_{\text{inner boundary}} - \rho_{\text{outer boundary}}}{r_{\text{inner boundary}} - r_{\text{outer boundary}}} \dots\dots(6b)$

$$b_s = \frac{\rho_{\text{inner}} r_{\text{outer}} - \rho_{\text{outer}} r_{\text{inner}}}{r_{\text{inner boundary}} - r_{\text{outer boundary}}} \dots\dots(6c)$$

- At $r_{il} \rightarrow$ the density is assumed still close to that of ρ_x ; i.e., that of the shock layer.
- At $r_{p1} \rightarrow$ the density is ρ_1 , the relative density measured at the "HeI peak"; from Table 5-ii.
- At $r_{p2} \rightarrow$ the density is ρ_2 , the relative density measured at the "HeII peak"; from Table 5-ii.
- At $r_{om} \rightarrow$ the density is $\rho_{2/2}$.

Inserting equations (6a,b,c) into equation (5) and integrating gives:

$$\begin{aligned} \rho_{TM} (r_{om}^2 - r_{im}^2) = & \rho_x (r_{il}^2 - r_{im}^2) + b_I (r_{p1}^2 - r_{il}^2) + \frac{2}{3} K_I (r_{p1}^3 - r_{il}^3) \\ & + b_{II} (r_{p2}^2 - r_{p1}^2) + \frac{2}{3} K_{II} (r_{p2}^3 - r_{p1}^3) \\ & + b_{III} (r_{om}^2 - r_{p2}^2) + \frac{2}{3} K_{III} (r_{om}^3 - r_{p2}^3) \end{aligned}$$

where b_I and K_I are functions of ρ_x . \dots\dots(7)

This is solved for ρ_x , which is tabulated below for 2,000 and 4,000 uHg initial pressures when the HeII peak passes the 5.08 and 3.18 cm. observation points.

Table 6-i Relative densities for shock fronts

Initial pressure	From Figs. 4.18 4.19		From Figs. 4.9, 4.10 and 4.15, 4.16			Densities (relative) Table 5-ii		Calculated shock average density (relative) ρ_x
	r_{im}	r_{om}	r_{il}	r_{p1}	r_{p2}	ρ_1	ρ_2	
2,000 uHg	0	4.03	2.5	2.93	3.18	2.28	1.37	5.51
	3.5	5.75	4.0	4.78	5.08	0.79	0.59	4.73
4,000 uHg	0	4.6	2.4	2.75	3.18	1.43	1.31	5.26
	3.0	5.6	4.5	4.83	5.1	0.69	0.69	3.76

$R_0 = 7.62$ cm.

↑ off-axis observation point

The results in Table 6-i show that the relative density across the shock front is quite close to 4. This is the value expected for plane shock fronts in which shock ionization is unimportant. It should, however, be kept in mind that this calculated density depends on the model used. A magnitude of approximately 5 for the 3.18 cm. radius is not surprising when one considers the effect of a cylindrical geometry.

In the luminous regions, the mass density can be less than that in the cold gas (ρ_1 or $\rho_2 \approx 0.7 \rho_0$). This result indicates that gas passing through the shock heated region into the luminous shell is heated by ohmic dissipation of the discharge current. This ohmic heating expands the gas and hence, reduces its density.

The main conclusion to be drawn from a study of the mass distribution in the collapsing shell is that the shell consists of two regions. Within the inner region the gas is shock heated, but very little kinetic energy of the cold gas is used in ionizing it. In the outer, luminous region the density is lower than that of the cold gas and indicates that Joule heating predominates here.

6.2 EVIDENCE OF GUDERLEY FLOW

The conclusions of the previous section can be confirmed by an application of the Guderley theory for shock formation in a compressible gas (Guderley, 1942). The influence of cylindrical geometry and ionization processes is manifested in the time behaviour of observable fronts. Limitations of this theory render it useful only in the analysis of higher initial pressure discharges (2,000 μHg and above). Using Jukes's (1958) theoretical development and Böttcher's (1963) notation, the Guderley flow theory can be summarized as follows.

Assume the discharge consists of a cylinder of ionized gas (Figure 6.3) in which, i) electrical conductivity is infinite;

- ii) the current rises instantaneously from zero to a finite value on a bounding cylindrical sheet (the subsequent time variation is to be found);
- iii) the thickness of a shock wave is very small compared with the dimensions of the discharge.

Figure 6.3 Discharge column and associated circuit

Consider the equations of the gas motion between the current sheet and the ingoing, cylindrical shockwave preceding it. From Courant (1948), these equations with cylindrical symmetry are:

$$\text{continuity:} \quad \frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial r} + \frac{\rho}{r} \frac{\partial (rv)}{\partial r} = 0 \quad \dots\dots(1)$$

$$\text{momentum:} \quad \rho \left(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} \right) = - \frac{\partial p}{\partial r} \quad \dots\dots(2)$$

and the equation of particle-isentropy:

$$\left(\frac{\partial}{\partial t} + v \frac{\partial}{\partial r} \right) (\rho \rho^{-\chi}) = 0 \quad \dots\dots(3)$$

where $\chi = \gamma = 5/3$, if the gas ionization processes do not occur. Böttcher introduces χ , to account for the possibility that shock ionization can occur.

$v(r,t)$ is the radial velocity of the gas;
 p , the pressure;
 ρ , the density.

One solution of these partial, differential equations is obtained by assuming a similarity, or "progressing wave" solution of the form:

$$\begin{aligned} v &= \frac{nr}{t} V(\xi) \\ p &= \left(\frac{nr}{t} \right)^2 P(\xi) \quad \rho = \Omega(\xi) \quad \dots\dots(4) \end{aligned}$$

where ξ is a parameter defined as:

$$\xi = \frac{r}{(-t)^n} \quad \dots\dots(5)$$

in which n is an eigenvalue (to be found), and the observed times $(-t)$ are represented with the pinch time (t_p) as origin $(t_p = 0)$.

The $v \frac{t}{r}$, $p \left(\frac{t}{r} \right)^2$, ρ appear constant to an observer moving on a trajectory of constant ξ . The velocity of the observer is $\frac{nr}{t}$. If the flow contains a moving shock front, ξ must remain constant on the trajectory of the front.

The relationship between the ratio of specific heats (denoted by g), χ and n is quite complicated, but can be summarized below:

Table 6-ii

Numerical relations for Guderley flow parameters

g	χ	n	source
1.667	1.667	0.817	Jukes
1.4	1.4	0.834	Guderley
1.31	1.235	0.8494	Bötticher for helium*
1.195	1.285	0.8568	
1.195	1.25	0.8588	

Jukes develops a criterion for the behaviour of the current. If the total current flowing on the cylinder is I , when its radius is r_c , the pressure at the cylinder wall p_c is just the magnetic pressure outside,

$$p_c = \frac{B_e^2}{2\mu_0} = \frac{\mu_0 I^2}{8\pi r_c^2} \dots\dots\dots(6)$$

In a Guderley flow p_c , r_c , and therefore, $I(t)$ are known as functions of time. From Figure 5.1, the applied field E , causing the flux change satisfies:

$$E = \frac{d}{dt} (L_d \times I) = \frac{d}{dt} \left(2 \log \frac{R_o}{r_c} \times I \right) \dots\dots\dots(7)$$

From equations (4) and (5), Jukes (1958) shows that for a Guderley flow:

I must decay almost linearly with time up to the first pinch, $L_d \times I$ increases almost linearly from zero.

This condition is satisfied by a constant applied field.

* Böttcher's (1961) experimental conditions are as follows (compare with Table 2-i).

