THE USE OF AN APPROXIMATE INTEGRAL METHOD TO ACCOUNT FOR INTRAPARTICLE CONDUCTION IN GAS-SOLID HEAT EXCHANGERS.

by

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B.Sc (Hons) , University of Wales ,U.K. 1983

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

in

FACULTY OF GRADUATE STUDIES

Department of Mechanical Engineering

We accept this thesis as conforming

to the required standard

UNIVERSITY OF BRITISH COLUMBIA

APRIL 1985

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Date: APRIL 1985

ABSTRACT

The mathematical equations describing transient heat transfer between the fluid flowing through a fixed bed and a moving bed of packing were formulated. The resistance to heat transfer within the packing due to its finite thermal conductivity was taken into account.

An approximate integral method was applied to obtain an analytical solution to transient response of the bed packing.Results for two cases of fixed and moving bed were obtained. The validity of the approximate method was checked against the more exact method employed by Handley and Heggs who obtained the results for a fixed bed of packing with a step change in fluid inlet temperature. It was concluded that the approximate mehod gives results that agree well with the more exact methods.

The method considered here provids a quick determination of the packing mean temperature in order to obtain the effectiveness.The other peculiarity of this method is that the effect of packing thermal conductivity can be examined very quickly since the solution is in analytical form. The analysis of the results revealed that as the thermal conductivity of the packing decreases the difference between its surface and mean temperature increases.A series of charts showing the comparison between the packing surface and mean temperatures for different thermal conductivities are presented.

The approximate method was then applied to the case of a moving bed of packing. It was concluded that the effect of packing thermal conductivity is more severe than expected. A series of charts representing the moving bed effectiveness versus dimensionless length for different thermal conductivities are presented.

Table of Contents

List of Tables

List of Figures

Acknowledgements

The author wishes to express his appreciation towards Prof.E.G.Hauptmann for all his help and advice.
The author would also like to thank Miss Afarin.Radjaie (for her moral support) and Mr Iraj.Riahi (for his financial support).

This work is dedicated to my parents.

Nomenclature

A	= Solid surface area per unit bed volume	m^2/m^3
A_b	= Bed cross-sectional area	m^2
B	= Porosity	-
Bi	= Biot number $=hR/K_s$	-
C	= Fluid specific heat capacity at constant pressure	J/kg K
C_s	= Solid specific heat capacity	J/kg K
d	= Matrix semi-thickness	m
D_{ev}	= Equivalent spherical diameter	m
F	= Normalised fluid temperature	$(T_f-T_i)/(T_{fi}-T_i)$
$f(\theta)$	= Prescribed heat flux at the solid surface	W/m^2
h	= Convective heat transfer coefficient	W/m^2K
i	= Number of time steps	-
K_s	= Matrix thermal conductivity	W/m K
y	= Distance from the bed entrance	m
L	= Bed length	m
M_s	= Bed density	kg/m^3
$\dot{m}$	= fluid mass flow rate / area	kg/m^2s
$\dot{m}_s$	= Solid mass flow rate / area	kg/m^2s
n	= Number of length steps along the bed	-
Nu	= Nusselt number $=hd/K_s$	-
P	= Period of fluid flow	s
Pr	= Prandtl number	-
Q	= Heat transfer	J
r	= Distance from the centre of sphere	m
R	= Radius of sphere	m

Re	=	Reynolds number	-
T	=	Solid temperature	K
T_s	=	Solid surface temperature	K
T_f	=	Fluid temperature	K
u	=	Fluid velocity	m/s
u_s	=	Solid velocity	m/s
V_b	=	Bed volume	m^3
x	=	Distance from the surface of the slab	m
z	=	Dimensionless thickness or radius	-

Greek symbols

α	=	Thermal diffusivity	m^2/s
δ	=	Penetration depth	m
δ_0	=	Dimensionless penetration depth	-
ϵ	=	Effectiveness	-
η	=	Dimensionless time$=hA(\theta-y/u)/M_sC_s$	-
θ	=	Time from the start of the operation	s
Λ	=	Dimensionless bed length $= \xi_{y=L}$	-
ν	=	Kinematic viscosity	m^2/s
ξ	=	Dimensionless distance along the bed$=hAy/\dot{m}C$	-
Π	=	Dimensionless period $= \eta_{\theta=P}$	-
ρ_f	=	Fluid mass density	kg/m^3
ρ_s	=	Solid mass density	kg/m^3
Ψ	=	Normalised solid temperature$=(T-T_i)/(T_{fi}-T_i)$	-

Subscripts

f = Fluid

i = Inlet or initial

m = Mean

0 = Outlet

s = surface

1. INTRODUCTION

1.1 GENERAL

Heat exchange between two fluid streams at different temperatures represents an important technological process in many branches of industry.
There are essentially two types of heat exchangers [1],[1]

1. Recuperators or conductance heat exchangers,in which the thermal conditions are assumed to be invariant with time.Thus the rates of heat flow are steady,with the convection from the hotter fluid continuously equal to the convection to the colder fluid and both equal to the steady rate of conduction through the separating heat transfer surface (Fig.1).

2. Capacitance heat exchangers or regenerators which make use of the thermal capacity of the heat transfer surface.

Regenerators are used extensively as air heaters.As an example consider MHD (Magnetohydrodynamic) power generation which uses the interaction of an electrically conducting fluid with a magnetic field to convert part of the energy of the fluid directly into electricity (Fig.2)[2].

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[1] numbers in square bracket refer to the Bibiliography

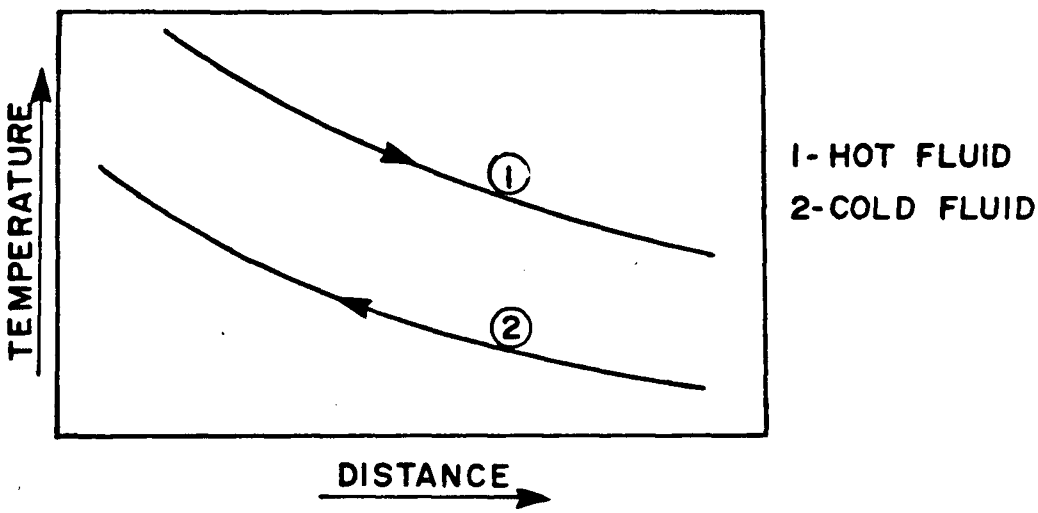


FIG.1(a)

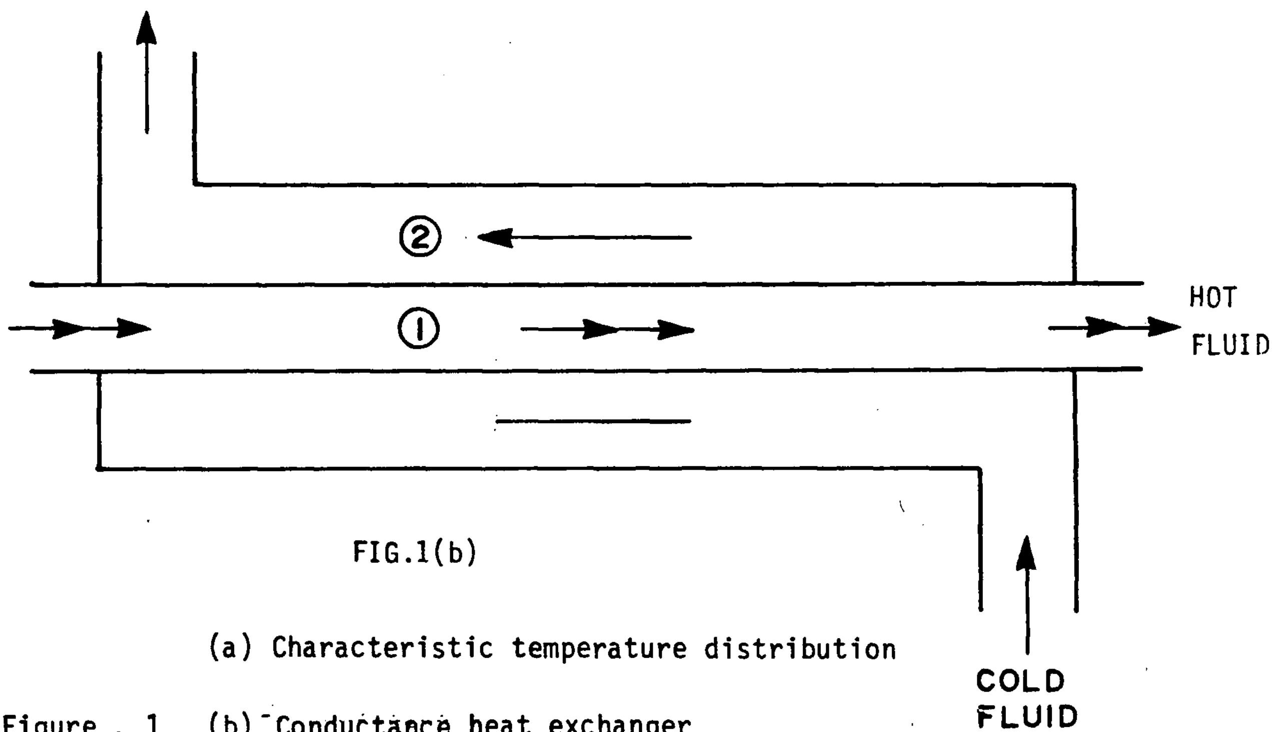


FIG.1(b)

(a) Characteristic temperature distribution

Figure . 1 (b) Conductance heat exchanger
(Counter flow)

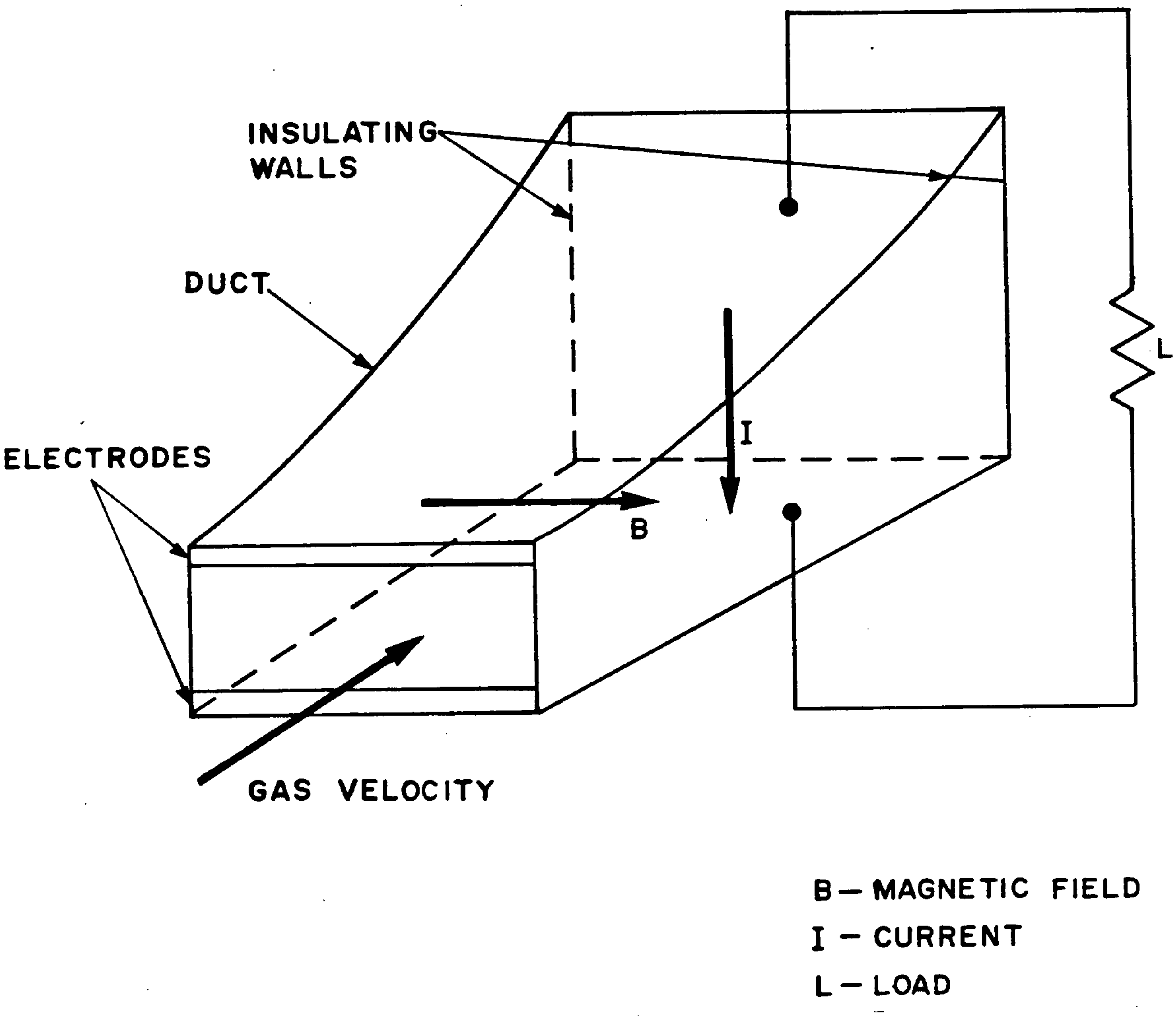


Figure 2. Schematic arrangement of an MHD duct.

In the fossil fired MHD system, the temperature of the fluid (combustion air) at the magnetic field inlet, which determines the conductivity and hence the power density at that point, depends on the performance of the combustion chamber. For optimum performance ,it is essential to preheat the combustion air to a temperature greater than 1200 ^{0}C. This is achieved by utilizing regenerative air preheaters.

Another area in which regenerators are finding increasing application is as exhaust heat exchanger in small gas turbines [3].In these engines ,it is essential to obtain a large heat transfer area within the smallest possible volume.This requires passages of small diameter and the large number of these can be very conveniently incorporated in a regenerator.

The peculiarity of a regenerator is that the heat transfer media (called the matrix) is alternatively heated by one gas and cooled by the other.This means that either the matrix must be moved periodically in and out of the gases or that the matrix be alternatively swept by the hotter fluid, when it absorbs heat, and then by the colder fluid to which the heat absorbed is then returned.The former is called a **continiuously operating** and the latter is called an **intermittent** regenerator, respectively.

There are two main problems associated with the mechanical design of these two types of regenerators [1]:

1. Providing means for changing either the gas flow or the matrix periodically to enable the latter to be heated and cooled by contact with both gases in turn and thus to exchange heat between them.
2. Sealing the two gas flows before,during and after the exchange of heat to prevent excessive leakage.

Problem 1 is easily solved in the case of continuously operating regenerators, but problem 2 is more difficult to solve.In the case of intermittent regenerators both problems can be easily avoided.

One of the major problems associated with the thermal design of regenerators is the transient response of the matrix.This problem arises when the regenerator is made up of matrices with low thermal conductivity. If the thermal conductivity of the matrix is sufficiently low then allowance must be made for the thermal gradient within the matrix; this is called the **intraconduction** effect. Consequently there is a combined convection-conduction(within the matrix) heat transfer in relation to uniquely convective heat transfer.The inclusion of the matrix conductivity makes the thermal design of the regenerator more complicated .There are a number of solutions to this problem.For example it is a common practice to adopt different numerical schemes (such as trapezoidal approximation, central difference approximation or Crank- Nicholson method) to solve the transient response of the regenerator matrix. However,these numerical schemes involve substantial expenditure of computational time and computer storage [4].Heggs and Carpenter [5] predicted that the programs which neglect the intraconduction typically require a sixth of the computing time of those including it.

The purpose of the present study is to propose an alternative solution to the problem of determining transient reponse of the matrix. The proposed method utilizes an approximate integral technique to obtain a solution in an analytical form.This avoids the use of numerical schemes and thereby enables the designer to take account of the intraconduction effect more efficiently.

The central features of the regenerator heat exchanger are regenerator matrices.The choice of the matrix is mainly affected by the way in which the periodic heating and cooling of this matrix is performed.It may consist of plates, wires ,spheres or broken solids of irregular shape [1].

Ideally the matrix material selected should possess high values of specific heat,density and melting point,good strength at elevated temperatures;also it should be easily available and cheap.It should not react chemically with the impurities present in the heating gases(usually combustion gas). In some cases the aerodynamic drag properties of the matrix are of great importance.Thus it is obvious that all these points should be considered before a decision to select the appropriate matrix is made.

The matrix can be brought into contact with the heat exchanging fluids (both hot and cold) in different ways.The following types of regenerators may therefore be distinguished (refer to Fig.3).

1.1.1 STATIONARY MATRIX

In this type of regenerator, the matrix is stationary while the change over from the heating to the cooling period of the matrix is performed by alternately changing the flow of the two fluids.

The rate of heat transfer to or from a surface is directly proportional to the area available for heat transfer.The overall dimension of the regenerator is related to the heat transfer area; increasing the surface area per unit bed volume decreases the overall dimension required for a prescribed amount of heat transfer.In the case of a stationary matrix, the two heat exchanging fluids flow alternately through one and the same matrix.Consequently the overall dimension is the same for both the heating and the cooling side (Fig.4).

Regenerators of this type are subdivided into different groups depending on the type of the matrix used.The pebble bed for example (Fig.4a) is one type whose matrix elements can be of any shape and may consist of light, inexpensive ceramic materials capable

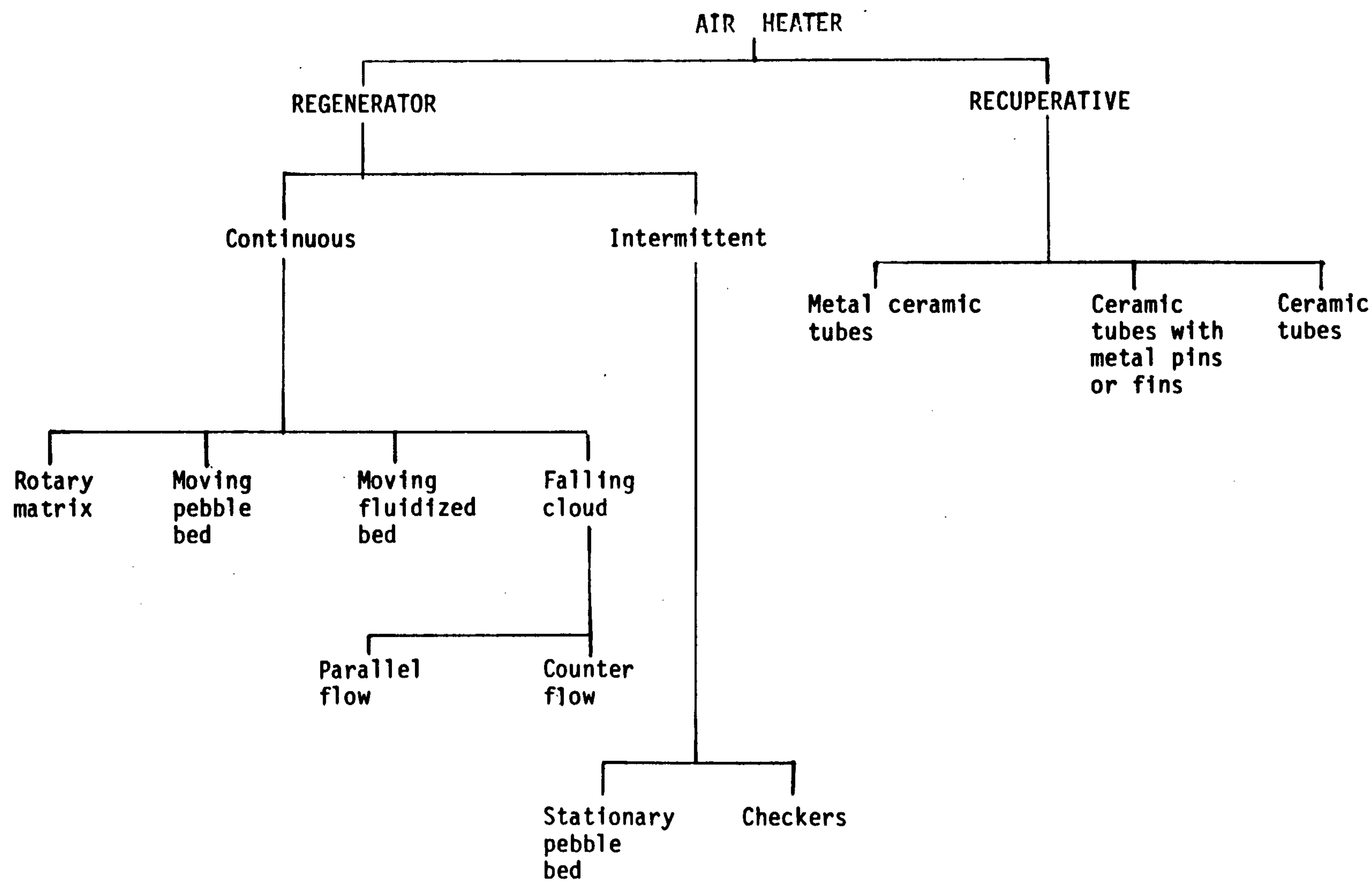


Figure 3. Types of air heater.

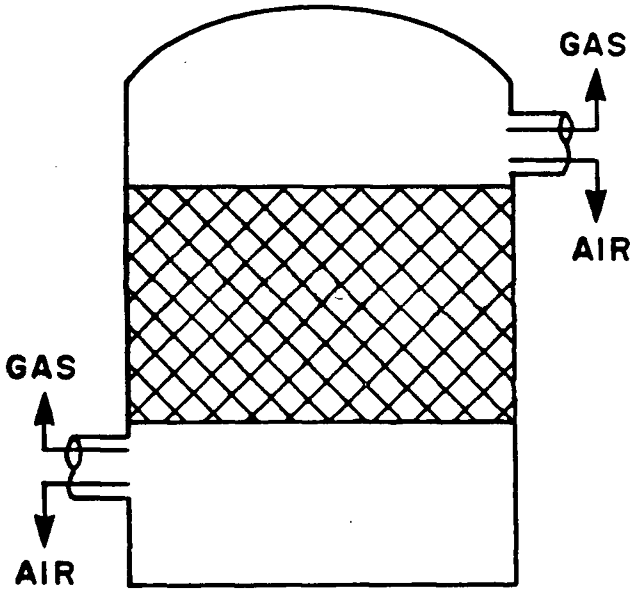


FIG.4(a)

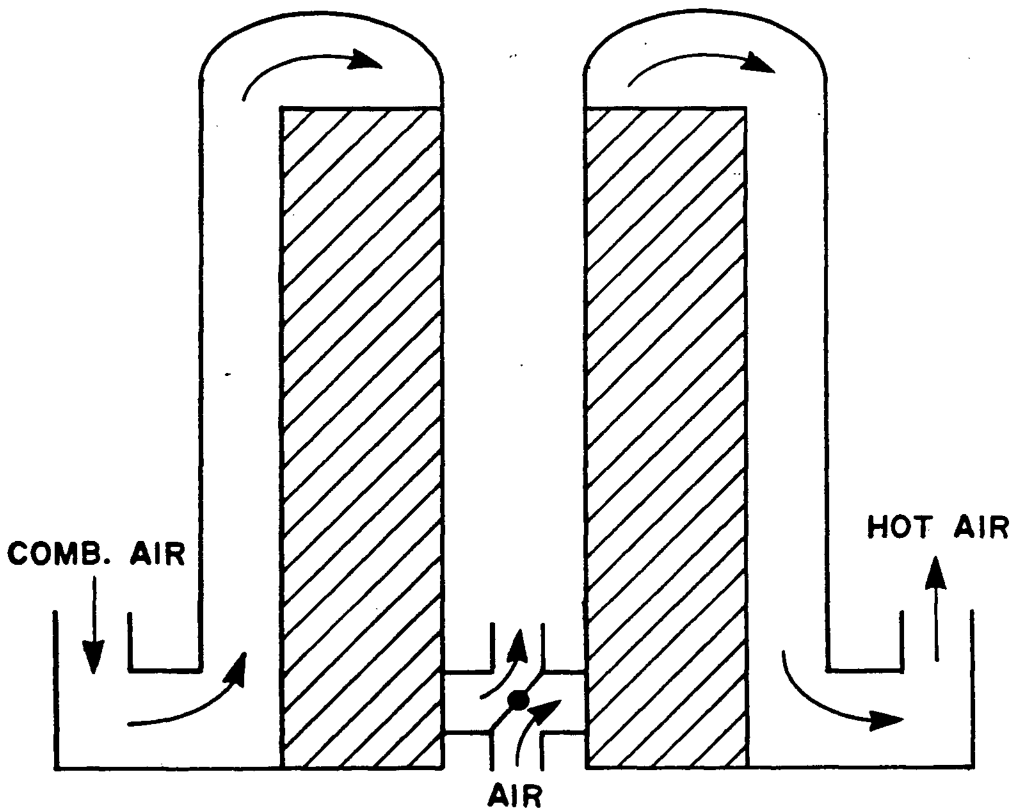


FIG.4(b)

Figure 4. Fixed bed regenerator.

of withstanding high temperatures[1].These matrices can be easily removed from their containers to be cleaned and if neccessary replaced.Possible kinds of elements are spheres or other irregular shapes.The elements can be supported by the walls of their container.For this type of regenerator, porosity (the ratio of empty volume to the total volume) is an important factor which comes into analysis.The porosity for a fixed bed is usually about 0.37 to 0.38.

The Cowper stove is an alternative type in which the matrix is made of refractory brick [6].The brick regenerator is used extensively in glass making and steel making industries, for preheating air to temperatures of the order of 900 to 1200 C (Fig.4b).

