A POTENTIAL FLOW THEORY
FOR AIRFOIL SPOILERS

by

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B.A.Sc. University of British Columbia, 1964

M.A.Sc. University of British Columbia, 1966

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

in the Department
of
Mechanical Engineering

We accept this thesis as conforming to the
required standard

THE UNIVERSITY OF BRITISH COLUMBIA

February, 1970

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ABSTRACT

A theory is presented for the calculation of the pressure distribution and lift for arbitrary thick airfoils fitted with normal upper surface spoilers in two dimensional incompressible flow. Airfoil shape and angle of attack and spoiler location and height are arbitrary and unrestricted. The theory uses a sequence of conformal transformations from a basic flow past a circle, with one or two sources on that part of the circle corresponding to the surface of the airfoil and spoiler exposed to the wake. The flow inside the separating streamlines is ignored, and the upper surface pressure downstream of the spoiler is taken as an empirical parameter, assumed constant. The sources in the wake permit satisfaction of Kutta conditions with the desired pressure at the spoiler tip and airfoil trailing edge. Features of the theory include good prediction of loading distribution, a finite wake width and a pressure distribution on the separating streamlines decreasing asymptotically towards the free stream value at infinity. The theoretical predictions are compared with lift and pressure measurements on a Joukowski airfoil of 11% thickness and 2.4% camber, and with lift measurements on a 14% thick Clark Y airfoil. Both airfoils were tested through a range of angle of attack with spoilers of 5 and 10% chord height, each at several locations. Good agreement is found.

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ACKNOWLEDGMENT

The author wishes to express his sincere gratitude to Professor G. V. Parkinson for his encouragement and valuable direction throughout this study. It has indeed been a privilege to work with him.

Mr. R. Hirschfield and Mr. M. A. Lundberg, summer research assistants, designed the wind tunnel models and assisted with the wind tunnel measurements. Their help is gratefully acknowledged.

The author would also like to thank the Computing Center of The University of British Columbia for the use of their facilities.

The research for this thesis was supported in part by the Defence Research Board of Canada, Grant number 9551-13.

LIST OF SYMBOLS

A_n	=	Fourier coefficients defined in equation (20)
B_n	=	Fourier coefficients defined in equation (20)
c	=	Chord of airfoil
C_D	=	Drag coefficient
C_L	=	Lift coefficient
C_P	=	Pressure coefficient
C_{PA}	=	Pressure coefficient at trailing edge
C_{Pc}	=	Pressure coefficient at spoiler tip
$C_{P\sigma}$	=	Incremental pressure coefficient in the wake
E	=	Spoiler chordwise location
F	=	Complex potential
H	=	Wind tunnel width
h	=	Spoiler height
i	=	$(-1)^{\frac{1}{2}}$
P	=	Pressure
P_∞	=	Free stream pressure
Q_L	=	Strength of lower source
Q_U	=	Strength of upper source
q_L	=	Nondimensional strength of lower source
q_U	=	Nondimensional strength of upper source
R_1	=	Radius of circle in the Z_2 plane
R_2	=	Distance between spoiler tip and center of circle in the Z_2 plane
U	=	Free stream velocity in the Z_1 plane

- V_2 = Free stream velocity in the Z_4 plane
 w = Complex velocity
 X, Y = Coordinates of airfoil in the Z_1 plane
 X_c, Y_c = Coordinates of spoiler tip in the Z_1 plane
 Z_0 = Location of center of circle in the Z_2 plane
 Z_1 = Complex variable defining the Z_1 plane
 Z_2 = Complex variable defining the Z_2 plane
 Z_3 = Complex variable defining the Z_3 plane
 Z_4 = Complex variable defining the Z_4 plane
 Z_L = Location of lower source in the Z_4 plane
 Z_U = Location of upper source in the Z_4 plane
 α = Angle of attack
 α_0 = Zero lift angle for basic airfoil
 α_{l_0} = Zero lift angle for airfoil-spoiler combination
 Γ = Circulation strength
 γ = Nondimensional circulation strength
 δ_L = Angular position of lower source in the Z_4 plane
 δ_U = Angular position of upper source in the Z_4 plane
 $\epsilon(\varphi)$ = Defined in equation (21)
 $\mathfrak{J}$ = Complex variable defining the true circle plane (Fig. 6)
 $\mathfrak{J}'$ = Complex variable defining the off circle plane (Fig. 6)
 θ = Angular variable in Z_4 plane
 θ_0 = Angle defining spoiler location in the Z_2 plane
 θ_A = Angle defining the trailing edge in the Z_4 plane
 θ_c = Angle defining the spoiler tip in the Z_4 plane
 μ = Angular variable in the $\mathfrak{J}'$ plane

$$\xi = R_2/R_1$$

$$\rho = \text{Free stream density}$$

$$\phi_0 = \text{Angle defined in Fig. 1a.}$$

$$\psi = \text{Angular variable in the } \xi \text{ plane}$$

$$\gamma = \text{Function related to the polar radius in the } \xi' \text{ plane}$$

$$\gamma_0 = \text{Constant related to the polar radius in the } \xi \text{ plane}$$

I INTRODUCTION

The current development of V/STOL aircraft has caused a renewed interest in the investigation of the aerodynamic characteristics of upper surface spoilers, or spanwise fences. These devices are used on wings for roll control, if deflected asymmetrically, or for high drag generation if deflected symmetrically. Knowledge of their sectional characteristics is fundamental to an understanding of their performance with finite span on wings. Two dimensional tests of airfoils with spoilers can be carried out in wind tunnels, but, as in all wing aerodynamics, a usable theoretical model is most desirable.

Since the upper surface flow downstream of a spoiler is separated, and since there are presently no theories available to correctly predict base pressure in such separated flows, clearly a theoretical model will require at least one empirical parameter. Also, although the transient performance of airfoils with spoilers after spoiler actuation, and the performance of spoilers on airfoils with slotted flaps, are of great interest, it is necessary to consider a simpler problem first, the steady two dimensional flow past a solid airfoil with a fixed spoiler.

The most successful of existing theoretical solutions to this problem is by Woods (1, 2), who uses a linear perturbation free streamline potential theory to predict the incremental pressure distribution and the lift, drag, and incremental pitching moment

on an airfoil with spoiler in subsonic flow as a function of airfoil incidence and spoiler height, angle to airfoil surface, chordwise position, and base pressure. As is usual with linear perturbation theories, it is restricted to thin airfoils at low incidence with small spoilers. Woods recognized that the airfoil boundary layer would reduce the effective height of a spoiler, and Barnes (3) used the results of wind tunnel experiments to devise an empirical modification to Woods' theory for incompressible flow in which the effective spoiler height is determined by the boundary layer displacement thickness on the basic airfoil at the spoiler location. Barnes also proposed an empirical equation for predicting spoiler base pressure from the airfoil-spoiler geometry, and demonstrated good agreement with wind tunnel measurements on two airfoils of the predicted lift and pitching moment by the modified Woods theory. Barnes' paper gives a useful list of references to other experimental and theoretical work on spoilers. An additional special but relevant problem of an airfoil with a split flap and suction has been treated by Mandl (4).

Although the present theory also uses conformal mapping of the two dimensional irrotational airfoil flow field, it is quite different in approach from that of Woods, and is not a perturbation theory, so that it offers the advantage that airfoil incidence, thickness, and camber as well as spoiler height and location are unrestricted. However, only normal spoilers on airfoils are considered. The flow is two dimensional and incompressible for which Laplace's equation is applicable. The effect of the airfoil boundary layer is not considered, although an empirical

modification like that of Barnes could be made. In the following sections, the theory is developed, and applied to a Joukowski airfoil of 11% thickness and 2.4% camber and to a 14% thick Clark Y airfoil.

II THEORY

2.1 Theory for Joukowski Airfoils

In the first part of the analysis, a thick cambered Joukowski airfoil is used to develop the theory, since the airfoil is mapped by a simple conformal transformation from a circle. The fact that Joukowski airfoils have a cusped trailing edge is also desirable for the theory, since this permits smooth separation, with a specified velocity, from the trailing edge. In the analysis airfoil thickness, camber and incidence as well as spoiler height and position are arbitrary and unrestricted. However, only normal spoilers are considered.

2.1.1 Transformations

A thick, cambered Joukowski airfoil in the Z_1 plane (Fig. 1a) is mapped from a circle of radius R_1 and center at Z_0 in the Z_2 plane by the well known Joukowski transformation:

$$Z_1 = Z_2 + \frac{1}{Z_2} \quad \dots(1)$$

The value of the complex quantity Z_0 is determined from the thickness and the camber of the given airfoil. The magnitude of the radius R_1 can be computed from the knowledge of Z_0 and geometrical considerations.

Figure 1a. Complex Transform Planes.

Figure 1b. Complex Transform Planes.

The spoiler on the airfoil is introduced in the Z_2 plane as a radial straight line segment, which when mapped onto the Z_1 plane becomes a normal spoiler with very slight, but not inappropriate curvature for practical spoiler heights.

The chordwise location of the spoiler in the Z_1 plane E is determined by the angular variable θ_0 in the Z_2 plane. This relation is shown in Fig. 2. Also the height of the spoiler in the Z_1 plane is related to the length of the straight segment in the Z_2 plane as well as the spoiler position E ; this is illustrated in Fig. 3 showing the height of the spoiler as a function of $\frac{R_2}{R_1}$ and E .

Next, the circle with spoiler is mapped onto a slit along the real axis in the Z_3 plane (Fig. 1b):

$$Z_3 = \frac{Z_2 - Z_0}{R_1 e^{i\theta_0}} + \frac{R_1 e^{i\theta_0}}{Z_2 - Z_0} \quad \dots(2)$$

This transformation is made up of a clockwise rotation through θ_0 , a translation to shift the center of the circle to the origin, a scaling to reduce the radius to unity and finally a Joukowski transformation which maps the complete contour onto the slit in the Z_3 plane.