Discharge vessel: 18 cm. diameter by 100 cm. long
Filling pressure: 6 Torr Helium
Condenser bank: 100 kJoule; charging voltage 35 kv,
initial inductance 4.5 mhenries
Maximum current: 500 kAmp.
Pinch time ~ 5 μ sec, followed by additional contrac
at 10 and 15 μ sec.

From the foregoing, it can be seen that a necessary, but not sufficient, condition for Guderley flow is that the current $I(t)$ decays almost linearly while the pinch is being formed. From the luminosity graphs, this occurs only for the 2,000 and 4,000 μHg cases (Figures 4.14 and 4.15). For the 500 and 1,000 μHg graphs (Figures 4.12 and 4.13), the discharge current increases while the pinch is being formed. This explains why, when $(\log r)$ vs. $(\log t)$ values were plotted for 500 and 1,000 μHg , slopes were obtained which did not fall in the range, $8.1 < n < 8.6$, predicted by the Guderley theory.

For 2,000 and 4,000 μHg initial pressures, according to equation (5), the slope of the line $(\log r)$ vs. $(\log t)$ should give n , the eigenvalue for the similarity solution. By using Table 6-ii connecting n and g , an estimate can be obtained of the "specific heat ratio", g . This procedure was tried for Tam's inner radii (Figures 4.18, 4.19). His values give an n which falls outside the range predicted by theory. This is not too surprising, since he used the half-height of his current peak as the radius for the inner boundary of the collapsing shell. The real shock front is probably a little further ahead than he predicts. From Sections 4.6 and 6.1 though, it seems reasonable that most of the mass within the shock layer is found behind this boundary.

The $(\log r)$ vs. $(\log t)$ graphs for the inner radius of the luminosity shell (r_{il}) are given in Figure 6.5. A "HeII peak" graph for the 2,000 μHg case is provided in Figure 6.4. The "HeI peak" front yields a graph with a similar slope and is therefore not included. Graphs for the HeI and HeII peak radiation curves in the 4,000 μHg case yield an n outside the range predicted by theory. This can be explained by noting that the motion of

the HeI and HeII peak radiation regions was assumed to be governed by Guderley flow. However, as will be demonstrated in the following section, these highly luminous shells act as a current piston which is brought to rest, i.e., the luminous region does not form an intrinsic part of the Guderley flow. The Guderley flow itself continues onward towards the discharge axis, giving rise to the current peak and axial continuum preceding the luminous shell of Figure 4.19. Essentially, in this case, the shock front is driven to the axis by its own momentum, leaving the current piston of the Jukes model behind.

According to Figure 6.5, a slope of $n = 0.83$ for the inner boundary of the luminosity shell indicates that:

$$1.4 < g \leq 1.667 \quad \text{.....(8)}$$

which would be characteristic for relatively little to no shock ionization.

The slope for the "HeII peak" in Figure 6.4 for the 2,000 μHg case gives $n = 0.86$, so that:

$$1.2 < g < 1.3 \quad \text{.....(9)}$$

which is the region Böttcher (1963) claims for ionization processes to occur in a helium plasma.

Although the accuracy of the graphs is not precise, they at least show a trend in g . Statement (8) shows a range of g characteristic of a relatively cold gas. This is in the region of the luminosity front. The range of g in statement (9) indicates higher temperatures occurring deeper within the luminosity shell ($g \approx 1.2$ for helium at 30,000° K and 1 atmosphere pressure). This agrees with the findings of Sections 5.3 and 6.1.

The inner edge of the luminosity shell at 4,000 μHg conforms to Guderley flow. This corresponds to no shock ionization. Since the current peak region does not exhibit Guderley flow in this case, the boundary between the current "piston" and shock heated gas is somewhere close to the edge of the luminous shell.

The occurrence of Guderley flow in the 2,000 μHg case is valuable from another point of view. This lies in the current decay criterion, and the constant applied voltage E required by equation (7).

For initial pressures $< 2,000 \mu\text{Hg}$, the current is still rising as the pinch is formed. Equations (3) and (6) are violated as the current "diffuses" into the collapsing shell and excites helium throughout the whole shell by direct electron-atom collisions.

Figure 6.4

2,000 μ , "HeII peak" vs. time

Figure 6.5 Inner luminosity radius r_{il} vs. time

6.3 THE PISTON MODEL OF SHOCK FORMATION

In the previous section, qualitative agreement was found to exist with the Jukes-Allen theory of Guderley flow at the inner edge of the luminous shell for the 2,000 and 4,000 μHg filling pressure cases. An implicit assumption of this theory is that the current-bearing layer can be regarded as distinct from the shock region upon which it acts as a piston.

From the logarithmic "radius versus time" graphs of Figure 6.5, it would appear that the value of parameter g_2 in the shock front is close to the specific heat ratio of a perfect monatomic gas: $\gamma = 5/3$. This is corroborated in Section 6.1, where it was found that

$$\frac{p_2}{p_1} = \frac{\gamma + 1}{\gamma - 1} \approx 4$$

Such a result is expected for a plane, non-ionizing shock propagating through an ideal gas. The cylindrical geometry of the collapsing current shell can therefore be relaxed to considerations of plane fronts at the 5.08 and 3.18 cm. off-axis observation points.

The pressure behind a plane shock front can be calculated from particle velocities observed in its wake. This serves as an independent check on spectroscopic and magnetic field measurements. If these values are comparable, then the conclusions arrived at in the previous two sections with regard to shock flow are confirmed as well.

Consider a semi-infinite cylindrical pipe, filled with gas, and terminated at one end by a piston (Figure 6.6). At an initial instant of time, the piston begins to move into the pipe with constant velocity U . The gas adjacent to the piston must move with the same velocity, while the

Figure 6.6 Piston driven shock front

gas farther ahead is compressed and accelerated. A shock wave is formed and moves along the pipe. At first, the shock and piston are close together. Since the shock velocity is slightly larger than U , the shock front subsequently separates from the piston and a region of gas lies between them (region 2). In front of the shock wave (region 1) the gas pressure is equal to its initial value p_1 and its velocity relative to the pipe is zero. In region (2), the gas moves with constant velocity, equal to the velocity U of the piston. The gas pressure P_2 between the piston and the shock wave can be expressed in the following manner (Landau and Lifshitz, 1959, p. 358):

$$\frac{P_2}{P_1} = 1 + \frac{(\gamma + 1) U^2}{4 c_1^2} + \frac{\gamma U}{c_1} \left[1 + \frac{(\gamma + 1)^2 U^2}{16 c_1^2} \right]^{\frac{1}{2}}$$

where $\gamma = 5/3$, and $c_1^2 = \gamma (p_1/\rho_1)$

These equations can be applied to our experiment. P_1 and ρ_1 are the initial filling pressure and density of the gas in the discharge tube. The velocity U of the gas in front of the piston is assumed to be equal to the velocity of the HeI radiation peak (Figures 4.9 and 4.10). With these assumptions, pressures P_2 can be calculated from the above equation and

compared with the pressures available from spectroscopic and magnetic probe data (Table 5-ii). The spectroscopic pressures (P_s) are determined from values of n_e and T_e at the HeI radiation peak. The magnetic pressure (P_m) is calculated from the maximum magnetic field which occurs at the outer edge of the current shell (Tam, 1967). All three measurements are independent and refer to different positions within the shell (see Figure 6.6).

