DESIGN, INTEGRATION, AND DYNAMICAL MODEL OF A MULTI-MODULE DEPLOYABLE MANIPULATOR SYSTEM (MDMS)

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We accept the thesis as conforming to the required standard

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Date 25 April 2009......
ABSTRACT

This thesis focuses on the mechanical design, development, mathematical dynamical model formulation, and demonstration of controlled operation of a Multi-module Deployable Manipulator System (MDMS). The new robot system can be used in ground-based as well as space-based operations. The system is composed of a mobile base supporting four modules connected in a chain topology. Each module consists of two links: one free to slew while the other permitted to deploy and retrieve. It is designed for future experimental investigations aimed at dynamics and control of this variable geometry manipulator by implementing different control algorithms to regulate its performance. The manipulator design involves the selection and sizing of actuators, the design of mounting and connecting components, and the selection of hardware as well as software for real-time control. The integration of computer control system components for the MDMS is also discussed in details. The governing equations of motion for the planar dynamics of the manipulator system are obtained using an $O(N)$ algorithm, based on the Lagrangian approach and velocity transformations. The $O(N)$ character is computationally efficient permitting real-time control of the system. A dynamical simulation program is written in C language. The accuracy of the formulation and the validity of the computer code can be verified through energy conservation tests. The classical Proportional-Integral-Derivative (PID) control strategy is implemented to illustrate satisfactory operation of the designed system.
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<td>$a$</td>
<td>acceleration of deployment</td>
</tr>
<tr>
<td>$C_i'$</td>
<td>equivalent viscous damping coefficient for the $i^{th}$ revolute joint</td>
</tr>
<tr>
<td>$C_v''$</td>
<td>equivalent viscous damping coefficient for the transverse mode of vibration for the payload</td>
</tr>
<tr>
<td>$d$</td>
<td>bearing bore diameter</td>
</tr>
<tr>
<td>$d_i$</td>
<td>translation vector of $F_i$ from the tip of the $(i-1)^{th}$ module, Fig. 3-4</td>
</tr>
<tr>
<td>$D_i$</td>
<td>inertial position vector to the frame $F_i$, Fig. 3-1</td>
</tr>
<tr>
<td>$dm_i$</td>
<td>mass of the infinitesimal element located on the $i^{th}$ body, Fig. 3-1</td>
</tr>
<tr>
<td>$EI$</td>
<td>flexural rigidity of the payload</td>
</tr>
<tr>
<td>$f_{N+1}$</td>
<td>displacement of the mass element located at $r_{N+1}$ due to the flexibility of the payload, Fig. 3-1</td>
</tr>
<tr>
<td>$F$</td>
<td>vector containing the terms associated with the centrifugal, Coriolis, gravitational, elastic and internal dissipative forces, Eq. (3.1)</td>
</tr>
<tr>
<td>$F_0$</td>
<td>inertial reference frame</td>
</tr>
<tr>
<td>$F_i$</td>
<td>reference frame attached to the $i^{th}$ module</td>
</tr>
<tr>
<td>$F_a$</td>
<td>thrust force component of the linear actuator to accelerate the mass</td>
</tr>
<tr>
<td>$F_f$</td>
<td>thrust force component of the linear actuator to overcome the friction between the moving load and the surface it is resting on</td>
</tr>
<tr>
<td>$F_g$</td>
<td>thrust force component of the linear actuator to overcome gravitational effect</td>
</tr>
<tr>
<td>$F_{th}$</td>
<td>design thrust force of the linear actuator, $F_a + F_f + F_g$</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity</td>
</tr>
<tr>
<td>$g_i$</td>
<td>position vector of the frame $F_i$ relative to the frame $F_{i-1}$</td>
</tr>
<tr>
<td>$h$</td>
<td>height of the cross-section of the module</td>
</tr>
<tr>
<td>$I^n$</td>
<td>$n \times n$ identity matrix</td>
</tr>
<tr>
<td>$J_{a,i}$</td>
<td>actuator rotor inertia</td>
</tr>
<tr>
<td>$J_{eq}$</td>
<td>equivalent moment of inertia at actuator output shaft</td>
</tr>
<tr>
<td>$J_M$</td>
<td>estimated module moment of inertia</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
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<td>--------</td>
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<tr>
<td>$K_{ai}$</td>
<td>stiffness of the $i^{th}$ revolute joint</td>
</tr>
<tr>
<td>$l_d, l_s$</td>
<td>length of deployable and slewing links, respectively</td>
</tr>
<tr>
<td>$l_i$</td>
<td>length of the $i^{th}$ module</td>
</tr>
<tr>
<td>$l_m$</td>
<td>half-length of the module, Eq. (2.5)</td>
</tr>
<tr>
<td>$l_p$</td>
<td>length of the payload</td>
</tr>
<tr>
<td>$L$</td>
<td>load on the rolling support</td>
</tr>
<tr>
<td>$m_{ai}$</td>
<td>mass of the actuator located at the $i^{th}$ revolute joint</td>
</tr>
<tr>
<td>$M$</td>
<td>coupled system mass matrix</td>
</tr>
<tr>
<td>$\tilde{M}$</td>
<td>decoupled system mass matrix</td>
</tr>
<tr>
<td>$M_{brg}$</td>
<td>friction moment of the roller bearing</td>
</tr>
<tr>
<td>$\tilde{M}_i$</td>
<td>decoupled mass matrix of the $i^{th}$ body</td>
</tr>
<tr>
<td>$M_{roll}$</td>
<td>friction torque from the rolling support</td>
</tr>
<tr>
<td>$n$</td>
<td>encoder resolution</td>
</tr>
<tr>
<td>$n_a$</td>
<td>number of system actuators</td>
</tr>
<tr>
<td>$n_c$</td>
<td>number of system constraints</td>
</tr>
<tr>
<td>$n_s$</td>
<td>number of generalised coordinates describing the system dynamics</td>
</tr>
<tr>
<td>$n_u$</td>
<td>number of generalised coordinates describing the dynamics of each module</td>
</tr>
<tr>
<td>$N$</td>
<td>number of modules</td>
</tr>
<tr>
<td>$P$</td>
<td>load on the bearing</td>
</tr>
<tr>
<td>$P^c$</td>
<td>matrix assigning the Lagrange multipliers to the constrained equations</td>
</tr>
<tr>
<td>$q$</td>
<td>set of generalised coordinates leading to the coupled mass matrix $M$</td>
</tr>
<tr>
<td>$\tilde{q}$</td>
<td>set of generalised coordinates leading to the decoupled mass matrix $\tilde{M}$</td>
</tr>
<tr>
<td>$\tilde{q}_i$</td>
<td>set of generalised coordinates associated with the $i^{th}$ body, leading to the decoupled mass matrix $\tilde{M}_i$</td>
</tr>
<tr>
<td>$q_s$</td>
<td>specified component of $q$</td>
</tr>
<tr>
<td>$Q$</td>
<td>vector containing the external non-conservative generalized forces</td>
</tr>
<tr>
<td>$Q^t$</td>
<td>matrix assigning inputs to the actuated variables</td>
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</table>
$r_i$ position vector of the elemental mass $dm_i$ with respect to the frame $F_i$

$R$ gear ratio

$R_{a_i}$ inertial position vector of the actuator located at the $i^{th}$ revolute joint

$R_{d}$ Rayleigh dissipation function for the whole system

$R_{dm_i}$ inertial position vector to an elemental mass $dm_i$ on the $i^{th}$ body

$R^{C,R}$ first velocity transformation matrices

$R^{V}$ second velocity transformation matrix

$s$ distance from rolling support to motor axis about which it moves

$t$ time

$T$ total kinetic energy of the system

$T_i$ rotation matrix mapping $F_0$ onto $F_i$

$T_a$ design torque for the actuator of the revolute joint

$T_r$ resistance torque for the actuator of the revolute joint

$u$ vector containing control inputs

$u_{N+1}$ longitudinal components of $f_{N+1}$

$U_e$ total strain energy of the system

$U_g$ total gravitational potential energy of the system

$v_{N+1}$ transverse components of $f_{N+1}$

$W$ load on the linear actuator

$x_i, y_i$ Cartesian components of $r_i$

Greek Symbols

$\alpha$ angular acceleration of the revolute joint

$\alpha_i$ rigid body angular motion of the revolute joint $i$

$\beta_i$ flexibility contribution to angular motion of the revolute joint $i$

$\delta_{N+1}$ generalized coordinate for the elastic deformation of the payload

$\Phi_{N+1}$ mode shape function matrix for the payload

$\phi$ inclination of the linear actuator
\( \varphi_{N+1} \)  
mode shape function for the transverse deformation of payload

\( \gamma \)  
coder multiplier

\( \gamma_i \)  
rotation of the \( i^{th} \) frame relative to the \((i^{th} - 1)\) frame

\( \Delta \)  
vector containing the Lagrange multipliers

\( \eta_i \)  
inertial orientation of the actuator rotor on the \( i^{th} \) joint

\( \eta_{\text{screw}} \)  
ball screw efficiency

\( \mu \)  
coefficient of friction for the bearing

\( \mu_s \)  
coefficient of friction for the rolling support

\( \theta_d \)  
desired position accuracy of the revolute joint

\( \tau_d \)  
design torque for the linear actuator servomotor

\( \Delta \tau \)  
time taken for maneuver

\( \tau \)  
time from start of maneuver

\( \psi_i \)  
inertial orientation of the frame \( F_i \)

A dot above a character refers to total differentiation with respect to time.
A boldface lower case character denotes a vector.
A boldface upper case character denotes a matrix.
Subscript 'd' corresponds to deployable link.
Subscript 's' refers to the slewing link or a specified coordinate.

**Abbreviations, Acronyms**

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<tr>
<td>CAD</td>
<td>Computer Aided Design/Drafting</td>
</tr>
<tr>
<td>CPU</td>
<td>Central Processing Unit</td>
</tr>
<tr>
<td>D/A</td>
<td>Digital/Analog</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>DSP</td>
<td>Digital Signal Processing</td>
</tr>
<tr>
<td>ERA</td>
<td>European Robotic Assembly</td>
</tr>
<tr>
<td>EVA</td>
<td>Extra-Vehicular Activities</td>
</tr>
<tr>
<td>IAL</td>
<td>Industrial Automation Laboratory</td>
</tr>
<tr>
<td>I/O</td>
<td>Input/Output</td>
</tr>
<tr>
<td>IRIS</td>
<td>Institute of Robotics and Intelligent Systems</td>
</tr>
<tr>
<td>ISA</td>
<td>Industry Standard Architecture</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
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<tr>
<td>ISS</td>
<td>International Space Station</td>
</tr>
<tr>
<td>JEM-RMS</td>
<td>Japanese Experimental Module Robotic Manipulator System</td>
</tr>
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<td>MDMS</td>
<td>Multi-module Deployable Manipulator System</td>
</tr>
<tr>
<td>MSS</td>
<td>Mobile Servicing System</td>
</tr>
<tr>
<td>NEMA</td>
<td>National Electrical Manufacturers Association</td>
</tr>
<tr>
<td>$O(N)$</td>
<td>Order- $N$</td>
</tr>
<tr>
<td>PC</td>
<td>Personal Computer</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional-Integral-Derivative</td>
</tr>
<tr>
<td>PMDI</td>
<td>Precision MicroDynamics Inc.</td>
</tr>
<tr>
<td>PPI</td>
<td>Pulse Power I</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse Width Modulation</td>
</tr>
<tr>
<td>SPDM</td>
<td>Special Purpose Dexterous Manipulator</td>
</tr>
<tr>
<td>SRMS</td>
<td>Shuttle Remote Manipulator System</td>
</tr>
<tr>
<td>SSRMS</td>
<td>Space Station Remote Manipulator System</td>
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<td>VGM</td>
<td>Variable Geometry Manipulator</td>
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1. INTRODUCTION

1.1 Preliminary Remarks

Robotic manipulators have been playing an important role in space exploration because of the harsh space environment and the challenges associated with it. It is an indispensable part of the International Space Station (ISS). Involving the United States, Canada, Russia, eleven European countries and Japan, the ISS is the largest cooperative venture in science and technology ever undertaken [1]. The first manipulator used in space was the Canadarm (Figure 1-1), officially known as the Shuttle Remote Manipulator System (SRMS), which was launched on 12th November 1981 for the first time. It heralded Canada’s involvement in a flight with crew. With the ISS, Canada is contributing an essential component of the Station – the Mobile Servicing System (MSS, Figure 1-2). It consists of three elements - the Mobile Remote Servicer Base System, the Space Station Remote Manipulator System (SSRMS), and the Special Purpose Dexterous Manipulator (SPDM). The SSRMS is capable of handling large payloads and assist with docking of the space shuttle. The SPDM is a smaller two-armed robot capable of handling delicate assembly operations, like the human hand. The tasks performed by the MSS would include:

- capture and release of spacecraft;
- maneuvering of payload;
- berthing and de-berthing;
- support of Extra-Vehicular Activities (EVA);
- construction, operation and maintenance of the space station.
Figure 1-1 The Canadarm.
Figure 1-2  Mobile Servicing System (MSS).
Other examples of space-based robotic manipulators are the Japanese Experimental Module Robotic Manipulator System (JEM-RMS) and European Robotic Assembly (ERA) [2-4]. These manipulator designs include solely revolute joints and fixed-length links to emulate the movement of a human arm.

In contrast to the purely revolute joints based design, manipulators with a combination of revolute and prismatic joints offer several useful characteristics with respect to the dynamics and control [5]. Such manipulators are called deployable manipulators, and with several modules connected in series, a Multi-module Deployable Manipulator System (MDMS) is evolved (Figure 1-3). The prismatic joint is located at each module thus permitting change in length of the deployable link. The ability to deploy and retrieve in addition to slew is the unique feature of the design. The MDMS offers several advantages over the conventional manipulator design that involves only revolute joints [5]:

- The decision making is simpler. For example, to reach a given point P in the x, y-plane would present two possible configurations for a two-link revolute joint manipulator (Figure 1-4). However, the deployable manipulator system has a unique solution.
- Inertia coupling, for the same number of joints, is significantly reduced as shown in Figure 1-5. Note, the inertia force due to deployment or retrieval does not create moment about joints A and B. This is not the case with the slewing motion of the link 2. In the figure, $F_{ij}$ refers to the inertia force, due to slewing or deploying maneuver of the link $i$, at the link $j$.
- It presents better facility to overcome obstacles, especially involving small gaps (Figure 1-6).
Figure 1-3  (a) Deployable manipulator on an orbiting platform with a pair of slewing and deployable links; (b) Multi-modules Deployable Manipulator System (MDMS).
Figure 1-4  Simpler decision making algorithm for the deployable manipulator.

Figure 1-5  Reduced inertia coupling with the deployable manipulator.
• reduced number of singular positions for equal number of joints as shown in Figure 1-7.

Although this novel, multi-module, variable geometry manipulator has the above-mentioned desirable features, it has received relatively less attention [6, 7]. This is particularly the case with reference to a satisfactory ground-based prototype system which can be used to assess, through real-time experiments, effectiveness of a variety of control procedures for their possible application to space-based systems. The main objective of the thesis is to undertake design, construction, integration and operation of such multi-module prototype manipulator with slewing and deployable links.

1.2 Brief Review of the Relevant Literature

Due to the increasing importance of the robotic manipulators in space missions, there has been considerable interest in understanding their complex dynamics and control, both through computer simulations and experimental set-ups. Relevant literature in these areas is briefly reviewed here.

1.2.1 Dynamical studies of slewing-deployable manipulators

As early as in the late 1970s, Lips and Modi [8, 9] studied at length the dynamics of spacecraft with a rigid central body to which deployable beam-type members were attached (Figure 1-8a). Modi and Ibrahim [10] presented a relatively general formulation for this class of problems involving a rigid body supporting deployable beam- as well as plate-type members (Figure 1-8b). Subsequently, Modi and Shen [11] extended the study to account for deployment as well as slewing of the appendages (Figure 1-8c). Lips [12], Ibrahim [13], and
Not possible to reach the target $P$

The deployable link can extend through gaps and reach target $P$

Figure 1-6  Better facility to overcome obstacles for the deployable manipulator.

Kinematics: $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = [J] \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}$

Here $[J]$ is the Jacobian matrix

Inverse Kinematics: $\begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} = [J]^{-1} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \frac{adj[J]}{\det[J]} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$

For singularity, $\det[J] = 0$.

Figure 1-7  Reduced number of singular positions for the deployable manipulator.
Shen [14] have reviewed this aspect of the literature in considerable detail in their theses. In the above-mentioned studies [8-14], each appendage, either involving slewing and/or deployment members, was directly connected to the central body; a manipulator-type chain geometry of the links or appendages was not involved. Moreover, the attention was entirely on the system dynamics in presence of external disturbances and manipulator maneuvers. There was no attempt at control of the system response to a desired pre-set criterion.

A new manipulator system involving slewing as well as deployable links, schematically shown in Figure 1-8(d), was first proposed for space application by Marom and Modi [15]. Planar dynamics and control of the one-unit mobile manipulator with rigid links but flexible joints, located on an orbiting flexible platform, were investigated. Results showed significant coupling effects between the platform and the manipulator dynamics. Control of the system during tracking of a specified trajectory, using the computed torque technique, proved to be quite successful. Modi et al. [16] and Hokamoto et al. [17-19] extended the study to the multi-module configuration, referred to as Variable Geometry Mobile Manipulator and the Mobile Deployable Manipulator (MDM) system, respectively. The model with an arbitrary number of modules accounted for the joint as well as link flexibility. A relatively general formulation for three-dimensional dynamics of the system in orbit was the focus of the study by Modi et al., while Hokamoto et al. explored a free-flying configuration. More recently, Hokamoto et al. [20] studied the control of the Mobile Deployable Manipulator with an arbitrary number of modules, each with two flexible links: one of them free to slew (revolute joint); and the other deployable (prismatic joint). They developed a numerical procedure for the inverse kinematics of the system and demonstrated
Figure 1-8  Schematic diagrams of space structure models: (a) Lips [12], rigid spacecraft with deployable beam-type members; (b) Ibrahim [13], rigid spacecraft with deployable beam- and plate-type members; (c) Shen [14], rigid spacecraft with slewing-deployable appendages; (d) Marom [15], flexible spacecraft with rigid slewing-deployable links and a rigid payload at the deployable end (flexible revolute joint).
that trajectory control of the end-effector using the resolved acceleration approach was quite successful even in the presence of flexibility.

