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THE DETECTION OF ALPHA PARTICLES
WITH SUPERCONDUCTING TUNNEL JUNCTIONS

by

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ABSTRACT

A superconducting thin film tunnel junction ($\text{Sn-SnO}_2\text{-Sn}$) of total thickness 4000 Å, area $7 \times 10^{-4} \text{ cm}^2$ and normal (4.2 K) resistance 77 mΩ was prepared on a glass substrate. When cooled to 1.2 K the junction was biased at 0.3 mV where, the Josephson supercurrent having been suppressed with a magnetic field, the junction dynamic resistance had its maximum value of 9.3Ω. The junction was then bombarded with 5.1 MeV alpha particles and the resulting pulses induced in the tunneling current were observed to have amplitudes up to 19 times the preamplifier-dominated rms output noise level.

For purposes of analysis, it was assumed that the induced current pulse had the form $i(t) = i_0 \exp(-t/\tau)$, $t \geq 0$. With this form of the current pulse and the known transfer function of the transmission line-amplifier system, it was calculated that for all pulses $\tau = (1.38 \pm .33) \times 10^{-7} \text{ sec}$ and that for the largest amplitude pulses, corresponding to an energy loss $\Delta E_\alpha \leq 2.75 \text{ MeV}$, i_0 lay in the range $20 \leq i_0 \leq 26 \text{ } \mu\text{A}$ with a most probable value of 22 μA .

With this value of i_0 and $\Delta E_\alpha = 2.75 \text{ MeV}$, an upper limit of $8.2 \times 10^{-3} \text{ eV}$ has been assigned to the value of $w(\text{Sn})$, the average energy expended by the alpha particle to excite a quasiparticle pair in superconducting tin at 1.2 K.

A tentative theory of the superconducting tunnel junction charged particle detector is given and the cryogenic and electronic apparatus required for the measurements are described.

Details related to thin film junction fabrication technology and interpretation of dc experimental results are discussed in four appendices.

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CHAPTER 1

INTRODUCTION

A. Motivation

Much of the present knowledge of the structure of the atomic nucleus has been obtained through the analysis of the energy distribution of nuclear reaction products. Because of the continuing need for more precise data, considerable effort has gone into the development of particle detectors, resulting in the evolution of a sophisticated technology of nuclear particle spectrometry. (See eg. Yuan, 1961; Ajzenberg-Selove, 1960; Dearnaley & Northrop, 1966). From the nuclear physicist's point of view, the ideal detector (or spectrometer) is one with high energy resolution--the ability to distinguish between particles having very nearly the same energy--with sharp time resolution--the ability to distinguish between particles arriving at the detector at very nearly the same time, and with high detection efficiency to maximize the rate of particle detection. (Note: in subsequent discussion the term "detector" is often used synonymously with "spectrometer".) This thesis describes the theory of and preliminary results from a new, fast, highly sensitive particle detector in the form of a superconducting tunnel junction--a "sandwich" consisting of two thin superconducting films separated by a very thin insulating layer (see figure 2-1). The detector shows promise of improved energy resolution, and comparable time resolution, but reduced detection efficiency compared with lithium drifted germanium detectors, which are currently the best nuclear detectors available.

When an energetic particle such as a proton or alpha particle penetrates a medium, it loses energy via a series of ionizing collisions with the atoms close to its path. For each given substance, (gas or solid), it is convenient to define a quantity w , the average energy which the charged particle must lose in that substance to produce a pair of excitations such as an ion pair or electron-hole pair. Consequently, if an energetic particle

loses energy ΔE in a substance, the number N of excitations ultimately produced (a measure of the signal amplitude) is $N = \Delta E/w$. If the energetic particle always had to lose exactly w to produce an excitation in a given substance, then a perfect detector made from that substance could distinguish between energies $\Delta E_1 = Nw$ and $\Delta E_2 = (N \pm 1)w$, and the energy resolution of the detector would be w . However, since w is an average over a very large number of excitations, most of which will involve energy losses differing from w , the number of excitations produced in real detecting media by a certain energy input is subject to fluctuations. In the simplest case where the excitations are considered to be independently produced and where N is large, the fluctuation may be taken to be the statistical fluctuation $N^{1/2}$. Hence, uncertainty of order $\sqrt{N}/N$ is introduced in determining the energy of a particle, for now $\Delta E = wN(1 \pm \sqrt{N}/N)$. In a more realistic case, which very often occurs, the excitation processes are not independent, but are correlated by the fact that the particle must lose all its energy in the detector; the statistical fluctuation is then given by $FN^{1/2}$ rather than $N^{1/2}$, where F , the Fano factor (Dearnaley and Northrop, 1966) lies between 0 and 1 and, for example, has the value $F \leq 0.16$ for excitations produced by 1-10 MeV electrons in germanium at 78 K. (Mann et al, 1966).

The uncertainty decreases with increasing N so that, from this viewpoint, the best energy resolution will be obtained from the detector in which for given energy loss, N is largest or, in other words, in which w is smallest.

The motivation for considering the superconducting tunnel junction as a particle detector is apparent from the comparison of $\sqrt{N}/N$ for several media in table 1-1.

Two other quantities of interest for comparing different detecting media have been included in the table. The specific rate of energy loss ($-dE/dx$) and the range, calculated for a 5 MeV alpha particle, are optimal in Pb which means that to stop particles of a given energy a smaller thickness of Pb would be needed than of any conventional detecting material. Smaller size, of course, generally implies faster response.

The idea of using superconductors to detect nuclear particles is not new; what is novel is the exploitation of the favourable characteristics of the superconducting tunneling junction for this purpose.

Andrews et al (1949) bombarded a thin strip of niobium nitride, held very close to its superconducting transition temperature, with alpha particles

Material	w(eV)	N for E=1 MeV	$\sqrt{N}/N$	-dE/dx (MeV/cm)	Range (5 MeV α)
Hydrogen-gas	37	2.7×10^4	6.1×10^{-3}	.18	15.5 cm
Krypton-gas	24	4.2×10^4	4.9×10^{-3}	1.25	2.81
Silicon-semi-con.	3.6	2.8×10^5	1.9×10^{-3}	1500	2.5×10^{-3}
Germanium-semi-con.	2.9	3.5×10^5	1.7×10^{-3}	2000	1.5×10^{-3}
Tin*-super-con.	.003*	3.3×10^8	5.5×10^{-5}	2300	1.6×10^{-3}
Lead*-super-con.	.005*	2×10^8	7.1×10^{-5}	2600	1.3×10^{-3}

Table 1-1: Particle Detection Parameters for Several Materials

* The magnitude of w used for the superconductors is based on the assumption that, in analogy with semiconductors, (Sherman, 1964) w may be 3-5 times the energy gap width--an assumption later shown to be consistent with the results of this experiment. (see section D of this chapter and chapter 7.)

and observed the resulting voltage pulses developed along the length of the strip. A similar experiment using evaporated narrow thin films rather than bulk strips were later proposed by Sherman (1962,B) and performed by Spiel et al (1965). When an alpha particle impinged on the superconductor a small, cylindrical region surrounding the track was driven normal. The diameter of this region increased as a function of time at a rate determined by the ambient current carried by the film and the energy lost by the alpha particle, until it became equal to film width; at this point, because of the finite resistance due to this transverse normal region, a pulse was observed in the voltage between the ends of the film. The basic differences between the superconducting single strip and the tunnel junction detectors will be outlined at the end of the next section; more detail on the superconducting strip detectors is given in section A, chapter 3.

It was Burstein et al (1961) who first proposed and analyzed the use of the superconducting tunnel junctions, which had recently been studied in the pioneering work of Giaever (1960,61), as detectors of electromagnetic radiation. Such a detector is of great practical interest because the frequency range to which it is sensitive, approximately 85 to 650 GHz,

is virtually inaccessible by conventional detection techniques. The two processes of interest in this device are the "optical" excitation of quasiparticles across the superconducting energy gap and "photon-assisted" tunneling. It turns out that tunnel junctions may also serve as generators and detectors of microwave sound (Lax and Vernon, 1965; Goldstein et al, 1965; and Eisenmenger and Dayem, 1967) via an analogous process known as "phonon-assisted" tunneling. A brief review of these applications is given by Taylor (1968).

The scene was now set for work to begin on the superconducting tunnel junction charged particle detector. On one side was the nuclear spectroscopist with his continuing demand for more sensitive detectors; on the other was the low temperature physicist with a new device having interesting physical properties and exciting potential for applications. The first steps in an attempt to adapt this potential and supply the demand are described in this thesis.

B. Principle of Operation

1. Phenomenological Treatment

For a given superconducting thin film tunnel junction at temperature T whose insulating layer (see figure 2-1) is suitably thin, there exists a well-defined relationship $I = I(V, T)$ --called the I - V characteristic--between the dc current I passing from one film to the other via electron tunneling and the voltage V developed across the insulating layer. At bias voltages $0 < V < 2\Delta(T)/e$, where e is the electronic charge and $2\Delta(T)$ is the temperature dependent superconducting energy gap width (discussed in chapter 2), the current is proportional to the density of thermally excited quasiparticles which depends approximately upon the temperature according to $\exp(-\Delta(T)/k_B T)$. (See also figure 6-1; k_B is Boltzmann's constant.) This statement neglects tunnel current contributions arising from the tunneling processes involving more than one particle; such additional contributions are discussed in Appendix B. Neglecting for the present the temperature dependence of Δ , it is evident that any stimulus, (eg. the energy lost by a nuclear charged particle in traversing or coming to rest in a tunnel junction), which increases the film temperature will increase the tunneling current. That such an effect existed was not in question. What was in question, however, was whether or not the change in tunneling current due to a 5 MeV alpha particle, for example, was observable

and, if so, did the effect show promise for charged particle energy measurements?

Before going further, the role played by the energy gap should be stressed. As mentioned earlier, low statistically limited energy resolution requires that Δ should be as small as possible ($\Delta_{\min}$). On the other hand, the "leakage" or thermal quasiparticle tunneling current through the junction should be minimal to reduce shot noise and this implies that Δ should be large ($\Delta_{\max}$). Clearly, for a given lowest operating temperature, a superconductor with a compromise value (Δ_c) such that $\Delta_{\min} < \Delta_c < \Delta_{\max}$ is required, but Δ_c should favour $\Delta_{\max}$ for the energy resolution is only degraded by a factor $(1 + \Delta_c/\Delta_{\min})^{1/2}$ whereas the thermal current is increased from the optimal value by a factor $\exp[\frac{1}{2}(\Delta_{\max} - \Delta_c)/k_B T]$.

When a tunnel junction is bombarded with charged particles, the particles traverse one or both of the films giving up some of their energy to the films predominantly via a series of ionizing collisions. The amount of energy deposited directly in the junction ΔE is given by $\Delta E = \int_0^{x_m} (dE/dx) dx$ where x_m is the path length through the superconducting layers and (dE/dx) is the specific rate of energy loss which depends on the initial energy and type of particle and the material from which the films are prepared. For purposes of analysis, it is assumed that the resulting disturbance of the temperature distribution away from its equilibrium value is a maximum immediately following the passage of the alpha particle and that it relaxes back to the equilibrium condition in a characteristic time τ . During this period an increase in the tunneling current occurs which, after appropriate amplification and shaping, appears as a signal pulse superimposed on the ambient thermal tunneling current. This pulse may be analyzed for information about the energy loss which produced it and the time at which it was produced, using standard nuclear data acquisition electronics. It is found experimentally (chapter 6), in agreement with theoretical predictions (chapter 3) that the signal to noise ratio for the overall system (detector plus amplifiers) is a maximum when the dynamic resistance $r = (\partial V/\partial I)_T$ is as large as possible. The dc operating current of the junction was therefore chosen so that $(\partial V/\partial I)_T$ was a maximum; the magnitude of I_0 was not critical, as the junction noise sources were small compared with preamplifier noise. Questions concerning the amplitude and duration of such a pulse are dealt with briefly in the following section

and discussed at length in chapters 3 and 7.

2. Microscopic Viewpoint

The purely phenomenological discussion given in the preceding section is necessitated by the extreme complexity of the microscopic picture sketched below.

Ultimately, the energy lost by a charged particle as it passes through the films of the tunnel junction leads to the generation of densities of electron and phonon excitations along the particle track which exceed those present when the films are in thermal equilibrium with a constant temperature bath. The energy is transported away from the track by these excitations and, from the work of Crittenden (1968), it seems that the transport processes might be adequately described by classical heat diffusion theory. As a consequence of the temperature distribution which obtains in the superconducting films, excited quasiparticle densities N_0 are produced which are in excess of the thermal equilibrium value. The excess is maximum immediately following the penetration of the alpha particle and it decays back to zero with the same characteristic time as does the temperature.

Detailed calculation of N_0 is made difficult by at least two complicating factors. First of all, the superconducting energy gap ($2\Delta(T)$) at a point in the superconductor is unlike the band gap in semiconductors in that it is strongly temperature dependent and vanishes, of course, at the superconducting-normal transition temperature T_c . Now large temperature excursions are anticipated in the vicinity of the track so that for periods of tens of nanoseconds (indicated by preliminary calculations carried out by Mr. G. May of this laboratory) the metal in this very small region is, in fact, in the normal state--no fixed value of Δ may therefore be assumed. Secondly, the phonons emitted when two excited quasiparticles combine to form a Cooper pair may not be ignored for they, in turn, may break up other pairs. Rothwarf and Taylor (1967) have shown that the rate at which these recombination phonons are lost from the energy range $\hbar\omega \geq 2\Delta(T)$ often determines the rate at which excess excited quasiparticle densities will decay. In a small volume like the tunnel junction, it is expected that transmission of the phonons to the glass substrate and superfluid helium will play an important part. (For further discussion, see chapter 7).

Certainly, the situation in superconducting tunnel junctions is even more complicated than in semiconductor detectors of practicable geometry where, although the energy gap may be considered constant and the effect of boundaries ignored, the analysis is quite involved (see eg. Shockley, 1961).

3. Difference from Intermediate State Bolometers

At this point a distinction can be made between the tunnel junction detector and the single superconducting strip used by previous workers. Essentially, the single strip acts like a switch. Particles with energy below some threshold value (E_{th})--determined by the bias current and strip dimensions, material and temperature--are unable to convert a sufficiently large region of the strip into the normal state so that no voltage pulse occurs; particles with energy $E > E_{th}$ do generate a pulse but its amplitude is insensitive to the actual magnitude of E and is not suitable for multichannel spectrometry. In the junction detector, on the other hand, an excess quasiparticle density is generated as long as E is greater than a few times $2\Delta(T)$, say, so that the energy threshold is set by the junction-amplifier noise and fluctuations in the signal. (Insufficient theoretical or experimental evidence is available yet to make any statements concerning the linearity between pulse amplitude and energy deposited in the junction.) By analogy, it might be said that the single superconducting strip is to the tunnel junction detector as the geiger counter is to the proportional counter.

C. Expected Signal

As a starting point, the time dependence of the current pulse resulting from the passage of an alpha particle through the junction is assumed to be

$$i(t) = i_0 \exp(-t/\tau) \quad , \quad t \geq 0 \quad (1-1)$$

where the peak current i_0 is proportional to the maximum excess excited quasiparticle density and τ is the characteristic time mentioned earlier. To evaluate i_0 in equation 1-1, it is convenient to define

$$i_0 = \delta I = (\partial I / \partial T)_V \delta T$$

$$N_0 = \delta n = (\partial n(T) / \partial T) \delta T \approx 2\Delta E / w \cdot (Vol)$$

where $n(T)$ is the thermal equilibrium quasiparticle density and Vol is the volume in which the energy ΔE is distributed (see chapter 3). Eliminating δT between these expressions gives

$$i_o = \left(\frac{\partial I}{\partial T} \right)_V \left(\frac{\partial T}{\partial n} \right) \frac{2\Delta E}{w^*(Vol)} \quad (1-2)$$

Rough estimates of the various parameters in equation 1-2 indicate that $i_o \approx 15 \mu A$ for $\Delta E = 0.5$ MeV in tin at 1.2 K. (It should be pointed out that the values of $(\partial I/\partial T)_V$ and $(\partial T/\partial n)$ used in this estimate are constant and do not represent in any way the true, complicated behaviour of these two quantities which are strongly temperature dependent.) In spite of these gross simplifications, this estimate of i_o is reasonably consistent with the most probable value of $22 \mu A$ deduced experimentally (see section E following and chapter 7).

Estimates of the time constant τ are on an even less firm footing. Decay of the excess quasiparticle density occurs either by tunneling (under suitable bias conditions) with probability W_T per second per excess quasiparticle pair or recombination with probability W_R per second per excess quasiparticle pair. (Recombination, see chapter 7, is taken to include those processes which irreversibly reduce the quasiparticle density in the superconducting films, it does not include merely the recombination of quasiparticles to form a Cooper pair with the emission of a phonon, since such a phonon may in turn create a new pair of quasiparticles from a Cooper pair, in which case the quasiparticle density has not been irreversibly reduced.) Hence the rate at which the excess quasiparticle density decays is $W = W_T + W_R = \tau^{-1}$.

W_T may be estimated via a method due to Ginsberg (1962). For realistic junction geometries and barrier thicknesses, see chapter 3, W_T ranges from 2×10^5 to $3 \times 10^6 \text{ sec}^{-1}$.

Because of the influence of the recombination phonons mentioned earlier, it is expected that W_R should not be estimated solely from the theory of quasiparticle recombination (see eg. Woo and Abrahams, 1968). Measurements of an effective recombination probability W_R' , which includes the effect of phonon loss from the films, have been made in thin film triode structures by several workers (see eg. Levine and Hsieh, 1968) who find that

for superconductors and temperatures of interest $2 \times 10^7 \text{ sec}^{-1} > W_R' > 5 \times 10^5 \text{ sec}^{-1}$. Although the steady state conditions under which W_R' was measured differ from those in tunnel junctions bombarded with charged particles, one has little recourse but to use $W_R' + W_T$ for an order of magnitude estimate of τ^{-1} .

Within the reliability of these estimates, τ ranges from 4×10^{-8} to $1.4 \times 10^{-6} \text{ sec}$.

In summary, the expected current is assumed to be of the form $i(t) = i_0 \exp(-t/\tau)$ with $i_0 \approx 15 \mu\text{A}$ (for energy $\Delta E = 0.5 \text{ MeV}$ deposited in the tin junction at 1.2 K) and $4 \times 10^{-8} < \tau < 1.4 \times 10^{-6} \text{ sec}$.

D. Noise

It was found (chapter 6) that the dominant source of noise in the present experiment was the current sensitive preamplifier and that the magnitude of the noise was $3.5 \mu\text{V(rms)}$ referred to the preamplifier input. (This noise level turned out to be equivalent to the pulse amplitude produced by a 100 keV energy loss in the particular junction used in the detection experiment.) When tunnel junctions having higher output impedances than the unit employed in this work ($\approx 9.3\Omega$) are fabricated, noise sources in the junction itself, as outlined in chapter 3, may assume an important role in determining junction performance.

The requirement that the amplifier sensitivity had to meet was that pulses having the parameters predicted above should produce an amplifier output equal to or greater than three times the amplifier noise level. An amplifier was constructed which met this criterion.

E. Preview of Results

The experiment reported in this thesis demonstrates for the first time that the superconducting thin film tunnel junction may indeed be used as a detector of charged particles. Specifically, it shows that the pulse of tunneling current resulting from the bombardment of a $\text{Sn-SnO}_2\text{-Sn}$ junction by 5.1 MeV alpha particles is readily observable. An upper limit of $8.2 \times 10^{-3} \text{ eV}$ has been placed on $w(\text{Sn})$, the average energy loss by a charged particle required to excite a quasiparticle pair in superconducting tin at 1.2 K which, with reference to table 1-1, may be seen to agree reasonably well with the value derived from a very naïve analogy with semiconductors.

(Because of ambiguity caused by heat diffusing back from the substrate to contribute to the pulses as discussed in chapter 7--this agreement may perhaps be fortuitous, and in fact $w(\text{Sn})$ may be smaller than the naïve analogy would suggest.)

The pulses observed (figure 6-8) had amplitudes up to 19 times the rms noise level. Because their amplitude decreased with decreasing junction dynamic resistance as predicted, because they could be turned off with a mechanical shutter placed between the junction and source, and because the pulse count rate agreed with that calculated from the source strength and the experimental geometry, it was concluded that the pulses certainly were the result of alpha particle bombardment of the junction itself and did not arise from bombardment of the lead-in films, or the substrate alone. A least squares fit of the theoretical pulse shape (modified by the transfer function of the amplifier system) to that recorded photographically yields $\tau = 138 \pm 33$ nsec where the errors correspond to a 90% confidence limit. With this range of τ and the known sensitivity of the amplifier system, i_o for the largest amplitude pulses observed is deduced to lie in the range $20 \leq i_o \leq 26$ μA with a most probable value of 22 μA . The value of ΔE corresponding to this value of i_o is discussed below. The total number N_o of quasiparticle pairs produced by the alpha particle is calculated to be

$$N_o = \frac{W}{W_T e} \int_0^{\infty} i(t) dt = i_o / W_T e \approx 3.4 \times 10^8$$

where W and W_T have been defined previously and e is the electronic charge. Considerable uncertainty is involved in estimating the total amount of energy ΔE_α that the alpha particle deposits in the junction. Taking 2.75 MeV as an upper bound (see the discussion in chapter 7) gives $w(\text{Sn}) \leq \Delta E_\alpha / N_o = 8.2 \times 10^{-3}$ eV as mentioned above.

These results must be regarded as preliminary, both insofar as the understanding of the basic physics involved is concerned, and insofar as an evaluation of these junctions as worthwhile nuclear radiation detectors is concerned. Further theoretical and experimental work, which is being continued in this laboratory by Mr. G. May and Dr. B. White, is required to elucidate the physical details of the processes involved and probe such subjects as energy resolution, linearity of response and reproducibility of performance (see chapter 8). In particular, investigations are being made

concerning the detailed mechanism for charged particle energy loss in superconductors and the manner in which the heat spike diffuses throughout the substrate and films; this topic is therefore treated very briefly in this thesis.

F. Thesis Outline

Chapter 2 introduces the subject of tunneling with a brief discussion of tunneling between normal metals. This is followed by a short review of topics from superconductivity theory which are of relevance to the tunnel junction charged particle detector. An outline of pertinent topics from quasiparticle and dc Josephson tunneling theory concludes the chapter.

The operating principles of the superconducting charged particle detector are given in chapter 3. To place the tunnel junction detector in perspective three commonly used devices for measuring charged particle energy spectra, the gas-filled, scintillation and semiconductor detectors, are quickly reviewed. Other detection applications of tunnel junctions, particularly microwave photons and phonons are examined because of their analogy to charged particle detection. It is established that the junction having the most favourable characteristics for particle detection is one composed of identical superconductors which have high stopping power and a large energy gap. The nature of the quasiparticle excitations ultimately generated in the superconductor by a traversing charged particle is discussed after which the parameters for a small signal equivalent circuit for the detector are derived and an estimate of the signal size obtained. This is followed by notes on the importance of the quasiparticle tunneling probability and the practical operation of the detector. Concluding the chapter is a survey of the noise sources in the tunnel junction detector which may ultimately limit its performance.

Chapters 4 and 5, which describe the cryogenic apparatus and electrical measurement techniques respectively, are primarily of technical interest and may be overlooked by the reader uninterested in these details.

A discussion of the experimental results may be found in chapter 6. Of chief concern are the dc characteristics of the Sn-Sn junctions used, the details connected with the observation and physical characteristics of pulses in the tunneling current which were induced by alpha particle

bombardment and the estimation of the junction capacitance from noise measurements.

Chapter 7 is an account of the analysis of the results dealing particularly with the determination of the parameters of interest τ , i_0 and w . A discussion of problems pertaining to a rigorous calculation of the amount of energy deposited in the junction by a 5.1 MeV alpha particle closes the section.

Finally, in chapter 8, the important results are summarized and placed in perspective by evaluating the tunnel junction as a possible charged particle spectrometer. Guidelines for future work bring the chapter and the main body of the thesis to a conclusion.

The four appendices deal with matters of specific technical interest. Appendix A outlines thin film tunnel junction preparation techniques; Appendix B analyzes the I-V characteristics of so-called "leaky" junctions; Appendix C is a synopsis of phenomena observed in the preparation of Pb-Pb junctions; Appendix D is an account of studies made to determine the effect of finite film resistance on the apparent sign of the resistance of the junction measured with a standard 4-terminal technique.

CHAPTER 2

THEORETICAL ASPECTS OF THE SUPERCONDUCTING TUNNEL JUNCTION

A. Introduction

In its most common form, the superconducting tunnel junction consists of two superimposed evaporated thin metal films separated by an extremely thin insulating layer, usually a thermally grown oxide on the bottom film (see figure 2-1). The "junction" is defined as the region of overlap of the films. Typically, the films are a few tenths of a millimeter wide and several thousand angstroms thick; the insulating layer thickness is usually 10 to 20 angstroms. The mechanism for current flow from one superconducting film to the other is, as the name implies, electron tunneling.

To introduce the essentials of the subject of tunneling, the problem of tunneling between normal metals is considered first (section B). Before going on to discuss tunneling between superconductors, a pause is made (section C) for the purpose of reviewing briefly those topics from the theory of superconductivity which are relevant to the superconducting tunnel junction (especially as a charged particle detector). The last three sections are devoted to a discussion of tunneling between superconductors: section D considers single quasiparticle tunneling, section E investigates Josephson or pair tunneling (because of its nuisance rather than intrinsic value to the tunnel junction detector) and, for the sake of completeness, section F looks briefly at multiparticle tunneling.

The reader interested in studying tunneling phenomena in general is referred to a very recent and comprehensive review edited by Burstein and Lundqvist (1969).

B. Tunneling Between Normal Metals

It was Sommerfeld and Bethe (1933) who first calculated the electron tunnel current through a metal-insulator-metal (M-I-M) tunneling

Figure 2-1: Thin Film Tunneling Junction

diode. During the past 36 years, considerable theoretical work on tunneling structures has been carried out--bibliographies to the literature on this subject can be found in Stratton (1962), Simmons (1963A,B), Kuhn (1966) and Duke (1969)--but the actual physical preparation and study of thin film tunnel junctions was delayed until 1960 when high vacuum and thin film technology had sufficiently advanced for Giaever (1960) to make $\text{Al-Al}_2\text{O}_3\text{-Pb}$ tunneling junctions. In this section, the general result for the tunneling current between normal metals is quoted followed by an approximation due to Simmons which finds application in relating the observed tunnel current to the insulator thickness. (Details concerning the calculations may be found in Harrison (1961) and the references cited above.)

1. Tunneling Current

The potential energy diagram for an ideal M-I-M tunnel junction is shown in figure 2-2. As long as one is interested only in the gross features of the I-V characteristic (of the order of hundreds of meV), it is permissible to use a non-interacting or free electron model of the tunneling process (see eg. Duke, 1969). (In such a model the interactions of the electrons both with themselves and the phonons in the metal and the barrier are neglected; the effect of the periodic potential of the lattice is approximated by an effective mass.) If a one dimensional barrier and specular transmission through it are assumed (ie. the transverse wave number k_t is the same for the initial and final electron states) and if it is further assumed that the matrix element for the transition from metal 1 to metal 2 is such that $|M_{12}|^2 = |M_{21}|^2 = |M|^2$, the tunnel current density in the x direction is (Kuhn, 1966 or Harrison, 1961)

$$j_1 = \frac{4\pi e}{h} \sum_{k_t} \int_{-\infty}^{\infty} |M|^2 \rho_{1x}(E) \rho_{2x}(E + eV) [f_1(E) - f_2(E + eV)] dE \quad (2-1)$$

In equation 2-1, e is the electronic charge, $h = \hbar 2\pi$ is Planck's constant, $f(E)$ is the Fermi distribution function and $\rho_x(E) \propto (\partial E(k)/\partial k_x)^{-1}$ is a density of states giving the number of electrons having "x directed" energy between E_x and $E_x + dE_x$ with fixed transverse momentum k_t . Within the approximations stated, equation 2-1 is the fundamental equation for tunneling through a one-dimensional barrier and applies equally well to

Figure 2-2: Potential Energy Diagram for Ideal M-I-M Structure (after Simmons, 1963A)

M-I-M structures, whether the metal is normal or superconducting. A distinction between the two cases must be made, as will be seen shortly, when the matrix element is evaluated.

Evaluating $|M|^2$ by assuming that the electrons were essentially localized to one side of the barrier or the other and using a WKB approximation through the barrier region, Harrison (1961) found

$$|M|^2 = T(E_x) [4\pi^2 \rho_{1x}(E) \rho_{2x}(E + eV)]^{-1}$$

Substitution in equation 2-1 gives

$$j_1 = \frac{2e}{h} \sum_{k_t} \int_0^\infty T(E_x) [f(E) - f(E + eV)] dE \quad (2-2)$$

where $T(E_x)$ is the WKB result for the barrier transmission of a free electron incident on the barrier.

Examination of equation 2-2 reveals that the density of states factors appearing in 2-1 have cancelled with those contained in the matrix element so that the tunneling current no longer depends on them. This result is consistent with the observed behaviour of normal M-I-M tunnel junctions but contrary to the behaviour of junctions in which one or both of the metals are superconducting (see eg. figure 2-6). Briefly, this absence of the density of states from the result for superconductors arises from the failure of the independent particle model to adequately represent superconductors. The discrepancy may be removed by assuming (Harrison, 1961) that the density of states entering equation 2-1 corresponds to that of quasiparticles but that $|M|^2$ is still satisfactorily given by the normal density of states. (The validity of this assumption has been established from microscopic theory, see eg. Douglas & Falicov, 1964.) As a result, cancellation of these factors is removed and agreement with experiment obtained. Further discussion of tunneling between superconductors is reserved for section D of this chapter.

With equation 2-2, the problem of finding the tunnel current between two normal metals is solved in principle but further approximations are required to obtain an explicit current voltage relation

for comparison to experiment. Of particular interest to the present experiment is the relation of the tunnel current to the thickness of the insulator.

2. Dependence of Tunnel Current Upon Barrier Parameters

It will be shown in chapter 3 that one condition for optimum performance of the tunnel junction as a charged particle detector is that the barrier be as thin as possible. Consequently, from an experimentalist's point of view, an expression relating effective barrier thickness to the measurable I-V characteristic will serve as a useful guide in fabricating tunnel junctions. Such calculations have been given by Simmons (1963A,B) who has related the current density to the thickness, average value of the insulator energy gap and dielectric constant of the barrier. It is the purpose of this paragraph to give his results for barrier parameters of interest to this experiment.

If the sum over k_t is changed to an integral over the appropriate surface in k space and it is assumed that the effective mass of the electron is the same whether in the conduction band of the metal or tunneling through the insulator, equation 2-2 may be re-written in the form (Kuhn, 1966; Simmons, 1963A--equation 7)

$$j = \frac{4\pi me}{h^3} \int_0^{E_m} T(E_x) dE_x \int_0^\infty [f(E) - f(E + eV)] dE_r \quad (2-3)$$

where m is the electron mass, E_m is the maximum energy of the electrons and $E_r = \frac{1}{2}m(v_y^2 + v_z^2)$. Evaluation of equation 2-3 to determine the current voltage relation explicitly is made difficult by the complicated behaviour of $T(E_x)$ which, with reference to figure 2-2, may be written

$$T(E_x) = \exp \left[\frac{-4\pi}{h} \int_{s_1}^{s_2} \{2m(\phi(x) - E_x)\}^{\frac{1}{2}} dx \right]$$

Simmons' contribution was to express $T(E_x)$ in an approximate form for various barrier shapes $\phi(x)$ and subsequently calculate the tunneling current density (J) at $T = 0$ K for low, intermediate and high voltages between electrodes of similar and dissimilar metals. For similar metals--

the type of junction used in this experiment--his expression for a rectangular barrier, with image forces included, in the region of low voltages (V) is

$$\frac{J}{V} = \frac{(2m)^{\frac{1}{2}}}{s} \left(\frac{e}{h} \right)^2 \phi_L^{\frac{1}{2}} \exp(-A\phi_L)^{\frac{1}{2}} \quad (2-4)$$

where

$$\phi_L = \phi_0 - 2 \left(\frac{1.15\lambda s}{s_2 - s_1} \right) \ln \left(\frac{s_2}{s_1} \right)$$

and

$$s_1 = \frac{1}{2}s[1 - (1 - 1.15\lambda/K\phi_0)^{\frac{1}{2}}] \approx 3(eV \text{ \AA})/K\phi_0,$$

$$K\phi_0 \gg 1.15\lambda$$

$$A \approx (4\pi s/h) (2m)^{\frac{1}{2}}$$

m = electron mass

e = electron charge

s = thickness of insulating region = $s_2 + s_1$

h = Planck's constant

ϕ_0 = mean barrier height

$\lambda = e^2 \ln(2) / 16\pi K \epsilon_0 s$

ϵ = permittivity of insulating film

ϵ_0 = permittivity of empty space

K = dielectric constant = ϵ/ϵ_0

Equation 2-4 has been evaluated for several values of K and ϕ_0 and plotted in figure 2-3. Such curves provide a convenient estimate of the insulator thickness once the room (or lower) temperature experimental junction conductance is known. ($\lambda = \frac{1}{2}$ the quantity used by Simmons, 1963A)

It is important to note that the thickness $S_T = s$ derived from tunneling resistance measurements (R_n) and equation 2-4 ($V/J = R_n \cdot A$) is the thickness of an ideal, uniform barrier whose area A is the geometrical area of the junction. The thickness of a real barrier, however, is not constant but varies stochastically from point to point (Hurych, 1966).

Now the thinner regions of the insulator will be emphasized because of the exponential dependence of resistance upon thickness so that the effective area of the barrier for tunneling is not necessarily equal to A (geometrical) nor is the average thickness $\langle S \rangle_A = \iint_A S(x,y) dx dy$ necessarily equal to S_T . This point is of relevance to chapter 3 where the dependence of tunneling probability on insulator thickness is discussed and to chapter 6 where the junction capacitance is estimated.

C. Review of Superconductivity Theory

As mentioned earlier, this section is devoted to those aspects of superconductivity theory which are germane to an understanding of the superconducting tunnel junction and its utilization as a detector of charged particles. Topics such as zero electrical resistance and the Meissner effect, while of interest in general, will therefore be ignored (apart from the fact that the standard 4-terminal network for measuring I-V characteristics of tunnel junctions is exact only in the limit of zero film resistance; see appendix D).

Many reviews are now available dealing with the subject of superconductivity in great detail so that no attempt will be made to justify or derive the results given in the succeeding paragraphs. The interested reader is referred to: Kuper (1968), Rickayzen (1965), Blatt (1964), Schrieffer (1964) or Bogoliubov (1962); concise presentations are given by Tinkham (1965) and Lynton (1964).

The prominent part played by the energy gap, as far as the tunnel junction detector is concerned, was stressed in chapter 1. This section therefore will be concerned primarily with the energy gap and its dependence upon temperature, specimen size and magnetic field. (A comprehensive theoretical and experimental review of the energy gap in superconductors has been given by Douglass and Falicov (1964).) Other important concepts such as the two fluid model, the quasiparticle density of states and the quasiparticle distribution function are introduced at this time for clarity and subsequent reference.

1. Two Fluid Model

The very successful Bardeen, Cooper, Schrieffer (BCS, 1957) theory views the superconductor as consisting of two interpenetrating fluids: a superfluid and a normal fluid. Making up the superfluid are

Cooper pairs which are pairs of electrons, loosely "bound" via a phonon-mediated interaction and occupying a pair of electron states $(\vec{k}\uparrow, -\vec{k}\downarrow)$ within a thin shell $(\pm(k_B\theta_D/E_F)k_F \approx \pm .004k_F)$ straddling the Fermi surface. ($\theta_D \approx 200$ K is a typical Debye temperature, k_B is Boltzmann's constant) It is the Cooper pairs which give the superconductor its characteristic long range order and its ability to carry current without dissipation. The normal fluid consists of quasiparticle excitations from the superconducting ground state, having characteristics related to electron and hole excitations in a normal metal. Much as in a normal metal, the quasiparticles are readily scattered and are able to change their energy by arbitrarily small amounts. The main difference between excitations in superconductors and in metals lies in the relationship between energy and momentum, as described below.

As the temperature of a metal capable of becoming a superconductor is lowered below a characteristic transition temperature T_c , the Fermi sea becomes unstable against the formation of Cooper pairs. Further decreasing of the temperature permits more and more quasiparticles to condense into the paired state until at absolute zero no unpaired quasiparticles, whose energies are within approximately $k_B\theta_D$ of the Fermi energy are present. This highly correlated BCS ground state of the superconductor leads naturally to an energy gap or band of forbidden energies in the single quasiparticle energy spectrum of a superconductor as will be seen shortly.

2. Energy Gap

At 0 K, the lowest excited state of the superconductor may be found by considering the effect of adding to the system an electron in the state $\vec{k}\uparrow$ where its mate $-\vec{k}\downarrow$ is assumed to be empty. The effect of this process is to remove the state $(\vec{k}\uparrow, -\vec{k}\downarrow)$ from the manifold of states participating in the pairing interaction responsible for binding together the Cooper pairs; removing this state requires a finite energy Δ_k . If the Bloch energy of the added electron is ϵ_k measured with respect to the Fermi energy, the energy required to create a quasiparticle excitation in $\vec{k}\uparrow$ in the superconductor is then given by:

$$E_k = (\epsilon_k^2 + \Delta_k^2)^{\frac{1}{2}} \quad (2-5)$$

In contrast to the situation in normal metals at 0 K, the effect of the pairing interaction in superconductors is to establish a finite probability that an electron may be added to a state below as well as above the Fermi surface. In either case, the excitation energy E_k is positive. Similarly, the energy required to extract an electron from $\vec{k}\uparrow$ in the ground state (or create a hole in $\vec{k}\uparrow$) is found to be given by equation 2-5 and to be positive whether $|\vec{k}|$ is greater or less than the Fermi momentum k_F . The minimum energy required to inject or extract an electron is then $\Delta k_F \equiv \Delta \approx 10^{-4} - 10^{-3}$ eV (see eg. table 2-1). This gap in the single quasiparticle excitation spectrum is that which would be observed in experiments which deposit or withdraw electrons from the superconductor (as in normal metal-insulator-superconductor tunnel junctions).

In experiments which do not inject or extract electrons (as in electromagnetic absorption work) the energy gap in the observed spectrum is $E_g = 2\Delta$ for now two quasiparticles must be created: (a) an electron is extracted from $\vec{k}_1\uparrow$ (or a quasihole is created in $\vec{k}_1\uparrow$) requiring energy E_{k1} ; (b) an electron is created in state $\vec{k}_2\uparrow$ requiring energy E_{k2} . The total excitation energy required is then $E_{k1} + E_{k2} \geq 2\Delta = E_g$. Paragraphs (a) to (c) following consider briefly the temperature, specimen size and magnetic field dependence of the energy gap.

