OPTIMIZATION OF MACHINING PARAMETERS IN MILLING

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Abstract

The cost of machining for milling is dependent on machining parameters such as spindle speed and feed per tooth. The competitiveness of manufacturing industries can be increased by optimization of machining parameters. A scientific method for the optimization of machining parameters for workpieces of continously varying radial widths is proposed in this thesis. The necessary mathematics for the proposed procedure is derived for both single pass and multi-pass milling operations. The computational results obtained on the basis of derived mathematical formulation are analysed and discussed. The cutting direction has a considerable influence on the cost of machining. Therefore, an algorithm to determine the influence of cutting direction on machining cost is also suggested. The best cutting directions for a number of workpieces of known geometry are ascertained on the basis of computational results.

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Nomenclature

K_s:	specific cutting pressure
a:	axial depth of cut
s_t:	feed per tooth
s_{max}:	maximum allowable feed per tooth
Z:	number of tooth
N:	spindle r.p.m.
d:	radial width of cut
R:	tool radius
v:	tool traverse rate
S_{eq}:	equivalent feed per tooth
ϕ:	instantaneous angle of immersion
ϕ_s:	swept angle of cut
C_h:	machine cost rate
C_t:	tool cost
T_{ct}:	tool change time
X:	thermal fatigue parameter
E_r:	range of thermal strain parameter
t_c:	cooling time
M:	moment on the cutter
T:	torque on the cutter
T_{max}:	maximum allowable torque on the cutter
σ_{max}:	maximum allowable shank stress

P_{max}:	maximum allowable power
Δt:	sampling interval
F_t:	tangential force
F_r:	radial force
F_x:	cutting force in the x-direction
F_y:	cutting force in the y-direction

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ਸੁਲਤਾਨੁ ਹੋਵਾ ਮੇਲਿ ਲਸਕਰ
ਤਖਤਿ ਰਾਖਾ ਪਾਉ ।।
ਹੁਕਮੁ ਹਾਸਲੁ ਕਰੀ ਬੈਠਾ
ਨਾਨਕਾ ਸਭ ਵਾਉ ।।

Chapter 1

Introduction

Low costs and high productivity are important requirements of competitive economies. Sophisticated manufacturing systems, such as Flexible Manufacturing Systems (FMS) and Computer Integrated Manufacturing (CIM), are quite effective in meeting these demands. Untended or minimally manned machining centres are the most versatile form of computerised manufacturing. These systems offer a significant technological advancement in terms of quality, design, time and costs, and may increase the user's competitive advantage.

The most advanced automatic manufacturing systems utilize computers as an integral part of their control. The era of advanced automation started with the introduction of numerical controlled machine tools. The term numerical control is commonly used for programmable automation; a demand for high accuracy in manufacturing and a desire to reduce the production time as well as the necessity to produce complex geometries were the primary motivations for the development of these machines. Numerical controlled machines are hardware based machines which use electronic hardware and digital circuit technology. In order to increase the flexibility of these systems controllers based upon general purpose computers rather than specialised hardware were introduced. The machines which use these controllers are called CNC (Computer Numerical control) machines. The schematic diagram of a CNC system is shown in Fig. (1.1). These machine tools use a computer to control the machine tool and eliminate some of the hardware circuits in the control cabinet of an NC machine tool. CNC systems are quite flexible

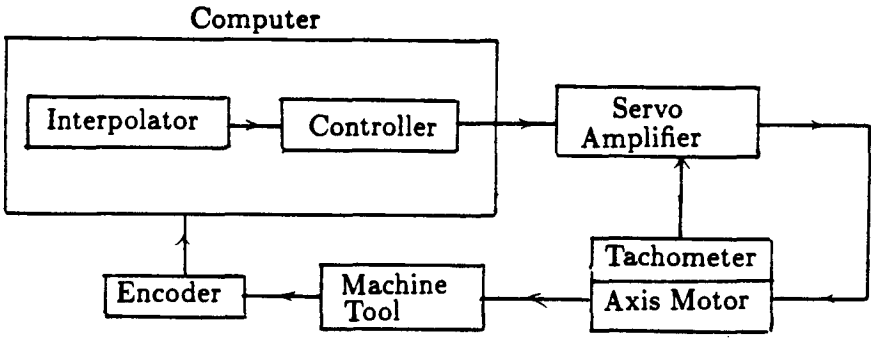

Figure 1.1: Schematic Diagram of a CNC System

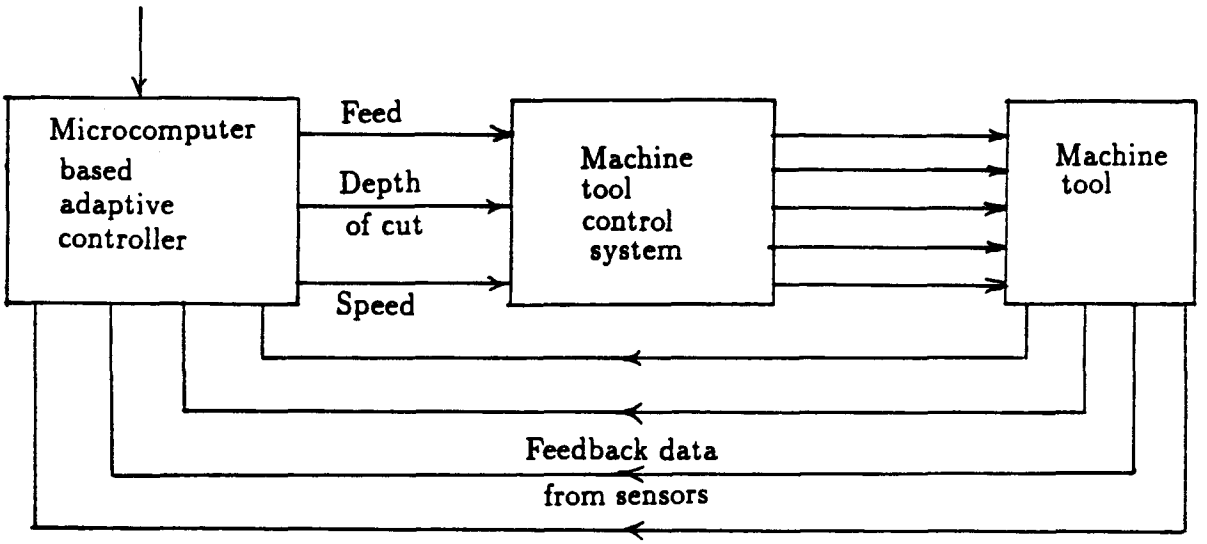

Figure 1.2: Adaptive Machine Tool Control System Block Diagram

and because of the declining costs of minicomputers and microcomputers the number of CNC systems has increased tremendously in last few decades.

The full potential of CNC systems can be realized only if there are realistic strategies of prescribing the operating parameters like speeds and feedrates of the machine tool. Quite often, the prescription of these parameters is based on the experience and knowledge of the part programmer. For the prevention of tool breakage and the safety of machine tool, the most adverse machining conditions, (which might not occur in reality), are taken into consideration. Therefore, the estimate of operating parameters tends to be conservative, which results in an under-utilization of machines and production losses. This common drawback of CNC systems can be overcome to some extent by the adaptive control strategy. In adaptive control, the operating parameters automatically adapt themselves to the existing conditions of machining in real time. Adaptive control systems for machine tools can be divided in two categories:

- Adaptive control with optimization (ACO)
- Adaptive control with constraints (ACC)

In ACO systems, the extremum of a specified performance index is obtained within process and system constraints. The performance index can be an economic function such as the process cost or profit. In ACC systems the maximum possible machining parameters within a prescribed region bounded by process and system constraints are selected. In reality most ACC systems use a single easily measured parameter as the only variable. The block diagram of an adaptive machine tool control is shown in Fig. (1.2).

Accurate measurement of tool wear or tool life is important for the implementation of adaptive machine tool control. The capability of the computing equipment to do all the calculations in real-time is also crucial for the performance of adaptive control systems.

Most of the objective functions used for the optimization of machining parameters are non-linear functions. Direct search techniques or hill climbing methods are required for the numerical solution of these problems. These techniques are time consuming and quite often it is not possible to find the optimal solution in real-time adaptive control. The implementation of adaptive control becomes easier if the optimization of machining parameters is carried out beforehand and the optimum values are stored in a database.

The primary focus of this thesis is to suggest procedures to determine the optimal machining parameters for milling with various workpiece geometries. Both single pass and multi-pass milling operations are considered. Keeping in view the strong influence of cutting direction on the optimization procedure, an algorithm for the computation of an optimal cutting direction is also suggested. Some researchers have proposed the adaptive control of the feedrate based on the maximum allowable force. This method is compared with the strategy proposed in this thesis.

1.0.1 Thesis Outline

A brief literature review on optimization of machining parameters, adaptive control and tool path planning for milling process is presented in chapter 2 of this thesis. Chapter 3 discusses the economics of a milling process. The tool life equation, the objective function and mathematical relations for the constraints are discussed in that chapter. Chapter 4 suggests a scientific basis for the selection of machining parameters for a workpiece of a given geometry. The optimization of machining parameters for single pass milling operations and a mathematical formulation for the evaluation of machining time is also included in that chapter.

Since, in reality, a given cutter can only cut a certain maximum radial width of cut, (because of physical or dynamic constraints), some practical guidelines have been suggested for the subdivision of the total machining surface of the workpiece. Chapter 5

contains the multiple pass optimization problem for milling operations.

Finally, in chapter 6 the problem of determination of the influence of cutting direction on cost has been addressed, an attempt has been made to determine the cutting direction that will minimize the cost for a generalized workpiece of given dimensions.

Chapter 2

Literature Review

The examination of the economics of the milling process is an important research topic. Optimization of machining parameters is a highly desirable goal of any economic study of milling process. Adaptive control and tool path planning also have a considerable influence on the process cost of milling. However, relatively little attention has been paid to this topic by manufacturing engineering researchers. This chapter presents a brief overview of some of the work done in the areas of optimization of machining parameters, tool path planning and adaptive control for milling process.

Yellowley and Desmit [1] have modelled the single pass optimization problem for milling and have developed a suitable algorithm for its solution. They consider variable cost as the single objective function. An algorithm which minimizes this objective function within four inequality constraints is used to suggest the optimum values of cutter diameter, feed per tooth and peripheral velocity for a given geometry of shoulder. An important conclusion for the selection of cutter diameter and number of teeth is expressed as follows:

"In general, it would seem that it is preferable to either use the smallest diameter of cutter available which is capable of machining the required width, or the first larger radius having a greater number of teeth."

Chang and Wysk [2] have proposed an optimization criterion based on the discrete transformation method. The objective function for this study is not process cost but

production time which is expressed as follows:

$$\begin{aligned} T_T &= T_c + T_t \\ T_T &= \frac{L}{V_f} + \frac{uL}{V_f^{p+1} f(n, d, R, Z)} \end{aligned}$$

where

T_c	actual cutting time
T_t	tool change time per unit of work
T_T	production time per piece (idle time excluded)
L	length of work piece
u	unit tool change time (could be a function of R and Z)
V_f	cutter traverse speed
n	spindle rotation rate
d	radial depth of cut
R	cutter radius
Z	number of teeth
p	constant coefficient

The tool life equation used in the study is of the following form:

$$T_L = V_f^p f(n, d, R, Z) \tag{2.1}$$

It is clear from the expression for production time per piece that the objective function contains five decision variables V_f, n, d, R and Z. Some of these variables like spindle speed and number of teeth belong to discrete sets such as S_n and S_z. This fact is used for the discrete transformation of the objective function. Consequently only three variables are left and the solution procedure is drastically simplified.

The optimization procedure suggested by Chang and Wysk is a good analytical exercise but, somehow, falls short of practical requirements. Minimization of production time is a desirable goal for those economies which have a scarcity of goods. The minimization of production time is a primary manufacturing objective in very rare situations. In a modern, free-market, with fiercly competitive economies the real challenge is of reducing costs and not of reducing production time. Also, the model proposed by Chang and Wysk does not take into account important constraints such as the tooth breakage and shank breakage constraint. An optimization procedure which does not safegaurd the cutting tool from breakage is of limited practical use.

Chatter and the physical dimensions of the tool impose a limit on the maximum radial width for a single pass. In cases where the amount of stock to be removed in a rough milling operation exceeds the allowable width, there is a need for a multipass operation. Yellowley and Gunn [3] have examined the problem of multipass milling operations. The following mathematical expression is used for the tool life equation:

$$T_L = \frac{360}{\phi_s} \frac{C_6}{X^m S_{eq}^b V^a} \tag{2.2}$$

where

C_6	is a constant
ϕ_s	is the swept angle of cut in radian
X	is the thermal fatigue parameter
S_{eq}	is the equivalent chip thichness in mm
V	is the peripheral velocity in mm per sec.
a	is the axial depth of cut in mm
a, b, m	are positive exponents

A similar tool life equation is discussed in detail in the next chapter.

The cutting time per unit length for each pass is given by

$$t_{co}(i) = C_7 f_2(d_i)^{(1-\beta)} f_1(d_i)^{\beta} \tag{2.3}$$

$f_1(d)$ and $f_2(d)$ are obtained from the following relations:

$$\begin{aligned} S f_2(d) &\leq C_3 \\ V T^{\alpha} S f_1(d)^{\beta} &= C_2 \end{aligned}$$

where

C_7	is a constant
V	is the cutting velocity
T	is the tool life
S	is the feed/rev.
C_2	is a constant
C_3	is a constant
d	is the radial width

and

$$0 < \alpha < \beta \leq 1 \tag{2.4}$$

The objective function is reduced to the following mathematical form:

$$\sum_{i=1}^{n} (f_2(d_i)^{(1-\beta)} f_1(d_i)^{\beta}) \tag{2.5}$$

$$0 \leq d_i \leq d_{max}$$
$$\sum_{i=1}^{n} d_i = d_{TOT}$$
$$0 < \alpha < \beta \leq 1$$

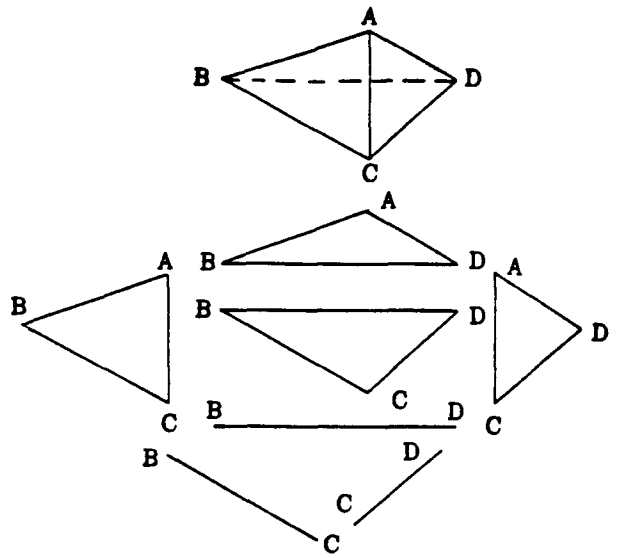

Figure 2.1: Boundary representation of a polyhedron

where

d_i	is the radial depth for the ith pass
d_{max}	is the chatter limited depth
d_{TOT}	is the total amount of stock to be removed

The objective function is minimized and an optimal solution is obtained. The optimal solution has the property that all passes except one should be taken at the maximum allowable width of cut, with the one other pass used to remove the required remaining amount of stock.

Some researchers have reasoned that the optimal metal removal rate should not be modelled independently of the cutter path selected for the operation. The choice of a proper cutter path can reduce the tool wear due to the lesser engagement of the tool with the job, thus resulting in process optimization.

Wang et al. [5] presented a mathematical model for computing an optimal tool cutter path for face milling. They utilized this model to identify the minimum length of cut

for face milling flat surfaces. At the highest level a polyhedron is represented as a set of surfaces. Then each surface is broken down into line segments which compose the surface. The lowest level consists of the starting and end points of vectors which are the vertices of surfaces. An illustration of boundary representation of a polyhedron is shown in Fig. (2.1). The equation of set of surfaces of a polyhedron would be of the following form:

$$
\begin{aligned}
A_1X + B_1Y + C_1Z &< (or >)D_1 \\
A_2X + B_2Y + C_2Z &< (or >)D_2 \\
\cdots \quad & \cdots \\
\cdots \quad & \cdots \\
\cdots \quad & \cdots \\
A_NX + B_NY + C_NZ &< (or >)D_N
\end{aligned}
$$

Each of the above surfaces can be represented in terms of their boundary vectors as follows:

$$
\begin{aligned}
A_{T1}X_T + B_{T1}Y_T &< (or >)D_{T1} \\
A_{T2}X_T + B_{T2}Y_T &< (or >)D_{T2} \\
\cdots \quad & \cdots \\
\cdots \quad & \cdots \\
\cdots \quad & \cdots \\
A_{TN}X_T + B_{TN}Y_T &< (or >)D_{TN}
\end{aligned}
$$

An N-sided polygon is divided into (N-2) triangles and the length of cut L_m for each triangle is computed from an analytical expression. Finally, the lengths L_m for all the (N-2) triangles are added together to obtain the total tool travel length. This procedure

is used for a range of sweep angles from 0 to 180 degrees and the angle corresponding to the minimum tool travel length is thus determined.

The same authors in a second paper [5] have examined the two commonly used approaches of stair case and window frame milling. Again a boundary representation scheme is used for the transference of input data about the workpiece geometries. A polyhedron is represented as a set of surfaces and each surface can be identified by the equation of its edges. Edges are recognized by their starting or end points.

Once the part geometry has been defined, the impact of the selection of a starting point and cutting orientation on tool path is studied for both stair case and window frame milling. The conclusions of this study are summarized as follows:

1. In window frame milling, the selection of a starting point does not significantly affect the length of cut, although a small amount of variation exists.

2. The cutting orientation in stair case milling produces a significant impact on the length of cut. The average variation is on the order of $5 - 10$ percent. The worst case can be as much as 100 percent.

3. There appears to be no correlation between the optimal cutting orientation and other control parameters, such as tool diameter and number of edges.

4. Based on the experimental results, the optimal length of cut generated by stair case milling is better than that generated by window frame milling. However, the average results from stair case milling are sometimes worse than those of window frame milling.

5. From the experimental results, the authors observed that, for stair case milling of regular polygons, the optimal cutting orientation is normally parallel to the longest edge of a given polygon.

Prabhu and Wang [6] have also developed a mathematical model representing the total tool path on an N-sided convex polygon surface. They have considered the staircase type of tool path. The mathematical formulation is complex and can not be solved by standard analytical or numerical methods. They have proposed an algorithm to find an optimal solution between 0 and 180 degrees. The conclusions of this study are stated as follows:

1. For a triangle the optimum sweep angle seems to be the one which makes the sweep path parallel to the largest side of the triangle, which is consistent with the findings of Wang et al. [5].

2. For a square or rectangle or parallelogram the optimum sweep angle is the one that makes the sweep parallel to any one of its sides.

3. If the square is divided into two triangles the sweep of the tool path parallel to the largest side of the triangles does not give an optimal solution.

4. A series of local optima of the objective function make global optimization difficult.

The above mentioned tool path planning studies for milling use length of cut as the only optimization criterion. The authors of these studies have indicated that shortest length of cut would minimise the tool wear. It is, however, not clear why these researchers have chosen this objective. In most practical situations minimization of process cost is the foremost consideration and in some rare cases minimization of production time is the primary goal of manufacturing planning. Minimization of tool wear may not fulfil any of these objectives. It has been proven by Yellowley [1] and Chang et al. [2] that both process cost and production time depend on the machining parameters. Therefore, a study on optimal tool path planning without any regard to machining parameters and constraints may not be too useful for manufacturing engineering.

It has been mentioned in chapter 1 that the adaptive control of machine tools is an effective way of selecting machining parameters and reducing costs. Several researchers have proposed the adaptive control of machine tools by varying the feed rate adaptivaly, and keeping the cutting forces below the limiting value.

Tlusty et al. [7] proposed the following relationhip between the cutting force and the workpiece traverse rate:

$$\begin{aligned} F_{act}(t) &= Cv_{act}(t-\tau) \\ F_{act}(s) &= Cv_{act}(s)e^{-\tau s} \end{aligned}$$

where

τ is the tooth period

τ represents the time delay between velocity change and force change. The above relations are for actual values of these parameters and not for commanded values. The proportionality constant C is expressed as follows:

$$C = Kba \tag{2.6}$$

where

K depends on the workpiece material

b is the axial depth of cut

a is the radial depth of cut

The force error is evaluated as follows:

$$e_f = \frac{F_{nom} - F_{act}}{F_{nom}} \tag{2.7}$$

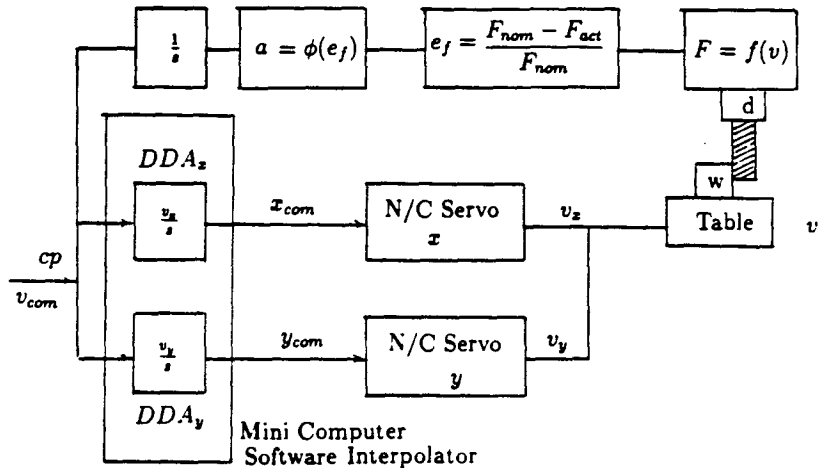

Figure 2.2: Block Diagram of Adaptive Control Loop (Tlusty)

where

F_{nom} is the desired cutting force which should

; be kept constant by adapting feedrate

The actual cutting force is measured by the dynamometer attached to the spindle. This force signal is compared with the nominal force F_{nom}. The result of the comparison is the relative force error e_f. Based on this force error a desired change of velocity is expressed as acceleration $a = \phi(e_f)$. Integration of this expression gives us the commanded velocity v.

The block diagram of the adaptive control scheme proposed by Tlusty is shown in Fig. (2.2).

Tomizuka et al. [9] based their study on the model reference adaptive control method. The milling process was treated as a first order dynamic process with time varying parameters. Daneshmend [10] used a similar strategy for the turning process. Due to

cutting process to a simple time varying gain with a time invariant pole.

Tomizuka et al. [8] treated the dynamics of the feed drive as a gain which is not valid for dc servo controlled machine tools. Moreover, none of these above mentioned studies took into account the inevitable nonlinearities associated with systems of these kind.

Altintas et al. [10] developed an adaptive control strategy based on linear dynamics of the plant with simple nonlinearities. The milling process to be controlled is considered to have two cascaded dynamic processes. The time-invariant feed drive servo control and the time variant cutting process dynamics. The discrete transfer function of the feed drive servo is expressed as:

$$\begin{aligned} G_s(z) &= \frac{c(k)[mm/tooth]}{u(k)[count/s]} \\ G_s(z) &= \frac{k_p z^{-1}(1 + z_1 z^{-1})}{1 + p_1 z^{-1}} \end{aligned}$$

where

k_p, z_1, p_1 are constants

The time variant cutting process dynamics is expressed by the following discrete transfer function:

$$F_p(k) = \frac{\beta z^{-1}}{1 + \alpha z^{-1}} c(k) + \frac{\gamma}{1 + \alpha z^{-1}} \tag{2.8}$$

The process parameters α, β and γ are time varying and functions of the workpiece geometry. The maximum cutting forces are regulated by estimating the time varying parameters α, β and γ at each sampling period. The method of Normalized Recursive Least Square is used to estimate these parameters. The block diagram of the adaptive control scheme proposed by Altintas is shown in Fig. (2.3).

None of these works on adaptive force control have indicated the method of calculating the maximum reference force. It is also not certain how the feedrates obtained on the

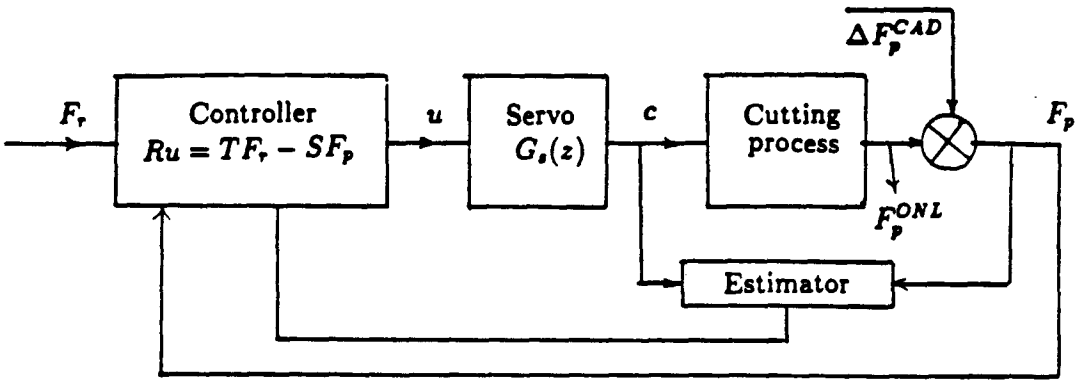

Figure 2.3: Block Diagram of Adaptive Control Loop (Altintas)

basis of this strategy are kept below the tooth breakage constraint. Therefore, there is a need to conduct a closer investigation of adaptive force control strategy so that a comparison between this strategy and the optimization strategy proposed in this thesis can be made. However, it has been decided not to include the research related to this topic in this thesis.

Chapter 3

The Economics of Milling Process

There are two kinds of industries in a free market economy - competitive and closed. The competitiveness of an industry can only be established by thoroughly studying all the details of the manufacturing cost and finding ways and examining means of reducing this cost. The cost of machining can only be reduced by the proper selection of machining parameters. The best selection is made when the value chosen for these parameters is such that cost is minimized or profit maximized.

Milling is an important metal cutting process, but relatively little attention has been paid to the economics of this process. The basic geometry of the milling process is shown in the Fig. (3.1). The machining parameters of interest in a milling process are:

- Tool radius R
- Spindle rotational speed N
- Feed per tooth s_t
- Radial width of cut d
- Axial depth of cut a
- Number of teeth Z

Optimization of these parameters can be based on several different objectives. Some of these objectives are:

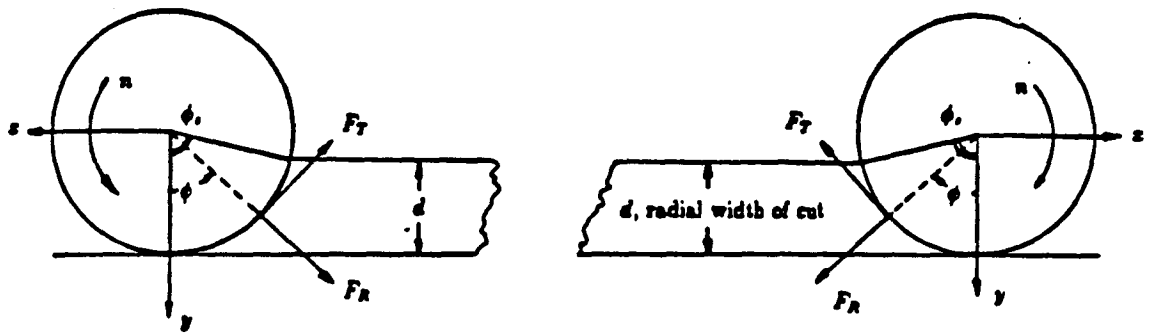

Figure 3.1: Basic Geometry of the Milling Process

- Minimization of process time
- Minimization of process cost
- Maximization of profit

Depending upon specific circumstances any one of these criteria may be important. Each criterion will typically lead to the selection of different conditions. Barrow has presented an analysis for each of the above mentioned objectives. Minimization of process cost is the most common objective and, therefore, the present work is based on this objective.

The cost of producing a part can be divided into fixed and variable costs. Fixed costs are independent of the machining process; these costs consist of machine centre set up costs and raw material costs. Therefore, the goal of an economic model which is of interest to manufacturing engineers is to minimize the variable costs. Figs. (3.2) and (3.3) show the variation of total variable cost and machining time with cutting speed

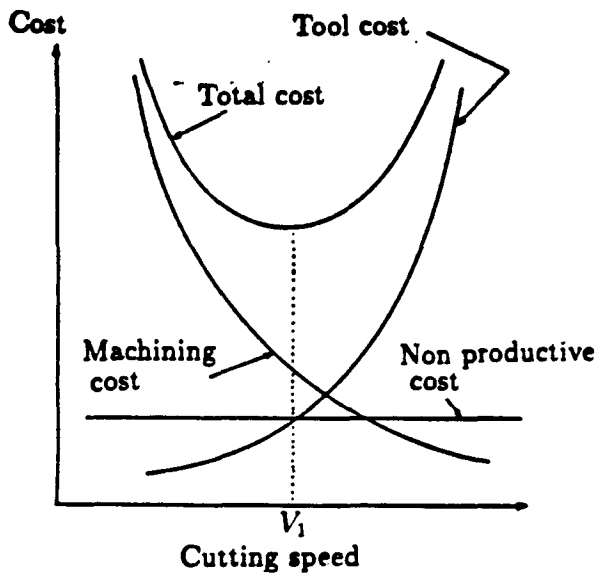

Figure 3.2: Variation of total variable cost with cutting speed

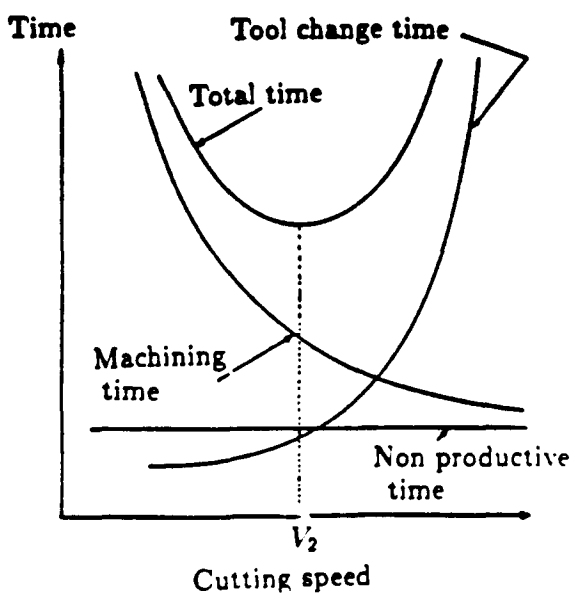

Figure 3.3: Variation of machining time with cutting speed

respectively. These graphs indicate that the velocity giving the minimum cost is less than the velocity giving minimum time corresponding to maximum production rate.

3.1 Cost Equation

Yellowley [1] has proposed an equation for the cost of milling per unit length at constant width and depth of cut. This equation can be expressed as follows:

$$C_l = \frac{C_h}{v} + \frac{C_t}{vT_L} + \frac{C_h T_{ct}}{vT_L} \tag{3.1}$$

where

C_l	is the process cost per unit length in dollars
C_h	is the machine cost rate in dollars per second
C_t	is the tool cost in dollars
T_{ct}	is the tool change time in seconds
v	is the tool traverse rate in mms per second
T_L	is the tool life in seconds

The values of economic parameters used in the above equation are site specific and are therefore dependent on the shop/machine/tool combination.

The machine cost rate includes labour, plant operating costs and machine operating costs. Tool change cost is calculated by multiplying tool change time by machining cost rate. Tool cost is determined by multiplying the tool life fraction used in the machining operation by the total cost of the tool.

