DEEP HOLE RESIDUAL STRESS MEASUREMENTS USING ELECTRONIC SPECKLE PATTERN INTERFEROMETRY

by

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Abstract

This research study presents the design, development and testing of a sensor which utilizes the techniques of Electronic Speckle Pattern Interferometry (ESPI) for making deep-hole residual stress measurements. ESPI is optical technique that can be used to determine deformation at the surface of an object by combining interference patterns before and after deformation occurs. Although the technique has previously been used for determining residual stresses close to the surface, it has not yet been applied to measuring stresses deep within the interior of a specimen.

The deep-hole method determines interior residual stresses in a specimen by making measurement of a reference hole. Overcoring of the reference hole results in stress relief leading to deformations at the hole surface. Pre- and post-deformation measurements of the reference hole are then used to determine the original stress state. In order to make these measurements a deep hole imaging sensor has been designed. The design focuses light down a long tube to illuminate the reference hole. The hole is then imaged onto a CCD camera and the optical data provides the necessary displacement measurements. A least squares technique utilizing a series of basis functions allows explicit calculation of all six components of the stress tensor from the displacement data.

The sensor was subjected to a series of tests to demonstrate its capability to make the desired optical measurements. The basic functionality of the sensor was verified through displacement tests where stable, distinctive ESPI fringe patterns were observed. However, the application of a known stress field produced large disturbances resulting in a de-correlation of the optical data. Based on the observed de-correlation, recommendations are provided for improving the rigidity and overall performance of the design.
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Chapter 1
Introduction

1.1 The Nature of Residual Stresses
The occurrence of residual stresses within manufactured products presents an important challenge to industry. Common manufacturing processes such as machining, surface hardening, heat treatment and welding all create residual stresses. Residual stress is a stress state that exists within material, independent of any external or applied load [1,2]. Internal force equilibrium is maintained by a balance of tensile and compressive stresses whose resultant force and moment are both zero. Tempered glass provides a common example of such an arrangement. A permanent state of surface compression is induced by rapid surface cooling of the glass in order to increase impact resistance. Complementary tensile stresses develop in the interior and balance the compressive stresses on the surface [3]. Figure 1.1 below depicts the distribution of residual stresses across the width of the glass.

![Stress distribution within tempered glass](image)

*Figure 1.1 Stress distribution within tempered glass*
1.1.1 Formation of Residual Stress

Two primary effects contribute to residual stress formation: transient temperature gradients and non-uniform plastic deformation. Transient temperature gradients are present in processes involving heating or thermo-chemical treatments. Examples include: welding, casting, quenching, laser and plasma heat treatments, case hardening and cooling of multiphase materials [1,3]. Welding provides a useful example. As heat is applied, a localized area becomes molten. Upon cooling, the weld bead is restrained from contracting by the solid surfaces of the adjoining metals. As a result, the weld bead develops a state of localized tension while the surrounding metal undergoes a state of local compression.

Another important example occurs in the non-uniform cooling of large ingots. While at a high temperature, the ingot is initially stress free. Upon cooling, solidification begins on the outer surface. Once the outer surface is solidified, the inner core begins to cool and contract. As a result, tensile stresses develop within the interior of the specimen balanced by compressive stresses nearer to the surface.

Many manufacturing processes also involve non-uniform plastic deformation. Examples of these include: forming, rolling, extruding, machining, grinding, and shot peening [1,2]. Most machining operations produce permanent elongation at the surface of the work piece [4]. This deformation is confined only to a very thin layer on the surface. Yet in order for this layer to match the bulk material of the work piece, the surface must undergo compression, while the remaining material tension. These forces result in a highly localized stress gradient near the surface.
1.1.2 Effects of Residual Stresses

The lasting effects of residual stresses can greatly influence the operational behavior and performance of parts. They remain present when the material is subjected to further stress and, in accordance with the principle of superposition, the effects of the addition may be beneficial or detrimental. For instance, it is well known that regions of compressive residual stresses retard crack growth and failure due to fatigue [5]. Shot peening is a common example in which compressive stresses are induced in the surface of a specimen in order to provide increased resistance to stress corrosion cracking. The effects of shot peening greatly enhance the operational performance of materials [2].

In other cases, residual stresses are detrimental and can lead to failure when combined with external stress conditions, or by increasing the susceptibility to failure mechanisms such as stress corrosion cracking [2]. In the case of fatigue, residual stress is superimposed as a mean stress with dynamic stress. A striking example is fatigue failure under applied compressive stress in the presence of tensile residual stresses [5]. The occurrence of residual stresses in stock material is also noteworthy. Residual stresses occur in large castings which are then used to produce stock. When the stock is subsequently used in engineering applications, the effects of the residual stresses can be profound. In one such instance, shown in Figure 1.2, a cargo ramp was machined from a sheet of aluminum containing residual stresses. As material was removed the re-equilibration of the internal stresses caused large deformation and curvature of the part, rendering it unusable. Given the widespread occurrence of these stresses and their corresponding consequences, it is essential to be able to quantify the nature and magnitude of these stresses.
1.2 Measurement Techniques for Residual Stress

There is a wide range of techniques available for measuring residual stresses. Both destructive and non-destructive methods exist and employ various techniques in order to make the necessary measurements. Non-destructive methods include: X-ray and neutron diffraction, ultrasonic, and Barkhausen noise; Destructive methods include: sectioning, layering, hole drilling, ring coring and deep hole drilling [1].

The choice of a particular method depends upon the specifics of the geometry and the measurement objectives. Non-destructive methods are based upon a relationship between various physical properties and residual stress. This in turn allows measurement of the in-situ stress state without any
damage to the specimen. X-ray diffraction, which involves measurement of crystal lattice spacing, is the best established non-destructive method for wide field applications. However it measures only surface stresses, which in many cases can be a significant limitation [6].

In contrast, destructive methods remove material, thereby disturbing the state of equilibrium within the material. Consequently, relaxation occurs as the stresses re-equilibrate. The original stress state must then be inferred through measurement of the strain or deformation change caused by this re-equilibration. In general, the residual stress evaluation involves three steps: creation of a new stress state through material removal; measurement of resulting strain/deformation; and calculation of residual stress from the measured strain or deformation.

Of the destructive methods, the hole drilling and ring core methods are the most common [1,6]. Due to their effectiveness and relative ease of implementation they have found great practical application. The principles of both methods are similar and involve circularly symmetric material removal: hole drilling involves measuring the deformations around a drilled hole; ring coring involves measuring deformations within an annulus. The hole drilling method was pioneered by Mathar, who used a mechanical extensometer to measure displacements around a hole drilled in a stressed plate [7]. In 1950, Soete and Vancrombrugge replaced the mechanical extensometer with strain gauges [8], as shown in Figure 1.3, and the procedure is now an ASTM standard test method [9]. A typical hole drilling arrangement involves drilling a small hole (1-4 mm) to a depth approximately equal to its diameter. A three element rosette provides a measurement of the relaxation resulting from the material removed in the formation of the hole.
The opposite arrangement was developed by Milbradt, in 1951, resulting in the creation of the ring coring method [10]. Ring-coring, also shown in Figure 1.3, involves the machining of an annulus rather than a hole. In contrast to hole drilling, ring coring provides almost complete relief of the surface strains and is therefore more sensitive [1]. This arrangement is also less subject to diametric errors or eccentricity of the annulus with respect to the strain gauge. However, the size of the ring is relatively large and more damaging than hole drilling and is therefore less commonly used.

![Figure 1.3 Hole Drilling and Ring Coring Techniques (Gary Schajer)](image)

It is important to note that hole drilling, ring coring and most other methods, focus on the determination of surface stresses. This is related to that fact that, often, residual stresses are highly localized near the surface of a part. However, in cases such as large castings, significant residual
stress may be present deep within the interior. Therefore depth of penetration is another important aspect of any residual stress measurement techniques. The depths of penetration in steel of the various destructive and non-destructive methods are summarized in Figure 1.4 below.

As illustrated, the depth of penetration varies greatly according to the method. Conventional non-destructive methods such as X-rays are restricted to depths of several micrometers into steel while ultrasonic methods are useful to depths of only 2 mm. Destructive methods, such as hole drilling and ring coring, are generally far more extensive, reaching depths up to 10 mm [11]. However methods for determine stresses deep within the interior of a specimen remain limited.
The most commonly used method for determining residual stresses at depths greater than 10 mm is deep-hole drilling. Deep-hole drilling has applications in rock mechanics, evaluation of welded joints and large castings. In the field of rock mechanics deep-hole techniques has been in use since the 1960's [12]. The method has also been adapted to the measurement of residual stress in metals. The work of Beaney [13] and Zhdanov and Gonchar [14] has provided the basis for through thickness stress measurement in thick steel components and weldments. The deep-hole method can be generalized into the following three steps: an initial reference hole is drilled in a specimen; stressed material around the hole is removed through overcoring; the resulting change in the dimensions of the hole is then measured.