1.1.2 MOVING MATRIX

The characteristic of this type of regenerator is that the switch over from the heating to the cooling period and vice-versa is due to the matrix movement.This type of regenerator is subdivided into different groups depending on the matrix and the movement of the matrix (Fig.3).The four common types are rotary matrix,moving pebble bed, falling liquid slag and falling solid particles [2].

The operation of a rotary regenerator relies on the thermal storage of a slowly rotating matrix.With each revolution of the matrix a cycle of heating by hot air and cooling by cold air is completed.Axial and radial flow are the two basic forms of matrix configuration (Fig.5). One of the major design problems with regenerators of this type is to prevent the high pressure gas leaking to the low pressure air.This is controlled with the aid of appropriate seals.

The second type of regenerator which incorporates a moving matrix is a falling cloud regenerator(Fig.6).This type of regenerator has two chambers,nominally a superior chamber in which the matrix is heated by the fluid and an inferior chamber in which the fluid is heated by the matrix. The matrix consists of small solid particles of a suitable heat transfer medium,such as potassium sulphate,which are continuously melted in the upper chamber.The molten material must then be pressurized by a slag pump (not shown in Fig.6) before injection to the lower chamber where it is atomized before falling through the rising cold fluid.In this chamber the molten droplets are solidified and then returned externally to the top of upper chamber for recycling.There are technical problems with the design of liquid slag regenerators such as development of the slag pump and atomization of the molten material [7].

An alternative to falling liquid slag is falling solid particles (Fig.7). The matrix consists of broken solid particles either of regular or irregular shape.The

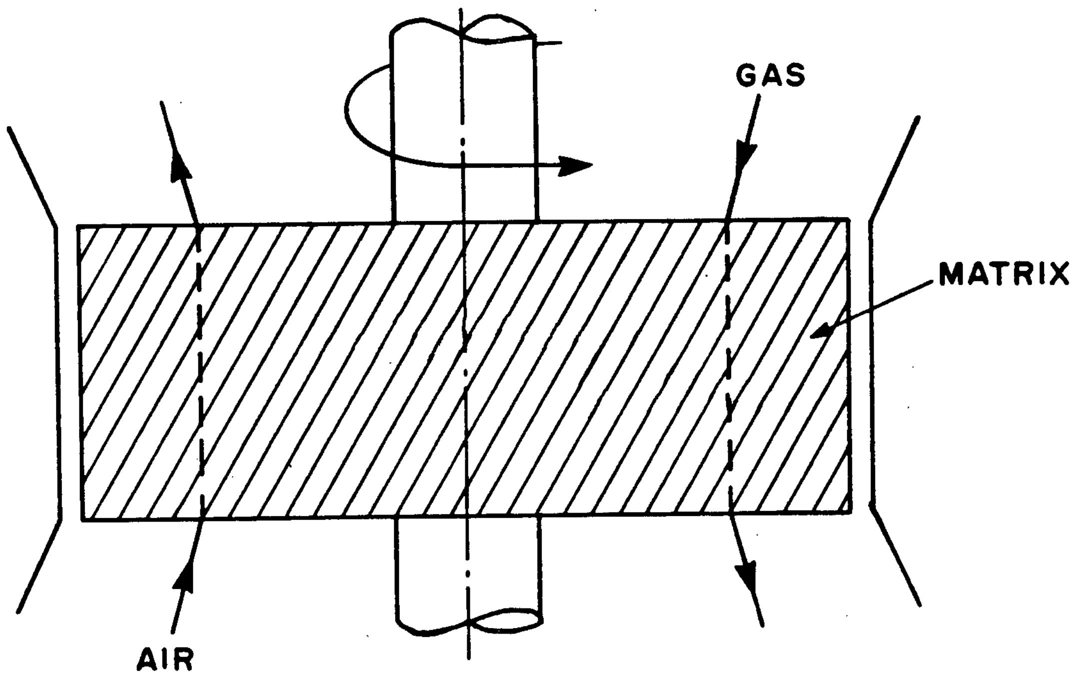
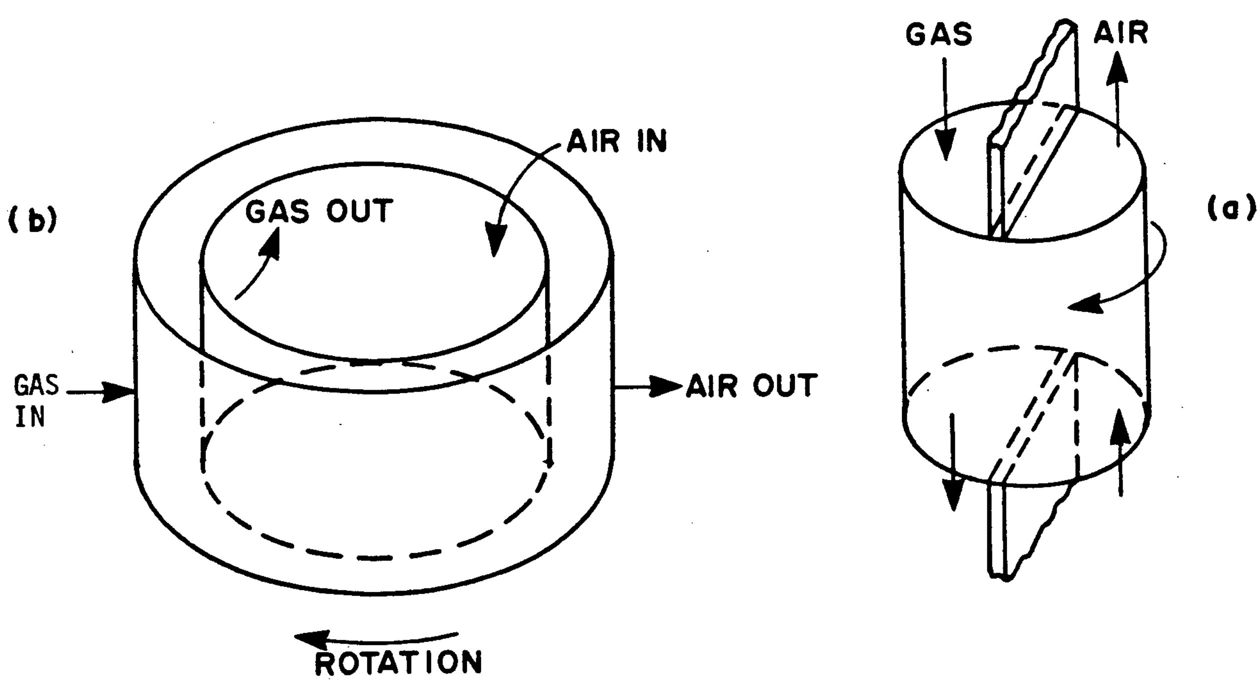


Figure 5(a) Schematic diagram of rotary regenerator.



Radial flow

Axial flow

Figure 5(b) Rotary regenerator flow arrangement .

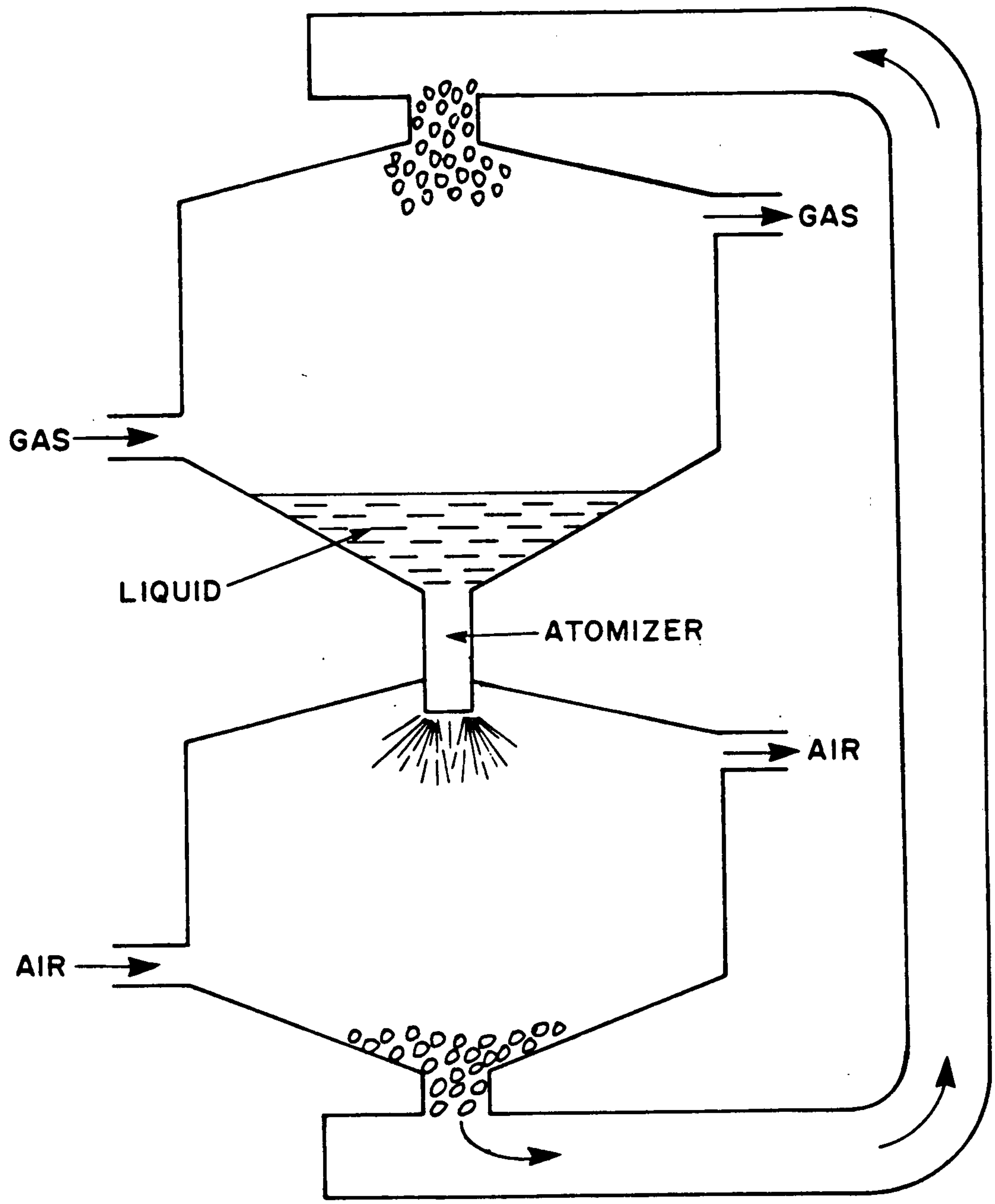


Figure 6. Falling cloud regenerator.

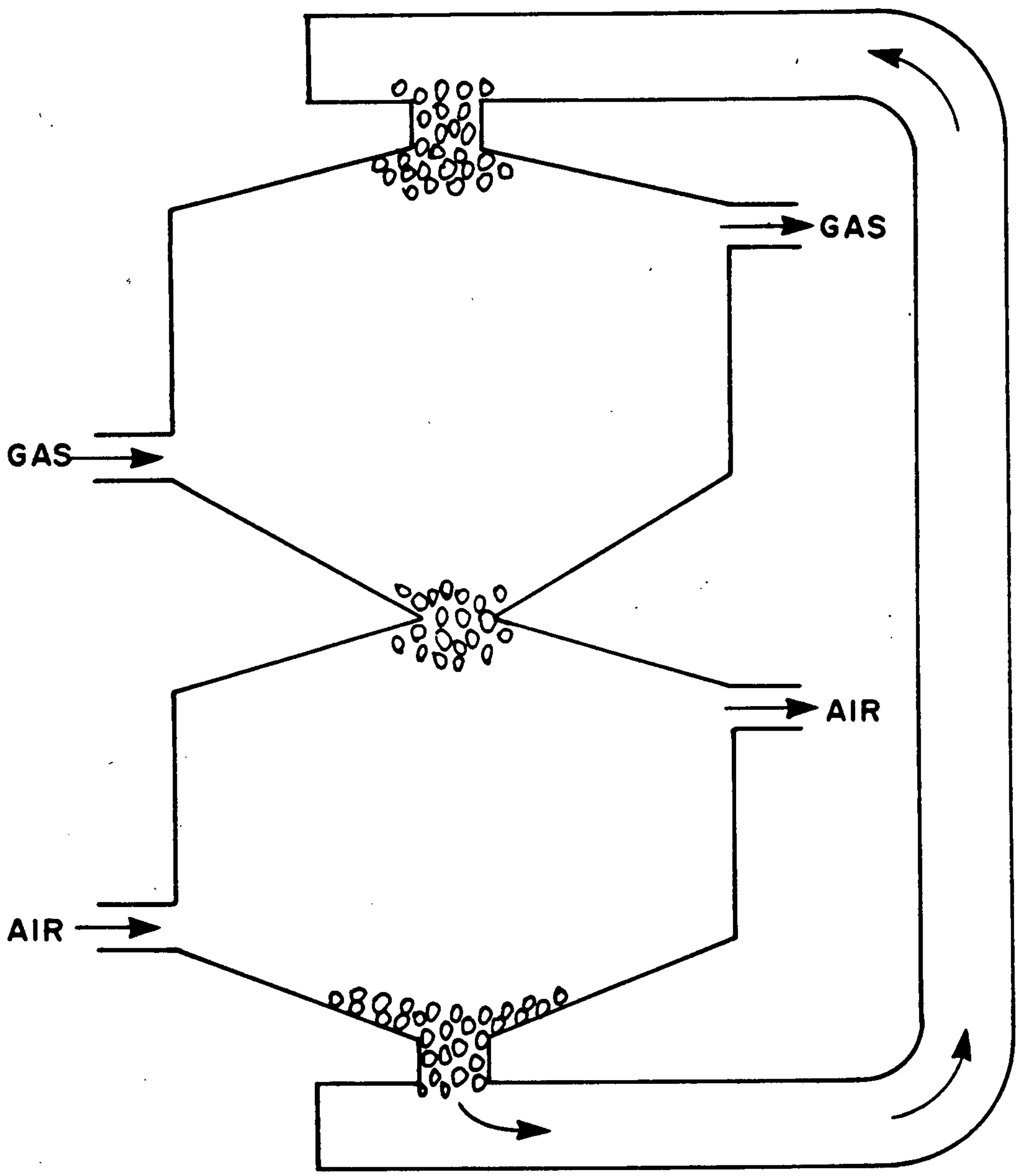


Figure 7. Moving pebble bed regenerator.

extension of this type is a moving pebble bed regenerator.One of the fundamental requirements is to ensure the uniform movement of the particles through the system , especially at inlet and outlet, to prevent the formation of dead sections. A new concept in the design of these regenerators is the use of a divergent bed in the direction of gas passage.It has been proven that such a design contributes to the uniform temperature profile in the gas behind the bed [8].

1.2 REVIEW OF PREVIOUS WORK

Regenerative heat exchange is one of the most common industrial processes.It is therefore of some importance to formulate the laws governing the rate of heat transfer in such a case,and if possible, to obtain a mathematical expression for the temperature distribution throughout such a system.

Much work has been done on developing mathematical models of regenerators [4-14].The theoretical considerations of the fundamental physics of heat transfer are complicated and certain simplifying assumptions must be made in order to obtain a useful mathematical model.

Mathematical analyses of regenerators are divided into three groups which are explained in the following sections.

1.2.1 SCHUMANN MODEL

The simplest mathematical model of a regenerator was first developed by Schumann in 1929 [13]. He suggested a model in which a fluid stream was allowed to flow through a packed bed of broken solids (Fig.4a). However his model can be employed for the case of fluid passing through the channels of brick matrix (often called chequer work) (Fig .4b).

Schumann's method of developing an exact mathematical treatment to the heat transfer problem in a regenerator assumes a bed consisting of crushed material at a uniform temperature; a fluid is allowed to pass lengthwise through the prism at a uniform rate of flow.The problem is to find the distribution of temperature in the bed and in the fluid for all time, assuming that

1. **The thermal properties of the system are independent of the temperature.**
2. **The axial conduction in either the fluid phase or the solid phase is negligible compared to the transfer of heat from solid to fluid.**
3. **The fluid flow rate does not vary along the bed.**
4. **There is no transverse thermal gradient within the particles at any instant.**

Based on these assumptions ,Schumann derived a pair of coupled differential equations (given in chapter 2) which determine the transfer of heat. With an appropriate set of boundary conditions the problem is solved completely.

Willmot [6] has presented a computer solution for the Schumann model.In an order of magnitude analysis ,he has shown that the axial conduction within the matrix is negligible provided that d/L^2 is small;where d is the semi-thickness of the matrix and L is the bed length.This ratio is negligible in most practical cases.

1.2.2 INTRACONDUCTION MODEL

One of the major disadvantages of the Schumann model is that it neglects the thermal gradient within the matrix.This simplifying assumption is justified provided that the matrix has a very high thermal conductivity. In many cases this is not so. Glass and ceramic (from which regenerator matrices are often made [12]) have a sufficiently low thermal conductivity that allowance must be made for the thermal gradient within the particles. The matrix thermal conductivity is taken into consideration in terms of a dimensionless parameter called Biot number which is defined as

$$Bi = hR/K_s \text{ (spherical geometry)} ,$$
$$= hd/K_s \text{ (planar geometry)} ,$$

where h=convective heat transfer coefficient ,

R=radius of sphere ,

d=semi-thickness of matrix ,

K_s=matrix thermal conductivity .

A model which includes the effect of matrix thermal conductivity is called an intraconduction model.Figure.8 shows the effect of conductivity (or Bi number)on the matrix temperature profile.

Previous studies have shown that intraconduction may have a significant effect on the thermal behavior of regenerators [4-13],although the inclusion of this effect makes the model more complicated. Different mathematical models have been proposed and have been solved either numerically or analytically.

Handely and Heggs [12] applied the Crank-Nicholson method in order to obtain a solution to transient response of the matrix in a fixed bed regenerator.Their theoretical results were in good agreement with their experimental observation.The authors also proposed a dimensionless group which predicts the dividing line between the Schumann and intraconduction models.

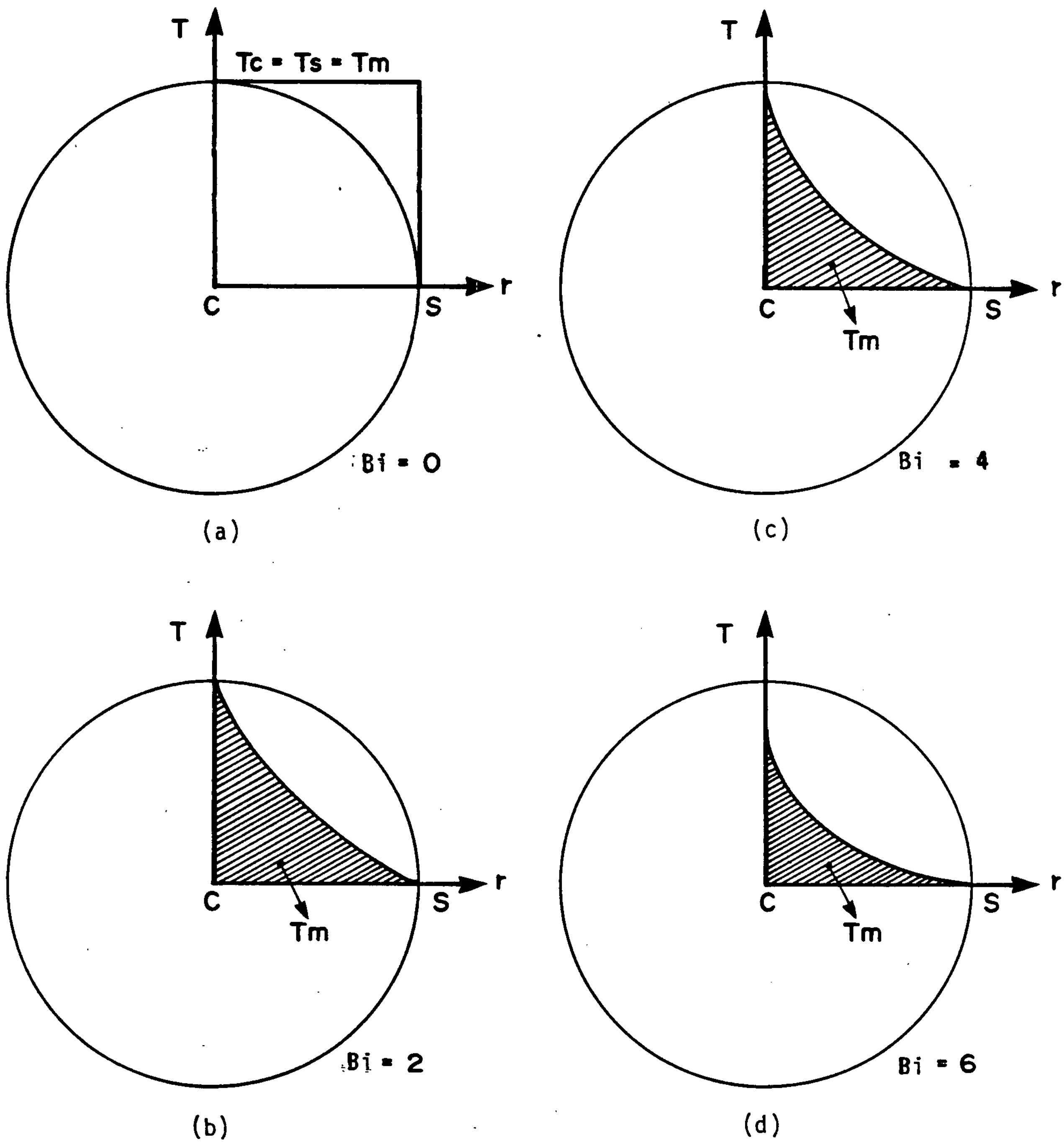


Figure 8. Effect of thermal conductivity of solid on the temperature profile, ($Bi = hR/K$).

Hausen [15] proposed a model in which the effect of matrix thermal conductivity is taken into consideration in terms of an overall heat transfer coefficient. This is the so called modified-infinite conduction model. The overall heat transfer coefficient is defined in terms of the actual convective heat transfer coefficient and a parameter which is called modification factor. A number of authors [4,6,15] have proposed different expressions for this modification factor.

An analytical solution to transient heat conduction in solids can be obtained by the approximate integral method.This method was first employed by Goodman [16] to solve a very simple problem of unsteady heat conduction in a semi-infinite slab.His results were in good agreement with the well established results of Carslaw and Jaeger [17].However, the elegant methods proposed by Carslaw and Jaeger will only be satisfactory if the thermal conductivity is independent of temperature; whereas the integral method can be extended to include the effect of temperature dependent thermal conductivity.Goodman applied the integral method to transient heat conduction in planar geometry. Lardner and Pohle [18] have demonstrated that the method is equally appropriate for spherical geometry.A more complete discussion of this method is given in the following chapter.

1.3 SCOPE OF THE PRESENT INVESTIGATION

The literature search confirms the fact that for a sufficiently low thermal conductivity, the intraconduction effect should be taken into consideration. The inclusion of this effect makes the analysis more involved. The solution to transient response of the matrix has to be determined.

The purpose of the present work is to propose an analytical solution to the matrix transient response by utilizing an approximate integral technique. The use of this method avoids lengthy computer programs; the method provides a quick determination of the solid mean temperature in order to obtain the effectiveness of the regenerator. The usual simplifying assumption of constant solid thermal properties can also be relaxed.

The first stage after the development of the proposed approximation is to examine its validity against more rigorous analyses.Once the accuracy of the method is established ,it can then be extended to include different types of regenerator and also the effect of matrix geometry on the regenerator performance. It should be emphasized that there are no published results for the moving bed regenerator.The development of the integral method provides the means to obtain design data for the moving bed regenerator.

2. THE GOVERNING EQUATIONS

2.1 DIMENSIONLESS PARAMETERS

It is a common practice in regenerator design to present the results in terms of a number of dimensionless parameters which are introduced to simplify the governing equations. These parameters were first introduced by Hausen [15].

Number of transfer units (NTU)

This parameter is also termed the reduced length (Λ) in the literature. It is defined in terms of the heat transfer coefficient,heat transfer area per unit bed volume and the fluid capacity rate[2] (refer to Fig.9) .

$$\xi = hAy/(\dot{m}C),$$

$$\Lambda = \xi_{y=L},$$

where h=convective heat transfer coefficient ,
A=heat transfer area/unit bed volume ,
$\dot{m}$=fluid flow rate/unit area ,
C=fluid specific heat capacity at constant pressure.

These terms are explained in more detail in Appendix A.

