STRUCTURAL FAILURE IDENTIFICATION AND CONTROL OF THE MULTI-MODULE DEPLOYABLE MANIPULATOR SYSTEM (MDMS)

by

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ABSTRACT

This thesis focuses on the study of dynamics and control, and failure detection and identification (FDI) for the multi-module deployable manipulator system (MDMS), which has been designed and developed in our laboratory. The interest in this unique class of manipulator systems originated mainly from Canada's contribution to the International Space Station (ISS). In contrast to all the existing space-based robotic manipulators, which have revolute (slew) joints only, the MDMS has a combined prismatic (deployable, telescopic) joint and a revolute joint in each module, and several such modules connected in series to form the manipulator system. This novel design possesses several attractive features in dynamics and control; for example, reduced number of singular configurations, reduced dynamic interactions, and improved obstacle-avoidance capability for a specified number of degrees of freedom (joints). Research has been performed in the dynamics and control of this class of manipulators, particularly by our laboratory, which has resulted in the design and development of a prototype MDMS. The research presented in this thesis extends this founding work to explore possible types of structural failure in the MDMS and their accurate identification, and the use of kinematic redundancy and controllability to adapt to such failures.

Structural failures of a robotic system are critical in remote and hazardous environments such as space and nuclear active sites. If a joint of the manipulator fails in the course of a manoeuvre, it is immensely desirable to be able to complete the specified task automatically without immediate human intervention. This capability will result in higher system reliability, increased safety, better maintainability, improved survivability, and greater cost effectiveness. In the present work, methodology is developed for
accurate identification of structural failures in the MDMS, which through the use of a
decision making strategy, effective control and kinematic redundancy is capable of
satisfactorily executing the intended robotic task even in the presence of structural failure.
The method of failure identification is based on Bayes hypothesis testing. First, a
possible set of failure modes is defined, and a hypothesis is associated with each
considered failure mode. The most likely hypothesis is selected depending on the
observations of the response of the manipulator and a suitable test. This test minimizes
the maximum risk of accepting a false hypothesis, and accordingly the identification
methodology is considered optimal. The implementation of this approach to the
detection of structural failures in MDMS is presented in detail. In the developed
methodology, a bank of discrete Kalman filters is used for the computation of the
hypothesis-conditioned information about the MDMS, which is required in the decision
logic.

The developed failure identification methodology is rather general, and any
structural failure mode can be included in the detection strategy. For the purpose of
illustration, three specific joint failure modes are considered: sensor failure, locked joint,
and freewheeling or free-sliding joint. Successful identification of these joint failure
modes using the developed approach is demonstrated by implementation on a single- and
a two-module version of the MDMS. Furthermore, the failure detection and
identification (FDA) scheme has been incorporated into the control system for safe
operation under failures. Computer simulations are used to demonstrate how a two-
module redundant deployable manipulator is able to successfully complete a targeting
and pointing task in the presence of a failure where the shoulder joint of the manipulator
is locked. The present study provides a good foundation for further studies in fault
tolerance or fault recovery for this class of novel robotic manipulators.
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### LIST OF SYMBOLS

- $a_i$: joint trajectory equation coefficients
- $B$: risk function
- $C$: discrete system output matrix
- $C_{ij}^J$: equivalent viscous damping coefficient for the $i^{th}$ revolute joint
- $C_{vp}^P$: equivalent viscous damping coefficient for the transverse mode of vibration for the payload
- $C$: cost function
- $C_{ij}$: cost of accepting $H_i$ when $H_j$
- $d_i$: translation vector of $F_i$ from the tip of the $(i-1)^{th}$ module
- $D_i$: inertial position vector to the frame $F_i$, Fig. 2-1
- $dm_i$: mass of the infinitesimal element located on the $i^{th}$ body, Fig. 2-1
- $e$: vector of joint position error
- $EI$: flexural rigidity of the payload
- $f_{N+1}$: displacement of the mass element located at $r_{N+1}$ due to the flexibility of the payload, Fig. 2-1
- $f_{Y/H}(y/H_j)$: conditional probability density function of the observation vector given that $H_j$ is true
- $F$: vector containing the terms associated with the centrifugal, Coriolis, gravitational, elastic and internal dissipative forces, Eq. (2.1)
- $F_0$: inertial reference frame
- $F_i$: reference frame attached to the $i^{th}$ module
- $F_i$: actuator torque of the slew joint of the $i^{th}$ module
- $g$: acceleration due to gravity
- $g_i$: position vector of the frame $F_i$ relative to the frame $F_{i-1}$
- $H_i$: $i^{th}$ hypothesis
- $J_{ai}$: actuator rotor inertia
- $K_d$: derivative gain matrix of the PID controller
\( K_i \) integral gain matrix of the PID controller

\( K_p \) proportional gain matrix of the PID or FLT controller

\( K_i \) stiffness of the \( i^{th} \) revolute joint

\( K_v \) derivative gain matrix of the FLT controller

\( l_d, l_s \) length of deployable and slewing links, respectively

\( l_i \) length of the \( i^{th} \) module

\( l_{N+1} \) length of the payload

\( L(\ddot{x}(k/j)) \) admissible loss function of the estimation error

\( m_{ai} \) mass of the actuator located at the \( i^{th} \) revolute joint

\( M \) number of hypotheses

\( M \) coupled system mass matrix

\( \tilde{M} \) decoupled system mass matrix

\( \tilde{M}_i \) decoupled mass matrix of the \( i^{th} \) body

\( n_c \) number of controlled coordinates

\( N \) number of modules

\( P(H_i, H_j) \) joint probability that \( H_i \) is accepted and \( H_j \) is true

\( P(H_i / H_j) \) conditional probability of accepting \( H_i \) given that \( H_j \) is true

\( P^c \) matrix assigning the Lagrange multipliers to the constrained equations

\( P_{H_j} \) apriori probability of occurrence of \( H_j \)

\( q \) set of generalized coordinates leading to the coupled mass matrix \( M \)

\( \tilde{q} \) set of generalized coordinates leading to the decoupled mass matrix \( \tilde{M} \)

\( q_d \) desired values of the controlled generalized coordinates

\( q_c \) controlled component of the generalized coordinates

\( \tilde{q}_i \) set of generalized coordinates associated with the \( i^{th} \) body, leading to the decoupled mass matrix \( \tilde{M}_i \)

\( q_s \) specified component of \( q \)

\( q_u \) uncontrolled component of the generalized coordinates
$Q$ vector containing the external non-conservative generalized forces

$Q^d$ variance of disturbance in the input forces and torques provided by the motors in failure mode identification algorithm

$Q^a$ matrix assigning inputs to the actuated variables

$r_i$ position vector of the elemental mass $dm_i$ with respect to the frame $F_i$

$R_{a_i}$ inertial position vector of the actuator located at the $i^{th}$ revolute joint

$R_d$ Rayleigh dissipation function for the whole system

$R_{dm_i}$ inertial position vector to an elemental mass $dm_i$ on the $i^{th}$ body

$R^C, R$ first velocity transformation matrices

$R$ second velocity transformation matrix

$R$ measurement noise covariance in failure mode identification algorithm

$S$ hypothesis conditioned error covariance matrix

$t$ time

$T$ total kinetic energy of the system

$T_i$ actuator force of the deployable joint of the $i^{th}$ module

$T_i$ rotation matrix mapping $F_0$ onto $F_i$

$u$ vector containing control inputs

$u(k)$ discrete system input vector

$U_e$ total strain energy of the system

$U_g$ total gravitational potential energy of the system

$v(k)$ zero mean Gaussian white noise process in system outputs

$V_i$ covariance matrix of the hypothesis conditioned measurement error $y - \hat{y}_i$

$w(k)$ zero mean Gaussian white noise process in system inputs

$x_i, y_i$ Cartesian components of $r_i$

$x_d, y_d$ desired target

$x(k)$ discrete system state vector

$\hat{x}(k/j)$ estimate of $x(k)$

$\tilde{x}(k/j)$ state estimation error
$y$ observation vector

$y(k)$ discrete system observation vector

$\hat{y}_i$ expected value of the measurement vector conditioned on hypothesis $H_i$

$\tilde{y}$ observation estimation error

$Z$ observation space

$Z_i$ $i^{th}$ observation subspace

Greek Symbols

$\alpha_i$ rigid body angular motion of the revolute joint $i$

$\beta_i$ flexibility contribution to angular motion of the revolute joint $i$

$\delta_{N+1}$ generalized coordinate for the elastic deformation of the payload

$\phi$ desired pointing direction

$\Phi(k+1,k)$ discrete system state matrix

$\varphi_{N+1}$ mode shape function for the transverse deformation of payload

$\gamma_i$ rotation of the $i^{th}$ frame relative to the $(i^{th} - 1)$ frame

$\Lambda$ vector containing the Lagrange multipliers

$\psi_i$ inertial orientation of the frame $F_i$

$\rho_s, \rho_d$ linear mass density of slewing link and deploying link, respectively

$\rho_s, \rho_{d_i}$ linear mass density of $i^{th}$ slewing link and deploying link, respectively

$\rho_{N+1}$ mass per unit length of the payload

$\Gamma(k+1,k)$ discrete system input matrix

A dot above a character refers to total differentiation with respect to time.

A boldface lower case character denotes a vector.

A boldface upper case character denotes a matrix.

Subscript 'd' corresponds to deployable link.

Subscript 's' refers to the slewing link or a specified coordinate.
<table>
<thead>
<tr>
<th>Abbreviations, Acronyms</th>
<th>Description</th>
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<tbody>
<tr>
<td>ERA</td>
<td>European Robotic Arm</td>
</tr>
<tr>
<td>FDI</td>
<td>Failure Detection and Identification</td>
</tr>
<tr>
<td>FLT</td>
<td>Feedback Linearization Technique</td>
</tr>
<tr>
<td>ISS</td>
<td>International Space Station</td>
</tr>
<tr>
<td>LQR</td>
<td>Linear Quadratic Regulator</td>
</tr>
<tr>
<td>MBS</td>
<td>Mobile Remote Base System</td>
</tr>
<tr>
<td>MDMS</td>
<td>Multi-module Deployable Manipulator System</td>
</tr>
<tr>
<td>$O(N)$</td>
<td>Order- $N$</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional-Integral-Derivative</td>
</tr>
<tr>
<td>ROV</td>
<td>Remotely Operated Vehicles</td>
</tr>
<tr>
<td>SPDM</td>
<td>Special Purpose Dexterous Manipulator</td>
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<tr>
<td>SRMS</td>
<td>Shuttle Remote Manipulator System</td>
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<tr>
<td>SSRMS</td>
<td>Space Station Remote Manipulator System</td>
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CHAPTER 1  INTRODUCTION

1.1 Preliminary Remarks

The research presented in this thesis concerns the accurate identification of an existing failure mode of a robotic manipulator, and the use of that information together with a control strategy and possibly redundant degrees of freedom to satisfactorily complete the intended robotic task. This objective is achieved through developments in modeling, control, and failure identification methodology that are somewhat general, and then applying the developed methodology to an innovative multi-module deployable manipulator system (MDMS) through computer simulation. A prototype of the MDMS has been designed and developed in our laboratory. Even though the computer simulations are carried out using the parameters of this prototype manipulator, the experiments of failure identification are carried out through computer simulations only. This is justified because the analytical model of the laboratory prototype has been thoroughly validated in the Master's research of the candidate [1]. Since any attempt to generate physical failures in the prototype manipulator can harm the newly constructed manipulator, experimental work on the subject have to be conducted very carefully, and is not included in the present work. In the future, as possible faults manifest, the developed scheme can be implemented on the prototype to diagnose such faults, enabling further validation of the procedure developed here.

Robotic manipulators play an important role in space exploration and other critical tasks such as mechanical operations in nuclear active areas and medical environments, because of the harsh, unstructured or delicate operating environment and the challenges associated with it. As the technologies related to the automatic control of
robotic manipulators advance and as the required robotic tasks become increasingly critical, the issue of reliability becomes more and more important. Fault tolerance in robotics or other automation systems has been gaining attention in the research community. Fault tolerance in these systems can be divided into three basic areas of study: fault detection, fault identification, and system recovery. The ability to detect and identify a fault or a failure mode in a system and to continue the execution of the specified tasks in a satisfactory manner without the need for immediate human intervention are desirable. This results in higher system reliability, better maintainability, improved survivability, increased efficiency, and greater cost effectiveness. As an example, Figure 1-1 shows one of the many similar tasks that are performed by the Space Station Remote Manipulator System (SSRMS), also known as Canadarm2. In this image, the Mobile Remote Base System (MBS) is moved by the Canadarm2, for installation in the International Space Station (ISS). The Canadarm2 has seven degrees of mobility, and working in three-dimensional (3-D) space it has one redundant degree of mobility for performing an intended task. Other examples of robotic manipulators intended for space applications are the Special Purpose Dexterous Manipulator (SPDM), which is part of the SSRMS, and the European Robotic Arm (ERA). Both manipulators are expected to perform tasks on the ISS. Similar to these manipulators, most robotic arms have revolute joints only and also possess kinematic redundancies. It should be noted that revolute joint manipulators with deployable links (i.e., hybrid manipulators with both revolute and prismatic joints) have several advantages compared to the purely revolute-joint manipulators (articulate robots) with the same number of links.
These advantages include:

- simpler decision making requirements;
- reduced inertia coupling or dynamic interaction;
- better capability to avoid obstacles;
- reduced number of singular positions.

Figure 1-1  Installation of the Mobile Remote Servicer Base System (MBS) by Canadarm2. (Source: NASA Imagery)

This design idea has been implemented on some applications, although as a simplified version. Some examples of existing robotic arms that employ this idea are:

- Underground storage tank telescopic arm by Numet Engineering in Peterborough, Ontario
- Hydraulic excavators by Gradall Excavators in New Philadelphia, Ohio
- Telescopic Spraying Arm by Sika Schweiz AG Tunneling & Mining in Switzerland.
1.2 Objectives

The current research project aims at studying the dynamics and control of the novel multi-module deployable manipulator system (MDMS), particular with regard to employing its kinematic redundancy for failure recovery once a failure is correctly identified. Redundant manipulators have more degrees of freedom than are required to complete a specified task. The prototype MDMS consists of four modules connected in series, and each module has a revolute (slew) joint and a prismatic (telescopic or deployable) joint, and free to operate in a horizontal plane. Accordingly, it has eight degrees of freedom. To locate a general point in a plane only two generalized coordinates (or degrees of freedom) are required. Then there will be six degrees of redundancy in this manipulator, for a positioning task in a plane. If pointing the end effector (or, manipulator tip) in a specified direction (i.e., a pointing operation) is required in addition to reaching a specified point (i.e., positioning operation), then a total of three degrees of freedom will be required for the task in a plane. In that case, the MDMS prototype will have five degrees of freedom.

The redundancy typically can be used to optimize a design criterion, such as singularity handling, obstacle avoidance, joint torques minimization, and dynamic interaction (e.g., base reaction) minimization [2]. Another criterion that can be dealt with by the redundancy is adaptation to structural failure, or failure recovery, or failure resolution. This is an important requirement for space robotics because it is difficult to carry out a major repair on a space robot after it is launched. In this context, it is desirable to correctly identify (diagnose) the existing failure condition of the manipulator and to complete the task through proper control and possibly with the use of any
redundant degrees of freedom, without immediate human intervention. The main research objective of the present work is to develop an identification system for structural failures in an MDMS and to incorporate it into the control system of the MDMS with the aim of completing the intended task in a satisfactory manner under existing condition of failure.

1.3 A Brief Review of Relevant Literature

Since the late 1970s, the dynamics and control of orbiting platform-based robotic manipulator systems have been studied by several research groups. A new manipulator system involving slewing as well as deployable links, schematically shown in Figure 1-2, was first proposed for space applications by Marom and Modi. Planar dynamics and control of a single-unit mobile manipulator with rigid links and flexible joints, located on an orbiting flexible platform, were investigated [3]. Results showed significant coupling effects between the platform and the manipulator dynamics. Control of the system during tracking of a specified trajectory, using the computed torque technique, proved to be quite successful. Modi et al. [4] and Hokamoto et al. [5] - [7] extended the study to the multi-module configuration, referred to as the Variable Geometry Mobile Manipulator (VGMM) and the Mobile Deployable Manipulator (MDM) system, respectively. The model with an arbitrary number of modules accounted for the flexibilities in the joints as well as the links. A relatively general formulation for three-dimensional dynamics of the system in orbit was the focus of the study by Modi et al., while Hokamoto et al. explored a free-flying configuration. More recently, Hokamoto et al. [8] studied the control of the Mobile Deployable Manipulator with an arbitrary number of modules, each with two flexible links: one of them free to slew (revolute joint); and the other deployable
(prismatic joint). They developed a numerical procedure for the inverse kinematics of the system and demonstrated that trajectory control of the end-effector using the resolved acceleration approach was quite successful even in the presence of flexibility.

Figure 1-2 Multi-module Deployable Manipulator System (MDMS).

Pradhan et al. [9] reported an elegant procedure for the $O(N)$ formulation describing the three-dimensional motion of a system, in tree topology, using the Lagrangian approach. Caron extended the $O(N)$ procedure to account for system flexibility as well as deployment and applied it to study planar dynamics as well as control of a formidable multi-module deployable manipulator system [10]. The dynamical parametric study [11] clearly showed complex interactions between flexibility, librational dynamics, and manipulator manoeuvres.
Dynamicists have been consistently interested in the discretization procedure that would satisfactorily capture the effect of system flexibility. Zhang et al. [12] attempted to address this issue by:

- assessing the effect of number of modes on the response of a single module manipulator, on an orbiting flexible platform, during a prescribed manoeuvre;
- studying the effect of variation in flexibility of the revolute joints as well as slewing and deployable links;
- varying the speed of manoeuvre.

Results showed that the fundamental mode was able to capture the system response quite accurately. The joint flexibility as well as the speed of manoeuvre substantially affected the system response. However, the effect of link flexibility was relatively negligible.

Of particular significance is the recent contribution by Chen [13]. He developed for the first time, an $O(N)$ formulation for the three-dimensional dynamics of an orbiting flexible manipulator, with an arbitrary number of modules, traversing a flexible platform. He illustrated the application of the formulation by studying the dynamics of a multi-module system in the plane of the orbit. Control of the rigid degrees of freedom for a single module manipulator using the Feedback Linearization Technique (FLT) was also investigated. Recently, Yang et al. [14] & [15] extended the study to a two-module system with control of rigid as well as flexible degrees of freedom using the FLT in conjunction with a Linear Quadratic Regulator (LQR).

The use of kinematic redundancy of a space manipulator for the minimization of dynamic interactions and particularly the loads transmitted between the manipulator base
and the space structure has been analytically developed and verified by de Silva [2]. His method uses proper design of the trajectory of the robot and the formulation of the inverse kinematics of a general, redundant manipulator, to minimize the loads transmitted between the manipulator and the supporting structure. The approach has been applied to a practical space manipulator system.