In cases where the reference hole is large enough, strain gauges can be mounted within the interior of the specimen [15]. More commonly, devices such air gauges and diameter gauges are used for making the physical measurements relating to the deformation. However these measurements are strictly diametric and do not include an out-of-plane component. The residual stress tensor, shown in Figure 1.5, is symmetric and contains six independent components. If out-of-plane strains are not measured then they must be inferred by assuming a plane stress condition, as shown in Figure 1.5. The accuracy of the plane stress assumption may vary according to the application, but is certainly not sufficient in all cases. Therefore it is highly desirable to overcome this limitation by means of an alternative measurement technique.
An innovative alternative to strain gauges is the use of optical techniques. Such techniques have been applied towards measurement applications since the 1970's [16]. In the 80's and 90's, Antonov, McDonach, Nelson and McCrickerd demonstrated the use of optical techniques for surface displacements around drilled holes [17, 18]. More recently Steinzig, Ponslet and Schajer and have applied Electronic Speckle Pattern Interferometry (ESPI) towards the development of full field calculations of hole drilling residual stresses [19,20,21]. In the ESPI procedure, interferograms created by laser speckles are combined in order to determine displacements at the surface of an object. The non-contact nature of the measurement avoids the significant time associated with the installation of strain gauges. More importantly, ESPI is capable of providing a very rich full field data set and, as such, is ideal for determination of the complete stress tensor residual stresses. However, it has not yet been applied toward deep-hole applications.
1.3 Objective

The objective of this research study is to design, manufacture and test a deep-hole sensor for determining the complete tensor stresses within large metal specimens using ESPI measurements. Current deep-hole methods face two primary challenges: traditional measurement techniques, which are strictly diametric, cannot determine all six components of the stress tensor; furthermore, due to size restrictions, the mounting of strain gauges in the interior of a specimen is often impractical and/or time consuming.

The deep hole sensor developed here seeks to overcome these challenges. The sensor will be used to determine residual stresses deep within the interior of an isotropic metallic specimen, while utilizing the principles of ESPI for making the necessary measurements. Particular applications would include large castings and forgings used to produce stock material.

1.3.1 Requirements

- The sensor shall be capable of producing sufficient data in order to determine all six independent components of the residual stress tensor
- The sensor shall be capable of making measurements in a 10 mm reference hole at a minimum depth of 100 mm into the interior of the specimen

1.3.2 Scope

- Design development, manufacture and assembly of the sensor
- Calculation method required to determine stress from optical displacement measurements
- Experimental testing and validation of the sensor
Chapter 2
Deep Hole Measurement

The previous chapter summarized some typical methods for measuring residual stresses, the majority of which focus on the determination of surface stresses. Of particular interest among these is the deep hole method, which is able to determine stresses deep within the interior of a specimen. This method has been used extensively in rock mechanics since the 1960’s [12]. In industrial applications the method is applied to interior residual stress measurements in welded joints and large castings [14, 22, 23]. The research described in this thesis further develops this work.

2.1 Deep-hole Procedure

Figure 2.1 illustrates the deep-hole method. The first step (A) is to drill a reference hole in the material specimen. For small weldments the hole, typically 6 mm in diameter, is gun drilled and honed to provide a good surface finish [22,23]. Rock mechanics measurements are carried out on a much larger scale, with a borehole diameter of around 1.5 in (36 mm) [15, 24, 25]. The second step (B) is to measure the diameter of the hole at various depths. The method for doing this may vary depending upon the application (see below). Once the diameter is measured, the next step (c) is to over core a ring of material from around the hole. This is accomplished either by cutting or EDM. In step (D) the diameter of the initial reference hole is again measured indicating the strain relief caused by the over-coring. In some cases the over-coring is repeated at various depths to establish stress variation with depth, shown in (E). In other cases, the reference hole is completely over-cored and the entire cylindrical column is measured along its diameter and height [22, 23].
2.2 Measurement Techniques

The method used for measuring the change in the dimensions of the reference hole can significantly impact the accuracy of the stress calculations. Three types of measurement techniques are commonly applied: diametric measurements, diametric & axial measurements and strain gauges.
2.2.1 Diametric Measurement

Diametric measurements are straightforward to make and can be applied to all deep-hole applications. In rock mechanics a three-component borehole deformation gage, illustrated in Figure 2.2, is commonly used [25]. The cylindrical device is placed in the reference hole, causing cantilevered tabs to make contact with the hole surface. Changes in the hole diameter results in displacement of the tabs and is measured via strain gauges contained within the device housing. The device is capable of simultaneously measuring deformation along three diameters (each at a different depth along the reference hole) 60 degrees apart.

In applications such as weld joints the scale is smaller and another instrument, called an air gauge, is commonly used [22, 23, 26]. The air gauge consists of a cylindrical plug that allows pressurized air to flow down the centre of the plug and escape thorough jets on the side of the plug. The plug is placed within the reference hole, and any change in the diameter of the hole results in pressure changes which are accurately measured via a transducer. An example of an air gauge is shown in Figure 2.3.

*Figure 2.2 Three component borehole gauge [27]*

*Figure 2.3 Air gauge [28]*
The major limitation of these devices is that they measure only diametric deformations. Neither measure axial deformation; therefore they cannot give any information concerning axial stresses. This limitation is overcome by assuming a plane stress condition, where the axial stress $\sigma_z = 0$. For the general case of two dimensional plane stress, shown in Figure 2.4, the relieved circumferential strain at the surface of the hole is given by:

$$\varepsilon_{\theta} = \frac{2P + 4Q \cos 2\theta + 4T \sin 2\theta - \nu \sigma_z}{E}$$  \hspace{1cm} (2.1)$$

where:

$$P = \frac{\sigma_x + \sigma_y}{2} \hspace{1cm} Q = \frac{\sigma_x - \sigma_y}{2} \hspace{1cm} T = \tau_{xy}$$ \hspace{1cm} (2.2, 2.3, 2.4)

$E$ is the Young’s modulus, $\nu$ is the Poisson’s ratio and $\theta$ is the angular coordinate. Assuming that $\sigma_z = 0$, the in-plane stresses ($P,Q,T$), can be determined by taking three diametric measurements at specific angles (e.g. $\theta = 0, 45, 90$ degrees).

*Figure 2.4 Plane Stress Element*
Measurement error can be reduced by taking measurements at numerous angles and using a least squares method [29]. While this provides a significant improvement with respect to noise, it does not remove the plane stress assumption, which is true only at the top free surface of the specimen.

### 2.2.2 Diametric & Axial Measurement

Away from the surface a condition of generalized plane strain exists. Therefore, in some applications an additional measurement in the axial (z) direction is achieved by removing the entire trepanned column and measuring the change in column length [22, 26]. In the case of generalized plane strain the circumferential and axial strains are:

\[
\varepsilon_\theta = \frac{2P + 4Q(1 - \nu) \cos 2\theta + 4T(1 - \nu) \sin 2\theta - \nu \sigma_z}{E} \\
\varepsilon_z = \frac{2P \nu - \sigma_z}{E}
\]

While the plane strain analysis is more realistic than the plane stress analysis, it presents two drawbacks: 1) it is significantly challenging to measure the change in length of the trepanned column to a high degree of accuracy; 2) even when the change in length is well known, the axial strain is not being measured locally and therefore the measured \( \varepsilon_z \) is an average value over the length of the column.

### 2.2.3 Strain Gauge Measurement

The most accurate calculations can proceed when the strains are measured locally in all three axes. In some rock mechanics applications, the hole size is large enough that strain gauges can be employed.
However placing strain gauges in the reference hole poses a significant challenge. While strain gauges can be placed manually, the difficulty in doing so increases with increasing depth and/or smaller hole diameter. Devices have been designed to assist in the placement of strain gauges. The *Borre probe*, developed by the Swedish State Power Board, is a device capable of placing strain gauges into deep (500m), water filled boreholes [15]. Once in place, the measurements provide up to nine strains spanning all three axes. Leeman and Hayes [24] provide a method for determining the complete stress tensor from the strain gauge data. For a specimen with a bore hole of radius = a, the components of stress can be denoted in terms of cylindrical coordinates as shown Figure 2.5.

![Figure 2.5 Stress Components in Cylindrical Coordinates](image-url)
The stresses in cylindrical coordinates \( S_r, S_\theta, S_z, S_{r\theta}, S_{\theta z}, S_{rz} \) near the surface of the hole can be described by the following six equations [adapted from 24]:

\[
S_r = P(1 - \alpha^2) + Q(1 + 3\alpha^4 - 4\alpha^2 \cos 2\theta + T(1 + 3\alpha^4 - 4\alpha^2 \sin 2\theta)
\]

\[
S_\theta = P(1 + \alpha^2) - Q(1 + 3\alpha^4 \cos 2\theta + \tau_{xy}(-1 - 3\alpha^4) \sin 2\theta)
\]

\[
S_{r\theta} = Q(-1 + 3\alpha^4 - 2\alpha^2 \sin 2\theta + \tau_{xy}(1 - 3\alpha^4 + 2\alpha^2) \cos 2\theta)
\]

\[
S_z = \sigma_z + Q(-4\nu\alpha^2 \cos 2\theta + T(-4\nu\alpha^2) \sin 2\theta)
\]

\[
S_{\theta z} = \tau_{xy}(-1 - \alpha^2 \sin \theta + \tau_{xy}(1 + \alpha^2) \cos \theta)
\]

\[
S_{rz} = \tau_{xy}(1 - \alpha^2) \cos \theta + \tau_{xy}(1 - \alpha^2) \sin \theta
\]

where \( \alpha = a/r \), the directions \( r, \theta, z \) are shown in Figure 2.5 and \( P, Q, T, \) are defined on page 14.