Pradhan et al. [21] reported an elegant procedure to the $O(N)$ formulation for three-dimensional motion of a system, in tree topology, using the Lagrangian approach. Caron [22] extended the $O(N)$ procedure to account for system flexibility as well as deployment and applied it to study planar dynamics as well as control of a formidable multi-module deployable manipulator system. The dynamical parametric study [23] clearly showed complex interactions between flexibility, librational dynamics, and manipulator maneuvers.

Dynamicists have always been interested in the discretization procedure that would satisfactorily capture the effect of system flexibility. Zhang et al. [24] attempted to address this issue by:

(i) assessing the effect of number of modes on the response of a single module manipulator, on an orbiting flexible platform, during a prescribed maneuver;

(ii) studying the effect of variation in flexibility of the revolute joint as well as slewing and deployable links;

(iii) varying the speed of maneuver.

Results showed that the fundamental mode was able to capture the system response quite well. The joint flexibility as well as the speed of maneuver affected the system response substantially. However, the effect of link flexibility was relatively negligible.

Of particular significance is the recent contribution by Chen [25]. He obtained for the first time, $O(N)$ formulation for the three-dimensional dynamics of an orbiting flexible
manipulator, with an arbitrary number of modules, traversing a flexible platform. He illustrated application of the formulation by studying dynamics of a multi-module system in the plane of the orbit. Control of rigid degrees of freedom for a single module manipulator using the Feedback Linearization Technique (FLT) was also investigated. Recently, Yang et al. [6, 7] extended the study to a two-module system with control of rigid as well as flexible degrees of freedom using the FLT in conjunction with the Linear Quadratic Regulator (LQR).

1.2.2 Experimental test facilities

Numerous laboratory scale experimental test facilities have been organized around the world to assess on ground different control strategies, which may eventually be implemented in space-environment. However, most facilities involve one or two links with revolute joint connections only. For example, an experimental flexible manipulator system was designed and developed at the University of Sheffield [26]. It is a single flexible link manipulator with one revolute joint and a point mass as the payload attached at the tip of the link. A five-axis planar macro-micro manipulator facility was developed, from design through model validation, by Van Vliet and Sharf [27]. It consists of five revolute joints: two degrees of freedom (dof) macro-manipulator with two flexible links at the tip of which is attached a three-dof micromanipulator with three rigid links. It should be emphasized that the above-mentioned manipulators consist of solely revolute joints, and their link lengths are fixed.

Recently, a two-module variable geometry manipulator, with revolute as well as prismatic joints, was designed and constructed at the University of British Columbia [5]. It is
shown in Figure 1-9. Its slewing arm length is 30cm, and the maximum extension of the deployable arm is 20cm. This manipulator is being tested on ground using several different control strategies. However, it has some limitations:

- massive and heavy;
- backlash and friction present at the joints;
- static downward deflection owing to the long, heavy body and the absence of any kind of support;
- limited slewing motion of $-135^\circ$ to $135^\circ$.

1.3 Scope of the Investigation

With the above as background, the present study focuses on the development of a prototype Multi-module Deployable Manipulator System for ground-based operations and experiments. This includes the mechanical design and construction of the manipulator as well as the selection and integration of a real-time computer control system. Furthermore, a dynamical model for such a system with an arbitrary number of modules is obtained. The model treats the system as composed of a mobile base, free to move in a plane, supporting the modules that are connected in a chain topology as shown in Figure 1-10. As indicated before, each module consists of two links: one free to slew while the other is permitted to deploy and retrieve. The links are considered to be rigid while the revolute joints are treated as flexible. The model closely simulates the prototype mentioned above. It will help assess dynamics of the manipulator as affected by the system parameters, to assist in its design, as well as effectiveness of various control strategies.
Figure 1-9  Variable Geometry Manipulator (VGM) developed at the University of British Columbia [5].
Figure 1-10  Schematic diagram of the MDMS.
Chapter 2 discusses the development of the prototype manipulator, currently operational in the IRIS Spacecraft Control Laboratory. Special features of the design as well as the construction and integration processes are described at length.

Formulation of the equations of motion is given in Chapter 3. It presents a dynamical model for the system to help assess system response and its control. Of particular interest is the $O(N)$ character of the Lagrangian formulation that is computationally efficient permitting real-time control of the system. The position of the mobile base, the slewing angle of the revolute joints, the deployment of the links controlled by the prismatic joints, as well as the deflection of the flexible payload are taken as generalized coordinates. The trajectories of these coordinates can be specified by introducing constraint relations associated with the Lagrange multipliers.

Chapter 4 focuses on the development of a C program to integrate the equations of motion for numerical simulations.

Finally, in Chapter 5, the operation of the prototype MDMS with a PID control is illustrated. The thesis ends with a summary of important results and conclusions based on them, lists main contributions, and provides recommendations for future studies (Chapter 6).
2. THE MDMS PROTOTYPE DESIGN

2.1 General Description

This chapter presents the design, construction and integration of the prototype Multi-module Deployable Manipulator System (MDMS). The objective is to develop a working system for real-time tests and control implementation. Figure 2-1 shows the integrated and operational manipulator presently located in the IRIS Spacecraft Control Laboratory of the Department of Mechanical Engineering, at the University of British Columbia, and the basic configuration of the manipulator is shown in Figure 2-2. It consists of a mobile base supporting a four-module, i.e. eight links, manipulator. Each module has two links: one capable of slewing while the other permitted to deploy and retract. The workspace of this manipulator system is a circle of ≈4.5m diameter. The rotational motions are achieved with revolute joints actuated by harmonic drive gearing DC servo motors. The deployment and retrieval of links are performed by ballscrew linear actuators. The actuators and optical encoders are connected to the 8-axis ISA bus servo I/O card and amplifiers, which interface the robotic manipulator with a PC compatible computer.

2.2 Design Requirements and Criteria

The design requirements are:

- eight-axis planar robotic manipulator with four modules, each consisting of one slewing link and one deployable link;
- full circle slewing motion for the first revolute joint at the base;
- rolling supports for the manipulator;
- acceleration of 0.08m/s² at design payload of 5kg;
Figure 2.1 The prototype of a Multi-module Deployable Manipulator System (MDMS).
Figure 2.2 Basic configuration of the prototype MDMS.
15cm (≈ 6in) maximum extension of each deployable link;

- maximum slew speed = 60°/s;
- maximum deployment speed = 4cm/s;
- PC-based system on which different control strategies can be implemented.

With an eight-axis manipulator working in a plane, the redundancy would permit operation with joint failure and ease of obstacle avoidance. In space, when an actuator fails, it would be difficult to repair or replace components, and tasks will have to be performed with the available actuators. In this situation, presence of redundant degrees of freedom provides alternatives to accomplish the desired task. Furthermore, the unit’s ability to deploy is quite an advantage in getting through a gap between two obstacles as pointed out in Chapter 1. The rolling supports are important because the length of the manipulator induces a high loading at each unit as well as at the base. This could affect the performance of the actuators, even damage them. The 15-centimetre extension of each unit provides sufficient change in length for demonstrating the better performance with prismatic joints. The slew and deployment speeds are comparable to the motions in space environment. The PC-based control system will provide the flexibility such that different algorithms can be implemented for the investigation of the manipulator’s regulated behavior as well as assess relative merit of different strategies.

The design criteria are:

- essentially rigid links;
• reduced size, weight and inertia compared to the existing two-module system (Figure 1-9);
• minimum machining, i.e. reduced machine shop-time;
• ease of construction and assembly.

Reduction of the manipulator mass was an important consideration during the design process. Any additional mass of the system would result in greater inertia thus adversely affecting its performance. In addition to the above criteria, the mechanical and electronic components were chosen based on their performance characteristics like speed, accuracy, reliability and robustness.

It is important to point out that the construction of the manipulator involved two types of components:

(a) those supplied by outside manufacturers;

(b) those designed and machined in-house in the department’s machine-shop by the author.

Appendix I gives details of the components in the category (a). Information about the suppliers is provided in Appendix II. Machined parts like the base, motor mounts and joint supports were made of Aluminum 6061 for its low density ($\rho_{Al} = 2710$ kg/m$^3$), strength (yield strength = 255MPa) and highly machineable character. CAD drawings for the prototype are included in Appendix III. It may be pointed out that the machining task proved to be quite challenging. It involved considerable time and effort ($\approx 400$ hours) beyond the design of the parts.
2.3 Preliminary Concept of the Manipulator Design

With these requirements and criteria, a preliminary concept of the manipulator was evolved as the starting point (Figure 2-3). Beginning from the base, where a DC servomotor is mounted and provides full circle slewing motion to the whole manipulator, a linear actuator is mounted on an angle bracket, which is connected to its motor shaft. Three identical units are connected to the first module in series, and revolute joint motors are installed at junctions of any two adjacent units. With the concept in mind, available actuators were evaluated, compared and finally appropriate selection made.

2.4 Main Components of the Manipulator System

The two main components of this robotic system are the manipulator and its control system. Basically, the manipulator consists of:

- rotary actuators with sensors for the revolute joints;
- linear actuators with sensors for the prismatic joints;
- rolling supports for the manipulator;
- test bench for the workspace of the manipulator;
- bearings and shaft couplings;
- mounting brackets and connecting components.

The control system comprises of:

- amplifiers and power supplies;
- motion control interface cards;
- computer software for programming controllers.
Figure 2-3 Preliminary concept of the prototype MDMS.
The selection of actuators, motion control interface cards and computer software are discussed in the following sections. As mentioned earlier, the companies referred to are listed in Appendix I.

2.5 Revolute Joint Actuators

For this experimental prototype, which carries out light duty tasks, electric drives are chosen as opposed to hydraulic or pneumatic actuators because, in general, electric drives are easy to control. As a matter of fact, the Canadarm and most space-based robotic manipulators employ electric drives for their revolute joint actuators. Three electric motor categories were narrowed down to, and their performance evaluated.

2.5.1 DC servomotor plus gearhead

The first option is the DC servomotor plus a gearhead. With NEMA standard or planetary gearhead, backlash (10 to 20 arc minutes) is present, and this contributes to the non-linearity of the system that makes its control rather challenging. One can employ an extremely low backlash gearhead, such as the Stealth planetary gearhead from Bayside, the leading manufacturer of mechanical motion products. It is an all-helical planetary design for high performance servo applications and has 3 to 5 arc minutes of backlash. However, it is 25% - 50% more expensive than the planetary spur gearheads. Besides, mounting of the gearhead onto the motor requires the installation of an adapter between the two by following a set of instructions. The adapter would increase the length and weight of the actuator, violating the design criterion on the system mass.
2.5.2 Direct drive servomotor

The second option is the direct drive servomotor that eliminates the problems of gear backlash and installation, since a direct drive transmission does not have a gearhead. However, the sizes and weights of the direct drive servomotors do not meet the design criteria. Moreover, they are more expensive and heavier compared to the first option.

2.5.3 Harmonic drive gearing actuator

The third option considered is that of the harmonic drive gearing actuator from HD Systems Inc. Featuring zero backlash, high positional accuracy and stiffness, these actuators provide precise motion control and large torque capacity in very compact packages. The integral package contains a servomotor, a harmonic drive gearhead, a tachometer and/or an optical encoder. These components are designed to be power-compatible, and are 10% - 30% cheaper than the first option. Furthermore, the option of flange output provides a flat surface for mounting the brackets directly to which the deployable link unit can be connected. The harmonic drive system was selected for implementation.

2.6 Prismatic Joint Actuators

For the deployable links, the first option investigated was that of the linear servomotor. It consists of a coil assembly that moves along a magnetic track with an air gap between the two. It has some advantages:

- no backlash because of direct drive;
- smooth motion because of no contacting surfaces to cause friction and stick-slip behavior, i.e. no wear;
• no lubrication needed other than air;
• virtually no maintenance required.

However, these motors are intended for high-speed and high-acceleration applications, which is not the case here. The assembly is heavy because it is made of magnetic iron, and hence does not satisfy the design criteria. Most importantly, for the same size as a ball-screw actuator, the linear motor generates a lower thrust. Furthermore, additional design work is required for the mounting of the coil and magnetic assemblies, as well as the linear encoder that comes as an extra unit.

Next, the linear actuators that employ ball-screws were considered. An integral package is available from Dynact Incorporated. It consists of a rotary servomotor, ball-screw and nut, a deploying shaft and an encoder. It is called the Pulse Power I (PPI) linear actuator, and can supply up to 400N (90 pounds) thrust with the standard motor. Customized motor can also be installed if more thrust, less weight, etc. are desired. Moreover, with the integral package, the machine-shop time would be greatly reduced. Therefore, this actuator unit is chosen for the deployable link.

2.7 Sizing of Actuators

The required torque, $T_a$, for each of the four revolute joints is computed using the basic equation from Newton’s second law [28],

$$T_a = T_r + J_{eq} \alpha,$$

(2.1)

where $T_r$ is the resistance torque; $J_{eq}$ is the equivalent moment of inertia (including rotor, load, gearing, dampers, etc.); and $\alpha$ is the angular acceleration to take the load to the
maximum speed. For this application, the resistance is contributed by the friction torque in
the bearings and from the rolling friction of the rolling supports at the revolute joints,

\[ T_R = M_{brg} + M_{roll}, \]  

(2.2)

where \( M_{brg} \) is the friction moment in the rolling bearing; and \( M_{roll} \) is the friction torque
from the rolling support. They can be computed using the following equations [29]:

\[ M_{brg} = 0.5 \mu P d; \]  

(2.3)

\[ M_{roll} = \mu_s L s; \]  

(2.4)

where \( \mu \) is the coefficient of friction for the bearing given by the bearing manufacturer; \( P \) is
the bearing load in Newton; \( d \) is the bearing bore diameter in millimetre; \( \mu_s \) is the
coefficient of friction for the rolling support; \( L \) is the load on the rolling support in Newton;
and \( s \), measured in metre, is the distance between the rolling support and the axis of slewing
motion. The coefficients of friction for deep groove ball bearing and thrust ball bearing are
0.0015 and 0.0013, respectively, and for the rolling support, it is 0.002 [29]. The equivalent
moment of inertia consists of the actuator inertia and the module inertia. Each module is
2.3kg maximum and 0.381m long when fully extended. These values are used for the
outermost revolute joint since it is actuating one module. For the other three joints, the mass
and length values are doubled, tripled and quadrupled accordingly while going towards the
base. The moments of inertia for the actuators are available from the manufacturer
catalogue, and for the modules can be calculated using the equation [30],

\[ J_M = M \left( \frac{4 \cdot l_m^2}{3} + \frac{h^2}{12} \right), \]  

(2.5)
where $M$ is the actuated mass in kilogram; $l_m$ is the half-length of the module in metre; and $h$ is the height of the cross-section of the module in metre. With the maximum slewing speed of 60°/s and using a typical trapezoidal velocity profile [31], the maximum acceleration is determined to be $2 \text{ rad/s}^2 (120°/s^2)$. This will increase the speed to its maximum in 0.5 second. Using a safety factor of 2, the design torque requirements for the four slewing joint motors, starting from the base, are calculated to be:

- slewing joint motor #1: 30 N-m;
- slewing joint motor #2: 13 N-m;
- slewing joint motor #3: 4 N-m;
- slewing joint motor #4: 0.5 N-m.

With these torque values and the speed requirement of 1 rad/s (10 rpm), harmonic drive gearing actuators are selected by referring to the torque-speed curves of the actuators provided by the manufacturer, and the results are listed in Table 2-1.

**Table 2-1** Selected actuators and their specifications for revolute joints.

<table>
<thead>
<tr>
<th>Joint 1</th>
<th>Joint 2</th>
<th>Joint 3</th>
<th>Joint 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>HD Systems</td>
<td>HD Systems</td>
<td>HD Systems</td>
</tr>
<tr>
<td></td>
<td>RFS-20-3007</td>
<td>RH-14C-3002</td>
<td>RH-11C-3001</td>
</tr>
<tr>
<td>Max output torque (Nm)</td>
<td>84</td>
<td>20</td>
<td>7.8</td>
</tr>
<tr>
<td>Max output speed (rpm)</td>
<td>40</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Diameter (m)</td>
<td>0.085</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>Length (m)</td>
<td>0.216</td>
<td>0.148</td>
<td>0.125</td>
</tr>
<tr>
<td>Mass (kg)</td>
<td>3.6</td>
<td>0.78</td>
<td>0.51</td>
</tr>
<tr>
<td>Inertia at output shaft (kgm²)</td>
<td>1.2</td>
<td>0.082</td>
<td>0.043</td>
</tr>
</tbody>
</table>
For the prismatic joints, the required thrust force is governed by the friction between the moving load and the surface it is resting on, force to accelerate the mass, and gravitational effects:

\[ F_{th} = F_f + F_a + F_g; \]  

\[ F_f = \mu_s W \cos \phi; \]  

\[ F_a = \frac{W}{g} \times a; \]  

\[ F_g = W \sin \phi; \]

where \( \mu_s \) is the coefficient of friction between the mass to be moved and the surface it rests on; \( W \) is the load to be moved; \( \phi \) is the angle between the actuator and the horizontal plane; and \( a \) is the maximum acceleration of the mass.

Since the linear actuators are mounted horizontally, the gravitational force is zero. Taking the mass of each unit to be 2.3kg (5 pounds), which is the maximum allowed, the total mass with a safety factor of 2 is 13.6kg (30 lbs) for the first, i.e. the innermost, module. The coefficient of rolling friction for steel-on-steel is 0.002. As a result, the total thrust force required by the actuator at the first module was computed to be 138N (31 lbs), which is a lot less than what the linear actuator with the standard servomotor can supply. The torque required by the servomotor can be calculated using the standard equation [32],

\[ \tau = \frac{F_a \times \text{lead}}{\eta_{\text{screw}} \times 2\pi}. \]  

Since the Pulse Power I has a screw lead of 3mm (0.125in) and a screw efficiency of 90%, the design torque from the servomotor with 10% torque margin was calculated to be
1kg-m (13 oz-in). Hence a customised motor can be selected and used. With the aid of the mechanical designer at Dynact, customised servomotors from Pittman that are smaller and lighter than the standard ones are chosen to drive linear actuators. Pittman Model 14203 motor can supply a continuous torque of 0.015kg-m (21 oz-in) at 2900rpm, which corresponds to a linear speed of 14.5cm/s. For the other three linear actuators, Pittman Model 14201 motor is chosen since, according to Dynact, it is the smallest Pittman motor that can be installed on Pulse Power I, and the size and weight are acceptable. This motor can provide a continuous torque of 0.007kg-m (10 oz-in) at 3650rpm, which converts to 18.25cm/s linear speed. Approximately 2/3 of 0.009kg-m (13 oz-in) torque is required at the second module to actuate the third and fourth modules. Pittman 14201 motor meets the requirement and hence it was selected.