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[2]capacity rate=$\dot{m}C$

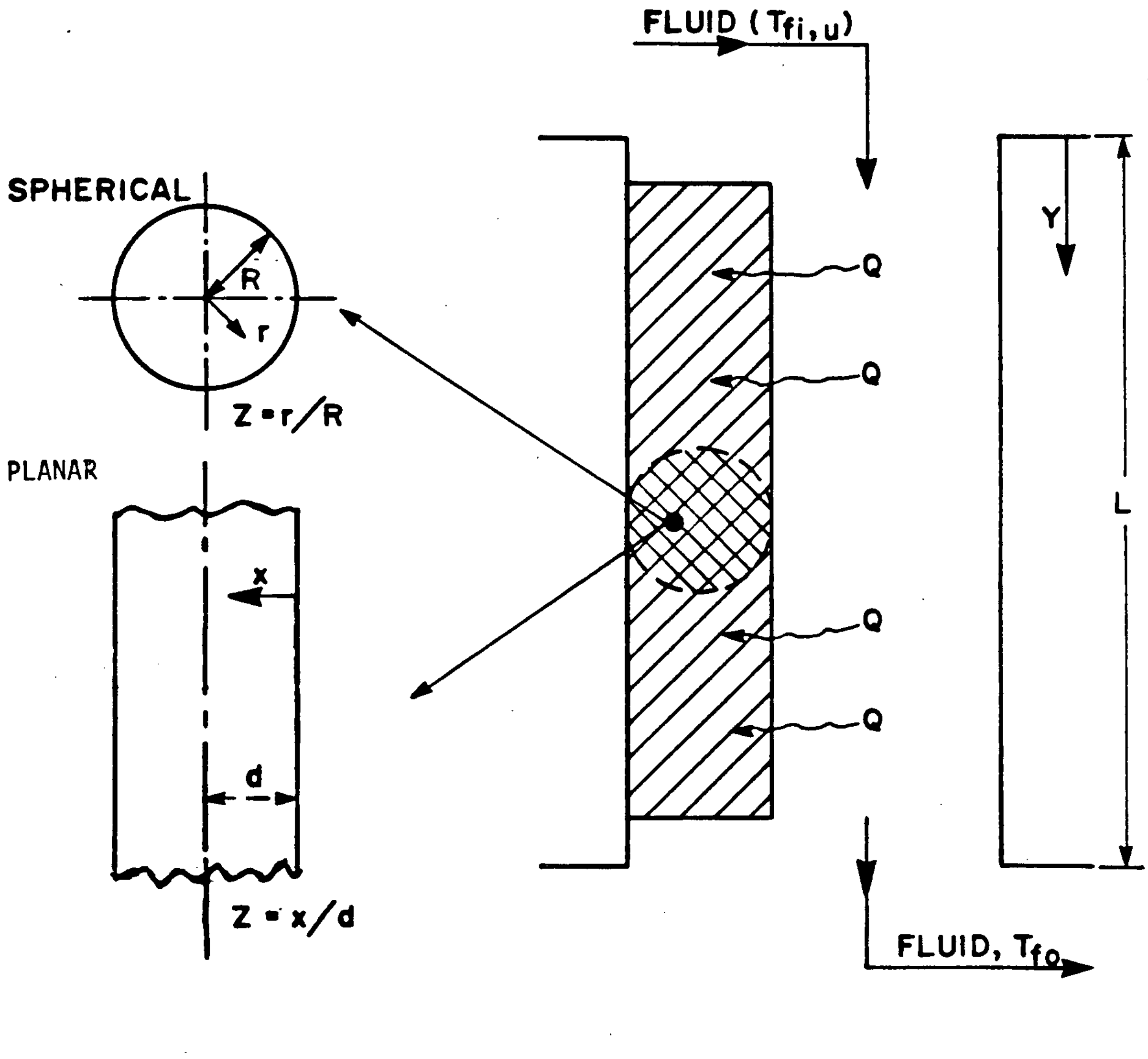


Figure 9. Typical dimensions of a regenerator.

Dimensionless period

The period of each heating and cooling cycle is nondimensionalised as

$$\eta = hA(\theta - y/u)/(M_s.C_s) \ ,$$

$$\Pi = hA(P - L/u)/(M_s.C_s) \ ,$$

where P=period of fluid flow ,

L=bed length ,

u=fluid velocity ,

M_s=bed density ,

C_s=solid specific heat capacity .

The term L/u which represents the fluid residence time is usually ignored.This is permissible in many regenerator applications because it is negligible compared to the period (p) [12]. In short cycling applications the residence time becomes of similar magnitude to the period and the effect of L/u can not be ignored.

Effectiveness

This is defined as the ratio of the actual rise in the matrix temperature to its maximum possible rise (refer to Fig.9), that is

$$\epsilon = [\dot{m}_s C_s (T_{ms0} - T_{si})/((\dot{m}C)_{min}).(T_{fi} - T_{si})] \ ,$$

where $\dot{m}_s$=solid flow rate/bed area ,

$(\dot{m}C)_{min}$=minimum of the two $\dot{m}C$,

$(\dot{m}_sC_s)/(\dot{m}C)_{min}$=capacity rate ratio.

It should be emphasized that the solid temperature is presented as a mean temperature ,which is different than the solid surface temperature.This is explained in more detail in Appendix D.

Dimensionless thickness or radius

The slab thickness and the sphere radius are nondimensionalised as (refer to Fig.9)

1. For the slab;

$z=x/d$, where d=thickness of the slab .

2. For the sphere ;

$z=r/R$, where R=sphere radius .

Normalised temperatures

The fluid and solid temperatures are normalized as

1. For the fluid

$$F=(T_f-T_{si})/(T_{fi}-T_{si}) \ .$$

2. For the solid

$$\Psi=(T_s-T_{si})/(T_{fi}-T_{si}) \ .$$

2.2 THE MATHEMATICAL MODELS

It was explained in the previous chapter that regenerators can be divided into different groups depending on the type of matrix employed.In the present work two types of regenerators are considered (fixed and moving bed) with two matrix geometries (planar and spherical).

The equations are developed for the rate of heat transfer between a group of solid particles(either moving or fixed)and the fluid moving countercurrently to the particles.

There are two heat transfer processes which take place in a thermal regenerator. However one may predominate depending on the assumptions made in developing the model. If it is assumed that the matrix thermal conductivity is infinite (Schumann model) the dominant heat transfer process is heat transfer across the surface of the matrix (or heat transfer to the fluid). On the other hand if the matrix thermal conductivity is assumed to be finite (Intraconduction model) ,the conduction also becomes important.

The two models are analysed in more detail in the next two sections.

2.2.1 SCHUMANN MODEL

This is the simplest model of a thermal regenerator and is based on the following simplifying assumptions:

a. **The thermal properties of the system are independent of temperature,**
b. **The transfer of heat by conduction in the fluid itself is small compared to the heat transfer by convection from the fluid to the solid,**
c. **The fluid flow rate does not vary along the bed,**
d. **There is no thermal gradient within the matrix.**

If there is no transverse thermal gradient within the matrix,then the matrix can be assumed to be at a uniform temperature and can be represented by a single temperature at any point along the regenerator (Fig.8a). Thus the only heat transfer process is the heat gained/lost by the fluid passing through the regenerator.

Fluid phase heat transfer equation

If $\dot{m}$ is the mass rate of fluid flow/unit bed area past a section a distance y from the entrance of the regenerator, then between y and y+dy the heat transferred from/to the fluid in time $d\theta$ will be (refer to Appendix A)

$$dQ=\dot{m}C[(\partial T_f/\partial y)_\theta+1/u.(\partial T_f/\partial \theta)_y]dy.A_b \ , \qquad (2.1.a)$$

$$=\dot{m}C(DT_f/Dy)dy.A_b \ , \qquad (2.1.b)$$

where $D/Dy=(\partial/\partial y)+1/u.(\partial/\partial\theta)$

Solid phase heat transfer equation

The total heat flow to the fluid must be equal to the heat lost by the matrix, that is

$$dQ=hA(T-T_f)dy.A_b \ . \qquad (2.2)$$

If $\dot{m}_s$ is the mass flow rate of solid/unit area, then between y and y+dy the heat lost from the matrix will be

$$dQ=\dot{m}_sC_s[(\partial T/\partial y)_\theta+1/u_s.(\partial T/\partial\theta)_y]dy.A_b \ . \qquad (2.3.a)$$

The above equation can also be written as

$$dQ=M_s C_s[u_s(\partial T/\partial y)_\theta+(\partial T/\partial \theta)_y]dy.A_b \qquad (2.3.b),$$

where $M_s=\dot{m}_s/u_s$,

if the bed is stationary ($u_s=0$) then equation (2.3.b) becomes

$$dQ=-M_s C_s(\partial T/\partial \theta)dy.A_b \ . \qquad (2.3.c)$$

The sign difference between equations (2.3.a) and (2.3.c) is due to the change of direction in which y is measured; for the moving bed y is measured from the solid entrance whereas for the fixed bed y is measured from the opposite end. Also in both of the equations the matrix temperature is represented by a single temperature T. This is because the model assumes that at any point along the regenerator the matrix is at a uniform temperature.

<u>Summary of the equations</u>

Combining equations (1),(2) and (3):

For the moving bed

$$hA(T-T_f)=\dot{m}_s C_s[(\partial T/\partial y)_\theta+1/u_s.(\partial T/\partial \theta)_y] \ , \qquad (2.4.a)$$
$$=\dot{m}_s C_s(DT/Dy) \ ,$$

and
$$hA(T-T_f)=\dot{m}C[(\partial T_f/\partial y)_\theta+1/u.(\partial T_f/\partial \theta)_y] \ , \qquad (2.4.b)$$
$$=\dot{m}C(DT_f/Dy) \ .$$

For the fixed bed: Only the solid phase equation changes,

$$hA(T-T_f)=-M_s C_s(\partial T/\partial \theta) \ . \qquad (2.5)$$

2.2.2 INTRAPARTICLE CONDUCTION MODEL

For regenerators composed of matrices with low thermal conductivity, eg. glass and ceramics, assumption (d) of the Schumann model is invalid. Thus allowance must be made for thermal gradient within the matrix. The temperature of the solid at any point along the regenerator can no longer be represented as one temperature. It is thus desirable to obtain the temperature distribution within the solid and represent the matrix temperature as a mean temperature.

There are thus two heat transfer processes for an intraconduction model ,

1. **Heat is gained/lost by the fluid passing through the regenerator.**
2. **Heat is transferred within the matrix.**

1. Fluid phase heat transfer equation

This equation is the same as that for the Schumann model.

2. Solid phase heat transfer equation

There are two stages of heat transfer in the solid phase.

(a) Heat transfer across the surface of solid:

This can be represented as

$$h(T_s - T_f) = K_s(\partial T/\partial x)_{x=0} \qquad \text{(Planar geometry)}, \qquad (2.5.a)$$

$$= -K_s(\partial T/\partial r)_{r=R} \qquad \text{(Spherical geometry)}. \qquad (2.5.b)$$

It should be emphasized that in this case T_s represents the solid surface temperature which might not necessarily be the same as mean solid temperature.

The heat lost by the solid can be represented in terms of the rate of change of its internal energy as (refer to Appendix A)

$$dQ = \dot{m}_s C_s dy.A_b[(\partial(\int_0^d T\,dx)/\partial y) + (\partial(\int_0^d T\,dx)/\partial\theta)/u_s]/d\ , \qquad (2.6)$$

where $(\int_0^d T\,dx)/d$ represents the mean solid temperature, and d is the semi-thickness of the matrix.

(b) Heat transfer within the solid:

The matrix thermal conductivity is finite and there is a temperature distribution within the matrix.Heat transfer within the matrix is represented by the diffusion equation.Assuming the problem is one dimensional we have

for planar geometry

$$\partial T/\partial \theta=\alpha(\partial^2 T/\partial x^2) \ , \qquad (2.7.a)$$

for spherical geometry

$$\partial T/\partial \theta=\alpha[(\partial^2 T/\partial r^2)+(2/r)(\partial T/\partial r)] \ . \qquad (2.7.b)$$

The above equations are coupled by the symmetry condition

planar ,

$$(\partial T/\partial x)_{x=d}=0 \quad d\text{=thickness of slab} \ , \qquad (2.8.a)$$

spherical ,

$$(\partial T/\partial r)_{r=0}=0 \quad R\text{=radius of sphere} \ . \qquad (2.8.b)$$

The distribution of gas and solid temperature along the regenerator is obtained by solving these equations with appropriate initial conditions

for a fixed bed , the initial condition is represented by a step change in the gas inlet temperature ,

$$T_f = T_{fi} \qquad \text{for } \theta \geq 0 \text{ and } y=0 \text{ ,}$$

$$T = T_i \text{ ,} \qquad \text{for } \theta = 0 \text{ and } y=0 \text{ .}$$

for a moving bed , the inlet temperatures are specified.

2.3 NON-DIMENSIONAL FORM OF GOVERNING EQUATIONS

The governing equations can be nondimensionalised in terms of dimensionless parameters ξ, η, Biot number and normalised temperatures defined in section 1. The equations in dimensionless form are (Refer to Appendix A)

Fluid phase

Equations (2.4.b) and (2.5) become (refer to Appendix A),

$$\partial F/\partial \xi = (\Psi_S - F) \text{ ,} \qquad \text{(fixed bed) ,} \qquad (2.9.a)$$

$$= (F - \Psi_S) \text{ ,} \qquad \text{(moving bed) .} \qquad (2.9.b)$$

Again the sign difference is due to the change of direction in which ξ (or y) is measured.

Solid phase

Planar geometry

Equations (2.5.a),(2.7.a)and (2.8.a) become (refer to Appendix A)

$$(\partial\Psi/\partial z)_{z=0}=Bi(\Psi_s-F) \ , \tag{2.10.a}$$

$$\partial\Psi/\partial\eta=[\partial^2\Psi/\partial z^2]/Bi \ , \tag{2.10.b}$$

$$(\partial\Psi/\partial z)_{z=1}=0 \ . \tag{2.10.c}$$

Spherical Geometry

Equations (2.5.b),(2.7.b)and(2.8.b) become(refer to Appendix A)

$$(\partial\Psi/\partial z)_{z=1}=-Bi(\Psi_s-F) \ , \tag{2.11.a}$$

$$\partial\Psi/\partial\eta=[(\partial^2\Psi/\partial z^2)+(2/z)(\partial\Psi/\partial z)]/(3Bi) \ , \tag{2.11.b}$$

$$(\partial\Psi/\partial z)_{z=0}=0 \ . \tag{2.11.c}$$

The initial conditions are nondimensionalised for the case of a fixed bed as ,

$$F=1 \quad \text{at } \xi=0 \text{ and } \eta\geq 0 \ ,$$

$$\Psi=0 \quad \text{at } \eta=0 \text{ and } \xi\geq 0 \ .$$

3. THE METHOD OF SOLUTION

3.1 INTRODUCTION TO THEINTEGRAL METHOD.

The inclusion of matrix thermal conductivity in the thermal design of regenerators makes the analysis more complicated.The diffusion equation (equations.2.10.b,2.11.b) must be solved to obtain the temperature distribution within the matrix.This can be done by employing numerical techniques (discussed in the review of previous work) which require lengthy computer programs.

An alternative solution is the application of the approximate integral technique.This method was first introduced by von Karman and Pohlhausen in order to solve the boundary layer problem in fluid mechanics.However,the method is equally appropriate for unsteady heat conduction in solids.Goodman [16] employed the technique to solve the diffusion equation,coupled with either linear or non-linear boundary conditions. The method makes use of two assumptions:

a. **The thermal properties(ie.conductivity,density etc) are usually assumed to be independent of temperature in order to linearize the diffusion equation.**

b. The solid is initially at a constant temperature.

Goodman subsequently has developed a technique to account for temperature dependent thermal properties.This is explained in more detail in section (3.4) of this chapter.

The integral method introduces a quantity $\delta(\theta)$ called the **penetration depth.**This is defined as a distance into which the heat flux at the surface penetrates the solid,and beyond which there is no heat transferred.Consequently there will be a temperature gradient inside the solid up to the penetration depth ,while the solid will be at a uniform temperature beyond this point (Fig.10).This is expressed mathematically as

$$(\partial T/\partial x)_{x=\delta}=0 \ .$$

The penetration depth is analogous to the boundary layer thickness in fluid mechanics.

The technique adopted by the approximate integral method can be explained as follows;

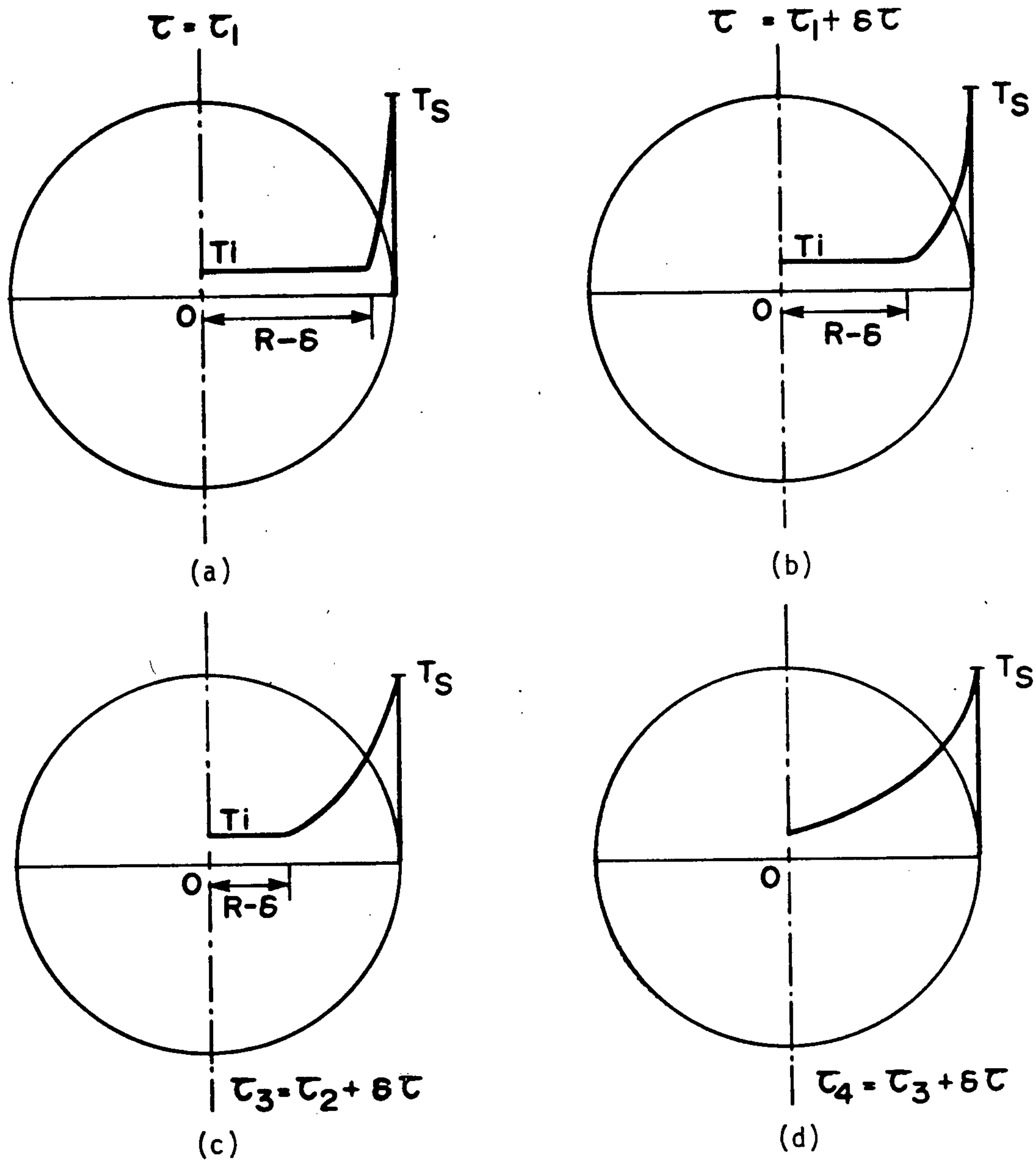


Figure 10. Schematic representation of penetration depth concept.

T_s = Surface temperature.

T_i = Initial temperature.

The solid temperature is represented by a polynomial in x (or r for spherical geometry) .The order of the polynomial is limited by the number of variable constraints (ie. boundary conditions, initial conditions etc) .The unknown coefficients are usually a function of time.

One way to improve the accuracy of the assumed profile is to increase the order of the polynomial.Each additional parameter which is thereby introduced is determined from an additional derived constraint.This may not always be possible.

Koh.y [19] has demonstrated that the temperature profile is better approximated by an exponential function than a polynomial, however the analysis becomes more involved. There is never a unique procedure to follow in using the integral method.The ultimate criterion for determining whether or not a particular profile is successful must involve an assessment of both its accuracy and simplicity.A simple polynomial profile has been found adequate for most engineering purposes.

3.2 PLANAR GEOMETRY

The solid temperature is represented by a polynomial of x,where x is the distance from the surface of the solid.Since the order of the polynomial is limited by the number of constraints,it would be advantageous to model the matrix as a semi-infinite slab extending in the x direction.In this way the order of the polynomial can be increased as will be seen later.

3.2.1 SEMI-INFINITE SLAB

The temperature distribution $T(x,\theta)$ inside the slab is to be calculated subject to the following constraints

$$T(\delta,\theta)=T_i \ , \qquad (3.2.1)$$

$$\partial T(\delta,\theta)/\partial x=0 \ , \qquad (3.2.2)$$

$$\partial T(0,\theta)/\partial x=h.(T_s-T_f)/K_s=-f(\theta) \ . \qquad (3.2.3)$$

There are three constraints so the profile must be represented by a second-order polynomial. The assumption that the slab is initially at a constant temperature can be utilized in deriving an additional constraint .Equation (3.2.1) is differentiated with respect to time and then substituted in the diffusion equation.The result is

$$\partial^2 T(\delta,\theta)/\partial x^2=0 \ . \tag{3.2.4}$$

This is usually called the smoothing condition.

The temperature distribution can now be represented by a cubic profile.The constraints are nondimensionalised in terms of z (dimensonless depth) ,Biot number ,normalised temperatures and δ_0, that is (refer to Appendix B)

$$\Psi=0 \text{ at } z=\delta_0 \ , \tag{3.2.5}$$

$$\partial\Psi/\partial z=0 \text{ at } z=\delta_0 \ , \tag{3.2.6}$$

$$\partial\Psi/\partial z=Bi.(\Psi_s-F)=-f(\eta) \text{ at } z=0 \ , \tag{3.2.7}$$

$$\partial^2\Psi/\partial z^2=0 \text{ at } z=\delta_0 \ , \tag{3.2.8}$$

where $\delta_0=\delta/d$,

is the dimensionless penetration depth. The cubic profile will take the form (refer to Appendix B)

$$\Psi=f(\eta).(\delta_0-z)^3/(3.\delta_0^2) \ . \tag{3.2.9}$$

The surface temperature is obtained by setting z=0 in equation (3.2.9) ,the result is

$$\Psi_s = f(\eta).\delta_0/3 \ . \qquad (3.2.10)$$

Equation (3.2.10) expresses a relationship between the solid surface temperature and the penetration depth.Consequently, once the penetration depth is calculated ,it will only require a simple algebraic manipulation to calculate the surface temperature or vice-versa.

The penetration depth is obtained by integrating the diffusion equation (eqn.2.10.b) from z=0 to $z=\delta_0$ and substituting for Ψ from equation (3.2.9),the result is (refer to Appendix B)

$$\delta_0 = [12.(\int_0^{\eta} f(\eta)\ d\eta\ )/(f(\eta).Bi)]^{0.5} \ . \qquad (3.2.11)$$

The surface temperature can then be obtained by substituting for δ_0 from equation (3.2.10).The result is

$$\Psi_s = [4.f(\eta).(\int_0^{\eta} f(\eta)\ d\eta)/(3.Bi)]^{0.5} \ . \qquad (3.2.12)$$

3.2.2 SLAB OF FINITE THICKNESS

Initially, the symmetry condition (eqn.2.10.c) does not affect the temperature distribution within the matrix. The matrix can thus be modeled as though it were semi-infinite.

However, at some later time the penetration depth reaches the centre of the matrix and the symmetry condition comes into effect. At this stage the penetration depth has no meaning and the model should be replaced by a slab of finite thickness whose far surface is insulated (representing the symmetry condition). The slab is subject to the following constraints

$$\partial T(d,\theta)/\partial x=0 \;, \qquad (3.2.13.a)$$

$$T(0,\theta)=T_s \;, \qquad (3.2.14.a)$$

$$\partial T(0,\theta)/\partial x=h.(T_s-T_f)/K_s \;. \qquad (3.2.15.a)$$

The above equations in dimensionless form are

$$\partial \Psi(1,\eta)/\partial z=0 \;, \qquad (3.2.13.b)$$

$$\Psi(0,\eta)=\Psi_s \;, \qquad (3.2.14.b)$$

$$\partial \Psi(0,\eta)/\partial z=Bi.(\Psi_s-F)=-f(\eta) \;. \qquad (3.2.15.b)$$

The second-order profile must then take the form

$$\Psi=\Psi_s-f(\eta).(z^2-2.z)/2 \ . \qquad (3.2.16)$$

The surface temperature Ψ_s will be obtained by integrating the diffusion equation (eqn.2.10.b) with respect to z.The solid temperature Ψ is replaced by the polynomial expression(eqn.3.3.4).As opposed to the previous case,the integration extends from z=0 to z=1. The result is (refer to Appendix B)

$$\Psi_s=[f(\eta)/3+\int_0^{\eta} f(\eta)\ d\eta\ /Bi\ ]+\text{constant} \ . \qquad (3.2.17)$$

Assume η_0 is the time at which the penetration depth reaches the far surface of the slab .The initial condition (ie. $\Psi_s(\eta_0)$) can be obtained by explicitly setting $\delta_0=1$ in equation (3.2.10).Setting $\Psi_s=\Psi_s(\eta_0)$ in equation (3.2.12) results in obtaining η_0. The constant of integration is then obtained by setting $\Psi_s=\Psi_s(\eta_0)$ and $\eta=\eta_0$ in equation(3.2.17).The result is

$$\Psi_s=f(\eta)/3 +\int_{\eta_0}^{\eta} f(\eta)\ d(\eta)/Bi \qquad \text{For } \eta\geq\eta_0 \ . \qquad (3.2.18)$$

In summary

If $\eta<\eta_0$ then equation (3.2.12) is used.

If $\eta\geq\eta_0$ then equation (3.2.18) is used.

3.3 SPHERICAL GEOMETRY

Lardner and Pohle [18] have demonstrated that for spherical geometry, the polynomial representation of the temperature profile is inappropriate.Since for spherical geometry the steady state solution is proportional to (1/r),they suggested a profile of the form

$$T=(\text{Polynomial in } r)/r \ . \tag{3.3.1}$$

Although Lardner and Pohle dealt with a spherical hole, the same method is applicable to the case of a solid sphere.The procedure adopted for spherical geometry is the same as that for planar geometry , that is , the original profile includes the penetration depth.As soon as the penetration depth reaches the centre of the sphere,a second profile should be used.