Strains are measured at the surface of the borehole where \( \alpha = 1 \). The strain gauge rosettes are arranged in the axial (\( \varepsilon_z \)) circumferential (\( \varepsilon_\theta \)) and 45° diagonal (\( \varepsilon_d \)) directions. By substituting Equations 2.5 and 2.7 into Hooke’s law and multiplying by -1 (corresponding to the relaxation of the strains) the surface strains \( \varepsilon_\theta \) and \( \varepsilon_z \) are given as:

\[
\varepsilon_\theta = \left(-2P + 4(1-\nu^2)Q \cos 2\theta + 4(1-\nu^2)T \sin 2\theta + \nu \sigma_z\right) / E
\]

\[
\varepsilon_z = (-2\nu P - \sigma_z) / E
\]

The 45° diagonal strain is derived from Mohr’s circle:

\[
\varepsilon_{45} = \frac{\varepsilon_z + \varepsilon_\theta + \gamma_{\theta z}}{2}
\]
where $\gamma_{yz}$ is found by substituting Equation 2.8 in Hooke's law to produce:

$$\gamma_{yz} = \left( -4(1 + \nu) T \sin \theta + 4(1 + \alpha^2)(1 + \nu) \tau_{xy} \cos \theta \right) / E$$

(2.16)

and the $45^\circ$ diagonal strain is given by:

$$\varepsilon_{45} = \left( -2 P(1 - \nu) + 2(1 - \nu^2) Q \cos 2\theta + 2(1 - \nu^2) T \sin 2\theta - \frac{1}{2}(1 - \nu) \sigma_z + 2(1 + \nu) \sin \theta \tau_{xz} - 2(1 + \nu) \cos \theta \tau_{yz} \right) / E$$

(2.17)

These equations can be inverted to determine the surface stresses from the strain gauge data [24]. Although the use of strain gauges is effective, the method is suitable only for the larger reference hole diameters typically involved in rock mechanics. Therefore, given the limitations associated with both diametric measurements and strain gauges, it is desirable to develop another measurement technique capable of determining the complete stress tensor and more suitable for smaller scale industrial applications. Optical methods such as ESPI provide such an opportunity, and are described in the following chapter.
Chapter 3
Electronic Speckle Pattern Interferometry

The use of optical methods is well established in residual stress applications. Techniques such as holographic interferometry have proven effective in determining surface displacement measurements [17, 18]. Recently, Electronic Speckle Pattern Interferometry (ESPI) has been successfully applied to hole-drilling applications [19, 20, 21]. The technique allows full field measurement of surface displacement to be collected and stored electronically. This gives an accurate description of the shape change that the object has undergone. The information can then be used to determine the corresponding residual stress. The principles of ESPI are described below.

3.1 Laser Interferometry Principles

ESPI like many optical techniques is based upon the interference of light waves, referred to as interferometry. When two light waves combine, the resultant light intensity depends on their individual intensities and their relative phase. If two light waves are in phase they constructively interfere and their individual intensities add. If on the other hand, the two waves are out of phase, they destructively interfere and their individual intensities subtract. This forms the basis of laser speckle formation illustrated in Figure 3.1.
Figure 3.1 Formation of Laser Speckles

Figure 3.1 shows an object whose surface is diffuse and illuminated by a coherent light source such as a laser. The reflection is scattered in multiple directions. At a given image plane, reflected light will arrive with varied phases: those reflected light rays which happen to be in phase with one another will produce a bright speckle; those that are out of phase with one another will produce a dark speckle. These speckles thus contain information regarding the surface height profile of the object. For a diffuse object the speckles will appear as a random distribution of bright and dark spots; any displacement of the surface will change the path length of the reflected light and the intensity of each speckle accordingly. Furthermore if some portion of the initial coherent light is directly aimed towards the image plane, without reflecting off the object, it can be used to establish a reference. This reference, when combined with the reflected beam, provides the means by which to precisely correlate object surface displacement with the corresponding change in speckle intensity.
3.2 ESPI System Setup

A typical ESPI system is arranged as shown in Figure 3.2. A laser light source is split into two beams, an illumination beam and reference beam. The illumination beam is used to illuminate the object. The reflected beam, referred to as the object beam, is then viewed by a CCD camera. As mentioned above, it is necessary to compare the reflected beam to a known reference. Hence within the CCD camera assembly, the object beam is then combined with the reference beam to form an interference image. The image is then recorded by the CCD and stored in the computer for processing.

![Figure 3.2 ESPI System Setup](image)

*Figure 3.2 ESPI System Setup [19]*
3.3 ESPI Data

Data from the ESPI procedure is derived from the aforementioned interference between the object beam and the reference beam which is recorded by the CCD camera. When the object and reference beams are combined, the intensity of the resultant light wave varies according to the interference expression given by [19]:

\[ I = I_{\text{ref}} + I_{\text{obj}} + 2\sqrt{I_{\text{ref}} I_{\text{obj}}} \cos(\theta - \phi) \]  

(3.1)

where \( I_{\text{ref}} \) is the intensity of the reference beam, \( I_{\text{obj}} \) is the intensity of the object beam, \( \theta \) is the phase distribution angle, and \( \phi \) is the relative initial angle. Light intensities are detected by the CCD and displayed pixel by pixel. Therefore this equation describes the resulting intensity of a single pixel on the image which, in turn, corresponds to a specific point on the object surface. The same equation with different parameter values applies to each of the other pixels.

3.4 ESPI Image Processing

Equation 3.1 gives the light intensity for a given phase difference, \( \phi \), of the object and the reference beams. In practice the light intensity is measured and the phase difference is the quantity to be determined. Equation 3.1 contains three unknowns: \( I_{\text{ref}} \), \( I_{\text{obj}} \), and \( \phi \). Therefore at least three measurements are necessary in order to evaluate them. These measurements are made by stepping the reference beam using a piezo-actuator to angles \( \theta = 0^\circ, 90^\circ, 180^\circ, \) and \( 270^\circ \). The benefit of taking four rather than three measurements is to simplify the calculation and provide some data averaging.
Figure 3.3 displays a general light wave produced by this interference pattern and the four intensity values resulting from the phase stepping.

\[ I = B + A(\cos(\theta - \phi)) \]  
\[ A = 2\sqrt{I_{\text{ref}}I_{\text{obj}}} \]  
\[ B = I_{\text{ref}} + I_{\text{obj}} \]

As mentioned, the four-measurement approach simplifies the analysis. When Equation 3.2 is evaluated at \( \theta = 0^\circ, 90^\circ, 180^\circ, 270^\circ \) the following four equations are produced:
\[ I_1 = B + A\cos\phi \quad (3.4) \]
\[ I_2 = B - A\sin\phi \quad (3.5) \]
\[ I_3 = B - A\cos\phi \quad (3.6) \]
\[ I_4 = B + A\sin\phi \quad (3.7) \]

Subtracting (3.7) from (3.5) and dividing by (3.4) – (3.6) yields:

\[ \phi = \tan^{-1}\left(\frac{I_2 - I_4}{I_1 - I_3}\right) \quad (3.8) \]

Once the initial set of four are taken the object then undergoes a hole-drilling or ring coring operation. After stress relaxation has occurred, a subsequent set of four images is taken. Figure 3.4 shows a typical set of 8 images before (pre image set) and after (post image set) the hole drilling operation.

![Figure 3.4 Pre and Post Deformation Images](image_url)
While all eight images shown in Figure 3.4 exhibit a similar random-looking pattern it is important to recall certain relationships: each pixel within an image is random with respect to any other pixel; however the same pixel within each set of four images is related through the phase stepping of the reference beam; additionally from one set to the other the same pixel is related through the stress relaxation and subsequent surface displacement of the object. Thus, from each set of images it is possible to determine the two initial angles, \( \phi_1 \) (pre deformation angle) and \( \phi_2 \) (post deformation angle) given by:

\[
\tan \phi_1 = \frac{(I_2 - I_4)}{(I_1 - I_3)} \tag{3.9}
\]

\[
\tan \phi_2 = \frac{(J_2 - J_4)}{(J_1 - J_3)} \tag{3.10}
\]

where the variables \( I \) and \( J \) are used to differentiate between pre- and post-deformation light intensities, respectively. The total phase change (\( \Omega \)) is calculated as the difference between the pre- and post-drilling initial angles given by:

\[
\tan \Omega = \tan(\phi_2 - \phi_1) \tag{3.11}
\]

Using trigonometric identities and substituting Equations 3.9 and 3.10 gives:

\[
\Omega = \tan^{-1} \left( \frac{(I_2 - I_4)(J_1 - J_3) - (I_1 - I_3)(J_2 - J_4)}{(I_1 - I_3)(J_1 - J_3) + (J_2 - J_4)(I_2 - I_4)} \right) \tag{3.12}
\]

Once the phase change is calculated, displacement can be determined by:

\[
D = \frac{\lambda}{2\pi} \Omega \tag{3.13}
\]

where \( \lambda \) is the wavelength of the laser light. Using these displacements the residual stresses are then calculated as described in Chapter 5.
Chapter 4
Prototype Sensor Design

The design of the ESPI sensor presented significant challenges. Numerous requirements, both optical and mechanical, needed to be satisfied. This chapter describes the relevant optical and mechanical features of the design. Figure 4.1 shows a 3D view of the design.

Figure 4.1 ESPI Deep Hole Sensor
4.1 Optical Design

The primary requirement for design of the sensor is the ability to make deep hole measurements in a 10 mm diameter reference hole. This required modification of the ESPI setup shown in Figure 3.2 to allow the object and illumination beams to travel along the same path. The optical arrangement was re-designed as shown in Figure 4.2 to meet this objective.