2.8 Assembly and Construction

The assembly and construction of the prototype was successfully accomplished with the aid of highly qualified technicians from the machine shop and the electronic laboratory of the Department of Mechanical Engineering. Working in collaboration with technicians, a systematic construction procedure was established. The following subsections discuss in detail integration of the manipulator components. As indicated before, the final versions of the design and assembly drawings are included in Appendix III.

2.8.1 Components of the final design

Table 2-1, presented earlier on page 28, lists actuators selected with specifications for the revolute joints based on the discussion in the previous sections. Basically, revolute joints use the harmonic drive gearing actuators from HD Systems Inc., and the linear actuators are
Pulse Power I (PPI) from Dynact Incorporated with servo motors from Pittman. Four Pittman motors, one Model 14203 and three Model 14201, are installed onto the Dynact PPI linear actuators. Including the mounting brackets and connectors, the overview of the CAD model is shown in Figure 2-4. The design of the module sub-assemblies is described in the following sections.

2.8.2 Manipulator base and test-bench

The mobile base of the manipulator consists of a box frame built from steel angles, called Dexion, and four swiveling casters installed on the bottom corners (Figure 2-5). With the casters, this mobile base can be moved into the middle of the test-bench (Figure 2-6) for performing experiments, and taken away when the test-bench is needed by the Variable Geometry Manipulator, the first prototype, mentioned earlier (p. 14, Figure 1-9). Thus the test-bench serves both the two-module manipulator designed and constructed by Chu [5] as well as the present four-module manipulator. The test-bench is built from square tubes, which are part of the link tube system, supplied by Freeman Wright Agencies Ltd. in Vancouver. These link tubes are integrated using their unique connectors, which do not require any fasteners, such as nuts and bolts. The construction of the bench using this link tube system is preferred since it is easier compared to the traditional steel angles, which require a large number of nuts, bolts and stiffeners. For the steel ball rolling supports to move on, the bench top is a piece of flat thin steel plate. To minimize the weight of material resting on the link tube frame, a steel plate of 10 gage thickness is used. Aluminum is not suitable for this because it is too soft. Preliminary experiments showed the steel ball to leave indentation marks damaging the smooth surface. A better surface finish is achieved by using cold rolled sheet steel. Furthermore, 16mm (5/8in) thick plywood is installed in each of
Figure 2.4 Overall CAD model for the MDMS prototype.

- RH-8 Harmonic Drive Actuator
- RH-14 Harmonic Drive Actuator
- RH-11 Harmonic Drive Actuator
- RFS-20 Harmonic Drive Actuator
- Module 1
- Module 2
- Module 3
- Module 4
- Frame Plate
Figure 2-5 Mobile base of the MDMS.

Steel angles (Dexion)

MDMS Base Plate Bolted on Top

Swiveling casters (4)
Figure 2-6  Manipulator workspace test-bench.
1.2x1.2m (4x4ft) frame section right underneath the steel top to provide support and to prevent the sagging of the steel top. The circular cut in the middle of the bench top provides the necessary clearance for the manipulator base.

2.8.3 First module

On the mobile base is mounted the frame plate which supports the entire manipulator. As shown in Figure 2-7, the first slewing joint motor (RFS-20) is bolted onto the motor mount, which is supported on the two angle legs using screws, and the legs are connected to the frame plate. The aluminium plate can be adjusted for horizontal planar accuracy. Using a stiffened aluminium angle, the first Pulse Power I linear actuator is then mounted on the flange face of RFS-20. Same as a shaft, the flange face rotates and gives the manipulator full-circle slewing motion. Therefore, the manipulator can reach to the same extent in all directions in the planar workspace.

2.8.4 Revolute joint

The design of the three modules that connect to the first module is basically identical with the exception of the harmonic drive actuator and bearing sizes. A detailed view of the revolute joint of the MDMS is shown in Figure 2-8. The module connector integrates the joint bracket to the end of the deploying shaft of the previous module using a taper pin. Two deep groove ball bearings are installed, one each at the top and bottom plates, to support the slewing link. The slewing link consists of the PPI linear actuator, which is based on the actuator mount, and top and bottom connectors between the mount and the bearings. The actuator mount is machined from a piece of channel aluminium. The top connector is attached to the harmonic drive gearing actuator and is secured in place with two setscrews.
Figure 2-8  Revolute joint details.
Two screws are required to prevent any possible slack during operation. Rolling support (Figure 2-9) is then attached to the bottom plate. It consists of an aluminium plate on which three ball transfers are installed. This plate is bolted onto the joint bottom plate as shown in Figure 2-8.

2.9 Computer Control System

Components of the motion control system for the manipulator include an IBM compatible host computer, motion control interface board, power amplifiers, DC servomotors as actuators and optical encoders as feedback sensors. Figure 2-10 shows the schematic of the servo control system. Only one axis is shown for clarity. The rationale for selection of specific hardware components is discussed in the following sections.

2.9.1 Optical encoders

Optical encoders that come with the servomotors are used as the feedback devices for the control system. They are digital transducers, thus the analogue-to-digital conversion, which may lead to quantization error, is eliminated. The optical methodology resulting in the no-contact feature with rotating discs makes the device durable and almost maintenance-free. The output signal from the encoder is in number of pulses. It relates to the position of the rotor, and hence the slew motion of the manipulator module as well as the position of the deployable link can be determined. According to the HD Systems, Inc.'s catalogue, the minimum encoder resolution can be determined using the following equation,

\[ n = 5 \cdot \frac{60 \cdot 360}{\theta_A \cdot R \cdot \gamma} \]  

(2.11)
Figure 2.9  Rolling support.
Figure 2-10  Servo control system schematic.
where $\theta_d$ is the desired position accuracy at the output in arc-minutes; $R$ is the gear reduction ratio; and $\gamma$ is the encoder multiplier, which is equal to 4 for the quadrature encoders employed in this design. Using the maximum position accuracy of the motors, the minimum encoder resolution for the four revolute joint motors is calculated to be 270, 135, 135 and 108 Pulses Per Revolution (PPR), respectively. The standard resolution with the harmonic drive gearing actuators is 1000PPR with 200, 360, and 500 as special options. Since the actuators have the same gear ratio, identical encoders are used for the four revolute joints giving the same resolution at the output shaft. To meet the minimum requirement, encoders with 360PPR resolution are used, and with the gear reduction ratio of 100, the resolution at the output shaft becomes 36000PPR. Therefore, the smallest step that can be measured is $\frac{1}{36000}$ revolution or 0.01°. As for the linear actuators, the standard encoder resolution is 500PPR. Therefore, with a screw lead of 3mm, the smallest linear motion that can be measured is 0.006mm. The signal with number of pulses from the optical encoders can be measured directly through the quadrature encoder inputs on the motion control interface board as discussed in the following section.

2.9.2 Motion control interface board

With the type of actuators and sensors selected, motion control interface cards, for communication between the manipulator and the computer, are to be chosen. To implement different control algorithms on this robotic manipulator system, an open architecture real-time control system is desired. With the real-time control system, the robotic manipulator will be asked to complete certain tasks within a period of time. Therefore, the importance is not only in getting the task done, but also in performing it within the prescribed time. The
two options explored are the motion control interface cards of the DSP-based and a PC-based system. With the DSP-based card, a digital signal processing (DSP) chip or board is required, and this would increase the total cost by approximately $2500. The processor is used to perform computations necessary to properly control the motor. The conventional thinking has been that the DSP chip is necessary because the main CPU in the PC does not have the processing power to perform the calculations in a reasonable amount of time. This was true in the past, but it is no longer so. Moreover, for many applications, an update rate of 1 millisecond is more than sufficient, and this makes an on-board processor unnecessary for most applications.

Based on the above consideration, the PC-based motion control interface card, MFIO-3A, from Precision MicroDynamics Inc. (PMDI) appeared attractive. This board was developed in Industrial Automation Laboratory (IAL) of the Department of Mechanical Engineering, at the University of British Columbia, and was commercialised by PMDI. The MFIO-3A has three channels of 16-bit D/A converters, three quadrature decoder channels, 24 digital I/O channels, a programmable interval timer and a watchdog timer. However, for the 8-axis manipulator system under consideration, three MFIO-3A cards would be required. It is desirable to use one board that can provide enough number of channels rather than having a bank of several cards. With a little search, an 8-axis ISA bus servo I/O motion control card from Servo To Go, Inc. was found. It is a PC-based motion control board that operates on the ISA bus of the computer. It has similar functionality as the MFIO-3A board, but more number of axes and less expensive. The input/output only approach of this board opens the system to other servo algorithms and control experimentation. The detailed specifications are included in Appendix I. With the Windows NT drivers available with the I/O board, the
8-axis ISA bus servo I/O card was selected to operate and control the manipulator in real-time.

2.9.3 Servo amplifiers and power supplies

Servo amplifiers are selected based on the power of the selected servo actuators. Two types of amplifiers are considered: linear and pulse width modulation (PWM). The PWM amplifiers are selected because:

- linear amplifiers are generally more expensive;
- linear amplifiers are meant for noise sensitive applications;
- PWM amplifiers are technologically more advanced;
- overheating is a problem for linear amplifiers.

Model 12A8 25A Series servo amplifiers from Advanced Motion Control in Camarillo, CA are selected. These PWM servo amplifiers are designed to drive brush type DC motors. Eight of them are required for the 8-axis manipulator. Series PS1600W unregulated power supply, which is designed to complement the selected amplifiers, is also selected. Two units of power supplies are required since each can operate a maximum of six amplifiers. Detailed specifications of the amplifiers and power supplies are listed in Appendix I.

2.9.4 Control system schematic

Components of the control system are schematically shown in Figure 2-11. Here S1 and D1 denote the slewing and deployable joints of module 1, respectively. The interface cards between the manipulator and the computer acquire position readings from the encoders,
Figure 2-11  Schematic diagram of the overall control system.
send them to the control program which generates control actions. The commands are conveyed to the actuators also through the interface cards.
3. DYNAMICAL MODEL OF THE MDMS

3.1 Preliminary Remarks

An analytical model of a robotic manipulator is valuable in several ways. It can be used in preliminary design evaluation, computer simulation, and model-based control. This chapter develops the equations of motion for the MDMS, a ground-based planar slewing and deployable robotic manipulator system, schematically presented earlier in Figure 1-10. Essentially, the system consists of a mobile base moving in a plane, a manipulator formed by an arbitrary number of modules or units attached to and supported by the base, and a flexible payload at the end of the last unit. Each unit comprises two links: one free to slew with a revolute joint at one end, while the other is permitted to deploy and retrieve through a prismatic joint at the end of the previous link. The second link carries a revolute joint at the other end for connection to the next unit. The links are considered rigid while the joints are taken to be flexible.

It is not sufficient for a mathematical model to be accurate in order to control the manipulator system. The computation will likely have to be done in real-time; hence its efficiency is a key requirement. As can be anticipated, the governing equations of motion turn out to be extremely lengthy, highly nonlinear, nonautonomous, and coupled, and can be expressed in the general form

$$M(q,t)\ddot{q} + F(q,q,\dot{q},t) = Q(q,q,\dot{q},t),$$

(3.1)

where $M(q,t)$ is the system mass matrix; $q$ is the vector of the generalized coordinates; $F(q,q,\dot{q},t)$ contains the terms associated with the centrifugal, Coriolis, gravitational, elastic, and internal dissipative forces; and $Q(q,q,\dot{q},t)$ represents generalized forces, including the
control inputs. Eq (3.1) describes the inverse dynamics of the system. For simulations, forward dynamics of the system is required, and Eq. (3.1) must be solved for $\ddot{q}$,

$$\ddot{q} = M^{-1}(Q - F).$$

(3.2)

The solution of these equations of motion generally requires $O(N^3)$ arithmetic operations, where $N$ represents the number of units considered in a study [21]. In other words, the number of computations required by an $O(N^3)$ algorithm will vary with the cube of the number of units. Clearly, the computational cost can become prohibitive for a large $N$. Therefore, $O(N)$ algorithm, where the number of arithmetic operations increases linearly with the number of units in the system, is employed in deriving the equations of motion. Such algorithms reduce the computational time and memory requirements considerably, making real-time applications possible.

In the following, the kinematic equations of the system are developed first followed by the modeling of the joint and payload flexibility. The kinetic energy of the system is then derived. Two velocity transformations are introduced in order to arrive at new forms of the kinetic energy. The derivation of expressions for the gravitational and strain potential energies follows. Energy dissipation mechanisms are presented as well, and generalized forces are discussed. The Lagrangian principle is then used to derive the equations of motion. The final section examines the use of Lagrange multipliers when some of the degrees of freedom are constrained.
3.2 Kinematics of the System

A serial manipulator, supported on a mobile base, can be represented as an open chain of rigid bodies (units, modules) as shown in Figure 3-1. There are \( N \) units and a flexible payload attached at the end of the \( N^{th} \) unit. Note that the length of the bodies 1 to \( N \) can vary in time while the \( (N+1)^{th} \) body is a flexible payload with fixed length. Furthermore, each body can rotate as well as translate with respect to its neighbours.

3.2.1 Reference frames

The inertial reference frame \( F_0 \) is located at the center of the planar workspace. The position of each body is described by assigning body-fixed frames to each element of the chain. Thus, the frame \( F_1 \) is located at the base of the first unit where the mobile base is attached. The body-fixed frame for each of the remaining units is attached to the joint at the base of the unit. In other words, the reference frame \( F_i \) is attached to the base of the \( i^{th} \) body. Since the payload is a beam-type body, the \( x_i \) axis is along the length of the beam; the \( y_i \) axis is perpendicular to the \( x_i \) direction, in the workspace plane.

3.2.2 Position vectors

The position of an infinitesimal mass element \( dm_i \), located on the \( i^{th} \) unit, can be described with respect to the inertial frame as

\[
R_{dm_i} = D_i + T_i r_i. \tag{3.3}
\]

For the \( (N+1)^{th} \) body or the flexible payload, the position vector can be expressed as
Figure 3-1 System model.
\[ R_{dm_{N+1}} = D_{N+1} + T_{N+1}[r_{N+1} + f_{N+1}(r_{N+1})] \]  \hspace{1cm} (3.4)

Here \( D_i \) defines the inertial position of the origin of the \( i^{th} \) body-fixed frame and is denoted by
\[
D_i = \begin{bmatrix} D_{x} \\ D_{y} \end{bmatrix}.
\hspace{1cm} (3.5)
\]

\( T_i \) is the rotation matrix which maps the components of \( r_i \), expressed in terms of the body-fixed coordinates, onto inertial coordinates. It is defined as
\[
T_i = \begin{bmatrix} \cos \psi_i & -\sin \psi_i \\ \sin \psi_i & \cos \psi_i \end{bmatrix},
\hspace{1cm} (3.6)
\]
where \( \psi_i \) describes the orientation of the frame \( F_i \) and corresponds to the angle formed between the \( x_i \) and \( x_0 \) axes. \( r_i \) is the position vector of the elemental mass \( dm_i \) with respect to \( F_i \) and is denoted by
\[
r_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}.
\hspace{1cm} (3.7)
\]

Finally, \( f_{N+1}(r_{N+1}) \) represents the elastic displacement of the mass element located at \( r_{N+1} \) due to the flexibility of the payload,
\[
f_{N+1}(r_{N+1}) = \begin{bmatrix} u_{N+1} \\ v_{N+1} \end{bmatrix},
\hspace{1cm} (3.8)
\]
where \( u_{N+1} \) and \( v_{N+1} \) are the longitudinal and transverse elastic deformation of the payload respectively, and will be presented in details in a later section.
3.2.3 Modeling of joint flexibility

All the revolute joints are considered flexible owing to the shaft and coupling flexibility. The joint flexibility is modeled by a linear torsional spring, which connects the rotor of the actuator to the slewing end of the module [33]. The stators of the motors are rigidly attached to the ends of the prismatic joints. Figure 3-2 shows details of a flexible revolute joint. $\alpha_i$ is the angular motion of the rotor with respect to the stator. $\beta_i$ is the deformation angle of the torsional spring with stiffness $K_{ai}$ and represents slewing motion with respect to the rotor. $\gamma_i$ gives the total angular displacement of the slewing link. Hence,

$$\gamma_i = \alpha_i + \beta_i, \quad i = 1, 2, \ldots, N.$$  \hfill (3.9)

3.2.4 Modeling of the flexible payload

The dynamics of a robotic manipulator with rigid links can be described by a set of ordinary differential equations. However, the time dependent elastic deformation of the flexible payload, with distributed parameters, requires partial differential equations for its description. Furthermore, such continuous system possesses infinite degrees of freedom. Therefore, it is necessary to approximate the elastic deformation of the payload using ordinary differential equations with a finite number of generalized coordinates.

The payload was modeled as an Euler-Bernoulli cantilever beam (the Thin Beam Theory), where the effects of rotary inertia and shear deformations, which become important for large deflections and high frequencies, are neglected. In the present study, the fundamental vibration mode of a cantilever beam is used during the dynamical simulations,
Figure 3-2  Representation of a flexible revolute joint and relative slew angle.
and the study focuses on the transverse displacements. Figure 3-3 shows model of the flexible half-payload as a beam.