Table 6-iii

Comparison of pressures determined from velocity, spectroscopic and magnetic field measurements

Initial pressure ($\mu\text{Hg.}$)	Observation point off-axis (cm.)	Velocity of HeI peak cm/usec	P_v velocity determined pressure	P_s spectroscopically determined pressure	P_m magnetic field determined pressure
2,000	3.18	1.42	1.18	1.55	1.54
	5.08	0.93	0.51	0.48	1.09
4,000	3.18	0.95	1.06	2.03	0.29
	5.08	0.80	0.75	0.88	0.99

Pressures in units of $\text{dyne/cm}^2 \times 10^{16}$

In the light of the approximations and experimental errors involved, the agreement between velocity and spectroscopically determined pressures at 5.08 cm. off-axis is quite reasonable (Table 6-iii). This supports the form of density dependence across the shock front assumed in Section 6.1. The larger magnetic pressures indicate an acceleration of the shell towards the discharge axis. Closer to the axis, a disparity in these values is apparent. For the lower initial pressure case, the agreement between spectroscopic and magnetic pressures shows that the shell is moving uniformly. The velocity determined pressure is lower by 20%, but this may be due to curvature effects not taken into consideration by the model. The lack of

agreement close to the axis for the higher initial pressure case might be explained by increased Joule heating of the shock heated gas which would raise the pressure above the level predicted by ideal shock theory.

The qualitative agreement between spectroscopically and velocity determined pressures, particularly far off the discharge axis, confirms the validity of the model proposed in Section 6.1. Hence, the conclusion that $\frac{\rho_2}{\rho_1} = 4$ has been reinforced. This result contrasts with Böttcher's (1965) findings in which he concludes that $\frac{\rho_2}{\rho_1} \approx 10$.

The evidence in the last three sections points to the following explanation for plasma formation in the Z-pinch discharge. The discharge current rapidly energizes gas in its immediate vicinity by Joule heating. The predominant species in this region is ionic, so that the build up of thermal pressure is counteracted on the outside edge of the plasma by a large magnetic pressure. The helium ions are therefore confined and are only free to move inwards, towards the axis of the discharge vessel. The gradient between magnetic and thermal pressures in this highly excited region is responsible for the inward motion of the plasma. Thus, the current layer acts as an inward moving piston which sets up a shock wave in front of itself. This shock in turn heats up the quiescent gas through which it passes, but does not possess sufficient energy or time to ionize it.

It should be noted that the model proposed above holds only for higher initial density plasmas (2,000 μHg and above). In the previous section, it was found that the Guderley theory does not provide an adequate description of low initial pressure discharges (500 to 1,000 μHg). This failure

suggests that Joule heating pervades the whole plasma shell. Since no adequate theory has been devised to take this energy input into account, the question of shock ionization would remain unsolved. Also, the high incidence of impurities observed in such low pressure discharges would render spectroscopic results doubtful in the best of circumstances.

7.0 SUMMARY AND PROPOSALS FOR FUTURE WORK

A linear Z-pinch has been studied using a framing camera, time-resolved spectrograph, and monochromator. The results of these optical investigations have been correlated with magnetic probe work conducted by Tam (1967). The pre-pinch stage of the discharge in helium emits very little light. Framing camera and time-resolved spectrographic observations yield meaningful results only if many discharges can be superimposed on one film record. This required the development of a switching and trigger system accurate to within $\pm 0.2 \mu\text{sec}$.

The structure of the collapsing plasma shell differs markedly with initial filling pressures. At low initial pressures the luminosity and current shells overlap almost completely. Peak HeI and HeII intensities coincide at the position of maximum current density. Intense continuum radiation appears when the leading edge of the luminous shell reaches the discharge axis.

At high filling pressures, the HeII region lags behind the HeI zone. This separation was first observed on framing camera records taken through interference filters. Confirmation was obtained by quantitative spectroscopic measurements which show higher temperatures at larger radii. Peak HeII radiation still coincides with the position of maximum current density. However, the current shell extends inward beyond the leading edge of the HeI radiation zone. Strong continuum radiation occurs in the axial region as the current shell reaches the discharge axis, even though the luminous zone remains several centimeters off axis. The early appearance of axial

continuum and the extension of the current shell beyond the visible plasma region indicate a strong perturbation of the gas ahead of the luminous plasma shell.

Pressure and density determinations behind this discontinuity show that it can be interpreted as a non-luminous shock front. The pressure was calculated from a simple model in which the maximum current density region acts as a piston. The velocity of this piston is assumed to be that of the HeI radiation peak. The results compare well with two independent measurements:

- i) the magnetic pressure $B^2/2\mu_0$
- ii) the pressure in the luminous region obtained from spectroscopic temperature and density measurements.

The density in the shock front was found from a model of the density profile across the observed current shell. The width of the current shell and the total mass contained within it are known. By determining the local density at two points within the current shell, the density in the shock front can be calculated with the aid of the model. The resulting density ratio of 4 is characteristic of a non-ionizing shock.

Future work might be directed at detecting the non-luminous shock wave using schlieren and interferometer techniques. The nature of the intensely radiating pinch continuum can be investigated

- a) at sufficiently short wavelengths so that electron densities can be obtained from an optically thin approximation;
- b) at a long wavelength for which the optically thick approximation will yield electron temperature.

The faint axial spot appearing ahead of the pinch continuum (Figure 4.4) is a subject worth further inquiry. Finally, information of the discharge action would be more complete if reliable measurements of the voltage waveform can be conducted simultaneously with the discharge current of the Z-pinch.

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THE CONTROL CIRCUIT

The following account is an enlargement on the discussion in Section 2.1. A photomultiplier monitors the speed of the rotating mirror. Since the discharge must occur when the mirror rotates at a preset speed (i.e., the "tripping frequency"), the control system must contain a frequency gate. The frequency gate is opened only when the time interval between pulses of the photomultiplier (T) reaches a required value t_t (the "tripping" time interval). As the mirror speed is increased from zero, the interval between pulses decreases, finally reaching the value t_t . The frequency gate must then issue a signal that initiates the Z-pinch discharge. However, this signal must be delayed a time T_d to allow the mirror to swing into the proper recording position. A block diagram of the control circuit is given in Figure 1, while the pulse sequences formed by it are shown in Figure 2. All figure numbers relate to this appendix unless stated otherwise.

a) The Frequency Gate

The pulse sequence emanating from the trigger beam photomultiplier (Figure 2a) is fed into a pulse inverter and shaper (Figure 1). This unit inverts, amplifies, and improves the rise time of the incoming pulses (Figure 2b). Each of these pulses is fed in turn to a system composed of a Tektronix type 162 wave form generator and a type 163 pulse generator (Figure 1). These units combine to form a delayed pulse and generate the sequences of Figures 2c and 2d respectively. The trailing edge of each pulse in 2d appears exactly t_t microseconds (the "tripping" frequency time interval) after the leading edge of each pulse in the sequence 2b.

The undelayed pulse of 2b and the delayed pulse of 2d emerging from the pulse generator are then fed into the coincidence unit (Figure 1). The coincidence circuit sends out a very small pulse (about 5 volts) when either input pulse appears separately at one of the two input points (Figure 2e). This occurs when the time interval T between pulses is greater than t_t . As the mirror speed increases, the pulses in Figure 2a to 2e crowd closer together. When the "tripping" frequency is reached, $T = t_t$, the trailing edge of the pulse in Figure 2d coincides with the leading edge of the next pulse in Figure 2b. Both input pulses to the coincidence unit now occur simultaneously, giving an output of 25 volts characterized by the shape in Figure 2f.

The first pulse from the coincidence unit in the sequence 2f triggers a delay unit (Figure 1). As the mirror speed increases, the pulse in Figure 2b "moves forward" along the pulse in Figure 2d causing the coincidence circuit to produce a pulse sequence of the form in Figure 2g.

b) The Delay System

The delay system setting is chosen so that the first visible light from the discharge hits the top of the spectrograph slit (Figure 3.1, Section 3.1). This corresponds to the time T_d required by the mirror to swing from the triggering to recording position (positions A and B, Figure 2.1, Section 2.0). This system is made up of a second pair of Tektronix type 162 and 163 units. Their action is the same as that described above in the frequency gate portion of this section. The pulse sequence from the coincidence circuit in Figure 2f takes on the role of the sequence in Figure 2b. This causes an output from the waveform

generator of the type 2c which in turn causes the pulse generator to form a pulse of the type in Figure 2d. In 2d the time delay $T_d = x$, and for the rotating mirror it is of the order of 120 microseconds. No observable jitter is apparent (i.e., < 0.08 microseconds).

