CONSTITUTIVE BEHAVIOUR OF ALUMINUM ALLOYS AA3104, AA5182, AND AA6111 AT BELOW SOLIDUS TEMPERATURES

by

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Abstract

The vertical direct chill (DC) casting process is a key production route for the fabrication of aluminum alloy sheet products. In the DC casting process, due to differential temperature, strain and strain rate conditions across the bottom and the side faces of sheet ingot, high thermal stresses are generated, that in turn can cause butt-curl, stability problems, cracking or melt bleed-outs. These 'defects', developed during the DC casting process have been a major quality concern since the invention of DC casting in the 1930s. To predict thermal stresses developed during DC casting process and optimize the casting process, thermomechanical models have been used. Development of these thermomechanical models for DC casting of light metals requires knowledge of the constitutive behaviour of the material under thermomechanical conditions that are typical (strain rates from $1 \times 10^{-1} \text{s}^{-1}$ to $1 \times 10^{-5} \text{s}^{-1}$) of those experienced during DC casting. This research work reviews the thermomechanical conditions experienced during DC casting and the use of an empirical model to predict the high and low temperature constitutive behaviour of aluminum alloys in the solid state under deformation conditions relevant for DC casting. The effect of temperature, strain and strain rate has been studied for three commercially important alloys, namely: AA3104, AA5182 and AA6111. A brief study on the effect of sample orientation in the ingot has also been conducted. To determine the material parameters necessary for the extended Ludwik equation, compression tests were conducted, on industrially supplied as-cast material, using the Gleeble 3500 available at the UBC. Correlations quantifying the material parameters for each alloy as a function of temperature were developed for each of the alloys studied. To validate the material parameters of the extended Ludwik equation, some different compression tests were
performed in which specimens were deformed while they were cooled in air after heating up to \( \sim 500^\circ\text{C} \). These complex thermomechanical history tests were simulated using the FE program ABAQUS where the constitutive behaviour of the materials was simulated using the extended Ludwik equation. In the complex history tests, the material experiences some recovery which is not accounted for in the extended Ludwik equation. To account for recovery, an empirical model has been suggested based on the work hardening parameter for the material.
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List of Symbols

\( \sigma \) Stress (MPa)
\( \varepsilon_p \) Plastic strain
\( T \) Temperature (°C, K)
\( \dot{\varepsilon}_p \) Plastic strain rate (s\(^{-1}\))
\( \sigma_o, \sigma_{YS}, \sigma_y \) Yield stress (MPa)
\( E \) Elastic Modulus (GPa)
\( \varepsilon_{el} \) Elastic strain
\( K \) Strength index (MPa)
\( n \) Strain hardening parameter
\( m \) Strain rate sensitivity
\( \dot{\varepsilon}_o \) Constant (=1 s\(^{-1}\))
\( \dot{\varepsilon}_p \) Constant (=1 x 10\(^{-4}\) s\(^{-1}\))
\( \varepsilon_p \) Constant (=1 x 10\(^{-6}\))
\( A \) Material constant (s\(^{-1}\))
\( \sigma_o \) Material constant (MPa)
\( n_H \) Material constant
\( \sigma_s \) Steady state stress (MPa)
\( Q \) Creep activation energy (J.mole\(^{-1}\))
\( R \) Universal gas constant (J.mole\(^{-1}\)'K\(^{-1}\))
\( \sigma_H \) Hardening stress (MPa)
\( k \) Material Constant
\( \alpha \) Hardening parameter in Sellars-Teggart-Hardenin Equation
\( Z \) Zener-Hollomon parameter
\( m_H \) Material parameter
\( \rho \) Dislocation density in the material
\( M \) Taylor’s factor
\( k_1, k_2 \) Material constants in Kocks-Mecking equation
\( \sigma_d \) Stress caused by hardening in the material (MPa)
\( \mu \) Shear modulus of the material (GPa)
\( \sigma_{yo} \) Yield stress in the material at 0 K
\( k \) Boltzman’s constant
\( b \) Burger’s vector
\( g_o, q_o, p_o, g_y, q_y, p_y \) Material constants
\( \theta_o \) Initial strain hardening rate in the material
\( \sigma_v \) Saturation stress (MPa)
\( \sigma_{vo} \) Saturation stress in the material at 0 K
\( \varepsilon_{\text{total}}, \dot{\varepsilon}_{\text{total}} \)  
Total strain, total strain rate (s\(^{-1}\))

\( \varepsilon_{\text{creep}}, \dot{\varepsilon}_{\text{creep}} \)  
Creep strain, creep strain rate (s\(^{-1}\))

\( \varepsilon_{\text{thermal}}, \dot{\varepsilon}_{\text{thermal}} \)  
Thermal strain, thermal strain rate (s\(^{-1}\))

\( \nu \)  
Poisson’s ratio

\( \alpha(T) \)  
Total thermal expansion coefficient (°C\(^{-1}\))

\( \alpha'(T) \)  
Thermal expansion coefficient at temperature \( T \) °C

\( D_o, D_{RT} \)  
Diameter (mm) at room temperature,

\( L_o \)  
Length (mm) at room temperature

\( D(t) \)  
Instantaneous diameter at time \( t \) s

\( \Delta D \)  
Instantaneous change in diameter (mm)

\( F(t) \)  
Load at time \( t \) s

\( A(t) \)  
Area at time \( t \) s

\( n_{RT} \)  
Strain hardening index at room temperature

\( LE11 \)  
Logarithmic strain component in 11 (X) direction

\( EE11 \)  
Elastic strain component in 11 direction

\( PE11 \)  
Plastic strain component in 11 direction

\( THE11 \)  
Thermal strain component in 11 direction

\( LE22 \)  
Logarithmic strain component in 22 (Y) direction

\( EE22 \)  
Elastic strain component in 22 direction

\( PE22 \)  
Plastic strain component in 22 direction

\( THE22 \)  
Thermal strain component in 22 direction

\( PEEQ \)  
Equivalent strain (Von-Mises equivalent of plastic strain in uniaxial compression)

\( \sigma_{11}, \sigma_{22} \)  
Stress (MPa) components in 11 and 22 directions

\( \Delta \varepsilon_{\text{creep}}^n \)  
Creep strain increment for time step \( \Delta t \) s

\( \Delta \varepsilon_{\text{plastic}}^n \)  
Plastic strain increment for time step \( \Delta t \) s

\( \varepsilon_{\text{plastic}}^n \)  
Accumulated plastic strain up to \( n^{th} \) time step

\( \varepsilon_{\text{creep}}^n \)  
Accumulated creep strain up to \( n^{th} \) time step

\( \varepsilon_{\text{plastic}}^n \)  
Accumulated plastic strain up to \( n^{th} \) time step considering recovery
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Chapter I - Introduction

The value of Canadian primary aluminum production was estimated at $6.0 billion dollars in 2005 making it one of the most important industries for Canada [1]. Currently Canada is the third largest producer of aluminum (2.9 Mt/y) behind China (7.7 Mt/y) and the USA (3.6 Mt/y) [1]. Some of the largest markets in the aluminum industry include transportation (AA5xxx and AA6xxx series), packaging (AA3xxx series) and building construction. Figure 1.1 shows typical applications for some aluminum alloys. The aluminum properties which contribute to its widespread use are: a high strength/weight ratio, high formability, high toughness at room temperature, high electrical conductivity, high reflectivity, good surface finish and good corrosion resistance.

Sheet aluminum is produced via a processing route which includes casting, homogenization, hot and cold rolling and heat treatment. In terms of casting, the semi-continuous direct chill (DC) casting processing technique has been used extensively for production of sheet-aluminum ingot during the past 60 years.

Figure 1.1 - Use of aluminum alloys for both packaging (AA3104 and AA5182) as well as automobiles (AA6111 and AA5182)
1.1 DC casting process

Figure 1.2 shows a schematic of a DC casting process. An aluminum dummy block (also known as a bottom block) is initially inserted into an open rectangular/circular aluminum mould. Liquid aluminum alloy is fed through the feeder into the assembly created by the mould and the bottom block. After the liquid alloy reaches a certain height in the mould, the bottom block is lowered into a pit at a certain rate (normally 1 – 1.5 mm.s$^{-1}$). This process is stopped when the cast ingot reaches a predetermined height of approximately 4 – 10 m [2].

After the desired ingot length is achieved, the DC casting processes is stopped. The ingot is allowed to cool and is removed from underneath the DC casting mould and the DC casting process is then re-started to produce a new ingot. This makes it a semi-continuous casting process.

The mould is called the primary cooling zone. This causes the liquid metal to start cooling in the mould itself. After the primary cooling zone, the partially solidified ingot is cooled using water that impinges on the surface of the ingot from the bottom of the mould. The water-cooling consists of an impingement zone as well as a parallel or free falling water zone. The impingement zone and the freefalling water zone together make-up the secondary cooling zone. During the start-up the ingot is cooled by the bottom block as well.
Figure 1.2 - Schematic of the DC casting process.

DC casting can be better explained by dividing it into three main phases [3-5] – (1) the start up phase or non-stationary stage during which the temperature profile, solidification front, ingot shape and liquid metal-air interface change with time, (2) semi-stationary phase, and (3) steady state process. Steady state occurs when approximately 1 meter of the ingot has been cast and a steady state thermal field (or a constant sump profile) is established [2, 4]. Figure 1.3 shows a schematic of the various cooling mechanisms during the DC casting process start-up phase.
1.2 Quality issues in DC casting process

During the DC casting of aluminum alloys, the majority of heat is removed in the secondary cooling zone where the hot ingot surface is exposed to water. This high rate of heat transfer causes a non-uniform temperature distribution within the cast body. At the center of the ingot the cooling rates are relatively slow ($<0.01 \text{ K.s}^{-1}$) as compared to ($>1\text{ K.s}^{-1}$) at the surface [6] which causes strain rates of the order of $10^5 \text{ s}^{-1}$. Thermal contraction and solidification shrinkage so produced may develop excessive stresses and strains which may cause many defects in the ingot body and geometry such as hot tearing, cold cracking, butt curl, butt swell, and rolling face pull-in [2, 4, 7, 8].
1.3 Thermomechanical Modelling for DC casting

To date, quality control and design modifications to the DC casting process have been done using a trial and error approach. In order to understand the development of defects and optimize the design of the casting process, mathematical modelling is used for DC casting. These thermomechanical models address the issues of heat transfer, fluid flow, microstructure evolution, and their integrated interaction causing thermal stresses to develop during DC casting. Critical input to these models is the mechanical behaviour of the material for the prevalent conditions of the strain, strain rate and temperature during DC casting.

In 2001-2002 the Materials Engineering department at the University of British Columbia started a project for the prediction of, heat transfer and stress development during vertical DC casting (VDC). The current work is a part of this UBC – VDC project which attempts to measure the material behaviour at deformation conditions of relevance to DC casting and then to develop constitutive behaviour models for as-cast aluminum alloys which can be used to predict the mechanical response of the material.
Chapter II – Literature Review

One of the continuing problems in the DC casting of aluminum and other non-ferrous alloys is the incidence of cracking and macroscopic deformation (i.e., butt curl and rolling face pull in) in the cast product [3, 4, 9]. These defects can lead to low recovery rates and low quality cast products. The main cause of these defects is the development of thermal stresses in the casting during solidification and subsequent cooling. One technique which can be used to understand the influence of processing parameters on the development of thermal stresses and hence cracking or macroscopic deformation of the cast product is to develop thermomechanical models of the process. Once developed, these models can be used to describe the stress/strain state of the material at any point in the casting process [3, 4, 10-14]. A key aspect to the development of these models is the availability of constitutive behavior models which accurately represent the material behavior in both the semi-solid as well as solid state under deformation conditions experienced during DC casting.

Although the stress-strain behaviour of wrought aluminum alloys have been studied fairly extensively, the stress-strain behaviour of as-cast aluminum alloys and how it varies under deformation conditions similar to those which occur during DC casting have received little attention. To date, a variety of models to predict the constitutive behavior of specific aluminum alloys in the solid state have been proposed. These models include: mechanistically based models [8, 15-17], empirically based models [13, 18, 19] and physically based models [20]. However, there has been no consensus on the best way to represent this behavior occurring during DC casting.
2.1 Thermomechanical history during DC casting

During the DC casting process the thermomechanical history experienced throughout the ingot varies significantly. As expected, during solidification of DC-cast products, the cooling rate decreases rapidly with increasing distance away from the product surface [21]. For example, at the surface of sheet ingots the cooling rates are relatively fast (\( \sim 1 \, ^\circ\text{C.s}^{-1} \)) whereas in the center of the ingot the cooling rates are relatively slow (\( \sim 0.25 \, ^\circ\text{C.s}^{-1} \)) [22]. The cooling rates at the surface can reach a maximum of \( \sim 10 \, ^\circ\text{C.s}^{-1} \) during DC casting [23]. Hence the rate of deformation experienced by the product during this process is complex and strain rates experienced by the material are of the order of \( 10^{-5} \, \text{s}^{-1} \).