Before going on to these topics, one important point should be stressed. Since quasiparticles must be created in pairs to conserve, on the average, the total number of electrons, it is tempting to consider these excitations as electron-like and hole-like in analogy with the situation in normal metals. Such an analogy must be used with caution however as the excitations from the ground state are really linear combinations of "holes" and "electrons" becoming hole-like or electron-like only in the limit of $|\vec{k} - \vec{k}_F| > (\Delta/2\epsilon_F) k_F$. (The excitations from the ground state are conveniently described by the Bogoliubov operators

$$\begin{aligned} \gamma_{k\uparrow}^+ &= u_k c_{k\uparrow}^+ - v_k c_{-k\downarrow} \\ \gamma_{-k\downarrow}^+ &= u_k c_{-k\downarrow}^+ + v_k c_{k\uparrow} \end{aligned} \quad (2-6)$$

which create quasiparticles in states $\vec{k}\uparrow$ and $-\vec{k}\uparrow$ respectively. In equations 2-6, c_k^+ and c_k are the creation and annihilation operators for Fermions, $v_k^2 = \frac{1}{2}(1 - \epsilon_k/E_k)$ is the probability that $(\vec{k}\uparrow, -\vec{k}\uparrow)$ is occupied and $u_k^2 = 1 - v_k^2$ is the probability that $(\vec{k}\uparrow, -\vec{k}\uparrow)$ is unoccupied. Clearly, for $k \gg k_F$, $v_k^2 \rightarrow 0$ and $\gamma_{k\uparrow}^+ \rightarrow c_{k\uparrow}^+$. The importance of this distinction will become evident in section D where the use of the so called semiconductor model to represent superconductors is briefly discussed.

(a) Temperature Dependence of the Energy Gap

At $T = 0$ K, BCS find the magnitude of the energy gap to be

$$2\Delta(0) = 3.5 k_B T_c$$

where T_c is the superconducting-normal transition temperature mentioned earlier. The variation of the ratio $\Delta(T)/\Delta(0)$ with temperature is plotted in figure 2-4 indicating that the gap decreases very slowly for $T < T_c/3$ so that

$$2\Delta(T) \approx 3.5 k_B T_c \quad 0 \leq T \leq 0.3 T_c$$

For temperatures near T_c , the gap width is approximated by

$$2\Delta(T) \approx 3.2 k_B T_c (1 - T/T_c)^{\frac{1}{2}}$$

Tabulations of $\Delta(T)/\Delta(0)$ are given by Parmenter and Berton (1964) and Muhlschlegel (1959).

Table 2-1 gives the transition temperatures as well as the theoretical and experimental values for the energy gap for four commonly used superconductors (cf. also Douglass & Falicov, 1964). For Sn at 1.2 K it is clear, from figure 2-4, that the gap width may be taken to be $2\Delta(0)$ with negligible error.

(b) Specimen-Size Dependence of the Energy Gap

Expressions derived from the BCS microscopic theory are all for systems of infinite volume; for thin films proper boundary conditions should be considered. Anderson's theory of the "dirty"

Figure 2-4: Temperature Dependence of Energy Gap parameter $\Delta(T)$

superconductor (Anderson, 1959) and the good agreement of energy gap

Superconductor	T_c (°K)	$2\Delta(0)$ meV	
		BCS	Exp't. ($T < \frac{1}{2}T_c$)
Al	1.19	0.36	0.35
In	3.41	1.03	1.05
Sn	3.72	1.13	1.12
Pb	7.18	2.18	2.54

Table 2-1: Comparison of Theoretical and Experimental Values of $2\Delta(0)$.

values whether determined experimentally from bulk or thin film samples (Lynton, 1964), indicate that in zero magnetic field, the boundary condition problem is not a serious one so that it is assumed henceforth that

$$\Delta(\text{films}) \approx \Delta(\text{bulk}), \quad H = 0$$

When $H \neq 0$, however, the situation becomes more complicated as seen in the following section.

(c) Magnetic Field Dependence of Energy Gap

It is well known (see eg. Lynton (1964) or Kuper (1968)) that for $T < T_c$, superconductivity in a bulk specimen may be quenched by applying a magnetic field of sufficient strength H_c so that, if $H_c = H_0$ for $T = 0$,

$$\frac{H_c(T)}{H_0} = 1 - \left(\frac{T}{T_c} \right)^2$$

As it has been shown, the presence of an energy gap is fundamental to the superconducting state which implies, that if superconductivity can be magnetically quenched for $T < T_c$, the energy gap width must depend on the magnetic field present.

No complete microscopic theory of the effect of a magnetic field on the energy gap is yet available. Experimental

evidence with thin films (Meservey and Douglass, 1964) taken at $T = 0.12T_c$ indicates that the gap goes continuously to zero with magnetic field H according to

$$\frac{\Delta(H)}{\Delta(0)} = 1 - (H/H_c)^2 \quad (2-7)$$

For their films, the ratio $d/\lambda = 1.5$ was less than the approximate value of 4 which is appropriate to the films used in the present experiment but, as described in section A, chapter 6, the 7% gap decrease for $H = 100$ Oe predicted by 2-7 agrees favourably with that which we measured for tin. (Here, d is the film thickness and λ is a characteristic penetration depth of the order of 500 Å in tin at 1.2 K.)

3. Distribution Function for Quasiparticles

BCS assume the quasiparticle excitations are independent and obey Fermi Dirac statistics with the result that the distribution function for the quasiparticles is the familiar Fermi distribution

$$g(E) = [1 + \exp(\beta E)]^{-1} \quad (2-8)$$

where $\beta = 1/k_B T$ and all the other symbols have been defined previously
(Note: $E \geq \Delta \geq 0$)

4. Density of States

The single quasiparticle density of states per unit energy per unit volume is given by BCS as

$$N_S(E) = N_m(0) |E| (E^2 - \Delta^2(T))^{-\frac{1}{2}}, \quad E \geq \Delta(T) \quad (2-9)$$

where $N_m(0)$ is the density of electron states of one spin per unit energy per unit volume at the Fermi surface of the normal metal. From equation 2-9, it is evident that $N_S(E)$ reduces to $N_m(0)$ for $E \gg \Delta(T)$ as it should. (The interaction responsible for superconductivity affects only those states within a few kT of the Fermi energy and the quasiparticle and normal Bloch states should coincide at energies far from the Fermi surface). The singularity at $E \approx \pm \Delta(T)$ arises from the fact that states "squeezed out"

of the gap must be accommodated at the edges, to satisfy the requirement that the total number of states be conserved.

D. Single Quasiparticle Tunneling

In the discussion of tunneling between normal metals given in section B it was pointed out that, in order to obtain agreement with the results obtained when one or both of the metals are superconducting, it was necessary only to substitute the quasiparticle density of states for that of the electrons in the normal metal. The purpose of this section is to briefly outline why this assertion is valid and why the semiconductor energy diagram may be safely used to interpret single quasiparticle tunneling. Concluding the section is an expression for the single quasiparticle tunneling current between superconductors derived on the basis of the semiconductor model.

The first experiments dealing with tunneling currents between a normal metal and a superconductor were reported by Giaever (1960). Soon afterward, Nicol et al (1960) and Giaever and Megerle (1961) extended the study to the case of tunneling between two superconductors. Since then a vast experimental and theoretical effort has gone into understanding and exploiting this phenomenon; the present state of the subject is summarized by Burstein and Lundqvist (1969).

1. Tunneling Between a Normal Metal and a Superconductor

Cohen et al (1962) computed the current I_{SM} in an M-S tunnel structure at finite temperature and found that, because of the twofold degeneracy of energy levels in superconductors, the coherence factors u_k and v_k from equation 2-6 disappeared yielding the simple result

$$I_{SM} = C \int_{-\infty}^{\infty} |M|^2 [f_M(E - eV) - f_S(E)] N_M(E - eV) N_S(E) dE \quad (2-10)$$

(The degeneracy arises from the fact that for each $|k_1| > k_F$ there is a $|k_2| < k_F$ such that $E_{k1} = E_{k2}$ and $u_{k1}^2 + u_{k2}^2 = 1 = v_{k1}^2 + v_{k2}^2$. In summing over all k , terms of the form $\sum_k u_k^2 h(k) + v_k^2 h'(k)$ reduce to $\sum_{|k| > k_F} h(k) + h'(k)$.) In equation 2-10, C is a constant, $|M|^2$ is the matrix element relating normal electrons on each side of the insulator, f_M is the Fermi factor for the normal metal, $N_M(E)$ is the density of electron

states in the normal metal, $N_S(E)$ is the density of quasiparticle states in the superconductor (equation 2-9) and V is the difference in the Fermi levels. The occupation probability $f_S(E)$ is defined such that

$$\begin{aligned} f_S(E) &= g(E) \text{ for } E > 0 \\ &= 1-g(|E|) \text{ for } E < 0 \end{aligned} \quad (2-11)$$

where $g(E)$ is the quasiparticle occupation probability given in equation 2-8.

The significance of this result obtained by Cohen et al is that it could have been derived from the general tunneling equation 2-1 simply by inserting the quasiparticle density of states for metal 1 say, remembering that the one dimensional density of states is proportional to the total density of states, and summing over k_t with the assumption of spherical symmetry of the dependence of E upon wave number. In other words, the only essential difference in the tunneling current when one of the metals is superconducting is in the density of states factor--the u_k and v_k may be ignored and the quasiparticle excitations in the superconductor regarded in one-to-one correspondence with the electron and hole excitations in the normal state. This view of the superconductor is, of course, the so-called semiconductor model first introduced by Giaever and Megerle (1961) to successfully explain their experimental results. Taking a slightly different approach to Cohen et al, Bardeen (1962) confirmed their result, providing further justification for using the semiconductor model in calculating simple single quasiparticle tunneling currents. (When the simultaneous tunneling of two or more quasiparticles is considered (see eg. Wilkins, 1969) the coherence factors u_k and v_k do not cancel and the simple independent quasiparticle model with a modified density of states is inadequate.)

2. Tunneling Between Two Superconductors

Following the approach of Cohen et al (1962), Josephson (1962) computed the tunneling current between two superconductors (S_1 - S_2) and found it to be of the form

$$J_{SS} = J_0 + \frac{1}{2} J_1 S_1^+ S_2 + \frac{1}{2} J_1^* S_2^+ S_1 \quad (2-12)$$

where S_i^+ is an operator defined in such a way that when acting on the ground state of superconductor 1 with N electrons it creates a new superconducting ground state of the system with $N + 2$ electrons. J_0 is similar to the expression obtained by Cohen et al and reduces because of cancellation of the coherence factors to a form like equation 2-10 with $N_M(E) \rightarrow N_S(E)$ and $f_M(E) \rightarrow f_{S1}(E)$. The other terms in equation 2-12 represent a new effect, now known as the Josephson effect, which is discussed further in section E of this chapter.

The important result from the foregoing discussion of single particle tunneling in M-S or S_1 - S_2 structures is that the tunnel current due to thermally excited quasiparticles may be satisfactorily derived from the simple semiconductor-type single particle approximation of a superconductor. Such a derivation is outlined in the next paragraph for an S_1 - S_2 junction as this is the situation of relevance to the superconducting tunnel junction to be used for charged particle detection.

3. The S_1 - S_2 Tunneling Current

When a superconducting tunnel junction like that of figure 2-1 is biased at a voltage V , it may be represented by figure 2-5. In the semiconductor model sketched there, the thermally excited quasiparticles in each superconductor are represented by the occupied states above and the unoccupied states below the gap. The density of the quasiparticle states is given by (cf. equation 2-9)

$$\begin{aligned} N_{S1}(E) &= N_1(0) \quad n_1(E) = N_1(0) |E| / (E^2 - \Delta_1^2(T))^{1/2}, \quad |E| > \Delta_1(T) \\ &= 0, \quad |E| < \Delta_1(T) \end{aligned} \quad (2-13)$$

and they are occupied with probability $f_S(E)$ given by equation 2-11. The single quasiparticle current flowing from side 1 to side 2 may then be written (see eg. Giaever and Megerle (1961) or Shapiro et al, (1962)) as

$$I_{1-2} = A' \int_{-\epsilon_{F2}}^{\infty} |M|^2 N_{S1}(E) f_{S1}(E) N_{S2}(E + eV) (1 - f_{S2}(E + eV)) dE$$

where A' is a constant, $|M|^2$ is the matrix element for the transition and

Figure 2-5: Quasiparticle Energy Diagram for S_1 -I- S_2 Junction

Figure 2-6: I-V Characteristic for Asymmetric Junction.
(after Shapiro et al. 1962)

ϵ_{F2} is the Fermi energy of metal 2. A similar result obtains for I_{2-1} so that by assuming the density of states $N_1(0)$ and $|M|^2$ (see Bardeen, 1961) are constant near the Fermi surface, the net current may be written

$$I = I_{1-2} - I_{2-1} = A|M|^2 \int_{-\infty}^{\infty} n_1(E) n_2(E+eV) [f_S(E) - f_S(E+eV)] dE \quad (2-14)$$

where A is a constant. (For mathematical convenience, the lower limit of integration has been extended to $-\infty$. Because of the action of the Fermi functions the important region of integration is confined to an interval of a few $k_B T$ about the Fermi level.) The similarity between the results of equation 2-14 derived from the phenomenological semiconductor picture and those of equation 2-10 derived from an exact microscopic approach is evident.

The agreement of equation 2-14 with experiment has been demonstrated by Shapiro et al (1962) who evaluated the equation numerically and obtained a close fit with their experimental data (see figure 2-6).

E. Josephson Tunneling

The tunneling current between two superconductors as calculated by Josephson (1962) was given in equation 2-12. Two new effects implicit in this result were indicated by Josephson at that time and later confirmed experimentally: (1) at zero bias voltage, the single quasiparticle current $J_0 = 0$, but a dc supercurrent up to a maximum of $|J_1|$ can occur; (2) at finite voltages the usual dc current J_0 appears but there is also a high frequency ac current of amplitude $|J_1|$ and frequency $2eV/h$ ($1\mu V$ corresponds to 483.6 MHz). The latter effect--the ac Josephson effect--is not relevant to the superconducting tunnel junction charged particle detector and will not be discussed further; the interested reader is referred to the book edited by Burstein and Lundqvist (1969).

The other effect--the dc Josephson effect--corresponds to the nondissipative transfer of Cooper pairs across the insulator in which the quasiparticle distribution in either superconductor is not disturbed. Physically, the effect arises when two superconductors are arranged sufficiently close together ($\approx 10 \text{ \AA}$) as to become weakly coupled in the sense that the relative phase between the Cooper pairs in each of the super-

conductors is no longer arbitrary. In other words, the Cooper pair correlations extend through the intervening distance so that the two superconductors behave in some respects as a single block. The Josephson effect is thus not necessarily a tunneling phenomenon but tunnel junctions have been used as a means of studying the effect because this is the physical situation for which detailed calculations are most readily made.

As mentioned in the introduction to this chapter, the dc Josephson supercurrent is of interest to the present experiment only because of its nuisance value. The way in which this effect is undesirable is outlined in paragraph 1. Paragraphs 2 and 3 briefly review the theory of the dc Josephson supercurrent and paragraph 4 considers how the supercurrent may be suppressed with a magnetic field.

In addition to the Burstein, Lundqvist book (1969) a recent concise review of the subject has been given by Anderson (1967); probably the most recent comprehensive bibliography is given by Schroen (1968).

1. Unfavourable Aspects of the Josephson Effect

With reference to figure 2-7 it is seen that the I-V characteristic of a junction exhibiting Josephson tunneling features a large supercurrent which may exceed the magnitude of the thermally-excited quasiparticle tunneling current discussed in section D and shown as the curve labelled "ideal" ($V < 2\Delta/e$) in figure 2-7), by a factor of 5 to 10. When the supercurrent reaches a critical value (I_{crit}) it is replaced by tunneling quasiparticles and a voltage $V = 2\Delta/e$ appears across the junction. Now it is evident that a Josephson junction biased at $I < I_{crit}$ might be made to switch from $V = 0$ to $V = 2\Delta/e$ when bombarded with a charged particle and so be useful as a particle sensor and energy threshold discriminator but its non-linear nature makes this response unsuitable for multichannel spectrometry.

It is shown in chapter 3 that the desired operating point of the tunnel junction detector (denoted by Q in figure 2-8(b)) is in the region of high differential resistance ($r = \partial V / \partial I$)_T on the quasiparticle tunneling curve. As this biasing point cannot be reached reproducibly in a junction exhibiting a Josephson supercurrent, a way must be found to prevent pair tunneling from occurring. Such a technique is outlined in paragraph 4 following.

Figure 2-7: I-V Characteristic for Junction Displaying dc Josephson Current

2. Theory of the dc Josephson Effect

The detailed microscopic theory of the Josephson effect (Scalapino (1969), Josephson (1965), Anderson (1964)) shows that the super-current density J passing between two superconductors comprising a tunnel junction is given by (cf. equation 2-12)

$$J = J_1 \sin \phi \quad (2-15)$$

where ϕ is the quantum mechanical phase difference between the pair wave-functions in the two superconductors and J_1 is a measure of the tunneling probability through the barrier. J_1 varies exponentially with the barrier thickness and also depends on the temperature. Ambegoakar and Baratoff (1963) have calculated J_1 for a symmetric junction--identical superconductors on either side of the barrier--and find it to be

$$J_1(T) = \frac{\pi \Delta(T)}{2eR_n A} \tanh \left[\frac{\Delta(T)}{2k_B T} \right]$$

where e is the electronic charge, R_n is the low temperature, low voltage resistance of the junction when in its normal state and A is the junction overlap area. When $T = 0$ K, J_1 is a maximum given by

$$J_1(0) = \frac{\pi \Delta(0)}{2eR_n A} \quad (2-16)$$

and for typical tin specimens used in this experiment,
 $(R_n \approx 0.1 \Omega, A = 7 \times 10^{-4} \text{ cm}^2)$ $I_1 = J_1 \cdot A \approx 10 \text{ mA}$.

In practice, the current passing through a junction is controlled by an external power supply--perhaps a battery in series with a variable resistor (see eg. figure 5-1)--and the phase difference ϕ adjusts itself as in equation (2-15) to carry this value of current. Consequently as the current determined by the external circuit is increased, ϕ increases until it reaches $\frac{1}{2}\pi$ at which point the junction is carrying the maximum or critical dc Josephson current $I_c = J_1(\text{max}) A$. For currents greater than I_c , the junction switches to the quasiparticle I-V curve as shown in the recorder tracing of figure 2-7. The dotted line on the characteristic of the figure indicates retrace behaviour common to almost all specimens.

When the current (I) through the junction is decreasing, the junction becomes unstable for $I < I_c$ and switches over to the zero-voltage or supercurrent condition with the slightest electrical or mechanical disturbance. The origin and detailed shape of this hysteresis loop are still an open question (Schroen, 1968). Basically, it is governed by the quasiparticle tunneling current and some workers (March and Blackford, 1967) suggest that the power dissipated in the junction may play an important role.

3. Effect of Magnetic Field on dc Josephson Current

A further result of the microscopic theory is that the dc Josephson current should depend on the vector potential since the quantum mechanical phase difference depends on the vector potential. Hence, in mks units, equation 2-15 should be (Jaklevic et al, 1965)

$$J = J_1 \sin \left[\phi - \frac{2e}{\hbar} \int_1^2 \vec{A} \cdot d\vec{\ell} \right] \quad (2-17)$$

where the line integral term from side 1 to side 2 is included to ensure that the current density is invariant under a gauge transformation. Equation 2-17 has been evaluated by Jaklevic et al, (1965) for a symmetric tunneling junction placed in a uniform magnetic field $\vec{B}$ which is parallel to the oxide layer; they find

$$J(x) = J_1 \sin [\phi(0) + (2e/\hbar) Bx(t + 2\lambda)] \quad (2-18)$$

where t is the oxide thickness, λ is the penetration depth of the magnetic field and x is the distance along the junction perpendicular to $\vec{B}$ ($x = 0$ at the center of the junction, see figure 2-8(a).) Equation 2-18, which gives the magnetic field dependence of the Josephson current, indicates that the current at one point can flow opposite to the way it flows at another. It follows from equation 2-18 therefore that for a junction of given size there are certain values of $\vec{B}$ for which the maximum dc Josephson current $I(\max)$ carried by the junction will be zero. This may be seen by integrating 2-18 from $-\frac{1}{2}W$ to $+\frac{1}{2}W$ (W is the length of the junction perpendicular to $\vec{B}$)

Figure 2-8(a): Schematic Cross-Section of Tunneling Junction

Figure 2-8(b): Dependence of $I_{\max}$ upon Applied Magnetic Field $\vec{B}$

and setting $I_1 = J_1 \cdot A$ so that

$$I = I_1 \frac{\sin(p\pi)}{p\pi} \cdot \sin \phi(0) \quad (2-19)$$

where

$$p = \Phi_j / \Phi_0$$

and

$$\Phi_j = B(2\lambda + t)W = \text{flux enclosed by effective cross-sectional area of junction}$$

$$\Phi_0 = h/2e = 2.07 \times 10^{-7} \text{ G cm}^2 = \text{quantum unit of magnetic flux}$$

As mentioned above, the phase difference ϕ adjusts to the experimental conditions so that maximum positive current flow is obtained with $\phi(0) = \pm \frac{1}{2}\pi$ making

$$I(\max) = I_1 \left| \frac{\sin(p\pi)}{p\pi} \right| \quad (2-20)$$

It is clear from (2-20) that $I(\max) = 0$ whenever Φ_j equals some integral multiple of $h/2e$ and a plot of $I(\max)$ against $\vec{B}$ will resemble the familiar Fraunhofer diffraction pattern (see figure 2-8(b)). Typically, $W \approx 0.03 \text{ cm}$ and $\lambda \approx 500 \text{ \AA}$ so that the period of $I(\max)$ in $\vec{B}$ is about 0.7 G. The existence of this effect was first confirmed by Rowell (1963) using symmetrical Pb-Pb junctions.

In junctions of small dimension or low critical current, the magnetic field due to the tunneling current can be neglected. Such is not the case if the junction width becomes comparable to or larger than a characteristic length $2\lambda_J$, the Josephson penetration depth, given by (in mks units)

$$\lambda_J = (h/4e\mu_0 J_1)^{\frac{1}{2}} \quad (2-21)$$

where μ_0 is the permeability of free space and it is assumed that $2\lambda + t \approx 2\lambda$. The significance of λ_J is that when the junction width W is large compared to λ_J , the applied magnetic field is screened from the interior of the junction due to the flow of the Josephson currents themselves with the result that most of the direct current flows within a distance $2\lambda_J$ of the junction edges (Ferrell and Prange, 1963). Estimates of λ_J for

typical junctions in the literature range from 0.1 to 1.0 mm which is sufficiently large compared to the junction dimensions used in this experiment, that uniform penetration of the magnetic field may be safely assumed. Instances where λ_J is important have been discussed by Rowell (1963), Owen and Scalapino (1967) and Pritchard and Schroen (1968).

4. Suppression of Josephson Supercurrent

Equation 2-20 and figure 2-8(b) suggest two alternative approaches for magnetically suppressing the supercurrent: either apply a precisely determined field such that p is an integer (ie. $B = n\phi_0 (W(2\lambda + t))^{-1}$, $n = 1, 2, \dots$) making $I(\max) = 0$ or apply a sufficiently large field that the $(p\pi)^{-1}$ dependence reduces $I(\max)$ to negligible proportions.

Immediately, the first option is ruled out because of the fact that, in the junctions used in this experiment, no sharp minima in which the supercurrent vanished were ever observed. (Even if such minima had been obtained, an extremely well-regulated magnetic field would, of course, be required to maintain that condition.)

Experimentally, it turns out (see section 1, chapter 6) that the second approach is the one to take in that magnetic fields of 30-100 G are sufficient to effectively suppress the supercurrent with a maximum reduction in the energy gap width of less than about 6%.

F. Multiparticle Tunneling Phenomena

For the sake of completeness, it should be mentioned here that higher order or multiparticle tunneling processes may also occur in superconducting tunneling junctions. As the name implies, multiparticle tunneling is the name given to the simultaneous transfer of more than one quasiparticle through the tunnel junction barrier. (Probably no higher order than two-particle tunneling has ever been observed (Wilkins, 1969).) This second order process (proportional to $|M|^4$) goes only if there is an interaction between the particles (as in a superconductor) and is manifested as a sharp increase in the current at $eV = \Delta$ (see Taylor and Burstein, 1963).

An excellent review of this topic has recently been given by Wilkins (1969); further discussion may also be found in appendix B of this thesis where defective or "leaky" junctions are considered.

CHAPTER 3

OPERATING PRINCIPLES OF THE SUPERCONDUCTING CHARGED PARTICLE DETECTOR

A. Introduction

This chapter outlines the principles and practical operation requirements of the superconducting tunnel junction as a charged particle detector. To place this application of tunnel junctions in perspective, it is convenient to review briefly: (1) three of the present techniques for measuring charged particle energy spectra and (2) charged particle detection applications of other superconducting devices. Because of its relation to the detection of charged particles, previous work on the detection of microwave radiation is also reviewed (section B).

The detection of ionizing radiation is considered briefly in section C for the purpose of stipulating the attributes which the tunnel junction's response to such radiation should have if the junction is to be useful as a spectrometer. In section D it is established that the most favourable tunnel junction configuration is the symmetric one with lead films as the superconductor. The nature of the quasiparticle excitations ultimately generated by an alpha particle in the superconducting tunnel junction is considered in section E. A small signal equivalent circuit for the detector is given in section F following which (section G) the assumed form of the resulting signal current pulse is stated along with estimates of its parameters.

Because of the restriction it places on the maximum acceptable insulating layer thickness, the tunneling probability W_T is dealt with in section H. A note on the practical operation of the detector is given in section I and, concluding the chapter, is a short survey of tunnel junction noise sources (section J).

1. Present Methods of Charged Particle Spectrometry

Broadly speaking, the methods can be divided into three major categories: the measurement of momentum, energy loss or velocity. General reviews of these methods can be found in books by Yuan (1961) and Ajzenberg-Selove (1960); a short survey is given by Wood (1965). Since the superconducting "detector" is essentially an energy-loss device, only those detectors of the "energy loss measurement" type will be discussed here.

When a charged particle passes through matter, it loses energy through the excitation and ionization of atoms close to the path of the particle. This property has been exploited in the development of several types of particle spectrometers. Three in common use are the gas-filled, scintillation, and semiconductor detectors.

Basically, the gas-filled detector is a metal or glass chamber, equipped with an anode and cathode, and filled with an accurately defined volume of gas. The passage of an energetic charged particle through a suitable "window" into the gas volume causes electrons to be removed from some of the gas molecules thereby producing electron-positive-ion pairs. Under the influence of the electric field, the electrons migrate to the anode and the positive ions to the cathode. Because the average amount of energy lost by the charged particle to produce an ion pair is approximately independent of the energy of the particle, the amplitude of the resulting charge pulse is proportional to the energy lost in the gas by the ionizing particle.

A typical scintillation detector consists of a scintillating phosphor--crystal, liquid, plastic solid or gas--optically coupled to a photomultiplier tube. When an ionizing particle strikes the phosphor, the energy dissipated causes loosely bound electrons in the material to be excited into the conduction band. Then, at imperfection sites in the crystal, a large fraction of the excited electrons fall back to their ground state and emit a photon in a process called fluorescence. The photons emitted during fluorescence strike the photocathode of the photomultiplier tube whose output signal is proportional to the number of photons generated or the energy lost by the charged particle in traversing the phosphor. In this case, proportionality arises from the approximate constancy of the

average energy loss per photon generated.

During the past ten years, semiconductor detectors have become almost universally preferred for charged particle detection. Books by Dearnaley and Northrop (1966) and Taylor (1963) deal thoroughly with these detectors giving details on theory and application; a recent review by Tavendale (1967) surveys the present state of detector technology. Essentially, the semiconductor detector consists of a p-n junction which is formed close to one face of a slab of germanium or silicon. Typically the junction is within 0.5μ of the one face and is very abrupt. When the junction is reverse biased, a depletion region is created which forms the sensitive volume of the detector. The electrons and holes generated by the passage of a charged particle through this region drift to the electrodes under the applied electric field thence giving rise to a charge pulse whose amplitude is linearly related to the particle energy, the linearity being due to the approximate energy independence of the average energy loss per electron-hole pair.

It is of interest to note that the average energies w required to produce electron-hole pairs in silicon and germanium are 3.6 and 2.9 eV respectively, compared to an average value of about 30 eV per ion pair in gases and about 300 eV per photoelectron at the photocathode of a scintillation counter. (Dearnaley and Northrop, 1966). This implies, therefore, that for a given particle energy, the available signal in a semiconductor is significantly larger than for the other counters and that the statistical fluctuations, expressed as a percentage of the signal (see chapter 1) are correspondingly smaller.

2. Charged Particle Detection Applications of Superconducting Devices

Although the detection of charged particles with superconducting tunneling junctions has never been reported prior to the present experiment, several workers have proposed and/or performed experiments involving the detection of alpha particles using other superconducting devices.

Andrews et al (1949) bombarded a strip of niobium nitride ($3.5 \times 0.4 \times 0.006$ mm, $T_c = 15.6$ K), with alpha particles from a polonium source and found that countable pulses were produced, one for each impact. The NbN strip was maintained at 15.5 K, just below its transition tem-

perature, a direct current was applied and the potential between the ends of the strip monitored with a pulse amplifier and oscilloscope. It was found that the counting rate was a maximum when the ambient current was 40 mA and the specimen was maintained at the mid point of its transition between the normal and superconducting states--a change of ± 0.1 K reduced the signal to the same level as the noise. It was estimated that the pulses lasted for about 10^{-4} sec with an amplitude of about 0.1 μ V. The maximum signal to noise ratio was 3 to 1. No theoretical explanation of their findings was given by the authors but, since the strip was presumably in the intermediate state, the temperature rise accompanying the passage of an alpha particle would be manifested as an increase in the strip resistance and therefore (assuming constant current biasing) as an increase in the voltage across the specimen.

A proposal to use the cryotron, or superconducting switch, to detect ionizing particles was put forward by Sherman (1962,A). The cryotron, it was suggested, should consist of two gate loops, made of dissimilar superconductors such as lead and tin with a control loop of niobium on the lead gate. If the device were maintained at a temperature just below the lowest of the three critical temperatures ($T_c = 3.72$ K for Sn), it was argued that the cryotron could be kept in a metastable state with the tin loop (1000 Å thick, 10 μ wide) carrying a persistent current slightly less than the critical current at which superconductivity would be quenched. The passage of a charged particle into that loop might then establish a small region of normal metal across the tin film and cause the current to be suddenly switched into the lead loop which, prior to this instant, would be in the superconducting state but not carrying any current. A pulse would then result on the niobium control winding which could be used to register the count and then reflected back to drive the Pb gate normal thereby switching the current back into the tin loop ready for the next particle.

Sherman (1962,B) made a second proposal for a superconducting nuclear particle detector which was to consist of a narrow (10 μ), thin (1000 Å) film of current-carrying superconductor cooled well below its transition temperature and placed in series with a small resistor. It was argued that an alpha particle impinging on a tin film of such dimensions would cause the resistance to change from zero to about 0.2 Ω by inserting a small transverse strip of normal metal in the path of the current. The

reduction of current would be detected across the load resistor as a voltage pulse estimated to be about 1 mV for a 5 mA transport current and $10\ \Omega$ load. The rise time of the pulse was estimated to be roughly 10^{-11} sec and the fall time to be about 10^{-6} sec.

Recently, Spiel et al (1965), successfully detected 5.3 MeV alpha particles using thin superconducting films in an experiment similar in spirit to that outlined by Sherman (1962,B) but different in detail in that the particle induced transitions were observed by means of the IR drop produced by the ambient current along the length of the film rather than in a load resistance. The effect was observed in both indium and tin films; typical films were $1000\ \text{\AA}$ thick and $34\ \mu$ wide. Similarly to the experiment reported by Andrews (1949), voltage pulses occurred only for temperatures near the critical temperature and transport currents near the critical current. For an In film carrying a current of 5.5 mA at 3.3 K, the voltage pulses had a peak amplitude of approximately 200 μ V rising in about 15 nsec and decaying in roughly 70 nsec. The radius of the cylindrical volume driven normal by the passage of an alpha particle was estimated to be $14\ \mu$ under these conditions. The switching action in these devices could be used as a particle detector and energy threshold discriminator but it is not suited to multichannel spectrometry.

A subject that might be re-investigated, following Andrews' lead, is the possibility of employing intermediate state bolometers as charged particle spectrometers, for alloys having high sensitivity for heat pulse detection are now being developed. For example, Ackerman et al (1969) report the development of $400\ \text{\AA}$ thick Zn-Cd films for temperature pulse detection with a sensitivity $(\Delta R/RAT) = 400$ near the centre of the normal-superconducting transition ($T_c \approx 0.5\ \text{K}$). (For comparison, $(\Delta R/RAT) \approx \frac{1}{4}$ for carbon films.) It is conceivable that such a film, mounted on a suitable substrate and biased electrically, magnetically, and thermally at its mid-transition point, could be made to respond linearly to heat pulses resulting from energetic charged particles impinging upon the substrate. As the cross-sectional area of these films must be kept small to make the resistance high, the volume available for dissipation of a concentrated heat pulse is limited. Consequently, to avoid switching the film completely normal, it would be necessary to dissipate the original heat pulses in a relatively large volume which, of course, increases the response

time. The stopping of low energy radiation in the film itself is another alternative but now the heating would no longer be macroscopic and the complex behaviour of mixed state domains in the presence of electrical, magnetic and thermal gradients must be considered.

B. Detection of Microwave Radiation with Tunnel Junctions

Since the pioneering work of Giaever (1960,1961) on tunneling between superconductors, much attention has been given to using this phenomenon to study the more fundamental aspects of superconductivity. In addition, however, the potential practical applications of superconductive tunneling were quickly realized with the result that many attempts have been made to develop useful devices based on the tunneling principle. A recent review by Taylor (1968) summarizes work that has been done in this area.

Of particular interest, because the processes involved are somewhat analogous to those taking place in the detection of charged particles with tunnel junctions, is the work which has been done on the detection of microwave photons and phonons. The aim of this section is to acknowledge the historical priority of these tunnel junction applications and to illustrate thereby those ideas and results from previous workers which have benefited the present experiment. For simplicity, a symmetric superconducting junction will be considered.

1. Microwave and Infrared Photon Detection

The feasibility of using tunneling junctions as microwave and far infra-red quantum detectors was first proposed and analyzed by Burstein et al (1961A,B). The process involved increasing the quasiparticle density by "optically" exciting quasiparticles across the energy gap of the superconductor in a manner analogous to the quantum detection of visible and near infra-red radiation by p-n semiconductor diodes. In the former case, the lower frequency (ω) limit of detectable radiation would be governed by the requirement that the energy of the radiation absorbed in the junction films is

$$\hbar\omega \geq 2\Delta(T)$$

which is the energy required to break up a Cooper pair into two quasiparticles, which could subsequently tunnel--as indicated schematically in

figure 3-1(a). Following a short burst of radiation, the extra current due to the tunneling of the "optically" excited quasiparticles would be in the form of a pulse superimposed on the background tunneling current which, for bias voltages $eV < 2\Delta(T)$ (as seen in chapter 2), is proportional to the density of thermally excited quasiparticles. The magnitude and duration of this pulse would depend upon such factors as the quasiparticle tunneling probability and lifetime. An important feature of such detectors is that the frequency range they cover, approximately 85 GHz (Al) to 650 GHz (Pb), is one which has hitherto been virtually inaccessible. By the same token, because of the paucity of laboratory generators in this range, the properties of the Burstein device have not been widely studied; the only claim to the effect being found experimentally seems to be the unpublished work of Dayem and Miller (see Taylor, 1968).

If the interaction shown in figure 3-1(a) were the only one possible, no change in the tunneling current would be observed for $\hbar\omega < 2\Delta(T)$. Dayem and Martin (1962) noticed experimentally, however, that considerable interaction between the microwave field and the junction did take place for $\hbar\omega < 2\Delta(T)$ via a process which has come to be known as photon-assisted tunneling (see figure 3-1(b)). As in the case for excitation across the gap described above, the radiation breaks up a Cooper pair into two quasiparticles one of which remains in the normal fluid in S_a while the other tunnels through the barrier to become part of the normal fluid of S_b . It is clear that the process indicated in figure 3-1(b) can proceed for radiation energies less than $2\Delta(T)$ and, in fact, for a junction biased at voltage V , the threshold for a one-photon process is

$$\hbar\omega = 2\Delta(T) - eV$$

In addition, multiphoton processes, having a threshold given by

$$n\hbar\omega = 2\Delta(T) - eV \quad (n \text{ an integer})$$

were observed by Dayem and Martin and more recently by Cook and Everett (1967).

Perhaps the most useful reference at the present to the

(a)

(b)

(c)

Figure 3-1: Schematic Energy Diagrams Depicting Effects of Microwave Photon or Phonon Absorption (a,b) and Phonon Generation (c)

subject of photon-assisted tunneling is the work of Cook and Everett cited above who give extensive experimental results--including, for example, the detection of as little as 2 μ W of power at 36 GHz--and improve on the earlier theoretical interpretations of Tien and Gordon (1963) and Cohen, et al (1963).

2. Microwave Phonon Detection

In the two-fluid picture of the superconductor ($0 < T < T_c$), a dynamic equilibrium exists between the quasiparticles and the Cooper pairs. Pairs are continually being broken up by absorbing thermal phonons of energy greater than or equal to $2\Delta(T)$ and quasiparticles are continually recombining into pairs via the emission of phonons of energy equal to $2\Delta(T)$ --according to Burstein, et al (1961) and Rothwarf and Cohen (1963) phonon emission rather than photon emission is the dominant decay mode.

Because of this property, tunnel junctions can be used as either detectors or generators of very high frequency phonons--several hundreds of GHz.

Detection takes place via a process analogous to "optical" excitation as shown in figure 3-1(a). The absorption of phonons of energy greater than $2\Delta(T)$ in a junction results in the dissociation of Cooper pairs into quasiparticles which tunnel and give rise to an extra current. Unfortunately, this effect is difficult to study as phonons in this frequency range are prohibitively difficult to generate by conventional means. Eisenmenger and Dayem (1967) overcame this difficulty and were able to observe the effect by using another tunnel junction as the source of phonons. Acoustic coupling was provided by evaporating the two Sn-SnO₂-Sn junctions on opposite ends of a 1 cm long sapphire crystal; the generating unit was biased at $eV \geq 2\Delta(T)$ and the receiving one at $eV < 2\Delta(T)$.

The details of phonon generation are illustrated schematically in figure 3-1(c). Quasiparticles tunneling from S_a into S_b undergo in general two relaxation processes. First, there is a relaxation to the top of the energy gap in a characteristic time τ_T with the emission of a phonon of energy $\hbar\omega = eV - 2\Delta$ followed by a recombination with another excited quasiparticle in a characteristic time τ_R accompanied by the emission of a phonon of energy $\hbar\omega = 2\Delta$. Recent experimental results (Miller & Dayem, (1967)

and Levine and Hsieh (1968)) indicate $5 \times 10^{-8} < \tau_R < 2 \times 10^{-6}$ sec, depending on the temperature, and $\tau_T < 10^{-1} \tau_R$.