The output pulse from the delay unit fires a bistable multivibrator (Figure 1).

c) The Bistable Multivibrator

The bistable multivibrator functions as a single shot device. When manually reset, it is ready to accept pulses above a discrimination level greater than 5 volts. This is necessary since smaller pulses emanating from the coincidence circuit might be passed through the delay unit, causing a premature discharge. The arrival of a 25 volt pulse from the delay unit sets the unit into its other stable operating mode, simultaneously producing a positive pulse of about 20 volts to fire the 2D21 thyatron pulse circuit (Figure 2.4, Section 2.2b).

Subsequent pulses fed into the multivibrator do not cause pulse output to the 2D21 thyatron circuit until the multivibrator is manually reset. This "single shot" feature increases the life span of the thyatron pulse generator which would otherwise be required to fire every t_t microseconds as long as the mirror rotates above the tripping frequency.

d) Operating Procedure

The sequence of operations in using the rotating mirror is as follows. The single shot unit is set manually and the mirror speed is increased gradually from zero. When the correct speed is reached, a trigger pulse

produced by the control circuit fires the bank. The air supply to the turbine which drives the rotating mirror is cut off immediately the bank fires. This reduces the mirror speed again to zero. The above sequence is repeated to give the desired number of exposures on the film strip or spectrographic plate.

It should be noted that the mirror sweeps past the film again at a time $T_d + t_t$, since by then it has turned a half-revolution ($t_t = 250$ microseconds for framing camera, ≈ 1 millisecond for the spectrograph). No complications arise from this because radiation from the plasma dies away in a time of the order $T_d + 150$ microseconds.

Appendix I, Figure 1

Block diagram of control circuit
(Letters refer to pulse trains in Figure 2, below)

Appendix I, Figure 2

Pulse sequences used in triggering the discharge

APPENDIX II

ROGOWSKI COIL AND INTEGRATOR CIRCUIT

The total current (I) through the discharge is measured by a Rogowski coil. An 11.4 cm. length of RG65 A/U delay line, with its outer ground shield removed, is placed between the insulated flat high current leads which conduct current to the discharge tube (Figure 1, this appendix).

If the total current (I) is uniformly distributed over the flat current leads, the magnetic flux (B) through the coil(L) is proportional to (I). The voltage (v^1) induced on the coil (L_1) is proportional to dB/dt and therefore, dI/dt . The output signal of the coil is integrated by the passive integration circuit producing an output signal (v) proportional to I. A schematic diagram of the circuit is given in Figure 2 of this appendix.

The usual method of calibrating the current is to put

$$I = kv \quad \text{.....(1)}$$

$$\int Idt = k \int v dt = Q = cv \quad \text{.....(2)}$$

where v = the output voltage of the integrator circuit,
I = the discharge current,
Q = the charge on the condenser bank,
v = the charging voltage of the condenser bank,
c = the capacitance of the condenser bank.

Integrating the area under the curves in Figure 2.9 (Section 2.3), a value of k can be obtained by applying equ'n (2) above. Using this method, it was found that

$$k = .094 \times 10^6 \text{ amp/volt} \pm 10\% \quad \text{.....(3)}$$

Because of the relatively short time constant of the integration circuit used, equ'n (1) must be replaced by:

$$I = k'(v + \frac{1}{R_i C_i} \int v dt) \quad \text{.....(1a)}$$

so that equ'n (2) becomes:

$$Q = k' \left[\int v dt + \frac{1}{R_i C_i} \iint v dt dt' \right] \quad \text{.....(2a)}$$

where, as before: Q = the charge on the condenser bank
 v = output voltage of integrator circuit

but R_i, C_i = the resistance and capacitance of the integrating circuit

k' = calibration constant for the current (equ'n (1))
 now modified for the short integration time.

To check what effect the additional term of equ'n (1a), (i.e., $\int \frac{v dt}{RC}$), would contribute to the measured oscilloscope voltage, both "uncorrected" and "corrected" Rogowski traces were plotted for the initial pressure 4,000 μ Hg helium (Figure 3, this appendix).

From Figure 3, the "uncorrected" trace gives an underestimate of a "true" current reading that varies:

from 10% at 6 μ sec.
 to 35% at 10 μ sec.

Since the times of interest for the lower initial pressures (i.e., 500, 1,000 and 2,000 μ Hg helium) occur sooner than the 4,000 μ Hg case, the percentage error of the "uncorrected" traces with respect to a proper current calibration would be even less (see Figure 2.9, Section 2.3).

Using the calibration constant k of equ'n (3), the maximum discharge current for each initial pressure can be calculated, i.e.,

$$I_{\max} = k v_{\max}$$

where $v_{\max}$ = the maximum oscilloscope voltage in the first half-cycle of the discharge.

The ringing period of the discharge and the maximum discharge current in the first half-cycle for the range of initial pressures is as follows (from Figure 2.9):

Pressure	$V_{\max}$	$I_{\max}$	Ringing period
500 μHg	2.3 volts	216×10^3 amps	37×10^{-6} sec.
1,000 μHg	2.5 volts	235×10^3 amps	37×10^{-6} sec.
2,000 μHg	2.75 volts	260×10^3 amps	35×10^{-6} sec.
4,000 μHg	3.0 volts	282×10^3 amps	34×10^{-6} sec.

For two discharge systems which differ markedly only in the size of capacitance C of the charging condenser bank, it can be shown that:

$$\frac{I_1}{I_2} = \frac{T_1}{T_2} = \sqrt{\frac{C_1}{C_2}} \quad \dots\dots(4)$$

where I = discharge current
 T = ringing period of the discharge
 C = capacitance of the condenser bank
and the indices 1,2 designate the two systems.

From Tam's thesis: $C_2 = 53 \mu\text{F}$, $I_{2,\max} = 200 \times 10^3$ amps, $T_2 = 22 \mu\text{sec}$.
The value of the capacitance in the discharge used here is $C_1 = 106 \mu\text{F}$.
The substitution of these values in equ'n (4) yields:

$$I_{1,\max} = \sqrt{2} \times 200 \times 10^3 = 283 \times 10^3 \text{ amps.}$$

$$T_1 = \sqrt{2} \times 22 \times 10^{-6} = 31 \times 10^{-6} \text{ sec.}$$

These results compare favourably with the 4,000 μHg case in the table above. A similar trend of increasing maximum discharge current and decreasing ringing period over a range of initial pressures is observed in argon (Daughney 1966, p. 62). In that case, the integration constant was 8 times that of the ringing period of discharge.

Despite the relatively short time constant of the integration circuit, the recorded values of the oscilloscope voltage can still give a reasonable measure of the discharge current. From Figure 3, it can be seen that for

the time of interest (0 μ sec to 10 μ sec), the recorded voltage closely follows the "true" integrated trace. With the proper choice of calibration constant k (equ'n 3), the discharge current can be estimated at worst by a 20% error. All current traces in the text are derived from the "uncorrected" voltage readings.

The frequency x gain response of the integrator circuit was found to be flat up to 1 megacycle (Figure 4, this appendix). Since the discharge current frequency is approximately 20 kc/sec, the integration of input signals is reliable to the accuracy discussed above. However, the high frequency minima superimposed on the discharge trace are not smoothed out by the integrator. If these minima are sharply peaked, then their occurrence in time can be regarded as accurate within the bounds of experimental error (i.e., less than 0.3 μ sec.).

