A STUDY OF B-SPLINES AND THEIR APPLICATIONS TO SURFACE DESIGN AND MANUFACTURE

by

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1982

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

in

THE FACULTY OF GRADUATE STUDIES

Department of Mechanical Engineering

We accept this thesis as conforming

to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

MARCH, 1984

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Date: March 7, 1984.

ABSTRACT

Compound curved surfaces are frequently encountered in engineering applications, such as marine propellers, turbines, ship hulls, aeroplane fuselages and automobile bodies. In the manufacture of such items the surface definition is required in a smoothly changing form without surface oscillations and irregularities. Frequently, however, the surfaces are defined by measured prototype data, which may be sparse in nature and have attendant measurement errors. In such instances the data has to be approximated and possibly smoothed by a series of curves of some known form. There are a number of types of curves available for curve fitting in CAD/CAM work.

In this work the basis and properties of B-spline curve fitting are established and evaluated in relation to other curve fitting techniques. A series of trial shapes, or bench marks, were devised which showed that B-splines generated the smoothest curves and are very suitable for computer-aided-design applications.

B-splines, with other types of spline curves, form the basis of a new computer-aided-machining program called G-SURF. The surface is defined within G-SURF as a grid of orthogonal space curves and is machined using an end milling cutter inclined at a predetermined angle to the surface normals. The end mill thus cuts on the leading or trailing edge (as appropriate) and is able to remove material very efficiently at a full and preselected material cutting speed. Within this work G-SURF has been used to design and produce a series of trial objects, including a 3ft. ship hull and a chain-saw guide bar. They have served to prove and test the G-SURF program.

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ACKNOWLEDGEMENTS

The author wishes to thank Dr. G.W. Vickers for his unending words of encouragement and for the guidance given throughout this work. He is also thankful to his colleagues for their valuable suggestions and criticism.

Finally, to his parents and sister, the author owes his deep gratitude for their unfailing encouragement throughout the work.

The investigation reported herein was supported by the Natural Sciences and Engineering Research Council of Canada.

CHAPTER 1

INTRODUCTION TO B-SPLINE CURVE FITTING AND G-SURF

1.1 Introduction

Compound curved surfaces are frequently encountered in engineering applications[1,2]. The performance and appearance of ship hulls, aeroplane fuselages , marine propellors, and automobile bodies, etc. depend largely on the smoothness of the surface[3].

In most of the cases, a prototype is first made and tested and the surface is subsequently measured from the prototype. In the case of marine propellors the surface is stored as aerofoil sections at different radius ratios, while ship hulls are defined at various sections called water-lines, station-lines and buttock-lines[3]. The surface data, however generated, is in essence a finite series of points. The points do not define the complete surface, they may be sparse in nature and frequently have measurement errors. Surface oscillations or irregularities are usually not acceptable as they become apparent in the finished product. A hood or fender of an automobile should "look" smooth when it is complete[2].

Doubly curved surfaces present a challenge in manufacturing as they cannot be easily machined by plane or cylindrical generators. However, if they can be defined as a series of curves smoothly translating through or among the given data points, the surface can be reproduced on a CNC machine.

1.2 G-SURF

G-SURF is a CAD-CAM package designed and developed at UBC for designing and manufacturing doubly curved surfaces of essentially low to moderate curvature[4]. G-SURF approximates a surface as a series of orthogonal space curves. The number and spacing of the curves determines the adequacy of the surface definition.

The orthogonal space curves used to define the surface may be B-spline or other types of spline curves. The orientation of the local or component coordinate reference frame across the surface is given in Figure 1.1. The tangent of the spline curve determines the local Y-axis while the tangent to the orthogonal spline curve determines the local X-axis.

As shown in Figures 1.2 and 1.3 the surface is machined with an end milling cutter inclined at a predetermined angle to the surface normal in the oscullating plane (i.e. the tangential plane to the curve along which the tool is travelling). The end mill thus cuts on the trailing or leading edge and operates at full cutting speed at all times during the machining operation. The greater the inclination of the end mill to the surface normal the smaller is the effective radius of cutting and the less likely is the cutter to remove essential surface material. Curvature and interference checks can be made within the program and cutter inclination adjusted accordingly.

G-SURF is designed for use on a mini-computer and is thus suitable for small CNC manufacturers with modest computing facilities. The output is applicable to 2-1/2, 3, 4 or 5 axis CNC machines. The 4 and 5 axis CNC machines are more flexible and enable a wider range of work to be completed.

1.3 Curve Fitting

A number of curve fitting methods can be used within G-SURF to define the orthogonal surface grids. The two basic choices when fitting a curve to discrete data points are interpolation or approximation. The former passes a curve through all the data points, however much in error they may be, while the latter passes a curve among the points. Approximation generally gives a smoother curve for CAD/CAM applications but where no adjustment can be made to the position of the specified data interpolation becomes a necessity[5].

The common types of curve fitting with their attendant characteristics are given below:

1.3.1 Least-mean-square curve fitting

The least-mean-square method of curve fitting minimizes the mean-square deviations of a polynomial function[6]. This fit is appropriate when the error in the data points have a normal distribution. Even one "wild" point can distort a least-mean-square fit to the extent that it is unacceptable. Least-mean-square fits frequently generate oscillations in the curve as shown in Figure 1.4. Wild points, sharp slopes or a scarcity of points at the end of the curve can start fluctuations, which make this fit unsuitable for CAD/CAM purposes.

1.3.2 n^{th} order polynomial fit

A polynomial can be made to pass through a given set of data points. The difficulty is that the order of the polynomial is determined by the number of points. Higher order polynomials frequently generate oscillations as shown in Figure 1.5 and are not suited for surface design in CAD/CAM applications[2].

1.3.3 Conic splines*

Splines are piecewise combinations of particular curve types which can be made to match in position and slope at the junction of each section[7]. They are the mathematical equivalent of the mechanical splines, or flexible elastic strips, used by draftsmen to smooth curves[8].

The general equation of a conic is given as:

$$ax^2 + bxy + cy^2 + dx + ey + j = 0 \quad .$$

This represents all two-dimensional sections through a right circular cone (i.e., line, circle, ellipse, parabola, hyperbola — all single curvature curves). With conic splines this equation can be used to fit the first three data points and then to fit the next three data points with position and slope continuity at each junction. Conic splines do not give curvature continuity at junctions. Changing any data point changes all the fitted equations. A worked example of a conic spline is given in Appendix I and summarised in Figure 1.6.

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* The conic equations are applied to advantage in CURVFIT[2] to span a set of closely spaced digitised points with widely spaced 'control points' at which position and slope are chosen and specified. Inflexion points are identified as additional control points. The method is capable of producing either approximation or interpolation fits.

1.3.4 Cubic spline curve fitting

These are a set of cubic polynomial equations of the form:[2]

$$ax^3 + bx^2 + cx + d = y \quad .$$

This equation is more flexible than the conic equation in that it generates double curvature. In this case the equation can be used to fit the first four data points and then to fit each additional data point with position, slope and curvature continuity at each junction. A worked example of the cubic spline curve fitting is given in Appendix II and the results shown in Figure 1.7. While any degree of polynomial curves can be used the cubic is most widely accepted in CAD/CAM applications. It is of sufficiently high order to give curvature continuity and yet low enough to prevent unnecessary oscillations.

As with conic splines all equations for a particular curve are altered if any data is changed. Also large vertical slopes create computational difficulties.

1.3.5 B-Splines curve fitting

B-splines are linear combinations of basis functions formed from part of the power series given by $y = (x-\varepsilon)^k$ where x is the independent variable, y the dependent variable , ε is any constant and k the degree of the polynomial. B-splines have the property of generating the least curvature (i.e. smoothest) of all curves, produce no oscillations and, are variationally diminishing (i.e. degenerate to the least power required to fit the data — a straight line is fit by an equation of a straight line)[5, 10, 11]. B-splines may be of any order although cubic B-splines are used within G-SURF. These span four intervals and alteration of any data points effects only the neighbouring four spans. The definition and formulation of B-splines are given in Chapter 2 of this work.

1.4 Objectives

The object of the present work is to examine the basis of B-spline curve fitting and to evaluate it in relation to other curve fitting techniques. A series of trial shapes, or bench marks, were devised for making these comparisons. In addition, it is required that the capabilities of G-SURF be investigated and developed by machining a series of test surfaces of increasing complexity. The applicability of the approach to local industrial problems is illustrated by two recent projects. One is the generation of smooth data and CNC tapes required in the manufacture of chain saw guide bars. The other is in lofting and machining a three-foot model ship hull.

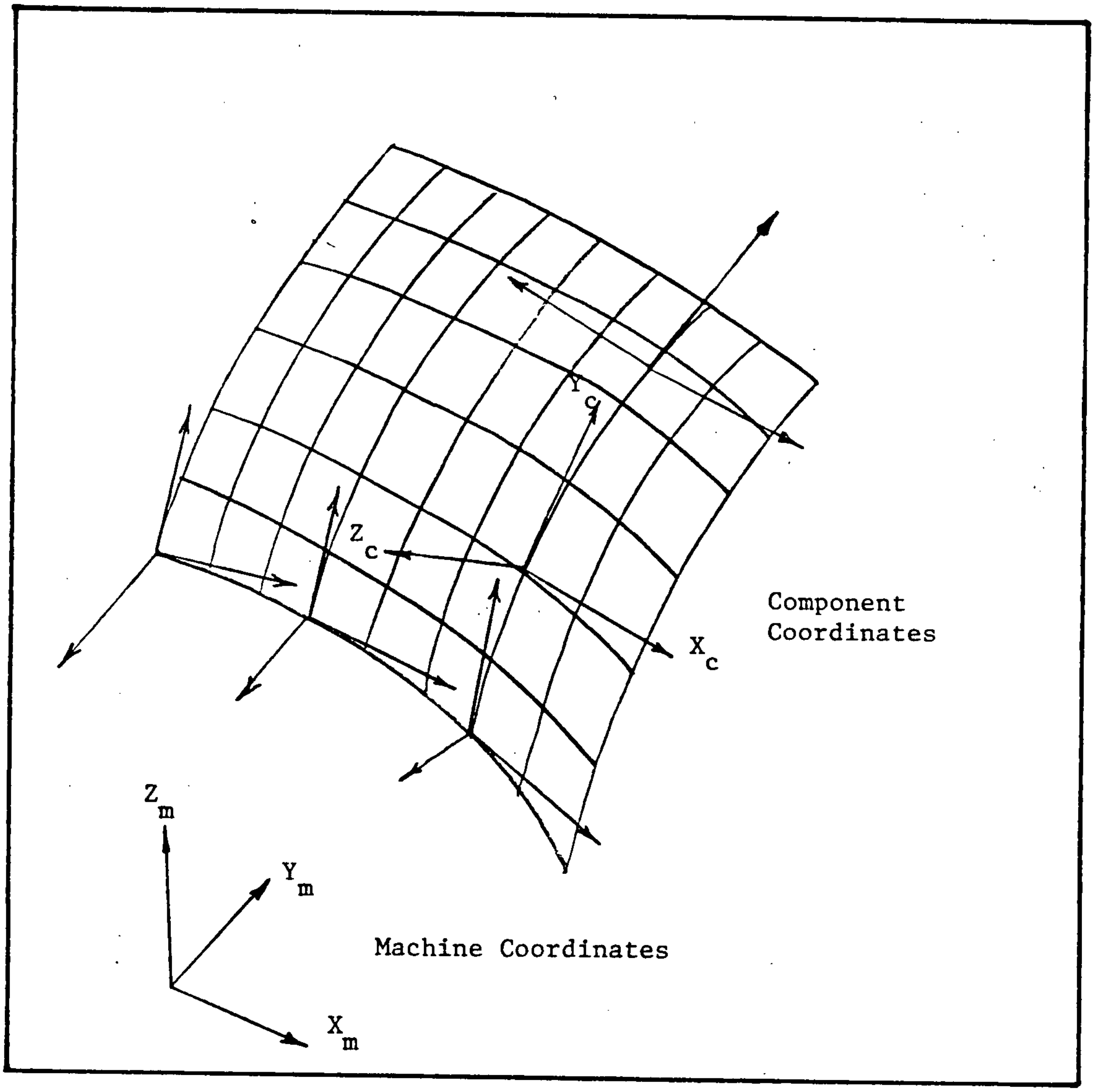


FIG. 1.1 Surface approximated by a set of curves.

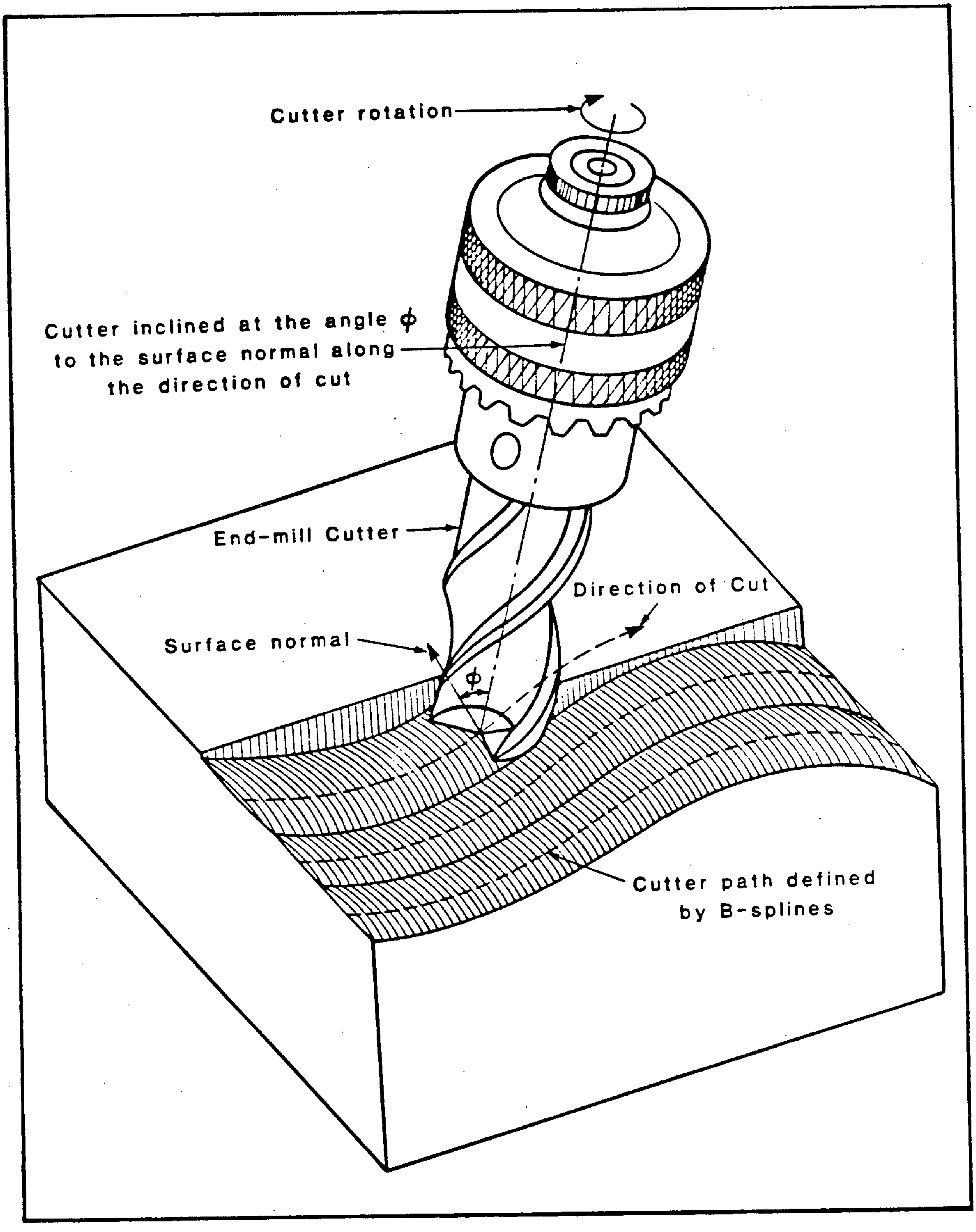
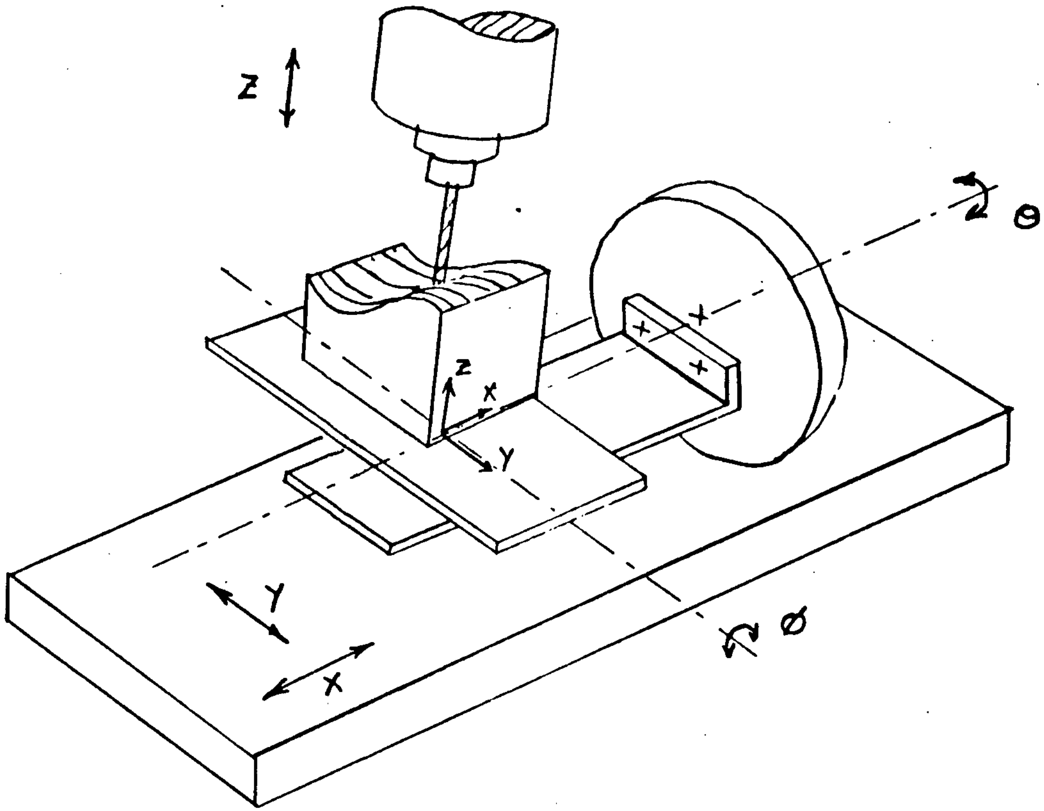


FIG. 1.2 G-SURF machining technique.



Rotary tables were designed by Dr.Shi-Gang Wang & Dr.J.P.Duncan.

FIG. 1.3 A schematic of a 5-axis C.N.C. machine.

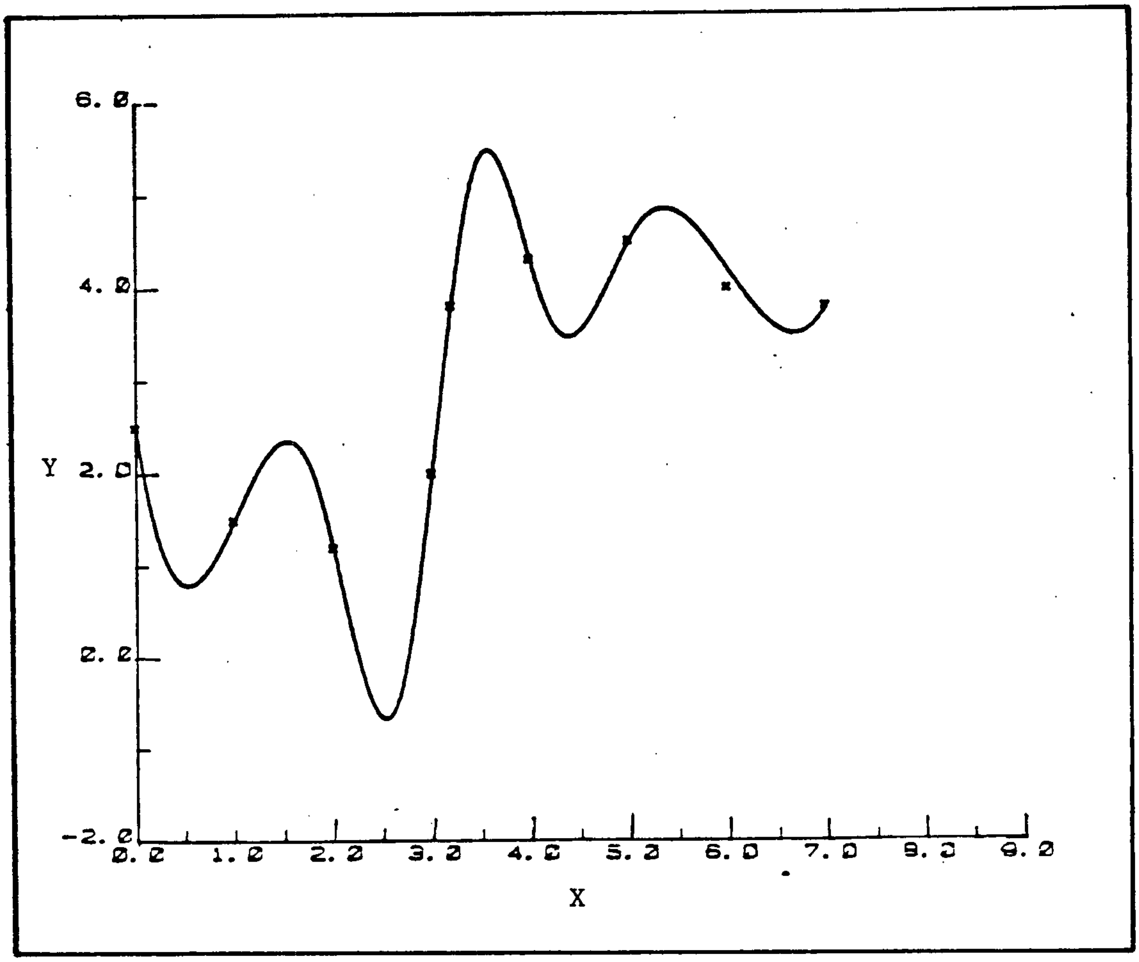


FIG.1.4 Curve fitting using Least-mean-squares.

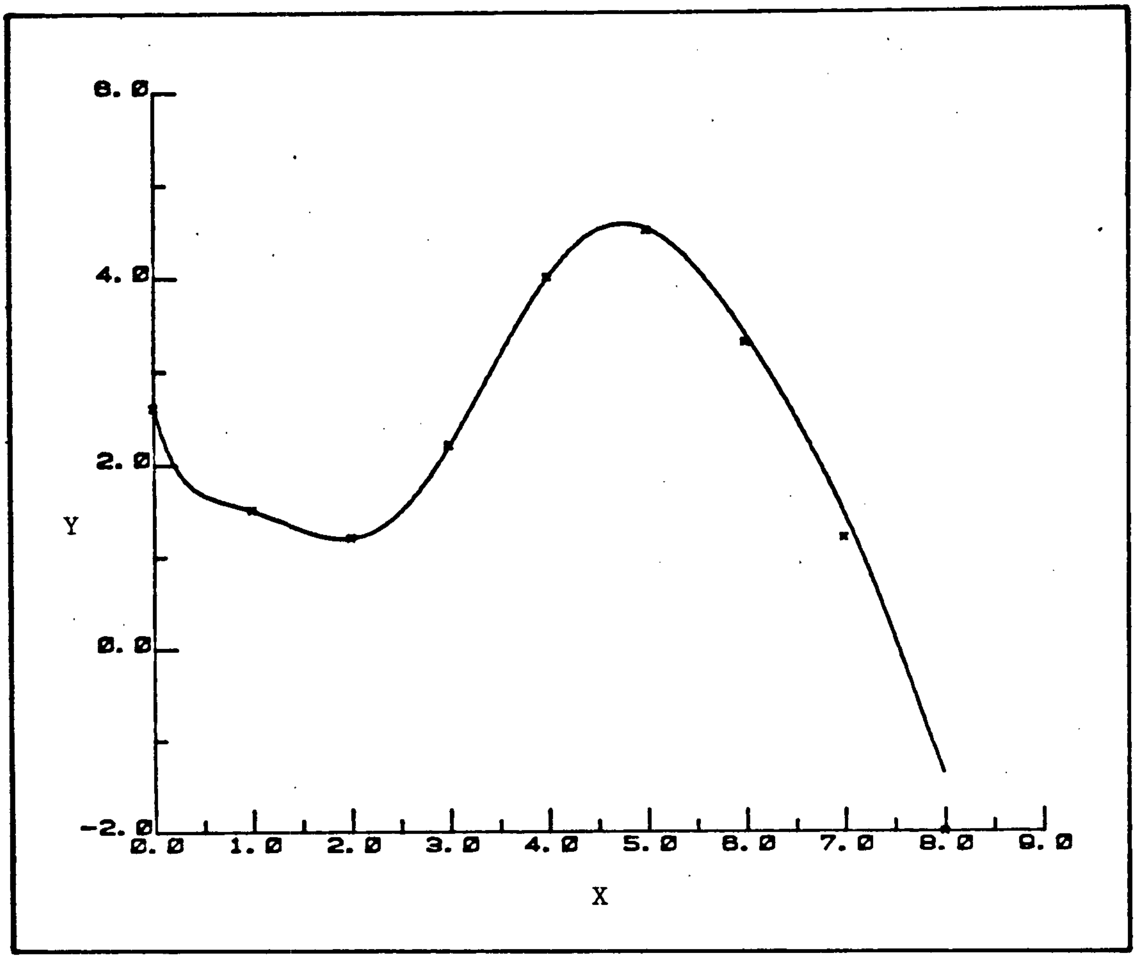


FIG. 1.5 Curve fitting using 9th order polynomial.

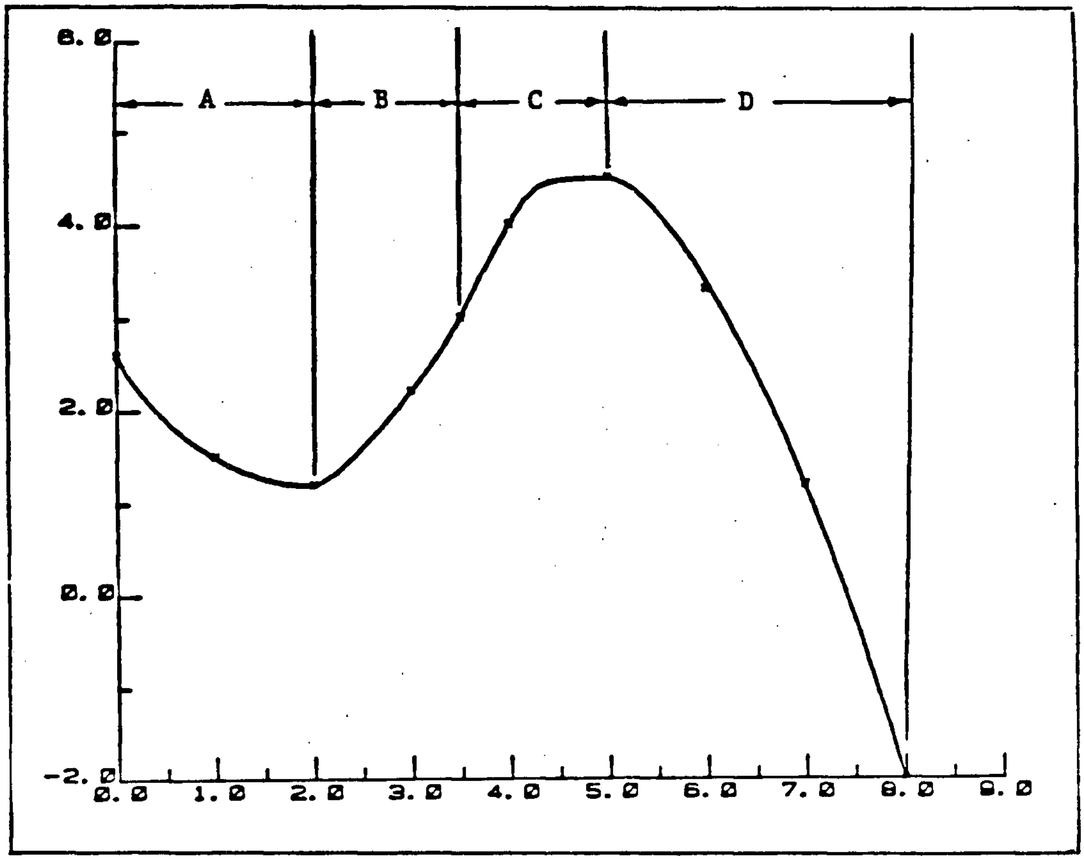


FIG. 1.6a Curve fitting using Conic splines, with inflection point at Y = 3.0.

Equations of curves in different regions are:-

Region A : $X^2 + 2.220*X*Y - 1.58*Y^2 - 1.333*X + 3.17*Y + 2.47 = 0.0$

Region B : $X^2 - 1.420*X*Y - 0.57*Y^2 - 2.29*X + 1.31*Y + 1.59 = 0.0$

Region C : $X^2 + 25.500*X*Y - 12.25*Y^2 - 124.75*X - 4.13*Y + 254.5 = 0.0$

Region D : $X^2 + 0.570*X*Y + 0.13*Y^2 - 12.60*X - 3.56*Y + 38.45 = 0.0$

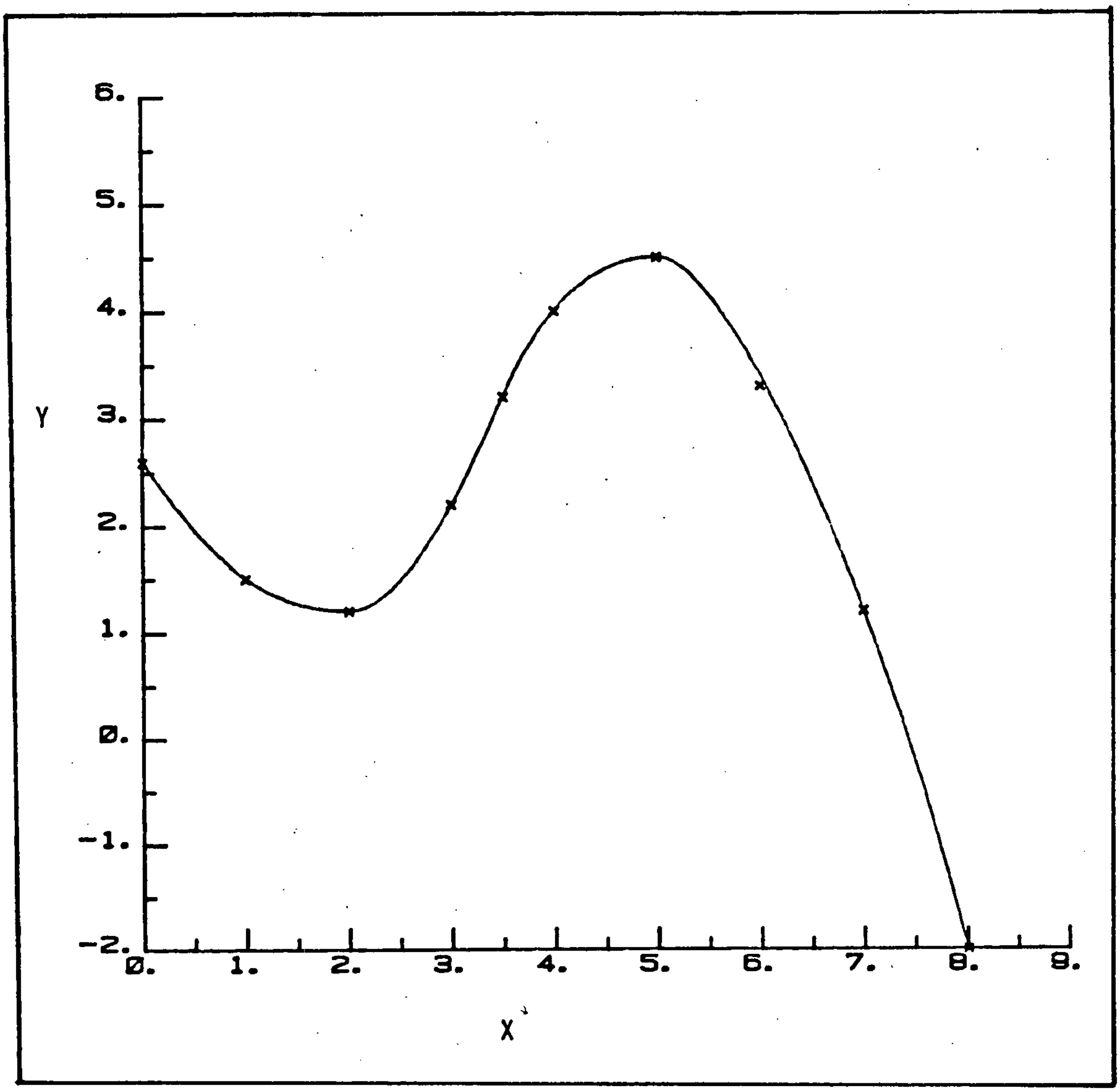


FIG. 1.6b Curve fitting using Conic splines with inflection point at Y = 3.2.