The feedback linearization technique (FLT) is a popular approach for the control of highly nonlinear systems over a large operating range. Application of this approach to robotic manipulators has been formulated and demonstrated by De Silva and MacFarlane [16]. Subsequently, De Silva developed a computationally efficient recursive approach based on FLT to linearize and decouple a manipulator system for effective control [17].

With respect to joint failure in robotic systems, Ting et al. investigated internal shock phenomena due to the failure of joint actuation, and a recovery algorithm for both serial and parallel mechanisms under such circumstances [18]. Simulation results illustrated the effectiveness of this recovery algorithm, which attempts to reduce the internal shock when failure occurs. Alekseev et al. applied an observer-based approach to solve the problems of on-line failure detection in thrusters and sensors of Remotely Operated Vehicles (ROV). A bank of single-output reduced-order adaptive observers was used, with each observer estimating only one measurable variable whose real value was obtained by using the actual sensor under diagnosis [19]. In 1998, Kimura et al. proposed a decentralized autonomous control algorithm for hyper-redundant manipulators that uses parallel processing with low-performance processors to achieve adaptation to partial failure in space robotics [20]. A number of manipulator joints were locked at a certain angle in a computer simulation and the adaptability of the control
algorithm to these failures was assessed. The success rate of the control algorithm in completing the positioning task was found to be greater than 90%. In the following year, Shin and Lee presented a fault detection strategy and a robust fault recovery control scheme for robot manipulators to overcome the actuator failures [21]. The free-swinging joint failure, which caused the loss of torque at the joint, was considered. Simulation results for a three-link planar robot arm with a failed joint were presented. In 2000, Goel et al. proposed a method to detect and identify faults in wheeled mobile robots using an adaptive estimation procedure. It predicted the outcome of several faults, and collectively learned from it a failure pattern [22]. Each of the multiple parallel Kalman Filter (KF) estimators was tuned to a particular fault and predicted the expected values for the sensor readings. The difference between the estimated signals and the actual sensor readings was processed by a backpropagation neural network [23], and the particular fault was identified. In 2001, Liu proposed an adaptive fault tolerant control method to maintain the desired performance in the absence of actuation degradation, while monitoring the degradation online for detecting possible actuator failures [24]. The actuator failures were represented by a significant deviation in the value of the actuator torque coefficient matrix, which was an identity matrix in the case of no actuator failure. Next year, Song et al. combined two classes of important fault diagnosis methods: mathematical model-based approach including parameter estimation and state estimation and the artificial intelligence-based approach including expert systems, neural networks and fuzzy logic [25]. Neural networks method used the difference between the outputs of the estimation model and the robot manipulator for training and identifying any faults that may occur in the robotic system. In addition, a fixed dead zone scheme was
proposed to overcome the weight drifting of the neural network due to small disturbances in the system during training. More recently, Goel, Maciejewski and Balakrishnan analyzed unidentified locked-joint failures in kinematically redundant manipulators [26]. They examined the convergence issues as the manipulator comes to rest, and the corresponding terminal position and orientation of the end-effector. Conditions under which the manipulator converges were explicitly defined, and the workspace regions for which task completion might be possible could be identified even with unidentified failures.

1.4 Scope of Investigation

In the present investigation, first a computationally efficient simulation program is developed for a multi-module deployable manipulator system (MDMS). A physical prototype with four modules is shown in Figure 1-3, which has been designed and developed in our laboratory. The developed simulation program is based on a relatively general system model, which is a serial manipulator with an arbitrary number \( N \) of modules. Each module can perform both slewing (revolute) operations and deployment (prismatic or telescopic) operations, by rotating the link and changing the link length separately through two motors. The dynamics of the manipulator model subject to different system and manoeuvre parameters is studied. Automatic control of the manipulator system is carried out by implementing two control methods: proportional-integral-derivative (PID) control and feedback linearization technique (FLT) to this model. The performance of the MDMS with control, under various operating conditions is studied.
Proper operation and control are preconditions for the intended satisfactory operation of the MDMS under structural failure, by using the methodology developed in the subsequent phase of the present research. To realize the main end objective of the present research, a scheme is developed for detecting and identifying (i.e., diagnosing) a failure mode in a multi-module deployable manipulator system. This methodology may be applied for the detection and identification of any possible failure mode in the manipulator system. The failure modes incorporated in the present work are: sensor failure, free-wheeling joint failure, and locked-joint failure, which are similarly applicable to both revolute and prismatic joints. These failure modes are simulated in the MDMS, and the developed method is used to demonstrate the capability to accurately detect and identify each failure mode at any time during a robotic task. The capability to satisfactorily complete a specified task, after a structural failure, is studied by incorporating the methodologies of failure identification, control, and kinematic redundancy. It is demonstrated that a two-module deployable manipulator system is able to complete a specified targeting and pointing task in the event that one revolute joint is locked during the manoeuvre.
Figure 1-3  Prototype multi-module deployable manipulator system (MDMS).
In Chapter 2, the derivation of the governing equations of motion for a multi-module deployable manipulator system (MDMS) using the Lagrangian procedure and $O(N)$ approach is outlined. Validity of the formulation and the computer simulation code is checked through the conservation of energy for several cases.

A parametric study of the dynamic performance of a ground-based deployable manipulator system is presented in Chapter 3. The physical parameters used in the study are the same as those of the laboratory prototype. The effects of initial joint flexibility, speed of manoeuvre, different joint stiffness values, and different point payload mass are assessed.

In Chapter 4, the performance of a nonlinear control strategy based on the Feedback Linearization Technique (FLT) is compared with proportional-integral-derivative (PID) control. This is accomplished by implementing these two control algorithms to the $O(N)$ simulation program. A geometric derivation of the inverse kinematics algorithm for a two-module deployable manipulator, which is used later in performing a targeting and pointing task, is also presented.

Chapter 5 focuses on the development of a failure detection and identification (FDI) strategy for the MDMS. It is based on Bayesian hypothesis testing and is combined with the use of a bank of Kalman filters for optimally estimating the response of the manipulator for various failure modes. A set of equations for failure detection and identification as well as response estimation is developed. Furthermore, numerical models, which simulate three different failure modes, are obtained. They will be used in the demonstration of the FDI strategy in the subsequent chapter.
In Chapter 6, the capability of the developed strategy to successfully detect and identify joint failure is demonstrated, using the examples of a slew-deployable arm and a two-module deployable manipulator system. Later in the chapter, it is shown how a two-module deployable manipulator system is able to successfully complete a targeting and pointing task in the event that one revolute joint locks during the manoeuvre.

Chapter 7 is the concluding chapter of the thesis. It summarizes the important tasks of the thesis, outlines the main contributions, and suggests possible future avenues for further research on the subject.
CHAPTER 2 DEVELOPMENT OF DYNAMIC MODEL

2.1 Introduction

This chapter outlines the development of a dynamic model of the multi-module deployable manipulator, which will be used for subsequent studies in dynamics and control and also for illustration of the failure detection strategy developed in the present work. A detailed formulation of the model has been carried out in the previous research of the author [1]. A validation of both the model formulation and the computer code will be presented in the present chapter. A distinctive feature of the considered manipulator is the use of a sequence of "modules" each of which consisting of combination of a revolute and a prismatic joint. This innovative design has received little attention in ground-based applications although space-based manipulator systems with this feature have been investigated by us to some extent.

The robotic system under study has the following key features:

(a) It is a planar ground-based system that consists of a mobile base moving in a plane, a manipulator formed by an arbitrary number of two-degree-of-freedom modules attached to and supported by the base, and a flexible payload at the end of the last unit (i.e., at the end effector).

(b) Each unit comprises two "links" having two degrees of freedom: one link (or joint) is free to slew (i.e., a revolute joint), which carries a prismatic joint, which is permitted to deploy and retrieve. The revolute joint of the second module can be attached to the prismatic joint of the first module.

(c) The links are considered rigid while the revolute joints are known to possess some flexibility. This flexibility is mainly attributed to the harmonic drive gearing in the motors of the revolute joints (of the developed prototype).
(d) The damping in the flexible joints and the payload is accounted for through Rayleigh’s dissipation function.

The model considered here is rather general and applicable to a large class of ground-based manipulator systems that have fixed or variable link lengths and revolute/prismatic combinations. The governing equations are derived using the Lagrangian approach with specified coordinate motions constrained through Lagrange multipliers. As can be anticipated, the equations of motion are rather lengthy (depending on the number of bodies or units in the system), highly nonlinear, non-autonomous, and coupled, and can be expressed in the general and compact form:

\[ M(q,t)\ddot{q} + F(q,q,t) = Q(q,q,t) \]  

(2.1)

where \(M(q,t)\) is the system mass matrix; \(q\) is the vector of generalized coordinates; \(F(q,q,t)\) contains the terms of centrifugal, Coriolis, gravitational, elastic, and internal dissipative forces; and \(Q(q,q,t)\) represents generalized forces, including the control inputs. Eq. (2.1) describes the inverse dynamics of the system. For simulation of the manipulator motion, the so-called forward dynamics of the system is required; specifically, Eq. (2.1) must be solved for \(\ddot{q}\) using

\[ \ddot{q} = M^{-1}(Q - F) \]  

(2.2)

Solving this equation requires \(O(N^3)\) arithmetic operations in the computation of the inverse of the inertia matrix, \(M^{-1}\), where \(N\) denotes the number of bodies (or units) included in the manipulator model [9]. The computational cost can become prohibitive for large \(N\). For example, for a four-module manipulator, the computation time for the
inverse of the inertia matrix will be on the order of 4 and 64 for the $O(N)$ and $O(N^3)$ algorithms, respectively. Specifically, the latter scheme requires 16 times more computational time for the system dynamics and as a result the system bandwidth (possible maximum operating speed) drops by a similar factor. In view of this, an $O(N)$ algorithm, where the number of arithmetic operations increases linearly with the number of bodies in the system, is employed in the numerical simulations.

2.2 Equations of Motion

The serial manipulator considered here, supported on a mobile base, can be represented by an open chain of bodies (units) as shown in Figure 2-1. There are $N$ bodies, and a flexible payload attached at the end of the $N^{th}$ unit. Note that the length of the bodies 1 to $N$ can vary in time, simulating the presence of a prismatic link embedded in each revolute link, while the $(N+1)^{th}$ body is a flexible payload with fixed length. Furthermore, each body can rotate as well as translate with respect to its neighbours.

By the Lagrangian approach, the methodology consists of deriving the system energy using the decoupled set of coordinates $\tilde{q}$, and then converting it to the more convenient coupled set $q$. Here, $\tilde{q}$ is composed of the inertial positions and orientations, and the lengths of the units, the inertial angular position of the rotors in the revolute joints, and the time dependent generalized coordinates for the transverse elastic displacement of the flexible payload. In obtaining the coupled set of coordinates, the inertial rectilinear position of a unit is converted into the translation of the unit from the tip of the previous unit, and the inertial angular position of a rotor is converted into the angular motion of the rotor with respect to the stator in the revolute joint.
The conversion is done through the use of velocity transformations. The general equations of motion are obtained using the Lagrangian procedure as

$$\ddot{\mathbf{q}} = M^{-1} \mathbf{Q} - M^{-1} \left( \dot{\mathbf{M}} \dot{\mathbf{q}} - \frac{1}{2} \frac{\partial (\mathbf{q}^T \mathbf{Mq})}{\partial \mathbf{q}} \right) + \frac{\partial \mathbf{U}_e}{\partial \mathbf{q}} + \frac{\partial \mathbf{U}_w}{\partial \mathbf{q}} + \frac{\partial \mathbf{R}_d}{\partial \dot{\mathbf{q}}}$$

(2.3)
where $U, U_e,$ and $R_d$ are the gravitational potential energy, the strain energy and the energy dissipation, respectively; $Q$ represents the non-conservative generalized forces; and $M$ is the system mass matrix.

The inverse and the time-derivative of the mass matrix are computed as follows:

$$M^{-1} = R^{-1}(I - R^e)(\bar{M}^{-1}(I - R^e)^T R^{-T})$$

(2.4)

$$\ddot{M} = (\dot{R}^e)^T \ddot{R} R^e + (\dot{R}^e)^T \dot{M} \dot{R}^e + (\dot{R}^e)^T \ddot{R} \dot{R}^e$$

(2.5)

where $R, R^e,$ and $R^r$ are the velocity transformation matrices, and $\bar{M}$ is the system mass matrix associated with the decoupled set of generalized coordinates. A detailed description of these matrices is provided in Appendix I. Note that both matrices being inverted in Eq. (2.4); i.e., $R$ and $\bar{M}$, are block diagonal. Consequently, their inversion is an $O(N)$ process. Furthermore, the structure of the remaining matrices in Eq. (2.4) allows their multiplication to be of $O(N)$ as well. Thus, inversion of the system matrix, in terms of the coupled set of generalized coordinates, is now an $O(N)$ process, which is computationally much more efficient.

The coordinates required to describe the system kinematics are taken to be generalized coordinates in order to make the formulation as general as possible. However, it is often useful to specify some of these generalized coordinates, depending on the executed task. They are prescribed through constraint relations, which are introduced in the equations of motion through Lagrange multipliers. When constrained, Eq. (2.1) takes the form
\[ M\ddot{q} + F = Q^d u + P^c A \] (2.6)

where \( u \) is a vector containing the actuator forces and torques; \( Q^d \) is the matrix which assigns the components of \( u \) to the actuated variables; \( A \) is the vector containing the Lagrange multipliers; and \( P^c \) is the matrix assigning the multipliers to the constrained equations. The Lagrange multipliers can be found by solving the following equation:

\[ \ddot{q}_s + F^s - F^u u = F^c A \] (2.7)

This equation may be expressed as

\[ A = (F^s)^{-1} (\ddot{q}_s + F^s - F^u u) \] (2.8)

where \( \ddot{q}_s \) denotes the specified coordinates; and \( F^s, F^g, \) and \( F^u \) are the specified parts of \( F^s = M^{-1} P^c, F^g = M^{-1} F, \) and \( F^u = M^{-1} Q^d \), respectively. Consequently, the generalized acceleration vector may be computed from the following equation:

\[ \ddot{q}_g = F^g A - F^g + F^u u \] (2.9)

where \( F^g, F^g, \) and \( F^u \) are the generalized part of \( F^s, F^g \) and \( F^u \), respectively.

### 2.3 Verification of the Computer Code

Even with the formulation for planar dynamics and the computationally efficient \( O(N) \) approach, the numerical code is found to be rather lengthy. It involves more than 4,000 lines. The size of the governing equations of motion, in addition to the number of operations required to derive them, can easily lead to errors in the formulation and the
computer program. To some extent, these errors can be avoided through careful and systematic derivation and programming. Furthermore, errors can manifest themselves when the computer code is compiled and its constituting parts are linked, thereby resulting in compiling and linking errors. However, some errors may be quite elusive and require precise checks. One acceptable verification procedure is to check the conservation of energy of the system, in absence of energy dissipation. Another alternative is to match the simulation results with the results for specific cases studied by other researchers. Although the energy conservation approach does not provide a complete validation, it does serve as a "feel good" exercise to show that the computer code does not have any bugs. An experimental validation of the analytical model has been carried out in the Master's work of the candidate [1].

2.4 Conservation of Energy

In the absence of damping and external, non-conservative, generalized forces, the total energy of the system must remain constant. Thus, a variation in the total energy of a conservative system would indicate an error in the simulation program and/or in the derivation of the equations of motion.

A thorough check of the conservation of energy is performed on several cases, involving a variety of system parameters, initial conditions, number of bodies, and manipulator configurations. A check is considered successful if and only if the variation of energy is found to correspond to the numerical noise. This phase of the program verification is considered complete when the selected cases produce constant total energy. The following test cases, which serve as selected examples, illustrate the methodology adopted and the validity of the computer code for widely differing situations.
The parameter values used in the simulation are based on the constructed prototype manipulator [27], and are listed below:

- Slewing link length, \( l_s = 0.29 \) m;
- Deploying link length, \( l_d = 0.18 \) m;
- Slewing link linear mass density, \( \rho_s = 3.21 \) kg/m
- Deploying link linear mass density, \( \rho_d = 1.122 \) kg/m
- Revolute joint actuator mass:
  - Module #1: \( m_{a1} = 3.6 \) kg
  - Module #2: \( m_{a2} = 0.78 \) kg
  - Module #3: \( m_{a3} = 0.51 \) kg
  - Module #4: \( m_{a4} = 0.31 \) kg
- Revolute joint actuator inertia:
  - Module #1: \( J_{a1} = 1.2 \) kg\(m^2\)
  - Module #2: \( J_{a2} = 0.0816 \) kg\(m^2\)
  - Module #3: \( J_{a3} = 0.043 \) kg\(m^2\)
  - Module #4: \( J_{a4} = 0.015 \) kg\(m^2\)
- Revolute joint stiffness:
  - Module #1: \( K_1 = 10000 \) Nm/rad
  - Module #2: \( K_2 = 8830 \) Nm/rad
  - Module #3: \( K_3 = 1823 \) Nm/rad
  - Module #4: \( K_4 = 712.3 \) Nm/rad

First, the simple case of a double pendulum motion is simulated using a two-module system. It may be pointed out that the following tests are conducted with the manipulator moving in a vertical plane where the only restoring force arises from the gravitational pull. The two modules #3 and #4, with their length specified and fixed at
0.32 m are connected by a revolute joint, allowing relative rotational motion, as shown in Figure 2-2. Initially the modules are aligned at $80^\circ$ with respect to the vertical. With zero initial velocity and acceleration, the system is released at time $t = 0$. After 0.3 seconds, module #3 has moved to a new position ($\psi_3 = 68^\circ$), where $\psi_3$ is the orientation of module #3 with respect to the horizontal, while module #4 is lagging behind due to higher inertia about the system support. At $t = 0.4$ s, the manipulator reaches its lowest position where it is closest to the ground and moving with the highest velocity; i.e., its kinetic energy reaches the peak while the potential energy is at minimum. After crossing this point, the system starts to decelerate as it reaches higher heights. Then module #4 leads module #3 because it has a higher momentum and hence it is able to better overcome the retarding force. The system executes one-half cycle reaching the highest point on the opposite side at $t \approx 0.8$ s from where the reverse cycle begins. The corresponding variation in position, velocity and acceleration during the three-second-cycle are shown in Figure 2-3. It also shows the variations of the kinetic, potential and total energies. The deviation in the total energy from its initial value is around $10^{-5}\%$.

![Figure 2-2](image.png)

**Figure 2-2  Free oscillation of the manipulator model with the initial conditions $\psi_3 = 10^\circ, \psi_4 = 10^\circ$.**
Figure 2-3  Time histories of the system parameters during the energy conservation test – Case 1: \( \psi_3(0) = 10^\circ, \psi_4(0) = 10^\circ \).

The second case considers the application of an initial displacement to both rotational joints, as shown in Figure 2-4. The initial angle of module #3 is taken to be 45° with respect to the vertical, and module #4 is initially aligning with the horizontal axis; i.e., \( \alpha_4 = -45^\circ \) and \( \psi_4 = 0^\circ \), where \( \alpha_4 \) is the rigid angular displacement of module #4 with respect to module #3. As in the previous case, the initial velocity and acceleration
are taken to be zero, and the physical characteristics of the manipulator are kept the same. The simulation is run for 3 seconds, and the time histories for both degrees of freedom are recorded. The position, velocity and acceleration plots show that the manipulator goes through an oscillation cycle similar to the one observed in Case 1. The results clearly illustrate that the total system energy is conserved.