3.2 Tool Life Equation

Milling is an extremely complex process. It is not only a discontinuous process, but is also affected by both mechanical and thermal shock. Milling is also characterized by a variable chip thickness during cut. Research workers in Germany are of the belief that the mechanical effects are more important whereas Japanese and Soviet workers consider thermal effects to be the more prevalent. Yellowley [11] has suggested that a realistic tool life equation can be obtained by considering only the thermal effects provided that only one mode of milling is considered and chip formation at exit is not problematic.

There are many ways of defining the useful tool life but the most common criterion is related to flank wear. A tool is considered to have reached the end of its life when it reaches the maximum limit of wearland width V_B^*. The rate of change of the width of wearland with respect to time can be approximated by the following relation:

$$\frac{dV_B}{dt} = \frac{V_B^*}{T_L} \tag{3.2}$$

where

T_L is the tool life corrresponding to V_B^*

The above equation may be used to evaluate an expression for the equivalent feed in milling in the following manner:

The relation between tool life and equivalent chip thickness is assumed to be of the following form:

$$hT_L^k = C \tag{3.3}$$

where

h is the chip thickness

k is a constant with value quite close to unity (3.4)

Therefore,

$$T_L = \frac{C}{h} \tag{3.5}$$

Consequently, the rate of change of wear land width takes the following form:

$$\frac{dV_B}{dt} = \frac{V_B^* h}{C} \tag{3.6}$$

Integrating the above expression to evaluate the total wear in a swept angle, we obtain:

$$V_B^t = \frac{V_B^*}{C} \int_0^{\phi_s} s_t \sin \phi d\phi \tag{3.7}$$

The expression for average wear rate is :

$$\frac{V_B^t}{\phi_s} = \frac{V_B^*}{C\phi_s} \int_0^{\phi_s} s_t \sin \phi d\phi \tag{3.8}$$

Yellowley has proposed the concept of this so called equivalent feed rate to combine the influence of cutter diameter, width of cut and feed per tooth on the milling process. The equivalent feed rate is defined as that constant feed rate which will yield the same average wear rate as the variable chip thickness in milling. Using this concept of equivalent feed in Eq. (3.6), the average wear rate can also be expressed as:

$$(\frac{dV_B}{dt})_{ave} = \frac{V_B^* S_{eq}}{C} \tag{3.9}$$

From Eqs. (3.8) and (3.9), we obtain:

$$\frac{V_B^* S_{eq}}{C} = \frac{V_B^*}{C\phi_s} \int_0^{\phi_s} s_t \sin \phi d\phi \tag{3.10}$$

or

$$S_{eq} = \frac{s_t}{\phi_s} \int_0^{\phi_s} \sin \phi d\phi \tag{3.11}$$

The influence of the intermittent nature of the milling process on tool life can be well represented by the thermal fatigue parameter, introduced by Yellowley [11] as :

$$X = E_r (Nx)^{\frac{1}{2}} \tag{3.12}$$

where

E_r range of thermal strain parameter

N rotational speed

x ratio of total cycle time to time in cut

It is clear from the above relation that the thermal fatigue parameter is dependent on both the range of thermal strain and the number of thermal strain cycles per unit cutting time. The range of thermal strain is a function of heating and cooling time. The values for this parameter do not vary too much for high speed steels and carbide tool materials for the same heating and cooling times. The mathematical expression for the range of thermal strain parameter is as follows:

$$E_r = 39 \log t_c - 23 \log t_h + 37.5 \tag{3.13}$$

where

t_c cooling time

t_h heating time

Both the concepts of equivalent feed and thermal fatigue parameter have been used in the tool life equation which is based on the allowable amount of flank wear in the tool. Some of the process constraints are aimed at preventing catastrophic failure of the tool. The active tool life of a milling cutter is defined as :

$$T_L = \frac{2\pi}{\phi_s} \frac{C_1}{X^m S_{eq}^n V^p a^q} \tag{3.14}$$

where

C_1 is a constant

ϕ_s	is the swept angle of cut in radian
X	is the thermal fatigue parameter
S_{eq}	is the equivalent chip thichness in mm
V	is the peripheral velocity in mm per sec.
a	is the axial depth of cut in mm
m, n, p, q	are positive exponents

This equation is only valid when there is no chip sticking and when the lag angle between leading and trailing edges is very small. We have not considered the effect of mechanical shock caused by entry and exit conditions in the above equation. Fortunately, in processes where chip sticking does not occur, the entry and exit conditions do not affect tool life in milling. The effect of the tool/workpiece materials is reflected in the values of the constants in the equation.

3.3 Process Constraints

The physical properties of the work/tool pair, the capacity of the driving motor, the dynamics of the machining process (e.g.chatter) and part design specifications such as surface finish impose constraints on the values of machining parameters. Therefore, any realistic economic model must take into account these constraints. The constraints which can influence the economics of the milling process are listed below:

- Tooth Breakage Constraint
- Shank Breakage Constraint
- Power and Torque Constraint
- Chatter Constraint

- Surface Finish Constraint

These constraints will be discussed one by one.

3.3.1 Tooth Breakage Constraint

Tooth breakage is defined as the loss of a significant portion of the edge of an individual tooth. A catastrophic tooth breakage will eventually result in damage to the workpiece and the machine. Therefore, in order to avoid this catastrophic tooth breakage it is essential to control the maximum cutting stress experienced by the cutting teeth. Yellowley has defined the tooth breakage constraint limit as:

$$s_t \sin \phi_s \leq s_{max} \qquad (d < R) \tag{3.15}$$

$$s_t < s_{max} \qquad (d \geq R) \tag{3.16}$$

where

ϕ_s is the swept angle in radians.

s_{max} is the maximum allowable feed per tooth in mm.

s_t is the feed per tooth in mm.

3.3.2 Shank Breakage Constraint

The shank of a tool may fail under the combination of bending and torsional working loads. To avoid failure, the maximum tensile stress allowed on the shank must be kept under a critical value. This critical value depends on the geometry and mechanical properties of the shank. Let σ represent the normal stresses on the tool and τ be the shearing stress. This state of stress can be represented by Mohr-circle diagram as shown in Fig. (3.4). The maximum tensile stress is given by the distance OA on this diagram.

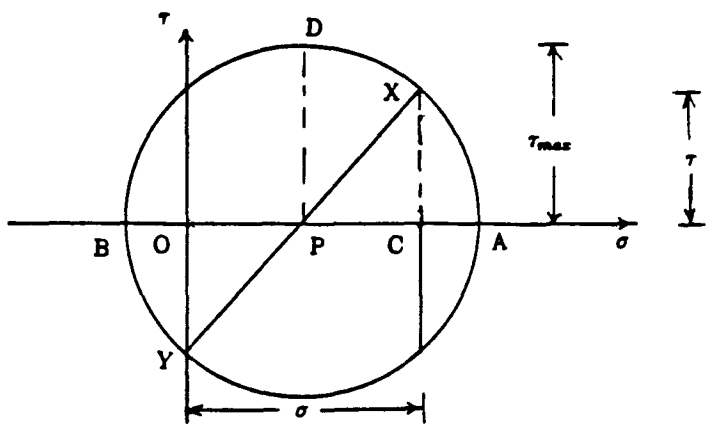

Figure 3.4: Mohr-circle Diagram for Shank Stresses

It is clear from the geometry of this figure that

$$
\begin{aligned}
OA &= OC + CA \\
OA &= OC + AP - PC \\
OC &= \sigma \\
PC &= \frac{\sigma}{2} \\
AP &= PX \\
PX &= \sqrt{(\tau)^2 + (\frac{\sigma}{2})^2}
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
OA &= \sigma + \sqrt{(\tau)^2 + (\frac{\sigma}{2})^2} - \frac{\sigma}{2} \\
OA &= \frac{\sigma}{2} + \sqrt{(\tau)^2 + (\frac{\sigma}{2})^2}
\end{aligned}
$$

From elementary mechanics of material we know that for a circular cross section,

$$\sigma = \frac{MR}{I}$$

$$\tau = \frac{TR}{J}$$

where

M	is the moment on the cutter
T	is the torque on the cutter
J	is the polar moment of inertia
I	is the moment of inertia
R	is the shank radius

Hence,

$$OA = \frac{MR}{2I} + \sqrt{(\frac{TR}{J})^2 + (\frac{MR}{2I})^2} \tag{3.17}$$

After some simplification we obtain:

$$\sigma_{OA} = \frac{2}{\pi R^3}[M + (M^2 + T^2)^{\frac{1}{2}}] \tag{3.18}$$

Therefore, the mathematical expression for the torque constraint is:

$$\frac{2}{\pi R^3}[M + (M^2 + T^2)^{\frac{1}{2}}] < \sigma_{max} \tag{3.19}$$

where

σ_{max}	is the max. allowable tensile stress on the tool shank

For a specific tool with defined geometry and material, the tensile stress on the shank can be controlled by varying the depth of cut (a), peripheral cutting speed (V) and cutter traverse rate (v).

3.3.3 Power and Torque Constraints

Power and torque are machine constraints which are imposed by the maximum capacity of the motor. The violation of these constraints may cause a serious damage to the power drive, spindle shaft or workpiece. These constraints can be represented by the following inequality relations:

$$Kavd < P_{max} \tag{3.20}$$

$$KaR(\frac{v}{V})d < T_{max} \tag{3.21}$$

where

K	is the specific cutting pressure
P_{max}, T_{max}	are the maximum allowable power and torque

3.3.4 Chatter Constraint

A chatter threshold limits the width of cut (d) and the depth of cut (a). This threshold must not be exceeded if instability of the milling process is to be avoided. It is extremely difficult to study the effect of width of cut on the occurrence of instability in milling. This is mainly due to the following reasons:

- The width of cut influences both the magnitude and direction of the average resultant force.
- The width of cut influences the frequency content of the milling force signal and the basic frequencies are dependent on the cutter diameter and number of teeth.

It is therefore extremely difficult to formulate a realistic chatter constraint without significant specific machine tool and work/tool data. Chatter is neglected in the optimization studies.

3.3.5 Surface finish Constraint

Surface finish imposes a constraint on the depth of cut and cutter traverse rate, especially for fine finishing. However, its influence is usually not considered in the study of rough milling operations. A theoretical estimate could of course be made based upon the kinematics of the process. This would not in most cases be indicative of actual finish because of radial run out, adhered material and dynamic effects. Therefore, surface finish constraints are also not included in the optimization studies.

Chapter 4

Optimization of Single Pass Milling Operations with Irregular Workpieces

The milling operation is an intermittent cutting process. The rotating cutter with one or more cutting teeth comes in contact with a translating workpiece producing a chip of variable thickness. The machining parameters for the milling process which can be optimized are mentioned in chapter 3, and are once again listed below :

- Tool radius R
- Spindle rotational speed N
- Feed per tooth s_t
- Radial width of cut d
- Axial depth of cut a
- Number of teeth Z

Yellowley and Desmit [1] have developed an algorithm for the selection of tool diameter, feed per tooth and peripheral velocity for a shoulder of given geometry. Many workpiece surfaces however have polygonal, circular or elliptic geometries. The radial widths for these workpieces vary continously throughout the tool traverse length because of which the optimum machining conditions also keep changing. The problem of determining the optimum machining parameters for these workpieces is, therefore, quite complex but of immense practical value.

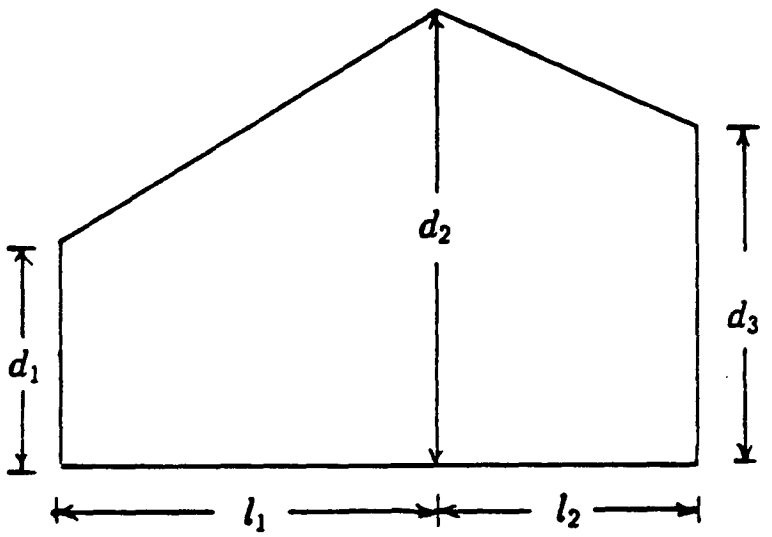

Figure 4.1: Generalized Workpiece Geometry (Top view)

In this chapter we present a model which can be used for the selection of machining parameters for a large variety of workpiece geometries. The general method and strategy on which the model is based is applied on a few specific workpiece geometries. However, the same approach is also valid for other geometries not discussed in this thesis.

4.1 Mathematical Formulation

Let us consider a workpiece geometry as shown in the Fig. (4.1). It is assumed that the maximum radial width encountered in the workpiece does not exceed the chatter limit and a tool which can machine the workpiece in a single pass is available in the machine shop. Also for simplicity the axial depth of cut is kept constant throughout this work unless otherwise specified.

For process economy and real-time process control, the feed rate needs to be varied with changing radial width as the tool traverse the workpiece length. This is done at a series of sampling intervals. It is assumed that the tool traverses the whole workpiece

length $(l_1 + l_2)$ in n sampling intervals.

let

d_i	be the radial width for the ith interval
x_i	be the distance traversed by the tool in the ith interval
Δt	is the sampling period
N_i	is the R.P.M. for the ith interval
s_{t_i}	is the feed per tooth for the ith interval

Here i varies from 1 to n. The whole workpiece length can be written as a summation of the distances traversed in all the sampling intervals as follows:

$$l_1 + l_2 = \sum_{i=1}^{n} x_i \tag{4.1}$$

Mathematically d_i can be obtained by one of the following two expressions:

$$d_i = d_1 + k_1(\sum_{k=1}^{i-1} x_k) \tag{4.2}$$

$$d_i = d_2 - k_2[\sum_{k=1}^{i-1}(x_k - l_1)] \tag{4.3}$$

where

$$\begin{aligned} \sum_{k=1}^{i-1} x_k &= \frac{Z\Delta t}{60}.(\sum_{k=1}^{i-1} s_{t_k} N_k) \\ k_1 &= \frac{d_2 - d_1}{l_1} \\ k_2 &= \frac{d_2 - d_3}{l_2} \end{aligned}$$

It is clear from Eqs. (4.2) and (4.3) that the radial width at each sampling interval is dependent on the workpiece geometry and the history of spindle speed and machine feed.

Using Eq. (3.1) the process cost of machining the ith interval of length x_i can be expressed as:

$$C_{p_i} = (C_h + \frac{C_t}{T_{L_i}} + \frac{C_h T_{ct}}{T_{L_i}})\Delta t \tag{4.4}$$

where

T_{L_i} is the tool life for the conditions at the ith interval

In terms of the basic machining parameters, the Eq. (4.4) can be expressed as :

$$C_{p_i} = f(d_i, N_i, s_{t_i}) \tag{4.5}$$

It has been proved by Yellowley that the highest allowable feed always results in the minimum cost. Therefore, for minimization of cost, we select the maximum allowable feed without exceeding the limit imposed by any of the constraints. Now consider the tooth breakage constraint, Eqs. (3.14) and (3.15). For any milling process, the tool is pre-selected and hence the tool radius is constant. Therefore, the feed per tooth is either a constant or a function of the radial width of cut as shown below:

$$\begin{aligned} s_{t_i} &= s_{max} && (d < R) \\ \text{or} \quad s_{t_i} &= g(d_i) && (d \geq R) \\ g(d_i) &= \frac{s_{max}}{\sin(\cos^{-1}(1 - \frac{d_i}{R}))} \end{aligned}$$

It has been shown earlier that the radial width of each time interval is a function of the workpiece geometry and the history of spindle speed and machine feed. It can be evaluated by using Eq. (4.2) or (4.3). Therefore, the feed which satisfies the torque constraint, Eq. (3.20), can be obtained from the above expressions. Equation (3.19) can then be used to find the spindle speed within the power constraint which results in a minimum process cost.

Symbol	Value
C_h	0.005 dollars/sec
C_t	$.0396.R.Z$
T_{ct}	120 sec
C_1	1179.36
m	2
n	1
p	2
q	0.5
P_{max}	7.5 K.W.
K_s	4140 N/sq. mm.
r_1	0.3
s_{max}	0.2 mm
σ_{max}	1242 N/sq. mm.

Table 4.1: Table of constants

The process cost of machining the whole workpiece is :

$$C_p = \sum_{i=1}^{n} C_{p_i} \tag{4.6}$$

For minimum process cost for the whole workpiece we will have :

$$C_{p_{min}} = \sum_{i=1}^{n} (C_{p_i})_{min} \tag{4.7}$$

The influence of cutter radii and number of teeth on cost for a generalized workpiece and a triangular workpiece when feeds and speeds are kept optimal for each interval is shown in Figs.(4.2) and (4.3) respectively. The optimal feeds and speeds for both a generalized workpiece and a triangular workpiece are graphically shown in Figs. (4.4), (4.5), (4.6) and (4.7). The values of economic parameters, tool life constant, tool life exponents, machine constraints, cutting constants and tool material properties used in this thesis are listed in table 4.1. It is obvious from Figs. (4.2) and (4.3) that it is preferable to use the smallest diameter cutter available which is capable of machining

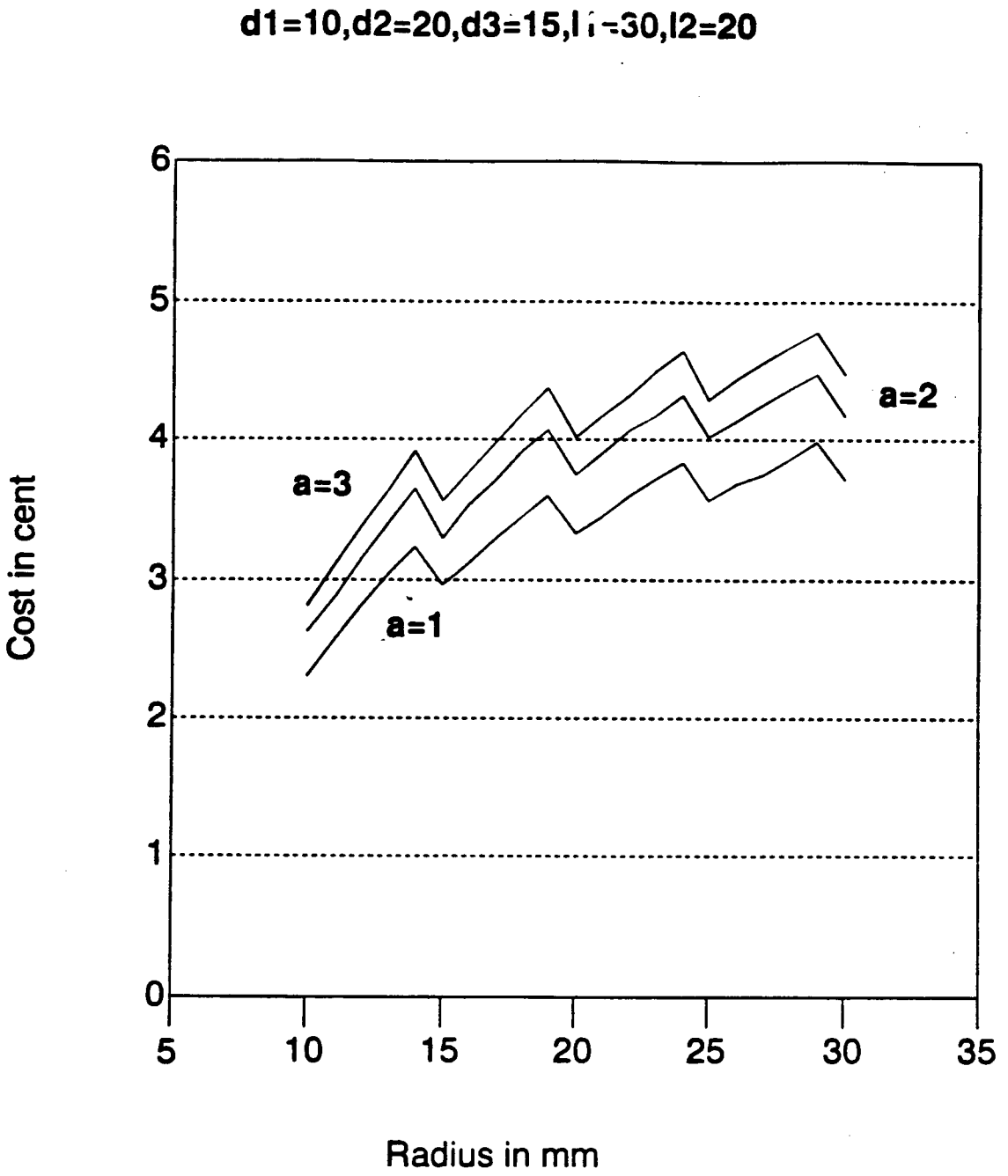

Figure 4.2: Influence of Tool Radius on Cost for a Generalized Workpiece

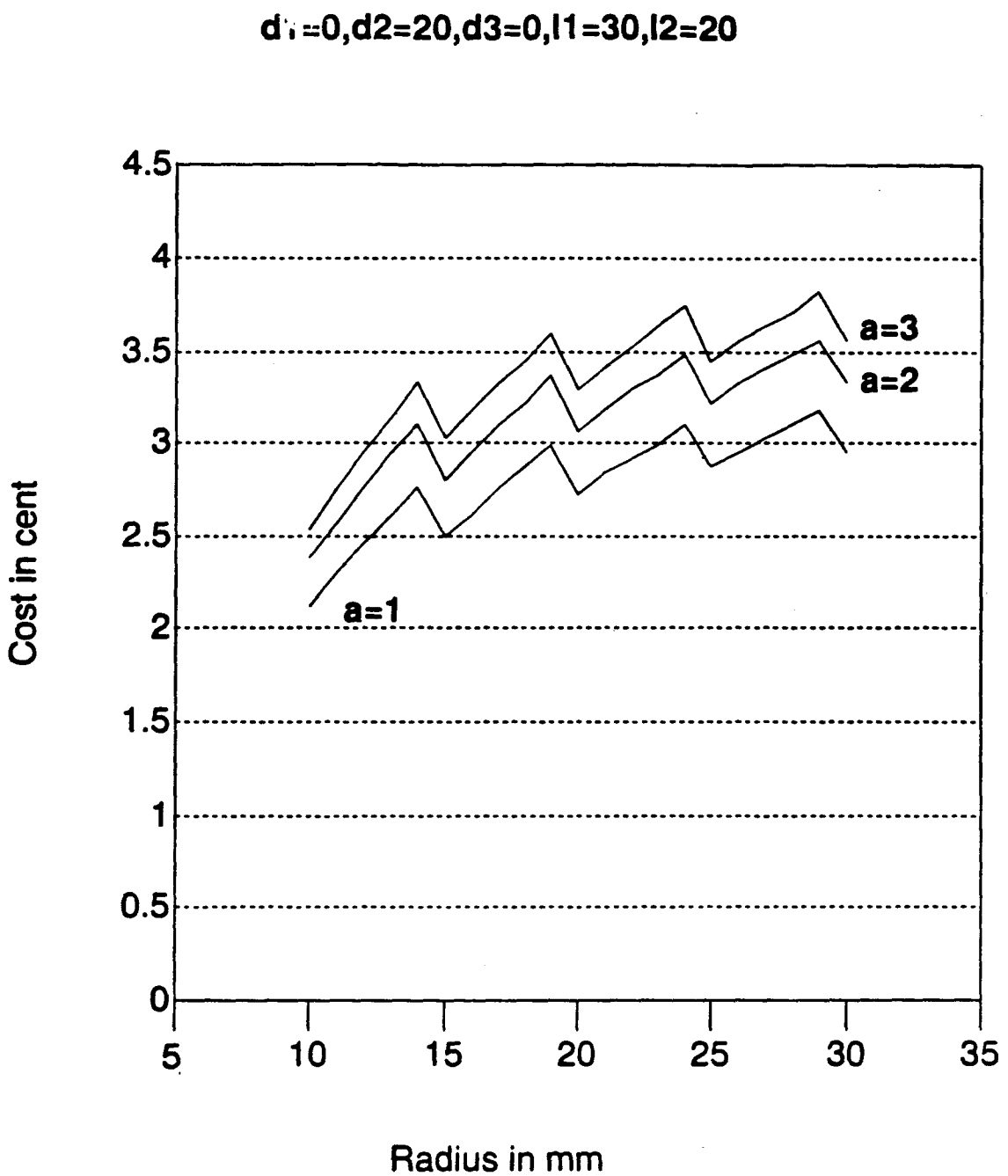

Figure 4.3: Influence of Tool Radius on Cost for a Triangle

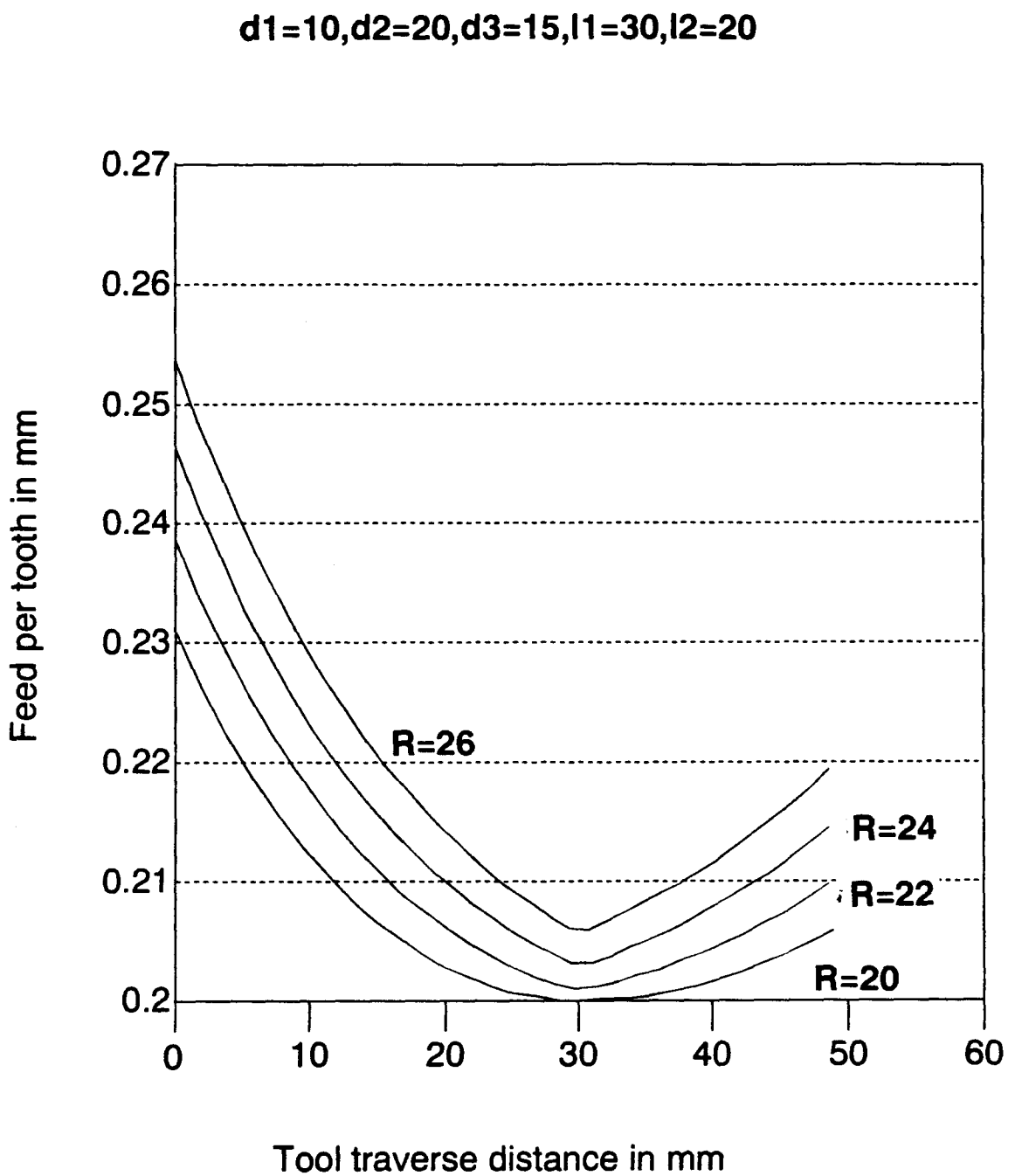

Figure 4.4: Optimum Feeds and Tool Traverse Distance For a Generalized Workpiece

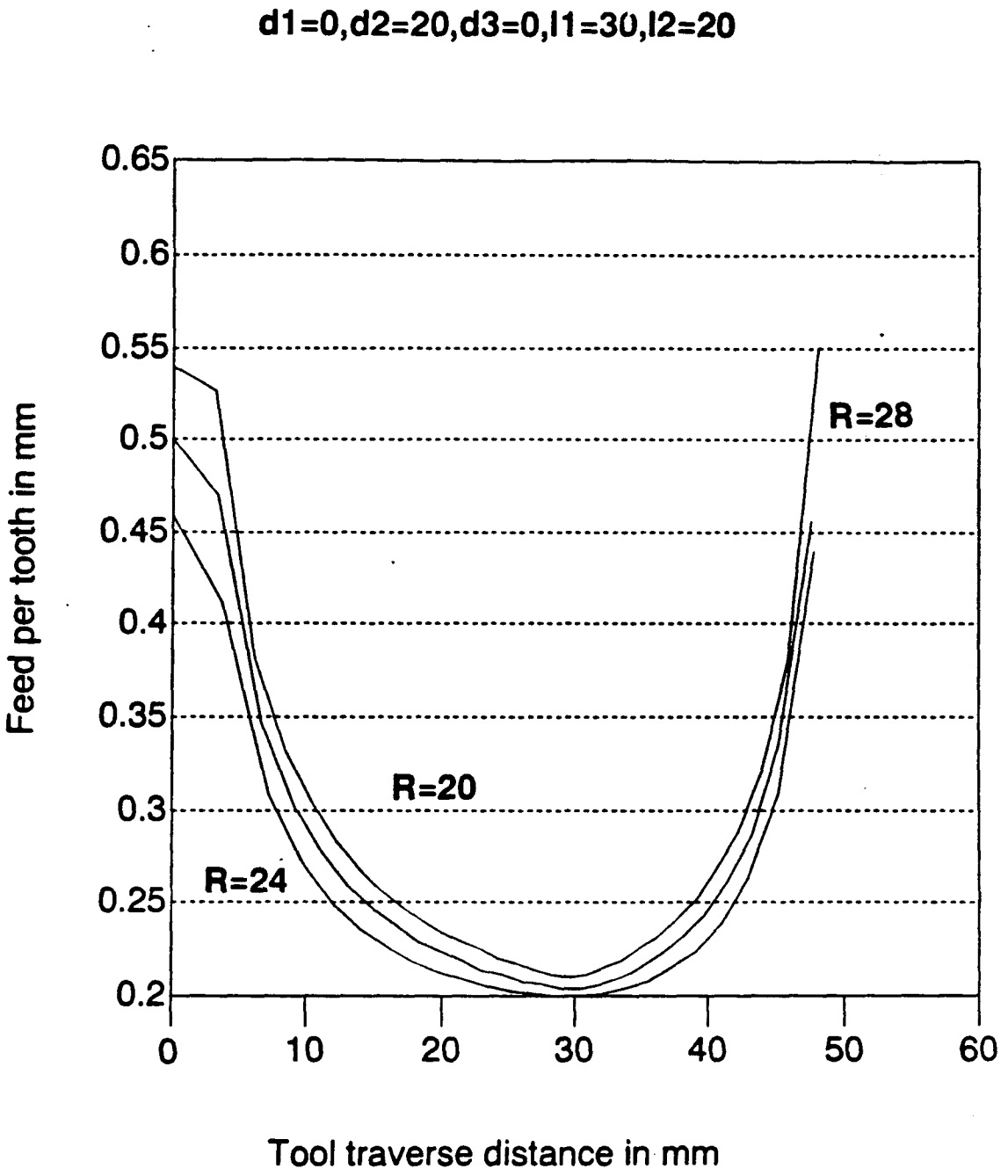

Figure 4.5: Optimum Feeds and Tool Traverse Distance For a Triangle

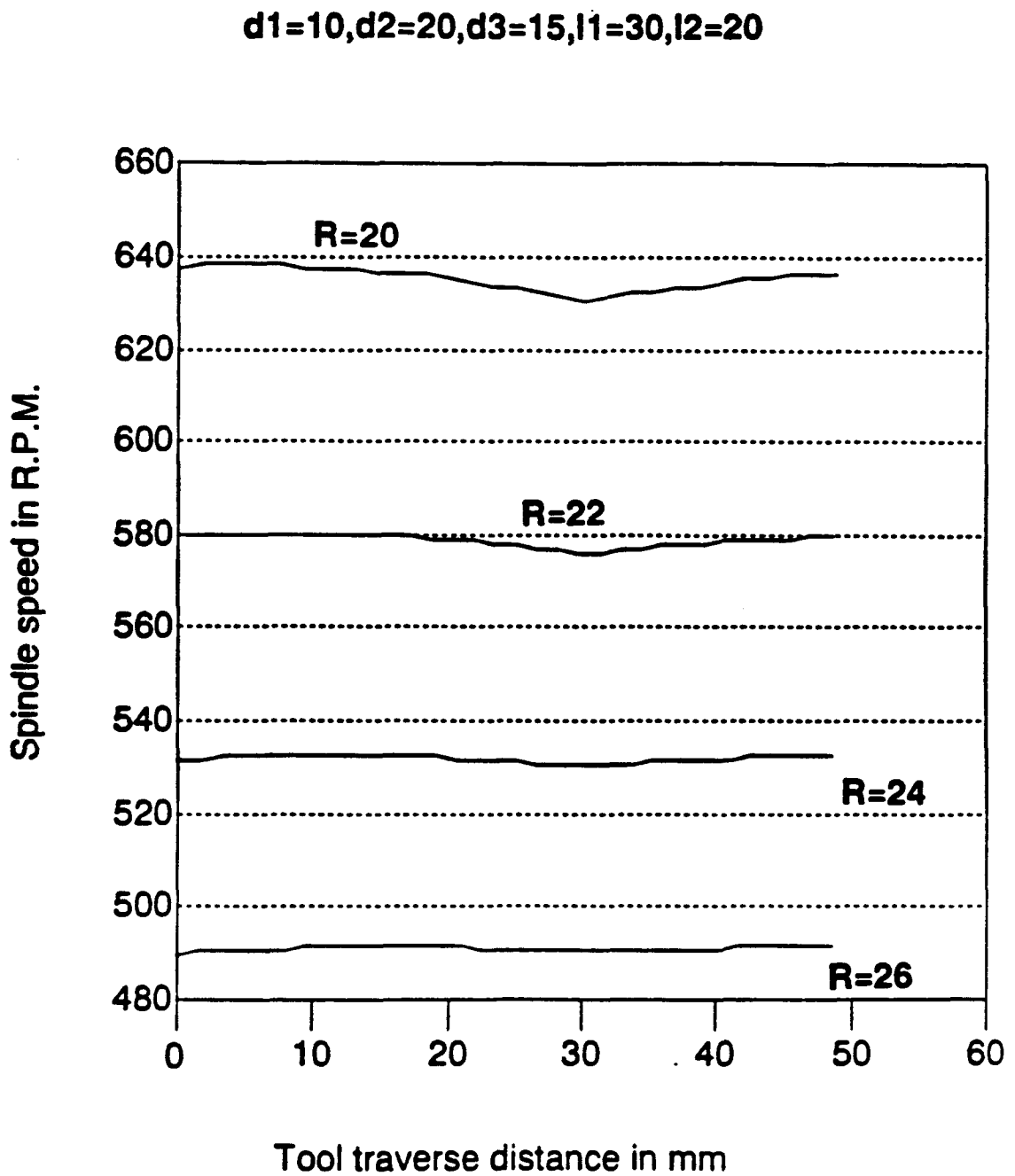

Figure 4.6: Optimum Speeds and Tool Traverse Distance For a Generalized Workpiece