Then the slit is mapped onto a circle of unit radius in the Z_4 plane where the uniform flow is parallel to the real axis:

Figure 2. Spoiler Geometry.

Figure 3. Spoiler Geometry.

$$Z_3 = \left[\frac{1}{2} \frac{\xi^2 + 1}{\xi} - 1 \right] + \frac{1}{2} \left[\frac{1}{2} \frac{\xi^2 + 1}{\xi} + 1 \right] \left[Z_4 e^{i(\alpha - \theta_0)} + \frac{e^{i(\theta_0 - \alpha)}}{Z_4} \right] \quad \dots(3)$$

where $\xi = \frac{R_2}{R_1}$. This transformation is made up of a translation, scaling, Joukowski, and rotation. In the Z_4 plane the spoiler becomes part of the circle and the flow separation points A, C are fixed on the perimeter.

The location of these separation points in the Z_4 plane is achieved as follows:

In the Z_2 plane

$$C = Z_0 + R_2 e^{i\theta_0}$$

and $A = +1$

In the Z_3 plane

$$C = \frac{\xi^2 + 1}{\xi}$$

and $A = 2 \cos \phi_0$

Finally, in the Z_4 plane

$$\theta_c = \theta_0 - \alpha \quad \dots(4)$$

$$\text{and } \theta_A = \theta_c - \cos^{-1} \left[\frac{2 \cos \phi_0 + \left(1 - \frac{1}{2} \frac{\xi^2 + 1}{\xi}\right)}{\left(1 + \frac{1}{2} \frac{\xi^2 + 1}{\xi}\right)} \right] \quad \dots(5)$$

Here θ_c and θ_A are the angular positions of the separation points as defined in Fig. 4. They can be calculated for a given airfoil-spoiler combination.

The combined transformation derivative $\frac{dz_1}{dz_4}$ can be evaluated readily:

$$\frac{dz_1}{dz_4} = \left[\frac{z_2^2 - 1}{z_2^2} \right] \times \left[\frac{R_1 e^{i\theta_0} (z_2 - z_0)^2}{(z_2 - z_0)^2 - R_1^2 e^{2i\theta_0}} \right] \times$$

$$\times \left[\frac{e^{i(\alpha - \theta_0)}}{2} \left(\frac{1}{2} \frac{z_4^2 + 1}{z_4^2} + 1 \right) \left(\frac{z_4^2 - e^{2i(\theta_0 - \alpha)}}{z_4^2} \right) \right] \quad \dots (6)$$

It can be observed that $\frac{dz_1}{dz_4}$ has three simple zeros and two simple poles in the region corresponding to the flow field and the airfoil boundary:

- 1 - a zero at $z_2 = +1$ (trailing edge)
 - 2 - a zero at $z_4 = e^{i(\theta_0 - \alpha)}$ (spoiler tip)
 - 3 - a pole at $z_2 = z_0 + R_1 e^{i\theta_0}$ (spoiler base)
 - 4 - a pole at $z_2 = z_0 - R_1 e^{i\theta_0}$
 - 5 - a zero at $z_4 = -e^{i(\theta_0 - \alpha)}$
- (on airfoil surface)

Hence points A and C are critical points of the combined transformation from Z_1 to Z_4 with simple zeros of $\frac{dz_1}{dz_4}$ at both

points. Accordingly, angles are doubled at points A and C, and stagnation streamlines at A and C in the Z_4 plane would become tangential separation streamlines at A and C in the Z_1 plane. Similarly, the pole at the spoiler base will result in the prediction of a stagnation point at D in the Z_1 plane. The remaining simple zero and simple pole on the airfoil surface coincide on the boundary, and cancel.

2.1.2 Mathematical Flow Model

The actual flow about the airfoil separates from the spoiler tip C and the trailing edge A, and the resulting wake is found, experimentally, to be at nearly constant pressure over the back face of the spoiler and the upper surface of the airfoil behind the spoiler. Since there are presently no theories available to correctly predict base pressure in such separated flows, clearly a theoretical model will require specification of the base pressure coefficient.

This situation suggests a free streamline model for the flow outside the wake of the type used by Woods (1) or by Roshko (5). However, the geometrical difficulties of the present problem appear to rule out this approach, and instead the flow exterior to the airfoil and its wake boundaries (the streamlines separating from the spoiler tip and the trailing edge) is modelled by adding to the basic flow past the transform circle in the Z_4 plane, suitable singularities inside the region corresponding to the wake to represent its effect on the outer flow.

There are advantages to an open wake representation, since

the bounding streamlines will probably give reasonable approximations to the trajectories of vortices formed from the actual bounding shear layers, and these trajectories are of interest because of the effect of wake vortices from wing spoilers on downstream aerodynamic surfaces. Accordingly, source singularities were chosen, and it was found that they had to be located on the body surface, in the wake region, to satisfy the separation pressure boundary conditions.

The flow model in the Z_4 plane (Fig. 4) consists of uniform flow parallel to the real axis past a circular cylinder of unit radius with separation points at A (the trailing edge) and C (the spoiler tip). Separation is achieved by adding either one or two sources on the surface of the cylinder between points A and C. Both possibilities have been considered, and they will be referred to as the 1-source and the 2-source models respectively. Circulation about the cylinder is introduced, for lift control, by adding a vortex at the center of the circle. When a source is placed on the surface, an image source must be added on the surface and a sink at the center of the circle in order to satisfy the usual boundary condition on the cylinder. This is equivalent to placing a double source at the surface and a sink at the center as a limiting case for a source outside with the image and the sink inside.

For the 1-source model, the strength of the lower source in the 2-source model is set equal to zero, consequently the dependence of the flow equations on the lower source and its position vanishes. The complex potential for the 2-source model is:

Figure 4. Singularities in Z_4 - plane.

$$\begin{aligned}
F(z_4) = & V_2 \left(z_4 + \frac{1}{z_4} \right) + i \frac{\Gamma}{2\pi} \ln(z_4) + \frac{Q_U}{\pi} \ln(z_4 - z_U) \\
& + \frac{Q_L}{\pi} \ln(z_4 - z_L) - \frac{(Q_U + Q_L)}{2\pi} \ln(z_4) \quad \dots(7)
\end{aligned}$$

and the complex velocity:

$$\begin{aligned}
W(z_4) = \frac{dF}{dz_4} = & V_2 \left(1 - \frac{1}{z_4^2} \right) + i \frac{\Gamma}{2\pi} \frac{1}{z_4} + \frac{Q_U}{\pi} \frac{1}{z_4 - z_U} \\
& + \frac{Q_L}{\pi} \frac{1}{z_4 - z_L} - \frac{(Q_U + Q_L)}{2\pi} \frac{1}{z_4} \quad \dots(8)
\end{aligned}$$

where V_2 is the velocity of the uniform incident flow in the Z_4 plane, related to the corresponding velocity in the physical plane as follows:

$$V_2 = \frac{R_1}{2} \left[1 + \frac{1}{2} \frac{\xi^2 + 1}{\xi} \right] U$$

Variables Γ , Q_U , Q_L , z_U , z_L are the circulation, source strengths and the source positions respectively. On the surface of the cylinder:

$$z_4 = e^{i\theta}, \quad z_U = e^{i\delta_U}, \quad z_L = e^{i\delta_L}$$

therefore the dimensionless complex velocity on the cylinder surface is:

$$\frac{W(z_4)}{V_2} = \frac{e^{-i\theta}}{z i} \left[-4 \sin \theta - 2 \gamma + q_u \cot\left(\frac{\theta - \delta_u}{2}\right) + q_L \cot\left(\frac{\theta - \delta_L}{2}\right) \right] \quad \dots (9)$$

where

$$q_u = \frac{Q_u}{\pi V_2}, \quad q_L = \frac{Q_L}{\pi V_2}, \quad \gamma = \frac{\Gamma}{2\pi V_2}$$

The total number of unknowns for the 2-source model is five ($\gamma, q_u, q_L, \delta_u, \delta_L$); and for the 1-source model is three (γ, q_u, δ_u).

2.1.3 Boundary Conditions

The flow given by equation (7) satisfies the boundary condition that the flow is uniform at infinity and the airfoil and spoiler boundary AEDC is a streamline. It remains to satisfy the condition of separation at points A and C in the Z_1 plane, corresponding to stagnation points at A and C in the Z_4 plane, and to specify the value of the pressure coefficient at these points in the Z_1 plane.

The condition of separation in the Z_4 plane is satisfied if:

$$W(z_4) = 0 \quad \text{for} \quad \theta = \theta_c \quad \text{and} \quad \theta = \theta_A.$$

This leads to

$$q_u \cot\left(\frac{\theta_c - \delta_u}{2}\right) + q_L \cot\left(\frac{\theta_c - \delta_L}{2}\right) - 4 \sin \theta_c - 2\gamma = 0 \quad \dots(10)$$

and

$$q_u \cot\left(\frac{\theta_A - \delta_u}{2}\right) + q_L \cot\left(\frac{\theta_A - \delta_L}{2}\right) - 4 \sin \theta_A - 2\gamma = 0 \quad \dots(11)$$

The pressure coefficient at the two separation points in the physical plane is specified through the use of Bernoulli's equation which applies to the flow outside the wake, including the separation streamlines, so that

$$P - P_\infty = \frac{\rho}{2} (U^2 - |W|^2)$$

or

$$C_p = \frac{P - P_\infty}{\frac{1}{2} \rho U^2} = 1 - \frac{|W|^2}{U^2} \quad \dots(12)$$

The values of C_{p_A} and C_{p_C} are made equal to the measured pressure coefficient in the wake, thus specifying the values of $\left| \frac{W(z_1)}{U} \right|_A$

and $\left| \frac{W(z_1)}{U} \right|_C$ by equation (12). This leads to two new boundary

conditions imposed on the flow field.

