DEVELOPMENT OF EMPIRICAL RIB PILLAR DESIGN CRITERION FOR OPEN STOPE MINING

By

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We accept this thesis as conforming to the required standard

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The design of open stope rib pillars has been done using many empirical methods, but none of the methods has been verified with a design survey. This thesis uses data collected in the "Integrated Mine Design Study" to develop an empirical rib pillar design method for open stope mining. The method is called the "pillar stability graph".

The design variables in the method are: the compressive strength of the intact pillar material, the average pillar load determined by numerical modelling, the pillar width and the pillar height. The graph has been refined with the use of more than 80 literature case histories of hard rock pillars in room and pillar mining.

The pillar stability graph and the pillar data base are used to examine the applicability of empirical methods commonly used in open stope rib pillar design. The investigation found the pillar strength curves developed by Hoek and Brown (1980) may be useful under some conditions for the design of open stope rib pillars but formulas by Hedley (1972), Obert and Duvall (1967) and Bieniawski (1983) are not applicable.

Guidelines, using the pillar stability graph method, are proposed for the design of permanent open stope rib pillars, stable temporary open stope rib pillars, and failing temporary open stope rib pillars.
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CHAPTER 1
INTRODUCTION

Open stope mining has been practiced in Canada since the 1930's. The design of open stope mines is centered around determining the largest stable stopes and the optimum size for pillars. Systematic methods to design open stopes and their separating "rib" pillars have not been confirmed in typical Canadian open stope mining conditions. In 1986, the Natural Sciences and Engineering Research Council (NSERC), Noranda Research and Falconbridge Limited agreed to sponsor the "Integrated Mine Design Project", a research project at the University of British Columbia under the supervision of Dr. H.D.S. Miller. The goal of the study was to investigate open stope mine design methods by confirming the validity of existing stope and rib pillar design methods or by developing new empirical methods. This thesis is a compilation and analysis of the information and data collected for the design of rib pillars in open stope mining.

The first section of this chapter is a summary of the contents of the thesis. The remainder of the chapter will introduce the problem of designing open stope rib pillars by describing open stope mining, and discussing the role of rib pillars in open stope mining.

1.1 Contents of the Thesis
This study begins by describing open stope mining and the role of rib pillars in open stope mining. In Chapter 2, the characteristics of progressive pillar failure are discussed and the factors that influence rib pillar stability are identified. Chapter 3 contains a review of the empirical and numerical design methods used for open stope rib pillars. The rib pillar data collected in the Integrated Mine Design Project is presented in Chapter 4. Chapter 5 discusses the use of boundary element numerical methods to determine the average stress in open stope rib pillars. The load induced on all of the data base pillars is estimated in this section. Chapter 6 shows the development of a new empirical pillar design method called the "Pillar Stability Graph", based on graphical analysis of the rib pillar data and data from literature. It also compares the new method with existing empirical design methods for open stope rib pillars. Chapter 7 briefly discusses the application of the pillar stability graph for the design of open stope rib pillars. A summary and conclusion of the thesis is found in Chapter 8.

1.2 Open Stope Mining

Open stope mining is a general name used to describe a highly varied mining method. There are many important features that make up the method, and many variations on each of the features. The following discussion of the definition, applicability, and description of open stope mining is taken largely from an unpublished paper on open stope mining methods,

1.2.1 Definition of Open Stoping

Three characteristics, common to all open stoping methods, make it distinct from other mining methods.

i) Open stoping is a non entry mining method. Once stope production has started, all activities requiring miners are done from the periphery of the stope. The open stope does not need to be entered and at no time are miners exposed to the production face.

ii) It is generally a naturally supported mining method (although some artificial support is occasionally used). Naturally supported means that displacement and deformation of the rock mass is limited to elastic orders of magnitude. The underground structures created are designed to be stable and self-supporting (in opposition to caving methods). Mining is done in a manner to ensure that unstable release of energy due to mining does not occur (from Brady 1981).

iii) Stopes are opened to their full dimensions before a stabilizing fill is introduced.

These three characteristics distinguish open stoping from all other underground methods. Cut and fill, longwall, room and pillar and shrinkage are all entry methods that require workers to enter the production face of the stope. Block caving and
sublevel caving induce large, unstable movements of rock and include the continual dissipation of energy as mining proceeds, so they can not be considered naturally supported methods. Methods such as AVOCA, which introduces fill during extraction to prevent stope instability, or shrinkage stoping, which keeps the stope full of broken ore, are excluded from open stoping because the stope is never fully open.

1.2.2 Applicability of Open Stoping

There are some orebody and geological limitations to the application of open stoping. Modifications of open stoping can be made to mine a wide variety of orebodies, but some conditions present difficult problems.

Open stoping is best suited to orebodies that are steep dipping. Stopes in the orebody must dip sufficiently above the angle of repose of the broken ore (above 50° to 55°) to permit gravity flow of the ore to the stope bottom. Open stoping can be successful in shallow dipping orebodies (approximately less than 30°) but the orebody must be quite thick (greater than about 15 metres in true thickness). If an orebody is not steep dipping or thick and flat, open stoping can not be used.

For mining a steep dipping orebody, the orebody outline must be fairly regular and the orebody needs to be greater than about 5 metres in width. Irregular orebodies are difficult to delineate and mine. Generally, at widths less than 5 metres, wall rock dilution due to drill hole deviation and blast damage
becomes too great to use open stoping effectively.

The rock mass strength of the orebody and the surrounding country rock is very important in open stoping. The stronger the rock, the larger the stopes can be made, and consequently, the more productive the method will be. At the least, fair rock mass strength is needed in the ore and wall rock to guarantee that the open stopes will be naturally supporting.

A final restriction on open stoping is the orebody must be reasonably large. This is necessary to get a few working faces (because open stoping is often a cyclical method), to take advantage of the large scale of the mining method, and to justify the cost of the development associated with open stope mining.

1.2.3 Description of Typical Open Stope Mining Methods

Open stoping methods are so dependent on the orebody shape, size and orientation that no two mines are exactly the same. Most open stope mining activities can be generalized into two basic stages: pre-mining development and production. Open stoping has a large amount of pre-mining development. Typical development usually includes:

- sublevel accesses such as ramps, man-way raises (figure 1, note A), and sublevel drifts (figures 1 and 2, note B),
- a drilling horizon which includes stope access drifts (figures 1 and 2, note C) and drill drives (figure 1, note D) or overcuts (figure 2, note E),
FIGURE 1. The elements of an idealized longitudinal longhole open stoping method showing the blasting, mucking and backfilling operations (after Hudyma 1988a).
FIGURE 2. The elements of an idealized transverse blasthole open stoping method showing the drilling, blasting, mucking and backfilling operations (after Hudyma 1988a).
- a mucking horizon, which may include:
  - a footwall haulage drift (figures 1 and 2, note F),
  - stope access undercuts (figure 2, note G) or drawpoints (figures 1 and 2, note H),
  - stope undercut scrams, V-cuts or collection cones (figure 1, note I),
  - the opening of a slot raise (figure 2, note J) by staging, drop raising, Alimak raise climber or by raise borer.

Production mining involves:
- using parallel drill holes to slash ore into the slot raise to form an expansion slot which is opened the full width of the stope,
- drilling production holes in parallel (figure 2, note K) or ring patterns (figure 1, note L). The holes are used to blast ore into the expansion slot.

Generally, the expansion slot is opened at one end of the stope and ore is slashed into the slot causing a gradual retreat of the production face. This retreat may be longitudinal (along the orebody, as in figure 1) or transverse (across the orebody, as in figure 2).

As a stope is blasted, ore is removed from the bottom of the stope. The ore is almost always removed with the use of trackless load-haul-dump equipment, and taken to an orepass system. There are a few mines using slusher/scaper equipment or continuous mining equipment to move the muck to an orepass, but these operations are quite rare. The ore pass system moves
the muck to a central collection point for transport out of the mine. When the stope is completely blasted, it may be filled with waste rock or classified mill tailings to permit recovery of pillars left between stopes (both figures 1 and 2 show the filling of stopes).

1.3 Role of Rib Pillars in Open Stope Mining

The most economic open stope method involves mining the entire orebody in one longitudinal stope. If the use of this full lens mining creates the potential for serious stope instability, major stope support such as rib pillars and backfill will likely be needed. The role of rib pillars in open stope mining is to provide stability to a mining block by limiting rock mass displacements and restricting the exposure of the rock mass in the stope back and walls.

In the past, if full lens mining was not possible, pillars had to be left to maintain overall mine stability. Recently, improvements in mining technology have caused a trend towards the sequencing of extraction so that pillars are never created, even in very large orebodies. However, of the 34 Canadian open stope mines investigated in this study (from 1986-1988), 27 used rib pillars to separate stopes in the orebody. These pillars varied in size from about 2000 m$^3$ up to 150,000 m$^3$, depending on factors such as: the orebody geometry, the type of open stoping method, and the mining sequence. The dimensions of the pillars in the data base are given in Chapter 4.1 (Table 5, page 70).
It is important that rib pillars perform their designed role. Mines using rib pillars may leave as much as half of the orebody reserves in temporary pillars. The consequences of poor pillar design can seriously affect the recovery of this ore. A pillar that does not perform its intended role may cause:

- excessive stope or pillar sloughing,
- difficult and expensive pillar recovery,
- loss of pillar access,
- the need for remedial measures such as development rehabilitation or artificial support,
- low productivity,
- or the loss of ore reserves.
CHAPTER 2
RIB PILLAR FAILURE

The first step in quantifying the variables that influence pillar stability is to describe pillar failure. While open stope rib pillar failure has not been deeply researched, some of the principles of failure in intact hard rock, soft rock and rock masses are applicable to open stope rib pillars. The objective of this chapter is to briefly discuss the characteristics of pillar instability and compare them to observations and documentation of failure in open stope rib pillars. Using these ideas about pillar failure, the factors that influence the stability of open stope pillars will be identified.

2.1 Failure Mechanisms and Characteristics

Rib pillar failure can be broken into two basic modes: progressive (stable) failure and bursting (unstable) failure. Progressive failure refers to gradual deterioration of a rock mass in a slow, non-violent manner. Bursting failure is the violent release of energy causing the instantaneous fracture of rock. Although the conditions associated with each may be very different, both modes of failure create serious difficulties for mining.

This thesis will describe and quantify progressive failure. Progressive failure is related to the in situ rock properties of the pillar and mine, and the static underground stress field.
Both of these factors are quantifiable with reasonable accuracy. Bursting failure is also related to in situ rock properties. However, it is also dependent upon factors such as local stress concentration, the energy released due to the mining and changes in the dynamic stress field. It is not intended to investigate these factors as they are not quantifiable with technology and budget available for this study. For this reason, the thesis will not attempt to describe or quantify unstable failure.

Although rib pillar failure in open stope mining is not uncommon, it is rarely well documented. A reason for the lack of documentation is that visual observation and monitoring of pillars is difficult in open stope mining and there is no universal method to describe the characteristics and effects of rib pillar failure. Another potential reason for the absence of documentation is that the failure of rib pillars is often not considered an immediate problem, especially with open stope mining methods using backfill. In the primary mining, rib pillar failure often does not cause operational problems that are serious enough to warrant changing the mining sequence. Consequently, the operational effects of rib pillar failure may not be experienced until pillar mining starts. This failure often results in low productivity, waste dilution, higher mining costs and possibly lost ore.

Several signs indicating pillar stability problems in open stope ribs have been identified. These signs of pillar distress
are:

- cracking and spalling of rock in rib pillar development and raises,
- audible noise heard in the pillars or microseismic events located with monitoring systems,
- deformed or plugged drill holes causing drill rods to be stuck and causing problems in loading holes,
- overdraw from primary stopes with the "free" muck being unblasted, oversize material from pillar walls,
- stress redistribution from rib pillars affecting nearby pillars and hanging wall and footwall drifts and raises,
- hourglassing and cracking of pillars seen from development,
- major displacements and changes in stress shown by instrumented monitoring systems such as extensometers, stress meters and sloughmeters.

No single sign necessarily denotes pillar failure, but these signs are commonly reported during pillar failure.

Progressive pillar failure is a gradual process. Problems may be minor at first, but get worse with time. Pillar damage and deterioration can occur through intact rock and along existing structural discontinuities. Although purely structurally controlled failures occur in pillars, the overall influence of geological structure in open stope pillars is not predominant. Stress, pillar loading and development of stress related fractures appears to be predominant. Consequently, the
discussion of rib pillar failure will focus on rock fracturing, pillar loading, and the subsequent loss of pillar load bearing ability.

2.1.1 Rock Fracturing

Rock fracturing is a primary indicator of pillar failure and is the ultimate reason for the loss of load bearing ability and pillar disintegration. Brady and Brown (1985) define fracturing as "... the formation of planes of separation in the rock material. It involves the breaking of bonds to form new surfaces." Fracturing generally starts at the pillar walls where the rock mass is weakest due to the lack of confinement of pillar material. As failure progresses, fractures propagate and develop in the central parts of the pillar and the size and intensity of existing fractures increases.

Krauland and Soder (1987) defined 6 stages to classify pillar failure based on visual observation of pillar fracturing in room and pillar mines. The stages defined are:

"0) No fractures.
1) Slight spalling of pillar corners and pillar walls, with short fracture lengths in relation to pillar height, subparallel to pillar walls.
2) One or a few fractures near surface, distinct spalling.
3) Fractures appear also in central parts of the pillar.
4) One or a few fractures occur through central parts of the pillar, dividing it into two or several parts, with rock falls from the pillar. Fractures may be parallel to pillar walls or diagonal, indicating emergence of an hour-glass-shaped pillar.
5) Disintegration of the pillar. Major blocks fall out and/or the pillar is cut off by well defined fractures. Alternatively, a well developed hour-glass shape may emerge, with central parts completely crushed."
Krauland and Soder also noted that although the appearance of pillar failure was highly variable due to geological inhomogeneities, the basic pattern of failure propagation remained constant for progressive failure. This is perhaps the best documentation and definition of an actual mine pillar failure mechanism. Use of the Krauland and Soder observational approach to classify open stope pillars is not generally possible due to the lack of visual access. However, the mode of failure described above is similar to that seen by the author in several open stope mines and is documented in a few open stope mines (Falmagne 1986; Bray 1967) where sufficient visual access was available. The only observation of Krauland and Soder that this author has not seen in open stope mining is the division of pillars into distinct regions due to fracturing. This part of the mechanism is not likely to occur in open stope pillars. The potential for a fracture to completely sever a pillar is much lower in open stope mining than in room and pillar mining due to the larger scale of open stope pillars. Fractures would have to be very continuous, flat and planar to transect and divide open stope pillars.

From personal observation and literature descriptions, some of the most common types of fracturing found in mine pillars are:

i) surface fracturing and spalling (figure 3a) is usually the first location of fracture development (Krauland and Soder 1987) and often a result of lack of pillar wall confinement
FIGURE 3a. Parallel fracturing and spalling due to a lack of confinement at the pillar walls (after Brady and Brown 1985).

FIGURE 3b. Internal splitting and axial cracking of a pillar due to deformable pillar layers or the propagation of parallel wall fractures (after Brady and Brown 1985).

FIGURE 3c. Diagonal crushing fractures may occur in confined or massive pillars (after Brady and Brown 1985).
(Fairhurst and Cook 1966).

ii) internal axial cracking (figure 3b) may be caused by highly deformable layers between the pillar and the adjacent wall rock (Brady and Brown 1985) or may be parallel surface fractures that propagate or develop in the centre of the pillar (Agapito 1974).

iii) diagonal crushing fractures (figure 3c) are often found in confined or massive pillars (Coates 1981).

2.1.2 Pillar Load-deformation Curve

Pillar loading can be hypothetically described using a load-deformation (stress-strain) curve (see figure 4). As a pillar is loaded, it compresses according to the line OA. At a load $P_{\text{max}}$, the maximum pillar load bearing capacity is reached. Beyond this point, post-failure deformation of the pillar will occur but at a reduced load. This peak load will be taken as the point of failure in a pillar. Bieniawski (1987) states, "... the ultimate strength is a state at which the rock specimen or the pillar changes from a gradually increasing load-bearing capacity to a constant or gradually decreasing load-bearing capacity."

Determining the actual load-deformation characteristics of a hard rock mine pillar is not possible. Curves for small hard rock laboratory specimens are easily determined and curves for small in situ coal pillars have been developed (Wagner 1974; Bieniawski and Van Heerden 1975), but it is not experimentally
FIGURE 4. A hypothetical load-deformation curve can be used to describe the stress-strain characteristics of a pillar. The pillar exhibits linear elastic deformation (along line OA) until the maximum load is reached ($P_{\text{max}}$). Pillar deformation continues (along line AB), but with a decreasing load bearing capacity (after Starfield and Fairhurst 1968).
practical to conduct load-deformation tests on large samples of jointed rock (Brady 1977). While this leaves the load-deformation curve of a hard rock mine pillar as a theoretical concept, it is a convenient method to describe pillar failure and the loss of pillar load bearing capacity.

2.1.3 Loss of Load Bearing Capacity

Ultimately, rock fracturing is the main reason for loss of pillar load bearing capacity. However, the onset of fracturing does not necessarily signify that the pillar has failed. Agapito (1974), in his study of oil shale pillars, found that fracturing started as minor spalling in the pillar perimeter and occurred at stress levels well below the ultimate load capacity of a pillar. He also noted that as fracturing occurred in the outer shell of the pillar, monitoring showed that stress concentrations built up in the pillar core. Wagner (1974) monitored the in situ stress distribution in more than 30 underground coal pillars using a series of hydraulic jacks. He found that at several stages of compression, the perimeter of the pillar carried relatively little stress compared to the central core of the pillar (figure 5). He noted that most of the load bearing capacity of a pillar is found in the core of the pillar and is largely dependent on the confinement provided by the pillar shell.

After failure of the pillar (due to serious internal and surface fracturing), Wagner (1974) found that a confined pillar
Wagner (1974) did a series of in situ load-deformation tests on coal pillars using hydraulic jacks. For this case, 25 jacks were put in a 5X5 pattern in a square pillar. The graph on the top shows the load-deformation characteristics of the pillar in general. The oblique diagrams give the relative load on each of the 25 jacks at four stages of pillar compression. The diagrams show that with increasing compression and increasing average pillar stress, the core of the pillar carries an increasing percentage of the load, while the unconfined periphery of the pillar carries less load. Diagram four shows that the pillar core carries a significant load despite the fact that the pillar is losing its overall load bearing capacity (redrawn from Wagner 1974).
core had a considerable load bearing capacity. Krauland and Soder (1987) wrote that loss of load bearing capacity in the post failure range of pillar loading depends largely upon the slenderness of the pillars and the presence of fill. This is also supported by the laboratory testing of rock specimens in "stiff-testing" machines. Starfield and Fairhurst (1968) demonstrated that if confining pressure on a sample is increased, the peak load capacity increases and the post failure load bearing capacity is greatly enhanced (see figure 6).

The loss of load bearing capacity in open stope rib pillars is also highly dependent on confinement of the pillar core. However, in open stope mining pillar walls can be very large. Once progressive failure starts, the fractured wall material will peel off, preventing confinement of the pillar core, and finally resulting in complete pillar disintegration. There are methods to prevent fractured wall material from becoming detached from the pillar. These methods include the use of backfill, installation of artificial support such as cable bolts, and leaving open stopes full of broken ore as long as possible to provide some confinement to the pillar walls. The author has seen several examples of failed rib pillars with a considerable load bearing capacity. In these cases, the pillar core had remained confined because the fractured pillar material was confined by backfill before it had the opportunity to slough from the pillar walls.
FIGURE 6. The stress-strain curves for laboratory specimens loaded under increasing confining pressures show an increase in peak load and an increase in the post-peak load bearing capacity (after Starfield and Fairhurst 1968).
2.2 Significant Variables in Open Stope Pillar Stability

Based on the failure characteristics described above, there are several variables that could be important in the design of rib pillars. This section will describe the variables and their potential influence.

2.2.1 Intact Rock Strength

With rock fracturing playing a large role in the stability of pillars, the resistance of the pillar material to fracturing and crushing is an important factor in pillar strength. The most common index for comparing the strength of different rock types is the uniaxial compressive test. The uniaxial compressive strength (UCS) is the maximum load that a standardized diameter drill core can sustain under uniaxial loading conditions. The UCS is variable upon specimen size, so the sample diameter is standardized to about 54 mm (NX size drill core). Further information about the uniaxial test can be found in a report by an International Commission on standardization of laboratory tests (ISRM Commission 1979).

2.2.2 Pillar Load

Pillar load is a primary factor in pillar deformation, rock fracturing and pillar failure. The distribution of stress in a pillar may have a significant effect on the performance and stability of the pillar. However, there is no conclusive method to determine stress in a pillar and there is no single value
that can used to describe the complete loading condition of a pillar.

The state of stress in a pillar varies upon the stress applied to the pillar as well as the location inside the pillar. The stress applied to a pillar varies on the pre-mining stress field and the size and location of stopes, underground workings and other pillars. The stress inside the pillar is dependent upon areas of weakness such as geological discontinuities, the proximity of excavations and the fracturing in the pillar. With these points kept in mind, determining the distribution of stress in a pillar with a high degree of precision is not possible.

For this thesis, it was necessary to find a value to represent the load on a pillar. The load was taken as the average stress found at several points along the pillar mid-height centerline, determined using numerical modelling techniques. The reason is that this location has the highest normal stresses in the pillar, and is frequently observed as the first area of failure. This choice of stress analysis location will be discussed in more detail in Chapter 5.2.2.