3.3.1 SPHERE OF INFINITE RADIUS

The penetration depth is measured from the surface of the sphere. The temperature profile is subject to the following constraints

$$\partial T(R-\delta,\theta)/\partial r=0 \ , \tag{3.3.2.a}$$

$$T(R-\delta,\theta)=T(i) \ , \tag{3.3.3.a}$$

$$\partial T(R,\theta)/\partial r=-h.(T_s-T_f)/K_s \ , \tag{3.3.4.a}$$

$$\partial^2 T(R-\delta,\theta)/\partial r^2=0 \ . \tag{3.3.5.a}$$

Equation(3.3.5.a)represents the smoothing condition. The above equations in dimensionless form are

$$\partial\Psi(1-\delta_0,\eta)/\partial z=0 \ , \tag{3.3.2.b}$$

$$\Psi(1-\delta_0,\eta)=0 \ , \tag{3.3.3.b}$$

$$\partial\Psi(1,\eta)/\partial z=-Bi.(\Psi_s-F)=-f(\eta) \ , \tag{3.3.4.b}$$

$$\partial^2\Psi(1-\delta_0,\eta)/\partial z^2=0 \ , \tag{3.3.5.b}$$

where $\delta_0=\delta/R$=dimensionless penetration depth.(3.3.6)

There are 4 constraints ,consequently the polynomial will be a cubic.Adopting the suggestion by Lardner and Pohle, the result is

$$\Psi=(A.z^3+B.z^2+C.z+D)/z \ . \qquad (3.3.7)$$

Applying the constraints,the profile must take the form

$$\Psi=-f(\eta).[z-(1-\delta_0)]^3/(\delta_0^2.(3-\delta_0).z) \ . \qquad (3.3.8)$$

The surface temperature is obtained by setting z=1,in above equation, the result is

$$\Psi_s=-f(\eta).\delta_0/(3-\delta_0) \ . \qquad (3.3.9)$$

The penetration depth is obtained by integrating the diffusion equation (eqn.2.11.b) after Ψ has been replaced by equation(3.3.8). The integration extends from $z=1-\delta_0$ to z=1,the result is (refer to Appendix B)

$$[(5\delta_0^2-\delta_0^3)/(3-\delta_0)]=20(\int_0^\eta f(\eta)\ d\eta)/(3Bi.f(\eta)) \ .(3.3.10)$$

The surface temperature is obtained by substituting for δ_0 from equation(3.3.9),the result is

$$3\Psi_s^2[5f(\eta)-2\Psi_s]=20(\Psi_s-f(\eta))^2(\int_0^\eta f(\eta)\ d\eta)/(3Bi) \quad (3.3.11)$$

3.3.2 SPHERE OF FINITE RADIUS

As in the case of planar geometry,the symmetry condition at the centre does not come into effect until the penetration depth reaches the centre.At this point the profile has to be changed to take account of symmetry condition.The new set of constraints are

$$\partial T(0,\theta)/\partial r=0\ , \quad (3.3.12.a)$$

$$\partial T(R,\theta)/\partial r=-h.(T_s-T_f)/K_s\ , \quad (3.3.13.a)$$

$$T(R,\theta)=T_s\ . \quad (3.3.14.a)$$

In dimensionless form the equations are

$$\partial \Psi(0,\eta)/\partial z=0\ , \quad (3.3.12.b)$$

$$\partial \Psi(1,\eta)/\partial z=-Bi.(\Psi_s-F)=-f(\eta)\ , \quad (3.3.13.b)$$

$$\Psi(1,\eta)=\Psi_s\ . \quad (3.3.14.b)$$

Again adopting the suggestion by Lardner and Pohle the profile will take the form

$$\Psi=(A.z^3+B.z^2+C.z)/z\ .$$

It should be emphasized that since the sphere is solid,the term 1/z should not appear in the final expression of the temperature profile.This is why there is no constant term included in the polynomial of z in the above equation.Applying the constraints the profile will take the form

$$\Psi=\Psi_s+f(\eta).(1-z^2)/2\ . \qquad (3.3.15)$$

Integration of the diffusion equation after equation(3.3.15)is substituted for Ψ will give

$$\Psi_s=-f(\eta)/5-(\int_0^{\eta}f(\eta)\ d\eta\ )/Bi\ +constant\ . \qquad (3.3.16)$$

The constant of integration is obtained by the procedure explained in previous section,the result is

$$\Psi_s=-f(\eta)/5-(\int_0^{\eta}f(\eta)\ d\eta)/Bi\ , \qquad (3.3.17)$$

where η_0 defines the end of the initial stage and the beginning of the second stage.

<u>In summary</u>

If $\eta<\eta_0$ Equation(3.3.11)should be used

If $\eta\geq\eta_0$ Equation(3.3.17)should be used

3.4 TEMPERATURE-DEPENDENT THERMAL PROPERTIES

The usual simplifying assumption of constant thermal properties made in developing the mathematical model of regenerators can be relaxed.When the thermal properties are temperature-dependent, the diffusion equation is replaced by

$$\rho.C.\partial T/\partial\theta=\partial(K.\partial T/\partial x)/\partial x \ . \qquad (3.4.1)$$

Both K and ρ.C are temperature-dependent.The procedure to obtain the temperature distribution will be somewhat different to that adopted for constant thermal properties.Goodman [20] has demonstrated that only the thermal properties at the surface enter the problem. This simplifies the analysis in a way that there will still be only one unknown,ie.the surface temperature.

The procedure is explained in more detail in Goodman's paper [20].

3.5 NUMERICAL PROCEDURE

The fluid and solid phase equations describe the intraparticle conduction model.The fluid phase equation is solved numerically by a finite difference approximation, while the solid phase equation is solved using the approximate integral method.The two unknowns are the fluid and the solid temperature throughout the bed.

The calculation is carried out for two types of regenerators, namely fixed bed and moving bed.

3.5.1 FIXED BED

The regenerator bed is represented by a 2 dimensional grid. The length of the bed y (or Λ in dimensionless form) is divided into n equal increments of δy(or $\delta\xi$), that is

$$\Lambda = n.\delta\xi \ . \tag{3.5.1}$$

The period P (or Π in dimensionless form) during which the fluid is passed through the bed is divided into i equal increments of $\delta\theta$ or ($\delta\eta$ in dimensionless form)

$$\Pi = i.\delta\eta \ . \tag{3.5.2}$$

At each point along the bed the fluid and solid temperatures are represented as F(n,i) and $\Psi_s(n,i)$ respectively. At each step point there are two unknown temperatures,ie F(n,i+1) and $\Psi_s(n,i+1)$, provided the temperatures at the (n,i) point are known.

The fluid phase equation (eqn.2.9.b) is represented by a central-difference approximation as

$$(1+\Delta\xi/2)F(n,i+1)-\Psi_s(n,i+1)\Delta\xi/2=(1-\Delta\xi/2)F(n-1,i+1)+\Delta\xi.\Psi_s(n-1,i+1)/2 \ .$$

The solid phase equations involve $\int f(\eta)\,d\eta$ .In order to represent the equations in numerical form,the integral term is approximated by the area under the curve $f(\eta)$ versus η ,that is

$$\int_0^{\Delta\eta} f(\eta)d\eta=Ar1(n,i+1)$$
$$=Ar1(n,i)+\Delta\eta[f(n,i+1)+f(n,i)]/2 \ ,(3.5.3.a)$$

and

$$\int_{\eta_0}^{\eta_0+\Delta\eta} f(\eta)d\eta=Ar2(n,i+1)$$
$$=Ar2(n,i)+\Delta\eta[f(n,i+1)+f(n,i)]/2,(3.5.3.b)$$

where initially

$$Ar1(n,0)=0 \quad , \qquad \text{and} \qquad Ar2(n,\eta_0)=0 \quad .(3.5.3.c)$$

The solid phase equations can now be written in their numerical form.

Planar geometry

For $\eta<\eta_0$,

$$\delta_0(n,i)=[12Ar1(n,i)/(Bi.f(n,i))]^{0.5} \quad , \qquad (3.5.4)$$

$$\Psi_s(n,i+1)=[4f(n,i+1).Ar1(n,i+1)/(3Bi)]^{0.5} \quad . \qquad (3.5.5)$$

For $\eta\geq\eta_0$,

$$\Psi_s(n,i+1)=Bi.f(n,i+1)/3+Ar2(n,i+1)/Bi \quad , \qquad (3.5.6)$$

where

$$f(n,i)=Bi.[F(n,i)-\Psi_s(n,i)] \quad . \qquad (3.5.7)$$

In order to determine which equation should be used to obtain the solid surface temperature,the penetration depth should be calculated first.If the penetration depth is less than the semi-thickness of the matrix ,equation (3.5.5) should be used, otherwise,equation(3.5.6)should be used.

Spherical geometry

The same procedure as for the planar geometry is adopted. The penetration depth and solid surface temperature are represented in their finite difference form as

For $\eta<\eta_0$,

$$5\delta_0^2(n,i)-\delta_0^3(n,i)$$
$$=20(3-\delta_0(n,i)).Ar1(n,i)/(3Bi.f(n,i)) \ , \quad (3.5.8)$$

$$3\Psi_s^2(n,i+1)(5f(n,i+1)-2\Psi_s(n,i+1))=$$
$$20(\Psi_s(n,i+1)-f(n,i+1))^2.Ar1(n,i+1)/(3Bi) \ . \quad (3.5.9)$$

For $\eta\geq\eta_0$,

$$\Psi_s(n,i+1)=-f(n,i+1)/5-Ar1(n,i+1)/Bi \ , \quad (3.5.10)$$

where

$$f(n,i)=Bi.[\Psi_s(n,i)-F(n,i)] \ , \quad (3.5.11)$$

where η_0 represents the end of first stage and the beginning of the second stage.

In order to determine which of the above equations should be used, the penetration depth at each step point should be calculated first.

At the (n,i) step point,the unknowns are $F(n,i+1)$ and $\Psi_s(n,i+1)$,provided the temperatures at the (n-1,i+1) and (n,i) points are known.

The starting values for the solution are obtained from the initial condition which in their finite difference form are

$$F(n,i)=1 \quad \text{At } n=0 \text{ and } i\geq 0 \,, \qquad (3.5.12.a)$$

$$F(n,i)=0 \quad \text{At } i=0 \text{ and } n>0 \,, \qquad (3.5.12.b)$$

$$\Psi_s(n,i)=0 \quad \text{At } i=0 \text{ and } n\geq 0 \,, \qquad (3.5.12.c)$$

$$\delta_0(n,i)=0 \quad \text{At } i=0 \text{ and } n\geq 0 \,. \qquad (3.5.12.d)$$

At the entrance (ie.n=0)the fluid temperature does not vary with time,that is

$$F(0,i+1)=F(0,i)=1 \;.$$

Consequently,there is only one unknown temperature,Ψ_s, which can be obtained from either equation (3.5.6) or (3.5.5).

3.5.2 MOVING BED REGENERATOR

In the case of a moving bed regenerator, the period of operation is not an independent characteristic of the system. It is defined as the time it takes a solid particle to traverse one full length of the regenerator chamber. Consequently, the period of the cycle is related to the regenerator height. The conditions are steady at the entrance and exit of the regenerator. Only the temperature distribution at different points along the regenerator bed during one cycle is of interest. To obtain a maximum efficiency, a contra-flow arrangement should be adopted.

The bed is divided into n equal increments of $\Delta\xi$. The direction ξ is measured from the solid entrance. This is because the penetration depth during the initial stages of the cycle must be obtained in order to determine which equation should be used for the solid temperature calculation.

The solid and fluid temperatures at each point along the regenerator are represented by $\Psi_s(n)$ and $F(n)$ respectively; $\Psi_s(0)$ represents the solid inlet temperature, whereas $F(0)$ represents the fluid outlet temperature.

The fluid phase equation (eqn.2.9.b) is represented by a central difference approximation as

$$F(n+1)-F(n)=\Delta\xi[F(n+1)-\Psi_s(n+1)+F(n)-\Psi_s(n)]/2. \quad (3.5.13)$$

The solid phase equations in their numerical form are shown below,

Planar geometry

For $\eta<\eta_0$,

$$\delta_0(n)=[12.Ar1(n)/(Bi.f(n))]^{0.5} , \quad (3.5.14)$$

$$\Psi_s(n+1)=[4.f(n+1).Ar1(n+1)/(3.Bi)]^{0.5} . \quad (3.5.15)$$

For $\eta\geq\eta_0$,

$$\Psi_s(n+1)=f(n+1)/3+Ar2(n+1)/Bi , \quad (3.5.16)$$

where

$$f(n)=Bi.[F(n)-\Psi_s(n)] , \quad (3.5.17.a)$$

$$Ar1(n+1)=\int_0^{\Delta\eta} f(\eta)\ d\eta , \quad (3.5.17.b)$$

$$Ar2(n+1)=\int_{\eta_0}^{\eta_0+\Delta\eta} f(\eta)\ d\eta . \quad (3.5.17.c)$$

<u>Spherical geometry</u>

For $\eta<\eta_0$,

$$5\delta_0^2(n)-\delta_0^3(n)=20(3-\delta_0(n))Ar1(n)/(3Bi.f(n)) \quad , \quad (3.5.18)$$

$$3\Psi_s^2(n+1)[5f(n+1)-2\Psi_s(n+1)]=20[\Psi_s(n+1)-f(n+1)]^2 .Ar1(n+1)/(3Bi) \quad . \quad (3.5.19)$$

For $\eta\geq\eta_0$,

$$\Psi_s(n+1)=-f(n+1)/5-Ar1(n+1)/Bi \quad , \quad (3.5.20)$$

where

$$f(n)=Bi.[\Psi_s(n)-F(n)] \quad , \quad (3.5.21.a)$$

$$Ar1(n+1)=\int_0^{\Delta\eta} f(\eta)\, d\eta \quad , \quad (3.5.21.b)$$

$$Ar2(n+1)=\int_{\eta_0}^{\eta_0+\Delta\eta} f(\eta)\, d\eta \quad . \quad (3.5.21.c)$$

It should also be emphasized that

$$\Delta\eta=\Delta\xi\dot{m}C/(M_sC_su_s) \quad . \quad (3.5.22)$$

At each point along the regenerator there are two unknowns, namely, $\Psi_s(n+1)$ and $F(n+1)$, provided that $\Psi_s(n)$ and $F(n)$ are known.

The analysis proceeds by using the average of the fluid and solid inlet temperatures as a first approximation for the fluid outlet temperature.The calculated inlet temperature is then compared with the actual given inlet temperature.If there is any discrepancy the initial approximation is adjusted and the analysis is then repeated.

4. RESULTS AND DISCUSSION

The first stage in analysing the results is to establish the validity of the approximate integral method.This is achieved by comparing the results obtained against the results published by different authors.

Unfortunately the published results are scarce.Most of the previous analyses have disregarded intraconduction effects.One source of results available are those published by Handley and Heggs [12] ,who employed the Crank-Nicholson scheme to solve the diffusion equation numerically.The results are mainly for a fixed bed regenerator with a spherical matrix.

4.1 FIXED BED

4.1.1 SPHERICAL GEOMETRY

The diffusion equation,coupled with the appropriate boundary conditions, were solved using the approximate integral method for different Biot numbers.The results where then compared with those published by Handley and Heggs. Figs.11,12,13 represent the fluid outlet temperature for different Biot numbers.An excellent agreement between the two methods can be seen.The comparison of the results were made for a range of dimensionless parameters $1 \le \Lambda \le 40$ and $0 < Bi \le 5$ which cover the design range of most industrial regenerators.

Experimental studies carried out by the same authors revealed an excellent accuracy of the results [12].

4.1.2 PLANAR GEOMETRY

Unfortunately no results could be found for comparing the numerical and integral method for the planar geometry.Most of the previous studies have concentrated on the spherical geometry (which is a more practical geometry).

The fluid outlet temperature profile is expected to follow a characteristic trend (S shaped profile) if the method is correct. It is apparent from Fig.14 that the fluid outlet temperature follows the expected trend.Thus it can be deduced that the integral method is equally applicable to the planar geometry.Fig.14 represents the fluid outlet temperature profile for different reduced lengths.

Once the validity of the method is established, the analysis can be extended to examine the severity of intraconduction effect for different geometries. This is achieved by comparing the difference between the solid surface and mean temperature for the two common geometries at different Biot numbers.

It is expected that as the Biot number is increased, the difference between the solid surface and mean temperature should increase.This is because as the Biot number increases(ie. K_s decreases), the depth into which the heat flux at the surface penetrates the solid decreases. Consequently, the surface temperature will be much higher than the temperature at the centre of the sphere.Thus the resulting mean temperature is smaller.(refer to Fig.8).

To show this,the graph of percentage difference between the solid surface and mean temperature was plotted for different Biot numbers (Figs.15,16,17). The percentage difference was obtained from

$$\text{Difference}=(\Psi_s-\Psi_m)/\Psi_s$$

Evidently,as the Biot number increases the maximum difference between the two temperatures increases, as expected.It can be seen (Fig.17) that for a moderate Biot number (Bi=2) a maximum difference between the two temperatures is about 9 percent for the planar geometry.Consequently, a model which neglects the intraconduction effect overestimates the effectiveness.

It is also evident from the same charts that for a fixed dimensioless group (Λ and Bi) ,the difference between the surface and mean temperature is greater for the planar geometry than that for the spherical geomtry.That is,the intrasphere conduction mechanism has less significance than intraplanar conduction.This is because for an equal Biot number and reduced length (Λ), the sphere radius has to be smaller than the slab thickness (for the same material). Carpenter and Heggs arrived at the same conclusion in their analysis [5].

Most of the investigators have tried to predict the dividing line between the Schumann and intraconduction models [5]. The dividing line is usually expressed in terms of dimensionless parameters (Λ, Π ,Bi) .The main reason for this is to reduce the computation time required to solve the model numerically.This of course is not important if the problem is solved analytically.

4.2 MOVING BED REGENERATOR

Since there are no results available for a moving bed regenerator, the validity of the results presented here are based upon the validity of the method.

As for the fixed bed, the difference between the solid surface and mean temperature increases as the Biot number increases (Fig.18). Consequently, at a fixed Λ, the mean solid temperature decreases as the Biot number increases.Since the regenerator effectiveness is defined in terms of the solid mean temperature, the reduction in the former is refelected in the reduction of the latter.So it can be deduced that as the Biot number is increased , the effectiveness should decrease.This is shown in Fig.19. It should be pointed out that for a fixed thermal conductivity, the Biot number reduces as the size of the sphere is reduced.So one would expect a higher effectiveness for a matrix of smaller size.But the size of the sphere is limited by its terminal velocity.That is, if the spheres are too small, they might be blown off the top by the oncoming fluid.

The analysis revealed that as the Biot number is increased ,the distance increment at which the solid and fluid temperatures are calculated must be reduced.This reduction in the distance increment ($\delta\xi$) might cause a stability problem at very high Biot numbers.The stability problem is due to the form of the temperature profile in the solid.The present derived profile is based on Lardner and Pohle [18] suggestion that $\Psi \propto 1/z$.

Further analysis revealed that the sphere should be assumed to be solid even at the initial stages where the penetration depth concept is applied, then the term (constant/z) must be omitted from the final expression for the solid temperature profile.This would results in a second form of profile which is of the form

$$\Psi=-f(\eta)[z-(1-\delta_0)]^3/(3\delta_0)^2 .$$

The integration of the diffusion equation will result in

For $\eta<\eta_0$,

$$27\Psi_s^4+54\Psi_s^3.f(\eta)+45\Psi_s^2.f(\eta)^2=20f(\eta)^3.(\int_0^\eta f(\eta)\, d\eta)/Bi .$$

For $\eta\geq\eta_0$,

$$\Psi_s=-.9\int_0^{\eta_0} f(\eta)\, d\eta/Bi -f(\eta)/5 -\int_{\eta_0}^{\eta} f(\eta)\, d\eta/Bi .$$

Surprisingly, for $\eta\geq\eta_0$, the solid temperature is almost identical to the original derived surface temperature based on Lardner and Pohle suggestion.The solution to the above equations were compared with the original results.The results were extremely close for low Biot numbers as is evident from Figs.20,21.However,the advantage of this second profile is that it is stable even for high Biot numbers (Bi≥8).This is shown by plotting the charts of effectiveness versus reduced length for different Biot numbers (Figs.22,23,24). It is therfore suggested that at high Biot numbers the second profile must be used.

It is evident from the charts of effectiveness versus reduced length that the effect of Biot number is very significant.For example, consider Fig.24 which shows the effectiveness versus reduced length for Bi=0.1 and Bi=10. It can be seen that in order to obtain the same effectiveness (for example 60 percent), the reduced length (or the bed length) has to be almost tripled in the case of Bi=10.This critical information had not been available prior to this work.

Finally ,from the definition of effectiveness, it is apparent that as the capacity rate ratio (R)[3] is reduced, the effectiveness reduces.This is shown in Fig.25 .

---

[3] $R=(\dot{m}_s.C_s)/(\dot{m}.C)_{min}$

5. CONCLUSION

The approximate integral method was employed to obtain an analytical solution to transient response of a regenerator matrix; the following points are concluded,

1. The approximate method gives results that agree well with the more exact methods employed for the case of a fixed bed regenerator.

2. The effect of Biot number is much more severe than expected in the case of a moving bed regenerator.

6. AREAS OF FURTHER RESEARCH

The intraconduction model is based on a number of simplifying assumptions .The use of an integral method enables one to relax some of these assumptions. In order to examine the effect of these assumptions the following is suggested,

1. The simplifying assumption of constant thermal properties (for the solid) should be relaxed. This can be easily achieved with the use of the integral method [19].

2. It is believed that the bed extension in the direction of gas flow contributes to a uniform temperature profile [8]. Thus the simplifying assumption of uniform fluid velocity should be relaxed .

