SPACE VEHICLE MOTION RECOVERY IN PRESENCE OF ACTUATOR FAILURE

by

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Abstract

Recovery control methods have been proposed to tolerate the failure of the thrusters used in the attitude control system of spacecraft. Thrusters are used, in pair, in the spacecraft control system to exert external pure torques on the spacecraft. These torques can either be directly used to perform the rotational maneuvers or can be used to remove the angular momentum built up in the momentum wheels of the spacecraft.

The problem of stabilizing a spacecraft subjected to disturbance torques with control torques about two of its principal axes is addressed for the first time in this research. It has been shown that a stable equilibrium, best matching the objectives of the mission, has to be found. Two control laws have been proposed to arrive at the newly defined stable equilibrium. The first law, a nonlinear kinematic control scheme, is based on the Lyapunov method. The second law, which linearizes the system about the equilibrium point, uses the pole placement method as a kinematic controller. In both control laws, after the kinematic controller is developed, the backstepping method is used to derive the control efforts at each instance of the spacecraft corrective motion.

Torque thrusters are used for momentum removal of the spacecraft momentum wheels. Malfunctioning of these thrusters therefore hinders momentum dumping process. The spinning speeds of the spacecraft momentum wheels increases by time until they become saturated and the control over the attitude of the spacecraft is lost. The momentum removal procedures using external control torques about two or even one principal axis of the spacecraft is proposed for the first time in this research.

All the proposed control methods have been examined using numerical simulations and are shown to achieve desired performance.

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Acknowledgements

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Chapter 1

Introduction to actuators and control concepts

1.1 Spacecraft navigation equipment

The motions of a spacecraft as a rigid body can be divided into two categories, translational and rotational. The translational motions are performed by exerting an external force on the spacecraft. This force can be either a gravitational force or a thruster force. The gravitational force is dependant on the mass of the spacecraft, the nearby planet and the position vector from the center of mass of the planet to the center of mass of the spacecraft. The gravitational force had a significant role in propulsion of Voyager [1]. Although gravitational force does not consume any fuel, it does not provide spacecraft with sufficient autonomy either. Therefore, the fine-tuning of the spacecraft path must still be done through other types of actuators. Figure 1.1.a shows the main thrusters of the Cassini spacecraft used for the propulsion tasks [2].

There are typically three types of actuators used for performing the attitude maneuvers: thrusters, momentum wheels and magnetorquers.

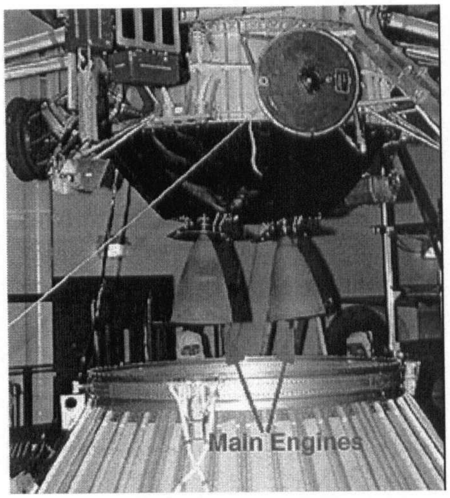

Figure1.1.a: Main thrusters of Cassini [2]

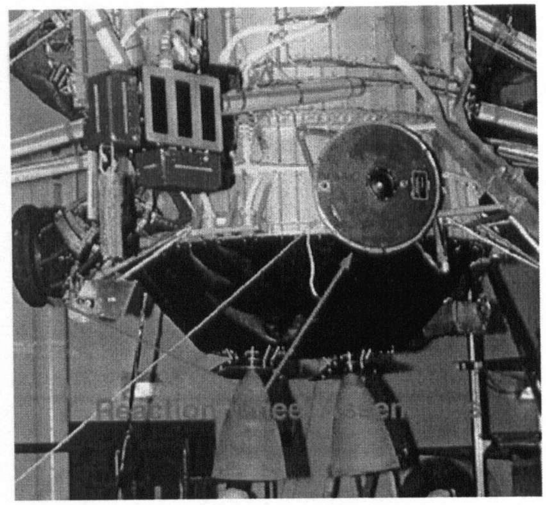

Figure1.1.b: Momentum wheels of Cassini

Thrusters of smaller size than those used for propulsion are used in pairs to exert external torques on the spacecraft. The generated torques using the thrusters are independent of the surrounding conditions, in contrast with magnetic torquers discussed later. Although the amount of torque is usually constant, by proper use of the pulse width modulation method any desired torque is achievable. Consider a desired torque over time. By the manipulation of the "on" duration of the thrusters, we can make the integral of the exerted torque over a small duration be equal to the integral of the desired torque over the same duration. The drawback of this method is the induced vibration due to the repeated switching of the exerted torque. An advantage of using the thrusters is their capability to generate external torque on the system. Consider some angular momentum being accumulated over time due to the disturbance torques on the spacecraft. By using the thrusters, we can compensate for this; i.e., "dump the momentum" of the spacecraft.

The disturbance torques acting on a spacecraft can be generated from different sources [3] including the translational thrusters, the magnetic torques, the gravity gradient torques, the aerodynamic torques, the solar radiation pressure and the frictional torques in the bearings. If the vector force generated by the thruster does not pass through the center of mass of the spacecraft there will be a torque acting on the system in addition to the exerted force. Electromagnetic torques, resulting from the interaction of the magnetic field generated by the spacecraft circuits and the external magnetic field, can act as disturbance torques. The gravity force changes nonlinearly with the distance. This causes the distribution of the gravity to differ from the distribution of mass over the spacecraft. If we represent the gravity force on the spacecraft by a net force vector, this vector is applied slightly off the center of mass and generates some torque. The gravity gradient torque tends to align the principal axis, associated with the lowest principal moment of inertia, along the direction of gravitational pull. Light carries momentum and when it is reflected from a surface an exchange of momentum occurs with the surface. This exchange of momentum results in a force being exerted on that surface. The solar panels of a spacecraft form the largest area of the exposure to the sunlight. As the solar panels are extended far from the center of mass the torque arm corresponding to the solar radiation force will be large and therefore a noticeable torque is generated. The frictional torques present in the bearings of the spacecraft, such as in the momentum wheels, act as internal disturbance torques. The friction torque does not change the total angular momentum of the spacecraft, but it can deviate the direction of the spacecraft in space. All the above disturbance torques are minute, but in the resistance-free environment they affect the rotational motions of the spacecraft.

The other instruments used for compensating the disturbance torques and performing the attitude maneuvers, are the reaction and momentum wheels, Figure 1.1.b. A reaction or momentum wheel is composed of a flywheel connected to the main spacecraft through an electric motor. The motor can exert torque on the flywheel, which in turn results in a reaction torque being exerted on the spacecraft. This reaction torque opposes the external disturbance torques. Therefore, the disturbance torques will affect flywheels instead of the spacecraft and the speed of the flywheel will increase by time. If the nominal spin rate of the flywheel is zero the assembly is called reaction wheel. The momentum wheels typically have some momentum bias, and spin at high rates, which slowly change to absorb the environmental torques [4]. Since the principles of dynamics and control of the reaction and momentum wheels are the same all the discussions ahead holds for both assemblies and from this point on we will use the terms interchangeably. A major advantage of using the momentum wheels is that they can exert accurate torques on the spacecraft. Another advantage of employing momentum exchange devices is that they use the electrical energy instead of consuming fuel. Electrical energy can easily be generated on the spacecraft using the solar panels in contrast with fuel, which is not recyclable. In fact, one of the key issues determining the life time of a spacecraft is the amount of fuel remaining on board. By using the electrical energy the life time of the spacecraft will be significantly increased. While using the momentum wheels for the attitude stabilization, the external torques on the spacecraft are accumulated and result in the speeding of the momentum wheels. There is an increasing relationship between the current passing through the electric motor and its torque, if the speed of the motor is below a specific limit. When the speed of the motor passes that limit, the motor is called saturated. The speed of the momentum wheel must then be reduced during a process called "momentum dumping". During the momentum dumping an external torque must be applied to the spacecraft to cease the total angular momentum accumulated over time. This external torque is usually provided by using the thrusters or the magnetorquers.

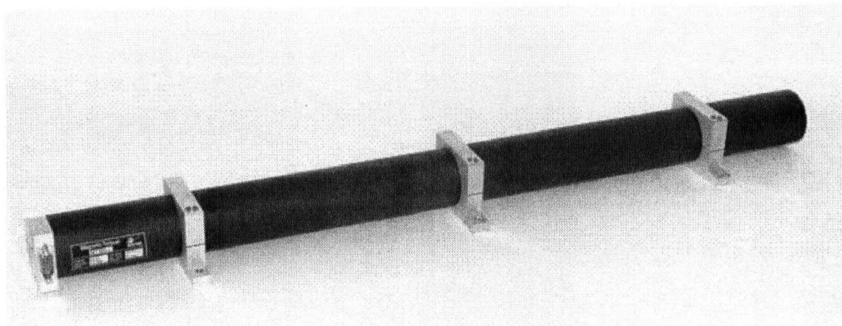

Figure 1.2: A magnetic torquer [5]

If a magnetic moment is generated in the spacecraft, the interaction between this magnetic moment and the magnetic field of the earth will result in a torque. This external torque can be used for the attitude stabilization, the attitude maneuvers or the momentum dumping of the momentum wheels. This is the principle of using the magnetorquers, Figure 1.2. The exerted torque will be dependant on both the magnetic moment of the spacecraft and the magnetic field of the earth present at the location of the spacecraft. Through momentum wheels we can generate external torques using the electrical energy. Therefore, without any fuel consumption or any need of momentum dumping we can perform the attitude maneuvers. The drawback of these instruments is their dependency on the outside magnetic field. This dependency is why they cannot be used in interplanetary spacecraft, which fly through the environments where there is no magnetic field present. The generated torques of the magnetic torquers are relatively small compared to other instruments and they cannot perform the fast attitude maneuvers. Since the earth magnetic field varies from point to point, the generated torque is not accurate. Therefore, the magnetorquers are used for the rough attitude maneuvers in near earth satellite.

1.2 Review of the stability and control concepts

1.2.1 Equilibrium and stability

By controlling a system we try to reach the desired states of the system and maintain them afterwards. The desired state should be a stable equilibrium point of the controlled system. In this section the mathematical definitions are given for the concepts of equilibrium and stability. The methods used for proving the stability are presented which can also be used for stabilizing an equilibrium of a system.

We represent our system in the state space form as

$$\dot{x} = f(x) \tag{1}$$

A state x^* is an equilibrium state (or an equilibrium point of the system) if once $x(t)$ equals x^* it remains equal to x^* for all the future time. [6]. The following equation should therefore be satisfied

$$f(x^*) = 0 \tag{2}$$

An equilibrium point of a system can be either stable or unstable which is defined as bellow.

The equilibrium state $x = 0$ is said to be stable (in the sense of Lyapunov) if, for any $R > 0$, there exists $r > 0$, such that if $\| x(0) \| < r$, then $\| x(t) \| < R$ for all $t \geq 0$. Otherwise, the equilibrium point is unstable. In the above definition $\| . \|$ represents the Euclidean norm of a vector. What is mostly expected from a controller is not only to keep the trajectory in the vicinity of an equilibrium point but also to make the system gradually converge to the desired point in time. This property is called the asymptotic stability; an equilibrium point, 0, is called asymptotic stable if it is stable and if in addition there exists some $r > 0$ such that $\| x(0) \| < r$ implies that

$x(t) \to 0$ as $t \to \infty$. Intuitively, the stability of an equilibrium means that the system returns to the equilibrium if slightly deviated from that equilibrium. As there are always disturbances in real systems, a real system almost never stays at an unstable equilibrium point. The definition of the stability and the asymptotic stability were given considering $x = 0$ as the equilibrium point. If the equilibrium point of the control system is not located at the origin we can still use the above definitions by mean of a change of variables. Consider x^* to be the equilibrium point of the system if we define $\tilde{x} = x - x^*$, since x^* is constant we will have $\dot{\tilde{x}} = \dot{x}$. Using equation (1) we obtain $\dot{\tilde{x}} = f(\tilde{x} + x^*) = g(\tilde{x})$. Now $\tilde{x} = 0$, which corresponds to $x = x^*$, will be the equilibrium of the system and the previous definition holds.

The above definitions were used to define the stability and the asymptotic stability of a system in a local sense. The behavior of a system while the states are in a neighborhood of the equilibrium points was discussed in the previous definitions. The global stability should be defined to address the behavior of the system, in a global manner. If the asymptotic stability holds for any initial state the equilibrium point is said to be asymptotically stable at large. It is also called globally asymptotically stable.

Now that the definitions of stability have been stated, we introduce some methods for checking the stability of a system. In the Lyapunov's linearization method the local stability of a system is checked by the behavior of its linearized model. Consider a system with the state space equations as (1). By linearization of this system, we mean finding the best A matrix in the equation (3) that can approximate the behavior of the system about the origin.

$$\dot{x} = Ax \tag{3}$$

Considering the first order terms in the Taylor series, matrix A can be expressed as

$$A = \left(\frac{\partial f}{\partial x} \right)_{x=0} \tag{4}$$

where, $\frac{\partial f}{\partial x}$ is the Jacobian of the function f with respect to the states of the system. According to the Lyapunov's linearization method for local stability [6],

- If the linearized system is strictly stable (i.e. all the eigenvalues of A are strictly in the left half of the complex plane) then the equilibrium point is asymptotically stable.
- If the linearized system is unstable (i.e. if at least one of the eigenvalues of A is strictly in the right half complex plane) then that equilibrium point of the nonlinear system is unstable.
- If the linearized system is marginally stable (i.e. all the eigenvalues of A are in left half of the complex plane but at least one of the poles is on the imaginary axis) then nothing can be concluded from the linear approximation. The nonlinear system can be stable, asymptotically stable or even unstable.

The most common method for proving the stability of a system, especially the global stability is the Lyapunov's direct method. In this method a function of the states, like $V(x)$, should be found such that it is positive definite while its time derivative negative is semi definite. A positive (/negative) definite function is a strictly positive (/negative) function of states vanishing only at the origin. A positive (/negative) semi-definite function is defined similarly to the positive (/negative) definite function except, it can be zero or positive (/negative) throughout the domain.

Lyapunov Theorem for local stability states that [6], if in a ball B_{R_0}, containing the equilibrium point $\mathbf{0}$ there exists a scalar function with continuous first partial derivatives such that

- $V(x)$ is positive definite (locally in B_{R_0})
- $\dot{V}(x)$ is negative semi-definite (locally in B_{R_0})

then the equilibrium point **0** is stable. If $\dot{V}(x)$ is locally negative definite in B_{R_0}, then the stability is asymptotic.

For proving global stability of a system the Lyapunov's global stability condition should be checked. Assume that there exists a scalar function V of the state x, with continuous first order derivatives such that

- $V(x)$ is positive definite
- $\dot{V}(x)$ is negative definite
- $V(x) \to \infty$ as $\| x \| \to \infty$

then the equilibrium at the origin is globally asymptotically stable.

It often happens that we can find a Lyapunov function with a negative semi definite time derivative, but we want to prove the global stability of the system. The powerful invariant set theorem attributed to La Salle can be used in this case. A set **G** is an invariant set for a dynamic system if every system trajectory which starts from a point in **G** remains in **G** for all future time. For instance, any equilibrium point is an invariant set. The global invariant set theorem can be stated as follows. Consider the system described in (1) with f being continuous. Let $V(x)$ be a scalar function with continuous first partial derivatives. Assume that

- $\dot{V}(x) \leq 0$ over the whole state space
- $V(x) \to \infty$ as $\| x \| \to \infty$.

Let **R** be a set of all points where $\dot{V}(x) = 0$, and **M** be the largest invariant set in **R**, then all the solutions globally asymptotically converge to **M** as $t \to \infty$.

1.2.2 Homogeneity concept and applications

When the controllability of a system is lost by linearization, the system has to be modeled nonlinearly. On the other hand, although we may have to keep the lower order nonlinear terms, the higher order terms can be neglected. Using the concept of homogeneity we can simplify the actual system to a lower order nonlinear system and develop a control law for the simplified system that stabilizes the actual system. In the following we define the homogeneous system and state how stability of the homogeneous approximation of the system is related to the stability of the entire system.

Dilation operator [7]: let $\lambda > 0$ and any set of positive scalars $r_i > 0$, $i = 1,\dots,n$, then the attitude dilation operator δ_λ is defined as

$$\delta_\lambda(x_1,\dots,x_n) = \left(\lambda^{r_1} x_1,\dots,\lambda^{r_n} x_n\right) \tag{5}$$

Where $r_i > 0$ are the weights of dilation. Now we can define the homogeneous functions based on the definition of the dilation. A function $h : \Re^n \to \Re$ is said to be positively homogeneous of degree k with respect to a given dilation δ_λ if

$$h(\delta_\lambda(x_1,\dots,x_n)) = \lambda^k h(x_1,\dots,x_n) \tag{6}$$

A vector field is a function $f(x)$ that assigns to each point $x \in \Re^n$ a vector $f(x) \in \Re^n$. For a vector field $f : \Re^n \to \Re^n$ being homogeneous means

$$f_i(\delta_\lambda(x_1,\dots,x_n)) = \lambda^{k+r_i} f_i(x_1,\dots,x_n) \tag{7}$$

where f_i stands for the i^{th} component of the vector field f. The following theorem states how homogeneity can be used similar to linearization in controlling systems.

Consider f to be a homogeneous vector field of degree k with respect to a given dilation δ_λ and let g be a continuous vector field both defined on $\Re^n$, such that for all $i = 1,...,n$

$$\lim_{\lambda \to 0} \frac{g_i(\delta_\lambda(x_1,...,x_n))}{\lambda^{k+r_i}} = 0 \tag{8}$$

The origin is an equilibrium of the system so the function f vanishes at the origin. If the trivial solution $x = 0$ of $\dot{x} = f(x)$ is locally asymptotically stable, the same is true for the trivial solution of the perturbed system $\dot{x} = f(x) + g(x)$. This fact can be used in controlling general systems like $\dot{x} = h(x)$. We should decompose the state space function to a function, $f(x)$ with degree of homogeneity of k and the remainder of higher order terms, $g(x)$. Therefore

$$h(x) = f(x) + g(x)$$

$$\lim_{\lambda \to 0} \frac{g_i(\delta_\lambda(x_1,...,x_n))}{\lambda^{k+r_i}} = 0$$

Then we continue designing a controller to stabilize the trivial solution of $\dot{x} = f(x)$. The origin will be a locally stable equilibrium point of the entire system as a result of the above theorem.

1.2.3 Lie derivative and Lie algebra

In this section the mathematical definitions of the Lie derivatives, the Lie brackets and the Lie algebra are stated [6]. This mathematical tool is needed for proving controllability of a nonlinear system, while using the feedback linearization method the Lie algebra is used for checking the necessary and sufficient condition of the feedback linearizability of the system. In the following, a vector function $f : \mathrm{IR}^n \to \mathrm{IR}^n$ is called a vector field. Given a smooth scalar function $h(x)$ the gradient of this function denoted by $\nabla h = \partial h / \partial x$ is a row vector defined as

$(\nabla h)_i = \partial h / \partial x_i$. Similarly given a vector field $f(x)$ the Jacobian of f denoted by ∇f is defined as $(\nabla f)_{ij} = \partial f_i / \partial x_j$.

Definition of the Lie derivative: let $h: \mathrm{IR}^n \to \mathrm{IR}$ be a smooth scalar function and $f: \mathrm{IR}^n \to \mathrm{IR}^n$ be a smooth vector field on IR^n then the Lie derivative of h with respect to f is a scalar function defined by $L_f h = \nabla h\ f$. Thus, the Lie derivative $L_f h$ is the directional derivative of h along the vector field f. The repeated Lie derivatives can be recursively defined as

$$L_f{}^0 h = h \tag{9}$$

$$L_f{}^i h = L_f(L_f{}^{i-1} h) \tag{10}$$

Based on the definition of the Lie derivative, the Lie bracket is defined as: Let f and g be two vector fields on IR^n. The Lie bracket of f and g is a third vector field defined by

$$[f, g] = \nabla g\ f - \nabla f\ g \tag{11}$$

The following notation is often used for the Lie product of two functions

$$ad_f g = [f, g] \tag{12}$$

Repeated Lie brackets are defined recursively as

$$ad_f{}^0 g = g \tag{13}$$

$$ad_f{}^i g = \left[f, ad_f{}^{i-1} g\right] \tag{14}$$

The above definitions are used in section 4.10 to examine the feasibility of controlling the underactuated spacecraft using feedback linearization method.

1.2.4 Feedback linearization

The traditional method of dealing with nonlinear systems has been to linearize the system (using for example the Taylor expansion) and then using the linear controller design methods. It may also be possible to define some parameters based on the states and inputs of the actual plant, such that the new parameters form states and inputs of a linear control system. The main point in the feedback linearization of a system is to find a mapping that can find a linear system resembling actual nonlinear plant. Consider the nonlinear system

$$\dot{x} = f(x,u) \tag{15}$$

where x represents the states vector and u is the inputs vector. The input-state linearization technique, the feedback linearization technique used in this thesis, finds a state transformation $z = \boldsymbol{z}(x)$ and an input transformation $u = \boldsymbol{u}(x,u)$ such that the nonlinear system dynamics is transformed to an equivalent linear time invariant system [6].

$$\dot{z} = Az + Bv \tag{16}$$

in which z is the new state vector of the system. Not every nonlinear system can be linearized using input-state linearization. Before going through the necessary and sufficient condition for the feedback-linearizability we state the definition of the vector field distribution. Given a set of smooth vector fields $X_1, X_2, \ldots, X_m$, a distribution $G(x)$ is defined [8] as

$$G(x) = \text{Span}\{X_1, X_2, \ldots, X_m\} \tag{17}$$

Equivalently

$$G(x) = \alpha_1 X_1 + \alpha_2 X_2 + \ldots + \alpha_m X_m \tag{18}$$

in which $\alpha_m(x)$ is a smooth function of x.