From the theory of the Z-pinch (Daughney 1966), the discharge current behaves as:

$$I(t) = \frac{V}{WL_t} \sin(Wt + at^3) \quad \text{.....(5)}$$

where, W = the ringing frequency
 a = a constant
 V = the applied external voltage
 L_t = the total inductance of the system.

For time $t' \ll \frac{2\pi}{W}$, $L_t \approx L_i$ = the total inductance of the system at the beginning of the discharge,

$$\text{so that } I(t') = \frac{V}{L_i} t' \quad \text{.....(6)}$$

By measuring the initial slope of the traces in Figure 2.13, an estimate of the initial total inductance can be made:

$$L_i = 0.09 \mu\text{Henries} \pm 10\%.$$

Appendix II, Figure 1

Placement of Rogowski coil

Appendix II, Figure 2

Circuit diagram of Rogowski coil and integrator

L — Rogowski coil

$L = 15 \mu\text{H}$

$C = 16.5 \times 10^{-12} \text{ f}$

} specific impedance = 950Ω

R_1 — "attenuating" impedance = 1500Ω

R_2 — cable terminating impedance = 47Ω

R_i — integrator resistor = 470Ω

c_i — integrator capacitance = $0.1 \times 10^{-6} \text{ f}$

} integrator time

constant = $47 \mu\text{sec.}$

v' — induced voltage $\propto dI/dt$

v — output voltage

Appendix II, Figure 3

Appendix II, Figure 4

Frequency response of integrator

ϕ phase shift $\rightarrow$ — Δ —
 $|A|$ gain $\rightarrow$ — $\circ$ —
 f frequency $\rightarrow$ — $\circ$ —

APPENDIX III

CORRELATION OF MONOCHROMATOR TRACES WITH TIME-RESOLVED SPECTROGRAPHIC PLATES

A relatively long time constant was discovered in the time constant of the circuit connecting the photomultiplier to the monitoring oscilloscope. Appendix V discusses the method of overcoming this difficulty. In correcting the photomultiplier traces, some information is lost. However, Figures 1, 2, and 3 of this appendix show that gross characteristics of the discharge, such as the appearance of continua, can still be determined.

The break in the first continuum for the 500 and 1,000 μHg cases is lost in the reduced traces. By correlating the time axis of the spectrographic plate with a monochromator trace, an estimate can be made of the time-duration of the "dark line" or break in the first continuum. Figures 3.6 to 3.9 are derived from this data reduction procedure. The sharp outline of this break implies a reproducibility of better than 0.25 $\mu\text{sec.}$ in the formation of the first continuum.

An estimate of the jitter in reproducibility of secondary continua can be obtained by analyzing the monochromator traces. In Figure 1 of this appendix, the 4695 $\text{\AA}$ and 4642 $\text{\AA}$ traces show the first secondary continuum to occur 0.286 $\mu\text{sec.}$ apart. Since the 4695 $\text{\AA}$ trace is close to the HeII 4686 line, it is affected by broadening from HeII. Therefore, a further inspection of the two traces for jitter in the second secondary continuum is unproductive. Figure 2 of this appendix shows the 1,000 μHg monochromator traces for two separate shots at 4640 $\text{\AA}$. There is good

correspondence for the first (pinch) continuum and also for the first secondary continuum. The second secondary continuum shows a jitter of 0.29 μ sec.

Hence, the first (pinch) continuum occurs within a time interval of less than 0.25 μ sec. from discharge to discharge. The appearance of secondary continua can be affected by a jitter of about 0.3 μ sec.

Appendix III, Figure 2 1,000 μHg filling pressure

Appendix III, Figure 3

2,000 μHg filling pressure

APPENDIX IV

MONOCHROMATOR-PHOTOMULTIPLIER CALIBRATION

a) Photomultiplier Responses

The photomultiplier tube (EMI, Type 9558B) was connected to a high voltage power supply with an output of 1450 volts. To check the linear response range of the tube, the monochromator was set at the HeII 4686 peak, and a set of neutral density filters was used to step down light from the discharge tube. The intensity of radiation transmitted by the filters is calculated from the relation:

$$I_t = I_i \times 10^{-D}$$

where I_i = incident intensity

D = n.d. filter "index"

I_t = transmitted intensity

Figure 1 of this appendix shows that "saturation" sets in for oscilloscope readings above 25 volts. Work done by others at this laboratory, with the same photomultiplier-power supply combination, shows that this linear response region extends well below 4 volts, down to about 10 millivolts.

To avoid saturation effects, a neutral density filter was used for cases in which the photomultiplier response approached the 20 volt region. The same filter was used throughout. Its relative transmission was calibrated with a tungsten lamp for the wavelength regions of interest.

Wavelength	<u>Incident radiation</u>	
	<u>Transmitted radiation</u>	
HeI 5876 Å		5.98
HeII 4686 Å		6.17
HeI 3889 Å		5.14

b) Instrumental Function and Response.

The reciprocal dispersion and instrumental broadening in the first-order spectrum of the apparatus were measured by using Geissler tubes for several elements. The monochromator was swept through accessible regions nearest the helium lines of interest by a continuous drive mechanism. The photomultiplier response was traced out by chart-recorder.

Arc	Region measured ($\text{\AA}$)	Reciprocal dispersion ($\text{\AA}/\text{mm.}$)
Iron	3865-3903	10.2
Argon	4259-4272	10.1
Mercury	5770-5791	9.9
Neon	7024-7032	10.0

For an estimate of instrumental broadening, the following lines were investigated:

Spectral line ($\text{\AA}$)	Full half-width ($\text{\AA}$)	Spectral line ($\text{\AA}$)	Full half-width ($\text{\AA}$)
HgI 3125.66	0.19	AI 4200.67	0.19
HgI 3131.55	(0.22)	HgI 4358.35	0.22
HgI 3131.83	0.19	HgI 5769.59	0.20
HgI 3650.15	0.24	HgI 5790.65	0.29
HgI 3662.88	0.20	NeI 5852.49	0.16
HgI 4046.56	0.23	NeI 5881.89	0.16
HgI 4077.81	0.20	NeI 6328.17	0.11 ← taken with
AI 4198.3	0.19	NeI 6532.88	0.19 He-Ne laser.

A helium Geissler tube proved unsatisfactory since considerable broadening of the lines takes place. HeI 5876 gave a full half-width of $0.7 \text{ \AA}$. The variation in half-width in the table above occurs because the lines scanned are not infinitesimally narrow. A reasonable approximation for the instrumental broadening can be taken as $0.2 \text{ \AA}$, which remains constant over the region of interest ($3,800\text{--}6,000 \text{ \AA}$).

c) Absolute Intensity Calibration.

A carbon arc was used for absolute intensity calibration. The results for the HeII 4686 and HeI 5876 regions are given in Figures 2 and 3 of this appendix. The operation of the arc is that outlined by Null and Lozier (1962) where:

$$\mathcal{P}(\lambda, T) = \frac{W_c(\lambda, T)}{W_{BB}(\lambda, T)} = 1 \quad \dots\dots(1)$$

for W_c = power radiated by carbon arc,
 W_{BB} = power radiated by black body;
 and the arc temperature $T = 3,800^\circ \text{ K}$, for a direct current
 10 amps, 150 volts.

The arc was a standard commercial model, made by Leybold, with the projection lens removed. The electrodes were oriented at 90° as prescribed by Null and Lozier. Ringsdorff spectroscopic carbons RW202 and RW401 served as the anode and cathode respectively. A 150 volt, 15 amp regulated direct current power supply (Sorensen Nobatron DCR 150-15) was employed in series with a high current, variable carbon resistor. The arc was operated just below the "hissing point" as described in the literature.