The evolution in the stress state of the material during the DC casting process is also very complex. As an example of this Figure 2.1(a) and Figure 2.1(b) shows the predicted surface temperature, as well as the evolution in the stress at the surface and centre of an ingot as it cools to room temperature using an uncoupled finite element (FE) thermomechanical model (strain rate independent plasticity model) of DC casting developed at the University of British Columbia [14]. As shown in Figure 2.1(b), stresses at the surface are initially tensile and towards the end of the process become compressive. At the center of the ingot the stress state initially is compressive and then towards the end of the process it changes to a state of tension.
Figure 2.1 - Model predictions of the thermomechanical history during DC casting showing: a) Predicted surface temperature at the centre of the broad face near the bottom of the ingot during the start-up phase of the DC casting process [14] and b) Predicted stress history at the surface and centre of the ingot as it cools to room temperature [14]
Figure 2.1 indicates the complex nature of the stress evolution in the material during DC casting. In order to model the stress state in the material accurately, it is imperative to have knowledge of the constitutive behaviour of the material in the solid state.

2.2 As-cast microstructure of the alloys

The three aluminum alloys investigated in this research include: AA3104, AA5182 and AA6111. This section will give some background on the as-cast microstructure for each of these alloys.

AA3104, is a non heat treatable Al-Mn-Mg alloy, that has found its main application in can-body stock and many other food-packaging applications [24]. The
evolution of the microstructure of AA3004 during solidification has been reported by Backerud [25]. The as-cast microstructure of AA3104 consists of primary aluminum and constituent particles. About 85% of the constituent particles correspond to Al$_6$(Fe,Mn) and the remainder to $\alpha$–Al$_{13}$(Fe,Mn)$_3$Si. Mn being in the matrix makes a super saturated solid solution [26].

AA5182 is a non-heat treatable (Al-Mg) aluminum alloy used frequently in the automobile industry for inner body parts as well as packaging applications i.e. the lid for beverage cans that do not need a good surface finish [27]. Evolution of different constituent particles and secondary phases during solidification of AA5182 has been reported in [28]. The as-cast microstructure of AA5182 consists of primary $\alpha$–aluminum and eutectic phases – Al$_6$(Fe,Mn), Al$_3$Fe, Al$_5$Mg$_3$ and Mg$_2$Si [6].

AA6xxx series alloys have a good combination of formability and final strength and excellent surface finish which enables them to be used for outer body panels [29]. AA6111 aluminum alloy contains Mg, Si, and Cu as the principle alloying elements and Mn and Fe are present in minor amounts. The large number of elements present in the alloy causes a variety of eutectic reactions during solidification such that the as-cast microstructure is quite complex. The as-cast microstructure of AA6111 consists of primary aluminum as well as a variety of intermetallic phases such as: $\alpha$-Al(Fe,Mn)Si, $\beta$-Al(Fe,Mn)Si, AlCuMgSi, Mg$_2$Si, Al$_2$Cu and primary Si particles [29]. Chen et al. [29] studied the solidification behaviour of DC cast AA6111 and characterized different phases present in the alloy from coherency temperature to above the solidus temperature.

One of the most important features of the microstructure of industrially cast aluminum alloys is the development of constituent particles. These are typically particles
that are quite large and are formed during the solidification process. Due to large amount of Fe present in aluminum alloys, complex or multi-component phases Al (Mg, Mn, Fe, Si) can form as constituent particles. The size and size distribution of the constituent particles depend on the rate of solidification which in turn is a function of the casting speed [21].

2.3 Constitutive behavior of aluminum alloys during DC casting

During DC casting the material experiences a complex variation of temperature as well as strain. To quantify the mechanical behaviour of the material during this process, it is critical to have a robust and germane constitutive behaviour model which is able to predict the response of the material under these complex thermomechanical conditions. This includes model which can predict the effect of strain rate at high temperatures as well as work hardening that will occur at lower temperatures and the de-convolution of these two effects at intermediate temperatures where both strain hardening and strain rate sensitivity play a role in the constitutive behavior.

Modelling of the DC casting process has evolved from simple heat transfer models [30, 31] in the early 1970's to complex thermal stress models which include the constitutive behavior of the material in both the semi-solid and solid state. The constitutive models used to simulate material behavior in the solid state range from simple power law equations that relate plastic strain to flow stress, to secondary creep based equations, and to complex internal state variable based models which provide knowledge of microstructure evolution of the alloy during cooling to room temperature.
One of the earliest thermal stress analyses during DC casting was done by Moriceau [32] in 1975. In this work he simulated the constitutive behaviour by adopting a temperature dependent elasto-plastic model of the form: \( \sigma = \sigma(\varepsilon_p, T) \). The earliest work on coupled thermal stress modeling was by Janin [33]. In this work a simple elastic-plastic constitutive approach which ignored strain rate sensitivity, was employed. Smelser and Richmond in 1988 [34] studied the air gap formation that occurred during the solidification of a cylinder. The constitutive model they incorporated was a visco-plastic internal state variable model developed by Sample and Lalli [20]. This model was developed based on hot forming data; hence the deformation conditions (strain rates and strains) were much larger than those experienced during DC casting. Early work by Mathew and Brody [35] used an interesting "overlay method" [36, 37] to combine the creep behaviour of the form of \( \sigma = \sigma(\dot{\varepsilon}_p, T) \) and the work hardening behaviour. This was used to conduct a 3D analysis of the steady state thermal stresses developed in solidifying cylindrical sections during continuous casting. In this method, the solid was assumed to consist of several layers of material each of which undergoes the same deformation. The stress field generated by the imposed deformation is determined by summing up the contribution of the individual overlays. By introducing a suitable number of layers and assigning different material characteristics to each layer, sophisticated composite material models can be developed [37].

Drezet and Rappaz [3] have described both 2D and 3D fully coupled thermal-stress models of DC casting. The material behavior in the solid state was modeled using the steady state creep law given by Garofalo's equation [38]. Farup and Mo [39] studied the effect of work hardening for similar thermomechanical histories as in DC.
casting, using Garafalo's steady state creep equation and have shown that it is critical to incorporate work hardening that occurs at lower temperatures in the model in order to get accurate predictions.

Sengupta et al. [40] describe the development of a 3D fully coupled thermal-stress model using the FE package ABAQUS for AA5182. In this work the material behavior was input into tabular form based on measured data for the variation in the stress as a function of strain, temperature and strain rate.

Fjaer and Mo [12, 13] have worked extensively on modeling of DC casting using the FE based ALSPEN they developed in-house. In this work they have modeled the material behavior using a modified Ludwik equation [13, 41-43]. Both of these approaches are essentially elastic rate-dependent plastic formulations.

Figure 2.2 illustrates the different approaches to modeling material constitutive behaviour that have been used in DC casting process models. As shown in this figure a number of different approaches have been used that range in complexity as well as applicability. The next section will describe the formulation of some of these models as well as the experimental measurements that have been done.
Figure 2.2 - Range of constitutive modeling approaches used in DC casting process modeling.

The constitutive equations used in DC casting modeling can be classified in four major categories:

1) Creep based models – They are applicable for high temperatures where material does not have any work hardening.

2) Ludwik equations – They have only 3 material parameters but need to be validated under conditions more closely simulating thermal histories during DC casting.

3) Unified constitutive equations – non separation of rate-independent and rate dependent plasticity. They contain many material parameters (one set of equations treat all deformation phenomena), but can be used with FE codes.
4) MATMOD equations - This is only a special type of unified constitutive equations. They contain many material parameters, primarily material constants are calculated using trial and error. They can be coupled with FE codes.

5) Measured data – There are inconsistencies in measured data. Need to create a lot of measured data so that interpolations are not too large.

2.4 Constitutive equations to model material behavior during DC casting

In this section, commonly used constitutive equations for modeling thermal stresses during the DC casting process and some other models used for modeling residual stresses are discussed in detail.

2.4.1 Ludwik Equation

Although the Ludwik equation [44] has not been reported in any reference for as-cast aluminum alloys it can be used to model the constitutive behaviour up to solidus temperature. Traditionally, this equation has been used to model the deformation behavior of a work-hardening rigid plastic material and is given in Equation 2.1:

\[ \sigma = \sigma_0 + Ke_p^n \]  

Where \( \sigma \) (MPa) is stress, \( \sigma_0 \) (MPa) is the yield strength of the material, \( K \) (MPa) is the strength index, \( e_p \) is the plastic strain and \( n \) is the work hardening parameter.

Equation 2.2 can be used to model the materials elastic behaviour based on Hooke’s law:

\[ \sigma = Ee_{el} \]  

Then material behaviour can be described in the elastic as well as plastic range using Equation 2.3:
As shown in Equations 2.1 - 2.3, the Ludwik equation can be used to model the change in the stress as a function of strain but strain rate effects are neglected.

2.4.2 Extended Ludwik Equation

From room temperature to the coherency temperature in the as-cast condition, the material behaviour can be described using the extended Ludwik equation \[19, 45, 46\]. As shown in Equation 2.4, this modified version of the Ludwik equation incorporates the effect of strain rate on the flow stress.

\[
\sigma = \begin{cases} 
E\varepsilon_d \left( \varepsilon_d \leq \frac{\sigma_y}{E} \right) \\
K\dot{\varepsilon}_p^n \left( \dot{\varepsilon}_p \geq \frac{\sigma_y}{E} \right)
\end{cases}
\]  

As shown in Equations 2.1 - 2.3, the Ludwik equation can be used to model the change in the stress as a function of strain but strain rate effects are neglected.

\[
\sigma = K\dot{\varepsilon}_p^n \left( \frac{\dot{\varepsilon}_p}{\varepsilon_o} \right)^m
\]

where \(\sigma\) is the stress (MPa) beyond the yield point, \(\dot{\varepsilon}_p\) is the plastic strain rate (s\(^{-1}\)), \(\varepsilon_o\) is a constant taken equal to 1 s\(^{-1}\), and \(m\) is the strain-rate sensitivity index. Coherency temperature is defined as the temperature above which the metal is treated as a liquid and below which it is treated as a solid. At coherency temperature material just starts developing stress \[47, 48\]. It is worth mentioning here that this equation can be used only if the material has been deformed plastically. This equation can be used in conjunction with Hooke’s law to model the elastic-plastic stress-strain behaviour of the material from room temperature to the solidus temperature. The effect of temperature on
the constitutive behaviour is taken into account through the dependency of terms \( K \), \( n \) and \( m \) on temperature.

Parameters \((K, n \text{ and } m)\) required for this equation can be determined by fitting experimentally measured data against equation so that a best fit is obtained. The typical variation in these parameters as a function of temperature is shown in Figure 2.3.

![Figure 2.3 - Parameters calculated for AA3104 by Haaften [19] for the extended Ludwik equation.](image)

As shown in Figure 2.3, the strength coefficient \( K \) and strain hardening index, \( n \), decrease with increasing temperature whereas, the strain rate sensitivity, \( m \) increases with increasing temperature.
The extended Ludwik equation has been found to approximate the measured data fairly well and has been used [19, 45] to model the constitutive behaviour of the material for the thermomechanical condition similar to DC casting. For high temperature and high strain rates, the extended Ludwik equation does not predict the stress well if used with parameters calculated using low strain rate measurements. Parameters of the extended Ludwik equation will not be able to predict data for a wide range of strain rates if the material parameters are calculated for specific strain rates or small range of strain rates.

2.4.3 Modified Ludwik Equation

Hannart et al. [10], Haaften et al. [41], Magnin et al. [42, 43], and Nedreberg [49] have also used a modified version of the Ludwik equation. This is essentially another version of an empirical elasto-visco-plastic model which has been called the modified Ludwik equation as shown in Equation 2.5:

$$\sigma = K \left( \varepsilon_p + \dot{\varepsilon}_p \right)^n \left( \dot{\varepsilon}_p + \dot{\varepsilon}_p \right)^m$$ (2.5)

Similar to the extended Ludwik equation, this equation is capable of predicting the constitutive behaviour of a strain rate sensitive material. Here $\sigma$ is stress (MPa), $K$ (MPa) is a material constant which is related to the strength of the material, $\dot{\varepsilon}_p$ is the equivalent plastic strain rate ($s^{-1}$), $m$ is the strain rate sensitivity coefficient, $n$ is the strain hardening coefficient, $\varepsilon_p$ is total plastic strain accumulated by the material at temperatures below 400°C. Above this temperature it is assumed that no strain hardening occurs, and the flow stress is purely dependent on temperature and strain rate. The two coefficients $\dot{\varepsilon}_{ps} (1 \times 10^{-4} \text{ s}^{-1})$ and $\dot{\varepsilon}_{pe} (1 \times 10^{-6} \text{ s}^{-1})$ which appear in Equation 2.5, have
been introduced in Hannart’s DC casting model [10] to give a non-zero yield stress at low temperatures. These constants have no physical significance and are small compared to $\dot{\epsilon}_p$ and $\varepsilon_p$.

A simplex method similar to the one developed by Kozlowski et al. [50] is typically used to determine the parameters $K$, $n$ and $m$ as a function of temperature by finding the minimum in the error between model predictions and experimental measurements.

The modified Ludwik equation can be applied for a continuous description of the mechanical behaviour from room temperature to the coherency temperature. A zero value of the strain hardening coefficient $n$ in Equation 2.5, gives the Norton Law as shown in Equation 2.6

$$\sigma = K \left( \dot{\epsilon}_p + \dot{\epsilon}_p J \right)^n$$  \hspace{1cm} (2.6)

whereas a zero value of the strain rate sensitivity $m$ gives the Hollomon Law as shown in Equation 2.7.

$$\sigma = K \left( \varepsilon_p + \varepsilon_p \right)^n$$ \hspace{1cm} (2.7)

2.4.3 Garofalo’s or Sellars-Tegart Equation

Garofalo’s or Sellars-Tegart equation [19, 39] has been used to describe the steady state creep behaviour of materials at high temperatures. This equation is given by:

$$\dot{\varepsilon} = A \left[ \sinh \left( \frac{\sigma}{\sigma_s} \right) \right]^{m} \exp \left( \frac{-Q}{RT} \right)$$ \hspace{1cm} (2.8)
where \( A \) (s\(^{-1}\)), \( \sigma_0 \) (MPa) and \( n \) are material constants, \( \sigma_s \) (MPa) is the steady state stress, 
\( Q \) (J.mole\(^{-1}\)) is the creep activation energy, \( R \) (J.mole\(^{-1}\).K\(^{-1}\)) is the universal gas constant, and \( T \) is the temperature (K).

