REDUCING ADDED RESISTANCE USING AN ANTI-PITCH FOIL

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Abstract

In this study, the use of a passive bow anti-pitch foil for the purpose of reducing the vertical motions and added resistance in waves of a small marine vehicle is investigated. The difficulties, limitations and assumptions inherent in studying ships in waves are discussed.

A mathematical model is developed incorporating linear strip theory, Joosen added resistance theory and oscillating hydrofoil theory in an attempt to predict the effect of a hypothetical anti-pitch foil on the heave and pitch motions and added resistance in head seas of the 12-metre yacht Canada II.

The results of regular head wave towing tank experiments on a 1:8 scale model of Canada II are presented for both the bare hull and for the hull fitted with a model anti-pitch foil to demonstrate the experimental effect of the foil on vertical motions and added resistance.

The results of the theoretical and experimental investigations show that the anti-pitch foil is in fact very effective in reducing the motions and added resistance of the Canada II hull. Experimental results show heave amplitude is reduced by as much as 15%, pitch amplitude by 22% and added resistance by as much as 40%.

The heave and pitch motion amplitude responses predicted by the linear strip theory for the hull with no foil, do not agree well with experiment, especially in the region around resonance. It is concluded that the linear strip theory used is not completely adequate in predicting the hydrodynamic coefficients and consequently the heave and pitch motions of Canada II due to non-linearities in her hull form. These non-linearities, in particular the flared topsides and large fore and aft overhangs, violate the linear assumptions of strip theory and result in increased hydrodynamic damping and speed dependent restoring
coefficients that are not accounted for by theory. The Joosen added resistance theory predictions for the bare hull do agree well with experiment despite the hull non-linearities.

The change or percentage reduction in heave and pitch amplitude response due to the addition of the anti-pitch foil is predicted well by the oscillating hydrofoil theory, in spite of the inaccuracies of the strip theory in predicting the actual motions. Also, the Joosen theory predictions of the percentage reduction in added resistance due to the addition of the foil agree very well with experiment.

It is concluded that the mathematical model provides a very good conservative estimate of the amount of reduction in heave and pitch motion and added resistance that can be expected from the addition of a passive bow anti-pitch foil. The model could be very useful as a preliminary design tool for naval architects, where no such tool existed before.

The findings of this study indicate that the concept of using a passive bow anti-pitch foil to reduce the vertical motions and added resistance in waves is well worth pursuing.
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Nomenclature

\[ A = \text{Aspect ratio of foil} \]
\[ A_{ij} = \text{Added mass coefficients} \]
\[ B = \text{Beam of ship} \]
\[ B_{ij} = \text{Damping coefficients} \]
\[ C_w = \text{Wave celerity} = g/\omega \]
\[ C_{ij} = \text{Restoring coefficients} \]
\[ C(\kappa) = \text{Theodorsen circulation function} \]
\[ F_3 = \text{Heave exciting force} \]
\[ F_5 = \text{Pitch exciting moment} \]
\[ J(\nu) = \text{Bessel function} \]
\[ K = \text{Lateral coefficient of accession to inertia} \]
\[ I_5 = \text{Moment of inertia for pitch} \]
\[ L = \text{Length of ship} \]
\[ M_{jk} = \text{Mass coefficients of ship} \]
\[ S = \text{Planform area of foil} = bc \]
\[ V = \text{Forward velocity of ship} \]
\[ V_{za} = \text{Vertical velocity amplitude} \]
\[ a_{ij} = \text{Two-dimensional sectional added mass coefficient} \]
\[ b = \text{Foil span} \]
\[ b(\xi) = \text{Sectional beam} \]
\[ b_{ij} = \text{Two dimensional sectional damping coefficient} \]
\( c \) = Chord of foil  
\( d(\xi) \) = Sectional draft of ship  
\( f(\xi) \) = Sectional exciting force  
\( g \) = Gravitational acceleration  
\( h \) = Depth of foil below waterline  
\( k \) = Wave number  
\( l \) = Distance of foil from LCG of ship  
\( m(\xi) \) = Sectional exciting moment  
\( p_v \) = 2-D pressure in phase with velocity  
\( p_a \) = 2-D pressure in phase with acceleration  
\( z \) = Heave displacement (positive up)  
\( z_a \) = Heave amplitude  
\( z_{p-p} \) = Experimental peak-to-peak heave  
\( \alpha_s \) = Steady angle of attack of foil (usually zero)  
\( \epsilon_z \) = Heave phase angle  
\( \epsilon_\theta \) = Pitch phase angle  
\( \zeta_a \) = Wave amplitude  
\( \zeta_{p-p} \) = Experimental wave height, peak-to-peak  
\( \eta_b \) = Angular and translatory displacements  
\( \theta \) = Pitch angle (positive bow down)  
\( \theta_a \) = Pitch amplitude  
\( \theta_{p-p} \) = Experimental peak-to-peak pitch
\( \kappa \) = Reduced frequency = \( \omega_e c / 2V \)

\( \lambda \) = Wavelength

\( \mu_e \) = Non-dimensional encounter frequency

\( \nu \) = \( \pi c / \lambda \)

\( \xi_c \) = Position of longitudinal centre of gravity

\( \rho \) = Mass density of water

\( \sigma_{aw} \) = Non-dimensional added resistance coefficient

\( \phi \) = \( 2\pi l / \lambda \)

\( \omega \) = Wave frequency

\( \omega_e \) = Frequency of encounter

\( \nabla \) = Volume of displacement
Acknowledgement

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Chapter 1

Introduction

1.1 Motivation

Frustration with the performance in waves of Canada II during the 1987 America’s Cup Elimination Trials in Fremantle Western Australia was the primary motivation behind this research. As a member of the sailing crew during the races, the author witnessed first hand the importance of seakeeping on the performance of a sailing yacht in a seaway. While Canada II was a solid performer in calm water conditions, her performance relative to the other yachts dropped drastically during rough wave conditions. The Canada II program was not alone in its lack of understanding of the problem. In fact, though millions of dollars were spent on hull research and development by the 13 challengers and 5 defenders vying for the Cup, very little of this was devoted to the problem of predicting added resistance in waves and subsequently performance in waves.

The majority of research dollars were spent optimizing the calm water performance of the hull through computer simulations and towing tank tests. In fact, this has been the general practice in 12-metre design for many years. Yacht designers have relied on calm water performance predictions when optimizing their designs and have been forced to assume that a design that is predicted to perform better than another in calm water will also perform better in waves [37].

It should be noted that the prediction of added resistance and performance of a yacht in waves is an extremely complex and difficult problem which helps explain the lack of
Chapter 1. Introduction

Figure 1.1: Canada II in Waves
useful research in this area. It should also be noted that until 1987, the America's Cup was held in Newport Rhode Island where wind conditions were predominantly light and performance in waves was not considered to be a crucial factor. On the other hand, in Fremantle during the 1987 Cup trials, the winds were mostly strong (up to 30 knots) and due to the shallow water and short fetch the waves conditions were quite extreme, making performance in waves a critical factor. As a result, the interest in performance prediction in waves is relatively recent.

It was the author's goal to shed more light on the problem of added resistance in waves and investigate possible solutions that might be useful for future Canadian America's Cup challenges. While the research is directed toward the problem of added resistance of 12-metre yachts, the findings should also be applicable to other types of small marine craft.

1.2 Needs Analysis

Before undertaking this research, it was first necessary to establish whether a bona-fide need exists to find ways of solving the problem of reducing added resistance in waves.

In order to determine how important a factor 12-metre designers felt that added resistance was and to ascertain how they incorporated it into their 1987 12-metre designs, the author distributed a questionnaire to most of the yacht designers involved in the 1987 America's Cup. A sample of the questionnaire appears in Appendix A. Response to the questionnaire was good including representatives from the Stars and Stripes, Heart of America, Eagle, New Zealand, Canada II, and U.S.A. syndicates.

A summary of the answers to the questionnaire is shown in Table 1.1. The results show that all designers felt that reducing added resistance due to waves would lead to a significant increase in upwind performance and was worth pursuing. In the highly irregular sea conditions encountered in Fremantle, designers estimated that the added resistance
Table 1.1: Summary of Designer Questionnaire

<table>
<thead>
<tr>
<th>Stars &amp; Stripes</th>
<th>USA</th>
<th>New Zealand</th>
<th>Eagle</th>
<th>Heart of America</th>
<th>Canada II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Is added resistance significant?</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>To what % of total resistance?</td>
<td>30-40</td>
<td>15-30</td>
<td>5-20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Did you have a design program for added resistance?</td>
<td>Yes - poor results</td>
<td>No</td>
<td>No - used experience</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Did you perform tank wave tests?</td>
<td>Some</td>
<td>Some</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Did you use tank test results?</td>
<td>No - unreliable</td>
<td>No</td>
<td>-</td>
<td>No - unreliable</td>
<td>-</td>
</tr>
<tr>
<td>Were computer ship motion studies part of your program?</td>
<td>Yes - in-conclusive</td>
<td>No</td>
<td>No</td>
<td>Yes - underestimated</td>
<td>No</td>
</tr>
<tr>
<td>Did you use any pitch reducing techniques in Fremantle?</td>
<td>Yes (not specified)</td>
<td>No</td>
<td>modified trim</td>
<td>No</td>
<td>Steering technique</td>
</tr>
</tbody>
</table>
accounted for up to 50% of the total upwind resistance, making it an extremely crucial factor.

In contrast, when asked about the importance placed on reducing added resistance in the actual design programs, few designers had committed any time to research and development in this area. Some had gone as far as to conduct towing tank wave tests and computer motion and added resistance computer studies, however not one had actually used these results in their final design. All the designers indicated that in the end they had to rely on their previous experience and intuition when addressing the seakeeping aspects of their designs. It is interesting to note that only Stars and Stripes (the eventual Cup winner) stated that they had used a pitch reducing technique to improve seakeeping between the start of the Trials and the final Cup races (though they would not mention the method!).

It is very clear from this polling of state of the art yacht designers that a definite need does indeed exist to gain a clearer understanding of added resistance of yachts in waves and to find ways of reducing it to improve performance in waves.

While researching added resistance in waves, it became apparent to the author that added resistance was a problem for many types of marine vehicles other than racing yachts including cargo vessels, naval ships, passenger ferries, fishing vessels, drill ships, etc. In fact the bulk of the research in the area of added resistance and seakeeping has been carried out on cargo ships and naval vessels (not yachts).

Some of the problems that arise as a result of inadequate seakeeping of these other forms of marine craft are:

- Speed loss due to inadequate horsepower to overcome added resistance.
- Voluntary speed reduction due to excessive pitching accelerations.
- Increased fuel consumption to maintain speed in waves.
• Passenger discomfort.

• Inability for workers to carry out duties.

• Inability for naval vessels to carry out operations (e.g., landing aircraft on a carrier in rough seas).

In some types of craft, the problems are quite minor such as passenger discomfort on ferries, however for vessels such as cargo ships or drill ships these problems can have major economic and scheduling implications costing millions of dollars. In the case of naval vessels, the inability to carry out operations due to rough water could, in the worst case, result in loss of life.

Though, to some, the need to reduce added resistance and improve seakeeping of racing yachts in order to win races may seem somewhat trivial, one can see that throughout the marine industry there is a definite pressing need to address this problem as improved seakeeping and reduced added resistance can lead to reduced costs for cargo carriers, increased passenger comfort, increase in worker productivity, improved fishing operations, and most importantly in naval situations decrease the frequency of life threatening situations.

By now the reader may have guessed that there is a clear relationship between the motions of a ship in waves and its added resistance. This relationship will be discussed quantitatively in later chapters; but for now let us state that, in general, a decrease in the motions of a ship subject to a certain sea condition will lead to a decrease in added resistance.
1.3 Purpose of Research

The preceding discussion leads to the primary purpose of this investigation which was to show that the use of a bow anti-pitch foil can reduce the motions and added resistance of a marine vehicle in waves. As the author's interest is primarily with racing yachts, the hull form to be investigated will be that of the 12-metre Canada II; however the arguments presented should apply to most conventional small marine vehicles.

To achieve the objective of the study, a mathematical model was developed from existing linear ship motion theory, added resistance theory and oscillating hydrofoil theory in an attempt to predict the effect of an anti-pitching foil on Canada II. A model foil was designed and implemented on a 1/8 scale model of Canada II and model experiments were conducted at the B.C. Research Ocean Engineering Centre towing basin. The results of the experiments, with and without the foil, are compared to demonstrate that the foil does in fact reduce the motions and added resistance. The experimental results are also compared to the theoretical predictions in order to determine if the theory could be useful as a preliminary design tool for naval architects considering the use of an anti-pitch foil.

1.4 Scope of Investigation

A literature search was conducted on the causes and mechanisms of added resistance in waves. Various methods of motion and resistance reduction were identified and studied leading to the choice of a bow anti-pitch foil as the most promising method for detailed study.

Historical theoretical and experimental research into the phenomena of ship motions and added resistance was then studied in order to more precisely define the scope this research should take.
1.4.1 Experimental and Theoretical Constraints

The study of motions and added resistance becomes extremely complex when considering sailing vessel dynamics due to the coupled interaction between the aerodynamic forces on the sails and the hydrodynamic forces on the hull and appendages [37, 32]. Thus, in order to carry out an experimental and theoretical investigation that would yield meaningful results, it became necessary to simplify the problem drastically in order to isolate the most important aspects of the problem.

It is not practical to study all of the conditions the vessel might encounter on the ocean therefore this study will look only at the worst case scenario under somewhat realistic conditions.