2.2.3 Pillar Shape

Chapter 2.1.3 described the role of confinement in pillar stability and the load bearing capacity. Pillar shape has a huge influence on confinement of the pillar core. It affects:

- the load-convergence characteristics of pillars at failure
(Hudson et al. 1971; Starfield and Fairhurst 1968),
- the post-failure deformation modulus of pillars (Hudson et al. 1971; Wagner 1974),
- the stress distribution in a pillar (Starfield and Fairhurst 1968; Wagner 1974),
- and the effect of geological structure and fracturing on pillar stiffness and failure (Sarkka 1984).
This confirms that pillar shape as a significant variable in pillar stability.

2.2.4 Structural Discontinuities in Pillars

The effect of geological structure on rib pillars depends upon whether the structure involves major discontinuities such as faults and shear zones or minor discontinuities like joint sets. Pillars intersected by a major structure must be analyzed based on the specific situation. The orientation and shear strength of the major structure will play a dominant role in stability. However, in open stoping, intersection of a major structure is not a common problem and design of such pillars is an exception rather than a regular occurrence. When possible, rib pillars are located to avoid intersection by major geological discontinuities.

Less prominent discontinuities such as jointing and local fracturing, are a much more common problem in pillar design. The influence of minor discontinuities on rib pillars depends upon the orientation, continuity, frequency and shear strength
of the structures. At the pillar central core, the effect of minor discontinuities on pillar stability is small because the triaxial state of confinement prevents rock movement along the joints. Geological discontinuities have a more significant effect on instability in unconfined regions of pillars. Allcott and Archibald (1981), Page and Brennan (1981), and Von Kimmelmann (1984) mention structurally controlled wedge failures from pillar walls. One would expect to find little or no confinement of the rock near pillar walls. Consequently, the influence of structure is best accounted for using wall stability analyses. An excellent method for wall stability analysis is described by Potvin et al. (1988a). The method quantifies the influence of geological structure, mining induced stress, and stope dimensions to predict the stability of each surface of an open stope. When the analysis predicts a stable pillar wall, the effect of minor structure on the stability of unfractured rib pillars will be small.

2.2.5 Effect of Pillar Volume

Pillars are made of blocks of intact rock separated by natural and mining induced discontinuities. So the influence of pillar volume on stability is really a function of two variables: the volume effect on the strength of intact rock, and the influence of the number of structural defects in the pillar.

Laboratory compressive testing of small samples has shown an
influence of specimen size on the compressive strength of intact rock (see figure 7). However, testing of large intact rock specimens has found that above a "critical" volume, the strength does not decrease significantly (see figure 8). This concept of asymptotic specimen strength is reported by Bieniawski (1975), Herget et al. (1984), and Pratt et al. (1972). These authors found the critical volume to be less than one cubic metre. With the volume of blocks in open stope pillars usually being much larger than this critical volume, there is a very limited influence of the volume effect of intact rock.

The number of structural discontinuities in a pillar will depend upon the volume of the pillar. Hoek and Brown (1980) suggest that this influence can be quantified through the use of rock mass classification methods. Hardy and Agapito (1977), Stephansson (1985), and other authors have suggested that correction factors to account for pillar volume be used in pillar strength determination. Both of these ideas will be investigated with open stope rib pillar case histories in Chapter 6.1.3.

2.2.6 Effect of Backfill

The use of fill is very important in current open stope mining methods. A survey by the Ontario Ministry of Labour (Campbell 1987) found that almost all Ontario open stope mines use cemented fill to aid in pillar recovery. The general purpose of fill is used to provide overall mine stability,
FIGURE 7. There is a very large influence of specimen size on the strength of intact rock, for small specimen diameters (after Hoek and Brown 1980).

FIGURE 8. Strength testing of samples of increasing specimen length shows a decreasing influence of size. Beyond a "critical" length, there is no significant decrease in specimen strength. This critical size is about 1 metre (after Bieniawski and Van Heerden 1975).
especially in stope hanging walls and footwalls, by limiting spans to stable dimensions and to permit a high ratio of extraction of the orebody with potentially minimal dilution.

The role of fill in pillar failure is much less dramatic. Singh (1976) finds that fill:

- provides lateral support to pillars to inhibit spalling and prevent collapse,
- acts as a retaining media to contain fractured rock, thereby retarding the development of failure in surrounding rock,
- and reduces energy release rates allowing rock to fail in a non-violent manner.

None of these effects of filling has a large influence on the rock fracturing mode of failure described above in Chapter 2.1.1. Fill does provide restraint and confinement to fractured rock to prevent sloughing of pillar material and consequently enhances the post-failure load bearing capacity of pillars. Thomas (1979) supports Singh's comments by writing that fill is not likely to provide stope wall support before unrealistic fill deformation (approximately 20%) has occurred. He finds that fill is most beneficial to mining when it provides rock confinement causing the rock mass to support itself.

Consolidated and cemented fills have been found more effective at aiding in underground stability (Bharti 1987). However, the main purpose of consolidated fills is to be self-supporting and free-standing during pillar recovery operations.
failure due to rock fracturing. It does give support to failed pillars to maintain their integrity and some load bearing capacity. This aids in overall mine stability and simplifies pillar recovery operations.

2.2.7 Effect of Blasting

Blasting practices are very important in the success of any mining method. Poor blasting practices can turn a stable and efficient design into a very inefficient design. Some of the effects of poor blasting in open stope mining include: poor fragmentation, overbreak beyond stope limits, need for frequent post-blast clean-up and development rehabilitation, development of blast induced fractures in the rock mass, and rock mass disturbance and instability in stope walls and pillars due to excessive vibrations.

Quantifying poor blasting in an empirical method is very difficult. There is no clear definition of poor blasting, and the consequences are highly varied. The best solution in describing blasting is to list some of the practices used to minimize the effect of blasting. These practices are often referred to as control blasting, and include: minimizing the charge weight per delay; using charge decking, decoupling, and/or low density explosives; using efficient hole location and blast sequencing; and blasting to a free face.

Although the significance of blasting practices is very great in mining, there are no criterion to quantify the effects
of blasting on mining. Consequently, blasting will not be discussed as a design variable in this thesis.

2.3 Chapter Summary

Progressive failure of open stope rib pillars is difficult to observe due to lack of visual access. Several indirect signs of pillar distress have been documented. These signs are directly associated with rock fracturing in the pillar. Fracturing generally starts at the pillar walls and propagates or develops in the pillar core as pillar deterioration progresses. Fractured rock loses some or all of its load bearing capacity, depending on the confinement of the material.

Pillar failure can be described as the state when a pillar changes from having an increasing load bearing capacity to a constant or decreasing load bearing capacity. Failure can hypothetically be described using a pillar load-deformation curve. The degree of confinement of a pillar has a large influence on the shape of that curve.

Open stope rib pillar design should be based on the conditions that influence pillar failure and load bearing capacity. These conditions are rock fracturing and pillar confinement. The conditions may be influenced by a number of factors, including: the intact strength of the pillar material, the pillar load, the shape of the pillar, the presence of structural discontinuities, and the volume of the pillar.
CHAPTER 3

REVIEW OF PILLAR DESIGN METHODS

There are two general approaches to current rib pillar design: empirical methods, and numerical methods. Empirical design is based on observation of case histories and previous experience in similar geotechnical conditions. Numerical design is largely based on measured parameters and material properties. However, there is not a clear division between the two approaches. Some numerical procedures are occasionally used in empirical design and some experience and observational information is used in numerical techniques.

This chapter will discuss the two approaches as they are applied to hard rock pillar design. It will briefly describe the background fundamentals in each method, and give a short discussion of their respective advantages, disadvantages and limitations.

3.1 Empirical Design Methods

Empirical design methods are characterized by the fact that they consider a pillar as one unit. It is assumed that there is no variation in stability within a pillar. The stability of that pillar is interpreted based on three variables:

i) pillar load,

ii) pillar strength,

iii) and safety factor.
Methods of calculating or determining each of these parameters are based upon quantifying underground observations and past experience. Typically, pillar load is determined using empirical rules of thumb or numerical tools. Pillar strength and an appropriate safety factor are calibrated with case histories and/or laboratory experiments.

The safety factor is defined as:

\[ S.F. = \frac{\text{pillar strength}}{\text{pillar load}} \]

It has three basic purposes:

- to expand the load and strength determination methods to different mining conditions,
- to make a design more conservative
- and to account for the inaccuracy in the input parameters.

For instance, a pillar in an entry mining method would be designed more conservatively than a pillar in a non-entry mining method. In order to use the same strength and load determination procedures for the design of both situations, a higher safety factor would be designed in the entry method because the degree of instability acceptable is less. The choice of safety factor is usually based on experience with the specific design method.

The following sub-sections will summarize the techniques developed for calculating pillar strength and pillar load and will list the safety factors suggested for these design procedures. Because there are a large number of different techniques used to determine pillar load and strength, emphasis
will be placed on those methods used for hard rock design. A more complete discussion of the empirical design methods is documented by Potvin (1985).

3.1.1 Pillar Strength Determination

There are many factors that may influence the strength of a mine pillar. These factors include:

- size and shape of the pillar,
- volume of the pillar,
- resistance of intact pillar material to crushing,
- presence of discontinuities,
- strength and orientation of the discontinuities,
- confinement and triaxial strength of the pillar rock mass,
- and the presence of groundwater.

The number of potentially significant variables makes pillar strength very difficult to determine analytically. Some of these variables are not significant under selected mining conditions. For such situations, pillar strength may be estimated empirically. The most commonly used empirical pillar strength methods in hard rock mining are:

- Salamon's formula,
- Hedley's formula,
- Obert and Duvall formula,
- and the Hoek and Brown pillar strength curves.

The first three of these methods are variations of the empirical
strength formulas developed for underground coal mines. Consequently, a brief discussion of the empirical coal formulas is helpful, although they see very limited use in hard rock pillar design.

3.1.1.1 Empirical Strength Formulas

A major area of pillar design research has been in underground coal mining. A basic premise of this work was that full size pillar strength could be determined by extrapolating the results from laboratory testing of coal specimens. Two forms of the empirical strength equation were developed:

- the size effect formula,
- and the shape effect formula.

A) The size effect formula is defined as:

\[ \sigma_p = K \times \left( \frac{w^a}{h^b} \right) \quad a \neq b \]

where:

- \( \sigma_p \) = pillar strength (psi),
- \( K \) = uniaxial compressive strength of one cubic foot of pillar material,
- \( w \) = pillar width,
- \( h \) = pillar height,
- \( a, b \) = unequal empirically defined constants.

This formula is based on the fact that rock strength is dependent on the size of the sample. This is due to the presence of discontinuities (such as joints, foliations, bedding, blast fractures, and mineralogy). As rock samples of a
constant shape increase in size, the strength of the sample decreases. This size effect is taken into account by giving a different weighting to the coefficients for \( w \) and \( h \) in a shape effect formula. Table 1 gives the constants \( a \) and \( b \) proposed by different authors.

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>( a )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Streat (1954)</td>
<td>0.5</td>
<td>1.00</td>
</tr>
<tr>
<td>Holland-Gaddy (1962)</td>
<td>0.5</td>
<td>1.00</td>
</tr>
<tr>
<td>Greenwald et al. (1939)</td>
<td>0.5</td>
<td>0.833</td>
</tr>
<tr>
<td>Hedley and Grant (1972)</td>
<td>0.5</td>
<td>0.75</td>
</tr>
<tr>
<td>Salamon and Munro (1967)</td>
<td>0.46</td>
<td>0.66</td>
</tr>
<tr>
<td>Bieniawski (1968)</td>
<td>0.16</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Table 1 (after Babcock, Morgan and Haramy 1981).

B) The shape effect formula, which is defined as:

\[
\sigma_P = K \times \left[ A + B \times \left( \frac{w}{h} \right) \right]
\]

or

\[
\sigma_B = K \times \left( \frac{w^a}{h^b} \right) \quad \text{a = b}
\]

where:

- \( \sigma_P \) = pillar strength (psi),
- \( K \) = uniaxial compressive strength of one cubic foot of pillar material,
- \( w \) = pillar width,
- \( h \) = pillar height,
A,B,a,b = empirically defined constants.

The shape effect formula denotes a difference in strength for pillars of different shape but equal cross-sectional area. The greater the pillar width to pillar height ratio, the greater the pillar strength. A change in mode of failure is one apparent cause of the shape effect on pillar strength. Slender pillars tend to fail along structural discontinuities in the rock mass. While for wide pillars, failure is likely to be caused by crushing of the pillar material. Tables 2 and 3 give the constants a,b,A,B proposed by different authors.

| Constants a and b used in the shape effect formula:  
| $\sigma_p = K \cdot \frac{w^a}{h^b}$ |
|---|---|---|
| SOURCE | a | b |
| Zern (1926) | 0.5 | 0.5 |
| Hazen and Artier (1976) | 0.5 | 0.5 |
| Holland (1956) | 0.5 | 0.5 |
| Morrison et al. | 0.5 | 0.5 |

Table 2 (after Babcock, Morgan and Haramy 1981).

| Constants A and B used in the shape effect formula:  
| $\sigma_p = K \cdot [A + B \cdot \frac{w}{h}]$ |
|---|---|---|
| SOURCE | A | B | w / h |
| Bunting (1911) | 0.700 | 0.300 | 0.5 - 1.0 |
| Obert et al. (1960) | 0.778 | 0.222 | 0.5 - 2.0 |
| Bieniawski (1968) | 0.556 | 0.444 | 1.0 - 3.1 |
| Van Heerden (1973) | 0.704 | 0.296 | 1.14 - 3.4 |
| Sorensen and Pariseau (1978) | 0.693 | 0.307 | 0.5 - 2.0 |

Table 3 (after Babcock, Morgan and Haramy 1981).
The constants and coefficients in each of these formulas were based on pillar case histories and laboratory testing of scale pillars. Three of the most prominent empirical pillar design studies and surveys provided formulas commonly used in hard rock pillar strength determination.

3.1.1.2 Salamon's Formula

In 1967, Salamon published a survey of stable and failed square coal pillars in South African mines. The study investigated 98 stable and 27 collapsed pillar areas. Using a size effect formula, and assuming the mean safety factor for all the failed cases was 1.0, the coefficients $K$, $a$ and $b$ were calibrated. This gave the formula:

$$\text{strength} = K \times w^{0.46} / h^{0.66}$$

where:

- $\text{strength} = \text{pillar strength (psi)},$
- $K = 1320 = \text{strength of one cubic foot of pillar material},$
- $w = \text{pillar width (feet)},$
- $h = \text{pillar height (feet)}.$

The complete database is commonly displayed in a histogram (see figure 9). To determine a suitable safety factor for this strength formula in entry mining methods, Salamon averaged the safety factor of the most dense concentration of 50% of the stable pillars to get an average of 1.57 (see figure 9). He felt that this safety factor was adequately conservative to
FIGURE 9. Histogram of the safety factors for stable and failed pillar case histories in South African bord and pillar coal mining. The range of safety factors for the most dense concentration of 50% of the stable cases is between 1.31 and 1.88. Salamon chose the mean of this range, 1.57, as adequately conservative to design stable, permanent pillars in room and pillar coal mining (after Salamon 1967).
ensure stability for pillars in room and pillar coal mines.

Despite the fact that the study is based on square pillars in bord and pillar coal mining in South Africa, Salamon's formula has been used for the design of hard rock open stope rib pillars. The factor to account for the strength of the pillar material is adjusted to the strength of one cubic foot of intact hard rock, but the coefficients and safety factor used are those originally proposed by Salamon.

3.1.1.3 Hedley's Formula

Hedley and Grant (1972) proposed a pillar strength formula based on data from hard rock room and pillar mining at Elliot Lake. They empirically calibrated a size effect formula similar to that proposed by Salamon (discussed above). The formula was defined as:

\[ Qu = k \times w^{0.5} / h^{0.75} \]

where:

\( Qu = \text{pillar strength (psi)} \),
\( k = 26,000 = \text{strength of one cubic foot of pillar material (psi)} \),
\( w = \text{pillar width (feet)} \),
\( h = \text{pillar height (feet)} \).

The data base to develop this formula consisted of 23 stable pillars, 2 partially failed pillars and 3 crushed pillars. For application of their pillar strength formula, Hedley and Grant suggested that pillars with a safety factor greater than 1.5 are stable and pillars with a safety factor near 1.0 are crushed.
These safety factors are based on interpretation of the graphical plot of the data base (see figure 10).

This strength formula has been further confirmed for room and pillar mining, through studies by Von Kimmelmann et al. (1984), and Townsend (1982). It is the only pillar strength formula developed based on hard rock mining case histories. So although no published study has confirmed its use for open stope pillars, it is widely used in open stope pillar design.

3.1.1.4 Obert and Duvall Shape Effect Formula

Obert et al. (1946) performed a series of compressive strength tests on specimen coal pillars with various shapes. It was determined that the shape effect of pillar strength follows the empirical relationship:

$$\sigma_p = \sigma_1 \ast [0.778 + 0.222(w/h)]$$

where:

- $\sigma_p =$ pillar strength,
- $\sigma_1 =$ uniaxial strength of a cubical pillar specimen,
- $w =$ pillar width,
- $h =$ pillar height.

The formula did not include any factor to account for the size effect on strength, but instead suggested a safety factor between 2 and 4 be used in pillar design.

In hard rock pillar design, this formula has been suggested to account for shape effect by several authors (Krauland and Soder 1987; Hedley et al. 1979; Herget et al. 1984). These authors used additional methods to account for pillar strength
FIGURE 10. The estimated stress and strength for case histories of pillars in room and pillar mining in the Elliot Lake uranium mining district. Safety factor lines have been drawn on the graph. The chart shows that all the case histories with a safety factor above 1.5 are stable (after Hedley and Grant 1972).
3.1.1.5 Hoek and Brown Pillar Strength Curves

Hoek and Brown (1980) proposed a series of curves for the estimation of pillar strength (see figure 11). The curves were developed based on numerical modelling and the distribution of failed rock inside pillars of different shapes and for a range of rock mass qualities, using the empirical rock mass failure criteria:

\[ \sigma_p = \sigma_3 + (m \cdot \sigma_C \cdot \sigma_3 + s \cdot \sigma_C^2)^{1/2} \]

where:

- \( \sigma_p \) = average pillar strength,
- \( \sigma_3 \) = minimum principal stress,
- \( \sigma_C \) = uniaxial compressive strength of the intact pillar material,
- \( m & s \) = empirical constants based on the rock mass quality of the pillar material.

Hoek and Brown proposed these pillar design curves assuming that a pillar has failed when the stress across the centre of the pillar exceeds the strength of the rock mass. They stated that a safety factor of 1.0 or less would imply that a pillar is theoretically unstable and that a safety factor in excess of 1.5 should be used for permanent pillars. However, these recommendations do not seem to be confirmed by case history back-analysis.

Each curve can be considered a pillar failure criterion for a specific rock mass quality. Hoek and Brown proposed that the
Intact samples of fine grained igneous crystalline rock
\[ m = 17, s = 1 \]

Very good quality rock mass
\[ m = 8.5, s = 0.1 \]

Good quality rock mass
\[ m = 1.7, s = 0.004 \]

Fair quality rock mass
\[ m = 0.34, s = 0.0001 \]

Poor quality rock mass
\[ m = 0.09, s = 0.000001 \]

influence of pillar volume and structural defects could be quantified through the use of rock mass classifications. Consequently, the m and s constants account for pillar volume and structural defects because they have been related to the two most common rock mass classification methods, CSIR by Bieniawski (1973) and NGI by Barton et al. (1974).

Originally, the strength curves were not supported by case histories, however practical application by Potvin (1985) and Page and Brennen (1982) has been successful for the good and fair rock mass quality curves.

3.1.2 Pillar Load

In underground mine design, it is difficult to determine the actual load that will be acting on a pillar. For most safety factor pillar design methods, two procedures are currently used. The first method, called the Tributary Area Theory, uses a simplified approach to underground stress redistribution. The other method, generally termed numerical modelling, involves the use of the theory of elasticity to determine stress redistribution. In contrast to the simplicity of the tributary area theory, numerical modelling requires the use of a computer due to the sophistication of the calculation process.

3.1.2.1 Tributary Area Theory

The Tributary Area Theory assumes that when stopes are opened there is an equal and symmetric stress redistribution regardless of the size and location of the pillars created. It
is often described using the analogy of a smooth flowing stream obstructed by bridge piers (see figure 12). To permit a continuous flow rate in the stream, streamlines are concentrated between the piers (i.e. between the stopes). This causes the flow velocity (stress) between the piers (in the pillars) to increase. The increase in flow velocity is generally dependent on the ratio of the width of the stream (width of the mining area) to the sum of the distances unobstructed by the piers (sum of the pillar widths).

So in a rock mechanics perspective, this theory describes the redistribution of principal stress flowlines into pillars. The average pillar load thus depends on the ratio of the total area extracted to the total area remaining in the pillar. Figure 13 shows the application of the Tributary Area Theory to several types of pillars (including rib pillars).

Due to the simplicity of this theory, some factors that fundamentally influence stress in pillars are ignored. These factors are:

- the number of pillars in the mining block (or the extent of the mining area),
- the location of the pillar in the mining block,
- the redistribution of stress into the abutments,
- and the shape of the pillar.

