PARAMETERIZATION OF SOLAR IRRADIATION UNDER CLEAR SKIES

by

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ABSTRACT

This study compares 5 existing insolation models on three levels of complexity. The model by Leckner represents a spectral integration model, the models by Bird & Hulstrom, Davies & Hay and by Hoyt are parameterization models and the model of A.S.H.R.A.E. is a simple seasonal model. The emphasis of the comparison was kept on the attenuation by atmospheric aerosols as well as on the aspect of the scattered radiation. No new model will be proposed; instead, several improvements to increase the accuracy or to make the application easier will be presented, notably for the determination of aerosol attenuation. Simplification has been achieved by using the correlation between horizontal ground visibility and aerosol attenuation, resulting in easy-to-handle equations. While most models received only minor changes it was necessary to restructure the A.S.H.R.A.E. algorithm substantially.
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Nomenclature

A  A.S.H.R.A.E. coefficient  [W/m²·m]

BA  Aerosol forward scattering ratio

AMS  Air mass

B  A.S.H.R.A.E. coefficient

C  A.S.H.R.A.E. coefficient

C to H  Factors as defined in text

DS  Double scattered diffuse radiation  [W/m²·m]

FR  Forward scattering ratio

I  unsubscripted: total irradiance;  [W/m²·m]
    subscripted: spectral irradiance  [W/m²·m·μm]

Idif  Diffuse irradiance  [W/m²·m]

Irr  Direct irradiance  [W/m²·m]

K  Davies' and Hay's turbidity parameter;
    Optical depth

M  Multiplicator in A.S.H.R.A.E. algorithm

N  Molecule density

P  Pressure  [kPa]

PR  Attenuation probability

SC  Solar constant  [W/m²·m]

SH  Scale height  [km]

SS  Single scattered diffuse radiation  [W/m²·m]

TS  Triple scattered diffuse radiation  [W/m²·m]

UG  Concentrated atmospheric gas  [cm]

UO  Concentrated atmospheric ozone  [cm]

UW  Precipitable water  [cm]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
<td>VIS</td>
<td>Visibility</td>
<td>[km]</td>
</tr>
<tr>
<td>WO</td>
<td>Ratio of (AS)/(AA+AS)</td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>Hoyt's Rayleigh scattering parameter</td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>Hoyt's aerosol scattering parameter</td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>Absorption coefficient</td>
<td>[1/cm]</td>
</tr>
<tr>
<td>ra</td>
<td>Atmospheric albedo</td>
<td></td>
</tr>
<tr>
<td>rg</td>
<td>Reflectivity of the ground (albedo)</td>
<td></td>
</tr>
<tr>
<td>s</td>
<td>Pathlength</td>
<td>[km, cm]</td>
</tr>
<tr>
<td>a</td>
<td>Absorptance</td>
<td></td>
</tr>
<tr>
<td>a a</td>
<td>Angström wavelength exponent</td>
<td></td>
</tr>
<tr>
<td>β</td>
<td>Angström particle number density</td>
<td></td>
</tr>
<tr>
<td>λ</td>
<td>Wavelength</td>
<td>[nm, μm]</td>
</tr>
<tr>
<td>τ</td>
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<td></td>
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<tr>
<td>θ</td>
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Values in Brackets

- (A) Aerosol
- (AA) Aerosol absorption
- (Abs) Absorption
- (AS) Aerosol scattering
- (C) Carbon dioxide
- (G) Gas
- (O) Ozone
- (OX) Oxygen
- (R) Rayleigh scattering
- (Uv) Ultra violet part of the spectrum
(Vi)  Visible part of the spectrum
(Sc)  Scattering
(tot)  total
(1).. General optical properties
...(4)

Subscripts

h  on horizontal surface
n  at normal incidence
0  Extraterrestrial value
\lambda  spectral value

Glossary

Airmass: The ratio of the actual pathlength versus the shortest possible pathlength of the radiation through the earth's atmosphere. (See page 6/7 and eq. (1-6).


Index of refraction: A complex number; the real part is a measure for the optical difraction, the imaginary part indicates the dielectric properties. This index is a function of the wavelength.
Insolation: The total solar radiation which reaches the ground.

Precipitable water: The height of the completely condensed water in a column from ground to the edge of the atmosphere (mostly in [cm]).

Scattering phase function: A function in three dimensions which gives the relative distribution of radiation after scattering.

Solar constant: The amount of radiation which would reach the earth in the absence of an atmosphere at mean sun-earth distance. The exact amount of the solar constant is subject of discussions in the scientific community.

Turbidity: 'Turbid' air refers to attenuation of radiation in the atmosphere from sources other absorbing gases.

Zenith angle: The angle between the normal to the ground and the position of the sun. The zenith angle and the solar altitude add up to 90 degrees.
I. Introduction

The popular interest in solar radiation and its research received an appreciable boost in the last decade. Although this has brought the aspects of solar irradiation to a broader attendance, it must be said that solar radiation was a subject of research throughout this century; in fact, research on extinction of radiation was done more than a century ago (for example: Lambert [1]).

The methods of prediction of solar irradiation find a wide range of applications in such fields as:
- calculation of cooling loads for air conditioners
- forestry and agriculture
- performance of solar cells
- heating of buildings
- material deterioration under sunlight
- thermal power generation

The solar radiation reaches the surface of the earth either as direct (beam) radiation or as diffuse radiation after scattering and reflection. While the beam radiation is of special interest to such applications as focusing devices, the diffuse radiation contributes a considerable percentage of the insolation and can not be neglected in most cases.

While it is satisfactory for many purposes to know the total amount of irradiation over the whole spectrum of wavelengths, other applications call for spectral values of
diffuse and direct irradiation - notably in the field of photovoltaics.

Often, the knowledge of irradiation under cloudless conditions is sufficient because peak loads occur under clear sky conditions. Therefore this study will be limited to irradiation under clear skies.

Five models of different complexity will be the basis of this study which has the goal to simplify the use and extend the applicability of various parameterization models. Every model will be described briefly, followed by suggestions for an improvement of the respective model. Fundamentals of the scattering and absorption of solar radiation through the earth's atmosphere are briefly discussed in the next chapter.
I.1 Outline of the Atmospheric Attenuation

The extinction of radiation in the atmosphere is not constant over the spectrum; this becomes evident through the fact that the diffuse radiation of the clean cloudless sky is blue and turns red for very high zenith angles, i.e. the sun very close to the horizon. Before an attempt can be made to model broadband attenuation, a short description of the spectral (as a function of wavelengths $\lambda$) attenuation phenomena seems necessary.

Solar radiation is not emitted homogeneously over the spectrum. It has a maximum at around 480 nm which is at the blue end of the visible spectrum. Fig. 1 shows the solar radiation arriving above the earth's atmosphere, as well as radiation after passing through the atmosphere. Also shown in this figure is a graph of the radiation which reaches the earth's surface after passing through a typical atmosphere. It becomes quite clear that in some bands of the spectrum the attenuation is very strong and in others it is rather weak.

The extraterrestrial radiation reaches the outer limits of the atmosphere (approx. 100 km above sea level) virtually without any attenuation, simply because the outer space provides an almost complete vacuum. On its way from the apparent solar surface to the earth's surface the quantum may be subject to either scattering or absorption (or both) when interference with particles occurs. The probability of interaction grows with the
increasing density of suspended matter. At a given particle density the probability of interaction increases with increasing pathlength.

The probability for non-interference is called transmittance and must be a fraction of unity. There is no interference in vacuum, therefore the transmittance becomes unity. If the medium is totally opaque and thus the mean free pathlength zero, the transmittance becomes zero as well. For most intermediate cases we expect an exponential decay of the amount of directly transmitted radiation at a particular wavelength.

The main sources of attenuation are gases, which absorb strongly in selected parts of the spectrum ("bands of absorption") and also scatter, and particles in the air, which mainly scatter, but have (weaker) absorptive properties as well. The term "absorption" describes the interaction where the quantum energy is converted to heat (or some other form of energy) and the quantum ceases to exist. "Scattering" means a change in direction of the quantum through an interaction with a corpuscle without any loss of the quantum energy.

Under the assumption that only one type of attenuator exists in a homogenous medium, the transmittance for any particular wavelength can be written as:

\[ \tau(1) = \exp\{-K(1)\} \]

\[ \lambda \]

The term \( \tau \) stands for transmittance and \( K \) for the optical depth. The irradiation can now be written as:
\[
I_{\text{rr}} = I \cdot \exp\{-K(1)\}
\]

another attenuators can be added to this atmosphere. The combination of the influences of two (or more) attenuators can be described by way of multiplication of the transmittances for the various attenuators:

\[
\tau(\text{tot}) = \frac{\tau(1) \cdot \tau(2) \cdot \tau(3) \cdot \ldots}{\lambda \lambda \lambda \ldots}
\]

or

\[
\tau(\text{tot}) = \exp\{-K(1) + K(2) + K(3) + \ldots\}
\]

In atmospheric applications those K's represent the optical depth of atmospheric constituents, such as ozone, dry air, water vapour, aerosols or mixed gases. This leads to the formulation of a transfer equation which describes the amount of incoming direct radiation:

\[
I_{\text{rr}} = I \cdot \exp\{-K(1) + K(2) + K(3) + \ldots\}
\]

Naturally one would like to define exactly the absorption and the scattering of solar radiation at every wavelength. Restrictions in the resolution of the measuring equipment and in the practicability of data handling make it necessary to devide the spectrum in a finite number of small spectral bands.