Microwave phonons can also interact with tunnel junctions to produce phonon-assisted tunneling which is analogous to the photon-assisted tunneling described above and shown in figure 3-1(b). Eisenmenger and Dayem (1967, see also Eisenmenger, 1969) found the effect to be negligible at the frequencies with which they were working but other workers, eg. Lax and Vernon (1965), Abeles and Goldstein (1965) and Goldstein et al (1966), using frequencies in the X band (9 GHz) have demonstrated the existence of the effect. Because the frequencies were too low to produce current steps at voltages $eV = 2\Delta(T) - n\hbar\omega$ as in the quantum photon-assisted tunneling experiments, the process was manifested simply as an excess current dependent on both the junction bias and acoustic power incident on the junction. Agreement with theory is only qualitative and several serious discrepancies between experiment and theory remain.

C. Detection of Ionizing Radiation

In nuclear particle spectroscopy, a particle is "detected" by measuring its kinetic energy and establishing an interval of time during which the particle interacted with the detecting medium. Thus, a distinction is made between devices which respond to nuclear radiation in the sense just defined and a device like the geiger counter which merely signals the presence of radiation, giving no measure of its energy other than it must exceed some threshold value. Photographic emulsions, on the other hand, yield information concerning the energy of individual particles but are not able to distinguish in time between the arrival of one particle and any other. To detect charged particles, then, the superconducting tunnel junction must respond in a manner that is not only proportional to the energy of the particles impinging on the junction but is sufficiently rapid and reversible as to minimize the time resolution--a measure of the smallest time interval which can occur between the arrival of two particles that are distinguished and recorded as separate entities.

The principle upon which the superconducting ionizing particle detector is based has been outlined in chapter 1. It is shown there and confirmed experimentally (see chapter 7) that τ , the characteristic time during which the signal current pulse relaxes back to zero, is of the order of $10^{-7} - 10^{-6}$ sec. As the current pulse is assumed to rise instantaneously,

(instantaneously meaning in times much shorter than τ) the overall pulse width is expected to be sufficiently short to make possible the resolution in time of individual events in fluxes as high as 10^6 particles/sec.

In the three types of particle detector described earlier in section A, there was some typical energy w required to generate the ion pairs, photoelectrons or electron-hole pairs which constituted the detection signal. This concept was carried over to the superconducting tunnel junction in chapter 1 where, if energy ΔE is deposited in the junction by a nuclear particle, the number of quasiparticle pairs excited is taken to be

$$N_o = \Delta E/w$$

Since in conventional particle spectrometers w does not vary strongly with particle energy, it is hoped (although there is no evidence yet to substantiate this hope) that the amplitude of the excess current pulse may be linearly dependent upon the particle energy loss in the detector.

From this brief heuristic discussion, it can be seen that in principle the superconducting tunnel junction may possibly satisfy the criteria established above for it to be considered a particle detector. The following sections will examine the attributes of the tunnel junction which determine its performance as a detector and describe how such a junction may be used in practice.

D. Optimum Junction Type and Constituent Material

1. Type of Junction to be Used as a Detector

There are four types or configurations of tunnel junction which could be of interest as charged particle detectors: the normal metal-normal metal junction (M-M), the normal metal-superconductor junction (M-S), the symmetric junction (S-S) with the same superconductor on either side of the barrier and the asymmetric junction (S_1 - S_2) composed of two different superconductors. (Because of the difficulty in dealing with the S_1 - S_2 junction mathematically and because the M-S and S-S junctions represent limiting cases of the S_1 - S_2 junction, it is not considered explicitly in the following discussion.) It turns out that the S-S junction is the most favourable.

The purpose of this section is to establish a quantitative basis for selecting the S-S configuration. First of all, general criteria for achieving maximum signal-to-noise ratio in the coupling of tunnel junctions to a common base preamplifier are introduced (paragraph a) after which the relevant parameters for each configuration are estimated from theory (paragraph b) and inter-compared (paragraph c). (A common base preamplifier was chosen over the common emitter configuration because the lower input impedance of the former was more compatible with the junction dynamic resistance $r = (\partial V / \partial I)_T$ which had been measured on various specimens and could be expected theoretically.)

It must be emphasized that this analysis in no way purports to be a general theory of the optimum coupling of tunnel junctions to an arbitrary amplifier nor is it implied that the common base transistor pre-amplifier used was the optimum preamplifier. While such a general theory of source-amplifier coupling is of interest for the ultimate, efficient use of tunnel junction detectors, it is of little value at the present rudimentary stage of investigation because of the lack of detailed information about signal pulse rise and fall times and because of the present inability to predict r accurately even for a junction made under specified conditions.

(a) Performance Criteria

Consider the small signal equivalent circuit of figure 3-2 where the tunnel junction detector is represented as a current generator in parallel with the junction dynamic resistance r . (The equivalent circuit for the detector is considered in more detail in a subsequent portion of this chapter; the overall detector-preamplifier circuit is examined closely in chapters 6 and 7.) In anticipation of the results from the noise calculation and measurements of chapter 6, noise sources associated with the detector, such as Johnson noise in the bias resistance and shot noise on the biasing current I , have been omitted as they are negligible in comparison to the transistor noise sources.

The signal current i_s is treated as being due to the momentary heating of the material on both sides of the insulating barrier from its ambient (bath) temperature T to $T + \delta T$ where $\delta T \ll T$, then

$$i_s = \left(\frac{\partial I}{\partial T} \right)_V \delta T$$

$$r_{in} = r_{e'} + r_{b'}(1-\alpha)$$

$$r_c \gg r_{b'} + r_{e'}$$

Figure 3-2: Equivalent Circuit of Detector and Preamplifier

where $I = I(V, T)$ is the thermal tunnel current for the junction biased at voltage V . (Note: throughout this and subsequent discussion, only those cases are considered in which both sides of the junction are at the same temperature. The restriction that $\delta T \ll T$ clearly does not hold in the particle track vicinity immediately following the passage of the particle. Hence, this small signal approach is applicable only for times sufficiently long following a particle bombardment that the energy lost by the particle may be considered to have diffused throughout a large enough volume that the average temperature $T + \delta T$ in this volume satisfies the restriction.)

If one were concerned with a detector to be used with noiseless amplifiers, the quantity of interest for comparing one type of junction with the other would be

$$\xi = \frac{(\partial I / \partial T)_V}{I^{\frac{1}{2}}}$$

Here, ξ is a measure of the fundamental signal-to-noise ratio of the detector and, of course, it is desirable that ξ should be as large as possible.

In practice, however, one deals with noisy amplifiers so that, while the criterion that ξ should be large is unchanged, the role played by the junction resistance r becomes important. With reference to the noise figure F for the detector-preamplifier system (see equation 6-6) it can be seen that, remembering M is proportional to rI , F decreases monotonically with r , decreasing to its lowest value in the limit of $r \rightarrow \infty$. Thus, from the point of view of minimizing F , the desired type of junction and biasing point are those where r is as large as possible. However, additional factors, (requiring considerations too detailed to be given in this preliminary work) having to do with the required value of the system response time and the specific value of the energy loss ΔE , enter into the determination of an optimum (presumably finite) r .

Nonetheless, since r_{opt} is probably far in excess of what one is able to obtain presently with tunnel junctions, the performance criteria may be adequately summarized by the statement that r and ξ should be maximum.

(b) Derivation of the Parameters Determining Performance

(i) M-M Junction

The tunnel current in the M-M junction may be found from the general result of equation 2-14 since the matrix element $|M|^2$ is taken to be the same whether the metals on either side of the insulator are normal or superconducting. When both metals are normal, $n_1(E) = n_2(E + eV) = 1$ so that

$$I = A|M|^2 \int_{-\infty}^{\infty} [f(E) - f(E + eV)] dE$$

It is convenient to redefine the constant $A|M|^2$ to be G/e so that, upon integration, the current is

$$I \approx \frac{G}{e\beta} \ln(e^{\beta eV}) = GV$$

where $\beta = 1/kT$ and e is the electronic charge. The constant G may thus be interpreted as the low temperature normal state conductance of the tunnel junction giving

$$I_{MM} = GV$$

$$r_{MM} = (\partial V / \partial I)_T = G^{-1} \quad (3-1)$$

$$(\partial I / \partial T)_V = 0$$

Immediately, this last result removes the M-M junction from contention as, to first order at least, the tunnel current is insensitive to temperature change making $i_s = 0$.

(ii) M-S Junction

Again, the tunnel current is given by equation 2-14 with say $n_2(E + eV) = 1$ but with $n_1(E)$ given by equation 2-13. Hence

$$I_{MS} = \frac{G}{e} \int_{-\infty}^{\infty} \frac{|E|}{(E^2 - \Delta^2)^{\frac{1}{2}}} [f(E) - f(E + eV)] dE$$

$$= \frac{G}{e} \int_{\Delta}^{\infty} n_1(E) [f(E - eV) - f(E + eV)] dE$$

To go further requires some approximations. At $T = 1.2 \text{ K}$ ($\beta \approx 10 \text{ (meV)}^{-1}$) the minimum value of E is $\Delta(1.2 \text{ K}) = 0.55 \text{ meV}$ (for Sn) so that, to a good approximation $f(E + eV) \approx \exp(-\beta(E + eV))$, $V > 0$ and, to within about 10%, $f(E - eV) \approx \exp(-\beta(E - eV))$, $0 < eV < \Delta/2$. Thus

$$I_{MS} = (2G/e) n_T \sinh(\beta eV)$$

where, for future reference, the thermal equilibrium density of quasiparticles in the superconductor is

$$n_o(T) = N_m(0) n_T = 2 N_m(0) \int_{\Delta}^{\infty} n(E) \exp(-\beta E) dE$$

$$= 2 N_m(0) e^{-\beta \Delta(T)} \int_0^{\infty} \frac{p + \Delta(T)}{(p^2 + 2p\Delta(T))^{\frac{1}{2}}} \cdot e^{-\beta p} dp \quad (3-2)$$

$$= 2 N_m(0) \Delta(T) K_1(\beta \Delta(T))$$

with K_1 a first order modified Bessel function of the second kind (Erdélyi, 1954). For purposes of this analysis, it is convenient to set $K_1(x) = \alpha(x) \exp(-x)$ where $\alpha(x)$ varies slowly with x (for $x = \beta \Delta \gg 1$) so that

$$I_{MS} = (2G/e) \Delta(T) \alpha(\Delta(T)/kT) \exp(-\Delta(T)/kT) \sinh(eV/kT),$$

$$0 < eV < \Delta/2$$

$$T \approx 1.2 \text{ K.}$$

As mentioned in chapter 2, for $T < T_c/3$, $\Delta(T) \approx \text{constant}$ so that for small temperature changes near $T_o \approx 1.2 \text{ K}$ one may take $\Delta(T) = \Delta = \text{constant}$. Since α varies only slightly with its argument, it may also be treated approximately as a constant giving

$$I_{MS} = (2G\Delta\alpha/e) \exp(-\beta\Delta) \sinh(\beta eV)$$

(3-3)

$$\frac{1}{r_{MS}} = 2G\Delta\alpha\beta \exp(-\beta\Delta) \cosh(\beta eV)$$

and

$$S_{MS} = \left(\frac{\partial I}{\partial T} \right)_V = \left(\frac{-2G\Delta\alpha}{ekT^2} \right) \exp(-\beta\Delta) [-\Delta \sinh(\beta eV) + V \cosh(\beta eV)]$$

(iii) S-S Junction

Once more, the tunnel current is given by equation 2-14 with $n_1(E)$ and $n_2(E + eV)$ given by 2-13 with the result

$$I_{SS} = \frac{G}{e} \int_{-\infty}^{\infty} \frac{|E| |E + eV| [f(E) - f(E + eV)] dE}{(E^2 - \Delta^2(T))^{\frac{1}{2}} [(E + eV)^2 - \Delta^2(T)]^{\frac{1}{2}}}$$

Using similar reasoning and notation to the M-S case, one finds I_{SS} (see section E (equation 3-8) of this chapter) to be given approximately by

$$I_{SS} = (2G\Delta\alpha/e) [1 - \exp(-\beta eV)] \exp(-\beta\Delta)$$

$$0 < T < 2 K$$

$$\Delta < eV < 2 \Delta$$

The other quantities of interest are

$$\frac{1}{r_{SS}} = 2G\Delta\alpha\beta \exp(-\beta(\Delta + eV)) \quad (3-4)$$

and

$$S_{SS} = \left(\frac{\partial I}{\partial T} \right)_V = \left(\frac{-2G\Delta\alpha}{ekT^2} \right) \exp(-\beta\Delta) [\exp(-\beta eV) (eV + \Delta) - \Delta]$$

(c) Comparison of Parameters

Because of the restrictions on V in equations 3-3 and 3-4, it is convenient to evaluate the MS parameters at $eV = \Delta/2$ and the SS

parameters at $eV = \Delta$. (This corresponds to their respective so-called mid bias points where the bias voltage is halfway between 0 and the level at which the large current jump occurs.) It is then easy to show that

$$\frac{I(MS)}{I(SS)} \approx \frac{1}{2} \exp(\frac{1}{2} \beta \Delta)$$

$$\frac{r(MS)}{r(SS)} \approx 2 \exp(-3\beta \Delta/2)$$

and

$$\frac{S(MS)}{S(SS)} \approx \frac{1}{4} \exp(\frac{1}{2} \beta \Delta) = \frac{1}{2} \frac{I(MS)}{I(SS)}$$

These ratios, which have been evaluated for Pb ($\Delta = 1.25$ meV) and Sn ($\Delta = 0.55$ meV), are given in table 3-1.

Parameter	Pb	Sn
$\frac{I(MS)}{I(SS)}$	260	7.8
$\frac{S(MS)}{S(SS)}$	130	3.9
$\frac{r(MS)}{r(SS)}$	1.4×10^{-8}	5.2×10^{-4}
$\frac{\xi(MS)}{\xi(SS)}$	8.1	1.4

Table 3-1: Comparison of Parameters for M-S and S-S Junctions

The important conclusion to be drawn from these results is that while the intrinsic signal-to-noise ratio ξ is slightly greater in the M-S junction this advantage is overwhelmingly offset by the relatively much lower dynamic resistance of the M-S junction. Balancing a slight loss of signal against more efficient coupling of that signal to the amplifier along with a significant reduction in amplifier noise, one is compelled to choose the symmetric junction as the most promising device. As confirmed

in the next section, it is also evident that the junction should be made of Pb films (or a superconductor having an even larger energy gap) for optimal performance.

2. Type of Superconductor

To provide a quantitative basis for comparing the response of different superconductors to nuclear particle bombardment, it is convenient to define a figure of merit

$$F' = N_O / (N_T + N_O)^{\frac{1}{2}}$$

where N_O is the number of quasiparticle pairs generated in a junction by the radiation and $N_T = n_O(T) \cdot \text{Vol}$, where $\text{Vol} = \text{Junction Volume}$, is the thermal equilibrium number of quasiparticles present in the junction at the ambient temperature T . For a good signal-to-noise ratio, F' should be as large as possible for the signal is proportional to N_O and the noise is proportional to $(N_T + N_O)^{\frac{1}{2}}$. To deal with F' in general, however, is inconvenient so that two limiting forms of F' will be considered instead.

$$(a) \quad \underline{N_O \ll N_T}$$

This case, which corresponds to the situation when a very small amount of energy is injected into the detector, may be studied with the use of

$$F = N_O / N_T^{\frac{1}{2}} \quad (3-5)$$

If, as suggested previously, an average energy loss by a charged particle w is required to dissociate a Cooper pair into two quasiparticles, then for a nuclear particle losing energy ΔE in the junction,

$$N_O = \Delta E / w \approx \Delta E / \gamma \Delta(T)$$

under the assumption (following Sherman, 1962,A) that w is some multiple of the gap energy. (In a semiconductor like silicon, for example, $w(\text{Si}) = \text{average energy loss per electron-hole pair} = 3.6 \text{ eV} = 3.2 E_{\text{gap}}$, Dearnaley and Northrop, 1966.) N_T is given by equation 3-2,

$$n_o(T) = N_T/Vol. = 2N_m(0) \Delta(T) K_1(\beta\Delta(T))$$

At the typical operating temperature of 1.2 K, $\beta \approx 10 \text{ (meV)}^{-1}$ so that $\beta\Delta = 5.5$ and 12.5 respectively for Sn and Pb, the two superconductors of interest. (see table 3-2.) As before, for arguments of this magnitude, it is useful to write $K_1(x) = \alpha(x) \exp(-x)$ which gives

$$F = \frac{\Delta E \exp(\frac{1}{2}\beta\Delta(T))}{\gamma\Delta^{3/2}(T) (2N_m(0) \cdot Vol \cdot \alpha(\beta\Delta(T)))^{\frac{1}{2}}}$$

The energy ΔE gained by a superconductor in slowing down a traversing nuclear particle depends on the particle's specific energy loss in that material. Considerable data in this area are available in nuclear data tables (eg. Marion, 1960). From these and other tables, Blackmore (1967) constructed a semi-empirical expression from which could be calculated, via a program written for the Van de Graaff PDP-8 computer, the energy loss of particles in passing through a thin foil. This program was used to estimate the energy loss of 5.1 MeV alpha particles in thin Sn and Pb films; the results, along with those for Al, are plotted in figure 3-3. (For practical reasons having to do with their ease of preparation as thin films and the working temperatures conveniently available, Pb and Sn were the only superconductors given serious consideration.)

No evidence is available yet concerning the variation of $\gamma = w/\Delta(T)$ from one superconductor to another but perhaps some guidance may be sought from the situation in semiconductors. Dearnaley and Northrop (1966) list the gap energy E_g and experimental values of w for six different solids (see table 1-1 for Ge and Si); the ratio of w/E_g varies only between 2.9 and 4.6. There appears therefore to be some justification in assuming the change in γ from superconductor to superconductor may be sufficiently small that γ can be taken as a constant.

$N_m(0)$, the energy density of states at the Fermi level when the metal is in the normal state, is similarly not very well known but, as Ziman (1965) points out, $N_m(0)$ for most metals except the transition ones is not far different from the free electron value $n(E) = \text{const} \times (E_F)^{\frac{1}{2}}$ where E_F is the Fermi energy. It will be assumed therefore that $N_m(0) \approx n(E_F)$ where $E_F = 4.4 \text{ eV}$ for both Pb and Sn.

Figure 3-3: Energy loss of 5.1 MeV α -Particle Traversing Film Thickness t

The relative values of F are sufficient for a comparison of the two superconductors; the relevant factors are summarized in table 3-2 for films of identical thickness and area.

Superconductor	ΔE (relative units) Fig. 3-3	$\Delta(T)$ meV	$\beta\Delta(T)$	$\alpha(\beta\Delta)$	F (relative to Sn)
Sn	3	.55	5.5	.57	1
Pb	4	1.25	12.5	.36	16.5

Table 3-2: Comparison of Figures of Merit for Sn and Pb.

Clearly, from this point of view, Pb is superior to Sn as a material from which to make a superconducting ionizing particle detector. The temperature 1.2 K was chosen as a basis on which to compare the superconductors because this is a temperature reached with relative ease experimentally. Lower temperatures would serve only to increase this figure of merit in favour of Pb whose transition temperature (T_c) is higher than Sn-- because

$$F \propto \exp(\frac{1}{2}\beta\Delta) = \exp(.875 T_c/T)$$

This last result could give the false impression that for optimum performance, the energy gap should be as large as possible and one should perhaps use semiconductors. That such is not true may be seen from considering the case when ΔE is very large so that $N_o \gg N_T$.

$$(b) \underline{N_o \gg N_T}$$

In this instance, the general figure of merit approaches

$$F'' = N_o^{\frac{1}{2}} = (\Delta E / \gamma \Delta(T))^{\frac{1}{2}}$$

so that

$$\frac{F''(Sn)}{F''(Pb)} \approx \left(\frac{3}{4} \cdot \frac{1.25}{.55} \right)^{\frac{1}{2}} = 1.3$$

and Sn is seen to be slightly favoured over Pb because of the former's smaller energy gap.

(c) Conclusions

It is evident that, in general, the choice of the optimum superconductor will depend upon the expected energy loss in the superconductors and the accessible operating temperatures. On these grounds, a symmetric Pb-Pb junction cooled to 1.2 K would appear to be the better choice for the particle energies of this experiment. Unfortunately, as discussed in Appendix C, Pb-Pb junctions with satisfactory tunneling characteristics proved prohibitively difficult to make so that an unhappy compromise was necessary; symmetric Sn-SnO₂-Sn junctions, therefore, were used for the detection experiments.

E. Excitations in the Tunnel Junction Charged Particle Detector

Now that it has been established that a symmetric superconducting junction is the most desirable configuration, it is necessary to consider the excitations produced in the superconducting films by the charged particle as it passes through.

Throughout its traversal of one or both of the films comprising the junction, the charged particle loses energy primarily by the ionization and excitation of electrons associated with the atoms along its path. It seems reasonable (as discussed in more detail in chapter 7, section F) that within a time the order of less than 10^{-9} sec after the passage of the particle, the resulting electron and phonon excitations in a slender cylinder co-axial with the particle track have relaxed to a common local temperature which is in excess of the junction equilibrium (or helium bath) temperature. The aim of this section is to consider the quasiparticle excitations which occur in the superconductors after this thermalization has taken place.

At this point it is useful to recall the phonon detection experiments described in section B of this chapter where the detection process was described in terms of the injected phonons dissociating Cooper pairs into quasiparticles which could subsequently tunnel. The present situation is analogous. Here, the phonons in excess of the thermal equilibrium number correspond to the injected phonons and, as long as these excess phonons remain in the energy range $\hbar\omega > 2\Delta(T)$, they may generate quasiparticles via the breaking up of pairs.

From the microscopic point of view, when one considers an individual event in which a pair is broken up by a phonon, the resulting excitation may be described as the creation of an electron-hole pair from a vacuum state containing the same number of coupled pairs before and after the (phonon-pair) interaction. Such a process is possible because even though the pair broken up (say $(\vec{k}\uparrow, -\vec{k}\downarrow)$, with $\vec{k}\uparrow$ going to $\vec{k}'\uparrow$) might have had $\vec{k}, \vec{k}' > \vec{k}_F$, the momentum state of an excitation is not sufficient, as it is in a normal metal or superconductor, to uniquely determine its electron or hole nature. (In the Bogoliubov representation mentioned in chapter 2 (equation 2-6) the quasiparticle excitations are written explicitly as a coherent sum of amplitudes for an electron and hole.) Because of the coherence factors v_k and u_k , an individual excitation process need not necessarily conserve the particle number in that it definitely creates an electron and definitely creates a hole. On the average, however, particle number must be conserved so that the average density of excited electrons and holes, must be the same. Using the semiconductor model of a superconductor, which is a thermal average not an individual excitation event model, one may view the quasiparticle excitations ultimately generated by the charged particle as equal numbers of electrons (above the gap) and holes (below the gap). These may subsequently tunnel, obeying Fermi's "Golden Rule" and the conservation of energy, as do the quiescent thermally excited quasiparticles considered in the derivation of the tunnel current between two superconductors given in chapter 2 (equation 2-14). It is this picture which is used in the signal size estimates to follow in section F.

Before leaving this section, the characteristic time τ with which this excess density of phonons and quasiparticle excitations decays to zero should be considered. As mentioned briefly in chapter 1 and more fully in chapter 7, τ depends primarily on the rate at which the excess quasiparticles tunnel and the rate at which phonons are lost from the energy range $\hbar\omega \geq 2\Delta(T)$; τ was estimated (chapter 1) to lie in the range $4 \times 10^{-8} < \tau < 1.4 \times 10^{-6}$ sec which includes the value of approximately 1.4×10^{-7} sec deduced from this experiment. (see chapter 7.)

F. Small Signal Equivalent Circuit

Logically, the next topic to be discussed should be the expected signal size but, to motivate and clarify such a discussion, it is convenient to interject at this point (1) a general technique for obtaining the signal

current and (2) an analysis of the small signal equivalent circuit of the tunnel junction detector.

1. General Treatment of Signal

Following a path often taken by electrical engineers in describing the characteristics of an active device like a triode or transistor, one may consider figure 3-4 where a typical family of tunnel junction dc I-V curves taken at various temperatures T is plotted. The load line, which passes through the chosen biasing point Q --the criterion for choosing Q being that $r = (\partial V / \partial I)_T$ should be maximum as mentioned earlier--has a negative slope equal to the reciprocal of the load resistance R_L seen by the tunnel junction. Using figure 3-4, one sees that as T is increased above T_0 , the corresponding tunnel current is uniquely determined.

A general macroscopic method of determining the characteristics of the current pulse arising from charged particle bombardment is now evident. From the work of Spiel et al (1965) and Crittenden (1968) it seems that the manner in which the heat energy diffuses away from the particle track is adequately described by the classical diffusion equation (see chapter 7). Thus, in principle, one may calculate $T(x, y, t)$ where x and y lie in the plane of the junction and t is the time elapsed since the "instantaneous" thermalization. For an isothermal junction at temperature T with (for the sake of simplicity) a perfectly uniform barrier of area A , the tunnel current density $J(T) = I(T)/A$ where $I(T)$ would be found from figure 3-4. It follows then that the signal tunneling current observed at time t is given by

$$I(t) = \int_A J(T(x, y, t)) dx dy$$

While straightforward in principle, such an analysis is complicated in practice because of problems in solving the diffusion equation (see chapter 7) and by the fact that a real barrier is not perfectly uniform making the tunnel current density a function of x and y as well as of T . Calculations along these lines have been initiated at U.B.C. by Dr. B. L. White and Mr. G. May and will not be pursued further in this thesis.

Lacking detailed information therefore about the time dependence of T , it will be necessary to assume some time dependence of the

Figure 3-4: Typical dc I-V Curves for Tunnel Junction for Various Bath Temperatures T_i

Figure 3-5: Small Signal Equivalent Circuit for Tunnel Junction

signal current $i(t)$ --as was done in chapter 1--and estimate its magnitude from a small signal analysis.

2. Small Signal Analysis

Consider figure 3-4 again where the junction is biased at Q which corresponds to some current $I_o = I(V_o, T_o)$. If the average peak temperature excursion (δT) of the junction following bombardment by a charged particle is small compared with T_o , and with the difference $T_c - T_o$, the change in current is given approximately by the first two terms of the Taylor's series expansion of $I(V, T)$

$$\delta I \approx (\partial I / \partial V)_{T_o} \delta V + (\partial I / \partial T)_{V_o} \delta T + \dots, \quad eV_o < 2\Delta(T)$$

where it is assumed, in consistency with experimental evidence, that $(\partial I / \partial V)_{T_o}$ and $(\partial I / \partial T)_{V_o}$ are reasonably constant over a small range of δV and δT . An equivalent form may be written

$$\delta V \approx (\partial V / \partial I)_{T_o} \delta I + (\partial V / \partial T)_{I_o} \delta T + \dots$$

Thus, by Thévenin's theorem, the junction may be represented as a voltage generator $\delta e = (\partial V / \partial T)_{I_o} \delta T$ in series with an impedance $r = (\partial V / \partial I)_T$ or, by Norton's theorem, as a current generator $\delta i = (\partial I / \partial T)_V \delta T$ in parallel with an impedance $r = (\partial V / \partial I)_T$. Both representations are shown in figure 3-5. (Note that the current source form was used in section D of this chapter in anticipation of this result.)

The importance of having both r and $(\partial I / \partial T)_{V_o}$ as large as possible was stressed in section D. It might be noted in passing that $(\partial I / \partial T)_{V_o}$ will be largest in those junctions exhibiting "pure" single quasiparticle tunneling. Those units having metallic filaments or "bridges" joining the films through imperfections in the insulating layer will pass a relatively large current at finite voltages which is temperature insensitive (see appendix B) making them useless as potential detectors of charged particles.

In practice, the parameters $(\partial I / \partial T)_V$ and r may be estimated experimentally from a series of I-V characteristics taken over a range of temperatures near T_o ; they may also be estimated roughly from quasiparticle

tunneling theory as outlined below.

3. Derivation of Small Signal Parameters

The current in a symmetrical junction (cf. equation 2-14) is, in the notation of chapter 2,

$$I = A|M|^2 \int_{-\infty}^{\infty} n_1(E)n_2(E+eV)(f(E) - f(E+eV)) dE \quad (3-7)$$

As was done in section D of this chapter, it is convenient to set $A|M|^2 = G/e$ and, as will be evident shortly, to use the approximation $f(E) = (1 + \exp(\beta E))^{-1} \approx \exp(-\beta E)$. The range of validity of $n(E)$ is restricted by equation 2-13 so that, for $eV < 2\Delta(T)$, 3-7 may be re-written as

$$I = \frac{G}{e} \left[\int_{\Delta+eV}^{\infty} \frac{E(E-eV)[(f(-E)-f(-E+eV))]dE}{(E^2-\Delta^2(T))^{\frac{1}{2}}((E-eV)^2-\Delta^2(T))^{\frac{1}{2}}} + \int_{\Delta}^{\infty} \frac{E(E+eV)[(f(E)-f(E+eV))]dE}{(E^2-\Delta^2(T))^{\frac{1}{2}}((E+eV)^2-\Delta^2(T))^{\frac{1}{2}}} \right]$$

Thus, to a good approximation for Sn with $0 < T < 2K$, the current is

$$I = \frac{G}{e} \left[(e^{\beta V} - 1) \int_{\Delta+V}^{\infty} \frac{E(E-V)e^{-\beta E}dE}{(E^2-\Delta^2)^{\frac{1}{2}}((E-V)^2-\Delta^2)^{\frac{1}{2}}} + (1-e^{-\beta V}) \int_{\Delta}^{\infty} \frac{E(E+V)e^{-\beta E}dE}{(E^2-\Delta^2)^{\frac{1}{2}}((E+V)^2-\Delta^2)^{\frac{1}{2}}} \right]$$

$$\equiv \frac{G}{e} (Int_1 + Int_2)$$

where, for convenience, it is understood that $V \rightarrow eV$ and $\Delta = \Delta(T)$. Setting $t = E-V-\Delta$ and performing some elementary manipulations yields

$$Int_1 = e^{-\beta\Delta}(1-e^{-\beta V}) \int_0^{\infty} \frac{(t+\Delta)e^{-\beta t}}{(t^2+2\Delta t)^{\frac{1}{2}}} \left[1 - \left(\frac{\Delta}{t+V+\Delta} \right)^2 \right]^{-\frac{1}{2}} dt$$

which, since $q = \left(\frac{\Delta}{t+V+\Delta} \right) < 1$, can be written

$$\text{Int}_1 = e^{-\beta\Delta}(1-e^{-\beta V}) \int_0^\infty \frac{(t+\Delta)e^{-\beta t} dt}{(t^2+2\Delta t)^{\frac{1}{2}}} \left(1 + \frac{1}{2} q^2 + \frac{3}{8} q^4 + \dots\right)$$

A similar substitution, $t = E - \Delta$, in Int_2 reveals that $\text{Int}_2 = \text{Int}_1$ or

$$I = \frac{2G}{e} \text{Int}_1$$

To obtain a tractable form for I , it is necessary to terminate the series after the first term which may only be done within about 30% precision by restricting V to be in the range $\Delta < V < 2\Delta$. With this approximation, the integral is that of equation 3-2 making (with eV explicitly re-inserted)

$$I = \frac{2G}{e} (1 - \exp(-\beta eV)) \Delta K_1(\beta\Delta) \quad , \quad \begin{matrix} 0 < T < 2K \\ \Delta < eV < 2\Delta \end{matrix} \quad (3-8)$$

and

$$r^{-1} = (\partial I / \partial V)_T = 2G\beta \exp(-\beta eV) \Delta K_1(\beta\Delta) \quad (3-9)$$

To calculate $(\partial I / \partial T)_V$, the identity

$$\frac{d}{dz}(K_1(z)) = -K_0(z) - z^{-1}K_1(z)$$

is used with the result

$$\left(\frac{\partial I}{\partial T}\right)_V = \frac{-2G\Delta\beta}{eT} \left[\frac{K_1(\beta\Delta)}{\beta} \left[(\beta eV + 1)e^{-\beta eV} - 1 \right] - \Delta K_0(\beta\Delta)(1 - e^{-\beta eV}) \right] \quad (3-10)$$

where K_0 is a zero order modified Bessel function of the second kind.

G. Estimate of the Signal Size

For purposes of analysis, it is assumed that the current pulse superimposed on the ambient tunneling current is of the form

$$i(t) = i_0 \exp(-t/\tau) \quad t \geq 0 \quad (3-11)$$

where i_0 is proportional to the magnitude of the excess quasiparticle density and τ (see section E of this chapter) is the characteristic time with which this density decays.

At the outset it must be emphasized that the actual time dependence is probably much more complicated than the simple exponential form of equation 3-11. As pointed out in the last section, our theoretical analysis has not yet sufficiently advanced to justify the use of a more complex form and, as it turns out in chapter 7, the quality of the data presently available does not seem to warrant the use of such a form. It must be remembered, however, that the form of $i(t)$ chosen will affect the value of i_0 deduced from the experimental measurements (chapter 7) and will ultimately affect, therefore the value found for w .

For definiteness, alpha particles of 5.1 MeV energy are assumed to be incident on the junction according to the geometry of figure 6-7.

1. Macroscopic or Phenomenological Approach

As discussed previously in this chapter, the small signal approximation is taken to apply at times sufficiently long after the alpha particle traversal that a small volume of the junction ($2\lambda A'$) may be considered to be at an average temperature $T_0 + \delta T$ where δT is suitably small. (A' is the area of the "hot" material and λ , typically 2000 Å, is the film thickness.) Now the parameters of the preceding section were derived on the assumption that the entire junction is at the same temperature and the total effective barrier area A (typically $4 \times 10^{-4} \text{ cm}^2$) is passing current; it seems reasonable then that the peak signal current may be written

$$i_0 = (\partial I / \partial T)_V \delta T \cdot \frac{A'}{A}$$

If $n_0(T)$ is the thermal equilibrium density of quasiparticles (equation 3-2) then the change in the quasiparticle density (in the volume $2\lambda A'$) due to the momentary temperature rise δT caused by the alpha particle is

$$\delta n = \left[\frac{\partial n_0(T)}{\partial T} \right] \delta T \approx \frac{2N_0}{2\lambda A'} = \frac{\Delta E}{w\lambda A'} \quad (3-12)$$

Solving 3-12 for δT yields

$$i_o = \left(\frac{\partial I}{\partial T} \right)_V \left(\frac{\partial T}{\partial n_o(T)} \right) \frac{\Delta E}{w\lambda A} \quad (3-13)$$

The maximum energy lost by the alpha particle directly in the tin is about 0.5 MeV which corresponds to an alpha particle incident on the junction at an angle of 80° with a path length in the tin of approximately 23,000 Å (cf. figure 3-3). Calculating $(\partial n_o(T)/\partial T)$ from equation 3-2 and $(\partial I/\partial T)_V$ from equation 3-10 at a temperature of 1.2 K for a junction having $G^{-1} = R_n = 0.1 \Omega$ and using the value $w(\text{Sn}) \approx .003 \text{ eV}$ as given in table 1-1 yields an expected maximum current amplitude

$$i_o \approx 15 \mu\text{A}$$

which is near the range of values found experimentally in chapter 7. The corresponding temperature increase is $\delta T \approx 0.12 \text{ K}$.

It should be noted that these estimates are based on the assumption that the energy deposited by the alpha particle in the substrate does not contribute to the expected signal. It turns out (see chapter 7) that this assumption is invalid; hence $\Delta E(\text{effective}) > \Delta E = 0.5 \text{ MeV}$ implying therefore that i_o is underestimated. Furthermore, the values of $(\partial I/\partial T)_V$ and $(\partial n_o(T)/\partial T)$ used are constant and are valid (cf. figure 3-4) only in the limit of small temperature changes.

The manner in which the amplifier sensitivity was checked to ensure detection of pulses with this magnitude and time dependence is outlined briefly in chapter 5.

[Note: an alternative method of estimating δT is to set (cf. equation 3-12)

$$\delta T = \Delta E / \rho c_p \lambda A'$$

where ρ is the density and c_p the heat capacity. Taking $\rho = 7.29 \text{ g cm}^{-3}$ and $c_p(1.2 \text{ K}) = 9 \times 10^{-6} \text{ J g}^{-1}(\text{degree K})^{-1}$ (Johnson, 1960) and the same values for the other quantities as before gives $i_o \approx 24 \mu\text{A}$ which agrees reasonably well with that found above. (This latter value of i_o is

probably an overestimate since c_p increases rapidly with increasing temperature).]

2. "Microscopic" Approach

The considerations of this paragraph provide some justification for the simple form of $i(t)$ which has been assumed.

As stated earlier, immediately following the passage of an alpha particle there exists a density N_0 of excited quasiparticle pairs which is in excess of $n_0(T)$. Decay of the excess density proceeds along two paths: (1) the quasiparticles tunnel through the insulating layer or (2) they recombine to form Cooper pairs and emit a phonon of energy $\hbar\omega > 2\Delta(T)$. Analysis is greatly simplified at this point if it is assumed that $N_0 \ll n_0(T)$ and that the recombination phonons may be ignored. As Rothwarf and Taylor (1967) state, however, this approximation is rarely valid and they have given an analysis of the general situation assuming steady-state quasiparticle injection and uniform junction geometry. Subsequently, they arrive at a pair of coupled equations describing the time dependence of N (the total number of quasiparticles) and N_ω (the total number of phonons with energy $\hbar\omega > 2\Delta$). Ideally, a similar analysis, modified to account for the transient energy input, should be applied to the tunnel junction particle detector but this would require careful treatment of the energy diffusion problem taking into account specifically the junction boundaries and thermal impedances at those boundaries. Such a program will not be undertaken here; it is mentioned at this point only to illustrate the limitations of the analysis to be given below.

As a starting point, one is interested in the simplest possible model for the pulse generator to be inserted in figure 3-5. Thus, one ignores the time dependence of the rate W_R at which the quasiparticles will recombine and assumes that the tunneling rate W_T is constant and sufficiently small with respect to W_R that they are effectively independent of one another. The decay of the excess quasiparticle density may then be viewed as sketched below.