This investigation is therefore limited to studying the motions and added resistance of the upright yacht when encountering head seas. The analysis was carried out for only one speed and one waveheight corresponding to realistic values encountered in Fremantle. Only the response of the hull and appendages are studied; the effect of the sails and rigging being neglected. The yacht is restricted to the vertical modes of motion, pitching and heaving, only. Motion and added resistance responses were measured and predicted for regular sine wave conditions only. Finally, practical considerations regarding full scale implementation of a bow anti-pitch foil such as induced yaw moment, fabrication, installation, vibration etc. are not addressed.

The validity of these simplifications, limitations and assumptions are addressed in subsequent chapters.
Chapter 2

Historical Background

2.1 Phenomenon of Added Resistance

Added resistance is the extra resistance, over and above the calm water resistance, that a ship encounters when under the influence of wind and waves. Thus a ship operating at a certain speed in wind and waves will usually require more power than the same ship operating at the same speed in perfectly calm water.

2.1.1 Causes of Added Resistance

The added resistance experienced by a ship is caused by the following factors [35]:

1. Wave reflection or diffraction of the incident wave system on the ship

2. The motion of the ship (rigid body motions). The vertical motions, heaving and pitching, have the largest effect.

3. Sideslip or drift angle of the ship due to the wind or wave system sideforce exerted on the ship.

4. Windage on the superstructure and hull and auxiliary propulsion device (such as sails).

5. Motion of the control surfaces. Rudder and other movable appendages will influence the motion of the ship.
6. Change in propeller efficiency because the propeller is operating in a fluctuating flow due to the motion of the ship. On a sailing yacht the efficiency of the sail driving force may be affected for the same reason.

As one can see, the added resistance experienced by a ship can be the result of many interacting components. This investigation deals only with components (1) and (2) with the motions being restricted to heave and pitch in head seas. Although the other components may be important they are difficult to study and separate. Also, previous researchers have identified items (1) and (2) as the largest contributing factors in the head sea case and the head sea case has been shown to produce the maximum added resistance [2, 35].

2.1.2 Mechanisms of Added Resistance in Head Seas

Several researchers have examined the mechanisms that lead to the added resistance of a ship when it is heaving and pitching in head seas. Vossers [38] and Hanaoka et al [15] suggested that the following three mechanisms are responsible:

1. Interference between incident waves and waves generated by the ship when heaving and pitching. Sometimes called the drift force.

2. Damping force associated with the forced heaving and pitching in calm water; for example under the exitation of some external mechanism such as an oscillator.

3. A component due to the wave system experiencing some wave reflection from the ship’s hull. This is sometimes called the diffraction force.

All three of these mechanisms involve energy transfer from the ship to the water. This energy is transmitted to the water and remains in the waves generated by the ship (except for a negligible amount consumed by viscous friction). In principle, these three wave
components can be superimposed. However, the forces developed by these mechanisms will be proportional to the waveheight squared and are therefore nonlinear. Therefore, in reality, these components are nonlinear and interrelated and cannot be separated.

Of the three mechanisms, the drift force has been shown to have the largest contribution to added resistance followed by the damping force and finally the least contribution is from the diffraction force [35, 39].

2.2 Theoretical Techniques for the Prediction of Added Resistance

Many formulations have been put forward over the years for predicting added resistance in waves. These follow one of three methods:

- Hull Pressure Methods
- Momentum and Energy Methods
- Radiated Energy Methods

In this section, three formulations are considered starting with Havelock [16], followed by Joosen [17] and culminating with the work of Gerritsma and Beukelman [10].

Before developing these theories it is necessary to first fix the coordinate system of a ship in waves for consistancy. Figure 2.2 identifies the variables and sign convention associated with a ship heaving and pitching in regular waves. $z$ is heave displacement, positive up, and $\theta$ is pitch angular displacement, positive bow down.

2.2.1 Havelock Theory

One of the first attempts to calculate added resistance of a ship in waves was carried out by Havelock [16] in 1932. He obtained a simple and easy to use formula for the mean added resistance due to a heaving and pitching ship in head seas at any given frequency
Figure 2.2: Co-ordinate System
of encounter. Havelock's method involved determining the mean value of the longitudinal component of the pressure forces integrated over the wetted part of the oscillating ship's hull. The water pressure was assumed to be the undisturbed pressure of the incident wave, implying the use of the Froude-Krylof hypothesis. The Havelock expression is:

\[ R_{aw} = -\frac{k}{2} (F_a z_a \sin \varepsilon_F + M_a \theta_a \sin \varepsilon_M) \]  

(2.1)

where \( k \) is the wave number, \( F_a \) and \( M_a \) are amplitudes of excitation force and moment in heave and pitch and \( z_a \) and \( \theta_a \) are the corresponding heave and pitch amplitudes. \( \varepsilon_F \) and \( \varepsilon_M \) are the phase angles between the exciting function and the response for heave and pitch respectively.

Through use of the Froude-Krylof hypothesis, Havelock avoided the difficult problem of evaluating the complicated diffracted waves. Thus, this solution was considered a first approximation only.

Even though Havelock neglects diffraction effects and pitch-heave cross coupling, the theory still provides some valuable insights into added resistance. Firstly, we see that both the motion amplitudes and their phases are important factors in calculating added resistance. Also, it is possible to note that the maximum added resistance is likely to occur in the region of pitch and heave resonance. Because of this direct pitch and heave motion relationship to added resistance found in the Havelock equation, it can now be stated categorically that hulls with poor motion characteristics will suffer from higher added resistance or conversely an improvement in motion characteristics will lead to less added resistance. This validates the general statement to this effect made in the introduction.

2.2.2 Joosen Theory

The foundation of Joosen's theory [17] developed in 1966 was to consider a control volume around the ship and then derive an energy or momentum balance. This approach was
first suggested by Maruo [23] in his analysis of the drifting force of a body in waves.

The basis of the solution is to subdivide the velocity potential into the incoming wavefield, the diffraction of the wave system by the body and that due to the oscillating body in a regular wave system. The resultant boundary value problems were solved by Maruo. Joosen then expanded Maruo's expression into an asymptotic series with respect to the slenderness ratio $L/B$ and kept only first order terms.

Joosen's simple expression for added resistance is given by:

$$R_{aw} = \frac{\omega^3}{2g} (N_2 z_a^2 + N_\theta \theta_a^2)$$

(2.2)

where $N_2$ and $N_\theta$ are the damping coefficients of heave and pitch. This slender body approximation is valid for short waves only and produced a speed independent added resistance. The theory was extended by Joosen to account for forward speed by substituting the wave encounter frequency $\omega_e$ for the wave frequency $\omega$ in equation 2.2 (though not entirely mathematically consistent).

This equation has been shown to be equivalent to Havelock's equation 2.1 and suffers again from neglecting the pitch-heave cross coupling and the diffraction effect [35]. However, a fundamental concept is inherent in its formulation as well. Equation 2.2 shows that the added resistance can be regarded as a result of the radiated damping waves. That is, the energy loss associated with the damping characteristics of the ship is related to the work that must be done to keep the ship motions in a constant phase relationship with the forcing functions. This energy is supplied by the ship and is dissipated in the damping waves radiating from the hull. Therefore, the added resistance of a ship in waves can be found by computing the total energy content of these radiated damping waves.
2.2.3 Gerritsma and Beukelman Theory

The concept of the direct relationship of the added resistance to the energy contained in the damping waves was developed further by Gerritsma and Beukelman [10]. Their approach was to determine the energy radiated from the hull during one wave encounter period and then equate this to the added work being done by the ship during the same period.

The energy radiated during one wave encounter period $T_e$ is given by:

$$P = \int_0^{T_e} \int_0^L b(x) V_z^2(x, t) \, dx \, dt$$  \hspace{1cm} (2.3)

where $b(x)$ is the velocity dependent damping coefficient of the ship at any longitudinal position.

$$b(x) = N(x) - V (dm(x)/dx)$$  \hspace{1cm} (2.4)

where $m(x)$ and $N(x)$ are the zero speed sectional added mass and damping coefficients respectively and $V$ is ship velocity.

$V_z$ in equation 2.3 is the vertical velocity of the ship section relative to the disturbed water surface elevation and is given by:

$$V_z(x, t) = z - x_b \hat{\theta} + V \theta - \zeta^*$$  \hspace{1cm} (2.5)

$\zeta^*$ is the effective vertical water displacement for a cross section. Here Gerritsma and Beukelman have made a dynamic correction to the Froude-Kryloff hypothesis in order to take into account the presence of the ship's hull. This correction takes the form of a modification of the incident wave amplitude by the hull surface and is given by:

$$\zeta^* = \zeta \left(1 - \frac{k}{y_w} \int_{-T}^{0} y_b e^{k z_b} \, dz_b\right)$$  \hspace{1cm} (2.6)

where $\zeta$ is the undisturbed wave height, $y_w$ is the half-width of the waterline and $y_b$ and $z_b$ are the offsets of the hull.
Chapter 2. **Historical Background**

If we assume that $V_z$ is a harmonic function of time of frequency $\omega_e$ it can be expressed as:

$$V_z = V_{za} \cos(\omega_e t + \epsilon) \quad (2.7)$$

and the time dependence of equation 2.3 can be integrated to yield:

$$P = \frac{\pi}{\omega_e} \int_0^L b(x)V_{za}^2(x) \, dx \quad (2.8)$$

The added work of the ship done by the towing force $R_{aw}$ can be given as [15]:

$$P = R_{aw}(V + c)T_e = R_{aw} \lambda \quad (2.9)$$

where $c$ is the wave celerity and $\lambda$ is the wavelength.

From equations 2.8 and 2.9 we obtain the final form of the Gerritsma and Beukelman equation:

$$R_{aw} = \frac{k}{2\omega_e} \int_0^L b(x)V_{za}^2(x) \, dx \quad (2.10)$$

This equation requires an accurate knowledge of the zero speed added mass and damping coefficient distribution as well as the pitching and heaving motions in order to calculate the added resistance. However the reader will note the elegant and simple derivation which does not require solving any hydrodynamic boundary conditions.

This approach attempts to correct for the Froude-Kryloff hypothesis and the pitch-heave cross coupling is inherent in its formulation since $V_z$ is a function of both $z$ and $\theta$. However other simplifying assumptions are still present. From equation 2.10, it is clear that the added resistance is assumed to vary as waveheight squared since $V_{za}$ is proportional to waveheight. Also, in head seas, transverse symmetry of all physical processes is assumed.

The Gerritsma and Beukelman theory overcomes some of the drawbacks of previous theories in that it attempts to account for the diffraction mechanism of added resistance in very short waves, which Havelock and Joosen neglect.
2.3 Ship Motion Theory

It is evident from all of the added resistance theories just discussed that an accurate knowledge of ship heaving and pitching motion amplitude and phases as well as added mass and damping coefficients is necessary in order to calculate added resistance. In this section, we will briefly summarize the classical linear hydrodynamic ship motion theory for the coupled heave-pitch equations of motion which allows us to predict these quantities.

2.3.1 Coupled Heave-Pitch Equations of Motion

The first ship motion theory suitable for numerical computations which had an adequate accuracy for engineering applications was the well known strip theory developed by Korvin-Kroukovsky and Jacobs in 1957 [20]. This theory was extended to predict motions in sway, yaw, and roll as well as pitch and heave by Salvesen et al in 1970 [26]. In doing so, improvements to the basic pitch-heave formulation of Korvin-Kroukovsky and Jacobs were realized.

Under the assumptions that the motion responses are linear and harmonic, the six linear coupled equations of motion can be written in the form:

\[ \sum_{k=1}^{6} [(M_{jk} + A_{jk})\ddot{\eta}_k + B_{jk}\dot{\eta}_k + C_{jk}\eta_k] = F_j e^{i\omega t} \quad j = 1, \ldots, 6 \quad (2.11) \]

where \( \eta_k \) are the translatory and angular displacements.

Under the further assumption of ship lateral symmetry, the six coupled equations of motion reduce to two sets of equations, one set of three coupled equations for pitch, heave and surge and another set for yaw, sway and roll. That is, for a ship with lateral symmetry surge, heave and pitch are not coupled to yaw, sway and roll.

If one assumes that the ship has a long slender hull form, it can be shown [26] that the hydrodynamic forces associated with the surge motion are very much smaller than the
forces associated with heave and pitch. Thus, under these assumptions, the three coupled equations of motion for pitch, heave and surge reduce to two coupled equations for heave and pitch.

The heave-pitch coupled equations can be written as:

\[
\begin{align*}
(M + A_{33}) \ddot{z} + B_{33} \dddot{z} + C_{33} z + A_{35} \ddot{\theta} + B_{35} \dot{\theta} + C_{35} \theta &= F_3 e^{i\omega_t} \\
A_{53} \ddot{z} + B_{53} \dddot{z} + C_{53} z + (I_5 + A_{55}) \ddot{\theta} + B_{55} \dot{\theta} + C_{55} \theta &= F_5 e^{i\omega_t}
\end{align*}
\]

where we have returned to the notation of Figure 2.2 for heave and pitch displacement and the subscript 3 denoting heave and 5 denoting pitch. $F_3$ and $F_5$ are the amplitudes of the exciting force and moment.

2.3.2 Determination of Hydrodynamic Coefficients

The relationships for the added mass and damping coefficients, $A_{jk}$ and $B_{jk}$, and the amplitude of the exciting force and moment, $F_3$ and $F_5$, involves the general hydrodynamic problem of solving for the total velocity potential of the fluid. The velocity potential must satisfy the Laplace equation subject to a free surface and hull boundary condition as well as a suitable radiation condition.