![Figure 4.2 Optical Arrangement](image)

The illumination beam is generated by a laser and provided to the sensor via a fiber optic. The beam is collimated by a lens and is split into two portions by a half-silvered mirror (plate beam splitter) oriented at 45 degrees to the incoming beam. Half of this light passes directly through the mirror where it reflects off a plane mirror attached to a piezo-electric actuator. The light is phase stepped and the reflected beam now acts as the reference beam. After reflection the light returns to the half-silvered mirror and is reflected 90 degrees towards the CCD.
The other half of the original beam is reflected 90 degrees along the forward path towards the object. The light illuminates the object and the object beam reflects along the return path and is imaged onto the CCD by an imaging lens. For pictorial clarity, the forward and return paths are shown displaced in Figure 4.2. At the far end of the return path the interference pattern created by the combination of the object beam and reference beam is viewed and recorded by the CCD.

4.1.1 Design of the Reflective End Piece

It is important to note that the 'object' in Figure 4.2 is the sidewall of the reference hole where displacement is measured. In order to view this surface the illumination beam must be directed radially outwards and the reflected light directed back towards the CCD. Therefore the reflector at the end of the tube is a one of the most critical aspects of the sensor design. The most important aspect of the reflector design is the information content of the optical data produced. Several designs were considered and after much experimental investigation and analytical modeling a three face prism reflector was selected. Chapter 5 provides a more comprehensive discussion of the selection process as it relates specifically to the stress analysis. Figure 4.3 shows the prism design.
Obtaining the prism was a difficult undertaking as no such product is commercially available. One potential option was identified: a retro-reflector cube. The backside corner of these cubes can be silvered and used as a three face prism reflector; however sizes are limited, custom silvering is cost prohibitive and the steepness of the corner (the angle that each face of the prism is oriented at with respect to the incoming light) is fixed at 35 degrees (the angle given by the corner of a cube).

In the absence of a suitable commercial product, the prism was custom built. The prism was machined from aluminum on a mill and ground flat. However, in order to provide a sufficiently reflective surface, three 1 mm thick front surface aluminized mirrors were cut to shape, beveled and glued on the faces of the prism. A custom jig was manufactured in order to cut the mirrors and bevel the edges to the precise angles required.

4.1.2 Arrangement of Lens and Half-silvered Mirror

Precise arrangement of the lens and half-silvered mirror is necessary in order to direct the laser and image the object on the CCD. Figure 4.4 shows the path of the illumination beam. For pictorial clarity the 90 degrees deflection produced by the mirror is omitted.

![Figure 4.4 Illumination Beam Path](image-url)
The laser exits the fiber optic as a cone of light with an arc of approximately 12 degrees. This light is
collimated by a 6.25 mm diameter, 50 mm focal length lens and focused along the forward path
towards the reflector. Placing the lens at one focal length from the end of the fiber optic results in a
beam that is collimated (e.g. all rays emerge from the lens parallel to the axis along which the beam
travels). However the collimating lens was placed at slightly less than one focal length ($D_c = 48 \text{ mm}$)
in order to diverge the outgoing light, increase the spot size of the illumination at the object plane and
fill the entire aperture of the reflector. Upon striking the reflector, the light is reflected at an angle of
40 degrees toward the sidewalls of the reference hole. Light reflecting from the reference hole is
redirected by the prism faces towards the CCD, and imaged through a 6.25 mm diameter, 60 mm
focal length lens, as shown in Figure 4.5.

![Figure 4.5 Object Beam Path](image)

On the forward path, the location of the imaging lens affects only the overall spot size at the object
plane. However, on the return path the location of the imaging lens is more important affecting both
the focus and magnification of the image viewed by the CCD. The first step in determining the object
and image distances, $D_o$ and $D_i$, involved matching speckle size to pixel size. It is essential that the
diameter of the speckles is greater than or equal to that of the pixels (~10 um) in order to ensure that each pixel corresponds to only one point on the object surface. Speckle size is directly related to the F-stop through the following relationship [30]:

\[
\text{Speckle Diameter} = (2.4) \times (\lambda) \times (\text{F-stop})
\]  

(4.1)

where the wavelength, \( \lambda \), of the laser is 532nm. The F-stop is in turn given by [29]

\[
\text{F-stop} = \frac{\text{(Focal length)}}{\text{(Clear aperture)}}
\]

(4.2)

Since the clear aperture is already constrained by the size of the reference hole and the inner diameter of the tube assembly, the longest available focal length lens was chosen. The object distance and image distance were then selected through a process of experimental trial and error by balancing four criteria: maximize magnification; maximize \( D_o \); minimize \( D_i \); maximize illuminated area of the prism reflector. \( D_o \) and \( D_i \) are related to the focal length through Equation 4.3, while the magnification (M) is given by Equation 4.4 [31]. The calculated distance are \( D_o = 180 \text{ mm} \), and \( D_i = 90 \text{ mm} \).

\[
\frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{f}
\]

(4.3)

\[
M = \frac{D_i}{D_o}
\]

(4.4)
4.1.3 Lens Tilting and Newton's Rings

Before the final design was completed, large scale testing was performed on an optical bench in order to assess the image quality of the optical arrangement. During this testing prominent interference patterns were observed, in the form of Newton's Rings. Figure 4.6 displays an example of these Newton's rings:

![Newton's Rings](image)

*Figure 4.6 Newton's Rings*

Extensive testing was conducted to identify the source of the interference. It was observed that when the spherical convex imaging lens is replaced with a cylindrical lens, the resulting interference pattern remains but takes on a different shape. Furthermore, when the lens is placed on the CCD side of the half-silvered mirror, no rings are observed. However, when the lens is placed on the object side of the mirror, requiring the light to pass through twice (once on the forward path, and again on the return path) the rings are produced. Therefore, the interference results from light passing through the imaging lens on the forward path. Tilting the lens deflects the rings out of the field of view of the CCD without significantly impairing image quality. Therefore, a tilt angle of approximately 8 degrees was designed into the lens mounting in order to remove the fringes.
4.2 Mechanical Design

The mechanical design of the sensor includes the base, tube assembly, lens mounts, gripper, and the CCD camera housing.

4.2.1 Sensor Base

The sensor base houses the half-silvered mirror, adapter mount for the fiber optic and the piezoelectric stepper mount as shown in Figure 4.7.

![Figure 4.7 Sensor Base](image)

The central portion of the base is a 40 x 40 x 40 mm cubic mirror mount supplied by Linos Photonics. In the centre of the cube is a half cylinder upon which the half-silvered mirror is mounted. This allows the mirror to be rotated to any desired angle. The cylinder can be fixed at a particular angle and constrained on its diameter using set screws.
On one side of the cube a custom adapter mounts the fiber optic to direct the illumination beam toward the center of the mirror. One end of the adapter mounts directly to the face of the cube through four cap screws, and the fiber optic couples directly to the other end. The collimating lens is mounted in the center of the mount; its axial position relative to the end of the fiber optic is determined by a set of spacer tubes.

A piezo actuator is housed on the opposing side of the cube within a custom mount. Although conducting the phase stepping within the main housing of the sensor requires an additional level of complexity, it has distinct advantages over providing the phase stepped reference beam externally through a fiber optic (as is the case in Figure 3.2). This arrangement improves the stability of the reference beam by eliminating any disturbances that may be encountered when traveling along the fiber optic. A further advantage of this arrangement was discovered during the large scale bench testing. The portion of the illumination beam that passes straight through the mirror reflects off the surfaces on the opposing end and back toward the CCD. The percentage of reflected light is great enough to cause substantial noise in the image. Therefore utilizing the reflection as the reference beam converts this problem into an elegant solution.

The piezo mount is attached to the cube using four cap screws. An end cap on the back end of the mount allows the piezo actuator to align concentrically with the central axis of the base assembly. One end of the piezo is glued to the surface of the end cap, and a small concave mirror is glued to the other end. The concavity allows the reflected light to be tightly focused onto the CCD providing full coverage over the desired spot size.
In order to maintain an optimal ratio between the intensity of the illumination beam and the reference beam, light passing through the half-silvered mirror must be attenuated before reflecting back to the CCD. This is accomplished through the use of two linear polarizers. Front surface reflection of the polarizers is significant, and in some instances may be greater than the desired level of attenuation. Therefore they must be mounted at 45 degrees to the incoming light. One polarizer is glued to the face of a truncated tube; the other is placed atop the first and the two are secured by heat shrink tubing around their circumference. This allows rotation of the top polarizer in order to adjust the level of attenuation while maintaining their position with respect to the oncoming beam.

