SIMULTANEOUS HEAT AND MASS TRANSFER IN WET WOOD PARTICLES

by

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ABSTRACT

A study is made of the simultaneous heat and mass transfer processes which occur within a finite wood cylinder of circular cross-section when it is convectively dried.

Governing transport equations are developed allowing for the functional dependence of both thermal and moisture diffusivities on moisture content and temperature. Derivative type boundary conditions are included in the analysis. The equations are formulated using a cylindrical coordinate system because it is well suited to modelling wood's anisotropy. In the case considered, the axial coordinate direction is aligned with the wood-grain.

Due to the coupling and non-linearity present in the transport and boundary equations, an implicit finite-difference solution scheme is formulated. The three-time-level scheme uses an equation splitting technique to simplify its solution on the computer.

A mathematical wood-model, as available in the literature, is refined and used to determine moisture and thermal diffusivities, and mass transfer boundary conditions. Results from this wood-model apply to softwoods below the fiber-saturation moisture content. Combined diffusivities of liquid and vapour are calculated for the radial and axial directions and results for the radial direction are compared to those found from diffusion experiments. Desorptional isotherms are used in the mass transfer boundary condition equations to relate surface humidity and moisture content.
The wood-model is used in the mass transfer equation to determine wood drying behaviour under isothermal conditions. Initial moisture contents are uniform and equal to the fiber-saturation value. Solutions are presented to give local and average moisture content as a function of time as well as moisture content profiles. The effect of drying temperature was investigated for one case of wood density and shrinkage.
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Simultaneous heat and mass transfer in wet wood occurs when wood is subjected to a drying process. Objectives of analysis of such drying problems are: (1) determination of drying times, (2) determination of drying rates, and (3) finding which variables control the drying process. Drying problems have been investigated for several decades, but only now is a comprehensive understanding of the drying process being developed for porous materials such as softwoods.

Drying problems were first analyzed theoretically during the 1920's. Sherwood (24,25,26)*, Newman (20), Tuttle (35) and many others were concerned with mass transfer in porous materials. Materials commonly considered at that time were wood, soap, and other manufactured products. Solutions to these drying problems were obtained by assuming the movement of moisture through porous materials was a diffusion phenomenon and the moisture diffusivity was constant. Experiments were required to determine an effective diffusion coefficient for each drying problem considered. Once it was known and substituted into the mass transfer equation, drying behaviour could be predicted.

It was during this initial work that the now classical drying periods were identified and attempts made to explain their cause. The drying periods are most readily identified by the rate at which

*Numbers in parentheses refer to literature cited in the list of references.
the body is drying. If the body is initially saturated, the first period will be constant rate drying. This period occurs when drying is controlled by external mass transfer and all evaporation takes place from a liquid film on the surface. Drying continues at a constant rate until a critical moisture content is reached. At this time, a liquid film cannot be maintained on the surface and the falling rate period begins. Drying during this period is principally controlled by internal mass transfer. A typical drying-rate curve is shown in Figure 1.

![Figure 1 - Drying Rate as a Function of Time](image-url)
During the drying process, heat is transferred from the surroundings to the drying body. Therefore, a complete drying model must consider heat as well as mass transfer. Equations for these simultaneous transport processes are coupled through the diffusion coefficients and the internal evaporation term present in the heat transfer equation. Simultaneous transport equations with constant moisture and thermal diffusivities have been solved numerically by Berger and Pei (2) and Harmathy (9). Although the solutions obtained are for the complete drying model they still require, as did the previous simpler models, knowledge of effective diffusivities.

Understanding of the fundamental mechanisms affecting moisture movement during the constant rate period is good and adequate calculations of drying rate can be made. Because the drying rate is externally controlled during this period, it can be calculated by equating the energy required for surface evaporation with the heat transferred from the surroundings to the surface. Standard methods of calculation are available in Reference (23).

The falling rate period is the most time consuming of the two drying periods and therefore is very important in the prediction of over-all drying times. Presently, drying behaviour must be determined using the governing transport equations with effective moisture and thermal diffusivities. The shortage of experimental effective diffusivities makes analysis of the falling rate period unsatisfactory.
Wood is a porous, anisotropic and hygroscopic material. Its heat and mass transfer diffusivities are functions of both moisture content and temperature. Use of effective diffusivities, as in constant coefficient analysis, leads to solutions of wood drying problems which are limited by their dependence on experimental inputs.

A large amount of research has been performed by Stamm (28), MacLean (17), and others to predict wood properties. Stamm's work in the 1930's, and its later refinements, have produced a mathematical model that can be used to predict the diffusivities of liquids and vapours in softwood. Research by MacLean has produced empirical thermal conductivity relationships for softwoods. Results of this past research can be combined into one complete wood model which then can be applied in the transport equations.

The objectives of the work contained in this thesis were to refine the existing wood model, to develop the heat and mass transfer equations relevant to convective drying of a finite circular wood cylinder, to formulate a numerical method to solve the simultaneous transport equations and finally, to substitute moisture diffusivities predicted by the wood model into a computerized solution of the mass transfer equation and obtain wood drying behaviour for select isothermal drying conditions.
II. BACKGROUND AND PREVIOUS WORK

A. Drying of Porous Solids

1. Periods of drying

   Drying involves the removal of water or some other adsorbed fluid from a wet porous solid. In the majority of drying situations the body is surrounded by a heated fluid of low moisture content. The body is heated by its surroundings, either by convection or radiation, or both, and moisture is removed in the vapour phase by convection.

   Typical moisture and temperature curves for a solid subjected to convective drying by a hot gas are shown in Figures 2 and 3, respectively.

2. Mass transfer mechanisms

   The movement of moisture through porous solids is a complex phenomenon. Past research has resulted in identification of several driving potentials which can cause internal mass transfer. For a particular drying problem, moisture content, temperature and character of the solid's porosity determine which potentials are most important. In current literature, porous solids are ideally modelled either as rows of long continuous capillaries (capillary porous), or as an assemblage of non-communicating cavities, or a combination of these. Different mechanisms of moisture movement are important for each case.
Figure 2 - Average moisture content as a function of time

Figure 3 - Average temperature as a function of time
There are four internal mass transfer driving potentials which may be important at different stages in the drying process. These are:

(i) Concentration gradient

Mass transfer due to a concentration gradient is analogous to heat transfer due to a temperature gradient. The mass flux is calculated using Fick's first law of diffusion,

\[ \dot{m} = -D \frac{\partial c}{\partial x}, \]

\[ \dot{m} = \text{mass flux (g/cm}^2\text{-sec)}, \]

\[ D = \text{diffusion coefficient (cm}^2\text{/sec)}, \]

\[ \frac{\partial c}{\partial x} = \text{concentration gradient in the x direction (g/cm}^4\text{)}. \]

Fick's relation was used by Sherwood (25,26) and others (20, 35) in their early analysis of drying problems and has subsequently been used extensively. Comparison of predictions with experiment has been good and this has led to general acceptance of concentration gradient diffusion as an important mechanism of moisture movement. Use of the equation requires knowledge of both liquid and vapour diffusivities.

(ii) Temperature gradient

Mass transfer as a result of a temperature gradient was first discovered in 1934 by Luikov (14) and now is referred to as moisture
thermal diffusion. The postulated relation for mass flux is

\[ \dot{m} = -\delta D \frac{\partial T}{\partial x}, \]

where

\[ \dot{m} = \text{mass flux (g/cm}^2\cdot\text{sec)}, \]
\[ \delta = \text{thermal diffusion coefficient (°C}^{-1}), \]
\[ D = \text{diffusion coefficient (cm}^2/\text{sec}), \]
\[ \frac{\partial T}{\partial x} = \text{temperature gradient in the x direction (°C/cm)}. \]

Soviet scientists (14) have investigated moisture thermal diffusion as it applies to drying problems and found that it was not important at low temperatures. In the case of wood drying, it can be neglected for wood temperatures less than 100°C.

(iii) Capillary potential

Capillary potential is important for capillary porous solids near saturation. For these solids, liquid exists in the larger capillaries and can migrate along the capillary if surface tension forces cause a sufficient net liquid pressure gradient. Capillary flow is a principal mass transfer mechanism during the constant rate period. Movement of moisture by this mechanism can continue until the capillaries are empty of liquid.

A thorough investigation of this type of mass transfer has been made by Luikov (15). Mathematical treatment is analogous to that for concentration gradient diffusion.
(iv) Lebedev (14) has found by experiment that mass transfer can occur in porous solids because of a gradient in total pressure. During his wood drying tests, total internal gas pressure was measured at several points throughout the sample. Significant vapour movement from the wood interior to the periphery caused by an advantageous pressure gradient occurred when the wood temperature was greater than 100°C.

The mass flux relation postulated for this mechanism is

\[ \dot{m} = - D_p \frac{\partial P}{\partial x}, \]

where

- \( D_p \) = coefficient of molar flow (sec),
- \( \frac{\partial P}{\partial x} \) = pressure gradient in the x direction (dyne/cm²).

For all the above internal mass transfer mechanisms, mass transfer from the surface to the surrounding fluid is considered to be analogous to convective heat transfer. The relationship used is

\[ \dot{m} = K_G (H_s - H_a) , \]

where

- \( \dot{m} \) = mass flux (g/cm²·sec),
- \( K_G \) = surface convective mass transfer coefficient (g/cm²·sec),
- \( H_s, H_a \) = surface and ambient relative humidity, respectively.
3. Heat transfer mechanisms

The two principal mechanisms of heat transfer for porous materials are conduction and convection. Conductive heat transfer is the only one commonly assumed for the interior of the body, while convective transfer is assumed at the body surface. Convective heat transfer within a capillary porous solid has been analyzed by Dyer and Sunderland (6). They considered steady state subliminal drying of a capillary porous solid containing frozen liquid when both internal conduction and convection occur.

4. Constant rate period

Constant rate drying has been considered extensively in the literature (7,8,23). Mass transfer during this first drying period is adequately understood for practical drying problems. The accepted method of calculation will be given here, but more complete discussion can be obtained in the previously cited literature.

During constant rate drying, internal resistance to mass transfer is small compared to external resistance, consequently drying rate is controlled by external variables: surface mass transfer coefficient and ambient humidity.

Moisture moves from the body interior to the surface because of capillary flow and concentration gradient diffusion. As the fluid is removed from the surface, the surface liquid retreats slightly into the capillaries and forms concave surfaces. Surface tension and the near-surface concentration gradient induce sufficient moisture move-
ment to keep the surface covered with a thin liquid film. Other mass transfer mechanisms make negligible contributions to total mass flux during this period.

At steady state, the heat input balances the heat required for surface evaporation and from a balance of these energy terms, the drying rate is determined. Allowance must be made for energy required to heat the vapour as it passes through the thermal boundary layer. The effect of this heat-sink on actual heat transfer to the solid has been investigated by Spalding (p. 72 of (12)) and several other researchers (21,22). Spalding recommends a correction factor that would adjust the heat transfer coefficient for a dry body to that for the same body with surface mass flux. Assuming this correction and equating all surface heat transfer terms, the relations for surface temperature and drying rate are

\[
T_s = T_a - \frac{\sigma K_G}{h} (H_s - H_a)
\]

and

\[
\dot{M} = AK_g (H_s - H_a)
\]

where

\[
\sigma = \text{latent heat of evaporation (cal/g),}
\]

\[
K_g = \text{convective mass transfer coefficient (g/cm}^2\text{-sec),}
\]

\[
h = \text{actual convective heat transfer coefficient for the solid surface (cal/cm}^2\text{-sec-°C),}
\]

\[
T_s, T_a = \text{surface and ambient temperature, respectively (°C),}
\]

\[
H_s, H_a = \text{surface and ambient relative humidity, respectively,}
\]
A = surface area (cm$^2$),
\(\dot{M}\) = drying rate (g/sec).

The equation for surface temperature has to be solved by iteration. The surface temperature will be the wet bulb temperature for pure convective drying or above this if other heat sources, such as radiation, are present.

5. Falling rate period

Ideally, the moisture content during the constant rate period is uniform throughout the body and moisture content decreases with time until a critical moisture content is reached. The critical moisture content depends on drying conditions and material properties and is therefore very difficult to predict analytically. Estimates of critical moisture contents for convective drying of various materials are given in Reference (23).

As the material's moisture content approaches the critical value, resistance to internal mass transfer becomes increasingly important. Mass transfer rate depends on the series resistances of internal and external transfer. Because each term affects the drying rate, the transition from constant to falling rate drying is gradual rather than abrupt. As surface moisture approaches its equilibrium value, the drying rate becomes completely controlled by internal transfer.
Internal mass transfer can be caused by any of the four means previously mentioned: concentration gradient, capillary potential, temperature gradient and pressure gradient diffusion. The types receiving most attention in the literature are concentration gradient diffusion and capillary flow. Other types of mass transfer are important if temperature above the fluid's boiling point are involved. Capillary flow requires the existence of liquid in the capillaries and is therefore more associated with the constant rate than the falling rate period.

Analysis of concentration gradient diffusion in porous solids was made by Sherwood and Newman. This diffusion phenomenon is analogous to heat conduction in a solid. Using this analogy, analytical solutions to the governing mass transfer equation are readily available in works on heat transfer. The necessary assumptions are constant moisture diffusivity and constant proportionality between surface moisture content and humidity. As an example, the solution for average moisture content of an infinite slab, convectively dried on both surfaces from a uniform initial moisture content, as adapted from reference (15), is shown below.

\[
\frac{\bar{U} - U_a}{U_i - U_a} = \sum_{n=1}^{\infty} B_n \exp \left\{ -\frac{2Dt}{L^2} \right\}, \tag{1}
\]

where

\[
\bar{U}, U_i, U_a = \text{average, initial and ambient moisture content, respectively, (mass water/mass dry wood)},
\]

\[
D = \text{moisture diffusivity, (cm}^2/\text{s)}.
\]
\[ L = \text{slab half thickness (cm).} \]

The coefficient \( \mu_n \) is obtained from the solution of the transcendental equation,

\[
\cot \mu_n = \frac{\mu_n}{B_i^m}.
\]

\( B_n \) is determined from the relationship,

\[
B_n = \frac{2B_i^2}{\mu_n(B_i^2 + B_i + \mu_n^2)}.
\]

Both involve the Biot mass number, \( B_i^m \), which is given by the equation,

\[
B_i = \frac{K_G L \text{ const}}{D \rho_D},
\]

where

\[
\text{const} = \frac{\text{surface moisture content}}{\text{surface relative humidity}},
\]

\[
K_G = \text{convective mass transfer coefficient (g/cm}^2\text{-sec)},
\]

\[
\rho_D = \text{density of dry solid (g/cm}^3\).
\]

Solutions for simple finite bodies such as blocks and finite cylinders can be obtained by taking products of solutions for related infinite bodies. Detailed analysis of these types of problems are given by Luikov (15). He also treats cases where initial moisture distributions are parabolic.

Drying rates can be determined by differentiating the relation for average moisture content with respect to time.
The difficulty in applying constant coefficient case analysis to actual drying problems is that actual diffusion coefficients may vary with moisture content and temperature. Effective diffusion coefficients for mass transfer are determined experimentally then used in the constant coefficient case analysis. The difficulty with this approach is the dependence on preliminary experimental results. An insufficient volume of experimental results are currently available to dependably determine the effective coefficient which applies to a particular drying problem.

6. Simultaneous mass and heat transfer

Determination of the thermal behaviour of a drying body requires solution of both the heat and mass transfer equations. The mass transfer equation is required to include the heat-sink caused by internal evaporation. Because the two equations are coupled, solutions are obtained by numerical procedures.

Berger and Pei (2) solved the simultaneous transport equations for an isotropically porous infinite slab. They assumed capillary flow of liquid and concentration gradient diffusion of vapour. Liquid and vapour concentrations were coupled using a general numerical approximation of sorptional isotherms. Moisture and thermal diffusivities were taken to be constant over the full drying process. Solutions indicate the effect of changes in Biot heat and mass numbers and Luikov and Kossovich numbers on drying rate for both the constant and falling rate periods. No experimental results were available that could be compared to their predictions.
Harmathy (9) considered diffusional heat and mass transfer in an isotropically porous infinite slab. He assumed all mass transfer occurred in the vapour phase and that moisture and thermal diffusivities were constant. Input constants were obtained either from handbooks or experiments. Theoretical and experimental moisture and temperature curves for the falling rate drying of clay brick were compared. Agreement between theory and experiment was good for moisture, but poor for temperature.

B. Wood Structure

Wood is a complex organic material comprised of many specialized structures necessary for its growth. A sketch of a softwood (coniferous species) is shown in Figure 4 along with the three principal structural dimensions. The principal structures are tracheids, rays, parenchyma, resin canals and pits. Tracheids extend in the axial direction of a tree and are long linear cells with a length as much as 75 times its diameter; they taper to a blunt tip when viewed radially and to a sharp tip when viewed tangentially. Rays are ribbon-like arrangements of cells principally oriented in the radial direction. Resin canals are large tubular spacings surrounded by nutrient storage cells and act to transport nutrients in both the axial and radial directions. Pits are thin areas of the tracheid walls which allow for inter-tracheid passage of nutrients. Parenchyma are cells used for nutrient storage and they extend in both the radial and axial directions.
FIGURE 4 - MAGNIFIED THREE DIMENSIONAL SKETCH OF A SOFTWOOD: tt, axial surface; tg, tangential surface; rr, radial surface; tr, tracheids; wr, wood ray; c, cavity; p, pit; sp, springwood; sm, summerwood; ar, annual ring; rd, resin duct.
Thorough discussion of each of the structures mentioned above and other less important ones are available in the literature (4,18).
III. DEVELOPMENT OF MASS AND HEAT TRANSFER EQUATIONS

A. Problem Definition

The problem is to develop heat and mass transfer equations for drying of the finite circular wood cylinder shown in Figure 5. The body is surrounded by a moving fluid of known temperature, moisture content, velocity and properties. Wood-grain direction is aligned with the axial direction.

B. Mass Transfer Analysis

Internal and external mass transfer processes included in the analysis are shown in Figure 6. Assumptions used in the equation development are:

(i) Fick's first law of diffusive mass transfer applies. This equation can be applied to all the mass transfer mechanisms previously discussed (pp. 7-9) and has been used in previous drying analysis.
(ii) Mass transfer due to temperature or pressure gradients is negligibly small. These mass transfer mechanisms have been investigated by Lebedev (14) for wood and were found to make an insignificant contribution to total mass transfer if the body temperature was below 100°C.
(iii) One diffusivity coefficient for combined movement of both liquid and vapour can be determined using the wood model.
FIGURE 5 - BODY COORDINATE SYSTEM AND DIMENSIONS

FIGURE 6 - INTERNAL AND EXTERNAL MASS TRANSFER PROCESSES
(iv) No coupling exists between mass transfer in each coordinate direction.
(v) Mass transfer from the surface can be proportionally related to the difference between surface and ambient vapour density using a surface convection coefficient.
(vi) Wood shrinkage and swelling can occur.

1. Mass transfer equation

To develop the mass transfer equation, the law of conservation of mass was applied to an elemental volume of the finite cylinder. Moisture movement was related to the liquid gradient through a combined diffusivity which includes the movement of both liquid and vapour phases. An order of magnitude estimate of liquid and vapour contributions to moisture capacitance was made for the control volume. It was found that the vapour contribution was $10^{-4}$ times that of the liquid (pp. 105-106, Appendix A). Because of this, the change in moisture content of the elemental volume with time was taken as the change in liquid content with time. The effect of wood shrinkage or swelling on the liquid gradient and on the moisture content in the elemental volume were included in the analysis.

In dimensional form, the mass transfer equation (Appendix A) is

$$\frac{1}{r} \frac{\partial}{\partial r} (r A_R(U,T) \frac{\partial U}{\partial r}) + \frac{\partial}{\partial z} (A_L(U,T) \frac{\partial U}{\partial z}) = B(U) \frac{\partial U}{\partial t} , \quad \ldots (2)$$

where

$$U = \text{moisture content (mass water/mass dry wood)},$$
\( T = \) temperature (°C),
\( t = \) time (sec),
\( r = \) radial coordinate (cm),
\( z = \) axial coordinate (cm),

\( A_R(U,T) = \) diffusivity of moisture in the radial direction as a function of moisture content and temperature (cm²/sec),

\( A_Z(U,T) = \) diffusivity of moisture in the axial direction as a function of moisture content and temperature (cm²/sec),

\( B(U) = \frac{\text{Volume of dry element (cm}^3\text{)}}{\text{Volume of wet element (cm}^3\text{)}} \).

For the case of constant diffusivity and no volumetric changes, this equation reduces to the one solved by Newman (20).

Equation 2 was made dimensionless to improve the size of the terms by using initial values of body dimensions and radial moisture diffusivity. The non-dimensional form of the equation is

\[
\frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* A_R(U,T) \frac{\partial U}{\partial r^*} \right) + \frac{\partial}{\partial z^*} \left( A_Z(U,T) \frac{\partial U}{\partial z^*} \right) = B(U) \frac{\partial U}{\partial t^*} \quad \ldots \ldots (3)
\]

where

\[
t^* = \frac{A_R(U,T) \cdot t}{R^2} \quad ,
\]

\[
r^* = \frac{r}{R_i} \quad ,
\]
\[ z^* = \frac{z}{L_i} , \]

\[ A_{R}(U,T) = \frac{A_R(U,T)}{A_R(U,T)_i} , \]

\[ A_{Z}(U,T) = \frac{(R/L)^2 A_z(U,T)}{A_R(U,T)_i} . \]

The important parameter arising from the two dimensionality of the mass transfer process is

\[ (R/L)^2_i \left( \frac{A_z(U,T)}{A_R(U,T)_i} \right) . \]

2. Mass transfer boundary conditions

The governing mass transfer equation has internal and external boundary conditions it must satisfy. The external boundary condition for the radial direction is of standard form (13) and is given by the dimensional equation,

\[ \rho_D A_R(U,T) \frac{aU}{\partial r} + K_{GR}(\rho_V - \rho_a) = 0 , \quad r = R; -L \leq z \leq L ; \]

where

- \( \rho_D \) = density of dry wood (g/cm\(^3\)),
- \( K_{GR} \) = convective mass transfer coefficient for the radial surface (cm/sec),
- \( \rho_V \) = surface water vapour density (g/cm\(^3\)),
- \( \rho_a \) = ambient water vapour density (g/cm\(^3\)).
Equation 4 is not consistent in moisture content, the desired dependent variable. It was necessary to use a relation, specific to the material being considered, that would relate water vapour density to moisture content. Relations which do this are sorptional isotherms. Since the final objective was to model wood drying, desorptional isotherms for Sitka spruce were used. These are shown in Figure 7. They give equilibrium relationships between local moisture content and local absolute humidity for different wood temperatures.

To use the desorptional isotherms it is necessary to assume that the rate of attaining equilibrium between surface moisture and local humidity is much greater than the rate of moisture diffusion. This assumption is required since non-equilibrium isotherms are not presently available.