By calculating the black body power emitted by the carbon arc, $W_c(\lambda)$ from the formula,

$$W_c(\lambda) = \frac{2hc^2}{\lambda^5} (e^{hc/\lambda kT} - 1)^{-1} \text{ erg/ sec.cm}^2 \text{ str.}(\Delta\lambda, \text{cm.}) \quad \dots\dots(2)$$

the monochromator response can be calibrated at the wavelengths of interest.

The experiment was designed originally without absolute intensity measurements in mind. The radiation calibration was done after time-resolved line profiles had been made of the discharge. The monochromator was realigned upon an optic axis outside the discharge tube and parallel to its axis. The carbon arc was placed at the focal plane of the quartz-water

achromat. A quartz window and screen, (corresponding to the viewport and low tension electrode of the discharge tube), were placed at appropriate positions along the optic axis to duplicate the light path followed by the radiation from the discharge. It was found that the anode spot image of the arc was smaller than the slit height of 2 mm. used when analyzing the discharge radiation. The slit height was stepped down during the calibration process to ensure that only the bright inner region of the anode spot was monitored. The difference in slit heights was taken into account when the power output of the discharge radiation was calculated. Hence, the monochromator response, in volts, can be related to the power radiated from Figures 2 and 3, and equ'ns (1), (2).

$$\text{HeI } 5876 \quad \left\{ \begin{array}{l} 5800-5876 \text{ \AA} = 4.26 \times 10^{14} \text{ erg/ sec. cm.}^2 \text{ str. volt } (\Delta\lambda, \text{cm.}) \\ 5876-5900 \text{ \AA} = 4.29 \times 10^{14} \text{ erg/ sec. cm.}^2 \text{ str. volt } (\Delta\lambda, \text{cm.}) \end{array} \right.$$

$$\text{HeII } 4686 \quad \left\{ \begin{array}{l} 4600-4686 \text{ \AA} = 5.53 \times 10^{14} \text{ erg/ sec. cm.}^2 \text{ str. volt } (\Delta\lambda, \text{cm.}) \\ 4686-4940 \text{ \AA} = 5.59 \times 10^{14} \text{ erg/ sec. cm.}^2 \text{ str. volt } (\Delta\lambda, \text{cm.}) \end{array} \right.$$

Appendix IV, Figure 1

Photomultiplier response V
(in volts)
as a function of intensity

Appendix IV, Figure 3

Instrument response
5876 Å region

APPENDIX V

RETRIEVAL OF INFORMATION FROM "SPOILED" MONOCHROMATOR TRACES

The relation between photomultiplier current, $I(t)$, and the voltage displayed on the oscilloscope, $V(t)$, can be shown to be

$$I(t) = \frac{V}{R} + C \frac{dV}{dt} \quad \text{.....(1a)}$$

$$= \frac{1}{R} \left[V(t) + \tau \frac{d}{dt} V(t) \right] \quad \text{.....(1b)}$$

where $\tau = RC$ = the time constant of the "integrator circuit" formed by the final stage resistor and cable capacitance (see Figure 1, this appendix).

For times of interest $\gg \tau$, equ'n (1b) becomes the standard formula

$I(t) = V(t)/R$. However, in measuring the rise time of the photomultiplier, it was found that $\tau = 5.2 \mu\text{sec}$. For times $< 5.2 \mu\text{sec}$., the second term of equ'n (1b) becomes significant.

The circuit was properly terminated and its response compared with the arrangement used under experimental conditions (Figure 2, this appendix).

Figure 1

Circuit involving final stage resistor and coaxial cable

R = final stage resistor of photomultiplier
 C = capacitance of cable

This provides a test of the value $\tau = 5.2 \mu\text{sec}$. The oscilloscope traces in Figure 2a,b (this appendix) were provided by S.S. Medley. They show monochromator responses taken perpendicular to the discharge axis in argon at 1,000 μHg .

Figure 3a shows an oscilloscope trace for a discharge in helium at 2,000 μHg . The observation was made 1" parallel to the discharge axis. Figure 3b displays the proper response after corrections for the neutral density filter and equ'n (1) are applied. With the aid of a ruled magnifying glass, the traces can be read to 0.05 μsec . for an oscilloscope display of 2 $\mu\text{sec./cm}$.

Figure 5.2 (Section 5.1) shows a typical line profile constructed from monochromator traces illustrated by Figure 3b. The multiplicity of observation points and the shape of the line near its half-height ensure a determination of "half-widths" by, at worst, 20% error. Intensity and half-width measurements were made from graphs approximately four times the size of Figure 5.2. As mentioned in Section 5.1, only two points in time were used from each trace; since a differentiation according to equ'n (1b) is involved at each of the 30 points used in constructing Figure 3b (this appendix). Although the procedure is lengthy, the circuit construction allowed an absolute intensity calibration to be made. It was noted in Appendix IVc that the experiment was not originally designed with absolute intensity measurements in mind. Had the circuit been terminated properly, the relatively weak carbon arc signal would not have registered above the photomultiplier noise signal.

a) Oscilloscope display of properly terminated photomultiplier circuit

b) Oscilloscope display of photomultiplier circuit as used in experimental conditions

c) Comparison of waveforms
 ---- trace a) enlarged by constant factor
 ——— trace b) corrected

The ordinate shows the true voltage response to the photomultiplier current $I(t)$

a)

HeI 5876 Å for helium
at 2,000 μ Hg, 1" off-axis.
Taken with neutral
density filter.

b)

Trace a) corrected
for "time integration" and
neutral density filter

Appendix V, Figure 3

Correction of monochromator trace

APPENDIX VI

PARTITION FUNCTIONS FOR HELIUM

The partition function (Z) is an expression giving the distribution of atoms occupying different energy states in a system:

$$Z = \sum_n g_n e^{-(E_n/kT)} \quad \text{.....(1)}$$

where the summation is taken over all energy states of the system.

g_n = the statistical weight of the n^{th} state
 E_n = the energy of the n^{th} state
 T = absolute temperature
 k = the Boltzmann constant

For neutral helium, the partition function is as follows (energies in electron volts):

$$\begin{aligned} Z_{\text{He}^0}(T) = & 1 + 3e^{-\frac{19.73}{kT}} + e^{-\frac{20.53}{kT}} + 9e^{-\frac{20.87}{kT}} + 3e^{-\frac{21.13}{kT}} \\ & + 3e^{-\frac{22.62}{kT}} + e^{-\frac{22.82}{kT}} + 9e^{-\frac{22.91}{kT}} + 3e^{-\frac{22.97}{kT}} \\ & + 20e^{-\frac{22.97}{kT}} + 3e^{-\frac{23.49}{kT}} + e^{-\frac{23.57}{kT}} + 9e^{-\frac{23.61}{kT}} \\ & + 51e^{-\frac{23.64}{kT}} + 3e^{-\frac{23.87}{kT}} + e^{-\frac{23.91}{kT}} + 96e^{-\frac{23.94}{kT}} \\ & + 144e^{-\frac{24.11}{kT}} + 196e^{-\frac{24.21}{kT}} + 256e^{-\frac{24.27}{kT}} \quad \text{.....(2)} \end{aligned}$$

where the summation is taken up to $n = 8$. This estimate was given by Wiese, based on energy values from Moore's tables (1959). It is applicable for L.T.E., but does not take into account the lowering of ionization energy (ΔE) due to electric microfields within the plasma.

These restrictions apply to the following formula for the partition function of singly ionized helium:

$$Z_{\text{He}^+} = 2 + \sum_{n=2}^{\infty} 2n^2 e^{\frac{E_+}{kT} [1 - (1/n^2)]} \quad \text{.....(3)}$$

with $E_+ = 54.4$ electron volts.