More than 98\% of the solar radiation is emitted between 290 nm and 4000 nm. For reasons to be explained later, most of the radiation outside the above limits does not reach the earth
and is therefore of no interest. This study uses the spectral division of Thekaekara [2] which divides the spectrum into 144 parts within the range of 290 nm to 4000 nm as outlined in Table III. The bandwidths are not as small as desirable - especially at the longer end close to 4000 nm - but a spectrum with narrower intervals would not allow the use of many sets of absorption coefficients available in the literature. Thekaekara's spectrum was adopted by NASA and many sets of absorption coefficients are based on it. Recently, a new spectrum, proposed by the World Radiation Center has been adopted by the World Meteorological Organization [34].

To understand the transfer equation, the mathematical handling of the spectral attenuation for the various atmospheric constituents has to be understood. In the absence of a complete and comprehensive set of measured data this study uses theoretical spectral transmittances for dry air scattering, aerosol attenuation, ozone absorption, mixed atmospheric gas absorption and water vapor absorption as criteria for any parameterization. These values were taken from Leckner [3].

It is important to determine the pathlength of radiation. The shortest pathlength possible for the solar radiation is with the sun in the zenith. Commonly the pathlength is expressed in non-dimensional form of the 'airmass'. At sea level, an airmass of unity is the shortest pathlength (zenith angle \( \theta = 0 \)); at zenith angles higher than zero the airmass increases. The simplest mathematical formulation to determine the airmass uses the following assumptions:

- there is no curvature of the earth
the index of refraction of the air is equal to unity.

Thus the airmass takes on the form:

$$AMS = \frac{1}{\cos(\theta)} = \sec(\theta)$$  \hspace{1cm} (I-6)

Other formulations that eliminate the restrictive assumptions are introduced in later sections whenever they are used.
upper curve: extraterrestrial spectrum
lower curve: spectrum at sea level for average atmospheric conditions

Figure 1, The Solar Spectrum
1.2 Transmittance Due To Dry Air Scattering (Rayleigh scattering)

Approximately 80% of the atmosphere consists of nitrogen and nitrogen affects radiation mainly by scattering. The theory of scattering by particles much smaller than the wavelength of the radiation - and molecules are just that - is well founded and goes back to the work of Lord Rayleigh. The next largest constituent of the atmosphere, the biatomic oxygen, is a molecular scatterer as well. It also absorbs radiation but the absorptive property is of second order between 290 and 4000 nm. Rayleigh found that the transmittance due to molecular scattering is wavelength dependent. Under the assumption that the scattering molecules are perfect spheres, the transmission through dry air with molecules much smaller than the wavelength of the radiation becomes an exact function of the fourth power of the wavelength and can be put into the following form:

$$r(R) = \exp\{K(R) \cdot \text{AMS}\}$$  \hspace{1cm} (I-7)

where the optical depth \( K(R) \) at unity AMS is given as (Penndorf [4]):

$$K(R) = \frac{2}{(n - 1)} \cdot N / (3 \cdot N \cdot \lambda^4)$$  \hspace{1cm} (I-8)

The term 'N' gives the molecule density with 'N subscript o' the density at sea level and 'n subscript s' the index of refraction.
Leckner [3] uses an exponent of \(-4.08\) to allow for the departure of molecules from the theory of perfect spheres (based on Rayleigh):

\[
\tau(R) = \exp\{k(R) \cdot (\lambda \cdot \text{AMS})\}^{\frac{-4.08}{\lambda}}
\]  

(I-9)

with: \( k(R) = -0.008735 \) \( \lambda \) in [\( \mu \text{m} \)]  

(I-10)

Thus the transmittance due to dry air scattering as used in this study is represented by Eq. (I-9) and Eq. (I-10). A plot of \( \tau(R,\lambda) \) against \( \lambda \) is shown in Fig. 2. As we can see from Fig. 2, the transmittance due to scattering by molecules increases with increasing \( \lambda \). Thus Rayleigh scattering has its strongest effect in the short end of the solar spectrum.
Figure 2, The Transmittance due to Rayleigh Scattering
I.3 Transmittance due to Ozone Absorption

Oxygen atoms not only come in pairs but also in triplets in form of ozone. Not only does ozone scatter (this is included in Eq. I-9), but it also absorbs radiation. Ozone absorbs radiation in the shorter wavelengths of the solar spectrum and it is mainly responsible for the absorption of the ultraviolet radiation, excessive amounts of such would harm the life on earth. The transmittance due to ozone can be described by the following equation:

\[ r(O) = \exp\left\{-\frac{k(O) \cdot UO \cdot \text{AMS}}{\lambda} \right\} \]  \hspace{1cm} (I-11)

The term 'UO' denotes the amount of ozone in a vertical column. Depending on season and latitude the value of 'UO' varies between UO=0.20 [cm] and UO=0.50 [cm] at NTP.

Furthermore, the vertical profile of the ozone concentration shows a distinct concentration at very high altitude with a peak at around 22 km above sea level for midlatitudes. The peak elevation decreases slightly towards the poles.

Ozone absorption coefficients k(O) used in this study were taken from Leckner [3] who in turn took the coefficients from Vigroux [5]. These coefficients are reproduced in Table III. The transmittance of ozone as a function of wavelength is shown in Fig. 3. As we see, the transmittance approaches zero for \( \lambda \)-values smaller than 300 nm. Below 290 nm no radiation is
transmitted due to the total absorption by ozone. This is the reason why no portion of radiation below 290 nm is taken into account as mentioned above in Section I.1.
Figure 3, Transmittance due to Ozone Absorption
I.4 Absorption by Atmospheric Mixed Gases

Among all the dry air gases (i.e. excluding water vapor), ozone has a distinct concentration profile in the vertical direction and it is due to this reason that the absorption by ozone was treated separately in Section I.3. All the remaining gases (such as oxygen and carbon dioxide, etc.) are more or less homogenously distributed in the atmosphere and their concentration does not vary greatly. In this study they are referred to as "mixed gases". Among these mixed gases, the main absorbers of solar radiation between 290 and 4000 nm are carbon dioxide and oxygen.

Fig. 4 shows the monochromatic transmittance due to absorption by atmospheric mixed gases. Most of the absorption occurs outside of the visible part of the spectrum with the exception of an oxygen band at 760 nm which is right at the threshold of human perception. The first expectation for an equation to formulate the transmittance due to mixed gases would probably be something like the following:

$$T(G) = \exp\{-K(G)\cdot\text{AMS}\}$$

But Fig. 4 shows that the attenuation by mixed gases occurs in narrow bands with steep flanks. The set of wavelength intervals is much too crude for the assumption that the attenuation and the irradiation within those intervals are constant. Absorption coefficients for mixed gases are therefore averaged over the intervals. There are many approaches to model the absorption by
mixed atmospheric gases (Fowle [6], Howard [7]). This study uses the approach of Leckner [3] who in part based his work on publications of Yamamoto [8], McClatchey [9] and Elterman [10].

The shape of absorption bands can be classified. Each class of absorption band theoretically calls for a different function to describe the extinction, as outlined by Goody [11] and Tiwari [12]. Within the spectral range of 290 to 4000 nm however, it is sufficiently accurate to use one function only; the transmittance due to the absorption by mixed atmospheric gases can then be described by the following equation:

$$
\tau(G) = \exp\left(-\frac{1.41 \cdot K(G) \cdot AMS}{\lambda} \left(\frac{1+118.3 \cdot K(G) \cdot AMS}{0.45}\right)\right)
$$  \hspace{1cm} (I-13)

The coefficients $K(G)$ used with this equation to produce Fig. 4 were taken from Leckner [3] and are reproduced in Table III.
Figure 4, Transmittance due to Absorption by Mixed Gases
I.5 Absorption by Water

The attenuation behaviour of water vapor is far more complex than the one of ozone and is similar to the one of the mixed atmospheric gases. Water absorbs in certain bands of the spectrum with a very steep rise of the absorption at the flanks of these bands which are often close to bands of almost total transparency. Similar to the mixed gases this study adopted the water absorption treatment of Leckner [3] who obtained the transmittance equation for water vapor as:

\[
\tau(W) = \exp\left(\frac{0.2385 \cdot \text{UW} \cdot k(W) \cdot \text{AMS}}{\lambda} \frac{0.45}{(1+20.07 \cdot \text{UW} \cdot k(W) \cdot \text{AMS})^{0.45}}\right) \tag{I-14}
\]

Eq. (I-14) is of the same form as the transmittance equation for mixed gases - Eq. (I-13) - with the extension of an element describing the varying amount of water. The set of water absorption coefficients used to produce Fig. 5 with Eq. (I-14) are reproduced in Table III; they were taken from Leckner [3].