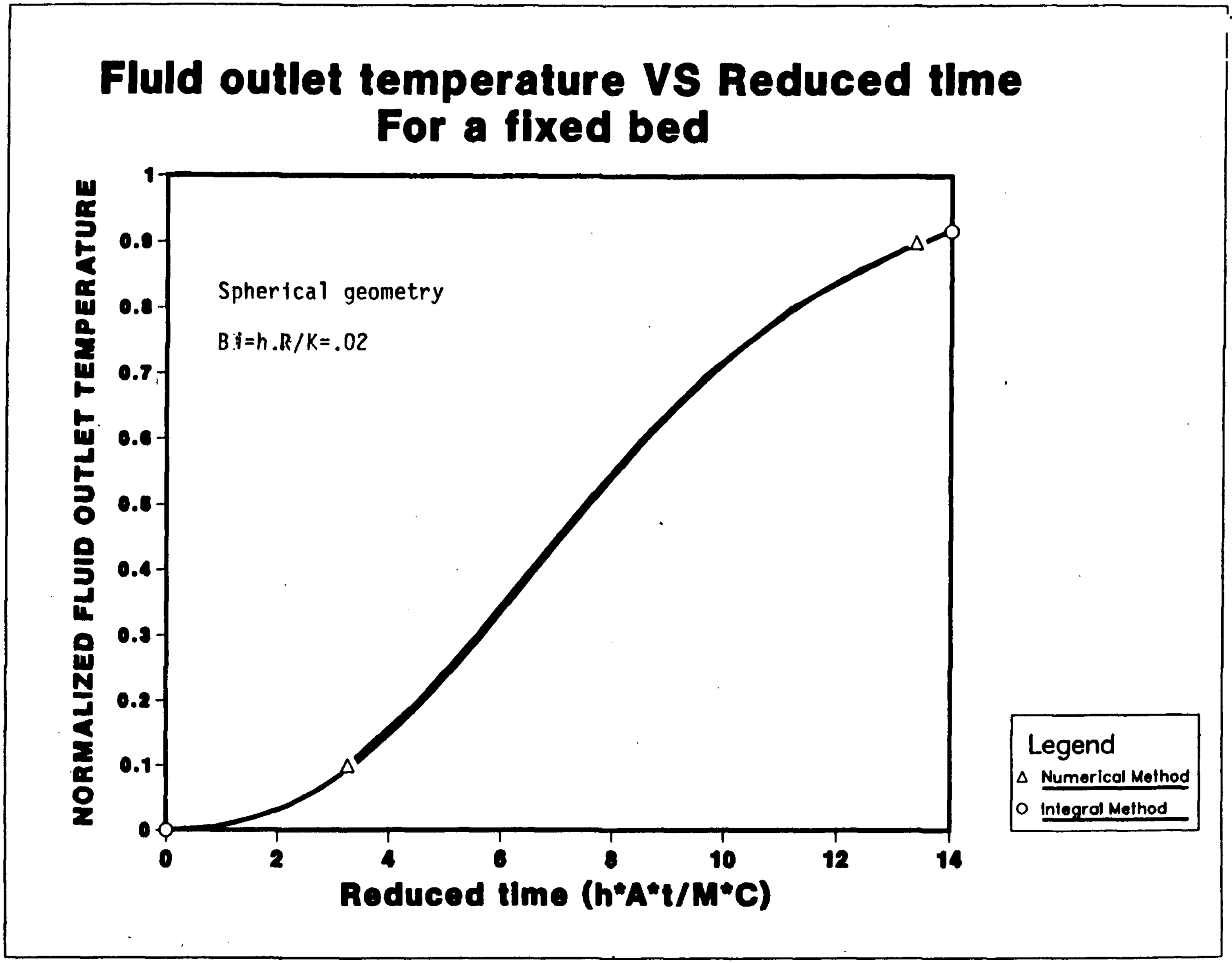


Figure 11. Comparison between the numerical and analytical method employed

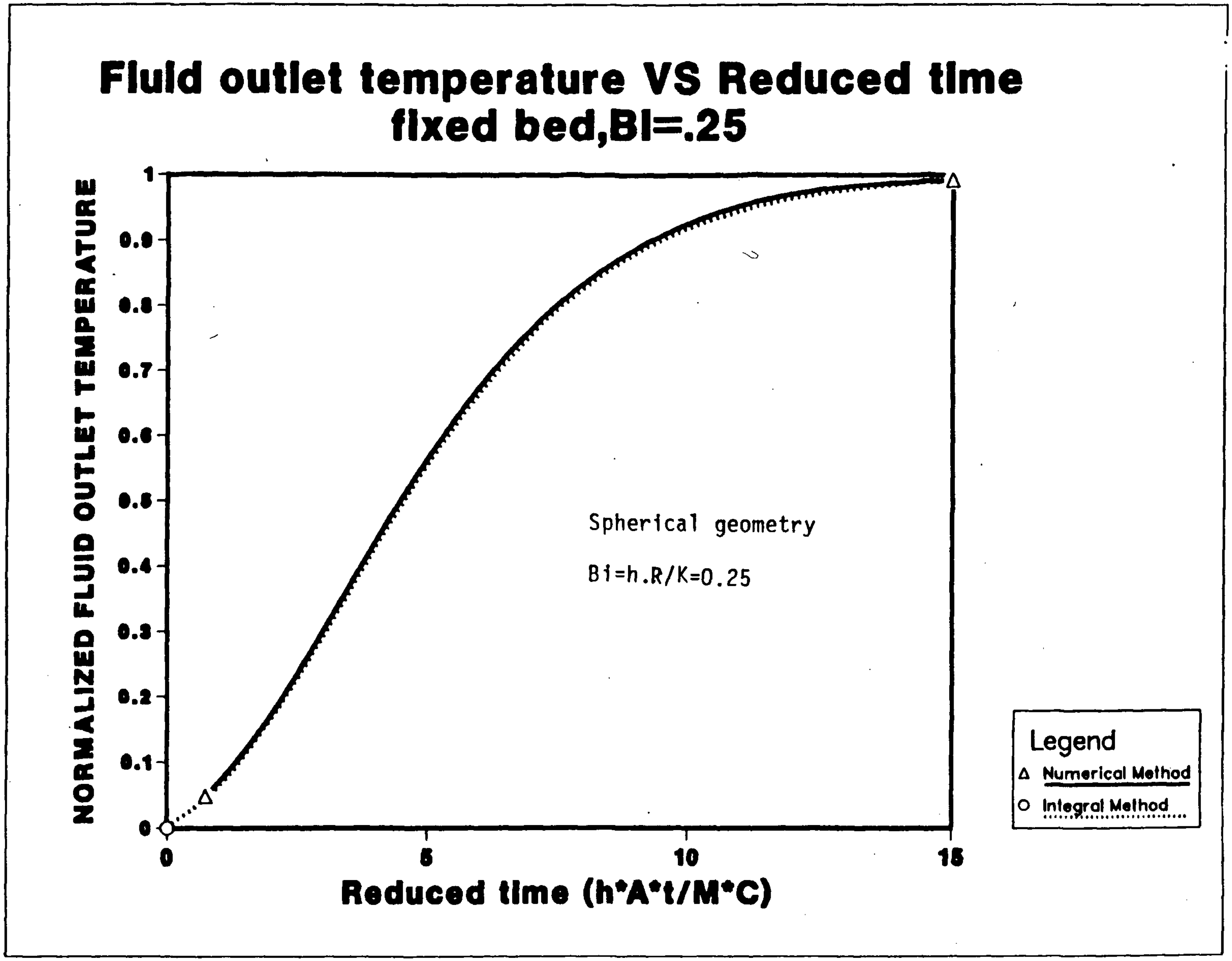


Figure 12. Comparison between the numerical and analytical method employed

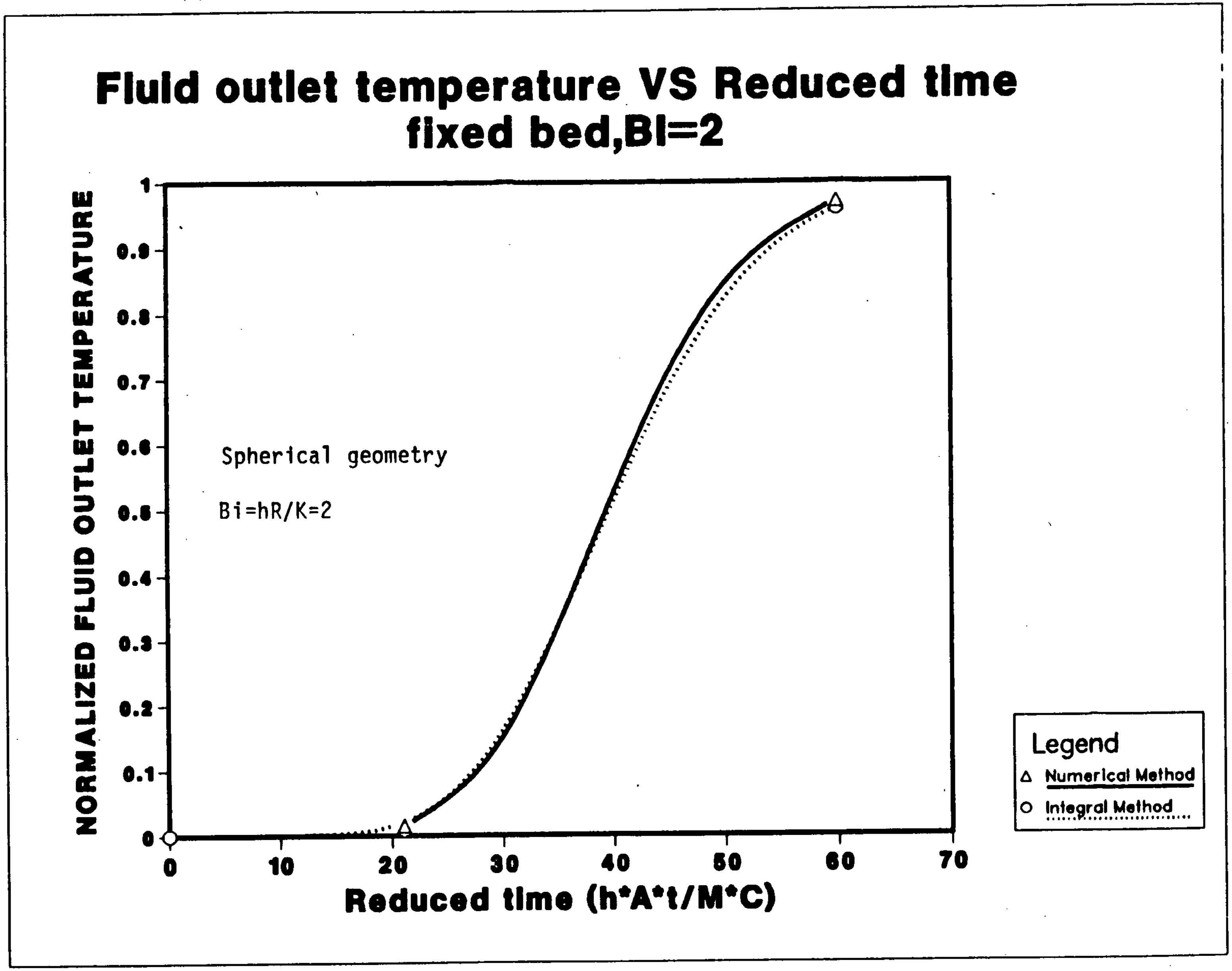


Figure 13. Comparison between the numerical and analytical method employed

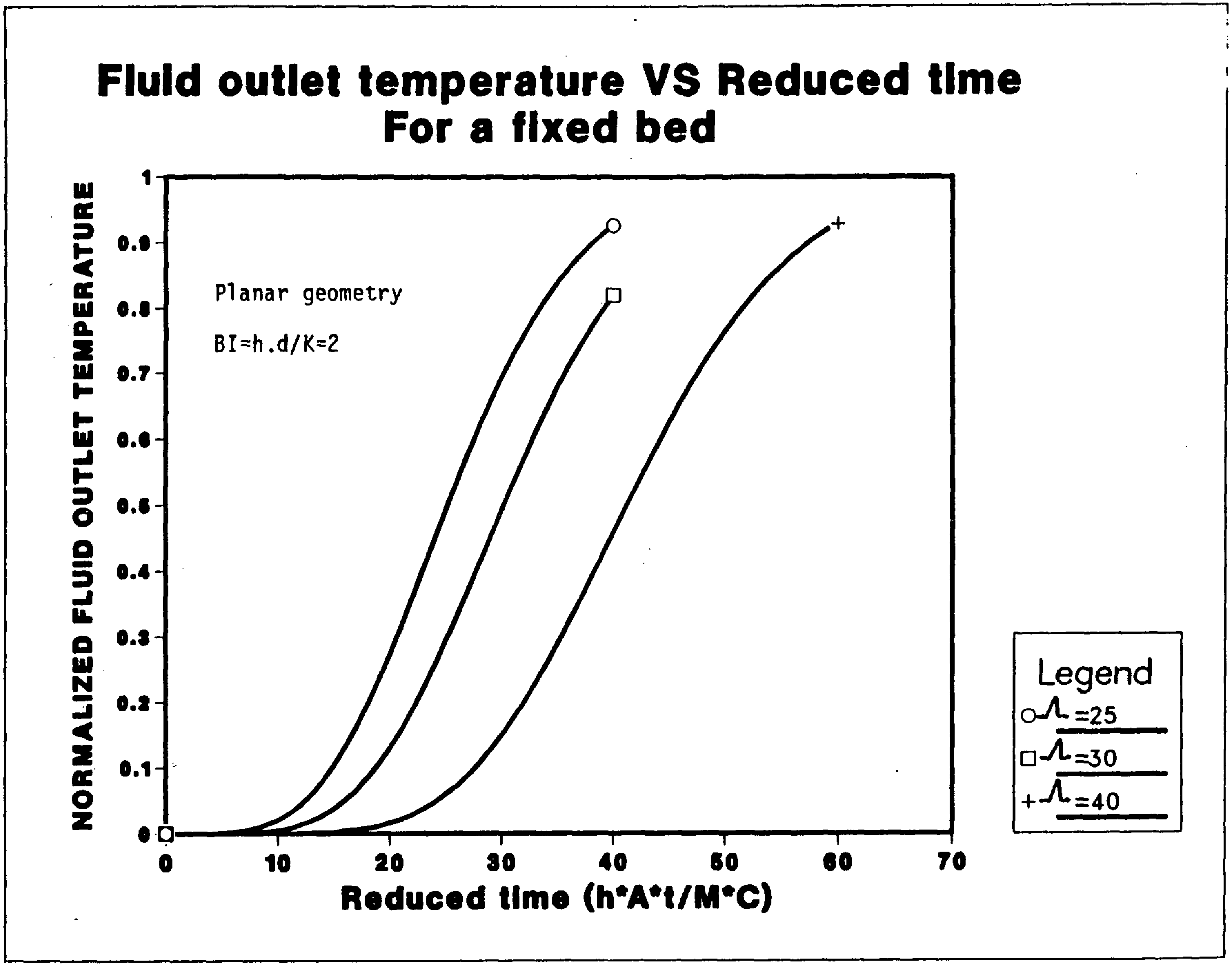


Figure 14. The characteristic S shaped curves of fluid outlet temperature profile for a fixed bed regenerator (various reduced length).

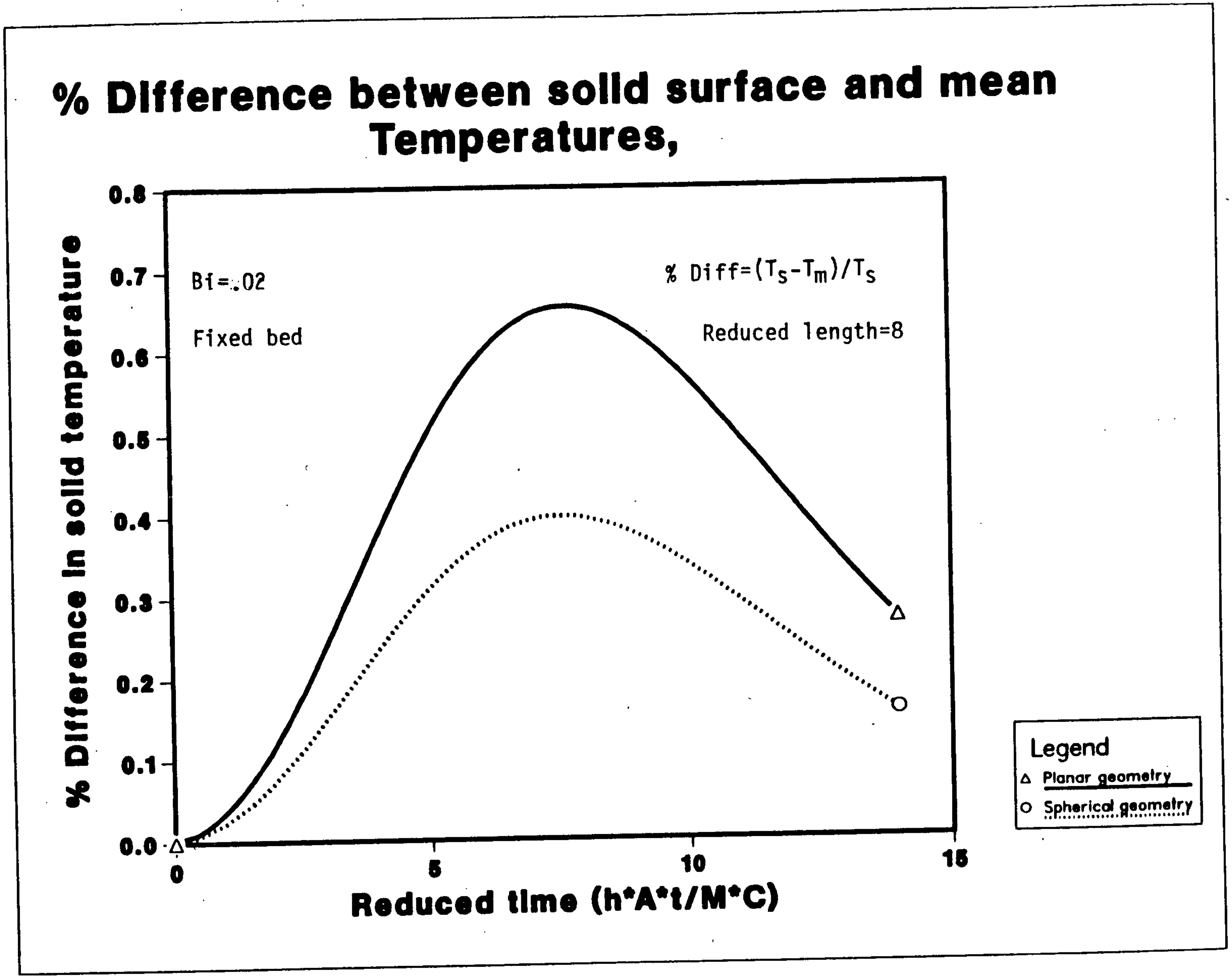


Figure 15. The effect of thermal conductivity on solid temperature profile (Bi=.02).

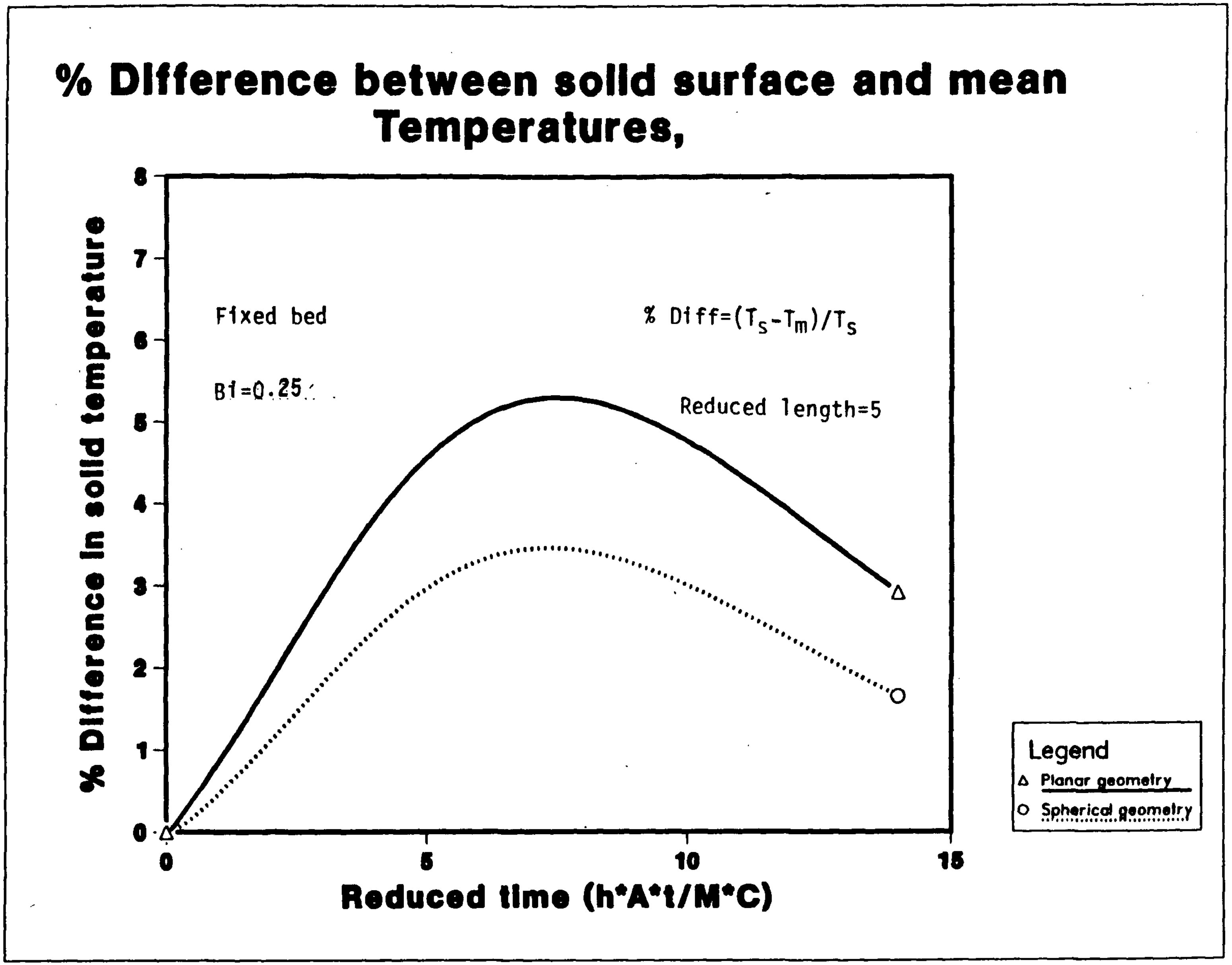


Figure 16. The effect of thermal conductivity on the solid temperature profile (Bi=0.25).

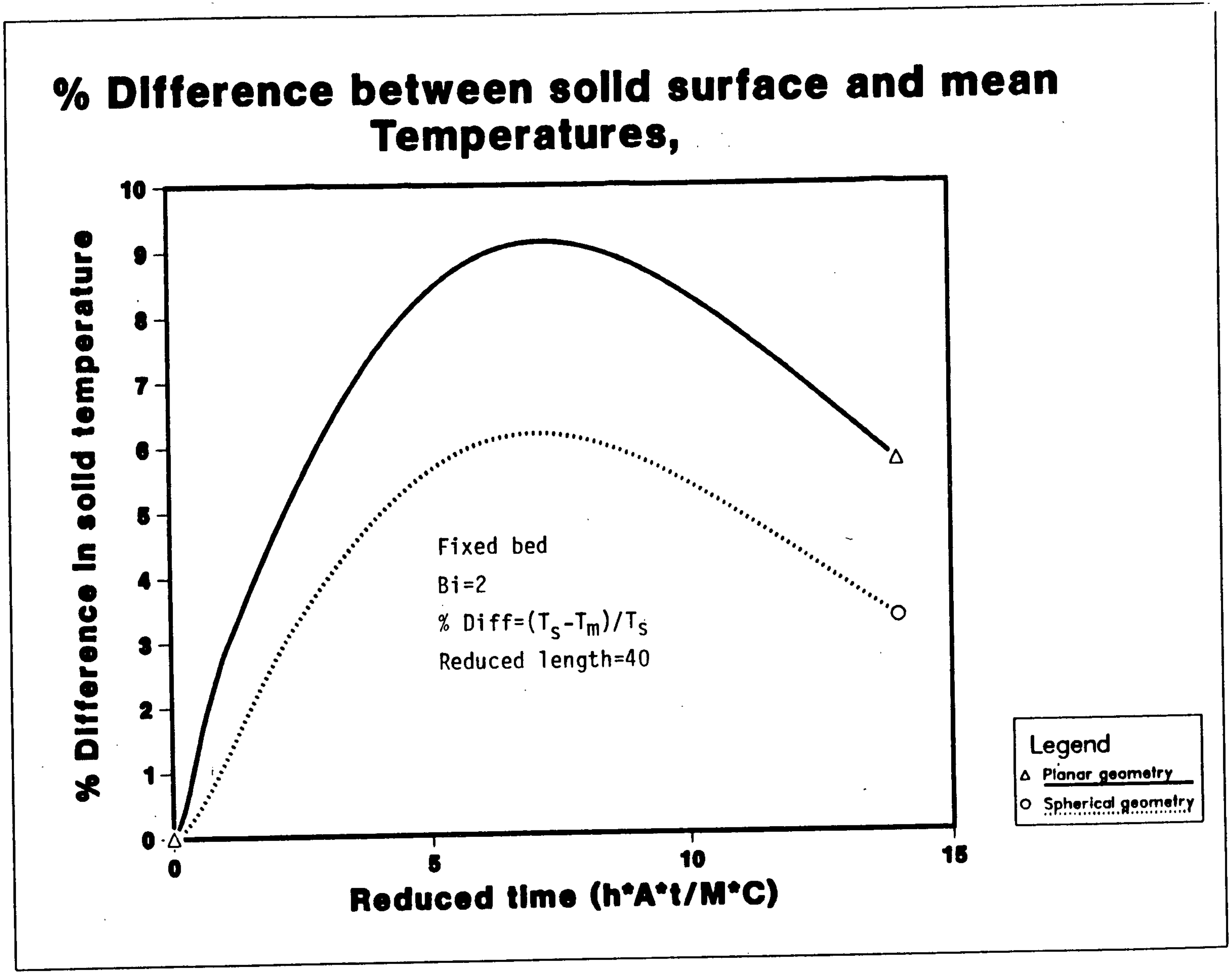


Figure 17. The effect of thermal conductivity on solid temperature profile (Bi=2).

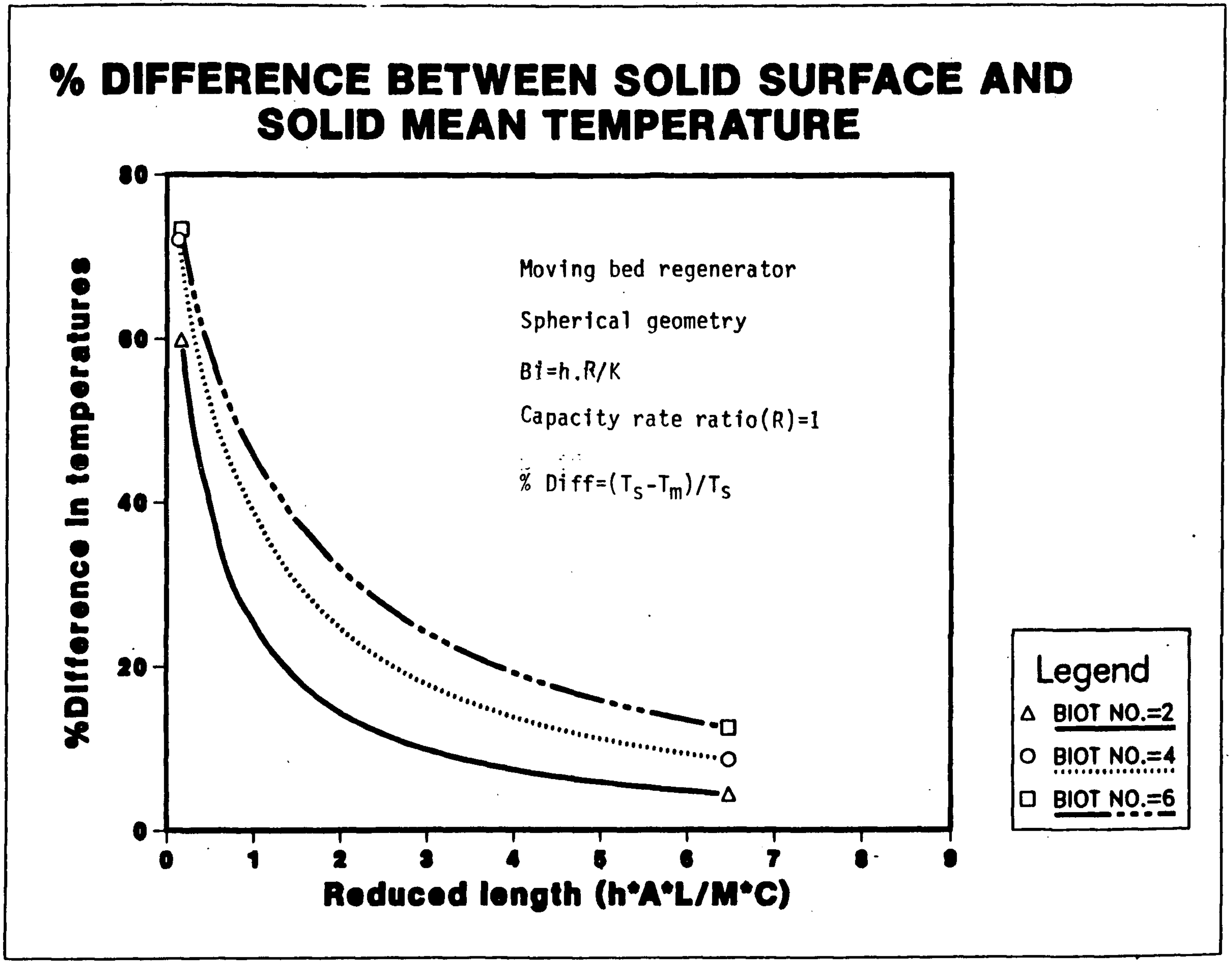


Figure 18. The effect of thermal conductivity on solid temperature profile (Moving bed).

