CULTURE, ECONOMIC STRUCTURE, AND THE DYNAMICS OF ECOLOGICAL ECONOMIC SYSTEMS

By

John M. Anderies

B.Sc., Colorado School of Mines, Golden, Colorado, U.S.A, 1987

M.Sc., University of British Columbia, 1996

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES

DEPARTMENT OF MATHEMATICS

INSTITUTE OF APPLIED MATHEMATICS

We accept this thesis as conforming

to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

July, 1998

In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Mathematics

The University of British Columbia
Vancouver, Canada

Date August 28, 1998

DE-6 (2/88)

Abstract

In this thesis several models are developed and analyzed in an attempt to better understand the interaction of culture, economic structure, and the dynamics of human ecological economic systems. Specifically, how does the ability of humans to change their individual behavior quickly and easily in response to changing environmental conditions (behavioral plasticity) alter the dynamics of human ecological economic systems? What role can cultural and social institutions play in affecting individual behavior and thus the dynamics of such systems? Finally, how do assumptions about the production and consumption of goods and services within human ecological economic systems affect their dynamics.

Much work concerning interacting economic and natural processes has focused on technical issues and problems with standard economic thought. Less attention has been paid to the role of human behavior. The work presented herein addresses both but emphasizes the latter. Three models are developed: a model of the Tsembaga of New Guinea which focuses on the roles of behavior, cultural practices and ritual on the dynamics of the Tsembaga ecosystem; a model of Easter Island where the linkage between economic models of utility and the resulting behavioral model is studied; and finally a model of a modern two sector economy with capital accumulation where the emphasis is evenly split between behavior and economic issues.

The main results of the thesis are: behavioral plasticity exhibited by humans can destabilize ecological economic systems and culture and social organization can play a critical role in offsetting this destabilizing force. Finally, the analysis of the two sector model indicates that there is a window of feasible investment levels that will lead to a

sustainable economy. The size of this window depends on culture and social organization, namely the way economic growth is managed and how the associated benefits are distributed. The two sector model clarifies the idea of a sustainable economy, and allows the possibility of reaching one to be clearly characterized.

Table of Contents

List of Tables

List of Figures

Acknowledgement

I would like to thank Dr. Colin Clark for his financial and moral support over the past 5 years; his many readings of my work and helpful ideas and comments. I would also like to thank my committee members for helpful comments and ideas as I developed the thesis, especially Leah Keshet and James Brander. Finally I am greatly indebted to my wife and friend Margaret; thanks, your turn.

Chapter 1

Introduction

Since the 1970's, the impact of human activities on ecosystems has been receiving more and more attention. Through this increased awareness, 'sustainability' – the basic question of whether and how human populations can continue to live on earth indefinitely without threatening the survival of all biological populations – has become an important international issue, and the focus of much research. Unfortunately there are deep divisions between different groups of people regarding the fundamentals of the sustainability issue.

Examples of such divisions are everywhere – in the popular media and in academic debates. For example, several authors have argued that the economic process is fundamentally influenced by entropic decay [27, 19] while others [67] argue that the entropy law is irrelevant because the earth is a thermodynamically open system. Some experts are very concerned about the degradation of agricultural ecosystems (soil erosion, etc.) [28, 49, 50] while others praise the power of technology to "liberate the environment" and give us "effectively landless agriculture" [6](p. 172) via "[a] cluster of innovations including tractors, seeds, chemicals, and irrigation, joined through timely information flows and better organized markets [that will] raise yields to feed billions more without clearing new fields" [6](p. 171).

The aim of this thesis is to address several aspects of this division. For this purpose, different views on sustainability can be divided in to two broad classes:

A. (expansionist view) Sustainability is mainly a technical issue. The present paradigm of economic growth can continue indefinitely as long as increases in efficiency offset increasing pressure on natural resources and ecological systems.

B. (steady state view) Sustainability involves a comprehensive understanding of the place of human populations within ecosystems. Achieving a sustainable world will require a fundamental paradigm shift concerning the way humans lead their lives.

There are two key points to note about these different positions. First, the existence of this difference hinders the development of effective policy to govern the relationship between human economic and ecological systems. Second, position A is the paradigm of choice in present policy formation without sufficient evidence that it is the "correct" view.

Clearly, the only way society can move toward a sustainable state is to extract important truths from both views and with them forge some strategy to guide future human environmental interactions. This is not an easy task for two reasons. First, human agro-ecosystems may be too complex to understand in enough detail to be useful in policy formation. Second the views of people on either side of the issue may be, as Rees [54] notes, based more " [on] differing fundamental beliefs and assumptions about the nature of human-kind-environment relationships" rather than fact. At the heart of the issue are assumptions that underly the models and arguments made in support of either view (see the forum in [7] for a collection of recent papers on the continuing debate).

I believe there are three fundamental questions the must be addressed before real progress can be made in resolving differences concerning the concept of sustainability. First, the expansionist view assumes that our ability to solve problems with technology is necessarily a good thing. Is this so? Second, how important are our cultural and social institutions in determining whether a human economic system is sustainable? Finally, how do assumptions that underly economic growth models used to support the expansionist position affect the dynamics of human ecological economic systems? The main thrust of this thesis is to develop a modeling framework to help answer these three questions.

My approach is to develop dynamical systems models to study humans as ecological populations. These models focus on how human behavioral and cultural systems interact with the environment, and they are deliberately stylized to avoid the trap of generating models that are too complicated with too many assumptions to be of practical use, e.g. [43, 44]. Only the most basic features of general human economic ecological systems are included. In attempting to answer the questions posed above I develop three different models of this type, two involving simple societies of anthropological interest and one modern economic system with capital accumulation, with the following objectives:

- The first model addresses the first two questions in the context of a simple human agro-ecosystem. The human ability to modify behavior quickly and over a wide range of different activities, (defined as behavioral plasticity), is emphasized. The role that behavioral plasticity plays in the dynamics of a human agro-ecosystem is studied in detail. Of special interest is the destabilizing effect of behavioral plasticity, and the stabilizing role culture and social organization may play.

- The second model is directed towards the third question. Here, a linkage between economic concepts and an evolving ecological economic system is developed. Economic models of behavior based on the optimization of some measure of utility are introduced. Utility measures that result in realistic behavior in the context of an evolving ecological economic system are identified. Again, the destabilizing effect of behavioral plasticity is highlighted.

- In the third model, the ideas developed in the first two models are combined to develop the model of the modern economic system. This model model addresses all three questions in the context of economic growth in a bounded environment.

In addition to shedding light on the three fundamental questions posed above, the models developed in this thesis provide tools to study operational aspects of sustainability. This is very useful since much of the problem with the sustainability concept is that it is easy to imagine what a sustainable state might be like, but few ask whether it is possible to get from our present state to a sustainable state. As Rees [54] notes: "....sustainability will require a 'paradigm shift' or a 'fundamental change' in the way we do business, but few go on to describe just what needs to be shifted...". Thinking about a sustainable world is pointless unless we can find a way to get there. In a recent article, Proops et al. [52] emphasize the need to formulate a goal of sustainability, set an intermediate target, and develop feasible paths toward this goal. The analytical framework developed in this thesis provides a flexible, simple, and precise means of studying (for a given set of assumptions) exactly what cultural attributes are sustainable or not, and more importantly, what key aspects affect the feasibility of potential paths to a sustainable human ecological economic system.

The structure of the thesis is as follows. Chapter 2 outlines the background, assumptions and basic structure of the modelling framework. Next, in Chapter 3 the modelling framework is applied to the society of the Tsembaga, a tribe that occupies the highlands of New Guinea. Next, the ideas developed in Chapter 3 are extended in Chapter 4 where a model proposed by Brander et. al [9] to explain the rise and fall of the Easter Island civilization is used to develop and study more advanced economic concepts typically used to model human consumptive and productive activities. These authors argue that the Polynesian culture that occupied Easter Island was mismatched to the ecosystem they found and thus perished. The authors also discuss the implications of their model for other societies that collapsed, and for our own society. The main point is that more complex economic models in which agents exhibit maximizing behaviors based on a certain utility function do not necessarily give rise to richer models behavior - indeed they can result in very simple, not very realistic behavioral patterns. Here we emphasize how non-substitutability in consumption fundamentally alters the behavior of the model and the nature of the approach to the sustainable state, and that realistic behavior depends on the inclusion of this aspect in utility functions.

Finally, pulling together the ideas of chapters 3 and 4, I develop a model of a two sector (a sector in economics is a grouping of associated productive activities) economy and embed it in a model ecosystem. The economy has an agricultural (bioresource) sector and a manufacturing sector. Economic agents (individuals who take part in productive and consumptive activities within the economy) can devote the productive capacity of the economy to four different activities: the consumption of agricultural, manufactured, investment, and resource goods. This model includes all the components that form the basis of the current debate about human environmental interaction: we rely on flows from the environment but we can use our productive capacity to substitute for these flows, increase efficiency, reduce waste, and help regenerate the environment. Those holding the steady state view emphasize the importance of the former while expansionists emphasize the power and importance of the latter. With the modelling framework developed herein, their interaction can be studied.

Chapter 2

The Modeling Framework

In this chapter, the background and assumptions underlying the modeling framework are addressed. The modeling approach is outlined, and the general model that is employed throughout the thesis is developed. Next, the important features of the models that are important to the questions posed in the introduction are discussed. Finally, the analytical techniques used to uncover these features are presented.

When trying to model the interaction between elements in a system, e.g. predators with prey, one competitor with another, an organism with its environment, one necessarily has to model the way each element affects how other elements change over time. The most common approaches are to write down differential equations, difference equations, functional differential equations (when age structure is important), or a stochastic process. Often several approaches are appropriate for a given problem so the choice of approach often depends on the intentions of the modeler.

The models I develop in this thesis are all deterministic dynamical systems. The advantage of this approach is that the models are clear and simple, allowing the underlying assumptions and concepts to be easily seen by inspecting the differential equations that constitute the model. Drawbacks are that implicit in deterministic models is the assumption that everything is "well mixed" and there are no spatial or random effects allowed. That is to say that each variable in the model necessarily represents an average value of a particular quantity. Clearly no real system is well mixed and deviations from the average can substantially alter the dynamics of the system in question. Fortunately, it is often the case that many aspects of a real system can be inferred from the structure of the "mean field" or average model given by the deterministic ordinary differential equation system.

Studying the dynamics of such models is a difficult task. If the model is simple enough it can be studied by analytical methods. The models in this thesis are too complex to study analytically. Fortunately, there are numerical techniques available that allow dynamical systems theory to be used on more complex systems. In the next section I will briefly discuss the application of dynamical systems type models to ecological systems and explain how I extend them for the special case of human economic ecological systems.

2.1 Dynamical Systems Models of Ecological Systems

Ecologists have long used simple systems of differential equations to model ecosystems so as to understand how different behavioral patterns may effect the dynamics between individuals that interact in the ecosystem. Because my interest is specifically with behavior and environmental constraints, the way behavior is modeled, and the way a model is placed in an ecological context are very important. I will illustrate this by way of a simple example.

Differential equation models of ecosystems often take form

$$\frac{dx}{dt} = f(x, p) \tag{2.1}$$

where $x \in \Re^n$ describes the state of the ecosystem and $p \in \Re^k$ is a parameter vector. This type of model has been extensively studied (e.g. [65, 26, 11, 21, 42]). In such models, the behavior of organisms is often modeled by a functional response that is completely determined by the state of the system. For example the simplest Lotka-Volterra predator prey model given by

$$
\begin{aligned}
\frac{dh}{dt} &= rh - \alpha ph && (2.2a)\\
\frac{dp}{dt} &= -\beta h + \gamma ph, && (2.2b)
\end{aligned}
$$

where $h(t)$ and $p(t)$ are the prey and predator population densities, respectively. This model exhibits unrealistic neutral oscillations where predator and prey numbers can take on arbitrarily large values. This is due to the fact that behavior is modeled too simply and there is no ecological context. Prey behavior is limited to eating and growing. They do nothing to avoid predators or carry out any other complex behavior. Predators die and eat prey; never changing their behavior whether they are hungry or full. The organisms are behaviorally rigid, or for our purposes, not behaviorally plastic. Almost all animals have some measure of behavioral plasticity, and this is especially true of humans. Ecologists often include more complex behavior by introducing a functional response term to model the way a predator consumes prey. At the very least, these models include some means of satiating the appetite of the predator. For example equations 2.2 could be modified by replacing the term αph in equation 2.2a with the functional response $g(h,p)$. Holling [34] proposed the functional response:

$$
g(h,p) = \frac{\alpha ph}{p+k} \tag{2.3}
$$

where k is the prey concentration at which the predator consumes at one-half its maximum rate. As p increases, the rate at which prey are removed approaches αh; each predator is consuming at a constant, maximum rate. Note that although some increased behavioral plasticity is added and the model is more realistic, the behavior or the predator is completely determined by the state of the system and not by any internal feedback. For example, if there are fewer prey and the predator becomes hungry, there is no mechanism in the model to allow the predator to change its strategy or work harder. If we attempt to model a human ecosystem, this is a key feature to include. Indeed, in chapter 3 we will see just how important this is. To properly model a system where individual organisms are behaviorally plastic, we have to add equations that model the internal state of the organisms and how they influence behavior. I will address this issue in a moment, but first let me turn to the second point mentioned above, the ecological context.

The predator prey model given by 2.2 is completely isolated from the environment. The equations model the system shown in figure 2.1. In reality, ecological systems are not isolated but are embedded in a physical environment and are *dissipative*; they continuously dissipate derivatives of solar energy.

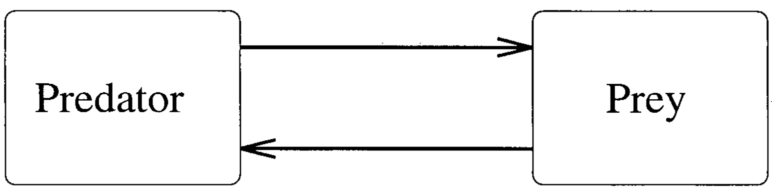


Figure 2.1: Isolated predator-prey model.

For a realistic model, we must include the fact that there is some abiotic component, x_a, the medium through which this dissipative process occurs. A recent paper addressing this point [61], suggests that the equations of motion be written this way:

$$\dot{x} = f(x_a, x, p, z(t), d) \tag{2.4}$$

where x_a are abiotic components, d describes the dissipative process, and $z(t)$ represents some external forcing. This is just a general mathematical statement that instead of modeling the system shown in figure 2.1 we must model the system shown in figure 2.2.

In such a model, the fundamental processes that make the interaction between predator and prey possible are included. In terms of equation 2.4, the abiotic components would include the soil structure of the ecosystem. The forcing might be the weather

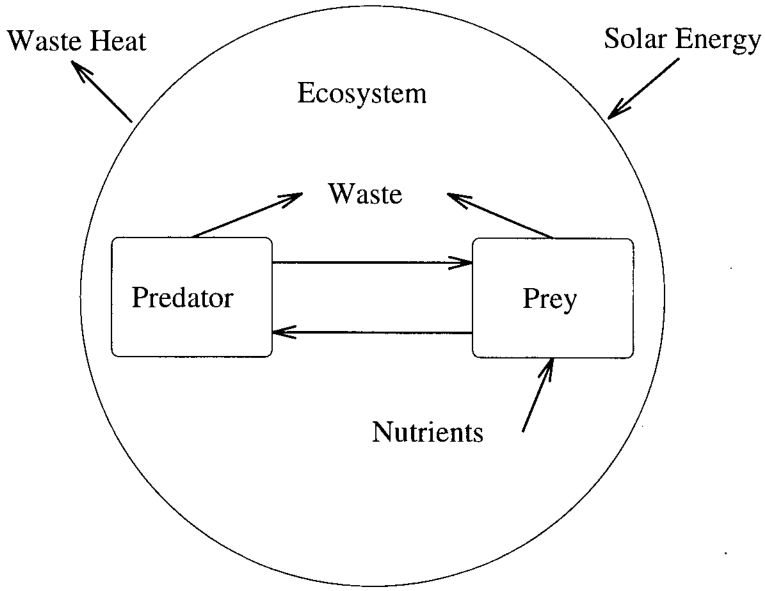


Figure 2.2: Predator-prey model embedded in an ecosystem where the dependence on abiotic compents and the dissipative processes of nutrient generation and waste assimilation fueled by the sun is considered.

patterns. The dissipative processes would include the metabolism of the plant community which generates nutrients, the animal metabolisms which convert the nutrients to energy and waste products, and the decomposer community that assimilates the waste and breaks it down for reuse. Only when these aspects are included can any ecosystem model be considered ecologically realistic. The most simple way that these important features can be included in a model is by introducing a "carrying capacity" term. In a predator prey model the carrying capacity is often defined as the maximum number of prey that can be supported in the given ecosystem thus lumping the dissipative process into one term. The model given by equations 2.2 could be modified to include this aspect along with more complex behavior to read

$$\frac{dh}{dt} = r(1-\frac{h}{K})h - \frac{\alpha ph}{p+k} \tag{2.5a}$$

$$\frac{dp}{dt} = -\beta h + \frac{\gamma ph}{p+k}, \tag{2.5b}$$

where K is the carrying capacity. This model yields a stable fixed point or a stable limit cycle. This is much more reasonable than the arbitrarily large fluctuations possible in the model specified by equation 2.2. The key point I wish to draw out is the importance of behavior and ecological context in ecological models. If we wish to extend this modeling framework to human ecological economic systems, these are key issues we need to address. Indeed, the issue of ecological context is fundamental in the debate about sustainable development.

2.2 Human economic ecological systems

2.2.1 Background

Most of the work on human economic ecological systems has been either in the context of (optimal) economic growth, or the optimal exploitation of resources. Unfortunately, economic models often lack ecological context. The example above shows that modeling without proper ecological context may lead to quite absurd results, and economic models are no exception.

For example, the model of Solow [58] in the context of optimal economic growth with exhaustible resources states that along an optimal growth path, constant net output can be maintained in the face of dwindling resource inputs. Later, when further analyzing Solow's work, Hartwick [31] presented the savings rule: invest all rents from exhaustible resources (in replenishable man-made capital) to maintain constant net output indefinitely. This result is based on a model like that shown in figure 2.3. The economic system is viewed as a circular flow of exchange between firms and households as shown on the left in figure 2.3 interacting with the physical world on the right. The physical world is often just viewed as a source of raw materials (to be optimally extracted as in the case of the Solow/Hartwick model) and a sink for wastes.

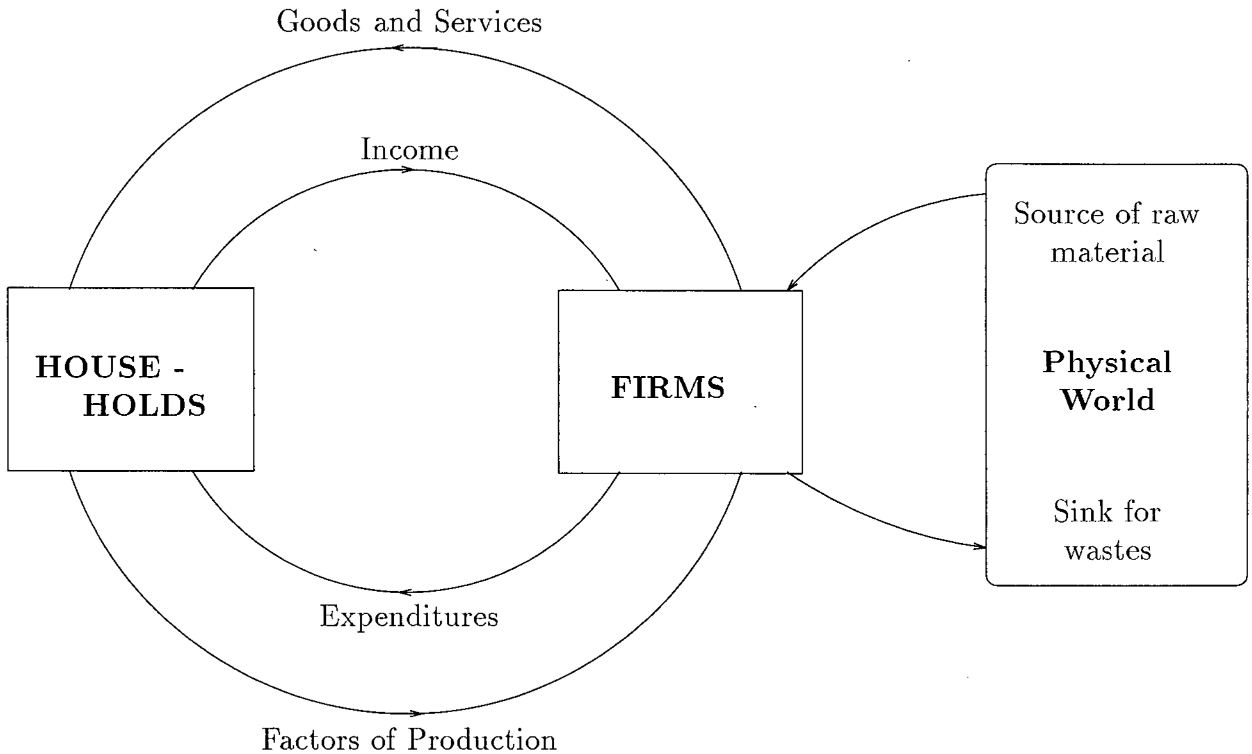


Figure 2.3: Schematic of the circular flow of exchange as perceived by standard economics. The connection to the real world, even as merely a source of raw materials and a waste bin, is seldom shown.

Clearly, the underlying assumptions in such models are critical to obtaining results such as those above. In the case above, it is assumed that the production of commodities, Y, is given by

$$Y = K^{\alpha} L^{\beta} N^{\gamma} \tag{2.6}$$

where K and L are man-made capital stocks and population respectively, N is a flow of natural resources, and α, β, and γ are parameters assumed to satisfy $\alpha+\beta+\gamma=1$. For the case where the population is held constant and there is no technological progress, the dynamical system for this optimal economic growth model is

$$\frac{dK}{dt} = AK^{\alpha}N^{\gamma} - C \tag{2.7a}$$

$$\frac{dN}{dt} = -\left(\frac{\alpha}{1-\gamma}\right)\frac{CN}{K}, \tag{2.7b}$$

where A is a constant representing the contribution to production of the fixed labor force, and C is total consumption of the population. The first equation states that capital, K, increases at a rate given by the total commodity production rate less what is consumed. The second equation states that the resource flow diminishes (optimally) as resources are used up. Now, C is always less than or equal to $AK^{\alpha}N^{\gamma}$ (you can't consume more than you make) thus $\frac{dK}{dt} \geq 0$. This implies that $K(t) \geq 0$ for all $t \geq 0$ which results in the right hand side of 2.7b being negative for all $t \geq 0$ forcing $N(t)$ to approach zero asymptotically as time tends toward infinity.

A glance at this model will reveal its similarity to 2.2 where K is analogous to the predator and N is analogous to (in this case a finite stock of) the prey. The parallel I wish to draw is the similarity in the growth function assumed for the predator and capital. The predator can still grow at very low prey levels if there are sufficiently many predators! Similarly, the capital can continue to grow with a very low resource flow, as long as there are sufficient capital stocks. The absurdity in the case of the predator model is obvious, and ecologists quickly modified this model as already discussed. The difficulty in the economic growth model is more difficult to see, and economists have been slower than ecologists to modify such models.

The Solow result depends on the assumption that the factors of production, man-made capital (a stock), and resources (a flow), are near perfect substitutes. Much of ecological economics is concerned with exposing the underlying physical problems associated with such models and developing more realistic models (for recent examples see [60, 12]). The emphasis of this work is the non-substitutability among different stocks and between stocks and flows. Even if these modifications were made to the Solow model, there is still no clear ecological context; the only connection to the physical world is through a finite stock of resources to optimally use up.

Herman Daly [18] and Nicholas Georgescu - Roegen [27] were among the first (ecologically minded) economists to recognize the need to study the system shown in figure 2.4 and to emphasize that in addition to the issue of finite resource stocks, there is the issue of ecological context: we are embedded in a natural world that is important to our survival regardless of its connection to the economic process. This is the type of model which is developed and analyzed in the rest of this thesis.

The other key component that governs the evolution of an ecological economic system, namely human behavior, has received much less attention in the literature than technical issues related to economic models and ideas. For example maximization of utility over the next twenty years is most often assumed as the primary goal driving behavior. This has two important consequences: this assumption has become ingrained in standard economics, encouraging this behavior within society whether natural or not; in policy formation the model implies that only the next few years are important. In defense of his model, Solow [59] makes this very point. He indicates that the main purpose of these models is for planning over the next 60 years. How feasible is this planning strategy?

Before turning our attention to the mathematical model, note two main points:

- Any realistic model of the interaction of organisms with their environment must address the role of individual behavior.

- Maintaining realism in the way that different inputs interact in the productive

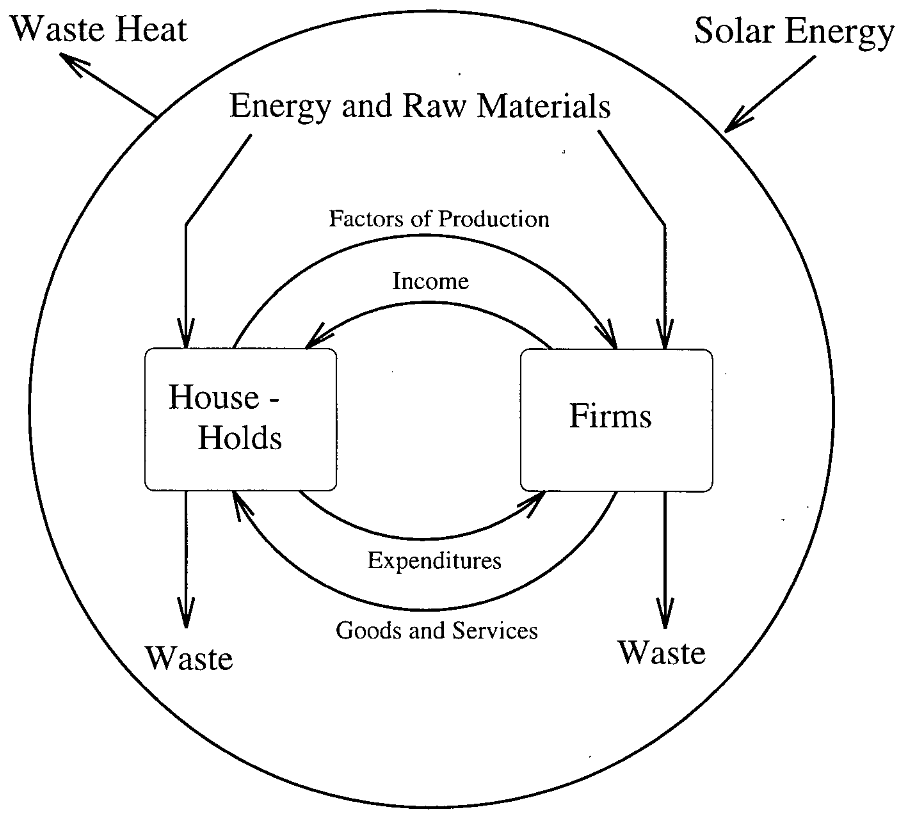


Figure 2.4: Schematic of the circular flow of exchange as perceived by standard economics embedded in the proper ecological context.

> process is important, but ecological context may be more so. Explicit modelling of the influence of organisms on the abiotic components and dissipative processes upon which they rely is crucial to capturing the dynamics of the system.

The topic of the next section is the mathematical expression of these ideas.

2.2.2 The general model

It is difficult to define a model that would be suitable to study a wide variety of ecological economic systems because of the variability of human cultural and social systems. Thus, the following is a general description of the model intended to emphasize basic structures common to human ecological economic systems. The general model will then be made specific in later chapters. State variables will be defined, a behavioral model is developed and the dynamics of the physical system are specified. Consistency with these definitions is maintained where possible, but there are slight notational differences between different models.

State variable definitions

The minimum ecological contextual variables are the productivity of the biophysical processes and the stock of low entropy material in the ecosystem. The only organisms explicitly modeled are humans. Unique to economic systems is the ability of humans to create capital which greatly enhances their ability to carry out productive activities. Thus, the following (stock) variables are necessary to track the state of the system:

$$\begin{aligned} h &= \text{Human population density,} \\ k_r &= \text{Stock of renewable natural capital,} \\ k_n &= \text{Stock of nonrenewable natural capital,} \\ k_h &= \text{Stock of man-made capital.} \end{aligned}$$

The precise definitions of the state variables and their units are as follows:

- Human population density. Units are people per cultivable hectare. These units were chosen because organisms are inextricably linked to some energy conversion process. A population of 100 people *occupying* 1,000,000 hectares would seem a low population density - but not if only 100 hectares of the total land were productive. Thus we are explicit about population per cultivable hectare. For comparison, this number might typically be 0.0001 for hunter-gatherers [51], 0.5 for swidden agriculturalists in New Guinea [51], and about 4 for the industrialized world [6].

• Renewable natural capital. It is difficult to assign units to capital, natural or man-made. Consider an example of man-made capital, the common passenger car. Should we measure the capital by a physical quantity? Should it be measured in tons of rubber, steel, or glass?? The entire heap of physical objects that comprise the car is totally useless without one quart of transmission fluid or some fuel. Clearly, we must define capital in terms of the service it provides per unit of input. Car engine capital could be defined as horsepower output per fuel input. Now an engine that has been used for 80,000 miles can be compared to a new one. The objects are almost physically indistinguishable, but the service they provide per unit of input is discernibly different. The case is similar for renewable natural capital. Renewable natural capital can be measured as the potential of natural systems to generate streams of biophysical processes that stabilize the biosphere's structure and function (natural income streams). The capital value of agricultural land, for example, is measured as its productivity per unit of input.

• Nonrenewable natural capital. Again there are difficulties with units but I simply define nonrenewable natural capital as any low entropy material such as iron ore, petroleum, etc. for which human society can find a use.

• Human made capital. As with natural capital, the units of human made capital are related to productivity, or ability to do work. In our model, capital is related to how much work can be accomplished per capita. In a community with no human made capital, the per-capita work potential is somewhere between 200 kcal/hour for light activity to 1000 kcal/hour for extremely hard work. For a highly capitalized society, the per-capita work potential would be 100-1000 times these values. I would like to stress the idea of work potential - for without fuel, the work potential provided by the capital stock is not realizable.

The behavioral model

The behavioral model consists of two components: a description of the population's allocation of available time and energy to different tasks, and a description of how a particular allocation would change in response to a change in the state of the system. The model is based on neo-classical theories of production and consumer behavior [32, 14, 64]. As already mentioned, these models often have no ecological context. To remedy this, these models are modified to reflect thermodynamic considerations and limits to substitutability that many economists and scientists stress [60, 13, 16, 17, 30, 28, 55, 18].

The basic model of behavior assumes that people act to maximize their utility, i.e. they solve the optimization problem:

$$\max \ U(y_1, y_2, ..., y_n; \vec{c}) \tag{2.8}$$

$$\text{s.t.} \qquad \textstyle\sum_{i=1}^{n} y_i p_i = w \tag{2.9}$$

where $U(y_1, y_2, ..., y_n)$ is the utility associated with the consumption of commodity y_i whose prices are p_i, $\vec{c}$ is a vector of parameters that describe the preferences (or culture) of the society being modeled, and w is the wage rate. The solution of this problem generates an expenditure system which specifies how much of each good will be purchased, and thus how many resources should be devoted to the production of each of these goods for any given set of prices. Prices are determined by firms trying to maximize profits in the face of a given demand with a certain technology specified by a production function of the form

$$y_i = f_i(x_1, .., x_m) \tag{2.10}$$

where y_i is the output of the i^{th} commodity and the x_j are inputs, or in the language of economics, factors of production. In economics, the "classic" factors of production were labor, land, and man-made capital. In my models, factors of production include labor, man-made capital, renewable natural capital, and nonrenewable natural capital. The inclusion of these latter two inputs links the productivity of the economy to the physical state of the system. Thus human preferences influence the nature of economic activity which in turn influences the ecosystem. This two step linkage connects human culture to the physical environment. The other component of the cultural model is to specify a decision process to cope with the situation when the optimal solution to the consumer problem is not feasible for the state of the physical system and current technology. Mathematically, this amounts to parameters that define the utility and production functions changing over time.

The nature of the utility function plays a very important role in the dynamics of the system as does the way the population changes its preferences over time. These issues are explored in detail in chapters 3, 4, and 5. The final element we must address in developing the model is the set of rules that govern the dynamics of the system.

Before describing the dynamics of the system, I would like to make clear the usage of the term "behavioral plasticity". As used in this thesis, behavioral plasticity refers individual behavior. Each individual can change their behavior in response to changing environmental conditions. The group behavior is then the result of the aggregation of individual behaviors. This is to be contrasted with behavioral plasticity at the group, or cultural level, i.e. cultural or social institutions changing with changing environmental conditions. This assumes that cultural process form with some purpose, an assumption with which I disagree. I view cultural processes as outgrowths of individual interactions, or "emergent variables". Whether or not a particular set of cultural processes (e.g. the ritual cycle of the Tsembaga) are adaptive is, to a large extent, accidental. Social institutions, on the other hand, can and do form in response to particular problems. They can be viewed as behaviorally plastic at the group level. I do not address this issue directly in the thesis, but propose some directions for further research in chapter 6.

System dynamics

The dynamics of the system are based on the following basic assumptions:

- All human activities require materials and energy and create waste flows - there are no 'free lunches'. Statements about feeding billions with clusters of innovations while sparing land are really about shifting our reliance from one resource to another and this must be recognized.

- Ecosystems provide flows of critical services - climate stabilization, waste assimilation, food production, etc.

- Man can, through capital creation, innovation and technical advances increase the efficiency with which both renewable and non-renewable resources are used.

- There are limits to substitution in both production and consumption.

- Human economic activity can degrade natural capital (e.g. pollution, soil erosion, etc.). Humans can offset this degradation to some extent by directing a portion of the economy's productive capacity toward this end.

- The dissipative nature of the system requires the constant input flow of energy to maintain a certain level of organization at a given level of technology (i.e. things wear out).

- As materials become more scarce, more work will be required to collect and transform them into useful objects.

In order to simplify notation, I represent the state of the system with a vector, i.e. let $\vec{s} = (h, k_r, k_n, k_h)$ –the human population density, the stock of renewable natural capital, nonrenewable natural capital, and man-made capital, respectively, at an instant in time. Then, a general model that embodies the assumptions listed above has the form:

$$\frac{dh}{dt} = g_h(\vec{s},\vec{c})h \tag{2.11a}$$
$$\frac{dk_r}{dt} = g_{k_r}(\vec{s},\vec{c}) - d_{k_r}(\vec{s},\vec{c}) \tag{2.11b}$$
$$\frac{dk_n}{dt} = g_{k_n}(\vec{s},\vec{c}) - d_{k_n}(\vec{s},\vec{c}) \tag{2.11c}$$
$$\frac{dk_h}{dt} = g_{k_h}(\vec{s},\vec{c}) - d_{k_h}(\vec{s},\vec{c}). \tag{2.11d}$$

All of the functions above depend on the state of the system, $\vec{s}$, and the preferences (culture) of the population as represented by $\vec{c}$.