A study by Salamon (1974) details the first three problems by comparing the average stress for a problem predicted by
FIGURE 12. The analogy of streamlines in a smoothly flowing stream obstructed by bridge piers is often used to describe the concentration of stress in pillars (after Hoek and Brown 1980).

\[ \sigma_p = \gamma z (1 + \frac{W_o}{W_p}) \]

RIB PILLARS

\[ \sigma_p = \gamma z (1 + \frac{W_o}{W_p})^2 \]

SQUARE PILLARS

\[ \sigma_p = \gamma z (1 + \frac{W_o}{W_p}) (1 + \frac{L_o}{L_p}) \]

RECTANGULAR PILLARS

IRREGULAR PILLARS

FIGURE 13. The tributary area theory, for average pillar load calculation, applied to several different pillar layouts (after Hoek and Brown 1980).
tributary area to those predicted by an electric analogue model. The stress in square room and pillar panels of three, seven and eleven square pillars (in each horizontal direction) were investigated. The average pillar load according to the tributary area theory is $4Q_{33}$ ($Q_{33}$ is the pre-mining stress component). Figure 14 shows the analogue results of these tests. Stress redistribution into the abutments results in the analogue predicted stress always being lower than the tributary area predicted load. As the panel widens (larger number of pillars), the load predicted by the analogue approaches the value of $4Q_{33}$. It is also demonstrated by this model that the location of the pillar in the panel has a significant effect on its load.

The influence of the shape of a pillar is documented in an investigation of the Tributary Area Theory and two dimensional boundary element modelling of rib pillars (Potvin et al. 1987). Figure 15 shows that as a pillar becomes more slender, the average pillar load predicted by modelling decreases. This effect is also discussed by Salamon (1974) and is attributed to decreasing pillar stiffness with increasing pillar slenderness.

In summary, the Tributary Area Theory provides a very quick solution for determining pillar load. However, the accuracy of the method is diminished if there are a small number of pillars, a small mining panel, or if the pillars are slender in shape. Bieniawski (1983) comments that in coal mining, the overestimation of pillar load by tributary area may be as much
FIGURE 14. Using an electric analogue model, Salamon (1974) showed the variation in pillar stress caused by increasing the number of pillars (N) in a mining panel. σ is the pillar stress, and Q_{33} is the premining stress. The triangular symbols correspond to the three pillars in panel (a), the circular symbols correspond to the seven pillars in panel (b), and the diamond symbols correspond to the eleven pillars in panel (c). The graph shows a distinct influence of the location of a pillar and the number of pillars on the stress induced.
FIGURE 15. A study using two dimensional boundary element numerical modelling shows the influence of pillar shape and the number of pillars on the average stress (after Potvin et al. 1987).
as 40%, while the author has found that the Tributary Area
Theory may overestimate the load in open stope rib pillars by as
much as 100% (Hudyma 1988b).

3.1.2.2 Numerical Modelling

Several types of numerical models are available to aid in
the calculation of pillar load. Each of these models has
different characteristics and a different means of calculation.
The models applicable to hard rock pillar design will be
discussed in chapter 3.2.

When used in empirical design methods, the capabilities of
numerical models include the ability to:
- analyze complex mining geometries,
- account for any number of pillars and any size of mining
  seam,
- recognize pillar location in a mining block,
- determine loads in individual pillars,
- and account for variations in pillar shape.

Numerical modelling removes many of the problems associated with
tributary area and is usually necessary to estimate the pillar
load. However, the use of numerical modelling is a skill that
takes a degree of knowledge, experience and calibration to use
efficiently in pillar design. These topics will all be
discussed in more depth in Chapter 3.2 and Chapter 4.

3.1.3 Safety Factor

Hoek and Brown (1980) state that, "A safety factor of 1.0
implies that the pillar is theoretically unstable and that the failure could propagate across the entire pillar ...". The safety factors suggested for various empirical design procedures in entry mining methods are listed in Table 4. The degree of instability acceptable in entry methods is much less than that in open stope methods. So, although there seems to be an agreement that a safety factor of about 1.5 is sufficient for pillar design in entry mining methods, this has not been verified for open stope mining.

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>SAFETY FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salamon (1967)</td>
<td>1.6</td>
</tr>
<tr>
<td>Hedley (1972)</td>
<td>1.5</td>
</tr>
<tr>
<td>Obert and Duvall (1967)</td>
<td>2 - 4</td>
</tr>
<tr>
<td>Hoek and Brown (1980)</td>
<td>1.5</td>
</tr>
<tr>
<td>Bieniawski (1983)</td>
<td>1.5 - 2.0</td>
</tr>
<tr>
<td>Stacey and Page (1986)</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 4. Safety factors suggested by various authors for pillar design in entry mining methods.

Stacey and Page (1986) state that for pillars in non-entry mining methods a minimum safety factor of 1.1 is necessary and to design pillars to yield or fail, a safety factor of less than 0.5 should be used. However, no data are presented to substantiate these values.

Ultimately, none of these formulas or safety factors is based on observation and experience in open stope mining. Using a factor of safety adds a conservative cushion against the potential error associated with empirical design methods.
However, a conservative design is not necessarily the most cost effective design. Using the safety factors suggested for an entry method pillars will likely give a stable design, but experience and calibration of an empirical design procedure will provide a better estimate of the safety factor needed.

3.2 Numerical Design Methods

In recent years, several numerical (or computational) methods have been developed specifically for use in underground rock mechanics design. The program codes were created to permit two dimensional or three dimensional stress and displacement investigations around excavations in rock.

In simplistic terms, numerical modelling can be described with figure 16. A region (R) is defined in a medium and loading conditions are applied to the region. Excavations (E) are then created in the medium. The principle function of numerical modelling is to calculate the magnitude and orientation of the stresses and displacements acting in the vicinity of these excavations. The redistribution of stresses may be based on elastic and/or plastic behaviour of the medium.

3.2.1 Types of Numerical Methods

Individual computational methods were developed to analyze problems with respect to specific properties of the medium. Brown (1987) grouped these properties into three broad categories:
FIGURE 16. An idealized sketch showing the principle of numerical modelling of underground excavations after Potvin et al. 1987).
- differential continuum methods,
- integral methods,
- and discontinuum methods.

Differential continuum methods (also called finite element and finite difference methods) require discretization of the medium within the region of interest, at the boundary of the problem and at a long distance from the boundary of the problem (also termed the far field). Continuum methods assume the problem to be solved will not be influenced by discontinuities in the rock mass (medium). This means the rock mass contains few or no significant discontinuities, or the discontinuities are so common and uniform that individually they have no effect on stress redistribution. Consequently, for continuum methods, it is assumed that the medium can be represented by "equivalent" continuum rock mass material properties. Differential continuum methods permit analysis using elastic and plastic theory. However, discretization inaccuracies at the boundary and the far field, extensive data preparation and high computing times make finite element methods less appealing for rock mechanics design. An extensive discussion of finite element methods is presented by Zienkiewicz (1977).

Integral methods (or boundary element methods) also use the continuum approach but only require approximations or discretization at the problem boundary. This greatly reduces the amount of data needed to describe the problem and consequently the amount of computer time needed to complete the
computations. However, they are best suited to linear and homogeneous (or piece-wise homogeneous) material behaviour. The use of boundary element methods and their application in rock mechanics is detailed in a book by Crouch and Starfield (1983).

Discontinuum methods are a special type of differential technique. They generally assume a rock mass can be modelled by a finite number of discontinuous blocks. The most common discontinuum approach is called the distinct element method. It uses rigid blocks and the laws of motion to determine the forces and displacements applied to the blocks. A good description of the basis of distinct element models and a general application in a rock mass is given in Cundall (1987).

The most appropriate numerical method for open stope pillar design depends on the in situ medium conditions and the form of stress response expected. As discussed in Chapter 2, pillars are not likely to be influenced by individual minor discontinuities and are loaded in a biaxial, elastic manner. Consequently, the numerical method best suited to open stope pillar design is a continuum approach using the theory of elasticity. The most efficient approach for these conditions is the integral method. Finite element methods could perform the computations adequately, but are not as efficient as boundary element methods in elastic stress analysis. As a result, all of the numerical modelling in this thesis will focus on the application of boundary element methods.
3.2.2 Interpretation of Boundary Element Results in Mining

The boundary element stress analysis technique has been developed to approximate the stress distribution around openings with irregular shapes oriented in a two dimensional or three dimensional stress field. However, boundary element methods do not directly determine failure. The stress distribution needs to be interpreted to determine the effect on underground stability. Many types of failure criterion have been applied in the analysis of stress distributions. This section will outline the common methods of boundary element interpretation used in pillar design. The methods of interpretation include:

(i) post-processing failure criterion,
(ii) interactive failure criterion,
(iii) and principal stress magnitudes.

3.2.2.1 Post-Processing Failure Criterion

Post processing failure criteria are applied to the solution after the stress analysis is complete. The failure criterion does not have any effect on the stress solution. Generally, continuum material properties, such as intact rock strength, rock mass strength, discontinuity shear strength, or rock mass characterization parameters, are estimated for the rock mass behaviour. The failure criterion is calibrated based on the estimated material properties and experience in similar rock conditions. Common failure criteria used in post processing were developed by:

- Murrell (1965) and Bieniawski (1974) for intact rock,
- Hoek and Brown (1980) for jointed rock masses,
- and Coulomb (1776) for discontinuities.

The failure criterion is applied to stresses at many points in a pillar. Based on the distribution of theoretically failed rock, pillar stability is determined and potential mining problems are delineated.

An example of the application of a post-processing failure criterion is described by Brady (1977) in the analysis of an experimental open stope pillar at the Mount Isa Mine in Queensland, Australia. A criterion was calibrated for the failure of pillar material based on a formula originally proposed by Murrell (1965). From the observation of local rock spalling, the following formula was developed:

\[ \sigma_1 = 9.34 \sigma_3^{0.75} + 94.0 \]

where,

\( \sigma_1 \) = the major principal stress at failure (MPa),

\( \sigma_3 \) = the minor principal stress (MPa).

The failure criterion was then applied to \( \sigma_1 \) and \( \sigma_3 \) stress distributions for stable and failed open stope pillar case histories for verification. Figure 17 shows the results of applying the criterion to a stable pillar. The points denoted by "F" in the figure representing the theoretical zone of failed rock in the pillar. The predicted zones of failed rock are small and isolated at the stope periphery, which corresponds well with the stable assessment. Figure 18 shows the criterion applied to the stress distribution of a pillar that failed. The
FIGURE 17. An empirical failure criterion has been applied to the two dimensional stress distribution of a stable open stope rib pillar. Points denoted by "F" represent the area of rock that has theoretically failed. For this pillar, the failure zones are small and isolated at the peripheries of the pillar. This corresponds to a generally stable assessment for the pillar (after Brady 1977).

FIGURE 18. The theoretical distribution of failed rock is much greater in this pillar. The actual pillar collapsed shortly after being reduced to this size (after Brady 1977).
zone of failed rock covers a significant portion of the pillar, which also agrees with the actual assessment.

Application of a failure criterion to the theoretical stress distribution around underground excavations is very common for the interpretation of boundary element solutions. However, it assumes that load is entirely carried by the pillar material and that there is no stress redistribution due to destressing of the failed rock mass. This assumption may not be correct for highly loaded pillars.

3.2.2.2 Interactive Failure Criterion

An interactive failure criterion works during the numerical computations by adjusting the stress in regions of the rock mass that fail due to high stress. This requires a criterion to determine the peak strength of the rock mass and the post failure rock mass characteristics. Calibration of this type of criterion is very involved and has a fundamental effect on the results.

Documentation of the use of an interactive failure criterion is given by Maconachie et al. (1981) at the C.S.A. mine, Cobar Mines Pty., New South Wales. The displacement discontinuity program "N-Fold" with an interactive failure criterion was used to investigate the stress condition of a sill pillar. The program considers non-linear deformation and brittle yielding of elements. The yield point and post failure deformation varies based on the confinement of the element. Figure 60 shows that for increasing confinement (increasing distance from a free
FIGURE 19. The peak strength, deformation characteristics, and effect of location used for investigating a pillar case history with a displacement discontinuity program (after Maconachie et al 1981).

FIGURE 20. The normal stress and the failed regions estimated with the displacement discontinuity program for a sill pillar case history (after Maconachie et al 1981).
face), the peak strength increases and the post-peak load bearing capacity of the rock improves. The calibration of the failure criterion was based on estimations of the in situ rock mass strength and laboratory material properties. The material properties were subsequently verified based on observation and monitoring of the sill pillar.

When applied to a longitudinal section of the sill pillar (figure 20), the zones of failed, and yielding rock were outlined and the magnitude of the normal stresses for rock under elastic deformation was determined. The failure criterion helped determine the best stope extraction sequence and indicated the need of a pendant pillar to maintain stability in the sill pillar.

While potentially very useful in pillar design, this type of failure criterion needs a large amount of calibration and verification before becoming a reliable tool. Generally, the more sophisticated and complex the program and failure criterion, the greater the number of assumptions introduced into the solution.

3.2.2.3 Principal Stress Magnitude.

The most common and simplistic method of boundary element interpretation is analysis of principal stress magnitudes. In pillars, stress distributions are plotted on mine plans or sections to reveal areas of high or low principal stress. Potential mining problems are then estimated based on the stress distributions.
A typical example of the use of principal stress magnitude analysis is given in a paper by Bywater et al. (1983), at the Mount Isa mine. It was determined through experience that areas with normal stress greater than 70 MN/m$^2$ generally exhibit spalling and are prone to local rock failure. A linear elastic displacement discontinuity code was used to analyze the potential stress distributions in a new mining block. Figure 21 shows two different extraction sequences for the mining block, with the predicted stresses corresponding to the legend. The analysis shows more overstressed areas being developed in the second sequence which would cause problems earlier in the pillar recovery.

When the rock mass strength has not been estimated, stresses are frequently normalized against the intact uniaxial compressive strength of the rock. Mining problems are likely to occur if the normalized major principal stress is greater than $1/3$ (Bawden et al. 1988) to $1/2$ (Mathews et al. 1980).

3.2.3 Limitations of Boundary Element Modelling

While boundary element modelling is a sophisticated design tool, it has several limitations and potential sources of inaccuracy in applied rock mechanics. The limitations can be grouped into two basic categories:

(i) limitations with respect to modelling a rock mass,

(ii) and limitations due to computational assumptions.

3.2.3.1 Modelling a Rock Mass
FIGURE 21. The distribution of normal stress in a mining block was estimated for two different mining sequences to determine the best stope extraction sequence (after Bywater et al. 1983).
A numerical modelling solution assumes the medium has perfect material properties. In reality, a rock mass is not a perfect material. A number of approximations and assumptions are usually necessary for the estimation of the properties describing the rock mass. The material properties of a rock mass have to be estimated assuming the rock mass behaves as an isotropic continuum. This means that the rock mass either has no significant discontinuities, or the discontinuities are sufficiently small, regular and frequent that they have no effect on stress. For minor structure such as rock joints, this may not be a serious limitation. However for major structure, especially faults that have moved substantially, the rock mass may not act as an isotropic continuum at all. This could invalidate any numerical solution that did not explicitly model the discontinuity.

Most boundary element methods give the rock mass linear elastic deformational characteristics. Laboratory measurements have found that over a range of loading conditions, hard rock samples exhibit some non-linear and plastic deformation. In addition, the post-failure load bearing behaviour of an in situ rock mass is dependent upon several variables that are not related to the elastic characteristics of the rock. For low to medium loading conditions, the use of linear elasticity is generally acceptable, but for a discontinuum, highly loaded, or failed rock mass, linear elastic behaviour is a poor assumption.

Parametric studies using boundary element models have shown
a large influence of the pre-mining stress regime. This is an expensive and difficult parameter to measure. The actual in situ stress field varies with depth and can be profoundly influenced by major structural discontinuities. Consequently, the virgin stress used in numerical methods will only be an approximation of the actual conditions.

It is important to be aware of these limitations and their possible effect on the numerical solution's ability to describe the condition of a stressed rock mass.

3.2.3.2 Computational Assumptions

Boundary element methods are numerical approximations of the solution to a boundary value problem. Only the simplest excavation geometries can be solved analytically, so for complicated geometries, a solution is determined through a numerical iteration process. This necessitates discretizing the boundary into segments and piecewise modelling of stresses and displacements on each segment. The result is:

- the interior solution (stresses off the boundary) may not be accurate very near the discretized boundary,
- and the numerical solution is only an approximation because the computation is completed when a specified convergence criterion is met.

Through the modelling of boundaries with known solutions, it has been found that the larger the number of elements on a boundary, the greater the accuracy of the numerical model with
respect to the known closed form solution. The magnitude of the
difference between the numerical model and the closed form
solution decreases with an increase in the number of elements,
so there is a practical limit to the influence of the number of
elements. Above this limit, the addition of extra elements does
little or nothing to improve the accuracy of the solution.

In summary, reading too much detail in a numerical solution
can be misleading. Calibration of numerical models with
experience and case histories can be as important as the type of
numerical model used or how the results are analyzed. It should
be kept in perspective that boundary element methods only
account for stress related failure. Structurally controlled
failure or failure due to the combination of stress and
structure may not be interpreted from numerical modelling stress
distributions.
CHAPTER 4
OPEN STOPE RIB PILLAR DATA BASE

The objective of this chapter is to present the rib pillar data collected during the Integrated Mine Design Study. This will be done by:
- discussing some of the general characteristics and information of the pillar case histories,
- presenting the background and physical information on each case history,
- defining the qualitative scale used to give an assessment to the case histories,
- and describing the signs of failure for all the case histories that experienced stability problems.

4.1 General Data Base Information

The original data used in this thesis has been collected in Canadian open stope mines. The 47 case histories are only a fraction of the total data collected during the "Integrated Mine Design Study". Some of the data was rejected because:

(i) geotechnical parameters including in situ stress, intact rock strength and the influence of geological structure could not be estimated with confidence,

(ii) the actual events of the case history could not be verified,

(iii) the stress conditions in the case history were too complex
to be back-analyzed with the means available at U.B.C.

Throughout the course of the study, several mines requested that their name not appear directly associated with data. To respect their anonymity, there is no specific reference to the site of any unpublished data in this thesis. Specific information about the mining environment, geotechnical parameters and case histories is presented through the use of mine numbers.

The data base is supplemented by information presented in U.B.C. theses that discuss open stope rib pillars, by Goldbeck (1985), Potvin (1985) and Pakalnis (1986).

A significant feature of many of the pillars in the data base is that they were stable at one time during the mining and later failed. The failure was caused by increased extraction near the pillar or mining portions of the pillar. Among the 47 case histories in the data base, 30 originate from 13 pillars at different stages of extraction. These "yielding pillars" will be very important to the development of a rib pillar design method.

4.2 Background Data

The background information concerning pillar dimensions, depth, mining environment including ratio of extraction and backfill, and an assessment of the pillar condition is given in Table 5. The dimensions and ratio of extraction are defined
**TABLE 5.** Background data for all the pillar case histories.
according to figure 22. The dimensions presented are the design dimensions. The actual dimensions may vary slightly for most cases due to blast induced damage. For pillars that have failed, the actual dimensions (especially pillar width) may be substantially smaller than the design dimensions, due to excessive sloughing. Justification for the assessment of the condition of each sloughing and failed pillar is given in chapter 4.3.

Specific information about the geological setting of each case history can be found in the isometric sketch corresponding to the mine number (see Appendix I). Each geological setting is comprised of:

- the underground stress regime,
- the hanging wall, footwall and orebody material properties and characteristics including,
  - rock type,
  - intact uniaxial compressive strength,
  - elastic modulus,
  - poisson's ratio,
  - NGI rock mass classification,
- the orebody shape and size,
- and the mining methods used in various parts of the orebody.

Several mines use very similar stope and pillar dimensions throughout the mine. Inclusion of this data would potentially
Lo1 = length of stope 1
Lo2 = length of stope 2
Wp = width of pillar
Hp = height of pillar, or stope breadth
Ho = stope height

EXTRACTION RATIO = \[
\frac{\left(\frac{Lo1 + Lo2}{2}\right)}{\left(\frac{Lo1 + Lo2}{2}\right) + Wp}
\]

FIGURE 22. This figure shows the geometrical definition for the stope and pillar dimensions used in this thesis.
double or triple the size of the data base. However, using several case histories with the exact same information would not broaden the capability of the data base to develop a design method. It would create problems in data presentation and dilute the influence of single case histories. As a result, only unique cases are presented.

4.3 Pillar Assessment

The signs of rib pillar instability are listed in Chapter 2.1. Based on these signs, three qualitative assessments have been chosen to categorize the condition of the pillars in the data base.

A stable assessment is given to pillars generally not showing any signs of instability. Any ground control problems are too small to have an effect on mining near the pillar.

A sloughing assessment is given to pillars showing one or more of the above signs, but the extent of deterioration is not severe and is reported in only a few areas of the pillar. The ground control problems associated with sloughing pillars have a limited effect on mining, such as: drilling problems, loss or difficulty in maintaining some drill holes, the need for development scaling and rehabilitation and some wall sloughing and pillar overbreak. The sloughing assessment is also used to describe pillars whose stability problems are time dependent, becoming more severe as mining continues. Several pillar case histories have been assessed as sloughing, but have used quick
backfilling to prevent complete pillar failure.

A failed assessment is given to pillars showing large and severe signs of instability. Their effects on mining, include:

- loss of ore,
- low productivity due to oversize material and overbreak created during mining, the need for frequent rehabilitation of development or the use of cable bolts to prevent loss of pillar development,
- and severe cracking, joint opening, and displacement often needing immediate stope filling to prevent complete pillar disintegration.

The assessment of pillars was based largely on documentation and description by on-site staff and some observations by the author. Justification of the assessment for all the sloughing and failed pillars is detailed below, by describing the most serious signs of instability for each case history:

CASE # 3
Assessment: Failure.
Pillar Condition: Sloughing of large slabs from pillar walls into primary stope drawpoints, problems in maintaining blastholes, wall sloughing intersected development in the middle of the pillar.