This equation has been fit using experimental data for tests with strain rates ranging from \((10^{-6} \text{ s}^{-1} \text{ to } 10^{-2} \text{ s}^{-1})\) and temperatures from 400 °C to 600 °C for an AA3103 alloy [51]. In Garofalo’s equation, unlike previous equations, the temperature shows explicitly and other terms are temperature independent and this equation is very suitable for use at temperatures where only creep processes occur. Equation (2.8) can be written in the form:

\[
\dot{\varepsilon} \exp \left( \frac{Q}{RT} \right) = A \left[ \sinh \left( \frac{\sigma_s}{\sigma_a} \right) \right]^{n}
\]

(2.9)

Where \( \dot{\varepsilon} \exp \left( \frac{Q}{RT} \right) \) is called the Zener-Hollomon parameter [52] and is denoted by \( Z \). By plotting \( Z \) Vs. \( \sigma_s \), the parameters of Garofalo’s creep equation can be calculated.

A steady state creep model such as this cannot be used to describe the mechanical behaviour of aluminum alloys from room temperature to the solidus temperature as it does not take into account strain hardening. Farup and Mo [39] have suggested that for temperatures above 400 °C a steady state creep law should be used and below that an absolute work hardening law should be used.
2.4.4 Hardening-Sellars-Tegart Equation

Garofalo’s or Sellars-Tegart equation can be extended for its application at lower temperatures. In doing so \( \sigma_s \) is replaced by \( \sigma_H \) (MPa) and is combined with a simple hardening law as follows:

\[
\sigma_H = (\sigma_0 + k \sqrt{\alpha(T)} \dot{e}) f(Z) \tag{2.10}
\]

where \( k \) is a constant and \( \alpha \) is a hardening parameter. The function \( f(Z) \) is defined as follows:

\[
f(Z) = \min \left( 1, \arcsin \left( \frac{Z}{A} \right)^{m_H} \right) \tag{2.11}
\]

where \( Z \) is the Zener-Hollomon parameter as defined earlier in paragraph 2.4.3, \( A \) is the same material constant as in Equation 2.9, and \( m_H = \frac{1}{n_H} \). The parameter \( \alpha \) represents the percentage of hardening occurring at a certain temperature compared to the hardening occurring at room temperature under the same load. At high temperatures, \( \alpha = 0 \), and at low temperatures, \( \alpha = 1 \). Between 250 °C and 400 °C, \( \alpha \) changes from 1 to 0 to provide a smooth transition between hardening-dominated behaviour to steady-state behaviour. The temperature dependency of \( \alpha \) is as follows:

\[
\alpha = \frac{1}{1 + \exp(a_0 + a_1 T)} \tag{2.12}
\]

in which \( a_0 \) and \( a_1 \) are fitting parameters. Equations 2.10 through 2.12 will be referred as the combined hardening–Sellars-Tegart (HST) equation [19, 45].

21
Although the extended Ludwik equation describes the data better, this adapted form of the Sellars-Tegart equation is easier to implement in the DC casting model because the temperature appears explicitly in this equation.

The parameters determined for HST equation suit better to the homogenized material as compared to as-cast material. This may be due to the fact that in the as-cast material due to differential cooling rate, strain hardening developed in the material is different as we move from surface to the core of DC-cast [19].

At high temperatures strain hardening becomes negligible. As a consequence at high temperatures and low stress levels (i.e. $\frac{\sigma}{\sigma_0} < 0.8$), the HST equation gets modified into a power-law equation. The same is valid for the extended Ludwik equation at high temperature because $n$ becomes zero. So under these circumstances, it is expected that $m$ equals $m_H$ and that $\frac{K}{\varepsilon_0^m}$ equals $\sigma_0 - \exp\left(\frac{m_H Q}{RT}\right)$. $m$ and $m_H$ approach each other above 350 °C [19].

The HST equation can also be used to predict the constitutive behaviour from room temperature to the solidus temperature but it is not as good as the extended Ludwik equation. The extended Ludwik equation is optimized for each temperature individually that is why it estimates material behaviour better than the HST equation which fits all the data value simultaneously. HST equation is a bit inconsistent in describing the experimental data in the intermediate temperature range where material behaviour is both strain hardening and strain rate dependent. One of the advantages of HST equation over the extended Ludwik equation is the direct dependence of the hardening parameter $\alpha$ on the temperature. This makes it easier to be implemented in the thermomechanical models.
of DC casting. One more very important advantage of the HST equation over the extended equation is that, it can be used for strongly varying strain rate conditions. Values of parameters determined for HST equation for low strain rates can predict the flow stresses for high strain rates fairly closely.

Table 2.1 below shows a summary of different constitutive equations found in literature for different aluminum alloys.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Aluminum Alloys</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extended Ludwik equation</td>
<td>AA1050, AA3104, AA5182 [19, 45, 46]</td>
</tr>
<tr>
<td>Modified Ludwik equation</td>
<td>AA2024 [10], Al- 4.5 % Cu [42, 43], AA3004 [41], AA6063 [13, 53]</td>
</tr>
<tr>
<td>Garofalo’s steady state creep equation</td>
<td>AA5182 [7], AA3103 [39, 51], AA3104 [54], 1201 [4]</td>
</tr>
<tr>
<td>Hardening-Sellar’s Tegart equation</td>
<td>AA1050, AA3104, AA5182 [19, 45, 46]</td>
</tr>
</tbody>
</table>

2.4.5 Equations based on Internal Variables

Mo and Holm [23] have described constitutive equations based on internal variables representing kinematic and isotropic strain hardening and recovery. These equations are called Miller’s MATMOD constitutive equations [55]. Parameters of latest form of MATMOD equations which account for solute effect and yield surface distortion,
have been determined for different aluminum alloys [56] for high strain and high strain rate deformation conditions.

Stress-strain responses predicted by integration of the MATMOD equations show that the steady state stress is reached after ~ 2 % strain at 400°C. That means, MATMOD equations if used for DC casting purpose, will underestimate stress.

One of the major problems with the internal variable equations is that, in case of instantaneous strain rate changes, the strain transient from one strain rate to the other strain rate, in the equations of less number of internal variable equations is very large. Moreover, if the number of internal variables is increased to predict the exact transient, that requires more material constants and in turn more complex experiments are required for determination of the material constants.

In DC casting, material is subject to non-proportional straining. In DC casting process, one should account for kinematic straining also, which is a tensor quantity representing loading in anisotropic conditions. This is applicable for DC casting case where in some regions of DC cast ingot, because for small plastic strains, there is large local pile up of mobile dislocations as compared to the total dislocation density [23].

2.5 Effect of Work Hardening

Farup and Mo [39] have shown from their continuous cooling experiments that straining above the solidus temperature does not affect the rheological behaviour below this temperature. In other words rheological behaviour of the alloy in the mushy zone does not induce strain hardening, but might affect the thermally induced deformations and tendency towards hot tearing. To study the effect of strain history on material
behaviour, Van Haaften et al. [19] prestrained the specimens to 1 to 2 % at high temperature but below solidus before straining them at 50 °C. It was found that prestraining at 500 °C and 400 °C did not affect the material behaviour at lower temperature significantly, whereas prestraining at 300 °C and below this temperature increased the yield stress at 50 °C. Prestraining below 200 °C does not depend on temperature. Above 400 °C, recovery due to annihilation of dislocations is faster due to climb (thermal activation) of dislocations. Whereas below 200 °C temperature dependency of strain hardening is not significant.

2.6 Kocks-Mecking Equation based on work hardening

Strain hardening is caused by accumulation of dislocations. Recovery is caused by annihilation of dislocation governed by self-diffusion of atoms and climb of dislocations. Recovery is thus a thermally activated mechanism and recovery is a function of temperature and current dislocation density. Hence it is a function of strain rate and temperature. When the dislocation density is very high, further deformation may enhance annihilation of dislocations [56-59]. The aforesaid can be explained in form of the following equations:

\[
\frac{d\rho}{d\varepsilon_p} = M \left( k_1 \sqrt{\rho} - k_2 \rho \right) \quad (2.13)
\]

where \( \rho \) is dislocation density in the material, \( \varepsilon_p \) is applied plastic strain. \( M \) is Taylor’s factor and \( k_1, \) and \( k_2 \) are material constants. The first part of the equation: \( k_1 \sqrt{\rho} \) accounts for the rate of accumulation of dislocations in the material with increasing plastic strain. Whereas the second part: \( k_2 \rho \) accounts for recovery in the material.
The above equation describes the evolution of dislocation density in the material as a function of plastic strain. This effect which is known as dynamic recovery is not of significant importance for thermally induced deformations because of small strains involved. Steady state creep is dominating mechanism in the solidification regime. At lower temperatures annihilation mechanism is low. Therefore large dislocation densities are needed to read equilibrium between creation and annihilation of dislocations characteristics of steady-state creep. Chen et al. [60] have modeled constitutive behaviour of a wrought aluminum alloy AA5182 using a mechanical threshold stress, also known as the MTS-Voce model. They have modeled the mechanical behaviour of AA5182 in the thermal recovery regime.

In DC cast alloys having very low concentration of elements, the strain hardening effect is mainly due to dislocation configurations [23]. Recently a modified form of the Kocks-Mecking and Voce equation, which is known as the MTS-Voce model, has been used for modeling of residual stresses in quenched W319 aluminum alloy [61]. We assume that distortions caused during DC casting are of same characteristics as residual stresses during thermal treatment of material. For this purpose MTS-Voce model which was developed for stage III hardening will be briefly introduced here.

The flow stress in the material \( \sigma \) (MPa) is written as the sum of three components: and athermal stress \( \sigma_a \) (MPa) which is considered to be zero for large grain size material such as cast W319 [61], yield stress \( \sigma_y \) (MPa) and stress caused by hardening in the material \( \sigma_d \) (MPa). This can be written in the form of the following equation:

\[
\sigma = \sigma_y + \sigma_d
\]  

(2.14)
The evolution equations for yield stress and stress caused by hardening in the material as given by the following expressions:

\[
\frac{\sigma_y}{\mu} = \frac{\sigma_{yo}}{\mu_o} \left\{ 1 - \left( \frac{kT}{\mu b^3 g_{o_y} \bar{\varepsilon}_p} \ln \frac{\dot{\varepsilon}_o}{\bar{\varepsilon}_p} \right)^{1/p_y} \right\}
\] (2.15)

and

\[
\sigma_d = \sigma_v \left( 1 - \exp \left( -\frac{\theta_o}{\sigma_v} \varepsilon_p \right) \right)
\] (2.16)

where symbols have following meanings:

- \(\mu\) (GPa) is the shear modulus of the material which is a function of the temperature.

\[
\mu = \mu_o \exp \left( \frac{D}{\exp \left( \frac{215}{T} \right) - 1} \right)
\] (2.17)

- \(\mu_o\) (GPa) is shear modulus of the material at reference temperature of 0 K. \(D\) (GPa) is a material constant.

- \(\sigma_{yo}\) is the yield stress in the material at 0 K. \(k\) and \(b\) are Boltzman’s constant and Burger’s vector of aluminum respectively. \(g_{o_y}\), \(q_v\), and \(p_v\) are material constants.

- \(\theta_o\) is initial strain hardening rate in the material and \(\varepsilon_p\) is plastic strain. \(\dot{\varepsilon}_o\) (1 x 10^7 s^-1) is a reference strain rate and \(\dot{\varepsilon}\) is the actual strain rate in the material. Evolution of the saturation stress \(\sigma_v\) (MPa) is given by an expression similar to Equation 2.17:

\[
\frac{\sigma_v}{\mu} = \frac{\sigma_{vo}}{\mu_o} \left\{ 1 - \left( \frac{kT}{\mu b^3 g_{o_v} \bar{\varepsilon}_p} \ln \frac{\dot{\varepsilon}_o}{\bar{\varepsilon}_p} \right)^{1/p_v} \right\}
\] (2.18)

\(\sigma_{vo}\) is the saturation stress in the material at 0 K. \(g_{o_v}\), \(q_v\), and \(p_v\) are material constants.
2.7 Consideration of parameters affecting microstructure

Several mechanisms are attributed to the strength of aluminum alloys that do not change necessarily during the casting process. Grain size which can be accounted for the flow stress via the Hall-Patch relation [62], remains almost constant during the DC casting process. Also no recrystallization and formation of subgrains occurs because of the low plastic strains involved [23]. The strength also depends on the number, size and size distribution of precipitates (particle strengthening) and number of alloying elements [23]. In DC casting due to fast cooling rate elements still remain in the solid solution and secondary precipitation can be neglected. Farup and Mo [39] have reported that due to low alloying element concentration (in AA3103) and low diffusivity of Mn in Al, the effect of alloying elements on the mechanical behaviour under DC casting conditions of transient temperature, strain and strain rate, can be neglected.