<table>
<thead>
<tr>
<th>Initial Conditions</th>
<th>Specified Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_3 = y_3 = x_4 = y_4 = 0$</td>
<td>$x_3 = y_3 = x_4 = y_4 = 0$</td>
</tr>
<tr>
<td>$\alpha_3 = \psi_3 = 45^\circ, \alpha_4 = -45^\circ, \psi_4 = 0^\circ$</td>
<td>$l_3 = l_4 = 0.32$</td>
</tr>
<tr>
<td>$\dot{\alpha}_3 = \dot{\psi}_3 = \dot{\psi}_4 = 0$</td>
<td>i.e. lengths are held fixed</td>
</tr>
</tbody>
</table>

Figure 2-4  Time histories of the system parameters during the combined slewing disturbance of $\psi_3(0) = 45^\circ$ and $\psi_4(0) = 0^\circ$. 

25
While the two cases given above simulate the dynamics in a vertical plane, the following case shows the manipulator motion in the horizontal plane. In this case, the initial displacement is applied to the flexible joint of module #4. Somewhat lower joint stiffness values are used; i.e., \( k_3 = k_4 = 5 \text{ Nm/rad} \), for a more significant vibration amplitude. Results are shown in Figure 2-5. The system energy is found to be conserved in this case as well, with the change of total energy in the order of \( 10^{-4} \% \).

The foregoing cases consider the system when subjected to an initial disturbance in terms of position. However, it would be beneficial to perform the energy conservation test with initial conditions involving parameters other than position alone.

Case 4 considers an initial impulse disturbance applied at the equilibrium point when the two-module manipulator system is initially aligned with the vertical, as shown in Figure 2-6. As in the previous cases, the lengths of the modules #3 and #4 are held fixed at 0.32m. An initial velocity input of \( \dot{x}_3 = \dot{y}_3 = 180 \text{ °/s} (\pi \text{ rad/s}) \) is given to the revolute joint of module #3, while keeping the second module undisturbed. At time \( t = 0 \), the kinetic energy is at its maximum value. In this position the manipulator is in its stable equilibrium orientation, where the potential energy is at minimum. The system swings upward reaching an elevation where its total kinetic energy is converted into potential energy; then, the reverse cycle begins. It should be noted that, even during such a complex motion, the system energy is conserved with a deviation in the order of just \( 10^{-5} \% \). These various cases provide considerable confidence in the accuracy and the completeness of both the formulation and the numerical integration code.
Figure 2-5  Verification of the conservation of energy through free vibration of the revolute joints in the horizontal plane.
Initial Conditions

- $x_3 = y_3 = x_4 = y_4 = 0$
- $\alpha_3 = \psi_3 = 90^\circ, \alpha_4 = 0^\circ, \psi_4 = 90^\circ$
- $l_3 = l_4 = 0.32$
- $\dot{\alpha}_3 = \dot{\psi}_3 = 180^\circ/s, \dot{\alpha}_4 = \dot{\psi}_4 = 0$
- i.e. lengths are held fixed

Specified Coordinates

- $x_3 = y_3 = x_4 = y_4 = 0$

Figure 2-6  Response of the system to an impulsive disturbance of $\dot{\alpha}_3 = \dot{\psi}_3 = 180^\circ/s$. 

---

**Figure 2-6  Response of the system to an impulsive disturbance of $\dot{\alpha}_3 = \dot{\psi}_3 = 180^\circ/s$.**
CHAPTER 3 STUDY OF THE SYSTEM DYNAMICS

3.1 Preliminary Remarks

The previous chapter outlined an approach for the development of a computer code to simulate the dynamics of a serial manipulator with an arbitrary number of slewing and deployable units (joints). The next logical step is to gain physical insight into the dynamics of this class of ground-based manipulators through parametric analysis and simulation, which is undertaken in the present chapter. Specifically, results are obtained and investigated from a study of uncontrolled system dynamics during a wide variety of parameters and initial conditions. This brings to light the coupling effects, namely the interaction dynamics, among the members of the robotic system. The intent is to establish useful behavioural characteristics of the time dependent variables of the system subjected to prescribed manoeuvres. For instance, data pertaining to the amplitude and frequency of vibrations at the joints will be useful in the design of the control algorithms and trajectory planning for the prototype manipulator. The rationale in the context of the present research is to gain a good insight into the dynamic behaviour of the manipulator system, which will lead to satisfactory controller development, and subsequently acceptable behaviour under faulty conditions by making use of the approach developed in the present research. The first set of simulation results focus on the effect of initial joint flexibility. Following that, various manoeuvre speeds, joint stiffness values and point payload masses are investigated.

The amount of information generated through this parametric study is considerable. For conciseness, only a few representative cases, illustrating the typical dynamic behaviour of the robot system, are reported here. It may be pointed out that
simulations presented in this chapter are carried out with the manipulator operating in the horizontal plane. Therefore, the parameter $g$ representing the Earth's gravitational acceleration is set to zero. In a sense, this would approximate the microgravity environment in space with the manipulator executing planar motion. Energy dissipation due to friction is not included for most cases in order to obtain the conservative system response.

3.2 Numerical Data

Unless otherwise specified, the following numerical values, which are based on the constructed prototype, are used during the simulation:

- Slewing link length, $l_s = 0.29$ m;
- Deploying link length, $l_d = 0.18$ m;
- Slewing link linear mass density, $\rho_s = 3.21$ kg/m
- Deploying link linear mass density, $\rho_d = 1.122$ kg/m
- Revolute joint actuator mass:
  - Module #1: $m_{a1} = 3.6$ kg
  - Module #2: $m_{a2} = 0.78$ kg
  - Module #3: $m_{a3} = 0.51$ kg
  - Module #4: $m_{a4} = 0.31$ kg
- Revolute joint actuator inertia:
  - Module #1: $J_{a1} = 1.2$ kgm$^2$
  - Module #2: $J_{a2} = 0.0816$ kgm$^2$
  - Module #3: $J_{a3} = 0.043$ kgm$^2$
  - Module #4: $J_{a4} = 0.015$ kgm$^2$
- Revolute joint stiffness:
Module #1: $K_1 = 10000$ Nm/rad
Module #2: $K_2 = 8830$ Nm/rad
Module #3: $K_3 = 1823$ Nm/rad
Module #4: $K_4 = 712.3$ Nm/rad

3.3 System Response

This section briefly summarizes principal findings of a comprehensive dynamical response study. The system variables are defined as shown in Figure 3-1. The developed simulation program is general such that it is capable of performing studies on manipulators with arbitrary number of modules; however, manipulator models of up to four modules were investigated since the constructed prototype manipulator system also has four modules.

Figure 3-1  Schematic representation of the manipulator model showing the system coordinates.
3.3.1 Initial Joint Flexibility Effect

The purpose of these simulation tests is to study the response behaviour of the manipulator when subject to initial angular deflection in any of the revolute joints. As a starting point, a single-module deployable arm with module #4 parameters was examined. An initial joint flexibility of 1 degree was given to the system, and the simulation ran for 0.2 second, which is selected to show a clear waveform of the response. In the absence of energy dissipation, and with the deployable link being held in place throughout the simulation, the single module oscillates about the revolute joint. Since there is not external input force or torque, the total energy of the system is conserved as expected and shown in Figure 3-2. It may be noted that this initial joint flexibility induces motion in the joint's rigid degree of freedom, which is initially zero. As the joint's flexible degree of freedom, \( \beta_4 \), goes through sinusoidal motion with 1° amplitude, the joint's rigid degree of freedom oscillates between 0° and approximately 1.4°. The resultant revolute joint motion is a sinusoidal oscillation between 1° and 0.4°. The last graph shows the kinetic, potential and total energy of the system. The interchange of kinetic and potential energy as well as the conservation of total energy can be observed.

System response with an initial joint flexibility of 5° is shown in Figure 3-3. Basically, the response is similar to the previous case, except higher amplitude of vibration. The maximum displacement in the rigid degree of freedom is 7°, or 1.4 times the initial joint flexibility as in the previous case. The total system energy is conserved throughout the simulation although the total system energy is higher because of higher initial system energy with greater value in the initial joint flexibility.
Figure 3-2 Response of the deployable arm to an initial joint flexibility of 1°: time histories of displacements and conservation of energy.
Figure 3-3  Response of the deployable arm to an initial joint flexibility of 5°: time histories of displacements and conservation of energy.

Furthermore, if the deployable link is let free to move, it will extend as the module oscillates. As the module extends, its mass moment of inertia about the axis of
rotation or oscillation increases, and this acts as a damping effect in the revolute joint motion. The time histories of the displacements and conservation of energy are shown in Figure 3-4. The recorded data shows that $\alpha_4$ is diverging while $\beta_4$ is decaying, both in a slow fashion, although it is not apparent from the plots. Nevertheless, the angular position of the module, $\psi_4$, is shown decaying with time as the damping effect from the extending module increases. The deployment of the module, $l_4$, is also shown, and the total energy of the system is conserved expectantly as shown in the bottom graph.

To study the dynamic coupling between modules, a two-module deployable manipulator system containing modules #3 and #4 is used in the following case. An initial joint flexibility of 5° was given to module #4, and the deployable links of both modules were free throughout the simulation. Results are shown in Figure 3-5. The initial joint flexibility in module #4 results in its own joint vibration as well as induced vibration in module #3 rotational motion, though of a smaller magnitude. Similar to the results in Figure 3-4, the deployable link of module #4 extends as the module oscillates. It is observed that the deployable link of module #3 retracts slightly although there is oscillation in its revolute joint. In fact, the centrifugal force extending the module #3 deployable link is overcome by the inertia force of module #4 slewing link when its deployable link extends. Conservation of total system energy is also observed.
Initial Conditions

\[ \alpha_4 = 0^\circ, \psi_4 = 5^\circ \]
\[ \dot{\alpha}_4 = \dot{\psi}_4 = 0 \]
\[ l_4 = 0.30 \]

Specified Coordinates

none

Figure 3-4  Response of the deployable arm to an initial joint flexibility of 5° with deployable link not held fixed.
Initial Conditions

\[ \alpha_3 = \alpha_4 = 0^\circ, \psi_3 = 0, \psi_4 = 5^\circ \]
\[ \dot{\alpha}_3 = \dot{\psi}_3 = \dot{\alpha}_4 = \dot{\psi}_4 = 0 \]
\[ l_3 = l_4 = 0.30 \]

Figure 3-5  Response of the two-module manipulator to an initial joint flexibility of 5° in module #4 with deployable links not held fixed.
3.3.2 Speed of Manoeuvre

The purpose of this second study is to investigate the effect of different speeds for a specified manoeuvre. The manoeuvre employed in this study is a 180° slew motion of modules #3 and #4, and is implemented using a sine-on-ramp profile. This profile ensures zero velocity and acceleration at the beginning and end of the manoeuvre. Modules #1 and #2 are held fixed in place. Two manoeuvre durations, 2 and 4 seconds, were simulated for comparison, and results are presented in Figure 3-6 and Figure 3-7, respectively. Time histories of modules #1 and #2 motion were omitted since these two modules were held fixed in place during the simulation. In both cases, the manoeuvre induces vibrations in the revolute joints, with higher magnitude in the case of faster manoeuvre, as shown in the plots of \( \beta_3 \) and \( \beta_4 \). Furthermore, these plots show that upon completion of manoeuvre, the vibrations in the revolute joints remain. However, in the prototype manipulator system with the presence of damping, these vibrations shall be reduced or diminish over time. Owing to the high stiffness in the revolute joints, the vibrations are negligible with respect to the joint positions as they are not observed in the graph of slew joint position.
Initial Conditions
\[
\begin{align*}
\alpha_1 &= \alpha_2 = \psi_1 = \psi_2 = 0 \\
\alpha_3 &= -90^\circ, \alpha_4 = 0 \\
\psi_3 &= -90^\circ, \psi_4 = -90^\circ \\
\dot{\alpha}_3 &= \dot{\psi}_3 = \dot{\alpha}_4 = \dot{\psi}_4 = 0 \\
l_1 &= l_2 = l_3 = l_4 = 0.30
\end{align*}
\]

Specified Coordinates
\[
\begin{align*}
\alpha_1 &= \alpha_2 = \psi_1 = \psi_2 = 0 \\
\alpha_3 &= 90^\circ, \alpha_4 = 0 \\
l_1 &= l_3 = l_4 = 0.30
\end{align*}
\]

Figure 3-6  Effect of manoeuvre speed - 180° slew manoeuvre in 2 seconds.
3.3.3 Different Joint Stiffness

The third system parametric study presented in this section aims at the investigation of different joint stiffness values. Same specified manoeuvre as in Figure 3-7. Effect of manoeuvre speed - 180° slew manoeuvre in 4 seconds.
3-6 was simulated with a different stiffness values. The stiffness values of all 4 revolute joints were set at 50 Nm/rad, which is much lower than the actual system parameters of the prototype. The joints are now more flexible, and higher magnitude in the flexible degrees of freedom shall be expected. Time histories of module #3 and #4 motion coordinates were graphed and shown in Figure 3-8. Significant difference in the flexible degrees of freedom was observed. The magnitude increases by approximately 10 times while the vibration frequency in the revolute joints is lower with smaller stiffness values.

3.3.4 Point Payload Mass

The last study performed in this research is on the effect of different point payload mass. Two cases are presented in this section: payload mass same as and five times one of the modules. The 4-module manipulator is simulated to go through the same manoeuvre as in Figure 3-6 for comparison. Results for the first case were shown in Figure 3-9. It was observed that in the presence of payload results in a larger magnitude of vibration in the revolute joints as the manipulator performs the manoeuvre. More significant vibration can be noted in the graph of \( \psi_3 \) and \( \psi_4 \) as compared to the case with no payload. Results for the second case with payload mass five times greater were graphed and shown in Figure 3-10. Compared to the first case, the revolute joints experience the same magnitude, but different frequency, of vibration.

In summary, initial joint flexibility in one revolute joint induces joint vibration in other joints, and results in extension or retraction in the deployable links through dynamic coupling. Higher manoeuvre speed results in lower magnitude, but higher frequency, of vibration in the joints. Lower joint stiffness values or more flexible joints yield higher magnitude, but lower frequency, of vibration in the joints. Finally, the presence of a
point mass payload induces vibration in the flexible joints. Furthermore, larger payload mass results in a lower frequency of vibration.

\[ K_{i=1,2,3,4} = 50 \text{ Nm/rad} \]

**Initial Conditions**

\[
\begin{align*}
\alpha_1 &= \alpha_2 = \psi_1 = \psi_2 = 0 \\
\alpha_3 &= -90^\circ, \alpha_4 = 0 \\
\psi_3 &= -90^\circ, \psi_4 = -90^\circ \\
\dot{\alpha}_3 &= \dot{\psi}_3 = \dot{\alpha}_4 = \dot{\psi}_4 = 0 \\
\ell_1 &= \ell_2 = \ell_3 = \ell_4 = 0.30
\end{align*}
\]

**Specified Coordinates**

\[
\begin{align*}
\alpha_1 &= \alpha_2 = \psi_1 = \psi_2 = 0 \\
\alpha_3 &\to 90^\circ, \alpha_4 = 0 \\
\ell_1 &= \ell_2 = \ell_3 = \ell_4 = 0.30
\end{align*}
\]

**Figure 3-8** Effect of joint stiffness - \( K = 50 \text{ Nm/rad} \) for all revolute joints.
payload mass = module mass

Initial Conditions
\[ \begin{align*}
\alpha_1 &= \alpha_2 = \psi_1 = \psi_2 = 0 \\
\alpha_3 &= -90^\circ, \alpha_4 = 0 \\
\psi_3 &= -90^\circ, \psi_4 = -90^\circ \\
\dot{\alpha}_3 &= \dot{\psi}_3 = \dot{\alpha}_4 = \dot{\psi}_4 = 0 \\
l_1 &= l_2 = l_3 = l_4 = 0.30
\end{align*} \]

Specified Coordinates
\[ \begin{align*}
\alpha_1 &= \alpha_2 = \psi_1 = \psi_2 = 0 \\
\alpha_3 &\to 90^\circ, \alpha_4 = 0 \\
l_1 &= l_2 = l_3 = l_4 = 0.30
\end{align*} \]

Figure 3-9  Effect of point payload mass — payload mass = module mass.
Figure 3-10 Effect of point payload mass – payload mass = 5 times module mass.
CHAPTER 4 CONTROLLED BEHAVIOUR OF THE MANIPULATOR

4.1 Introduction

The prescribed slew manoeuvres, as presented in the previous chapter, are achieved using constraint relations enforced through Lagrange multipliers. The generalized coordinates are instructed to follow specified paths with the assumption that the actuators can deliver instantaneously the forces and torques required to execute the maneuver. In other words, the assumption of an ideal system is made whose actuators possess the necessary power deliver the required forces and torques as functions of time. The simulation presented in the previous chapter proved useful in understanding the dynamic behaviour of the manipulator system under various operating conditions. A manipulator is required to perform a desired task with a specified level of accuracy. This requires the operation of the system using a controller. In reality, then, the performance of any robotic system is governed by among other things the capability of its actuators and the performance of the controller which regulates its dynamics.

This chapter presents the implementation of the controllers in the \( O(N) \) deployable manipulator model, and evaluation of the resulting performance. A rationale is that in order to somehow achieve satisfactory performance of the manipulator under a faulty condition, the non-faulty manipulator must exhibit good controlled behaviour as a precondition. Two control laws, Proportional-Integral-Derivative (PID) control and Feedback Linearization Technique (FLT), are considered for assessment of the controller performance and for comparative evaluation. These two methods are utilized to control
the rigid degrees of freedom, which are specified explicitly or computed from specified manoeuvres in the workspace using the pertinent inverse kinematics algorithm.

4.2 System Response to Manoeuvring Commands

This section presents the manoeuvring operations of joint position tracking of the manipulator system first using a PID controller and then incorporating the FLT. For joint position tracking, a sine-on-ramp displacement profile, which assures zero velocity and zero acceleration both at the beginning and the end of the manoeuvre, is used to represent the reference input.