In general the complex velocity in the Z_1 plane is related to the complex velocity in the Z_4 plane through the transformation derivative:

$$W(z_1) = \frac{W(z_4)}{dz_1/dz_4}$$

This expression is an indeterminate form at the spoiler tip and the trailing edge, since at these points, both $W(z_4)$ and $\frac{dz_1}{dz_4}$ are zero.

At the spoiler tip C the critical term in the expression for

$\frac{dz_1}{dz_4}$ in equation (6) is given by

$$(z_4 - e^{i(\theta_0 - \alpha)})$$

Using equation (4) and the fact that on the cylinder in the Z_4 plane $Z_4 = e^{i\theta}$, the critical term becomes

$$\begin{aligned} (e^{i\theta} - e^{i\theta_c}) &= e^{i(\frac{\theta + \theta_c}{2})} \left(e^{i(\frac{\theta - \theta_c}{2})} - e^{-i(\frac{\theta - \theta_c}{2})} \right) , \\ &= 2i e^{i(\frac{\theta + \theta_c}{2})} \sin\left(\frac{\theta - \theta_c}{2}\right) . \end{aligned}$$

Since it is the magnitude of the velocity at separation that relates to the pressure coefficient, the variation of $\frac{dz_1}{dz_4}$ in the neighbourhood of C can be written

$$\frac{dz_1}{dz_4} \propto z \sin\left(\frac{\theta - \theta_c}{2}\right)$$

Thus the velocity near the spoiler tip in the physical plane is

determined by the proportionality

$$\left| \frac{W(z_1)}{U} \right| \propto \left| \frac{-2 \sin \theta - \gamma + \frac{q_u}{2} \cot\left(\frac{\theta - \delta_u}{2}\right) + \frac{q_L}{2} \cot\left(\frac{\theta - \delta_L}{2}\right)}{2 \sin\left(\frac{\theta - \theta_c}{2}\right)} \right|$$

which is an indeterminate form at $\theta = \theta_c$. Using L'Hospital's rule and letting $\theta \rightarrow \theta_c$

$$\left| \frac{W(z_1)}{U} \right|_c \propto \left| 2 \cos \theta_c + \frac{q_u}{4} \csc^2\left(\frac{\theta_c - \delta_u}{2}\right) + \frac{q_L}{4} \csc^2\left(\frac{\theta_c - \delta_L}{2}\right) \right|$$

The constant of proportionality is made of the remainder of the transformation derivative evaluated at point C and the ratio $\frac{V_z}{U}$, giving the magnitude of the velocity of separation from the spoiler tip in the physical plane as

$$\left| \frac{W(z_1)}{U} \right|_c = \left[\frac{\xi^2 - 1}{2 \xi^2} \right] \times \left| \frac{(R_1 \xi e^{i\theta_0} + z_0)^2}{(R_1 \xi e^{i\theta_0} + z_0)^2 - 1} \right| \times$$

$$\times \left| 2 \cos \theta_c + \frac{q_u}{4} \csc^2\left(\frac{\theta_c - \delta_u}{2}\right) + \frac{q_L}{4} \csc^2\left(\frac{\theta_c - \delta_L}{2}\right) \right| \quad \dots(13)$$

Similarly, at the trailing edge A the critical term in the

expression for $\frac{dz_1}{dz_4}$ in equation (6) is given by $(Z_2 - 1)$. In order to evaluate the indeterminate form, Z_2 must be related to the angular variable θ in the Z_4 plane. From equation (2)

$$Z_2 = \frac{R_1 e^{i\theta_0}}{2} \left[Z_3 - i\sqrt{4 - Z_3^2} \right] + Z_0$$

where Z_3 is given by equation (3) for points on the slit as follows

$$Z_3 = \left[1 + \frac{\xi^2 + 1}{2\xi} \right] \cos(\theta - \theta_c) + \left[\frac{\xi^2 + 1}{2\xi} - 1 \right] \quad \dots(14)$$

Thus the variation of $\frac{dz_1}{dz_4}$ in the neighbourhood of A can be written as

$$\frac{dz_1}{dz_4} \propto \frac{R_1}{2} e^{i\theta_0} \left[Z_3 - i\sqrt{4 - Z_3^2} \right] + Z_0 - 1$$

where Z_3 is defined by equation (14). The velocity near the trailing edge in the physical plane is determined by the proportionality

$$\left| \frac{W(z_1)}{U} \right| \propto \left| \frac{-z \sin \theta + \frac{q_0}{2} \cot\left(\frac{\theta - \delta_0}{2}\right) + \frac{q_L}{2} \cot\left(\frac{\theta - \delta_L}{2}\right) - \gamma}{\frac{R_1}{2} e^{i\theta_0} \left[Z_3 - i\sqrt{4 - Z_3^2} \right] + Z_0 - 1} \right|$$

Using L'Hospital's rule to evaluate the indeterminacy at $\theta \rightarrow \theta_A$

$$\left| \frac{w(z_1)}{U} \right|_A \propto \left| \frac{2 \cos \theta_A + \frac{q_U}{4} \csc^2 \left(\frac{\theta_A - \delta_U}{2} \right) + \frac{q_L}{4} \csc^2 \left(\frac{\theta_A - \delta_L}{2} \right)}{2 \left[1 + \frac{\xi^2 + 1}{2\xi} \right]} \right|$$

Again the constant of proportionality is the remainder of the transformation derivative evaluated at point A and the ratio $\frac{V_2}{U}$. Hence, the magnitude of the velocity of separation from the trailing edge in the physical plane is given by:

$$\left| \frac{w(z_1)}{U} \right|_A = \frac{\sin^2 \phi_0}{R_1 \sin^2(\theta_c - \theta_A)} \times \left| \frac{2 \cos \theta_A + \frac{q_U}{4} \csc^2 \left(\frac{\theta_A - \delta_U}{2} \right) + \frac{q_L}{4} \csc^2 \left(\frac{\theta_A - \delta_L}{2} \right)}{\left[1 + \frac{\xi^2 + 1}{2\xi} \right]} \right| \quad \dots (15)$$

Equations (10), (11), (13) and (15) satisfy the requirements that smooth separation occurs at the spoiler tip and at the trailing edge with the pressure specified at both points to be equal to that measured experimentally. These equations are now used to solve for the five unknowns q_U , q_L , δ_U , δ_L , γ , so that another condition is needed to solve for all the unknowns in the 2-source model. Such a condition is introduced in section 2.3.1.

The 1-source model removes the ambiguity by dropping what

appears to be the least significant boundary condition, the specification of the pressure coefficient at the trailing edge. It is found that the airfoil pressure distribution, even quite close to the trailing edge on the underside, is not strongly dependent on the value of C_{p_A} , which is therefore left unspecified. In the previous equations q_L is taken to be zero, so q_L and δ_L are eliminated and equations (10), (11) and (13) are used to solve for q_u , δ_u , γ . Equation (15) merely gives the value of C_{p_A} , which is found to be more positive than the empirical value assumed to apply over surface ABC. Thus, there is a pressure discontinuity at A predicted by the 1-source model.

2.2 Theory for Arbitrary Airfoils

A logical extension to the theory would be its application to arbitrary thick airfoils, if it is to have some practical value for design purposes. This is achieved by the use of Theodorsen's transformation (6), which will map any airfoil onto a circle. The problem, then, becomes similar to that of a Joukowski airfoil as viewed in the Z_2 plane (Fig. 1a). In general, practical airfoils do not have a cusped trailing edge. However, since the theory does require a cusp at the trailing edge to satisfy the condition of smooth separation, the trailing edge must be artificially modified into a cusp. This modification is applied to the upper surface of the airfoil aft of the spoiler. Thus, the altered portion of the airfoil is completely within the wake, caused by the spoiler, and has no effect on the outer flow field. It is suggested that a third order polynomial be used to replace the

upper surface of the airfoil to span the last 10% of the chord. This will permit the location of spoilers upto the 90% chord station, as measured from the leading edge. Spoilers positioned aft of this are of little practical interest. The four coefficients of the polynomial are determined by specifying both ordinate and slope of the two end points, to match those of the airfoil lower surface at the trailing edge and the airfoil upper surface at the 90% chord station.

As a specific application, it was decided to consider a 14% thick Clark Y airfoil, since force measurements on such an airfoil fitted with normal spoilers were available for comparison. The airfoil surface is defined by a finite number of points tabulated in Riegels (7). Both the basic and the modified airfoils are shown in Fig. 5.

2.2.1 Transformations

A 14% thick modified Clark Y airfoil is generated in the Z_1 plane so that the chord is aligned with the real axis. The trailing edge is located at $X = +2$, and the midpoint between the leading edge and its center of curvature at $X = -2$, so as to resemble the orientation of a Joukowski airfoil. The Joukowski transformation will map the airfoil in the Z_1 plane onto the z' plane in Fig. 6.

$$Z_1 = z' + \frac{1}{z'} \quad \dots(16)$$

CLARK Y AIRFOIL
14% THICK
(MODIFIED TRAILING EDGE)

Figure 5. Clark Y Airfoil with Spoiler.

TRUE-CIRCLE PLANE

OFF-CIRCLE PLANE

Figure 6. Complex Transform Planes.

The resulting curve in the ζ' plane will be nearly circular in shape, since most wing sections have a general resemblance to each other and to the Joukowski airfoil.

The coordinates of the contour in the ζ' plane are defined by the relation:

$$\zeta' = e^{\psi(\mu) + i\mu}$$

and the corresponding points in the Z_1 plane are found by using equation (16):

$$Z_1 = e^{\psi(\mu) + i\mu} + e^{-\psi(\mu) - i\mu}$$

Since

$$e^{i\mu} = \cos \mu + i \sin \mu$$

then it can be shown that:

$$Z_1 = 2 \cosh \psi(\mu) \cos \mu + 2i \sinh \psi(\mu) \sin \mu$$

In the Z_1 plane, the coordinates X , Y of the airfoil surface are known. Hence, it is possible to obtain expressions for ψ and μ in terms of X and Y as follows:

$$\left. \begin{aligned} \sinh \psi(\mu) &= \frac{Y}{2 \sin \mu} \\ \cosh \psi(\mu) &= \frac{X}{2 \cos \mu} \end{aligned} \right\} \dots(17)$$

Eliminating $\psi(\mu)$ from the above relations gives:

$$2 \sin^2 \mu = \left[1 - \left(\frac{X}{2} \right)^2 - \left(\frac{Y}{2} \right)^2 \right] + \left\{ \left[1 - \left(\frac{X}{2} \right)^2 - \left(\frac{Y}{2} \right)^2 \right]^2 + Y^2 \right\}^{1/2} \dots(18)$$

Because $\sin \mu$ is known in terms of the airfoil coordinates X and Y , the value of $\psi(\mu)$ can be found from equation (17).