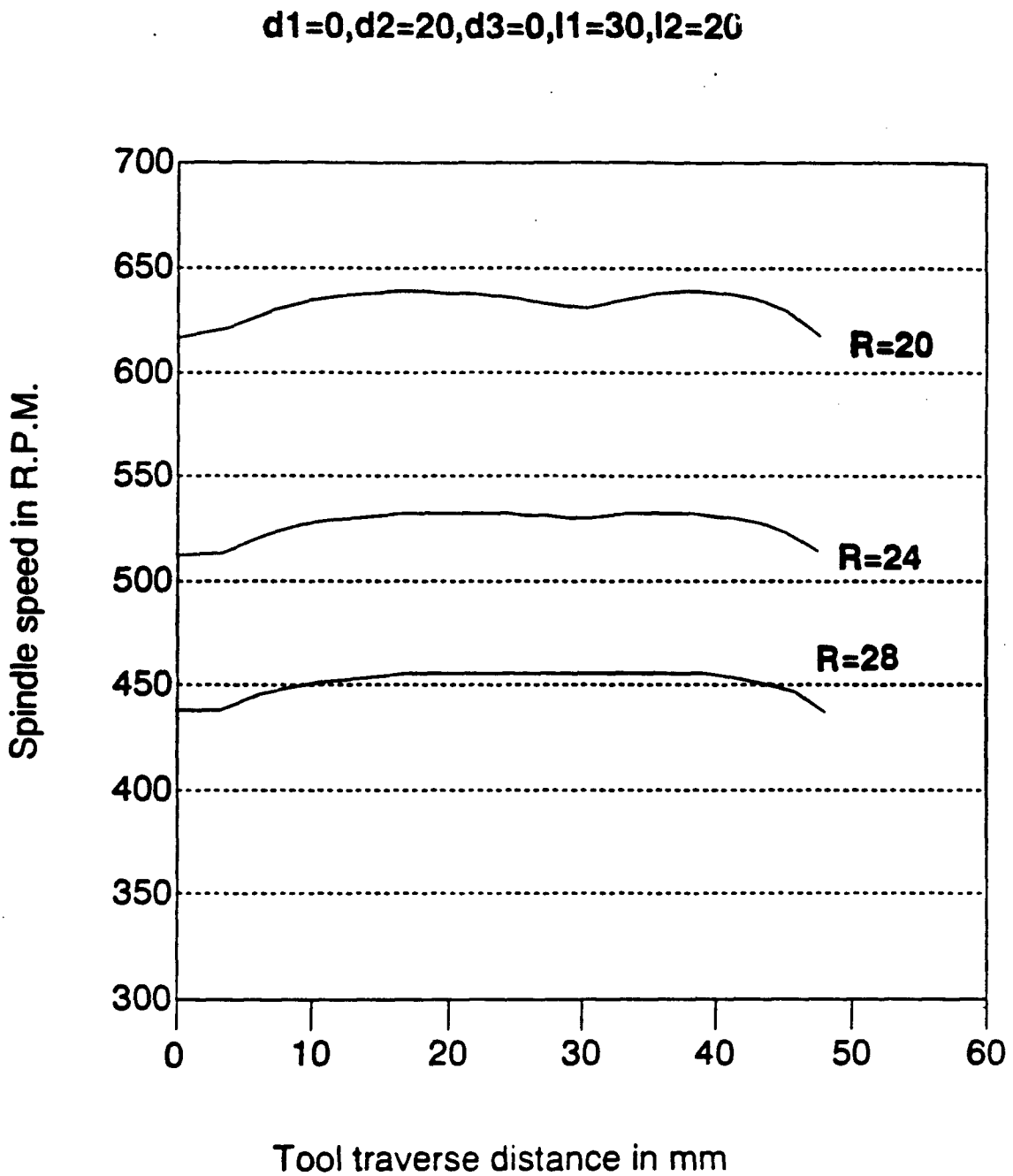

Figure 4.7: Optimum Speeds and Tool Traverse Distance For a Triangle

the workpiece or the next larger diameter with more number of teeth. These graphs also indicate that the cost of machining a larger axial depth is higher as would intuitively be expected. Graphs (4.4) and (4.5) indicate that the allowable feed per tooth decreases with increasing radial depths and also the value of allowable feed per tooth for each radial width increases with the tool diameter. No clear trend is available from the graphs of optimum spindle speeds.

It is thus clear that with the help of the optimization strategy we have employed, it is possible to determine the optimal values of feed and speed for any kind of workpiece for any cutting direction if we can obtain the radial widths either as a function of the workpiece geometry or by some other means. The optimal values of feed and speed will result in an optimal process cost.

4.1.1 Evaluation of Machining Time

The estimation of total processing time is important for efficient process planning and scheduling of manufacturing activities. The total processing time consists of the following:

- Set up time
- Loading unloading time
- Machining process time

Machining process time consists of manual time and machining time. An analytical expression for the machining time in milling when the feed is maintained at an optimal value can be derived as follows:

Machining time is a function of the workpiece geometry, tool diameter, number of teeth and the tool traverse rate. The tool diameter and the number of teeth can be

selected on the basis of previous analysis and availability. The tool traverse rate is dependent on the machining constraints. Therefore, it is possible to mathematically evaluate the machining time for many workpiece geometries by making use of the constraint relations.

Let us again consider the geometry of Fig. (4.1). Tooth breakage can be avoided by restricting the maximum chip thickness encountered to some constant value according to the inequality relations of Eqs. (3.14) and (3.15). Rewriting those relations, we have

$$s_t \sin \phi_s \leq s_{max} \qquad (d < R) \tag{4.8}$$

$$s_t < s_{max} \qquad (d \geq R) \tag{4.9}$$

where

ϕ_s is the swept angle in radians.

s_{max} is the maximum allowable feed per tooth in mm.

s_t is the feed per tooth in mm.

Case 1:$d < R$

The tooth breakage constraint for this case is governed by equation (4.8).Rewriting equation (4.8)

$$s_t \sin \phi_s \leq s_{max} \qquad (d < R) \tag{4.10}$$

This equation gives:

$$s_t \leq \frac{s_{max}}{\sin \phi_s} \tag{4.11}$$

The tool traverse rate in mms per second can be obtained from the relation:

$$\frac{dx}{dt} = \frac{s_t N Z}{60} \tag{4.12}$$

where

N is the spindle r.p.m.

Z is the number of teeth.

For optimum conditions, feed per tooth should be as large as possible. Therefore (4.11) assumes the form:

$$s_t = \frac{s_{max}}{\sin \phi_s} \tag{4.13}$$

Combining (4.12) and (4.13) we obtain:

$$\frac{dx}{dt} = \frac{s_{max} N Z}{60 \sin \phi_s} \tag{4.14}$$

A constant spindle R.P.M. which does not exceed the constraints can be determined for the given workpiece geometry. The selection of the tool is made before the start of machining so that the tool radius R and the number of teeth Z are constant during machining. The maximum permissible value of s_{max} is also constant for a tool/workpiece combination. Therefore, the tool traverse rate can be represented by the relation:

$$\frac{dx}{dt} = \frac{C}{\sin \phi_s} \tag{4.15}$$

where

$$C = \frac{s_{max} N Z}{60} \tag{4.16}$$

From (4.15), we obtain the following relation:

$$dt = \frac{1}{C} \sin \phi_s dx \tag{4.17}$$

We can integrate this equation to evaluate the time taken to traverse a specified length.

let

t_1 is the time taken to traverse length l_1

t_2 is the time taken to traverse length l_2

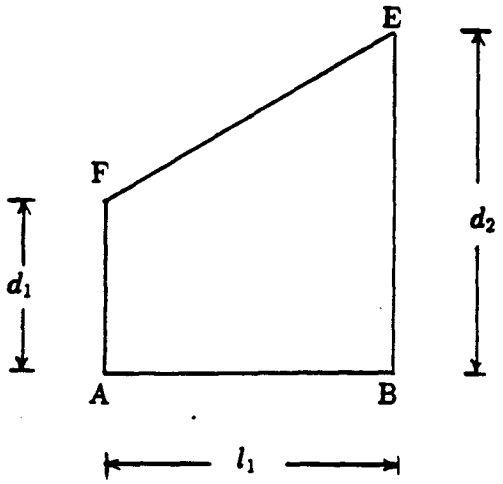

Figure 4.8: Geometry of Portion ABEF (Top View)

Then for portion ABEF Fig. (4.8),we obtain:

$$\int_0^{t_1} dt = \frac{1}{C}\int_0^{l_1} \sin\phi_s dx \tag{4.18}$$

The swept angle ϕ_s can be represented in mathematical form as follows:

$$\phi_s = \cos^{-1}(1 - \frac{d}{R}) \tag{4.19}$$

The radial depth of tool at an arbitrary position during its traverse can be found from the following expression:

$$d = d_1 + k_1 x \tag{4.20}$$

The slope k_1 can be represented by the following relation:

$$k_1 = \frac{d_2 - d_1}{l_1} \tag{4.21}$$

here

x is the distance traversed by the tool

Using Eqs. (4.19), (4.20) and (4.21) in Eq. (4.18) we obtain the following expression:

$$\int_0^{t_1} dt = \frac{1}{C}\int_0^{l_1} \sin\cos^{-1}[1 - \frac{d_1 + k_1 x}{R}]dx \tag{4.22}$$

let

$$\begin{aligned} m_1 &= 1 - \frac{d_1}{R} \\ m_2 &= \frac{k_1}{R} \end{aligned}$$

Integrating the left hand side of Eq. (4.22) and using the expressions for m_1 and m_2 in that equation, we obtain:

$$t_1 = \frac{1}{C}\int_0^{l_1} \sin\cos^{-1}(m_1 - m_2 x)dx \tag{4.23}$$

let

$$\cos^{-1}(m_1 - m_2 x) = y_1 \tag{4.24}$$

then

$$\begin{aligned} m_1 - m_2 x &= \cos y_1 \\ x &= \frac{1}{m_2}(m_1 - \cos y_1) \\ dx &= \frac{\sin y_1 dy_1}{m_2} \end{aligned}$$

Substituting the variable y_1 in place of x by using the above relations, equation (4.23) takes the following form:

$$t_1 = \frac{1}{Cm_2}\int \sin^2 y_1 dy_1 \tag{4.25}$$

The new limits of integration can be evaluated as follows:

when

$$\begin{aligned} x &= 0 \\ y_1 &= \cos^{-1} m_1 \end{aligned}$$

and when

$$\begin{aligned} x &= l_1 \\ y_1 &= \cos^{-1}(m_1 - m_2 l_1) \end{aligned}$$

Equation (4.25) can also be written in the form:

$$t_1 = \frac{1}{2Cm_2}\int(1 - \cos 2y_1)dy_1 \tag{4.26}$$

Integrating the right hand side of this expression, we obtain:

$$t_1 = \frac{1}{2Cm_2}(y_1 - \frac{\sin 2y_1}{2}) \tag{4.27}$$

Applying the limits of integration to the above expression we obtain:

$$\begin{aligned} t_1 &= \frac{1}{2Cm_2}[\cos^{-1}(m_1 - m_2 l_1) - \frac{1}{2}\sin(2(\cos^{-1}(m_1 - m_2 l_1))) \\ &\quad - \cos^{-1} m_1 + \frac{1}{2}\sin 2(\cos^{-1} m_1)] \end{aligned}$$

Rearranging the terms,

$$\begin{aligned} t_1 &= \frac{1}{2Cm_2}[\cos^{-1}(m_1 - m_2 l_1) - \cos^{-1} m_1] \\ &\quad + \frac{1}{4Cm_2}[\sin(2\cos^{-1} m_1) - \sin 2(\cos^{-1}(m_1 - m_2 l_1))] \end{aligned}$$

The time of traverse for any distance x between zero and l_1 can be obtained from the following expression:

$$\begin{aligned} t_x &= \frac{1}{2Cm_2}[\cos^{-1}(m_1 - m_2 x) - \frac{1}{2}\sin 2(\cos^{-1}(m_1 - m_2 x))) \\ &\quad - \cos^{-1} m_1 + \frac{1}{2}\sin 2(\cos^{-1} m_1)] \end{aligned} \tag{4.28}$$

where

t_x is the time of traverse for a distance x between length l_1

x is the distance traversed

let

$$\begin{aligned} m_3 &= \frac{1}{2Cm_2} \\ m_{11} &= \cos^{-1}(m_1 - m_2 x) \\ m_{12} &= cos^{-1} m_1 \\ m_{13} &= sin[2(m_{11})] \\ m_{14} &= sin[2(m_{12})] \end{aligned} \tag{4.29}$$

With these substitutions, equation (4.28) becomes:

$$t_x = m_3(m_{11} - m_{12}) + \frac{m_3}{2}(m_{14} - m_{13}) \tag{4.30}$$

The above expression is valid for any distance x between zero and l_1

Similarly we can integrate equation (4.17) for portion BCDE Fig (4.9) as follows:

$$\int_0^{t_2} dt = \frac{1}{C}\int_0^{l_2} \sin\phi_s dx \tag{4.31}$$

where

ϕ_s is the swept angle as before

From Fig. (4.9), the radial depth at an arbitrary point along length l_2 can be determined from the following relation:

$$d = d_2 - k_2 x \tag{4.32}$$

where

$$k_2 = \frac{d_2 - d_3}{l_2} \tag{4.33}$$

Substituting the above expressions for d and k_2 in equation (4.31), we have:

$$\int_0^{t_2} dt = \frac{1}{C}\int_0^{l_2} \sin[\cos^{-1}(1 - \frac{d_2 - k_2 x}{R})]dx \tag{4.34}$$

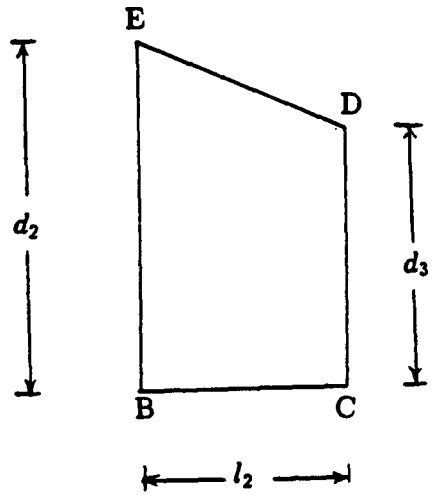

Figure 4.9: Geometry of Portion BCDE (Top View)

let

$$\begin{aligned} m_5 &= 1 - \frac{d_2}{R} \\ m_6 &= \frac{k_2}{R} \end{aligned}$$

Integrating left hand side of Eq. (4.34) and using m_5 and m_6 in that equation, we obtain:

$$t_2 = \frac{1}{C}\int_0^{l_2} \sin[\cos^{-1}(m_5 + m_6 x)]dx \tag{4.35}$$

let

$$\cos^{-1}(m_5 + m_6 x) = y_2 \tag{4.36}$$

then

$$\begin{aligned} m_5 + m_6 x &= \cos y_2 \\ x &= \frac{1}{m_6}(\cos y_2 - m_5) \\ dx &= -\frac{\sin y_2 dy_2}{m_6} \end{aligned}$$

Substituting the variable y_2 in place of x in Eq. (4.35) we have:

$$t_2 = -\frac{1}{Cm_6}\int(\sin^2 y_2)dy_2 \tag{4.37}$$

The new limits of integration can be evaluated as follows:

when

$$\begin{aligned} x &= 0 \\ y_2 &= \cos^{-1} m_5 \end{aligned}$$

and when

$$\begin{aligned} x &= l_2 \\ y_2 &= \cos^{-1}(m_5 + m_6 l_2) \end{aligned}$$

Writing Eq. (4.37) in a slightly different form:

$$t_2 = -\frac{1}{2Cm_6}\int(1 - \cos 2y_2)dy_2 \tag{4.38}$$

Integrating the right hand side of this expression, we obtain:

$$t_2 = \frac{1}{2Cm_6}(\frac{\sin 2y_2}{2} - y_2) \tag{4.39}$$

Applying the limits of integration :

$$\begin{aligned} t_2 &= \frac{1}{2Cm_6}[\frac{1}{2}\sin 2(\cos^{-1}(m_5 + m_6 l_2)) \\ &\quad - \cos^{-1}(m_5 + m_6 l_2) - \frac{1}{2}\sin 2(\cos^{-1}(m_5)) + \cos^{-1} m_5] \end{aligned}$$

Rearranging the terms,

$$\begin{aligned} t_2 &= \frac{1}{2Cm_6}[\cos^{-1} m_5 - \cos^{-1}(m_5 + m_6 l_2)] \\ &\quad + \frac{1}{4Cm_6}[\sin 2(\cos^{-1}(m_5 + m_6 l_2) - \sin 2(\cos^{-1} m_5)] \end{aligned}$$

The time of traverse for any distance x between l_1 and l_2 can be obtained from the following expression:

$$\begin{aligned} t_x &= \frac{1}{2Cm_6}[\cos^{-1} m_5 - \cos^{-1}(m_5 + m_6 x)] \\ &+ \frac{1}{4Cm_6}[\sin 2(\cos^{-1}(m_5 + m_6 x)) - \sin 2(\cos^{-1} m_5)] \end{aligned}$$

where

t_x is the time of traverse for a distance x between length l_1 and length l_2.

x is the distance traversed.

let

$$\begin{aligned} m_7 &= \frac{1}{2c_1 m_6} \\ m_{17} &= \cos^{-1}(m_5 + m_6 x) \\ m_{18} &= \cos^{-1} m_5 \\ m_{19} &= sin[2(m_{17})] \\ m_{20} &= sin[2(m_{18})] \end{aligned}$$

Substituting the new nomenclature in equation (4.40) we have:

$$t_x = m_7(m_{18} - m_{17}) + \frac{m_7}{2}(m_{19} - m_{20}) \tag{4.40}$$

The above equation is valid for any distance between l_1 and l_2

Case2:$d \geq R$

The tooth breakage constraint for this case is governed by the following equation:

$$s_t < s_{max} \qquad (d \geq R) \tag{4.41}$$

let the operating feed s_{t_o} be slightly less than the maximum allowable feed s_{max}, then the tool traverse rate in mms per second can be obtained from the relation:

$$\frac{dx}{dt} = \frac{s_{t_o} N Z}{60} \tag{4.42}$$

here

s_{t_o} is a constant operating speed

Integration of the above expression yields:

$$t_x = \frac{x}{C} \tag{4.43}$$

where

t_x is the time of traverse for a distance x between length $(l_1 + l_2)$.

x is the distance traversed.

and

$$C = \frac{s_{t_o} N Z}{60} \tag{4.44}$$

It is clear from Eqs. (4.30), (4.40) and (4.43) that the machining time for milling is a function of the workpiece geometry, tool diameter, number of teeth and the tool traverse rate. For many industries such as aircraft industry increasing the production rate is a highly desirable goal. The analytical expressions for machining time can be used to determine the machining parameters which minimize the production time for specific workpiece geometries. Since minimization of process cost is the focus of this thesis it has been decided not to explore the criterion of minimization of production time any further.

Chapter 5

Optimization of Multi-Pass Milling Operations

The maximum allowable radial width of cut for a single pass is dependent on the chatter constraint and the size of the milling cutters available in the machine shop. When the radial width of cut for machining is larger than the maximum allowable value, there is a need for multi-pass milling operations. This need often arises in rough milling operations. For these cases, it is therefore necessary to select an appropriate radial width for each pass. This must be carried out in a manner which minimizes cost. Yellowley and Gunn [2] have given a mathematical formulation for multipass milling operations, and established an optimal selection procedure for the radial width of cut for each pass. This chapter deals with the optimal selection of radial widths for various workpiece geometries.

Let us once again consider the generalized workpiece, Fig (4.1), of chapter 3. Rewriting Eq. (4.5)

$$C_{p_i} = f(d_i, N_i, s_{t_i}) \tag{5.1}$$

It was shown in chapter 3 that the objective for a single pass milling operation is to minimize the value of C_p^i for each sampling interval. The radial width of the workpiece d_i for each sampling interval is fixed. For a constant cutter radius the feed per tooth s_{t_i} is obtained from the tooth breakage and torque constraints. The spindle speed N_i which does not exceed the limit imposed by the power constraint and results in the minimum cost is selected for each sampling interval. The process cost of machining the whole

workpiece is obtained from the following expression:

$$C_p = \sum_{i=1}^{n} C_{p_i} \tag{5.2}$$

For a multipass operation the radial width d_i for each interval can be varied by changing the subdivision of the machining area. Our objective is then to minimize the following expression:

$$C_{p_{i_j}} = min. \sum_{j=1}^{k} (\sum_{i_j=1}^{n_j} f(d_{i_j}, N_{i_j}, s_{t_{i_j}}) \tag{5.3}$$

Subscript i denotes the number of intervals and j denotes the number of passes. For example d_{i_j} represents the radial width at the ith interval and jth pass. Mathematical formulation and computational results for two pass milling operations are presented below. These results will then be used to make conclusions about a general multi-pass milling operation.

5.1 Mathematical Formulation

The generalized workpiece of Fig. (4.1) can be subdivided into three different geometric shapes depending on the maximum radial width d_r selected for the first pass. A suitable range for the selection of max. radial width of the first pass is chosen.

let

d_r be the maximum radial width of the first pass

d_q be the maximum radial width of the second pass

R be the radius of the cutter for both the passes

$d_{r_{min}}$ be the lower limit in the range of max. radial width of the first pass

$d_{r_{max}}$ be the upper limit in the range of max. radial width of the first pass

λ_{1_1} be the workpiece length for the first pass

λ_2 be the workpiece length for the second pass

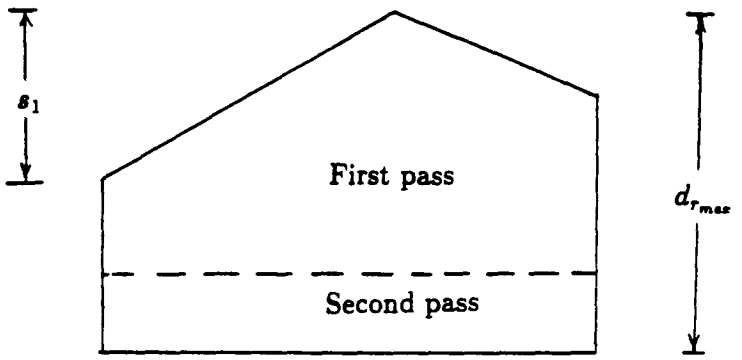

Figure 5.1: $s_1 \leq d_r \leq d_{r_{max}}$ (Top view)

here

$$
\begin{aligned}
d_q &= d_2 - d_r \\
\lambda_1 &= \sum_{i_1=1}^{n_1} x_{i_1} \\
\lambda_2 &= \sum_{i_2=1}^{n_2} x_{i_2}
\end{aligned}
$$

Let us introduce the following geometric constants:

$$
\begin{aligned}
s_1 &= d_2 - d_1 \\
s_2 &= d_2 - d_3
\end{aligned}
$$

Figs. (5.1), (5.2) and (5.3) show the three different ways in which the workpiece can be subdivided when d_r moves in its range from $d_{r_{min}}$ to $d_{r_{max}}$. It is obvious that with the change in the value of d_r, the dimensions of each subdivision also undergo a change. Therefore, our aim is to determine the dimensions of each part at every value of d_r within the selected range and evaluate the machining cost. We can thus ascertain the best subdivision of the workpiece for minimum machining cost.

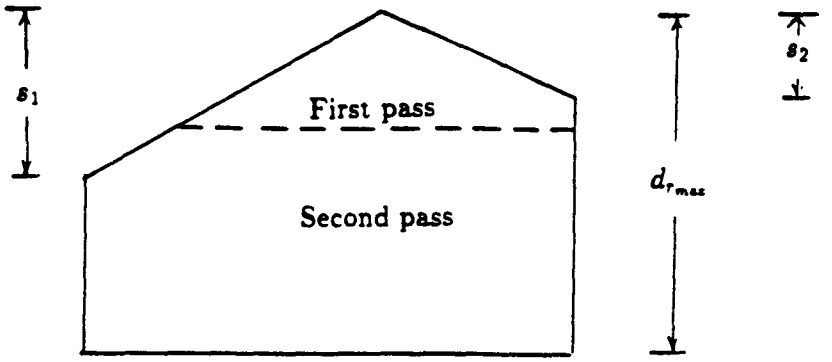

Figure 5.2: $s_2 \leq d_r \leq s_1$ (Top view)

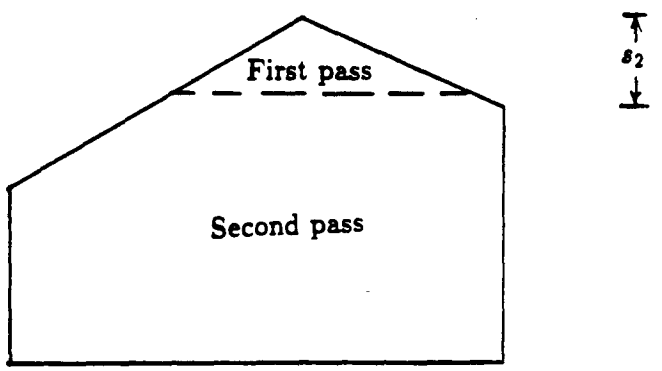

Figure 5.3: $d_{r_{min}} \leq d_r \leq s_2$ (Top view)

Using Eq. (4.4), the process cost of machining the ith interval of jth pass of length x_{i_j} can be expressed as:

$$C_{p_{i_j}} = (C_h + \frac{C_t}{T_{L_{i_j}}} + \frac{C_h T_{ct}}{T_{L_{i_j}}})\Delta t \tag{5.4}$$

where

$T_{L_{i_j}}$ is the tool life for the conditions at the ith interval of jth pass

Δt is the sampling period (5.5)

In terms of the basic machining parameters, Eq. (5.4) can be expressed as:

$$C_{p_{i_j}} = f(d_{i_j}, N_{i_j}, s_{t_{i_j}}) \tag{5.6}$$

The process cost of machining the whole workpiece is :

$$C_p = \sum_{j=1}^{k}(\sum_{i_j=1}^{n_j} C_{p_{i_j}} \tag{5.7}$$

For minimum process cost, we should have:

$$C_{p_{min}} = \sum_{j=1}^{k}(\sum_{i_j=1}^{n_j}(C_{p_{i_j}})_{min} \tag{5.8}$$

In order to determine the influence of d_r on cost, we should be able to express the machining cost as a function of d_r. It has been shown earlier (Eqs. (4.2) and (4.3)) that the radial width is a function of the workpiece geometry and the history of spindle speed and machine feed. In addition, in a two-pass milling operation, the radial width would also depend on the way the workpiece has been subdivided . In other words the radial width can be represented as a function of d_r, workpiece geometry and the history of spindle speed and feed. Workpiece geometry and the history of spindle speed and feed are known beforehand. Therefore, it is possible to determine the influence of d_r on cost by using Eq.(5.6).

Let us mathematically derive the relationships for the radial width for the three cases represented by figures (5.1), (5.2) and (5.3).