Here:

\[ \boldsymbol{D}_{N+1} = \text{position vector of the payload reference frame } F_{N+1} \text{ with respect to the inertial frame}; \]

\[ dm_{N+1} = \text{mass element on the payload beam}; \]

\[ x_{N+1} = \text{position of the mass element along the } X_{N+1}-\text{axis}; \]

\[ v_{N+1}(x_{N+1}, t) = \text{deflection of the mass element along the } Y_{N+1}-\text{axis}; \]

\[ l_{N+1} = \text{length of the payload}. \]

The vibration of the payload is expressed in the form of a product of spatially varying shape functions \( \varphi_{N+1} \) and the time dependent generalized coordinates \( \delta_{N+1} \). Thus, the transverse elastic displacement \( v_{N+1} \) of the payload is described by

\[ v_{N+1} = \varphi_{N+1}(x_{N+1}, t) \]

(3.10)

where \( \varphi_{N+1} \), the fundamental transverse mode of the cantilever payload, can be written as

\[ \varphi_{N+1}(x_{N+1}) = \sin \lambda x_{N+1} - \sinh \lambda x_{N+1} + 1.3622(\cosh \lambda x_{N+1} - \cos \lambda x_{N+1}). \]

Here, \( \lambda = \frac{1.875}{l_{N+1}} \) for the first mode [34].

3.2.5 Velocity vectors

The inertial velocity of a mass element \( dm_i \) on the \( i^{th} \) module is obtained by taking the time derivative of Eq. (3.3) giving

\[ \dot{\mathbf{R}}_{dm_i} = \dot{D}_i + PT_i \dot{r}_i + T_i \dot{r}_i, \]

(3.11)
Figure 3-3 Planar deformation of a flexible beam-type payload.
Similarly, the inertial velocity of a mass element \(dm_{N+1}\) on the payload is given by:

\[
\dot{R}_{dm_{N+1}} = \dot{D}_{N+1} + PT_{N+1} (r_{N+1} + \Phi_{N+1} \delta_{N+1}) \psi_{N+1} + T_{N+1} \Phi_{N+1} \dot{\delta}_{N+1}.
\]  

(3.13)

Actuators regulating the slewing of the manipulator units are also modeled. Since they are located at the manipulator joints, their inertial velocity, defined as \(\dot{R}_{a,i}\), is described by

\[
\dot{R}_{a,i} = \dot{D}_{i},
\]  

(3.14)

while the inertial angular velocity of the rotor of the \(i^{th}\) actuator is given by \(\dot{\eta}_{i}\).  

### 3.2.6 Generalized coordinates

The position and orientation of the body-fixed frames are known relative to the inertial frame, in addition to the length \(l_i\) of each body. In Eqs. (3.3) and (3.4), the position of \(F_i\) is specified directly relatively to the inertial frame by \(D_i\), and similarly, its orientation by the inertial angle \(\psi_i\) (Figure 3-1). Corresponding velocities can be established by differentiation with respect to time.

The kinematics of each manipulator unit is described by \(n_u = 5\) generalized coordinates: \(D_x, D_y, \eta_i, \psi_i, l_i\). It should be noted that \(l_i\) is included as a generalized coordinate for each unit to account for deployment. Therefore, the set of generalized coordinates describing the kinematics of each manipulator unit is defined as

\[
P = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}
\text{ and } \dot{r}_i = \begin{bmatrix} i_i \\ 0 \\ 0 \\ 0 \end{bmatrix}
\]

for deploying section of the unit; for slewing section of the unit.
with \( i = 1, 2, \ldots, N \). Now, the set of generalized coordinates associated with the payload is,

\[
\tilde{q}_{N+1} = \begin{bmatrix} D_{N+1} \\
\psi_{N+1} \\
\delta_{N+1}
\end{bmatrix} \in \mathbb{R}^4.
\]  

Thus the set of generalized coordinates required for the complete description of the system kinematics can be written as

\[
\tilde{q} = \begin{bmatrix} \tilde{q}_1 \\
\tilde{q}_2 \\
\tilde{q}_3 \\
\vdots \\
\tilde{q}_N \\
\tilde{q}_{N+1}
\end{bmatrix} \in \mathbb{R}^{n_s},
\]  

where \( n_s = Nn_a + 4 \).

### 3.3 Kinetic Energy of the System

The total kinetic energy of the system can be written as

\[
T = \sum_{i=1}^{N+1} \frac{1}{2} \left( \dot{\mathbf{r}}_{pm}, \dot{\mathbf{r}}_{pm}, d_m + \sum_{j=1}^{N} \frac{1}{2} (m_{a_j} \dot{\mathbf{D}}_i \cdot \dot{\mathbf{D}}_i + J_{a_j} \dot{\mathbf{\hat{\eta}}}_i^2) \right),
\]  

where the first term represents the kinetic energy of the manipulator units and the payload, while the second term assesses the contribution from the actuators at the manipulator joints, which have a mass \( m_{a_j} \) and a rotary inertia \( J_{a_j} \). Rewriting Eqs. (3.11) and (3.13) in matrix form:
\[
\begin{align*}
\dot{R}_{dm_i} &= \begin{bmatrix} I^2 & 0 & PT_i r_i & T_i \frac{\partial r_i}{\partial x_i} \end{bmatrix} \dot{q}_i; \\
\dot{R}_{dm_{N+1}} &= \begin{bmatrix} I^2 & v_{il} & v_{i2} \end{bmatrix} \dot{q}_{N+1};
\end{align*}
\]

where \( I^2 \) is a 2 \times 2 identity matrix and

\[
\begin{align*}
v_{il} &= PT_{N+1} \left( r_{N+1} + \begin{bmatrix} 0 \\
\phi_{N+1} \delta_{N+1} \end{bmatrix} \right); \\
v_{i2} &= T_{N+1} \begin{bmatrix} 0 \\
\phi_{N+1} \end{bmatrix}.
\end{align*}
\]

Therefore, the total kinetic energy of the system can be expressed as

\[
T = \sum_{i=1}^{N+1} \frac{1}{2} \dot{q}_i^T \int_{m_i} \left[ \begin{bmatrix} I^2 & 0 & PT_i r_i & T_i \frac{\partial r_i}{\partial x_i} \end{bmatrix} \right]^T \begin{bmatrix} I^2 & 0 & PT_i r_i & T_i \frac{\partial r_i}{\partial x_i} \end{bmatrix} dm_i \dot{q}_i \\
+ \sum_{i=1}^{N+1} \frac{1}{2} \dot{q}_i^T \begin{bmatrix} m_{ai} I^2 & 0 & 0 & 0 \\
0 & J_{ai} & 0 & 0 \\
0 & 0 & 0 & 0 \end{bmatrix} \dot{q}_i + \frac{1}{2} \dot{q}_{N+1}^T \int_{m_{N+1}} \left[ \begin{bmatrix} I^2 \\
v_{il}^T \\
v_{i2}^T \end{bmatrix} \right] \begin{bmatrix} I^2 & v_{il} & v_{i2} \end{bmatrix} dm_{N+1} \dot{q}_{N+1}.
\]

Eq. (3.22) can be written in a compact form as

\[
T = \sum_{i=1}^{N+1} \frac{1}{2} \dot{q}_i^T \tilde{M}_i \dot{q}_i.
\]

A detailed description of \( \tilde{M}_i \) is provided in Appendix IV. When the summation in Eq. (3.23) is expressed in matrix form, a quadratic expression is obtained for the total kinetic energy of the system,

\[
T = \frac{1}{2} \dot{q}^T \tilde{M} \dot{q},
\]
where \( \tilde{M} \) is the \( n_s \times n_s \) mass matrix of the system,

\[
\tilde{M} = \begin{bmatrix}
\tilde{M}_1 & 0 & 0 & \cdots & 0 \\
0 & \tilde{M}_2 & 0 & \cdots & 0 \\
0 & 0 & \tilde{M}_3 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & \tilde{M}_{N+1}
\end{bmatrix}.
\] (3.25)

### 3.4 Alternate Expressions for the Kinetic Energy

Eq. (3.24) gives the total kinetic energy of the system in terms of the set of generalized coordinates \( \tilde{q} \). With this set, where \( D_i \) and \( \eta_i \) are taken as generalized coordinates, the dynamics of each body is described independently, without referring to adjacent bodies. This results in an elegant mass matrix, given by Eq. (3.25). As a result, the dynamics of each body is decoupled from that of the other bodies. Clearly, this is not the case in reality as the motion of a given body is affected by that of the others. Therefore, the constraint forces between adjacent bodies must be incorporated into the final equations of motion for this system. This process can be simplified through the use of another set of generalized coordinates, \( q \), which is now developed.

#### 3.4.1 Alternate set of generalized coordinates

As pointed out, it is necessary to define a second set of generalized coordinates to account for constraints imposed by the adjacent bodies. While the second set also considers \( \psi_i \), \( l_i \), and \( \delta_{N+1} \) as generalized coordinates, the position and orientation of the body-fixed frames are specified relative to the previous frame instead of the inertial frame. With this
approach, the frame $F_i$ is related to the frame $F_{i-1}$. $F_i$ can be obtained by translating $F_{i-1}$ along $g_i$ and rotating it about the $z_i$-axis by an angle $\gamma_i$, as shown in Figure 3-2. Hence,

$$D_i = D_{i-1} + g_i,$$

(3.26)

and

$$\psi_i = \psi_{i-1} + \gamma_i.$$  

(3.27)

Figure 3-4 shows that the vector $g_i$ consists of the sum of two vectors: $l_{i-1}$, which denotes the length of the $(i-1)^{th}$ body, or $l_{i-1} = [l_{i-1}^T 0]^T$; and $d_i$, which represents the translation of $F_i$ from the tip of the $(i-1)^{th}$ body due to a prismatic joint. Therefore,

$$g_i = T_{i-1}(l_{i-1} + d_i), \quad i = 2, \ldots N + 1.$$  

(3.28)

The above expression is pre-multiplied by $T_{i-1}$ so that the components of $g_i$ are in terms of inertial coordinates.

**Figure 3-4** Description of a body-fixed frame relative to the preceding frame.
The rotation $\gamma_i$ of the frame $F_i$ with respect to the frame $F_{i-1}$ has two contributions (Figure 3-2, Eq. (3.9)). Note,

$$\eta_i = \psi_{i-1} + \alpha_i.$$  

(3.29)

Thus, the position of the body-fixed frame $F_i$ is expressed in terms of that of $F_{i-1}$, which itself is related to the location of $F_{i-2}$. This referencing with respect to the preceding frame continues until the frame $F_1$ is reached, which is directly described relative to the inertial frame by $D_i$ and $\psi_1$. Thus

$$D_i = D_1 + \sum_{j=2}^{i} g_j.$$  

(3.30)

Hence, a second set of generalized coordinates, $q$, is available for the complete description of the orientation of the system,

$$q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_N \\ q_{N+1} \end{bmatrix} \in \mathbb{R}^{n},$$  

(3.31)

where $q_1 = \tilde{q}_1$,

$$q_i = \begin{bmatrix} d_i \\ \alpha_i \\ \psi_i \\ l_i \end{bmatrix} \in \mathbb{R}^{n}, \quad i = 2, \ldots, N,$$  

(3.32)

and
The decoupled set of generalized coordinates, $\bar{q}$, leads to a simple expression for the system's kinetic energy. On the other hand, the second set, $q$, simplifies the specification of the constraint forces. Therefore, the methodology here consists in deriving the system energy using the decoupled set of coordinates, $\bar{q}$, and then converting it to the more convenient coupled set, $q$. This can be done through the use of the two velocity transformations which are derived now.

As mentioned earlier, both $\dot{\bar{q}}_i$ and $\dot{q}_i$ use $\dot{\psi}_i$, $\dot{\xi}_i$, and $\dot{\delta}_{N+1}$, as generalized velocities. Hence, to find a relationship between $\dot{\bar{q}}_i$ and $\dot{q}_i$, the task consists of finding expressions for $\dot{D}_i$ and $\dot{\eta}_i$ in terms of the second set of generalized coordinates $q$. For both the velocity transformations, the time derivative of Eq. (3.30) is required,

$$\dot{\eta}_i = \dot{\psi}_i + \dot{\xi}_i.$$  

(3.34)

In matrix form,

$$\dot{\eta}_i = [0 \ 0 \ 0 \ 1 \ 0] \dot{\bar{q}}_{i-1} + [0 \ 0 \ 1 \ 0 \ 0] \dot{q}_i,$$

(3.35)

which is equivalent to

$$\dot{\eta}_i = [0 \ 0 \ 0 \ 1 \ 0] \dot{\bar{q}}_{i-1} + [0 \ 0 \ 1 \ 0 \ 0] \dot{q}_i.$$  

(3.36)

The case of $D_i$ is now addressed.
(a) First velocity transformation

For the first velocity transform, the time derivative of Eq. (3.26) is needed,

\[ \dot{D}_i = D_{i-1} + \dot{g}_i, \]  \hspace{1cm} (3.37)

where

\[ \dot{g}_i = P_{g_i} \dot{\psi}_{i-1} + T_{i-1} \dot{i}_{i-1} + T_i \dot{d}_i. \]  \hspace{1cm} (3.38)

Eq. (3.38) can be rewritten as

\[ \dot{D}_i = \begin{bmatrix} 0 & 0 & \dot{g}_i \end{bmatrix} \begin{bmatrix} D_{i-1} \\ \dot{i}_{i-1} \\ \dot{\psi}_{i-1} \end{bmatrix} + [T_{i-1} \ 0 \ 0] \begin{bmatrix} \dot{d}_i \\ \alpha_i \\ \psi_i \end{bmatrix}. \]  \hspace{1cm} (3.39)

Using Eq. (3.37) and Eq. (3.40), the following matrix relation can be obtained,

\[ \ddot{q}_i = R^C_i \dot{q}_{i-1} + R_i \dot{q}_i. \]  \hspace{1cm} (3.40)

The details of the \( R_i^C \) and \( R_i \) matrices can be found in Appendix V. Combining Eq. (3.40) for \( i = 1, \ldots, N + 1 \), a relation between \( \ddot{q}_i \) and \( \dot{q}_i \) is obtained as

\[ \ddot{q} = R^C \ddot{q} + R \dot{q}, \]  \hspace{1cm} (3.41)

where:

\[ R^C = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ R^C_i & 0 & 0 & 0 & \cdots & 0 \\ 0 & R^C_2 & 0 & 0 & \cdots & 0 \\ 0 & 0 & R^C_i & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \]  \hspace{1cm} (3.42)
Rewriting Eq. (3.42) yields the first velocity transformation,

\[ \dot{\mathbf{q}} = (I^w - R^C)^{-1} R \dot{q} . \]  

(b) Second velocity transformation

The second velocity transformation requires the time derivative of Eq. (3.31),

\[ \mathbf{D}_i = \mathbf{D}_i + \sum_{j=2}^{i} \dot{q}_j . \]  

Rewriting the above equation in matrix form,

\[ \dot{\mathbf{D}}_i = \sum_{j=1}^{i-1} \begin{bmatrix} T_{j,1} & 0 & P g_{j+1} & T_j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \dot{q}_j + [T_{i-1} 0 0 0] \dot{q}_i . \]  

Using Eq. (3.36) and Eq. (3.47) leads to:

\[ \dot{\mathbf{q}}_i = \sum_{j=1}^{i-1} R^p_j \dot{q}_j + R^A_{i-1} \dot{q}_{i-1} + \mathbf{R}_i \dot{q} , \quad i = 2, \ldots, N ; \]  

\[ \dot{\mathbf{q}}_{N+1} = \sum_{j=1}^{N-1} R^p_j \dot{q}_j + R^A_N \dot{q}_N + \mathbf{R}_{N+1} \dot{q}_{N+1} . \]  

The details of the \( \mathbf{R}_i \), \( R^A_i \), and \( R^p_i \) matrices are given in Appendix V. Collecting Eq. (3.47) for \( i = 1, \ldots, N+1 \) leads to the second velocity transformation as

\[ \dot{\mathbf{q}} = R^w \dot{q} , \]  

where
3.4.3 Two alternate expressions for the kinetic energy

Using the two velocity transformations derived in the previous section, the kinetic energy can be expressed in terms of $\dot{q}$. Therefore, making use of Eq. (3.49), the kinetic energy expression takes the form

$$ T = \frac{1}{2} \dot{q}^T R \nu^T \tilde{M} R \nu \dot{q}. $$

But, using Eq. (3.45), one has

$$ T = \frac{1}{2} \dot{q}^T R^T \left( I^n - R^C \right)^T \tilde{M} \left( I^n - R^C \right)^{-1} R \dot{q}, $$

where the kinetic energy of the system, as well as the mass matrix $M$, are both expressed in terms of the coupled set of generalized coordinates. Note that the inverse of the mass matrix, as defined in Eq. (3.51), has the form

$$ M^{-1} = R^{-1} \left( I^n - R^C \right) \tilde{M}^{-1} \left( I^n - R^C \right)^T R^{-T}. $$

Now the matrices inverted in Eq. (3.52), i.e. $R$ and $\tilde{M}$, are both block diagonal, thus their inversion is an $O(N)$ process. Furthermore, the structure of the remaining matrices in
Eq. (3.51) allows their multiplication to be also of $O(N)$. Thus, inversion of the system mass matrix, in terms of the coupled set of generalised coordinates, is now an $O(N)$ process.

3.5 Potential Energy of the System

Contribution to the potential energy, $U$, arises from two sources: gravitational potential energy ($U_g$); and the strain energy ($U_e$) due to the deformations of the elastic members. The details of the potential energy terms and their derivatives are given in Appendix VI.

3.5.1 Gravitational potential energy

With the inertial $x_0,y_0$-plane oriented vertically and $y_0$ being the vertical axis, the total gravitational potential energy of the system can be written as

$$U_g = -\left\{ \sum_{i=1}^{N+1} g(j_0 \cdot R_{d_{mi}})im_i + \sum_{i=1}^{N} g(j_0 \cdot R_{a_i})m_{a_i} \right\}.$$  \hspace{1cm} (3.53)

The first term is the gravitational potential energy for the units and the payload, and the second term is for the actuators at the revolute joints. As the prototype manipulator system is operating in a horizontal plane, the inclusion of gravitational potential energy is solely for the purpose of verification of the simulation program through conservation of energy of a nondissipative system.