In equation 2.11a, $g_h(\vec{s},\vec{c})$ represents the per-capita growth rate of the population. It will depend on, among other things, per-capita consumption of commodities, and per-capita birth rates. Similarly in equation 2.11b, $g_{k_r}(\vec{s},\vec{c})$ defines the natural regeneration of bioresources. A common form for $g_{k_r}(\vec{s},\vec{c})$ might be the logistic function, or Gompertz function commonly used in fisheries [15]. The growth of nonrenewable natural capital modeled by g_{k_n} is associated with the continued discovery of new reserves, new materials, and new and better ways to use materials. Finally, the growth in man-made capital stocks, g_{k_h} is the result of new investment.

The term $d_{k_r}(\vec{s},\vec{c})$ models decreasing quality of renewable natural capital as nutrients are removed and soil structure is damaged through agricultural activities. The function $d_{k_n}(\vec{s},\vec{c})$ represents the simple fact that flows of resources are required to produce economic output, while $d_{k_h}(\vec{s},\vec{c})$ captures the simple fact that machines wear out.

Associated with each dynamical system for the physical state space outlined by equations 2.11a through 2.11d is one for the cultural state space. The cultural dynamics are very specific to a particular model realization and are impossible to state in general. In a pure labor economy for example, the cultural dynamics might simply consist of how the population changes its work effort over time. In an economy with capital accumulation, work effort, desired capital to output ratio, and savings rate might constitute the cultural state space. In each of the models discussed in chapters 3, 4, and 5 the cultural models are slightly different.

2.3 Analytical methods

A given family of models specified by equations 2.11 can be cataloged by a parameter space in which each point represents a realization of the model. The main objective of studying this family of models is to divide this parameter space into regions where the model has the same qualitative behavior. When a boundary between these regions is crossed, the behavior of the model fundamentally changes–i.e. a bifurcation occurs. An example is a parameter space divided into two regions, one where the model exhibits a stable equilibrium (sustainable economy), and one where the model exhibits only large amplitude cyclical behavior (unsustainable economy). The nature of these regions generally depends on key parameters or ratios of parameters. For example, in the specific application of the model in chapter 3, the nature of the model behavior depends on three parameters, the work level of the population and the marginal rates of technical substitution of land and labor. Parameter combinations where the model exhibits a sudden change of behavior generate the boundaries between regions in parameter space.

The two basic model features of stable equilibrium and cyclical behavior relate to whether an economy can *attain* a sustainable state. In both cases, one can describe a stationary point where each of the state variables remains constant. Such a description would correspond to one for a sustainable economy where human population, natural, and man-made capital stocks are constant. This says nothing of whether the system can sustain the flows of materials necessary to maintain this state. This is directly related to the difficult question of the meaningfulness of assessing sustainability using the idea of natural capital versus flows of materials [33]. The analysis applied herein illustrates the importance of both measures. If the steady state is stable, then the flows of materials necessary to maintain it are feasible. If it is not, the steady state is unattainable. The bifurcation from a steady state to limit cycle marks the boundary between these possibilities. Figure 2.5 illustrates this point.

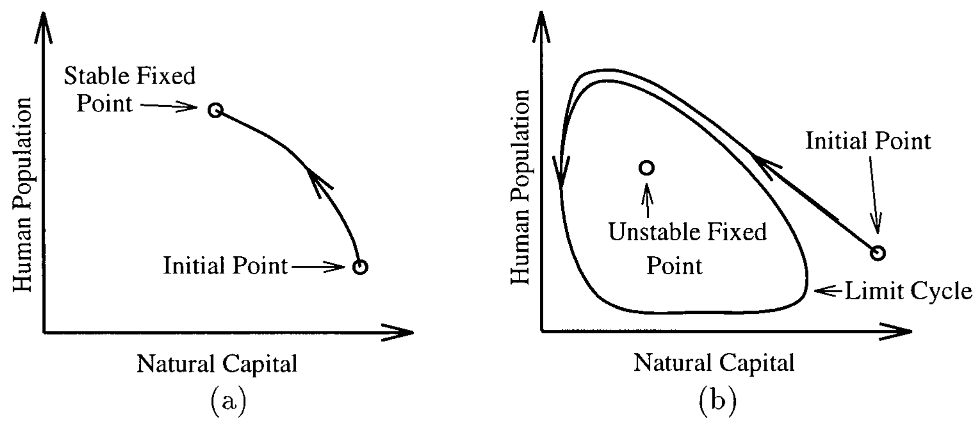


Figure 2.5: Two main model structures: (a) attainable steady state, (b) unattainable steady state.

In graph (a), any reasonable initial condition with high renewable natural capital and low population will evolve to a sustainable state. In graph (b), on the other hand, no reasonable initial condition with high renewable natural capital and low population will evolve to a sustainable state. In this case, the difference between equilibrium natural capital stocks might not provide enough information to discriminate between the two cases as [33] points out. The modelling framework developed herein does.

Unfortunately, computing the boundary between the behavior exhibited in graph (a) from that shown in graph (b) is a difficult task in general. If the system is of low dimension, standard analytic methods of dynamical systems theory can be applied reasonably easily [39]. For large dimensional systems, such analysis becomes impractical. The main tool I employ is a numerical technique known as pseudo arclength continuation available in the software package Auto [20]. The analysis amounts to starting at a known fixed point of the system and tracking its behavior in very small steps. By locating points where the stability of the fixed point changes, we can detect local bifurcations and use these to divide the parameter space as mentioned above.

The main transition we encounter in the models presented in this thesis is called a Hopf bifurcation. Hopf bifurcations occur when a stable fixed point changes to an unstable fixed point surrounded by a stable limit cycle. In mathematical terms, two eigenvalues of the Jacobian of the system in question occur as complex conjugates, and all other eigenvalues have negative real parts. When a parameter is varied, if the real parts of the eigenvalues that occur as complex conjugates change from negative to positive, then the steady state changes from being locally stable to locally unstable, and a periodic orbit develops around the steady state. It is the detection of these Hopf bifurcation and the tracking of their dependence on parameter values using the software package Auto that helps us to study the underlying structure of the models presented herein.

Chapter 3

Culture and human agro-ecosystem dynamics: the Tsembaga of New Guinea

In his classic ethnography of the Tsembaga of New Guinea, *Pigs for the Ancestors*, Roy Rappaport [53] proposed that the cultural practices and elaborate ritual cycle of these tribal people was a mechanism to regulate human population growth and prevent the degradation of the Tsembaga ecosystem. This is probably the best known work in applying ecological ideas, especially systems ecology [45], in anthropology. Rappaport treated the Tsembaga ecosystem as an integrated whole in which the the ritual cycle was a finely tuned mechanism to maintain ecosystem integrity.

Although Rappaport provided detailed ethnographic and ecological information to support his claim, many aspects of his model were subsequently criticized. The main points of criticism were that his work ignored historical factors and the role of the individual, relied on the controversial concept of group selection, and focused too much on the idea of equilibrium. Several simulation models of the Tsembaga ecosystem were constructed to test Rappaport's hypothesis [57, 23] and evaluate possible alternatives, e.g. [24]. The basic conclusions were that it was possible to develop models supporting Rappaport's hypothesis but they were extremely sensitive to parameter choices, and other simpler population control mechanisms might be more likely [10, 24].

Rappaport's original work and associated modeling work by others provide an excellent context in which to apply the modeling framework outlined in chapter 2. The Tsembaga system is a perfect example by which to address the first two questions proposed in the introduction: What role does behavioral plasticity play in this ecosystem?

Does it cause problems or solve them? Do cultural processes play as important a role as Rappaport suggested, and if so how?

To answer these questions, the model is developed in three stages. After summarizing the relevant information for the model in the next section, a physical model for a simple human agro-ecosystem is developed and calibrated based on quantitative information provided by Rappaport [53]. Behavior (in terms of the effort devoted to agriculture) is fixed, and the focus is on the importance of the food production function and associated feedbacks on the dynamics of the physical system. Next, the model is extend to allow for changing levels of work effort in agriculture based on the needs of the human and pig populations (i.e the behavioral plasticity of the population is increased). Finally, more complex behavioral dynamics representing the ritual cycle of the Tsembaga are added.

3.1 The ecological and cultural system of the Tsembaga

The Tsembaga occupy a rugged mountainous region in the Simbai and Jimi River Valleys of New Guinea along with several other Maring speaking groups with whom they engage in some material and personnel exchanges through marriages and ritual activity. These groups each occupy semi-fixed territories that intersperse in times of plenty and become more rigidly separated in times of hardship. Outside these interactions, the Tsembaga act as a unit in ritual performance, material relations with the environment, and in warfare.

The Tsembaga rely on a simple swidden (slash-and-burn) agricultural system as a means of subsistence. At the time of Rappaport's [53] field work they occupied about 830 ha, 364 of which were cultivable. The Tsembaga also practice animal husbandry (the most prominent domesticated animal being pigs) but derive little energetic value from this activity. Pork probably serves as a concentrated source of protein for particular segments of the population as it is rarely eaten other than on ceremonial occasions, and several taboos surround its consumption that seem to direct it to women and children who need it most.

Much of the activity of the Tsembaga is related to the observance of rituals tied up with spirits of the low ground and the red spirits. The spirits of the low ground are associated with fertility and growth while the red spirits which occupy the high forest forbid the felling of trees. The ritual activity that is the focus here is the Kaiko. The Kaiko is a year long pig festival where a host group entertains other groups which are allies to the host group in times of war. The Kaiko serves to end a 5 to 25 year long ritual cycle that is coupled with pig husbandry and warfare. It is this ritual cycle that Rappaport hypothesized acted as self-regulatory mechanism for the Tsembaga population preventing the degradation of their ecosystem.

The three main ingredients of the ritual cycle, pig husbandry, the Kaiko itself, and the subsequent warfare, are intricately interwoven with the political relationships between the Tsembaga and the neighboring groups. The Tsembaga maintain perpetual hostilities with some groups and are allied with other groups without whose support they will not go to war. There are two important aspects of pig husbandry: raising pigs requires more energy than is derived from their consumption; pigs are the main source of conflict between neighboring groups because they invade gardens. From this perspective the keeping of pigs is completely nonsensical. However, the effort required to raise pigs is a strong information source about pressure on the ecosystem. The greater the pig population, the greater the chance an accidental invasion of neighboring gardens will occur. Each time a garden is invaded, there is a chance that the person whose garden was invaded will kill the owner of the invading pig. Records are kept of such deaths which must be avenged during the next ritually sanctioned bout of warfare. From this perspective, pigs provide a meter of ecological and human population pressure and help "measure" the right amount of human population reduction required to prevent the degradation of their ecosystem. The Kaiko, when all but a few of the pigs in the herd of the host group are slaughtered, helps facilitate material transfers with other groups, allows the host group to assess the support of its allies, and resets the pig population.

The ritual cycle as the homeostatic mechanism proposed by Rappaport operates as follows: human and pig populations grow until the work required to raise pigs is too great. A Kaiko is called and most of the pig herd is slaughtered for gifts to allies and to meet ritual requirements. The Tsembaga then uproot the rumbim plant in an elaborate ritual and thus release themselves from taboos prohibiting conflict with neighbors. Warfare, motivated by the requirement of each tribe to exact blood revenge for all past deaths caused by the enemy tribe, begins with a series of minor "nothing fights" where casualties are unlikely then escalates to the "true fight" where axes are the weapons of choice and casualties are much more likely. Periods of active hostilities seldom end in decisive victories but rather when both sides have agreed on "enough killing" related to blood revenge from past injustices. The ritual cycle then begins anew with both the pig and human populations reduced to (hopefully) levels that will not cause ecological degradation. As the model is developed I will fill in the relevant details of each of the components summarized here.

An obvious question is if the ritual cycle does play such and important role in the Tsembaga ecosystem, how did it come about? It is this point that has received much attention in subsequent literature regarding Rappaport's hypothesis. In this thesis, the focus is not how the Tsembaga cultural system evolved, but rather on the more general question of how behavioral plasticity (i.e. the very presence of humans) and associated cultural practices affect the structure and dynamics of agroecosystems. For more on the issue of the evolution of group behavior (culture) versus individual behavior, and how a cultural system such as the Tsembaga might come about, see Anderies [4, 3] and Alden Smith [2].

3.2 The model

3.2.1 Definitions

Following the framework set out in chapter 2, the following physical state variables apply to the Tsembaga:

$h(t)$: Tsembaga population density in persons per cultivable hectares. At the time of Rappaport's [53] study the Tsembaga numbered 204 and occupied 364 cultivable hectares, thus $h = \dfrac{204}{364} = 0.56$.

$k_r(t)$: Renewable natural capital in the Tsembaga ecosystem. Here, renewable natural capital is related to the productive potential of the 364 hectares upon which the Tsembaga rely for their survival. The variable k_r should be thought of as an index of productivity, i.e. productivity per unit of land per unit of effort directed to agriculture.

Similarly, the appropriate cultural state variables are:

$c_1(t)$: Tsembaga per capita birthrate.

$c_2(t)$: Fraction of population devoting 1 man year of energy (2000 hours at 350 kcal/hr) to horticulture each year. Thus the total energy devoted to horticulture at time t is given by $c_2(t) \cdot h(t) \cdot A_c$ man years of energy per year, where A_c is the total number of cultivable hectares available to the population.

We then specify the dynamics for each of the variables based on the interaction of human activities and the energy flows through the system. We define the function that governs human population growth as $f_1(h, k_r, c_1, c_2)$ - the formal statement that population growth depends on the human population, land productivity, per capita birthrate, and work effort directed to cultivating the land. Similarly, the biophysical regenerative process of forest recovery is defined as $f_2(h, k_r, c_1, c_2)$. The functions f_1 and f_2 represent the change in the human population and renewable natural capital over time which leads to the two dimensional dynamical system:

$$\frac{dh}{dt} = f_1(h, k_r, c_1, c_2) \tag{3.1a}$$

$$\frac{dk_r}{dt} = f_2(h, k_r, c_1, c_2). \tag{3.1b}$$

In the next two sections, we explicitly define the forms of f_1 and f_2 based on the ecology of the Tsembaga system. Major considerations are: the nutritional requirements of the Tsembaga population, soil properties and the food production process of the Tsembaga that couples them to the land.

3.2.2 Tsembaga subsistence and the population growth rate, f_1

The canonical way to represent f_1 is

$$f_1 = (b - d)h \tag{3.2}$$

where b and d are the per capita birth and death rates respectively. We are specifically interested in how these rates depend on food production and nutrition, so we separate influences on birth and mortality into a constant component not associated with food intake and a component that does depend on food intake. First we define the food production of the population as $e(h, k_r)$, then f_1 can be written as:

$$f_1 = (b_n(c_1) - d_n(e(h, k_r, c_2)))h. \tag{3.3}$$

The term b_n is the "net birth rate" which is the natural (culturally dependent) birth rate less the natural death rate and **does not** depend on food intake. The term $d_n(e(h, k_r, c)$ is the "net death rate" which is the difference between the portions of fertility and mortality that **do** depend on food intake.

The form of d_n is inferred from the subsistence pattern of the Tsembaga who rely almost completely on fruits and vegetables (99% by weight) for their usual daily intake, the greatest portion of which come from their gardens. Of this non-animal intake, taro, sweet potato, and fruits and stems constitute the largest part (over 60%) of the diet. These starchy staples combined with a wide variety of leafy vegetables and grains, including protein rich hibiscus leaves, combine to provide adequate calories for the entire population and adequate protein for all but the young children. At low levels of production, below a minimum requirement of around 2500 kcal/day, the net per capita death rate increases quickly due to malnutrition. Buchbinder [10] proposed that the mechanism linking malnutrition and mortality could be increased malaria infection due to reduced immunity. Above this minimum, the net death rate of the population can be decreased through the improved nutrition associated with better quality animal protein that improves characteristics such as sexual development, immunity, etc. This decrease in net death rate is, however, small compared with the increase in net death rate associated with malnutrition.

The simplest way to represent $d_n(\cdot)$ mathematically is to assume that once the per capita food requirements are met, $d_n(\cdot)$ approaches 0 asymptotically. Below this minimum requirement, $d_n(\cdot)$ rises quickly. If we choose the units of $e(h, k_r, c_2)$ to be energy requirements per person per year then the quantity $e(h, k_r, c_2)/h$ represents the relative level of nutrition of the population. If this ratio is one, the nutritional needs of the population are just being met. If this ratio is larger than one, the population is producing more than it needs. It devotes the excess to pig husbandry and receives the benefits in terms of increased intake of concentrated protein and fat. The ratio being less than one has the obvious implications. A convenient function with the desired properties is the exponential, and we can represent the mortality,$d_n(\cdot)$ as

$$d_n(e(\cdot)) = a \exp\left(-\alpha \frac{e(\cdot)}{h}\right) \tag{3.4}$$

where the parameter a characterizes the speed at which people die due to malnutrition and α indicates the response to nutrients. For example if $a = 3$ and there is no nutrient intake, 40% percent of the population would be dead within two months, and 78% would perish by 6 months. In the model, I have chosen α and a in the interval $[1, 10]$. There are many reasonable choices but the behavior of the model is qualitatively unchanged by any reasonable combination of these parameters. We can now define $f_1(h, k_r, c_2)$ completely as

$$f_1(h, k_r, c_1, c_2) = (b_n(c_1) - a \exp\left(-\alpha \frac{e(h, k_r, c_1, c_2)}{h}\right))h. \tag{3.5}$$

3.2.3 The ecology of slash-and-burn agriculture

The Tsembaga agricultural system amounts to a piece of land being cleared, cultivated for one year and then left fallow for 15 to 25 years. The gardens are cut in the wetter season in May and early June, allowed to dry, then burned in the dryer season between June and September, and planted immediately thereafter. Because the Tsembaga live on a fixed amount of land, the fallow period and amount of land in production at any one time are directly related. For the Tsembaga, the 15 to 25 year fallow period correlates to about 19 hectares or a little over five percent of the available land being cultivated at any one time.

The dynamics of slash and burn agriculture can be viewed as a cycle with two phases: the cultivation phase and the fallow recovery phase. During the cultivation phase, nutrients contained in the biomass of the forest are released into the soil through burning, a portion of which are subsequently removed through cultivation. In addition to direct nutrient removal, gardening has other negative effects on soil quality, especially on soil structure. Juo et al. [37] have cataloged some of these indirect effects:

- The removal of ground cover exacerbates erosion.

- Increased frequency of clearing and cultivation causes the gradual destruction of soil macropore system due to increased foot traffic and tilling.

- Burning and cultivation lead to the gradual destruction of the root mat, the decomposition of humidified organic matter, and the reduction of the contribution of organic and microbial processes to nutrient cycling.

Frequency and intensity of cultivation probably both effect recovery times (Szott et al. [62]) and the negative effects of agriculture on soil productivity probably increases nonlinearly with food production. I assume, probably conservatively, that these effects increase linearly with food production.

During the subsequent fallow phase, the nutrient cycling process shown schematically in Figure 3.1 is reestablished through forest succession. The rate of the cycling process and the associated rate at which nutrients are recycled and fixed in the soil depends on the four processes depicted in Figure 3.1: litter fall, decomposition, mineralization, and uptake [47]. Uptake and litter fall are related to standing biomass which, of course, depends on soil nutrients. Thus, the rate of change of soil nutrients depends on the level of nutrients in the soil. Finally, the nutrient cycling process is governed by the characteristics of the community of decomposing and mineralizing organisms in the soil which set an upper limit on the amount of nutrients in the soil. The simplest way to capture this behavior is by the well known logistic function. This is obviously an oversimplification for a very complex process. However, if compared to a detailed, much more complex model for this process [35], the qualitative behavior is captured reasonably well by the logistic. Combining the effects of biophysical regeneration and degradation due to agriculture, the rate of change of renewable natural capital is

$$f_2(h, k_r, c_2) = n_r k_r (1 - k_r / k_r^{max}) - \beta e(h, k_r, c_2) \tag{3.6}$$

where n_r is the maximum regeneration rate, k_r^{max} is the maximum soil nutrient level for the ecosystem, and β is the appropriate conversion factor relating food production to productivity.

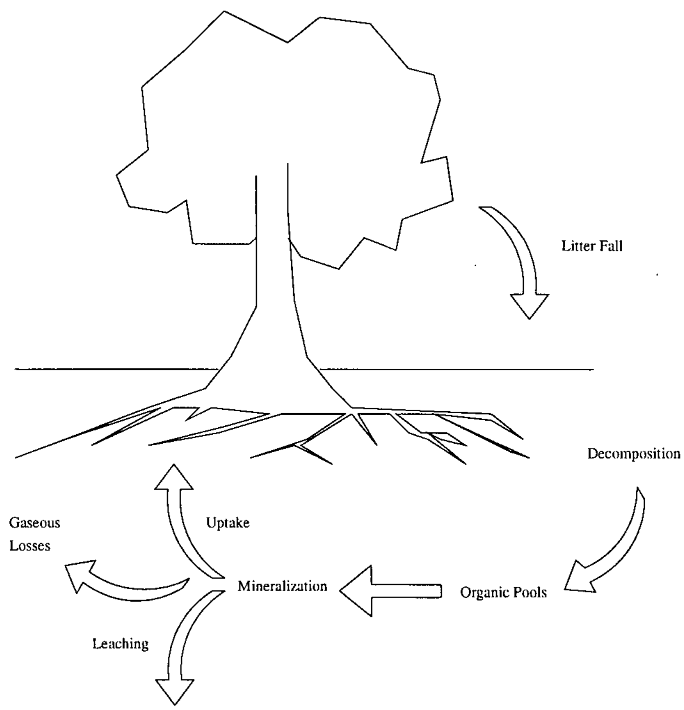
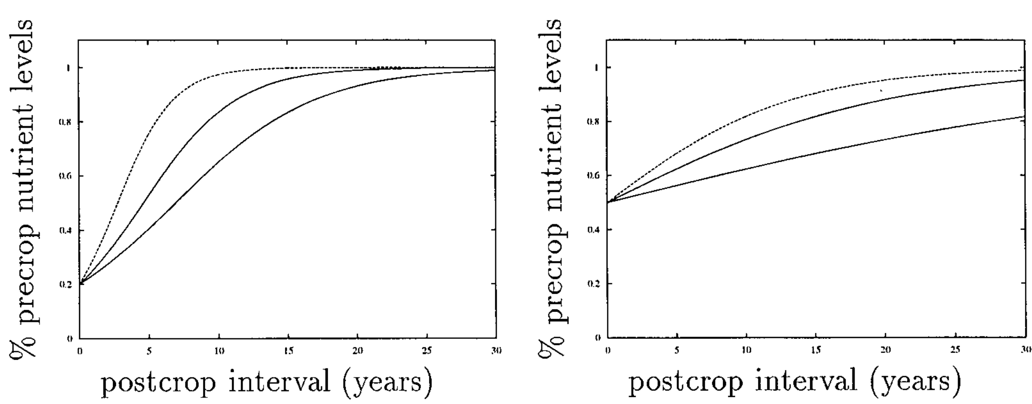


Figure 3.1: Graphical representation of nutrient cycling process in a forest. Adapted from ([47])

There is some difficulty associated with the determination of the intrinsic regeneration rate, n_r, for the forests the Tsembaga occupy. It is possible, however, to get an idea of the order of magnitude n_r from other studies. The time of successional recovery from slash and burn to stable litter falls ranges from seven years in the plains of the United States [56] to 14-20 years in the tropics [22]. The numbers for Guatemala closely match the fallow periods for the Tsembaga in New Guinea, so we can scale n_r for a characteristic recovery time of 15 to 25 years if the forest is left undisturbed. Figure 3.2 shows recovery curves for different values of n_r and different initial conditions for $k_r(0)$. Since we do not know $k_r(0)$ we can only bracket reasonable values of n_r in the following way. If enough nutrients are removed to reduce k_r to 20% of its maximum value, we examine recovery curves from this value (graph (a) in Figure 3.2) to see that if $n_r = 0.3$ or 0.5, the system recovers too fast. The recovery time for this initial condition and $n_r = 0.2$ is reasonable so we take 0.2 to be the upper bound for n_r. If cropping does not reduce soil nutrients so drastically, say to a level of 50%, lower values of n_r are reasonable. Graph (b) in Figure 3.2 shows the results for $n_r = 0.05$, 0.1, and 0.15 respectively, suggesting that 0.05 might be taken as a lower bound for n_r. Thus we assume that $n_r \in [0.05, 0.2]$. This range could be significantly narrowed from a quantitative measurement of soil parameters before and after cropping. Unfortunately, it seems that when these measurements have been attempted, the range of error of measurement exceeds the magnitude of the variables themselves.



(a)- post crop nutrient levels: 20% of original

(b)- post crop nutrient levels: 50% of original

Figure 3.2: Recovery curves for different values of the condition of the soil after cropping and recovery rate n_r. In figure (a), the values of n_r coresponding to curves of increasing steepness are 0.2, 0.3, and 0.5. Likewise, in figure (b), these values are 0.05, 0.1, and 0.15.

With f_1 and f_2 now completely defined, we can rewrite the dynamical system representation of the Tsembaga ecosystem defined by Equations 3.1a and 3.1b as

$$\frac{dh}{dt} = (b_n(c_1) - a\exp\left(-\alpha\frac{e(h,k_r,c_1,c_2)}{h}\right))h \tag{3.7a}$$

$$\frac{dk_r}{dt} = k_r n_r(1 - k_r/k_r^{max}) - \beta e(h,k_r,c_2). \tag{3.7b}$$

Given the problems with associating units to renewable natural capital, it is convenient to rescale the model by k_r^{max} by letting $k_r = \widetilde{k_r} \cdot k_r^{max}$, with $\widetilde{k_r} \in [0,1]$. Now, $\widetilde{k}_r$ represents the mean productivity index per hectare of the land the population is occupying, one being maximum productivity, zero being barren. We also drop the explicit dependence of b_n on c_1 by assuming b_n is a linear function of c_1 and treating b_n as a parameter. The rescaled equations are (dropping the tilde notation):

$$\frac{dh}{dt} = (b_n - a\exp\left(-\alpha\frac{e(h,k_r,c_1,c_2)}{h}\right))h \tag{3.8a}$$

$$\frac{dk_r}{dt} = k_r n_r(1 - k_r) - \beta e(h,k_r,c_2). \tag{3.8b}$$

Our final task is the specification of $e(\cdot)$.

3.2.4 The food production function

For Equation 3.8b of the model, we need an explicit form of the food production function, $e(h,k_r,c_2)$. Unfortunately, although several simple causal relationships are understood, there is no fundamental scientific understanding of how nutrients, soil processes, and crop output are related. Examples of work on this problem include France and Thornley's [25] development of plant growth models and Keulen and Heemst's [38] empirical work on crop response to the supply of macronutrients. Economic approaches that focus on energy inputs and resource degradation can be found in work by Cleveland [16, 17] and Giampietro et al. [28]. Econometric work on determining the form of production functions has been carried out by many authors, see for example [1, 48].

Several functional forms have been suggested for modeling crop output in the work just mentioned, but two are of interest for the model: the von Liebig and the Cobb-Douglas. The von Liebig function is based on von Liebig's law which states that crop output is a function of the most limiting resource. The functional form is

$$y = A_{sw} \min_{i \in I} [f_i(x_i)] \tag{3.9}$$

where y is output, A_{sw} is the yield plateau set by the soil and weather, x_i is the total availability of the i^{th} nutrient, and each f_i is a concave function from $\mathcal{R}$ to $[0, 1]$. Lanzer and Paris [40] proposed to use this functional form in place of the commonly used polynomial forms and in a later paper, Ackello-Ogutu, Paris, et al. [1] tested the von Liebig crop response against polynomial specifications and were able to reject the hypothesis that crop response is polynomial. Further, they could not reject that crop response was of the minimum or von Liebig type.

Paris et al. [48] estimated the von Liebig function for cotton lint response to the input of water and nitrogen. They assumed that f_N and f_W were linear and lumped all other scarcities into one variable m, to get

$$y = \min_{N,W} [\alpha_N + \beta_N N, \alpha_W + \beta_W W, m]. \tag{3.10}$$

Note that α_N and α_W represent nutrients already present, while the other terms represent applied nutrients. The production surface for this production function is shown in Figure 3.3.

The key point to note is that the variable m places a constraint on production due to all the other variables not accounted for.

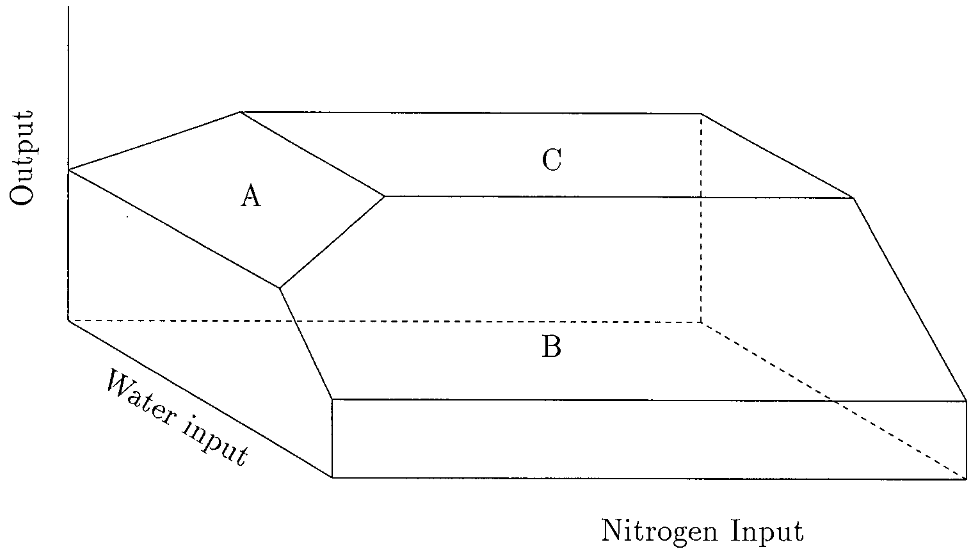


Figure 3.3: The production surface for cotton lint as modeled by the von Liebig production function. A, B, and C are the Nitrogen, Water, and "m" limiting planes respectively.

Although the von Liebig function may be the best representation of reality, the fact that it is not smooth will cause difficulties when analyzing the dynamical system. Instead, a commonly used production function from economics, the Cobb-Douglas given by

$$y = k \prod_{i=1}^{n} x_i^{a_i} \tag{3.11}$$

where x_i is the i^{th} input and a_i are constants is used as an approximation. The problem with this function is that it allows infinite substitutability. That is, if the inputs were land and water, this function says that productivity can be maintained in the face of a drought by bringing more land under cultivation. This is clearly absurd. If on the other hand, the inputs of interest are not physical quantities, for example energy input, the situation is different.

If the general form of the von Liebig function given by Equation 3.9 is used to model output where the input variable is human work energy, the physical inputs f_i(energy in) may be nonlinear. This is definitely the case for the Tsembaga with regard to the amount of land brought into cultivation for a given amount of labor. Here, the Cobb-Douglas is not such a bad approximation to the von Liebig as shown in Figure 3.4.

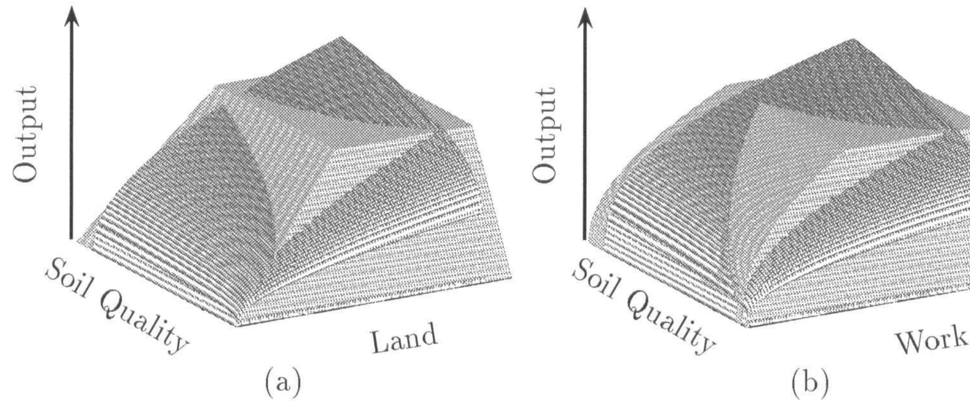


Figure 3.4: The Cobb Douglas production function overlayed on the von Liebig function. Case (a) - inputs are physical quanities. Case (b)-one input is a nonphysical quantity, work, upon which the physical input, land depends in a non-linear way.

The two inputs to agriculture accounted for in my model are human energy and renewable natural capital. Other inputs such as rainfall and solar energy input are assumed to be fairly constant, which based on the indications of the Tsembaga, is accurate. They indicate that the weather never fluctuates significantly enough to influence crop output, at least not in their lifetimes. Under these assumptions, the food energy production function is of the form:

$$e(h, k_r, c_2) = k(w(h, c_2))^{a_1} k_r^{a_2} \tag{3.12}$$

where $w(h, c_2)$ is the amount of energy the population directs towards agriculture, a_1 and a_2 are the output elasticity of energy and renewable natural capital respectively, and k is a proportionality constant. Fortunately Rappaport [53] made detailed measurements of the energy input per unit area of land cultivated along with the associated output.

Using this information we can calibrate the food energy production function, i.e., for a given choice of a_1 and a_2, Rappaport's data can be used to compute an estimate of k as follows.

Rappaport indicates that when the human population was 204 and the pig population was 169 animals weighing between 120- and 150 pounds, the amount of land cultivated was about 18 hectares or 6% of the total cultivable land, leaving 94% fallow. The trophic requirements of pigs are similar to those of humans, and their population can thus be converted into equivalent Tsembaga numbers. The average Tsembaga weighs 94 pounds so their 169 pigs would have the same trophic demands as 240 Tsembaga. Thus, the 18 hectares supported approximately 444 Tsembaga equivalents.

Based on his energetic analysis, one person year (2000 hours at 350 kcal/hr) of energy input is sufficient to clear, burn, cultivate, and harvest one hectare of land. Using energy units in human annual energy requirements, 18 man years of energy input produced 444 units of total energy output or 1.22 energy units per hectare. Now, making a guess at the stage of recovery the secondary forest when brought into cultivation, we can estimate k. Supposing the nutrient level is 80% that of a mature forest, we have

$$1.22 = k(18)^{a_1} 0.8^{a_2} \Rightarrow k = \frac{1.22}{(18)^{a_1} 0.8^{a_2}}. \tag{3.13}$$

Then, given the definition of c_2, the work devoted to agriculture is $w(h, c_2) = hc_2 A_c$. For the situation described above, $c_2 = 0.09$, and $A_c = 364$.

Assuming that the villagers do not waste labor, a certain work effort is roughly correlated to the amount of land being cultivated. If the relationship were linear, increased effort would increase land under cultivation proportionately. If an additional proportional amount of land of equal quality is is brought under cultivation, one would expect that output would increase proportionately. This situation would be modeled by choosing

$a_1 = 1$. Given the terrain of the Tsembaga, however, increased work input will not increase the amount of land cultivated proportionately. Each marginal unit of land brought into cultivation requires further travel distances which may require substantial elevation gains, and the passage of natural barriers such as ridges and rivers. This suggests that $a_1 < 1$ but not substantially. Estimating a reasonable value for a_2 is more difficult and will be discussed later. The model is now fully specified:

$$\frac{dh}{dt} = (b_n - a\exp\left(-\alpha\frac{k(c_2 h A_c)^{a_1} k_r^{a_2}}{h}\right))h \tag{3.14a}$$

$$\frac{dk_r}{dt} = k_r n_r(1 - k_r) - \beta k(c_2 h A_c)^{a_1} k_r^{a_2}, \tag{3.14b}$$

and we can now study its behavior.

