THREE DIMENSIONAL HEAT FLOW IN THE DIRECT
CHILL CASTING OF NON-FERROUS METALS

by

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ABSTRACT

A three dimensional mathematical model has been developed to study heat flow and solidification in the Direct Chill casting of non-ferrous metals with rectangular as well as irregular cross-sections. The model which is based on an alternating direction, implicit finite-difference numerical method is capable of simulating heat flow both in the steady state and transient part of the casting operation. The validity of the model has been verified by comparing predicted pool profiles and pool depths with industrial measurements.

The model has been used to study the importance of heat flows in the various directions, and the limitations of using two-dimensional heat flow models are brought out. This study has shown that a two-dimensional model which neglects heat flow normal to the narrow face can be used to simulate the solidification of slabs with aspect ratios greater than 2.5, cast under conditions of conventional D.C. cooling. Further it was demonstrated that with reduced secondary cooling, a two-dimensional model that neglects axial heat conduction is preferable. Model calculations show that in cooling large sections the unsteady state can occupy around 25% of the total casting cycle.
The formation of cracks in jumbo ingots of Prime Western Grade zinc has been investigated with the aid of the mathematical model. It has been shown that the cracking is caused by reheating of the surface below the spray cooling zone, if this zone is short and characterized by a high water flux. The surface reheating generates tensile strains at the solidification front where cracking is aided by the presence of lead rich liquid in the inter-dendritic regions. A new spray assembly has been designed which attempts to cool the casting more uniformly from the top of the mould to the bottom of the liquid pool. This new spray system has been tested in-plant for casting Prime Western Grade zinc and results have proven its effectiveness in preventing crack formation.
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Chapter 1

INTRODUCTION

Over the last half century a number of new processes have been developed in both the ferrous and non-ferrous industries. The majority of these new processes have been brought about to increase production and to effect improvement in overall efficiency. Examples are the continuous casting of steel, Direct Chill casting of non-ferrous metals, BOF and Q-BOP steel making etc. Although these processes have all been commercialised, improvement in the efficiency of production and the quality of the product is continually sought. This requires an understanding of the process and the relationships between fundamental and process variables. Mathematical models are valuable tools in this regard as they provide an inexpensive way of learning about the process and of studying the effect of process variables on the overall operation.

A number of mathematical models have been developed for the investigation of heat flow in ferrous and non-ferrous continuous casting. However only a handful of these have been used beyond the developmental stage to predict solidification structure and to determine conditions for the elimination of internal cracks. Almost all
the models developed in this area to date are either one or two dimensional in nature. Although they are adequate for the simulation of the continuous casting of steel, their use is rather limited for the analysis of heat flow in D.C. casting of rectangular sections of non-ferrous metals with low aspect ratios. Three-dimensional heat flow models are required under these conditions. Rapid developments in the field of high-speed digital computers have made it possible to develop these models.

1.1 Objectives of the Present Work

The primary objective of this study was to develop a fully three-dimensional heat flow model to simulate the Direct Chill casting of non-ferrous metals. Because of the semi-continuous nature of this process, it was also desired to include the transient initial portion of the casting in the calculations. The second objective of this study was to validate the model developed with industrial measurements and then use the model to demonstrate the importance of the different components of heat flows under different casting conditions. Finally the most important objective of this study was to use the model to assist in solving a cracking problem in the D.C. casting of Prime Western Grade zinc.
Chapter 2

REVIEW OF THE LITERATURE

2.1 Introduction

Direct Chill casting, more commonly known as D.C. casting, was developed in the late 1930's and, since that time, has become the "work horse" of a modern non-ferrous casting plant. In addition to being reliable, it has proved to be a very economical production technique for casting non-ferrous metals such as aluminium, copper, magnesium and zinc. Most sections that are D.C. cast are either circular for extrusion applications or rectangular for rolling applications; but other shapes are also produced. Examples are the T-ingots in the aluminium industry and the jumbo ingot of special shape in the zinc industry. Modern day D.C. casting machines have enormous production capacities. As an example, the slab casting facility at Alcan in Oswego, New York can produce 5, 7 or 9 slabs at a time of 460 mm thickness, 2200 mm maximum width and 5100 mm length, equivalent to a cast weight per drop ranging from 35 to 45 tonnes (3).

It is interesting to compare D.C. casting of non-ferrous metals with the continuous casting of steel.
Two important differences that can be seen are the length of the mould and the casting speed. D.C. casting moulds are characterized by their short lengths. It is very common to find moulds only 20 to 50 mm long for casting aluminium and zinc in comparison with 600-900 mm long moulds used in the case of steel. The casting speeds encountered in D.C. castings are an order of magnitude lower than those in steel. One other factor which distinguishes D.C. casting is the semi-continuous nature of its operation, although in recent years a continuous horizontal caster has been developed (4-7). The use of horizontal D.C. casting however is restricted to casting smaller sections and alloys of low strength. The model developed in this study has been used for the analysis of heat flow in vertical D.C. casting operations.

2.2 D.C. Casting

A schematic diagram of the Direct Chill casting process is shown in Fig. 2.1. As is obvious from the name the surface of the casting in this operation is chilled by the mould cooling water which exits the mould and impinges directly on the solidifying metal. Basically, cooling of the casting in a conventional D.C. casting process is carried out in three stages: primary cooling in the water jacketed mould, secondary cooling by the flood
Fig. 2.1 A schematic diagram of the openhead Direct Chill Casting Process.
water impinging on the casting and finally tertiary cooling by the stagnant water pool in a water-collection pit. In some special applications minor variation of the above practice has been noted (28-30, 33).

The sequence of operations carried out during D.C. casting are as follows. The stool or bottom block is raised into the mould to cover the opening and liquid metal is pumped into the mould. Once the liquid metal reaches a certain level the bottom block is lowered at a controlled rate. The level of metal in the mould is very critical from the standpoint of surface quality of the casting and is maintained constant through the use of a float valve assembly. The exact value of the metal head in the mould will depend on a number of factors including the casting speed, the alloy cast, the section size and the pouring temperature. When the length of the casting reaches the bottom of the pit the casting is terminated. The entire operation is repeated once the casting is taken out of the pit. In a semi-continuous D.C. casting operation lubricant is normally applied on an intermittent basis to the mould at the beginning of a casting. However continuous application of lubricant is practiced in some horizontal machines.

Even though the description given in the preceding
paragraph is a typical D.C. casting sequence, the procedure is often modified for special cases. For example, while casting aluminium slabs for deep drawing application the casting is subjected to a reduced secondary spray cooling in place of conventional flood cooling to obtain a larger cell structure (28-30, 33). Similarly in a zinc D.C. casting operation the water jacketed mould is replaced by a solid mould which is cooled by water sprays.

A number of papers have been published dealing with the various aspects of the D.C. casting operation (1-48). Emley (2) has recently published a complete survey of the continuous casting of aluminium including D.C. casting. A similar account for the case of copper and its alloys has been published by Kreil et al (17).

Since the surface and sub-surface quality of the D.C. cast ingots determines the amount of scalping to be carried out prior to fabrication, considerable research effort has been directed towards improving the surface quality (13, 18, 20-22, 24, 44). The use of shorter moulds, which results in a higher quality surface is made possible by insulating the top portion of the normal mould with insulating marinite lining. Recently an electromagnetic mould has been developed in Russia (18) in which
the metal does not contact the mould surface. This has been shown to yield high quality ingots which could be fabricated without the intermediate scalping operation. Adoption of this novel technique in a casting plant in Switzerland has been described by Meier et al (44).

Like other industries the influence of computers has also been felt in D.C. casting operations, where for example micro-processors are being used for on-line control (43).

2.3 Review of Mathematical Models in D.C. casting

The first mathematical model of heat flow in D.C. casting was published by Roth (26) in 1943. In order to solve the mathematical equations analytically this author had to make several simplifying assumptions, like negligible heat transfer in the mould region of the casting and constant surface temperature below the mould. Because of the simplistic nature of this model very poor agreement was obtained between predicted and measured shell thicknesses. However in spite of the rather crude nature of this model, credit must be given for attempting to place the casting operation within a mathematical framework.

A more refined model employing a numerical solution was proposed by Adenis et al (27) in the early 1960's. Their
model was developed to simulate steady state heat flow in casting cylindrical magnesium alloy ingots. Because of the rather long mould used in their study (240 mm), the heat flow in the mould region was divided into three zones, the top zone having perfect contact between the molten metal and mould, the intermediate zone having a film of oil between the casting and inside of the mould and finally the bottom zone having an air gap. These authors reported good agreement between measured pool depths and values predicted using their model. However they have used the liquidus temperature rather than the solidus in this comparison. The main uncertainty in their model was associated with characterizing boundary conditions in the long mould region.

Kroeger and Ostrach (31) have simulated the steady-state temperature as well as fluid flow resulting from natural convection during the casting of a cylindrical ingot of a pure metal. Assuming constant temperature as the boundary condition on the surface of the ingot for the heat flow model, these authors have shown that substantial fluid velocities can develop from natural convection. However it was also shown that the strong velocity field had negligible effect on the location of the solid-liquid interface.
The mathematical model described by Peel and Pengelly (28, 29) was also developed for heat flow in cylindrical ingots as well as steady-state conditions. Because of the long moulds used and small diameters cast, the bottom of the pool was very close to the bottom of the mould. Since a substantial portion of the total heat was removed in the mould region, these authors have formulated a variable resistance model for describing heat flow through the air gap. Experimental results were later used for "fine tuning" the model. An attempt to predict the cell structure using the average cooling rate in the liquidus-solidus temperature range was not successful although better agreement was obtained while using the gradient of the cooling curve at the solidus temperature. Even with this new technique a match between the measured and calculated values of cell size could only be considered qualitative. The importance of the proper characterization of boundary conditions has been well emphasized in their work.

Mathew (32) has made an analysis of both heat flow and thermal stresses during continuous casting. This model was formulated to calculate the steady-state temperature and stress distribution in a cylindrical ingot. Unlike the previous models a finite-element numerical procedure was adopted for solving the heat flow and stress equations; and
the elegance of the finite element method in handling various types of boundary conditions has been stressed in this work. Although the model was set up for calculating heat flow in a cylinder, it has also been used for square sections. The stress calculations carried out in this study are of doubtful value in a quantitative sense because of the uncertainty in the high temperature mechanical properties of metals near their melting point.

Continuing on the same line as Peel and Pengally (28, 29), Beattie (30, 33) has developed a model for calculating heat flow in rectangular slabs. Based on steady-state principles his model takes into account heat flow normal to the broad face and in the axial direction. Careful experimental work was undertaken to characterize boundary conditions. The temperatures measured by implanting thermocouples at various locations in the mould were used in calculating the heat flux boundary conditions. The parameter employed for predicting the cell structure is different from that reported by Peel and Pengelly. Instead of using the gradient at the solidus temperature, this author has used the gradient at a temperature in the solidus-liquidus range, in order to obtain a good match between calculated and predicted cell sizes. The mesh size employed in his numerical model had to be much finer than that used by
Peel and Pengelly (2.5 mm instead of 10 mm) to obtain reasonable cell size predictions. In spite of these refinements major discrepancies were observed in predicting cell structures for a rectangular slab, 690 x 250 mm, subjected to reduced secondary cooling. This may arise because heat flow perpendicular to the narrow face has been neglected.

Fossheim and Madsen (48) have developed heat flow models for cylindrical as well as rectangular ingots. Their rectangular model again corresponded to a two dimensional heat flow in the axial and one transverse direction. Space discretization in their model was based on a box integration method and the time integration on an exponential transformation of the heat conduction equation with an alternating direction implicit technique. As will be shown in Chapter 5, the use of a two-dimensional model for simulating heat flow in a rectangular slab 381 x 250 mm (aspect ratio 1.5) can lead to considerable error. In addition to this, looking at their contour profiles it appears that the authors have made an incorrect choice regarding the second dimension for heat flow. Instead of considering the smaller of the two sections, namely 250 mm, they have performed the calculations for a thickness of 381 mm.
Jovic et al (35) have recently developed a two-dimensional model for calculating the temperature field in continuously cast rectangular slabs of aluminium alloys, as well as predicting the cell structure. Although considerable care was taken in characterizing the boundary conditions these authors have made an incorrect choice in one of their assumptions. In calculations involving a rectangular slab 360 x 1600 mm, they have chosen to neglect heat flow in the axial direction and considered only the two transverse directions. Since the casting speed employed in the simulation was very low (1 mm/s), axial heat conduction cannot be neglected especially when casting a material like Al - 1% Mn which has a high thermal conductivity. A better choice would have been to neglect heat flow in a direction normal to the narrow face since the aspect ratio was fairly high (4.4).

Weckman et al (45) have also developed a steady state model based on axisymmetry for cylindrical ingots. The model uses the finite-element method and can only treat pure metals and eutectic alloys. The pool profile predicted by the model has been compared to experimental pool profiles obtained during the continuous casting of a zinc ingot with a square cross-section. In order to make the comparison meaningful, the cross-sectional areas of the cylindrical and square
ingots were set equal to each other. A simple calculation of the surface area to volume ratios indicates that there is a difference of 11% between the two cases. Thus this model can only have limited use in studying heat flow in rectangular sections.

Szargut et al (36) have published a paper on steady-state heat flow in the continuous casting of a cylindrical copper ingot. Other than the fact that their model is based on an explicit finite-difference scheme with its associated stability problems, no new material has been presented in their paper.

Recently Jensen (37) has developed a model for simulating heat flow in cylindrical ingots including the unsteady-state part of the casting operation. The results of a simulation for the casting of 381 mm diameter ingot are reported but very little information is given regarding the numerical procedure adopted in the calculation. Further no comparison has been made to check the validity of the model calculations.

In summary, a number of mathematical models have been written to simulate heat flow in D.C. casting. The majority of these models are based on assumptions of steady-state operation and cylindrical geometry. The remaining models
which apply to rectangular sections are also only two dimensional with heat flow neglected either in one of the transverse directions or the axial direction.

The present work has been undertaken to develop a truly three dimensional model to simulate both unsteady and steady state heat flow in casting square and rectangular sections. Further as will be shown this model has been used for testing the validity of using a two dimensional version under special limiting cases.
Chapter 3

DEVELOPMENT OF THE HEAT FLOW MODEL

3.1 Introduction

The problem of developing a model for the analysis of heat flow in D.C. casting essentially involves the solution of a partial differential equation describing unsteady heat conduction. Because of the slow casting speeds employed in this operation and the high thermal conductivity of the material cast the normal assumption of negligible heat conduction in the axial direction, made in the models for the continuous casting of steel is not valid. The problem of considering heat flow in all three directions is made simpler in the case of a cylindrical ingot. If axisymmetry can be assumed the heat flow problem is reduced to two directions namely radial and axial (27-29, 31, 32, 36, 45).

When developing heat flow models for rectangular sections such a simplification is not possible. However in order to reduce the problem also to two dimensions it is usual to neglect heat flow in the direction normal to the narrow face (30, 37, 48). Although this is not a bad assumption for slabs which are thin and wide, it leads to
considerable error for sections with a small aspect ratio (ratio of the transverse dimensions), as will be seen in Chapter 5. In such cases it is necessary to consider heat flow in all three directions.

Because of the semi-continuous nature of the D.C. casting operation, it is interesting also to study the unsteady-state portion of the casting as it could occupy a sizeable fraction of the total casting time. Thus the model undertaken in this study is based on unsteady state, three-dimensional heat flow.

3.2 Assumptions Made in the Model

The following assumptions have been made in the development of the model.

1. In the case of the simulation of rectangular or square sections two-fold symmetry has been assumed for the mid face planes, and the calculations are performed only for one quarter of the casting. In the case of jumbo zinc ingots, because of the presence of only one symmetry plane, calculations are made for one-half of the casting.

2. Mixing in the liquid pool has been neglected, i.e. a stagnant pool has been assumed. This has been
shown experimentally (29) and theoretically (31) to be a reasonable assumption for non-ferrous castings with shallow pools. The assumption may not be valid however in situations involving electro-magnetic stirring. In such cases the thermal conductivity of the liquid could be increased to reflect the stirring as has been done in the case of the continuous casting of steel (71).

3. In the simulation of zinc-jumbo casting the thermal conductivity of zinc has been assumed constant and the same for both liquid and solid. From the thermophysical properties given by Touloukian (51), the thermal conductivity of solid zinc has a value twice that of the liquid. Thus assigning a constant solid thermal conductivity value for the liquid as well as the solid corresponds to a very mild stirring in the liquid pool. The error introduced by assuming constant thermal conductivity for solid zinc is small since it changes only by 20% between room temperature and its melting point. Moreover Peel and Pengelly (29) have claimed that the effect of thermal conductivity is negligible in the case of aluminium casting. In this study appropriate constant values have been used for solid and liquid regions. The value used in the mushy region on
the relative amount of solid and liquid fractions assuming equilibrium solidification. It is possible to take into account varying thermal conductivity through minor modification of the computer program.

4. The specific heat is allowed to vary as a function of temperature. However no iterative calculations are made within a time interval. The value of specific heat is evaluated at the beginning of a time interval based on the temperatures obtained from the previous interval and is kept constant during that particular interval. The temperature dependence of specific heat is adequately described through use of small time intervals.

3.3 Heat Flow Equation and Boundary Conditions

The unsteady-state, heat-conduction equation in three dimensions with constant thermal conductivity for a Cartesian co-ordinate system can be written as,

$$\frac{k}{\partial^2 T}{\partial x^2} + \frac{k}{\partial^2 T}{\partial y^2} + \frac{k}{\partial^2 T}{\partial z^2} = \rho c \frac{\partial T}{\partial t} \tag{3.1}$$

where $T$ denotes the temperature $x, y, z$, the three directions $k$, the thermal conductivity
\[ p, \text{ the density} \]
\[ c, \text{ the specific heat} \]
\[ t, \text{ the time.} \]

A detailed list of the symbols used is also given before Appendix 1. In order to solve Eq. (3.1) initial and boundary conditions are required.

**Initial condition:**

Physically, the initial condition describes the conditions existing at the beginning of casting operation when the bottom block is raised to cover the mould opening, and the mould is filled with molten metal. Since the filling of the mould is carried out in a very short time interval (usually around 1 minute), it is assumed there is negligible heat loss during this procedure. Thus the initial temperature is taken to have a constant uniform value throughout the casting and equal to the pouring temperature.

\[ T = T_p \text{ at } t = 0, 0 \leq x \leq X, 0 \leq y \leq Y, 0 \leq z \leq Z \ldots 3.2 \]

where \( T_p \) is the pouring temperature.

**Top Boundary Condition:**

When the casting operation is started, the flow of liquid metal into the mould is adjusted to match the casting speed. Since this metal normally comes from a holding
furnace, the temperature of the metal remains steady during a casting operation. Thus to simulate this top boundary condition, the top portion of the casting is always kept at the pouring temperature.

\[ T = T_p \quad t > 0, \quad z = 0, \quad 0 \leq x \leq X, \quad 0 \leq y \leq Y \]  \[ \quad \ldots \quad 3.3 \]

It is possible to have

\[ T = T_p (x,y), \quad t > 0, \quad z = 0 \]  \[ \quad \ldots \quad 3.4 \]

if the temperature distribution at the top is known more accurately.

**Bottom Boundary Condition:**

This involves the transfer of heat from the bottom of the ingot to the platten with which it is in contact. Since the platten is not provided with any cooling, only a small amount of heat flows through it. In the present work this boundary condition has been handled through the use of a heat-transfer coefficient.

\[ -k \frac{\partial T}{\partial z} = h (T - T_b), \quad t > 0, \quad z = Z, \quad 0 \leq x \leq X, \quad 0 \leq y \leq Y \]  \[ \quad \ldots \quad 3.5 \]

A constant value of .209 kW/m K (.005 cal/cm K °C s) has been employed through most of the calculations presented in this work. Although this heat-transfer coefficient might have some effect during the start up of the casting, it has
no effect on the steady-state operation. A further discussion of the subject will be reserved for a later section on the importance of unsteady heat flow in Chapter 5.

Side Boundary Conditions:

This is the most important of all the boundary conditions since the ingot is cooled only from the sides. Here again a heat-transfer coefficient type of boundary condition has been employed.

\[-k \frac{\partial T}{\partial x} = h(z) (T - T_w) \quad t > 0, \quad x = X, \quad 0 \leq y \leq Y, \quad 0 \leq z \leq Z \quad ... \quad 3.6\]

\[-k \frac{\partial T}{\partial y} = h(z) (T - T_w) \quad t > 0, \quad y = Y, \quad 0 \leq x \leq X, \quad 0 \leq z \leq Z \quad ... \quad 3.7\]

where \( h(z) \) is the overall heat-transfer coefficient which is a function of the position along the \( z \)-axis. Thus the heat-transfer coefficient used in the mould will have a different value than the one used in the spray region. Characterization of this heat-transfer coefficient is very important to the accuracy of the simulations. The exact value used will be given in the section dealing with the validation of the model.

Centre Boundary Conditions:

Wherever possible, symmetry conditions were applied in simplifying the problem. Thus in the case of a
rectangular slab with two symmetry planes, zero heat flux has been assumed at the mid-face planes and the calculations are performed only for one quarter of the casting, as mentioned earlier.

\[-k \frac{\partial T}{\partial x} = 0 \quad \text{for } x = 0, \quad 0 \leq y \leq Y, \quad 0 \leq z \leq Z \quad \ldots \quad 3.8\]

\[-k \frac{\partial T}{\partial y} = 0 \quad \text{for } y = 0, \quad 0 \leq x \leq X, \quad 0 \leq z \leq Z \quad \ldots \quad 3.9\]

The differential equation Eq. (3.1) together with the initial and boundary conditions Eqs. (3.2) to (3.9) comprise a complete mathematical statement of the problem.

3.4 Method of Solution

The use of analytical methods to solve Eq. (3.1) is precluded by the complex nature of the boundary conditions as well as the growth of the ingot in the casting direction. Similarly semi-analytical methods, like the integral-profile method developed by Hills (80), although useful in modelling heat flow in the continuous casting of steel, have limited use in the present problem due to the importance of axial conduction and the unsteady state. A numerical method has therefore been adopted for solving the partial differential equation. Because of the considerable amount of time and effort involved in the development of the computer program, the initial choice of the numerical
method is very important. The method chosen should be able to yield a reasonable level of accuracy with moderate computer requirements.

Two of the most popular methods for solving problems related to heat flow and solidification are finite element and finite-difference techniques. The use of finite-element methods for solving solidification problems are discussed by several authors (32, 72-74). Although this method is elegantly suited for handling steady state problems, and has an edge over finite-difference method in handling complex geometry, it has a rather limited use in three dimensional transient problems from the standpoint of the cost of computation. Emery and Carson (74) have compared finite-element and finite-difference methods for the solution of a two dimensional heat flow problem. It is clear from this analysis that the finite-difference method exceeds the finite-element methods in efficiency for transient problems in three dimensions. Thus it was decided to use a finite-difference method for solving Eq. (3.1).

A good treatment of the finite difference methods can be found in any standard text book on numerical methods (64, 65, 78, 79). Finite-difference methods can in general be classified into three broad categories namely explicit,
The explicit finite difference methods are the simplest of the three types and do not require simultaneous solution of equations. In this method the future temperature of a node is calculated based on the present temperature of that particular node and surrounding nodes. In a three-dimensional problem, an interior node will be surrounded by six nodes. The main disadvantage of the explicit method is the restriction imposed on the time steps that can be used for a given node size, for proper stability of the numerical method. The restriction imposed on the time interval increases by a factor of three for the three dimensional case as compared to the one dimensional problem, if the node size is kept the same. This is illustrated for the case of zinc and for a small node size used in this work. For a Dirichlet type boundary condition, the stability criterion is

\[ \frac{k \Delta t}{\rho c} \left( \frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2} \right) \leq \frac{1}{2} \]

where \( k \) is the thermal conductivity
\( \rho \) is the density
\( c \) is the specific heat
\( \Delta x, \Delta y, \Delta z \) are node sizes in \( x, y \) and \( z \) dimensions
\( \Delta t \) is the time interval.
Substituting the appropriate values for the different variables,

\[ k = 113 \text{ W/m K}, \quad \rho = 7140 \text{ kg/m}^3, \quad c = 0.3830 \text{ J/gK} \]

\[ \Delta x = 15.24 \text{ mm}, \quad \Delta y = 15.24 \text{ mm}, \quad \Delta z = 20 \text{ mm} \]

one obtains \( \Delta t \leq 1.08 \text{ s} \).

When the above-mentioned calculations are repeated with a convection type boundary condition a lower value, 0.55 s, (see end of Appendix 1 for this calculation) is obtained for \( \Delta t \). Other finite difference methods described below are not subject to any stability criterion and values of \( \Delta t \), eight fold greater than that imposed by explicit methods have been commonly used in this work. In conclusion it can be said that the explicit methods have a limited role to play in three dimensional problems.

In the fully implicit method, unlike the explicit method, the future temperature of a node is related to the present temperature of that node and the future temperature of the surrounding nodes. Since these surrounding temperatures are not known a priori the implicit method essentially involves the solution of a system of simultaneous equations. For a three-dimensional problem with ten nodes in each direction, it would mean solving one thousand equations simultaneously. Although most of the elements in the
coefficient matrix will be zeros (there will be only seven non zero terms per row) and need not be stored, the best solution procedure like the Gauss-Seidel method would still take up considerable computer time as the solution would have to be iterated. In this regard the fully implicit methods could be compared to the finite element methods. However there are no stability problems associated with fully implicit methods.

In order to overcome the difficulties of stability conditions encountered in explicit methods and problems associated with solving a large number of simultaneous equations in implicit methods, special procedures called alternating direction implicit methods have been developed (75-77). The procedure adopted in this work was originally proposed by Brian (77). It has an unconditional stability and converges with discretization error of the order $0 \left[ (\Delta x)^2 + (\Delta t)^2 \right]$.

In this method within each time interval $\Delta t$, the calculations are performed in three stages. In the first stage the calculations are made implicit in the $x$-direction and explicit in the $y$ and $z$ directions. This is followed by similar procedures in the $y$ and $z$ direction. Finally the new temperatures at the end of the time interval is
calculated using an explicit formula.

\[ \frac{T^* - T_n}{\Delta t/2} = \delta^2_x T^* + \delta^2_y T_n + \delta^2_z T_n \] \hspace{1cm} ... 3.10

\[ \frac{T^{**} - T_n}{\Delta t/2} = \delta^2_x T^* + \delta^2_y T^{**} + \delta^2_z T_n \] \hspace{1cm} ... 3.11

\[ \frac{T^{***} - T_n}{\Delta t/2} = \delta^2_x T^* + \delta^2_y T^{**} + \delta^2_z T^{***} \] \hspace{1cm} ... 3.12

\[ \frac{T_{n+1} - T_n}{\Delta t} = \delta^2_x T^* + \delta^2_y T^{**} + \delta^2_z T^{***} \] \hspace{1cm} ... 3.13

where \( \delta^2_x, \delta^2_y \) and \( \delta^2_z \) are the central difference operators defined by

\[ \delta^2_x T_{i,j,k} = \frac{T_{i-1,j,k} - 2T_{i,j,k} + T_{i+1,j,k}}{\Delta x^2} \]

\[ \delta^2_y T_{i,j,k} = \frac{T_{i,j-1,k} - 2T_{i,j,k} + T_{i,j+1,k}}{\Delta y^2} \]

\[ \delta^2_z T_{i,j,k} = \frac{T_{i,j,k-1} - 2T_{i,j,k} + T_{i,j,k+1}}{\Delta z^2} \]
i, j, k being the letters used for numbering the nodes in the x, y and z directions.

$T^*$, $T^{**}$, $T^{***}$ are the intermediate fictitious temperatures calculated which do not have any special meaning.

$T_n$ and $T_{n+1}$ are the temperatures at the beginning and the end of a time interval $\Delta t$. $\Delta x$, $\Delta y$ and $\Delta z$ are the distance between nodes in x, y and z directions.

Substitution of finite difference equations Eqs. 3.10 to 3.13 in place of the partial differential equation results in a set of simultaneous equations involving tridiagonal coefficients for which a very efficient solution procedure exists. Thus in spite of the three sets of calculations to be carried out within each time interval, this is a more efficient procedure than the fully implicit method. Details of the procedure by which the casting is divided into different nodes and the setting up of the nodal equations are presented in Appendix 1. For a simple case of a rectangular parallelepiped with regular node size in all the three dimensions there will be twenty-seven different types of nodal equations.
One of the problems with the alternating implicit technique is the treatment of the radiation boundary condition. Introduction of the radiation boundary condition directly into the heat balance equation renders the equations non-linear. In order to overcome the problem, the boundary condition can be linearized using the relationship

\[ h_{av} = \sigma \varepsilon \left( \frac{T^4 - T_{a}^4}{T - T_{a}} \right) \] ... 3.14

where the average heat transfer coefficient \( h_{av} \) is calculated assuming an average value for the surface temperature. Here \( T_{a} \) is the temperature of the ambient medium, \( \sigma \) is the Stefan-Boltzmann constant and \( \varepsilon \), the emissivity. Because of the low temperatures encountered in aluminium and zinc casting, this boundary condition will have a negligible effect.

The growth of the ingot is simulated through periodic addition of a set of nodes at the pouring temperature to the top portion of the casting, as has been carried out by Ballantyne (66). The time interval over which a row of nodes is added is calculated from the casting speed and the node size in the casting direction. Thus for a casting speed of \( v \) and node size \( \Delta z \), an addition is made every \( \Delta t_{a} \) given by

\[ \Delta t_{a} = \frac{\Delta z}{v} \] ... 3.15
The time interval $\Delta t$ for model calculations is selected such that $\Delta t_a$ will be an integral multiple of $\Delta t$

$$
\Delta t_a = N (\Delta t) \quad \ldots 3.16
$$

where $N$ is an integer. Thus there is no independent control of $\Delta t$ and $\Delta z$. Initially numerical calculations were performed to check the effect of $N$, on the predicted results. The effect was found to be very small and thus the value of $N$ was arrived at, based on casting speed, node size and the cost of computation. Values of $N$ used in this work range from 2 to 8.

The release of latent heat during solidification poses some problems in the implicit finite difference technique. To overcome this problem, a technique commonly employed by other workers (29, 33, 66) has been adopted in this study. The latent heat is released linearly over the liquidus-solidus temperature range as follows:

$$
c_m = c + \frac{L}{T_L - T_s} \quad \ldots 3.17
$$

where $c_m$ denotes the specific heat in the mushy region, $c$ the average specific heat evaluated at the solidus and liquidus temperatures, $L$ the latent heat of solidification, $T_L, T_s$ the liquidus and solidus temperatures.
This method requires that the temperature of nodes undergoing solidification fall within the liquidus-solidus interval at some point during the calculation. If however large heat flows take place it is possible for some nodes to jump from above the liquidus to below the solidus within one time interval. This problem is often encountered in heat flow calculations of D.C. casting because of the high heat transfer coefficients resulting from flood cooling and the high thermal conductivity of the material cast. When this happens the latent heat will not be released from that node. This point can be appreciated from the simple calculation that latent heat amounts to over 60% of the total heat removed from the top of the casting to the bottom of the pool.

In order to avoid this problem a post-iterative correction procedure has been employed. In this technique the temperature of the nodes which jumped from one phase to another is modified using a heat balance approach. An example for a node going from above the liquidus to below the solidus is given below.

Let $T_1$ be the temperature of node before a time interval $T_L$ the liquidus temperature $T_S$ the solidus temperature $T_2$ the temperature after the time interval
$c_1$, the specific heat evaluated at $T_1$

$\rho$, the density of the material

$c_2$, $c_3$ the specific heat of the material in the mushy zone and evaluated at the solidus temperature

$v$, the volume of the node

The net change in the heat content of that node

$$= v \rho c_1 (T_1 - T_2)$$

The following comparison decides whether the node will end up in the mushy region or in the solid region. If

$$\rho v c_1 (T_1 - T_2) > \rho v c_1 (T_1 - T_e) + v \rho c_2 (T_e - T_s)$$

then the node will end up in the solid zone. Otherwise the corrected temperature will be in the mushy region. The corrected temperature when the above-mentioned inequality is satisfied is given by

$$T_3 = T_s - \left[ (T_e - T_2) \frac{c_1}{c_3} - (T_e - T_s) \frac{c_2}{c_3} \right] \quad \ldots \quad 3.18$$

Similar equations are obtained for nodes jumping from the liquid to mushy and mushy to solid phases. In cases where the casting undergoes reheating this procedure is repeated in reverse.