3.5.2 Strain energy

Potential energy is also stored in the form of elastic deformations of the system. This contribution is given by
\[ U_v = \sum_{i=1}^{N} \frac{1}{2} K_{ai} \beta_i^2 + \frac{1}{2} \int_{x_{N+1}}^{x_{N+1}} EI(x_{N+1}) \left[ \frac{\partial^2 \left( \phi_{N+1} \delta_{N+1} \right)}{\partial x_{N+1}^2} \right]^2 dx_{N+1} , \] (3.54)

where the first term represents the contribution from the deformation of the joints; the second term corresponds to the elastic deformation of the payload in the transverse direction; \( K_{ai} \) is the joint stiffness; \( \beta_i = \psi_i - \alpha_i - \psi_{i-1} \); and \( EI \), the structural stiffness of the payload in the transverse direction, is permitted to vary along the length of the payload.

3.6 Energy Dissipation

Dissipation of energy was included in the model. A Rayleigh dissipation function is used [35], and is defined as one-half the instantaneous rate of change of mechanical energy occurring in the system,

\[ R_a = \sum_{i=1}^{N} \frac{1}{2} C_t^i \beta_i^2 + \frac{1}{2} \int_{x_{N+1}}^{x_{N+1}} C_v^i I(x_{N+1}) \left[ \frac{\partial^2 \left( \phi_{N+1} \delta_{N+1} \right)}{\partial x_{N+1}^2} \right]^2 dx_{N+1} \] (3.55)

where the first term corresponds to the viscous dissipation at the joints, while the second term represents the dissipation due to the structural damping in the transverse direction; \( C_t^i \) and \( C_v^i \) are the equivalent viscous damping coefficients for the joint vibration, and the transverse mode of vibration of the payload, respectively; and \( I(x_{N+1}) \) is the moment of inertia of cross-sectional area about the neutral axis at \( x_{N+1} \). The details of the energy dissipation terms and their derivatives are given in Appendix VII.

3.7 Equations of Motion

The equations of motion are obtained using the Lagrangian procedure

66
\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} + \frac{\partial R_d}{\partial q} = \mathbf{Q},
\]  

(3.56)

where \( \mathbf{Q} \) corresponds to the non-conservative generalized forces. They can be written as

\[
\ddot{q} = \mathbf{M}^{-1} \mathbf{Q} - \mathbf{M}^{-1} \begin{pmatrix} \dot{M} \dot{q} - \frac{1}{2} \frac{\partial (\dot{q}^T \dot{M} \dot{q})}{\partial q} + \frac{\partial U_g}{\partial q} + \frac{\partial U_e}{\partial q} + \frac{\partial R_d}{\partial q} \end{pmatrix},
\]

(3.57)

where \( U_g, U_e, \) and \( R_d \) are defined in equations (3.53), (3.54), and (3.55), respectively; \( \mathbf{M}^{-1} \) is given by Eq. (3.52), and \( \mathbf{M} \) is as described in Eq. (3.50). The terms \( \dot{M} \) and \( \frac{\partial (\dot{q}^T M \dot{q})}{\partial q} \) are described in detail in Appendix VIII.

### 3.8 Generalized Forces

The generalized forces \( \mathbf{Q} \) correspond to the torques applied by the revolute joint actuators and the forces exerted by the linear actuators for link deployment. The actuator located at the \( i^{th} \) joint of the manipulator provides the \( i^{th} \) body with a \( T_i \) which results in slewing motion of the unit. Furthermore, each unit is equipped with a linear actuator responsible for its deployment and retrieval. This actuator provides a force \( F_i \) along the length of the \( i^{th} \) unit. Therefore, the set of actuator generalized forces can be written as

\[
\mathbf{u} = [T_1, F_1, T_2, F_2, \ldots, T_N, F_N]^T,
\]

(3.58)

where \( \mathbf{u} \in \mathbb{R}^{n_a} \), with \( n_a = 2N \).

The components of \( \mathbf{Q} \) represent the contributions of all the external forces to the equations of motion corresponding to each generalized coordinate. These contributions can be established through the principle of a virtual work which leads to the equation
where \( Q_i \) and \( q_i \) represent the \( i^{th} \) components of \( Q \) and \( q \), respectively; and \( F_{ej} \) symbolize the \( j^{th} \) external force applied at \( R_j \). Eq. (3.60) is used to derive the relationship between \( Q \) and \( u \),

\[
Q = Q^d u,
\]

where:

\[
Q^d = \begin{bmatrix}
Q^d_1 & 0 & 0 & \cdots & 0 \\
0 & Q^d_2 & 0 & \cdots & 0 \\
0 & 0 & Q^d_3 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & Q^d_{N+1}
\end{bmatrix} \in \mathbb{R}^{n \times m(u)};
\]

with

\[
Q^d_i = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 0 \\
0 & 1
\end{bmatrix} \in \mathbb{R}^{n \times 2}, \text{ for } i = 1, \ldots, N; \text{ and } Q^d_{N+1} = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix} \in \mathbb{R}^{4 \times 2}.
\]

3.9 Specified Coordinates

In this study, the coordinates required to describe the system kinematics are taken to be generalized coordinates in order to make the formulation as general as possible. However, it is often useful to specify some of these generalized coordinates. For instance, in the particular case of the manipulator studied, \( d_i = 0 \) for \( i = 2, \ldots, N + 1 \). In other words, manipulator units from 2 to \( N + 1 \) are attached to the tip of the previous unit in the chain. To
put it differently, connection to the mobile base is confined only to body 1. Furthermore, cases where the length of the units is varied in a specified manner, or where joint rotors are locked in place at a specified angle, would require the use of specified coordinates. These coordinates are prescribed through constraint relations which are introduced in the equations of motion through Lagrange multipliers. Therefore, when constrained, Eq. (3.1) takes the form

\[ M\ddot{q} + F = Q^d u + P^c \Lambda, \]  

(3.63)

where: \( u \in \mathbb{R}^{n_a} \) is a vector containing the \( n_a \) actuator forces and torque; \( Q^d \) is the matrix assigning the components of \( u \) to the actuated variables \( (Q = Q^d u) \); \( \Lambda \in \mathbb{R}^{n_c} \) is the vector containing the \( n_c \) Lagrange multipliers; and \( P^c \) is the matrix assigning the multipliers to the constrained equations. In order to find the values of the Lagrange multipliers and achieve the desired constraints, the above equation can be rewritten in the form

\[ \ddot{q} + F^s - F^u u = F^s \Lambda, \]  

(3.64)

where \( F^s = M^{-1} F \), \( F^u = M^{-1} Q^d \), and \( F^s = M^{-1} P^c \). Separating the specified variables \( (q_s) \) from the generalized ones \( (q_q) \) gives

\[
\begin{bmatrix}
\ddot{q}_s \\
\ddot{q}_s \\
\end{bmatrix} +
\begin{bmatrix}
F^s \\
F^s \\
\end{bmatrix}
-
\begin{bmatrix}
F^u \\
F^u \\
\end{bmatrix} u
=
\begin{bmatrix}
F^s \\
F^s \\
\end{bmatrix} \Lambda.
\]  

(3.65)

From the equation associated with the specified coordinates, the Lagrange multipliers can be determined,

\[ \ddot{q}_s + F^s_s - F^u u = F^s_s \Lambda, \]  

(3.66)

i.e.

\[ \Lambda = F_s^{-1} (\ddot{q}_s + F_s^s - F^u u). \]  

(3.67)
Therefore, the generalised acceleration vector, $\ddot{q}_g$, is calculated from the following equation,

$$\ddot{q}_g = F_g^s A - F_g^g + F_g^u u.$$  \hspace{1cm} (3.68)

It may be pointed out that the equations of motion still retain their $O(N)$ character, even in the presence of constraints, as the Lagrange multipliers can be obtained recursively [29]. Thus, in the case where the $j^{th}$ variable is constrained to be constant at its initial value, $\dot{q}_{s,j} = 0$. In the case of prescribed manoeuvres, $\dot{q}_{s,j}$ is simply defined as the desired acceleration profile. In the present study, a sinusoidal acceleration profile is adopted for prescribed manoeuvres. It assures zero velocity and acceleration at the beginning and end of the manoeuvre, thereby reducing the structural response of the system. The manoeuvre time history considered is as follows,

$$q_{s,j}(\tau) = \frac{\Delta q_{s,j}}{\Delta \tau} \left[ \tau - \frac{\Delta \tau}{2\pi} \sin \left( \frac{2\pi}{\Delta \tau} \tau \right) \right],$$  \hspace{1cm} (3.69)

where $q_{s,j}$ is the constrained coordinate; $\Delta q_{s,j}$ is its desired variation; $\tau$ is the time; and $\Delta \tau$ is the time required for the manoeuvre. The time history of $q_{s,j}$, $\dot{q}_{s,j}$, $\ddot{q}_{s,j}$ are plotted, for the case $\Delta q_{s,j} = 1$ and $\Delta \tau = 1$, in Figure 3-5.
Figure 3-5  Normalized time histories of the sinusoidal maneuvering profile showing displacement, velocity and acceleration.
4. NUMERICAL INTEGRATION CODE

4.1 Preliminary Remarks

The equations governing the planar dynamics of a general $N$-module ground-based manipulator system were derived in the previous chapter. With the equations of motion cast in the form given in Eq. (3.1), the mathematical model is said to describe the inverse dynamics of the system. If the desired values of the system's generalised coordinates as well as their first two time derivatives are specified, the model can be used to estimate the generalised forces required from the control actuators. However, in the present case, forward dynamics of the system is required, and Eq. (3.57) must be used. The acceleration vector $\ddot{q}$ must be integrated twice over time to obtain the time history $q(t)$ of the various degrees of freedom. Knowledge of $q(t)$ then leads to a complete description of the system's motion.

The simulation of the system's dynamics requires Eq. (3.57) to be solved. While the solution of this equation is conceptually simple, the integration of $\ddot{q}$ is not. For simple models, a closed-form solution can be sought using existing linear and nonlinear analytical approaches. However, the full set of coupled nonlinear differential equations requires a numerical approach. The problem is further complicated by the fact that the equations of motion form a stiff set, i.e. the time-scales involved show large differences.

This chapter discusses the C program written for the dynamical simulation of the manipulator system with a flexible payload. The aim was to develop an efficient computer code capable of dealing with a wide range of system parameters. The structure of the computer code is introduced, and the main features of the program are discussed.
4.2 Structure of the Computer Code

As mentioned earlier, the simulation of the system’s dynamics requires Eq. (3.2) to be solved, where \( F \) is defined in Eq. (3.57). Therefore, the main task of the program consists essentially in finding \( M^{-1} \), \( F \), \( Q \), and thus \( \dot{q} \), for each time-step. The system dynamics is then cast in the first order form,

\[
\dot{x} = \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} \dot{q} \\ M^{-1}(Q - F) \end{bmatrix},
\]

where \( x = [q^T \quad \dot{q}^T]^T \). \( \dot{x} \) is integrated at each desired point in time using Gear’s method, which is essentially a backward differentiation formula for stiff equations, employing an implicit linear multiple step method of the predictor-corrector type.

The architecture of the program performing these tasks is shown in Figure 4-1. The program was developed in a highly modular fashion to facilitate modifications. Initially, it asks the user if a flexible payload is present. If the payload is absent, the generalized coordinates associated with it are omitted. Then the program reads various input parameters from user or text files:

- number of modules from file ‘size.txt’;
- links’ masses and lengths, joint stiffness and damping parameters from file ‘mdms.txt’;
- payload mass, length, stiffness and damping coefficients from file ‘mdms.txt’;
- initial conditions for generalised coordinates, i.e. \( x(0) = [q(0)^T \quad \dot{q}(0)^T]^T \), from user;
Figure 4-1  Outline of the simulation program structure.
- relative error tolerance, initial integration step-size, and number of time-steps for the simulation from user;
- starting time, duration, and amplitude of joint slewing, deployment, and mobile base motion from user.

To compute the state for the next time-step, the *IMSL DGEAR* Fortran subroutine is called in as a compiled Fortran object file ‘hdgear.obj’ from the Main Program. It features a version of Gear’s method where the selection of the integration step-size is automatic, being based on the user-specified relative error-bound. In the DGEAR subroutine, the *C* function *FCN*, which calculates the derivative of the current state, is called in. This calculation requires an extensive computer programming based on the model formulation described in the previous chapter. As one may notice, the formulation involves algebraic manipulations of matrices. This is handled by a utility file ‘matrix.c’ which contains functions for performing the creation, multiplication and inverse of matrices. The *FCN* function is called as many times as the number of iterations required by Gear’s method to converge to the solution, the next state, within the specified error-bounds. The solution \( x(t_{i+1}) \) is recorded in an output file and becomes the new current state for the next time-step. This procedure is repeated at subsequent time-steps, until the end of the specified computation period. All the solution states are recorded in output files.

4.3 Computation of \( \dot{x} \) Vector

The *FCN* function constitutes the largest portion of the program, and the detailed flow diagram of the function call is shown in Figure 4-2. The direction of the arrow suggests calling of the function being pointed to by the previous entry. In order to obtain the time
Figure 4-2 Flow diagram of simulation program.
derivatives of the current states, $\mathbf{x}$ in Eq. (4.1), $\dot{q}_g$ is computed from $QgDotDot()$ function.

This acceleration vector is converted to the first order form, $\mathbf{x}$, before it is sent back to the DGEAR subroutine for the iteration procedure. According to Eq. (3.68), this requires the formulation of several matrices associated with the corresponding functions: $Fsg()$ for $F^s_g$; $LM()$ for $A$, $Fug()$ for $F^u_g$, and $Fgg()$ for $F^g_g$. The separation of a matrix into two parts: specified and generalised, as in Eq. (3.65), is achieved by creating two extraction matrices in the functions $ExtractS()$ and $ExtractG()$, respectively, based on the data from the text file “lagrange.txt”. For example, $F^s_g$ is computed by pre-multiplying the generalised extraction matrix by $F^s$. $Fsg()$ calls $Fs()$ that calculates $F^s$, which requires $M^{-1}$ and $P^e$.

The inverse of the system inertia matrix is computed based on Eq. (3.52), and this involves the following functions for the corresponding matrix formulation:

- $RR_I_T()$ for $R^{-T}$, $RR_I()$ for $R^{-1}$ and $R^{-1}$; $R_I$;
- $IRRcT()$ for $(I - R^e)^T$, $IRRc()$ for $I - R^e$, $RRc()$ for $R^e$, and $Rc()$ for $R^e$;
- $MMhat_I()$ for $\tilde{M}^{-1}$, $Mhat()$ for $\tilde{M}^{-1}$, $Mhat()$ for $\tilde{M}_i$.

Based on the information regarding the specified coordinates obtained from the file ‘lagrange.txt’, the multiplier-assigning matrix, $P^e$, can be constructed. For the Lagrange multipliers in Eq. (3.67), the following functions are called within $LM()$:

- $Fus()$ for $F^u$;
- $u()$ for $u$;
- $QsDotDot()$ for $\dot{q}_s$;
\* \* Fgs() for \( F_s^g \); \\
\* Fss_I() for \( F_s^{s^{-1}} \).

Similarly, \( Fu() \) function calculates \( F^u \) that requires \( M^{-1} \) and \( Q^d \), which is defined in Eq. (3.62), and \( Fg() \) computes \( F^g \) that involves \( M^{-1} \) and \( F \) vector. The \( F \) vector in Eq. (3.57) is calculated by calling a number of functions, as shown in Figure 4-2:

\* \( MMDot() \) for \( \dot{M} \), \\
\* \( RvDotT() \) for \( \dot{R}^yT \), \( MMhat() \) for \( \tilde{M}^T \), \( Rv() \) for \( \dot{R}^r \), \( RvT() \) for \( \dot{R}^yT \), \\
\* \( MMhatDot() \) for \( \dot{\tilde{M}} \), \( MhatDot() \) for \( \dot{M} \), \( RvDot() \) for \( \dot{R}^r \);

\* \( qDot() \) for \( \dot{q} \);

\* \( dMdq() \) for \( \frac{\partial M}{\partial q} \), \\
\* \( \Rightarrow MhatPsi() \) for \( \frac{\partial \tilde{M}}{\partial \phi} \), \( RvPsi() \) for \( \frac{\partial R^y}{\partial \phi} \), \( qDotT() \) for \( \dot{q}^T \), \( Mhatl() \) for \( \frac{\partial \tilde{M}}{\partial l} \), \( Rvl() \) for \( \frac{\partial R^y}{\partial l} \), \( Rvdx() \) for \( \frac{\partial R^y}{\partial d_x} \), \( Rvdy() \) for \( \frac{\partial R^y}{\partial d_y} \), \( MhatDelta() \) for \( \frac{\partial \tilde{M}}{\partial \delta} \).

The computations leading to the acceleration vector \( \ddot{q} \) require a significant number of matrix products and additions. Several of the matrices involved have a number of constituent block submatrices as zero. Efficient subroutines were designed specifically for each matrix product. An effort was made not to multiply the zero elements, thereby considerably reducing the number of computations.
Even with the formulation for planar dynamics and computationally efficient $O(N)$ approach, the numerical code was rather lengthy. It involved around 4,000 lines ($\approx 50$ pages). For brevity, the code is not included in the thesis, however, it is available by contacting the author. A comment concerning checks on the validity of the computer code would be appropriate. The size of the governing equations of motion, in addition to the number of operations required to derive them, can easily lead to formulation and programming errors. To some extent, these errors can be avoided through careful and systematic derivation and programming. Furthermore, errors can reveal themselves when the computer code is compiled and its constituting parts are linked, thereby resulting in compiling and linking errors. However, some errors may be quite elusive and require precise checks. One convenient verification procedure is to check the conservation of energy in absence of dissipation. Similarly, the conservation of angular momentum can also be verified. Another alternative is to match simulation results for particular cases studied by other researchers. Assessment of the computer code is in progress.
5. OPERATION OF THE MDMS PROTOTYPE

5.1 Preliminary Remarks

In Chapter 2, the design and construction of the MDMS prototype as well as the conceptual computer control system integration were described. This chapter discusses the integration of the control system components in detail and presents some representative experimental results for the controlled motion of the manipulator. The Servo To Go S8 Servo I/O board, used to interface the manipulator’s sensors and actuators to the host computer, has library functions for setting PID gains and performing the servo control. One can also choose among three types of motion trajectory generation: position only, position + velocity, or position + velocity + acceleration interpolation (a trapezoidal velocity profile) through the library functions. It also has the flexibility of letting the user to implement desired control strategies and select different motion profiles. In this case, for the MDMS, only a few library functions are needed to set the sampling rate, acquire encoder signals, and send out command signal to the amplifier through DAC channels. A computer program for the PID controller was written in C language. It serves as manager for data exchange and coordinator of the above functions (Figure 2-10). A copy of the source code is included in Appendix IX.