The total amount of the precipitable water in a vertical column can be estimated from a number of observations such as dew point temperature or partial pressure of water vapor (see Leckner [3]).
Figure 5, Transmittance due to Water Absorption
I.6 Attenuation by Atmospheric Aerosol

The accurate mathematical description of the attenuation by atmospheric aerosols is extremely difficult. This is shown by the variety which exists in the most important parameters which influence the determination of aerosol attenuation:

- the particle size and its distribution
- the dielectric properties which determine the absorption
- the shape of a particle
- the wavelength
- the number of aerosol particles per unit volume
- aerosols both scatter and absorb.

Probably the most difficult task is the determination of the particle size distribution and its optical properties. Over the decades there have been numerous suggestions to determine the number and the size distribution of aerosols in a given volume of the atmosphere—none of them entirely satisfactorily. One difficulty is the cut-off point: The smaller the particle the more difficult it becomes to prove its existence.

The Rayleigh theory of scattering does not describe the scattering by aerosols because the aerosol particles (or most of them) are substantially larger than the wavelength of the solar radiation within the spectrum of interest. Gustav Mie [13] developed a theory which treated the scattering by particles larger than the wavelength of the incident radiation but this
theory is also restricted to spherical particles. Nevertheless it proved to be a valuable tool for the description of aerosol scattering, though a treatment within the range of engineering applications requires some simplifying assumptions.

Under the assumption that the particle diameter (or radius) distribution follows a power law and that absorption is negligible or is nonexistent, one can apply Mie's theory in the form of a rather simple relation of wavelength dependence. Negligible absorption means that the complex part of the index of refraction has to be small or zero which in turn means that the aerosols have to have dielectric properties. The fact that water condenses around particles increases the applicability of this assumption. A still widely used approach to implement these assumptions was first published by Angström [14,15]. His formulation for the spectral transmittance due to aerosol attenuation takes on the following form:

\[
\tau(\lambda) = \exp\left\{ \beta \cdot \frac{a}{\lambda} \right\}
\]

The constants \(a\) and \(\beta\) are obtained with filter measurements at two wavelengths. With only two equations to determine \(a\) and \(\beta\) the result is a set of two constants. At a given wavelength the value of \(a\) is a function of the particle size. It becomes quite clear, that for every wavelength and for every class of particle size a unique \(a\) is valid. An average constant \(a\) is thus an averaged value over 2 parameters. The particle size distribution of the atmospheric aerosols usually follows a power law quite closely. Exceptions from this rule have been observed for
maritime aerosols and for unusual accumulations of dust from a single source like volcanic eruptions or forest fires of large extension. The average $a$ over the whole spectrum between 290 and 4000 nm has a value between 0.9 and 2.0 with the most frequent observations around $a=1.3$. Fig. 6 shows the relationship of the turbidity parameters at particular wavelengths and for various radii of the aerosol particles (from McCartney [16]).

Angström called the constant $\beta$ 'Particle Number Density'. This might be misleading because $\beta$ is not only a function of the particle numbers per unit volume but even more a function of the aerosol mass density.

For applications of spectral irradiance values it might be of interest to increase the number of wavelengths at which filter measurements are taken to determine $\beta$ not only as a constant but as a function of the wavelength. This study's main concern lays with the parameterization models and the exact value of the spectral transmittance is not as important as the average transmittance value over the whole spectrum. To determine the latter it is sufficient to obtain $\beta$ as a constant and it seems that Angström's choice of wavelengths was a good one in respect to the accuracy of the broadband aerosol transmittance. Fig. 7 shows the transmittance due to aerosol attenuation as a function of wavelength for average $\beta=0.1$ and an $a$ of unity.
Figure 6, Optical Properties of Aerosols as a Function of Wavelength. (From McCartney [16])
Figure 7, Transmittance due to Aerosol Attenuation
I.6.1 The Aspect of Visibility

The determination of the transmittance due to aerosols from turbidity measurements is quite a formidable task. It would be desirable to link the aerosol transmittance to a more easily measureable quantity like the horizontal visibility. The first problem arises with the definition of visibility. Linke [17] defines the threshold of human perception as a contrast difference of 2%. If the apparent brightness of an ideal black body is more than 0.98 times the apparent brightness of a diffuse background, the object is considered invisible for the human eye, which has a maximum sensitivity at $\lambda=550$ nm. The linking of visibility to a fixed quotient of contrast has the definite advantage that the horizontal optical depth follows immediately. Neglecting any amount of molecular scattering:

$$k(A,0) = \ln(1-0.98)/\text{VIS} \quad [1/\text{km}]$$  \hspace{1cm} (I-16)

If Rayleigh scattering is taken into account, the optical depth for Rayleigh scattering (0.01162 1/km) [10] has to be subtracted and we obtain:

$$k(A,0) = (3.912/\text{VIS}) - 0.01162 \quad [1/\text{km}]$$  \hspace{1cm} (I-17)

A variety of authors have established relations between the vertical aerosol density distribution and the aerosol density
near the ground. A very convenient way to do so was published by McClatchey [9] who established the 'scale height' of an aerosol distribution. 'Scale height' is the equivalent pathlength at ground level aerosol density to obtain the same aerosol attenuation as for the vertical path through the atmosphere:

$$SH = \frac{K(\text{vertical path})}{k(\text{ground, horiz})}$$ (I-18)

It is quite clear that the scale height becomes higher with decreasing pollution because the aerosol pollution shows the highest density close to the ground. Under normal circumstances - no aerosol of one single source, i.e. volcanic eruption, forest fire - it was found that the scale height $SH$ increases more or less linearly with increasing visibility. Buckius and King [18], with data from McClatchey [10], found the scale height at a visibility of 23 km to be $SH(23)=1.577$ km and at a visibility of 5 km to be $SH(5)=1.132$ km. These results find support in data from Zuev [19]. With Eqs. (I-17) and (I-18), $\beta$ can now be written as [18]:

$$\beta = (0.55) \cdot \left( \frac{3.912}{V\text{I}S} - 0.01162 \right) \cdot \left( \frac{(1.577-1.132) \cdot (V\text{I}S - 5)}{23 - 5} + 1.132 \right)$$ (I-19)

Although a guess has to be made about the value of the particle size distribution exponent '$a$', the equation proves very valid under usual aerosol distributions where '$a'$ is commonly around 1.3. For values of '$a'$ different from 1.3 however, the use of Eq. (I-19) in conjunction with Eq. (I-15) yields diverging results for equal visibilities, which can be attributed to the fact that the maximum sensitivity of the eye (at 550 nm) does
not coincide with the maximum spectral irradiance (at 480 nm). This study therefore suggests a slight modification of Eq. (I-19) so that equal transmittances for equal visibilities can be obtained:

$$\beta = (3.912/VIS - 0.01162) \cdot ((16.2385 + VIS) \cdot (F - G \cdot a) + H) \quad (I-20)$$

where

$$F = 2.3575 \times 10^{-2} \quad (I-20a)$$

and

$$G = 9.387 \times 10^{-3} \quad (I-20b)$$

and

$$H = 0.278863 \quad (I-20c)$$

These Eqs. (I-19) and (I-20) do not cover visibilities in fog where the particles become very big.

The link between ground visibility and aerosol optical depth will be used in the parameterization models as outlined in the next chapter.
II Treatment of Direct Radiation

There are currently 3 levels of models to estimate direct (and diffuse) radiation on clear days. Models which use spectral transmittances and irradiances and perform numerical integrations represent the highest level. This study presents a rather simple model of this kind by Leckner [3] as an example for this approach.

The middle level is represented by less complex parameterization models which do not perform integrations over the spectrum but still split the determination of the extinction into various parts representing atmospheric constituents. This study presents three models of this kind from Davies and Hay [20], Hoyt [21] and from Bird and Hulstrom [22].

The lowest level of models simulates insolation through just one equation with airmass as a variable and constants valid for entire months not considering any changes of the atmosphere. The most widely known such model is the A.S.H.R.A.E. algorithm [31] which will be presented in Section II.5 of this study.

The basic problem of modeling direct insolation is the solution of the transfer equation (Eq. I-5) for a particular wavelength and its integration over the whole spectrum:

\[
\text{Irr(tot)} = \frac{4000}{290} \int_{0n\lambda}^{\lambda} I \cdot \tau(tot) \cdot d\lambda \quad (II-1)
\]

For a model of the top level the main problem is the definition of the spectral transmittances for the various attenuators over the whole spectrum. Sections I.1 through I.6 have shown one way
of solving this task. Other models - not presented here - divide the atmosphere into a number of homogeneous layers for which the attenuating properties are defined individually [24,25].

The transfer equation for a particular wavelength or with enough accuracy for a very narrow band is given below:

\[ \tau = \tau(R) \cdot \tau(A) \cdot \tau(W) \cdot \tau(G) \cdot \tau(O) \]  \hspace{1cm} (II-2)

However, it is mathematically not sound to formulate the broadband transmittance as follows:

\[ \tau = \tau(R) \cdot \tau(A) \cdot \tau(W) \cdot \tau(G) \cdot \tau(O) \]  \hspace{1cm} (II-3)

Despite this obvious mathematical inconsistency all models of the middle level presented in this study force the determination of the spectrally integrated irradiation values into the form of Eq.(II-3) or similar expressions. Eq.(II-3) is therefore called the 'broadband transfer equation', although there is an inherent error in this formulation.