3.3 Dynamic behavior of the model

Equations 3.8a and 3.8b represent a family of models parameterized by c_2, a_1, and a_2. Applying the techniques described in chapter 2 to our model system allows us to assess its sensitivity to the structure of the food production function and the work level of the population. Over a wide range of physically meaningful values for b_n, a, α, n_r, and β, the model exhibits a (locally) asymptotically stable equilibrium population density of around 0.6 when $c_2 = 0.09$ which agrees with the demographic data previously discussed. The corresponding equilibrium renewable natural capital value is around 0.75; quite reasonable given that cultivated land is rotated so at any one time at least 10% of the land has just been cultivated and other land is in various stages of recovery.

The model's qualitative behavior is sensitive to c_2, a_1,and a_2. If we fix $a_1 = 0.7$ and $a_2 = 0.3$ representing the case where bringing more land under cultivation is more marginally productive than increasing renewable natural capital (soil quality), the model exhibits a Hopf bifurcation when c_2 is varied as shown in the bifurcation diagram in Figure 3.5. Points on the solid line represent stable equilibria while those on the dotted line represent unstable equilibria. The large solid circles represent stable limit cycles. For c_2 less than approximately 0.1354 the system will exhibit a stable equilibrium. For c_2 greater than 0.1354, the equilibrium becomes unstable, and a stable limit cycle with a period of about 300 years appears in which population builds and reaches its maximum after about 250 years then declines over the next 20 to 30 years. When the population density is extremely low, the land recovers over the next 20 to 30 years and the process begins again.

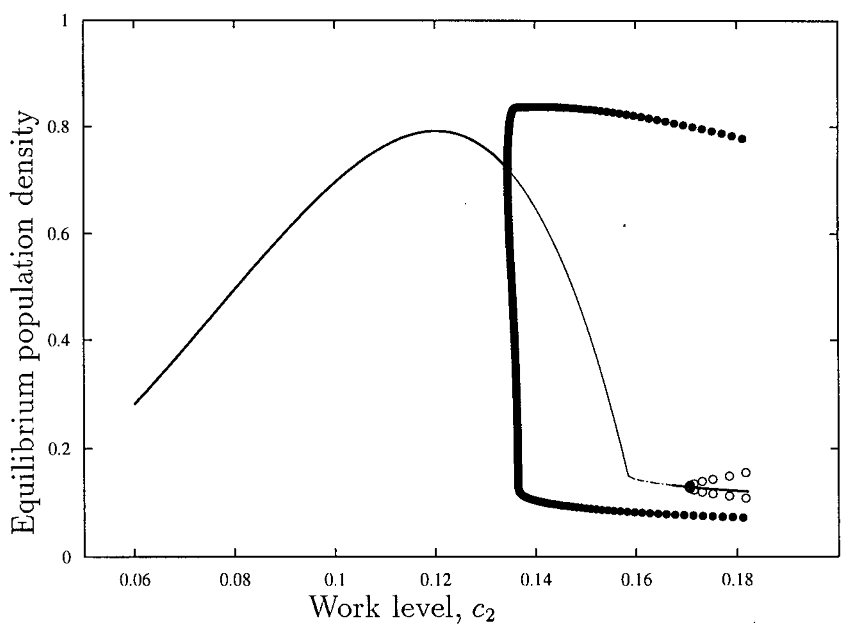


Figure 3.5: Bifurcation diagram for swidden agriculture with $a_1 = 0.7$ and $a_1 = 0.3$. The heavy solid line represents stable equilibria points while the thin line represents unstable equilibrium points. The dark circles represent the maximum and minimum values taken on by x_1 on the stable limit cycle, i.e. as the system goes through one cycle, x_1 varies from 0.1 to 0.8 people/cultivable hectare.

The key point is that if the population works at a level $c_2 = 0.09$ as it was during

Rappaport's field work, the ecosystem is very stable.

More interesting is the model's dependence on the relative marginal productivities of soil and labor. If we make the common assumption that $a_1 + a_2 = 1$ (the economic implications of which will be discussed later), then the effect of the output elasticity of soil and labor on the dynamics of the model can be studied by varying one parameter, either a_1 or a_2. It turns out that there is a relationship between the output elasticity of energy input versus renewable natural capital as is made clear by comparing Figure 3.6 with Figure 3.5.

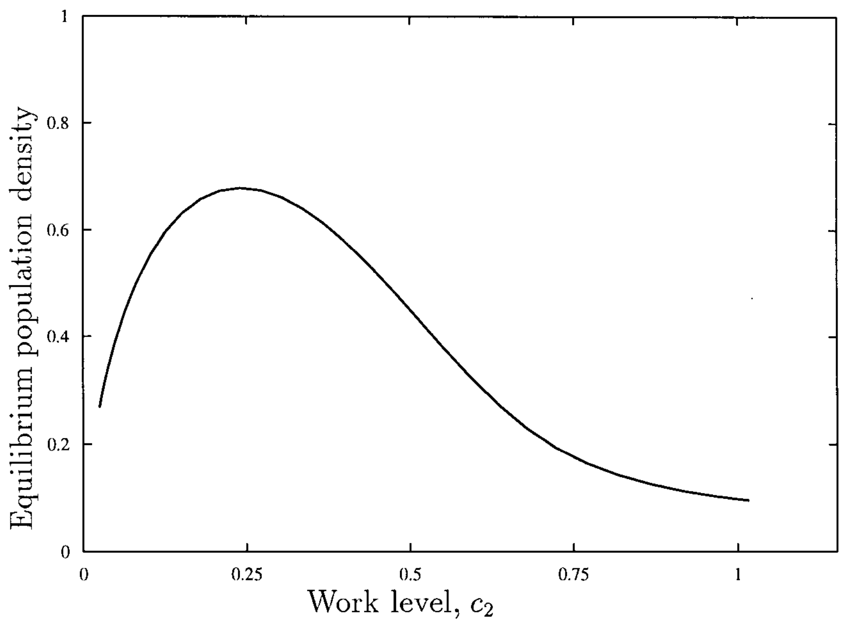


Figure 3.6: Bifurcation diagram for swidden agriculture with $a_1 = 0.4$ and $a_2 = 0.6$. As in figure (3.5) the solid line represents stable fixed points.

When $a_1 = 0.7$ a bifurcation occurs near $c_2 = 0.1354$ as previously noted but when $a_1 = 0.4$, no bifurcation occurs for any value of c_2 as indicated by Figure 3.6.

In order to understand this behavior, we create a two parameter bifurcation diagram, Figure 3.7, that shows all the combinations of c_2 and a_1 for which a Hopf bifurcation occurs. The curve generated by these points separates c_2 - a_1 parameter space into regions with qualitatively different behaviors shown in Figure 3.8. Curves for two different cases are shown, one where the population is more and less susceptible to death due to malnutrition as indicated on the diagram. In each case there is a threshold value of a_1 below which no bifurcation occurs, i.e. the system remains stable for any level of work. This phenomenon has an interesting ecological interpretation.

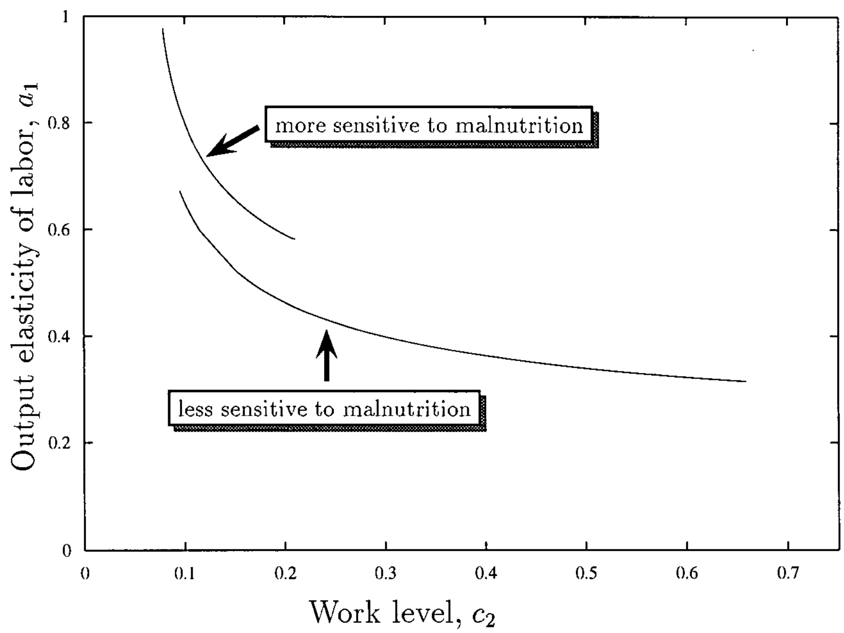


Figure 3.7: Two parameter bifurcation diagram for the swidden agriculture model. The curves represent parameter combinations at which a Hopf bifurcation occurs.

In any ecological model, the relative strengths and timing of feedbacks between state variables governs model stability. In our case, the agriculturalists receive feedback from the land in terms of productivity per unit effort and the land receives feedback from the agriculturalists in the form of population density.

Given that $e(h, k_r, c_2) = k(c_2 h A_c)^{a_1} k_r^{a_2}$, the marginal productivity of each input is

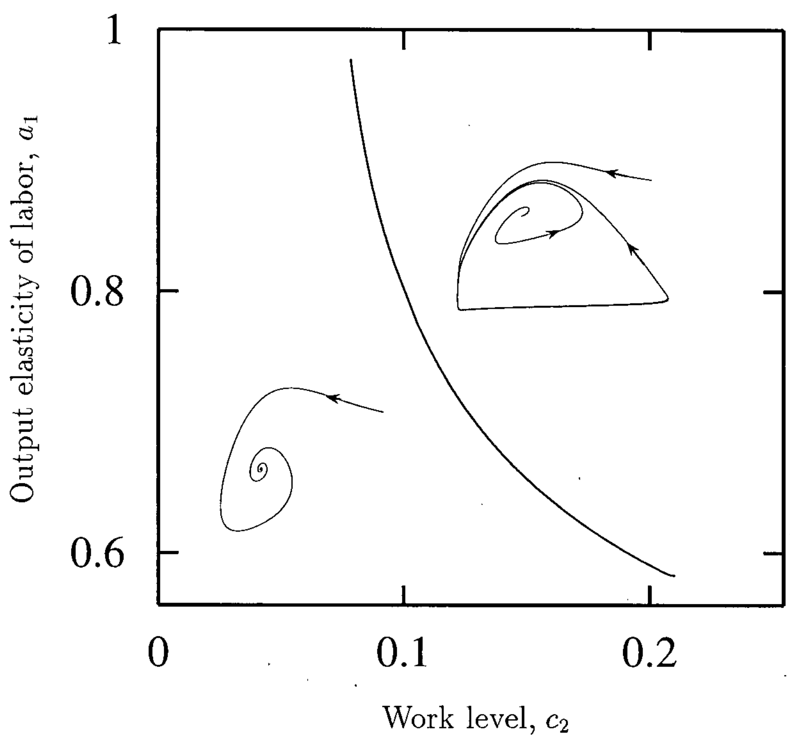


Figure 3.8: Change in dynamics as the bifurcation boundary is crossed. The system goes to a stable equilibrium for parameter values to the left and below the curve while for those above and to the right, the system exhibits stable, cyclic behavior.

defined as

$$\frac{\partial e(h, k_r, c_2)}{\partial h} = \frac{a_1 e(h, k_r, c_2)}{c_2 h A_c}, \tag{3.15}$$

and

$$\frac{\partial e(h, k_r, c_2)}{\partial k_r} = \frac{a_2 e(h, k_r, c_2)}{k_r}, \tag{3.16}$$

respectively. The parameters a_1 and a_2, called output elasticities in economics, are measures of proportional increase in productivity associated with increasing work effort and renewable natural capital respectively. If the output elasticity of labor is higher than the output elasticity of natural capital, it will pay to bring more lower quality land into production (shorter fallow periods) as opposed to preserving soil quality. The declining natural capital feedback is weakened by the stronger feedback of increased yields due to increased cultivation effort. Under these circumstances, the ecological system exhibits a bifurcation from a stable to an unstable system if the work level becomes too high.

If on the other hand, the output elasticity of labor is lower and that of renewable natural capital correspondingly higher, the possibility of bifurcating from a stable to an unstable system is reduced. The feedback from decreased renewable natural capital is now stronger and exerts more pressure on the population. This pressure keeps the population in check before natural capital is degraded to the point below which the population can not be supported. From the agriculturalists' point of view, the gains from cultivating more land are more than offset by the productivity losses associated with reduced soil quality and nutrient levels resulting from the shorter fallowing periods, a strong feedback to avoid working the land too hard.

Notice that the curve for the case where the population is less sensitive to malnutrition and disease extends to lower values of a_1 for which a bifurcation occurs. Malnutrition and disease is the mechanism through which reduced agricultural productivity affects the population. If this mechanism is weakened, the stabilizing influence of reduced natural capital is also weakened. This has the effect of making the model unstable for wider range of values of a_2. The critical point to take away from this analysis is that as output elasticity of labor is increased and the relationship of malnutrition and disease to mortality in the population is weakened, the **potential** for ecosystem instability increases. Whether or not that potential is realized depends on how behaviorally plastic the population is, the issue to which we now turn our attention.

3.4 Behavioral plasticity

In general, in models of animal population dynamics, behavior, although state dependent, is relatively inflexible. Dynamics and stability characteristics are determined by physical aspects of the ecosystem coupled with the fixed behaviors of organisms that occupy it. Mechanisms that might cause a change in the qualitative behavior of such a system might be changes in the external environment (e.g. [8]) , or evolutionary dynamics (e.g. [29]).

In an ecological model involving humans, the situation is quite different. The system can move in and out of regimes of stability and instability very quickly with changing behavior. For example, the amount of land that the Tsembaga put into cultivation (the value of c_2) is not constant–it depends on the human and pig population. To investigate the effect this has on the model, we now treat c_2 not as an exogenously set parameter, but rather, as an endogenously determined quantity by allowing the population to adjust c_2 to attempt to meet nutritional requirements. The work level is governed by the difference between actual food production and desired food production and the availability of additional labor. A simple expression for the dynamics of c_2 is:

$$\frac{dc_2}{dt} = \lambda_{c_2}\left(d_f - \frac{e(h, k_r, c_2)}{h}\right)(c_2^{max} - c_2) \tag{3.17}$$

where d_f is the food demand, c_2^{max} is the upper limit on the fraction of the population working full time cultivating the land, and λ_{c_2} is the speed of response of the population to changes in demand.

The food demand is culturally set, and I define it as follows: if the minimum food requirements of the population are being met on average (about 3000 calories per day), then $d_f = 1$. Significant deviations away from one are possible, as human populations exist on a daily caloric intake ranging from around 2000 up to 6000 calories. The parameter c_2^{max} could be culturally set or set by physical limitations. The parameter λ_{c_2} is a measure of the behavioral plasticity of the population, setting the time scale on which behavioral change can occur. As λ_{c_2} increases, the population can change its behavior on shorter time scales. If we append Equations 3.8a and 3.8b with Equation 3.17 we have a three dimensional dynamical system that describes the human agroecosystem. This system exhibits a steady state if either food demand is met ($\frac{e(h,k_r,c_2)}{h} = 1$), or the population is working at the maximum permissible level ($c_2 = c_2^{max}$).

By treating c_2^{max} as a bifurcation parameter, we can explore the behavior of the system defined by Equations 3.8a, 3.8b, and 3.17. The results are shown in Figure 3.9. If $c_2^{max} < 0.1354$ the model exhibits a stable equilibrium. The stable equilibrium vanishes when $c_2^{max} > 0.1354$ and a stable limit cycle develops.

If the population is somehow limited in the maximum effort it devotes to agriculture, the nutrition and disease population control mechanism proposed by Buchbinder [10] would effectively stabilize the system. From the description of their computer simulation model, it seems that Foin and Davis [24] set an upper limit on "cultivation intensity" which would explain their conclusion supporting Buchbinder's hypothesis.

If, on the other hand, the maximum effort the Tsembaga could devote to agriculture if necessary is above the critical level, (which is reasonable to believe since, for example, this would only require that 15% of the population be willing to work in agriculture if necessary) the stabilizing mechanism proposed by Buchbinder would not be sufficient to stabilize the system. Thus, if there is any hope of the ecological system being stable, some other mechanism, perhaps cultural, must come into play.

If we let $c_2^{max} = 0.25$, meaning one fourth of the population could devote a person year of energy to agriculture if necessary, the population could work hard enough to meet food demand and then c_2 is dynamically set by the relation

$$1 = \frac{e(h, k_r, c_2)}{h}. \tag{3.18}$$

Then from Equation 3.8a and 3.8b, for equilibrium we must have

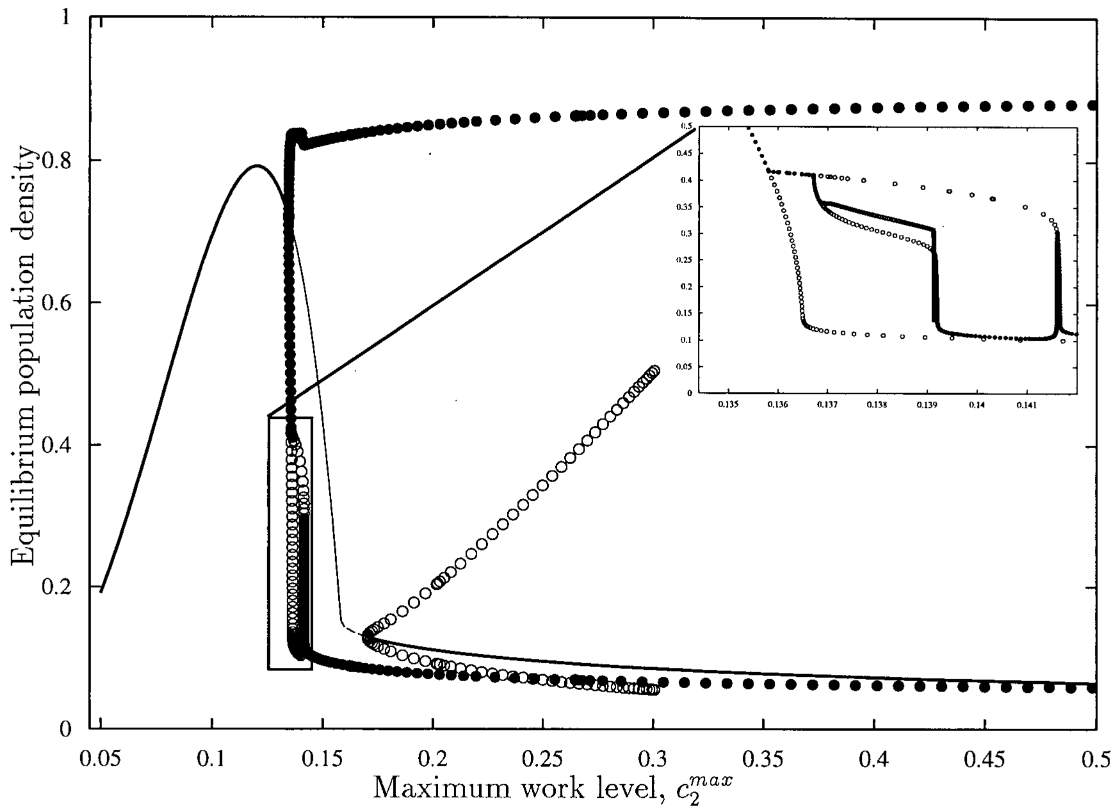


Figure 3.9: Bifurcation diagram with c_2^{max} as the bifurcation parameter in the swidden agriculture model. The upper inset is an exploded view of the boxed region in the main bifurcation diagram showing the increase in complexity of the dynamics when culture is added to the system. These dynamics occur over an extremely narrow parameter range, thus having a low probability of being observed in the physical system.

$$b_n - a\exp(-\alpha \cdot 1) = 0 \tag{3.19a}$$

$$k_r n_r (1 - k_r) - \beta h = 0. \tag{3.19b}$$

If the parameters b_n, a, and α are such that Equation 3.19a is satisfied, the nonlinear system defined by Equation 3.18 and 3.19b defines a one dimensional manifold of fixed points in $\Re^3$. The equilibrium population, natural capital level, and work level depend on initial conditions. Of interest to us is how the net birthrate must be exactly balanced by the net death rate associated with the nutritional level achieved when food demand is met. If the population could, through some cultural mechanism such as infanticide or some other type of birth control, match these rates, the system would be (neutrally) stable. Here, we see how extreme behavioral plasticity can destabilize a system by nullifying the feedback control of resource limitation and transferring the responsibility of ecosystem regulation from environmental to cultural mechanisms.

It is probable that the net growth rate of the population is positive when food demand is met which violates the stability condition given by 3.19a. In this case the ecosystem exhibits cyclic behavior. It is very interesting to compare the limit cycle behavior of the cases with and without behavioral plasticity. Figure 3.10 shows the limit cycles that develop in the system where the work level is treated as a parameter (inner cycle) set constant at $c_2 = 0.14$ and those that develop when the work level is dynamically set with $c_2^{max} = 0.25$ (outer cycle). Figure 3.11 shows the work level and food production over time. Several interesting points are worth making about these figures.

First, the period of the outer cycle where the work level is dynamically set is about twice that of the case were the work level is constant. The reason for this can be seen in Figure 3.11. The initial work level is very low, around 0.05, because if the population is low and renewable natural capital is high at $t = 0$ little effort is required to meet food demands. The population does not over exploit its environment just because it can, and just meets food demand. With the case where the work level is constant at 0.14, the population exploits the environment at a constant rate. When renewable natural capital is high, the population can produce an abundance of food which increases the growth rate of the population. Thus, when the level of renewable natural capital is high, a population that just meets food demand grows more slowly than a population with a constant work level. The difference is indicated in Figure 3.10 by the difference in time required for the population to reach a maximum: 240 versus 720 years for the constant

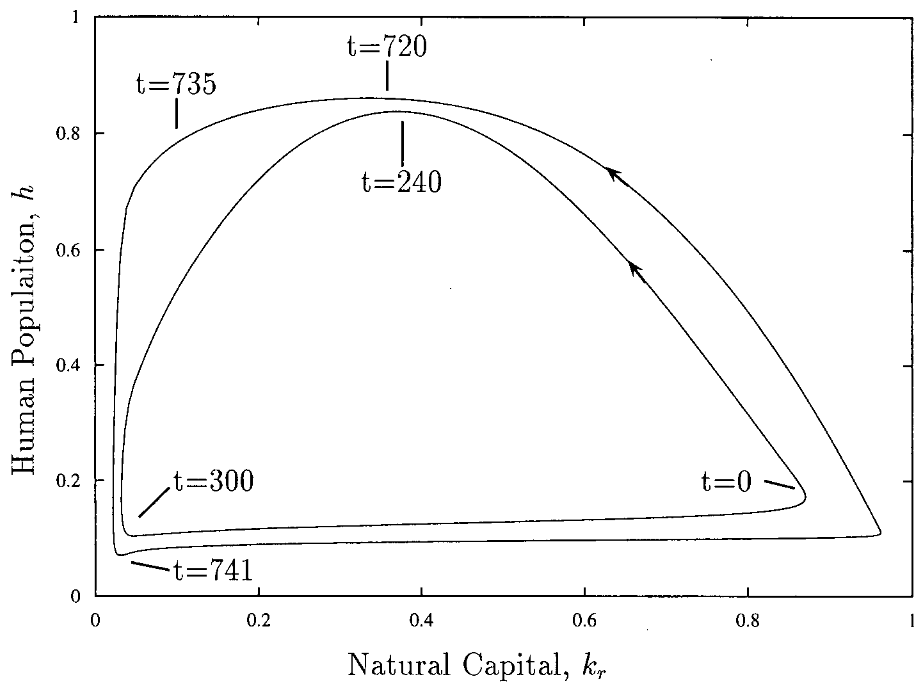


Figure 3.10: Limit cycles that develop as the the system becomes unstable. The inner cycle is for the case where the work level is constant at 0.14. The outer cycle represents the case where the work level is set by demand.

and dynamic work level cases, respectively.

Next, notice that in the constant work level case, after the population reaches a maximum, it begins to decline immediately. This decline to the lowest population level takes about 60 years. In the dynamic work level case, by increasing work level dramatically as shown in Figure 3.11 around $t = 720$, the population is able to delay the precipitous decline in population for about another 15 years. In doing so, however, the population puts itself into a more precarious position of very high population density in a very degraded environment. The precipitous decline now takes 6 years instead of 60!

Since the Tsembaga do adjust their work level, the model suggests that unless some mechanism intervenes, their ecosystem is doomed to crash. This could be avoided by maintaining the knife edge set of parameters required for stability in 3.19a by controlling

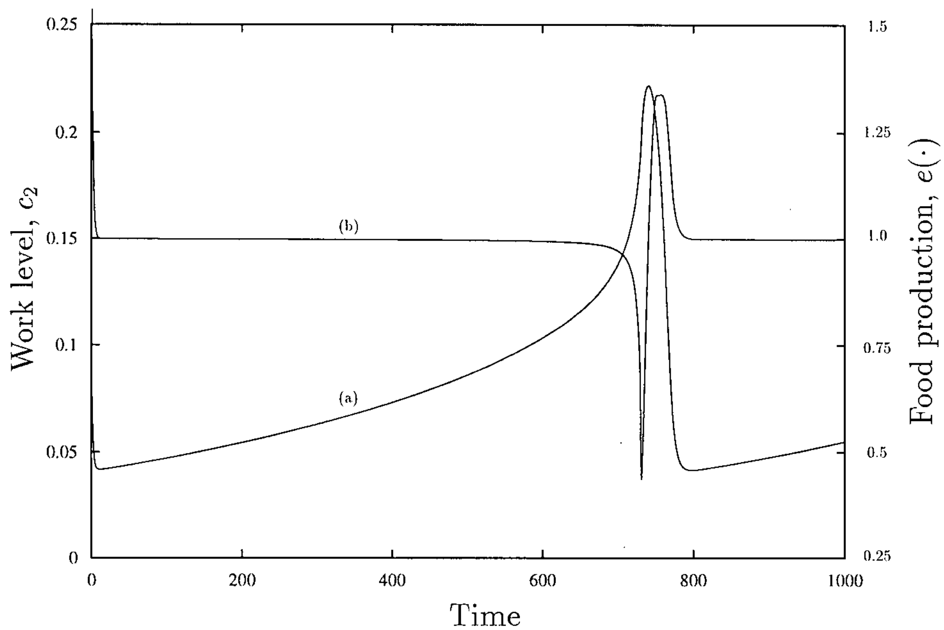


Figure 3.11: Work level (curve (a)) and food production (curve (b)), over time.

birth and death rates within the population, or possibly by the ritual cycle. It seems that the former is not the case; the Tsembaga actively seek to be as "fertile" as possible as evidenced by their rituals to improve fertility. In the next section, we add the dynamics of the ritual cycle and determine the conditions under which it could maintain a balance in and prevent the degradation of the Tsembaga ecosystem.

3.5 Modelling the ritual cycle

The ritual cycle dynamics are added in two parts. First we address pig husbandry to find that even without the ritual cycle, pig husbandry alone can help stabilize the system. Next we add the ritual cycle to show that under certain assumptions the ritual cycle can stabilize the system, and that stability is not as sensitive to parameter choices as it is to how the number of people who ought to be killed during warfare is related to pig invasions.

3.5.1 The parasitism of pigs

The bulk of the responsibility of keeping pigs falls on Tsembaga women. They do most of the work in planting, harvesting and carrying the crops used to feed the pigs. In this sense, the pigs can be viewed as parasitizing Tsembaga women. They benefit from energy derived from the ecosystem but do not contribute to obtaining that energy. It turns out that this relationship, in and of itself, is enough to *help* stabilize the ecosystem. The mechanism is related to the fact that working too hard is a major factor in destabilizing the ecosystem. If the human population is the sole benefactor of its agricultural effort, it grows in number, produces a larger labor pool, and the per-capita work level remains constant. If, on the other hand, the population keeps pigs, as the pig population grows relative to the human population, the per-capita work level *increases.* In this way, the pigs act as an ecosystem monitoring device.

This is clearly illustrated by the model. In all the previous investigations, it was assumed that the Tsembaga devoted a constant 55% of their harvest (based on demographic information at a point in time) to pigs maintained a constant pig to person ratio (no ritual cycle). By treating this ratio as a parameter, r_p, we can generate a figure similar to Figure 3.8 where the parameters of interest are the percentage of food being consumed by humans and c_2^{max}. Figure 3.12 is the result. The curve in graph (a) separates regions in parameter space of stability and instability. Notice that the more food the humans eat themselves, i.e. $r_p \to 1$, the lower the level of c_2^{max} at which the system becomes unstable. Recall that with $r_p = 0.45$, the system goes unstable when $c_2^{max} = 0.1354$. This represents only a 50% increase in work effort which is plausible. Now consider the case where $r_p = 0.3$, the system remains stable until c_2^{max} reaches approximately 0.22. This represents a more than doubling of work effort which may be intolerable to the

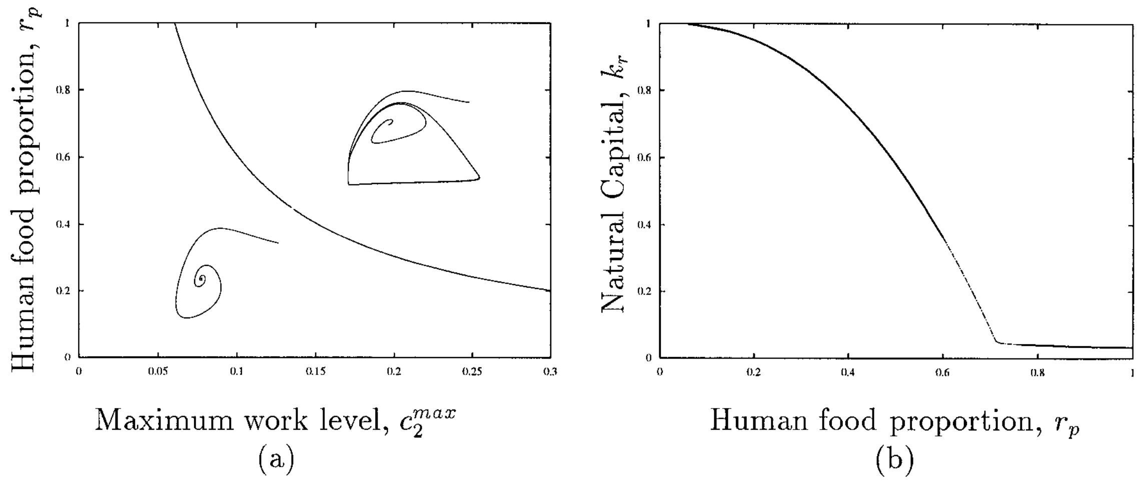


Figure 3.12: The influence of pigs on system dynamics. Figure (a) shows the bifurcation boundary in c_2^{max}-r_p parameter space. Figure (b) shows the equilibrium natural capital level as a function of r_p.

population. Thus, just by being there, the pigs help stabilize the system. Note that this stability comes at the expense of human nutrition. In this model, food is first fed to the pigs and the remainder is fed to the population. This is not what happens; the Tsembaga eat the best food first and give the rest to their pigs. This difference requires the more elaborate ritual cycle mechanism to stabilize the system.

3.5.2 The ritual cycle

The ritual cycle consists of periods of ritually sanctioned truces separated by warfare. The rituals that mark the transitions between the phases are the Kaiko that marks the end of the truce period and the planting of a plant called rumbim (*cordyline fruticosa*) that marks the beginning of the next truce. Figure 3.13 is a representation of the cycle.

The length of the arcs on the circle is loosely representative of the times between events. The Kaiko itself lasts one year. Warfare lasts for a matter of months. The time between planting the rumbim that signifies truce and the Kaiko (typically about

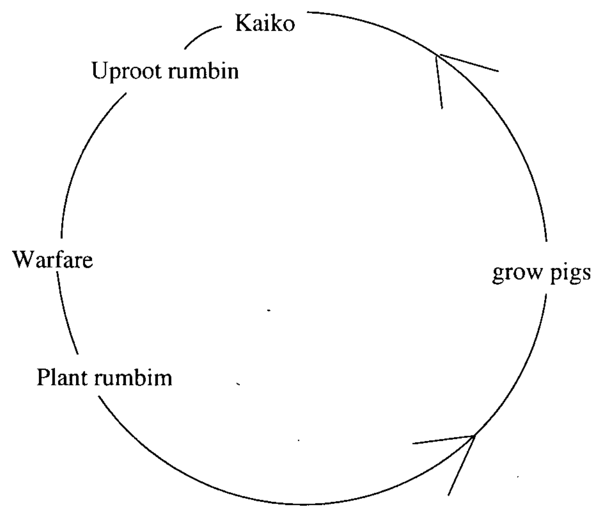


Figure 3.13: The ritual cycle of the Tsembaga

12 years) depends on the demographics of the pigs. In this period enough pigs must be grown to satisfy ritual requirements, but the staging of the Kaiko also depends on when women get tired of being parasitized by pigs. The question mark between the uprooting of the rumbim and the beginning of warfare indicates uncertainty about the timing of the onset of warfare, although Rappaport indicated that fighting had usually resumed within 3 months of the uprooting of the rumbim.

After a truce, the populations return to tending gardens and pigs. As the pig population increases, work load on the women also increases. Rappaport computed that there were an average of 2.4 pigs of the 120- to 150-pound size to each mature female at the outset of the 1962 Kaiko. This translates into a pig to person ratio (in terms of biomass) of about 1.2. The range of the number of pigs kept was 0 to 6. Rappaport observed only one woman keeping 6, and four keeping 5 and postulates that these figures may represent the maximum physically possible. When females are burdened with this many pigs, their complaints to their husbands become more frequent. The husbands then call for the Kaiko to be staged during which the pig herd is drastically reduced via ritual sacrifice.

To model this we add variables for the pig population (p) and the "harvest" (q) level of pigs. When p is less than the level tolerable by the Tsembaga women, q is very low. When p reaches a critical level of about 2-3 pigs per woman, the Kaiko "breaks out" and q increases very rapidly. The dynamics of this type of system can be modeled by a dynamical system of the form:

$$\frac{dq}{dt} = \tau(p/h - g(q)) \tag{3.20a}$$

$$\frac{dp}{dt} = (r - q)p \tag{3.20b}$$

where r is the intrinsic growth rate of the pig population and the function $g(q)$ has the form in Figure 3.14, and τ, which is relatively large, is the relaxation time. The trajectory in the phase plane generated by the dynamics in 3.20a and 3.20b is superimposed on $g(q)$. When the quantity p/h is between 0.2 and 1.2, Equation 3.20a forces q to track the function $g(x)$ very closely. Once outside these limits, the difference between p/h and $g(q)$ grows causing q to change very quickly, as shown in Figure 3.15.

After the staging of the Kaiko, the ritually sanctioned truce between hostile groups is ended by the uprooting of the rumbim plant. Hostilities are then allowed to, but do not necessarily, resume. If hostilities can be avoided through two ritual cycles, lasting peace between the two hostile groups can be established. Rappaport notes, however, that hostilities are generally resumed by three months after the Kaiko and can last up to six months.