2.8 Summary

Material behaviour during DC casting can be divided into three main regimes: a) below 200 °C, material work hardens substantially and is not sensitive to strain rate changes, b) 200 °C – 400 °C, material work hardens and is strain rate sensitive as well, c) above 400 °C, material does not work harden and is very sensitive to strain rate.

Extended Ludwik equation and Hardening-Sellars-Tegart equations have been used recently for modeling of constitutive behaviour of DC cast materials. Their use in a DC casting thermomechanical process simulation has not been reported yet. HST equation has advantage of having direct influence of temperature on the flow stress and that makes it easy for use in DC casting simulations. Modified Ludwik equation is also
similar to extended Ludwik equation but the two empirical constants $\dot{\varepsilon}_p$ and $\varepsilon_p$, only add to the number of parameters and the complexity of the parameter determination.

MTS-Voce equation which was originally developed for stage III work hardening behaviour can be used fundamentally for modeling of thermal stresses. One advantage is the direct dependence of temperature on the yield stress and steady state stress. Similar to the HST equation, for this equation parameters are calculated for the whole temperature regime.
Chapter III – Objectives

3.1 Objectives

Direct chill casting has been the main processing technique for the last 60 years, for the production of commercially significant aluminum alloys. In order to develop thermomechanical models of the process, accurate knowledge of the material response over the temperature regime from the coherency temperature down to room temperature is essential.

The objectives of this project include:

a) Characterize the as-cast solid-state material behaviour under temperature (25 °C to 500 °C), strain and strain rate (1 x 10^{-1} s^{-1} to 1 x 10^{-5} s^{-1}) conditions similar to those experienced during DC casting. This was done for three industrially significant aluminum alloys, AA3104, AA5182 and AA6111.

b) Develop and validate a model to mathematically represent the material constitutive behaviour in the solid state during the DC casting process.

Experimental measurements included:

a) Constant temperature uniaxial compression tests to measure the stress-strain response

b) Continuous cooling experiments coupled with deformation in uniaxial compression for model validation purpose.

3.2 Methodology

The objectives of this work were achieved using a multi-faceted approach to the research including: experimental measurements using the Gleeble 3500 under constant
temperature conditions, analysis of the measured data, modeling using the commercial FE code ABAQUS† and validation of the developed models predicting complex thermomechanical history experiments on the Gleeble* 3500. Uniaxial compression tests were conducted under temperature range of 25 °C – 500 °C, and strain rates (1 x 10^{-1} s^{-1} to 1 x 10^{-5} s^{-1}) to reflect typical conditions the material would be exposed to during the DC casting operation at both the center and surface of the sheet ingot. During these tests temperature was constant. Data from uniaxial compression tests was fitted against the extended Ludwik equation and the parameters for the equation were calculated using a least squares method.

3.2.1 Model validation

Validation of the model was performed using experiments in which specimens were continuously cooled coupled with applied compressive force. These tests reflected more accurately the type of stress history the material would be exposed to during DC casting. The stress developed in the material during these continuous cooling experiments was predicted by a commercial FE package ABAQUS in which material behaviour was characterized using the extended Ludwik equation. Figure 3.1 shows schematically how the experimental, modeling and validation activities within the project were linked.

† ABAQUS is a trademark of Hibbit, Karlsson & Sorensen Inc., Pawtucket, RI, USA.

* Gleeble is a trademark of Dynamic Systems, Inc., Poestenkill, NY.
Figure 3.1 - Schematic showing major steps in the current research project.
Chapter IV – Experimental

4.1 Start material

The experimental work was carried out on three industrially significant as-cast aluminum alloys namely: AA3104, AA5182 and AA6111. The material was received in the as-cast condition from the Alcan International Arvida Research and Development Laboratories (ARDC) located in Jonquiere, Quebec. The chemical compositions for each of the alloys are shown in Table 4.1.

Table 4.1 - Chemical composition (wt %) of the alloys used in this study

<table>
<thead>
<tr>
<th>Alloy*</th>
<th>Mg</th>
<th>Mn</th>
<th>Si</th>
<th>Cu</th>
<th>Cr</th>
<th>Fe</th>
<th>Ti</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA3104</td>
<td>1.15</td>
<td>0.88</td>
<td>0.20</td>
<td>0.20</td>
<td>0.001</td>
<td>0.38</td>
<td>0.003</td>
</tr>
<tr>
<td>AA5182</td>
<td>4.63</td>
<td>0.49</td>
<td>0.06</td>
<td>0.043</td>
<td>0.007</td>
<td>0.17</td>
<td>0.006</td>
</tr>
<tr>
<td>AA6111</td>
<td>0.76</td>
<td>0.22</td>
<td>0.63</td>
<td>0.75</td>
<td>0.001</td>
<td>0.25</td>
<td>0.036</td>
</tr>
</tbody>
</table>

* Balance is aluminum.

4.1.1 Characterization

Initial characterization of the as-cast material was carried out via optical microscopy. This included quantifying the grain size. This work was carried out using: an optical microscope and a Clemex 2.1 image analyzer.

Figure 4.1 shows typical micrographs of as-cast AA3104, AA5182 and AA6111 as seen using an optical microscope. For optical microstructure characterization, specimens were polished to 0.05 μm. The specimens were then anodized for 40-80 s in a
solution of 4 mL HBF$_4$ (48 %) and 200 mL H$_2$O. During anodization, a 99.9 % pure aluminum strip acts as the cathode and the specimen works as the anode. During this procedure, 20 V direct current or 0.2 A / cm$^2$ was used [63]. For grain size measurements, the line intercept method was employed.

(a) AA3104  
(Average Grain Size 100 µm )  
(b) AA5182  
(Average Grain Size 200 µm )  
(c) AA6111  
(Average Grain Size 100 µm )

Figure 4.1 - Optical micrographs for: a) AA3104, b) AA5182 and c) AA6111.
4.2 Measurement of constitutive behaviour

4.2.1 Gleeble 3500

Uniaxial compression tests were performed on a Gleeble 3500 thermomechanical simulator available in the Department of Materials Engineering at the University of British Columbia (UBC). The Gleeble 3500 can perform deformation tests under very controlled conditions of prescribed strain, strain rate and temperature. Figure 4.2 shows the Gleeble 3500 which was used to perform the experiments for this study.
4.2.2 Compression tests

The development of the models used to predict the constitutive behaviour of the aluminum alloys was done based on measured stress-strain data under a variety of deformation conditions. This data was generated by performing uniaxial compression tests under constant temperature conditions. The dimensions of the cylindrical compression specimens were 10 mm in diameter and 15 mm in length. Compression specimens were taken both in the parallel and perpendicular directions with respect to the DC casting direction as shown in Figure 4.3. Specimens with their length parallel to DC
casting direction will be referred as "Parallel" specimens. Specimens with their length perpendicular to DC casting direction will be referred as "Perpendicular" specimens. The location and orientation of the test specimens taken from the as-cast aluminum ingots as well as the number of specimens taken are shown in Table 4.2.

Figure 4.3 - Schematic of DC casting ingot showing parallel and perpendicular orientations of the specimens considered in this study.
Table 4.2 - Location and orientation of test specimens taken from the as-cast ingot.

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Location</th>
<th>Orientation</th>
<th>No. of specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA3104</td>
<td>6 cm below rolling face</td>
<td>Parallel</td>
<td>8</td>
</tr>
<tr>
<td>AA3104</td>
<td>6 cm below rolling face</td>
<td>Perpendicular</td>
<td>20</td>
</tr>
<tr>
<td>AA5182</td>
<td>6 cm below the surface and 6 cm from bottom face</td>
<td>Parallel</td>
<td>7</td>
</tr>
<tr>
<td>AA5182</td>
<td>6 cm below the surface and 24.6 cm from bottom face</td>
<td>Perpendicular</td>
<td>21</td>
</tr>
<tr>
<td>AA6111</td>
<td>6 cm below rolling face</td>
<td>Parallel</td>
<td>10</td>
</tr>
<tr>
<td>AA6111</td>
<td>6 cm below rolling face</td>
<td>Perpendicular</td>
<td>14</td>
</tr>
</tbody>
</table>

The majority of the tests were performed by heating the specimen up to the test temperature and then deforming it at a constant strain rate and temperature to a known total strain. In some instances the strain rate was varied during the test so that a quantitative estimate of the strain rate sensitivity could be made. Figure 4.4 shows an example of a constant strain rate test and a variable strain rate test highlighting the calculation of the strain rate for the two types of strain histories. For each test, the actual strain rate experienced by the sample was calculated based on the derivative of the most linear segment of the strain history plot.
Figure 4.4 - Strain rate history experienced by samples during uniaxial compression testing showing both constant and variable strain rate tests. Data shown here is for test no. 1 and test no. 4 in Table A.2 in Appendix A.

The experimental procedure using the Gleeble 3500 included heating the samples at 5 °C.s\(^{-1}\) to the test temperature, holding at the test temperature for 60 s, followed by compressive deformation under a constant temperature and constant or variable strain rates to a total plastic strain of 0.03 - 0.3. In the Gleeble 3500, heating of the specimen occurs via resistance heating hence, it is possible to effect very rapid heat-up rates. For the compression tests, deformation temperatures ranged from room temperature up to 500 °C and strain rates ranged from \(1 \times 10^{-1}\) s\(^{-1}\) to \(1 \times 10^{-5}\) s\(^{-1}\). These values were chosen as they represented the range of deformation conditions typically experienced by the
material as it is cooled in the solid state during DC casting. During each test, the temperature was controlled using a thermocouple spot welded to the surface of the specimen at the center of the specimen along its length. The thermocouple used in this study was an alumel-chromel thermocouple. During the deformation, the instantaneous strain was measured in the specimen based on diametral strain, which was measured using a C-gauge mounted at the center of the specimen. Figure 4.5 shows a typical temperature history during the constant temperature tests.

![Figure 4.5 - Typical thermal history employed for the constant temperature compression tests.](image)

Figure 4.5 - Typical thermal history employed for the constant temperature compression tests.
4.2.3 Determination of stress-strain data

The true strain in the specimen was calculated using the following expression:

\[ e = -2\ln \left( \frac{D(t)}{D_o} \right) \]  \hspace{1cm} (4.1)

where,

- \( D_o \) is the initial diameter (mm) of the specimen,
- \( D_{RT} \) is the diameter at room temperature, and \( \Delta D_o \) is the increase in the diameter due to thermal expansion.
- \( D(t) \) is the instantaneous diameter at time \( t \) s. and is calculated as:

\[ D(t) = D_{RT} + \Delta D_{def} \]  \hspace{1cm} (4.3)

\( \Delta D_{def} \) is the measured deformation at any time \( t \) sec.

The true stress in the specimen was calculated using the following expression:

\[ \sigma = \frac{F(t)}{A(t)} \]  \hspace{1cm} (4.4)

Where \( F(t) \) is the instantaneous force measured by the load cell on the Gleeble 3500 and \( A(t) \) is the instantaneous cross section area of the specimen.

4.2.3.1 Determination of yield stress

The yield stress \( \sigma_{YS} \) was measured using the 0.2 % offset strain from the true stress-strain plots. The modulus used to do these calculations was based on literature
values of the elastic modulus ($E$) at the test temperature [64]. Figure 4.6 shows a typical stress-strain response, indicating how the yield stress was assessed.

Figure 4.6 - Typical stress-strain response for AA3104 aluminum alloy at 25 °C and a strain rate of $1 \times 10^{-3}$/s, indicating calculation of yield stress using 0.2 % off-set method. Data shown here is for test no. 1 in Table A.2 in Appendix A.

4.2.3.2 Effect of specimen size

The effect of the specimen size on the measured stress-strain data was determined by measuring the stress-strain data of AA5182 specimens which were 11.11 mm diameter by 16.67 mm length (~ 11 %) bigger then the standard sample size. See Table A.5 in Appendix A for details of the test conditions used for these tests.
4.3 Validation of constitutive behaviour models

In order to validate the constitutive behaviour models developed, tests were performed using the Gleeble 3500 that more closely simulated the type of thermal-mechanical history which would be experienced by the material in the solid state during DC casting.

These tests consisted of heating ($5^\circ \text{C s}^{-1}$) the test specimen to temperatures just above 500 $^\circ\text{C}$, holding them for 60 s (or without hold), then naturally air-cooling them from this temperature such that they experienced a continuous cooling condition and during this air cool applying compressive deformation such that the total strain experienced by the sample is low (0.03 – 0.07). Figure 4.7 shows a schematic indicating the typical thermomechanical history experienced by the test specimen during a continuous cooling test.

![Figure 4.7 - Typical thermal history employed for the continuous cooling compression tests.](image-url)
Figure 4.7 shows two types of thermal histories: one with a 60 s hold period before the specimen was naturally cooled, and the other with no such hold period. The former thermal history will be referred as “Hold”. The later thermal history will be referred to as “No-Hold”.
Chapter V – Mathematical Modelling

Constant temperature uniaxial compression tests as well as continuous cooling uniaxial compression tests performed in the experimental part of this research, were simulated by using a mathematical model using the commercial finite element method (FEM) package, ABAQUS. ABAQUS is a comprehensive, general-purpose finite element analysis package that can be used to simulate highly non-linear thermal and mechanical evolution in a material under a given set of conditions. Our motivation for using ABAQUS to simulate these tests was the fact that this FEM package has been used by a number of researchers including those at UBC to develop a 3-dimensional (3D) thermal stress model of DC casting.

