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Abstract

The subjects of tool-life, tool-wear and machinability have attracted extensive research interest most of which have been directed toward turning operations. The added complexity of milling operations, resulting from both the discontinuous nature of the process and the varying chip thickness during cut, are the reason for reduced research interest in comparison to turning. Although several researchers have examined the milling process, considerable controversy still surrounds the basic factors influencing tool-life in milling. In the study of the basic factors involved in the milling of both high strength steel and titanium alloy work materials, the basic form of the equations which result tend to support a thermal fatigue process as the most likely source of difference between continuous and discontinuous cutting. To validate this hypothesis further, special tests have been conducted within an inert atmosphere to ascertain the influence of oxidation on wear rate. It is also found that the influence of exit conditions may be critical in some circumstances, and experiments have again been carried out to examine this phenomenon. Finally, from a consideration of the tracking of tool-wear in milling operations, a comprehensive scheme which allows both identification of cutting conditions and tracking of wear on end mills and face mills is presented.
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Dedication

To 'Loye and Mairo..........

..........the sources of my pride and my joy.
INTRODUCTION

The phenomenon of Tool-Wear in cutting tools is extremely complex, involving and affected by a variety of disparate parameters and cutting conditions. Since the type and amount of wear to which a tool is subjected ultimately determine its total usefulness, Tool-Life is of extreme importance in any consideration of the economics of machining and the quality of finished products.

The economic benefits that can be achieved through a thorough knowledge of the subject are extensive: An understanding of process economics can result in the selection of machining conditions to optimize the life of the tool. This means that fewer tools are expended on any given component and consequently fewer tool changes are required per batch. Significant savings in tooling, and in machine and manpower costs thus result. These savings may well determine the profitability of any machine shop, be it small or large.

The Milling process is more complex than a process such as turning. Of increased complexity however is the subject of tool-wear in milling. Due to the complex nature of this machining process, the formulation of a realistic and useful mathematical model which accurately describes the process and gives a reliable, unique solution is difficult. The lack of previous research directed toward tool-wear in milling, in contrast to tool-wear in turning, in which substantial work has been carried out, may be attributed to the complex nature of the milling process itself (the variables affecting tool-life in turning are only half
the number in milling). Previous workers [1] have suggested that the main technical problem presented by milling as distinct from turning arises from the variation in chip thickness during cut and the interruption which occurs in the cutting cycle.

Of economic importance also is the subject of Tool-wear Monitoring or Sensing. The impetus for research in this area is the need to prevent premature tool failure which is devastating to surface finish, can lead to increased tooling costs, and in the worst case can cause extensive damage to the machine tool. With wear monitoring, the large safety factors normally applied in practice to the useful life of a cutting tool to prevent premature breakage (either by reducing the time of use or by reducing the cutting speed) become unnecessary. Under such circumstances, the tool in general cuts longer so that savings in tooling costs result.

Needless to say, it would be economically beneficial if useful tool-life information could be achieved on-line with no machine stoppage or tool removal. If force is accepted as an indirect measure of wear, then information on the condition of the tool should be obtainable from real-time force measurements, such measurements, of course, necessitating the use of dynamometers with reasonable bandwidth.

This thesis presents work performed in the areas mentioned. Chapter 1 discusses previous research in the general area of tool-wear and machinability. Chapter 2 describes the experimental work carried out in the investigation of tool-wear in milling and discusses the results in the light of previous work. Chapter 3 discusses force modeling, tool-wear monitoring and related concepts,
describes the experimental work carried out in this area, and presents the results obtained. Chapter 4 presents conclusions and recommendations.
Milling is a very important metal removal operation, yet a relatively small amount of effort has been directed towards better understanding of the process. While recent work from many research groups [36] have attempted to examine the increased efficiency in real milling operations, their work is generally based upon a constraint-driven approach; in most cases, metal removal rates are maximized consistent with constraints such as edge breakage, shank breakage, chatter, etc. Such investigations, however, have no basis in terms of the traditional approach to process optimization which requires detailed tool-life and wear relationships. Without doubt, the complexity of the process and the cost and time involved in the study of tool-life in milling has discouraged most groups, after an initial flurry of interest in the 1960's and 70's. This chapter reviews the basic concepts of milling, tool-life and tool-wear, reviews earlier work in milling which is related to the present research, and attempts to indicate the areas in which considerable doubt remain.

1.1 Milling

Milling is the name given to a metal removal process or operation involving the use of a tool with one or more teeth rotating about a fixed axis, each tooth
removing material from a workpiece being fed past the tool. Fig. 1.1 shows a schematic representation of the process:

![Fig. 1.1. Simple Schematic of Milling Operation](image)

There are basically two modes of milling: up- (or conventional) milling and down- (or climb) milling. When the cutter is rotating so that its peripheral velocity is in the opposite direction to the direction in which the workpiece is being fed, the mode of milling is termed up-milling. It is characterized by a zero value of undeformed chip thickness at entry into the workpiece and a finite value at exit from the workpiece. In down-milling, the reverse is the case. The cutter rotation is such that the peripheral velocity is in the same direction as that in which the workpiece is being fed, and this operation is characterized by a finite value of undeformed chip thickness at entry and a zero value at exit. Owing to the ambiguity involved in determining the direction of the peripheral velocity for different widths of cut, these definitions are valid only for widths of cut less than the radius of the cutter.
The major distinct characteristics of milling, which lead to a significant increase in complexity over turning, are its discontinuous or intermittent nature and the variable feed experienced by each tooth in cut (see Fig. 1.1).

1.1.1 Classification of Milling Operations

Milling operations can be classified into two types:

- Peripheral (or slab) milling
- Face (or end) milling

Fig. 1.2. Milling Operations: (a) Peripheral Milling; (b) Face Milling (after Armarego and Brown)

In peripheral milling, (see Fig. 1.2a), cutting occurs on the teeth at the periphery of the cutter, and the generated surface is a plane parallel to the cutter axis. The teeth can either be straight cutting edges, helical or form teeth.

In face milling however, cutting is performed by the edges on the periphery and the face (or end) of the cutter, and the generated surface is a plane at right angles to the cutter axis (see Fig. 1.2b).
1.1.2 Geometrical and Force Nomenclature in Milling

With reference to Fig. 1.3, the following are definitions of variables commonly used in milling terminology:

- **a** - Axial depth of cut
- **d** - Radial width of cut
- **R** - Cutter radius
- **d/R** - Immersion; (2=full immersion, 1=1/2 immersion, 1/2=1/4 immersion)
- **V** - Peripheral velocity
- **v** - Feed-rate
- **S_t** - Feed per tooth
- **φ_s** - Swept angle of cut
- **φ** - Instantaneous angle of rotation
- **RPM** - Spindle speed
- **F_R** - Radial component of cutting force
- **F_T** - Tangential component of cutting force
- **F_X,F_Y,F_Z = f(F_R,F_T,φ)** - X, Y, and Z components of cutting force
1.2 Tool-Wear

Cutting tools, due to the nature of the cutting process, are subjected to extremely severe rubbing. They are in metal-to-metal contact with both the workpiece and the chip under conditions of very high stress, and at high temperature. The result is gradual breakdown of the tool material in the region of cutting; this is termed "tool-wear". It is noteworthy that temperature plays a significant role in the wearing process and that wear behavior may vary significantly between one workpiece/tool combination and another.

1.2.1 Types of Wear

Tool-wear is of three major types: flank wear (or wear land), crater wear and notch wear. Depending on the circumstance, a tool can be affected by one, two, or all three types of wear.

- Flank wear (or wear land): this wear occurs on both the major and minor cutting edges under essentially all cutting conditions. It gives rise to what is
universally known as the "wear land" which is more or less a uniform wear zone on the flank of the tool as shown in Fig. 1.4.

According to Yellowley [2], flank wear can largely be attributed to a mechanical process initiated when primary particles are released from the tool face by a temperature-dependent process. As these primary particles are released and travel down the flank, they scour its surface and in the process release secondary particles by abrasion. These secondary particles also act similarly to further abrade the rest of the flank. Progressively, this process results in a wear land being formed on the flank. The effect of this wear land when it occurs on the major cutting edge (which is responsible for bulk metal removal) is increased cutting forces and higher temperatures which, if left unchecked, can lead to vibration of the tool and workpiece as well as to a condition yielding inefficient cutting. On the minor cutting edge, which determines machining accuracy and surface finish, oversized products with poor surface finish can result. However, under most practical conditions, a tool will
fail due to major flank wear before the minor flank wear is large enough to result in the manufacture of an unacceptable component. Flank wear is quantified in terms of the wear land width, \( VB \).

- **Crater wear**: this wear, found on the rake face of the tool, is caused by high contact stresses and high chip-tool interface temperatures which lead to localized pitting of the tool face some distance up the face. Diffusion is the major wear mechanism at play, but other mechanisms also play a role. The phenomenon, usually referred to as cratering, is commonly expressed in terms of the maximum depth of crater.

![Crater Wear on an Insert](image-url)

*Fig. 1.5. Crater Wear on an Insert (after Mills and Redford)*

Previous work [3] has shown that the maximum depth of crater occurs at a substantial distance from the cutting edge and that the crater curvature corresponds to the chip's radius of curvature. Under most practical cutting conditions, crater wear is less severe than flank wear, particularly at lower cutting speeds. However, since the rake face temperature is known to increase more rapidly than that of the flank face with increased cutting speed, and higher cutting speeds are known to generally lead to larger wear in a shorter time,
naturally, with increased cutting speed, crater wear can become quite significant. As crater wear progresses, it will eventually intersect the wear land so that a major fracture results. Nonetheless, crater wear is now of much less concern because rake face coatings (TiC, TiN, AlO₂) have been developed to drastically retard its growth.

- **Notch wear**: this occurs at the end of the major flank wear land where the tool is in contact with the uncut workpiece surface as shown in Fig. 1.6. It appears as a pronounced wear or notch at this point, and is caused by localized effects such as a hardened layer on the uncut surface possibly resulting from work-hardening introduced by a previous cut or an oxide scale. Although the notch will not significantly affect the cutting properties of the tool, it can become severe and relatively deep. As cutting continues, fracture of the tool will likely result.

![Fig. 1.6. Notch Wear on an Insert (after Mills and Redford)](image-url)
1.2.2 Mechanisms of Wear

While the scope of this work does not include examination of wear mechanisms in detail, a brief description is useful. There are five major mechanisms through which wear may occur:

- Wear by abrasion: the most common type of wear, in which relative motion, both between the underside of the chip and the face, as well as between the newly cut surface and the flank, causes the tool to wear even though the newly cut workpiece surface and the chip may be much softer than the tool material. In many cases, the reason for this is that, even though the workpiece and the chip may be relatively soft, hard inclusions or precipitates arising from the manufacturing process or from heat treatment will be present in the workpiece. Hard particles can also result from breakdown of work hardened, unstable built-up-edges. Abrasive wear is normally involved in the development of a wear land on the flank.

- Wear by adhesion: this is a wear mechanism which is active on the face of the tool at low cutting speeds. It has been shown that pressure welding exists between the face of the tool and the underside of the chip under essentially all cutting conditions and that this gives rise to welded junctions being formed between the chip and the tool. The relative motion between the faces causes these welded junctions to be sheared broken and often the shearing occurs beyond the tool face. The shearing action results in tool material being removed with the chip. Additionally, for those cases in which a built-up-edge (bue) is present, although adhesion will occur, the adhesion will not result in tool
material being removed if the bue is stable. For unstable bues, if strong bonding occurs between the bue and the tool material, it is likely that, when the bue detaches itself from the face, it will carry with it small quantities of the tool material.

- **Wear by diffusion**: diffusion is a temperature-dependent process in which atoms are transferred (or diffuse) in the opposite direction to the concentration gradient. The more recent concepts of wear consider diffusion to be an integral part of other wear mechanisms, contrary to previous claims that wear was caused purely by diffusion at contacting asperities. An example of a circumstance under which diffusion may be classified as a part of, for example the abrasion wear mechanism, is in the cutting of steel with a tungsten carbide tool. The chemical affinity between the steel and the cobalt binder in the tungsten carbide leads to diffusion of the cobalt out of the tool and into the steel. The result is the formation of a weakened surface layer on the tool face which is highly susceptible to abrasive wear. The inclusion of alloying elements such as titanium and tantalum help to retard diffusion in tool carbides.

- **Wear by fatigue**: this will only be significant when adhesive and abrasive wear rates are small. A tool surface will gradually fail owing to this mechanism if it is repeatedly subjected to loading and unloading as in intermittent cutting. Previous research has also demonstrated that the motion of an asperity on a surface rubbing across a tool face causes alternating stresses which eventually lead to fatigue failure. Fatigue cracking is unlikely to occur, however, if the stress is below a certain limit. Since the contact pressures are determined by
the yield properties of the workpiece material, fatigue wear can be reduced by
the use of cutting tools which are appreciably harder than the workpiece.

1.3 Tool-Life

Tool-life is a term used to describe the useful cutting life of a tool and
depends on the type and amount of wear to which the tool is subjected and the
quality specification of the part being machined. The type and amount of wear
are determined by the cutting conditions under which the tool is operating.
Under all cutting conditions, tool-life information is of extreme significance in the
prevention of economically undesirable catastrophic tool failure.

1.3.1 Determination of Tool-Life

As briefly mentioned, tool-life information is of extreme significance during
cutting operations if sudden tool failure is to be avoided. The economic
repercussions of sudden tool failure are large and undesirable. For this reason,
the usual practice in most manufacturing environments assumes the tool has
reached the end of its life long before the onset of catastrophic failure. The tool
is then replaced.

Fig. 1.7 shows a typical relationship between tool-wear and cutting time for
the case of flank wear. There are three major regions of interest; namely the
primary wear, secondary wear and tertiary wear zones.
Initially, with a new tool, the wear rate is high as is evident from the primary wear zone in Fig. 1.7. As the tool ages or "wears in", the wear rate slows considerably in what is called the secondary wear zone. At the end of this region by which time the flank wear is usually far greater than that recommended as the criteria for failure, the wear rate rapidly increases again in the tertiary zone. If this continues, it will rapidly lead to tool failure.

To prevent catastrophic tool failure, the usual practice is to change the tool well before the end of the secondary wear zone, i.e., well before the critical value of wear land width, $VB^*$, is attained. This end point, however, is actually a function of the final tool-life. Since even for one tool-workpiece combination tool-wear and final tool-life are highly variable, it follows that judgment, expertise and experience are necessary to determine the point at which the tool is changed. At this point of tool change, the tool is said to have reached the end of its useful life and the time elapsed termed the tool-life.
1.3.2 Tool-Life in Turning

As stated previously, owing to the relative simplicity of this cutting process over milling, considerably more research has investigated tool-life in the area. One of the pioneering workers was Taylor [8] who developed the well known "Taylor tool-life equation":

\[ V \times T^\alpha = \text{constant} \]

where \( V \) is the cutting speed,
\( T \) is the tool-life, and
\( \alpha \) is an empirical constant dependent on tool geometry.

Several years later, workers in the area modified the Taylor tool-life equation to reflect the influence of the feed-rate and the depth of cut:

\[ V \times T^\alpha \times S^\beta \times a^\gamma = \text{constant} \]

where \( V \) and \( T \) are as before,
\( S \) is the feed-rate,
\( a \) is the depth of cut, and
\( \beta \) and \( \gamma \) are empirical constants.

Because the empirical constants are typically less than one, Eqn. 1.2 clearly shows that the influence of cutting speed is the most significant on tool-life. This is particularly true because of the practical cutting speeds at which turning operations are performed. Fig. 1.8 shows a typical tool-life against cutting speed relationship.
Fig. 1.8. Typical Relationship Between Tool-Life and Cutting Speed

The practical cutting speeds at which turning operations are performed fall in the region c to d, while milling operation are typically performed at lower speeds in the region a to b. Owing to the characteristics of the relationship, the influence of cutting speed on tool-life in turning is shown to be more marked than in milling.