5.2 Control System Integration

Besides the design of the manipulator, the integration of the control system components requires certain amount of research work in finding appropriate terminals and connectors for the communications among pieces of equipment. The following subsections discuss several aspects of this issue.
5.2.1 Optical encoders

With the I/O board in the host computer sitting outside the workspace table and 2.1m (7 ft) away from the manipulator base, the 50cm loose wires from the encoders need to be extended. A Molex Micro-Fit 3.0™ Wire-to-Wire Receptacle, with female terminals, is installed at the end of each set of wires. The receptacle, which mates with a plug from the same supplier, provides positive latching to the plug, fully isolates contacts, and has an integral pull tabs for ease in unmating. The plug with male terminals is then installed on a wire with 5 or 8 connections depending upon the type of encoder. The other end of this wire, of suitable length, is connected to one of the quadrature encoder channels where the signal is acquired. With Molex receptacles and plugs, the manipulator actuators can be easily detached from the computer control system for maintenance and replacement of encoders or actuators. The connections at the I/O board side is discussed in a later subsection.

5.2.2 Power supplies and amplifiers

Two power supplies from Advanced Motion Controls, one for 2 axes and another for 6 axes, are used to drive the amplifiers and actuators. Each amplifier is fixed onto the power supply base with two screws. For the smaller power supply, two pairs of wires are available to connect to the two amplifiers. However, for the 6-axis power supply, only one pair of power lead is available, and the supply of power to the amplifiers is through a bank of screw terminals. These terminals are divided into two groups: power and ground. In order to distribute the power to six amplifiers, pieces of sheet metal acting like bridges
are installed between the adjacent terminals in each group. The amplifiers are connected
to the terminals with 20AWG wires and ring-type solderless connectors.

5.2.3 Servo To Go servo I/O board

The servo I/O board, installed in one of the ISA slots in the host computer, has
five ribbon cable connectors:

- P1: 24 bits of Opto-22 compatible digital I/O;
- P2: 8 channels of analog input, 8 bits of user I/O, and 8 motor direction bits;
- P3: encoder input and analog output for axis 1-4;
- P4: encoder input and analog output for axis 5-8;
- P5: battery backup input.

Basically, only P3 and P4 connectors are used for this project to acquire encoder
information and send out voltage command signals. Two 50-pin ribbon cables are
required to create connections to encoders and amplifiers. However, neither of these two
connectors has a required 5V power supply for the encoders. Since there is a 5V power
supply at P1 (and P2), another 50-pin ribbon cable is installed at P1. With a socket board
(also known as bread board), this power is supplied to the encoders. In addition, two
Phoenix Contact VARIOFACE 50-position interface modules are used between the
ribbon cables and the wires from the encoders and the amplifiers.

5.3 Operation of the MDMS with PID Control

The operation of the MDMS was carried out with a Proportional-Integral-
Derivative (PID) controller. PID controllers are used in virtually all types of systems
because of their simplicity, robustness and ease of implementation. A variety of maneuvers through independent joint command were performed to assess the ability of the controller to follow a prescribed path. Sine-on-ramp position profile was employed as the reference joint trajectory for both slew and deployment maneuvers.

For example, the revolute joint of the first module was tested with a command of 0 to 45° in 5 seconds. All the modules were aligned with each other to form a straight line initially. Due to the high gear ratio of the harmonic drive actuators, the other three revolute joints remained fixed. Figure 5-1 shows the joint response with reference to the desired joint trajectory. The controller gains were chosen arbitrarily, i.e., no tuning was carried out in this test.

For the deployment, the prismatic joint of the first module was tested with a command of 0 to 0.05m in 5 seconds. Due to the presence of static friction, the response of the first few tests showed some delays. To compensate for friction, a constant value was included in the control command signal. Figure 5-2 shows the improved joint response with reference to the desired trajectory. Note the presence of limit cycle type oscillations close to the end of the maneuver. It seems to resemble chattering and may be attributed to friction induced relaxation type of oscillations. This suggests a need for tuning of the controller. In general, similar performance results were obtained for other modules suggesting working condition of the designed manipulator. In summary, the prototype MDMS promises to fulfill its role in testing performance of different control strategies during execution of maneuvers and tracking of trajectories.
Figure 5-1  Comparison between desired and actual joint trajectories during a 45° slew maneuver at module 1.
Figure 5-2  Comparison between desired and actual joint trajectories during a 0.05m deployment maneuver at module 1.
6. CONCLUDING REMARKS

6.1 Contributions

The main contributions of this project can be summarised as follows:

- A novel Multi-module Deployable Manipulator System (MDMS) has been designed, integrated and is operational. Detailed design and operation of such a manipulator have not been reported in open literature.

- A relatively general mathematical model is developed to simulate the two-dimensional dynamics of the ground-based MDMS. The model accounts for flexibility of the payload and interconnecting revolute joints between modules. The non-recursive $O(N)$ Lagrangian formulation promises to be computationally efficient permitting real-time control of the system.

- A computer code under development will be able to simulate controlled dynamics of a manipulator consisting of an arbitrary number of modules. Such versatile and efficient numerical code for a novel manipulator has not been reported. It will help assess relative merit of various control procedures.

- Application of the classical PID controller to the prototype system represents an innovative and fresh beginning in controlling this four-module robotic manipulator.
6.2 Conclusions Based on Design

Based on the design and integration of the MDMS, the following general conclusions can be drawn:

- For the revolute joint of the MDMS, harmonic drive gearing actuator is the best option compared to regular DC servomotor with a gear head and direct drive servomotor, in terms of the size, mass, torque output, and cost.
- For the prismatic joint, ball-screw linear actuator is better than linear servomotor because the latter is quite bulky, heavy and supplies less amount of force for the same size and mass as the linear actuator.
- The length of each module is usually 2.5 to 5cm longer than the required deployment of the link to accommodate other components, such as shaft coupling and bearing. This limits the reduction of the mass as well as the inertia of the module.
- Installed using unique connectors and no fasteners, the link tube system is one of the most convenient and economical ways of building a test-bench of the size equal to the workspace for the MDMS.
- At a competitive cost, the ISA bus servo I/O board from Servo To Go Inc. provides good functionality, programming flexibility, and ease of use for the control of the MDMS.
- The prototype manipulator appears quite promising in meeting its objectives of controlled maneuvers and trajectory tracking.
6.3 Recommendations for Future Work

Based on the validation results and limited operation of the prototype manipulator, several recommendations for future work can be suggested:

- Add a thrust bearing at each of the rolling supports to permit its swiveling motion and reduce the resistance while the manipulator is in motion.
- Investigate and implement improved rolling or sliding support for the prototype manipulator.
- Perform dynamical studies using the simulation program for the $O(N)$ model to assess validity of the formulation and the computer code. Conservation of energy for nondissipative system and comparison with reported results can be used to advantage to that end.
- Simulate controlled behaviour with the $O(N)$ model and compare with the experiments performed on the prototype manipulator.
- Extend the present model to account for the out-of-plane motion, i.e. develop a full three-dimensional model.
- Carry out path planning and inverse kinematics with emphasis on obstacle avoidance; investigate the effect of redundancy on system performance; complete a given task with one or more joints not operational.
- Implement different control strategies, such as optimal, adaptive and intelligent control on the prototype manipulator to regulate the rigid and flexible dynamics of the system.

- Design and create a computer program for the animation of simulation results for visual appreciation of the physics of the problem.
LIST OF REFERENCES


APPENDIX I: COMPONENTS OF THE MDMS PROTOTYPE

1. Model S8 8-Axis ISA Bus Servo I/O Card

- Manufacturer/Supplier: Servo To Go, Inc., 1716 221st Place NE, Redmond, WA 98053, USA.

- Specifications:
  - Up to 8 channels of encoder input (A, B, and Index signal inputs) with 24 bit counters;
  - Single-ended or differential (RS422 compatible) encoder input signals;
  - Up to 8 channels of analog output with +10 Volt to -10 Volt span and 13 bit resolution;
  - Sign bit digital output for each analog output channel;
  - 32 bit digital I/O, configurable in various input and output combinations;
  - 8 channels of analog input with 13 bit resolution, configurable as +/- 10V or +/- 5V spans;
  - Interval timers capable of interrupting the PC;
  - Timer interval is programmable to 10 minutes in 25 microsecond increments.
  - Battery backup input used to maintain encoder-counting capability in event of a power failure so that re-initialization is not required when power is removed.
  - Board address detection with IRQ software selectable used to determine the board base address automatically without user interaction;
  - IRQ number is software selectable - no board jumper required;
  - Watchdog timer can be used to initiate servo shutdown in the case of a catastrophic computer malfunction.
2. **Model 12A8 Brush Type PWM Servo Amplifier**

- Manufacturer: Advanced Motion Controls, 3629 Vista Mercado, Camarillo CA 93012, USA.
- Supplier: Electromate Industrial Sales Limited, 4300 Steeles Avenue West, Unit #39, Woodbridge, Ontario L4L 4C2.

- Specifications:
  - DC supply voltage = 20 to 80 V;
  - Peak current = ± 12 A;
  - Maximum continuous current = ± 6 A;
  - Minimum load inductance = 200 μH;
  - Switching frequency = 36 kHz;
  - Heatsink (base) temperature range = -25° to +65° C;
  - Power dissipation at continuous current = 10 W;
  - Over-voltage shut-down = 86 V;
  - Bandwidth = 2.5 kHz;
  - Screw terminal power connector;
  - Molex connector signal connector;
  - Size = 75.8 x 129.3 x 25.1 mm (2.98 x 5.09 x 0.99 inches);
  - Weight = 0.28 kg (10 oz.).

3. **Model PS16L30 Power Supply**

- Manufacturer: Advanced Motion Controls
- Supplier: Electromate Industrial Sales Limited

- Specifications:
• Unregulated DC power sources;
• 120 VAC primary winding;
• 30 V output transformer;
• 53 A nominal current rating;
• Peak current for 2 seconds at 2 times nominal rating with a 20% duty cycle;
• Size = 13.00" x 10.50" x 6.00"
• Can host up to six 25A series amplifiers;
• Weight = 25 lb.

4. **Model PS2X300W-24V Power Supply**

- Manufacturer: Advanced Motion Controls
- Supplier: Electromate Industrial Sales Limited
- Specifications:
  - Unregulated DC power sources
  - 120 VAC 50/60 Hz primary winding;
  - 24 VDC output transformer;
  - 12 A nominal current rating;
  - Peak current for 2 seconds at 2 times nominal rating with a 20% duty cycle;
  - Size = 9.00" x 5.75" x 3.47"
  - Can host two 25A Series servo amplifiers;
  - Weight = 9 lb.

5. **Model RFS-20-3007-E050DO Harmonic Drive Gearing Actuator**

- Supplier: Electromate Industrial Sales Limited
Specifications:

- Max. continuous stall torque = 14 Nm (122 in-lb);
- Maximum output speed = 40 rpm;
- Voltage = 75 V;
- Torque constant = 21.0 Nm/A (182 in-lb/A);
- Back EMF constant = 2.15 V/rpm;
- Armature resistance = 3.4 Ω;
- Armature inductance = 2.7 mH;
- No-load running current = 0.8 A;
- Peak current = 4.8 A.

6. Model RH-14C-3002-E050DO Harmonic Drive Gearing Actuator

- Supplier: Electromate Industrial Sales Limited

Specifications:

- Max. continuous stall torque = 7.8 Nm (69 in-lb);
- Maximum output speed = 50 rpm;
- Voltage = 24 V;
- Torque constant = 5.76 Nm/A (51 in-lb/A);
- Back EMF constant = 0.6 V/rpm;
- Armature resistance = 2.7 Ω;
- Armature inductance = 1.1 mH;
- No-load running current = 0.91 A;
- Peak current = 4.1 A.
7. Model RH-11C-3001-E050DO Harmonic Drive Gearing Actuator

- Manufacturer: HD Systems, Inc.
- Supplier: Electromate Industrial Sales Limited
- Specifications:
  - Max. continuous stall torque = 4.4 Nm (39 in-lb);
  - Maximum output speed = 50 rpm;
  - Voltage = 24 V;
  - Torque constant = 4.91 Nm/A (43 in-lb/A);
  - Back EMF constant = 0.5 V/rpm;
  - Armature resistance = 4.7 Ω;
  - Armature inductance = 1.6 mH;
  - No-load running current = 0.55 A;
  - Peak current = 2.1 A.

8. Model RH-8C-3006-E050DO Harmonic Drive Gearing Actuator

- Manufacturer: HD Systems, Inc.
- Supplier: Electromate Industrial Sales Limited
- Specifications:
  - Max. continuous stall torque = 14 Nm (122 in-lb);
  - Maximum output speed = 40 rpm;
  - Voltage = 75 V;
  - Torque constant = 21.0 Nm/A (182 in-lb/A);
  - Back EMF constant = 2.15 V/rpm;
  - Armature resistance = 3.4 Ω;
• Armature inductance = 2.7 mH;
• No-load running current = 0.8 A;
• Peak current = 4.8 A.

9. **Model #14203 LO-COG® Brush-Commutated Motor**

- Manufacturer: Pittman, 343 Godshall Drive, Harleysville, PA 19438 USA.
- Supplier: Christenson-Bellows Valvair Ltd., 110A-81 Golden Drive, Coquitlam, BC V3K 6R2.
- Specifications:
  - Peak torque (stall) = 159 oz-in;
  - No-load speed = 3456 rpm;
  - Voltage = 24.0 V;
  - Torque constant = 9.26 oz-in/A;
  - Back EMF constant = 6.85 V/krpm;
  - Resistance = 1.38 Ω;
  - Inductance = 2.26 mH;
  - No-load current = 0.24 A;
  - Peak current (stall) = 17.4 A;
  - Weight = 31.2 oz.

10. **Model #14201 LO-COG® Brush-Commutated Motor**

- Manufacturer: Pittman
- Supplier: Christenson-Bellows Valvair Ltd.
- Specifications:
  - Peak torque (stall) = 62.8 oz-in;
- No-load speed = 4230 rpm;
- Voltage = 24.0 V;
- Torque constant = 7.44 oz-in/A;
- Back EMF constant = 5.50 V/krpm;
- Resistance = 2.79 Ω;
- Inductance = 2.54 mH;
- No-load current = 0.26 A;
- Peak current (stall) = 8.6 A;
- Weight = 20.8 oz.

11. PP1-B8(ZB)-D1-ZZ-06-MS(2) Pulse Power I Linear Actuator

- Manufacturer: Dynact, Incorporated, 11 Centre Drive, Orchard Park, NY 14127.
- Supplier: Christenson-Bellows Valvair Ltd.
- Specifications:
  - 8 pitch ball screw (90% efficiency) with 0.0005 backlash nut;
  - 1:1 direct drive;
  - Motor-frame for non-standard motor;
  - 6" stroke;
  - Magnetic reed switches (2);
  - 47 lbs thrust with Motor #14203; 32 lbs thrust with Motor #14201;
  - Weight = 2.5 lbs.
APPENDIX II: LIST OF HARDWARE COMPONENTS MANUFACTURERS

1. Advanced Motion Controls
   Product supplied: amplifiers and power supplies
   Location: 3629 Vista Mercado, Camarillo, CA 93012
   Ph.: 805-389-1935
   Fax: 805-389-1165
   Web site: http://www.a-m-c.com

2. Bayside Motion Group
   Product supplied: gear heads
   Location: 27 Seaview Boulevard, Port Washington, NY 11050 USA
   Ph.: 1-516-484 5353
   Fax: 1-516-484 5496
   Web site: http://www.bmgnet.com

3. B C Conveying Machinery Ltd
   Product supplied: Mathews ball transfers
   Location: 69 W69th Ave., Vancouver, BC
   Ph.: 604-321-2331
   Fax: 604-321-2320

   Product supplied: amplifiers and power supplies
   Location: 829A Middlesex Turnpike, Billerica, MA 1821 3954 USA
   Ph.: 216-524-8800
   Fax: 978-663-6662

5. Dynact Incorporated
   Product supplied: Pulse Power I linear actuator
   Location: 11 Centre Drive, P.O. Box 466, Orchard Park, NY 14127
   Ph.: 716-667-7079
   Fax: 716-622-1966
   Web site: http://www.dyhact.com

6. HD Systems, Inc.
   Product supplied: harmonic drive gearing actuators
   Location: 89 Cabot Court, Hauppauge, NY 11788
   Ph.: 800-231-HDSI, 516-231-6630
   Fax: 516-231-6803
   Web site: http://www.HDSystemsInc.com
7. **Parker Automation Group**
   Product supplied: direct drive motor
   Location: 6035 Parkland Blvd., Cleveland, Ohio 44124
   Ph.: 216-896-3000
   Fax: 216-896-4000
   Web site: http://www.parker.com/automationgroup

8. **Pittman**
   Product supplied: servomotors
   Location: 343 Godshall Drive, Harleysville, PA 19438
   Ph.: 215-256-6601
   Fax: 215-256-1338
   Web site: http://www.pittmannet.com

9. **Precision MicroDynamics, Inc.**
   Product supplied: MFIO-3A motion control interface cards
   Location: #3-512 Frances Ave., Victoria, B.C., V8Z 1A1 Canada
   Ph.: 250-382-7249
   Fax: 250-382-1830
   Web site: http://www.pmdi.com

10. **Servo To Go, Inc.**
    Product supplied: S8 8-axis ISA bus servo I/O card
    Location: 1716 221st Place NE, Redmond, WA 98053
    Ph.: 425-868-4636
    Fax: 425-868-5323
    Web site: http://www.servotogo.com

11. **SKF Canada Ltd.**
    Product supplied: rolling bearings
    Location: 3665 Wayburne Drive, Burnaby, B.C., V5G 3L3
    Ph.: 604-291-9921
    Fax: 604-291-2965
    Web site: http://www.skf.ca
APPENDIX III: CAD DRAWINGS OF THE MDMS PROTOTYPE

(Note: Drawings are scaled down to fit within the specified margins)
MULTI-MODULE DEPLOYABLE MANIPULATOR SYSTEM (MDMS)

TITLE: SI MOTOR MOUNT   ITEM NO.: 2   DWN BY: K. WONG
DATE: 13 JULY 1999   SCALE: 1 : 1.5   DRAWING NO.: MDMS-01-01

IRIS SPACE ROBOTICS LABORATORY
UNIVERSITY OF BRITISH COLUMBIA
Vancouver, B.C.