5.1 Simulation of constitutive behaviour in ABAQUS

For simulation of the validation experiments ABAQUS was used. The primary reason for using ABAQUS was to integrate the constitutive equation with the ABAQUS model. Various constitutive behaviour models based on creep and work hardening are available in ABAQUS to simulate a wide range of material behaviors subject to loading. Constitutive models within ABAQUS can be used to quantify the state of stress at a material integration point during experiments based on the state of strain, strain rate and temperature.

The mechanical constitutive models provided in ABAQUS consider both the elastic and inelastic response of the material. The inelastic response of the material can be
modeled using either strain rate-dependent or strain rate-independent inelastic models and can be defined with a yield surface (plastic) or without a yield surface (creep).

In ABAQUS, it is assumed that the total deformation is made up of elastic and inelastic components, and in the case of a thermal stress analysis, a thermal strain also. Various sources of strain are assumed to be additive according to Equation 5.1.

$$\varepsilon_{\text{total}} = \varepsilon_{\text{el}} + \varepsilon_{\text{pl}} + \varepsilon_{\text{creep}} + \varepsilon_{\text{thermal}}$$  \hspace{1cm} (5.1)

where $\varepsilon_{\text{el}}$, $\varepsilon_{\text{pl}}$, $\varepsilon_{\text{creep}}$ and $\varepsilon_{\text{thermal}}$ are elastic, plastic, creep and thermal strain components respectively. Differentiating Equation 5.1, a similar relation can be shown in terms of strain rate as shown in Equation 5.2.

$$\dot{\varepsilon}_{\text{total}} = \dot{\varepsilon}_{\text{el}} + \dot{\varepsilon}_{\text{pl}} + \dot{\varepsilon}_{\text{creep}} + \dot{\varepsilon}_{\text{thermal}}$$  \hspace{1cm} (5.2)

The stress state of the material can be expressed based on a constitutive law which uses these strains and strain rates as input or displacement versus time and temperature.

5.2 Material properties

5.2.1 Elastic deformation

The elastic response of the material is modeled within ABAQUS using Hooke’s law whereby there is a linear relationship between stress and strain in this region as shown in Equation 5.3:

$$\sigma = E \varepsilon_{\text{el}}$$  \hspace{1cm} (5.3)

where $E$ is the modulus of elasticity i.e. the Young’s modulus.
Young’s modulus for AA3104, AA5182 and AA6111 were taken from the literature and varied as a function of temperature as shown in Table 5.1. Constant value of Poisson’s ratio of 0.30 was used in the analysis.

Table 5.1 - Elastic properties used in the ABAQUS simulations.

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Temperature (T °C)</th>
<th>Modulus of Elasticity, ( E ) (GPa)</th>
<th>Poisson’s ratio, ( \nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA3104</td>
<td>25 ( \leq T \leq ) 500</td>
<td>( E = 79.748 - 2.3798 \times 10^{-2} T - 2.5267 \times 10^{-5} T^2 ) [64]</td>
<td>0.3 [64]</td>
</tr>
<tr>
<td>AA5182</td>
<td>25 ( \leq T \leq ) 500</td>
<td>( E = 71.600 - 3.135 \times 10^{-2} T - 3.452 \times 10^{-5} T^2 ) [65]</td>
<td>0.3 [64]</td>
</tr>
<tr>
<td>AA6111</td>
<td>25 ( \leq T \leq ) 500</td>
<td>( E = 79.748 - 2.3798 \times 10^{-2} T - 2.5267 \times 10^{-5} T^2 ) [64]</td>
<td>0.3 [64]</td>
</tr>
</tbody>
</table>

5.2.2 Inelastic deformation

As stated earlier, inelastic deformation behaviour can be modeled using a number of different constitutive models available within ABAQUS. These models include: strain rate-independent plasticity models, strain rate-dependent plasticity or strain rate-dependent creep models. Plasticity models require the definition of a yield criterion. In the strain rate-dependent plasticity option, ABAQUS uses direct tabular data in which the material property data is provided as tables indicating the variation in flow stress as a function of strain for various equivalent plastic strain rates and temperatures. The yield
stress at a given strain, strain rate and temperature is then interpolated in a linear fashion directly from these tables.

Alternatively, strain rate dependent inelastic deformation can be handled within ABAQUS by using a subroutine called UHARD in which a specific model describing the constitutive behaviour of the material can be integrated. For the extended Ludwik equation that has been used in the present work, the subroutine UHARD requires following variables to be defined:

a) SYIELD: \( \sigma_o \) - Yield Stress for isotropic plasticity (in Pa).

b) HARD(1): Variation of \( \sigma_o \) with respect to the equivalent plastic strain, \( \frac{\partial \sigma_o}{\partial \varepsilon^{pl}} \);

where \( \varepsilon^{pl} \) is the equivalent plastic strain

c) HARD(2): Variation of \( \sigma_o \) with respect to the equivalent plastic strain rate, \( \frac{\partial \sigma_o}{\partial \dot{\varepsilon}^{pl}} \);

where \( \dot{\varepsilon}^{pl} \) is the equivalent plastic strain rate (s\(^{-1}\))

d) HARD(3): Variation of \( \sigma_o \) with respect to temperature (\( \theta \)), \( \frac{\partial \sigma_o}{\partial \theta} \)

The variables HARD(1), HARD(2), HARD(3) for the extended Ludwik equation

\[ \sigma_o = K \left( \varepsilon^{pl} \right)^n \left( \dot{\varepsilon}^{pl} \right)^m \]

are given below. \( K \) (MPa) is a strength parameter of the material, \( n \) is the strain hardening index, and \( m \) is the strain rate sensitivity index.

\[
HARD(1) = K.n.\left( \varepsilon^{pl} \right)^{n-1}.\left( \dot{\varepsilon}^{pl} \right)^m
\]

\[
HARD(2) = K.\left( \varepsilon^{pl} \right)^n.m.\left( \dot{\varepsilon}^{pl} \right)^{m-1}
\]

\[
HARD(3) = \frac{\partial K}{\partial \theta}.\left( \varepsilon^{pl} \right)^n.\left( \dot{\varepsilon}^{pl} \right)^m + K.\left( \varepsilon^{pl} \right)^n.\left( \dot{\varepsilon}^{pl} \right)^m.\log \dot{\varepsilon}^{pl} \frac{\partial m}{\partial \theta} + K.\left( \varepsilon^{pl} \right)^n.\left( \dot{\varepsilon}^{pl} \right)^m.\log \varepsilon^{pl} \frac{\partial n}{\partial \theta}
\]

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The determination of the yield stress as well as the parameters $K$, $n$ and $m$ and their dependency on temperature for each of the alloys is discussed in Chapter VI.

5.2.3 Thermal contraction/expansion properties

ABAQUS requires the use of a total thermal expansion coefficient. This thermal expansion coefficient can be calculated as the average or equivalent thermal expansion coefficient over the range of interest. It is defined as shown in Equation 5.4

$$\alpha(T) = \frac{1}{T - T^0} \int_{T^0}^{T} \alpha'(T) dT$$

(5.4)

where $\alpha(T)$ is the effective total thermal expansion and $\alpha'(T)$ is the thermal expansion coefficient. $T^0$ is the reference temperature above which material is assumed to have zero thermal strain. In the present study $T^0$ has been taken as 577 °C. Table 5.2 shows total (remove total) thermal expansion coefficient calculated from Ref. [64].

<table>
<thead>
<tr>
<th>Table 5.2 – Thermal Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alloy</td>
</tr>
<tr>
<td>AA3104, AA5182 and AA6111</td>
</tr>
</tbody>
</table>

5.3 Simulation of Gleeble experiments using ABAQUS

The reason for developing a simple uniaxial compression model was to examine the validity of the constitutive behaviour model developed in the solid state to simulate
more complex cooling and deformation conditions. The model requires specification of the sample geometry, boundary conditions (including the nature of loading) and the material response (constitutive behaviour).

5.3.1 Geometry

The model developed in ABAQUS to simulate the Gleeble experiments was a 2-dimensional (2D) axisymmetric model. Due to the circumferential symmetry the model yields a domain with a height of 15 mm and radius of 5 mm. The finite element mesh consisted of 1200, 2D 4 noded isoparametric axisymmetric elements, and had a total of 1281 nodes. ABAQUS evaluates the material response at each material integration point of which there are 4, located at the nodal positions.

5.3.2 Boundary conditions

The boundary conditions used for the 2D axisymmetric model are illustrated in Figure 5.1. Due to symmetry, displacement at \( r=0 \) is constrained. At the bottom of the sample (i.e. \( z=0 \)) displacement in the axial direction is set to zero so as to avoid rigid body motion. In Figure 5.1 displacement in the axial direction is represented by “\( V \)” and displacement in the radial direction is represented by “\( U \)”. It is shown in Figure 5.1 that boundaries \( V_1 \) and \( U_1 \) are fixed, and boundaries \( V_2 \) and \( U_2 \) are free.

To exactly simulate the Gleeble experiments the elements were subjected to displacement vs. time history and temperature vs. time history determined from the experimental data. This type of input in ABAQUS code is called an amplitude-curve. For the constant temperature tests, the displacement was specified as a function of time.
whereas for the continuous cooling tests the displacement as well as thermal history was specified.

Figure 5.1 - FE model of uniaxial compression sample (1200 elements and 1281 nodes) simulated in ABAQUS showing mechanical boundary conditions.
Chapter VI – Results and Discussion

This section of the thesis will outline the major results from this investigation.

The results of experiments done to study the effect of specimen size discussed in Chapter IV, have been shown in the Appendix A. The results show that there is no significant effect of specimen size on the stress-strain response of the material.

Results and discussion about the effect of orientation of the specimen are also shown in the Appendix A.

6.1 Typical stress –strain response of as-cast aluminum alloys

Using the methodology described in Chapter III for calculation of stress-strain and yield stress, the stress-strain response of all the alloys studied was determined. Appendix A gives data from each test of the calculated yield stress as well as the actual temperature and strain rate experienced by the test sample.

6.1.1 Effect of temperature

As expected, as the temperature increased the strain hardening and overall flow stress of the material decreased. This was seen for all the alloys studied as shown in Figure 6.1.
Figure 6.1 - Effect of temperature on the stress-strain response for: a) AA3104 (strain rate ~ $1 \times 10^{-3} \text{s}^{-1}$) b) AA5182 (strain rate ~ $1 \times 10^{-2} \text{s}^{-1}$) and c) AA6111 (strain-rate ~ $1 \times 10^{-3} \text{s}^{-1}$).
(b) AA5182

(c) AA6111
As shown in Figure 6.1, all of the alloys studied show high work hardening rates at low temperatures. As the deformation temperatures increase, the work hardening decreases due to dynamic recovery.

6.1.2 Effect of strain rate

To understand the effect of strain rate, compared stress-strain responses for different strain rates at same temperatures are shown in Figure 6.2.

![Graph showing stress-strain responses for different strain rates at various temperatures.](a) AA3104

Figure 6.2 - Effect of strain rate with increasing temperature: (a) AA3104 (b) AA5182 (c) AA6111.
(b) AA5182

(c) AA6111
Figure 6.2 shows that for all the alloys, as the strain rate decreases the flow stress decreases, and vice-versa. This change of stress in the material due to change in strain rate is due to the strain rate sensitivity of the material. As the temperature of the material increases this strain rate sensitivity also increases. This sensitivity to strain rate was not seen until the temperature was ~200 °C for the AA3104 and AA5182 alloys. In AA6111, strain rate sensitivity start from ~ 30 °C.

To assess the effect of strain rate (strain rate sensitivity) at various temperatures, strain rate jump tests were performed. These tests involved changing the strain rate very quickly during a test and recording the flow stress of the material. Some strain-rate jump tests are shown in Figure 6.2(a) through Figure 6.2(c). In these tests strain rate was increased or decreased to the effect of strain rate on the flow stress which increases as the temperature increases.

If the material shows a "positive" strain rate sensitivity, an increase in strain rate will cause it to respond with an increase in the flow stress of the material. Similarly a decrease in strain rate will cause the flow stress to decrease. e.g. In Figure 6.2(c) at point ‘a’ strain rate was decreased from $1 \times 10^{-2} \text{s}^{-1}$ to $1 \times 10^{-3} \text{s}^{-1}$ and at point ‘b’ strain rate was increased from $1 \times 10^{-3} \text{s}^{-1}$ to $1 \times 10^{-2} \text{s}^{-1}$. Due to strain rate effect the flow stress decreases and increases respectively.

6.2 Determination of material parameters in extended Ludwik equation

The extended Ludwik equation, given by Equation 6.1 was used to determine the material based constants from the measured stress-strain data. The reason for choosing
this equation to model the constitutive behavior of the alloys studied in this research was its simplicity and low number of material parameters.

\[ \sigma = K \varepsilon_p^n \dot{\varepsilon}_p^m \]  

(6.1)

Where \( \sigma \) is the flow stress in MPa, \( \varepsilon_p \) is the plastic strain, \( \dot{\varepsilon}_p \) is the plastic strain rate, \( K \) is a strength parameter (MPa), \( n \) is the strain hardening index of the material and \( m \) is the strain rate sensitivity of the material. This equation is used to predict the flow stress of the material in the plastic region once it has yielded. If a plastic strain of 0.002 is input to this equation it will predict the yield stress for the material in the temperature regime where material is strain-rate sensitive.