Equation (3) is based on the hydrogenic behaviour of HeII.

Values for $Z_{\text{He}^0}(T)$ and $Z_{\text{He}^+}(T)$ are tabulated on the following pages for $10,000^\circ < T < 60,000^\circ \text{ K}$. They are compared with more detailed results published by Drawin (1965), where the effect of ΔE is taken into account. The calculations based on equations (2) and (3) were obtained from a computer program furnished by Mr. R.N. Morris.

Depression of the ionization limit (ΔE), due to the Stark effect, can be ignored since for

$$\left. \begin{array}{l} T_e = 40,000^\circ \text{ K} \\ n_e = 1.7 \times 10^{17} \text{ cm}^{-3} \end{array} \right\} \Delta E = 0.19 \text{ e.v.}$$

The tables show the calculated partition functions to be consistent with a ΔE in the region of 0.20 electron volts for an error of about 5%.

Table (i)

Partition function (Z) for neutral helium

Temperature	Z	Z (Drawin, 1965)					
		$\Delta E = 0.10$ (e.v.)	$\Delta E = 0.25$ (e.v.)	$\Delta E = 0.50$ (e.v.)	$\Delta E = 1.0$ (e.v.)	$\Delta E = 2.0$ (e.v.)	$\Delta E = 3.0$ (e.v.)
17500	1.0001	1.000	1.000	1.000	1.000	1.000	1.000
18000	1.0002						
18500	1.0003	1.001	1.000	1.000	1.000	1.000	1.000
19000	1.0004						
19500	1.0006	1.001	1.000	1.000	1.000	1.000	1.000
20000	1.0008						
20500	1.0011	1.002	1.001	1.000	1.000	1.000	1.000
21000	1.0015						
21500	1.0021	1.004	1.002	1.001	1.000	1.000	1.000
22000	1.0028						
22500	1.0037	1.008	1.003	1.001	1.001	1.000	1.000
23000	1.0048						
23500	1.0062	1.014	1.005	1.003	1.001	1.001	1.001
24000	1.0079						
24500	1.0100						
25000	1.0125	1.025	1.009	1.004	1.002	1.001	1.001
25500	1.0155						
26000	1.0190	1.048	1.014	1.007	1.003	1.002	1.002
26500	1.0232						
27000	1.0282						
27500	1.0339	1.072	1.024	1.012	1.005	1.003	1.003
28000	1.0406						
28500	1.0482						
29000	1.0570	1.117	1.038	1.018	1.008	1.004	1.004
29500	1.0670						
30000	1.0782						
30500	1.0910	1.186	1.060	1.029	1.011	1.006	1.006
31000	1.1053						
31500	1.1213						
32000	1.1391	1.289	1.093	1.044	1.017	1.009	1.009
32500	1.1588						
33000	1.1807						
33500	1.2047	1.440	1.140	1.066	1.025	1.013	1.013
34000	1.2311						
34500	1.2600						
35000	1.2915	1.656	1.208	1.096	1.036	1.019	1.019
35500	1.3259						
36000	1.3631						
36500	1.4034						

Table (i) (cont'd)

		Z (Drawin, 1965)					
Temperature	Z	$\Delta E = 0.10$ (e.v.)	$\Delta E = 0.25$ (e.v.)	$\Delta E = 0.50$ (e.v.)	$\Delta E = 1.0$ (e.v.)	$\Delta E = 2.0$ (e.v.)	$\Delta E = 3.0$ (e.v.)
37000	1.4469	1.961	1.302	1.138	1.051	1.026	1.026
37500	1.4937						
38000	1.5441						
38500	1.5980						
39000	1.6558	2.332	1.431	1.196	1.071	1.036	1.035
39500	1.7174						
40000	1.7831						
40500	1.8530	2.954	1.605	1.273	1.097	1.049	1.047
41000	1.9271						
41500	2.0057						
42000	2.0889						
42500	2.1767	3.717	1.837	1.375	1.130	1.065	1.063
43000	2.2694						
43500	2.3669						
44000	2.4695						
44500	2.5773	4.719	2.140	1.507	1.173	1.085	1.063
45000	2.6902						
45500	2.8086						
46000	2.9324						
46500	3.0617						
47000	3.1966	6.017	2.529	1.676	1.228	1.110	1.107
47500	3.3373						
48000	3.4838						
48500	3.6362						
49000	3.7945						
49500	3.9589	7.671	3.024	1.888	1.295	1.141	1.137
50000	4.1294						

Table (ii)

Partition function (Z) for singly ionized helium

Temperature	Z	Z (Drawin, 1965)					
		$\Delta E = 0.10$ (e.v.)	$\Delta E = 0.25$ (e.v.)	$\Delta E = 0.50$ (e.v.)	$\Delta E = 1.0$ (e.v.)	$\Delta E = 2.0$ (e.v.)	$\Delta E = 3.0$ (e.v.)
36000	2.000	T = 36,182					
36500	2.000	2.000	2.000	2.000	2.000	2.000	2.000
37000	2.000	T = 37,991					
37500	2.000	2.001	2.000	2.000	2.000	2.000	2.000
38000	2.000	T = 39,891					
38500	2.000	2.001	2.000	2.000	2.000	2.000	2.000
39000	2.000	T = 41,885					
39500	2.000	2.001	2.000	2.000	2.000	2.000	2.000
40000	2.000	T = 43,980					
40500	2.000	2.003	2.001	2.000	2.000	2.000	2.000
41000	2.000	T = 46,179					
41500	2.000	2.006	2.002	2.001	2.001	2.000	2.000
42000	2.000	T = 48,488					
42500	2.000	2.021	2.007	2.003	2.002	2.001	2.001
43000	2.000	T = 50,912					
43500	2.000	2.039	2.012	2.006	2.003	2.002	2.002
44000	2.001	T = 53,458					
44500	2.001	2.069	2.022	2.011	2.006	2.003	2.003
45000	2.001						
45500	2.001						
46000	2.001						
46500	2.001						
47000	2.001						
47500	2.001						
48000	2.002						
48500	2.002						
49000	2.002						
49500	2.002						
50000	2.003						
50500	2.003						
51000	2.003						
51500	2.004						
52000	2.004						
52500	2.005						
53000	2.005						
53500	2.006						

Table (ii) (cont'd)

Temperature	Z	Z (Drawin, 1965)					
		E = 0.10 (e.v.)	E = 0.25 (e.v.)	E = 0.50 (e.v.)	E = 1.0 (e.v.)	E = 2.0 (e.v.)	E = 3.0 (e.v.)
54000	2.006						
54500	2.007						
55000	2.008						
55500	2.008			T = 56,131			
56000	2.009	2.121	2.038	2.018	2.009	2.006	2.004
56500	2.010						
57000	2.011						
57500	2.012						
58000	2.013			T = 58,937			
58500	2.014						
59000	2.015	2.205	2.065	2.030	2.015	2.009	2.007
59500	2.017						
60000	2.018						
60500	2.020						
61000	2.021						
61500	2.023			T = 61,884			
62000	2.025	2.340	2.106	2.049	2.024	2.014	2.011
62500	2.027						
63000	2.029						
63500	2.031						
64000	2.034			T = 64,978			
64500	2.036						
65000	2.039	2.551	2.171	2.078	2.038	2.022	2.017