Knowing the relation between moisture content and humidity can be obtained from the desorptional isotherms, Equation 4 was rewritten as

$$\frac{3U}{3r^*} + \left\{ \frac{K_{GR} R \rho_{sat} (H)}{A_R(U,T) \rho_D (U)} \right\} U = \frac{K_{GR} R \rho_a}{A_R(U,T) \rho_D} ;$$

$$r^* = 1 ; -1 < z^* < 1 . \quad \cdots (5)$$

The equation is now in dimensionless form and consistent in the moisture content variable, $U$. Letting

---

1 Sorptional isotherms refer to isotherms for either an adsorption or desorption process.
Figure 7 - Desorption moisture content-relative vapour pressure curves for Sitka spruce at different temperatures (32)
The boundary condition equation is

\[ \frac{3U}{3r^*} + B_{mR}(U,T) U = \lambda_R ; \quad r^* = 1 ; \quad -1 \leq z^* \leq 1 ; \]

\[ \tag{6} \]

where

\[ B_{mR}(U,T) = \text{radial Biot mass number as a function of } U \text{ and } T, \]

\[ \lambda_R = \text{ambient condition applying to the radial surface.} \]

Because of the symmetry of both the body and drying conditions, internal boundary conditions have a simple form. No mass transfer will occur across the cylinder axis or across the radial plane situated at \( z^* = 0 \). For the radial direction, the internal boundary condition is given by the equation,

\[ \frac{3U}{3r^*} = 0 ; \quad r^* = 0 ; \quad -1 \leq z^* \leq 1 . \]

\[ \tag{7} \]

Boundary conditions for the axial direction are of identical form to those for the radial direction. The external and internal boundary conditions are, respectively,
\[ \frac{\partial U}{\partial z} + B_{imZ}(U, T) U = \lambda_Z \] 

...(8)

and

\[ \frac{\partial U}{\partial z} = 0. \] 

...(9)

Here

\[ B_{imZ} = \text{Biot mass number for the axial direction} \]

\[ = \frac{K_GZ L \rho_{\text{sat}} H}{A_z(U, T) \rho_D U} \]

and

\[ \lambda_Z = \text{ambient condition for the axial direction} \]

\[ = \frac{K_GZ L \rho_a}{A_z(U, T) \rho_D} \]

where

\[ K_{GZ} = \text{convective mass transfer coefficient for the axial surface (cm/sec)}. \]

C. Heat Transfer Analysis

The heat transfer equation was developed by performing a heat balance on an elemental volume of the finite cylinder. Heat transfer processes included in the analysis are shown in Figure 8. Assumptions used in the equation development are:

(i) Internal heat transfer occurs by conduction and obeys Fourier's law.

(ii) Thermal conductivities are given by the wood model and are effective values which take porosity into account.
HEAT-SINK FROM INTERNAL EVAPORATION

RADIAL CONDUCTION

CONVECTION FROM RADIAL SURFACE

AXIAL CONDUCTION

CONVECTION FROM AXIAL SURFACE

FIGURE 8 - INTERNAL AND EXTERNAL HEAT TRANSFER PROCESSES
(iii) Internal evaporation occurs and has a constant latent heat.
(iv) There is no coupling between heat transfer in each coordinate direction.
(v) Wood shrinkage and swelling can occur.
(vi) Heat transfer from the surface can be proportionally related to the difference between surface and ambient temperature using a surface convection coefficient.

1. Heat transfer equation

The heat transfer equation, in dimensional form, (Appendix B) is

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r K_R(U,T) \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left( K_Z(U,T) \frac{\partial T}{\partial z} \right)
\]

\text{conductive heat transfer}

\[
\rho_D \sigma \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r f R A_R(U,T) \frac{\partial U}{\partial r} \right) + \frac{\partial}{\partial z} \left( f_z A_z(U,T) \frac{\partial U}{\partial z} \right) - B(U) \frac{\partial U}{\partial t} \right]
\]

\text{heat-sink caused by internal evaporation}

\[
= \rho_D C(U,T) B(U) \frac{\partial T}{\partial t}
\]

\text{rate of change of internal energy}

\[
\text{(10)}
\]

where

\[
K_R(U,T) = \text{thermal conductivity in the radial direction (cal/cm-sec-°C)},
\]

\[
K_Z(U,T) = \text{thermal conductivity in the axial direction (cal/cm-sec-°C)},
\]
f_R = fraction of radial diffusion that is liquid,

f_Z = fraction of axial diffusion that is liquid,

\( \sigma \) = latent heat of evaporation (cal/g),

C(U,T) = specific heat of wet wood (cal/g-°C).

Equation 10 was made dimensionless by using initial values of \( K_R(U,T), A_R(U,T) \) and \( C(U,T) \), as well as the defined non-dimensional temperature,

\[ T^\ast = \frac{T_a - T}{T_a - T_i} \]

where

\( a \sim \) ambient

and

\( i \sim \) initial.

The non-dimensionalized form of Equation 10 is

\[
\frac{1}{r^\ast} \frac{\partial}{\partial r^\ast} \left( r^\ast K^\ast_R(U,T) \frac{\partial T^\ast}{\partial r^\ast} \right) + \frac{\partial}{\partial z^\ast} \left( K^\ast_Z(U,T) \frac{\partial T^\ast}{\partial z^\ast} \right)

+ (K_{0_i} L_{u_i}) \left\{ \frac{1}{r^\ast} \frac{\partial}{\partial r^\ast} \left( r^\ast f^\ast_R A^\ast_R(U,T) \frac{\partial U}{\partial r^\ast} \right) \right.

+ \frac{\partial}{\partial z^\ast} \left( f^\ast_Z A^\ast_Z(U,T) \frac{\partial U}{\partial z^\ast} - B(U) \frac{\partial U}{\partial t^\ast_m} \right) \}

= B(U) C^\ast(U,T) \frac{\partial T^\ast}{\partial t^\ast_h} \]

\[ \ldots\ldots(11) \]
where

\[ K^*_R(U,T) = \frac{K_R(U,T)}{K_R(U,T)_i}, \]

\[ K^*_Z(U,T) = \left(\frac{R}{L_i}\right)^2 \frac{K_Z(U,T)}{K_R(U,T)_i}, \]

\[ C^*(U,T) = \frac{C(U,T)}{C(U,T)_i}, \]

\[ K_{0i} = \text{initial Kossovich number}, \]

\[ \sigma = \frac{\alpha}{(T_a - T_i)C(U,T)_i}, \]

\[ L_{ui} = \text{initial Luikov number}, \]

\[ = \frac{A_R(U,T)_i}{\alpha_R(U,T)_i}, \]

\[ \alpha_R(U,T)_i = \text{initial radial thermal diffusivity}, \]

\[ = \frac{K_R(U,T)_i}{C(U,T)_i \rho_D}, \]

\[ t^*_h = \alpha_R(U,T)_i \frac{t}{R_i^2}. \]

2. Heat transfer boundary conditions

Equation 11, the governing heat transfer equation, must satisfy external and internal boundary conditions. The external boundary condition includes a heat-sink term to account for liquid to vapour phase-change at the surface. For the radial direction, the external
The internal boundary condition for heat transfer mathematically states that temperature profiles are symmetric about the cylinder axis and across the radial plane situated at $z^* = 0$. The internal boundary condition for the radial direction is
\[ \frac{\partial T^*}{\partial r} = 0 ; \quad r^* = 0 ; \quad -1 \leq z^* \leq 1 . \quad \ldots \quad (14) \]

External and internal boundary equations were developed for the axial direction in the same manner as shown for the radial direction. These are

\[ \frac{\partial T^*}{\partial z^*} + B_i Z^* T^* + \kappa \frac{L_u}{L_Z} \frac{\partial U}{\partial z^*} = 0 ; \quad z^* = 1 ; \quad 0 \leq r^* \leq 1 , \quad \ldots \quad (15) \]

and

\[ \frac{\partial T^*}{\partial z^*} = 0 ; \quad z^* = 0 ; \quad 0 \leq r^* \leq 1 ; \quad \ldots \quad (16) \]

where

- \( B_i Z \) = axial Biot number
  \[ = h_Z L / K_Z(U,T) , \]
- \( \kappa \) = \( \sigma / C(U,T)(T_a - T_i) \),
- \( L_u L_Z = f_Z A_Z(U,T) / \alpha_Z(U,T) \),
- \( \alpha_Z(U,T) = K_Z(U,T) / \rho_B C(U,T) . \)

The mass and heat transfer equations and boundary conditions given by Equations 3, 6-9, 11 and 13-16 completely describe the mass and heat transport within the cylinder. What is now required are the moisture diffusivities and thermal conductivities for wood and a method to solve these non-linear transport equations.
IV. WOOD MODEL

The complete wood model is developed in three sections, each concerned with a specific wood characteristic required for the previous simultaneous heat and mass transfer analysis. Shrinking and swelling of wood is considered first, followed sequentially by moisture diffusivity, thermal conductivity and specific heat. All results are restricted to softwoods with a moisture content not greater than the fiber-saturation value, $U_f$.

A. Shrinking and Swelling of Wood

Wood volume is dependent on moisture content, internal structure and density. There is comprehensive experimental research available which indicates how these variables affect wood volume (4,10, 29).

When wet wood is dried, no shrinkage occurs until the moisture content is below the fiber saturation point. If all moisture is removed the bulk volume change may range from 6% to 20% of the dry volume. Shrinkage is greatest in the tangential direction, varying from 5% to 12% (based on dry volume). It is usually from 2% to 7% in the radial direction and less than 1% in the axial direction. The radial wood rays, for the radial direction, and, the tracheids, for the axial direction, decrease shrinkage in those directions by acting as structural reinforcement. Local volume change is influenced by
surrounding as well as local moisture content. If a moisture gradient exists in the body, differential shrinkage will occur.

Wood substance is the material forming the tracheid walls and is predominately lignified cellulose fibrils. Below the fiber saturation point, water is adsorbed in the wood substance producing a solid solution and reducing, but not eliminating, wood-substance cohesive forces. As moisture is adsorbed or desorbed, the fibrillar structure of the wood substance inhibites motion of the tracheid inner wall and subsequently keeps voidage volume relatively constant. Change in bulk volume is caused by the change in tracheid wall volume. Since water forms a solid solution when it is adsorbed in the tracheid wall, the change in bulk volume is almost directly proportional to the volume of liquid adsorbed.

A linear relation between wood volume and moisture content was measured by Stamm (29) from drying experiments conducted using soft-woods (Figure 9). The approximation used to model this volume change behaviour is

$$\frac{V_D}{V_S} = \frac{1}{(1 + kU/U_f)}$$  \hspace{1cm} (17)

where

- $V_D$ = volume of dry wood (cm$^3$),
- $V_S$ = volume of wood at moisture content $U$ (cm$^3$),
- $k$ = shrinkage factor.
FIGURE 9 - VOLUMETRIC SHRINKAGE OF LOBLOLLY PINE OF DIFFERENT DENSITIES
The shrinkage factor $k$ is determined by fitting Equation 17 to experimental data for the particular wood species being considered. Because of the form of Equation 17, $k$ is actually the maximum fractional change in volume based on dry volume.

B. Moisture Diffusivities

1. Simplified wood structure

A sketch of a softwood section is shown in Figure 10. The majority of softwoods contain all the structures shown, but quantity will vary according to species. Tracheids are the principal structural component of softwood occupying greater than 90% of the volume. Table I gives the proportion of total wood volume occupied by the various wood components.

The cross-sectional area and wall thickness of a tracheid depends upon the part of the growing season in which it developed. In summer, tracheids have a rectangular cross-section and thick walls while in spring they have a square cross-section and thin walls. A sketch of a tracheid is shown in Figure 11 to illustrate its tapered ends.

To simplify treatment of tracheids in the diffusion analysis they are modelled as uniformly sized cells of square cross-section. Their length is taken as $3/4$ of the mean tip to tip length, $0.38$ cm, measured by Stamm (28) for a variety of softwoods. The effective length is taken as $3/4$ of the average length to account for the tapered ends. Typical tracheid dimensions are given in Table I for several American softwoods.
FIGURE 10 - MAGNIFIED THREE DIMENSIONAL SKETCH OF A SOFTWOOD: tt, axial surface; tg, tangential surface; rr, radial surface; tr, tracheids; wr, wood ray; c, cavity; p, pit; sp, springwood; sm, summerwood; ar, annual ring; rd, resin duct.
<table>
<thead>
<tr>
<th>Species</th>
<th>Tracheids</th>
<th>Rays</th>
<th>Longitudinal Parenchyma</th>
<th>Resin Canals</th>
<th>Length,mm</th>
<th>Tangential Diameter μ range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eastern white pine</td>
<td>94.0</td>
<td>5.3</td>
<td>0.7</td>
<td>3.0</td>
<td>1.6</td>
<td>5.0</td>
</tr>
<tr>
<td>Ponderosa pine</td>
<td>93.0</td>
<td>6.7</td>
<td>0.3</td>
<td>3.6</td>
<td>1.5</td>
<td>5.0</td>
</tr>
<tr>
<td>Sitka spruce</td>
<td>92.5</td>
<td>7.2</td>
<td>0.3</td>
<td>5.6</td>
<td>3.6</td>
<td>7.3</td>
</tr>
<tr>
<td>Douglas fir</td>
<td>92.5</td>
<td>7.3</td>
<td>0.2</td>
<td>3.9</td>
<td>1.7</td>
<td>7.0</td>
</tr>
<tr>
<td>Western hemlock</td>
<td>91.2</td>
<td>8.8</td>
<td></td>
<td>4.2</td>
<td>1.8</td>
<td>6.0</td>
</tr>
<tr>
<td>Balsam fir</td>
<td>94.3</td>
<td>5.7</td>
<td></td>
<td>3.5</td>
<td>1.9</td>
<td>5.6</td>
</tr>
<tr>
<td>Western red-cedar</td>
<td>93.1</td>
<td>6.9</td>
<td>Trace</td>
<td>3.5</td>
<td>1.4</td>
<td>5.9</td>
</tr>
</tbody>
</table>

Reproduced from Reference 4.
FIGURE 11 - SCHEMATIC OF TRACHEID

FIGURE 12 - SCHEMATIC OF PIT-PAIR
Inter-tracheid communicating pits are thin areas in the tangential and radial walls, usually situated near the ends of the tracheid. Bordered pits, as shown in Figure 12, are the type most commonly found in softwoods. Microscopic holes called pit pores exist in the pit membrane. Relevant pit membrane and pit pore dimensions as well as estimates of the number of pits per tracheid for American softwoods are given in Table II.

**TABLE II**

Structural Data for Pits of American Softwoods

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average thickness of pit membrane</td>
<td>.5 to 1.2 μ</td>
</tr>
<tr>
<td>Average fraction of cavity wall made up of pit openings</td>
<td>.014</td>
</tr>
<tr>
<td>Average number of pits per wood fiber</td>
<td>130</td>
</tr>
<tr>
<td>Average diameter of pit pore</td>
<td>30 μ</td>
</tr>
<tr>
<td>Average diameter of pit chamber</td>
<td>2 to 10 μ</td>
</tr>
</tbody>
</table>

Data obtained from Reference 28.

Stamm (28) has shown that the minor constituents such as resin ducts and wood rays have negligible effect on moisture movement. They were therefore not included in the simplified wood-structure model.
Treating the tracheids as cells and including the pits on the tracheid walls, Stamm (28) proposed the simplified wood model shown in Figure 13. This structure applied to all structural directions, but different values for internal parameters must be used for each. Mean values of the necessary structural dimensions required to apply the model to the radial or axial directions are given in Table III.

2. Diffusion network for movement of liquids and vapours in the simplified wood structure

The movement of liquid and vapour is modelled as a concentration gradient diffusion phenomenon. Diffusion paths for moisture movement through the simplified wood structure are shown in Figure 14. Movement of liquid through the wood substance, often referred to as bound water diffusion, occurs in the continuous cell walls with diffusivity $D_1$, in the discontinuous cell walls with diffusivity $D_3$ and in the pit membrane with diffusivity $D_5$. The possible paths for vapour movement are through the cell cavity, with diffusivity $D_2$, through the pit chamber, with diffusivity $D_4$, and through the pit pore, with diffusivity $D_6$.

A general solution to the diffusion network can be determined in terms of the six pathway diffusivities. This solution gives the combined diffusivity of liquid and vapour through wood. The combined diffusivity, $D_c$, was determined by the author to be

$$D_c = D_1 + \frac{D_2 y}{D_2 + y}, \quad \ldots \ldots (18)$$
FIGURE 13 - SIMPLIFIED WOOD STRUCTURE USED IN WOOD MODEL

FIGURE 14 - MOISTURE DIFFUSION NETWORK FOR WOOD MODEL
<table>
<thead>
<tr>
<th>Property or Dimension</th>
<th>Symbol</th>
<th>Value</th>
<th>Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Void fraction</td>
<td>$\varepsilon$</td>
<td>.65 to .72</td>
<td>$1 - \frac{\rho_D}{(1 + kU/U_f)}\left(\frac{1}{\rho_{WS}} + \frac{U}{\rho_D}\right)$</td>
</tr>
<tr>
<td>Density of dry wood</td>
<td>$\rho_D$</td>
<td>.4</td>
<td></td>
</tr>
<tr>
<td>Density of wood substance</td>
<td>$\rho_{WS}$</td>
<td>1.46 g/cm³</td>
<td></td>
</tr>
<tr>
<td>Density of adsorbed water</td>
<td>$\rho_0$</td>
<td>1.11 g/cm³</td>
<td></td>
</tr>
<tr>
<td>Average thickness of double cell walls</td>
<td>$L_w$</td>
<td>5.2$\mu$ - 7.2$\mu$</td>
<td>$\frac{1 - \sqrt{\varepsilon}}{n_t}$</td>
</tr>
<tr>
<td>Average cavity width</td>
<td>$d_c$</td>
<td>27$\mu$</td>
<td></td>
</tr>
<tr>
<td>Average tracheid length</td>
<td>$L_t$</td>
<td>.38 cm</td>
<td></td>
</tr>
<tr>
<td>Average effective cavity length</td>
<td>$L_C$</td>
<td>.285 cm</td>
<td>$3/4 L_t$</td>
</tr>
<tr>
<td>Average pit membrane thickness</td>
<td>$L_p$</td>
<td>1$\mu$</td>
<td></td>
</tr>
<tr>
<td>Number of fiber cavities traversed per cm in the axial direction</td>
<td>$n_f$</td>
<td>3.5</td>
<td></td>
</tr>
<tr>
<td>Number of cell walls traversed per cm in the axial direction</td>
<td>$n_l$</td>
<td>2.5</td>
<td>$n_f - 1.0$</td>
</tr>
<tr>
<td>Number of cell walls traversed in the radial direction</td>
<td>$n_t$</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>Effective fraction of cross-sectional area covered by pit membrane pores for axial direction</td>
<td>$q_{\ell}$</td>
<td>.0038</td>
<td></td>
</tr>
<tr>
<td>Effective fraction of cross-sectional area covered by pit membrane pores for radial direction</td>
<td>$q_t$</td>
<td>.00052</td>
<td></td>
</tr>
<tr>
<td>Fraction of cross-sectional area covered by pits</td>
<td>$q_p$</td>
<td>.014</td>
<td></td>
</tr>
</tbody>
</table>

Data obtained from Reference (28).
where
\[ Y = D_3 + \frac{D_4(D_5 + D_6)}{D_4 + D_5 + D_6} \] ....(19)

The separate pathway diffusivities, \( D_{1-6} \), must be determined with respect to the same concentration gradient. In this analysis a gradient in liquid concentration (mass of liquid per volume of wet wood) was used.

When \( D_1 \) is small and \( Y \) is approximately equal to \( D_3 \), diffusion primarily occurs by an evaporation-condensation process; liquid evaporates from one cell wall, diffuses across the cavity, and condenses on the opposite cell wall. Elimination of the pit system from the wood model removes diffusivities \( D_4 \) to \( D_6 \) with the result that \( Y = D_3 \).

3. Diffusivity of liquid in the wood substance

Diffusivity of bound water in wood is a function of moisture content and temperature. When water is adsorbed in the wood substance it establishes weak hydrogen bonds with the available hydroxyl groups of the cellulose. Bonding forces decrease exponentially with distance from the adsorption sites. Hence, diffusivity increases in an exponential manner with increases in moisture content. The effect of moisture content on bound water diffusivity in the axial direction is shown in Figure 15 (30). In this figure diffusion coefficients, averaged over a complete adsorption experiment, are plotted versus an effective moisture content. The effective moisture content is the point after which the adsorption process is unaffected by initial conditions and
Figure 15: Axial bound water diffusivity as a function of effective moisture content for Sitka spruce at 25°C
was felt, by the researchers, to be the moisture content to which the measured diffusivity best applied. The effective moisture content was calculated using the relationship,

$$U_e = U_i + \frac{2}{3} (U_u - U_i)$$  

where

$$U_i, U_u, U_e = \text{initial, ultimate, and effective moisture contents, respectively.}$$

The results shown in Figure 15 are the best presently available for axial diffusion and were used in this work. Because of the absence of results for radial bound water diffusion, the effect of moisture on axial diffusion was assumed to also apply to the radial direction.

The effect of temperature on radial bound water diffusion has been correlated by Yao (40) using the Arrhenius rate equation

$$D_{BR} = D_0 \exp \left( \frac{-E}{RT} \right) ,$$

where

$$D_{BR} = \text{radial bound water diffusivity at absolute temperature } T, (\text{cm}^2/\text{sec}),$$

$$D_0 = \text{constant (cm}^2/\text{sec}),$$

$$R = \text{gas constant (cal/mole}^\circ\text{K}),$$

$$E = \text{activation energy (cal/mole).}$$
His relation for $D_{BR}$, on the basis of a gradient in mass of water per volume of wood substance, is

$$D_{BR} = 0.353 \exp\left(-\frac{8,184}{RT}\right) \text{cm}^2/\text{sec} . \quad (21)$$

A relation was postulated by the author that would account for the effect of moisture content and temperature on bound water diffusivity. Its general form is

$$D_B = fn(U)fn(T) .$$

The function of moisture content, $fn(U)$, was determined using the experimental results for axial bound water diffusivity. In Figure 15, the effective moisture content was taken as a first approximation to actual moisture content and used in a polynomial fit to the diffusivity curve. This polynomial was combined with Yao's result given in Equation 21 to obtain the final relationship,

$$D_B' = (46.84U^2 - 410.93U^3 + 1106.11U^4)\exp\left(-\frac{8,184}{RT}\right) , \quad (22)$$

where

$$D_B' = \text{bound water diffusivity for a gradient in mass of water per volume wood substance (cm}^2/\text{sec}).$$

Equation 21 was transformed to apply to a gradient in mass of water per volume of wet wood. The relationship between $D_B'$ and desired
bound water diffusivity, $D_B$, is given by the following equation:

$$D_B = \left( \frac{\rho_D}{\rho_{WS}} \right) \frac{D'_V}{(1 - \epsilon)^2} \quad \cdots \cdots (23)$$

where

$$\rho_{WS} = \text{density of wood substance (g/cm}^3\text{),}$$

and

$$\epsilon = \text{void fraction.}$$

The derivation of Equation 23 is in Appendix C.

4. Diffusivity of water vapour

Diffusion of water vapour in the wood voids was taken to be the same as in free air. This diffusivity was calculated using the empirical Equation (11),

$$D'_V = .22(T/273)^{1.75}(760/P) \quad \cdots \cdots (24)$$

where

$$D'_V = \text{diffusivity of water vapour in air with respect to a density gradient (cm}^2/\text{sec),}$$

$$T = \text{absolute temperature (}^\circ \text{K),}$$

$$P = \text{pressure (mm Hg).}$$

The diffusivity, $D'_V$, applies to a different gradient than the one used for the bound water diffusivity. To be compatible, $D'_V$ was converted to a gradient in mass of water per volume of swollen wood. The final
diffusivity relation, as shown in Appendix C, is given by

\[ D_V = \frac{\rho_{\text{sat}}}{\rho_D} (1 + \frac{kU}{U_f})^2 \frac{\partial H}{\partial U} D_V \]  

\[ \text{......}(25) \]

where

- \( D_V \) = diffusivity of water vapour with respect to a gradient in mass of water per volume of wet wood (cm\(^2\)/sec),
- \( \rho_{\text{sat}} \) = saturation density of water vapour (g/cm\(^3\)),
- \( \frac{\partial H}{\partial U} \) = local slope of desorptinal isotherm.