Because the 'broadband transfer equation' is widely used despite the obvious mathematical deficiency, it seems appropriate to assess the error inherent in this formulation. The problem can be split into two separate questions:

1.) To what extent can a "broadband transmittance", averaged over the whole spectrum, be established for a single attenuator.
2.) To what extent can the "broadband transmittances" for attenuators be used multiplicatively in conjunction with each other and with the solar constant.

The averaged transmittance for a single attenuator - an assumed ideal condition where only one kind of attenuation occurs over the whole spectrum - is now defined by means of the following equation:

\[
\tau(1) = \frac{\int_{290}^{4000} I \cdot \tau(1) \cdot d\lambda}{\int_{290}^{4000} I \cdot d\lambda}
\]

where:

\[
\int_{290}^{4000} I \cdot d\lambda = SC
\]

It is possible to integrate this equation and define the respective \( \tau \) as a function of the airmass and the density of the attenuator. This function - which will most likely not be in a closed form - can then be parameterized with any desired accuracy as a function of the airmass and the density of the attenuating medium. All models of the middle level use this approach.

The second question is not so readily answered: For an easier understanding the example will be restricted to two attenuators. The effect of three or more attenuators can then be extrapolated. For one attenuator the transmittance is described by Eq. (II-4) above. For two attenuators the correct equation
becomes:

\[ \tau_{\text{tot}} = \frac{1}{290} \int_{0}^{4000} \frac{I \cdot \tau(1) \cdot \tau(2) \cdot d\lambda}{\ln \lambda} \]

\[ \text{SC} \]

(II-6)

It is now quite obvious, that the product of Eq.(II-4) for two different attenuators is not the same as Eq.(II-6) because the integral of a product of two functions is not the same as the product of two integrals of the same two functions, unless one (or both) of those functions degenerates to a constant.

Despite this mathematical verdict most parameterization models use this approach. The errors involved are rather small because over very large parts of the spectrum only one of the attenuators is dominant or complete extinction occurs through one attenuator, where another would be quite strong.

However, the user of parameterization models is cautioned not to use single elements of the models because the performance of a single transmittance might be less accurate than the product.

On the lowest level no attempt is made to solve the transfer equation; instead, a simple power law with the airmass as a variable is adopted. The next sections will outline these models in detail.
II.1 The Spectral Model by Leckner

This study presents the solar insolation model by Leckner [3] as an example for the models of the top level, where numerical integration methods are used to calculate the solar insolation incident on the earth's surface. The elements of Leckner's model are presented in parts in the sections I.2 through I.6.

The basic equation of Leckner's model of solar insolation has the following form:

\[
I_{rr} = 4000 \int_{n \to 290} \frac{I_{I} \cdot \tau(R) \cdot \tau(A) \cdot \tau(W) \cdot \tau(G) \cdot \tau(O) \cdot d\lambda}{n \cdot \lambda}\quad (II-7)
\]

The various spectral transmittances are calculated with the respective expressions discussed earlier in chapter I, sections 2 to 6. The performance of Leckner's model for various amounts of water vapor, ozone and turbidity is shown in Fig's. 8 to 11 (at the end of Chapter II). Also shown are other models as a comparison. The vertical axis shows the irradiance in watts per square meter. Mean sun to earth distance was assumed. Turbidity values were obtained with Eq. (I-19). The horizontal axis shows the zenith angle; unless otherwise noted for a particular model, Eq. (I-6) was used to obtain the airmass. Leckner's model tends to predict slightly higher values than the other models except for conditions of high turbidity.

While the model of Leckner exceeds the possibilities of a small calculator, it is nevertheless easy to use on a computer.
Because of the good performance (compared to more elaborate models) combined with a tolerable amount of required computer time the Leckner model has been chosen as a standard for the development of components for the other models presented in this study.

If further studies should show some significant systematic deviations of Leckner's model from reality, corrections on the elements of the other models should be easy to perform. The next sections present these models and some changes suggested to improve their applicability and/or practicability.
II.2 The Model by Davies and Hay

Davies and Hay [20] present a model to calculate solar irradiation on horizontal surfaces. The basis for this model (from now on referred to as Model A) is the "transfer equation for broadband transmittances" as outlined at the beginning of Chapter II (Eq. II-3) with a slight modification. Because water vapor and mixed gases absorb in parts of the spectrum where no ozone absorption occurs, the transmittances for water vapor (and gases) and ozone are not multiplied and the following approach is used instead:

\[ \text{Irr} = SC \cdot \cos(\theta) \cdot [\tau(O) \cdot \tau(R) - a(W)] \cdot \tau(A) \quad (II-8) \]

The absorptance due to ozone and the resulting transmittance are given as follows (Lacis and Hansen [26]). For the ozone absorption band in the ultra violet:

\[ a(O, Uv) = \frac{0.02118 \cdot UO}{1 + 0.042 \cdot UO + 3.23 \cdot 10^{-4} \cdot (UO)^2} \quad (II-9a) \]

and for the band in the visible part of the spectrum:

\[ a(O, Vi) = \frac{1.082 \cdot UO}{(1+138.6 \cdot UO)} + \frac{0.0658 \cdot UO}{1+(103.6 \cdot UO)^3} \quad (II-9b) \]

and the absorption due to ozone as the sum of the above:

\[ a(O) = a(O, Uv) + a(O, Vi) \quad (II-9c) \]

and finally for the transmittance due to ozone:
\( \tau(0) = 1 - \alpha(0) \) \hspace{1cm} (II-9)

The combined absorptance by water vapor and mixed atmospheric gases in Model A was taken from Lacis and Hansen [26] as

\[
\alpha(W) = \frac{2.9 \cdot UW}{(1+141.5\cdot UW) + 5.925\cdot UW} \hspace{1cm} (II-10)
\]

This function approaches asymptotically the value 0.49 for large airmasses. The transmittance \( \tau = (1-\alpha) \) will therefore never be less than 0.5.

The transmittance due to scattering by dry air (Rayleigh scattering) was presented in tabulated form. The authors also published a polynomial expression to fit the tabulated transmittance data:

\[
\tau(R) = 0.972 - 0.08262AMS + 0.00933AMS - 0.00095AMS + 0.0000437AMS \hspace{1cm} (II-11)
\]

The above expression fits the table well within reasonable limits of the airmass.

Davies and Hay circumvented the difficult task of establishing a parameterization for the aerosol transmittance. Instead, following a suggestion by Houghton [27], they employed the simple relationship:

\[
\tau(A) = K \hspace{1cm} (II-12)
\]

No procedure was given to determine \( K \) and the user was left with the suggestion to use \( K = 0.95 \) "as a global average value". This value would be matched by "clean Atlantic air". For anticyclonic days a value of \( K = 0.88 \) was suggested; the authors
further recommend "local calibration against measurement".

It is obvious that the weak point of the model of Davies and Hay for any application is the uncertainty involved in the determination of the aerosol transmittance. This study therefore attempts to improve Davies' and Hay's model by replacing their transmittance equation for aerosols.

Integrated values were obtained from the model of Leckner [3] in form of the following equation:

\[
\tau(A) = \frac{\int_{290}^{4000} I \cdot \exp\left(-\beta \cdot \lambda \right) \cdot d\lambda}{\int_{290}^{4000} I \cdot d\lambda} \quad (\text{II-13})
\]

As was shown in chapter II, Eqs. II-4 to II-6, the broadband transmittance for an attenuator with inhomogenous attenuation characteristics over the spectrum can never be represented by a power function.

As a simplifying step, the fiction of a power law was upheld to obtain a formulation which is easy to handle, although this approach necessarily means a loss of accuracy at higher airmasses. Because of the significance of peak radiation values, the constants of the following Eq.(II-14) were chosen to fit Eq.(II-13) at low airmasses (AMS = 1 and AMS = 2):

\[
\tau(A) = C + D \cdot \exp(-E \cdot \beta \cdot \text{AMS}) \quad (\text{II-14})
\]

where:

\[
C = (0.12445 \cdot a - 0.0162) \quad (\text{II-14a})
\]
\[
D = (1.003 - 0.125 \cdot a) \quad (\text{II-14b})
\]

and

\[
E = (1.089 \cdot a + 0.5123) \quad (\text{II-14c})
\]

The Eq. (II-14) makes the assumption that the user has access to measured values of \( \beta \). The determination of \( \beta \) - as outlined in section I.6.1 - makes filter measurement a necessity. To obtain reasonably accurate values of \( \beta \) without filter measurements, it is suggested that Eq. (I-20) be used in conjunction with Eq. (II-14). The performance of Model A is shown in Figs. 8 to 11 against other parameterizations. The results of this model correspond well with other models except for high turbidity.
The model by Hoyt (from now on called Model B) uses a derivative of the broadband transfer equation. Hoyt obtains the direct part of the solar radiation as:

$$\text{Irr} = S \cdot C \cdot \cos(\theta) \cdot \left(1 - \sum_{i=1}^{5} a_i \right) \cdot \tau(AS) \cdot \tau(R)$$  \hspace{1cm} (II-15)