This corrective procedure is very handy in using relatively long time intervals allowed by the unconditional
stability of the alternating direction implicit finite difference scheme. However a large increase in time interval to decrease the cost of computation is not recommended on two counts. Firstly the values of thermo-physical properties that change with temperature are evaluated at the beginning of a time interval and kept constant during that interval. Thus having a very large time step may counteract the accuracy sought with varying thermophysical properties. Secondly when a large time interval is used, the post-iterative correction procedure alters the temperature field so much that the program goes into an unstable mode.

Thus although there are no constraints regarding the size of time intervals arising from the solution procedure, caution should be exercised in selecting the time interval both from the viewpoint of the accuracy of solution and the cost of computation. In this study, time intervals ranging from 2 to 10 seconds have been used. The exact value selected depended on the metal cast, size of section, casting speed and severity of cooling conditions.

3.5 Mathematical Check for Internal Consistency of the Computer Program

Although a majority of mathematical models require a numerical method to obtain a solution, analytical methods
are very useful to check out the numerical techniques. In a complex problem using finite differences, limiting cases which often have analytical solution can be used for comparison to the numerical results. They are also very valuable tools for debugging the computer program in the initial stages of model development. It should be noted that in this check, solidification (latent heat) and growth phenomena of the model are not included.

The solution of the three dimensional heat flow equation defined in the region \(-a < x < a, -b < y < b, -c < z < c\) with zero initial temperature and unit surface temperature is given by Carslaw and Jaeger (83) as

\[
T = 1 - \frac{64}{\pi^3} \sum_{\ell=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{\ell+m+n}}{(2\ell+1)(2m+1)(2n+1)} \cos \frac{(2\ell+1)\pi x}{2a} \cos \frac{(2m+1)\pi y}{2b} \cos \frac{(2n+1)\pi z}{2c} e^{-\beta \ell, m, n t}
\]

where \(\beta = \frac{\alpha \pi^2}{4} \left[ \left( \frac{2\ell+1}{a} \right)^2 + \left( \frac{2m+1}{b} \right)^2 + \left( \frac{2n+1}{c} \right)^2 \right] \) ... 3.19

and \(\alpha\) is the thermal diffusivity, \(t\) is the time, \(2a, 2b, 2c\) the dimensions of rectangular parallelepiped, \((x, y, z)\) is the location of where the temperature is calculated.
In this specific example heat flow in a cube of side 609.6 mm was considered. The thermal diffusivity of the material used is 12.95 mm²/s. Initially the material is uniformly at a temperature of 260°C and subsequently for all time \( t > 0 \) the surfaces of the cube are maintained at 537.7°C. The calculated temperatures at the centre of the cube are presented in Table I for both numerical and analytical methods. The results shown in this table have been calculated for a constant time interval of 36 s, but different node sizes. As can be seen the numerically calculated values approach the analytical results as the number of nodes used in the calculation increases. Some of the results from this table have been plotted in a different fashion in Fig. 3.1. The percent error on the y axis has been calculated as the percent difference between the numerical and analytical results. It can be seen that for this time interval of 36 s, using 21 nodes per side, the numerical solution yields numbers very close to the analytical results.

The influence of the time interval on the comparison between numerical and analytical method is presented in Table II. For these runs the node size has been kept constant. It can be seen from this table that for the node size considered, the time interval had a very small effect
<table>
<thead>
<tr>
<th>Time (s)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
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<td>N = 17</td>
<td>N = 21</td>
<td></td>
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<td>263.73</td>
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<tr>
<td>720</td>
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<td>305.41</td>
<td>303.31</td>
<td>302.31</td>
<td>300.57</td>
</tr>
<tr>
<td>1080</td>
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<td>362.17</td>
<td>360.83</td>
<td>360.21</td>
<td>357.48</td>
</tr>
<tr>
<td>1440</td>
<td>414.07</td>
<td>412.06</td>
<td>411.37</td>
<td>411.05</td>
<td>410.51</td>
</tr>
<tr>
<td>1800</td>
<td>450.67</td>
<td>449.69</td>
<td>449.35</td>
<td>449.20</td>
<td>448.95</td>
</tr>
<tr>
<td>2160</td>
<td>476.01</td>
<td>476.57</td>
<td>476.42</td>
<td>476.35</td>
<td>476.24</td>
</tr>
<tr>
<td>2520</td>
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<td>495.33</td>
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<td>508.44</td>
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<td>523.76</td>
<td>523.79</td>
<td>523.80</td>
<td>523.81</td>
</tr>
</tbody>
</table>

Table I  Comparison between numerical and analytical values of temperatures (°C) at the centre of a cube, as a function of time for different sizes of nodes.
Fig. 3.1 The effect of the number of nodes on percent error at the end of different time intervals.
<table>
<thead>
<tr>
<th>Time (s)</th>
<th>$\Delta t = 180s$</th>
<th>$\Delta t = 36s$</th>
<th>$\Delta t = 18s$</th>
<th>Analytical</th>
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<td>263.73</td>
<td>263.70</td>
<td>262.66</td>
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<tr>
<td>720</td>
<td>300.83</td>
<td>302.31</td>
<td>302.35</td>
<td>300.57</td>
</tr>
<tr>
<td>1080</td>
<td>359.52</td>
<td>360.21</td>
<td>360.23</td>
<td>357.48</td>
</tr>
<tr>
<td>1440</td>
<td>410.80</td>
<td>411.05</td>
<td>411.06</td>
<td>410.51</td>
</tr>
<tr>
<td>1800</td>
<td>449.12</td>
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<td>449.20</td>
<td>448.95</td>
</tr>
<tr>
<td>2160</td>
<td>476.32</td>
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<td>476.35</td>
<td>476.24</td>
</tr>
<tr>
<td>2520</td>
<td>495.30</td>
<td>495.30</td>
<td>495.30</td>
<td>497.27</td>
</tr>
<tr>
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<td>508.44</td>
<td>508.44</td>
<td>508.44</td>
</tr>
<tr>
<td>3240</td>
<td>517.52</td>
<td>517.52</td>
<td>517.52</td>
<td>517.54</td>
</tr>
<tr>
<td>3600</td>
<td>523.80</td>
<td>523.80</td>
<td>523.79</td>
<td>523.81</td>
</tr>
</tbody>
</table>

Table II  Comparison between numerical and analytical values of centre temperatures ($^\circ$C) of a cube, as a function of time, for different time steps.
on the results of the calculation. However this comment holds good only with respect to centre temperatures. If on the other hand comparisons were made for temperatures away from the centre then the calculations done with finer time intervals will show appreciable difference to that performed with coarser time steps.

Convective Boundary Conditions:

The above-mentioned comparisons helped in checking the equations developed for all interior nodes. In order to check the equations of the surface node a different procedure was followed. The heat transfer coefficient was set equal to zero in two directions and a finite value inserted in the third direction. The result obtained from the three dimensional model should yield a zero temperature gradient in two of the directions.

The analytical solution for one-dimensional heat flow in a slab of thickness 2L, initially at a uniform temperature subjected to convective type boundary condition is given by (83)

\[
\frac{\theta(x,t)}{\theta_0} = 2 \sum_{n=1}^{\infty} e^{-\frac{\delta_n^2}{\omega} (\alpha t / L^2)} \frac{\sin \delta_n \cos (\delta_n x / L)}{\delta_n + \sin \delta_n \cos \delta_n} ... 3.20
\]

\[
\delta_n \tan \delta_n = \frac{hL}{k} ... 3.21
\]
\[ \theta(x,t) = T(x,t) - T_\infty \]

\[ \theta_0 = T_0 - T_\infty \]

Where \( T_\infty \) and \( T_0 \) are the initial and ambient temperatures and \( h \), the heat transfer coefficient, the value of \( x \) is measured with respect to centre of the thickness. In this check again the thermal diffusivity of the material used is 12.95 \( \text{mm}^2/\text{s} \). Other values used are \( T_0 = 260^\circ \text{C} \), \( T_\infty = 537.7^\circ \text{C} \), \( L = 305.8 \text{ mm} \), \( h = 565.4 \text{ W/m}^2\text{K} \), \( k = 46.7 \text{ W/m K} \).

Comparison between the numerical and analytical results are presented in Fig. 3.2. It can be seen that the difference between the numerical and analytical methods increase in going from the centre of the slab to the surface. However with shorter time intervals very close agreements can be obtained between the two. The use of longer time intervals may generate oscillations at the surface. This is shown in Fig. 3.3. Unlike the explicit finite difference methods these oscillations are stable and will dampen after some time. The time intervals used in this study were selected to avoid this problem in most of the cases.

The procedure mentioned above was repeated for the other two directions yielding identical results and thereby verifying the equations developed for all the nodes,
Fig. 3.2 Comparison between analytical and numerical calculations of temperatures in the slab.
Fig. 3.3 Stable oscillatory nature of the numerically calculated surface temperatures for large values of time interval.
including the surface nodes.

3.6 Flow Chart of the Computer Program

A flow chart of the computer program is given in Figs. 3.4a and 3.4b. The program has been written in the Fortran IV language. The basic version developed originally was for the analysis of heat flow in rectangular or square shaped castings. However this has been modified substantially for use in irregularly shaped zinc jumbo ingots. A copy of the source program for jumbo shaped casting is presented in Appendix 2.

The program has been written such that it is possible to stop the computer run at any intermediate point, study the results and restart from the same point. This procedure was very useful in detecting abortive runs from the standpoint of saving computer time. The computer used was an Amdahl 470/V-6-II under the MTS system. The program requires roughly 0.8 Mega Bytes of memory and takes about 80 seconds of CPU time in performing 160 iterations. In this particular case the array dimensions varied from \((10, 16, 3)\) to \((10, 16, 43)\) at the end of the run.
Fig. 3.4(a) Flow chart of the computer program.
1. Implicit calculations in the X direction → Boundary conditions
   implicit calculations in the Y directions → Boundary conditions
   implicit calculations in the Z direction → Boundary conditions

   Compute new temps.

   Check whether any node jumps from one phase to another. Assign nodes solids, liquid & mushy

2. Yes → Print output
   No → PLOT req'd?
       Yes → Obtain plot
       No → End of calculations

   Add an extra slice of node in Z direction

   No → New set of nodes to be added?
       Yes → Stop

Fig. 3.4(b)  Flow chart of the computer program (continued from Fig. 3.4(a))
Chapter 4

VALIDATION OF THE RESULTS
FROM THE MATHEMATICAL MODEL

4.1 Introduction

Before the mathematical model could be used with confidence in a predictive mode it required careful validation. This was accomplished by comparing model-predicted pool profiles to industrial measurements obtained under identical casting conditions. A similar validation technique has been reported in other studies (28-30, 36, 48, 66). In D.C. casting the pool profile may be obtained experimentally by adding an alloy towards the end of casting when steady state has been reached. Later the contour is revealed after sectioning, polishing and etching the cast ingot. In some cases the pool profiles can be seen immediately after sectioning without surface preparation because of the difference in the machinability of the tracer alloy and the parent metal.

In addition to tracer addition, dip-stick measurements have been made in which a rod was lowered into the molten pool, to obtain the maximum pool depth. This is a simple but very useful technique and the value of dip-stick
measurements increases considerably when applied to large ingots which require considerable cutting and machining.

Three separate validations have been made in this study. Two involve the D.C. casting of aluminium ingot using conventional flood cooling and reduced secondary cooling in the sub-mould region respectively. The third validation was made for the casting of zinc jumbos of special shape.

4.2 Conventional D.C. Casting of Aluminium - Alcan

4.2.1 Aluminium Ingots : 381 x 991 mm

In this conventional form of D.C. casting the surface of the casting is flooded with the cooling water from the mould, thereby producing intense cooling in the sub-mould region. In all the simulations presented, the material cast is pure aluminium, the thermophysical properties of which are given in Table III. The working length of the mould used is 63.5 mm so that with a node thickness of 15.87 mm in the casting direction, four node slices are contained in the mould. The heat-transfer coefficient used for the top two slices in the mould are 1256 W/m$^2$K and 1047 W/m$^2$K respectively. The use of these heat-transfer coefficients result in heat flux values
<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Heat of Solid</td>
<td>0.934 J/g K</td>
</tr>
<tr>
<td>Specific Heat of Liquid</td>
<td>0.934 J/g K</td>
</tr>
<tr>
<td>Latent Heat of Fusion</td>
<td>387 J/g</td>
</tr>
<tr>
<td>Liquidus Temperature</td>
<td>631 °C</td>
</tr>
<tr>
<td>Solidus Temperature</td>
<td>630 °C</td>
</tr>
<tr>
<td>Density of Solid</td>
<td>2700 kg/m³</td>
</tr>
<tr>
<td>Density of Liquid</td>
<td>2700 kg/m³</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>209.3 W/m K</td>
</tr>
<tr>
<td>Ambient Temperature</td>
<td>15 °C</td>
</tr>
</tbody>
</table>

Table III  Thermophysical Properties of Aluminium used in Conventional Flood Cooling Simulations.
obtained from temperature measurements in the mould (33, 84). In order to account for the air-gap formation in the lower part of the mould, the heat-transfer coefficient was decreased to 209 W/m²K for the surface nodes in the remaining two slices inside the mould. For all surface nodes below the mould, a very high heat-transfer coefficient has been used to reflect the intense cooling within the flood-water zone. The value of this heat-transfer coefficient which is 14.65 kW/m²K was arrived at, by trial and error methods, by comparing measured and predicted pool depths for a section 381 x 991 mm cast at 1.778 mm/s. From these runs it was very clear that the heat-transfer coefficient because of its high value, had a very small effect on the pool depths. Thus it was possible to check the model by comparing the pool depths obtained for different casting speeds for the same set of boundary conditions. Finally in these calculations the latent heat was released over a 1°C interval.

For the 381 x 991 mm section the casting speeds ranged from 1.185 mm/s to 2.116 mm/s. Fig. 4.1 shows the comparison between calculated and measured pool profiles for a casting speed of 1.778 mm/s. The measured profiles have been obtained by adding zinc
Fig. 4.1 Comparison between the predicted and measured pool profiles for 381 x 991 mm aluminium ingot cast at 1.778 mm/s (obtained at the mid-plane parallel to the narrow face).
and lead tracer (84) to the molten pool. As can be seen there is an excellent agreement between the two profiles. It should be noted that the staircase pattern of the calculated pool profile which results from the coarseness of the finite-difference mesh and the narrow range over which the latent heat is released has been smoothened out in Fig. 4.1. It may also be noted that the effect of the high heat-transfer coefficient in the sub-mould region is felt higher up in the casting because of axial heat conduction. Of the total heat removed from the start of cooling to the bottom of the pool, less than 5% is removed in the mould region.

The three-dimensional temperature distribution in the casting calculated from the model is presented in Table A3.I of Appendix 3. Fig. 4.2 shows pool profiles obtained at the longitudinal mid-face planes after steady state conditions have been reached. The notation used for the various directions are as follows. The casting direction was always taken as z-axis. In the transverse plane x-axis was taken perpendicular to the broad face and y-axis perpendicular to the narrow face. In all the calculated pool profiles shown in this work, the top two node slices are at the pouring temperature; and this should be taken
Fig. 4.2 Steady state pool profiles obtained at the longitudinal mid-planes for 381 x 991 mm aluminium ingot cast at 1.778 mm/s.
note of when calculating the pool depth from these contours. The profile obtained in the longitudinal mid-plane parallel to broad face is in agreement with the bucket shaped pool observed across the width of the section.

In order to study the effect of heat conduction in the second transverse direction, on the overall heat transfer, the computer program was run in a two-dimensional mode. Here heat flow perpendicular to the narrow face (y-direction) was neglected. The calculations were performed by setting the heat-transfer coefficient equal to zero in the y-direction and at the same time reducing the number of nodes in the y-direction to a minimum of three. Fig. 4.3 shows the comparison between the two and three dimensional calculations, obtained in the longitudinal mid-plane parallel to the narrow face. It is seen that there is negligible difference between the two pool profiles. As will be shown in Chapter 5, if the aspect ratio exceeds 2.5 for conventional D.C. casting, there is no need to include the heat flow in a direction perpendicular to the narrow face (y-direction) in the heat-flow calculations. This is an important result because computing costs for the three
Fig. 4.3 Comparison of the pool profiles from the two-dimensional and three-dimensional calculations for casting 381 x 991 mm aluminium ingot at 1.778 mm/s.
dimensional model are about 5 to 8 times more than for the two dimensional case. Therefore considerable computer costs can be saved by safely making the assumption of two dimensional heat conduction. In all the validation runs presented in this chapter for conventional D.C. casting, wherever the aspect ratios are greater than 2.5, the model has been run only in a two dimensional mode.

Fig. 4.4 shows a comparison between the measured and calculated pool depths as a function of casting speed for 381 x 991 mm aluminium section. In all the cases the pool depth has been measured by using a steel wire to probe the bottom of the pool; the accuracy of these measurements is within ±10 mm (84). The agreement between the calculated and measured pool depths are excellent. The pool profiles obtained at the different casting speeds are shown in Fig. 4.5.

4.2.2 Aluminium Ingots : 457 x 1143 mm

The boundary conditions used here are identical to those given in section 4.2.1. This is the largest of the four sections simulated under this category. The casting speeds used range from 0.974 mm/s to
Fig. 4.4 Comparison between the measured and calculated pool depths for 381 x 991 mm aluminium ingot cast at different speeds.
Fig. 4.5 Calculated pool profiles for 381 x 991 mm aluminium ingot cast at different speeds.
2.117 mm/s. Fig. 4.6 shows the comparison between the calculated and measured pool depths for the five different casting speeds. Although the match is not perfect the difference between the calculated and measured pool depths was only 30 mm or 4% of the measured sump depth. For this section size with an aspect ratio of 2.5, a small difference was observed in the pool depths obtained between a two dimensional and three dimensional calculations at a speed of 2.117 mm/s. However no differences were observed at lower speeds.

4.2.3 Aluminium Ingots: 305 x 1010 mm.

For this section size the comparison between the calculated and measured pool depth is shown in Fig. 4.7; and like the previous cases, close match is obtained between the two.

4.2.4 Aluminium Ingot: 229 x 813 mm

This is the smallest of the four sections simulated and higher casting speeds have been employed, the maximum of which is 2.794 mm/s. Fig. 4.8 shows the comparison between the measured and calculated pool depths.
aluminum ingot - 457 mm x 1143 mm
conventional d.c. cooling
• calculated
○ measured

Fig. 4.6 Comparison between the calculated and measured pool depths for 457 x 1143 mm aluminium ingot cast at different speeds.
Fig. 4.7  Comparison between the calculated and measured pool depth for 305 x 1010 mm aluminium ingot cast at different speeds.
Fig. 4.8 Comparison between the calculated and measured pool depths for 229 x 813 mm aluminium ingot cast at different speeds.
4.2.5 Summary of Conventional D.C. Casting Simulations

Table IV summarises the calculated and measured pool depths obtained for the various casting conditions. Thus in the case of conventional D.C. cooling, comparison has been made for four different sections ranging in size from 229 x 813 mm to 457 x 1153 mm. In addition for each section at least four different casting speeds have been used to check the validity of the model. In all cases good agreement has been obtained between measured and predicted values of the pool depths.

Figure 4.9 shows a plot of the time required to reach steady state against the casting speed for the various section sizes. The time to reach steady state is defined as the time taken for the molten pool to cease growing from the start of casting. For 229 mm and 381 mm thick sections the casting speed has no effect on the time to reach steady state. However for thicker sections increasing casting speed causes a small increase in this time. Going from 229 to 457 mm thick sections, the time to reach steady state increases from 125 s to 450 s. It should be noted that in any normal casting operation the maximum casting speed is reached not instantaneously but over a finite amount of time, usually around 1
<table>
<thead>
<tr>
<th>Section Size</th>
<th>Casting Speed (mm/s)</th>
<th>Pool Depth Measured (mm)</th>
<th>Pool Depth Calculated (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>229mm x 813mm</td>
<td>1.524</td>
<td>184</td>
<td>175</td>
</tr>
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<td></td>
<td>1.905</td>
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<td></td>
<td>2.794</td>
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<td>2.159</td>
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<td>396</td>
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<td></td>
<td>1.439</td>
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<td></td>
<td>1.778</td>
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<td></td>
<td>2.116</td>
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<td></td>
<td>2.117</td>
<td>762</td>
<td>778</td>
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</table>

Table IV  Comparison between calculated and measured pool depths for Aluminium sections cast at different speeds.
Fig. 4.9 Time required for the pool profiles to reach steady state for aluminium ingots of various sizes as a function of casting speed.
to 2 minutes. Thus in the case of thinner sections by the time the steady casting speed is reached, the casting would have settled down to steady state conditions. The importance of unsteady state increases with increasing section thicknesses. A further discussion of this is made in Chapter 5.

4.3 Reduced Secondary Cooling - British Aluminium

In this cooling practice the mould water is not applied immediately below the mould. In its place a mild cooling is effected by the use of fine air-atomised sprays. This results in a very coarse cell structure suitable for deep drawing applications. Beattie et al (30) have measured the heat-transfer coefficients for these sprays under various operating conditions in the laboratory. The thermophysical properties employed in this simulation are given in Table V, while boundary conditions used are presented in Table VI. The computer program has been altered to accommodate the different values of thermal conductivity for the solid and liquid regions.

The section size used in this simulation is 254 x 690 mm, while the casting speed is 0.833 mm/s. The development of the molten pool computed as a function of time is presented in Figs. 4.10 to 4.12, which show the pool
<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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<tbody>
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<td>Specific Heat of Solid</td>
<td>1.13 J/g K</td>
</tr>
<tr>
<td>Specific Heat of Liquid</td>
<td>1.13 J/g K</td>
</tr>
<tr>
<td>Latent Heat of Fusion</td>
<td>387 J/g</td>
</tr>
<tr>
<td>Liquidus Temperature</td>
<td>658°C</td>
</tr>
<tr>
<td>Solidus Temperature</td>
<td>635°C</td>
</tr>
<tr>
<td>Density</td>
<td>2700 kg/m³</td>
</tr>
<tr>
<td>Thermal Conductivity of Solid</td>
<td>222 W/mK</td>
</tr>
<tr>
<td>Thermal Conductivity of Liquid</td>
<td>105 W/mK</td>
</tr>
</tbody>
</table>

Table V  Thermophysical Properties of Aluminium used in Reduced Secondary Cooling Simulations.
<table>
<thead>
<tr>
<th>Zone No.</th>
<th>Position Below Liquid Surface (mm)</th>
<th>Heat Transfer Coefficient (W/m² K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (mould)</td>
<td>0 - 52</td>
<td>921</td>
</tr>
<tr>
<td>2</td>
<td>52 - 78</td>
<td>1842</td>
</tr>
<tr>
<td>3</td>
<td>78 - 104</td>
<td>938</td>
</tr>
<tr>
<td>4</td>
<td>104 - 156</td>
<td>663</td>
</tr>
<tr>
<td>5</td>
<td>156 - 208</td>
<td>622</td>
</tr>
<tr>
<td>6</td>
<td>208 - 286</td>
<td>580</td>
</tr>
<tr>
<td>7</td>
<td>286 - 364</td>
<td>538</td>
</tr>
<tr>
<td>8</td>
<td>364 - 442</td>
<td>953</td>
</tr>
<tr>
<td>9</td>
<td>442 - 520</td>
<td>290</td>
</tr>
<tr>
<td>10</td>
<td>520 - downwards</td>
<td>166</td>
</tr>
</tbody>
</table>

**Table VI**  
Heat Transfer Coefficients used as a function of position below the liquid surface, for reduced secondary cooling.
Fig. 4.10  Liquidus and solidus profiles obtained at the longitudinal mid-planes of 254 x 690 mm aluminium ingot cast under reduced cooling conditions at 0.833 mm/s, after 312 s from start.
Fig. 4.11  Liquidus and solidus profiles obtained at the longitudinal mid-planes of 254 x 690 mm aluminium ingot cast under reduced cooling conditions at 0.833 mm/s, after 624 s from start.
Fig. 4.12 Liquidus and solidus profiles obtained at the longitudinal mid-planes of 254 x 690 mm aluminium ingot cast under reduced secondary cooling conditions at 0.833 mm/s, after 1248 s from start.
profiles at the longitudinal mid-face planes. The steady state pool depth obtained was 470 mm in comparison with the measured value of 510 mm. The three-dimensional temperature distributions computed from the model are presented in Table A3-II in Appendix 3. The effect of neglecting heat conduction in a direction normal to the narrow face (y-direction) is shown in Fig. 4.13, in the form of pool profiles obtained in the longitudinal mid-plane parallel to the narrow face. Unlike the case of conventional D.C. cooling an effect owing to heat conduction in both the transverse directions is seen even though the aspect ratio is 2.79. The calculations revealed that for this rather small section, steady state is reached only 15 minutes from the start of cooling. A three dimensional view of the pool surface is shown in Fig. 4.14.

An attempt to simulate an increased casting speed of 1.266 mm/s with the same boundary condition was less successful. Here the pool depth obtained from the simulation was 1014 mm. When experiments were carried out at British Aluminium to cast this 254 x 690 mm section at 1.266 mm/s difficulties were encountered in obtaining a stable pool (85). Even in three of the more successful experiments the pool depths ranged from 760 mm to 860 mm. Thus the calculated value from the model falls short of the highest pool depth measured.
Fig. 4.13 Comparison of steady state pool profiles for the two-dimensional and the three-dimensional calculations of 254 x 690 mm aluminium ingot cast under reduced secondary cooling conditions at .833 mm/s.
Fig. 4.14 Three-dimensional visualization of liquid pool surface of 254 x 690 mm aluminium ingot cast of .833 mm/s under reduced secondary cooling, as seen from the different angles.
The reason for the difference between the measured and calculated pool depths can be attributed to the uncertainty in the precise value of the heat-transfer coefficients. Numerical calculations performed by varying the value of the heat-transfer coefficients showed that the pool depth is a very sensitive function of this variable. This is in contrast to the conditions in conventional D.C. cooling, where the heat transfer coefficient had a very small effect on the pool depth. The uncertainty in the boundary condition stems from the fact that, in this simulation the surface of the casting stays at 360°C even at the bottom of the strand. Thus for considerable length of the casting from the top of the mould, the heat transfer mechanism would be in the film boiling regime. Depending on the temperature at which the mechanism changes to nucleate boiling, the heat-transfer coefficient will change in magnitude. It is felt that under these conditions it is very difficult to predict the heat-transfer coefficients a priori from the laboratory experiments, and measurements would have to be carried out in plant to characterize the boundary conditions properly.

The model calculations show that under these slow cooling conditions, it is essential to consider heat flow in both transverse directions and should not be neglected in one of the directions as in the model of Beattie (30, 32), even though the aspect ratios are greater than 2.5.
4.4 Zinc - Jumbo Casting - Cominco

Fig. 4.15 shows the cross-section of a jumbo ingot and the various dimensions of the casting. Because of the absence of symmetry elements in one of the transverse direction, calculations must be performed for one-half of a casting. In this simulation a full three dimensional model is required because, the aspect ratio is 1.07. The material cast is high grade zinc for which the thermo-physical properties are given in Table VII.

The use of the finite-difference method for irregular geometries such as that of the jumbo does not pose any special problems in principle. However a new set of equations had to be included in the computer program to take care of the irregularly shaped nodes. The manner in which the casting has been discretized is presented in Fig. 4.16. Since it was required to perform the calculations for one-half of a casting, in order to cut down the computer costs slightly, a variable node size has been used in this simulation. The sub-division of the jumbo has been made such that a finer node exists at the surface and coarser nodes at the centre.

The boundary conditions for this simulation were obtained by freezing in thermocouples near the surface of
Fig. 4.15 A cross section of the jumbo ingot with the various dimensions given in mm.
Fig. 4.16  Discretization of the jumbo ingot for the finite-difference calculations.
<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density of Liquid</td>
<td>6620 kg/m$^3$</td>
</tr>
<tr>
<td>Density of Solid</td>
<td>6981 kg/m$^3$</td>
</tr>
<tr>
<td>Specific Heat of Liquid</td>
<td>-480 J/g K</td>
</tr>
<tr>
<td>Specific Heat of Solid</td>
<td>Zinc - 0.343 + 0.154 (10^{-3}) T J/g K</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>113 W/m$^2$ K</td>
</tr>
<tr>
<td>Latent Heat of Fusion</td>
<td>113 J/g</td>
</tr>
<tr>
<td>Liquidus Temperature</td>
<td>420°C</td>
</tr>
<tr>
<td>Solidus Temperature</td>
<td>410°C</td>
</tr>
</tbody>
</table>

**Table VII** Thermophysical Properties of Zinc used in Jumbo Ingot Simulation.
the jumbo (~20 mm) during a casting and letting them drop with the descending ingot. The exact position of the thermocouple tip was located subsequently by sectioning the ingot. From the temperature-time plot measured, the surface heat-transfer coefficient was back-calculated by a trial and error procedure. Table VIII gives the heat transfer coefficient used in this simulation. Fig. 4.17 compares the measured temperature with the calculated values.

The pool profile was obtained for a casting speed of 1.27 mm/s by adding an alloy of 10% copper-in-zinc to the sump after steady state conditions had been reached. After the addition, the pool was stirred using a paddle, driven by a power drill. The ingot was later sectioned longitudinally at the mid-plane perpendicular to the two non-notched faces. The surface then was gently polished to remove the machine marks. The pool boundary was finally delineated by etching with a 4% Nital solution. The longitudinal sections where the contour profiles have been obtained from the model are shown in Fig. 4.18, as shaded. Figs. 4.19 to 4.21, shows the contour profiles obtained at these two sections, at different times from the start of the casting. Fig. 4.21 correspond to steady state conditions whereby the pool profile remains the same with
<table>
<thead>
<tr>
<th>Distance from the top of Mould (mm)</th>
<th>Heat Transfer Coefficients (W/m² K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 212</td>
<td>20934</td>
</tr>
<tr>
<td>212 - 216</td>
<td>4187</td>
</tr>
<tr>
<td>1450 - 1454</td>
<td>125</td>
</tr>
</tbody>
</table>

Between 212 and 1454 mm the Heat Transfer Coefficient has been dropped in an exponentical fashion.

Table VIII Heat Transfer Coefficients used in Zinc Jumbo Casting.
Fig. 4.17 Comparison of the measured and calculated temperature profiles for the zinc jumbo ingot cast at 1.69 mm/s.
Fig. 4.18 A three-dimensional view of the jumbo ingot showing the longitudinal sections where the pool profiles have been obtained.
Fig. 4.19 Pool profiles obtained at the longitudinal sections shown in Fig. 4.18 for casting zinc jumbo ingot at 1.27 mm/s, 278 s after the start.
Fig. 4.20  Pool profiles obtained at the longitudinal sections shown in Fig. 4.18 for casting zinc jumbo ingot at 1.27 mm/s, 557 s after the start.
Fig. 4.21  Steady state pool profiles obtained at the longitudinal sections shown in Fig. 4.18 for casting zinc jumbo ingot at 1.27 mm/s.
further increase in time. The three dimensional steady state temperature distribution is given in Table A3-III.

Comparison between measured and calculated pool profiles for the zinc jumbo is seen in Fig. 4.22. As can be seen there is a good agreement between the two. The shifting of the bottom of the sump to an asymmetrical position can be clearly seen in this figure. A three dimensional view of the pool surface is presented in Fig. 4.23.

The development of the pool for this casting speed has also been monitored by dipping a steel rod into the pool. The results obtained are presented in Fig. 4.24. In this graph the pool depth is plotted against the length of the casting at any instant. Note that the shell does not really commence growth at the bottom until after 500 mm of casting. This is due to the fact that the heat has to diffuse from the centre of the casting only through the sides as no cooling is provided on the bottom.

Fig. 4.25 shows a similar plot obtained for a higher casting speed of 1.693 mm/s. As in the previous case, good agreement is obtained between the calculated and measured values. The time required for steady state conditions can be calculated by dividing the cast length
Fig. 4.22 Comparison between the calculated and measured pool profiles for the casting of zinc jumbo ingot at 1.27 mm/s.
Fig. 4.23  Three-dimensional visualization of the liquid pool surface of zinc jumbo ingot cast at 1.27 mm/s.
Fig. 4.24 Comparison between the calculated and measured pool depths obtained at different times from the start of casting of zinc jumbo ingot cast at 76 mm/min.
Fig. 4.25 Comparison between the calculated and measured pool depths obtained at different times from the start of casting of zinc jumbo ingot cast at 102 mm/min.
where the curve becomes flat by the casting speed. It is seen from this and the previous Fig. 4.24, that it takes just over 10 minutes for steady state conditions to prevail. Since the total duration of the casting is only around 40 minutes, this means that for 25% of the total casting time the casting is in an unsteady state.