The value of \( \frac{\partial H}{\partial U} \) was determined using the desorptinal isotherms for Sitka spruce. These were approximated with a relation of the form,

\[ H = f\left(\frac{U}{U_f}\right) + g(T) h\left(\frac{U}{U_f}\right) \]  

\[ \text{......}(26) \]

where \( f(U/U_f) \) and \( h(U/U_f) \) are functions of the moisture content variable \( U/U_f \) and \( g(T) \) is a function of temperature. Details of these three functions are given on pages 113 to 116 of Appendix C. Results of Equation 26 for three temperatures, 25°, 60° and 100°C are compared with experimental results for Sitka spruce in Figure 16.

Diffusion of water vapour through pit pores cannot be calculated by assuming it is the same as for free air. Pore diameters and the mean free path of a water vapour molecule are of the same order of magnitude and passage of gas is therefore not a pure diffusion process.
FIGURE 16 - DESORPTION ISOTHERMS FOR SITKA PINE AT DIFFERENT TEMPERATURES
Tarkow and Stamm (33,34) have estimated diffusivities for water vapour diffusion through pit pores by fitting the diffusion model to experimental results for carbon dioxide diffusion. From their work they recommended a diffusivity $1/40$ of the free air value should be used for pore diffusion of water vapour. This approximation was used.

5. Pathway diffusivities for the diffusion network

The diffusivities for each path in the diffusion network (Figure 14) were calculated using the bound water and vapour diffusivities given by Equations 23 and 25, respectively. Calculations were made by considering the fractional area and fractional total length which applied to the path. The general relation used is

$$D_{\text{path}} = \frac{(\text{Area of path})}{(\text{Total area of cross-section})} \cdot \frac{(\text{Total length})}{(\text{Path length})} D_{B,V}$$

...(27)

where

$$D_{\text{path}} = \text{pathway diffusivity (cm}^2/\text{sec)},$$

$$D_{B,V} = \text{diffusivity of bound water or water vapour, which ever applied (cm}^2/\text{sec}).$$

Path diffusivities results for the radial and axial directions are given in Appendix C. Stamm's recommended values of wood parameters for the path diffusivity relations are given in Table III.
6. Diffusivity relations used in the mass transfer equation

Moisture diffusivities as obtained from the diffusion network equations, Equations 18 and 19, apply to a gradient in liquid concentration of wet wood. The standard mass flux relation appropriate for this diffusivity is

\[ \dot{m} = -D_c(U,T) \frac{\partial}{\partial x} (\rho_s U) \text{ g/cm}^2\text{-sec} \]  \hspace{1cm} (30)

The density of wet wood, \( \rho_s \), can be related to the density of dry wood using Equation 17. Substituting this relation into Equation 30 and using the chain rule of differentiation the mass flux relation becomes

\[ \dot{m} = \frac{D_c(U,T)}{(1 + kU)^2} \frac{\partial U}{\partial x} \text{ g/cm}^2\text{-sec} \]  \hspace{1cm} (31)

Letting

\[ A(U,T) = \frac{D_c(U,T)}{(1 + kU)^2} \text{,} \]

Equation 31 has the simple form

\[ \dot{m} = -A(U,T) \frac{\partial U}{\partial x} \text{ g/cm}^2\text{-sec} \]  \hspace{1cm} (32)

which is the one used in the development of the mass and heat transfer equations. \( A_R(U,T) \) and \( A_Z(U,T) \) apply to Equation 32 when the radial and axial directions, respectively, are being considered.

The radial and axial diffusivities were calculated for different temperatures, dry-wood densities and shrinkages. The effect of temperature on \( A_R(U,T) \) and \( A_Z(U,T) \) is shown in Figure 17. \( A_Z(U,T) \) is a strong function of temperature as a result of the influence of vapour
diffusion. The maximum in the curve for the axial diffusivity and the lower inflection point in the curve for the radial diffusivity both result from the combined influence of vapour diffusion and the shape of the desorbational isotherms. These changes in the curves are situated where there is a maximum in \( \partial H / \partial U \). This is approximately at \( U/U_f = .25 \). The effect of dry-wood density is shown in Figure 18 for the radial direction and in Figure 19 for the axial direction. Each diffusivity behaves differently to changes in density. Axial diffusivity, because of its dependency on vapour diffusion, is most sensitive to changes in wood density. Several different shrinkages (ranging from 0% to 12%), were used in the calculation of diffusivities for a wood temperature of 25°C. There was a slight increase in diffusivity with increased shrinkage, but these results were not plotted because maximum changes in diffusivity were less than 4%.

Relative contributions of the different paths in the diffusion network are given in Figure 20. From this figure it is evident that continuous bound water diffusion plays a minor role except near fiber saturation. At higher temperatures than the one shown, bound water diffusion has less effect. The higher sensitivity of axial diffusion to vapour movement and desorptional isotherms is shown by the curve for \( (1 - D_1/D_c)Z \). Near \( U_f \), \( \partial H / \partial U \) approaches 0 and thereby increases the relative contribution made by bound water. The curves for \( D_3/Y \) show the importance of the pit system near zero moisture.

\( A_R(U_f, T) \) is shown in Figure 21. These are the diffusivities used to non-dimensionalize the mass transfer equation.

The computer program used to calculate the diffusivities is given on pp. 158 and 159 of Appendix H. The relationship for \( U_f \) is given on p.117 of Appendix C.
\begin{align*}
\rho_D & \geq 0.4 \text{ g/cm}^3 \\
k & = 0.1
\end{align*}

Figure 17.1 - Radial Diffusivity as a Function of Moisture Content at Different Temperatures
FIGURE 18 - RADIAL DIFFUSIVITY AS A FUNCTION OF MOISTURE CONTENT AT DIFFERENT DRY-WOOD DENSITIES.
FIGURE 19 - AXIAL DIFFUSIVITY AS A FUNCTION OF MOISTURE CONTENT AT DIFFERENT DRY-WOOD DENSITIES
\[ \rho_D = 0.4 \text{ g/cm}^3 \]
\[ k = 0.1 \]
\[ T = 25^\circ C \]

FIGURE 20 - FRACTION OF DIFFUSION OCCURRING ALONG PRINCIPAL PATHWAYS
FIGURE 21 - RADIAL DIFFUSIVITY AT FIBER SATURATION AS A FUNCTION OF TEMPERATURE

\[ A_R(U_p,T) \times 10^5 \text{ cm}^2/\text{sec} \]

\[ \rho_D = 0.4 \text{ g/cm}^3 \]

\[ k = 0.1 \]
C. Thermal Conductivity and Specific Heat

The thermal conductivity of wood depends principally on the three variables: (1) direction of heat transfer, (2) moisture content, and, (3) wood density. Of secondary importance are the relative proportion of spring and summer wood, defects present in the wood structure and the kind and quantity of chemical substances such as gums, tannins and oils.

Wangaard (38), from heat conductivity determinations on both hardwoods and softwoods, found that for hardwoods, thermal conductivity was significantly greater in the radial direction than in the tangential direction, while for softwoods, there was negligible difference between these conductivities. In other work, Wangaard (37) examined the ratio between axial and transverse (radial or tangential) conductivities. He found the ratio for Douglas fir ranged from 2.28 to 3.8 with a mean of approximately 3.0. MacLean (17), in tests with Douglas fir and Red oak, found this ratio to be between 2.25 and 2.75.

MacLean conducted thermal conductivity experiments on 32 species comprised of both softwoods and hardwoods. Radial conductivities were measured then correlated to wet-wood density (dry mass/wet volume) and moisture content. The relationship he obtained applies to hardwoods and softwoods which have a moisture content less than 40%. Equation 17 relating wet and dry wood volumes was used to put his relationship in a form compatible with relationships used in this work, that is, in terms of dry wood density. The final equation for radial conductivity is
\[ K_R(U,T) = \frac{\rho_D}{(1 + \frac{kU}{U_f})} (.2 + .4 U) + .024 \text{watts m}^0\text{K} \] ....(33)

In a large group of experimental tests made on samples with moisture contents varying from 0 to 33%, radial conductivities calculated from MacLean's equation were compared to experimental values. The agreement was good; 78% of all calculated conductivities were within 10% of the experimental value.

The axial conductivity of wet wood was approximated using Equation 31 for radial conductivity and Wangaards finding that the ratio of axial to radial conductivity is approximately 3.0. The relationship for axial conductivity is therefore,

\[ K_Z(U,T) = \frac{\rho_D}{(1 + \frac{kU}{U_f})} (.6 + 1.2 U) + .071 \text{watts m}^0\text{K} \] ....(34)

Empirical equations for the specific heat of dry wood are available in reference (27). These relationships have been determined experimentally for several American softwoods and give specific heat as a function of temperature. Also given in this reference is a method commonly used to account for moisture effect. Moisture is included in the relation by assuming a specific heat contribution proportional to water volume.
V. NUMERICAL SOLUTION OF THE MASS AND HEAT TRANSFER EQUATIONS

A finite-difference approximation technique was used to formulate a solution method for the two-dimensional transport equations. A very stable scheme was desired because of the unstable character of coupled equations with variable coefficients and boundary conditions. To satisfy the requirement of good stability, an implicit three-time-level scheme was chosen. In this method, moisture content and temperature at advanced time are calculated using their values at present and previous time. A grid system was established within the body and the advanced-time values of moisture content and temperature, for each grid point, were obtained by solution of a matrix equation.

Boundary condition equations are included in the formulation of the matrix equation. Solution of the matrix equation consequently satisfies all necessary boundary conditions.

The matrix equation arising from an implicit numerical solution to a two-dimensional problem is poorly conditioned and can be difficult to solve on a digital computer. An alternating direction method as given in Mitchell (19) was used to split the one matrix equation into two equations. This two-step solution method involves solution of two well-conditioned tri-diagonal matrix equations along lines first parallel to one then parallel to the other coordinate axis. The tri-diagonal form makes these equations readily solvable.

Symmetry of the mass and heat transfer processes in the body enables the use of a rectilinear grid system. The grid system,
as shown in Figure 22 applies at any angle of revolution about the cylinder axis. Grid point indices, K and KP, are used when solving along lines parallel to the radial and axial coordinates, respectively.

A. Mass Transfer

The finite difference form of the non-dimensional mass transfer equation, Equation 3, is

\[
\frac{1}{r^2} \delta_R [r A_R(U,T) \delta_R] U^n + \delta_Z [A_Z(U,T) \delta_Z] U^n = \frac{1}{\xi_m} B(U) \left( \frac{u^{n+1} - u^{n-1}}{2} \right)
\]

where

\( \delta_R = \) central difference operator for the radial direction,

\( \delta_Z = \) central difference operator for the axial direction,

\( U^{n-1}, U^n, U^{n+1} = U \) at previous, present and advanced time levels, respectively,

\( \xi_m = \frac{\text{non-dimensional time step}}{(\text{non-dimensional space step})^2} = \frac{\Delta t_m}{S^2} \).

The term, \( \xi_m \), is the mesh ratio of the finite difference approximation. The central difference operators determine the difference between the
FIGURE 22 - GRID SYSTEM FOR NUMERICAL METHOD
dependent variable one-half space step either side of a grid point.
Mathematically, these are given by the relations,

\[ \delta_R U^n_K = U^n_{K+\frac{1}{2}} - U^n_{K-\frac{1}{2}} \]  \hspace{1cm} \ldots \ldots (36)

and

\[ \delta_Z U^n_{KP} = U^n_{KP+\frac{1}{2}} - U^n_{KP-\frac{1}{2}} \]  \hspace{1cm} \ldots \ldots (37)

where \( K \) and \( KP \) are as given in Figure 22 and \( U_K \) is the moisture content at grid point \( K \).

For a three-time-level scheme, the substitution

\[ U^n = \frac{U^{n+1} + U^n + U^{n-1}}{3} \]

is used. Use of this substitution in the general difference formula, Equation 35, with \( A^*_R(U,T) = A^*_Z(U,T) = B(U) = 1 \) leads to an unconditionally stable scheme in the sense of von Neumann (19). The three-time-level form of Equation 35 is

\[
\left\{ 1 - \frac{2\varepsilon_m}{3B(U)} \left[ \frac{\delta_R(r^*A^*_R(U,T)\delta_R)}{r^*} + \delta_Z(A^*_Z(U,T)\delta_Z) \right] \right\} U^n_K
\]

\[ = \frac{2\varepsilon_m}{3B(U)} \left[ \frac{\delta_R(r^*A^*_R(U,T)\delta_R)}{r^*} + \delta_Z(A^*_Z(U,T)\delta_Z) \right] U^n_K \]

\[ + \left\{ 1 + \frac{2\varepsilon_m}{3B(U)} \left[ \frac{\delta_R(r^*A^*_R(U,T)\delta_R)}{r^*} + \delta_Z(A^*_Z(U,T)\delta_Z) \right] \right\} U^{n-1}_K \]

\[ \ldots \ldots (38) \]
Equation 38, when expanded to matrix form, is poorly conditioned and for cases where large numbers of grid points are used, digital solutions are excessively time consuming.

The equation splitting technique was used on Equation 38. A fourth order operator term was added to both sides so that the left hand side could be factored. Removing the parameter lists from the coefficients, the factored form of the equation is

\[
\begin{bmatrix}
1 - \frac{2\varepsilon_m}{3Br^*} \delta_R(r*A_Z^*\delta_R) \\
1 - \frac{2\varepsilon_m}{3B} \delta_Z(A_Z^*\delta_Z)
\end{bmatrix}
U^{n+1}_K
\]

\[
= \frac{2\varepsilon_m}{3B} \left[ \frac{\delta_R(r*A_Z^*\delta_R)}{r^*} + \delta_Z(A_Z^*\delta_Z) \right] U^n_K
\]

\[
+ \left\{ 1 + \frac{2\varepsilon_m}{3B} \left[ \frac{\delta_R(r*A_Z^*\delta_R)}{r^*} + \delta_Z(A_Z^*\delta_Z) \right] \right\} U^{n-1}_K
\]

\[
+ \frac{4\varepsilon_m^2}{9B^2r^*} \delta_R(r*A_Z^*\delta_R)\delta_Z(A_Z^*\delta_Z)(\alpha U^n_K + \beta U^{n-1}_K). 
\]

\[ \ldots \ldots (39) \]

The two parameters, \( \alpha \) and \( \beta \), arise from addition of the fourth order terms to the right hand side of Equation 38. They are relaxation parameters and must satisfy the condition,

\[ \alpha + \beta = 1, \]

to have the same order of accuracy as the original formula.
Defining an intermediate value of $U$ as $U^{(n+1)*}$, Equation 39 can be split into the two desired equations:

1. \[ [1 - \frac{2E_m}{3BR^*} \delta_R(r^*A^*\delta_R)] U_K^{(n+1)*} = \frac{2E_m}{3B} \left[ \frac{\delta_R(r^*A^*\delta_R)}{r^*} + \delta_Z(A^*\delta_Z) \right] U_K^n \]
\[
+ [1 + \frac{2E_m}{3B} \left( \frac{\delta_R(r^*A^*\delta_R)}{r^*} + \delta_Z(A^*\delta_Z) \right) U_K^{n-1} \]
\[
+ \frac{2E_m}{3B} \delta_Z(A^*\delta_Z)(\alpha U_K^n + \beta U_K^{n-1}) \quad \ldots(40) \]

2. \[ [1 - \frac{2E_m}{3B} \delta_Z(A^*\delta_Z)] U_{KP}^{n+1} = U_K^{(n+1)*} \]
\[
+ \frac{2E_m}{3B} \delta_Z(A^*\delta_Z)(\alpha U_{KP}^n + \beta U_{KP}^{n-1}) \quad \ldots(41) \]

Since the left side of each equation involves only one operator, the matrices formed are tri-diagonal. The solution for moisture content at the advanced time level, $(n+1)$, is obtained by solving first for $U^{(n+1)*}$ then for $U^{(n+1)}$.

Equations 40 and 41 apply only to internal grid points because they do not include boundary conditions. Knowledge of the boundary conditions at previous, present and advanced time levels are required
Values for the advanced time levels are not known, but for this work, were approximated by those from the present time level.

B. Mass Transfer Boundary Conditions Approximation

Use of the central difference operators at grid points along internal or external boundaries requires special treatment. The difficulty which arises can be seen by expanding a finite difference equation about any grid point. For the radial direction, using the $k$ index, the expansion is

$$\frac{1}{r^*} \delta_R (r^* A^*_R \delta_R) U_k = \frac{1}{r^*} \left[ r^*_k \frac{1}{2} A^*_R (U_{k+1} - U_k) \right] - r^*_k \frac{1}{2} A^*_R (U_k - U_{k-1})$$

For the axial direction, using the $k_p$ index, the expansion is

$$\delta_z (A^*_z \delta_z) U_{k_p} = A^*_z (U_{k_p+1} - U_{k_p})$$

$$- A^*_z (U_{k_p-1} - U_{k_p-1})$$

When considering a grid point situated on an internal boundary, terms $U_{k-1}$ and $U_{k-\frac{1}{2}}$, for the radial direction (Equation 42), and terms $U_{k_p-1}$ and $U_{k_p-\frac{1}{2}}$, for the axial direction (Equation 43), are outside the defined grid area. Similarly at the external boundaries, the terms
$U_{K+1}$, $U_{K+\frac{1}{2}}$, $U_{KP+1}$ and $U_{KP+\frac{1}{2}}$ are outside the defined grid area. These terms are necessary for the complete matrix equation and must be estimated using approximate internal and external boundary condition equations. The equations used for each boundary will now be considered.

1. Internal boundary

Radial and axial internal boundary conditions given by Equations 7 and 9, respectively, were approximated using the finite difference technique. The final relations are

$$U_{K-1} = U_K; \quad r^* = 0; \quad 0 \leq z^* \leq 1,$$

and

$$U_{KP-1} = U_{KP}; \quad 0 \leq r^* \leq 1; \quad z^* = 0.$$

Values of the half-step terms, $U_{K-\frac{1}{2}}$ and $U_{KP-\frac{1}{2}}$ were determined by averaging adjacent values. For example,

$$U_{K-\frac{1}{2}} = \frac{U_{K-1} + U_K}{2} = U_K.$$

2. External boundary

The general form for the external boundary condition is given by the relationship

$$\frac{\partial U}{\partial x^*} + Bi_m x(U,T) U = \lambda x.$$
where \( x^* \) is either space coordinate. The derivative \( \frac{\partial U}{\partial x^*} \) is approximated as the change in \( U \) across two grid spaces. Mathematically, for each direction, these derivatives are

\[
\frac{\partial U}{\partial r^*} = \frac{U_{K+1} - U_{K-1}}{2S} \quad \text{.....(48)}
\]

and

\[
\frac{\partial U}{\partial z^*} = \frac{U_{KP+1} - U_{KP-1}}{2S} \quad \text{.....(49)}
\]

Equations 48 and 49 were substituted into the appropriate form of Equation 47, depending on the coordinate direction, to obtain the following approximations:

\[
U_{K+1} = U_{K-1} - 2SBi_m R U_K + 2S\lambda_R; \quad r^* = 1; \quad 0 \leq z^* \leq 1 \quad \text{.....(50)}
\]

and

\[
U_{KP+1} = U_{KP-1} - 2SBi_m Z U_{KP} + 2S\lambda_Z; \quad 0 \leq r^* \leq 1; \quad z^* = 1 \quad \text{.....(51)}
\]

Values of half-step terms, such as \( U_{K+\frac{1}{2}} \) and \( U_{KP+\frac{1}{2}} \), were approximated by averaging values at adjacent grid points. Considering the radial direction only,
C. Heat Transfer

The governing heat transfer equation, in dimensionless form, (Equation 11) was approximated using finite difference operators. The result, as given below, is very similar to the mass transfer equation.

\[
\frac{1}{r^\*} \delta_R (r^*k^*(U,T)\delta_R) T^*n + \delta_Z (k^*_Z(U,T)\delta_Z) T^*n
\]

\[+ (KoLu)_i \left\{ \frac{1}{r^*} \delta_R (r^*f_R A^*_R(U,T)\delta_R) U^n \right. \]

\[+ \delta_Z (f_Z A^*_Z(U,T)\delta_Z) U^n - \frac{B(U)}{\xi_m} \left( \frac{U^{n+1} - U^{n-1}}{2} \right) \}

\[= \frac{B(U)C^*(U,T)}{\xi_h} \frac{(T^*(n+1) - T^*(n-1))}{2} \quad \text{(53)}\]

where

\[T^*(n-1), T^*n, T^*(n+1) = T^* \text{ at previous, present and advanced time levels, respectively,} \]

and

\[\xi_h = \frac{\Delta t^*_h}{\Delta z^2} .\]

Letting
\[ I = \frac{1}{r^*} \delta_R r^* A^* (U, T) \delta_R U^n + \delta_Z f_Z A^* (U, T) \delta_Z U^n \]

\[ - \frac{B(U)}{\varepsilon_m} \frac{(U^n+1 - U^{n-1})}{2} \]  

\[ \text{and} \]

\[ T^* n = \frac{T^*(n+1) + T^* n + T^*(n-1)}{3} , \]

the heat transfer equation, neglecting coefficient parameter lists, becomes

\[ \left\{ 1 - \frac{2 \varepsilon_n}{3BC^*} \left[ \frac{\delta_R r^* K^* \delta_R}{r^*} + \delta_Z (K^* \delta_Z) \right] \right\} T^*(n+1) \]

\[ = \frac{2 \varepsilon_n}{3BC^*} \left[ \frac{\delta_R r^* K^* \delta_R}{r^*} + \delta_Z (K^* \delta_Z) \right] T^* n \]

\[ + \left\{ 1 + \frac{2 \varepsilon_n}{3BC^*} \left[ \frac{\delta_R r^* K^* \delta_R}{r^*} + \delta_Z (K^* \delta_Z) \right] \right\} T^*(n-1) \]

\[ + \frac{2 \varepsilon_n}{BC^*} (KoLu) \varepsilon I \]  

\[ \text{Adding fourth order operator terms Equation 55 can be split into the two equations:} \]

\[ 1. \left[ 1 - \frac{2 \varepsilon_n}{3BC^*} \frac{\delta_R r^* K^* \delta_R}{r^*} \right] T^*(n+1) \]

\[ = \frac{2 \varepsilon_n}{3BC^*} \left[ \frac{\delta_R r^* K^* \delta_R}{r^*} + \delta_Z (K^* \delta_Z) \right] T^* n \]
\[ + \left\{ 1 + \frac{2\varepsilon_h}{3BC^*} \left[ \frac{\delta R(r^*K_r\delta R)}{r^*} + \delta_Z(K_Z\delta Z) \right] \right\} T_k^*(n-1) \]

\[ + \frac{2\varepsilon_h}{3BC^*} \delta_Z(K_Z\delta Z)(\alpha T_k^n + \beta T_k^{n-1}) \]

\[ + \frac{2\varepsilon_h}{BC^*} (KoLu)_i I \]  

\[ \ldots \ldots (56) \]

\[ 2. \left[ 1 - \frac{2\varepsilon_h}{3BC^*} \delta_Z(K_Z\delta Z) \right] T_k^{n+1} = T_k^{(n+1)*} \]

\[ + \frac{2\varepsilon_h}{3BC^*} \delta_Z(K_Z\delta Z)(\alpha T_k^n + \beta T_k^{n-1}) \]  

\[ \ldots \ldots (57) \]

As for the development of the mass transfer equation, \( \alpha \) and \( \beta \) are relaxation parameters. They must satisfy the condition

\[ \alpha + \beta = 1 \]

for Equations 56 and 57 to have the same order of accuracy as Equation 55.

The solution for \( T_k^{(n+1)*} \) is found in two steps. First, Equation 56 is solved for \( T_k^{(n+1)*} \) then the desired advanced-time value of temperature, \( T_k^{(n+1)} \), is found from Equation 57. But, before the solution can be calculated, the two approximating equations must be modified to include boundary conditions. Approximations for internal and external boundary conditions are considered in the next section.
The heat-sink term, $I$, accounts for internal liquid evaporation. It was included on the right hand side of Equation 56 because of the assumption that the mass transfer equation would have previously been solved.