CASE # 8
Assessment: Failure.
Pillar Condition: Severe axial cracking in pillar development requiring cable bolting to maintain overcut and undercut stability, several feet of overbreak beyond blastholes and hourglass sloughing in the middle of pillar walls.
CASE # 17
Assessment: Sloughing.
Pillar Condition: Shears and joints opening in pillars, sloughing of pillar walls into primary stopes. Some problems in drilling and maintaining drill holes.

CASE # 20
Assessment: Sloughing.
Pillar Condition: Progressive sloughing of pillar walls into adjacent stopes.

CASE # 23
Assessment: Failure.
Pillar Condition: Severe sloughing of pillar walls into adjacent stopes.

CASE # 25
Assessment: Failure.
Pillar Condition: Major shear displacement extending over two levels 45 metres apart, sloughing of pillar walls.

CASE # 28
Assessment: Sloughing.
Pillar Condition: Severe ground fracturing causes abandonment of pillar development.

CASE # 30
Assessment: Failure.
Pillar Condition: Pillar crushes violently after nearby pillar is recovered by blasting.

CASE # 33
Assessment: Failure.
Pillar Condition: Extensive cracking of pillar, followed by the sloughing of 2 rings of drill holes and major collapse of the upper half of the pillar into adjacent stopes.

CASE # 34
Assessment: Sloughing.
Pillar Condition: Extensive cracking of the pillar reported, with some sloughing into nearby stopes.
CASE # 36
Assessment: Failure.
Pillar Condition: West side of the pillar sloughs into adjacent stope causing breakthrough to a pillar cross-cut.

CASE # 37
Assessment: Failure.
Pillar Condition: Wall sloughing creates a hole completely through the pillar.

CASE # 42, 45
Assessment: Failure.
Pillar Condition: Severe cracking, spalling and joint opening in pillar development with wooden cribs and cable bolting needed to limit development closure and collapse, heavy overbreak on production blasts.

CASE # 47
Assessment: Sloughing.
Pillar Condition: One vibrating wire stressmeter shows decrease in stress through pillar.
(reference: Goldbeck 1985).

CASE # 48
Assessment: Failure.
Pillar Condition: All vibrating wire stressmeters show decrease in stress through pillar.
(reference: Goldbeck 1985).

CASE # 49
Assessment: Failure.
Pillar Condition: Sharp decrease in pillar stress shown by vibrating wire stressmeters.
(reference: Goldbeck 1985).

CASE # 56
Assessment: Sloughing.
Pillar Condition: Serious axial cracking in pillar as stopes retreated to pillar.

CASE # 59
Assessment: Sloughing.
Pillar Condition: Axial cracks in pillar develop and open after recovery of a nearby pillar.
CASE # 60
Assessment: Sloughing.
Pillar Condition: Axial cracks in pillar develop and open after recovery of a nearby pillar.

CASE # 61
Assessment: Sloughing.
Pillar Condition: Sloughing of pillar walls as far as centre of pillar, overbreak from pillars during primary mining and severe overbreak during secondary stope mining.
CHAPTER 5
BOUNDARY ELEMENT METHODS IN RIB PILLAR DESIGN

Boundary element numerical methods are an effective way to estimate the stress at any point in a rib pillar (for reasons described in Chapter 3.2.3). For each of the case histories presented in Chapter 4, a direct integral two dimensional program (BITEM) and a pseudo-three dimensional displacement discontinuity program (MINTAB) method will be used to estimate the average pillar stress. For many rib pillar geometries, these programs give adequate results. However, BITEM and MINTAB have limitations that may cause serious inaccuracies when applied to some three dimensional problems. Ideally, a three dimensional method would be used to determine the stress distribution in each case history. However, true three dimensional analysis is very new technology and the programs have large setup and run times, need quite sophisticated computing facilities, and are limited in program size.

To better define these limitations, a three dimensional boundary element code (BEAP) and BITEM and MINTAB will be used to investigate the average stress in typical rib pillar geometries. This comparison will be used to approximate the error associated with the application of the two dimensional and displacement discontinuity methods to 3D problems.
5.1 Boundary Element Methods Used

The following general description of the boundary element methods and numerical codes involved in the study is taken largely from an unpublished paper written at U.B.C. (Hudyma 1988b).

5.1.1 BITEM

The 2D direct boundary integral model "BITEM" is based on the program "BITE" developed by P.C. Riccardella at the Carnegie-Mellon university in 1973. It was expanded to perform piece-wise homogeneous elasticity analyses by CSIRO (Commonwealth Scientific and Industrial Research Organization, Australia) in 1978. The program was subsequently modified for the U.B.C. mainframe computer by R. Pakalnis in 1983 and later for an IBM compatible computer by CANMET under the program name PCBEM (Pakalnis 1987).

The boundary integral technique is designed for problems that have one long dimension and a constant cross sectional shape. It requires the discretization of all excavation surfaces into segments connected by nodes (see figure 23). An explicit solution is selected to represent the medium's in situ stress conditions. These field stresses can be constant or can vary linearly with position. When excavations are created, the stress perpendicular to the boundary nodes becomes zero. BITEM then calculates tractions and displacements at all the nodes of all the boundaries. The boundary solution is determined through
FIGURE 23. Isometric view of an opening that is long in one direction and the discretization of the boundary used in two dimensional modelling (after Hudyma 1988b).
an iterative procedure in which the stress and displacement at each node influences the stress and displacement of the other nodes of the boundary. This procedure ends when the difference between the last two iterations is less than a user defined convergence criterion. Once a boundary solution has been determined, stresses and displacements internal to the problem boundary can be determined using the boundary solution and stress-strain relationships. A more detailed description of the boundary integral technique is found in Brady and Bray (1978).

5.1.2 MINTAB

Mintab is a pseudo-three dimensional displacement discontinuity boundary element program. The original code was written by Dr. S.L. Crouch in South Africa. The program has had several major modifications resulting in several different program names, including: MINSIM, MINTAB, BESOL and N-FOLD. Each variation has special features such as the inclusion of backfill elements, use of a semi-infinite domain (can account for the surface of the earth), use of multiple en echelon seams, faults and folds in the seams and a program interactive failure criterion with post-failure rock mass characteristics. The version used for this study is CANMET's MINTAB version 4.0 (1983) which performs only linear elastic analysis of one planar seam, in an infinite domain, and with no built in failure criterion.

MINTAB uses the displacement discontinuity method to solve
stresses, strains and displacements in three dimensions around excavations in tabular orebodies. In MINTAB, the orebody is discretized into a grid of square two dimensional elements (see figure 24). Each element represents mined or unmined area in the reef. The third dimension is the width of the seam. To give an accurate solution, the seam width must be small in relation to the overall size of the problem. The definition of a small seam and the limitations of displacement discontinuity modelling will be discussed in Chapter 5.4.

For practical purposes, the reef can be considered as two parallel planes. Creating excavations in the grid induces movement of the planes. Relative movements between the two planes are broken into two components. Ride components act parallel to the plane boundaries and closure components act normal to the planes. The seam elements are subjected to a three dimensional stress field (see figure 24). Displacement discontinuity components in three dimensions are associated with each element and represent relative displacement between the two planes. If the two planes do not come in contact due to displacement, the tractions \( \sigma_{zz}, \sigma_{yz}, \) and \( \sigma_{xz} \) are all zero. Displacements and stresses at unmined points in the seam are calculated as a linear combination of the displacement discontinuities of all the elements in the seam. A more detailed description of the displacement discontinuity method is given by Starfield and Crouch (1973).
FIGURE 24. Oblique view of the MINTAB seam geometry and the stress applied locally on each element in the reef.
5.1.3 BEAP

BEAP is a three dimensional boundary element program developed by J.A.C. Diering as a PhD thesis, at Pretoria University (1987), in conjunction with CANMET, INCO (Thompson Division) and GEMCOM (Pty.) Limited. Version 1.0, used in this project, is due for public release in the fall of 1988.

Excavation boundaries are generally discretized by quadrilateral elements (see figure 25). The problem is subject to an arbitrarily oriented stress field. The stress and displacements on the boundary elements vary quadratically and are non-conforming. This means displacements and tractions on each element are assumed to vary according to a quadratic polynomial, and the displacements between adjacent elements are discontinuous. The resulting numerical model has some powerful abilities in mining related stress analysis, including:

- the need for fewer elements to discretize an excavation than other three dimensional boundary element models,
- the ability to accommodate up to five zones with different material properties,
- the use of lumping to reduce data storage requirements,
- and the ability to determine stresses and displacements very close to an excavation boundary.

Further details about BEAP can be found in Diering (1987) and Diering and Stacey (1987).

5.2 Open Stope Rib Pillar Modelling
FIGURE 25. A typical BEAP geometry showing the boundary of the excavations defined by two dimensional quadratic, non-conforming elements in a three dimensional stress field (after Hudyma 1988b).
Boundary element numerical modelling of hard rock excavations relies largely on the problem geometry and the magnitude and orientation of the pre-mining stress. This section describes a consistent method to specify the stope and pillar dimensions and to determine the average load on rib pillars in open stope mining.

5.2.1 Defining the Open Stope Geometry

In this thesis, the dimensions of stopes and pillars will be defined according to figure 26. Pillar dimensions are defined with respect to the direction of the greatest induced stress. The pillar height is typically defined as parallel to the direction of greatest induced load. Induced load, in any direction, is mostly a function of the size and shape of the excavation surface perpendicular to that load. For small excavation surfaces, the stress redistribution is small. For large excavation surfaces, the stress redistribution will be much larger. In horizontal orebodies (where most of the original pillar design research was done), the greatest induced load is vertical and the pillar height is vertical (see figure 27a). In steep dipping orebodies, the largest induced load is horizontal and the pillar height is horizontal (see figure 27b). For inclined orebodies, the pillar height is defined as the direction perpendicular to the orebody.

5.2.2 Defining the Average Pillar Stress
Lo1 = length of stope 1
Lo2 = length of stope 2
Wp = width of pillar
Hp = height of pillar, or stope breadth

FIGURE 26. This figure defines the dimensions for stopes and pillars, and the orientation for the in situ stress regime for this thesis.
FIGURE 27a. A rib pillar in a horizontal seam loaded by the weight of the overburden.

FIGURE 27b. The direction of loading on a pillar in a vertical orebody.
For an idealized open stope rib pillar in a vertical orebody, the in situ stress acts in three basic directions: $\sigma_x$, $\sigma_y$, and $\sigma_z$ (see figure 28). Pillar stress is a result of the pre-mining stress that is concentrated because of adjacent excavations. Stress concentration in a direction is generally proportional to the size and shape of the stope surfaces normal to that stress direction. In pillar design, the direction of greatest importance is usually the direction that has the highest stress.

Inside rib pillars, the stress acting in the $\sigma_x$ direction is the lowest because it is parallel to the orebody strike which causes it to be shadowed by the open stopes. The induced load in the $\sigma_y$ direction is almost always larger than in the $\sigma_z$ direction, because the pre-mining stress in the $\sigma_y$ direction is typically much greater than in the $\sigma_z$ direction. In addition, for sub-vertically dipping orebodies, the stope surface normal to the $\sigma_y$ direction is much larger than those perpendicular to the $\sigma_z$ direction. This means the pillar stress in sub-vertical orebodies is almost always highest in the $\sigma_y$ direction.

There is a large variation in the $\sigma_y$ stress field in a rib pillar. The best location to determine the average $\sigma_y$ stress is the pillar centerline at the middle of the stope height (also called the pillar "mid-height centerline"), see figure 28. The reasons for this location are:

- it is the region of highest normal stress ($\sigma_y$ direction),
- it is the region of lowest confining stress ($\sigma_x$ direction),
FIGURE 28. The mid-height plane and centerline for tall open stope geometries.
- it is often observed to be one of the first areas of instability in a pillar,
- the effect of the excavation corners and stope ends are at a minimum,
- this is usually the plane of analysis when two dimensional modelling (in plane strain) is used.

However, there may be a large variation in the $\sigma_y$ stress at the mid-height centerline. Hoek and Brown (1980) show that as a pillar becomes more slender (taller and narrower), the stress distribution across the mid-height of the pillar becomes more uniform. In a squat pillar, the stress distribution varies significantly across the pillar mid-height centerline. They suggest that the average pillar stress should be the average value of the maximum principal stress (in the $\sigma_y$ direction) across the pillar. So for this thesis, the average pillar stress for open stope rib pillars will be calculated as the average stress along the mid-height centerline of the pillar.

5.3 2D Modelling of 3D Excavation Geometries

Numerical modelling of underground excavations with 3D methods is a time consuming and expensive procedure. Two dimensional numerical modelling can be used effectively to estimate the stress found in some of the planes of a 3D pillar geometry, and at a much lower cost than 3D numerical methods. One of these planes is at the mid-height of tall open stopes,
which is of primary concern in open stope rib pillar design. This sub-section will discuss how 2D modelling can be used to estimate the average pillar stress in open stope rib pillars. It will also estimate the difference between 2D and 3D numerical modelling for various open stope mining geometries.

5.3.1 Plane Strain Solution

To estimate the stress around open stopes, the plane strain solution is generally used. Plane strain conditions assume that around an excavation all the mining induced displacements occur in the plane of the orebody cross-section and the displacements are the same for all cross-sections. In a typical geometry, a stope is modelled in the xy plane (see figure 28). The assumption is that in the 3D situation, the stope ends have no influence on the cross-section plane. Brown (1985) notes that:

"For uniform excavation cross-sections, other than those with extreme axial ratios, the plane strain boundary stresses usually approximate the correct three-dimensional stress to within less than ten, and sometimes five, per cent at locations removed by at least two excavation 'diameters' from intersections, excavation ends or changes of cross-section."

In applying plane strain conditions to open stope rib pillar design, the subject of interest is the influence of the stope ends on the stress at the mid-height centerline of the pillar. If the mid-height plane is not sufficiently removed from the
stope ends, some of the mining induced stress redistribution will occur into the abutments at the stope ends, rather than into the pillar. This means that the stress at the mid-height plane is greatest when there is no influence of the stope ends, which is the case for the 2D plane strain solution. This is confirmed in work done by Watson and Cowling (1985) at Mt. Isa and is observed in the results to be discussed in Chapter 5.3.2.

5.3.2 Comparison of 2D and 3D Numerical Modelling Results

A comparison of several different stope geometries was done with the 3D model BEAP, and the 2D model BITEM in plane strain. The objective was to investigate in more depth the degree of overestimation predicted by BITEM for different stope and pillar geometries.

The size of the plane normal to the $\sigma_p$ stress (shaded plane, figure 29) has the greatest influence on stress concentration at the pillar mid-height centerline. To check the influence of the stope ends on the mid-height plane, the ratio of stope height to stope length was varied. Four tests, comprised of a total of 12 different stope geometries, were modelled with BITEM and BEAP. The first test checked the average pillar stress as the height was increased for stopes with a square cross-section. The second test checked the average pillar stress for stopes with a constant height and an increasing longitudinal stope length. The third test checked the average pillar stress as the height was increased for stopes with a constant longitudinal cross-
FIGURE 29. The shaded plane has the greatest influence on the mid-height \( \sigma_y \) stress.

<table>
<thead>
<tr>
<th>TEST</th>
<th>STOPE LENGTH (L)</th>
<th>STOPE BREADTH (B)</th>
<th>STOPE HEIGHT (H)</th>
<th>PILLAR WIDTH (Wp)</th>
<th>BEAP AVE. PILLAR STRESS INCREASE</th>
<th>BITEM H:L RATIO</th>
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| LONGITUDINAL STOPE CROSSSECTION | 100              | 10                | 60               | 50                | 1.5                              | 0.6 : 1        | 2.2                              |
|                                   | 60               | 10                | 60               | 30                | 1.75                             | 1 : 1          | 2.45                             |
|                                   | 30               | 10                | 60               | 15                | 2.05                             | 2 : 1          | 2.5                              |
|                                   | 10               | 10                | 60               | 10                | 1.7                              | 6 : 1          | 1.8                              |

| LONGITUDINAL STOPE CROSSSECTION | 30               | 10                | 30               | 15                | 1.65                             | 1 : 1          | 2.5                              |
|                                   | 30               | 10                | 60               | 15                | 2.05                             | 2 : 1          | 2.5                              |
|                                   | 30               | 10                | 120              | 15                | 2.3                              | 4 : 1          | 2.5                              |

| TRANSVERSE STOPE CROSSSECTION    | 10               | 10                | 40               | 10                | 1.6                              | 4 : 1          | 1.8                              |
|                                   | 10               | 20                | 40               | 10                | 1.4                              | 4 : 1          | 1.6                              |
|                                   | 10               | 40                | 40               | 10                | 1.2                              | 4 : 1          | 1.4                              |

TABLE 6. Comparison of BEAP and BITEM for four sets of different orebody geometries.
section. The final test checked the average pillar stress as stopes of a constant height and length were increased in breadth.

Table 6 shows the stope and pillar dimensions for each run (the dimensions are defined in figure 29). Table 6 also shows the average pillar stress increase for BEAP and BITEM and the stope height:length ratio. The average pillar stress increase is defined as the average pillar stress divided by the pre-mining stress in that direction (ie. $\sigma_y$ in figure 28).

In all 12 cases, the average pillar stress at the mid-height centerline was higher for the 2D plane strain (BITEM) models than the 3D BEAP models. The overestimation of BEAP by BITEM is shown for each geometry in figure 30. The dashed line on figure 30 is an estimate of the maximum overestimation of BEAP by 2D plane strain modelling for various stope height to stope length ratios. As the stope height to stope length ratio increases, the average pillar stress predicted by the 3D models is closer to the 2D plane strain solution. As the stope height to length ratio increased over 4:1, the 3D stress induced in the horizontal plane essentially remained the same and converged to levels similar the stress predicted by plane strain modelling. Brown's comment (above) that a stope cross-section needs to be at least two excavation "diameters" from the stope end, for good agreement between 2D plane strain and 3D modelling results, would correspond to a stope height to stope length ratio of 4:1. His estimation of less than 10 % difference between 2D plane
FIGURE 30. Overestimation of average pillar load by the 2D "BITEM" boundary element method for the 12 runs in the four tests.
strain and 3D modelling agrees well with the results presented in figure 30.

5.4 Displacement Discontinuity Modelling of 3D Stope Geometries

For excavations with irregular cross-sections or small stope length to stope height ratios, the 2D plane strain method can not effectively predict the average stress at the mid-height centerline of a pillar. The displacement discontinuity (DD) boundary element method MINTAB may be useful in these conditions. The DD code can be used to predict three dimensional stress redistribution around thin, tabular orebodies. For MINTAB analysis, the orebody must be a single seam with negligible variation in strike, dip and thickness. In addition, the thickness of the seam must be small compared to the length of excavations made in the seam. The following subsections will investigate the effect of the seam thickness on MINTAB's ability to predict stresses at the mid-height centerline of open stope rib pillars.

5.4.1 Seam Thickness Limitations

To help discuss the influence of the thickness of the reef, the ratio of the shortest stope dimension to the seam thickness is defined as the "seam thickness ratio". In open stope mining, where stopes are typically taller than they are long, the seam thickness ratio will usually be the ratio of stope length to stope breadth (see figure 31). Other authors have discussed the
FIGURE 31. The dimensions and geometry of the MINTAB/BEAP comparison tests.

<table>
<thead>
<tr>
<th>TEST</th>
<th>STOPE LENGTH (L)</th>
<th>STOPE BREADTH (B)</th>
<th>STOPE HEIGHT (H)</th>
<th>PILLAR WIDTH (Wp)</th>
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TABLE 7. Comparison of BEAP and MINTAB for the four different tests.
influence of the seam thickness ratio. Crouch (1986) states that 3D displacement discontinuity programs:
"...can be used to analyze any excavation that has a breadth:thickness ratio of 3 or more."

When investigating stress distributions around different excavation geometries with the pseudo-3D displacement discontinuity method, Brady (1978) was more conservative in finding that a,
"...comparison with results from independent three-dimensional analyses of these excavation shapes, indicate that the method is satisfactory for openings where the span/height ratio is greater than 5."

The influence of the seam thickness ratio on average pillar stress will be checked through the use of the tests described in Chapter 5.3.2.

5.4.2 Comparison of Displacement Discontinuity and 3D Numerical Modelling

A comparison was made between the three dimensional average pillar stress results from the BEAP runs in Chapter 5.3.2 and the average pillar stress predicted by MINTAB for the same stope geometries. The goal was to determine the influence that the seam thickness ratio has on the accuracy of displacement discontinuity modelling. The 12 stope geometries for the four tests are summarized in table 7. This table shows the stope and pillar dimensions, the seam thickness ratio for each geometry
and the average pillar stress for each BEAP and MINTAB run (average pillar stress increase is calculated as the ratio of the average pillar stress to the pre-mining stress).

The difference between the two models for the various seam thickness ratios is given in figure 32. A very rough estimate of the maximum difference between MINTAB and BEAP is shown in figure 32. This dashed envelope is based on the absolute magnitude of the difference (for all the points), and plotted as a mirror image above and below the 0% line. In the majority of the tests, there is little difference between the average pillar stresses predicted by BEAP and MINTAB. At a seam thickness of 1.0, there is less than 10% difference for all five tests. There is less than a 5% difference for the five tests having a seam thickness ratio equal to or greater than 3.0. Overall, only one test showed a difference of greater than 10%. However, there are only two tests with a seam thickness ratio of less than one. Many more tests are needed before any conclusions can be drawn about the ability of MINTAB to model stope and pillar geometries with low seam thickness ratios.