Further work in the area of tool-life in turning has allowed a reduction in the number of variables in the modified tool-life equation (1.2). By introducing a variable called the "equivalent chip thickness", \( h_e \), into the equation to replace the feed-rate and the depth of cut, it is possible to account for the influences of the replaced variables and, in addition, two other important variables: the approach angle of the turning tool and its nose radius (see Fig. 1.9). The simplified equation is:
where $h_e$ is the equivalent chip thickness, $A$ is the undeformed area of cut, $L_a$ is the length of active cutting edge, $\psi$ is the approach angle of the turning tool, $r$ is the nose radius of the turning tool, and $\delta$ is an empirical constant.

Fig. 1.9. Schematic of Turning Operation Showing Undeformed Area of Cut

1.3.3 Influence of Various Parameters on Tool-Life

In examining the complexity of the milling operation, Yellowley [1] listed a number of variables which had been observed to influence tool-wear and hence life. These were: work material, tool material, tool geometry, cutting speed, feed
per tooth, depth of cut, width of cut, time in cut, time out of cut, cutter diameter, entry conditions and exit conditions. The effect of any one of these factors on tool-life may be direct or through its effect on another variable. This section discusses the effect on tool-life, of some variables as investigated by previous workers in the field.

• **Tool geometry:** this affects tool-life through other variables such as number of teeth in cutter, cutter diameter and rake face angle. Kuljanic [4] investigated the influence of the number of cutter teeth on tool-life in face milling. The research was motivated by a desire to show that multi-tooth cutter life could not be predicted by single tooth tool-life tests as had previously been the practice. His experiments showed that under constant milling conditions, the tool-life, \( T \), is a nonlinear decreasing function of the number of cutter teeth, \( z \). He approximated this function by a polynomial in \( z \) over the range 1-6 (see Fig. 1.10).

Fig. 1.10. *Tool-Life as a Function of Number of Cutter Teeth (after Kuljanic)*
By introducing a 'heat generated ratio', Kuljanic concluded that thermal
effects were the main reason for the results obtained in his tests. He had shown
that there was a significant increase in workpiece and cutter temperatures with
an increased number of teeth in the cutter and that this had a dominant effect on
tool-life over the effect of mechanical shock or impact resulting from two or more
teeth in the cutter. His recommendation was that, since the largest differences in
tool-life were obtained between one and two teeth in the cutter, tool-life tests to
predict multi-tooth cutter life should be performed with at least a three tooth
cutter.

The effect of rake face angle is observable from results of work performed
by Tlusty et al. [5] although no reference was made to this aspect of the work in
their report. One-quarter, one-half and full immersion experiments were carried
out with tools having rake face angles of -5°, 0° and +5°. The results showed
that the tool with the +5° rake face angle generally gave the longest tool-life
values. The one of the other two geometries which gave a higher tool-life
depended on the immersion and mode of milling.

- **Cutting speed**: the influence of this variable was considered by Tlusty et
  al. [5] in their investigation of wear in the peripheral milling of low carbon steel. A
  set of experiments had been performed at a typical practical cutting speed of
  120 m/min while influences of various variables on tool-wear and life were
  investigated. One of the experiments was repeated at twice this cutting speed to
  observe any significant differences arising. The results showed that, in general,
tool-life was not significantly affected by the gross increase in cutting velocity.
However, the previously observed differences between up- and down-milling
were vastly reduced, the down-milling results being worse at the higher velocity while the up-milling results were almost identical. The absence of chip adhesion at exit in up-milling with the higher cutting speed was also noticed. The author notes that, following the explanation given in Sec. 1.3.2 with regard to the practical cutting speeds in milling and the attendant influence on tool-life, the result obtained by these workers is to be expected. If milling operations were carried out at the same cutting speeds as turning operations, the influence of the cutting speed would be more pronounced than that experienced by the workers. For this same reason, typically, milling operations are performed at lower speed ranges (see Fig. 1.8), i.e., to improve tool-life. No explanation was offered by Tlusty et al. [5] for the reduced tool-life values obtained in down-milling at the higher speed. However, based on the author's experience, it can probably be attributed to the effect on the tool of increased thermal shock resulting from the higher temperatures at the higher cutting speed and/or mechanical shock resulting from the increased severity of impact during entry into the workpiece at the higher cutting speed.

- **Feed per tooth**: the influence of this parameter on tool-life was also investigated by Tlusty et al. [5]. An experiment was set up to examine tool-wear rates at various values of feed per tooth. The results showed that the feed per tooth had negligible influence on the wear rate and hence tool-life.

- **Width of cut**: this variable affects tool-life predominantly through its influence on times in and out of cut. These parameters directly determine the ratio of active cutting time to total cutting time, i.e., as the width of cut decreases, since the time the tool spends in actual cut decreases, naturally this ratio would
decrease. Therefore, for a constant active cutting time, the number of cycles undergone by the tool would increase and, at the same time, the range of thermal strain in the cycle increases. Increases in the number of cycles undergone and the range of thermal strain have actually been proven to be detrimental to tool-life by Yellowley et al. [6]. The workers introduced the concept of a thermal fatigue parameter which characterizes the influence of the width of cut. This parameter is discussed in more detail later in this chapter. Nevertheless, results of experiments conducted by Tlusty et al. [5] seem to show that, generally, for the same mode of cut, the smaller widths of cut yield shorter tool-life values. Experiments discussed in Chapter 2, however, verify this.

- **Mode of milling**: according to Tlusty et al. [5], this affects tool-life principally through its effect on the entry and exit conditions, which influence the severity of both mechanical and thermal shock. Through their experiments they had generally observed that the down-milling mode led to higher active cutting lives than up-milling, that the mechanical damage or chipping of the cutting edge was more severe in up-milling than in down-milling, and that the active cutting life of the tool did not appear to be greatly influenced by the entry conditions in down-milling.

These results illustrate the significance of chip adhesion in influencing tool-wear. In down-milling in which the tooth enters with a finite chip thickness and exits with zero chip thickness, the likelihood of chip adhesion, in comparison to up-milling in which the maximum chip thickness is encountered close to exit, is minimal. It is evident that the severity of damage which may be inflicted on the tooth on any one entry would be closely related to the intensity and extent of
adhesion in the preceding exit [5]. As a result, it would be expected that up-milling would yield higher wear rates and thus shorter tool-life values than down-milling.

Nevertheless, as had been shown in earlier work [1], the adhesion of chips to the cutting edge is most significant only when machining work-hardening work materials. Formation of tensile cracks in the tool surface during the final shearing process as the tool point approaches the free surface may be attributable to the ability of the work material to work-harden: according to Yellowley [7] and on the basis of milling experiments performed on a titanium alloy, with the tool experiencing a finite chip thickness as it exits from the work material, the shear plane rotates to coincide with the cutting velocity vector (see Fig. 1.11) so that the final action is similar to a blanking and piercing operation [11]. This rotation of the shear plane causes the material to work-harden, and the resulting tensile forces cause cracking of the tool. The chip or 'foot' formed in the process sticks to the tool and, as mentioned earlier, proves detrimental to the tool on the succeeding entry.
Later work by Pekelharing et al. [12-14] into exit failure of cutting tools suggested that the shear plane actually rotated past the cutting velocity vector to a negative position and that the forming foot, also rotating during this time, tried to take the chip with it in its rotation. The result is a dangerous change in the pressure distribution in the chip-tool contact zone; all remaining cutting forces concentrate near the cutting edge thus chipping the tool. According to the workers, depending on the exit angle of the workpiece, the phenomenon could either be aggravated or suppressed. It is noteworthy that these suggestions were made on the basis of interrupted cutting lathe experiments (performed on steel work material) being used to simulate the milling operation. Irrespective, it is expected that these conflicting explanations will be verified by experiments to be conducted in the course of this research.

An exception to the supposedly harmless entry condition in down-milling was observed by Yellowley [7] in the carbide milling of stainless steel. He pointed out that chip adhesion could be critical in the initial stages of down-milling if the cutter was traversing into a sharp corner and therefore experienced a finite value of chip thickness at exit until it had traveled a distance into the workpiece equal to its radius. In such a case, chip adhesion comparable in
intensity to that in up-milling occurs; as a result of the severe entry conditions of down-milling, the resulting mechanical damage is catastrophic. The phenomenon would no doubt be even more pronounced with a work-hardening work material, resulting in almost immediate tool failure. In verifying, he demonstrated that the phenomenon was non-existent when a cusp of the same shape as the cutter was machined to replace the sharp corner before cutting commenced.

One of the earliest workers to investigate the influence of the mechanical effect of impact, at tool entry, on tool-wear was Kronenberg [15]. He proposed that the life of the tool was influenced by the location and magnitude of the initial impact of the tool with the workpiece for a given tool geometry. Fig. 1.12 shows the schematic of tool contact at entry into the workpiece proposed by Kronenberg.

![Fig. 1.12. Schematic of Initial Contact at Tooth Entry (after Kronenberg)](image)
Fig. 1.12 shows that initial contact between workpiece and tool may occur either as a point, a line, or a full area contact, depending on the tool geometry and the positioning of the tool. For prevention of premature tool breakage at entry, impact should be kept away from the cutting point S. In an attempt to quantify the influence of impact at any point, Kronenberg defined the partial time of penetration, $T_S$, to be the time which elapses between the initial contact and the contact of the tool at point S. He concluded by suggesting that the most desirable condition to yield optimum tool-life could be obtained by careful coordination of axial and radial rake angles, corner angle, chamfer angle and angle of engagement. Later workers in the area (Opitz et al. [16]) considered the parameter $T_S$ to be inadequate and went further to define the partial area of engagement, $F_S$, as the area which is crossed by the index line before the point S is reached (the index line being the line formed by the intersection of the plane STUV and the plane in which the rake face of the tool lies). They further showed that the higher the partial area of engagement, the higher the resulting tool-life.

In reviewing the work of these early researchers in the light of results obtained in the course of his own work, Yellowley [1] noted that their work had been undertaken for the face milling operation, in which a change in cutter offset affected several variables simultaneously, and he demonstrated how the relationship between tool-life and cutter offset could vary in such a situation. Yellowley noted that in his own down-milling experiments on a titanium alloy, the results did not show any discernible differences in tool-life, irrespective of whether the tool-work contact was a line or an area or, in the case of the line, whether the line was formed at the edge or across the face of the tool.
With respect to thermal shock, however, Yellowley [7] suggested that the influence of thermal shock would be more severe in down-milling. The problem was simulated by the application of a three-step increase in temperature to the rake face with this increase similar to that which would be experienced by the rake face in an up-milling operation. The rake face was then allowed to cool before the same three temperatures in reverse order were applied to simulate down-milling conditions. The rake face was finally then allowed to cool (see Fig. 1.13).

![Fig. 1.13. Influence of Up- and Down-Milling on Rake Face Temperature (after Yellowley and Barrow)](image)

As these results confirm, the level of compressive strain after a typical cooling time was similar for both up- and down-milling cases. However, with the maximum value of compressive strain at the start of the next heating cycle higher in the down-milling case, the range of thermal strains would naturally be higher in down-milling than in up-milling. In conclusion, the literature reviewed showed that the mechanical influence of chip adhesion, where present, has a predominant effect on tool-life over the thermal shock effect.
1.4 Concept of Equivalent Feed

As already pointed out, one characteristic problem associated with milling is that of the varying feed experienced by each tooth in cut. Fig 1.14 shows a schematic of the variation in feed seen by a tooth over one cutter revolution for both one-half and full immersion cutting.

![Diagram of Feed Variation in Milling](Image)

**Fig. 1.14. Variation of Feed in Milling Over One Cutter Revolution**

Fig. 1.15a shows the approximately linear relationship between tool-wear and time when cutting is performed at either of constant feed-rates $S_1$, $S_2$, and $S_3$, as would be the case in turning. The points corresponding to the varying wear seen by the milling tooth at each of points a, b and c (with feed-rate values $S_1$, $S_2$ and $S_3$ respectively) in Fig. 1.15b are shown on Fig 1.15a. The broken line passing through these points describes the typical tool-wear - time relationship for milling.
To cope with the problem of the varying feed in milling during tool-life analyses, Yellowley et al. [1] introduced the concept of an equivalent feed, $S_{eq}$. This was defined as that constant feed which, if applied, would give the same wear rate as in the actual case in which the feed is continuously varying. Fig. 1.16 shows a schematic plot of tool-wear versus active cutting time for a milling operation over three revolutions of the cutter. The equivalent feed is that feed which would give a constant wear rate equivalent to the dotted line shown passing through the 'arc ends' of the plot. The equivalent feed is therefore a function of the feed per tooth, the width of cut and the radius of the cutter,

\[ S_{eq} = f(S_t, d, R) \]

The concept of the equivalent feed allows the two variables (feed per tooth, $S_t$, and width of cut, $d$) to be combined and results in significant savings in machinability testing time.
As discussed, the influence of feed per tooth on tool-life was negligible during the tests performed by Tlusty et al. [5]. For some combinations of tool and workpiece materials, this has been found to be true. In such cases, the influence on tool-life of the highly emphasized varying undeformed chip thickness in milling may be ignored. According to Yellowley et al. [6], under these conditions, any variation in tool-life may result from either mechanical or thermal effects or a combination of both. Until their work was performed, opinion on the relative influence of the two effects had been divided. To enhance isolation of the two (mechanical and thermal) effects during experimentation, their work dealt with the end milling process only so that either of the entry or exit conditions could be kept constant while the influence of the thermal effect(s) was investigated. The following discussion will show further that the work of

1.5.1 Tool-Life in Half and Full Immersion Tests

The geometry of the end milling process shows that a changed width of cut (1/2 immersion or full immersion cutting) will alter the relationships between the time in and out of cut. This change would appear to be the major reason for the tool-life differences observed between 1/2 and full immersion cutting.

Yellowley et al. [6] in performing a series of 1/2 and full immersion tests to investigate tool-life differences, employed a carbide end mill on EN28 (AISI 4140 mild steel) work material. Fig. 1.17 shows the results of the tests conducted.

![Fig. 1.17. The Comparison Between End Mill Slotting and Half Immersion Tests (after Yellowley and Barrow)](attachment://image.png)

The results of the 1/2 immersion tests carried out for both up- and down-milling conditions showed that tool-life values obtained in both cases were comparable during all tests. Workers noted that this was to be expected since
previous work had indicated that the influence of the mode of milling was only significant when machining high strength or heavily work-hardening materials. However, a marked difference was observed between the 1/2 immersion tests and the slotting tests (see Fig. 1.17); a considerable increase in tool-life was obtained with the slotting experiment. Considering that, for the same cutting time, the active cutting time in slotting is twice that in 1/2 immersion, i.e., in slotting, the tool is in cutting contact with the workpiece for twice as long as it is in 1/2 immersion, these results therefore suggested the influence of another factor unaccounted for prior to this time.

In further investigation of these results, Yellowley et al. [6] repeated the slotting tests with a 1/16 inch gap in the workpiece on the cutter center line (see Fig. 1.18) to verify whether the bad exit conditions of up-milling and the bad entry conditions of down-milling influenced the results. The new results were comparable to the previous results obtained without the gap in the workpiece. It was therefore concluded that the differences in heating and cooling times experienced by the cutter in slotting and 1/2 immersion were the reason for the differences observed in tool-life values for both cases. The workers realized the need for a parameter which would characterize the influence of the repeated heating and cooling, or thermal fatigue, of the workpiece during cutting. One objective of this research will be to determine if another influence besides thermal fatigue, presently unaccounted for, is responsible for the observed phenomenon.
1.5.2 Explanation of Thermal Fatigue Phenomenon

Milling cutter teeth are subjected to a low cycle, high plastic strain process, owing to the characteristic intermittent cutting which results in repeated heating and cooling of the rake face. An explanation of the phenomenon would consider the face of the milling tool to consist of several parallel layers of tool material. On entering the workpiece, the surface layer of the tool is rapidly heated to its maximum temperature; the time required for this has been shown by workers in the area [9], [10] to be small when compared with the cutting cycle times normally encountered in milling. As a result, the layer begins to expand immediately but, owing to restraint from subsequent layers below which have not attained the same temperature (see Fig. 1.19a), it is put into compression. The magnitude of this compressive stress will decrease as the duration of heating increases and the temperature gradient between successive layers decreases. Upon exiting the workpiece, the surface of the tool is rapidly cooled. According to Yellowley et al. [6], the temperature distribution within the tool is now as shown in Fig. 1.19b. This distribution indicates that, with increasing cooling time,
the temperature gradient within the tool will fall and result in reduced compressive stresses in the surface layers of the tool.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{temperature_distribution_graph}
\caption{Temperature Distribution Within Tool During Heating and Cooling Cycles (after Yellowley and Barrow)}
\end{figure}

During the heating cycle, the tool material is believed to yield in compression thereby giving rise to residual tensile stresses within the surface layers when the tool is cooled. The result of these residual tensile stresses is weakening and eventual cracking of the tool surface. As the number of cycles completed by the tool increases, the tool surface is further weakened by the fatigue process and is thus more susceptible to mechanical wear.