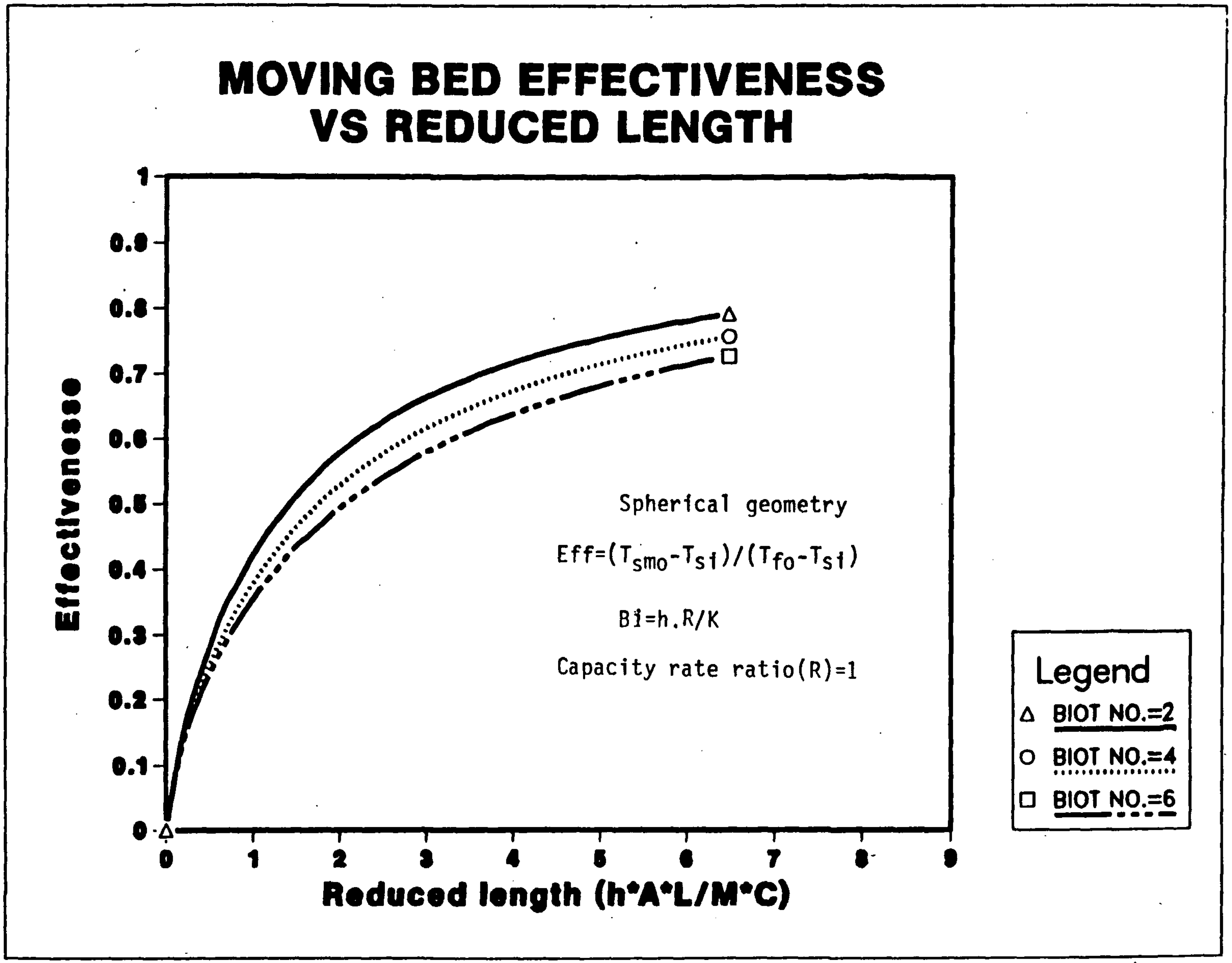


Figure 19. The effect of thermal conductivity on regenerator effectiveness (original profile)

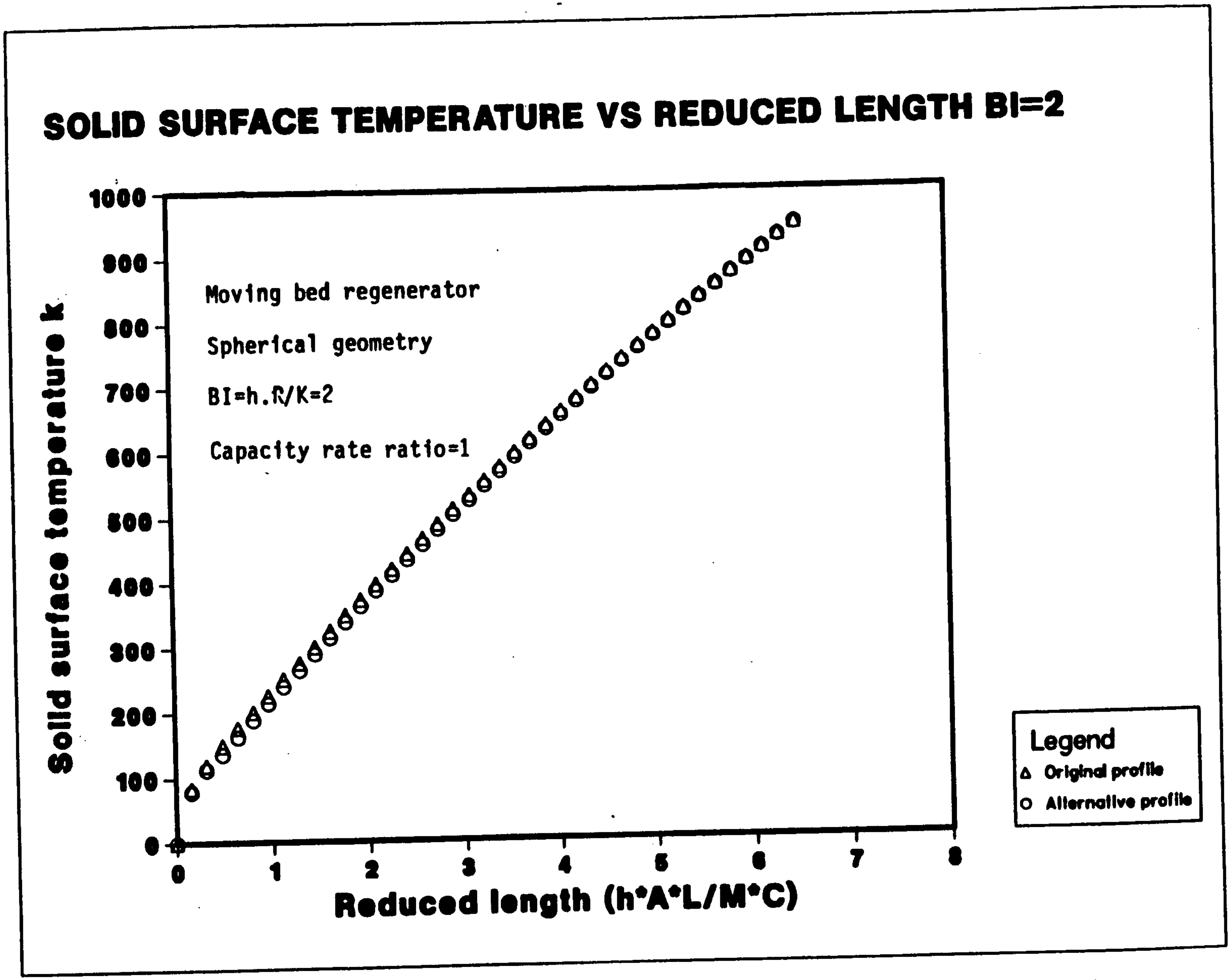


Figure 20. The comparison between the original profile and the alternative profile.

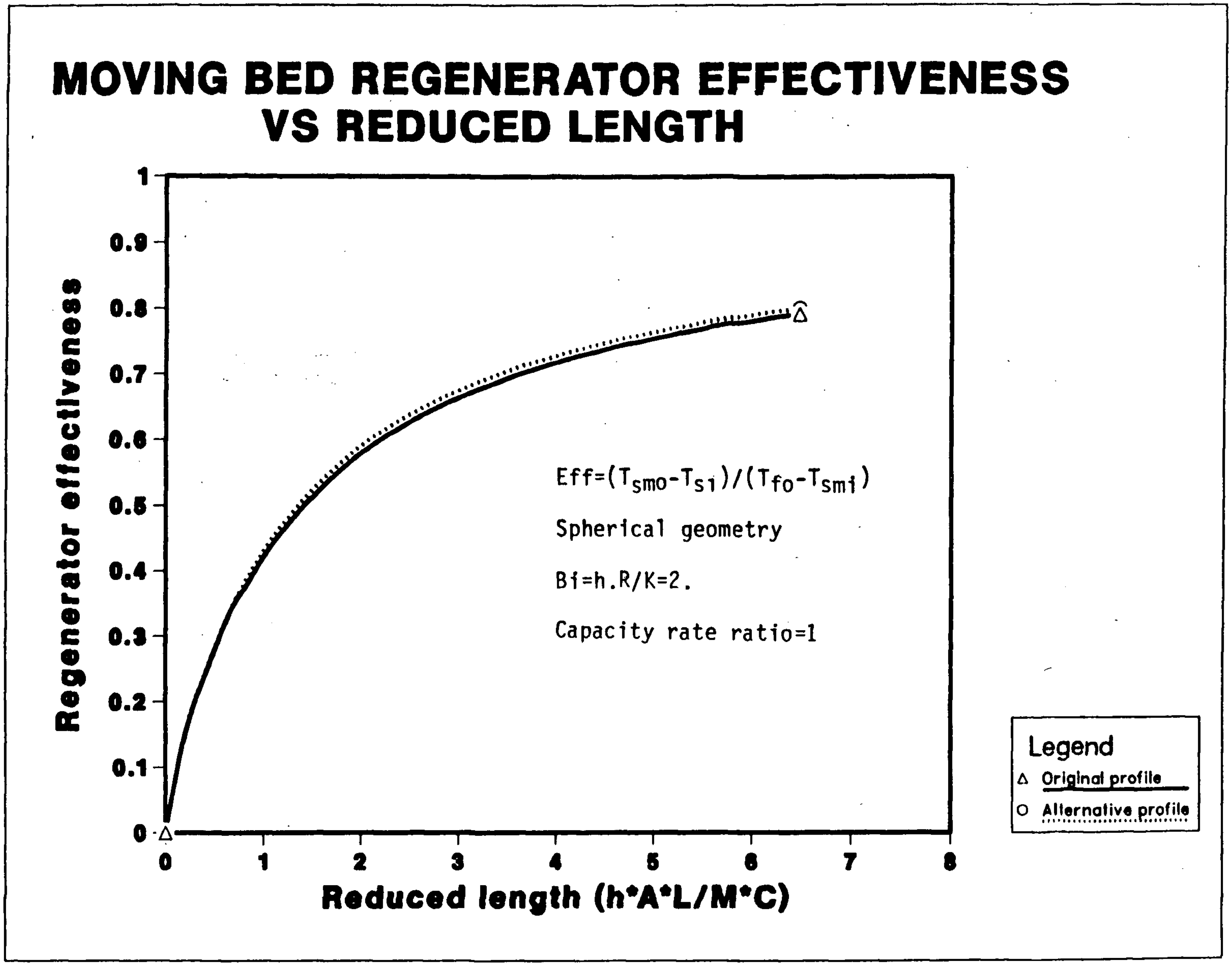


Figure 21. The comparison between the effectiveness based on the two profiles (Bi=2).

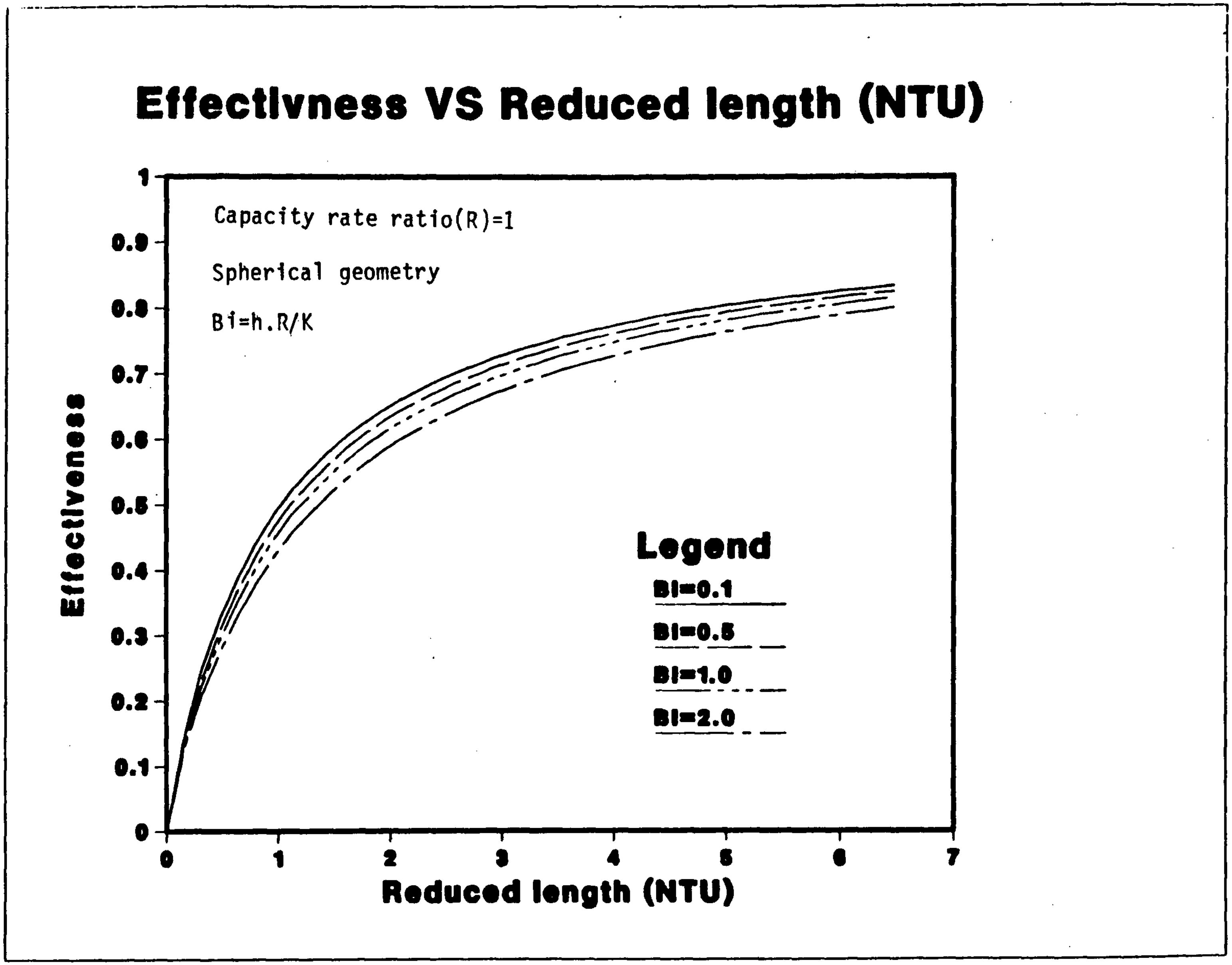


Figure 22. Moving bed regenerator - effectiveness based on the alternative profile.

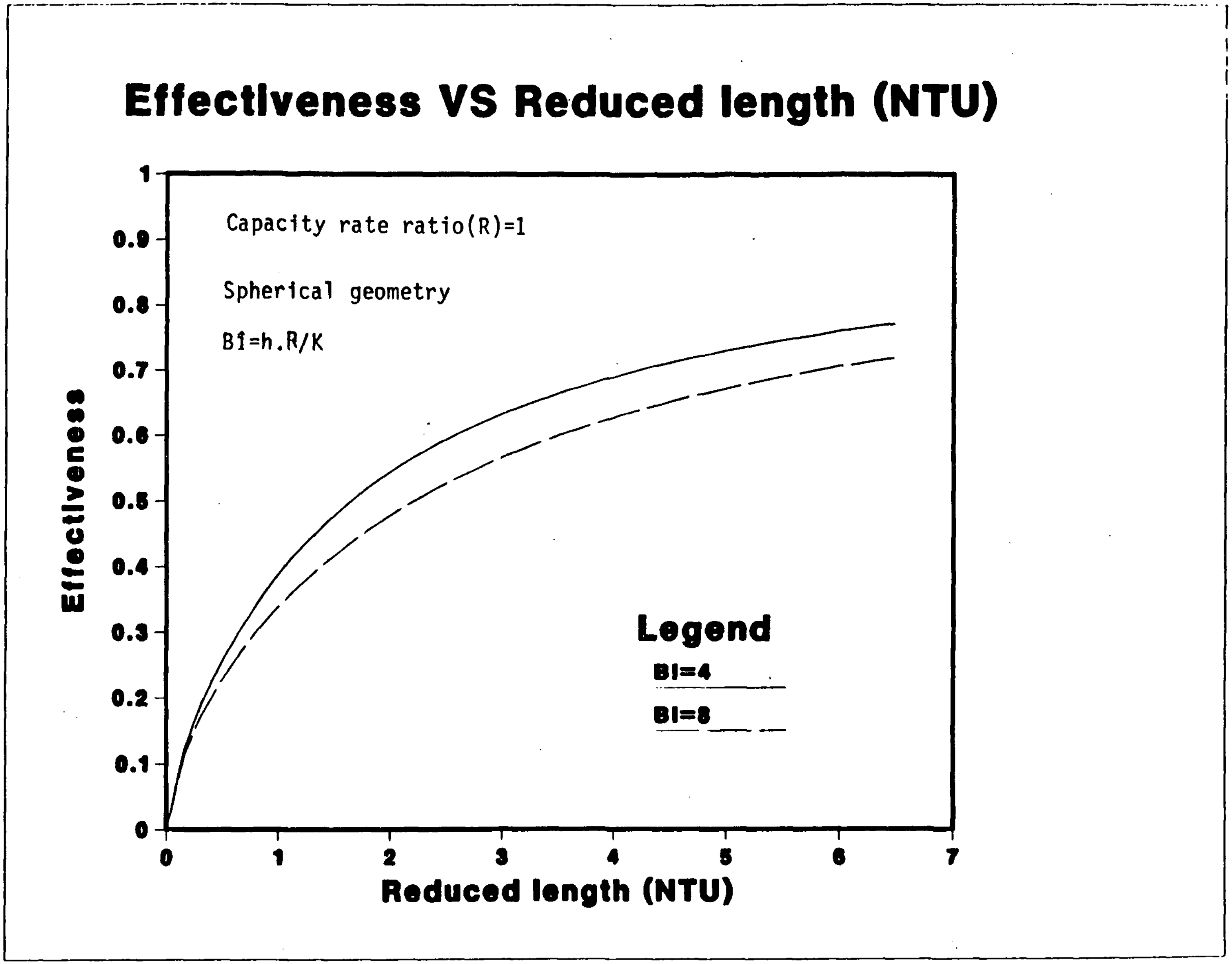


Figure 23. Moving bed regenerator effectiveness based on the alternative profile.

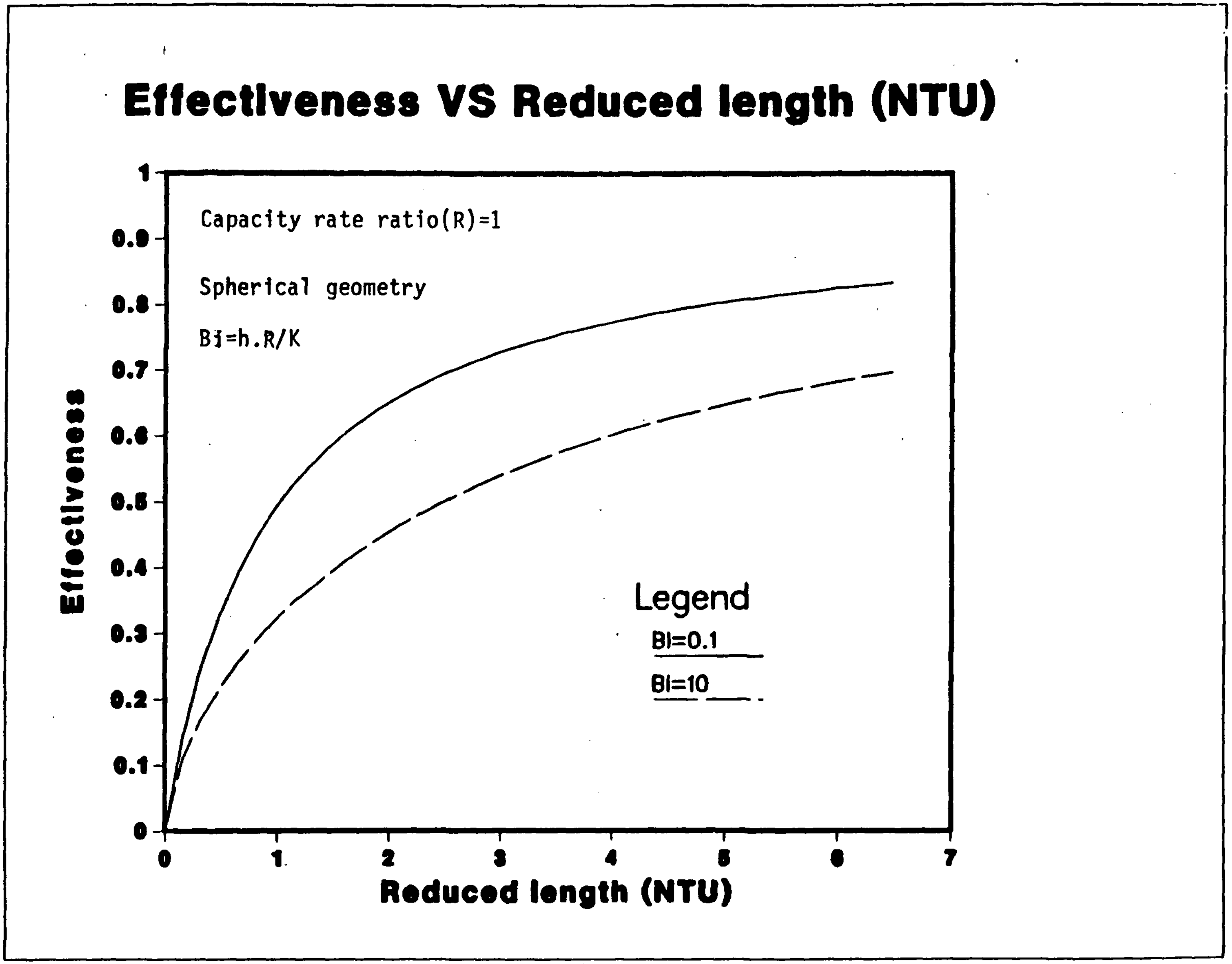
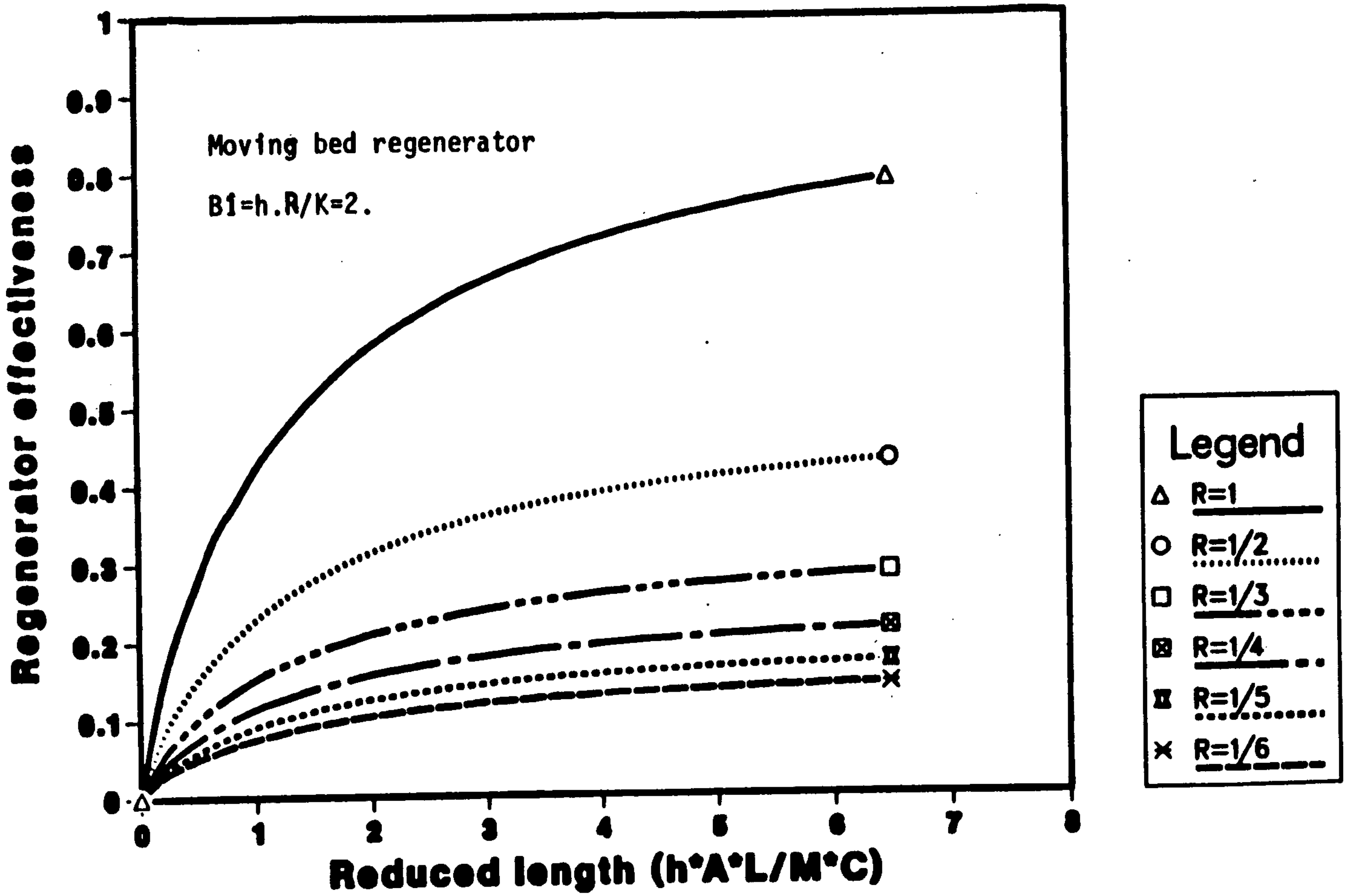


Figure 24. Moving bed regenerator effectiveness based on the alternative profile.

Figure 25.

THE EFFECT OF CAPACITY RATE RATIO ON EFFECTIVENESS
R=M1*C1/M2*C2



BIBLIOGRAPHY

1. Hryniszak.W. "Heat exchangers", 1958

2. Womack.H. "Open cycle MHD power generation" ,1969

3. Bayley.F.J Owen.J and Turner.A "Heat transfer", 1972

4. Carpenter,K.J. and Heggs.P.J. "Prediction of dividing line between conduction and convection effects in regenerator design" , Trans.I.Chem.Eng. vol 56,1978

5. Carpenter.K.J. and Heggs.P.J. "The effects of packing material, size and arrengment on the performance of thermal regenerators ", Int.Heat trnsfer.Conf 6-th, Toronto 1978

6. Willmot.A.J "The regenerative heat exchanger computer representation" J.Heat mass transfer. Vol 12,P 997, 1969

7. Horn.G Sharp.A and Hryniszak.W "Air heaters and seed recovery from MHD plant" ,Phil.Trans.Roy.Soc. Vol 261,1967

8. Schneller.J and Halvacka.V "Moving bed heat exchangers and regenerators" ,Heat exchanger design and theory source book, Chap27

9. Carpenter.K.J and Heggs.P.J. "A modification of the thermal regenerator infinite conduction model to predict the effects of intraconduction", Trans.I.Chem.Eng.Vol 57,1979

10. Cserveny.I. "Contributions to the thermal design of moving pebble bed regenerative air heaters"

,Revue.roumanie des sciences techniques, Vol 22 ,No 1, P125,1977

11. Furnas.C. "Heat transfer from a gas stream to a bed of broken solids ", Indus.Eng.Chem, 1930

12. Handley.D and Heggs.P.J. "The effect of thermal conductivity of the packing material on transient heat transfer in a fixed bed " ,J.heat mass transfer. Vol 12.P549 1969

13. Schumann.T.E "A liquid flowing through a porous prism" ,J of Franklin Inst.Vol 208, 1929

14. Saez.A and Mccoy.B.J "Transient analysis of packed bed thermal storage system" Int.J.Heat mass transfer. Vol 26, No1,1983

15. Hausen.H "vervollstandigate Berechnung des Warmeaustauches in regeneratoren". Z.Ver.Dt.Ing.No2,31-1942.

16. Goodman.R.T "The heat balance integral and its application to problems involving a change of phase", Heat transfer and fluid mechanics Institute, California institute of technology, Pasadena June 1957, p383.