Ø 0.196 THRU HOLE (6) FOR NO. 10 SCREWS
Ø 0.363 THRU HOLE (4) FOR M8 HEX HD BOLTS
Ø 1-24 UNC TYP., 4 HOLES

MAT'L: 1/4" ALUMINUM
QTY: 1
MULTI-MODULE DEPLOYABLE MANIPULATOR SYSTEM (MDMS)

TITLE: D1 ACTUATOR MOUNT
ITEM NO.: 3
DWN BY: K. WONG
DATE: 13 JULY 1999
SCALE: 1 : 2
DRAWING NO.: MDMS-01-02

IRIS SPACE ROBOTICS LABORATORY
UNIVERSITY OF BRITISH COLUMBIA
Vancouver, B.C.
HOLE FOR 3/0 TAPER PIN

Ø 0.250" SLOT CENTERED AT 0.700" DIA., 10" LONG

MAT'L: 2" DIA. ALUMINUM
QTY: 3

IRIS SPACE ROBOTICS LABORATORY
UNIVERSITY OF BRITISH COLUMBIA
Vancouver, B.C.

MULTI-MODULE DEPLOYABLE MANIPULATOR SYSTEM (MDMS)

TITLE: MODULE CONNECTOR
ITEM NO.: 7
DWN BY: K. WONG
DATE: 27 APRIL 1999
SCALE: 1:1
DRAWING NO.: MDMS-02-01
\( \phi 0.266 \text{ TYP., 6 HOLES FOR 1/4" BOLTS} \)

\( \phi 1.375 \)

\( \phi 0.75 \)

\( B10-24 (4) \)

\( 1.678 \)

\( 2.75 \)

MAT'LT: C4X1-5/8X3/16 ALUMINUM
QTY: 3

IRIS SPACE ROBOTICS LABORATORY
UNIVERSITY OF BRITISH COLUMBIA
Vancouver, B.C.

MULTI-MODULE DEPLOYABLE MANIPULATOR SYSTEM (MDMS)

TITLE: JOINT BRACKET
ITEM NO.: 5
DWN BY: K. WONG
DATE: 16 JULY 1999
SCALE: 1 : 1.5
DRAWING NO.: MDMS-02-02
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**ESTIMATED TOTAL: 2.81**
APPENDIX IV: DECOUPLED MASS MATRIX

The decoupled mass matrix of the system was derived in Section 3.2. It can be expressed as

\[
\tilde{M} = \begin{bmatrix}
0 & 0 & \cdots & 0 \\
0 & \tilde{M}_2 & \cdots & 0 \\
0 & 0 & \tilde{M}_3 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & \tilde{M}_{N+1}
\end{bmatrix}
\] (IV.1)

Here, \( \tilde{M}_i, i = 1, \ldots, N \), represents the mass matrix of the units while \( \tilde{M}_{N+1} \) corresponds to the mass matrix of the payload. Therefore,

\[
\tilde{M}_i = \left[ \begin{array}{c}
1 \\
0 \\
0 \\
\vdots \\
0
\end{array} \right] \begin{bmatrix}
0 & 0 & \cdots & 0 \\
0 & \tilde{M}_i & \cdots & 0 \\
0 & 0 & \tilde{M}_i & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & \tilde{M}_i
\end{bmatrix}
\begin{bmatrix}
1 \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix} + \begin{bmatrix}
m_{ai} & 0 & 0 & 0 & 0 \\
0 & m_{ai} & 0 & 0 & 0 \\
0 & 0 & J_{ai} & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

With \( dm_i = \rho_s dx_i \) for the \( i^{th} \) slew link section and \( dm_i = \rho_d dx_i \) for the \( i^{th} \) deployable link section, the integral is divided into two parts,

\[
\tilde{M}_i = \left[ \begin{array}{c}
m_{ai} \\
0 \\
0 \\
0
\end{array} \right] \begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix} + \rho_s \int_0^{l_{si}} \begin{bmatrix}
1 & 0 & 0 & -x_i \sin \psi_i & 0 \\
0 & 1 & 0 & x_i \cos \psi_i & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
x_i \\
y_i \\
z_i \\
\psi_i \\
\phi_i
\end{bmatrix} dx_i
\]
\[
\eta = \int_{l_i}^l \left[
\begin{array}{cccc}
1 & 0 & 0 & -x_i \sin \psi_i & \cos \psi_i \\
0 & 1 & 0 & x_i \cos \psi_i & \sin \psi_i \\
0 & 0 & 0 & 0 & 0 \\
-x_i \sin \psi_i & x_i \cos \psi_i & 0 & x_i^2 & 0 \\
\cos \psi_i & \sin \psi_i & 0 & 0 & 1
\end{array}
\right] dx_i,
\]

where \( \rho_{si} \) and \( \rho_{di} \) are the masses per unit length for the slewing link and deploying link, respectively, and \( l_{si} \) and \( l_{di} \) are the lengths of the slewing link and deploying link, respectively.

The computed decoupled mass matrix of the manipulator unit becomes

\[
\tilde{M}_i = \begin{bmatrix}
M_{11} & 0 & 0 & M_{14} & \rho_{di} l_{di} \cos \psi_i \\
0 & M_{22} & 0 & M_{24} & \rho_{di} l_{di} \sin \psi_i \\
0 & 0 & J_{ai} & 0 & 0 \\
M_{41} & M_{42} & 0 & M_{44} & 0 \\
\rho_{di} l_{di} \cos \psi_i & \rho_{di} l_{di} \sin \psi_i & 0 & 0 & \rho_{di} l_{di}
\end{bmatrix},
\]

where:

\[
M_{11} = M_{22} = \rho_{si} l_{si} + \rho_{di} l_{di} + m_{ai};
\]

\[
M_{14} = M_{41} = -\sin \psi_i \left( \frac{\rho_{si} l_{si}^2}{2} + \rho_{di} l_{di} - \frac{\rho_{di} l_{di}^2}{2} \right);
\]

\[
M_{24} = M_{42} = \cos \psi_i \left( \frac{\rho_{si} l_{si}^2}{2} + \rho_{di} l_{di} - \frac{\rho_{di} l_{di}^2}{2} \right);
\]

\[
M_{44} = \frac{\rho_{si} l_{si}^3}{3} + \rho_{di} \left( l_{di}^2 l_{di} - l_{di} l_{di}^2 + \frac{l_{di}^3}{3} \right);
\]

For the payload:

\[
\tilde{M}_{N+1} = \int_{m_{N+1}} \left[ \begin{bmatrix} I^2 \\
\nu_{il}^T \\
\nu_{il}^T \\
\nu_{il}^T \\
\nu_{il}^T \end{bmatrix} \right] dm_{N+1};
\]
\[ \nu_{i1} = P T_{N+1} \left( r_{N+1} + \begin{bmatrix} 0 \\ \phi_{N+1} \delta_{N+1} \end{bmatrix} \right); \quad (IV.7, a) \]

\[ \nu_{i2} = T_{N+1} \begin{bmatrix} 0 \\ \phi_{N+1} \end{bmatrix}. \quad (IV.7, b) \]

With \( dm_{N+1} \) equal to \( \rho_{N+1} dx_{N+1} \), where \( \rho_{N+1} \) is mass per unit length for the payload, the payload inertia matrix is computed to be

\[
\tilde{M}_{N+1} = \begin{bmatrix}
\rho_{N+1} l_{N+1} & 0 & \tilde{M}_{N+1(1,3)} & \tilde{M}_{N+1(1,4)} \\
0 & \rho_{N+1} l_{N+1} & \tilde{M}_{N+1(2,3)} & \tilde{M}_{N+1(2,4)} \\
\tilde{M}_{N+1(3,1)} & \tilde{M}_{N+1(3,2)} & \tilde{M}_{N+1(3,3)} & 0.569 \rho_{N+1} l_{N+1}^2 \\
\tilde{M}_{N+1(4,1)} & \tilde{M}_{N+1(4,2)} & 0.569 \rho_{N+1} l_{N+1}^2 & 1.713 \rho_{N+1} l_{N+1}^2 \\
\end{bmatrix}, \quad (IV.8)
\]

where:

\[
\tilde{M}_{N+1(1,3)} = \tilde{M}_{N+1(3,1)} = -\rho_{N+1} \left( \frac{\sin \psi_{N+1} l_{N+1}^2}{2} + \frac{\cos \psi_{N+1} l_{N+1} \delta_{N+1}}{1.277} \right); \quad (IV.9, a)
\]

\[
\tilde{M}_{N+1(2,3)} = \tilde{M}_{N+1(3,2)} = \rho_{N+1} \left( \frac{\cos \psi_{N+1} l_{N+1}^2}{2} - \frac{\sin \psi_{N+1} l_{N+1} \delta_{N+1}}{1.277} \right); \quad (IV.9, b)
\]

\[
\tilde{M}_{N+1(3,3)} = \rho_{N+1} \left( \frac{l_{N+1}^3}{3} + 1.713 l_{N+1} \delta_{N+1}^2 \right); \quad (IV.9, c)
\]

\[
\tilde{M}_{N+1(1,4)} = \tilde{M}_{N+1(4,1)} = -\rho_{N+1} \sin \psi_{N+1} l_{N+1}; \quad (IV.9, d)
\]

\[
\tilde{M}_{N+1(2,4)} = \tilde{M}_{N+1(4,2)} = \frac{\rho_{N+1} \cos \psi_{N+1} l_{N+1}}{1.277}. \quad (IV.9, e)
\]
APPENDIX V: VELOCITY TRANSFORMATION MATRICES

The order-\( N \) or \( O(N) \) algorithm used in the thesis relies on two velocity transformations, which factorize the system mass matrix. The details of the transformation matrices involved are given here.

The first velocity transformation can be expressed as

\[
\tilde{q} = (I^n - R^c)^{-1} R \dot{q},
\]  

where:

\[
R = \begin{bmatrix}
R_1 & 0 & \cdots & 0 & 0 \\
0 & R_2 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & R_N & 0 \\
0 & 0 & \cdots & 0 & R_{N+1}
\end{bmatrix} \in \mathbb{R}^{n\times n};
\]  

and

\[
R^c = \begin{bmatrix}
0 & 0 & 0 & 0 & \cdots & 0 \\
R_1^c & 0 & 0 & \cdots & 0 \\
0 & R_2^c & 0 & \cdots & 0 \\
0 & 0 & R_3^c & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 0
\end{bmatrix} \in \mathbb{R}^{n\times n}.
\]  

Here:

\[
R_i = I^n; \]

\[
R_i = \begin{bmatrix}
T_{i-1} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\cos \psi_{i-1} & -\sin \psi_{i-1} & 0 & 0 & 0 \\
\sin \psi_{i-1} & \cos \psi_{i-1} & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}, \quad i = 2, \ldots, N; \]
and

\[
R_{N+1} = \begin{bmatrix}
T_N & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix} = \begin{bmatrix}
\cos \psi_N & -\sin \psi_N & 0 & 0 \\
\sin \psi_N & \cos \psi_N & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix} ;
\] (V.6)

\[
R_i^c = \begin{bmatrix}
I^2 & 0 & P_i & T_i \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & -(l_i + d_{(i+1)x})\sin \psi_i - d_{(i+1)y} \cos \psi_i & \cos \psi_i \\
0 & 1 & (l_i + d_{(i+1)x})\cos \psi_i - d_{(i+1)y} \sin \psi_i & \sin \psi_i \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix},
\]
for \( i = 1, \ldots, N-1 \); (V.7)

\[
R_N^c = \begin{bmatrix}
I^2 & 0 & P_{N+1} & T_N \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & -(l_N + d_{(N+1)x})\sin \psi_N - d_{(N+1)y} \cos \psi_N & \cos \psi_N \\
0 & 1 & (l_N + d_{(N+1)x})\cos \psi_N - d_{(N+1)y} \sin \psi_N & \sin \psi_N \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix} ;
\] (V.8)

The second velocity transformation is defined as

\[
\tilde{q} = R^v q ,
\] (V.9)

where:

\[
R^v = \begin{bmatrix}
\bar{R}_1 & 0 & 0 & \ldots & 0 & 0 & 0 \\
R_1^a & \bar{R}_2 & 0 & \ldots & 0 & 0 & 0 \\
R_1^b & R_2^b & \bar{R}_3 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
R_1^b & R_2^b & R_3^b & \ldots & \bar{R}_{N-1} & 0 & 0 \\
R_1^p & R_2^p & R_3^p & \ldots & R_{N-1}^a & \bar{R}_N & 0 \\
R_1^p & R_2^p & R_3^p & \ldots & R_{N-1}^p & R_N & \bar{R}_{N+1} \\
\end{bmatrix} \in \mathfrak{g}^{n \times n} ;
\] (V.10)
$$\bar{R}_i = I^x; \quad \text{(V.11)}$$

\[
R_j^s = \begin{bmatrix}
T_{j,1} & 0 & P_{g_{j+1}} & T_j & [1] \\
0 & 0 & 0 & 0 & [0]
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\cos \psi_{j-1} & -\sin \psi_{j-1} & 0 & -(l_j + d_{(j+1)x})\sin \psi_j - d_{(j+1)y} \cos \psi_j & \cos \psi_j \\
\sin \psi_{j-1} & \cos \psi_{j-1} & 0 & (l_j + d_{(j+1)x})\cos \psi_j - d_{(j+1)y} \sin \psi_j & \sin \psi_j \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

for \( j = 1, \ldots, N - 2; \quad \text{(V.12)}\)

\[
R_i^A = \begin{bmatrix}
T_{i,1} & 0 & P_{g_{i+1}} & T_i & [1] \\
0 & 0 & 1 & 0 & [0]
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\cos \psi_{i-1} & -\sin \psi_{i-1} & 0 & -(l_i + d_{(i+1)x})\sin \psi_i - d_{(i+1)y} \cos \psi_i & \cos \psi_i \\
\sin \psi_{i-1} & \cos \psi_{i-1} & 0 & (l_i + d_{(i+1)x})\cos \psi_i - d_{(i+1)y} \sin \psi_i & \sin \psi_i \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

for \( i = 1, \ldots, N - 1; \quad \text{(V.13)}\)

\[
\bar{R}_i = \begin{bmatrix}
T_{i,1} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos \psi_{i-1} & -\sin \psi_{i-1} & 0 & 0 \\
\sin \psi_{i-1} & \cos \psi_{i-1} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}, \quad i = 2, \ldots, N; \quad \text{(V.14)}
\]
\[
R_j^p = \begin{bmatrix}
T_j \quad 0 \\
0 \quad P_{j+1} \quad T_j \\
0 \quad 0 \\
0 \\
0 \quad 0
\end{bmatrix}
\]
\[
= \begin{bmatrix}
\cos \psi_{j-1} & \sin \psi_{j-1} & 0 - (l_j + d_{(j+1),p}) \sin \psi_j - d_{(j+1),p} \cos \psi_j & \cos \psi_j \\
\sin \psi_{j-1} & \cos \psi_{j-1} & 0 & \sin \psi_j \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix},
\]
\[j = 1, \ldots, N-1; \quad (V.15)\]

\[
R_N^a = \begin{bmatrix}
T_{N-1} \quad 0 \\
0 \quad P_{N+1} \quad T_N \\
0 \quad 0 \\
0 \\
0 \quad 0
\end{bmatrix}
\]
\[
= \begin{bmatrix}
\cos \psi_{N-1} & -\sin \psi_{N-1} & 0 - (l_N + d_{(N+1),p}) \sin \psi_N - d_{(N+1),p} \cos \psi_N & \sin \psi_N \\
\sin \psi_{N-1} & \cos \psi_{N-1} & 0 & \cos \psi_N \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix},
\]
\[ (V.16)\]

\[
\bar{R}_{N+1} = \begin{bmatrix}
T_N \quad 0 \\
0 \quad 1 \\
0 \quad 0 \\
0 \quad 0 \quad 1
\end{bmatrix}
\]
\[
= \begin{bmatrix}
\cos \psi_N & -\sin \psi_N & 0 & 0 \\
\sin \psi_N & \cos \psi_N & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},
\]
\[ (V.17)\]
APPENDIX VI: POTENTIAL ENERGY

The total gravitational potential energy of the system can be written as

\[ U_g = -\left\{ \sum_{i=1}^{N+1} g(j_o \cdot R_{dm_i}) dm_i + \sum_{i=1}^N g(j_o \cdot R_\alpha) m_\alpha \right\} , \tag{VI.1} \]

where

\[ j_o \cdot R_{dm_i} = \begin{cases} D_{iy} + x_i \sin \psi_i + y_i \cos \psi_i, & i = 1, 2, \ldots N \\ D_{(N+1)y} + x_{N+1} \sin \psi_{N+1} + \varphi_{N+1} \delta_{N+1} \cos \psi_{N+1}, & i = N + 1 \end{cases} \tag{VI.2} \]

Thus, for the \( N \) slewing links,

\[ U_{gs} = -\sum_{i=1}^N \rho_i \cdot l_i \cdot g \left( D_{iy} + \frac{l_i \sin \psi_i}{2} \right) . \tag{VI.3} \]

Similarly, for the \( N \) deploying links,

\[ U_{gd} = -\sum_{i=1}^N \rho_i \cdot l_i \cdot g \left( D_{iy} + \frac{l_i \sin \psi_i}{2} \right) . \tag{VI.4} \]

As for the payload,

\[ U_{gp} = -\rho_{N+1} g \left( D_{(N+1)y} l_{N+1} \frac{\sin \psi_{N+1}}{2} \right) \]

\[ -\frac{\rho_{N+1} g \delta_{N+1} \cos \psi_{N+1}}{\lambda} \left( \sinh \lambda l_{N+1} - \sin \lambda l_{N+1} - 0.7341 (\cosh \lambda l_{N+1} + \cos \lambda l_{N+1}) \right) \tag{VI.5} \]

\[ + \frac{1.4682 \rho_{N+1} g \delta_{N+1} \cos \psi_{N+1}}{\lambda} . \]