5.1.1 Case 1: $d_{r_{min}} \leq d_r \leq s_2$

For this case the radial width of the first pass is given by one of the following two relations:

$$d_{i_1} = k_1(\sum_{k=1}^{i_1-1} x_k) \qquad (0 < \sum_{k=1}^{i_1-1} x_k < \frac{d_r}{k_1})$$

$$d_{i_1} = d_r - k_2(\sum_{k=1}^{i_1-1} x_k) \qquad (\frac{d_r}{k_1} < \sum_{k=1}^{i_1-1} x_k < \frac{d_r}{k_1} + \frac{d_r}{k_2})$$

where

$$\sum_{k=1}^{i_1-1} x_k = \frac{Z\Delta t}{60}.(\sum_{k=1}^{i_1-1} s_{t_k} N_k)$$

$$k_1 = \frac{d_2 - d_1}{l_1}$$

$$k_2 = \frac{d_2 - d_3}{l_2}$$

The radial width of the second pass can be obtained from one of the following three relations:

$$d_{i_2} = d_1 + k_1(\sum_{k=1}^{i_2-1} x_k) \qquad (0 < \sum_{k=1}^{i_2-1} x_k < l_1 - \frac{d_r}{k_1})$$

$$d_{i_2} = d_2 - d_r \qquad (l_1 - \frac{d_r}{k_1} < \sum_{k=1}^{i_2-1} x_k < l_1 + \frac{d_r}{k_2})$$

$$d_{i_2} = (d_2 - d_r) - k_2(\sum_{k=1}^{i_2-1} x_k) \qquad (l_1 + \frac{d_r}{k_2} < \sum_{k=1}^{i_2-1} x_k < l_1 + l_2)$$

$$\lambda_1 = \frac{d_r}{k_1} + \frac{d_r}{k_2}$$

$$\lambda_2 = l_1 + l_2$$

5.1.2 Case 2: $s_2 < d_r \leq s_1$

For this case the radial width of the first pass is given by one of the following two relations:

$$d_{i_1} = k_1(\sum_{k=1}^{i_1-1} x_k) \qquad (0 < \sum_{k=1}^{i_1-1} x_k < \frac{d_r}{k_1})$$

$$d_{i_1} = (d_3 - (d_2 - d_r)) - k_2 \sum_{k=1}^{i_1-1}(x_k - l_1) \qquad (\frac{d_r}{k_1} < \sum_{k=1}^{i_1-1} x_k < \frac{d_r}{k_1} + l_2)$$

where

$$\sum_{k=1}^{i_1-1} x_k = \frac{Z\Delta t}{60}.(\sum_{k=1}^{i_1-1} s_{t_k} N_k)$$
$$k_1 = \frac{d_2 - d_1}{l_1}$$
$$k_2 = \frac{d_2 - d_3}{l_2}$$

The expressions for the radial width of the second pass are :

$$d_{i_2} = d_1 + k_1(\sum_{k=1}^{i_2-1} x_k) \qquad (0 < \sum_{k=1}^{i_2-1} x_k < l_1 - \frac{d_r}{k_1})$$
$$d_{i_2} = d_2 - d_r \qquad (l_1 - \frac{d_r}{k_1} < \sum_{k=1}^{i_2-1} x_k < l_1 + l_2)$$

The length of the workpiece for the two passes :

$$\lambda_1 = \frac{d_r}{k_1} + l_2$$
$$\lambda_2 = l_1 + l_2$$

5.1.3 Case 3: $s_1 < d_r \leq d_{r_{max}}$

The radial width of first pass can be obtained by one of the following two relations:

$$d_{i_1} = (d_1 - (d_2 - d_r)) + k_1(\sum_{k=1}^{i_1-1} x_k) \qquad (0 < \sum_{k=1}^{i_1-1} x_k < l_1)$$
$$d_{i_1} = (d_3 - (d_2 - d_r)) - k_2 \sum_{k=1}^{i_1-1}(x_k - l_1) \qquad (l_1 < \sum_{k=1}^{i_1-1} x_k < l_1 + l_2)$$

where

$$\begin{aligned}
\sum_{k=1}^{i_1-1} x_k &= \frac{Z\Delta t}{60}.(\sum_{k=1}^{i_1-1} s_{t_k} N_k) \\
k_1 &= \frac{d_2 - d_1}{l_1} \\
k_2 &= \frac{d_2 - d_3}{l_2}
\end{aligned}$$

The radial width of the second pass is:

$$d_{i_2} = d_2 - d_r \qquad (0 < \sum_{k=1}^{i_2-1} x_k < l_1 + l_2) \tag{5.9}$$

The length of the workpiece for the two passes :

$$\begin{aligned}
\lambda_1 &= l_1 + l_2 \\
\lambda_2 &= l_1 + l_2
\end{aligned}$$

5.1.4 Evaluation of Cost

Feed per tooth for a constant tool radius can be obtained from the tooth breakage constraint from the following relations:

$$\begin{aligned}
s_{t_{i_j}} &= s_{max} && (d < R) \\
\text{or} \qquad s_{t_{i_j}} &= g(d_{i_j}) && (d \geq R) \\
g(d_{i_j}) &= \frac{s_{max}}{\sin(\cos^{-1}(1 - \frac{d_{i_j}}{R}))}
\end{aligned}$$

The feed calculated from the above formula is reduced until it satisfies the torque constraint. Eqs. (5.4), (5.6) and (5.7) can then be used to find that spindle speed within the power constraint which gives the minimum machining cost. The cost characteristics for a rectangle, triangle, an equilateral triangle, a symmetric and an unsymmetric generalized workpiece are shown in Figs. (5.4), (5.5), (5.6), (5.7) and (5.8) respectively. These cost characteristics are for a two pass milling operation when the speeds and the

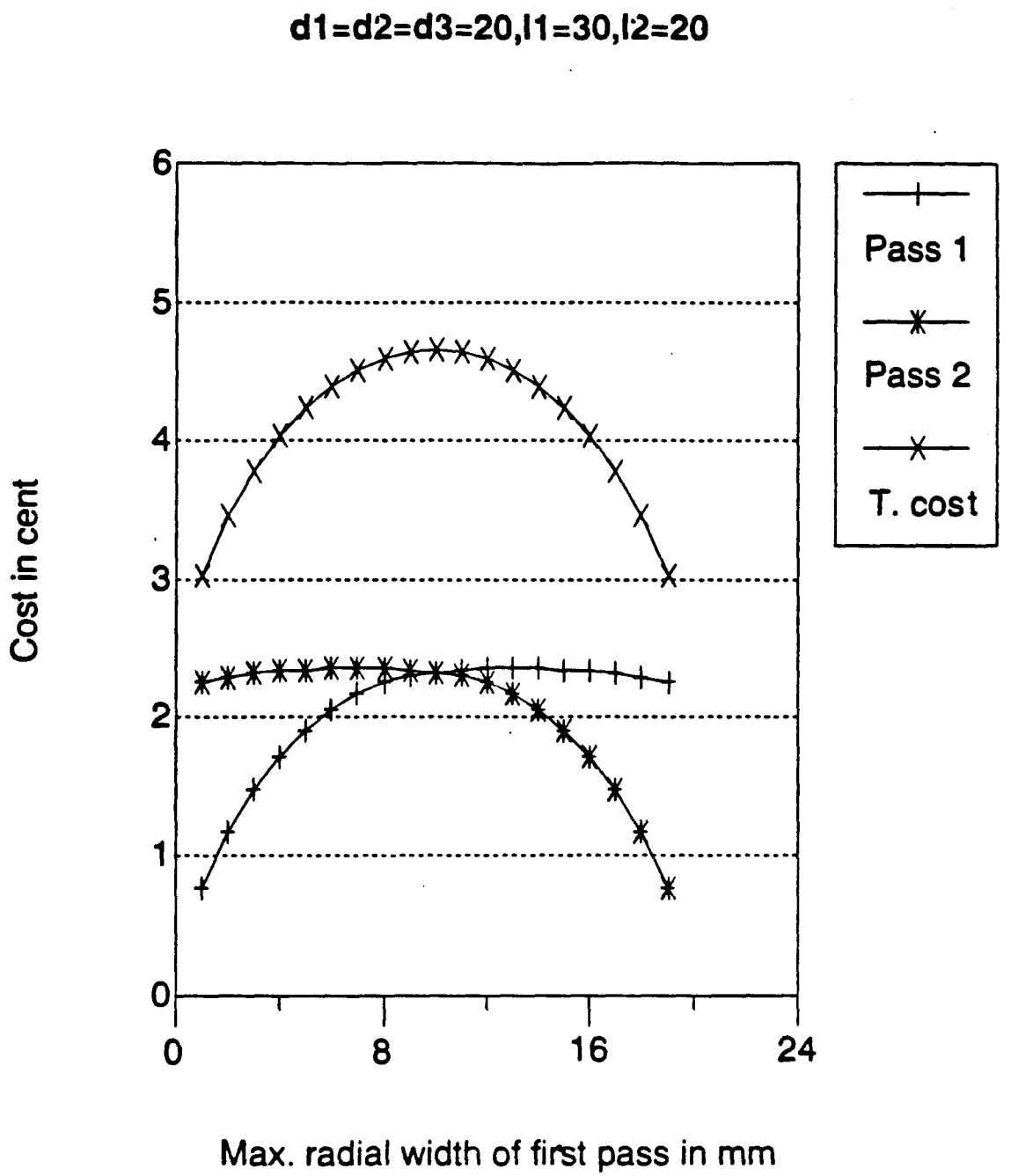

Figure 5.4: Cost Characteristics of a Rectangle

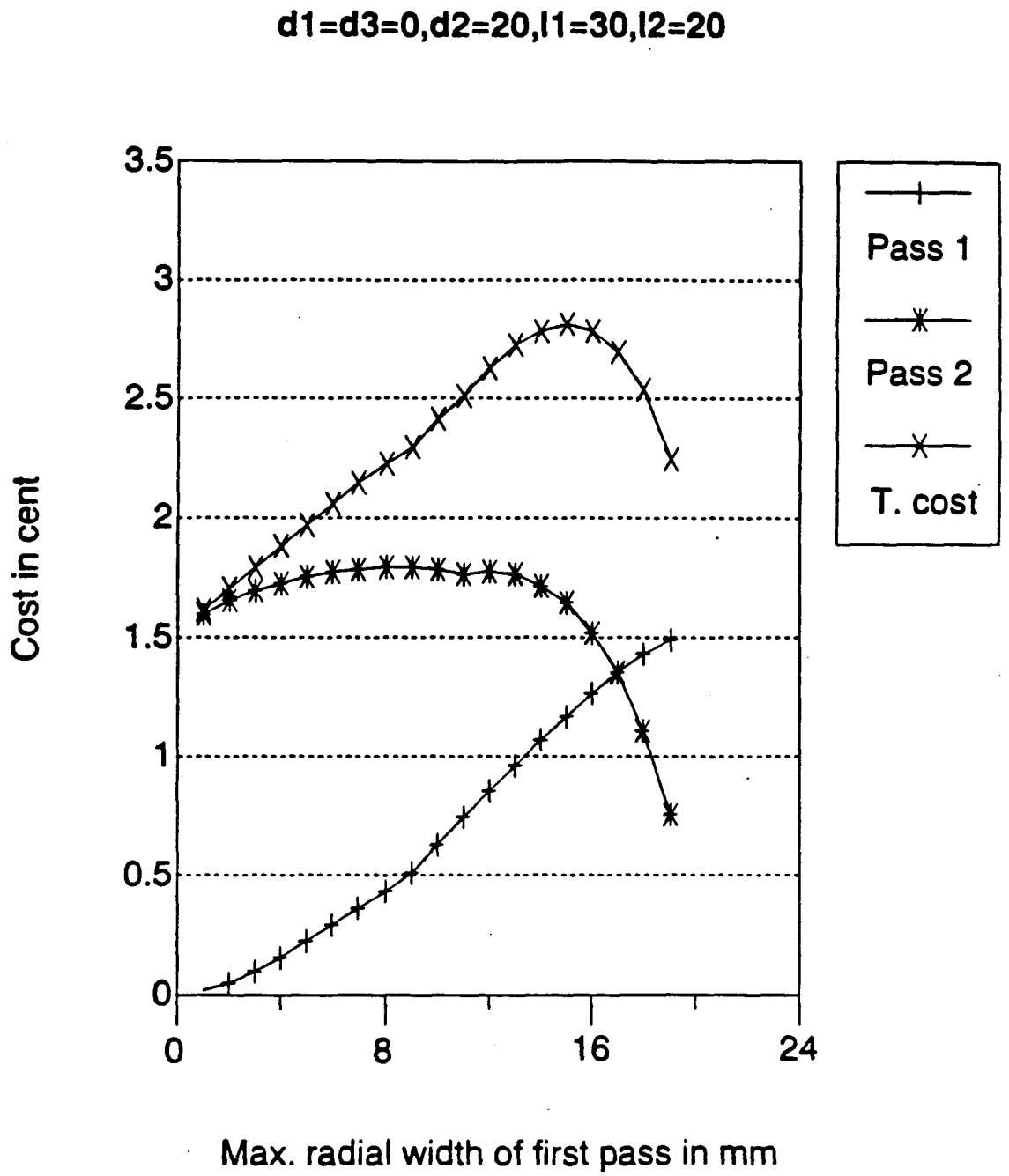

Figure 5.5: Cost Characteristics of a Triangle

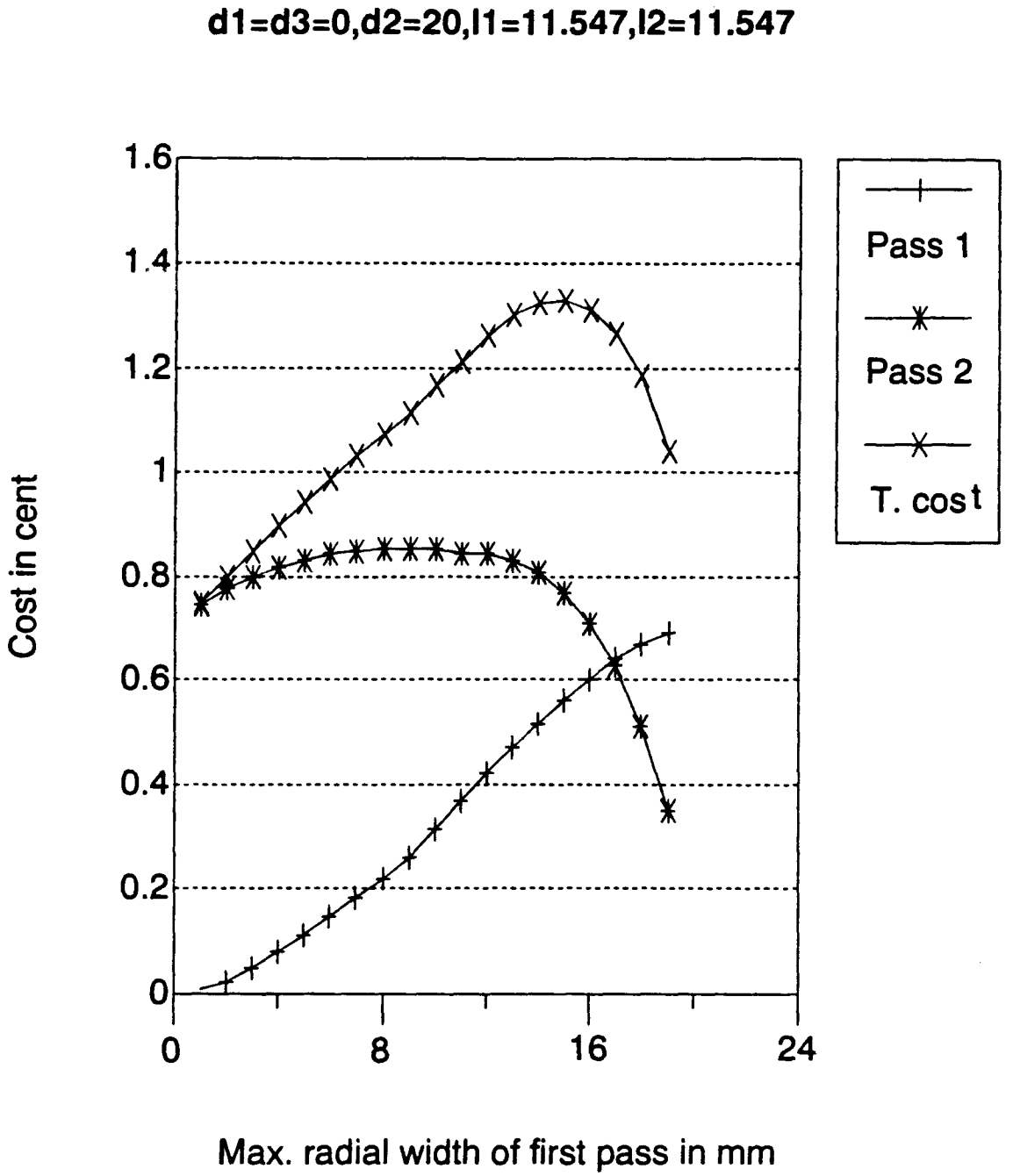

Figure 5.6: Cost Characteristics of an Equilateral Triangle

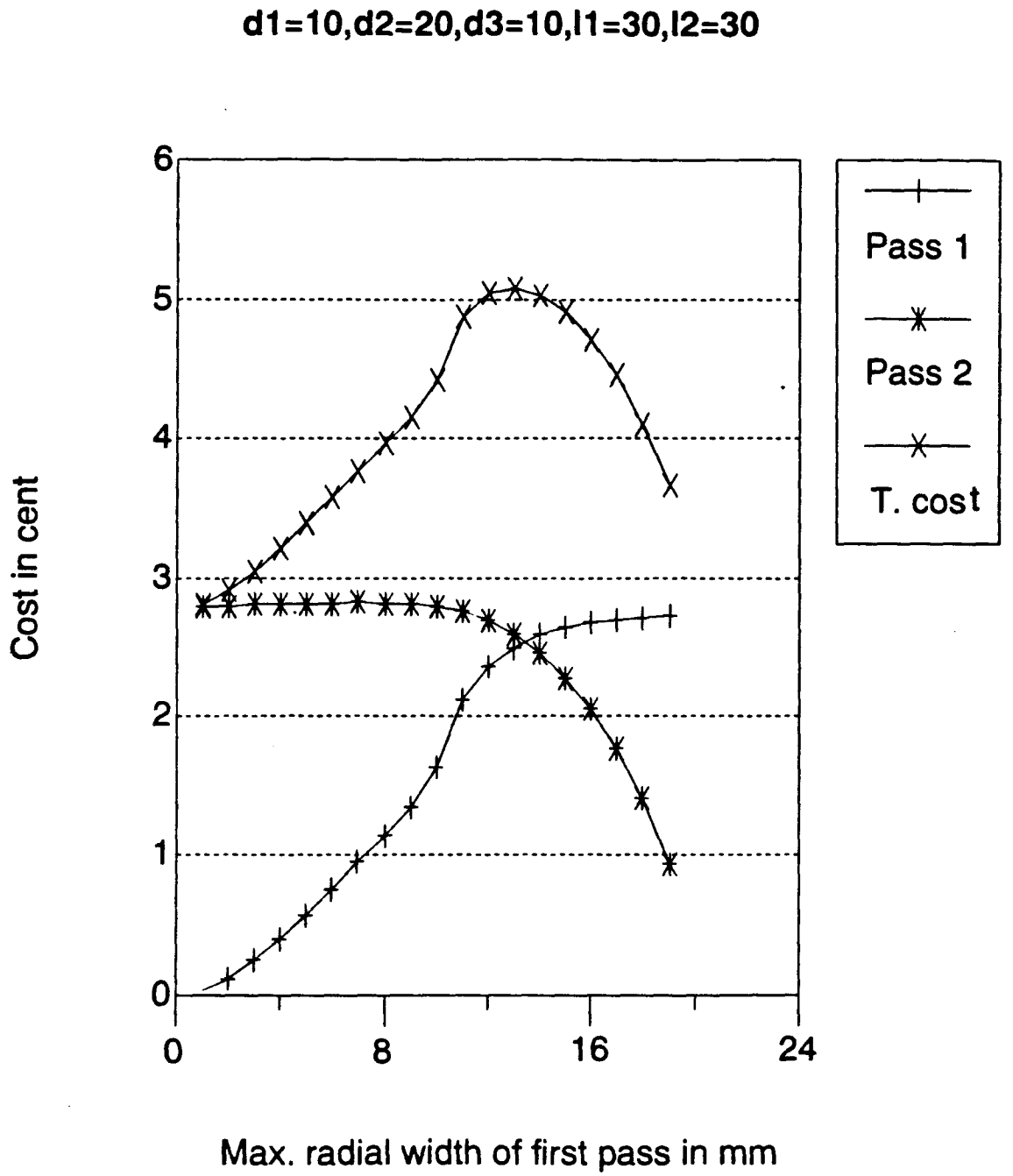

Figure 5.7: Cost Characteristics of a Symmetric Generalized Workpiece

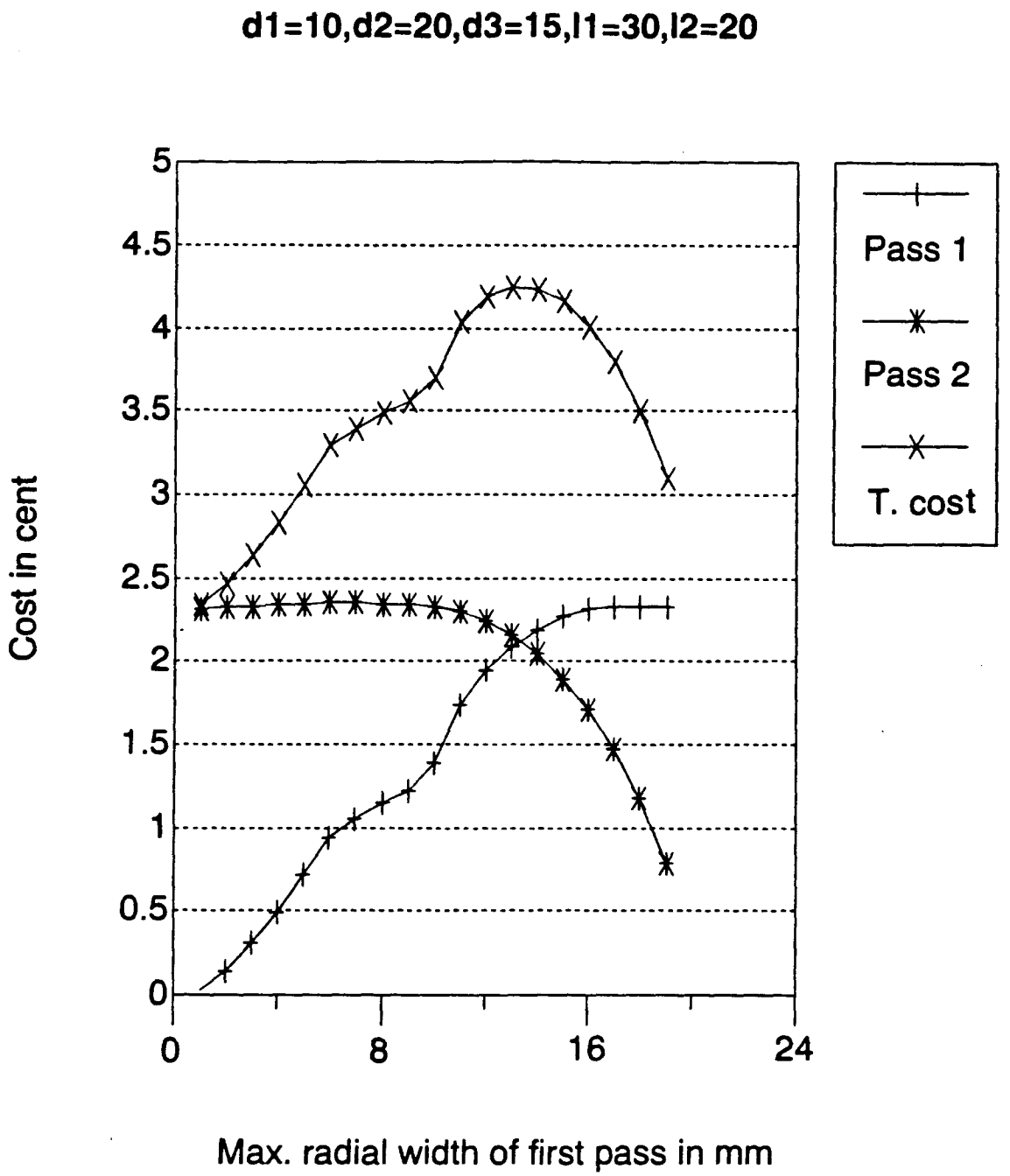

Figure 5.8: Cost Characteristics of an Unsymmetric Generalized Workpiece

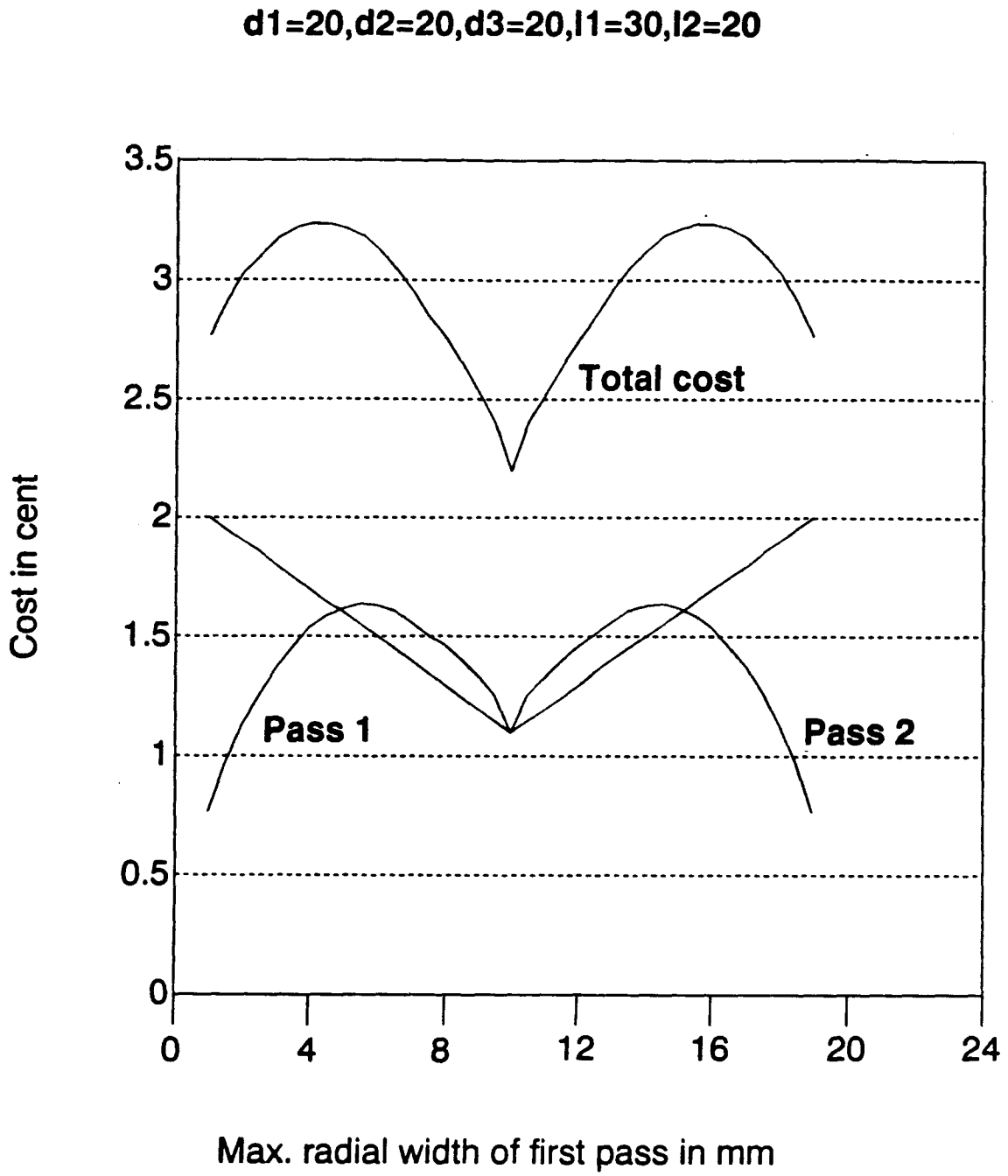

Figure 5.9: Cost Characteristics of a Rectangle (varying tool diameter)

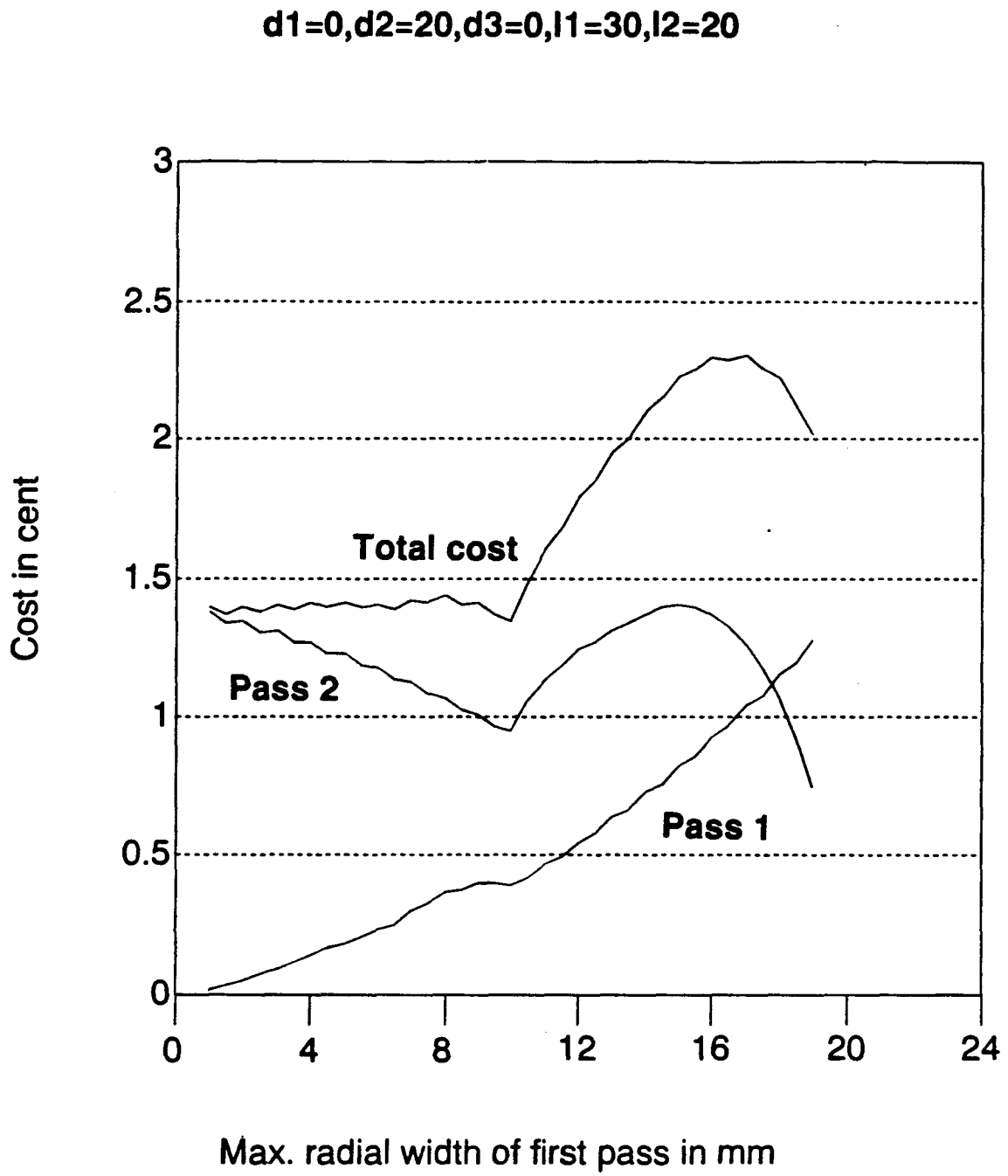

Figure 5.10: Cost Characteristics of a Triangle (varying tool diameter)