4.2.2 Lens Mounting, Tube Assembly and Light Absorption

The tube assembly contains the path along which the illumination beam and object beam travel. The housing tube of the assembly mounts directly to one face of the cube via the tube adapter mount. Within the center tube and inside the fiber optic mount the collimating lens and imaging lens are located in place using a series of spacer tubes as shown in Figure 4.8. This is an effective method for mounting lenses as their axial location can easily be achieved and adjusted by cutting precision lengths of tube [31]. Position of the imaging lens is maintained by a tightly fitted end cap. As mentioned previously, the imaging lens was tilted at an angle of 8 degrees in order to eliminate the effects of Newton’s rings on the images. A custom jig was manufactured to bevel the ends of the imaging lens spacer tubes precisely.
The importance of light management also became apparent during initial testing of the assembled sensor. Along the forward and return path more than 90% of the illumination beam is dispersed and absorbed with only small amount returning to form the actual image of the hole. As a result undesirable internal reflections within the base and tube assembly can be of the same magnitude as that of the image. Extensive efforts were undertaken to reduce their effects: front surface reflection off the end cap was reduced by adjusting the collimating lens location so as to minimize the spot size of the beam at the entrance to the tube assembly; all interior surfaces of the base assembly, as well as the spacer tubes were painted black; the end cap was manufactured from non-reflective black Delrin; finally, the inner diameter of the housing tube was lined with a light absorbing sleeve. These efforts greatly improved the quality of the images.
4.2.3 Mechanical Gripper and Reflector Mount

An important and challenging aspect of the mechanical design is the gripping feature at the end of the tube. Once the tube has been placed into the reference hole it is essential that it remain in position before and after the ring coring. Movements on the order of 1/2 the size of a pixel (~5um) significantly impair the accuracy of the data produced by the interference pattern. The gripper minimizes such movements by installing a preload, thus fixing the end of the tube to the hole. A section view of the gripper design is shown in Figure 4.9.

![Figure 4.9 Gripper Feature](image-url)
The gripper is actuated by a custom nut which threads onto the tube adapter. The rim of one end of the nut is sandwiched between two washers which are press-fit onto the gripper tube. Turning the nut causes the gripper tube to move longitudinally with respect to the housing tube. At the far end of the tube assembly the gripper tube is slotted resulting in three long tabs. The housing tube is flared by 0.5 mm in diameter. Rotation of the nut causes the gripper tube to move down the length of the center tube and expands the three tabs which make contact with the reference hole. After the pre- and post-deformation images have been taken, the tube can be removed from the reference hole by turning the nut in the opposite direction and withdrawing the tabs.

The distance between the point of contact with the reference hole and the reflector must be known precisely in order to ensure the accuracy of the calculations. Therefore the gripper tube was machined to allow the reflector to be mounted directly to it as shown in Figure 4.10. This ensures that the distance is maintained as the sensor takes measurements at different locations within the reference hole.

*Figure 4.10 Reflector Mount*
4.2.4 CCD Camera Housing

The CCD is a Cohu 1100 series ½ inch format monochrome camera. It consists of two components: a main electronics board and an optical sensor. The CCD is mounted within an injection molded plastic housing. On one side panel of the housing a BNC and TNC style connector provide the camera with power input and a video signal out to the computer. A cross section of the Camera Housing is shown Figure 4.11.

![Figure 4.11 CCD Camera Housing](image)

The optical sensor includes a c-mount interface allowing it to be threaded directly to the c-mount tube. The c-mount tube serves as the interface between the housing and the cubic base. An adapter plate with a 25 mm through hole is mounted directly to one face of the cubic base. The c-mount tube can move axially along the hole and is secured along its diameter by two 8-32 cap screws. This allows
precise adjustment of the axial position of the optical sensor in order to focus the image. The through hole in the adapter plate is also offset by 1.5 mm in order to accommodate for the shift in the light rays which occur at the half-silvered mirror, as shown in Figure 4.12.

4.3 Operational setup
Once assembled the sensor operates in conjunction with a laser generator and control software. Figure 4.13 below shows the assembled sensor and laser generator. The laser generator provides a 532nm continuous wave green laser through a fiber optic. The fiber optic carrying the illumination beam is connected to the fiber coupler on the sensor base. The laser generator also provides power to the piezo actuator. The power in and video signal out to the CCD camera is connected directly to the computer containing the control software (not shown). The laser-generator is also connected to the computer and receives commands from the serial port which actuate the laser and the phase stepping of the piezo.
Figure 4.13 Assembled sensor and laser generator

During testing the end of the tube is inserted into the material specimen and moved to the desired location. The actuator nut is turned, engaging the tabs and establishing preload. The focus of the image is adjusted by moving the camera assembly along the axial direction to the desired location and securing it in place with the aforementioned cap screws. Based upon the reflectivity of the hole
surface the reference beam can be adjusted by rotating the polarizers until the desired illumination to reference beam ratio is achieved. Once the desired focus and intensity ratio are achieved testing can commence. In operation, the test procedure follows the steps outlined in Chapter 2 for the deep hole method. The data is taken in accordance with the four image phase stepped procedure described in Chapter 3. The final step requires the analysis in order to determine the desired stresses from the optical data. This is described in the following chapter.
Chapter 5
Determination of Stress

The basic elements required for the calculation of stress have already been introduced: Chapter 2 briefly discusses the constitutive equations relating stress and strain; Chapter 3 describes the mathematical basis for the optical data acquired through the ESPI method. In this chapter these concepts are incorporated into a method which determines the complete 3-dimensional stress tensor from the given data.

5.1 Determining Displacements

Raw optical data is provided by the CCD in the form of light intensity and recorded by the frame grabbing software. The first task is to convert intensity values into displacements. Figure 5.1 provides an overview of the steps involved in calculating displacement.

![Diagram](Figure 5.1 Displacement Calculation Procedure)

**Figure 5.1 Displacement Calculation Procedure**
The camera detects light intensity at each one of the CCD detector cells. These light intensity values are written to a matrix consisting of 640 x 480 elements (corresponding to the computer screen pixels); 4 pre and 4 post drilling images are taken resulting in 8 matrices (A). The 8 matrices are combined as described in Section 3.4. Using these 8 matrices in conjunction with Equation 3.12, a single matrix consisting of the “wrapped” phase change Ω for each pixel is calculated (B). Since the phase change is calculated using the arctan function, it can only evaluate angles within the range -180° < Ω < 180°. Angles outside this range are “wrapped” in multiples of 360° into the range. For example an angle of 450° would be evaluated as 90°. The phase is unwrapped using an algorithm that compares the angles evaluated at adjacent pixels and adds multiples of +/- 360° as needed to create a smooth angular variation (C). Next, using Equation 3.13, the displacements are determined from the phase change (D). Stresses are then calculated from the measured displacements, d_i, as described below.

5.2 Determining Stress from Displacements

Vector d_i contains the measured displacements in the direction of the sensitivity vector (defined below) for each pixel location i. To determine stress, a least-squares method is proposed using the theoretical displacement responses from the components of the stress tensor as basis functions. Each basis function encompasses the linear elastic stress-displacement solution for all the pixels and the geometry of the reflective end piece. The matrix of measured displacements is then compared to the superposition of these basis functions and a least squares analysis is used to find the stresses.
5.3 Basis Functions

It is necessary to first develop a total of seven basis functions, corresponding to each of the six components of the stress tensor, $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \tau_{xy}, \tau_{xz}, \tau_{yz}$ and a rigid body motion $\Delta$. The latter rigid body motion occurs mainly because of small temperature changes in the equipment. To simplify the calculations these components are defined in terms of stresses $P$, $Q$, $T$, $\sigma_z$, $\tau_{xz}$, $\tau_{yZ}$, and $\Delta$ where

\[
P = (\sigma_x + \sigma_y)/2, \quad Q = (\sigma_x - \sigma_y)/2 \quad \text{and} \quad T = \tau_{xy}\]

shown in Figure 5.2 and 5.3.

![Figure 5.2 In-plane stress components](image1)

![Figure 5.3 Out-of-plane stress components](image2)
5.3.1 Stress-Displacement Solutions

The first step involves determining stress-displacement solutions for a finite hole contained in an "infinite" body. The solutions are derived for a linear elastic, isotropic specimen by a method of superposition; the radial (u), circumferential (v) and axial (w) displacements are constructed by considering the individual contributions to the displacements stemming from each stress component individually. Begin by considering the radial (u) and circumferential (v) displacements due only to a uni-axial stresses $\sigma_0$ acting on a plate weakened by a circular hole of radius $a$ [32]:

$$u_o = \frac{\sigma_o}{8\mu r} \left[ (\kappa - 1)r^2 + 2R^2 + 2\left[R^2(\kappa + 1) + r^2 - \frac{R^4}{r^2}\right]\cos 2\theta \right] \tag{5.1}$$

$$v_o = -\frac{\sigma_o r}{4\mu} \left[ R^2(\kappa - 1) + r^2 + \frac{R^4}{r^2}\right] \sin 2\theta \tag{5.2}$$

where $R$ is the radius of the hole. For plane strain the lame constants $\mu$, and $\kappa$ can be written in terms of the elastic modulus (E) and poisons ratio ($\nu$), as [32]:

$$\mu = \frac{E}{2(1 + \nu)} \quad \kappa = 3 - 4\nu \tag{5.3, 5.4}$$

Substituting Equations 5.3 and 5.4 into 5.1 and 5.2 and evaluating the expressions for $\sigma_0 = \sigma_x$ and $r = a$ gives:

$$u_x = \frac{\sigma_x (1-\nu^2)a}{E} \left[ 1 + 2\cos 2\theta \right] \tag{5.5}$$

$$v_x = -\frac{2\sigma_x a(1-\nu^2)}{E} \sin 2\theta \tag{5.6}$$
Similarly, evaluating Equation 5.1 and 5.2 for expressions for $\sigma_o = \sigma_y$ and substituting $\theta$ with $90+\theta$ gives:

$$u(x) = \frac{\sigma_y a(1-v^2)}{E} [1 - 2 \cos 2\theta]$$

$$v(x) = \frac{2\sigma_y a(1-v^2)}{E} \sin 2\theta$$

Superposition of Equations 5.5 and 5.7 yields the radial displacement due to a combined $\sigma_y$ and $\sigma_z$:

$$u_{(x+y)} = \frac{2a(1-v^2)}{E} P + \frac{4a(1-v^2)}{E} Q \cos 2\theta$$

Similarly, the superposition of Equations 5.6 and 5.8 yields the circumferential displacement:

$$v_{(x+y)} = -\frac{4a(1-v^2)}{E} Q \sin 2\theta$$

Substituting $\theta$ with $45+\theta$ into Equations 5.10 and 5.11 yields the radial and circumferential displacements caused by the in-plane shear stress $T$:

$$u_{(in-plane\_shear)} = \frac{4a(1-v^2)}{E} T \sin 2\theta$$

$$v_{(in-plane\_shear)} = \frac{4a(1-v^2)}{E} T \cos 2\theta$$

Equations 5.10-5.13 provide the radial and circumferential components of displacement produced by in-plane stresses. To these must be added the displacements resulting from the axial stress ($\sigma_{zz}$) and the out of plane shear stresses ($\tau_{xz}$, $\tau_{yz}$).
The radial displacement due to the axial stress can be determined from Poisson’s ratio:

\[ u_{\text{axial\_stress}} = -\frac{a \nu \sigma_z}{E} \]  \hspace{1cm} (5.14)