Fig. 4.26 shows the freezing of the ingot as seen from a transverse section. Here the solidus isotherm for different position along the casting direction have all been compressed on to the plane of the paper. The shape of the freezing front obtained in an actual cast can be seen as faint rings in Fig. 4.27. The rings in Fig. 4.27 are caused by the intermittent stirring of the pool during the casting of a dilute alloy of lead in zinc.

Finally the importance of including the notch in the calculation is shown in Fig. 4.28, which shows pool profiles calculated with and without the notch. As can be seen there is a substantial effect of the notch on the pool depths.

4.5 Summary of Validation Runs

The model has been validated using measurements from
Fig. 4.26 Freezing profiles as seen on a jumbo cross-section.
Fig. 4.27 Macrostructure of the zinc jumbo cross-section showing the freezing lines seen in Fig. 4.26.
Fig. 4.28 Comparison of the calculated pool profiles obtained with and without the notch for casting zinc at 1.69 mm/s.
three D.C. casting operations: two for the case of aluminium and one for the case of zinc. Excellent agreement is obtained between calculated and measured pool profiles in the aluminium ingots cooled by conventional flood cooling. In the case of aluminium blocks subjected to reduced secondary cooling the simulation from the model was less successful for higher casting speeds. This has been attributed to uncertainty in characterizing the boundary conditions from the results obtained in the laboratory experiments. In the case of zinc jumbo good agreement has been observed between measured and predicted pool profiles.
Chapter 5

EFFECT OF CASTING VARIABLES ON HEAT FLOW

5.1 Introduction

In this chapter a description is given of how the model, developed in the previous section, was used in a predictive mode to study the effect of different casting variables on the overall heat flow. In order to make these calculations meaningful, all the simulations presented in this section have been carried out under casting conditions obtainable in industrial practice. In this way the importance of considering heat flow in the transverse dimensions is brought out, with specific examples involving the casting of various sections of aluminium and zinc. The importance of axial conduction in comparison with bulk motion of the casting is demonstrated by considering different casting speeds. The effect of thermal conductivity on the overall heat flow is studied by comparing simulations of aluminium and zinc casting. Finally some comments are made regarding variables which have negligible effect on heat flow but a very profound influence on the metallurgical structure.
5.2 Effect of Aspect Ratio*

In the modelling of heat flow in rectangular slabs it has been common practice to neglect heat flow in the transverse direction, that is, parallel to the broad face (30, 33, 48). In this way the programming effort and the computational costs are reduced considerably as compared to the case of a three-dimensional model.

The importance of heat flow in the transverse direction has been investigated with specific examples of the casting of aluminium and zinc ingots with different aspect ratios. In these calculations the cooling conditions used are similar to those discussed in Chapter 4 pertaining to conventional D.C. cooling. The casting speed employed was 1.778 mm/s for all the calculations.

Fig. 5.1 shows the steady-state pool depth obtained in casting aluminium ingots with various section sizes. The bottom curve corresponds to sections of 381 x 381 mm, 381 x 571 mm, 381 x 762 mm and finally a 381 mm thick aluminium slab of infinite width. The latter two-dimensional calculation was carried out by reducing the number of nodes

*The aspect ratio is defined as the ratio between the two transverse dimensions with the larger value taken as the numerator.
Fig. 5.1 Effect of aspect ratios on the pool depths in casting 381 mm and 457.2 mm thick aluminium slabs at 1.778 mm/s.
in the width direction to three while the heat-transfer coefficient in this direction was set equal to zero. Thus no gradients are imposed in the width direction and identical temperatures are calculated by the program for the three rows of nodes. From Fig. 5.1 it can be clearly seen that the effect of the second transverse dimension diminishes as the aspect ratio exceeds 2.5. Thus for heat flow calculations in rectangular slabs with aspect ratios greater than 2.5, it is adequate to consider two dimensions only.

The top curve in Fig. 5.1 corresponds to sections of 457 x 457 mm, 457 x 686 mm, 457 x 914 mm, 457 x 1143 mm and a 457 mm thick slab of infinite width. Compared to the lower curve the transition from three-dimensional to two-dimensional heat flow occurs at slightly higher values of the aspect ratio. But even in this case the effect is negligible beyond an aspect ratio of 2.5. This conclusion is valid for all casting speeds lower than the one used in this calculation, namely 1.778 mm/s, as well as for sections smaller than 457 mm in thickness. In addition this will also apply to cooling conditions which are more intense than those used in the present calculations.

Fig. 5.2 shows the calculated steady-state pool
Fig. 5.2 Effect of aspect ratios on the pool depths in casting 381 mm thick zinc slabs at 1.78 and 2.2 mm/s.
depth plotted against the aspect ratio for zinc cast at two speeds. The thickness of the section considered is 381 mm. The top curve corresponds to a casting speed of 2.2 mm/s while the bottom curve corresponds to 1.778 mm/s. In comparison with aluminium the on-set of two-dimensional heat flow is seen at even lower aspect ratios. Thus it is adequate to consider two-dimensional heat flow for aspect ratios exceeding 2.0. The same result also holds for higher casting speeds.

The above discussion on the relative importance of two-and three-dimensional heat flow has been made with respect to conventional D.C. cooling. However the application of a reduced secondary cooling practice such as is followed at British Aluminium (30, 85) changes this picture.

Two-and three-dimensional calculations have been performed on a 250 x 690 mm section (aspect ratio of 2.76) with these cooling conditions. Table IX presents the pool depth obtained from the two sets of calculations for two different casting speeds: 0.833 mm/s and 1.26 mm/s. It is seen that there is a significant difference between the two calculations even though the aspect ratio exceeds 2.5 and the discrepancy increases as the casting speed is increased. Thus error introduced into the pool-depth calculations increases from 10% at 0.833 mm/s to 20.5% at 1.26 mm/s.
Table IX  Comparison between three dimensional and two dimensional pool depths for reduced secondary cooling of 254 x 690 mm aluminium ingot.

<table>
<thead>
<tr>
<th>Speed mm/s</th>
<th>Pool Depth 3-Dimensional</th>
<th>Pool Depth 2-Dimensional</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.833</td>
<td>468 mm</td>
<td>520 mm</td>
</tr>
<tr>
<td>1.266</td>
<td>1014 mm</td>
<td>1222 mm</td>
</tr>
</tbody>
</table>
As will be seen in the following section it appears better for this cooling condition to neglect axial conduction but consider heat flow in both transverse directions.

5.3 Effect of Axial Conduction

It has been traditional practice in almost all the models developed to date for non-ferrous casting, to include the effect of axial conduction. In order to check the importance of the z-component of heat conduction, the computer program was modified by bypassing the implicit calculations in the z-direction. In these calculations there is only one slice in the z-direction and the solidification is followed as this slice travels from the meniscus downwards at the casting speed. This is the procedure followed in models of the continuous casting process for steel.

Fig. 5.3 shows the surface temperature at the mid-face of a 457 mm square aluminium ingot vs distance below the meniscus calculated with and without axial conduction. As expected, the curve obtained by including the axial conduction is the smoother of the two. The rebound of the surface temperature for the case of no axial conduction is caused by the formation of an air gap in the mould.
Fig. 5.3 Calculated surface temperature profiles obtained at the mid-face of 457 x 457 mm aluminium ingot cast at .974 mm/s, with and without the axial conduction.
Furthermore the curve without the axial conduction also shows a much steeper temperature gradient corresponding to direct chilling in the secondary zone.

Fig. 5.4 shows the centre temperature of a 457 mm square aluminium ingot vs distance below the meniscus. As before the two conditions considered are with and without axial conduction. It can be seen that by not including the axial conduction the pool depth increases from 254 to 302 mm, that is by about 20%.

Table X shows the importance of including axial conduction for various casting conditions. Several important points can be drawn from this table. It is seen that the importance of axial conduction decreases with increase in casting speed. Thus for 457 mm square aluminium ingot the differences between the two calculations decrease from 20% at a casting speed of 0.974 mm/s, to 3% at 2.116 mm/s. In the case of slabs the effect of casting speed is much less pronounced. For a 457 mm thick aluminium slab the difference between the two calculations is 9% at 0.974 mm/s and 4% at 2.116 mm/s. Since billets are cast at faster speeds than slabs, this means it is possible to neglect axial conduction in the billet case. Thus it would have been a better assumption for Jovic et al. (35) in modelling
Fig. 5.4 Calculated centre temperature profiles obtained for 457 x 457 mm aluminium ingot cast at .974 mm/s, with and without the axial conduction.
<table>
<thead>
<tr>
<th>Material</th>
<th>Section Size (mm)</th>
<th>Cooling</th>
<th>Casting Speed (mm/s)</th>
<th>Pool Depth with Axial Conduction (mm)</th>
<th>Pool Depth without Axial Conduction (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A^z</td>
<td>457 x 457</td>
<td>Conventional</td>
<td>0.974</td>
<td>254</td>
<td>302</td>
</tr>
<tr>
<td>A^z</td>
<td>457 x 457</td>
<td>Conventional</td>
<td>2.116</td>
<td>572</td>
<td>587</td>
</tr>
<tr>
<td>A^z</td>
<td>457 x ∞</td>
<td>Conventional</td>
<td>0.974</td>
<td>365</td>
<td>397</td>
</tr>
<tr>
<td>A^z</td>
<td>457 x ∞</td>
<td>Conventional</td>
<td>2.116</td>
<td>794</td>
<td>826</td>
</tr>
<tr>
<td>Zn</td>
<td>457 x 457</td>
<td>Conventional</td>
<td>0.974</td>
<td>746</td>
<td>810</td>
</tr>
<tr>
<td>A^x</td>
<td>254 x 690</td>
<td>Reduced Secondary</td>
<td>0.833</td>
<td>468</td>
<td>494</td>
</tr>
</tbody>
</table>

Table X  Steady state pool depths for aluminium and zinc ingots calculated with and without the axial conduction.
heat flow in a 360 x 1600 mm section to include axial conduction, but neglect conduction in a direction perpendicular to the narrow face. The effect of thermal conductivity is to decrease the importance of axial conduction with a decrease in thermal conductivity. Thus for 457 mm square ingots of aluminium and zinc cast at 0.974 mm/s, the differences between the case of axial conduction and no axial are 20% and 9% respectively.

Finally the importance of axial heat conduction for the case of reduced secondary cooling has been studied for the casting of a 254 x 690 mm aluminium section at 0.833 mm/s. The boundary conditions employed are given in Table VI, in Chapter 4. Compared to previous cases the difference between the two pool depths, namely 468 mm and 494 mm, is only 5% even at this low casting speed. At higher speeds the differences would be even less significant.

The results obtained in this part of the work are opposite to the effects reported in section 5.2 where neglecting conduction in one of the transverse directions was discussed. In that case it was seen that an increase in the casting speed increased the error in the two-dimensional calculations. Therefore when developing
two-dimensional models in these systems it is safer to
neglect axial conduction but consider heat flow in both
transverse directions.

5.4 Importance of Unsteady State

In most of the models developed to date, including
those for cylindrical shapes with axial symmetry, steady-
state conditions have been assumed. However it is important
to note that unlike the continuous casting of steel where
the casting operation is truly continuous, the vertical D.C.
casting is only semi-continuous in nature. Therefore the
initial transient part of the casting may be a significant
fraction of the casting cycle. Since the model developed
in this work can be used to study transient effects the
importance of unsteady state has been investigated.

The unsteady state has been studied experimentally
by Sergerie and Bryson (14) who have discussed the problem
of ingot "bowing" observed in the transient section of
large sheet ingot casting. This results from the use of
short moulds in which the first metal freezing on the stool
cap feels the strong effects of a direct quench sooner than
it would if larger moulds were used. During 'bowing' which
occurs as the butt emerges from the mould, the ends of the
butt shrink upwards off the stool cap and inwards away from
the ends of the mould. The authors have patented a "pulsed cooling" method to reduce the heat transfer rate in the transient portion by using a pulsed water spray.

Recently Yu (43) has tackled the same problem by drastically changing the heat transfer mechanism at the ingot surface through the use of water containing dissolved carbon dioxide. As the cooling water exits from the mould the dissolved gas evolves as micron-size bubbles forming a temporary, effective insulation layer on the surface of the casting, thereby reducing the heat transfer. The importance of cooling in the transient portion can be appreciated from the fact that both of the above-mentioned ideas have been patented.

The temperature field in the ingot at the start of the cast is different from the temperature in the steady-state part of the casting which follows. Thus special measures may need be taken to prevent cracking in the early transient part of the cast, since the crack, once initiated, can continue to propagate in the steady state even though the steady-state cooling conditions may not generate cracks per se. Fig. 5.5. shows the calculated surface temperature profiles at the mid-plane on the bottom face of a jumbo cross-section. The metal cast is aluminium at a speed of
Fig. 5.5 Calculated surface temperature profiles for the initial and steady state slices for casting aluminium jumbo ingot (at the bottom mid-face of a jumbo section).
1.35 mm/s. The two profiles correspond to the temperature histories of two slices leaving the mould at different times, one corresponding to the first slice exiting the mould (dashed line) and the other corresponding to a slice cast under steady-state conditions (solid line). It should be noted that steeper axial temperature gradients are set-up in the initial slice compared to the steady-state slice; and this possibly could initiate surface cracks.

The time required for the pool profile to reach steady state will depend on a number of factors including the metal cast, section size and casting speed. This transient time was presented as a function of casting speed for casting aluminium slabs of various thicknesses in Fig. 4.6 of Chapter 4. It was seen that for smaller sections, casting speed had no effect on this time while for thicker sections it had a small effect.

Fig. 5.6 shows the transient time for casting various sections of aluminium and zinc. Because of the lower value of thermal conductivity in the case of zinc it takes a longer time for steady state conditions to be achieved.

The effect of bottom heat-transfer coefficient on the transient portion was studied with the model. The
Fig. 5.6 Time required for the pool profiles to reach steady state for 381 mm thick aluminium and zinc slabs of different aspect ratios cast at 1.778 mm/s.
value of heat-transfer coefficients as high as 418 \text{w/m}^2\text{K} had negligible effect in conventional D.C. cooling, while the effect was greater with reduced secondary cooling. The value of bottom heat-transfer coefficient did not affect the steady state temperature field.

5.5 Effect of Section Size

The effect of section size has been investigated in casting square sections of aluminium and zinc. The section sizes considered were 305 x 305 mm, 381 x 381 mm and 457 x 457 mm. In all the cases the casting has been maintained at 1.778 mm/s. Fig. 5.7 shows the steady-state pool depths obtained in casting sections mentioned above under D.C. cooling conditions. Comparing aluminium and zinc it is found that the low thermal conductivity of zinc is responsible for the much steeper increase in the pool depth with increasing section size. The dramatic effect of thermal conductivity on the pool shape can be appreciated from Fig. 5.8, where the pool profiles for 381 x 381 mm sections of aluminium and zinc are also presented. Fig. 5.9 shows the time required for steady state to be achieved. It can be seen that in the case of aluminium, because of its high thermal conductivity, the transient time keeps pace with section size resulting in a linear relationship between the two.
Fig. 5.7 Steady state pool depths for square sections of aluminium and zinc cast at 1.778 mm/s.
Fig. 5.8 Calculated steady state pool profiles for 381 mm square sections of aluminium and zinc cast at 1.778 mm/s.
Fig. 5.9 Time required for the pool profiles to reach steady state for square sections of aluminium and zinc cast at 1.778 mm/s.
5.6 Effect of Super Heat

In order to compensate for the drop in temperature of the molten metal during transfer into the mould, a quantity of superheat is provided to the liquid metal. In this analysis it has been found that super heat has a very minor effect on the location of the solidus isotherm. It was also seen that most of the superheat was removed in the very top portion of the casting.

5.7 Effect of Cooling Conditions

The effect of sub-mould cooling conditions has already been considered in Chapter 4 in connection with validation of the model where the difference between conventional D.C. cooling and reduced secondary cooling of aluminium slabs was seen. Fig. 5.10 shows the relationship between the heat-transfer coefficient below the mould and the steady-state pool depth for the casting of 610 x 546 mm zinc sections at 1 mm/s. In these calculations, the heat-transfer coefficient is kept constant, at the appropriate value, for the entire length of the strand. It is observed that as the heat-transfer coefficient is decreased, there is a steep increase in the pool depth.
Fig. 5.10 Effect of the heat-transfer coefficient on the steady state pool depths for casting 610 x 546 mm zinc ingot at 1 mm/s.
5.8 Effect of Latent Heat Release on Model Calculations

In the section dealing with the development of the mathematical model a detailed account was given of the manner in which latent heat is released. A number of computer runs were undertaken in the initial stages of model development to test the sensitivity of the calculations to this parameter. In all the calculations it was seen that the temperature range over which the latent heat was released had a small effect on the location of the solidus isotherm. However, as might be expected, the location of the liquidus isotherm was changed considerably. For example, in two calculations of the casting of zinc where the latent heat was released over a range of 10°C between 420°C and 410°C and over a range of 1°C between 419 and 420°C respectively, there was a difference of only 10 mm in the position of the bottom of the solidus isotherm, whereas there was close to 110 mm difference in the case of the liquidus isotherm. The effect of the manner of latent heat release decreases with increasing thermal conductivity of the metal.

It was discussed in Chapter 3 that while simulating heat flow in the D.C. casting of non-ferrous metals with high thermal conductivity, it is possible for nodes to jump from above the liquidus to below the solidus. A post iterative correction procedure has been used to overcome
this problem. However use of this correction routine in the program in conjunction with having a large time interval can create stability problems. A wider spread between the liquidus and the solidus temperature helps in improving the stability problems. In most of the calculations the latent heat has been released over a range of either 1°C or 10°C. In the case of simulations dealing with the casting of reduced secondary cooled aluminum slabs a wider range of 23°C has been used.

5.9 Summary

In this chapter, the results of the model study on the importance of the different heat flow variables have been reported for D.C. casting under a range of casting conditions. It was shown that in D.C. casting involving conventional secondary cooling, where the aspect ratio exceeds 2.5, it is adequate to consider heat flow in two dimensions, namely, the shorter of the transverse dimensions and the axial direction. In the case of reduced secondary cooling the axial conduction was seen to play a minor part compared to the transverse heat flows. Thus in modelling heat flows in these systems, the complexity of the problem may be reduced considerably by neglecting the axial heat conduction. The importance of the unsteady state and the
The effect of section size and cooling conditions have been demonstrated using specific examples of casting aluminium and zinc.
USE OF THE HEAT FLOW MODEL TO SOLVE A CRACKING PROBLEM IN THE D.C. CASTING OF PRIME WESTERN GRADE JUMBO INGOTS

6.1 Introduction

Formation of cracks has long been recognized as a problem in the D.C. casting of metals such as high strength aluminium alloys (12, 42, 47). Unlike the continuous casting of steel where many cracks are generated mechanically, e.g. by mould oscillation and bending and straightening operations, cracks in non-ferrous D.C. casting are most often caused by thermal stresses generated during casting.

Kreil et al (17) discuss the various cracks formed in casting copper and its alloys. Similarly Dieffenbach (12) proposed remedial measures for solving the cracking problems faced in casting different aluminium alloys. There are also other papers dealing with internal cracks in aluminium alloys (32, 40, 42, 47, 61-63, 67) and copper alloys (55, 56). No references have been found in the literature however concerned with the formation of cracks in the D.C. casting of zinc and its alloys.
6.2 Internal Cracks in the D.C. Casting of Prime Western Grade Zinc

Prime Western Grade zinc is an alloy of zinc containing approximately 1 wt% lead which is used in galvanizing applications. It has been found that D.C. casting of this particular alloy can give rise to severe internal cracking problems. In order to solve this problem it was necessary to study the cracks in detail, e.g. crack location and frequency, morphology of crack surfaces and thermal conditions prevailing at the time of crack formation.

Thus an experimental campaign was conducted in-plant at Cominco Ltd., Trail where a two strand D.C. casting machine has been in operation for many years for the production of Special High Grade zinc jumbos with a notched cross-section (Fig. 4.15). In this study a total of seven castings of Prime Western Grade zinc were made with the existing cooling assembly which is characterised by a short intense cooling zone below the mould; and the details of the various runs are presented in Table XI. Four of the runs were cast at 1.69 mm/s and the other three at 1.27 mm/s. These correspond to normal and slightly below normal casting speeds respectively that apply for the casting of High Grade zinc (99.99%). In the existing
<table>
<thead>
<tr>
<th>Run No.</th>
<th>Casting Speed</th>
<th>Pouring Temp</th>
<th>I Spray Pressure</th>
<th>II Spray Pressure</th>
<th>III Spray Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.69 mm/s</td>
<td>427°C</td>
<td>289.5 kPa</td>
<td>262 kPa</td>
<td>0*</td>
</tr>
<tr>
<td>2</td>
<td>1.27 mm/s</td>
<td>426°C</td>
<td>289.5 kPa</td>
<td>262 kPa</td>
<td>0*</td>
</tr>
<tr>
<td>3</td>
<td>1.69 mm/s</td>
<td>430°C</td>
<td>303.4 kPa</td>
<td>310.3 kPa</td>
<td>310.3 kPa**</td>
</tr>
<tr>
<td>4</td>
<td>1.27 mm/s</td>
<td>424°C</td>
<td>310.3 kPa</td>
<td>310.3 kPa</td>
<td>317.2 kPa**</td>
</tr>
<tr>
<td>5</td>
<td>1.69 mm/s</td>
<td>425°C</td>
<td>275.8 kPa</td>
<td>275.8 kPa</td>
<td>344.7 kPa</td>
</tr>
<tr>
<td>6</td>
<td>1.69 mm/s</td>
<td>425°C</td>
<td>248.2 kPa</td>
<td>262.0 kPa</td>
<td>262.0 kPa</td>
</tr>
<tr>
<td>7</td>
<td>1.27 mm/s</td>
<td>422°C</td>
<td>206.8 kPa</td>
<td>206.8 kPa</td>
<td>275.8 kPa</td>
</tr>
</tbody>
</table>

* Existing spray arrangement in both strands A and B
** Only 8 of the 16 nozzles in the third ring used
3 - 7 No sprays in the third ring for strand B
3, 4, 5 Third set of sprays placed 457 mm below the centre of second ring, in strand A
6, 7 Third set of sprays placed 125 mm below the centre of the second ring in strand A

Table XI  Casting speed conditions for the different runs during the experimental campaign.
cooling assembly each of the two strands is cooled by a set of two spray rings one impinging directly on the mould (flat spray) and the second located just below the mould. In some of the runs extra cooling was applied to one of the strands by a third spray ring. In order to obtain the thermal conditions that exist in the casting machine, thermocouples were inserted in the liquid pool from the top and frozen in near the surface of the casting during steady-state operation, and then were allowed to descend with the jumbo. Thus it was possible to characterize the heat extraction rates in the mould and sub-mould region as a function of distance below the mould. In most of the runs the length of the jumbo cast was about 3600 mm. The casting was finally cut into sections 610 mm long and the transverse sections were inspected for the presence of cracks. Whenever a major crack was seen its location and length were measured. Tables XII to XVII give the results of these measurements for each run respectively. All the cracks mentioned in these tables had a crack width ranging from 0.5 mm to 2.0 mm. In addition to these, fine hairline cracks were also observed but are not considered important, and therefore are not included in Tables XII to XVII. Sections containing cracks were also collected and taken to the laboratory for metallographic examination. Some of the sections were macro-etched to study the grain structure.
<table>
<thead>
<tr>
<th>Section No.</th>
<th>Bottom</th>
<th>Notch(Left)</th>
<th>Notch(Right)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A2</td>
<td>Surface - 160 mm</td>
<td>Surface - 160 mm</td>
<td>-</td>
</tr>
<tr>
<td>1A5</td>
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<td></td>
<td>30 - 125 mm</td>
<td>-</td>
</tr>
<tr>
<td>1A7</td>
<td>35 - 125 mm</td>
<td>30 - 120 mm</td>
<td>-</td>
</tr>
<tr>
<td>1B2</td>
<td>Surface - 155 mm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1B3</td>
<td>30 - 150 mm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1B4</td>
<td>30 - 140 mm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1B5</td>
<td>28 - 140 mm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1B6</td>
<td>28 - 147 mm</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table XII  Location of cracks in jumbo cross-sections taken at various points along the length of strands A and B (Run 1)
Table XIII Location of cracks in jumbo cross-sections taken at various points along the length of strands A and B (Run 2).

<table>
<thead>
<tr>
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<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2A4</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>2A5</td>
<td>48 - 112 mm</td>
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<td>35 - 150 mm</td>
</tr>
<tr>
<td>2A7</td>
<td>-</td>
<td>35 - 125 mm</td>
<td>30 - 132 mm</td>
</tr>
<tr>
<td>2B2</td>
<td>50 - 115 mm</td>
<td>-</td>
<td>55 - 105 mm</td>
</tr>
<tr>
<td>2B3</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2B4</td>
<td>-</td>
<td>-</td>
<td>40 - 135 mm</td>
</tr>
<tr>
<td>2B6</td>
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<td>-</td>
<td>45 - 145 mm</td>
</tr>
<tr>
<td>2B7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Section No.</td>
<td>Bottom</td>
<td>Notch(Left)</td>
<td>Notch(Right)</td>
</tr>
<tr>
<td>------------</td>
<td>------------</td>
<td>-------------</td>
<td>--------------</td>
</tr>
<tr>
<td>3A2</td>
<td>40 - 150 mm</td>
<td>Surface - 120 mm</td>
<td>40 - 130 mm</td>
</tr>
<tr>
<td>3A3</td>
<td>40 - 150 mm</td>
<td>-</td>
<td>40 - 135 mm</td>
</tr>
<tr>
<td>3A4</td>
<td>40 - 155 mm</td>
<td>-</td>
<td>35 - 140 mm</td>
</tr>
<tr>
<td>3A5</td>
<td>-</td>
<td>-</td>
<td>25 - 185 mm</td>
</tr>
<tr>
<td>3A6</td>
<td>-</td>
<td>Surface - 75 mm</td>
<td>30 - 120 mm</td>
</tr>
<tr>
<td>3A7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3B2</td>
<td>Surface - 150 mm</td>
<td>Surface - 125 mm</td>
<td>-</td>
</tr>
<tr>
<td>3B3</td>
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<td>-</td>
</tr>
<tr>
<td>3B4</td>
<td>35 - 160 mm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3B5</td>
<td>25 - 150 mm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3B6</td>
<td>30 - 140 mm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3B7</td>
<td>-</td>
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</table>

Table XIV  Location of cracks in jumbo cross-sections taken at various points along the length of strands A and B (Run 3).
<table>
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<th>Notch(Right)</th>
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</thead>
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<td>4A2</td>
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<td>Surface - 140 mm</td>
<td>Surface - 145 mm</td>
</tr>
<tr>
<td>4A3</td>
<td>-</td>
<td>Surface - 130 mm</td>
<td>Surface - 140 mm</td>
</tr>
<tr>
<td>4A5</td>
<td>-</td>
<td>Surface - 120 mm</td>
<td>30 - 140 mm</td>
</tr>
<tr>
<td>4A6</td>
<td>-</td>
<td>Surface - 100 mm</td>
<td>40 - 115 mm</td>
</tr>
<tr>
<td>4A7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4B2</td>
<td>-</td>
<td>Surface - 90 mm</td>
<td>-</td>
</tr>
<tr>
<td>4B4</td>
<td>-</td>
<td>Surface - 90 mm</td>
<td>-</td>
</tr>
<tr>
<td>4B5</td>
<td>-</td>
<td>Surface - 85 mm</td>
<td>-</td>
</tr>
<tr>
<td>4B6</td>
<td>-</td>
<td>Surface - 85 mm</td>
<td>-</td>
</tr>
<tr>
<td>4B7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table XV** Location of cracks in jumbo cross-sections taken at various points along the length of strands A and B (Run 4).
<table>
<thead>
<tr>
<th>Section No.</th>
<th>Bottom</th>
<th>Notch(Left)</th>
<th>Notch(Right)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5A1</td>
<td>Surface – 90 mm</td>
<td>Surface – 75 mm</td>
<td>Surface – 75 mm</td>
</tr>
<tr>
<td>5A2</td>
<td>Surface – 150 mm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5A3</td>
<td>Surface – 140 mm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5A4</td>
<td>Surface – 140 mm</td>
<td>-</td>
<td>Surface – 140 mm</td>
</tr>
<tr>
<td>5A5</td>
<td>Surface – 140 mm</td>
<td>-</td>
<td>Surface – 120 mm</td>
</tr>
<tr>
<td>5A6</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5B1</td>
<td>Surface – 90 mm</td>
<td>Surface – 90 mm</td>
<td>Surface – 80 mm</td>
</tr>
<tr>
<td>5B2</td>
<td>-</td>
<td>Surface – 105 mm</td>
<td>-</td>
</tr>
<tr>
<td>5B3</td>
<td>-</td>
<td>40 – 140 mm</td>
<td>-</td>
</tr>
<tr>
<td>5B4</td>
<td>-</td>
<td>45 – 150 mm</td>
<td>-</td>
</tr>
<tr>
<td>5B5</td>
<td>-</td>
<td>Surface – 110 mm</td>
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</tr>
<tr>
<td>5B6</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table XVI  Location of cracks in jumbo cross-sections taken at various points along the length of strands A and B (Run 5).
<table>
<thead>
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<th>Section No.</th>
<th>Bottom</th>
<th>Notch (left)</th>
<th>Notch (Right)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6A1</td>
<td>25 - 160 mm</td>
<td>50 - 120 mm</td>
<td>25 - 50 mm</td>
</tr>
<tr>
<td>6A2</td>
<td>25 - 165 mm</td>
<td>30 - 155 mm</td>
<td>30 - 150 mm</td>
</tr>
<tr>
<td>6A3</td>
<td>30 - 148 mm</td>
<td>30 - 135 mm</td>
<td>30 - 150 mm</td>
</tr>
<tr>
<td>6A4</td>
<td>-</td>
<td>35 - 130 mm</td>
<td>-</td>
</tr>
<tr>
<td>6A5</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6A6</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6B1</td>
<td>-</td>
<td>Surface - 135 mm</td>
<td>Surface - 150 mm</td>
</tr>
<tr>
<td>6B2</td>
<td>-</td>
<td>25 - 130 mm</td>
<td>Surface - 165 mm</td>
</tr>
<tr>
<td>6B3</td>
<td>-</td>
<td>25 - 110 mm</td>
<td>Surface - 135 mm</td>
</tr>
<tr>
<td>6B4</td>
<td>-</td>
<td>20 - 135 mm</td>
<td>Surface - 130 mm</td>
</tr>
<tr>
<td>6B5</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6B6</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table XVII Location of cracks in jumbo cross-sections taken at various points along the length of strands A and B (Run 6).
A few general comments can be made regarding the occurrence of the large cracks, an example of which can be seen in the transverse section of a Prime Western Grade jumbo ingot shown in Fig. 6.1. It is seen that these cracks tend to form normal to the surface predominantly in three areas near the two notches and next to the bottom surface. These cracks are similar in many respects to the mid-way cracks often seen in transverse sections of continuously cast steel billets (86, 87). After macro-etching the transverse surfaces of the jumbo sections, these cracks were seen to occupy inter granular regions, between columnar grains of zinc. Although the cracks observed in Fig. 6.1 penetrate to the surface, this was not always the case, but only when the cracking was very severe.

Sixty-five sections were visually inspected for cracks and fifty-one, or 78%, were found to contain one or more severe cracks. In some runs a crack could be seen running through the entire length of the casting.

6.3 Heat Flow Analysis

A heat flow analysis of the jumbo casting was performed by using the heat flow model in conjunction with the measured temperature profile obtained with the frozen-in thermocouples. Since it was not possible to freeze in
Fig. 6.1  A cross-section of Prime Western Grade jumbo ingot showing the internal cracks.
the thermocouple at the surface of the casting, the surface heat-transfer conditions existing in the machine were back calculated using the three-dimensional heat flow model by trial and error. The boundary conditions were adjusted until a match was obtained between the measured and calculated temperature profiles.

Fig. 6.2 shows the temperature profiles calculated from the model for the various nodes as shown in the insert. These have been obtained for a casting speed of 1.69 mm/s with heat-transfer conditions in the existing cooling system with a short intense spray in the sub-mould region. It can be seen that the surface of the jumbo undergoes reheating beyond 120 s. Translating the time axis into a distance axis using the casting speed, this corresponds to the bottom of the second spray ring below the mould.

Surface reheating below the sprays is important because it results in crack formation in the following way. Reheating, which is a maximum at the surface causes the surface to expand more than the interior region of the solidified shell; and thus the surface is constrained and put into compression, while a tensile strain is generated at the solidification front. These tensile strains are responsible for the formation of the cracks. A similar
Fig. 6.2  Calculated temperature profiles for the different nodes for zinc jumbo ingot cast at 1.69 mm/s.
mechanism has been proposed for the formation of mid-way cracks in continuously cast steel billets (86).