The method of Salvesen et al [26] is to first linearize the hull and free surface boundary conditions. With the boundary conditions linearized appropriately, it is possible to solve for the velocity potential. The next step is to obtain the hydrodynamic forces and moments acting on the hull using Bernoulli's equation. By solving Bernoulli's equation, an expression can be obtained for the hull added mass and damping coefficients as well as the exciting force and moment. It is possible to reduce this three dimensional potential flow problem to a sum of two dimensional problems, with the result that we need only solve for the two dimensional added mass and damping coefficients and the two dimensional exciting force and moment along the length of the hull.
Chapter 2. Historical Background

A complete mathematical derivation of the hydrodynamic coefficients and exciting force and moment appears in the Appendix of Salvesen et al [26] if the reader wishes a more detailed treatment.

2.3.3 Assumptions

In the context of this study, it is more important to clearly state the assumptions that are made in the derivation rather than examine the derivation itself as these assumptions are very significant in the application of the theory.

First of all, in order to make it a potential flow problem, it is assumed that all viscous effects can be disregarded hence the only damping considered is the energy loss associated with creating surface waves. This is valid for heave and pitch motion as the viscous damping is very small in this case [35]. To have a potential flow problem, one must also assume incompressible, irrotational flow.

In order to linearize the potential flow problem, it is necessary to assume that the wave resistance perturbation potential and all its derivatives are small enough to be ignored in the formulation of the motion problem. Physically, this means the free surface waves created by the ship advancing at constant speed in calm water are assumed to have no effect on the motions. This appears to be a reasonable assumption for fine slender hull forms.

Finally, in order to reduce the three dimensional problem to a summation of two dimensional problems, it is necessary to assume that the wave frequency is relatively high. This means that the waves created by the ship's oscillations should have a wave length of the order of the ship beam rather than the ship length. This is a critical assumption in that in reality the maximum motion responses are in the fairly low frequency long wavelength range in which the theory appears to be invalid. However, the pitch and heave motions in the low frequency range are dominated by the hydrostatic restoring forces so that these
apparent inaccuracies in the hydrodynamic coefficients in the low frequency range only have a minor effect on the predicted motion characteristics.

### 2.3.4 Added Mass and Damping Coefficients

The speed and frequency dependent added mass and damping coefficients derived by Salvesen, Tuck and Faltinsen [26] are:

\[
\begin{align*}
A_{33} &= \int a_{33} \, d\xi \\
B_{33} &= \int b_{33} \, d\xi \\
A_{35} &= -\int (\xi - \xi_c) a_{33} \, d\xi - \frac{V}{\omega_e^2} B_{33} \\
A_{53} &= -\int (\xi - \xi_c) a_{33} \, d\xi + \frac{V}{\omega_e^2} B_{33} \\
B_{35} &= -\int (\xi - \xi_c) b_{33} \, d\xi + V A_{33} \\
B_{53} &= -\int (\xi - \xi_c) b_{33} \, d\xi - V A_{33} \\
A_{55} &= \int (\xi - \xi_c)^2 a_{33} \, d\xi + \frac{V^2}{\omega_e^2} A_{33} \\
B_{55} &= \int (\xi - \xi_c)^2 b_{33} \, d\xi + \frac{V^2}{\omega_e^2} B_{33}
\end{align*}
\]

The reason for the underlined term in equation 2.21 is explained later in Chapter 3.

The hydrostatic restoring coefficients are assumed to be independent of frequency and forward speed and are given by:

\[
\begin{align*}
C_{33} &= \rho g \int b(\xi) \, d\xi \\
C_{35} &= C_{53} = -\rho g \int (\xi - \xi_c) b(\xi) \, d\xi \\
C_{55} &= \rho g \int (\xi - \xi_c)^2 b(\xi) \, d\xi
\end{align*}
\]
2.3.5 Exciting Force and Moment

The final form of the exciting force and moment expressions derived by Salvesen et al including an empirical assumption suggested by Korvin-Kroukovsky [20] are:

\[
F_3 = \text{Re}\{\zeta_3 \int f(\xi)e^{ik(\xi_\ell - \xi_\ell)}\,d\xi\} 
\]

(2.25)

\[
F_5 = \text{Re}\{\zeta_5 \int m(\xi)e^{ik(\xi_\ell - \xi_\ell)}\,d\xi\} 
\]

(2.26)

where

\[
f(\xi) = [\rho gb(\xi) - \omega(\omega_3 a_{33} - ib_{33})]e^{-d(\xi)\sigma(\xi)k} 
\]

(2.27)

\[
m(\xi) = -(\xi - \xi_\ell) \left[ f(\xi) - \frac{V}{i\omega_3} \omega(\omega_3 a_{33} - ib_{33}) \right] e^{-d(\xi)\sigma(\xi)k} 
\]

(2.28)

2.4 Experimental Work on Ship Motions and Added Resistance

A great deal of experimental work has been carried out in an effort to validate ship motion theoretical models and their assumptions. This section summarizes the experimental techniques used and the conclusions made from experimental work with respect to the added resistance and motions of ships and yachts.

2.4.1 Experimental Techniques

Two different methods of measuring model scale added resistance and motions of ships are commonly used in towing tanks. These are [28, 35]:

1. Constant Thrust Method

2. Constant Velocity Method

In the constant thrust method, the model is towed by a constant weight and the resultant speed of the model is measured. The average speed is then recorded over an
integer number of waves. This enables one to determine the speed loss (as compared to calm water) associated with the constant towing force. Using this method, the model is restricted in roll, yaw, and sway but is free to heave, pitch and surge.

In the constant velocity method, the model is firmly attached to the resistance dynanometer and no speed variations are allowed. The model is again allowed to heave and pitch but is restricted to surge in addition to being restricted to yaw, sway and roll.

The quantities measured in both methods of testing include resistance, ship speed, wave frequency, wave amplitude, heave and pitch amplitudes and heave and pitch phase.

Motion and added resistance tests can be carried out in regular sine waves or in irregular wave spectra. For regular sinusoidal wave tests, in order to obtain a complete frequency response spectrum, the model should be tested for at least ten wavelengths for each speed under consideration. At a particular wavelength the tests should be carried out at a number of different waveheights.

Heave and pitch motion response spectra can be obtained directly from the measured heave and pitch motions found at each encounter frequency. The motion spectra are usually presented in non-dimensional form by dividing heave and pitch by wave amplitude and wave slope respectively.

Added resistance is presented in the form of the non-dimensional added resistance coefficient given by:

$$\sigma_{aw} = \frac{R_{aw}}{\rho g \zeta_a^2 (B^2 L)}$$

(2.29)

where $R_{aw}$ is the average resistance in waves less the calm water resistance at the corresponding speed and $\zeta_a$ is the measured wave amplitude. Note that the assumption that the added resistance is proportional to wave amplitude squared is inherent in this expression.
The regular wave motion responses and added resistance are plotted against the non-dimensional frequency of encounter given by:

\[ \mu_e = \omega_e \sqrt{\frac{L}{g}} \]  

(2.30)

where the dimensional frequency of encounter \( \omega_e \) is related to the wave frequency \( \omega \) by:

\[ \omega_e = \omega \left( 1 + \frac{V}{g} \right) \]  

(2.31)

This results in completely non-dimensional plots.

To obtain the predicted ship response in irregular waves, the measured regular wave motion and added resistance response spectra are applied to a given wave spectrum using the principle of superposition (assuming this is valid). Conversely, the ship response spectrum can be measured directly for one particular irregular wave spectrum by running the model through that spectrum.

2.4.2 Experimental Findings for Ships and Yachts

Most experimental research on motions and added resistance has been carried out on cargo ships and naval vessels which tend to have long slender slab-sided hull forms. Tests conducted on these hull forms have resulted in the following findings and conclusions:

- Principle of linear superposition is valid for motion and added resistance tests [13].
- Ship motions are proportional to wave amplitude [35].
- Added resistance is proportional to wave amplitude squared [35].
- Scaling of added resistance can be carried out under Froude’s Law [2].
- Strip theory of computing added mass and damping is valid [39].
Researchers studying the motions and added resistance of yachts have tried to extend these findings for regular hull forms to include yacht hull forms, however yacht hulls tend to be very non-linear in form causing deviations from the assumptions which are made in strip theory calculations for cargo ship motions and added resistance [8]. These non-linearities include [19]:

- Yachts are rarely slab-sided.
- L/B and L/D ratios are low in comparison to ships.
- Heeled shapes with appendages are difficult to model analytically.
- Due to long overhangs and heel, the dynamic waterline may be significant especially in terms of damping and restoring force [12].

In addition, when testing yachts for motions and added resistance there are extra difficulties that are not present when testing ships including [19]:

- yachts operate at large heel and leeway angles.
- yachts operate in oblique waves.
- the vortices shed by a yacht hull as a consequence of sideforce generation may influence the motion response.
- sails may influence the damping and the sail driving forces may in turn be affected by the motions due to deformation of the rig and the sails.

With these difficulties and limitations in mind, researchers were still able to make some useful conclusions regarding motions and added resistance of yachts.
Chapter 2. Historical Background

Research on Yachts

In 1967 Spens et al [32] laid the groundwork for all future studies on yachts in waves. They studied the motions and added resistance of yachts in regular oblique and head waves resulting in the following findings:

- Response in oblique waves is in general agreement with response in head seas of corresponding frequency.
- Change in leeway angle due to waves is negligible.

Thus they concluded that tests in head waves could be used to analyse a yacht's performance in oblique waves.

Gerritsma [8] expanded on the work of Spens, arriving at the following conclusions:

- Yacht motions are proportional to wave amplitude.
- The waveheight squared law is not always valid and the relationship between added resistance and waveheight can be frequency dependent.
- Unlike cargo ship hulls, for yachts the restoring force and moment in pitch and heave increase with forward speed due to its own wave system.
- Present linear strip theory methods are not quantitatively accurate when applied to yacht hull forms.
- Motions and resistance for 20 degrees heel do not differ substantially from the upright condition values.

Pedrick [25] in 1974 analysed the results of yacht tests in oblique waves determining that force testing in oblique waves can be used to make predictions of motions and speed
to windward accounting for both added resistance and side force effects in any specified sea condition.

A recent study was carried out by Klaka and Penrose [19] in 1987 for the Kookaburra syndicate. They determined through model tests that:

- Winged keels have little effect on the motions and added resistance of 12-metre hull forms.

- Heel angle had little effect on motions but caused a significant decrease in added resistance (approx. 50%).

- Upright yacht testing alone is not adequate for prediction of speed loss due to waves.

These final two conclusions disagree with the results of Gerritsma [8] which found that heel angle had only a small effect on added resistance (though for a very different hull form). However, a very recent study just completed by Gerritsma [11] tends to agree with Klaka and Penrose that for heavy displacement yachts, heel angle can affect added resistance.

2.5 Reducing Motions and Added Resistance in Waves

To reduce the motions of a vessel in a seaway, motion stabilization devices and methods are often adopted. Using these methods, the amplitude, rate and accelerations of the motions as well as the associated dynamic effects such as deck wetness and slamming can be reduced considerably. Since added resistance is directly related to ship motions, one might expect that motion stabilization may also be effective in reducing added resistance in a seaway. With this in mind, this section will examine the various methods of motion reduction and the experimental findings in this area.
2.5.1 Methods of Motion Reduction

With reference to the equations of motion (equations 2.12 and 2.13 in section 2.3.1) one can see that there are three mechanisms by which one can reduce forced motions [2].

1. By increasing the damping coefficient. (damping stabilization)

2. By reducing the natural frequency of the ship to shift the region of resonance. (tuning stabilization)

3. Reducing the total exciting force or moment by applying a stabilizing moment that is opposite in phase to the wave exciting moment. (equilibrium stabilization)

Some of the devices and methods that are used for motion reduction and their associated mechanisms are:

- Changing mass distribution (tuning)
- Movement of weight (equilibrium)
- Passive and active tanks or other vibration absorber (tuning and equilibrium)
- Anti-motion fins or foils (damping)
- Gyroscopic stabilizers (equilibrium)
- Passive stabilizers such as bilge keels for roll (damping)

Most motion reduction techniques have been developed for roll stabilization such as anti-roll tanks, bilge keels, anti-roll fins etc. This is because the forces and moments involved with rolling are comparatively small. Also, normal vessels have inherently low damping properties with respect to roll resulting in a sharply tuned resonant region.
The problem of reducing pitch motions is quite different from that of roll stabilization. The fact that pitch is already heavily damped by the vessel’s hull means that very large forces are required for any further increase in damping. The pitch exciting moment is also comparatively large requiring very large forces in order to counteract it.

It is quite impractical to generate these large forces by the means of internal devices generally associated with roll stabilization such as vibration absorbers, moving weights or gyros. Thus most attempts at pitch stabilization have involved the use of external devices capable of generating and sustaining the large forces required. The external devices that have been investigated for pitch stabilization include fixed bow foils, movable stern fins and, to a lesser degree, fixed stern fins and controllable bow foils [14].

2.5.2 Experimental Findings on Anti-Pitch Foils

Many studies have been done on the use of anti-pitch foils on large cargo ships and naval vessels such as aircraft carriers or destroyers. Most of this work has been concerned only with reducing the pitching motions of the ship so that it can maintain speed or maintain its ability to operate in a seaway, with little attention being paid to the reduction of added resistance.

Abkowitz [1] showed in 1957 that the pitching motion of ships could be reduced to one-half normal values by the use of reasonable sized anti-pitching fins at the bow, particularly at speeds in the vicinity of synchronism or in irregular seas with synchronous components. Spens in 1962 [31] found that:

- Fixed fins were effective at the bow but not at the stern.
- Controlled fins are not necessary at the bow.
- No change in phase relationship of pitch to waves appears possible.
Stefun in 1959 [33] carried out experiments to optimize the parameters of fixed bow fin design. He found that:

- Fins of approximately 2% of the waterplane area of the hull were most effective for their size.
- Fins of aspect ratio 2 showed the best results.
- The use of tip fences causes an additional reduction in pitch of 5% over those without fences.