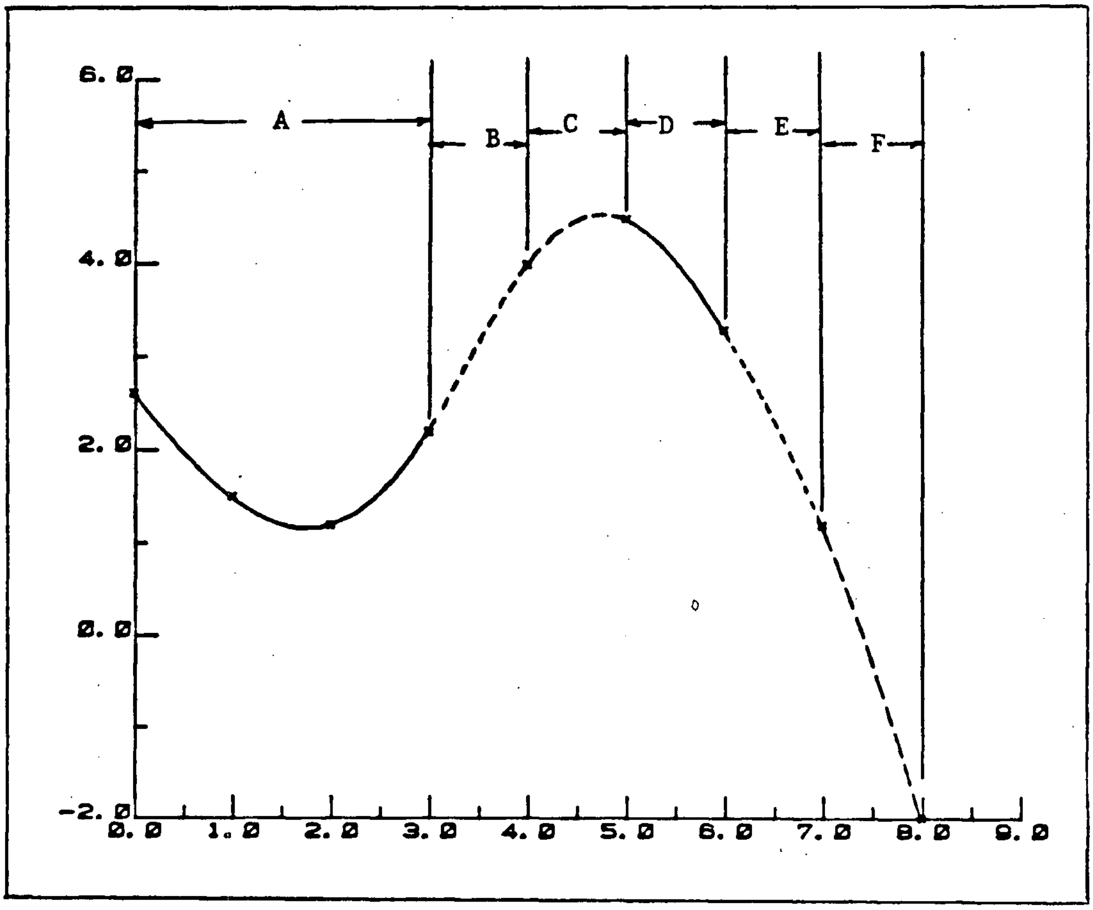


FIG. 1.7 Curve fit using Cubic splines.

Equations of curve in different regions are :-

<u>Region A</u> : $2.6 - 1.33*X + 0.15*X^2 + 0.08*X^3 = Y$

<u>Region B</u> : $29.6 - 23.33*X + 9.15*X^2 - 0.91*X^3 = Y$

<u>Region C</u> : $-124.2 + 87.02*X - 19.69*X^2 + 1.49*X^3 = Y$

<u>Region D</u> : $740.7 - 431.93*X + 84.10*X^2 - 5.43*X^3 = Y$

<u>Region E</u> : $-356.51 + 72.67*X + 3.66*X^2 - 0.96*X^3 = Y$

<u>Region F</u> : $-11353.3 + 4785.59*X - 669.6*X^2 + 31.1*X^3 = Y$

CHAPTER 2

B-SPLINE THEORY AND EXAMPLES

B-splines are defined[5] as linear combinations of basis functions formed from the truncated power series $y = (x-\varepsilon)^k$ where x is the independent variable, y the dependent variable, ε is any constant and k the degree of the polynomial. The k^{th} order basis functions are appropriately scaled k^{th} divided differences of the truncated power series.

Given a knot sequence $[t_1, t_n]$ a B-spline approximation to a set of data points is given by:

$$F(x) = \sum_{i=1}^{n} \alpha_i B_{i,j}(x) \qquad (1)$$

where $F(x)$ is the function value of the curve at x,

α_i is the coefficient of the i^{th} basis function,

$B_{i,j}$ is the value of the i^{th} basis function or order j.

An example of a B-spline fit and basis function is given in Figures 2.1 and 2.2.

2.1 Knots

For the cubic B-spline used in G-SURF the basis function consists of four cubic polynomial arcs stretching across five knots. Each is normalised and non-zero over four intervals and zero everywhere else[13]. At most, four basis functions of order four are non-zero at any point on a curve as shown in Figure 2.3 for the interval 3 to 4.

The point at which a basis function starts is called the knot point. These points are usually coincident with the data points. To incorporate end-slope control, additional knots are added at the end points of the curve[5]. The number of additional knots required at each end is (k-1) where k is the order of the B-spline.

A knot sequence may contain identical knots up to a multiplicity k. The effect of a knot occuring with a multiplicity n, i.e.

$$t_i = t_{i+1} = \cdots = t_{i+k-1} \tag{2}$$

to decrease the degree of differentiability of the basis function $B_{i,k}$ at x to $C^{(k-n-1)}$, i.e., if any two knots coincide the curve loses its differentiability by 1 at that point. The formation of a double knot may be viewed as being a coalescence of two adjacent knots as shown in Figure 2.4.

2.2 Basis Functions

Given a knot sequence $\tau_i = [t_i, \cdots, t_n]$ the basis function of order k (degree k-1) may be defined[5, 12] as

$$B_{i,k}(x) = \frac{x - t_i}{t_{i+k-1} - t_i} B_{i,k-1}(x) + \frac{t_{i+k} - x}{t_{i+k} - t_{i+1}} B_{i+1,k-1}(x) \tag{3}$$

where the initial conditions are defined as

$$B_{i,1}(x) = \begin{cases} 1 & \text{if } t_i \leq x < t_{i+1} \\ 0 & \text{otherwise} \end{cases} . \tag{4}$$

In order to get a better understanding of the basis function the following table is derived, using Equations (3) and (4), for a knot sequence $\tau = [t_1, \cdots, t_n]$ in the region $t_4 \leq x < t_5$.

k = 1	k = 2	k = 3	k = 4
$B_{1,1} = 0$			
	$B_{1,2} = 0$		
$B_{2,1} = 0$		$B_{1,3} = 0$	
	$B_{2,2} = 0$		$B_{1,4} = (1-x)^3$
$B_{3,1} = 0$		$B_{2,3} = (1-x)^2$	
	$B_{3,2} = (1-x)$		$B_{2,4} = \frac{x(2-x)}{2} \left\{(1-x) + \frac{2-x}{2}\right\}$
$B_{4,1} = 1$		$B_{3,3} = x(1-x) + \frac{x(2-x)}{2}$	
	$B_{4,2} = x$		$B_{3,4} = \frac{x^2}{2}\left\{(1-x) + \left(\frac{2-x}{2}\right)\right\} + (3-x)\frac{x^2}{6}$
$B_{5,1} = 0$		$B_{4,3} = \frac{x^2}{2}$	
	$B_{5,2} = 0$		$B_{4,4} = \frac{x^3}{6}$
$B_{6,1} = 0$		$B_{5,3} = 0$	
	$B_{6,2} = 0$		
$B_{7,1} = 0$			

Each column in the above table represents the basis functions of different orders.[12]

(i) The first column contains equations of basis functions of order one. The basis function is given in Figure 2.5. If $B_{j,1}$ is used to fit a set of data points the resulting fit will be a histogram.

(ii) The second column gives equations of polynomial pieces representing the basis function of order two between t(4) & t(5). The basis function is given in Figure 2.6. If a curve is fit using $B_{i,2}$, the output will be a linear interpolation with only point continuity.

(iii) The third column contains the equation of polynomial pieces forming basis function of order three in the interval t(4) to t(5). The basis polynomials are shown in Figure 2.7. Curve fitted using $B_{i,3}$ is continuous in point and slope.

(iv) The fourth column contains equations of polynomial pieces of order four in the range t(4) to t(5). The basis polynomials are given in Figure 2.8. The curve obtained using $B_{i,4}$ is continuous in point, slope and curvature. This is the order of B-splines used in the G-SURF program.

A worked example of B-spline curve fitting is given in Appendix III. For computational purposes a recurrence algorithm developed by Carl de Boor[5] is used to evaluate the basis function.

2.3 <u>Nodes</u>

Once the basis functions have been defined, the B-spline <u>approximation</u> of order k to any arbitrary function[10], or set of data, may be stated as:

$$F(x) = \sum_i \alpha(\varepsilon_i)\, B_{i,k}(x) \tag{4}$$

where

$$\varepsilon_i = \frac{1}{(k-1)} \left(t_{i+1} + t_{i+2} + \cdots + t_{i+k-1} \right) . \tag{5}$$

ε_i's are called the nodes and the value of the function at the nodes gives us the coefficient of B-spline basis functions. This is shown in detail in Appendix III.

In B-spline <u>interpolation</u>, however, the procedure to determine α_i's is slightly different. In matrix form B-splines basis function may be defined as:

$$\{F\} = [B]\{\alpha\}$$

or (6)

$$\{\alpha\} = [B]^{-1}\{F\} \quad .$$

Knowing both the matrices on the right hand side, the matrix $\{\alpha\}$ can be easily determined. There are other features about $[B]$ and $\{F\}$ which make the calculation of $[\alpha]$'s very easy. $[B]$ is a banded matrix and can be inverted without pivoting by Gauss's method[5]. This approach is used within (BSPIN) for B-spline interpolation.

2.4 Geometric Interpretation of B-Spline Approximation

B-spline approximation (BSAPP) can be fitted to a set of data points without calculating the basis function. Carl de Boor has proved that

$$F_{i,k}(x) = \begin{cases} F_i & \text{(for } k = 0) \\ \lambda F_{i,k-1} + (1-\lambda)F_{i-1,k-1} & \text{(for } k > 0) \end{cases} \qquad (7)$$

$$\lambda = \frac{x - t_i}{t_{i-k+M} - t_i} \quad .$$

where

M is order of B-spline, $k = M-1$,

F_i is the value of the i th data point,

$F_{i,k}(x)$ is the value of B-spline fit of degree k at the point x.

Consider a knot sequence $\tau = [t_0 \; .... \; t_{14}] = [111123456789999]$.

The value of the B-spline is to be found for $x = 7.6$. Since

$t_9 < 7.6 < t_{10}$, $i = 9$.

Using the recursive relation, given in equation 7, we get

$$F_{9,3}(7.6) = \lambda\, F_{9,2}(7.6) + (1-\lambda)\, F_{8,2}(7.6)$$

$$\lambda = \frac{(7.6 - 7)}{(1)} = 0.6$$

$$F_{9,2}(7.6) = \lambda\, F_{9,1}(7.6) + (1-\lambda)\, F_{8,1}(7.6)$$
$$\lambda = \frac{(7.6 - 7.0)}{(4-2)} = 0.3$$

$$F_{8,2}(7.6) = \lambda\, F_{8,1}(7.6) + (1-\lambda)\, F_{7,1}(7.6)$$
$$\lambda = \frac{(7.6 - 6.0)}{(4-2)} = 0.3$$

$$F_{9,1}(7.6) = \lambda\, F_9 + (1-\lambda)\, F_8$$
$$\lambda = \frac{(7.6 - 7.0)}{(4-1)} = 0.2$$

$$F_{8,1}(7.6) = \lambda\, F_8 + (1-\lambda)\, F_7$$
$$\lambda = \frac{(7.6 - 6.0)}{(4-1)} = 0.53$$

$$F_{7,1}(7.6) = \lambda\, F_7 + (1-\lambda)\, F_6$$
$$\lambda = \frac{(7.6 - 5.0)}{(4-1)} = 0.87$$

The value of the data points F_6, F_7, F_8, F_9 can be put into the above relations and the value of B-spline determined. This is shown as a graphical solution in Figure 2.9. A geometerical fit to the S-curve at the point x = 5.5 is shown in Figure 2.10.

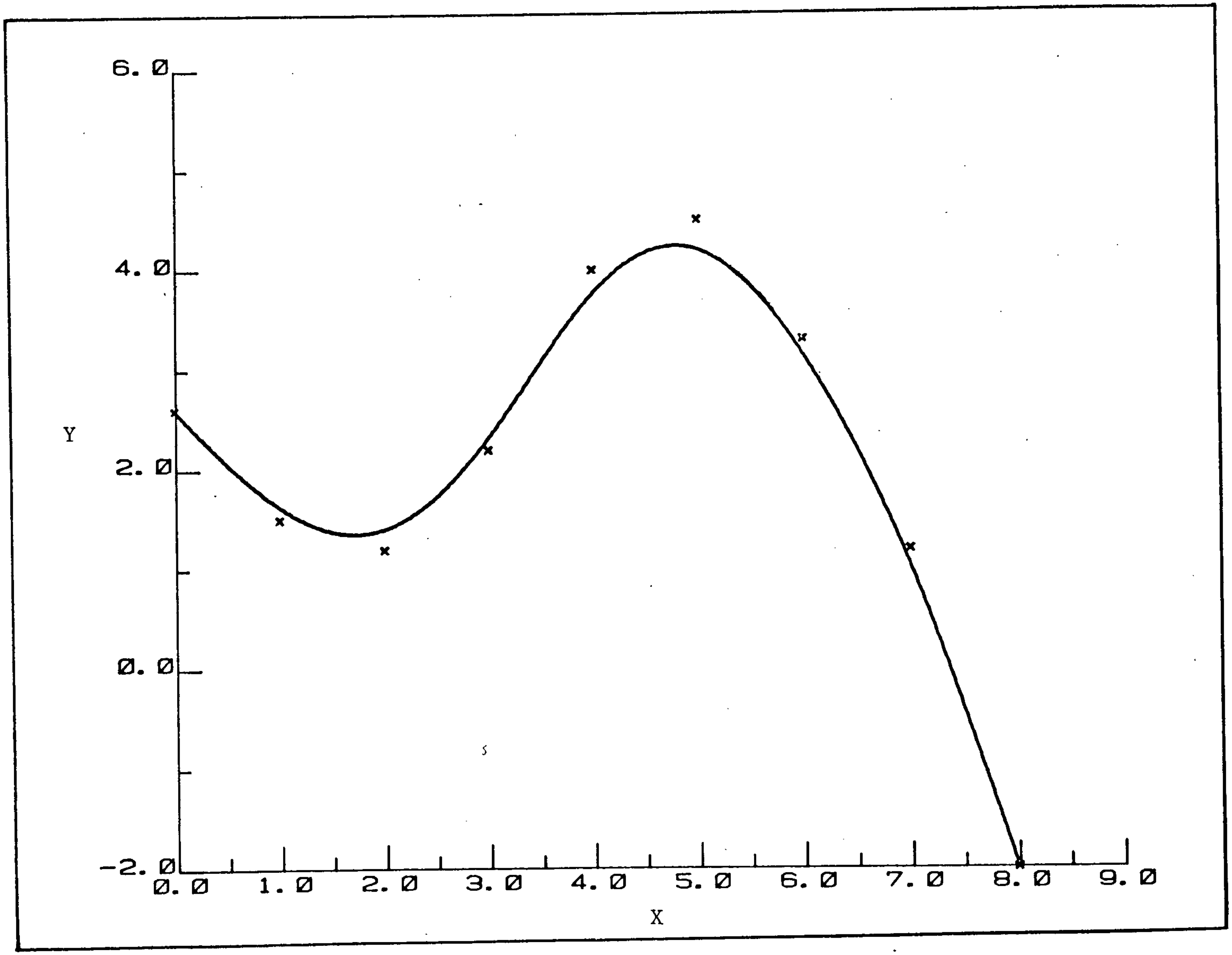


FIG. 2.1 A B-Spline fit to a given set of data points.

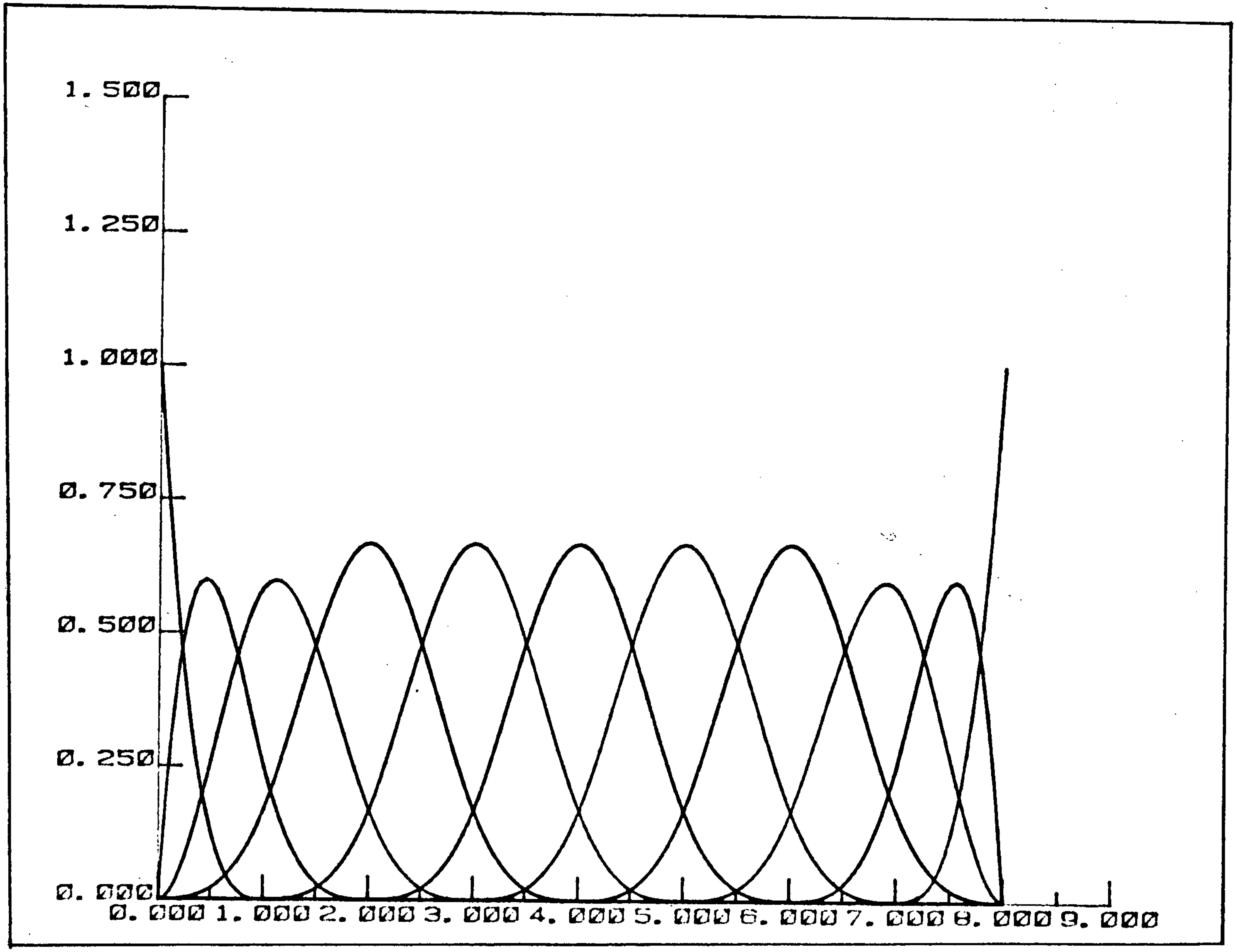


FIG. 2.2 Basis function for a set of equally spaced knots.

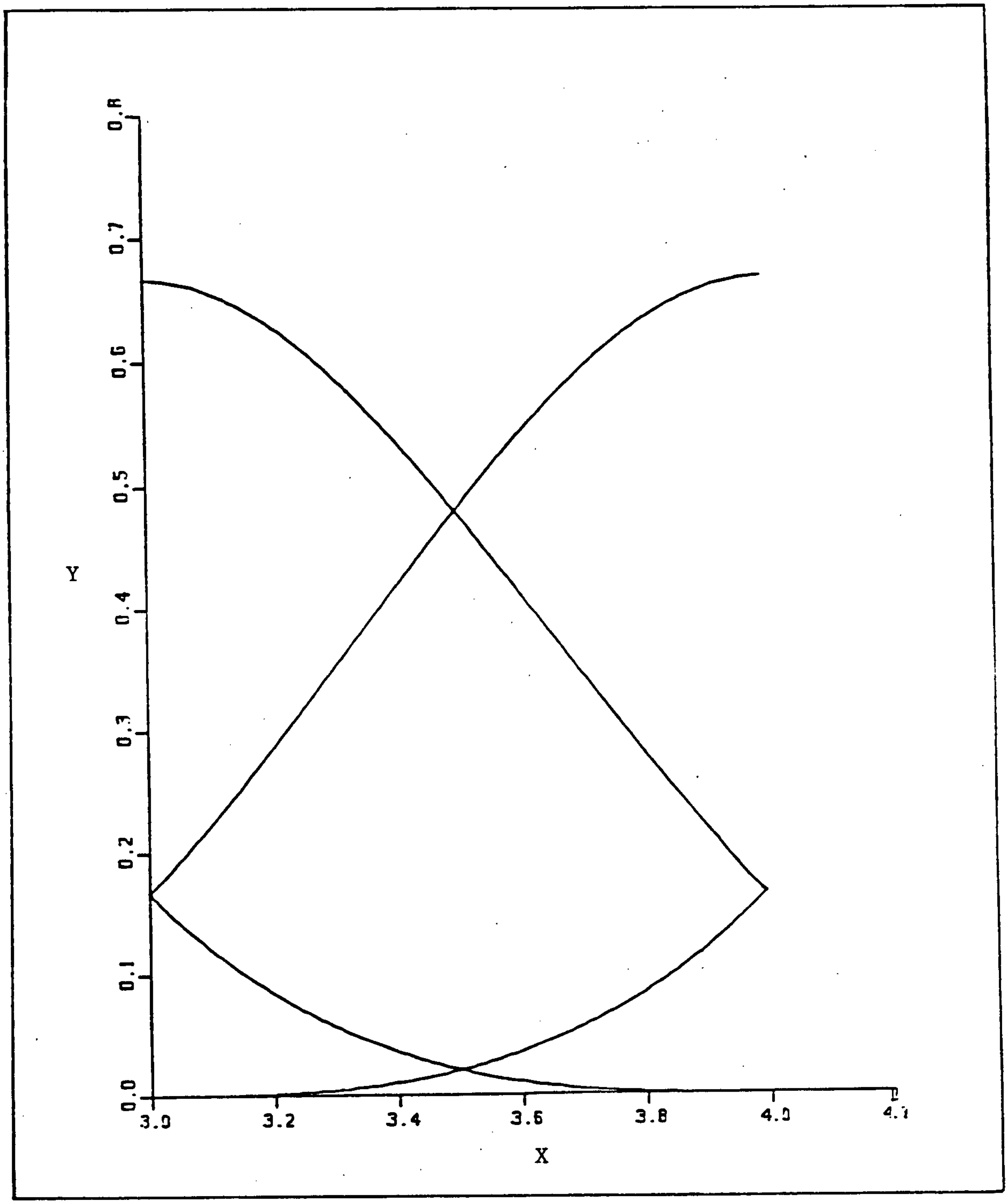


FIG. 2.3 Basis function between two equally spaced knot points.

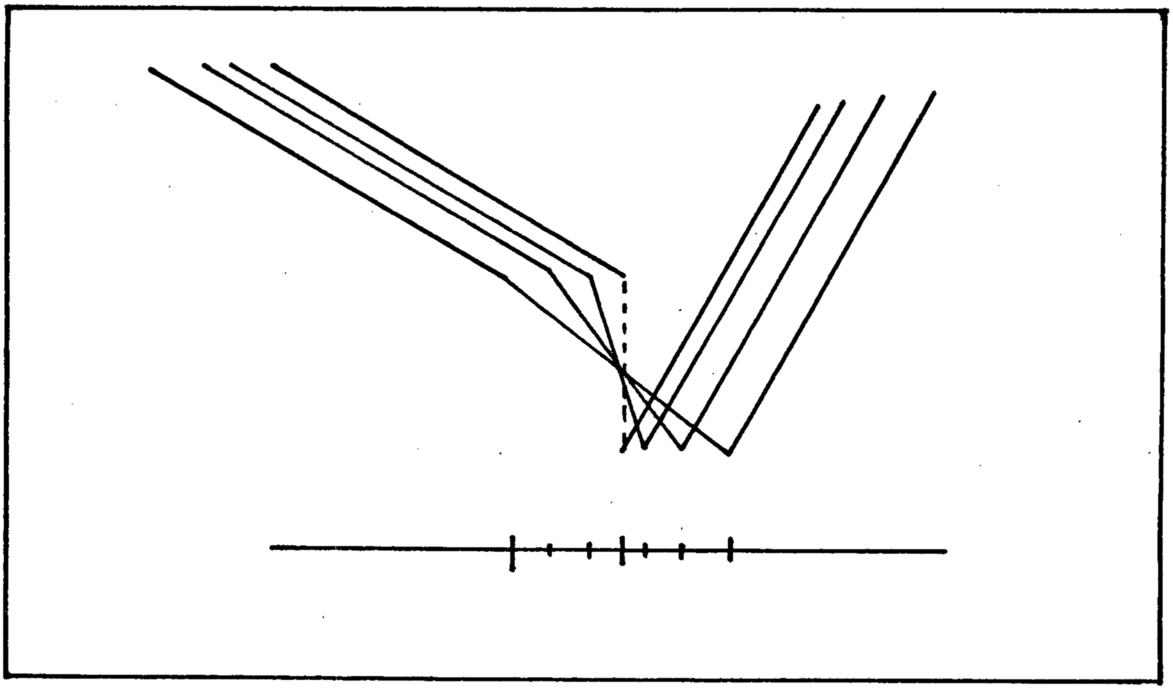


FIG.2.4 Knot coalescence.

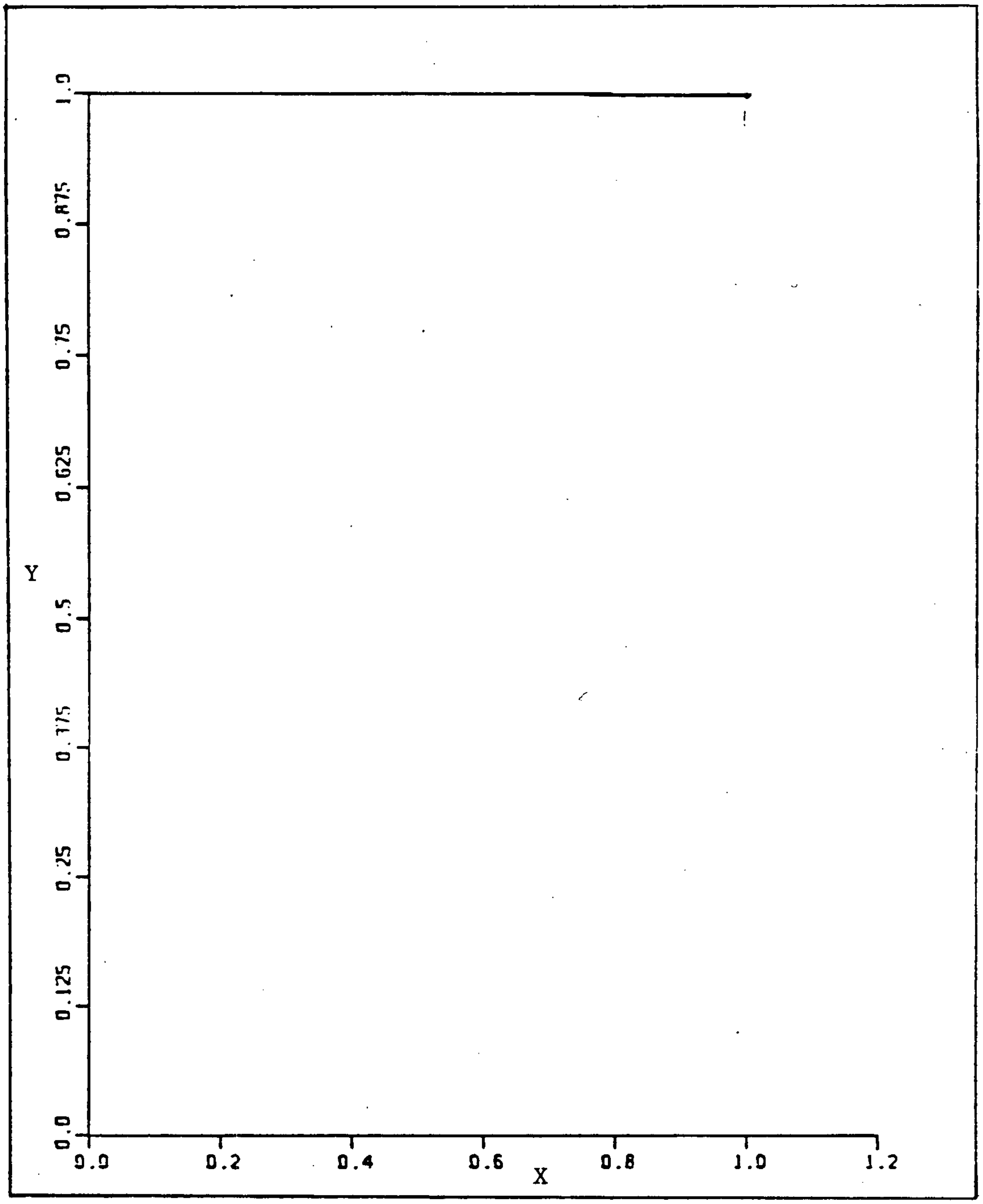


FIG. 2.5 Basis function of order one.

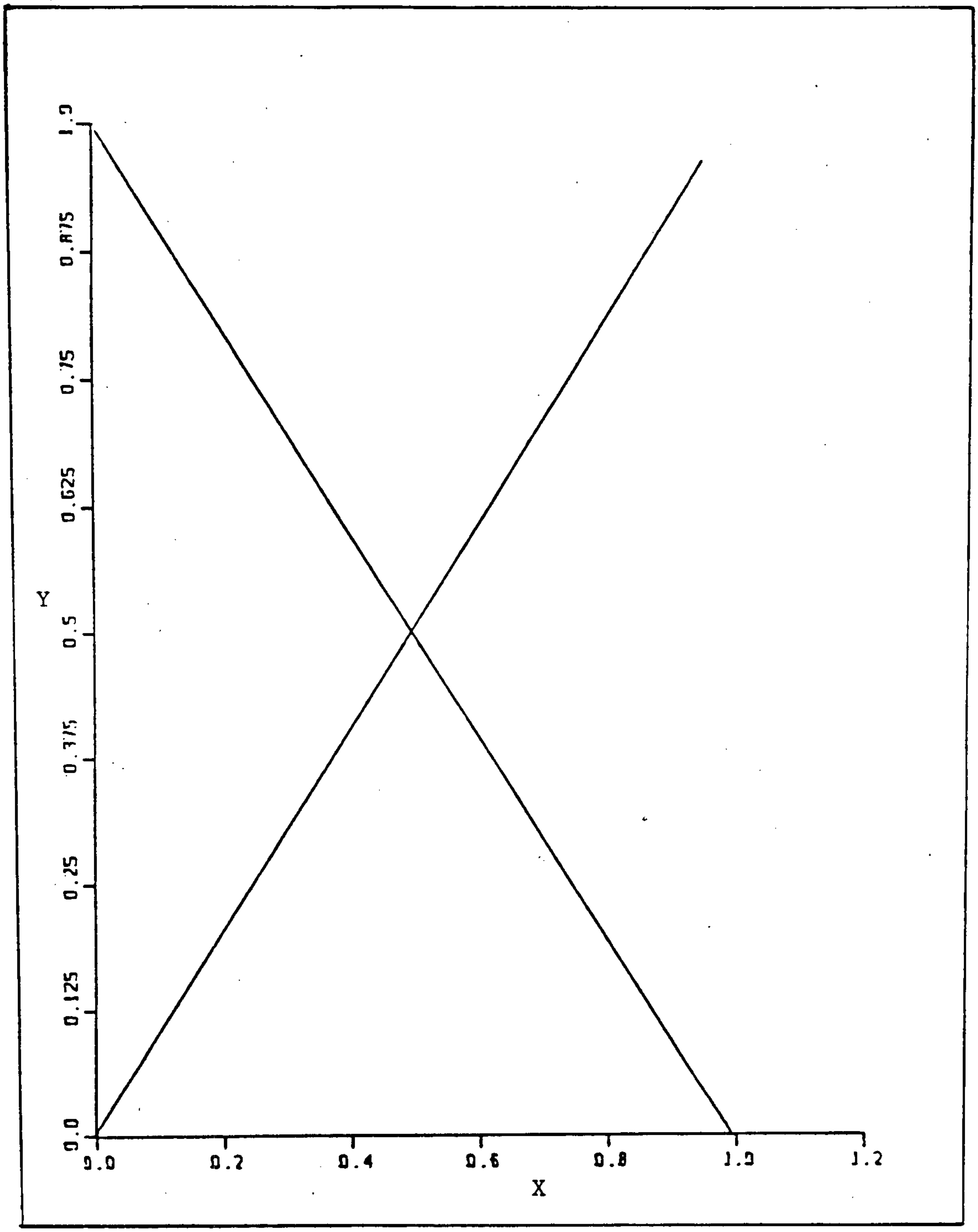


FIG. 2.6 Basis function of order two.