The actual nonlinear system is represented in the following form

$$\dot{x} = f(x) + g(x)u \tag{19-a}$$

$$y = \lambda(x) \tag{19-b}$$

In which $x \in \Re^n$ is the state vector, $u \in \Re^m$ is the control effort vector and y is the output vector. Therefore, $g(x)$ is an $n \times m$ matrix, with $g_1, g_2, ..., g_m$ being each of its columns. We define the following distributions [9].

$$\begin{aligned} G_0 &= \text{span}\{g_1, ..., g_m\} \\ G_1 &= \text{span}\{g_1, ..., g_m, ad_f g_1, \ldots, ad_f g_m\} \\ &\ldots \\ G_i &= \text{span}\{ad_f{}^k g_j : 0 \le k \le i, 1 \le j \le m\} \end{aligned} \tag{20}$$

If x^0 is the desired equilibrium of the system (19), the system can be feedback linearized if and only if

i. For each $0 \le i \le n-1$, the distribution G_i has constant dimension near x^0

ii. The distribution G_{n-1} has dimension n

iii. For each $0 \le i \le n-2$ the distribution G_i is involutive.

A distribution G is called involutive if for any two vector fields $\tau_1, \tau_2 \in G$ their Lie bracket $[\tau_1, \tau_2] \in G$ [8].

The feedback linearization method is used in chapter 5 for deriving the attitude changing controller. The feasibility of using this method for controlling underactuated spacecraft is studied in section 4.10.

1.2.5 Backstepping method

The backstepping is a useful method in controlling cascade systems. Consider the system

$$\dot{x} = f(x) + g(x)y, \ \ f(0) = 0, x \in \Re^n, y \in \Re \tag{21-a}$$

$$\dot{\xi} = m(x,\xi) + \beta(x,\xi)u,\ y = h(\xi),\ \xi \in \Re^q, u \in \Re \tag{21-b}$$

n and q are the integer numbers representing the dimension of the domains. Consider the states of the system to be $[x,\xi]^{\mathrm{T}}$. The states x are not directly related to the effort term vector u and are affected by y, a function of all the states and the effort vector. The subsystem in the equation (21-a) is considered first. The parameter y is considered as the effort term of this system and the rule $y = \alpha(x)$ is derived to stabilize the origin of the subsystem. Using the backstepping method we derive a control rule for u such that the vector y converges to the prescribed value $\alpha(x)$ and even if y is still not close to the prescribed values the entire system does not diverge. Since the subsystem described by the equation (21-a) is stable a Lyapunov function, V, can be found. If the function V is positive definite, its time derivative is negative semi-definite and if $V(x)$ is bounded provided x is also bounded, we say that the "condition 1" is preserved. According to the nonlinear backstepping theorem [10] if $\alpha(x)$ is a differentiable function and the condition 1 is preserved there exists a feedback control law which guarantees the global boundedness and convergence of state vector to the largest invariant set contained in the set

$$E_a = \left\{ \begin{bmatrix} x \\ \xi \end{bmatrix} \in \Re^{n+q} \,\middle|\, \dot{V}(x) = 0, y = \alpha(x) \right\} \tag{22}$$

One particular choice provided by the backstepping method [10] is

$$u = \left(\frac{\partial h}{\partial \xi} \beta(x,\xi) \right)^{-1} \left\{ -c(y - \alpha(x)) - \frac{\partial h}{\partial \xi} m(x,\xi) + \frac{\partial \alpha}{\partial x} [f(x) + g(x)y] - \frac{\partial V}{\partial x} g(x) \right\}, c > 0 \tag{23}$$

This control low is expanded to multi input systems and to the systems with nonlinear effort state relation in the next chapters. The backstepping method is used for deriving the controller for underactuated spacecraft in chapter 4.

Chapter 2

State of the art in control of underactuated spacecraft and momentum dumping

2.1 Literature survey for control of underactuated spacecraft

There are usually three control torques provided for controlling and correcting the attitude of a spacecraft. These torques can be applied using different types of actuators including thrusters and momentum wheels. As long as the actuators are all functional and we can change the attitude using a combination of the three linearly independent control torques, the spacecraft is referred to as "fully actuated". In case of a failure in any of these actuators, the spacecraft must be controlled using two or even one control torque. In this case, since the number of the control torques is less than the number of the attitude describing parameters, the spacecraft is called "underactuated".

Attitude control of an underactuated spacecraft has been under attention for a long while and numerous methods have been used for stabilizing a spacecraft. Crouch [11] for the first time discussed the controllability of an underactuated spacecraft using either thrusters or momentum wheels and showed that the controllability is preserved with less than three pairs of thrusters while it is lost even if one of the three momentum wheels becomes defective. Attitude stabilization of underactuated spacecraft, using two pairs of thrusters, has been considered in [12] and has been shown that the spacecraft cannot be locally stabilized to a static equilibrium using a smooth feedback control law. The control method proposed in [13] and the first control method in [14] bring the spacecraft to the desired orientation using a sequence of maneuvers. The first step of this sequence makes the angular velocity of the spacecraft zero and later the desired attitude is reached by performing other maneuvers, each designed so that at the end of each maneuver the angular velocity of the spacecraft is returned zero. Therefore, if zero velocity is an unstable state of the system or if we want to track a time varying attitude, we cannot use either of these two methods. Stabilization of a spacecraft has also been approached using the concept of homogeneity [15], [16], [17]. The general idea is to divide the states in the dynamic equations to a main part and a set of perturbation terms, which are of higher orders than the main terms. The main part is then globally stabilized and the entire system is shown to be locally asymptotically stable. This method works only if the initial states are in a region close enough to the desired values. Another shortcoming of this method is the limit over the control gains; if we want to reach the desired values faster by increasing the control gains, there will be the danger of the trajectory entering the unstable zone. Intelligent control methods have also been used for controlling the underactuated spacecraft. Ge and Chen [18] used the genetic algorithm to optimize an objective function corresponding to the trajectory error and the control efforts. The derived control inputs are then used for the open loop controlling of the system. In addition to the inability of dealing with the modeling errors and the disturbances, the optimized control inputs are not generic and are highly dependent on the initial conditions. This means that the optimization must be done before performing any maneuvers; this demands either a huge computational capability on board or a high speed and steady earth-based communication, which is hard to achieve especially for the interplanetary spacecraft.

A new attitude parameterization has been introduced in [18] and [19]. Based on this approach a new control method has been developed in [20] and later modified [21], [22] to counter the bounded inputs. However, only the attitude stabilization of an axisymmetric spacecraft has been discussed. Bacconi et al [23] have used an internal supervisor to choose between the control laws in [17] and [24] at each instance based on the appropriate performance condition of each control law. Recently, a control method [25] has been developed using the quaternions for the attitude parameterization. We used the same parameterization in deriving the control laws in chapter 5. Although a rigid (not necessarily axisymmetric) spacecraft has been considered for deriving the control law in the kinematics level, but with respect to the dynamics, the spacecraft has been assumed axisymmetric.

In the first phase of this research, we consider a practical case where the control of an underactuated spacecraft is investigated. If any of the auxiliary thrusters responsible for the compensation of the disturbance torques becomes dysfunctional, not only the spacecraft will become unactuated about one of the principal axes, but also there will be a constant torque about the unactuated axis. By using the Lyapunov control method, we employ the remaining two thrusters to bring the spacecraft into a rotational motion about any axis, and set that axis in the desired direction. If this axis is chosen to be the main propulsion thruster axis then, the translational motion of the spacecraft will be unaffected by the failure. We divide the governing equations of the spacecraft into two subsystems. In controlling the first subsystem, a control law is developed to enable the spacecraft reach the desired motion by prescribing the values for the angular velocities about the actuated axes. This control law is shown to make the desired motion of the spacecraft globally asymptotically stable. We then use a modification of the backstepping method for the systems with nonlinear effort-state relation, derived in the appendix, to find the control torques. Finally, several numerical simulations are performed which illustrate the robustness of the system.

2.2 Literature survey for momentum dumping

The angular momentum accumulated in the momentum wheels can cause some trouble for the spacecraft. As discussed before, the spacecraft is subjected to the disturbance torques. The effect of these torques is compensated by the implementation of the momentum wheels which transfers the angular momentum caused by the disturbance torque to the wheels and leaves the spacecraft unaffected. The motors of the momentum wheel assemblies are torque controlled and the increasing relation between the torque implemented by a motor and the electrical current passing through it only holds if the motor is spinning at low speeds. When the spinning speed of a momentum wheel becomes so high that the torque control is lost, it is called saturated. In addition, the gyroscopic torques generated during the attitude maneuvers is proportional to the angular momentum of the spacecraft. When the spinning speed of the momentum wheels increases the gyroscopic torques intensify and more electrical energy is consumed during the attitude maneuvers. There are typically two approaches to prevent saturation of the momentum wheels; the momentum management and the momentum dumping.

The first method is to try to have the speed of the momentum wheels zero at the end of each cycle of the periodic motion of a satellite. This approach is called momentum management. The momentum management of the spacecraft has two aspects. First, if there are no disturbance torques acting on the spacecraft and the initial spinning speed of the momentum wheels has been zero, then at the end of each attitude changing maneuver the spinning speeds should vanish. If the spacecraft uses three momentum wheels with linearly independent main axes, this condition is preserved according to the principle of conservation of the angular momentum. To avoid lack of control over the spacecraft in case of a momentum wheel failure, usually four momentum wheels are placed in the spacecraft. All four wheels are simultaneously implemented and it is possible that the total angular momentum of the spacecraft is zero but the momentum wheels are spinning. The momentum management of spacecraft in this case means adjusting the control rule to minimize the momentum wheels spin rate at the end of the maneuver, as discussed in [26]. The second aspect of the momentum management is to diminish the total momentum change of the spacecraft at the end of each cycle of the periodic maneuvers. Consider a satellite orbiting the earth. As mentioned before there are several disturbance torques acting on the satellite including the gravity gradient torques, the solar radiation pressure, the earth's magnetic filed effects and the aerodynamics drags. All these torques have periodic and secular components when considering their variation when the satellite orbits the earth. The periodic components are cancelled out when integrating their effect over an orbiting period. The secular terms however, build momentum in the reaction wheels. The gravity gradient torque depends on the orientation of the spacecraft. Some external torques can also be generated implementing the magnetic torquers. So, by deviating the spacecraft slightly form its desired attitude or by continuously using the magnetic torquers it is possible to make the net momentum change vanish at the end of each rotation of the spacecraft about the earth.

The gravity gradient torque has been used extensively in the momentum management of the space stations. The momentum management of the Sky lab space station has been performed by implementing the gravity gradient torque [27], [28]. The momentum management and the attitude stabilization control laws are designed together in [29] and [30] using the optimal linear quadratic design methods. The control law proposed in [29] was later modified to a digital controller to be implemented on the Freedom space station [31]. μ synthesis from modern control theory has been applied to design the momentum dumping control law proposed in [32]. In [33] the pole placement techniques have been used to design the attitude controller and the momentum management controller for a multi-body and flexible spacecraft. Johnson and Skelton [34] took into consideration all the disturbance torques applied on the spacecraft. They used an estimator to measure online the disturbance torque being applied on the spacecraft and designed an optimal controller to make use of this disturbance torque to desaturate the momentum wheels.

The magnetic torques have been implemented for a long while in the spacecraft attitude control and momentum management. Glaese et al [35] implemented the magnetic torquers in the low cost pointing control system for the space telescope. [36] and [37] addressed using the magnetic torquers for the momentum management of the earth pointing Global Positioning Satellite (GPS). Camilo and Markley [38] discussed the bang-bang and linear controllers for a spacecraft controlled with the magnetic torques. They came up with a control effectiveness ratio between the orbit-averaged magnetic torques and the disturbance torques acting on the spacecraft. This ratio should be greater than one for the satellite's entire life time. The ratio can be used in the preliminary studies instead of the detailed simulations. An adaptive control law for unloading the angular momentum of a spacecraft was proposed by Burns and Flashner [39]. They made use of the magnetic torques in addition to the aerodynamic and gravity gradient torques for controlling the spacecraft. The magnetic torquers were used together with one momentum wheel to control the attitude of a small satellite [40]. Two complimentary control strategies were implemented to maximize the controllability in the varying Earth magnetic field. Steyn [41] made a detailed comparison between the optimal desaturation algorithms and the conventional cross product law. While there is a significant work on momentum management of the spacecraft, the problem of momentum dumping using less that three actuators has not previously been discussed.

Chapter 3

Problem definition and significance

3.1 Failure scenarios

The focus of this research is on the motion recovery of the spacecraft in case of the thruster failures. Thrusters play two main roles in the attitude control of spacecraft; they can be used for providing control torques needed in the attitude maneuvers and they are necessary in removing the accumulated momentum in the momentum wheels.

The thrusters are placed in a special configuration on the spacecraft to generate pure external torques. The torques are determined by the attitude control system based on the attitude and angular velocity feedbacks and the estimation of the disturbance torque on the spacecraft. If the control torque about one of the principal axes is lost, we still desire to control the attitude of the spacecraft. In such a situation the spacecraft must be controlled using the control torques about two principal axes. The situation becomes worse if we have a disturbance torque about the *unactuated* axis which we cannot compensate with the thruster torques. In such a situation the disturbance torque kicks the spacecraft out of the stability. It is shown in the next chapter that having a constant control torque about the unactuated axis causes no static attitude to be stable. Therefore, in order be stable, the spacecraft must rotate. For the first time a control law has been developed to deal with this disturbance torque and to bring the spacecraft to a stable condition in which an arbitrary axis of the spacecraft is fixed in space and the spacecraft rotates with constant angular velocity about that axis. This arbitrary axis may be the main thruster, the communication antenna, the camera or a specific sensor of the spacecraft.

The thrusters have the key importance in performing the momentum dumping procedures. Any external torque, including the disturbance torques, change the total angular momentum of the spacecraft. When the momentum wheels are implemented for controlling the spacecraft this change of angular momentum is transferred to the momentum wheels, altering their spinning speed, so that the main body of spacecraft remains unaffected. The total disturbance torque consists of the persistent and periodic components. The periodic torques are cancelled out while being integrated over a period of time. The secular torques cause the momentum wheels to accelerate until their angular velocity passes the saturation limit. In this case the momentum wheel is saturated. The thrusters are then used to remove the angular momentum accumulated in the flywheels. The momentum dumping of the spacecraft using two or one external control torques is addressed for the first time in this research. Inability to remove the angular momentum of the momentum wheels results in lack of control over the spacecraft, and eventually ends its functional life. Therefore, using the proposed control method keeps the spacecraft functional in spite of the loss of the external torque about one or even two of the spacecraft axes.

3.2 Probability of actuation failure

Considering the harsh conditions under which a spacecraft operates, the likelihood of a failure is not insignificant. These conditions include the very low ambient pressure, the severe temperatures, the significant temperature differences between the opposite sides of the spacecraft, and the solar flares. The spacecraft is built on earth and usually at the ambient pressure. If there is any air bubble trapped in the spacecraft structure, it can cause significant stresses in the structure during the mission. As there is little or no matter in the outer space there is essentially no heat transfer with the spacecraft through conduction or convection. The temperature of the spacecraft is therefore dependant on the amount of radiation from the planets or stars [42]. In case of a satellite facing the earth while not being exposed to the sunlight this temperature can be as low as -200°C [43]. Considering that the spacecraft has been manufactured at the room temperature, this is a significant temperature change. This temperature variation results in the joints loosening or tightening which may disable the joints. Since different materials have different expansion coefficients, this great temperature differential can also cause deflections in the structure of the spacecraft. On the other hand, since the only way of heat transfer is radiation there can be huge temperature differences between the two sides of the spacecraft, which can lead to severe deflection in the weak elements such as the antennas or the solar panels [44].

Another source of failure in spacecraft is the effect of the solar flares. A solar flare is an electromagnetic blast at the sun releasing the energy equivalent to tens of millions of hydrogen bombs. The energy is released in a wide spectrum of electromagnetic waves, ranging from the long wavelength radio to the short wavelength gamma rays [45]. Since there is no atmosphere protecting the spacecraft from the flares, they can cause severe damages to the spacecraft. For example, a solar flare has been identified as the main cause of the failure of the Nozomi spacecraft [46]. As a whole, the undesirable and unexpected perturbation can lead to the structural failure, the equipment seizures and the electric noises.

The unpredictability of the situation is another factor that can result in a failure in the spacecraft systems. Interplanetary spacecraft go through new frontiers to gather some basic information about the new planets. Typically, the situation happening to the spacecraft is not well known and many scenarios can happen to the spacecraft which have not been experienced on earth before.

There are three solutions for continuing the mission while a failure has occurred. The first one is to repair the spacecraft using another spacecraft or by the human intervention in space. In this case the faulty part must have been identified by the main spacecraft itself and that specific part must be replaced using the external help. Due to the complexity of such a process and the need for specialized instruments, this is a rare option. Sending a repairing spacecraft together with the spare part and conducting the repair action could cost more than the initial spacecraft. Therefore, the repair using external help is not feasible except in the case of the extraordinary expensive and close to earth spacecraft such as the Hubble telescope.

The second solution is to resort to redundancy of the key parts of the spacecraft. There should be a fault detection, diagnosis and reconfiguration system that identifies the faulty part and replaces it with the corresponding redundant part. The problem with this method is the additional weight of the spacecraft due to the weight of the redundant parts. The weight is a key factor in designing a spacecraft. The launching equipment usually weights ten times the actual spacecraft. Therefore, by doubling the weight of any module on the spacecraft the weight of the launching equipment will proportionally increase and imposes huge numbers on the cost of the mission.

The third solution is the software manipulation of the system so that the desired performance is obtained in spite of the failure. This method will impose no additional cost and is the most desirable solution. The only problem is that it is not always possible to find such a recovery algorithm. Simply, if the mission could be accomplished without using a part then what has been the reason for putting such a part in the spacecraft? The recovery algorithms are usually not efficient and will impose some performance constraints on the spacecraft. In practice, a combination of the last two methods has proven to be the most practical.

In this research, the recovery solutions have been proposed to determine the most desirable maneuvers to help accomplish the mission in the case of actuator failures.

Chapter 4

Attitude control of underactuated spacecraft

There are three independent parameters needed to uniquely identify the attitude of the spacecraft. The angular velocity vector of the spacecraft is a vector in the three-dimensional space and therefore has three components. A spacecraft is called *fully actuated* if there are at least three linearly independent control torque vectors available for controlling the spacecraft. Otherwise, if because of actuator failure, less than three linearly independent control torques are available, the spacecraft is called *underactuated.*

4.1 Governing equations

Different notations can be used to represent the orientation of a rigid body. In this section the notation introduced in [20] is used. The orientation of a body-fixed coordinate system relative to the inertial frame is described using a real number, *z*, and an imaginary number, w. The inertial coordinate system can be transferred to the body coordinate system using two rotations. The first rotation is about the inertial z-axis with an amount of z radians. Let $(\hat{i}_1, \hat{i}_2, \hat{i}_3)$ represent the inertial coordinate system; $(\hat{b}_1, \hat{b}_2, \hat{b}_3)$ be the principal axes of the body coordinate system; and $(\hat{i}'_1, \hat{i}'_2, \hat{i}'_3)$ represent the intermediate coordinate system obtained by the rotation of the inertial coordinate system about its z-axis. Both $(\hat{b}_1, \hat{b}_2, \hat{b}_3)$ and $(\hat{i}'_1, \hat{i}'_2, \hat{i}'_3)$ are expressed in the inertial frame. If $\hat{i}_3' = \alpha' \hat{b}_1 + \beta' \hat{b}_2 + \gamma' \hat{b}_3$ then the complex attitude coordinate, w, is defined in [20] as $w = \dfrac{\beta' - i\alpha'}{1+\gamma'}$, in which $i = \sqrt{-1}$. The second rotation that takes the intermediate coordinate system to the body coordinate system is about the unit vector $\hat{u} = \dfrac{2w_1}{2|w|}\hat{i}_1' + \dfrac{-2iw_2}{2|w|}\hat{i}_2'$ with the rotation angle $\theta = \arccos\left(\dfrac{1-|w|^2}{1+|w|^2}\right)$. In which if $w = w_1 + iw_2$, $\overline{w} = w_1 - iw_2$ and $|w|^2 = w\overline{w}$. The most significant advantage of this method of representing kinematics over the other methods is the relation between the kinematic parameters and the angular velocities; we have [24]

$$\dot{w} = -i\omega_3 w + \frac{\omega}{2} + \frac{\overline{\omega}}{2} w^2 \tag{24}$$

$$\dot{z} = \omega_3 + \mathrm{Im}(\omega\overline{w}) \tag{25}$$

In which ω_3 is the component of the angular velocity of the body about its z-axis, and Im(.) stands for the imaginary part. According to [20], equation (24) can also be written as

$$\frac{d}{dt}|w|^2 = \left(1+|w|^2\right)\mathrm{Re}(\omega\overline{w}) \tag{26}$$

or

$$\dot{w}_1 = w_2(\omega_2 w_1 + \omega_3) + \frac{1}{2}\omega_1\left(1 + w_1^{\ 2} - w_2^{\ 2}\right) \tag{27-a}$$

$$\dot{w}_2 = w_1(\omega_1 w_2 - \omega_3) + \frac{1}{2}\omega_2\left(1 - w_1^{\ 2} + w_2^{\ 2}\right) \tag{27-b}$$

In which Re(.) stands for the real part of a number. The advantage of these equations over the kinematic relations in the Euler representation is that they are simple polynomial equations rather than trigonometric functions. The equations governing w are independent from the equation for z, which we will use from this point on in our proposed control law.