Considering the minimum seam thickness ratios of 3 and 5 suggested by Crouch and Brady, the difference in average pillar stress between BEAP and MINTAB is much less than expected. Reasons why these authors suggest conservative seam thickness ratios may be:

- a high level of agreement between the DD and 3D solutions was sought in the analyses done by Crouch and Brady,
COMPARISON: DD AND 3D NUMERICAL METHODS

INFLUENCE OF THE SEAM THICKNESS RATIO

SEAM THICKNESS RATIO (LENGTH:BREADTH)

- 3D TESTS
- using the average of several elements to determine the average pillar stress has the effect of "smoothing out" large differences at individual elements in the pillar,
- or the open stope rib pillar geometries analyzed in the 12 tests are much simpler and more amenable to DD numerical modelling than the excavation geometries analyzed in the verifications by Crouch and Brady.

While complex mining geometries have not been investigated, the results of the comparison suggest that using a seam thickness ratio of three will give very good agreement between MINTAB and BEAP for open stope rib pillars. Further checks of the influence of the seam thickness ratio will be done in Chapter 5.5 using case histories from the data base.

5.5 Pillar Load Calculations for the Open Stope Data Base

There is no absolute method that can determine the average stress or load in a mine pillar. As discussed above, and in Chapter 3, linear elastic numerical modelling can often give consistent approximations of the pre-failure load in hard rock mine pillars. For pillars that have a sloughing or deteriorating condition, load determined by linear elastic numerical modelling may be a considerable overestimate. This can be attributed to the local loss of load bearing capacity due to rock fracturing and pillar deformation. For failed pillars, the linear elastic load will not be representative of the stress conditions. A failed pillar will have lost some, or nearly all
of its load bearing capacity, resulting in stress redistribution into nearby competent pillars or abutments. The inability of linear elastic modelling to determine an approximate load for sloughing and especially failed pillars presents difficulties in developing a reliable method of predicting pillar failure.

5.5.1 Assumptions

In order to set a consistent method for determining loading conditions for all pillar assessments, it will be assumed that pillars are infinitely elastic in their deformation characteristics. This means that pillars will not lose their load bearing capacity regardless of their physical condition. While not being technically accurate to the actual problem, this assumption will permit the investigation of the stress and geometrical conditions that existed before failure and a rudimentary look at the conditions that have resulted in failure of open stope pillars. Ultimately, it will provide the basis for predicting conditions that are associated with pillar failure.

5.5.2 Pillar Load Results

The ability of BITEM and MINTAB to model each problem geometry in the data base was evaluated. If a program could not adequately account for the excavations affecting the stress conditions of the pillar, numerical analysis was not done. This situation occurred for BITEM when the geometries of all the
significant excavations could not be included in the plane of the problem. MINTAB was not used to investigate a stope and pillar geometry when en-echelon stopes were part of the problem geometry, or the orebody had significant changes in thickness or significant changes in direction. For each case history, Table 8 shows:

- the pre-mining stress normal to the orebody,
- the limiting geometrical ratios associated with the applicability of MINTAB (the seam thickness ratio) and BITEM (the stope height to length ratio),
- the average stress predicted for the pillar by each numerical method and the best estimate of the average pillar stress,
- the estimated error associated with the best load due to assumptions associated with modelling three dimensional stope and pillar geometries with numerical methods that are not three dimensional,
- the average pillar load calculated using the tributary area theory (chapter 3.1.2.1),
- and the error in the tributary area load compared to the numerically determined load.

The best estimate of the average pillar load was chosen based on the limiting ratios for BITEM and MINTAB. If a case history had a high stope length to stope width ratio, the BITEM load was used. If a case history had a high seam thickness ratio, the MINTAB load was used. If the stope geometry did not
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<th>BITEM STRESS (MPa)</th>
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<th>% ERROR</th>
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<td>58</td>
<td>34%</td>
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<td>57</td>
<td>0.8</td>
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<td>1.8</td>
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<td>0.7</td>
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<td>NA</td>
<td>7.0</td>
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<td>0.7</td>
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<td>36</td>
<td>&lt;10%</td>
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<td>NA</td>
<td>SLOUGH</td>
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<tr>
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<td>18</td>
<td>5.6</td>
<td>43</td>
<td>3.0</td>
<td>46</td>
<td>46</td>
<td>&lt;10%</td>
<td>50</td>
<td>82%</td>
<td>STABLE</td>
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<td>55</td>
<td>18</td>
<td>5.6</td>
<td>43</td>
<td>3.0</td>
<td>46</td>
<td>46</td>
<td>&lt;10%</td>
<td>54</td>
<td>17%</td>
<td>SLOUGH</td>
</tr>
<tr>
<td>56</td>
<td>30</td>
<td>1.1</td>
<td>59</td>
<td>0.4</td>
<td>48</td>
<td>59</td>
<td>25-45%</td>
<td>69</td>
<td>17%</td>
<td>SLOUGH</td>
</tr>
<tr>
<td>57</td>
<td>30</td>
<td>5.8</td>
<td>38</td>
<td>0.2</td>
<td>46</td>
<td>46</td>
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<td>45</td>
<td>18%</td>
<td>SLOUGH</td>
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<tr>
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<td>30</td>
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<td>40</td>
<td>0.2</td>
<td>45</td>
<td>45</td>
<td>&lt;10%</td>
<td>48</td>
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<td>0.8</td>
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<td>0.2</td>
<td>54</td>
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<td>95</td>
<td>31%</td>
<td>SLOUGH</td>
</tr>
<tr>
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<td>30</td>
<td>0.6</td>
<td>82</td>
<td>0.7</td>
<td>53</td>
<td>53</td>
<td>&gt;45%</td>
<td>119</td>
<td>65%</td>
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<td>NA</td>
<td>70</td>
<td>70</td>
<td>&lt;10%</td>
<td>88</td>
<td>25%</td>
<td>SLOUGH</td>
</tr>
</tbody>
</table>

**TABLE 8.** Pillar load information for all the open stope rib pillar case histories using BITEM, MINTAB and the Tributary Area Theory.
fit either limiting ratio, the BITEM load was used. BITEM is used to estimate the average pillar stress in these situations because it accounts for the geometry of these problems better than MINTAB, and the error associated with BITEM (from figure 30), is better understood than the error associated with MINTAB (from figure 32).

The error associated with the best load is based on the comparisons of MINTAB and BITEM to a true three dimensional numerical method presented in chapters 5.3.2 and 5.4.2. The results in Table 8 show that stable case histories and primary stoping geometries tend to have a lower degree of error associated with the predicted load. This is because primary stoping geometries are more regular than secondary and tertiary geometries and consequently fit the modelling constraints (ie. the limiting ratios) better. The error in the best pillar load is an estimation of the maximum possible error based on figures 30 and 32. The actual error is smaller for many of the pillars. For this reason, the load applied in the development of a pillar design method will not be adjusted for the estimated error.

Table 8 shows that the tributary area theory has highly varied results compared to the best load estimated by numerical modelling. It can be assumed that tributary area overestimates the average pillar load. It is also apparent that the greater the stope height:length ratio of the case history geometry, the better the agreement between the tributary area theory load and the load predicted by numerical modelling. In general, the
overestimation of the predicted stress makes the tributary area theory very unreliable in the estimation of the average load in open stope rib pillars.

5.5.3 Numerical Model Comparison Using the Case Histories

Chapters 5.3 and 5.4 gave a detailed comparison of two dimensional and displacement discontinuity numerical modelling against a three dimensional method. Analysis of the data base case histories provides further information for comparison.

Two case histories fit the MINTAB seam thickness ratio limitation of 3 or greater and also have a large height to length ratio (greater than 4) making good BITEM cases. For both of these case histories, Table 9 shows the stope height to length ratio, the seam thickness ratio, the average pillar stress predicted by BITEM and MINTAB and the difference in the predicted stress. This comparison shows good agreement between the average pillar load for the two methods when a stope and pillar geometry meets both of the limiting ratios.

<table>
<thead>
<tr>
<th>CASE NUMBER</th>
<th>BITEM</th>
<th>MINTAB</th>
<th>PERCENT DIFFERENCE BETWEEN BITEM AND MINTAB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HEIGHT: LENGTH RATIO</td>
<td>AVERAGE PILLAR LOAD (MPa)</td>
<td>SEAM THICKNESS RATIO</td>
</tr>
<tr>
<td>54</td>
<td>5.6</td>
<td>43</td>
<td>3.0</td>
</tr>
<tr>
<td>55</td>
<td>3.4</td>
<td>44</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Table 9. Comparison of MINTAB and BITEM results when both programs limitations are satisfied.
Three case histories satisfy the MINTAB seam thickness constraint, but do not have a large height to length ratio. These are good MINTAB geometries, but not favorable for BITEM modelling. BITEM will overestimate the average pillar load. For each of these case histories, Table 10 shows the stope height to length ratio, the seam thickness ratio, the average pillar stress predicted by BITEM and MINTAB and the overestimation by BITEM of the MINTAB predicted pillar load. When these three cases are compared against the BITEM overestimation of BEAP graph developed in Chapter 5.3.2, they plot slightly above the maximum over-estimation documented in chapter 5.3.2 (see figure 33). However, considering a potential error of up to 10% for the MINTAB case histories, the results are not very far above the limit found in Chapter 5.3.2.

<table>
<thead>
<tr>
<th>CASE NUMBER</th>
<th>BITEM</th>
<th>MINTAB</th>
<th>PERCENT DIFFERENCE BETWEEN BITEM AND MINTAB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HEIGHT: LENGTH RATIO</td>
<td>AVERAGE PILLAR LOAD (MPa)</td>
<td>SEAM THICKNESS RATIO</td>
</tr>
<tr>
<td>18</td>
<td>2.0</td>
<td>90</td>
<td>3.0</td>
</tr>
<tr>
<td>19</td>
<td>1.7</td>
<td>91</td>
<td>3.3</td>
</tr>
<tr>
<td>31</td>
<td>0.9</td>
<td>60</td>
<td>3.8</td>
</tr>
</tbody>
</table>

Table 10. Comparison of BITEM and MINTAB, when the MINTAB limitation is met, but the BITEM limitation is not met. The overestimation by BITEM is in the range estimated in Chapter 4.
FIGURE 3. Overestimation of average pillar load by the 2D "BITEM" boundary element method for the comparison tests and 3 case histories, and 3D tests.
Many of the case histories investigated had a large stope height to length ratio (making them good geometries for BITEM modelling), but do not fit the seam thickness criterion needed for accurate MINTAB modelling. By using both numerical methods, the effect of a low seam thickness ratio can be compared against the satisfactory pillar load results given by BITEM. Table 11 shows the stope height to length ratio, the seam thickness ratio, the BITEM and MINTAB average pillar stress, and the difference in the average pillar stress for thirteen different geometries. The results of the MINTAB analysis vary up to ± 25 % with the BITEM results. For the geometries with larger seam thickness ratios (≥1.0 but <3.0), the difference in average pillar stress between the two methods is less. The maximum difference in pillar load is slightly higher than the 12 runs in Chapter 5.4.2, when plotted on the graph of percent difference in pillar stress versus seam thickness ratio (see figure 34). The envelope showing the maximum error has been redrawn in figure 34.

5.6 Chapter Summary

The three boundary element models (BITEM, MINTAB and BEAP), used in investigating open stope rib pillar load, have been briefly described. Conventions for defining open stope rib pillar geometries and determining the average pillar stress have been presented.

The use of three dimensional boundary element modelling is
<table>
<thead>
<tr>
<th>CASE NUMBER</th>
<th>BITEM</th>
<th>MINTAB</th>
<th>PERCENT DIFFERENCE BETWEEN MINTAB AND BITEM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HEIGHT: LENGTH RATIO</td>
<td>AVERAGE PILLAR LOAD (MPa)</td>
<td>SEAM THICKNESS RATIO</td>
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<tr>
<td>7</td>
<td>4.5</td>
<td>55</td>
<td>0.6</td>
</tr>
<tr>
<td>8</td>
<td>4.5</td>
<td>69</td>
<td>0.6</td>
</tr>
<tr>
<td>17</td>
<td>4.0</td>
<td>29</td>
<td>0.9</td>
</tr>
<tr>
<td>21</td>
<td>4.0</td>
<td>28</td>
<td>0.7</td>
</tr>
<tr>
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<td>26</td>
<td>1.0</td>
</tr>
<tr>
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<td>38</td>
<td>1.1</td>
</tr>
<tr>
<td>34</td>
<td>6.3</td>
<td>31</td>
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</tr>
<tr>
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<td>75</td>
<td>0.5</td>
</tr>
<tr>
<td>44</td>
<td>5.0</td>
<td>76</td>
<td>0.7</td>
</tr>
<tr>
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<td>5.0</td>
<td>102</td>
<td>0.6</td>
</tr>
<tr>
<td>57</td>
<td>5.8</td>
<td>38</td>
<td>0.2</td>
</tr>
<tr>
<td>58</td>
<td>4.4</td>
<td>40</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 11. Comparison between good BITEM and poor MINTAB geometries shows the average pillar stress varying up to ± 25%. 

FIGURE 34. The difference between the average pillar stress predicted by MINTAB and the average pillar stress predicted by BEAP for the comparison tests and 13 case histories.
not possible for the case histories in the data base. This is due to: high program set up and run times, and program space limitations. The 2D plane strain and pseudo-3D displacement discontinuity (DD) methods have been used to estimate the load for each pillar case history. Both of these programs have geometrical limitations that may introduce error into the average pillar load. The geometrical limitations have been described and the error associated with 2D plane strain and DD methods has been quantified using 12 test runs and some case histories from the data base. Figure 33 shows the potential error associated with 2D plane strain modelling for open stope rib pillar geometries. Figure 34 shows the potential error associated with the displacement discontinuity method for open stope rib pillar geometries.
CHAPTER 6
DEVELOPMENT OF A PILLAR DESIGN METHOD

It was stated in Chapter 3 that a design method for open stope rib pillars has not been developed or confirmed. Other authors have shown that the best way to develop and verify a design procedure is to conduct a survey and confirm a method with case histories. There are many examples of pillar design studies, the most notable being: Salamon (1967) in South African coal mines, Hedley and Grant (1972) in Canadian hard rock room and pillar mining, and Bieniawski (1983) in United States coal mines. Each of these studies used experience and calibration to develop a method for mining specific conditions.

The number of mines visited in the "Integrated Mine Design Study" has resulted in the collection of a substantial amount of data of stable and failed rib pillars from Canadian open stope mines. This data will be used to develop an empirical design method for rib pillars in open stope mining. In addition, a wealth of data from hard rock room and pillar mines has been found in literature to help confirm the new empirical method.

Specifically, the intention of this chapter is to:
- verify the variables significant in open stope rib pillars based on the data available,
- present a method that explains the results of the case histories in the data base,
- use case histories from literature (mostly from room and
pillar mining), to verify the design concept and refine the method,
and compare the new method to some of the open stope design procedures commonly used in the past.

6.1 Choice of Variables

Chapter 2.3 discussed variables that may be significant in the failure of open stope rib pillars. These variables were: intact rock strength, pillar load, pillar shape and confinement, structural discontinuities, and pillar volume. They will be quantified through the use of:
- uniaxial compressive strength for intact rock strength,
- boundary element numerical modelling to determine pillar load,
- pillar height and width to account for pillar shape and confinement,
- empirical rock mass classification methods to account for structural discontinuities,
- and the pillar dimensions (from table 5, page 70) can be used to determine the pillar volume.

No attempt will be made to quantify the effect of backfill. In Chapter 2, backfill was not considered significant in preventing the failure of pillars, although its presence may have a large influence in preventing pillar disintegration if failure occurs.

6.1.1 Applicability of Statistical Methods
Ideally, the data base presented in Chapter 4.2 could be used to test the significance of each variable in the stability of pillars. Some of the variables are obviously significant. Pillar load, pillar width and the strength of the intact rock are known to have a large influence on the stability of a pillar. However, the influence of pillar height, pillar volume and minor rock mass discontinuities (such as joints) in open stope rib pillars is not obvious. The use of statistics to test the significance of these three variables was considered, but was later rejected for a couple of reasons.

The first reason is the assessment of pillar stability can not be quantified into a numerical value. The pillar case histories were assessed with qualitative categories of stable, sloughing and failed. These categories limit the use of regression and factorial design methods, because the categories can not be quantified numerically. A system of giving the stable, sloughing and failed assessments an arbitrary numerical value and using regression techniques on these values also would not work well. The wide range of instability signs and characteristics that are exhibited by the failed pillars can not be quantified by a single arbitrary value and there is no satisfactory criterion to determine a representative value for failed pillars.

The second reason why the use of statistics is not feasible is related to the yielding pillar case histories. These are pillars that were originally stable but eventually became
unstable due to stopes or pillars being mined in the vicinity or robbing of the pillar. For the yielding pillars, the uniaxial compressive strength (UCS), pillar height, rock mass characterization and pillar volume do not change significantly from the stable to failed cases. Consequently, a statistical method would find that these variables have no significant influence of pillar condition. The only variables that change significantly for yielding case histories are the average pillar stress and the pillar width. Removing the yielding pillars from the data base reduces the number of case histories to 12 stable pillars, 3 sloughing pillars, and 1 failed pillar. This is too small a data base to reach confident statistical conclusions about significant variables.

The last major problem with using statistical methods in the data base, is the lack of precision in the estimation of some of the data. Chapter 5.5 discusses the determination of average pillar load for each case history. The potential error associated with this variable varies from less than 10% to greater than 45% (see Table 8, page 105) and implies that a large degree of accuracy should not be used. It is not a precise variable and would present significance problems if included in a statistical technique.

6.1.2 Design Variables

The most important variables in open stope pillar design are pillar width and the average pillar load. There is more
flexibility in choosing and designing these two variables than any of the others. The intact rock uniaxial compressive strength, rock mass quality and pillar height (orebody width) are all a function of the geological setting and can not be controlled or changed. Pillar width has a large influence on the lateral confinement of the pillar core, the pillar stiffness, and the modulus of deformation of failing pillars. The magnitude of pillar load has a direct influence on the degree of fracturing in a pillar. However, both of these variables need to be normalized before information from different mining conditions can be compared. Pillar load is frequently normalized by comparing it against the intact rock strength (discussed in Chapter 3.2.2.3). This gives a good measure of the state of stress and fracturing in a pillar. Pillar width is typically normalized through the use of the ratio of the pillar width/height. Pillar width/height is used by many authors to account for the effects of pillar shape (see Chapter 3.1.1).

6.1.3 Discounted Variables

Two variables have been discounted for design. Pillar volume and the influence of geological discontinuities may be significant in general pillar design, but their importance has not been proven for open stope pillars. Using methods proposed by other authors and information from the data base, it will be shown that the two discounted variables have a relatively small
variation in magnitude in the open stope pillar data base, and consequently could only have a minor effect on pillar stability.

6.1.3.1 Pillar Volume

Several authors (Hoek and Brown 1980; Agapito and Hardy 1982; Stephansson 1985) have proposed the use of a factor to account for the effect of pillar volume. The reasoning was the rock mass strength decreases with an increase in pillar volume, due to a larger number of flaws and discontinuities in the rock mass. Consequently, the volume effect is an indirect means of accounting for the effect of discontinuities.

Agapito and Hardy (1982) suggested the following equation to relate the unconfined uniaxial compressive strength from laboratory testing with in situ unconfined compressive pillar strength:

\[ \sigma_O = \sigma_C \left( \frac{V_1}{V_I} \right)^a \]

where,

- \( \sigma_O \) = unconfined compressive strength of the pillar,
- \( \sigma_C \) = average laboratory uniaxial compressive strength,
- \( V_1 \) = volume of the laboratory specimen,
- \( V_I \) = volume of the pillar,
- \( a \) = coefficient of volume reduction,
  - = 0.12 for coal,
  - = 0.08 for oil shale,
  - = 0.06 for good quality, hard quartzite.

Using the formula, we can compare the influence of the variation of pillar volumes in the data base. For this data
base, the smallest open stope pillar has a volume of about 2500 cubic metres, and the largest open stope pillar has a volume of about 150,000 cubic metres.

\[
\frac{\sigma_{2,500}}{\sigma_{150,000}} = \frac{\sigma_C (V_1 / 2500)^{0.06}}{\sigma_C (V_1 / 150000)^{0.06}} = 1.28
\]

So, for the full range of pillar volumes in the data base, this formula shows only a small influence (less than 30%).

The lack of sensitivity of volume is only part of the problem with using this coefficient of volume reduction method. Any method to account for the influence of flaws or discontinuities in a rock mass should be based on an assessment of the quality of the rock mass. The frequency, orientation, continuity and shear strength of discontinuities in a rock mass should be considered when estimating the effect of discontinuities. This formula does not consider any rock mass characteristics and as a result, it does little to account for the influence of discontinuities in pillar strength.

6.1.3.2 Structural Discontinuities

As mentioned above, to account for the influence of geological discontinuities in pillar strength, the characteristics of the rock mass must be quantified. Currently, the most effective method of describing a rock mass is with empirical rock mass classifications. The two most common classifications are the NGI system, developed by Barton, Lien
and Lunde of the Norwegian Geotechnical Institute (1974), and the CSIR system, developed by Bieniawski of the South African Council for Scientific and Industrial Research (1976).

Data for the CSIR rock mass classification was collected in the "Integrated Mine Design Study." Herget et al. (1984) and Stacey and Page (1986) suggest using rock mass classifications as strength reduction factors by applying them against the uniaxial compressive strength of rock. For instance, if the in situ intact rock strength is $\sigma_0$ and the rock mass has a CSIR rock mass rating of 75%, then the in situ rock mass strength is $(0.75 \times \sigma_0)$.