If $R(N, n; t)$ is the joint probability distribution function that at time t , there are still N excess quasiparticle pairs left and n quasiparticle pairs have tunneled, then

$$\begin{aligned} R(N, n, t + dt) = & R(N, n; t) (1 - NWdt) \\ & + R(N+1, n; t) (N+1) W_R dt \\ & + R(N+1, n-1; t) (N+1) W_T dt \end{aligned}$$

Solving

$$\frac{dR}{dt} = R_{N+1, n} (N+1) W_R + R_{N+1, n-1} (N+1) W_T - R_{N, n} NW$$

yields

$$\langle N(t) \rangle = N_0 e^{-Wt} \quad (3-14)$$

$$\langle n(t) \rangle = \frac{N_0 W_T}{W} (1 - e^{-Wt}) \quad (3-15)$$

The current pulse due to the fraction of the excess quasiparticle pairs which have tunneled in time t is thus

$$i(t) = e \frac{d\langle n(t) \rangle}{dt} = e N_0 W_T \exp(-Wt) \quad (3-16)$$

where e is the electronic charge. [Note: the current is not $2e d\langle n(t) \rangle / dt$ because, at the usual biasing point $eV \approx \Delta$, only one member of each quasiparticle pair can tunnel to the other superconductor, due to the action of the energy gaps.] Setting $e N_0 W_T = i_0$ yields

$$i(t) = i_0 \exp(-t/\tau), \quad t \geq 0$$

which, of course, is the form assumed earlier (equation 3-11) without

justification.

As it should be, the quantity i_o just introduced is the same as that found by the phenomenological treatment (equation 3-13). This may be seen by substituting for W_T via equation 3-17 to give

$$i_o = \frac{G}{eN_m(0)} \cdot \frac{\Delta E}{\omega \lambda A}$$

which is identical to equation 3-13 for simple, approximate forms of $n_o(T)$ (equation 3-2) and $I(V = \Delta/e, T)$ (equation 3-8).

H. Tunneling Probability

From equation 3-16, it can be seen that, ignoring the bandwidth limitations of real amplifiers, the maximum signal i_o is obtained when the tunneling probability per sec W_T is maximum. (N_o is fixed by the choice of superconductor and the energy lost by a charged particle in traversing the tunnel junction). On the other hand, from the small signal equivalent circuit and equation 3-10, it is evident that the signal $i_s = (\partial I / \partial T)_V \delta T$ is largest for maximum junction conductance G .

This section will demonstrate the equivalence of these two requirements and give an insight into the restrictions they place on the acceptable insulating layer thickness.

A rough estimate of W_T may be made using a method suggested by Ginsberg (1962). For bias voltages $eV \gg 2\Delta(T)$, the current should be

$$I = e^2 N_m(0) A \lambda W_T V$$

where $N_m(0)$ is the normal state density of free electron states per unit volume near the Fermi surface and the other symbols have their usual meaning. In this range of bias voltage, the differential resistance is very nearly constant and equal to the low temperature, normal state resistance (R_n) of the junction so that

$$W_T = (e^2 N_m(0) \lambda A R_n)^{-1} = G (e^2 N_m(0) \lambda A)^{-1} \quad (3-17)$$

and, like $(\partial I / \partial T)_V$, W_T is proportional to G . The number of free electron states per unit volume per unit energy range for both spins is

$$N(\epsilon) d\epsilon = 4\pi (2m)^{3/2} \epsilon^{1/2} h^{-3} d\epsilon \equiv k\epsilon^{1/2} d\epsilon \quad (3-18)$$

where m is the electron mass and h is Planck's constant, so that the average value of $N(\epsilon)$ near the Fermi surface ($\epsilon = \epsilon_F$) in a range $\delta\epsilon \ll \epsilon_F$ is

$$\bar{N}(\epsilon_F) = \frac{1}{\delta\epsilon} \int_{\epsilon_F - \delta\epsilon/2}^{\epsilon_F + \delta\epsilon/2} k\epsilon^{1/2} d\epsilon \approx k\epsilon_F^{1/2} \quad (3-19)$$

Now, the number of free electrons N_f is simply

$$N_f = \int_0^{\epsilon_F} N(\epsilon) d\epsilon = 2k\epsilon_F^{3/2} / 3 \quad (3-20)$$

which gives, upon combining equation 3-19 and 3-20,

$$N_m(0) = \bar{N}(\epsilon_F) = \frac{3}{2} N_f / \epsilon_F \quad (3-21)$$

Equation 3-21 may then be evaluated by taking

$$N_f = nN_A \rho / A_t \quad (3-22)$$

and (Wilson, 1965)

$$\epsilon_F = (h^2 / 8m) (3N_f / \pi)^{2/3} \quad (3-23)$$

where

n = number of free electrons per atom

N_A = Avogadro's number

ρ = density

A_t = atomic weight

For Sn, $n = 1.1$ (Wilson, 1965) so that

$$N_f = 4.05 \times 10^{22} \text{ els cm}^{-3}$$

$$\epsilon_F = 4.4 \text{ eV} \quad (3-24)$$

$$N_m(0) = 1.4 \times 10^{22} \text{ eV}^{-1} \text{ cm}^{-3}$$

Typical specimens have $\lambda = 2000 \text{ \AA}$, $A = 0.2 \text{ mm} \times 0.2 \text{ mm} = 4 \times 10^{-4} \text{ cm}^2$ and $R_n = G^{-1} \approx 0.1 \Omega$ yielding $W_T = 5.6 \times 10^5 \text{ sec}^{-1}$.

So far, it has been stated qualitatively that the maximum signal i_o is greatest for W_T "large". To place this statement on a quantitative basis, consider equations 3-15 and 3-16 which may be rewritten as

$$\langle n(t) \rangle = N_o \frac{W_T}{W_R + W_T} [1 - \exp(-t(W_T + W_R))]$$

$$i(t) = e N_o W_T \exp(-t(W_T + W_R))$$

respectively. Assuming that the statistical fluctuations in the signal go according to $\langle n(t) \rangle^{\frac{1}{2}}$, it is imperative that $\langle n(t) \rangle$ be as large as possible which means W_T should at least be comparable to W_R . (W_R is the rate at which the quasiparticle population is reduced by processes other than tunneling, see chapter 7.) Two practical considerations place an upper bound on W_T ; (1) minimum practicable thickness of insulating layer (a 1 or 2 mono-layer thick insulator is consistent with $W_T \approx 7 \times 10^7 \text{ sec}^{-1}$); (2) finite rise time of real amplifiers (if $W_T > 10^9 \text{ sec}^{-1}$, the signal would decay before an amplifier could respond).

Unfortunately, no firm theoretical estimate of W_R is available yet for tunnel junctions subjected to charged particle bombardment. For purposes of establishing the orders of magnitude involved, however, the value of $2 \times 10^7 \text{ sec}^{-1} > W_R > 2 \times 10^5 \text{ sec}^{-1}$ as determined experimentally by the steady state injection of quasiparticles in superconductors (Miller and Dayem, 1967 and Levine and Hsieh, 1968--see also chapter 1) may be used.

By inspection of equation 3-17, it is clear that for a given type of superconductor, W_T may be increased by decreasing either the junction

volume (λA) or the tunneling resistance. The former possibility is ruled out by the requirement that, for detection efficiency, the volume should be as large as possible. Junctions with lower tunneling resistance may be produced by decreasing the thickness of the insulating layer (see table 3-3) but only at the cost of giving rise to an unwanted dc Josephson current and running the risk of the oxide being patchy (at $10 \text{ \AA}$ it is only 2 or 3 monolayers thick) and/or pierced by metallic filaments which become superconducting shorts. The first objection is not serious as a reasonably low magnetic field will quench the supercurrent (see chapters 2 or 6); the second problem is very serious and has proved to be a formidable obstacle to making a usable detector.

Table 3-3, calculated for a typical junction of dimensions given above, illustrates that tunneling thicknesses (as derived from figure 2-3) no greater than $10 \text{ \AA}$ are permissible if W_T is to be comparable to W_R . (Here, S_T is the thickness of an ideal, uniformly thick insulating layer)

$S_T (\text{\AA})$	$G^{-1} = R_n \Omega$	$W_T \text{sec}^{-1}$
6	8×10^{-4}	7.0×10^7
8	1.6×10^{-2}	3.5×10^6
10	0.27	2.1×10^5
12	5.4	1.0×10^4

Table 3-3: Tunneling Probability per sec for Several Tunneling Thicknesses.

I. Practical Operation of Detector

It is convenient at this juncture to insert a brief description of the manner in which the superconducting tunnel junction would operate in practice.

The simplest method of bombarding a tunnel junction with alpha particles is to mount an appropriate radioactive source (Pu^{249} , 5.13 MeV alphas was the one chosen) inside the helium dewar in close proximity to the junction (see figures 4-1 and 4-3). Now the energy loss per unit path

length (dE_{α}/dx) of a 5.1 MeV alpha particle in liquid helium is roughly 140 MeV/cm calculated on the basis of the stopping power $\epsilon = 6.4 \times 10^{-15}$ eV cm² for helium (Whaling, 1958) and density $\rho = 0.145$ g cm⁻³ for liquid helium at 1.2 K (Wilks, 1967). As a shutter between the source and junction is desirable, the minimum practical source-junction distance would be about 2 or 3 mm; hence, 5 MeV alpha particles would be stopped in the liquid.

Consequently, the specimen and source must be raised above the helium bath level, being maintained at 1.2 K by a thin film of superfluid helium and by contact with a copper cold finger whose lower end is immersed in the liquid. At 1.2 K, the vapour pressure above the bath is about 0.5 Torr which leads to a specific energy loss of approximately 25 keV/cm; the energy lost by 5 MeV alpha particles in traversing this attenuated helium atmosphere would therefore be a negligible 0.5%. Of course, the alpha particles must also pass through the superfluid film which would cover the junction but this film is very thin, the order of 4×10^{-6} cm at a height of 0.5 cm above a 1.2 K helium bath (Wilks, 1967), and the corresponding energy loss is only about 0.6 keV. For all practical purposes therefore, the alpha particles striking the junction may be considered monoenergetic, having energy 5.1 MeV.

Previous discussions have emphasized that the junction must be fabricated in such a way as to have a high tunneling conductance and therefore large $(\partial I/\partial T)_V$. Furthermore, it has been observed that the operating temperature should be as low as possible and the operating bias voltage must be chosen such that r is as large as possible. Finally, to suppress the unwanted dc Josephson supercurrent which junctions with a sufficiently thin insulating layer may exhibit, it might be necessary to supply a moderate magnetic field (<100 G) in the plane of the junction. (A detailed account of problems associated with attaining these operating conditions in practice is given in section A, chapter 6.)

J. Noise

This section outlines very briefly the noise sources that should be considered in describing the performance of the tunnel junction as a detector.

In the present experiment the magnitude of the junction noise is not important for it is shown (section C, chapter 6) that the dominant

noise source was the common base preamplifier. As technology improves and lower noise preamplifiers are designed--perhaps operating inside the helium dewar at 4 K--the noise associated with the tunnel junction will become more important and for that reason is discussed at this time.

1. Shot Noise on Biasing Current (I)

The shot noise, which arises from the corpuscular nature of the current, is given by

$$\overline{i_s^2} = 2eI\delta f$$

where δf is the bandwidth interval.

2. Johnson (Thermal) Noise

For a tunnel junction in thermal equilibrium, the Johnson or thermal noise is given by the expression

$$\overline{i_J^2} = 4kT\delta f/r_o$$

where r_o is the zero-voltage junction dynamic resistance, k is Boltzmann's constant and T is the absolute temperature of the junction. Note: this expression is valid only for an isothermal junction at zero bias voltage V ; when $V \neq 0$ the noise is given by $\overline{i_s^2}$ above.

3. Generation-Recombination (GR) Noise

GR noise is due to fluctuations in the density of thermally excited quasiparticles brought about by variations in the generation and recombination rates.

4. Variations in Insulation Thickness

In any real tunnel junction, the insulating layer thickness

is not uniform but may be thought of as varying stochastically (Hurych, 1966) over the junction area. As discussed in section H the tunneling probability depends strongly on barrier thickness which implies that the stochastic variation in insulator thickness will be manifested as fluctuations in the ambient tunneling current. Calculations of the magnitude of this effect are in progress at U.B.C. and will be reported later.

5. Fluctuations in Signal

Fluctuations in the signal arise principally from the following sources, which are due to the same mechanisms which produce noise in the ambient current: generation and recombination noise in the current due to radiation-excited quasiparticles, variation in oxide layer thickness, and fluctuations in the number of quasiparticles originally excited by the bombarding charged particle.

K. Summary

Beginning the chapter was a very brief description of the conventional gas-filled, scintillation and semiconductor charged particle detectors. The use of single strip superconducting bolometers as radiation detectors was then outlined followed by an account of the use of tunneling junctions as detectors of microwave photons and phonons.

In discussing the principles upon which the tunnel junction charged particle detector is based, it was shown that the device having the best characteristics was a Pb-Pb symmetric superconducting junction. (These units could not be used in practice however, because of the difficulty in fabricating them with satisfactory characteristics.)

The quasiparticle excitations which are generated by the alpha particle in the superconductors were considered next and it was concluded that, on the average, they could be regarded as equal numbers of electrons and holes appearing above and below the energy gap respectively in the semiconductor model.

Following a brief discussion of the general approach to analyzing the signal from the detector, a small signal equivalent circuit of the junction detector was found and expressions for the parameters $(\partial I / \partial T)_V$ and r were derived from the tunneling current $I = I_{SS}(V, T)$ between similar

superconductors.

The current pulse $i(t)$, which follows the passage of a charged particle through the junction and is superimposed on the thermal tunneling current, was assumed to be of the form

$$i(t) = i_0 \exp(-t/\tau) \quad , \quad t \geq 0 \quad (3-11)$$

This form of $i(t)$ was found to be that given by a very simple model in which the excess quasiparticle density decayed via two decoupled and time independent channels--tunneling and recombination.

The size of the signal current i_0 was estimated on the basis of typical junction characteristics and operating conditions to be approximately 15 μ A. That the amplifier sensitivity was sufficient to detect pulses of that form and magnitude was checked experimentally.

The importance and means of obtaining a high tunnel probability was then discussed on the basis of a simple estimate of W_T due to Ginsberg (1967)

$$W_T = G(e^2 N_m(0) \lambda A)^{-1} \quad (3-17)$$

A short description of the practical operation of the junction was then given emphasizing the need to have the actual overlap area slightly above the helium bath level and the need of a magnetic field in the plane of the junction to bias out the dc Josephson current.

Finally, a survey of noise sources in the tunnel junction detector is given. They are not considered in depth as it turns out experimentally that the preamplifier was the principal source of noise.

CHAPTER 4

CRYOGENIC APPARATUS

A. Introduction

This chapter is of technical interest only and may be omitted by the reader uninterested in experimental details.

In essence, the cryogenic equipment consists of a double-walled glass vessel containing liquid helium 4--referred to hereafter as the helium dewar--sitting inside a larger, similar vessel filled with liquid nitrogen, referred to as the nitrogen dewar (see figure 4-1). Liquid nitrogen serves both as a pre-coolant and as a heat shield for the liquid helium bath. The specimens are immersed in the helium bath whose temperature can be varied from about 1.2 K to 4.2 K through regulation of the vapour pressure above the liquid.

B. Dewars and Dewar Cap

Both the nitrogen and helium dewars, constructed by the departmental glassblower, are of conventional design except that a pyrex tee is affixed to the top of the double-walled portion of the helium dewar. The tee was necessitated by the fairly unconventional manner in which the dewars were installed. They were mounted in a firmly anchored table, the top of the tee being about four feet above floor level, thereby making it possible to easily install or remove the specimens without disturbing the dewars. One port of the tee thus provided an opening for pumping on the helium dewar; the other port provided a readily demountable vacuum coupling for the dewar cap.

The brass dewar cap provides a vacuum tight cover for the helium dewar with ports for the transfer siphon, pressure sensing, electrical feedthrough and, of course, the specimens. Because it was necessary to use two different specimen holders--discussed in a later section--the cap was designed to accept either one.

Figure 4-1: Cryogenic Apparatus (Schematic) (see page following for key).

KEY TO FIGURE 4-1

- G1 Marsh Dial Gauge, Type 100 0-760 Torr
G2 Edwards Dial Gauge, Type C.G.3 0-20 Torr
G3 Edwards Vacustat, Model 2E, 0-1 Torr, Lowest Reading Sensitivity = 0.001 Torr
G4 Marsh Dial Gauge, (no type No.) 0-760 Torr
G5 Ashcroft Dial Gauge, Type 1850 0-760 Torr (vac), 0-380 Torr (press. gauge)
1. Sample mount.
 2. Electrical lead feedthroughs.
 3. Sample mount support and control tubes.
 4. Dewar cap.
 5. Pyrex Tee (3 x 3 x 2 in.).
 6. Helium Dewar Interspace pumpout port.
 7. Helium Dewar, glass, 76 mm i.d., 90 mm o.d., approx. 48 in. long.
 8. Nitrogen Dewar, glass, 116 mm i.d., 150 mm o.d., approx. 38 in. long.
 9. Liquid Nitrogen.
 10. Liquid Helium.
 11. Sample.
 12. Helmholtz coils.
 13. He return line.
 14. Welch Rotary Pump, Model 1402B, 5 cfm.
 15. Stokes Pump, Model 49-10, 80 cfm.
 16. Welch Rotary Pump, roughing purposes, Model 1405.
 17. Mercury manostat (0-100 Torr).
 18. Bottled dry N₂ gas; flushing purposes.
 19. Transfer siphon port.

C. Pumps

The vacuum pumps used to reduce the vapour pressure of the liquid helium are shown schematically in figure 4-1. Temperatures of about 2 K could be obtained with the Welch 1402B rotary pump and about 1.2 K with the Stokes, Model 49-10, Low Temperature laboratory "community" pump.

D. Pressure-Temperature Measurement

Temperatures are measured by observing the helium vapour pressure and relating it to the absolute temperature with a standard table. The vapour pressure-temperature relationship of liquid helium 4, the subject of extensive investigation, can be found in many standard references, eg. Mendelssohn (1960). A mercury vacustat and two dial gauges, see figure 4-1, provide adequate coverage of the pressure range from 0.12 Torr (1.0 K) to 760 Torr (4.2 K).

E. Sample Mounts

In spite of the great care taken in their fabrication, as described in Appendix A, it was conceded that probably no more than 25-50% of the tunneling junctions prepared would have the suitable characteristics, as discussed in an earlier chapter, for a sensitive test of their response to charged particle bombardment. For this reason, two types of specimen mount are needed: one to check the I-V curves and magnetic field behaviour of all five junctions at once, thereby detecting the good specimens; the other to investigate the charged particle response of individual specimens. A unique feature of the two mounts is that, in order to change samples, only these pieces of apparatus need be disturbed.

The multiple-junction sample holder is shown in figure 4-2. Permutation of the twelve electrical leads connected to the substrate permits four-terminal testing of any of the five junctions.

Figure 4-3 shows the important features of the single junction sample holder and alpha particle source support. Provision is made for eight electrical leads, the glass-to-metal seals being obtained by using the base of a discarded octal electronic tube. The vacuum tight sliding seal permits rotation of the control tube which activates the shutter separating the radioactive source from the tunnel junction. A small 2 K Ω Spectrol potentiometer, Model 60-1-1, serves primarily as a bearing for the control tube but can be introduced as a circuit element if required. The shutter is a thin piece of stainless steel

Figure 4-2: dc Sample Holder

Figure 4-3: Pulse Test Holder

to which a thin layer of Pb was soldered to ensure the stoppage of 5 Mev alpha particles and to make the stable shutter position the vertical, or closed, one by lowering the shutter's centre of gravity below the pivot point. A source of 5.13 Mev alpha particles (Pu 239) was prepared on a thin, smooth piece of stainless steel (1/4" x 1/4") which was then soldered to the copper support as shown in the diagram. The strength of the source was measured to be 1.1×10^7 dpm into 4π steradians or about 5 μ Ci. Pivoting the support about a vertical axis made it possible to vary the angle at which the alpha particles impinged on the sample.

Because the source-sample distance is roughly 1.0 cm--much greater than the range of alpha particles in liquid helium (see chapter 3)--it is necessary that the helium bath level be kept below the junction when tests are carried out. On the other hand, the bath must be kept as close as possible to the junction for maximum cooling. A compromise between these conditions was reached in the following manner.

The sample substrate was clamped to a copper bar of high thermal conductivity--electrolytic tough pitch grade--most of which was immersed in liquid helium to serve as a cold finger. In addition, the four electrical leads are passed around the bottom of the sample mount before connection to the sample, so that heat transferred down the electrical leads and support tubes is dumped directly into the bath. Consequently, the junction may be somewhat above the bath level but its temperature should be but slightly greater than that of the bath because it is thermally anchored to the bath via the copper bar. Superfluid flow over the junction serves as an additional cooling agent.

The level of the helium bath falls, of course, during an experiment so that a sliding vacuum-tight seal was designed--not shown on the sketch--which, when interposed between the sample mount flange and dewar cap, permitted the junction to be lowered in step with the bath level.

The shielded parallel pair transmission line for the voltage leads as shown in the diagram insert, consists of two parallel .007 in. diam copper wires spaced an average distance of .06 in. apart and kept central in the shield by beads of Q-dope placed every few inches along its entire length of 1.62m. It was not practicable to use the more obvious co-axial line configuration of a copper centre wire and stainless steel shield because the

resulting contact potential would be larger than the bucking voltage available with the dc voltmeter monitoring the junction bias. Using a small diameter commercial cable, such as RG No. 174/U, was ruled out because of its relatively large heat leak. A comparison of these three types of transmission line is given in table 4-1.

TYPE OF TRANSMISSION LINE	ESTIMATED HEAT LEAKAGE		CHARACTERISTIC IMPEDANCE (Ω)
	WATTS	* ℓ /hr boil off of liquid He	
Shielded parallel pair (.125 in. diam S. Steel tube, 2 x .0071 in. Cu wire)	7.8×10^{-3}	.011	170
Co-axial (.125 in. diam S. Steel tube, 1 x .0071 in. Cu wire)	5.8×10^{-3}	.008	150
Co-axial, RG 174/U	80×10^{-3}	.115	50

Table 4-1: Comparison of Transmission Line Characteristics

*White (p.198) and Hoare, et al (p. 142) give the figure 1.43 ℓ /hr/watt as the boil off rate of liquid helium.

CHAPTER 5.

ELECTRICAL MEASUREMENT TECHNIQUES

A. dc Measurements

1. Introduction

Before attempting to use any thin-film junction to detect charged particles, it is necessary as discussed in Chapter 3 that its dc characteristics be well known. Of particular interest is the dynamic resistance, $r = (\partial V / \partial I)$, $0 < V < 2\Delta/e$, and the magnetic field dependence of any supercurrent which the junction may pass. Examination of the dc I-V curves of various specimens thus serves a double purpose: to eliminate quickly those junctions having undesirable characteristics and to provide a basis for deciding optimal use of specimens having suitable characteristics.

The major components of the dc circuitry are, as shown in figure 5-1, a battery power supply, current- and volt-meters with recorder outputs and an X-Y recorder.

2. Power Supply

The power supply is energized by a battery because tunneling junctions, especially those exhibiting the Josephson effect, are highly non-linear devices making it imperative that the ripple in the biasing current be absolutely minimal. Potentiometer R1 serves as a coarse current control; potentiometer R2 serves as a voltage divider such that only a fraction of the noise generated in the ten-turn potentiometer appears across the specimens.

For some "leaky" specimens, the maximum output current (16 mA) from the regular supply was insufficient to obtain meaningful I-V curves. It was to test such junctions that the simple, high current (150 mA) auxiliary supply was employed.

3. Shielding

In practice, the voltage applied to the samples is very

Figure 5-1: dc Circuitry (Schematic)

small--one or two millivolts--and comparable to that which might be induced by fluctuating stray electrical fields. Consequently, care was taken to encase the entire circuitry in shielding (see figure 5-1), the only exception being a one-foot section of patch-cords used to connect the power supply and voltmeter to the multi-strand shielded cable leading to the cryostat.

4. Display of I-V Characteristics

A Moseley X-Y recorder, Model 2DR-2, was used to plot out the current-voltage curves. Because its maximum sensitivity was only 0.5 mV/in.--the maximum junction voltage of interest for a Sn specimen would be about 1.5 mV--it was necessary to provide some amplification, ie. a Hewlett Packard 425A voltmeter with recorder output, for a reasonable display. (see figure 5-1). The current was measured with a Keithley, Model 600A, battery-operated electrometer which provided a voltage output signal for the Y axis of the recorder. A battery powered ammeter was employed because passive meters having suitably wide current ranges were insufficiently sensitive and reasonably priced line-powered meters were not adequately isolated from ground. Low frequency ripple, mostly 60 Hz, present on the output of both the voltage and current meters introduced hysteresis in the curves traced out by the recorder. The amount of hysteresis was reduced to negligible proportions by placing a low-pass filter, 7.5 K Ω and 64 μ F, across the outputs of both meters but the introduction of this 0.5 sec time constant required that the curves be traced out quite slowly and uniformly.

B. Helmholtz Coils for Magnetic Biasing

As pointed out in Chapter 3, it is necessary to provide a magnetic field of up to 100 G in the plane of the junction to suppress the Josephson tunneling current. Helmholtz coils were chosen as being the simplest and most economical means of producing a uniform, low level magnetic field. Space limitations required that the coils be placed outside the nitrogen dewar setting therefore a lower limit of about 6.5 inches on the coil radius.

Consequently, a pair of coils were designed and constructed with the following specifications:

Power Supply.....Hewlett-Packard, Model 6268A, 40V-30A,
operated in constant voltage mode.

Figure 5-2: Calibration of Helmholtz Coils (Series Mode)

Maximum Field.....100 G, Series mode
200 G, Parallel mode
Power Dissipation.....330 watts, series mode
1150 watts, parallel mode
Number of Turns.....298 per coil
Coil Diameter.....14 in. i.d., 17 in., o.d.
Coil Width.....1.31 in.
Coil Form.....Aluminum
Wire.....13 gauge, heavy Formel covered copper.

The power supply was operated in the constant voltage rather than the constant current mode because the quoted ripple and noise (1 μ V rms) was much less in the former.

The coils were wound in such a way that current could be passed through all 298 turns connected in series or two 149 turn superimposed coils connected in parallel (see inset of figure 5-2). In this way, provision was made for fields up to almost 200 G, without having to use a higher voltage supply.

Calibration of the centre field region, carried out with a Bell, Model 240, Incremental Gaussmeter, showed the coils to meet design specifications. Figure 5-2 contains the calibration curve for the coils connected for series operation.

C. Pulse Detection Electronics

1. Biasing and Pulse Circuitry

Junctions to be used for alpha particle detection were mounted on the so-called "pulse test" sample holder described in figure 4-3; the circuitry required to provide dc bias and connection to the pulse amplifier is shown in figure 5-3. The filter box serves to isolate the dc amplifiers from unwanted ac signals and provide a low impedance path to the pulse amplifier. Discussion of the electrical properties of the shielded parallel pair transmission is postponed until section E of this chapter.

2. Preamplifier Design

Figure 5-4 is a schematic of the low noise, current sensitive common base (CB) preamplifier, with a common collector second stage serving

Note:
Capacitances in μF
Resistances in Ω unless
otherwise noted.

Figure 5-4: Preamplifier Schematic

as impedance transformer. (The common emitter (CE) preamplifier ($Z_{in} = 400 \Omega$) shown in the diagram was constructed on the same chassis to provide for voltage sensitive amplification should the need have arisen.) The input impedance ($Z_{in} = R_{in} + jX_{in}$) of the CB preamplifier, determined directly with a Hewlett-Packard RF Vector Impedance Meter (Model 4815A), is plotted as a function of frequency in figure 5-5. An independent check of the input impedance of the preamplifier and the preamplifier-filter box combination was made (see figure 5-5) with an oscilloscope and signal generator by measuring the phase and amplitude relations between input current and voltage. Some of the inductive reactance may be attributed to the $1\mu F$ coupling capacitors in the filter box and preamplifier which, when checked separately, were found to look like $1\mu F$ in series with $0.5\mu H$, having a self resonating frequency of about 0.9 MHz.

The 10-90% risetime (τ_R) was determined by observing the response of the amplifier to an input pulse rising in less than 1 nsec. It was found that $\tau_R \approx 15$ nsec which is consistent with the measured upper 3db frequency of 28 MHz.

3. Ancillary Electronics

Apart from the preamplifiers and veto discriminator, the remaining amplifying and pulse handling devices used were of commercial manufacture; figure 5-6 is a block diagram illustrating the arrangement of the various instruments. Table 5-1 lists the individual units.

A check of the amplifier system sensitivity was performed by using it to amplify known amplitude pulses developed by a pulse generator in a lumped constant model of the small signal equivalent circuit of the specimen. The system was considered to be sufficiently sensitive when the so-called "minimum detectable" input pulse--that pulse whose corresponding output signal was equal to 3 times the rms noise voltage--was comparable to the anticipated pulses (see section G, Chapter 3) originating in the tunnel junction due to alpha particle bombardment.

D. Interference Vetoing System

Such a high gain amplifier system is, unfortunately, highly sensitive to electro-magnetic interference. In spite of careful shielding, spurious signals were detected which were found to originate in such sources as fluorescent lights, electric motors and the Helmholtz coil power supply.

Figure 5-5: Preamplifier Input Impedance ($Z_{in} = R_{in} + jX_{in}$)

Figure 5-6: Electronics Arrangement; Electronic Units are listed in Table 5-1

The amplitude and number of these unwanted signals were such that extracting information concerning the legitimate pulses coming from the junction detector by discriminator level setting or statistical analysis

TABLE 5-1 Electronic Units Used on the Experiment

- A - LeCroy Research Systems, Dual Linear Amplifier, Model 107F
- B - Hewlett-Packard, Linear Amplifier, Model 462A, (2)
- C - Ortec, Linear Amplifier, Model 410
- D - Hewlett-Packard, Linear Amplifier, Model 460A
- E - Nuclear-Data, Pulse Height Analyzer, Model 101 (Note: This unit was out of order when the pulse data were actually acquired from specimen.)
- F - Keithley, Wideband VTVM, Model 120
- G - Ortec, Discriminator-Scaler, Model 429
- H - Ortec, Scaler, Model 430
- I - Tektronix, Oscilloscope, Model 454
- J - Ortec, Pulse Stretcher, Model 411
- K - U.B.C., Interference Veto Discriminator, Circuit Diagram, fig. 5-7
- L - Datapulse, Pulse Generator, Model 106A, (not shown on fig. 5-6)

was not feasible (see also the brief discussion of the Veto system in Chapter 6). Consequently an "Interference Vetoing" system was devised (see figure 5-6) which operated in parallel with the main amplifier system--with appropriate antennae to pick up the same interference as the main system--and put out gating pulses to paralyze the scaler and pulse height analyzer whenever the interference signal exceeded a threshold value. (The threshold level is conveniently set by stopping the alpha particles before they reach the junction by lowering it below the helium bath surface and adjusting the discriminator until "zero" counts are observed on the gated scaler.)

Figure 5-7 is a schematic of the veto discriminator unit designated by "K" in figure 5-6.

E. Electrical Properties of Transmission Line

1. Introduction

A brief physical description of the so called shielded parallel pair transmission line used in this experiment was given in chapter 4. This section summarizes the electrical characteristics of the

Figure 5-7: Schematic of Veto Discriminator

line needed for the noise and pulse shape analysis of chapter 7 and describes how they were determined.

To a reasonable approximation, the line (assumed to be linear) may be represented by lumped constants for its length ($\ell \approx 1.6\text{m}$) is much less than the minimum wavelength of 85m corresponding to the upper 3db-down frequency of the amplifier system (3.5 MHz). The impedance Z_i seen looking into such a line when it is terminated in $Z_t = R_t + jX_t$ is simply $Z_i = R_\ell + R_t + j(X_\ell + X_t)$ where $Z_\ell = R_\ell + jX_\ell$ is the effective series impedance of the line.

The following two paragraphs describe the measurement of R_ℓ and X_ℓ and the establishment of Z_i for practical transmission line-load configurations. Paragraph 5 is an investigation made to check the consistency between those parameters measured and those which may be estimated approximately from theory.

2. Effective Line Impedance

Although Z_ℓ may have been estimated from theory, little confidence could be placed in the results because of the inhomogeneity of the line (see paragraph 5) and the temperature range it spanned. (One end was immersed in 1.2 K liquid helium while the other end was thermally anchored at room temperature 295 K.) Consequently, R_ℓ and X_ℓ were determined experimentally over the amplifier passband by shorting the "detector end" of the transmission line (making $R_t = X_t = 0$) and measuring $Z_i = Z_\ell = R_\ell + jX_\ell$ with a Hewlett-Packard Model 4815A RF Vector Impedance Meter (Z meter) for three different temperature conditions, namely: complete line at room temperature (295 K), one end at 295 K with the remaining 90% cooled to 77 K with helium gas, one end at 295 K with the other at 1.2 K as in the actual experiment. The results are plotted in figure 5-8 where the error bars shown are typical and arise from the accuracy limits of the Z meter. (For a reading of $Z = |Z| \exp(j\phi)$, the accuracy limits quoted by the manufacturer are $\Delta Z = \pm 4\%$ F.S.D. and $\delta = \Delta\phi = \pm 3^\circ$ so that $R = |Z| \cos \phi (1 \pm \Delta Z/|Z| \pm \delta \tan \phi)$ and $X = |Z| \sin \phi (1 \pm \Delta Z/|Z| \pm \delta \cot \phi)$.)

(a) Effective Resistance

The effective line resistance is clearly both temperature and frequency dependent. A least squares fit of $R_\ell = R_{\ell 0} + R_{\ell 1} \cdot f$ to the data

in figure 5-8 yields the following values. (For future reference, when the line is said to be at temperature T, it is to be understood that one end is at 295 K and the other at T; eg. $R_{\ell 0}(77 \text{ K}) = 1.73 \pm .34\Omega$).

Temperature Range (K)	$R_{\ell 0}(\Omega)$	$R_{\ell 1}(\Omega \text{ MHz}^{-1})$
295-295	$2.47 \pm .49$	$.875 \pm 0.17$
295-77	$1.73 \pm .34$	$.534 \pm .10$
295-1.2	$1.60 \pm .32$	$.557 \pm .11$

(b) Effective Inductance

Within experimental accuracy, the reactive component of Z_i is temperature independent and purely inductive so that it may be written $X_{\ell} = 2\pi f L_{\ell}$ where L_{ℓ} is the effective inductance of the line over the bandwidth of interest. Setting $2\pi L_{\ell}$ equal to the slope of the best straight line through the experimental points for all three temperatures yields

$$L_{\ell} = 1.63 \pm .13\mu\text{H}$$

This value of L_{ℓ} agrees closely with the value of $1.6\mu\text{H}$ obtained by terminating the line in a calibrated, (925 pF) high frequency capacitor and finding the resonant frequency $\omega_o^2 = (LC)^{-1}$.

3. Line Input Impedance with Practical Loads

On the basis of the preceding results, it may be said that, for the frequencies used in this experiment, the transmission line terminated in Z_t may be regarded as a two-terminal network whose input impedance is $Z_i = R + jX$ where, for temperature T

$$\begin{aligned} R &\approx R_t + R_{\ell 0}(T) + R_{\ell 1}(T) \cdot \omega / 2\pi \\ X &\approx X_t + j\omega L_{\ell} \end{aligned} \tag{5-1}$$

If equation 5-1 is to be used to describe the input impedance looking into

Figure 5-8: Resistance & Reactance of Transmission Line.

either end of the line, it must be assumed that the effective line impedance $Z_{\ell} = R_{\ell}(T) + j\omega L_{\ell}$ is the same whether determined from the warm or "cold" end. For $T = 295$ K, this assumption was verified experimentally but it could not be confirmed, of course, at $T = 77$ or 1.2 K.

Two specific transmission line-load configurations are of interest to the interpretation of the experimental results. For pulse shape analysis, the desired impedance is that seen by the 1.2 K junction as it looks into the cold end of the transmission line when it is terminated with the preamplifier at 295 K (similarly to inset, figure 5-9); this impedance will be denoted by $Z_{i,cold} = Z_{ic}$. For noise calculations, the required impedance is that seen by the preamplifier as it looks into the warm end of the transmission line when it is terminated with a cold tunnel junction (see figure 7-1); this impedance will be denoted by $Z_{i,warm} = Z_{iw}$.

To test the accuracy of equation 5-1 in predicting Z_{ic} and Z_{iw} for the actual junction, studies were made (at 295 K) of the line terminated with the preamplifier and with the line terminated in a lumped constant model of a tunnel junction. To illustrate these tests, the results with the preamplifier as load are given below.

(a) Z_i for Line with Preamplifier as Load

The preamplifier input impedance $Z_{in} = R_{in} + jX_{in}$ was determined independently (see figure 5-5) so that by equation 5-1, the predicted impedance looking into the line is

$$Z_i = R_{in} + R_{\ell}(295 \text{ K}, \omega) + j(X_{in} + \omega L_{\ell}) \quad (5-2)$$

This may be compared to $Z'_i = R' + jX'$ obtained with the Z meter by connecting the preamplifier to the line as shown in the inset of figure 5-9. If the model of equation 5-1 is valid it is expected that $Z_i \approx Z'_i$.

The results are given in figure 5-9. Clearly, the measured reactance X' agrees well with that obtained from equation 5-2. The agreement between the corresponding resistive components of the impedance is not as good but may be made consistent by using the slightly smaller average value of $R_{in} = 7.5\Omega$ instead of 7.9Ω as determined from figure 5-5. (Such a change is within the accuracy limits stipulated by the Z meter

Figure 5-9: Impedance of Transmission Line with Preamplifier as Load.

manufacturer.)

(b) Z_i for "Cold" Transmission line + Preamplifier

On the basis of the Z_ℓ symmetry assumption stated earlier and the agreement between measurement and the model of the line represented by equation 5-1, Z_{ic} may be written

$$Z_{ic} = R_{in} + R_\ell(1.2 K, \omega) + j\omega L_c = R_{ic} + jX_c \quad (5-3)$$

where $R_{in} = 7.5\Omega$ and $L_c = L_\ell + L_{preamp.} = 1.69\mu H$. For reference, R_{ic} is plotted in figure 5-9.

(c) Z_i for Transmission line + Tunnel Junction

By the same reasoning as given above, Z_{iw} may be written

$$Z_{iw} = R_j + R_\ell(1.2 K, \omega) + j(X_j + \omega L_\ell) \quad (5-4)$$

where $Z_j = R_j + jX_j$ is the tunnel junction output impedance.

4. Characteristic Impedance

The effective line resistance (R_ℓ) and inductance (L_ℓ) which have been deduced are all that is required to represent the line in the lumped constant series impedance approximation that has been chosen. To aid in understanding the line and to ensure that the measured parameters for the line are self-consistent, the characteristic impedance Z_o of the line was also determined.

Z_o was found by terminating the transmission line in a purely resistive load R_L and adjusting R_L until the input impedance of the line (Z_i) as measured by the Z meter exhibited no reactive component. (Note: "purely resistive" and "no reactive component" mean that within the $\pm 1\frac{1}{2}^\circ$ empirically determined phase angle sensitivity of the instrument near zero, the observed phase angle in the two impedances was zero.) The value of Z_o thus deduced was

$$Z_o = 168 \pm 17\Omega$$

where the errors arise from the accuracy limits on the phase angle (as just

stated) and on the magnitude (as given in paragraph 2) for both R_L and Z_i .