The dynamic equations, describing the evolution of the angular velocities, are the well known Euler equations. The spacecraft if subjected to the disturbance torques about three principal axes and the control torques. In this study, we assume that because of a failure in thrusters we have lost the control over one of the spacecraft principal axes. Therefore, only control torques about two of the spacecraft principal axes are available. Without loss of generality, it is assumed that the third principal axis has become unactuated. Therefore, the torque about the third principal axis will be the only component corresponding to the disturbance torques. This component which is assumed to be constant is called M_{3c}. The net torque applied about the first and second principal axes are respectively M_1 and M_2. The Euler equations written in the principal coordinate system of the spacecraft is

$$I_1\dot{\omega}_1 = (I_2 - I_3)\omega_2\omega_3 + M_1 \quad (28\text{-a})$$

$$I_2\dot{\omega}_2 = (I_3 - I_1)\omega_3\omega_1 + M_2 \quad (28\text{-b})$$

$$I_3\dot{\omega}_3 = (I_1 - I_2)\omega_1\omega_2 + M_{3c} \quad (28\text{-c})$$

In which I_1, I_2, I_3 *are* the moments of inertia about $\hat{b}_1, \hat{b}_2$ and $\hat{b}_3$ and M_1, M_2, M_{3c} are the moments about each axis, respectively. From equations (28) we have

$$\dot{\omega}_1 = A\omega_2\omega_3 + u_1' \quad (29\text{-a})$$

$$\dot{\omega}_2 = B\omega_1\omega_3 + u_2' \quad (29\text{-b})$$

$$\dot{\omega}_3 = C\omega_1\omega_2 + u_{3c} \quad (29\text{-c})$$

In the above equations $A = \frac{I_2 - I_3}{I_1}, B = \frac{I_3 - I_1}{I_2}, C = \frac{I_1 - I_2}{I_3}$ are the differential inertial ratios and $u_1' = \frac{M_1}{I_1}, u_2' = \frac{M_2}{I_2}$, $u_{3c} = \frac{M_{3c}}{I_3}$ are the intermediate effort terms. Now we introduce the new parameterization

$$\tilde{\omega}_j = \omega_j - \omega_{dj} \Rightarrow \omega_j = \tilde{\omega}_j + \omega_{dj} \qquad j = 1,2,3$$

In which ω_{dj} is the desired angular velocity about the j^{th} principal axis in the body coordinate system. Since the desired angular velocities are constant, we have $\dot{\tilde{\omega}}_j = \dot{\omega}_j$. By redefining the control inputs, we can linearize the dynamic equations for the first two angular velocities using the feedback linearization.

$$u_1 = A\omega_2\omega_3 + u_1' \tag{30-a}$$

$$u_2 = B\omega_1\omega_3 + u_2' \tag{30-b}$$

Therefore, we have

$$\dot{\omega}_1 = \dot{\tilde{\omega}}_1 = u_1 \tag{31-a}$$

$$\dot{\omega}_2 = \dot{\tilde{\omega}}_2 = u_2 \tag{31-b}$$

So far we have feedback linearized the relation between ω_1, ω_2 and the control torques about the actuated principal axes.

4.2 Finding the equilibrium point

Having a constant control torque about the unactuated axis causes no static attitude to be stable. During the typical missions of a spacecraft it is required that the spacecraft points to a specific location on the earth or in space. Therefore, the attitude of the spacecraft changes slowly and the nominal attitude can be regarded constant which implies zero angular velocities. Having ω_1 and ω_2 zero equation (28-c) results,

$$I_3\dot{\omega}_3 = M_{3c} \tag{32}$$

Since M_{3c} is the nonzero third component of the disturbance torque, equation (32) implies that $\dot{\omega}_3$ is also nonzero. Therefore, the angular velocity of the spacecraft changes by time; as a result the spacecraft attitude deviates from the nominal value and the nominal attitude become unstable. Since no constant attitude can be chosen as the desired stable direction, a rotational motion should be chosen as the desired trajectory, which is both stable and best, agrees with the spacecraft mission. At each stage of the spacecraft mission, one of its instruments plays most crucial role among all the sensors and actuators on board. In transmitting data, the main antenna, for example must point towards the earth-based dishes. While taking the satellite images the camera(s) should be pointing exactly at the target(s). In the translational motion it is the main thruster engine that should fire in the correct direction. So, although we cannot hold the spacecraft stationary in any direction we may still be able to make the spacecraft rotate about one of its instruments so that the axis of the crucial instruments points a specific constant direction in space. Consider the translational maneuver of the spacecraft. The proposed desired equilibrium motion, while having a failure, is the net angular velocity vector of the spacecraft being collinear with the main thruster's direction. This means that the spacecraft will rotate about the main thruster's axis of symmetry so that the direction of the exerted force on the spacecraft will not change during the motion. Assume the direction of the thruster be $[a,b,c]^{\mathrm{T}}$ in the coordinate system attached to the spacecraft and $a^2+b^2+c^2=1$. We want the angular velocity of the spacecraft to be in the same direction as the thruster's axis so the components of the desired angular velocity will be

$$\begin{bmatrix} \omega_{d1} \\ \omega_{d2} \\ \omega_{d3} \end{bmatrix} = k \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{33}$$

in which k is a scalar and $\omega_{d1}, \omega_{d2}, \omega_{d3}$ represent the desired angular velocities in body coordinate system. If k is constant, using equations (28-c) and (33) we have

$$k^2 ab(I_2 - I_1) = M_{3c} \Rightarrow k^2 = \frac{M_{3c}}{ab(I_2 - I_1)} \tag{34}$$

Therefore, to be able to have the spacecraft rotate about its thruster axis with a constant angular velocity we require a positive value for $\frac{M_{3c}}{ab(I_2 - I_1)}$ which is not always the case. Even by admitting a time varying desired speed from equations (28-c) and (33) we will have

$$\dot{k} = \frac{ab(I_1 - I_2)k^2}{I_3 c} + M_{3c} \tag{35}$$

The above equation describes the steady state motion of the spacecraft. When equation (35) corresponds to a stable system the coefficient of k^2 must be non-positive so if we want the spacecraft to be able to function in spite of having any one of its auxiliary thrusters out of order the following equations must be simultaneously true,

$$\begin{aligned} \frac{I_2 - I_3}{I_1} &\leq 0 \\ \frac{I_3 - I_1}{I_2} &\leq 0 \\ \frac{I_1 - I_2}{I_3} &\leq 0 \end{aligned} \tag{36}$$

As all the moments of inertia are positive, the above equations will never hold simultaneously. So we will proceed with the assumption that k is constant and $\frac{M_{3c}}{ab(I_2 - I_1)}$ is positive, hence from (34) we obtain

$$k = \sqrt{\frac{M_{3c}}{ab(I_2 - I_1)}} \tag{37}$$

So far, we have found the corrective angular velocity for making the spacecraft rotate about its thruster axis. In addition to this condition, the axis of the spacecraft's thruster must lie in a specific direction. Without loss of generality, we assume this direction to be the z-axis of the inertial coordinate system. This means that the vector $[0,0,1]^T$ in the inertial coordinate system lies along the vector $[a,b,c]^T$ in the body coordinate system as depicted in Figure 4.1.

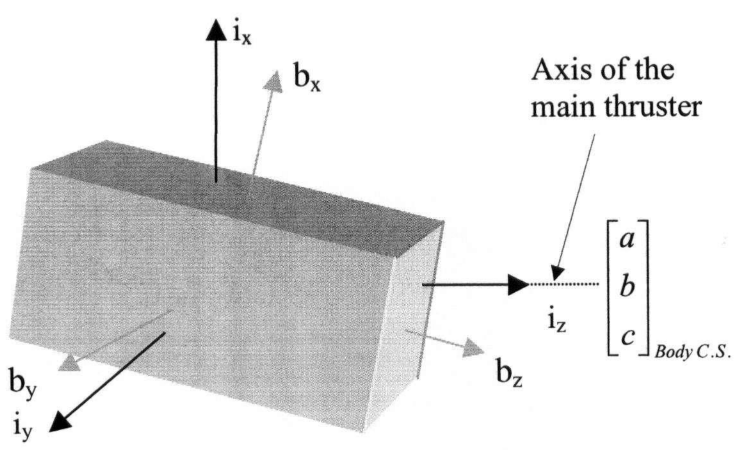

Figure 4.1:. Inertial and body coordinate systems in desired orientation

We have [24]

$$R=\frac{1}{1+|w|^2}\begin{bmatrix}\mathrm{Re}\left[(1+w^2)e^{iz}\right] & \mathrm{Im}\left[(1+w^2)e^{iz}\right] & -2\,\mathrm{Im}[w]\\ \mathrm{Im}\left[(1+\overline{w}^2)e^{-iz}\right] & \mathrm{Re}\left[(1-\overline{w}^2)e^{-iz}\right] & 2\,\mathrm{Re}[w]\\ 2\,\mathrm{Im}[we^{iz}] & -2\,\mathrm{Re}[we^{iz}] & 1-|w|^2\end{bmatrix} \tag{38}$$

R is the rotation matrix that takes the vectors in inertial coordinate system to the body coordinate system. Inertial z-axis is equivalent to body $[a,b,\mathrm{c}]^{\mathrm{T}}$ vector so we have

$$\begin{bmatrix}a\\b\\c\end{bmatrix}=R\begin{bmatrix}0\\0\\1\end{bmatrix}=\frac{1}{1+|w|^2}\begin{bmatrix}-2\,\mathrm{Im}(w)\\2\,\mathrm{Re}(w)\\1-|w|^2\end{bmatrix}$$

$$c=\frac{1-|w|^2}{1+|w|^2}\Rightarrow 1-|w|^2=c+c|w|^2\Rightarrow |w|^2=\frac{1-c}{1+c}$$

$$b=\frac{2\,\mathrm{Re}(w)}{1+|w|^2}\Rightarrow 2w_1=b\left(1+\frac{1-c}{1+c}\right)\Rightarrow w_1=\frac{b}{1+c}$$

$$a=\frac{-2\,\mathrm{Im}(w)}{1+|w|^2}\Rightarrow -2w_2=a\left(1+\frac{1-c}{1+c}\right)\Rightarrow w_2=\frac{-a}{1+c}$$

We can now see that by this choice of coordinate representation colinearity of the inertial z axis with the thruster's axis is satisfied if $w_{1d}=\dfrac{b}{1+c}$ and $w_{2d}=\dfrac{-a}{1+c}$ without any specific conditions on the attitude representation parameter, z.

4.3 Tracking of the desired rotation using linearization

The equations (25), (27), (29-c) and (31) describe the kinematics and dynamics of the spacecraft. At the kinematics level we have to manipulate the orientation so that the actual parameters w_1 and w_2 are equal to w_{d1} and w_{d2}. We do not have any specific desired value for z, but if we only control the first two kinematic parameters, w_1 and w_2, there is a possibility that z tends to infinity during the motion. As z is an angle, its going to infinity is not an issue. Effectively it means a continuous rotation. Therefore, we continue with controlling only the parameters w_1 and w_2 from the orientation coordinates (27). The equation (25) will become redundant. We now categorize the remaining five equations into two groups each described in the equation sets (39) and (40), respectively;

$$\dot{w}_1 = w_2(\omega_2 w_1 + \omega_3) + \frac{1}{2}\omega_1\left(1 + w_1^{\ 2} - w_2^{\ 2}\right) \tag{39-a}$$

$$\dot{w}_2 = w_1(\omega_1 w_2 - \omega_3) + \frac{1}{2}\omega_2\left(1 - w_1^{\ 2} + w_2^{\ 2}\right) \tag{39-b}$$

$$\dot{\omega}_3 = C\omega_1\omega_2 + u_{3c} \tag{39-c}$$

$$\dot{\omega}_1 = u_1 \tag{40-a}$$

$$\dot{\omega}_2 = u_2 \tag{40-b}$$

We will first try to bring w_1, w_2, ω_3 to the desired values by using ω_1 and ω_2 as control inputs. If w_1, w_2 gain the desired constant values, the spacecraft will be in desired direction and spinning about the thruster's axis. If in addition ω_3 reaches its desired value, the spin rate of the spacecraft, and ω_1, ω_2 will also reach their desired values. In the next step, we try to find the appropriate values for u_1, u_2 so that now the entire system converges to the desired state. We use the backstepping method in the second step. If the spacecraft reaches the desired motion, the parameters w_1, w_2, ω_3 will be constant and by using equations (39) we will have

$$w_{2d}(\omega_{2d} w_{1d} + \omega_{3d}) + \frac{1}{2}\omega_{1d}\left(1 + w_{1d}^{\ 2} - w_{2d}^{\ 2}\right) = 0 \tag{41-a}$$

$$w_{1d}(\omega_{1d} w_{2d} - \omega_{3d}) + \frac{1}{2}\omega_{2d}\left(1 - w_{1d}^{\ 2} + w_{2d}^{\ 2}\right) = 0 \tag{41-b}$$

$$C\omega_{1d}\omega_{2d} + u_{3c} = 0 \tag{41-c}$$

Considering the error $\widetilde{w}_1 = w_1 - w_{1d}$, $\widetilde{w}_2 = w_2 - w_{2d}$, $\widetilde{\omega}_3 = \omega_3 - \omega_{3d}$ and using (41) we obtain

$$\dot{\widetilde{w}}_1 = w_{2d}\left[\omega_{2d}\widetilde{w}_1 + \widetilde{\omega}_2 w_{1d} + \widetilde{\omega}_3\right] + \widetilde{w}_2\left[\omega_{2d} w_{1d} + \omega_{3d}\right] + \frac{1}{2}\omega_{1d}\left[2w_{1d}\,\widetilde{w}_1 - 2w_{2d}\widetilde{w}_2\right]$$

$$+\frac{1}{2}\widetilde{\omega}_1\left[1 + w_{1d}^{\ 2} - w_{2d}^{\ 2}\right] + O^2(\varepsilon) \tag{42-a}$$

$$\dot{\widetilde{w}}_2 = w_{1d}\left[\omega_{1d}\widetilde{w}_2 + \widetilde{\omega}_1 w_{2d} - \widetilde{\omega}_3\right] + \widetilde{w}_1\left[\omega_{1d} w_{2d} - \omega_{3d}\right] + \frac{1}{2}\omega_{2d}\left[-2w_{1d}\,\widetilde{w}_1 - 2w_{2d}\widetilde{w}_2\right]$$

$$+\frac{1}{2}\widetilde{\omega}_2\left[1 - w_{1d}^{\ 2} + w_{2d}^{\ 2}\right] + O^2(\varepsilon) \tag{42-b}$$

$$\dot{\widetilde{\omega}}_3 = C\widetilde{\omega}_1\omega_{d2} + C\omega_{d1}\widetilde{\omega}_2 + O^2(\varepsilon) \tag{42-c}$$

In the above equations if $\varepsilon = [\omega_1, \omega_2, \omega_3, w_1, w_2, w_3]$, $O^2(\varepsilon)$ is a polynomial containing terms which are of the second or higher orders of ε. We define

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \tilde{\omega}_3 \\ \tilde{w}_1 \\ \tilde{w}_2 \end{bmatrix} \tag{43-a}$$

$$F = \begin{bmatrix} 0 & 0 & 0 \\ w_{2d} & w_{2d}\omega_{2d} + w_{1d}\omega_{1d} & w_{1d}\omega_{2d} + \omega_{3d} - w_{2d}\omega_{1d} \\ -w_{1d} & -w_{1d}\omega_{2d} - \omega_{3d} + w_{2d}\omega_{1d} & w_{2d}\omega_{2d} + w_{1d}\omega_{1d} \end{bmatrix} \tag{43-b}$$

$$g_1 = \begin{bmatrix} C\omega_{d1} \\ \dfrac{1 + w_{1d}^{\;2} - w_{2d}^{\;2}}{2} \\ w_{1d}w_{2d} \end{bmatrix} \quad g_2 = \begin{bmatrix} C\omega_{d2} \\ w_{1d}w_{2d} \\ \dfrac{1 - w_{1d}^{\;2} + w_{2d}^{\;2}}{2} \end{bmatrix} \tag{43-c}$$

Introducing v_1 and v_2 to be the prescribed values for $\tilde{\omega}_1$ and $\tilde{\omega}_2$ which make the subsystem (39) stable, $G = [g_1 \mid g_2]$ and $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ the equation (42) can be stated as

$\dot{X} = FX + Gv$

We now form the controllability matrix for the above system; $\hat{C} = [G \mid FG \mid F^2G]$. Using the symbolic math software the rank of $\hat{C}$ was shown to be always three. This means that the linearized control system is always controllable.

The control law proposed considers an input proportional to the states i.e. $v = -KX$. In which K is a 2 by 3 gain matrix. To determine the gain matrix, the method proposed in [47] is used. In this method, we first compute the following matrix functions for each desired eigenvalue of the system, λ_i.

$$\Phi_{3\text{x}3}(\lambda_i) = (\lambda_i I_3 - F)^{-1} \tag{44-a}$$

$$\Psi_{3\times 2}(\lambda_i) = \Phi(\lambda_i)G_{3\times 2} \tag{44-b}$$

In the next step the matrix E is formed as

$$E = [\Psi(\lambda_1) \mid \Psi(\lambda_2) \mid \Psi(\lambda_3)]$$

We should then choose three independent columns of E. If we call these independent columns η_i, $i = 1,2,3$ the following equations can be used to determine K:

$$K\eta_i = -e_i,\ i = 1,2,3 \Rightarrow K[\eta_1 \quad \eta_2 \quad \eta_3] = -[e_1 \quad e_2 \quad e_3] \Rightarrow K = -[\eta_1 \quad \eta_2 \quad \eta_3]^{-1}[e_1 \quad e_2 \quad e_3] \tag{45}$$

In the above equations each e_i is an arbitrary unit vector. We choose the following normal vectors.

$$e_1 = e_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ e_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

To make the K matrix have three different eigenvalues each column vector η_i must be chosen from the corresponding $\Psi_{3\times 2}(\lambda_i)$ function. Then we can compute K from the equations (43)-(45).

By finding the matrix K we can determine the desired first and second angular velocities at each instance so that the remaining subsystem has the specified eigenvalues and so $\tilde{w}_1, \tilde{w}_2, \tilde{\omega}_3$ go to zero if time approaches infinity.

$$v = -KX \tag{46-a}$$

$$\dot{v} = -K\dot{X} \tag{46-b}$$

In the next step, we derive the control law for the entire system based on the subsystem control law (46) and the backstepping method. For the system

$$\dot{X} = FX + G\xi \tag{47-a}$$

$$\dot{\xi} = u \tag{47-b}$$

If the subsystem (47-a) is stable provided $\xi = v$, and the Lyapunov function V can be used for proving this stability, considering $z = \xi - v$ and using the following control law ensures the stability of the entire system [10].

$$u = -C_0 z + \dot{v} - G^T \nabla V_X \tag{48}$$

In which C_0 is the positive gain coefficient and ∇V_x is the gradient of V with respect to X. The linearized governing equations, (40) and (42), should be expressed in the form of equations (47). To this end the X and ξ vectors are chosen as

$$X = \begin{bmatrix} \tilde{\omega}_3 \\ \tilde{w}_1 \\ \tilde{w}_2 \end{bmatrix} \tag{49-a}$$

$$\xi = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} \tag{49-b}$$

The matrices F and G are then defined as previously defined in (43) and the effort vector is defined as $u = [u_1, u_2]^{\mathrm{T}}$. The Lyapunov function used for proving the stability of the subsystem (47-b) is

$$V = \frac{1}{2}\left(\tilde{w}_1^{\,2} + \tilde{w}_2^{\,2} + \tilde{\omega}_3^{\,2}\right) = \| X \|^2 / 2 \tag{50}$$

Since all the eigenvalues of the controlled system are placed in the negative half plane of the complex plane, the time derivative of the above Lyapunov function is always negative. Taking the gradient of the Lyapunov function with respect to the X vector, ∇V_X is equal to

$$\nabla V_X = \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \tilde{\omega}_3 \end{bmatrix} \tag{51}$$

At the beginning of the implementation of the control law there is a substantial difference between the actual angular velocities and the prescribed angular velocities derived from the control law for the first subsystem. This difference multiplied by the gain factor C_0 will result in a huge control torque, which typically exceeds the torque limit of the thrusters. To prevent this happening we set C_0 to be a function of the error term:

$$C_0 = \frac{C_1}{1+\sqrt{(\omega_1 - v_1)^2 + (\omega_2 - v_2)^2}} \tag{52}$$

The property of the above modified gain factor is when there is a great difference between ω and v, such as at the beginning of the implementation of the control law, the gain factor will be reduced to prevent the thruster saturation. Meanwhile, the performance will not be affected when the angular velocity errors are small.

Having u_1 and u_2 the control torques can be found from

$$M_1 = I_1(u_1 - A\omega_2\omega_3) \tag{53-a}$$

$$M_2 = I_2(u_2 - B\omega_1\omega_3) \tag{53-b}$$

4.4 Feasibility of gain optimization in linear control of the underactuated spacecraft

The energy consumed during the attitude maneuver is a critical factor in assigning the control gains. This energy is associated with the fuel consumption during the mission. Since the refueling of the spacecraft is a rare option, the less energy is consumed during the maneuvers the longer is the life span of the spacecraft. In this section, we consider the flexibility in assigning the control gain matrix while having the locations of the poles of the control system unchanged.

It has been proven [47] that if a linear control system is completely controllable, any set of desired closed loop poles can be achieved using a constant gain matrix. To study the flexibility in assigning the control gain matrix controllability of the system using one control torque is examined. In controlling the system $\dot{X} = FX + Gv$ using the first component of the effort vector, v, the open-loop system turns into

$$\dot{X} = FX + g_1 v_1 \tag{54}$$

The controllability matrix of the control system in (54) is

$$\hat{C}_1 = \left[g_1 \mid Fg_1 \mid F^2 g_1 \right] \tag{55}$$

Using the Symbolic Math Toolbox of the Matlab® software it has been derived that in case of nonzero disturbance torque, the rank of the controllability matrix, $\hat{C}_1$, is three. This implies that the controllability is preserved even for controlling the system using the first component of the angular velocity. If only the second component of effort vector is used for controlling the system $\dot{X} = FX + Gv$, the system becomes

$$\dot{X} = FX + g_2 v_2 \tag{56}$$

The controllability matrix corresponding to the control system (56) is

$$\hat{C}_2 = \left[g_2 \mid Fg_2 \mid F^2 g_2 \right] \tag{57}$$

Similarly, using the Symbolic Math Toolbox of the Matlab® software the rank of the controllability matrix, $\hat{C}_2$, was found to be three, if the disturbance torque was nonzero. Therefore, the desired eigenvalues of the controlled system can be achieved by prescribing any of the first two components of the angular velocity vector. If we set the second component of the angular velocity vector to zero and assign the first component to achieve the poles of the closed loop system the gain matrix will be in the form

$$K = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ 0 & 0 & 0 \end{bmatrix} \tag{58}$$

Since

$$v = Kx \tag{59}$$

the second component of vector v is always zero. Similarly, if the second component of the angular velocity is assigned to determine the poles of the closed loop system, the gain matrix will be in the following form

$$K = \begin{bmatrix} 0 & 0 & 0 \\ K_{21} & K_{22} & K_{23} \end{bmatrix} \tag{60}$$

Therefore, the constant gain matrix that results in the desired poles of the system is not unique and can be optimized to be associated with the least energy consumption.