4.2.1 Proportional-Integral-Derivative Control

A Proportional-Integral-Derivative (PID) controller of the following form is implemented to the $O(N)$ model of the manipulator system:

$$ u = K_p e + K_i \sum e \Delta t + K_d \dot{e} $$  \hspace{1cm} (4.1)

In the controller Eq. (4.1), $e$ and $\dot{e}$ are vectors of joint position and velocity errors, respectively; $\Delta t$ is the control rate; $u = [T_1, F_1, T_2, F_2, \ldots, T_n, F_n]^T$, where $T_i$ and $F_i$ are the actuator torque and force of the slew and the deployable joints, respectively of the $i^{th}$ module; and $K_p$, $K_i$, $K_d$ are the proportional, integral and derivative gain matrices, respectively. The gain matrices are $n \times n$ diagonal matrices with the non-zero terms equal to the gain values for the slew and the deployable joints, given in the same sequence as in $u$. The controller gains are chosen and tuned manually to achieve satisfactory performance.
In the first simulation, the shoulder joint of a two-module manipulator is to commanded to go from $0^\circ$ to $120^\circ$ in 5 seconds following a sine-on-ramp displacement profile, while keeping the whole manipulator straight; i.e., $\alpha_4 = 0$. Initially, both modules are aligned at $0^\circ$, and during the manoeuvre, the deployable joints in both modules are specified as being locked. Figure 4-1 shows the time histories of the controlled slew manoeuvres of both shoulder and elbow joints. The joint trajectory tracking errors and the controller commands are shown as well. With some initial experience, the PID gains for both revolute joints are set at 50, 0, and 10, respectively. It is noted from the results that the joint trajectory tracking for both revolute joints are fairly accurate. In the graph showing the $\alpha_3$ tracking, the desired and the controlled joint trajectories are so close that they appear to overlap. The elbow joint position deviates from the desired value owing to the dynamic coupling when the shoulder joint is in motion. However, with the employed set of controller gains the deviation is quite small. The maximum errors for the shoulder and the elbow joints are $0.2^\circ$ and $0.06^\circ$, respectively. The maximum actuator torque of the shoulder joint is much higher than that of the elbow joint because the former is executing a dynamic manoeuvre while the latter is maintained at the original position. With manually tuned controller gains, the required actuating torque in either joint is within the capability of the actuators in the constructed prototype manipulator.

Next, the two-module manipulator is commanded to bend its elbow by slewing the elbow joint through $120^\circ$ in 5 seconds. The parameters of the modules #3 and #4 as presented in Chapter 3 are used in this simulation, and the results are shown in Figure 4-2. The PID gains used in this simulation are 50, 0, and 20, respectively. To maintain the
module #3 in its original position, a maximum torque of approximately 0.05Nm is required, limiting the deviation to within 0.05°. For the manoeuvre of the outer module, a maximum torque of less than 0.03Nm is required. As in the previous case, the required maximum torque values are within the capability of the servo actuators of the laboratory prototype. The tracking of both joints are successful, with the maximum errors limited to 0.05° and 0.03°, respectively.

Summarizing, a PID controller has been successfully implemented in the general $O(N)$ model of the manipulator system, and the ability to track joint trajectories with a high level of accuracy has been demonstrated through simulation studies for a two-module deployable manipulator.
Initial Conditions
\[
\begin{align*}
\alpha_3 &= \alpha_4 = \psi_3 = \psi_4 = 0 \\
\dot{\alpha}_3 &= \dot{\psi}_3 = \dot{\alpha}_4 = \dot{\psi}_4 = 0 \\
l_i &= l_2 = l_3 = l_4 = 0.30 \\
\dot{l}_i &= \dot{l}_2 = \dot{l}_3 = \dot{l}_4 = 0
\end{align*}
\]

Controlled Coordinates
\[
\alpha_3 \rightarrow 120^\circ, \alpha_4 = 0
\]

Specified Coordinates
\[
l_i = l_2 = l_3 = l_4 = 0.30
\]

Figure 4-1  A 120° manoeuvre by a two-module manipulator system using PID control.
Initial Conditions
\[ \alpha_3 = \alpha_4 = \psi_3 = \psi_4 = 0 \]
\[ \dot{\alpha}_3 = \dot{\alpha}_4 = \dot{\psi}_3 = \dot{\psi}_4 = 0 \]
\[ l_1 = l_2 = l_3 = l_4 = 0.30 \]
\[ \dot{l}_1 = \dot{l}_2 = \dot{l}_3 = \dot{l}_4 = 0 \]

Controlled Coordinates
\[ \alpha_3 = 0, \alpha_4 \rightarrow 120^\circ \]

Specified Coordinates
\[ l_1 = l_2 = l_3 = l_4 = 0.30 \]

Figure 4-2  A 120° manoeuvre by the elbow joint of a two-module manipulator system using PID control.
4.2.2 Control Using Feedback Linearization Technique

Since a satisfactory dynamic model of the manipulator is available, as introduced in Chapter 2, the Feedback Linearization Technique (FLT) may be employed to realize a linear behaviour in the nonlinear manipulator system. Then a linear compensator may be used to achieve the desired system response. This method has been used by others for the control of robots; for example by Spong [28] for manipulators with flexible joints. The advantages of the approach are: (i) the linearization is global and not local about an operating point; (ii) the control algorithm based on the FLT is straightforward; and (iii) the compensator design for a feedback linearized model is similar to that for an originally linear system. In fact, in the present work, the resulting uncoupled and linearized equations of motion are controlled through a conventional proportional-derivative (PD) controller. This control strategy is implemented in the $O(N)$ model for controlling the motion of the slewing and deployable joints.

The equations of motion of the $O(N)$ manipulator model as given in section 2.2 may be expressed in the compact form

$$M\ddot{q} + F = Q^d u + P^e A$$

(4.2)

It is noted that the offset positions of the manipulator units ($\mathbf{d}_i, i = 2, 3, \ldots, N$) are constrained to remain fixed, and this requires the use of Lagrange multipliers. The controlled degrees of freedom do not require any constraints, and the flexible revolute joints are left free during motion. The substitution of $A$, which is solved using (2.8), into (4.2) yields the form
\[ M\ddot{q} + F' = Q'_d u \]  

(4.3)

where \( F' = F - P^c (F_s^s)^{-1}(\dot{q} + F_s^s) \) and \( Q'_d = Q^d - P^c (F_s^s)^{-1} F_s^u \). The solution for \( \ddot{q} \) gives

\[ \ddot{q} = -\bar{F}' + \bar{Q}'_d u \]  

(4.4)

where \( \bar{F} = M^{-1}F' \) and \( \bar{Q}'_d = M^{-1}Q'_d \). The vector \( \ddot{q} \) containing the generalized accelerations can be divided into a controlled component \( \ddot{q}_c \) and an uncontrolled component \( \ddot{q}_u \). It should be noted that the uncontrolled component also includes the specified degrees of freedom in \( \ddot{q} \). Thus, Eq. (4.4) can be rewritten as

\[
\begin{bmatrix}
\ddot{q}_c \\
\ddot{q}_u 
\end{bmatrix} = -\begin{bmatrix} 
\bar{F}' \\
\bar{F}_u' 
\end{bmatrix} + \begin{bmatrix}
\bar{Q}'_d \\
\bar{Q}'_d u 
\end{bmatrix} u 
\]  

(4.5)

Considering only the equation of controlled dynamics, one has

\[ \ddot{q}_c = -\bar{F}_c + \bar{Q}_d u \]  

(4.6)

In theory, the desired control input can be obtained by solving Eq. (4.6) for \( u \), thus

\[ u = (\bar{Q}_d)^{-1}(\bar{Q}_d + \bar{F}_c) \]  

(4.7)

where \( \ddot{q}_d \) represents the desired value of \( \ddot{q}_c \), and the term \( \bar{F}_c \) offsets the nonlinear effects of the system. However, it can be shown from Eq. (4.5) that the uncontrolled degrees of freedom in the controller model affect and are affected by the control action. In order to ensure a robust behaviour of the controller, \( \ddot{q}_c \) is taken to have the form [29]
\[
\dot{q}_c = \dot{q}_d + K_v (\dot{q}_d - \dot{q}_c) + K_p (q_d - q_c)
\]  

(4.8)

where \(q_c\) consists of the controlled generalized coordinates, \(q_d\) consists of their desired values, and \(K_v\) and \(K_p\) are the derivative and proportional gain matrices, respectively:

\[
K_v = \begin{bmatrix}
K_{v1} & 0 & 0 & \cdots & 0 \\
0 & K_{v2} & 0 & \cdots & 0 \\
0 & 0 & K_{v3} & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & K_{v_{n_c}}
\end{bmatrix}
\]  

(4.9)

and

\[
K_p = \begin{bmatrix}
K_{p1} & 0 & 0 & \cdots & 0 \\
0 & K_{p2} & 0 & \cdots & 0 \\
0 & 0 & K_{p3} & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & K_{p_{n_c}}
\end{bmatrix}
\]  

(4.10)

Here \(n_c = 2N\) is the number of controlled coordinates. The diagonal elements of both matrices are scalars selected in order to satisfy the control specifications; for example, the prescribed requirements of settling time, maximum permissible overshoot, and so on. By equating (4.6) and (4.8) and solving for the control input \(u\), one obtains

\[
u = (\tilde{Q}_{dc})^{-1} [\bar{F}_c + \tilde{q}_d + K_v (\dot{q}_d - \dot{q}_c) + K_p (q_d - q_c)]
\]  

(4.11)

A block diagram of the FLT controller is shown in Figure 4-3.
The performance of the FLT controller is now assessed using several test cases involving a two-module deployable manipulator. A suitable performance requirement for the controller gains $K_p$ and $K_v$ would be to achieve a critically damped response $q_c$ as well as reduction of the position error to 2% of its initial value in 5 seconds for the slew and the deployment degrees of freedom. With these requirements, the controller parameter values $K_{pi} = 1.326$ and $K_{vi} = 2.652$ for $i = 1, 2, \ldots, n_c$ are used in the simulations.

In the first case, the controlled response of the system is studied for a sine-on-ramp manoeuvre involving a slew from $\alpha_4 = 50^\circ$ to $\alpha_4 = 60^\circ$ with the outer module being extended from $l_4 = 0.3$ m to $l_4 = 0.35$ m. Figure 4-4 and Figure 4-5 demonstrate the
effectiveness of the FLT controller in this combined manoeuvre. The control inputs and the response of the controlled degrees of freedom are presented. It is noted that the controller is able to hold the module #3 in its original position. The deviations in both revolute and deployable joints #3 are negligible. As for the trajectory tracking in the joints of module #4, it is noted that the joints follow the desired trajectories with high precision. The maximum errors in the slew and deployable joints are 0.008° and 0.015mm, respectively. This level of performance is achieved with neither trial and error nor tuning of the controller gains, which makes the simulation studies quite easy to perform.

The second case illustrates the controlled response of the system during a 120° slewing manoeuvre of the inner module of a two-module manipulator. The two modules are to be held aligned at all times during the manoeuvre. This case is identical to the PID controlled case shown in Figure 4-1. Again, a sine-on-ramp profile is used as the desired joint trajectories. The results are shown in Figure 4-6. It is seen that the tracking error of the shoulder joint manoeuvre is 0.1°, an improvement from 0.2° for PID controller with tuned controller gains. Furthermore, the elbow joint is held within 0.09° of its original position. In addition, the controller has been able to stop any motion in the deployable joints.

The final case illustrates the controlled response of the system during a 120° slewing manoeuvre of the elbow joint. This case is comparable to the PID controlled case shown in Figure 4-2. The results are shown in Figure 4-7. Since both PID and FLT controllers perform successfully, as clear from the results presented in this chapter, either
of them may be used in the control of joint motions in the presence of joint failure, in conjunction with the approach developed in the next chapter.

<table>
<thead>
<tr>
<th>Initial Conditions</th>
<th>Controlled Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_3 = 0, \alpha_4 = 50^\circ )</td>
<td>( \alpha_3 = 0, \alpha_4 \to 60^\circ )</td>
</tr>
<tr>
<td>( \psi_3 = 0, \psi_4 = 50^\circ )</td>
<td>( l_3 = 0, l_4 \to 0.35m )</td>
</tr>
<tr>
<td>( \dot{\alpha}_3 = \dot{\psi}_3 = \dot{\alpha}_4 = \dot{\psi}_4 = 0 )</td>
<td>( )</td>
</tr>
<tr>
<td>( l_1 = l_2 = l_3 = l_4 = 0.30 )</td>
<td>( )</td>
</tr>
<tr>
<td>( \dot{l}_1 = \dot{l}_2 = \dot{l}_3 = \dot{l}_4 = 0 )</td>
<td>( )</td>
</tr>
</tbody>
</table>

---

**Figure 4-4** Controlled response of module #3 of the two-module manipulator system during a 10° slew and 0.05 m deployment: control inputs and response of the controlled degrees of freedom.
Figure 4-5  Controlled response of module #4 of the two-module manipulator system during a 10° slew and 0.05 m deployment: control inputs and response of the controlled degrees of freedom.
Initial Conditions
\[ \alpha_3 = \alpha_4 = \psi_3 = \psi_4 = 0 \]
\[ \dot{\alpha}_3 = \dot{\psi}_3 = \dot{\alpha}_4 = \dot{\psi}_4 = 0 \]
\[ l_1 = l_2 = l_3 = l_4 = 0.30 \]
\[ \dot{l}_1 = \dot{l}_2 = \dot{l}_3 = \dot{l}_4 = 0 \]

Controlled Coordinates
\[ \alpha_3 \to 120^\circ, \alpha_4 = 0 \]

Specified Coordinates
\[ l_1 = l_2 = l_3 = l_4 = 0.30 \]

Figure 4-6  FLT control of a two-module manipulator system with its shoulder joint executing a 120° manoeuvre.
Initial Conditions
\[ \alpha_3 = \alpha_4 = \psi_3 = \psi_4 = 0 \]
\[ \dot{\alpha}_3 = \dot{\psi}_3 = \dot{\alpha}_4 = \dot{\psi}_4 = 0 \]
\[ l_i = l_2 = l_3 = l_4 = 0.30 \]
\[ \dot{l}_i = \dot{l}_2 = \dot{l}_3 = \dot{l}_4 = 0 \]

Controlled Coordinates
\[ \alpha_3 = 0, \alpha_4 \rightarrow 120^\circ \]

Specified Coordinates
\[ l_i = l_2 = l_3 = l_4 = 0.30 \]

Figure 4-7  FLT control of a two-module manipulator system with its elbow joint executing a 120° manoeuvre.
4.3 Inverse Kinematics Algorithm

Chapters 5 and 6 develop an approach to demonstrate the ability of a manipulator to recover or properly adjust in the incident of a joint failure, by successfully identifying the failure. In order to implement the approach, the inverse kinematics of the manipulator has to be formulated. In this section, an inverse kinematics algorithm for a two-module deployable manipulator is presented. The two-module manipulator has four degrees of freedom. Three degrees of freedom are needed to carry out tasks of targeting (positioning) and orienting the end effector in a plane. Hence, if one of the joints would fail, the manipulator could still complete the task provided that it is achievable. When a failure is detected in any one of the joints, with the inverse kinematics algorithm, a new desired joint trajectory can be computed for the three operational links and provided for the controller. For demonstration purposes, only the algorithms for no failure and failed shoulder joint are given here.

When both modules of the manipulator system are fully operational, the system has one redundant degree of mobility for planar tasks. Rather than tackling a redundant system, here the length of the first deployable link is specified. The other three joint variables are then solved geometrically. Figure 4-8 shows a schematic diagram of a two-module deployable manipulator performing a targeting and pointing task. With \((x_d, y_d)\) and \(\phi\) being the desired target and the pointing direction, respectively and \(l_1\) fixed at an arbitrary value, one can express the elbow joint locus with respect to the manipulator tip as:

\[
(x_d - l_2 \cos \phi)^2 + (y_d - l_2 \sin \phi)^2 = l_1^2
\] (4.12)
This is a quadratic equation in \( l_2 \), which, upon solving, yields the following results:

\[
l_2 = (x_d \cos \phi + y_d \sin \phi) \pm \sqrt{(x_d \cos \phi + y_d \sin \phi)^2 - (x_d^2 + y_d^2 - l_1^2)}
\]  

(4.13)

\[
\theta_i = \arctan\left(\frac{y_d - l_2 \cos \phi}{x_d - l_2 \sin \phi}\right)
\]

(4.14)

and

\[
\theta_2 = \phi - \theta_i
\]

(4.15)

Figure 4-8  Targeting and pointing tasks for a two-module deployable manipulator.

Out of the two possible solutions for \( l_2 \), the smaller value is preferred for the reasons of less movement and lower inertia of module #2, and consequently less effort for the slew manoeuvre of this module. This computed \( l_2 \) is also checked to guarantee that it is greater than 0.3m, the fully retracted length of each of the modules of the constructed deployable manipulator system. One sample case of this algorithm, point-to-
point manoeuvre, is shown in Figure 4-9. Initially, the two modules are aligned with the x-axis. The desired target is at +0.1m in both x and y directions, and the desired final pointing direction is 15°. With the desired final tip position and orientation, the desired joint values for the three operational links are computed, and they are reached using a sine-on-ramp trajectory. The computed trajectory in the workspace is computed from the joint coordinates, and the pointing direction is just the sum of $\theta_1$ and $\theta_2$. With a sine-on-ramp trajectory, the joints are ensured to have zero velocities as well as finite accelerations at the beginning and the end of the manoeuvre.

The second configuration considered here is where the shoulder joint of the two-module manipulator has failed. When this happens, the revolute shoulder joint is locked at some position, and the other three joint positions, which are required to complete the desired manoeuvre are determined using the inverse kinematics algorithm presented below.

When $\theta_1$ is fixed, in order to achieve the desired orientation, $\theta_2$ has to be equal to $\phi - \theta_1$. The corresponding deployed lengths of the modules can be computed by expressing the x and y elbow joint coordinates in terms of $\theta_1$ and $l_1$, and $\theta_2$ and $l_2$, respectively, and equating them to the x and y coordinate expressions:

\[
\begin{align*}
\text{x - coordinate:} & \quad l_1 \cos \theta_1 = x_d - l_2 \cos \phi \\
\text{y - coordinate:} & \quad l_1 \sin \theta_1 = y_d - l_2 \sin \phi
\end{align*}
\]

Solving these two equations simultaneously yields the following expressions for $l_1$ and $l_2$. 

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\[ l_1 = \frac{x_d - l_2 \cos \phi}{\cos \theta_2} \]  
\[ l_2 = \frac{y_d - x_d \tan \theta_2}{\sin \phi - \tan \theta_2 \cos \phi} \]  

The same manoeuvre as the one shown in Figure 4-9 is carried out with the shoulder joint held fixed at some position. The resulting joint trajectories for the other three degrees of mobility are shown in Figure 4-10. The shoulder is kept locked at the
initial position; i.e., $\theta_1 = 0^\circ$, throughout the manoeuvre. The s ine-on-ramp trajectories of the other three joints are shown in the figure. Upon solving the required deployed lengths for both modules, they are checked against the physical joint limits of the existing prototype manipulator modules. Inverse kinematics algorithms for other joint failure scenarios are solved in a similar manner. For illustration purposes, only two cases have been presented here.