17. Carslaw.H.S. and Jauger.J.C. "Conduction of heat in solids" Oxf.Univ.Press. london 1959.

18. Lardner.T.J and Pohle.F.V ,J.app.Mech Vol 28,1961

19. Goodman.R.T "The heat balance integral-further consideration and refinements" ,Trans.ASME. (J.heat trans), 1961

20. Koh.C.Y. "One dimensional heat conduction with arbitrary heating rate and variable properties ",J.of Aerospace Scince,1961

21. McConnachie.T and Thodos.G "Transfer process in the flow of gases through packed and distended bed of spheres" ,A.I.CH.E journal 1963

22. Rowe.P.N Claxton.K.J and Lewis.T.B "Heat and mass transfer from a single sphere in an extensive flowing fluid" ,Trans.Inst.Chem.Eng,Vol 43,1965

23. Kreyszig.E "Advanced engineering mathematics" 1967

24. Nakada.T ,Nakamura.N ,Narita.Y and Taira.T "Studies of falling particles regenerator", 5-th Int.Conf.MHD. 1971

APPENDIX A:DERIVATION AND DIMENSIONAL ANALYSIS

A.1 Dimensionless parameters

It is a common practice in regenerators thermal design to represent the results in terms of a group of dimensionless parameters. The governing equations will also be simplified by transforming the independent variables y, θ and x(or r)into these dimensionless parameters.

Dimensionless length

This is defined in the following manner

$$\xi = h.A.y/(\dot{m}.C) \quad . \tag{A.1.1}$$

The number of transfer unit(N.T.U) is equivalent to ξ at y=L, that is

$$\Lambda = h.A.L/(\dot{m}.C) \quad . \tag{A.1.2}$$

The number of transfer unit is also called the reduced length.

<u>**Dimensionless time**</u>

This is defined as

$$\eta=h.A.(\theta-y/u)/(M_s.C_s) \;. \tag{A.1.3}$$

The dimensionless period of one cycle operation, Π, is equivalent to η at θ= period, that is

$$\Pi=h.A.(P-L/u)/(M_s.C_s) \;. \tag{A.1.4}$$

For moving bed regenerators,the period of one cycle is defined as the time required by the solid to travel one full length of regenerator.The dimensionless time is expressed in terms of the dimensionless length.

The time θ required by the solid to travel a full length of regenerator is

$$\theta=y/u_s$$

Substituting for θ in equation (A.1.3) and ignoring the fluid residence time y/u ,the dimensionless time will be

$$\eta=h.A.y/(M_s.C_s.u_s) \;, \tag{A.1.5.a}$$

but from (A.1.1)

$$y=\dot{m}.C.\xi/h.A\ , \tag{A.1.5.b}$$

so

$$\eta=\xi.\dot{m}.C/(M_s.C_s.u_s)\ . \tag{A.1.5}$$

<u>Porosity (B)</u>

The porosity or void fraction is defined as

$$B=(V_b-n.V_s)/V_b\ , \tag{A.1.6.a}$$

where V_b is the bed volume and n is the number of solid particles in the bed.Porosity can also be defined as

$$B=(\rho_s-M_s)/\rho_s\ . \tag{A.1.6.b}$$

<u>Heat transfer area</u>

The heat transfer area is defined as

A=(particles surface area)/bed volume ,

or

$$A=(n.A_s)/V_b\ , \tag{A.1.7}$$

but from (A.1.6.a)

$$V_b=(n.V_s)/(1-B)\ ,$$

so substitute for V_b in equation (A.1.7),the result is

$$A=A_s.(1-B)/V_s \quad . \tag{A.1.8}$$

For planar geometry equation (A.1.8) reduces to

$$A=(1-B)/d \quad d=\text{semi-thickness} \quad . \tag{A.1.9.a}$$

For spherical geometry equation (A.1.8) reduces to

$$A=3.(1-B)/R \quad R=\text{radius} \quad . \tag{A.1.9.b}$$

In practice the matrix is not of regular geometry.The regenerator is usually composed of broken rocks. It is therefore required to express the characteristic size of the matrix in terms of an equivalent spherical diameter.This is defined as

$$D_{ev}^3=[6.\text{net volume of rocks}/\pi.n] \quad . \tag{A.1.10}$$

If the rocks are all of the same shape and size, the above equation can be written as

$$D_{ev}^3=[6.V_s/\pi] \quad . \tag{A.1.11}$$

Heat transfer coefficient

It is a common practice to represent the heat transfer coefficient in terms of flow Reynolds number.There are a number of different correlations suggested for this purpose,some of which are listed in Table.1 .

Experimental studies have shown that the degree of packing (or porosity) has a very large influence on the heat transfer coefficient [9] .Consequently,the correlations which include such effect are more conservative. There are two ways to account for porosity effect,these are

1.It is suggested [8,20] that the porosity effect should be included in Reynolds number calculation.Reynolds number is then redefined as

$$Re=u.D_{ev}/[(1-B).\nu]$$ [4] ,

$$Re=u.D_{ev}/(B.\nu)$$ [5] ,

these are so called modified Reynolds number.

2.The porosity can be taken into consideration as an independent variable. Most of the correlations listed in Table.1 are based on such a consideration.

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[4] ref 20
[5] ref 8

It is readily shown [21] that in the absence of natural convection, the Nusselt number of a single sphere in an extensive flow approaches 2 ,when the Reynolds number approaches 0.The correlations that are based on such finding are thus more accurate.

In the present study,the correlation due to I.S.Cservery [10] was utilized for heat transfer calculation.

Correlations	Author	Comments
$Nu=0.332Pr^{1/3}Re^{0.5}$		This is used for chequer work matrix
$Nu=(.255/B)*Pr^{1/3}Re^{2/3}$	Handley and Heggs	
$Nu=(.29A/B)PrRe^{0.7}$	Schneller.J	Modified Reynolds number
$Nu=.016Pr^{0.67}Re^{1.3}$	Frantz.J	Porosity effect ignored
$Nu=2+0.69Pr^{1/3}Re^{0.5}$	Rowe.P.N	for a single sphere
For Re>100 $Nu=2+0.6(3.25-2.25B)Re^{0.5}Pr^{1/3}$ For Re<100 $Nu=2+6(3.25-2.25B)Pr^{1/3}(Re/100)^{1.69}$	Cserveny.I	An own inter - polation formula

Table. 1. Correlations for the convective heat transfer coefficient.

A.2 Derivation of Governing equations

The governing equations are developed for a general case of moving bed regenerators. These are then modified for the case of fixed bed regenerator accordingly.

The total heat flow to the fluid with mass flow rate $\dot{m}$, between y and y+dy (where y is measured from fluid entrance) comprise two components. The first component is the heat transferred to the mass $(\dot{m}.A_b.d\theta)$ in moving between y and $y+\delta y$, that is

$$dQ1=(\dot{m}.d\theta.A_b).C.(\partial T_f/\partial y)_\theta.dy \quad \text{(A.2.1.a)}$$

The second component is the heat transferred to the fluid enclosed between y and y+dy as its temperature changes with time, that is

$$dQ2=\rho.A_b.dy.C.(\partial T_f/\partial\theta)_y.d\theta \ , \quad \text{(A.2.1.b)}$$

but $\dot{m}=\rho.u$, so

$$dQ2=(\dot{m}.A_b/u).C.dy.(\partial T_f/\partial\theta)_y.d\theta \ . \quad \text{(A.2.1.c)}$$

The total heat transferred to the fluid is then

$$dQ=\dot{m}.C.A_b.[(\partial T_f/\partial y)+(\partial T_f/\partial\theta)/u].dy.d\theta \ . \quad \text{(A.2.2)}$$

This is equal to the heat lost from the solid, that is

$$dQ=h.(A.dy.A_b).(T_s-T_f).d\theta \ . \tag{A.2.3}$$

Equating the total heat loss from the solid to the total heat gain by the fluid ,the result is

$$\dot{m}.C.[(\partial T_f/\partial y)+(\partial T_f\ /\partial\theta)/u]=h.A.(T_s-T_f) \ . \tag{A.2.4}$$

From chain rule

$$DT_f/Dy=\partial T_f/\partial y+(\partial T_f/\partial\theta)/u \ ,$$

so equation (A.2.4) can be written as

$$\dot{m}.C.dT_f/dy=h.A.(T_s-T_f) \ . \tag{A.2.5}$$

The heat lost from the solid can also be expressed in terms of its rate of change of internal energy.Defining U as the internal energy /unit area then

$$U=\rho_s.C_s.\int_0^d T\ dx \qquad \text{for one solid ,} \tag{A.2.6.a}$$

$$U=M_s.C_s.\int_0^d T\ dx \qquad \text{for the whole bed .} \tag{A.2.6.b}$$

As for the fluid the total change in solid internal energy comprises two components,that is

$$h.A.(T_s-T_f)=(dU/d\theta)/d \ , \tag{A.2.7}$$

where

$$dU/d\theta = M_s C_s [u_s . \partial(\int_0^d T\ dx)/\partial y + \partial(\int_0^d T\ dx)/\partial\theta] . \qquad (A.2.8)$$

If the bed is stationary then $u_s=0$ and

$$h.A(T_s - T_f) = -(M_s . C_s/d) . \partial(\int_0^d T\ dx)/\partial\theta . \qquad (A.2.9)$$

Note that the sign difference between equations (A.2.9) and (A.2.8) is due to the direction in which y is measured.

It can be seen that if the solid temprature is uniform throughout ,then equations (A.2.8 and A.2.9) reduce to the solid phase equations for the Schumann model.

In the present study, the two equations used to model the regenerator are equations (A.2.5) and the diffusion equation,which is

$$\rho_s C_s (\partial T/\partial\theta) = K_s (\partial^2 T/\partial x^2) \qquad \text{Planar geometry ,} \quad (A.2.10.a)$$

$$= K_s [\partial^2 T/\partial r^2 + (2/r) . \partial T/\partial r] \quad \text{Spherical .} \quad (A.2.10.b)$$

The diffusion equation is coupled with the following boundary conditions

1.The symmetry condition

$$(\partial T/\partial x)_{x=d}=0 \qquad \text{planar ,} \qquad (A.2.11.a)$$

$$(\partial T/\partial r)_{r=0}=0 \qquad \text{Spherical .} \qquad (A.2.11.b)$$

2.Heat flux at the surface

$$K_s.(\partial T/\partial x)_{x=0}=h.(T_s-T_f) \qquad \text{planar ,} \qquad (A.2.12.a)$$

$$K_s.(\partial T/\partial r)_{r=R}=h.(T_f-T_s) \qquad \text{spherical .} \qquad (A.2.12.b)$$

The above equations are nondimensionalised in terms of ξ, η,Bi and normalised temperatures.From the definition of these parameters the following can be deduced ,

$$dT_f=(T_{f0}-T_i).dF \text{ ,}$$

$$dT=(T_{f0}-T_i).d\Psi \text{ ,}$$

$$\partial\theta=M_s.C_s.\partial\eta/(h.A) \text{ ,}$$

$$\partial y=\dot{m}.C.\partial\xi/(h.A) \text{ ,}$$

$$T_s-T_f=(\Psi_s-F).(T_{f0}-T_i) \text{ ,}$$

$$dz=dr/R \qquad \text{For spherical geometry ,}$$

$dz=dx/d$ For planar geometry .

Equation (A.2.5) can now be written in dimensionless form as

$$\Psi_s - F = dF/d\xi \ . \tag{A.2.13}$$

For moving bed regenerators, y is measured from the solid entrance. Consequently ,equation (A.2.13) must be altered accordingly,the result is

$$F - \Psi_s = dF/d\xi \ . \tag{A.2.14}$$

The diffusion equation(A.2.10) can also be written in dimensionless form as

<u>planar geometry</u>

$$\rho_s . C_s . h . A . (\partial\Psi/\partial\eta)/(M_s . C_s) = K_s . (\partial^2\Psi/\partial z^2)/d^2 \ ,$$

from the definition of porosity (B) , $M_s=\rho_s.(1-B)$,

from equation (A.9.a) $A=(1-B)/d$ .So substitute for A and M_s in above equation, then

$$\partial\Psi/\partial\eta = K_s . (\partial^2\Psi/\partial z^2)/(h.d) \ ,$$

or

$$\partial\Psi/\partial\eta = (\partial^2\Psi/\partial z^2)/Bi \ . \tag{A.2.15}$$

The corresponding constraints are

1.Symmetry condition

$$(\partial\Psi/\partial z)_{z=1}=0 \qquad (A.2.16)\ .$$

2.The heat flux at the surface

$$K_s.(\partial\Psi/\partial z)/d=h.(\Psi_s-F)\ ,$$

that is

$$(\partial\Psi/\partial z)_{z=0}=Bi.(\Psi_s-F)\ . \qquad (A.2.17)$$

<u>spherical geometry</u>

The diffusion equation in dimensionless form will be

$$\rho_s C_s h.A(\partial\Psi/\partial\eta)/M_s C_s=K_s[(\partial^2\Psi/\partial z^2)/R^2+(2/z.R)(\partial\Psi/\partial z)/R]\ ,$$

but $\quad M_s=\rho_s.(1-B)\ ,$

and $\quad A=3.(1-B)/R\ ,$

so $\quad 3(\partial\Psi/\partial\eta).h/R=k_s/R^2[\partial^2\Psi/\partial z^2+(2/z).\partial\Psi/\partial z]\ ,$

or $\quad \partial\Psi/\partial\eta=[\partial^2\Psi/\partial z^2+(2/z).(\partial\Psi/\partial z)]/(3Bi)\ . \qquad (A.2.18)$

The corresponding constraints are

1.Symmetry condition

$$(\partial\Psi/\partial z)_{z=0}=0\ . \tag{A.2.19}$$

2.Heat flux at the surface

$$(K_s/R).(\partial\Psi/\partial z)=h.(F-\Psi_s)\ ,$$

that is

$$(\partial\Psi/\partial z)_{z=1}=Bi.(F-\Psi_s)\ . \tag{A.2.20}$$

APPENDIX B:INTEGRAL METHOD

B.1 Planar geometry

The matrix (usually chequrework) is modelled in two stages

Semi-infinite slab

The diffusion equation in dimensionless form is

$$\partial\Psi/\partial z=(\partial^2\Psi/\partial z^2)/Bi \ . \tag{B.1.1}$$

The penetration depth,defined in chapter 3,has a characteristic property such that the solid is at an equilibrium temperature (initial temperature) at any point beyond penetration depth.

The matrix is subject to the following dimensionless boundary conditions

$$\Psi(\delta_0,\eta)=\Psi_i=0 \ , \tag{B.1.2}$$

$$\partial\Psi(\delta_0,\eta)/\partial z=0 \ , \tag{B.1.3}$$

$$\partial\Psi(0,\eta)/\partial z=Bi.(\Psi_s-F)=-f(\eta) \ . \tag{B.1.4}$$

Differentiating equation (B.1.2) with respect to η and substituting back into the diffusion equation results in an extra boundary condition called the smoothing condition, that is

$$\partial^2\Psi(\delta_0,\eta)/\partial z^2=0 \ . \tag{B.1.5}$$

There are 4 constraints. The matrix temprature is thus represented by a cubic profile, that is

$$\Psi=A+Bz+Cz^2+Dz^3 \ . \tag{B.1.6}$$

Applying the constraints, there will be 4 equations

$$A+B\delta_0+C\delta_0^2+D\delta_0^3=0 \ ,$$

$$B+2C.\delta_0+3D.\delta_0^2=0 \ ,$$

$$B=-f(\eta) \ ,$$

$$2C+6D.\delta_0=0 \ .$$

The 4 unknowns are found by solving the above simultaneous equations , the cubic profile is then

$$\Psi=f(\eta).(\delta_0-z)^3/(3.\delta_0^2) \ . \tag{B.1.7}$$

The surface temprature Ψ_s is obtained by setting z=0 in equation (B.1.7), the result is

$$\Psi_s = f(\eta).\delta_0/3 \ . \tag{B.1.8}$$

The diffusion equation is now integrated with respect to z, that is

$$\int_0^{\delta_0} (\partial\Psi/\partial\eta)\ dz = (\int_0^{\delta_0} (\partial^2\Psi/\partial z^2)\ dz)/Bi \ . \tag{B.1.9}$$

From Liebnitz theorem

$$\int_0^{\delta_0} (\partial\Psi/\partial z)\ dz = d(\int_0^{\delta_0} \Psi\ dz)/d\eta - \Psi(\delta_0,\ \eta)(d\delta_0/d\eta)\ , \tag{B.1.10}$$

but by definition $\Psi(\delta_0,\eta)=0$, so equation (B.1.9) becomes

$$d(\int_0^{\delta_0} \Psi\ dz)/d\eta = [(\partial\Psi/\partial z)_{z=\delta_0} - (\partial\Psi/\partial z)_{z=0}]/Bi \ . \tag{B.1.11}$$

Applying equations (B.1.3) and (B.1.4)to the above equation , the following is obtained

$$d(\int_0^{\delta_0} \Psi\ dz)/d\eta = f(\eta)/Bi \ . \tag{B.1.12}$$

Substituting for Ψ from (B.1.7), the result is

$$d(\int_0^{\delta_0} [f(\eta).(\delta_0 - z)^3/(3\delta_0^2)]\ dz)/d\eta = d(f(\eta).\delta_0^2/12)/d\eta \ . \tag{B.1.13}$$

So equation (B.1.12) becomes

$$d[f(\eta).\delta_0^2/12]/d\eta = f(\eta)/Bi \ . \tag{B.1.14}$$

The solution to the above differential equation is obtained by integration with respect to η,the result is

$$\delta_0=[12.(\int_0^{\eta} f(\eta)\ d\eta\ )/(Bi.f(\eta)]^{0.5}\ , \tag{B.1.15}$$

where $\delta_0=0$, at $\eta=0$.

The surface temperature is computed by substituting for δ_0 from (B.1.8),the result is

$$\Psi_s=[4.f(\eta).(\int_0^{\eta} f(\eta)\ d\eta)/(3.Bi)]^{0.5}\ . \tag{B.1.16}$$

Slab of finite thickness

At time $\eta=\eta_0$, the penetration depth reaches the centre of the matrix.From this point on the penetration depth has no meaning and the matrix has to be remodelled.This is achieved by modelling the matrix as a finite slab (thickness d) subject to the following dimensionless constraints

$$\Psi(0,\eta)=\Psi_s\ , \tag{B.1.17}$$

$$\partial\Psi(0,\eta)/\partial z=Bi.(\Psi_s-F)=-f(\eta)\ , \tag{B.1.18}$$

$$\partial\Psi(1,\eta)/\partial z=0\ . \tag{B.1.19}$$

There are 3 constraints.So the temperature profile must be a second-order polynomial,that is

$$\Psi = A + Bz + Cz^2 \ . \tag{B.1.20}$$

The constants A,B and C are found from constraints.The profile will then take the form

$$\Psi = \Psi_s + f(\eta).(z^2 - 2z)/2 \ . \tag{B.1.21}$$

The diffusion equation is now integrated with respect to z. After applying the Liebnitz theorem,the result is

$$d(\int_0^1 \Psi \, dz)/d\eta = [(\partial\Psi/\partial z)_{z=1} - (\partial\Psi/\partial z)_{z=0}]/Bi \ . \tag{B.1.22}$$

Applying equations (B.1.18) and (B.1.21) to the above equation, the following is obtained

$$d(\Psi_s - f(\eta)/3)/d\eta = f(\eta)/Bi \ . \tag{B.1.23}$$

The solution to the above differential equation is obtained by integration with respect to η,the result is

$$\Psi_s - f(\eta)/3 = (\int_0^\eta f(\eta) \, d\eta)/Bi + \text{Constant} \ . \tag{B.1.24}$$

At time $\eta=\eta_0$, $\delta_0=1$,ie.the penetration depth has reached the thickness of the slab.So from (B.1.8)

$$\Psi_s(\eta_0) = f(\eta_0)/3 \ . \tag{B.1.25}$$

Substitute for $\Psi_s(\eta_0)$ in equation(B.1.24) ,the result is

$$\Psi_s(\eta_0)-f(\eta_0)/3=(\int_0^{\eta_0} f(\eta)\, d\eta)/Bi+Constant\ ,$$

so

$$Constant=-(\int_0^{\eta_0} f(\eta)\, d\eta)/Bi\ . \qquad (B.1.26)$$

Substitute for Constant in equation (B.1.24),the result is

$$\Psi_s=f(\eta)/3+(\int_{\eta_0}^{\eta} f(\eta)\, d\eta)/Bi\ . \qquad (B.1.27)$$

B.2 Spherical geometry

As for the planar geometry,the spherical matrix is modelled in two stages

Sphere of infinite radius

The diffusion equation for spherical geometry in dimensionless form is

$$\partial\Psi/\partial\eta=[\partial^2\Psi/\partial z^2+(2/z).\partial\Psi/\partial z]/(3.Bi)\ . \qquad (B.2.1)$$

The penetration depth concept is employed to approximate the solid temperature profile.The penetration depth is measured from the surface of the sphere.The matrix is subject to the following dimensionless constraints

$$\Psi(1-\delta_0,\eta)=\Psi_i=0 \ , \tag{B.2.2}$$

$$\partial\Psi(1-\delta_0,\eta)/\partial z=0 \ , \tag{B.2.3}$$

$$\partial\Psi(1,\eta)/\partial z=-Bi.(\Psi_s-F)=-f(\eta) \ . \tag{B.2.4}$$

Equation (B.2.2) is differentiated with respect to η and then substituted back in the diffusion equation.This results in an extra boundary condition of the form

$$\partial^2\Psi/\partial z^2+(2/z).(\partial\Psi /\partial z)=0 \ . \tag{B.2.5}$$

Applying equation (B.2.3) to the above equation,it reduces to the smoothing condition,that is

$$\partial^2\Psi(1-\delta_0,\eta)/\partial z^2=0 \ . \tag{B.2.6}$$

The suggested profile is of the form [17]

$$\Psi=(\text{polynomial in } z)/z \ . \tag{B.2.7}$$

The reason for this form of the profile is that it resembles the steady state solution of the diffusion equation.The steady state solution being

$$\Psi \propto (1/z) \ .$$

The polynomial of z is a cubic. So the final profile will take the form

$$\Psi=Az^{2}+Bz+C+D/z \ . \tag{B.2.8}$$

Applying the constraints and solving for A,B,C and D ,the final expression will be

$$\Psi=-f(\eta).[z-(1-\delta_0)]^{3}/(\delta_0^{2}.(3-\delta_0).z) \ . \tag{B.2.9}$$

The surface temprature is obtained by setting z=1 in the above equation,that is

$$\Psi_s=-f(\eta).\delta_0/(3-\delta_0) \ . \tag{B.2.10}$$

The diffusion equation (B.2.1) is rewritten as

$$\partial(\Psi.z)/\partial\eta=[\partial^{2}(\Psi.z)/\partial z^{2}]/(3.Bi) \ . \tag{B.2.11}$$

The above equation is integrated with respect to z. After applying the Liebnitz theorem,the result is

$$d(\int_{1-\delta_0}^{1}\Psi.z\ dz)/d\eta=(\int_{1-\delta_0}^{1}\partial^{2}(\Psi.z)/\partial z^{2}\ dz)/(3.Bi) \ . \tag{B.2.12}$$

The right hand side of the above equation can be simplified by applying equations (B.2.2),(B.2.3) and (B.2.4).Equation (B.2.12) reduces to

$$d\left(\int_{1-\delta_0}^{1} \Psi.z\, dz\right)/d\eta = -f(\eta)/[Bi.(3-\delta_0)] \ . \tag{B.2.13}$$

It can be seen that the above differential equation is highly non-linear. The non-linearity can be reduced to some extent by readjusting the diffusion equation.Equation (B.2.1) is now written as

$$\partial(\Psi.z^2)/\partial\eta = [\partial(z^2.\partial\Psi/\partial z)/\partial z]/(3.Bi) \ . \tag{B.2.14}$$

The above equation is now integrated with respect to z,the result is

$$d\left(\int_{1-\delta_0}^{1} \Psi.z^2\, dz\right)/d\eta = [z^2(\partial\Psi/\partial z)]_{1-\delta_0}^{1}/(3Bi) \ . \tag{B.2.15}$$

After applying the constraints,the above equation reduces to

$$d\left(\int_{1-\delta_0}^{1} \Psi.z^2\, dz\right)/d\eta = -f(\eta)/(3Bi) \ . \tag{B.2.16}$$

It can be seen that equation (B.2.16) is less non-linear in comparison with equation (B.2.13). Substituting for Ψ from equation (B.2.9) and performing the integral,the result is

$$d[(\delta_0^3-5\delta_0^2).f(\eta)/(3-\delta_0)]/d\eta = -20f(\eta)/(3Bi) \ . \tag{B.2.17}$$

The above differential equation is solved by integration with respect to η, with the initial condition ($\delta_0=0$ at $\eta=0$) the result is

$$[(5\delta_0^2-\delta_0^3)/(3-\delta_0)]=20(\int_0^\eta f(\eta)\ d\eta)/(3Bi.f(\eta)) \ . \qquad (B.2.18)$$

The surface temprature is obtained by substituting for δ_0 from equation (B.2.10),the result is

$$3\Psi_s^2[5f(\eta)-2\Psi_s]=20[\Psi_s-f(\eta)]^2.(\int_0^\eta f(\eta)\ d\eta)/(3Bi) \ . \qquad (B.2.19)$$

<u>Sphere of finite radius</u>

At some time η_0,the penetration depth reaches the centre of the sphere.At this time the penetration depth concept should be disregarded due to the symmetry effect.The matrix is now subject to the following dimensionless constraints

$$\partial\Psi(0,\eta)/\partial z=0 \ , \qquad (B.2.20)$$

$$\partial\Psi(1,\eta)/\partial z=-Bi.(\Psi_s-F)=-f(\eta) \ , \qquad (B.2.21)$$

$$\Psi(1,\eta)=\Psi_s \ . \qquad (B.2.22)$$

The profile will take the form [17]

$$\Psi=(\text{polynomial in } z)/z \ . \qquad (B.2.23)$$

It should be noted that since the sphere is subject to a boundary condition at the centre,the polynomial should not include a constant term .So the profile will take the form

$$\Psi=Az^2+Bz+C \ . \tag{B.2.24}$$

The unknowns A,B and C are found by applying the constraints, the result is

$$\Psi=\Psi_s+f(\eta).[1-z^2]/2 \ . \tag{B.2.25}$$

The modified diffusion equation (B.2.14) is now integrated with respect to z,the result is

$$d(\int_0^1 \Psi.z^2\ dz)/d\eta=[\int_0^1 \partial(z^2.\partial\Psi/\partial z)/\partial z\ dz]/(3Bi) \ ,$$

$$=-f(\eta)/(3Bi) \ . \tag{B.2.26}$$

Substituting for Ψ from equation (B.2.25) and performing the integration,the result is

$$d[\Psi_s/3+f(\eta)/15]/d\eta=-f(\eta)/(3Bi) \ . \tag{B.2.27.a}$$

The above differential equation is solved by integration with respect to η,the result is

$$\Psi_s+f(\eta)/5=-(\int_0^\eta f(\eta)\ d\eta)/Bi+Constant \ . \tag{B.2.27}$$

At $\eta=\eta_0$, $\delta_0=1$ .So $\Psi_s(\eta_0)$ can be obtained by setting $\delta_0=1$ in equation (B.2.10),the result is

$$\Psi_s(\eta_0)=-f(\eta_0)/2 \ . \tag{B.2.28}$$

Substituting for $\Psi_s(\eta_0)$ in equation (B.2.27) the constant of integration is obtained,that is

$$\text{Constant}=-3f(\eta_0)/10+(\int_0^{\eta_0} f(\eta)\ d\eta)/Bi \ . \tag{B.2.29}$$

The term $f(\eta_0)$ can be computed by setting $\delta_0=1$ in equation (B.2.18),the result is

$$f(\eta_0)=10(\int_0^{\eta_0} f(\eta)\ d\eta)/(3Bi) \ . \tag{B.2.30}$$

From equations (B.2.30) and (B.2.29),it can be deduced that the constant of integration is zero.So equation (B.2.27) becomes

$$\Psi_s=-f(\eta)/5-(\int_0^{\eta} f(\eta)\ d\eta)/Bi \ . \tag{B.2.31}$$

APPENDIX C: EFFECTIVENESS COMPUTATION

In regenerators the thermodynamically perfect situation occurs when the matrix at the exit from the regenerator is at the same temperature of the entering hot fluid. Obviously, this is not possible in practice because of the resistance to the heat transfer between the two media. To measure the performance of a regenerator against the idealised situation, a parameter called **effectiveness** is employed. This is defined as the ratio of the actual rise (or drop) in the matrix temperature to the maximum possible rise. In mathematicall form ϵ is defined as

$$\epsilon=\dot{m}_s.C_s.(T_{ms0}-T_{msi})/[(\dot{m}.C)_{min}.(T_{fi}-T_{msi})] , \qquad (C.1)$$

where $(\dot{m}.C)_{min}$=minimum of the two capacity rates,

and T_{ms}=Mean solid temperature .