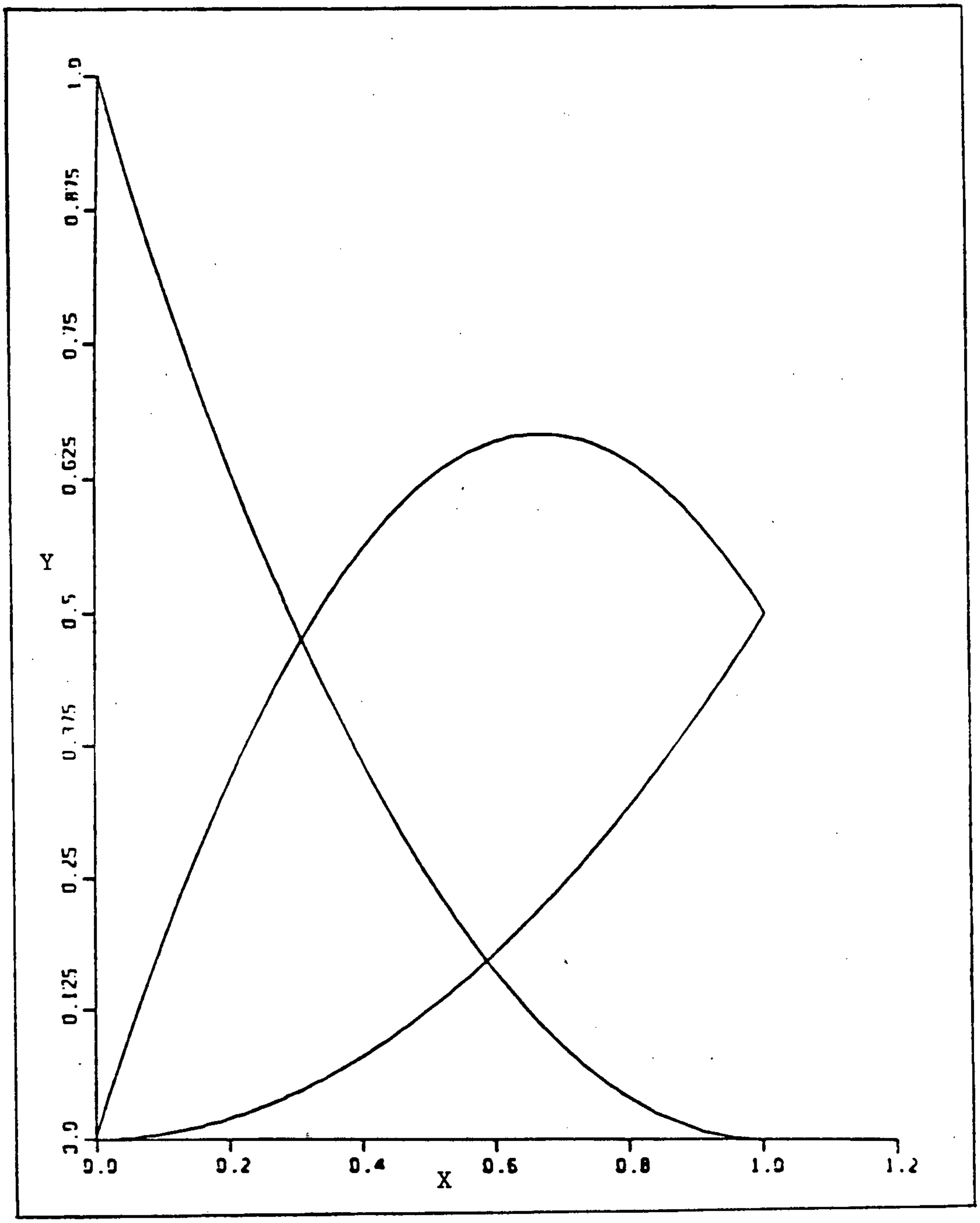


FIG. 2.7 Basis function of order three.

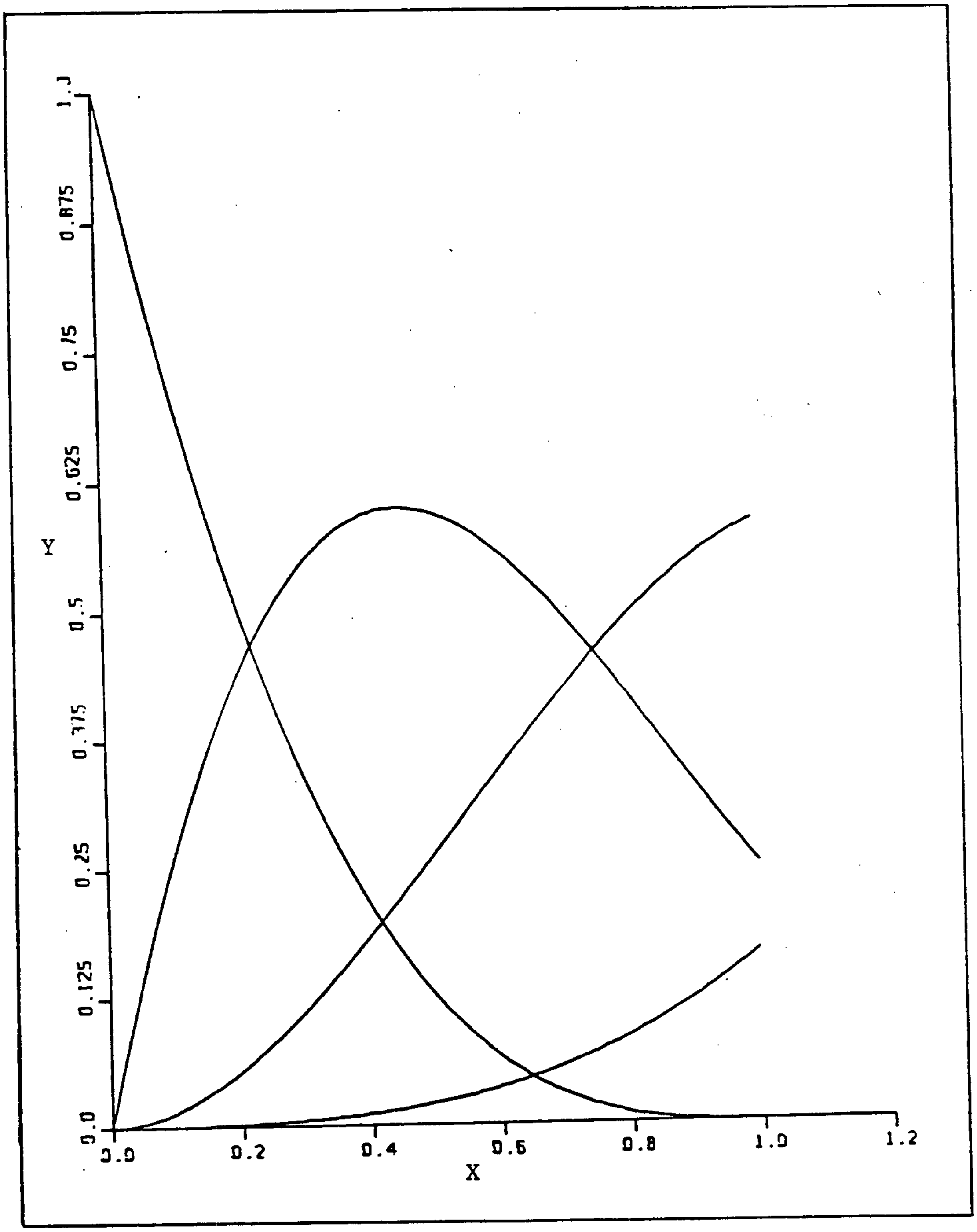


FIG.2.8 Basis function of order four.

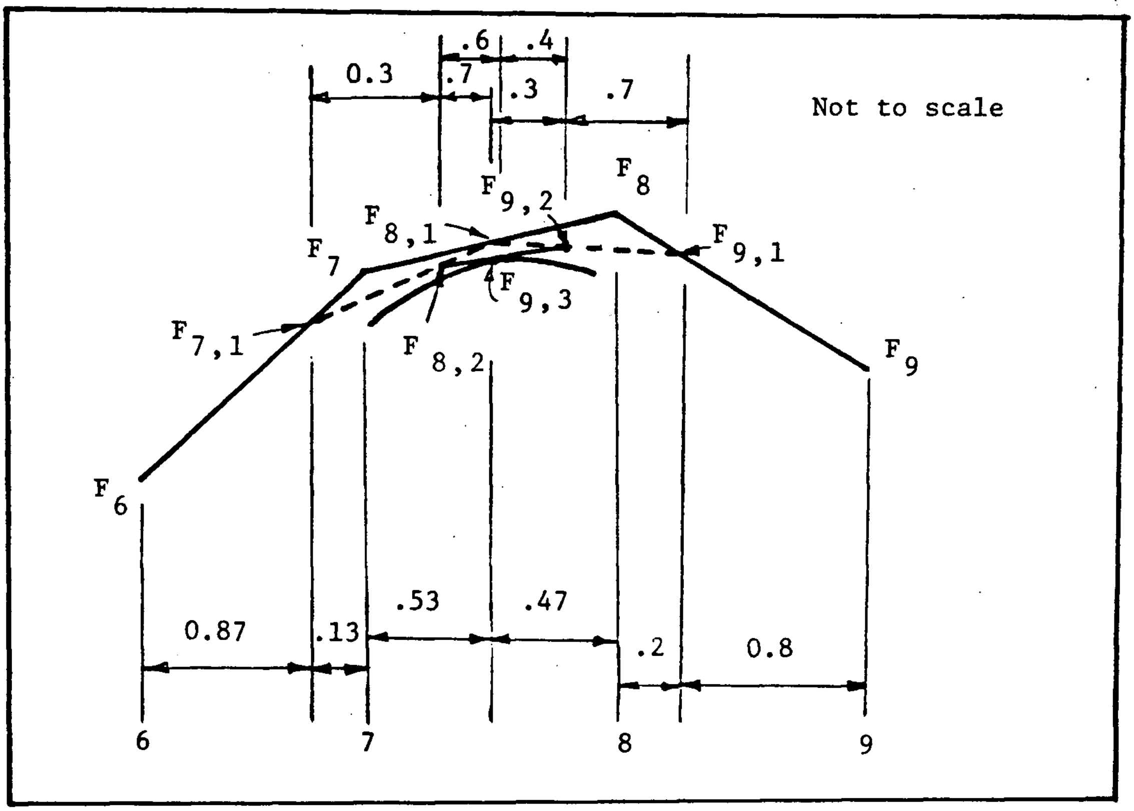


FIG. 2.9 Geometeric explanation of B-Splines.

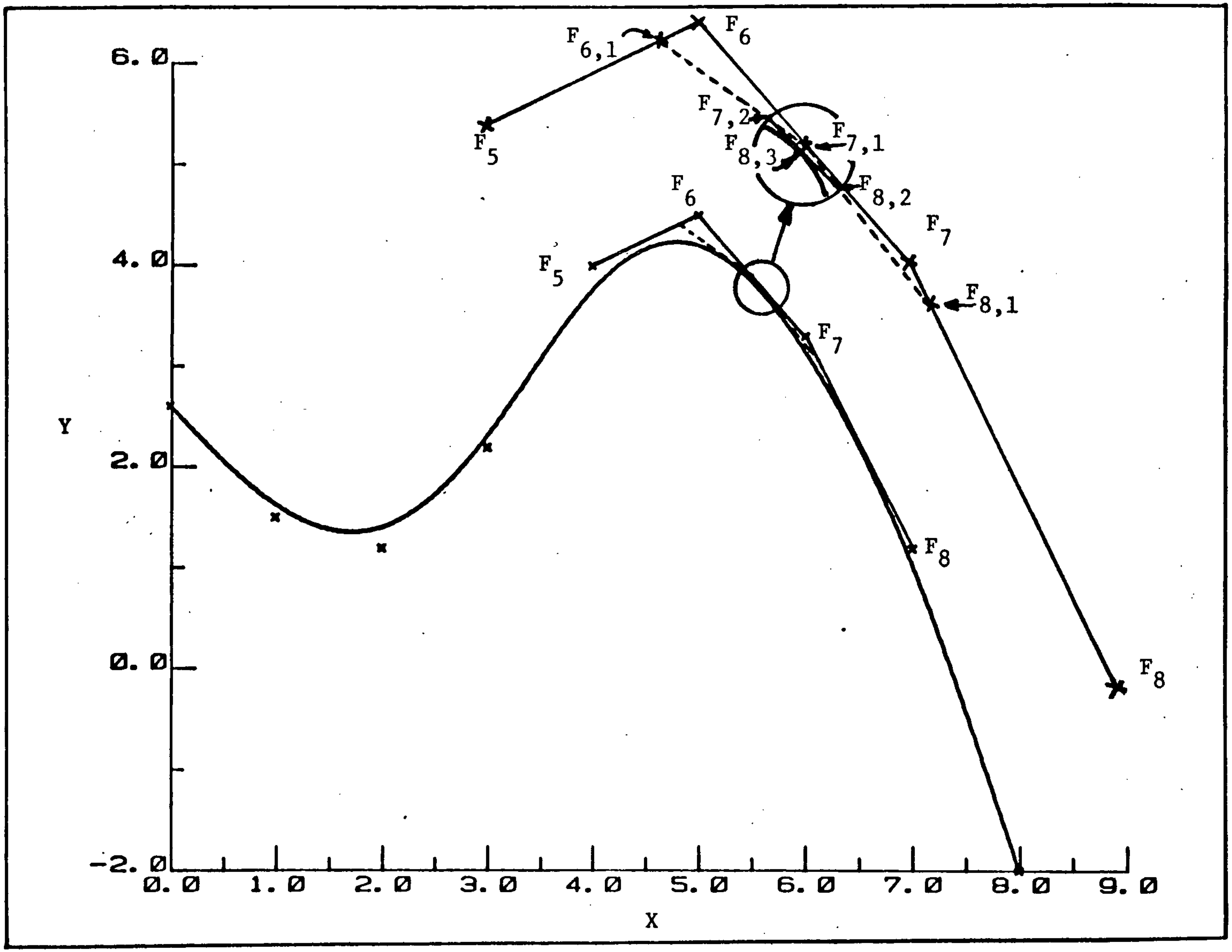


FIG. 2.10 B-Spline (BSAPP) geometeric construction.

CHAPTER 3

CURVE FITTING COMPARISONS

3.1 Test Curves and Curve Fitting Methods

To evaluate B-spline curves in relation to other curve fitting methods a series of trial shapes, or bench marks were devised. The three chosen standard data sets are given in Figures 3.1 to 3.3 and comprise of:

1. The S-curve: used to evaluate the smoothing capabilities of each method. Figure 3.1.
2. The two-level curve: used to evaluate the treatment of high slopes. Figure 3.2.
3. The off-set point: used to evaluate the capability of each method to deal with an off-set point and thereafter follow a straight line. Figure 3.3.

The curve fitting methods implemented on the PDP 11/34 and used for comparison with B-splines are as follows:

1. BSAPP — B-spline approximation.
2. BSPIN — B-spline interpolation.
3. CON.SPL — conic polynomial spline.
4. CUB.SPL — cubic polynomial spline.
5. LMS — least-mean-squares approximation.
6. 9TH POLY — 9th order polynomial curve.
7. UNIGRAPHIC — general fitting routine from McAuto Unigraphics package (installed locally at National Research Council).

3.2 Comparison of Curve Fitting Methods

3.2.1 The S-Curve

The resulting curves for BSAPP, BSPIN, CUB.SPL, CON.SPL, LMS, 9TH POLY and UNIGRAPHIC are given in Figures 3.4 to 3.10 respectively. This set of data points were fitted well by most of the methods but some differences are evident.

BSAPP generated the smoothest curve, although as discussed previously, this did not pass through all the data points. However, the fitted curve always lies within the convex hull of the enclosing polygon as shown in Figure 3.4.

BSPIN and CUB.SPL fitted the data points with a smooth curve which passed through all the points as shown in Figures 3.5 and 3.6. CON.SPL fitted the data points with a fairly smooth curve. However, it was not as smooth as BSPIN and CUB.SPL.

Further, CON.SPL required the most manual intervention and some prior knowledge of the curve. The user is required to define all inflection points and slopes as well as end point slopes. Manipulation of the inflection point slopes can be used to improve the curve but this depends on user's skill and the result is not unique.

Results from LMS given in Figure 3.8 show unwanted oscillations. It was found that suitable results were obtained only when a large number of data points were used. Even then there is no assurance that oscillations will not occur.

The 9TH POLY given in Figure 3.9 also shows some oscillations. With less smooth data sets these oscillations can become very significant.

Results from UNIGRAPHIC curve fitting are given in Figure 3.10 The results are not unlike those of CUB.SPL shown in Figure 3.6.

3.2.2 The two-level curve

This set of data highlights the capabilities of a method to deal with relatively sudden changes of slopes without overshoot. The resulting curves for BSAPP, BSPIN, CUB.SPL, CON.SPL and UNIGRAPHIC are given in Figures 3.11 to 3.15 respectively.

BSAPP fitted by far the smoothest curve, and had no overshoots as shown in Figure 3.11.

BSPIN, CUB.SPL and CON.SPL all generated serious oscillations in the fitted curve as shown in Figures 3.12 to 3.14. BSPIN and CUB.SPL gave similar results. The results for CON.SPL were better than BSPIN and CUB.SPL. However, the operator time required for CON.SPL was significant and probably unacceptable in practice.

The spline fitting routine on UNIGRAPHIC also gave oscillations as shown in Figure 3.15.

3.2.3 The off-set point

This set of data highlights the capabilities of a method to deal with off-set points. The resulting curves for BSAPP, BSPIN, CUB.SPL, CON.SPL and UNIGRAPHIC are given in Figures 3.16 to 3.21 respectively.

BSAPP smoothed the data to a remarkable extent, still keeping within the convex hull of the enclosing polygon. The straight line portion was exactly reproduced as shown in Figure 3.16.

BSPIN and CUB.SPL fitted a highly oscillating curve to the given set of data points as shown in Figures 3.17 and 3.18. Oscillations created in the offset curve were carried down into the straight line portion of the curve.

CON.SPL had great difficulty in fitting this set of data points. Additional inflection points, data points and slopes were tried but even so the most acceptable curve is shown in Figure 3.19.

The conic spline and spline general from UNIGRAPHIC give identical and unsatisfactory results shown in Figures 3.20 and 3.21. In the case of the conic spline the curve turns on itself, while in spline general oscillations are encountered.

3.3 Other Evaluation Factors

While the technical suitability of a curve fitting method to a particular application is of prime concern, other factors such as:

1. the ease of use;
2. computational size and application to mini-computers; and
3. speed of computation

need to be exercised. A general assessment of these factors is given in Table 3.1. From the standpoint of ease of use BSAPP and CUB.SPL were rated 'good' while CON.SPL was rated 'poor' as it required the most manual intervention and some prior knowledge of the curve shape. The task image size, which is the memory size required to store and run the object code for the compiled version is given in Table 3.1. It can be seen that BSPIN and CUB.SPL require approximately the same memory sixe (32kB). BSAPP requires a somewhat larger memory,(50kB), although it gives a graphic terminal display. If this feature were removed it would be approximately the same size as BSPIN and CUB.SPL. CON.SPL requires approximately twice the amount of memory as the other methods.

The overall assessment is that BSAPP, BSPIN and CUB.SPL are all easy to use and require similar computer memory space. BSAPP gives by far the smoothest curves and is the only method to suitably handle all the bench mark curves. BSPIN and CUB.SPL both give good interpolation results. CON.SPL requires the most memory capacity and the most user input, but is still suitable for some applications. The other methods LMS and n^{th} order polynomial are not considered suitable for CAD/CAM applications.

TABLE 3.1

PROPERTY	BSAPP	BSPIN	CON.SPL	CUB.SPL
1. Task image size	25088 Words 50 K Byte	16192 Words 32 K Byte	30528 Words 61 K Byte	17248 Words 34 K Byte
2. Graphic's ability	Graphic display, Paper plot	Paper plot	Paper plot	Paper plot
3. System dependence	IGL, PLOT10	PLOT10	PLOT10	PLOT10

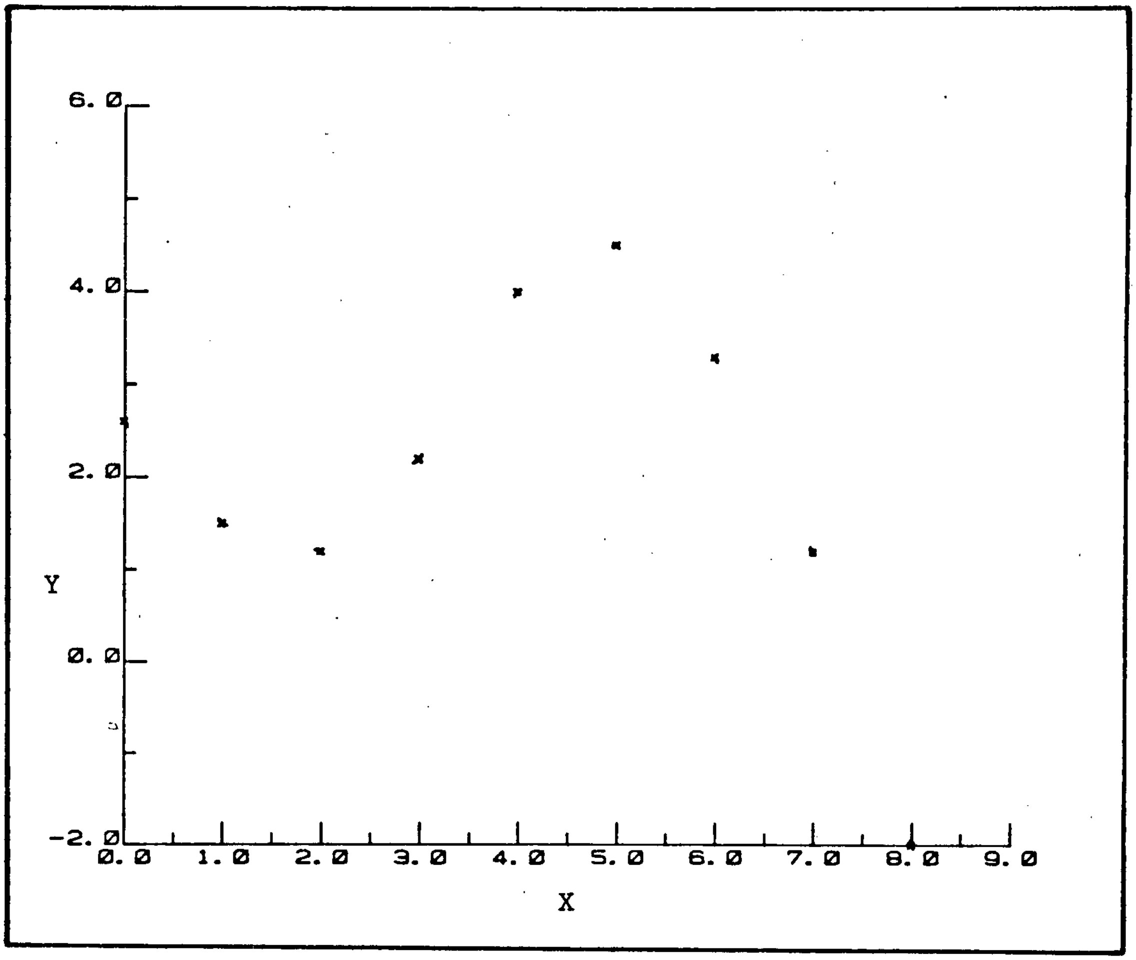


FIG. 3.1 Data points fot the S-curve.

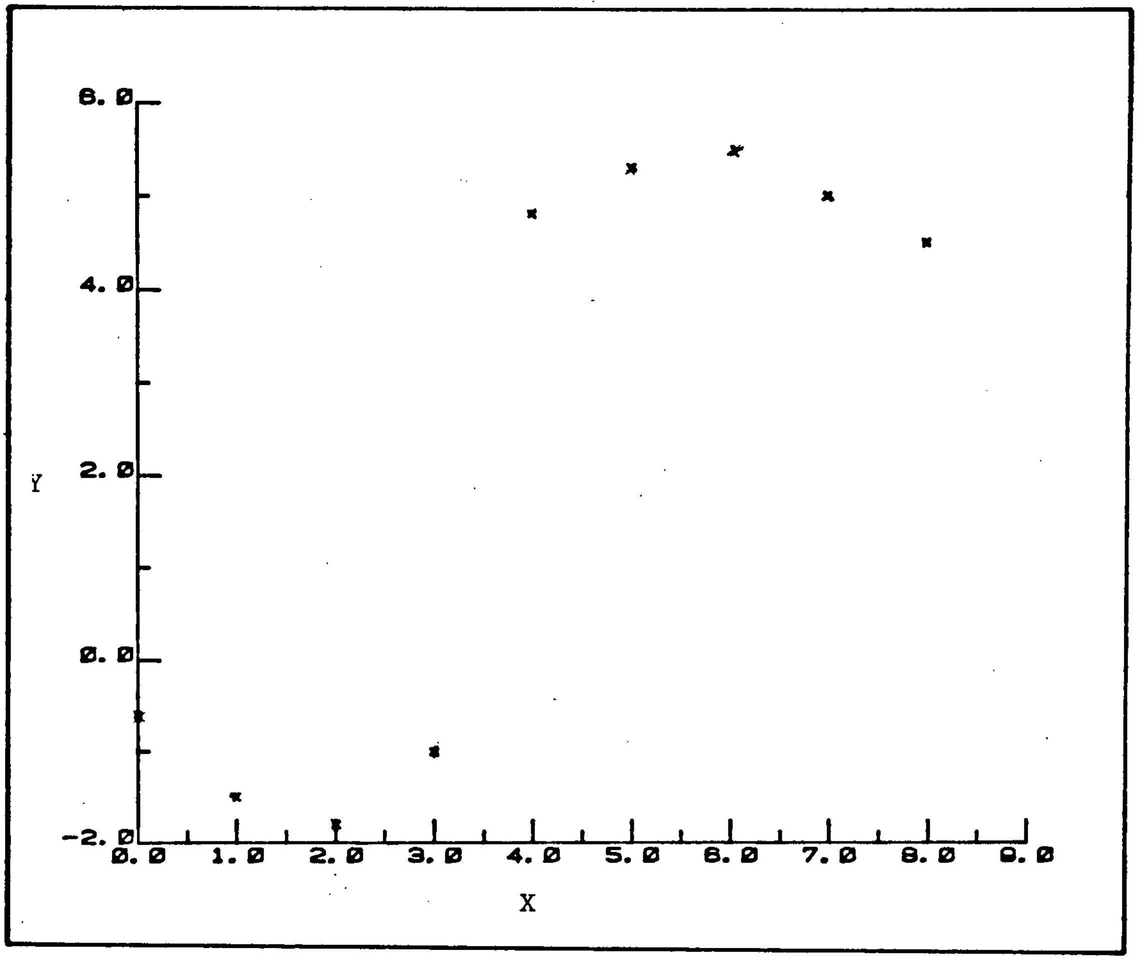


FIG. 3.2 Data points for the two level curve.

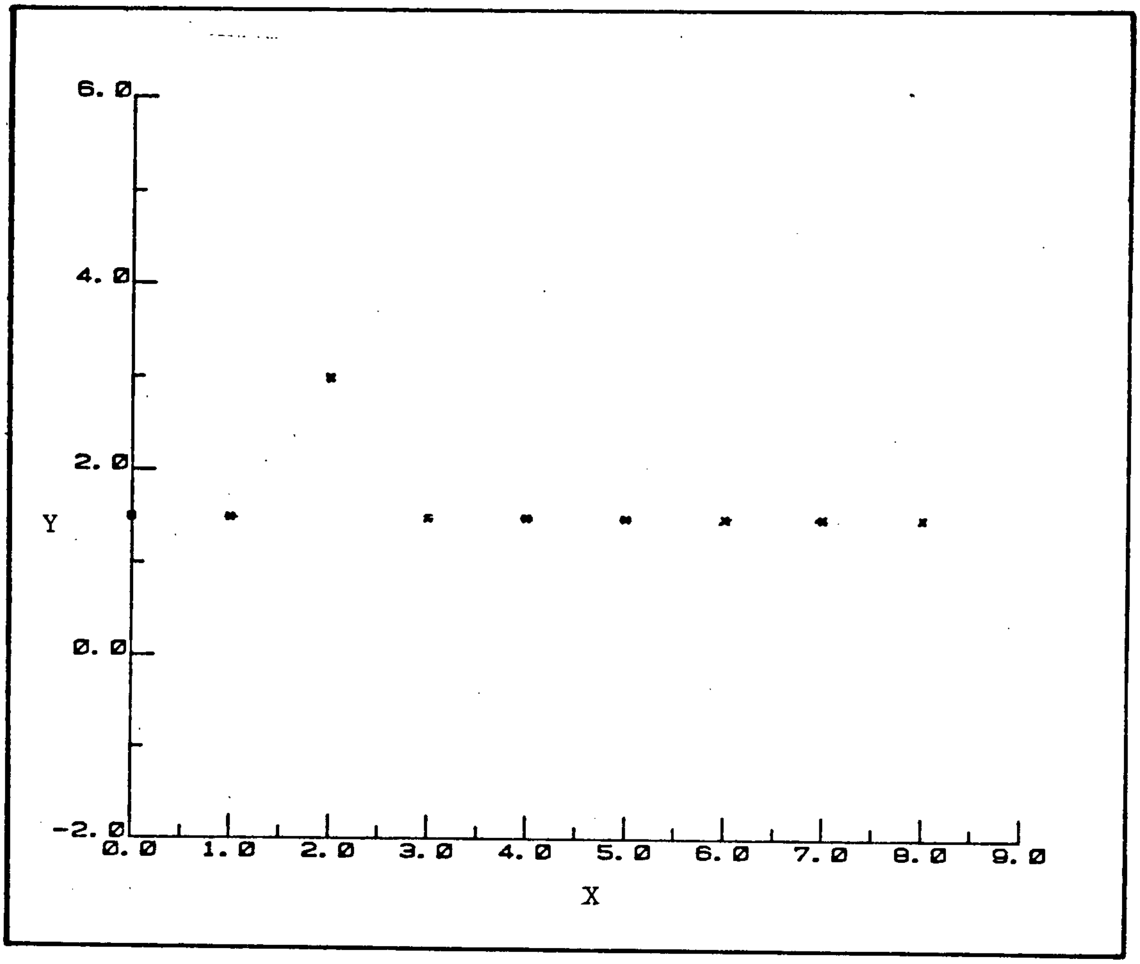


FIG. 3.3 Data points for the off-set point.

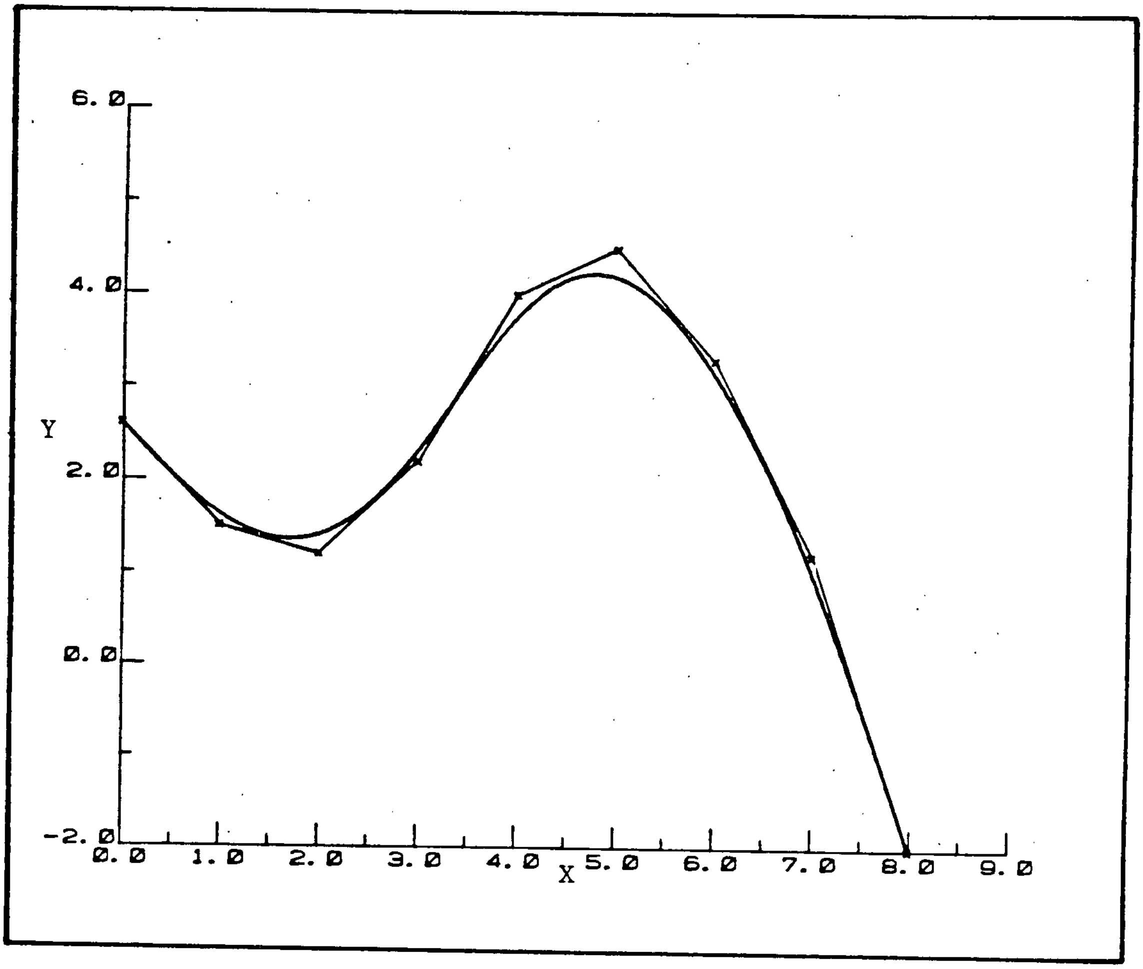


FIG. 3.4 A B-Spline (BSAPP) fit to the S-Curve.