![Graphs of trajectories](image)

**Figure 4-10** An illustrative result of inverse kinematics computation — slew link #1 fixed.
CHAPTER 5  STRUCTURAL FAILURE DETECTION

5.1 Introduction

The previous chapters presented the foundation for the research presented in this thesis. In particular, an analytical model was developed for an innovative multi-module deployable manipulator system (MDMS), which has been designed and developed in our laboratory. Using this model, the performance of the manipulator system was evaluated through computer simulation, using the parameter values of the prototype manipulator. Subsequently, two control schemes: proportional-integral-derivative (PID) and feedback linearization technique (FLT) were implemented on the manipulator model and the resulting performance was evaluated. Also, the inverse kinematics formulation of the analyzed manipulator was developed. This foundation work is needed for carrying out the main goal of the present work: structural failure identification for the MDMS. The reason is proper performance of the non-faulty manipulator system is a precondition for the expected satisfactory performance of the manipulator system under structural malfunctions, with the use of the developed approach.

This chapter presents the development and formulation of the methodology for structural failure identification for the multi-module deployable manipulator system (MDMS). The possible structural failure modes in the MDMS include the following:

- Total failure
  - Locked joint (due to jamming of transmissions or bearings, etc.)
  - Free-swinging of a revolute joint or free sliding of a prismatic joint (due to disengagement of transmission, loss of power in actuator, etc.)
  - Data acquisition failure at joint
• Sensor failure at joint

• Partial failure

• Slip in joint coupling (due to increased backlash, partial disengagement of transmission, wear, actuator drive malfunction, etc.)

• High resistance in joint motion (due to misalignment in transmission and bearings, lubrication problems, etc.)

• Increased vibration (due to malfunction in gear, actuator, or bearings, etc.)

The identification process involves two major tasks: decision and estimation. A decision as to which failure mode currently exists in the manipulator system is made in an optimal way on the basis of the costs associated with various decisions (correct and incorrect). The development starts with the introduction of the general Bayesian hypothesis-testing method for the detection of a failure mode. A more specialized case using the assumption of a Gaussian problem, which is implemented and used in the computer simulation, is also included. The Gaussian assumption is justified in view of the familiar central limit theorem [30], and is commonly used in the engineering systems of the category considered in the present work. The formulated decision logic requires the measurement errors and the corresponding covariance matrices, conditioned on each hypothesis at each sampling instant. This information is obtained by estimation using a bank of Kalman filters. Section 5.4 reviews the discrete filter theory and presents the algorithm used to obtain the hypothesis conditioned information. The last section discusses the consideration of numerical simulation with emphasis on joint failure detection. Numerical simulation of several failure modes, which will be used for detection, is also presented.
5.2 Bayesian Hypothesis Testing

The main goal of the research presented in this thesis is to correctly detect the operating state of a manipulator system, whether fully operational or experiencing some structural failure, and satisfactorily bring the task to a conclusion using the detected information and proper control. In applying Bayes hypothesis testing, first the system is assumed to take one of several, say $M$, disjoint states each of which is associated with a considered failure mode. Then, the most likely hypothesis is selected depending on the observations and based upon a suitable test. Common tests used for this purpose are:

- Bayes test
- Likelihood ratio test
- Minimax test

The Bayes test is a general method whereas the other two are special cases derived from it. In the likelihood ratio test, it is assumed that different hypotheses have the same apriori probabilities of occurrence. The minimax test is derived by minimizing the maximum risk. The main task here is to find the Bayes risk function corresponding to the least favourable apriori probability distribution. Then the test is derived by using the general Bayesian procedure.

The general Bayesian hypothesis testing algorithm is derived by first defining $M$ disjoint states or hypotheses, denoted by $H_1, H_2, H_3, \ldots, H_M$. The observation space $Z$ is partitioned into $M$ subspaces, denoted by $Z_1, Z_2, Z_3, \ldots, Z_M$. Then

$$Z = \bigcup_{i=1}^{M} Z_i$$
If the observation vector falls into a particular observation subspace $Z_i$, then $H_i$ is chosen as the most likely hypothesis. Symbolically, this may be expressed as

$$y \in Z_i \Rightarrow \text{accept } H_i$$ (5.1)

The subspaces $Z_i$ are determined by minimizing the risk function $B$, which is defined as the expected value of the cost function, denoted by $C(H, H)$. This cost function specifies the penalty or loss or cost of making an incorrect decision. Here $H$ denotes the hypothesis set $\{H_1, H_2, \ldots, H_M\}$. Particularly, $C_y$ is the cost of accepting $H_i$ when $H_j$ is true. The risk function can be expressed as

$$B = E[C(H, H)] = \sum_{i=1}^{M} \sum_{j=1}^{M} C_y P(H_i, H_j)$$ (5.2)

where $P(H_i, H_j)$ is the joint probability that $H_i$ is accepted and $H_j$ is true:

$$P(H_i, H_j) = P(H_i / H_j) P_{H_j}$$ (5.3)

and

$$P(H_i / H_j) = \int_{\hat{z}_i} f_{y/H} (y / H_j) \, dy$$ (5.4)

where

$P(H_i / H_j)$ is the conditional probability of accepting $H_i$ given that $H_j$ is true,

$P_{H_j}$ is the apriori probability of occurrence of $H_j$. 

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and

\[ f_{Y_i|H}(y/H_j) \] is the conditional probability density function of the observation random vector given that \( H_j \) is true. The substitution of the above equations into (5.2) yields

\[
B = \sum_{i=1}^{M} \sum_{j=1}^{M} C_{ij} P_{H_j} \int_{Z_j} f_{Y_i|H} (y/H_j) \, dy
\]  

(5.5)

Since

\[
\sum_{i=1}^{M} \int_{Z_j} f_{Y_i|H} (y/H_j) \, dy = 1
\]

one has

\[
0 = \sum_{j=1}^{M} C_{ij} P_{H_j} - \sum_{j=1}^{M} \sum_{i=1}^{M} C_{ij} P_{H_j} \int_{Z_j} f_{Y_i|H} (y/H_j) \, dy
\]

Adding this to (5.5), the risk function may be expressed as

\[
B = \sum_{j=1}^{M} C_{ij} P_{H_j} + \sum_{i=1}^{M} \left( \sum_{j=1}^{M} \beta_y \right) \, dy
\]

(5.6)

where

\[
\beta_y = (C_y - C_{ij}) P_{H_j} f_{Y_i|H} (y/H_j)
\]

(5.7)

The first term on the right hand side of (5.6) is a constant and therefore can be disregarded in the minimization. In addition, from (5.7) it is clear that
\[ \beta_i = 0 \text{ for all } i \]

and

\[ \beta_j > 0 \text{ for } i \neq j \]

because, for a realistic problem with non-zero probability density functions, one has

\[ C_j > C_i \text{ for any } i \text{ and all } j \neq i \]

and

\[ P_{H_j, f_{Y/H}}(y/H_j) > 0 \]

Consequently, the risk function may be simplified to give

\[ B^* = \sum_{i=1}^{M} \int \left( \sum_{j=1, j \neq i}^{M} \beta_j \right) dy \] (5.8)

For a particular observation vector \( y \), if the minimum of \( \sum_{j=1, j \neq i}^{M} \beta_j \) corresponds to \( i = l \), then we accept \( H_l \) as the most likely hypothesis because then and only then each integral in the summation over \( i \) will be at its minimum, hence resulting in the minimum of \( B^* \).

Suppose that the apriori probabilities of occurrence of different hypotheses are the same; specifically,

\[ P_{H_j} = P_{H} \text{ for all } j \]

and the cost of selecting a correct hypothesis is zero:

\[ C_j = 0 \text{ for } i = j \]

\[ = C > 0 \text{ for } i \neq j \]
Then

\[ \sum_{j=1, j \neq i}^{M} \beta_{ij} = CP_{H} \sum_{j=1, j \neq i}^{M} f_{Y_i / H_j}(y / H_j) \]

Since the minimum of this expression is obtained if and only if the largest term in the summation is eliminated, the decision logic becomes

Accept \( H_i \) if \( f_{Y_i / H_i}(y / H_i) \geq f_{Y_i / H_j}(y / H_j) \) for all \( i \neq l \) \hspace{1cm} (5.9)

5.3 Gaussian problem

Commonly, the input to the manipulator system and the measurements through the sensors contain disturbances and noise, especially in the presence of any structural failure or malfunction. A common assumption is that the disturbances and noise are independent zero mean Gaussian white noise vectors. This is a common assumption for engineering systems, and is justified in view of the central limit theorem [30]. If the system initial states are Gaussian, then the state vector is also Gaussian because it is a linear combination of the initial state and the input disturbances. Accordingly the measurement vector is also Gaussian. Therefore, one has

\[ f_{Y_i / H_j}(y / H_j) = \frac{1}{(2\pi)^{m/2} |V_i|^{1/2}} e^{-\frac{1}{2}(y - \hat{y}_i)^T V_i^{-1}(y - \hat{y}_i)} \] \hspace{1cm} (5.10)

where

\( \hat{y}_i \) is the expected value of the measurement vector conditioned on hypothesis \( H_i \),

and
$V_j$ is the covariance matrix of the hypothesis conditioned measurement error \( y - \hat{y}_j \).

Since logarithm is a monotonic function, the test (5.9) is equivalent to the following:

\[
\text{Accept } H_i \text{ if } \ln f_{y/Hj}(y/H_j) > f_{y/Hj}(y/H_j) \text{ for all } i \neq l
\]

Applying (5.10) yields

\[
\text{Accept } H_i \text{ if } \alpha_i \leq \alpha_i \text{ for all } i \neq l \tag{5.11}
\]

where

\[
\alpha_i = \ln |V_j| + \frac{1}{2} (y - \hat{y}_j)^T V_j^{-1} (y - \hat{y}_j) \tag{5.12}
\]

This test is used in the studies carried in this thesis.

5.4 Hypothesis Conditioned Information

The decision logic developed in the previous section requires the information of the measurement errors \( y - \hat{y}_j \) and the corresponding covariance matrices \( V_j \) conditioned on each hypothesis \( H_j \) at every instant when the system is checked for any failure and a decision is made. These can be scheduled to happen either at each sampling instant or after a number of observations is accumulated. The latter may produce a more accurate decision because more information is available at each hypothesis testing stage. On the other hand, if a wrong decision were made, the system would operate erroneously for a longer period of time before the next decision stage when a correction might be made. In
the current research, a decision is made at every sampling instant. The hypothesis conditioned errors and the covariance matrices are obtained from a bank of $M$ Kalman filters, using the algorithm presented in this section.

Consider the manipulator system with dynamics given by the following discrete state-space equations

$$x(k+1) = \Phi(k+1,k)x(k) + \Gamma(k+1,k)u(k) + \Gamma(k+1,k)w(k) \quad (5.13)$$

$$y(k+1) = Cx(k+1) + v(k+1) \quad (5.14)$$

where $w$ and $v$ are independent zero mean Gaussian white noise processes so that

$$E[w] = 0 \quad (5.15)$$

$$E[v] = 0 \quad (5.16)$$

$$E[ww^T] = 0 \quad (5.17)$$

$$E\left[w(j)w^T(k)\right] = \delta_{jk}Q \quad (5.18)$$

$$E\left[v(j)v^T(k)\right] = \delta_{jk}R \quad (5.19)$$

The initial state vector is a Gaussian process, which is independent of the input disturbances and measurement noise, and the mean and the covariance matrix are given by

$$E[x(0)] = \bar{x}(0) = x_0 \quad (5.20)$$
Considering the estimate $\hat{x}(k/j)$ of $x(k)$, given the set of past observations \( \{y(1), y(2), \ldots, y(j)\} \), the optimal estimate is obtained by minimizing the cost function $E[L(\hat{x}(k/j))]$, and is expressed as

$$\hat{x}(k/j) = E[x(k)/y(1), y(2), \ldots, y(j)]$$  \tag{5.22}$$

Here $L(\hat{x}(k/j))$ is an admissible loss function of the estimation error $\hat{x}(k/j) = x(k) - \hat{x}(k/j)$.

From (5.13), one has

$$x(k) = \Phi(k, j)x(j) + \sum_{i=j+1}^{k} \Phi(k, i) \Gamma(i, i-1)[u(i-1) + w(i-1)].$$

By taking the conditional expectation of both sides given $\{y(1), y(2), \ldots, y(j)\}$ and with the assumption that $w(i)$ is independent of $y(j)$ for $i > j$, the optimal estimate of (5.22) is derived as

$$\hat{x}(k/j) = \Phi(k, j)\hat{x}(j/j) + \sum_{i=j+1}^{k} \Phi(k, i) \Gamma(i, i-1) u(i-1)$$  \tag{5.23}$$

The conditional distribution of a Gaussian random vector $x$ given another one $y$, is Gaussian with mean and the covariance matrix as the following [31]:

$$E[x/y] = \bar{x} + P_{xy} P_{yy}^{-1}(y - \bar{y})$$  \tag{5.24}$$
\[ P_{x/y} = P_{xx}^{-1} P_{xy} P_{yy}^{-1} \]

(5.25)

where

\[ E[x] = \bar{x} \]
\[ E[(x - \bar{x})(y - \bar{y})^T] = P_{xy} \]

Considering the Gaussian random vectors \( x, y, \) and \( z \), one has

\[ E[x/y,z] = E[x/y,z] = E[x/y] + E[x/z] - \bar{x} \]

(5.26)

where

\[ \bar{z} = z - E[z/y] \]

Since \( x(0) \) and \( w(i) \) are Gaussian, \( x(k) \) is merely a sum of Gaussian random vectors, and therefore is Gaussian. Similarly, \( y(k) \) is also Gaussian because \( v(i) \) and \( x(i) \) are Gaussian.

Now, using (5.22) and (5.26), one can obtain,

\[ \hat{x}(k+1/k+1) = E[x(1)/y(1), y(2), \ldots, y(k+1)] \]
\[ = E[x(1)/y(1), \ldots, y(k)] \]
\[ + E[x(1)/\tilde{y}(k+1/k)] - \bar{x}(k+1) \]

(5.27)

where

\[ \tilde{y}(k+1/k) = y(k+1) - E[y(k+1)/y(1), \ldots, y(k)] \]

(5.28)
Substitution of (5.23) into (5.27) yields

\[
\hat{x}(k+1/k+1) = \Phi(k+1,k)\hat{x}(k/k) + \Gamma(k+1,k)u(k) + E[\hat{x}(k+1) / \hat{y}(k+1/k)] - \bar{x}(k+1)
\]

(5.29)

By taking the expected value on both sides of (5.28) and from a fundamental result on conditional expectation, the following result is obtained:

\[
E[\hat{y}(k+1/k)] = 0
\]

(5.30)

From this result and (5.24), the expectation expression in (5.29) becomes

\[
E[\hat{x}(k+1) / \hat{y}(k+1/k)] = \bar{x}(k+1) + K(k+1)\hat{y}(k+1/k)
\]

(5.31)

where

\[
K(k+1) = P_{x\hat{x}}P_{\hat{y}\hat{y}}^{-1}
\]

(5.32)

Also, from (5.28), (5.14) and (5.16), one has

\[
\hat{y}(k+1/k) = y(k+1) - C\hat{x}(k+1/k) = C\hat{x}(k+1/k) + v(k+1)
\]

(5.33)

Furthermore, from (5.30), (5.16) and (5.19), one has

\[
P_{\hat{y}\hat{y}} = E[(C\hat{x}(k+1/k) + v(k+1))(C\hat{x}(k+1/k) + v(k+1))^T]
\]

\[
= CP(k+1/k)C^T + R
\]

(5.34)

where
\[ P(k/j) = E[\tilde{x}(k/j)\tilde{x}^T(k/j)] \] (5.35)

Since, from (5.13) and (5.23),

\[ \tilde{x}(k + 1/k) = \Phi(k+1,k)\tilde{x}(k/k) + \Gamma(k+1,k)w(k), \]

and \( \tilde{x}(k/k) \) is independent of \( w(k) \), the expected value in (5.35) may be expressed as

\[ P(k + 1/k) = \Phi(k+1,k)P(k/k)\Phi^T(k+1,k) \]
\[ + \Gamma(k+1,k)Q\Gamma^T(k+1,k) \] (5.36)

Another term in (5.32) is derived, based on the independence between the estimation error and the optimal estimate, as

\[ P_{\hat{y}} = E[(\tilde{x}(k+1/k) + \hat{x}(k+1) - \tilde{x}(k+1))(C\tilde{x}(k+1/k) + v(k+1))^T] \]
\[ = P(k+1/k)C^T \] (5.37)

The equations derived in this section are used to compute the following hypothesis conditioned data:

the hypothesis conditioned error from (5.33) and (5.23):

\[ \tilde{y}(k + 1/k) = y(k+1) - \hat{y}(k+1/k) \]
\[ = y(k+1) - C(\Phi\hat{x}(k/k) + \Gamma u(k)) \]

the hypothesis conditioned error covariance matrix from (5.34):

\[ S(k+1) = P_{\hat{y}} = CP(k+1/k)C^T + R \]
5.5 Joint Failure Modes

The identification technique derived in the previous section is verified by demonstrating joint failure detection, although other failure modes mentioned in section 5.1 may be identified with a further and similar procedure. Three joint failure modes are considered in this research: sensor failure, locked joint, and free joint. These three specific types of faults were chosen for good reason. Sensor failure is somewhat common while joint failure is rather critical when occurs. It can be argued that if the approach works for these three somewhat complex and critical cases of failure, then the approach should work satisfactorily for other common or critical faults as well. In any event, once a failure hypothesis and a failure model are established for each such fault, the subsequent approach is quite similar. The errors and the error covariance matrices conditioned on different hypotheses, which are associated with the considered failure modes, require system models that represent the corresponding failure modes. These models have been developed and verified through computer simulation, in Chapter 3. To start with, the deployable arm shown in Figure 5-1, which is a single module version of MDMS with rigid joints, is considered. Although damping is included in the simulation model, the presented cases primarily omit damping because the undamped system represents a more critical situation. Its model is formulated using the Lagrangian approach, linearized about an operating point and then discretized. The resulting equations are cast in the form of (5.13) and (5.14). The formulated nonlinear equations of motion are checked before they are linearized. A simple pendulum-like motion is simulated for the single module using Matlab. The conservation of system energy and the comparison with the $O(N)$ model has suggested that the formulation is correct. The
system equations are then linearized by expanding them using Taylor series and neglecting the higher order terms. Involving differentiation, this task is performed with the aid of Maple mathematical tools. The formulated model is of the form given in (5.13). The details of the matrices are included in Appendix II. The response of this system is simulated and verified by comparison with the nonlinear model. This model is modified to represent different failure modes, and is used as a plant model in simulation as well as for the computation of hypothesis conditioned information. These models are described in the following sections.

5.5.1 Sensor Failure

In the mode of sensor failure, the optical encoder (motion sensor) of a revolute joint or a deployable link fails. In its non-faulty operation the optical encoder provides a reading in number of pulses, which corresponds to the angular or linear position of the joint. In the faulty mode it is assumed that the sensor gives a zero output although the
joint motion continues according to the actuator input signal. In this case, the model is modified such that the row of the measurement (output) matrix $C$ corresponding to the sensor that has failed is replaced by a null row. A torque of 0.01Nm and a force of 0.01N are applied to the revolute and deployable joints, respectively. Figure 5-2 shows the simulation results for the response (output) of a linearized and discretized model that has its revolute joint sensor failed for a period of time – 2.5 to 5 seconds in this case. The time histories of both joints are shown. Their magnitude increases with time, as the manipulator moves with the constant torque and force applied to the joints. Consider the system output (response) vector $y = [\delta \theta, \delta l]^T$ shown in the figure. Output 1 drops to zero upon the sensor failure while the output 2 is not affected. Sensor failure models for other manipulator systems with a different number of modules can be derived in a similar way, by replacing the row of the output matrix corresponding to a particular sensor with a null row. These models can be used in the hypothesis testing algorithm for sensor failure detection. Representative results for various cases of failure are presented in the next chapter.