1.5.3 Definition of Thermal Fatigue Parameter

Following extensive research and experimentation involving heating and cooling times of tools, Yellowley et al. [6] developed an expression for a thermal fatigue parameter, $X$. The preceding discussion indicates that such a parameter should account for the temperature range traversed as a result of the heating and cooling of the tool, the proportion of cycle time over which the tool is being intermittently heated and cooled, and the influence of cycling:
\[ X = f(E_{r}, RPM, x); \quad E_{r} = f(t_{c}, t_{h}) \]

where \( E_{r} \) is the range of thermal strain parameter,
\( RPM \) is the rotational speed of the cutter,
\( x \) is the ratio of total cutting time to active cutting time, and
\( t_{c}, t_{h} \) are the cooling and heating times respectively.

With the development of this parameter, the workers could use it to obtain a more realistic tool-life equation for milling. Experiments showed that the tool-life varying inversely with the thermal fatigue parameter would explain why, in general, smaller widths of cut (which give larger values of thermal fatigue parameter) yield smaller tool-life values.

1.5.4 Tool-life Equation for Milling

According to Yellowley et al. [1], the simplest tool-life equation for milling is of the Taylor type. This equation can be written to include the influence of the velocity and the depth of cut as:

\[ T_{a} = \frac{\text{constant}}{V^{p} \times a^{q}} \quad \text{1.4} \]

where \( T_{a} \) is the active tool-life,
\( V \) is the cutting velocity,
\( a \) is the depth of cut, and
\( p \) and \( q \) are empirical constants.
With the development of the equivalent feed, $S_{eq}$, and the thermal fatigue, $X$, their influences could be included in the tool-life equation. The tool-life equation for milling was finally presented by Yellowley et al. [1] in the form:

$$T_s = \frac{\text{constant}}{X^n \times S_{eq}^{1/3} \times V_p \times a_q^{1.5}}$$

where $n$ and $\beta$ are empirical constants.

Experiments verifying the validity of this tool-life equation for milling suggest that the equation is limited to one mode of milling and should not be used when chip-sticking conditions prevail. Furthermore, the equation is not strictly valid when the angle of lag between leading and trailing edges of the cutter become appreciable compared to the swept angle of cut.

The work to be conducted by the author during the present research are:

- Tool-life experiments on steel and titanium materials to obtain tool-life data.
- Tool-life analyses to obtain tool-life equations for both materials and to permit an evaluation of the influence of various parameters on tool-wear.
- Argon influenced tool-life test to verify the role of oxidation in tool-wear.
- Experiments to verify the occurrence of exit failure resulting from sharp corner conditions in down-milling.
- Development of an experimental set-up to investigate the processes occurring at tool-exit from the workpiece which result in exit failure in materials affected by the phenomenon.
- Experiments to investigate the validity of a force model proposed for milling.
- Experiments to enable the development of a wear tracking parameter devoid of the problems associated with previously developed wear tracking parameters.
Chapter 2

Tool-Life Investigation: Experimentation and Analysis

This chapter describes a detailed investigation of tool-life. The initial series of experiments examine the validity of previously derived equations during the determination of tool-life equations for milling of high strength steel and titanium workpieces. The later sections of the chapter are concerned with the examination of the influences of oxidation and exit conditions on tool-life.

2.1 Tool-Life Experimentation

The first series of tests examined tool-life when milling steel and titanium work materials under different cutting conditions, these data were used to obtain tool-life equations for the respective materials and to verify the influences of chosen cutting parameters on tool-life. As mentioned earlier, because the variables affecting tool-life in milling are numerous, isolation of the individual influences of these variables is difficult. On the basis of work reviewed, it would seem the most significant influences are likely to be exerted by the width of cut (through times in and out of cut), the cutter diameter (through thermal cycling), the feed-rate, and the mode of milling. The tool-life tests therefore which were designed to investigate the separate influences of these variables on tool-life, isolated each variable in turn, while keeping the other variables constant. The depth of cut and the peripheral velocity remained constant during each series of
tests. The influence of the width of cut on tool-life was isolated by tests that employed different widths of cut at the same feed-rate (or more correctly, equivalent feed). The influence of feed-rate was isolated by tool-life tests conducted with the same width of cut at different feed-rates. Identical tests using two cutters of different diameters permitted isolation of the influence of cycling frequency. In this case, the change in peripheral velocity that would have resulted from the change in cutter diameter was prevented by changing the spindle speed accordingly. To investigate mode of milling, up- and down-milling experiments were performed under the same cutting conditions to determine their influence on tool-life.

Spindle speeds selected for the experiments described used average values within suggested ranges\(^1\) for the respective work materials. The feed-rates were chosen so that the resulting feed-per-tooth values lay between the averages 0.03 and 0.1 mm/tooth.

For each experiment, the wear on the cutting tool used was monitored and recorded at frequent intervals during cutting. When the wear on a cutting tool had exceeded a preset limit, the tool was considered to have reached the end of its life and removed from cut. A graph of wear against cutting time could then be plotted for the tool, and from it the time elapsed before the wear reached the preset limit read off and recorded as the tool-life of the tool. With the tool-life data collected in this way, an analysis for both work materials was possible.

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\(^1\)Machinist's Handbook
The analysis presented is based on the validity of thermal fatigue as the proven major influence on tool-wear. However, this validity has been questioned by other workers [37] who suggested that perhaps the influence of this phenomenon on wear was not quite as significant. On this basis, the possibility of another major influence being at play, namely oxidation, was investigated during this research. The aim was to determine whether any major improvement in tool-life could be achieved by excluding oxygen from the immediate region surrounding the tool during cutting. Any major improvement would confirm that oxidation was responsible for the influence of cut time ratio on tool-wear and thus dispel the validity of thermal fatigue being the major influence; otherwise, the validity of thermal fatigue would be corroborated. Further confirmation would be obtainable from an experimental design that permits cutting in an environment that would alter the temperature range within which the tool is alternately heated and cooled during cutting. Similar work has however been carried out by previous workers [38].

2.1.1 Equipment Setup and Test Procedure

The milling machine used for all the tests in this research was a vertical spindle, 5.5 kW, Bridgeport 2S machine capable of 18 spindle speed changes and 18 longitudinal and cross feed changes. Other equipment for the tool-life tests consisted of: an end mill chuck with a set of collets; a five pocket face mill cutter; a hand-held microscope with a graduated reticle, retrofitted with a holder and a magnetic stand; a two axis, moving table, measuring laboratory microscope fitted with micrometers on each axis and a stop watch. The cutting
tools used comprised 1/2 and 1/4 inch three flute High Speed Steel (HSS) end 
mills and tungsten carbide (H13A grade) inserts. As stated, the work materials 
were AISI 4140 low alloy steel and a titanium alloy. The end mills were 
employed on the steel work material and the face mill inserts on the titanium 
alloy work material. Fig. 2.1 shows the milling machine set up for tests on the 
steel work material. Visible are the end mill cutter, the steel workpiece and the 
hand-held microscope for measuring the tool-wear.

Fig. 2.1. Equipment Set-Up for Tool-Life Tests on Steel Work Material
The chief differences between this set-up and that used for the tests on the titanium alloy were the replacement of the end mill with a face mill, and the use of the laboratory microscope rather than the hand-held microscope to measure the wear on the inserts. Fig. 2.2 shows the laboratory microscope with an insert in place.

---

*Fig. 2.2. Two Axes Measuring Laboratory Microscope Used for Tests on Titanium Alloy, and Stopwatch Used for All Tests*
The experimental procedure to obtain the tool-life data was:

• for the experiments on the steel work material, for any chosen end mill size, mode of milling, width of cut, feed-rate and spindle speed, the tool was allowed to cut over one pass (approximately 16 inches long). Prior to commencing the test, a cusp, cutter-radius distant into the workpiece, was machined so as to prevent premature tool-wear caused by exit failure (see Sec. 2.2).

• the hand held microscope was then positioned to measure the wear on all three flutes.

• with the wear recorded and the wear pattern on each flute sketched, the tool was put into cut over another pass and checked again.

• the process was continued until the wear was observed to have exceeded the preset limit; in the case of the steel experiments, 0.01 inch.

• in the manner described earlier, the time taken for the wear to reach exactly 0.01 inch was then read off as the tool-life of the tool for that one test.

In the case of the tests carried out on the titanium alloy, only one insert was used in the face mill cutter, in order to conserve work material, and the insert was removed and its wear checked often (thirty seconds to two minute intervals), depending on the cutting speed and feed-rate in operation. It was important to check the tool more often during these tests because of the high wear rates associated with titanium. The occurrence of sudden tool deterioration was higher with titanium than with steel. In the case of titanium, the wear limit was set at 0.015 inch.
The procedure to quantify the tool-wear differed slightly for both test cases. Irregularity in the wear patterns on the end mill flutes were observed in the steel experiments; for this reason mean values were estimated and compared against the preset limit at each point at which the tool was checked. For titanium for which the wear pattern on the insert was rather uniform, the maximum wear value was used instead. Provided consistency was maintained, this difference in measuring procedure was not expected to influence the tool-life analysis.

2.1.2 Tool-life Data For Steel and Titanium Work Materials

Tables 2.1 and 2.2 present the tool-life data obtained during the series of tests conducted. It is noteworthy to state that the tool-life values shown in both tables represent averages of data obtained over three series of tests, in the case of the steel, and as many as five series of tests in the case of the titanium.
Chapter 2  Tool-Life Investigation: Experimentation and Analysis

Table 2.1. Tool-Life Data for AISI 4140 Low Alloy Steel Work Material

<table>
<thead>
<tr>
<th>Test #</th>
<th>Spindle Speed [rpm]</th>
<th>Mach. Feed [mm/min.]</th>
<th>Feed per Tooth, ( S_t ) [mm/tooth]</th>
<th>Cutter Dia. [inch]</th>
<th>Width of Cut</th>
<th>Mode of Milling</th>
<th>Tool-Life, ( T_s ) [minutes]</th>
<th>Active Tool-Life, ( T_a ) [minutes]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>912</td>
<td>102</td>
<td>0.037</td>
<td>0.5</td>
<td>1/2 imm. Up</td>
<td>25.76</td>
<td>6.44</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>912</td>
<td>102</td>
<td>0.037</td>
<td>0.5</td>
<td>Slotting</td>
<td>34.91</td>
<td>17.455</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>912</td>
<td>168</td>
<td>0.061</td>
<td>0.5</td>
<td>Slotting</td>
<td>25.77</td>
<td>12.865</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1800</td>
<td>168</td>
<td>0.031</td>
<td>0.5</td>
<td>Slotting</td>
<td>19.28</td>
<td>9.64</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>912</td>
<td>264</td>
<td>0.0965</td>
<td>0.5</td>
<td>Slotting</td>
<td>15.63</td>
<td>7.815</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>912</td>
<td>264</td>
<td>0.0965</td>
<td>0.5</td>
<td>1/2 imm. Up</td>
<td>11.33</td>
<td>2.833</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>912</td>
<td>264</td>
<td>0.0965</td>
<td>0.5</td>
<td>1/2 imm. Down</td>
<td>14.5</td>
<td>3.625</td>
<td></td>
</tr>
</tbody>
</table>

- Cutter type: 3 flute HSS end mill
- Depth of cut: 1.0 mm

\[ T_s = T_i \times \frac{\Phi_s}{360^\circ} \]; where \( \Phi_s \) is the swept angle of cut

Table 2.2. Tool-Life Data for Titanium Alloy Work Material

<table>
<thead>
<tr>
<th>Test #</th>
<th>Spindle Speed [rpm]</th>
<th>Mach. Feed [mm/min.]</th>
<th>Feed per Tooth, ( S_t ) [mm/tooth]</th>
<th>Width of Cut</th>
<th>Tool-Life, ( T_s ) [minutes]</th>
<th>Active Tool-Life, ( T_a ) [minutes]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>576</td>
<td>62.4</td>
<td>.1083</td>
<td>1/2 imm.</td>
<td>5.182</td>
<td>1.296</td>
</tr>
<tr>
<td>2</td>
<td>576</td>
<td>30.0</td>
<td>.0521</td>
<td>1/2 imm.</td>
<td>12.644</td>
<td>3.161</td>
</tr>
<tr>
<td>3</td>
<td>576</td>
<td>24.0</td>
<td>.0417</td>
<td>1/2 imm.</td>
<td>34.91</td>
<td>8.728</td>
</tr>
<tr>
<td>4</td>
<td>576</td>
<td>62.4</td>
<td>.1083</td>
<td>Slotting</td>
<td>6.105</td>
<td>3.053</td>
</tr>
<tr>
<td>5</td>
<td>576</td>
<td>62.4</td>
<td>.1083</td>
<td>3/4 imm.</td>
<td>7.17</td>
<td>2.39</td>
</tr>
<tr>
<td>6</td>
<td>576</td>
<td>62.4</td>
<td>.1083</td>
<td>1/4 imm.</td>
<td>11.0</td>
<td>1.833</td>
</tr>
</tbody>
</table>

- Mode of milling: Down
- Cutter type: 5 pocket face mill, Dia. 3.15" with tungsten carbide insert
- Depth of cut: 1.0 mm

2.1.3 Data Analysis

This section demonstrates the logic underlying the derivation of an expression for the equivalent feed in milling and links this concept with the idea
developed by previous workers of the thermal fatigue parameter to allow the development of tool-life equations for various work/tool pairs. The validity of previous work is also reviewed in the light of new data obtained during this research.

The equivalent feed was defined earlier as that constant feed which, if applied, would give the same wear rate as the continuously varying feed in milling. Fig. 2.3 illustrates the constant wear rates resulting from constant feed-rates S1, S2, and S3, as would be the case in, for example, turning. The corresponding times T1, T2, and T3 represent the active tool-life values obtained before attaining the wear limit, $VB_0$, under these respective conditions.

As a result of the linearity, the wear rate can be expressed as:

$$\frac{dVB}{dt} = \frac{VB_0}{T_a}$$

where $VB$ is the instantaneous wear, and $T_a$ is the active cutting life.
If a Taylor type relationship is assumed between feed, $S$ and active tool-life, $T_a$:

$$S \times T_a^\beta = \text{constant} = C$$

where $\beta$ is an empirical constant for milling

therefore

$$\Rightarrow T_a = \left( \frac{C}{S} \right)^{\frac{1}{\beta}}$$  \hfill (2.3)

Substituting Eqn. 2.3 into 2.1:

$$\frac{dVB}{dt} = VB_0 \cdot \left( \frac{S}{C} \right)^{\frac{1}{\beta}}$$  \hfill (2.4)

A milling tooth subjected to wear during one pass through the work is shown in Fig. 2.4a. Fig. 2.4b shows how the wear would vary during the period in cut.

![Fig. 2.4. Milling Tooth and Associated Wear Over One Cycle](image)
The total wear, $VB_1$, over this pass can be expressed as:

$$VB_1 = \int_0^{t_{cut}} \frac{dVB}{dt} \, dt$$  \hspace{1cm} 2.5

where $t_{cut}$ is the time spent in cut.

Substituting Eqn. 2.4 into 2.5:

$$VB_1 = \frac{VB_0}{C^{\frac{1}{\beta}}} \int_0^{t_{cut}} (S_t \cdot \sin \phi)^{\frac{1}{\beta}} \, d\phi$$  \hspace{1cm} 2.6

From Fig. 2.4, the average wear rate over the pass can be expressed as:

$$\frac{VB_1}{t_{cut}} = \frac{VB_0}{C^{\frac{1}{\beta}}} \cdot \frac{1}{\phi_s} \int_\theta^{\phi_s} (S_t \cdot \sin \phi)^{\frac{1}{\beta}} \, d\phi$$  \hspace{1cm} 2.7

where $S_t$ is the feed per tooth, and

$\phi_s$ is the swept angle of cut.