5. Consistency Check of Measured Values with Theory

The preceding paragraphs have shown that all the parameters needed to characterize the line in the analysis of succeeding chapters may be determined experimentally. No faith could be placed in these quantities calculated from theory because the line operated over a significant range of temperatures and bore but faint resemblance to the ideal, simple configurations for which calculations are readily made (see eg. Bleil, 1957). This section first describes the very non-ideal construction of the line and then compares the measured parameters with those estimated for the shielded parallel pair (spp) transmission line--the geometry which most closely approximates the actual line.

Unlike the ideal spp line, the experimental line was not operated in balanced mode for both the "shield" and one of the wires were grounded. Being homemade, it had a non-uniform structure. For example, because of space limitations, the "shield" was only 2/3 as long as the center conductors; the two wires were somewhat unevenly spaced both with respect to each other and to the "shield"; three dielectrics were present: (1) a thin layer (≈ 0.001 in.) of Formel varnish which covered the wires, (2) quasi-continuous lumps of Q-dope, which occupied perhaps 20-40% of the volume inside the "shield", (the Q-dope was used to space the wires) and (3) room air or helium which filled the remaining volume.

Table 5-2 summarizes the comparison between the experimentally and theoretically estimated parameters. Considering the inhomogeneous nature of the line, the agreement is quite reasonable.

6. Propagation Velocity

From the foregoing discussion, it is evident that a reasonable representation of the line has been obtained in that impedances measured with this model are self-consistent and the parameters of the model agree fairly well with those estimated from theory. However, one aspect of the transmission line remains which is not yet completely understood but which, fortunately, is not essential to the analysis of later chapters--the propagation velocity $u = (LC)^{-\frac{1}{2}} = Z_o/L \approx (.56 \pm .1)c$ seems to

be inordinately low.

This value of $u = c(K'K'_m)^{-\frac{1}{2}}$ implies that $K'K'_m = 3.2^{+1.5}_{-.9}$ where $K' = \epsilon'/\epsilon_0$ is the effective dielectric constant and $K'_m = \mu'/\mu_0$ is the effective relative permeability. It is difficult to perceive how such

Parameter	Experiment	Theory (Model)
Resistance (R, 3 MHz)	$3.1 \pm .6\Omega/\text{m} *$	$2.1\Omega/\text{m}$ (spp)
Inductance (L)	$1.01 \pm .08\mu\text{H}/\text{m} *$	$1.3\mu\text{H}/\text{m}$ (pp) †
Characteristic Impedance Z_0	$168 \pm 17\Omega$	190Ω (spp) §

Table 5-2: Comparison of Experimental and Theoretical Transmission Line Parameters.

* The physical length of the line is 1.62 m.

† (pp) denotes parallel pair--no estimate of L is available for the spp.

§ This value of Z_0 depends on the value of effective dielectric constant and effective relative permeability assumed for the line--see paragraph 6.

a large product obtains in view of the fact that the major dielectric in the line, by volume, is air and there are no obviously magnetic materials present.

Consider first the effective dielectric constant. In addition to air, the other dielectrics present in the line were Formel (a polyvinyl acetate resin) with $K = 2.92$ (von Hippel, 1954) and Q-dope (mostly polystyrene) with $K = 2.52$ – 2.65 as stated by the manufacturer (GC Electronics, Rockford, Illinois) and checked experimentally. The frequency stability of both dielectrics is illustrated by the fact that K (polystyrene) is essentially independent of frequency over the range 1 – 3000 MHz and K (Formel) decreases by only 5% over the same range (von Hippel, 1954).

Although the Formel and Q-dope occupy only a relatively small fraction of the volume inside the "shield", their contribution to the effective dielectric constant is larger than their contribution to the volume. This arises from the fact that these substances are located close

to the wire surfaces where the electric field, and therefore the stored energy density, is largest. (To illustrate this point, consider a line with simple co-axial geometry with radii a (wire) and c (shield) and a dielectric K surrounding the wire out to a radius b such that for $a < r \leq b$, $K > 1$ and for $b < r \leq c$, $K = 1$. The effective dielectric constant of such an arrangement is $K' = \ln(c/a) (K^{-1} \ln(b/a) + \ln(c/b))^{-1}$; setting $a = .0035$ in., $b = .03$ in., $c = .06$ in. and $K = 2.6$, as a very rough approximation to the situation in the actual transmission line, yields $K' \approx 1.86$ which differs significantly from the value $K'(\text{volume}) = 1.37$ calculated on the assumption that each dielectric contributes in proportion to its relative volume.) It might be expected that this effect would be even more pronounced in the twin wire line because the electric field lines would tend to be concentrated in the Q-dope filled region keeping the wires apart. Thus, an effective dielectric constant as large as 2.3 (the lower limit on $K'K'_m$) is perhaps conceivable.

There seems to be no way in which the transmission line could exhibit an effective relative permeability significantly greater than one. It is possible that the relative permeability of the type 304 stainless steel used in the shield was greater than one--due to cold working effects and the irreversible partial transformation from the austenitic to the martensitic phase which takes place in this type of stainless steel as it is repeatedly cooled to helium temperatures (Reed and Mikesell, 1960)--but because of its relatively small volume, which is further essentially reduced by the skin effect, the contribution of the "shield" to the overall permeability of the line is expected to be negligible. (Because it was not possible to test K_m for the tubing used as the "shield" without destroying the line, a piece of similar "off the shelf" tubing (not known to have been cooled to helium temperatures) was checked but it gave no evidence of having $K_m > 1$.)

From these considerations, it appears that the propagation velocity deduced from Z_0 and L is not convincingly consistent with reasonable values of the effective dielectric constant and relative permeability but that the maximum value of u allowed by the errors does approach a value which is consistent with theory. This problem was not pursued further as it was not considered central to the purpose of the experiment. To satisfactorily resolve the anomaly would require a careful study of the actual line which, because of its unwieldy geometry and horrendous mixture of dielectric interfaces, presents a formidable challenge.

F. Summary

The apparatus for tracing out the dc I-V curves, magnetically biasing the specimens and observing the pulses output from the detector has been described. Quantities of particular interest to subsequent chapters are:

1. Preamplifier + Filter Input Impedance (Z'_{in})

$$Z'_{in} = R'_{in} + jX'_{in} \text{ where, in the amplifier bandpass,}$$

$$R'_{in} \approx 7.9\Omega = R_{in} \text{ and } X'_{in} = \omega L - (\omega C)^{-1} \quad (5-5)$$

with $L = 0.23\mu\text{H}$ and $C = 0.5\mu\text{F}$.

2. Transmission Line + Load Equivalent Circuit

For the transmission line at temperature T terminated in $Z_t = R_t + jX_t$, the effective input impedance is approximately $Z_i = R + jX$ where

$$R = R_t + R_\ell(\omega, T)$$

$$X = X_t + j\omega L_\ell$$

with ω being the angular frequency, $L_\ell = 1.63\mu\text{H}$ the effective line inductance and $R_\ell = R_{\ell 0} + R_{\ell 1} \cdot \omega/2\pi$ the frequency and temperature dependent effective line resistance described in section E, paragraph 2.

CHAPTER 6

RESULTS

A. dc Characteristics

As mentioned in earlier chapters, it was necessary to obtain the dc I-V characteristics of each junction at helium temperatures to determine its suitability as a detector. The quantities of particular interest, as outlined in Chapter 3, were the dynamic resistance $r = (\partial V / \partial I)_T$ and the degree to which the supercurrent could be suppressed by the application of a magnetic field. This last mentioned test particularly discriminates against "leaky" junctions--those with electrical shorts through the insulating layer--as the resultant supercurrent is very much less sensitive to a magnetic field than is the dc Josephson supercurrent. "Leaky" junctions and their characteristics are described briefly in Appendix B.

1. Temperature Dependence of Tunneling Current

Figure 6-1 illustrates the variation in the dc I-V characteristic with change in temperature for a so called "good" Sn-Sn junction. The marked decrease in current with decreasing temperature and the fact that a field of 100 G has clearly suppressed the zero-voltage current indicate that the observed current flow is that of tunneling single quasiparticles. The distinction between this I-V characteristic and one obtained from a typical leaky junction, as shown in Appendix B, is thus readily apparent.

In addition, it may be seen that the slope $(\partial I / \partial V)_T$ of the I-V curve decreases as the temperature is lowered. This points out the need to operate the junction at the lowest convenient temperature so that the dynamic resistance r may be as large as possible to maximize the signal to noise ratio in accord with the theory outlined in Chapter 3.

2. Magnetic Field Dependence of Tunneling Current

The observed variation of the junction current with a magnetic

field parallel to the plane of the junction is shown in the composite X-Y recorder tracings of figure 6-2. (In practice, plots of this sort were more useful diagnostically than the conventional $I(\text{crit})$ -B or Fraunhofer type plot shown in figure 2-8(b)). In particular, three features are evident:

(a) Magnitude of the Supercurrent ($\vec{B}=0$)

For a given junction at a fixed temperature, the observed maximum zero-voltage current (I_c) was found to vary from tracing to tracing and to be only a small fraction of the theoretical maximum supercurrent (see equation 2-16) given by

$$I_{cm} = J_o(T) \cdot A \approx \pi \Delta(0) / 2eR_n$$

The observations are summarized in table 6-1.

SPECIMEN	THEORETICAL I_{cm}	AVERAGE OBSERVED I_{cm}	$\frac{I_{cm} \text{ OBS.}}{I_{cm} \text{ THY.}}$
H-4	3mA	.04±.02mA	.013
H-5	4.5mA	.045±.005mA	.01
J-5	11.2mA	.35±.15mA	.031

Table 6-1: Comparison of Experimental & Theoretical Maximum Supercurrent.

That the observed I_c is almost two orders of magnitude smaller than the theoretical maximum for zero magnetic field may be attributed to the fact that no attempt was made to magnetically shield the specimens. Workers studying the dc Josephson effect and detecting I_c as high as 50-90% of I_{cm} (theoretical), report (Jaklevic et al, 1965) finding it necessary to cool their junctions while shielded from the earth's magnetic field. Otherwise, flux was apparently trapped in the films or junctions which severely attenuated the maximum Josephson current. In addition, the condition " $\vec{B} = 0$ " in the present experiment indicates only that the field due to the Helmholtz coils cancelled out the component of the earth's field lying in the junction plane (see figure 5-2). Most likely, therefore, at " $\vec{B} = 0$ " stray magnetic fields were present which, as

discussed in Chapter 2, need only be of small magnitude to have a profound effect on I_c .

(b) Magnetic Field Dependence of I_c

As outlined in chapter 2, I_c should decrease with increasing magnetic field according to $\left| \frac{\sin(\pi XB)}{\pi XB} \right|$ where B is the field magnitude and X is a term characteristic of a given junction and typically of the order 1.5 G^{-1} . (For $B \approx 70 \text{ G}$, $I_c \leq I_{cm}/100\pi$). Qualitative agreement with theory is clearly evident in figure 6-2 which indicates that the "good" junctions do indeed exhibit the dc Josephson effect. The important point, as far as the present experiment is concerned, is that the zero-voltage supercurrent can be effectively suppressed by the application of a magnetic field which is much smaller than the critical field of the superconducting films. (See paragraph 5 of this section.)

(c) Variation of $\partial V/\partial I$ with Magnetic Field

For a given magnetic field B and temperature T, there is a point (V_m, I_m) at which $r = \partial V/\partial I$ ($eV \leq 2\Delta$) is a maximum. It is evident in figure 6-2 that $r_m = (\partial V/\partial I)_{V_m, I_m}$ varies somewhat with magnetic field, decreasing sharply at about 70 G as shown in figure 6-3. This decrease in r_m , the consequence of an increasingly significant reduction in the superconducting energy gap width effected by the magnetic field, is similar to the decrease in r_m brought about by increasing the temperature--thereby reducing the gap--as is evident in figure 6-1.

(d) Optimum Biasing Point

The criteria for and means of determining the optimum operating point are now clear. From a series of curves like those in figure 6-2, taken at the lowest accessible temperature, the values of I, V and B are found at which r_m is maximum and the supercurrent is minimum. For example, had specimen H-4 not degenerated with thermal cycling--to be discussed in paragraph 4 of this section--the biasing point chosen would have been $I = 9.5\mu\text{A}$, $V = 0.22\text{mV}$ and $B = 70 \text{ G}$ where $r_m \approx 100\Omega$ at 1.2 K.

3. Determination of Dynamic Resistance ($\partial V/\partial I$) as a Function of Voltage

No provision was made for obtaining $\partial V/\partial I$ directly on an X-Y plot. For purposes of selecting the operating point it was sufficiently accurate to draw tangents to the I-V curves and determine their slope. It

Figure 6-3: Variation of maximum dynamic resistance with magnetic field

Figure 6-4: Dynamic resistance vs voltage for specimen J-5

turned out, however, that in order to understand the manner in which the noise varied with bias voltage--see section D of this chapter--a more precise evaluation of $r = \partial V / \partial I$ was necessary for sample J-5, the sample with which the alpha particles were detected.

A fourth order polynomial $I = \sum_{n=0}^4 a_n V^n$ was least squares fitted to each of two experimental curves ($0 \leq V \leq 0.8$ mV) obtained at $T = 1.2$ K and $B = 30$ G for this specimen. For each curve, $r = (\partial I / \partial V)^{-1}$ was calculated with the mean value being shown in figure 6-4.

4. Deterioration of Specimens after Thermal Cycling

Because of the high percentage of faulty junctions, the testing apparatus was designed (see Chapter 4) so that five specimens could be "dc" tested at one time in the so called "dc holder" to determine the specimen with the most desirable characteristics. After warming slowly (24-36 hours) up to room temperature in a helium atmosphere, the specimens were removed from the "dc holder", the best specimen was separated from the others and mounted in the "ac" or "pulse test holder" (see figure 4-2 and 4-3). This extra handling of the specimens was necessitated by the fact that the "pulse test holder", which was equipped with the alpha source and shutter, was originally designed to take only one 1 cm x 2 cm substrate with its four electrical leads.

It was discovered--when Sn specimens were finally made having satisfactory dc characteristics--that upon being warmed up to room temperature, installed in the "pulse test holder", and cooled again to helium temperatures, a specimen suffered considerable, apparently irreversible, changes in its tunneling properties rendering it useless as a detector. Of the 6 samples for which data are available, the general symptoms were a decrease in the maximum dynamic resistance, little or no temperature dependence of the current (unlike figure 6-1) and reduced sensitivity of the supercurrent to a magnetic field (unlike figure 6-2). Table 6-2 lists the properties of one such junction (H-4) which was cycled in the above manner. These "after cycling" characteristics are very similar to those of the "leaky" specimens discussed in Appendix B which implies that during the cycling procedure, minute metallic filaments or superconducting shorts had developed through the insulating oxide. The fact that these

filaments produced no significant change in the 4.2 K normal state junction resistance may be explained as follows. If R_n is the normal state resistance

SPECIMEN Property	H-4		J-5	
	Before Cycling	After	Before Cycling	After
R_n (4.2 K)	.286 Ω	.289 Ω	.077 Ω	.076 Ω
I_c (B)	.0003mA(100G)	.1mA(100G)	.001mA(40G)	.01mA(40G)
$r_m = \partial V / \partial I$	64 Ω	4 Ω	70 Ω	9.3 Ω
Temp. Dependence	Fits Theory	Insensitive to Temp.	Not Measured	Reasonable
B Dependence	I_c quenchable	I_c not quenchable	I_c quenched	I_c quenched

Table 6-2: Specimen Characteristics Before and After Thermal Cycling.

before and R_n' is the normal resistance after cycling, then

$$\frac{1}{R_n'} = \frac{1}{R_n} + \frac{1}{R_{fil}}$$

where R_{fil} is the resistance of the filaments when in their normal state. Rough estimates of R_{fil} obtained by assuming that the current carried by the filaments I_{fil} is given by

$$I_{fil} = I(V_o, T_o, \text{ after cycling}) - I(V_o, T_o, \text{ before cycling}) = \frac{V_o}{R_{fil}}$$

indicate $R_{fil} \approx 3-6\Omega$ which means that, within experimental error, $R_n' \approx R_n$.

In an attempt to prevent deterioration, a new method was devised for cycling sample J-5 which was successful insofar as the ensuing changes in the properties of the junction were not so severe as to prevent

its being successfully used as a detector. This new technique consisted of warming the specimens up rather rapidly (about 1 1/2 hours) from 4.2 K by slowly raising them out of the helium bath into a helium-filled plastic bag kept at room temperature. Specimen J-5 was then installed in the "pulse holder" and the reverse procedure carried out. In this way the turn around time was reduced by an order of magnitude and the time of exposure to "high" temperatures minimized.

The mechanism by which the metallic filaments appear is not clear. It is interesting to note that following the initial thermal cycling, specimen J-5 was kept at a temperature of 4.2 K or less for some 40 hours with no change in performance which would seem to exonerate the testing and operating procedures. Great care was always taken to avoid the condensation of water on any junction as the deleterious effect of water on thin films is well known. Three possible causes of the filament appearance which cannot be ruled out however on the basis of accumulated evidence are: (1) Strain in the junction region caused by differential contraction of the films and substrate, (2) Contamination by water vapour during the re-mounting time, (3) Contamination by vapour from remnants of solder flux left on the "pulse holder" electrical terminals.

The obvious solution to the deterioration problem in future work is to avoid thermal cycling by building a specimen holder in which several specimens can be tested at once and the best ones selected for pulse measurements without ever being removed from the helium bath.

5. Magnetic Field Dependence of Energy Gap

The observed magnetic field dependence of the superconducting energy gap is shown in figure 6-5. If the value of the gap is chosen by extrapolation to the voltage axis, one finds

$$\frac{2\Delta(B = 100)}{2\Delta(B = 0)} = \frac{1.05 \text{ meV}}{1.11 \text{ meV}} = 0.945$$

which is consistent with the findings of other workers (Douglass & Falicov, 1964) for films at these temperatures in that their empirical relation

Figure 6-5: Magnetic Field Dependence of Energy Gap

(equation 2-7) yields

$$\frac{\Delta(H)}{\Delta(0)} = 1 - \left(\frac{H}{H_{cf}} \right)^2 \approx 1 - \left(\frac{100}{382} \right)^2 = .93$$

Here, $H_{cf} = 382$ Oe is the critical field for a thin Sn film at temperature T calculated from (Meservey & Douglass, 1964)

$$H_{cf}(T) = H_c(0) (1 - (T/T_c)^2) (1 - (2\lambda/d) \tanh(d/2\lambda))^{-\frac{1}{2}}$$

with

$$T = 1.2 \text{ K}$$

$$T_c = 3.72 \text{ K (Critical temperature)}$$

$$H_c(0) = 306 \text{ Oe (Bulk Critical field at } T = 0 \text{ K)}$$

$$\lambda = 500 \text{ \AA (Penetration Depth)}$$

$$d = 2000 \text{ \AA (Film Thickness)}$$

The magnetic field used with the detector junction (J-5) was only 30 G so that the decrease in the energy gap due to magnetic biasing may be taken to be less than 5% and may be safely neglected in further analysis.

B. Observation of Pulses from Alpha Particles

Following the dc tests and re-mounting procedure described in the previous section, sample J-5 was cooled to 1.2 K and the optimum operating point determined as discussed previously. (see figure 6-6)

1. Counting of Pulses

With the electronics set up as in figure 5-6, the bandwidth having been adjusted for equal rise and fall times of 100 nsec, counts were observed on the discriminator-scaler. That the pulses were produced by bombardment of the junction area itself was evident from the observation that the pulses could be controlled by closing the mechanical shutter (shown in figure 4-3) or by lowering the substrate until the overlap area was just below the level of the helium bath.

Figure 6-6: Detector Biasing Conditions

(a) Count Rate

The observed count rate, with shutter open and the discriminator set at 5 times the rms noise output from the amplifiers (33 mV for this bandwidth), was 222 ± 3 counts per minute (cpm). This agrees reasonably well with the particle flux at the junction predicted from the source strength and junction-source geometry illustrated in figure 6-7. If $N_o = 1.1 \times 10^7$ is the particle flux from the source then N_j , the number of alpha particles striking the junction per minute, is $N_j = N_o A \cos(\theta) / 4\pi l^2 = 198$.

A reduced number of counts, about 65 cpm, was also observed when the shutter was in the "closed" position--a disturbing result which fortunately has a ready explanation. In order to maximize the path length of the alpha particles through the junction, the source was located to one side of the substrate (see figure 6-7) so that the shutter, which moved in the plane of the junction, when "closed" or pressed against the source left about 35% of the source still exposed to the junction. The predicted count rate with the shutter "closed" would thus be $.35 N_j = 69$ cpm which is consistent with the observed rate of 65 cpm. Some contamination was detected on the shutter but the number of alpha particles coming from this source were a negligibly small fraction, roughly 5×10^{-4} , of those coming from the main source.

(b) Influence of Veto System

Chapter 5 contains a description of an interference-vetoing system which was designed to paralyze the discriminator-scaler whenever bursts of electro-magnetic interference were present thereby preventing these being counted as meaningful pulses. Thus, only those pulses from the detector amplifying system were counted whose amplitude exceeded the scaler discriminator level (V_{disc}) and whose arrival in time was anti-coincident with an interference pulse. All other pulses, whose amplitude exceeded V_{disc} , were counted on another scaler--see figure 5-6--and the count rate for these ranged from 13,000 to 18,000 cpm. Clearly, with such a high interference count level, the small count rate of 200 cpm due to legitimate pulses from the detector itself would have been undetectable without an efficient vetoing system.

Fig. 6-7: Junction-Source Geometry
(Shutter not shown for clarity)

Fig. 6-8: Pulses observed at optimum bandwidth;
 Vertical - $1.75 \mu\text{A}/\text{div.}$; Horiz. - $200 \text{ nsec}/\text{div.}$
 (Vertical scale calibrated for a square input
 pulse as described in section B.2 (p.125).)

Even with such a relatively high interference pulse count rate, the corresponding "dead time" or percentage of time during which the discriminator-scaler was paralyzed and unable to accept pulses from the detector was negligibly small, being less than about 0.5%.

2. Pulse Characteristics

While it is interesting and satisfying to observe that the count rate agrees with the predicted alpha particle flux at the junction, the key to understanding the physical processes taking place in the detector lies in the interpretation of the characteristics of the actual pulses.

The pulses were recorded photographically on a wide-band oscilloscope (2.4 nsec rise time) which was connected to the amplifier and vetoing systems as shown in figure 5-6. Optimization of the amplifier system bandwidth (τ_{int} and τ_{diff} "out" on Ortec amplifier) led to the observation of pulses (shown in figure 6-8) whose amplitudes were up to 19 times the rms noise level. The upper and lower 3-db points of the amplifier system at this bandwidth setting were later measured to be 0.1 MHz and 3.5 MHz respectively which is consistent with the observed response (100 nsec rise time and 1.7 μ sec fall time) of the system to square pulses injected from a pulse generator.

To determine its current sensitivity, the amplifier system was used to measure pulses of known amplitude produced by a pulse generator in a lumped constant model of the junction small signal equivalent circuit (see figures 6-9 and 3-5)

$$(I_o)_{step} = \frac{V_{in}}{39 + \left(\frac{1}{10} + \frac{1}{R_{in}} \right)^{-1}} = \frac{V_{pg}}{(\text{attenuation})(44\Omega)}$$

For $V_{pg} = 110$ mV and attenuation of 50 db, $(I_o)_{step} = 8\mu A$ which corresponds to an output voltage of $V_{out} = 2.3$ V making the amplifier system sensitivity

$$S = 1V/3.5 \mu A \quad (6-1)$$

The signal to noise ratio was a maximum for the operating point shown in figure 6-6 at which $r = (\partial V / \partial I)_T$ was maximum. Reducing r

Figure 6-9: Current Sensitivity Calibration

Figure 6-10: Pulse Amplitude vs Dynamic Resistance

by shifting the bias voltage above or below 0.3 mV caused both the noise level to increase, as shown in figure 6-12, and the pulse amplitude to decrease as recorded by a series of photographs taken at various bias voltages. This variation of pulse amplitude with dynamic resistance is plotted in figure 6-10 along with the theoretical variation calculated on the basis of the small signal equivalent circuit of the junction depicted in the inset. (cf. Figure 3-5). If $r = (\partial V / \partial I)_T$ and $R = 10.3\Omega$ is the input impedance of the transmission line-preamplifier system (see Chapter 5) then $\frac{i_s}{\delta I} = \frac{r}{R+r}$ and

$$\frac{V_{out}(r = r_i)}{V_{out}(r = r_o)} = \frac{R + r_o}{r_o} \cdot \frac{r_i}{R + r_i}$$

which is consistent with the measured values.

3. Pulse Height Spectrum

Figure 6-11 is a histogram showing the number of times pulses of a given amplitude were observed plotted against pulse amplitude. As no multi-channel pulse height analyzer was available, this pulse height spectrum was obtained by examining a series of pulse photographs (similar to that of figure 6-8) and simply counting the number of pulses falling within the ranges denoted by the small division lines of the oscilloscope graticule. (All photographs so analyzed were taken under identical amplifier conditions with the junction biased at the optimum operating point.) The sharp cut-off at the low amplitude end of the spectrum denotes the pulse triggering level.

At first glance, this wide range in pulse amplitudes is surprising for the alpha particles striking the junction are essentially monoenergetic (5.10-5.15 MeV). It turns out, as discussed in more detail in Chapter 7, that the amplitude variation may be attributed to two causes: (1) the energy lost by the particles in passing through the films varied from 0.5 to 0.14 MeV depending on the angle of incidence (figure 6-7); (2) the junction is thermally coupled to the substrate and some of the energy expended in the substrate as the particle comes to rest diffuses back to

Figure 6-11: Pulse Height Spectrum

the junction in sufficient time to contribute to the pulse.

C. Noise

1. Observed Noise

The noise output from the detector-amplifier system was measured with a Keithley Model 120 wideband (10 Hz-100 MHz) voltmeter--calibrated to read the rms value of a true sine wave--connected as shown in figure 5-6. (Strictly speaking, because the meter is calibrated to read the rms of sinusoidal, not random, signals, noise voltages indicated by the meter should be multiplied by 1.13 to obtain their true rms value (Partridge, 1958).) Of special interest was the observed variation of noise with junction dynamic resistance plotted in figure 6-12. (These data will be considered again in Section D for purposes of estimating the junction capacitance, C_J .)

Noise readings were also taken with a number of capacitors (C_{ext}) connected in parallel to the junction. As it was not possible during the course of the run to place them in close proximity to the junction, they were connected at room temperature in the "filter box"--see figure 5-3. Unfortunately, only the nominal values of these capacitors are known as the actual components used were accidentally mislaid before they could be calibrated. Other capacitors of similar manufacture were, by measurement, found to be of typically $\pm 20\%$ precision at 1 kHz but self-resonating at about 9 MHz. This means that in the amplifier bandpass (0.1 to 3.5 MHz) the actual capacitors used in the test would probably exhibit a reactance corresponding to much less than the nominal capacitance which makes the noise measurements taken as a function of C_{ext} of little value other than a consistency check (see figure 6-13).

2. Origin of Noise

This section will show that the dominant source of noise in the experiment was the common base preamplifier and not the superconducting tunnel junction particle detector.

(a) Representation of Detector as Noise Generator

The equivalent circuit including noise generators for the detector as biased and connected to the amplifying system (see figures 5-3 and 5-6) is given in figure 6-14(a) where

Figure 6-12: Output Noise vs junction Dynamic Resistance

Figure 6-13: Output Noise vs External Capacitance

Figure 6-14(a): Detector Equivalent circuit with Noise Generators

Figure 6-14(b): Detector noise Generator equivalent Circuits

$r_{e'}$ = emitter resistance, $r_{b'}$ = base resistance, $r_{c'}$ = collector resistance

Figure 6-14(c): CB Preamplifier with Noise

$$i_b^2 = \text{Johnson noise in bias resistance } (R_b) = \overline{I_{Rb}^2} = 4kTB/R_b$$

$$i_b^2 = \text{shot noise on bias current } (I) = \overline{I_B^2} = 2eIB$$

with T the room temperature (295 K), k Boltzmann's constant and B the system bandwidth. In practice, $R_b \approx 20 \text{ K}\Omega \gg r$, hence the equivalent circuit may be reduced to the forms shown in figure 6-14(b), where, being uncorrelated, the noise currents add quadratically. Thus, over the bandwidth B,

$$\begin{aligned} v_d^2 &= r^2 \left(\frac{4kTB}{R_b} + 2eIB \right) \\ &= 4kTBr \left(\frac{r}{R_b} + \frac{2eIr}{4kT} \right) \\ &\equiv 4kTBr(M) \end{aligned} \quad (6-2)$$

At the operating point $I \approx 70 \text{ }\mu\text{A}$ which, along with the other values cited, makes $M \approx 1.35 \times 10^{-2}$.

(b) Equivalent Circuit for CB Transistor with Noise

The "low frequency" equivalent circuit for a transistor operating in common base (CB) mode with white noise generators as given by Woll and Herscher (1962) is shown in figure 6-14(c). In this case, the noise generators are:

$$\begin{aligned} v_{e'}^2 &= \overline{I_e^2} r_{e'}^2 = 2eI_E Br_{e'}^2 = 2kTBr_{e'}^2 ; & r_{e'} &= \frac{kT}{eI_E} \\ v_{b'}^2 &= 4kTBr_{b'} \end{aligned} \quad (6-3)$$

and

$$\overline{v_c^2} = 2eI_E \alpha(1-\alpha) r_c^2 B = 2kT\alpha(1-\alpha) r_c^2 B/r_{e'}$$

where $\overline{I_e^2}$ is the shot noise on the emitter current I_E and $H_{FE} = \frac{\alpha}{1-\alpha} \approx 40$ is the measured common emitter forward current gain. (The "low frequency" representation is used as the upper frequency of the amplifying system

bandpass is about 3.5 MHz whereas the upper cutoff frequency f_T for the 16G2 transistor used in the preamplifier is quoted by the manufacturer as 1.1 GHz.)

(c) Calculation of Noise Figure

To appreciate the relative importance of the detector and preamplifier as noise sources, it is convenient to define the following noise figure

$$F = \frac{\text{total output noise power}}{\text{output noise power due to noise of detector}}$$

From the circuit for the preamplifier and detector shown in figure 6-15 it is evident that the noise power is proportional to the mean square voltage e_o^2 appearing across R_L . Taking the open loop case for simplicity (ie. $R_L = \infty$), one finds:

$$i_e = \frac{v_d + v_{e'} + v_{b'}}{(r + r_{e'} + r_{b'})} \quad (6-4)$$

and

$$e_o = -v_b + v_c + i_e(r_{b'} + \alpha r_c)$$

Substituting i_e into e_o , collecting terms and taking the mean square value of both sides yields

$$e_o^2 = \frac{\alpha^2 r_c^2 (\overline{v_d^2} + \overline{v_b^2} + \overline{v_{e'}^2}) + \overline{v_c^2} (r + r_{e'} + r_{b'})^2}{(r + r_{e'} + r_{b'})^2} \quad (6-5)$$

where it has been assumed that the noise generators are uncorrelated, $\alpha r_c \gg r_{b'}$, and $\alpha r_c \gg r_{e'} + r$.

The output noise power due to the noise of the detector is found by setting $\overline{v_{b'}^2} = \overline{v_{e'}^2} = \overline{v_c^2} = 0$ in 6-5 which gives

$$F = 1 + \frac{\overline{v_{b'}^2} + \overline{v_{e'}^2}}{\overline{v_d^2}} + \frac{\overline{v_c^2} (r + r_{e'} + r_{b'})^2}{\alpha^2 r_c^2 \overline{v_d^2}}$$

Figure 6-15: Equivalent circuit for Detector and Amplifier with noise

Figure 6-16: Equivalent circuit for Signal to Noise Ratio estimate

Using equations 6-2 and 6-3 makes

$$F - 1 = \frac{1}{r_M} \left[r_{b'} + \frac{1}{2} r_{e'} + \frac{(r + r_{e'} + r_{b'})^2}{2r_{e'} H_{FE}} \right] \quad (6-6)$$

The transistor bias current (I_E) was 10 mA, so that by 6-3 $r_{e'} = 2.6 \Omega$.

Angell (1967) gives the input impedance of a CB transistor as

$R_{in} = r_{e'} + r_{b'}(1-\alpha)$ making $r_{b'} = 216 \Omega$ using the measured (see chapter 5) input impedance of approximately 8Ω . Putting these values into equation 6-6 yields $F = 3840$ clearly identifying the preamplifier as the major source of noise.

(d) Equivalent Input Noise (for Circuit Used)

It is convenient to represent the preamplifier noise generators by an equivalent voltage generator (v_g) across the junction dynamic resistance. (see figure 6-14(b) and 6-15). Thus $F = v_g^2/v_d^2$ or

$$v_g = (4kTBrMF)^{\frac{1}{2}} = 5.2 \mu V(\text{rms}), \quad r = 9.3 \Omega$$

(e) Signal to Noise Ratio Estimate

From the pulse photograph (figure 6-8), the maximum signal is seen to correspond to about $8.1 \mu A$ which can be represented by a voltage generator $V_s = 8.1 \mu A \cdot r = 75 \mu V$ across the junction dynamic resistance. (see figures 6-14(b) and 6-16). The two generators v_g and V_s are thus located at the same point in the detector-preamplifier equivalent circuit so that the quasi-theoretical signal to noise ratio (S/N) is simply $S/N = V_s/v_g = 14.5$ in reasonable agreement with the measured value of 19.

(f) Noise with Lumped Constant Models of Specimens

To confirm experimentally that the observed output noise depends essentially only on the source impedance seen by the pre-amplifier and not on noise sources associated with the detector, a lumped constant model of a tunnel junction, consisting of a 9.6Ω resistor in parallel with 3000 pF, was substituted for the actual specimen. The resultant noise, measured with the amplifier gains unchanged from measurements with the actual specimen, was indeed equal to that obtained with the operating junction. To some extent, this precise agreement is fortuitous because the output noise was observed (see figure 6-12,13) to vary with junction

resistance and parallel capacitance and yet the lumped constant components of 9.6Ω and 3000 pFs. were only approximations to the actual values. Nonetheless, because the absence of the detector has not significantly altered the noise, it is reasonable to conclude that the preamplifier was in fact the dominant noise source.

D. Determination of Junction Capacitance

For mathematical simplicity the presence of C_J was ignored in the noise investigations of the previous section. The experiments mentioned above in which the noise was measured with a resistor and a capacitor substituted for the real specimen justified this simplified approach in that they were consistent with the conclusion (about the amplifier being the main noise source) arrived at theoretically with $C_J = 0$.

In order to proceed with the analysis of the results carried out in chapter 7, however, a reasonably close estimate of C_J is required. The purpose of this section is to satisfy this requirement, first by calculating C_J from a parallel plate model and second by a detailed analysis of the noise measurements taking the effect of C_J into account. It turns out that the range of values found by the two approaches are reasonably compatible.

1. Parallel Plate Model

A rough estimate of the junction capacitance may be obtained by regarding the device as a simple parallel plate capacitor having thin Sn film electrodes and a dielectric of SnO_2 .

(a) Dielectric Constant of SnO_2

Van Daal (1968) gives the static or zero frequency dielectric constant ($K = \epsilon/\epsilon_0$) of bulk SnO_2 as lying between 9 and 14 depending on the crystal orientation. (Other workers, eg. Coon and Fiske (1965), have used $K = 5$ without qualification--presumably this is an approximation to the optical dielectric constant (which is inappropriate here) quoted by some authors, eg. Summitt and Borrelli (1965) and Arai (1960), as being approximately 4.) It is not readily apparent that the dielectric constant for bulk material may be meaningfully carried over to very thin layers--as in a tunnel junction--although there is some work by Kohn (1958) and Mead (1961) which would seem to justify such an extrapolation.

(b) Insulator Thickness

The thickness of the insulating layer is estimated from the measured low temperature 4.2 K, low voltage, normal state tunneling resistance of the junction, $R_n = 0.077 \pm .001 \Omega$. To derive the effective tunneling thickness (S_T) from figure 2-3, it is necessary to know, in addition to K, the mean barrier height of the insulator ϕ_o and the normalized resistance $\sigma = R_n \cdot A$ where $A = 7 \times 10^{-4} \text{ cm}^2$ is the junction overlap area.

The magnitude of the forbidden energy band in SnO_2 (E_{gap}) is given by Arai (1960) and Summitt et al (1964) as 4 eV. Assuming that the Fermi surface lies within the band gap, ϕ_o must be in the range $0 < \phi_o < E_{\text{gap}}$. A precise value of ϕ_o cannot be specified because the presence of impurities, non-stoichiometry and defects in the insulator make the position of the Fermi surface in the band gap uncertain.

Taking $\sigma = 5.4 \times 10^{-3} \Omega \text{ mm}^2$, $1 < \phi_o < 4 \text{ eV}$ (for example), and $9 \leq K \leq 14$ yields S_T lying between 7 and 14 Å (cf. figure 2-3). (Note: S_T is the thickness of an ideal, uniformly thick insulating layer of area A whose tunneling resistance is R_n and is not to be confused with some average thickness $\langle S \rangle A = \iint_A S(x,y) dx dy$ where $S(x,y)$ is the actual insulator thickness at x,y.)

Chow (1963) and Hurych (1966) point out, however, that the effective thickness (S_c) to be used for capacitance calculations is 1.5 to 3 times greater than S_T . This difference is attributable to the fact that the thickness of the insulating layer is not constant but varies stochastically from point to point. Such non-uniformities will strongly affect the tunnel current--because of its exponential dependence upon the thickness--but only weakly affect the capacitance.

For $9 \leq K \leq 14$ and $10 \leq S_c \leq 42 \text{ Å}$, the calculated value of C_J is in the range $1300 \leq C_J \leq 8000 \text{ pF}$. Since $\phi_o < 1 \text{ eV}$ implies $S_c > 42 \text{ Å}$ and $C_J < 1300 \text{ pF}$, all that can be concluded is that for $0 < \phi_o < 4 \text{ eV}$

$$0 < C_J \leq 8000 \text{ pF}$$

A more direct and more precise estimate was obtained from the noise measurements as described in the next section.

2. Noise Measurements

In section C of this chapter, the noise output from the junction-amplifier system (V_n) was discussed at some length and was found to be a function of the impedance in the preamplifier input loop exemplified by the observed variation of noise with r (the junction dynamic resistance) for unknown constant junction capacitance C_J (actual) = C_{Ja} ; this quantity will be denoted as $V_n(r, C_{Ja}, \text{Meas.})$. From the small signal equivalent circuit of the detector-amplifier system, the output noise may be calculated as a function of r and C_J ; this quantity will be denoted as $V_n(r, C_J, \text{Thy.})$. The purpose of this section is to estimate the junction capacitance by determining the range of C_J for which $V_n(r, C_J, \text{Thy.})$ is consistent with the experimentally measured noise $V_n(r, C_{Ja}, \text{Meas.})$.