D. Heat Transfer Boundary Conditions

Boundary conditions were included in the heat transfer matrix equation in the same way they were included in the mass transfer equation. This was done to compensate for grid points used by the central difference operators at the boundaries which were outside the established grid system. The heat transfer internal boundary conditions are of identical form to the mass transfer internal boundary conditions. External boundary conditions are of different form to their mass transfer counter-parts because of a term added to account for surface evaporation.

1. Internal boundary

Approximation of internal boundary condition Equations, 14 and 16, produce the two equations,

$$T_{K-1}^* = T_K^* ; \quad r^* = 0 ; \quad 0 \leq z^* \leq 1 . \quad (58)$$

and

$$T_{KP-1}^* = T_{KP}^* ; \quad 0 \leq r^* \leq 1 ; \quad z^* = 0 . \quad (59)$$
2. External boundary

External boundary condition approximations for the radial and axial directions, using Equations 13 and 15, are

$$T_{K+1}^* = T_{K-1}^* - 2SBi_R T_K^* - 2K0Lu_{LR}(U_K - U_{K-1}) ;$$

\[ r^* = 1 ; 0 \leq z^* \leq 1 , \quad \ldots \ldots (60) \]

and

$$T_{KP+1}^* = T_{KP-1}^* - 2SBi_Z T_{KP}^* - 2K0Lu_{LZ}(U_{KP} - U_{KP-1}) ;$$

\[ 0 \leq r^* \leq 1 ; z^* = 1 . \quad \ldots \ldots (61) \]

E. Solvable Form of the Mass and Heat Transfer Matrix Equations

The mass transfer boundary conditions, Equations 44, 45, 50, and 51, when combined with the finite difference approximation for the mass transfer equation, Equations 40 and 41, produce two equations for each grid point. One equation is for the \((n+1)^*\) time level, the other for the \((n+1)\) time level. Writing these for each grid point produces a set of simultaneous equations which form a matrix equation. The order of the matrices is equal to the number of grid points.

The two matrix equations for mass transfer are:

1. \[ [F] \{U\}^{(n+1)*} = \{W\} , \quad \ldots \ldots (62a) \]
Values of $U$, for each time level and grid point, are held in vectors. $[F]$ and $[G]$ are square, tri-diagonal arrays. In the above equations all, but $U^{(n+1)}$ and $U^{(n+1)*}$, are known. Details of the arrays shown are given in Appendix D.

The general matrix equations for heat transfer are identical to those for mass transfer except for the presence of the heat-sink term, $\{A\}$. These equations are:

1. $[FT] \{T\}^{(n+1)*} = \{WT\} + \{A\}$ , ........(63a)

2. $[GT] \{T\}^{(n+1)} = \{XT\} + [YT] \{T\}^{(n+1)*}$ . ........(63b)

$\{T\}$ terms are vectors with the number of entries equal to the number of grid points. $[FT]$ and $[GT]$ are square, tri-diagonal arrays. All terms are known except for $\{T\}^{(n+1)*}$ and $\{T\}^{(n+1)}$. Details of all arrays in Equation 63 are located in Appendix E.

F. Calculation of Drying Rate and Average Moisture

In order to obtain complete results for wood drying, drying rate and average moisture content were calculated for each successive
time level. Drying rate was calculated from the rate of convective surface mass transfer. This calculation requires a surface mass transfer coefficient for the radial and axial surfaces, and surface and ambient humidities. Integration over the complete surface was performed using a Simpson's rule approximation (p. 24 of Reference 5). Drying rates for the \( n^{th} \) time level were determined using humidities from the same time level. Humidities were calculated using surface moisture contents in conjunction with desorptional isotherms. The computer program written for the drying rate calculation is given on pp. 159 to 161 of Appendix H.

Average moisture content was determined, depending on the particular drying problem, by one of two methods. For the well behaved cases, those cases not using moisture diffusivities obtained from the wood model, the average moisture content was found by integrating the drying rate with respect to time. This integration was performed for each successive time interval. The amount of mass removed during the time interval was subtracted from the previous moisture content to obtain an up-dated value. The computer program used for these calculations is given on p. 161 of Appendix H.

For the drying cases where moisture diffusivities from the wood model were used, average moisture content was determined by integrating over the interior volume. Integration was achieved using a Trapezoid rule approximation at each time level. Details of the computer program are given on p. 161 of Appendix H.
G. Computer Program to Solve the Mass Transfer Equation

A computer program to solve the mass transfer equation as given by Equation 62 was written in Fortran IV for an IBM 370 digital computer. Terms of the equations were generated using the method and equations given in Appendix D. Solutions to the matrix equations were achieved using a Computer Science Department standard program called TRISLV. This program solves tri-diagonal matrix equations and all necessary calling procedures are given in the University of British Columbia usage document reproduced in Appendix I. A listing of the complete computer program is given in Appendix H. Detailed explanation of operation procedures are given in Appendix G.

Exact and numerical solutions to a drying problem were compared. For this comparison, moisture diffusivities and boundary conditions were assumed constant and moisture content was uniformly distributed throughout the body. The exact solution was obtained by adapting results from Luikov (16) for transient heat conduction. The comparison, shown in Figure 23, indicates good agreement between the two solutions. The numerical solution method was also successfully tested on a case where \( Bi_{mR} = 20 \) and \( Bi_{mZ} = 20 \).
FIGURE 23 - COMPARISON OF EXACT AND NUMERICAL SOLUTIONS
VI. DISCUSSION OF RESULTS

A. Moisture Diffusivities Predicted by the Wood Model

Predicted radial diffusivities were compared to experimental results of Biggerstaff (3) and Veljkovic (36). Biggerstaff calculated integral radial diffusivities at various wood temperatures from drying tests with thin Eastern Hemlock slabs. Initial moisture content was uniform and equal to that at fiber saturation. Radial diffusivities were calculated by comparing the experimental results to an approximate solution applicable to constant diffusivity drying of an infinite slab. This solution applies when drying is internally controlled and Fourier numbers are less than 0.1 (19). This method of calculating diffusivities has been used extensively (30, 31) because of its simplicity. Veljkovic determined tangential and radial diffusivities for Western Hemlock. In her work, diffusivities were calculated by best-fitting a constant diffusivity solution to experimental results. Wood samples, 2 in x 4 in x 12 in containing 70% to 100% moisture (dry basis), were dried in a fluidized bed of sand. Internal temperature and moisture content were monitored at several internal locations. Wood temperatures, at the beginning of the region used to calculate the diffusivities, were approximately 20% below bed temperatures. For purpose of comparison with wood model results, the bed temperatures were taken as wood temperatures.

Radial diffusivities predicted by the wood model are functions of both temperature and moisture content. An effective moisture content,
as obtained from Equation 20, was used to eliminate the influence of moisture content. The value used was 1/3 of fiber saturation moisture content appropriate to drying temperature.

The comparison of wood-model predictions and previously cited work is shown in Figure 24. Also shown are theoretical predictions made by Wirakusumah (39) who used Stamm's wood model, but in all calculations considered bound water diffusivity to be constant. Results of our work compare very well with the experimental work shown. This good agreement concerning the effect of temperature is primarily due to the relationship included in the analysis which allows for the dependency of bound water diffusion on temperature.

Unfortunately, experimental results were not available with which the predicted effect of moisture content and diffusion direction could be compared. In our results, the ratio of axial to radial diffusivity evaluated at \( 1/3 U_f \) ranged from 20 at 25°C to 41 at 100°C. Bateman et al. (40) measured an axial diffusivity of approximately \( 8.5 \times 10^{-5} \text{ cm}^2/\text{sec} \) for wood with a dry wood density of 0.4 g/cm³ dried at 40°C. This compares reasonably well with the predicted value of \( 11.5 \times 10^{-5} \text{ cm}^2/\text{sec} \) calculated at the effective moisture content.

Yao (40), from structural considerations, determined that radial diffusivities in woods of different densities should be inversely related to the square of their densities. In equation form, this relationship is

\[
\frac{D_{c@\rho_1}}{D_{c@\rho_2}} = \left( \frac{\rho_2}{\rho_1} \right)^2 .
\]  

\ldots(64)
Figure 24 - Comparison of predicted and experimental radial diffusivities for the effect of temperature.
He validated this relationship by comparing predicted results to experimental diffusivities measured by Stamm (31). Agreement was within approximately 15%. Results for radial diffusivity at different densities as given by the wood model also have the interrelationship shown in Equation 64. This can be explained with the use of relevant terms of the diffusivity equations. In the radial direction, bound water diffusion through the discontinuous cell walls controls the diffusion through the cavity-wall-cavity wood structure. With this consideration and neglecting pit and continuous wall diffusion, Equation 18 simplifies to

\[ D_C \approx D_3. \]

From calculations for the pathway diffusivities, \( D_3 \propto (\rho_{\text{WS}}/\rho_D)^2 \). Therefore, for constant \( \rho_{\text{WS}} \), combined diffusivities can be related to wood density as in Equation 64. This relationship does not hold for diffusion in the axial direction because of the importance of vapour diffusion.

**B. Numerical Solution Method**

The mass transfer equation with constant diffusivities and boundary conditions was solved for several different values of grid spacing. Results of the tests are shown in Figures 25 and 26. From Figure 25 it is evident that space increment effects the accuracy of
FIGURE 25 - EFFECT OF SPATIAL STEP SIZE ON NUMERICAL SOLUTION FOR DRYING CURVES

GRID INTERVALS
- 3
- 5
- 10
- 16

\[ \frac{A_R}{A_Z} (\frac{R}{L})^2 = 2.5, \]

\[ \text{Bi}_{mR} = 4.84 \]

\[ \text{Bi}_{mZ} = 0.97 \]
FIGURE 26 - EFFECT OF SPATIAL STEP SIZE ON NUMERICAL SOLUTION FOR MOISTURE PROFILES
the solution over the full drying time. The solution for center moisture content is most affected because of the approximation used for the internal boundary condition. Although accuracy improves with decreased grid spacing, improvements were negligible for cases where the number of grid intervals exceeded twice the largest Biot mass number. This estimate for grid spacing needs more investigation since it is likely that there is a more optimal criterion of selection especially for Biot numbers greater than 20. Due to space limitations the maximum number of grid intervals the computer program was designed to accept is 44; this selection produces 1936 grid points.

The effect of grid spacing on moisture profiles is shown in Figure 26. These results indicate the solution for the axial direction is more sensitive to grid spacing than is the solution for the radial direction. This difference is caused by the equation splitting technique. The split equations solve first for the radial direction then for the axial direction. This preferential treatment is sufficient to make the solution for the radial direction more accurate for all values of grid spacing.

Results for the investigation of grid spacing effects were stable and acceptably smooth as a result of adequate choice of the time increment. Solutions become oscillatory when large time increments are used. If very large ones are used and/or the differential equation is highly non-linear, it is possible to obtain unstable oscillatory solutions. An example of an oscillatory drying curve is shown in Figure 27. Here it can be seen that oscillations of the surface
FIGURE 27 - OSCILLATORY NUMERICAL SOLUTION FOR DRYING CURVE
solutions may not significantly affect the solution for the center moisture content. From solutions generated for this work, it was found that smooth solutions were obtained when the mesh ratio, $\xi_m$, was less than unity; smaller time steps produce smoother solution curves.

Numerical solutions of the mass transfer equation, for the case of constant diffusivity and boundary conditions, were obtained using three pairs of the relaxation parameters, $\alpha$ and $\beta$. Keeping within the condition that their sum equal unity, the values chosen were 0.5, 0.5; 2, -1; 1, 0. The difference between the three solutions was of the order of 0.02%. This is an insignificant difference and for simplicity, subsequent runs used values of $\alpha = 0.5$ and $\beta = 0.5$.

C. Combination of Wood Model and Mass Transfer Equation

1. Effect of boundary conditions

Real boundary conditions, as given by the S-shaped isotherms of Figure 16, and linearized boundary conditions, as given by the straight lines of Figure 16, were each used to calculate drying curves. This was done to determine if the solution to the mass transfer equation for wood could be simplified by using linearized boundary conditions without unduly affecting accuracy. Moisture diffusivities were taken to be constant.

The difference between results for the two isotherm shapes tested was found to be dependent on the magnitude of the Biot mass
number. Figure 28 shows results for relatively small Biot mass numbers. Since for a majority of surface moisture contents the S-shaped isotherms give larger surface humidities than do the linearized isotherms, drying rates will be correspondingly higher. It is this larger drying rate which causes faster drying for the real boundary conditions, as shown in Figure 28. For large Biot mass numbers, surface moisture content quickly approaches ambient moisture content. When ambient moisture is low, the difference between results using the actual and linearized isotherms should be small. This behaviour was investigated by considering a case with $\text{Bi}_{mR} = 21$ and $\text{Bi}_{mZ} = 210$. For this case, there was negligible difference between drying results using the two types of boundary conditions.

Use of the linearized isotherms has an advantage when considering mass transfer with constant moisture diffusivities. The mass transfer equation can be written in standard non-dimensional form; moisture content $U$ becomes the new variable $U^*$ where $U^* = (U - U_a)/(U_i - U_a)$. Using $U^*$, the mass transfer equation can be solved exactly in the form of a product solution. Mathematical treatment of this type of problem can be adapted from Luikov (16). An example is given in this work on pages 13 and 14.

2. Wood drying

Wood model results were substituted into the numerical solution procedure for the mass transfer equation to simulate isothermal drying of softwood. Initial moisture contents were set equal to the fiber
Figure 28 - Comparison of Drying Curves from Real and Linearized Boundary Conditions

\[ \frac{A_Z}{A_R} \left( \frac{R}{L} \right)^2 = 2.5 \]

\[ (Bi_{mR})_1 = 4.85 \]

\[ (Bi_{mZ})_1 = 0.97 \]

\[ \lambda_R \bar{\phi} = 0.0 \]

\[ \lambda_Z \bar{\phi} = 0.0 \]

\[ 0.01 \leq t^* \leq 1.0 \]
saturation value appropriate for the drying temperature and taken to be uniformly distributed. Ambient vapour density was zero. Convective mass transfer coefficients were calculated from an empirical equation which applies to heat transfer from an infinite cylinder placed perpendicular to a flowing fluid (13). This relation assumes heat and mass transfer from the surface are analogous processes. It was necessary to assume that the convection coefficients calculated for the cylinder wall applied also to the cylinder ends. The final relationship (Appendix F) is a function of Reynolds number and the ratio of the Schmidt and Prandtl numbers. In all calculations the ratio of the Schmidt and Prandtl numbers was taken as unity and the length to radius ratio of the body was 2.0.

A plot of center moisture content versus time for drying at 40°C and 60°C is shown in Figure 29. Also shown is a result for drying with constant moisture diffusivity. The curve for constant diffusivity can be considered to consist of three regions. The first region ends when the moisture content has decreased to 60% of its original value. This region is the one where drying is strongly influenced by initial conditions. The second region is the linear portion of the curve. While in this region, drying is an exponential function of time. The slope of this line is commonly used by experimenters to determine effective diffusivity because it is proportional to diffusivity. The third section indicates the body is coming to equilibrium with its surroundings.
Figure 29 - Center moisture content as a function of time for softwood at temperatures of 40°C and 60°C
Contrasting the constant and wood model diffusivity cases, it can be seen that the second and third regions, for the latter case are not clearly defined. The first region still exists until the 60% moisture content point, but instead of a linear second region, the slope of the line continuously decreases. This behaviour indicates the existence of a continuously decreasing effective diffusivity, a result which is in qualitative agreement with experimental findings.

Average moisture content versus time for 60°C is shown in Figure 30. Oscillations existed in the computer results used to generate this curve because of limitations of the numerical method. These oscillations were eliminated, for the purpose of plotting, by averaging between successive time intervals. Dimensionless time intervals were less than .002 and therefore justify this simple averaging procedure.

Drying rate curves for the two temperatures considered are shown in Figure 31. Falling rate drying occurs over the full drying process. Drying rate falls off rapidly, showing that the drying process is internally controlled. For $t^*_m$ greater than 0.1, surface moisture content has reached near its equilibrium value.

The effect of moisture diffusivities on moisture content profiles is shown in Figure 32. The profile in the radial direction has the usual parabolic character. This is expected since radial moisture diffusivity, when compared to axial diffusivity, is relatively constant. The bell shaped form of the axial diffusivity is the cause of the reverse in curvature seen in the axial moisture profile.
FIGURE 30 - AVERAGE MOISTURE CONTENT AS A FUNCTION OF TIME FOR SOFTWOOD AT A TEMPERATURE OF 60°C

\[ A_R(U_f, 60^\circ C) = 0.3022 \times 10^{-4} \text{cm}^2/\text{s} \]

\[ T = 60^\circ C \]

\[ \frac{L}{R} = 2.0 \]

\[ \text{Re} = 20 \]

\[ \frac{Sc}{Pr} = 1.0 \]

\[ \rho_d = 0.4 \text{ g/cm}^3 \]

\[ k = 0.1 \text{ cm}^3/\text{G} \]

\[ \rho_a = 0.0 \text{ g/cm}^3 \]
FIGURE 31 - DIMENSIONLESS DRYING RATE AS A FUNCTION OF TIME FOR SOFTWOOD AT TEMPERATURES OF 40°C AND 60°C
FIGURE 32 - MOISTURE PROFILES FOR SOFTWOOD AT A TEMPERATURE OF 60°C

\[ A_R (\bar{\nu}, 60^\circ C) = 0.3022 \times 10^{-4} \text{ cm}^2/\text{sec} \]

\[ \frac{L}{R} = 2.0 \]

\[ \text{Re} = 20 \]

\[ \frac{Sc}{Pr} = 1.0 \]

\[ \rho_D = 0.4 \text{ g/cm}^3 \]

\[ k = 0.1 \]

\[ \rho_a = 0.0 \text{ g/cm}^3 \]
Results of the wood drying simulation are subject to limitations of the wood model and the external boundary conditions. The relationship used for convective mass transfer applies adequately for the Reynolds number range 20 to 4000 and requires that convective heat and mass transfer are completely analogous. The restrictions on the wood model, other than those included in the assumptions made for its development, are: (1) the wood structure must stay structurally intact; breakdown of the wood structure could occur because of internal stresses; pyrolysis occurs when wood temperatures are greater than 100°C, (2) the cylinder dimensions are greater than the minimum values for a tracheid, i.e., the body is large enough to apply the average quantities used in the wood model.

D. Heat Transfer

Numerical solution of the heat transfer equation is very similar to the mass transfer equation solution. The computer program used to solve the mass transfer equation could be modified by adding the necessary steps to calculate the heat-sink term \( A \). Appropriate substitutions of coefficients between the two cases would allow the rest of the solution method programming to remain the same. The principal difficulties would be the lack of computer storage space and the cost of computer time.
E. Scope of Future Work

Experimental results are required to compare with the wood drying simulation. Results of such a comparison would enable improvement of the wood model or, if satisfactory agreement was found, the wood model would greatly assist in specifying softwood diffusion coefficients and in understanding wood drying behaviour.

A more thorough investigation of the behaviour of the program is required so that optimal selection of time and space integration increments can be made. It would also lead to a better understanding of the solutions sensitivity to the various input parameters such as ambient moisture content, Reynolds number and the Schmidt number - Prandtl number ratio.

The computer program should be reorganized so that the section which contains the numerical solution method is a separate subroutine. This would improve the program's generality and ease its application to other than drying problems.

Conditional to verification of the mass transfer part of the wood model, the heat transfer equation could be solved. This would require a certain amount of computer programming, but the existing numerical solution scheme could be used in its present form.
VII. CONCLUSIONS

1. The principal variables in the wood model which control radial and axial diffusivity are dry-wood density, temperature and moisture content. Wood shrinkage and swelling has only a slight effect on diffusivity. All other wood parameters which apply to softwoods are internally specified and from the agreement with experimental results (Figure 24) are adequate for softwoods in general.

2. The wood model diffusivities agree well with experimental results on the effect of temperature, but there are insufficient experimental results available to substantiate the predicted effect of moisture content. The dependency of radial diffusivity on wood density agrees with experimental results.

3. The numerical solution method can be used to solve two-dimensional non-linear parabolic partial differential equations. For the solutions, the mesh ratio should be less than unity and the number of grid intervals should, if possible, be twice the largest Biot mass number.

4. Drying times for the body analyzed in this work are sensitive to drying temperature. The time to dry the body to the same center moisture content was 40% less for the 60°C case than the 40°C case.
5. For bodies with large length to radius ratios, the small magnitude of $A_R(U,T)$ produces a large Biot mass number and drying behaviour of wood will therefore be relatively insensitive to values of Reynolds number and ratio of Schmidt number to Prandtl number.
REFERENCES


The analysis uses a control volume of the finite cylinder. No moisture gradient exists in the angular direction so that mass transfer occurs in the radial and axial directions only.

The unsteady-state mass transfer equation was obtained by equating the net diffusion out of the control volume to the decrease in mass of moisture within the control volume. The result is

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( rD \frac{\partial c}{\partial r} \right) + \frac{\partial}{\partial z} \left( D \frac{\partial c}{\partial z} \right) = \frac{1}{V_S} \frac{\partial M}{\partial t} \quad \text{.....(A-1)}
\]

where

\[V_S = \text{volume of wet control volume,}\]
\[= 2\pi r dr dz (\text{cm}^3),\]
\[c = \text{liquid concentration,}\]
\[= \text{mass of water/volume of wet wood (g/cm}^3),\]
\[M = \text{mass of moisture within the control volume (g)}.\]

Liquid concentration, \(c\), was related to liquid moisture content, \(U\), using the wood model relationship which relates wood volume and moisture content. The definition of liquid concentration is

\[c = \frac{\text{mass of liquid}}{\text{volume of wet wood}},\]
The wet and dry volumes of wood were related using Equation 17 to produce, from Equation A-2, the following relationship:

\[ c = \frac{\rho_d U}{(1 + kU/U_f)} \]  

Substituting Equation A-3 into Equation A-1 and using the chain rule of differentiation, the mass transfer equation becomes

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r D_R}{1 + kU/U_f} \frac{\partial U}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{D_k}{1 + kU/U_f} \frac{\partial U}{\partial z} \right) = \frac{1}{\rho_D V_S} \frac{\partial M}{\partial t} \]  

The moisture capacitance term, \( \frac{\partial M}{\partial t} \), is defined by the relationship,

\[
\frac{\partial M}{\partial t} = \frac{\partial}{\partial t} \left\{ \rho_D V_D U + \varepsilon \rho_V V_S \right\} 
\]

where

- \( \rho_D \) = density of dry wood (g/cm³),
- \( V_D \) = volume of dry wood (cm³),
- \( \varepsilon \) = voidage fraction,
- \( \rho_V \) = density of water vapour (g/cm³),
- \( V_S \) = volume of wet control volume (cm³).
The magnitude of the vapour term, $\varepsilon \rho V_S$, in Equation A-5, was compared to the magnitude of the liquid term, $\rho_D V_D U$. Using Equation 17, this ratio,

$$\frac{\varepsilon \rho V_S}{\rho_D V_D U},$$

can be written as

$$\frac{\varepsilon \rho_{\text{sat}} H}{\rho_D U} (1 + kU/U_f) \quad \ldots \ldots (A.6)$$

By considering the possible magnitudes of each term in Equation A-6, it was determined that

$$\frac{\varepsilon \rho_{\text{sat}} H}{\rho_D} (1 + kU/U_f) \bar{\varepsilon} \rho_{\text{sat}}/\rho_D \quad \ldots \ldots (A-7)$$

Typical values of $\rho_D$ and $\rho_{\text{sat}}$ are:

$$\rho_{\text{sat}} @ 100^\circ C = 4.5 \times 10^{-5} \text{ g/cm}^3,$$

$$\rho_D = 0.4 \text{ g/cm}^3.$$

These typical values of $\rho_{\text{sat}}$ and $\rho_D$ were substituted into Equation A-7 and it was found that the ratio of mass of vapour to mass of liquid in the control volume is of the order of $10^{-4}$. This result was used
as justification for neglecting the vapour term present in Equation A-5.