Full scale tests have been carried out using anti-pitching fins with mixed results. Although in several cases significant pitch reduction was achieved, severe vibrations caused their removal [3, 14]. Other findings of researchers in this area include:

- Deeper submergence of fins, greater fin span and tip fences can help reduce structural loadings caused by vorticity effect.
- If fins remain a minimum distance below the water surface (depending on hull form) an air cavity will not form and vibration will not occur.
- Maximum pitch reduction can be achieved by fins located in the forward 10% of the ship.

Stefun [33] was the only one to measure added resistance as well as motions during his anti-pitch fin tests and he found that although the fins caused a 10% to 15% increase in the calm water resistance, this was more than offset by a decrease in motion-induced resistance due to the effect of the anti-pitching fins. Conolly [3] also noticed in full scale trials that the RPM of a ship equipped with foils tended to be lower than that for a normal ship when travelling at the same speed in identical seas.
Chapter 2. Historical Background

No-one has previously researched the effect of an external appendage such as a bow anti-pitch foil on the motions and added resistance of a yacht hull. The idea of wings on rudders of yachts as an anti-pitch device was mentioned briefly by Cox and Whitaker [4] but has not yet been pursued. The only research that has been carried out on reducing motions and added resistance of yachts in waves was conducted by Spens [32] and Gerritsma and Beukelman [12] and dealt only with changing mass distribution. Both found that decreasing the longitudinal radius of gyration of the hull will improve the seakeeping performance of the yacht. This agrees with the usual practice of yachtsmen to concentrate the weight of their yachts in the centre of the boat whenever possible. Gerritsma showed further that a light displacement yacht tends to show better seakeeping qualities than a heavy displacement yacht of similar hullform [11, 12].

The results of the research on anti-pitch foils on ships led the author to believe that even though anti-pitching fins did not seem practical for large ships due to structural and vibrational problems, a properly placed and sized anti-pitch foil could be useful for reducing the added resistance and motions of yachts and other small marine craft.
Chapter 3

Mathematical Model Formulation

From the historical work on ship motions and added resistance theory presented in Chapter 2, a mathematical model was developed to predict the heave and pitch motions and added resistance of the Canada II hull form. This theory was then extended to include the effect of a bow anti-pitch foil.

In this chapter, the equations used in the numerical calculations are presented and the results of the predicted motions and added resistance for Canada II with and without the anti-pitch foil are compared.

3.1 Principal Parameters and Limitations

The principal particulars for the hull form of Canada II are listed in Table 3.2. The offsets for Canada II were obtained by digitizing the lines drawings which were kindly supplied by the designer Bruce Kirby.

It should be noted at this point that Canada II's hull form, as with most yacht hull forms (see section 2.4.2) deviates from the linear assumptions associated with linear motion theory. The hull is not slab-sided, has a relatively low $L/B$ ratio and possesses long fore and aft overhangs not accounted for in linear theory. In addition, the theory neglects the effect of deck wetness and forefoot emergence on the motions and added resistance. Thus, the validity of the use of the linear motion theory may be suspect if one were expecting accurate quantitative results. However since the object of this theoretical investigation is to detect the relative difference of the hull with the anti-pitch foil and
Table 3.2: Canada II Principal Particulars

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Waterline</td>
<td>45 ft.</td>
</tr>
<tr>
<td>Maximum Beam on Waterline</td>
<td>11.36 ft.</td>
</tr>
<tr>
<td>Draft to Waterline (Hull)</td>
<td>4.96 ft.</td>
</tr>
<tr>
<td>Draft to Waterline (Keel)</td>
<td>9.08 ft.</td>
</tr>
<tr>
<td>Displacement</td>
<td>59,360 lbs. (S.W.)</td>
</tr>
<tr>
<td>Length/Beam Ratio</td>
<td>3.96</td>
</tr>
<tr>
<td>Length/Draft Ratio</td>
<td>9.07</td>
</tr>
<tr>
<td>Beam/Draft Ratio</td>
<td>2.29</td>
</tr>
<tr>
<td>Waterplane Area</td>
<td>356.9 sq. ft.</td>
</tr>
<tr>
<td>Wetted Surface Area</td>
<td>695.0 sq. ft.</td>
</tr>
<tr>
<td>Vertical Center of Gravity</td>
<td>3.67 ft. below waterline</td>
</tr>
<tr>
<td>Longitudinal Center of Gravity</td>
<td>2.04 ft. aft of midships</td>
</tr>
</tbody>
</table>

without, it was hoped that the linear theory used along with added resistance theory would be sufficient in a qualitative context and therefore would still be useful as a design tool. As shown by Scragg et al [27] of the Stars and Stripes syndicate, even though their wave resistance studies showed disappointing quantitative results, the theory was still very useful in showing the relative difference between candidate designs.

In order to make direct comparisons to later experimental results the theoretical scope was constrained to the following:

- Zero heel angle only.
- Head seas only.
- One speed and one waveheight (8 knots and 2.5 ft. full scale)
- Neglect effect of sails and rigging.
3.2 Frank Close-Fit Motion Program

To predict the heave and pitch motions of Canada II, the Frank Close-Fit Computer Program [7] was used. This program is based on the theory of Salvesen et al [26] presented in section 2.3.

The coupled equations for heave and pitch are repeated here as:

\[
(M + A_{33})\ddot{z} + B_{33}\dot{z} + C_{33}z + A_{35}\ddot{\theta} + B_{35}\dot{\theta} + C_{35}\theta = F_3 e^{i\omega t} \tag{3.32}
\]

\[
A_{53}\ddot{z} + B_{53}\dot{z} + C_{53}z + (I_6 + A_{55})\ddot{\theta} + B_{55}\dot{\theta} + C_{55}\theta = F_5 e^{i\omega t} \tag{3.33}
\]

with the added mass and damping coefficients being identical to those given in section 2.3.4 except for the expression for the pitch damping coefficient \(B_{55}\). The Frank program does not include the underlined forward speed term shown in the expression for \(B_{55}\) in equation 2.21 of section 2.3.4. This forward speed correction has been included in this study to be consistant with the Salvesen theory and with the conclusions of Smith [30].

3.2.1 2-D Added Mass and Damping

The Frank program solves for the 2-D sectional added mass and damping coefficients \(a_{33}\) and \(b_{33}\) using the close-fit method developed by Frank [7] which is a form of boundary element method. The geometric shape of the section is mathematically represented by a given number of offset points with straight line segments between the points.

The 2-D velocity potential is then obtained for a distribution of source singularities over the submerged surface of the cylinder with constant strength over each of the segments. These source singularities are such that each one of them satisfies the 2-D Laplace equation, the linearized free surface condition and the infinity radiation conditions. The cylinder wall boundary condition is finally satisfied by solving for the appropriate strength for each of the source segments. This gives the velocity potential for the boundary value
problem.

By using the linearized Bernoulli equation, the hydrodynamic pressures on the cylinder wall can be obtained. Integration of these pressures over the immersed portion of the cylinder yields the hydrodynamic force and therefore the added mass and damping coefficients.

This method is very accurate, however it breaks down at higher frequencies. Thus there is a limiting frequency at which the added mass and damping can no longer be calculated for a section. For a rectangular cylinder with beam \( b \) and draft \( d \) it can be shown that the first frequency for which the method fails is [7]:

\[
\mu_1 = \left(\pi L/b\right) \coth\left(\pi d/b\right)^{1/2}
\]  

(3.34)

where \( \mu_1 \) is non-dimensional with respect to ship length \( L \). Thus the program only computes motion results for non-dimensional encounter frequencies less than that given by equation 3.34. \( \mu_1 \) is directly proportional to ship beam \( b \) and for \( b \to \infty \) the lower limit of \( \mu_1 \) is \( \sqrt{L/d} \). Therefore, as a general rule, the close-fit method is applicable for all encounter frequencies;

\[
\mu_e < \sqrt{L/d}
\]

(3.35)

3.2.2 2-D Exciting Force and Moment

The complex 2-D sectional exciting force and moment \( f(\xi) \) and \( m(\xi) \) are also computed by the close-fit method. The resulting expressions differ from those given in equations 2.27 and 2.28 in section 2.3.5 as those equations use an empirical assumption. The Frank program expressions for sectional exciting force and moment are:

\[
f(\xi) = 2 \int_{0}^{b(\xi)} \left\{ 1 - \frac{\mu}{\mu_e} \left[ p_a(\eta) + i p_v(\eta) \right] \right\} e^{k \xi(\eta)} d\eta
\]

(3.36)

\[
m(\xi) = -\left(\xi - \xi_e\right)f(\xi) - 2i \frac{V\mu}{\mu_e^2} \int_{0}^{b(\xi)} \left[ p_a(\eta) + i p_v(\eta) \right] e^{k \xi(\eta)} d\eta
\]

(3.37)
where \((\eta, \zeta)\) are the sectional co-ordinates with the \(\zeta\)-axis positive upward and the \(\eta\)-axis at the undisturbed free surface. Here \(p_a\) and \(p_v\) are the 2-D hydrodynamic pressures in phase with the acceleration and the velocity respectively. This method is more correct than that of equations 2.27 and 2.28 since the vertical variation in the pressures is included very accurately. Equations 2.25 and 2.26 are then used to find \(F_3\) and \(F_5\).

The geometric parameters for Canada II were input to the Frank Close-Fit Computer Program for a speed of 8 knots. The resultant coefficients and exciting force and moments were then input to a Lotus spreadsheet and equation 2.21 was used to correct the pitch damping coefficient \(B_{65}\) for forward speed. The equations of motion (equations 3.32 and 3.33) were then solved for pitch and heave.

### 3.3 Modelling Effect of Anti-Pitch Foil

#### 3.3.1 Foil Parameters

From the findings of Stefun [33] and others listed in section 2.5.2, a set of near optimal foil parameters was developed for a hypothetical bow anti-pitch foil fitted to the Canada II hull. The relevant parameters of the hypothetical foil are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foil Span</td>
<td>4.0 ft.</td>
</tr>
<tr>
<td>Foil Chord</td>
<td>2.0 ft.</td>
</tr>
<tr>
<td>Depth Below Free Surface</td>
<td>6.0 ft.</td>
</tr>
<tr>
<td>Initial Incidence Angle</td>
<td>0.0 degrees</td>
</tr>
<tr>
<td>Distance From LCG of Ship</td>
<td>24.5 ft.</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>2.0</td>
</tr>
<tr>
<td>Planform Area</td>
<td>8.0 sq. ft.</td>
</tr>
</tbody>
</table>
3.3.2 Anti-Pitch Foil Theory

The problem of a horizontal hydrofoil undergoing oscillatory motion is analogous to the flutter problem of a wing in aerodynamics. If we assume potential flow, it is essentially a problem of solving for the unsteady lift and moment experienced by the foil. To investigate the effect of the hypothetical foil on Canada II, the method of Spens [32] which is based on the work of Kaplan [18] was used. By this method, it is assumed that the foil is a flat plate of large aspect ratio in a potential flow field undergoing linear harmonic oscillations.

The theoretical (unsteady motion) hydrodynamic lift $L$ on a hydrofoil of large aspect ratio and its moment $M$ about the center of gravity of the system are, when it is undergoing oscillatory motion in still water [18]:

$$L = \frac{\rho \pi SV^2}{1 + \frac{2}{A}} \left[ -\theta - \frac{\dot{z}}{V} - \left( \frac{1}{4} - \frac{l}{c} \right) \frac{c}{V} \dot{\theta} \right] C(\kappa) + K \rho \frac{\pi}{4} Sc \left[ -V \dot{\theta} - \ddot{z} + i \dot{\theta} \right]$$  \hspace{1cm} (3.38)

$$M = \frac{\rho \pi SV^2}{1 + \frac{2}{A}} \left( \frac{1}{4} - \frac{l}{c} \right) \left[ -\theta - \frac{\dot{z}}{V} - \left( \frac{1}{4} - \frac{l}{c} \right) \frac{c}{V} \dot{\theta} \right] C(\kappa) + Kc \left( \frac{1}{4} - \frac{l}{c} \right) \rho \frac{\pi}{4} Sc V \dot{\theta} - Kl \rho \frac{\pi}{4} Sc \left[ -\ddot{z} + l \dot{\theta} \right] + \frac{K \rho \pi Sc^3}{128} \ddot{\theta}$$ \hspace{1cm} (3.39)

Note that $\frac{\rho \pi SV^2}{1 + \frac{2}{A}}$ is the steady state damping force of the foil per unit vertical velocity by aerodynamic theory. Although limited to fairly large aspect ratios it may still be considered to be applicable to the case of aspect ratio 2. The term $K \rho \frac{\pi}{4} Sc$ is the virtual or added mass of the flat plate (the actual mass of the foil is insignificant).

When evaluating the lift and moment of the foil attached far from the LCG of the system, the quarter chord $c/4$ of the foil can be neglected as insignificant when compared to the distance $l$ to the LCG. Equations 3.38 and 3.39 can therefore be reduced to:

$$L = \frac{\rho \pi bcV^2}{1 + \frac{2}{A}} C(\kappa) \left[ -V \theta - \ddot{z} + l \dot{\theta} \right] + K \rho \frac{\pi}{4} bc^2 \left[ -V \dot{\theta} - \ddot{z} + \dot{\theta} \right]$$ \hspace{1cm} (3.40)

$$M = -lL$$ \hspace{1cm} (3.41)
In these equations \( C(\kappa) \) is the Theodorsen circulation function for a two dimensional oscillating hydrofoil evaluated from tables in Luke and Dengler [22] and \( K \) is the coefficient of accession to inertia from Munk [24].

### 3.3.3 Modification of Coefficients in Equation of Motion

By separating the terms associated with \( z, \theta \) and their derivatives in equations 3.40 and 3.41 one can obtain expressions that must be added to the coefficients in the heave-pitch equations of motion.