If a constant equivalent feed, $S_{eq}$, is defined so that it results in a wear rate equal to the average in the milling operation:

$$\Rightarrow \frac{dVB}{dt} = VB_0 \cdot \left( \frac{S_{eq}}{C} \right)^{\frac{1}{\beta}} = \frac{VB_0}{C^{\frac{1}{\beta}}} \cdot \frac{1}{\phi_s} \int_\theta^{\phi_s} (S_t \cdot \sin \phi)^{\frac{1}{\beta}} \, d\phi$$  \hspace{1cm} 2.8

$$\therefore S_{eq} = \left[ \frac{\frac{1}{\phi_s} \int_\theta^{\phi_s} (S_t \cdot \sin \phi)^{\frac{1}{\beta}} \, d\phi}{\theta} \right]^{\beta}$$  \hspace{1cm} 2.9
Or \( \frac{S_{eq}}{S_t} = \left[ \frac{1}{\phi_s} \int_0^\phi (\sin \phi)^{\frac{1}{2}} d\phi \right]^b \) \hspace{1cm} 2.10

For any given width of cut, the right hand side of Eqn. 2.10 results in a constant. Fig. 2.5 illustrates how the ratio \( \frac{S_{eq}}{S_t} \) typically varies with the ratio of width of cut to cutter radius (or immersion), \( \frac{d}{R} \):

![Graph showing variation of \( \frac{S_{eq}}{S_t} \) with \( \frac{d}{R} \) in Milling.]

As a result of the symmetry between full and half immersion cutting, the ratio \( \frac{S_{eq}}{S_t} \) is the same for both cases, i.e., for the same feed-per-tooth, \( S_f \), the equivalent feed, \( S_{eq} \), is the same for both full and half immersion cutting. This information is useful in the ensuing analysis.
The tool-life equation for milling developed by Yellowley et al. [1], as presented in equation 1.5, is expressed as:

\[ T_a = \frac{constant}{X^n \times S_{eq}^{1/\beta} \times V^p \times a^q} \]  \hspace{1cm} 2.11

where \( T_a \) is the active tool-life,

\( V \) is the cutting velocity,

\( a \) is the depth of cut, and

\( n, \beta, p \) and \( q \) are empirical constants.

If the influence of the cutting velocity and the depth of cut are considered less significant as mentioned earlier, these variables are thus kept constant and the equation can be rewritten as:

\[ T_a = \frac{constant}{X^n \times S_{eq}^{1/\beta}} \]  \hspace{1cm} 2.12

As a result of direct proportionality (Eqn. 2.10), the equivalent feed, \( S_{eq} \), can be replaced by the feed-per-tooth, \( S_f \) in Eqn. 2.12.

\[ \Rightarrow \quad T_a = \frac{constant}{X^n \times S_f^{1/\beta}} \]  \hspace{1cm} 2.13

Taking the logarithm of both sides of the expression:

\[ \log T_a = \log constant - \log X^n - \log S_f^{1/\beta} \]  \hspace{1cm} 2.14

For experiments performed at the same immersion but different feed-rates, the thermal fatigue parameter, \( X \), is constant.
If a graph of $\log T_\alpha$ is plotted against $\log S$, the exponent $1/\beta$ is defined by the slope of the graph.

For full and half immersion experiments performed at the same feed-rate, as pointed out earlier, the equivalent feed is the same.

$$\Rightarrow \log T_\alpha = -\frac{1}{\beta} \log S + \text{constant} \quad 2.15$$

If $\log T_\alpha$ is plotted against $\log X$, the exponent $n$ may be determined from the slope of the graph. The validity of the theory as well as the data obtained will be verified by the accuracy of these graphs.

Working back from the value of the constant given in Eqn. 2.15 enables determination of the constant in the tool-life equation (Eqn. 2.12). This constant is given by the value of the intercept on the same $\log T_\alpha$ against $\log S$, plot used to determine $\beta$.

An expression for the thermal fatigue parameter, $X$, developed by Yellowley et al. [6], can be written as:

$$X = E_r \cdot (RPM \cdot x)^{\frac{1}{2}} \quad 2.17$$

where $E_r$ is the range of thermal strain parameter, $RPM$ is the spindle speed, and $x$ is the ratio of cutting time, $t$, to active cutting time, $t_a$. 
An expression for the range of thermal strain parameter was also developed [6] and can be written as:

\[ E_r = 39 \cdot \log t_c - 23 \cdot \log t_h + 37.5 \]  \hspace{1cm} 2.18

where \( t_c \) is the cooling time in milliseconds, and \( t_h \) is the heating time in milliseconds.

This expression assumes that the rake face temperature is constant during the heating cycle. However, since the parameter is simply intended to characterize the relative influences of heating and cooling times at constant peripheral velocity, equivalent feed and depth of cut (and for one mode of milling), the assumption was considered justified. A series of tests conducted by the workers also verified the validity of the expression.

In Eqn. 2.18, the heating and cooling times \( t_h \) and \( t_c \) can be obtained if the time taken for one revolution of the cutter (cycle time) is split in the ratio of time in cut to time out of cut. The heating time will be given by the time in cut and the cooling time by the time out of cut. The cycle time of the cutter is given by the inverse of the spindle speed, and the ratio itself depends on the immersion of the cutter. With this knowledge and employing the expressions presented earlier, the thermal fatigue parameter may be computed for the different immersions employed in cutting the steel and titanium work materials. The following tables present the results of these computations.
Table 2.3. Thermal Fatigue Parameter Values for Steel

<table>
<thead>
<tr>
<th>SPINDLE SPEED [rpm]</th>
<th>CYCLE TIME [msec.]</th>
<th>IMMERSION</th>
<th>( t_h ) [msec.]</th>
<th>( t_c ) [msec.]</th>
<th>( E_R )</th>
<th>( x )</th>
<th>( X )</th>
</tr>
</thead>
<tbody>
<tr>
<td>912</td>
<td>65.79</td>
<td>Half</td>
<td>16.45</td>
<td>49.34</td>
<td>75.57</td>
<td>4</td>
<td>4564.04</td>
</tr>
<tr>
<td>912</td>
<td>65.79</td>
<td>Full</td>
<td>32.89</td>
<td>32.89</td>
<td>61.77</td>
<td>2</td>
<td>2638.26</td>
</tr>
</tbody>
</table>

Table 2.4. Thermal Fatigue Parameter Values for Titanium

<table>
<thead>
<tr>
<th>SPINDLE SPEED [rpm]</th>
<th>CYCLE TIME [msec.]</th>
<th>IMMERSION</th>
<th>( t_h ) [msec.]</th>
<th>( t_c ) [msec.]</th>
<th>( E_R )</th>
<th>( x )</th>
<th>( X )</th>
</tr>
</thead>
<tbody>
<tr>
<td>576</td>
<td>104.17</td>
<td>One Quarter</td>
<td>17.36</td>
<td>86.81</td>
<td>84.55</td>
<td>6</td>
<td>4973.04</td>
</tr>
<tr>
<td>576</td>
<td>104.17</td>
<td>Half</td>
<td>26.04</td>
<td>78.13</td>
<td>78.76</td>
<td>4</td>
<td>3780.40</td>
</tr>
<tr>
<td>576</td>
<td>104.17</td>
<td>Three Quarter</td>
<td>34.72</td>
<td>69.44</td>
<td>73.85</td>
<td>3</td>
<td>3071.55</td>
</tr>
<tr>
<td>576</td>
<td>104.17</td>
<td>Full</td>
<td>52.08</td>
<td>52.08</td>
<td>64.97</td>
<td>2</td>
<td>2205.06</td>
</tr>
</tbody>
</table>

Determination of constants in tool-life equation

The procedure for the determination of these constants is the same for both work materials. It begins by determining the exponent \( 1/\beta \) of the equivalent feed in the manner previously described. With \( \beta \) determined, the ratio \( \frac{S_{eq}}{S_i} \) can be evaluated, and therefore the equivalent feed computed. The other exponent \( n \) of the thermal fatigue parameter and the constant in the tool-life equation can then both be determined as described previously.
Fig. 2.6 shows the plot of the slotting data used in the determination of $\beta$ for steel. The predicted data points (linear fit points) and the original data points are both shown, and the agreement is seen to be good.

![Graph showing log $T_a$ vs. log $S_t$ for slotting data to determine $\beta$ for steel](image)

*Fig. 2.6. Plot of log $T_a$ Vs. log $S_t$ for Slotting Data to Determine $\beta$ for Steel*

The line fit is achieved with a linear regression, and the inverse of the slope obtained from the regression data gives $\beta = 1.198$. 
With the same slope, similar plots for the other cutting conditions given in Table 2.1 may be constructed (see Fig. 2.7). The same slope is applicable to all the test conditions on this table since $\beta$ is constant for the work/tool pair.

Fig. 2.7. $\log T_a$ Vs. $\log S_t$ Plots for All Test Conditions on Steel Using Known $\beta$
In the same manner as that employed for the steel (in this case using half immersion data taken from Table 2.2), \( \beta \) can be determined for the titanium work material (see Fig. 2.8):

\[
\log T_a \text{ Vs. } \log S_t
\]

From the regression data, \( \beta = 0.554 \) for the titanium work material.

Fig. 2.8. Plot of \( \log T_a \) Vs. \( \log S_t \) for Half Immersion Data to Determine \( \beta \) for Titanium
With the same slope as the previous plot, plots for the other cutting conditions employed on the titanium may be located (see Fig. 2.9):

![Plot of log T_a Vs. log S_t for All Test Conditions on Titanium](image)

**Fig. 2.9. Plot of log T_a Vs. log S_t for All Test Conditions on Titanium**

With the exponent $\beta$ determined, evaluation of the expression for the ratio $\frac{S_{eq}}{S_i}$ given in Eqn. 2.10 is possible for both work materials. However, to preclude the need for a numerical integration procedure, the exponent $\beta$ is assumed such that the value of $1/\beta$ is integer. Because of the magnitude of the error in $\frac{S_{eq}}{S_i}$ introduced in making this assumption (less than 2.5%), the simplification is not expected to significantly influence the result of the integration. Table 2.5 summarizes the integration results for the different immersions employed:
Table 2.5. Calculated $S_{eq}/S_t$ Values for Steel and Titanium

<table>
<thead>
<tr>
<th>Work Material</th>
<th>Actual $\beta$</th>
<th>Assumed $\beta$</th>
<th>Assumed $1/\beta$</th>
<th>$S_{eq}/S_t$ for Various Immersions Using Assumed $\beta$ values.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>1.198</td>
<td>1.0</td>
<td>1</td>
<td>Full $(\phi_S = \pi)$ 3/4 $(\phi_S = 2\pi/3)$ 1/2 $(\phi_S = \pi/2)$ 1/4 $(\phi_S = \pi/3)$</td>
</tr>
<tr>
<td></td>
<td>0.554</td>
<td>0.5</td>
<td>2</td>
<td>0.637 - 0.637 - 0.707 0.777 0.707 0.542</td>
</tr>
</tbody>
</table>

As expected, the ratio $\frac{S_{eq}}{S_t}$ is equal in full and half immersion cutting.

The determination of the exponent $n$ for both the steel and the titanium materials can now be achieved with the aid of Figs. 2.10 and 2.11 respectively:

**Fig. 2.10. Plot of $\log T_\alpha$ Vs. $\log X$ to Determine $n$ for Steel**

The exponent $n$ is given by the slope of the plot. Each of the three sets of test conditions plotted above, however, yield slightly different slopes:
Slope of plot 1 = -1.82
Slope of plot 2 = -1.85
Slope of plot 3 = -1.40

The value of $n$ for steel is therefore taken to be the average of the slopes of plots 1 and 2, i.e., $n = 1.835$.

![Graph](image)

*Fig. 2.11. Plot of log $T_a$ Vs. log $X$ to Determine $n$ for Titanium*

Evaluation of the slope of this plot gives $n = 1.59$ for the titanium work material. Data points shown for both plots represent averages of values obtained over a series of tests.

Finally, with the exponents obtained, the tool-life *constant* for both materials may be computed. Results of such computations give the values of the *constant*
for steel and titanium to be respectively 1503210 and 5652 and lead to final tool-life equations for both materials in the form:

\[
T_s = \frac{1503210}{X^{1.835} \times S_{eq}^{0.196}}
\]

\[
T_s = \frac{5652}{X^{1.59} \times S_{eq}^{0.554}}
\]

These equations are valid for feed-rates within the ranges employed for both sets of tool-life tests performed.

2.1.4 Discussion of Results.

The results obtained from the tool-life tests and the attendant analysis show that the original data are reproducible. The trends exhibited by both sets of data generally conform with results obtained during earlier research and discussed in Chapter 1. First, regarding the influence of feed per tooth (or equivalent feed) based on the values of the exponent 1/\(\beta\) obtained for both materials: the results of the analysis suggest that the influence of this variable on tool-life would be significantly stronger for the titanium (1/\(\beta\)=1.805) than for the steel material (1/\(\beta\)=0.835). These results agree with the findings of Yellowley et al. [1] and Tlusty et al. [5] during the cutting of titanium and steel respectively. The former's experimental data show significant changes in the tool-life with change in feed-rate while milling titanium. Tlusty reports insignificant changes in tool-life with change in the feed during the milling of steel. Second, regarding the influence of thermal fatigue (or immersion): the value of the exponent \(n\) obtained
for the steel \((n=1.835)\) suggests a stronger influence of this variable than that of the feed-rate on tool-life. In contrast, however, the titanium result \((n=1.59)\) suggests that the influence of thermal fatigue on tool-life for this material is less significant than that of feed-rate. These results show that the influence of thermal fatigue is somewhat stronger for the steel than for the titanium material.

Considering the tool-life data obtained, for both materials, slotting leads to higher tool-life values than half immersion cutting as a result of thermal fatigue. Regarding the other immersions employed on titanium, at the same feed-per-tooth, the 3/4 and 1/4 immersion tests both yield higher tool-life values than the 1/2 immersion test yields. On the basis of the thermal fatigue theory, it is to be expected that 3/4 immersion would result in a higher tool-life over 1/2 immersion; however, this theory is contradicted by the increased tool-life value of 1/4 immersion over 1/2 immersion. This contradiction is likely attributable to the domination of the influence of equivalent feed over that of thermal fatigue under these circumstances (there is a 24% decrease in equivalent feed from 1/2 to 1/4 immersion). The influence of cutter diameter is reflected in the steel data, which show that a smaller diameter yields a shorter tool-life than does an equivalent test employing a tool of twice the former's diameter and cutting conditions which yield a larger value of equivalent feed. As explained earlier, the logical explanation for this is thermal cycling. The influence of mode of milling is reflected in the steel data in which down-milling generally leads to higher tool-life values under otherwise identical cutting conditions. This is consistent with the conclusion reached in Chapter 1. The likely chip adhesion occurring at exit from the workpiece in up-milling, even during the milling of mild steel which cannot be
considered to be a severely work-hardening material, proves more detrimental to the life of the tool than any thermal or mechanical shock the tool may experience at entry into the workpiece in down-milling.

2.1.5 Tool-Life Tests Within an Inert Atmosphere

Previous sections have emphasized the significance of the thermal fatigue theory in explanation of tool-life phenomena. This section describes tests designed and conducted to verify the theory further.

Under some circumstances, oxidation plays a major role in the wear of cutting tools. Therefore, if the effect of oxidation could be excluded from the region surrounding the tool, it is expected that improved tool-life would be attainable. Since the level of significance apportioned to the oxidation factor would therefore depend on the extent of the improvement observed, it should be possible to determine whether oxidation is responsible in any way for the influence of cut time ratio on tool-life, or if in fact thermal fatigue is the primary factor.

In the experiments conducted, oxygen was displaced from the tool region by pumping argon into an enclosure surrounding the tool, thus creating an inert atmosphere. Argon was chosen to facilitate the creation and preservation of the inert atmosphere within the cutting zone.

The apparatus comprised the milling machine in the form used for previous tool-life tests but with the addition of a sheet metal enclosure designed to fit around the collar of the machine and surround the cutting environment (see Fig.
2.12). Other equipment included a pressurized argon bottle with an attached flow meter.