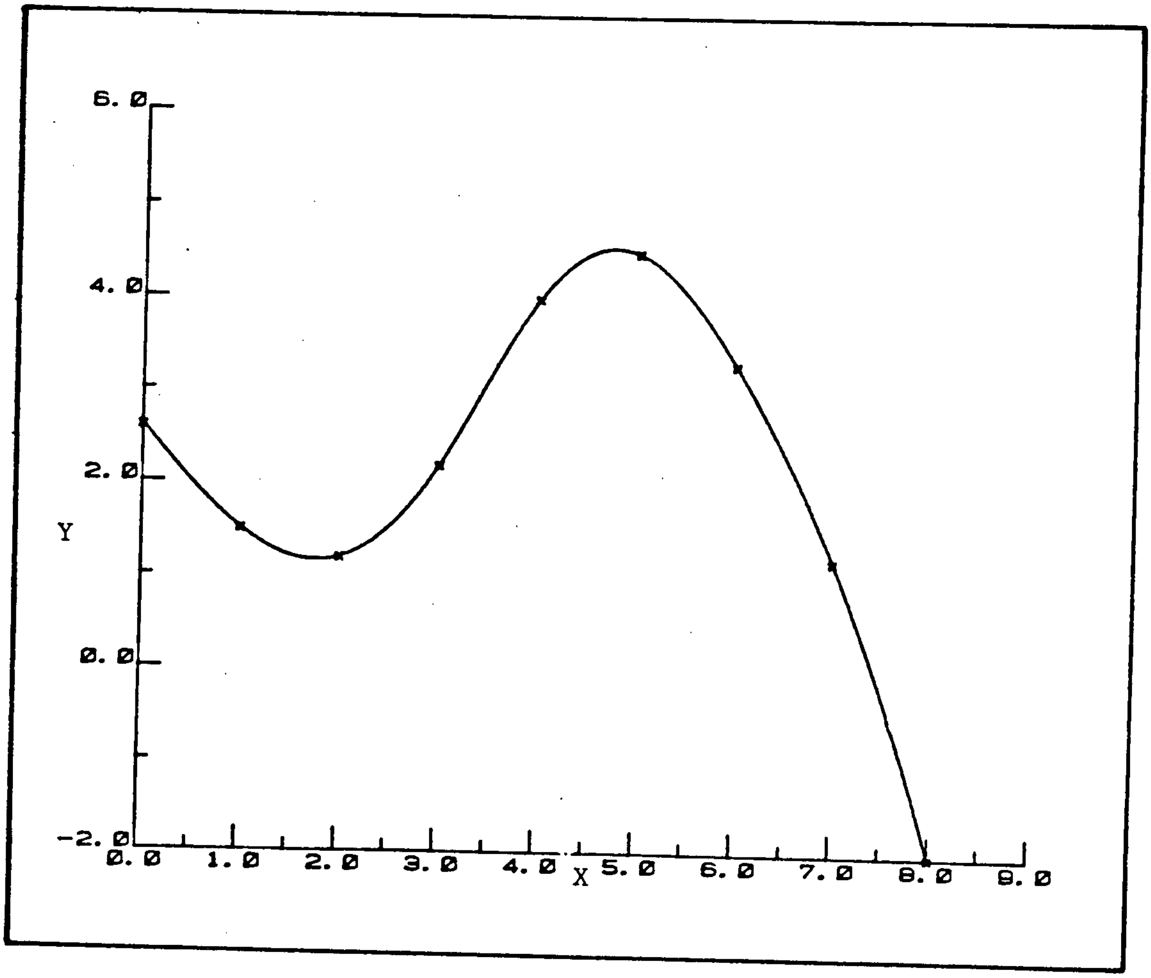


FIG. 3.5 A B-Spline (BSPIN) fit to the S-Curve.

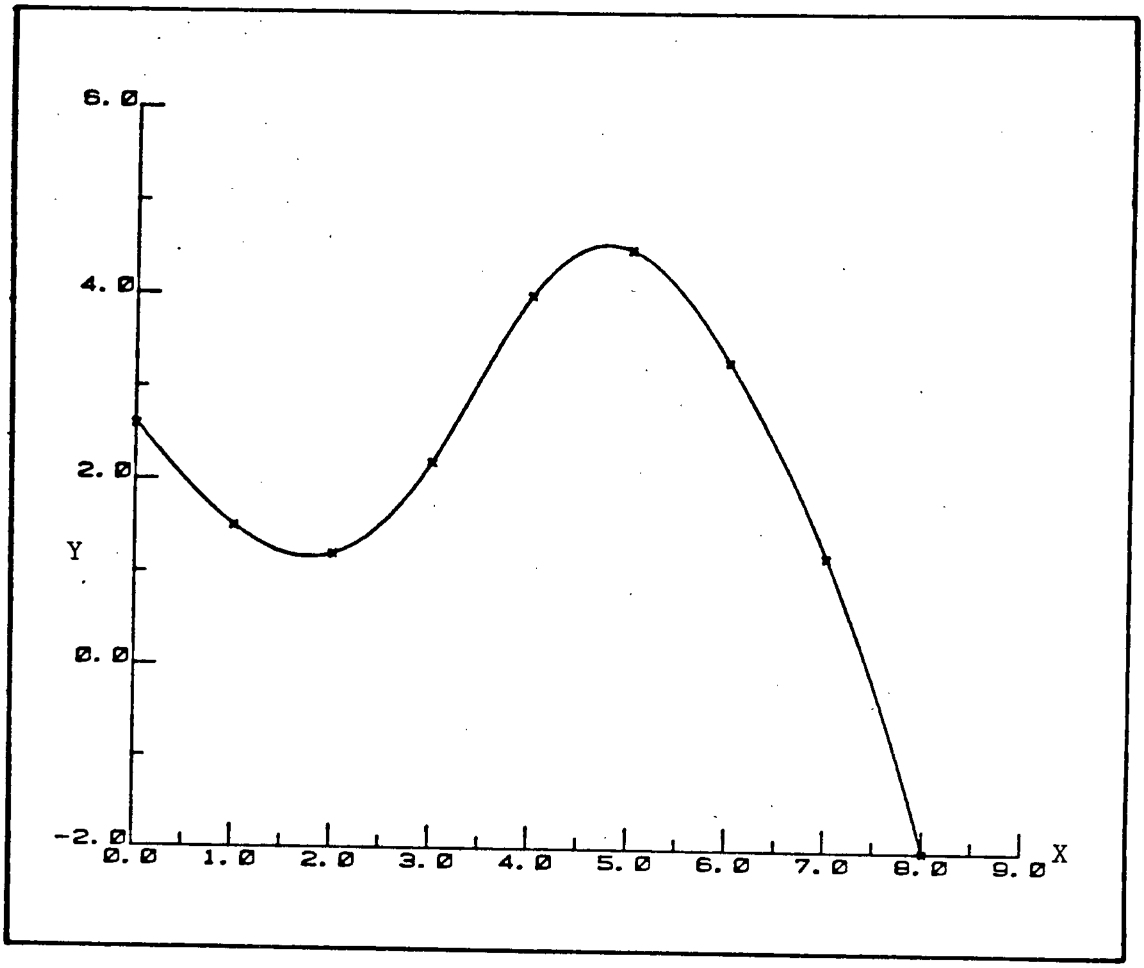


FIG. 3.6 A cubic spline (CUB.SPL) fit to the S-Curve.

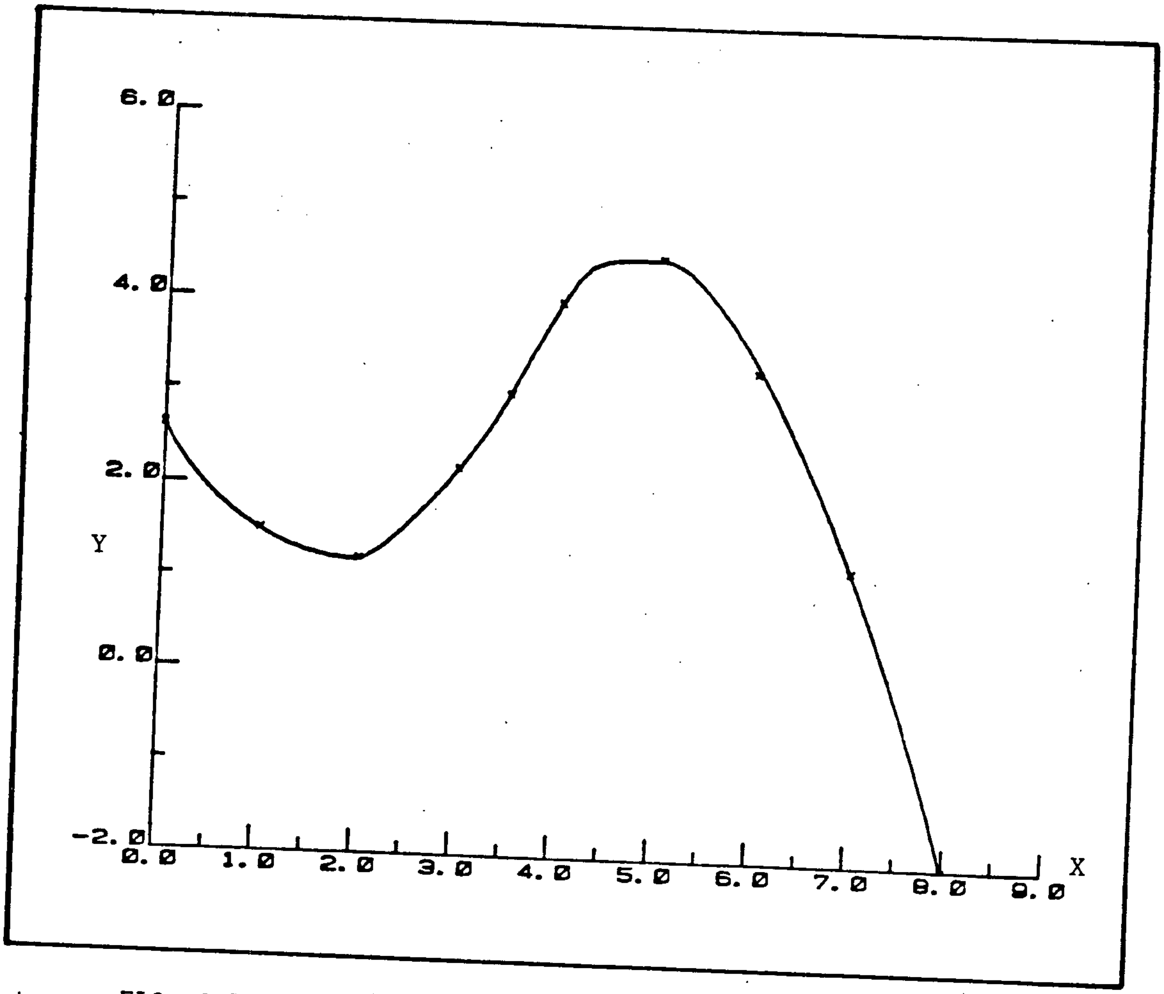


FIG. 3.7 A conic spline (CON.SPL) fit to the S-Curve.

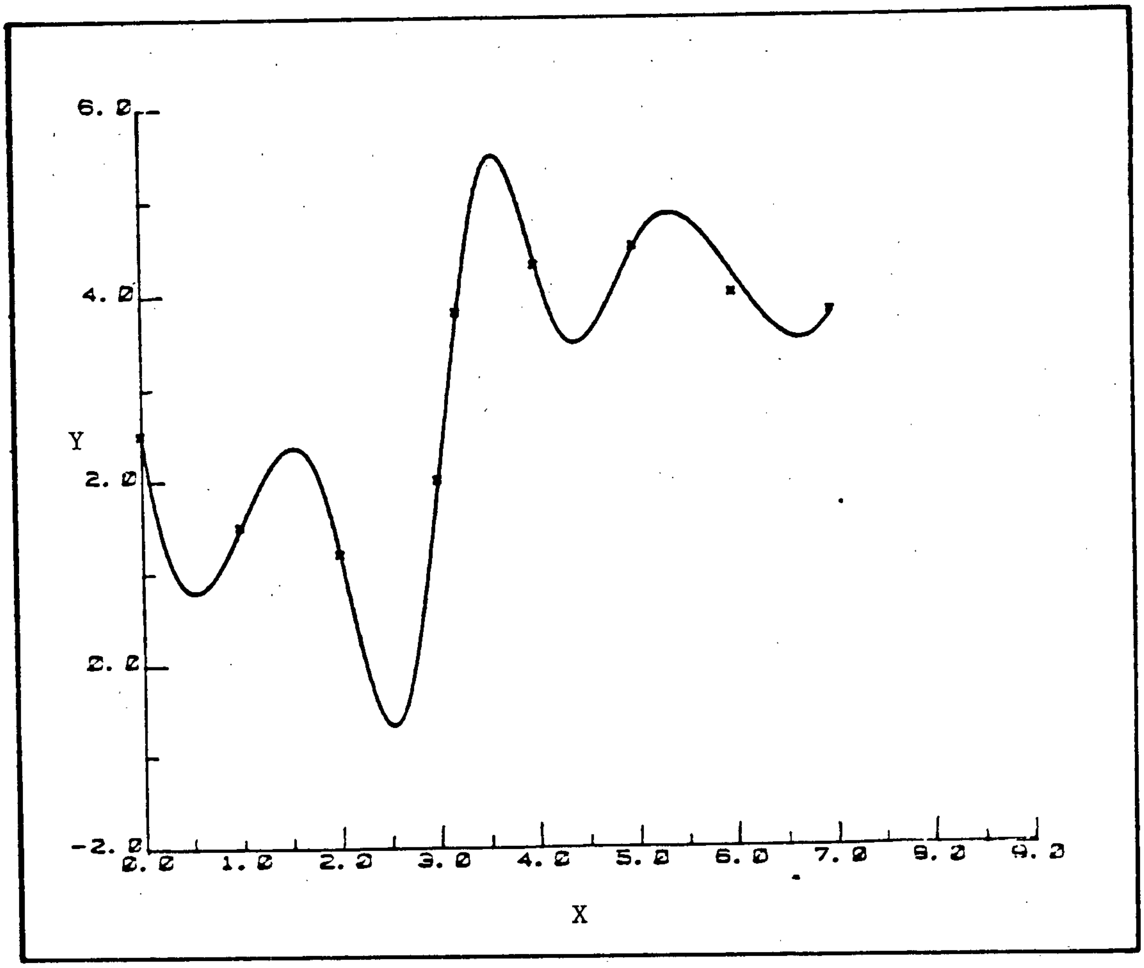


FIG. 3.8 A least-mean-squares (LMS) fit to the S-Curve.

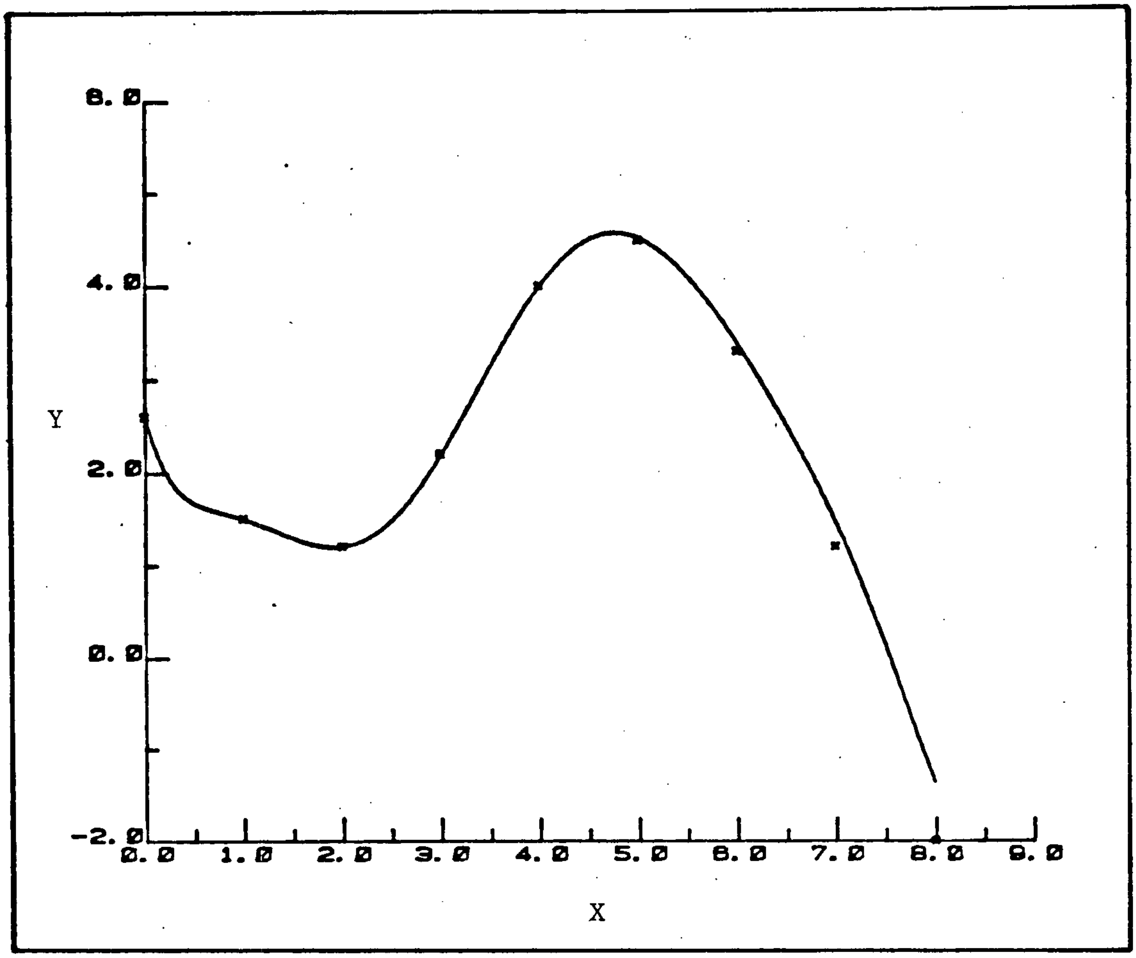


FIG. 3.9 A 9th order polynomial (9thPOLY) fit to the S-Curve.

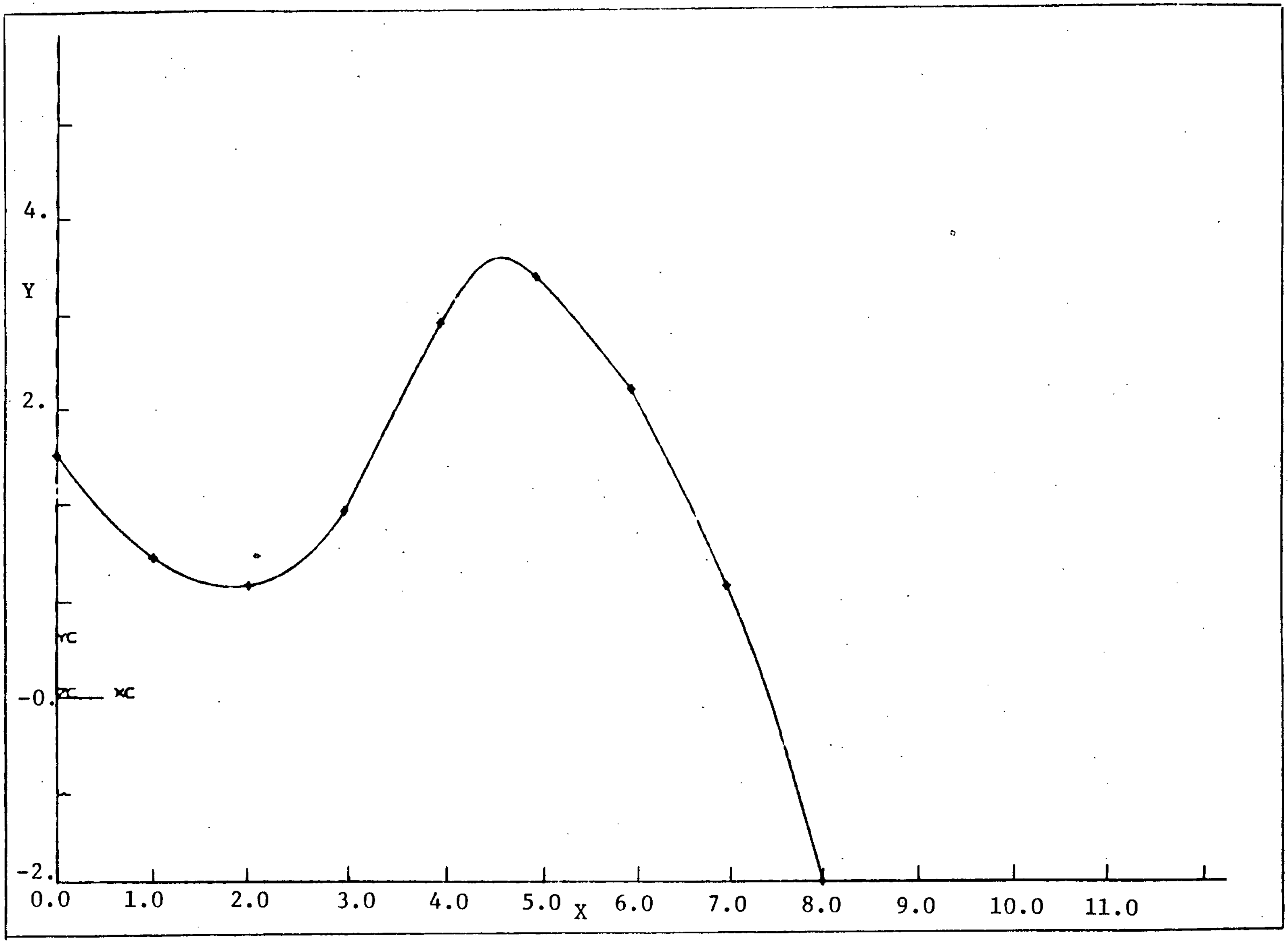


FIG. 3.10 UNIGRAPHIC'S fit to the S-Curve.

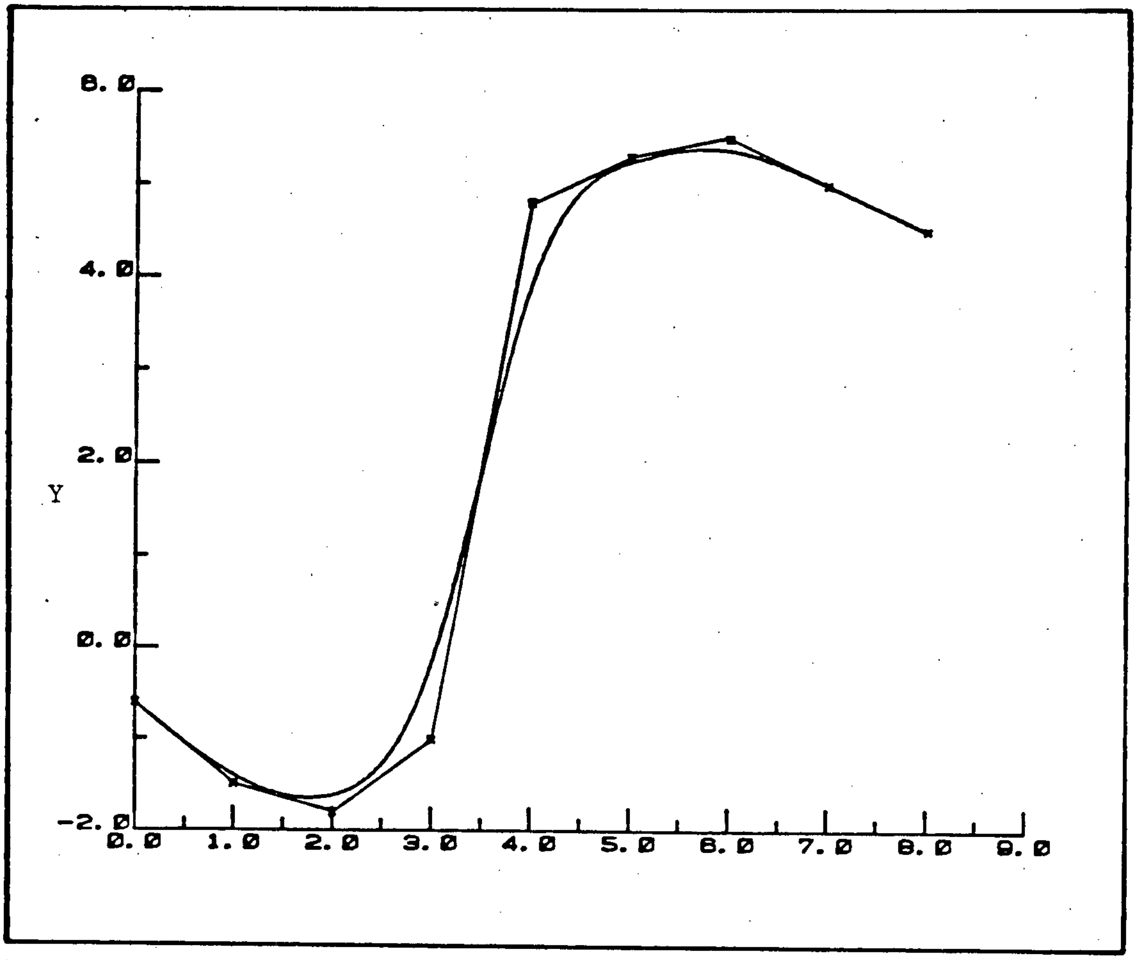


FIG. 3.11 A B-Spline(BSAPP) fit to the two level curve.

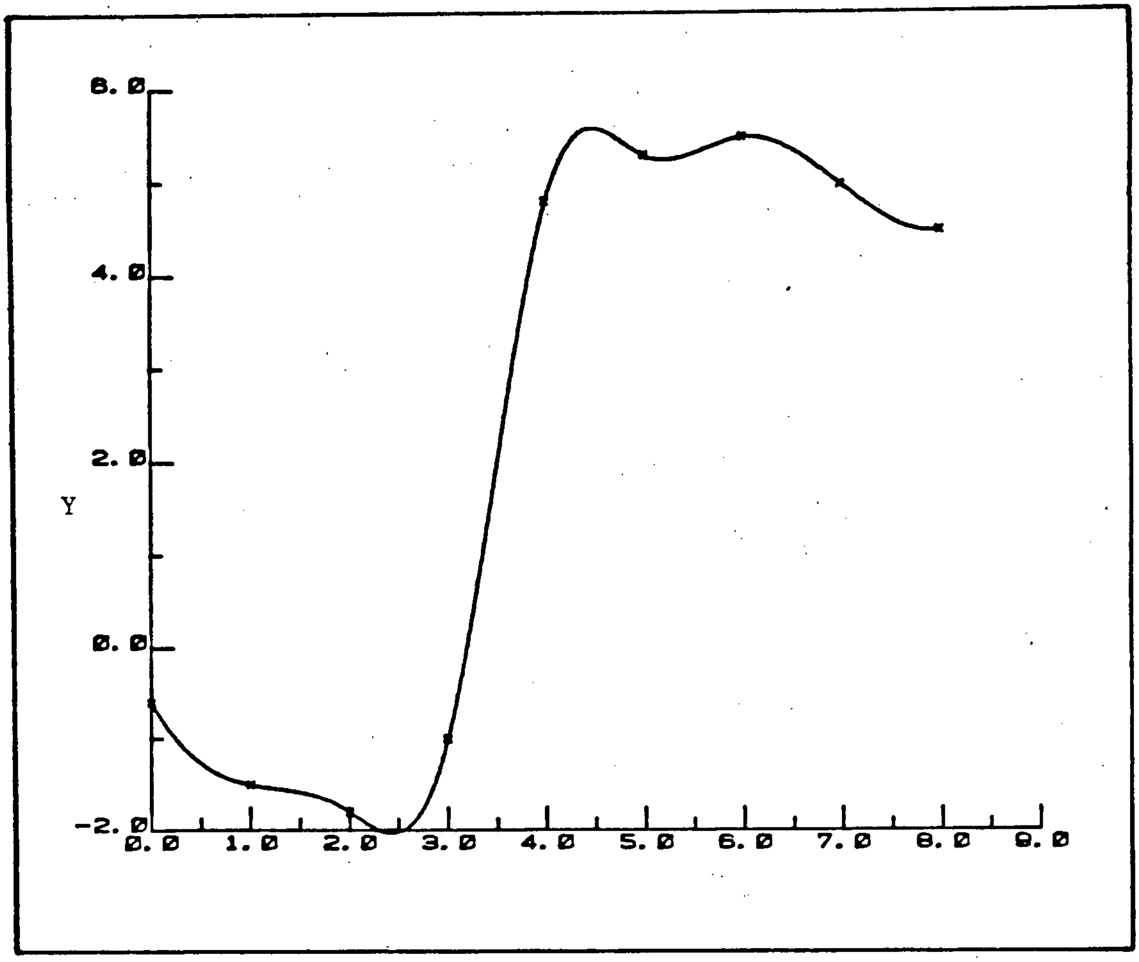


FIG. 3.12 A B-Spline(BSPINT) fit to the two level curve.

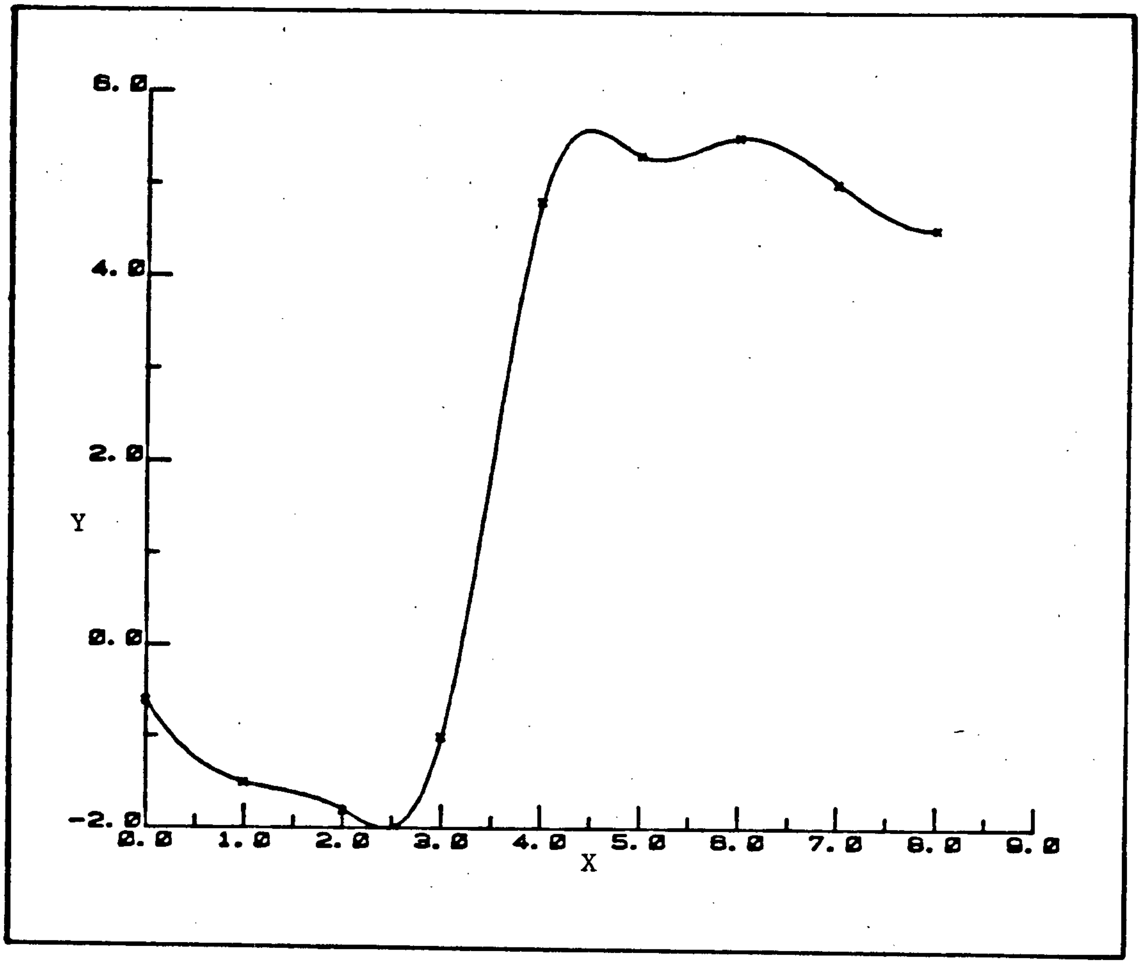


FIG. 3.13 A Cubic spline(CUB.SPL) fit to the two level curve.