Table 5 (page 70) shows the CSIR geomechanics rating "RMR" (acronym for rock mass rating) for the pillars in the open stope data base. The mean RMR is 69.6, with a standard deviation of 4.8. This small range in rock mass ratings is not unrealistic because the source of the majority of the information in the data base is mines in the Canadian shield. The classification methods are designed to characterize a much wider range of rock masses. With this small a range of rock mass quality, however, it is not possible to verify that the inclusion of a rock mass strength reduction factor would adequately account for any influence of discontinuities in the design of open stope rib pillars.

Using a strength reduction variable could be an effective method to account for the influence of structural discontinuities in a rock mass. However, the available data
could only prove this over a small range of rock mass conditions. Rather than include a variable whose influence can not be effectively calibrated or verified, the effect of structural discontinuities has been omitted. A large amount of data from a much wider variety of rock mass conditions is needed to confirm and calibrate the significance of a strength reduction factor.

6.2 Pillar Stability Graph

The methodology for the development of an open stope rib pillar design criterion is based on the graphical comparison of the significant variables discussed above and the assessment of pillar case histories. The y-axis of the graph has been chosen to represent the normalized pillar load, while the x-axis is defined by the pillar width to pillar height ratio. Stable pillars from the data base are plotted with square symbols, sloughing pillars are represented by cross shaped symbols, and failed pillars are located with diamond symbols (see figure 35). By arranging the graph in this form (and not including correction factors for volume and rock mass quality), the graph stays intuitively simple. The influence of varying the design variables is clear-cut and explicit. This graph will be referred to as the "pillar stability graph".

6.2.1 Graphical Data Analysis

Comparison of the shape and the loading condition of
FIGURE 35. The pillar stability graph showing the open stope rib pillar data base.
pillars, using the pillar stability graph, exposes a trend in rib pillar behaviour. The graph shows squat pillars under low stress conditions as stable (bottom right region of the graph in figure 35). Pillars become less stable as their graphical position is located more towards the upper left corner of the graph, which represents highly stressed, slender, and failure prone pillars.

The graph has be divided into two zones based on this data (see figure 36). The upper left side of the graph denotes conditions in which pillars have failed. The bottom right side of the graph shows conditions in which pillars have not suffered any serious instability. The two zones are separated by a transition area. The location of this area has been approximated based on the graphical location and physical condition of the case histories. No statistical methods have been used to locate the transition area. The bottom line of the transition area corresponds to the region where major pillar stability problems are first encountered. Only one sloughing pillar, no failed pillars, and all but four of the stable pillars plot below this line. This bottom line does not necessarily signify pillar failure, but rather the onset of mining problems due to pillar instability. Sloughing or deteriorating pillars could carry an even greater load (as reported by Goel and Page 1981), but displacement, rock fracturing and pillar deformation will increase. The top line roughly defines a criterion where pillar failure has been
FIGURE 36. The pillar stability graph showing the stable and failed zones and the transition area.
observed in the case histories of the data base. No stable case histories, four of the nine sloughing pillars and all but three of the failed pillars are found above this line. Pillars plotting above this line generally have:

- started to lose load bearing capacity,
- suffered a large amount of fracturing,
- experienced large displacements of rock,
- and had severe sloughing of pillar walls (unless confined by backfill).

In regions of the graph where sufficient rib pillar data is not available to locate the transition zone, it has been approximated with dashed lines.

6.2.2 Influence of Pillar Load Approximations

In Chapter 5.5.2, the maximum error in the average pillar load was estimated for each case history. To check the influence of this error, the average pillar load is decreased by the maximum amount of the error shown in Table 8 (page 105). The reason for the decrease is that the majority of pillar loads are estimated by BITEM, which overestimated the actual pillar load. Data in which the error could not be reasonably estimated were omitted. This occurred for 6 of the 47 data points.

Figure 37 is a plot of the pillar stability graph using the reduced average pillar load with the original transition area. The modified data still fits the graph well, with only three sloughing cases and one failed case below the transition zone.
FIGURE 37. The pillar stability graph with the pillar load reduced for all the data points by the maximum amount listed in Table 8. The reduction in load does not have a large influence on the location of the transition zone.

PILLAR STABILITY GRAPH
OPEN STOPE RIB PILLAR DATA

LOAD/UCS

0.60

0.50

0.40

0.30

0.20

0.10

0.00

0.0

0.4

0.8

1.2

1.6

2.0

2.4

PILLAR WIDTH/PILLAR HEIGHT

□ STABLE

+ SLoughing

○ FAILURE

FAILED

STABLE
It should be kept in mind that the adjusted load was decreased by an estimate of the maximum error, and most cases will have an error smaller than the maximum.

We can conclude that the error in the average pillar load does not significantly change the method proposed. It also demonstrates the fact that the pillar loading conditions has less of an effect on pillar stability than the pillar shape (width/height ratio).

6.2.3 Importance of Yielding Pillar Case Histories

As discussed in the data base description (Chapter 4.1), there are 13 pillars that were stable and subsequently failed due to mining. These pillars comprise 30 of the 47 case histories in the open stope data base. The yielding pillar case histories are very useful in developing a design method because the stable and failed cases should plot in their respective zones separated by the transition area. Figure 38 is a plot of the entire data base with the stages of each yielding pillar joined by a solid line. The yielding pillar endpoints correspond well to the stable and failed zones which reinforces the location of the transition area. As a pillar fails, its location moves from the stable zone, through the transition area, and into the failed zone. The yielding pillars also demonstrate the sensitivity of the graph to predict pillar failure.
FIGURE 38. The pillar stability graph with all the case histories of the 13 yielding pillars joined by solid lines. This reinforces the location of the transition zone and shows the sensitivity of the method to predict failure.
6.2.4 Limitations of the Pillar Stability Graph

There are a few comments to be made concerning the limitations of the pillar stability graph. Firstly, the data in and near the transition zone shows a variety of behaviour. This suggests that a great degree of precision is not inherent to the graph. This lack of precision is a function of inaccuracy in the input data and the broad assessments used to categorize pillars. The size of the transition zone could be considered a measure of the accuracy of the pillar stability graph.

It should be emphasized that this is an empirically developed relationship and is more reliable when applied in conditions similar to those in the data base. Specifically, the range of the various data is:

\[
70 \text{ MPa} < \text{UCS} < 316 \text{ MPa},
\]
\[
9 \text{ metres} < W_p < 45 \text{ metres},
\]
\[
60 < R_M R < 78
\]

where,

\[
\text{UCS} = \text{the intact rock uniaxial compressive strength},
\]
\[
W_p = \text{the pillar width},
\]
\[
R_M R = \text{a measure of the rock mass competency using the CSIR rock mass classification}.
\]

A final note about the pillar stability graph is that there are almost no stable pillars with an (average pillar load/UCS) ratio greater than 0.5, and very few stable pillars with an (average load/UCS) ratio greater than 0.33. This suggests that there is a practical limit to the maximum normalized load for a
stable open stope rib pillar. These values correspond well with suggestions by Mathews et al. (1980) and Bawden et al. (1988), of the maximum normalized major principal stress allowable before stress related mining problems become excessive.

6.3 Data from Literature

Very few open stope pillar case histories found in literature provide sufficient information that they can applied to the pillar stability graph. Fully documented room and pillar mining case histories are more common. Three studies of hard rock pillar design have been found which contain the pillar load, uniaxial compressive rock strength, pillar shape information and an assessment of the pillar stability. The two largest studies deal with room and pillar mining while the third is a smaller and more detailed study that deals with open stope rib pillar design.

6.3.1 Data from Canadian Room and Pillar Mining

In the 1960's, a major rock mechanics investigation was undertaken in the Elliot Lake uranium mining district to determine stable stope and pillar configurations. One of the results was a pillar strength formula (described in Chapters 3.1.1.3, and 6.4.1). The details of the formula development and the data base were published by Hedley and Grant (1972). Their data base consisted of 23 stable pillars, 2 pillars that were partially failed, and 3 pillars that were crushed. Pillars in
the uranium mines are very long in one direction which is the same shape as pillars in open stope mines. However, the pillar dimensions and volume are substantially lower in room and pillar mining.

Using the data in the paper (Hedley and Grant 1972), the case histories were plotted on the pillar stability graph (see figure 39). The Elliot Lake data fits the pillar stability graph quite well with all of the stable pillars plotting below the transition area, and most of the partially failed and crushed pillars plotting in the transition area. Ideally, for this data, the transition zone would probably be slightly lower. This would give a better separation between the stable and unstable pillars. However, there is not sufficient data near the transition zone to warrant adjusting its location.

The rock mass quality for the Elliot Lake mines is similar to that found in the "Integrated Mine Design Study". A discussion on pillar stability at the Denison Mine (Townsend 1982), which is one of the mines in Hedley's study, gives the pillars an NGI rock mass quality of 45. This is roughly equivalent to a CSIR rating of 78, based on a relationship proposed by Bieniawski (1976). An RMR of 78 is within the range of the rock mass qualities found in the open stope pillar data base. Due to the variable nature of a rock mass, it is wrong to assume an RMR of 78 for all pillars in the Elliot Lake data base. However, it can be concluded that the general rock mass conditions between the two data bases are similar.
FIGURE 39. The pillar stability graph showing the data from room and pillar mining published by Hedley and Grant (1972) in their study on the development of a pillar strength formula.
An interesting observation can be made concerning the influence of pillar volume. The volume of an average pillar in the Elliot Lake data base is approximately 25 to 50 times smaller than the average volume of the open stope data base ($\approx 1000 - 2000 \text{ m}^3$ for room and pillar, and $\approx 50,000 \text{ m}^3$ for open stoping). A relative increase of pillar strength due to the smaller volume should raise the relative position of the transition zone. This does not correspond with the cases of partially failed and crushed Elliot Lake pillars. According to the Hedley data, the transition zone should probably be slightly lower. Based on this observation, there appears to be little difference in the influence of pillar volume between pillars in open stope and room and pillar mining.

6.3.2 Data from a Botswana Room and Pillar Mine

A paper by Von Kimmelmann et al. (1984) discussed the development of a pillar design criterion at BCL Limited in Botswana. Back analysis of a large number of existing pillars was performed using the pseudo-three dimensional displacement discontinuity numerical method.

Pillar deterioration was assessed with the following criterion:

"Group A (intact pillars) displayed minor spalling particularly associated with any overbreak into the hanging wall or footwall gneisses. No joint opening was observed. Group B pillars exhibited prominent spalling generally
associated with structural features. Slight opening of the joints into the pillar was also noted.

Group C pillars displayed severe spalling of intact rock, pronounced opening of joints and deformation of drill holes."

The Group A assessment corresponds reasonably well with stable pillars, Group B with sloughing pillars, and Group C with failed pillars. Table 12 gives the pillar classification, pillar shape, pillar load, and remarks on the pillar stability for the complete data base presented by Von Kimmelmann (1984).

Two different types of pillars were investigated. Pillars that were near square (when viewed in plan) and pillars that were very long in one dimension (see figure 40). The long pillars were applied directly to the pillar stability graph (see figure 41). Using the transition zone for the open stope pillars, one stable pillar is above the transition zone and five stable case histories are in the transition zone. These case histories suggest the transition zone could be located slightly higher making the current pillar stability graph a bit conservative for this data.

The square pillars can not be directly applied to the pillar stability graph. Several authors have noted that rectangular pillars are significantly stronger than square pillars (Wagner 1974; Salamon 1983; Kersten 1984; Stacey and Page 1986). To account for the difference during design, these authors have suggested the use of an effective pillar width:
### Classification of Square Pillars

<table>
<thead>
<tr>
<th>PILLAR NO.</th>
<th>CLASSIFICATION</th>
<th>W/H</th>
<th>ESTIMATED PILLAR STRESS (MPa)</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B</td>
<td>0.80</td>
<td>28</td>
<td>Opening of Joints</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>1.70</td>
<td>26</td>
<td>Joints tight</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>1.70</td>
<td>30</td>
<td>Minor spalling</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>1.24</td>
<td>34</td>
<td>Spalling in gneiss</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>1.00</td>
<td>34</td>
<td>Spalling in M.S. Joints tight</td>
</tr>
<tr>
<td>6</td>
<td>B</td>
<td>1.30</td>
<td>35</td>
<td>Fractured M.S. Assoc. with jointing</td>
</tr>
<tr>
<td>7</td>
<td>C</td>
<td>1.20</td>
<td>55</td>
<td>Severe spalling</td>
</tr>
<tr>
<td>8</td>
<td>C</td>
<td>0.96</td>
<td>55</td>
<td>Severe spalling</td>
</tr>
<tr>
<td>9</td>
<td>C</td>
<td>1.00</td>
<td>58</td>
<td>Severe spalling</td>
</tr>
<tr>
<td>10</td>
<td>C</td>
<td>1.50</td>
<td>58</td>
<td>Severe spalling &amp; opening of Joints</td>
</tr>
<tr>
<td>11</td>
<td>C</td>
<td>0.50</td>
<td>58</td>
<td>Failed pillar</td>
</tr>
<tr>
<td>12</td>
<td>C</td>
<td>1.26</td>
<td>53</td>
<td>Marked hangingwall deterioration</td>
</tr>
<tr>
<td>13</td>
<td>B</td>
<td>1.40</td>
<td>58</td>
<td>Hangingwall deterioration</td>
</tr>
<tr>
<td>14</td>
<td>C</td>
<td>1.40</td>
<td>58</td>
<td>Severe spalling</td>
</tr>
<tr>
<td>15</td>
<td>C</td>
<td>0.76</td>
<td>50</td>
<td>Slabbing Assoc. with fault</td>
</tr>
<tr>
<td>16</td>
<td>A</td>
<td>1.10</td>
<td>37</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>B</td>
<td>1.74</td>
<td>40</td>
<td>Spalling Assoc. with structural features</td>
</tr>
<tr>
<td>18</td>
<td>A</td>
<td>2.50</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>C</td>
<td>0.60</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>C</td>
<td>0.90</td>
<td>48</td>
<td>Severe spalling Assoc. with deterioration of hangingwall</td>
</tr>
<tr>
<td>22</td>
<td>C</td>
<td>0.60</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>C</td>
<td>0.60</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>B/C</td>
<td>1.32</td>
<td>55</td>
<td>H/W instability + deformed boreholes</td>
</tr>
<tr>
<td>25</td>
<td>B</td>
<td>1.50</td>
<td>47</td>
<td>Spalling</td>
</tr>
<tr>
<td>26</td>
<td>B</td>
<td>1.67</td>
<td>48</td>
<td>Joints opening</td>
</tr>
<tr>
<td>27</td>
<td>A</td>
<td>1.60</td>
<td>35</td>
<td>Minor spalling in footwall gneiss</td>
</tr>
<tr>
<td>28</td>
<td>A</td>
<td>2.00</td>
<td>35</td>
<td>Minor spalling</td>
</tr>
<tr>
<td>29</td>
<td>C</td>
<td>1.00</td>
<td>59</td>
<td>Severe spalling</td>
</tr>
<tr>
<td>30</td>
<td>C</td>
<td>1.00</td>
<td>59</td>
<td>Assoc. with bad hangingwall conditions</td>
</tr>
<tr>
<td>31</td>
<td>C</td>
<td>1.00</td>
<td>59</td>
<td>Severe spalling</td>
</tr>
<tr>
<td>32</td>
<td>C</td>
<td>1.00</td>
<td>59</td>
<td>Failed</td>
</tr>
<tr>
<td>33</td>
<td>B</td>
<td>0.80</td>
<td>54</td>
<td>Large pillar</td>
</tr>
<tr>
<td>34</td>
<td>B/C</td>
<td>0.92</td>
<td>55</td>
<td>Assoc. with bad hangingwall conditions</td>
</tr>
<tr>
<td>35</td>
<td>B/C</td>
<td>1.20</td>
<td>54</td>
<td>Severe spalling</td>
</tr>
<tr>
<td>36</td>
<td>C</td>
<td>1.00</td>
<td>55</td>
<td>Severe spalling</td>
</tr>
<tr>
<td>37</td>
<td>B</td>
<td>0.92</td>
<td>55</td>
<td>Spalling and local slabbing</td>
</tr>
<tr>
<td>38</td>
<td>C</td>
<td>0.60</td>
<td>60</td>
<td>Failed</td>
</tr>
<tr>
<td>39</td>
<td>B/C</td>
<td>1.30</td>
<td>56</td>
<td>Spalling of gneiss overbreak</td>
</tr>
<tr>
<td>40</td>
<td>C</td>
<td>2.27</td>
<td>60</td>
<td>Severe spalling</td>
</tr>
<tr>
<td>41</td>
<td>C</td>
<td>2.22</td>
<td>63</td>
<td>Severe spalling</td>
</tr>
<tr>
<td>42</td>
<td>B/C</td>
<td>1.50</td>
<td>63</td>
<td>Severe spalling</td>
</tr>
<tr>
<td>43</td>
<td>C</td>
<td>2.00</td>
<td>59</td>
<td>Severe spalling</td>
</tr>
<tr>
<td>44</td>
<td>B</td>
<td>1.20</td>
<td>56</td>
<td>Spalling Assoc. with Joint opening</td>
</tr>
<tr>
<td>45</td>
<td>B/C</td>
<td>1.40</td>
<td>63</td>
<td>Prominent spalling in gneiss and M.S.</td>
</tr>
<tr>
<td>46</td>
<td>C</td>
<td>1.80</td>
<td>53</td>
<td>Spalling</td>
</tr>
<tr>
<td>47</td>
<td>A</td>
<td>2.60</td>
<td>60</td>
<td>Minor spalling</td>
</tr>
</tbody>
</table>

### Classification of Long Pillars (L≥W)

<table>
<thead>
<tr>
<th>PILLAR NO.</th>
<th>CLASSIFICATION</th>
<th>W/H</th>
<th>ESTIMATED PILLAR STRESS (MPa)</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>1.00</td>
<td>25</td>
<td>V. minor spalling</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>1.50</td>
<td>29</td>
<td>V. minor spalling</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>1.25</td>
<td>40</td>
<td>Joints opening</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>0.43</td>
<td>35</td>
<td>Spalling</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>0.40</td>
<td>50</td>
<td>Spalling</td>
</tr>
<tr>
<td>6</td>
<td>A</td>
<td>0.50</td>
<td>28</td>
<td>Minor spalling in M.S.</td>
</tr>
<tr>
<td>7</td>
<td>A</td>
<td>1.00</td>
<td>45</td>
<td>Slight movement on hangingwall contact</td>
</tr>
<tr>
<td>8</td>
<td>A</td>
<td>1.48</td>
<td>48</td>
<td>Minor spalling Assoc. with joints</td>
</tr>
<tr>
<td>9</td>
<td>A</td>
<td>1.30</td>
<td>50</td>
<td>No borehole deformation</td>
</tr>
<tr>
<td>10</td>
<td>A</td>
<td>1.26</td>
<td>47</td>
<td>Stable</td>
</tr>
</tbody>
</table>

**TABLE 12.** Data used by Von Kimmelmann et al. (1984) in the development of a pillar failure criterion.
FIGURE 40. A plan view of room and pillar mining at BCL Limited, showing the use of long pillars and square pillars (after Von Kimmelmann 1984).
FIGURE 41. The pillar stability graph showing the long pillar data presented by Von Kimmelmann et al. (1984).
\[ W_{\text{eff}} = \frac{4 \times A}{C} \]

where:

- \( W_{\text{eff}} \) = the effective pillar width,
- \( A \) = pillar cross sectional area,
- \( C \) = pillar circumference.

The reasoning is that for very long pillars (and open stope rib pillars), a pillar is effectively exposed on only two walls and consequently stronger than square pillars, which are exposed on four walls. Using this concept, square pillars have half the effective width of a long pillar having the same pillar height to width ratio. In figure 42, the square pillars at BCL Limited have been plotted on the pillar stability graph using their effective pillar width/height ratio (i.e. the actual width/height ratio).

The adjusted square pillar data agrees reasonably well with the original transition zone on the pillar stability graph. Three stable square pillar case histories plot above the failure line on the transition zone, while all the yielding and failed pillars plot above the transition area. The effective width adjustment for square pillars on the pillar stability graph adequately explains the assessment for this data. As with the long pillars, the adjusted square pillar data suggests the transition zone could be located slightly higher. However, this inaccuracy is on the conservative side for stable pillar design.

6.3.3 Data from an Australian Open Stope Mine
FIGURE 42. The square pillar data presented by Von Kimmelmann et al. (1984) is plotted on the stability graph using an effective width in the H/W ratio.
A test open stoping block at Mt. Isa is described in depth by Brady (1977). The objective of the trial mining block was to obtain information for rib pillar design and calibrate a failure criterion for the rock mass. Figure 43 shows the test broken into five stages.

Stage 1 (problem No. 1) shows the development of two slot raises, and the S86 raise to observe pillar conditions.
Stage 2 contains the opening of the S85 stope.
Stage 3 shows the opening of S87 stope which creates the S86 pillar.
Stage 4 is the expansion of the S87 stope, with the S86 pillar remaining stable and intact.
Stage 5 shows the robbing of the S86 pillar which resulted in failure of the pillar.

Brady presented sufficient information that stages 3, 4 and 5 could be modelled with BITEM to determine the average pillar load. The stope height:length ratio for all three cases is greater than 3, so less than 20% error is expected in the average pillar load determined by BITEM (the error is estimated using figure 33, page 109). The modelling results were in good agreement with a principal stress contour diagram in the original paper.