Next, the contour in the ζ' plane is mapped onto a circle of radius e^{ψ_0} in the ζ plane (Fig. 6). The transformation relating the ζ' to the ζ plane is the general transformation:

$$\zeta' = \zeta e^{\sum_{n=1}^{\infty} \frac{A_n + i B_n}{\zeta^n}} \dots(19)$$

where the values of the real coefficients A_n and B_n are to be determined from the airfoil shape.

The coordinates of points on the circle in the ζ plane are defined by:

$$\zeta = e^{\psi_0 + i\varphi}$$

Relating the two complex variables ζ and ζ' for points on the contours will give the identity:

$$\zeta' = \zeta e^{(\psi(\mu) - \psi_0) + i(\mu - \varphi)}$$

When this is compared with equation (19), it becomes obvious that:

$$(\psi(\mu) - \psi_0) + i(\mu - \varphi) = \sum_{n=1}^{\infty} (A_n + iB_n) \frac{1}{\zeta^n}$$

Expressing ζ in polar form on the circle:

$$\zeta = e^{\psi_0} (\cos \varphi + i \sin \varphi)$$

will yield:

$$(\psi(\mu) - \psi_0) + i(\mu - \varphi) = \sum_{n=1}^{\infty} e^{-n\psi_0} (A_n + iB_n) (\cos n\varphi - i \sin n\varphi)$$

Equating the real and imaginary parts gives the two Fourier expansions:

$$\psi(\mu) - \psi_0 = \sum_{n=1}^{\infty} (A_n \cos n\varphi + B_n \sin n\varphi) e^{-n\psi_0} \quad \dots(20)$$

and

$$\epsilon(\varphi) = \varphi - \mu = \sum_{n=1}^{\infty} (A_n \sin n\varphi - B_n \cos n\varphi) e^{-n\psi_0} \quad \dots(21)$$

Here, ψ_0 is related to the radius of the circle in the ζ plane. Consequently, in order for the deviation of the near circle from the true circle to be a minimum, the value of ψ_0 is taken to be

$$\psi_0 = \frac{1}{2\pi} \int_0^{2\pi} \psi(\mu) d\varphi \quad \dots(22)$$

In order to obtain the values of the coefficients A_n and B_n , equations (19), (20) and (21) are approximated by a finite number of terms. Since equation (20) requires an expression of ψ in terms of φ , and since ψ is ordinarily known as a function of μ from the airfoil coordinates, this leads to an iterative process, which converges rapidly to give the values of $\psi(\varphi)$ and $\epsilon(\varphi)$. A first approximation is to set $\epsilon(\varphi) = 0$ and to solve for ψ_0 from equation (22), then to evaluate an initial set of

coefficients A_n and B_n using equation (20), and finally to use these coefficients in equation (21) to obtain the second approximation to $\epsilon(\varphi)$. This process is repeated until no change in the outcome of $\epsilon(\varphi)$ is observed. When this occurs, the final values of the coefficients A_n and B_n are known, and can be used in equation (19) to define the mapping function relating the ζ' and ζ planes.

Now that mapping an arbitrary airfoil in the Z_1 plane onto a circle in the ζ plane has been achieved, the problem becomes similar to that of the Joukowski airfoil as viewed in the Z_2 plane. As before, the spoiler on the airfoil is introduced in the ζ plane as a radial straight line segment, which when mapped onto the physical plane becomes a normal spoiler with very slight curvature.

The chordwise location of the spoiler on the airfoil (E) is determined by the angular variable θ_0 in the ζ plane. Here, θ_0 is the angle between the positive real axis and the radial segment representing the spoiler. The dependence of E on θ_0 is shown in Fig. 7. Also, the height of the spoiler is shown to be a function of ζ and E in Fig. 8.

The transformation relating the ζ and Z_3 planes becomes:

$$Z_3 = \frac{\zeta}{R_1 e^{i\theta_0}} + \frac{R_1 e^{i\theta_0}}{\zeta} \quad \dots(23)$$

Figure 7. Spoiler Geometry.

Figure 8. Spoiler Geometry.

Here, the value of R_1 is taken to be equal to the radius of the circle in the ζ plane,

$$R_1 = e^{i\psi_0}$$

The mapping of the slit in the Z_3 plane onto the circle in the Z_4 plane is unaltered and is given by equation (3). Similarly, the location of the separation points A and C is given by equations (4) and (5). Care must be taken when using equation (4) for the Clark Y airfoil, since α is the angle of attack measured from the chordline and not from the lower surface as is conventional for this class of airfoils.

The combined transformation derivative $\frac{dz_1}{dz_4}$ can be written as follows:

$$\frac{dz_1}{dz_4} = \left[\frac{dz_1}{d\zeta'} \right] \left[\frac{d\zeta'}{d\zeta} \right] \left[\frac{d\zeta}{dz_3} \right] \left[\frac{dz_3}{dz_4} \right]$$

This can be evaluated readily:

$$\begin{aligned} \frac{dz_1}{dz_4} = & \left[\frac{\zeta'^2 - 1}{\zeta'^2} \right] \times \frac{\zeta'}{\zeta} \left[1 - \sum_{n=1}^{\infty} n \frac{(A_n + iB_n)}{\zeta^n} \right] \times \\ & \times \left[\frac{R_1 e^{i\theta_0} \zeta^2}{\zeta^2 - R_1^2 e^{2i\theta_0}} \right] \left[\frac{e^{i(\alpha - \theta_0)}}{2} \left(\frac{1}{2} \frac{\zeta^2 + 1}{\zeta} + 1 \right) \left(\frac{z_4^2 - e^{2i(\theta_0 - \alpha)}}{z_4^2} \right) \right] \dots (24) \end{aligned}$$

This expression for $\frac{dz_1}{dz_4}$ has the same number of simple zeros and simple poles as did the expression in equation (6). Consequently, the spoiler tip and the trailing edge are critical points of the mapping and permit the satisfaction of smooth separation. If the airfoil trailing edge had been left unaltered,

$\frac{dz_1}{dz_4}$ would still have had a zero at point A, but it would not have been a simple zero. The order of the zero would be less than unity, so that the velocity term would dominate, and a stagnation point would appear at the trailing edge. (When the basic Clark Y airfoil is mapped onto the ζ' plane, the resulting contour is indicated by the dotted line in Fig. 6.)

2.2.2 Boundary Conditions

The flow model for the Clark Y airfoil is the same as that shown in Fig. 4 for the Joukowski airfoil. Consequently, expressions for the complex potential $F(Z_4)$, and the complex velocity $W(Z_4)$ are given by equations (7) and (8) respectively. However, it will be convenient to rewrite the expression for the complex velocity as follows:

$$\frac{W(Z_4)}{V_2} = \frac{1}{Z_4} \left\{ \left(Z_4 - \frac{1}{Z_4} \right) + \frac{q_0}{z} \left[\frac{Z_4 + Z_0}{Z_4 - Z_0} \right] + \frac{q_L}{z} \left[\frac{Z_4 + Z_L}{Z_4 - Z_L} \right] + i\gamma \right\} \quad \dots (25)$$

Again, V_2 is the incident flow in the Z_4 plane and is related to the uniform flow in the Z_1 plane by:

$$V_2 = \frac{e^{\psi_0}}{2} \left[1 + \frac{1}{2} \frac{\xi^2 + 1}{\xi} \right] U.$$

The boundary conditions for the arbitrary airfoil are the same as those for the Joukowski airfoil, in that smooth separation must occur at the spoiler tip and the trailing edge with a specified value of the pressure coefficient at both points. The condition of separation in the Z_1 plane at points A and C is satisfied by setting $W(Z_4)$ equal to zero at the corresponding points. This results in the two expressions given by equations (10) and (11). In order for the pressure coefficient to be specified at the separation points, it is necessary to obtain expressions for $W(Z_1)$ at these points. These were shown to be of an indeterminate form; consequently, it is appropriate to consider them separately.