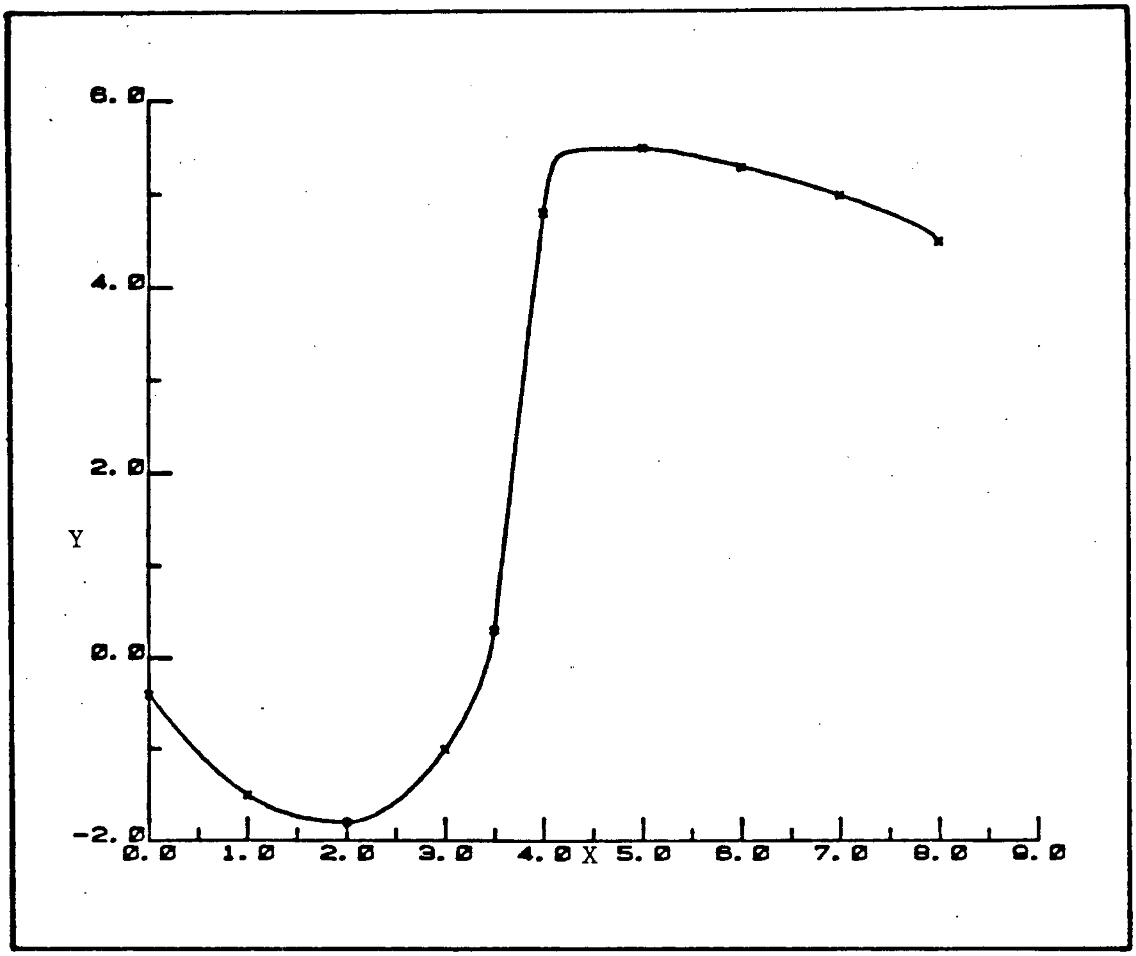


FIG. 3.14 A Conic spline(CON.SPL) fit to the two level curve.

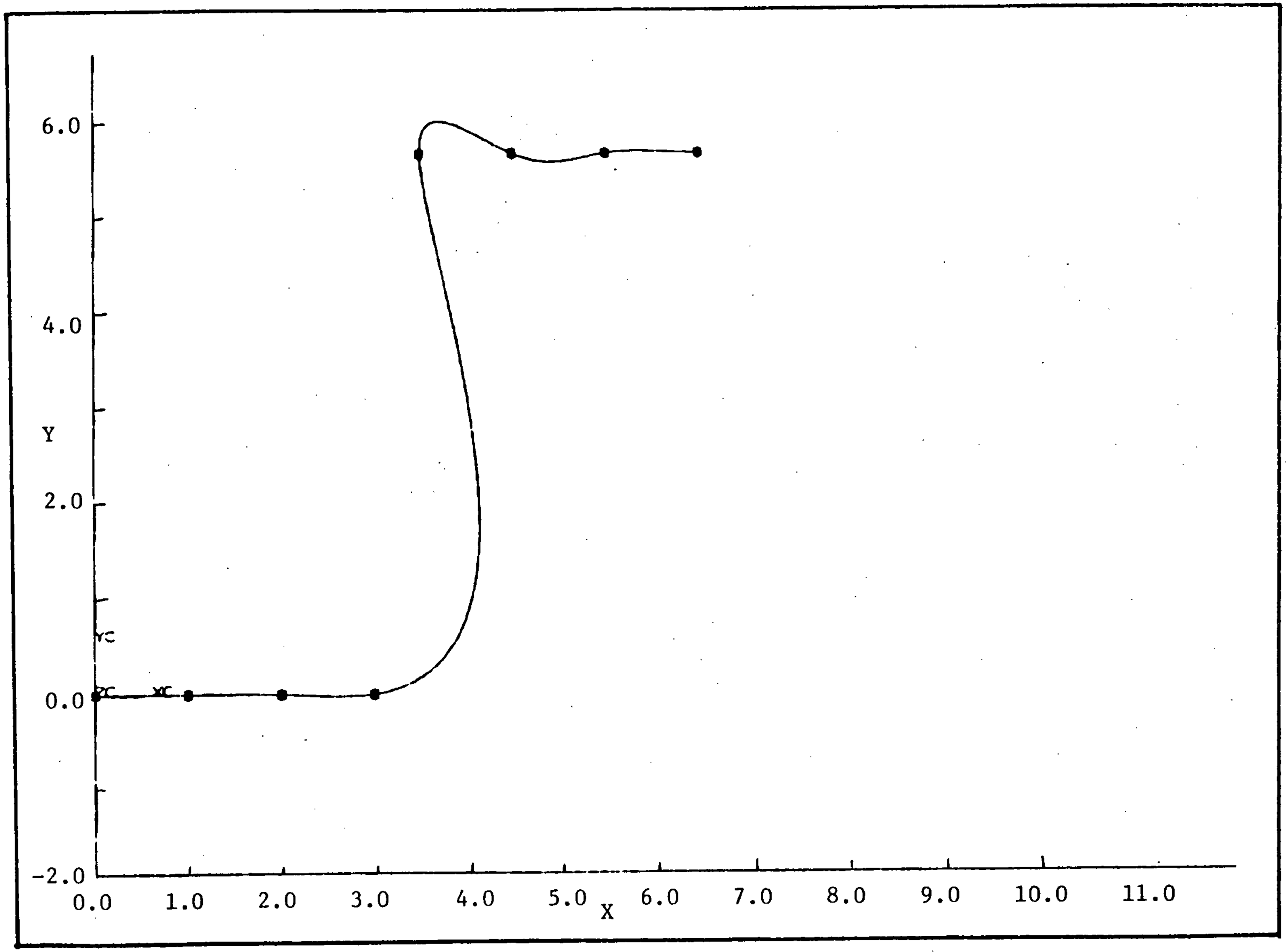


FIG. 3.15 UNIGRAPHICS fit to the two level curve.

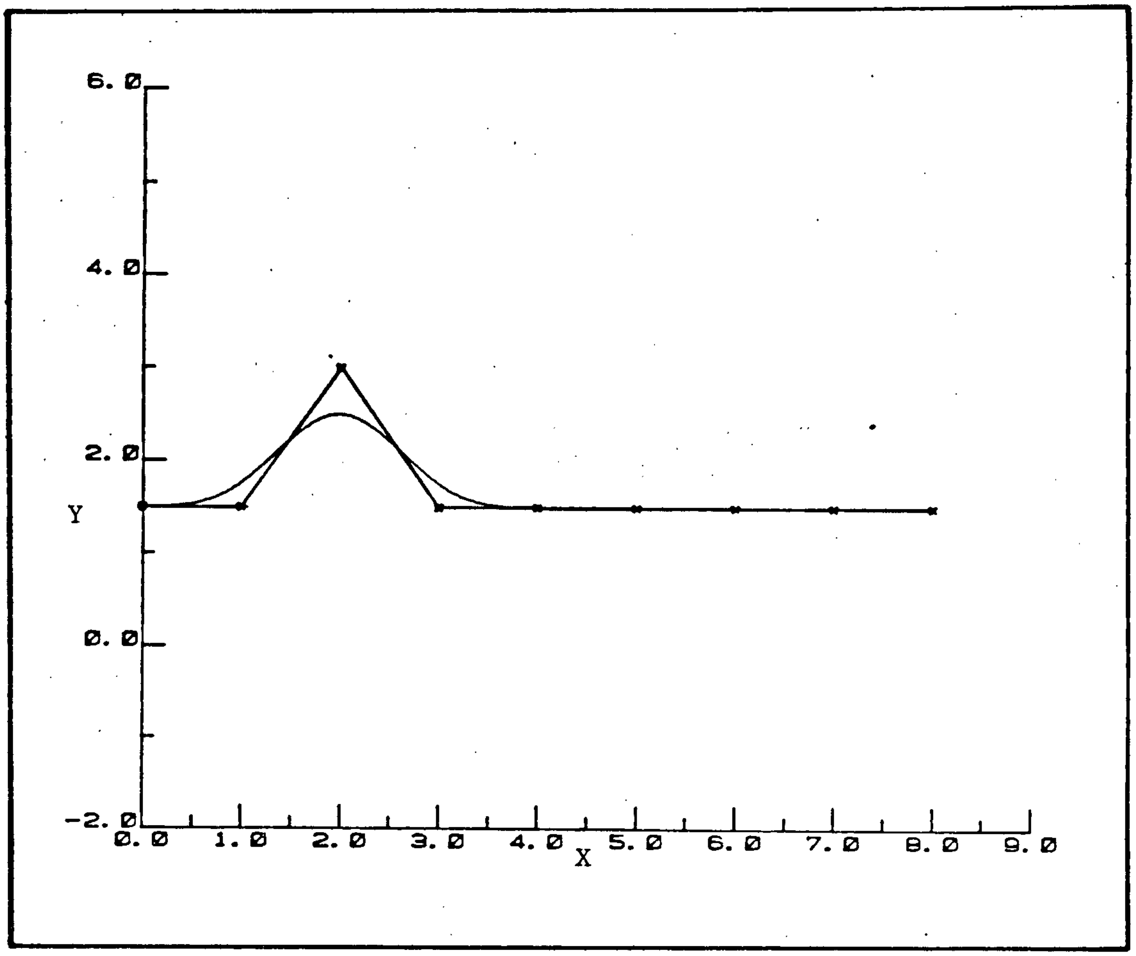


FIG. 3.16 A B-Spline(BSAPP) fit to the off-set point.

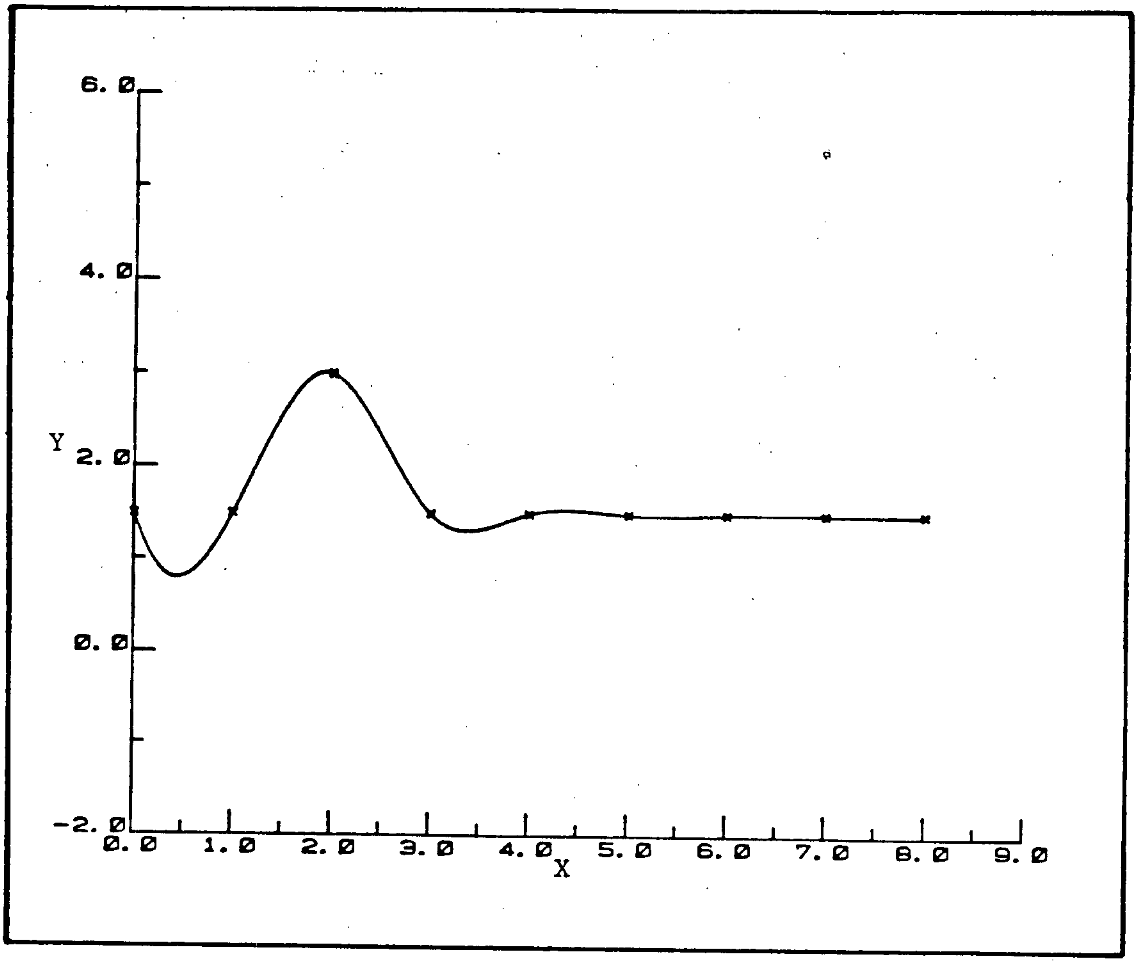


FIG. 3.17 A B-Spline(BSPINT) fit to the off-set point.

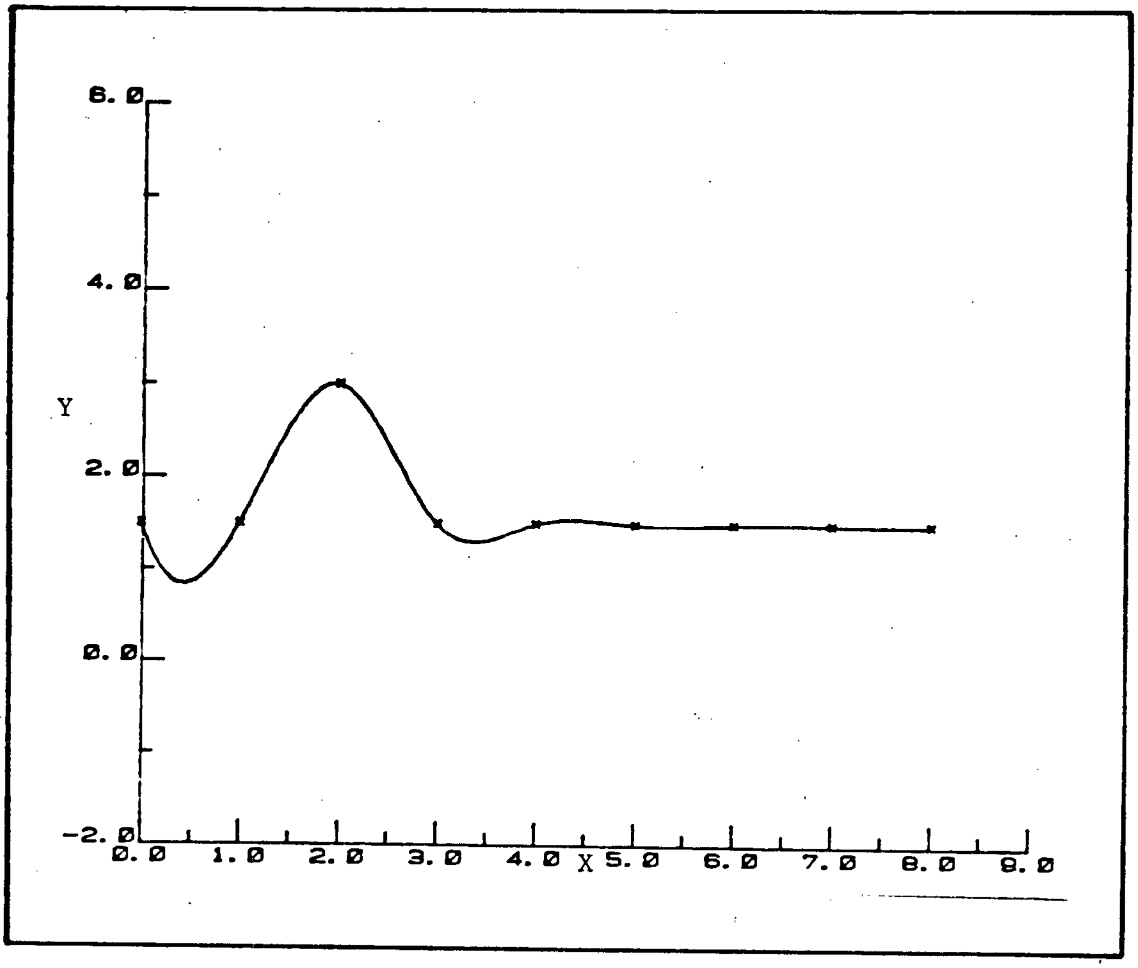


FIG. 3.18 A cubic spline(CUB.SPL) fit to the off-set point.

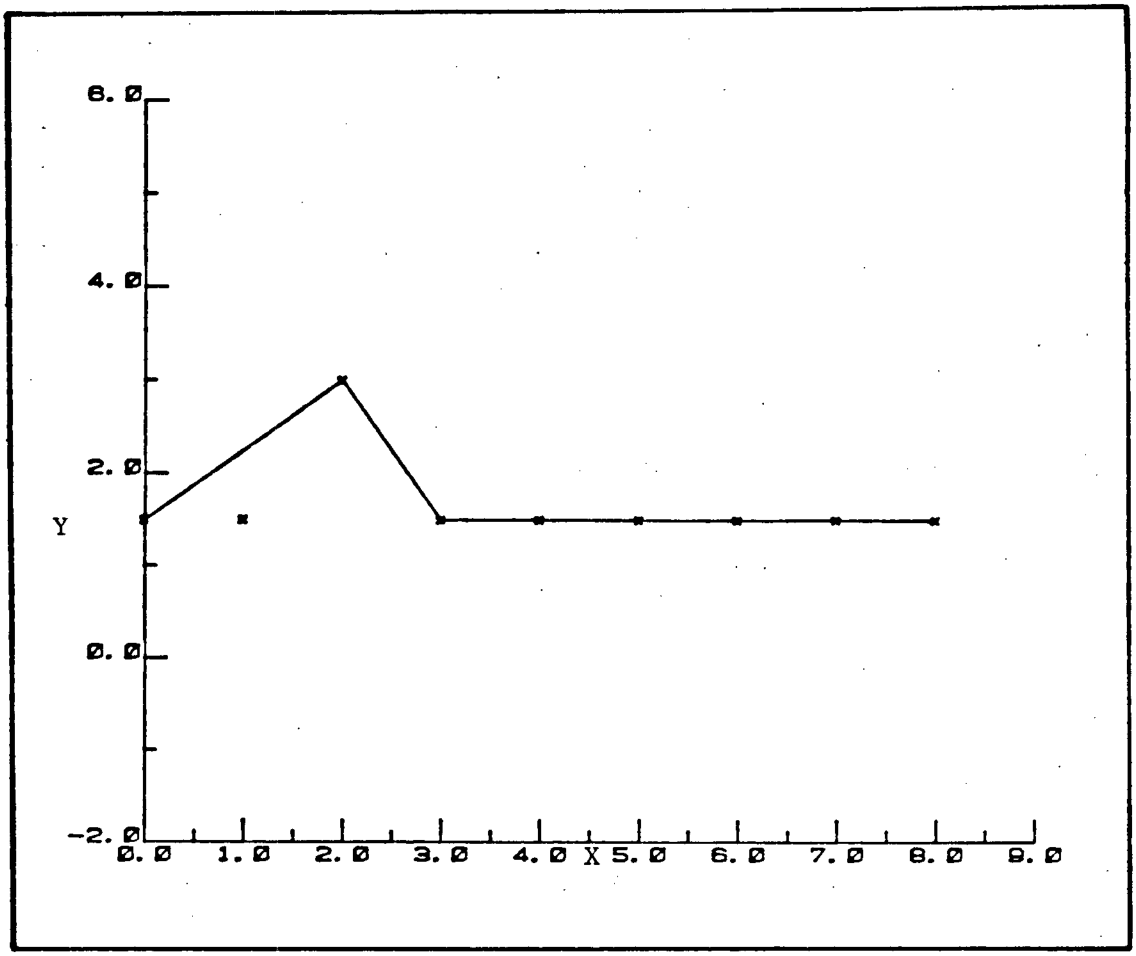


FIG. 3.19 A conic spline(CON.SPL) fit to the off-set point.

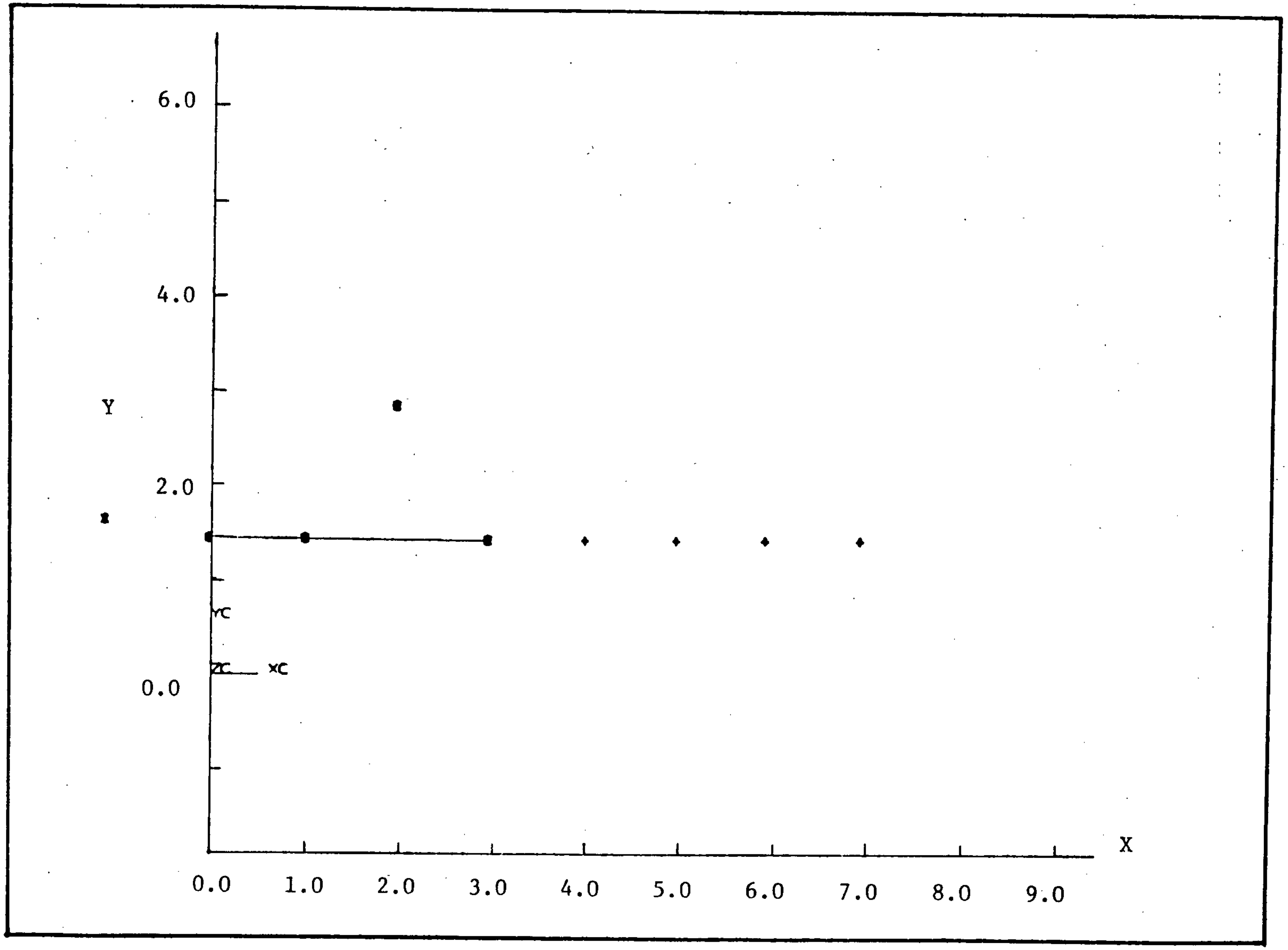


FIG. 3.20 UNIGRAPHICS conic fit to a off-set point.

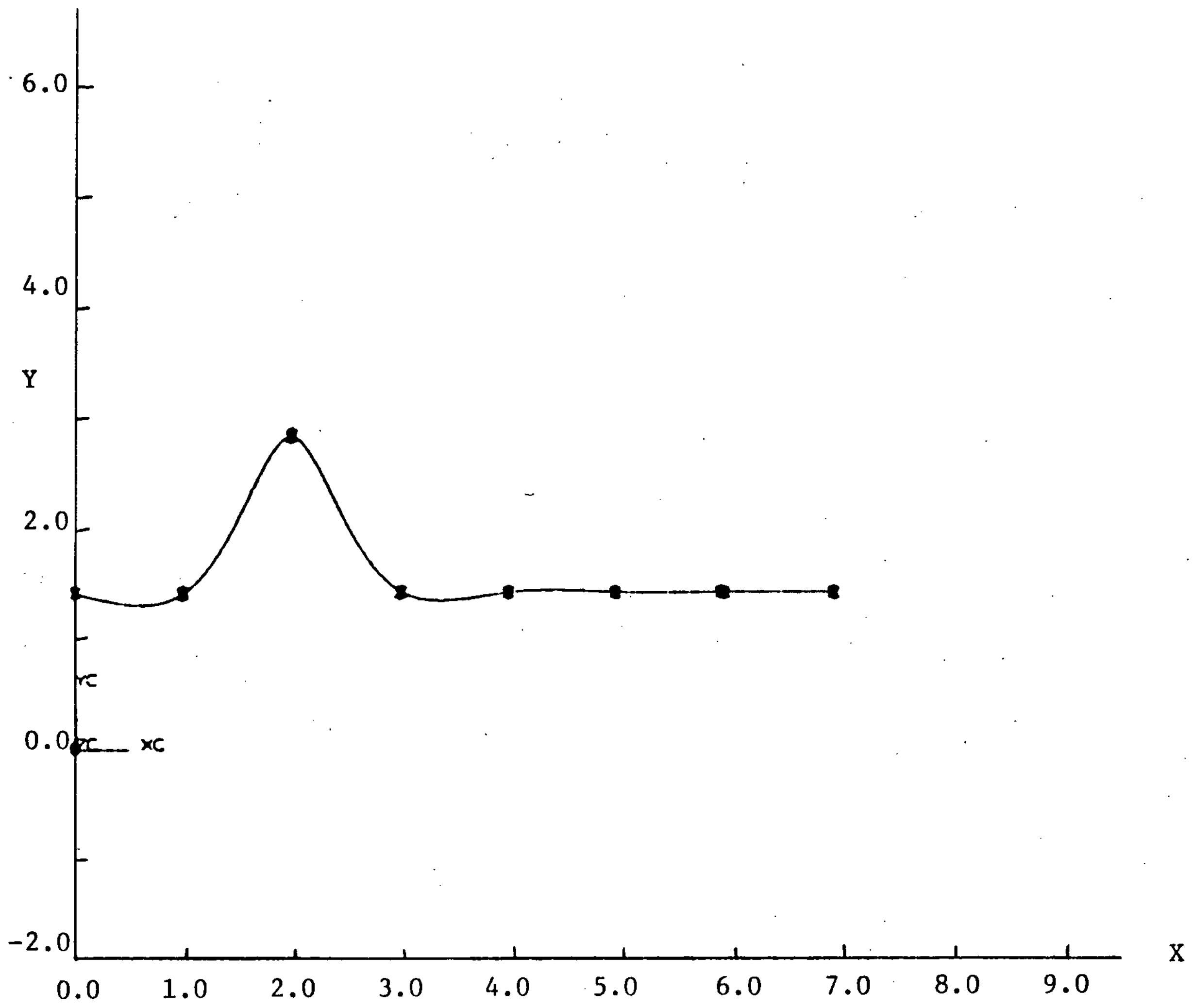


FIG. 3.21 UNIGRAPHICS spline fit to a off-set point.

CHAPTER 4

B-SPLINE AND G-SURF APPLICATIONS

4.1 Introduction

B-spline and G-SURF have been discussed in the preceding chapters. G-SURF is a Fortran computer program which utilizes B-splines, and other spline curves, to define and machine surfaces. G-SURF exploits the smoothing property of B-splines to generate smooth surface shapes. Two test objects and two industrial components were machined using this approach and are discussed below. The process of machining these objects served to prove the versatility of the approach and at the same time highlighted some interesting features.

The objects that were machined are:

1. Twisted Surface,
2. Sine curve,
3. Chain-saw guide bar, and
4. Ship hull.

4.2 Twisted Surface

This doubly curved surface can be obtained by twisting the opposite edges of a square flat plate. The surface was generated by G-SURF from a set of equi-spaced points lying on equi-spaced lines as shown in Figure 4.1. B-splines fitted straight lines to this set of points and a predetermined number of machine data points and data surface normals were generated. The object was machined on a 5-axis machine in time share mode from a 2-1/2 axis controller. The cylindrical end milling cutter is programmed through G-SURF to move along the generating lines in such a way that it has a constant inclination (10 degrees) to the surface normal in the local tangent plane, as shown in Figure 1.2. A photograph of the final machined component is given in Figure 4.2 and shows that each of the cuts is straight although each had been generated with a full range of 5-axis movements. As the cutter axis was inclined at a small angle to the local surface normal the resulting blend of adjacent cuts gave an excellent surface finish.

It was noted during machining that material removal was accomplished very effectively using the full cutting speed of the end mill.

4.3 Sine Curve

A series of sine curves were selected to generate a surface with both convex and concave sections. To give the best possible access to these shapes the G-SURF program was adapted to cut on either the leading or trailing edges as shown in Figure 4.3. Thus, deeper concave surfaces can be accessed without the spindle or the tool interfering with the object. Figure 4.3 shows G-SURF's mode of changing its cutting edge at every change in curvature.

The computer plot of the surface is shown in Figure 4.4 and photograph of the machined component in Figure 4.5. During the course of machining it became apparent that alignment of all the five axes was very important. The first few attempts to machine the sine curve produced a step at the highest and lowest point. This step was eventually explained by misalignment in the y-z plane. A computer simulation of a misalignment in this plane is given in Figure 4.6 for an error of 2 mm in the y-axis.

4.4 Chain Saw Guide Bar

For the CNC machining of a chain saw guide bar a set of digitized data points was supplied from an existing bar as shown in Figure 4.7. A major requirement was that the final guide bar should be smooth and have a minimum negative curvature.

A cubic spline curve was initially fitted to this set of data on the UNIGRAPHIC system with an instantaneous slope curve, as shown in Figures 4.8 and 4.11 respectively. Subsequently, a B-spline was fitted to the same set of data points as shown in Figure 4.9. The results clearly show that B-splines give a much smoother fit although by no means perfect.

An iterative procedure was adopted in which the original set of data was replaced by points from the last B-spline fit. The result of the 2^{nd} iteration gives a smoother curve, as is shown in Figures 4.9 and 4.10, and the derivative plots are given in Figures 4.12 and 4.13.

4.5 Ship Hull

A model of a ship hull was made to be used for stability tests in the B.C. research ship testing facility. The data for station lines was obtained for the Eastwood hull as shown in Figure 4.14. On the full size ship the station lines were 10 feet apart and the whole length of the ship was 106 feet. The selected model length was 30 inches. The ship hull surface was developed with B-splines and the final shape of the hull is given in Figure 4.15. A photograph of the completed oak model is shown in Figure 4.16.

The ship hull was machined using only the 3-axis capabilities of the CNC machine.

LEAF 4.4 MISSED IN NUMBERING.

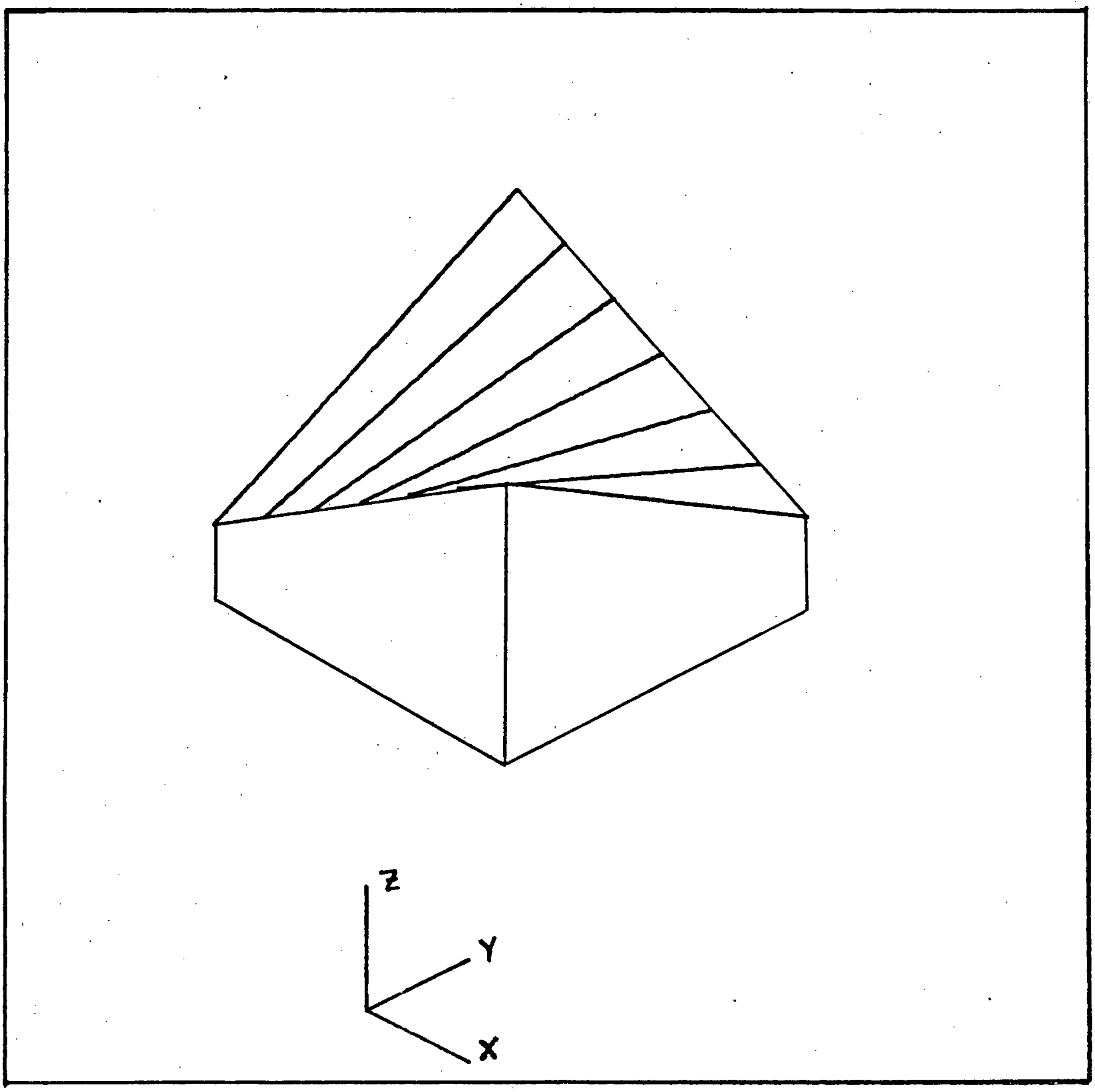


FIG. 4.1 The Twisted plane.