### 5.5.2 Locked Joint Failure

In this mode, the relative motion of a joint is stopped due to some form of jamming, an obstacle or seizure in a joint component, such as the gear transmission or the bearings. The model for this failure mode is derived by reducing the total degrees of freedom of the manipulator system by one, by fixing the generalized coordinate (relative motion of the joint) corresponding to the locked joint. In this manner, the equation corresponding to the coordinate for the locked joint is left out from the original system of nonlinear equations of motion. Once locked, the position of the locked joint stays the
same as that at the instant just prior to the locking, and its relative velocity drops to zero and maintains at this value throughout the locked state.

The nonlinear model is given the input torque and force of 0.01 Nm and 0.01 N to its revolute and deployable joints, respectively, for a time period of 5 seconds, and locking is set to occur from $t = 2\,\text{s}$ to $4\,\text{s}$. Simulation results of the system response are shown in Figure 5-3. An abrupt change in the velocity of the deployable joint 1 is noticed at the instant the shoulder is locked.

![Graphs of Input Torque, Input Force, Slew Angle, Deployment, and System Output 1 and 2 vs. Time](image)

**Figure 5-2** Deployable arm with the failed sensor in its revolute joint.
As shown in Figure 5-4, the position of the elbow joint increases in the positive direction and then decreases as the moment of inertia of module #2 increases due to the extension of its deployable link. Through dynamic coupling, the shoulder joint rotates in the opposite direction to the elbow joint. In addition, it moves in a slower pace because of the higher load and inertia, which it is driving. The linearized and discretized version of this model is used in the hypothesis testing algorithm for detecting locked shoulder joint in a two-module manipulator system.

Figure 5-3  Locking of shoulder joint during $t = 2 - 4$ s for a two-module deployable manipulator – module #1 inputs and response
5.5.3 Freewheeling Joint Failure

The last failure mode considered in this study is the free swinging of a revolute joint. This can happen when the connection between the motor and the slew link is lost, for example due to a broken link, slipping or disengagement in the transmission, or loss of torque in the actuator as a result of malfunction in the drive system or the actuator. When this happens, the corresponding actuator torque drops to zero, and has no effect on the manipulator motion. However, in this case, the motion of the free joint is not constrained and not known in advance, and it depends on the dynamics of the other three
joints. Furthermore, the system order is not reduced like it did in the previous case. Figure 5-5 and Figure 5-6 show the simulation results for this failure mode. The dashed lines in the joint coordinate plots represent the response in the absence of failure. When the shoulder joint is set free, it swings further and faster in the negative direction because the positive torque is not present anymore to counteract the negative torque applied through dynamic coupling. At the same time, the faster motion of the shoulder joint induces increased deployment and higher velocity in the deployable (prismatic) joint #1.

![Graphs showing simulation results](image)

**Figure 5-5** Shoulder joint set free during $t = 2 - 4$ s in a two-module deployable manipulator – module #1 inputs and response.
As for module #2, in spite of the constant slew joint torque, magnitudes of the position and velocity are larger compared to the case of no failure. The increase in the displacement and the velocity in the slew joint 1 in turn imposes the same influence in the slew joint 2. The corresponding response of the deployable joint 2 is also shown. Again, this model will be used as the plant to demonstrate the detection of freewheeling of the shoulder joint as well as in the estimation module of the hypothesis testing algorithm after it is linearized and discretized, as discussed in Chapter 6.

![Graphs showing torque, position, velocity, and force over time for a two-module deployable manipulator with module #2 inputs and response.](image)

Figure 5-6 Shoulder joint set free during $t = 2 - 4$ s for a two-module deployable manipulator – module #2 inputs and response.
CHAPTER 6  FAULT DIAGNOSIS AND CONTROL

6.1  Introduction

This chapter brings together the techniques developed and verified in the previous chapters with the goal of accomplishing accurate diagnosis of a structural fault in a manipulator system and satisfactorily controlling the system under faulty conditions. This is done through computer simulation of the overall problem and carrying out illustrative examples. The numerical results are computed using a nonlinear model of the multi-module deployable manipulator system (MDMS) formulated using the Lagrangian approach, with the parameters values identical to those of the prototype MDMS which has been design and built in our laboratory. Models for the considered failure modes are linear and consequently are not valid when large output variations are present. This is overcome by updating the linear models at each identification step when the hypothesis testing is carried out. Failure diagnosis simulations for a slew-deployable arm and for a two-module deployable manipulator system are presented. The last section of the chapter presents the execution of targeting and pointing tasks in the presence of failure in one of the links.

6.2  Joint Failure Identification

The failure mode identification algorithm using hypothesis testing is implemented in a slew-deployable manipulator system for detecting joint failures. Two different system configurations are considered in the present study: a single-module manipulator and a two-module manipulator. To begin with, detection of the modes of sensor failure and locked joint are implemented in a single-module slew-deployable arm without activating closed-loop automatic control. The purpose is to gain confidence in the
detection algorithm by means of this relatively simple fourth order system. Next, in order to demonstrate the ability of the manipulator system to complete a targeting and pointing task in the presence of failure in one of the links, the same algorithm with appropriate modifications is implemented in a two-module slew-deployable manipulator system, which is an eighth order system.

The sampling period for the linearized and discretized models is chosen to be equal to the simulation time step. The outputs used in this work are the slew and deployable joint positions. They are measured with the optical encoders mounted on the joint motor shafts. The inputs are the forces and torques provided by the motors. The $Q$ matrix in Eq. (5.18) of Chapter 5 has to be selected so as to reflect the variance of the disturbance in the input forces and torques, and the $R$ matrix in Eq. (5.19) gives the measurement noise covariance, which depends on the specific failure mode.

In the present simulation three possible failure states are considered. Including the state of no failure, the four hypotheses which will be tested are:

1) $H_1$: No failure
2) $H_2$: Sensor failure
3) $H_3$: Locked joint
4) $H_4$: Free joint

The $R$ matrix has to be selected such that there is a higher noise covariance associated with a failed sensor. The actual values of $Q$ and $R$ depend on the sensors used and the engineering units, which will vary from system to system. However, the relative values must reflect the reality of the simulated situation.
6.2.1 Slew-Deployable Arm

This section presents simulation results for failure detection of a slew-deployable arm, a single module version of the MDMS. The detection of individual failure modes is implemented in separate simulation programs before they are combined into a single program to detect any one of the failure modes at a time.

Figure 6-1 shows the detection of failed sensor in the slew and deployable joints at different time of operation. A constant torque of 0.1Nm and a constant force of 0.1N are applied to the slew and the deployable joints, respectively, throughout the simulation. The resulting displacements and velocities for both joints are plotted with respect to time. Both joints displace in the positive direction, and the velocity of the slewing joint drops due to the increased moment of inertia of the arm as it elongates when the deployable joint moves. The graphs in the third row of Figure 6-1 show the simulated outputs in the presence of sensor failure, in either the slew or the deployable joint. Here, they mimic sensor failure in the slew joint during the time period from 0.5s to 1.5s, and then in the deployable joint during the time period from 2.5s to 3.5s. It is clear that the fault identification algorithm developed in the present work has been able to successfully identify the failure incidents in both time periods, as represented by the detection indicators: 1 => slew joint sensor failure, 2 => deployable joint sensor failure, and 3 => no failure.

Figure 6-2 shows the identification of locked slew joint in the robotic arm. The same torque and force inputs as in the previous case are applied to the joints in the present case. The simulated joint motions in the presence of locked slew joint are plotted in the four graphs shown in the top half of the figure. The locked slew joint during the
time period from 0.5s to 2s is simulated in this case. Specifically, the joint position is maintained fixed during this time period, with no relative motion; i.e., $\dot{\theta} = 0$, and the joint motion is resume at the time instant of 2s. The outputs from the plant are shown by the graphs in the third row, which are entered into the hypothesis testing algorithm. The last graph indicates the algorithm has successfully detected the failure state.
Figure 6-1 Detection sensor failure in the slew-deployable arm.
Figure 6-2  Detection of locked slew joint in the slew-deployable arm.

The last failure scenario studied with the single-module deployable arm is the case when the revolute joint loses the actuating torque. This failure corresponds to free-
swinging revolute joint, which can occur when the connection between the actuator and the link is broken or when the actuator loses power and the braking system malfunctions. The simulation results obtained for this case are shown in Figure 6-3. As before, constant torque and force inputs of 0.1Nm and 0.1N are applied to the revolute and deployable joints, respectively, for the entire time period of operation. The simulated joint motions in the presence of free slewing joint are given by the four graphs in the top half of the figure. In the simulation, free slewing joint is imitated during the time period of 1s to 2.5s. During this time, the revolute joint moves freely in the absence of actuator torque, and it continues to slew rather than stop due to its momentum. The response of the manipulator is shown by the graphs in the third row of Figure 6-3, and is entered into the hypothesis testing algorithm. The last graph indicates that the algorithm has been able to successfully identify the failure.
6.2.2 Multi-module Deployable Manipulator System (MDMS)

In this section, simulation results are presented for the detection of locked shoulder joint in a two-module deployable manipulator system. The implemented
algorithm for the slew-deployable arm was expanded for this two-module system. The slew and deployable joints were given constant arbitrary torque and force values of 0.1Nm and 0.1N, respectively, for the whole simulation time period. The plant, which was represented by a non-linear model, was simulated to have its shoulder seized during the time of \( t = 2 \) s to \( t = 4 \) s in the 5-second simulation. While the manipulator is in motion, it is checked for the happening of locked shoulder joint at each time step during the simulation. Figure 6-4 shows the simulated joint motions for the four links with the shoulder joint being locked from \( t = 2 \) s to \( t = 4 \) s, and the corresponding motion response for the other three joints. Starting from rest, with the applied torque and through dynamic coupling, the shoulder joint slew to approximately 53° in the opposite direction to the applied torque in the first 2 seconds. Then it was held at this position, simulating a locked joint failure mode, for the next 2 seconds. The shoulder joint velocity dropped to zero, as shown in the graph of slew joint #1 velocity. The elbow joint and the two deployable joints also experience a sudden change in acceleration. Then the differential motion of the joints, shown in the top four graphs in Figure 6-5, was input to the detection mechanism, and the locked shoulder joint failure mode was successfully detected as shown in the detection indicator graph in the same figure.
Figure 6-4 Detection of locked shoulder joint in a two-module manipulator: simulated plant.
6.3 Targeting and Pointing Task with Locked Shoulder Joint

In this final section, it is demonstrated how the two-module deployable manipulator is able to reach and point at a given target in the presence of locked shoulder joint failure. The accomplishment of this task requires at least three of the available four manipulator joints. When there is no failure, the deployable link of the second module is held fixed at its initial position, and the remaining three joints are employed to complete the tasks. Given the target location in the 2-dimensional workspace and the desired
pointing direction, the required final positions of these three joints are calculated through inverse kinematics, which incorporate the fixed length of the second module. Then desired joint trajectories are computed with constraints on the initial and final positions, velocities and accelerations. When no failure is present or detected, the sine-on-ramp profile described in Chapter 4, which assumes zero velocities and accelerations at the beginning and the end, may be used to plan the joint trajectories. However, when a locked-joint failure is detected, the joint trajectories need to be re-computed, and the sine-on-ramp profile is no longer feasible since the joint velocities at the failure instant are non-zero and unknown. In order to overcome this problem, quintic polynomials are used to plan the joint displacement trajectories. The following set of equations with unknown coefficients represents the motion of each joint as a function of time:

\[ q(t) = a_5 t^5 + a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0 \]  
\[ \dot{q}(t) = 5a_5 t^4 + 4a_4 t^3 + 3a_3 t^2 + 2a_2 t + a_1 \]  
\[ \ddot{q}(t) = 20a_5 t^3 + 12a_4 t^2 + 6a_3 t + 2a_2 \]

When locked shoulder joint or some other failure mode is detected, the six coefficients are computed using these equations and the joint trajectories are updated for each of the three employed joints. Consequently, smooth joint velocity and continuous acceleration profiles are guaranteed.

Once the desired joint trajectories are obtained in this manner, the joints are controlled to follow these profiles using a proportional-integral-derivative (PID)
controller for each joint independently. In the present example, the controller gains are
determined by trial and error, resulting in satisfactory performance.

As an illustrative example, a two-module deployable manipulator, initially fully-
retracted and aligned with the x-axis, is commanded to reach a target located at 0.1 m and
0.5 m in the x- and y-directions, respectively, from the starting tip location, and then to
point the tip (end effector) at 60° with respect to the x-axis. Locked shoulder joint failure
is assumed to occur at $t = 3$ s prior to which the second deployable link is held fixed at
the initial position and the motion is carried out by the remaining 3 joints. Figure 6-6
shows the controlled joint motion and the resultant pointing direction of the manipulator
tip. In each plot, the desired or planned trajectory is plotted using dashed lines while the
controlled trajectory is plotted using solid lines. When the locking incident happens and
is detected, the desired joint trajectories are updated based on the actual positions,
velocities and accelerations of the joints. In the plot of slew joint #1 position, it is seen
that there is a step change in the planned trajectory at the instant of joint locking. This is
because the desired trajectory is updated using the actual joint position at that particular
time instant as the initial value. Strictly speaking, this section of the trajectory is not
computed, but rather determined by the locking position of the joint. It is also noted that
for improved tracking of the slew and deployable joints of the module #1, further tuning
of the controller gains is required.

From the position plot of the deployable joint #2, it is seen that the joint is
controlled to remain at the initial deployed length when no locking is detected. Then, at
$t = 3$ s, its trajectory is updated and the joint is commanded to deploy to a length of 0.464
m from 0.35 m in the next 2 seconds, in order to complete the original task. From the
simulation results it is found that the tracking error in the final position is only 0.4%, and the pointing direction is off the desired angle by just 0.6° or 1%.

Figure 6-7 shows the manoeuvre in the workspace. With both modules fully-retracted, the elbow joint starts at the location (0.35, 0), and the manipulator tip starts at the location (0.7, 0). At the point where the joint locking occurs, module #1 is locked at an angle of 9.8°. It is seen that the elbow joint, after locking occurs, moves outward with the extension of the deployable joint #1 and then in an opposite direction to its final location at (0.568, 0.098). It is also seen clearly that the tip path is composed of two curves, one before and the other after the shoulder joint is locked. From the simulation results it is found that the final tip position is (0.809, 0.500) compared to the target location at (0.8, 0.5). The simulation results show successful detection of the failure mode of locked shoulder joint and subsequent completion of the targeting and pointing task, merely with the help of a simple PID controller,
Figure 6-6  Tracking of planned joint trajectories using PID control and the corresponding pointing direction of the tip.
Figure 6-7  A targeting and pointing manoeuvre of a two-module deployable manipulator in the presence of a locked-shoulder-joint failure.
CHAPTER 7 CONCLUDING REMARKS

7.1 Introduction

This thesis concerns the identification of structural failures in a multi-module deployable manipulator system (MDMS) and satisfactory operation of the faulty manipulator by making use of the correct failure identification and proper control. It is assumed that some redundant degrees of freedom are available in the manipulator. In this concluding chapter, the primary contributions of the thesis are summarized. The main conclusions drawn as a result of the presented research are indicated. Suggestions are made for the directions of further work in the subject area.

7.2 Contributions

The main contributions of the thesis may be summarized as follows:

(i) A numerical code was developed for a relatively general formulation of a novel ground-based multi-modular robotic manipulator system, with slewing and deployable links in each module. Such a computationally efficient simulation program based on a non-recursive $O(N)$ Lagrangian approach, for studying dynamics and control of this class of manipulator systems has not been developed before.

(ii) A comprehensive study was carried out on the planar dynamics and control of a flexible multi-module, ground-based manipulator using the developed computationally efficient simulation and two types of control – proportional-integral-derivative (PID) and feedback linearization technique (FLT). This study provided a thorough understanding of the
effect of various system parameters on the dynamics of the manipulator, and its controlled performance. Verification of good performance of the manipulator in the absence of a structural failure is a precondition for achieving satisfactory performance under structural failures, by employing the approach developed in the present work.

(iii) A rather general methodology for detection and identification of specified types of structural failure in a multi-module deployable manipulator system was developed. Based on Bayesian hypothesis testing and using a bank of Kalman filters, the developed methodology was able to accurately identify the structural failures in MDMS. Such a failure identification method has not been developed for this class of manipulators before.

(iv) The developed method of failure detection and identification was implemented on a single-module (4th order) manipulator and a two-module (8th order) deployable manipulator. The demonstration through computer simulation, of the successful completion of a specified task by a redundant deployable manipulator in the presence of joint failure, is an important contribution, which leads to many practical applications.

7.3 Summary of Conclusions

A multi-module deployable manipulator system (MDMS) each module of which has slewing and deployable links, has been designed and developed in our laboratory. The research presented in this was aimed at carrying out a comprehensive study of the dynamics and control of this novel and versatile ground-based manipulator system, with the end objective of using the gained knowledge to develop methodologies for structural
failure identification and control. The emphasis was on the detection and identification of somewhat representative set of structural failures in the system and utilization of the knowledge of accurate identification, possible redundancy of the manipulator system, and proper control in the completion of certain tasks even in the presence of a structural failure. Based on the investigation, the following general conclusions can be made:

(i) A computationally efficient simulation program, which has a high degree of modularity, was developed for the dynamics of the novel deployable manipulator system. The program was verified and validated through conservation of the total system energy, and found to be highly accurate with the variation of the total energy in the order of $10^{-5}\%$. This program is useful in a wide variety of dynamic performance studies of the particular class of manipulators.

(ii) In the dynamical studies using the developed program, the system response was analyzed for different initial joint flexibilities, speeds of manoeuvre, joint stiffness values and point payload masses. More significant coupling was observed in the presence of higher initial joint flexibility, faster manoeuvre, lower joint stiffness values and larger point payload mass. The excitation of the flexible degrees of freedom of the manipulator at large magnitudes leads to the deterioration the motion accuracy of the manipulator, particularly near the end of a manoeuvre. Parametric studies of this nature are crucial in the investigation of the dynamic performance of the manipulator system.
(iii) In view of the high modularity of the simulation program, various control algorithms can be easily implemented and the controlled performance may be studied under a variety of conditions. Different configurations of the manipulator system may be studied in this manner since the system model represents a general N-module manipulator. Two relatively simple control algorithms were implemented to study the controlled behaviour of the MDMS. Although both methods produced good joint tracking results, the feedback linearization technique (FLT) control required less time and effort in tuning the control parameters. The joint flexibility, which was not modeled in the FLT implementation, had virtually no effect on the performance of the MDMS under FLT control. It was able to regulate the elastic degrees of freedom rather well as a result of dynamic interactions (coupling).