At the spoiler tip C the critical term in the expression for $\frac{dz_1}{dz_4}$ in equation (24) is given by:

$$(z_4 - e^{i(\theta_0 - \alpha)})$$

In section 2.1.3 it was shown that the variation of $\frac{dz_1}{dz_4}$ in the neighbourhood of C can be written as:

$$\frac{dz_1}{dz_4} \propto z \sin\left(\frac{\theta - \theta_c}{2}\right)$$

Using the expression for the complex velocity given by equation (9), the velocity in the physical plane is determined by the proportionality:

$$\left| \frac{W(z_1)}{U} \right| \propto \left| \frac{-2 \sin \theta - \gamma + \frac{q_U}{2} \cot\left(\frac{\theta - \delta_U}{2}\right) + \frac{q_L}{2} \cot\left(\frac{\theta - \delta_L}{2}\right)}{2 \sin\left(\frac{\theta - \theta_c}{2}\right)} \right|$$

Evaluating this equation at $\theta = \theta_c$ using L'Hospital's rule gives:

$$\left| \frac{W(z_1)}{U} \right|_c \propto \left| 2 \cos \theta_c + \frac{q_U}{4} \csc^2\left(\frac{\theta_c - \delta_U}{2}\right) + \frac{q_L}{4} \csc^2\left(\frac{\theta_c - \delta_L}{2}\right) \right|$$

Introducing the constant of proportionality gives the magnitude of the velocity of separation from the spoiler tip in the physical plane as:

$$\left| \frac{W(z_1)}{U} \right|_c = \left[\frac{\xi^2 - 1}{2\xi^2} \right] \times \left| \left[\frac{\xi}{\xi^2 - 1} \frac{1}{\left(1 - \sum_{n=1}^{\infty} n \frac{A_n + iB_n}{\xi^n} \right)} \right]_c \right| \times \left| 2 \cos \theta_c + \frac{q_U}{4} \csc^2\left(\frac{\theta_c - \delta_U}{2}\right) + \frac{q_L}{4} \csc^2\left(\frac{\theta_c - \delta_L}{2}\right) \right| \quad \dots(26)$$

Similarly, at the trailing edge A the critical term in the expression for $\frac{dz_1}{dz_4}$ in equation (24) is given by:

$$(\xi' - 1)$$

so that the variation of $\frac{dz_1}{dz_4}$ in the neighbourhood of A is written as:

$$\frac{dz_1}{dz_4} \propto (\xi' - 1)$$

Using the expression for the complex velocity $W(Z_4)$ given by equation (25), the velocity in the Z_1 plane near the trailing edge is determined by the proportionality:

$$\left| \frac{W(z_1)}{U} \right| \propto \left| \frac{(z_4 - \frac{1}{z_4}) + \frac{q_0}{z} \left[\frac{z_4 + z_0}{z_4 - z_0} \right] + \frac{q_L}{z} \left[\frac{z_4 + z_L}{z_4 - z_L} \right] + i\gamma}{\xi' - 1} \right|$$

which is an indeterminate form at $\xi' = +1$. Using L'Hospital's rule by differentiating the numerator and the denominator with respect to the variable ξ' , and letting $\xi' \rightarrow +1$ will give:

$$\left| \frac{W(z_1)}{U} \right|_A \propto \left| \left[1 + \frac{1}{z_4^2} - q_0 \frac{z_0}{(z_4 - z_0)^2} - q_L \frac{z_L}{(z_4 - z_L)^2} \right]_A \right| \times \left| \left[\frac{dz_4}{d\xi'} \right]_A \right|$$

where

$$\left[\frac{dz_4}{d\zeta'} \right]_A = \left[\frac{dz_4}{dz_3} \right]_A \left[\frac{dz_3}{d\zeta} \right]_A \left[\frac{d\zeta}{d\zeta'} \right]_A$$

In the Z_4 plane the trailing edge corresponds to:

$$z_4 = e^{i\theta_A}$$

Hence, the velocity in the physical plane is given by:

$$\left| \frac{W(z_1)}{U} \right|_A \propto \left| z \cos \theta_A + \frac{q_U}{4} \csc^2 \left(\frac{\theta_A - \delta_U}{2} \right) + \frac{q_L}{4} \csc^2 \left(\frac{\theta_A - \delta_L}{2} \right) \right| \times \left| \left[\frac{dz_4}{d\zeta'} \right]_A \right|$$

Introducing the constant of proportionality gives the magnitude of the velocity of separation from the trailing edge in the physical plane as:

$$\begin{aligned} \left| \frac{W(z_1)}{U} \right|_A &= \left[\frac{e^{\frac{\gamma_0}{2}}}{4} \left(1 + \frac{\zeta^2 + 1}{2\zeta} \right) \right] \times \left| \left[\frac{dz_4}{d\zeta'} \right]_A \right|^2 \times \\ &\times \left| z \cos \theta_A + \frac{q_U}{4} \csc^2 \left(\frac{\theta_A - \delta_U}{2} \right) + \frac{q_L}{4} \csc^2 \left(\frac{\theta_A - \delta_L}{2} \right) \right| \quad \dots (27) \end{aligned}$$

Equations (26) and (27) when used together with equation (12) specify the value of the separation pressures at the spoiler tip and the trailing edge. The magnitude of this pressure is taken equal to that measured experimentally.

Thus, equations (10), (11), (26) and (27) are used to solve for the unknowns q_u , q_l , δ_u , δ_l , γ . Again the need for another condition is evident for a unique solution to the 2-source model. This will be discussed in section 2.3.1.

As was the case with the Joukowsky airfoil, the 1-source model has only three unknowns to completely specify the flow field, and equations (10), (11), and (26) are used to determine these unknowns, namely q_u , δ_u , γ . Also, the pressure discontinuity at A will be present when using the 1-source model for the Clark Y airfoil.

2.3 Calculations

Since the four equations resulting from the previously mentioned boundary conditions are not sufficient to determine the five unknown flow parameters defining the 2-source model, it is necessary to impose an additional condition on the flow for this model.

2.3.1 Additional Boundary Condition

During the early stages of this work, the lower source position in the 2-source model was left unspecified to serve as a free parameter and to permit the determination of the remaining unknowns. It was observed that the lift, for any airfoil-spoiler

combination, predicted by the 2-source model was consistently lower than that predicted by the 1-source. It was also found that as the position of the lower source approached point A, corresponding to the trailing edge in the Z_4 plane, the strength of the source decreased monotonically and became zero at point A. Thus, the effect of the lower source on the flow field became vanishingly small as its position approached point A. In particular, when $\delta_L = 0.95 \theta_A$, the pressure distribution given by the 2-source model was indistinguishable from that predicted by the 1-source model except near the trailing edge on the underside of the airfoil where the 1-source model will always result in a pressure discontinuity at the trailing edge.

When comparing the lift results from the 1-source model with those obtained experimentally, it was found that the theoretical values were almost always larger than the measured ones. This suggests a criterion for choosing a value for the lower source position for the 2-source model in order to provide better agreement between theory and experiment. However, to match the value of the lift coefficient for the 2-source model with experiment for every angle of attack, spoiler position and height, would make the theory too empirical and useless for the prediction of the loading distribution before any experiment is performed.

A study of the experimental lift as a function of angle of attack for the two airfoils tested with spoilers shows that the zero lift angle, for a given airfoil with a normal spoiler, depends very little on the spoiler position when it is varied between the 50% and 90% chord locations. This, together with the

fact that the 1-source model gave predictions for values of $\frac{dC_L}{d\alpha}$ in good agreement with experiment, will serve as a criterion for choosing the value of δ_L in the 2-source model. Thus, by using the value of $\frac{dC_L}{d\alpha}$ predicted by the 1-source model and by specifying the value of the zero lift angle for a given airfoil-spoiler combination, the linear relation between lift and incidence is known for the 2-source model. Then, the value of δ_L is chosen to give the appropriate lift specified by this relation.

The two theoretical models differ only by the presence of an additional source which has a secondary effect on the flow field. Thus, to say that both models should have the same value of $\frac{dC_L}{d\alpha}$ is not at all unreasonable, particularly when the 1-source model provides good agreement with experiment for values of $\frac{dC_L}{d\alpha}$.

In general when using conventional potential flow theory to solve for the flow over a given airfoil, it is observed that the lift is always overestimated by the theory for values of incidence in the normal operating range for the given wing section. This is due to the vorticity within the boundary layer. The discrepancy in lift becomes smaller as the zero lift angle is approached, no doubt due to the reduction in vorticity in the boundary layer, hence the better the agreement in the overall circulation about the airfoil. By this observation, it seems justified to specify the value of the zero lift angle for the 2-source model to match that measured experimentally. Thus, the present theory requires the specification of the wake pressure

coefficient for both models, and in addition, it requires the knowledge of the zero lift angle for the 2-source model.

It was observed that varying the pressure coefficient in the wake had little effect on the distribution over the rest of the airfoil, but produced a proportional change in the resulting value of lift. It was also found that the experimentally measured pressure in the wake, for the two airfoils tested, was very nearly the same for equivalent spoiler height and location and airfoil incidence. Thus, in the absence of experimental values for the wake pressure for a given airfoil section, it is possible to make a reasonable estimate of this value based on data obtained from other airfoils.

Similarly, in the absence of experimental values for the zero lift angle, needed for the 2-source model, it is possible to make a reasonable estimate of α_{l_0} using a linearized theory developed by Woods (2) and later modified by Barnes (3). In his report, Barnes gives an expression for the lift on any airfoil fitted with a normal spoiler as follows:

$$C_L = \frac{\pi}{2} (1 + \sqrt{E/c})^2 (\alpha - \alpha_0) - \frac{\pi}{2} (1.06) (E/c + \sqrt{E/c})^{1/2} (2 h/c \sqrt{1 - C_{Pc}})^{1/2}$$

$$- \frac{1}{4} (1 + \sqrt{E/c})^2 \left\{ \cosh^{-1} \left[1 - 2 \frac{\sqrt{E/c} - 1}{\sqrt{E/c} + 1} \right] + \left[\left(1 - 2 \frac{\sqrt{E/c} - 1}{\sqrt{E/c} + 1} \right)^2 - 1 \right]^{1/2} \right\} C_{P\sigma} \dots (28)$$

where $C_{P\sigma}$ is an incremental spoiler base pressure coefficient introduced by Woods, and given by Barnes as an empirical formula applicable to all airfoils as follows:

$$C_{P\sigma} = 2 \left[\frac{Y_c}{c} + \frac{(2 - X_c)}{c} \alpha \right] - 2.5 \frac{h}{c} - 0.18$$

In equation (28) α_0 is the zero lift angle for the airfoil without spoiler, and (X_c, Y_c) are the coordinates of the spoiler tip in the Z_1 plane. To obtain the zero lift angle for the airfoil-spoiler combination, C_L is made to vanish and α becomes α_{l_0} . Thus the equation reduces to:

$$(\alpha_{l_0} - \alpha_0) - (1.06)(E/c)^{1/4} \frac{(2h/c \sqrt{1 - C_{Pe}})^{1/2}}{(1 + \sqrt{E/c})^{3/2}} - \frac{1}{2\pi} \left\{ \cosh^{-1} \left[1 - 2 \frac{\sqrt{E/c} - 1}{\sqrt{E/c} + 1} \right] + \right. \\ \left. \left[\left(1 - 2 \frac{\sqrt{E/c} - 1}{\sqrt{E/c} + 1} \right)^2 - 1 \right]^{1/2} \right\} \left[2 \left(\frac{Y_c}{c} + \frac{(2 - X_c)}{c} \alpha_{l_0} \right) - 2.5 \frac{h}{c} - 0.18 \right] = 0 \quad \dots (29)$$

This is a linear equation in α_{l_0} which can easily be solved for any airfoil-spoiler combination. All the incidence variables are measured from the chord line which is the same as the real axis in the Z_1 plane. The predicted values for α_{l_0} are compared in Table I with experimental values. It is clear that this linearized theory gives poor predictions for spoilers mounted at the 50% chord station, but the agreement improves as the spoiler is moved back to the 90% chord location. It is suggested that the predicted

value for α_{l_0} for the 70% chord station be used to represent the average of the relatively constant values measured experimentally. This would give a value for α_{l_0} reasonably near the observed quantities, and would serve as an input parameter for the 2-source model during the initial stages of an investigation into the loading characteristics of an airfoil-spoiler combination.