He defines five different absorptances $a$ for water, ozone, carbon dioxide, oxygen and aerosol and two transmittances due to scattering by particles (aerosol) and by dry air (Rayleigh). This Eq. (II-15) is basically the 'broadband transfer equation' as outlined in chapter II "Treatment of Direct Radiation". This equation has been modified by Hoyt by substituting the transmittances $\tau$ by $\tau = (1-a)$ and by then neglecting all terms of second or higher order:

Based on:

$$\tau = \tau_1 \cdot \tau_2 \cdot \tau_3$$

he substitutes

$$\tau = (1-a_1) \cdot (1-a_2) \cdot (1-a_3)$$

which is

$$\tau = 1 - a - a \cdot a + a \cdot a + a \cdot a - a \cdot a \cdot a$$

and is simplified to

$$\tau = 1 - a - a - a$$  \hspace{1cm} (II-16)
The influence of this substitution and simplification is small for small absorptances but increases with higher airmasses where absorption may become quite large. Hoyt's equation easily delivers results 5% smaller than if he had used the original "broadband transfer equation" with the respective absorption values. Hoyt's absorptance values are therefore adjusted to his equation and cannot be used properly with other models at high airmasses. The absorptances for Model B are

for water:
\[ a(W) = 0.110 \cdot (6.31 \cdot 10^{-4} + U_W) - 0.0121 \]  \hspace{1cm} (II-17)

for ozone:
\[ a(O) = 0.045 \cdot (8.34 \cdot 10^{-4} + U_O) - 3.1 \cdot 10^{-3} \]  \hspace{1cm} (II-18)

for carbon dioxide:
\[ a(C) = 0.00235 \cdot (0.0129 + U_C) - 7.5 \cdot 10^{-4} \]  \hspace{1cm} (II-19)

for oxygen:
\[ a(OX) = 7.5 \cdot 10^{-3} \cdot AMS \]  \hspace{1cm} (II-20)

for aerosol:
\[ a(AA) = (1 - wo) \cdot \{g(\beta)\} \]  \hspace{1cm} (II-21)

Unfortunately, Hoyt did not give closed form formulas for the calculation of the two scattering components and the absorption by aerosol. Instead, he used the following relationship to obtain the transmittances due to scattering:
transmittance due to Rayleigh scattering:

$$\tau(R) = \{f(\text{AMS})\} \quad (\text{II-22})$$

transmittance due to aerosol scattering:

$$\tau(\text{AS}) = \{g(\beta)\} \quad (\text{II-23})$$

The two functions $f(\text{AMS})$ and $g(\beta)$ were given in tabulated form only without a parameterization function (see below). While the

<table>
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<th>$f(\text{AMS})$</th>
<th>$\beta$</th>
<th>$g(\beta)$</th>
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</table>

Table I

Table II

table for $g(\beta)$ extends to fairly large turbidity values, the usefulness of the table for $f(\text{AMS})$ is restricted because it extends only to airmass 4 which is approximately equivalent to a zenith angle $\theta = 75$ degrees. Even under equatorial conditions the table restricts the use of the model to times of more than one hour before sunset or after sunrise. Although the curve for both functions $f(\text{AMS})$ and $g(\beta)$ are smooth enough to allow considerable extrapolation, the applicability of the model is
hampered.

This study tries to open this otherwise well performing model (graphical results in comparison with other models are shown in Fig. 8 to 11) to wider use by replacing the tables for $f(AMS)$ and $g(\beta)$ through two parameterization formulas which extend the range of application beyond the present limits of $AMS=4$ and $\beta=0.32$. However, as Fig. 11 shows, at very high turbidity the results of this model are well below the results of other models.

This study presents a parameterization not of $f(AMS)$ but of $r(ray)=f(AMS)**AMS$:

$$r(R) = (0.375566) \cdot \exp(-0.221185 \cdot AMS) + 0.615958 \quad (\text{II-24})$$

The formulation for $r(R)$ (Eq. II-24) never deviates more than 0.2% from the values given by Hoyt. To estimate the range of reasonable accuracy for extrapolated values beyond $AMS=4$, the UBC Computer Center Software "*OLSF" was used to compare with Eq. (II-24). The range of confidence (error less than 2%) extends as far as $AMS=8$. This reduces the limited time range around sunrise/sunset to less than half of its previous value. The table for $g(\beta)$ will be replaced by a parameterization of similar construction as follows:

$$g(\beta) = 1.909267 \cdot \exp(-0.6670236 \cdot \beta) - 0.914 \quad (\text{II-25})$$

This equation never yields a deviation from Hoyt's tables
of more than 0.32%. As for the transmittance function an attempt of extrapolation has been made with a powerful software (UBC *OLSF). Against this standard, confidence in Eq. (II-25) can be maintained up to turbidity values of $\beta=0.5$.

These modifications of Model B open it to applications of atmospheres of high turbidity at small solar altitudes.
II.4 The Model by Bird and Hulstrom

Bird and Hulstrom published a model [22] to calculate direct insolation and later extended their model to diffuse radiation and incorporated some minor changes of the direct part of the model to improve the performance [23]. This study presents the model of Bird and Hulstrom as published in [23] (from now on called "Model C") which is its most recent form.

Model C uses the "broadband transfer equation" almost in its original form; only a constant factor is used:

\[
I_{rr} = S \cdot \cos(\theta) \cdot 0.9662 \cdot \tau(R) \cdot \tau(A) \cdot \tau(W) \cdot \tau(G) \cdot \tau(O) \quad (II-26)
\]

The use of the factor '0.9662' stems from the fact that Bird and Hulstrom used a spectrum for their parameterization that only considered the range from 300 nm to 3000 nm. Because there is some transmission past the above mentioned spectral boundaries the said factor was incorporated. The transmission equations are parameterized as follows:

\[
\tau(R) = \exp\left\{-0.0903 \cdot (AMS') \cdot (1 + AMS' - (AMS')^{1.01})\right\} \quad (II-27)
\]

\[
\tau(O) = 1 - 0.1611 \cdot XO \cdot (1 + 139.48 \cdot XO) - 0.3035 \cdot (0.002715 \cdot XO)/(1 + 0.044 \cdot XO + 0.0003 \cdot XO) \quad (II-28)
\]
Bird and Hulstrom used Kasten's [28] formulation for the airmass as given below in Eq. (II-33):

\[ AMS = \frac{1}{\cos\theta + 0.15 \cdot (93.885 - \theta)} \]  

For Rayleigh scattering and the mixed gas absorption a pressure corrected airmass was used:

\[ AMS' = AMS \cdot \frac{P}{101.3} \]  

Eqs. (II-33) and (II-33a) are also valid in other parameterization models. As seen in Eqs. (II-31) and (II-32) above, Bird and Hulstrom use atmospheric turbidity values at two wavelengths (380 nm and 500 nm); turbidity values at these wavelength are measured by the U.S. National Weather Service
(Flowers et al. [29]) for some locations on a routine basis. The use of two weighted spectral turbidities gives a much more accurate picture of the atmospheric turbidity and its influence on broadband solar insolation. But for most locations this procedure calls for either a substantial amount of filter measurement at two wavelength of the visible spectrum or guesswork.

This study attempts to make the Model C more accessible and therefore suggests to replace the parameterization for the aerosol transmittance. In all cases where detailed turbidity data from filter measurements is not available, the following equation allows the calculation of the aerosol transmittance:

\[
T(A) = 10.97 - 1.265 \cdot \text{VIS}^{-0.66} \left( \frac{\text{AMS}}{0.9} \right)
\]  

The Eq. (II-34) was derived by using the link between horizontal visibility and the turbidity as outlined in Section I.6.1. It may be used to substitute for other aerosol transmittances of models of the same level (Model A and Model B) but not for the model described in the following Section II.5.
II.5 The A.S.H.R.A.E Model

This model on the lowest level uses a very simple algorithm [31]. The extinction over the whole spectrum is assumed to follow a power law function with the airmass. All attenuative influences at an airmass of one as well as the solar constant and its variation over the year are compounded into two sets of 12 constants, one set for every month.

Only the variation of the water vapour content over the year was taken as variable to determine these constants, which are reproduced in App.IV. The irradiance equation has the following form:

\[
I_{rr} = A \cdot \cos(\theta) \cdot \exp\{-B \cdot AMS\} \tag{II-35}
\]

On a log-normal plot this function transforms to a straight line with an intercept of 'A' for AMS = 1/cos(\(\theta\)) = 1.

The major advantage of the A.S.H.R.A.E. algorithm is its easy handling and the major drawback is its inelasticity to any change of concentration of atmospheric constituents. Because the influence of the varying concentration of ozone is small and the concentration of gaseous absorbers does not change very much (except water vapour), the main targets to increase the elasticity of the model are the absorption by water vapor and the scattering by aerosols.