The position below the liquid level at which the internal cracks are generated can be determined from the heat flow model if it can be assumed that the cracks form close to the solidification front, because then the depth of the crack beneath the surface gives the shell thickness at the time of crack formation. The shell thickness calculated for a 1.69 mm/s casting speed is shown in Fig. 6.3, and the accompanying diagram gives the surface temperature profile. The band on the left-hand side has been drawn from the measured location of the inner tip of the crack for the various sections inspected in the test campaign. It can be seen from this figure that cracks are always initiated after the reheat event has taken place.

The magnitude of reheating required to cause cracks is believed to be 45-50°C which is lower than the value of 100-150°C quoted for steel (86). This comparison is based on the coefficient of linear expansion which for polycrystalline zinc is 39.7 \( (10^{-6})/°C \) as compared to 17 \( (10^{-6})/°C \) for steel. The critical value of tensile strains that cause hot-tearing in steel has been estimated at around 0.2%. If a similar criterion is applied for zinc, then the reheat of
Fig. 6.3 Growth of the shell as a function of distance in the axial direction for zinc jumbo ingot cast at 1.69 mm/s. Figure on the right shows the surface temperature profile at the bottom mid-face of a jumbo section.
45°C observed in the present work is adequate for hot tears to occur. This is only a rough comparison because the critical strain that is required would very much depend on the cohesion between grains which in turn is affected by the presence of liquid films between columnar dendrites.

6.4 Metallographic Analysis

In order to investigate the mechanism of crack formation further a metallographic examination was carried out on the cracked surface. Fig. 6.4 shows a macro-photograph of the cracked surface. In this particular sample the crack was seen to go all the way through the thickness of the section (100 mm). The bottom face of the jumbo section is on the left-hand side of the figure. The growth of dendrites parallel to the direction of heat flow is quite evident; obviously the axial component of heat conduction is very important owing to the intense spray cooling below the mould.

Fig. 6.5 shows a scanning electron micrograph of the cracked surface taken at a higher magnification. The interdendritic nature of these cracks is clear from this picture. The identity of the white particles observed on the fractured surface was investigated in some detail.

Fig. 6.6 (a) shows the scanning electron micrograph
Fig. 6.4  A macro-photograph of the cracked surface. Magnification 1.3 X.
Fig. 6.5  Scanning electron micrograph of a cracked surface. Magnification 200 X.
Fig. 6.6(a) Scanning electron micrograph of a cracked surface revealing the smooth nature of the surface. Magnification 1000 X.

Fig. 6.6(b) Pb x-ray picture of Fig. 6.6(a).
of the cracked surface at a much higher magnification. The smooth nature of the surface strongly points to the presence of liquid films at the interface and hot tearing. Fig. 6.6 (b) shows the Pb x-ray scan obtained from the same area; and thus the white particles seen in Fig. 6.6 (a) and 6.5 are a lead-rich second phase. Similar observations were also made with respect to other areas of the surface (Fig. 6.7a and b). The presence of the lead-rich phase can be explained by examining the phase diagram for the Zn-Pb system shown in Fig. 6.8. It can be seen that zinc has an extremely low solid solubility for lead, e.g. 0.5 - 0.9 wt % at the monotectic temperature of 417.8°C. Thus in Prime Western Grade zinc it is possible to have lead-rich liquid present in the inter-dendritic region, thereby drastically decreasing cohesion between grains.

6.5 Mechanism of Crack Formation

Based on the preceding results the following mechanism can be proposed for crack formation in the D.C. casting of Prime Western Grade zinc. The primary cause for the formation of cracks is incorrect cooling practice below the mould. It is seen that the short intense cooling adopted in the current practice leads to reheating of the jumbo surface below the second spray ring which results in the expansion of the surface. Because the surface heats and expands more than
Fig. 6.7(a) Scanning electron micrograph of a cracked surface revealing the smooth nature of the surface. Magnification 1000 X.

Fig. 6.7(b) Pb x-ray picture of Fig. 6.7(a).
Fig. 6.8  Phase diagram of Pb-Zn system (89).
the interior of the solidified shell, a compressive strain is generated at the surface and a tensile strain at the solidification front. The strain is sufficient (−.2 to .3%) to cause dendrites separated by liquid films of lead near the solidification front to open up and form a crack. The morphology of the crack surfaces is indicative of such a hot tearing mechanism.

Table XVIII shows the maximum surface reheat calculated using the model at the various locations of the casting. The nodes referred to as top and bottom correspond to the surface nodes at the mid-plane of non-notched surfaces and the notch corresponds to the node at the bottom of the notch. Compared to other surface nodes these three show the maximum reheat. Further it can be noted from this table that the reheat of the top node was much less compared to the bottom and notch nodes. These predictions match the observations of crack locations; cracks were confined to the mid-plane region adjacent to the bottom and notched faces.

Fig. 6.9 shows the effect of casting speed on the surface reheat phenomenon as predicted by the heat flow model. The three speeds considered are 1.69 mm/s, 1.27 mm/s and 0.85 mm/s. It can be seen that the reheating is reduced considerably when casting at lower speeds. This suggests that cracking should be less severe at lower speeds.
Table XVIII Calculated values of reheat at different points on the surface of the jumbo section. Top and bottom correspond to mid-face on the non-notched surfaces.

<table>
<thead>
<tr>
<th>Node Location</th>
<th>Reheat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom (1)</td>
<td>64°C</td>
</tr>
<tr>
<td>Top (2)</td>
<td>35.5°C</td>
</tr>
<tr>
<td>Notch (3)</td>
<td>53°C</td>
</tr>
</tbody>
</table>
Fig. 6.9 Effect of casting speed on the surface re-heating at the bottom mid-face of a jumbo section.
which is in line with operator experience. Although it is possible to reduce the severity of the cracking problem by decreasing the casting speed, it is not a practicable solution owing to the lower production rates: further the quality of the surface deteriorates at lower casting speeds. Cold shuts and surface laps are often seen on the surface of the ingots cast at low speeds.

6.6 Design of New Cooling System

Having thus ascertained that surface reheating below the sprays was the cause of cracks in Prime Western Grade zinc jumbos the design of a new spray system which would minimize this phenomenon was undertaken. A number of runs were made using the heat-flow model and the effect of heat-transfer coefficient at various points along the strand on the reheat values was studied. Fig. 6.10 shows the surface temperature profiles obtained with different heat-transfer coefficients in the sub-mould region. The values of heat-transfer coefficients used were 20.93 kW/m²K corresponding to the existing set-up, 10.46 kW/m²K and 41.86 kW/m²K. The value of heat transfer coefficients used below the second spray have been kept the same for all the three runs. It can be seen that decreasing the heat-transfer coefficient in the sub-mould region results
Fig. 6.10  Effect of the heat-transfer coefficient in the sub-mould region on the surface reheating at the bottom mid-face of a jumbo section.
in a decrease in the reheat. It was very clear from these runs that the reheat could be minimized by decreasing the intensity of the sprays in the sub-mould region and by increasing the cooling below the second spray ring with additional sprays.

Thus the new cooling arrangement was designed to maintain uniform cooling all the way down to the bottom of the liquid pool to ensure that reheating of the surface was minimized prior to complete solidification. This was accomplished by redistributing the total amount of water used over a wide area of the surface. The arrangement of the sprays in the new cooling assembly is shown in Fig. 6.11. This corresponds to the bottom face of the non-notched jumbo section. Other faces were also provided with similar arrangement. As in the existing practice a flat spray nozzle was used in the top ring impinging on the mould to ensure adequate solidification in the mould and a minimum of break-outs. However in order to decrease the quantity of water which ultimately falls from the mould through the sub-mould sprays the nozzle selected for the top ring had one-half the capacity of the existing flat spray nozzle. For cooling in the sub-mould region a total of four spray rings were designed with 8 nozzles per spray ring (two per face). The nozzles selected for these four
Fig. 6.11 Arrangement of spray nozzles in the new cooling assembly for the bottom surface of a jumbo section.
rings were different from the nozzles used in the old design. The new nozzles are of the wide angle type which provide a uniform water flux distribution over a given face (88). Thus the use of new spray nozzles results in a water flux of 0.9 \( \ell/\text{m}^2\text{s} \) in comparison with 4.8 \( \ell/\text{m}^2\text{s} \). Although there is a large difference between the two values, the total amount of water in the new design is only marginally less than the old value. The new spray assembly was constructed using 38 mm diameter galvanized pipes with threaded connections. A few minor modifications were required to the casting assembly to accommodate the new spray system.

6.7 Testing of the New Cooling System

The new spray assembly was tested in-plant by making a total of four runs. In the first run the material cast was high grade zinc at 1.27 mm/s. This run was carried out essentially to check out the new assembly as well as to gain confidence of the operators for casting the more difficult Prime Western Grade zinc alloy. Since this initial run did not pose any problem, Prime Western Grade zinc was cast at speeds of 1.27 mm/s and 1.48 mm/s in three runs. As before transverse sections were inspected for the presence of cracks. Of the twenty-two sections inspected between two strands in two runs no section showed any major cracks as seen in the previous campaigns. As before in
some sections, extremely fine hairline cracks were also observed. Fig. 6.11 shows a section from the new campaign. It should be noted that the fine hairline crack present was less visible before etching with hydrochloric acid. It was the opinion of the operating people that the quality of the Prime Western Grade zinc cast using the new design was at least as good as the quality of high grade zinc jumbos cast with the old set-up.

6.8 Summary

The cracking problem encountered in casting Prime Western Grade zinc has been investigated, and shown to be the result of the short intense spray cooling practice employed which generates surface reheating and tensile strains at the solidification front. Opening of cracks between adjacent columnar dendrites under the influence of these strains is enhanced by the presence of lead-rich liquid films.

A new spray system has been designed with the aid of the three-dimensional heat flow model to overcome the problem of surface reheating by maintaining uniform cooling to the bottom of the liquid pool. The new assembly has been tested in-plant for the casting of Prime Western Grade zinc and shown to totally eliminate the severe cracks.
Fig. 6.12  A cross-section of Prime Western Grade jumbo ingot with the new cooling system.
Chapter 7

SUMMARY AND CONCLUSIONS

A fully three dimensional model has been developed to simulate heat flow and solidification in the Direct Chill casting of non-ferrous metals with rectangular as well as irregular notched cross-sections. The model employs an alternating-direction implicit finite-difference method to solve the governing heat conduction equation and can take into account both steady-state operation as well as initial transient conditions. The model has been tested extensively for internal consistency and its validity has been checked by comparing predictions of pool profiles and pool depths with industrial data for the D.C. casting of aluminium and zinc. The model has been used to investigate the relative importance of the individual components of heat conduction and has revealed the following:

1. For thick aluminium slabs e.g. 381 mm and 457 mm thickness, subjected to conventional D.C. cooling, a two-dimensional model in which heat flow parallel to the broad-face is neglected can be used when the aspect ratio exceeds 2.5.

2. For thick zinc slabs of 381 mm thickness the same
two-dimensional model is adequate for aspect ratios exceeding 2.0.

3. For the case of reduced secondary cooling it is preferable to neglect heat flow in the axial direction, but consider both the two transverse directions when formulating a two-dimensional model even though the aspect ratio exceeds 2.5.

4. The model has shown the first 25% of the total casting cycle in the production of zinc jumbo sections is in the unsteady state. The transient part of the casting is also very important when reduced secondary cooling is employed. For conventional D.C. cooling of aluminium the unsteady state is important only in casting large sections, e.g. 457 x 1143 mm.

A cracking problem encountered in the D.C. casting of Prime Western Grade zinc has been investigated with the aid of the mathematical model. It has shown that the cracking is caused by the use of short-intense spray cooling in the sub-mould region. Temperature measurements and model calculations have revealed that this cooling practice results in the reheating of the surface below the spray region, which in turn generates tensile strains at the solidification
front where films of lead-rich liquid separate dendrites and facilitate the opening up of cracks.

Based on this analysis a new spray cooling assembly has been designed for casting Prime Western Grade zinc jumbo ingots. This design attempts to cool the surface of the jumbo more uniformly over the entire length of the liquid pool and also prevent surface reheating. Experiments have been undertaken in-plant to test the new system, and results have shown that the new cooling assembly is effective in preventing cracks.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>specific heat</td>
<td>Jg⁻¹K⁻¹</td>
</tr>
<tr>
<td>h</td>
<td>heat transfer coefficient</td>
<td>Wm⁻²K⁻¹</td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity</td>
<td>Wm⁻¹K⁻¹</td>
</tr>
<tr>
<td>L</td>
<td>latent heat of solidification</td>
<td>Jg⁻¹</td>
</tr>
<tr>
<td></td>
<td>thickness of slab</td>
<td>mm</td>
</tr>
<tr>
<td>T, T₁, T₂, T*, T**, T***</td>
<td>temperature</td>
<td>°C</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
<td>s</td>
</tr>
<tr>
<td>v</td>
<td>casting speed</td>
<td>mm s⁻¹</td>
</tr>
<tr>
<td></td>
<td>volume of a node</td>
<td>mm³</td>
</tr>
<tr>
<td>x</td>
<td>x-direction</td>
<td>dimensionless</td>
</tr>
<tr>
<td>y</td>
<td>y-direction</td>
<td>dimensionless</td>
</tr>
<tr>
<td>z</td>
<td>z-direction</td>
<td>dimensionless</td>
</tr>
<tr>
<td>X</td>
<td>length in X-direction</td>
<td>mm</td>
</tr>
<tr>
<td>Y</td>
<td>length in Y-direction</td>
<td>mm</td>
</tr>
<tr>
<td>Z</td>
<td>length in Z-direction</td>
<td>mm</td>
</tr>
<tr>
<td>α</td>
<td>thermal diffusivity</td>
<td>mm²s⁻¹</td>
</tr>
<tr>
<td>Δx</td>
<td>distance step in x-direction</td>
<td>mm</td>
</tr>
<tr>
<td>Δy</td>
<td>distance step in y-direction</td>
<td>mm</td>
</tr>
<tr>
<td>Δz</td>
<td>distance step in z-direction</td>
<td>mm</td>
</tr>
<tr>
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<td>time step</td>
<td>s</td>
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</tr>
<tr>
<td>σ</td>
<td>Stefan-Boltzmann constant</td>
<td>kWm⁻²</td>
</tr>
<tr>
<td>ρ</td>
<td>density</td>
<td>kgm⁻³</td>
</tr>
<tr>
<td>θ</td>
<td>temperature</td>
<td></td>
</tr>
</tbody>
</table>
Subscripts

£ - liquidus
s - solidus
m - mushy
n - present time interval
n+1 - future time interval
i - node identification in x-direction
j - node identification in y-direction
k - node identification in z-direction
b - Base
av - average


84. Sutherland, J.G., Personal Communications.


88. Prabhakar, B., Personal Communications.

APPENDIX 1

DEVELOPMENT OF FINITE DIFFERENCE EQUATIONS
Al.1 Alternating Direction Finite Difference Equations for Three Dimensional Problems.

The finite difference equations which replace the unsteady partial differential equation have been obtained using a heat balance approach. In this method the material being analysed is divided into a number of discrete elements of finite dimensions. In a three dimensional heat flow problem of a rectangular parallelepiped, calculations are performed only for one quarter of the casting as indicated in Fig. Al-1, because of the symmetry present at the mid-planes. In the z-direction which is also the casting direction, the whole casting had to be analysed because of the different boundary conditions involved at the top and bottom. The sub-division of the casting into a number of elements is shown in Fig. Al-2. Half nodes are present at the surface and centre with respect to x and y directions and top and bottom with respect to z-direction. In Fig. Al-2 only the surface nodes are visible. In this particular problem there are altogether 27 different types of nodes depending on their location in the casting.
oo'x'x, oyy'o' - zero heat flux boundary condition
yy'd'd, dd'x'x, o'y'd'x' - heat-transfer coefficient boundary condition
oydx - constant temperature boundary condition

Fig. A1.1 Dotted region is the volume over which calculations are performed.
Fig. A1.2 Discretization of the rectangular parallelepiped showing the surface nodes.
The generation of the simultaneous equations is illustrated below for the case of an interior node and a surface node.

**Interior Node:**

Let \( i,j,k \) be the indices of the node under investigation and \( \Delta x, \Delta y, \Delta z \) be its dimensions, as well as distance between nodes.

**Stage I: Implicit in x-Direction**

- **Rate of Heat in by conduction in x-direction**
  \[
  = -k\Delta y\Delta z \left( \frac{T^{*}_{i,j,k} - T^{*}_{i-1,j,k}}{\Delta x} \right)
  \]

- **Rate of Heat out by conduction in x-direction**
  \[
  = -k\Delta y\Delta z \left( \frac{T^{*}_{i+1,j,k} - T^{*}_{i,j,k}}{\Delta x} \right)
  \]

- **Rate of Heat in in the y-direction**
  \[
  = +k\Delta x\Delta z \left( \frac{T_{i,j-1,k} - T_{i,j,k}}{\Delta y} \right)
  \]

- **Rate of Heat out in the y-direction**
  \[
  = -k\Delta x\Delta z \left( \frac{T_{i,j+1,k} - T_{i,j,k}}{\Delta y} \right)
  \]

- **Rate of heat in in the z-direction**
  \[
  = +k\Delta x\Delta y \left( \frac{T_{i,j,k-1} - T_{i,j,k}}{\Delta z} \right)
  \]
Rate of Heat out in the z-direction

\[ = - k \Delta x \Delta y \frac{T_{i,j,k+1} - T_{i,j,k}}{\Delta z} \]

Rate of heat consumption = 0
Rate of heat generation = 0
Rate of heat accumulation = \( \rho c \frac{\Delta x \Delta y \Delta z}{\Delta t/2} (T^*_{i,j,k} - T_{i,j,k}) \)

\( T^* \) is the temperature at the end of time Step \( \Delta t/2 \)

From Energy Balance,

\[ (\text{Rate of heat in}) - (\text{Rate of heat out}) + (\text{Rate of heat generation}) - (\text{Rate of heat consumption}) = \text{Rate of Accumulation} \]

\[ k \frac{\Delta y \Delta z}{\Delta x} (T^*_{i-1,j,k} - 2T^*_{i,j,k} + T^*_{i+1,j,k}) + \]

\[ k \frac{\Delta x \Delta z}{\Delta y} (T^*_{i,j-1,k} - 2T^*_{i,j,k} + T^*_{i,j+1,k}) + \]

\[ k \frac{\Delta x \Delta y}{\Delta z} (T^*_{i,j,k-1} - 2T^*_{i,j,k} + T^*_{i,j,k+1}) \]

\[ = 2 \frac{\Delta x \Delta y \Delta z}{\Delta t} \rho c (T^*_{i,j,k} - T_{i,j,k}) \]

Since only \( T^* \)'s are unknown, they are kept on the left hand side and everything else moved to the right. Simplifying we get,
The form in which Eq. Al-1 is presented is very useful in cases where it is desired to have a change in the node sizes in the different directions.

Implicit in y-direction:

Following on the same lines as before results in,

\[- T^*_{i,j-1,k} \left( \Delta x \Delta z \right) + T^*_{i,j,k} \left( \frac{2 \Delta x \Delta y \Delta z \rho c}{k \Delta t} + 2 \frac{\Delta y \Delta z}{\Delta x} \right) + T^*_{i,j+1,k} \left( \frac{2 \Delta x \Delta y \Delta z \rho c}{k \Delta t} + 2 \frac{\Delta x \Delta z}{\Delta y} \right) (T_{i-1,j,k} - 2T_{i,j,k} + T_{i+1,j,k}) \ldots \text{Al-1} \]

where \( T^* \) are the unknown temperatures
Implicit in the z-direction:

\[ -T_{i,j,k-1}^{***}\left(\frac{\Delta x\Delta y}{\Delta z}\right) + T_{i,j,k}^{***}\left(\frac{2\Delta x\Delta y}{\Delta z} + \frac{2\Delta x\Delta y\Delta z \rho c}{k\Delta t}\right) \]

\[ -T_{i,j,k+1}^{***}\left(\frac{\Delta x\Delta y}{\Delta z}\right) \]

\[ = \left(\frac{2\Delta x\Delta y\Delta z \rho c}{k\Delta t}\right) T_{i,j,k} + \frac{\Delta y\Delta z}{\Delta x} (T_{i-1,j,k}^{*} - 2T_{i,j,k}^{*} + T_{i+1,j,k}^{*}) + \frac{\Delta x\Delta z}{\Delta y} (T_{i,j-1,k}^{**} - 2T_{i,j,k}^{**} + T_{i,j+1,k}^{**}) \]

... AI-3

where \( T^{***} \) are the unknown temperatures.

Explicit formula for calculating the new temperature at the end of a time interval \( \Delta t \) is given by

\[ T_{i,j,k}^{n+1} = \left\{ \begin{array}{l}
\frac{\Delta y\Delta z}{\Delta x} (T_{i-1,j,k}^{*} - 2T_{i,j,k}^{*} + T_{i+1,j,k}^{*}) + \\
\frac{\Delta x\Delta z}{\Delta y} (T_{i,j-1,k}^{**} - 2T_{i,j,k}^{**} + T_{i,j+1,k}^{**}) + \\
\frac{\Delta x\Delta y}{\Delta z} (T_{i,j,k-1}^{***} - 2T_{i,j,k}^{***} + T_{i,j,k+1}^{***}) \end{array} \right\} / \\
\left(\frac{\Delta x\Delta y\Delta z \rho c}{\Delta t}\right) + T_{i,j,k} \]

... AI-4

where \( T_{i,j,k}^{n+1} \) is the new temperature.
If the nodes were indexed $i = 1, 2 \ldots L$ in the $x$ direction, $j = 1, 2 \ldots M$ in the $y$ direction, and $k = 1, 2 \ldots N$ in the $z$ direction, then the above equations are valid for nodes having the indices $i, j, k$ where

- $i = 2, 3 \ldots L-1$
- $j = 2, 3 \ldots M-1$
- $k = 2, 3 \ldots N-1$

Equations for a node lying on the boundary with respect to $x$ direction, but interior with respect to $y$ and $z$ directions.

Dimensions of the node $\Delta x, \Delta y, \Delta z$

Nodal indices $(L,j,k)$

Implicit in the $x$-direction:

$$- T_{L-1,j,k}^{*} (\frac{\Delta y \Delta z}{\Delta x}) + T_{L,j,k}^{*} \left( \frac{\Delta y \Delta z}{\Delta x} + \frac{x \Delta y \Delta z \rho c}{k \Delta t} \right) +$$

$$\frac{h \Delta y \Delta z}{k} = \frac{\Delta x \Delta y \Delta z \rho c}{k \Delta t} T_{L,j,k} + h \frac{T_{A}}{k} \frac{\Delta y \Delta z}{2 \Delta y} +$$

$$h \frac{\Delta x \Delta z}{2 \Delta z} (T_{L,j-1,k} - 2T_{L,j,k} + T_{L,j+1,k}) +$$

$$\frac{\Delta x \Delta y}{2 \Delta z} (T_{L,j,k-1} - 2T_{L,j,k} + T_{L,j,k+1}) \ldots \text{A1-5}$$
Implicit in y-direction:

\[-T^{**}_{L,j-1,k} \left( \frac{\Delta x \Delta z}{2 \Delta y} \right) + T^{**}_{L,j,k} \left( \frac{\Delta x \Delta z}{\Delta y} \right) + \frac{\Delta x \Delta y \Delta z \rho C}{k \Delta t}\]

\[-T^{**}_{L,j+1,k} \left( \frac{\Delta x \Delta z}{2 \Delta y} \right)\]

\[= \frac{\Delta x \Delta y \Delta z}{k \Delta t} T_{L,j,k} + \frac{\Delta z \Delta y}{\Delta x} \left( T^{*}_{L-1,j,k} - T^{*}_{L,j,k} \right)\]

\[-h \frac{\Delta z \Delta y}{k \Delta x} \left( T^{*}_{L,j,k} - T_{A} \right) + \frac{\Delta x \Delta y}{2 \Delta z} \left( T_{L,j,k-1} - T_{L,j,k} \right)\]

\[-2T_{L,j,k} + T_{L,j,k+1}\]

... A1-6

Implicit in z-direction:

\[-T^{***}_{L,j,k-1} \left( \frac{\Delta x \Delta y}{2 \Delta z} \right) + T^{***}_{L,j,k} \left( \frac{\Delta x \Delta y}{\Delta z} \right) + \frac{\Delta x \Delta y \Delta z \rho C}{k \Delta t}\]

\[-T^{***}_{L,j,k+1} \left( \frac{\Delta x \Delta y}{2 \Delta z} \right)\]

\[= \frac{\Delta x \Delta y \Delta z}{k \Delta t} T_{L,j,k} + \frac{\Delta z \Delta y}{\Delta x} \left( T^{*}_{L-1,j,k} - T^{*}_{L,j,k} \right)\]

\[-h \frac{\Delta z \Delta y}{\Delta k} \left( T^{*}_{L,j,k} - T_{A} \right) + \frac{\Delta x \Delta z}{2 \Delta y} \left( T^{**}_{L,j-1,k} - T^{**}_{L,j,k} \right)\]

\[-2T^{**}_{L,j,k} + T^{**}_{L,j+1,k}\]

... A1-7
Explicit formula

\[
T_{L,j,k}^{n+1} = \left\{ \begin{array}{l}
\frac{\Delta x \Delta z}{\Delta z} \left( T_{L-1,j,k}^* - T_{L,j,k}^* \right) + h \frac{\Delta x \Delta z}{k} \left( T_{L,j,k}^* - T_A \right) + \frac{\Delta x \Delta y}{2 \Delta y} \left( T_{L,j-1,k}^{**} - 2T_{L,j,k}^{**} \right) + T_{L,j+1,k}^{**} + \frac{\Delta x \Delta y}{2 \Delta y} \left( T_{L,j,k-1}^{***} - 2T_{L,j,k}^{***} + T_{L,j,k+1}^{***} \right) \\
\frac{\Delta x \Delta y \Delta z}{2 \Delta t} \end{array} \right\} + T_{L,j,k} + \ldots \text{A1-8}
\]

where \( h \) is the heat transfer coefficient at the boundary in the \( x \) direction, and \( T_A \) is the ambient temperature.
Stability Criterion for Explicit Finite Difference Using Convective Type Boundary Conditions.

The most stringent criterion will be applied for a surface node, which is on the bottom corner of the ingot. This particular node has three of its six faces subjected to heat transfer with the outside.

Let LMN stand for the node identification.

Doing a heat balance for this node in explicit finite difference form

\[
\frac{\Delta T}{\Delta t} = \frac{k}{\Delta x} \left( T_{L-1,M,N} - T_{L,M,N} \right) + \frac{k}{\Delta y} \left( T_{L,M,N-1} - T_{L,M,N} \right) + \frac{k}{\Delta z} \left( T_{L,M,N-1} - T_{L,M,N} \right) - h \left( T_{L,M,N} - T_A \right) + \frac{\Delta x \Delta y \Delta z}{8 \Delta t} (T_{L,M,N}^{n+1} - T_{L,M,N}^n)
\]

... A1-9
where $h_1$, $h_2$, $h_3$ are the heat transfer coefficients in the $x$, $y$ and $z$ direction.

Here the stability condition for explicit method is

\[
(1 - 2 \frac{\Delta t k}{\Delta x^2 \rho c} - 2 \frac{\Delta t k}{\Delta y^2 \rho c} - 2 \frac{\Delta t k}{\Delta z^2 \rho c}) \geq 0
\]

\[
\frac{2h_1 \Delta t}{\rho c \Delta x} - \frac{2h_2 \Delta t}{\rho c \Delta y} - \frac{2h_3 \Delta t}{\rho c \Delta z} \geq 0
\]

substituting $k = 113$ W/m K
\[\rho = 7140\] kg/m$^3$
\[c = .3830\] J/g K
\[h_1 = 9210.9\] W/m$^2$K
\[h_2 = 9210.9\] W/m$^2$K
\[h_3 = 209.34\] W/m$^2$K
\[
\Delta x = \Delta y = 15.24\] mm
\[\Delta z = 20\] mm

we get $\Delta t = < 0.55$ seconds
APPENDIX 2

SOURCE LISTING OF THE COMPUTER PROGRAM
A FORTRAN PROGRAM FOR SIMULATING THREE-DIMENSIONAL HEAT FLOW AND SOLIDIFICATION IN CASTING JUMBO SECTIONS OF ZINC. BECAUSE OF THE SYMMETRY, THE CALCULATIONS HAVE BEEN PERFORMED ONLY FOR ONE HALF OF THE CASTING.

NSTART = 0 CORRESPONDS TO STARTING FROM THE BEGINNING

NITF - NUMBER OF TIMES CALCULATIONS ARE PERFORMED

NFLT - NUMBER OF TIMES FLOATS ARE REQUIRED

WNUM - NUMBER OF STEPS AFTER WHICH A NEW SLICE IS ADDED TO THE Z-DIRECTION

Z - THICKNESS OF THE SLICE IN THE Z DIRECTION

RAIY - THERMAL CONDUCTIVITY OF THE MATERIAL IN CGS

DT - TIME INTERVAL OVER WHICH CALCULATIONS ARE DONE

TAU - THE TOTAL CASTING TIME AT THE END OF EACH CALCULATION

L,M,N - THE NUMBER OF NODES IN THE X, Y, Z DIRECTION - 1 FOR JUMBO CALCULATIONS L=9 AND M=15 - THE DISCRETIZATION IS DONE AS PER DATA SUBROUTINE AREFL

TEF1 - FLOW TEMPERATURE DEG C

TEF2 - SOME SMALL TEMPERATURE AS A DUMMY VALUE TO FILL THE GRID IN THE NOTCH AREA FOR THE PLOTTING SUBROUTINE

DPL - DENSITY OF THE LIQUID G/CM3

DENS - DENSITY OF THE SOLID G/CM3

TLC - LIQUIDUS TEMPERATURE DEG C

TSCL - SOLIDUS TEMPERATURE DEG C

CF1 - SPECIFIC HEAT OF THE LIQUID CAL/G C

BLHT - LATENT HEAT OF SOLIDIFICATION CAL/G

H3 - Eddy heat transfer coefficient CGS UNITS SPECIFIC HEAT OF SOLID INCORPORATED AS A FUNCTION OF TEMPERATURE IN THE FUNCTION SUBROUTINE CP

DIMENSION T(10,16,91),T(10,16,91),T2(10,16,91),T3(10,16,91),T4(10,16,91),T5(10,16,91)

DIMENSION A(101),E(101),C(101),D(101),T1ME(101)

DIMENSION AEX(16,11,2),AEY(11,16,2),AEXZ(11,16,2)

DIMENSION TS(38,30,50)

COMMON/C1/DX,EY,CZ,DT,BKAY

COMMON/C2/SN(10,16,91)

COMMON/C4/L,M,N

COMMON/C5/NCH,TEMP1

COMMON/C6/H3

COMMON/C7,T,T1,T2,T3,TN

COMMON/C8/PHY(10,16,91)

COMMON/C9/PHY,PHYS,PHY1

COMMON/C10/TL10,TSCL,DENS,DENL

COMMON/C11/CFL,EFL,NUMB,RLHT

COMMON/C12/BINP(20),CFL(20)

COMMON/C13/TAS

COMMON/C16/NNUM,NUM2

COMMON/C17/ARX,ARY,ARZ
BEADING INPUT DATA TO THE PROGRAM

BEAT (5, 200) NUBKUN
BEAT (5, 90) (BINPT (I), I=1, 20)
BEAT (5, 95) NSTART, NITE, NFLT, NNUM

FORMAT (14)
BEAT (5, 100) Z, BKAJ, ET, TAU
BEAT (5, 20C) L, F, W
BEAT (5, 100) TEMP1, TEMP2
BEAT (5, 10C) DENS, DENS, TLIQ, TSOL, CPL, BLHT, H3
BEAT (5, 90) (CPF (I), I=1, 20)
FORMAT (2044)
CALL GSET (TS, 1040, 50, 1040, 0.)