![Fig. 2.12. Schematic of Arrangement Used for Argon Influenced Tests](image)

The flow rate of the argon was set at the maximum possible (50 ft³/hour) and the gas was kept flowing for the duration of cutting. The procedure for determining tool-life data was the same as for previous tests and the data obtained are shown in Table 2.6.

**Table 2.6. Tool-Life Data for Argon Influenced Milling of Steel and Titanium**

<table>
<thead>
<tr>
<th>Work Material</th>
<th>Spindle Speed [rpm]</th>
<th>Machine Feed [mm/min.]</th>
<th>Width of Cut</th>
<th>Mode of Milling</th>
<th>Tool-Life, $T_I$ [minutes]</th>
<th>Improvement Over 'Non-Argon' Test [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>912</td>
<td>264</td>
<td>Slotting</td>
<td>-</td>
<td>19.6</td>
<td>25.4</td>
</tr>
<tr>
<td>Steel</td>
<td>912</td>
<td>264</td>
<td>1/2 imm</td>
<td>Down</td>
<td>16.37</td>
<td>12.9</td>
</tr>
<tr>
<td>Titanium</td>
<td>576</td>
<td>62.4</td>
<td>1/2 imm</td>
<td>Down</td>
<td>6.437</td>
<td>24.2</td>
</tr>
</tbody>
</table>
These results do not show a particularly significant improvement in the tool-life values. The improvements observed are much smaller than those associated with a large change in immersion. Oxidation is evidently therefore not the major factor influencing the wearing of cutting tools in milling and it must be concluded that thermal fatigue is indeed the dominant factor influencing tool-life in milling.

2.2 Investigation of Exit Failure in Milling

Exit failure is the phenomenon by which a milling tool is mechanically damaged as it exits from the workpiece. The phenomenon is more likely to occur during the milling of heavily work-hardening materials and can occur during either up-cut or down-cut milling, depending on the cutting conditions.

Conflicting theories have been advanced to explain the occurrence of this phenomenon (see Chapter 1). In this section, the problem is further investigated, and an experiment designed and conducted to verify the validity of the theories is presented.

2.2.1 Sharp Corner Test

One factor influencing damage of the tool as it exits from the workpiece is the thickness of the chip at exit. A finite chip thickness at exit is more likely to cause chip-sticking and consequent damage upon exit and subsequent re-entry than does a zero chip thickness. For this reason the phenomenon has been observed to occur more commonly in up-milling than in down-milling. The rare
occasion in which it occurs in down-milling was observed by Yellowley [7] while traversing into a corner of a stainless steel workpiece as illustrated in Fig. 2.13.

![Fig. 2.13. Schematic Illustration of Sharp Corner Wear](image)

Under these circumstances, before the tool has traveled a distance equal to its radius into the workpiece (or before a cusp, given by the dashed arc 2, is fully formed in the workpiece), the tool sees a finite chip thickness at exit from the workpiece, i.e., arc 1, and the result is catastrophic damage to the tool upon the subsequent re-entry.

An experiment on titanium alloy verified this phenomenon during investigation of the present research into exit failure. Test 1 in Table 2.2 was repeated without first machining the cusp into the workpiece as had been the practice for previous tests. Fig. 2.14 shows graphically a comparison of both tests:
2.2.2 Investigation of Behavior at Tool Exit Leading to Failure

The influence of exit conditions on tool chipping and life has been discussed. The two main explanations are in fact similar: the early work by Yellowley [7] considered that the shear plane would rotate until the shear angle was approximately zero and that on work-hardening materials the final failure would occur through crack propagation at an angle of 45 degrees to the direction of maximum shear stress (the mechanism has been described in detail by Noble.
and Oxley [11] in relation to blanking and piercing operations). In later work, Pekelharing identified negative shear angles; the evidence shows that these negative angles are created by a combination of a reducing shear angle as postulated by Yellowley, and a bending rotation of the workpiece free surface which unlike a sheared part does not have a support at the free surface.

The former explanation is supported by the result of an upper bound analysis of the problem. A simple schematic of the shear plane region of a cutting operation performed with a tool having zero degree rake is shown in Fig. 2.15a. The attendant velocity vector polygon is shown in Fig. 2.15b:

![Diagram](image_url)

**Fig. 2.15.** (a) Schematic of Shear Plane Region of Cutting Operation; (b) Associated Velocity Vector Polygon

Plastic Work \( = V_s \cdot A_s + V_w \cdot A_w \)  \( \text{2.21} \)

where \( V_s \) is the shear plane velocity,

\( V_w \) is the resultant chip velocity, and

\( A_s \) and \( A_w \) are the shear plane and chip cross-sectional areas.
respectively.

Assuming unit width into the plane of the paper,

\[ A_x = \frac{x}{\cos \theta}, \text{ and } A_w = h. \]

also,

\[ V_x = \frac{V_0}{\cos \theta}, \text{ and } V_y = V_0 \cdot \tan \theta \]

where \( V_0 \) is the workpiece approach velocity.

Substituting into the expression for plastic work yields:

\[
\text{Plastic Work} = V_0 \left( \left( \frac{x}{\cos \theta} \cdot \frac{1}{\cos \theta} \right) + (h \cdot \tan \theta) \right) \tag{2.22}
\]

Eqn. 2.22 shows that the minimum work will occur when \( \theta = 0 \). Since shearing will always take place along the direction of least work, the shear plane will obviously rotate into this position as the tool approaches exit as suggested by Yellowley.

The major differences between the two theories, however, result from the proposed influences of exit conditions on tool chipping. While Yellowley had associated this phenomenon with the removal of the formed foot in the succeeding impact, Pekelharing associated the damage with the increased forces arising in the final stages of tool exit. However, no matter which explanation is used, no damage would be expected at the extremes of exit angle (see Fig. 2.16).
Fig. 2.16. Typical Exit Angles

At the position shown in (a), the chip will likely continue to be formed until exit, while with that shown in (c) a burr will be formed due to the bending away of the free surface. The position shown in (b) however will lead inevitably to one of the two possible mechanisms described.

One more recent rigorous experimental investigation of the influence of chipping at exit on tool failure was by van Luttervelt et al. [14]. This investigation confirmed that the influence is restricted to a narrow zone of exit angles. While the data from van Luttervelt's work showed the danger of such exits, it also showed that several other factors associated with the geometry of cut and the time in cut significantly influenced the tool damage. The most notable finding was that smaller diameter cutters performed significantly better than larger diameters; no explanation was suggested since the authors could not identify any changes to similar parameters in the original work of Pekelharing. Later experiments attempted to examine the influence of reducing time in cut for the larger cutters. Differences in tool-life were observed; however, the change in time in cut was associated with a change in entry conditions, thus making definitive statements difficult.
The data from the later paper is believed to be in agreement with the original work of Yellowley in which both thermal and exit influences were considered. The major influence of cutter diameter as far as the exit failure is concerned is considered due to the decreased time out of cut (at the same peripheral speed) which leads to reduced welding of the adhered chip foot. The worst damage situation would be expected to occur if the adhered chip is made to traverse a wide chip thickness upon re-entry rather than being removed instantaneously upon impact as would likely be the case with reduced welding, i.e., the entry conditions will also be important.

Great difficulty is clearly involved in the experimental verification of any one of the theories presented, considering that the exit phenomenon at normal practical conditions lasts for only a few milliseconds. The use of high speed photography or even the capture of dynamic forces under these conditions is impractical. The author has attempted to examine qualitatively the processes occurring during the last few milliseconds of cut using a new test set-up shown in Fig. 2.17. The idea is to examine the output of the accelerometer which is attached to the aluminum wedge.
The wedge is positioned so that impact into the aluminum will occur at a distance of 0.25 mm (0.01 inch) after the cutting edge passes the undeformed outside boundary of the titanium workpiece. In this manner the exit of the tool from the workpiece can be easily timed. Since the whole apparatus is mounted on top of the dynamometer, the output of the accelerometer can not be directly associated with dynamic force signal; nevertheless, a sharp increase or decrease in force at exit should lead to useful qualitative information regarding the processes occurring at exit.
The majority of the tests were carried out at practical cutting speeds, and the results obtained are reproducible. The Y component of the cutting force and the output of the accelerometer (acceleration in the Y direction) were both captured on an oscilloscope, the sampling time being either 100 or 200 microseconds (depending on the spindle rotational frequency). Both signals were also filtered before entering the oscilloscope.

Fig. 2.18a shows the overall pattern. The accelerometer signal shows little activity during the initial impact; however, both at exit from the titanium workpiece and at entry into the aluminum wedge, activity is pronounced.
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Fig. 2.18a. Traces for Exit Failure Investigation with Cutting Conditions 288 rpm and 24 mm/min

The behavior of the signals during exit were examined over a range of cutting conditions and are shown in Figs. 2.18b to d. As in the previous figure, at exit there is a sudden drop in force which is appreciable considering the accelerometer is not attached directly to the titanium workpiece. The accelerometer then records the impact into the aluminum wedge almost instantaneously (time period ranging approximately between 0.2 and 0.8 milliseconds for the respective spindle speeds) after exit from the titanium.
Fig. 2.18b. Traces for Exit Failure Investigation with Cutting Conditions 180 rpm and 14.4 mm/min
Fig. 2.18c. Traces for Exit Failure Investigation with Cutting Conditions 144 rpm and 14.4 mm/min
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---

Fig. 2.18d. Traces for Exit Failure Investigation with Cutting Conditions 72 rpm and 14.4 mm/min

The time between the force changes is approximately equal to that required for the tool to traverse the 0.25 mm between the undeformed boundary of the workpiece and the aluminum wedge. The dynamic response of the wedge then dominates the accelerometer signal. In general, the behavior is consistent for all the test cases; however, at the very low speed of 72 rpm, the influence of built-up-edge is apparent with considerable perturbation in force levels both at
entry and exit. In all cases, as a result of the low pass filter employed, the output of the dynamometer shows no large dynamic effects.

Despite the qualitative nature of the tests, it is the author's belief that the timing and sign of the accelerometer output lend some credibility to the possibility of brittle fracture at exit as opposed to the continual shearing with rotation assumed by Pekelharing. It is noteworthy that in this experiment as well as in the tool-life tests, the adhesion of chip roots and associated ribbon chips comprising several individual elements welded on impact were observed. This was reported by Yellowley in those cases in which exit conditions were problematic and more recently by Ghani and Barrow [16]. Pekelharing, in his experiments, reported seeing no such adherence. It is possible that, depending upon the work-hardening capacity of the work material, the behavior at exit varies and both mechanisms may play a role. In the case of the work material examined here, however, the major problem appears to be adhesion and subsequent damage upon re-entry.
Chapter 3

Force Modeling and Tool-Wear Tracking in Milling

This chapter discusses: a) the development of a force model for the milling of titanium work material and experiments performed to verify the validity of the model and b) tool-wear tracking (or monitoring), employing force data obtained from sharp and worn tool tests, with the aim of developing techniques for the real time identification of wear in complex milling operations.

3.1 The Modeling of Instantaneous Forces in Milling

In the general case of peripheral milling, the forces acting on a single tooth milling cutter in up- and down-milling are shown in Fig. 3.1. Normal practice resolves both the radial ($F_R$) and tangential ($F_T$) components of force into the feed direction ($X$) and perpendicular to the feed direction ($Y$).

Fig. 3.1. Forces Acting on a Single Tooth in Up- and Down-Milling
Chapter 3  Force Modeling and Tool-Wear Tracking in Milling

The normal approach to force estimation assumes that $F_T$ may be obtained from the product of the instantaneous area of cut ($A$) and the specific cutting pressure ($K_s$); it is also usually assumed that the ratio of radial to tangential force component ($r_i$) is known and constant. While in the simplest case $K_s$ and $r_i$ are assumed constant, later work has assumed these parameters to be known functions of the mean chip thickness [18].

In an early consideration of the problem, Yellowley [17] suggested that, since the chip thickness in peripheral milling was generally low, as a first improvement on a linear model of cutting forces, the edge forces (nose and flank), which are known to constitute a significant proportion of total forces at such conditions, should be considered. A model which considers $F_T$ to be made up of two components was developed: one directly proportional to the undeformed area of cut and a second directly proportional to the length of cutting edge engaged.

A general form for the tangential force component may be expressed as:

$$F_T = K_1 \cdot a \cdot S_t \cdot \sin \phi + K_2 \cdot L_a$$

where $K_1$ is the cutting component of the specific cutting pressure,

- $K_2$ is the edge force per unit length,
- $a$ is the depth of cut,
- $S_t$ is the feed-per-tooth,
- $\phi$ is the instantaneous angle of rotation, and
- $L_a$ is the length of cutting edge engaged.
The corresponding expression for the radial component is given by:

\[ F_r = r_i \cdot F_T \]  \hspace{1cm} (3.2)

where \( r_i \) is the ratio of radial to tangential force component.

The instantaneous area of cut is expressed as:

\[ A = a \cdot S_i \cdot \sin \phi \]  \hspace{1cm} (3.3)

and the equivalent chip thickness, \( h_e \) is given by:

\[ h_e = \frac{A}{L_s} \]  \hspace{1cm} (3.4)

therefore Eqn. 3.1 can be re-expressed in the form:

\[ \frac{F_T}{A} = K_1 + \frac{K_2}{h_e} \]  \hspace{1cm} (3.5)

Since the ratio of radial to tangential force components on the rake face and flank/nose of the tool is expected to be different, the ratio must be broken into two components: \( r_1 \) for the cutting forces and \( r_2 \) for the edge forces. Typical values for the ratios range between \( 0.2 < r_1 < 0.5 \) and \( 1 < r_2 < 4 \). The radial force can therefore be expressed in terms of these and the R.H.S of Eqn. 3.5 as:

\[ \frac{F_r}{A} = r_1 \cdot K_1 + r_2 \cdot \frac{K_2}{h_e} \]  \hspace{1cm} (3.6)

Eqns. 3.5 and 3.6 are final forms of the model equations to be used in the current analysis.
From a geometrical evaluation of Fig. 3.1, expressions can be written for the instantaneous forces as follows.

For up-milling:

\[
F_x = F_T \cdot \cos \phi + F_R \cdot \sin \phi \tag{3.7}
\]

\[
F_y = F_T \cdot \sin \phi - F_R \cdot \cos \phi \tag{3.8}
\]

and for down-milling:

\[
F_x = F_R \cdot \sin \phi - F_T \cdot \cos \phi \tag{3.9}
\]

\[
F_y = F_R \cdot \cos \phi + F_T \cdot \sin \phi \tag{3.10}
\]

In terms of the tangential and radial force components, these equations may be re-expressed as:

For up-milling:

\[
F_T = F_x \cdot \cos \phi + F_y \cdot \sin \phi \tag{3.11}
\]

\[
F_R = F_x \cdot \sin \phi - F_y \cdot \cos \phi \tag{3.12}
\]

and for down-milling:

\[
F_T = F_y \cdot \sin \phi - F_x \cdot \cos \phi \tag{3.13}
\]

\[
F_R = F_y \cdot \cos \phi + F_x \cdot \sin \phi \tag{3.14}
\]

A validation of the model has been attempted in this work in two inter-related ways. A series of cutting force tests conducted on a titanium alloy under up-, down- and centerline-milling (slotting with workpiece width less than cutter diameter) conditions employed the same set-up used for the exit failure
Investigation tests. Using the force data \((F_X, F_Y)\) from the up- and down-milling tests, the tangential and radial force components were obtained from the equations presented.

For a tool with a nose radius such as that used in these tests, \(h_e\) is given by the following expression:

\[
\frac{1}{h_e} = \frac{1}{S \cdot \cos \psi} + \frac{1}{2 \cdot a} + \frac{0.57 \cdot (r \cdot \cos \psi)}{a \cdot S} \tag{3.15}
\]

where \(S\) is the instantaneous feed (in milling),
\(\psi\) is the approach angle (zero for tooling used in these tests),
\(r\) is the nose radius of the insert, and
\(a\) is the depth of cut.

(for the geometry of the milling insert used, \(r = 1.194\) mm.)