The increments that must be added due to the foil are:

\[
\begin{align*}
\Delta A_{33} &= K \rho \frac{\pi}{4} bc^2 \\
\Delta A_{55} &= (\Delta A_{33})l^2 \\
\Delta A_{35} &= \Delta A_{53} = -(\Delta A_{33})l \\
\Delta B_{33} &= \left( \frac{\rho \pi bcV}{1 + \frac{2}{A}} \right) C(\kappa) \\
\Delta B_{55} &= \Delta B_{33}l^2 - \Delta A_{33}Vl \\
\Delta B_{35} &= -\Delta B_{33}l + \Delta A_{33}V \\
\Delta B_{53} &= -\Delta B_{33}l \\
\Delta C_{35} &= \Delta B_{33}V \\
\Delta C_{55} &= -\Delta B_{33}Vl \\
\Delta C_{33} &= \Delta C_{53} = 0
\end{align*}
\]

These account for the effect of the system oscillating in still water. A change in lift and moment due to the wave orbital motion acting on the foil as the system moves through regular head waves must also be included. These result in increments to the right hand
side of the equations of motion as follows [31]:

\[
\Delta L_w = \rho \pi S V^2 |\hat{w}_0| (1 - i \alpha_a) e^{i(\omega_c t + \phi)} \left\{ \left[ \frac{J_0(\nu) - i J_1(\nu)}{1 + \frac{2}{A}} \right] C(\kappa) + i K \left( 1 + \frac{C_w}{V} \right) J_1(\nu) \right\}
\]

\[
\Delta M_w = -l(\Delta L_w)
\]

where

\[
|\hat{w}_0| = \zeta_a \left( \frac{2\pi C_w}{\lambda V} \right) e^{-2\pi h/\lambda}
\]

|\hat{w}_0| is the ratio of the vertical component of the orbital velocity of the wave to the velocity along the x-axis.

The increments to the motion coefficients and the exciting force and moment were calculated for Canada II with an anti-pitch foil, for the foil parameters stated in Table 3.3 for a speed of 8 knots into head seas. These calculations were integrated into the existing Lotus spreadsheet and the modified equations of motion were solved for heave and pitch for the hull with foil.

### 3.4 Added Resistance Calculation

After reviewing the literature on added resistance theories, it was decided to use a modified version of the Joosen equation presented in section 2.2.2. Joosen extended his work on equation 2.2 to account for pitch-heave cross coupling [17]. It should be restated that this theory assumes a slender body approximation for the drift forces and is valid only for short waves. Also, the wave diffraction effect is neglected; therefore the theory also breaks down for very short waves where the diffraction effect becomes dominant. Since in this investigation we are concerned mostly with the resonant region, these assumptions should not present a problem.

The final form of the modified Joosen equation is [35]:

\[
\sigma_{aw} = E_1 + E_2 + E_3
\]
where

\begin{align}
E_1 &= C_0 \overline{B_{33}} (z_a / \zeta_a)^2 \\
E_2 &= C_0 (2\pi L/\lambda)^2 \overline{B_{35}} (\theta_a / k\zeta_a)^2 \\
E_3 &= -2C_0 (2\pi L/\lambda) \overline{B_{35}} (z_a / \zeta_a) (\theta_a / k\zeta_a) \cos \epsilon \\
C_0 &= \frac{1}{16} \frac{L^2}{B^2} \left( \omega_c \sqrt{\frac{L}{g}} \right)^3 \nabla / L^3 \\
\epsilon &= |\epsilon_x - \epsilon_\theta|
\end{align}

The damping coefficients appearing in the above equations are given by:

\begin{align}
\overline{B_{33}} &= (1/\omega_c \nabla)(g/L)^{1/2} \int b_{33}(\xi) d\xi \\
\overline{B_{35}} &= (1/\omega_c \nabla)(g/L)^{1/2} \int (\xi - \xi_c)b_{33}(\xi) d\xi \\
\overline{B_{55}} &= (1/\omega_c \nabla)(g/L)^{1/2} \int (\xi - \xi_c)^2 b_{33}(\xi) d\xi
\end{align}

The Joosen method was chosen over the Gerritsma and Beukelman method (see section 2.2.3) for its ease of implementation. The Gerritsma and Beukelman method requires an accurate knowledge of the distribution of the sectional added mass and damping while the Joosen method requires only the final total damping coefficients of the hull, which are more readily available. Again, since only a relative measure is required in this study, it was decided that though the Gerritsma and Beukelman method might be more accurate (due to its more accurate handling of diffraction effects and cross coupling) its increased complexity did not warrant its use. It is recommended that if more accurate predictions of added resistance are desired in future investigations, one should consider using the Gerritsma and Beukelman method.

The predicted added resistance for Canada II with and without the foil was calculated using equation 3.53 using the spreadsheet solutions to the equations of motion and the calculated values of the damping coefficients.
3.5 Results of Theoretical Calculations

In this section, the predicted heave motion and pitch motion responses and added resistance resulting from the theoretical calculations are presented for Canada II with and without the bow anti-pitch foil.

3.5.1 Heave Motion

The calculated heave response for Canada II is shown in Figure 3.3. Examining the bare hull results first, one notices that the heave response follows the wave amplitude at low frequencies of encounter as one would expect, then begins to rise rapidly as $\mu_e$ increases into the resonant region. Heave resonance for the bare hull is predicted to occur at $\mu_e = 3.2$ which corresponds to a wavelength $\lambda$ of 77 ft. or $1.17L$ for Canada II at a speed of 8 knots. At resonance, the nondimensional heave response peaks at 1.27 indicating that heave is quite heavily damped. As $\mu_e$ increases beyond resonance the heave response drops rapidly.

Unfortunately, the close-fit method for calculating added mass and damping breaks down for Canada II's hull form at $\mu_e = 3.6$ (see section 3.2.1). Hence, no calculations can be made beyond this frequency. This corresponds to $\lambda = 65$ ft. or $1.45L$ which is still a relatively long wave, however it is safe to assume that the heave response will decrease monotonically beyond the cutoff frequency since at very high frequencies the heave response will tend toward zero.

If one examines the heave response of Canada II with the anti-pitch foil attached, one finds that the response is quite different from that for the bare hull. For $\mu_e < 2.7$ the heave response with the foil is predicted to be slightly higher. Above $\mu_e = 2.7$, near the resonant region, the response with foil does not rise as rapidly as the bare hull, reaching a peak of only 1.16 at $\mu_e = 3.0$. The resonant frequency of $\mu_e = 3.0$ is slightly lower than
Theoretical Heave Response

Figure 3.3: Theoretical Heave Amplitude Response
Chapter 3. Mathematical Model Formulation

that of the bare hull at $\mu_e = 3.2$. Above resonance the response can again be expected to drop monotonically. In the resonant region and beyond the predicted decrease in heave response as a result of the foil is up to 11% which is quite significant.

Though at first one might be surprised to find that for $\mu_e < 2.7$ the heave response with the foil is actually higher than for the bare hull, it can be explained as follows. If one examines the changes in the coefficients of motion from the foil (see section 3.3.3) one sees that $\Delta A_{33}$, the added mass of the foil, is a constant while $\Delta B_{33}$, the damping due to the foil, is a function of $C(\kappa)$ which is a function of frequency and vertical velocity. At the lower frequencies of encounter the magnitude of $C(\kappa)$ is quite small with the result that the added mass term dominates. This results in a greater heave response since extra added mass reduces the damping ratio. As $\mu_e$ increases beyond 2.7, $C(\kappa)$ increases and the damping term starts to dominate the heave response causing the response to cross over the bare hull response.

The slightly lower resonant frequency of the hull with foil can be attributed to the increase in total added mass due to the foil. Since the restoring coefficient is assumed to be constant, the natural frequency would be expected to decrease.

3.5.2 Pitch Motion

Referring to Figure 3.4, one sees the predicted pitch response. At first glance one notices immediately a significant reduction in the non-dimensional pitch amplitude for the hull with foil compared to the bare hull throughout the frequency spectrum. As with heave, the pitch response is predicted to rise for the low encounter frequencies up to the resonant frequency. However, unlike heave, the rise is much more gradual and results in a less pronounced resonant hump. Again, we see a slight decrease in the resonant frequency with the addition of the foil, shifting from $\mu_e = 3.0$ for the bare hull to $\mu_e = 2.8$ for the hull with foil.
Chapter 3. Mathematical Model Formulation

Theoretical Pitch Response

Figure 3.4: Theoretical Pitch Amplitude Response
For the bare hull, the peak non-dimensional pitch amplitude at resonance is 1.33. With the addition of the foil this peak drops to 1.14 at resonance representing over a 15% predicted decrease in pitch amplitude. Even at the lower encounter frequencies the foil is predicted to decrease pitch amplitudes by 10% implying that, in theory at least, the anti-pitch foil is expected to live up to its name.

3.5.3 Added Resistance

The predicted added resistance of Canada II, with and without anti-pitch foil, is shown in Figure 3.5. For the bare hull the non-dimensional added resistance coefficient $\sigma_{aw}$ is predicted to be zero for very long waves (low encounter frequency) then rises rapidly to a peak of $\sigma_{aw} = 3.4$ at a resonant frequency of $\mu_e = 3.4$. Due to the break down of the close-fit method for the ship motions when $\mu_e > 3.6$, the added resistance cannot be calculated beyond this frequency either, since added resistance is calculated directly from the heave and pitch motions. Note, however, that the added resistance drops quickly between $\mu_e = 3.4$ and $\mu_e = 3.6$ and can be expected to continue to drop. Note also that the added resistance curve has a narrow and steep resonant peak which corresponds closely to the heave and pitch resonant frequencies, as expected.

Turning now to the added resistance response of the hull with foil attached, one sees that the resonant frequency is essentially the same and $\sigma_{aw}$ is predicted to drop significantly at this point from 3.4 for the bare hull to 2.35 for the hull with foil. This represents a 31% predicted reduction in added resistance due to the addition of the foil.

If one examines the modified Joosen equation (equation 3.53) closely, this large reduction is predicted. The additional damping provided by the foil does not affect the values of the purely hydrodynamic damping coefficients $\overline{B}_{33}, \overline{B}_{55}$ and $\overline{B}_{35}$ in the expressions for $E_1, E_2$ and $E_3$. However, the effect of the foil is to reduce the predicted heave and pitch motion amplitude significantly as we have seen. Since the expressions for $E_1, E_2$ and $E_3$
Theoretical Added Resistance Response Spectrum

![Diagram of Theoretical Added Resistance Response Spectrum with legend showing non-dimensional added resistance coefficient against non-dimensional encounter frequency.](image)

Figure 3.5: Theoretical Added Resistance
are second order in motion amplitudes the reduced heave and pitch amplitudes translate into an even higher percentage reduction in added resistance.

It should be noted that in the calculation of added resistance with the foil, the extra hydrodynamic drag of the foil due to the lift it creates has not been included. However it is expected that this extra drag is a small percentage of the total added resistance (especially at resonance).
Chapter 4

Experimental Program

In this chapter, the anti-pitch foil experimental program carried out by the author is described including a description of the test facilities, model preparation, method of testing, test program, observations and results.

4.1 Experimental Set-up

4.1.1 Facilities
All experiments were conducted at the B.C. Research Ocean Engineering Centre towing tank. The tank measures 67.0m x 3.7m x 2.4m and the hydraulic drive towing carriage is capable of speeds up to 4.5 m/s. A hydraulically operated, computer controlled flap type wavemaker is installed at one end of the tank and is capable of producing regular or irregular waves up to a height of 0.25m or just over 10 ins. for wave frequencies between 0.3 and 1.5 Hertz (see Figure 4.6). Models are attached to the carriage and towed by a single heave post arrangement as shown in Figure 4.7. Resistance is measured by a shear beam load cell dynamometer attached at the bottom of the heave post and is accurate to ±0.05 lbs. Average speed of the carriage for the run is measured using a digital encoder wheel. Pitch angle is measured directly from the pivot point of the heave post by an angular deflection potentiometer with an accuracy of ±0.05 degrees. Heave displacement is measured using a rolling wheel potentiometer fixed to the carriage and resting against the free-to-heave post. Heave accuracy is ±0.1 inches.
Figure 4.6: Wave Paddle
Figure 4.7: Heave Post Arrangement
Waveheight is measured using a Keiseko electromechanical water level meter with an accuracy of ±0.01 inches. The wave probe is mounted on a cantilever bridge that extends to near the midbreadth of the tank located near the midpoint of the length of the tank. Figure 4.8 shows the waveprobe set-up.

The data acquisition system consists mainly of a ST41B signal conditioner and a Digital Electronics MINC 11 mini-computer with analogue to digital module. The signal conditioner provides the required excitation voltage to the measurement transducers as well as amplification and filtering of the returned signal. The MINC 11 computer contains an analogue to digital conversion unit for storing the test data in a binary digital format. Figure 4.9 shows the data acquisition equipment on the towing carriage.

4.1.2 Model Parameters

A wooden model of Canada II shown in Figure 4.10 was kindly made available by the designer Bruce Kirby. The model was constructed to 1:8 scale according to a lines plan supplied by the designer. The model was fitted with a lead ballast keel for proper location of the VCG. The principal dimensions for the model are given in the Table 4.4.

It was attempted to place the heave post attachment point at the VCG of the model; however this was not possible as the VCG is actually in the keel. As a result, the heave post pivot point was placed 1 inch above the VCG which was as close as was practical. It was felt that this would not significantly affect the motions of the model.