ECHO INPUT

WRITE (6, LISTA)
WRITE (6, LISTB)

100 FORMAT (5E10.4)
200 FORMAT (3(14,1X) )
FLCATL=L
FLCATM=M
DFMAT=M
CS= (ET*NNUM)/(1L)
L = L+1
B = M+1
N = N+1

CALCULATION OF THERMO PHYSICAL PROPERTIES

PBE BUSY ZONE

DENS = (DENS+DENL)/2.
CPH = (CP(TSOL)+CP(TSOL))/2. + (BLHT/(ILIQ-TSOL))
EPHY = DENS*CPH
PHYS = DENS*CP(TSOL)

INITIALISATION PROCEDURE

IF (NSTART, NE, 0) GO TO 105
NUM2 = 0
CALL INITI2 (T1, TEMPl, TEMP2)
CALL INITI2 (T1, TEMPl, TEMP2)
CALL INITI2 (T2, TEMPl, TEMP2)
CALL INITI2 (T3, TEMPl, TEMP2)
DO 101 I = 1, L
DO 101 J = 1, M
DO 101 K = 1, N
IF (I, J, K) = 1
CONTINUE
GO TO 114
101 CONTINUE
105 CONTINUE
DC 108 K=1,N
DO 108 J=1,N
REAL (8) (T(I,J,K),I=1,L),(LPS(I,J,K),I=1,L)
108 CONTINUE
DC 113 I=1,L
DO 113 J=1,M
T1(I,J,1)=T(I,J,1)
T2(I,J,1)=T(I,J,1)
T3(I,J,1)=T(I,J,1)
Tn(I,J,1)=T(I,J,1)
113 CONTINUE
NUM2=0
MEC=1
114 CONTINUE
CALL OUTPUT(T)
CALL SORTI
C
C NCSING
C
CALL NSOET
LI=L-1
MI=M-1
NI=N-1
C
C DISCRETIZATION OF THE CASTING
C
CALL AREWOL
111 CONTINUE
CALL OUTFST2
C
C START CF CALCULATIONS
C
DO 1001 KJI=1,NFLOT
DO 1002 KJI=1,NITE
TAE=TAU+DT
CALL EHTPEP
DO 600 J=1,4
DO 600 K=2,N
CALL CEEFII(I,L,J,K,A,B,C,D)
CALL TEDAG(I,L,A,B,C,D,TPE1)
DO 550 I=1,L
TI(I,J,K)=TPEI(I)
550 CONTINUE
600 CONTINUE
C CALCULATION AT THE NOTCH
C
II=6
JI=5
DO 615 KJI=1,4
DC 610 K=2,N
CALL CEEFII(I,J,K,A,B,C,D)
CALL TEDAG(I,J,A,B,C,D,TPEI)
DO 630 I=1,J
TI(I,J,K)=TPEI(I)
630 CONTINUE
610 CONTINUE
JI=JI+1
IJ=IJ+1
615 CONTINUE
C CALCULATION BELOW THE NOTCH
C
DO 640 J=9,M
DO 645 K=2,N
CALL CCEFFJ(1,L,J,K,A,B,C,D)
CALL TIBIAG(1,L,A,B,C,D,TFBIME)
DO 660 J=1,L
T1(I,J,K)=TFBIME(I)
660 CONTINUE
645 CONTINUE
640 CONTINUE
C IMPLICIT WITH RESPECT TO J DIRECTION
C
DC ECC I=1,6
DC 800 K=2,N
CALL CCEFFJ(1,M,I,K,A,B,C,D)
CALL TIBIAG(1,M,A,B,C,D,TFBIME)
DO 750 J=1,M
T2(I,J,K) = TFBIME(J)
750 CONTINUE
700 CONTINUE
C CALCULATION AT THE NOTCH
C
DO 760 K=2,N
DO 770 I=7,L
CALL CCEFFJ(1,4,I,K,A,B,C,D)
CALL TIBIAG(1,4,A,B,C,D,TFBIME)
DO 780 J=1,4
T2(I,J,K) = TFBIME(J)
780 CONTINUE
770 CONTINUE
760 CONTINUE
IJI=7
JII=6
DC 799 JKI=1,4
DO 790 K=2,N
CALL COEFFJ(JII,B,IJJ,K,A,B,C,D)
CALL TIBIAG(JII,B,IJJ,K,A,B,C,D,TFBIME)
DO 765 J=JII,M
T2(IJJ,J,K) = TFBIME(J)
765 CONTINUE
795 CONTINUE
JII=JII+1
IJJ=IJJ+1
799 CONTINUE
C C
C IMPLICIT WITH RESPECT TO Z DIRECTION
C
DO 900 I=1,L
DO 900 J=1,4
CALL CCEFFK(2,N,I,J,A,B,C,D)
CALL TIBIAG(2,N,A,B,C,D,TFBIME)
DO 850 K=2,N
T3(I,J,K) = TFBIME(K)
850 CONTINUE
900 CONTINUE
C CALCULATION AT THE NOTCH
C
IJK=6
JIK=5
DC 910 KKK=1,4
DO 915 I=1,IJK
CALL COEFF(2,N,I,JIK,A,B,C,D)
CALL TRIDAG(2,N,A,B,C,D,TPRIME)
DC 920 K=2,N
T3(I,JIK,K)=TPRIME(K)
920 CONTINUE
915 CONTINUE
IJK=IJK+1
JIK=JIK+1
910 CONTINUE
C CALCULATION BELOW THE NOTCH
DO 940 I=1,L
DO 950 J=9,H
CALL COEFF(2,N,I,J,A,B,C,D)
CALL TRIDAG(2,N,A,B,C,D,TPRIME)
DO 960 K=2,N
T3(I,J,K)=TPRIME(K)
960 CONTINUE
950 CONTINUE
940 CONTINUE
C COMPUTE THE TEMPERATURES AT THE END OF A TIME INTERVAL
C
CALL COMPUT(1,L,1,4,N)
CALL COMPUT(1,6,5,5,N)
CALL COMPUT(1,7,6,6,N)
CALL COMPUT(1,8,7,7,N)
CALL COMPUT(1,9,8,8,N)
CALL COMPUT(1,L,9,9,N)
C CORRECT THE TEMPERATURES CALCULATED FOR THE
C RELEASE OF LATENT HEAT
C
CALL LATHET(1,L,1,4,N)
CALL LATHET(1,6,5,5,N)
CALL LATHET(1,7,6,6,N)
CALL LATHET(1,8,7,7,N)
CALL LATHET(1,9,8,8,N)
CALL LATHET(1,L,9,9,N)
C REINITIALISE T MATRIX
C
DC 1110 I=1,L
DO 1110 J=1,N
DO 1110 K=2,N
T(I,J,K)=IN(I,J,K)
1110 CONTINUE
CALL CHECK(MCH)
IF(MCH.EQ.0)GO TO 109
CALL KEENO
CALL COMPUT(T)
CALL SUBFT
109 CONTINUE
1002 CONTINUE
C

IMPLICIT CALCULATIONS IN THE X DIRECTION

CALCULATIONS OF THE TRIDIAGONAL COEFFICIENTS

SUBROUTINE COEFF(I,J,K,A,B,C,D)
DIMENSION A(1),E(1),C(1),D(1)
DIMENSION AX(11,16,2),ARY(11,16,2),ABZ(11,16,2)
DIMENSION T(10,16,91),T1(10,16,91),T2(10,16,91),T3(10,16,91),T4(10,16,91),T5(10,16,91)
COMMON/C1/X1,Y1,Z1,T1,T2,T3,T4,T5
COMMON/C2/S4(10,16,91)
COMMON/C3/NTPY(10,16,91),LPS(10,16,91)
COMMON/C4/NC8,TEMP1
COMMON/C5/H3
COMMON/C6/PHY(10,16,91)
COMMON/C7/ARX(11,16,2),ARY,ARZ
COMMON/C8/S4(10,16,91)
COMMON/C9/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C10/NCH,TIHP1
COMMON/C11/PHY(10,16,91)
COMMON/C12/PHY(10,16,91)
COMMON/C13/ARX(11,16,2),ARY,ARZ
COMMON/C14/S4(10,16,91)
COMMON/C15/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C16/NCH,TIHP1
COMMON/C17/PHY(10,16,91)
COMMON/C18/PHY(10,16,91)
COMMON/C19/NCH,TIHP1
COMMON/C20/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C21/PHY(10,16,91)
COMMON/C22/PHY(10,16,91)
COMMON/C23/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C24/NCH,TIHP1
COMMON/C25/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C26/PHY(10,16,91)
COMMON/C27/PHY(10,16,91)
COMMON/C28/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C29/NCH,TIHP1
COMMON/C30/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C31/PHY(10,16,91)
COMMON/C32/PHY(10,16,91)
COMMON/C33/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C34/NCH,TIHP1
COMMON/C35/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C36/PHY(10,16,91)
COMMON/C37/PHY(10,16,91)
COMMON/C38/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C39/NCH,TIHP1
COMMON/C40/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C41/PHY(10,16,91)
COMMON/C42/PHY(10,16,91)
COMMON/C43/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C44/NCH,TIHP1
COMMON/C45/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C46/PHY(10,16,91)
COMMON/C47/PHY(10,16,91)
COMMON/C48/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C49/NCH,TIHP1
COMMON/C50/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C51/PHY(10,16,91)
COMMON/C52/PHY(10,16,91)
COMMON/C53/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C54/NCH,TIHP1
COMMON/C55/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C56/PHY(10,16,91)
COMMON/C57/PHY(10,16,91)
COMMON/C58/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C59/NCH,TIHP1
COMMON/C60/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C61/PHY(10,16,91)
COMMON/C62/PHY(10,16,91)
COMMON/C63/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C64/NCH,TIHP1
COMMON/C65/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C66/PHY(10,16,91)
COMMON/C67/PHY(10,16,91)
COMMON/C68/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C69/NCH,TIHP1
COMMON/C70/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C71/PHY(10,16,91)
COMMON/C72/PHY(10,16,91)
COMMON/C73/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C74/NCH,TIHP1
COMMON/C75/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C76/PHY(10,16,91)
COMMON/C77/PHY(10,16,91)
COMMON/C78/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C79/NCH,TIHP1
COMMON/C80/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C81/PHY(10,16,91)
COMMON/C82/PHY(10,16,91)
COMMON/C83/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C84/NCH,TIHP1
COMMON/C85/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C86/PHY(10,16,91)
COMMON/C87/PHY(10,16,91)
COMMON/C88/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C89/NCH,TIHP1
COMMON/C90/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C91/PHY(10,16,91)
COMMON/C92/PHY(10,16,91)
COMMON/C93/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C94/NCH,TIHP1
COMMON/C95/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C96/PHY(10,16,91)
COMMON/C97/PHY(10,16,91)
COMMON/C98/NTYPE(10,16,91),LPS(10,16,91)
COMMON/C99/NCH,TIHP1
COMMON/CAII GEAEfc
COMMON/CAII FILEIN(4)
COMMON/CAII ELCTND
COMMON/STCF
COMMON/ENI

189
\[1\]
\[190\]
\[3\] \((AEZ(I,J)) \cdot (T(I,J,K-1)-2 \cdot T(I,J,K) + T(I,J,K+1)) \cdot S4(I,J,K) \cdot EHY(I,J,K) \cdot T(I,J,K) \]

\[14\]
\[A(I) = -AEZ(I,J,1) \]
\[E(I) = (AEY(I,J,1)) + S4(I,J,K) \cdot EHY(I,J,K) \]

\[15\]
\[A(I) = -ABZ(I,J,1) \]
\[B(I) = ARX(I,J,1) \cdot ARX(I,J,2) + S4(I,J,K) \cdot EHY(I,J,K) \]

\[16\]
\[E(I) = (AEZ(I,J,1)) + S4(I,J,K) \cdot EHY(I,J,K) \]

\[17\]
\[A(I) = -AEZ(I,J,1) \]
\[B(I) = ARX(I,J,1) \cdot ARX(I,J,2) \]

\[18\]
\[A(I) = -ARX(I,J,1) \]
\[B(I) = ARX(I,J,1) \cdot ARX(I,J,2) \]

\[19\]
\[A(I) = -ARX(I,J,1) \]
\[B(I) = (ARX(I,J,1)) + S4(I,J,K) \cdot EHY(I,J,K) \]

\[20\]
\[A(I) = -ARX(I,J,1) \]
\[B(I) = (ARX(I,J,1)) + S4(I,J,K) \cdot EHY(I,J,K) \]
21  \[ A(I) = \frac{-AX(J, I, 1)}{2}. \]

\[ E(I) = (ARX(J, I, 1) + BRX(J, I, 2)) / 2. \]

\[ D(I) = \left( \begin{array}{c}
\frac{H_1(K)}{2} \\
HRX(J, I, 1)
\end{array} \right) \]

\[ T(FY(J, I, 2)) = (T(I, J, K) + T(I, J, K)) \]

\[ (AFZ(J, I)) \]

\[ H_4(AJ, K) \]

\[ S4(I, J, K) \]

\[ 21 \text{ TO 1000} \]

22  \[ B(I) = (AX(J, I, 2) + BRX(J, I, 1)) / 2. \]

\[ C(I) = \frac{-AX(J, I, 2)}{2}. \]

\[ D(I) = \left( \begin{array}{c}
\frac{H_1(K)}{2} \\
HRX(J, I, 1)
\end{array} \right) \]

\[ T(FY(J, I, 2)) = (T(I, J, K) + T(I, J, K)) \]

\[ (AFZ(J, I)) \]

\[ H_4(AJ, K) \]

\[ S4(I, J, K) \]

\[ 22 \text{ TO 1000} \]

23  \[ A(I) = \frac{-AX(J, I, 1)}{2}. \]

\[ B(I) = (ARX(J, I, 1) + BRX(J, I, 2)) / 2. \]

\[ C(I) = \frac{-AX(J, I, 2)}{2}. \]

\[ D(I) = \left( \begin{array}{c}
\frac{H_1(K)}{2} \\
HRX(J, I, 1)
\end{array} \right) \]

\[ T(FY(J, I, 2)) = (T(I, J, K) + T(I, J, K)) \]

\[ (AFZ(J, I)) \]

\[ H_4(AJ, K) \]

\[ S4(I, J, K) \]

\[ 23 \text{ TO 1000} \]

24  \[ A(I) = \frac{-AX(J, I, 1)}{2}. \]

\[ E(I) = (ARX(J, I, 1) + BRX(J, I, 2)) / 2. \]

\[ D(I) = \left( \begin{array}{c}
\frac{H_1(K)}{2} \\
HRX(J, I, 1)
\end{array} \right) \]

\[ T(FY(J, I, 2)) = (T(I, J, K) + T(I, J, K)) \]

\[ (AFZ(J, I)) \]

\[ H_4(AJ, K) \]

\[ S4(I, J, K) \]

\[ 24 \text{ TO 1000} \]

25  \[ E(I) = (ARX(J, I, 1) + BRX(J, I, 2)) / 2. \]

\[ D(I) = \left( \begin{array}{c}
\frac{H_1(K)}{2} \\
HRX(J, I, 1)
\end{array} \right) \]

\[ T(FY(J, I, 2)) = (T(I, J, K) + T(I, J, K)) \]

\[ (AFZ(J, I)) \]

\[ H_4(AJ, K) \]

\[ S4(I, J, K) \]

\[ 25 \text{ TO 1000} \]

26  \[ A(I) = \frac{-AX(J, I, 1)}{2}. \]

\[ B(I) = (ARX(J, I, 1) + BRX(J, I, 2)) / 2. \]

\[ C(I) = \frac{-AX(J, I, 2)}{2}. \]

\[ D(I) = \left( \begin{array}{c}
\frac{H_1(K)}{2} \\
HRX(J, I, 1)
\end{array} \right) \]

\[ T(FY(J, I, 2)) = (T(I, J, K) + T(I, J, K)) \]

\[ (AFZ(J, I)) \]

\[ H_4(AJ, K) \]

\[ S4(I, J, K) \]

\[ 26 \text{ TO 1000} \]

27  \[ A(I) = \frac{-AX(J, I, 1)}{2}. \]

\[ B(I) = (ARX(J, I, 1) + BRX(J, I, 2)) / 2. \]

\[ C(I) = \frac{-AX(J, I, 2)}{2}. \]

\[ D(I) = \left( \begin{array}{c}
\frac{H_1(K)}{2} \\
HRX(J, I, 1)
\end{array} \right) \]

\[ T(FY(J, I, 2)) = (T(I, J, K) + T(I, J, K)) \]

\[ (AFZ(J, I)) \]

\[ H_4(AJ, K) \]

\[ S4(I, J, K) \]

\[ 27 \text{ TO 1000} \]

28  \[ A(I) = \frac{-AX(J, I, 1)}{2}. \]

\[ B(I) = (ARX(J, I, 1) + BRX(J, I, 2)) / 2. \]

\[ C(I) = \frac{-AX(J, I, 2)}{2}. \]

\[ D(I) = \left( \begin{array}{c}
\frac{H_1(K)}{2} \\
HRX(J, I, 1)
\end{array} \right) \]

\[ T(FY(J, I, 2)) = (T(I, J, K) + T(I, J, K)) \]

\[ (AFZ(J, I)) \]

\[ H_4(AJ, K) \]

\[ S4(I, J, K) \]

\[ 28 \text{ TO 1000} \]
$$2 \left( A B Y (I, J, I) / 2 \right) = (T(I, J-1, K) - T(I, I, K)) +$$

$$3 \left( A E Z (I, J) \right) = (T(I, J, K-1) - T(I, J, K)) -$$

$$4 \left( E Z (K) \right) = (T(I, J, K) - A M E (K)) -$$

$$5 \left( A B Y (I, J) \right) = (T(I, J, K) - A M B (K))$$

GO TO 1000

28

$$A (I) = - A E X (J, J, I)$$


1 + \left( B 7 (K) * A R E A 1 \right)$$

$$C (I) = - A R X (J, J, I)$$

$$D (I) = A R Y (I, J, 1) * (T(I, J-1, K) - T(I, J, K)) + A R Y (I, J, 2) +$$

$$1 \left( T(I, J+1, K) - T(I, J, K) \right) +$$

$$2 A E Z (I, J) \left( T(I, J, K-1) - T(I, J, K) \right) +$$


$$3 \left( B 7 (K) * A R E A 1 * A M B (K) \right) +$$

$$4 \left( A B Y (I, J) \right) = (T(I, J, K) - A M E (K))$$

GO TO 1000

29

$$A (I) = - A R X (J, J, I) / 2$$

$$B (I) = (A R X (J, J, I)) * A E X (J, J, I) / 2 + S 4 (I, J, K) * P H Y (I, J, K)$$

1 + \left( B 7 (K) * A R E A 1 / 2 \right)$$

$$C (I) = - A R X (J, J, I) / 2$$


$$1 \left( A E Y (I, J, 1) / 2 \right) + A E Y (I, J, 2) +$$

$$1 \left( T(I, J+1, K) - T(I, J, K) \right) +$$

$$2 A E Z (I, J) \left( T(I, J, K-1) - T(I, J, K) \right) +$$

$$3 \left( H 7 (K) \right) = (A B Y (I, J) * E Z / E R A Y) + (T(I, J, K) - A M E (K))$$

GO TO 1000

30

$$A (I) = - A R X (J, J, I)$$

$$B (I) = (A R X (J, J, I)) + S 4 (I, J, K) * P H Y (I, J, K) +$$

1 \left( B 7 (K) * A R E A 5 \right)$$

$$2 + \left( H 6 (K) * A R E A 2 * S I N (T H) / 2 \right)$$

$$D (I) = (B 7 (K) * A A E A 5 * A M E (K)) +$$

$$1 S 4 (I, J, K) * P H Y (I, J, K) * T(I, J, K) +$$

$$2 (A E Y (I, J, 1) / 2) + (T(I, J-1, K) - T(I, J, K)) +$$

$$2 (A E Y (I, J, 2) / 2) + (T(I, J+1, K) - T(I, J, K)) +$$

$$3 A E Z (I, J) \left( T(I, J, K-1) - T(I, J, K) \right) +$$

$$4 \left( H 6 (K) \right) = (A R E A 2 * S I N (T H) * A M B (K))$$

GO TO 1000

31

$$A (I) = - A R X (J, J, I) / 2$$

$$E (I) = (A R X (J, J, I) / 2) + S 4 (I, J, K) * P H Y (I, J, K) +$$

1 \left( E 7 (K) * A R E A 2 \right)$$

$$2 + \left( H 6 (K) * A A E A 2 / 2 \right)$$


$$1 (A E Y (I, J, 1) / 2) + (T(I, J-1, K) - T(I, J, K)) +$$

$$1 (A E Y (I, J, 2) / 2) + (T(I, J+1, K) - T(I, J, K)) +$$

$$2 A E Z (I, J) \left( T(I, J, K-1) - T(I, J, K) \right) +$$

$$3 (H 3 * A E Y (I, J) * E Z / E R A Y) + (T(I, J, K) - A M B (K))$$

GO TO 1000

32

$$A (I) = - A R X (J, J, I)$$


1 + \left( C 7 (I) * A R E A 5 \right)$$

$$C (I) = - A R X (J, J, I)$$


$$1 (A E Y (I, J, 1) / 2) + (T(I, J-1, K) - T(I, J, K)) +$$

$$2 A E Z (I, J) \left( T(I, J, K-1) - T(I, J, K) \right) +$$

$$3 (H 5 (K) * A R E A 4) * (A M B (K) - T(I, J, K))$$

GO TO 1000
\[ A(1) = -\frac{\text{ARX}(J,I,1)}{2}. \]
\[ B(1) = \text{ARX}(J,I,1) + \text{ARX}(J,I,1)/2.\]
\[ C(1) = -\frac{\text{ARX}(J,I,1)}{2}. \]
\[ D(1) = \frac{\text{SH}(J,I,K)}{2}.\]

1. \[ (E5(K)) \text{#EY}(J,I,1)*Y3/BKAY/2. \]
2. \[ (E6(K)) \text{#EBZ}(J,I,1)*Z2/BKAY. \]
3. \[ (E7(K)) \text{#EY}(J,I,1)*Z2/BKAY. \]

\[ A(2) = -\frac{\text{ARX}(J,I,1)}{2}. \]
\[ B(2) = \text{ARX}(J,I,1)/2.\]
\[ C(2) = \frac{\text{ARX}(J,I,1)}{2}. \]
\[ D(2) = \frac{\text{SH}(J,I,K)}{2}.\]

1. \[ (E5(K)) \text{#EY}(J,I,1)*Y3/BKAY/2. \]
2. \[ (E6(K)) \text{#EBZ}(J,I,1)*Z2/BKAY. \]
3. \[ (E7(K)) \text{#EY}(J,I,1)*Z2/BKAY. \]
39 A(I) = -AEX(J,I,1)/2.
B(I) = (ARX(J,I,1)/2) * S4(I,J,K) * FHY(I,J,K) * H(I)
  + AEX(J,I,1) * X1/BKAY/2.
D(I) = (S4(I,J,K) * FHY(I,J,K) * T(I,J,K)) +
  1 (AEZ(I,J)) * (T(I,J,K-1) - T(I,J,K)) +
  2 (ABY(I,J,2)) * X1/BKAY * AME(K)/2 +
  3 (AEY(I,J,2)) * (T(I,J,K-1) - T(I,J,K)) +
  4 (AEY(I,J,2)) * Y3/BKAY * AME(K) +
  5 (H3*ARZ(I,J) * E2/BKAY) * (T(I,J,K) - AME(K)

GO TO 100

1000 CONTINUE
RETURN
END

IMPLICIT CALCULATIONS IN THE Y DIRECTION

SUBROUTINE COEFFS(IE,IE,I,K,A,B,C,D)
DIMENSION A(1),E(1),D(1),C(1)
DIMENSION ABX(16,11,2),ABY(11,16,2),ABZ(11,16)
DIMENSION T(10,16,91),T1(10,16,91)
1. T2(10,16,91),A3(10,16,91),IN(10,16,91)
CCCMEN/C1/EX, IY, EY, EY, BKAY
CCCMEN/C17/EX, IY, EY, AEY
COMMON/C2/S4(10,16,91)
CCCMEN/C3/4TYPE(10,16,91),LF(10,16,91)
COMMON/C5/CEL, TOMP
CCCMEN/C6/H3
COMMON/C18/X1, Y1, Z1, Y2, Y3, Y4, I4, TH
CCCMEN/C19/AEB1, AEB2, AEB3, AEB4, AEB5, AEB6
DC 2000 J=16,IE
JJ= TYPE(I,J,K)
GO TO (110,111,112,113,114,115,116,117,118,119,120,121,122,123,124,125,126,127,128,129,130,131,132,133,134,135)

110 B(J) = (ABY(I,J,2)) + S4(I,J,K) * FHY(I,J,K)
  1 + ABY(I,J,2) * S4(1,J,K) * FHY(I,J,K)
C(J) = -ABY(I,J,2)
  1 (AEZ(I,J)) * (T(I,J,K-1) - T(I,J,K) + T(I,J,K+1)) +
  2 (ABY(I,J,2)) * X1/BKAY * AME(K) +
  3 (AEY(I,J,2)) * Y3/BKAY * AME(K) +
GO TO 2000

111 E(J) = ABY(I,J,2) + FHY(I,J,K) * S4(I,J,K)
C(J) = -ABY(I,J,2)
  1 (AEZ(I,J)) * (T(I,J,K-1) - T(I,J,K) + (I,J,K+1)) +
  2 (ABY(I,J,2)) * X1/BKAY * AME(K) +
  3 (AEY(I,J,2)) * Y3/BKAY * AME(K) +
GO TO 2000

112 E(J) = (ABY(I,J,2)) + S4(I,J,K) * FHY(I,J,K)
C(J) = -ABY(I,J,2)
1 \( (AFZ(I,J)) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •
2 \( (BX(J,I,1)) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •
3 \( (E1(K) \times EX(J,I,1)) \times T(I,J,K) \) •
4 \( (AFZ(I,J)) \times T(I,J,K) \) •
5 \( (AFZ(I,J)) \times T(I,J,K) \) •

GO TO 2000

113
\[ A(J) = -AEY(I,J,1) \]
\[ B(J) = ABY(I,J,1) + ARY(I,J,2) + S4(I,J,K) \times PHY(I,J,K) \]
\[ C(J) = -AEY(I,J,2) \]
\[ D(J) = S4(I,J,K) \times PHY(I,J,K) \times T(I,J,K) \] +
1 \( (AEZ(I,J)) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •
2 \( AFZ(J,I,2) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •
3 \( AFZ(J,I,1) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •

GO TO 2000

114
\[ A(J) = -AEY(I,J,1) \]
\[ B(J) = ABY(I,J,1) + AFY(I,J,2) + S4(I,J,K) \times PHY(I,J,K) \]
\[ C(J) = -AEY(I,J,2) \]
\[ D(J) = S4(I,J,K) \times PHY(I,J,K) \times T(I,J,K) \] +
1 \( (AEZ(I,J)) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •
2 \( AFZ(J,I,1) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •
3 \( AFZ(J,I,2) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •

GO TO 2000

115
\[ A(J) = -AEY(I,J,1) \]
\[ B(J) = ABY(I,J,1) + AFY(I,J,2) + S4(I,J,K) \times PHY(I,J,K) \]
\[ C(J) = -AEY(I,J,2) \]
\[ D(J) = S4(I,J,K) \times PHY(I,J,K) \times T(I,J,K) \] +
1 \( (AEZ(I,J)) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •
2 \( AFZ(J,I,1) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •
3 \( AFZ(J,I,2) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •

GO TO 2000

116
\[ A(J) = -AEY(I,J,1) \]
\[ B(J) = (AEY(I,J,1) + S4(I,J,K) \times PHY(I,J,K)) \times Y1/BKAY \]
\[ C(J) = (AEY(I,J,1) + S4(I,J,K) \times PHY(I,J,K)) \times T(I,J,K) \] +
1 \( (AFZ(J,I,1)) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •
2 \( AFZ(J,I,1) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •
3 \( (E2(K) \times AFY(I,J,1)) \times Y1/BKAY \) •

GO TO 2000

117
\[ A(J) = -AEY(I,J,1) \]
\[ B(J) = (AEY(I,J,1) + S4(I,J,K) \times PHY(I,J,K)) \times Y1/BKAY \]
\[ C(J) = (AEY(I,J,1) + S4(I,J,K) \times PHY(I,J,K)) \times T(I,J,K) \] +
1 \( (AFZ(J,I,1)) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •
2 \( AFZ(J,I,1) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •
3 \( (E2(K) \times AFY(I,J,1)) \times Y1/BKAY \) •

GO TO 2000

118
\[ A(J) = -ARY(I,J,1) \]
\[ B(J) = (ARY(I,J,1) + S4(I,J,K) \times PHY(I,J,K)) \times Y1/BKAY \]
\[ C(J) = -ARY(I,J,2) \]
\[ D(J) = S4(I,J,K) \times PHY(I,J,K) \times T(I,J,K) \] +
1 \( (AEZ(I,J)) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •
2 \( AFZ(J,I,1) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •
3 \( AFZ(J,I,1) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •

GO TO 2000

119
\[ B(J) = (ARY(I,J,1) + S4(I,J,K) \times PHY(I,J,K)) \times Y1/BKAY \]
\[ C(J) = -ARY(I,J,2) \]
\[ D(J) = S4(I,J,K) \times PHY(I,J,K) \times T(I,J,K) \] +
1 \( (AEZ(I,J)) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •
2 \( AFZ(J,I,1) \times (T(I,J,K-1) - 2 \times T(I,J,K) + T(I,J,K+1)) \) •
3 (E3*ARZ(I,J)*E2/BKAY) * (T(I,J,K) -AMB(K))
8 * (AFY(I,J,K)*E4(K)*Y2*AME(K)/BKAY/2.)

GO TO 200

120 E(J) = (AFY(I,J,2)/2.) * (S4(I,J,K)*PHY(I,J,K))
1 + (AFY(I,J,2)*E4(K)*Y2/BKAY/2.
C(J) = -AFY(I,J,2)/2.
1 (AFY(J,I,1)/2.) * (T(I-1,J,K) - T(I,J,K)) -
2 (E4(K) * AFY(J,I,J) * X1/BKAY/2.) * (T(I,J,K) - T(I,J-1,K)) +
3 (E3*ARZ(I,J) *E2/BKAY) * (T(I,J,K) -AMB(K))
8 + (AFY(I,J,2)*Y4(K)*Y2*AME(K)/BKAY/2.)

GO TO 200

121 E(J) = (AFY(I,J,2)/2.) + S4(I,J,K) * PHY(I,J,K)
1 + (AFY(I,J,2)*Y4(K)*Y2/BKAY/2.
C(J) = -AFY(I,J,2)/2.
1 (AFY(J,I,1)/2.) * (T(I-1,J,K) - T(I,J,K)) -
2 (E4(K) * AFY(J,I,J) * X1/BKAY/2.) * (T(I,J,K) - T(I,J-1,K)) -
3 (E3*ARZ(I,J) *E2/BKAY) * (T(I,J,K) -AMB(K))
8 + (AFY(I,J,2)*Y4(K)*Y2*AME(K)/BKAY/2.)