During actively hostile periods, actual combat is frequently halted for the performance of rituals associated with casualties and for pigs and gardens to be tended. Warfare comes to a halt with another ritual truce when both sides feel that enough killing has taken

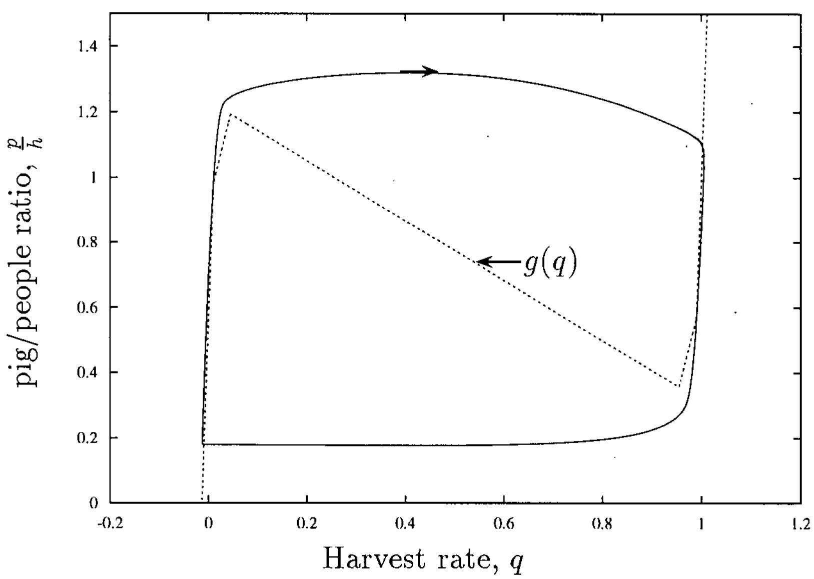


Figure 3.14: Form of $g(x)$ in equation (3.20a) and the associated limit cycle.

place or combatants simply tire of fighting. Since the fighting forces are composed of principal combatants and their allies, as time goes on, the support of allies becomes more difficult to maintain which increases pressure to bring hostilities to an end. To model this we use the fact that after several casualties have occurred, the people to pig ratio begins to decrease. As this happens, the per-person work level begins to increase and daily living activities become more pressing. The pig to person ratio acts as a proxy for this increased work effort and the warfare outbreak dynamics can be expressed by:

$$\frac{dw}{dt} = \tau(h/p - \gamma g(w) + \delta) \tag{3.21}$$

where w is the per-capita death rate due to war and γ and δ merely scale and shift the ratio of people to pigs where the outbreak of war and ritual truce occur. The human and pig population dynamics under this scenario are shown in Figure 3.16.

The most critical aspect of the model for the ritual cycle and its effect on the human population is the set assumptions made about the effect of warfare on the population.

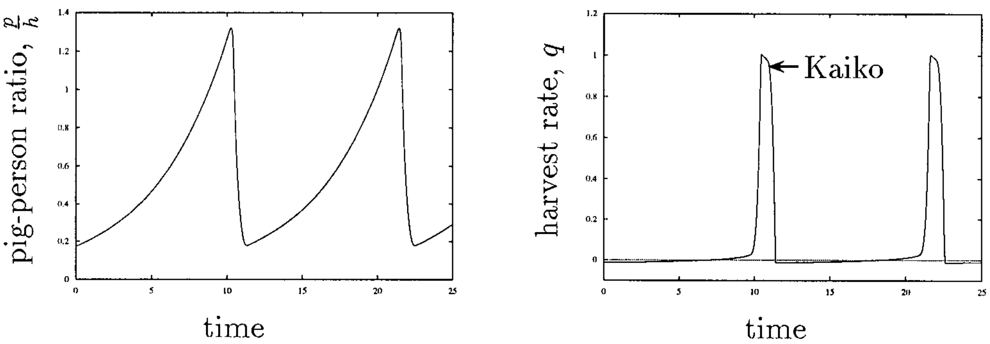


Figure 3.15: The dynamics of the ritual cycle. These represent the time plots of the limit cycle shown in figure (3.14). Between Kaikos, the harvest rate is very low. When the pig to person ratio exceeds the tolerable level, the harvest rate increases dramatically representing the pig slaughter associated with the Kaiko as shown in the graph on the right.

Unfortunately, data on warfare-related mortality are not rich - estimates range from two to eight percent of the population [23]. This is not an important issue with regard to stability, however. The key point is the assumption that the number of deaths due to warfare is a constant fraction of the population. If we make this assumption then the human population dynamics would be given by

$$\frac{dh}{dt} = (b_n - a \exp\left(-\alpha \frac{e(h, k_r, c_1, c_2)}{h}\right) - w)h \tag{3.22}$$

If the system is to evolve to a stable limit cycle, the parameters that govern the dynamics of w must be chosen such that the average value over one cycle of the quantity

$$(b_n - a \exp\left(-\alpha \frac{e(h, k_r, c_1, c_2)}{h}\right) - w) \tag{3.23}$$

vanishes. Since the cultural dynamics act to drive $e(h, k_r, c_1, c_2)$ toward 1, the growth rate of the human population is nearly constant and only very weakly dependent on the *physical state* of the system over most of a cycle. The average war mortality over a cycle must be balanced against essentially a constant growth rate, and there is no mechanism by which the model can "seek" an equilibrium population level. In this case the ability

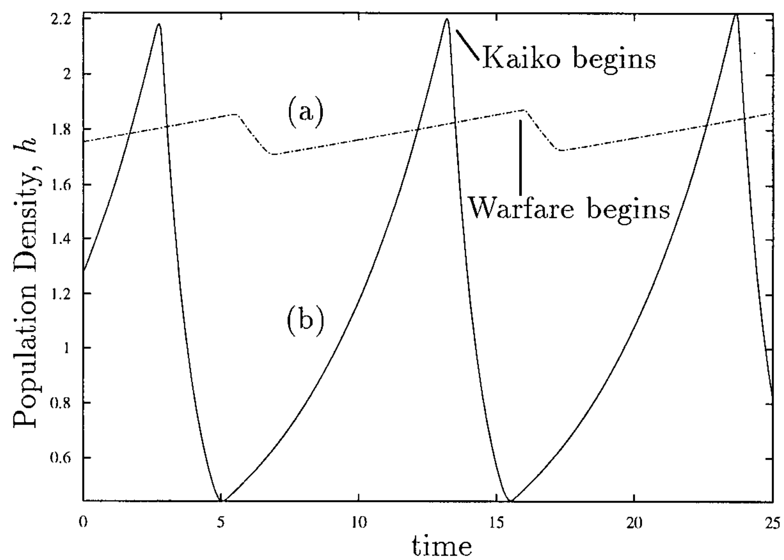


Figure 3.16: An example of of the human (a), and pig (b), population trajectories under cultural outbreak dynamics. After the Kaiko when the pig population drops drastically (curve (b)) warfare resumes and the the human population drops (curve (a)). As people are killed, the human pig ratio drops until a cutoff is reached and a truce is called.

of the Kaiko to stabilize the system is very sensitive to parameter choices. This may help explain why the model due to Shantzis and Behrens [57] was neutrally stable and, of course, why when Foin and Davis [23] used different parameters (making the counterpart of expression 3.23 in their model positive in mean over one cycle) found that the Kaiko would not stabilize the system. Here, there is no mechanism by which the model can "seek" an equilibrium population level.

If, on the other hand, we assume that mortality due to warfare increases nonlinearly with the population size, the Kaiko can stabilize the system. Rappaport actually indicated that this was the case. As there are more pigs, people, and gardens there are more ways for pigs to invade gardens and cause conflict, increasing the number of required blood revenge deaths during an active period of warfare. The number of ways a pig might invade an enemy's garden rises much faster than linearly with increases in pig and garden numbers. If we assume that number of war moralities behaves roughly as the square of the population size, the human population dynamics are given by

$$\frac{dh}{dt} = (b_n - a \exp\left(-\alpha \frac{e(h, k_r, c_1, c_2)}{h}\right) - wh)h. \tag{3.24}$$

We then define the full ecological system by the physical component defined by Equations 3.24, 3.14b, and 3.20b and the cultural component defined by Equations 3.20a, 3.17, and 3.21 to arrive at the following dynamical system:

$$\frac{dh}{dt} = (b_n - a \exp\left(-\alpha \frac{k(c_2 h A_c)^{a_1} k_r^{a_2}}{h}\right) - wh)h \tag{3.25a}$$

$$\frac{dk_r}{dt} = k_r n_r (1 - k_r) - \beta k (c_2 h A_c)^{a_1} k_r^{a_2} \tag{3.25b}$$

$$\frac{dp}{dt} = (r - q)p \tag{3.25c}$$

$$\frac{dc_2}{dt} = \lambda_{c_2} \left(d_f - \frac{k(c_2 h A_c)^{a_1} k_r^{a_2}}{h}\right)(c_2^{max} - c_2) \tag{3.25d}$$

$$\frac{dq}{dt} = \tau(p/h - g(q)) \tag{3.25e}$$

$$\frac{dw}{dt} = \tau(h/p - \gamma g(w) + \delta). \tag{3.25f}$$

3.5.3 The behavior of the full system

The dynamics of the ritual variables are confined to stable limit cycles and the work level follows food demand forcing the overall system behavior to be cyclic. With the human population dynamics defined by 3.24, the ritual warfare acts to drive the system to equilibrium keeping the human population in check. Figure 3.17 illustrates the behavior of several trajectories beginning from different reasonable initial conditions. They all collapse onto a very small amplitude stable limit cycle. Projections of this cycle onto the $h - p$ and $k_r - h$ planes are shown in Figure 3.18.

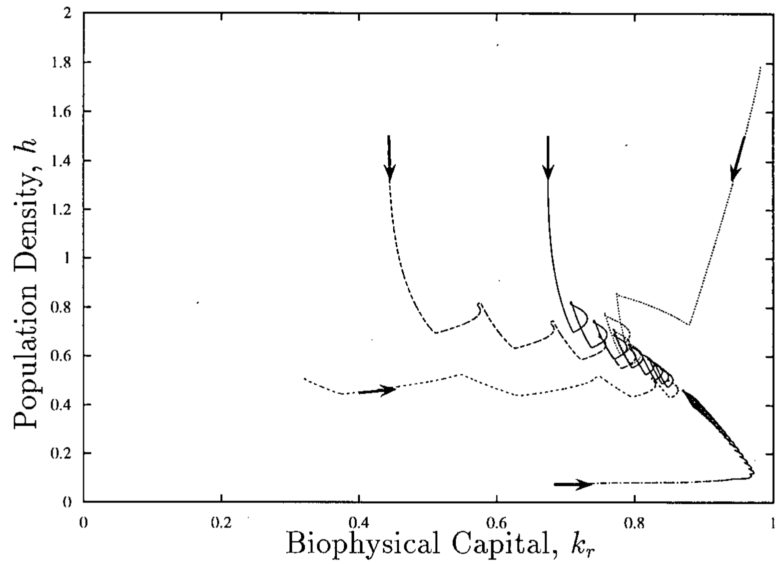


Figure 3.17: Sample trajectories for the full model. Any time that the human population is large compared to biophysical capital, the pig to people ratio will be high and warfare will break out. This drives the population to a more stable (or sustainable) region in the state space whence the system collapses onto the very low amplitude limit cycle shown in figure(3.18).

The ritual cycle effectively keeps the human population density in the interval $(0.41, 0.49)$ and the natural capital in the interval $(0.86, 0.89)$. Compare these fluctuations to the case without the ritual cycle (see Figure 3.10). The model predicts that if the Tsembaga attempt to meet food demand, it is possible that the ritual cycle could play a critical role in stabilizing the ecosystem.

3.6 Conclusions

The dynamical system model for the Tsembaga ecosystem based on the ethnographic work of Rappaport [53] developed in this paper suggests that behavioral plasticity, feedback from the land, and the relationship between people and pigs are the main factors affecting ecosystem stability. Behavioral plasticity, in the form of the ability of the

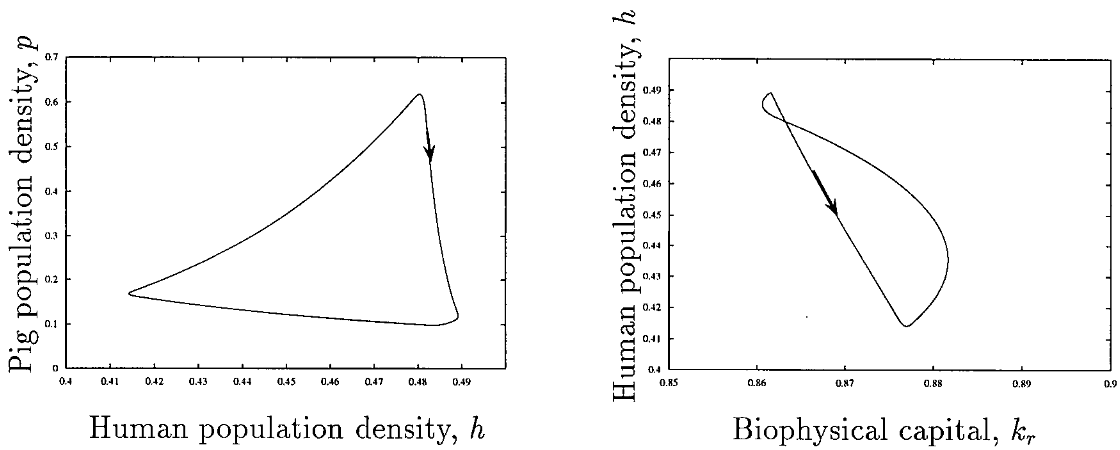


Figure 3.18: Limit cycle for the full model projected into the $x_1 - x_3$ and $x_2 - x_1$ planes respectively.

Tsembaga to adjust food production based on demand, is strongly destabilizing because it allows people to attempt to overcome nutritional deficiencies that would otherwise help stabilize the system. Critical to the effect behavioral plasticity has on the model is the relative productivity of labor. If the increased nutritional intake generated by increased effort more than offsets the soil productivity loses due to the associated shorter fallow periods, the model stability structure is sensitive to changes in effort directed to agriculture. Increased output elasticity of the soil (sensitivity of soil productivity to increased effort) has a stabilizing influence, reducing the importance of behavioral plasticity in determining the stability of the system.

If the output elasticity of labor (in the short run) is higher than that of soil (probably reasonable) then the destabilizing effect of behavioral plasticity can be so strong as to nullify the stabilizing effect of malnutrition and disease proposed by Buchbinder [10] opening up the possibility of temporally violent oscillations in population numbers. By extending the model, it was shown that pig husbandry, in and of itself, helped stabilize the system. Finally, pig husbandry combined with the ritual cycle can act as a homeostatic mechanism to stabilize ecosystem as proposed by Rappaport *if war mortality is density dependent.* This runs contrary to earlier results [23, 24, 57] that emphasized sensitivity to parameters. The model presented here is fairly robust to changes in parameters and suggests that the key factors are the structure of the food production function and density dependence of war related mortality.

Many of the original criticisms of Rappaport's work centered on the problem of explaining how the Tsembaga cultural system might have come about, and the appropriateness of the ecosystem concept as he applied it. Of course, no model can explain the evolution of behavior, at best it can only shed light on how certain behavior could be adaptive. The focus of this paper was to study the effects humans and their cultural practices can have on an ecosystem. We found that culture can be both destabilizing (how hard a population decides to work) and stabilizing (the ritual cycle). The model presented here supports the claim that a cultural mechanism such as the Tsembaga ritual cycle can operate to prevent ecosystem degradation. If an individual can do better by participating in the existing cultural "environment" rather than going against it, any cultural construct that prevents ecosystem destruction could have adaptive value for the individual. In this sense the ritual cycle of the Tsembaga could have adaptive value as Rappaport originally proposed. The model also highlights the destructiveness of a society that directs ever increasing quantities of energy to agriculture in the face of continually degrading soil quality, and the importance of the role "sustainable culture" might play in both past and present sustainable human agroecosystems.

The main point to take away from this model is that the human ability to modify behavior to overcome short term resource shortages does not, as many economists believe, help the society reach a sustainable state. It has the opposite effect: it makes the sustainable state harder to achieve. The model suggests that collective social action is more critical making a sustainable world a reality. Also, it must be emphasized that this social action can not be "soft" by which I mean actions that focus on trying to continue what we are doing with less. The social action has to be an *emergent property* of individual beliefs. Think, for example, if excessive individual wealth accumulation and greed were viewed with as much indignation and disgust as say incest or rape, we might be faced with a quite different present and future world. Simple economic and technological fixes that are not accompanied by cultural change might do nothing more than help paint us into a corner. This will be illustrated in chapter 5 with regard to investment and wealth distribution practices.

Chapter 4

Non-substitutibility in consumption and ecosystem stability

If we wish to extend the modelling framework to more complex economic systems with a wider range of possible activities and more state variables, defining how the linkage between them operates becomes the main challenge. The main question is how do people decide to allocate energy to the different activities and how do feedbacks from the environment influence this allocation. Economists have dealt with this problem in great detail through the use of the market, where the main feedbacks from the environment are prices, and utility functions determine how income is allocated among available activities.

The aim of this section is to examine in detail the implications of assuming a standard economic model for the interaction between behavior and environment, i.e. how certain assumptions about utility generate very specific cultural structures. We accomplish this by studying and extending a model of the economic system of Easter Island developed by Brander et al. [9]. In this model the authors develop the hypothesis that the culture and economic system of the invading Polynesians were incompatible with the physical properties of Easter Island. This mismatch between cultural and ecological systems lead to the eventual collapse of the system. This is an excellent example of the importance of studying culture and economic systems within an ecological context.

4.1 The Easter Island model

Brander et al. [9] developed a simple general equilibrium model to characterize the collapse of the society on Easter Island that created the stone monuments for which the

Island is so well known. The model has two state variables:

$S(t)$: Renewable resource stock ($\equiv k_r$ in my notation)

$L(t)$: Available labour in the population ($\equiv h$ in my notation)

The renewable resource stock would include agricultural output and fish catch potential. As is traditional with economic models, the population is modeled as a labor pool that is proportional to the physical population. The dynamics of the Easter Island ecosystem according to Brander et al. [9] are then given by

$$\frac{dS}{dt} = G(S) - H(S, L) \tag{4.1a}$$

$$\frac{dL}{dt} = (b - d + F(H, L))L \tag{4.1b}$$

where $G(s)$ is the intrinsic growth rate of the renewable resource (food and wood), $H(S, L)$ is the harvest rate of the resource, b and d are the constant birth and death rates for the labor force (population) and $F(H, L)$ is the variable growth rate of the population that depends on resource use. The cultural subsystem is associated with the determination of $H(S, L)$ and $F(H, L)$. The cultural system is modeled by treating the inhabitants of Easter Island rational economic agents attempting to maximize utility through the consumption of material goods. This cultural structure, of course, determines a large part of the model's behavior, just as it did in the Tsembaga case. This provides an example of how cultures can be compared. Tsembaga ritual culture (non economic behavior) stabilized the system while if the culture commonly ascribed to modern industrial man prevailed on Easter Island, they would be doomed to "overshoot and collapse".

Within this cultural model, the population consumes two goods - bioresource goods (agricultural output and fish), H, and manufactured goods (tools, housing, and artistic output), M. The cultural dynamics, i.e. the way the population decides to partition available energy among possible activities of producing and consuming goods are then determined by solving a constrained maximization problem. Brander et al. use a Cobb-Douglas utility function,

$$u(h,m) = h^{\beta} m^{1-\beta} \tag{4.2}$$

where h and m are per capita consumption rates of the bioresource and manufactured goods respectively, and β defines the preferences for these goods. If w is the wage rate, the budget constraint is

$$p_h h + p_m m = w, \tag{4.3}$$

p_h and p_m being the respective prices of the two goods. By the choice of units Brander et al. set $p_m = 1$ (M is defined as the numeraire good whose price is the benchmark by which all prices are measured). Solving this maximization problem results in the following per-capita demand functions:

$$h = \frac{\beta w}{p_h} \quad \text{and} \quad m = w(1-\beta). \tag{4.4}$$

Equation 4.4 thus defines the demand side of the economy. To model the supply side, we must employ production functions to link demands with physical possibilities. The production functions chosen by Brander et al. are

$$H = \alpha S L_H \tag{4.5a}$$

$$M = L_M. \tag{4.5b}$$

Equation 4.5a asserts that the quantity of H produced is proportional to the product of the size of the resource stock and the quantity of labor devoted to obtaining it, L_H. Such production functions are commonly used in fisheries [15]. Equation 4.5b states that M depends on labor alone, L_M and by choice of units, one unit of labor produces one unit of M.

The link between the supply and demand side is, of course, the market. The market will equilibrate when the supply prices equal the demand prices. Assuming that the economic processes are much faster than natural processes, Brander et al. assume that the market is always in equilibrium so that linking the supply and demand sides of the economy reduces to solving a set of algebraic equations. Assuming that the only costs of production are due to labor, the per-unit supply prices are given by

$$p_h = \frac{wL_H}{H} \tag{4.6a}$$

$$p_m = \frac{wL_M}{M}. \tag{4.6b}$$

From equation 4.5b we see that $\frac{L_M}{M} = 1$ and since $p_m = 1$ we must have that the wage rate is also 1. Combining this fact with equations 4.5a and 4.6a we see that

$$p_h = \frac{1}{\alpha S} \tag{4.7}$$

which merely says as the resource stock decreases, its supply price increases. Substituting the supply prices and wage rate into equation 4.4 yields the actual per-capita amounts of H and M produced:

$$h = \alpha\beta S \tag{4.8a}$$

$$m = 1 - \beta \tag{4.8b}$$

In order to extend this model and illustrate how the choice of utility functions relates to the level of behavioral plasticity exhibited by the populations we express culture as the amount of energy devoted to each available activity. This requires relating the per-capita consumption to the energy required to produce it. We will accomplish this in the same manner as with the Tsembaga model. Let us assume that the available labor is a fraction of the total population, i.e.

$$L = \gamma N \tag{4.9}$$

where N is the total population at time t. Brander et al. assume that γ is equal to 1 (again by choice of units) thus $N = L$. By definition, the total demand for H and M is the per-capita demand multiplied by the total population:

$$H = Nh = Lh = L\alpha\beta S \quad \text{and} \quad M = Nm = Lm = L(1-\beta) \tag{4.10}$$

Now, using the production functions once again, we can determine the energy (or labor) required to meet these demands, i.e. we set the total production equations equal to the total demand equations:

$$L\alpha\beta S = L_H \alpha S \;\Rightarrow\; L_H = \beta L \tag{4.11a}$$

$$L(1-\beta) = L_M \tag{4.11b}$$

Thus, the Easter Island Culture as characterized by this economic model is one in which a constant proportion, β, of the labor force is directed towards producing bioresource goods, while the remaining portion of the labor force, $1-\beta$, directs its energy towards the production of manufactured goods.

The final aspect of the model to be specified is how the fertility function F depends on the per-capita intake of bioresource goods (nourishment). Here Brander et al. make the assumption that net fertility increases linearly with per-capita consumption of bioresources, i.e. the better life is the higher the propensity to reproduce. Thus they let

$$F = \phi \frac{H}{L} \tag{4.12}$$

where ϕ is a positive constant and the ratio of H to L represents the actual per-capita intake of bioresource goods. Thus the culture of Easter Island can be completely specified by two parameters: β, its taste for bioresource goods and ϕ, its fertility response coefficient.

With the cultural sub-model specification complete, we are left to quantify the physical aspect of the model; the growth rate of the bioresource, $G(S)$. Here Brander et al. assume the common logistic function: $G(S) = rS(1 - S/K)$ where r is the intrinsic growth rate and K is the carrying capacity. The planar dynamical system we wish to study is then given by:

$$\frac{dS}{dt} = rS(1 - S/K) - \alpha\beta SL \tag{4.13a}$$

$$\frac{dL}{dt} = (b - d + \phi\alpha\beta S)L. \tag{4.13b}$$

4.2 Model Critique

A glance at equations 4.13a reveals that they are equivalent to a Lotka-Volterra predator-prey system with density-dependent prey growth rate. The behavior of such systems is well known and I will not discuss it here (see [11]). Rather, I will focus on how assumptions about culture affect the model - especially focusing on the role of behavioral plasticity.

The model specified by equations 4.13a has one non-trivial equilibrium point (S^*, L^*) that satisfies $S^* > 0$, $L^* > 0$ and

$$\frac{dS(S^*, L^*)}{dt} = 0 \tag{4.14a}$$

$$\frac{dL(S^*, L^*)}{dt} = 0. \tag{4.14b}$$

This equilibrium point is globally asymptotically stable, the proof of which relies on a simple application of a theorem due to Kolmogorov relating to planar systems of this type (see [42] or [21]). Beginning from any interior initial condition, the system will converge to the steady state. Depending on parameter values, the steady state will either be a node or a spiral which will force the system to converge to the equilibrium either monotonically or through a series of damped oscillations. Of interest to Brander et al. is that for certain parameter values representative of the situation on Easter Island, the system will exhibit transitory oscillatory behavior which manifests itself in overshoot and collapse. Figure 4.1 shows the human population and resource stock trajectories for an initial condition of 40 humans landing on Easter Island with the resource stock at carrying capacity (The units for the resource are a matter of scaling. Brander et al. [9] choose a carrying capacity of 12,000 units for convenience.)

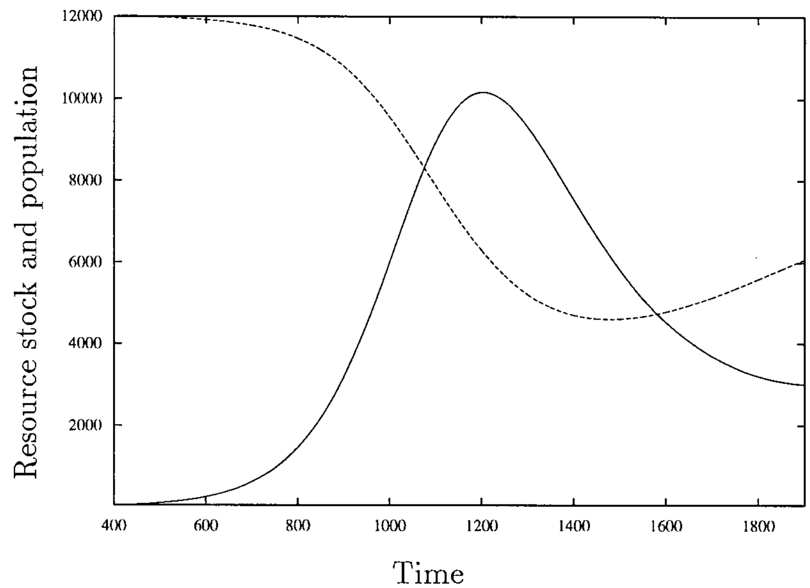


Figure 4.1: Population and resource stock trajectories for Easter Island model from ([9])

The archaeological record indicates the first presence of humans at around 400 AD. The population increases which is accompanied by a decrease in resource stock. The population (and available labor) peaks at around 1250 AD corresponding to the period of intense carving in the archaeological record. The population subsequently declines due to resource depletion. The model predicts a population of about 3800 in 1722, close to the estimated value of 3000. The model thus gives a reasonable qualitative picture of what may have happened to the culture on Easter Island. The culture became very productive and able to undertake the construction of major monuments, i.e. the labor force increased thus making L_M large enough to complete such a large scale project. The population subsequently declined due to resource degradation which left the small population who knew nothing of the origin of the great monuments to meet the Dutch ships in the eighteenth century. The discussion in Brander et al. [9] is very interesting and I refer the reader there for more detail.

4.2.1 Behavioral plasticity and collapse

In this section we examine how the nature of the population collapse depends on the level of behavioral plasticity exhibited by the population. The nature of the collapse can be more clearly understood by examining the per-capita growth rate over a time scale meaningful to a member of the population. Figure 4.2 shows the annual per-capita net growth rate of the population from the time of initial colonization to the time of the Dutch ships arrived in the eighteenth century.

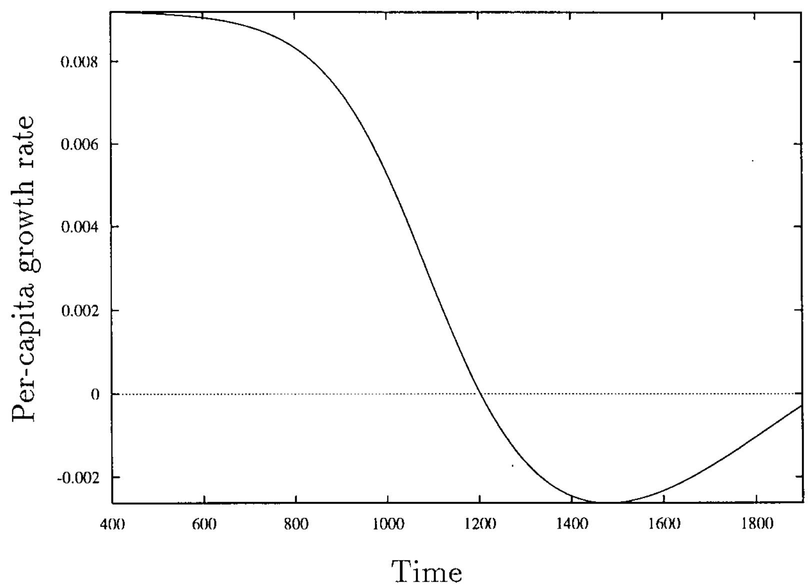


Figure 4.2: Per-capita growth rate from the time of initial colonization to the time of first European contact.

The population exhibits positive growth up to approximately 1200 AD when it peaks at around 10,000 individuals. The maximum per-capita annual growth rate is around

0.92%–very low by today's standards. Similarly, the minimum net growth rate is -0.262% which implies that even under the most extreme resource shortage conditions the population is decreasing very slowly. It takes 600 years for population to drop from 10,000 to 3800. Compare this to populations doubling every 40 years at present. Next consider the perceived change in an individual's standard of living over a life span of say 70 years from the year 1000 AD to 1070 AD when it is decreasing most rapidly . In this period one would experience a 12% decrease in bioresource intake over an entire lifetime. Although the quality of life is going down, it is not changing catastrophically. From our present day point of view the manner in which the population adjusts to the environment depicted by the model might not be that bad.

We can now investigate the role behavioral plasticity has to play in the nature of the collapse. Recall from equations 4.11 we deduced that the population directs a constant proportion β of the labor force towards the bioresource sector while what is left is directed to the manufacturing sector. Further, equations 4.8 indicate that the per-capita rate of consumption of the manufactured goods is constant, no matter what quantity of bioresources are being consumed. This implies that as the bioresource stock is depleted and becomes more expensive to produce, individuals continue to consume the same amount of manufactured goods and consume less and less bioresources. The population could be starving, yet the utility maximizing strategy is to keep the proportion of labor directed to each activity constant.

The problem here is substitutability. Cobb-Douglass utility functions allow for one input to be substituted for another without affecting utility. Based on this model, the optimal strategy in the face of a resource good shortage is to increase consumption of cheaper manufactured goods. This is reasonable in some cases, but not where bioresource goods that sustain one's very life are concerned. In short, the standard Cobb-Douglass utility function cannot capture the possibility that labor could be shifted from one sector to the other–the structure of the economic system is fixed over time.

The only aspect of the model that allows for behavioral flexibility is the fertility function, and this depends on how it is interpreted. If the change in per-capita growth is due to active choices on the part of individuals depending on "quality of life" as measured as per-capita intake of bioresource goods then these changes would be considered the result of behavioral plasticity. If on the other hand, these changes are due to indirect effects and not active choice, then there is no behavioral plasticity built into the model.

4.3 Adding behavioral plasticity to the Easter Island model

There are two aspects of the Easter Island model where behavioral plasticity might manifest itself, either in the structure of the economy, or in the overall effort expended by each individual in the population. One way to introduce the possibility for structural change in the economy is to modify the utility function. I do so by utilizing a Stone-Geary type utility function which assumes that there is a minimum amount of bioresource goods (subsistence level) at which utility is zero, i.e.:

$$U(h,m) = (h - h_{min})^{\beta} m^{1-\beta} \tag{4.15}$$

where $h > h_{min}$. Modifying the model so that overall work effort can change is accomplished by changing γ from equation 4.9 from a constant to a state variable. As before, we can determine the optimal consumption of resources by maximizing $U(h,m)$ as defined by 4.15 subject to the income constraint

$$p_h h + p_m m \leq \gamma w \tag{4.16}$$

where w is the wage paid per unit of labor. The resulting optimal consumption levels are:

$$h \;=\; (1-\beta)h_{min} + \frac{\gamma w \beta}{p_h} \tag{4.17a}$$

$$m = (1-\beta)(\frac{\gamma w - p_h h_{min}}{p_m}) \tag{4.17b}$$

Now we have that the optimal consumption level of h consists of a price dependent and a price independent portion. This is more realistic as it says to spend excess income on certain proportions of h and m only after meeting minimum nutritional requirements. Equations 4.17 only make physical sense when

$$p_h \le \frac{\gamma w}{h_{min}}, \tag{4.18}$$

but this condition will always be satisfied if $h > h_{min}$. Substituting equation 4.7 for p_h into equation 4.18 and assuming as before that $w = 1$ and $p_m = 1$, we see that the condition for the system to make physical sense reduces to

$$h_{min} \le \gamma\alpha S \tag{4.19}$$

which simply says that if the demand h_{min} can be met at the present work level, use the optimality conditions given by 4.17 to divide excess capacity to the tasks of producing m and h.

If 4.18 is not met, the optimality conditions do not say what to do. Common sense suggests that if people are tying to meet minimum nutritional requirements, they would produce all the bioresource goods possible, i.e.

$$h = \gamma\alpha S. \tag{4.20}$$

Finally, we can, by combining the above equations with the production functions given by 4.5a and 4.5b, compute the amount of labor (available work) the population should devote to producing bioresource goods and manufactured goods:

$$L_h = \begin{cases} \frac{N(1-\beta)h_{min}}{\alpha S} + N\gamma\beta & \text{if } h_{min} \le \gamma\alpha S \\ N\gamma & \text{otherwise} \end{cases} \tag{4.21a}$$

$$L_m = \begin{cases} (1-\beta)N(\gamma - \frac{h_{min}}{\alpha S}) & \text{if } h_{min} \leq \gamma\alpha S \\ 0 & \text{otherwise} \end{cases} \tag{4.21b}$$

The "culture" defined by 4.15 combined with the physical system defined by 4.5a and 4.5b generates the decision process defined by 4.21. Notice that in contrast to the original model, the division of labor is no longer fixed. As the price of bioresource goods increases, labor is shifted out of the production of manufactured goods into the bioresource sector - i.e. there is *structural* change in the economy. Finally, the population has the option to increase the work level γ in an effort to meet its needs, just as in the Tsembaga model. I assume that the population will increase its work level only after all labor is shifted into producing bioresource goods. This leads to the new system we wish to analyze:

$$\frac{dS}{dt} = rS(1 - S/K) - \alpha S L_h \tag{4.22a}$$

$$\frac{dN}{dt} = (b - d + \phi\alpha S L_h)N \tag{4.22b}$$

$$\frac{d\gamma}{dt} = \lambda(h_{opt} - h_{prod})(\gamma_{max} - \gamma). \tag{4.22c}$$

where $h_{prod} = \gamma\alpha S$ is the quantity of bioresource goods actually produced. When condition 4.18 is met, $h_{opt} \leq h_{prod}$ and the amount of bioresource goods the population is capable of making will exceed the amount it wishes to make so work levels will decrease to the optimal level. If, on the other hand, condition 4.18 is not met, the population will try to increase its work level to meet optimal demand. We can now analyze how the dynamics of the model change under these conditions.