		Joukowsky		Clark Y	
Spoiler Geometry		Measured From Chord Line		Measured From Lower Surface	
E/c	h/c	Theory	Expt.	Theory	Expt.
0.5	.05	5.94	3.00	2.88	1.40
0.7	.05	4.45	2.70	0.89	1.40
0.9	.05	4.29	2.60	0.23	0.00
0.5	.10	10.16	6.60	7.09	5.20
0.7	.10	8.00	6.10	4.38	4.70
0.9	.10	7.41	6.10	3.35	3.90

Table I: Zero Lift Angle Comparisons

2.3.2 Method of Solution

Theoretical solutions were obtained for a Joukowski airfoil of 11% thickness and 2.4% camber both with and without spoiler, and for a 14% thick Clark Y airfoil both with and without spoiler.

Solutions for the basic Joukowski airfoil were obtained by the classical method in which the Kutta condition is satisfied at the trailing edge, determining the circulation about the airfoil. The lift and pressure distribution were computed for a range of angle of attack. The value of the complex quantity Z_0 corresponding to the given camber and thickness is $(-0.09 + 0.05i)$.

When using the 1-source model to solve for the flow over the Joukowski airfoil fitted with a normal spoiler, equations (10), (11) and (13) are modified by setting $q_L = 0$. It is obvious that variables q_u , and γ appear linearly in equations (10) and (11), consequently it was possible to obtain expressions for q_u and γ in terms of the remaining unknown δ_u . These expressions are then used in equation (13) to obtain a lengthy equation in

δ_u , which is solved by Newton's iterative method. With the flow parameters q_u , δ_u and γ known, the airfoil surface velocity over the contour AEDC is found from:

$$\left| \frac{w(z_1)}{U} \right| = \frac{R_1}{4} \left[1 + \frac{z^2 + 1}{2z} \right] \left| \frac{-4 \sin \theta - z\gamma + q_u \cot\left(\frac{\theta - \delta_u}{2}\right)}{dz_1/dz_4} \right| \quad \dots (30)$$

where $\frac{dz_1}{dz_4}$ is given by equation (6), and the corresponding value of the pressure coefficient is found from equation (12).

The pressure coefficient was evaluated for 98 points on the airfoil surface over contour AEDC, and was taken to be a constant over contour CBA, corresponding to the measured wake pressure. The resulting pressure coefficient distribution is integrated numerically using the trapezoidal rule to obtain the value of lift coefficient C_L . The above procedure was carried out for a range of angle of attack α for the airfoil fitted with spoilers of 5 and 10% chord height, each at 50, 70 and 90% chordwise locations. Values of $dc_L/d\alpha$ for each of the airfoil-spoiler configurations, used for solving for the unknowns in the 2-source model, are obtained from the resulting values of C_L as a function of α .

Thus the complete solution to the 1-source model is a prerequisite to obtaining the values of the flow parameters in the 2-source model. As a first step for this solution it is necessary to define the lift as a function of incidence. This is achieved by using the value of $dc_L/d\alpha$ predicted by the 1-source model for the same configuration, and then specifying the value of α_{l_0} . The zero lift angle is either obtained experimentally or an estimate of its value can be derived from equation (29). Next, equations (10), (11) and (13) are used to obtain expressions for q_u , q_L and γ in terms of the remaining two unknowns δ_u and δ_L . These are used in equation (15) to give a single equation in δ_u and δ_L . To solve for these, a rapidly converging process is established in which an initial choice for δ_L is made. Then δ_u is obtained by using the secant iterative method. Once an initial set of values for the flow parameters is

determined, the airfoil surface velocity over the contour AEDC is found from:

$$\left| \frac{W(z_1)}{U} \right| = \frac{R_1}{4} \left[1 + \frac{\xi^2 + 1}{z\xi} \right] \left| \frac{-4\sin\theta - z\gamma + q_u \cot\left(\frac{\theta - \delta_u}{2}\right) + q_l \cot\left(\frac{\theta - \delta_l}{2}\right)}{dz_1 / dz_4} \right| \quad \dots(31)$$

where dz_1/dz_4 is given by equation (6), and the corresponding value of the pressure coefficient is found from equation (12). The resulting pressure distribution is integrated giving the lift. This value of lift is then compared with the previously specified lift curve, and if it is found that the calculated lift is too low, δ_l is moved closer to point A in the Z_4 plane, and vice versa. The whole procedure is repeated until the resulting lift agrees with the specified value. This was carried out for a range of incidence for the airfoil fitted with spoilers of 5 and 10% chord height, each at 50, 70 and 90% chordwise locations. In the case of spoilers at the 90% chord station, it was found that the lift curve predicted by the 1-source model was already lower than the experimental curve. In these situations it is impossible to make the 2-source model agree better with measured values, since it can only be made to produce smaller lift values than those predicted by the 1-source. However, it remains desirable to obtain a solution to the 2-source model, since it removes the pressure discontinuity at the trailing edge. Thus, it is suggested that a value for the lower source position be taken as $\delta_l = 0.95 \theta_A$. This will not alter the lift appreciably, nor will it change the

pressure distribution except near the trailing edge.

To obtain solutions for the basic Clark Y airfoil, it is necessary to calculate values for coefficients A_n and B_n defining the mapping from the ζ' to the ζ planes. This is achieved by representing the surface of the airfoil by 100 distinct points separated appropriately to account for rapid curvature change. These are then used in equations (17) and (18) to compute the corresponding values of μ and $\psi(\mu)$. Values of coefficients A_n and B_n are then computed from equation (20) by the iterative process described in section 2.2.1. These were calculated for values of n up to 40, since it was felt that this would give sufficient accuracy in the determination of the mapping function.

Thus, the flow about the basic Clark Y airfoil is determined in the ζ plane with the Kutta condition again satisfied at the trailing edge. This determines the circulation about the airfoil and results in the prediction of a stagnation point at the trailing edge due to the fact that it is not cusped. The lift and pressure distribution were computed for a range of incidence.

When a spoiler is fitted on the upper surface of the airfoil it becomes necessary to modify the upper surface near the trailing edge to produce a cusp at A. This is done by following the procedure outlined in section 2.2. Next, a new set of coefficients A_n and B_n must be evaluated, since the modified trailing edge will have an effect on the mapping function relating the ζ' and ζ planes. With the mapping function defined it becomes possible to solve for the flow over the airfoil-spoiler combination.

Using the 1-source model, equations (10), (11) and (26) are

modified by setting $q_L = 0$. Then, following the same procedure outlined for the Joukowski airfoil, flow parameters q_u , δ_u and γ are evaluated and the airfoil surface velocity over contour AEEDC is found from:

$$\left| \frac{w(z_1)}{U} \right| = \frac{e^{\gamma_0}}{4} \left[1 + \frac{z^2}{z_3^2} \right] \left| \frac{-4\sin\theta - 2\gamma + q_u \cot\left(\frac{\theta - \delta_u}{2}\right)}{dz_1/dz_4} \right| \quad \dots(32)$$

where dz_1/dz_4 is given by equation (24). As in the Joukowski airfoil case, the lifts and pressure distributions were computed for a range of incidence and for spoilers of 5 and 10% chord height, each at 50, 70 and 90% chordwise positions, again resulting in predictions for $dc_L/d\alpha$ for the various configurations.

For the 2-source model, the same procedure as outlined for the Joukowski airfoil is again used to determine the flow parameters, and the value for $dc_L/d\alpha$ predicted by the 1-source model together with specification of α_{l_0} provides the additional condition needed to solve for all the unknowns. The velocity over the surface of the airfoil is given by:

$$\left| \frac{w(z_1)}{U} \right| = \frac{e^{\gamma_0}}{4} \left[1 + \frac{z^2}{z_3^2} \right] \left| \frac{-4\sin\theta - 2\gamma + q_u \cot\left(\frac{\theta - \delta_u}{2}\right) + q_L \cot\left(\frac{\theta - \delta_L}{2}\right)}{dz_1/dz_4} \right| \quad \dots(33)$$

where dz_1/dz_4 is given by equation (24). Values of lift and pressure distributions were computed for a range of angle of attack

and for spoilers of 5 and 10% chord height, each at 50, 70 and 90% chordwise locations.

III EXPERIMENTS

3.1 Joukowski Airfoil

A Joukowski airfoil of 27 in. span, 12.08 in. chord, 11% thickness and 2.4% camber was constructed. Due to the theoretical cusped trailing edge, some construction modifications in the neighbourhood of the trailing edge were needed. Since this airfoil was to be used with normal spoilers mounted on the upper surface it was possible to modify the upper surface near the trailing edge so that the last part of the airfoil had an approximately constant thickness of $1/8$ in. In the presence of spoilers the modified part of the upper surface is completely within the wake and has no effect on the outer flow field. The original and modified profiles are shown in Fig. 9.