(iv) The inverse kinematics algorithm and a joint trajectory planning scheme were developed using geometric considerations for a two-module manipulator system for subsequent utilization in the failure detection and identification procedure.

(v) A somewhat general methodology for failure detection and identification (FDI) based on Bayesian hypothesis testing principles was developed for the novel deployable manipulator system. The derivation method confirms that this scheme may be applied for any type of failure although only a limited number of structural failure modes were considered in this work.
(vi) In particular, the developed approach for joint failure detection and identification was demonstrated using a single-module deployable arm and a two-module manipulator system through computer simulation. Three scenarios of joint failure: sensor failure, locked joint failure and freewheeling joint failure were detected and identified successfully.

(vii) A targeting and pointing task was successfully completed by a two-module manipulator system in the presence of a specific joint failure. This illustrated the power of the developed methodology and indicated its potential application under other scenarios of failure.

7.4 Recommendations for Future Work

The present thesis serves as a good initial step in the research and analysis of an important and novel class of manipulators, MDMS. The particularly emphasis has been in the failure detection and identification (FDI) and the exploration of the redundancy of MDMS, with appropriate control, to successfully complete a task under structural failure. The general formulation as well as the computationally efficient simulation program for the multi-module deployable manipulator system will be a useful tool in future studies of dynamics and control of the MDMS. There are several issues which remain unexplored or demand further investigation. Some of the more challenging and useful issues are listed below:

(i) Dynamical studies of the ground-based manipulator system model with a larger number of modules. This should lead to the exploration of other dynamic issues and stability considerations, for example.
(ii) Implementation of other, probably more advanced, control methodology in the numerical model to perform control studies and evaluate the performance using a computationally efficient program for a general number of modules. This should lead to methodologies for controller design and selection depending on the robotic task to be carried out.

(iii) Extension of the inverse kinematics algorithm to account for a larger number of modules or $N$-modules in general. A recursive algorithm may be sought for this purpose, with the resulting computational advantages.

(iv) Expansion of failure modes to other scenarios, such as data acquisition failure, slip in joint coupling due to increased backlash, wear, or actuator loading, high resistance to joint motion (e.g., due to joint misalignment), and increased vibration due to malfunctioned gears, actuators, or bearings.

(v) Numerical simulation of the failure detection and identification (FDI) method for a more general multi-module deployable manipulator system, for recovery to complete a complex task, possibly with the incorporation of an extended inverse kinematics algorithm.

(vi) Use of extended Kalman filters in the FDI algorithm, in the case of a fully nonlinear and general manipulator system

(vii) Implementation of the FDI methodology in the prototype multi-module deployable manipulator system (MDMS), which has been designed and developed in our laboratory and carry out an experimental investigation.
BIBLIOGRAPHY


APPENDIX I  THE $O(N)$ MODEL MATRICES

The dynamic simulation of the multi-module deployable manipulator system (MDMS), as carried out in the present thesis, uses a computationally-efficient $O(N)$ formulation of the robot dynamic equations. This formulation is presented in this appendix. The Langrangian approach is used in this formulation to obtain the equations of motion.

I.1 Decoupled Mass Matrix

The decoupled mass matrix of the numerical model given in section 2.2 can be expressed as

$$M = \begin{bmatrix}
M_1 & 0 & 0 & \cdots & 0 \\
0 & M_2 & 0 & \cdots & 0 \\
0 & 0 & M_3 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & M_{N+1}
\end{bmatrix}$$

Here, $M_i$, $i = 1, \ldots, N$, represents the mass matrix of the manipulator units while $M_{N+1}$ corresponds to the mass matrix of the payload. Then

$$\tilde{M}_i = \int \left[\begin{bmatrix}
1 & 0 \\
0 & 1 \\
0 & 0 \\
\partial r_i^T & T_i^T & P_i^T \\
\frac{\partial r_i}{\partial x_i} & T_i^T
\end{bmatrix} \right] \begin{bmatrix}
1 & 0 & 0 & PT_i r_i & T_i \frac{\partial r_i}{\partial x_i}
\end{bmatrix} dm_i + \begin{bmatrix}
m_{a_i} & 0 & 0 & 0 & 0 \\
0 & m_{a_i} & 0 & 0 & 0 \\
0 & 0 & J_{a_i} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad (I.1.2)$$
With \( dm_i = \rho_{s_i} dx_i \) for the \( i^{th} \) slew link section and \( dm_i = \rho_{d_i} dx_i \) for the \( i^{th} \) deployable link section, the integral is divided into two parts:

\[
\tilde{M}_i = \begin{bmatrix}
    m_{a_i} & 0 & 0 & 0 \\
    0 & m_{a_i} & 0 & 0 \\
    0 & 0 & J_{a_i} & 0 \\
    0 & 0 & 0 & 0
\end{bmatrix}
+ \rho_{s_i} \int_0^{l_{s_i}} \begin{bmatrix}
    1 & 0 & 0 & -x_i \sin \psi_i & 0 \\
    0 & 1 & 0 & x_i \cos \psi_i & 0 \\
    -x_i \sin \psi_i & x_i \cos \psi_i & 0 & 0 \\
    0 & 0 & 0 & 0
\end{bmatrix} dx_i,
\]

\[
+ \rho_{d_i} \int_{l_{d_i}}^{l_{s_i}} \begin{bmatrix}
    1 & 0 & 0 & -x_i \sin \psi_i & \cos \psi_i \\
    0 & 1 & 0 & x_i \cos \psi_i & \sin \psi_i \\
    -x_i \sin \psi_i & x_i \cos \psi_i & 0 & 0 \\
    \cos \psi_i & \sin \psi_i & 0 & 0 & 1
\end{bmatrix} dx_i,
\]

where \( \rho_{s_i} \) and \( \rho_{d_i} \) are the masses per unit length for the slew link and the deploy link, respectively; and \( l_{s_i} \) and \( l_{d_i} \) are the lengths of the slew link and the deploy link, respectively.

The computed decoupled mass matrix of the manipulator unit becomes

\[
\tilde{M}_i = \begin{bmatrix}
    M_{11} & 0 & 0 & M_{14} & \rho_{d_i} l_{d_i} \cos \psi_i \\
    0 & M_{22} & 0 & M_{24} & \rho_{d_i} l_{d_i} \sin \psi_i \\
    0 & 0 & J_{a_i} & 0 & 0 \\
    M_{41} & M_{42} & 0 & M_{44} & 0 \\
    \rho_{d_i} l_{d_i} \cos \psi_i & \rho_{d_i} l_{d_i} \sin \psi_i & 0 & 0 & \rho_{d_i} l_{d_i}
\end{bmatrix},
\]

where:

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\[ M_{11} = M_{22} = \rho_s l_{s1} + \rho_d l_{d1} + m_{a1}; \quad (I.1.5, a) \]

\[ M_{14} = M_{41} = -\sin \psi_i \left( \frac{\rho_s l_{s1}^2}{2} + \rho_d l_{d1}^2 - \frac{\rho_d l_{d1}^2}{2} \right); \quad (I.1.5, b) \]

\[ M_{24} = M_{42} = \cos \psi_i \left( \frac{\rho_s l_{s1}^2}{2} + \rho_d l_{d1}^2 - \frac{\rho_d l_{d1}^2}{2} \right); \quad (I.1.5, c) \]

\[ M_{44} = \frac{\rho_s l_{s1}^3}{3} + \rho_d \left( l_{s1} l_{d1}^3 - l_{d1}^2 + \frac{l_{d1}^3}{3} \right). \quad (I.1.5, d) \]

For the payload:

\[ \tilde{M}_{N+i} = \int_{m_{N+i}} \begin{bmatrix} I^2 \\ v_{i1} \\ v_{i2} \end{bmatrix} \begin{bmatrix} I^2 & v_{i1} & v_{i2} \end{bmatrix} dm_{N+i}; \quad (I.1.6) \]

\[ v_{i1} = PT_{N+i} \left[ r_{N+i} + \begin{bmatrix} 0 \\ \phi_{N+i} \delta_{N+i} \end{bmatrix} \right]; \quad (I.1.7, a) \]

\[ v_{i2} = T_{N+i} \begin{bmatrix} 0 \\ \phi_{N+i} \end{bmatrix}. \quad (I.1.7, b) \]

With \( dm_{N+i} \) equal to \( \rho_{N+i} dx_{N+i} \), where \( \rho_{N+i} \) is mass per unit length for the payload, the payload inertia matrix may be expressed as

\[ \tilde{M}_{N+i} = \begin{bmatrix}
\rho_{N+i} l_{N+i} & 0 & \tilde{M}_{N+i}(1,3) & \tilde{M}_{N+i}(1,4) \\
0 & \rho_{N+i} l_{N+i} & \tilde{M}_{N+i}(2,3) & \tilde{M}_{N+i}(2,4) \\
\tilde{M}_{N+i}(3,1) & \tilde{M}_{N+i}(3,2) & \tilde{M}_{N+i}(3,3) & 0.569 \rho_{N+i} l_{N+i}^2 \\
\tilde{M}_{N+i}(4,1) & \tilde{M}_{N+i}(4,2) & 0.569 \rho_{N+i} l_{N+i}^2 & 1.713 \rho_{N+i} l_{N+i}^3 
\end{bmatrix}, \quad (I.1.8) \]

where:

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\[ \tilde{M}_{N+1,(3,3)} = \rho_{N+1} \left( \frac{l_{N+1}^3}{3} + 1.713 l_{N+1} \delta_{N+1}^2 \right) ; \]  

(I.1.9, c)

\[ \tilde{M}_{N+1,(4,4)} = \frac{\rho_{N+1} \sin \psi_{N+1} l_{N+1}}{1.277} ; \]  

(I.1.9, d)

\[ \tilde{M}_{N+1,(2,4)} = \frac{\rho_{N+1} \cos \psi_{N+1} l_{N+1}}{1.277} . \]  

(I.1.9, e)

### I.2 Velocity Transformation Matrices

The order- \( N \) or \( O(N) \) algorithm used in the thesis relies on two velocity transformations, which factorize the system mass matrix. The details of the transformation matrices involved are given now.

The first velocity transformation can be expressed as

\[ \tilde{\dot{q}} = (I^n - R^C)^{-1} \dot{R} \dot{q} , \]  

(I.2.1)

where:

\[ R = \begin{bmatrix} R_1 & 0 & \ldots & 0 & 0 \\ 0 & R_2 & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & R_N & 0 \\ 0 & 0 & \ldots & 0 & R_{N+1} \end{bmatrix} \in \mathbb{R}^{n_x \times n_y} ; \]  

(I.2.2)
and

\[
R^C = \begin{bmatrix}
0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 \\
0 & R_2^C & 0 & 0 & \cdots & 0 \\
0 & 0 & R_3^C & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0
\end{bmatrix} \in \mathbb{R}^{n \times n}. \tag{1.2.3}
\]

Here:

\[
R_i = I^{n} ; \tag{1.2.4}
\]

\[
R_i = \begin{bmatrix}
T_{i-1} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \\
\begin{bmatrix}
\cos \psi_{i-1} & -\sin \psi_{i-1} & 0 & 0 \\
\sin \psi_{i-1} & \cos \psi_{i-1} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} , \quad i = 2, \ldots, N ; \tag{1.2.5}
\]

and

\[
R_{N+1} = \begin{bmatrix}
T_N & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} = \begin{bmatrix}
\cos \psi_N & -\sin \psi_N & 0 & 0 \\
\sin \psi_N & \cos \psi_N & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} ; \tag{1.2.6}
\]
The second velocity transformation is defined as

\[ \dot{q} = R^{v} \ddot{q}, \]  

(1.2.9)

where:

\[ R^{v} = \begin{bmatrix}
R_1 & 0 & 0 & \cdots & 0 & 0 & 0 \\
R_1^A & R_2 & 0 & \cdots & 0 & 0 & 0 \\
R_1^B & R_2^B & R_3 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
R_1^B & R_2^B & R_3^B & \cdots & R_{N-1}^A & R_N & 0 \\
R_1^B & R_2^B & R_3^B & \cdots & R_{N-1}^A & \bar{R}_N & 0 \\
R_1^P & R_2^P & R_3^P & \cdots & R_{N-1}^P & R_N^A & \bar{R}_{N+1} \\
\end{bmatrix} \in \mathbb{R}^{n \times n}; \]  

(1.2.10)
\[
R_j^R = \begin{bmatrix}
T_{j,1} & 0 & P_{g_{j+1}} & T_j \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
= \begin{bmatrix}
\cos \psi_{j-1} & -\sin \psi_{j-1} & 0 & -\left(l_j + d_{(j+1)x}\right)\sin \psi_j - d_{(j+1)y} \cos \psi_j & \cos \psi_j \\
\sin \psi_{j-1} & \cos \psi_{j-1} & 0 & \left(l_j + d_{(j+1)x}\right)\cos \psi_j - d_{(j+1)y} \sin \psi_j & \sin \psi_j \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]
for \( j = 1, \ldots, N - 2; \) 

\[
R_i^A = \begin{bmatrix}
T_{i,1} & 0 & P_{g_{i+1}} & T_i \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
= \begin{bmatrix}
\cos \psi_{i-1} & -\sin \psi_{i-1} & 0 & -\left(l_i + d_{(i+1)x}\right)\sin \psi_i - d_{(i+1)y} \cos \psi_i & \cos \psi_i \\
\sin \psi_{i-1} & \cos \psi_{i-1} & 0 & \left(l_i + d_{(i+1)x}\right)\cos \psi_i - d_{(i+1)y} \sin \psi_i & \sin \psi_i \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]
for \( i = 1, \ldots, N - 1; \) 

\[
\bar{R}_i = \begin{bmatrix}
T_{i,1} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
= \begin{bmatrix}
\cos \psi_{i-1} & -\sin \psi_{i-1} & 0 & 0 & 0 \\
\sin \psi_{i-1} & \cos \psi_{i-1} & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix},
\]
for \( i = 2, \ldots, N; \) 

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$$R_j^p = \begin{bmatrix} T_{j-1} & 0 & P_{g_{j+1}} & T_j \ 0 & 0 & 0 & 0 \ 0 & 0 & 0 & 0 \ \end{bmatrix}$$

$$= \begin{bmatrix} \cos \psi_j & \sin \psi_j & 0 & -\left(t_j + d_{(j+1)x}\right)\sin \psi_j - d_{(j+1)y} \cos \psi_j \cos \psi_j & \\
\sin \psi_j & \cos \psi_j & 0 & \left(t_j + d_{(j+1)x}\right)\cos \psi_j - d_{(j+1)y} \sin \psi_j \sin \psi_j & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

\(j = 1, \ldots, N - 1;\) 

(1.2.15)

$$R_N^d = \begin{bmatrix} T_{N-1} & 0 & P_{g_{N+1}} & T_N \ 0 & 0 & 0 & 0 \ 0 & 0 & 0 & 0 \ \end{bmatrix}$$

$$= \begin{bmatrix} \cos \psi_{N-1} & -\sin \psi_{N-1} & 0 & \left(t_N + d_{(N+1)x}\right)\sin \psi_N - d_{(N+1)y} \cos \psi_N \sin \psi_N \\
\sin \psi_{N-1} & \cos \psi_{N-1} & 0 & \left(t_N + d_{(N+1)x}\right)\cos \psi_N - d_{(N+1)y} \sin \psi_N \cos \psi_N \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \end{bmatrix},$$

(1.2.16)

$$\overline{R}_{N+1} = \begin{bmatrix} T_N & 0 & 0 \ 0 & 1 & 0 \ 0 & 0 & 1 \ \end{bmatrix} = \begin{bmatrix} \cos \psi_N & -\sin \psi_N & 0 & 0 \\
\sin \psi_N & \cos \psi_N & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \end{bmatrix}.$$ 

(1.2.17)
## 1.3 Potential Energy

The total gravitational potential energy of the manipulator system may be written as

\[
U_g = -\sum_{i=1}^{N+1} \int_{m_i} g \left( \mathbf{j}_0 \cdot \mathbf{R}_{dm_i} \right) dm_i + \sum_{i=1}^{N} g \left( \mathbf{j}_0 \cdot \mathbf{R}_{a_i} \right) m_{a_i},
\]

where

\[
\mathbf{j}_0 \cdot \mathbf{R}_{dm_i} = \begin{cases} 
D_{\psi} + x_i \sin \psi_i + y_i \cos \psi_i, & i = 1, 2, \ldots, N \\
D_{(N+1)\psi} + x_{N+1} \sin \psi_{N+1} + \phi_{N+1} \delta_{N+1} \cos \psi_{N+1}, & i = N+1
\end{cases}
\]

Then, for the \( N \) slewing links, one has

\[
U_{gs} = -\sum_{i=1}^{N} \rho_s l_s g \left( D_{\psi} + \frac{l_s \sin \psi_i}{2} \right).
\]

Similarly, for the \( N \) deploying links

\[
U_{gd} = -\sum_{i=1}^{N} \rho_d l_d g \left( D_{\psi} + l_i \sin \psi_i - \frac{l_d \sin \psi_i}{2} \right).
\]

For the payload, one has

\[
U_{gp} = -\rho_{N+1} g \left( D_{(N+1)\psi} l_{N+1} + \frac{l_{N+1}^2 \sin \psi_{N+1}}{2} \right) - \rho_{N+1} g \delta_{N+1} \cos \psi_{N+1} \left( \sinh \lambda l_{N+1} - \sin \lambda l_{N+1} - 0.7341 \left( \cosh \lambda l_{N+1} + \cos \lambda l_{N+1} \right) \right) + \frac{1.4682 \rho_{N+1} g \delta_{N+1} \cos \psi_{N+1}}{\lambda}.
\]

Finally, for the \( N \) actuators

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Thus the total gravitational potential energy is

\[ U_g = U_{gi} + U_{ga} + U_{gp} + U_{ga}. \]  

The partial derivatives of the gravitational potential energy with respect to the generalized coordinates are obtained with Maple V mathematics software, and are listed below:

\[ \frac{\partial U_g}{\partial D_{ix}} = 0; \]  

\[ \frac{\partial U_g}{\partial D_{iy}} = -g \left[ \sum_{i=1}^{N} \left( \rho_{s} l_{s} + \rho_{d} l_{d} + m_{a} \right) + \rho_{N+1} l_{N+1} \right]; \]  

\[ \frac{\partial U_g}{\partial \alpha_i} = 0, \quad i = 1, \ldots, N; \]  

\[ \frac{\partial U_g}{\partial \psi_i} = \frac{1}{2} g \cos \psi_i \left( \rho_d l_d^2 - \rho_s l_s^2 \right) \]  

\[ + \sum_{j=1}^{N} \left[ -g \left( \rho_{s_j} l_{s_j} + \rho_{d_j} l_{d_j} + m_{a_j} \right) l_i \cos \psi_i + d_{(i+1)x} \cos \psi_i + d_{(i+1)y} \sin \psi_i \right] \]  

\[ - \rho_{d} l_{d} g l_i \cos \psi_i - \rho_{N+1} g l_{N+1} \left( l_i \cos \psi_i + d_{(i+1)x} \cos \psi_i - d_{(i+1)y} \sin \psi_i \right) \]  

\[ i = 1, \ldots, N; \]  

\[ \frac{\partial U_g}{\partial l_i} = -\rho_{d} l_{d} g \sin \psi_i - \sum_{j=1}^{N} \left[ g \sin \psi_i \left( \rho_{s_j} l_{s_j} + \rho_{d_j} l_{d_j} - m_{a_j} \right) - \rho_{N+1} g \sin \psi_i l_{N+1} \right] \]  

\[ i = 1, \ldots, N; \]
\[
\frac{\partial U_g}{\partial d_{tg}} = -\sum_{j=i}^{N} \left[ g \sin \psi_{i-1} \left( \rho_{s_j} l_{s_j} + \rho_{d_j} l_{d_j} + m_{a_j} \right) \right] - \rho_{N+1} \sin \psi_{i-1} l_{N+1},
\]

\[i = 2, \ldots, N + 1; \quad \text{(I.3.8, f)}\]

\[
\frac{\partial U_g}{\partial d_{ty}} = -\sum_{j=i}^{N} \left[ g \cos \psi_{i-1} \left( \rho_{s_j} l_{s_j} + \rho_{d_j} l_{d_j} + m_{a_j} \right) \right] - \rho_{N+1} \cos \psi_{i-1} l_{N+1},
\]

\[i = 2, \ldots, N + 1; \quad \text{(I.3.8, g)}\]

\[
\frac{\partial U_g}{\partial \psi_{N+1}} = -\frac{\rho_{N+1} g}{\lambda} \left[ \frac{1}{2} l_{N+1}^2 \cos \psi_{N+1} \lambda \right.