The reader uninterested in the following mathematical details may wish to skip on to section E where the acceptable range of C_J values obtained from the noise measurements, along with the other principal results from this chapter, are summarized.

(a) Calculation of $V_n(r, C_J, \text{Thy.})$

Figure 6-17 shows the equivalent circuit of the tunnel junction--transmission--line--preamplifier system (see figures 3-5, 6-14 and 6-16) with the preamplifier noise generators $v_{e'}$, v_b , and v_c , (equation 6-3) in place. In keeping with the conclusions of section C, the junction noise generators have been neglected. Assuming $r_c \gg R_L + r_b$, it may be shown that the mean square noise voltage across R_L (see figure 6-15) is given by

$$\overline{e_o^2} = \frac{R_L^2 \alpha^2 4kTB}{(r+r_{e'}+r_b(1-\alpha))^2} \left(r_b + \frac{1}{2} r_{e'} + \frac{(r+r_{e'}+r_b)^2}{2H_{FE}r_{e'}} \right)$$

To extend this result to the circuit of figure 6-17, one may recall equation 5-4 and replace r by the transmission line-detector impedance $Z = Z(r, C_J) = R + jX$ where

Figure 6-17: Equivalent Circuit of Junction-Preamplifier System with Noise Sources only (no alpha pulses)

$$R = \frac{r}{1 + (\omega r C_J)^2} + R_L(1.2 K, \omega)$$

$$X = \frac{-r(\omega C_J r)}{1 + (\omega r C_J)^2} + j\omega L; \quad L = 1.63 \mu H.$$

Since $1 K\Omega > 470\Omega \gg r_e + r_b(1-\alpha)$, the filter box-CB transistor combination may be replaced by a series impedance $Z'_{in} = R'_{in} + jX'_{in}$ where (cf. figure 5-5 and equation 5-5)

$$R'_{in} = 7.9\Omega = r_e + (1 - \alpha)r_b,$$

$$X'_{in} = j(\omega L - 1/\omega C) = X_L - X_C$$

with $L = 0.23 \mu H$ and $C = 0.5 \mu F$. (Noise generators corresponding to the Johnson noise in the parallel resistors R_F and R_B in figure 6-17 are ignored as their contribution to the net output noise voltage may readily be shown to be negligible.)

For detector impedances Z of interest, $r_b \gg r_e + |Z|$, which means that over a bandwidth df

$$\begin{aligned} \tilde{e}_o^2 df &= R_L^2 \alpha^2 4kTdf (r_b + \frac{1}{2}r_e + r_b^2/H_{FE}r_e) (Z + Z'_{in})^{-2} \\ &\equiv K df (Z + Z'_{in})^{-2}, \quad |Z| \ll r_b, \end{aligned} \quad (6-7)$$

The magnitude of the total noise voltage for particular values (r' , C_J') of the junction equivalent circuit parameters is therefore

$$e_o^2(r', C_J') = \int_{f_1}^{f_2} \tilde{e}_o^2 df = K \int_{f_1}^{f_2} df / |Z(r', C_J') + Z_{in}'|^2$$

where the limits of integration are the amplifying system 3db-down points, $f_1 = 0.1$ MHz and $f_2 = 3.5$ MHz. Now $V_n(r, C_J, \text{Thy.}) \propto e_o(r, C_J)$ so that one may form the noise voltage ratio

$$\gamma(\text{Thy.}) = \frac{V_n(r, C_J, \text{Thy.})}{V_n(r', C_J', \text{Thy.})} = \left[\frac{e_o^2(r, C_J)}{e_o^2(r', C_J')} \right]^{\frac{1}{2}} \quad (6-8)$$

With $C_J' = C_J$ and $r' = 9.3\Omega$, $\gamma(\text{Thy.})$ was evaluated by Simpsons' rule integration for an appropriate range of values for r, C_J thereby generating the family of curves shown in figure 6-18.

(b) Computation of $V_n(r, C_{Ja}, \text{Meas.})$

From equation 6-7 it may be seen that the theoretical output noise voltage depends upon the preamplifier input loop impedance $Z + Z_{in}'$. The measured output noise $V_n(\text{Meas.})$, however, includes contributions from the postamplifiers whose noise is insensitive to $Z + Z_{in}'$ so that it is convenient to write

$$V_n(\text{Meas.}) = [V^2(Z + Z_{in}') + V_o^2]^{\frac{1}{2}} \quad (6-9)$$

Here $V(Z + Z_{in}')$ is the component of output noise voltage depending on $Z + Z_{in}'$ and V_o is the "open input loop" output noise voltage. The quantity of interest for comparison to theory is therefore

$$V_n(r, C_{Ja}, \text{Meas.}) = [V_n^2(\text{Meas.}) - V_o^2]^{\frac{1}{2}} \quad (6-10)$$

where, by experiment, $V_o \approx 36$ mV. For comparison to $\gamma(\text{Thy.})$, the ratio

$$\gamma(\text{Meas.}) = \frac{V_n(r, C_{Ja}, \text{Meas.})}{V_n(9.3, C_{Ja}, \text{Meas.})}$$

is computed from the data of figure 6-12 and equation 6-9 and plotted in figure 6-18. (The circles around the various $\gamma(\text{Meas.})$ points are a measure

Figure 6-18: Theoretical and Measured Preamplifier Noise Output vs Dynamic Resistance of Junction

of the experimental uncertainty.)

(c) "Best" value of C_J

Inspection of figure 6-18 shows reasonable agreement of theory to experiment for a junction capacitance

$$1500 \leq C_J \leq 4500 \text{ pF}$$

which lies inside the range of values predicted from the parallel plate model in section 1 of this chapter.

(d) Consistency Check

Within experimental error, this value of the junction capacitance agrees with that obtained graphically from setting

$$\frac{V_n (r = 0, C_J = 0, \text{Thy.})}{V_n (r = 9.3, C_J, \text{Thy.})} = \frac{V_n (\text{transmission line shorted, Meas.})}{V_n (r = 9.3, C_{Ja}, \text{Meas.})} \quad (6-11)$$

The left hand side of 6-11 was computed similarly to equation 6-8; the right hand side is the ratio of the output noise measured with the transmission line terminated with a dc short (at 1.2 K) to the noise measured with the transmission line terminated with the operating tunnel junction.

E. Summary

This chapter has described in some detail the results obtained in the course of detecting alpha particles with tunnel junctions. The significant points are listed below:

(1) The dc I-V characteristics were consistent with theory and the results of other workers. From these characteristics, the maximum $r = (\partial V / \partial I)_T$ was found to be 9.3Ω .

(2) With the junction biased at its optimum operating point, which was found, as predicted theoretically, to correspond to those values of I, V and B for which r was a maximum and the supercurrent was minimum, pulses were detected whose count rate agreed with that predicted from the source strength and the junction geometry. This observation, together with the fact that the pulses could be turned off by interposing a mechanical shutter between the source and junction, led to the conclusion that the

pulses were produced by alpha particle bombardment of the junction itself.

(3) The pulses observed, at optimum amplifier bandwidth ($\tau_{\text{rise}} = 100 \text{ nsec}$, $\tau_{\text{fall}} = 1.7 \text{ } \mu\text{sec}$), had amplitudes up to 19 times the preamplifier-dominated rms noise level of roughly 100 keV referred to the input.

(4) From noise measurements, the junction capacitance C_J was found to lie in the range

$$1500 \leq C_J \leq 4500 \text{ pF}$$

which is consistent with the range of values predicted from a parallel plate model.

(5) Of minor significance to the reader but of major concern to the experimenter was the observed tendency of tunneling junctions to deteriorate upon thermal cycling.

CHAPTER 7

ANALYSIS OF RESULTS

A. Introduction

As discussed in Chapter 1, the quantity of most interest to be obtained from the present experiment is w --the average energy loss by a charged particle to excite a quasiparticle pair--for comparison with the corresponding quantity in other types of detector and, eventually, for comparison with theory when it is sufficiently well developed. To determine w from the measured pulse response of the superconducting tunnel junction, it is necessary to know the form of the current pulse $i(t)$ which is superimposed on the thermal equilibrium tunneling current following each alpha particle bombardment. Once $i(t)$ is known, the number N_0 of quasiparticles created by the alpha particle having lost energy ΔE in the junction may be estimated, giving $w = \Delta E/N_0$.

In this experiment, the results were not sufficiently precise to determine directly the form of $i(t)$. Therefore, it is assumed that the current pulses have the form (equation 3-11) $i(t) = i_0 \exp(-t/\tau)$, ($t \geq 0$) where the peak amplitude i_0 is proportional to the magnitude of the excess density of quasiparticles generated by the passage of the alpha particle and τ is a characteristic time during which this excess density decays to zero. Admittedly this model has no direct theoretical basis, being assumed for the sake of convenience in analysis, but our theoretical analysis is not yet sufficiently developed to justify a more complex time dependence. (A theoretical analysis of $i(t)$ based on classical heat diffusion theory is being carried out by Mr. George May of this laboratory.)

The purpose of this chapter is to estimate w by deducing the parameters τ and i_0 from the amplifier output pulse data. In order to deduce these parameters and their errors, the effects of the transfer function of the junction-transmission line-amplifier system must be taken into account

(section B), and the effects of noise in the system must be allowed for (sections C and D).

Having determined τ and i_o , one may easily find the total amount of charge which actually tunneled. An estimate of ΔE then makes it possible to give (section E) a figure for $w(\exp)$, the average energy loss per tunneling quasiparticle, and w the average energy loss per quasiparticle pair excited. A discussion (section F) of problems pertaining to a rigorous calculation of ΔE concludes the chapter.

B. Derivation of Detector-Transmission Line-Amplifier Transfer Function

A fundamental concept of linear systems analysis (see eg. Bohn, 1963) is the system transfer function $H(s)$ defined as

$$H(s) = v_o(t)/v_i(t)$$

where $v_i(t)$ and $v_o(t)$ are the input and output signals respectively, $s=j\omega$ is the complex frequency and t is the time. The aim of this section is to derive $H(s)$ from the known characteristics of the detector--transmission line-amplifier system and find $v_o(t)$ using an assumed form of $v_i(t)$.

1. Small Signal Equivalent Circuit

Figure 7-1(a) is a schematic of the pulse amplifying network and figure 7-1(b) is the corresponding small signal equivalent circuit where the amplifier is now assumed noiseless. Moving from left to right, $v_i(t)$ is the input voltage signal obtained from the assumed form of $i(t)$ and application of Thévenin's theorem to the circuit to the left of P-P in figure 7-1(a); thus

$$v_i(t) = i(t) Z_d = i_o \exp(-t/\tau) \frac{r}{1 + j\omega r C_J} \quad (7-1)$$

Incorporating the results of chapter 5, (equation 5-3), the effective input impedance of the "cold" transmission line-filter box-preamplifier may be represented approximately as a series inductance L and resistance R where $L = 1.69 \mu\text{H}$ and $R = 10.3\Omega$ (From figure 5-9, $R = R_{ic}$ is seen to range from 9.5Ω to 11.6Ω over the system bandwidth but, in order to keep the transfer function in a tractable form, the mid-band value of $R=10.3\Omega$

(a)

(b)

Figure 7-1: Equivalent Circuit of Pulse Detection System

has been chosen as a suitable average.) The transfer function of this part of the network is then

$$Y(s) = i_e(t)/v_i(t) \quad (7-2)$$

The CB transistor preamplifier transforms the input signal current $i_e(t)$ at low impedance level into output signal αi_e at high impedance level such that

$$v'(t) = \alpha i_e(t) R_L = i_e(t) R' \quad (7-3)$$

where R' is taken to be frequency independent.

It was pointed out in chapter 6 that the optimum amplifier system bandwidth for pulse detection corresponded to a 10-90% rise time of 100 nsec. To take this time constant into account, an integrating circuit is included where the amplifier rise time $\tau_a = R_o C_o$. For this portion of the circuit

$$W(s) = v''(t)/v'(t) \quad (7-4)$$

and, representing the gain of the amplifying system by G ,

$$v_o(t) = G v''(t) \quad (7-5)$$

The desired transfer function $H(s)$ may thus be obtained from equations 7-2 to 7-5

$$H(s) = \frac{v_o(t)}{v_i(t)} = GR' W(s) Y(s) = \frac{V_o(s)}{V_i(s)} \quad (7-6)$$

where, by definition (see, eg. Bohn, 1963), $V(s)$ is the Laplace transform of $v(t)$ written

$$V(s) = \mathcal{L}[v(t)] = \int_0^{\infty} v(t) \exp(-st) dt$$

Thus,

$$V_o(s) = GR'V_i(s) W(s) Y(s)$$

and the desired quantity $v_o(t)$ may be found by taking the inverse transform

$$v_o(t) = \mathcal{L}^{-1}[V_o(s)]$$

All that remains now is to determine $V_i(s)$, $W(s)$ and $Y(s)$ from figure 7-1(b).

2. Laplace Transform Representation

(a) $V_i(s)$

Writing the impedances in terms of their transforms and applying the definition of $V(s)$ to equation 7-1 yields

$$V_i(s) = \frac{i_o}{C_J} \cdot \frac{1}{s+a} \cdot \frac{1}{s+d} \quad (7-7)$$

where $a = \tau^{-1}$, $d = (rC_J)^{-1}$.

(b) $W(s)$

Inspection of the integrating circuit shows

$$W(s) = \frac{1}{C_o s} \cdot \frac{1}{R_o + 1/C_o s} = \frac{b}{s+b} \quad (7-8)$$

with $b = (\tau_a)^{-1} = (R_o C_o)^{-1}$.

(c) $Y(s)$

Somewhat more complicated algebraically than the other terms, $Y(s)$ is given by

$$\begin{aligned} Y(s) &= [R + Ls + r/(1 + srC_J)]^{-1} \\ &= (s + d)/L(s^2 + a_1s + a_0) \end{aligned} \quad (7-9)$$

where $a_1 = (R + Ld)/L$, $a_0 = (RC_Jd + 1)/LC_J$.

Combining equations 7-7 to 7-9 gives

$$V_o(s) = \frac{GR'bi_o}{C_JL} \cdot \frac{1}{s+a} \cdot \frac{1}{s+b} \cdot \frac{1}{s^2 + a_1s + a_0} \quad (7-10)$$

3. Inverse Transform

To obtain the inverse transform of 7-10 it is convenient to set $GR'bi_o/C_JL = \text{constant} = A$, where A is independent of time, and to factor the quadratic term such that

$$\frac{V_o(s)}{A} = \begin{cases} \frac{1}{(s+a)} \cdot \frac{1}{(s+b)} \cdot \frac{1}{(s+\alpha_1)(s+\alpha_2)}, & a_1^2 \geq 4a_0 \\ & (\text{or } C_J \leq C_{J0}) \\ \frac{1}{(s+a)} \cdot \frac{1}{(s+b)} \cdot \frac{1}{(s+\alpha)^2 + \gamma^2}, & a_1^2 \leq 4a_0 \\ & (\text{or } C_J \geq C_{J0}) \end{cases}$$

Both forms of $V_o(s)$ must be considered as C_{J0} , the value of C_J which satisfies $a_1^2 = 4a_0$, is about 3300 pF which lies right in the middle of the accepted range of values for C_J as determined from noise measurements in chapter 6. The inverse transforms, obtained from standard Laplace

Transform Tables [eg. Erdélyi (1954) and McCollum and Brown (1965)],
are as follows:

$$\begin{aligned} \frac{v_o(t)}{A} = & - \frac{\exp(-at)}{(a-b)(a-\alpha_1)(a-\alpha_2)} + \frac{\exp(-bt)}{(a-b)(b-\alpha_1)(b-\alpha_2)} - \\ & - \frac{\exp(-\alpha_1 t)}{(\alpha_1 - \alpha_2)(a - \alpha_1)(b - \alpha_1)} + \frac{\exp(-\alpha_2 t)}{(a - \alpha_2)(b - \alpha_2)(\alpha_1 - \alpha_2)}, \end{aligned}$$

$C_J \leq C_{J0}$
 $t \geq 0$
 [7-11(a)]

$$\begin{aligned} \frac{v_o(t)}{A} = & \frac{\exp(-bt)}{(a-b)[(\alpha-b)^2 + \gamma^2]} - \frac{\exp(-at)}{(a-b)[(\alpha-a)^2 + \gamma^2]} + \\ & + \frac{\exp(-at)\cos(\gamma t + \phi)}{\gamma[(\alpha-b)^2 + \gamma^2]^{\frac{1}{2}}[(\alpha-a)^2 + \gamma^2]^{\frac{1}{2}}} \end{aligned}$$

$C_J \geq C_{J0}$
 $t \geq 0$
 [7-11(b)]

where $\phi = \tan^{-1} \frac{\gamma}{\alpha-a} - \tan^{-1} \frac{\alpha-b}{\gamma}$

$$\alpha_1 = 2a_o(a_1 + (a_1^2 - 4a_o)^{\frac{1}{2}})^{-1} = \frac{a_o}{\alpha_2}$$

$$\alpha = \frac{a_1}{2} = \frac{R + Ld}{2L}$$

$$\gamma = (a_o - \frac{1}{4}a_1^2)^{\frac{1}{2}}$$

$$A = GR'bi_o/C_JL$$

4. Justification of Approximate Transmission Line Treatment

In chapter 5, the justification for representing the input impedance of the transmission line terminated with the preamplifier as $R_{ic} + jX_{ic}$ (equation 5-3), was based on the agreement between this model and the measured impedance. To check the validity of this approximation in describing the impulse response of the system, square pulses from a pulse generator were developed in a lumped constant model of the specimen and the output waveform from the preamplifier was photographed on an oscilloscope for comparison to the theoretical waveform. (The arrangement was similar to figure 6-9 except that four different capacitances (C) were employed in separate tests.)

The theoretical waveform was calculated in the manner described above using the average room temperature resistance and inductive components of the line ($R = 13 \Omega$, $L = 1.69 \mu H$, see figure 5-9). In all four cases, the agreement between the theoretical and experimental output waveforms was sufficiently good (and sufficiently sensitive to variations in R or L) that treating the line and its load by the more precise, but algebraically complicated successive-reflections technique (Bohn, 1963) seemed unwarranted.

C. Determination of Current Pulse Parameter τ

The object of this section is to estimate τ from a least squares fit of the theoretical pulse shape (equations 7-11) to the pulse shape observed on photographs (see eg. Figure 6-8). This statistical approach was made necessary by the noise superimposed on the signal pulses. Thus, following Orear (1958), if x_i are the theoretical values characterizing the pulse shape and $\bar{x}_i$ are the corresponding measured values, assumed to be Gaussian distributed with standard deviation σ_i , the least squares function to be minimized is

$$M^* = \sum_{i=1}^N (x_i - \bar{x}_i)^2 / \sigma_i^2 \quad (7-12)$$

It turns out (Orear, 1958) that M^* is merely the χ^2 distribution of (N-n) degrees of freedom where n is the number of parameters solved for and N is the number of experimental points. The probable value of M^* is $\bar{M}^* = N-n$ with the standard deviation $\Delta M^* = [2(N-n)]^{\frac{1}{2}}$.

(1) Extraction of the $\bar{x}_i$ and σ_i from Photographs

Consider the typical pulse photograph in figure 6-8.

Let $t = t_0 \approx 250$ nsec be the time at which a given pulse reaches its maximum amplitude, then the pulse amplitude $a(t)$ at N other times (t_i), for $0 \leq t_i \leq 2t_0$, may be measured directly from the photograph (for the same pulse) to form the ratio

$$y_i = a(t_i)/a(t_0) \quad , i = 1, \dots, N$$

Because the amplifying system is linear, the pulse shape (as characterized by y_i) depends only on the time constants of the system and not on the absolute amplitude--provided of course that the amplitude is sufficiently greater than the rms noise voltage. Consequently, maximum amplitude pulses on 5 different photographs were chosen to be analyzed in this fashion to arrive at mean values

$$\bar{x}_i = \frac{1}{5} \sum_{j=1}^5 y_{ij} \quad , i = 1, \dots, 7$$

with variance (Evans, 1955)

$$\sigma_i^2 = \frac{1}{4} \sum_{j=1}^5 (y_{ij} - \bar{x}_i)^2 \quad , i = 1, \dots, 7$$

where y_{ij} is the measured ratio $a(t_i)/a(t_0)$ on the j th pulse.

The remaining quantities needed to calculate M^* are the corresponding theoretical ratios calculated from equations 7-11,

$$x_i = v_0(t_i)/v_0(t_0) \quad , i = 1, \dots, 7$$

which, as it should be, is independent of i_0 .

At this juncture, a word is in order concerning the precision with which the times t_i are known. In practice, since the zero of time for a particular pulse is not clearly discernible from the photograph, the t_i must be measured with respect to t_0 , which can only be estimated from photographs to lie within a range of values $t_0 \pm \Delta t_0$. Hence,

$t_i = t_o + t'_i \pm \Delta t_o \pm \Delta t'_i$ where $-t_o \leq t_i \leq t_o$ and $\Delta t'_i$ is the error involved in marking off time intervals t'_i . To take this uncertainty into account, M^* must be calculated over a range of t_o .

(2) M^* Calculation

Inspection of equations 7-11 reveals that τ is the only unknown variable in the x_i but that the value of τ which minimizes M^* will depend upon the particular values chosen for C_J and t_o . It was shown in section B that

$$1500 \leq C_J \leq 4500 \text{ pF}$$

and t_o may be estimated from figure 6-8, by extrapolation of the maximum amplitude pulse to the ac ground level, to lie in the range

$$245 \leq t_o \leq 265 \text{ nsec}$$

A computer program was written to evaluate equations 7-11 over the stipulated ranges of C_J and t_o yielding the typical plot of figure 7-2. As a measure of the uncertainty in the so-called most probable τ --i.e. that value which minimizes M^* for a particular C_J and t_o --it is convenient, although arbitrary, to accept all τ which fall within a 90% confidence level. In this context "90% confidence level" means that if the experiment were repeated under exactly the same conditions, $\tau = \tau(C_J, t_o)$ is such that the probability is 90% that the newly measured $\bar{x}_i$ would give $M^* \leq Q$ where $Q=10.6$ is the value for Q for a χ^2 distribution with $N-n = 7-1 = 6$ degrees of freedom (see eg. χ^2 tables, Handbook of Physics and Chemistry). Thus, for example (see figure 7-2) a 90% confidence limit on $\tau = \tau(C_J = 3000 \text{ pF}, t_o = 260 \text{ nsec})$ gives $123 \leq \tau \leq 167 \text{ nsec}$ with a most probable value of roughly 142 nsec.

Figure 7-3 summarizes the results of the M^* calculations. The region shown in heavy black outline is the "90% confidence volume" in C_J, t_o, τ space compatible with the uncertainty in t_o and C_J described above; the surface of most probable values of τ lies roughly midway between the upper and lower extremes.

Clearly, it is not meaningful to speak of a unique τ

Figure 7-2: M^* as Function of τ , t_o and C_J .

Figure 7-3: 90% Confidence Volume in C_J , t_o , τ Space

as giving the best fit of the theoretical to measured pulse shapes. For purposes of estimating i_o , however, it is convenient to take a central value (shown in figure 7-3) of $\tau = \tau(3000 \text{ pF}, 255 \text{ nsec}) \approx 138 \text{ nsec}$.

(3) Validity of the Assumed Input Current Pulse

The values of τ chosen by the least squares fit are based on the assumption that the current pulse $i(t) = i_o \exp(-t/\tau)$ and the measured transfer function constitute a sufficient basis from which to calculate the form of the observed pulse. Some information about the validity of this assumption may also be obtained from the M^* calculations.

In particular, if (C_J', t_o', τ') minimizes M^* then $M^*(C_J', t_o', \tau')$ should lie in the range $\bar{M}^* \pm \Delta M^* \approx 6 \pm 3.5$ if the theoretical pulse shape is adequate (Braddick, 1966) whereas $M^*(C_J', t_o', \tau') < 2.5$ would imply τ is not determined by the measurement and $M^*(C_J', t_o', \tau') > 9.5$ would imply more parameters should have been used. The plot of figure 7-4 shows that the calculated M^* (minimum) do indeed fall generally within the desired range indicating not only that τ is meaningful but that a higher order parameter form of $i(t)$ is not justified with the present data.

D. Estimate of Current Pulse Amplitude (i_o)

A result which will be needed later is the current pulse amplitude i_o corresponding (for reasons to be discussed in section E following) to the maximum amplitude observed pulses. The method of obtaining this parameter from the photographed pulses (figure 6-8) and the amplifier system sensitivity calibration is outlined below.

Equations 7-11 contain i_o implicitly in that

$$A = \frac{GR' i_o b}{C_J L} = \frac{v_o(t)}{f(t)}$$

where $v_o(t_o)$ is the peak pulse amplitude measured photographically from the oscilloscope (in output volts $V(\text{output})$) at time t_o and $f(t_o)$ is the right hand side of equations 7-11 evaluated at $t = t_o$. All that remains is to find the amplifier system transfer constant GR' .

This quantity is determined, as shown in the next two paragraphs, from the amplifier sensitivity calibration described in chapter 6 where very

Figure 7-4: $M^*(\text{minimum})$ v/s Relaxation time (τ)

fast rising (step) current pulses of known amplitude were injected into a lumped constant model of the small signal equivalent circuit of the specimen (see figure 6-9) and the output pulse $v_{oc}(t)$ was recorded photographically on an oscilloscope. (The gain and bandwidth settings of the amplifiers were of course unchanged from the signal pulse measurements.)

1. Transfer Function for Input Current Step Pulse

The fast rising input current pulse may be represented approximately by $i_c(t) = i_{oc} u(t)$ where

$$u(t) = \begin{cases} 0, & t < 0 \\ \frac{1}{2}, & t = 0 \\ 1, & t > 0 \end{cases}$$

and i_{oc} is the measured maximum amplitude input calibrating current. Substituting $i_c(t)$ into equation 7-1 and transforming gives

$$V_{ic}(s) = \frac{i_{oc}}{C_{Jc}} \cdot \frac{1}{s} \cdot \frac{1}{s+d}$$

which is the same as equation 7-7 with $a = 0$. ($C_{Jc} = 3500$ pF is the "dummy" specimen value of C_J .) The forms of both $W(s)$ and $Y(s)$ are unchanged from the discussion in section B so that the appropriate transfer function is given by 7-11(b)--since $C_{Jc} > C_{J0}$ --with $a = 0$. Thus

$$\frac{v_{oc}(t)}{B} = - \frac{\exp(-bt)}{b[(\alpha-b)^2 + \gamma^2]} + \frac{1}{b(\alpha^2 + \gamma^2)}$$

(7-13)

$$+ \frac{\exp(-\alpha t) \cos(\gamma t + \phi)}{\gamma[(\alpha-b)^2 + \gamma^2]^{\frac{1}{2}} (\alpha^2 + \gamma^2)^{\frac{1}{2}}}$$

where

$$\phi = \tan^{-1} \frac{\gamma}{\alpha} - \tan^{-1} \frac{\alpha-b}{\gamma}$$

and

$$B = \frac{GR' i_{oc} b}{C_{Jc} L}$$

Quantities α and γ , as before, are given in terms of a_0 , a_1 and d which must be suitably redefined for the lumped constant model values of

$C_J = C_{Jc}$, $r = r_c = 10 \Omega$ and $R = R_c = 12 \Omega$. (R_c is the midband room temperature value of the transmission line-preamplifier series resistance; L and b are unchanged.)

2. Evaluation of GR'

From section B, $b = 10^7 \text{ sec}^{-1}$ and the values of α and γ pertinent to 7-13 are about $1.8 \times 10^7 \text{ sec}^{-1}$ and $0.74 \times 10^7 \text{ sec}^{-1}$ respectively. These values, which make the denominators of all three terms of the equation the same order of magnitude, combined with the fact that the times of interest for measured $v_{oc}(t)$ are the order of $1 \mu\text{sec}$, drastically simplify equation 7-13. Hence $v_{oc}(t \approx 10^{-6} \text{ sec}) \rightarrow v_{oc}(t = \infty)$ or

$$\frac{v_{oc}(t_{\infty})}{B} \approx \frac{1}{b(\alpha^2 + \gamma^2)} = \frac{1}{ba_0}$$

Remembering the definitions of B and a_0 given earlier yields

$$GR' = \frac{v_{oc}(t_{\infty})}{i_{oc}} \cdot \frac{R_c + r_c}{r_c}$$

From chapter 6, $v_{oc}(t_{\infty})/i_{oc} = 1 \text{ V}/3.5 \mu\text{A}$ so that

$$GR' = \frac{1 \text{ V}}{3.5 \mu\text{A}} \cdot \frac{22}{10} = 0.628 \quad \frac{\text{V(output)}}{\mu\text{A(input)}}$$

3. Evaluation of i_o

From the definition of A given earlier, the signal current pulse amplitude may now be written in terms of known quantities:

$$i_o = \frac{v_o(t_o) C_J L}{f(t_o) GR' b}$$

Table 7-1 contains the values of i_o obtained for the maximum amplitude pulse

of figure 6-8 (corresponding to $v_o(t_o) \approx 2.35$ V(output)) and $f(t_o)$ and C_J evaluated for extreme values of C_J , t_o , τ consistent with figure 7-3.

C_J (pF)	t_o (nsec)	τ (nsec)	i_o (μ A)
1500	245	153	21.2
1500	245	120	24.8
2600	265	160	20.6
3000	265	171	19.7
3500	245	114	24.9
3900	249	110	25.5
4000	265	169	19.6
4500	250	105	26.2
4500	265	160	20.3
*3000	255	138	22.3

Table 7-1: Estimate of i_o for maximum Amplitude Pulse.

* Central values of C_J , t_o , τ

The uncertainty in C_J , t_o and τ is seen to lead to a spread in i_o so that for the maximum amplitude observed output pulse in question, i_o may be said to lie in the 90% confidence interval $20 \leq i_o \leq 26\mu$ A with a most probable value of 22μ A.

E. Energy Loss per Quasiparticle

1. Number of Quasiparticles Produced

Once i_o is known, the total number N_o of excited quasiparticle pairs produced by the alpha particle may be found by recalling the expression $\langle n(t) \rangle$, the expected number of quasiparticles tunneling in time t , from equation 3-15. Setting

$$\langle n(\infty) \rangle = \frac{N_o W_T}{W} = \frac{1}{e} \int_0^{\infty} i_o \exp(-t/\tau) dt \quad (7-14)$$

gives

$$N_o = i_o / W_T e \quad (7-15)$$

where $i(t) = i_0 \exp(-t/\tau)$ is the form assumed for the current pulse superimposed on the ambient tunneling current.

Before going on to evaluate 7-15 it is necessary to estimate W_T and discuss the significance of $W_R = W - W_T$.

(a) Tunneling Probability

From equation 3-17, $W_T = G(e^2 N_m(0) \lambda A)^{-1}$ where G is the low temperature, normal state conductance of the specimen, e is the electronic charge, $N_m(0)$ is the normal state density of free electron states per unit volume near the Fermi surface, λ is the film thickness and A the junction overlap area. For the sample (J-5) with which this analysis is concerned,

$$G = (R_n(4.2 \text{ K}))^{-1} = (.077 \Omega)^{-1}$$

$$\lambda = 2030 \pm 280 \text{ \AA} \quad (7-16)$$

$$A = 7.1 \times 10^{-4} \text{ cm}^2$$

Using equations 3-18 to 3-23, with the number of free electrons per atom $n = 1.1$ as derived from anomalous skin effect measurements (Wilson, 1965), one finds $N_f = 4.05 \times 10^{22} \text{ els/cm}^3$, $E_F(\text{Sn}) = 4.4 \text{ eV}$ making

$$N_m(0) = 1.4 \times 10^{22} \text{ eV}^{-1} \text{ cm}^{-3} \quad (7-17)$$

The value finally obtained for the tunneling probability is therefore

$$W_T = 4.1 \times 10^5 \text{ sec}^{-1} \quad (7-18)$$

(b) Recombination Process

The population of excess quasiparticles generated by the alpha particle is reduced as these quasiparticles recombine with others to form Cooper pairs and rejoin the superfluid (see chapter 2). This recombination takes place primarily by emission of phonons of energy

$\hbar\omega \geq 2\Delta(T)$ rather than photons, as the rate of the former exceeds that of the latter by a factor of about 10^7 (Rothwarf and Cohen, 1963). Subsequently these phonons, if not lost from the energy range $\hbar\omega \geq 2\Delta(T)$ by diffusion to the substrate (Jones and Pennebaker, 1963) or the superfluid helium (Wilks, 1967), may create additional quasiparticles by Cooper pair excitation so that eventually a dynamic equilibrium is restored in which pairs are continually being dissociated by thermal phonons of energy $\hbar\omega \geq 2\Delta(T)$ and quasiparticles are continually recombining into pairs via the emission of phonons.

Rothwarf and Taylor (1967) have shown that for quasiparticles injected into a superconducting film at a constant rate, the observed recombination rate W'_R is dominated by the rate $W_Y = \tau_Y^{-1}$ at which phonons are lost from the range $\hbar\omega \geq 2\Delta(T)$ and not by the rate $W_R = \tau_R^{-1}$ at which individual quasiparticles recombine into Cooper pairs.

From these considerations, it is apparent that the recombination rate W'_R deduced from the observed characteristic decay rate $W = W_T + W'_R(W_R, W_Y) = \tau^{-1}$ is not simply that calculated from the theory of quasiparticle recombination (W_R , see eg. Woo and Abrahams, 1968) but is also a function of the phonon loss processes which go according to W_Y .

(c) Effect of the Energy Gap Parameter

It should perhaps be noted that as the temperature distribution in the tunnel junctions relaxes back to the equilibrium situation, the magnitude of the energy gap $2\Delta(T)$ increases and will affect the manner in which the excess quasiparticle population decays. Some appreciation for the influence of $\Delta(T)$ may be gained from the following approximate calculations.

For a superconductor at temperature T , the thermal equilibrium density of quasiparticles $n(T)$ is given by equation 3-2:

$$n(T) = N_m(0) \Delta(T) K_1(\beta\Delta(T)), \quad T < T_c \quad (3-2)$$

$$\approx N_m(0) \Delta(T) \exp(-\Delta(T)/k_B T) \quad (3-3)$$

(Here, $N_m(0)$ is the normal state density of free electron states per unit

volume near the Fermi surface ($E = 0$), $\Delta(T)$ is the energy gap parameter and k_B is Boltzmann's constant. For convenience $K_1(x)$, the first order modified Bessel function of the second kind, is approximated as (cf. p.55) $K_1(x) = \alpha(x) \exp(-x) \approx \exp(-x)$ as $\alpha(x)$ varies slowly with x and is the order of one.) Thus, as the temperature in the tunnel junction decreases to the equilibrium level (T_0), the change in the excess quasiparticle density $\delta n = n(T) - n(T_0)$, ignoring the recombination phonons, would be given by (for $T_0 < T < T_c$)

$$\frac{\partial(\delta n)}{\partial T} = N_m(0) \exp\left[-\frac{\Delta(T)}{k_B T}\right] \left[\left[1 - \frac{\Delta(T)}{k_B T}\right] \frac{\partial \Delta(T)}{\partial T} + \frac{\Delta^2(T)}{k_B T^2} \right]$$

From chapter 2, it may be recalled that for $T < \frac{1}{2} T_c$, $2\Delta(T) \approx 3.5 k_B T_c$ so that in this range of temperatures, $\partial(\delta n)/\partial T$ would be governed by a Boltzmann factor $\exp(-1.75 T_c/T)$ similar to that governing the density of excited carriers in a semiconductor. Near T_c , however, $\Delta(T)$ varies as $(1 - T/T_c)^{\frac{1}{2}}$ resulting in considerable departure from the straightforward exponential decay.

(d) Evaluation of N_0

The number of quasiparticle pairs initially created by the passage of an alpha particle may now be calculated from equation 7-15 and the estimate that $W_T = 4.1 \times 10^5 \text{ sec}^{-1}$ (equation 7-18). It was shown in section D that the most probable value of i_0 corresponding to a maximum amplitude observed pulse was $22 \mu\text{A}$; therefore

$$N_0 = i_0 / W_T e = 3.4 \times 10^8 \quad (7-19)$$

2. Energy Loss Corresponding to Maximum Amplitude Pulse

The remaining quantity needed to calculate $w = \Delta E / N_0$ is ΔE , the total amount of energy deposited in the superconducting tunnel junction by the alpha particle. Ideally, for purposes of nuclear spectroscopy, ΔE should be equal to the total alpha particle energy $E_\alpha = 5.1 \text{ MeV}$ but this condition could not be achieved with the thin film junctions used.

The alpha particle path length through the film of total thickness 2λ is $x = 2\lambda / \cos\theta$ where θ is the angle of incidence. From figure 6-7 the most oblique angle of incidence was 80° which corresponds to a path length of 2.3×10^{-4} cm. Thus, with the use of figure 3-3, the maximum amount of energy lost directly ΔE (direct) in the tin films is seen to be only about 0.5 MeV.

The remainder of E_α , which was deposited in the glass substrate as the alpha particle comes to rest, may not however, be neglected in determining w . No attempt was made to thermally decouple the films from the substrate with the result that, perhaps as much as one half of the energy deposited in the glass may have diffused back into the tin sufficiently rapidly to contribute to the pulse. For purposes of the present calculation therefore,

$$\Delta E \approx \Delta E(\text{direct}) + h (E_\alpha - \Delta E(\text{direct})) \quad (7-20)$$

where $0 \leq h \leq 0.5$ making $\Delta E \leq 2.75$ MeV. The relation of equation 7-20 to the observed pulse spectrum (fig. 6-11) is considered in section F of this chapter.

3. Estimate of w

Taking $\Delta E \leq 2.75$ MeV as the total energy deposited in the junction corresponding to the maximum amplitude pulse and the result for N_o from equation 7-19 gives an upper limit

$$w(\text{tin}) \leq \frac{\Delta E}{N_o} = 8.2 \times 10^{-3} \text{ eV/q.p. pair} \quad (7-21)$$

which is roughly 7.5 times the value of the energy gap in superconducting tin at 1.2 K (see table 3-2). Purely a phenomenological quantity, w is the average amount of energy which must be deposited in a superconductor to create an excited quasiparticle pair; it is not to be confused with $E_g = 2\Delta(T)$ derived from microscopic theory which is the minimum amount of energy required to break up a Cooper pair at temperature T .

Another quantity of interest for comparing the energy resolution of the tunnel junction detector to conventional semiconductor

spectrometers is $w(\text{exp})$, the energy loss per quasiparticle which actually tunneled. Thus, from equation 7-15

$$w(\text{exp}) \leq \frac{\Delta E}{\langle n(\infty) \rangle} = \frac{\Delta E}{N_0} \cdot \frac{W}{W_T} \approx .145 \text{ eV} \quad (7-22)$$

F. Energy Loss and Diffusion

This section outlines some considerations relevant to the calculation of the manner in which the energy lost by the alpha particle diffuses through the junction and substrate giving rise to a heat pulse.