4.3.1 Model analysis

We begin the analysis by first letting $\lambda = 0$ and focusing our attention on the effect that h_{min} has on the model. If we take $w(0) = 1$ and and $h_{min} = 0$, we retrieve the original model. For the parameters chosen by Brander et al., we know there is globally stable equilibrium point at $N = 4791.7$ and $S = 6250$. We can again use pseudo-arclength continuation to investigate the nature of this equilibrium point as h_{min} is varied. Figure 4.3 is the result of this exercise.

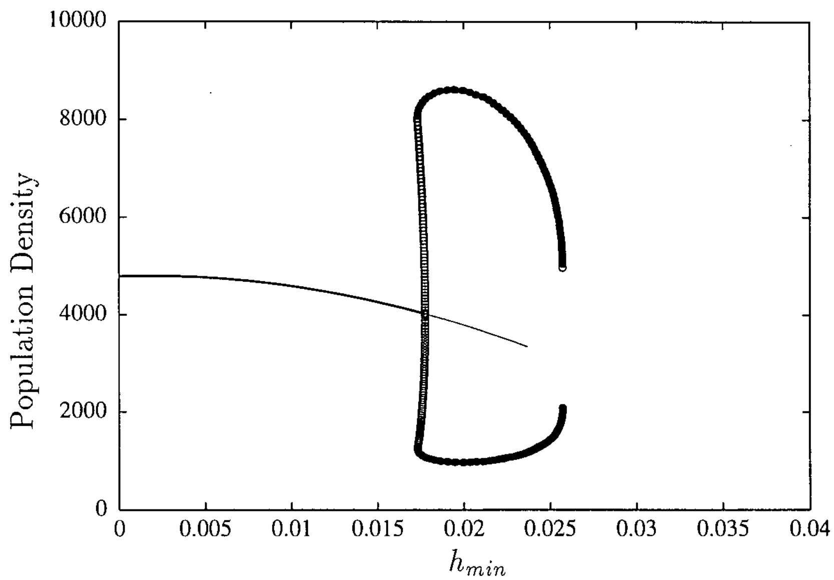


Figure 4.3: Bifurcation diagram for modified Easter Island model.

As with the Tsembaga model, the way in which the population partitions its energy profoundly affects the dynamics of the model ecosystem. We see from figure 4.3 that a stable equilibrium point persists up to a value of h_{min} near 0.017 where a Hopf-bifurcation occurs. For values of h_{min} beyond the bifurcation point, not only does the system lose stability, but the nature of the dynamics far from the singular point change as well. Figure 4.4 shows the change in the dynamics as well as the role behavioral plasticity has to play.

The figure to the left shows the population trajectories for the original model and for the modified model with $h_{min} = .02$ The figure to the right shows how the structure of the economy evolves over time. initially, the two trajectories are roughly the same. For

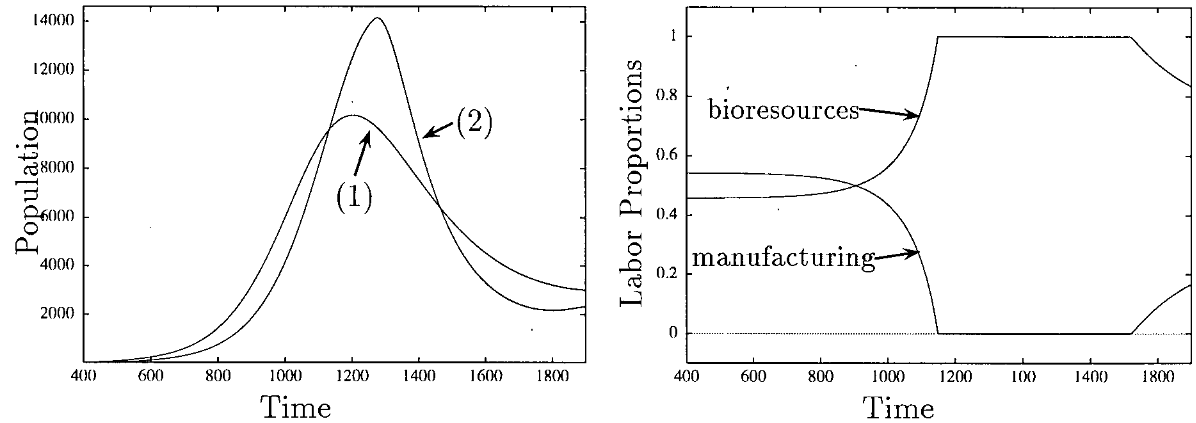


Figure 4.4: Trajectories for population and proportions of labor in each sector over time. In the leftmost graph, curve (1) is for the original model as proposed by Brander while (2) is from the modifed model.

the first 400 years the structure of the economy remains fairly stable with approximately 48% of the labor force working in the bioresource sector and the remainder in the manufacturing sector. As bioresources become more scarce, the economic structure begins to change and labor is shifted into the bioresource sector until all of the population is working in this sector by between 1100 and 1200 AD. This shifting of available work into the bioresource sector enables the population to grow about 100 years longer than in the original model up to a peak of around 14,000 as compared to 10,000. Also evident is the much more rapid decline that the more behaviorally plastic population must endure after it has pushed its ecosystem too far. Here, behavioral plasticity enabled the population to maintain its positive growth trajectory longer resulting in a more dramatic decline.

The final aspect of this model to be discussed is the effect of allowing the population to decide to work harder, i.e. set $\lambda > 0$. Figure 4.5 shows the results for $w_{max} = 3$, i.e. the population is willing to triple its work effort if necessary.

The graph on the right in figure 4.5 shows the structure of the economy changing over

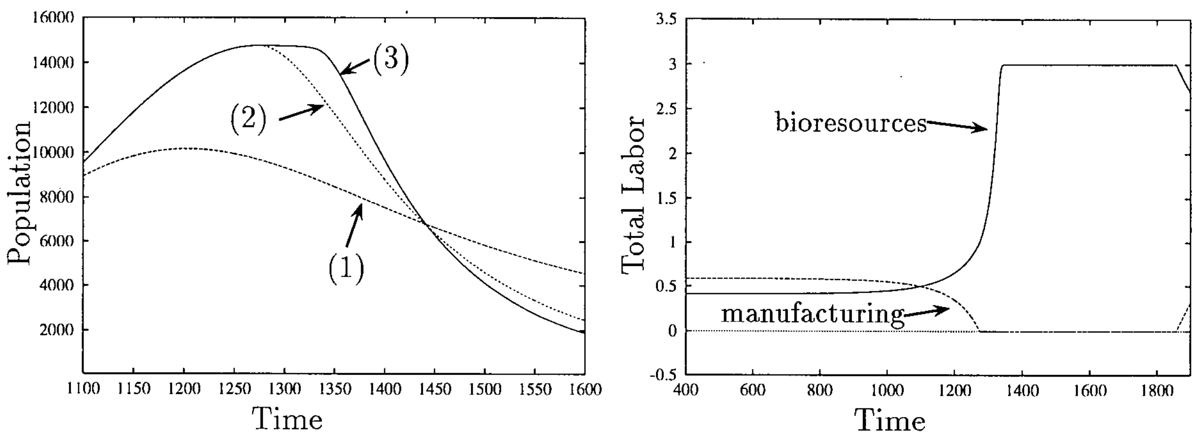


Figure 4.5: Trajectories for population and total labor in each sector over time for the case $\lambda \neq 0$. In the leftmost graph, curve (1) is for the original model as proposed by Brander, (2) is from the modifed model with $\lambda = 0$, and (3) is the case for the modified model with $\lambda \neq 0$.

time as bioresources become more scarce. In this case, when all the labor force has shifted into the bioresource sector the population begins to increase its work effort. Trajectory (3) in the figure to the left shows the case where the population increases its work effort. By doing so, the population averts a further decrease in the intake of bioresources (and thus quality of life) for about 80 years. Unfortunately, this decision ultimately increases the price the population has to pay in the rate of decrease of the population when it finally does collapse. The rate of decrease is four times that of the original model and two times that of the modified model with a fixed work level. This type of scenario is very reminiscent of our situation today. We are increasing the amount of work we do as we attempt to maintain our standard of living. Obviously, we may be simply buying ourselves a little time and increasing the ultimate price we will have to pay.

4.4 Conclusions

In this section we have studied the interaction between culture and ecosystems in the context of a model where the economy is more complex. The model I proposed where both the structure and the overall work level of the economy were allowed to change experienced a bifurcation from a stable steady state to a limit cycle which produced more dramatic changes in population dynamics. The key point to observe is that, as with the Tsembaga model, increased behavioral plasticity *decreased* the stability of the system. In this light, the ability of modern economies to change their structure quickly in response to changing environmental conditions so frequently lauded by the expansionist view, might not be such a positive asset in achieving sustainability.

Obviously one can argue that this model is not rich enough to capture our ability to become more efficient, to utilize different goods to perform certain tasks, to generate capital, and to try to improve natural capital before it degrades, thus averting the collapse experienced by the simple model and enabling a transition to sustainability. Examining such a model is the focus of the next chapter of this thesis.

Chapter 5

The dynamics of a two sector ecological economic system

In this chapter, I will extend the concepts I have developed so far to study the dynamics of a model of a two sector economy with capital accumulation. This is a much harder problem than we have addressed so far. The Tsembaga and Easter Island models were both pure labor economies. The only decisions taking place in these economies were how hard to work and what portion of available labor to devote to each activity. In an economy with labor and capital, the decisions are more complex. Here we have firms that are trying to utilize resources efficiently while consumers are simultaneously trying to maximize utility. In order to tackle this problem, we will have to develop more sophisticated economic concepts for modeling economic growth.

To this end, this chapter is organized as follows. In the first section, I summarize important concepts from the theory of economic growth that are important for this model. Next, I outline the relevant concepts from production and utility theory and related issues such as non-substitutability of consumer goods that we investigated in chapter 4 and the importance of the nature of the production function that we encountered in chapter 3 that are used to construct the model economic growth system. Finally, I develop the ecological system in which the economic growth system is embedded. The final step is then to analyze the dynamics of the resulting system.

5.1 Simple economic growth models

Jensen [36] gives an exhaustive treatment of simple economic growth models with two state variables: labor and capital. Such simple models have received much attention in the economic literature, often focusing on the steady state growth trajectory of an economy. This steady state trajectory corresponds to a constant capital-labor ratio with economic output growing with capital and labor growth. An economic growth model necessarily consists of three components: relationships that describe the dynamics of labor and capital over time, a relationship between economic output and a given level of capital and labor (factors of production), and information specifying what society does with economic output. Mathematically, the model consists of a dynamical system coupled with algebraic equations governing production and consumption.

A common example of a simple economic growth model with a single production sector would be:

$$dL/dt = nL \tag{5.1}$$

$$dK/dt = sY \tag{5.2}$$

where L is labor (generally viewed as the number of workers in a population), K is the quantity of capital, n is the per-capita growth rate of the population, Y is the physical output of the economy and s is the proportion of output that is saved. The output of the economy is typically given by a function of the form $Y = f(L, K)$ where $f(L, K)$ is assumed to satisfy the following conditions: $f(L, 0) = f(0, K) = 0$, $\forall K$ and L, $\frac{\partial f}{\partial L} > 0$, $\frac{\partial f}{\partial K} > 0$, $\frac{\partial^2 f}{\partial L^2} < 0$, $\frac{\partial^2 f}{\partial K^2} < 0$. The behavioral dynamics of the population modeled here are obviously quite simple - a constant proportion s of output is devoted to savings and $(1 - s)Y$ units of output are consumed. Clearly, the behavior of such a system hinges on the assumptions about the production function and the behavior of the population.

It is easy to see that for the conditions normally placed on f, the behavior of the above system is very simple. Using simple differential inequalities one can see that any trajectory beginning in the first quadrant (both capital and labor are positive) will remain there for all time and both state variables will grow without bound. Thus, the population, capital stocks, and productivity all grow exponentially. To address economic growth in a bounded ecosystem the dynamical system has to be extended to include dynamic resource constraints and economic model must be extended to accommodate more complex behavior. In order to develop such a model, some additional concepts from production and utility theory must be employed, which I will briefly review in the next section.

5.1.1 Basic laws of production and the theory of the firm

Very basic to an economic growth model is the specification of the laws of production or the production technology of the economy. Some specific examples of production functions were discussed in the model for agricultural output in the Tsembaga ecosystem (Chapter 3). The production technology is represented by a production function, $Y = f(x_1, x_2, ..., x_n)$, that characterizes technological alternatives for the inputs x_i and the *maximal* output Y obtainable for a given choice of these inputs. The characteristic of the production function most important for this model is the possibility of technical substitution between inputs.

The technical substitution possibilities specified by a particular production function refers to what extent one input may be substituted for another to maintain a fixed level of output. As we already saw, the Cobb-Douglas allows infinite substitutability between inputs, an assumption that may be completely unrealistic. Problems associated with such assumptions have received much attention in the ecological economics literature (e.g. see [60] for a review). At the opposite end of the spectrum is the Leontief production function usually written as

$$Y = \min_{i=1,..,n} \{\frac{x_i}{\beta_i}\} \tag{5.3}$$

where β_i is the requirement of input i per unit of output.

This is the analogue of the von-Liebig function used to describe agricultural production that we have already met. Here, there is absolutely no possibility for substitution between inputs. Clearly, neither extreme is entirely realistic, and different levels of substitutability are to be found for different types of inputs and outputs. For example, land can't be substituted for water to maintain productivity during a drought. A sewing machine and electrical energy can be substituted for a person with needle and thread in the construction of a garment. In my model, I assume that the overall production technology is of the Leontief form for physical inputs but capital and labor are substitutable to carry out productive activity in the production process. That is, let x_i be the ith *physical input* and let $\xi(L,K)$ represent productive activity where L is labor in hours and K represents services provided by capital, then

$$Y = \min \left\{ \frac{\xi(L,K)}{\beta_a}, \min_{i=1,..,n} \left(\frac{x_i}{\beta_i}\right) \right\}. \tag{5.4}$$

I represent $\xi(L,K)$ with a Cobb-Douglas production function i.e. $\xi(L,K) = L^{\alpha}K^{\beta}$. The resulting production function given by equation 5.4 allows infinite substitution between capital and labor, but no substitution between labor and capital (stocks), and raw materials (flows). This production function would not allow labor to be substituted for aluminum in the production of a bicycle, but it does allow a frame jig to be substituted for a human hand to hold the frame in place as it is welded.

Recall from Chapter 3 that α and β measure the marginal productivities of labor and capital respectively. It is commonly assumed that $\alpha + \beta = 1$ or that the production function has constant returns to scale (or the elasticity of scale is 1). Elasticity of scale (ϵ_s) is a measure of the proportionate change in output associated with a proportionate change of all inputs. If $\epsilon_s = 1$, doubling all inputs exactly doubles output. If $\epsilon_s > 1$, doubling of all inputs more than doubles output, etc. In my model I assume that productive activity exhibits constant returns to scale.

Next, I assume perfect competition (individual firms cannot affect prices by their choices of output levels) and that firms are making decisions in the "short run". In the economics literature, time scales are resolved to the "short run" and the "long run". This distinction is related to what managers are able to change as they make decisions. It is assumed that in the short run, managers can't change capital stocks. Thus for short run decisions, managers are faced with a fixed capital stock and will select the optimal labor input. In the long run, managers can adjust both capital and labor stocks in response to the conditions in the labor and capital markets. In my model, there is no explicit modeling of investment supply and demand, managers make only short run decisions and capital growth is determined completely by savings rates.

Finally I assume that firms will make full and efficient utilization of available factors of production. They will attempt to fully utilize capital stocks and select the optimal labor and output levels to minimize cost (or maximize profit). For an economy with multiple firms, full and efficient utilization means the total capital is divided optimally among the firms and then optimal labor is selected within each industry. The final aspect of firm behavior important to this model is the labor market. The optimal labor input for a given industry depends on the relationship of the cost of labor (wage) to the cost of capital. Thus given the cost of capital as fixed, the availability and cost of labor will determine the optimal combination of labor and capital.

5.1.2 Consumer behavior

The behavior of consumers is modeled using the standard approach from neo-classical economics: consumers maximize utility subject to an income constraint. We have already seen the importance the form of the utility function plays in ecosystem dynamics in Chapter 4. We saw with the Easter Island model that restricted substitutability between bioresources and manufactured goods was destabilizing. The Stone-Geary utility function is given by

$$\log u = \sum_{i=1}^{n} \log \left(q_i - q_i^{min}\right) \tag{5.5}$$

where u is utility, q_i are commodities, and q_i^{min} are the minimum amounts of a commodity required. This function is intuitively appealing. If the economy is capable of production levels above minimum requirements, people will substitute among favorite goods, trading off nightly fillet mignon for a better quality compact disc player. However, starving people won't try to ease their suffering by making bead necklaces, simply because there is no food and there are beads. The Stone-Geary utility function nicely captures this behavior as demonstrated in chapter 4.

5.2 The ecological economic model

The model that is the focus of the rest of this thesis is a two sector economic model coupled with an ecological model. The economy has an agricultural and non farm business sector (manufacturing). This choice of division for economic activities is motivated by the fact that we wish to model the effects of economic activity on two basic stocks: renewable natural capital and nonrenewable natural capital. A more common division of economic activity is between the agricultural, manufacturing, and service sectors. In my model I have vertically integrated the manufacturing and service sectors with the idea that the provision of services relies heavily on manufactured goods (insurance agents use cars, cell phones, computers, fuel, paper, etc. to do their jobs) and that the impact of these activities tend to be more focused on nonrenewable natural capital.

The economic ecological system model is shown schematically in figure 5.1. There are two basic flows in the model: the flow of raw materials and services from the state variables into the economic system and the flow of goods and services out of the economic system. The economic system represented by the non-farm business and agricultural sectors draw flows of low entropy materials from the stock of nonrenewable natural capital and services from labor, man-made capital, and renewable natural capital converts them to a flow of goods and services. The arrows between the two sectors represent the inter-industry transfer of goods and services. The human population, based on its preferences, can decide to consume goods and services, direct them towards investment, or increasing nonrenewable natural capital stocks through research and development for new materials, recycling, more efficient use of materials, or more efficient extraction techniques.

The model attempts to capture as simply as possible the fundamental aspects of both sides of the argument about sustainable development. All of the processes by which many believe we will continue to avert environmental degradation are included: ever-increasing efficiency, better material use, etc., but the achievement of these ends all require flows of economic goods and services and generate their own impact on the ecosystem. A perfect example is recycling. Recycling reduces the environmental impact of some production processes but requires capital, labor, energy input, and generates a waste stream, i.e. it merely transfers ecological stress from one form to another.

5.2.1 The economic system

In this section I will solve the simultaneous consumer and firm optimization problems in order to specify how labor and capital are allocated to each sector. We begin by specifying the technology in each of the sectors. Should the need arise, please refer to the table provided at the end of the chapter for an easy reference for the definitions of symbols.

As we have seen before, agriculture is best modeled with the von-Liebig or Leontief

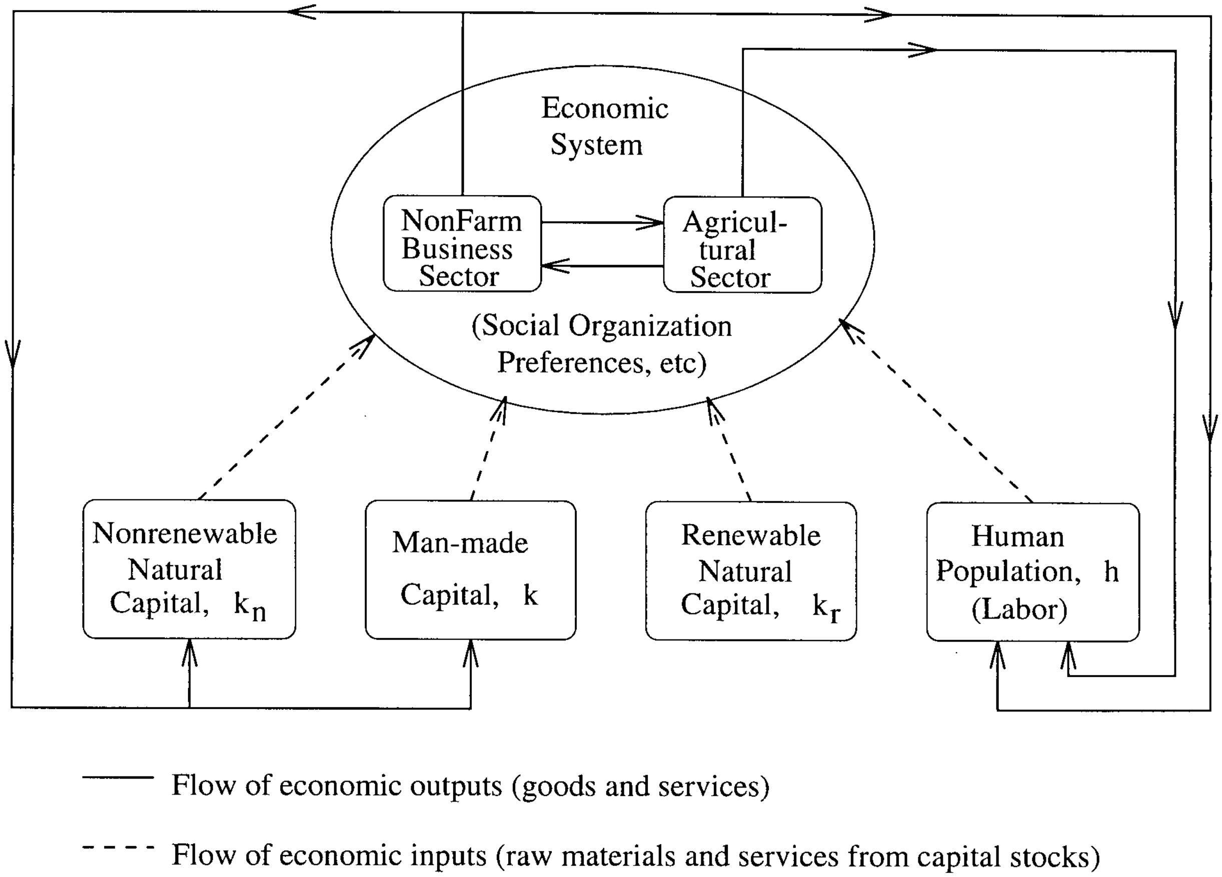


Figure 5.1: Schematic of two sector ecological economic model.

function. I assume that

$$Y_a = E_a(k_r)min\{\frac{\xi_a}{\beta_{\xi_a}}, \frac{l}{\beta_l}, \frac{N}{\beta_N}\} \tag{5.6}$$

where Y_a is annual agricultural output, $E_a(k_r)$ is a measure of efficiency related to soil and weather and is a function of the stock of natural capital, k_r. The inputs are productive activity ξ_a, land l, and nutrients N (phosphorus, nitrogen, potassium, etc.). The β's are the per unit input requirements per unit of output. Efficient utilization implies that

$$\frac{\xi_a}{\beta_{\xi_a}} = \frac{l}{\beta_l} = \frac{N}{\beta_N} \tag{5.7}$$

thus for a given amount of land, there is a set nutrient requirement and a physically determined amount of work required to carry out the production process. The population will decide how much productive activity (ξ_a) to direct to agricultural production via the optimal combination of capital (K_a) and labor (L_a) based on the production function

$$\xi_a = L_a^{a_a} K_a^{b_a}. \tag{5.8}$$

In the model, natural capital provides several free services and could be called an economic sector in a sense. Among other things, it generates soil and soil nutrients, assimilates waste, and irrigates via the solar water pump. In equation 5.6 this is reflected by the fact that efficiency is a function of the stock of natural capital, but also through the nutrient input required for a given level of output. The required nutrients can be supplied by the "natural sector" as is the case in the Tsembaga ecosystem, or by the manufacturing sector (fertilizer, etc.).

Thus at low levels of agricultural output, natural nutrient production is sufficient to meet demand. As output increases, nutrients in the form of fertilizer, pesticides, and genetically engineered seed must be provided from the manufacturing sector. Let R_{ma} be the manufactured goods required per unit of agricultural output. As agricultural production increases R_{ma} increases from zero up to some maximum where most of the nutrients for agriculture are supplied by the manufacturing sector. It is a messy bookkeeping and computational problem to try to relate R_{ma} directly to agricultural output. Instead, the ratio of population density to renewable natural capital, $\frac{h}{k_r}$ is used as an indirect measure of agricultural output. The higher this ratio, the more pressure is being put on k_r and more nutrients must be injected into the system from the manufacturing sector. The functional relationship is

$$R_{ma}(x) = \frac{\beta_N x^3}{x^3 + {\beta_{half}}^3} \tag{5.9}$$

where β_N is the nutrient requirement per unit of agricultural output, and β_{half} is the level of $\frac{h}{k_r}$ at which R_{ma} is one-half the maximum. This function has the property that below a certain threshold value of x, $R_{ma}(x)$ is very small (nutrients are being provided by natural capital). As x increases above the threshold, $R_{ma}(x)$ begins to increase rapidly up to a maximum where all nutrient inputs come from the manufacturing industry.

Choosing the units so that $\beta_{\xi_a} = 1$, and assuming efficient factor utilization we have

$$Y_a = E_a(k_r)L_a^{a_a}K_a^{b_a}. \tag{5.10}$$

with nutrient demand from the manufacturing sector, Y_{ma} given by

$$Y_{ma} = R_{ma}(\frac{h}{k_r})Y_a. \tag{5.11}$$

The story is similar for manufacturing (= non farm business sector) except that here, the manufacturing industry includes the production of inputs and the finished product. This is necessary to avoid including a third sector in the model for the production of raw materials. Thus we can write manufacturing production in terms of the productive activity directed towards the process of extracting raw materials and using them to deliver goods and services:

$$Y_m = E_m(k_n)\xi_m \tag{5.12}$$

where Y_m is manufacturing output. The efficiency of the manufacturing process, E_m, depends on the stock of nonrenewable natural capital, k_n, because as stocks of low entropy materials go down (e.g. metal per ton of ore, reservoir petroleum saturation, etc.), more and more work is required to extract raw materials. As in the agricultural sector $\xi_m = L_m^{a_m}K_m^{b_m}$ thus we have

$$Y_m = E_m(k_n)L_m^{a_m}K_m^{b_m}. \tag{5.13}$$

If we define the capital-labor ratio $\eta_i = \frac{K_i}{L_i}$, and assume constant returns to scale, equations 5.10 and 5.13 can be rewritten in the form

$$Y_a \;=\; E_a(k_r)L_a\eta_a^{b_a} = E_a(k_r)\eta_a^{-a_a}K_a \tag{5.14a}$$

$$Y_m \quad = \quad E_m(k_n) L_m \eta_m^{b_m} = E_m(k_n) \eta_m^{-a_m} K_m \tag{5.14b}$$

which we will employ later. Equations 5.10 and 5.13 determine how agricultural and manufacturing outputs are related to labor and capital devoted to them. The question remains: how does society decide how much to consume of each product and how much labor and capital should be devoted to each activity?

To answer the first question, we assume that society directs energy to producing agricultural, manufactured, investment, and resource goods. The first three require no explanation. Resource goods would consist of any effort to find more raw materials, improve material efficiency or develop new materials. Consumers then solve the following constrained maximization problem:

$$\max\ U(q_a, q_m, q_i, q_r) = (q_a - q_a^*)^{c_a} (q_m - q_m^*)^{c_m} q_i^{c_i} q_r^{c_r} \tag{5.15}$$

$$\text{subject to: } P_a q_a + P_m q_m + P_i q_i + P_r q_r \leq I \tag{5.16}$$

where $q_a, q_m, q_i,$ and q_r are the per-capita consumption rates of agricultural, manufacturing, investment, and resource goods, P_a, P_m, P_i, and P_r are their respective prices, I is per-capita income, and c_a through c_r are the cultural parameters that characterize the preference for each good. As in the Easter Island model, there are minimum intake levels of certain commodities below which the population will alter its behavior. Here we assume that there is a minimum level of agricultural goods q_a^* set by human nutritional requirements and a minimum quantity of manufactured goods, q_m^* necessary to meet housing, clothing, and minimal capital requirements such as very simple tools. There is no minimum investment or resource-good levels - when faced with merely surviving, the population concentrates on the bare essentials.

By applying the technique of Lagrange multipliers, we can solve the problem specified by 5.16. Define supernumery income, I_s by

$$I_s = I - P_a q_a^* + P_m q_m^* \tag{5.17}$$

then we obtain the following first order conditions for the optimal per-capita consumption levels :

$$q_a = q_a^* + \frac{c_a I_s}{P_a} \tag{5.18a}$$

$$q_m = q_m^* + \frac{c_m I_s}{P_m} \tag{5.18b}$$

$$q_i = \frac{c_i I_s}{P_i} \tag{5.18c}$$

$$q_r = \frac{c_r I_s}{P_r} \tag{5.18d}$$

Equations 5.18 are interpreted as follows. After meeting minimum demands of agricultural and manufactured goods, a proportion of the income left over, the supernumery income I_s is devoted to each of the four activities. This defines the demand side of the economy.

The supply side of the economy is characterized by firms maximizing profits. The profit functions for the agricultural and manufacturing sectors (=non-farm business) are

$$\Pi_a(L_a, K_a) = P_a Y_a - wL_a - rK_a - Y_a R_{ma} P_m \tag{5.19a}$$

$$\Pi_m(L_m, K_m) = P_m Y_m - wL_m - rK_m - Y_m R_{am} P_a \tag{5.19b}$$

where w and r are the per-unit costs of labor and capital respectively, R_{ma} is the rate at which manufacturing goods are utilized by the agricultural industry, and R_{am} is the rate at which agricultural goods are utilized by the manufacturing industry. I assume that labor and capital decisions made in one industry will not affect prices in the other so firms will maximize profits by finding the optimal labor-capital inputs via first order conditions given by (for example in agriculture)

$$\frac{\partial \Pi_a(L_a, K_a)}{\partial L_a} = \frac{a_a Y_a}{L_a}(P_a - R_{ma} P_m) - w = 0 \tag{5.20a}$$

$$\frac{\partial \Pi_a(L_a, K_a)}{\partial K_a} = \frac{b_a Y_a}{K_a}(P_a - R_{ma} P_m) - r = 0 \tag{5.20b}$$

with an analogous set of equations for the manufacturing industry. These two equations determine the optimal capital labor ratio:

$$\eta_a^{opt} = \frac{K_a^{opt}}{L_a^{opt}} = \frac{wb_a{}'}{ra_a} \tag{5.21}$$

which says that the optimum factor inputs depend on the labor to capital cost ratio and the factor productivities. Next, by adding equations 5.20a and 5.20b we arrive at the zero profit condition:

$$P_a Y_a = wL_a^{opt} + rK_a^{opt} + Y_a R_{ma} P_m, \tag{5.22}$$

which says that, at optimum, the revenue generated by the production and sale of agricultural goods exactly covers the production costs. This relationship is true for any CRS technology. Until further notice, all the quantities I will be referring to are the optimal quantities (where this makes sense), and I will drop the superscript. Equations 5.18 characterize the demand for goods while 5.21, and 5.22 along with their counterparts for the manufacturing industry characterize the demand.

5.2.2 Computing the general equilibrium

Computing the general equilibrium reduces to setting the aggregate demand equations equal to the aggregate supply equations. The demand for agricultural goods is composed of the per-capita consumption multiplied by the population level plus the agricultural goods used in the manufacturing industry, i.e.

$$Y_a^D = hq_a + Y_m^D R_{am} \tag{5.23}$$

where h is the human population, and the superscript indicates "demanded". The demand for manufactured goods is composed of the demands of consumption, investment, and resource goods all of which are produced by the manufacturing sector, plus the manufactured goods consumed by the agricultural sector. Thus

$$Y_m^D = hq_m + hq_i + hq_r + Y_a^D R_{ma}. \tag{5.24}$$

The demands for agricultural and manufactured goods are easily computed by dividing equation 5.22 and the counterpart for manufacturing through by the appropriate prices. Setting the results equal to the right hand sides of equation 5.23 and 5.24 yields the general equilibrium equations:

$$\begin{aligned} P_a hq_a + P_a Y_m R_{am} &= wL_a + rK_a + Y_a R_{ma} P_m && (5.25a) \\ P_m hq_m + P_m hq_i + P_m hq_r + P_m Y_a R_{ma} &= wL_m + rK_m + Y_m R_{am} P_a && (5.25b) \end{aligned}$$

Equations 5.25 specify the equilibrium with *efficient factor utilization.* Recall that in the model *full factor utilization* is enforced. This requires that

$$\begin{aligned} L_a + L_m &= L && (5.26a) \\ K_a + K_m &= K && (5.26b) \end{aligned}$$

where L and K are the total labor and capital available, respectively. Equations 5.21, 5.25 and 5.26 constitute a system of five equations (of which three are nonlinear because prices and output are nonlinear functions of capital and labor) and six unknowns: L_a, L_m, K_a, K_m, r, and w. Thus given any one variable, all other equations could be solved for the other variables. Since in this model money acts only as a numeraire, the system is closed by fixing r (the factor cost of a unit of capital) as the numeraire good and measuring prices in terms of r.

There are several problems with this approach. First and most obvious is the problem of existence and uniqueness of solutions to systems of nonlinear equations. Then, supposing there is a unique solution, there is the difficulty of locating it. The algebraic system of equations that characterize the economic system is coupled with a dynamical system that characterizes the ecosystem – i.e. the human population, capital stocks, natural capital stocks, and so on. Thus, the economic system equations must be solved continuously as the physical system evolves. If there is no explicit solution to the economic model as was the case for the models in Chapter 4, the ecological economic system model is a set of differential algebraic equations (DAE) Although there are techniques to solve DAE's (i.e. collocation, [5]), dynamical system and bifurcation analysis tools such as XPPaut and Auto are not set up to handle this situation. Thus, in order to study the structure of the model, we must reformulate the general equilibrium problem.