Eqns. 3.5 and 3.6 indicate that linear relationships exists between \(\frac{F_r}{A}\) and
\(\frac{1}{h_e}\), and also \(\frac{F_k}{A}\) and \(\frac{1}{h_e}\). With these equations, sets of data values may be computed for these parameters at regular intervals (of \(\phi\)) through the workpiece, for both modes of milling. Plots can then be made of \(\frac{F_r}{A}\) versus \(\frac{1}{h_e}\), and \(\frac{F_k}{A}\) versus \(\frac{1}{h_e}\), from which \(K_1, K_2\) and then \(r_1, r_2\) may be determined for each mode of milling.

The simulation of a centerline-milling operation can be achieved by combining the dynamics of an up-milling operation with that of a down-milling
operation (in that order). By substituting the four constants $K_1, K_2, r_1$ and $r_2$ for the respective milling modes in Eqns. 3.5 and 3.6, the tangential and radial force components for both the up-milling side and the down-milling side of an equivalent centerline-milling operation may be simulated. Comparison of the simulated data with the actual experimental data gives a measure of the accuracy of the simulation and therefore of the validity of the model employed.

Figs. 3.2, 3.3 and 3.4 respectively show plots of the experimental data obtained from up-, down- and centerline-milling tests conducted at 288 rpm and 14.4 mm/min. In summary the first two plots are used to determine the constants which are used to simulate the third plot. The simulation compared to the actual plot verifies the validity of the model used.
Fig. 3.2. Experimental Force Data from Up-Milling Test on Titanium
Fig. 3.3. Experimental Force Data from Down-Milling Test on Titanium
Fig. 3.4. Experimental Force Data from Centerline-Milling Test on Titanium

The data derived from the experiments illustrated by Figs. 3.2 and 3.3 are shown below in Tables 3.1 and 3.2 respectively:
Determination of the model constants for each milling mode was achieved using a regression analysis performed between the relevant variables of the respective tables as explained earlier. The results of these analyses are shown in Table 3.3.

**Table 3.3. Determined Force Model Constants for Titanium Alloy**

<table>
<thead>
<tr>
<th>Model Constant</th>
<th>Up-Milling</th>
<th>Down-Milling</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_1 ) [N/mm²]</td>
<td>2511</td>
<td>2302</td>
</tr>
<tr>
<td>( K_2 ) [N/mm]</td>
<td>29.18</td>
<td>10.87</td>
</tr>
<tr>
<td>( r_1 )</td>
<td>0.52</td>
<td>0.53</td>
</tr>
<tr>
<td>( r_2 )</td>
<td>1.42</td>
<td>2.9</td>
</tr>
</tbody>
</table>
The substitution of these constants gives the simulated force data, so that a comparison between the actual data and the simulated data is possible. The graphical illustrations that follow (Figs. 3.5 and 3.6) show these comparisons.

*Fig. 3.5. Comparison of Actual and Simulated Force Data (Tangential and Radial Components)*
Fig. 3.6. Comparison of Actual and Simulated Force Data (X and Y Components)

These last two figures show that, in general, the simulated data are in good agreement with the experimental data (within normal limits of error). In general, the down-milling data compare more favorably than do the up-milling data. This occurrence can probably be attributed to the previously discussed phenomena associated with up-milling, and in particular with work-hardening work materials (chip-sticking, built-up-edge, exit failure etc.), that likely introduce error into the data.
On the basis of this one data set, the model appears valid and provides a good representation of cutting forces.

Further validation of the force model can be achieved if the model is employed to simulate, on the basis of Fourier Series (F.S.) modeling, cutting forces obtained during machining of the titanium alloy work material. The cutting forces are filtered before display on the oscilloscope; this is the reason for the F.S. approach to the problem, i.e., to account for the effect of filtering on the forces. The theoretical basis for F.S. modeling was first presented by Yellowley [19]. Later work by Hosepyan [20] detailed the derivation of F.S. equations for a particular work/tool set-up. The derivations of modified general equations used for the current simulation (based upon Yellowley's equations) are given in Appendix 1. Despite this reference to Appendix 1, the following comments should be noted with respect to the derivations: a) the constants $K_1$, $K_2$, $r_1$ and $r_2$ are assumed constant when actually they are not; previous workers have shown that this is a reasonable assumption; b) only up-milling has been considered although a similar analysis could be carried out for down-milling.

The final forms of the derived expressions follow. From Eqns. A1.20 and A1.21 in Appendix 1:

$$F_{x1} = K_1 \cdot a \cdot S_r \cdot \left[ a_{x0} + \sum_{k=1}^{\infty} (a_{xk} \cdot \cos k\phi + b_{xk} \cdot \sin k\phi) \right]$$  \hspace{1cm} (3.16)

$$F_{y1} = -K_1 \cdot a \cdot S_r \cdot \left[ a_{y0} + \sum_{k=1}^{\infty} (a_{yk} \cdot \cos k\phi + b_{yk} \cdot \sin k\phi) \right]$$  \hspace{1cm} (3.17)

From Eqns. A1.54 and A1.55:
\[ F_{x2} = K_1 \cdot L_a \cdot h^* \left[ a_{w0} + \sum_{k=1} b_{wk} \cdot \cos k\phi + b_{wk} \cdot \sin k\phi \right] \]  

\[ F_{y2} = -K_1 \cdot L_a \cdot h^* \left[ a_{wy0} + \sum_{k=1} b_{wyk} \cdot \cos k\phi + b_{wyk} \cdot \sin k\phi \right] \]  

from which the total X and Y axes forces can be expressed as:

\[ F_x = F_{x1} + F_{x2} \]  

\[ F_y = F_{y1} + F_{y2} \]

A simple program (see Appendix 2) was written on the basis of this theory, to simulate the cutting forces obtained during up-milling experiments on titanium alloy. The simulation and the actual experimental results were then compared to confirm again the validity of the underlying model employed. The constants \( K_1, r_1 \) and \( r_2 \), supplied to the program, are obtained from the up-milling data in Table 3.3. The comments in the program listing explain the determination of the remaining constants supplied to the program.

The comparisons between the simulated cutting forces and the actual cutting forces are shown in Figs. 3.7 and 3.8:
Fig. 3.7. Simulated (Top) and Actual (Bottom) Cutting Force Comparison For Fourier Series Based Model Investigation; 288 rpm, 14.4 mm/min
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Fig. 3.8. Simulated (Top) and Actual (Bottom) Cutting Force Comparison For Fourier Series Based Model Investigation; 288 rpm, 30 mm/min
The simulated force data in both cases provide a good representation of the actual cutting forces within normal limits of experimental error. This good representation emphasizes the validity of the F.S. approach and, in particular, the underlying model used.

On the basis of the results obtained from these analyses it is concluded the model is valid for force representation in metal cutting in general and milling in particular.

3.2 Tracking of Machining Condition and Tool-Wear in Milling

This section will commence by discussing previous related research conducted by other workers. It will then discuss work (past and current) carried out within the Manufacturing Engineering Laboratory at the University of British Columbia [21] toward the development of tracking techniques for machining condition and tool-wear. The experimental and analytical contributions of the author to the research are also detailed.

Several authors have attempted to derive relationships between cutting forces and both wear and breakage [22], [23]; the majority of effort has, not surprisingly, been directed toward the monitoring of turning operations. Previous approaches which have demonstrated considerable promise have been based upon either the tracking of a single force component [26], or the tracking of a ratio of force components (ratio of thrust to tangential component) [27], [28]. However, before any of such approaches may be adopted, any bias which may have been introduced due to changes in cutting conditions must be removed from the forces.
In contrast to turning, only recently have serious attempts been directed toward the use of forces or other physical manifestations of the cutting process to track tool-wear in milling. Among interesting approaches related to real time monitoring were those of Szabo [29] who examined the influence of wear on the damping ratio of a major machine-tool/workpiece mode and Dornfeld [30] who examined the influence of wear on acoustic emissions from the cutting process. Recent publications concerned with the influence of wear on the mean and dynamic force components resulting from the cutting process include two papers from one research group which used force parameters based upon the low frequency force components. The first of these papers [31] examined the influence of wear on the total harmonic power in the low frequency range of the milling spectrum, while the second paper [25] attempted to identify critical ratios of components which were most influenced by wear and suggested the use of an identification technique to determine width of cut. The authors also suggested the construction of an intelligent system to assess likely force component ratios and to identify the current wear land through simulation of the force equations in the frequency domain.

The main aim of the research at the Manufacturing Engineering Laboratory in this area has been the development of methods or techniques that permit in-process (or real time) identification of wear in milling operations employing easily obtainable force information from the cutting process. The methods considered are based upon earlier research conducted by Yellowley [17], [19] and more recent work in the area of tool-wear monitoring by Hosepyan [20]. Although force is an indirect measure of wear, the monitoring of wear through force will
continue to be one of the most practical and direct of approaches until direct wear measurement in real time is available and proven.

The model used in the current work is based upon a Fourier Series formulation (see Sec. 3.1). The applicable equations are therefore as presented previously. Since in this application, the interest is in a limited frequency range, the use of the F.S. approach appears reasonable. As low a bandwidth as possible is required for the transducers since the use of existing measurement techniques and monitoring devices at conventional milling frequencies can then be permitted.

The quasi-mean resultant force, $F_{qm}$, has proven useful in previous work concerned with tool breakage detection in milling operations. The quasi mean resultant force (qrf) is the square root of the sum of the squares of the mean values of forces in two perpendicular directions in the plane perpendicular to the axis of the cutter, i.e.,

$$F_{qm} = \sqrt{\left(\bar{F}_x(v)\right)^2 + \left(\bar{F}_y(v)\right)^2}$$ 3.22

The qrf will be equal to the mean of the actual resultant force only if the forces are constant. Its most notable property is its independence of the cutting direction. Yellowley [19] has shown that, in those cases in which the metal cutting component of the total force largely exceeds the parasitic component, the qrf is directly related to the magnitude of the fundamental coefficients of the tangential force component for a single tooth cutter (determined from torque measurements). i.e.,
\[ F_{cm} = \frac{1}{2} \cdot (1 + r^2)^{1/2} \cdot (a_1^2 + b_1^2)^{1/2} \cdot \cos(\psi_e) \]  

where \( a_1 \) and \( b_1 \) are the fundamental coefficients for the tangential force, 
\( r \) is the mean value for the force ratio, and 
\( \psi_e \) is the effective approach angle.

All tests were conducted on a Bridgeport universal milling machine, the same one used for previous tests. Cutting forces were measured with the piezoelectric based three component dynamometer also used for previous tests (the bandwidth of the dynamometer with workpiece attached is greater than 500 Hz). In the first set of tests conducted by Hosepyan [20], a four tooth, inserted carbide, square shoulder face mill (0° axial rake, 7° radial rake) was used to machine a hardened low alloy steel (AISI 4140, 380 BHN) with only one insert in the cutter (to facilitate signal observation). The force data were low pass filtered at a frequency above the fundamental frequency of a cutter with four teeth. The digitized data for the single tooth were then spaced in time and summed to simulate a hypothetical four tooth cutter (all teeth identical) the simulation of which could be analyzed. The second set of tests conducted by the author used three tooth standard length HSS end mills (1/2 inch diameter) again to machine AISI 4140 work material but with a lower hardness than before (300 BHN). As in the previous set of tests, the force data were low pass filtered above the fundamental tooth frequency.
### 3.2.1 Tracking Techniques for the Recognition of Machining Condition

The problem of tracking width of cut in milling has been treated by Yellowley [17], [19], [32]. These research efforts showed that the most efficient manner of tracking machining conditions in milling is to use a mean value of one parameter and the corresponding magnitude of the fundamental component to establish width of cut. The mean value alone may then be used to establish axial depth of cut, providing prior cutting data are available. Alternatively in the case of tools with a finite nose radius, depth of cut may be identified with the use of the ratio of the two orthogonal components of the thrust force. The latter technique is dangerous if large amounts of concentrated wear are present.

The following are the most promising parameters for use in machine condition identification procedures; they are all essentially independent of tool condition and are direction insensitive:

1. The mean and fundamental magnitude of torque. The theoretical values of the ratio of these two have been shown to be a reasonable indicator of swept angle of cut although with a single tooth the ratio is relatively insensitive [19].

2. The mean and fundamental magnitude of the axial force. The ratio of these two parameters will produce essentially the same result as the method using torque values in cases in which the nose radius is large. It is, however, not applicable to carbide inserted cutters with large facets.

3. The deviatoric component of force and the quasi-mean resultant force. The ratio of these two parameters was shown by Altintas and Yellowley [32] to
have potential. Unfortunately, significant computation is required. In cases in which only the mean and fundamental components are present in the signal, the ratio becomes more amenable to analysis.

Since this final parameter is independent of cutting direction, depth of cut, cutting constants ($r$ and $K$), and is only slightly influenced by tool condition, it has been selected for further study in this work. The physical significance of this immersion tracking parameter may be appreciated by consideration of the force locus created in milling; a typical one is shown in Fig. 3.9.

![Milling Force Locus](image)

**Fig. 3.9. Milling Force Locus**

This illustration shows the quasi-mean resultant force vector, $F_{qm}$, and the deviatoric vector, $F_{dev}$, which rotates through one revolution for each tooth period. The mean value of the deviatoric component would be expected to decrease as immersion increases; at the same time, the qrf would be expected
to increase. The ratio of the two would be expected to provide an effective immersion tracking parameter.

The X and Y components of force for a cutter with $N$ teeth in terms of the mean and fundamental F.S. coefficients can be expressed as:

$$F_x = N \cdot a_{x0} + N \cdot \{a_{xN} \cdot \cos(N\omega t) + b_{xN} \cdot \sin(N\omega t)\}$$  \hspace{1cm} 3.24

$$F_y = N \cdot a_{y0} + N \cdot \{a_{yN} \cdot \cos(N\omega t) + b_{yN} \cdot \sin(N\omega t)\}$$  \hspace{1cm} 3.25

where $\omega$ is the spindle rotational frequency,

$t$ is the time, and

$a_{xN}, b_{xN}, a_{yN}$ and $b_{yN}$ are coefficients of the F.S. representation of the forces in the X and Y direction.

Since a parameter related to the area swept out by the deviatoric force component, $F_{dev}$ is desired, expressions for the X and Y components of $F_{dev}$ may also be given as:

$$F_{devx}^2 = \left[ \left( \frac{a_{x1}^2 + b_{x1}^2}{2} \right) + \left( \frac{a_{x1}^2 - b_{x1}^2}{2} \right) + a_{x1} \cdot b_{x1} \cdot \sin(2N\omega t) \right]$$  \hspace{1cm} 3.26

and

$$F_{devy}^2 = \left[ \left( \frac{a_{y1}^2 + b_{y1}^2}{2} \right) + \left( \frac{a_{y1}^2 - b_{y1}^2}{2} \right) + a_{y1} \cdot b_{y1} \cdot \sin(2N\omega t) \right]$$  \hspace{1cm} 3.27

The magnitude of the mean squared deviatoric force can be given by:

$$F_{dev}^2 = \left( \frac{a_{xN}^2 + b_{xN}^2 + a_{yN}^2 + b_{yN}^2}{2} \right)$$  \hspace{1cm} 3.28
At this point, the immersion tracking parameter, \( Q \), can be defined. \( Q \) is proportional to the ratio of the area swept out by the mean deviatoric force and the square of the qrf. It is both non-dimensional and direction insensitive. It is independent of the average force ratio, \( r \), and therefore should be insensitive to flank wear, i.e.,

\[
Q = \frac{(a_{xN}^2 + b_{xN}^2 + a_{yN}^2 + b_{yN}^2)}{(a_{x0}^2 + a_{y0}^2)} = 2 \cdot \left( \frac{F_{dev}}{F_{qm}} \right)^2
\]

3.29

The real time computation involved in tracking the parameter \( Q \) would be relatively minimal. Since only the fundamental and mean values are required, simple signal processing techniques and equipment are all that are necessary to extract data.

The effectiveness of the immersion tracking parameter, \( Q \), is illustrated in Fig. 3.10 in which experimental data for both three and four tooth cutters are given together with expected values:
Fig. 3.10. Relationship Between Immersion Parameter $Q$ and Immersion for Experimental and Predicted Values

It is noteworthy that both sharp and worn tool data ($0<V_B<0.35$ mm) are shown and that a variety of feed-rates were used for the three tooth cutter. The helix angle of the three tooth end mill would only significantly influence $Q$ in cases in which the axial depth of cut is high. In these tests, the axial depth of cut was 2 mm, therefore the influence was negligible and could be neglected.