4.5 Numerical simulations for the linear controller and discussions

The numerical simulations have been performed to evaluate the performance of the proposed control law. Inspired by the Venus Express spacecraft [48] the simulated spacecraft is assumed to have a mass of 1400 Kg with the moments of inertia about the first, second and third principal axes of 600, 640 and 500 Kg m^2, respectively.

The undesired constant torque about the unactuated axis is assumed to be 10 Nm. Initially the spacecraft is considered to be quite off the desired orientation, $\widetilde{w}_{1,init} = -0.4$, $\widetilde{w}_{2,init} = 0.5$, while rotating with angular velocity, $\omega_{initial} = [0,0,\omega_{desired}]$. After several trial and error simulations, the eigenvalues of the controlled subsystem were selected as [0.3, 0.4, 0.5] and $C_1 = 10$ which results in recovery of the motion within 35 seconds. These values were not explicitly optimized as we only want some reasonable values for the runs. The variations of the attitude parameters, the angular velocities and the implemented torques for a representative simulation are illustrated in Figure 4.2. It can be seen that the states converge smoothly to their desired values. The effect of the gain modification expressed in (52) can be seen as torque fluctuation at the beginning of simulations. This modification has successfully reduced the initial control torques from 200 KNm to 600 Nm.

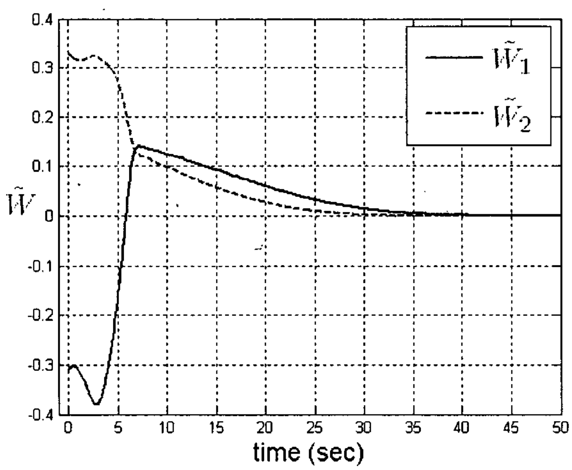

Figure 4.2a: Orientations errors variations

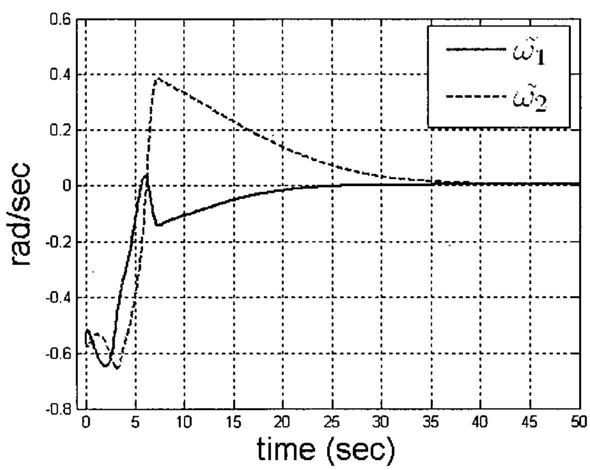

Figure 4.2.b: Variation of the errors of the first and second components of angular velocities

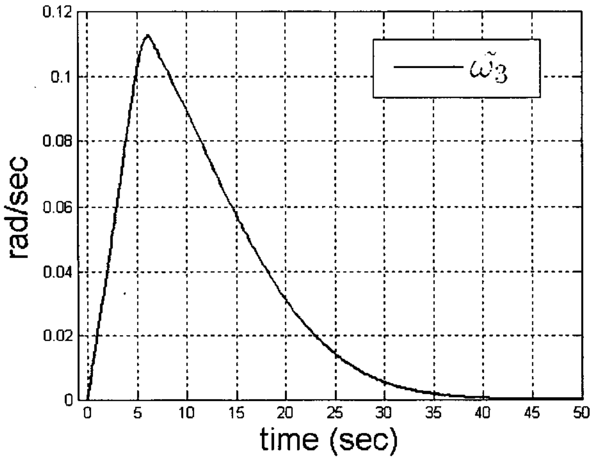

Figure 4.2c: Variation of the error of the third component of angular velocity

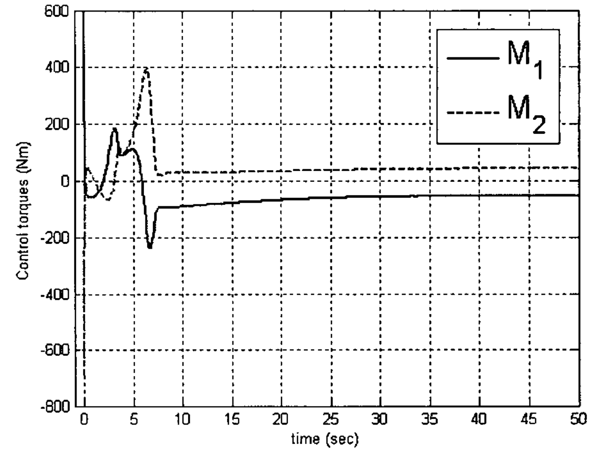

Figure 4.2d: The control Torques

4.6 Tracking of desired rotation using nonlinear control methods

In this section the system is stabilized without linearization. This can improve the robustness of the controller in sense of convergence over a broader range of initial conditions. The equations (25), (27), (29-c) and (31) describe the kinematics and dynamics of the spacecraft. At the kinematics level we have to manipulate the orientation so that the actual parameters w_1 and w_2 are equal to their desired values, w_{d1} and w_{d2}. Since z is associated with the rotations about the non-rotating axis of the spacecraft, we do not have any specific desired value for z, but if we only control the first two kinematic parameters, w_1 and w_2, there is a possibility that z goes to infinity during the motion. As z is an angle, its going to infinity is not an issue. Effectively it means a continuous rotation. Therefore, we continue to control only the parameters w_1 and w_2 from the orientation coordinates, and as a result equation (25) becomes redundant. We now categorize the remaining five equations into two groups each described in the equation sets (61) and (62) respectively;

$$\dot{w}_1 = w_2(\omega_2 w_1 + \omega_3) + \frac{1}{2}\omega_1\left(1 + w_1^{\ 2} - w_2^{\ 2}\right) \tag{61-a}$$

$$\dot{w}_2 = w_1(\omega_1 w_2 - \omega_3) + \frac{1}{2}\omega_2\left(1 - w_1^{\ 2} + w_2^{\ 2}\right) \tag{61-b}$$

$$\dot{\omega}_3 = C\omega_1\omega_2 + u_{3c} \tag{61-c}$$

$$\dot{\omega}_1 = u_1 \tag{62-a}$$

$$\dot{\omega}_2 = u_2 \tag{62-b}$$

We will first try to bring w_1, w_2, ω_3 to the desired values by using ω_1 and ω_2 as the control inputs. If w_1, w_2 have the desired constant values the spacecraft will be in the desired direction and spinning about the desired axis. If in addition, ω_3 reaches its desired value the spin rate of the spacecraft will be the desired spin rate and ω_1, ω_2 will have their desired values as well. Let α_1, α_2 be the prescribed values for ω_1, ω_2 resulting from controlling the equation set (61). In the next step, we attempt to find the appropriate values for the control effort parameters u_1 and u_2 so that the entire system converges to the desired values. We use the backstepping method in this step.

For controlling the first subsystem, we use the Lyapunov stability concepts. If *V(x)* is a strictly positive function of the states except at the origin and $\dot{V}(x)$ is always negative the states will asymptotically converge to zero when time goes to infinity. Consider the following deviation measuring parameters.

$$\tilde{\omega}_3 = \omega_3 - \omega_{3d}$$

$$\tilde{w}_j = w - w_{jd} \quad j = 1,2$$

We introduce the following Lyapunov function for the subsystem described in (61)

$$V_s = |\tilde{w}|^2 / 2 + \tilde{\omega}_3{}^2 / 2 \tag{63}$$

It can be easily seen that V_s is strictly positive except at the origin, where w_1, w_2 and ω_3 reach their desired values. In the first step we should define the control laws α_1, α_2 such that the time derivative of the Lyapunov function, $\dot{V}_s$, is negative except when w_1, w_2, ω_3 are all zero. From (63) we have

$$\dot{V}_s = \tilde{w}_1 \dot{\tilde{w}}_1 + \tilde{w}_2 \dot{\tilde{w}}_2 + \dot{\tilde{\omega}}_3 \tilde{\omega}_3 \tag{64}$$

If we prescribe the control inputs such that $\tilde{w}_1 \dot{\tilde{w}}_1 + \tilde{w}_2 \dot{\tilde{w}}_2 = -K|\tilde{w}|^2$ in which K is a positive real number, and $\dot{\tilde{\omega}}_3 = -c_0 \tilde{\omega}_3$ where c_0 is also a constant positive number, then the time derivative of the Lyapunov function will be $\dot{V}_s = -K|\tilde{w}|^2 - c_0 \tilde{\omega}_3{}^2$ which is a negative definite function. In this case, w_1, w_2, ω_3 converge to their desired values by time. As w_{1d}, w_{2d} are constant $\dot{\tilde{w}}_1 = \dot{w}_1, \dot{\tilde{w}}_2 = \dot{w}_2$. If we set $\omega_1 = \alpha_1$ and $\omega_2 = \alpha_2$ by using (61) we will have

$$\tilde{w}_1 \dot{\tilde{w}}_1 + \tilde{w}_2 \dot{\tilde{w}}_2 = m\alpha_1 + n\alpha_2 + p \tag{65}$$

In which

$$m = (1 + w_1{}^2 - w_1{}^2)\tilde{w}_1 / 2 + w_1 w_2 \tilde{w}_2 \tag{66-a}$$

$$n = w_1 w_2 \tilde{w}_1 + (1 - w_1{}^2 + w_2{}^2)\tilde{w}_2 / 2 \tag{66-b}$$

$$p = \tilde{w}_1 w_2 \omega_3 - \tilde{w}_2 w_1 \omega_3 \tag{66-c}$$

In order to have $\tilde{w}_1 \dot{\tilde{w}}_1 + \tilde{w}_2 \dot{\tilde{w}}_2 = -K|\tilde{w}|^2$ while $\omega_1 = \alpha_1, \omega_2 = \alpha_2$ we should have

$$m\alpha_1 + n\alpha_2 = -K|\tilde{w}|^2 - p \tag{67}$$

Since ω_{3d} is constant $\dot{\omega}_3 = \dot{\tilde{\omega}}_3$. Then from (39-c) we have $\dot{\tilde{\omega}}_3 = C\omega_1\omega_2 + u_{3c}$. Therefore, to have $\dot{\tilde{\omega}}_3 = -c_0\tilde{\omega}_3$ we should have

$$\alpha_1\alpha_2 = d \tag{68}$$

Where

$$d = -(c_0\tilde{\omega}_3 + u_{3c})/C \tag{69}$$

By multiplying (67) by α_1 and using (68), we will have

$$m\alpha_1^2 + (K|\tilde{w}|^2 + p)\alpha_1 + nd = 0 \tag{70}$$

The condition for the equation (70) to have real roots is that

$$\Delta = (K|\tilde{w}|^2 + p)^2 - 4mnd \geq 0 \tag{71}$$

If $mnd \geq 0$, $K|\tilde{w}|^2 + p \geq 2\sqrt{mnd}$ results in having a nonnegative discriminant. As K must be a positive real number, we set the lower limit for K to be K_0, a positive gain, later tuned during the simulations.

$$K = \max\{(2\sqrt{mnd} - p)/|\tilde{w}^2|, K_0\} \Rightarrow \Delta \geq 0$$

Setting $K = (2\sqrt{mnd} - p)/|\tilde{w}^2|$ results in vanishing of the discriminant. In which case the numerical errors may cause the discriminant to become "slightly" negative, which results in unacceptable imaginary values. Since increasing K will always increase the stability of the system, to avoid the numerical errors causing problems we consider ten percent safety buffer and set

$$K = 1.1 \times \max\left\{(2\sqrt{mnd} - p)/|\tilde{w}^2|, K_0\right\} \qquad , mnd \geq 0 \tag{72}$$

If $mnd<0$ the discriminant of the equation (70) will always be negative. In this case, K must be chosen so that there is no discontinuity in α_1 and α_2 if mnd changes sign. This means that K must vary smoothly while mnd passes through the origin. One way is to make K an even function of mnd:

$$K = 1.1 \times \max\left\{(2\sqrt{-mnd} - p)/|\tilde{w}^2|, K_0\right\} \qquad mnd < 0 \tag{73}$$

We can now compute α_1 from (70):

$$\alpha_1 = \frac{-(K|\tilde{w}|^2 + p) \pm \sqrt{(K|\tilde{w}|^2 + p)^2 - 4mnd}}{2m} \tag{74}$$

and from (68)

$$\alpha_2 = \frac{d}{\alpha_1} \tag{75}$$

The equations (74) and (75) yield two sets of roots:

$$\alpha_1 = \frac{-(K|\tilde{w}|^2 + p) - \sqrt{(K|\tilde{w}|^2 + p)^2 - 4mnd}}{2m}, \alpha_2 = \frac{-(K|\tilde{w}|^2 + p) + \sqrt{(K|\tilde{w}|^2 + p)^2 - 4mnd}}{2n} \tag{76-a}$$

$$\alpha_1 = \frac{-(K|\tilde{w}|^2 + p) + \sqrt{(K|\tilde{w}|^2 + p)^2 - 4mnd}}{2m}, \alpha_2 = \frac{-(K|\tilde{w}|^2 + p) - \sqrt{(K|\tilde{w}|^2 + p)^2 - 4mnd}}{2n} \tag{76-b}$$

$(K|\tilde{w}|^2 + p)$ is a positive number, so for the first set of roots, (76-a), we have

$$\lim_{m \to 0} \alpha_1 = \infty, \lim_{n \to 0} \alpha_2 \neq \infty$$

while for the second set of roots, (76-b),

$$\lim_{m\to 0}\alpha_1 \neq \infty, \lim_{n\to 0}\alpha_2 = \infty$$

So if we use only one of the answer sets, there is a possibility that when m or n approaches zero either α_1 or α_2 goes to infinity. We will decide which solution to consider based on observing which coefficient is going to vanish. If $|m/n| < .5$ the probability of vanishing m is higher, so the second solution set must be chosen. Similarly, if n approaches zero, $|m/n|$ increases in magnitude until it passes 2. In which case, we switch and use the solution described in (76-a). In other cases, the control law will remain unchanged to prevent the unnecessary switchings. Therefore, we will have a hysteresis shape kinematic control as depicted in the Figure 4.3.

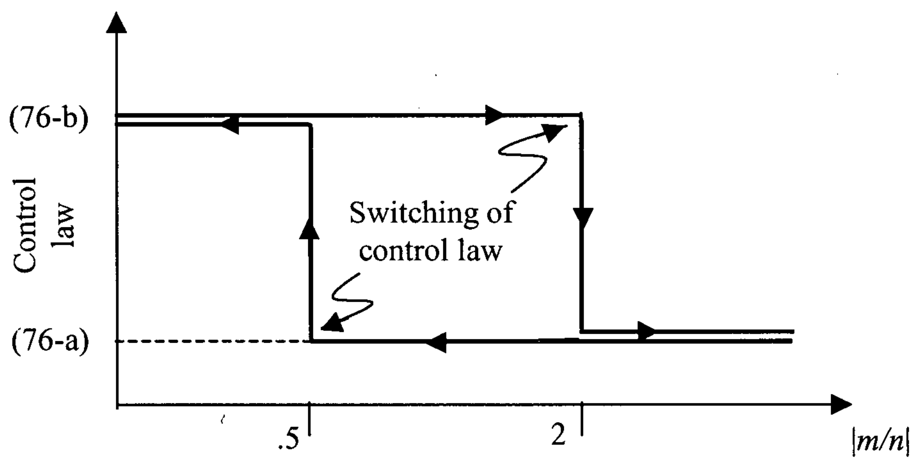

Figure 4.3: Solution choice for kinematic control Law

Note that as can later be seen in the simulations some switchings occur in the beginning stages of control and this is needed regardless of the initial choice of the control law. It is preferred not to have any discontinuities in the control law. If we want to eliminate the discontinuities generated by the switching, the term $(K\,|\,\tilde{w}\,|^2 + p)^2 - 4mnd$ in (74) must be zero. This means that

$$K\,|\,\tilde{w}\,|^2 + p = \pm\sqrt{4mnd} \tag{77}$$

The above equation leads to a real *K* only if *mnd>0*, which it not always the case. Even if (77) results in a real *K*, this answer can be negative which is not acceptable since it makes the time derivative of the attitude error positive.

If we define, $f = K\,|\,\tilde{w}\,|^2 + p$ the time derivatives of α_1 and α_2 will be

$$\dot{\alpha}_1 = \frac{m\left(-\dot{f} \pm \dfrac{2\dot{f}f - 4(\dot{m}nd + m\dot{n}d + mn\dot{d})}{2\sqrt{f}}\right) - \dot{m}\left(-f \pm \sqrt{f^2 - 4mnd}\right)}{2m^2} \tag{78}$$

$$\dot{\alpha}_2 = \frac{\dot{d}\alpha_1 - \dot{\alpha}_1 d}{\alpha_1^{\,2}} \tag{79}$$

in which

$$\dot{m} = (w_1\dot{w}_1 - w_2\dot{w}_2)\tilde{w}_1 + \frac{1 + w_1^{\,2} - w_2^{\,2}}{2}\dot{\tilde{w}}_1 + \dot{w}_1 w_2 \tilde{w}_2 + \dot{w}_2 w_1 \tilde{w}_2 + w_1 w_2 \dot{\tilde{w}}_2 \tag{80-a}$$

$$\dot{n} = (-w_1\dot{w}_1 + w_2\dot{w}_2)\tilde{w}_2 + \frac{1 - w_1^{\,2} + w_2^{\,2}}{2}\dot{\tilde{w}}_2 + \dot{w}_1 w_2 \tilde{w}_1 + \dot{w}_2 w_1 \tilde{w}_1 + w_1 w_2 \dot{\tilde{w}}_1 \tag{80-b}$$

$$\dot{p} = \dot{\tilde{w}}_1 w_2 \omega_3 + \tilde{w}_1 \dot{w}_2 \omega_3 + \tilde{w}_1 w_2 \dot{\omega}_3 - \dot{\tilde{w}}_2 w_1 \omega_3 - \tilde{w}_2 \dot{w}_1 \omega_3 - \tilde{w}_2 w_1 \dot{\omega}_3 \tag{80-c}$$

$$\dot{d} = -\frac{c_0 \dot{\tilde{\omega}}_3}{C} \tag{81}$$

We use these time derivatives to design the control law for the stabilization of the entire system.

4.7 Modification of the nonlinear kinematic control law

Until now we have developed a control law that guarantees that the time derivatives of both $|\tilde{w}|^2/2$ and $\tilde{\omega}_3^{\,2}/2$ are negative except at the origin. A problem arises when $|\tilde{w}|$ goes to zero faster than $\tilde{\omega}_3$ and we get $|\tilde{w}|^2=0$ while $\tilde{\omega}_3$ has not vanished. Having $|\tilde{w}|^2=0$ for all the future time means that the spacecraft is rotating about its thruster axis. In other words, the first and second components of the angular velocity are related to the third one through (33) so we have

$$\alpha_1 = a\omega_3/c \tag{82-a}$$

$$\alpha_2 = b\omega_3/c \tag{82-b}$$

in which $[a,b,c]^{\mathrm{T}}$ is the direction of the main thruster in the spacecraft coordinate system that should now be the direction of the angular velocity in spacecraft coordinate system. Therefore, we can no longer set α_1 and α_2 independent from ω_3. In addition, the desired rotation of the spacecraft is about the thruster's axis:

$$\omega_{1d} = a\omega_{3d}/c \tag{83-a}$$

$$\omega_{2d} = b\omega_{3d}/c \tag{83-b}$$

The angular velocity in the desired rotation is constant so from (29-c) we have

$$u_{3c} = -C\omega_{1d}\omega_{2d} \tag{84}$$

Using the equations (29-c), (82), (83) and (84) we have

$$\frac{d}{dt}\left(\frac{\tilde{\omega}_3^{\,2}}{2}\right)=\tilde{\omega}_3\dot{\tilde{\omega}}_3=\frac{Cab}{c^2}\tilde{\omega}_3\left((\tilde{\omega}_3+\omega_{3d})^2-\omega_{3d}^{\,2}\right)=\frac{Cab}{c^2}\tilde{\omega}_3^{\,2}(\tilde{\omega}_3+2\omega_{3d}) \tag{85}$$

Only if the right hand side of the equation (85) is negative then $\tilde{\omega}_3$ converges to zero. The right hand side of (85) depends on $\tilde{\omega}_3$ and can become positive which can lead the system to diverge from the desired angular velocity. Thus, we suggest using a two-stage control law. In the first stage of which the desired angular velocity is reached and in the second stage the spacecraft is moved to the desired orientation for which we use the control law in (76). In the first stage if we set $\dot{\tilde{\omega}}_3 = -c_0\tilde{\omega}_3$ then we have $\frac{d}{dt}\left(\frac{\tilde{\omega}_3^{\ 2}}{2}\right) = -c_0\tilde{\omega}_3^{\ 2} < 0$, from (68) and (69) we obtain

$$\alpha_1\alpha_2 = -(c_0\tilde{\omega}_3 + u_{3c})/C \tag{86}$$

The assignment of α_1 and α_2 must be done in such a way that the spacecraft eventually rotates with the desired angular velocity. This means that the first and second components of the angular velocity, α_1 and α_2, must also converge to ω_{1d} and ω_{2d}. If α_1 and α_2 have the same ratio as ω_{1d} and ω_{2d}, not only the convergence is achieved but also it is assured that the magnitude of α_1 and α_2 are appropriately proportional to each other. In other words, it is assured that the magnitude of α_1 is neither much larger nor much smaller than the magnitude of α_2. Using (86) and the stated assignment rule we obtain

$$\alpha_1 = a\sqrt{\left|\frac{c_0\tilde{\omega}_3 + u_{3c}}{Cab}\right|} = \sqrt{\left|\frac{a(c_0\tilde{\omega}_3 + u_{3c})}{Cb}\right|} = \omega_{1d}\sqrt{\left|\frac{c_0\tilde{\omega}_3 + u_{3c}}{u_{3c}}\right|} \tag{87-a}$$

$$|\alpha_2| = \left|\frac{-(c_0\tilde{\omega}_3 + u_{3c})}{C\alpha_1}\right| = \sqrt{\left|\frac{b(c_0\tilde{\omega}_3 + u_{3c})}{Ca}\right|} = |\omega_{2d}|\sqrt{\left|\frac{c_0\tilde{\omega}_3 + u_{3c}}{u_{3c}}\right|} \tag{87-b}$$

A problem arises when the terms under the square root vanish. In which case the derivatives of α_1 and α_2 will go to infinity. To avoid this possibility ten percent safety buffer was added to the assignment:

$$\alpha_1 = \omega_{1d}\sqrt{\frac{0.1|u_{3c}| + |c_0\tilde{\omega}_3 + u_{3c}|}{1.1|u_{3c}|}} \tag{88-a}$$

$$\alpha_2 = \frac{-(c_0\tilde{\omega}_3 + u_{3c})}{C\alpha_1} \tag{88-b}$$

Time derivative of the kinematic control law parameters will then be

$$\dot{\alpha}_1 = \frac{\omega_{1d}\,\text{sign}(c_0\tilde{\omega}_3 + u_{3c})c_0\dot{\tilde{\omega}}_3}{\sqrt{1.1\,|\,u_{3c}\,|\times 2\sqrt{0.1\,|\,u_{3c}\,|+|\,c_0\tilde{\omega}_3 + u_{3c}}}} \tag{89-a}$$

$$\dot{\alpha}_2 = -\frac{c_0\dot{\tilde{\omega}}_3\alpha_1 - \dot{\alpha}_1(c_0\tilde{\omega}_3 + u_{3c})}{C\alpha_1^2} \tag{89-b}$$

which will be used for computing the control torques.