Using the simplified form of Equation A-5, the mass transfer equation (Equation A-4) becomes

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r D_R}{(1 + kU/U_f)^2} \frac{\partial U}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{D_z}{(1 + kU/U_f)^2} \frac{\partial U}{\partial z} \right) = \frac{1}{(1 + kU/U_f)} \frac{\partial U}{\partial t}
\]

\[\text{(A-8)}\]

or, in terms of appropriately defined coefficients, this equation can be written as

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r A_R(U,T) \frac{\partial U}{\partial r} \right) + \frac{\partial}{\partial z} \left( A_Z(U,T) \frac{\partial U}{\partial z} \right) = B(U) \frac{\partial U}{\partial t}
\]

\[\text{(A-9)}\]
APPENDIX B
DEVELOPMENT OF HEAT TRANSFER EQUATION

Analysis for the development of the heat transfer equation was analogous to that performed for the mass transfer equation.

The net heat transfer out of the control volume, including the heat-sink caused by internal evaporation, was equated to the decrease in internal energy of the control volume. The result was

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r K_y(U,T) \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left( K_z(U,T) \frac{\partial T}{\partial z} \right) - \sigma \dot{m}_{\text{evap}} = \rho D(U) C_s(U,T) \frac{\partial T}{\partial t} \]

\[ \ldots (B-1) \]

where

- \( C_s(U,T) = \) specific heat of wet wood (cal/g°C),
- \( B(U) = \) volume of dry control volume / volume of wet control volume
- \( \dot{m}_{\text{evap}} = \) rate of evaporation within the control volume (g/sec-cm³),
- \( \sigma = \) latent heat of evaporation for water (cal/g).

The rate of evaporation, \( \dot{m}_{\text{evap}} \), was calculated by subtracting the net accumulation of liquid within the control volume from the net diffusion of liquid into the control volume. In equation form, the relationship desired for Equation B-1 is
\[ \dot{m}_{\text{evap}} = \rho_D \left\{ \frac{1}{r} \frac{\partial}{\partial r} (rf_R A_R(U,T) \frac{\partial U}{\partial r}) + \frac{\partial}{\partial z} (f_Z A_Z(U,T) \frac{\partial U}{\partial z}) \right\} - B(U) \frac{\partial U}{\partial t} \] 

\[ \text{......(B-2)} \]

where

\[ f_R = \text{fraction of radial diffusion that is liquid} \]

and

\[ f_Z = \text{fraction of axial diffusion that is liquid}. \]

The terms \( f_R \) and \( f_Z \) are determined using results of the wood-model diffusivity equations. They equal the fraction of moisture diffusion which doesn't pass through the pit system.

To obtain the complete heat transfer equation, Equation B-2 must be substituted into Equation B-1.
1. Relation between $D'_B$ and $D_B$

The bound water diffusivity, $D'_B$, applies when a gradient in mass of water per volume wood substance is used. This diffusivity is the one shown in Figure 15 and given by Yao's empirical relationship, Equation 21. To determine mass flux using $D'_B$ the relationship used is

$$m = - D' \frac{\partial}{\partial x} \left( \frac{\text{mass water}}{\text{volume wood substance}} \right). \quad \ldots \ldots (C-1)$$

The concentration term of Equation C-1 can be written as

$$\left( \frac{\text{mass water}}{\text{mass dry wood}} \right) \left( \frac{\text{mass dry wood}}{\text{volume wet wood}} \right) \left( \frac{\text{volume wet wood}}{\text{volume wood substance}} \right).$$

Therefore, in equation form, the relationship for liquid concentration in Equation C-1 is

$$\frac{\text{mass water}}{\text{volume wood substance}} = U \frac{1}{\rho_S \left(1 - \varepsilon \right)}.$$

Voidage, $\varepsilon$, is given by the relationship in Table III. Substituting for $\varepsilon$ gives

$$\frac{\text{mass water}}{\text{volume wood substance}} = \frac{U}{\left( \frac{1}{\rho_{WS}} + \frac{U}{\rho_0} \right)} \quad \ldots \ldots (C-2)$$
Substituting Equation C-2 into Equation C-1 and expanding, the following relationship was obtained:

$$\dot{m} = -D_B \frac{D'}{B} \rho_{ws} (1 + \frac{\rho_{ws}}{\rho_0} U)^2 \frac{\partial U}{\partial x}.$$  \hspace{1cm} \text{......(C-3)}

The desired form for the mass transfer equation uses a diffusivity $D_B$ and a gradient in moisture concentration. This appropriate mass flux relationship is

$$\dot{m} = -D_B \frac{\partial C}{\partial x}.$$  \hspace{1cm} \text{......(C-4)}

Using Equation A-3, Equation C-4 becomes,

$$\dot{m} = -D_B \rho_D \frac{D'}{B} \rho_{ws} \frac{\partial U}{(1 + \frac{kU}{U_f})^2 \frac{\partial U}{\partial x}}.$$  \hspace{1cm} \text{......(C-5)}

Now, equating Equations C-3 and C-5 and rearranging gives

$$D_B = \frac{\rho_D \frac{D'}{B} \rho_{ws}}{(1 + \frac{kU}{U_f})^2 \left( \frac{1}{\rho_{ws}} + \frac{U}{\rho_0} \right)^2}.$$  \hspace{1cm} \text{......(C-6)}

Using the equation for voidage, Equation C-6 becomes,

$$D_B = \frac{\rho_D D_B^{'}}{\rho_{ws} (1 - \epsilon)^2}.$$  \hspace{1cm} \text{......(C-7)}
2. Relation between $D'_V$ and $D_V$

The diffusivity of water vapour in air given by Equation 24 was modified to be applicable to a gradient in liquid concentration.

The relation for unidirectional vapour phase diffusion with respect to a vapour density gradient is

$$\dot{m} = -D'_V \rho_{sat} \frac{\partial H}{\partial x}, \quad \ldots (C-8)$$

where

- $\dot{m}$ = mass flux (g/cm\(^2\)-sec),
- $D'_V$ = diffusivity of water vapour in air (cm\(^2\)/sec),
- $H$ = absolute humidity,
- $\rho_{sat}$ = saturation water vapour density (g/cm\(^3\)).

The desired relationship has the form,

$$\dot{m} = -D_V \frac{\partial C}{\partial x}, \quad \ldots (C-8)$$

where $C$ is as given by Equation A-3. Substituting for $C$ and expanding gives

$$\dot{m} = -\frac{D_V \rho_D}{(1 + kU/U_f)^2} \frac{\partial U}{\partial x} \quad \ldots (C-9)$$
Rewriting Equation C-8 in terms of a gradient in $U$,

$$\dot{m} = -D_v \rho_{\text{sat}} \frac{\partial H}{\partial U} \frac{\partial U}{\partial x}.$$ 

.....(C-10)

Equate Equations C-9 and C-10 to get

$$D_v = \frac{\rho_{\text{sat}}}{\rho_D} (1 + kU/U_f)^2 \frac{\partial H}{\partial U} D_v.$$ 

.....(C-11)

The term $\frac{\partial H}{\partial U}$ was calculated from desorptional isotherms. In this work, isotherms for Sitka Spruce were used. The relation for $\frac{\partial H}{\partial U}$ is developed in the next section.

3. Curve fit for desorptional isotherms of Sitka spruce.

The experimentally determined isotherms used are reproduced from data of Loughborough in Figure 16. In these curves the humidity, $H$, is a function of both moisture content and temperature. To simplify curve-fitting, the isotherms were re-plotted as humidity versus the fraction of fiber saturation moisture content, $U/U_f$.

A function was postulated to fit these curves. This function has the form,

$$H = f \left(\frac{U}{U_f}\right) + g(T)h\left(\frac{U}{U_f}\right).$$ 

.....(C-12)
The function \( f(U/U_f) \) was determined by fitting the isotherm for a temperature of \( T = 25^\circ C \). The result was

\[
f(U/U_f) = \sum_{i=0}^{5} P_i(U/U_f)^i,
\]

.....(C-13)

where

\[
P_0 = 7.464428 \times 10^{-3},
\]

\[
P_1 = 3.32111 \times 10^{-1},
\]

\[
P_2 = 9.870049,
\]

\[
P_3 = -2.362482 \times 10^{-1},
\]

\[
P_4 = 2.110410 \times 10^{1},
\]

\[
P_5 = -6.68861.
\]

The function \( h(U/U_f) \) is the difference in humidity, at any moisture content, between the 100°C and 25°C isotherm. A 7th order polynomial was fitted to the isotherm for 100°C to obtain:

\[
H(T = 100^\circ C) = \sum_{i=0}^{7} Q_i(U/U_f)^i,
\]

.....(C-14)

where

\[
Q_0 = 1.601415 \times 10^{-4},
\]

\[
Q_1 = 1.376967 \times 10^{1},
\]
\[ Q_2 = 1.093299 \times 10^1, \]
\[ Q_3 = -4.669788 \times 10^1, \]
\[ Q_4 = -8.222108 \times 10^1, \]
\[ Q_5 = -7.635399 \times 10^1, \]
\[ Q_6 = 3.665102 \times 10^1, \]
\[ Q_7 = -7.130367. \]

Subtracting Equation C-14 from Equation C-15 gives the desired relation,

\[ h(U/U_f) = \sum_{i=0}^{7} Q_i (U/U_f)^i - \sum_{i=0}^{5} P_i (U/U_f)^i. \quad \text{.....(C-15)} \]

The temperature function \( g(T) \) was determined using the data given in Table C-I. This data was taken from the experimental isotherms of Figure 16. A good fit for \( g(T) \) was obtained using the polynomial,

\[
\begin{array}{|c|c|}
\hline
\text{Temperature} & \text{Variation of Equilibrium Humidity with Temperature} \\
\text{T(°C)} & \text{H(T) - H(T = 25°C)} & \text{H(T = 100°C) - H(T = 25°C)} \\
25 & 0.0 & \\
40 & 0.09 & \\
50 & 0.18 & \\
60 & 0.29 & \\
80 & 0.60 & \\
90 & 0.80 & \\
100 & 1.00 & \\
\hline
\end{array}
\]
\[ g(T) = \sum_{i=1}^{3} J_i (T - 25)^i \quad \ldots \ldots \text{(C-16)} \]

where

\[ J_1 = 0.3815 \times 10^{-2} \]
\[ J_2 = 0.13636 \times 10^{-3} \]
\[ J_3 = 0.1173 \times 10^{-6} \]

and \( T \) was measured in degrees Celsius.

Experimental values for the fiber saturation moisture content at temperatures of 25°, 40°, 50°, 60°, 80° and 100°C were obtained from the data of Loughbourough (p. 59 of Reference 28). This data is reproduced in Table C-II along with predictions of the linear approximation used in this work.

**TABLE C-II**

Fiber Saturation Point as a Function of Temperature

<table>
<thead>
<tr>
<th>Temperature ( T(°C) )</th>
<th>Fiber Saturation Point</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental (g/g)</td>
</tr>
<tr>
<td>25</td>
<td>0.306</td>
</tr>
<tr>
<td>40</td>
<td>0.292</td>
</tr>
<tr>
<td>50</td>
<td>0.283</td>
</tr>
<tr>
<td>60</td>
<td>0.272</td>
</tr>
<tr>
<td>80</td>
<td>0.251</td>
</tr>
<tr>
<td>100</td>
<td>0.229</td>
</tr>
</tbody>
</table>
\[ U_f = 0.308 - 0.00107 (T - 25^\circ C) \text{ g/g} \quad \ldots \ldots \text{(C-17)} \]

A similar approximation was used by Berger and Pei [2].

The slope of the isotherms was required for the moisture diffusivity calculations. \( \frac{\partial H}{\partial U} \) was determined by differentiating Equation C-12 with respect to \( U \). The result was

\[
\frac{\partial H}{\partial U} = \frac{1}{U_f} \left\{ \sum_{i=1}^{5} P_i (U/U_f)^{i-1} + g(T) \left( \frac{1}{2} \sum_{i=1}^{7} Q_i (U/U_f)^{i-1} \right) - \sum_{i=1}^{5} P_i (U/U_f)^{i-1} \right\}, \quad \ldots \ldots \text{(C-18)}
\]

where \( P_i, Q_i \) and \( g(T) \) are as previously given. \( \frac{\partial H}{\partial U} \) for temperatures of 25°C and 100°C are shown in Figure C-I.

4. Path Diffusivities

Both the radial and axial diffusivity equations were derived using Stamm's simplified wood model. A sketch of this model including important structural dimensions is shown in Figure 13. All softwood structural data and definitions necessary for the derived equations are given in Table III.
FIGURE C-1 - SLOPE OF DESORPTION ISOTHERMS AS A FUNCTION OF MOISTURE CONTENT FOR TEMPERATURES OF 25°C AND 100°C
The wood cells, as viewed axially, are considered to have a square cross-section. Double-cell-wall thickness, $L_w$, is given by the relationship,

$$L_w = \frac{1 - \sqrt{\varepsilon}}{n_t} \quad \ldots \ldots \text{(C-19)}$$

Path diffusivities were determined using Equation 27. The multiplier terms of Equation 27 are identically positioned in the equations below.

Path 1: Continuous cell wall.

Radial -

$$D_1 = (1 - \sqrt{\varepsilon}) \times 1 \times D_B$$

Axial -

$$D_1 = (1 - \varepsilon) \times 1 \times D_B$$

Path 2: Cell cavity.

Radial -

$$D_2 = (1 - n_t L_w) \frac{1}{(1 - n_t L_w)} D_V$$

$$= D_V$$

Axial -

$$D_2 = \varepsilon D_V$$
Path 3: Discontinuous cell wall.

Radial -

\[ D_3 = (\sqrt{\varepsilon} - q_t - q_p) \times \frac{1}{n_t L_w} D_B , \]

\[ = \frac{\sqrt{\varepsilon} - q_t - q_p}{1 - \sqrt{\varepsilon}} D_B . \]

Axial -

\[ D_3 = (\sqrt{\varepsilon} - q_L - q_p) \frac{1}{n_L W} D_B , \]

\[ = \frac{\sqrt{\varepsilon} - q_L - q_p}{n_L / n_t (1 - \sqrt{\varepsilon})} D_B . \]

Path 4: Pit chamber.

Radial -

\[ D_4 = q_p \times \frac{1}{n_t (l_w - l_p)} D_V , \]

\[ = \frac{q_p}{(1 - \sqrt{\varepsilon} - n_t L_p)} D_V . \]

Axial -

\[ D_4 = q_p \times \frac{1}{n_L (l_w - l_p)} D_V , \]
\[
= \frac{q_p D_p}{n_t (1 - \sqrt{\epsilon}) - n_s L_p}
\]

Path 5: Pit membrane

Radial -
\[
D_5 = q_p \times \frac{1}{n_t L_p} D_B
\]

Axial -
\[
D_5 = q_p \times \frac{1}{n_s L_p} D_B
\]

Path 6: Pit pore

Radial -
\[
D_6 = q_t \times \frac{1}{n_t L_p} (D_p/40)
\]

Axial -
\[
D_6 = q_t \times \frac{1}{n_s L_p} (D_p/40)
\]
APPENDIX D
NUMERICAL SOLUTION OF MASS TRANSFER EQUATION

The matrix terms present in Equation 62 are themselves comprised of several matrices. Separation of the matrix terms in this way helped to minimize redundant calculations in the computer program.

The first equation of the two step solution method,

$$[F] \{U_K\}^{(n+1)*} = \{W\} \quad \ldots \ldots (D-1)$$

will be considered first. This equation was developed by applying the operator form of the finite difference mass transfer equation to each grid point. Boundary condition equations were included.

In Equation 1, $\{U_K\}^{(n+1)*}$ is a moisture content vector with an entry for each grid point. The entries are numbered using the K index of Figure 22 in ascending order from 1 to NK.

The other two terms of Equation D-1 are given by the relationships:

$$[F] = [FIR] - [FF] + [\theta]^n \quad \ldots \ldots (D-2)$$

and

$$\{W\} = ([FF] - [\theta]^n)\{U_K\}^n + ([FIR] + [FF] - [\theta]^{n-1})\{U_K\}^{n-1} + ([G] - [\phi]^n)(1 + \alpha)\{U_K\}^n$$

$$+ ([G] - [\phi]^{n-1})(1 + \beta)\{U_K\}^{n-1} + \{V\} \quad \ldots \ldots (D-3)$$
[FIR] is a square diagonal array, NK by NK. In the computer program, this array was treated as a vector of length NK. The array form of [FIR] is

\[
[FIR] = \frac{3}{2\varepsilon_m} \\
\begin{bmatrix}
0 \\
B(U_2)S \\
B(U_3)2S \\
\vdots \\
B(U_{NJ})R \\
B(U_{NJ+2})S \\
\vdots \\
B(U_{2NJ})R \\
\vdots \\
B(U_{NK})R \\
\end{bmatrix}
\]

\[\ldots (D-4)\]

[FF] is the square (NK x NK) tridiagonal array,

\[
[FF] = \\
\begin{bmatrix}
BF(AF + CF) \\
AF BF CF \\
\vdots \\
AF BF CF \\
AF + CF BF 0 \\
0 BF AF + CF \\
AF BF CF \\
\vdots \\
(AF + CF) BF \\
\end{bmatrix}
\]

- radial internal boundary

- radial external boundary

\[\ldots (D-5)\]
where

\[ AF = r^*_K - \frac{1}{2} A^*(U^-_K, T^-_K) , \]  
\[ CF = r^*_K + \frac{1}{2} A^*(U^+_K, T^+_K) , \]  
and 

\[ BF = -(AF + CF) . \]

The necessary definitions for Equations D-6, D-7 and D-8 are as follows:

\[ r^*_K \pm \frac{1}{2} = \text{Radius of grid point } (K \pm S/2) , \]  
\[ U^\pm_K = \frac{U_K + U_{K \pm 1}}{2} , \]  
\[ T^\pm_K = \frac{T_K + T_{K \pm 1}}{2} . \]

\([G]\) is a square tri-diagonal array \(NK\) by \(NK\). The diagonals are not side by side as in the array \([FF]\). If the rows are indicated by the \(i\) index and columns by the \(j\) index, the diagonals are located at sites where

\[ i = j , \]
\[ i = j + NJ \]

and

\[ j = i + NJ . \]

\([G]\) is given by the relationship,
where:

\[ \begin{align*}
AG &= r^*_K A^+_Z (U^-_K, T^-_K), \\
CG &= r^*_K A^-_Z (U^+_K, T^+_K), \\
BG &= -(AG + CG).
\end{align*} \]

Moisture contents \( U^+_K \) and \( U^-_K \) and temperatures \( T^+_K \) and \( T^-_K \) are given by Equations D-10 and D-11. In Equations D-13 and D-14,

\[ r^*_K = \text{Radius of grid point } K. \]

[\( \theta \)] and [\( \phi \)] hold information on the grid point Biot mass numbers for the radial and axial surfaces, respectively. Present and previous values of [\( \theta \)] and [\( \phi \)] are used in the method. The arrays are square with all entries on the principal diagonal.
In the computer program these arrays are each held in NK x 2 vectors. In array form they are

\[
[\theta]^\ell = 2S
\]

- radial external boundary

\[
[\phi]^\ell = 2S
\]

- axial external boundary

where \( \ell \) indicates the \( n \) or \( (n - 1) \) time level.
\[
\begin{bmatrix}
0 \\
\vdots \\
0 \\
\text{CF} \left(2\lambda^n_R + \lambda^{n-1}_R\right) \\
0 \\
\vdots \\
0 \\
\text{CF} \left(2\lambda^n_R + \lambda^{n-1}_R\right) \\
\text{CG} \left((1 + \alpha)\lambda^n_Z + (1 + \beta)\lambda^{n-1}_Z\right)
\end{bmatrix}
\]

- radial external boundary

\[
\{V\lambda\} = 2S
\]

\[
\begin{bmatrix}
\text{CF} \left(2\lambda^n_R + \lambda^{n-1}_R\right) \\
\text{CG} \left((1 + \alpha)\lambda^n_Z + (1 + \beta)\lambda^{n-1}_Z\right)
\end{bmatrix}
\]

- axial external boundary

\[
\begin{bmatrix}
\text{CF} \left(2\lambda^n_R + \lambda^{n-1}_R\right) \\
\text{CG} \left((1 + \alpha)\lambda^n_Z + (1 + \beta)\lambda^{n-1}_Z\right)
\end{bmatrix}
\]

- radial and axial external boundary

The second part of the two step solution is

\[
[J] \{U_{KP}\}^{n+1} = \{X\} + [Y] \{U_{KP}\}^{(n+1)*} 
\]

\[\ldots (D-19)\]

In Equation D-19,

\[
[J] = [FI] - ([JJ] - [\zeta]^n), 
\]

\[\ldots (D-20)\]

\[
\{X\} = ([JJ] - [\zeta]^n)\alpha\{U_{KP}\}^{n} + ([JJ] - [\zeta]^{n-1})\beta\{U_{KP}\}^{n-1} + \{\eta\} 
\]

\[\ldots (D-22)\]
and

\[ [Y] = [F_1] . \] ..........(D-22)

\( \{U_{KP}\} \) is a vector containing moisture contents for grid points sequential from 1 to \( NK \) in the KP index.

\[
\begin{bmatrix}
B(U_1) \\
B(U_2) \\
\vdots \\
\vdots \\
B(U_{NK})
\end{bmatrix}
\]

\[ [F_1] = \frac{3}{2E_m} \] ..........(D-23)

\( U_1, U_2, \ldots, U_{KP} \), in the above and following arrays, is moisture content for a grid point using the KP index.
where

\[ AJ = A^+_Z(U^+_{KP}, T^+_{KP}) \]  
\[ CJ = A^+_Z(U^+_{KP}, T^+_{KP}) \]  
\[ BJ = - (AJ + CJ) \]

\( U^\pm_{KP} \) and \( T^\pm_{KP} \) are determined by replacing \( K \) with \( KP \) in Equations D-10 and D-11.

\([\zeta]^n_{\text{in}}\) and \([\zeta]^n_{\text{in-1}}\) are square arrays, NK by NK, with all non-zero entries on the principal diagonal. They contain the axial mass Biot numbers for the \( n \) and \( n-1 \) time levels, respectively, and, are held in one NK by 2 vector in the computer program.
The boundary condition vector \([\eta]\) is given by the equation,

\[
\{\eta\} = 2S \\
0 \\
\vdots \\
0 \\
CJ((1-\alpha)\lambda^n_Z - \beta\lambda^{n-1}_Z) \\
0 \\
\vdots \\
0 \\
CJ((1-\alpha)\lambda^n_Z - \beta\lambda^{n-1}_Z)
\]

\[
\ldots \ldots (D-29)
\]
APPENDIX E

NUMERICAL SOLUTION OF HEAT TRANSFER EQUATION

The finite difference solution equations for heat transfer, 63a and 63b, are in a form very similar to the equations for mass transfer. Use was made of this similarity to reduce duplication of arrays as much as possible.