Turbulence stimulation was provided on the hull by cylindrical studs of 0.125 inch diameter and 0.1 inch high spaced 1 inch apart on station 2. On the keel, a similar row of studs were installed 0.75 inches aft of the leading edge [34].
Figure 4.8: Wave Probe Set-up
Figure 4.9: Data Acquisition Equipment on Carriage
Figure 4.10: Canada II Model
Table 4.4: Canada II 1:8 Scale Model Particulars

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Waterline</td>
<td>5.6 ft.</td>
</tr>
<tr>
<td>Maximum Beam on Waterline</td>
<td>1.42 ft.</td>
</tr>
<tr>
<td>Draft to Waterline (Hull)</td>
<td>0.62 ft.</td>
</tr>
<tr>
<td>Draft to Waterline (Keel)</td>
<td>1.14 ft.</td>
</tr>
<tr>
<td>Displacement</td>
<td>113.04 lbs.</td>
</tr>
<tr>
<td>Length/Beam Ratio</td>
<td>3.96</td>
</tr>
<tr>
<td>Length/Draft Ratio</td>
<td>9.07</td>
</tr>
<tr>
<td>Beam/Draft Ratio</td>
<td>2.29</td>
</tr>
<tr>
<td>Waterplane Area</td>
<td>5.58 sq. ft.</td>
</tr>
<tr>
<td>Wetted Surface Area</td>
<td>10.86 sq. ft.</td>
</tr>
<tr>
<td>Vertical Center of Gravity</td>
<td>5.5 inches below waterline</td>
</tr>
<tr>
<td>Longitudinal Center of Gravity</td>
<td>3.06 inches aft of midships</td>
</tr>
</tbody>
</table>

4.1.3 Model Foil Design

A 1:8 scale model anti-pitch foil was designed and built corresponding to the full scale foil parameters used in the theoretical investigation (see Table 3.3). A NACA 0012 foil section of rectangular planform was used for the main foil because of its high lift to drag ratio and relatively high stall angle [32]. Originally, it was planned to attach the foil to the hull using a single non-surface piercing strut attached just below the waterline, aft of the forward perpendicular. However, it was determined that with this arrangement it would be too cumbersome to take the foil on and off and would require a through-hole in the model, which was undesirable. Instead, a double surface piercing strut arrangement was designed that could easily be attached and removed with the model in the water and required only three screwholes in the deck of the model. The model foil assembly drawings are shown in Appendix B. Note that the struts are foil shaped to reduce drag. Also, tip fences were attached to the end of the foils to reduce end effects as suggested by Stefun [33].

Figure 4.11 shows the Canada II model as it was for testing with the foil attached. An
underwater sideview of the model with foil attached appears in Figure 4.12. The model foil parameters are listed in the Table 4.5.

Table 4.5: Model Anti-Pitch Foil Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foil Span</td>
<td>6.0 inches</td>
</tr>
<tr>
<td>Foil Chord</td>
<td>3.0 inches</td>
</tr>
<tr>
<td>Depth Below Free Surface</td>
<td>9.0 inches</td>
</tr>
<tr>
<td>Initial Incidence Angle</td>
<td>0.0 degrees</td>
</tr>
<tr>
<td>Distance From LCG of Ship</td>
<td>3.06 ft.</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>2.0</td>
</tr>
<tr>
<td>Planform Area</td>
<td>18.0 sq. inches</td>
</tr>
</tbody>
</table>

4.2 Test Program

The model test program consisted of calm water resistance tests, calm water natural frequency tests and regular sinusoidal head wave tests.

4.2.1 Calm Water Tests

In order to obtain the experimental added resistance it is necessary to first determine the calm water resistance since:

$$ R_{aw} = R_{wave} - R_{calm} $$

Thus calm water resistance tests were conducted for the Canada II model for both the bare hull and the hull with anti-pitch foil attached. In addition to determining the calm water resistance at the target speed of 8 knots, it was of interest to see the effect of the foil on the calm water resistance at all speeds. Therefore the tests were done for model speeds corresponding to 6-9 knots full scale.

In order to obtain the natural frequencies in heave and pitch for the Canada II hull, forced oscillation tests were conducted on the bare hull. This involved forcing the model
Figure 4.11: Model Anti-Pitch Foil Fitted For Testing
Figure 4.12: Underwater Sideview of Model Anti-Pitch Foil
to a large heave or pitch displacement then releasing it and measuring the period of
oscillation. These tests were done at zero speed and at a model speed corresponding
to 8 knots in order to determine if forward speed affected the heave and pitch natural
frequencies.

4.2.2 Regular Wave Tests

Before carrying out regular wave tests, the wave paddle had to be calibrated so that
a reasonably constant waveheight could be produced at any frequency in the frequency
range.

All regular wave tests were carried out at a constant model speed corresponding to
8 knots full scale and at a constant waveheight corresponding to 2.5 feet full scale. These
were chosen by the author as realistic conditions encountered in Fremantle. All tests were
done using the constant velocity method of wave testing described in section 2.4.1. Using
this method the model is free to pitch and heave but restricted in surge, sway, yaw, and
roll. Regular wave tests were completed for the bare hull and the hull with foil for 15
regular spaced non-dimensional encounter frequency intervals of 0.25 between \( \mu_e = 1.25 \)
and \( \mu_e = 4.75 \). Calibration of all equipment was carried out before each day of testing.
All runs were videotaped for later examination of motions.

The procedure for the each run of the wave tests was as follows:

1. Take transducer zero values.

2. Start wavemaker.

3. Allow 3-4 waves to pass waveprobe then begin measuring waveheight.

4. Start model just before waves get to it.

5. When model is at speed, begin data acquisition.
6. Stop waveheight measurements before model passes waveprobe.

7. Stop data acquisition before decelerating model.

This test procedure is designed to maximize run length and minimize the starting response transient damping time [6].

4.3 Observations and Data Reduction

4.3.1 Test Observations

Several difficulties surfaced during the course of testing. Firstly, it proved to be very difficult to maintain a constant waveheight over all of the runs. The waveheight seemed very sensitive to small changes in the tank depth. As a result, waveheights were found to vary over the course of testing by about ±0.3 inches from the target waveheight of 3.75 inches.

It was observed that at very low frequencies, the waves did not have a sinusoidal waveform. This was probably due to the small depth of the wave paddle flap. As a result, no data could be gathered for non-dimensional encounter frequencies of less than 1.25 which corresponds to a full scale wavelength of over 300 ft. This a relatively long wave therefore well out of the resonant range and should not limit the usefulness of the data.

At certain frequencies, it was observed that the water behind the wave paddle would slosh around violently. When this occurred, water would escape around the sides of the wave flap into the main tank and adversely affect the wave form at these frequencies. This problem was most apparent at the wave paddle frequencies corresponding to $\mu_e = 4.25$.

During the calm water natural frequency tests, it was observed that the hull tended to damp out oscillations after only 1 to 2 cycles indicating very heavy damping.

Several interesting observations were noted on the models motions during the wave
tests from the video footage. Firstly, it was visually clear that there was a reduction in pitch motions with the addition of the foil especially at resonance where motion amplitudes were greatest. It also appeared that the phase of the pitch motion with respect to the waves was different when the foil was present than when it was not. Any difference in heave motions was not clearly apparent from the video.

At $\mu_e = 3.0$ the pitching motion was observed to be especially large and violent, especially for the bare hull. In fact, the bare hull frequently submerged its bow into the crest of the next wave at this frequency causing green water to be shipped over the deck. Figure 4.13 is a photograph of the bare hull at $\mu = 3.0$. Notice the bow underwater and the splash rail that had to be fitted to divert the water running over the deck. It was also observed at these frequencies that the flat aft counter tended to slap down on the water. Bow submergence was not so evident in the tests of the model with the foil attached however did occur from time to time. Figure 4.14 is a photograph of a test at $\mu = 3.0$ with the foil attached.

4.3.2 Data Reduction

As mentioned before, the quantities measured during testing in waves were model speed, waveheight, pitch angle, heave displacement and resistance force.

Pitch, heave and resistance data was collected on three channels at a sampling frequency of 40 samples per second for an average run time of 15 seconds. This data was multiplexed and stored as one binary data file per run. Speed was recorded directly off the carriage drive console and was assumed to be constant over the length of the run for which data was gathered. Wave height was measured separately, again at 40 samples per second for at least ten wave cycles before the model passed the wave probe location.

The raw data files for heave, pitch and resistance were then demultiplexed into separate files. These files, along with the waveheight datafile, were then visually edited using a
Figure 4.13: Bow Immersion of Canada II Bare Hull
Figure 4.14: Bow Immersion of Canada II Hull With Anti-Pitch Foil
program developed by the author. Each file was inspected and a window of at least 10 cycles were visually selected that most closely resembled steady state harmonic oscillation.

From the selected window for resistance, the average value of the resistance over an integer number of oscillations within the window was calculated to find $R_{\text{wave}}$ for each run. From the selected windows for heave, pitch and waveheight the average peak-to-peak value was calculated over the number of steady state oscillations within the window giving $z_{p-p}$, $\theta_{p-p}$ and $\zeta_{p-p}$ for each run.

Added resistance for each run was then calculated as:

$$ R_{aw} = R_{\text{wave}} - R_{\text{calm}} $$

(4.62)

where $R_{\text{calm}}$ is measured from the calm water resistance results at the model speed of the wave run. The non-dimensional added resistance coefficient $\sigma_{aw}$ is then calculated using:

$$ \sigma_{aw} = \frac{4R_{aw}}{\rho g z_{p-p}^2 B^2 L} $$

(4.63)

Note that the added resistance is assumed to vary as waveheight squared over the range of waveheights of the tests.

The average heave peak-to-peak value for each run was non-dimensionalized by the waveheight:

$$ \text{Non-dimensional heave amplitude} = \frac{z_{p-p}}{\zeta_{p-p}} $$

(4.64)

and the average pitch peak-to-peak value for each run is nondimensionalized by wave slope:

$$ \text{Non-dimensional pitch amplitude} = \frac{\theta_{p-p}}{\left(\omega^2/g\right)\zeta_{p-p}} $$

(4.65)

where $\omega$ is the wave frequency measured from the wave data.

The results were tabulated for a total of 30 runs, 15 for bare hull and a corresponding 15 for hull with foil and appear in full in Appendix C. The added resistance coefficient,
non-dimensional heave amplitude and non-dimensional pitch amplitude were then plotted against non-dimensional frequency of encounter (see section 2.4.1).

4.4 Experimental Results

4.4.1 Calm Water Tests

Figure 4.15 shows a plot of the calm water resistance for the Canada II model with and without the anti-pitch foil for model speeds corresponding to 6 to 9 knots full scale speed. Notice that the foil causes a significant increase in the calm water resistance at all speeds tested, as would be expected. At the speed of interest for the wave tests, around 4.8 ft/sec or 8 knots full scale, the increase in resistance due to the foil is 25%. It is thought that this rather larger than expected increase is due mostly to the two surface piercing struts rather than the foil itself, though misalignment of the foil may also have contributed. The wave resistance of the struts may be quite significant. The original idea of a single non-surface piercing strut would probably result in a smaller increase in the calm water resistance due to the foil assembly. This difference in calm water resistance does not affect the added resistance as the calm water effect is subtracted out as shown by equation 4.62. However in terms of the overall performance of the boat, it is desirable to have the lowest calm water resistance possible.

Table 4.6 presents the results of the heave and pitch natural frequency tests for the bare hull. In both types of motion, one sees that the non-dimensional natural frequency, \( \mu_e \), increases with an increase in speed from 0 to 8 knots. This shows that the natural frequencies of heave and pitch are strongly influenced by speed. It is probable that the restoring force and moment increase with forward speed due to the ship's own wave system as a result of the non-linear form of the hull. Linear motion theory assumes that the restoring coefficients in the equation of motion are \textit{not} speed dependent and this has
Chapter 4. Experimental Program

Experimental Calm Water Resistance

Figure 4.15: Calm Water Model Resistance For Canada II
Table 4.6: Calm Water Pitch and Heave Natural Frequencies

<table>
<thead>
<tr>
<th></th>
<th>0 Knots</th>
<th>8 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heave</td>
<td>2.80</td>
<td>3.16</td>
</tr>
<tr>
<td>Pitch</td>
<td>3.15</td>
<td>3.55</td>
</tr>
</tbody>
</table>

been shown to be a valid assumption for regular cargo ship hull forms [8]. However it appears that some sort of speed correction to the restoring coefficients is necessary for yacht hull forms.

4.4.2 Heave Motion

Figure 4.16 shows the non-dimensional heave amplitude plotted against the non-dimensional frequency of encounter for the Canada II model without and with the foil. The resonant frequency in heave is at $\mu_e = 3.16$ from Table 4.6. For the bare hull the heave amplitude rises slowly from 1.0 to a maximum of 1.12 at $\mu_e = 3.01$ then begins to drop steeply to 0.14 at $\mu_e = 4.7$.

The heave amplitude response of the hull with foil behaves quite differently. It rises quite rapidly from 1.0 at $\mu_e = 1.0$ to a peak of 1.13 at $\mu_e = 2.25$ which is well below the resonant frequency. It then starts to drop and crosses the bare hull response at $\mu_e = 2.8$. It then continues to drop steeply to 0.1 at $\mu_e = 4.7$.

Before the crossover frequency the heave response of the hull with foil is up to 5% higher than that of the bare hull. Above the crossover frequency of 2.8 the response of the hull with foil is 15-20% lower than the bare hull response.
Figure 4.16: Experimental Heave Amplitude Response
4.4.3 Pitch Motion

Figure 4.17 shows a plot of non-dimensional pitch amplitude from the experiments. The resonant frequency according to the calm water natural frequency tests is $\mu_e = 3.55$.

Looking at the bare hull results, one sees that the pitch response at the lowest tested encounter frequency is just over 1.0. The response then unexpectedly peaks at 1.22 for $\mu = 1.5$ however then drops again and stays relatively level at around 1.1 up to $\mu_e = 2.75$. The pitch amplitude response then drops rapidly from 1.13 at $\mu_e = 2.75$ to 0.125 at $\mu_e = 4.75$.