4.8 Complete model extension

Now that we have α_1, α_2 and their time derivatives, we can apply the backstepping control method to make the entire system stable. The method used for deriving the control law is discussed in section 4.9. From the equations (61) we have

$$\frac{d}{dt}\begin{bmatrix}\tilde{w}_1\\ \tilde{w}_2\\ \tilde{\omega}_3\end{bmatrix} = \begin{bmatrix}w_2\omega_1\\ -w_1\omega_3\\ u_{3c}\end{bmatrix} + \begin{bmatrix}\left(1+w_1^{\,2}-w_2^{\,2}\right)\omega_1/2 + w_1w_2\omega_2\\ w_1w_2\omega_1 + \left(1-w_1^{\,2}+w_2^{\,2}\right)\omega_2/2\\ C\omega_1\omega_2\end{bmatrix} \tag{90}$$

Now we introduce the following vectors

$$x = \begin{bmatrix}\tilde{w}_1\\ \tilde{w}_2\\ \tilde{\omega}_3\end{bmatrix},\quad \xi = \begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix},\quad g(\xi) = \begin{bmatrix}\left(1+w_1^{\,2}-w_2^{\,2}\right)\omega_1/2 + w_1w_2\omega_2\\ w_1w_2\omega_1 + \left(1-w_1^{\,2}+w_2^{\,2}\right)\omega_2/2\\ C\omega_1\omega_2\end{bmatrix} \tag{91}$$

We then have

$$g' = \left.\frac{\partial g}{\partial \xi}\right|_{\xi=\alpha} = \begin{bmatrix} (1+w_1^{\ 2}-w_2^{\ 2})/2 & w_1 w_2 \\ w_1 w_2 & (1-w_1^{\ 2}+w_2^{\ 2})/2 \\ C\alpha_1 & C\alpha_2 \end{bmatrix} \tag{92}$$

In which ω_1 and ω_2 have been set to their prescribes values of α_1 and α_2. Since $V_s = |\tilde{w}|^2/2 + \tilde{\omega}^2/2$ and $x = [\tilde{w}_1, \tilde{w}_2, \tilde{\omega}_3]$ we have

$$\frac{\partial V_s}{\partial x} = \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \tilde{\omega}_3 \end{bmatrix} \tag{93}$$

Using equations, (92), (93) and (101), we obtain the control inputs, u, as

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = -C_0 \begin{bmatrix} \omega_1 - \alpha_1 \\ \omega_2 - \alpha_2 \end{bmatrix} + \begin{bmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \end{bmatrix} - \begin{bmatrix} (1+w_1^{\ 2}-w_2^{\ 2})/2 & w_1 w_2 & C\alpha_1 \\ w_1 w_2 & (1-w_1^{\ 2}+w_2^{\ 2})/2 & C\alpha_2 \end{bmatrix} \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \tilde{\omega}_3 \end{bmatrix} \tag{94}$$

in which C0 is a constant positive gain factor. After finding u the actual torques can be found as

$$M_1 = I_1(u_1 - A\omega_2\omega_3) \tag{95-a}$$

$$M_2 = I_2(u_2 - B\omega_1\omega_3) \tag{95-b}$$

4.9 Numerical simulations

Two sets of numerical simulations have been performed to illustrate the performance of the proposed control law. The Simulink® simplified scheme of the controller is illustrated in Figure A.2.1. Inspired by the Deep Impact Flyby spacecraft [49] the spacecraft in our simulations is assumed to be 601 Kg with the moments of inertia about first, second and third principal axes to be 690, 810 and 410 Kg m^2, respectively.

In the first simulation, the third component of the initial angular velocity is the same as its final desired value. While the initial orientation and the first two components of the angular velocity differ from the final desired values. This simulation aims to illustrate the part of control law that corrects the orientation of the spacecraft. The results of the simulation are shown in Figure 4.4.

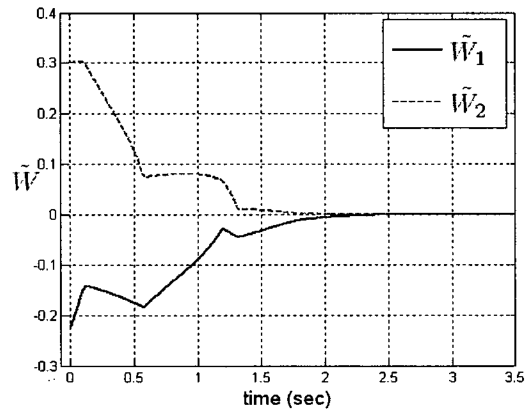

Figure 4.4.a: Orientation error variations

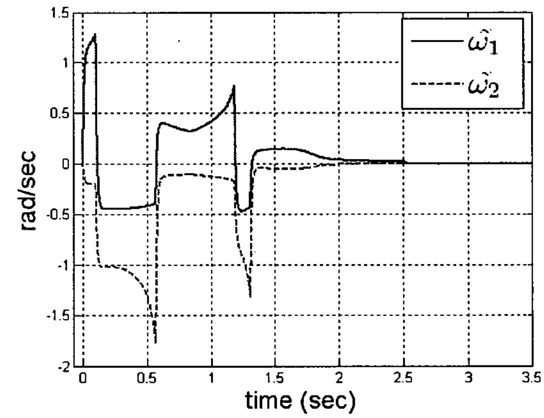

Figure 4.4.b: Angular velocity error variations

The spacecraft is oriented into the desired direction rotating with the constant angular velocity in less than three seconds. The control coefficients used here are $K = .5, c_0 = 1, C_0 = 50$. The rapid changes in the angular velocities and the slope of the attitude coordinates correspond to the switchings in the control law for the first subsystem. It can be seen that the switchings are essential in controlling the spacecraft since the switchings are more than one and none of the kinematic laws in (76) can control the system individually.

The second simulation resembles a typical fault occurrence situation. The spacecraft is initially stationary and in the desired orientation so that the thruster is aligned with the inertial z-axis. It is assumed that it takes fault detection and diagnosis systems of the spacecraft 10 seconds to identify the fault and switch to the proposed control law for the recovery of the spacecraft motion. This is considered ample time. The recovery is done by first reaching the desired angular velocity about the unactuated axis and then putting the thruster in the desired orientation as illustrated in Figures 4.5.a – 4.5.c.

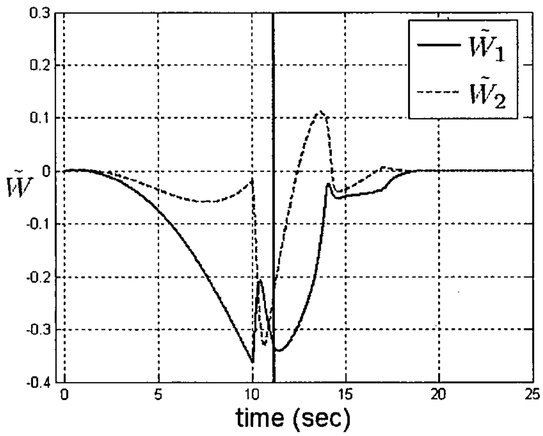

Figure4.5.a: Orientation error variations

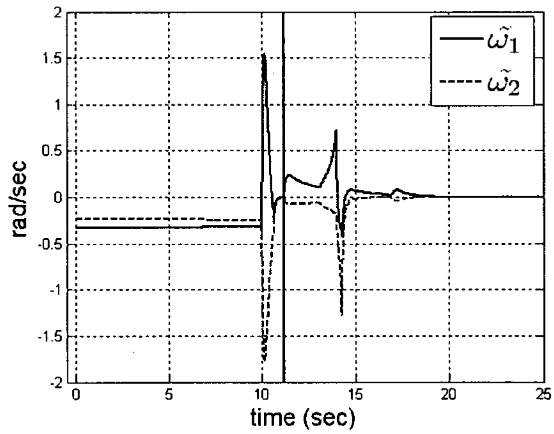

Figure 4.5.b: variation of error in first and second components of angular velocity

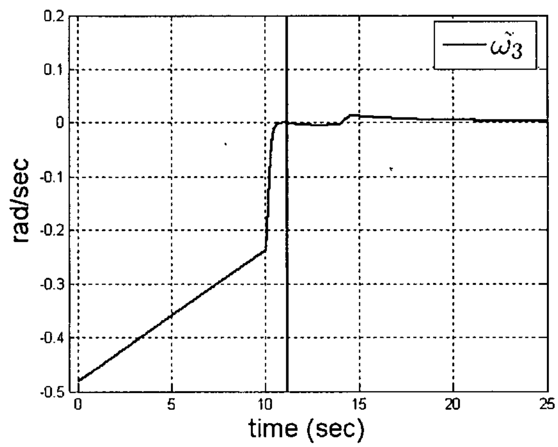

Figure 4.5.c: Variation of the error in the third component of angular velocity

The total motion illustrated in the above figures consist of three parts; from time zero for 10 seconds the recovery control law has not been implemented, so the spacecraft is only subjected to a constant torque about the unactuated axis. In this phase, the third component of angular velocity increases and the orientation of the spacecraft deviates from the desired direction. After 10 seconds, the recovery law is implemented. It takes about 1.5 seconds to reach the desired value of the third component of the angular velocity. The solid vertical lines in the figures illustrate the beginning of the third phase of the simulation during which the spacecraft reaches its desired orientation. With the gain parameters being $c_0 = C_0 = 10$ for the ω_3 correction phase and $K_0 = 0.2$, $C_0 = 10$, $c_0 = 0.2$ for the attitude stabilization phase, the entire recovery of spacecraft takes about 8 seconds. By increasing the control gain factors the recovery takes less time while demanding more torques. The rapid changes in the angular velocities and the attitude representation parameters correspond to either the switchings between the different phases of the simulation, as those at 10 and 11.2 seconds, or the intrinsic switchings in the attitude stabilization control law. It can be seen that the error value for the third component of angular velocity is always decreasing except during the switchings, which is then compensated shortly after the switching is completed.

To illustrate the effect of the moments of inertia on the performance of the controller, another simulation has been performed. The results of this simulation are illustrated in Figure 4.6. The spacecraft under consideration is the same, but the unactuated axis is the y-axis of the body coordinate system. Considering the equations (66) and (69) it can be seen that the only parameter not related to initial conditions that affects the response of the system is $C = (I_1 - I_2)/I_3$. Computing the parameter C for the two different choices of the unactuated axis, it can be seen that in the current simulation $C = 0.35$ while in the previous simulations it has been 0.3. Therefore, the difference in the choice of the unactuated axis does not have a significant effect on the response of the system from the inertia variation point of view.

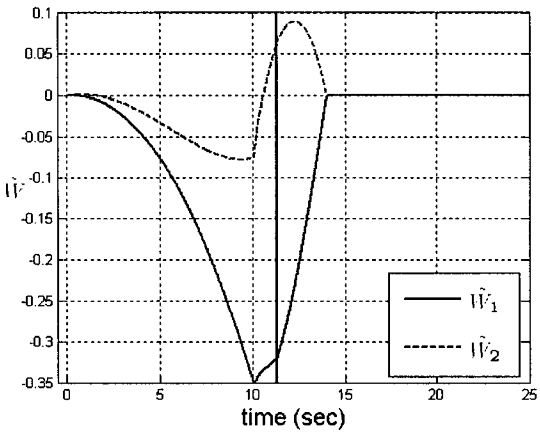

Figure4.6.a: Orientation error variations

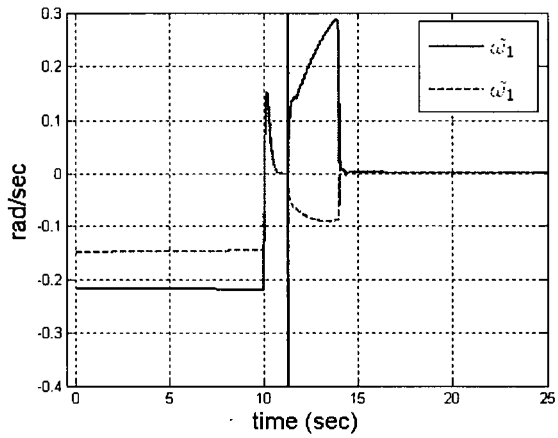

Figure 4.6.b: Variation of error in first and second components of angular velocity

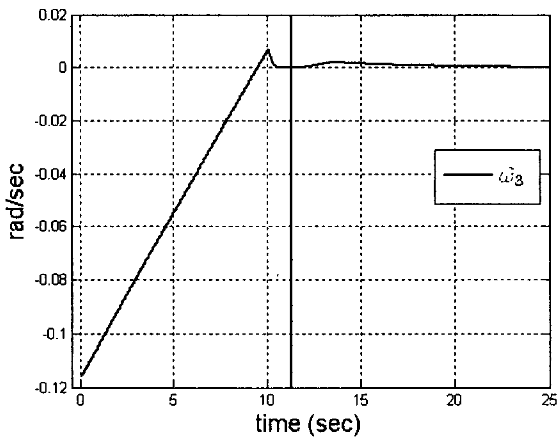

Figure 4.6.c: Variation of the error in the third component of angular velocity

The results of the simulations, however, differ from the previous one in the number of switchings and the duration of the convergence. The initial conditions for both simulations are the same. The recovery controller is implemented after 10 seconds from the start of the simulation. During these 10 seconds the attitude and angular velocities of the spacecraft vary. These variations differ for each simulation since the disturbance torque is applied about different axis. Therefore, the initial conditions of the controlled systems vary. Since the change of the inertia related factors are not significant, the variations in the controlled system response can be associated with the change of the initial value of the states. This simulation shows that the trend and duration of approach can be different for different initial conditions.

Because of the complete modeling of the system, i.e. the avoidance of using the simplification methods such as the concept of homogeneity, and the globally stable control law for the first subsystem, the proposed control law is highly robust. Since the initial errors in the orientation and the angular velocity only show up in the governing equations for the first subsystem, which has become globally stable, they do not have any effect on the stability of the system. On the other hand, linearization has been used for derivation of the control law for the entire system. During the switchings, the second subsystem may not be able to catch up to the prescribed values, which can result in the instability. This issue can be solved by increasing the gain factor, C_0, at the cost of higher control torques. Being globally stable is the main advantage of the proposed control law over other methods such as that in [50].

4.10 Generalization of the backstepping method to systems with nonlinear effort-state relation

In this section the theoretical base for deriving the control law for the entire system, having the kinematic control law is presented.

Consider the system

$$\dot{x} = f(x) + g(\xi) \tag{96}$$

$$\dot{\xi} = u \tag{97}$$

In which x is the state vector corresponding to the first subsystem and u is the control effort vector. Assume that $x=0$ is a global asymptotical equilibrium of (96) required $\xi=\alpha$. If we define the parameter $v=\xi-\alpha$ we have $\dot{v}=u-\dot{\alpha}(x)$ on the other hand using the first terms of the Taylor's expansion we have $\dot{x}=f(x)+g(v+\alpha)\cong f(x)+g(\alpha)+\frac{\partial g}{\partial v}v$

We now define g' as $g'=\frac{\partial g}{\partial v}=\frac{\partial g}{\partial \xi}$ so we have

$$\dot{x}\cong f(x)+g(\alpha)+g'(x)v \tag{98}$$

Suppose that the subsystem described by $\dot{x}=f(x)+g(\alpha)$ is stable, so there will be a Lyapunov function V_s which is positive except at zero and has a negative time derivative. We now introduce the Lyapunov function for the entire system

$$V_e=V_s+|v|^2/2 \tag{99}$$

So

$$\dot{V}_e=\dot{V}_s+v.\dot{v}=\frac{\partial V}{\partial x}\bullet\left(f(x)+g(\alpha)+g'(x)v\right)+v\bullet(u-\dot{\alpha})$$

where "$\bullet$" represents the dot-product.

Since if $\xi=\alpha$ the (96) subsystem will be stable

$$\frac{\partial V_s}{\partial x}\bullet\left(f(x)+g(\alpha)\right)=-W(x)\leq 0$$

in which $W(x)$ is a positive function except at the origin

$$\dot{V}_e=-W(x)+\frac{\partial V_s}{\partial x}\bullet g'(x)v+v\bullet(u-\dot{\alpha})=-W(x)+\left(\frac{\partial V_s}{\partial x}\right)^T g'(x)v+(u-\dot{\alpha})^T v$$

$$=-W(x)+\left[\left(\frac{\partial V_s}{\partial x}\right)^T g'(x)+\left(u^T-\dot{\alpha}^T\right)\right]v \tag{100}$$

If we define the control input, u, such that $(\partial V_s / \partial x)^T g'(x) + \left(u^T - \dot{\alpha}^T\right) = -C_0 v^T$ the Lyapunov function of the entire system will become negative definite and the system will be stable. Therefore, we define the control input u as

$$u^T = -C_0 v^T - (\partial V_s / \partial x)^T g'(x) + \dot{\alpha}^T \Rightarrow u = -C_0 v + \dot{\alpha} - g'(x)^T (\partial V_s / \partial x) \tag{101}$$

in which

$$g' = \left.\frac{\partial g}{\partial \xi}\right|_{\xi=\alpha} \tag{102}$$

This introduced control law has been used to derive the control law for the entire system from the kinematic controller in section 4.7.

4.11 Feasibility of controlling underactuated spacecraft using nonlinear feedback linearization method

One of the most common methods in the nonlinear control of the systems is the feedback linearization method. In this section it is examined whether the underactuated spacecraft system can be controlled using the feedback linearization method. The feedback linearization method has been briefly discussed in section 2.2.4. To be able to check whether the system is feedback linearizable, the governing equations of the system should be organized into the form of (19). Equations (25), (27) and (29) are the governing equations. Choosing the state vector of the system to be

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_6 \end{bmatrix} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ w_1 \\ w_2 \\ z \end{bmatrix} \tag{103}$$

Using equations (25), (27), (29) and (103) the governing equations of the system will turn into

$$\begin{aligned}
\dot{x}_1 &= Ax_2x_3 + u_1' \\
\dot{x}_2 &= Bx_1x_3 + u_2' \\
\dot{x}_3 &= Cx_1x_2 \\
\dot{x}_4 &= x_5(x_2x_4 + x_3) + \frac{x_1}{2}\left(1 + x_4^2 - x_5^2\right) \\
\dot{x}_5 &= x_4(x_1x_5 - x_3) + \frac{x_2}{2}\left(1 - x_4^2 + x_5^2\right) \\
\dot{x}_6 &= x_2x_4 - x_1x_5 + x_3
\end{aligned} \tag{104}$$

Equations (104) can be written in the form of (19) as

$$\dot{x} = f(x) + g_1(x) + g_2(x) \tag{105}$$

where

$$f(x) = \begin{bmatrix} Ax_2x_3 \\ Bx_1x_3 \\ Cx_1x_2 \\ x_5(x_2x_4 + x_3) + \frac{x_1}{2}\left(1 + x_4^2 - x_5^2\right) \\ x_4(x_1x_5 - x_3) + \frac{x_2}{2}\left(1 - x_4^2 + x_5^2\right) \\ x_2x_4 - x_1x_5 + x_3 \end{bmatrix} \tag{106}$$

$$g_1(x) = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{6\times 1} \quad \text{and} \quad g_2(x) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{6\times 1}$$

The next step is to form the feedback linearizability matrix

$$G_5 = \begin{bmatrix} g_1 & g_2 & ad_f g_1 & ad_f g_2 & \cdots & ad_f^5 g_1 & ad_f^5 g_2 \end{bmatrix} \tag{107}$$

The Lie algebra should be computed as discussed in section 2.2.3. Computing the Lie derivatives and the Lie brackets using the codes written in Matlab® symbolic math, it can be seen that the rank of the matrix G_5, when the first three components of the state vector vanish is four. Vanishing of the first three components of the state vector corresponds to vanishing of the angular velocity of the spacecraft. Since this rank is less than the rank of the state space, which is six, the system cannot be feedback linearized when the desired attitude is fixed. For control of the underactuated spacecraft subjected to the disturbance torques, the desired motion involves rotation of the spacecraft. Therefore, the first three components of the state vector are not zero at the desired equilibrium and the system can be feedback linearized. Therefore, the feedback linearization is a globally stable alternative to the linear controller introduced before. Two points must be considered in applying feedback linearization. First, the trajectory must not pass through the states with zero angular velocity. This means that the spacecraft must never stop rotating. A stop causes loss of controllability and the spacecraft cannot be guided to the desired attitude afterwards. The second point is that feedback linearization cannot be used if the disturbance torques are not present and the spacecraft is supposed to reach a non-rotating attitude.