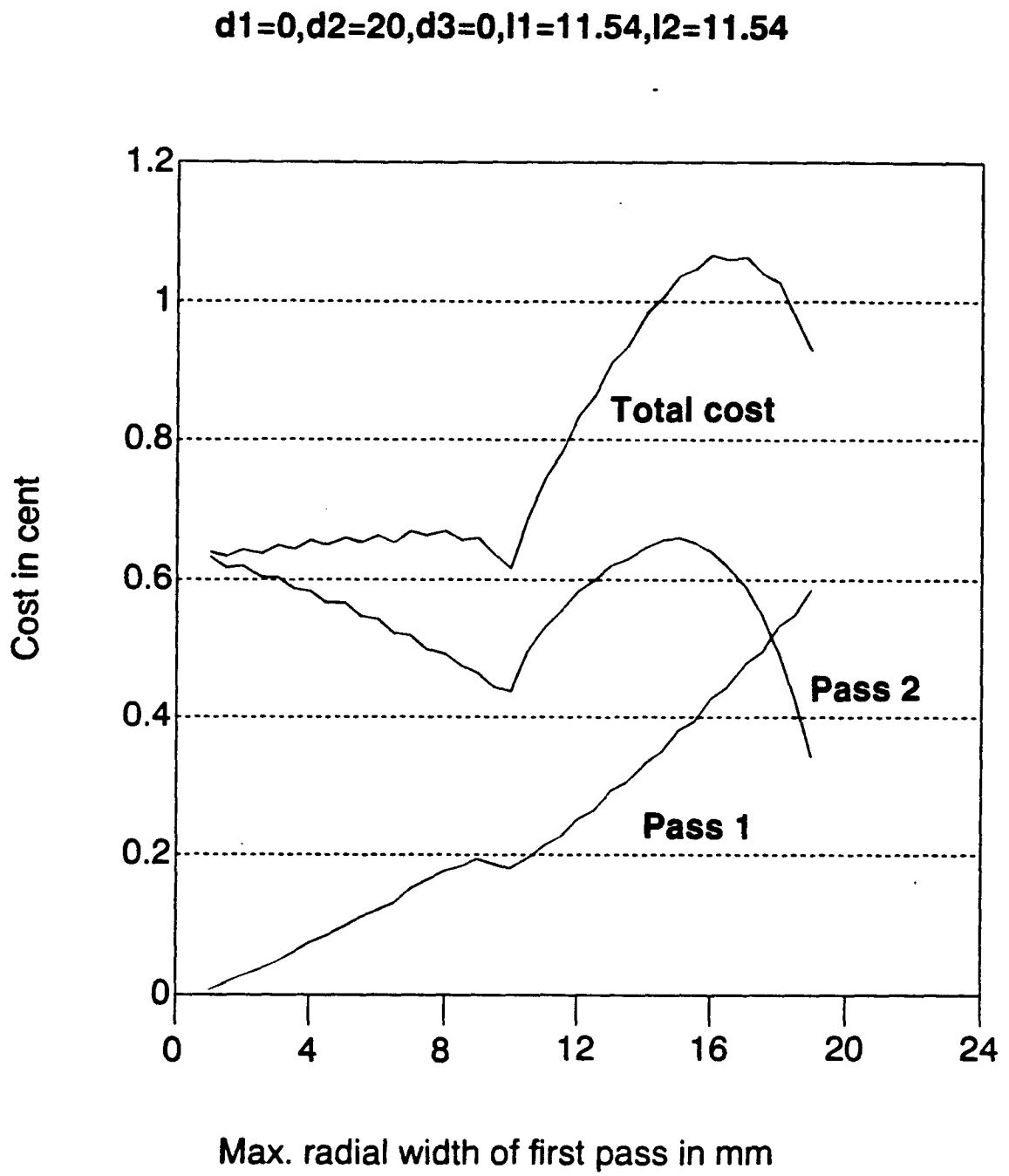

Figure 5.11: Cost Characteristics of an Equilateral Triangle (varying tool diameter)

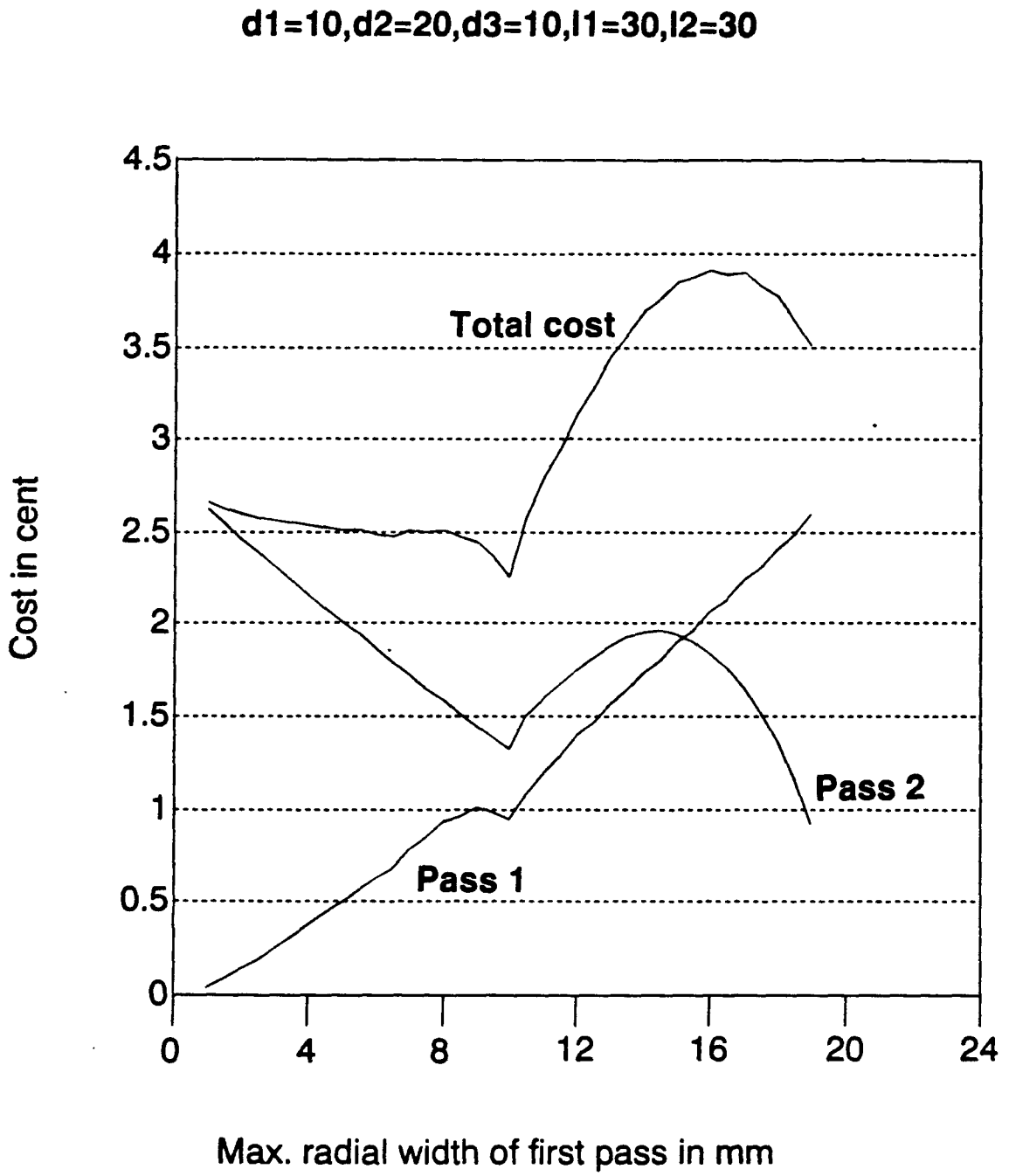

Figure 5.12: Cost Characteristics of a Symmetric Generalized Workpiece (varying tool diameter)

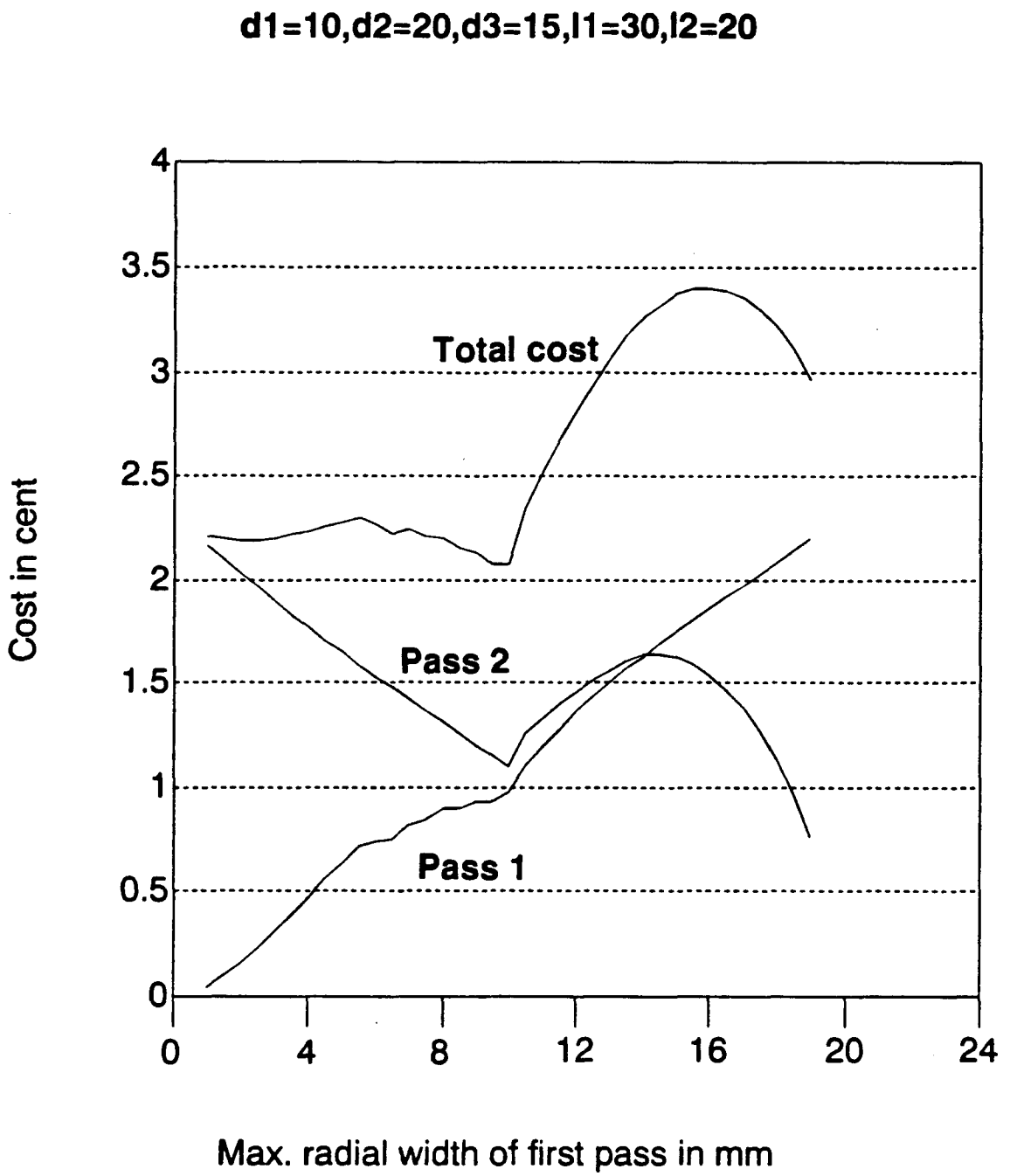

Figure 5.13: Cost Characteristics of an Unsymmetric Generalized Workpiece (varying tool diameter)

feeds are selected in an optimal manner. Three different cutter radii are used and the axial depth of cut is kept constant for simplicity.

In the above discussion a tool of constant radius has been used for all values of d_r. In actual practice, a tool with a diameter equal to the larger of the two values of d_r and d_q is capable of machining both the parts at a lower cost. This feature is included in graphs (5.9), (5.10), (5.11), (5.12) and (5.13).

The machining costs for single pass operations for a few workpiece geometries are listed in table (5.1). The following conclusions can be drawn from the above mentioned data :

- Single pass operations should be preferable to multi pass milling operations provided the chatter limit is not encountered during the single pass operation.
- The cost increases at a faster rate with increasing radial widths for smaller radial widths.
- The cost decreases at a much slower rate with decreasing radial widths for larger radial widths.
- The machining cost for an area with larger radial widths will be less than that of an equal area with smaller radial widths.
- It is preferable to first machine the area with largest radial widths of cut within the chatter limit and take the remaining area in next passes.
- The cost of machining is highest when the whole area is divided in two equal halves.

Workpiece geometry	d_1	d_2	d_3	l_1	l_2	Cost in cents
Unsymmetric generalized	10	20	15	30	20	2.208859
Symmetric generalized	10	20	10	30	30	2.708165
Unsymmetric triangle	0	20	0	30	20	2.036704
Equilateral triangle	0	20	0	11.54	11.54	0.660608
Rectangle	20	20	20	30	20	2.122573

Table 5.1: Machining costs for single pass milling operations

5.2 Evaluation of Centre to Centre Tool Traverse Distance

The machining time is dependent on the centre to centre tool traverse distance. This distance is a function of the workpiece geometry and the tool radius. The accurate evaluation of this distance is important for a realistic path planning and machine control. The detailed mathematics required to evaluate the centre to centre tool traverse distance is included in this section for the sake of completeness.

5.2.1 Case 1: $d_{r_{min}} \leq d_r \leq s_2$

From the geometry of Fig. (5.14), the following relations can be obtained:

$$\angle 2 = \angle 3 = \angle 4 = \angle 5 = \angle \theta_1 \tag{5.10}$$

$$\begin{aligned}
\angle 6 &= 90 - \angle \theta_1 \\
\frac{pq}{R} &= \sin \angle 6 \\
pq &= R \sin(90 - \theta_1) \\
pq &= R \cos \theta_1 \qquad (5.11) \\
as &= d_2 - (d_1 + d_r) \\
qa &= R - pq \qquad (5.12) \\
\frac{qa}{nq} &= \sin \angle 5
\end{aligned}$$

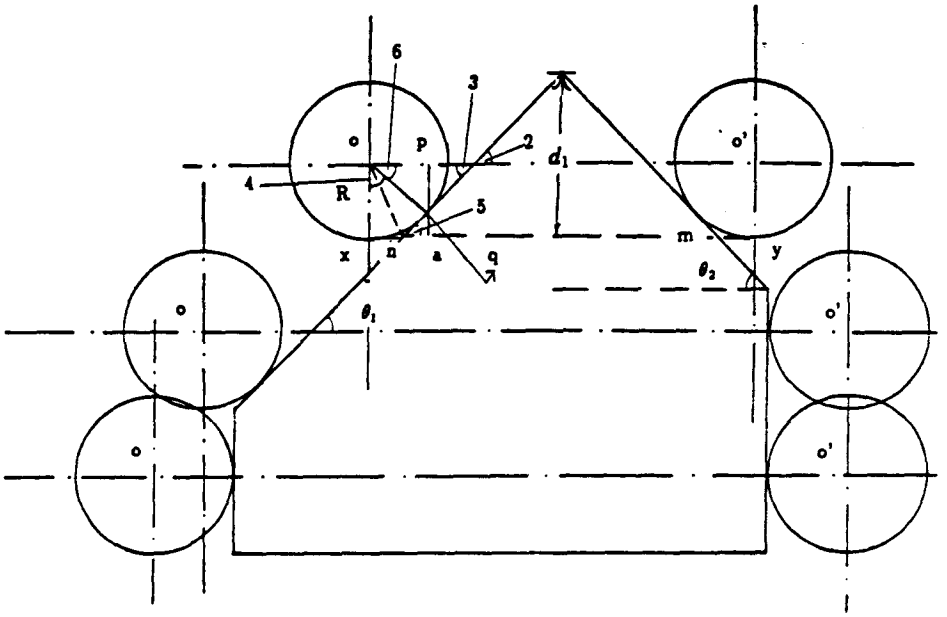

Figure 5.14: Geometry for Centre to Centre Tool Traverse Distance

$$nq = \frac{qa}{\sin\theta_1} \tag{5.13}$$

$$on^2 = R^2 + nq^2 \tag{5.14}$$

$$xn^2 = on^2 - R^2 \tag{5.15}$$

Using the above relations, we obtain:

$$\begin{aligned}
xn^2 &= R^2 + nq^2 - R^2 \\
xn^2 &= (\frac{qa}{\sin\theta_1})^2 \\
xn^2 &= (\frac{R - R\cos\theta_1}{\sin\theta_1})^2 \\
xn &= \frac{R - R\cos\theta_1}{\sin\theta_1} \\
xn &= R(\frac{1 - \cos\theta_1}{\sin\theta_1})
\end{aligned}$$

where

$$\theta_1 = tan^{-1}(\frac{d_2 - d_1}{l_1})$$

Similarly

$$ym = R(\frac{1 - \cos\theta_2}{\sin\theta_2}) \tag{5.16}$$

where

$$\theta_2 = tan^{-1}(\frac{d_2 - d_3}{l_2})$$

The total centre to centre distance moved by the tool for the first pass is:

$$L_1 = \lambda_1 + xn + ym \tag{5.17}$$

where

$$\lambda_1 = \frac{d_r}{\tan\theta_1} + \frac{d_r}{\tan\theta_2}$$

5.2.2 Case 2: $s_2 \leq d_r \leq s_1$

From Fig. (5.14), it is clear that the centre to centre tool traverse distance for the first pass is:

$$L_1 = \lambda_1 + xn + R \tag{5.18}$$

where

$$\lambda_1 = \frac{d_r}{\tan\theta_1} + l_2$$

The centre to centre distance moved by the tool for the second pass for all the three cases is:

$$L_2 = \lambda_2 + 2R$$

$$\lambda_2 = l_1 + l_2$$

Chapter 6

Optimal Cutting Direction for N-sided Polygonal Surfaces

The selection of cutting direction is an extremely important issue which has a considerable effect on a machining process. Tool wear is dependent on the tool/workpiece engagement time which is directly influenced by the selection of direction of tool motion. The cutting direction also determines the geometry of the machining surface and hence the other parameters like machine feed and spindle speed. The influence of workpiece geometry, spindle speed and machine feed on cost was discussed in the preceding chapters. In this chapter the selection of the best cutting direction for minimum machining cost for an N-sided polygon is discussed.

6.1 Mathematical Formulation

Let

$$(x_i, y_i) \qquad\qquad i = 1, 2, 3, \ldots n \tag{6.1}$$

be the n vertices of an n-sided convex polygon.

The equation of n edges of the polygon can be obtained from the following expressions:

$$\begin{aligned} y - y_i &= m_i(x - x_i) \\ i &= 1, 2, 3, \ldots n \end{aligned}$$

where

$$\begin{aligned} m_i &= \frac{y_i - y_{i+1}}{x_i - x_{i+1}} \\ i &= 1, 2, 3, \ldots (n-1) \end{aligned}$$

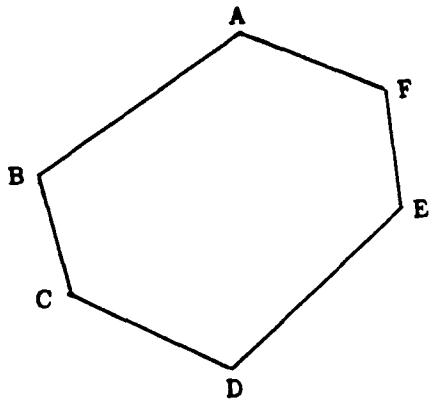

Figure 6.1: Geometry of a Generalized 6-sided Polygon (Top view)

It is assumed that the tool centre to centre workpiece length l_θ is traversed in p sampling intervals and the distance traversed in the jth sampling interval is x_j.

then

$$l_\theta = \sum_{j=1}^{p} x_j \tag{6.2}$$

also

$$l_\theta = l_{max} + 2R_\theta \tag{6.3}$$

where

l_{max} is the maximum workpiece length in a given direction

R_θ is the cutting tool radius

The cutting tool can be represented by the equation of a circle as follows:

$$\begin{aligned} (x - h_j)^2 + (y - g_j)^2 &= R_\theta^2 \\ j &= 1, 2, 3, \ldots p + 1 \end{aligned}$$

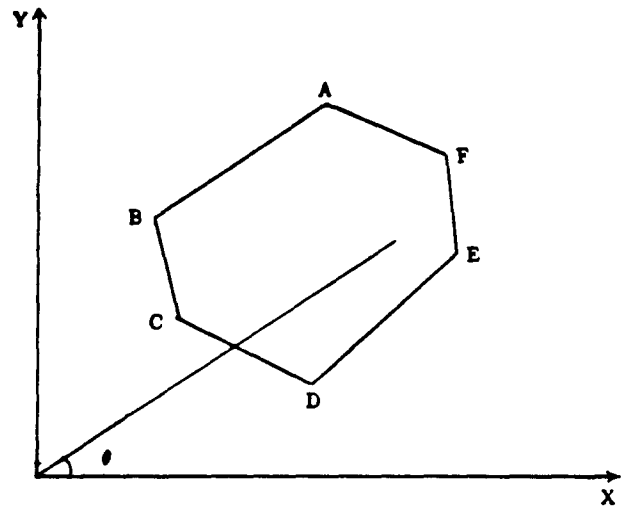

Figure 6.2: Cutting Direction θ (Top view)

where

(h_j, g_j) are $p+1$ tool centres for p sampling intervals

Our aim, as before, is to determine the cutting parameters such as spindle speed and feed for each sampling period and to evaluate the machining cost. The machining cost can then be obtained for a range of cutting directions and the cutting direction which gives the minimum machining cost is finally selected. The cutting direction is defined as the angle which the direction of tool traverse makes with the positive x-axis as shown in the Fig.(6.12). As the tool traverses the length l_θ, the swept angle of cut changes continously. It is, therefore, necessary to determine the swept angle of cut for each sampling interval. This can be accomplished geometrically and is discussed below.

The points of engagement and disengagement of the tool with the workpiece lie on the edges of the workpiece. There can be a maximum of $2n$ points of intersection between a circle and n straight lines. Out of all the points of intersection only two points of intersection which fall on the boundary of the workpiece represent the entry and exit

points. The angle these two points subtend at the centre of the circle is the swept angle of cut for a particular position and the moving direction of the tool.

let

θ		be the cutting direction
(x'_i, y'_i)		be the vertices of the polygon in the new coordinate system
(h'_j, g'_j)		be the tool centres in the new coordinate system
i	$=$	$1, 2, 3, \ldots n$
j	$=$	$1, 2, 3, \ldots p+1$

The mathematical formulas for coordinate transformation are derived in appendix A.

The maximum width of the workpiece d_{max} perpendicular to the cutting direction θ is the largest of the following distances:

$$
\begin{array}{ll}
abs(y'_i - y'_{i+1}) & i = 1, 2, 3, \ldots (n-1) \\
abs(y'_i - y'_{i+2}) & i = 1, 2, 3, \ldots (n-2) \\
abs(y'_i - y'_{i+3}) & i = 1, 2, 3, \ldots (n-3) \\
\ldots & \ldots \\
\ldots & \ldots \\
\ldots & \ldots \\
abs(y'_i - y'_{i+(n-1)}) & i = 1
\end{array}
$$

The maximum workpiece length l_{max} in the θ direction is the largest of the following distances:

$$
\begin{array}{ll}
abs(x'_i - x'_{i+1}), & i = 1, 2, 3, \ldots (n-1) \\
abs(x'_i - x'_{i+2}), & i = 1, 2, 3, \ldots (n-2)
\end{array}
$$

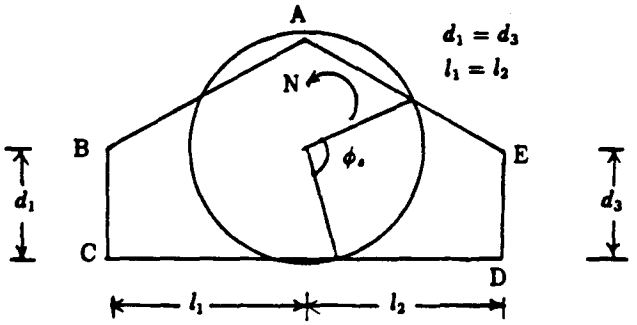

Figure 6.3: Swept angle for a 5-sided Symmetric Polygon

$$
\begin{array}{ll}
abs(x'_i - x'_{i+3}), & i = 1, 2, 3, \ldots (n-3) \\
\ldots & \ldots \\
\ldots & \ldots \\
\ldots & \ldots \\
abs(x'_i - x'_{i+(n-1)}), & i = 1
\end{array}
$$

let

$(a'_1, b'_1), (a'_2, b'_2)$ be the coordinates corresponding to the maximum width

$(c'_1, d'_1), (c'_2, d'_2)$ be the coordinates corresponding to the maximum workpiece length

R_θ be the tool radius for the θ direction (6.4)

$$R_\theta = \frac{d_{max}}{2} \tag{6.5}$$

The coordinates of the tool centre at the start of machining (h'_1, g'_1) can be determined

from the expressions:

$$h'_1 = c'_1 - R$$

$$h'_1 = c'_2 - R$$

$$g'_1 = \frac{b'_1 + b'_2}{2}$$

The subsequent tool centres can be obtained from the following relations:

$$\begin{aligned} h'_j &= h'_1 + \sum_{k=1}^{j-1} x_k \\ h'_j &= h'_1 - \sum_{k=1}^{j-1} x_k \\ g'_j &= g'_1 \\ j &= 2,3,4,\ldots p+1 \end{aligned}$$

Two relations for h'_1 and h'_j are for two sides of the workpiece.

The points of intersection between the tool and the workpiece edges can be obtained from the following relations:

$$\begin{aligned} \psi'_{2i-1} &= \frac{-\beta_i + \sqrt{\beta_i^2 - 4\alpha_i\gamma}}{2\alpha_i} \\ i &= 1,2,3,\ldots n \\ \psi'_{2i} &= \frac{-\beta_i - \sqrt{\beta_i^2 - 4\alpha_i\gamma}}{2\alpha_i} \\ i &= 1,2,3,\ldots n \end{aligned}$$

where

$$\begin{aligned} \beta_i &= 2m'_i b'_i - 2m'_i g'_j - 2h'_j \\ \gamma_i &= (b'_i)^2 + (g'_j)^2 + (h'_j)^2 - 2b'_i g'_j - R_\theta^2 \\ \alpha_i &= 1 + (m'_i)^2 \end{aligned}$$

$$
\begin{aligned}
b'_i &= y'_i - m'_i x'_i \\
m'_i &= \frac{y'_i - y'_{i+1}}{x'_i - x'_{i+1}} \\
i &= 1, 2, 3, \ldots n
\end{aligned}
$$

Out of these $2n$ points of intersection, a pair of points which lie on the boundary of the polygon and fall in the cutting region are selected. These two points correspond to the entry and exit positions.

let

(e'_j, f'_j) be the coordinates of the entry point

(u'_j, v'_j) be the coordinates of the exit point

The distance between these two point is:

$$D_\theta = \sqrt{(e'_j - u'_j)^2 + (f'_j - v'_j)^2} \tag{6.6}$$

The angle these two points subtend at the centre of the tool is the swept angle of cut. This is shown in Fig. (6.3). The swept angle can be obtained from the cosine law of triangles as follows:

$$\phi_s = \cos^{-1}\left(\frac{2R_\theta^2 - D_\theta^2}{2R_\theta^2}\right)$$

Once we know the swept angle of cut for a particular sampling interval, we can find the optimal feed, spindle speed and cost by making use of the optimization strategy described earlier. Costs for all the sampling intervals can be summed to obtain the total cost of machining at a cutting direction of θ.

This procedure has been used to compute costs for a range of angles from 0 degree to 180 degrees. The graphs between cutting direction and the cost for a few shapes are shown in Figs. (6.4)-(6.11).