It is apparent from equation (C.1) that the solid temperature at inlet and outlet are represented as mean temperatures. Consequently, the first step towards the effectiveness computation, is the calculation of the solid mean temperature.

C.1 Planar geometry

The mean solid temperature in planar geometry is defined as

$$T_m=(\int_0^d T\ dx)/d\ , \qquad (C.2)$$

or in dimensionless form

$$\Psi_m=\int_0^1 \Psi\ dz\ . \qquad (C.3)$$

There are two expressions for Ψ depending on the penetration depth.These were obtained in the previous Appendix.

For $\eta<\eta_0$,

$$\Psi=f(\eta).(\delta_0-z)^3/(3.\delta_0^2)\ . \qquad (B.1.7)$$

For $\eta\geq\eta_0$,

$$\Psi=\Psi_s+f(\eta).(z^2-2.z)/2\ . \qquad (B.1.21)$$

The corresponding mean temperatures are then

For $\eta<\eta_0$,

$$\Psi_m=f(\eta).[\int_0^1(\delta_0-z)^3\ dz]/(3.\delta_0^2)\ ,$$

but

$$\int_0^1 \Psi\ dz=(\int_0^{\delta_0}\Psi\ dz)+(\int_{\delta_0}^1 \Psi\ dz)\ ,$$

by definition the second integral is equal to zero and the mean temperature will be

$$\Psi_m=f(\eta).[\int_0^{\delta_0}(\delta_0-z)^3\ dz]/(3.\delta_0^2)\ ,$$

that is

$$\Psi_m=f(\eta).\delta_0^2/12\ . \tag{C.4}$$

For $\eta\geq\eta_0$,

$$\Psi_m=\int_0^1[\Psi_s+f(\eta).(z^2-2.z)/2]\ dz\ ,$$

performing the integration,

$$\Psi_m=\Psi_s-f(\eta)/3\ , \tag{C.5}$$

where $\quad f(\eta)=Bi.(F-\Psi_s)\ .$ (C.6)

C.2 Spherical geometry

The mean temperature in cartesian coordinate (3 dimensional) is defined as

$$T_m=(\int\int\int T\ dx\ dy\ dz)/(\int\int\int dx\ dy\ dz)\ . \tag{C.7}$$

The above equation can be transformed to sherical coordinate system by applying the Jacobian transformation [21].

Defining $x=r.\cos\theta.\sin\phi$,

$y=r.\sin\theta.\sin\phi$,

$z=r.\cos\phi$.

The Jacobian transformer will be

$$J=\begin{vmatrix} \partial x/\partial r & \partial x/\partial\theta & \partial x/\partial\phi \\ \partial y/\partial r & \partial y/\partial\theta & \partial y/\partial\phi \\ \partial z/\partial r & \partial z/\partial\theta & \partial z/\partial\phi \end{vmatrix} =-r^2\sin\phi\ . \tag{C.9}$$

so $\int\int\int dx\ dy\ dz=\int\int\int -r^2\ \sin\phi\ dr\ d\theta\ d\phi$. (C.10)

Substitute the above,in equation (C.7)

$$T_m=\int_0^R(-Tr^2)dr.\int_{-\pi/2}^{\pi/2}d\theta.\int_0^{2\pi}\sin\phi\ d\phi/\int_0^R -r^2\ dr.\int_0^{2\pi}\sin\phi\ d\phi.\int_{-\pi/2}^{\pi/2}d\theta$$

or

$$T_m=\int_0^R(T.r^2)\ dr/(\int_0^R r^2\ dr)\ . \tag{C.11}$$

The mean temperature in dimensionless form is

$$\Psi_m = 3.\int_0^1 \Psi.z^2\ dz\ . \tag{C.12}$$

There are two expressions for Ψ,depending on the penetration depth,these are

For $\eta<\eta_0$,

$$\Psi = -f(\eta).[z-(1-\delta_0)]^3/[(3-\delta_0)\delta_0^2 z]\ . \tag{B.2.9}$$

The mean temperature is obtained by substituting the above expression for Ψ in equation (C.12).First it is noted that

$$\Psi_m = 3\int_{1-\delta_0}^{1} (\Psi.z^2)\ dz + 3\int_0^{1-\delta_0} (\Psi.z^2)\ dz\ , \tag{C.13}$$

and by definition $\Psi=0$ beyond penetration depth.So the second integral in equation (C.13) is zero and (C.13) reduces to

$$\Psi_m = 3.\int_{1-\delta_0}^{1} (\Psi.z^2)\ dz\ . \tag{C.14}$$

Substituting for Ψ from equation (B.2.9) in above equation,the result is

$$\Psi_m = -3.f(\eta).\int_{1-\delta_0}^{1} z.[z-(1-\delta_0)]^3\ dz/[(3-\delta_0).\delta_0^2\ ]\ ,$$

so

$$\Psi_m=-3.f(\eta).(5\delta_0^2-\delta_0^3)/[20.(3-\delta_0)] \ . \tag{C.15}$$

It is interesting to note that from equation (B.2.18) the mean temperature can also be written as

$$\Psi_m=-\int_0^{\eta} f(\eta)\ d\eta/Bi \ . \tag{C.16}$$

For $\eta\geq\eta_0$,

$$\Psi=\Psi_s+f(\eta).(1-z^2)/2 \ . \tag{B.2.25}$$

Substituting for Ψ in equation (C.12), the result is

$$\Psi_m=3\int_0^1[\Psi_s.z^2+f(\eta).(z^2-z^4/2)]\ dz \ ,$$

that is

$$\Psi_m=\Psi_s+f(\eta)/5 \ , \tag{C.17}$$

where $\quad f(\eta)=Bi.(\Psi_s-F)$. $\quad$ (C.18)

APPENDIX D :SAMPLE CALCULATION

D.1 Governing equations

The governing equations are written in their numerical form as

Fluid phase

$$(1+\Delta\xi/2)F(n,i+1)-\Delta\xi/2\Psi_s(n,i+1)=\Delta\xi/2\Psi_s(n-1,i+1)+(1-\Delta\xi/2)F(n-1)$$

Solid phase

1. Planar geometry

For $\eta<\eta_0$,

$$\delta_0(n,i)=[12Ar1(n,i)/(Bi.f(n,i))]^{0.5} , \quad (D.2)$$

$$\Psi_s(n,i+1)=[4f(n,i+1).Ar1(n,i+1)/(3Bi)]^{0.5} . \quad (D.3)$$

For $\eta\geq\eta_0$,

$$\Psi_s(n,i+1)=f(n,i+1)/3+Ar2(n,i+1)/Bi , \quad (D.4)$$

where

$$f(n,i)=Bi[F(n,i)-\Psi_s(n,i)] \ , \tag{D.5}$$

$$Ar1(n,i+1)=Ar1(n,i)+\Delta\eta[f(n,i+1)+f(n,i)]/2 \ , \tag{D.6}$$

$$Ar2(n,i+1)=Ar2(n,i)+\Delta\eta[f(n,i+1)+f(n,i)]/2 \ , \tag{D.7}$$

note that $Ar1(n,0)=0$ and $Ar2(n,\eta_0)=0$.

2.Spherical geometry

For $\eta<\eta_0$,

$$5\delta_0^2(n,i)-\delta_0^3(n,i)=20[3-\delta_0(n,i)]Ar1(n,i)/(3Bi.f(n,i)) \ , \tag{D.8}$$

$$3\Psi_s^2(n,i+1)[5f(n,i+1)-2\Psi_s(n,i+1)]=20[\Psi_s(n,i+1)-f(n,i+1)]^2 .Ar1(n,i+1)/(3Bi) \ . \tag{D.9}$$

For $\eta\geq\eta_0$,

$$\Psi_s(n,i+1)=-f(n,i+1)/5-Ar1(n,i+1)/Bi \ , \tag{D.10}$$

where

$$f(n,i)=Bi[\Psi_s(n,i)-F(n,i)] \ , \tag{D.11}$$

$$Ar1(n,i+1)=Ar1(n,i)+\Delta\eta[f(n,i+1)+f(n,i)]/2 \ . \tag{D.12}$$

D.2 Method of solution

D.2.1 Fixed bed

The analysis begins by first computing $\delta_0(n,i)$ at each step point.If $\delta_0(n,i)<1$,then equations (D.3 or D.9) are used to compute the solid temperature.If however,$\delta_0(n,i)\geq 1$,then equations (D.4) or(D.10) are used to compute the solid temperature.

There is a step change in the fluid temperature at the entrance to the regenerator (ie.$F(0,i)=F(0,i+1)$) .Thus essentially there is only one unknown temperature which is $\Psi_s(n,i+1)$.This can be computed by solving equations (D.3) or (D.9) .At any other point the unknown temperatures can be computed by solving the two simultaneous equations representing the fluid phase and solid phase ,these are

For $\eta<\eta_0$,

Equations (D.1) and (D.3) or (D.9) should be solved to compute the unknowns $\Psi_s(n,i+1)$ and $F(n,i+1)$.This is done by substituting for $F(n,i+1)$ from (D.1) in (D.3) or (D.9) and solving for $\Psi_s(n,i+1)$.It should be clear that $\Psi_s(n,i+1)$ lies within the limit

$$0\leq\Psi_s(n,i+1)\leq 1 \qquad \text{(D.13)}$$

For $\eta \geq \eta_0$,

The linear equations (D.1) and (D.4) or (D.10) are gathered together in a matrix form

$$U[F(n,i+1),\Psi_s(n,i+1)]=H \ , \tag{D.14}$$

where U and H are the following matrices.

1.For planar geometry

$$U=\begin{bmatrix} 1+\Delta\xi/2 & -\Delta\xi \\ -Bi/3-\Delta\eta/2 & 1+Bi/3+\Delta\eta/2 \end{bmatrix} . \tag{D.15.a}$$

$$H=\begin{bmatrix} \Delta\xi(\Psi_s(n-1,i+1))/2+(1-\Delta\xi/2)F(n-1,i+1) \\ Ar2(n,i)/Bi+\Delta\eta[F(n,i)-\Psi_s(n,i)]/2 \end{bmatrix} . \tag{D.15.b}$$

2.For spherical geometry

$$U=\begin{bmatrix} 1+\Delta\xi/2 & -\Delta\xi/2 \\ \\ -Bi/5-\Delta\eta/2 & 1+Bi/5+\Delta\eta/2 \end{bmatrix}. \qquad \text{(D.16.a)}$$

$$H=\begin{bmatrix} \Delta\xi(\Psi_s(n-1,i+1))/2+(1-\Delta\xi/2)F(n-1,i+1) \\ \\ -Ar1(n,i)/Bi+\Delta\eta[F(n,i)-\Psi_s(n,i)]/2 \end{bmatrix}. \qquad \text{(D.16.b)}$$

The solution is merely the inversion of (D.14),that is

$$[F(n,i+1),\Psi_s(n,i+1)]=U^{-1}.H \ . \qquad \text{(D.17)}$$

The starting values for the iteration are

$$F(0,i)=1 \quad \text{For } i\geq 0 \ , \qquad \text{(D.18.a)}$$

$$\delta_0(n,0)=0 \quad \text{For } n\geq 0 \ , \qquad \text{(D.18.b)}$$

$$\Psi_s(n,0)=0 \quad \text{For } n\geq 0 \ . \qquad \text{(D.18.c)}$$

The initial fluid temperature at any position other than entrance is computed from

$$\partial\Psi/\partial z=-f(\eta)\ , \tag{D.19.a}$$

but $\Psi(n,0)=0$ for all n, so

$$F(n,0)=0 \quad \text{for } n>0\ . \tag{D.19.b}$$

D.2.2 Moving bed

The fluid phase need to be altered for this case,since ξ has to be measured from the solid entrance.Also for moving bed regenerators $\Delta\eta$ is expressed in terms of $\Delta\xi$. Consequently, the unknowns are $F(n+1)$ and $\Psi_s(n+1)$. The equations will be identical to the previous case (fixed bed), if $\Delta\xi$ is replaced by $-\Delta\xi$.

The solid and fluid inlet temperatures are known.In order to start the iteration ,one need to know the fluid outlet temperature. This is initially approximated as the mean solid and fluid inlet temperatures,that is

$$F(0)=[F(\Lambda)+\Psi_s(0)]/2\ . \tag{D.22}$$

The fluid and solid temperatures at each step point are then computed using the procedure outlined for the fixed bed.The calculated fluid inlet temperature is then compared with the actual given temperature.If there is any discrepency, the initial approximation is readjusted and the procedure is then repeated until the two values coincide.

The U and H matrices are identical to the previous case except that

$$\Delta\xi_{moving} = -\Delta\xi_{fixed} \,,$$

also the problem is one dimensional, that is the unknowns are F(n+1) and $\Psi_s(n+1)$.

APPENDIX E :THE COMPUTER PROGRAM

Three programs ;fixedplan ,fixedsph and moving were written for handling the thermal design of fixed and moving bed regenerators respectively. The intraconduction effect was included in all the three programs. The programs were written in Fortran language and were run on Amdahl 470.

The first two programs were written for the purpose of examining the validity of the integral method.This was achieved by comparing the analytical results obtained (using the integral method) with the published results (using numerical methods).The third program was written in order to obtain a set of charts for moving bed regenerators.

The input data for the first two programs included the parameters required to calculate the reduced length and dimensionless period for the fixed bed.However,for a moving bed regenerator it was only neccessary to compute the reduced length.

A suitable time and distance increments were chosen(0.2 for both).The fluid and solid surface temperatures at each step point were then computed. For a moving bed regenerator the fluid and solid surface temperatures at each length increment along the whole length were obtained .

Two external subroutines were used in all three programs,namely

1. **Zerol**

This external subroutine was used to solve the two non-linear equations for the penetration depth and solid surface temperature, when $\delta_0<1$.

2. **SLE**

This external subroutine was used to solve the two linear simultaneous equations for the fluid and solid surface temperatures.The subroutine uses the matrix inversion technique to solve the equations

The list of all three programs are included at the end of this section.

```
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 LEN,LENU
      REAL*4 RELEU
C
C
C     THIS IS A PROGRAM TAKING ACCOUNT OF,
C     INTRAPARTICLE CONDUCTION FOR SPHERICAL COORDINATE
C
C     THE NAME OF THIS PROGRAM IS REGE
C
      DIMENSION  TGU(100),TSU(100),HTU(100),TSM(100)
      DIMENSION A1U(5,5),B1U(5),TU(5,5),X1U(5),IPERM(10)
      DIMENSION ARU(100),DIFF(100)
      DIMENSION PENEU(100),EFF(100)
      COMMON /UPPER/ DTU,BIU
      COMMON /UPER1/ DIS,RL2,RL3
      COMMON /DELTA/ TG1,TG2,TS1,TS2,AR1
      COMMON /SOLTE2/TG6,TS5,AR3
C
C
C     COMMON BLOCK DELTA DEFINES THE VARIABLES USED IN SUBROUTINE FN
C     IN WHICH THE PENETRATION DEPTH IS CALCULATED.
C
C
C     COMMON BLOCK /SOLTE2/ DEFFINES THE VARIABLES USED IN SUBROUTINE
C     FCN IN WHICH TS(N,I+1),TG(N,I+1) ARE CALCULATED.
C
C
C
C
C
C     HT=HEAT TRANSFER       AR=AREA UNDER CURVE HT VS TIME
C     ARE=AREA UNDER THE CURVE F*PENETRATION VS TIME
C     X=SOLUTION OF NON-LINEAR EQUATION
C     X1=SOLUTION OF LINEAR EQUATION SOLVED BY MATRIX INVERSION
C     A1=MATRIX OF COEFFICIENTS
C     B1=MATRIX OF RIGHT HAND SIDE OF EQUATION A.X=B
C     DT=INCREMENT OF TIME
C     CONST=F(TIME) WHEN T=T0
C     PENE=PENETRATION DEPTH
C
      READ(5,500)SOLK,DENSOL,CPSOL,DIASOL,PORO,SOLFLO
  500 FORMAT(E7.2,F7.1,F4.1,E8.1,F4.1,F4.2)
      READ(5,5100)GAMTU,LENU,BEDAU,VOLFU
 5100  FORMAT(F6.1,F5.2,F4.1,F5.2)
      CPGU=0.917+(2.58E-4)*GAMTU-(3.98E-8)*GAMTU**2
      VISGU=1.46E-6*GAMTU**1.5/(110.+GAMTU)
      DENGU=353./GAMTU
      GASKU=6.16E-7*GAMTU**0.68
      VKIGU=DENGU/VISGU
      DENU=DENSOL/DENGU-1.
      GASFU=VOLFU*DENGU
       VTERU=SOLFLO/(DENSOL*(1.-PORO)*BEDAU)
       UGASU=GASFU/(DENGU*BEDAU)
      REYNU=UGASU*DIASOL*VKIGU
      PRANU=VISGU*CPGU/GASKU
      VOID=3.25-2.25*PORO
      IF(REYNU.GE.100.) GO TO 8000
```

```
      Y4U=200.
      EXTERNAL FN2U
      LOGICAL LZ2U
C
C
C
C      X(1)ANDX(2) ARE THE INITIAL GUESSES FOR TG(N,I+1)
C       AND TS(N,I+1).
C
C
      TG6=TGU(NU)
      TS5=TSU(NU)
      AR3=ARU(NU)
C
C       THE VALUES OF TG(N,I+1),TS(N,I+1) ARE NOW CALCULATED
C
C
      CALL ZERO1(X4U,Y4U,FN2U,E2,LZ2U)
      TGU(NU+1)=RL2*TG6/RL3-DIS*(X4U+TS5)/(2.*RL3)
      TSU(NU+1)=X4U
      ARU(NU+1)=ARU(NU)-(TGU(NU+1)+TGU(NU)-TSU(NU+1)-TSU(NU))*RLU
      HTU(NU)=BIU*(TGU(NU)-TSU(NU))
      EXTERNAL FNU
      LOGICAL LZU
      E1=5.E-7
      X2U=0.
      Y2U=2.
      TG1=TGU(NU)
      TG2=TGU(NU+1)
      TS1=TSU(NU+1)
      TS2=TSU(NU)
      AR1=ARU(NU)
C
C
C     THE VALUE OF PENETRATION DEPTH IS NOW CALCULATED
C
C
      CALL ZERO1(X2U,Y2U,FNU,E1,LZU)
      PENEU(NU+1)=X2U
      RL5=(5.*PENEU(NU)**2-PENEU(NU)**3)/(3.-PENEU(NU))
      TSM(NU)=-3.*BIU*(TSU(NU)-TGU(NU))*RL5/20.
      WRITE(8,2560)PENEU(NU+1),TSU(NU+1),TGU(NU+1),DTU,BIU,TGU(2)
 2560 FORMAT(6F12.4)
      IF (NU.NE.2) GO TO 2570
      DIFF(NU)=0.
      GO TO 2580
 2570 DIFF(NU)=100.*(TSU(NU)-TSM(NU))/TSU(NU)
 2580 EFF(NU)=R*(TSM(NU)-TSU(2))/(TGU(NU)-TSU(2))
      GO TO 901
C
C
C
C     IF T>=T0 USE MATRIX INVERSION TO SOLVE 2 LINEAR EQUATIONS
C     AT THIS STAGE PENETRATION DEPTH IS EQAL TO THICKNESS OF PLAT
C
C
 2590 PENEU(NU+1)=PENEU(NU)
```

```
      NDIMA=5
      NDIMT=5
      NDIMBX=5
      NN1=2
      NSOL=1
C
C
C
C     NN1=NO.OF EQUATIONS, A1=MATRIX OF COEFFICIENTS
C     B1=MATRIX OF KNOWN RIGHT HAND SIDE  A*X=B
C
C
C
      A1U(1,1)=RL3
      A1U(1,2)=DIS/2.
      A1U(2,1)=-BIU/5.-DTU/2.
      A1U(2,2)=1.+BIU/5.+DTU/2.
      B1U(1)=RL2*TGU(NU)-DIS*TSU(NU)/2.
      B0=DTU*TSU(NU)/2.
      B1U(2)=-ARU(NU)/BIU+DTU*TGU(NU)/2.-B0
      CALL SLE(NN1,NDIMA,A1U,NSOL,NDIMBX,B1U,X1U,
     @ IPERM,NDIMT,TU,DET,JEXB)
      TGU(NU+1)=X1U(1)
      TSU(NU+1)=X1U(2)
      ARU(NU+1)=ARU(NU)-RLU*(TGU(NU)+TGU(NU+1)-TSU(NU+1)-TSU(NU))
      HTU(NU)=BIU*(TGU(NU)-TSU(NU))
      TSM(NU)=-ARU(NU)/BIU
      EFF(NU)=R*(TSM(NU)-TSU(2))/(TGU(NU)-TSU(2))
      DIFF(NU)=100.*(TSU(NU)-TSM(NU))/TSU(NU)
 901  MU=NU-2
 1000 CONTINUE
      DTEMP=TGU(NU)-(TGINU-TSINU)
      IF(DABS(DTEMP).GE.0.1) GO TO 1001
      DO 1050 NU=2,NNU
      MU=NU-2

      IF(MU.GT.0)GO TO 1601
      DMU=DIS*MU
      GO TO 1602
 1601 DMU=DMU+DIS
 1602 WRITE(6,3301)DMU,EFF(NU)
 3301 FORMAT(F4.2,F6.3)
      WRITE(7,3302)DMU,TSU(NU)
 3302 FORMAT(F4.2,F8.3)
 1050   CONTINUE
     GO TO 999
 1001 IF(DTEMP.GT.0.) GO TO 1002
     TGU(2)=TGU(2)+DABS(DTEMP/4.)
     GO TO 1
 1002 TGU(2)=TGU(2)-DABS(DTEMP/4.)
     GO TO 1
 999  STOP
    END
    FUNCTION FNU(X)
C                 PENETRATION DEPTH IS CALCULATED IN THIS FUNCTION
C
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON /UPPER/ DTU,BIU
```

```
      COMMON /DELTA/ TG1,TG2,TS1,TS2,AR1
      P2=TS1-TG2
      P1=BIU*DTU*(TS1+TS2-TG1-TG2)/2.+AR1
      FNU=(5.*X**2-X**3)*P2*BIU-20.*(3.-X)*P1/(3.*BIU)
      RETURN
      END
      FUNCTION FN2U(X)
C
C             TS(N,I+1) AND TG(N,I+1) ARE CALCULATED IN THIS SUBROUTIN
C             USING THE NON LINEAR EQUATION FOR TS
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /UPPER/ DTU,BIU
      COMMON /UPER1/ DIS,RL2,RL3
      COMMON /SOLTE2/ TG6,TS5,AR3
      X1=RL2*TG6/RL3-DIS*(X+TS5)/(RL3*2.)
      P4=5.*BIU*(X-X1)-2.*X
      P2=X-BIU*(X-X1)
      P1=AR3+BIU*DTU*(X-X1+TS5-TG6)/2.
      IF(P2.GT.0) GO TO 10
      FN2U=3.*X**2*P4-20.*(-P2)**2*P1/(3.*BIU)
      GO TO 20
 10   FN2U=3.*X**2*P4-20.*P2**2*P1/(3.*BIU)
 20   RETURN
      END
```