4.12 Comparison of the proposed control methods

In comparison, nonlinear controller is insensitive to the initial conditions, while the linear controller is smooth and therefore requires much less actuation torques. Linear controller has been derived by approximating the system by a linear system. This approximation differs from the reality as we get further from the desired point where the system has been linearized about.

The controller may not converge if the initial conditions are far from the desired values or the control gains are set to high values. The other point about linearization is that it cannot be performed if the desired angular velocity is zero. If the angular velocity vanishes the controllability over the unactuated axis is lost.

Nonlinear controller stabilizes the desired states whatever the initial conditions are. It is also applicable for stabilization of a non-rotating direction. A non-rotating direction is chosen as the desired state if there is no disturbance torques acting on the spacecraft. However as proved in [12] a spacecraft controlled by two pairs of gas jet actuators cannot be stabilized to a static equilibrium using a *smooth* feedback control law. The fluctuations in the control law demands high control torques to make the system keep up with the prescribed values. The backstepping method used in deriving the controller results in global stability only if the kinematic law is smooth. To make the overall system stable, the trajectory must be kept as close to the prescribed non-smooth values as possible. This prevents from reducing the control torques by reducing the gains. Overall, the nonlinear control method works under difficult conditions but it demands high control torques.

The best way of controlling an underactuated spacecraft subjected to disturbance torques is to use a combination the of control laws. The nonlinear controller should be used if the trajectory is far from the origin or if the desired state is a non-rotating direction. However, when the spacecraft is close to the desire orientation using linear controller rotates the spacecraft to the desired attitude smoothly. If the desired angular velocities are not zero feedback linearization methods can be used for controlling the spacecraft as shown in section 4.10. This method results in smooth globally stable controller stabilizing the desired motion. Therefore, it is a globally stable alternative for the linear controller.

Chapter 5

Momentum dumping of a spacecraft using less than three external control torques

If the spacecraft is controlled using the momentum wheels, the external torques applied on the spacecraft, including disturbance torques, are accumulated. This will cause the angular velocity of the momentum wheels to increase and to prevent their saturation this accumulated momentum must be eliminated. The vanishing of the accumulated momentum in momentum wheels is called *momentum dumping* and it is achieved through the use of external control torques. Momentum dumping is a crucial stage in the attitude control of the spacecraft. If the momentum wheels saturate, the attitude controller of the spacecraft will become unstable. Therefore, the spacecraft will no longer point to the desired directions and will naturally be out of service.

The external torques can be generated using the thrusters or the magnetic torquers. If one of the actuators is out of order due to a failure, the principal axis corresponding to that actuator will become unactuated. The angular momentum of the system will be affected by the external torques as

$$\dot{\vec{H}} = T \tag{108}$$

in which $\vec{H}$ is the angular momentum vector of the spacecraft and T is the net external torques applied. If we are not able to apply external torque about any of the principal axes, according to (108) that component of the angular momentum cannot be changed. In this section, a control method has been developed to diminish the angular velocities of all the momentum wheels using two or even one external torque actuator.

5.1 Governing equations

The angular momentum of the spacecraft is derived [51] as

$$H = [I^*]\omega + [A]\Omega \tag{109}$$

in which $\left[I^*\right]$ is the inertia tensor of the spacecraft and the momentum wheels with all the wheels locked. Matrix $[A]$ contains the components of the inertia tensor of the momentum wheels that contribute to the total angular momentum after multiplication by the spinning speed of the wheels.

$$\Omega = \begin{bmatrix} \vdots \\ \Omega_i \\ \vdots \end{bmatrix} \tag{110}$$

in which Ω_i is the spinning speed of the i^{th} momentum wheel. The angular momentum vector of the i^{th} momentum wheel due to the spinning of the motor, described in the spacecraft coordinate system is $A_i\Omega_i$. Therefore, the matrix A can be formed as

$$A = [\ldots \quad A_i \quad \ldots] \tag{111}$$

Considering the coordinate system attached to the center of mass of the spacecraft and rotating with that, from (108), we will have.

$$\dot{\vec{H}}_c = T_c \tag{112}$$

in which $\vec{H}_c$ is the angular momentum vector of the spacecraft and T_c is the net external torques applied, both about the center of mass of the system, C. $\vec{H}_c$ has been described in the body coordinate system, which is rotating with the angular velocity ω. Considering the effect of the rotation of the body coordinate system, we have [52]

$$\dot{\vec{H}}_c = \frac{d}{dt}\left([I^*]\omega + [A]\Omega\right) = \left[I^*\right]\dot{\omega} + [A]\dot{\Omega} + \omega \times \left([I^*]\omega + [A]\Omega\right) \tag{113}$$

In the above equation, the term $\omega \times \left([I^*]\omega + [A]\Omega\right)$ corresponds to the effect of the rotation of the coordinate system. Defining the skew symmetric matrix $[\tilde{a}]$

$$[\tilde{a}] = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{114}$$

Using the above definition we can obtain [53]

$$a \times b = [\tilde{a}]b \tag{115}$$

We can now rewrite the equation (113) into the matrix form

$$\dot{\vec{H}}_c = \frac{d}{dt}\left([I^*]\omega + [A]\Omega\right) = \left[I^*\right]\dot{\omega} + [A]\dot{\Omega} + [\tilde{\omega}]H_c \tag{116}$$

Rewriting (112) using (116) we will have

$$\left[I^*\right]\dot{\omega} + [A]\dot{\Omega} + [\tilde{\omega}]\vec{H}_c = T_c \tag{117}$$

in which $[\tilde{\omega}]$ is the skew symmetric matrix defined as in (114).

Now we obtain the governing equations of the spinning of each of the momentum wheels.

The flywheel of the momentum wheel assembly is carefully balanced to be symmetric. Therefore, from the three principal moments of inertia the two, which correspond to the axes perpendicular to the axis of symmetry, are the same. We call the moment of inertia about the axis of symmetry $I_{\parallel}$ and the moment of inertia about the other two axes $I_{\perp}$. Using the Eller's equation [52] we get

$$I_{\parallel}\dot{\omega}_{\parallel} = (I_{\perp} - I_{\perp})\omega_{\perp 1}\omega_{\perp 2} + T_{\parallel} \tag{118}$$

In which $\omega_{\perp 1}$ and $\omega_{\perp 2}$ are the angular velocity components perpendicular to the axis of symmetry and $T_{\parallel}$ is the torque exerted by the momentum wheel motor. $\omega_{\parallel}$ is the component of the angular velocity of the flywheel along the axis of the momentum wheel, viewed by an inertial observer. The spinning velocity of the motor of the i^{th} momentum wheel assembly is Ω_i and $B_i = [B_{i1}, B_{i2}, B_{i3}]^{\mathrm{T}}$ is the unit vector along the axis of that flywheel in the body coordinate system. The component of the angular velocity of the flywheel about the motor axis is computed as

$$\omega_{\parallel i} = \Omega_i + B_i \bullet \omega = \Omega_i + B_i^{\mathrm{T}}\omega \tag{119}$$

We can represent all the momentum wheel governing equations in a single matrix relation

$$[J]\{\dot{\Omega} + [B]\dot{\omega}\} = u \tag{120}$$

In the above equation $[\Omega] = [\ldots, \Omega_i, \ldots]^{\mathrm{T}}$ and $[u] = [\ldots, u_i, \ldots]^{\mathrm{T}}$, where u_i is the actuation torque of the i^{th} momentum wheel. The moment of inertia of the i^{th} momentum wheel about its motor axis is J_i. The matrices $[J]$ and $[B]$ are defined below

$$[J] = \begin{bmatrix} \ddots & & 0 \\ & J_i & \\ 0 & & \ddots \end{bmatrix} \tag{121}$$

$$[B] = \begin{bmatrix} \vdots \\ B_i^{\mathrm{T}} \\ \vdots \end{bmatrix} \tag{122}$$

We can rewrite the equation (120) as

$$\dot{\Omega} = -[B]\dot{\omega} + [J]^{-1}u \tag{123}$$

Then substituting (123) into (117) we obtain

$$\dot{\omega} = \{[I^*] - [A][B]^{-1}\}\{-[A][J]^{-1}u - [\tilde{\omega}]H_c + T_c\} \tag{124}$$

The angular momentum of the i^{th} wheel caused by the spinning of the motor will be $J_i\Omega_i B_i$. These vectors should be summed over all of the momentum wheels to result in the angular momentum vector of the spacecraft due to spinning of all the momentum wheels, this will be

$$\begin{bmatrix} \ddots & & 0 \\ & J_i & \\ 0 & & \ddots \end{bmatrix} \begin{bmatrix} \ldots & B_i & \ldots \end{bmatrix} \begin{bmatrix} \vdots \\ \Omega_i \\ \vdots \end{bmatrix} \tag{125}$$

Comparing this to equation (109) and considering the definition (122) we get

$$A = B^{\mathrm{T}} J \tag{126}$$

Therefore, the equation (124) can be rewritten as

$$\dot{\omega} = \{[I^*] - [B^{\mathrm{T}}]J[B]\}^{-1}\{-[B^{\mathrm{T}}]u - [\tilde{\omega}]H_c + T_c\} \tag{127}$$

or

$$\dot{\omega} = [I^{\Delta}]^{-1}\{-[B^{\mathrm{T}}]u - [\tilde{\omega}]H_c + T_c\} \tag{128}$$

In which I^{Δ} is defined as

$$[I]^{\Delta} = \{[I^*] - [B^{\mathrm{T}}] J [B]\} \tag{129}$$

The quaternion vector is used for representing the attitude of the spacecraft. Consider $[D]$ to be the direction cosine matrix corresponding to the coordinate transformation between the inertial coordinate system and the body-fixed coordinate system. This direction cosine matrix is known [26] to have three eigenvalues one of them being equal to unity. If e is the eigenvector corresponding to the unit eigenvalue, we have $[D]e = e$. The inertial coordinate system can be brought to the body coordinate system by a pure rotation about the eigenaxis of the direction cosine matrix, e, with the amount of α radians. The quaternion vector describing the attitude of the spacecraft is now defined as [54]

$$\boldsymbol{q} = [q_1, q_2, q_3, q_4]^{\mathrm{T}} \tag{130}$$

In which

$$\begin{aligned} q_1 &= e_1 \sin(\alpha/2) \\ q_2 &= e_2 \sin(\alpha/2) \\ q_3 &= e_3 \sin(\alpha/2) \\ q_4 &= \cos(\alpha/2) \\ e &= [e_1, e_2, e_3]^{\mathrm{T}} \end{aligned} \tag{131}$$

If d_{ij} is the element of the direction cosine matrix located at the i^{th} row and the j^{th} column, the quaternion elements can be found as follows,

$$q_4 = \pm 0.5\sqrt{1 + d_{11} + d_{22} + d_{33}}$$

$$q_1 = 0.25(d_{23} - d_{32}) / q_4$$

$$q_2 = 0.25(d_{12} - d_{21}) / q_4$$

$$q_3 = 0.25(d_{12} - d_{21}) / q_4$$

The kinematic equation for the quaternion changes is

$$\frac{d}{dt}\boldsymbol{q} = \frac{1}{2}\,\Omega'\boldsymbol{q} \tag{132}$$

In which

$$\Omega' = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix} \tag{133}$$

The equation (132) can be rewritten in the following form [54]. Equation (132) is the kinematic equation of the system and together with the dynamic equations (128) and (123) forms the governing equations of the system.

It is assumed that there are three acting momentum wheels, installed to rotate about the principal axes of the spacecraft. The B-matrix introduced in (122) will then be

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \tag{134}$$

Therefore the equations (123), (128) and (129) will change to

$$\dot{\Omega} = -\dot{\omega} + [J]^{-1} u \tag{135}$$

$$\dot{\omega} = \left[I^{\Delta}\right]^{-1} \{-u - [\tilde{\omega}] H_c + T_c\} \tag{136}$$

$$[I]^{\Delta} = \left\{ \left[I^{*} \right] - J \right\} \tag{137}$$

Now that we have the governing equations, we can move on to deriving the control scheme.

5.2 Momentum dumping maneuvers

During the momentum dumping process the angular velocity of the momentum wheels of the spacecraft must be removed. So, the initial conditions of the momentum dumping process are identified as the spacecraft being in the desired orientation while the momentum wheels are rotating near the saturation velocities. The desired final condition is having the spacecraft in the desired orientation while the angular velocity of the wheels have vanished.

5.2.1 Momentum dumping with two control torques

If we have three acting external torques, the momentum dumping can be performed while having the spacecraft in the desired direction throughout the process [26]. If one of the thrusters is out of order due to a failure, the orientation has to be temporarily changed for facilitating the momentum dumping; without loss of generality, we assume the failed thruster to be the one incorporated with the third principal axis. If the spacecraft is stationary then $\omega = 0$, $\dot{\omega} = 0$ and $[\tilde{\omega}] = 0$. From the equation (136), we have $0 = \left[I^{\Delta}\right]^{-1} \{-u - [0] H_c + T_c\}$. Since the third thruster is out of order $T_{c3} = 0$, so $u_3 = 0$. J is defined to be a diagonal matrix so J^{-1} is also diagonal and therefore the third component of $[J]^{-1}u$ will be zero. Then, from the equation (135) we have $\dot{\Omega}_3 = 0$. This means that the angular velocity of the third momentum wheel remains unchanged and cannot be dumped. Therefore, the attitude of the spacecraft has to be temporarily changed during the momentum dumping procedure.

Since the orientation of the spacecraft must be changed for momentum dumping of the underactuated spacecraft, there is a possibility that the spacecraft will not be in the desired direction by the end of momentum dumping maneuver. We should now see if correcting the attitude of the spacecraft accumulates speed in the momentum wheels. Consider that the spacecraft is stationary at the orientation Q with all the momentum wheels not rotating (the common situation after the momentum dumping). The angular momentum of the spacecraft from (109) is computed to be zero. We then rotate the spacecraft to the desired orientation R, using the momentum wheels. Since the spacecraft will be stationary at the orientation R, $\omega = 0$. From the conservation of the angular momentum we have $H = [I^*][0] + [A]\Omega = 0$ therefore $[A]\Omega = 0$. From (126) and (134) we have $[A] = [J]$. Thus, $[A]$ is not singular and $\Omega = 0$. Therefore, none of the momentum wheels will be rotating after the attitude maneuver is performed. This means that correcting the attitude of the spacecraft does not accumulate speed in the momentum wheels.

As a momentum dumping procedure, if the angular velocity of the third momentum wheel is vanished we can dump the angular momentum in the rest of the momentum wheels while the spacecraft is stationary. The motors of the momentum wheels apply internal torque which does not affect the total angular momentum of the system. If the spacecraft is rotationally stationary and its angular momentum vector lies in the x-y plane of the spacecraft, $H_3 = 0$.

Therefore, the third component of $A[\Omega]$ will be zero. Since $[A]=[J]$ is a diagonal matrix $\Omega_3 = 0$. the maneuver of rotating the spacecraft using its momentum wheel, such that the angular momentum vector lies in the x-y plane is proposed for vanishing the third momentum wheel angular velocity. In that final orientation, the z-axis of the spacecraft will be perpendicular to the angular momentum vector, $\vec{H}$. We call this intermediate stage attitude Q.

The shortest rotation from the initial orientation to Q is done about the vector $\vec{Z}_0 \times \vec{H}$. In which $\vec{Z}_0$ is the z-axis of the initial body coordinate system. In this rotation, the z-axis of the spacecraft coordinate system remains in the plane normal to $\vec{Z}_0 \times \vec{H}$.

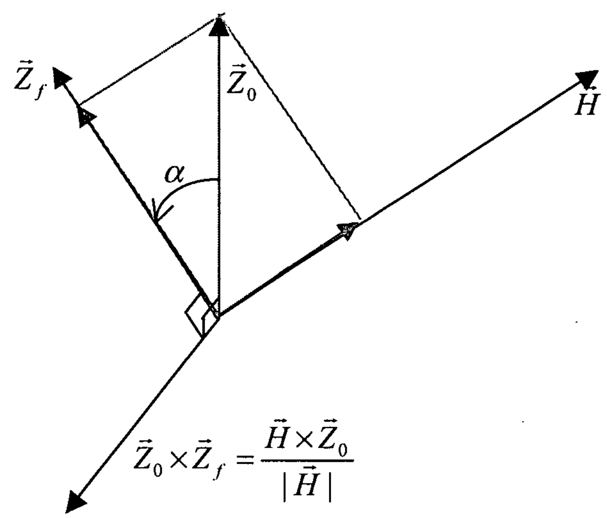

Figure 5.1: Momentum dumping intermediate attitude

It can be seen from the Figure 5.1 that the final orientation of the z-axis, $\vec{Z}_f$, should be a unit vector along the normal component of $\vec{Z}_0$ to $\vec{H}$. Therefore, we have

$$\vec{Z}_f = \vec{Z}_0 - \frac{\vec{H} \bullet \vec{Z}_0}{|\vec{H}|} \frac{\vec{H}}{|\vec{H}|} \tag{138}$$

In the above equation $\frac{\vec{H}}{|\vec{H}|}$ is the unit vector along $\vec{H}$. The value $\frac{\vec{H} \bullet \vec{Z}_0}{|\vec{H}|}$ is the component of $\vec{Z}_0$ along the vector $\vec{H}$. Therefore, in the equation (138) we have subtracted the component of $\vec{Z}_0$ along the angular momentum vector from $\vec{Z}_0$ to get to the normal component. Since before the momentum dumping the spacecraft has been in the desired direction, the spacecraft coordinate system is assumed to coincide with the inertial coordinate system at the beginning of the motion. The spacecraft coordinate system is then rotated about $\vec{Z}_0 \times \vec{Z}_f$ with an angle of α so that the z-axis reaches $\vec{Z}_f$. $\vec{Z}_f$ is along one of components of $\vec{Z}_0$, so the angle of rotation will be $-\pi/2 < \alpha < \pi/2$. Therefore, the angle of rotation can be computed as

$$\alpha = \sin^{-1}\left(\frac{\vec{Z}_0 \times \vec{Z}_f}{|\vec{Z}_0||\vec{Z}_f|}\right) \tag{139}$$

We know the vector which rotation has been about and the angle of rotation, so the quaternions describing the attitude of the spacecraft in the final orientation can be derived as follows.

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \left(\vec{Z}_0 \times \vec{Z}_f\right)\sin(\alpha/2) \tag{140}$$

$$q_4 = \cos(\alpha/2) \tag{141}$$

5.2.2 Momentum dumping with one external control torque

The accumulated momentum in the momentum wheels can be removed using even one external control torque. This means, even if the control over the external torques about two of the principal axes of the spacecraft is lost, the momentum dumping process can be performed and the spacecraft will be able to perform the attitude maneuvers afterwards. The general solution is to first rotate the spacecraft so that the actuated axis aligns with the direction of the total angular momentum vector. This causes the angular velocity of the momentum wheels incorporated with the other two principal axes to vanish. In the next step the available thrusters are used to exert a torque in the reverse direction of the momentum vector, which is now in the same direction as the actuated axis. While this external torque is applied the attitude stabilization method using the momentum wheels is active. As a result, the attitude of spacecraft does not change while the angular velocities of the momentum wheels diminish by time. After all the angular momentum of the spacecraft has been removed the spacecraft is returned to its initial attitude implementing the momentum wheels and using the same control scheme for the attitude stabilization as that of the previous two steps. We should now quantify the attitude in which the actuated axis lies in the direction of the angular velocity vector. Without loss of generality, the x-axis is assumed to be the external torque actuated axis of the spacecraft. The fastest and shortest way of bringing $\hat{e}_1 = [1,0,0]^{\mathrm{T}}$, the unit vector along the x-axis, along the $\vec{H}$, the angular momentum vector, is the rotation about their cross product $\hat{e}_1 \times \vec{H}$. Therefore, the inertial frame can be taken to the intermediate frame, in which the intermediate x-axis lie in the direction of the angular momentum vector, by a rotation about $\hat{e}_1 \times \vec{H}$ with the angle of α. $\sin(\alpha)$ and $\cos(\alpha)$ can be found from the magnitude of the cross and dot products as follows

$$\sin(\alpha) = \frac{|\hat{e}_1 \times \vec{H}|}{|\vec{H}|} \tag{142}$$

$$\cos(\alpha) = \frac{\hat{e}_1 \bullet \vec{H}}{|\vec{H}|} \tag{143}$$

The unit vector along $\hat{e}_1 \times \vec{H}$ is the unit vector determining the first three quaternion coordinates. Therefore, the quaternions representing the intermediate coordinate system are

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \frac{\hat{e}_1 \times \vec{H}}{|\hat{e}_1 \times \vec{H}|} \sin(\alpha) \tag{144}$$

$$q_4 = \cos(\alpha) \tag{145}$$

Now that the quaternions associated with the desired attitude have been derived, a control law must be implemented to bring the attitude of the spacecraft to the desired setting using the momentum wheels.

5.3 Attitude changing control methods

In this section, we present two control methods for performing the attitude changing in the momentum dumping maneuvers. Having a look at equation (136) we find that the momentum wheels torque vector, u, if negated, will act similarly as the external torque terms, T_c. Therefore, the control laws for controlling the attitude of the spacecraft using the thrusters can be generalized to control of the spacecraft using the momentum wheels in addition to the control methods developed for controlling spacecraft using the momentum wheels.