It is quite interesting to assess the influence of water vapor concentration on the broadband transmittance of solar radiation. This has been done by using Leckner's [3] formulation
for transmittance through water vapor in the following form:

\[
\tau(W) = \frac{4000}{290} \int_{0}^{\lambda} I \cdot \exp\left(\frac{-0.3 \cdot UW \cdot K(W) \cdot AMS}{\lambda}\right) \cdot (1 + 20.07 \cdot UW \cdot K(W) \cdot AMS)^{-0.45} \cdot d\lambda
\]

The calculations were performed for a multitude of values of precipitable water and the results are shown in Fig. 12 at the end of Chapter II.

From Fig. 12, it is quite obvious that the first few millimeters of precipitable water have a very strong influence on the transmittance while a difference of one millimeter at higher values of water concentration does not account for very much change in transmittance.

The monthly values of 'A' were determined by using average water vapor values for the United States [33]. The A.S.H.R.A.E. Model also assumed that 200 dry dust particles per cubic centimeter were present with no variation over the year. The algorithm to establish the transmitted radiation was taken from Moon [30]. Because of his use of 'particles per volume' instead of 'aerosol mass concentration' the changing water content in the atmosphere also had an influence on the aerosol transmittance calculation due to condensation around the nuclei.

The other parameter with a great spectrum of variation is the aerosol content. Similar to Eq. (II-36) above, the influence
of turbidity on the broadband transmittance was determined by using Leckner's formulation of aerosol optical depth in the following form to obtain an (ideal) transmittance through aerosols only (same as (11-13):

\[
\tau(\lambda) = \frac{4000}{290} \frac{\int_{0}^{\lambda} I_0 \cdot \exp\{-\beta \cdot \lambda^{-\alpha}\} \cdot d\lambda}{\int_{0}^{\lambda} I_0 \cdot d\lambda}
\]

The amount of turbidity was varied by varying \( \beta \) over a wide range. The results are shown in Fig.13. From this figure it becomes obvious that the transmittance as a function of turbidity is much more linear than it is the case for water transmittance. While we can observe a weakened increase of water vapor absorption at higher water vapor concentrations, it is quite obvious that the variation of the turbidity has an effect on the transmittance, regardless how much aerosol there is in the atmosphere.

Unfortunately, the values for dust and water concentration, which were used to establish the A.S.H.R.A.E algorithm, represent extremely clear situations which are rarely observed. While this might seem to be of advantage for the calculation of peak insolation values, over-estimation for monthly averages result from the use of it.

This fact calls for a change in the A.S.H.R.A.E. algorithm that would make the varying turbidity its main parameter of influence. This study suggests the use of the ground visibility
as the variable in a modified A.S.H.R.A.E. model. If a further increase of accuracy is desired, an additional parameter accounting for the variation of the humidity can be included. At any rate, as seen from Fig. 13, an error of less than 4% results from a permanent water content of 1.5 cm.

With a best fit method versus other models described in this study, the incident radiation can be put in this form:

\[
I_{rr} = SC \cdot r(A) \cdot M(W) \cdot (0.775)^{0.5}
\]

(II-38)

with:

\[
r(A) = (1 - 1.3 \cdot \text{VIS})^{0.85}
\]

(II-39)

and with:

\[
M(W) = (1.0223 - 0.0149 \cdot U_W)
\]

(II-40)

In most cases, \( M(W) \) can be set to unity which is its exact value for \( U_W = 1.5 \) cm. The Eq. (II-39) above for the transmittance due to aerosol is very similar to the one presented in the Bird and Hulstrem model. It has, however, been tailored to fit into the context of Eq. (II-38) together with Eq. (II-40) and should therefore not be used with other parameterization models. If Eq. (II-39) is replaced by Eq. (II-34), the term \( 0.775 \) in (II-38) has to be replaced by \( 0.745 \). This may create errors of up to 2%.

The Eq. (II-38) also does not allow a variation of the ozone absorption or the rayleigh scattering. An amount of 0.3 cm of concentrated ozone in the air was assumed and that the
simulation takes place at sea level. Any adjustment for elevations different from sea level has to be made via the AMS relation.

In the event of precipitable water differing strongly from UW=1.5 cm, it is suggested that corrections are made either by referring to Figure 12 or by using Eq. (II-40) as a multiplier for equation (II-38).

The suggested changes to the A.S.H.R.A.E algorithm, which extend the range of application to turbid air, are definitely worth the slightly increased amount of calculations to use the model. Performance of Eq. (II-38) is shown in Fig. 8 to 11; the performance of Eq. (II-38) versus the existing A.S.H.R.A.E. model is shown in Fig. 14. These graphs show that the suggested changes make the A.S.H.R.A.E. model comparable with other parameterization values. Figure 14 shows that the values of the old A.S.H.R.A.E. model are linked to extremely low turbidities.
visiblity 105 km
ozone content 0.31 cm
water content 2.93 cm

Figure 8, Performance of five models.
Figure 9, Performance of five models.

visibility 23 km
ozone content 0.34 cm
water content 1.42 cm
Figure 10, Performance of five models.
Figure 11, Performance of five models.
Figure 12, Transmittance due to water vapor only.
Figure 13, Transmittance due to aerosol attenuation.
Figure 14, Performance of the old and new ASHRAE models.
III The Treatment of Diffuse Radiation

All the presented models contain ways to calculate the amount of diffuse radiation. Diffuse radiation is the radiation that reaches the earth's surface not directly but after (possibly: multiple) scattering and/or reflection. Often a distinction is made between scattered radiation from the sky ("sky radiation") and "multiply reflected radiation" - reflected from the earth back to the sky and then scattered backward to the earth. There is no way to distinguish these two components of the diffuse radiation by way of measurement. This distinction is purely for the convenience of the mathematical treatment. Generally the determination of the diffuse radiation under cloudless sky conditions is more difficult than the treatment of the direct radiation because:

- the complexity of the scattering phase function renders the closed formula mathematical treatment of single scattering extremely difficult and that of multiple scattering becomes practically impossible;
- the inhomogenous nature of the aerosol distribution makes the determination of the scattering versus absorption ratio very difficult;
- the reflectivity of the earth's surface (called "albedo") is variable and its determination difficult.

Simplifying models are generally necessary to calculate the diffuse radiation. In the next sections the extent of these simplifications will be shown and what results can be expected
in return.
III.1 General Approach for Single Scattering

Most models make use of the simplifying assumption that all scattering occurs once only. Secondary or multiple scattering is neglected. This simplification makes the calculation of Rayleigh scattering — scattering by particles much smaller than the wavelength of the radiation — very easy: The phase function for Rayleigh scattering is symmetrical, provided the fiction of perfectly spherical particles is upheld. This means that 50% of the scattered radiation is scattered forward and 50% backward.

The treatment of Mie (aerosol) scattering is not as easy as the Rayleigh scattering because the phase function is not symmetrical but shows a strong bias in the forward direction.

To determine the amount of diffuse radiation, further assumptions are necessary, i.e. regarding the sequence of the scattering. They will be introduced with the respective models.
III.2 Multiple Scattering

None of the models presented in this study approaches the task to calculate the diffuse radiation under consideration of multiple scattering or absorption of scattered radiation. Some models like the one of Lacis and Hansen [26] or the Lowtran Model [25] - not discussed in this study because of the excessive amount of computation time - use algorithms to calculate the diffuse irradiation on the basis of possible multiple scattering. These models divide the inhomogenous atmosphere into many layers (as many as 50) which are homogenous in themselves. These models obtain a high degree of accuracy but demand an excessive computational effort.

One cannot expect a large gain in accuracy for low airmasses and low concentrations of scattering matter in the atmosphere because the amount of multiply scattered radiation - a term of second order - becomes very small if there is not very much scattered radiation. But it proves to be worthwhile to include the effects of multiple scattering for very turbid air at low solar altitude. Under these conditions the fraction of diffuse irradiation which is multiply scattered increases drastically.

To reduce the errors at large zenith angles and to put the calculation of diffuse radiation more in tune with the physical reality, this study presents a comparatively easy procedure to include the effects of double and triple scattering into the
spectral Model of Leckner (and into the parameterization models if so desired). The following assumptions form the basis of this work:

- The single scattered radiation has a pathlength that is 50% longer than the direct radiation.
- The twice scattered radiation follows a path that is three times the pathlength of the direct radiation.
- Radiation which is scattered more than twice follows a path 5 times as long as the path of the direct radiation.
- The scattering model of homogenous layers will be replaced by a model in which scattering probabilities are defined according to the function of transmittance.

The above assumptions are the result of a semi-empirical approach; iterations were performed to obtain a best fit against other models which incorporate multiple scattering.