Finally, for the \( N \) actuators,

\[ U_{ga} = -\sum_{i=1}^N g D_{iy} m_\alpha . \tag{VI.6} \]

Thus the total gravitational potential energy is

\[ U_g = U_{gs} + U_{gd} + U_{gp} + U_{ga} . \tag{VI.7} \]
The partial derivatives of the gravitational potential energy with respect to the generalized coordinates are obtained with Maple V mathematics software and listed below:

\[ \frac{\partial U_g}{\partial D_{ix}} = 0; \]  
\[ \text{(VI.8, a)} \]

\[ \frac{\partial U_g}{\partial D_{iy}} = -\frac{g}{2} \sum_{i=1}^{N} \left[ \rho_{s_i} l_{s_i} + \rho_{d_i} l_{d_i} + m_{a_i} \right] \]  
\[ \text{(VI.8, b)} \]

\[ \frac{\partial U_g}{\partial \alpha_i} = 0, \quad i = 1, \ldots, N; \]  
\[ \text{(VI.8, c)} \]

\[ \frac{\partial U_g}{\partial \psi_i} = \frac{1}{2} g \cos \psi_i \left( \rho_{d_i} l_{d_i}^2 - \rho_{s_i} l_{s_i}^2 \right) \]  
\[ + \sum_{j=1}^{N} \left[ -g \rho_{s_j} l_{s_j} + \rho_{d_j} l_{d_j} + m_{a_j} \right] \left[ l_i \cos \psi_i + d_{(i+1)x} \cos \psi_i + d_{(i+1)y} \sin \psi_i \right], \]  
\[ - \rho_{d_i} l_{d_i} g l_i \cos \psi_i - \rho_{N+1} g l_{N+1} \left( l_i \cos \psi_i + d_{(i+1)x} \cos \psi_i - d_{(i+1)y} \sin \psi_i \right) \]  
\[ i = 1, \ldots, N; \]  
\[ \text{(VI.8, d)} \]

\[ \frac{\partial U_g}{\partial l_i} = -\rho_{d_i} l_{d_i} g \sin \psi_i - \sum_{j=1}^{N} \left[ g \sin \psi_i \left( \rho_{s_j} l_{s_j} + \rho_{d_j} l_{d_j} - m_{a_j} \right) \right] - \rho_{N+1} g \sin \psi_i l_{N+1} \]  
\[ i = 1, \ldots, N; \]  
\[ \text{(VI.8, e)} \]

\[ \frac{\partial U_g}{\partial d_{ix}} = -\sum_{j=1}^{N} \left[ g \sin \psi_{i-1} \left( \rho_{s_i} l_{s_i} + \rho_{d_i} l_{d_i} + m_{a_i} \right) \right] - \rho_{N+1} \sin \psi_{i-1} l_{N+1}, \]  
\[ i = 2, \ldots, N+1; \]  
\[ \text{(VI.8, f)} \]

\[ \frac{\partial U_g}{\partial d_{iy}} = -\sum_{j=1}^{N} \left[ g \cos \psi_{i-1} \left( \rho_{s_i} l_{s_i} + \rho_{d_i} l_{d_i} + m_{a_i} \right) \right] - \rho_{N+1} \cos \psi_{i-1} l_{N+1}, \]  
\[ i = 2, \ldots, N+1; \]  
\[ \text{(VI.8, g)} \]
The total elastic potential energy of the system can be written as

\[ U_e = \sum_{i=1}^{N} \frac{1}{2} K_{\alpha i} \beta_i^2 + \frac{1}{2} \int_{0}^{l_N+1} EI(x_N^+) \left[ \frac{\partial^2 (\psi_{N+1})}{\partial x_{N+1}^2} \right]^2 dx_{N+1}. \]  

(VI.9)

Rewriting the above expression in terms of the generalized coordinates gives

\[ U_e = \sum_{i=1}^{N} \frac{1}{2} K_{\alpha i} (\psi_i - \alpha_i - \psi_{i-1})^2 + 6.18 \frac{EI(x_N^+)}{l_{N+1}^3} \delta_{N+1}. \]  

(VI.10)

The partial derivatives of the elastic potential energy with respect to the generalized coordinates are listed below:

\[ \frac{\partial U_e}{\partial D_{ix}} = \frac{\partial U_e}{\partial D_{iy}} = 0; \]  

(VI.11, a)

\[ \frac{\partial U_e}{\partial \alpha_i} = -K_{\alpha i} (\psi_i - \alpha_i - \psi_{i-1}), \quad i = 1, \ldots, N; \]  

(VI.11, b)

\[ \frac{\partial U_e}{\partial \psi_i} = K_{\alpha i} (\psi_i - \alpha_i - \psi_{i-1}) - K_{\alpha i+1} (\psi_{i+1} - \alpha_{i+1} - \psi_i), \quad i = 1, \ldots, N; \]  

(VI.11, c)

\[ \frac{\partial U_e}{\partial \psi_{N+1}} = K_{\alpha N+1} (\psi_{N+1} - \alpha_{N+1} - \psi_N); \]  

(VI.11, d)

\[ \frac{\partial U_e}{\partial l_i} = 0, \quad i = 1, \ldots, N; \]  

(VI.11, e)
\[
\frac{\partial U_e}{\partial d_{ix}} = \frac{\partial U_e}{\partial d_{iy}} = 0, \quad i = 2, \ldots, N+1; \quad \text{(VI.11, f)}
\]

\[
\frac{\partial U_e}{\partial \delta_{N+1}} = 12.362 \frac{IE\delta_{N+1}}{l_{N+1}^3}. \quad \text{(VI.11, g)}
\]
APPENDIX VII: ENERGY DISSIPATION

The total energy dissipation in the system can be expressed using the Rayleigh function as

\[ R_d = \sum_{i=1}^{N} \frac{1}{2} C_i \dot{\beta}_i^2 + \frac{1}{2} \int_{0}^{\frac{l_N}{2}} C_p I(x_{N+1}) \left[ \frac{\partial^2 (\varphi_{N+1} \delta_{N+1})}{\partial x_{N+1}^2} \right]^2 dx_{N+1}. \]  

(VII.1)

Rewriting the above expression in terms of the set of generalized coordinates gives

\[ R_d = \sum_{i=1}^{N} \frac{1}{2} C_i \dot{\psi}_i^2 \left( \dot{\psi}_i - \dot{\alpha}_i - \dot{\psi}_{i-1} \right)^2 + 6.18 \frac{C_p I_N}{l_{N+1}^3} \delta_{N+1}. \]  

(VII.2)

The partial derivatives of the energy dissipation with respect to the rate of change of the generalized coordinates are computed symbolically using Maple V and listed below:

\[ \frac{\partial R_d}{\partial \dot{D}_{i_x}} = 0; \]  

(VII.3, a)

\[ \frac{\partial R_d}{\partial \dot{\alpha}_i} = -C_i \left( \dot{\psi}_i - \dot{\alpha}_i - \dot{\psi}_{i-1} \right), \quad i = 1, \ldots, N; \]  

(VII.3, b)

\[ \frac{\partial R_d}{\partial \dot{\psi}_i} = C_i \left( \dot{\psi}_i - \dot{\alpha}_i - \dot{\psi}_{i-1} \right) - C_{i+1} \left( \dot{\psi}_{i+1} - \dot{\alpha}_{i+1} - \dot{\psi}_i \right), \quad i = 1, \ldots, N; \]  

(VII.3, c)

\[ \frac{\partial R_d}{\partial \dot{\psi}_{N+1}} = 0; \]  

(VII.3, d)

\[ \frac{\partial R_d}{\partial \dot{l}_i} = 0, \quad i = 1, \ldots, N; \]  

(VII.3, e)

\[ \frac{\partial R_d}{\partial \dot{d}_{i_x}} = 0, \quad i = 2, \ldots, N+1; \]  

(VII.3, f)

\[ \frac{\partial R_d}{\partial \delta_{N+1}} = 12.362 \frac{IC_p \delta_{N+1}}{l_{N+1}^3}. \]  

(VII.3, g)
APPENDIX VIII: DERIVATIVES OF THE MASS MATRIX

In the derivation of the equations of motion, it is necessary to obtain the derivative of the coupled system mass matrix. The differentiation procedure is now briefly outlined.

When the coupled mass matrix $M$ is required, the velocity transformation

$$\dot{q} = R^v \dot{q}$$

(VIII.1)

can be used to obtain

$$M = R^v T \tilde{M} R^v.$$  

(VIII.2)

Hence,

$$\dot{M} = R^v T \tilde{M} R^v + R^v T \dot{\tilde{M}} R^v + R^v T \tilde{\dot{M}} R^v,$$

(VIII.3)

where a dot over a matrix signifies that the time derivative of each component in the matrix must be taken. Using Maple V, the entries of $\dot{M}$ in matrix form are formulated to be as:

$$M_{ij} = t\{K^T M_k R_{vkj} + R: r M_{i} + K^T M_k R_{j} + R: r M_{j} \}.$$  

(VIII.4)

Furthermore,

$$\frac{\partial (q^T M q)}{\partial q} = \frac{\partial (q^T R^v T \tilde{M} R^v q)}{\partial q}.$$  

(VIII.5)

Expanding the above expression gives

$$\frac{\partial (q^T M q)}{\partial q} = \begin{bmatrix} \dot{q}^T \left( \frac{\partial R^v T \tilde{M} R^v}{\partial q_{11}} + R^v T \frac{\partial \tilde{M}}{\partial q_{11}} R^v + R^v T \tilde{M} \frac{\partial R^v}{\partial q_{11}} \right) q \\ \vdots \\ \dot{q}^T \left( \frac{\partial R^v T \tilde{M} R^v}{\partial q_{g}} + R^v T \frac{\partial \tilde{M}}{\partial q_{g}} R^v + R^v T \tilde{M} \frac{\partial R^v}{\partial q_{g}} \right) q \\ \vdots \\ \dot{q}^T \left( \frac{\partial R^v T \tilde{M} R^v}{\partial q_{N+1,4}} + R^v T \frac{\partial \tilde{M}}{\partial q_{N+1,4}} R^v + R^v T \tilde{M} \frac{\partial R^v}{\partial q_{N+1,4}} \right) q \end{bmatrix},$$

(VIII.6)
where \( q_{ij} \) corresponds to the \( j^{th} \) generalized coordinate associated with the \( i^{th} \) unit. Using the fact that each component within the square brackets is a scalar, Eq. (VIII.6) can be rewritten in the form

\[
\frac{\partial (q^T M q)}{\partial q} = \begin{bmatrix}
q^T \left( 2 \frac{\partial R^T}{\partial q_{11}} \tilde{M} R^T + R^T \frac{\partial \tilde{M}}{\partial q_{11}} R^T \right) \dot{q} \\
q^T \left( 2 \frac{\partial R^T}{\partial q_{ij}} \tilde{M} R^T + R^T \frac{\partial \tilde{M}}{\partial q_{ij}} R^T \right) \dot{q} \\
\vdots \\
q^T \left( 2 \frac{\partial R^T}{\partial q_{N+1,4}} \tilde{M} R^T + R^T \frac{\partial \tilde{M}}{\partial q_{N+1,4}} R^T \right) \dot{q}
\end{bmatrix}
\]

(VIII.7)

The matrices \( \tilde{M} \) and \( R^T \) were described in details in Appendix IV and V, respectively. The explicit differentiation of the mass matrix is not presented as the result is quite lengthy and does not provide much insight into the equations of motion.
APPENDIX IX: CONTROL PROGRAM FOR THE MDMS
void main(void)
{
    short iStep;          // step count
    float fSamPeriod;     // sampling period
    float fTime, fManuTime, fExpTime; // time, maneuver duration, experiment duration
    float fPI;
    long iInitEnc[8], iEnc[8]; // initial and current encoder readings
    long iDAC;            // DAC output signal
    float fSetPoint;
    float fDesired, fMeasured;
    float fControl;
    float fError, fErrorSum, fErrorDiff;
    float fPrevError;
    float fKp, fKi, fKd; // PID gains
    float fConvert[8];   // conversion factors for encoders
    char cMode;
    // slewing or deploying
    short iLinkNo;
    unsigned short iPort;
    unsigned short iAxis;
    short i;

    FILE *fpResults;
printf("\n\n***** Operation of the MDMS with PID Control *****\n\n");

// input maneuver duration [sec]
printf("Enter maneuver duration [sec]: ");
scanf("%f", &fManuTime);

// total experiment duration
fExpTime = fManuTime + 5;

// sampling period
//printf("Enter sampling period [ms] (0.5,1,2,3,4,5,10,100,1000): ");
//scanf("%f", &SamPeriod);

for (i = 0; i <= 7; i++) iInitEnc[i] = 0;

iDAC = 0;

fSamPeriod = 1.0;  // 1ms

fPI = (float)3.141592654;

fTime = 0.0;

iStep = 0;

// sine-on-ramp maneuvers
printf("\n\nSINE-ON-RAMP MANEUVER:
\n");

// input which axis in motion
do {
  printf("Which module (1,2,3 or 4)? ");
  scanf("%d", &iLinkNo);
  if (iLinkNo != 1 && iLinkNo != 2 && iLinkNo != 3 && iLinkNo != 4)
    printf("Invalid Input. Please try again.\n");
} while (iLinkNo != 1 && iLinkNo != 2 && iLinkNo != 3 && iLinkNo != 4);

do {
  printf("(S)lew or (D)eployment maneuver? ");
  cMode = getche();
  if (cMode != 'S' && cMode != 's' && cMode != 'D' && cMode != 'd')
    printf("Invalid Input. Please try again.\n");
} while (cMode != 'S' && cMode != 's' && cMode != 'D' && cMode != 'd');

if (cMode == 'S' || cMode == 's')
{
  iAxis = (iLinkNo-1)*2;
  printf("\nPlease enter slewing angle [20deg max.]: ");
  scanf("%f", &fSetPoint);
}

else if (cMode == 'D' || cMode == 'd')
{
  iAxis = iLinkNo*2-1;
  printf("\nPlease enter length of deployment [0.1m max.]: ");
  scanf("%f", &fSetPoint);
}

switch(iAxis)
{
  case 0:
    iPort = axis0;
break;
case 1:
iPort = axis1;
break;
case 2:
iPort = axis2;
break;
case 3:
iPort = axis3;
break;
case 4:
iPort = axis4;
break;
case 5:
iPort = axis5;
break;
case 6:
iPort = axis6;
break;
case 7:
iPort = axis7;
break;
default:
break;
}
printf("\nAxis = %d\n", iAxis+1);

/********************************************************
***********
//
count-to-degree conversion factor for revolute joints
// revolute joint #1 encoder resolution = 500 CPR
// revolute joint #2 encoder resolution = 500 CPR
// revolute joint #3 encoder resolution = 500 CPR
// revolute joint #4 encoder resolution = 1000 CPR
//
// gear ratio is 100 for all the revolute joint actuators
//
// Counts-To-Degrees = 360/(GearRatio*EncoderResolution)
/********************************************************
***********
fConvert[0] = (float)(360.0/(4*100*500));
fConvert[2] = (float)(360.0/(4*100*500));
fConvert[4] = (float)(360.0/(4*100*500));
fConvert[6] = (float)(360.0/(4*100*1000));

/********************************************************
***********
// counts to metres conversion factor for prismatic joints
// lead = 0.125" = 3.125mm = 0.003125m
// encoder resolution = 500 CPR
// Counts-To-Metres = lead/(4*EncoderResolution)
/********************************************************
***********
fConvert[1] = (float)(0.003125/(4*500));
fConvert[3] = (float)(0.003125/(4*500));
fConvert[5] = (float)(0.003125/(4*500));
fConvert[7] = (float)(0.003125/(4*500));

//******************************************************************************
// PID gains
//******************************************************************************
printf("PID gains [Kp,Ki,Kd]: ");
scanf("%f,%f,%f", &fKp, &fKi, &fKd);

//******************************************************************************
// Initialization Procedure for Servo-To-Go S8 board
//******************************************************************************

if (OpenStgDriver() == STG_FAILURE)
{
    printf("Unable to open path to driver.\n");
    exit(0);
}
fErrorSum = 0;
prevError = 0;

//******************************************************************************
// CONTROL LOOP
//******************************************************************************

// infinite control (for) loop: need break to exit according to the experim
ent time

if ((fpResults = fopen("results.txt","w"))==NULL)
    printf("Unable to open: results.txt\n");
fprintf(fpResults, "\n\n***** Operation of the MDMS with PID Control *****");
fprintf(fpResults, "\n\nManeuver duration = %.3fs", fManuTime);
fprintf(fpResults, "\n\nSampling period = %fms", fSamPeriod);
fprintf(fpResults, "\n\nKp = %f, Ki = %f, Kd = %f\n", fKp, fKi, fKd);
fprintf(fpResults, "\n\nTime Desired Measured Error u DAC\n");
SetDAC(iPort, 0);
ZeroEncodersAll();

for (; ;)
{
    if (fTime > fExpTime)
    {
        SetDAC(iPort, 0);
        break;
    }
    // WHEN SAMPLING INTERVAL IS REACHED
    iStep = iStep + 1;
fTime = fTime + fSamPeriod/1000;

if (fTime <= fManuTime)
    fDesired = (float)((fSetPoint/fManuTime)*(fTime-fManuTime)*sin(2*fPI *fTime/fManuTime)/(2*fPI));
else
    fDesired = fSetPoint;

GetEncoderAll(iEnc);
for (iAxis = 0; iAxis < iNAX; iAxis++)
    fMeasured = (float)(iEnc[iAxis]*fConvert[iAxis]);
fError = fDesired - fMeasured;
fErrorSum = fErrorSum + fError;
fErrorDiff = fError - fPrevError;
fControl = fKp*fError+fKi*fSamPeriod*fErrorSum/1000+fKd*fErrorDiff*1000/fSamPeriod;

fPrevError = fError;

if (fControl > 10)
    iDAC = 4096;
else if (fControl < -10)
    iDAC = -4095;
else
    iDAC = (long)((fControl) * 409.6 + .5);

SetDAC(iPort,iDAC);

// save data to file
fprintf(fpResults,"%.3f	%.2f	%.2f	%.2f	%d\n",fTime,fDesired,fMeasured,fError,fControl,iDAC);

CloseStgDriver(); // CloseStgDriver() defined in "stg_io.c"

******
// end of main program
******