GO TO 200

122 E(J) = -AFY(I,J,1)/2.
B(J) = (AFY(I,J,1) + AFY(I,J,2))/2. + S4(I,J,K) * PHY(I,J,K)
C(J) = -AFY(I,J,2)/2.
1 (AFY(J,I,1)/2.) * (T(I-1,J,K) - T(I,J,K)) -
2 (E4(K) * AFY(J,I,J) * X1/BKAY/2.) * (T(I,J,K) - T(I,J-1,K)) -
3 (E3*ARZ(I,J) *E2/BKAY) * (T(I,J,K) -AMB(K))

GO TO 200

123 A(J) = -AFY(I,J,1)/2.
B(J) = (AFY(I,J,1) + AFY(I,J,2))/2. + S4(I,J,K) * PHY(I,J,K)
C(J) = -AFY(I,J,2)/2.
1 (AFY(J,I,1)/2.) * (T(I-1,J,K) - T(I,J,K)) -
2 (E4(K) * AFY(J,I,J) * X1/BKAY/2.) * (T(I,J,K) - T(I,J-1,K)) -
3 (E3*ARZ(I,J) *E2/BKAY) * (T(I,J,K) -AMB(K))

GO TO 200

124 A(J) = -AFY(I,J,1)/2.
B(J) = (AFY(I,J,1) + AFY(I,J,2))/2. + S4(I,J,K) * PHY(I,J,K)
C(J) = -AFY(I,J,2)/2.
1 (AFY(J,I,1)/2.) * (T(I-1,J,K) - T(I,J,K)) -
2 (E4(K) * AFY(J,I,J) * X1/BKAY/2.) * (T(I,J,K) - T(I,J-1,K)) -
3 (E3*ARZ(I,J) *E2/BKAY) * (T(I,J,K) -AMB(K))

GO TO 200

125 A(J) = -AFY(I,J,1)/2.
B(J) = (AFY(I,J,1) + AFY(I,J,2))/2. + S4(I,J,K) * PHY(I,J,K)
C(J) = -AFY(I,J,2)/2.
1 (AFY(J,I,1)/2.) * (T(I-1,J,K) - T(I,J,K)) -
2 (E4(K) * AFY(J,I,J) * X1/BKAY/2.) * (T(I,J,K) - T(I,J-1,K)) -
3 (E3*ARZ(I,J) *E2/BKAY) * (T(I,J,K) -AMB(K))

GO TO 200

126 A(J) = -AFY(I,J,1)/2.
B(J) = (AFY(I,J,1) + AFY(I,J,2))/2. + S4(I,J,K) * PHY(I,J,K)
198

4 \((H3*ABZ(I,J) \cdot EZ/\text{BKAY}) \cdot (T(I,J,K) - \text{AM}(K))\)
5 \(- (H6(K) \cdot \text{AREA}3^{*}\text{SIN}(TH)/2.) \cdot (T1(I,J,K) - \text{AM}(K))\)
6 \(+ H6(K) \cdot \text{AREA}2^{*}\text{COS}(TH) \cdot \text{AM}(K)/2.\)

GO TO 2000

132

A(J) = -AF(I,J,1)
E(J) = (AF(Y(I,J,1)) \cdot \text{PHY}(I,J,K) \cdot S4(I,J,K) +
1 \((E5(K) \cdot AFY(I,J,1) \cdot Y3/\text{BKAY})\)
C(J) = S4(I,J,K) \cdot \text{PHY}(I,J,K) \cdot T(I,J,K) +
1 \((AGX(J,1,1) \cdot (T(1-1,J,K) - T1(I,J,K))\)
1 \((AGX(J,1,2) \cdot (T1(I+1,J,K) - T1(I,J,K))\)
2 \((AGZ(I,J)) \cdot (T(I,J,K-1) - T(I,J,K)) \cdot (I(I,J,K) - \text{AM}(K)) \cdot
3 \((E5(K) \cdot AFY(I,J,1) \cdot Y3/\text{BKAY} \cdot \text{AM}(K))\)

GO TO 2000

133

A(J) = -AFY(I,J,1)/2.
B(J) = (AFY(I,J,1)/2.) \cdot (S4(I,J,K) \cdot \text{PHY}(I,J,K) +
1 \((E5(K) \cdot AFY(I,J,1) \cdot Y3/\text{BKAY}/2.)\)
D(J) = S4(I,J,K) \cdot \text{PHY}(I,J,K) \cdot T(I,J,K) +
1 \((AGZ(J,1,1)/2.) \cdot (T(1-1,J,K) - T1(I,J,K)) \cdot
e2 \((AGZ(J,1,2)/2.) \cdot (T(1+1,J,K) - T1(I,J,K)) \cdot
e3 \((H3*AGZ(J,1)) \cdot \text{EZ}/\text{BKAY} \cdot (T(I,J,K) - \text{AM}(K)) \cdot
4 \((H5(K) \cdot AFY(I,J,1) \cdot Y3/\text{BKAY} \cdot \text{AM}(K)/2.)\)

GO TO 2000

134

E(J) = (AFY(I,J,2)) \cdot S4(I,J,K) \cdot \text{PHY}(I,J,K)
1 \(+ AFY(I,J,2) \cdot H6(K) \cdot Y4/\text{BKAY}\)
C(J) = -AFY(I,J,2)
D(J) = S4(I,J,K) \cdot \text{PHY}(I,J,K) \cdot T(I,J,K) +
1 \((AFZ(J,1)) \cdot (T(1,J,K-1) - 2 \cdot (T(I,J,K) + T(I,J,K+1)) \cdot
e2 \((AFZ(J,1,2)/2.) \cdot (T(1,J,K+1) - T1(I,J,K)) \cdot
e3 \((E5(K) \cdot AFZ(J,1,1) \cdot \text{EZ}/\text{BKAY}) \cdot (T(I,J,K) - \text{AM}(K)) \cdot
4 \((F5*KAFZ(J,1) \cdot Y4/\text{BKAY}) \cdot (T(I,J,K) - \text{AM}(K)) \cdot
5 \((E5(K) \cdot AFY(I,J,1) \cdot Y3/\text{BKAY} \cdot \text{AM}(K)/2.)\)

GO TO 2000

135

B(J) = (AFY(I,J,2)/2.) \cdot S4(I,J,K) \cdot \text{PHY}(I,J,K)
1 \(+ AFY(I,J,2) \cdot H6(K) \cdot Y4/\text{BKAY}/2.\)
C(J) = -AFY(I,J,2)/2.
D(J) = S4(I,J,K) \cdot \text{PHY}(I,J,K) \cdot T(I,J,K) +
1 \((AFZ(J,1,1)/2.) \cdot (T(1,J,K-1) - 2 \cdot (T(I,J,K) + T(I,J,K+1)) \cdot
e2 \((AFZ(J,1,2)/2.) \cdot (T(1,J,K+1) - T1(I,J,K)) \cdot
e3 \((E5*KAFZ(J,1,1) \cdot \text{EZ}/\text{BKAY}) \cdot (T(I,J,K) - \text{AM}(K)) \cdot
4 \((E5*KAFZ(J,1,2)/2.) \cdot (T(1,J,K) - \text{AM}(K)) \cdot
5 \((H5(K) \cdot AFZ(J,1,1) \cdot Y4/\text{BKAY} \cdot \text{AM}(K)/2.)\)

GO TO 2000

136

B(J) = (AFY(I,J,2)) \cdot S4(I,J,K) \cdot \text{PHY}(I,J,K)
2 \(+ (H6(K) \cdot \text{AREA}3^{*}\text{COS}(TH))\)
C(J) = -AFY(I,J,2)
D(J) = S4(I,J,K) \cdot \text{PHY}(I,J,K) \cdot T(I,J,K) +
1 \((AFZ(J,1)) \cdot (T(I,J,K-1) - 2 \cdot (T(I,J,K) + T(I,J,K+1)) \cdot
e2 \((AFZ(J,1,1)) \cdot (T(I,J,K+1) - T1(I,J,K)) \cdot
e3 \((H1(K) \cdot \text{AREA}6) \cdot (T(I,J,K) - \text{AM}(K)) \cdot
e4 \((H6(K) \cdot \text{AREA}3^{*}\text{COS}(TH) \cdot \text{AM}(K))\)
5 \(- (H6(K) \cdot \text{AREA}3^{*}\text{SIN}(TH)) \cdot (T1(I,J,K) - \text{AM}(K))\)

GO TO 2000

137

B(J) = (AFY(I,J,2)/2.) \cdot S4(I,J,K) \cdot \text{PHY}(I,J,K)
1 \(+ (H6(K) \cdot \text{AREA}3^{*}\text{COS}(TH)/2.)\)
C(J) = -AFY(I,J,2)/2.
D(J) = S4(I,J,K) \cdot \text{PHY}(I,J,K) \cdot T(I,J,K) +
1 \((AFZ(J,1,1)/2.) \cdot (T(1,J,K-1) - T1(I,J,K)) \cdot
e2 \((AFZ(J,1,2)/2.) \cdot (T1(I,J,K) - T1(I,J,K)) \cdot
e3 \((AFZ(J,1,1)) \cdot (T(I,J,K-1) - T(I,J,K)) \cdot
e4 \((E5*AFZ(J,1) \cdot \text{EZ}/\text{BKAY}) \cdot (T(I,J,K) - \text{AM}(K))\)

GO TO 2000
1 \cdot (F6(K) \cdot \text{AREA}3 \cdot \cos \theta + 2 \cdot A\text{MB}(K) )

2 = (B6(K) \cdot \text{AREA}3 \cdot \sin \theta + (T1(I, J, K) - A\text{MB}(K) )

\text{GO TO 2000}

138 \quad A(J) = -ABY(I, J, 1)

B(J) = (ABY(I, J, 1)) + (S4(I, J, K) \cdot \text{PHY}(I, J, K))

1 + (B5(K) \cdot \text{AREA}(I, J, 1) \cdot Y3/Ykay)

D(J) = S4(I, J, K) \cdot \text{PHY}(I, J, K) \cdot T(I, J, K)

1 \cdot (AEX(I, J, 1)) \cdot (T1(I-1, J, K) - T1(I, J, K)) -

2 \cdot (B1(K) \cdot AEX(I, J, 1) \cdot X1/Ykay) \cdot (T1(I, J, K) - AMB(K))

3 \cdot (AEZ(I, J, 1)) \cdot (T(I, J, K-1) - 2 \cdot T(I, J, K) + T(I, J, K+1))

4 \cdot (H5(K) \cdot ABY(I, J, 1) \cdot Y3/Ykay) \cdot \text{AMB}(K)

\text{GO TO 2000}

139 \quad A(J) = -ABY(I, J, 1)/2.

B(J) = (ABY(I, J, 1)/2.) + (S4(I, J, K) \cdot \text{PHY}(I, J, K))

1 + (B5(K) \cdot \text{AREA}(I, J, 1) \cdot Y3/Ykay/2.)

D(J) = S4(I, J, K) \cdot \text{PHY}(I, J, K) \cdot T(I, J, K)

1 \cdot (AEX(I, J, 1)/2.) \cdot (T1(I-1, J, K) - T1(I, J, K)) -

2 \cdot (B1(K) \cdot AEX(I, J, 1) \cdot X1/Ykay/2.) \cdot (T1(I, J, K) - AMB(K))

3 \cdot (AEZ(I, J, 1)) \cdot (T(I, J, K-1) - T(I, J, K))

4 \cdot (H3 \cdot AxeZ(I, J) \cdot X2/Ykay) \cdot (T(I, J, K) - ABE(K))

5 \cdot (BEZ(K) \cdot ABY(I, J, 1) \cdot Y3/Ykay) \cdot \text{AMB}(K)/2.)

\text{GO TO 2000}

\text{2000 CONTINUE}

\text{END}

\text{I N E L I C I T C A L C U L A T I O N S I N T H E Z D I R E C T I O N}

\text{S U N E X E C U T I N K (I E, I E, I, J, A, B, C, D)}

\text{D I M E N S I O N A(1), E(1), C(1), D(1)}

\text{D I M E N S I O N ABX(16, 11, 2), ARX(11, 16, 2), AMZ(11, 16)}

\text{D I M E N S I O N T(10, 10, 91), T1(10, 10, 91)}

1, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30,

2 121, 212, 213, 214, 215, 216, 217, 218,

1 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230,

2 231, 232, 233, 234, 235, 236, 237, 238, 239, 240,

21 \quad A(K) = -ARB(I, J, K)

B(K) = (AEZ(I, J) \cdot 2.) \cdot S4(I, J, K) \cdot \text{PHY}(I, J, K)

C(K) = A(K)

B(K) = S4(I, J, K) \cdot \text{PHY}(I, J, K) \cdot T(I, J, K)

1 \cdot (AEX(I, J, 2)) \cdot (T1(I+1, J, K) - T1(I, J, K))

2 \cdot (AEX(I, J, 2)) \cdot (T2(I+1, J, K) - T2(I, J, K))

6 \cdot (ABy(I, J, 2) \cdot S4(K) \cdot Y2/Ykay) \cdot (\text{AMB}(K) - T2(I, J, K))

\text{IF (K \cdot EC) \cdot D(K) = 0(K) + (AEZ(I, J)) \cdot T(I, J, 1)}

\text{GO TO 3000}
211  \[ A(K) = -ABZ(I,J) \]
\[ B(K) = ABZ(I,J) * 2. * S4(I,J,K) * PHY(I,J,K) \]
\[ C(K) = A(K) \]
\[ D(K) = S4(I,J,K) * PHY(I,J,K) * T(I,J,K) * \]
\[ 1 \] (AFX(J,J,1)) * (T1(I-1,J,K) - T1(I,J,K)) * \]
\[ 1 \] (AFX(J,J,2)) * (T1(I+1,J,K) - T1(I,J,K)) * \]
\[ 2 \] (AFF(I,J,2)) * (T2(I,J+1,K) - T2(I,J,K)) \]
\[ 8 \] * (AFY(I,J,2) * E4(K) * T2/EKAY) * (AMB(K) - T2(I,J,K)) \]
\[ IF (K.EQ.2) D(K) = E(K) * (ABZ(I,J) * T(I,J,1)) \]
\[ GO TC 3000 \]

212  \[ A(K) = -AFZ(I,J) \]
\[ B(K) = AEZ(I,J) * 2. * S4(I,J,K) * PHY(I,J,K) \]
\[ C(K) = A(K) \]
\[ D(K) = S4(I,J,K) * PHY(I,J,K) * T(I,J,K) * \]
\[ 1 \] (AFX(J,J,1)) * (T1(I-1,J,K) - T1(I,J,K)) * \]
\[ 2 \] (AFY(I,J,1)) * (T2(I,J+1,K) - T2(I,J,K)) \]
\[ 8 \] * (AFY(I,J,2) * E4(K) * T2/EKAY) * (AMB(K) - T2(I,J,K)) \]
\[ IF (K.EQ.2) D(K) = E(K) * (AFZ(I,J) * T(I,J,1)) \]
\[ GO TC 3000 \]

213  \[ A(K) = -AEZ(I,J) \]
\[ B(K) = AEZ(I,J) * 2. * S4(I,J,K) * PHY(I,J,K) \]
\[ C(K) = A(K) \]
\[ D(K) = S4(I,J,K) * PHY(I,J,K) * T(I,J,K) * \]
\[ 1 \] (AFX(J,J,1)) * (T1(I-1,J,K) - T1(I,J,K)) * \]
\[ 2 \] (AEY(I,J,1)) * (T2(I,J+1,K) - T2(I,J,K)) \]
\[ 8 \] * (AEY(I,J,2) * E4(K) * T2/EKAY) * (AMB(K) - T2(I,J,K)) \]
\[ IF (K.EQ.2) D(K) = E(K) * (AEZ(I,J) * T(I,J,1)) \]
\[ GO TC 3000 \]

214  \[ A(K) = -AEZ(I,J) \]
\[ B(K) = AEZ(I,J) * 2. * S4(I,J,K) * PHY(I,J,K) \]
\[ C(K) = A(K) \]
\[ D(K) = S4(I,J,K) * PHY(I,J,K) * T(I,J,K) * \]
\[ 1 \] (AEY(I,J,1)) * (T2(I,J+1,K) - T2(I,J,K)) * \]
\[ 2 \] (AEY(I,J,2)) * (T2(I,J+1,K) - T2(I,J,K)) \]
\[ 8 \] * (AEY(I,J,2) * E4(K) * T2/EKAY) * (AMB(K) - T2(I,J,K)) \]
\[ IF (K.EQ.2) D(K) = E(K) * (AEZ(I,J) * T(I,J,1)) \]
\[ GO TC 3000 \]

215  \[ A(K) = -ABZ(I,J) \]
\[ B(K) = ABZ(I,J) * 2. * S4(I,J,K) * PHY(I,J,K) \]
\[ C(K) = A(K) \]
\[ D(K) = S4(I,J,K) * PHY(I,J,K) * T(I,J,K) * \]
\[ 1 \] (AEY(I,J,1)) * (T2(I,J+1,K) - T2(I,J,K)) * \]
\[ 2 \] (AEY(I,J,2)) * (T2(I,J+1,K) - T2(I,J,K)) \]
\[ 8 \] * (AEY(I,J,2) * E4(K) * T2/EKAY) * (AMB(K) - T2(I,J,K)) \]
\[ IF (K.EQ.2) D(K) = E(K) * (ABZ(I,J) * T(I,J,1)) \]
\[ GO TC 3000 \]

216  \[ A(K) = -ABZ(I,J) \]
\[ B(K) = (AEZ(I,J) * 2.) * S4(I,J,K) * PHY(I,J,K) \]
\[ C(K) = A(K) \]
\[ D(K) = S4(I,J,K) * PHY(I,J,K) * T(I,J,K) * \]
\[ 1 \] (AEY(I,J,1)) * (T2(I,J+1,K) - T2(I,J,K)) * \]
\[ 2 \] (AEY(I,J,2)) * (T2(I,J+1,K) - T2(I,J,K)) \]
\[ 8 \] * (AEY(I,J,2) * E4(K) * T2/EKAY) * (AMB(K) - T2(I,J,K)) \]
\[ IF (K.EQ.2) D(K) = E(K) * (ABZ(I,J) * T(I,J,1)) \]
\[ GO TC 3000 \]

217  \[ A(K) = -AEZ(I,J) \]
\[ B(K) = AEZ(I,J) * 2. * S4(I,J,K) * PHY(I,J,K) \]
$1 \left( AEX(J,I,1)/2 \right) \left( T(1-I,J,K)-T(I,J,K) \right) +$ 
$1 \left( AEX(J,I,2)/2 \right) \left( T(1-I,J,K)-T(I,J,K) \right) +$ 
$2 \left( AFY(I,J,1)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$2 \left( AFY(I,J,2)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$3 H3*AEZ(I,J)*IZ/BRKAY*AE(K)$ 
GO TO 3000

224 $A(K) = -ARZ(I,J)$ 
$B(K) = \left( AEZ(I,J) \right) \left( S4(I,J,K) \right) \left( PHY(I,J,K) \right) +$ 
$1 H3*AEZ(I,J)*IZ/BRKAY$

C(K) = $S4(I,J,K) \left( PHY(I,J,K) \right) \left( T(I,J,K) \right) +$ 
$1 \left( AEX(J,I,1)/2 \right) \left( T(1-I,J,K)-T(I,J,K) \right) +$ 
$2 \left( AEX(J,I,2)/2 \right) \left( T(1-I,J,K)-T(I,J,K) \right) +$ 
$3 \left( AFY(I,J,1)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$4 \left( AFY(I,J,2)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$4 \left( AFY(I,J,2)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$5 (E3*ARZ(I,J)*IZ/BRKAY*AFE(K))$
GO TO 3000

225 $A(K) = -ARZ(I,J)$ 
$B(K) = \left( AEZ(I,J) \right) \left( S4(I,J,K) \right) \left( PHY(I,J,K) \right) +$ 
$1 H3*AEZ(I,J)*IZ/BRKAY$

C(K) = $S4(I,J,K) \left( PHY(I,J,K) \right) \left( T(I,J,K) \right) +$ 
$1 \left( AEX(J,I,1)/2 \right) \left( T(1-I,J,K)-T(I,J,K) \right) +$ 
$2 \left( AEX(J,I,2)/2 \right) \left( T(1-I,J,K)-T(I,J,K) \right) +$ 
$3 \left( AFY(I,J,1)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$4 \left( AFY(I,J,2)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$5 \left( AFY(I,J,2)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$4 \left( E3*AEZ(I,J)*IZ/BRKAY*AFB(K) \right)$
GO TO 3000

226 $A(K) = -AEZ(I,J)$ 
$B(K) = \left( AEZ(I,J) \right) \left( S4(I,J,K) \right) \left( PHY(I,J,K) \right) +$ 
$1 H3*AEZ(I,J)*IZ/BRKAY$

C(K) = $S4(I,J,K) \left( PHY(I,J,K) \right) \left( T(I,J,K) \right) +$ 
$1 \left( AEX(J,I,1)/2 \right) \left( T(1-I,J,K)-T(I,J,K) \right) +$ 
$2 \left( AEX(J,I,2)/2 \right) \left( T(1-I,J,K)-T(I,J,K) \right) +$ 
$3 \left( AFY(I,J,1)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$4 \left( AFY(I,J,2)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$5 \left( AFY(I,J,2)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$4 \left( E3*AEZ(I,J)*IZ/BRKAY*AFB(K) \right)$
GO TO 3000

227 $A(K) = -AEZ(I,J)$ 
$B(K) = \left( AEZ(I,J) \right) \left( S4(I,J,K) \right) \left( PHY(I,J,K) \right) +$ 
$1 H3*AEZ(I,J)*IZ/BRKAY$

C(K) = $S4(I,J,K) \left( PHY(I,J,K) \right) \left( T(I,J,K) \right) +$ 
$1 \left( AEX(J,I,1)/2 \right) \left( T(1-I,J,K)-T(I,J,K) \right) +$ 
$2 \left( AEX(J,I,2)/2 \right) \left( T(1-I,J,K)-T(I,J,K) \right) +$ 
$3 \left( AFY(I,J,1)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$4 \left( AFY(I,J,2)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$4 \left( AFY(I,J,2)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$5 \left( E3*AEZ(I,J)*IZ/BRKAY*AFB(K) \right)$
GO TO 3000

228 $A(K) = -AEZ(I,J)$ 
$B(K) = \left( AEZ(I,J) \right) \left( S4(I,J,K) \right) \left( PHY(I,J,K) \right) +$ 
$1 H3*AEZ(I,J)*IZ/BRKAY$

C(K) = $S4(I,J,K) \left( PHY(I,J,K) \right) \left( T(I,J,K) \right) +$ 
$1 \left( AEX(J,I,1)/2 \right) \left( T(1-I,J,K)-T(I,J,K) \right) +$ 
$2 \left( AEX(J,I,2)/2 \right) \left( T(1-I,J,K)-T(I,J,K) \right) +$ 
$3 \left( AFY(I,J,1)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$4 \left( AFY(I,J,2)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$4 \left( AFY(I,J,2)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$4 \left( AFY(I,J,2)/2 \right) \left( T(1-J,K)-T(2,I,J,K) \right) +$ 
$5 \left( E3*AEZ(I,J)*IZ/BRKAY*AFB(K) \right)$
GO TO 3000

229 $A(K) = -AEZ(I,J)$ 
$E(K) = \left( AEZ(I,J) \right) \left( S4(I,J,K) \right) \left( PHY(I,J,K) \right) +$ 
$1 H3*AEZ(I,J)*IZ/BRKAY$
\[ D(K) = S_4(I,J,K) \cdot \text{PHY}(I,J,K) \cdot T(I,J,K) \]
\[ A(K) = -A_{RZ}(I,J) \]
\[ B(K) = A_{E2}(I,J) \cdot S_4(I,J,K) \cdot \text{PHY}(I,J,K) \]
\[ C(K) = A(K) \]
\[ D(K) = S_4(I,J,K) \cdot \text{PHY}(I,J,K) \cdot T(I,J,K) \]
\[ E(K) = s_4(I,J,K) \cdot \text{PHY}(I,J,K) \cdot T(I,J,K) \]
\[ F(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ G(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ H(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ I(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ J(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ K(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ L(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ M(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ N(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ O(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ P(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ Q(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ R(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ S(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ T(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ U(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ V(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ W(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ X(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ Y(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ Z(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]

\[ A(K) = -A_{RZ}(I,J) \]
\[ B(K) = A_{E2}(I,J) \cdot S_4(I,J,K) \cdot \text{PHY}(I,J,K) \]
\[ C(K) = A(K) \]
\[ D(K) = S_4(I,J,K) \cdot \text{PHY}(I,J,K) \cdot T(I,J,K) \]
\[ E(K) = s_4(I,J,K) \cdot \text{PHY}(I,J,K) \cdot T(I,J,K) \]
\[ F(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ G(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ H(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ I(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ J(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ K(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ L(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ M(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ N(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ O(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ P(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ Q(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ R(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ S(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ T(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ U(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ V(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ W(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ X(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
\[ Y(K) = A_{E2}(I,J) \cdot T(I,J,K) \]
\[ Z(K) = A_{RZ}(I,J) \cdot A_{E2}(I,J) \cdot T(I,J,K) \]
8 * (ARY(I,J,2) * E6(K) * Y4 /BKAY) * (AMB(K) - T2(I,J,K))
IF (K.EC.2) D(K) = I(K) + (ABZ(I,J)) * T(I,J,1)
GO TC 3000

235
A(K) = -ABZ(I,J)
B(K) = (ABZ(I,J)) + (S4(I,J,K) * PHY(I,J,K))
C(K) = A(K)
D(K) = S4(I,J,K) * PHY(I,J,K) * T(I,J,K)
1 (EX(I,J,1)) + (T1(I,J,K) - I1(I,J,K))
2 (E1(K) * ABX(I,J) /2.) + (T2(I,J+1,K) - E2(I,J,K))
3 (F1(K) * ARZ(I,J) * D3 /BKAY)
4 * (H5(K) * ARX(I,J) * E5 /BKAY) * (AMB(K) - T2(I,J,K))
5 - (H5(K) * ABY(I,J) * Y3 /BKAY) * (AMB(K) - T2(I,J,K))
IF (K.EC.2) D(K) = I(K) + (ABZ(I,J)) * T(I,J,1)
GO TC 3000

236
A(K) = -ABZ(I,J)
B(K) = (ABZ(I,J)) + (S4(I,J,K) * PHY(I,J,K))
C(K) = A(K)
D(K) = S4(I,J,K) * PHY(I,J,K) * T(I,J,K)
1 (EX(I,J,1)) * (T1(I,J,K) - I1(I,J,K))
2 (E1(K) * ABX(I,J) /2.) * (T2(I,J+1,K) - E2(I,J,K))
3 (F1(K) * ARZ(I,J) * D3 /BKAY)
4 * (H5(K) * ARX(I,J) * E5 /BKAY) * (AMB(K) - T2(I,J,K))
5 - (H5(K) * ABY(I,J) * Y3 /BKAY) * (AMB(K) - T2(I,J,K))
IF (K.EC.2) D(K) = I(K) + (ABZ(I,J)) * T(I,J,1)
GO TC 3000

237
A(K) = -ABZ(I,J)
B(K) = (ABZ(I,J)) + (S4(I,J,K) * PHY(I,J,K))
C(K) = A(K)
D(K) = S4(I,J,K) * PHY(I,J,K) * T(I,J,K)
1 (EX(I,J,1)) + (T1(I,J,K) - I1(I,J,K))
2 (E1(K) * ABX(I,J) /2.) + (T2(I,J+1,K) - E2(I,J,K))
3 (F1(K) * ARZ(I,J) * D3 /BKAY)
4 * (H5(K) * ARX(I,J) * E5 /BKAY) * (AMB(K) - T2(I,J,K))
5 - (H5(K) * ABY(I,J) * Y3 /BKAY) * (AMB(K) - T2(I,J,K))
IF (K.EC.2) D(K) = I(K) + (ABZ(I,J)) * T(I,J,1)
GO TC 3000

238
A(K) = -ABZ(I,J)
B(K) = (ABZ(I,J)) + (S4(I,J,K) * PHY(I,J,K))
C(K) = A(K)
D(K) = S4(I,J,K) * PHY(I,J,K) * T(I,J,K)
1 (EX(I,J,1)) + (T1(I,J,K) - I1(I,J,K))
2 (E1(K) * ABX(I,J) /2.) + (T2(I,J+1,K) - E2(I,J,K))
3 (F1(K) * ARZ(I,J) * D3 /BKAY)
4 * (H5(K) * ARX(I,J) * E5 /BKAY) * (AMB(K) - T2(I,J,K))
5 - (H5(K) * ABY(I,J) * Y3 /BKAY) * (AMB(K) - T2(I,J,K))
IF (K.EC.2) D(K) = I(K) + (ABZ(I,J)) * T(I,J,1)
GO TC 3000

239
A(K) = -ABZ(I,J)
B(K) = (ABZ(I,J)) + (S4(I,J,K) * PHY(I,J,K))
C(K) = A(K)
D(K) = S4(I,J,K) * PHY(I,J,K) * T(I,J,K)
1 (EX(I,J,1)) + (T1(I,J,K) - I1(I,J,K))
2 (E1(K) * ABX(I,J) /2.) + (T2(I,J+1,K) - E2(I,J,K))
3 (F1(K) * ARZ(I,J) * D3 /BKAY)
4 * (H5(K) * ARX(I,J) * E5 /BKAY) * (AMB(K) - T2(I,J,K))
5 - (H5(K) * ABY(I,J) * Y3 /BKAY) * (AMB(K) - T2(I,J,K))
IF (K.EC.2) D(K) = I(K) + (ABZ(I,J)) * T(I,J,1)
GO TC 3000

3000
CONTINUE
RETURN
END

C
C
SUBROUTINE COMPUTE(L1, L2, M1, M2, K1)
SUBROUTINE TO CALCULATE THE TEMPERATURES AT THE
END OF A TIME STEP

DIMENSION T(10, 16, 91), T1(10, 16, 91), T2(10, 16, 91),
T3(10, 16, 91), T4(10, 16, 91), T5(10, 16, 91)
DIMENSION A(10), B(10), C(10), D(10), EPRIME(10)
DIMENSION AF(16, 11, 2), AE(11, 16, 2), AB(11, 16)
COMMON/C1/EX, EY, EZ, ET, EKAY
COMMON/C2/S4(10, 16, 91)
COMMON/C3/NTYPE(10, 16, 91), LFS(10, 16, 91)
COMMON/C4/L, M, K
COMMON/C5/NC, TEMPERATURE
COMMON/C6/H3
COMMON/C7/I, J, T1, T2, T3, T4
COMMON/C8/P1X, P1Y, P1Z, P1, P2, P3, P4, P5, P6
COMMON/C9/PH, EPH, EPH, EPH, EPH, EPH, EPH, EPH, EPH, EPH
COMMON/C10/L1Q, L1S, L2, L3, L4, L5, L6
COMMON/C11/MT, M2, M3, M4, M5, M6, M7, M8, M9
COMMON/C12/S4, S4, S4, S4, S4, S4, S4, S4, S4, S4
COMMON/C13/S4, S4, S4, S4, S4, S4, S4, S4, S4, S4
COMMON/C14/S4, S4, S4, S4, S4, S4, S4, S4, S4, S4
COMMON/C15/S4, S4, S4, S4, S4, S4, S4, S4, S4, S4
COMMON/C16/S4, S4, S4, S4, S4, S4, S4, S4, S4, S4
COMMON/C17/S4, S4, S4, S4, S4, S4, S4, S4, S4, S4
COMMON/C18/S4, S4, S4, S4, S4, S4, S4, S4, S4, S4
DO 1000 I=L1, L2
DO 1000 J=M1, M2
DO 1000 K=2, K1
IJK=I+JK*10+K
GO TO 310
231
310 IN(I,J,K) = (ARX(J,1,2)) * (T1(I+1,J,K) - T1(I,J,K)) +
1 (ARY(I,J,2)) * (T2(I,J+1,K) - T2(I,J,K)) +
2 (AEZ(I,J)) * (T3(I,J,K-1) - T3(I,J,K)) +
3 (S4(I,J,K))/2. * PHY(I,J,K) * T(I,J,K) +
GO TO 1000
311 TN(I,J,K) = (ARX(I,J,1)) * (T1(I-1,J,K) - T1(I,J,K)) +
1 (AFX(I,J,1)) * (T1(I+1,J,K) - T1(I,J,K)) +
2 (AFFY(I,J,1)) * (T1(I,J+1,K) - T1(I,J,K)) +
3 (AFZ(I,J)) * (T1(I,J,K-1) - T1(I,J,K)) +
GO TO 1000
312 TN(I,J,K) = (AFX(I,J,1)) * (T1(I-1,J,K) - T1(I,J,K)) +
1 (AFFY(I,J,1)) * (T1(I,J+1,K) - T1(I,J,K)) +
2 (AFFY(I,J,1)) * (T1(I,J+1,K) - T1(I,J,K)) +
3 (AFZ(I,J)) * (T1(I,J,K-1) - T1(I,J,K)) +
GO TO 1000
313 TN(I,J,K) = (AFX(I,J,1)) * (T1(I-1,J,K) - T1(I,J,K)) +
1 (AFFY(I,J,1)) * (T1(I,J+1,K) - T1(I,J,K)) +
2 (AFFY(I,J,1)) * (T1(I,J+1,K) - T1(I,J,K)) +
3 (AFZ(I,J)) * (T1(I,J,K-1) - T1(I,J,K)) +
GO TO 1000
\[ T_I (J, K) = ( (AEX (J, I, 1)) \times (T_1 (I-1, J, K) - T_1 (I, J, K)) + 1 \times (AEX (J, I, 2)) \times (T_1 (I+1, J, K) - T_1 (I, J, K)) \times (T_2 (I, J-1, K) - T_2 (I, J, K)) + 2 \times (AEX (I, J, 1)) \times (T_2 (I, J+1, K) - T_2 (I, J, K)) + 2 \times AEZ (I, J) \times (T_3 (I, J, K-1) - T_3 (I, J, K) + T_3 (I, J, K+1)) / \] 
\[ (S_4 (I, J, K) / 2) \times PBY (I, J, K) \times T (I, J, K) \]
\[
2 \left( \frac{f (Y_{I,J}^2)}{2} \right) \left( T_2 (I,J+1,K) - T_2 (I,J,K) \right)
\]
\[
2 \left( \frac{ABZ (I,J)}{2} \right) \left( T_3 (I,J,K) - T_3 (I,J,K) \right)
\]
\[
3 \left( \frac{B_3 \times AEZ (I,J)}{2} \right) \left( T_3 (I,J,K) - AHB (K) \right)
\]
\[
\left( \frac{f (Y_{I,J}^2)}{2} \right) \times \left( T_1 (I,J,K) \right)
\]