The first equation of the two step solution method for heat transfer was considered first. This equation is

$$[FT] \{T^*_K\}^{(n+1)*} = \{WT\} + \{\Lambda\} \quad \ldots \ldots \text{(E-1)}$$

$\{T^*_K\}^{(n+1)*}$ is a vector containing non-dimensionalized temperatures for grid points from 1 to NK in terms of the K index. These values are entered in ascending order starting at K=1. The other arrays present are determined using the following equations:

1. $$[FT] = [FIRT] - [FFT] + [\Theta T]^n \quad \ldots \ldots \text{(E-2)}$$

2. $$\{WT\} = \left[ (FFT) - (\Theta T)^n \right] \{T^*_K\}^n + \left[ (FIRT) + \left( FFT \right) - (\Theta T)^{n-1} \right] \{T^*_K\}^{n-1} + \left[ (GT) - (\Theta T)^n \right] (1 + \alpha) \{T^*_K\}^n + \left[ (GT) - (\Theta T)^{n-1} \right] (1 + \beta) \{T^*_K\}^{n-1} + \{VT\} \quad \ldots \ldots \text{(E-3)}$$
3. \[
\{A\} = 3(KoLu)_i \left\{ \left[ f_R \right] \left[ (FF) - (\theta)^n \right] + [f_Z][(G) - (\phi)^n] \right\} \{u_K^n + \{a\} - 1/3[FIR] \{u_K^{n+1} - u_K^{n-1}\} \right\}
\]

\[
.....(E-4)
\]

[FIRT] is of identical form as [FIR] and can be calculated by substituting

\[
\xi_m = \xi_h
\]

and

\[
B(U_K) = B(U_K) C^*(U_K, T_K)
\]

\[
.....(E-5)
\]

\[
.....(E-6)
\]

into Equation D-4.

Similarly for [FFT], the equation for [FF] can be used. The substitutions to make in Equation D-5 are

\[
AF = r^*_{K-1} K^*_R (U^-_K, T^-_K)
\]

\[
.....(E-7)
\]

\[
CF = r^*_{K+1} K^*_R (U^+_K, T^+_K)
\]

\[
.....(E-8)
\]

and

\[
BF = -(AF + CF)
\]

\[
.....(E-9)
\]

\[
U^+_K \text{ and } T^+_K \text{ are calculated using Equations D-10 and D-11.}
\]

[GT] is of the same form as [G] and can be calculated from Equation D-12 by letting

\[
AG = r^*_K K^*_Z (U^-_K, T^-_K)
\]

\[
.....(E-10)
\]
\[ CG = r^*_K K^* (U^+_K, T^+_K) \]  
\[ BG = -(AG + CG) \]

\( r^*_K \) is the radius of grid point \( K \).

\([\Theta T]^n\) and \([\Theta T]^{n-1}\) can be calculated from the relations for \([\Theta]^n\) and \([\Theta]^{n-1}\), respectively, by using CF as given by Equation E-8 and replacing \( Bi_{mR}^{\ell} \) with \( Bi_{R}^{\ell} \), where \( \ell \) indicates the appropriate time level. Similarly for \([\phi T]^n\) and \([\phi T]^{n-1}\), Equation D-17 for \([\phi]\) can be used. In this case, \( CG \) is given by Equation E-11 and \( Bi_{mZ}^{\ell} \) is replaced by \( Bi_{Z}^{\ell} \). The other terms present in Equation E-3 and E-4 are not of the same form as any terms in the mass transfer equation and will now be considered in detail.

In the relationship for \( \{A\} \) three terms, not previously considered, are present. These are \( \{f_R\} \), and \( \{f_Z\} \) and \( \{a\} \). For their development, the values of \( f_R \) and \( f_Z \) were assumed constant throughout the elemental volume and to have a value equal to that at the grid point. Using this assumption, the arrays are

\[
[f_R] = \begin{bmatrix}
  f_R(U_1) \\
  f_R(U_2) \\
  \vdots \\
  f_R(U_NK) 
\end{bmatrix} \quad \text{(E-13)}
\]

\[
[f_Z] = \begin{bmatrix}
  f_Z(U_1) \\
  f_Z(U_2) \\
  \vdots \\
  f_Z(U_NK) 
\end{bmatrix} \quad \text{(E-14)}
\]
In Equation E-15, CF and CG for each grid point are obtained from Equations D-7 and D-14. $\lambda_R$ and $\lambda_Z$ are determined for each grid point from Equations 6 and 8, respectively.
The final term, \( \{V \lambda T\} \), is calculated using the following equation:

\[
\begin{align*}
\{V \lambda T\} &= 2S \\
&= \begin{bmatrix}
0 \\
\vdots \\
0 \\
CF(2\mu_R^n + \mu_R^{n-1}) \\
0 \\
\vdots \\
0 \\
CF(2\mu_R^n + \mu_R^{n-1}) \\
0 \\
\vdots \\
0 \\
CF(2\mu_R^n + \mu_R^{n-1}) \\
CG((1+\alpha)\mu_Z^n + (1+\beta)\mu_Z^{n-1}) \\
\vdots \\
\vdots \\
CF(2\mu_R^n + \mu_R^{n-1}) + \\
CG((1+\alpha)\mu_Z^n + (1+\beta)\mu_Z^{n-1})
\end{bmatrix}
\end{align*}
\]

where

\[
\mu_R^\ell = \left[ KoLu_R(\lambda_R - Bi_{mR}U_K) \right]^\ell 
\]

\[
\mu_Z^\ell = \left[ KoLu_Z(\lambda_Z - Bi_{mZ}U_K) \right]^\ell 
\]

and

\( \ell = n \) or \((n-1)\) time level.

In Equation E-16, \( CF \) is calculated from Equation E-8 and \( CG \) is calculated from Equation E-11. The second part of the heat transfer equation solution is given by the equation,
\[ [J_T] \{T_K^*\}^{n+1} = \{X_T\} + [Y_T] \{T_K^*\}^{(n+1)*} \quad \ldots \ldots (E-19) \]

In Equation \(E-19\),

\[ [J_T] = [F_I] - [J_J] + [\zeta_T]^n \quad \ldots \ldots (E-20) \]

\[ \{X_T\} = ([J_J] - [\zeta_T]^n)\alpha \{T_K^*\}^n \]
\[ + ([J_J] - [\zeta_T]^{n-1})\beta \{T_K^*\}^{n-1} \div \{\eta_T\} \quad \ldots \ldots (E-21) \]

and

\[ [Y_T] = [F_I] \quad \ldots \ldots (E-22) \]

\([F_I]\) is determined from Equation \(D-23\) for \([F_I]\) by making the substitutions,

\[ \xi_m = \xi_h \quad \ldots \ldots (E-23) \]

and

\[ B(U_K) = B(U_{KP})C*(U_{KP}, T_{KP}) \quad \ldots \ldots (E-24) \]

\([J_J]\) is determined from Equation \(D-24\) for \([JJ]\) using the relationships,

\[ A_J = K_Z^{-1}(U_{KP}, T_{KP}) \quad \ldots \ldots (E-25) \]

\[ C_J = K_Z^{-1}(U_{KP}, T_{KP}) \quad \ldots \ldots (E-26) \]

and

\[ B_J = -(A_J + C_J) \quad \ldots \ldots (E-27) \]
\([\zeta T]^n\) and \([\zeta T]^{n-1}\) are of the same form as \([\zeta]^n\) and \([\zeta]^{n-1}\) given by Equation D-28. They are determined by replacing \(B_i^mZ\) and \(B_i^{m-1}\) with \(B_i^n\) and \(B_i^{n-1}\), respectively.

\(\{\eta T\}\) is determined by the following relation.

\[
\begin{bmatrix}
0 \\
\vdots \\
0 \\
CJ((1-\alpha)\mu^n_Z + \beta\mu^{n-1}_Z)
\end{bmatrix}
\begin{aligned}
\{\eta T\} &= 2S \\
&= CJ((1-\alpha)\mu^n_Z + \beta\mu^{n-1}_Z) \\
&= CJ((1-\alpha)\mu^n_Z + \beta\mu^{n-1}_Z)
\end{aligned}
\]

- axial external boundary

Equation E-28

CJ is given by Equation E-26 while \(\mu_Z\) is given by Equation E-18.
General convective heat and mass transfer relations for a circular cylinder were obtained from Reference 13. These relations use the Colburn j factors. For mass transfer,

\[ \text{Sh} = j_m \text{Re } \text{Sc}^{1/3} \quad \ldots \ldots (F-1) \]

and for heat transfer

\[ \text{Nu} = j_h \text{Re } \text{Pr}^{1/3} \quad \ldots \ldots (F-2) \]

where

- \( \text{Sh} \) = Sherwood number, \( K_G L/D_v \),
- \( \text{Sc} \) = Schmidt number, \( \nu/D_v \),
- \( \text{Nu} \) = Nusselt number, \( hL/K \),
- \( \text{Pr} \) = Prandtl number, \( \nu/\alpha \),
- \( \text{Re} \) = Reynolds number, \( 2VR/\nu \),
- \( j_m \) = Colburn factor for mass transfer,
- \( j_h \) = Colburn factor for heat transfer.

In many cases \( j_m = j_h \) [13] and Equations F-1 and F-2 can be used to relate \( \text{Sh} \) and \( \text{Nu} \). This was done to produce the relationship

\[ \text{Sh} = (\text{Sc}/\text{Pr})^{1/3} \text{Nu} \quad \ldots \ldots (F-3) \]

A relation for average Nusselt number of a circular cylinder placed normal to fluid flow was obtained [13];
\[ \bar{Nu} = C \text{Re}^n \]  \hspace{1cm} \text{(F-4)}

where \( C \) and \( n \) are listed in the table below.

<table>
<thead>
<tr>
<th>Re_D</th>
<th>C</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 - 40</td>
<td>.821</td>
<td>.385</td>
</tr>
<tr>
<td>40 - 4,000</td>
<td>.615</td>
<td>.466</td>
</tr>
<tr>
<td>4,000 - 40,000</td>
<td>.174</td>
<td>.618</td>
</tr>
</tbody>
</table>

The coefficients for the Reynolds number range 40 - 4,000 was used in Equation F-4 to give the following form of Equation F-3:

\[ \bar{Sh} = \left( \frac{Sc}{Pr} \right)^{1/3} \cdot 0.615 \cdot \text{Re}^{0.466} \]  \hspace{1cm} \text{(F-5)}

using the definition of the Sherwood number,

\[ \bar{Sh} = \frac{K_G \cdot L}{D_y} \]

and the diffusivity relation for diffusion of water vapour in free air,

\[ D_y = 0.22 \left( \frac{T}{273} \right)^{1.75} \left( \frac{760}{P} \right) \text{cm}^2/\text{s} \]  \hspace{1cm} \text{(F-6)}

where \( T \) and \( P \) are in units of °K and mm Hg, respectively, the convective coefficient was determined. The final relationship for both the radial and axial surfaces is
\[ K_G = \frac{\tilde{\text{Sh}} D_v}{2R} \]
APPENDIX G
COMPUTER PROGRAM OPERATION PROCEDURE

The computer program is sufficiently documented throughout to follow all the arithmetic manipulations and the program logic. Input and other important variables are defined in the program.

There are three different cases which can be handled by this computer program. The desired case is selected through the assignment of an appropriate value to the variable ICASE. The three cases considered and the necessary value of ICASE for each are given below:

1. Humidity = \( U/U_f \)
   Shrinkage > 0
   Diffusivities \( D_R \) and \( D_Z \) are constant.
   ICASE = 0

2. Humidity = \( f(U/U_f) + g(T)h(U/U_f) \)
   Shrinkage > 0
   Diffusivities \( D_R \) and \( D_Z \) are constant.
   ICASE = 1

3. Full wood model
   ICASE > 1

There are three alternative choices for program output and these are set by the control character ISPEW. Details of the effect of ISPEW are given in the comments situated at the start of the program (Lines 138 to 150).
For all cases, 21 variables must be read into the computer program. This reading process is commanded by lines 179 through 185 of the program. Input variables, their Fortran symbolic name, their number type and their format input code are given in Table G-I.

For the cases where ICASE $\leq 1$, the program will read the radial diffusivity, RDIFFN, and the axial diffusivity, ZDIFFN. Units of both are cm/sec$^2$ and they are read using scientific notation. The format code is E10.4 and the exponent must be placed at the far right of the field.

Output quantities are labelled, including units, when printed and are explained in the program comments (Lines 170 to 177).
<table>
<thead>
<tr>
<th>Variable</th>
<th>Fortran Symbolic Name</th>
<th>Number Type</th>
<th>Format Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial radius (cm)</td>
<td>R</td>
<td>Real</td>
<td>F10.4 1</td>
</tr>
<tr>
<td>Initial halflength (cm)</td>
<td>SCRL</td>
<td>Real</td>
<td>F10.4</td>
</tr>
<tr>
<td>Initial moisture content (g/g)</td>
<td>UO</td>
<td>Real</td>
<td>F10.4</td>
</tr>
<tr>
<td>Isothermal temperature (°K)</td>
<td>T</td>
<td>Real</td>
<td>F10.4</td>
</tr>
<tr>
<td>Relaxation parameter, α</td>
<td>ALPHA</td>
<td>Real</td>
<td>F10.6</td>
</tr>
<tr>
<td>Relaxation parameter, β</td>
<td>BETA</td>
<td>Real</td>
<td>F10.6</td>
</tr>
<tr>
<td>Initial dimensionless time step</td>
<td>DT</td>
<td>Real F</td>
<td>F10.6</td>
</tr>
<tr>
<td>Number of grid intervals</td>
<td>NH</td>
<td>Integer</td>
<td>I5 2</td>
</tr>
<tr>
<td>Program control character</td>
<td>ICASE</td>
<td>Integer</td>
<td>I5</td>
</tr>
<tr>
<td>Program control character</td>
<td>ISPEW</td>
<td>Integer</td>
<td>I5</td>
</tr>
<tr>
<td>Reynolds number of flow</td>
<td>RENLD</td>
<td>Real</td>
<td>F10.4</td>
</tr>
<tr>
<td>Schmidt number/Prandtl number</td>
<td>SCPR</td>
<td>Real</td>
<td>F10.4</td>
</tr>
<tr>
<td>Ambient water vapour density (g/cm³)</td>
<td>RHOINF</td>
<td>Real</td>
<td>F10.4</td>
</tr>
<tr>
<td>Density of drywood (g/cm³)</td>
<td>RHODW</td>
<td>Real</td>
<td>F10.4</td>
</tr>
<tr>
<td>Shrinkage factor, k</td>
<td>FK</td>
<td>Real</td>
<td>F10.4</td>
</tr>
<tr>
<td>Time end condition</td>
<td>RUUF</td>
<td>Real</td>
<td>F10.4</td>
</tr>
<tr>
<td>Secondary time step</td>
<td>STEP1</td>
<td>Real</td>
<td>F10.6</td>
</tr>
<tr>
<td>Tertiary time step</td>
<td>STEP2</td>
<td>Real</td>
<td>F10.6</td>
</tr>
<tr>
<td>Final time step</td>
<td>STEP3</td>
<td>Real</td>
<td>F10.6</td>
</tr>
<tr>
<td>Number of iterations to end of STEP1</td>
<td>LOC1</td>
<td>Integer</td>
<td>I5</td>
</tr>
<tr>
<td>Number of iterations to end of STEP2</td>
<td>LOC2</td>
<td>Integer</td>
<td>I5</td>
</tr>
</tbody>
</table>

10 indicates available field width; 4 indicates the number of digits to the right of the decimal point.

25 indicates the field width. The integer value must be placed at the far right of the field.
APPENDIX H

COMPUTER PROGRAM LISTING

C **** SOLUTION FOR THE GENERAL PARTIAL DIFFERENTIAL EQUATION
C **** GOVERNING ISOTHERMAL CONVECTIVE DRYING OF A FINITE
C **** CIRCULAR CYLINDER OF SOFTWOOD. THE EQUATION IS OF
C **** THE FORM:

C **** (1/R)D/DR(R AA1 DU/DR)+D/DZ(AA2 DU/DZ)=BB DU/DT

C **** AA1=FUNCTION OF MOISTURE ,U, AND TEMPERATURE,T.
C **** AA2=FUNCTION OF MOISTURE ,U, AND TEMPERATURE,T.
C **** BB= FUNCTION OF MOISTURE ,U,.

C **** THE SOLUTION OF THE GENERAL DIFFERENTIAL EQUATION IS
C **** NOT DEPENDENT ON THE FORM OF THESE FUNCTIONS SO
C **** WILL GIVE A SOLUTION ONCE THERE FORM IS SPECIFIED.

C **** BOUNDARY CONDITIONS FOR EACH EXTERNAL SURFACE ARE

C **** DU/DR + QEARNU = LAMBDA

C **** DU/DZ + Q*U = LAM1

DIMENSION LAM(200,2), LAMB(200,2), Q(200,2), QBAR(200,2),

C ***** (AF, BF, CF), (AG, BG, CG), (AJ, BJ, CJ), ARE THE TERMS
C ***** WHICH MAKE UP MATRICES F, G, J RESPECTIVELY.
C ***** I, J, INDICATE GRID POINT POSITION BY LONG., RADIAL
C ***** DIRECTIONS RESPECTIVELY.
C ***** K IS THE GRID POINT INDEX, KP IS AN ALTERNATIVE INDEX.

C ***** FI, FIR ARE THE IDENTITY MATRIX TIMES SCALERS
C ***** BC1, BC2, BC3, BC4, FIRSTT
C ***** BC1 = LOGICAL INTERNAL BOUNDARY, RADIAL DIRECTION
C ***** BC2 = LOGICAL EXTERNAL BOUNDARY, RADIAL DIRECTION
C ***** BC3 = LOGICAL INTERNAL BOUNDARY, AXIAL DIRECTION
C ***** BC4 = LOGICAL EXTERNAL BOUNDARY, AXIAL DIRECTION

C ***** EXAMPLE GRID NK=16, NI=NJ=4
C > <Z> <
C > BC4 <
C > 13 14 15 16 <
C > BC1 9 10 11 12 BC2 <
C > 5 6 7 8 <
C > K=1 2 3 4 --> R <
C > BC3 <
C > <

REAL LAMO, LAMOBR, LAM1, LAM1BR, LM1BR0, LAM10
C ***** SOLUTION SCHEME EQUATIONS:
C N  N+1*  N  N
C 1. (FIR -(F-THETA ))U = (F-THETA ) U +
C  N-1 N-1 N  N
C (FIR +F-THETA ) U + (G-PHI ) (1+ALPHA)U +
C  N-1 N-1 N
C (G-PHI ) (1+BETA)U +VLAM
C N+1  N
C 2. (FI-(J-ZETA ))U = FI*U
C  N  N
C ((J-ZETA )ALPHA*U +(J-ZETA )*BETA*U ) +VNETA
C ***** DEFINITIONS:
C N-1 INDICATES PREVIOUS TIME
C " PRESENT "
C (N+1)* " INTERMEDIATE TIME"
C N+1 " ADVANCED TIME"
C
COMMON /BC/ BC1, BC2, BC3, BC4, I, J, NI, NJ, FIRSTT
COMMON /ACON/ RDIFFN, ZDIFFN, ICASE
COMMON /CH/STEP1, STEP2, STEP3, LOG1, LOC2
COMMON /A/C1, C2, G, UF, RHOSAT, ART
COMMON /M/ FKCR, FKCZ, RHOINF, RO
COMMON /PAP/ R, S CURL, FK, RHO0W
COMMON /AVERG/ RMASS, RLAST, TIMCOR
1050 FORMAT(4F10.4)
1051 FORMAT(2E10.4)
1052 FORMAT(3F10.6,3I5)
1060 FORMAT(4X, 'K', T11, 2X, 'F(K,1)', T22, 3X, 'F(K,2)',
* T33, 2X, 'F(K,3)', T44, 1X, 'RAJ(K,1)', T55, 2X,
* 'RAJ(K,2)', T66, 1X, 'RAJ(K,3)', T77, 2X, 'G(K,1)'
C ** START OF MAIN PROGRAM: **

1068 FORMAT(' ',T21,15,T30,31F10.4,IX))
1069 FORMAT(' ',T21,15,T30,31F10.4,IX))
1070 FORMAT(' ',T21,15,T30,31F10.4,IX))
1071 FORMAT(' ',T21,15,T30,31F10.4,IX))
1072 FORMAT(' ',T21,15,T30,31F10.4,IX))
1073 FORMAT(' ',T21,15,T30,31F10.4,IX))
1074 FORMAT(' ',T21,15,T30,31F10.4,IX))
1075 FORMAT(' ',T21,15,T30,31F10.4,IX))
1076 FORMAT(' ',T21,15,T30,31F10.4,IX))
1077 FORMAT(' ',T21,15,T30,31F10.4,IX))
1078 FORMAT(' ',T21,15,T30,31F10.4,IX))
1079 FORMAT(' ',T21,15,T30,31F10.4,IX))
1080 FORMAT(' ',T21,15,T30,31F10.4,IX))
1081 FORMAT(' ',T21,15,T30,31F10.4,IX))
1082 FORMAT(' ',T21,15,T30,31F10.4,IX))
1083 FORMAT(' ',T21,15,T30,31F10.4,IX))
1084 FORMAT(' ',T21,15,T30,31F10.4,IX))
1085 FORMAT(' ',T21,15,T30,31F10.4,IX))
1086 FORMAT(' ',T21,15,T30,31F10.4,IX))
1087 FORMAT(' ',T21,15,T30,31F10.4,IX))
1088 FORMAT(' ',T21,15,T30,31F10.4,IX))
1089 FORMAT(' ',T21,15,T30,31F10.4,IX))
1090 FORMAT(' ',T21,15,T30,31F10.4,IX))
1091 FORMAT(' ',T21,15,T30,31F10.4,IX))
1092 FORMAT(' ',T21,15,T30,31F10.4,IX))
1093 FORMAT(' ',T21,15,T30,31F10.4,IX))
1094 FORMAT(' ',T21,15,T30,31F10.4,IX))
1095 FORMAT(' ',T21,15,T30,31F10.4,IX))
1096 FORMAT(' ',T21,15,T30,31F10.4,IX))
1097 FORMAT(' ',T21,15,T30,31F10.4,IX))
1098 FORMAT(' ',T21,15,T30,31F10.4,IX))
1099 FORMAT(' ',T21,15,T30,31F10.4,IX))
1100 FORMAT(' ',T21,15,T30,31F10.4,IX))
1101 FORMAT(' ',T21,15,T30,31F10.4,IX))
1102 FORMAT(' ',T21,15,T30,31F10.4,IX))
1103 FORMAT(' ',T21,15,T30,31F10.4,IX))
1104 FORMAT(' ',T21,15,T30,31F10.4,IX))
1105 FORMAT(' ',T21,15,T30,31F10.4,IX))
1106 FORMAT(' ',T21,15,T30,31F10.4,IX))
1107 FORMAT(' ',T21,15,T30,31F10.4,IX))
1108 FORMAT(' ',T21,15,T30,31F10.4,IX))
1109 FORMAT(' ',T21,15,T30,31F10.4,IX))
1110 FORMAT(' ',T21,15,T30,31F10.4,IX))
1111 FORMAT(' ',T21,15,T30,31F10.4,IX))
1112 FORMAT(' ',T21,15,T30,31F10.4,IX))
1113 FORMAT(' ',T21,15,T30,31F10.4,IX))
1114 FORMAT(' ',T21,15,T30,31F10.4,IX))
1115 FORMAT(' ',T21,15,T30,31F10.4,IX))
1116 FORMAT(' ',T21,15,T30,31F10.4,IX))
1117 FORMAT(' ',T21,15,T30,31F10.4,IX))
1118 FORMAT(' ',T21,15,T30,31F10.4,IX))
1119 FORMAT(' ',T21,15,T30,31F10.4,IX))
1120 FORMAT(' ',T21,15,T30,31F10.4,IX))
1121 FORMAT(' ',T21,15,T30,31F10.4,IX))
1122 FORMAT(' ',T21,15,T30,31F10.4,IX))
1123 FORMAT(' ',T21,15,T30,31F10.4,IX))
1124 FORMAT(' ',T21,15,T30,31F10.4,IX))
1125 FORMAT(' ',T21,15,T30,31F10.4,IX))
1126 FORMAT(' ',T21,15,T30,31F10.4,IX))
1127 FORMAT(' ',T21,15,T30,31F10.4,IX))
1128 FORMAT(' ',T21,15,T30,31F10.4,IX))
1129 FORMAT(' ',T21,15,T30,31F10.4,IX))
1130 FORMAT(' ',T21,15,T30,31F10.4,IX))
1131 FORMAT(' ',T21,15,T30,31F10.4,IX))
1132 FORMAT(' ',T21,15,T30,31F10.4,IX))
1133 FORMAT(' ',T21,15,T30,31F10.4,IX))
1134 FORMAT(' ',T21,15,T30,31F10.4,IX))
1135 FORMAT(' ',T21,15,T30,31F10.4,IX))
1136 FORMAT(' ',T21,15,T30,31F10.4,IX))

C **** DEFINITION OF PRINCIPAL VARIABLES: **

C ***** ALL VARIABLES NOT NOTED AS INTEGERS ARE REAL+4.