At a particular wavelength, the direct radiation is defined by the transfer equation (1-5). The difference between the extraterrestrial irradiation and the beam radiation is the amount that is depleted in the atmosphere; it includes the absorbed radiation and the scattered radiation, be it scattered to the earth or back to the space. It also includes radiation which was scattered and absorbed afterwards. Because the assumption of a layered atmosphere was dropped, the question arises how to divide the depleted radiation into the scattered part and the absorbed part. This is done by establishing the probability of attenuation for a single quantum. The probability for a quantum to be absorbed or scattered (possibly more than
The probability for a quantum to undergo Rayleigh scattering is defined by the quotient of the optical depth for Rayleigh scattering over the sum of the optical depths for all attenuating processes:

\[
PR(R) = \frac{PR \cdot \log(\tau(R))}{\log(\tau(tot))} \tag{III-2}
\]

where:

\[
\tau(tot) = \tau(R) \cdot \tau(A) \cdot \tau(W) \cdot \tau(G) \cdot \tau(O) \tag{III-3}
\]

The probability for a single quantum to undergo only one occurrence of Rayleigh scattering (and not more) becomes:

\[
PR(R,1) = PR(R) \cdot \tau(tot) \tag{III-4}
\]

Accordingly, the probability for a particle to undergo Rayleigh and then one or more further attenuating processes becomes:

\[
PR(R,2+) = PR(R) \cdot (1 - \tau(tot)) \tag{III-5}
\]

which is:

\[
PR(R,2+) = PR \cdot \frac{\log(\tau(R))}{\log(\tau(tot))} \tag{III-6}
\]

The Figure 15 illustrates the process of multiple scattering in a nonlayered atmosphere.
Explanation:
PR(Abs) = $x = \log(Tm)/\log(TaTrTm)$  \hspace{1cm} Ta=$\tau$(AS)
PR(R) = $y = \log(Tr)/\log(TaTrTm)$  \hspace{1cm} Tr=$\tau$(R)
PR(A) = $z = \log(Ta)/\log(TaTrTm)$  \hspace{1cm} Tm=$\tau$(AA)$\cdot$$\tau$(W)$\cdot$$\tau$(G)$\cdot$$\tau$(O)

For brevity, no multiplicator was used on this page.

Figure 15, Multiple Attenuation Pattern
Because 50% of the Rayleigh scattered radiation is scattered forward but an amount larger than 50% of the Mie scattered radiation is scattered forward, further assumptions have to be made to simplify the incorporation of multiple scattering:

- The radiation which is scattered into the halfsphere facing the earth with a plane parallel to the earth's surface is considered 'forward scattered'.

- The curvature of earth and atmosphere is neglected.

In the following Fig. 16, the approach to include multiple scattering into the spectral model will be graphically outlined. The forward scattering ratio 'FR' of the scattered radiation can be defined as:

\[
FR = \frac{0.5 \cdot \text{Idif}(R) + BA \cdot \text{Idif}(A)}{\text{Idif}(R) + \text{Idif}(A)}
\]  

(III-7)

The term 'Idif' denotes the diffuse radiation from the respective scatterer and BA is the aerosol forward scattering ratio, i.e. the fraction of Idif(A) which is scattered forward [20].
Figure 16, Multiple Scattering Pattern
The total 'sky diffuse radiation' then becomes the sum of the single, the double and the triple scattered radiation:

$$Idif(S) = SS + DS + TS$$  \hspace{1cm} (III-8)

with:

$$SS = I \cdot \{(1 - \tau(Sc)') \cdot \tau(Abs)' \cdot FR \cdot \tau(Sc)''\} \cdot 0h\lambda$$

$$DS = I \cdot \{(1 - \tau(Sc)') \cdot \tau(Abs)' \cdot (1 - \tau(Sc)'') \cdot \tau(Sc)'''\} \cdot 0h\lambda \cdot \{FR \cdot FR + (1 - FR) \cdot (1 - FR)\}$$

$$TS = I \cdot \{(1 - \tau(Sc)') \cdot \tau(Abs)''' \cdot (1 - \tau(Sc)'''') \cdot (1 - \tau(Sc)''''\} \cdot 0h\lambda \cdot \{FR \cdot FR \cdot FR + (1 - FR) \cdot (1 - FR) \cdot FR \cdot 3\}$$

where the number of primes corresponds to the class of scattering. As outlined previously in this Section, longer pathlengths of the scattered radiation require higher airmasses.

Finally, there is a small amount of radiation, which is reflected from the earth back to the sky and scattered back to the earth. This radiation is:

$$I(MR) = (Irr + SS + DS + TS) \cdot rg \cdot ra$$ \hspace{1cm} (III-9)

The term 'rg' stands for the reflectivity of the ground (albedo) and ra is the atmospheric albedo. The atmospheric albedo can be defined as:
ra = \frac{(SS^* + DS^* + TS^*)}{Irr} \quad (\text{III-10})

The total diffuse radiation can now be defined as:

I(tot) = I_{\text{Rr}} + I_{\text{dif}(S)} + I_{\text{dif}(MR)} \quad (\text{III-11})

This semi-empirical approach to model multiple scattering was used with the spectral transmittance functions of Leckner. The results are presented in Figures 18 to 21 as "Leckner, modified". The results are very encouraging and bring the spectrally integrated values close to the parameterization models.
III.3  Treatment of Diffuse Radiation with the Models

III.3.1  The Model of Leckner

The part of Leckner's Model to calculate diffuse radiation uses the following basic assumptions:

1.) The phase function for Mie scattering is assumed to be symmetric. Therefore it can be treated like Rayleigh scattering.

2.) The absorption by aerosol will be neglected and thus it is assumed that the aerosol attenuation is by means of scattering only.

3.) The reflectivity of the earth is zero. No multiply reflected radiation has to be considered.

Leckner's model defines the scattered radiation as the difference between the direct radiation and a fictitious beam that has only been subjected to absorption under exclusion of any aerosol absorption. Therefore the formulation of the diffuse radiation takes on the following form:

\[ \text{Idif} = 0.5 \cdot \cos(\theta) \cdot \frac{4000}{290} \int_0^n \int_0^\infty (1 - \tau(G) \cdot \tau(W) \cdot d\lambda) - \text{Irr} \]  

where the irradiance for normal incidence is taken from Eq. (II-7). The value for FR (forward scattering ratio) is assumed to be 0.5 because the phase functions for scattering are taken as symmetric.
Eq. (III-12) delivers fairly good results for low turbidity values and low ground reflectivity: Most assumptions make sacrifices on the accuracy of the aerosol scattered radiation; neither aerosol absorption nor phase function have significant influence under conditions of low turbidity. The performance of Leckner's approach is shown in comparison with other models in Figs. 18 to 21: Leckner's diffuse values are very low. This has to be expected because Leckner's approach does not account for secondary diffuse radiation.

For the same assumptions as above and with the added possibility of multiple scattering, Berlage [32] published a semi-exact mathematical treatment as early as 1928. Probably because of the language (German), this work seems to have gone unnoticed in the Anglophone world and it is well worth presenting his results. Berlage shows, that the spectral diffuse radiation becomes:

\[
Idif = 0.5 \cdot \frac{\int_{0n\lambda}^{4000} I \cdot \cos \theta \cdot \tau(AB) \cdot \left\{ 1 - \tau(R) \right\} \cdot d\lambda}{1 - 1.4 \cdot T \cdot \ln(\tau(R))} 
\]

(III-13)

where the 'T' is Linke's Trübungstindex (turbidity index) which is defined as:

\[
\tau(R) \cdot \tau(A) = \tau(R)
\]

(III-14)

Therefore (III-13) can be written as:

\[
Idif = 0.5 \cdot \frac{\int_{0n\lambda}^{4000} I \cdot \cos \theta \cdot \tau(AB) \cdot \left\{ 1 - \tau(R) \cdot \tau(A) \right\} \cdot d\lambda}{1 - 1.4 \cdot \ln(\tau(R) \cdot \tau(A))}
\]

(III-15)
As shown in section III.2, it is possible to change Leckner's diffuse model by implementing the multiple scattering algorithm with Leckner's transmittance functions.
III.3.2 The Model by Davies and Hay

The basic idea of the model by Davies and Hay is a division of the diffuse radiation into three parts:

1.) The diffuse radiation caused by Rayleigh scattering.
2.) The diffuse radiation caused by aerosol scattering.
3.) The diffuse radiation caused by reflection of all radiation from the earth's surface back to the sky and the backscattering from the sky to the earth.

An assumption must be made regarding the sequence of the attenuation processes. Davies and Hay use a layered model - the exact construction of which is shown in Fig. 17: no attenuation occurs concurrently. With this simplification the equations for the primary diffuse components become:

\[
I_{\text{dif}}(R) = S \cdot C \cdot \cos(\theta) \cdot [\tau(O) \cdot (1-\tau(R)) \cdot \tau(A) \cdot 0.5] \quad (III-17)
\]

and

\[
I_{\text{dif}}(A) = S \cdot C \cdot \cos(\theta) \cdot [(\tau(O) \cdot \tau(R) - a(W)) \cdot (1-\tau(A)) \cdot WO \cdot BA] \quad (III-18)
\]

The term 'WO' denotes the ratio between scattering and total attenuation due to aerosol (taken as 0.95) and 'BA' the forward scattering ratio. Note that in this model by Davies and Hay the diffuse radiation caused by Rayleigh scattering is not subject to attenuation by water vapor and the gases!
For the multiply reflected radiation, Davies and Hay are using the following equation:

\[ I_{\text{dif(MR)}} = (I_{\text{rr}} + I_{\text{dif(R)}} + I_{\text{dif(A)}} \cdot rg \cdot ra / (1 - rg \cdot ra) \]  

The values for the ground reflectivity 'rg' are usually around 0.2 but can go as high as 0.9 for fresh snow.