5.3.1. Control law with quaternion sign indifference

Consider the control law in which the momentum wheels control torque is derived as

$$-u=[K][q]-[D][\omega]+[\tilde{\omega}]H_c \tag{146}$$

in which $[K]$ and $[D]$ are the proportional and derivative gain matrices. Inspired by [54] these matrices are assigned as

$$[K]=k\left[I^{\Delta}\right] \tag{147-a}$$

$$[D]=d\left[I^{\Delta}\right] \tag{147-b}$$

$[q]$ contains the three components of the quaternion vector corresponding to the spacecraft coordinate system.

$$[q]=\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{148}$$

It has been shown in appendix 1 that using this control law the spacecraft undergoes an eigenaxis rotation. This means that it will rotate about the Euler axis corresponding to the initial attitude. The Euler axis corresponding to an attitude is the eigenvector corresponding to the unit eigenvalue of the direction cosine matrix. Note that since the quaternion vector is a unit vector, the absolute value of the fourth component of the quaternion vector can be derived having the first three components. Therefore, no information is lost by not including q_4 in the control law. In the quaternion attitude representation $[\boldsymbol{q}]$ and $-[\boldsymbol{q}]$ represent the same orientation. Therefore, in deriving the quaternion parameter from the direction cosine matrix, the output of the attitude sensors, there is a danger of the negation of the quaternion vector. To prevent this negation passing on to the control torques we modify the previous control method inspired by [51].

$$-u=[K]q_4[q]-[D][\omega]+[\tilde{\omega}]H_c \tag{149}$$

In the modified control law, if the sign of $[\boldsymbol{q}]$ changes the sign of both $[q]$ and q_4 will change and the sign of $q_4[q]$ will remain unchanged. Therefore, there will be no abrupt changes in the applied torques. The control law presented in (149) stabilizes the angular velocity zero and the quaternion vector $[\boldsymbol{q}]=[0,0,0,1]^{\mathrm{T}}$. To make the attitude represented by $[\boldsymbol{q}_d]$ stable, the quaternion representing the orientation of the spacecraft relative to the frame fixed at the desired orientation must be computed. This quaternion is called $[\tilde{\boldsymbol{q}}]$ and then $[\tilde{q}]$ and $\tilde{q}_4$ are defined similarly. The control law is now modified to

$$-u=[K]\tilde{q}_4[\tilde{q}]-[D][\omega]+[\tilde{\omega}]H_c \tag{150}$$

This modification make the spacecraft reach the attitude represented by $[\boldsymbol{q}_d]$.

The stability of the proposed control method is proved using the Lyapunov method [6]. We consider the following Lyapunov function.

$$V=\frac{1}{2}\omega^{\mathrm{T}}[K]^{-1}\left[I^{\Delta}\right]\omega+\tilde{q}_1^2+\tilde{q}_2^2+\tilde{q}_3^2=\frac{1}{2}\omega^{\mathrm{T}}[K]^{-1}\left[I^{\Delta}\right]\omega+1-\tilde{q}_4^2 \tag{151}$$

The matrices $[K]^{-1}$ and $\left[I^{\Delta}\right]$ are positive definite, so $\frac{1}{2}\omega^{\mathrm{T}}[K]^{-1}\left[I^{\Delta}\right]\omega$ will always be a positive scalar except at $\omega=0$, where it vanishes. The introduced V function is a positive function only vanishing if $\omega=0$, $\tilde{q}_4=1$ and $\tilde{q}_1=\tilde{q}_2=\tilde{q}_3=0$, which corresponds to the spacecraft being stationary at the desired orientation.

Now we should check to see if the time derivative of V is always negative.

$$\dot{V}=\frac{1}{2}\dot{\omega}^{\mathrm{T}}[K]^{-1}\left[I^{\Delta}\right]\omega+\frac{1}{2}\omega^{\mathrm{T}}[K]^{-1}\left[I^{\Delta}\right]\dot{\omega}-2\tilde{q}_4\dot{\tilde{q}}_4 \tag{152}$$

From the definitions (137) and (147-a) it is known that both $\left[I^{\Delta}\right]$ and $[K]$ matrices are symmetric therefore $[K]^{-1}\left[I^{\Delta}\right]$ will be a symmetric matrix as well. As a result

$$\omega^{\mathrm{T}}[K]^{-1}[I^{\Delta}] = \left([K]^{-1}[I^{\Delta}]\omega\right)^{T} \tag{153}$$

Then having

$$\left([K]^{-1}[I^{\Delta}]\omega\right)^{T}\dot{\omega} = \left([K]^{-1}[I^{\Delta}]\omega\right)\bullet\dot{\omega} = \dot{\omega}^{\mathrm{T}}[K]^{-1}[I^{\Delta}]\omega \tag{154}$$

we obtain

$$\omega^{\mathrm{T}}[K]^{-1}[I^{\Delta}]\dot{\omega} = \dot{\omega}^{\mathrm{T}}[K]^{-1}[I^{\Delta}]\omega \tag{155}$$

Now we can rewrite equation (152) as

$$\dot{V} = \dot{\omega}^{\mathrm{T}}[K]^{-1}[I^{\Delta}]\omega - 2\tilde{q}_4\dot{\tilde{q}}_4 \tag{156}$$

Using the equation (136) and (150) we get

$$[I^{\Delta}]\ \dot{\omega} = -[D]\omega - [K]\tilde{q}_4 q \tag{157}$$

The governing equation for the fourth component of the quaternions will be [54]

$$\dot{\tilde{q}}_4 = -\frac{1}{2}\omega^{\mathrm{T}}\tilde{q} \tag{158}$$

Substituting the equations (157) and (158) into (156) we get

$$\dot{V} = \omega^{\mathrm{T}}[K]^{-1}\left(-[D]\omega - [K]\tilde{q}_4\tilde{q}\right) + \omega^{\mathrm{T}}\tilde{q}\tilde{q}_4 \tag{159}$$

So we obtain

$$\dot{V} = -\omega^{\mathrm{T}}[K]^{-1}[D]\omega \tag{160}$$

Both $[K]^{-1}$ and $[D]$ are negative definite matrices. Therefore, $[K]^{-1}[D]$ is a positive definite matrix and the right hand side of the equation (160) will always be negative except when $\omega = 0$. The vanishing of the $\dot{V}$ function does not necessarily occur at the origin, whatever the attitude is, if the angular velocity vector vanishes, $\dot{V}$ will vanish as well. Therefore, the time derivative of the Lyapunov function is a positive semi-definite function of the states. According to the Lyapunov theorem for the local stability [6], this implies local stability of the origin.

For proving the global stability, we use the global invariant set theorem, described in section 1.2.1. According to (132), if the attitude of the spacecraft does not change the angular velocity vector of the spacecraft, ω, is zero. The ω vector being constantly zero implies $\dot{\omega}=0$. Using the equation (157), this condition is preserved if either $q=[0,0,0]^{\mathrm{T}}$ or $q_4=0$. $q=\boldsymbol{0}_{3\times 1}$ corresponds to the spacecraft being at the desired attitude, however the vanishing of q_4 does not correspond to the desired attitude. Therefore, to be able to prove the global stability it must be shown that the equilibrium corresponding to $q_4=0$ is unstable. Stability of the attitudes identified by $q_4=0$ and $\omega=0$ is determined using the Lyapunov's linearization method. The state vector of the system will be

$$x=[q_1,q_2,q_3,q_4,\omega_1,\omega_2,\omega_3]^{\mathrm{T}} \tag{161}$$

Using the equations (132) and (157), and linearizing the equations as described in section 1.2.1 the controlled system can be linearized to

$$\dot{x}=Ax$$

$$A=\begin{bmatrix} \multicolumn{2}{c}{\multirow{4}{*}{$\frac{1}{2}[G(\omega)]_{4\times 4}$}} & 0 & 0 & 0 \\ & & 0 & 0 & 0 \\ & & 0 & 0 & 0 \\ & & 0 & 0 & 0 \\ \left[-I^{\Delta^{-1}}kq_4\right]_{3\times 3} & \left[-I^{\Delta^{-1}}kq\right]_{3\times 1} & \multicolumn{3}{c}{\left[-I^{\Delta^{-1}}D\right]_{3\times 3}} \end{bmatrix} \tag{162}$$

Form the seven eigenvalues of the matrix A four are zero at $q_4=\omega=0$ and the others are the eigenvalues of the matrix $-I^{\Delta^{-1}}D$. Since I^{Δ} and D are both positive definite matrices $-I^{\Delta^{-1}}D$ will also be a positive definite matrix with three strictly positive eigenvalues. Therefore, the matrix A has three positive eigenvalues which makes the equilibrium $q_4=0$ unstable. As a result, the system converges to the only stable invariant attitude, which is the desired attitude.

5.3.2. Control law with quaternion sign dependency

Considering the dynamic equations of the spacecraft (136) it can be seen that, provided we use the feedback linearization method to compensate the gyroscopic terms, the external torques and the momentum wheel motor torques play the same role. Although applying internal control torques changes the angular momentum of the wheels, as far as the gyroscopic term, $\omega \times \vec{H}_c$, is compensated it does not have any effect on the behavior of the system. Here a well known control method for deriving external control torques [54] is expanded to the momentum wheel actuation case.

$$-u = [K][q] - [D][\omega] + [\tilde{\omega}]H_c \tag{163}$$

in which $[K]$ and $[D]$ are the proportional and derivative gain matrices assigned as

$$[K] = k[I^{\Delta}] \tag{164-a}$$

$$[D] = d[I^{\Delta}] \tag{164-b}$$

Using (136) the dynamic equation of the system is

$$I^{\Delta}\omega = -[K]q - [D]\omega \tag{165}$$

To prove the stability of the system we choose the following Lyapunov function

$$V' = 1/2\ \omega^{T}[K]^{-1}I^{\Delta}\omega + q_1^2 + q_2^2 + q_3^2 + (q_4 - 1)^2 = 1/2\ \omega^{T}[K]^{-1}I^{\Delta}\omega + 2(1 - q_4) \tag{166}$$

The time derivative of this Lyapunov function is determined similar to the previous case as

$$\dot{V}' = \omega^{T}[K]^{-1}I^{\Delta}\dot{\omega} - 2\dot{q}_4 \tag{167}$$

Provided $[K]^{-1}I^{\Delta}$ is a symmetric matrix. This condition is preserved by the choices for gain matrices (164). Using (165) and (158) we obtain

$$\dot{V}' = \omega^{T}[K]^{-1}(-[K]q - [D]\omega) + \omega^{T}q = -\omega^{T}[K]^{-1}[D]\omega \tag{168}$$

Since both $[K]^{-1}$ and $[D]$ are positive definite matrices $\dot{V}'$ will be a negative semi-definite function of states. Using the Lyapunov's theorem for local stability the function is proven to be locally stable. Using (165) the invariant attitudes of a spacecraft controlled using this method can only be the origin which corresponds to the desired attitude. Therefore, the global invariant set theorem implies that the origin is globally stable.

5.3.3 Comparison of the attitude changing control methods

Both control rules are proven globally stable and can be used for the attitude maneuvers of the spacecraft. One of the advantages of the control law with the quaternion sign indifference over the other method is its independence from the sign of the quaternion fourth element. As stated before by multiplying q_4 by q the control torques will be the same for assigning either $[\boldsymbol{q}]$ or -$[\boldsymbol{q}]$ to a single attitude. This can prevent sudden changes in the applied control torques and the vibrations resulted. But the main advantage of this control method is its intrinsic gain scheduling. If the spacecraft attitude is close to the desired direction, q_4 will be close to one. This means that while close to the desired attitude both control laws result in the same control torques except the sign compensation that is done in the former method. The total proportional gain in the sign insensitive method is equal to $-q_4[K]$ which is q_4 times the control gain in the sign sensitive method. While far from the desired direction q_4 is a small number increasing as we approach to the desired attitude. Therefore, the total proportional gain varies and increases as the spacecraft approaches the desired attitude. This variation prevents having the significant control torques at the beginning of the maneuver, where since the error vector is large, the torques can be huge if treated with the same gain coefficient. On the other hand, the behavior of the system remains unchanged in the vicinity of origin, which makes sure that the approach behavior of the system is not affected. Due to the two stated advantages, we choose the sign insensitive control method for performing the attitude maneuvers.

5.4 Angular momentum removal using external control torques

In the previous sections we determined the attitude at which the angular momentum removal should be performed and we derived the control laws to bring the spacecraft to that desired attitude implementing the momentum wheels. In this section, we derive a control law for deriving the external control torques to cease the angular momentum of the spacecraft. The main idea is to stabilize the attitude of spacecraft using the momentum wheels, while exerting the appropriate external torques to reduce the angular momentum. When the spacecraft is stabilized using the momentum wheels, all the external torques are transferred to the momentum wheels to prevent the rotation of the spacecraft. We assign this external torque to be negatively proportional to the angular momentum vector

$$T_c = -k_{dump}\vec{H}_c \tag{169}$$

In the above equation k_{dump} is the positive scalar gain. Using the equations (112) and (169) we obtain

$$\dot{\vec{H}}_c = \left[-k_{dump} I_{3\times3}\right]\vec{H}_c \tag{170}$$

Since $\left[-k_{dump} I_{3\times3}\right]$ is a negative definite matrix, with three eigenvalues being equal to $-k_{dump}$, the angular momentum of the spacecraft converges exponentially to zero. If the direction of the angular momentum of the spacecraft changes during the momentum removal process the attitude of the spacecraft must be altered so that the angular momentum vector be always in the plane or direction which the external torque actuation is available. Since the direction of the time derivative of the angular momentum vector is in the same direction as the vector (refer to equation (170)), its direction remains constant and attitude changes meanwhile the momentum removal is not necessary. Figure 5.2 illustrates the block diagram of the attitude control and momentum management control system.

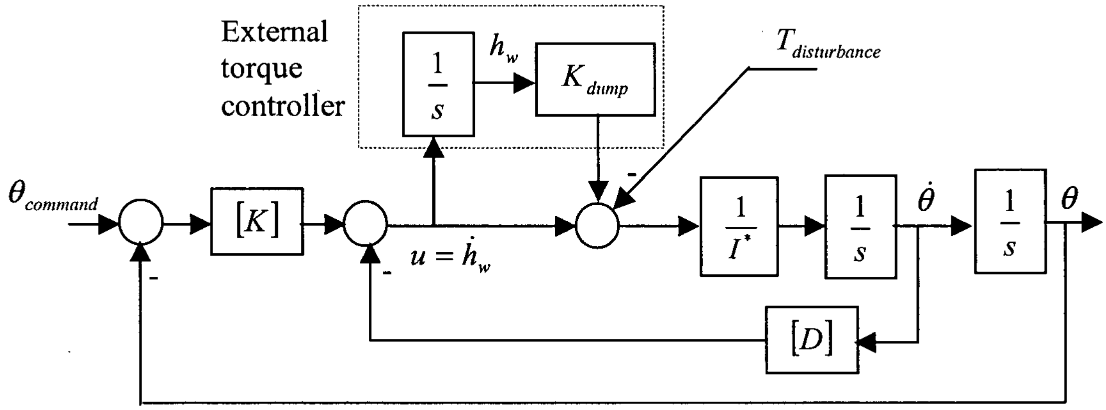

Figure 5.2: Momentum removal system

In the above figure $h_w = [A]\Omega$ is the angular momentum of the momentum wheels. The other parameters have been defined in section 5.1, 5.3 and 5.4. The external torque controller in the control system is augmented to the attitude control system introduced in (163). Therefore, while the attitude is stabilized using the momentum wheels, an external torque proportional to the angular momentum vector of the momentum wheels is implemented on the spacecraft to remove the accumulated angular momentum.

5.5 Numerical simulations and discussions

The numerical simulations have been performed to illustrate the performance of the control laws in action. The Simulink® simplified scheme of the controller is illustrated in Figure A.2.2. Inspired by [51] the specifications of the spacecraft is assumed to be

M = the total mass of the spacecraft and the momentum wheels 515 kg

I_1 = the moment of the inertia of the spacecraft without the momentum wheels about x-axis

=86.215 kg m^2

I_2 = the moment of the inertia of the spacecraft without the momentum wheels about y-axis

=85.07 kg m^2

I_3 = the moment of the inertia of the spacecraft without the momentum wheels about z-axis

=113.565 kg m^2

J_a = axial moment of inertia of the wheel =0.5 kg m^2

The momentum wheels are assumed to be mounted in the direction of the principal axes and the thrusters are assumed to exert external torques about the principal axes. Figure 5.3 illustrates the simulation results for the momentum dumping of the spacecraft using the available thruster torques about two axes.

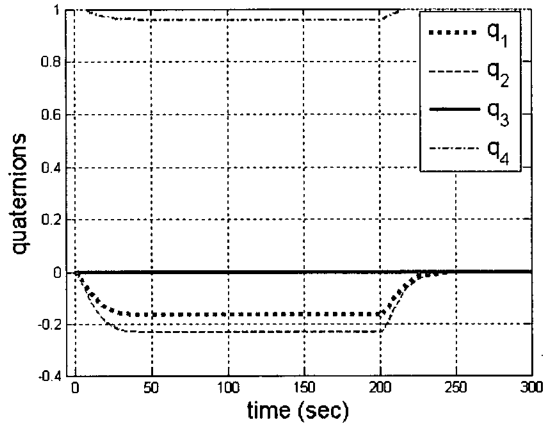

Figure 5.3.a: The attitude variations

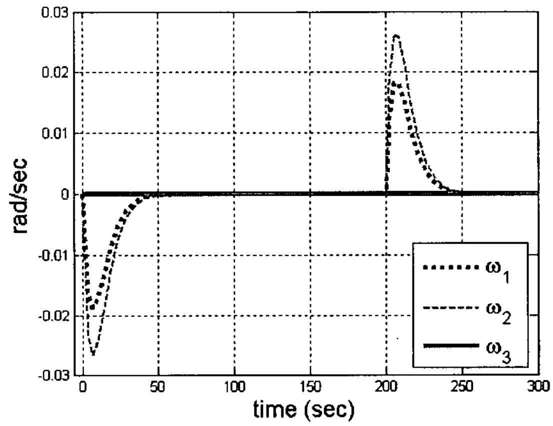

Figure 5.3.b: The angular velocity variations

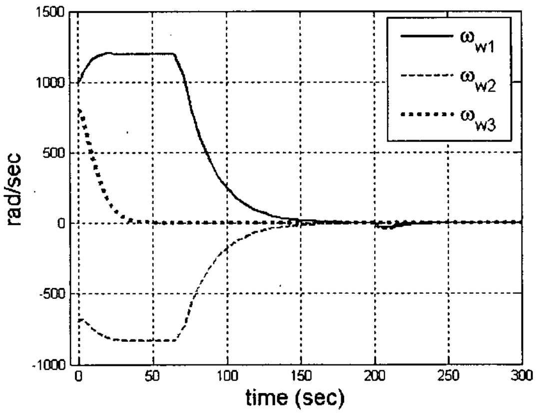

Figure 5.3.c: The spin rate of the M/Ws variations

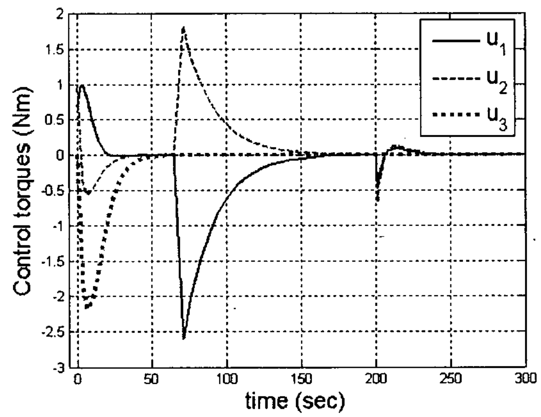

Figure 5.3.d: The torques of the M/W motors

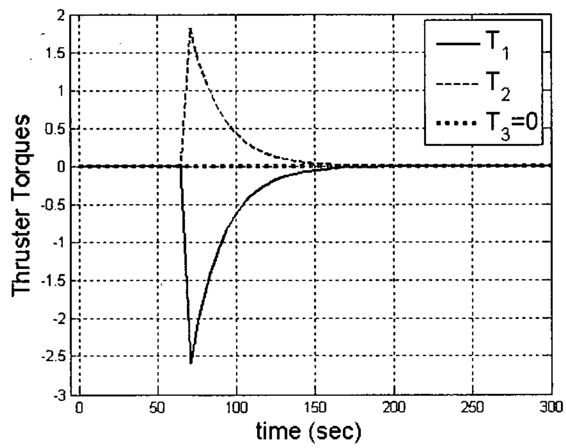

Figure 5.3.e: The thruster control torques

The initial condition is considered as the spacecraft being coincident with the inertial coordinate system, while the momentum wheels rotating with high spin rates. The maneuver is composed of three phases. In the first phase starting at time zero and lasting for about 70 seconds the spacecraft is brought to the intermediate attitude using the momentum wheels. In this orientation the angular momentum vector lies in the xy-plane of the spacecraft coordinate system and therefore the spin rate of the momentum wheel corresponding to the z-axis vanishes. The second step is angular momentum removal which starts from about the 70th second and last till the 250th second. The external torques proportional to the angular momentum of the spacecraft are being exerted. The attitude of the spacecraft should remain unchanged and since the disturbance torques are negligible compared to the thruster torques during the momentum removal phase, the momentum wheel torques are equal to the external thruster torques. The last phase of the maneuver is bringing the spacecraft coordinate system back to the inertial coordinate system using the momentum wheels. Note that the momentum wheel control torques are significantly smaller than those in the first phase. This difference corresponds to the gyroscopic torques that are compensated in the feedback linearized attitude control system. Since the spin rate of the momentum wheels were high in the first phase after being cross product with the spacecraft angular velocity vector it will result in huge gyroscopic toques.

The effect of the sizing of the momentum wheels has been illustrated by simulating the response of a spacecraft with five times the mass of the spacecraft in previous simulation and the same momentum wheels. The principal moments of inertia of the spacecraft will be five times more than that in the previous simulation. The results are illustrated in the Figure 5.4.