The airfoil was constructed out of two spanwise sections of wood joined at the center with an aluminum portion containing a total of 37 pressure taps. Twenty-four of the taps were distributed on the upper surface and the remainder were on the lower surface. The chordwise locations of these taps are given in Table II and their positions on the airfoil surface are shown in Fig. 9.

The airfoil was built with end plates to allow for the mounting of two spanwise spoilers, one having a height of 5% chord and the second a height of 10% chord. Each of these spoilers could be mounted in 5 different chordwise positions:

JOUKOWSKY AIRFOIL

11% THICKNESS

2.4% CAMBER

Figure 9. Joukowski Airfoil with 10% Spoiler.

UPPER SURFACE		LOWER SURFACE	
Tap No.	Chordwise Position X	Tap No.	Chordwise Position X
1	-2.027	25	-1.960
2	-2.010	26	-1.860
3	-1.960	27	-1.694
4	-1.894	28	-1.527
5	-1.827	29	-1.194
6	-1.740	30	-0.860
7	-1.627	31	-0.427
8	-1.482	32	+0.006
9	-1.280	33	+0.439
10	-1.079	34	+0.873
11	-0.878	35	+1.306
12	-0.676	36	+1.639
13	-0.475	37	+1.839
14	-0.274		
15	-0.073		
16	+0.129		
17	+0.330		
18	+0.531		
19	+0.733		
20	+0.934		
21	+1.135		
22	+1.337		
23	+1.538		
24	+1.739		

Table II: Pressure Tap Positions on Joukowski Airfoil

50, 60, 70, 80 and 90% chord. The gap between airfoil and spoiler was sealed with tape for each configuration.

The airfoil was mounted on a six-component strain gauge balance system at the 1/4-chord position. Measurements for lift, drag and pitching moment were recorded for a range of incidence. The pressure taps were connected to a multitube manometer bank filled with alcohol. The pressure readings on the airfoil surface and on the 10% spoiler were recorded for angles of attack α_{ℓ} , $\alpha_{\ell} + 4^{\circ}$, $\alpha_{\ell} + 8^{\circ}$ for all possible spoiler positions. Pressure taps were not built into the 5% spoiler because of its small size. The test Reynolds number was 4.4×10^5 .

3.2 Clark Y Airfoil

Measurements for lift, drag and pitching moment for a 14% thick Clark Y airfoil were available before the theoretical solutions were computed. They were obtained by Mr. M. A. Lundberg, a summer research assistant working under the direction of Professor G. V. Parkinson, to investigate the applicability of the standard wind tunnel wall corrections to airfoil-spoiler configurations.

Four airfoils of 9", 14", 19" and 24" chord were constructed. They were built with end plates to allow for the mounting of two spanwise spoilers having heights of 5% and 10% chord. The support system for each airfoil was at the mid-chord position. No pressure taps were installed in any of these airfoils.

Again, the six-component strain gauge balance system was used to measure lift, drag and pitching moment for all the airfoils

fitted with a 10% spoiler for a range of incidence and spoiler locations at 50%, 70% and 90% chord. Only the 24 in. model was used with a 5% spoiler. Pressure measurements in the wake region were obtained by inserting a Pitot tube behind the spoiler and recording the value of pressure at several locations within the wake. The test Reynolds number was 3×10^5 .

All the measurements for both airfoils were made in the low speed wind tunnel of the Mechanical Engineering Department of The University of British Columbia. This tunnel has a test section of 3 by 2 1/4 ft., over a length of 8 2/3 ft., and produces a very uniform flow, with turbulence level less than 0.1 percent over a wind speed range 0 - 150 fps.

There exists a certain amount of controversy over the use of established methods to correct for wind tunnel wall interference. The present configuration posed an additional complication in that separation occurs at the spoiler tip and at the trailing edge with the formation of a broad wake. It was observed that when using the corrections established by Pope and Harper (8), in which the wake blockage term is taken equal to $1/2 (c/H) C_D$, the lift curves for the different sizes of model did not collapse and a tendency to over-correct was evident, particularly for the larger models. It was further noticed that a value for the wake blockage term equal to $1/4 (c/H) C_D$, as suggested by Pankhurst and Holder (9), would be more suitable for the present configuration and would give a better collapse for the data.

Thus, using expressions for the correction of lift coefficient and angle of attack given by Pope and Harper together with

the modified wake blockage term, the lift curves for the Joukowski airfoil and the 14 in. Clark Y airfoil were corrected for tunnel wall effects. Next, the pressure coefficient distribution was corrected as follows:

$$\frac{1 - [(C_p)_{\text{CORR.}}]_{\alpha}}{1 - [(C_p)_{\text{UNCORR.}}]_{\alpha}} = \frac{[(C_L)_{\text{CORR.}}]_{\alpha}}{[(C_L)_{\text{UNCORR.}}]_{\alpha}} \quad \dots (34)$$

where α is the angle of attack at which the distribution was measured. The pressure coefficient in the wake was also corrected using equation (34), rather than using an expression suggested by Maskell (10) appropriate for separated flows, since the two methods of correction produced results that are very nearly the same.

IV RESULTS AND COMPARISONS

4.1 Joukowsky Airfoil

To check that the experimental airfoil behaved like a Joukowsky airfoil, it was tested without a spoiler, and the resulting C_p -distribution for $\alpha = 6^\circ$, uncorrected for tunnel wall effects, is compared in Fig. 10 with theoretical solutions. The dashed curve shows the distribution for $\alpha = 6^\circ$. As expected, it predicts a higher lift than is actually observed because the theory does not account for the reduction in circulation caused by the boundary layer vorticity. However, if comparison is made at equal lift (corresponding to $\alpha = 5.1^\circ$ for the theoretical calculation, as shown by the solid curve), the agreement is seen to be excellent, and the small bump in the experimental distribution caused by the artificially thickened trailing edge is evident. The variation of lift coefficient with angle of attack is shown in Fig. 11, in which agreement between theory and experiment, here corrected for wall effects, improves as the zero lift angle is approached, due to the reduction in boundary layer vorticity.

Figure 12 shows the effect of a 5% spoiler when mounted at 50, 70 and 90% chord positions. As the spoiler is moved forward the value of $dC_L/d\alpha$ is reduced, implying an increase in spoiler effectiveness at the forward position. However, no appreciable change in the value of the zero lift angle is observed. Figures 13, 14 and 15 show comparisons of theoretical and experimental

Figure 10. Pressure Distribution for Basic Joukowski Airfoil.

Figure 11. Lift Coefficient for Basic Joukowski Airfoil.

Figure 12. Experimental Lift Coefficient for Joukowski Airfoil with Spoiler.

Figure 13. Lift Coefficient for Joukowski Airfoil with Spoiler.

Figure 14. Lift Coefficient for Joukowski Airfoil with Spoiler.

Figure 15. Lift Coefficient for Joukowski Airfoil with Spoiler.

lift coefficient curves for a 5% spoiler. In each figure the 1-source model is seen to give excellent prediction for values of $dc_L/d\alpha$. However, as the spoiler is moved forward, the lift is over-estimated by the model. The 2-source model, on the other hand, offers considerable improvement for the 50% spoiler position, and none for the 90% location because the 2-source model can never predict a larger lift than that of the 1-source. Thus, in Fig. 15, the lift coefficient curves for the two theoretical models become indistinguishable.

In Fig. 16 the effect of a 10% spoiler located at 50, 70 and 90% chord is shown. Again, no appreciable change in the zero lift angle is observed. However, the spoiler effectiveness is increased when the location is moved forward, resulting in a decrease in values of $dc_L/d\alpha$. Figures 17, 18 and 19 show comparisons of theoretical and experimental lift coefficient curves for a 10% spoiler. In each case, the 1-source model seems to result in good prediction for values of $dc_L/d\alpha$, but, as for the 5% spoiler case, the lift is over-estimated by the model for the forward locations of spoilers. The 2-source model, in comparison, offers considerable improvement for the 50% location, and some for the 70%, but none for the 90% chord position.

Most of the experimental lift coefficient curves for airfoil-spoiler configurations have a slight curvature, which is due to the formation of a separated bubble ahead of the spoiler. The extent of this bubble becomes larger and its pressure less positive as the angle of attack is increased, thus resulting in a larger contribution to the lift. At the higher values of incidence

Figure 16. Experimental Lift Coefficient for Joukowski Airfoil with Spoiler.

Figure 17. Lift Coefficient for Joukowski Airfoil with Spoiler.

Figure 18. Lift Coefficient for Joukowski Airfoil with Spoiler.

Figure 19. Lift Coefficient for Joukowski Airfoil with Spoiler.

it is desirable to have the measured lift larger than the theoretical value in order to produce better agreement in the pressure distribution over the unseparated part of the airfoil.

Figures 20, 21 and 22 show comparisons of theoretical and experimental pressure coefficient distributions for different chordwise positions of a 5% spoiler with the airfoil at $\alpha = 11^\circ$ and with C_{p_C} matched. The experimental values have been corrected for tunnel wall effects. In each figure the 1-source model is compared with the experimental distribution and it is seen to give reasonable agreement. However, as the spoiler is moved forward, the upper surface suction and the lower surface pressures are over-estimated by the model, and the effect of C_{p_A} not being specified becomes more noticeable. In the three figures, it is seen that the 2-source model gives much better agreement with the experimental values over the airfoil surface.

One inevitable discrepancy between theory and experiment is evident in all the C_p -distribution figures. The theory predicts a stagnation point at the base of the upstream spoiler surface, so that $C_{p_D} = 1$. Actually, the adverse pressure gradient upstream of the spoiler causes boundary layer separation from the airfoil, with reattachment on the spoiler face. The constant pressure separation bubble can be clearly identified in the figures, and its extent is particularly well defined in Fig. 20 by comparison of the experimental variation with the curve for the 2-source model.