The axial stress produces no contribution to the circumferential displacement. The radial and circumferential displacements for the out-of-plane shear are given as: [33]

\[ u_{(xz\_shear)} = \frac{\tau_{xz} z}{G} \cos \theta \]
\[ u_{(yz\_shear)} = \frac{\tau_{yz} z}{G} \sin \theta \] \hspace{1cm} (5.15, 5.16)

\[ v_{(xz\_shear)} = -\frac{\tau_{xz} z}{G} \sin \theta \]
\[ v_{(yz\_shear)} = \frac{\tau_{yz} z}{G} \cos \theta \] \hspace{1cm} (5.17, 5.18)

The entire radial displacement, \( u \), is given by adding Equations 5.10, 5.12, 5.14, 5.15 and 5.16:

\[ u = \frac{2a(1-\nu^2)}{E} P + \frac{4a(1-\nu^2)}{E} Q \cos 2\theta + \frac{4a(1-\nu^2)}{E} T \sin 2\theta - \frac{a \nu}{E} \sigma_z + \frac{z}{G} \tau_{xz} \cos \theta + \frac{z}{G} \tau_{yz} \sin \theta \] \hspace{1cm} (5.19)

Similarly, the circumferential displacement \( v \) is composed from Equations 5.11, 5.13, 5.17 and 5.18:

\[ v = -\frac{4a(1-\nu^2)}{E} Q \sin 2\theta + \frac{4a(1-\nu^2)}{E} T \sin 2\theta - \frac{z}{G} \tau_{xz} \sin \theta + \frac{z}{G} \tau_{yz} \cos \theta \] \hspace{1cm} (5.20)

The axial displacement \( w \) is found by first considering the following stress-strain relationship [32]:

\[ \varepsilon_z = \frac{1}{E} \left( \sigma_z - \nu (\sigma_x + \sigma_y) \right) \] \hspace{1cm} (5.21)
Substituting the definition of \( P \) and recognizing that \( \varepsilon_z = \frac{w}{z} \), the axial displacement due to the hydrostatic stress \( P \) and the axial stress \( \sigma_z \) is given as:

\[
W_{(\text{hydrostatic+axial})} = -\frac{2z\nu}{E} P + \frac{z}{E} \sigma_z
\]  \hspace{1cm} (5.22)

The axial component of displacement due to the out-of-plane shear is [33]:

\[
W_{(xz\text{-shear})} = \frac{a}{G} \tau_{xz} \cos \theta \hspace{1cm} W_{(yz\text{-shear})} = \frac{a}{G} \tau_{yz} \sin \theta
\]  \hspace{1cm} (5.23, 5.24)

The total displacement is found by adding Equations 5.22, 5.23 and 5.24.

\[
w = -\frac{2z\nu}{E} P + \frac{z}{E} \sigma_z + \frac{a}{G} \tau_{xz} \cos \theta + \frac{a}{G} \tau_{yz} \sin \theta
\]  \hspace{1cm} (5.25)

### 5.3.2 Geometry of the End Piece

The geometry of the end piece produces a transformation from the actual to the observed displacements which must be combined with the stress-displacement solutions. For a prism the transformation for each side of the prism can be derived by considering the simple case of a plane mirror inclined at an angle \( \beta \) to the incoming light ray, shown in Figure 5.4.
A light ray parallel to the axis of the hole will be reflected toward the sidewall of the hole at an angle of $2\beta$. When viewed from the top, the ray impinges on the mirror at a point with coordinates $x, y$, measured from the center of the hole (note: for pictorial clarity the mirror is not shown in the top view). The light is deflected in the $x$ direction to a point $x^*, y$ on the sidewall of the hole forming the angle $\phi$, where:

$$\phi = \sin^{-1}\left(\frac{y}{a}\right)$$  \hspace{1cm} (5.26)
Sidewall displacements in the direction of the sensitivity vector result in phase changes which in turn cause changes in the light intensity. The sensitivity vector is in the direction of the reflected light ray denoted by angle $2\beta$ to the sidewall surface in Figure 5.4. Consider a displacement $\delta$ in the direction of the sensitivity vector. In the radial direction (along the x-axis in Figure 5.4) $\delta$ transforms according to two factors: $\sin 2\beta$ (as seen from the side view); $\cos \phi$ (from the top view). The circumferential displacements, as seen from the top view, transform according to $-\sin \phi$. Displacements in the z direction, as seen from the side view, transform according to $\cos 2\beta$. The complete transformation between observed displacement $d_i$ and the three-dimensional displacements $u,v,w$, are provided by:

$$d_i = u \sin 2\beta \cos \phi - v \sin \phi + w \cos 2\beta$$  \hspace{1cm} (5.27)

where $u$, $v$ and $w$ are given by Equations 5.19, 5.20, and 5.25 respectively.

### 5.4 Least Squares Analysis

Equation 5.27 applies to each pixel within the illuminated area. It has two important characteristics: 1) the left hand side of the equation is what is measured, while the right hand side contains the desired quantities, thus presenting an inverse problem; 2) The matrix of observed displacements contain a substantial amount of data (640 x 480 pixels) from which only 7 values must be extracted. Seven independent basis functions can be developed from Equation 5.27 by substituting a unit stress for each of the components $P$, $Q$, $T$, $\sigma_{zz}$, $\tau_{xz}$, $\tau_{yz}$ and the rigid body displacement $\Delta$ and setting the remaining components to zero. Equation 5.27 can be rewritten as:
or in shorthand notation:

\[ \mathbf{B} \mathbf{s} = \mathbf{d} \]  \hspace{1cm} (5.29)

where \( \mathbf{B} \) is a matrix whose columns contain the basis functions, \( \mathbf{s} \) is a vector containing the stresses to be evaluated, and \( \mathbf{d} \) is a vector containing the desired stresses. Following the least squares method, both sides of Equation 5.29 are pre-multiplied by the transpose of matrix \( \mathbf{B} \), giving the normal equations:

\[ \mathbf{B}^T \mathbf{B} \mathbf{s} = \mathbf{B}^T \mathbf{d} \]  \hspace{1cm} (5.30)

Solving Equation 5.30 yields the coefficients of the \( \mathbf{s} \) vector: \( P \), \( Q \), \( T \), \( \sigma_{zz} \), \( \tau_{xz} \), \( \tau_{yz} \) and \( \Delta \).

### 5.5 Calculation Stability

The stability of the least squares calculation is highly dependant upon the quality of the data. Displacement data contain contributions from all six components of the stress tensor, each of which must be separated from the others and also from rigid body effects. Greatest calculation stability occurs when all of the basis functions are mutually orthogonal. If they are not orthogonal, they must at least be linearly independent and preferably greatly different from each other. In-plane shear effects \( \mathbf{(Q, T)} \) are rotated 90° to one another with \( \cos \theta \) and \( \sin \theta \) variation; Out-of-plane shear effects \( \mathbf{(\tau_{xz}, \tau_{yz})} \).
\( \tau_{\nu} \) are likewise related with \( \cos \theta \) and \( \sin \theta \) variation and consequently easy to distinguish. However the hydrostatic (P) and axial stress (\( \sigma_z \)) components are similar to one another and difficult to separate.

In mathematical terms calculation stability can be assessed in terms of the condition number of the matrix \( B^T B \) in Equation 5.30 [34]. The condition number describes the amplification of relative error in the data \( d \), that could appear in the resulting stress solution \( s \). A small condition number is desirable because it minimizes sensitivity to noise in the stress solution. In the case where basis functions are similar to each other, the condition number greatly increases and the stress solution becomes very sensitive to data error. In order to minimize the error, a reflector design should be chosen whose shape is as dissimilar as possible from the other basis functions.

### 5.5.1 Reflective End Piece Design

As indicated by Equation 5.27, the geometry of the end piece greatly affects the form of the basis functions. Thus the importance of calculation stability and data quality prompted the selection of the prism geometry. Initially, four potential reflector designs, shown in Figure 5.5, were considered: single angled mirror, two-mirror 'roof', three-face prism, spherical mirror. A fifth option, a conical mirror, was rejected at an early stage because it gives a basis function for the hydrostatic stress \( P \) that is identical to that of the rigid body motions.
The angled mirror design is simple to construct but has the disadvantage of illuminating less than half of the sidewall circumference. It also introduces an angular variation similar to \( \cos \theta \) or \( \sin \theta \) which is undesirable in its resemblance to the basis functions for \( \tau_{xz}, \tau_{yz} \). The two-mirror "roof" illuminates more of the sidewall circumference; however, it introduces an angular variation similar to \( \cos 2\theta \) or \( \sin 2\theta \) which is undesirable in its resemblance to the basis functions for \( Q \) and \( T \).