I reformulate the problem by adding a labor market and writing the five equation system as one explicit algebraic equation and one differential equation. First, we substitute the values of Y_a, Y_m, and q_a given by 5.14a, 5.14b, and 5.18a respectively into 5.25a to get

$$P_a h q_a^* + c_a h I - c_a h P_a q_a^* - c_a h P_m q_m^* + P_a R_{am} E_m(k_n)\eta_m^{-a_m} K_m = \\ w L_a + r K_a + P_m R_{ma} E_a(k_r)\eta_a^{-a_a} K_a. \quad (5.27)$$

Then, from equation 5.21 and its counterpart for the manufacturing industry, we get a set of coupled equations for the optimal prices:

$$P_a = \frac{L_a w + K_a r}{E_a(k_r)\eta_a^{-a_a} K_a} + R_{ma} P_m \quad (5.28a)$$

$$P_m = \frac{L_m w + K_m r}{E_m(k_n)\eta_m^{-a_m} K_m} + R_{am} P_a. \quad (5.28b)$$

We can again use equation 5.21 to eliminate capital and labor from equations 5.28, i.e., at optimum we have:

$$L_a w = \frac{K_a r a_a}{b_a} \quad (5.29)$$

thus

$$\frac{L_a w + K_a r}{E_a(k_r)\eta_a^{-a_a} K_a} = \frac{\frac{K_a r a_a}{b_a} + K_a r}{E_a(k_r)\eta_a^{-a_a} K_a} = \frac{r(1 + \frac{a_a}{b_a})}{E_a(k_r)\eta_a^{-a_a}} = \frac{r\eta_a^{a_a}}{E_a(k_r) b_a}. \quad (5.30)$$

A similar relation holds for the manufacturing sector, enabling us to write equations 5.28 as

$$
\begin{aligned}
P_a &= \frac{r\eta_a^{a_a}}{E_a(k_r)b_a} + R_{ma}P_m && (5.31a)\\
P_m &= \frac{r\eta_m^{a_m}}{E_m(k_n)b_m} + R_{am}P_a. && (5.31b)
\end{aligned}
$$

Solving these coupled equations for the prices yields:

$$
\begin{aligned}
P_a &= \frac{r}{1-R_{ma}R_{am}}\left(\frac{\eta_a^{a_a}}{E_a(k_r)b_a} + \frac{R_{ma}\eta_m^{a_m}}{E_m(k_n)b_m}\right) && (5.32a)\\
P_m &= \frac{r}{1-R_{ma}R_{am}}\left(\frac{\eta_m^{a_m}}{E_m(k_n)b_m} + \frac{R_{am}\eta_a^{a_a}}{E_a(k_r)b_a}\right). && (5.32b)
\end{aligned}
$$

Notice the upward effect decreasing efficiencies and increasing inter-industry transfers have on prices. It is important to include this aspect in the model to capture the important fact of the heavy reliance of modern agriculture on manufacturing inputs. Notice that the prices in 5.32 depend only on physical constants, the per unit capital cost, and the capital-labor ratios η_a, and η_m. At optimum, the capital labor ratio can be replaced by the factor cost ratio via 5.21. Thus, given the factor cost ratio, optimal prices are determined up to the constant r. Thus equations 5.32 can be rewritten as

$$
P_a = rf_a(\omega) \quad \text{and} P_m = rf_m(\omega) \qquad (5.33)
$$

where $\omega \equiv \frac{w}{r}$ and

$$
\begin{aligned}
f_a(\omega) &= \frac{1}{1-R_{ma}R_{am}}\left(\frac{\omega^{a_a}(b_a/a_a)^{a_a}}{E_a(k_r)b_a} + \frac{R_{ma}\omega^{a_m}(b_m/a_m)^{a_m}}{E_m(k_n)b_m}\right) && (5.34a)\\
f_m(\omega) &= \frac{1}{1-R_{ma}R_{am}}\left(\frac{\omega^{a_m}(b_m/a_m)^{a_m}}{E_m(k_n)b_m} + \frac{R_{am}\omega^{a_a}(b_a/a_a)^{a_a}}{E_a(k_r)b_a}\right). && (5.34b)
\end{aligned}
$$

By writing the prices this way, we will see that r cancels and the equilibrium labor and capital devoted to agriculture and manufacturing depend only on the factor cost ratio ω. Finally, if I, the per-capita income of the economy could be written in terms of ω, equations 5.34 and 5.27 can be combined to write K_a as an explicit function of ω. Since hI, the total income of the economy, is equal to the sum of the total income generated by labor and capital, respectively, (factor rewards) we have,

$$hI = Lw + Kr = (L_a + L_w)w + (K_a + K_m)r \tag{5.35}$$

and using 5.29 we can eliminate the labor terms arriving at

$$hI = \frac{rK_a}{b_a} + \frac{rK_m}{b_m}. \tag{5.36}$$

Here we see that income and prices both depend on r. Fixing r is equivalent to choosing units for the money in the system - i.e. r is a numeraire. Since we are only including the dynamics of the labor market, we fix $r = 1$, then $\omega = w$. Finally, combining equations 5.25, 5.36, 5.26b, and 5.21 we arrive at at explicit formula for K_a in terms of K, h, and w:

$$K_a(K,h,w) = \frac{(1-c_a)hf_a(w)q_a^* + \dfrac{c_aK}{b_m} + R_{am}E_m(k_n)\left(\dfrac{a_m}{wb_m}\right)^{a_m} f_a(w)K - hc_af_mq_m^*}{\dfrac{1-c_a}{b_a} + \dfrac{c_a}{b_m} + R_{am}E_m(k_n)\left(\dfrac{a_m}{wb_m}\right)^{a_m} f_a(w) + R_{ma}E_a(k_r)\left(\dfrac{a_a}{wb_a}\right)^{a_a} f_m(w)}. \tag{5.37}$$

Thus given the total capital endowment of the economy, the human population, and the wage rate (= factor cost ratio), the optimal amount of capital to devote to agriculture is easily computed by 5.37. Then using 5.26b, and 5.21, the optimal levels of capital and labor to manufacturing and labor to agriculture can be computed. The problem is that the optimal labor quantities computed this way may not be equal to the labor endowment of the economy, that is: $L_a + L_m \neq L$ in general and the economy is out of equilibrium. This is where the role of the labor market comes into play. The labor market will link wages to available labor and force the economy to tend towards equilibrium. Before discussing the labor market, however, I would like to make a critical point about equation 5.35. This equation says that a certain portion of the revenue generated by the productive process is paid to workers in the form of wages while the remainder is paid to 'capital' in the form of interest, dividends, etc. It says nothing however about the *distribution* of income. I will address this point in more detail later.

Several (nonlinear) algebraic relationships have been proposed to relate labor supply, demand, and wages, e.g. [66], but I will employ a simple linear (in labor supply and demand) differential equation to model wage dynamics. The assumptions are basic: an oversupply of labor will put downward pressure on wages while and under-supply will have the opposite effect. This simple-minded model does nothing to address important labor market issues such as union activity and so on, but is sufficient for a start. Thus we have

$$\frac{dw}{dt} = \lambda_w(L - L_a(w) - L_m(w)) \tag{5.38}$$

where λ_w is the speed of response of wages to disparities between labor supply and demand. Equation 5.38 coupled with 5.37 comprise a fast efficient method forcing the economy to seek equilibrium in a dynamically evolving system. The alternative of solving a set of coupled nonlinear equations for the equilibrium is not only slower and more difficult, but also artificial. Economies are never in equilibrium, and equation 5.38 captures this fact. Further, we can actually adjust "out of equilibriumness" via the factor λ_w and study its effect on the dynamics of the system.

In order to illustrate the operation of the economic system, I have computed the equilibrium with arbitrary initial capital and labor endowments of 100 units each. Parameters are: $a_a = 0.3$, $a_m = 0.8$, $q_a^* = 0.5$, $q_m^* = 0.1$, $c_a = 0.2$, $c_m = 0.8$, $c_i = c_r = 0$, and $E_a = E_m$ are constant and set equal to 1. Figure 5.2 shows the results of this exercise.

The initial guess at the wage rate is 0.5 so each unit of labor is half as costly as a corresponding unit of capital. With such cheap labor, it is optimal to use well over 200

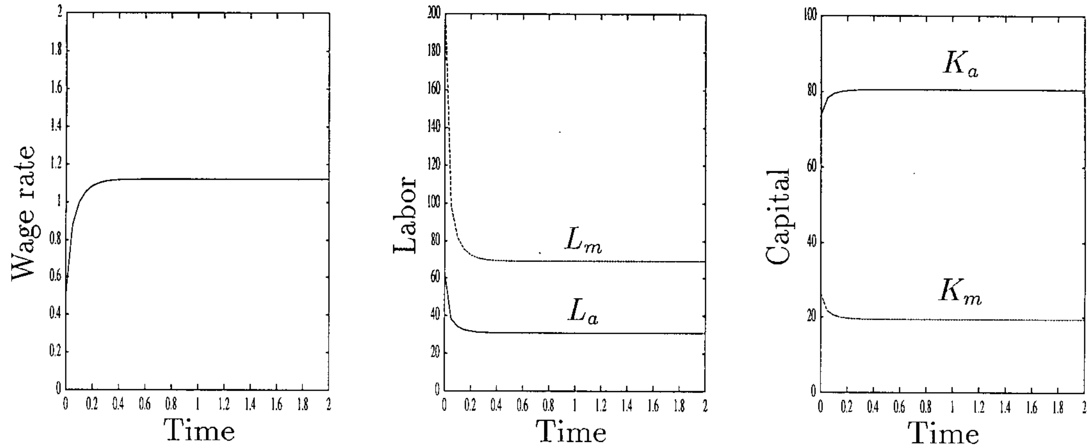


Figure 5.2: Trajectories of wages, capital, and labor as the economy adjusts.

units which far exceeds labor availability. Upward pressure on wages drives the system very quickly to the equilibrium state with $w = 1.121$, $K_a = 80.574$, $K_m = 19.426$, $L_a = 30.767$, $L_m = 62.233$. The question is, is this solution unique and optimal? Figure 5.3 helps put this question in perspective; it shows the utility function and the optimum solution above.

Note that the utility function is strictly convex inside the region where the economy can exceed its minimum demands of $q_a^* = 0.5$ and $q_m^* = 0.1$. The inset figure on the upper right is a contour plot of the surface on the lower left showing the optimum with a white dot, the region where minimum demands can't be met with available labor and capital endowments (white area), and where they can (grey scale area). For values of labor and capital in the grey scale region, it is tedious but not difficult to show that the necessary condition for optimality given by 5.18 is sufficient and the solution is unique.

In the region in the $L_a - K_a$ plane where minimum needs cannot be met, the utility function is defined to be identically 0. In this case there is no optimum solution so some other mechanism must be defined to allocate available resources to different activities. I accomplish this by assuming that if minimum needs cannot be met, the economy

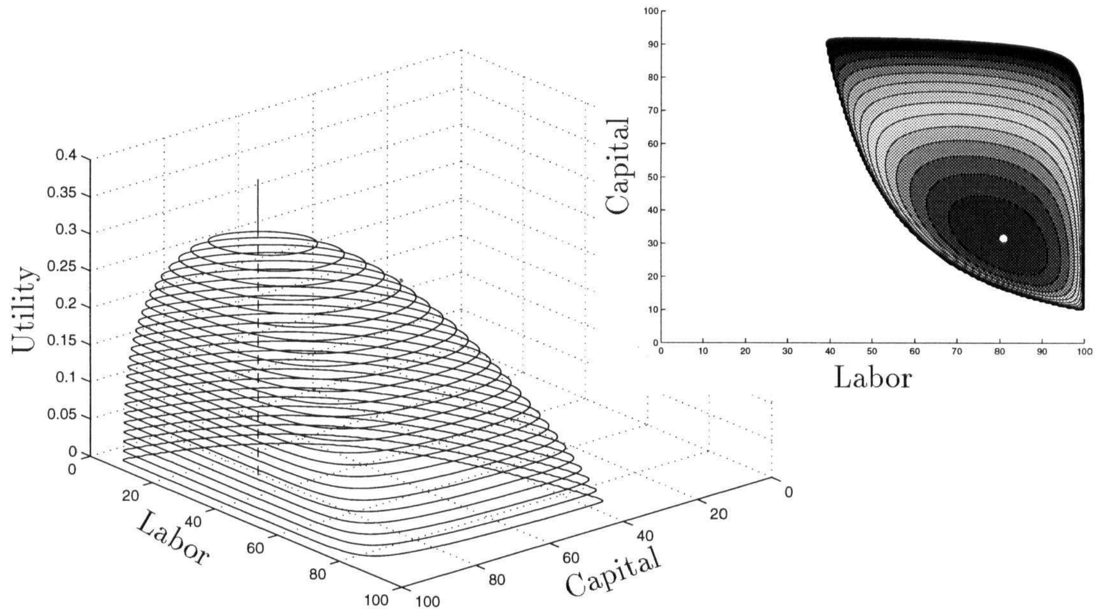


Figure 5.3: Surface plot of utility function showing optimal combination of labor and capital to agriculture.

will first attempt to meet food needs and devote what is left over to other activities. Mathematically, this translates to:

$$q_a = \begin{cases} q_a^* + \frac{c_a I_s}{P_a} & I_s \geq 0 \\ q_a^* & I_s < 0 \text{ and } I - P_a q_a^* > 0 \\ E_a(k_r) L^{a_a} K^{b_a} & otherwise \end{cases} \tag{5.39a}$$

$$q_m = \begin{cases} q_m^* + \frac{c_m I_s}{P_m} & I_s \geq 0 \\ \frac{I - P_a q_a^*}{P_m} & I_s < 0 \text{ and } I - P_a q_a^* > 0 \\ 0 & \text{otherwise} \end{cases} \tag{5.39b}$$

$$q_i = \begin{cases} \frac{c_i I_s}{P_m} & I_s \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{5.39c}$$

$$q_r = \begin{cases} \frac{c_r I_s}{P_m} & I_s \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{5.39d}$$

Before turning our attention to the physical system, I would like to emphasize two important aspects of the economic system: the effect of inter-industry transfers, and the (sensible) way the economy evolves when it becomes more difficult to meet minimum demands (i.e. how equations 5.39 work) . I do this by examining the evolution of the economy as the amount of manufactured goods purchased by the agricultural sector increases. Figure 5.4 shows how the consumption and expenditure patterns change under these conditions.

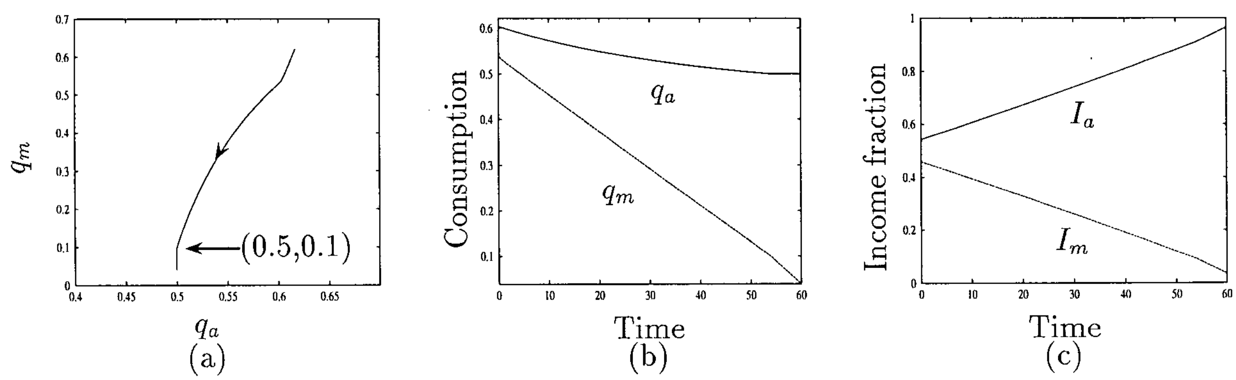


Figure 5.4: Graph (a) shows q_m versus q_a. Notice that consumption evolves toward (q_a^*, q_m^*). Graph (b) shows q_m (dotted) and q_a (solid) over time. Graph (c) shows the proportion of income devoted to purchasing manufacturing and agricultural goods, I_m and I_a respectively.

Figure 5.4(a) plots q_m versus q_a and illustrates how the economy moves to the point (q_m^*, q_a^*). Beyond this point, the economy first meets agricultural needs and uses what is left for manufactured goods as illustrated by the vertical line. Figure 5.4(b) shows consumption over time - large sacrifices in the consumption of manufactured goods are necessary to maintain agricultural production. Finally, figure 5.4(c) shows how increased reliance on manufactured inputs in agriculture will cause relative price increases for agricultural goods. With the economic system model complete, we now turn to the final task of specifying the physical system.

5.3 The ecological system model

The cultural (distributional) component of the model is contained in the economic system in the four parameters: c_a, c_m, c_i, and c_r that govern how the productive capacity of the economy is portioned to the different activities of consuming food, manufactured goods, investment goods, and resource goods respectively. We are left to specify how these activities interact with the state variables h, k_h, k_n, and k_r as defined in chapter 2. The dynamical system that we will analyze for the remainder of this chapter is:

$$\frac{dh}{dt} = (b(q_m) - d(q_a))h \tag{5.40a}$$

$$\frac{dk_h}{dt} = e_{k_r,i} h q_i - \delta k_h \tag{5.40b}$$

$$\frac{dk_n}{dt} = -e_{k_n,m} Y_m + e_{k_n,r} h q_r \tag{5.40c}$$

$$\frac{dk_r}{dt} = k_r n_r (1 - k_r) - e_{k_r,a} Y_a \tag{5.40d}$$

where $b(q_m)$ is the per capita birth rate as a function of per capita consumption of manufactured goods which incorporates the idea of "demographic transition", $d(q_a))$ is the nutrition dependent death rate function just as in the Tsembaga model, the $e_{i,j}$ are (conversion) factors measuring the effect of the jth process on the ith state variable, i.e. $e_{k_r,a}$ measures the effect of agriculture on renewable natural capital, δ is the rate of depreciation of man-made capital, and n_r is the (possibly dependent on economic output or the state of the system) regeneration rate of renewable natural capital.

The model specified by 5.40 is perhaps the simplest possible that incorporates all the key features that are debated in the literature. For example, equation 5.40a taken with equation 5.40b with $\delta = 0$ and $b - d$ held constant is a typical example of an economic growth model with no connection to the physical world. This would correspond to the model in figure 2.3. Figure 5.5 shows the evolution of a model economy under these circumstances. Graph (a) shows the trajectory of the economy in phase space from different initial capital and labor endowments. In this case, capital and labor grow without bound, converging to a fixed capital labor ratio determined by the level of investment of the economy, c_i as shown in graph (b). While the capital labor ratio is below the long run equilibrium level, standard of living increases up to a maximum as indicated in graph (c). After the long run equilibrium is reached, economic output grows exponentially, with per capita consumption constant.

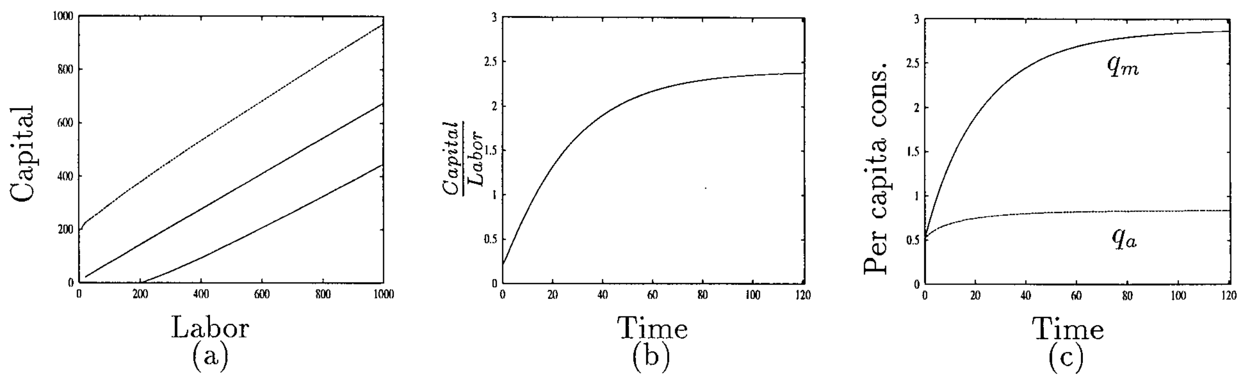


Figure 5.5: Graph (a) shows capital versus labor for the simple economic growth model corresponding to figure (2.3) and equations (5.2). Notice each trajectory has the same slope. Graph (b) shows the capital-labor ratio. Graph (c) shows the per capita consumption of manufactured (dotted) and agricultural goods (solid) over time.

Exponential economic growth is unrealistic in the long run, and the model incorporates important implications of entropic considerations called for by authors such as [27, 18] by allowing things to wear out - i.e. $\delta \neq 0$ in equation 5.40b, and including the physical reality that producing goods can degrade both renewable and nonrenewable natural capital in equations 5.40c and 5.40d.

Now, if one sets the right hand sides of equations 5.40 to zero to find the steady state(s), this would correspond to locating a steady state economy in phase space. Indeed, setting the equations above to zero and reading off the conditions for this to be true matches our intuitive idea about what a sustainable human agro- ecosystem is, i.e. at a steady state, birth rates will be depressed by changing economic structure (improved living standards and the increased marginal cost of children); investment rates will just offset depreciation (entropic decay) keeping capital stocks constant; and recycling, more efficient resource use, and reduced waste streams will offset degradation of natural capital. So what can be gained studying a complicated dynamical system? The verbal description does not say anything about the magnitudes of the state variables at equilibrium, nor does it say anything about whether the equilibrium is attainable, i.e. under what conditions can a system arrive at a sustainable state. It is one thing to characterize a sustainable state, but another to study its structure, the task to which we now turn our attention.

5.4 Analysis of the Model

Because the model structure is very rich, it will be explored a piece at a time. The first issue we will explore with the model is the interaction of investment, evenness of economic growth, and the distribution of wealth in an economy that relies on renewable natural capital - i.e. one step up from the most basic economic growth model involving only labor and capital. Complexity will then be added step by step, finishing with the analysis of the full model.

5.4.1 Investment, distribution of wealth, and ecosystem stability

Intuitively, the process of investment by which productive capacity is increased should make everyone's life better off. It is possible however to invest too much whereby, for example, the capital stock may grow to such a point that its maintenance puts such a drain on the economy that the standard of living is reduced. Another problem with too much investment is associated with overexploitation of resources due to being too efficient. In our model, investment helps productivity not only in the manufactured goods sector, but also in agriculture. This increased productivity in agriculture may destabilize the system by allowing the population to grow far beyond the level that an ecosystem could bear without degradation. One mechanism that might halt this process is behavioral changes associated with changing economic structure sometimes referred to as the "demographic transition". As the structure of the economy changes, the roles children play in the economy change which in turn suppresses birth rates. We investigate the interplay between these two process by analyzing the dynamics of the model while two parameters are varied: c_i - the investment level, and b_c - a parameter that relates how sensitive the birth rate is to per capita consumption of manufactured goods which I will explain in a moment. In this analysis, we assume that the efficiency in the manufacturing sector is constant and does not depend on the availability of low entropy materials. This leaves only three physical state variables: h, k_h, and k_r.

The function $b(x)$ relates the birth rate to per capita consumption of manufactured goods. As economic structure changes, there are several factors that might influence birth rates. First, the marginal cost of children increases as economic complexity increases. In simple rural economies, children can produce more than they consume at a young age (below 10 years). In a complex industrial economy, children are a financial burden to their parents for a much longer time. Values might also shift - the enjoyment of having children and of family life might be replaced with other leisure activities aided by having fewer children . What ever the mechanism, changing economic structure and the associated increased economic productivity seem to depress birth rates. It is this rationale that leads to the idea that continued economic development is the best policy if we wish to guide the global economy to a sustainable state. Again, although this argument is very attractive, there is the question of under what circumstances this goal is attainable. To capture this, I assume that $b(x)$ has the form

$$b(x) = b_0 \exp(-b_c x) \tag{5.41}$$

where b_0 is the per-capita birth rate when no manufactured goods are consumed and b_c measures the sensitivity of birth rates to the level of consumption. For large values of b_c, births decrease very rapidly with increased per capita consumption of manufactured goods and vice versa. The physical interpretation of b_c could be either that each individual in the population has a certain response to consumption or it could measure the distribution of income, or more precisely, the evenness of economic development. The latter is of most interest to us. Notice that the argument of $b(x)$ is q_m which is the *average* per capita consumption of manufactured goods. If economic development is not even, some individuals might enjoy certain benefits that reduce mortality with out experiencing other aspects of the development process that might suppress birth rates. In this case the response of the birth rate to consumption levels would be weak. This situation is modeled by a low value of b_c. If, on the other hand, economic growth is more even and income is distributed evenly, birth rates would fall off more quickly as consumption increased because more individuals in the population would reduce births for the same level of per capita intake. It turns out that for an economy that decides to invest, how evenly the the economy develops and distributes income is an important factor for its survival.

To illustrate, we examine the structure of the model as the parameters c_i and b_c are varied. To set the stage, suppose that economic growth is even and income is distributed very well within the economy. The system is then integrated with the following parameter values:

- Economic parameters: for the marginal productivities of labor in each industry we take $a_a = 0.3$ and $a_m = 0.8$. The value for manufacturing is based on some empirical work that suggests that values in the range of 0.7 to 0.8 are reasonable [32]. The value for agriculture is more speculative and is based on the heavy reliance on capital in modern agriculture. We take $q_a^* = 0.5$ and $q_m^* = 0.1$ which are arbitrary and depend on scaling and choice of units in the rest of the model. The only important thing is that agricultural goods become relatively more important in times of scarcity. The cultural parameters are $c_a = 0.05$, $c_m = 0.9$, $c_i = 0.05$, $c_r = 0$. I selected these values based on consumer data from the 1994 Statistical Abstract of the United States [46]. I simply adjusted the parameters until the proportion of income spent in each category generated by the model roughly matched those for the U.S., roughly 11 percent to food, 13 percent to investment, and the rest to personal consumption (manufactured goods). Next I set $E_a = 10k_r$ and $E_m = 1$. The efficiency in agriculture is based on energy data for agricultural production [51]. In this case, I assume that the efficiency of manufacturing is constant and unity and that there are no interindustry transfers - assumptions that will be relaxed later.

- Ecological parameters: $\delta = 0.03$, $e_{k_r,i} = 0.35$, $e_{k_n,m} = 0$. The parameter $e_{k_n,r}$ is irrelevant because no income is directed toward resource goods. Finally, $e_{k_r,a} = 0.005$, and $n_r = 0.1$. These parameters merely scale time in the model (i.e. just specify the units of measurement). The key physical parameters are b_0 and b_c. For example if $b_0 = 0.05$, at low levels of consumption, a couple (on average) would have around 6 births over a lifetime. Now we can study how the parameter b_c affects the model.

With these assumptions, we are left to analyze the following dynamical system:

$$\frac{dh}{dt} = (0.05\exp(-b_c q_m) - 7\exp(-10q_a))h \tag{5.42a}$$
$$\frac{dk_h}{dt} = 0.35hq_i - 0.03k_h \tag{5.42b}$$
$$\frac{dk_r}{dt} = 0.1k_r(1-k_r) - 0.005Y_a \tag{5.42c}$$
$$\frac{dw}{dt} = 0.1(h - L_a(w) - L_m(w)) \tag{5.42d}$$

where the following set of algebraic constraints apply. The optimal capital levels to devote to agriculture and manufacturing are

$$K_a = \begin{cases} 0.054\frac{h}{k_r}w^{0.3} + 0.156k_h - 0.005hw^{0.8} & K_a < k_a \\ k_h & \text{otherwise} \end{cases} \tag{5.43a}$$
$$K_m = k_h - K_a. \tag{5.43b}$$

Then equations 5.21, 5.14a, 5.14b, and 5.32 allow the optimal labor, output, and price levels to be computed:

$$L_a = 0.429\frac{K_a}{w} \qquad L_m = 4\frac{K_m}{w} \tag{5.44}$$

$$Y_a = 7.76k_r w^{-0.3} \qquad Y_m = 3.03w^{-0.8} \tag{5.45}$$

$$P_a = 0.184w^{0.3}k_r^{-1} \qquad P_m = 1.649w^{0.8}. \tag{5.46}$$

Recall that $L = L_a + L_m$ so per capita income and supernumery income can be computed:

$$I = \frac{k_h + wL}{h} \qquad I_s = I - 0.5P_a - 0.1P_m. \tag{5.47}$$

Finally, the per capita consumption levels are given by

$$q_a = \begin{cases} 0.5 + \frac{0.05 I_s}{P_a} & I_s \geq 0 \\ 0.5 & I_s < 0 \text{ and } I - 0.5 P_a > 0 \\ 10 k_r h^{a_a} k_h^{b_a} & \text{otherwise} \end{cases} \tag{5.48a}$$

$$q_m = \begin{cases} 0.1 + \frac{0.9 I_s}{P_m} & I_s \geq 0 \\ \frac{I - 0.5 P_a}{P_m} & I_s < 0 \text{ and } I - 0.5 P_a > 0 \\ 0 & \text{otherwise} \end{cases} \tag{5.48b}$$

$$q_i = \begin{cases} \frac{0.05 I_s}{P_m} & I_s \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{5.48c}$$

and the model is fully specified.

Figure 5.6 shows the trajectories of the model in phase space for $b_c = 3$ (relatively even economic development and wealth distribution). Graph (a) shows the population versus natural capital. As population grows, natural capital is reduced, but the system comes to stable equilibrium, i.e. a sustainable state. Graph (b) shows the population versus man-made capital. Notice that when the population is low, capital and labor grow maintaining a constant ratio (i.e. the labor versus capital curve is a straight line) as is common for simple economic growth models. However, as the system grows, it encounters limitations in natural capital which restricts human population and, in turn, capital growth. The capital-labor trajectory tends away from the linear growth trajectory (that would continue on indefinitely in a simple economic growth model including just labor and capital) and comes to equilibrium. Here we see the distinct difference embedding the economic growth model in a physical environment makes - population and capital cannot grow indefinitely.

Nonetheless, the outcome of the model under these conditions is very positive. If economic growth is even and wealth is reasonably distributed, the economy settles down

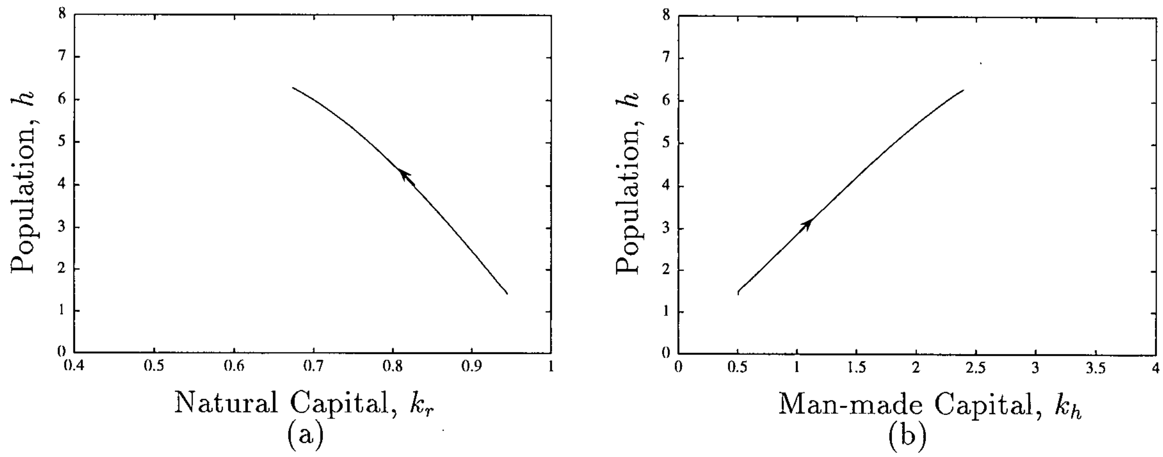


Figure 5.6: Graph (a) shows h versus k_r. Graph (b) shows h versus k_h.

to a steady state with each individual enjoying a high standard of living. The population equilibrates at a little over 6 people per (cultivated) hectare, with natural capital at about 65 % of the maximum. Figure 5.7 shows the evolution of capital, labor, and consumption over time.

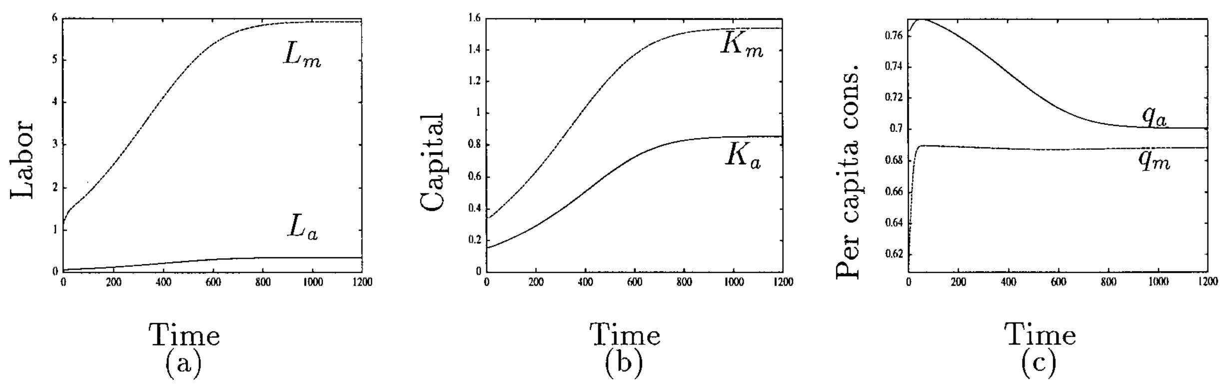


Figure 5.7: Graphs (a) and (b) show the distribution of labor and capital to agriculture and manufactuing respectively. Graph (c) shows the per capita consumption of manufactured and agricultural goods over time.

The bulk of the labor and capital are directed towards non farm business, consistent with what would be observed in a modern economy. The population consumes around 0.7 units of agricultural goods and manufactured goods respectively, both above their minimum values -i.e. life is quite good.

Now suppose we reduce b_c. Figure 5.8 is a bifurcation diagram showing the effect this has on the model. As b_c is reduced, a sub-critical Hopf bifurcation occurs at $b_c \approx 1.5$ Below this point the steady state is unstable, and the system undergoes large amplitude oscillations. This is to say that if the system begins from an initial condition with a value of b_c below 1.5, there is a barrier that precludes the system from arriving at a "sustainable state".

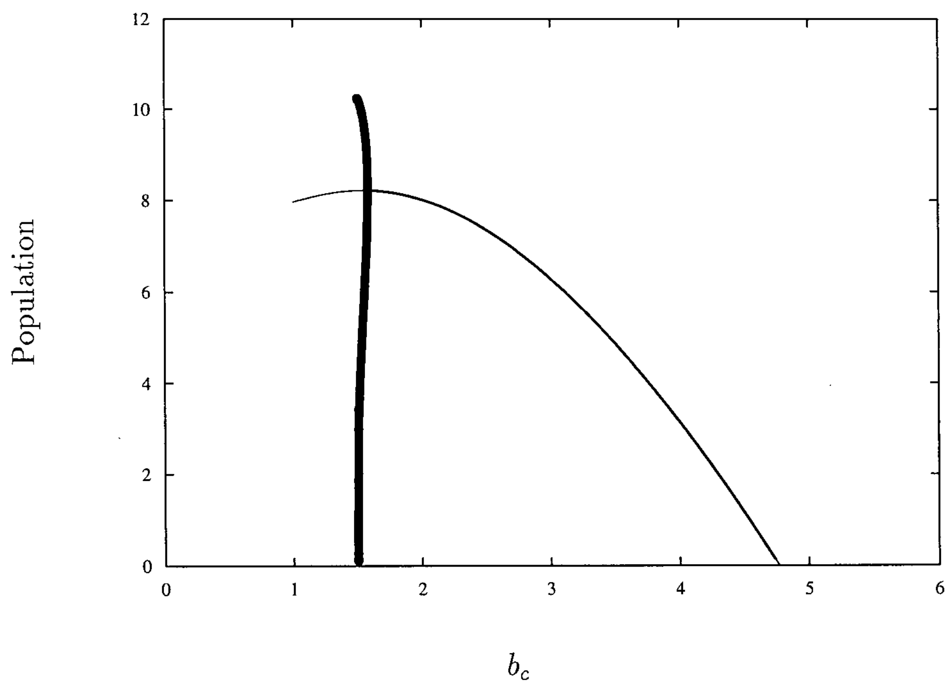


Figure 5.8: Bifurcation diagram for simplified model.

It turns out that there is an explicit relationship between investment, evenness of economic growth and distribution of wealth, and system stability that we can elucidate by performing a two-parameter continuation with b_c and c_i. Figure 5.9 is the result. For combinations of c_i and b_c in the region below the bifurcation boundary (more even development and wealth distribution for a given level of investment) there is always an **attainable** sustainable state. For combinations of c_i and b_c in the region above the bifurcation boundary (less even development wealth distribution for a given level of investment) the steady state is **unattainable**. The steady state is surrounded by a stable limit cycle which forms a boundary between any initial state outside the limit cycle and a sustainable economy.