Figures 23 and 24 show C_p -distributions for the airfoil with a 10% spoiler near the zero lift angle. The 1-source model gives

Figure 20. Pressure Distribution for Joukowski Airfoil with Spoiler.

Figure 21. Pressure Distribution for Joukowski Airfoil with Spoiler.

Figure 22. Pressure Distribution for Joukowski Airfoil with Spoiler.

Figure 23. Pressure Distribution for Joukowski Airfoil with Spoiler.

Figure 24. Pressure Distribution for Joukowski Airfoil with Spoiler.

good agreement for the 90% spoiler position and can not be improved upon by using the 2-source model. For the 50% spoiler location in Fig. 23, it is seen that the 2-source gives much better agreement than the 1-source and that the extent of the separated bubble is reduced for the lower incidence.

Figures 25, 26 and 27 give comparisons of the airfoil at $\alpha = 13^\circ$ with three positions of the 10% spoiler. Both the 1-source model and the 2-source model are presented and compared with experiment, and similar comments to those made about Figs. 20, 21 and 22 can be made here. It can be seen that the separation bubbles caused by the 10% spoiler are larger than those for the 5% spoiler.

In Fig. 28 the separation streamlines are plotted for about one chord length downstream for the case of a 5% spoiler at 90% chord with the airfoil at $\alpha = 8^\circ$. Both the 1-source and the 2-source models are shown. C_p -distributions along the upper streamline for each model are also plotted on the figure. It is seen that the streamlines become almost parallel a short distance downstream. The apparent shift in the streamlines of the two models is due to the larger overall circulation about the airfoil as predicted by the 1-source model resulting in the downward shift for the corresponding streamlines. The asymptotic separation of these streamlines is the quotient of the total wake source strength and the free stream velocity. The C_p -distribution along the streamlines decays gradually towards zero.

Figure 25. Pressure Distribution for Joukowski Airfoil with Spoiler.

Figure 26. Pressure Distribution for Joukowski Airfoil with Spoiler.

Figure 27. Pressure Distribution for Joukowski Airfoil with Spoiler.

Figure 28. Positions of and Pressure Distribution along Separation Streamlines.

4.2 Clark Y Airfoil

The pressure distribution for the basic Clark Y airfoil at $\alpha = 6^\circ$ is shown in Fig. 29. The effect of the finite angle at the trailing edge becomes evident in that a stagnation point will always be present at the trailing edge. The variation of lift coefficient with angle of attack is shown in Fig. 30. As usual, agreement between theory and experiment is improved as the zero lift angle is approached due to the reduction in the boundary layer vorticity.

Figure 31 shows the variation of the measured lift coefficient with angle of attack for the airfoil fitted with a 5% spoiler at three chordwise positions. The effect is the same as was observed before, in that the forward locations of the spoiler correspond to the lower values of $dc_L/d\alpha$. For this airfoil there seems to be a noticeable shift in the value of the zero lift angle for the 90% chord location. However, it is important to recall that the data for the 5% spoiler were obtained by using a 24 in. chord model, which represents a blockage ratio of 0.667, and the applicability of the wind tunnel wall corrections becomes somewhat questionable. Naturally, the values of $dc_L/d\alpha$ are also affected by this large blockage ratio. Figures 32, 33 and 34 show comparisons of theoretical and experimental lift coefficient curves for a 5% spoiler. In each figure the 1-source model is seen to give quite good predictions for values of $dc_L/d\alpha$, except for the 50% location (Fig. 32). However, as the spoiler position is moved forward the lift is over-estimated by this model, and the 2-source model seems to provide only a slight improvement at

Figure 29. Pressure Distribution for Basic Clark Y Airfoil.

Figure 30. Lift Coefficient for Basic Clark Y Airfoil.

Figure 31. Experimental Lift Coefficient for Clark Y Airfoil with Spoiler.

Figure 32. Lift Coefficient for Clark Y Airfoil with Spoiler.

Figure 33. Lift Coefficient for Clark Y Airfoil with Spoiler.

Figure 34. Lift Coefficient for Clark Y Airfoil with Spoiler.

the 50% location due to the disagreement in the value of $dc_L/d\alpha$. Better agreement between the 2-source model and experiment is observed at the 70% location (Fig. 33), but no improvement over the 1-source model is shown at the 90% position (Fig. 34).

Figure 35 shows the effect of a 10% spoiler located at three chordwise positions, and similar comments to those made earlier can be made here, except that values of $dc_L/d\alpha$ are more reliable, since the results are from a 14 in. model. Figures 36, 37 and 38 show comparisons of experimental and theoretical lift coefficient curves for the 10% spoiler. Again, the excellent prediction by the 1-source model for values of $dc_L/d\alpha$ is evident. Also, the 2-source model seems to give progressive improvement over the 1-source for values of the lift as the spoiler is moved forward.

No experimental distributions of the pressure coefficient were available for the Clark Y airfoil, hence, in the following figures, only comparisons between the 1- and the 2-source models are shown. Figures 34, 40 and 41 show the pressure distributions for the airfoil at $\alpha = 10^\circ$ fitted with a 5% spoiler at three different locations. The 2-source model, because it gives lower lift, has lower suction peaks than the 1-source, also less pressure over the lower surface for the 50% and 70% locations. For the 90% position the two distributions are identical except near the trailing edge, where the 1-source will have a pressure discontinuity.

Figures 42 and 43 show C_p -distributions for the airfoil with a 10% spoiler near the zero lift angle. The tendency for the two models to approach each other as the spoiler is moved back is

evident and is consistent with the general trend for all pressure distributions.

Figures 44, 45 and 46 give comparisons for the airfoil at $\alpha = 12^\circ$ with three positions of the 10% spoiler, and again comments similar to those made for the 5% spoiler are applicable here.

Figure 35. Experimental Lift Coefficient for Clark Y Airfoil with Spoiler.

Figure 36. Lift Coefficient for Clark Y Airfoil with Spoiler.

Figure 37. Lift Coefficient for Clark Y Airfoil with Spoiler.

Figure 38. Lift Coefficient for Clark Y Airfoil with Spoiler.

Figure 39. Pressure Distribution for Clark Y Airfoil with Spoiler.

Figure 40. Pressure Distribution for Clark Y Airfoil with Spoiler.

Figure 41. Pressure Distribution for Clark Y Airfoil with Spoiler.

Figure 42. Pressure Distribution for Clark Y Airfoil with Spoiler.

Figure 44. Pressure Distribution for Clark Y Airfoil with Spoiler.

Figure 45. Pressure Distribution for Clark Y Airfoil with Spoiler.

Figure 46. Pressure Distribution for Clark Y Airfoil with Spoiler.

V CONCLUSIONS

It has been demonstrated that the 1-source model gives good agreement with experiment for the lift and pressure distribution when spoilers are mounted near the trailing edge. This model also predicts values for $dc_L/d\alpha$ that are in excellent agreement with experiment for all possible airfoil-spoiler configurations. For the other locations of spoilers (nearer the mid-chord), the 2-source model gives much better agreement with experimental lift and pressure distributions. In general, the 2-source model is more desirable since it removes the pressure discontinuity at the trailing edge.

The only discrepancy in the pressure distribution, which is common to both theoretical models, is the experimentally evident separation bubble ahead of the spoiler. This is a boundary layer separation phenomenon that potential flow cannot cope with, and agreement between theory and experiment in that region is not expected.

The choice of specifying the zero lift angle as an additional condition for the 2-source model is not an uncommon step towards making potential flow agree more closely with experiment. Pinkerton (11), working with NACA Four Digit airfoils, suggested that the Kutta condition, determining the circulation and consequently the lift, be dropped and that instead the circulation be determined from the measured lift. Naturally this procedure produces

infinite suction at the trailing edge. However, it does not alter the distribution appreciably over the rest of the airfoil.

The significant contribution of the present theory is the prediction of loading distributions that are in good agreement with experiment. Linearized theories, such as those of Woods and Barnes, give good predictions for lift, but as expected, not very good pressure distributions, hence the need for a nonlinear theory not restricted to low incidence and small spoilers.

It should be possible to obtain better agreement between theoretical and experimental pressure distributions by finding an empirical relation for the pressure in the separated bubble as a function of the geometrical parameters for the airfoil-spoiler configuration. Since this pressure is constant for a given configuration, the present theoretical prediction over the separated part of the airfoil ahead of the spoiler could be replaced by the constant value obtained from the empirical relation.

REFERENCES

1. Woods, L. C. "The Theory of Subsonic Plane Flow" Cambridge University Press, 1961.
2. Woods, L. C. "Theory for Airfoil Spoilers" A.R.C. R.&M. 2969, May, 1953.
3. Barnes, C. S. "A Developed Theory of Spoilers on Airfoils" A.R.C. C.P. 887, July, 1965.
4. Mandl, P. "Potential Flow about an Airfoil with a Split Flap" Canadian Aeronautical Journal, Vol. 6, April, 1960, pp. 123-125.
5. Roshko, A. "A New Hodograph for Free Streamline Theory" NACA TN 3168, 1954.
6. Abbott, I. H., and von Doenhoff, A. E. "Theory of Wing Sections" Dover, 1949, pp. 53-63.
7. Riegels, F. W. "Airfoil Sections; Results from Wind Tunnel Investigations, Theoretical Foundations" Translated from the German by D. G. Randall, London, Butterworths, 1961.
8. Pope, A. and Harper, J. J. "Low-Speed Wind Tunnel Testing" John Wiley and Sons, 1966, Chapter 6.
9. Pankhurst, R. C. and Holder, D. W. "Wind-Tunnel Technique" Pitman and Sons Ltd., London, 1952, Chapter 8.
10. Maskell, E. C. "A Theory of the Blockage Effects on Bluff Bodies and Stalled Wings in a Closed Wind Tunnel" R.&M. No. 3400, November, 1963.
11. Pinkerton, R. M. "Calculated and Measured Pressure Distribution over the Midspan Section of the NACA 4412 Airfoil" NACA Report No. 563, 1936.