GO 10 1000

323
\[
T_N (I,J,K) = \left( \frac{ABX (J,I,1)}{2} \right) \left( T_1 (1-I,J,K) - T_1 (I,J,K) \right)
\]
\[
1 \left( \frac{f (Y_{I,J}^2)}{2} \right) \left( T_2 (I,J,K) - T_1 (I,J,K) \right)
\]
\[
2 \left( \frac{ABZ (I,J)}{2} \right) \left( T_2 (I,J,K) - T_2 (I,J,K) \right)
\]
\[
3 \left( \frac{B_3 \times AEZ (I,J)}{2} \right) \left( T_3 (I,J,K) - AHB (K) \right)
\]

GO 10 1000

324
\[
T_N (I,J,K) = \left( \frac{ABX (J,I,2)}{2} \right) \left( T_1 (1-I,J,K) - T_1 (I,J,K) \right)
\]
\[
1 \left( \frac{f (Y_{I,J}^2)}{2} \right) \left( T_2 (I,J,K) - T_2 (I,J,K) \right)
\]
\[
2 \left( \frac{ABZ (I,J)}{2} \right) \left( T_2 (I,J,K) - T_2 (I,J,K) \right)
\]
\[
3 \left( \frac{B_3 \times AEZ (I,J)}{2} \right) \left( T_3 (I,J,K) - AHB (K) \right)
\]

GO 10 1000

325
\[
T_N (I,J,K) = \left( \frac{ABX (J,I,2)}{2} \right) \left( T_1 (1-I,J,K) - T_1 (I,J,K) \right)
\]
\[
1 \left( \frac{f (Y_{I,J}^2)}{2} \right) \left( T_2 (I,J,K) - T_2 (I,J,K) \right)
\]
\[
2 \left( \frac{ABZ (I,J)}{2} \right) \left( T_2 (I,J,K) - T_2 (I,J,K) \right)
\]
\[
3 \left( \frac{B_3 \times AEZ (I,J)}{2} \right) \left( T_3 (I,J,K) - AHB (K) \right)
\]

GO 10 1000

326
\[
T_N (I,J,K) = \left( \frac{ABX (J,I,1)}{2} \right) \left( T_1 (1-I,J,K) - T_1 (I,J,K) \right)
\]
\[
1 \left( \frac{f (Y_{I,J}^2)}{2} \right) \left( T_2 (I,J,K) - T_2 (I,J,K) \right)
\]
\[
2 \left( \frac{ABZ (I,J)}{2} \right) \left( T_2 (I,J,K) - T_2 (I,J,K) \right)
\]
\[
3 \left( \frac{B_3 \times AEZ (I,J)}{2} \right) \left( T_3 (I,J,K) - AHB (K) \right)
\]

GO 10 1000

327
\[
T_N (I,J,K) = \left( \frac{ABX (J,I,1)}{2} \right) \left( T_1 (1-I,J,K) - T_1 (I,J,K) \right)
\]
\[
1 \left( \frac{f (Y_{I,J}^2)}{2} \right) \left( T_2 (I,J,K) - T_2 (I,J,K) \right)
\]
\[
2 \left( \frac{ABZ (I,J)}{2} \right) \left( T_2 (I,J,K) - T_2 (I,J,K) \right)
\]
\[
3 \left( \frac{B_3 \times AEZ (I,J)}{2} \right) \left( T_3 (I,J,K) - AHB (K) \right)
\]

GO 10 1000

328
\[
T_N (I,J,K) = (ABX (J,I,1)) \left( T_1 (1-I,J,K) - T_1 (I,J,K) \right)
\]
\[
1 \left( \frac{f (Y_{I,J}^2)}{2} \right) \left( T_2 (I,J,K) - T_2 (I,J,K) \right)
\]
\[
2 \left( \frac{ABZ (I,J)}{2} \right) \left( T_2 (I,J,K) - T_2 (I,J,K) \right)
\]
\[
3 \left( \frac{B_3 \times AEZ (I,J)}{2} \right) \left( T_3 (I,J,K) - AHB (K) \right)
\]

GO 10 1000

329
\[
T_N (I,J,K) = \left( \frac{ABX (J,I,2)}{2} \right) \left( T_1 (1-I,J,K) - T_1 (I,J,K) \right)
\]
\[
1 \left( \frac{f (Y_{I,J}^2)}{2} \right) \left( T_2 (I,J,K) - T_2 (I,J,K) \right)
\]
\[
2 \left( \frac{ABZ (I,J)}{2} \right) \left( T_2 (I,J,K) - T_2 (I,J,K) \right)
\]
\[
3 \left( \frac{B_3 \times AEZ (I,J)}{2} \right) \left( T_3 (I,J,K) - AHB (K) \right)
\]

GO 10 1000
330 \( T(I,J,K) = \left( \text{ABS}(J,J_1) \right)^* \left( T(I-1,J,K) - T(I,J,K) \right) - \\
1 \left( E7(K) \text{ABS}E5 \right)^* \left( T(I,J,K) - \text{AMB}(K) \right) + \\
2 \left( \text{AFY}(I,J,1) \right)^* \left( T2(I,J-1,K) - T2(I,J,K) \right) + \\
3 \left( \text{AFY}(I,J,2) \right)^* \left( T2(I,J+1,K) - T2(I,J,K) \right) + \\
4 \left( \text{AFZ}(I,J) \right)^* \left( T3(I,J,K-1) - 2^* T3(I,J,K) + T3(I,J,K+1) \right) \) / \\
5 \left( S4(I,J,K)/2. \right)^* \text{PHY}(I,J,K) + T(I,J,K) \) \\
GO TO 1000

331 \( T(I,J,K) = \left( \text{ABS}(J,J_1) \right)^* \left( T(I-1,J,K) - T(I,J,K) \right) - \\
1 \left( H6(K) \text{AREA2}^* \text{SIN}(TH) \right)^* \left( T1(I,J,K) - \text{AMB}(K) \right) + \\
2 \left( \text{H6}(K) \text{AREA2}^* \text{COS}(TH) \right)^* \left( \text{AMB}(K) - T2(I,J,K) \right) + \\
3 \left( \text{AFZ}(I,J) \right)^* \left( T3(I,J,K-1) - 2^* T3(I,J,K) + T3(I,J,K+1) \right) \) / \\
4 \left( S4(I,J,K)/2. \right)^* \text{PHY}(I,J,K) + T(I,J,K) \) \\
GO TO 1000

332 \( T(I,J,K) = \left( \text{ABS}(J,J_1) \right)^* \left( T(I-1,J,K) - T(I,J,K) \right) - \\
1 \left( \text{H6}(K) \text{AREA2}^* \text{SIN}(TH) \right)^* \left( T1(I,J,K) - \text{AMB}(K) \right) + \\
2 \left( \text{H6}(K) \text{AREA2}^* \text{COS}(TH) \right)^* \left( \text{AMB}(K) - T2(I,J,K) \right) + \\
3 \left( \text{AFZ}(I,J) \right)^* \left( T3(I,J,K-1) - 2^* T3(I,J,K) + T3(I,J,K+1) \right) \) / \\
4 \left( S4(I,J,K)/2. \right)^* \text{PHY}(I,J,K) + T(I,J,K) \) \\
GO TO 1000

333 \( T(I,J,K) = \left( \text{ABS}(J,J_1) \right)^* \left( T(I-1,J,K) - T(I,J,K) \right) - \\
1 \left( \text{H6}(K) \text{AREA2}^* \text{SIN}(TH) \right)^* \left( T1(I,J,K) - \text{AMB}(K) \right) + \\
2 \left( \text{H6}(K) \text{AREA2}^* \text{COS}(TH) \right)^* \left( \text{AMB}(K) - T2(I,J,K) \right) + \\
3 \left( \text{AFZ}(I,J) \right)^* \left( T3(I,J,K-1) - 2^* T3(I,J,K) + T3(I,J,K+1) \right) \) / \\
4 \left( S4(I,J,K)/2. \right)^* \text{PHY}(I,J,K) + T(I,J,K) \) \\
GO TO 1000

334 \( T(I,J,K) = \left( \text{ABS}(J,J_1) \right)^* \left( T(I-1,J,K) - T(I,J,K) \right) - \\
1 \left( \text{H6}(K) \text{AREA2}^* \text{SIN}(TH) \right)^* \left( T1(I,J,K) - \text{AMB}(K) \right) + \\
2 \left( \text{H6}(K) \text{AREA2}^* \text{COS}(TH) \right)^* \left( \text{AMB}(K) - T2(I,J,K) \right) + \\
3 \left( \text{AFZ}(I,J) \right)^* \left( T3(I,J,K-1) - 2^* T3(I,J,K) + T3(I,J,K+1) \right) \) / \\
4 \left( S4(I,J,K)/2. \right)^* \text{PHY}(I,J,K) + T(I,J,K) \) \\
GO TO 1000

335 \( T(I,J,K) = \left( \text{ABS}(J,J_1) \right)^* \left( T(I-1,J,K) - T(I,J,K) \right) - \\
1 \left( \text{H6}(K) \text{AREA2}^* \text{SIN}(TH) \right)^* \left( T1(I,J,K) - \text{AMB}(K) \right) + \\
2 \left( \text{H6}(K) \text{AREA2}^* \text{COS}(TH) \right)^* \left( \text{AMB}(K) - T2(I,J,K) \right) + \\
3 \left( \text{AFZ}(I,J) \right)^* \left( T3(I,J,K-1) - 2^* T3(I,J,K) + T3(I,J,K+1) \right) \) / \\
4 \left( S4(I,J,K)/2. \right)^* \text{PHY}(I,J,K) + T(I,J,K) \) \\
GO TO 1000

336 \( T(I,J,K) = \left( \text{ABS}(J,J_1) \right)^* \left( T(I-1,J,K) - T(I,J,K) \right) - \\
1 \left( \text{H6}(K) \text{AREA2}^* \text{SIN}(TH) \right)^* \left( T1(I,J,K) - \text{AMB}(K) \right) + \\
2 \left( \text{H6}(K) \text{AREA2}^* \text{COS}(TH) \right)^* \left( \text{AMB}(K) - T2(I,J,K) \right) + \\
3 \left( \text{AFZ}(I,J) \right)^* \left( T3(I,J,K-1) - 2^* T3(I,J,K) + T3(I,J,K+1) \right) \) / \\
4 \left( S4(I,J,K)/2. \right)^* \text{PHY}(I,J,K) + T(I,J,K) \) \\
GO TO 1000
337 \[ T(I, J, K) = \frac{(ABX(J, I, 1)}{2.} \times (T1(I - 1, J, K) - T1(I, J, K)) - \]
\[ B1(K) \times (T1(I, J, K) - T1(I - 1, J, K)) \]
\[ B5(K) \times (T3(I, J, K) - T3(I, J, K - 1)) \]
\[ (S4(I, J, K)/2.) \times (T4(I, J, K)/2.) \times (T4(I, J, K) - T4(I, J, K)) \]
\[ (H6(K) \times \cos(Th)/2.) \times \cos(Th) \times (T1(I, J, K) - T1(I, J, K)) \]
\[ (H6(K) \times \sin(Tb)/2.) \times \sin(Tb) \times (T1(I, J, K) - T1(I, J, K)) \]
\[ (H6(K) \times \cos(Th)/2.) \times \cos(Th) \times (T1(I, J, K) - T1(I, J, K)) \]
\[ (H6(K) \times \sin(Tb)/2.) \times \sin(Tb) \times (T1(I, J, K) - T1(I, J, K)) \]
\[ B3 \times (T5(I, J, K) - T5(I, J, K)) \times (T3(I, J, K) - T3(I, J, K)) \]
\[ B3 \times (T5(I, J, K) - T5(I, J, K)) \times (T3(I, J, K) - T3(I, J, K)) \]
\[ (S4(I, J, K)/2.) \times (T4(I, J, K) - T4(I, J, K)) \]

338 \[ T(I, J, K) = \frac{(ABX(J, I, 1)}{2.} \times (T1(I - 1, J, K) - T1(I, J, K)) - \]
\[ B1(K) \times (T1(I, J, K) - T1(I - 1, J, K)) \]
\[ B5(K) \times (T3(I, J, K) - T3(I, J, K - 1)) \]
\[ (S4(I, J, K)/2.) \times (T4(I, J, K)/2.) \times (T4(I, J, K) - T4(I, J, K)) \]
\[ (H6(K) \times \cos(Th)/2.) \times \cos(Th) \times (T1(I, J, K) - T1(I, J, K)) \]
\[ (H6(K) \times \sin(Tb)/2.) \times \sin(Tb) \times (T1(I, J, K) - T1(I, J, K)) \]
\[ (H6(K) \times \cos(Th)/2.) \times \cos(Th) \times (T1(I, J, K) - T1(I, J, K)) \]
\[ (H6(K) \times \sin(Tb)/2.) \times \sin(Tb) \times (T1(I, J, K) - T1(I, J, K)) \]
\[ B3 \times (T5(I, J, K) - T5(I, J, K)) \times (T3(I, J, K) - T3(I, J, K)) \]
\[ B3 \times (T5(I, J, K) - T5(I, J, K)) \times (T3(I, J, K) - T3(I, J, K)) \]
\[ (S4(I, J, K)/2.) \times (T4(I, J, K) - T4(I, J, K)) \]

339 \[ T(I, J, K) = \frac{(ABX(J, I, 1)}{2.} \times (T1(I - 1, J, K) - T1(I, J, K)) - \]
\[ B1(K) \times (T1(I, J, K) - T1(I - 1, J, K)) \]
\[ B5(K) \times (T3(I, J, K) - T3(I, J, K - 1)) \]
\[ (S4(I, J, K)/2.) \times (T4(I, J, K)/2.) \times (T4(I, J, K) - T4(I, J, K)) \]
\[ (H6(K) \times \cos(Th)/2.) \times \cos(Th) \times (T1(I, J, K) - T1(I, J, K)) \]
\[ (H6(K) \times \sin(Tb)/2.) \times \sin(Tb) \times (T1(I, J, K) - T1(I, J, K)) \]
\[ (H6(K) \times \cos(Th)/2.) \times \cos(Th) \times (T1(I, J, K) - T1(I, J, K)) \]
\[ (H6(K) \times \sin(Tb)/2.) \times \sin(Tb) \times (T1(I, J, K) - T1(I, J, K)) \]
\[ B3 \times (T5(I, J, K) - T5(I, J, K)) \times (T3(I, J, K) - T3(I, J, K)) \]
\[ B3 \times (T5(I, J, K) - T5(I, J, K)) \times (T3(I, J, K) - T3(I, J, K)) \]
\[ (S4(I, J, K)/2.) \times (T4(I, J, K) - T4(I, J, K)) \]

GO TO 1000

1000 CONTINUE
RETURN
END

SUBROUTINE FOR SOLVING A SYSTEM OF LINEAR SIMULTANEOUS EQUATIONS HAVING A TRIDIAGONAL COEFFICIENT MATRIX

SUBROUTINE TRIDAG(IF, L, A, B, C, D, V)
DIMENSION A(1), E(1), C(1), D(1), V(1), BETA(101), GAMMA(101)

COMPUTE INTERMEDIATE ARRAYS OF BETA AND GAMMA

BETA(IF) = E(IF)
GAMMA(IF) = D(IF) / BETA(IF)
IF = IF + 1
DO 1 I = IF, L
1 BETA(I) = B(I) - A(I) * C(I - 1) / BETA(I - 1)
GAMMA(I) = (C(I) - A(I)) * GAMMA(I - 1) / BETA(I)

COMPUTE FINAL SOLUTION OF VECTOR V

V(I) = GAMMA(I)
LAST = L - IF
DO 2 K = 1, LAST
I = L - K
2 V(I) = GAMMA(I) - C(I) * V(I + 1) / BETA(I)
RETURN
END

SUBROUTINE TO CALCULATE THE DIFFERENT ARE AND VOLUME TERMS FOR THE ELEMENTS
DISCRETIZATION OF THE CASTING IS ALREADY BUILT IN THROUGH THE DATA STATEMENTS
C
SUEFOUTINE ABEVCL
DIMENSION ARX(16,11,2),ABY(11,16,2),ABZ(11,16)
DIMENSION XY(10,2),XY(16,2)
DIMENSION XX(16),YY(10),ZZ(16,10)
CCPECN/C1/DX,IX,EZ,IT,URAY
CCPECN/C2/XX,XY,ZZ
CCPECN/C17/ABX,ABY,ABZ
CCPECN/C2/S4(10,16,91)
CCPECN/C3/NITYP(10,16,91),LFS(10,16,91)
CCPECN/C4/L,M,N
CCPECN/C16/X1,Y1,Z1,Z2,Y2,Y3,Y4,TH
CCPECN/C19/AREA1,AAREA2,AAREA3,AAREA4,AAREA5,AAREA6
DATA XY/0.,1.975,8*1.425,2*1.975,7*1.425,0./
DATA YX/0.,1.75,2.,2.05,9*1.75,3*1.75,2.,2.05,
19*1.75,2*1.19,0./
TH=6.88741
LL=L-1
MM=L-1
NN=N-1
C CALCULATING THE COORDINATES OF THE X AND Y
C GRID POINTS
XX(1)=0.
DO 1 J=2,L
XX(J)=YY(J-1,2)+YY(J,1)+XX(J-1)
1 CONTINUE
YY(1)=0.
DO 2 I=2,N
YY(I)=XY(I-1,2)+XY(I,1)+YY(I-1)
2 CONTINUE
DO 1400 J=1,L
DC 1300 I=1,LL
ABX(J,I,2)=(XY(J,1)+XY(J,2))*DZ/(XY(I,2)+XY(I+1,1))
1400 CONTINUE
DC 1500 I=2,L
DO 1600 J=1,M
ABY(J,I,1)=ABX(J,I-1,2)
1600 CONTINUE
DC 1700 J=1,MM
DO 1800 I=1,L
ABY(I,J,2)=(XY(I,1)+XY(I,2))*DZ/(XY(J,2)+XY(J+1,1))
1800 CONTINUE
DC 1900 J=2,M
DO 2000 I=1,L
ABY(I,J,1)=ABY(I,J-1,2)
2000 CONTINUE
DC 2100 I=6,LL
ABX(4,I,2)=ABX(4,I,2)/2.
2100 CONTINUE
DC 2200 I=7,L
ABX(4,I,1)=ABX(4,I-1,2)
2200 CONTINUE
ABY(6,4,2)=ABY(6,4,2)/2.
ABY(6,5,1)=ABY(6,5,1)/2.
ABF1=XY(4,2)*DZ/BRAY
AREA4=XY(6,2)*DZ/BRAY
A: 0 = 1, L
B: 0 = 1, M

\[[\begin{array}{cc}
X & Y \\
Z & W
\end{array}\] = \[[\begin{array}{cc}
A & B \\
C & D
\end{array}\] \times \[[\begin{array}{cc}
E & F \\
G & H
\end{array}\]]

\text{CONTINUE}

\text{END}
C
C HEAT TRANSFER COEFFICIENT SUBROUTINES. THE
C VARIOUS SUBROUTINES H1 THROUGH H7 ARE FOR
C DESCRIBING HTC AT THE VARIOUS LOCATION CF
C THE SURFACE OF THE CASTING. THEY ALL HAVE
C BEEN S11 FOR SYMMETRICAL CONFIGURATION
C
C
FUNCTION H1(K)
COMMON/C1/DX, LY, EZ, DT, BKAY
IF (K.GT.13) GO TO 10
H1= .5
RETURN
10 H1=0.34178*EXP(-(((K-1)*DZ-(DZ/2.))*0.02435))
RETURN
END
FUNCTION H2(K)
COMMON/C1/DX, LY, EZ, DT, BKAY
IF (K.GT.13) GO TO 10
H2= .5
RETURN
10 H2=0.34178*EXP(-(((K-1)*DZ-(DZ/2.))*0.02435))
RETURN
END
FUNCTION H4(K)
COMMON/C1/DX, LY, EZ, DT, BKAY
IF (K.GT.13) GO TO 10
H4= .5
RETURN
10 H4=0.34178*EXP(-(((K-1)*DZ-(DZ/2.))*0.02435))
RETURN
FUNCTION H5(K)
COMON/C1/EX,DT,DZ,DKAY
IF (K.GT.13) GO TO 10
H5 = .5
RETURN
10 H5 = .34178*EXP(-(((K-1)*DZ - (DZ/2.))*0.02435))
RETURN

FUNCTION H6(K)
COMON/C1/EX,DT,DZ,DKAY
IF (K.GT.13) GO TO 10
H6 = .5
RETURN
10 H6 = .34178*EXP(-(((K-1)*DZ - (DZ/2.))*0.02435))
RETURN

FUNCTION H7(K)
COMON/C1/EX,DT,DZ,DKAY
IF (K.GT.13) GO TO 10
H7 = .5
RETURN
10 H7 = .34178*EXP(-(((K-1)*EZ - (DZ/2.))*0.02435))
RETURN

FUNCTION AHB(K)
ABE = 5.
RETURN

INITIALISATION ROUTINE

SUBROUTINE INITIA(T,T1,T2)
DIMENSION T(10,16,91)
COMMON/C4/L,H,N
II = I-1
MM = M-1
NN = N-1
DO 100 I = 1,LL
DO 100 J = 1,MM
DO 100 K = 2,NN
T(I,J,K) = T1
100 CONTINUE
DO 150 I = 1,LL
DO 150 J = 1,MM
T(I,J,1) = T1
T(I,J,M) = T1
150 CONTINUE
DO 160 K = 2,NN
DC 160 J = 1,MM
T(I,J,K) = T1
160 CONTINUE
DO 170 I = 1,LL
DC 170 K = 2,NN
T(I,M,K) = T2
170 CONTINUE
   DO 190 J=1,MM
      T(I,J,1) = T1
      T(I,J,N) = T1
190  CONTINUE
   DO 210 I=1,LL
      T(I,E,1) = T2
      T(I,H,N) = T2
210  CONTINUE
   DO 220 K=2,NN
      T(1,1,K) = T2
      T(L,1,K) = T1
      T(I,E,K) = T2
220  CONTINUE
   X(I,1,1) = I1
   T(L,H,1) = T2
   T(1,B,1) = T2
   T(L,1,N) = T1
   T(I,1,N) = T2
   T(I,H,N) = T2

SUBROUTINE LATHET(I1,L2,M1,M2,K1)

SUBROUTINE TO RELEASE THE LATENT HEAT OF SOLIDIFICATION
AT THE DIFFERENT NODES
THIS ALSO CHARACTERIZES THE PHYSICAL STATE OF EACH NODE
BEING LIQUID, MUSEY OR SOLID REGION

DIMENSION T(10,16,91),T1(10,16,91),T2(10,16,91)
DIMENSION T3(10,16,51),TN(10,16,91)
COMMON/C7/T,T1,T2,T3,TN
COMMON/C3/WTYPE(10,16,91),LFS(10,16,91)
COMMON/C8/PHY(10,16,91)
COMMON/C9/PEYL,PHYS,PHY1
COMMON/C10/TLIC,TSLON,DENS,DENL
COMMON/C4/L,M,N
DO 1000 I=L1,L2
DO 1000 J=M1,M2
DO 1000 K=2,K1
LNE = LFS(I,J,K)
GO TO (10,20,1000),LND

10   IF(TN(I,J,K),GE.TLIC) GO TO 1000

CHANGE CF STATE

DOES FINALE TEMP END IN MUSEY ZONE?

IF((TLIC-TN(I,J,K))*PHY1-(TLIC-TSCL)*PHYM.GE.0.) GO TO 40

NODE ENDS UP IN MUSEY REGION

TN(I,J,K) = TLIC - (TLIC-TN(I,J,K) )*PHY1/PHYM
LPS(I,J,K) = 2
GO TO 1000

NODE MOVES INTO SOLID

T(I,J,K) = TSCL - (TILQ - TN(I,J,K)) * EHYL - (TILQ - TSOL) * EEYB / EHYG
LPS(I,J,K) = 3
GO TO 1000

NODE INITIALLY IN THE MUSHY REGION

IF (T(I,J,K) - G1 - TSOL) GC TO 1000

NODE MOVES INTO SOLID

T(I,J,K) = TSOL - (TSOL - TN(I,J,K)) * EHYG / EHYG
LES(I,J,K) = 3

1000 CONTINUE
RETURN
END

SUBCUTINE OUTINT(I1,NN)

SUBCUTINE TO PRINT OUT THE THREE DIMENSIONAL ARRAY CONTAINING INTEGER NUMBERS

DIMENSION I1(10,16,31)
COMMON/C14/L,M,N
DO 1000 K=1,N,NN
WRITE(6,900) K
900 FORMAT(5X,'K=',I2,2X,14/)
DO 850 J=1,M
WRITE(6,850) (I(I,J,K),I=1,L)
850 FORMAT(5X,11(I1,J,K),I=1,L)
1000 CONTINUE
RETURN
END

FUNCTION CP(T)

FUNCTION ECUHINE CALCULATES THE SPECIFIC HEAT
OF ZINC AT ANY PARTICULAR TEMPERATURE IN
UNITS OF CAL/G.C

TK = 1*273
CP = 0.08184 + 0.0367 * 1.E-03 * TK
RETURN
END

SUBCUTINE PHYREP

CALCULATES THE PRODUCT OF DENSITY AND SPECIFIC HEAT FOR ALL THE NODES AND
STORES IT IN THE ARRAY PH.

CP = CP * LH
RETURN
END

FUNCTION CP(T)

FUNCTION ECUHINE CALCULATES THE SPECIFIC HEAT
OF ZINC AT ANY PARTICULAR TEMPERATURE IN
UNITS OF CAL/G.C

TK = 1*273
CP = 0.08184 + 0.0367 * 1.E-03 * TK
RETURN
END
DIMENSION T(10, 16, 91), T1(10, 16, 91), T2(10, 16, 91)
DIMENSION T3(10, 16, 91), T4(10, 16, 91)
COMMON/C7/I, T1, T2, T3, TN
COMMON/C4/L, E, N
COMMON/C3/NTYPE(10, 16, 91), LFS(10, 16, 91)
COMMON/C8/PHY(10, 16, 91)
COMMON/C9/PEY, PHYS, PHY1
COMMON/C10/TLIC, TSOL, DEN1, DEN
DO 1000 I = 1, L
DO 1000 J = 1, M
DO 1000 K = 2, N
IND = IFS(I, J, K)
GO TO (10, 20, 30), IND

1000 CONTINUE
RETURN
END

C
NODES ABOVE LIQUIDUS
10 PHYS(I, J, K) = PHY1
GO TO 1000

C
NODES IN THE MUSHY REGION
20 PHYS(I, J, K) = PHYM
GO TO 1000

C
NODES IN THE SOLID REGION
30 PHYS(I, J, K) = C(T(I, J, K)) * DEN
RETURN
EN1

C
SUBROUTINE AIISC
SUBROUTINE FOR ADDING ONE SET OF NODES
RESULTING FROM THE GROWTH OF THE INGOT IN
THE Z DIRECTION. ALSO SOME INITIALIZATION

C
DIMENSION T(10, 16, 91), T1(10, 16, 91), T2(10, 16, 91)
DIMENSION T3(10, 16, 91), T4(10, 16, 91)
COMMON/C7/I, T1, T2, T3, TN
COMMON/C4/L, E, N
COMMON/C3/NTYPE(10, 16, 91), LFS(10, 16, 91)
COMMON/C8/PHY(10, 16, 91)
COMMON/C9/PEY, PHYS, PHY1
COMMON/C10/TLIC, TSOL, DEN1, DEN
COMMON/C2/S4(10, 16, 91)

C
MAKE THE TOP SLICE IN THE PREVIOUS INTERVAL
AS THE SECOND SLICE FROM THE TOP FOR
THIS TIME STEP
C
DO 10 K = 1, N
DO 10 J = 1, M
DO 10 I = 1, L
NC = K + 2 - K
T(I, J, NC) = T(I, J, NC - 1)
LFS(I, J, NC) = LFS(I, J, NC - 1)
CONTINUE
NB = K - 1
DO 15 K=1,2
DO 15 J=1,L
DO 15 I=1,N
CC 15 CONTINUE

S4(I,J,NC)=S4(I,J,NC-1)
NTYPE(I,J,NC)=NTYPE(I,J,NC-1)

CONTINUE

C
C INITIALISATION OF THE NEWLY ADDED SLICE
C
DO 20 1=1,L
DC 20 J=1,N
T(I,J,1)=T(I,J,2)
L=I,J,1)=LPS(I,J,2)

CONTINUE

N=N+1
RETURN
END

C
C COPYING OF THE TEMPERATURE FIELD IN THE
C CASTING IN EINARY FOR SUBSEQUENT USE IN
C STARTING THE PROGRAM
C
SUBROUTINE FILEIN(NI)
DIMENSION T(10,16,91),T1(10,16,91),T2(10,16,91)
DIMENSION T3(10,16,91),TN(10,16,91)
COMMON/T,T1,T2,T3,TN
COMMON/C4/L,N,K
COMMON/C3/NTYPE(10,16,91),LPS(10,16,91)
DO 10 K=1,N
DC 10 J=1,N
WRITE(NI),(T(I,J,K),I=1,L),(LPS(I,J,K),I=1,L)

CONTINUE
RETURN
END

C
C SUBROUTINE GRAPH
C
THIS SUBROUTINE PLOTS THE CONTINUOUS PROFILES OF
C THE TEMPERATURE FIELD
C
DIMENSION T(10,16,91),T1(10,16,91),T2(10,16,91)
DIMENSION XX(16),XY(10),Z2(16,10)
DIMENSION T3(10,16,91),TN(10,16,91)
DIMENSION ZP1(16),ZP2(10,16),ZF(101),ZF1(11),ZF2(16)
COMMON/C4/XX,YY,ZZ
COMMON/C1/XX,YY,ZZ
COMMON/C2/T,N
COMMON/C3/L,N,K
COMMON/C4/L,N,K
COMMON/C5/LIQ,TSOL,DENS,DENL
COMMON/C6/CPL,CSP,NUMFUN,BLOCK
DO 10 I=1,L
DC 10 J=1,N
ZF1(J,I)=T(I,J,K)

CONTINUE

DO 20 I=1,L
DC 20 J=1,N
ZF2(J,I) = T(I,I,J)

CONTINUE

C SCALING THE Y AXIS (CORRESPONDS TO ORIGINAL X AND Y AXES)

DYY1F=4.
DYY2F=4.
SY1=YY(10)/DYY1F
SY2=XX(16)/DYY2F
DO 15 I=1,L
YP(I)=YY(I)/DYY1P
CONTINUE

DC 16 J=1,M
YP2(J)=XX(J)/DYY2P
CONTINUE

C SCALING THE X AXIS (ORIGINALLY Z AXIS)

DXX=Z/4.
SX=XX*FLOAT(N-1)
DXXI=4.
XP(1)=0.
NX=N-1
DO 40 I=1,NX
XP(I+1)=XP(I)*CXX
CONTINUE

C CCOUNT BEGIN FROM SECTION PERPENDICULAR TO Y AXIS XZ PLANE

CALL FLTCL('XSIZE',128)
CALL FLTCL('YSIZE',66)
CALL FRAME1
CALL CNTCUR(XP,N,YP1,L,ZF1,101,TLIC,3.,TLIC)
CALL CNTCUR(XP,N,YP1,L,ZF1,101,TSCL,3.,TSCL)
XM=SX+4.
CALL PLOT(XM,0.,-3)

C CCOUNT BEGIN FROM SECTION PERPENDICULAR TO X AXIS YZ PLANE

CALL FRAME2
CN=TLIC
CALL CNTCUR(XP,N,YP2,M,ZP2,101,TLIC,3.,TLIC)
CALL CNTCUR(XP,N,YP2,M,ZP2,101,TSCL,3.,TSCL)
XM=EX+4.
CALL ELCT(XM,0.,-3)
CALL XFLAN
CALL COUTNE
XM1=SY2+4.
CALL ELCT(XM1,0.,-3)
RETURN
END