The \( K \) and \( n \) parameters in this equation were determined using a least square error method in Solver utility of Microsoft® Excel. This method has been described by some other researchers for similar problems [66]. The strain rate sensitivity index, \( m \), was calculated graphically using data from the stress-strain response in which the strain-rate was changed instantaneously at a constant temperature. The effect of the change in strain rate on the flow stress can then be used to determine the material strain rate sensitivity, \( m \).

This can be shown mathematically using the following expressions.

Equation 6.2 can be written in the following logarithmic form:

\[ \ln \sigma = \ln K + n \ln \varepsilon_p + m \ln \dot{\varepsilon}_p \]  

(6.2)

Differentiating Equation 6.2 both sides with respect to \( \dot{\varepsilon}_p \) the strain rate sensitivity can be calculated.

\[ m = \frac{\partial \ln \sigma}{\partial \ln \dot{\varepsilon}_p} \]  

(6.3)
An example of graphical calculation of strain rate sensitivity parameters $m$ is shown in Figure 6.3.

![Figure 6.3 - Graphical calculation of strain rate sensitivity index $m$. Data shown here is for test no. 12 in Table A.2 in Appendix A.](image)

The strain hardening parameter, $n$, was also determined graphically at 25°C where the strain rate sensitivity was negligible. This parameter was calculated using the Considère criteria. This criteria states that for strain rate insensitive materials, the rate of strain hardening becomes equal to the stress at the point of instability (in tensile test) as shown in Equation 6.4:

\[
\frac{\partial \sigma}{\partial \dot{\varepsilon}_p} = \sigma
\]  

(6.4)
The above expression shows that strain hardening rate and stress curves intersect each other at the point of instability at which $\varepsilon_p = n$. Assuming that stress-strain response in tension and compression is same, we have applied Equation 6.4 here. An example of the graphical solution is shown in Figure 6.4. The strain hardening plot shown in Figure 6.4 was calculated based on differentiating the stress-strain data and applying a 30-point adjacent averaging routine to the data.

![Graph showing strain hardening rate and stress curves]

**Figure 6.4 - Determination of strain-hardening parameter ($n$) from strain hardening rate.** Stress-strain response shown in this plot is for AA3104 at 25 °C for strain rate $= 1 \times 10^{-3} \text{ s}^{-1}$. Data for test no. 1 in Table A.2 in Appendix A.
6.2.1 Extended Ludwik equation parameters determined for alloys AA3104, AA5182 and AA6111

Table A.2, Table A.4, and Table A.7 provide the calculated parameters of the $K$, $n$, and $m$ for the extended Ludwik equation for the three alloys studied in this investigation. Parameters for the parallel specimens have also been calculated and are shown in Table A.1, Table A.3, and Table A.6.

After determining the individual parameters for each temperature, the trends in these parameters were idealized using linear segments. This was done so that implementation of these results into an FE code is much easier. Figure 6.5, Figure 6.6, and Figure 6.7 show the calculated parameters for the extended Ludwik equation determined respectively for AA3104, AA5182, and AA6111.

In Figure 6.5, Figure 6.6, and Figure 6.7, the discrete symbols show calculated parameters, and solid lines shows idealized parameter trends using linear segments.
Figure 6.5 - Parameters of extended Ludwik equation for AA3104: (a) $K$ (b) $n$ (c) $m$
Figure 6.6 - Parameters of extended Ludwik equation for AA5182: (a) $K$ (b) $n$ (c) $m$
Figure 6.7 - Parameters of extended Ludwik equation for AA6111: (a) $K$ (b) $n$ (c) $m$
Trends of parameter vs. temperature determined from the extended Ludwik equation are in good agreement with the ones reported by other workers in [19]. Strain rate sensitivity of AA3104 at 500 °C, is an exception. From Figure 6.5, Figure 6.6, and Figure 6.7 it is clear that the $K$ parameter decreases with increasing temperature. The strain hardening index $n$ decreases with increasing temperature and then becomes zero at a certain temperature (~ 400 °C). The strain rate sensitivity index $m$ remains negligible up to a certain temperature (~ 200 °C) and then increases. As seen in the plots, there is a lot of scatter in the parameters. One of the possibilities for this scatter is the different strain rates used in the tests for which the parameters were calculated. Haaften [45], it is mentioned that the parameters calculated for low strain rates are not suitable for prediction of low strain rate stress responses. This will be further discussed when we
discuss applicability of this model. Also from Figure 6.5, Figure 6.6, and Figure 6.7 it is clear that the behaviour of the three alloys can be divided into three temperature regimes:

(A) Temperature range in which the material has substantial work hardening behaviour and has very low strain rate sensitivity. (25 °C to ~ 200 °C).

(B) Temperature range in which the material work hardens and is sensitive to strain rate changes as well (200 °C to ~ 400 °C).

(C) Temperature range in which the material does not work-harden at all and is very sensitive to strain rate changes (400 °C to ~ 500 °C).

Table A.9, Table A.10, and Table A.11 in Appendix A show the idealized linear segment correlations determined for each of the parameters and alloys studied.

6.3 Validation of the extended Ludwik equation parameters

6.3.1 Validation using uniaxial compression data

Figure 6.8, Figure 6.9, and Figure 6.10 show comparisons of measured and predicted data by the correlations presented in Table A.9, Table A.10, and Table A.11.
Figure 6.8 - Comparison of measured and predicted stress-strain response of AA3104. Measured stress-strain response is shown by discrete symbols and predicted responses are shown by solid curves.
Figure 6.9 - Comparison of measured and predicted stress-strain response of AA5182. Measured stress-strain response is shown by discrete symbols and predicted responses are shown by solid curves.
Figure 6.10 - Comparison of measured and predicted stress-strain response of AA6111. Measured stress-strain response is shown by discrete symbols and predicted responses are shown by solid curves.

Figure 6.8 through Figure 6.10 show that the extended Ludwik equation is able to approximate the experimental curves. These predictions have been made using the correlations of parameters and temperatures, which are not always a perfect representative of individual temperature. Definitely if predicted using a discrete parameter set for a given temperature this model gives far better results. Figure B.1 in Appendix B shows that the extended Ludwik equation is able to predict the flow stress for strain-rate jump tests also.
6.3.2 Comparison of measured and predicted yield stress using the equation

The extended Ludwik equation has been used for development of the correlations for yield stress. In case of AA3104 and AA5182, the yield stress was correlated to temperature using a linear regression up to certain temperature, because it is supposed to be dependent only on temperature and is not affected much by strain rate. Above this temperature, the yield stress was modeled using extended Ludwik equation. As shown by our calculations, AA6111 has significant strain rate sensitivity $m$ even below 200°C. For this reason the yield stress for AA6111 was modeled using extended Ludwik equation only. Figure 6.11 shows a comparison of the calculated and predicted yield stress by the extended Ludwik equation.

Figure 6.11 - Yield stress of (a) AA3104 (b) AA5182, and (c) AA6111 shown as a function of temperature, predicted by extended Ludwik equation and using the parameters-temperature correlations.
6.3.3 Comparison to literature data

In this section we compare the prediction by our parameters of extended Ludwik equation against the stress-strain response reported by Haaften and co-workers in [19].

![Stress-strain response comparison](image)

(a) AA3104

Figure 6.12 - Comparing stress-strain response of (a) AA3104 and (b) AA5182 in the current research work to that reported by Haaften et al. [19]. Discrete symbols show data from the work of Haaften et al. [19], and solid curves show predictions by our model parameters.
Figure 6.12 compares some of the stress-strain responses of AA3104 and AA5182 studied in this research work to that studied by Haaften et al. [19]. From Figure 6.12(a) and Figure 6.12(b) it appears that for both alloys the high temperature data matches reasonably well however the low temperature data is quite different. In particular it appears that the yield stresses are quite different for the two alloys. Differences in the low temperature yield stress are probably due to the location in the ingot where the specimens were taken. Wan et al. [67], in a study of AA7050 alloy, showed that yield strength, grain size and tensile strength all changed from the surface to the center of the ingot; with the yield stress being lower at the center due to the slower cooling rates. A similar study for AA3104, has been done by Haaften et al. in [19] which shows that yield strength decreases rapidly as distance increases from surface to center. Table 6.1
compares the low temperature yield stresses determined in this research work with that calculated in the [19].

Table 6.1 - Comparison of yield stress values for AA3104 and AA5182 at 100°C in this research to those reported by Haaften et al. [19]

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Yield stress (MPa) (this study)</th>
<th>Yield stress (MPa) (Haaften [19])</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA3104</td>
<td>~111</td>
<td>60</td>
<td>1.85</td>
</tr>
<tr>
<td>AA5182</td>
<td>~139</td>
<td>88</td>
<td>1.58</td>
</tr>
</tbody>
</table>

Table 6.1 shows that the low temperature yield stresses in this study are ~1.6 – 1.9 times higher than the ones reported by Haaften. Previous work [67] indicated that the variation in yield stress could be as much as ~1.5 times higher at close to the surface as compared to the center which appears to explain this discrepancy.

6.3.4 Continuous cooling compression tests

As mentioned in Chapter IV – Experimental, Continuous Cooling Tests (CCT) were performed for validation of the parameters of the extended Ludwik equation under more complex strain and thermal histories which more closely simulated the strain rates and temperature history trends as in DC casting process.

Figure 6.13 below shows the measured true strain in radial direction (LEII) and thermal histories during a typical continuous cooling test for AA3104. The figure shows the complete test which included: heating of the specimen at 5 °C.s⁻¹, a hold at the start
temperature for \( \sim 60 \) s then continuous cooling in air. The strain history shows that during the heat-up some small strain is experienced by the sample due to the expansion of the sample during heating. The strain increases slightly due to restricted expansion during the hold. As the applied deformation does not start immediately after the hold is completed the initial contraction in the sample can be seen followed by a leveling off as the contraction of the sample and applied deformation are almost equal which ends at \( \sim 238 \) s and finally an increase in the strain as the amount of deformation applied exceeds the contraction of the sample. Once the rate of the applied deformation exceeds the contraction rate of the specimen, after \( \sim 238 \) s, the specimen is actually expanding in the radial direction while the sample cools.

Figure 6.13 - Strain history in the radial (\textit{LE11}, true strain in radial direction in the specimen) direction and thermal history during one of the continuous cooling test of AA3104.
In order to model this test in ABAQUS, it was necessary to input the correct strain and thermal history experienced by the sample. As ABAQUS applied the boundary condition in the longitudinal direction, the measured strain history in the radial (11 direction) was converted to a strain in the longitudinal (22 direction). This was done by assuming constant volume during the applied mechanical deformation. The ABAQUS simulation was done just after the hold period was completed during the air cool of the sample. This point is indicated in Figure 6.13 as “start point for ABAQUS simulations”.

Figure 6.14 shows a comparison between the longitudinal strain predicted during the ABAQUS simulation as well as those calculated based on the measured results. A description of analytical expressions used to calculate the various strain components viz. elastic strain, thermal strain, plastic strain is described in Appendix C. As can been seen both the strain and thermal histories used in ABAQUS match what occurred in the experiment very well. Strain and thermal histories during the other continuous cooling tests are similar. Figure D.1 in Appendix D shows the evolution of strain rates during the continuous cooling tests.
Figure 6.14 - Comparison of the true strain in the longitudinal direction (LE22) and temperature variation during continuous cooling test with ABAQUS simulation.

Figure 6.15 shows a comparison between the measured stress evolution in the specimen as compared to that predicted using ABAQUS for each of the alloys studied.
Figure 6.15 - Comparison of model predictions against measurements of the stress evolution in the sample during continuous cooling tests for: a) AA3104 b) AA5182 and c) AA6111.
b) AA5182

(c) AA6111
As can be seen in Figure 6.15 the predictions using the extended Ludwik constitutive model are reasonably good as compared to the measurements during the continuous cooling tests. However, there appears to be a consistent trend for the model to over predict the stress in the sample. This over prediction appears to start in the region where work hardening of the material begins at lower temperatures. In the high temperature region where creep will occur the predictions are quite good. Figure D.2 in Appendix D shows the % difference between measurement and prediction during continuous cooling tests. For the AA3104, the deviation occurs at temperatures below 350 °C and reaches a maximum error ~ 25% at ~ 250 °C. For the AA5182 alloy, the model predictions are quite good in the high temperature regime but then the model starts to under predict the measured stress at temperatures below ~ 400 °C. The error in these temperature regions for the AA5182 is ~ 10%. For the AA6111 alloy, the model over prediction occurs at temperatures below ~ 425 °C. Below this temperature, the model consistently over-predicts the stress by ~ 20 %. For the AA3104 and AA5182 alloys the deviation between the measured and predicted results (~15%) occur at temperatures in the mixed mode region (temperature ~ 400 °C to 200 °C) where both strain rate sensitivity and work hardening play a role.

Figure 6.15 show that the extended Ludwik equation can predict complex thermomechanical histories in which strain and strain rates both vary with varying temperature. However, there are some consistent differences in the CCT tests between the model predictions and measurements. These differences occur for a variety of reasons some of which include:
(1) Effect of thermal history on metallurgical response of the material during a CCT test as compared to a constant temperature test and

(2) Recovery (no accumulation of plastic strain) during a CCT.

6.3.5 Recovery during continuous cooling tests

Consideration of recovery may be important during continuous cooling tests where the material behaviour is affected by the previous temperature and strain history.

The ABAQUS model calculates the strain experienced by the material based solely on geometrical considerations, hence metallurgical phenomena such as recovery and recrystallization will not be taken into account. It is important to account for metallurgical processes as they will significantly alter the accumulated strain in the sample and predicted stress. In order to determine the role recovery plays during these tests, it was necessary to first divide the strain experienced by the material into a “creep” component where recovery occurred and no plastic strain would be accumulated and a “non-creep” component where no recovery occurred and plastic strain would be accumulated.