- \delta_{N+1} \sin \psi_{N+1} \left( \sinh (\lambda l_{N+1}) - \sin (\lambda l_{N+1}) \right)\]

- \left. 0.7341 \left( \cosh (\lambda l_{N+1}) + \cos (\lambda l_{N+1}) \right) - 1 \right] \];

\[\text{(I.3.8, h)}\]

\[
\frac{\partial U_g}{\partial \delta_{N+1}} = \frac{\rho_{N+1} g}{\lambda} \cos \psi_{N+1} \left\{ - \sinh (\lambda l_{N+1}) + \sin (\lambda l_{N+1}) \right. + 0.7341 \left[ \cosh (\lambda l_{N+1}) + \cos (\lambda l_{N+1}) \right] + 1 \right\}. \]

\[\text{(I.3.8, i)}\]

The total elastic potential energy of the system can be written as

\[
U_e = \sum_{i=1}^{N} \frac{1}{2} K_{\alpha_i} \beta_i^2 + \frac{1}{2} \int_{x_{N+1}}^{x_{N+2}} EI(x_{N+1}) \left( \frac{\partial^3 (\phi_{N+1} \delta_{N+1})}{\partial x_{N+1}^3} \right)^2 dx_{N+1}. \]

\[\text{(I.3.9)}\]

By rewriting the above expression in terms of the generalized coordinates, one obtains

\[
U_e = \sum_{i=1}^{N} \frac{1}{2} K_{\alpha_i} (\psi_i - \alpha_i - \psi_{i-1})^2 + 6.18 \frac{EI(x_{N+1}) \delta_{N+1}}{l_{N+1}^3}. \]

\[\text{(I.3.10)}\]

The partial derivatives of the elastic potential energy with respect to the generalized coordinates are expressed below:
\[ \frac{\partial U_e}{\partial D_{ix}} = \frac{\partial U_e}{\partial D_{iy}} = 0; \quad (1.3.11, \text{a}) \]

\[ \frac{\partial U_e}{\partial \alpha_i} = -K_{ai}(\psi_i - \alpha_i - \psi_{i-1}), \quad i = 1, \ldots, N; \quad (1.3.11, \text{b}) \]

\[ \frac{\partial U_e}{\partial \psi_i} = K_{ai}(\psi_i - \alpha_i - \psi_{i-1}) - K_{ai+1}(\psi_{i+1} - \alpha_{i+1} - \psi_i), \quad i = 1, \ldots, N; \quad (1.3.11, \text{c}) \]

\[ \frac{\partial U_e}{\partial \psi_{N+1}} = K_{a_{N+1}}(\psi_{N+1} - \alpha_{N+1} - \psi_N); \quad (1.3.11, \text{d}) \]

\[ \frac{\partial U_e}{\partial l_i} = 0, \quad i = 1, \ldots, N; \quad (1.3.11, \text{e}) \]

\[ \frac{\partial U_e}{\partial d_{ix}} = \frac{\partial U_e}{\partial d_{iy}} = 0, \quad i = 2, \ldots, N + 1; \quad (1.3.11, \text{f}) \]

\[ \frac{\partial U_e}{\partial \delta_{N+1}} = 12.362 \frac{IE\delta_{N+1}}{l_{N+1}^3}. \quad (1.3.11, \text{g}) \]

### 1.4 Energy Dissipation

The total energy dissipation in the system can be expressed using the Rayleigh function as

\[ R_d = \sum_{i=1}^{N} \frac{1}{2} C_i \dot{\theta}_i^2 + \frac{1}{2} \int_{x_{N+1}}^{x_N} C_{N+1} I(x_{N+1}) \left[ \frac{\partial^2 (\psi_{N+1} \delta_{N+1})}{\partial x_{N+1}^2} \right]^2 \, dx_{N+1}. \quad (1.4.1) \]

By rewriting the above expression in terms of the set of generalized coordinates, one obtains
\[ R_d = \sum_{i=1}^{N} \frac{1}{2} C_i \left( \dot{\psi}_i - \dot{\alpha}_i - \dot{\psi}_{i-1} \right)^2 + 6.18 \frac{C_v \delta_{N+1}^3}{l_{N+1}^3}. \] 

(1.4.2)

The partial derivatives of the energy dissipation with respect to the rate of change of the generalized coordinates are computed symbolically using Maple V, and are given below:

\[ \frac{\partial R_d}{\partial D_{ix}} = \frac{\partial R_d}{\partial D_{iy}} = 0; \] 

(1.4.3, a)

\[ \frac{\partial R_d}{\partial \dot{\alpha}_i} = -C_i \left( \dot{\psi}_i - \dot{\alpha}_i - \dot{\psi}_{i-1} \right), \quad i = 1, \ldots, N; \] 

(1.4.3, b)

\[ \frac{\partial R_d}{\partial \dot{\psi}_i} = C_i \left( \dot{\psi}_i - \dot{\alpha}_i - \dot{\psi}_{i-1} \right) - C_{i+1} \left( \dot{\psi}_{i+1} - \dot{\alpha}_{i+1} - \dot{\psi}_i \right), \quad i = 1, \ldots, N; \] 

(1.4.3, c)

\[ \frac{\partial R_d}{\partial \dot{\psi}_{N+1}} = 0; \] 

(1.4.3, d)

\[ \frac{\partial R_d}{\partial \dot{l}_{i}} = 0, \quad i = 1, \ldots, N; \] 

(1.4.3, e)

\[ \frac{\partial R_d}{\partial \dot{d}_{ix}} = \frac{\partial R_d}{\partial \dot{d}_{iy}} = 0, \quad i = 2, \ldots, N+1; \] 

(1.4.3, f)

\[ \frac{\partial R_d}{\partial \delta_{N+1}} = 12.362 \frac{IC_v \delta_{N+1}^3}{l_{N+1}^3}. \] 

(1.4.3, g)

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I.5 Derivatives of the Mass Matrix

In the derivation of the equations of motion of the manipulator system, it is necessary to obtain the derivative of the coupled system mass matrix. The associated differentiation procedure is outlined now. When the coupled mass matrix $M$ is required, the velocity transformation

$$\dot{q} = R^\nu \dot{q}$$  \hspace{1cm} (I.5.1)

may be used to obtain

$$M = R^\nu T \bar{M} R^\nu.$$ \hspace{1cm} (I.5.2)

Hence

$$\dot{M} = \dot{R}^\nu T \bar{M} R^\nu + R^\nu T \bar{\dot{M}} R^\nu + R^\nu T \bar{\dot{M}} R^\nu,$$ \hspace{1cm} (I.5.3)

where a dot over a matrix indicates that the time derivative of each component in the matrix is taken. Using Maple V, the entries of $\dot{M}$ in the matrix form are formulated as follows:

$$\dot{M}_{ij} = \sum_{k=1}^{N} \left( \dot{R}_{ik}^\nu \bar{M}_{k} R_{kj}^\nu + R_{ik}^\nu \dot{\bar{M}}_{k} R_{kj}^\nu + R_{ik}^\nu \bar{\dot{M}}_{k} R_{kj}^\nu \right).$$ \hspace{1cm} (I.5.4)

Furthermore

$$\frac{\partial(q^T \dot{M} q)}{\partial q} = \frac{\partial(q^T R^\nu T \bar{M} R^\nu \dot{q})}{\partial q}.$$ \hspace{1cm} (I.5.5)

By expanding the above expression, one obtains
where \( q_j \) corresponds to the \( j^{th} \) generalized coordinate associated with the \( i^{th} \) unit.

Using the fact that each component within the square brackets is a scalar, Eq. (1.5.6) can be rewritten in the form

\[
\frac{\partial (\dot{q}^T M \dot{q})}{\partial q} = \begin{bmatrix}
\dot{q}^T \left( \frac{\partial R^T}{\partial q_{11}} \tilde{M} R^T + R^T \frac{\partial \tilde{M}}{\partial q_{11}} R^T + R^T \tilde{M} \frac{\partial R^T}{\partial q_{11}} \right) \dot{q} \\
\vdots \\
\dot{q}^T \left( \frac{\partial R^T}{\partial q_j} \tilde{M} R^T + R^T \frac{\partial \tilde{M}}{\partial q_j} R^T + R^T \tilde{M} \frac{\partial R^T}{\partial q_j} \right) \dot{q} \\
\vdots \\
\dot{q}^T \left( \frac{\partial R^T}{\partial q_{N+1,4}} \tilde{M} R^T + R^T \frac{\partial \tilde{M}}{\partial q_{N+1,4}} R^T + R^T \tilde{M} \frac{\partial R^T}{\partial q_{N+1,4}} \right) \dot{q} 
\end{bmatrix},
\tag{1.5.7}
\]
APPENDIX II DISCRETE LINEARIZED STATE-SPACE MODEL FOR A MANIPULATOR MODULE

A linearized model of the manipulator system is required particularly in the implementation of the Kalman filters for failure identification. Derivation of this linear model, discretized for computer simulation, is outlined in this appendix. The model is linearized about an operating point, which itself is not fixed, and depends on the specific operating point.

II.1 State Matrix

The discrete state matrix for the rigid deployable arm module (two degrees of freedom) is a $4 \times 4$ matrix with the following entries:

$$
\mathbf{\Phi} = 
\begin{bmatrix}
\phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} \\
\phi_{21} & \phi_{22} & \phi_{23} & \phi_{24} \\
\phi_{31} & \phi_{32} & \phi_{33} & \phi_{34} \\
\phi_{41} & \phi_{42} & \phi_{43} & \phi_{44}
\end{bmatrix}
$$  (II.1.1)

$$
\phi_{11} = 1 + \frac{1}{2} h^2 \left( -m_1 g l_1 \cos \theta_1 + m_2 g (l_1 - l_1) \cos \theta_1 \right) m_{i1}^{-1} 
$$  (II.1.1, a)

$$
\phi_{12} = \frac{1}{2 m_{i1}^2} h^2 \begin{bmatrix}
m_{i1} \left( -2 m_2 \delta l_1 \delta \theta_1 + m_2 g \sin \theta_1 \right) \\
-2 m_2 (l_1 - l_2) \left( -\left( m_1 g l_1 - m_2 g (l_1 - l_1) \right) \sin \theta_1 \right)
\end{bmatrix} 
$$  (II.1.1, b)

$$
\phi_{13} = h - \frac{h^2 m_2 (l_1 - l_2) \delta l_1}{m_{i1}} 
$$  (II.1.1, c)
\[
\phi_{14} = -\frac{\hbar^2 m_2 (l_1 - l c_2) \delta \theta_i}{m_{11}} \quad (I I .1 , d)
\]

\[
\phi_{21} = -\frac{1}{2} \hbar^2 g \sin \theta_i \quad (I I .1 , e)
\]

\[
\phi_{22} = 1 + \frac{1}{2} \hbar^2 \delta \theta_i^2 
\quad (I I .1 , f)
\]

\[
\phi_{23} = \hbar^2 (l_1 - l c_2) \delta \theta_i 
\quad (I I .1 , g)
\]

\[
\phi_{24} = \hbar 
\quad (I I .1 , h)
\]

\[
\phi_{31} = \frac{\hbar g \cos \theta_i}{m_{11}} (-m_1 l c_1 + m_2 (l_1 - l c_2)) 
\quad + \hbar^2 m_2 (l_1 - l c_2) \left[ \frac{\delta l_1 \cos \theta_i}{m_{11}^2} (-m_1 l c_1 + m_2 (l_1 - l c_2)) + \frac{\delta \theta_i \sin \theta_i}{m_{11}} \right] 
\quad (I I .1 , i)
\]

\[
\phi_{32} = \frac{\hbar}{m_{11}^2} \begin{bmatrix}
\frac{m_1}{m_{11}} ( -2 \delta l_1 \delta \theta_i^2 + \delta \theta_i) \\
-2m_2 (l_1 - l c_2) \left( \tau - 2m_2 (l_1 - l c_2) \delta l_1 \delta \theta_i - m_1 g l c_1 \sin \theta_i \right) \\
-2m_2 (l_1 - l c_2) \delta l_1 \\
+ \frac{1}{2} \hbar^2 m_{11}^3 \left( \frac{m_1}{m_{11}} ( -2 \delta l_1 \delta \theta_i + \delta \theta_i) \\
-2m_2 (l_1 - l c_2) \delta l_1 \\
2m_2 (l_1 - l c_2) \delta \theta_i^3 \\
\right)
\end{bmatrix} 
\quad (I I .1 , j)
\]
\[ \phi_{33} = 1 - \frac{2hm_2 (l_1 - lc_2)}{m_1} \delta l_1 \]
\[ + \frac{1}{2} \frac{h^2}{m_1} \left( \frac{g \cos \theta_1 (m_1 l_1 + m_2 (l_1 - lc_2))}{m_1} \right) \]
\[ + \frac{4m_2^2 (l_1 - lc_2)^2 \delta \theta_1^2}{m_1^2} - \frac{4m_2 (l_1 - lc_2)^2 \delta \theta_1^2}{m_1} \] (II.1.1, k)

\[ \phi_{34} = -\frac{2hm_2 (l_1 - lc_2) \delta \theta_1}{m_1} \]
\[ + \frac{1}{2} \frac{h^2}{m_1} \left( \frac{m_1 m_2 (-2 \delta l_1 \delta \theta_1 + g \sin \theta_1)}{m_1^2} \right) \]
\[ + \frac{4m_2^2 (l_1 - lc_2)^2 \delta \theta_1 \delta \theta_1}{m_1^2} \] (II.1.1, l)

\[ \phi_{41} = -hg \sin \theta_1 + \frac{h^2 (l_1 - lc_2)}{m_1} \delta \theta_1 \left( m_1 g l_c \cos \theta_1 + m_2 g (l_1 - lc_2) \cos \theta_1 \right) \] (II.1.1, m)

\[ \phi_{42} = h \delta \theta_1^2 \]
\[ + h^2 (l_1 - lc_2) \delta \theta_1 \left( \frac{m_1 m_2 (2 \delta l_1 \delta \theta_1 + g \sin \theta_1)}{m_1^2} \right) \]
\[ + \frac{2m_2 (l_1 - lc_2)}{m_1^2} \left( \frac{m_1 m_2 (-2 \delta l_1 \delta \theta_1 + g \sin \theta_1)}{m_1^2} \right) \] (II.1.1, n)

\[ \phi_{43} = 2h \delta \theta_1 (l_1 - lc_2) + \frac{1}{2} h^2 \left( -g \sin \theta_1 - \frac{4m_2 (l_1 - lc_2)^2 \delta \theta_1^2}{m_1} \right) \] (II.1.1, o)

\[ \phi_{44} = 1 + \frac{1}{2} h^2 \left( \delta \theta_1^2 - \frac{4m_2 (l_1 - lc_2)^2 \delta \theta_1^2}{m_1} \right) \] (II.1.1, p)

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II.2 Input Matrix

The input distribution matrix of the linearized state-space model of a manipulator module is given by

\[ \Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \\ \gamma_{31} & \gamma_{32} \\ \gamma_{41} & \gamma_{42} \end{bmatrix} \]  \hspace{1cm} \text{(II.2.1)}

\[ \begin{align*}
\gamma_{11} &= \frac{h^2}{m_1} - \frac{h^3 m_2 (l_1 - lc_2) \delta l_1}{m_1^2} \\
\gamma_{12} &= -\frac{h^3 (l_1 - lc_2) \delta \theta_1}{m_1} \\
\gamma_{21} &= \frac{h^3 (l_1 - lc_2) \delta \theta_1}{m_1} \\
\gamma_{22} &= \frac{h^2}{m_2} \\
\gamma_{31} &= \frac{h}{m_1} \left( 1 - \frac{2hm_2 (l_1 - lc_2) \delta l_1}{m_1} \right) + \frac{1}{2m_1} \left( -m_1 g l c_1 \cos \theta_1 + m_2 g (l_1 - lc_2) \cos \theta_1 \right) + \frac{4m_2^2 (l_1 - lc_2)^2 \delta \theta_1^2}{m_1^2} \\
\gamma_{32} &= \frac{4m_2 (l_1 - lc_2)^2 \delta \theta_1^2}{m_1} \right) \right) \\
\end{align*} \]  \hspace{1cm} \text{(II.2.1, e)}
\[
\gamma_{32} = \frac{\hbar}{m_2} \left[ -\frac{2hm_2 (l_1 - lc_2) \delta \theta_1}{m_1} + \frac{1}{2} \left( \frac{m_1 m_2 (-2 \delta l_1 \delta \theta_1 + g \sin \theta_1)}{m_1^2} - \frac{2m_2 (l_1 - lc_2)}{m_1^2} \left( \tau - 2m_2 (l_1 - lc_2) \delta l_1 \delta \theta_1 - m_1 g l c_1 \sin \theta_1 \right) \right) \right]
\]

\[
\gamma_{41} = \frac{h}{m_2} \left[ 2h(l_1 - lc_2) \delta \theta_1 + \frac{1}{2} h^2 \left( -g \sin \theta_1 - \frac{4m_2 (l_1 - lc_2)^2 \delta l_1 \delta \theta_1}{m_1^2} \right) \right]
\]

\[
\gamma_{42} = \frac{\hbar}{m_2} \left[ 1 + \frac{1}{2} h^2 \left( \delta \theta_1^2 - \frac{4m_2 (l_1 - lc_2)^2 \delta \theta_1^2}{m_1^2} \right) \right]
\]