For the hull with foil attached, one sees that the pitch amplitude response again is just over 1.0 for the lowest tested frequency. It then peaks at a level of 1.14 for $\mu_e = 1.5$. For $\mu_e > 1.5$ the pitch amplitude drops steadily to a value of 0.1 for $\mu_e = 4.68$.

Notice that throughout the frequency range tested, the pitch amplitude response is lower when the foil is fitted than for the bare hull. At lower frequencies this reduction is in the order of 5% while in the resonant region near $\mu_e = 2.75$ and above the reduction in pitch amplitude due to the foil is higher than 20%.

4.4.4 Added Resistance

Figure 4.18 shows the measured added resistance from the experiments. For the bare hull at low frequencies, $\sigma_{aw}$ is very small. At $\mu_e = 2.25$ the added resistance begins to rise rapidly to a peak of $\sigma_{aw} = 3.75$ at $\mu_e = 3.66$. Above this frequency $\sigma_{aw}$ drops rapidly but then hits a secondary local peak of 3.07 at $\mu_e = 4.21$. After this, the added resistance again decreases with increasing $\mu_e$.

Looking at the results of the tests of the hull with foil, one sees that at the lowest frequency $\sigma_{aw}$ is very similar to that of the bare hull. As encounter frequency increases to $\mu_e = 2.25$, the added resistance of the hull with foil is slightly less than that of the bare
Experimental Pitch Response

Figure 4.17: Experimental Pitch Amplitude Response
Chapter 4. Experimental Program

Experimental Added Resistance

Figure 4.18: Experimental Added Resistance
hull. At $\mu_e = 2.25$, $\sigma_{aw}$ begins to rise steeply to a much lower peak than the bare hull of 2.3 at $\mu_e = 3.5$. It then drops and again rises unexpectedly to a second peak of 2.3 at $\mu_e = 4.25$. Above $\mu_e = 4.25$ the added resistance drops rapidly with increasing $\mu_e$. In the region between $\mu_e = 3.25$ and $\mu_e = 4.0$ the reduction in added resistance due to the foil is up to 37%. Notice that the region of maximum added resistance is in the same region of resonance for pitch and heave as one would have expected.

The appearance of a second peak in the added resistance results at $\mu_e = 4.25$ might be connected to the wave form at this frequency. Recall from section 4.4 that near this frequency it was observed that the wave form was adversely affected by the sloshing water behind the wave flap spilling around the flap into the main tank. This might be the cause of the extra resistance on the model.
Chapter 5

Comparison of Theoretical and Experimental Results

In this chapter, the theoretical results calculated in Chapter 3 and the experimental results presented in Chapter 4 are compared and discussed. This is done to determine the applicability of the mathematical model to the problem of predicting the effect of an anti-pitch foil on the motions and added resistance of Canada II.

The validity of the linear strip theory in predicting the heave and pitch motions of the bare hull is discussed first. This is followed by a discussion of the effectiveness of the anti-pitch foil theory in predicting the reduction in motions. Finally, the added resistance results are compared to determine if the Joosen theory gives a useful estimate of the reduction in added resistance that can be expected from the foil.

5.1 Ship Motion Comparison For Bare Hull

Before dealing with the anti-pitch foil at all, it is useful to examine if the strip theory model for ship motions agrees with the experimental results for the Canada II hull itself.

Comparing the heave motion results for the bare hull, shown in Figure 5.19, one finds that for $\mu_e < 2.0$ the theory matches the experimental results very well. For $\mu_e > 3.6$ the theory can also be said to match experiment reasonably, if one extrapolates the theoretical results to continue to drop monotonically. In the mid-frequency range between $\mu_e = 2.0$ and $\mu_e = 3.6$, however, the theory overpredicts the heave amplitude response significantly. Strip theory predicts a pronounced resonant hump at $\mu_e = 3.2$ while the experimental resonant hump is barely noticable.
Chapter 5. Comparison of Theoretical and Experimental Results

Heave Response For Bare Hull
Theory vs. Experiment

Figure 5.19: Heave Amplitude Response For Bare Hull: Theory vs. Experiment
This discrepancy indicates to the author that the Canada II hull is more heavily damped than the theory is capable of predicting, resulting in much lower heave amplitude values at resonance.

It is thought that this extra damping is a result of the non-linearities in the Canada II hull form. In calculating the damping coefficient, $B_{33}$, in the equation of motion, strip theory assumes (among other things) that the sections are slab-sided at the waterline and the hull does not extend out beyond the fore and aft perpendiculars. The Canada II hull form violates these assumptions. Firstly, there is considerable flare to the topsides of the hull, leading to a dramatic change in the section shapes as the hull heaves. In addition, Canada II has extremely long fore and aft overhangs that are not accounted for by the theory. When the hull is heaving, these overhangs become immersed causing large fluctuations in the damping distribution over the hull.

The effect of these non-linearities is small at low frequencies as the heave motion with respect to the wave is also small; therefore not as much of the overhangs or flared topsides would be immersed. However, in the resonant frequency range where the heave motions, and therefore the vertical velocities of the hull, are at their greatest, a large portion of the flared topsides and overhangs would become immersed. Recall from the calm water heave natural frequency test, that the model was observed to be very heavily damped in heave.

Comparing now the pitch motion of the bare hull, shown in Figure 5.20; with the exception of one point at $\mu_e = 1.5$, the theory tends to overpredict the pitch amplitude response throughout the frequency spectrum, especially at the resonant frequency and above.

The strip theory calculation predicts a steadily rising response in the low frequency range up to resonance, while the experiment shows the pitch amplitude response to be relatively flat in this region. It appears from this comparison that strip theory has severely
Chapter 5. Comparison of Theoretical and Experimental Results

Pitch Response For Bare Hull
Theory vs. Experiment

Non-dimensional pitch amplitude

Non-dimensional encounter frequency

Legend
- Experiment—no foil
- Strip Theory—no foil

Figure 5.20: Pitch Amplitude Response For Bare Hull: Theory vs. Experiment
Chapter 5. *Comparison of Theoretical and Experimental Results*

underpredicted the pitch damping, even more so than for heave.

Once again, the large overhangs of the Canada II hull are suspected of being responsible for the difference in pitch motion response as the strip theory does not include their effect on the pitch damping coefficient $B_{55}$. When the hull pitches, the fore and aft overhangs will alternately immerse, changing the underwater shape of the hull and moving the distribution of buoyancy fore and aft. The damping and added mass distribution will also be continually changing with more of it being found in the ends of the ship, well away from the LCG. When the pitch motion amplitudes are large, and the overhangs are immersed even further, the added damping will be increased substantially resulting in the pitching motion being damped out.

This proposal of increased hydrodynamic damping due to the overhangs can be justified by the fact that damping waves were observed to radiate out from the overhangs when they were immersed during the wave tests and the calm water natural frequency test. As has been discussed earlier, these radiating waves indicate additional damping. Note also that at resonance, the bow was observed to fully immerse and water was shipped on deck. This would contribute even further damping at resonance. This helps to explain the experimental results. Recall again that the calm water natural frequency tests showed that pitch was very heavily damped. In fact, it was difficult to measure the natural pitch period as the pitch motion was damped out within 1-2 cycles.

The calm water natural frequency tests also show another interesting result in that the natural frequency of heave and pitch increased with forward speed. This indicates that the restoring coefficients $C_{33}$ and $C_{55}$ are speed dependent. This increase is unexpected and not accounted for by strip theory, which assumes that the restoring coefficients are speed and frequency independent. The added masses, $A_{33}$ and $A_{55}$, increase with speed as shown in section 2.3.4; therefore according to strip theory the natural frequency should *decrease* with speed. The increase in the natural frequency shown by experiment is probably due
Chapter 5. *Comparison of Theoretical and Experimental Results*

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to the ship's own wave system effect on the restoring moment and force [12].

The conclusion of this comparison and discussion of experimental and strip theory results for ship motion is that the strip theory does not seem to be completely adequate as used for predicting the pitch and heave motions of a non-linear type hull form such as Canada II. It appears that in order to overcome the effect of the non-linearities of the hull form, some sort of method is needed that calculates the added mass and damping and exciting force amplitude distributions for the immersed portion of the hull at any instant in time and then time steps through the oscillations to calculate the heave and pitch motions. A method of this sort was developed by Parissis [32] however is extremely time-consuming and not generally used. It also appears that some kind of velocity effect needs to be incorporated into the restoring coefficients.

Although the strip theory results are not very encouraging quantitatively, they do provide us with a first approximation of the heave and pitch motion amplitude response spectrums.

### 5.2 Comparison of Effect of Anti-Pitch Foil on Motions

In this section, the usefulness of the anti-pitch foil mathematical model in predicting the *change* in the heave and pitch motion due to the foil is discussed. Note that the emphasis here has shifted from predicting the amplitudes of the actual motions themselves, to predicting the *change* in motion due to the foil.

As shown in section 3.3.3, the effect of the foil according to the theory, is to change the coefficients in the equation of motion. Thus, even though the strip theory does not give good agreement with experiment on motion amplitudes, it is hoped that the relative effect on the motions due to the foil can still be predicted reasonably using the anti-pitch foil theory. This would be a very useful preliminary design tool for a naval architect
considering the use of an anti-pitch foil.

Figure 5.21 shows the experimental and theoretical results for heave of both the bare hull and the hull with foil. In looking at the effect of the foil, the mathematical model agrees with the experiment in predicting that the heave amplitude response for the hull with foil will be higher than that for the bare hull for $\mu_e < 2.8$. The crossover point agrees quite well. Visually, it appears that above the crossover frequency, the magnitude of the decrease in heave amplitude between the bare hull and the hull with foil is similar for theory and experiment.

Looking at this reduction due to the foil more closely, Figure 5.22 shows the percentage difference in heave amplitude response between the bare hull case and when the foil is attached for both experiment and theory. This figure shows that the theoretical prediction of the reduction in heave motion due to the foil agrees reasonably with experiment. Even though the strip theory does not give a good estimate of the heave motion itself, the effect of the foil on the heave amplitude response is predicted reasonably accurately.

A comparison of the experimental and theoretical results for pitch motion of both the bare hull and the hull with foil attached is shown in Figure 5.23. Notice that the theory matches the experiment in predicting that the pitch amplitude decreases over the entire frequency range as a result of the addition of the foil. A similar plot to Figure 5.22 is shown for pitch in Figure 5.24. Though not as good a match as the heave results, the theory again shows reasonable agreement with experiment concerning the amount of reduction in pitch amplitude that can be expected by adding the anti-pitch foil. In fact, near the resonant region there is actually more reduction in pitch shown by experiment, indicating that the theory gives a conservative estimate of the reduction due to the foil which could be useful as a preliminary design tool.

The result of this analysis is that the mathematical model gives reasonable estimates of the effect of the foil in reducing the heave and pitch motions, in spite of the fact the
Figure 5.21: Heave Amplitude Response With and Without Foil: Theory vs. Experiment
Heave Response Difference
Theory vs. Experiment

Figure 5.22: Heave Amplitude Difference: Theory vs. Experiment
Chapter 5. Comparison of Theoretical and Experimental Results

Experimental and Theoretical Pitch Response

Figure 5.23: Pitch Amplitude Response With and Without Foil: Theory vs. Experiment
Chapter 5. Comparison of Theoretical and Experimental Results

Pitch Response Difference
Theory vs. Experiment

Figure 5.24: Pitch Amplitude Difference: Theory vs. Experiment
strip theory predictions of actual motions are inadequate.

5.3 Added Resistance Comparison

Figure 5.25 shows a comparison of the theoretical and experimental added resistance response for the bare hull. At $\mu_e > 2.2$, the theory matches the experiment fairly well, though in the experimental results there is a small shift to the right in the response spectrum. This shift is probably due to the change in restoring coefficients due to forward speed, as discussed earlier, and the resultant effect of increasing the heave and pitch natural frequencies. This would cause a shift to the right in the experimental added resistance response as its natural frequency would also be increased.

At resonance, the theory underpredicts the peak value of $\sigma_{aw}$. One might have expected that the experimental value of $\sigma_{aw}$ at resonance would be less than the theoretically calculated value since both heave and pitch amplitudes at resonance are much lower in the experiment than predicted by strip theory. However, as noted in section 4.3.1 the bow of the model was frequently observed to immerse under each wave at resonance causing a large amount of green water to be shipped on deck. Energy is expended in carrying and accelerating this water on deck to the velocity of the model and then dumping it again. This would show up as an increase in the resistance force on the model when the bow is submerged which explains the higher values of $\sigma_{aw}$ from experiment.

To examine how the added resistance theory predicts the effect of the foil on the added resistance refer to Figure 5.26 which shows the experimental and theoretical results for the bare hull and the hull with foil. From the figure, the difference between the bare hull and hull with foil cases appears to be similar in theory and experiment. Notice that the theory tends to be more accurate in predicting the height of the resonant peak in the foil case. This is probably because with the foil attached the pitch motions were reduced to
Chapter 5. Comparison of Theoretical and Experimental Results

Figure 5.25: Added Resistance For Bare Hull: Theory vs. Experiment
Experimental and Theoretical
Added Resistance Response

Legend
- ■ Experiment—no foil
- □ Experiment—foil
- ● Joosen—no foil
- ○ Joosen—foil

Figure 5.26: Added Resistance With and Without Foil: Theory vs. Experiment
the point that little bow submergence was observed.

Figure 5.27 shows the percentage difference between the added resistance with the foil attached and that of the bare hull for theory and experiment. Notice that the added resistance is usually reduced by a larger amount in the experiments than the theory predicts. However the theory tends to give a reasonably conservative envelope of the amount of reduction in added resistance that can be expected from the foil. Again, this could be very useful to naval architects when trying to determine the effect of an anti-pitch foil on the performance of a ship in waves.