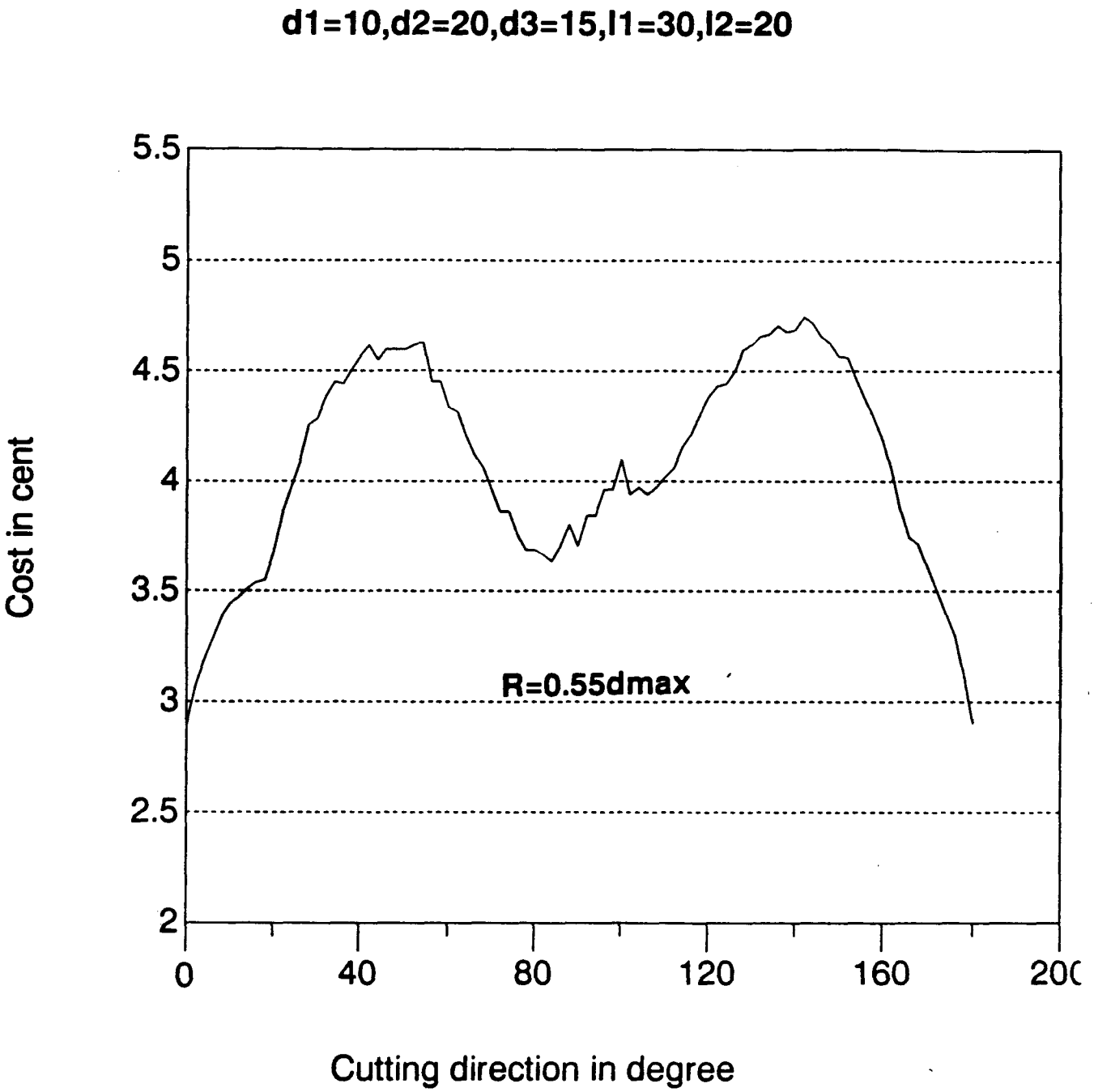

Figure 6.4: Cost characteristics for a 5-sided Unsymmetric Polygon

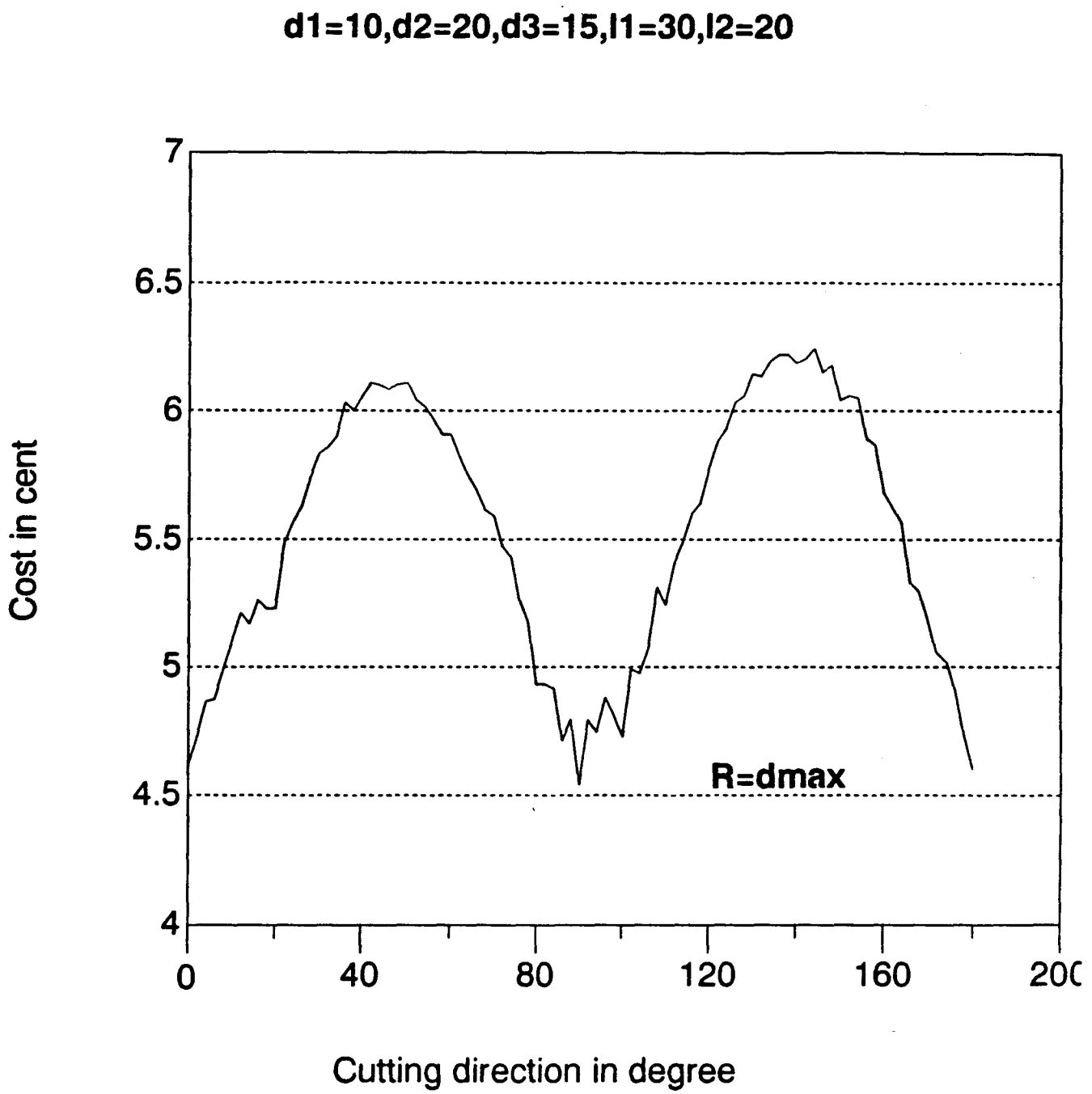

Figure 6.5: Cost characteristics for a 5-sided Unsymmetric Polygon

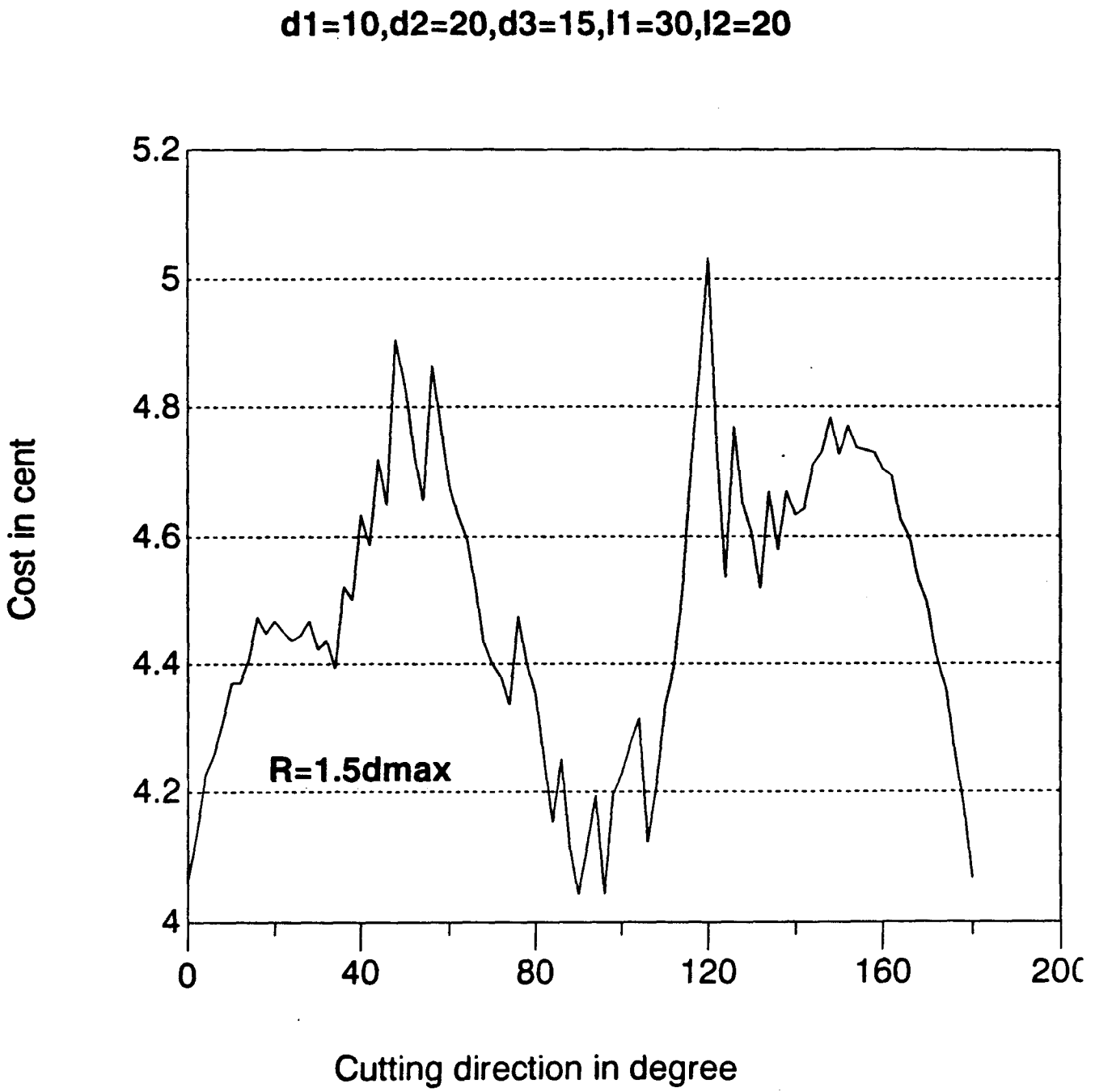

Figure 6.6: Cost characteristics for a 5-sided Unsymmetric Polygon

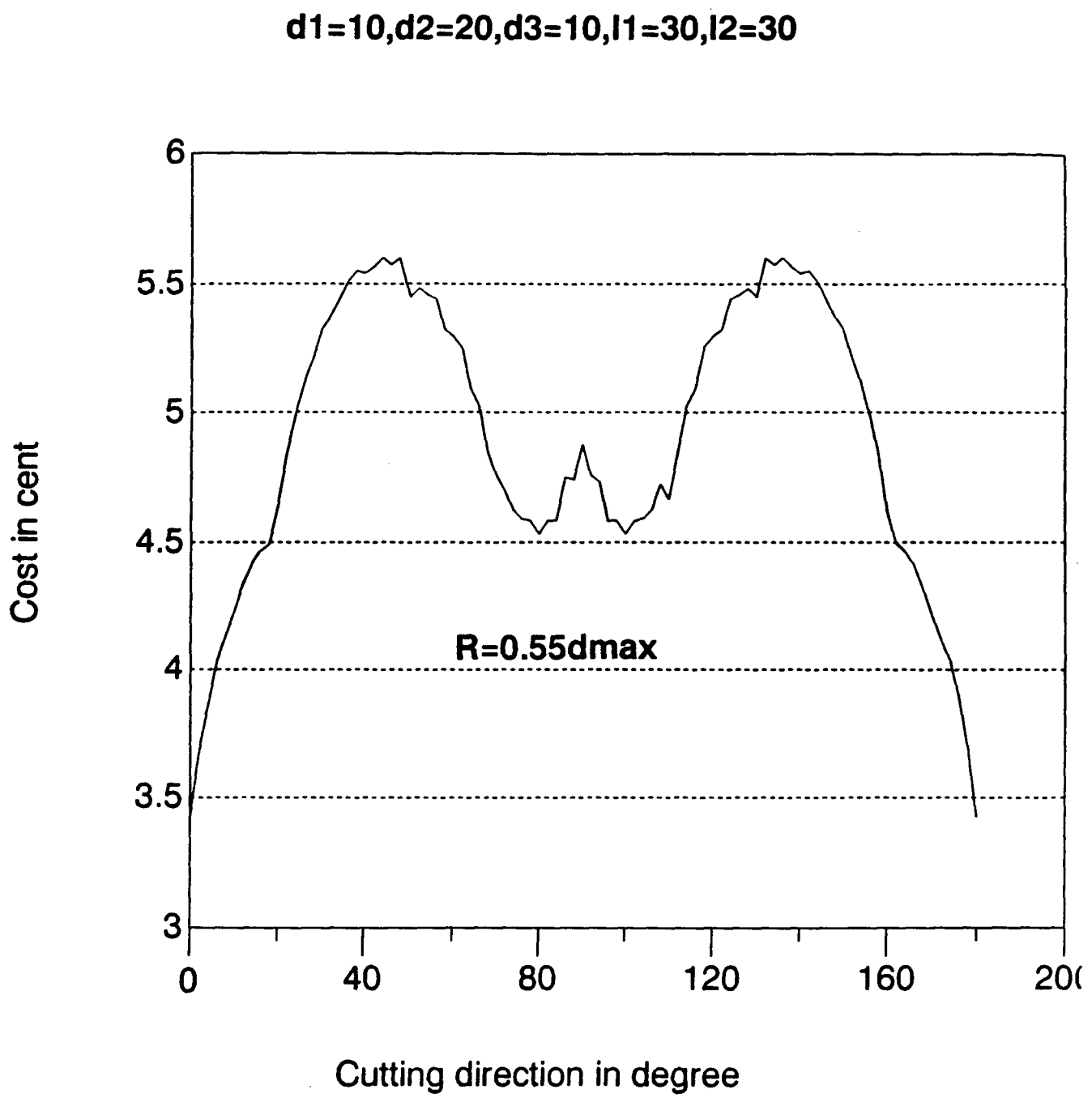

Figure 6.7: Cost characteristics for a 5-sided Symmetric Polygon

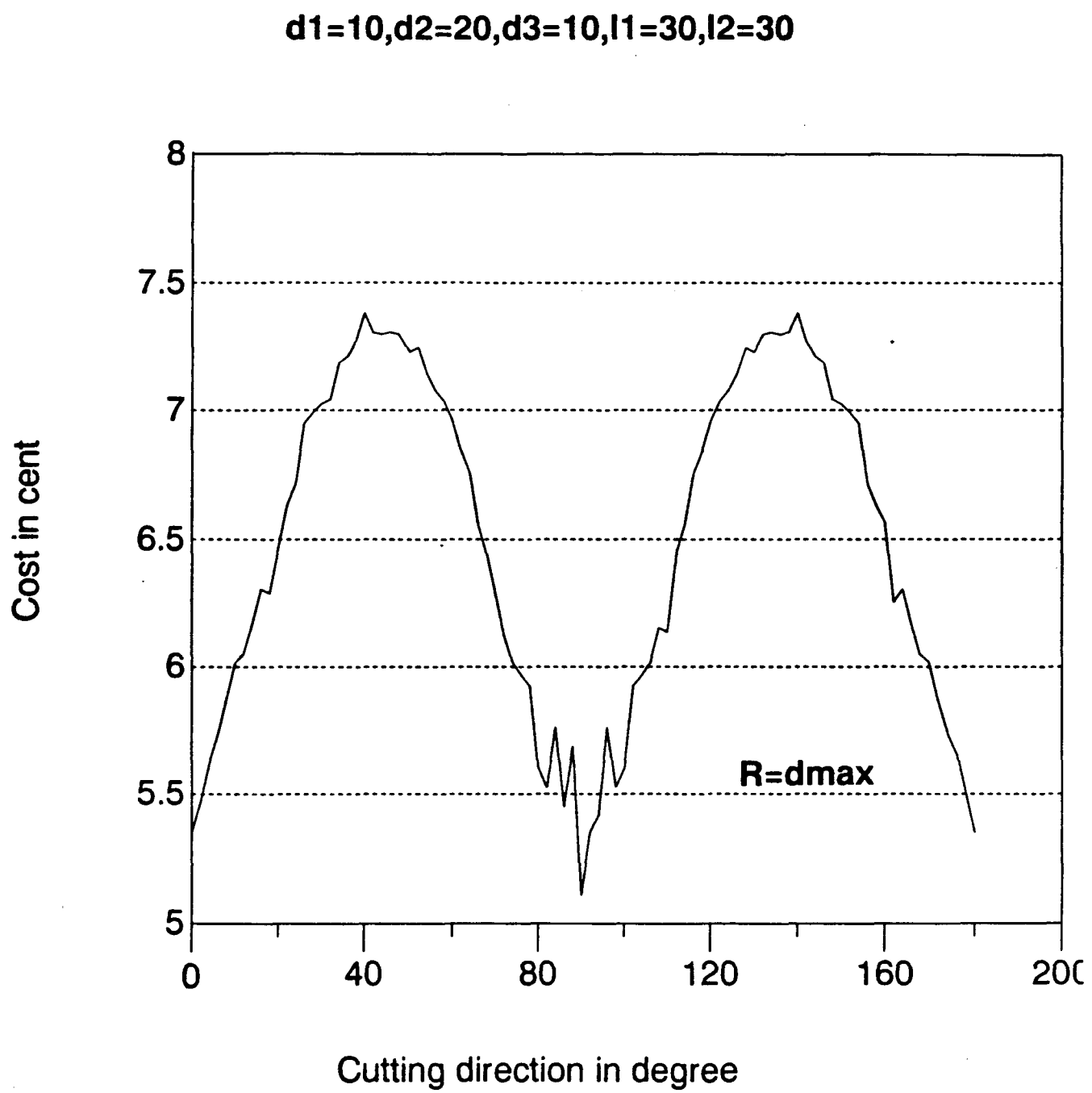

Figure 6.8: Cost characteristics for a 5-sided Symmetric Polygon

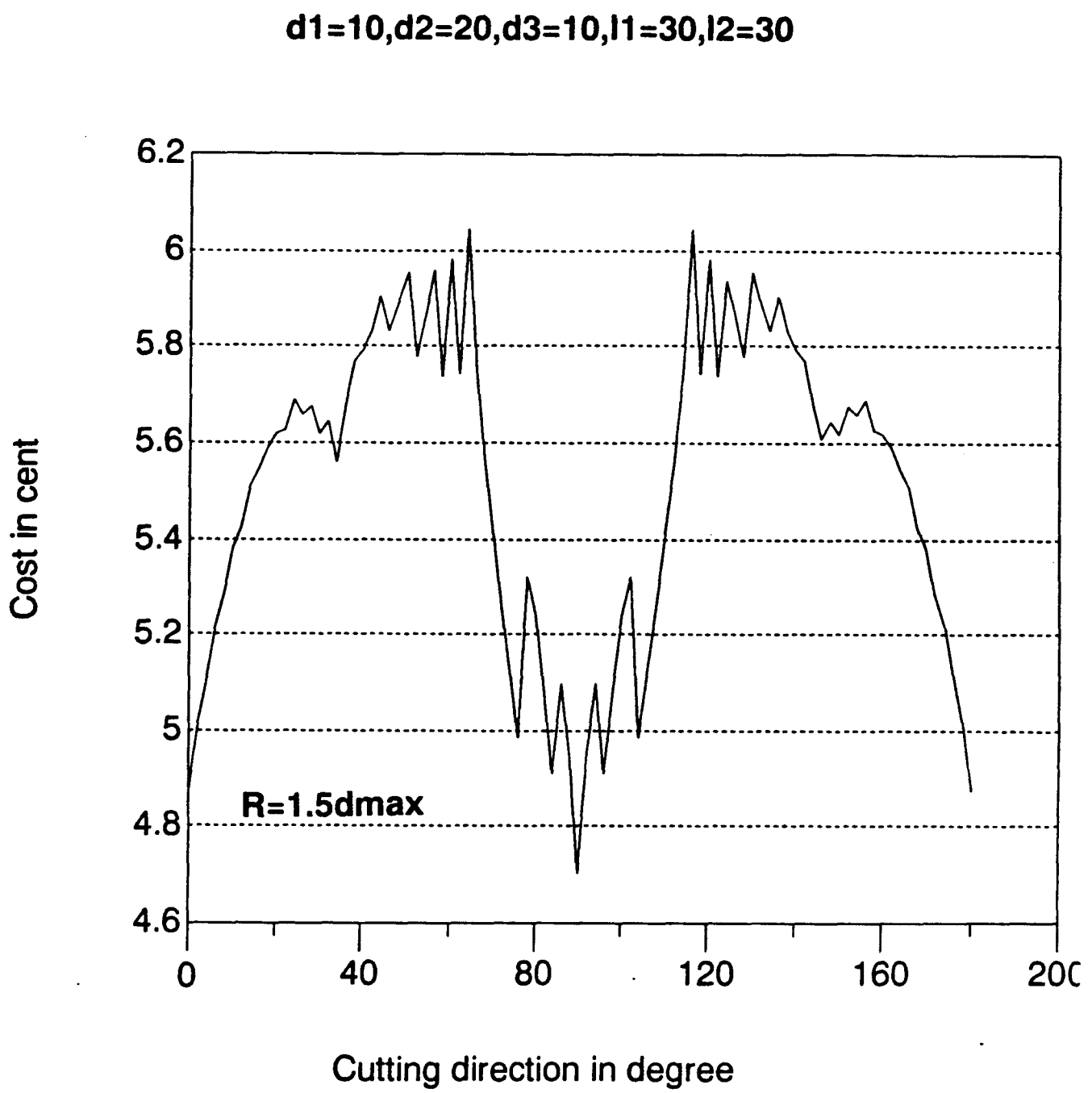

Figure 6.9: Cost characteristics for a 5-sided Symmetric Polygon

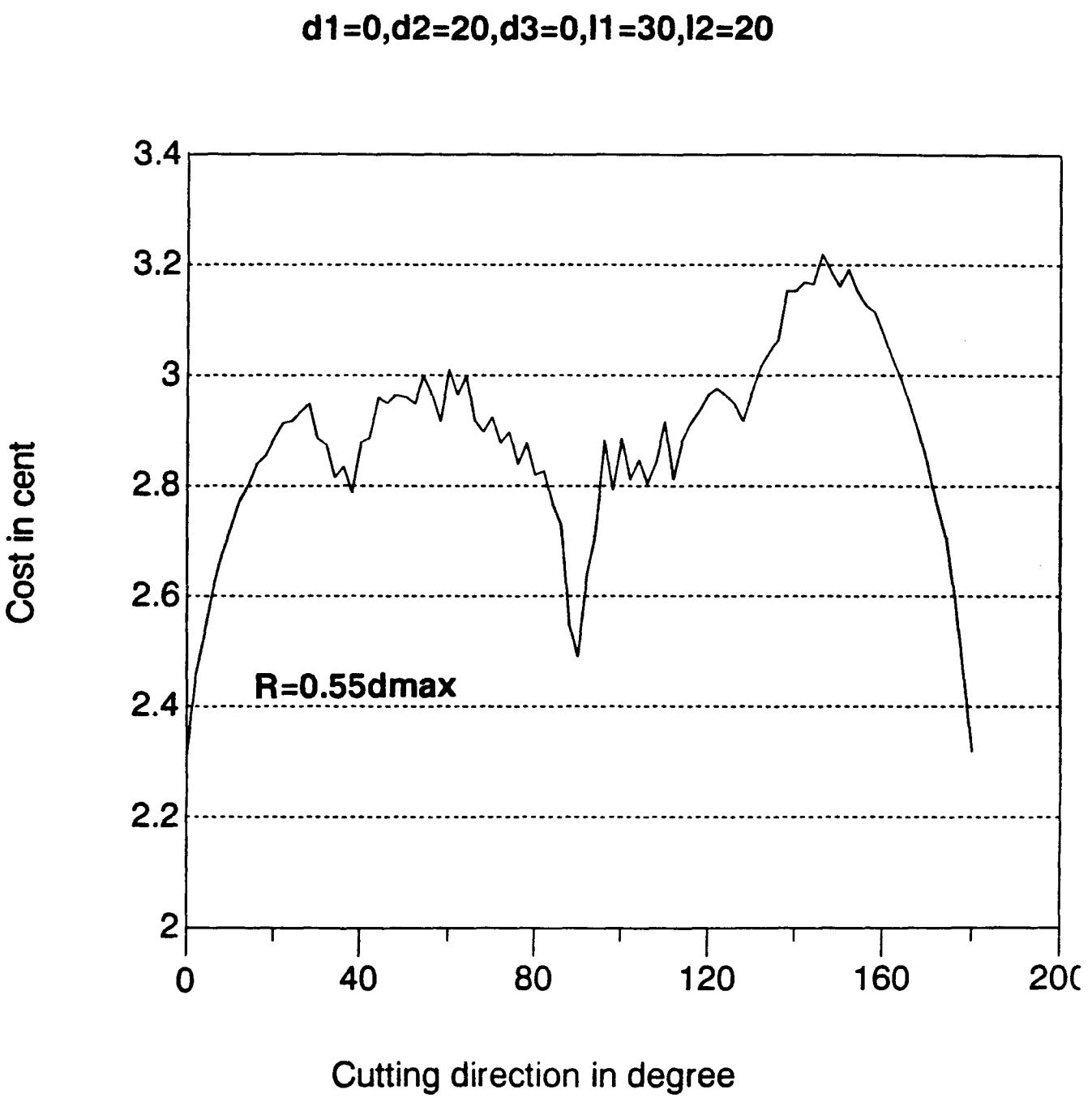

Figure 6.10: Cost characteristics for a Non-equilateral Triangle

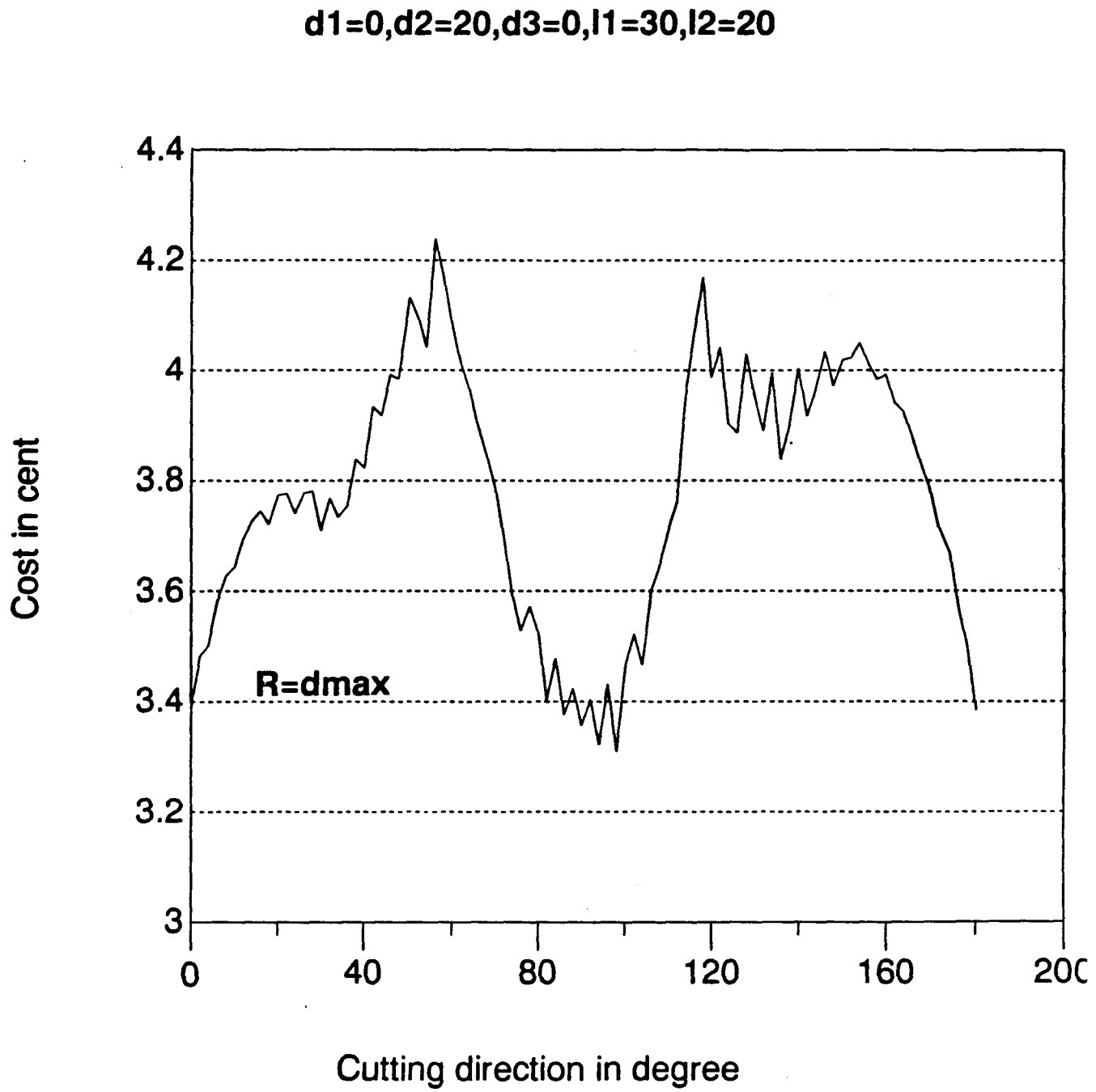

Figure 6.11: Cost characteristics for a Non-equilateral Triangle

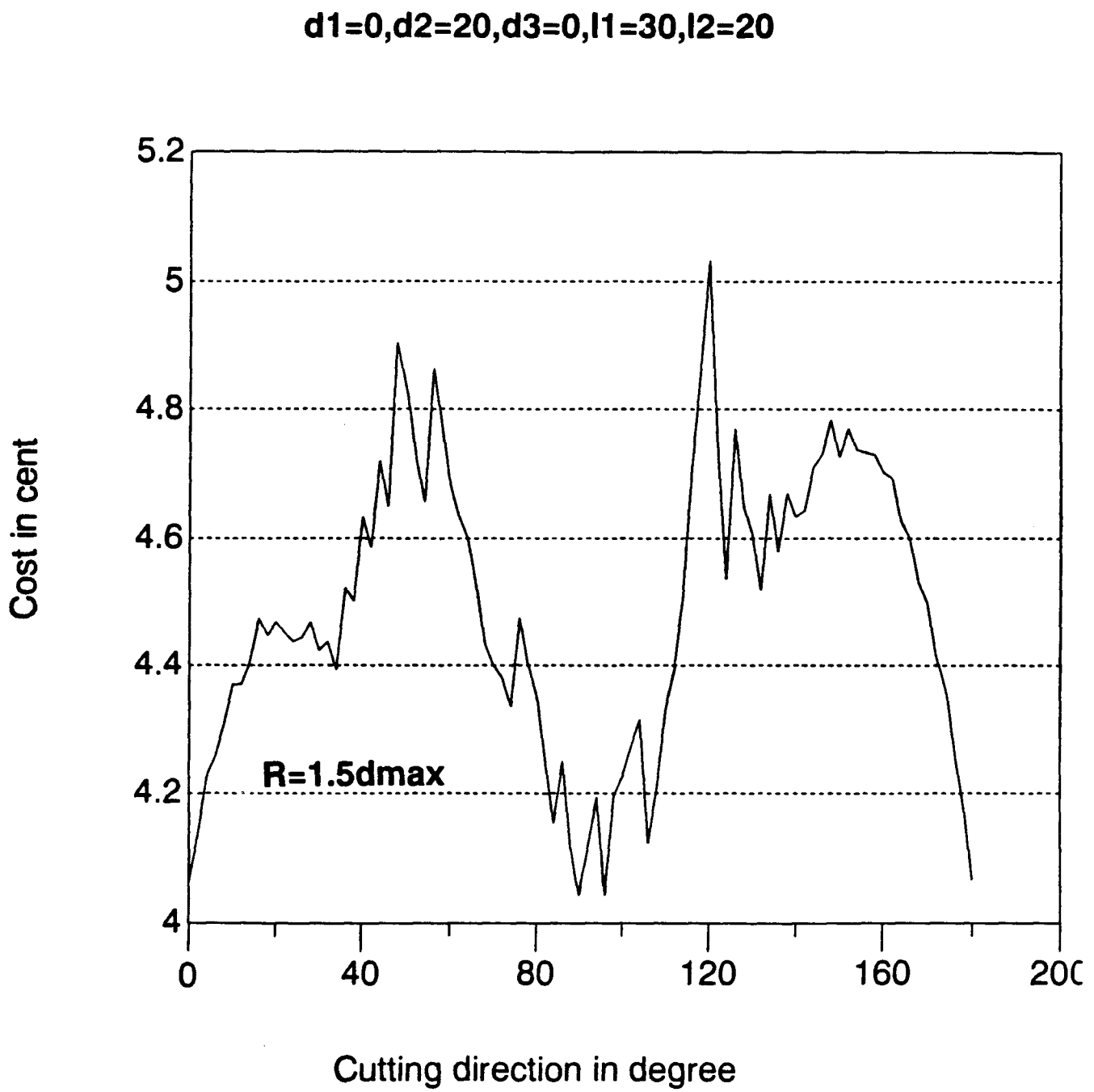

Figure 6.12: Cost characteristics for a Non-equilateral Triangle

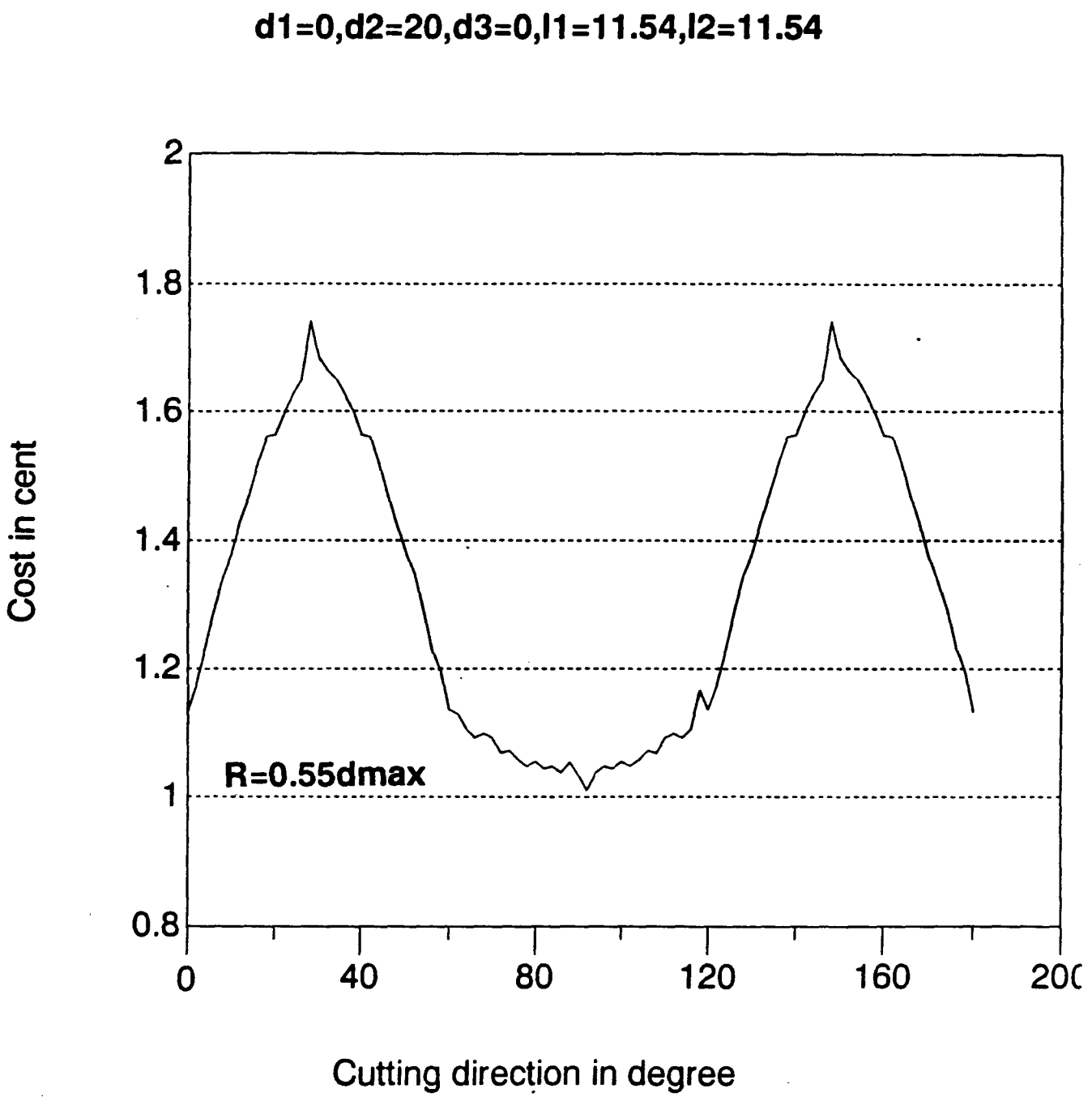

Figure 6.13: Cost characteristics for an Equilateral Triangle

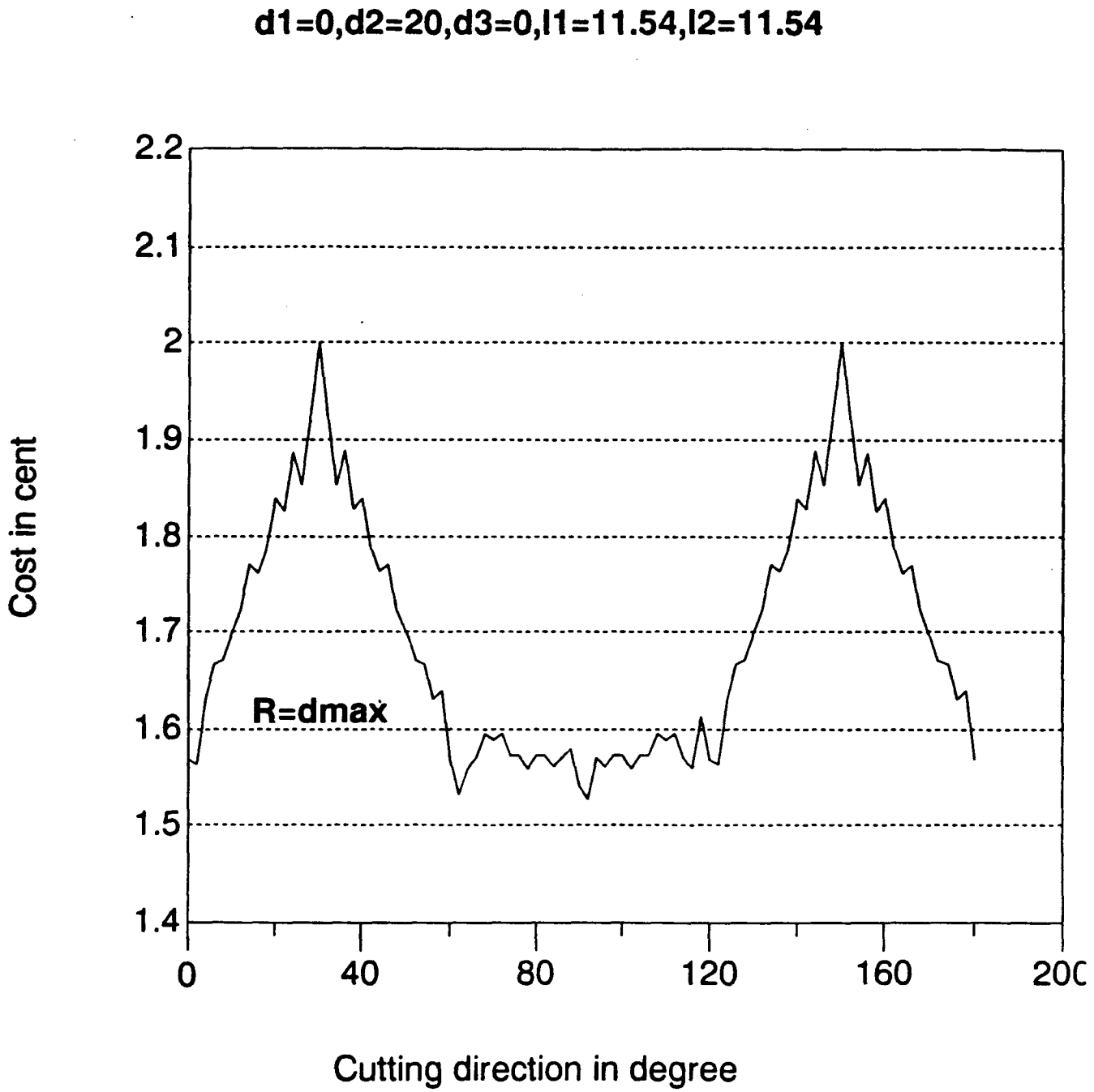

Figure 6.14: Cost characteristics for an Equilateral Triangle

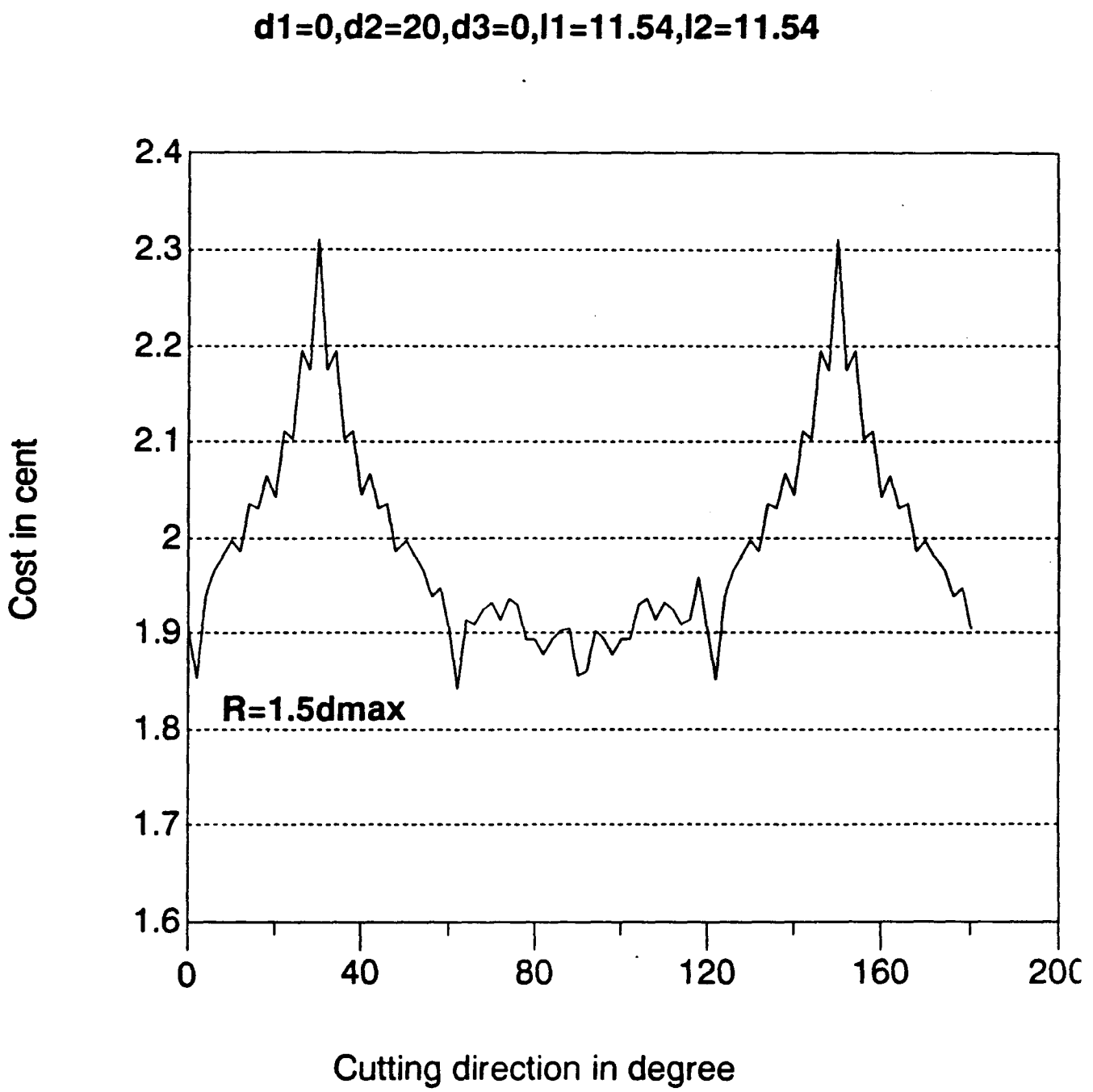

Figure 6.15: Cost characteristics for an Equilateral Triangle

6.1.1 Analysis of Results

The following conclusions can be drawn from the computational results:

1. Symmetric workpieces give symmetric graphs with small computational errors.

2. When the tool radius is slightly larger ($R = 0.55d_{max}$) than the maximum radial width encountered during machining; the cutting directions of 0 degree and 180 degree are the optimal directions for all the workpieces.

3. When the tool radius is twice ($R = d_{max}$) or thrice as large ($R = 1.5d_{max}$) as the maximum radial width encountered during machining; the cutting direction of 90 degree is the optimal direction for all the workpieces.

4. The results are dependent on the tool diameter and workpiece geometry.

5. The graphs for larger tool diameter with respect to the maximum radial width encountered during cut are shifted upwards. In other words, the cost increases for the selection of larger cutters.

6.1.2 Conclusions

This chapter describes a general method of evaluating the best cutting direction for a workpiece of known geometry. Even though the results have been obtained for only a few workpieces under certain site specific conditions, the method is valid for almost any workpiece geometry. A change in the assumptions and site specific conditions would alter the results. Software based on the proposed mathematical formulation can be developed which would give the best cutting direction for any set of conditions and any workpiece geometry.

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Appendix A

Coordinate Transformation

Let the coordinate system $X-Y$ be rotated by an angle of θ degrees. The new coordinate system is $X'-Y'$ represented by X' and Y' axis. The coordinate (x, y) in the old system is (x', y') in the new sytem. The line joining the origin with the coordinate (x, y) make an angle of α. The coordinates (x, y) and (x', y') can be mathematically related by the following equations (see Figs. 6.5- 6.9):

A.0.3 From Old to New

$\theta < \frac{\pi}{2}$

$\theta < \alpha$

$$\begin{aligned} \alpha &= \tan^{-1}\frac{y}{x} \\ y &= p\sin\alpha \\ p &= \frac{y}{\sin\alpha} \\ y' &= p\sin(\alpha-\theta) \\ x' &= p\cos(\alpha-\theta) \end{aligned}$$

$\theta < \frac{\pi}{2}$

$\theta > \alpha$

$$\alpha = \tan^{-1}\frac{y}{x}$$

$$
\begin{aligned}
y &= p\sin\alpha \\
p &= \frac{y}{\sin\alpha} \\
y' &= -\ p\cos(\frac{\pi}{2} - (\alpha - \theta)) \\
x' &= p\sin(\frac{\pi}{2} - (\alpha - \theta))
\end{aligned}
$$

$\theta = \frac{\pi}{2}$

$\theta < \alpha$

$$
\begin{aligned}
x' &= y \\
y' &= x
\end{aligned}
$$

$\theta = \pi$

$\theta < \alpha$

$$
\begin{aligned}
y' &= -y \\
x' &= -x
\end{aligned}
$$

A.0.4 From New to Old

$\theta < \frac{\pi}{2}$

$\theta < \alpha$

$$
\begin{aligned}
\tan(\alpha - \theta) &= \frac{y'}{x'} \\
\alpha &= \tan^{-1}\frac{y'}{x'} + \theta \\
\frac{y'}{p} &= sin(\alpha - \theta) \\
p &= \frac{y'}{\sin(\alpha - \theta)} \\
y &= p\sin\alpha \\
x &= p\cos\alpha
\end{aligned}
$$

$\theta < \frac{\pi}{2}$

$\theta > \alpha$

$$\begin{aligned}
\tan(\frac{\pi}{2} - (\theta - \alpha)) &= \frac{x'}{y'} \\
\frac{\pi}{2} - (\theta - \alpha)) &= abs(\tan^{-1}(\frac{x'}{y'})) \\
\alpha &= \theta - \frac{\pi}{2} + abs(\tan^{-1}(\frac{x'}{y'})) \\
\frac{x'}{p} &= sin(\frac{\pi}{2} - (\theta - \alpha)) \\
p &= \frac{x'}{\sin(\frac{\pi}{2} - (\theta - \alpha)} \\
y &= p \sin \alpha \\
x &= p \cos \alpha
\end{aligned}$$

$\theta = \frac{\pi}{2}$

$\theta < \alpha$

$$\begin{aligned}
x &= y' \\
y &= x'
\end{aligned}$$

$\theta = \pi$

$\theta < \alpha$

$$\begin{aligned}
y &= -y' \\
x &= -x'
\end{aligned}$$

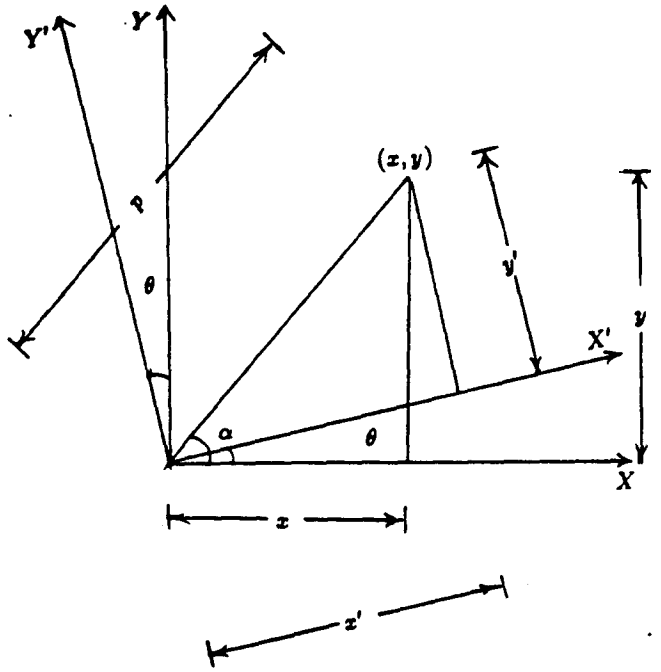

Figure A.1: Coordinate Transformation:$\theta < \frac{\pi}{2}, \theta < \alpha$

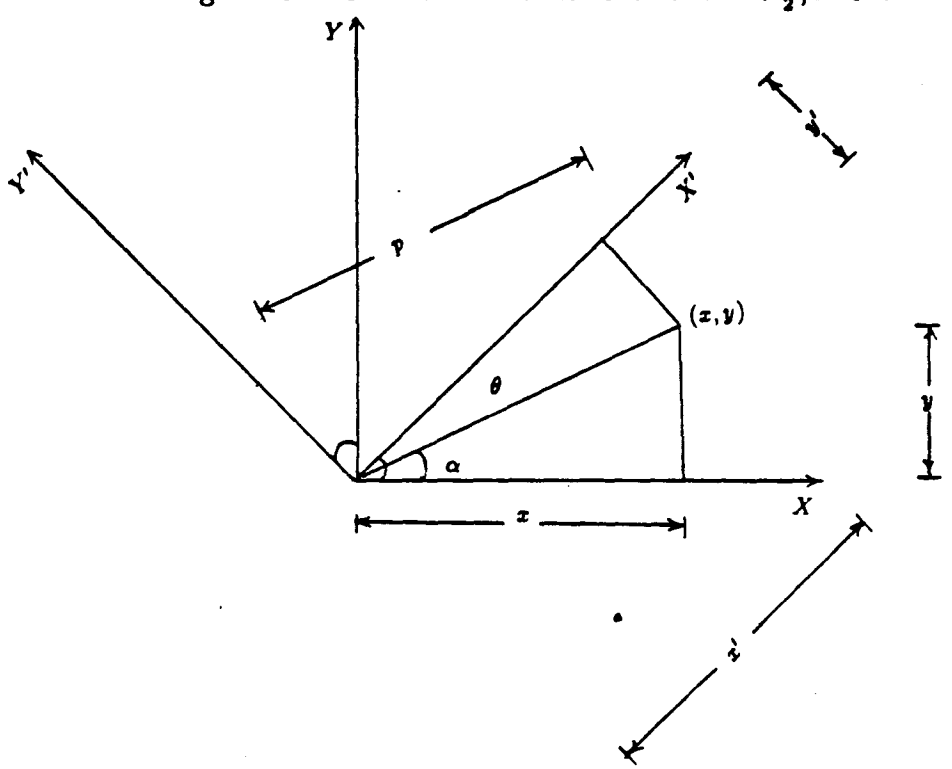

Figure A.2: Coordinate Transformation:$\theta < \frac{\pi}{2}, \theta > \alpha$

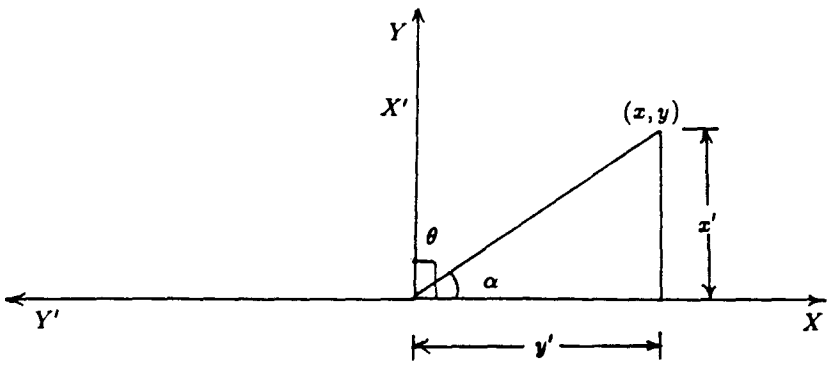

Figure A.3: Coordinate Transformation:$\theta = \frac{\pi}{2}, \theta > \alpha$

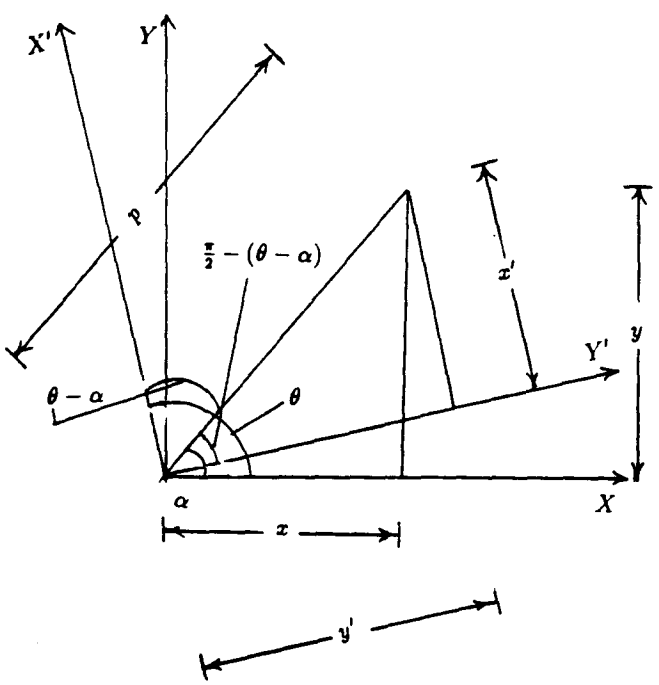

Figure A.4: Coordinate Transformation:$\theta > \frac{\pi}{2}, \theta > \alpha$

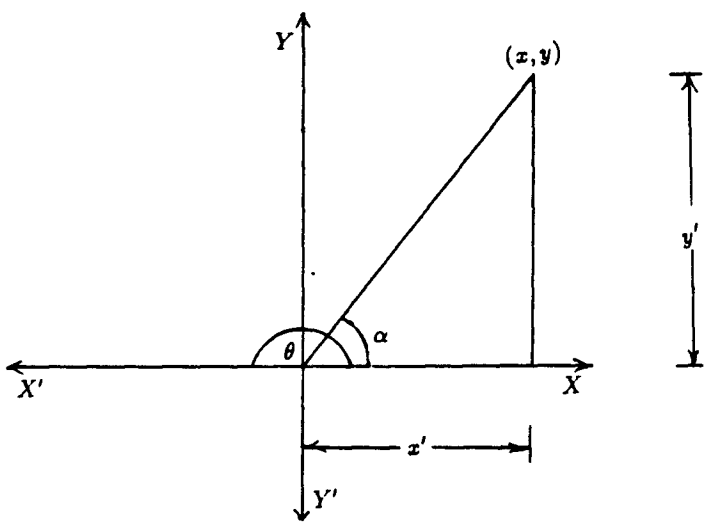