3.2.2 Tracking Techniques for the Recognition of Tool Condition

Minimizing the influence of changing cutting conditions on the parameter chosen to track tool condition is desirable. As stated earlier, the use of the ratio of thrust force to main cutting force has proven reliable as an indicator of tool
condition in turning; for this reason extended use of the same parameter to the monitoring of milling operations is attempted. Unfortunately, two complications make this approach difficult in milling:

1. The thrust force required, when an inserted tooth carbide cutter is used, is the vector sum of the axial and radial force components. While the axial component is easily measured, the instantaneous value of the radial component is not. It follows that a situation arises in which both torque and axial force are additive but cancellation of radial force components makes the straightforward application of the force ratio impossible.

2. In milling with carbide or ceramic cutting tools, there is a significantly greater likelihood of sustaining edge chipping. Under such circumstances, the simple use of the axial force and torque to infer a global state of wear around the tool is likely to cause problems, especially in those cases in which the approach angle of the tool is small or when chipping occurs on the primary cutting edge as opposed to the nose radius.

Generally, in the case of inserted carbide milling cutters or indeed any cutter having a radius or chamfered geometry, use of the axial component of force and the cutting torque to evaluate the condition around the radius is only possible if the following conditions are met:

1. The axial depth of cut should be significantly greater than the axial projection of the radius or chamfer, otherwise, a tracking parameter which is able to identify actual depth of cut should be used.
2. No damage which is significant enough to mask the influence of wear on the tool radius (through a compensating increase in torque) should occur to the primary (straight or fluted) portion of the cutting edge.

For an assessment of tool condition along the primary edge, an indirect approach based upon Eqn. 3.23 is employed, i.e.,

\[
\frac{F_{q_m}^2}{F_T(\text{mean})^2} = \frac{1}{4} \cdot \cos^2(\psi_s) \cdot (1 + r^2) \cdot \frac{a_1^2 + b_1^2}{a_0^2}
\]

where \( F_T(\text{mean}) \) is the mean value of the tangential component of force, and all other variables are as defined earlier.

A wear tracking parameter, \( P \), may be defined as:

\[
P = (1 + r^2) = \frac{4}{\cos^2(\psi_s)} \cdot \left( \frac{F_{q_m}}{F_T(\text{mean})} \right)^2 \cdot \frac{a_0^2}{(a_1^2 + b_1^2)}
\]

While the qrf and torque values are easily measured, the mean and fundamental components of tangential force for a single tooth cutter will not be available within the torque signal for a multiple tooth cutter. The use of this tracking parameter is therefore dependent on the capacity to monitor width of cut (allowing the ratio of the torque, or tangential, F.S. coefficients to be calculated), and axial depth (allowing the effective approach angle to be estimated). The problem which inevitably arises is that the F.S. coefficients are also influenced by wear. While the influence is small, changes in the parameter being tracked are also small; the result is problems particularly at high values of immersion. Fig. 3.11 shows this difficulty with simulated values of \((1+r^2)\), calculated in two
ways: first, with a knowledge of the influence of wear on the coefficients of the torque series, and second, with the assumption of no influence and that sharp tool values may be applied:

Fig. 3.11. Theoretical Values of the Wear Tracking Parameter as a Function of Immersion. (Values Denoted as \((F_{qm}/F_T)^2\) are Calculated Without Knowledge of Changes in F.S. Coefficients Due to Wear)

Fig. 3.11 shows that \(P\) is only likely to be effective for those immersions which are less than the cutter radius. An alternate strategy must be devised to cope with higher values of immersion.

The initial series of tests performed by Hosepyan [20], employing a four tooth carbide cutter upon a tempered low alloy steel material, provided a good
indication of the problems involved in wear tracking. Two sets of data were acquired: one in half immersion and the other in slotting. In both cases the depth of cut was 1.5 mm and the nose radius of the tool was 0.75 mm. The potential difficulties were apparent in both sets of data. In the case of slotting, the tool nose sustained considerably greater wear than did the primary straight edge (approximately two times greater, based on maximum values). In the half immersion tests, localized chipping of the edge, occurring along the straight portion, caused considerable force changes and a distortion of the tool tracking parameter for the nose portion. The use of the axial force based tracking parameter in the full immersion test is shown on one Y axis in Fig. 3.12. The pronounced wear around the nose radius enhances the ability of the parameter to indicate clearly the progression of wear. In contrast however, the tracking parameter based upon the qrf (shown on the other Y axis) only starts to show an influence at relatively high values of wear land.

![Fig. 3.12. The Performance of the Wear Tracking Parameters in Slotting](image-url)
The same wear tracking parameters are again shown in Fig. 3.13 for half immersion:

![Graph showing wear tracking parameters in half immersion](image)

**Fig. 3.13. The Performance of the Wear Tracking Parameters in Half Immersion**

In this case, the insert developed relatively minor edge chipping on the primary (straight) cutting edge. This damage is reflected clearly in the progression of the parameter based upon the qrf; however, the change in torque induced by the chipping masked the increasing axial force (due to flank wear on the nose) in the other parameter based on this force. In order to assess the effectiveness of the parameters at higher wear values, a second tool, pre-worn in full immersion, was tested in half immersion. Fig. 3.13 shows the pronounced wear evident in both tracking parameters.
To better understand the paradox at play, Fig. 3.14 shows a consideration of the characteristics of the axial force. This illustration suggests that the actual behavior of the axial force is considerably more complex than that of the simple model used to derive the tracking parameters:

![Graph showing Influence of Wear on the Axial Component of Force in Slotting with a Single Tooth Carbide Milling Cutter](image)

**Fig. 3.14. Influence of Wear on the Axial Component of Force in Slotting with a Single Tooth Carbide Milling Cutter**

In the case of the sharp tool, a considerable discrepancy exists between forces in the up and down portions of the cut (similar behavior is exhibited by both the radial and tangential components of force). In the case of the worn tool, the rubbing/cutting transition is clearly visible; it is likely that the force ratio on the flank changes markedly between the two regimes and that the higher frequency
terms influenced by this transition may distort methods which rely on higher frequency components.

The previous discussion indicates that the search for a simple force based parameter to allow the tracking of tool condition at high values of immersion has proved difficult. Initial attention was focused upon the shape of the force locus shown in Fig. 3.9 and parameters such as the amplitude of the deviatoric force component about its mean value were also examined. Even the phase angle between X and Y components of force was considered. While the use of the former parameter would be ideal (owing to its direction insensitivity), simulation showed that it is insensitive to wear with three and four tooth cutters in full immersion. Alternatively, very recently in the author's own research, the search examined the ratio of X and Y forces and experiments were carried out to obtain data upon which investigation could be conducted. For analysis of this consideration, the predicted mean, amplitude and maximum values of the X and Y forces are shown in Figs. 3.15 and 3.16:
Fig. 3.15. Predicted Values of the Magnitude of the Mean and Fundamental Coefficients of the Force in the Feeding Direction
Fig. 3.16. Predicted Values of the Magnitude of the Mean and Fundamental Coefficients of the Force Perpendicular to the Feeding Direction

These figures show that the use of a ratio of peak forces should lead to a parameter which varies little over the region of interest (from half to full immersion). This ratio is shown in Fig. 3.17 and actual experimental data obtained from the experiments (employing a three tooth cutter) are shown in Fig. 3.18.
Fig. 3.17. Predicted Influence of Wear on the Force Ratio $F_X(\text{max})/F_Y(\text{max})$
As illustrated, the sensitivity of the parameter to wear is not desirably marked; however, the parameter indicates the presence of wear and can be measured relatively simply.

The result of the investigation is that two parameters are required for the tracking of wear on the primary edge and a single parameter is required for tracking the wear on the radius or facet. In those instances in which the milling cutter has a large approach angle, the single parameter which is more readily measured for the nose will give a balanced representation of tool condition. Since in practice, however, a very large number of face mills and almost all carbide and high speed steel end mills use a zero approach angle (to permit
square shoulder cutting), any approach to tool condition monitoring should use information based upon the ratio of both the axial and radial components with the tangential force.
The tool-life experiments discussed in previous chapters yielded workable data for analyses and permitted the determination of tool-life equations for the respective steel and titanium work materials considered. Compared to those obtained from similar previous work, the data were reproducible and consistent and permitted confirmation of theories involving the influences of cutting parameters on tool-wear and life.

The problem of ascertaining through which phenomenon the cut time ratio most influenced tool-wear so that a level of significance could be allotted to the phenomenon was accomplished through experimentation with machining in an inert atmosphere. The results removed any doubts that thermal fatigue was the most significant phenomenon at play and therefore confirmed a lesser level of significance for oxidation.

The process occurring at tool exit from the workpiece which led, under certain conditions, to the problem of exit failure (of the tool) was investigated qualitatively with an experimental set-up designed to detect the point of exit from the workpiece while monitoring the cutting force. The enlightening results permitted definite conclusions to be reached about previous theories that had been put forward in explanation of the phenomenon. The potential for further research in this subject area is most promising. Future work should attempt to
isolate the accelerometer from the redundant dynamics of the cutting process to
which it is subjected as a result of being in indirect contact with the
dynamometer. This isolation will permit cleaner impact detection signals to be
obtained (see Sec. 2.2.2). Improved testpiece and experimental designs to
achieve greater sensitivity in determining force behavior in the exit region can
also be pursued.

A model which had been proposed earlier for force representation in milling
was investigated for validity. Model constants determined from experimental
(up- and down-milling) force data were combined and used to simulate the
forces in a center-line milling operation. The comparison obtained between the
actual and simulated forces was good and therefore confirmed the validity of the
model. A Fourier Series program based on the model was also written and used
to simulate forces in up-milling cutting experiments (see Appendix 2). The
validity of the model was confirmed again by the close comparison obtained
between the actual and simulated data.

Previous detailed work on the tracking of wear and machining condition in
milling was reviewed. The problems associated with the parameters suggested
for wear tracking were identified. Alternate wear tracking parameters which
employ X and Y force components were considered and the results showed that
given the simplicity of measuring these forces and despite the lack of
pronounced sensitivity, a tracking parameter based upon these components had
potential. The development of direct wear sensors on inserts, if pursued, would
provide the best and most realistic wear tracking technique.
Consider first, the cutting component of Eqn. 3.1, and let it be given by $F_{T_1}$, a component of $F_T$:

$$F_{T_1} = K_i \cdot a \cdot S_i \cdot \sin \phi$$  \hspace{1cm} A1.1

The periodic function ($\sin \phi$) in the expression can be expressed in the form of an F.S. as follows:

$$\sin \phi = \left[ a_0 + \sum_{k=1}^\infty \left[ a_k \cdot \cos k\phi + b_k \cdot \sin k\phi \right] \right]$$  \hspace{1cm} A1.2

Substituting this in Eqn. A1.1 gives:

$$F_{T_1} = K_i \cdot a \cdot S_i \cdot \left[ a_0 + \sum_{k=1}^\infty \left[ a_k \cdot \cos k\phi + b_k \cdot \sin k\phi \right] \right]$$  \hspace{1cm} A1.3

Determination of the Fourier coefficients is achieved by integrating the following standard equations for F.S.

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \sin \phi \cdot d\phi$$  \hspace{1cm} A1.4

$$a_1 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \sin 2\phi \cdot d\phi$$  \hspace{1cm} A1.5
Appendix 1 Derivation of Fourier Series Expressions for Force Modeling

\[ b_1 = \frac{1}{\pi} \int_{\phi_1}^{\phi_2} \sin^2 \phi \cdot d\phi \quad \text{A1.6} \]

\[ a_k = \frac{1}{\pi} \int_{\phi_1}^{\phi_2} \sin \phi \cdot \cos k\phi \cdot d\phi \quad \text{A1.7} \]

\[ b_k = \frac{1}{\pi} \int_{\phi_1}^{\phi_2} \sin \phi \cdot \sin k\phi \cdot d\phi \quad \text{A1.8} \]

Integration of these expressions results in the following general forms for the F.S. coefficients:

\[ a_0 = \frac{1}{2\pi} \left[ \cos \phi_1 - \cos \phi_2 \right] \quad \text{A1.9} \]

\[ a_1 = \frac{1}{4\pi} \left[ \cos 2\phi_1 - \cos 2\phi_2 \right] \quad \text{A1.10} \]

\[ b_1 = \frac{1}{2\pi} \left[ (\phi_2 - \phi_1) - \frac{1}{2} (\sin 2\phi_2 - \sin 2\phi_1) \right] \quad \text{A1.11} \]

\[ a_k = \frac{1}{2\pi} \left[ \frac{\cos(1+k)}{1+k} \phi_1 - \frac{\cos(1+k)}{1+k} \phi_2 - \frac{\cos(k-1)}{k-1} \phi_1 + \frac{\cos(k-1)}{k-1} \phi_2 \right] \quad \text{A1.12} \]

\[ b_k = \frac{1}{2\pi} \left[ \frac{\sin(1+k)}{1+k} \phi_1 - \frac{\sin(1+k)}{1+k} \phi_2 + \frac{\sin(k-1)}{k-1} \phi_2 - \frac{\sin(k-1)}{k-1} \phi_1 \right] \quad \text{A1.13} \]

Substitution of Eqn. 3.2 into Eqns. 3.7 and 3.8 results in the following general expressions for \( F_x \) and \( F_y \) as functions of \( F_T, \phi \) and \( r \):
\[ F_x = F_T (\cos \phi + r \cdot \sin \phi) \]  
A1.14

\[ F_y = F_T (\sin \phi - r \cdot \cos \phi) \]  
A1.15

Expressing these in terms of \( F_{T1} \) and \( r_1 \) gives:

\[ F_{x1} = F_{T1} (\cos \phi + r_1 \cdot \sin \phi) \]  
A1.16

\[ F_{y1} = F_{T1} (\sin \phi - r_1 \cdot \cos \phi) \]  
A1.17

Further substitution of \( F_{T1} \) in terms of its F.S. coefficients into the last two equations gives:

\[ F_{x1} = C \cdot (\cos \phi + r_1 \cdot \sin \phi) \left[ a_0 + \sum_{k=1}^{\infty} [a_k \cdot \cos k\phi + b_k \cdot \sin k\phi] \right] \]  
A1.18

\[ F_{y1} = C \cdot (\sin \phi - r_1 \cdot \cos \phi) \left[ a_0 + \sum_{k=1}^{\infty} [a_k \cdot \cos k\phi + b_k \cdot \sin k\phi] \right] \]  
A1.19

where \( C = K_1 \cdot a \cdot S \).