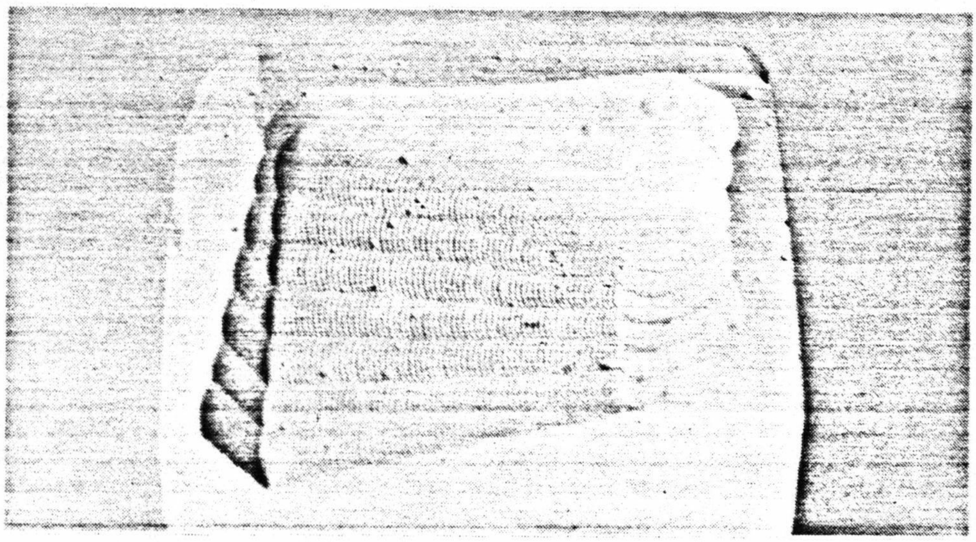


FIG.4.2 Photograph of the twisted plane.

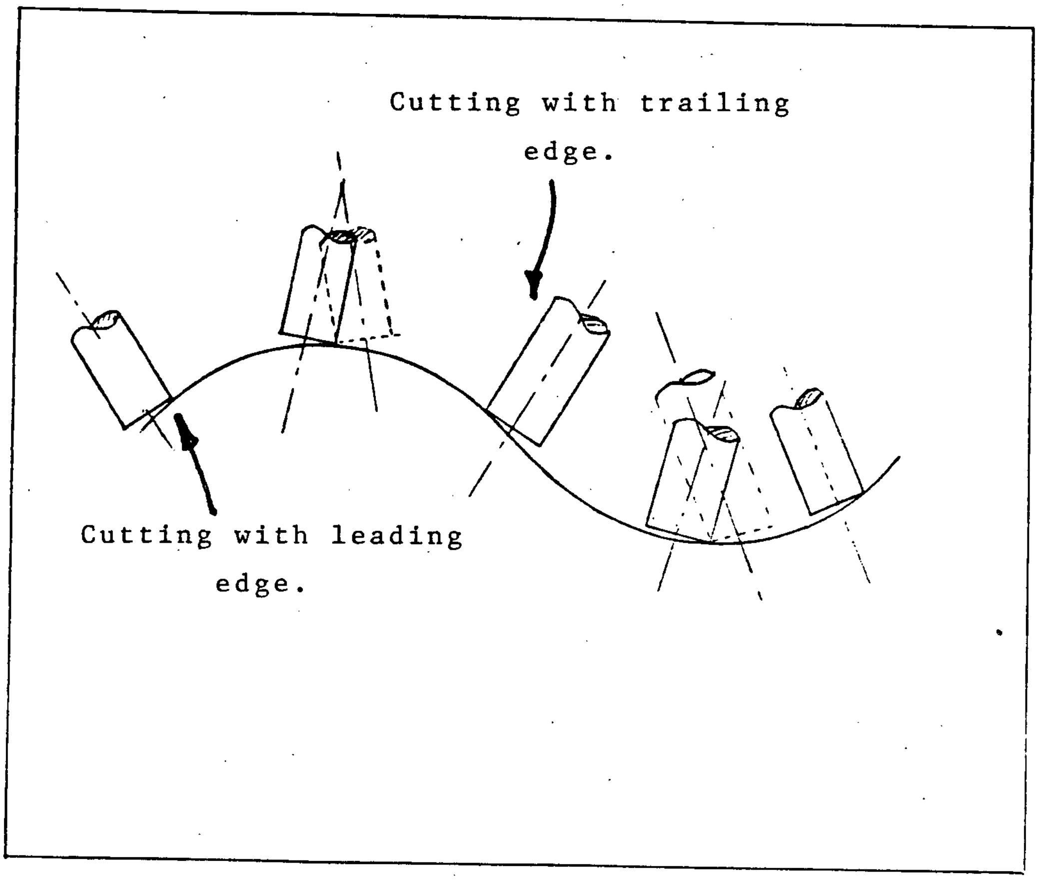


FIG. 4.3 G-SURF's mode of changing cutting edge.

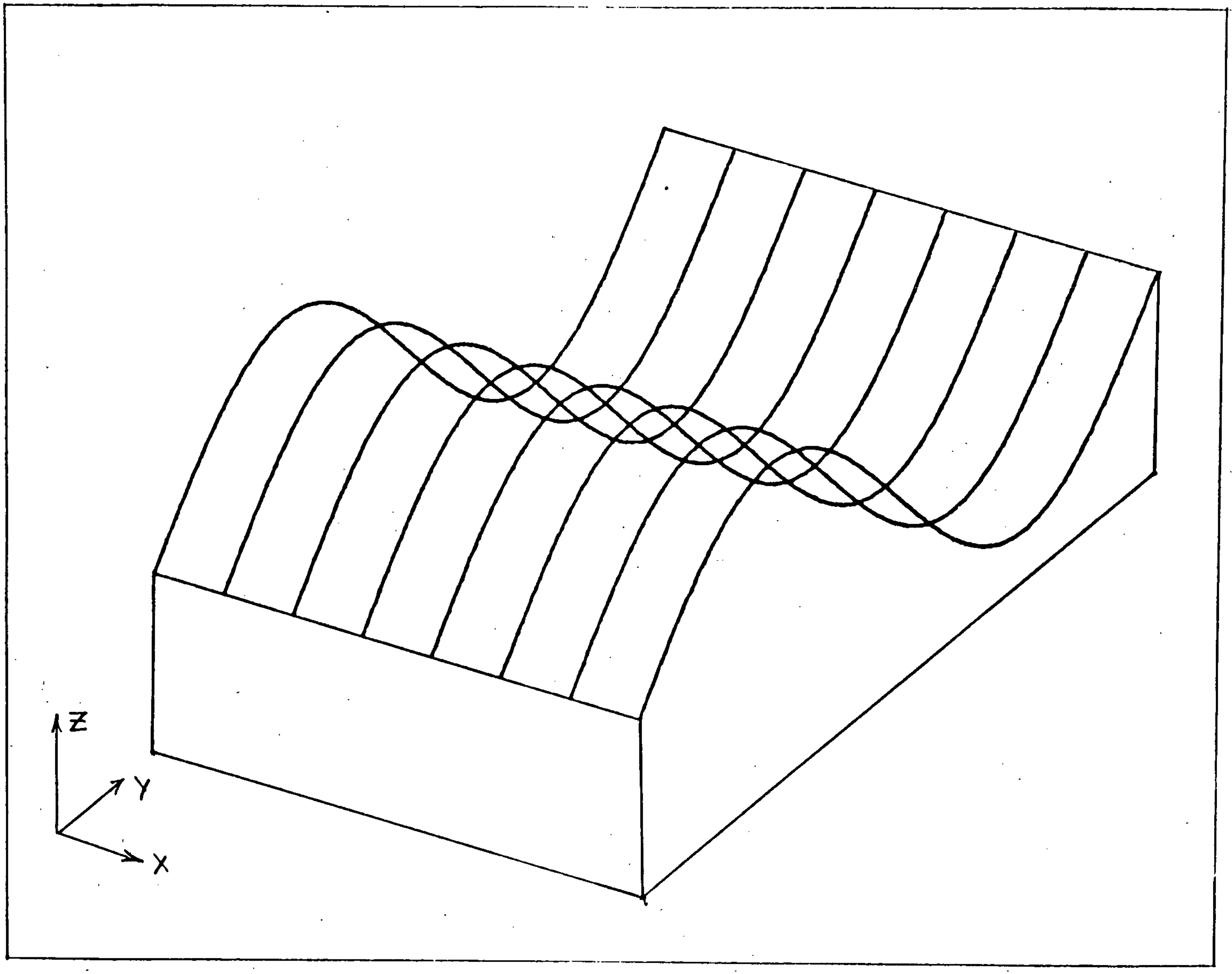


FIG. 4.4 The sine curve surface.

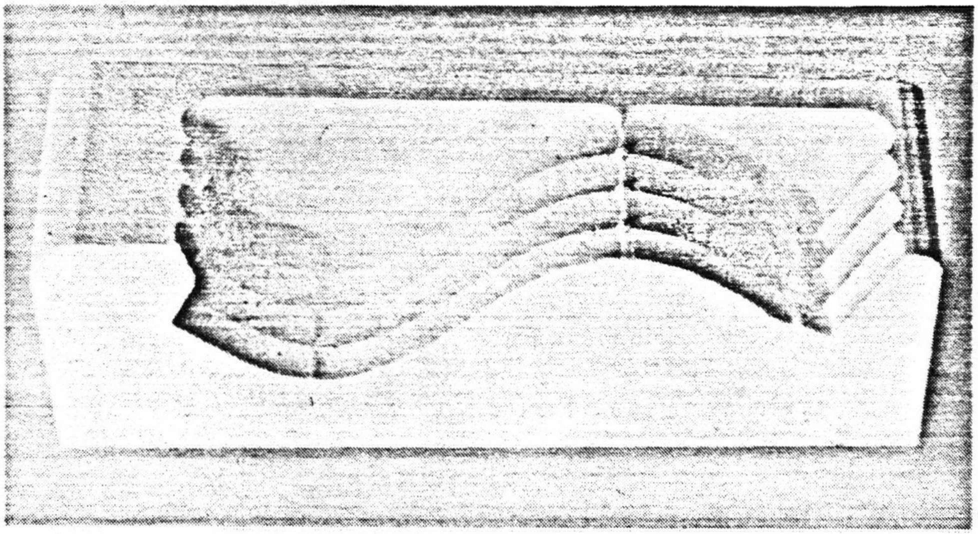


FIG. 4.5 Photograph of the sine curve surface.

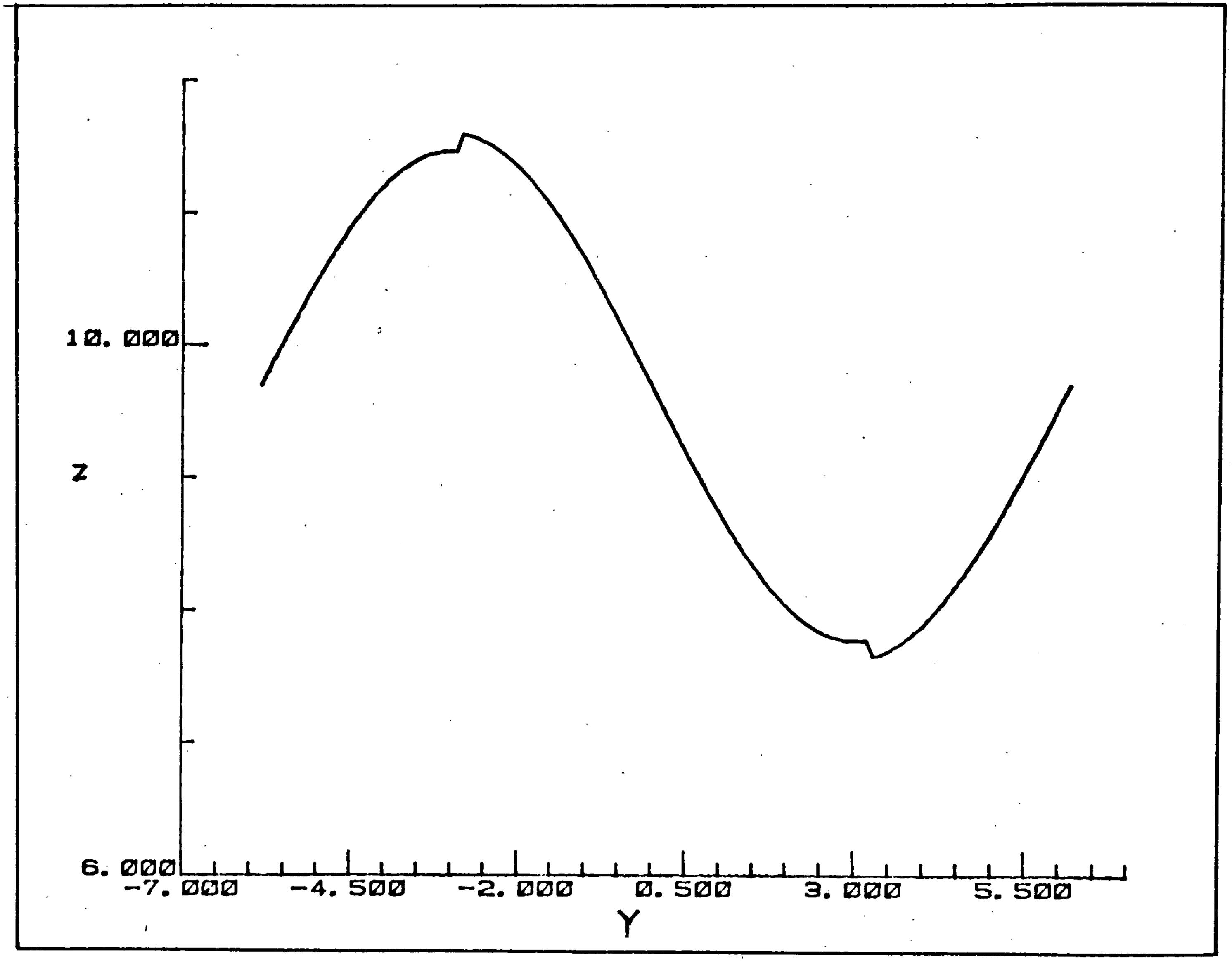


FIG. 4.6 Effect of misalignment in X-axis.

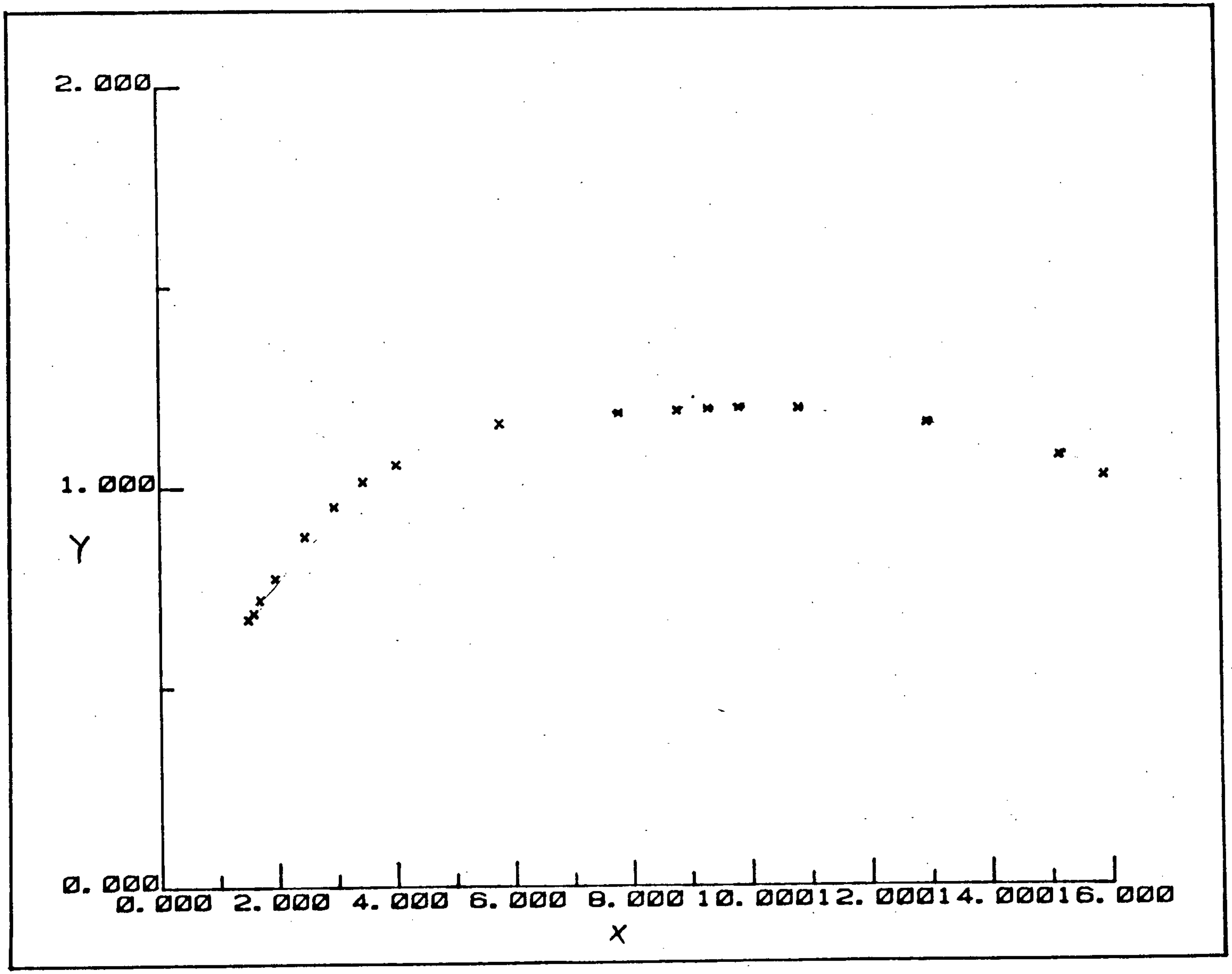


FIG. 4.7 Data points for chain saw blade.

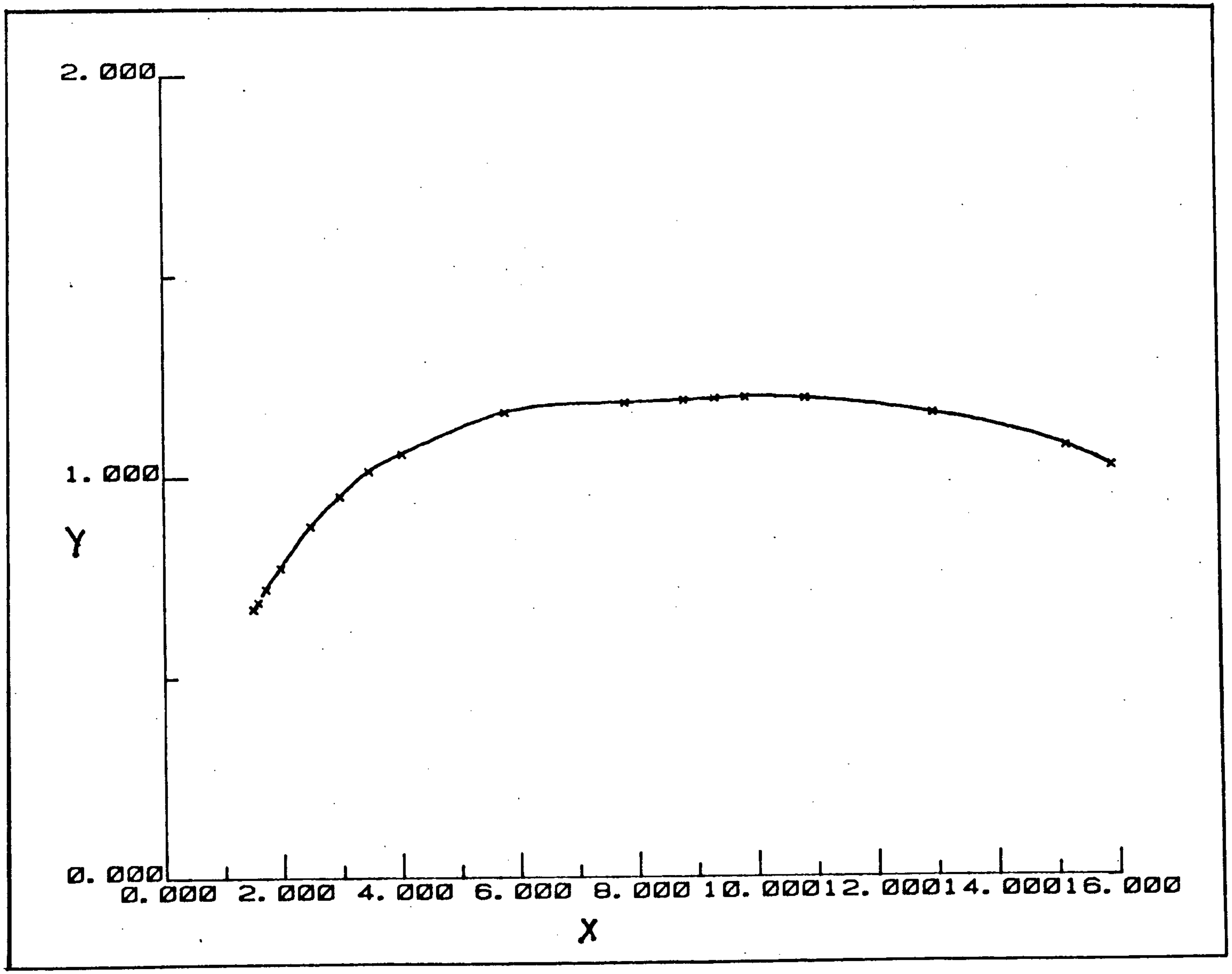


FIG. 4.8 Cubic spline fit to chain saw blade.

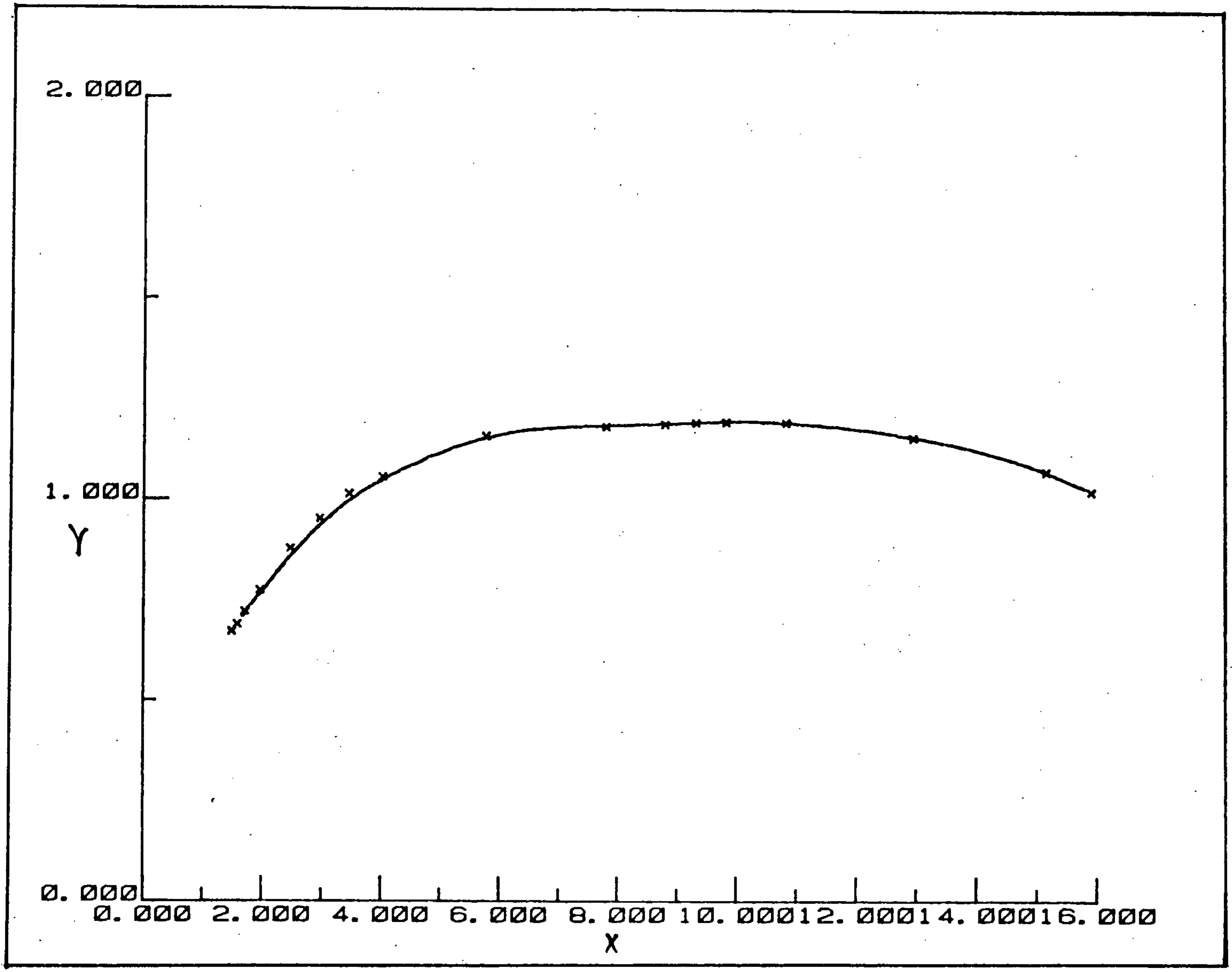


FIG. 4.9 B-Spline fit to chain saw blade.

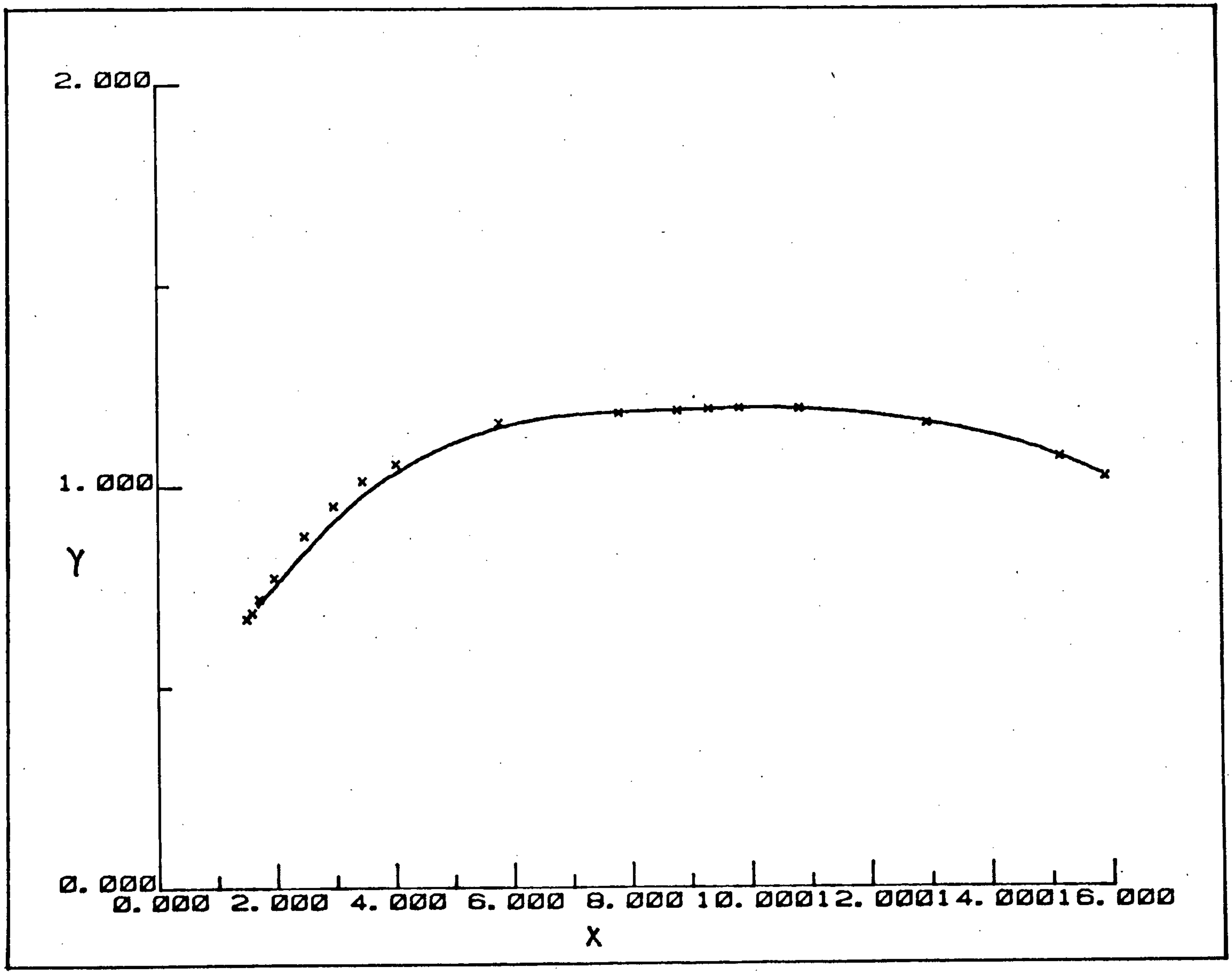


FIG. 4.10 B-Spline fit to chian saw blade, 2nd iteration.

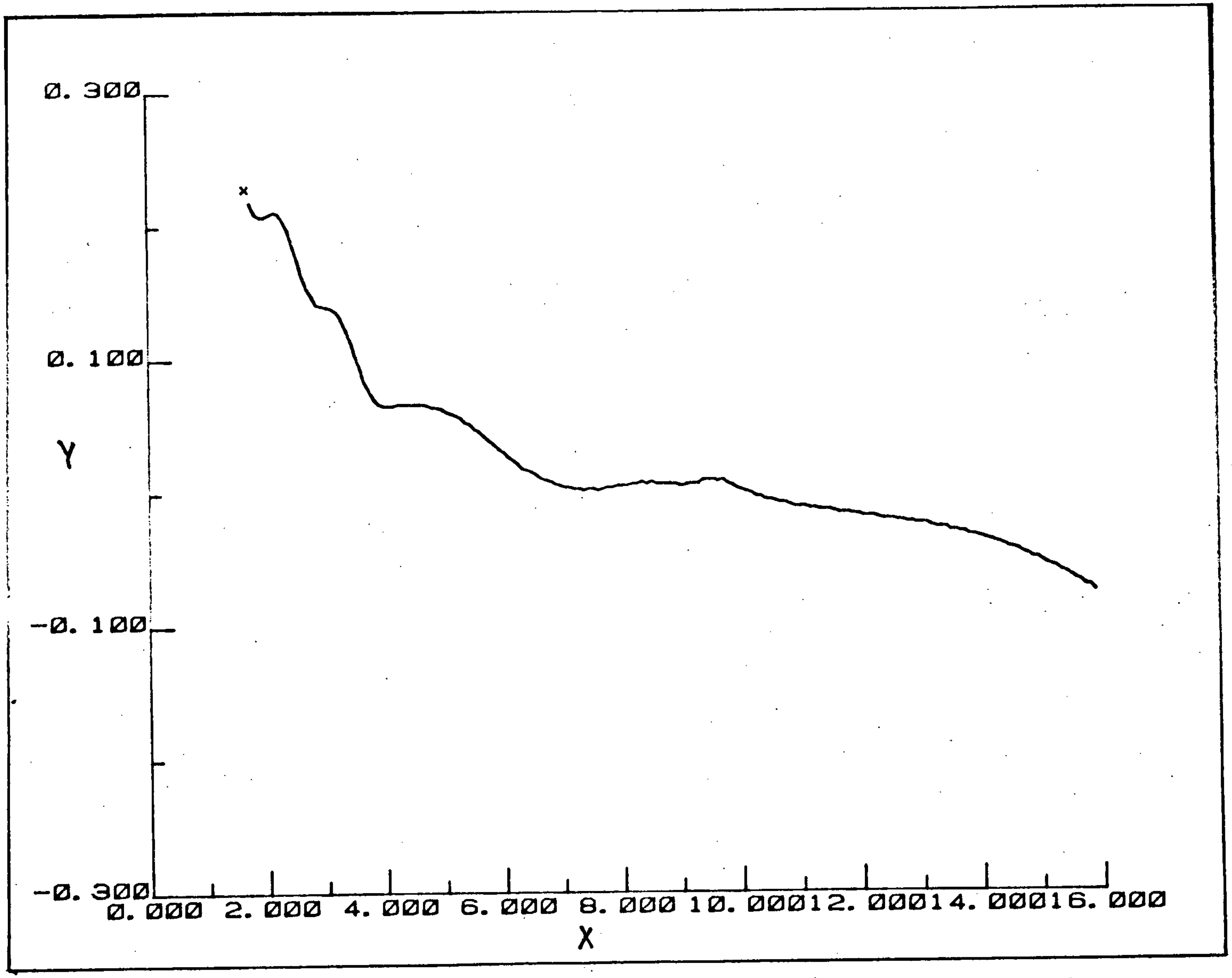


FIG.4.11 First derivative plot of cubic spline fit to chain saw blade.