C ***** R = INITIAL RADIUS, CM

C ***** SCRL = SCRIPT L = 1/2 INITIAL LENGTH, CM

C ***** UF = INITIAL MOISTURE CONTENT AT FIBER SATURATION

C ***** UO = INITIAL MOISTURE CONTENT, G WATER/ G DRYWOOD

C ***** IF(UO,GT,UF) UO=UF IN PROGRAM.

C ***** ALL WOOD MODEL RELATIONS DEFINED ONLY FOR 0.5E.UF.LE.1

C ***** U = LOCAL MOISTURE CONTENT, GRAM WATER/GRAM DRYWOOD

C ***** T = ISOTHERMAL TEMPERATURE, OEG K

C ***** CONTINUED BELOW.

1684 FORMAT(' ', 'INITIAL RADIAL BIOT MASS TRANSFER NUMBER = ',
1685 * F10.4,'/INITIAL LONGITUDINAL BIOT MASS TRANSFER NUMBER = ',
1686 * F10.4,'/INITIAL NON-DIMEN. AMBIENT COND. FOR RADIAL WALL = ',
1687 * F10.4,'/INITIAL NON-DIMEN. AMBIENT COND. FOR CYL. ENDS = ',
1688 * F10.4,'/INITIAL MASS TRANSFER RATE G/(AR*T/R**2) = ',E10.4)
C ***** ALPHA AND BETA ARE RELAXATION PARAMETERS, ALPHA+BETA=1.
C ***** DT = INITIAL DIMENSIONLESS TIME INCREMENT, AR/1.ME/T/R**2
C ***** NH = NUMBER OF INTERVALS FOR R* CR Z*, (INTEGER)
C ***** ICASE=0 HUMIDITY=U/UF FK=0,FK>0. (INTEGER)
C ***** ICASE=1 HUMIDITY=F(U/UF)+G(T)*H(U/UF) FK=0,FK>0. (INTEGER)
C ***** ICASE>1 FULL WOOD MODEL
C ***** ISPW<0 DON'T WRITE MATRICES
C ***** ISPW=0 WRITE MATRICES FOR FIRST TIME
C ***** ISPW>0 WRITE MATRICES EVERY TIME
C ***** ISPwl STOP PROGRAM AFTER FIRST TIME
C ***** ISPWX10 WRITE OUT U(I) AND U(NI) ONLY FOR EVERY
C ***** TIME STEP AND ALL U FOR EVERY 100TH.
C
1685 FORMAT('  ', 'INITIAL DIMENSIONLESS MOISTURE CONTENT = 1.0',
1686 FORMAT(' ', T72, 2((=10.4, IX)/))

C
C RENLD= REYNOLD'S NUMBER OF FLOW PAST CYLINDER
C SCPR= RATIO OF SCMIDT NO. TO PRANDTL NO.
C RHOINF = AMBIENT VAPOUR DENSITY, G/CM**3
C RHODW = DENSITY OF DRY WOOD, G/CM**3
C FK = SHRINKAGE PARAMETER (WET VOL/DRY VOL=1.+FK*UO/UF
C RUUF= TIME END CONDITION
C
C
C
C
C
C
C
C
C
C
C READ(5,1050) R, SCRL, UO, T
C READ(5,1052) ALPHA, BETA, DT, NH, ICASE, ISPW
C READ(5,1050) RENLD, SCPR, RHOINF
C READ(5,1050) RHODW, FK
C READ(5,1050) RUUF
C READ(5,1052) STEP1,STEP2,STEP3,LOC1,LOC2
C IF(ICASE .LE. 1) READ(5,1051)RDIFFN,ZDIFFN
C IF(ICASE.GT.0) GOTO 4
C WRITE(6,1673)RDIFFN,ZDIFFN
C GOTO 5
C
C 4 CONTINUE
C IF(ICASE.GT.1) GOTO 5
C WRITE(6,1680)RDIFFN,ZDIFFN
C
C 5 CONTINUE
C WRITE(6,1674)R,SCRL,UO,T
C WRITE(6,1675)ALPHA,BETA,DT,NH,ICASE,ISPW
C WRITE(6,1676)RENLD,SCPR,RHOINF
WRITE(6,1678)RUUF
WRITE(6,1679)STEP1,STEP2,STEP3,LOC1,LOC2
NI=NH+1
NJ=NH+1
NK=NI*NJ

C ***** CALCULATE NON-DIMENSIONAL SPACE STEP.
   FH=1./NH
10 CONTINUE

C ***** CALCULATE SURFACE MASS TRANSFER COEFFICIENTS
C ***** FKCR=COEFFICIENT FOR CYLINDER WALLS
C ***** FKZ=COEFFICIENT FOR CYLINDER ENDS
CALL SURFTR(T,RENLD,SCPR,R,FKCR,FKZ)

NTIME=1
IFLAG=0

C ***** START REAL TIME
   RTIME=0.
   FIRST=.TRUE..
C ***** POSITIONS IN U(K,J):(N+1)DT»U(K,1),(N+1)*DT»U(K,2)
C ***** (N)DT»U(K,3), (N-1)DT»U(K,4)
   Y=T-298.16
   UF=.308-.00107*Y
   IF(UO.GT.UF)  UO=UF

C ***** INITIALIZE BOUNDARY CONDITION AND MOISTURE CONTENT VECTORS
   DO 20  I=1,NK
      VLAM(I)=0.
   20 CONTINUE
   DO 22  JJ=1,NJ
      LAM1(JJ,2)=0.0
   22 CONTINUE
   DO 24  11=1,NI
      LAM1BP.(II,2)=0.0
   24 CONTINUE
   Rhosat=(1.E-3/(461.5209*T))**61.04-68.5*ALOG(T))
   C ***** RHOS= DENSITY OF WOOD SUBSTANCE AS DETERMINED BY STAMM
   C ***** USING HELIUM DISPLACEMENT, G/CC.
   C ***** RH0= DENSITY OF ADSORBED WATER, G/CC.
   RHOS=1.46
   RH0=1.11
   C1=RHWS/RH0
   C2=RHODW/RHWS
   RADFAC=1.+FK*U0
   TIMCOR=1.
   C ***** ARI= NONDIMENSIONALIZING RADIAL DIFFUSIVITY
   ARI=1.
257 \[ \text{ARI} = AA(1, U_0, T) \]
258 \[ R_0 = R \]
259 \[ SCRL = SCRL \]
260 \[ \text{CONTINUE} \]
261 \[ K = 0 \]
262 \[ RR = DT/(HH*HH) \]
263 \[ RAT1 = 3/(2*RR) \]
264 \[ \text{DO 180} \]
265 \[ \text{I} = 1, NI \]
266 \[ \text{DO 180} \]
267 \[ \text{J} = 1, NJ \]
268 \[ K = K + 1 \]
269 \[ KP = (J-1)*NI + I \]
270 \[ \text{CALL BOUND} \]
271 \[ \text{IF (.NOT. (BC2 .OR. BC4)) GO TO 50} \]
272 \[ \text{IF (ICASE .GT. 0) GO TO 30} \]
273 \[ \text{C ***** SET UP BOUNDARY CONDITIONS FOR ICASE = 0} \]
274 \[ \text{C} \]
275 \[ IK = I + NJ - J \]
276 \[ H(IK) = U(K,3)/UF \]
277 \[ \text{GO TO 34} \]
278 \[ \text{30 CONTINUE} \]
279 \[ \text{C ***** SET UP BOUNDARY CONDITIONS FOR ICASE > 0} \]
280 \[ \text{C ***** CALCULATE LAM1, LAM1R, C, AND QBAR.} \]
281 \[ \text{C} \]
282 \[ IK = I + NJ - J \]
283 \[ \text{IF (FIRSTT) GOTO 31} \]
284 \[ \text{C ***** REAPPROXIMATE VALUES OF U ON EXTERNAL BOUNDARIES.} \]
285 \[ \text{IF (.NOT. (BC4)) U(K,3) = (U(K-1,3) + HH*LAM1R(I,1)) / (1 + HH*Q3AR(I,1))} \]
286 \[ \text{IF (BC4) U(K,3) = (U(K-NJ,3) + HH*LAM1(J,1)) / (1 + HH*Q(J,1))} \]
287 \[ \text{31 CONTINUE} \]
288 \[ X = U(K,3)/UF \]
289 \[ \text{C ***** MINIMIZE FIT ERROR FOR HUMIDITY.} \]
290 \[ \text{IF (X .GT. 0) GO TO 32} \]
291 \[ \text{C ***** SAVE H FOR DRYING RATE CALCULATION} \]
292 \[ H(IK) = X \]
293 \[ \text{GO TO 34} \]
294 \[ \text{32 CONTINUE} \]
295 \[ \text{C ***** CALCULATE DESORPTIONAL ISOTHERMS} \]
296 \[ HT25 = 7.464428E-03 + 3.32111E-01*X + 5.870049*X**2 \]
297 \[ * -2.362482E+00*X**3 + 2.110415E+01*X**4 + 6.686810*X**5 \]
298 \[ 300 H100 = 1.604156E-04 + 1.376967E+00*X + 1.053299E+01*X**2 \]
299 \[ * -4.669788E+01*X**3 + 8.222108E+01*X**4 - 7.635399E+01*X**5 \]
300 \[ * +3.665102E+02*X**6 - 7.130367*X**7 \]
301 \[ HRUUF = HT100-HT25 \]
302 \[ \text{IF (HRUUF .LT. 0.0) HRUUF = 0.0} \]
303 \[ \text{C **** SAVE H.} \]
304 \[ H(IK) = HT25 + GT*HRUUF \]
305 \[ \text{C ***** SET UP APPROPRIATE BOUNDARY CONDITIONS} \]
306 \[ \text{34 CONTINUE} \]
307 \[ \text{IF (.NOT. BC4) GO TO 35} \]
308 \[ \text{C ***** DU/DZ + Q*U = LAM1} \]
309 \[ ZDIF = AA(3, U(K,3), T) \]
310 \[ RAT2 = R**2*FKCZ/(ARI*SCRL*RHODW*ZDIF) \]
311 \[ LAM1(J,1) = RAT2*RH0INF \]
312 \[ Q(J,1) = RAT2*RHOSAT*H(IK)/U(K,3) \]
313 \[ \text{35 CONTINUE} \]
314 \[ \text{IF (.NOT. BC2) GO TO 50} \]
C ***** DU/DR + QBAR*U = LAMBDR
RDIF=AA(1,1U(K,3),T)
RAT3=R*FKCR/(AKI*RHODW*RDIF)
LAMBDR(I,1)=RAT3*RDINF
QBAR(I,1) = RAT3*HOSAT*HI(IK)/U(K,3)

50 CONTINUE
C
C ***** GENERATE A, B, C, FOR EACH MATRIX *****
C ***** B IS ON DIAGONAL, OTHERS RELATE AS WRITTEN.
C
IF (BC2) GO TO 60
CF(K)=(J-1./2)*HH*AA(1,(U(K,3)+U(K+1,3))/2,T)
AF(K+1)=CF(K)

55 CONTINUE
AF(K) = AF(K+1)
GO TO 65

60 CONTINUE
IF(.NOT. FIRSTT ) GOTO 62
C ***** SUDOUF(J) IS U AT GRID POINT OUTSIDE BODY.
SUDOUF=U(K,3)
GOTO 64

62 CONTINUE
SUDOUF=U(K-1,3)+U(K,3)*(1-2*HH*QBAR(I,1))
SUDOUF=(SUDOUF+2*HH*LAMBDR(I,1))/2.
64 CONTINUE
CF(K)=(J-1./2)*HH*AA(1,SUDOUF,T)
THETA(K,1)=2*CF(K)*HH*QBAR(I,1)

65 CONTINUE
IF (BC4) GO TO 75
CJ(K)=AA(2,1U(K,3)+U(K+NJ,3))/2,T)
AJ(K+NJ)=CJ(K)
CG(K)=(J-1)*HH*CJ(K)
AG(K+NJ)=CG(K)
IF (BC3) GO TO 70

70 CONTINUE
AJ(K) = AJ(K+NJ)
AG(K) = AG(K+NJ)
GOTO 80

75 CONTINUE
IF(.NOT. FIRSTT ) GOTO 77
SUDOUJ=U(K,3)
GOTO 79

77 CONTINUE
SUDOUJ=U(K-NJ,3)+U(K,3)*(1-2*HH*Q(J,1)+2*HH*LAM1(J,1)
SUDOUJ=SUDOUJ/2.
79 CONTINUE
CJ(K)=AA(2,SUDOUJ,T)
CG(K)=(J-1)*HH*CJ(K)
PHI(K,1)=2*CG(K)*HH*Q(J,1)
ZETA(KP,1)=2*CJ(KP)*HH*Q(J,1)

80 CONTINUE
BF(K)=-(AF(K)+CF(K))
BG(K)=-(AG(K)+CG(K))
BJ(K)=-(AJ(K)+CJ(K))

C
C ***** GENERATE F, G, J, MATRICES FOR SOLUTION METHOD *****
C
F(K,2) = BF(K)
IF (BC1) GO TO 85
IF (BC2) GO TO 90
F(K,1) = AF(K)
F(K,3) = CF(K)
GO TO 95

85 CONTINUE
F(K,3) = - BF(K)
GO TO 95

90 CONTINUE
F(K,1) = - BF(K)

95 CONTINUE
G(K,2) = BG(K)
RAJ(KP,2) = BJ(K)
IF (BC3) GO TO 100
IF (BC4) GO TO 105
G(K,1) = AG(K)
G(K,3) = CG(K)
RAJ(KP,1) = AJ(K)
RAJ(KP,3) = CJ(K)
GO TO 110

100 CONTINUE
G(K,3) = - BG(K)
RAJ(KP,3) = - BJ(K)
GO TO 110

105 CONTINUE
G(K,1) = - AG(K)
G(K,2) = BG(K)
RAJ(KP,1) = - BJ(K)

110 CONTINUE

C
C ***** GENERATE BOUNDARY CONDITION VECTOR VLAM *****
C
IF (FIRSTT) GOTO 118
IF (RHOINF EQ. 0) GOTO 162
118 CONTINUE
IF (BC4) GO TO 130
IF (BC2) GO TO 125
GO TO 162

125 CONTINUE
VLAM(K) = CF(K) * ((1 + ALPHA) * LAM1(J,1) + (1 + BETA) * LAM1(J,2))
GO TO 160

130 CONTINUE
IF (BC1) GO TO 162
IF (BC2) GO TO 140
VLAM(K) = CG(K) * ((1 + ALPHA) * LAM1(J,1) + (1 + BETA) * LAM1(J,2))
GO TO 160

140 CONTINUE
VLAM(K) = CF(K) * ((2 * LAM1BR(I,1) + LAM1BR(I,2)) * CG(K) * ((1 + ALPHA) * 
* LAM1(J,1) + (1 + BETA) * LAM1(J,2))
GO TO 160

160 CONTINUE
VLAM(K) = VLAM(K) * 2 * HH

162 CONTINUE

C
C ***** GENERATE BOUNDARY CONDITION VECTOR VNETA *****
C
IF (FIRSTT) GOTO 164
IF (RHOINF EQ. 0) GOTO 179

164 CONTINUE
IF (BC4) GO TO 170
GO TO 179

170 CONTINUE

VNETA(KP)=CJ(K)*(LAM1(J,1)*(1-ALPHA)-BETA*LAMI(J,2))

175 CONTINUE

VNETA(KP)=VNETA(KP)*2*HH

179 CONTINUE

180 CONTINUE

C

C***** GENERATE FI AND FIR *****

C

185 CONTINUE

K=0

DO 190 I=1,NI

DO 190 J=1,NJ

K=K+1

KP=(J-1)*NI+I

FI(KP)=BB(U(K,3),FK)*RAT1

FIR(K) = F(KP)*((J-1)*HH

190 CONTINUE

C

C ***** WRITE OUT ALL THE MATRICES THAT EXIST TO NOW (IF DESIRED)

C

IF (ISPEW) 300,200,205

200 IF (.NOT. FIRSTT) GO TO 300

205 CONTINUE

WRITE(6,1060)

K=0

DO 285 I=1,NI

DO 285 J=1,NJ

K = K + 1

CALL BOUND

IF (BC1) GO TO 255

IF (BC2) GO TO 260

WRITE(6,1061) (F(K,IY), IY=1,3), (RAJ(K,IY), IY=1,3)

GO TO 265

255 CONTINUE

WRITE(6,1062) (F(K,IY), IY=2,3), (RAJ(K,IY), IY=2,3)

GO TO 265

260 CONTINUE

WRITE(6,1063) (F(K,IY), IY=1,2), (RAJ(K,IY), IY=1,2)

265 CONTINUE

IF (BC3) GO TO 270

IF (BC4) GO TO 275

WRITE(6,1064) (G(K,IY), IY=1,3)

GO TO 280

270 CONTINUE

WRITE(6,1065) (G(K,IY), IY=2,3)

GO TO 280

275 CONTINUE

WRITE(6,1066) (G(K,IY), IY=1,2)

280 CONTINUE

WRITE(6,1067) K, FI(K), FIR(K)

285 CONTINUE

WRITE(6,1068)

DO 290 K=1,NK

WRITE(6,1069) K, VLAM(K), VNETA(K), (THETA(K,L), L=1,2), (PHI(K,L), L=1,2), (ZETA(K,L), L=1,2)

290 CONTINUE

300 CONTINUE
C ***** MANIPULATE ALL MATRIX ENTRIES TO PUT MATRIX EQUATION 1
C ***** IN SOLVABLE FORM AND EQUATION 2 IN MOST REDUCED FORM.

K=0
DO 370 I=1,NI
DO 370 J=1,NJ
K=K+1
KP=(J-1)*NI+I
CALL BCUND
IF (BC1) GO TO 310
IF (BC2) GO TO 325
IF (BC3) GO TO 340
IF (BC4) GO TO 345
BG(K)=G(K,1)*(1+ALPHA)*U(K-NJ,3)+F(K,1)*U(K-1,3)+
* (F(K,2)+G(K,2)*(1+ALPHA))*U(K,3)+
* F(K,3)*(1+ALPHA)*U(K,NJ,3)+
* G(K,1)*(1+ALPHA)*U(K-NJ,4)+F(K,1)*U(K-1,4)+
* (F(K)+F(K,2)+G(K,2)*(1+ALPHA))*U(K,4)+
* F(K,3)*(1+ALPHA)*U(K+NJ,4)+
CG(KP)=(RAJ(KP,1)*U(K-NJ,3)+RAJ(KP,2)*U(K,3)+
* RAJ(KP,3)*U(K+NJ,3))*ALPHA+(RAJ(KP,1)*U(K-NJ,4)+
* RAJ(KP,2)*U(K,4)+RAJ(KP,3)*U(K+NJ,4))*BETA
GO TO 350
BG(K)=G(K,1)*(1+ALPHA)*U(K-NJ,3)+F(K,1)*U(K-1,3)+
* (F(K,2)+G(K,2))*(1+ALPHA)*U(K,3)+
* (F(K)+F(K,2)+G(K,2)*(1+ALPHA))*U(K,4)+
* F(K,3)*(1+ALPHA)*U(K+NJ,4)+
CG(KP)=(RAJ(KP,1)*U(K-NJ,3)+RAJ(KP,2)*U(K,3)+
* RAJ(KP,3)*U(K+NJ,3))*ALPHA+(RAJ(KP,1)*U(K-NJ,4)+
* RAJ(KP,2)*U(K,4)+RAJ(KP,3)*U(K+NJ,4))*BETA
GO TO 350
BG(K)=(F(K,2)-THETA(K,1)*(1+ALPHA))*U(K,3)+
* F(K,3)*(1+ALPHA)*U(K,3)+
* (F(K)+F(K,2)-THETA(K,2)*(1+ALPHA))*U(K,4)+
* F(K,3)*(1+ALPHA)*U(K+NJ,4)+
CG(KP)=(RAJ(KP,2)-ZETA(KP,1)*U(K,4)+RAJ(KP,3)*U(K,NJ,4)+
* (RAJ(KP,2)-ZETA(KP,2))*U(K,4)+RAJ(KP,3)*U(K-NJ,4))*BETA-
* VNETA(KP)
GO TO 350
BG(K)=(F(K,2)-THETA(K,1)+(1+ALPHA))*U(K-1,3)+
* G(K,1)*(1+ALPHA)*U(K-NJ,3)+F(K,1)*U(K-1,3)+
* (F(K,2)+G(K,2)*(1+ALPHA))*U(K,3)+
* G(K,3)*(1+ALPHA)*U(K+NJ,3)+
* G(K,1)*(1+ALPHA)*U(K-NJ,4)+F(K,1)*U(K-1,4)+
* (F(K)+F(K,2)-THETA(K,2)+G(K,2)*(1+ALPHA))*U(K,4)+
* G(K,3)*(1+ALPHA)*U(K+NJ,4)+VLAM(K)
GO TO 305
BG(K)=F(K,1)*U(K-1,3)+
* (F(K,2)-THETA(K,1)+G(K,2)*(1+ALPHA))*U(K,3)+
* G(K,3)*(1+ALPHA)*U(K+NJ,3)+
* F(K,1)*(1+ALPHA)*U(K-1,4)+
* (F(K)+F(K,2)-THETA(K,2)+G(K,2)*(1+ALPHA))*U(K,4)+
GO TO 305
* G(K,3)*(1+BETA)*U(K+NJ,4)+VLAM(K)
GO TO 315

335 CONTINUE
BG(K)=G(K,1)*(1+ALPHA)*U(K-NJ,3)+F(K,1)*U(K-1,3)+
* (F(K,2)-THETA(K,1)+G(K,2)-PHI(K,1))*(1+ALPHA)*U(K,3)+
* G(K,1)*(1+BETA)*U(K-NJ,4)+F(K,1)*U(K-1,4)+
* (FIR(K)+F(K,2)-THETA(K,2)+(G(K,2)-PHI(K,2))*(1+BETA)*
U(K,4)+VLAM(K)
GO TO 320

340 CONTINUE
BG(K)=F(K,1)*U(K-1,3)+
* (F(K,2)+G(K,2)*(1+ALPHA))*U(K,3)+
* F(K,3)*U(K+1,3)+G(K,3)*(1+ALPHA)*U(K+NJ,3)+
* F(K,1)*U(K-1,4)+
* (FIR(K)+F(K,2)+G(K,2)*(1+BETA))*U(K,4)+
* F(K,3)*U(K+1,4)+G(K,3)*(1+BETA)*U(K+NJ,4)
GO TO 315

345 CONTINUE
BG(K)=G(K,1)*(1+ALPHA)*U(K-NJ,3)+F(K,1)*U(K-1,3)+
* (F(K,2)+G(K,2)-PHI(K,1))*(1+ALPHA)*U(K,3)+
* G(K,1)*(1+BETA)*U(K-NJ,4)+F(K,1)*U(K-1,4)+
* (FIR(K)+F(K,2)+G(K,2)-PHI(K,2))*(1+BETA)*U(K,4)+
* F(K,3)*U(K+1,4)+VLAM(K)
GO TO 320