Figure A.5: Coordinate Transformation:$\theta = \pi, \theta > \alpha$

Appendix B

Pseudo-code for single pass milling operations

;Inputs:

Geometrical parameters of workpiece	$:d_1, d_2, d_3, l_1, l_2$
Economic parameters	$:C_h, C_t, T_{ct}$
Tool life constant and exponents	$:C_1, m, n, p, q$
Machine constraints	$:P_{max}, V_1$
Cutting constants	$:K_s, r_1$
Range of cutter radii	$:R_{min}, R_{max}$
Small increment in cutter radius	$:\Delta R$
Range of spindle speeds	$:V_{min}, V_{max}$
Cutting conditions	$:s_{max}, a$
Tool material properties	$:\sigma_{max}, etc.$
Milling type	:Upmilling or downmilling
Sampling interval	$:\Delta t$

;Outputs:

Cutter radius and Machining cost	$:R, C_{p_{min}}$

;

;Variables:

;

Big$k = \frac{R_{max}-R_{min}}{\Delta R}$	;Number of steps to cover the range
;	of cutter radii

$R_t = \frac{R_{max}-R_{min}}{4}$	;Increase in radius after which
;	the number of teeth change
$k_1 = \frac{d_2-d_1}{l_1}$	;Radial width slope upto length l_1
$k_2 = \frac{d_2-d_3}{l_2}$	;Radial width slope after length l_1
$E_1 = \frac{P_{max}(.75*10^6)}{K_s a}$	;Power constraint constant
;	
$i = 0$ *to* Big$k$	;Loop for range of radii
$R(i+1) = R_{min} + i\Delta R$	;Next cutter radius
if $R_{min} \leq R(i+1) < (R_{min} + R_t)$	;Selection of cutter teeth
then	
$z = 2$	
;	
if $(R_{min} + R_t) \leq R(i+1) < (R_{min} + 2R_t)$	
then	
$z = 3$	
;	
if $(R_{min} + 2R_t) \leq R(i+1) < (R_{min} + 3R_t)$;	
then	
$z = 4$	
if $R(i+1) \geq (R_{min} + 3R_t)$	
then	
$z = 6$	
;	
$L(i+1) = 6R(i+1)$	;Flute length

$T_{max} = .75 * 10^6 * \frac{P_{max}R(i+1)}{V_1}$	;Maximum allowable Torque
$C_p = 0.0$	;Initialize the process cost
$x = 0.0$	;Initialize total tool traverse distan
Delta= 500	;Arbitrary number for sampling
;	;intervals
$j = 1$ to Delta	;Loop for sampling intervals
if $x < l_1$	;Evaluation of radial depth of cut
then	
$d(j) = d_1 + k_1 x$	
else	
$d(j) = d_2 - k_2(x - l_1)$	
$\phi_s(j) = \cos^{-1}(1 - \frac{d(j)}{R(i+1)})$	;Swept angle of cut
;	
if $d(j) = 0$	;Initial radial depth for triangle
then	
$d(j) = 2$	
31 if $R(i+1) > d(j)$	;Feed per tooth
then	;from tooth breakage constraint
$s_t(j) = \frac{s_{max}}{\sin(\phi_s(j))}$	
else	
$s_t(j) = s_{max}$	
$\gamma(j) = \frac{K_s a z s_t(j)}{2\pi}$	Cutting force constant
$F_x(j) = \gamma(j)[(1 - \cos(2\phi_s(j))) + r_1(2\phi_s(j) - \sin(2\phi_s(j)))]$	;Average cutting forces

$F_y(j) = \gamma(j)[r_1(1 - \cos(2\phi_s(j))) - (2\phi_s(j) - \sin(2\phi_s(j)))]$;for upmilling

$F_x(j) = \gamma(j)[r_1(2\phi_s(j) - \sin(2\phi_s(j))) - (1 - \cos(2\phi_s(j)))]$;Average cutting forces

$F_y(j) = \gamma(j)[r_1(1 - \cos(2\phi_s(j))) + (2\phi_s(j) - \sin(2\phi_s(j)))]$;for downmilling

;

$F_R(j) = \sqrt{F_x^2 + F_y^2}$;Resultant cutting force

$M(j) = F_R(j)(L(i+1) - \frac{a}{2})$;Resulting moment on the cutter

$T(j) = \gamma d(j)$;Resulting torque

;

$\sigma(j) = \frac{2}{piR(i+1)^3} M(j)\sqrt{M(j)^2 + T(j)^2}$;Shank stresses

;

if $\sigma(j) < \sigma_{max}$;Shank breakage constraint

then

goto 41 ;Check torque constraint

else

$s_{max} = s_{max} - 0.01$;Reduce maximum feed

goto 31 ;Calculate feed again

;

41 if $T_{max} > T(j)$;Torque constraint check

then

goto 51 ;Calculate equivalent feed

else

$s_{max} = s_{max} - 0.01$;Reduce maximum feed

goto 31 ;Calculate feed again

;

$S_{eq}(j) = \frac{s_t(j)d(j)}{R(i+1)\phi_s(j)}$;Equivalent feed per tooth

$N_{max}(j) = \frac{V_{max}}{2\pi R(i+1)}$;Range of

$N_{min}(j) = \frac{V_{min}}{2\pi R(i+1)}$	;r.p.m.
;	
$N_{max_p}(j) = \frac{60E_1}{s_t(j)z}$	;Maximum r.p.m. allowed by
;	;Power constraint
$\Delta N = 100$	;Initial increment for r.p.m.
61 Omega=integer value of $\frac{N_{max}-N_{min}}{\Delta N}$	;Upper limit of the index
;	for r.p.m. optimization loop
;	
$k = 0$ to Omega	;Loop for optimization of r.p.m.
$N(k+1) = N_{min} + k\Delta N$	;Next spindle r.p.m.
;	
while $N_{max_p} > N(k+1)$	;Power constraint check
$N(k+1) = N(k+1) - 1$	;Reduce spindle r.p.m.
;	
$V(k+1) = \frac{2\pi R(i+1)N(k+1)}{60}$	;Spindle peripheral velocity
;	
$T_h(k+1) = \frac{60000*R(i+1)\phi_s(j)}{V(k+1)}$	;Heating time in ms
;	
$T_c(k+1) = \frac{60000*R(i+1)(2\pi-\phi_s(j))}{V(k+1)}$	;Cooling time in ms
;	
$smallx(k+1) = \frac{2\pi}{\phi_s(j)}$	;Ratio of total cycle time to
;	actual cutting time per cycle
$E_r(k+1) = 39\log(T_c(k+1)) - 23\log(T_h(k+1)) + 37.5$	;Range of thermal strain
;	
$X(k+1) = E_r(k+1)(\frac{N(k+1)*xsmall(k+1)}{60})^{\frac{1}{2}}$	;Thermal fatigue parameter

;

$T_L(k+1) = \frac{2\pi}{\phi_s(j)} \frac{C_1}{X(k+1)^m S_{eq}(j)^n V(k+1)^p a^q}$;Tool life

;

$\Delta C_p(k+1) = (C_h + \frac{C_t}{T_L(k+1)} + \frac{C_h T_{ct}}{T_{L(k+1)}})\Delta t$;Process cost for one interval

;

if $k > 1$ then ;Search of optimum r.p.m.

if $\Delta C_p(k+1) > \Delta C_p(k)$ then ;by comparing the cost

if $\Delta N > 1$ then ;with the previous value,

$N_{max} = N(k+1)$;narrowing the speed range,

$N_{min} = N(k-1)$;setting new limits for r.p.m

$\Delta N = \frac{\Delta N}{10}$ and new r.p.m. increment

goto 61 ;New upper limit of loop index

else

$N_{min} = N(1)$;

$\Delta N = 1$;

if $k = 0$ then ;

$\Delta C_{min} = \Delta C_p(k+1)$;Initialization of minimum cost

$N_{min_o} = N(k+1)$;Initialization of optimum r.p.m.

;

if $\Delta C(k+1) < \Delta C_{min}$ then Comparison with previous minimum cost

$\Delta C_{min} = \Delta C(k+1)$;New minimum cost

$N_{min_o} = N(k+1)$ New optimum r.p.m.

next k ;Try the next r.p.m.