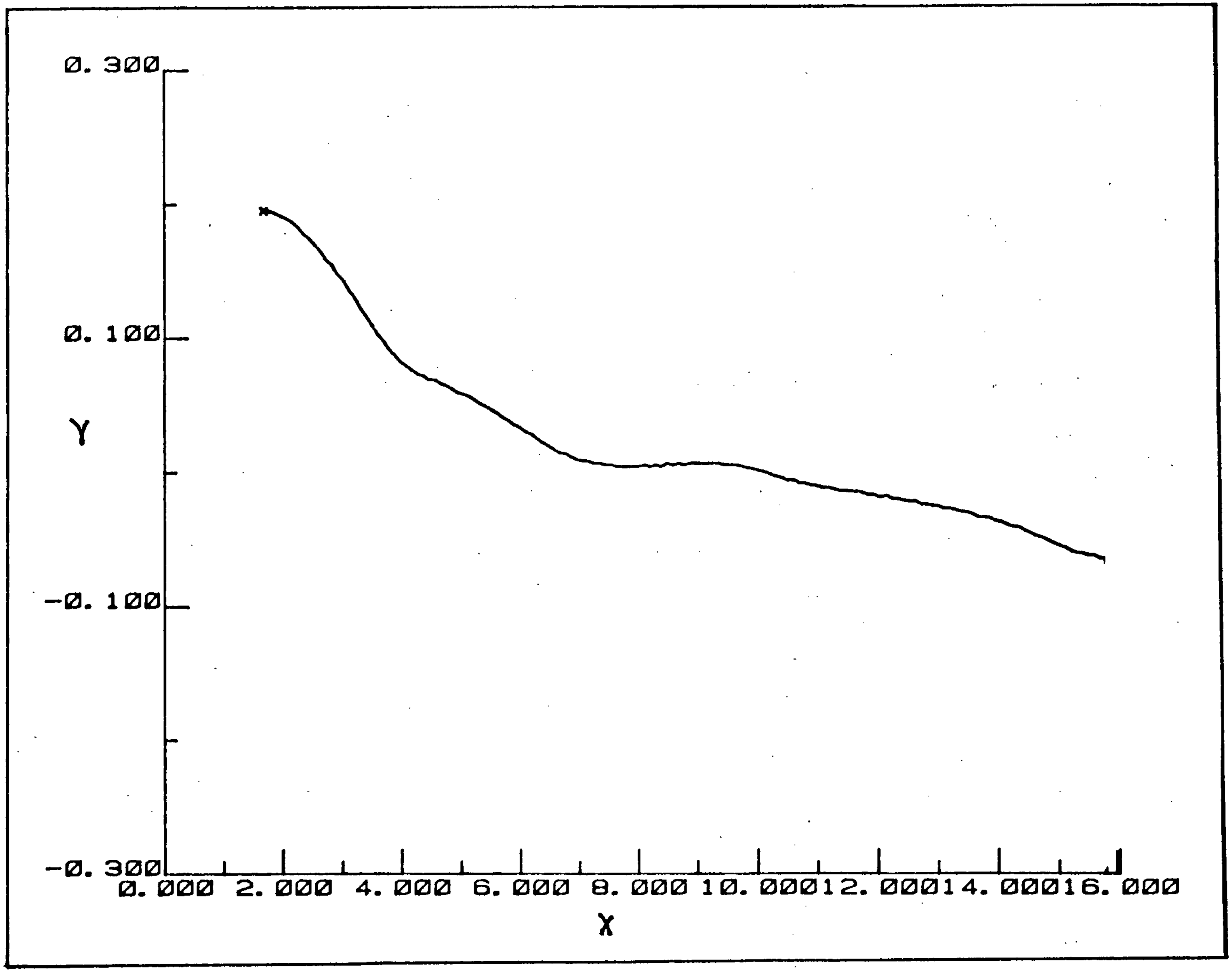


FIG. 4.12 First derivative plot of B-Spline fit to chain saw blade.

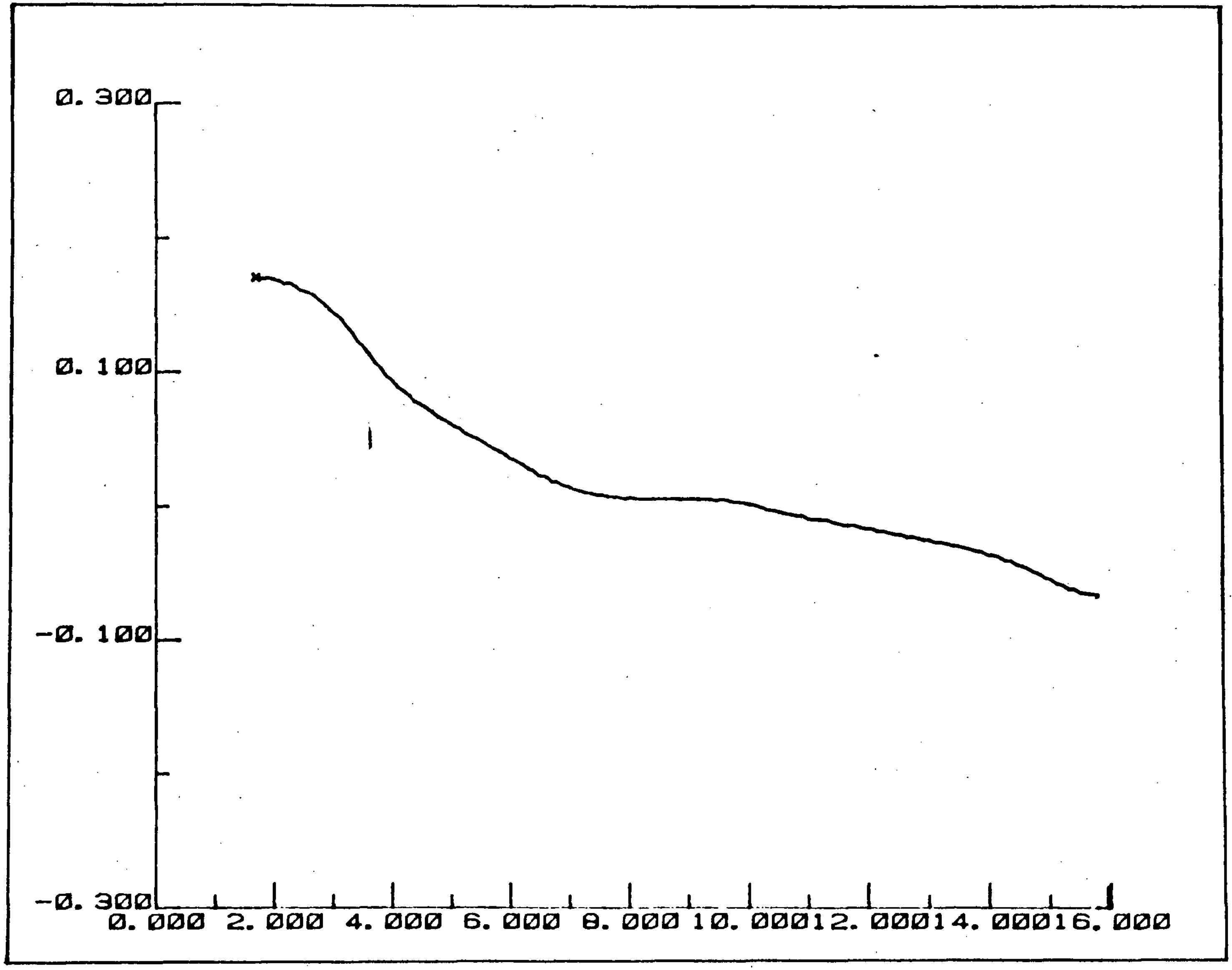


FIG. 4.13 1st derivative plot of B-spline fit (2nd iteration) to chain saw blade.

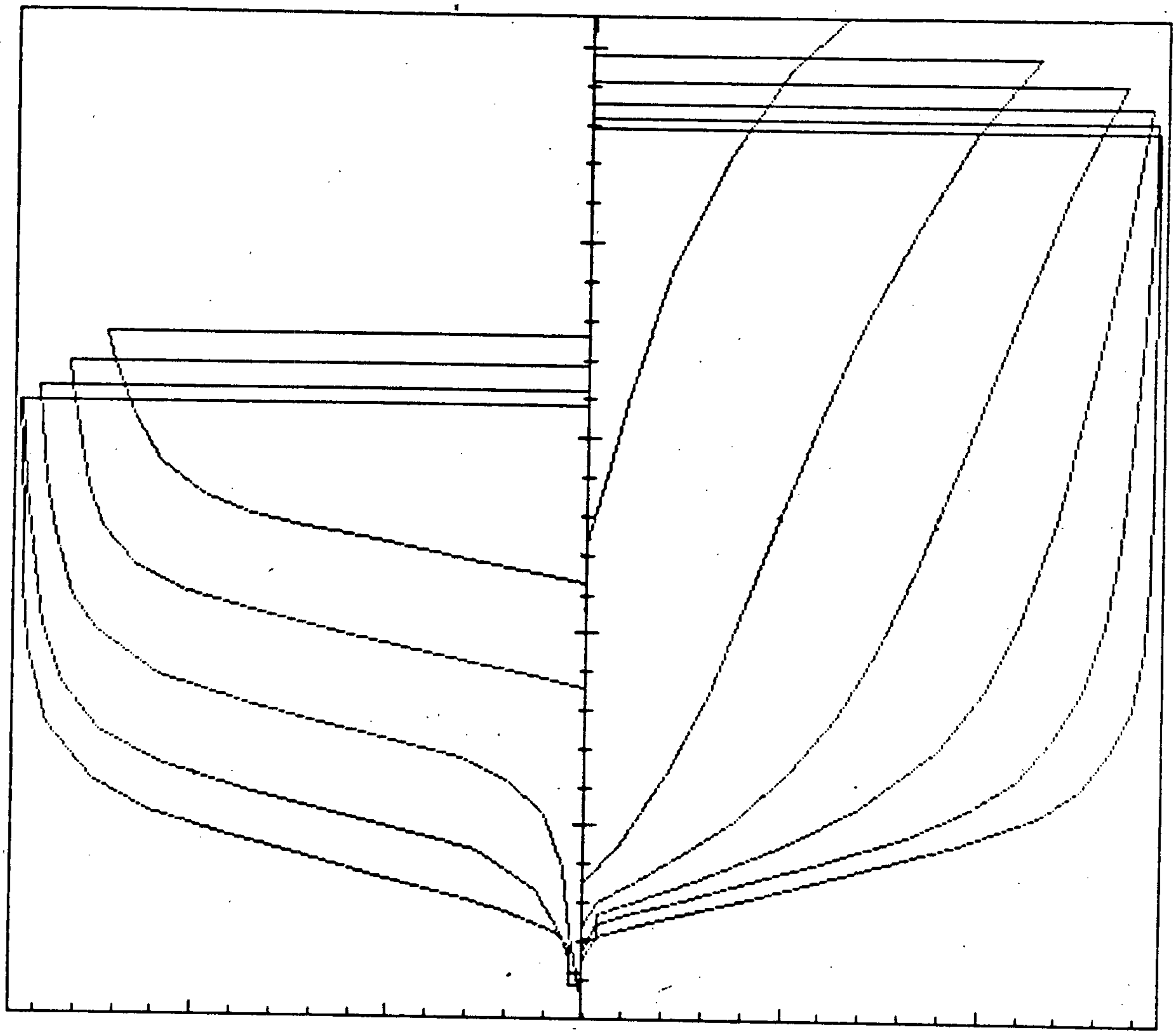


FIG. 4.14 Station lines for East-wood-hull.

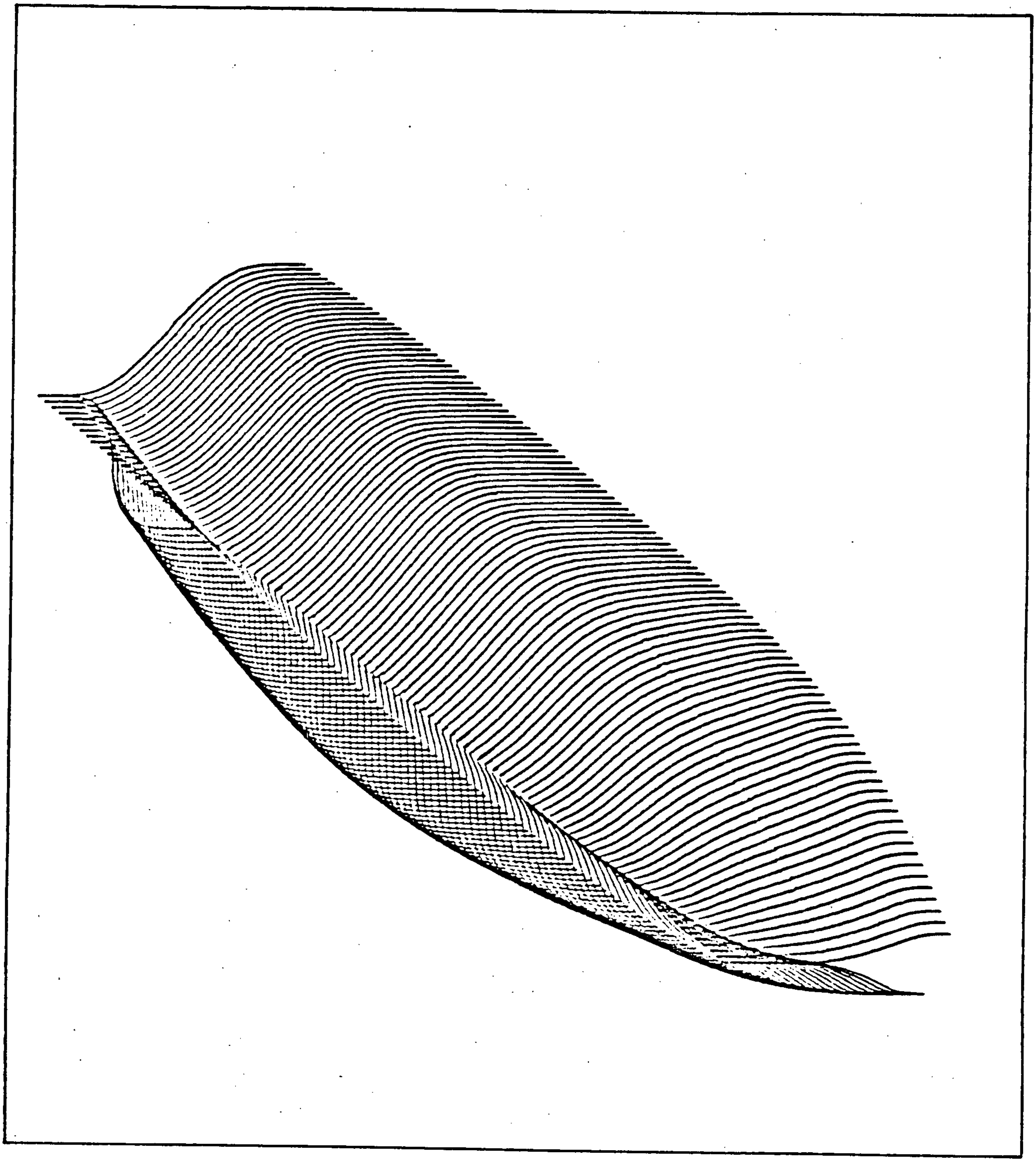


FIG.4.15 Tool path for East-Wood hull.

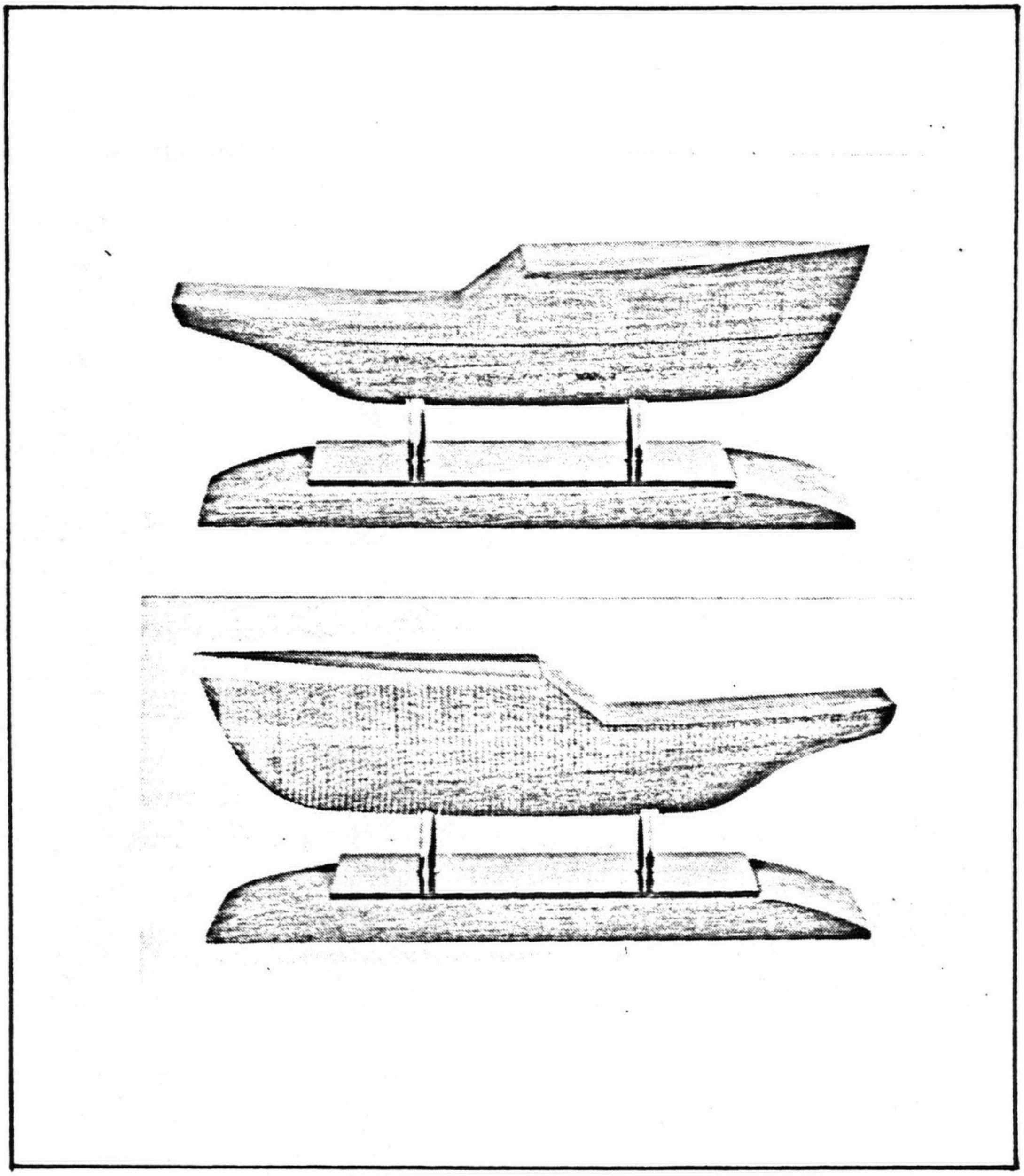


FIG. 4.16 Photograph of the ship hull.

CHAPTER 5

CONCLUSIONS

The basis of B-spline curve fitting has been examined and evaluated in relation to other curve fitting techniques. It is found that:

(a) B-spline approximation (BSAPP) fits, to a given set of data points, the smoothest curve which lies within the convex hull of the enclosing polygon.

(b) B-spline approximation (BSAPP) is variationally diminishing, i.e. it fits a straight line with a linear equation.

(c) B-spline interpolation (BSPIN) gives results very similar to cubic splines (CUB.SPL).

(d) B-spline interpolation (BSPIN) and cubic splines can fail when faced with high slopes or off-set points but otherwise they give good all purpose interpolation results.

(e) Conic spline (CON.SPL) requires the most manual intervention and in that sense is not very user friendly.

(f) Least-mean-square (LMS) and n^{th} order polynomial (9TH POLY) are not generally suitable for CAD/CAM applications.

The capabilities of G-SURF as demonstrated during the design and manufacture of the machined models are:

(a) Cutting with an end milling cutter is a very efficient means of material removal and gives good surface finish.

(b) Inclination of the end milling cutter to the surface normal enables the cutter to be positioned to minimize surface interference.

c) Positioning the end mill so that it cuts on either the leading or trailing edge increases the accessibility of the tool to deep concavities.

(d) Alignment of all the five axes on a CNC machine is critical.

(e) The computer CPU task file size required for G-SURF is approximately 55 kB which will fit on to any Fortran based mini computer.

5.1 Proposed Future Work

(a) The next phase of this work should include the study of B-spline surfaces and their comparison with other common surfaces, such as Coon's Patch, UNISURF, Ferguson's Patch, etc.

(b) Graphic relationships between different machining parameters to increase the efficiency of G-SURF.

APPENDIX I

A WORKED EXAMPLE OF CONIC SPLINE CURVE FITTING

The curve fitting routine called 'CON.SPL' fits a given set of data with piecewise conics of the general form

$$ax^2 + bxy + cy^2 + dx + ey + f = 0 \quad . \qquad \text{AI.1}$$

The curve can be guided by specifying slope at the end points or at inflection points. In the present example the data points for the S-curve are as follows:

x	0	1	2	3	4	5	6	7	8
y	2.6	1.5	1.2	2.2	4.0	4.5	3.3	1.2	2.0

The slope at the end points was specified by additional points as:

(-1.0, 4.0) for the starting slope

(8.5, -4.0) for the end slope.

The inflection point was chosen to be at (3.5, 3.0) and the slope point as (-1.8, 8.0).

To fit the first part of the spline curve the first three data points are used together with the starting slope at x = 0 and the weighted mean slope at x = 2.

At the first data point, x = 0, the slope is given as

$$\frac{dy}{dx} = \frac{4.0 - 2.6}{-1.0 - 0.0} = -1.4 \qquad \text{AI.2}$$

At the third point the weighted mean slope is

$$S_{w_3} = [S_2(x_4 - x_3) + S_3(x_3 - x_2)] / (x_4 - x_2) \qquad \text{AI.3}$$

where

$$S_2 = \frac{y_3-y_2}{x_3-x_2} = -0.3$$

$$S_3 = \frac{y_4-y_3}{x_4-x_3} = 1$$

so that at $x = 2.0$, $\frac{dy}{dx} = 0.35$.

The resulting equations for the first part of the spline curve are

$$0 + 0 + 6.76c + 0 + 2.6e + f = 0$$

$$0 + 2.66b - 7.28c + d - 1.4e + 0 = 0$$

$$1 + 1.5b + 2.25c + d + 1.5e + f = 0 \qquad \text{AI.4}$$

$$4 + 2.4b + 1.44c + 2d + 1.2e + f = 0$$

$$4 + 1.9b + .84c + d + 0.35e + 0 = 0 \ .$$

The solution of this set of simultaneous equations is

$$x^2 - 2.22xy - 1.59y^2 - 1.33x + 3.17y + 2.48 = 0 \ . \qquad \text{AI.5}$$

Similarly the equations for the other parts of the spline curve taking position and slope continuity at each junction are:

between $x = 2$ and $x = 3.5$

$$x^2 - 1.42xy + 0.58y^2 - 2.29x + 1.31y + 1.59 = 0 \ . \qquad \text{AI.6}$$

between $x = 3.5$ and $x = 5.0$

$$x^2 + 25.5xy - 12.25y^2 - 124.75x + 4.12y + 254.5 = 0 \qquad \text{AI.7}$$

between $x = 5.0$ and $x = 8.0$

$$x^2 + 0.58xy + 0.13y^2 - 12.603yx - 3.56y + 38.45 = 0 \ . \qquad \text{AI.8}$$

The plot of the conic spline curve is shown in Figure 1.6. In the last interval 4 points are fitted instead of three because a fit to the 1st, 3rd and 4th points in that interval, lies within the acceptable error limit at the 2nd point. Hence, for computational purposes the 2nd point is ignored.

In fitting curves with CON.SPL, it was found that the definition of the endpoint slope critically affected the curve fitting. If for example the end point slope was defined as being greater than or equal to the slope between the first two data points then a curve was not generated in this region as shown in Figure AI.1 and AI.2. However, if the end point slope is less than the slope between the first two points a suitable fit is obtained as shown in Figure 1.6.

UNIGRAPHIC conic fit has similar difficulties as shown in Figure AI.3, where the curve tends to curl back on itself in spite of the slope being less than the slope between the first two points. In figure AI.4 a UNIGRAPHIC conic fit to an end slope larger than the slope between the first two points is shown. The fit is poor.

In fitting curves with CON.SPL, inflection points and slope at the inflection point play a critical role. If the inflection point slope is less than slopes between the two sets of points, then a curve is not fit to the two adjacent segments of the curve as shown in Figure AI.5. If however the inflection point slope is in between the slopes of the two sets of points only one segment is fitted with a curve as shown in Figure AI.6. For an acceptable fit the inflection point slope is greater than the slopes between the two sets of points, as shown in Figure 1.6.

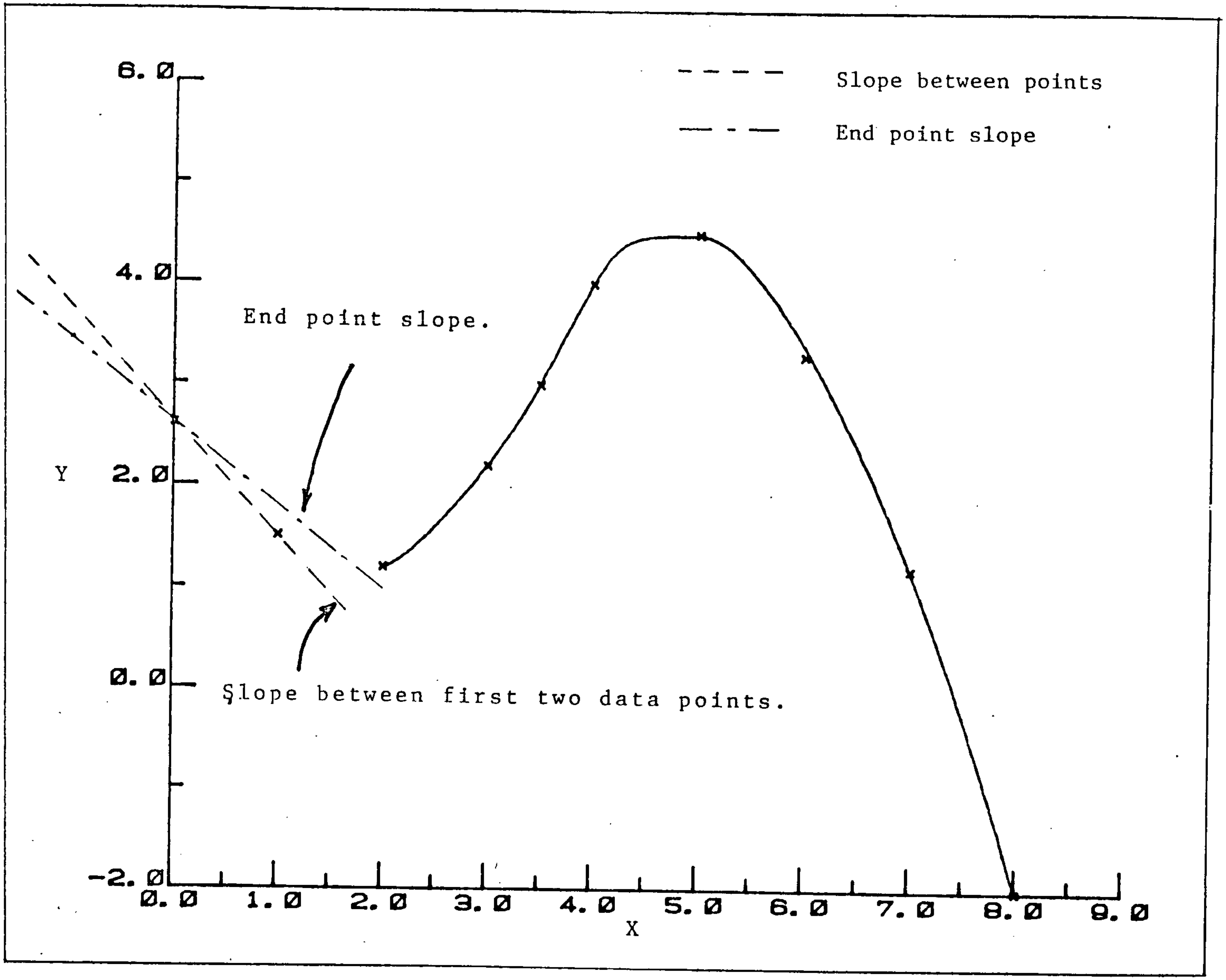


FIG. AI.1 Conic spline fit when end slope greater than slope between points.

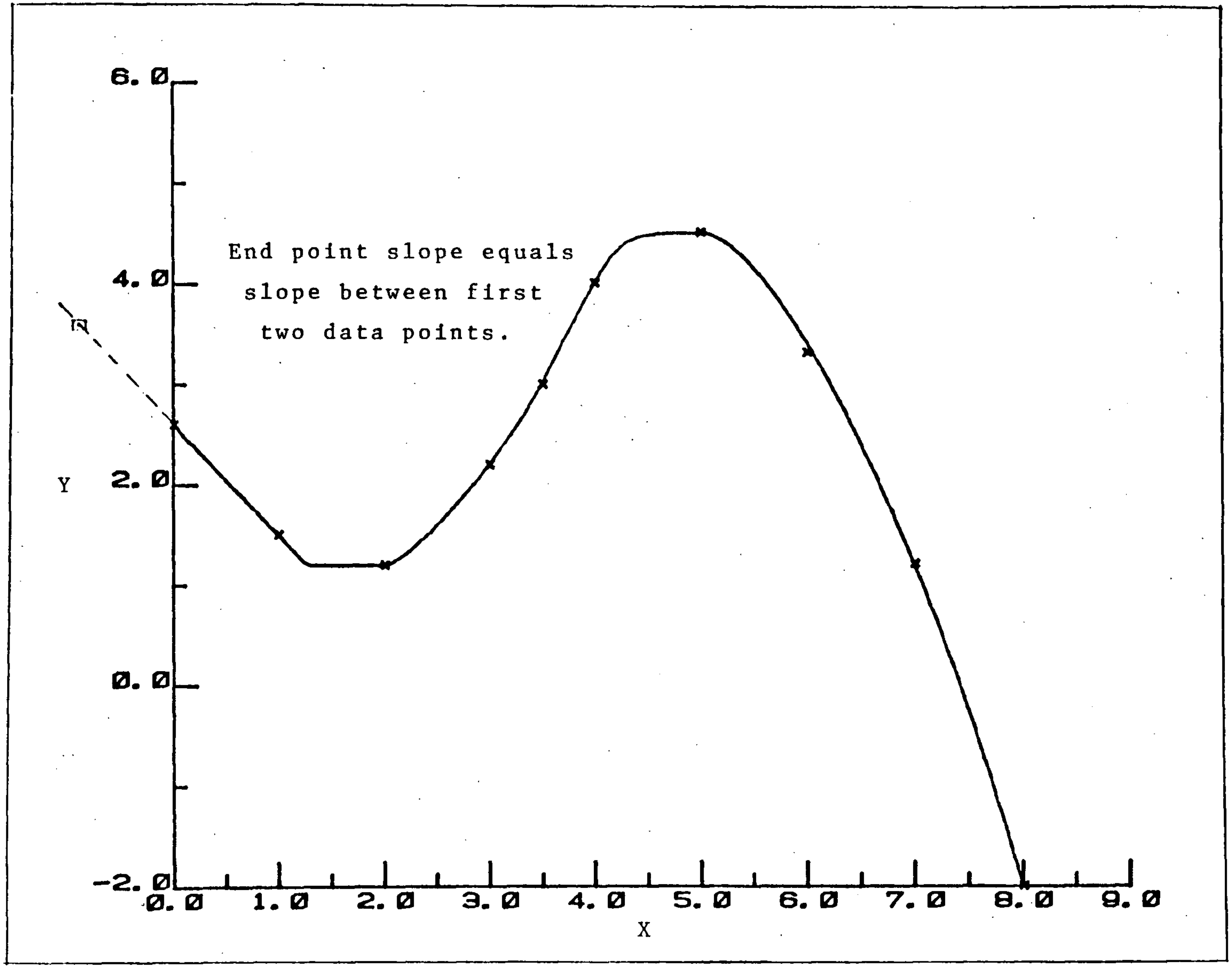


FIG.AI.2 Conic spline fit when end slope equals slope between points.