It should be noted that the larger discrepancy shown in the added resistance difference results as compared to the heave and pitch difference results may be due to variance in the waveheights that was observed during the experiments. This variance should not affect the motion responses as they have been shown to be proportional to waveheight for yacht hull forms as stated in section 2.4.2. However the variance in waveheight may affect the added resistance results adversely as added resistance has been shown to violate the waveheight squared law for some yacht hull forms.
Chapter 5. Comparison of Theoretical and Experimental Results

Added Resistance Response Difference
Theory vs. Experiment

Figure 5.27: Added Resistance Difference: Theory vs. Experiment
6.1 Conclusions

This research has shown through a theoretical and experimental investigation that a passive bow anti-pitch foil is very effective in substantially reducing the heave and pitch motions and added resistance in head seas of the Canada II hull.

Certain characteristics of the Canada II hull such as the flared topsides and extremely long overhangs, violate the linear assumptions made in ship motion theory. These nonlinearities lead to additional hydrodynamic damping that is not accounted for by the linear theory. As a result, ship motion theory overpredicts the response amplitudes of the bare hull for both heave and pitch, especially in the region of resonance.

Experiments show an increase in the heave and pitch natural frequencies with increased speed indicating speed dependent restoring coefficients. Linear theory on the other hand, assumes speed independent restoring coefficients. It appears that the restoring forces are dependent on the dynamic waterline resulting from the ship's own wave system.

It is therefore concluded that linear strip theory is not completely adequate for accurately predicting the hydrodynamic coefficients and consequently the heave and pitch motions of the Canada II bare hull. However, strip theory does provide us with a good first approximation of these motion responses.

The percentage reduction in heave and pitch motions due to the addition of the anti-pitch foil is predicted reasonably well by the oscillating hydrofoil theory. The relative
effect of the foil is in good agreement with experiment in spite of the fact that the actual
heave and pitch amplitudes, as predicted by strip theory, are not in adequate quantitative
agreement.

The Joosen theory gives a good prediction of the added resistance of the bare hull
despite the limitations of strip theory. The percentage reduction in added resistance due
to the addition of the foil is also predicted well by the Joosen theory.

Experimental findings for added resistance may have been adversely affected by prob­lems with the wave making apparatus. Difficulties in maintaining constant wave height
and bad waveforms at certain frequencies may have resulted in experimental error.

It is concluded that the mathematical model is very effective in predicting the per­
centage difference in motion amplitudes and added resistance due to the addition of a
bow anti-pitch foil.

For both heave and pitch motions and added resistance, the model tends to provide a
conservative envelope of the expected reduction due to the foil. The mathematical model
therefore provides a very useful preliminary design tool for naval architects, where no such
tool existed before.

6.2 Recommendations

This study was deliberately constrained in order to obtain useful conclusions. Therefore,
recommendations for future study mostly involve the freeing of these constraints.

The effect of heel angle on added resistance should be investigated as previous re­
searchers have obtained conflicting results, especially in recent studies. Heel angle may
also affect the efficiency of the anti-pitch foil with respect to reducing the heave and pitch
motions.

The motion and added resistance response spectra for regular waves should be applied
to actual coastal sea spectra to determine the predicted effectiveness of the foil in real random sea conditions. Irregular wave model tests would also be useful in this respect.

The effect of yaw angle on the foil should be studied to determine if the strut and foil produce yaw induced moments that might adversely affect steering and directional stability of the vessel.

It is recommended that work be done to optimize the strut and foil design in order to reduce the rather large increase in calm water resistance that was observed in this study. It is thought that one non-surface piercing strut rather than two surface piercing struts would go far in solving this problem. Any decrease in calm water resistance will improve overall performance which is a primary goal of the naval architect.

In terms of future theoretical work, the use of the Gerritsma and Beukelman technique of calculating added resistance should be considered as it may provide more accurate results (at the expense of increased complexity).

A theoretical investigation into the non-linear effect of long fore and aft overhangs on the hydrodynamic coefficients and the dynamic waterline effect on the restoring coefficients is recommended in order to improve the quantitative predictions of heave and pitch amplitudes.

It would be useful to extend the theoretical results to higher frequencies. Use of the finite element method may overcome the problem of breaking down at high frequencies that is inherent in the formulation of boundary element methods.

This study has provided the first step towards the possible use of bow anti-pitch foils to reduce motions and added resistance of small marine vehicles, in particular racing yachts. Of course there are several practical problems to consider before attempting a full scale trial of an anti-pitch foil. For instance, the effect of the weight of the system near the bow, the structural integrity of the hull and foil assembly and the effect of the foil on steering and directional stability, to name a few. In addition, the performance in
calm water can not be forgotten as the foil can only adversely affect this. Perhaps a form of retractable foil that is only deployed in rough water would be useful in this respect.

In summary, it appears from the findings of this study that the concept of using a passive bow anti-pitch foil on racing yachts and other small marine vehicles to reduce vertical motions and added resistance in waves is well worth pursuing.
Bibliography


California, 1971.


Appendix A

Sample Designer Questionnaire

What do you feel are the most important boat parameters and factors that contribute to the added resistance of a yacht due to waves?

In approximate order of importance:

1) Sailing length
2) Volume distribution
3) Weight distribution
4) Freeboard
5) Hull flare

These are all areas where correct characteristics will reduce added resistance due to waves; most of them are self-evident, for instance greater length will generally result in less motion, hence less added resistance.

Do you think a reduction in added resistance due to waves would result in a significant increase in upwind performance or is it too small a factor?

In a significant seaway, added resistance due to waves is certainly important. Obviously, the converse is also true; in no waves, there is no added resistance.
Appendix A. Sample Designer Questionnaire

What would you estimate the added resistance due to waves to be of the total resistance of a yacht travelling upwind at 8 knots into a Fremantle Seaway? (%)

Roughly 20%.

Did you have a segment of your design program aimed specifically at reducing added resistance of the yacht due to waves?

No. Money and time constraints prevented it.
How important a factor was added resistance due to waves in the development of your 12-metre design? What aspects of your design specifically addressed this problem?

Not too important. Unable to test, we relied on flat water performance and tried to ensure sufficient flare and freeboard to keep most of the waves from coming aboard.

Did you perform towing tank waves tests during your 12-metre design program?

No.

If so, to what extent did you actually use the results of these tests in your design process?

N/A
Do you feel the relative performance of different models in towing tank tests in calm water can be extrapolated to rough water with any confidence?

No. But in a highly restricted class like Twelve Meters, where weight distribution is pretty well controlled and length tends to get stereotyped, there are not too many controllable factors left to alter relative performance in waves. Offshore designs are probably a more fertile (if less well financed) field for advances in the reduction of added resistance due to waves.

Did you carry out computer ship motion prediction simulations as part of your design program?

No. Time and money again.

If so, what programs did you use and were the results useful or inconclusive?

N/A
Did you use or contemplate using any sort of pitch reducing technique during sailing in Fremantle?

Yes. Buddy was careful to steer around the waves (later in the regatta, when both the wave height and the crew's ability had increased significantly over the early rounds).

Do you have any papers on the subject of yacht performance in waves? If so, please list the titles and authors and in what publication I would find it.

The Performance of Sailing Yachts in Oblique Seas by David Pedrick
Marine Technology, October 1974

Surfing: Motions of a Vessel Running in Large Waves by John s. Letcher, Jr.
The Third Chesapeake Sailing Yacht Symposium (1977)
Appendix B

Model Foil Drawings
MODEL FOIL ASSEMBLY
### Appendix C

**Experimental Data**

Results of experiments on Canada 2 with and without foil

| RUN NUMBER | WAVE FREQ HZ | WAVE FREQ RAD/SEC | WAVE HEIGHT INS | WAVE LENGTH FT | WAVEMODEL VELOCITY M/S | VELOCITY FREQUENCY RAD/S | FREQUENCY WTS HZ | MEASURED CALC. | ENCLOSED ENCLOSED VELOCITY RAD/S WTS SHIP VELOCITY KNOTS |
|------------|--------------|--------------------|-----------------|---------------|------------------------|--------------------------|-----------------|--------------|--------------|---------------|----------------|-----------------|
| 19         | 0.349        | 2.19               | 3.767           | 42.08         | 1.46                   | 4.79                     | 0.463           | 0.475        | 2.98         | 8.02          |                 |                 |
| 20         | 0.41         | 2.58               | 3.742           | 30.49         | 1.46                   | 4.79                     | 0.567           | 0.574        | 3.61         | 8.02          |                 |                 |
| 21         | 0.453        | 2.85               | 3.755           | 24.37         | 1.46                   | 4.79                     | 0.645           | 0.659        | 4.14         | 8.02          |                 |                 |
| 22         | 0.505        | 3.17               | 3.795           | 20.10         | 1.46                   | 4.79                     | 0.743           | 0.751        | 4.72         | 8.02          |                 |                 |
| 23         | 0.552        | 3.47               | 3.839           | 16.82         | 1.46                   | 4.79                     | 0.837           | 0.843        | 5.30         | 8.02          |                 |                 |
| 24         | 0.597        | 3.75               | 3.862           | 14.38         | 1.46                   | 4.79                     | 0.930           | 0.928        | 5.83         | 8.02          |                 |                 |
| 25         | 0.638        | 4.01               | 3.746           | 12.59         | 1.46                   | 4.79                     | 1.018           | 1.023        | 6.43         | 8.02          |                 |                 |
| 26         | 0.682        | 4.29               | 3.777           | 11.02         | 1.46                   | 4.79                     | 1.117           | 1.119        | 7.03         | 8.02          |                 |                 |
| 27         | 0.722        | 4.54               | 2.747           | 9.83          | 1.46                   | 4.79                     | 1.209           | 1.21         | 7.60         | 8.02          |                 |                 |
| 28         | 0.758        | 4.76               | 3.886           | 8.92          | 1.46                   | 4.79                     | 1.295           | 1.36         | 8.55         | 8.02          |                 |                 |
| 29         | 0.799        | 5.02               | 3.834           | 8.03          | 1.46                   | 4.79                     | 1.396           | 1.393        | 8.75         | 8.02          |                 |                 |
| 30         | 0.837        | 5.26               | 4.045           | 7.32          | 1.46                   | 4.79                     | 1.492           | 1.484        | 9.32         | 8.02          |                 |                 |
| 31         | 0.871        | 5.47               | 4.212           | 6.76          | 1.46                   | 4.79                     | 1.580           | 1.567        | 9.85         | 8.02          |                 |                 |
| 32         | 0.907        | 5.70               | 3.747           | 6.22          | 1.46                   | 4.79                     | 1.676           | 1.661        | 10.44        | 8.02          |                 |                 |
| 33         | 0.939        | 5.90               | 4.011           | 5.81          | 1.46                   | 4.79                     | 1.763           | 1.754        | 11.02        | 8.02          |                 |                 |
| 34         | 0.35         | 2.20               | 3.8              | 41.84         | 1.44                   | 4.72                     | 0.463           | 0.463        | 2.91         | 7.91          |                 |                 |
| 35         | 0.405        | 2.54               | 3.8              | 31.24         | 1.47                   | 4.82                     | 0.559           | 0.563        | 3.54         | 8.07          |                 |                 |
| 36         | 0.455        | 2.86               | 3.803           | 24.75         | 1.41                   | 4.62                     | 0.642           | 0.654        | 4.11         | 7.74          |                 |                 |
| 37         | 0.503        | 3.16               | 3.873           | 20.26         | 1.49                   | 4.89                     | 0.744           | 0.745        | 4.68         | 8.18          |                 |                 |
| 38         | 0.551        | 3.46               | 3.9              | 16.88         | 1.46                   | 4.79                     | 0.835           | 0.838        | 5.27         | 8.02          |                 |                 |
| 39         | 0.6          | 3.77               | 4.086           | 14.24         | 1.43                   | 4.69                     | 0.929           | 0.927        | 5.82         | 7.85          |                 |                 |
| 40         | 0.642        | 4.03               | 3.938           | 12.43         | 1.44                   | 4.72                     | 1.022           | 1.016        | 6.38         | 7.91          |                 |                 |
| 41         | 0.681        | 4.28               | 3.906           | 11.05         | 1.46                   | 4.79                     | 1.114           | 1.121        | 7.04         | 8.02          |                 |                 |
| 42         | 0.721        | 4.53               | 2.851           | 9.66          | 1.46                   | 4.79                     | 1.207           | 1.195        | 7.51         | 8.02          |                 |                 |
| 43         | 0.759        | 4.77               | 4.027           | 8.90          | 1.44                   | 4.72                     | 1.290           | 1.295        | 8.14         | 7.91          |                 |                 |
| 44         | 0.798        | 5.01               | 3.883           | 8.65          | 1.44                   | 4.72                     | 1.385           | 1.381        | 8.68         | 7.91          |                 |                 |
| 45         | 0.836        | 5.25               | 4.103           | 7.33          | 1.47                   | 4.82                     | 1.494           | 1.474        | 9.26         | 8.07          |                 |                 |
| 46         | 0.874        | 5.49               | 4.293           | 6.71          | 1.46                   | 4.79                     | 1.588           | 1.574        | 9.69         | 8.02          |                 |                 |
| 47         | 0.909        | 5.71               | 3.707           | 6.20          | 1.45                   | 4.76                     | 1.676           | 1.654        | 10.39        | 7.96          |                 |                 |
| 48         | 0.942        | 5.92               | 3.897           | 5.78          | 1.46                   | 4.79                     | 1.771           | 1.742        | 10.95        | 8.02          |                 |                 |
### Appendix C. Experimental Data

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### Appendix C. Experimental Data

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