APPENDIX VII

SELF ABSORPTION EFFECTS

The following analysis pertains to an absorbing, homogeneous, L.T.E. plasma. If absorption and emission of radiation are considered to occur as in a black body cavity, the observed intensity $I(\lambda)$ can be expressed in general as:

$$I(\lambda) = B(\lambda) [1 - e^{-\tau(\lambda)}] \quad \dots\dots(1)$$

$$\text{where } \tau(\lambda) = \frac{E(\lambda)\ell}{B(\lambda)} \quad \dots\dots(2)$$

with $\tau(\lambda)$ = the optical depth at wavelength λ
 $B(\lambda)$ = black body radiation at wavelength λ
 $E(\lambda)$ = intensity of radiation emitted per unit volume
 ℓ = the geometrical depth of the plasma along the line of observation.

a) Considerations of Optical Depth

If $\tau(\lambda) \ll 1$, the plasma is said to be optically thin to radiation of wavelength λ . From equ'ns (1) and (2), the observed intensity in the optically thin case $I_{\text{thin}}(\lambda)$ is

$$I_{\text{thin}}(\lambda) = B(\lambda)\tau(\lambda) = E(\lambda)\ell \quad \dots\dots(3)$$

If $\tau(\lambda) \sim 1$, partial absorption occurs for radiation at wavelength λ . From equ'ns (1), (2), and (3), the ratio of observed radiation intensity $I(\lambda)$ to the intensity emitted in the optically thin case $I_{\text{thin}}(\lambda)$ is

$$\frac{I(\lambda)}{I_{\text{thin}}(\lambda)} = \frac{I(\lambda)}{E(\lambda)\ell} = \frac{1 - e^{-\tau(\lambda)}}{\tau(\lambda)} \quad \dots\dots(4)$$

For $\tau(\lambda) \gg 1$, the plasma is said to be optically thick with respect to radiation of wavelength λ . In this case, equ'n (1) becomes

$$I(\lambda) = B(\lambda) = \frac{2hc^2}{\lambda^5} \left[e^{hc/\lambda kT} - 1 \right]^{-1} \quad \dots\dots(5)$$

and the plasma emits radiation as a black body.

b) Self Absorption for Lorentzian Line Profiles

The optical depth at line centre $\tau(\lambda_0)$ for a Lorentzian line profile is (Griem, 1964; Cooper, 1966):

$$\tau(\lambda_0) = \tau_0 = l r_0 f_{mn} N_n (1 - e^{-hc/\lambda_0 kT}) \frac{\lambda_0^2}{W} \dots\dots(6)$$

where l = length of the discharge tube
 λ_0 = wavelength of the line investigated (cm.)
 W = half of the true line half-width (cm.)
 f_{mn} = absorption oscillator strength for the line
 N_n = number of atoms in the lower state n (cm⁻³)
 $r_0 = 2.82 \times 10^{-13}$ cm.

For radiation emanating at wavelength $\lambda \neq \lambda_0$, in the line profile, the optical thickness $\tau(\lambda)$ can be expressed as:

$$\tau(\lambda) = \frac{\tau_0}{1 + \frac{(\lambda - \lambda_0)^2}{W^2}} \dots\dots(7)$$

If the observed intensity $I(\lambda)$ is found to fall to half its maximum value at $\lambda_0 + \delta$, then from equ'ns (1) and (7)

$$1 - e^{-\tau_0} = 2 \left\{ 1 - \exp \left[\frac{-\tau_0}{1 + \left(\frac{\delta}{W} \right)^2} \right] \right\}$$

where δ is half of the observed half-width of the line centred at λ_0 .

The expression above, therefore, yields the ratio of the observed half-width (2δ) to its true value ($2W$)

$$\frac{\delta}{W} = \left\{ \frac{\tau_0}{\ln \left[\frac{2}{1 + \exp(-\tau_0)} \right]} - 1 \right\}^{\frac{1}{2}} \dots\dots(8)$$

The effect of self absorption on the observed total intensity of the line (I_t) can be calculated by integrating equ'n (1) over all wavelengths

$$I_t = B(\lambda_0) \int_0^\infty \left\{ 1 - e^{-\tau(\lambda)} \right\} d\lambda$$

$$= B(\lambda_0) \int_0^{\infty} \left\{ \tau(\lambda) - \frac{1}{2} [\tau(\lambda)]^2 + \dots \right\} d\lambda \quad \dots\dots(9)$$

since $B(\lambda)$ varies slowly over the line profile. Integrating the right hand side of equ'n (9) term by term, and using equ'n (2) for $\lambda = \lambda_0$, gives:

$$\frac{I_t}{E(\lambda_0) \ell} = \left[1 - \frac{\tau_0}{4} + \frac{1}{16} \tau_0^2 - \frac{5}{384} \tau_0^3 + \dots \right] \quad \dots\dots(10)$$

The evaluation of equ'ns (8), (10) and Figure 1 of this appendix were provided by Messrs. H.D. Campbell and H.G. James.

Assuming an L.T.E. distribution within an ionic species, N_n of equ'n (6) can be computed by the standard formula

$$N_n = N_0 e^{-E_n/kT} \frac{g_n}{Z(T)}$$

where N_0 is determined in Table 5-i, Section 5.2. The absorption oscillator strengths (f_{mn}) are taken from Griem (1964).

Spectral line	Transition	f_{mn}	g_n
HeII 4686	$3^2D - 4^2F$	1.016	18
HeI 5876	$2^3P - 3^3D$	0.62	9
HeI 3889	$2^3S - 3^3P$	0.066	3

c) Absolute Continuum Intensities

Cooper (1966) provides a complete discussion of contributions to the absolute continuum intensity from bremsstrahlung and recombination radiation. For our purposes, we consider only the bremsstrahlung component which, for a hydrogenic plasma (Conrads, 1966), exhibits an optical depth $\tau_B(\lambda)$ for wavelength λ :

$$\tau_B(\lambda) = 3.44 \times 10^{-26} \ell Z^2 n_i n_e g_B^Z \lambda^3 \left(\frac{E_H}{kT_e} \right)^{\frac{1}{2}} \exp\left(-\frac{\Delta E_\infty^{Z-1}}{kT_e}\right) \left[1 - \exp\left(-\frac{hc}{\lambda kT_e}\right) \right] \dots\dots(1)$$

where $n_i(n_e)$ = the number of ions (electrons) per cm^{-3}

E_H = the ionization energy for hydrogen

ℓ = the geometrical depth of the plasma along the line of observation

$\Delta E_\infty^{Z-1} = \frac{Ze^2}{4\pi\epsilon_0} \left(\frac{kT_e}{4\pi e^2 n_e} \right)^{-\frac{1}{2}} =$ lowering of ionization energy due to electric microfields within the plasma

$g_B^Z(T_e, \lambda) = \frac{\sqrt{3}}{\pi} \ln(2.25 \frac{\lambda kT_e}{hc}) =$ the Gaunt factor for bremsstrahlung at large wavelengths.

e, ϵ_0, k, h, c = the customary constants in c.g.s. units.

For a plasma, optically thin to radiation of wavelength λ , the power radiated per unit volume $E_B(\lambda)$ from bremsstrahlung radiation becomes:

$$E_B(\lambda) = 6.36 \times 10^{-47} n_i n_e Z^2 \frac{1}{(kT)^{\frac{1}{2}}} g_B^Z(T_e, \lambda) \frac{c}{\lambda^2} \dots\dots(2)$$

Hence, for an appropriate choice of wavelength (λ), the density of ions (or electrons) can be calculated from equ'n (2). For $n_i \approx 10^{18}$, $T_e \approx 80,000^\circ\text{K}$, which would be expected in the plasma column at the pinch stage, the wavelengths most suitable for this investigation would be in the near ultra-violet range of the spectrum, i.e., $\lambda < 3,000 \text{ \AA}$.

Appendix VII, Figure 1

Ratio of self absorbed halfwidth δ to optically thin halfwidth ω
for Lorentzian line shapes

τ_0 = optical depth at line centre