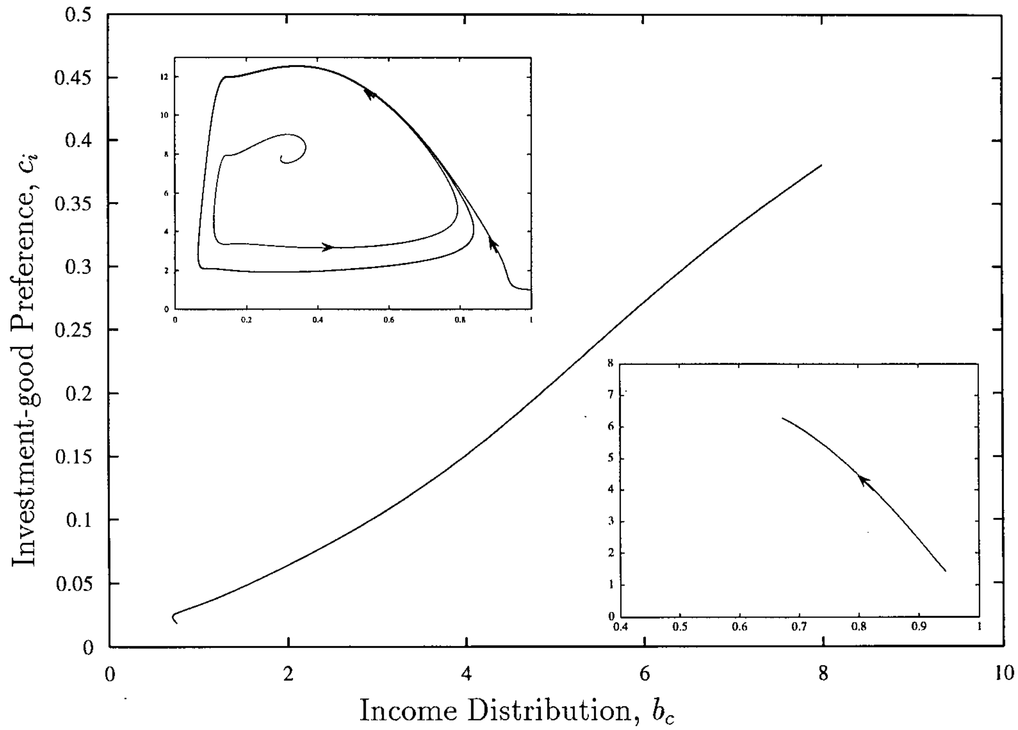


Figure 5.9: Change in dynamics as the bifurcation boundary is crossed. The system goes to a stable equilibrium (sustainable economy for parameter values to the right and below the curve (lower investment and better income distribution). For parameter combinations above and to the left , (higher investment and less even economic development and wealth distribution) the system undergoes stable, large amplitude fluctuations.

Figure 5.10 shows the trajectories for the model in phase space for $b_c = 1$, and $c_i = 0.1$ Graph (a) shows the population versus natural capital. As population grows, natural capital is reduced but in this case the population does not come to a steady state.

Instead, after the human population density reaches a maximum, continued increase in capital stocks and efficiency in agricultural production allows the population to be maintained for a short time while natural capital continues to decline. Figure 5.11 shows the evolution of labor, capital and consumption over time. Then we see both labor and capital being shifted out of manufacturing into agriculture in an attempt to maintain agricultural output. This corresponds to the flat portion of the curve in $k_r - h$ phase space on the left in figure 5.10. Increased productivity that accompanies capital growth masks the degradation of natural capital allowing the population to grow far beyond the capacity of the environment to support it. Finally, the population cannot maintain either agricultural or manufacturing output and capital stocks fall as shown in figure 5.10. Notice that in graph (c) in figure 5.11, per capita output of agricultural and manufactured goods are maintained up to the point when the system collapses suggesting that the signals to consumers about environmental degradation through the market system would not be strong enough to cause them to change their habits. Thus the first prediction of the model is that investment must be accompanied by efforts to insure that economic growth is even and and its associated benefits are evenly distributed to have any hope of reaching a "sustainable economy".

There are several other points that could be addressed here. For example how does changing the productivities of labor in agriculture and manufacturing change the structure of the model? One might also argue that the model does not really correctly characterize the nature of the the agricultural sector because it does not take into consideration measures that might preserve natural capital. On the other hand, both sectors are perfectly non-polluting. Also the manufacturing sector has a constant efficiency which does not capture the negative effects of dwindling resource supplies or the positive effects of innovation. Are the model predictions of any value then?

I believe so. The model predictions relate to a general phenomenon that transcends

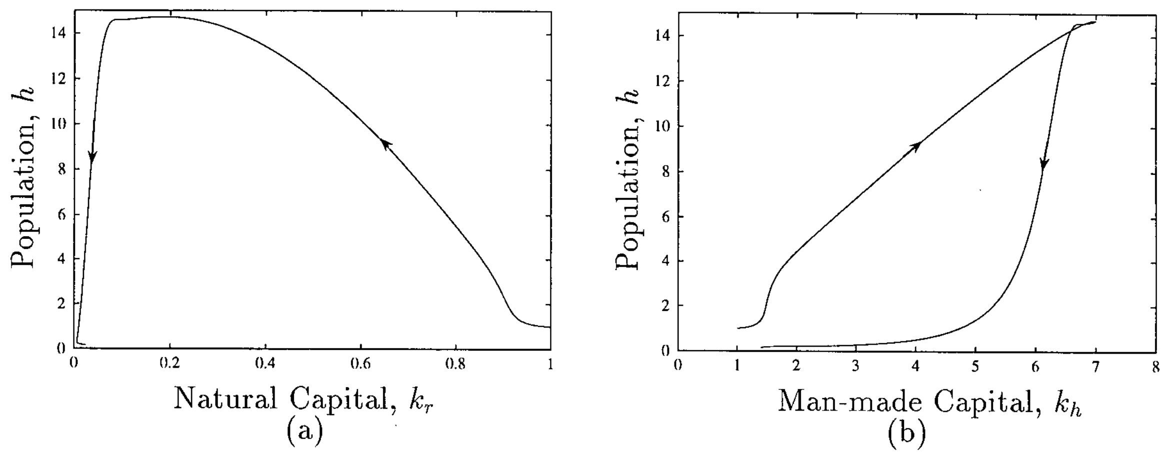


Figure 5.10: Graph (a) shows h versus K_r. Graph (b) shows h versus K_m.

the actual assumptions about the organization of a particular social system. That phenomenon is when the society can no longer bear *increased complexity* and must necessarily collapse. As Joseph Tainter [63] puts it, the marginal benefits of increased complexity approach zero. In our simplified model, as the society increases in complexity (manufactured capital increases) it receives positive benefits in terms of improved standard of living. If, however, the society moves into a position where it can no longer maintain the complex structure it has created, it becomes a burden and may cause the society to collapse. In our simple model, this occurs when all capital and labor is shifted into agriculture in an attempt to feed the population. When this occurs, capital stocks are neglected and decay - i.e. the society can no longer maintain its complex structure.

The point is, in one case increasing complexity leads to a sustainable economic ecological system and in the other case, increasing complexity leads to collapse. This emphasizes the important role that evenness of economic development and the management of the benefits of increased complexity play in the evolution of an economy. In *Collapse of Complex Societies* [63], Joseph Tainter describes several societies that he believes went

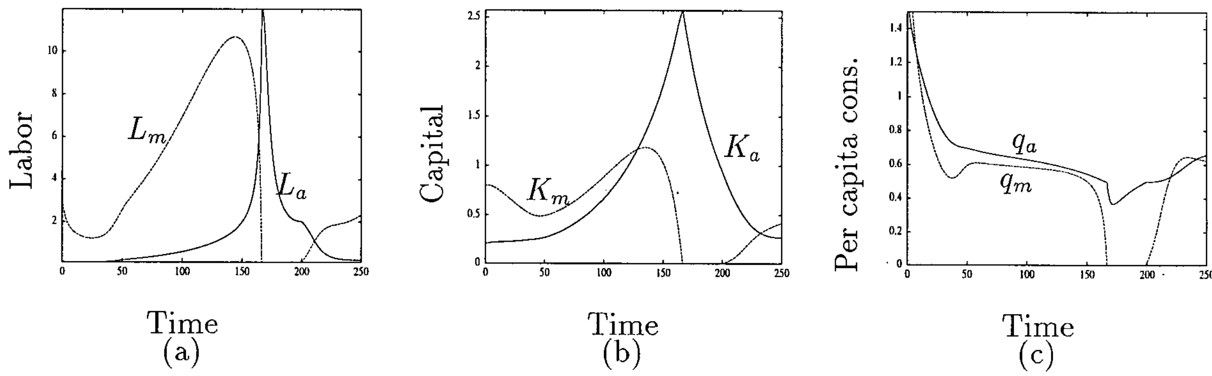


Figure 5.11: Graphs (a) and (b) show the distribution of labor and capital to agriculture and manufactuing respectively. Graph (c) shows the per capita consumption of manufactured and agricultural goods over time.

through a process of increasing societal complexity reaching a point where this increasing complexity became a burden and forced the society to collapse. Perhaps how well these societies managed the benefits of increased complexity is related to their subsequent collapse. The full model given by equations 5.40 can help explore this idea further.

5.4.2 Nonrenewable natural capital, efficiency, and flows between industries

In the previous example, it was assumed that the depletion of the nonrenewable natural capital had no effect on manufacturing efficiency which was assumed constant. It was also assumed in the previous example that neither industry relied on output from the other, i.e. there were no interindustry transfers of goods and services. Finally, the efficiency of agricultural output was modeled as a linear function of the renewable natural capital stock. In this section these unrealistic assumptions are relaxed. First, resource scarcity is explicitly modeled by making the parameters $e_{kn,m}$, and $e_{kn,mr}$ nonzero. The dynamics of the model are then explored under different assumptions about how society responds to resource shortages. Next, the effect of the relationship between natural capital stocks and the efficiency of production in the two sectors on the model is explored in more detail. Finally, the role of interindustry transfers (i.e. the dependence of agriculture on a flow of manufactured goods and services) on the model is investigated.

First, consider the role of nonrenewable natural capital depletion as modeled by equation 5.40d. At equilibrium, we must have

$$hq_r = \frac{e_{kn,m}}{e_{kn,r}} Y_m. \tag{5.49}$$

Since the amount of manufacturing output devoted to maintaining nonrenewable natural capital stocks (through such activities as exploration and technological development) is a fraction of the total output Y_m, the ratio $\frac{e_{kn,m}}{e_{kn,r}}$ must be less than 1. This simply means that the output used to find new nonrenewable resources has to more than replace those used in producing that output.

The next question is how society allocates output to the activity of generating new nonrenewable natural capital stocks. A simple way to model this process is to let the preference for resource goods increase as these stocks become more scarce. A reasonable function representing this relationship is

$$c_r = \frac{1 - c_a - c_i}{\lambda_{kn} k_n + 1}. \tag{5.50}$$

As resources become more scarce, society shifts its preference for consumption of goods and services to replacing sources of raw materials. Since the preferences must add up to one, the maximum value of c_r is $1 - c_a - c_i$, the preference "remainder" after food and investment needs are met. λ_{kn} is a measure of how responsive society is to resource shortages. Figure 5.12 depicts the relationship between k_n and c_r for different values of λ_{kn}. The lower λ_{kn}, the more responsive the society is to raw material shortages. If λ_{kn} is large, society will not devote output to replacing raw material stocks until the actual stock is quite low.

Finally, before exploring the implications of resource scarcity on the model, the dependence of the efficiency of the manufacturing and agricultural sectors on resource stocks

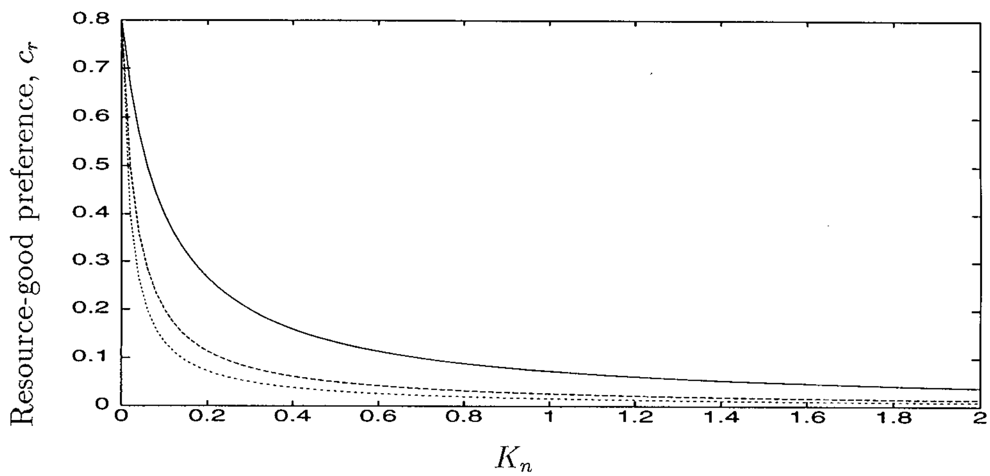


Figure 5.12: Resource good preference versus K_n for different values of λ_{kn}. From top to bottom, the values for λ_{kn} are 10, 30, and 50.

must be modeled. Above a certain level, the relative abundance of raw materials has little effect on manufacturing efficiency because only a small portion of total economic output must be directed towards their procurement. As they become more scarce, more economic output must be directed towards obtaining raw materials which reduces the overall efficiency of the production process. A simple function that captures this effect is

$$E_m(k_n) = \frac{k_n}{k_n + \overline{k}_n} \tag{5.51}$$

where $\overline{k}_n$ is the resource level at which efficiency is half the maximum. A similar functional form is used for productivity in agriculture, but is scaled so that when $k_r = 1$, $E_r(k_r) = 10$. The result is

$$E_a(k_r) = \frac{10k_r(1 + \overline{k}_r)}{k_r + \overline{k}_r}. \tag{5.52}$$

Figure 5.13 illustrates the form of these relationships. Graph (a) shows the manufacturing efficiency for $\overline{k}_n = 0.1$. Efficiency is mildly reduced until $k_n = 0.5$ (one-half of the original endowment) after which it falls off rapidly. Graph (b) shows the analogous relationship between E_r and k_r for different values of $\overline{k}_r$. In the following example,

$\overline{k}_r = 1$, $\overline{k}_n = 0.1$. This choice is arbitrary, with the only motivation being to capture the effects of nonlinearities in efficiency that are consistent with common sense. The effects of these parameters on the structure of the model are addressed in the next section where the full model is analyzed.

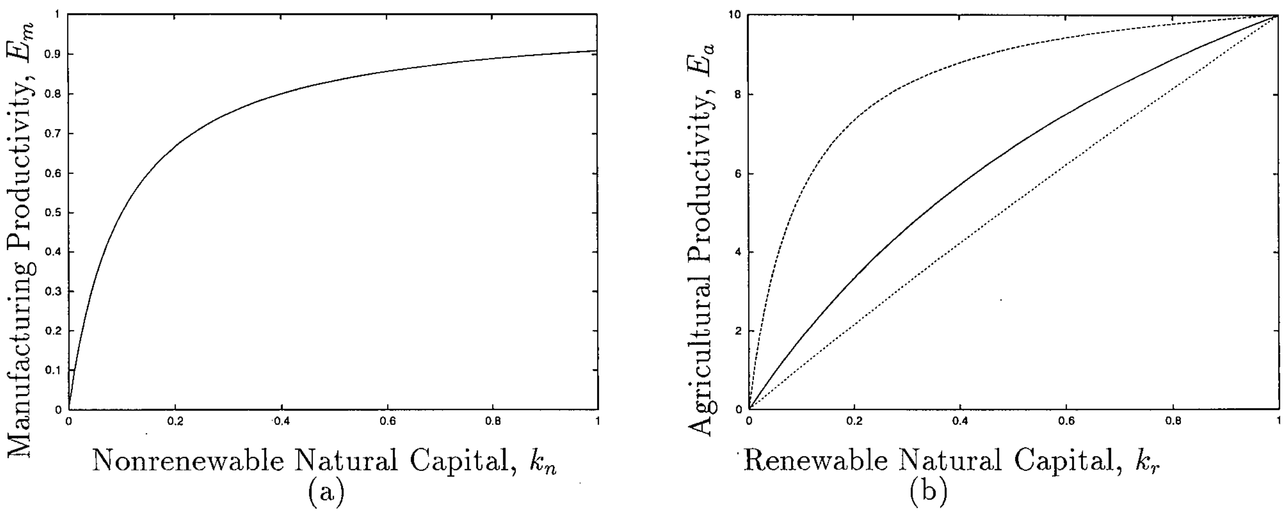


Figure 5.13: Graph (a) shows E_m versus k_n with $\bar{k}_n = 0.1$. Graph (b) shows E_r versus k_r for three different values of $\bar{k}_r$: 10, 1, 0.1 with decreasing values corresponding to increased curvature.

Nonrenewable Natural Capital

Here it is assumed that $e_{kn,m} = 0.01$, $e_{kn,r} = 0.1$, and $b_c = 3$. In this analysis, the assumption of no interindustry transfers is maintained. The dynamical system analyzed in this section is given by equations 5.42 appended with the expression for nonrenewable natural capital,

$$\frac{dk_n}{dt} = -0.01Y_m + 0.1hq_r. \tag{5.53}$$

Also, now that $c_r \neq 0$, the per capita consumption equations given by 5.48 must be appended with an expression for q_r:

$$q_r = \begin{cases} \frac{c_r I_s}{P_m} & I_s \geq 0 \\ 0 & \text{otherwise,} \end{cases} \tag{5.54}$$

where

$$c_r = \frac{0.9}{\lambda_{kn} k_n + 1}. \tag{5.55}$$

Finally, using the definitions of $E_m(k_n)$, and $E_a(k_r)$ given by equations 5.51 and 5.52, equations 5.43, 5.45 and 5.46 are replaced by

$$K_a = \begin{cases} 0.296hP_a + 0.156k_h - 0.0031hP_m & K_a < k_a \\ k_h & \text{otherwise} \end{cases} \tag{5.56a}$$

$$K_m = k_h - K_a, \tag{5.56b}$$

and

$$Y_a = \frac{15.52k_r w^{-0.3}}{1 + k_r} \qquad Y_m = \frac{3.03k_n w^{-0.8}}{0.1 + k_n} \tag{5.57}$$

$$P_a = \frac{0.092(1 + k_r)w^{0.3}}{k_r} \qquad P_m = \frac{1.649(0.1 + k_n)w^{0.8}}{k_n}. \tag{5.58}$$

Figure 5.14 shows the state variable trajectories for the case for $\lambda_{kn} = 10$. This corresponds to the society being relatively responsive to resource shortages and the raw material replacement process being able to generate ten times the raw materials it consumes. As long as society devotes economic output to replacing raw material stocks, the economic system can reach a sustainable steady state $(h, k_r, k_h, k_n) \approx (8, 0.6, 1.6, 0.68)$. The economic system is still subject to the problem of over-exploiting renewable natural capital and collapsing. The problem introduced by nonrenewable natural capital occurs when investment is too low, or stocks are allowed to dwindle to a low level before efforts are made to replace them (high value for λ_{kn}).

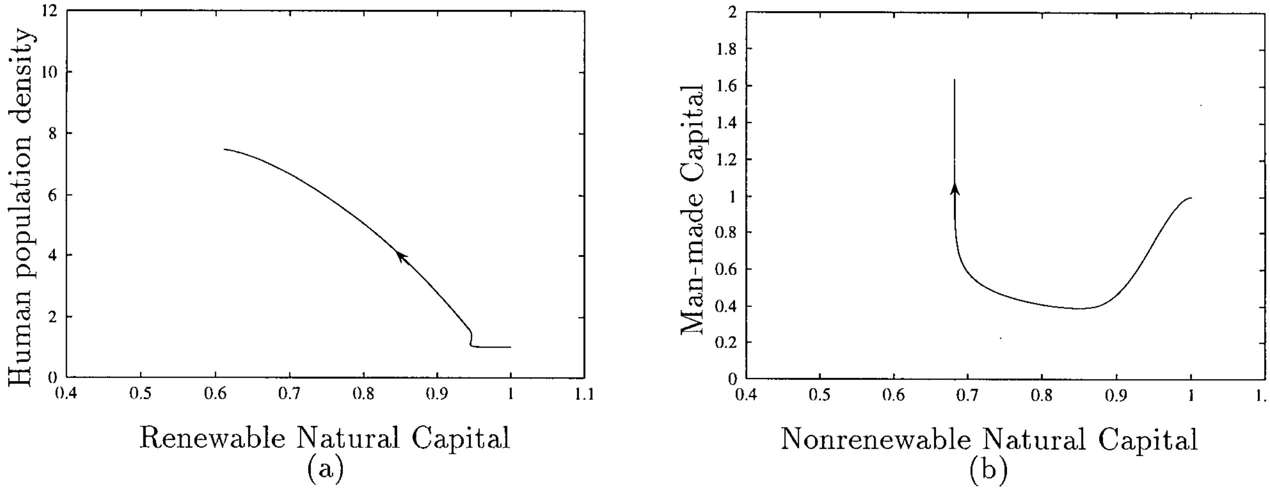


Figure 5.14: Graph (a) shows human population versus renewable natural capital. Graph (b) shows man-made capital versus nonrenewable natural capital.

Notice in figure (b) how nonrenewable natural capital is transformed into man-made capital as the economy develops. Once the economy is sufficiently developed, new sources of raw materials are being found (via improvements in efficiency, using new materials, using materials in new ways, etc) as fast as they are used in the production of goods and services. After this point, nonrenewable natural capital remains constant as the economy continues to develop towards its final state. If λ_{kn} is large, the situation is different. Figure 5.15 shows the equilibrium human population and man-made capital levels for different values of λ_{kn}.

As long as λ_{kn} is below about 45, the economy will reach a sustainable stable equilibrium state. As λ_{kn} is increased, equilibrium values of man-made capital decreases because society waits too long before addressing resource scarcity. When it finally does, manufacturing efficiency is low, more economic output must be directed towards maintaining raw material flows, and less can be directed to increasing man-made capital stocks. In this case the economy begins to develop just as with low levels of λ_{kn} but reaches a level

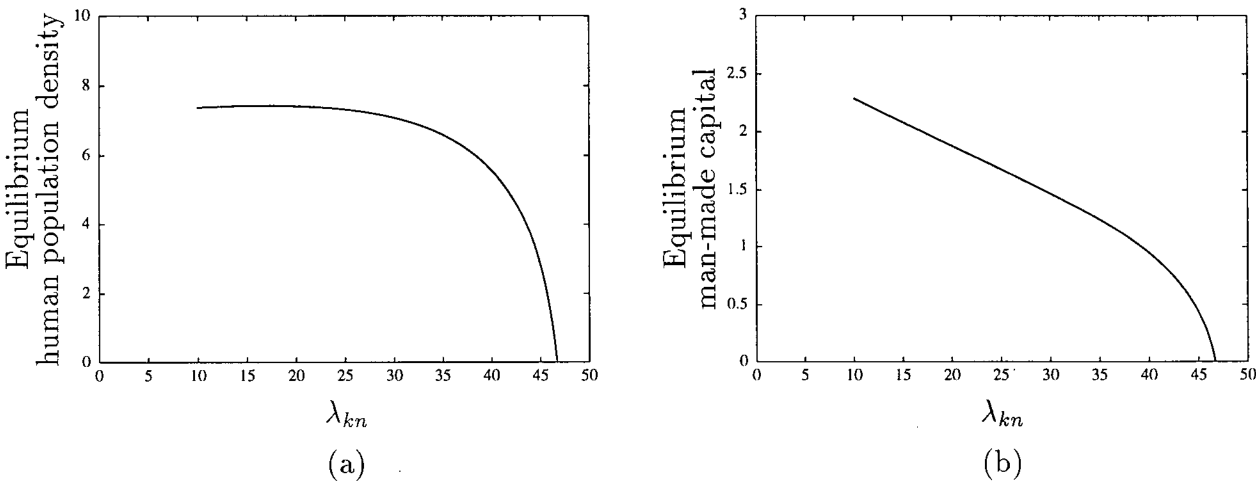
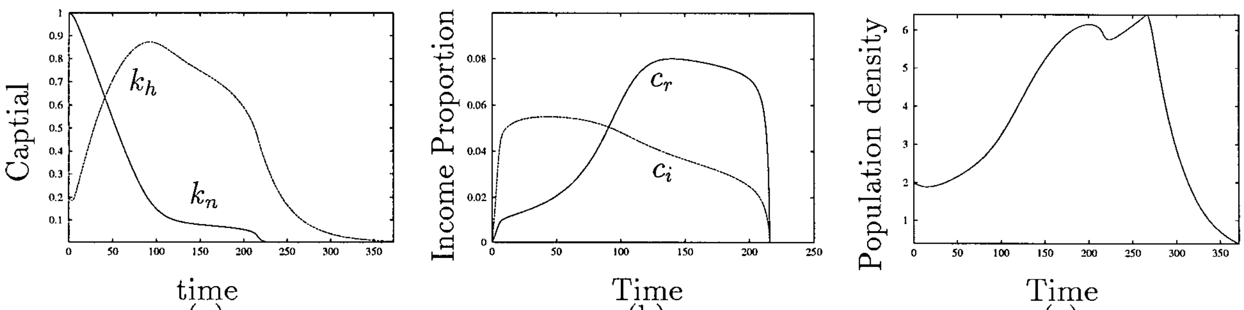


Figure 5.15: Graph (a) shows the stable equilibrium human population versus λ_{kn}. Graph (b) shows the stable equilibrium man-made capital versus λ_{kn}.

of complexity where it can no longer maintain agricultural and manufacturing output as well as look for new sources of raw materials. Figure 5.16 shows the transient dynamics for $\lambda_{kn} = 60$, and $c_i = 0.07$.

Graph (a) shows the evolution of man-made and nonrenewable natural capital over time. As with the previous example, nonrenewable natural capital is depleted as it is transformed into man-made capital. Here however, nonrenewable natural capital stocks are quite low (around 0.1 versus 0.7 in the example with $\lambda_{kn} = 10$) before society responds and begins to replace these stocks (around $t = 100$). Between $t = 100$ and $t = 200$ nonrenewable natural capital stocks are maintained by directing more economic output towards their replacement at the expense of new investment (as well as consumption but to a lesser degree) as shown in graph (b). The problem is that the effort to find replacements for nonrenewable natural capital stocks comes too late. At around $t = 225$, the cost of maintaining economic infrastructure, feeding the population, and replacing nonrenewable natural capital becomes to high for society to bear. All remaining factors



(a) (b) (c)

Figure 5.16: Graph (a) shows man-made and nonrenewable natural capital over time. Graph (b) shows resource and investment-good preferences over time. Graph (c) shows the human population density over time.

of production are then directed to feeding the population which is maintained for another 50 years and then the populations crashes as shown in graph (c).

As with the model where overexploitation of renewable natural capital was the cause of collapse, here we have a period of economic development by which the economic-ecological system reaches a bottleneck. Society attempts to negotiate the bottleneck by changing economic structure, but subsequently collapses. In the first case, economic development proceeds to a point where flows from renewable natural capital are insufficient to maintain the structure of the system. This "road to collapse" sets an upper bound on investment. In the second case, it is lack of flows from man-made capital that ultimately causes collapse. This "road to collapse" sets a lower bound on investment. The higher λ_{kn}, the higher the level of investment required to develop economic infrastructure to cope with resource scarcity *before it is too late.* This increased investment, on the other hand, might cause collapse due to natural capital overexploitation. These facts pose an interesting problem for a developing economy: there is a safe window of investment below which non-renewable natural scarcity poses the greatest threat to achieving sustainability and above which, overexploitation of renewable natural capital is the limiting factor.

The problem of finding the appropriate window to grow fast enough to overcome limitations in man made capital yet slow enough to avoid destroying natural capital is illustrated in figure 5.17.

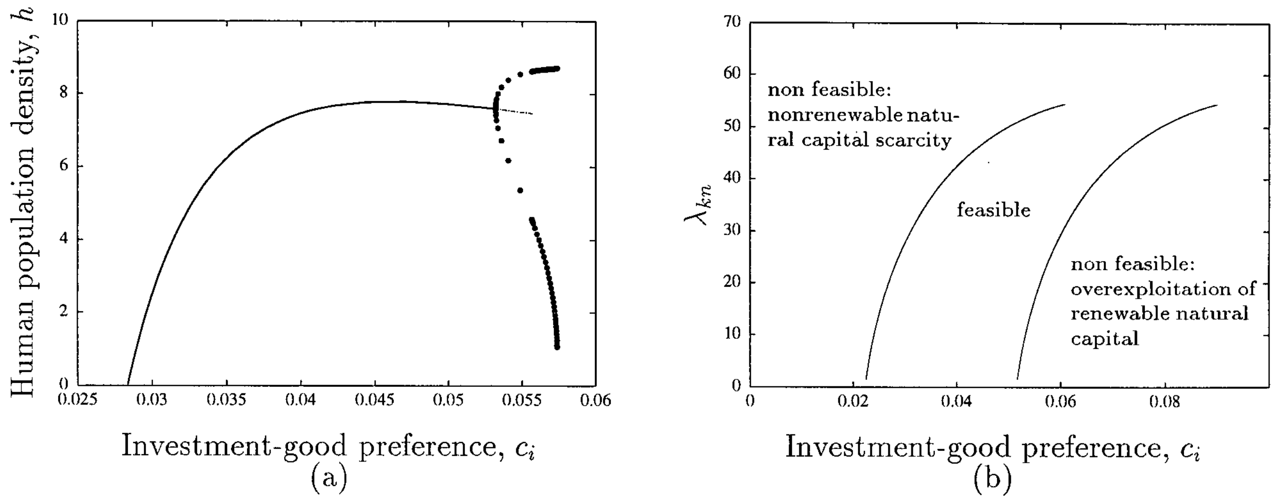


Figure 5.17: Graph (a) shows the bifurcation structure for $\lambda_{kn} = 10$. Graph (b) is the two parameter bifurcation diagram for λ_{kn} versus investment good preference.

Graph (a) shows the bifurcation structure for $\lambda_{kn} = 1$, i.e. society is relatively responsive to resource shortages. The window of feasible investment-good preference is quite narrow. The economy will evolve to a sustainable steady state if investment good preference is between 0.028 and 0.053. Investment good preferences outside this range will give rise to an economic development path that leads to collapse due to resource shortages or overexploitation of natural capital respectively. Graph (b) shows the dependence of this result on the responsiveness of society to resource shortages. The curve on the right depicts all the combinations of λ_{kn} and investment-good preference for which a Hopf bifurcation occurs. For a given λ_{kn} the corresponding value for investment-good preference is an upper bound for the feasible level of investment-good preference that will lead to a sustainable steady state economic ecological system. The curve on the left is the corresponding lower bound for investment-good preference to prevent resource shortages.

The region between these two curves defines the feasible region of investment-good preferences that will lead to a sustainable economy. Given that the range of possible values for investment-good preferences is from 0 to $1 - c_a$ (=0.95 in the example above), the width of the feasible region (about 0.025 in the example above) is quite narrow. Of course, these numbers should not be taken as representative of those a modern economy might face, but in the context of the model, they do indicate that the possibility of attaining a sustainable economic ecological system may be very sensitive to investment patterns.

Efficiency and feasible investment patterns

The nature of the relationship between investment patterns and feasible paths can depend on many things. Two key aspects of the model that affect this relationship are the relationships between efficiency and capital stocks and the transfer of goods between industries. In the above example, recall that $\overline{k}_n = 0.1$, and $\overline{k}_r = 1$. A low value like this for $\overline{k}_n$ corresponds to the fact that if an economy has a stock of raw materials available for productive activities, the size of that stock does not affect these activities until it is reduced to a level where some portion of productive capacity must be diverted to maintaining the stock. The lower $\overline{k}_n$, the more dramatic this transition. The significance of the relative nonlinearity in the relationship between k_r and E_a is more difficult to imagine. It could correspond roughly to the idea of ecosystem resilience. If an ecosystem is not resilient, productivity would decline rapidly due to agricultural disturbances (high value for $\overline{k}_r$). If an ecosystem is resilient, it might remain fairly productive even with a high level of disturbance, but break down more rapidly after some threshold level of disturbance is surpassed. The question is, how do different values for $\overline{k}_n$ and $\overline{k}_r$ affect the results shown in figure 5.17?

To investigate this, the model is analyzed by fixing $\lambda_{k_n} = 10$ and varying $\overline{k}_n$, and $\overline{k}_r$, leaving the rest of the model assumptions unchanged from the previous section. Thus, we now have

$$c_r = \frac{0.9}{10k_n + 1}, \tag{5.59}$$

and

$$Y_a = \frac{7.76k_r(1+\overline{k}_r)w^{-0.3}}{\overline{k}_r + k_r} \qquad Y_m = \frac{3.03k_n w^{-0.8}}{k_n + \overline{k}_n} \tag{5.60}$$

$$P_a = \frac{0.184(\overline{k}_r + k_r)w^{0.3}}{k_r(1+\overline{k}_r)} \qquad P_m = \frac{1.649(\overline{k}_n + k_n)w^{0.8}}{k_n}. \tag{5.61}$$

It turns out that increasing $\overline{k}_n$ shifts the feasible region to the right but does not significantly affect the width of the region. This is consistent with intuition: increasing $\overline{k}_n$ makes manufacturing efficiency more sensitive to resource shortages requiring more investment to avoid them. Also, reduced efficiency associated with increased $\overline{k}_n$ puts a drag on the economy slowing the growth process. This allows for a higher level of investment without overexploiting renewable natural capital. Thus both the minimum and maximum feasible values for investment-good preference are increased, shifting the feasible region to the right.

The model is much more sensitive to $\overline{k}_r$. This sensitivity is illustrated in figure 5.18 which shows a two parameter bifurcation diagram for investment-good preference versus $\overline{k}_r$. As ecosystems become less resilient (higher $\overline{k}_r$) , the system can tolerate more investment. This seems a bit counter intuitive, but is similar in nature to the Tsembaga model where increased productivity of renewable natural capital had a stabilizing tendency.