To better assess the sphere and prism designs, a mathematical model was developed which calculates the basis functions for each type of geometry and produces sample displacement images based on a unit stress. Figure 5.6 shows sample images for the sphere and the prism. Table 5.1 below shows the condition numbers for the matrix \( B^T B \) for simulated data corresponding to the four reflector designs shown in Figure 5.5.
Three-faced Prism

Spherical Mirror

Figure 5.6 Displacement Samples

Table 5.1 Reflector Design Condition Numbers

<table>
<thead>
<tr>
<th>Reflector Type</th>
<th>B(^T)B Condition Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angled mirror</td>
<td>4,500,000</td>
</tr>
<tr>
<td>Two-mirror roof</td>
<td>540,000</td>
</tr>
<tr>
<td>Three-faced prism</td>
<td>25,000</td>
</tr>
<tr>
<td>Spherical Mirror</td>
<td>6,000</td>
</tr>
</tbody>
</table>

The simulation reveals that the sphere provides the most favorable data (lowest condition number) due to the wide range of viewing angles. Furthermore, it illuminates the entire sidewall circumference and can be simple to construct and mount. Initially, the sphere design was incorporated into a larger...
scale model and tested. Figure 5.7 displays a hole lined with white graph paper imaged by the spherical reflector. As can be observed the corresponding variations in light intensity are large; high intensity reflections occur at the center of the sphere and intensity is reduced to nearly zero at the outermost perimeter. The uneven nature of the illumination proved to be too detrimental and after significant testing, the sphere design was rejected.

![Figure 5.7 Image of lined hole using a spherical mirror](image)

The next most desirable option is the three faced prism, whose condition number is not substantially greater than that of the sphere. Although difficult to construct, it introduces an angular variation similar to $\cos 30$ or $\sin 30$, which is wholly distinct from all other basis functions. Furthermore, it provides an even reflectivity and illuminates almost the entire sidewall circumference. Therefore, the three face prism was incorporated into the sensor design and used in a series of tests described in the following chapter.
Chapter 6
Experimental Validation

6.1 Imaging Tests
Once the sensor was assembled the first step was to ensure both object and reference beams were properly functioning. A tube with a 10 mm ID was lined with white, 1 mm increment graph paper to provide a reflective surface with easily identifiable reference points. With the piezo actuator removed, the sensor tube was placed into the hole and the side walls were imaged as shown in Figure 6.1. The three dark lines converging to the center are the 0.5 mm thick non-reflective edges of the three mirrors. The entire image is roughly 5 mm in diameter, and the faces of the prism illuminate a depth of around 3 - 4 mm. Next, the experiment was repeated with an aluminum specimen. A 10 mm hole was drilled into the aluminum and the surface was roughened using a fine grit sand paper. The illumination is slightly non uniform, with greater light intensity in the middle of each prism face due to the curvature of the hole. However it is sufficiently even to provide adequate illumination to all portions of the image.

![Lined Tube](image1)
![Aluminum Specimen](image2)

*Figure 6.1 Object Beam Test*
The next step was to ensure that the reference beam was properly functioning. The object path was blocked by a dark absorbent material and an image of only the reference beam was taken as shown in Figure 6.2. Small spots due to dust and an interference pattern due to internal reflection from the plate beam splitter are present. However the image has an even illumination and a sufficient area to ensure that the reference beam combines with the entire image beam.

![Figure 6.2 Reference Beam Test](image)

### 6.2 Functionality Test

With both the reference and object beam operating, tests were performed to verify the basic functionality of the sensor. The objective was to achieve distinct and stable fringes. The test was conducted with the sensor secured firmly to the optical bench, as shown in Figure 6.3. The end of the sensor was placed into an aluminum tube with an ID of 11.5 mm. A specially constructed mounting apparatus was used to secure the tube at both ends using set screws along the diameter.
Four sets of preliminary images were taken and stored. The setscrews were then tightened, resulting in a very small displacement of the tube which simulated deformation. The slightly oversized ID allowed the movement to occur without disturbing the sensor while the post displacement images were taken. Fringe patterns, as shown in Figure 6.4, were observed on all three faces of the prism and were maintained stable indefinitely thus verifying the functionality of the design. Subsequent tests were run by varying the light intensity and illumination to reference beam ratios. Prior experience with the ESPI equipment [19] indicates that the desired ratio of the reference beam to the object beam intensity should be around 1.8 - 2.5. However in this instance it was found that a ratio between 1.2 – 1.8 results in a more stable arrangement with the largest amount of functioning pixels within the illuminated area.
6.3 Stress Tests

With the ESPI sensor functioning the next step was to determine whether the observed displacements accurately indicate stress. This is achieved by subjecting a specimen to known loading conditions and comparing the observed displacements with the basis function displacement contours described in Chapter 5. Figure 6.5 below depicts three tests corresponding to specific components of the stress tensor: The first displays a specimen subjected to a uni-axial load, resulting in a combined hydrostatic and shear stress (Q); The second depicts a simply supported beam with the axis of the hole perpendicular to the applied load, resulting is a pure shear stress (T). The third test involves the simply supported beam with the axis parallel to the applied load, resulting in a shear stress ($\tau_{xz}$).
6.3.1 Compression Test

The first series of compression experiments were conducted using a Tinius-Olson uniaxial tension/compression test machine, shown in Figure 6.6. The machine consists of two large power screws mounted to a flat tabletop base and a rigid movable loading platform. The loading platform translates vertically and is actuated via the power screws which can apply a load of up to 55,000 lbs. Force transducers on each of the screws provide indication of the load to within +/- 0.01 lbs. A computerized data acquisition system monitors these signals, records the data and displays the loads.
The tests were conducted on a single specimen prepared from 6061 T6 aluminum and machined flat on all sides. The specimen is 2.0” x 3.9” x 10.2” and contains a through hole in the center of its width and one-quarter of the distance (2.6”) from one side. The hole was drilled and then precision reamed to a 25/64” diameter. The hole was roughened in order to provide a uniformly diffuse surface. Additionally four strain gauges were placed at the same height as the hole on two opposing faces of the specimen. The gauges serve two important functions: verifying the magnitude of the applied loads, and ensuring that loads are applied uniformly over the specimen so as to prevent unwanted bending moments.
The first test, corresponding to uni-axial loading, was conducted as shown in Figure 6.6. Since the testing required that the sensor be placed on its side, a custom designed stiff platform was glued directly to the specimen and the sensor was firmly clamped in place to it. Mounting the sensor directly to the specimen in this way minimizes any disturbances to the measurements during testing. Prior to testing, the specimen was loaded in increments of 1,000 lbs up to 15,000 lbs and the readings from the strain gauges were recorded. Subsequently, small pieces of .002 inch shim stock were placed as necessary at each of the four corners of the loading surface until the readings from each of the strain gauges were within 5% of one another. This ensured that the loading provided an even uni-axial stress and did not induce significant bending moments.

The sensor was placed into the hole of the specimen until the reflector prism was approximately halfway along the thickness and a live image was produced. Once a suitable reference/image beam ratio was achieved the sensor was clamped in place and the griping mechanism was engaged. The specimen was then loaded to a nominal load of about 500 lbs. It was necessary to wait a period of about 30 seconds in order to settle the effects of any gross movement. Once the live image had stabilized the pre-deformation set of four images was taken and stored. The machine was then loaded at the smallest possible rate of 0.02 linear inches per minute up to a load of 25000 lbs.

The loading of the specimen resulted in an almost immediate de-correlation (loss of speckle-to-pixel registration) of the image. De-correlation will occur when lateral deflections, exceeding half the size of a pixel, cause the speckles to "jump" from one pixel to the next. The de-correlation was confirmed visually as well as from the fluctuation of the calculated post-deformation step angles. The test was repeated several times with the same result. The test was then performed in reverse, by loading the
specimen to 25,000 lbs, taking an initial measurement and then relaxing the load. However the relaxation also resulted in an immediate de-correlation. Two other adjustments were made in hopes of improving the stability. First, the sensor was secured using an additional set of clamps and both the forward and reverse loading tests were performed. However de-correlation was again observed with little noticeable improvement. Second, a 2.5 mm diameter aperture of the tube assembly was placed in front of the imaging lens. Reducing the clear aperture increases the size of the speckles, thus resulting in an increased tolerance to disturbance. However the change produced little noticeable effect and de-correlation was still observed.

6.3.2 Optical Bench Tests

Once it was concluded that the disturbances produced by Tinius Olson testing machine were too large, a series of tests were designed to be conducted on the optical bench. It was surmised that the lower loads and additional stiffness afforded by the optical bench would result in a more stable arrangement. Figures 6.7 and 6.8 show the configurations for the optical bench tests and the test setup. The loads are applied by a loading plate, four ¼-20 cap screws and a precision ground drill rod. This loading arrangement ensures that no shear loads are transferred to the specimen. In configuration 1, a compressive load is distributed evenly over the top surface of the specimen by a load distribution plate. In configuration 2 a simple support with a centre load results in pure shear at the hole. Three new specimens were prepared: a 2” x 2” x 12”, UHDP; 2” x 3” x 3” polycarbonate; and 2” x 2” x 2”, 6061 T6 Al. The aluminum and polycarbonate were tested in configuration 1 while the UHDP was tested in configuration 2. The smaller surface area of the aluminum specimen and lower elastic modulus of the UHDP and polycarbonate ensured that sufficient displacements could be provided by the four ¼-20 screws.