The rock mass, as described in the paper by Brady, has similar characteristics and quality to the typical rock mass
FIGURE 43. The five stages of the S86 pillar in an open stope pillar test at Mt. Isa (after Brady 1977).
found in the data base. The volume of the Australia pillars is also similar to that in the data base. So, these two variables are not likely to have a significant influence in plotting the data on the pillar stability graph.

The three stages are plotted on a pillar stability graph in figure 44. The S86 pillar plots in the stable region for stages 3 and 4, and plots in the failure zone after stage 5. This agrees very well with Brady's description of the pillar during the test.

6.3.4 Summary of All the Data

A plot of the open stope data and all the data from literature is given in figure 45. The data from literature helps confirm the location of the transition area over a greater area. In the entire data base of 135 pillars, four stable case histories are found above the transition zone and one sloughing pillar is found below the transition zone. Consequently, the solid design lines for the transition zone have been extended. The success of the pillar stability graph in separating the different pillar assessments supports the decision to discount the influence of pillar volume and rock mass quality as insignificant in hard rock pillar design.

6.4 Comparison Against Other Design Methods

A number of empirical design methods are frequently used for rib pillars. However, none of these methods was based on open
FIGURE 44. The third, fourth and fifth stages of the S86 open stope rib pillar, presented by Brady (1977), are shown on the pillar stability graph. The data agrees very well with the stability graph.
FIGURE 45. The pillar stability graph showing the open stope rib pillar data and the literature data presented by Hedley (1972), Von Kimmelmann (1984), and Brady (1977).
Stope mining case histories. The following comparison of the methods against the pillar stability graph, shows the applicability of the other methods to the design of open stope rib pillars. Any negative evaluation should not be taken as a criticism of other methods, but rather it serves to show the limitations of these methods when applied to the design of open stope rib pillars.

6.4.1 Hedley's Pillar Strength Formula

The pillar strength formula developed by Hedley and Grant (1972) was based on data from room and pillar mining at Elliot Lake and has been discussed in Chapters 3.1.1.3 and 6.3.1. The formula is defined as:

\[ Qu = k \cdot \frac{w^a}{h^b} \]

where:

- \( Qu \) = pillar strength
- \( k \) = strength of 1 ft. cube (UCS\(_{12}\))
- \( w \) = pillar width (ft)
- \( h \) = pillar height (ft)
- \( a \) = empirical constant = 0.5
- \( b \) = empirical constant = 0.75

To get UCS\(_{12}\), several authors have used a scaling factor from the compressive strength of a 2 inch diameter specimen (UCS\(_2\)):
UCS_{12} \approx 0.7 \times UCS_2

This relationship has been found in works by Hedley and Grant (1972), Hedley et al. (1979), Hoek and Brown (1980), and Von Kimmelmann et al. (1984).

Hedley's formula is a size effect formula, which means that it accounts for the actual dimensions of a pillar and not just the pillar shape. To apply this to open stope rib pillars, the size of typical open stope rib pillars must be determined. The range of rib pillar sizes seen in 17 different Canadian open stope mines is presented in figure 46. The dimensions of permanent pillars are denoted by the symbol "P" and the dimensions of pillars in mining methods using backfill and temporary pillars are denoted by the symbol "B". The dashed lines give the upper and lower bound of pillar dimensions used in the 17 Canadian open stope mines. For various pillar width to pillar height ratios (ie. pillar strike length to orebody width ratios), the minimum and maximum pillar dimensions can be determined and applied to Hedley's size effect formula.

For application of their pillar strength formula, Hedley and Grant suggest that pillars with a safety factor greater than 1.5 are stable and pillars with a safety factor near 1.0 are crushed. Rearrangement of the safety factor formula,

\[
S.F. = \frac{Qu}{\sigma_p} = \frac{0.7 \times UCS_2 \times w^a}{\sigma_p \times h^b}
\]
FIGURE 46. The range of rib pillar dimensions seen in 17 Canadian open stopes. The letter "B" denotes pillars that were recovered with the use of backfill. The letter "P" denotes permanent pillars.
permits plotting of safety factor lines for 1.0 and 1.5, for the maximum and minimum rib pillar sizes observed in open stoping, against the data base (figure 47). The upper shaded zone shows the possible location of open stope rib pillars when designed with a safety factor of 1.0. The lower zone shows the possible location of open stope rib pillars when designed with a safety factor of 1.5. Size effect formulas assume that smaller pillars are stronger than large pillars. So, the upper line of each zone corresponds to the minimum pillar sizes seen in Canadian open stope mines, while the lower line of each zone corresponds to the maximum pillar sizes seen in Canadian open stope mines.

The graph shows that, for open stope rib pillar design, Hedley's formula is conservative relative to the pillar stability graph. In defense of Hedley's formula, it was designed for much smaller pillars and it is less conservative when applied in room and pillar mining (due to the nature of the size effect formula).

Comparison of the pillar stability graph against Salamon's formula (Chapter 3.1.1.2) would give a similar conclusion. Hedley and Salamon used the same method to determine the strength variable "K" and Salamon has very similar values for the empirical constants (a=0.46 and b=0.66). Salamon's method is actually a bit more conservative than Hedley's formula because Salamon recommended the use of a safety factor of 1.6 to ensure stable design and used a pillar height coefficient of
FIGURE 47. Comparison of the pillar stability graph and Hedley's formula for two safety factors. Hedley's formula is a size effect formula, so there is a range of pillar strength for each safety factor based on the size of open stope rib pillars observed in 17 Canadian mines.
b=0.66 compared to b=0.75 suggested by Hedley.

6.4.2 Hoek and Brown Pillar Strength Curves

Hoek and Brown (1980) proposed a series of curves (figure 11, page 44) for the estimation of pillar strength. These curves are discussed in more depth in Chapter 3.1.1.5. The curves were developed based on numerical modelling, rock mass failure distributions inside pillars of different shapes, and for a range of rock mass qualities, using the failure criteria:

\[
\sigma_p = \sigma_3 + (m \sigma_C \sigma_3 + s \sigma_C^2)^{1/2}
\]

where:

- \(\sigma_p\) = average pillar strength
- \(\sigma_3\) = minimum principle stress
- \(\sigma_C\) = uniaxial compressive strength of intact pillar material
- \(m \& s\) = empirical constants based on the rock mass quality.

The \(m \& s\) empirical constants have been related to the NGI and CSIR rock mass classifications.

Hoek and Brown proposed these pillar design lines assuming that a pillar is failed when the stress across the centre of the pillar exceeds the strength of the rock mass. Each curve corresponds to a failure line for a different rock mass quality.
Since Hoek and Brown used input parameters similar to those in the pillar stability graph, it was possible to reproduce some of their design curves on the design chart (see figure 48). The first observation is that Hoek and Brown design line for good rock mass quality (RMR \approx 60 - 80) corresponds reasonably well with the transition zone of the pillar stability graph. The majority of pillars in the open stope rib data base have a good rock mass quality. However, Hoek and Brown suggest a safety factor of 1.5 for permanent mine pillars. While this safety factor may be needed for the design of permanent pillars in entry mining methods, use of this safety factor would make Hoek and Brown curves quite conservative for open stope rib pillar design.

Hoek and Brown suggest a very large influence of the rock mass quality on pillar strength. The design curve for a fair rock mass quality is well below the transition zone of the pillar stability graph and the design curve for a very good rock mass quality is far above the transition zone. There are very few pillar case histories with fair or very good rock mass qualities in the data base, so the applicability of these curves for pillar design can not be verified. A substantial number of case histories of pillars in fair and very good rock masses are needed before these two curves could be used confidently in open stope rib pillar design.

6.4.3 Pillar Shape Effect Formulas
FIGURE 48. Three of the Hoek and Brown (1980) pillar strength curves plotted on the pillar stability graph. The transition zone of the pillar stability graph and the good rock mass quality curve are very close to each other.
There are several variations of the shape effect formula (see Chapter 3.1.1.1). Two of the most common variations were developed by Obert and Duvall (1967) and Bieniawski (1983).

Obert and Duvall (1967) presented a formula to account for the influence of pillar shape. It is based on compressive testing of coal specimen pillars of various shape by Obert et al. (1946). The proposed relationship was:

\[ \sigma_p = \sigma_1 \times [A + B \times (w / h)] \]

where:

- \( \sigma_p \) = pillar strength,
- \( \sigma_1 \) = uniaxial strength of a cubical pillar,
- \( w \) = pillar width,
- \( h \) = pillar height,
- \( A \) = empirical constant = 0.778
- \( B \) = empirical constant = 0.222.

The formula has been used by several authors (listed in Chapter 3.1.1.4) to account for the shape effect in hard rock pillar design.

The formula assumes that the strength of a cubical pillar (\( \sigma_1 \)) is known. If we assume the maximum cubical pillar strength on the pillar stability graph is found at the intersection of \( w/h = 1 \) and the failure line (top of the transition zone), the Obert and Duvall formula can be compared to the pillar stability graph and the data base. Figure 49 shows the Obert and Duvall formula plotted on the pillar stability graph. It does not compare well with the pillar data or the location of the
FIGURE 49. Comparison between the pillar stability graph and the Obert and Duvall (1967) shape effect formula applied with a safety factor of 1.0.
transition zone. The Obert and Duvall formula assumes a much higher strength for slender pillars than that shown by the case histories and the pillar stability graph transition zone. There are many failed and sloughing pillars below the failure line proposed by Obert and Duvall. This formula is not applicable to the design of open stope rib pillars.

A major coal pillar design study was carried out by Bieniawski (1983) at Pennsylvania State University in the late 1970's. One of the major results of the study was the development of a shape effect pillar strength formula. Bieniawski used a formula similar to that proposed by Obert and Duvall. Bieniawski's formula is:

\[ \sigma_p = K \times [0.64 + \left( \frac{0.36 \times W}{H} \right)] \]

where:

\( \sigma_p = \) the pillar strength,
\( K = \text{UCS}_{12} = \) the compressive strength of 1 cubic foot of intact pillar material,
\( W = \) pillar width,
\( H = \) pillar height.

Assuming \( \text{UCS}_{12} \approx 0.7 \times \text{UCS}_2 \) (shown in Chapter 6.4.1), a failure line can be plotted on the pillar stability graph. Bieniawski's formula is plotted in figure 50, for a safety factor of 1.0, 1.5 and 2.0. This formula does not compare well with the pillar data or the transition zone. For each safety factor, there are many pillar case histories that can not be
The shape effect formula proposed by Bieniawski (1983) applied with three different safety factors is compared against the pillar stability graph.
explained by Bieniawski's formula.

The conditions under which these formula were developed can explain their inadequacy for open stope rib pillar design. Both of the formulas is more applicable for pillars with a width/height ratio of much greater than one. For pillars with a width/height ratio of less than one, the shape effect formulas will overestimate pillar strength by large amounts. Generally, these formulas are not well suited to open stope rib pillar design.

6.5 Chapter Summary

The variables that are significant for open stope rib pillar design are: the pillar width and pillar height (defined according to figure 26, page 87), the compressive strength of the intact rock material and the load induced on the pillar. The volume of a pillar and the presence of geological discontinuities do not appear to be significant for open stope rib pillar design, over the range observed for these variables in Canadian open stope mines.

A pillar design chart has been developed based on open stope rib pillars and verified and refined based on hard rock room and pillar mining data found in literature. The total data base consists of 135 pillar case histories. The pillar stability graph contains stable and failed design areas separated by a transition zone, which shows a variety of pillar behaviour. The
transition zone is represented by a solid line where its location is well defined by data. The transition zone is represented by dashed lines where its exact location is not verified by the data.

The compatibility of a number of existing open stope rib pillar design methods with the pillar stability graph and the complete pillar data base was checked. Hedley's size effect formula (1972) was found to be quite conservative for open stope rib pillar design. The Hoek and Brown (1980) pillar strength curve for a good rock mass quality agreed well with the pillar stability graph. However, the applicability of the strength curves for the other rock mass qualities could not be verified. The pillar shape effect formulas proposed by Obert and Duvall (1967) and Bieniawski (1983) are not applicable to open stope rib pillar design.
The design of rib pillars depends on the duration of the support to be provided. Rib pillars may be designed to give permanent support to provide long-term stability to open stopes, to provide regional stability to the ore block and to protect access to the stopes. Conversely, ribs may be designed to give temporary support to a mining block until stope support is provided by backfill. The pillar is then recovered.

The decision to use permanent or temporary pillars is largely based on economics. In a relatively low grade orebody, a permanent pillar may be the most economical form of support because of the high costs associated with backfill and pillar recovery methods. In higher grade orebodies, temporary pillars are typically used because the cost of backfilling can be justified and the maximum extraction of the orebody is desired. This is shown explicitly in a comparison of the approximate value of ore per ton found in Canadian open stope mines using permanent and temporary pillars (Table 13). The average value per tonne in the mines using temporary pillars and fill is almost double that of the mines using permanent pillars.

Because permanent and temporary pillars have different purposes, their designs can be quite different. The following chapter will discuss the design of permanent and temporary
<table>
<thead>
<tr>
<th>MINES USING BACKFILL</th>
<th>APPROXIMATE VALUE OF ORE ($US/ton)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NORITA</td>
<td>$ 88</td>
</tr>
<tr>
<td>MATTAGAMI LAKE</td>
<td>$ 60</td>
</tr>
<tr>
<td>MINES GASPE</td>
<td>$ 68</td>
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<td>WESMIN</td>
<td>$ 128</td>
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<td>CORBET</td>
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<td>$ 125</td>
</tr>
<tr>
<td>CENTENNIAL</td>
<td>$ 54</td>
</tr>
<tr>
<td>SELBAIE - ZONE B</td>
<td>$ 100</td>
</tr>
<tr>
<td>FALCONBRIDGE</td>
<td>$ 129</td>
</tr>
<tr>
<td>MEAN</td>
<td>$ 98</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MINES USING PERMANENT PILLARS</th>
<th>APPROXIMATE VALUE OF ORE ($US/ton)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RUTTAN</td>
<td>$ 43</td>
</tr>
<tr>
<td>ALGOMA</td>
<td>$ 25</td>
</tr>
<tr>
<td>HEATH STEELE</td>
<td>$ 92</td>
</tr>
<tr>
<td>SELBAIE - ZONE A</td>
<td>$ 47</td>
</tr>
<tr>
<td>MEAN</td>
<td>$ 52</td>
</tr>
</tbody>
</table>

pillars in Canadian open stope mines and suggest some guidelines for using the pillar stability graph method. An example of the use of temporary pillars is also given.

7.1 Permanent Pillars

The maximum possible orebody extraction around permanent pillars is about 80%. Any remaining ore will be left in place, so ideally, permanent pillars should be located in low grade ore or waste. Oversize (conservative) pillar dimensions are permissible under these conditions. However, the design of permanent pillars in ore must be a compromise between conservative dimensions, to maintain the stability of the mining block for a long period of time, and non-conservative dimensions, to minimize the loss of ore in the pillar.

In a preliminary design, it is suggested that permanent pillars should plot below the transition zone, in the stable area of the pillar stability graph. The distance below the transition zone should be a function of the degree of confidence in the input data (especially the uniaxial compressive strength of the rock and the induced stress). The less confident the input data, the further below the transition zone a pillar should plot.

Ultimately, the best design for permanent pillars is optimised according to mining experience in the local ground conditions. A good example of using local experience in pillar design is documented by Pakalnis (1986) at Ruttan. The rib
pillars gradually fail as the longitudinal stopes are opened to their planned limits. However, the pillars retain sufficient rock mass competency that they remain relatively intact, without the use of backfill, and continue to provide stope support and regional mine support. In most mines, fracturing due to pillar failure would combine with geological structure to cause severe pillar sloughing and eventually complete pillar disintegration. At Ruttan, sloughing of failed pillar material is not a problem, so the rib pillars can be designed to gradually fail because they will remain intact and still provide the necessary stope support.

7.2 Temporary Pillars

Temporary rib pillars are used when it is intended to recover the entire orebody. This type of open stope mining involves the use of backfill and the extraction of ore must be carefully sequenced. An optimum mining sequence gives a high rate of mining, while avoiding stope and pillar instability.

One of the primary concerns for the design of temporary pillars is the ease of recovery of the pillar. Small pillars are more difficult and more expensive to recover. Figure 51 shows the range of temporary pillar dimensions used in 14 Canadian open stope mines. Generally, temporary rib pillars are designed with a strike length of greater than 8-10 metres and less than 25 metres. Pillar height (i.e. orebody width) varies from less than 5 metres to 60 metres.
FIGURE 51. The range of temporary rib pillar dimensions used in 14 Canadian open stope mines. The maximum strike length (pillar width) is about 25 metres, while the maximum orebody width (pillar height) is about 60 metres.
The design of temporary pillars depends on whether the pillar is intended to be stable or to fail. Both approaches are used in open stoping in Canada, and there are different recommendations that can be made in the design of each type of temporary pillar.

7.2.1 Stable Temporary Pillars

The majority of temporary rib pillars are designed to be stable. However, the mine operator's philosophy plays a large role in determining the size of temporary rib pillars. Pillars may be designed larger than necessary, or their dimensions may be minimized. Use of oversize rib pillars permits easier pillar recovery. In addition, the stability provided by the extra size means that the primary stopes may not need immediate filling, leaving some flexibility in the filling cycle. However, minimizing temporary pillar dimensions gives a higher primary mining volume and a quicker payback on capital and development costs. Minimizing pillar dimensions can become costly if the pillars fail unexpectedly or if the pillars are difficult to recover due to their small size. The consequences of failed temporary pillars may include:

- the loss of reserves,
- a high mining cost,
- the need for remedial stability measures such as cable bolting,
- regional instability such as hanging wall and back caving,
Cases of rib pillar instability and recovery problems are documented by many authors including Falmagne (1986), Brady (1977) and Bray (1967).

7.2.2 Failed Temporary Pillars

A relatively new concept in open stoping is to design the rib pillars to fail. The consequences of failed pillars described above can often be minimized if failure is planned. The advantage is that in a high stress environment, the pillar will not become overstressed and will be easier to recover. Designing rib pillars to destress or fail has been documented at INCO's Frood mine (Grace and Taylor 1985) and Falconbridge's Strathcona mine (Bharti 1987). To design a failing pillar with the pillar stability graph, it is suggested that a pillar plot well above the transition area, have a low pillar width to height ratio while having a pillar width large enough to permit easy recover. Although there are no pillars designed to fail in the data base, several pillar case histories that were discarded from the data base fit the above design suggestions.

There are a few qualitative recommendations and comments to add to the design of failing pillars:

- it is very important to fill the surrounding stopes as quickly as possible. The fill will provide lateral constraint on the pillar walls and will reduce sloughing of the fractured pillar
material,
- control blasting should be used near the pillar walls to minimize wall damage due to blast vibrations,
- development in pillars will likely need full friction artificial support such as cable bolts and grouted rebar, as pillar fracturing could substantially affect development stability,
- and drill hole closure and displacement could cause severe problems for longhole (small diameter drill hole) open stoping methods. Large diameter blastholes will likely be needed for recovery of failed pillars.

7.3 Case Example: Transverse Rib Pillars at Norita

7.3.1 Geology and Mining Method

The Norita mine is located in the Mattagami mining district in north western Quebec. The geological setting for the copper-zinc orebody is shown in Mine #19 in Appendix I. More detail can be found in papers by Bawden and Milne (1987), Chauvin (1986), and Goodier and Dube (1984). In recent years, the mine has converted to a transverse blasthole open stoping method. This case history will focus on the transverse pillars in the open stoping between levels 9 and 11 of the orebody (figure 52).

The mining block was divided into two levels with 17 stopes per level. The basic sequence of extraction for the mining block is shown by the roman numbers on figure 52. Primary
FIGURE 52. Isometric view of transverse blasthole open stoping at Norita. The basic sequence of stope extraction is shown in roman numbers.
Stopes were extracted every fourth stope. Stopes were filled with a 30:1 ratio of mill tailings and waste rock to cement. Temporary pillars (composed of three consecutive unmined stopes) are formed by the extraction of the primary stopes. Stability problems were not reported during the primary mining and the temporary pillars have been assessed as stable.

The next phase in the mining was to extract the middle (secondary) stope of the temporary pillars. The blasting of the secondary stopes was done carefully using control blasting methods. Explosives were decked with a maximum detonation per delay of 90 kilograms. After the stopes were emptied, they were backfilled with a 30:1 ratio of mill tailings and waste rock to cement.

With the commencement of primary mining between levels 9 and 10 (stage IV) and tertiary stope mining between levels 10 and 11 (stage III), deterioration of drawpoint pillars on level 10 necessitated frequent rehabilitation. Mining of the tertiary stopes encountered heavy blast overbreak (3.3 metre pull on 2 metre rounds). The ore was described as badly broken and fractured. This damage was induced by mining since the ground was classified as very good (Q ≈ 40 and RMR ≈ 75) before mining had started. With continued mining between levels 9 and 11 (stages III, IV, and V), development drifts in the 8-8 sill pillar (directly above level 9) deteriorated due to stress shedding from the transverse mining area. This was confirmed by stress cells installed near the transverse pillars.
Extensometer monitoring of the 8-8 sill pillar showed that the deterioration was directly related to mining events in the transverse mining block. Based on these observations (documented by Bawden and Milne 1987), the tertiary pillars in the transverse mining area were assumed to have failed.