The quantification of the creep strain was estimated based on the strain hardening parameter “n” at each temperature in the test as compared to the strain hardening at room temperature where no strain rate sensitivity or creep is observed for these alloys. In cases where n is zero the strain experienced by the sample was considered to be entirely creep and hence there would be no accumulation of plastic strain. As the material cooled, the amount of creep strain was calculated according to Equation 6.1 based on the n parameter at the temperature in comparison to n at room temperature ($n_{RT}$).
\( \varepsilon_{\text{creep}} = \varepsilon_{\text{plastic}} \left( 1 - \frac{n}{n_{RT}} \right) \)  

(6.1)

The % recovery can be calculated as shown in Equation 6.2 given by:

\[ \left( 1 - \frac{n}{n_{RT}} \right) \times 100 \]  

(6.2)

To explore the merit of using Equation 6.1 to account for recovery in the material some two step compression tests were performed in which the specimen was deformed at two different temperatures. These tests were designed so that the first deformation step took place at a relatively high temperature where substantial recovery would occur and were followed by deformation at a much lower temperature where no recovery and only work hardening would occur. Figure 6.16 shows the stress-strain response in two step tests Test 1, and Test 2, in which the specimen was first deformed at a relatively high temperature followed by deformation at 150 °C. As can be seen in Figure 6.16(a), the response of the material during deformation at 150 °C is not influenced by the prior thermomechanical history it experienced at 450 °C; the previous strain experienced by the material is completely recovered. In contrast, Figure 6.16(b) shows that deformation at 150 °C after deformation at 300 °C will have some accumulated strain from the first step as during deformation at 300 °C some small amount of work hardening occurs.

It is clear from Figure 6.16 that it is critical to include a recovery component into the constitutive behaviour law for materials which experience thermal histories that include high temperatures where recovery and recrystallization can occur. If the recovery is not included, than the constitutive models will tend to over predict the stresses in cases where the material is being cooled from high to low temperatures. It is worth mentioning that in
these tests we have used very high strains i.e. 0.15, and 0.125 relative to the strains experienced during DC casting (0.02-0.03).

Figure 6.16 - Comparison of predicted stress strain response as compared to measured during two-step deformation tests of AA5182 for: (a) Test 1 (1\textsuperscript{st} deformation at 450 °C and 2\textsuperscript{nd} deformation at 150 °C) (b) Test 2 (1\textsuperscript{st} deformation at 300 °C and 2\textsuperscript{nd} at 150 °C).
Figure 6.17 shows the impact including recovery would have on the continuous cooling test model validation experiments. As can be seen including recovery for these predictions has an effect in reducing the predicted stress such that in most cases it is much closer to what was actually measured. This figure indicates the importance of including this metallurgical phenomena into the constitutive behaviour models even under the low strains experienced during DC casting. The formulation of creep strain as employed for the continuous cooling tests, has been shown in Appendix D.
Figure 6.17 - Showing prediction of stress using recovery modeled as in Equation 6.1 during continuous cooling tests: (a) AA3104 (b) AA5182 (c) AA6111. The dotted lines show the predictions considering recovery.
Figure D.2 in Appendix D also shows the % difference between the measured and predicted stress evolution considering the recovery phenomena during continuous
cooling tests performed in this work. Figure D.3 in Appendix D shows the amount of % incremental recovery as a function of time and temperature during the continuous cooling tests for AA3104, AA5182, and AA6111.

6.3.6 Secondary creep model

As indicated in the literature review, a common technique used to model the constitutive behavior during DC casting are secondary creep based models [3, 11, 19]. To assess the importance of incorporating work hardening into the constitutive behavior model, a comparison of the measured stress evolution to the predicted using the Extended Ludwik equation which includes work hardening and using Garafalo's secondary creep equation was done. These results are shown in Figure 6.18. The constants required in the secondary creep equation for the alloys studied in this investigation were taken from the literature [6, 11]. Constants for these are equations are given in Table E.1 in Appendix E. For AA3104, constants determined for AA3103 were used.
Figure 6.18 - Predictions using a steady state creep based model. (a) AA3104 (b) AA5182
Figure 6.18 shows that the steady state creep law matches the measured data well at high temperatures but begins to under predict the measured stress response as the specimen cools and work hardening becomes more important.

6.4 Applicability of our model

The set of parameters that have been calculated for extended Ludwik equation has been determined for specimens taken close to the surface of the ingot and perpendicular to the casting direction. As shown in this chapter, the room temperature yield stress of the ingot can vary from the surface to the centre of the ingot as well as based on the sample
orientation. Although the model developed can be used at high temperatures, in order to predict accurate mechanical behavior throughout the ingot at low temperatures, it is necessary to include a spatial and orientation variable into the model.
Chapter VII - Summary and Conclusions

7.1 Conclusions

A critical input to modeling the thermomechanical behaviour during DC casting is the material constitutive behaviour in both the semi-solid and solid state. In this investigation, the constitutive behaviour of three commercially important aluminum alloys (AA3104, AA5182, and AA6111) in the solid state was investigated. A large part of the study consisted of measuring the stress-strain response of the material using the Gleeble 3500 thermomechanical simulator at UBC for a wide range of deformation temperatures (room temperature to 500 °C) and strain rates (1 x 10^{-1} s^{-1} to 1 x 10^{-5} s^{-1}). Based on these measurements, equation parameters $K$ (strength coefficient), $n$ (strain hardening index), and $m$ (strain rate sensitivity) were determined for the extended Ludwik equation. The parameters for each of the alloys exhibited similar trends with temperature; the $K$ parameter decreased as the temperature increased, the $n$ parameter stayed constant and then decreased until it eventually became zero and the $m$ parameter was zero initially and increased as a function of temperature. However the values and temperatures at which these transitions occurred varied slightly for each alloy studied.

Extensive validation was done for the extended Ludwik equation developed. This included comparisons of the model predictions to constant temperature uniaxial tests. In addition some cooling tests combined with mechanical deformation were conducted to more closely simulate the complex thermomechanical history experienced during DC casting.

It was determined that the extended Ludwik equation was able to approximate the material behaviour reasonably well however it tended to over predict the stress at
temperatures which included some work hardening for the complex cooling and deformation validation tests. To determine why this over-prediction was occurring, the effect of metallurgical phenomena such as recovery at high temperatures on the predicted stress was assessed.

The effect of recovery was modeled by comparing the amount of work hardening at each temperature to the work hardening that occurs at room temperature. This approach was validated by performing some two step deformation steps in which the first step was done at a relatively high temperature where some amount of recovery would occur. The second deformation step was then done at a temperature where no recovery is expected to occur. It was found that recovery can play an important role in the stress prediction and its importance increases as the amount of deformation at high temperatures increases. It was found that recovery did significantly lower the stress predictions during the complex cooling and deformation validation tests.

Our main conclusions from this study are as follows:

1. The extended Ludwik equation is able to reproduce reasonably well the material behavior under both simple constant temperature compression tests as well as complex cooling in combination with deformation tests.

2. The constitutive behaviour model developed predicts the high temperature constitutive behavior for AA5182 and AA3104 for other investigations found in the literature. However there is some deviation at the low temperatures. The reason for this is most probably related to the location in the ingot where the samples were taken.
3. The worst fit for the model in the complex cooling conditions is for the AA6111 alloy where the model begins to over predict the stress at temperatures as high as 450 °C. This is most probably due to the complex chemistry in this alloy as well as the complex precipitation reactions which occur.

4. An empirical approach to model recovery during complex thermomechanical histories has been used which estimates the amount of recovery as function of temperature based on a comparison of the work hardening rate at the test temperature as compared to the work hardening rate at room temperature. This approach appears reasonable when compared to experimental data and indicates that recovery does appear to play a significant role during DC casting.

Constitutive behaviour models need to incorporate both ingot location and orientation for low temperature behaviour as there is a large difference in the yield stress from the surface of the ingot to the centre and for different orientations. The effect of orientation was most pronounced for the AA6111 alloy. In AA6111 some other factors may add to that anisotropy of mechanical behaviour e.g. shape of the precipitates, preferred orientation of the precipitates with respect to the casting direction.

7.2 Future Work and Suggestions

Based on our research work we recommend the following future work:

1. Further investigation on the effect of sample orientation on constitutive behaviour should be done, especially for the AA6111 alloy.
2. The continuous cooling tests we have used are good to validate the model under complex thermomechanical histories, but they are not true representative of the DC casting thermomechanical conditions. The reason addressed here is related to the strain rate change during the continuous cooling tests. It is noteworthy that during a real DC casting process, at the rolling face, strain rate is higher in the beginning of the casting process and it decreases towards the end of the process. In the continuous cooling tests we have performed, the strain rate is low in the beginning of the test and it increases towards the end of the process. We recommend validating our model using a continuous cooling test of tensile specimen with no externally applied deformation.

3. Although the mechanical threshold model described by Kocks and Mecking is primarily used to model stage III work hardening, it may be worthwhile to attempt to use it to model the constitutive behaviour as it needs only temperature and strain rate values for prediction once all the material parameters are determined. The main advantage of this model is that parameters for different temperatures are not calculated individually, and the total number of parameters are less than any other available physically based constitutive model.
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APPENDIX A

A.1 Test matrices for uniaxial compression tests, showing modulus of elasticity \((E)\), yield stress \((YS)\), and parameters \(K\), \(n\), and \(m\) determined for various temperatures.

Table A.1 - Test matrix for parallel specimens of AA3104

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Temperature (°C)</th>
<th>Modulus of Elasticity (E) (GPa)</th>
<th>Yield Stress (Y_S) (MPa)</th>
<th>Strain Rate / s</th>
<th>(K) (MPa)</th>
<th>(n)</th>
<th>(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>70.41</td>
<td>137</td>
<td>9.0E-04</td>
<td>295</td>
<td>0.125</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>70.41</td>
<td>139</td>
<td>1.0E-03</td>
<td>318</td>
<td>0.133</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>65.16</td>
<td>149</td>
<td>9.0E-04</td>
<td>268</td>
<td>0.099</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>62.84</td>
<td>124</td>
<td>9.5E-03</td>
<td>277</td>
<td>0.137</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>250</td>
<td>60.39</td>
<td>130</td>
<td>1.0E-03</td>
<td>183</td>
<td>0.060</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>300</td>
<td>57.82</td>
<td>103</td>
<td>1.0E-02</td>
<td>179</td>
<td>0.060</td>
<td>0.028</td>
</tr>
<tr>
<td>7</td>
<td>300</td>
<td>57.82</td>
<td>102</td>
<td>1.1E-03</td>
<td>118</td>
<td>0.011</td>
<td>0.006</td>
</tr>
<tr>
<td>8</td>
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<td>52.29</td>
<td>112</td>
<td>1.1E-03</td>
<td>122</td>
<td>0</td>
<td>0.010</td>
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</tbody>
</table>
Table A.2 - Test matrix for perpendicular specimens of AA3104

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Temperature (°C)</th>
<th>Modulus of Elasticity $E$ (GPa)</th>
<th>Yield Stress (MPa)</th>
<th>Strain Rate / s</th>
<th>$K$ (MPa)</th>
<th>$n$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>70.41</td>
<td>124</td>
<td>9.0E-04</td>
<td>341</td>
<td>0.179</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>67.36</td>
<td>111</td>
<td>9.0E-04</td>
<td>319</td>
<td>0.178</td>
<td>0</td>
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<tr>
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<td>150</td>
<td>65.16</td>
<td>125</td>
<td>9.3E-03</td>
<td>282</td>
<td>0.140</td>
<td>0.002</td>
</tr>
<tr>
<td>4</td>
<td>175</td>
<td>64.02</td>
<td>112</td>
<td>9.0E-04</td>
<td>315</td>
<td>0.176</td>
<td>0</td>
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<tr>
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<td>200</td>
<td>62.84</td>
<td>124</td>
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<td>200</td>
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<td>6.0E-04</td>
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<td>7</td>
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<td>61.63</td>
<td>101</td>
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<td>8</td>
<td>250</td>
<td>60.39</td>
<td>100</td>
<td>9.0E-03, 9.0E-4</td>
<td>231</td>
<td>0.113</td>
<td>0.029</td>
</tr>
<tr>
<td>9</td>
<td>300</td>
<td>57.82</td>
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<td>8.0E-06, 8.0E-4</td>
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<td>0.105</td>
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<tr>
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<tr>
<td>Test No.</td>
<td>Temperature (°C)</td>
<td>Modulus of Elasticity $E$ (GPa)</td>
<td>Yield Stress (MPa)</td>
<td>Strain Rate / s</td>
<td>$K$ (MPa)</td>
<td>$n$</td>
<td>$m$</td>
</tr>
<tr>
<td>---------</td>
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<td>-----------</td>
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</tr>
<tr>
<td>13</td>
<td>400</td>
<td>52.29</td>
<td>54</td>
<td>8.3E-03, 9.0E-04</td>
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<td>0.023</td>
<td>0.123</td>
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<td>14</td>
<td>400</td>
<td>52.29</td>
<td>39</td>
<td>1.1E-03</td>
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<td>0.022</td>
<td>0.027</td>
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<td>52.29</td>
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<td>16</td>
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<td>9.9E-03, 2.0E-04</td>
<td>81</td>
<td>0</td>
<td>0.144</td>
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<td>28</td>
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<td>78</td>
<td>0.023</td>
<td>0.126</td>
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Table A.3 - Test matrix for parallel specimens of AA5182