64
Configuration 1

- Loading plate
- Precision drill rod
- Load distribution plate
- Specimen
- ¼-20 screws

Configuration 2

Figure 6.7 Optical bench test configurations

Figure 6.8 Optical bench test setup
Tests were conducted on all three specimens as follows: the sensor was firmly clamped to the optical bench and inserted into the specimen; the specimen was loaded with an initial preload; the gripping mechanism was engaged and an initial image was taken; the load was increased by tightening the four cap screws; post deformation images were taken. Despite the additional stiffness of the optical bench setup, de-correlation was still observed in both configurations and all three specimens. Furthermore the low reflectivity and high light absorption of the two plastic specimens made it difficult to achieve a large percentage of functioning pixels.

6.4 Discussion

Although significant effort was made to provide the most stable testing arrangement possible, the observed de-correlation highlights the need for increased rigidity within the sensor design and/or a less disruptive test setup. The functionality test described in Section 6.2 confirms that the sensor meets the most the basic requirements: the ability to make measurements of a hole surface at a significant depth; the ability to observe stable fringes with enough functioning pixels to perform the necessary calculations. However, the challenge is to achieve the required stiffness within the desired envelope. The long, slender geometry of the device and size of the measurement hole (10 mm) place challenging constraints on the lateral rigidity that can be achieved. Furthermore the sizes and dimensional tolerances were at, and perhaps at times beyond, those available in a conventional machine shop. Optical imagery, requires not only demanding tolerances, but also precision and cleanliness in the assembly process. Given the experience gained here and the availability of more specialized manufacturing facilities, there is a strong confidence that a second generation prototype of the type described here will be successful in making the desired measurements. This serves as the objective of future work as described in the following chapter.
A deep hole sensor has been developed for measuring the residual stresses within the interior of large metallic specimens. The specific objective involves gathering sufficient data to explicitly calculate all six components of the 3-dimensional stress tensor. This capability goes significantly beyond the 2-dimensional plane stress measurements that are available using current techniques. The inclusion of out-of-plane stresses provides a more realistic picture of the residual stresses contained within the interior of a specimen. This measurement capability is important for manufacturing process control, where it is desired to adapt processes to reduce harmful residual stresses.

The deep hole technique involves drilling an initial reference hole, inserting the sensor, and measuring the displacements of the hole as the surrounding material is overcored. These displacements involve a mixture of circumferential, radial, and axial components. The displacement measurement principle selected is based on Electronic Speckle Pattern Interferometry (ESPI). While ESPI has been demonstrated in applications involving surface stresses (i.e. hole drilling and ring coring), it had not yet been applied to deep interior stresses. In this study, the imaging technique was successfully adapted to the specific geometry required for deep hole applications.

The “full field” character of the ESPI technique enables displacement measurements to be made over an area comprising many thousands of data points. Therefore it provides a very rich data set from which all six desired stress components can be identified. The calculation of these stress components
from such a large set of data presents a significant mathematical challenge. The calculation technique proposed here uses a least squares method. The procedure involves evaluating the expected ESPI measurements corresponding to the unit values of the six residual stress components, each acting independently. These so called “basis functions” are evaluated analytically from close form stress-displacement solutions and the geometry of the reflector design. The least squares calculation identifies the six stress components from the best fit superposition of the basis functions within the measured data.

The basic functionality of the deep hole sensor was verified by displacement testing where stable, distinctive ESPI fringes were observed. So far, a similar level of functionality has not been achieved in conjunction with the application of known stresses. The sensor, as constructed, appears to be insufficiently rigid in order to maintain the required sub-micron positional accuracy during stress testing. As such, the sensor is susceptible to small disturbances which result in a de-correlation of the optical measurements. This issue remains to be resolved by a future re-design, in which the mechanical construction and manufacturing tolerances of the sensor will be adjusted to improve rigidity.

7.1 Key Findings
Although the last step of demonstrating the sensor under stress testing has yet to be resolved, this study has been successful in a number of key areas. Several issues were anticipated and addressed in the initial design; others came to light during the prototype testing and were resolved through design adjustment. The following paragraphs describe some of the principal challenges that were faced and overcome during the sensor development.
7.1.1 The occurrence of Newton's rings and the need for lens tilting

The occurrence of Newton's rings, as described in section 4.1.3, presented an unexpected obstacle to the ESPI sensor operation. The interference patterns result from the fact that light is passing through the imaging lens on both the forward path and return path. This is an unavoidable arrangement given the need to maximize the useable length of the tube assembly. Although the lens contains an antireflective coating, the interference is still large enough to impair the functionality of the design. Tilting the lens by an angle of 8 degrees provided an easy yet effective method for removing the Newton's rings pattern without significantly impairing the image quality or functionality of the sensor.

7.1.2 The importance of the reflector design and advantages and disadvantages of a sphere vs. prism reflector

As seen from Equation 5.18, the geometry of the reflector significantly affects the form of the basis functions. Therefore it is important to select a design which will insure calculation stability. As described in Section 5.5.1, computer modeling reveals that the sphere design presents the most theoretically attractive design, due to the large range of viewing angles. However experiments revealed that the large variation of light intensity in the sphere design so seriously impairs the optical image quality that useful measurements were not practical. Although difficult to manufacture, the three-faced prism design gives a more even illumination while still providing optical data suitable for individual stress identification.
7.1.3 The effect of internal reflection, light absorption and illumination and reference beam intensity

As noted in Section 4.2.2 testing revealed that more than 90 percent of the original illumination beam is dispersed and absorbed along the forward and return path. Given that only a small amount returns to form the actual image, the magnitude of internal light reflection within the base assembly can be on the order of or greater than that of the desired image. Therefore light management is a crucial aspect of the design and stray reflections on extraneous surfaces must either be avoided or minimized. This can be accomplished in a number of ways: collimating the light so that it impinges on as few surfaces as possible; inclining surfaces so that stray reflections are deflected in harmless directions; painting all surfaces black to maximize absorption and minimize reflection. Furthermore, the amount of light available for the reference beam is significantly larger than that of the image beam, and therefore it is necessary to provide a means of attenuating this light. As mentioned in Section 6.2, testing also revealed that for this design, a reference to illumination beam intensity ratio of 1.2 - 1.8 results in a higher number of functioning pixels.

7.1.4 The magnitude of the required stiffness

Although efforts were undertaken to insure a rigid design from the onset, the magnitude of the required stiffness did not become apparent until testing. As described in Section 6.3.1 and 6.3.2, testing revealed that small disturbances, such as those produced during specific loading, result in immediate pixel de-correlation. Given that the desired measurements are on the order of $\frac{1}{2}$ micron the sensor must be extremely stiff in order to maintain stability during operation. It is likely that the stiffness is most compromised by the long slender geometry of the tube assembly, the gripping mechanism, and the attachment of the tube assembly to the sensor base.
7.2 Future Work

The experience developed during this research study will greatly contribute to the success of a second generation prototype. The following list includes future work and design suggestions to be incorporated into the next generation prototype.

7.2.1 Explore alternative methods for conducting stress testing

The loading methods applied during the stress testing produce deflections and rigid body movements unlikely to be present during actual operation. Therefore, effort should be put towards designing a less disruptive test to apply a known stress field. Suggestions include: testing the sensor in a vertical configuration with the back end unconstrained; applying loads by non mechanical means (i.e. a shrink fit specimen); testing a residual stress specimen and using an alternate measurement method to correlate the results given by the sensor.

7.2.2 Increase the stiffness of the design in critical areas

The stiffness of the design can be improved in critical areas by a few simple methods: Replace the housing and gripping tube with a stiffer material and increase the thickness of the housing tube; replace press fit washers with a more secure means to transfer load to the gripper tube (i.e. a shoulder or groove in the gripper tube); mount the optics directly within the base assembly and increase the stiffness of the connection between the housing tube and the sensor base. Construct a stiff mounting bracket which firmly secures the base assembly to the test specimen.
7.2.3 Improve the manufacture of the reflective prism

The viewable area of the image can be increased by manufacturing the prism so that there are no unreflective edges. This can be accomplished by custom grinding a glass rod to the desired prism dimensions and then aluminizing the faces to provide the reflective surface.

7.2.4 Utilize a thinner half silvered mirror with antireflective coating

The use of a pellicle or beam splitter cube would eliminate the offset due to the thickness of the half silvered mirror (which is currently a plate beam splitter). Otherwise, an antireflective coating applied optimized for a 532 nm laser would also help reduce potential internal reflections occurring from the plate beam splitter.

7.2.5 Add reference beam attenuation and aperture adjustment features

Two additional features would greatly improve the functionality of the device. The attenuation feature should be redesigned so that attenuation can be achieved easily and without removing the piezo actuator. Also a feature should be added allowing easy adjustment of the aperture at the location of the imaging lens. This would facilitate experiments to determine the effect of changing the speckle size.

7.2.6 Manufacturing and assembly accuracy

The demanding optical tolerances and size of the manufactured parts required are difficult to achieve in a conventional machine shop. Enhanced precision in both the manufacturing and assembly process should significantly improve the sensor performance.
Bibliography


[33] G.S. Schajer, 2005. Personal communication