Expansion of the two expressions above gives:

\[ F_{x1} = C \cdot \left[ a_{x0} + \sum_{k=1}^{\infty} [a_{xk} \cdot \cos k\phi + b_{xk} \cdot \sin k\phi] \right] \]  
A1.20

\[ F_{y1} = -C \cdot \left[ a_{y0} + \sum_{k=1}^{\infty} [a_{yk} \cdot \cos k\phi + b_{yk} \cdot \sin k\phi] \right] \]  
A1.21

The coefficients for these modified F.S. can be expressed in terms of the coefficients of the previous F.S., derived earlier:
Appendix 1 Derivation of Fourier Series Expressions for Force Modeling

\[ a_{x0} = \frac{a_0 + r_i \cdot b_i}{2} \quad \text{A1.22} \]

\[ a_{x1} = a_0 + \frac{a_2 + r_i \cdot b_2}{2} \quad \text{A1.23} \]

\[ b_{x1} = r_i \cdot a_0 + \frac{b_2 - r_i \cdot a_2}{2} \quad \text{A1.24} \]

\[ a_{nk} = \frac{1}{2} \left[ a_{(k-1)} - r_i \cdot b_{(k-1)} + a_{(k+1)} + r_i \cdot b_{(k+1)} \right] \quad \text{A1.25} \]

\[ b_{nk} = \frac{1}{2} \left[ b_{(k-1)} + r_i \cdot a_{(k-1)} + b_{(k+1)} - r_i \cdot a_{(k+1)} \right] \quad \text{A1.26} \]

And similarly,

\[ a_{y0} = \frac{r_i \cdot a_1 - b_1}{2} \quad \text{A1.27} \]

\[ a_{y1} = r_i \cdot a_0 + \frac{r_i \cdot a_2 - b_2}{2} \quad \text{A1.28} \]

\[ b_{y1} = -a_0 + \frac{r_i \cdot b_2 - a_2}{2} \quad \text{A1.29} \]

\[ a_{yk} = \frac{1}{2} \left[ r_i \cdot a_{(k-1)} + b_{(k-1)} + r_i \cdot a_{(k+1)} - b_{(k+1)} \right] \quad \text{A1.30} \]

\[ b_{yk} = \frac{1}{2} \left[ r_i \cdot b_{(k-1)} - a_{(k-1)} + a_{(k+1)} + r_i \cdot b_{(k+1)} \right] \quad \text{A1.31} \]
The parasitic force component of Eqn. 3.1 must now also be expressed as an F.S.. The instantaneous feed, $S$ in milling is given by:

$$S = S_c \sin \phi$$  \hspace{1cm} A1.32

By substituting this, Eqn. 3.1 can be re-expressed as:

$$F_T = K_1 \cdot a \cdot S + K_2 \cdot L_a$$  \hspace{1cm} A1.33

Assuming that a critical feed $S^*$ is defined as that which yields a total cutting force comprising of equal components of cutting and parasitic force; then from Eqn. A1.33:

$$K_1 \cdot a \cdot S^* = K_2 \cdot L_a$$  \hspace{1cm} A1.34

or $$K_2 = \frac{K_1 \cdot a \cdot S^*}{L_a}$$  \hspace{1cm} A1.35

In reality, no cutting occurs to cause this situation; however, the concept is useful for modeling the component of total cutting force resulting from progressive wear of the tool. Substituting the last expression back into Eqn. A1.33 gives:

$$F_T = K_1 \cdot a \cdot S + K_1 \cdot a \cdot S^*$$  \hspace{1cm} A1.36

or $$F_T = K_1 \cdot a \cdot S + K_1 \cdot L_a \cdot h^*$$  \hspace{1cm} A1.37

where $h^*$ is the critical value of $h_e$ when cutting and parasitic forces are equal.
From here it can be seen that the parasitic term yields a rectangular force pulse. In the form shown in Eqn. A1.37, let this term be expressed as $F_{T2}$, the second component of the total tangential force $F_T$; i.e.,

$$F_{T2} = K_1 \cdot L \cdot h^*$$  \hspace{1cm} (A1.38)

The F.S. coefficients for this term can be determined from the following standard integral expressions:

$$a_{w0} = \frac{1}{2\pi} \int_{\phi_1}^{\phi_2} d\phi$$  \hspace{1cm} (A1.39)

$$a_{w1} = \frac{1}{\pi} \int_{\phi_1}^{\phi_2} \cos\phi \cdot d\phi$$  \hspace{1cm} (A1.40)

$$b_{w1} = \frac{1}{\pi} \int_{\phi_1}^{\phi_2} \sin\phi \cdot d\phi$$  \hspace{1cm} (A1.41)

$$a_{w2} = \frac{1}{\pi} \int_{\phi_1}^{\phi_2} \cos2\phi \cdot d\phi$$  \hspace{1cm} (A1.42)

$$b_{w2} = \frac{1}{\pi} \int_{\phi_1}^{\phi_2} \sin2\phi \cdot d\phi$$  \hspace{1cm} (A1.43)

$$a_{wk} = \frac{1}{\pi} \int_{\phi_1}^{\phi_2} \cos k\phi \cdot d\phi$$  \hspace{1cm} (A1.44)
\[ b_{w1} = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \sin k\phi \cdot d\phi \]  
\[ \text{A1.45} \]

These integrals can be evaluated to give the following general expressions:

\[ a_{w0} = \frac{[\phi_2 - \phi_1]}{2\pi} \]  
\[ \text{A1.46} \]

\[ a_{w1} = \frac{[\sin \phi_2 - \sin \phi_1]}{\pi} \]  
\[ \text{A1.47} \]

\[ b_{w1} = \frac{[\cos \phi_1 - \cos \phi_2]}{\pi} \]  
\[ \text{A1.48} \]

\[ a_{w2} = \frac{[\sin 2\phi_2 - \sin 2\phi_1]}{2\pi} \]  
\[ \text{A1.49} \]

\[ b_{w2} = \frac{[\cos 2\phi_1 - \cos 2\phi_2]}{2\pi} \]  
\[ \text{A1.50} \]

\[ a_{wk} = \frac{[\sin k\phi_2 - \sin k\phi_1]}{k\pi} \]  
\[ \text{A1.51} \]

\[ b_{wk} = \frac{[\cos k\phi_1 - \cos k\phi_2]}{k\pi} \]  
\[ \text{A1.52} \]

The parasitic force component can now be expressed in terms of its F.S. coefficients as:

\[ F_{r2} = K_1 \cdot L_a \cdot h^+ \left[ a_{w0} + \sum_{k=1}^{\infty} \left[ a_{wk} \cdot \cos k\phi + b_{wk} \cdot \sin k\phi \right] \right] \]  
\[ \text{A1.53} \]
Subsequent substitutions in the respective equations, as performed for the cutting component earlier, result in the final forms of the X and Y axes force components as:

\[
F_{x2} = K_1 \cdot L_a \cdot h^* \left[ a_{w0} + \sum_{k=1}^{\infty} \left( a_{wk} \cdot \cos k\phi + b_{wk} \cdot \sin k\phi \right) \right] \quad \text{A1.54}
\]

\[
F_{y2} = -K_1 \cdot L_a \cdot h^* \left[ a_{w0} + \sum_{k=1}^{\infty} \left( a_{wy} \cdot \cos k\phi + b_{wy} \cdot \sin k\phi \right) \right] \quad \text{A1.55}
\]

Again, the F.S. coefficients for these last two equations can be expressed in terms of the previously derived coefficients:

\[
a_{w0} = \frac{a_{w1} + r_2 \cdot b_{w1}}{2} \quad \text{A1.56}
\]

\[
a_{w1} = a_{w0} + \frac{a_{w2} + r_2 \cdot b_{w2}}{2} \quad \text{A1.57}
\]

\[
b_{w1} = r_2 \cdot a_{w0} + \frac{b_{w2} - r_2 \cdot a_{w2}}{2} \quad \text{A1.58}
\]

\[
a_{wk} = \frac{1}{2} \left[ a_{w(k-1)} - r_2 \cdot b_{w(k-1)} + a_{w(k+1)} + r_2 \cdot b_{w(k+1)} \right] \quad \text{A1.59}
\]

\[
b_{wk} = \frac{1}{2} \left[ b_{w(k-1)} + r_2 \cdot a_{w(k-1)} + b_{w(k+1)} - r_2 \cdot a_{w(k+1)} \right] \quad \text{A1.60}
\]

And

\[
a_{wy0} = \frac{r_2 \cdot a_{w1} - b_{w1}}{2} \quad \text{A1.61}
\]
Appendix 1 Derivation of Fourier Series Expressions for Force Modeling

\[
a_{w1} = r_2 \cdot a_{w0} + \frac{r_2 \cdot a_{w2} - b_{w2}}{2} \tag{A1.62}
\]

\[
b_{w1} = -a_{w0} + \frac{r_2 \cdot b_{w2} - a_{w2}}{2} \tag{A1.63}
\]

\[
a_{wK} = \frac{1}{2} \left[r_2 \cdot a_{w(k-1)} + b_{w(k-1)} + r_2 \cdot a_{w(k+1)} - b_{w(k+1)}\right] \tag{A1.64}
\]

\[
b_{wK} = \frac{1}{2} \left[r_2 \cdot b_{w(k-1)} - a_{w(k-1)} + a_{w(k+1)} + r_2 \cdot b_{w(k+1)}\right] \tag{A1.65}
\]

With the coefficients defined, the total forces can now be expressed simply as sums of the components:

\[
F_x = F_{x1} + F_{x2} \tag{A1.66}
\]

\[
F_y = F_{y1} + F_{y2} \tag{A1.67}
\]
Appendix 2

Program Listing for Fourier Series Based Cutting Force Simulation

REM SIMPLE PROGRAM THAT SIMULATES CUTTING FORCES USING A FOURIER SERIES BASED MODEL.
REM AUTHOR-SUNMBOLA OYAWOYE, DATE-JULY 1992
OPEN "C:\SUMBI\DATA.DAT" FOR OUTPUT AS #1
DIM A(500)
DIM AX(500)
DIM AY(500)
DIM B(500)
DIM BX(500)
DIM BY(500)
DIM AW(500)
DIM AWX(500)
DIM AWY(500)
DIM BW(500)
DIM BWX(500)
DIM BWY(500)

PI = 3.141593
PHI1 = 0
PHI2 = 1.353
REM PHI1 AND PHI2 ARE WORKPIECE ENTRY AND EXIT ANGLES RESPECTIVELY

DC = 2
REM DC IS THE DEPTH OF CUT

INPUT PHI, INC, K, R1, R2, ST, H, K1, LA
REM PHI IS THE START POINT FOR THE INSTANTANEOUS ROTATION ANGLE
REM INC IS THE PERIODIC INCREMENT OF THE ROTATION ANGLE
REM K1, R1, R2 ARE THE CONSTANTS OBTAINED FROM ACTUAL CUTTING DATA
REM K IS THE NUMBER OF FOURIER SERIES TERMS CONSIDERED
REM ST IS THE FEED-PER-TOOTH
REM H IS THE CRITICAL VALUE OF EQUIV. CHIP THICKNESS ALSO OBTAINED
REM FROM ACTUAL CUTTING DATA
REM LA IS THE LENGTH OF ACTIVE CUTTING EDGE
REM DETERMINATION OF F.S. COEFFICIENTS FOR CUTTING COMPONENT OF FORCE

10 A(0) = (COS(PHI1) - COS(PHI2)) / (2 * PI)
A(1) = (COS(2 * PHI1) - COS(2 * PHI2)) / (4 * PI)
B(1) = ((PHI2 - PHI1) - .5 * (SIN(2 * PHI2) - SIN(2 * PHI1))) / (2 * PI)
AX(0) = (A(1) + (R1 * B(1))) / 2
AY(0) = ((R1 * A(1)) - B(1)) / 2

SUMX = AX(0)
SUMY = AY(0)

REM SUMX AND SUMY ARE RUNNING SUMMATIONS OR TOTALS FOR F.S. TERMS

FOR J = 2 TO (K + 1)

A(J) = (((COS((1 + J) * PHI1)) / (1 + J)) - ((COS((1 + J) * PHI2)) / (1 + J)) -
((COS((J - 1) * PHI1)) / (J - 1)) + ((COS((J - 1) * PHI2)) / (J - 1))) / (2 * PI)

B(J) = (((SIN((1 + J) * PHI1)) / (1 + J)) - ((SIN((1 + J) * PHI2)) / (1 + J)) -
((SIN((J - 1) * PHI1)) / (J - 1)) + ((SIN((J - 1) * PHI2)) / (J - 1))) / (2 * PI)

IF J = 2 THEN

AX(J - 1) = A(0) + (A(J) + (R1 * B(J))) / 2
BX(J - 1) = (R1 * A(0)) + (B(J) - (R1 * A(J))) / 2
AY(J - 1) = (R1 * A(0)) + ((R1 * A(J)) - B(J)) / 2
BY(J - 1) = (-1 * A(0)) + ((R1 * B(J)) - A(J)) / 2

ELSE

AX(J - 1) = (A(J - 2) - (R1 * B(J - 2)) + A(J) + (R1 * B(J))) / 2
BX(J - 1) = (B(J - 2) + (R1 * A(J - 2)) + B(J) - (R1 * A(J))) / 2
AY(J - 1) = ((R1 * A(J - 2)) + B(J - 2) + (R1 * A(J)) - B(J)) / 2
BY(J - 1) = ((R1 * B(J - 2)) - A(J - 2) + (R1 * B(J)) + A(J)) / 2
END IF

RSUMX = (AX(J - 1) * COS((J - 1) * PHI)) + (BX(J - 1) * SIN((J - 1) * PHI))
RSUMY = (AY(J - 1) * COS((J - 1) * PHI)) + (BY(J - 1) * SIN((J - 1) * PHI))

REM INCREMENT RUNNING TOTALS
SUMX = SUMX + RSUMX
SUMY = SUMY + RSUMY

NEXT J

REM COMPUTE CUTTING FORCE COMPONENTS
FX1 = K1 * DC * ST * SUMX
FY1 = (-1 * K1) * DC * ST * SUMY

REM DETERMINATION OF PARASITIC FORCE COMPONENTS

AW(0) = (PHI2 - PHI1) / (2 * PI)
AW(1) = (SIN(PHI2) - SIN(PHI1)) / PI
BW(1) = (COS(PHI1) - COS(PHI2)) / PI
AWX(0) = (AW(1) + (R2 * BW(1))) / 2
AWY(0) = ((R2 * AW(1)) - BW(1)) / 2

WSUMX = AWX(0)
WSUMY = AWY(0)
REM WSUMX AND WSUMY ARE RUNNING TOTALS FOR THE PARASITIC F.S. TERMS

FOR J = 2 TO (K + 1)

\[ AW(J) = \frac{\sin(J \cdot \Phi_2) - \sin(J \cdot \Phi_1)}{J \cdot \pi} \]

\[ BW(J) = \frac{\cos(J \cdot \Phi_1) - \cos(J \cdot \Phi_2)}{J \cdot \pi} \]

IF J = 2 THEN

\[ AWX(J - 1) = AW(0) + \frac{AW(J) + (R^2 \cdot BW(J))}{2} \]

\[ BWX(J - 1) = \frac{R^2 \cdot AW(0)}{2} + \frac{BW(J) - (R^2 \cdot AW(J))}{2} \]

\[ AWY(J - 1) = \frac{R^2 \cdot AW(0)}{2} + \frac{(R^2 \cdot BW(J)) - BW(J)}{2} \]

\[ BWY(J - 1) = \frac{-1 \cdot AW(0)}{2} + \frac{(R^2 \cdot BW(J)) + AW(J)}{2} \]

ELSE

\[ AWX(J - 1) = \frac{AW(J - 2) - (R^2 \cdot BW(J - 2)) + AW(J) + (R^2 \cdot BW(J))}{2} \]

\[ BWX(J - 1) = \frac{BW(J - 2) + (R^2 \cdot AW(J - 2)) + BW(J) - (R^2 \cdot AW(J))}{2} \]

\[ AWY(J - 1) = \frac{(R^2 \cdot AW(J - 2)) + BW(J - 2) + (R^2 \cdot AW(J)) - BW(J)}{2} \]

\[ BWY(J - 1) = \frac{(R^2 \cdot BW(J - 2)) - AW(J - 2) + (R^2 \cdot BW(J)) + AW(J)}{2} \]

END IF

RWSUMX = \[ (AWX(J - 1) \cdot \cos((J - 1) \cdot \Phi)) + (BWX(J - 1) \cdot \sin((J - 1) \cdot \Phi)) \]

RWSUMY = \[ (AWY(J - 1) \cdot \cos((J - 1) \cdot \Phi)) + (BWY(J - 1) \cdot \sin((J - 1) \cdot \Phi)) \]
REM INCREMENT SUMMATIONS
WSUMX = WSUMX + RWSUMX
WSUMY = WSUMY + RWSUMY
NEXT J

REM COMPUTE PARASITIC FORCE COMPONENTS
FX2 = K1 * LA * H * WSUMX
FY2 = (-1 * K1) * LA * H * WSUMY

REM COMPUTE TOTAL FORCES
FX = FX1 + FX2
FY = FY1 + FY2

REM PRINT TO FILE AND SCREEN
WRITE #1, "FX=", FX, "FY=", FY, "PHI=", PHI
PRINT USING "FX= ####.###  FY= ####.###  PHI= ####.###"; FX; FY; PHI

REM INCREMENT INSTANTANEOUS ROTATION ANGLE
PHI = PHI + INC

REM TERMINATION CONDITION
IF PHI <= 1.4 THEN GOTO 10
CLOSE #1
END
References


[21] I. Yellowley, Y. Hosepyan and O. Oyawoye; "The Identification of Machining Condition and Tracking of Tool Wear in Milling Using Machining Forces." To be Published.