C CCOUNT END

SUBROUTINE OUTPUT2
COPPON/C10/TLIC,TSCL,DEKS,DEKL
COPPON/C1/EX,DX,DX,LT,RELY
COMMON/C4/L,M,N
COMMON/C11/CPL,CSP,NUMBUN,BL'H
COMMON/C6/H3
COMMON/C12/BINPI(20),CPF(20)
WRITE(6,10) (BINPI(I),I=1,20)
10 FORMAT('1','26X,20A4//')
WRITE(6,20) NUMBUN
20 FORMAT(55X,'FUN NO ',2X,14//)
WRITE(6,30)
30 FORMAT(5X,'THERMO PHYSICAL PROPERTIES')
WRITE(6,40) TLIC
40 FORMAT(10X,'LIQUIDUS TEMPERATURE = ',1X,F7.1,1X,'DEG C')
WRITE(6,50) TSLC
50 FORMAT(10X,'SOLIDUS TEMPERATURE = ',1X,F7.1,'DEG C')
WRITE(6,60) DENL
60 FORMAT(10X,'DENSITY OF THE LIQUID = ',1X,F5.1,1X,'G/CM3')
WRITE(6,70) DENS
70 FORMAT(10X,'DENSITY OF THE SOLID = ',1X,F5.1,'G/CM3')
WRITE(6,80) CEL
80 FORMAT(10X,'SPCIFIC HEAT OF THE LIQUID = ',1X,F5.2,'CAL/GM')
WRITE(6,90) (CPF(I),I=1,20)
90 FORMAT(10X,'SPCIFIC HEAT OF THE SOLID = ',1X,F5.2,'CAL/GM')
WRITE(6,100) RLHT
100 FORMAT(10X,'LATENT HEAT OF SOLIDIFICATION = ',F6.1,'CAL/GM')
WRITE(6,120) RRAY
120 FORMAT(10X,'THERMAL CONDUCTIVITY OF THE LIQUID = ',F5.2,
1X,'CAL/CM.DEG.C.SEC')
WRITE(6,130) RRAY
130 FORMAT(10X,'THERMAL CONDUCTIVITY OF THE SOLID = ',F5.2,
1X,'CAL/CM.DEG.C.SEC')
WRITE(6,140)
140 FORMAT(10X,'CASTING CONDITIONS = ',F5.2,'CMS/SEC')
WRITE(6,150) CSP
150 FORMAT(10X,'CASTING SPEED = ',F5.2,'CMS/SEC')
WRITE(6,160)
160 FORMAT(10X,'HEAT TRANSFER COEFFICIENTS USED')
WRITE(6,170) H3
170 FORMAT(15X,'BOTTOM HEAT TRANSFER COEFFICIENT = ',2X,F10.4)
RETURN
END

SUBROUTINE FRAME1
COMMON/C14/SX,SY1,SY2,DXX,DYY1,DYY2,DXXP,DYY1P,DYY2P
COMMON/C11/CPL,CSP,NUMBUN,BL'H
COMMON/C13/TAU
CALL FLCTFL('METRIC',1)
CALL AXCTFL('SIDE',-1)
CALL AXCTFL('DIGITS',1)
CALL AXFLGT('DIST ALONG Z-AXIS (CMS) = ',0.,SX,0.,DXXP)
CALL AXFLGT('SIDE',1)
CALL AXFLGT('DIST ALONG Z-AXIS (CMS) = ',90.,ST1,0.,DYY1P)
CALL FLT(0.,0.,3)
CALL AXCTFL('YORIGIN',SY1)
CALL AXCTFL('SIDE',1)
CALL AXFLGT('DIST ALONG Z-AXIS (CMS) = ',0.,SX,0.,DXXP)
CALL FLT(0.,0.,3)
CALL AXCTFL('YORIGIN',0.)
CALL AXCTFL('XORIGIN',SX)
CALL AXCTEL('SIDE',-1)
CALL AXFLCT('DIST ALONG X-AXIS (CBS);',90.,SY1,0.,DYY1P)
CALL AXCTEL('XCEIGIN',0.)
CALL AXCTEL('YOSIGIN',0.)
FCAT=NUMFUN
CALL SYMECL(1.,1.,0.4,'FUN';90.,4)
CALL NUMEER(1.,2.5,0.4,FLOAT,90.,-1)
CALL SYMEOL(1.5,1.0,0.4,'TIME=';50.,5)
CALL NUMEER(1.5,3.0,0.4,TAU,90.,0)
RETURN
END

SUBROUTINE FRAME2
COMMON/C14/SX,SY1,SY2,DXX,DYY1,DYY2,DXXP,DYY1P,DYY2P
COMMON/C11/CFL,CSP,NUMFUN,UNIT
COMMON/C13/TAU
CALL ELCTEL('METRIC',1)
CALL AXCTEL('SIDE',-1)
CALL AXCTEL('DIGITS',1)
CALL AXFLCT('DIST ALONG Z-AXIS (CBS);',0.,SX,0.,DXXP)
CALL AXCTEL('SIDE',1)
CALL AXFLCT('DIST ALONG Y-AXIS (CMS);',90.,SY2,0.,DYY2P)
CALL FLCT(0.,0.,3)
CALL AXCTEL('YCEIGIN',SY2)
CALL AXCTEL('SIDE',1)
CALL AXFLCT('DIST ALONG Z-AXIS (CMS);',0.,SX,0.,DXXP)
CALL FLCT(0.,0.,3)
CALL AXCTEL('YOSIGIN',0.)
CALL AXCTEL('XCEIGIN',SX)
CALL AXCTEL('SIDE',-1)
CALL AXFLCT('DIST ALONG Y-AXIS (CMS);',90.,SY2,0.,DYY2P)
CALL AXCTEL('XOSIGIN',0.)
CALL AXCTEL('YCEIGIN',0.)
FCAT=NUMFUN
CALL SYMECL(1.,1.,0.4,'FUN';90.,4)
CALL NUMEER(1.,2.5,0.4,FLOAT,90.,-1)
CALL SYMEOL(1.5,1.0,0.4,'TIME=';50.,5)
CALL NUMEER(1.5,3.0,0.4,TAU,90.,0)
RETURN
END

SUBROUTINE CHECK(MCE)
COMMON/C16/NUM,NUM2
NUM2=NUM2+1
IF (NUM2.EQ.NUM) GO TO 10
MCE=0
RETURN
10 MCE=1
NUM2=0
RETURN
END
SUBROUTINE TO PRINT THE MATRIX

SUBROUTINE TOUT1(T)

DIMENSION T(10,16,91)
COMMON/C4/L, M, N
COMMON/C3/IX, IY, IZ, DT, BAY
COMMON/C13/TAU
COMMON/C10/TIC, TSL, DES, DENI

WRITE(6,100) TAU

100 FORMAT(10X,'TEMPERATURES AT THE END OF TIME',
1 TE-1, I1, 'SECTIONS'
2 ZT-1, I1, 'CAT (N-T)

WRITE(6,145) T

145 FORMAT(10X,'SIZE OF THE INGOT IN Z DIRECTION', F10.2, 'CMS')

150 FORMAT(5X,K=' ',1X,I3)

DO 501 K=1, N

WRITE(6,150) T

501 CONTINUE

DO 502 J=1, M

WRITE(6,180) (T(I,J,K), I=1, L)

502 CONTINUE

CONTINUE

160 CONTINUE

IF (K.EQ.100) GC TO 1

1

WRITE(6,150) K

DO 503 J=1, M

WRITE(6,180) (T(I,J,K), I=1, L)

503 CONTINUE

CONTINUE

DO 504 K=20, 21

WRITE(6,150) K

504 CONTINUE

CONTINUE

END

SUBROUTINE XIPLAN

SUBROUTINE TO PLOT CONTOURS IN THE XY PLANE

NOTE THAT THE CONTOUR PROGRAM LOSES NOT WORK IF

THE GRID IS IRREGULARLY SHAPED. THIS CAN BE

OVERCOME BY GIVING DUMMY VALUES TO NONEXISTING

GRID POINTS

DIMENSION T(10,16,91), T1(10,16,91), T2(10,16,91)

DIMENSION T3(10,16,91), T4(10,16,91)

DIMENSION T5(16), T6(10), T7(16,10)

DIMENSION T8(16), T9(10), T10(16,10)

COMMON/C20/I1, IY, ZZ

COMMON/C14/SX, SY1, SY2, DX1, DYY1, DYY2, DX1P, DYY1P, DYY2P

COMMON/C7/I1, I2, T3, IN

COMMON/C4/L, M, N

COMMON/C10/TIC, TSL, DES, DENI

COMMON/C13/TAU

COMMON/C11/CPL, C3P, NUMER, ELET

GRID POINTS ON X AXIS (ORIGINALLY Y AXIS)
C SCALING
   DO 100 J=1,M
   XX1(J)=XX(J)/DYY2P
100 CONTINUE
   DC 200 I=1,L
   YY1(I)=YY(I)/DYY1P
200 CONTINUE
C DRAWING A CROSS SECTION OF THE JUMBO INGOT
C
   CALL FLCTRL('METRIC',1)
   CALL AXFTFL('SIZE',0)
   CALL AXFLCT('DIST ALONG Y AXIS (CMS);',0.,XX1(16),0.,DYY2P)
   CALL AXFLCT('DIST ALONG X AXIS (CMS);',90.,YY1(10),0.,DYY1P)
   CALL FLCT(XX1(1),YY1(10),3)
   CALL PLOT(XX1(4),YY1(10),2)
   CALL PLOT(XX1(4),YY1(6),2)
   CALL PLOT(XX1(5),YY1(6),2)
   CALL PLOT(XX1(9),YY1(10),2)
   CALL PLOT(XX1(16),YY1(10),2)
   CALL PLOT(XX1(16),YY1(1),2)
   CALL PLOT(XX1(1),YY1(1),3)
   FLCT=NUMRUN
   CALL SYMECL(1.,1.,0.,4.,'RUN ',90.,4.)
   CALL NUMER(1.,2.5,0.4,'FLOA1,90.,-1)
   CALL SYMBOL(1.5,1.0,0.4,'TIEE=',50.,5)
   CALL NUMER(1.5,3.0,0.4,TAU,90.,0)
RETURN
C FILLING THE GRID WITH TEMPERATURE VALUES
C
   ENTITY CUTLINE
   K2=(N/2)+1
   DO 350 I=1,L
      DO 350 J=1,4
         ZZ1(J,I)=T(I,J,K2)
   350 CONTINUE
   300 CONTINUE
   DO 400 I=1,6
      ZZ1(5,I)=T(I,6,K2)
   400 CONTINUE
   DO 450 I=1,7
      ZZ1(6,I)=T(I,6,K2)
   450 CONTINUE
   DO 500 I=1,8
      ZZ1(7,I)=T(I,7,K2)
   500 CONTINUE
   DO 600 I=1,9
      ZZ1(8,I)=T(I,8,K2)
   600 CONTINUE
   DC 700 I=1,L
   DO 800 J=1,16
      ZZ1(J,I)=T(I,J,K2)
   800 CONTINUE
   CALL CNTOUTE(XX1,16,YY1,10,ZZ1,16,TILQ,3.,TILQ)
   CALL CNTOUT5(XX1,16,YY1,10,ZZ1,16,TSGL,3.,TSGL)
RETURN
END
C
SUBROUTINE INITI2(I,TEMF,DUMMY)
C SUBROUTINE TO INITIALIZE THE TEMPERATURE FIELD
C IN THE CASE OF A JUMBO INGOT. NOTE THAT THE C ELEMENTS OUTSIDE THE JUMBO HAVE BEEN INITIALIZED C TO A DUMMY VALUE TO ENABLE THE USE OF CONTINUE C PROGRAM
DIMENSION T(10,16,51)
COMMON/C4/L,M,N
CALL GSFT(T,160,91,160,DUMMY)
DO 5999 K=1,N
DO 150 I=1,L
DO 250 J=1,4
T(I,J,K)=TEMF
150 CONTINUE
DO 250 I=1,6
T(I,5,K)=TEMF
250 CONTINUE
DO 300 I=1,7
T(I,6,K)=TEMF
300 CONTINUE
DO 400 I=1,8
T(I,7,K)=TEMF
400 CONTINUE
DO 500 I=1,9
T(I,8,K)=TEMF
500 CONTINUE
DO 600 L=1,N
DO 650 J=9,L
T(L,J,K)=TEMF
600 CONTINUE
9999 CONTINUE
RETURN
END
C
C SUBROUTINE TO SET OUT THE DIFFERENT TYPES OF NODES C
C
SUBROUTINE NUCOUT
COMMON/C3/NUTYPE(10,16,91),LPS(10,16,91)
COMMON/C4/L,M,N
LL=L-1
MN=M-1
DO 9999 I=1,L
DO 9999 J=1,M
DO 9999 K=1,N
IF (I.EQ.1) GO TO 100
IF (I.EQ.L) GO TO 200
IF (J.EQ.1) GO TO 300
IF (J.EQ.M) GO TO 400
IF (K.EQ.N) GO TO 500
NUTYPE(I,J,K)=5
GO TO 500
500 NUTYPE(I,J,K)=14
GO TO 9999
100 IF (J.EQ.1) GO TO 600
9999 CONTINUE
IF (J.EQ.M) GO TO 700
IF (K.EQ.N) GO TO 800
NTYEE(I,J,K) = 4
GO TO 9999
600 NTYPE(I,J,K) = 13
GO TO 9999
200 IF (J.EQ.I) GO TO 900
IF (J.EQ.M) GO TO 1000
IF (K.EQ.N) GO TO 1100
NTYEE(I,J,K) = 6
GO TO 9999
1100 NTYPE(I,J,K) = 15
GO TO 9999
300 IF (K.EQ.N) GO TO 1200
NTYEE(I,J,K) = 2
GO TO 9999
1200 NTYPE(I,J,K) = 11
GO TO 9999
400 IF (K.EQ.N) GO TO 1300
NTYEE(I,J,K) = 8
GO TO 9999
1300 NTYPE(I,J,K) = 17
GO TO 9999
600 IF (K.EQ.N) GO TO 1400
NTYEE(I,J,K) = 1
GO TO 9999
1400 NTYPE(I,J,K) = 10
GO TO 9999
700 IF (K.EQ.N) GO TO 1500
NTYEE(I,J,K) = 7
GO TO 9999
1500 NTYPE(I,J,K) = 16
GO TO 9999
900 IF (K.EQ.N) GO TO 1600
NTYEE(I,J,K) = 3
GO TO 9999
1600 NTYPE(I,J,K) = 12
GO TO 9999
1000 IF (K.EQ.N) GO TO 1700
NTYEE(I,J,K) = 9
GO TO 9999
1700 NTYPE(I,J,K) = 18
GO TO 9999
9995 CONTINUE
DC 1800 K=1,NN
NTYEE(6,4,K) = 19
1800 CONTINUE
DC 1350 K=1,NN
NTYEE(6,5,K) = 21
1350 CONTINUE
NTYEE(6,4,N) = 20
NTYEE(6,5,N) = 22
DC 1900 K=1,NN
DO 2000 I=7,II
NTYEE(I,4,K) = 23
2000 CONTINUE
1900 CONTINUE
DO 2100 I=7,II
NTYEE(I,4,N) = 24
2100 CONTINUE
DC 2200 K=1,NN
NTYPE (7,6,K) = 25
NTYPE (8,7,K) = 25
NTYPE (9,8,K) = 25

2200 CONTINUE
NTYPE (7,6,N) = 26
NTYPE (8,7,N) = 26
NTYPE (5,6,N) = 26
DO 2300 K = 1, NN
NTYPE (1,9,K) = 27

2300 CONTINUE
NTYPE (1,9,N) = 28
DO 2400 K = 1, NN
NTYPE (1,4,K) = 29

2400 CONTINUE
NTYPE (1,4,N) = 30
RETURN
END

C

SUBROUTINE SUMT
C
C S T A T E F C C H I O N O F T H E C A S T I N G.
C

DIMENSION T(10,16,91),T1(10,16,91),T2(10,16,91)
DIMENSION T3(10,16,91),T4(10,16,91),TS(30,30,50)
COMMON/C4,L,K,N
COMMON/C7/T,T1,T2,T3,TN
COMMON/C21/TS
IF (K.GT.52) GO TO 100
K1 = K - 2
IF (K.GT.30) GO TO 10
K2 = 8
GO TO 20

10 K2 = 30

20 DC 100 KK = 1, K2
I = 1
K3 = 1 - KK + 1
DO 21 J = 1, 10
TS (I, KK, K1) = T (J, 1, K3)
I = I + 1

21 CONTINUE
DO 25 J = 2, 4
TS (I, KK, K1) = T (10, J, K3)
I = I + 1

25 CONTINUE
DO 30 J = 1, 4
I1 = 10 - J
TS (I, KK, K1) = T (I1, 4, K3)
I = I + 1

30 CONTINUE
TS (I, KK, K1) = T (6, 5, K3)
I = I + 1
TS (I, KK, K1) = T (7, 6, K3)
I = I + 1
TS (I, KK, K1) = T (8, 7, K3)
I = I + 1
TS (I, KK, K1) = T (5, 8, K3)
I = I + 1
DC 35 J = 9, 16
TS(I, K, K1) = T(10, J, K3)  
I = I + 1
35 CONTINUE
DO 40 J = 1, 9
I2 = 10 - J
TS(I, K, K1) = T(I2, 16, K3)
I = I + 1
40 CONTINUE
100 CONTINUE
RETURN
END

C
C SUBROUTINE TSARRAY
C SUBROUTINE TO COPY THE SURFACE TEMPERATURE ARRAY
C INTO A FILE IN BINARY
C
DIMENSION TS(38, 30, 50)
COMMON/C4/L, M, W
COMMON/C21/TS
DC 10 K = 1, N
DC 1C J = 1, 30
WRITE(7) (TS(I, J, K), J = 1, 36)
10 CONTINUE
RETURN
END
APPENDIX 3

THREE DIMENSIONAL TEMPERATURE DISTRIBUTION IN THE CASTING FOR THE DIFFERENT RUNS
### Three-dimensional temperature distribution in 381 x 991 mm. aluminium ingot cast at 1.775 mm/s (conventional cooling).

#### Dist. from the mould 0.0 CMS

<table>
<thead>
<tr>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>700. 700. 700. 700. 700. 700. 700. 700. 700. 700.</td>
</tr>
</tbody>
</table>

#### Dist. from the mould 6.3 CMS

<table>
<thead>
<tr>
<th>Temperature</th>
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</thead>
</table>

#### Dist. from the mould 12.7 CMS

<table>
<thead>
<tr>
<th>Temperature</th>
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<tbody>
<tr>
<td>676. 675. 672. 667. 659. 646. 631. 571. 461. 320. 149.</td>
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</tbody>
</table>

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*Table A3.1*
<table>
<thead>
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<th>DIST. FROM THE MOULD</th>
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<tbody>
<tr>
<td>656.</td>
<td>657.</td>
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<tr>
<td>655.</td>
<td>654.</td>
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<tr>
<td>642.</td>
<td>642.</td>
</tr>
<tr>
<td>631.</td>
<td>631.</td>
</tr>
<tr>
<td>402.</td>
<td>400.</td>
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<tr>
<td>104.</td>
<td>104.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DIST. FROM THE MOULD</th>
<th>25.4 CMS</th>
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</thead>
<tbody>
<tr>
<td>643.</td>
<td>642.</td>
</tr>
<tr>
<td>643.</td>
<td>642.</td>
</tr>
<tr>
<td>643.</td>
<td>642.</td>
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<tr>
<td>642.</td>
<td>641.</td>
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<tr>
<td>641.</td>
<td>640.</td>
</tr>
<tr>
<td>639.</td>
<td>637.</td>
</tr>
<tr>
<td>631.</td>
<td>631.</td>
</tr>
<tr>
<td>320.</td>
<td>317.</td>
</tr>
<tr>
<td>64.</td>
<td>84.</td>
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</tbody>
</table>

<table>
<thead>
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<th>DIST. FROM THE MOULD</th>
<th>31.7 CMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>624.</td>
<td>633.</td>
</tr>
<tr>
<td>624.</td>
<td>633.</td>
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<tr>
<td>624.</td>
<td>633.</td>
</tr>
<tr>
<td>624.</td>
<td>633.</td>
</tr>
<tr>
<td>255.</td>
<td>255.</td>
</tr>
<tr>
<td>70.</td>
<td>69.</td>
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<tr>
<td>DIST. FROM THE MOULD</td>
<td>38.1 CMS</td>
</tr>
<tr>
<td>----------------------</td>
<td>----------</td>
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<tr>
<td>631. 631. 630. 571. 514. 452. 385. 313. 237. 158. 76.</td>
<td></td>
</tr>
<tr>
<td>631. 631. 630. 571. 514. 452. 385. 313. 237. 156. 76.</td>
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</tr>
<tr>
<td>631. 631. 630. 568. 510. 449. 382. 310. 235. 156. 76.</td>
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</tr>
<tr>
<td>631. 631. 619. 564. 505. 442. 376. 306. 231. 154. 75.</td>
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</tr>
<tr>
<td>631. 631. 556. 545. 489. 428. 364. 295. 223. 149. 73.</td>
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</tr>
<tr>
<td>630. 589. 549. 503. 452. 396. 337. 274. 206. 139. 68.</td>
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</tr>
<tr>
<td>210. 208. 201. 191. 176. 159. 138. 115. 90. 63. 36.</td>
<td></td>
</tr>
<tr>
<td>55. 58. 57. 55. 51. 47. 43. 38. 32. 26. 20.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>DIST. FROM THE MOULD</th>
<th>44.4 CMS</th>
</tr>
</thead>
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<tr>
<td>631. 601. 556. 507. 454. 397. 337. 273. 207. 138. 66.</td>
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<tr>
<td>574. 555. 520. 477. 429. 376. 319. 259. 197. 132. 65.</td>
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</tr>
<tr>
<td>489. 476. 454. 422. 383. 338. 289. 236. 179. 120. 60.</td>
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</tr>
<tr>
<td>287. 284. 274. 259. 236. 213. 184. 152. 118. 81. 43.</td>
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<tr>
<td>173. 171. 166. 157. 145. 131. 114. 96. 75. 54. 32.</td>
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<tr>
<td>51. 50. 49. 47. 44. 41. 37. 33. 29. 24. 19.</td>
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<th>50.8 CMS</th>
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<td>554. 541. 510. 471. 425. 373. 318. 258. 196. 131. 65.</td>
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<tr>
<td>545. 530. 504. 466. 421. 370. 315. 250. 194. 130. 64.</td>
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</tr>
<tr>
<td>525. 516. 492. 455. 412. 362. 309. 251. 190. 128. 63.</td>
<td></td>
</tr>
<tr>
<td>326. 317. 306. 289. 266. 237. 205. 169. 130. 89. 47.</td>
<td></td>
</tr>
<tr>
<td>237. 234. 227. 214. 196. 178. 154. 126. 100. 69. 38.</td>
<td></td>
</tr>
<tr>
<td>143. 142. 138. 121. 110. 96. 81. 64. 47. 29.</td>
<td></td>
</tr>
<tr>
<td>44. 44. 43. 41. 39. 36. 33. 30. 26. 22. 18.</td>
<td></td>
</tr>
</tbody>
</table>
DIST. FROM THE MOULD 57.1 CMS

| 455 | 449 | 432 | 405 | 370 | 329 | 282 | 230 | 175 | 116 | 59 |
| 453 | 447 | 430 | 404 | 369 | 327 | 281 | 229 | 175 | 118 | 59 |
| 446 | 440 | 424 | 398 | 364 | 323 | 277 | 226 | 172 | 116 | 59 |
| 432 | 426 | 411 | 386 | 353 | 314 | 269 | 220 | 168 | 113 | 57 |
| 406 | 403 | 385 | 366 | 335 | 308 | 274 | 236 | 194 | 149 | 101 |
| 372 | 368 | 355 | 335 | 308 | 274 | 236 | 194 | 149 | 101 | 52 |
| 324 | 321 | 310 | 293 | 270 | 241 | 208 | 172 | 132 | 90 | 47 |
| 265 | 263 | 254 | 241 | 222 | 195 | 173 | 143 | 111 | 77 | 42 |
| 157 | 195 | 189 | 179 | 166 | 145 | 130 | 108 | 85 | 60 | 34 |
| 120 | 119 | 115 | 110 | 102 | 93  | 82  | 69  | 56  | 41  | 26 |
| 35  | 38  | 36  | 35  | 32  | 30  | 27  | 24  | 21  | 16  | 10 |

DIST. FROM THE MOULD 63.5 CMS

| 384 | 380 | 367 | 346 | 319 | 285 | 245 | 202 | 155 | 105 | 54 |
| 362 | 377 | 365 | 344 | 317 | 283 | 244 | 201 | 154 | 104 | 53 |
| 375 | 376 | 358 | 336 | 311 | 278 | 240 | 197 | 151 | 103 | 53 |
| 361 | 356 | 346 | 327 | 301 | 269 | 232 | 151 | 103 | 51 |
| 341 | 337 | 326 | 308 | 284 | 254 | 219 | 181 | 139 | 95  | 49 |
| 310 | 307 | 297 | 281 | 259 | 232 | 201 | 166 | 128 | 88  | 46 |
| 271 | 266 | 259 | 246 | 227 | 204 | 177 | 146 | 113 | 78  | 42 |
| 222 | 219 | 213 | 202 | 187 | 168 | 146 | 121 | 95  | 66  | 37 |
| 165 | 163 | 158 | 150 | 139 | 126 | 110 | 92  | 73  | 52  | 31 |
| 101 | 101 | 96 | 93 | 87 | 75 | 70 | 60 | 49 | 37 | 24 |
| 34 | 34 | 34 | 33 | 31 | 29 | 27 | 25 | 23 | 20 | 17 |

DIST. FROM THE MOULD 69.8 CMS

<p>| 326 | 324 | 314 | 297 | 274 | 246 | 213 | 176 | 135 | 93 | 48 |
| 325 | 322 | 312 | 295 | 272 | 244 | 211 | 175 | 135 | 92 | 48 |
| 319 | 315 | 305 | 289 | 267 | 239 | 207 | 171 | 132 | 90 | 47 |
| 307 | 303 | 294 | 278 | 257 | 231 | 200 | 165 | 127 | 86 | 46 |
| 286 | 285 | 276 | 261 | 242 | 217 | 188 | 150 | 120 | 83 | 44 |
| 262 | 255 | 251 | 238 | 220 | 198 | 172 | 142 | 110 | 77 | 42 |
| 226 | 226 | 219 | 208 | 192 | 173 | 151 | 125 | 98 | 68 | 38 |
| 187 | 185 | 180 | 171 | 158 | 143 | 125 | 104 | 82 | 58 | 34 |
| 155 | 138 | 134 | 128 | 119 | 107 | 94 | 79 | 63 | 46 | 28 |
| 67 | 66 | 64 | 60 | 58 | 56 | 52 | 43 | 33 | 23 |
| 31 | 31 | 30 | 30 | 28 | 27 | 25 | 23 | 21 | 19 | 17 |</p>
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<td>283. 280. 271. 257. 236. 214. 185. 154. 119. 82. 44.</td>
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<td>211. 278. 269. 255. 236. 212. 184. 152. 118. 82. 44.</td>
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<tr>
<td>274. 272. 263. 249. 231. 207. 180. 149. 116. 80. 43.</td>
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<td>263. 261. 253. 239. 222. 199. 173. 144. 111. 77. 42.</td>
</tr>
<tr>
<td>247. 244. 237. 224. 206. 187. 163. 135. 105. 73. 40.</td>
</tr>
<tr>
<td>224. 222. 215. 204. 189. 170. 148. 123. 96. 67. 38.</td>
</tr>
<tr>
<td>195. 193. 187. 178. 165. 145. 130. 108. 85. 60. 34.</td>
</tr>
<tr>
<td>166. 156. 154. 146. 136. 123. 107. 90. 71. 54. 31.</td>
</tr>
<tr>
<td>120. 119. 115. 110. 102. 93. 82. 69. 56. 41. 26.</td>
</tr>
<tr>
<td>75. 75. 73. 70. 65. 60. 53. 46. 38. 30. 22.</td>
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<td>29. 28. 28. 27. 26. 25. 24. 22. 20. 18. 16.</td>
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</table>

<table>
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<tr>
<td>246. 245. 238. 225. 209. 186. 163. 136. 106. 73. 40.</td>
</tr>
<tr>
<td>246. 243. 236. 224. 207. 186. 162. 135. 105. 73. 40.</td>
</tr>
<tr>
<td>236. 227. 221. 209. 194. 175. 152. 126. 99. 69. 38.</td>
</tr>
<tr>
<td>155. 193. 187. 178. 165. 145. 130. 109. 85. 60. 34.</td>
</tr>
<tr>
<td>170. 166. 163. 155. 144. 130. 114. 95. 75. 54. 32.</td>
</tr>
<tr>
<td>135. 138. 134. 127. 119. 107. 94. 80. 63. 46. 28.</td>
</tr>
<tr>
<td>105. 104. 101. 96. 90. 82. 72. 62. 50. 38. 25.</td>
</tr>
<tr>
<td>67. 66. 64. 62. 58. 53. 48. 42. 35. 28. 21.</td>
</tr>
</tbody>
</table>

<table>
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<th>DIST. FROM THE MOULD 88.9 CMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>220. 217. 211. 200. 185. 167. 146. 121. 95. 67. 37.</td>
</tr>
<tr>
<td>214. 212. 206. 195. 181. 163. 142. 115. 93. 65. 37.</td>
</tr>
<tr>
<td>205. 203. 197. 187. 173. 156. 136. 114. 89. 63. 36.</td>
</tr>
<tr>
<td>151. 190. 184. 175. 162. 146. 128. 107. 84. 59. 34.</td>
</tr>
<tr>
<td>174. 172. 167. 159. 147. 133. 116. 97. 77. 55. 32.</td>
</tr>
<tr>
<td>151. 150. 145. 138. 126. 116. 102. 86. 68. 49. 30.</td>
</tr>
<tr>
<td>124. 123. 120. 114. 106. 96. 85. 72. 58. 43. 27.</td>
</tr>
<tr>
<td>54. 53. 50. 51. 51. 40. 87. 65. 56. 46. 35. 24.</td>
</tr>
<tr>
<td>60. 60. 58. 56. 53. 49. 44. 39. 33. 26. 20.</td>
</tr>
<tr>
<td>25. 25. 25. 24. 23. 22. 22. 20. 19. 18. 16.</td>
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<tr>
<td>DIST. FROM THE MOULD</td>
</tr>
<tr>
<td>----------------------</td>
</tr>
<tr>
<td>204. 202. 196. 186. 172. 155. 135. 113.</td>
</tr>
<tr>
<td>157. 195. 185. 180. 167. 150. 131. 110.</td>
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<tr>
<td>168. 187. 181. 172. 160. 144. 126. 105.</td>
</tr>
<tr>
<td>176. 174. 165. 161. 149. 135. 118. 99.</td>
</tr>
<tr>
<td>159. 158. 153. 146. 135. 123. 107. 90.</td>
</tr>
<tr>
<td>135. 136. 134. 127. 118. 107. 94. 79.</td>
</tr>
<tr>
<td>114. 113. 110. 105. 98. 89. 79. 67.</td>
</tr>
<tr>
<td>57. 86. 84. 80. 75. 68. 61. 52.</td>
</tr>
<tr>
<td>56. 56. 54. 52. 49. 46. 41. 36.</td>
</tr>
</tbody>
</table>
Table A3.II  Three-dimensional temperature distribution in 254 x 690 mm aluminium ingot cast at 0.833 mm/s (Reduced Secondary Cooling).
### DIST. FROM THE MOULD 31.2 CMS

<table>
<thead>
<tr>
<th>658</th>
<th>657</th>
<th>653</th>
<th>637</th>
<th>607</th>
<th>577</th>
<th>549</th>
</tr>
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<tbody>
<tr>
<td>658</td>
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<td>653</td>
<td>637</td>
<td>606</td>
<td>577</td>
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<td>652</td>
<td>627</td>
<td>599</td>
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</tr>
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<td>642</td>
<td>607</td>
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<td>480</td>
<td>468</td>
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<td>401</td>
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### DIST. FROM THE MOULD 41.6 CMS

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<td>558</td>
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<td>616</td>
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<td>572</td>
<td>542</td>
<td>509</td>
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<td>598</td>
<td>577</td>
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<td>547</td>
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Table A3.III  Three-dimensional temperature distribution in zinc jumbo ingot cast at 1.27 mm/s.
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<td>LIST FFCM THE MOULD 113.3 CMS</td>
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LIST FROM THE MCULD 141.6 CMS

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59. 58. 54. 49. 43. 36. 26. 20. 17. 15.
67. 65. 61. 57. 51. 45.
73. 72. 68. 63. 57. 51. 45.
79. 77. 73. 68. 62. 56. 49. 43.
82. 81. 76. 72. 66. 59. 52. 46. 39.
84. 82. 78. 73. 68. 61. 54. 48. 42. 37.
83. 81. 77. 73. 67. 61. 55. 49. 43. 38.
79. 78. 74. 70. 65. 60. 54. 48. 42. 38.
74. 73. 70. 66. 61. 56. 51. 45. 40. 36.
67. 66. 62. 60. 56. 52. 47. 42. 37. 33.
61. 60. 56. 55. 51. 47. 43. 38. 34. 31.
56. 55. 53. 50. 47. 43. 39. 35. 32. 28.
51. 50. 48. 46. 43. 39. 36. 32. 29. 26.