1. Energy Loss Processes

The primary means by which the alpha particle loses energy is by the ionization and excitation of the electrons associated with the atoms along its path. (In metals, semiconductors and insulators, the particle gives up only about 0.1% of its energy directly to the lattice--creating phonons and dislocations--with the remainder going to the electrons (Seitz and Koehler, 1956).) As the energy transfer at each ionizing collision is generally a small fraction (approximately the order of 10^{-3}) of the particle energy (Dearnaley and Northrop, 1966), the deflections from a straight line path are slight. The particle comes to rest in about 10^{-12} to 10^{-11} sec with roughly 90% of its path (see figure 7-5) being in the glass substrate making it necessary therefore to consider the energy loss processes in both metals and insulators.

In a metal, the primary electron excitations are sufficiently energetic that the usual argument (Jones, 1956) stating that electron-electron interactions are negligible compared with electron-phonon interactions does not hold; first because the $\vec{k}$ space restrictions on scattering due to the existence of the Fermi sphere are greatly reduced and second because the phonon density is small as a result of the factor of 1000 in relative excitation probabilities referred to above. (The argument given by Jones depends on there being a state of thermal equilibrium in which energy is shared equally between electron and lattice degrees of freedom.) Therefore the coupling between electron and phonon excitations is much weaker than either the electron-electron or phonon-phonon couplings, so that the two types of excitation may be considered to thermalize

Figure 7-5: Particle Track Geometry

Figure 7-6: Theoretical Pulse Height Spectrum, $h=0$ (Un-normalized)

individually in times of the order of 10^{-12} - 10^{-11} sec (Dearnaley and Northrop, 1966) with the electron temperature approximately 200 times greater than the lattice temperature. Finally, the weak electron-phonon coupling causes the two types of excitation to relax to a common temperature in about 10^{-9} sec (Seitz and Koehler, 1956).

In an insulator like glass, the electrons may not be considered free. In this case the time required for the electrons and lattice to relax to some common temperature is probably the order of 10^{-12} sec (as estimated for semiconductors by Dearnaley and Northrop, 1966).

It seems reasonable to assume therefore that within a time the order of 10^{-9} sec it is meaningful to speak of a local electron-lattice temperature, in a narrow cylinder co-axial with the particle track, which is in excess of the junction equilibrium temperature. The next paragraph considers the manner in which this heat energy diffuses outward from the track.

2. Energy Transfer

The heat energy is subsequently transported away from the particle track by electron and phonon excitations in a way which may be described by classical heat diffusion theory (Crittenden, 1968). A complete analytic solution of the diffusion equation $\rho c(\partial T/\partial t) = \nabla \cdot K \nabla T$ where ρ is the density, c the specific heat and K the thermal conductivity, is precluded by the following complicating factors:

(a) Temperature Dependence of c and K

Because of the wide range of temperatures occurring in the vicinity of the track, the temperature dependence of the specific heat and thermal conductivity may not be safely neglected. (For example, published data for glass (Johnson, 1960) indicate a variation in K and c of 2 and 5 orders of magnitude respectively over the temperature range from 1 K to 300 K.) With $c = c(T)$ and $K = K(T)$, however, the diffusion equation is non-linear making numerical methods imperative.

(b) Track Geometry

The distance which the alpha particle penetrates the glass substrate is given by R where R satisfies

$$\Delta E'_\alpha = E_\alpha - \Delta E_\alpha(\text{direct}) = \int_0^R \left(\frac{dE_\alpha}{dx} \right) dx.$$

Here, x is the distance measured along the track from the tin-glass interface, E_α is the initial alpha particle energy (5.1 MeV) and $\Delta E(\text{direct})$ is the energy lost by the alpha particle in the tin films. An approximate estimate of R may be obtained by taking $(dE_\alpha/dx) = \text{constant}$ in glass so that $R = \Delta E'_\alpha / (\partial E_\alpha / \partial x)$. The rate of alpha particle energy loss $(\partial E_\alpha / \partial x)$, estimated from the atomic stopping cross sections of the constituents of soda lime glass, is found to be roughly 1.6×10^3 MeV/cm. Now $\Delta E'_\alpha$ depends only slightly on the angle at which the alpha particles strike the junction (see figure 6-7) with the result that R ranges from 2.9 to 3.1×10^{-3} cm which, for all practical purposes, may be taken as $R \approx 3 \times 10^{-3}$ cm independent of angle of incidence. Consequently, as sketched in figure 7-5, about 90% of the track exists in the substrate.

(c) Boundary Conditions

With reference to figure 7-5, it is clear that the heat flow across two interfaces should be considered.

Although the acoustic impedances of tin and glass are quite close (American Institute of Physics Handbook), the reflection co-efficient for phonons at the tin-glass interface is sufficiently large that a finite thermal impedance exists. Little (1959) has calculated that at low temperatures the heat flow between pyrex and copper is

$$dQ/dt = 3.7 \times 10^5 (T_1^4 - T_2^4) \text{ erg sec}^{-1} \text{ cm}^{-2}$$

which, in the limit of small temperature differences becomes

$dQ/dt = 1.5 T^3 \Delta T \text{ W cm}^{-2}$ where ΔT is the temperature discontinuity at the interface. This corresponds then to a thermal impedance $Z_T = .67 T^{-3} \text{ K cm}^2 \text{ W}^{-1}$ which is presumably not far different from a tin-soda lime glass boundary.

The situation at the tin-superfluid helium (He^{II}) interface is more difficult to handle theoretically as the observed thermal impedance cannot be accounted for by consideration of the large acoustical mismatch alone. Other factors which have been suggested (Little, 1961) are the coupling of the conduction electrons in the metal to the totally

reflected phonons of the liquid and the adsorption of a thin layer of helium atoms on the metal interface. This layer of solid helium would provide an acoustic match between the liquid and metal because of its intermediate acoustic impedance. Experimental measurements (Gittleman and Bozowski, 1962) for the normal tin-liquid helium interface give $Z_T = 5.48 \text{ T}^{-3} \text{ K cm}^2 \text{ W}^{-1}$. The value of Z_T for a superconducting tin-liquid helium interface was some 10.6% greater.

From this brief discussion, it is apparent that the problem of the heat pulse diffusion through the tin-glass system is sufficiently complex to merit detailed consideration and, as mentioned earlier, such work is being continued in this laboratory.

3. Evidence for Heat Contribution from Substrate

The observed pulse height spectrum in figure 6-11 contains some information concerning the heat fed back into the junction from the substrate which will now be considered.

It was stated earlier that the total energy contributing to the pulse should be of the form

$$\Delta E \approx \Delta E(\text{direct}) + h(E_\alpha - \Delta E(\text{direct})) \quad (7-20)$$

where h is an experimental parameter assumed to lie in the range $0 \leq h \leq 0.5$.

The upper limit of $h = 0.5$ was inferred from a rough calculation which shows that the minimum time required for phonons, with their initial velocity components directed away from the glass-tin interface, to be perfectly reflected from the far side of the glass and return to the interface is $t \approx 2s/v_p = 2 \times 10^{-3} \text{ m} / 2 \times 10^3 \text{ m sec}^{-1} = 10^{-6} \text{ sec}$. (Here, v_p is the propagation velocity (Klemens, 1951) and the phonon mean free path for glass at low temperatures, which is known to be "long" (Ziman, 1965 and Kittel, 1956), is assumed to be of the order of the substrate thickness s .) From figure 6-8, it can be seen that the actual heat pulse must be less than $0.5 \times 10^{-6} \text{ sec}$ in width so it is not expected that the "reflected" heat pulse, corresponding to roughly one-half of the original energy input, will contribute significantly to the observed pulse.

To see the effect of the heat contribution from the substrate, it is convenient to consider two limiting cases: $h = 0$ and $h = 0.5$.

If $h = 0$, which implies there is no feedback of heat energy from the substrate, the pulse height spectrum should be of the form sketched in figure 7-6, for the energy $\Delta E(\text{direct})$ deposited in the film is inversely proportional to $\cos \theta$ where θ is the angle of incidence. Contrary to what was observed (figure 6-11), this spectrum predicts that lower energy pulses are very much more probable (up to 2 order of magnitude) than higher energy ones. It must be concluded therefore that heat diffusing back from the substrate, the amount of which is less sensitive to θ , was responsible to a large extent for the observed pulses. Thus $h = 0$ may be ruled out.

On the other hand if $h = 0.5$ and is independent of θ so that the junction acts mainly as a thermometer sitting on top of the glass, the predicted pulse height spectrum would be a "spike" centered around the 2.3 MeV point on the energy axis--far removed from the 0.5 MeV pulses due to $\Delta E(\text{direct})$ of figure 7-6. Such a distribution is incompatible with the observed spectrum which therefore rules out the simple case of $h = 0.5$ for all θ .

Intuitively, it is expected that h will depend upon θ , perhaps as the projection ($\sin \theta$) of the track on the glass-tin interface so that from these simple considerations, $h = h(\theta)$ and $0 < h(\theta) \leq 0.5$.

It might be argued that the minimum value of h would be the one yielding ΔE such that $w = \Delta E/N_0 = 2\Delta(T)$ where $\Delta(T)$ is the energy gap parameter at the bath temperature T . This criterion is difficult to justify at present as a detailed theory relating secondary ionization and microplasma phenomena in superconductors to an energy gap model is not yet available.

Although more sophisticated analysis might be used to establish narrower limits on h , it was felt such efforts were unjustified because of the paucity of data presently available. In addition, this problem of heat feedback will be obviated when the experiment is repeated in this laboratory by Dr. B. White and Mr. G. May with a phonon barrier between the substrate and tunnel junction.

G. Summary

For convenience, a résumé of the important results of this chapter is given below.

It was shown that the input impedance of the transmission line-preamplifier system could be written as $Z = R + j\omega L$ where $R = 10.3\Omega$, $L = 1.69\mu H$ and ω is the angular frequency. From the small signal equivalent circuit of the complete system, the output pulse, corresponding to an input current pulse of the assumed form $i(t) = i_0 \exp(-t/\tau)$ was derived using Laplace Transform techniques.

By least-squares fitting the theoretical output pulse shape to that observed on photographs, the value of τ was deduced to be $\tau = 138 \pm 33$ nsec where the errors are those corresponding to a 90% confidence limit.

Knowing the amplifier system transfer constant GR' from a sensitivity calibration, one could then calculate

$$20 \leq i_0 \leq 26\mu A$$

for maximum amplitude pulses.

The total number of quasiparticle pairs produced by the alpha particle N_0 was found to be

$$N_0 = i_0 / W_T e \quad (7-15)$$

where $W_T = 4.1 \times 10^5 \text{ sec}^{-1}$ is the estimated tunneling probability per excess quasiparticle per second and e is the electronic charge.

Taking into account that up to one-half of the energy deposited in the substrate by the alpha particle might have contributed to the observed pulses, the total energy corresponding to the maximum amplitude pulses was estimated to be $\Delta E \leq 2.75 \text{ MeV}$.

The average energy loss per quasiparticle pair is then

$$w = \Delta E / N_0 \leq 8.2 \times 10^{-3} \text{ eV/q.p.pair} \quad (7-21)$$

Another quantity of interest, the average energy loss per quasiparticle (quasielectron or quasihole) which actually tunneled $w(\text{exp})$, was found to be

$$w(\text{exp}) \leq 0.145 \text{ eV}$$

(7-22)

Concluding the chapter is a short discussion of problems concerning detailed calculation of the way in which the heat energy deposited along the alpha particle track diffuses through the glass-tin-liquid helium system.

CHAPTER 8

CONCLUSIONS

This chapter summarizes the important findings from the present work (section A) and attempts to place them in perspective through evaluation of the superconducting thin film tunnel junction as a nuclear spectrometer (section B). Section C outlines the direction in which future work in this area should proceed to improve both the understanding and performance of this device.

A. Major Results

The experiment has demonstrated that the superconducting thin film tunnel junction may be used as a detector of charged particles. In particular, it has shown that the pulse of tunneling current resulting from the bombardment of the junction by 5.1 MeV alpha particles was of sufficient amplitude and duration to be readily observable. Fabrication refinements required to improve the tunnel junction characteristics and so surpass the measured signal to noise ratio of 19 are, as outlined in section C, within the scope of present technology.

An upper limit of 8.2×10^{-3} eV has been placed on $w(\text{Sn})$, the average energy loss by a charged particle required to excite a quasiparticle pair in superconducting tin at 1.2 K. This phenomenological quantity, which hitherto had not been measured in superconductors, is of interest primarily to nuclear spectroscopists for the comparing of one detecting medium with another (see section B). This value of $w(\text{Sn})$ is approximately 7.5 times the energy gap (2Δ) in superconducting tin at 1.2 K which may be compared with the average energy loss per electron-hole pair created in a semiconductor (eg. silicon) where $w(\text{Si}) = 3.6$ eV which is 3.3 times the energy gap in silicon. (Dearnaley and Northrop, 1966)

B. Evaluation of the Device as a Nuclear Spectrometer

1. Energy Resolution

A convenient definition of statistically limited energy resolution R is (Dearnaley and Northrop, 1966)

$$R = \sigma/\bar{q} = N^{-\frac{1}{2}} = (w/E)^{\frac{1}{2}}$$

where $\bar{q} = Ne$ is the signal charge collected, $\sigma/e = N^{\frac{1}{2}}$ is the standard deviation in N , (the number of carriers produced by the charged particle), w is the mean energy required to generate a carrier, E is the particle energy and e is the electronic charge.

To compare the tunnel junction detector to another spectrometer the value of w to be used is $w(\text{exp}) = 0.145$ eV where $w(\text{exp})$ is the maximum value of the energy loss per quasiparticle which actually tunneled (see section E, chapter 7). Thus, for the same particle, it may be inferred that the statistically limited energy resolution of the present superconducting tunnel junction is a factor $(0.145/3.6)^{\frac{1}{2}} = 0.20$ smaller than that obtained with a silicon semiconductor detector, assuming the same Fano factor for both materials.

It is conceivable, with foreseeable improvements in junction fabrication technology, that this resolution factor could be improved to converge perhaps on the approximate limit $[E_{\text{gap}}(\text{super})/E_{\text{gap}}(\text{semi})]^{\frac{1}{2}} = 0.03$. For example, the tunneling probability W_T may be improved by decreasing the normal state junction resistance by the use of insulators with smaller dielectric constants and energy gaps than SnO_2 . (The normal state junction resistance may also be decreased, of course, by minimizing the insulator thickness, but a practical lower limit of about 10° Å precludes significant advances from this approach.) Furthermore, the recombination probability W_R --discussed in chapter 7--could be decreased by surrounding the junction with a phonon insulator, such as a crosslinked polymer (see section C), to reduce the loss of recombination phonons to the substrate and superfluid helium film.

The actual, rather than the statistical, limit of energy resolution is determined by other factors beside w . As discussed in

chapter 3, stochastic variations in insulator thickness across the junction overlap area imply that W_T is not uniform across the junction and fluctuations will occur in the number of quasiparticles which tunnel for a given input of energy. At present, the preamplifier noise dominates junction noise and degrades resolution but the fabrication of junctions with higher output impedances will improve this situation.

2. Linearity

Another point of concern is the dependence of w upon the type of particle, its energy (E) and the geometry of the junction. In the ideal spectrometer, $w(E)$ would be a constant k so that the signal charge collected $\bar{q} = \Delta E/k$ is linearly related to the energy ΔE which a particle of arbitrary initial energy loses in the system. This situation obtains approximately, for lightly ionizing radiation like protons and alpha particles, in practical configurations of several gas and semiconductor detectors (see eg. Dearnaley and Northrop, 1966). The reasons for this are that the ionization potential in the gas and the energy gap in the semiconductor remain effectively constant as the primary particle slows down and that the counters are sufficiently large that the interaction of their boundaries with processes taking place near the particle track may be safely ignored. As discussed below, neither condition holds in the superconducting tunnel junction detector.

The energy gap in the superconductor is, of course, temperature dependent such that it vanishes in the hot region near the particle track center, finally reappearing and increasing in magnitude as the temperature of the superconducting films relaxes back to that of the bath. In a sense, the situation is analogous to that in semiconductor detectors when the disorder left in the wake of a densely ionizing fission fragment permanently alters the band gap resulting in a departure from linearity often referred to as the "fission defect".

In chapter 7, attention was drawn to the fact that the rate at which phonons escaped from the tunnel junction played an important role in determining the temperature relaxation time τ and consequently w . It would seem therefore that conditions at the boundaries of the tunnel junction detector and the location of the particle tracks will affect the measured value of w .

Clearly, considerably more theoretical and experimental evidence is required to establish the variation of w with particle type and energy in superconducting tunnel junctions having a practicable geometry.

3. Stopping Efficiency

If the junction is to be useful as a spectrometer, it should be sufficiently massive to stop ionizing radiation so that all its energy is deposited in the junction. The stopping efficiency of a single tunnel junction is limited by its mass $\rho\lambda A$ which, for the present specimen, is approximately 2×10^{-7} g--a factor 10^8 smaller than for contemporary semiconductor detectors. The area A cannot be made much larger than at present because the junction capacitance would become sufficiently large to prevent the current pulse from being observed; λ could be increased but only at the expense of decreasing W_T (equation 3-17), the fraction of excited quasiparticles which tunnel and the speed of response of the detector system; ρ could be increased, but only some 55%, by using lead instead of tin.

It is conceivable however, with the present rapid growth of micro-electronic and junction fabrication techniques, (Schroen, 1968) that it may be possible to overcome this handicap of thin film junctions by manufacturing them in three-dimensional arrays.

4. Advantages over Conventional Spectrometers

In principle, the superconducting tunnel junction should exhibit superior energy, time and spatial resolution over conventional semiconductor spectrometers in the measurement of nuclear and X radiation which can be stopped in, say, 10,000 Å of lead. The results for the tin junction reported in this thesis indicate the feasibility of attaining this goal with a reasonable amount of further developmental effort. (see section C)

As mentioned in paragraph 2, stopping efficiency might be increased so to be comparable to semiconductors, by fabricating the junction in three-dimensional arrays. It remains to be calculated, however, how seriously the fluctuations in insulator thickness across the many junctions required to stop an energetic charged particle would deteriorate the overall energy resolution.

5. Disadvantages for Operation as a Spectrometer

An obvious shortcoming is the operating temperature of 1.2 K which necessitates a liquid helium cryostat. Even in laboratories where a supply of helium is available, the bulk and complexity of a cryogenic system would probably deter all but the most strongly motivated potential users.

To place a superconducting detector on line with an accelerator-produced beam involves still other difficulties. Because of the tiny junction area, the solid angle subtended by the junctions to an external source is very small indeed. Good spatial resolution is certainly possible but only at the expense of detection efficiency. Introducing the radiation into the cryostat in such a way as to minimize energy straggling and intensity attenuation could well prove to be a challenging cryogenic and nuclear engineering problem. Too high a particle flux would tend to overheat the junction and reduce its sensitivity if not render it inoperative.

A further hindrance to routine use is the tendency of thin film tunnel junctions to change their properties upon storage or thermal cycling (cf. Chapter 6). Workers in the area of development of tunnel junction arrays as computer elements (eg. Schroen, 1968) appear to have overcome this difficulty, however, by carefully defining the junction area with photo-resist and growing the oxide layer with an oxygen glow discharge.

Although none seems insuperable, these obstacles will no doubt impede the development of a practicable superconducting spectrometer.

C. Future Work

1. Use of Junctions with Larger Dynamic Resistance ($r = \partial V / \partial I$)

The signal to noise ratio of a junction-current sensitive preamplifier system benefits doubly from the use of junctions with reasonably large (500-1000 Ω) output impedances, for not only does the signal increase but the amplifier noise is a minimum (Woll and Herscher, 1962). In the present work, the magnitude of r was limited by the restriction of working with tin and the deterioration of tunneling characteristics after thermal cycling.

From equation 3-9, it may be seen that $r \propto \exp(\Delta(T)/kT)$

which immediately suggests two ways in which higher dynamic resistance junctions could be realized in future: the use of materials with larger energy gaps ($2\Delta(T)$) and the use of lower operating temperatures. Lead and niobium, for instance, have relatively large energy gaps (2.7 and 3.0 meV respectively, Douglass and Falicov, 1964) and have been used in tunnel junction fabrication (Pb fairly routinely, see eg. Rowell (1963), but Nb less so (Giaever, 1963)). Lower operating temperatures than 1 K are readily obtained with liquid He³ but, of course, at the expense of considerable complication in the ancillary cryogenics.

The problems associated with thermal cycling are easily obviated by design of a system in which the specimens are kept at helium temperatures from the time their preparation is complete until they are no longer of interest. In fact, should it be necessary, there is no fundamental reason why an evaporator could not be built into a cryostat so that a set of junctions need never be exposed to anything but vacuum or inert helium atmosphere.

2. Thermal Decoupling of Junctions from Substrate

Two benefits would accrue from thermally decoupling the tunnel junction from the substrate: the recombination probability W_R would be decreased and the ambiguity in w caused by the diffusion of heat from the substrate to the junction would be eliminated. In connection with experiments done on single superconducting strip particle detectors (Spiel et al, 1965), Crittenden (1968) has successfully thermally isolated a thin film strip from the substrate with a phonon insulator consisting of about 300 Å of electron-polymerized varnish. This technique, based on the work of Christy (1960), consists of bombarding the substrate (prior to film evaporation) with 100-200 eV electrons in the presence of silicon oil vapour.

3. Preamplifier Improvements

From the detailed analysis of chapter 7 in which the pulse relaxation time τ was extracted from photographic pulse data, it is evident much of the uncertainty could be reduced by the use of a wider bandwidth amplifier permitting direct measurement of the current pulse waveform to much shorter times. Furthermore, the preamplifier used was the dominant noise source so that any innovation reducing this factor would, of course, be beneficial as well.

APPENDIX A

JUNCTION PREPARATION

A. Substrate Preparation

The substrates used were ordinary microscope slides cut on one side to have overall dimensions 2 cm x 7.6 cm and scored to divide it into five areas of 1 cm x 2 cm as shown in figure A-1(a). In this way, all five junctions could be evaporated and dc tested at once and then the substrate could readily be broken up into the five smaller substrates for pulse testing of individual junctions (cf. Chapter 4).

Though originally "process cleaned" by the manufacturer (Corning), the slides had to be cleaned again because of the handling involved in cutting and scoring the packaged 1 in. x 3 in. slides. Cleaning was done with "Teepol" detergent after which the substrates were rinsed sequentially in hot, cold and distilled water followed by hot methanol. Fairly thick, probably 5,000-10,000 Å, silver contacts were then evaporated through an appropriate mask in the geometry shown in figure A-1(a) following which the substrates were stored in a dessicator until needed. Prior to being used in the actual junction fabrication, a completed substrate was rinsed again in hot methanol and mounted in the evaporator.

B. Evaporation Procedure

1. Base Film (figure A-1(b))

Evaporation took place in a CVC (Model CVE-15) oil diffusion pump evaporator with a base pressure of less than 5×10^{-7} Torr. The 2000 Å thick bottom film was evaporated at a pressure of $2-5 \times 10^{-6}$ Torr and a rate of approximately 2000 Å/min. A shutter placed between the vapour source and the substrate masked the substrate from the evaporant until the source was uniformly hot, impurities were driven off and evaporation was well under way.

Figure A-1 Junction Preparation

2. Oxidation

Some 30 minutes after the base film evaporation was complete, the diffusion pump was valved off and dry oxygen was slowly leaked into the bell jar until the pressure was about 1/3 atmosphere. The substrate holder, (see figure A-2), a copper block with internal electrical heater, was then warmed slowly (in about one hour) up to 110°C and kept at that temperature for 15-20 hours. After cooling to room temperature in the oxygen atmosphere, the specimens were exposed to ambient room air for about one minute following which the bell jar was evacuated. Presumably the brief exposure to room air, typically with 40-50% relative humidity, provided the catalytic action required to make the oxidation complete--the singular lack of success with tin junctions not submitted to this phase of the procedure emphasizes its importance. (It should be pointed out that the beneficial effect of a brief exposure to room air was not discovered until rather late in the experimental program so that its role was not carefully investigated. Those familiar with the vagaries of thin films will appreciate that one hesitates to tamper with a recipe that works.)

3. Top Film Evaporation

After the pressure had been reduced to the usual base level of less than 5×10^{-7} Torr, the 2000 Å thick top film was evaporated at the same pressure and rate as the lower film in the geometry shown in figure A-1(d). The substrate was at room temperature throughout both evaporations. Samples fabricated following this procedure were thus in a reasonably controlled environment from the time the substrate was installed until the completed junctions were removed from the evaporator to have the electrical leads attached.

A photomicrograph of a completed Sn-Sn junction is shown in figure A-3. (The white speck on the middle right is a dust particle).

4. Attachment of Electrical Leads and Mounting

Electrical connection to the thin films was made with fine (0.003 in.) gold wires indium-soldered to the silver contacts. (None of these joints, easily made with a moderately warm, very clean soldering iron (free of any flux), was ever observed to fail). The gold leads were then indium-soldered to the terminals of the dc or pulse test holder (see figures 4-2 and 4-3) after which the junctions were mounted in the helium dewar

ready for pre-cooling with liquid nitrogen in a dry nitrogen or helium gas atmosphere. Typically, the time required to effect the transfer of specimens from evaporator to helium dewar with all leads attached and electrical continuity checked was about 45 minutes; throughout this period the junctions were, of necessity, exposed to room air.

C. Evaporation Apparatus

1. Masks

The masks used to delineate the evaporated strips were made from 0.003 in. Beryllium-Copper shim stock which combines ease of photoetching with the desirable properties of low thermal expansion, low gas emissivity in vacuum and resistance to creasing. To ensure that the masks would fit closely and uniformly against the substrates, the masks rested on a 1/4 in. thick stainless steel table (ground flat on one side to ± 0.001 in.) and the substrate holder was so designed that the glass slides lay directly on the mask, held in place by the weight of the holder. (see figure A-2) The substrate holder moved only in a vertical plane; changing of masks while under vacuum was achieved by rotation of the mask table in a horizontal plane.

The motivation for this fairly sophisticated design was the desire to minimize the penumbra region on the films which, according to Rowell (1968), may give rise to various unfavourable junction characteristics. Inspection of photomicrographs of specimens made with this apparatus (figure A-3) indicates this goal was largely achieved.

2. Substrate Holder

The substrate holder served a two-fold purpose: to align and firmly press the glass slides on top of the masks and to heat the substrate uniformly to roughly 100°C in vacuum. As constructed, see figure A-2, the holder was machined from copper with an internal nichrome wire (.4 Ω /ft) heater. Mica sheets provided the required electrical insulation. Temperature measurement was effected with a copper-constantan thermocouple with room temperature reference, mounted directly on the holder body (visible on the photograph in figure A-2). Energized by a filtered dc power supply, the heater was capable of maintaining $110 \pm 2^\circ\text{C}$ for periods of 16-20 hours.

Fig. A-2: Evaporation Apparatus

Fig. A-3: Photomicrograph of Sn-Sn Tunnel Junction
(Magnification: X100)

3. Evaporation Sources

Commercial evaporation sources or "boats", obtained from R. D. Mathis Co., 1345 Gaylord St., Long Beach, California, were used latterly for sample preparation. Made of W, Mo or Ta stock, the preferable form of source was the so-called "open" type (No. S5) which consisted of a single piece of material (approximately 0.005 x 1 x 4 in.) folded longitudinally and bowed in the middle as in the plan view sketched below.

All three refractory metals were used with little to choose among them. Because of its brittleness, W was difficult to bend into a shape compatible with the evaporator electrodes but, apart from that, performed quite acceptably.

A second type of source found to be less satisfactory was the "pinhole" type (No. S17A, see sketch)

which tended to give non-uniform coatings. The trouble arises from the fact that the "hot spot" of evaporation wanders periodically along the length of the boat severely distorting the conical pattern over which the vapour is ideally distributed.

4. Film Thickness Measurement

A quartz crystal oscillator was used to measure film thickness and rate of evaporation. The principle of operation and practical details are discussed in a review by Behrndt (1966). There, it is shown that the shift in the fundamental frequency f of a crystal on which a film of thickness δs has been deposited is given by

$$\delta f = \frac{f^2 \rho \delta s}{N \rho_q} \quad (A-1)$$

where ρ is the density of the material being deposited, ρ_q is the density of quartz and N is the frequency constant which, for AT-cut crystals, is 1670 kHz-mm. Substituting the appropriate values for the constants in A-1 yields the useful result

$$\delta f = \frac{\rho}{1.78} \frac{\delta s}{\text{Hz}} \quad (\text{A-2})$$

for ρ in g cm^{-3} and δs in Angstroms.

The commercial unit used--a Sloan Instruments Corporation Deposit Thickness Monitor, Model DTM-3--included crystal, crystal oscillator hermetically sealed in sensor head (not shown in figure A-2), variable frequency oscillator and frequency meter.

In practice, because geometrical corrections must be applied to relate $(\delta s)_{\text{crystal}}$ to $(\delta s)_{\text{substrate}}$ it is more convenient to calibrate the crystal directly in terms of substrate thickness. Thus, one has

$$\delta f = \alpha (\delta s)_{\text{substrate}} \quad (\text{A-3})$$

where α is a constant which depends on ρ and the crystal-substrate-evaporation source geometry.

To evaluate $\alpha(\text{Pb})$, two independent methods of calibration were used. One method consisted of weighing the Pb films with a micro-balance and determining their thickness from the film area (as measured with a travelling microscope) and the bulk density; the other method involved the direct measurement of the film thickness with a Na light interferometer (a Sloan Instruments "Angstrometer", Model M100). The results are summarized below:

Method	$\alpha(\text{Pb}) \text{ Hz}/\text{\AA}$
Gravimetric	3.1
Interferometric	3.2

Applying approximate geometrical corrections to $\alpha(\text{Pb})$ yields $7.3 \text{ Hz}/\text{\AA}$

which is consistent, within experimental error, with the value $\rho/1.78 = 6.4 \text{ Hz/\AA}^\circ$ predicted from equation A-2.

The value of $\alpha(\text{Sn})$ was derived from $\alpha(\text{Pb})$ by using equation A-2, giving

$$\alpha(\text{Sn}) = \alpha(\text{Pb}) \rho_{\text{Sn}}/\rho_{\text{Pb}} = 2.02 \text{ Hz/\AA}^\circ$$

APPENDIX B

dc CHARACTERISTICS OF LEAKY SPECIMENS

A. Introduction

This appendix is a summary of work carried out in attempting to understand and systematize the experimental results obtained with so-called "leaky" superconducting tunnel junctions. (In this context, "leaky" is used to describe any junction whose dc I-V characteristic is incompatible with the well-known quasiparticle and Josephson tunneling characteristics described in Chapter 2).

If such junctions are mentioned at all in the literature, it is usually only in passing so that the new worker in the field, who prepares a set of specimens and finds their characteristics to differ markedly from the "typical" results given in most articles, is left to wonder what went wrong. The primary function of this section then is to attempt to fill this gap in the literature for the newcomer by (1) describing some of the little-publicized "bad" tunnel junction results and (2) giving a guide to places in the literature where individual points are discussed.

Part B is devoted to a presentation of the results and part C to their analysis. Some of the unresolved problems in analyzing the leaky junction characteristics are outlined in part D.

B. Results

1. I-V Characteristic

A representative I-V characteristic of a leaky Sn-SnO₂-Sn tunnel junction is given in figure B-1. (Similar results were obtained for the Pb-Pb O_x-Pb specimens discussed in Appendix C.) The 4 terminal network by which these X-Y plots were obtained is similar to that already described in Chapter 5. It should be noted that all these curves are taken in zero applied magnetic field but no effort was made to shield the specimens from the earth's field of approximately 0.2 G.

Figure B-1: Typical I-V Characteristics for "Leaky" Specimens.

2. Magnetic Field Dependence of Supercurrent

The I-V characteristics of most specimens showed some supercurrent, ie. the junction could pass a finite current without developing a voltage between the films. Because both the dc Josephson effect and superconducting metallic filaments through the insulating layer could account for such currents, a valuable diagnostic test is to determine the magnetic field dependence of the maximum supercurrent (I_c) (see figure B-2). (The circuit for this test was like that of figure 5-1, except that the input to the X axis was a voltage proportional to the current passing through the Helmholtz coils producing the magnetic field parallel to the plane of the junction.) To produce the I_{crit} -B plot, the magnetic field was set at some level B_i with the junction current I equal to zero; I was then slowly increased until the voltmeter was observed to jump discontinuously from its zero value, the vertical line traced out by the recorder then being proportional to $I_{crit}(B_i)$. The field was then increased to $B_i + 1$ and the operation repeated thereby eventually generating the curve of figure B-2.

C. Analysis of Results

For purposes of discussion it is convenient to consider the curves of figure B-1 in two parts: (a) $0.1 \text{ mV} < V$ and (b) $0 \leq V \leq 0.1 \text{ mV}$.

1. I-V Characteristic for $V > 0.1 \text{ mV}$

Theory predicts and experiments show (see Chapters 2 and 6) that the single quasiparticle tunneling current for bias voltages $V < 2\Delta/e$ should decrease markedly with decreasing temperature. It is immediately evident from a comparison of figures B-1 and 6-1 that such behaviour is not true for the leaky specimens. This suggests that the current in this region of the characteristic does not arise from tunneling quasiparticles but is instead carried primarily by metallic filaments which have been driven normal but whose resistance is still considerably less than that of the insulating layer when both films are normal (R_n).

One may therefore naïvely think of the junction as a perfect diode in parallel with numerous filaments whose total resistance is R_f . The I-V characteristic of such a model is illustrated in figure B-3. It would seem that such a model explains the general shape of the observed

Figure B-2: Typical I_{crit} - B Plot for "Leaky" Specimens

Figure B-3: I-V Characteristic for Model of Ideal Junction in Parallel with Metallic Filaments

characteristic but there still exists an excess current the origin of which, as discussed in section D, is not fully understood.

That some of the current passing between the films was due to tunneling is indicated by the sharp increase in current at bias voltages corresponding to twice the superconducting energy gap (see Chapter 2). The temperature dependence of the energy gap exhibited by these specimens was checked by plotting $\Delta(T)/\Delta(0)$ against T/T_c , where $\Delta(T)$ is one-half the energy gap at temperature T obtained experimentally by extrapolating to the voltage axis. Using the generally accepted values of $\Delta(0) = 1.1$ meV and transition temperature $T_c = 3.72$ K, gave fair agreement with the theoretical curve (figure 2-2).

2. Supercurrent ($V = 0$, Figure B-1)

As mentioned earlier, I_{crit} -B plots like that of figure B-2 were made in an attempt to estimate the relative magnitudes of the dc Josephson and metallic filament supercurrents. The reasoning behind this test is that the dc Josephson current is highly magnetic field dependent (see Chapter 2) and may be quenched for very small values of $\vec{B}$. On the other hand, the critical field for the metallic filament supercurrent should be very high (Anderson, 1964) as the flux quanta are expected to pass through the insulating spaces between the filaments leaving them superconducting shorts. Upon this premise, the magnitude of the supercurrent due to superconducting metallic filaments I_f in the junctions of figure B-2 was taken to be that supercurrent present at the lowest minimum detected in the I_{crit} -B plot and the remainder was assumed to be dc Josephson current I_J .

(a) Josephson Current

To confirm the assumption that dc Josephson current was indeed being observed, the magnitude of the maximum observed zero field current $I_J = I_{J0}$ and the periodicity of the magnetic field dependence of I_J were compared with theory.

The theoretical maximum dc Josephson current is given by (cf. Chapter 2)

$$I_{JM} = \pi\Delta(0)/2eR_n$$

(For the junctions under consideration here, the low temperature normal tunneling resistance R_n was estimated by decomposing the actual I-V characteristic as shown in figure B-3 and setting $(R_n)^{-1}$ equal to the slope which the "ideal" junction characteristic approached asymptotically for $eV \gg 2\Delta$). The ratio $\gamma = I_{JO}/I_{JM}$ was in the range $.07 < \gamma < 0.5$ which is consistent with the results of other experiments. (Langenberg et al, 1966).

The periodicity T' of the structure in $I_{crit} - B$ plots like figure B-2 may be estimated from the number of minima N' occurring in an interval of magnetic field ΔB so that $T' = \Delta B/N'$. From Chapter 2, the theoretical period between successive minima of the Josephson current is $T = \Phi_0 / 2\lambda W$ where $\Phi_0 = h/2e = 2 \times 10^{-7} \text{ G cm}^2$ is the quantum unit of flux, λ is the penetration depth of the magnetic field ($\approx 500 \text{ \AA}$ in Sn) and W is the dimension of the junction normal to B (see inset, Figure B-4). Both T' and T are plotted in figure B-4 as a function of W^{-1} and it is evident that, for the three specimens considered, the agreement between T' and T is quite poor. It must be remembered, however, that the sampling density in the $I_{crit} - B$ plots--ie. the distance between the vertical lines on figure B-2--corresponded to relatively large magnetic field increments of $b \approx 0.23 \text{ G}$ so that one does not expect to be able to resolve all the minima, especially when T is close $\frac{b}{2}$. This point is illustrated for a very simple case in the sketch below. There, it is assumed that $b = 2T/3$ and it is

quite evident that N' , the number of minima observed, would be about $1/2$ the actual number N . When the data of figure B-4 are corrected approximately to take this effect into account, the agreement between experiment and theory is quite reasonable.

In the light of these findings, it may be concluded that dc Josephson supercurrent was present although its precise magnitude remains somewhat obscured.

(b) Metallic Filament Current

The area A_f of metallic filaments or "superleaks"

Fig.B-4: Josephson Current Period vs (Junction Width) $^{-1}$ in Appl. Magnetic Field

Fig.B-5: Structure in I-V Characteristic Near $V = 0$

required to account for the observed supercurrent is an extremely small fraction of the geometrical junction area and it is entirely plausible that imperfections of this extent could occur in the insulating layer.

Consider, for example, the junction of figure B-2 and B-3. According to Anderson and Rowell (1963), the critical currents of finely divided specimens never exceeds 10^7 A cm^{-2} which, for $I_f = 1.7 \text{ mA}$ (see figure B-3) places a lower bound of about $1.7 \times 10^{-10} \text{ cm}^2$ on A_f . This is compatible with the value $A_f = L/\sigma R_f = 1.8 \times 10^{-10} \text{ cm}^2$ where $L = 10 \text{ Å}$ is the assumed length of the filaments, $\sigma = 10^4 \text{ } \Omega^{-1} \text{ cm}^{-1}$ is the assumed filament conductivity (Anderson and Rowell, 1963) and R_f is the filament resistance deduced from analysis of the I-V characteristic in figure B-3. The geometrical area A of that particular junction was $2.4 \times 10^{-3} \text{ cm}^2$ so that $A_f/A \approx 10^{-5}$ and there are no grounds for believing that this fraction is atypical of the leaky junctions studied.

Bardeen (1962) finds that the critical current of small specimens at temperature T is proportional to $(1 - t^2)^{3/2}$ where $t = T/T_c$. It might be expected that I_f should behave similarly as a function of temperature but unfortunately $I_{\text{crit}}-B$ plots were made at only one temperature (1.2 K) making it impossible to check that hypothesis.