7.3.2 Back Analysis Using the Pillar Stability Graph

Back analysis will focus on representative pillars in the mining block. After the primary mining between levels 10 and 11 was completed, stopes 10-5 and 10-9 had been extracted leaving a stable temporary pillar made of stopes 10-6, 10-7, and 10-8 (see figure 53). The pillar dimensions were: 55-60 metres in (stope) height, 33 metres in (pillar) width and 23 metres in (pillar) height (according to the convention in figure 26, page 87). The average load was estimated by two dimensional plane strain modelling (BITEM) at 75 MPa (case 43 from Table 8, page 105). The pillar plots well inside the stable zone of the pillar stability graph (figure 54).

During secondary mining stope 10-7 was extracted leaving the tertiary pillars 10-6 and 10-8 (figure 53), which were given a failed assessment. The pillar dimensions were: 55-60 metres (stope) height, 11 metres in (pillar width) and 23 metres in (pillar) height. The theoretical average pillar load on the 10-8 pillar was estimated at 99 MPa (case 42 from Table 8, page 165). This is a theoretical average pillar load because in practice the pillar has failed and destressed and therefore will
FIGURE 53. A longitudinal section of the blasthole open stoping block at Norita showing the pillar case histories (10-6, 10-7, and 10-8) used in this case history analysis (after Goodier and Dube 1984).
FIGURE 54. The pillar stability graph showing the location of the stable and failed transverse pillar case histories at Norita.
have a much lower actual load (see Chapter 5.5.1 for a more complete discussion of this assumption). The pillar plots above the transition zone in the failed area (see figure 54). This agrees very well with the failed assessment for the tertiary stopes.

7.3.3 Comments Concerning the Pillar Design

This yielding pillar case history illustrates the use of failed pillars in open stope mining. There are several comments and observations to make that are a consequence of the pillar design:

1 - Cable bolting of the tertiary stope backs was necessary, due to severe cracking and joint opening.

2 - Heavy overbreak during the pillar (tertiary stope) mining was encountered.

3 - Blastholes (6\frac{1}{2} inch diameter) were used for the entire mining block and were necessary to avoid the loss of drill holes due to crushing and fracturing during the tertiary pillar recovery.

4 - The stopes were filled quickly with waste rock and cemented mill tailings.

5 - The mining of the failed pillars was generally successful.
CHAPTER 8
SUMMARY AND CONCLUSIONS

8.1 Summary

The purpose of this study is to investigate the stability of rib pillars in open stope mining and develop guidelines for the optimization of rib pillar dimensions. This is accomplished through four major steps:

- description of the failure mechanism in open stope rib pillars,
- investigation of the methods currently used in open stope rib pillar design,
- quantification of the significant design variables,
- and formulation and verification of a new method based on open stope rib pillar data and case histories.

8.1.1 Open Stope Rib Pillar Failure

There are two basic types of failure in hard rock pillars. Progressive failure refers to gradual deterioration of a pillar in a slow, non-violent manner. Bursting failure is characterized by the violent release of energy causing instantaneous fracture of rock. This thesis only investigates progressive failure.

Open stope rib pillar instability is a progressive mechanism. Pillar failure is defined as the point at which progressive failure causes a pillar to start losing its load
bearing capacity. The decrease in load bearing capacity is largely due to fracturing of the rock mass in the pillar. Several signs of increasing pillar instability have been identified, including:
- cracking and spalling of rock in pillar development,
- audible noise heard in the pillars or microseismic events detected with monitoring systems,
- deformed or plugged drill holes,
- overdraw from stopes consisting of unblasted, oversize ore,
- stress redistribution from pillars affecting nearby pillars or development,
- hourglassing and cracking of pillars,
- and displacements or changes in stress shown by instrumentation.

8.1.2 Current Pillar Design Methods

Design methods used for open stope rib pillars were based on empirical pillar design studies or the use of numerical modelling and empirical failure criterion. Empirical pillar design methods were developed based on laboratory testing and/or investigation of actual mine pillars. These methods were developed for specific mining conditions and are not necessarily applicable for open stope rib pillar design. Numerical methods basically assume elastic and/or plastic rock mass behaviour to determine stress redistribution and rock mass displacement around underground excavations. Empirical failure criterion are
applied to the stress or displacement results to determine rock mass failure. However, it is difficult to verify an in situ rock mass failure criterion. Consequently, numerical design methods need extensive site calibration before they can be used effectively to design rib pillars in open stope mining.

The design methodology chosen for this thesis is a combination of numerical and empirical methods. Numerical techniques are used to determine pillar load, while pillar failure is determined from empirical back analysis of open stope rib pillar case histories.

8.1.3 Identification and Quantification of the Design Variables

Based on the data and case histories collected in the Integrated Mine Design Project, the factors that are significant for open stope rib pillar design are:

- the compressive strength of intact pillar material (UCS),
- the average pillar stress (determined with boundary element numerical modelling),
- the pillar height,
- and the pillar width.

Three factors were discounted as insignificant in rib pillar failure: the presence of minor geological discontinuities (such as joints), the effect of pillar volume, and the effect of backfill. The open stope rib pillar data and case histories did not prove these factors as being important in pillar failure. The background information for all the pillars in the data base
is presented in Table 5 (page 70) and the geological settings of all of the pillar case histories are shown in oblique orebody diagrams in Appendix I.

Three of the four design variables are quite easy to quantify. The UCS can be determined by laboratory testing of intact rock samples or estimated with the point load test. The pillar height and pillar width are measured from mine plans. The most difficult factor to quantify is the average pillar stress. A method to determine average pillar stress is proposed in Chapter 5. The two dimensional boundary element code BITEM and the pseudo-three dimensional displacement discontinuity boundary element model "MINTAB" have been used to estimate the pillar load for all the pillars in the data base. However, these methods have limitations when modelling some pillar geometries. The major geometrical limitations associated with two dimensional (2D) and displacement discontinuity (DD) numerical modelling have been identified. In addition, a rough error associated with these limitations is given in figure 33, page 109 (for 2D modelling), and in figure 34, page 112 (for DD modelling). These error estimates are based on a comparison of the 2D and DD models to 12 runs of the three dimensional boundary element code "BEAP".

8.1.4 Development of the Pillar Stability Graph

The open stope rib pillar data collected has been empirically analyzed and a pillar design graph has been developed
(figure 36, page 125). The design chart has been called the "Pillar Stability Graph". It contains stable and failed design areas separated by a transition zone. The pillar stability graph has been verified and refined based on more than 80 hard rock room and pillar case histories from literature. The complete data base of about 135 pillars is shown in figure 45 (page 145). The design chart explains the stability condition of the data base case histories very well and is quite sensitive in predicting pillar failure.

Empirical design methods used for open stope rib pillar design have been compared to the complete pillar data base and the pillar stability graph. The good rock mass quality design line of the pillar strength curves proposed by Hoek and Brown (1980) agrees quite well with the data base and pillar stability graph. However, the Hedley and Grant (1972) size effect pillar strength formula and the shape effect pillar strength formula's by Obert and Duvall (1967) and Bieniawski (1983) do not compare well with the pillar data or the pillar stability graph.

Open stope rib pillars may be designed to be permanent and stable, temporary and stable, or temporary and failing. Guidelines have been suggested for the design of each type of rib pillar using the pillar stability graph. A case history discussing the use of stable and failed temporary rib pillars is also presented.
8.2 Conclusions

8.2.1 Applicability of the Method

The pillar stability graph uses factors that are relatively easy to quantify data to predict the stability of open stope rib pillars. The method is most effective when rough predictions of stability are required. Minor problems such as local fracturing will not be predicted, but gross changes in pillar stability are recognized. The method is designed to predict failure of open stope rib pillars, but can be applied to some other types of pillars. It should be applicable for the design of open stope sill pillars, and rib and sill pillars in non-entry methods such as Vertical Crater Retreat. The mechanism of pillar failure for these types of pillars is the same as the mechanism of failure in open stope rib pillars.

This design method has not been developed or confirmed for pillars in entry methods such as shrinkage and room and pillar mining. The pillar stability graph would likely need the development of a safety factor before it could be applied to pillar design in entry mining methods.

8.2.2 Limitations of the Method

An empirical design method is more reliable when applied to conditions similar to those found in the original work. Consequently, the following limitations are suggested for the pillar stability graph:
70 MPa < UCS < 316 MPa,
9 metres < Wp < 45 metres,
60 < RMR < 78,
(Average Pillar Load / UCS) < 0.5.

where,
UCS = the intact rock uniaxial compressive strength,
Wp = the pillar width,
RMR = a measure of the rock mass competency using the
CSIR rock mass classification,
Average Pillar Load is determined using two dimensional or
displacement discontinuity boundary element numerical
modelling.
The pillar stability graph method may work satisfactorily
outside these limitations, but the current open stope data base
generally does not extend outside these limits.

Finally, it should be kept in perspective that this is a
preliminary design method. The assumptions and potential error
associated with the variables and design chart limit the
usefulness of the pillar stability graph for final design.

8.2.3 Design of Open Stope Rib Pillars

The design of open stope rib pillars is dependent upon the
role of that pillar in the stability of the mine. Rib pillars
may be designed to be give permanent support to open stopes, or
they may be designed to give temporary stope support until
backfill is in place. This decision is largely one of
Low grade orebodies cannot be mined using backfill and pillar recovery methods due to the higher mining cost. Medium and high grade mines can afford the cost of backfill and pillar recovery, so temporary pillars can be designed. In some instances, temporary pillars have been designed to fail to avoid stress build up. There are a few consequences of designing pillars to fail, including:

- the need for quick backfilling after the stope is extracted,
- the use of artificial support in pillar development,
- and the use of large diameter drill holes and control blasting practices.

8.3 Future Work

There is a limit to the value of collecting further general pillar case histories to refine the pillar stability graph. More cases of open stope pillars are not likely to significantly improve the accuracy of the existing graph or reduce the size of the transition area. This is not to say that pillar design at specific sites can not be aided by case histories from that site or from similar ground conditions. Past experience is the best way to refine pillar design methods to local conditions.

The understanding of one of the possible design factors may be improved by collecting specific case histories. The influence of rock mass characteristics was not found to be significant (Chapter 6.1.3.2), but varied over only a small range of rock mass qualities. Analysis of pillar case histories
in fair or very good quality rock masses may show that the quality of the rock mass is significant in open stope rib pillar design. If this can be proven, a correction factor to the existing pillar stability graph could be developed to account for the effect of rock mass quality.

Assessment of the pillars in the data base was sometimes difficult, and a substantial amount of data could not be applied because a reliable assessment could not be determined. A more detailed investigation into pillar failure mechanisms and in situ rock mass fracturing could improve pillar design methods. A better definition of failure can be developed through systematic in situ pillar monitoring using visual techniques (as shown by Krauland and Soder 1987) or through the use of instrumentation such as stressmeters (as shown by Agapito 1974), extensometers (as shown by Allcott and Archibald 1981) or microseismics. The use of microseismic systems for in situ monitoring shows great potential through quantifying the decrease of rock mass quality due to rock fracturing, and monitoring the changes in the load bearing condition of pillars. Both of these topics can be investigated with the microseismic technology currently available.
REFERENCES


Falmagne, V. 1986. Récupération des piliers de la lentille #3 à la mine Corbet.


Wagner, H. 1974. Determination of the complete load-deformation characteristics of coal pillars. Advances in Rock


APPENDIX I

Specific information about the geological setting of each case history can be found in the isometric sketch corresponding to the mine number. Each geological setting is comprised of:

- the underground stress regime,
- the hanging wall, footwall and orebody material properties and characteristics including (when available):
  - rock type,
  - intact uniaxial compressive strength,
  - elastic modulus,
  - poisson's ratio,
  - NGI rock mass classification,
  - stereonet of the major joint sets,
- the orebody shape and size,
- and the mining methods used in various parts of the orebody.

Mine #22 does not have an isometric orebody sketch due to the complexity of the orebody and the variability of the material properties and stress field.
MINE No. 2

ORE (LENS 2 & 3)

Rock Type: Massive Sulphide

\[ \gamma = 4.2 \, \text{t/m}^3 \]
\[ \sigma_c = 200 \, \text{MPa} \]
\[ E = 61.0 \, \text{GPa} \]
\[ v = 0.3 \]
\[ Q' = 7 \]

BANGING WALL & ROOF (LENS 2)

Rock Type: Andesite

\[ \gamma = 2.9 \, \text{t/m}^3 \]
\[ \sigma_c = 109 \, \text{MPa} \]
\[ E = 63.0 \, \text{GPa} \]
\[ v = 0.25 \]
\[ Q' = 4 \]

ORE (LENS 2 & 3)

Rock Type: Massive Sulphide

\[ \gamma = 4.2 \, \text{t/m}^3 \]
\[ \sigma_c = 200 \, \text{MPa} \]
\[ E = 61.0 \, \text{GPa} \]
\[ v = 0.3 \]
\[ Q' = 7 \]

BANGING WALL & ROOF (LENS 2)

Rock Type: Altered Andesite

\[ \gamma = 3.0 \, \text{t/m}^3 \]
\[ \sigma_c = 87 \, \text{MPa} \]
\[ E = 84.0 \, \text{GPa} \]
\[ v = 0.28 \]
\[ Q' = 0.9 \]
MINE No. 6

ORE
Rock Type: Breccia & Massive Sulphide
- $\gamma = 3.1 \text{ t/m}^3$
- $G_c = 125 \text{ kPa}$
- $E = 94.0 \text{ GPa}$
- $\nu = 0.22$
- $Q' = 9$

NORTH WALL
Rock Type: Norite
- $\gamma = 2.9 \text{ t/m}^3$
- $G_c = 113 \text{ kPa}$
- $E = 56.0 \text{ GPa}$
- $\nu = 0.17$
- $Q' = 9$

SOUTH WALL
Rock Type: Granite
- $\gamma = 2.7 \text{ t/m}^3$
- $G_c = 184 \text{ kPa}$
- $E = 73.0 \text{ GPa}$
- $\nu = 0.23$
- $Q' = 25$

Depth 1050 m
MINE No. 8

ORE

Rock Type: Syenite

\( \gamma = 2.85 \text{ t/m}^3 \)
\( q_c = 215 \text{ MPa} \)
\( E = 32.5 \text{ GPa} \)
\( v = 0.1 \)
\( Q' = 39 \)

FOOT WALL

Rock Type: Massive Tuff

\( \gamma = 2.9 \text{ t/m}^3 \)
\( q_c = 194 \text{ MPa} \)
\( E = 32.2 \text{ GPa} \)
\( v = 0.08 \)
\( Q' = 7 \)

HANGING WALL

<table>
<thead>
<tr>
<th>Good</th>
<th>Weak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mafic Tuff</td>
<td>Schist Tuff</td>
</tr>
<tr>
<td>( \gamma = 2.9 \text{ t/m}^3 )</td>
<td>( \gamma = 2.9 \text{ t/m}^3 )</td>
</tr>
<tr>
<td>( q_c = 183 \text{ MPa} )</td>
<td>( q_c = 155 \text{ MPa} )</td>
</tr>
<tr>
<td>( E = 28.2 \text{ GPa} )</td>
<td>( E = 30.0 \text{ GPa} )</td>
</tr>
<tr>
<td>( v = 0.22 )</td>
<td>( v = 0.15 )</td>
</tr>
<tr>
<td>( Q' = 35 )</td>
<td>( Q' = 57 )</td>
</tr>
</tbody>
</table>
MINE No. 10

ORE
Rock Type: Siliceous Ore

\[ \gamma = 3.0 \text{ t/m}^3 \]
\[ \sigma_c = 41 - 69 \text{ MPa} \]
\[ E = 44 - 53 \text{ GPa} \]
\[ v = 0.21 - 0.40 \]
\[ Q^* = 6 \]

HANGING WALL
Rock Type: Siliceous Sediment

\[ \gamma = 2.8 \text{ t/m}^3 \]
\[ \sigma_c = 60 \text{ MPa} \]
\[ Q^* = 6 \]

FOOT WALL
Rock Type: Siliceous Schist

\[ \gamma = 2.8 \text{ t/m}^3 \]
\[ \sigma_c = 140 \text{ MPa} \]
ORE

Rock Type: Porphyry

\[ \gamma = 2.72 \text{ t/m}^3 \]
\[ \sigma_c = 148 \text{ kPa} \]
\[ E = 18.5 \text{ GPa} \]
\[ v = 0.20 \]
\[ Q' = 30 \]
MINE No. 16

ORE
Rock Type: Massive Sulphide
\[ \gamma = 4.6 \text{ t/m}^3 \]
\[ \sigma_c = 176 \text{ MPa} \]
\[ E = 119.0 \text{ GPa} \]
\[ v = 0.24 \]
\[ Q' = 20 \]

HANGING WALL
Rock Type: Quartz Porphyry
\[ \gamma = 3.9 \text{ t/m}^3 \]
\[ \sigma_c = 91 \text{ MPa} \]
\[ E = 68.7 \text{ GPa} \]
\[ v = 0.19 \]
\[ Q' = 42 \]

FOOT WALL
Rock Type: Chlorite Tuff
\[ \gamma = 2.9 \text{ t/m}^3 \]
\[ \sigma_c = 84 \text{ MPa} \]
\[ E = 68.5 \text{ GPa} \]
\[ v = 0.25 \]
\[ Q' = 40 \]

LONGITUDINAL LONGHOLE STOPES
PERMANENT RIB PILLAR
MINE No. 17

LONGITUDINAL LONGHOLE OPEN STOPING

ORE
Rock Type: Massive Sulphide
\[ \gamma = 5.3 \, \text{t/m}^3 \]
\[ \sigma_c = 100 \, \text{MPa} \]
\[ E = 103 \, \text{GPa} \]
\[ v = 0.31 \]
\[ Q' = 19 \]

WALL
Rock Type: Gneiss
\[ \gamma = 2.7 \, \text{t/m}^3 \]
\[ \sigma_c = 52 \, \text{MPa} \]
\[ E = 105 \, \text{GPa} \]
\[ v = 0.20 \]
\[ Q' = 18 \]
MINE No. 19

MINED OUT & BACKFILLED TO SURFACE

LONGITUDINAL SUB-LEVEL RETREAT

TRANSVERSAL BLASTHOLE STOPES

ORE

Rock Type: Massive Sulphide

NORTH WALL (90°)

Rock Type: Basaltic Tuff

SOUTH WALL (90°)

Rock Type: Rhyolitic Tuff

\[
\begin{align*}
\sigma_c &= 316 \text{ MPa} \\
E &= 232.2 \text{ GPa} \\
v &= 0.16 \\
Q^* &= 44 \\
\end{align*}
\]

\[
\begin{align*}
\sigma_c &= 118 \text{ MPa} \\
E &= 95.0 \text{ GPa} \\
v &= 0.26 \\
Q^* &= 4.0 \\
\end{align*}
\]

\[
\begin{align*}
\sigma_c &= 98 \text{ MPa} \\
E &= 67.9 \text{ GPa} \\
v &= 0.15 \\
Q^* &= 2.2 \\
\end{align*}
\]
ORE
Rock Type: Massive Sulphide
\( \sigma_c = 100 \text{ MPa} \)
\( E = 88 \text{ GPa} \)
\( \nu = 0.20 \)
\( Q' = 10-20 \)

HANGING WALL & FOOTWALL
Rock Type: Quartz Meta Sediments
\( \sigma_c = 50-135 \text{ MPa} \)
\( E = 50-75 \text{ GPa} \)
\( \nu = 0.12-0.34 \)
\( Q' = 0.1-50 \)
MINE No. 23

CUT & FILL

LONGITUDINAL
VERTICAL
CRATER
RETREAT

ORE
Rock Type: Massive Sulphide
UCS = 310 MPa
Q' = 20

FOOTWALL & HANGING WALL
Rock Type: Argylite
UCS = 75 MPa
Q' = 0.6
ORE
Rock Type: Massive Sulphide
\( \gamma = 3.3 \, \text{t/m}^3 \)
\( \sigma_0 = 160 \, \text{MPa} \)
\( E = 80 \, \text{GPa} \)
\( \nu = 0.21 \)
\( Q' = 22 \)

HANGING WALL
Rock Type: Rhyolite
\( \gamma = 2.7 \, \text{t/m}^3 \)
\( \sigma_0 = 120-150 \, \text{MPa} \)
\( E = 80 \, \text{GPa} \)
\( \nu = 0.14 \)
\( Q' = 13-30 \)

FOOTWALL
Rock Type: Andesite/Diorite
\( \gamma = 3.0 \, \text{t/m}^3 \)
\( \sigma_0 = 160 \, \text{MPa} \)
\( E = 85 \, \text{GPa} \)
\( \nu = 0.23 \)
\( Q' = 14 \)
MINE No. 31

ORE
\[ \gamma = 3.5 \, \text{t/m}^3 \]
\[ \sigma_c = 265 \, \text{MPa} \]
\[ E = 63 \, \text{GPa} \]
\[ Q' = 25-40 \]

HANGING WALL
Rock Type: TUFF
\[ \gamma = 2.8 \, \text{t/m}^3 \]
\[ \sigma_c = 195 \, \text{MPa} \]
\[ E = 44 \, \text{GPa} \]
\[ Q' = 25-40 \]

FOOTWALL
Rock Type: Iron Formation
\[ \gamma = 2.9 \, \text{t/m}^3 \]
\[ \sigma_c = 275 \, \text{MPa} \]
\[ E = 51 \, \text{MPa} \]
\[ Q' = 25-40 \]

\[ \sigma_y = \gamma H \]

\[ \sigma_y = 8 + 1.6 \gamma H(\text{m}) \] (isostatic)

TYPICAL MINE CROSS SECTION
LONGITUDINAL LONGHOLE OPEN STOPING

DEPTH 850m