The key is that the feedback from ecosystems is stronger if they are less resilient. Unlike $\overline{k}_n$, increasing $\overline{k}_r$ widens the feasible range. For $\overline{k}_r = 10$ and $\overline{k}_n = 0.1$ the feasible values for investment-good preference lie between 0.028 and 0.082, about double the range for the case with $\overline{k}_r = 1$. As $\overline{k}_r$ is reduced, ecosystems remain productive at higher levels

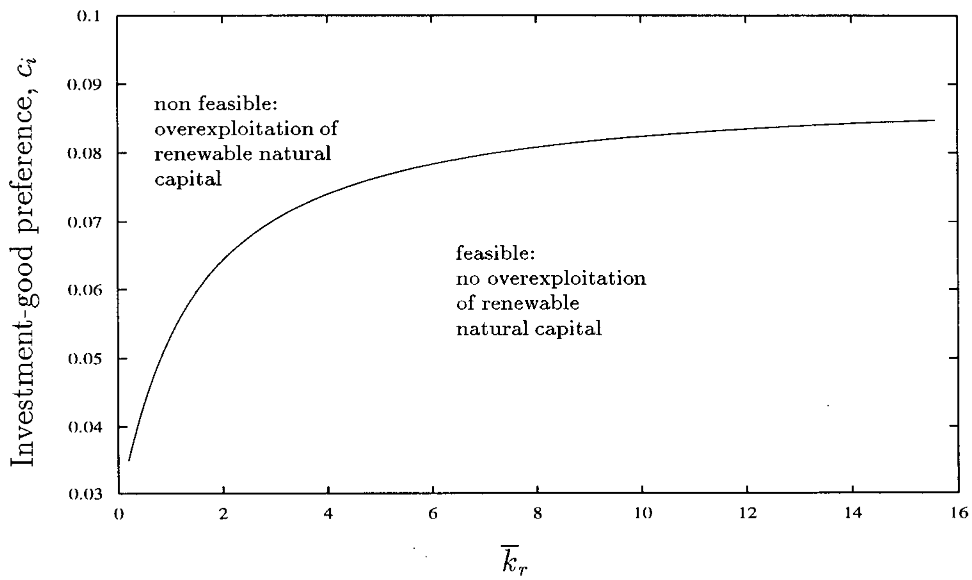


Figure 5.18: Two parameter bifurcation digram for investment-good preference and $\bar{k}_r$.

of agricultural disturbance. This weakens the feedback from natural systems and allows the human economic system to develop beyond the capacity of ecosystem to support it. Thus the more resilient ecosystems are, the more likely it is for human economic systems develop into situations from which they cannot extricate themselves. Thus the human propensity to try to fix things through attempting to increase productivity may be the worst development strategy possible.

The effect of interindustry transfers

The final aspect of the model that we address in this section is the role of interindustry transfers. In the previous examples, each industry was assumed to operate independently of the other. Neither sector relied on the other for raw material inputs. This is unrealistic for modern agriculture which relies heavily on manufactured products, most notably chemicals. Similarly, the manufacturing sector relies on fibers from the agricultural sector. In order to study the effects of interindustry transfers, we examine the effect that the parameters β_N, β_{half}, and R_{am} have on the model. All other parameters are fixed and the model assumptions remain unchanged from previous sections, i.e. the dynamical system is given by equations 5.42 and equation 5.53, optimal consumption by equations 5.48 and 5.54, output by 5.57, labor by 5.44, income by 5.47, and resource-good preference by 5.59. Because R_{am} and R_{ma} are not zero, no simplifications occur for the optimal capital and price levels. The full equations for the optimal capital and price levels given by 5.37 and 5.34, respectively, must be used.

Recall that β_N measures the quantity of nutrient inputs required per unit of agricultural output (a unit conversion factor) while β_{half} measures the productivity of natural capital. As β_{half} is increased, the higher the ratio of $\frac{h}{k_r}$ can be before nutrients produced by biological processes are no longer sufficient to meet demand. It turns out that the effect of material transfers from manufacturing to agriculture has a stabilizing effect. This is illustrated by the two parameter bifurcation diagram in figure 5.19 with $\beta_{half} = 6$ (meaning as population density per hectare approaches a typical value for a modern industrial economy, depending on the level of degradation of natural capital, a substantial amount of manufactured inputs would be required to meet food demand). As β_N increases, there is more pressure on the manufacturing sector which allows for increased investment without overexploiting renewable natural capital. Again, the harder natural capital is to exploit, the more stable the model.

Interestingly, changing β_N does not affect the minimum investment level necessary to avoid raw material shortages in the manufacturing sector. For example for $c_i = 0.05$ and $\beta_N = 0.1$, the feasible window for investment-good preference is 0.02838 to 0.1034. For $\beta_N = 0.2$, the feasible window for investment good preference is 0.02838 to 0.2462. This result is slightly counterintuitive. One would think that increased demand for manufactured goods in the agricultural sector would divert productive capacity away from investment and nonrenewable natural capital replacement. Avoiding resource shortages

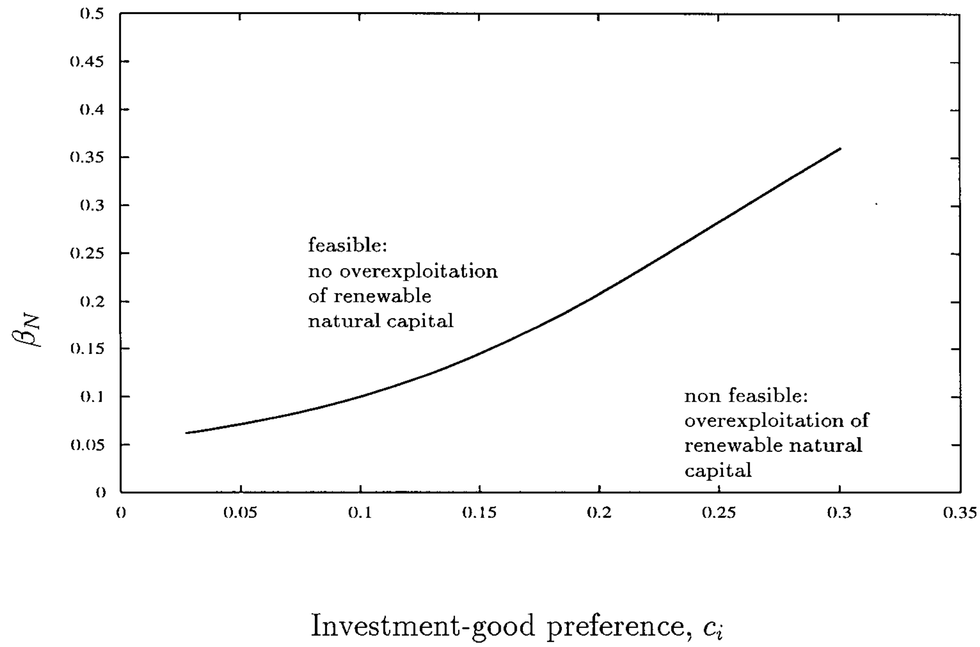


Figure 5.19: Two parameter bifurcation digram for investment-good preference and β_N.

would then require a higher investment-good preference. The reason why this is not the case is related to the pattern of economic growth associated with different values of β_N.

For each of the cases above, the equilibrium levels of per capita output of goods and services are very similar with $q_a = 0.64$, $q_m = 0.48$, $q_i = 0.024$, and $q_i = 0.06$ which translates into 16.6, 71, 3.6, and 8 percent of income spent on food, consumption, investment, and nonrenewable resource replacement respectively. What does change is the equilibrium levels of the state variables with $(h, k_h, k_n, k_r) = (5.199, 1.484, 0.632, 0.789)$ for $\beta_N = 0.1$ and $(h, k_h, k_n, k_r) = (3.93, 1.119, 0.627, 0.852)$ for $\beta_N = 0.2$. For larger values of β_N, equilibrium population and man made capital levels are lower, the renewable natural capital level is higher, and the non renewable natural capital level is almost unchanged. During the initial growth period of the economy, the increased price of food due to inputs from the manufacturing sector causes consumers to shift spending away from food. The lower food intake slows population growth slightly which, in turn, slows man-made capital growth. The overall growth of the economy is slowed so it equilibrates with a smaller human population and man-made capital stock. The result is that the scale of the final economy is smaller, putting less pressure on both non-renewable and renewable natural capital stocks. Thus the lower bound for feasible investment remains unchanged while the upper bound increases.

It is interesting how the two cases above which differ only very slightly in terms of their development over time and equilibrium economic output differ much more significantly in the equilibrium scale of the economy and levels of state variables. A drag on the economy that slows economic growth, which is often considered bad, may in the long run produce the same economic outcome as faster growth. The only difference is that the final scale of the slower growing economy is smaller, and the quality of renewable natural capital higher. If the state of the natural environment is related to quality of life, then the slower growing economy produces the better end result. This should be a major concern when considering how policy affects economic growth.

Next, we turn our attention to the role that transfers from the agricultural to the manufacturing sector have on the model. These transfers simply put more pressure on renewable natural capital for a given level of economic output. Figure 5.20 illustrates the relationship between the minimum and maximum feasible investment-good preference and R_{am}.

The maximum feasible investment-good preference is more sensitive to increases in R_{am} than is the minimum. This causes the feasible region to narrow as R_{am} is increased. Thus the more taxing the manufacturing sector is on the agricultural sector, the smaller the feasible investment region and the more difficult achieving sustainability is. For example, the model predicts that our reliance on paper products and wood fiber for use in the manufacturing sector may significantly reduce the range of feasible investment for our economy.

Another important aspect of the manufacturing industry is the pollution it generates.

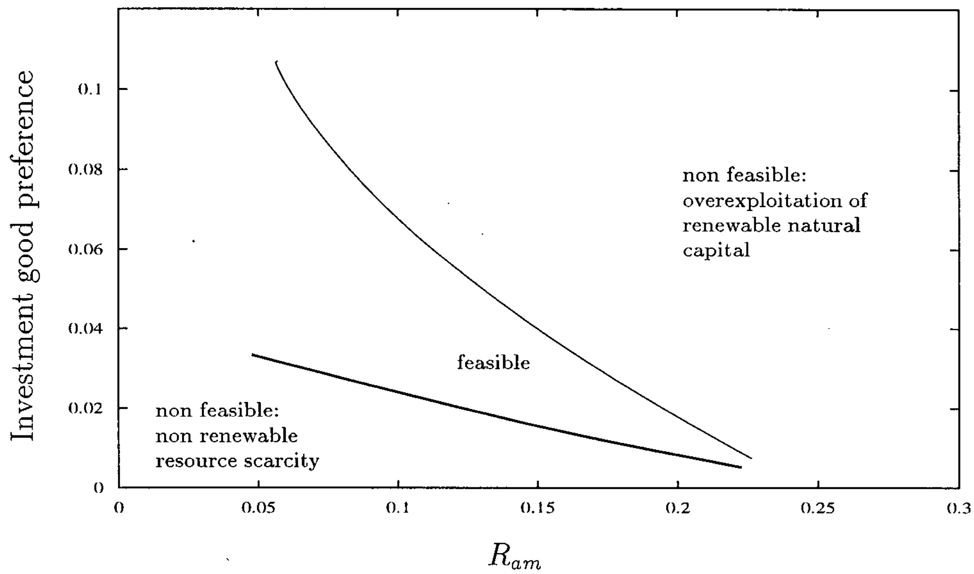


Figure 5.20: Two parameter bifurcation digram for investment good preference and R_{am}.

Although I have not addressed pollution directly (eg. as a state variable) its effect on the dynamics of the system can be studied indirectly. One key aspect of pollution in an ecological system is its negative effect on the operation of ecosystems. This can be modeled as a reduction in renewable natural capital associated with economic activity. This is similar to the effect R_{am} has on the economy - when manufacturing puts increased pressure on renewable natural capital, whether by compromising its operation through contamination or direct removal of nutrients, attaining sustainability is made more difficult.

5.5 Conclusions

In this chapter we have developed and studied the dynamics of a model for a two sector ecological economic system. The main results of this modelling exercise are that increases in efficiency (or more generally, productivity) do not necessarily increase the likelihood that a human ecological economic system can attain a sustainable state. Increasing productivity through capital growth (increased investment), and increasing the efficiency of the utilization of nonrenewable resources both make achieving a sustainable state less likely. This and the similar result in the Tsembaga model are mounting evidence that the answer to the first question posed in the introduction is "No". Our ability to solve problems is not necessarily a good thing.

Next, cultural parameters, like in the case of the Tsembaga, do play a key role in achieving sustainability. Here, key cultural parameters are investment good preference and how society manages economic growth and distributes its benefits. These results suggest that the answer to the second question posed in the introduction is "Very". Culture is very important in determining whether a human economic system is sustainable. These two points taken together suggest that the requirement for a sustainable ecological system are the right kind of values and cultural institutions, not the right technological fixes.

Finally, recall that nonsubstitutability in consumption is very destabilizing as demonstrated in chapter 4. The two sector model suggests that nonsubstitutability in production, on the other hand, can have both positive and negative impacts on the possibility of achieving a sustainable ecological economic system. Difficulty in finding substitutes for agricultural goods used in manufacturing dramatically reduces the possibility of achieving a sustainable ecological economic system. The possibility of substituting manufactured products for nutrients generated by renewable natural capital can have a stabilizing effect. The mechanism is the fact that diverting output from the manufacturing sector to agriculture can slow overall economic growth.

Several specific points that came to light through the analysis of the two sector model are:

- There is a critical relationship between the level of investment (speed of economic growth) an ecological economic system can tolerate and the evenness of economic growth. If an ecological economic system is to attain a sustainable state, for a certain level of investment, there is a minimum evenness of growth and distribution of wealth that must be maintained. If not, the system will grow beyond a point where the renewable natural capital can renew itself while providing sufficient flows of goods and services to maintain economic complexity, and the system will crash. Thus for a given value of b_c (which measures evenness of economic growth), the possibility of overexploiting renewable natural capital sets an upper bound on feasible levels of investment.

- If an economic system relies on flows of raw materials from non renewable natural capital stocks, there is a minimum level of investment and willingness to address resource shortages in a timely manner to attain a sustainable state. If not, the system will collapse because economic output is insufficient to maintain man-made capital and simultaneously maintain raw material flows. This possibility sets a lower bound on feasible levels of investment.

- The window of feasible levels of investment set by natural capital constraints is affected by the nature or the dependence of efficiency of production on natural capital stocks. If this relationship is highly nonlinear, and efficiency remains relatively high as stocks decline but then declines rapidly when stocks are below a certain threshold level, the window for feasible investment significantly narrows.

- The window of feasible levels of investment set by natural capital constraints is affected by the structure of the economic system. If the agricultural sector relies heavily on inputs from the manufacturing sector, the upper bound for feasible investment increases while the lower bound remains unchanged and the feasible window is widened. If the manufacturing sector relies on the agricultural sector for inputs, pressure on renewable natural capital increases and the feasible investment window is narrowed.

These aspects of the model structure have several interesting policy implications:

- Any policy that affects the rate of economic growth should be assessed as to its affect on the evenness of growth and the distribution of the benefits of that growth. How will the benefits of economic growth affect different segments of the population? Any economic activity that provides benefits from economic growth without the associated societal context associated with that economic growth should be viewed as highly suspect and fundamentally destabilizing. An example might be the green revolution which provides products to enhance agricultural production to groups who live outside the technologically based social structure that produces those goods. The result: potentially improved nutrition and increased birth rates without the increased marginal cost of children or other factors that might reduce birth rates.

- How much can we rely on market signals for resource scarcity? The market may signal shortages, but depending on the relationship between efficiency and resource stocks, the market signal may come too late. This is not due to a failure of the market, but rather to fundamental "unknowability" in the behavior of complex systems.

- Feedback generated by economic activity regarding the health of renewable natural capital stocks may be very weak and this fact must be built in to management policies. Such a scenario corresponds to graph (b) in figure 5.13 for $\overline{k}_r = 0.1$ (highest curvature), which recall was highly destabilizing and narrowed the range of feasible investment-good preference. This type of situation has been receiving more attention with respect to the specific renewable natural capital stock of marine fisheries [41]. Although terrestrial ecosystems are more easily observed than marine ecosystems, they are no less complex. Their artificially maintained productivity masks the continued degradation of agricultural resources due to erosion, loss of soil structure, and contamination, which may eventually cause a crash in productivity similar to what has been witnessed in marine fisheries.

- Any process that puts a drag on economic growth should not be viewed as necessarily bad in terms of the big picture of reaching a sustainable ecological economic system. Indeed, the model predicts that the propensity of humans to view these drags negatively and attempt to remove them through improvements in efficiency is fundamentally destabilizing and may severely reduce our chances of ever achieving a sustainable ecological economic system. This runs directly counter to the argument that increased efficiency will rescue us from ecological disaster. Further, any manufacturing process that puts pressure on renewable natural capital severely restricts the amount of economic growth an ecological system can endure. Thus any argument that proposes increased economic productivity as improving chances for achieving a sustainable ecological economic system without specifically addressing the pressure this economic activity places on ecosystems is flawed.

In this chapter we have studied not sustainable economic growth, but rather, feasible economic growth paths that will lead to a sustainable ecological economic system. The first implies that there is some way to grow sustainably (such as through environmentally friendly consumption). Admittedly, it seems economic growth is a necessary part of the particular evolutionary trajectory the human race is presently on, but we need economic growth of a very special kind. We need economic growth where the benefits and responsibilities of growth are evenly distributed among the participants in the economic system. Thus the concept of sustainable growth is not very useful. The concept of feasible economic growth paths generated by the two sector model we have studied in this chapter is. Such models help clarify critical relationships that may help in the design of policy to direct future development down such paths. Granted, the work presented here is speculative, but I believe that it is an important step in the right direction. I have only begun to explore the basic structure of the model. There are many directions to go from here to gain more understanding about economic growth in a bounded environment. I outline some directions for future research in the final chapter.

SYMBOL	INTERPRETATION
a_a	Marginal productivity of labor in agriculture
a_m	Marginal productivity of labor in manufacturing
b_a	Marginal productivity of capital in agriculture
b_m	Marginal productivity of capital in manufacturing
$b(\cdot)$	Per-capita birth rate. Depends on per-capita consumption of manufactured goods.
b_0	Maximum per-capita birth rate
b_c	Response of birth rate to per-capita consumption of manufactured goods.
c_a	Agricultural good consumption preference
c_i	Investment good consumption preference
c_m	Manufactured good consumption preference
c_r	Resource good consumption preference
$d(\cdot)$	Per-capita death rate. Depends on per-capita consumption of agricultural goods.
$E_a(\cdot)$	Agricultural sector production efficiency. Depends on renewable natural capital stock, k_r.
$E_m(\cdot)$	Manufacturing sector production efficiency. Depends on non-renewable natural capital stock, k_r.
$e_{i,j}$	Effect (conversion factor) of j-th process on i-th state variable
h	Human population density
I	Per-capita income
I_s	Supernumery per-capita income. (Income left over after basic needs have been met.)
k_h	Man-made capital stock
K_a	Man-made capital devoted to agriculture
K_m	Man-made capital devoted to manufacturing
k_n	Nonrenewable natural capital
k_r	Renewable natural capital
$\overline{k}_n$	Nonrenewable natural capital level at which efficiency is half of the maximum
$\overline{k}_r$	Measure of the nonlinearity in the relationship between renewable natural capital and efficiency in the agricultural sector.
n_r	Intrinsic regeneration rate of renewable natural capital

Table 5.1: Table of important symbols

SYMBOL	INTERPRETATION
P_a	Per-unit price of agricultural goods
P_i	Per-unit price of investment goods
P_m	Per-unit price of manufactured goods
P_r	Per-unit price of resource goods
q_a	Per-capita consumption of agricultural goods
q_i	Per-capita consumption of investment goods
q_m	Per-capita consumption of manufactured goods
q_r	Per-capita consumption of resource goods
q_a^*	Minimum tolerable per-capita consumption of agricultural goods
q_m^*	Minimum tolerable per-capita consumption of manufactured goods
$R_{ma}(\cdot)$	Manufactured goods required per unit of agricultural goods produced
$R_{am}(\cdot)$	Agricultural goods required per unit of manufactured goods produced
r	Per-unit cost of man-made capital
$U(\cdot)$	Utility
w	Per-unit cost of labor (wage rate)
Y_a	Output of agricultural goods
Y_m	Output of manufactured goods
η_a	Man-made capital to labor ratio in agriculture
η_m	Man-made capital to labor ratio in manufacturing
ω	Factor cost ratio
δ	Depreciation rate of Man-made capital
λ_w	Speed of response of wages to differences between labor supply and demand
λ_{kn}	Speed of response of resource-good preference to resource scarcity

Table 5.2: Table of important symbols, continued

Chapter 6

Reflections and future Research

In this thesis I have tried to develop the fundamental idea that the extreme behavioral plasticity of humans can be a fundamentally destabilizing force in the ecosystems they inhabit. It seems that the most stabilizing force is also related to this plasticity; our ability to generate culture and social organizations. For the Tsembaga, this was the ritual cycle. What stabilizing forces are available for modern industrial economies is unclear. What does modern industrial society and its associated culture have to offer to counter its own destabilizing tendencies?

I also tried to put the idea of behavioral plasticity and social structure in the context of neoclassical economic theory by addressing the affects that different assumptions about utility and production have on the evolution of ecological economic systems. I addressed non substitutability in consumption in the Easter Island model and non substitutability in both consumption and production in the two sector model. Finally I attempted to address the relative importance that cultural versus physical parameters play in the evolution of ecological economic systems.

The analysis of these models seem to point in the direction that social organization and cultural practices may be more influential than technical prowess in attaining a sustainable ecological economic system. Recall that if society directs enough economic output to replacing non renewable resources, the system will reach a sustainable equilibrium. This result is in a similar vein as that of Solow [58] and Hartwick [31] in the context of the theory of economic growth. My result is conservative; it assumes that efforts directed towards finding new resources or substitutes and improving efficiency are always successful. The problem in my model of a two sector economy it not too little investment, but rather too much investment and too much efficiency. In this case, social organization and cultural practices must play a role in reaching a sustainable state. They must *offset* destabilizing forces of investment and increasing efficiency.

Critics would argue that the model did not include the possibility of substituting man-made capital for renewable natural capital, the possibility of investing in natural capital, or intergenerational equity. Future research should focus on three main areas:

Simplifying the model

Based on the results of the analysis of the two sector model, we have a good idea of what the most important aspects of the model are, namely the over exploitation of natural capital. If we assume that society invests enough to avoid non renewable natural capital scarcity we can simplify the model considerably. We can drop equation 5.40c. If interindustry transfers could be neglected, this would simplify the model considerably, but we saw the significant effect that transfers from the agricultural sector to the manufacturing sector had on the model. We could retain this aspect of the model by including the negative effects of manufacturing processes on the environment directly rather than through the economic system. The simplification of the economic system would allow the temporary equilibrium wage rate to be computed directly, eliminating the need for equation 5.38. The model would then consist of only three differential equations for which it might be possible to obtain closed form analytical results for feasible investment paths.

Investing in natural capital

What if society set aside a reserve of renewable natural capital? By adding the possibility of society directing some portion of economic output to maintaining such a reserve or enhancing the quality of renewable natural capital being exploited we can explore this question. The idea of maintaining such reserves in fisheries has recently been addressed [41].

Culture versus Social Institutions

Recall that throughout the thesis, behavioral plasticity referred to individuals. At this level, I concluded that behavioral plasticity could be a very destabilizing force. Whether or not the culture of a particular group offsets this destabilizing force is accidental. On the other hand, behavioral plasticity can operate at the group level when a group decides to set up an institution in response to changing environmental conditions with a particular purpose in mind. A very important question is whether social institutions be set up to mediate human environmental interactions even though the underlying culture is destabilizing. For example, can social institutions stop the degradation of an ecosystem inhabited by a group where cultural practices attach social status to hoarding? This question could be addressed by extending the model to include both individual behavior and the behavior modifications induced by institutions.

Optimal economic growth

Given the possibility of investing in renewable natural capital (resource good preference), society would now have the following problem: What is the best set of preferences for consumption, investment, and resource goods and evenness of economic development? This depends on the definition of best. One definition might be a path that would provide the highest per-capita consumption levels over time with the least degraded environment possible. Table 6.3 shows some equilibrium levels of consumption of agricultural and manufactured goods and renewable natural capital for the model with no interindustry transfers. The first line of the table shows that lower levels of b_c, low levels of invest-

b_c	γ	q_a	q_m	K_r
3	0.04	0.636	0.472	0.612
6	0.06	0.832	0.562	0.955
6	0.08	0.849	0.593	0.638
8	0.01	0.990	0.624	0.731

Table 6.3: Equilibrium consumption and renewable natural capital levels versus b_c.

ment seriously degrade renewable natural capital resulting in low equilibrium levels of consumption and natural capital. In this case, people would have low standards of living and to add insult to injury would be living in a degraded environment. With more even economic growth, increased investment is possible resulting in higher standards of living with much better environmental quality as shown on line 2. More is not necessarily better in the case of investment. For $b_c = 6$ increasing investment good preference from 0.06 to 0.08 increases consumption levels but significantly degrades the environment. Thus for a given level of b_c there is in some sense an optimal level of investment.

By increasing both b_c and γ consumption levels can be increased still further and shown on line 4 of the table but to make the model realistic, there would have to some negative aspect of high b_c. This is not difficult to envision looking back on the different economic experiments of this century. It is often argued that the possibility of making it big fosters entrepreneurship which in turn drives improvements in efficiency. If wealth is distributed very equally, there may be no incentive for entrepreneurship. Thus if b_c increased too much and efficiency began to decline, there would be reason to tolerate a certain amount of distributional inequity that would make everyone better off.

In the model, these cultural parameters are constant over the evolution of the system. Certainly, culture changes over time, and an interesting optimal control problem would be to determine the optimal time paths of $b_c(t)$, $\gamma(t)$ and $\nu(t)$. Early in the evolution of an ecological economic system investment in man-made capital may be the most important activity while later, evenness of growth and wealth distribution along with investment in natural capital might be more important to utility maximization. If it were possible to obtain a feedback control for this system, then it could be used to develop optimal future policies given the present state of our system. Given the incredible challenges that lie ahead for the world ecological economic system, I am hopeful that future work in this area might provide some insight into possible means of dealing with them.

Bibliography

[1] C. Ackello-Ogutu, Q. Paris, and W. Williams, *Testing a von liebig crop response function against polynomial specifications*, American Journal of Agricultural Economics, 67 (1985), pp. 873–80.

[2] E. Alden Smith, *Anthropology, evolutionary ecology, and the explanatory limitations of the ecosystem concept*, in The Ecosystem Concept in Anthropology, Westview Press, Boulder, CO, 1982.

[3] J. Anderies, *Culture and human agro-ecosystem dynamics: the tsembaga of new guinea.*, Journal of Theoretical Biology, 192 (1998).

[4] J. M. Anderies, *An adaptive model for predicting !kung reproductive performance: A stochastic dynamic programming approach*, Ethology and Sociobiology, 17 (1996), pp. 221–245.

[5] U. M. Ashcer and R. J. Speteri, *Collocation software for boundary value differential-algebraic equations*, SIAM J. Sci. Comput., 15 (1994), pp. 938–952.

[6] J. H. Ausubel, *The liberation of the environment*, American Scientist, 84 (1996).

[7] S. Authors, *Forum section on resources and economic growth*, Ecological Economics, 22 (1997).

[8] W. Baltensweiler, *The relevance of changes in the composition of larch bud moth populations for the dynamics of its numbers*, in The Dynamics of Populatons, P. V. den Boer and G. R. Gradwell, eds., Wageningen: Center for agricultural publishing and documentation, 1970.

[9] J. A. Brander and M. S. Taylor, *The simple economics of easter island: a ricardo-malthus model of renewable resource use*, American Economic Review, (forthcoming, 1997).

[10] G. Buchbincer, *Nutritional stress and post-contact population decline among the maring of new guinea*, in Malnutrition, Behavior and Social organization, C. S. Greene, ed., Academic Press, 1977.

[11] M. Bulmer, *Theoretical Evolutionary Ecology*, Sinauer Associates, Sunderland, Mass., 1994.

[12] M. Cabeza Gutés, *The concept of weak sustainability*, Ecological Economics, 17 (1996).

[13] P. F. Chapman and F. Roberts, *Metal Resources and Energy*, Butterworths, London, 1983.

[14] J. W. Chung, *Utility and production functions : theory and applications*, Blackwell, 1994.

[15] C. W. Clark, *Mathematical Bioeconomics: The Optimal Management of Renewable Resources*, J. Wiley, New York, 1990.

[16] C. Cleveland, *The direct and indirect use of fossil fuels and electricity in usa agriculture, 1910-1990*, Agriculture, Ecosystems & Environment, 55 (1995), pp. 111–121.

[17] ——, *Resource degradation, technical change, and productivity of energy in u.s. agriculture*, Ecological Economics, 13 (1995), pp. 185–201.

[18] H. E. Daly, *Steady-State Economics : the Economics of Biophysical Equilibrium and Moral Growth*, San Francisco, W. H. Freeman, 1977.

[19] H. E. Daly and K. N. Townsend, *Is the entropy law relevant to the economics of natural resource scarcity? - yes, of course it is!; comment*, Journal of Environmental Economics and Management, 23 (1992), pp. 91–100.

[20] E. Doedel, *A program for the automatic bifurcation analysis of autonomous systems*, Cong. Num., 30 (1981), pp. 265–284.

[21] L. Edelstein-Keshet, *Mathematical models in biology*, Random House, New York, 1988.

[22] J. Ewel; *Litter fall and leaf decompostion in a tropical forest succession in eastern guatmala*, Guatamalan Journal of Ecology, 64 (1986), pp. 293–308.

[23] T. C. Foin and W. G. Davis, *Ritual and self-regulation of the tsembaga maring ecosystem in the new guinea highlands*, Human Ecology, 12 (1984), pp. 385–412.

[24] ——, *Equilibrium and nonequilibrium models in ecological anthropology: an evaluation of 'stability' in maring ecosystems in new guinea*, American Anthropology, 89 (1987), pp. 9–31.

[25] J. France and J. H. M. Thornley, *Mathematical Models in Agriculture*, Butterworths, London, 1984.

[26] H. I. FREEDMAN, *Deterministic Mathematical Models in Population Ecology*, M. Dekker, New York, 1980.

[27] N. GEORGESCU ROEGEN, *Entropy Law and Economic Process.*, Harvard Univ. Press, Cambridge, Mass., 1971.

[28] M. GIAMPIETRO, G. CERRATELLI, AND D. PIMENTEL, *Energy analysis of agricultural ecosystem management*, Agriculture, Ecosystems & Environment, 38 (1992), pp. 219–244.

[29] M. E. GILPIN, *Group Selection in Predator-Prey Communities*, Princeton University Press, Princeton, N.J., 1975.

[30] H. GROSCURTH, R. KUMMEL, AND V. G. W., *Thermodynamic limits to energy optimization*, Energy, 14 (1989), pp. 241–258.

[31] J. M. HARTWICK, *Intergenerational equity and the investing of rents from exhaustible resources*, American Economic Review, 67 (1977), pp. 972–974.

[32] D. F. HEATHFIELD AND S. WIBE, *An Introduction to Cost and Production Functions*, Macmillan Education, 1987.

[33] F. HINTERBERGER, F. LUKS, AND F. SCHMIDT-BLEEK, *Material flows vs. 'natural capital':what makes an economy sustainable?*, Ecological Economics, 23 (1997), pp. 1–14.

[34] C. S. HOLLING, *The functional response of predators to prey density and its role in mimicry and populations regulation.*, Mem. Entomol. Soc. Can., 45 (1965), pp. 1–60.

[35] R. J. HUGGETT, *Modelling the Human Impact on Nature : Systems Analysis of Environmental Problems*, Oxford University Press, Oxford ; New York, 1993.

[36] B. S. JENSEN, *The Dynamic Systems of Basic Economic Growth Models*, Kluwer Academic, 1994.

[37] A. JUO AND A. MANU, *Chemical dynamics in slash-and-burn agriculture*, Agriculture, Ecosystems & Environment, 58 (1996), pp. 49–60.

[38] H. V. KEULEN AND H. D. J. V. HEEMST, *Crop response to the supply of macronutrients*, tech. report, Pudoc, Wageningen, 1982.

[39] I. A. KUZNETSOV, *Elements of applied bifurcation theory*, Springer-Verlag, 1995.

[40] E. LANZER AND Q. PARIS, *A new analytical framework for the fertilization problem*, American Journal of Agricultural Economics, 63 (1981), pp. 93–103.

[41] T. Lauck, C. Clark, M. Mangel, and G. Munro, *Implementing the precautionary principle in fisheries management through marine reserves*, Ecological Applications, 8 (1998), pp. S72–S78. Supplement.

[42] R. M. May, *Stability and Complexity in Model Ecosystems*, Princeton University Press, Princeton, N.J, 1973.

[43] D. H. Meadows et al., *The Limits to growth; a report for the Club of Rome's project on the predicament of mankind*, Universe Books, New York, 1972.

[44] D. H. Meadows, D. Meadows, and J. Randers, *Beyond the limits: Confronting Global Collapse, Envisioning a Sustainable Future*, Chelsea Green Pub. Co., 1992.

[45] E. Odum, *Fundamentals of Ecology*, Saunders, Philidelphia, 1953.

[46] U. D. of Commerce, Economics, S. Administration, and B. of the Census, *Statistical Abstract of the United States*, U.S. Department of Commerce, 1994.

[47] C. Palm, M. Swift, and P. Woomer, *Soil biological dynamics in slash-and-burn agriculture*, Agriculture, Ecosystems & Environment, 58 (1996), pp. 61–74.

[48] Q. Paris and K. Knapp, *Estimation of von liebig response functions*, American Journal of Agricultural Economics, 71 (1989), pp. 178–86.

[49] D. Pimentel, *Ecological systems, natural resources, and food supplies*, in Food and Natrual Resources, D. Pimentel and C. Hall, eds., Academic Press, 1989.

[50] D. Pimentel, *World Soil Erosion and Conservation*, Cambridge University Press, 1993.

[51] D. Pimentel and M. Pimentel, *Food, Energy and Society*, Colorado University Press, Niwot Colorado, 1996.

[52] J. Proops, M. Faber, R. Manstetten, and F. Jost, *Acheiving a sustainable world*, Ecological Economics, 17 (1996).

[53] R. A. Rappaport, *Pigs for the ancestors : ritual in the ecology of a New Guinea people*, Yale University Press, 1968.

[54] W. E. Rees, *Achieving sustainability*, Journal of Planning Literature, 9 (1995).

[55] M. Ruth, *Integrating Economics, Ecology, and Thermodynamics*, Kluwer Academic, Dordrecht ; Boston, 1993.

[56] M. SCOTT, *The importance of old field succession on biomass increments to shifting agriculture*, Rocky Mountain Geological Journal, 6 (1976), pp. 318–327.

[57] S. B. SHANTZIS AND W. W. BEHRENS, *Population control mechanisms in a primitive agricultural society*, in Towards global equilibrium, D. L. Meadows and D. H. Meadows, eds., Wright-Allen Press, 1973.

[58] R. SOLOW, *Intergenerational equity and exhaustible resources*, in Symposium on the Economics of Exhaustible Resources, 1974, pp. 29–45.

[59] ——, *Georgescu-roegen versus solow/stiglitz: Reply*, Ecological Economics, (1997).

[60] D. . STERN, *Limits to substitution and irreversibility in production and consumption: A neoclassical interpretation of ecological economics*, Ecological Economics, 21 (1997).

[61] M. STRAŠKRABA, *Cybernetic theory of complex ecosystems*, in Complex Ecology : The Part-Whole Relation in Ecosystems, B. C. Patten and S. E. Jorgensen, eds., Prentice Hall, Englewood Cliffs, N.J., 1995.

[62] L. SZOTT, C. PALM, AND C. DAVEY, *Biomass and litter accumulation under managed and natural tropical fallows*, Forest Ecology and Management, 67 (1994), pp. 177–190.

[63] J. A. TAINTER, *The collapse of complex societies*, Cambridge University Press, 1988.

[64] H. R. VARIAN, *Microeconomic analysis*, Norton, 1992.

[65] R. WALTMAN, *Competition Models in Population Biology*, SIAM, 1983.

[66] A. L. WRIGHT, *An introduction to the theory of dynamic economics : a theoretical study in long run disequilibrium*, Oxford University Press, 1984.

[67] J. T. YOUNG, *Is the entropy law relevant to the economics of natural resource scarcity?*, Journal of Environmental Economics and Management, 21 (1991), pp. 169–179.