Further evidence of the influence of metallic filaments is the constant current jump, shown in figure B-1 at about 1.0 mA, which is examined in the next section.

3. I-V Characteristic for $0 < V \leq 0.1 \text{ mV}$

When I_{J0} is exceeded in an ideal tunneling junction exhibiting the dc Josephson effect, the transition from $V = 0$ to the ordinary quasi-particle tunneling characteristic should be a single discontinuous step as shown in figure 2-7. Such behaviour was never observed in leaky junctions as summarized in figure B-5. The manner in which the voltage increased from zero as the critical current was reached, fell into the two categories labelled (a) and (b) in figure B-5. The small jump, called type (a), will be considered briefly here; type (b), in which no discrete jump is detected, is discussed in section D.

A qualitative explanation of the small constant-current

voltage jump may be found by considering the I-V characteristic of a superconducting wire which is driven normal by a current in excess of its critical current i_c . London (1950) shows that the voltage drop V per unit length of an infinitely long cylindrical wire carrying current $i \geq i_c$ is

$$V = (1/2)\Omega_0 i [1 + (1 - (i_c/i)^2)^{\frac{1}{2}}]$$

where $\Omega_0 = \rho/\pi a^2$ is the low temperature normal state resistance per unit length of the wire. This relation predicts that when $i = i_c$ a voltage jump of $(1/2)\Omega_0 i_c$ occurs and that the characteristic asymptotically approaches the straight line $V = i\Omega_0$. The point of this discussion is that the low voltage region of such a characteristic would resemble the initial tracing of type (a), figure B-5. The significance of this similarity is difficult to assess in view of the fact that, geometrically, the metallic filaments being considered are the antithesis of an "infinite" wire. Lacking better evidence, however, one is led to the tentative conclusion that the small constant-current voltage jump, which appears only at the lowest temperatures, is the result of switching along the metallic filament intermediate state load line.

D. Unresolved Problems in the Analysis

This section outlines two areas of difficulty in interpreting the I-V characteristics of leaky junctions which are not yet fully understood. Specifically, these are the so-called "excess current" of figure B-3 and the structure near $V = 0$ in figure B-5.

1. Excess Currents

In the leaky specimens a current was observed which exceeded that which could be accounted for on the basis of a model consisting of an ideal quasiparticle tunnel junction in parallel with metallic filaments. Careful examination of the I-V characteristic of figure B-3 in the region $0.15 < V < 2\Delta/e$ shows that in addition to the main current jump at $eV = 2\Delta$, there are additional singularities located approximately at $2\Delta/n$, $n = 2, 3, 4, 5$. Comparison of curves taken at 1.2 K and 2 K shows little dependence of the amplitude of the singularities upon temperature. These singularities have been seen by other workers as well and this section is a synopsis of the explanations they have proposed.

Taylor and Burstein (1963) first noticed the singularity at $eV = \Delta$ and, in an accompanying paper, Schrieffer and Wilkins (1963) interpreted its origin in terms of second order tunneling effects involving double particle tunneling via pair dissociation or recombination. The ratio of the magnitude of the discontinuity in the current (J_{Δ}) at $eV = \Delta$ for the double particle processes to the magnitude of the current discontinuity ($J_{2\Delta}$) at $eV = 2\Delta$ for the single quasiparticle processes is (Schrieffer and Wilkins, 1963)

$$\frac{J_{\Delta}}{J_{2\Delta}} = \frac{\exp(-4\langle k_{\perp} \rangle d)}{\exp(-2\langle k_{\perp} \rangle d)} \quad (B-1)$$

In B-1, $\langle k_{\perp} \rangle$ is an average component of wave number k perpendicular to the barrier and d is the barrier thickness. For an ideal, uniform barrier $J_{\Delta} \approx J_{2\Delta}/10$, but, as the authors point out, the effective d for the two processes need not be equal for a patchy oxide layer. The thin regions of the film would be emphasized more by the double particle than the single particle process because of the exponential factors in J_{Δ} and $J_{2\Delta}$. The double particle processes are essentially independent of the quasiparticle densities and are consequently, in agreement with experiment, essentially independent of temperature.

Yanson et al (1965) reported the higher order singularities $2\Delta/n$, for n up to 6, in Sn-Sn junctions at 1.61 K in a field of 33 Oe applied to suppress the dc Josephson current. Strong evidence was also presented claiming their junctions to be free of metallic filaments across the insulator implying therefore that the singularities are associated with multiple particle tunneling through the barrier and not with processes involving the presence of superleaks.

The magnetic field dependence of the $2\Delta/n$ singularities in Pb-Pb tunnel junctions was studied by Marcus (1966, A,B) whose specimens exhibited supercurrents due to both Josephson tunneling and metallic filaments. He found that for a magnetic field normal to the plane of the junction ($B_{\perp}$), the amplitude and sharpness of the singularities decreased

in discrete steps with increasing B_1 in the range $10 < B_1 < 100$ G. The conclusion that Marcus draws from this evidence, in contradiction to Yanson et al, is that the $2\Delta/n$ singularities are due to a tunneling process taking place in the region of the junction where the metallic filaments exist.

Zawadowski (1966), on the other hand, interprets the data of Rochlin and Douglass (1966), who observed several series of $2\Delta/n$ singularities in Pb-Pb junctions, as being due to the break up of P pairs and the subsequent tunneling or transition through metallic filaments of n electrons in the same higher order quantum mechanical processes with a voltage threshold $2\Delta P/n$. Taking appropriate values of P and n , he was able to fit all the singularities observed by Rochlin and Douglass.

Still another explanation of the subharmonic tunneling phenomenon is suggested by Zawadowski in the same paper. It is well known that tunneling junctions act as a microwave generator at finite applied voltage because of the ac Josephson current; it is conceivable therefore that the singularities could be completely or partially the result of microwave photon-assisted quasiparticle tunneling.

In a later paper, Zawadowski (1967) considers briefly the problem of higher order tunneling processes concluding that processes of this type may be much more intense than expected if the transitions of the quasiparticles through the barrier occur at some defect in the barrier. Since Yanson et al have observed multiple particle tunneling in junctions apparently free of metallic filaments, it would seem that the superleaks in the samples of Marcus and those considered here are important only in the sense of being defects in the barrier at which higher-order tunneling is enhanced.

2. Structure near $V = 0$

Interesting behaviour in the I-V characteristic near zero voltage was observed in all leaky junctions; it appears to fall into the two general categories of figure B-5. Retrace loops, whose area varied approximately inversely with temperature, were always present but in (a) supercurrent was restored at $I = I_2 < I_{crit}$, and in (b) the supercurrent was restored at $I = I_{crit}$. Similar structure to case (a) has been observed by Yanson et al (1965) with their metallic filament-free junctions at 2 K

in zero magnetic field. The same workers also found structure resembling that of case (b) when they first applied a magnetic field of tens of oersteds, turned it off and ran an I-V curve at zero field. No mention is made whether the change is permanent. Yanson et al attribute their results to magnetic flux being partially trapped by inhomogeneities at the edges of the superconducting films; this explanation can probably be extended to the present junctions for the metallic filaments would serve mainly to increase the flux trapping. The influence of flux trapping on the detailed behaviour of tunnel junctions has been felt by other workers as well. Jakelevic et al (1965), for instance, disclose that they found it necessary to shield their junctions from even the earth's field while cooling them to prevent serious attenuation of the dc Josephson current.

As mentioned in paragraph 1, singularities were observed at $eV = 2\Delta/n$ with $n = 1, 2, 3$ and as high as 5 in some specimens. At bias voltages $eV < 2\Delta/5 \approx 0.22$ mV, however, these singularities were obscured by a much more complicated structure (see figures B-3 and B-5(b)) consisting partially of constant-voltage and constant-current steps; again, similar behaviour has been reported by Yanson et al (1965). It seems quite reasonable that these steps are evidence of the ac Josephson current coupling with the resonant modes of the junction which would act as an open ended parallel plate resonator. This effect was first studied by Fiske (1964, 1965) and Eck et al (1964, 1965); it is also discussed at some length by Langenberg, et al (1966).

(Note added in proof: Since this appendix was written, the author has become aware of a recent review (Wilkins, 1969) of the subject of multiparticle tunneling which deals with some of the problems raised here.)

APPENDIX C

OBSERVATIONS ON Pb-Pb TUNNEL JUNCTIONS

It was shown in Chapter 3, that the most desirable tunnel junctions for use as a charged particle detector is one with lead films on each side of the insulating layer (Pb-Pb). This appendix outlines some of the obstacles encountered in the course of attempting to develop suitable Pb-Pb junctions and explains why it was not possible to use these junctions in the detector experiment.

Section A is a summary of the low temperature results obtained with Pb-Pb specimens and the conclusions derived therefrom. Section B describes the phenomenon of nodule growth on the lead films and recounts the steps taken to understand and eliminate this undesirable side effect. Section C contains a very short survey of the literature concerning the oxidation of metals in general and of lead in particular; the plasma oxidation technique is suggested as an attractive alternative to the conventional thermal oxidation procedure.

A. Tunnel Junction Results

1. Junction Preparation

The recipe followed in preparing these junctions, derived from that used by workers in the Bell Telephone Laboratories (Rowell, 1968), is essentially the same as that given in Appendix A for Sn-Sn specimens. In this case, the insulating layer is a thermally grown oxide of lead--probably PbO, see section C. (The technique of Taylor and Burstein (1963) of evaporating a very thin layer of aluminum on top of the base film and allowing it to oxidize to form the tunneling barrier was rejected because of the possibility of incomplete conversion.)

Variations on the recipe that were tried in an effort to make reliable junctions will be pointed out in the discussion of section B.

2. Low temperature dc Characteristics

Without exception, all Pb-Pb tunnel junctions tested at liquid helium temperatures--about 50 altogether--were leaky in the sense defined in Appendix B. In many cases the Pb-Pb junctions were worse than the leaky Sn-Sn junctions in that no single-quasiparticle tunneling was evident, as denoted by the absence of a sharp increase in the current at $V = 2\Delta/e$. Furthermore, the large supercurrent exhibited by the Pb-Pb specimens was almost all due to superconducting metallic filaments (or superleaks) as it was in excess of the expected maximum theoretical dc Josephson critical current and, unlike the dc Josephson current, showed no significant dependence upon the applied magnetic field. (The critical current for some specimens exceeded 150 mA--the maximum current output available from the dc supply used, see chapter 5--and in several junctions, the thin films serving as current leads were observed to go normal and literally "burn out" before the junction would switch from the supercurrent characteristic.)

During an extensive experimental program aimed at improving the tunnel junction performance, the effects of varying all the obvious parameters such as evaporation rate, film thickness, oxidation procedure and film edge sharpness were studied but without success.

3. Conclusions from Pb-Pb Junction Studies

The obvious conclusion, of course, is that the lead oxide formed on the base film was very patchy so that, when the top film was evaporated to complete the junction, a significant fraction of the overlapping area consisted of two lead films in intimate contact forming an essentially continuous superconducting strip. In spite of the enormous effort put into growing a suitable oxide layer and the assertions of other workers concerning the ease of doing so, the dc measurements indicate this goal was never reached. Therefore, because no Pb-Pb junctions could be prepared in which the supercurrent could be suppressed nor a single quasiparticle tunneling characteristic of suitably high differential resistance obtained, the idea of using Pb-Pb junctions in the alpha particle detection experiment had to be abandoned.

One avenue of investigation which was fruitlessly followed in this study was the effect on junction performance of nodules observed to

have developed on the surface of the thin films. The next section recounts the details of this investigation.

B. Investigation of Nodule Growth on Lead Films

Microscopic examination of Pb-Pb tunnel junctions, see figures C-1 and C-2, revealed that nodules, having one dimension as large as 4μ , were randomly distributed over the Pb films. (Tin films, see eg. figure A-3, never formed these blemishes). Now if these growths are merely surface features of the completed junction they are not very important but, if they form on the base film before the top film is evaporated and consequently project through it, a mechanism for the formation of superleaks is clearly suggested. The investigations outlined below showed that the nodules did indeed form on the base film but when it later became possible to fabricate Pb-Pb junctions which were apparently nodule-free, they proved to be just as leaky as their predecessors. It seems, therefore, that the presence of nodules must contribute to the formation of superleaks but it is not the fundamental reason why the oxide patchiness persists. (Oxide growth on thin films is discussed briefly in section C.)

1. Description of Nodules

That the white specks of figure C-1, photographed with reflected light, were indeed growths on the surface of the films and not pinholes in the films was established by the observation at higher magnification (figure C-2) of the shadows they cast and by the fact that the specks were opaque to transmitted light. An attempt was made with an electron microscope to determine whether the nodules were metal that had been squeezed from surface fissures or large islands of metal oxide which had formed on the surface but the project proved futile because of the difficulty in detaching the lead films from the glass substrate without destroying them. Without this information, probably no fundamental understanding of these growths can be realized; the purpose of the present study was simply to learn how to prepare films free of these defects.

2. Origin of Nodules-The Result of Thermal Treatment

Surprisingly, the nodules were found to form whether a lead film was heated to 395 K in about 500 Torr of oxygen or cooled to 1.2 K in a helium atmosphere. To observe this, a set of junctions was prepared according to the schedule shown in figure A-1 and the films were examined microscopically after the completion of stages b, c, d and a liquid helium

(a)

(b)

Fig. C-1: Photomicrographs of Pb-Pb Tunnel Junction (X100)

(a) Before exposure to Liquid Helium

(b) After exposure to Liquid Helium

Fig. C-2: Photomicrograph showing Nodules
(Magnification X1220)

exposure. No nodules were visible on the recently formed base film (stage b) but they were after the base film was heated to expedite oxide formation (stage c). The nodules persist, being visible all along the base film, and appear to penetrate the crossed film when it is evaporated (stage d). (Photomicrograph C-1(a) was taken at this point; the vertical film is the base film.) Subsequent cooling of the entire junction to liquid helium temperatures caused further growth of the nodules on both films as illustrated in figure C-1(b). As an additional check, five other specimens were prepared simultaneously on one substrate and then separated such that three were heated and two were cooled (to 77 K); no significant difference between the two groups, with regard to the surface density of nodules formed, was discernible.

This result raises two problems. From a practical viewpoint, a superconducting tunnel junction which deteriorates upon cooling is intolerable; from a theoretical viewpoint, a chemical process that goes equally well at roughly 1 and 400 K is hard to envisage.

One explanation suggested by Young (1968) is that the evaporated films are in compression and that the nodules result from the differential expansion and contraction of the glass and lead upon thermal cycling. Although the argument is weakened somewhat by the absence of reports of similar effects by other workers who have also used glass substrates for Pb-Pb junctions, it is consistent with the results of the next paragraph where it is seen that the nodules did not form in films of thickness less than 1000 Å. Such films, presumably, would flow more freely with the glass than thicker ones and be less susceptible to ruptures which could give rise to nodules.

3. Tests Made to Isolate Nodule-Producing Parameters

There are, of course, a host of parameters which affect a given evaporation. No claim is made that the list of parameters tested here is exhaustive for one is concerned only with those which could conceivably give rise to the nodule growth. A "test" of the effect of a parameter change consisted of preparing a set of junctions (usually five) under the appropriate conditions, examining the films microscopically for the presence of nodules immediately following evaporation, cooling the junctions to 77 K in a helium atmosphere and re-examining them for nodule growth. (Once the thermal-

treatment origin of the nodules was established, cooling was chosen as the means of producing nodule growth for test purposes, as the heating cycle during the oxidation stage could conceivably be omitted in fabricating practical junctions but never the cooling cycle in using them. In other words, a thin film that is subject to nodule production at low temperatures is useless whether or not nodules are produced at elevated temperatures.)

First, the parameter variations of interest are listed and explained; the results are summarized in the final paragraph. (Within experimental limitations only one parameter was changed at a time.)

(a) Evaporant Purity

Two grades and forms of Pb were used as evaporant: 99.999% pure bar stock from Cominco and 99.99% pure pellets from Fisher Chemical. Similar evaporation sources were used in each instance.

(b) Evaporation Source

Evaporations were performed with "boats" of similar geometry (type S5, Appendix A) but made of Ta, Mo or W.

(c) Order of Film Deposition

To test the hypothesis that the nodules were due to contamination evolved from the initial heating of a new charge of evaporant, the usual order of evaporating the films, see figure A-1, was reversed.

(d) Substrate Preparation

To observe the effect of different cleaning techniques, a glass substrate of the type described in Appendix A was prepared in which 1/3 of the scored area had a final cleaning with chromic acid and 1/3 with very dilute hydrofluoric acid. The remaining 1/3 had been cleaned in the usual manner which did not involve the use of acids.

(e) Rate of Evaporation

The effect of evaporation rate on nodule growth is somewhat obscured by the unavoidable variations in rate which occur during the course of a given evaporation. In general, an average deposition rate of about 1000 Å/min was achieved.

(f) Film Thickness

The importance of film thickness to nodule formation

was studied first of all with a "pin-hole" source (see Appendix A) which gave films whose thickness varied continuously along the length of the substrate but which otherwise were formed under identical conditions. Later, junctions were made in which the thickness of one film was greater than 1000 Å and the thickness of the other less than 1000 Å.

(g) Summary

The results of all the tests except the last were negative in the sense that the nodule population produced was apparently invariant--as recorded photographically--to those changes of evaporation parameter. It was found, as stated earlier, that the nodules did not form, either when the film was heated or cooled, in a film of thickness less than 1000 Å. Films thicker than 1000 Å, prepared simultaneously to the thinner ones, formed nodules in the usual manner.

4. Conclusions from Tests

The consequences of this discovery with regard to the eventual use of Pb-Pb junctions for charged particle detection are serious. High detection efficiency demands that the films of the tunnel junction be as thick as possible (see Chapter 8) and a maximum practical film thickness of 1000 Å would be a severe handicap.

It should also be mentioned that these tests do not rule out the possibility that the nodules are an hydroxide of lead formed when the junctions are transported in room air. Personal experience has shown that water has a very corrosive effect upon thin lead films and other workers (eg. Rowell (1968) and Schroen (1968)) have emphasized the need of avoiding exposure of lead films to water vapour. As there is no apparent reason why this process would discriminate against films thinner than 1000 Å, it may be that hydrolysis is supplementary to the differential expansion-contraction mechanism suggested earlier, the points of rupture being more susceptible to corrosion than the surrounding "oxidized" surface.

The corrosive effect of mercury on metals like lead is also well known (see eg. Smithells 1967) and it is possible, though unlikely, that mercury contamination in the evaporator may not be negligible in this respect. (Although the evaporator used for these samples had an oil diffusion pump, hardware from a mercury diffusion pumped system had been temporarily installed in it at an earlier time, and the possibility of

residual mercury vapour contamination exists.)

C. Oxide Growth on Lead Films

One question which is not yet understood and upon which the preceding work sheds no light is the reason why it proved impossible to grow a continuous layer of oxide on the lead films. Compounding the problem is the fact that few data are available on the oxidation of lead at temperatures below its melting point. The purpose of this section is to cite some of the publications which are available and give a very brief summary of their findings which are relevant to tunnel junction fabrication.

Anderson and Tare (1964) prepared their 2500 Å thick lead films by evaporation from a well outgassed 99.999% pure lead charge at the relatively slow rate of about 250 Å/min. Doses of oxygen were subsequently injected into the evaporating vessel and the initial uptake of the oxygen by the films was found to be very rapid, (complete in less than one minute). An important result was that the presence of water vapour or carbon dioxide in the oxidizing atmosphere had little effect on the oxidation rate and none on the oxide structure which, as tested by electron-diffraction techniques, was found to be orthorhombic lead monoxide (PbO). Apart from the slow evaporation rate used by these workers compared to the 1000 Å/min rate of the present experiment, no special precautions appear to have been taken to ensure the growth of PbO that have been omitted in work here.

A later paper by the same authors (1965) extends the work on the oxidation rate of pure lead films and investigates the reduction in rate brought about by adding to the lead impurities such as bismuth and thallium.

Several papers dealing generally and theoretically with oxidation problems in metals are available as well. (see eg. the four articles by T. N. Rhodin et al, J. F. Chittum, D. W. Pashley and H. Fischmeister in a conference report arranged by Benard, 1965). Of particular usefulness is a review by Uhlig (1967) which includes a fairly extensive bibliography. Briefly, he states that the interaction of oxygen with a clean metal surface follows the sequence (1) physical adsorption of O₂ followed in most instances by dissociation of O₂ and chemisorption of O atoms, (2) nucleation of metal oxides at discrete sites and (3) formation and growth of a continuous oxide layer. The time required for this to happen is quite short. For example,

a continuous Cu_2O layer is formed in about 6 sec in 1 Torr of O_2 at 550°C ; even shorter times are required at higher pressures or lower temperatures. Certainly the 15-20 hours during which the lead films of this experiment were heated to 100°C and exposed to 500 Torr of O_2 would seem to be ample time to form a continuous oxide layer.

The glow-discharge oxidation technique described by Schroen (1968) is perhaps the best way to avoid the uncertainties inherent in thermally grown oxides. While admitting that present theoretical understanding of the processes involved is based upon tentative models, Schroen claims that this plasma method has been refined to the point that reproducible and stable tunnel junctions are routinely made.

APPENDIX D

EFFECT OF FINITE FILM RESISTANCE ON TUNNEL JUNCTION CHARACTERISTICS

A. Introduction

This appendix, of concern primarily to those who are interested in problems connected with thin film tunnel junction fabrication, describes two little-publicized effects which were observed early in the experiment as well as the theoretical and experimental steps taken in attempting to understand these effects. Though an interesting sidelight, the problems discussed here are not germane to an understanding of the tunnel junction charged particle detector.

The first effect observed (Section B) was that many crossed-film samples, when tested with the conventional 4-terminal technique sketched in figure D-1(a) displayed an I-V curve with negative slope. Pedersen and Vernon (1967) observed similar behaviour in the parallel-film type of junction (Sn-Sn) sketched in figure D-1(b).

The second effect discovered was that the slope of the I-V curve depended on ϕ , the angle of intersection of the films.

Theoretical (Section C) and experimental model studies (Sections D and E) of these phenomena show that they may be attributed to the finite resistance of the thin films constituting the junction.

B. Experimental Observations of Effects with Tunnel Junctions

1. "Negative Resistance"

Figure D-1(c) shows the I-V characteristics obtained with two typical Sn-Sn crossed-film tunnel junctions; similar results were obtained with Al-Pb and Pb-Pb junctions. It is evident that at certain temperatures the slope of the I-V curve is negative, (with I and V measured as shown in figure D-1(a)), so that the junction exhibits "negative resistance." In addition, the magnitude of the slope is temperature dependent so that the junction resistance goes from a large negative value at 296 K, through

Figure D-1(a): Crossed-Film Junction

Figure D-1(b): Parallel-Film Junction

Figure D-1(c): Typical I-V Characteristics of 2 Sn Tunneling Junctions

zero around 4 K to a small positive average value at 3 K and below.

These results are incompatible with the simple one-dimensional equivalent circuit often used to describe four terminal measurement of tunnel junctions as shown in figure D-2. In the figure, R_1 and R_2 are the

Figure D-2 Four-Terminal Equivalent Circuit of Tunnel Junction

resistance of thin films 1 and 2 respectively and R_T is the total tunneling resistance. For the polarities shown, which are identical with those of figure D-1(a), it is clear that the measured junction resistance

$R_j = V_{D-B}/I_{C-A} = R_T > 0$ for all temperatures $T > T_c$. (Below the transition temperature T_c , the I-V characteristics of tunnel junctions composed of superconducting films have small regions of negative slope which arise from the relative magnitudes of the energy gaps of the superconductors involved (see eg. Figure 2-6); such effects are not of interest in the present discussion). As there is no way in which the circuit of figure D-2 can explain the results shown in figure D-1(c) a more sophisticated model is required: the theoretical approach is discussed in section C and the experimental approach, consisting of junction simulation studies, is discussed in section D.

Several other features in the I-V curves of figure D-1(c) should be noted. Breaks appearing in the I-V curve at 3 K between 0.6 and 1.1 mV are caused by the films being temporarily driven normal by the measuring current. The reversal of slope shown to occur at about 1.2 mV for sample II is caused by the measuring current having driven the films completely normal. The fact that the resistance is approximately the same at 1.25 K and 3.0 K for both specimens indicates metallic filaments were shorting out the insulating layer so that only a very small component of the current flowing between the films was due to tunneling. It must be emphasized, however, that while the so-called "negative" resistance is

associated with low junction resistance, as shown in section C, it is not confined solely to junctions in which metallic shorts are present. Evidence of this comes from the results of Pedersen and Vernon (1967) whose junctions displayed conventional tunneling behaviour below the transition temperature.

2. Strip Intersection Angle-Dependence of Slope

Let $\phi (\neq 90^\circ)$ be the angle of intersection of the two thin film strips serving as current terminals (see figure D-1(a)). Thus, depending on the choice of terminals used in the measurements, a given junction may be characterized by two resistances, $R_j(\phi)$ and $R_j(180-\phi)$. Ideally, with reference to figures D-1(a) and D-2, $R_j(\phi) = R_j(180-\phi)$ as the effective resistance should be the tunneling resistance R_T which depends on the overlap area and is independent of the choice of current terminals. Experimentally, it was found that at room temperature, $R_j(\phi)$ was not equal to $R_j(180-\phi)$. Table D-1 summarizes the experimental results where

$$\rho = R_j(\phi \text{ large})/R_j(\phi \text{ small})$$

is introduced as a convenient measure of the junction resistance asymmetry. This angle dependence of the resistance was investigated with an experimental model study, the results of which are discussed in section E.

Sample (Pb-Al) No.	$R_j(\Omega)$ $\phi = 125^\circ$	$R_j(\Omega)$ $\phi = 55^\circ$	$\rho = \frac{R_j(\phi = 125^\circ)}{R_j(\phi = 55^\circ)}$
66-3	-.2	-.008	25
66-11	-.2	-.004	50
66-15	-.1	+.01	-10
66-17	+.24	+.3	.8
66-18	-.2	-.003	66
66-19	-.1	+.01	-10
66-21	+.11	+.17	.65
66-29	+.016	+.03	.53
16 others	.3-130	.3-130	1.0

Table D-1: Dependence of Junction Resistance on ϕ

C. Theoretical Investigations

The simplest junction to consider in detail is the parallel-film type for, if uniform thickness and width are assumed for the films and

insulator, the analysis is one-dimensional. It turns out that the behaviour of this junction can be described analytically whereas the behaviour of the crossed-film junction must be investigated numerically.

As shown in figure D-1(b), the current I is fed into terminal A on film 1 and removed from terminal C on film 2 with the junction voltage V_j being measured from terminal D to B. This case, which corresponds to $\phi = 180^\circ$ (cf. figures D-1(a) and (b)), has been analyzed by Pedersen and Vernon (1967) who find that $R_j \equiv V_j/I$ is given by

$$R_j(180^\circ) = \alpha \ell R_T [\text{csch}(\alpha \ell) + 2R_1 R_2 (R_1 + R_2)^{-2} \tanh(\alpha \ell / 2)] - R_1 R_2 / (R_1 + R_2); \quad (D-1)$$

where ℓ is the length of the junction and

$$\alpha^2 = g (r_1 + r_2)$$

$$R_1 = r_1 \ell$$

$$R_2 = r_2 \ell$$

$$R_T = (g \ell)^{-1}$$

Here, g is the tunnel conductance per unit length and r_1, r_2 are the resistances per unit length of films 1 and 2 respectively; R_T is the total tunneling resistance.

First of all, it is clear that when both films are superconducting, $R_1 = R_2 = 0 = \alpha$, the measured resistance R_j becomes the actual tunneling resistance R_T . Obviously, for finite values of R_1 and R_2 , $R_j \neq R_T$ and furthermore, because of the negative term, there will exist values of R_1 and R_2 for which R_j is negative.

The effect of finite film resistance on R_j is more readily observed if equation D-1 is simplified to the case of a symmetric junction, ie. $R_1 = R_2 = R$. Then

$$\alpha \ell = (R/2R_T)$$

and

$$R_j(\text{sym}) = (RR_T/2)^{\frac{1}{2}} \coth(R/2R_T)^{\frac{1}{2}} - R/2, \quad \phi=180^\circ \quad (D-2)$$

The plot of equation D-2, in figure D-3 shows that for $R/R_T = 2.88$, $R_j(\text{sym})$ is zero and, for larger ratios of R/R_T , $R_j(\text{sym})$ is negative.

The validity of this model was checked by Pedersen and Vernon with excellent agreement being obtained between the experimentally measured thin film junction resistance and that calculated from equation D-1 using experimental values for R_T , R_1 and R_2 .

Another junction configuration that may be treated as a one dimensional problem is the $\phi = 0^\circ$ case (cf. figure D-1(b)) when the current I is fed into terminal A on film 1 and removed from terminal B on film 2 with the junction voltage V_j being measured from terminal D to C. Using the same equivalent circuit as Pedersen and Vernon (1967) and applying the appropriate boundary conditions yields, for a symmetric junction $R_1 = R_2 = R$,

$$R_j(0^\circ) = V_j/I = (2RR_T)^{\frac{1}{2}} \text{csch} (2R/R_T)^{\frac{1}{2}} \quad (\text{D-3})$$

Equation D-3, plotted in figure D-3, shows that the measured resistance R_j becomes the tunneling resistance as $R \rightarrow 0$ and that $R_j \rightarrow 0$ in the limit of large film resistance.

Unfortunately, it is not possible to deal with the crossed-film junction in such a straightforward manner as the lumped equivalent circuit is a three-dimensional network. Consequently, the voltage $V(x,y)$ between the films, cannot be expressed in a simple, closed form and it would be necessary to resort to some form of numerical analysis such as the Finite Difference or the Monte Carlo method. (Binnis and Lawrenson, 1963).

While differing in detail from those of the crossed-film junction, the results of the $\phi = 180^\circ$ parallel-film case lead to some general conclusions for limiting cases concerning the effect of finite film resistance on the measured crossed film junction resistance. For $R \ll R_T$, which implies a relatively thick insulating layer, films 1 and 2 are effectively insulated from one another so that terminals D and B have essentially the same potential as A and C respectively making $R_j = (V_D - V_B)/I > 0$; for $R \gg R_T$, films 1 and 2 are virtually a single sheet with, as is confirmed experimentally in

Figure D-3: Effect of Film Resistance (R) on Measured Junction Resistance (R_j). R_T = constant

the following section, terminal D being negative with respect to B such that $R_j = (V_D - V_B)/I < 0$.

D. Experimental Simulation of Crossed-Film Junctions

1. Graphite Coated Paper

To simulate the junction in which $R \gg R_T$, a sheet of colloidal graphite coated paper, which is conducting on one side, was cut into a shape resembling two films "crossing" at an acute angle. (see figure D-4; this model corresponds in fact to $R_T = 0$). Equipotential lines were plotted by supplying a steady current as shown and measuring the voltage at every intersection of a reasonably fine grid laid out on the graphite surface. The distance from the intersection to the end of the "films" was about 10 times the "film" width so that by taking care to place the current electrodes centrally on terminals A and C a uniform current flow was obtained. Voltage was measured with a Fluke Differential voltmeter, Model 881A, having essentially infinite input impedance at the null point. It is clear from figure D-4 that $R_j = (V_D - V_B)/I_{A-C} < 0$. A resistance of the same sign but greater magnitude was obtained by passing the current from A to B and grounding terminal C. For this "junction"

$$\rho = \frac{R(102^\circ)}{R(78^\circ)} = \frac{-135\Omega}{-99\Omega} = 1.34$$

This simple test provides further confirmation that the reason that actual tunnel junctions displayed an angle dependent negative resistance (Table D-1) was because their tunnel resistance was sufficiently small as to make them look electrically like a single conducting sheet.

2. Soldered Manganin Strips

As a closer approximation to a real specimen in which $R \gg R_T$, two thin strips of manganin (6 x 1 x .012 in.) were soldered together at their middle region with care being taken to make the solder layer as uniform as possible. (A rough estimate shows $R/R_T \approx 10^6$ for this "junction")

Equipotential lines mapped out using the technique outlined above were similar to those of the conducting sheet. Thus, the predictions of the parallel-film structure in the region of very small tunneling resistance are borne out in models of crossed-film junctions.

Figure D-4: Graphite Coated Paper "Junction", ($R_T = 0$)

3. Compressed Nichrome Strips

In order to study the behaviour of crossed-film junctions for other values of R/R_T , a lucite jig was prepared in which two nichrome strips ($1/8 \times 4 \times .021$ in.) could be constrained to lie one on top of the other and intersect at an angle of 53° . The "tunneling resistance" between the two strips could then be varied by exerting pressure normally on the intersection with a small bakelite plunger. (see figure D-5) (A Budd Strain Indicator, Model P-350, was used to measure the load developed by the departmental workshop hydraulic press.) Before being placed in the jig, the surfaces of the strips which were to be superimposed were cleaned and polished with fine carborundum paper (No. 4/0).

The nichrome strip "junction" resistance R_j was measured with the 4-terminal network shown in figure D-5 for the two configurations $\phi = 53^\circ$ and $\phi = 127^\circ$. It was found that $R_j = V_{D-B}/I_{A-C}$ was positive for small loads on the plunger and negative for larger loads in both configurations. To understand this result qualitatively, it is assumed that the effective "tunneling resistance" R_T between the two strips is inversely related to the load pressing the surfaces together and certainly much more sensitive to changes in pressure than the bulk resistance R of the nichrome strips. For large loads (small R_T), the strips are essentially a continuous piece like the graphite coated paper and R_j is negative; for small loads (large R_T), the strips are insulated from each other making R_j positive.

The adequacy of the parallel film model to explain these crossed nichrome strip results quantitatively was explored in the following manner. Figure D-3 indicates that in the limit of $\phi = 0$, $R_j/R_T > 0$ for all R/R_T and in the limit $\phi = 180^\circ$, $R_j/R_T < 0$ for $R_j/R_T > 2.88$. It seems reasonable to assume therefore that for strips crossing at arbitrary ϕ , ($0 \leq \phi \leq 180^\circ$), the measured "junction" resistance R_j may be expressed as a linear sum of the two limiting cases (equations D-2 and D-3). Thus, in simplified form,

$$B = (1-\beta(\phi))[(A/2)^{\frac{1}{2}} \coth(A/2)^{\frac{1}{2}} - A/2] + \beta(\phi)(2A)^{\frac{1}{2}} \operatorname{csch}(2A)^{\frac{1}{2}}, \quad (D-4)$$

where $A = R(\phi)/R_T$, $B = R_j/R_T$ and the mixing parameter $\beta(\phi)$ lies in the range $0 \leq \beta(\phi) \leq 1$ with limiting values $\beta(0) = 1$ and $\beta(180) = 0$. The effective bulk strip resistance $R = R(\phi)$ is taken to be a free parameter

Figure D-5: Pressed Nichrome Strip "Junctions"

because the current distribution in the strips in the overlap region at arbitrary ϕ is not known; in the extreme cases, $R(0) = R(180) = 1.9 \text{ m}\Omega/\text{sq.}$ for the strips used. To relate equation D-4 to the simulated junction measurements it is necessary to make some assumption concerning the relation between R_T and the compressing load L . The form chosen, which is simplest to treat mathematically and yet has the correct asymptotic behaviour, is $R_T = k(\phi)/L$ where $k=k(\phi)$ is a parameter to be determined experimentally. (The ϕ dependence of k , like that of $R(\phi)$, arises from the unknown angular dependence of the current density in the "junction" region.) For a given load L_i , therefore

$$A_i = R(\phi)L_i/k(\phi) \text{ and } B_i = R_{ji}(\phi)L_i/k(\phi).$$

With equal weight assumed for the experimental values $B_i(\text{expt}) - R_{ji}(\text{expt}, \phi)L_i/k(\phi)$, equation D-4 was least squares fitted to the data obtained for the two nichrome strip "junction" configurations. The values of the parameters β , R and k which minimized

$$\sum_i [B_i(\text{expt}) - B_i(\text{theory})]^2$$

are given in table D-2.

ϕ	β	$R(\text{m}\Omega)$	$k(\text{m}\Omega \text{ lb.})$
53°	.96 $\pm$.01	2.8 $\pm$.1	71 $\pm$ 2
127°	.05 $\pm$.02	3.3 $\pm$.1	100 $\pm$ 10

Table D-2 Least Squares Fit Parameters

Figure D-6 summarizes the results. The solid curves represent equation D-4 evaluated for the values of β given in table D-2. (For reference purposes, the limiting behaviour of equation D-4 is shown by the dashed curves.) The experimental results have been reduced, as shown on the graph, by the appropriate best fit values of $R(\phi)$ and $k(\phi)$.

Figure D-6: Comparison of Parallel film theory with Crossed Nichrome Strip "Junction" data.

It is evident that this simple model succeeds quite well in interpreting the nichrome strip "junction" measurements. Particularly satisfying are the results (1) that the effective strip resistance R is fairly consistent in the two configurations, being reasonably close to the value $1.9 \text{ m}\Omega/\text{sq.}$ calculated for the parallel film geometry, and (2) that only a small change in β from the $\phi = 0$ and $\phi = 180^\circ$ configuration values is required to fit the 53° and 127° data.

The important conclusion to be drawn from these studies is that the nichrome strip "junction" can simulate the behaviour of thin film junctions and clearly demonstrate the interaction of the effective film and tunneling resistances in determining the sign and magnitude of the measured junction resistance for a given terminal configuration. The angular dependence of the junction resistance is merely another manifestation of the same effect.

E. Angular Dependence of the Thin Film Junction Resistance

This section demonstrates that the observed ϕ dependence of the magnitude of the thin film junction resistance (see Table D-1) may be understood from the nichrome strip simulations.

A convenient measure of the asymmetry in the measured junction resistances due to angle dependence is the quantity ρ which is defined in section B and appears in the right hand column of Table D-1. (In the limit of $R \ll R_T$, the films are essentially equipotentials so that R_j is expected to be independent of the choice of current and voltage terminals and ρ should tend to 1 as it does for 16 of the specimens of Table D-1. In the limit of $R \gg R_T$, the continuous sheet approximation, it is expected that ρ will be different from 1, its precise value depending on the junction geometry and tunneling conductance.)

By drawing smooth curves through the data obtained from two different nichrome strip "junctions" (samples a and b), it is possible to find ρ as a function of the load--see figure D-7. Three regions are in evidence, depending on the sign of $R_j(\phi)$. The significant part of this result is that, as set out in Table D-3, all the various asymmetries observed in actual thin film junctions can be simulated by the nichrome strip model.

It may be concluded therefore that, as in the nichrome strip

Figure D-7: ρ vs Load for Nichrome Strip "Junctions"

"junctions", the angular dependence of R_j in thin film junctions is simply

Region	Tunnel Junctions Exhibiting Same Asymmetry
I	66-17,-21,-29
II	66-15,-19
III	66-3,-11,-18

Table D-3: Junction Resistance Asymmetry Observed.

another demonstration of the interrelation between film and tunneling resistances.

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