350 CONTINUE
IF (K .EQ. NK) GO TO 355
F(K,3)=-F(K,3)
RAJ(KP,3)=-RAJ(KP,3)
CF(K)=F(K,3)
CJ(KP)=RAJ(KP,3)
CJ(KP)=RAJ(KP,3)

355 CONTINUE
IF (K .EQ. 1) GO TO 360
F(K,1)=-F(K,1)
RAJ(KP,1)=-RAJ(KP,1)
AF(K)=F(K,1)
AJ(KP)=RAJ(KP,1)

360 CONTINUE
F(K,2)=FIR(K)-F(K,2)*THETA(K,1)
RAJ(KP,2)=FIR(KP)-RAJ(KP,2)+ZETA(KP,1)
C
C ***** EQUATIONS ARE NOW IN THE FORM:
C 1. F * U(N+1) = BG
C 2. J * U(N+1) = FI * U(N+1) + CG
C ***** SET UP FOR TRISLV *****
C
BF(K)=F(K,2)
AJ(KP)=RAJ(KP,2)

370 CONTINUE
C
C ***** WRITE OUT MATRICES TO BE SOLVED (IF DESIRED) *****
C
IF (ISPEW) 400,375,380
C
375 IF (.NOT.*FIRSTT) GO TO 400
C
380 CONTINUE
WRITE(6,1663)
K=0
DO 395 I=1,N1
DO 395 J=1,NJ
K=K+1
CALL BOUND
IF (BC1) GO TO 385
IF (BC2) GO TO 390
WRITE(6,1660) K, (F(K,IY), IY=1,3), (RAJ(K,IY), IY=1,3),
*BG(K),CG(K)
GO TO 395
385 CONTINUE
WRITE(6,1661) K, (F(K,IY), IY=2,3), (RAJ(K,IY), IY=2,3),
*BG(K),CG(K)
GO TO 395
390 CONTINUE
WRITE(6,1662) K, (F(K,IY), IY=1,2), (RAJ(K,IY), IY=1,2),
*BG(K),CG(K)
GO TO 395
C
C ***** SOLVE FIRST EQUATION TO FIND U(N+1)*
C ***** AF = LOWER DIAGONAL
C ***** BF = DIAGONAL
C ***** CF = UPPER DIAGONAL
C ***** BG = RIGHT HAND SIDE VECTOR ON ENTRANCE
C ***** CG = U(N+1)* ON EXIT
C
400 CONTINUE
CALL TRISLV(NK, AF, BF, CF, BG, 2, 6405)
GO TO 410
405 CONTINUE
WRITE(6,1671)
STOP
410 CONTINUE
C
C ***** ADD U(N+1)* TO CG TERM TG REDUCED FORM OF EQUATION 2 *****
C
415 CONTINUE
C
C ***** SOLVE SECOND EQUATION *****
C
CALL TRISLV(NK, AJ, BJ, CJ, CG, 2, 6420)
C ***** SOLUTION FOR U(N+1) HELD IN CG
CG TO 425
420 CONTINUE
WRITE(6,1672)
STOP
425 CONTINUE
C
C ***** CALCULATE MASS TRANSFER RATE, RMDOT ****
C
C
CALL MDOTT(H,RMDCT1,RMDOT)
IF(FIRSTT) GOTO 182
C
C ***** CALCULATE AVERAGE MOISTURE CONTENT IN ONE OF TWO WAYS.
C USE AVGU IF ICASE .LE. 1 (ITS FASTEST) OTHERWISE USE USUM.
C
IF(ICASE .LE. 1) GOTO 427
DO 426 I=1,NK
AG(I)=U(I,3)
CONTINUE
CALL USUM(AG,NH,UO,UAVG)
GOTO 428
CALL AVGUDT(LAST,RMDCT1,RMDOT,UAVG)
WRITE(6,1686)RMDOT,UAVG
GOTO 183
182 CONTINUE
UAVG=1.
RLAST=1.
WRITE(6,1685)ARI
WRITE(6,1684)QBAR(1,1),Q(1,1),LAM1BR(1,1),LAM1(1,1),RMDOT
183 CONTINUE
CONTINUE
C ***** ALLOW FOR CHANGE OF BODY DIMENSIONS CAUSED BY SHRINKAGE.
C ASSUME ISOTROPIC SHRINKAGE.
OZ=((1.+FK*UO*UAVG)/RACFAC)**(1./3.)
R=R0*OZ
SCRLO=SCRL*QZ
TIMCOR=OZ**2
CONTINUE
CONTINUE
CONTINUE
CONTINUE
C ***** WRITE OUT MOISTURE CONTENT
IF(IABS(ISPEW) .EQ. 10) GO TO 430
WRITE(6,1664) K, U(K,2), U(K,1),UU0
GO TO 445
CONTINUE
IF(FLAG .NE. 100) GOTO 435
CONTINUE
CONTINUE
CONTINUE
C ***** COMPENSATE TIME TO ALLOW FOR SHRINKAGE.
RLTIME=RLTIME+(CT*TIMCOR)
WRITE(6,1666) RLTIMF
IF(ISPEW .EQ. 1) STOP
CONTINUE
CONTINUE
CONTINUE
CONTINUE
C ***** SET UP U(K,J) FOR NEXT TIME STEP
DO 450 J=1,NJ
U(J,3)=U(J,1)
CONTINUE
CONTINUE
CONTINUE
CONTINUE
C ***** SHIFT BOUNDARY CONDITION TERMS
I2=NK-(NJ-1)
DO 451 I=1,NK
PHI(I,2)=PHI(I,1)
CONTINUE
DO 452 I=NJ,NK,NJ
THETA(I,2)=THETA(I,1)
CONTINUE
DO 453 I=NI,NK,NI
ZETA(I,2)=ZETA(I,1)
CONTINUE
IF(RHOINF.EQ.0 .AND. .NOT.FIRSTT) GOTO 459
DO 456 I=1,NI
LAMI BR(I,2)=LAMI BR(I,1)
CONTINUE
DO 458 J=1,NJ
LAM1(J,2)=LAM1(J,1)
CONTINUE
IF(I EQL.0) IFLAG=0
FIRSTT = .FALSE.
NTIME=NTIME+1
C ***** TEST TIME
IF (RLTIME.GE.RUUF) STOP
C ***** SEE IF SPECIAL CASE REDUCE CALCULATIONS FOR
C ***** LATER RUNS.
IF (ICASE.GT.0 .OR. FK.GT.0.) GO TO 480
CALL BOUND
IF(.NOT.(BC2.0R.BC4)) GOTO 460
IK=I+NJ-J
HI IK)=U(K,3)/UF
CONTINUE
IF (K.EQ.NK > GO TO 462
F(K,3)=-F(K,3)
RAJ(K,3)=-RAJ(K,3)
CONTINUE
F(K,1)=-F(K,1)
RAJ(K,1)=-RAJ(K,1)
F(K,2)=FIR(K)-F(K,2)+THETA(K,1)
RAJ(K,2)=FI(K)-RAJ(K,2)+ZETA(K,1)
CONTINUE
DT LAST=DT
DTP=CHGTM(NTIME)
IF(DTP .EQ. DT) GOTO 475
DT=DTP
RP=DT/(HH*HH)
RAT=3./[12*RR]
C
C ***** AA1,AA2,BB,AND BOUNDARY CONDITIONS ARE CONSTANT.
C ***** FK=0.
C
GO TO 185
C
475 CONTINUE
C
C ***** DT,AA1,AA2,BB,AND, BOUNDARY CONDITIONS ARE CONSTANT
C ***** FK=0.
GO TO 300

C

**** ICASE>0 OR FK >0

C

480 CONTINUE

DLAST=DT

C

**** IF DESIRED CHANGE TIME STEP.

DT=CHGETIM(TIME)

GO TO 25

C

END

FUNCTION AA(L,U,T)

C

**** CALCULATION OF COMBINED DIFFUSIVITIES OF WATER IN

**** WOOD. AA1 IS THE RADIAL DIFFUSIVITY, AA2 IS THE

**** LONGITUDINAL DIFFUSIVITY. ON EXIT:

AA1 = AA1/ARI

AA2 = AA2*(R/SCRL)**2/ARI

COMMON /PAR/R,SCRL,FK,RHODW

COMMON /ACON/RDIFFN.ZDIFFN,ICASE

IF( ICASE .EQ. 0 .OR. ICASE .EQ. 1) GO TO 30

IF(U < 0.) U=0.

VOID=1-C2M(l+Cl*U)/(1+FK*U))

SVOID=SQRT(VOID)

DBPRI=(46.84*U**2-410.93*U**3+1106.11*U**4)*

* EXPI-4120.43/T)

DB=C2*DBPRI/(1-V0ID)**2

X=U/UF

C

***** USE FIT FOR H VERSUS U/UF TO FIND

SLOPE OF ISOTHERM,CHOU.

DHDU25=3.32111E-01+19.740098*x-7.087446*1*01**2

* +8.441645*01*X**3-32.44305*X**4

DELH=1.044856+2.125882*X-69.21918*X**2+24.46792*X**3

* -348.3269*X**4+219.9061*X**5-49.91257*X**6

DHDU=DHDU25+GT*DELH

DHDU=3HDU25+GT*DELH

DHDU=DHDU25+GT*DELH

DHDU=3HDU25+GT*DELH

DHDU=DHDU25+GT*DELH

DHDU=DHDU25+GT*DELH

IF (L.EQ.2) GO TO 10

DVPRI=.22*<T/273 )**1.75

DV=RHOSAT/RHODW*(1+F  K*U)** 2*DVPRI*DHDU

IF (L.EQ.2) GO TO 10

C

**** CALCULATE DIFFUSION COEFFICIENTS FOR RADIAL DIRECTION ***

D1=(1-SVID)*DB

C2=OV

D3=((SV0ID-.01452)/(1-SVOID))*DB

D4=(4.66E-5/((1-SVOID)/300)-1.E-4))*DV

D5=.466*DB

D6=.43E-4*DV

GO TO 20

C

**** CALCULATE DIFFUSION COEFFICIENTS FOR AXIAL DIRECTION ***

D1=(1-SVOID)*DB

D2=VCIO*DV

D3=((SV0ID-.0178)/(1-SVOID)*.0083))*DB

D4=.014*DV/((1-SVOID)*.0083-2.5E-4)

D5=56*DB
C **** CALCULATE DIFFUSIVITIES FOR CONSTANT COEFFICIENT CASE
30 CONTINUE
IF(L.EQ.2) GO TO 40
C **** CALCULATE RADIAL DIFFUSIVITY
DO=RDIFFN
GO TO 50

40 CONTINUE
C **** CALCULATE AXIAL DIFFUSIVITY
DD=ZDIFFN
50 CONTINUE
IF(L.EQ.2)DD=DD*(R/SCRL)**2
AA=DD/(1+FK*U)**2
IF(I.FQ.LICASE.LE.1) AA=DD
AA=AA/AR I
RETURN

END
FUNCTION BB(U,FK)
BB=1./(1.+FK*U)
RETURN
END
SUBROUTINE BOUND
C ***** DETERMINE IF ON BOUNDARY( SEE GRID AT START OF PROGRAM.)
COMMON /BC/ BC1, BC2, BC3, BC4, I, J, NI, NJ,FIRSTT
LOGICAL BC1, BC2, BC3, BC4, FIRSTT
BC1=.FALSE.
BC2=.FALSE.
BC3=.FALSE.
BC4=.FALSE.
IF (J .EQ. 1) BC1=.TRUE.
IF (J .EQ. NJ) BC2=.TRUE.
IF (I .EQ. 1) BC3=.TRUE.
IF (I .EQ. NI) BC4=.TRUE.
RETURN
END
FUNCTION CHGTIM(NX)
C ***** CHANGE TIME INCREMENT IN STEPS,( SEE DEFINITIONS AT START.
COMMON /CH/STEP1,STEP2,STEP3,LOC1,LOC2
IF(NX.GT.LOC1) GO TO 10
CHGTIM=STEP1
GO TO 40
10 CONTINUE
IF(NX.GT.LOC2) GO TO 20
CHGTIM=STEP2
GO TO 40
20 CONTINUE
CHGTIM=STEP3
40 CONTINUE
RETURN
END
SUBROUTINE MOOTT(H, RMDOT1, PM00T1
C **** SIMPSON'S RULE INTEGRATION OF SURFACE MASS TRANSFER
C TO FIND TOTAL RATE OF MASS TRANSFER FROM BODY (RMDOT)
COMMON /M,FKCR,FKCZ,RHOINF,RO
COMMON /A/C1,C2,GT,UF,RHOSAT,ARI
COMMON /PAR/R,SCRL,FK,RHODW
COMMON /EC/ BC1,BC2,BC3,BC4,I,J,NI,NJ,FIRSTT
LOGICAL BC1,BC2,BC3,BC4,FIRSTT
DIMENSION H(200)

C ***** H CONTAINS VALUES OF HUMIDITY FOR CYLINDRICAL SURFACE
C FROM 1 TO NI AND FOR END SURFACE FROM PERIMETER TO CENTER.
C CENTER H HELD AS H(NI)
C MINIMUM NI OR NJ IS 3
NJK=NI+NJ-1
NJ1=NI-1
NJ2=NI-2
NI1=NI+1
NI2=NI+2
NJK2=NJK-2

DO 1 IJ=1,NJ2
IK=M+IJ
H(IK)=(1.-1.*IJ/NJ1)*H(IK)
1 CONTINUE
NI2=NI-2
NL1=NI+1
NI2=NI-2
NJK2=NJK-2

C ***** TEST FOR ODD OR EVEN NUMBER OF INTERVALS
C HBARR=INTEGRAL FROM 0 TO 1 OF H*AZ
C HBARZ=INTEGRAL FROM 0 TO 1 OF R*H*DR

IF (NI/2*2.EC.NI ) GO TO 10
GO TO 20
5 CONTINUE

IF (NJ/2*2.EC.NJ) GO TO 30
GO TO 40

10 CONTINUE
SUM=H(1)*2+H(NI)
DO 15 IK=2,NI2,2
SUM=SUM+2*H(IK)+4*H(IK + 1)
15 CONTINUE

HBARR=(1./(NI-1))/3*SUM
GO TO 5

20 CONTINUE
SUM=H(1)*2*H(2)+H(NI)
DO 25 IK=3,NJ2,2
SUM=SUM+2*H(IK)+4*H(IK + 1)
25 CONTINUE

HBARR=(1./(NI-1))/3*SUM
GO TO 5

30 CONTINUE
SUM=H(NI)*2
DO 35 IK=NI1,NIK2,2
SUM=SUM+2*H(IK)+4*H(IK+1)
35 CONTINUE

HBRZ=(1./(NJ1))/3*SUM
GO TO 5

40 CONTINUE
SUM=H(NI)*4*H(NI)
DO 45 IK=NI1,NIK2,2
SUM=SUM+2*H(IK)+4*H(IK+1)
45 CONTINUE

HBRZ=(1./(NJ1))/3*SUM
GO TO 5

50 CONTINUE

C ***** RMDOT= TOTAL MASS TRANSFER FROM FINITE CYLINDER SURFACE
C ***** FORMULA APPLIES FOR CONSTANT TEMPERATURE CASE ONLY
SUBROUTINE SURFTR(T, RENLD, SCPR, R, FKCR, FKCRZ)
C**** CALCULATE SURFACE MASS TRANSFER COEFFICIENTS, FKCR, FKCRZ.
DMOLE = .22*((T/273.)**1.75)
SHBAR = .615*{SCPR***(1./3.))*(RENLD**.466)
SH = SHBAR
FKCR = DMOLE*SHBAR/(2.*R)
FKCRZ = DMOLE*SH/(2.*R)
RETURN
END

SUBROUTINE AVGU(DT, RMDOT1, RMDOT, UAVG)
COMMON /BC/BC1, BC2, BC3, BC4, I, J, NI, NJ, FIRSTT
COMMON /AVERG/RMASS, RLAST, TIMCOR
UAVG = UAVG - RMDOT1*DT*TIMCOR*(RLAST+RMDOT)/RMASS
RLAST = RMDOT
RETURN
END

SUBROUTINE USUM(P, NH, U0, UAVG)
DIMENSION P(2C00)
NI = NH+1
NJ = NI
NENDI = NH
HHH = 1./NH
NLAST = NENDI*NJ
RINT = 0.
DO 10 J = 2, NJ
SUM = 0.
DO 20 I = 10, J-2, NJ
K = (I-1)*NJ+J
SUM = SUM + P(K)
20 CONTINUE
SUM = SUM + (P(J) + P(NLAST+J))/2.
SUM = SUM*HHH
IF(J .EQ. NJ) SUM = SUM/2.
RINT = (J-1)*HHH*SUM + RINT
10 CONTINUE
RINT = RINT*HHH*2.
UAVG = UAVG/RINT/U0
RETURN
END
APPENDIX I

TRISLV

Purpose

To find the solution to a system of linear equations of the form $Ax=b$ where $A$ is a tri-diagonal matrix (a form often found in the numeric solution of partial differential equations).

Availability

Available in the public file *NUMLIB.

How To Use

CALL TRISLV(N,DL,DD,DU,RH,MODE,&nn)

where:

- $N$ is the number of equations.
- $DL$ is a one-dimensional array, dimensioned at least $N$. Its type depends on the value of MODE. On entry, it contains the lower diagonal coefficients starting at $DL(2)$. The contents of $DL$ are not changed by TRISLV and $DL(1)$ is not referenced.
- $DD$ is a one-dimensional array, dimensioned at least $N$. Its type depends on the value of MODE. On entry, it contains the diagonal coefficients.
- $DU$ is a one-dimensional array, dimensioned at least $N$. Its type depends on the value of MODE. On entry, it contains the upper diagonal coefficients. $DU(N)$ is set equal to zero by TRISLV.
- $RH$ is a one-dimensional array, dimensioned at least $N$. Its type depends on the value of MODE. On entry, it contains the right hand side, and on exit, the solution vector.
- $MODE$ indicates the type of the arrays $DL$, $DD$, $DU$, and $RH$ and the kind of matrix being entered.
  - $0$ if the arrays are REAL*8 and the original matrix, $A$, is entered.
  - $1$ if the arrays are REAL*8 and the decomposition of the matrix, $A$, is entered.
  - $2$ if the arrays are REAL*4 and the original
matrix, A, is entered.

\[ =3 \quad \text{if the arrays are REAL*4 and the decomposition of} \]
\[ \text{the matrix, A, is entered.} \]

nn is the statement number in the calling program to which control is returned if no solution is found.

**Comment**

For the \( I \)\(^{th} \) equation \( (1 < I < N) \) and MODE even:

\[ DL(I) \times X(I-1) + DD(I) \times X(I) + DU(I) \times X(I+1) = RH(I) \]

**Accuracy**

The accuracy depends on the condition of the matrix. The routine performs best on a diagonally dominant matrix. The sample output illustrates this point.

**Timing**

An estimate of the CPU time on the 370/168 in microseconds is:

<table>
<thead>
<tr>
<th>MODE</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>16*N</td>
</tr>
<tr>
<td>1</td>
<td>8*N</td>
</tr>
<tr>
<td>2</td>
<td>11*N</td>
</tr>
<tr>
<td>3</td>
<td>6*N</td>
</tr>
</tbody>
</table>
NOMENCLATURE

A(U,T)  diffusivity of moisture with respect to a gradient in mass of water per volume dry wood (cm²/sec)

A_R(U,T)  \frac{A_R(U,T)}{A_R(U,T)_i}

A_Z(U,T)  \frac{(R/L)_i^2 \cdot A_Z(U,T)}{A_R(U,T)_i}

B(U)  ratio of drywood volume to wet wood volume as given by Equation 17

Bi  \frac{hL}{K(U,T)}, Biot number

Bi_m  \frac{K_{GL} \rho_{sat} H}{A(U,T) \rho_D U}, Biot mass number

C(U,T)  specific heat of wet wood (cal/g·°C)

C*(U,T)  \frac{C(U,T)}{C(U,T)_i}

D_i  i = 1, 2, ..., 6 pathway diffusivities

D_B  diffusivity of liquid in wood substance with respect to a gradient in mass of water per volume of wet wood (cm²/sec)

D_C  diffusivity of moisture with respect to a gradient in mass of water per volume of wet wood (cm²/sec)

D_V  diffusivity of water vapour with respect to a gradient in mass of water per volume wet wood (cm²/sec)

f  fraction of moisture diffusion that is liquid

h  convective mass transfer coefficient (cal/cm²·sec·°C)

H  \frac{\rho_V}{\rho_{sat}}, relative humidity

I  net internal evaporation per unit volume as defined by Equation 54

K  grid point index

k  shrinkage factor as defined by Equation 17

K(U,T)  thermal conductivity of wet wood (cal/cm·sec·°C)
\(K_R(U,T)\)  
\(K_R(U,T)/K_R(U,T)_i\)

\(K_Z(U,T)\)  
\((R/L)_i^2 K_Z(U,T)/K_R(U,T)_i\)

\(K_G\)  
convective mass transfer coefficient (cm /sec)

\(\sigma\)  
\((T_a - T_i) C(U,T)\)  
Kossovich number

\(Ko\)  
\(\frac{\sigma}{(T_a - T_i) C(U,T)}\)

\(K_P\)  
grid point index

\(L\)  
cylinder half length (cm)

\(L\)  
characteristic length; \(L_i\) for the axial direction and \(R_i\) for the radial direction (cm)

\(Lu\)  
\(A(U,T)/\alpha(U,T)\), Luikov number

\(\dot{m}\)  
mass flux (g/cm²·sec)

\(\dot{M}^*\)  
actual drying rate/initial drying rate

\(NJ\)  
number of grid points in the radial or axial directions

\(NK\)  
total number of grid points

\(P\)  
pressure

\(r\)  
radial coordinate (cm)

\(r^*\)  
\(r/R_i\)

\(R\)  
cylinder radius (cm)

\(S\)  
1/(NJ-1), dimensionless grid spacing

\(t\)  
time (sec)

\(t_m\)  
\(A_R(U,T)_i t/R_i^2\), dimensionless time for mass transfer

\(t_h\)  
\(K_R(U,T)_i t/R_i^2\), dimensionless time for heat transfer

\(T\)  
temperature (°C)

\(T^*\)  
\(\frac{T_a - T}{T_a - T_i}\), dimensionless temperature

\(U\)  
mass water/mass dry wood, moisture content
$U_f$  moisture content at fiber saturation

$V$  volume (cm$^3$)

$z$  axial coordinate (cm)

$z^*$  $z/L_1$

$\alpha$  relaxation parameter

$\alpha(U,T)$,  

$K(U,T)/\rho_0C(U,T)$  

$\beta$  relaxation parameter

$\delta$  central difference operator

$\Delta$  difference increment in quantity

$\varepsilon$  void fraction (volume void/volume wetwood)

$\lambda$  dimensionless ambient condition as given by Equations 5 and 8

$Q$  of order of

$\xi$  $\Delta t^*/S^2$, mesh ratio of numerical method

$\rho$  density (g/cm$^3$)

$\sigma$  latent heat of evaporation (cal/g)

Subscripts

$a$  ambient condition of drying media

$B$  bound water

$D$  dry wood

$h$  indicates variable applies to heat transfer

$i$  initial condition

$K$  grid point location

$KP$  grid point location
L  variable applies to liquid
m  variable applies to mass transfer
R  radial direction
S  wet wood
sat saturated water vapour
V  water vapour
ws  wood substance
Z  axial direction

**Superscripts**

—  average quantity
n-1 previous time
n  present time
(n+1)* intermediate time
(n+1) advanced time