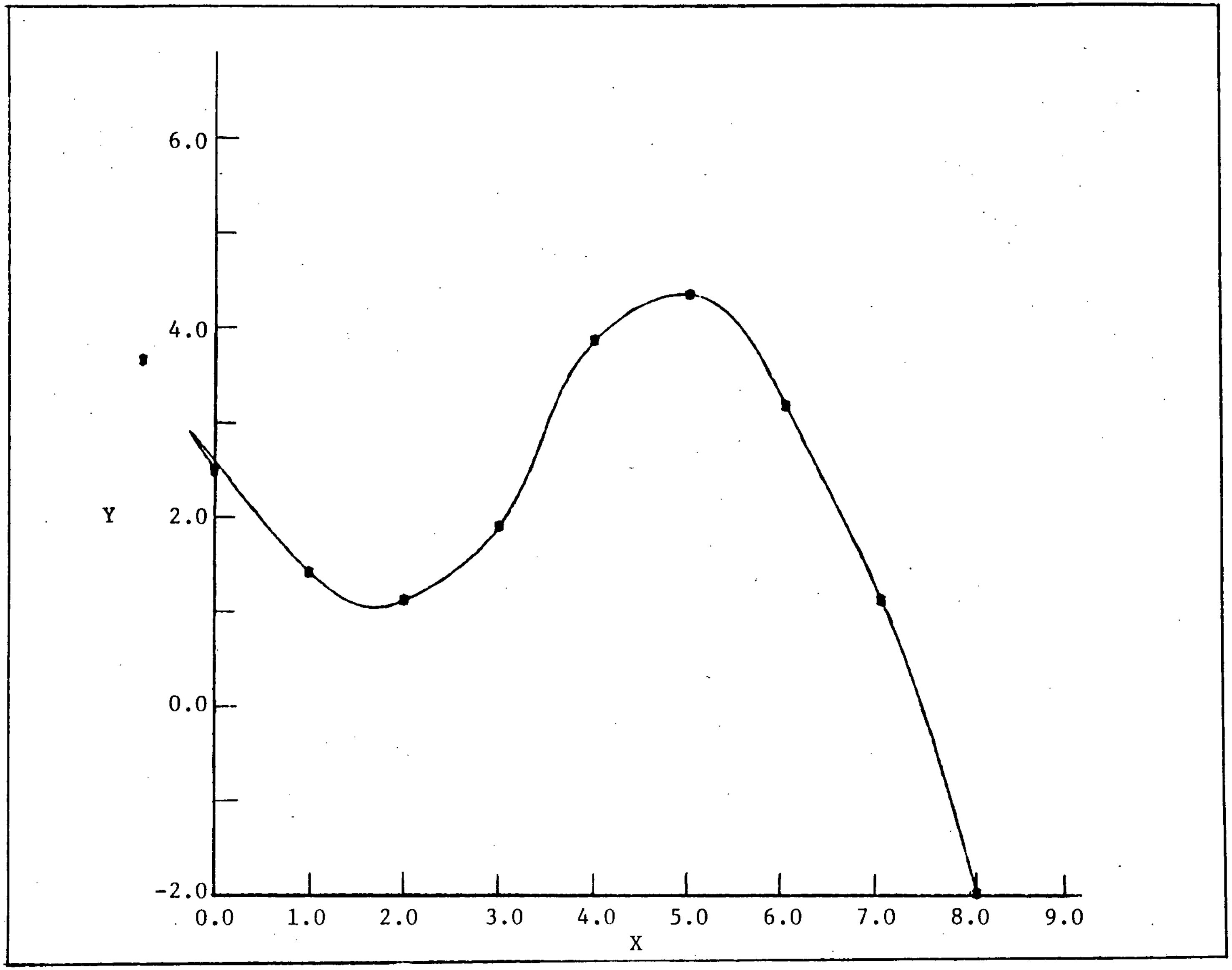


FIG. AI.3 UNIGRAPHICS Conic fit to end slope greater than slope between points.

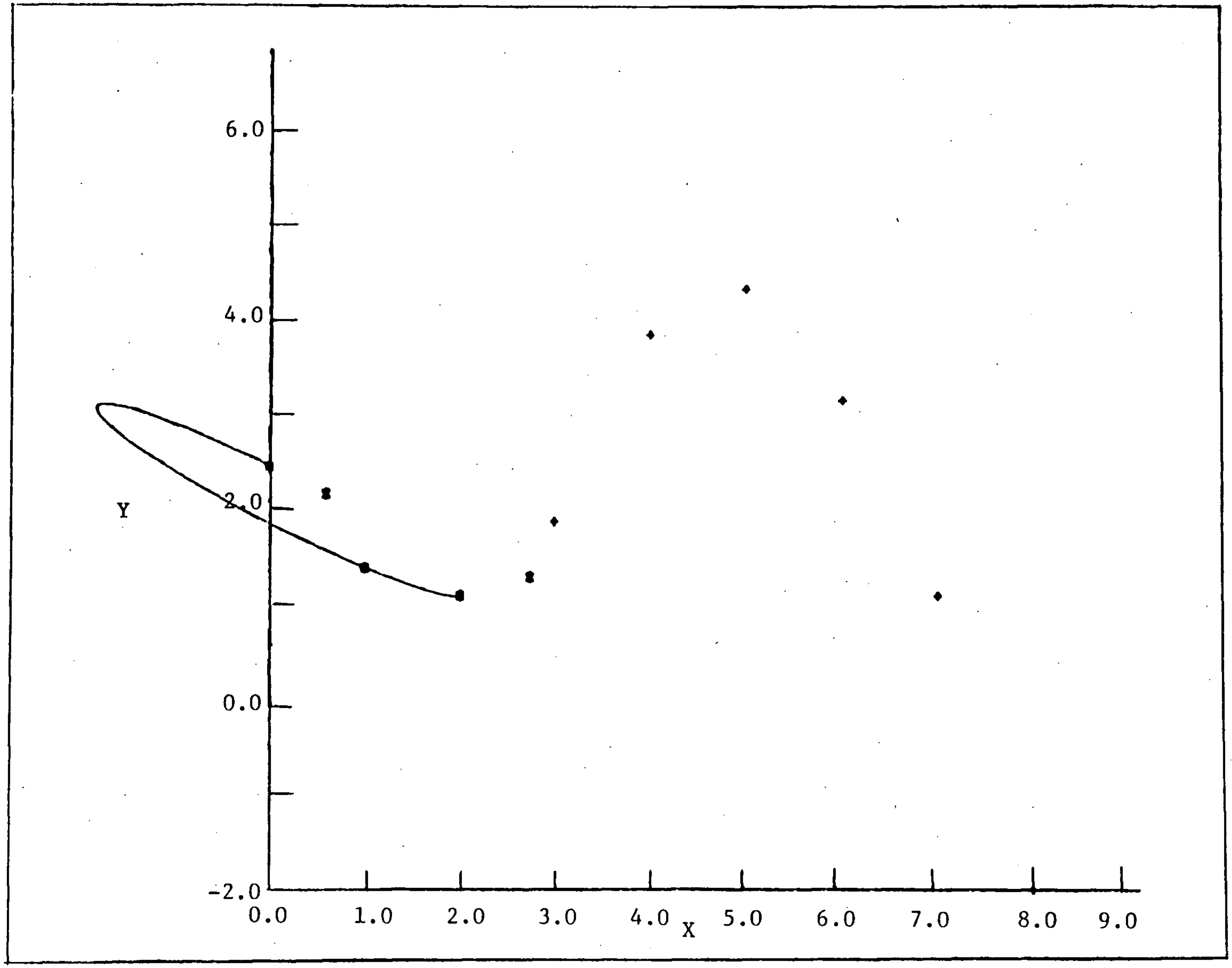


FIG.AI.4 UNIGRAPHIC Conic fit to end slope less than slope between points.

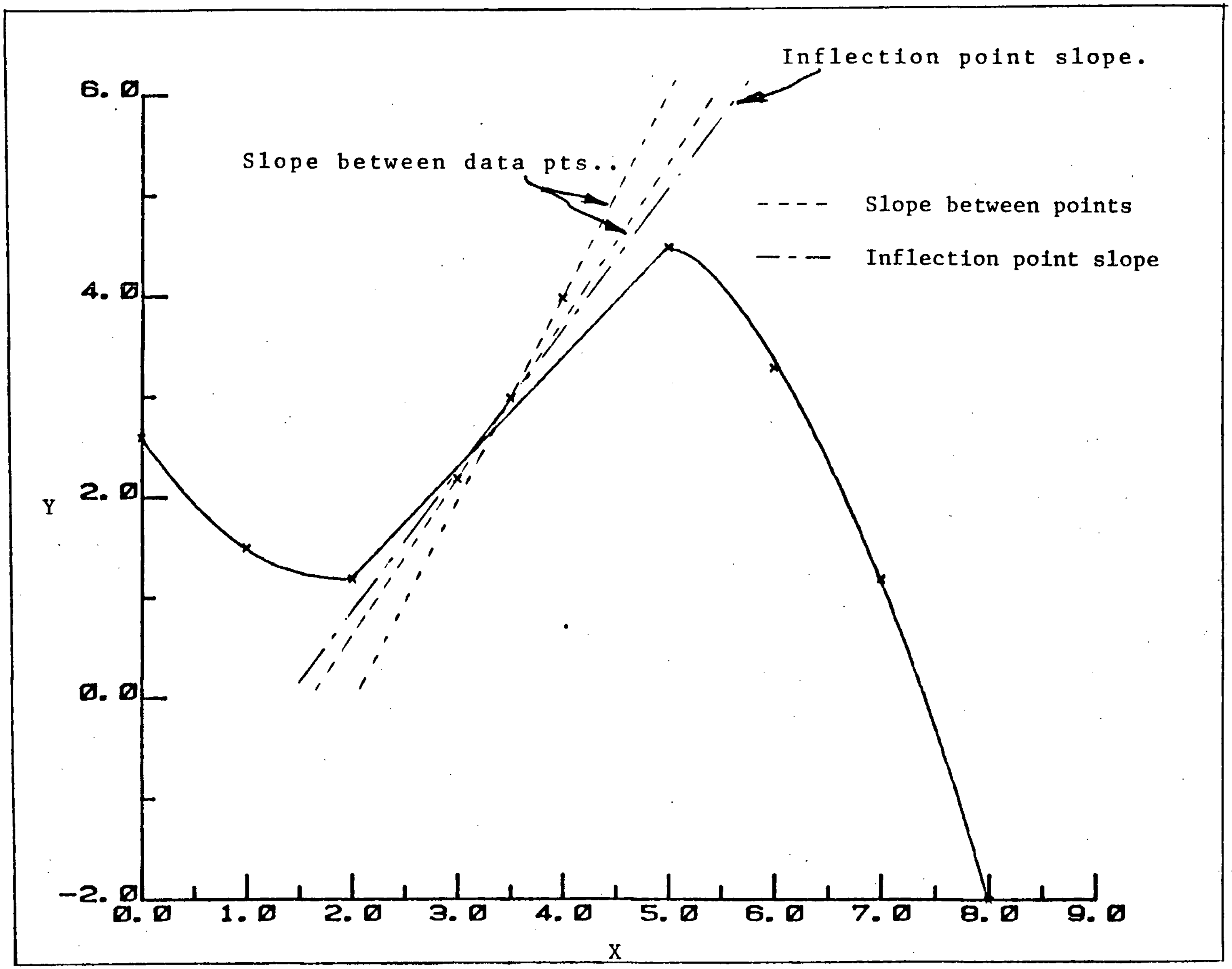


FIG. AI.5 Conic spline fit to inflection point slope less than slope between points.

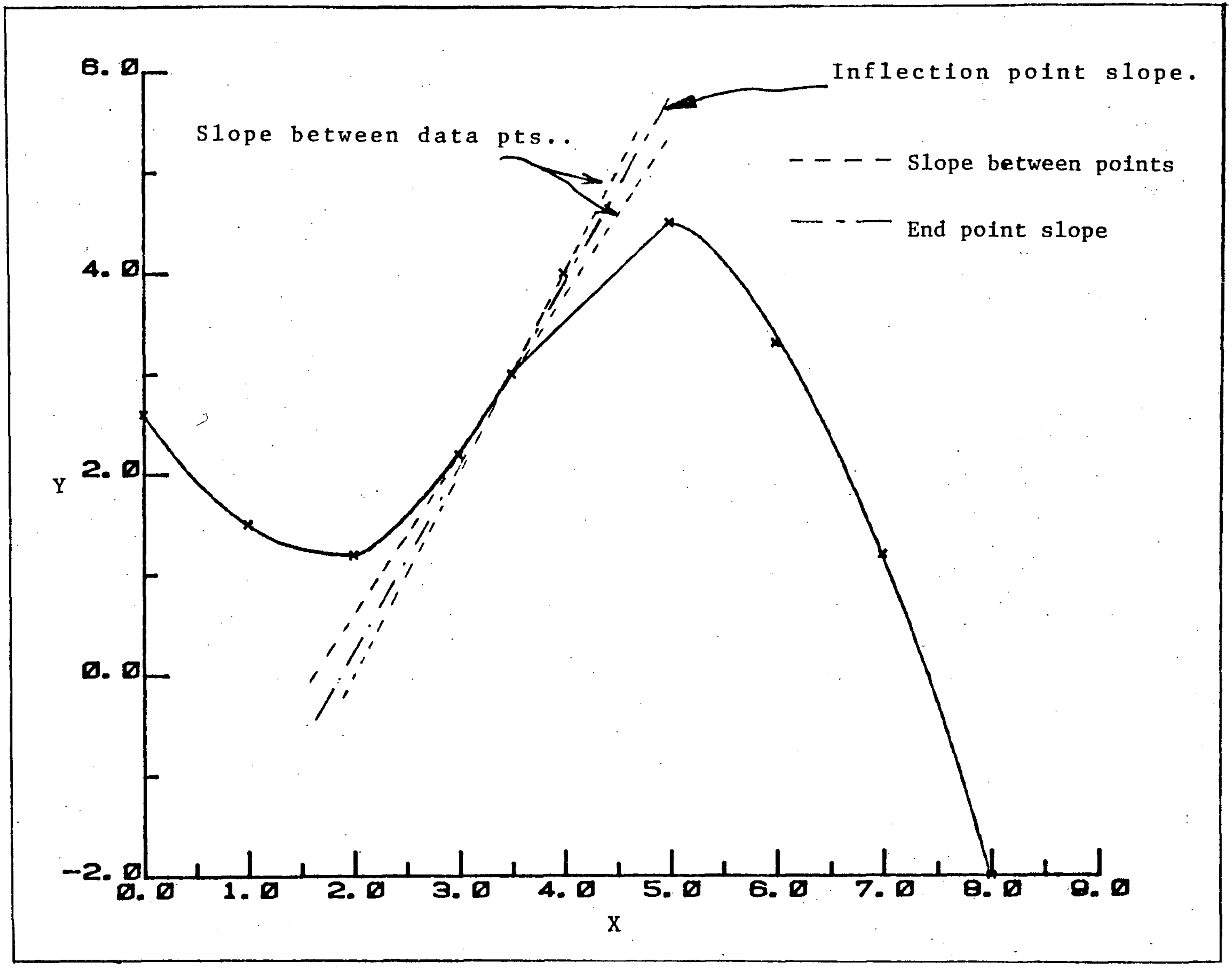


FIG. AI.6 Conic spline fit for inflection point slope between slope of data points.

APPENDIX II

A WORKED EXAMPLE OF CUBIC SPLINE CURVE FITTING

The curve fitting routine called 'CUB.SPL' fits a given set of of data with piecewise cubics of the general form

$$g(x) = a_i + b_i(x-x_i) + c_i(x-x_i)^2 + d_i(x-x_i)^3 \qquad \text{AII.1}$$

where

$$x_i \leq x < x_{i+1} \ .$$

Given a set of ordinates $x_1 < x_2 \cdots < x_n$ a cubic spline can be fit over the region $[x_1,x_n]$ in small cubic parabolas which have the above general equation and at junctions x_i have point, slope and curvature continuity.

$$g(x) = a_i + b_i(x-x_i) + c_i(x-x_i)^2 + d_i(x-x_i)^3$$

$$g'(x) = b_i + 2c_i(x-x_i) + 3d_i(x-x_i)^2 \qquad \text{AII.2}$$

$$g''(x) = 2c_i + 6d_i(x-x_i) \ .$$

Let us take the following set of data points for the S-curve.

x	0	1	2	3	4	5	6	7	8
y	2.6	1.5	1.2	2.2	4.0	4.5	3.3	1.2	-2

Since the slope and curvature are not specified at the ends, we fit a cubic polynomial to the first three intervals (four data points) and thereafter we fit cubic polynomials in each subsequent interval ensuring point, slope and curvature continuity.

To fit a curve between x_1 and x_4

$$g(x) = a_1 + b_1x + c_1x^2 + d_1x^3$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \end{bmatrix} \times \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix} = \begin{bmatrix} 2.6 \\ 1.5 \\ 1.2 \\ 2.2 \end{bmatrix} \qquad \text{AII.3}$$

$$a_1 = 2.6$$

$$b_1 = -1.3333$$

$$c_1 = .15$$

$$d_1 = .083.$$

To fit a curve between x_4 and x_5.

$$g(x_4) = g(3) = 2.2 .$$

$$g'(x_4) = b_1 + 2c_1x + 3d_1x^2 = 1.816$$

$$g''(x_4) = 2c_1x + 6d_1x = 1.8$$

$$g(x_5) = 4.0$$

Using these four conditions the following equations are obtained and can be solved to give the four coefficients a_2, b_2, c_2 and d_2.

$$\begin{bmatrix} 1 & 3 & 9 & 27 \\ 0 & 1 & 6 & 27 \\ 0 & 0 & 2 & 18 \\ 1 & 4 & 16 & 64 \end{bmatrix} \times \begin{bmatrix} a_2 \\ b_2 \\ c_2 \\ d_2 \end{bmatrix} = \begin{bmatrix} 2.2 \\ 1.8166 \\ 1.8 \\ 4.0 \end{bmatrix} \qquad \text{AII.4}$$

$$a_2 = 29.5984$$

$$b_2 = -28.3316$$

$$c_2 = 9.1494$$

$$d_2 = -0.9166.$$

The cubic equation for the curve between x_4 and x_5 is given as

$$g(x) = 29.5984 - 28.3316\ x + 9.1494\ x^2 - 0.9166\ x^3 \quad . \qquad \text{AII.5}$$

Similarly the equations for regions x_5 to x_9 can be fitted as:

region $\underline{x_5 - x_6}$

$$g(x) = -124.206 + 87.0247\ x - 19.6897\ x^2 + 1.4866\ x^3 \qquad \text{AII.6}$$

region $\underline{x_6 - x_7}$

$$g(x) = 740.724 - 431.9298\ x + 84.1005\ x^2 - 5.4327\ x^3 \qquad \text{AII.7}$$

region $\underline{x_7 - x_8}$

$$g(x) = -356.5129 + 72.6732\ x + 3.66795\ x^2 - 0.96422\ x^3 \qquad \text{AII.8}$$

region $\underline{x_8 - x_9}$

$$g(x) = -11353.3236 + 4785.5914\ x - 669.6061\ x^2 + 31.0964\ x^3 \qquad \text{AII.9}$$

A plot of these cubic spline curves is shown in Figure 1.7.

APPENDIX III

A WORKED EXAMPLE OF B-SPLINE CURVE FITTING

The curve fitting routines called 'BSAPP' and 'BSPIN' fit a given set of data with piecewise polynomials as defined in the text.

The data points for the S-curve are as follows:

x	0	1	2	3	4	5	6	7	8
y	2.6	1.5	1.2	2.2	4	4.5	3.3	1.2	-2.0

Both B-spline curve fitting methods go through the following sequence:

(a) Generate a knot sequence

There are 9 data points.

Hence, there are 15 knots.

The knot sequence t_1 to t_{15} is

0, 0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8, 8, 8, 8

(b) Generate the basis functions

Once the knot sequence has been generated the basis functions can be obtained from equations 3 and 4. The divided difference recurrence relationship is given in the text. However, for computational ease and computing efficiency, Carl de Boor's algorithm is used within G-SURF[5]. This algorithm goes through the following steps to produce the equations of the basis function in each interval.

Given a knot sequence $[t_i \cdots\cdots t_n]$ and k, the order of the B-spline to be generated, the basis functions can be obtained by the following steps:

1. $b_1 := 1$

2. for $j = 1, \cdots k-1$ do.

2.1 $\delta_j^R := t_{i+j} - x$.

2.2 $\delta_j^L := x - t_{i+1-j}$.

2.3 Saved := 0 .

2.4 for r = 1, ••• j, do.

2.4.1 Term := $b_r/(\delta_r^R + \delta_{j+1-r}^L)$.

2.4.2 b_r := Saved + δ_r^R * Term.

2.4.3 Saved := δ_{j+1-r}^L * Term.

2.5 b_{j+1} := Saved.

The basis functions can be generated using the de Boor algorithm.

Let $0 \leq x < 1$

then $i = 4$, because $t_4 \leq x < t_5$

$b_1 = 1$

$j = 1$

$\delta_1^R = (1-x)$

$\delta_1^L = x$

$r = 1$

$$\text{Term} = \frac{1}{1-n+n} = 1$$

$b_1 = (1-x)$

Saved $= x$

$b_2 = x$

$j = 2$

$\delta_2^R = (2-x)$

$\delta_2^L = x$

$r = 1$

$$\text{Term} = \frac{1-x}{1-x+n} = (1-x)$$

$b_1 = (1-x)^2$

$\text{Saved} = x(1-x)$

$r = 2$

$$\text{Term} = \frac{x}{(2-x)+x} = \frac{x}{2}$$

$b_2 = x(1-x) + (2-x)\frac{x}{2}$

$\text{Saved} = \frac{x^2}{2}$

$b_3 = \frac{x^2}{2}$

$j = 3$

$\delta_3^R = (3-n)$

$\delta_3^L = x$

$r = 1$

$$\text{Term} = \frac{(1-x)^2}{(1-x) + n} = (1-x)^2$$

$\underline{b_1 = (1-x)^3}$

$\text{Saved} = x(1-x)^2$

$r = 2$

$$\text{Term} = \frac{x(1-x) + \frac{x}{2}(2-x)}{(2-x) + x}$$

$\underline{b_2 = x(1-x^2 + (2-x)(1-x)\frac{x}{2} + (2-x)^2\frac{x}{4}}$

$\underline{\text{Saved} = \frac{x^2}{2}(1-x) + \frac{x^4}{4}(2-x) + (3-x)\frac{x^6}{6}}$

$r = 3$

$\text{Term} = \frac{x^2}{6}$

$$\underline{b_3 = \frac{x^2}{2}(1-x) + \frac{x^2}{4}(2-x) + (3-x)\frac{x^2}{6}}$$

$$\underline{b_4 = \frac{x^3}{6}}$$

So the equations of the basis functions in the region $0 \leq x < 1$ are

$$B_{1,4} = (1-x)^3$$

$$B_{2,4} = x(1-x)^2 + (2-x)(1-x)\frac{x}{2} + (2-x)^2\frac{x}{4}$$

$$B_{3,4} = \frac{x^2}{2}(1-x) + \frac{x^2}{4}(2-x) + (3-x)\frac{x^2}{6}$$

$$B_{4,4} = \frac{x^3}{6} .$$

Similarly solving for B-spline basis function equations in the region $1 \leq x < 2$ we obtain:

$$B_{2,4} = \frac{(2-x)^3}{4}$$

$$B_{3,4} = \frac{x}{4}(2-x)^2 + \frac{(3-x)(2-x)x}{6} + \frac{(3-x)^2(x-1)}{6}$$

$$B_{4,4} = \frac{x^2}{6}(2-x) + \frac{x}{6}(3-x)(x-1) + (4-x)\frac{(x-1)^2}{6}$$

$$B_{5,4} = \frac{(x-1)^3}{6}$$

For region $2 \leq x < 3$ we obtain:

$$B_{3,4} = \frac{(3-x)^3}{6}$$

$$B_{4,4} = \frac{x}{6}(3-x)^2 + \frac{(4-x)(x-1)(3-x)}{6} + \frac{(4-x)^2(x-2)}{6}$$

$$B_{5,4} = \frac{(x-1)^2(x-3)}{2} + \frac{(x-1)(4-x)(x-2)}{2} + \frac{(x-2)^2(5-x)}{6}$$

$$B_{6,4} = \frac{(x-2)^3}{6}$$

For region $3 \leq x < 4$ we obtain:

$$B_{4,4} = \frac{(4-x)^3}{6}$$

$$B_{5,4} = \frac{1}{6}\left[(x-1)(4-x)^2 + (5-n)(x-2)(4-x) + (5-x)^2(x-3)\right]$$

$$B_{6,4} = \frac{1}{6}\left[(x-2)^2(4-x) + (x-2)(5-n)(x-3) + (6-n)(x-3)^2\right]$$

$$B_{7,4} = \frac{(x-3)^3}{6} .$$

Similarly the basis function in the intervals $4 \leqslant x$ can be found.

(c) <u>Generating the Two Extra B-Spline Coefficients</u>

The B-spline approximation to any function g(x) is given by

$$g(x) = \sum_{i=1}^{n} \alpha_i B_{i,j} .$$

We know $B_{i,j}$ but we need to find α_i.

There are two methods which can be used to find α_i:

1. As there are (m+2) basis functions, we need (m+2) conditions to find all the α_i's. There are m data points, so we need two more conditions. These two conditions are obtained by making a linear interpolation in the first and last interval at the 0.333 and 0.666 of interval length respectively.

In this example the two generated data points are:

for the first interval

$$\left(0.333,\quad 2.6 + \left(\frac{1.5-2.6}{1-0}\right) *3333\right)$$

$$= \left(0.333,\quad \underline{2.234}\right)$$

and for the last interval

$$\left(7.666,\ 1-2 + \left(\frac{-2.0-1.2}{8-7}\right) * \frac{2}{3}\right)$$

$$= \left(7.666,\quad \underline{-\ 1.0666}\right) .$$

Thus α_i can be evaluated quite easily. Carl de Boor has proved that the α_i's are equal to the value of the data points[5]. Thus, the α_i may be given as:

$\alpha_1 = 2.6$

$\alpha_2 = \underline{2.234}$

$\alpha_3 = 1.5$

$\alpha_4 = 1.2$

$\alpha_5 = 2.2$

$\alpha_6 = 4.0$

$\alpha_7 = 4.5$

$\alpha_8 = 1.2$

$\alpha_9 = \underline{-1.0666}$

$\alpha_{10} = -2.0$.

To illustrate how the B-spline basis functions add up to form the desired fit the expanded region $3 \leq x < 4$ is shown in Figure AIII.1 with the basis functions drawn in there normalized form. In Figure AIII.2 basis functions are shown after they have been multiplied by their respective coefficients. This method is used in B-spline approximation (BSAPP).

2. α_i's can also be found by solving the matrix equality (6) green in the text. This method is used in B-spline interpolation (BSPIN).

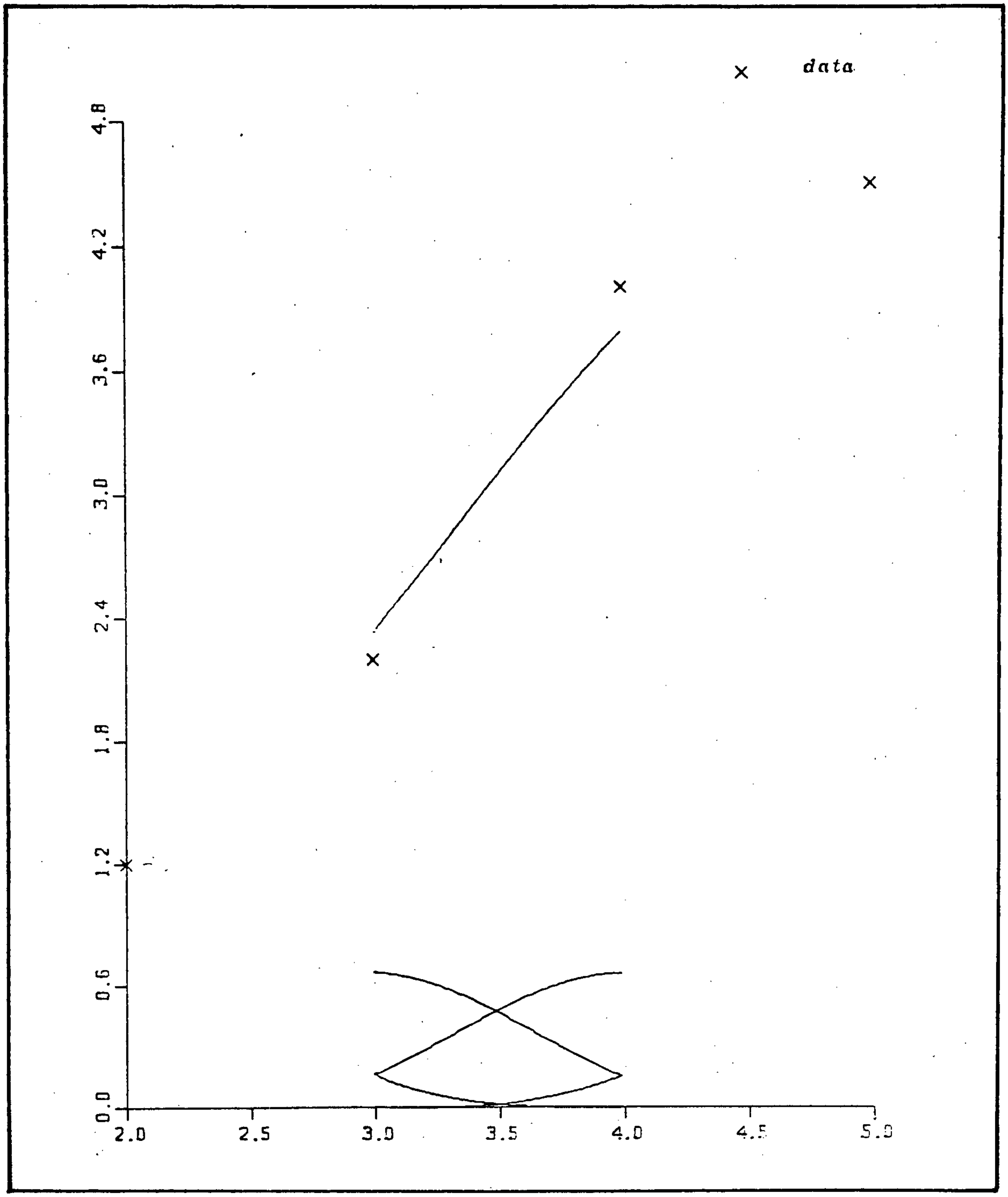


FIG. A3.1 Basis function add up to form curve.

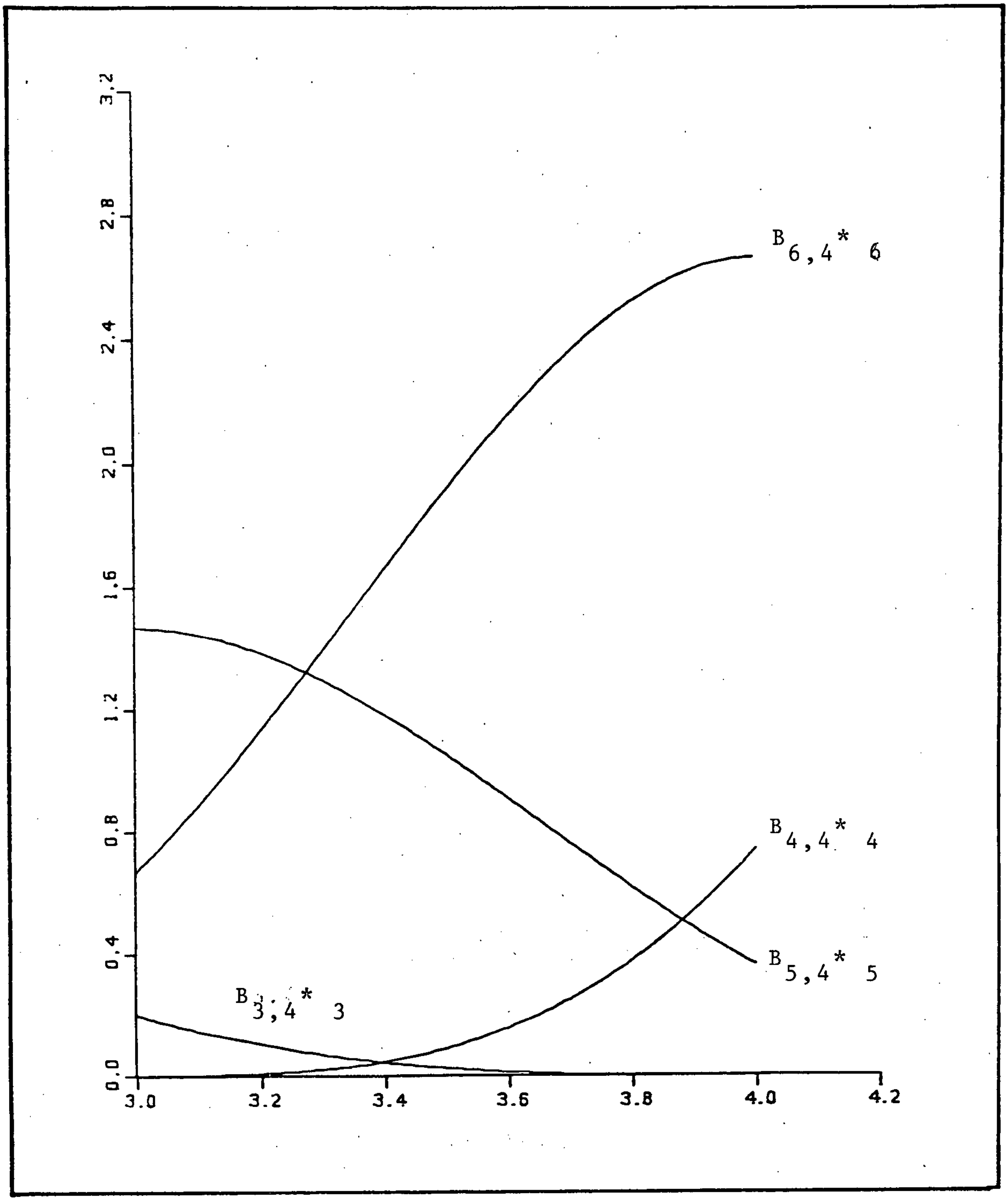


FIG. A3.2 Basis function after being multiplied by there coefficents.

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