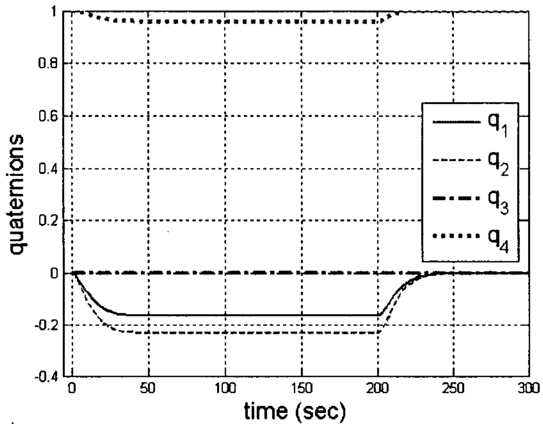

Figure 5.4.a: The attitude variations

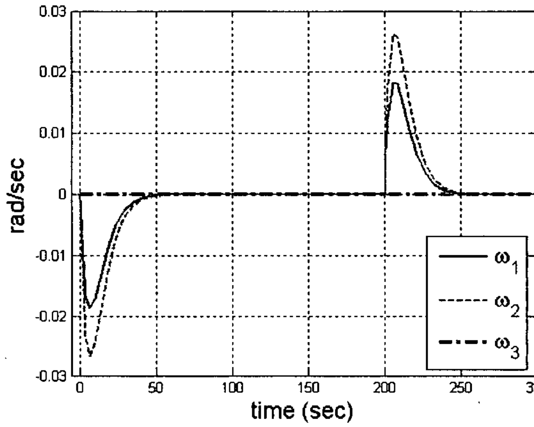

Figure 5.4.b: The angular velocity variations

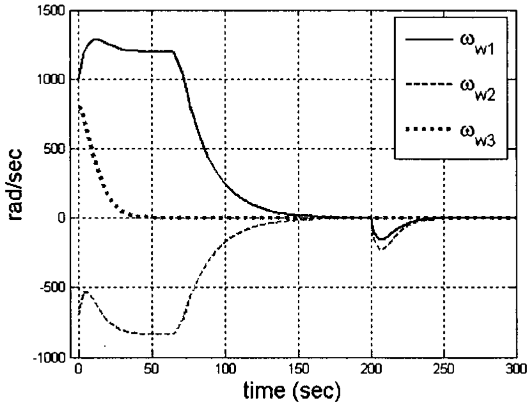

Figure 5.4.c: The spin rate of the M/Ws variations

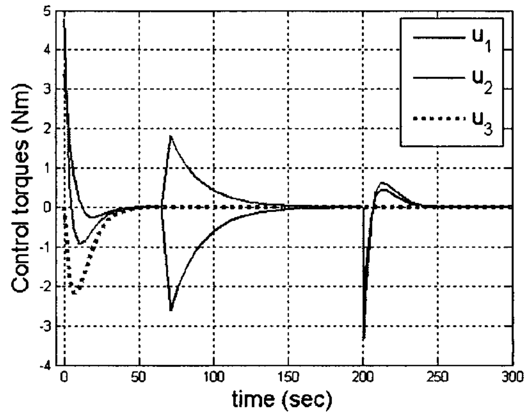

Figure 5.4.d: The torques of the M/W motors

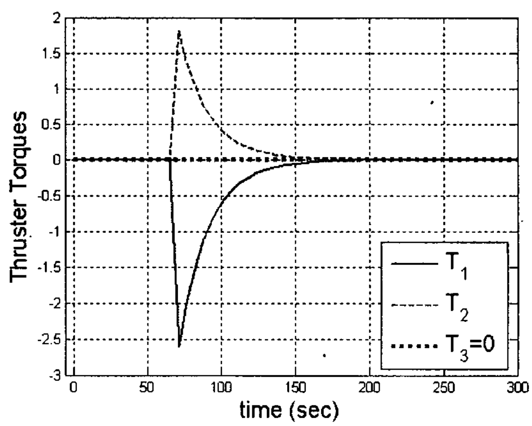

Figure 5.4.e: The thruster control torques

From equation (175) it can be seen that the dynamic equations of the controlled system is independent of the moment of inertia of either the spacecraft or the momentum wheels. This implies that the attitude and the angular velocity response of the spacecraft should not alter from the previous simulation. The moment of inertia and the initial spinning speed of the momentum wheels is the same as those in the previous simulation. The angular momentum stored in the momentum wheels is therefore the same. The thruster torques during the momentum dumping are determined based on the angular momentum stored in the momentum wheels. Therefore, the thrusters' torques should not vary from those in the previous simulation either. Figures 5.4.a, 5.4.b and 5.4.e are identical to Figures 5.3.a, 5.3.b and 5.3.e as predicted. The momentum wheels control torques have increased in the current situation which is necessary to give the larger spacecraft the same angular accelerations. It can be observed in Figure 5.4.d that the magnitude of the control torques corresponding to the second attitude alteration at time 200 second, is closer to that magnitude for the initial attitude change. In the initial attitude change the momentum wheels are spinning rapidly and this spin induces some gyroscopic toques during the direction variations. The closeness of the magnitude of the control torques in the initial and final attitude maneuvers implies that the control torques needed for rotating the body of the spacecraft are larger than those caused by the gyroscopic effects.

The angular momentum of the spacecraft can also be removed by having the external torque actuation about even one principal axis. Figure 5.5 illustrates the maneuver for this momentum dumping maneuver.

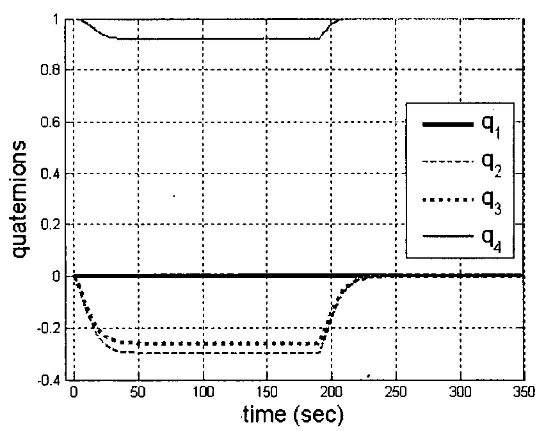

Figure 5.5.a: The attitude variations

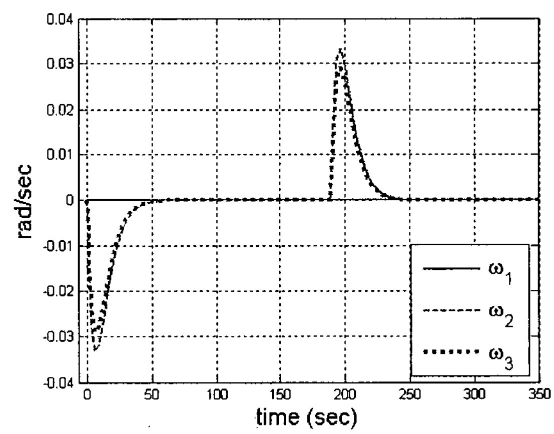

Figure 5.5.b: The angular velocity variations

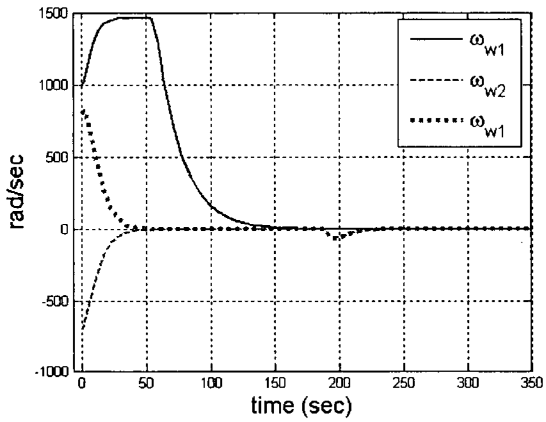

Figure 5.5.c The spin rate of M/Ws variations

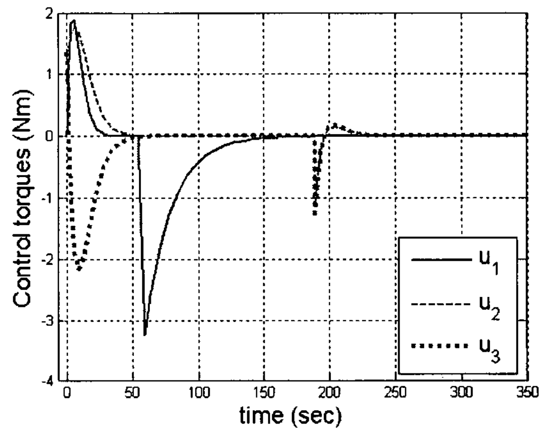

Figure 5.5.d The torques of The M/W motors

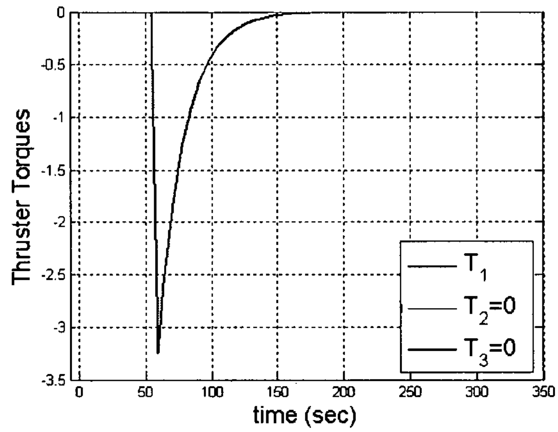

Figure 5.5.e Thruster control torques

The only external torque actuated axis of the spacecraft is considered the x-axis. The spacecraft is initially assumed coincident with the inertial coordinate system while the momentum wheels are spinning at high rates. The momentum dumping procedure consists of three steps. In the first step, lasting about 70 seconds, the spacecraft is rotated so that the spacecraft x-axis coincides with the direction of the total angular momentum vector remaining unchanged by time. The second phase is the spacecraft net angular momentum removal, which is performed by exerting torque about the x-axis which is kept in the direction of the angular momentum vector. This phase lasts for about 180 seconds. Finally, after the angular momentum of the spacecraft has been removed it is brought back to the inertial coordinate system.

The successive performance of the proposed procedure and the low-level control laws have been illustrated in the numerical simulations. The missions were accomplished in about six minutes. The duration of the procedure can be reduced by increasing the gains at the cost of increasing the control torques.

Chapter 6

Conclusions and future directive

6.1 Conclusions

This research was performed to develop the attitude control methods that can tolerate the spacecraft actuator failures. Two main roles are considered for the thrusters on board of the spacecraft; the attitude maneuvers and the momentum dumping. Fault tolerant control laws have been developed for accomplishing each of these tasks.

In performing the attitude maneuvers using thrusters, the effect of the disturbance torque was considered for the first time. At first, the desired equilibrium motion was defined to be ultimately tracked by the controller. A kinematic control law was derived to find the prescribed angular velocities that guide the spacecraft to the desired attitude. The Lyapunov control method was used to derive the two solutions for the kinematic controller. As predicted in the literature none of the solutions alone could stabilize the desired attitude. A switching criterion was then developed to employ the proper solution for the kinematic controller at each instance of the corrective maneuver. The backstepping method was finally used to derive the thruster torques to track the prescribed angular velocities determined by the kinematic controller.

Through this research the following contributions were made in controlling an underactuated spacecraft subjected to a constant torque about the unactuated axis:

- The equilibrium motion best matching the mission objectives was determined.
- The backstepping control method was extended for controlling the multi-input multi-output systems with a nonlinear effort-state relation.
- A nonlinear control method was developed to stabilize the decided equilibrium motion using the devised backstepping method.
- A linear control method was developed to locally stabilize the equilibrium motion.
- The numerical simulations were performed demonstrating the satisfactory performance of the control laws.

Fault tolerant control methods were developed for the first time for dumping the angular momentum of the spacecraft using less than three external torque actuators. The angular momentum accumulated in the momentum wheels, corresponding to the unactuated axes, was ceased by an attitude maneuver. During this maneuver, the angular momentum was transferred to the momentum wheels corresponding to the axes with active external torque actuation. The thruster torques were then implemented to remove the angular momentum of the spacecraft. The spacecraft was finally rotated back to the original orientation to continue its mission. The following contributions were made in momentum dumping of spacecraft.

- An attitude control plan was developed leading to the momentum dumping of the spacecraft implementing two or one external control torques.
- The common control laws for the rotational maneuvers were considered and the global stability of two of the methods was proved for the first time using the Lyapunov stability, invariant set and the Lyapunov local stability theorems.

- Implementing the most appropriate attitude maneuver methods in the attitude control plan, a globally stable control method was developed to remove the angular momentum of a spacecraft in case of having less than three external torque actuators.
- Numerical simulations were performed illustrating the robust performance and the effectiveness of the proposed control method.

6.2 Future scope

- The nonlinear control method proposed encounters some essential switchings in the kinematic control law. These switchings result in some significant control torques and may result in the saturation of the thrusters. Some scheduling methods can be used to implement the nonlinear control method while the trajectory is far from the set point that switches the control law to the smooth linear controller when being close to the desired values. This control scheme can make use of the robustness of the nonlinear control method while avoiding the subsequent switchings when it is not essential.
- Other nonlinear control methods such as the state feedback linearization can be used to develop a control law. The only problem is these methods are not applicable if the disturbance torque vanishes. In that case, the desired angular velocities will be zero and the system is not feedback linearizable as shown in chapter 5.
- The control laws proposed assumes having the exact values of the moment of inertia matrix of the spacecraft. The adaptive control methods can be incorporated for handling the inexactness of the inertia tensor.
- In the momentum dumping control method the external control torques applied by the thrusters were used for computing the torque of the momentum wheels. The thruster

torques can be computed using an estimator to make the system insensitive to the inaccuracies of the thruster torques.

- In the numerical simulations the gain factors of the controllers were set to roughly compromise between the duration of the mission and the control efforts. The value of the controller gains relative to each other determines the trend of the approach and is a factor to be decided as well. These gains can more precisely be determined through using the optimal control methods. The objectives of the spacecraft mission must be precisely known so that the objective function can be determined. The control gains should then be determined to maximize this objective function throughout the maneuver.

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Appendix 1

Spacecraft eigenaxis rotations using the control law proposed in [54]

The control law proposed in (146) and (147) was primary developed to perform eigenaxis rotation for attitude changing maneuvers. It was later proved that this method is globally stable and can be used to stabilize a desired orientation even if the initial condition is such that the eigenaxis rotation cannot be undertaken. In this appendix it is proven that that the transition from one orientation to the desired attitude is performed trough an eigenaxis rotation, provided the initial angular velocity is zero.

Each orientation can be represented by a Direction Cosine Matrix. The eigenvector associated with the unit eigenvalue of this matrix is called the eigenaxis corresponding to that orientation. The frame attached to the current orientation can then be performed through a single rotation about the eigenaxis. The eigenaxis of an orientation can simply be derived from the quaternion corresponding to that attitude. Recalling equation (131) we have

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \sin(\alpha / 2) \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \tag{171}$$

in which $[e_1, e_2, e_3]^{\mathrm{T}}$ is the eigenaxis of the spacecraft. Therefore, the eigenaxis, e, is the unit vector in the direction of the first three components of the quaternion vector.

Kinematic equations for quaternion representation, (132), can be written in the following form [54]

$$[\dot{q}] = \frac{1}{2}\omega \times [q] + \frac{1}{2}q_4\omega \tag{172}$$

$$\dot{q}_4 = -\frac{1}{2}\omega^{\mathrm{T}}[q] \tag{173}$$

in which

$$[q] = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{174}$$

Substituting the control law, equations (146) and (147), into the dynamic equations, equation (128), we get

$$\dot{\omega} = -d\omega - k[q] \tag{175}$$

Equations (172), (173) and (175) form the differential equation of the controlled system.

If the angular velocity has the same direction as the $[q]$ vector since $\dot{\omega}$ is a linear combination of ω and $[q]$, both in the direction of eigenaxis, it will be in the direction of eigenaxis as well. This implies that the angular velocity vector does not deviate from the eigenaxis direction once aligned with that.

It should also be checked weather the direction of the eigenaxis itself changes while the angular velocity is along e. If e and consecutively $[q]$ are collinear with ω, $\omega \times [q] = 0$. Then, from equation (172) we have

$$[\dot{q}] = \frac{1}{2} q_4 \omega \tag{176}$$

Since $[\dot{q}]$ is equal to angular velocity vector scaled by a factor of $\frac{q_4}{2}$, it will be collinear with ω and consecutively along $[q]$. This implies the direction of the eigenaxis does not change provided the angular velocity vector is along the eigenaxis.

Yet, we have proven that once the angular velocity vector is aligned with the eigenaxis neither the direction of eigenaxis nor the direction of angular velocity changes afterwards. In the rest-to-rest rotational maneuver the angular velocity of the spacecraft is zero at the beginning. It can be considered as $\omega(t = 0) = 0 = 0 \times e$. Therefore, the initial angular velocity of the spacecraft is along the eigenaxis. This implies that the direction of angular velocity remains along the eigenaxis and the direction of the eigenaxis remains constant throughout the maneuver. In other words, the direction of the spacecraft is altered by a rotation about the fixed eigenaxis of the initial orientation.

Appendix 2

Simulink® simplified schemes of the controllers

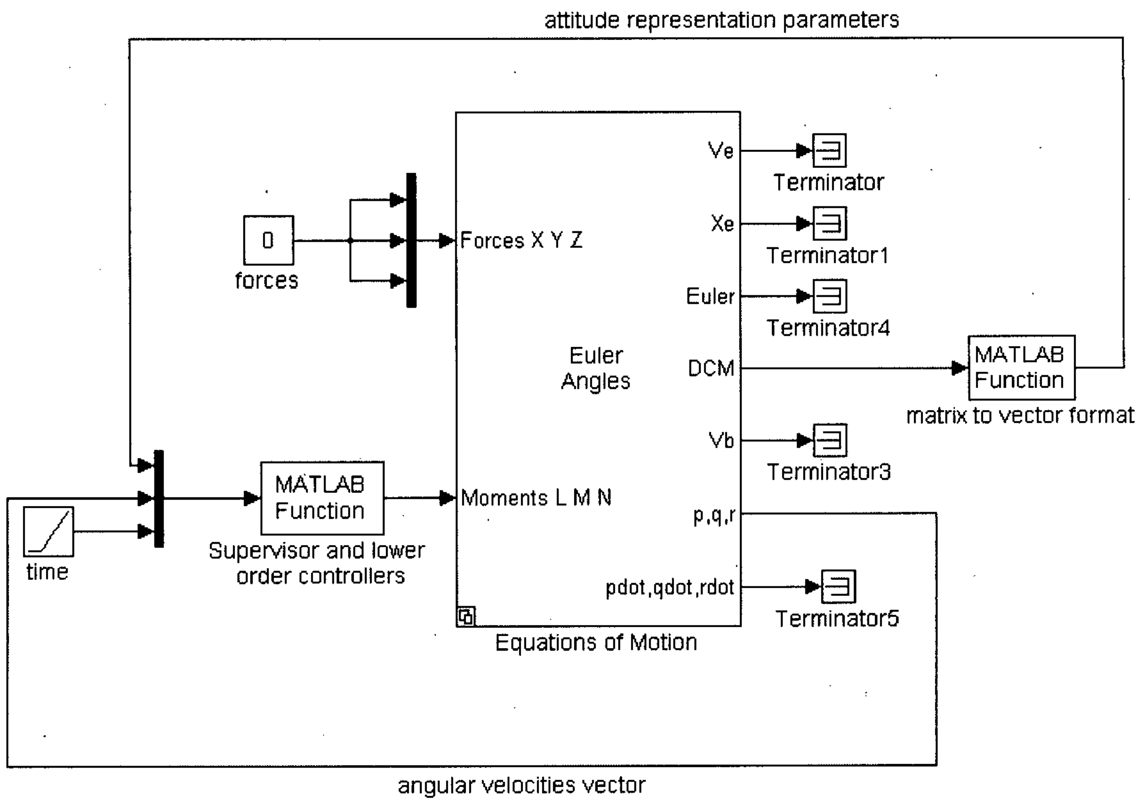

Figure A.2.1 Simplified underactuated spacecraft Simulink® attitude control scheme

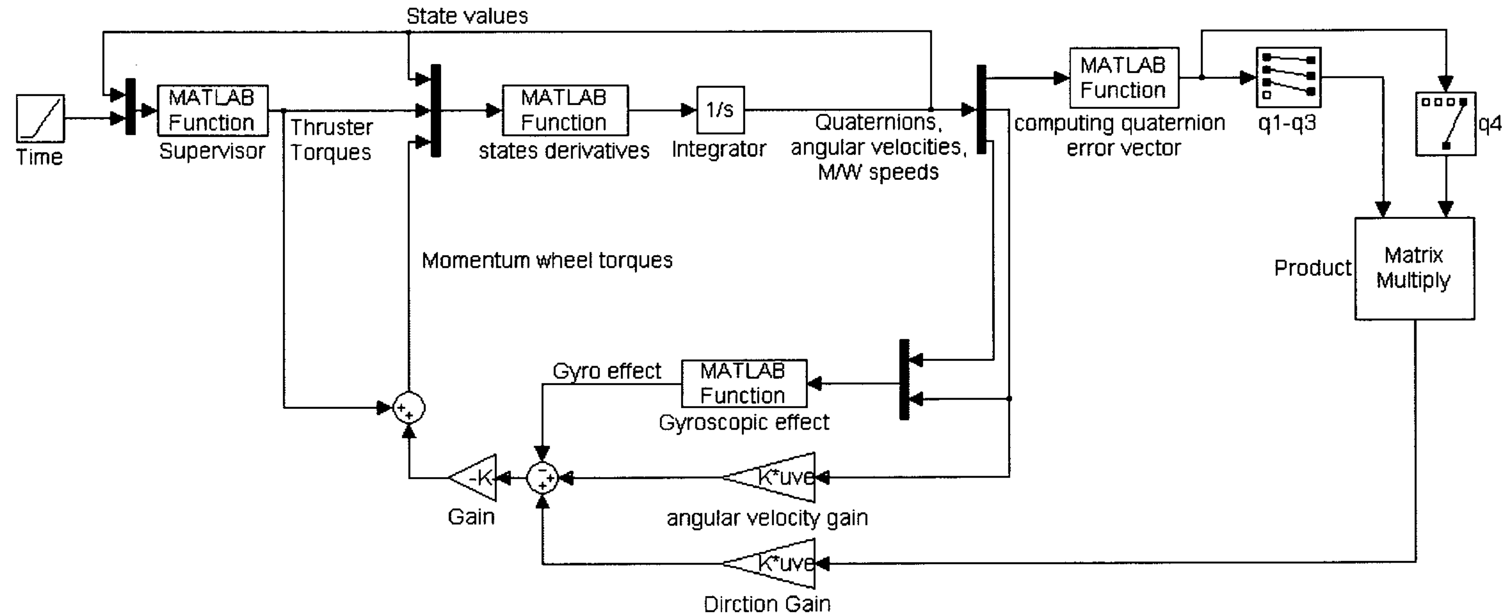

Figure A.2.2 Simplified spacecraft Simulink momentum dumping scheme