The atmospheric albedo employed by Davies and Hay was taken from Lacis and Hansen [26] and is given as:

\[ ra = 0.0685 + 0.17(1 - r(A)') - WO \]

The value of \( r(A)' \) is calculated as \( r(A) \) for a zenith angle \( \theta = 57 \) degrees.

The total radiation can now be determined by adding up the three parts of the diffuse radiation and the direct radiation and it can be brought into the following form:

\[ I_{\text{tot}} = I_{\text{rr}} + I_{\text{dif(R)}} + I_{\text{dif(A)}} + I_{\text{dif(MR)}} \]

The inclusion of the multiple scattering algorithm as outlined in Section III.2 is possible. With the absence of measured data it can not be considered beneficial because of the substantially increased complexity without a sure measure of any improvement. An increased data base might change the facts and reverse this recommendation. The performance is shown in Figs 18 to 21: While high at small zenith angles, the results come in low for zenith angles over 60 degrees. This fact suggests that the chosen function of the zenith angle might not be ideal.
III.3.3 The Model by Hoyt

The model by Hoyt basically uses the elements of the direct model as outlined in II.3. Hoyt also splits the diffuse radiation into three parts as did Davies and Hay. The main difference to the previously outlined Model A is the assumption that the scattered radiation from Rayleigh scattering and from Mie scattering is subject to the same absorptive influences as the direct radiation. Thereby the equations to determine the diffuse radiation become:

\[
I_{\text{dif}}(R) = SC \cdot \cos(\theta) \cdot \left( 1 - \sum_{i} a_i \right) \cdot [0.5 \cdot (1 - \tau(R))] \tag{III-21}
\]

and

\[
I_{\text{dif}}(A) = SC \cdot \cos(\theta) \cdot \left( 1 - \sum_{i} a_i \right) \cdot [.75 \cdot (1 - \tau(AS))] \tag{III-22}
\]

The Rayleigh forward scattering ratio was taken as 0.5 while the aerosol forward scattering ratio was not given as a function but as a constant \( BA = 0.75 \).

Hoyt's approach to determine the multiply reflected diffuse (MR) radiation resembles somewhat the one by Davies and Hay:

\[
I_{\text{dif}}(MR) = (I_{\text{rr}} + I_{\text{dif}}(R+A)) \cdot rg \cdot \left( 1 - \sum_{i} a_i \right) \cdot
\]

\[
[.5 \cdot (1 - \tau(R)') + (1 - \tau(AS)') \cdot .25] \tag{III-23}
\]

But it has one important difference; It is obviously assumed that the multiply reflected diffuse radiation is subject to
further attenuation. Physically and mathematically, this treatment is more accurate than the one outlined in the previous section. The total radiation is the sum of the direct part, the sky diffuse part and the multiply reflected radiation:

\[ I(\text{tot}) = \text{Irr} + \text{Idif}(R) + \text{Idif}(A) + \text{Idif}(MR) \] (III-24)

As for the inclusion of the multiple scattering algorithm, the comments to Davies' and Hay's model are valid for this model too. The performance is shown in Figs 18 to 21: The results of this model as well are reasonably close to the results of Model C.
III.3.4 The Model by Bird and Hulstrom

Like the models we previously dealt with, Bird and Hulstrom's model divides the diffuse radiation: One part describes the radiation which reaches the earth after scattering; the other part is made up of diffuse radiation that was reflected from the earth and backscattered to the earth's surface. The equation for the 'sky diffuse' part of the irradiance is given as:

\[
I_{\text{diff}} = I_s \cos(\theta) \cdot (0.79) \cdot \tau(O) \cdot \tau(W) \cdot \tau(G) \cdot \tau(A) \cdot 1.02 \cdot \left\{ \frac{0.5 \cdot (1-\tau(R)) + BA \cdot (1-\tau(AS))}{1-AMS+(AMS)} \right\}
\]

where:

\[
BA = 0.82
\]

and

\[
\tau(AS) = \frac{\tau(A)}{\tau(AA)}
\]

and:

\[
\tau(AA) = 1-KS \cdot (1+AMS - AMS) \cdot (1-\tau(A))
\]

The total radiation, including the multiply reflected radiation becomes thus:

\[
I_{\text{(tot)}} = \frac{I_{\text{R}} + I_{\text{diff}}}{1-rg\cdot rs}
\]

where
rs = 0.0685+(1-BA)·(1-τ(AS)) \hspace{1cm} (III-29)

The factor $KS$ is dependent on the aerosol size distribution; Bird and Hulstrom used a value of $KS=0.0933$ for all their calculations. The diffuse part of Model C can be changed to the 'multiple scattering' pattern as developed earlier in this study; the difference in performance, however, is so small that it cannot be considered an improvement. This study suggests therefore to use the diffuse part of Model C unchanged. The performance of the model is shown in Fig's 18 to 21: Model C was developed with the aid of more elaborate models and in the absence of better standards can be considered "primary standard" for the diffuse part of this study.
III.3.5 Diffuse Radiation with the A.S.H.R.A.E. Model

The A.S.H.R.A.E algorithm [31] uses a simple factor 'C' to multiply the direct radiation:

\[ \text{Idif} = C \cdot \text{Irr} \]

where:

\[ \text{Irr} = A \cdot \exp\{-B \cdot \text{AMS}\} \]

While it is accurate within reasonable limits to determine the diffuse radiation as a constant fraction 'C' of the direct radiation, the constants proposed by A.S.H.R.A.E. are very low and represent fixed turbidity values of unusually clear days. Therefore an underestimation occurs for all but the most clear atmospheric circumstances. Because the proposed change of the A.S.H.R.A.E formula (Eqs.II-38/39) allows variable visibilities, it is necessary to adapt the calculation of the diffuse radiation as well. Comparison with other models, notably the Model of Bird and Hulstrom, lead to the following simple parameterization:

\[ \text{Idif} = \text{Irr} \cdot (3/\text{VIS} + 0.1) \]

The performance of this equation is shown in Figs. 18 to 22: The results are in very good agreement with the results of Model C
Figure 17; Davies and Hay Layer Model; Two Stream Approximation of Diffuse Radiation
Figure 18: Performance of 5 Models, Diffuse Radiation

visibility 23 km
ozone content 0.34 cm
water content 1.42 cm
Figure 19; Performance of 5 Models, Diffuse Radiation
Figure 20: Performance of 5 Models, Diffuse Radiation
Figure 21: Performance of 5 Models, Diffuse Radiation

visibility 5 km
ozone content 0.31 cm
water content 2.93 cm
Figure 22: The ASHRAE Models, Diffuse Radiation
IV Concluding Remarks

It was the goal of this study to put together some of the wealth of data and achievements in the field of clear sky solar radiation to the user with an engineering background. The use of the horizontal meteorological range - the visibility - as the main parameter for turbidity calculations has to be seen in light of this goal: Local conditions have a strong influence on the correlation between visibility and turbidity.

This deficiency is outweighted, however, by the simplicity and availability of this method which promises much better results with only slightly increased complexity.

The turbidity is a main contributor to diffuse radiation. The proposed new A.S.H.R.A.E algorithm for diffuse radiation incorporates this for the first time.

The proposed treatment of multiple scattering also has the applied sciences in mind: A tool is offered to assess the influence of atmospheric components on multiple scattering without outlining a mathematical approach that disallowed general applications.
V Further Work

At present, extrapolations from low and medium range visibilities to high visibilities are used in this study. It would be most desirable to have an extensive data base on the correlation between visibility and turbidity for various locations, conditions and seasons. Studies in this field are comparatively simple to conduct, although the availability of advanced data processing capacities is considered essential.

It would be in the best interest of the solar radiation research to contact simultaneous measurements of the effects of all attenuators, notably water, ozone, aerosols and various gases as well as simultaneous measurements of quantities with possible high correlation such as the meteorological range. Such studies would allow to verify many existing models and other work that has been based on extensive simulation, including much of this study.

Further work is also recommended in the field of diffuse radiation, both broadband and spectral diffuse radiation. The existing models still show wide discrepancies and measured data is sketchy at best.
References


[17] Linke, F; "Handbuch der Geophysik", Vol. 9, 621ff, Berlin (1941)


[24] Selby, J.E. et al.; "Atmospheric transmittance from 0.25μm to 28.5 μm: Computer code LOWTRAN 3", AFCRL-75-0255; (1975)


[34] Eighth Session of The Commission for Instruments and Methods of Observation; WMO (1981)
Table III: spectrum, absorption coefficients

Col 1: Interval number
Col 2: $\Delta \lambda$ in [nm]
Col 3: Center of wavelength interval, [\mu m]
Col 4: Ozone absorption coefficients for Eq. (I-11)
Col 5: Water absorption coefficients for Eq. (I-14)
Col 6: Mixed gases absorption coefficients for Eq. (I-13)
Col 7: Fraction of Solar Constant within interval, [W/(m\cdot m\cdot \mu m)]

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Table IV

Coefficients A,B,C for A.S.H.R.A.E algorithm [31]

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