;

```
C_p = C_p + ΔC_min                    ;Summation of costs
Δx(j) = (s_t(j) N_min_o z)/60 Δt      ;Distance moved
x = x + Δx(j)                         ;Total tool traverse distance
;
if x ≥ l_1 + l_2 then                 ;Check whether the
goto 91                               ;whole workpiece is machined
else
81  next j                            ;Next sampling interval
;
91  C_pmin(i + 1) = C_p               ;Cost with the selected radius
;
next i                                ;Next tool radius
;
;Plot C_pmin for different cutter radii R(i + 1)
;stop
end
```

Appendix C

Pseudo-code for two-pass milling operations

;Inputs:

Geometrical parameters of workpiece	$:d_1, d_2, d_3, l_1, l_2$
Economic parameters	$:C_h, C_t, T_{ct}$
Tool life constant and exponents	$:C_1, m, n, p, q$
Machine constraints	$:P_{max}, V_1$
Cutting constants	$:K_s, r_1$
Range of maximum radial width	$:d_{r_{min}}, d_{r_{max}}$
for the first pass	;
Small increment in maximum	$:\Delta d_r$
radial width of first pass	;
Tool geometry	$:R, z$
Range of spindle speeds	$:V_{min}, V_{max}$
Cutting conditions	$:s_{max}, a$
Tool material properties	$:\sigma_{max}, etc.$
Milling type	:Upmilling or downmilling
Sampling interval	$:\Delta t$

;Outputs:

Maximum radial width of	$:d_r, C_{p_{min}}$
first pass and Machining cost	;

;

;Variables:

;

Big$k = \frac{d_{r_{max}} - d_{r_{min}}}{\Delta d_r}$	;Number of steps to cover the range
;	of max. radial width of first pass
$s_1 = d_2 - d_1$	;Geometric
$s_2 = d_2 - d_3$	;constants
$k_1 = \frac{d_2 - d_1}{l_1}$	;Radial width slope upto length l_1
$k_2 = \frac{d_2 - d_3}{l_2}$	;Radial width slope after length l_1
$E_1 = \frac{P_{max}(.75*10^6)}{K_s a}$	;Power constraint constant
;	
$i = 0$ *to* Big$k$	;Loop for range of maximum
;	radial width of the first pass
$d_r(i+1) = d_{r_{min}} + i\Delta d_r$	;Next maximum radial width
;	of the first pass
$L = 6R$	;Flute length
$T_{max} = .75 * 10^6 * \frac{P_{max} R(i+1)}{V_1}$	;Maximum allowable Torque
$C_p = 0.0$	;Initialize the process cost
$x = 0.0$	;Initialize total tool traverse distance
$flag = 1$	;Initialization of pass number
;	
Delta= 1000	;Arbitrary number for sampling intervals
;	
$j = 1$ to Delta	;Loop for sampling intervals

;

;Case 1

;

if $d_{r_{min}} \leq d_r(i+1) \leq s_2$;

then

$\lambda_1 = \frac{d_r(i+1)}{k_1} + \frac{d_r(i+1)}{k_2}$;Length of first pass

$\lambda_2 = l_1 + l_2$;Length of second pass

;

if $x < \frac{d_r(i+1)}{k_1}$ and flag= 1 ;

then

$d(j) = k_1 x$;Radial depth of first

;

if $\frac{d_r(i+1)}{k_1} < x < \lambda_1$ and flag= 1

then

$d(j) = k_2(x - \frac{d_r(i+1)}{k_1})$;pass of machining

;

if $x < (l_1 - \frac{d_r(i+1)}{k_1})$ and flag= 2 ;

then

$d(j) = d_1 + k_1 x$;Radial depth of second

;

if $(l_1 - \frac{d_r(i+1)}{k_1}) < x < (\lambda_2 - \frac{d_r(i+1)}{k_2})$ and flag= 2 ;

$d(j) = d_2 - d_r(i+1)$;pass of machining

;

if $x > (\lambda_2 - \frac{d_r(i+1)}{k_2})$ and flag= 2 ;

then

$d(j) = (d_2 - d_r(i+1)) - k_2[x - (l_1 + \frac{d_r(i+1)}{k_2})]$;

;

;Case 2

;

if $s_2 < d_r(i+1) \leq s_1$;

then

$\lambda_1 = \frac{d_r(i+1)}{k_1} + l_2$;Length of first pass

$\lambda_2 = l_1 + l_2$;Length of second pass

;

if $x < \frac{d_r(i+1)}{k_1}$ and flag= 1 ;

then

$d(j) = k_1 x$;Radial depth of first

;

if $\frac{d_r(i+1)}{k_1} < x < \lambda_1$ and flag= 1

then

$d(j) = d_r(i+1) - k_2(x - l_1)$;pass of machining

;

if $x < (l_1 - \frac{d_r(i+1)}{k_1})$ and flag= 2 ;

then

$d(j) = d_1 + k_1 x$;Radial depth of second

;

if $x > (l_1 - \frac{d_r(i+1)}{k_1})$ and flag= 2 ;

$d(j) = d_2 - d_r(i+1)$;pass of machining

;

```
;Case 3
;
if s_1 < d_r(i + 1) ≤ d_{r_max}                ;
then
λ_1 = l_1 + l_2                                ;Length of first pass
λ_2 = l_1 + l_2                                ;Length of second pass
;
if x < l_1 and flag= 1                         ;
then
d(j) = d_1 − (d_2 − d_r(i + 1)) + k_1 x        ;Radial depth of first
;
if l_1 ≤ x < λ_1 and flag= 1
then
d(j) = d_3 − (d_2 − d_r(i + 1)) − k_2(x − l_1) ;pass of machining
;
if x < λ_2 and flag= 2                         ;Radial depth of second
then
d(j) = d_2 − d_r(i + 1)                        ;pass of machining
;
if d_1 = d_2 = d_3                             ;Rectangular workpiece
if x < λ_1 and flag= 1                         ;Radial depth of first
then
d(j) = d_r(i + 1)                              ;pass of machining
;
```

if $x < \lambda_2$ and flag= 2	;Radial depth of second
then	
$d(j) = d_2 - d_r(i+1)$	;pass of machining
;	
$\phi_s(j) = \cos^{-1}(1 - \frac{d(j)}{R})$	;Swept angle of cut
if $d(j) = 0$	;Initial radial depth for triangle
then	
$d(j) = 2$	;
;	
31 if $R > d(j)$	;Feed per tooth
then	;from tooth breakage constraint
$s_t(j) = \frac{s_{max}}{\sin(\phi_s(j))}$	
else	
$s_t(j) = s_{max}$	
;	
$\gamma(j) = \frac{K_s a z s_t(j)}{2\pi}$	Cutting force constant
;	
$F_x(j) = \gamma(j)[(1 - \cos(2\phi_s(j))) + r_1(2\phi_s(j) - \sin(2\phi_s(j)))]$	;Average cutting forces
$F_y(j) = \gamma(j)[r_1(1 - \cos(2\phi_s(j))) - (2\phi_s(j) - \sin(2\phi_s(j)))]$	;for upmilling
$F_x(j) = \gamma(j)[r_1(2\phi_s(j) - \sin(2\phi_s(j))) - (1 - \cos(2\phi_s(j)))]$	;Average cutting forces
$F_y(j) = \gamma(j)[r_1(1 - \cos(2\phi_s(j))) + (2\phi_s(j) - \sin(2\phi_s(j)))]$	;for downmilling
;	
$F_R(j) = \sqrt{F_x^2 + F_y^2}$	;Resultant cutting force
$M(j) = F_R(j)(L - \frac{a}{2})$	;Resulting moment on the cutter
$T(j) = \gamma d(j)$	;Resulting torque
;	

$\sigma(j) = \frac{2}{piR^3} M(j)\sqrt{M(j)^2 + T(j)^2}$	;Shank stresses
;	
if $\sigma(j) < \sigma_{max}$	;Shank breakage constraint
then	
goto 41	;Check torque constraint
else	
$S_{max} = S_{max} - 0.01$	;Reduce maximum feed
goto 31	;Calculate feed again
;	
41 if $T_{max} > T(j)$	;Torque constraint check
then	
goto 51	;Calculate equivalent feed
else	
$S_{max} = S_{max} - 0.01$	;Reduce maximum feed
goto 31	;Calculate feed again
;	
$S_{eq}(j) = \frac{s_t(j)d(j)}{R\phi_s(j)}$	;Equivalent feed per tooth
;	
$N_{max}(j) = \frac{V_{max}}{2\pi R}$	;Range of
$N_{min}(j) = \frac{V_{min}}{2\pi R}$	;r.p.m.
;	
$N_{max_p}(j) = \frac{60E_1}{s_t(j)z}$	;Maximum r.p.m. allowed by
;	;Power constraint
$\Delta N = 100$	;Initial increment for r.p.m.
61 Omega=integer value of $\frac{N_{max}-N_{min}}{\Delta N}$	;Upper limit of the index
;	for r.p.m. optimization loop

$k = 0$ to Omega	;Loop for optimization of r.p.m.
$N(k+1) = N_{min} + k\Delta N$	;Next spindle r.p.m.
;	
while $N_{max_p} > N(k+1)$	;Power constraint check
$N(k+1) = N(k+1) - 1$	;Reduce spindle r.p.m.
;	
$V(k+1) = \frac{2\pi RN(k+1)}{60}$	;Spindle peripheral velocity
;	
$T_h(k+1) = \frac{60000*R\phi_s(j)}{V(k+1)}$	;Heating time in ms
;	
$T_c(k+1) = \frac{60000*R(2\pi-\phi_s(j))}{V(k+1)}$	;Cooling time in ms
;	
$smallx(k+1) = \frac{2\pi}{\phi_s(j)}$	;Ratio of total cycle time to
;	actual cutting time per cycle
$E_r(k+1) = 39\log(T_c(k+1)) - 23\log(T_h(k+1)) + 37.5$	;Range of thermal strain
;	
$X(k+1) = E_r(k+1)(\frac{N(k+1)*xsmall(k+1)}{60})^{\frac{1}{2}}$	;Thermal fatigue parameter
;	
$T_L(k+1) = \frac{2\pi}{\phi_s(j)} \frac{C_1}{X(k+1)^m S_{eq}(j)^n V(k+1)^p a^q}$	;Tool life
;	
$\Delta C_p(k+1) = (C_h + \frac{C_t}{T_L(k+1)} + \frac{C_h T_{ct}}{T_{L(k+1)}})\Delta t$	;Process cost for one interval
;	

if $k > 1$ then	;Search of optimum r.p.m.
if $\Delta C_p(k+1) > \Delta C_p(k)$ then	;by comparing the cost
if $\Delta N > 1$ then	;with the previous value,
$N_{max} = N(k+1)$	;narrowing the speed range,
$N_{min} = N(k-1)$	;setting new limits for r.p.m
$\Delta N = \frac{\Delta N}{10}$	and new r.p.m. increment
goto 61	;New upper limit of loop index
else	
$N_{min} = N(1)$	;
$\Delta N = 1$	;
;	
if $k = 0$ then	;
$\Delta C_{min} = \Delta C_p(k+1)$	;Initialization of minimum cost
$N_{min_o} = N(k+1)$	;Initialization of optimum r.p.m.
;	
if $\Delta C(k+1) < \Delta C_{min}$ then	Comparison with previous minimum cost
$\Delta C_{min} = \Delta C(k+1)$	;New minimum cost
$N_{min_o} = N(k+1)$	New optimum r.p.m.
next k	;Try the next r.p.m.
;	
$C_p = C_p + \Delta C_{min}$	;Summation of costs
$\Delta x(j) = \frac{s_t(j) N_{min_o} z}{60} \Delta t$	;Distance moved
$x = x + \Delta x(j)$	;Total tool traverse distance
;	
if $x \geq \lambda_1$ and flag= 1 then	;End of first pass
flag= 2	;Start of second pass

```
x = 0                                        ;Initialization of tool traverse distance
;
if x ≥ λ2 and flag= 2 then                   ;End of second pass
goto 91                                      ;
next j                                       ;Next sampling interval
;
91  C_pmin(i + 1) = C_p                      ;Cost with the selected radius
;
next i                                       ;Next maximum radial width for
;                                            ;first pass
;Plot C_pmin for maximum radial widths d_r(i + 1)
;                                            ;of first pass
stop
end
```

Appendix D

Pseudo-code for optimal cutting direction

;Inputs:

Geometrical parameters of workpiece	$:d_1, d_2, d_3, l_1, l_2$
Economic parameters	$:C_h, C_t, T_{ct}$
Tool life constant and exponents	$:C_1, m, n, p, q$
Machine constraints	$:P_{max}, V_1$
Cutting constants	$:K_s, r_1$
Range of cutting orientations	$:\theta_{min}, \theta_{max}$
Small increment in cutting orientation	$:\Delta\theta$
Tool geometry	$:z$
Number of sides of polygon	:Big$L$
Co-ordinates of one vertex	$:x(3), y(3)$
Shape of workpiece	$:Shape$
Range of spindle speeds	$:V_{min}, V_{max}$
Cutting conditions	$:s_{max}, a$
Tool material properties	$:\sigma_{max}, etc.$
Milling type	:Upmilling or downmilling
Sampling interval	$:\Delta t$

;Outputs:

Cutting direction	$:\theta, C_{p_{min}}$
and Machining cost	;

;Variables:

;

Big$k$ = Integer of $\frac{\theta_{max}-\theta_{min}}{\Delta\theta}$;Number of steps to cover the range

; of cutting orientations

$E_1 = \frac{P_{max}(.75*10^6)}{K_s a}$;Power constraint constant

;

;Prescribing the coordinates of polygons ;

;

if $shape = 1$ then ;Five sided polygon

;

$x(1) = l_1 + x(3)$;

$x(2) = x(3)$;

$x(3) = x(3)$;

$x(4) = l_1 + l_2 + x(3)$;

$x(5) = l_1 + l_2 + x(3)$;

$y(1) = d_2 + y(3)$;

$y(2) = d_1 + y(3)$;

$y(3) = y(3)$;

$y(4) = y(3)$;

$y(5) = d_3 + y(3)$;

;

if $shape = 2$ then ;Three sided polygon

;

$x(1) = l_1 + x(3)$;

$x(2) = x(3)$	;
$x(4) = l_1 + l_2 + x(3)$	;
$x(5) = l_1 + l_2 + x(3)$	;
$y(1) = d_2 + y(3)$	;
$y(2) = y(3)$	;
$y(3) = y(3)$	;
;	
$i = 0$ *to* Big$k$	;Loop for range of cutting
;	orientations
$\theta_d(i+1) = \theta_{min} + i\Delta\theta$	;Cutting orientation in degrees
$\theta(i+1) = \frac{\pi\theta_d(i+1)}{180}$	;Cutting orientation in radians
;	
$d_{max} = 0.0$	;Maximum workpiece width
;	;perpendicular to cutting direction
$l_{max} = 0.0$	;Maximum workpiece length
;	;along cutting direction
;	
$L = 0$ *to* Big$L$	;Loop for transformation of co-ordinates
;	
$\alpha(L) = \tan^{-1}(\frac{y(L)}{x(L)})$	;Angles of lines through origin
;	;and vertices in radians

```
P(L) = P(L)/sin(α(L))                    ;Lengths of lines joining
;                                        origin and vertices
;
;Transformation of vertex co-ordinates   ;
;
if θ(i+1) < π/2 and θ(i+1) < α(L) then   ;
y'(L) = P(L) * sin(α(L) − θ(i+1))        ;
;
if θ(i+1) < π/2 and θ(i+1) > α(L) then   ;
y'(L) = −P(L) * sin(θ(i+1) − α(L))       ;
;
if θ(i+1) < π/2 then                     ;
x'(L) = P(L) * cos(abs(α(L) − θ(i+1))    ;
;
if θ(i+1) = π/2 then                     ;
x'(L) = y(L)                             ;
y'(L) = x(L)                             ;
;
if θ(i+1) > π/2 and θ(i+1) < π then      ;
if π/2 > (θ(i+1) − α(L)) then            ;
y'(L) = −P(L) * cos(π/2 − (θ(i+1) − α(L))) ;
x'(L) = P(L) * sin(π/2 − (θ(i+1) − α(L))) ;
;
```

```
if θ(i + 1) > π/2 and θ(i + 1) < π then          ;
if π/2 < (θ(i + 1) − α(L)) then                  ;
y'(L) = −P(L) * cos(θ(i + 1) − α(L) − π/2)       ;
x'(L) = −P(L) * sin(θ(i + 1) − α(L) − π/2        ;
;
if θ(i + 1) = π then                             ;
y'(L) = −y(L)                                    ;
x'(L) = −x(L)                                    ;
;
;Maximum workpiece width and length              ;
;
if L > 1 then                                    ;
y20(L) = abs(y'(L) − y'(L − 1))
x20(L) = abs(x'(L) − x'(L − 1))
;
if y20(L) > d_max then                           ;
d_max = y20(L)                                   ;
b'_1 = y'(L)                                     ;Co-ordinates corresponding
b'_2 = y'(L − 1)                                 ;to the maximum width
a'_1 = x'(L)                                     ;
a'_2 = x'(L − 1)                                 ;
;
if x20(L) > l_max then                           ;
l_max = x20(L)                                   ;
c'_1 = x^prime(L)                                ;Co-ordinates corresponding
c'_2 = x'(L − 1)                                 ;to the maximum length
;
```

```
if L > 2 then                          ;
y21(L) = abs(y'(L) - y'(L - 2))        ;
x21(L) = abs(x'(L) - x'(L - 2))        ;
;
if y21(L) > d_max then                 ;
d_max = y21(L)                         ;
b'_1 = y'(L)                           ;
b'_2 = y'(L - 2)                       ;
a'_1 = x'(L)                           ;
a'_2 = x'(L - 2)                       ;
;
if x21(L) > l_max then                 ;
l_max = x21(L)                         ;
c'_1 = x'(L)                           ;
c'_2 = x'(L - 2)                       ;
;
if shape = 1 then                      ;
if L > (BigL - 2) then                 ;
y22(L) = abs(y'(L) - y'(L - 3))        ;
x22(L) = abs(x'(L) - x'(L - 3))        ;
;
if y22(L) > d_max then                 ;
d_max = y22(L)                         ;
b'_1 = y'(L)                           ;
b'_2 = y'(L - 3)                       ;
a'_1 = x'(L)                           ;
```

```
a'_2 = x'(L - 3)                    ;
;
if x22(L) > l_max then              ;
l_max = x22(L)                      ;
c'_1 = x'(L)                        ;
c'_2 = x'(L - 3)                    ;
;
if shape = 1 then                   ;
if L > (BigL - 1) then              ;
y23(L) = abs(y'(L) - y'(L - 4))     ;
x23(L) = abs(x'(L) - x'(L - 4))     ;
;
if y23(L) > d_max then              ;
d_max = y23(L)                      ;
b'_1 = y'(L)                        ;
b'_2 = y'(L - 4)                    ;
a'_1 = x'(L)                        ;
a'_2 = x'(L - 4)                    ;
;
if x23(L) > l_max then              ;
l_max = x23(L)                      ;
c'_1 = x'(L)                        ;
c'_2 = x'(L - 4)                    ;
;
next L                              ;
;
R_θ(i + 1) = d_max/2                ;Tool radius for full immersion
```

```
R(i+1) = 1.1 * R_θ(i+1)                ;10 percent tolerance
g'_1(i+1) = (b'_1 + b'_2)/2            ;
;
if c'_1 < c'_2 then                    ;
h'_1(i+1) = c'_1 - R(i+1)              ;
else                                   ;
h'_1(i+1) = c'_2 - R(i+1)              ;
;
l_θ(i+1) = h'_1(i+1) + l_max + 2R(i+1) ;
;
;Slope of edges of polygon             ;
;
L = 1  to  BigL                        ;
;
if L < Big L then                      ;
sden(L) = x'(L+1) - x'(L)              ;Denominator in slope expression
snum(L) = y'(L+1) - y'(L)              ;Numerator in slope expression
else                                   ;
sden(L) = x'(1) - x'(L)                ;
snum(L) = y'(1) - y'(L)                ;
;
if abs(sden(L)) < abs(1) then          ;
m'(L) = 200                            ;Edge is perpendicular to cutting direction
;
if abs(snum(L)) < abs(1) then          ;Edge is parrallel to cutting direction
```

$m'(L) = 0$	;
;	
if $abs(sden(L)) \geq abs(1)$ then	;
if $abs(snum(L)) \geq abs(1)$ then	;
$m'(L) = \frac{snum(L)}{sden(L)}$	;
;	
$b'(L) = y'(L) - m'(L) * x'(L)$	;y intercept for edges
;	
next L	;
;	
$L(i+1) = 6R(i+1)$	;Flute length
$T_{max} = .75 * 10^6 * \frac{P_{max}R(i+1)}{V_1}$	;Maximum allowable Torque
$C_p = 0.0$	;Initialize the process cost
Delta= 1000	;Arbitrary number for sampling intervals
$s_{t_i} = 0.02$	;Initial feed
$N_i = 200$	;Initial speed
$x = \frac{s_{t_i}N_i z}{60}\Delta t$	;Initial distance moved
;	
$j = 1$ to Delta	;Loop for sampling intervals
;	
$h'_j(j) = h'_1(i+1) + x$	;Subsequent co-ordinates of tool centre
$g'_j(j) = g'_1(i+1)$	;
$v'_j = -1000$	;Initialization of y co-ordinate of exit point
$f'_j = 1000$	;Initialization of y co-ordinate of entry point
$countp = 0.0$	;Counter of entry and exit points

```
;Points of intersection                                  ;
;
L = 1  to  BigL                                          ;
;
if m'(L) = 200 then                                      ;
bh(j) = -2 * g'_j(j)
chp(j) = (x'(L))^2 + (h'_j(j))^2 + (g'_j(j))^2 - R(i+1)^2-  ;
2 * x'(L) * h'_j(j)                                      ;
ah(j) = 1                                                ;
disc(j) = bh(j)^2 - 4 * ah(j) * chp(j)                   ;
;
k_o = 1  to  2                                           ;Loop for entry and exit points
;
if m'(L) ≠ 200 then                                      ;
if disc(j) ≥ 0 then                                      ;
if k_o = 1 then                                          ;
gp(k_o) = (-bh(j) + sqrt(disc(j)))/(2*ah(j))             ;
hp(k_o) = m'(L) * gp(1) + b'(L)                          ;
else                                                     ;
gp(k_o) = (-bh(j) - sqrt(disc(j)))/(2*ah(j))             ;
hp(k_o) = m'(L) * gp(k_o) + b'(L)                        ;
;
if m'(L) = 200 then                                      ;
if disc(j) ≥ 0 then                                      ;
if k_o = 1 then                                          ;
gp(k_o) = x'(L)                                          ;
```

```
hp(k_o) = (-bh(J) + sqrt(disc(j)))/2      ;
else                                      ;
gp(k_o) = x'(L)                           ;
hp(k_o) = (-bh(J) - sqrt(disc(j)))/2      ;
;
if disc(j) < 0 then                       ;
gp(k_o) = 1000000                         ;
hp(k_o) = 1000000                         ;
gp(k_o) = integer of gp(k_o)              ;
hp(k_o) = integer of hp(k_o)              ;
if L < Big L then                         ;
if x'(L) > x'(L + 1) then                 ;
fxlim = x'(L)                             ;Upper and lower
sxlim = x'(L + 1)                         ;limits of abscissas for edges
else                                      ;
fxlim = x'(L + 1)                         ;Upper and lower limits
sxlim = x'(L)                             ;of ordinates for edges
;
if y'(L) > y'(L + 1) then                 ;
fylim = y'(L)                             ;
sylim = y'(L + 1)                         ;
else                                      ;
fylim = y'(L + 1)                         ;
sylim = y'(L)                             ;
;
if L =BigL then                           ;
```

```
if x'(L) > x'(1) then                                  ;
fxlim = x'(L)                                          ;
sxlim = x'(1)                                          ;
else                                                   ;
fxlim = x'(1)                                          ;
sxlim = x'(L)                                          ;
;
if y'(L) < y'(1) then                                  ;
fylim = y'(1)                                          ;
sylim = y'(L)                                          ;
else                                                   ;
fylim = y'(L)                                          ;
sylim = y'(1)                                          ;
;
if gp(k_o) ≥ h'_j(j) then                              ;Check for entry
if m'(L) ≠ 200 and m'(L) ≠ 0 then                      ;and exit points
if gpint(k_o) ≤ fxlim and gpint(k_o) ≥ sxlim then      ;
if hpint(k_o) ≤ fylim and hpint(k_o) ≥ sylim then      ;
if abs(hp(k_o) − (m'(L) * gp(k_o) + b'(L))) ≤ 2 then   ;
countp = countp + 1                                    ;Increase the count by one
if hp(k_o) ≥ v'_j then                                 ;
v'_j = hp(k_o)                                         ;Update the
u'_j = gp(k_o)                                         ;exit point
;
```

```
if hp(k_o) < f'_j then                                  ;
f'_j = hp(k_o)                                          ;Update the
e'_j = gp(k_o)                                          ;entry point
goto 28                                                 ;
;
if gp(k_o) ≥ h'_j(j) then                               ;
if m'(L) = 200 then                                     ;
if gpint(k_o) = fxlim and gpint(k_o) = sxlim then       ;
if hpint(k_o) ≤ fylim and hpint(k_o) ≥ sylim then       ;
countp = countp + 1                                     ;
;
if hp(k_o) ≥ v'_j then                                  ;
v'_j = hp(k_o)                                          ;
u'_j = gp(k_o)                                          ;
;
if hp(k_o) < u'_j then                                  ;
f'_J = hp(k_o)                                          ;
e'_j = gp(k_o)                                          ;
goto 28                                                 ;
;
if gp(k_o) ≥ h'_j(j) then                               ;
if m'(L) = 0 then                                       ;
if gpint(k_o) ≤ fxlim and gpint(k_o) ≥ sxlim then       ;
if hpint(k_o) = fylim and hpint(k_o) = sylim then       ;
countp = countp + 1                                     ;
;
```

```
if hp(k_o) ≥ v'_j then   ;
v'_j = hp(k_o)           ;
u'_j = gp(k_o)           ;
;
if hp(k_o) < f'_j then   ;
f'_j = hp(k_o)           ;
e'_j = gp(k_o)           ;
next k_o                 ;
;
next L                   ;
;
if countp = 0 then       ;No tool workpiece
g1p(j) = 0.0             ;engagement
h1p(j) = 0.0             ;
g2p(j) = 0.0             ;
h2p(j) = 0.0             ;
;
if countp = 1 then       ;Tool just touches
g1p(j) = 0.0             ;the workpiece but no
h1p(j) = 0.0             ;cutting
g2p(j) = 0.0             ;
h2p(j) = 0.0             ;
;
if countp > 1 then       ;Entry and exit points
g1p(j) = u'_j            ;
h1p(j) = v'_j            ;
```

$g2p(j) = e'_j$	;
$h2p(j) = f'_j$	;
;	
$dp(j) = \sqrt{(g1p(j) - g2p(j))^2 + (h1p(j) - h2p(j))^2}$	;Distance between entry
;	and exit points
$exper(j) = \frac{2*R(i+1)^2 - dp(j)^2}{2*R(i+1)^2}$	;
$\phi_s(j) = \cos^{-1}(exper(j))$	;Swept angle of cut
;	
if $\phi_s(j) = 0$ then	;
next j	;
$d(j) = R(i+1)(1 - \cos(\phi_s(j)))$	;Radial width
if $d(j) = 0$	;Initial radial depth for triangle
then	
$d(j) = 2$	;
;	
31 if $R > d(j)$	;Feed per tooth
then	;from tooth breakage constraint
$s_t(j) = \frac{s_{max}}{\sin(\phi_s(j))}$	
else	
$s_t(j) = s_{max}$	
;	
$\gamma(j) = \frac{K_s a z s_t(j)}{2\pi}$	Cutting force constant
$F_x(j) = \gamma(j)[(1 - \cos(2\phi_s(j))) + r_1(2\phi_s(j) - \sin(2\phi_s(j)))]$	;Average cutting forces
$F_y(j) = \gamma(j)[r_1(1 - \cos(2\phi_s(j))) - (2\phi_s(j) - \sin(2\phi_s(j)))]$	;for upmilling

$F_x(j) = \gamma(j)[r_1(2\phi_s(j) - \sin(2\phi_s(j))) - (1 - \cos(2\phi_s(j)))]$;Average cutting forces

$F_y(j) = \gamma(j)[r_1(1 - \cos(2\phi_s(j))) + (2\phi_s(j) - \sin(2\phi_s(j)))]$;for downmilling

;

$F_R(j) = \sqrt{F_x^2 + F_y^2}$;Resultant cutting force

$M(j) = F_R(j)(L - \frac{a}{2})$;Resulting moment on the cutter

$T(j) = \gamma d(j)$;Resulting torque

;

$\sigma(j) = \frac{2}{piR^3} M(j)\sqrt{M(j)^2 + T(j)^2}$;Shank stresses

;

if $\sigma(j) < \sigma_{max}$;Shank breakage constraint

then

goto 41 ;Check torque constraint

else

$s_{max} = s_{max} - 0.01$;Reduce maximum feed

goto 31 ;Calculate feed again

;

41 if $T_{max} > T(j)$;Torque constraint check

then

goto 51 ;Calculate equivalent feed

else

$s_{max} = s_{max} - 0.01$;Reduce maximum feed

goto 31 ;Calculate feed again

;

$S_{eq}(j) = \frac{s_t(j)d(j)}{R\phi_s(j)}$	;Equivalent feed per tooth
;	
$N_{max}(j) = \frac{V_{max}}{2\pi R}$	;Range of
$N_{min}(j) = \frac{V_{min}}{2\pi R}$	;r.p.m.
;	
$N_{max_p}(j) = \frac{60E_1}{s_t(j)z}$	;Maximum r.p.m. allowed by
;	;Power constraint
$\Delta N = 100$	;Initial increment for r.p.m.
61 Omega=integer value of $\frac{N_{max}-N_{min}}{\Delta N}$	;Upper limit of the index
;	for r.p.m. optimization loop
;	
$k = 0$ to Omega	;Loop for optimization of r.p.m.
$N(k+1) = N_{min} + k\Delta N$	;Next spindle r.p.m.
;	
while $N_{max_p} > N(k+1)$	;Power constraint check
$N(k+1) = N(k+1) - 1$	;Reduce spindle r.p.m.
;	
$V(k+1) = \frac{2\pi RN(k+1)}{60}$	;Spindle peripheral velocity
;	
$T_h(k+1) = \frac{60000*R\phi_s(j)}{V(k+1)}$	;Heating time in ms;
$T_c(k+1) = \frac{60000*R(2\pi-\phi_s(j))}{V(k+1)}$	;Cooling time in ms
;	

$smallx(k+1) = \frac{2\pi}{\phi_s(j)}$	;Ratio of total cycle time to
;	actual cutting time per cycle
$E_r(k+1) = 39\log(T_c(k+1)) - 23\log(T_h(k+1)) + 37.5$	;Range of thermal strain
;	
$X(k+1) = E_r(k+1)(\frac{N(k+1)*xsmall(k+1)}{60})^{\frac{1}{2}}$	;Thermal fatigue parameter
;	
$T_L(k+1) = \frac{2\pi}{\phi_s(j)} \frac{C_1}{X(k+1)^m S_{eq}(j)^n V(k+1)^p a^q}$	;Tool life
;	
$\Delta C_p(k+1) = (C_h + \frac{C_t}{T_L(k+1)} + \frac{C_h T_{ct}}{T_{L(k+1)}})\Delta t$	;Process cost for one interval
;	
if $k > 1$ then	;Search of optimum r.p.m.
if $\Delta C_p(k+1) > \Delta C_p(k)$ then	;by comparing the cost
if $\Delta N > 1$ then	;with the previous value,
if $\Delta C_p(k+1) > \Delta C_p(k)$ then	;by comparing the cost
if $\Delta N > 1$ then	;with the previous value,
$N_{max} = N(k+1)$	;narrowing the speed range,
$N_{min} = N(k-1)$	;setting new limits for r.p.m
$\Delta N = \frac{\Delta N}{10}$	and new r.p.m. increment
goto 61	;New upper limit of loop index
else	
$N_{min} = N(1)$	;
$\Delta N = 1$	;
if $k = 0$ then	;

```
ΔC_min = ΔC_p(k + 1)                    ;Initialization of minimum cost
N_min_o = N(k + 1)                      ;Initialization of optimum r.p.m.
;
if ΔC(k + 1) < ΔC_min then              Comparison with previous minimum cost
ΔC_min = ΔC(k + 1)                      ;New minimum cost
N_min_o = N(k + 1)                      New optimum r.p.m.
next k                                  ;Try the next r.p.m.
;
C_p = C_p + ΔC_min                      ;Summation of costs
Δx(j) = (s_t(j) N_min_o z / 60) Δt      ;Distance moved
x = x + Δx(j)                           ;Total tool traverse distance
;
if x ≥ (l_θ(i + 1) − R(i + 1)) then     ;Machining finished
goto 91                                 ;
next j                                  ;Next sampling interval
;
91  C_p_min(i + 1) = C_p                ;Cost with the selected radius
;
next i                                  ;Next cutting direction
;
;Plot C_p_min for cutting directions θ_d(i + 1)
;
stop
end
```

Appendix E

Flow Chart for Single Pass Optimization