<table>
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<tr>
<th>Test No.</th>
<th>Temperature (°C)</th>
<th>Modulus of Elasticity $E$ (GPa)</th>
<th>Yield Stress (MPa)</th>
<th>Strain Rate / s</th>
<th>$K$ (MPa)</th>
<th>$n$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>70.41</td>
<td>104</td>
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<td>150</td>
<td>65.16</td>
<td>126</td>
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<td>297</td>
<td>0.148</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>63.95</td>
<td>108</td>
<td>8.4E-03, 1.0E-03</td>
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<td>0.139</td>
<td>0.032</td>
</tr>
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<td>250</td>
<td>61.61</td>
<td>104</td>
<td>1.0E-03</td>
<td>212</td>
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<td>0.002</td>
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<td>59.09</td>
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<tr>
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<td>300</td>
<td>59.09</td>
<td>103</td>
<td>1.0E-03</td>
<td>198</td>
<td>0.031</td>
<td>0.064</td>
</tr>
<tr>
<td>7</td>
<td>400</td>
<td>53.54</td>
<td>41</td>
<td>1.0E-03</td>
<td>42</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Test No.</td>
<td>Temperature (°C)</td>
<td>Modulus of Elasticity $E$ (GPa)</td>
<td>Yield Stress (MPa)</td>
<td>Strain Rate / s</td>
<td>$K$ (Mpa)</td>
<td>$n$</td>
<td>$m$</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------</td>
<td>-------------------------------</td>
<td>------------------</td>
<td>-----------------</td>
<td>-----------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>1</td>
<td>25</td>
<td>70.79</td>
<td>144</td>
<td>6.0E-04</td>
<td>359</td>
<td>0.162</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>70.79</td>
<td>145</td>
<td>8.0E-04</td>
<td>371</td>
<td>0.162</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>68.11</td>
<td>138</td>
<td>1.0E-03</td>
<td>307</td>
<td>0.136</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>150</td>
<td>66.12</td>
<td>101</td>
<td>7.2E-03, 8.0E-04</td>
<td>353</td>
<td>0.200</td>
<td>0.001</td>
</tr>
<tr>
<td>5</td>
<td>150</td>
<td>66.12</td>
<td>119</td>
<td>1.2E-03</td>
<td>289</td>
<td>0.131</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>200</td>
<td>63.95</td>
<td>129</td>
<td>5.7E-03</td>
<td>281</td>
<td>0.116</td>
<td>0.012</td>
</tr>
<tr>
<td>7</td>
<td>200</td>
<td>63.95</td>
<td>134</td>
<td>1.2E-03</td>
<td>229</td>
<td>0.087</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>200</td>
<td>63.95</td>
<td>125</td>
<td>7.0E-05</td>
<td>224</td>
<td>0.093</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>225</td>
<td>62.8</td>
<td>124</td>
<td>8.2E-03</td>
<td>302</td>
<td>0.127</td>
<td>0.023</td>
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<tr>
<td>10</td>
<td>250</td>
<td>61.61</td>
<td>125</td>
<td>5.0E-04</td>
<td>291</td>
<td>0.075</td>
<td>0.051</td>
</tr>
<tr>
<td>11</td>
<td>325</td>
<td>57.77</td>
<td>89</td>
<td>8.0E-04, 1.0E-04</td>
<td>252</td>
<td>0</td>
<td>0.138</td>
</tr>
<tr>
<td>12</td>
<td>350</td>
<td>56.4</td>
<td>91</td>
<td>9.9E-03, 1.1E-03</td>
<td>245</td>
<td>0.036</td>
<td>0.161</td>
</tr>
<tr>
<td>13</td>
<td>400</td>
<td>53.54</td>
<td>71</td>
<td>7.2E-03, 9.0E-04</td>
<td>180</td>
<td>0.002</td>
<td>0.187</td>
</tr>
<tr>
<td>14</td>
<td>425</td>
<td>52.04</td>
<td>32</td>
<td>1.1E-03, 1.0E-04</td>
<td>122</td>
<td>0</td>
<td>0.187</td>
</tr>
<tr>
<td>Test No.</td>
<td>Temperature (°C)</td>
<td>Modulus of Elasticity $E$ (GPa)</td>
<td>Yield Stress (MPa)</td>
<td>Strain Rate / s</td>
<td>$K$ (Mpa)</td>
<td>$n$</td>
<td>$m$</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------</td>
<td>-------------------------------</td>
<td>-------------------</td>
<td>----------------</td>
<td>----------</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>15</td>
<td>425</td>
<td>52.04</td>
<td>37</td>
<td>1.4E-03, 1.0E-04</td>
<td>133</td>
<td>0</td>
<td>0.197</td>
</tr>
<tr>
<td>16</td>
<td>450</td>
<td>50.5</td>
<td>47</td>
<td>1.0E-02, 9.0E-04</td>
<td>116</td>
<td>0</td>
<td>0.198</td>
</tr>
<tr>
<td>17</td>
<td>450</td>
<td>50.5</td>
<td>31</td>
<td>9.0E-04, 9.9E-03</td>
<td>120</td>
<td>0</td>
<td>0.193</td>
</tr>
<tr>
<td>18</td>
<td>500</td>
<td>47.3</td>
<td>39</td>
<td>1.1E-01, 1.0E-02, 0.001.1E-03</td>
<td>60</td>
<td>0</td>
<td>0.179</td>
</tr>
<tr>
<td>19</td>
<td>500</td>
<td>47.3</td>
<td>27</td>
<td>1.1E-02, 1.1E-03</td>
<td>71</td>
<td>0</td>
<td>0.213</td>
</tr>
<tr>
<td>20</td>
<td>500</td>
<td>47.3</td>
<td>21</td>
<td>9.0E-04, 9.8E-03</td>
<td>72</td>
<td>0</td>
<td>0.176</td>
</tr>
<tr>
<td>21</td>
<td>500</td>
<td>47.3</td>
<td>13</td>
<td>7.0E-06, 1.0E-03</td>
<td>46</td>
<td>0</td>
<td>0.090</td>
</tr>
</tbody>
</table>
Table A.5 - Test matrix for perpendicular specimens AA5182 of 11 % bigger size

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Temperature (°C)</th>
<th>Modulus of Elasticity $E$ (GPa)</th>
<th>Yield Stress (MPa)</th>
<th>Strain Rate / s</th>
<th>$K^*$</th>
<th>$n^*$</th>
<th>$M^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150</td>
<td>66.1</td>
<td>120</td>
<td>9.0E-04</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>450</td>
<td>50.5</td>
<td>29</td>
<td>9.0E-04</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

* For these tests parameters were not calculated.

Table A.6 - Test matrix for parallel specimens of AA6111

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Temperature (°C)</th>
<th>Modulus of Elasticity $E$ (GPa)</th>
<th>Yield Stress (MPa)</th>
<th>Strain Rate / s</th>
<th>$K$</th>
<th>$n$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>67.36</td>
<td>166</td>
<td>9.0E-05</td>
<td>347</td>
<td>0.117</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>150</td>
<td>65.16</td>
<td>167</td>
<td>1.1E-02, 1.0E-03</td>
<td>306</td>
<td>0.079</td>
<td>0.009</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>62.84</td>
<td>160</td>
<td>1.0E-02</td>
<td>305</td>
<td>0.072</td>
<td>0.033</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>62.84</td>
<td>152</td>
<td>1.0E-02, 1.0E-03</td>
<td>351</td>
<td>0.112</td>
<td>0.025</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
<td>57.82</td>
<td>103</td>
<td>1.0E-02</td>
<td>188</td>
<td>0.058</td>
<td>0.041</td>
</tr>
</tbody>
</table>
Table A.7 - Test matrix for perpendicular specimens of AA6111

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Temperature ($^\circ$C)</th>
<th>Modulus of Elasticity $E$ (GPa)</th>
<th>Yield Stress (MPa)</th>
<th>Strain Rate /s</th>
<th>$K$ (MPa)</th>
<th>$n$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>70.41</td>
<td>140</td>
<td>9.0E-04</td>
<td>368</td>
<td>0.158</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>67.36</td>
<td>118</td>
<td>3.0E-04, 6.0E-05</td>
<td>358</td>
<td>0.148</td>
<td>0.023</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>65.16</td>
<td>109</td>
<td>9.0E-04</td>
<td>382</td>
<td>0.181</td>
<td>0.024</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>62.84</td>
<td>98</td>
<td>8.0E-04</td>
<td>340</td>
<td>0.172</td>
<td>0.028</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>62.84</td>
<td>113</td>
<td>7.0E-04</td>
<td>312</td>
<td>0.125</td>
<td>0.033</td>
</tr>
<tr>
<td>Test No.</td>
<td>Temperature (°C)</td>
<td>Modulus of Elasticity $E$ (GPa)</td>
<td>Yield Stress (MPa)</td>
<td>Strain Rate / s</td>
<td>$K$ (Mpa)</td>
<td>$n$</td>
<td>$m$</td>
</tr>
<tr>
<td>---------</td>
<td>------------------</td>
<td>---------------------------------</td>
<td>--------------------</td>
<td>----------------</td>
<td>------------</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>6</td>
<td>250</td>
<td>60.39</td>
<td>125</td>
<td>9.0E-03, 1.0E-03</td>
<td>229</td>
<td>0.068</td>
<td>0.036</td>
</tr>
<tr>
<td>7</td>
<td>250</td>
<td>60.39</td>
<td>85</td>
<td>1.0E-03, 1.0E-02</td>
<td>251</td>
<td>0.130</td>
<td>0.042</td>
</tr>
<tr>
<td>8</td>
<td>300</td>
<td>57.82</td>
<td>71</td>
<td>1.0E-03, 1.0E-04</td>
<td>176</td>
<td>0.055</td>
<td>0.080</td>
</tr>
<tr>
<td>9</td>
<td>300</td>
<td>57.82</td>
<td>56</td>
<td>7.0E-06</td>
<td>111</td>
<td>0.039</td>
<td>0.037</td>
</tr>
<tr>
<td>10</td>
<td>350</td>
<td>55.11</td>
<td>62</td>
<td>1.0E-01, 1.0E-02, 1.0E-03</td>
<td>18</td>
<td>0</td>
<td>0.085</td>
</tr>
<tr>
<td>11</td>
<td>450</td>
<td>49.33</td>
<td>31</td>
<td>1.1E-02, 1.0E-03</td>
<td>52</td>
<td>0</td>
<td>0.115</td>
</tr>
<tr>
<td>12</td>
<td>450</td>
<td>49.33</td>
<td>22</td>
<td>1.1E-03, 1.0E-02</td>
<td>47</td>
<td>0</td>
<td>0.102</td>
</tr>
<tr>
<td>13</td>
<td>500</td>
<td>46.25</td>
<td>17</td>
<td>1.1E-03</td>
<td>36</td>
<td>0</td>
<td>0.110</td>
</tr>
<tr>
<td>14</td>
<td>500</td>
<td>46.25</td>
<td>17</td>
<td>1.0E-01, 1.0E-02, 1.0E-03</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>
A.2 An example of typical error in the experiments

![Graph comparing Test No. 1 and Test No. 2 in Table A.1.1 to show repeatability (~ 5 %)]

Figure A.1 - Comparing Test No. 1 and Test No. 2 in Table A.1.1 to show repeatability (~ 5 %)

A.3 Effect of specimen size

As mentioned in experimental section Chapter 4, due to the large grain size of the as-cast material, the effect of specimen size on the measured stress-strain response was assessed. This was done to ensure that a larger specimen size did not significantly alter the measured stress-strain curve. Table A.8 shows a comparison of the estimated number of grains in the two specimen sizes of AA5182 tested. This calculation was done assuming cubic grains with edge size equal to average grain size ~ 200 \( \mu \)m. Figure A.2
shows a comparison of the stress-strain response for the two sizes of test specimens under the same test conditions.

Table A.8 - Estimated numbers of grains in the cross-section of the two specimens based on the measured average grain size.

<table>
<thead>
<tr>
<th>Specimen Size</th>
<th>Number of estimated grains (based on measured grain size: ~ 200 μm, and assuming every grain to be a cube of edge size ~ 200 μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard (10 mm OD)</td>
<td>~ 1963</td>
</tr>
<tr>
<td>Large (11.1 mm OD)</td>
<td>~ 2422 (23 % increase)</td>
</tr>
</tbody>
</table>

It is clear from the stress-strain response that there is no identifiable effect of specimen size on the stress-strain response. The difference is within the repeatability limit of the experiments. For tests details see Test no. 5 and 17 in Table A.4, and Test No. 1 and 2 in Table A.5 in Appendix A.
Figure A.2 - Comparison of stress-strain response between standard and larger compression test specimens. Strain rate is \( \sim 1 \times 10^{-03} \) s\(^{-1}\). Discrete symbols show data for standard size specimen, and continuous curves show data for larger size specimen.

A.4 Effect of specimen orientation in the ingot

As discussed in the experimental section, Chapter 4, the effect of test sample orientation on the measured results was determined. To do this, specimens were taken parallel and perpendicular to the casting direction as well and tested. These results are shown in Figure A.3. It is clear from Figure A.3 that there are some differences in the stress-strain response of parallel and perpendicular specimens. In particular, the yield stress of the samples taken parallel to the casting direction is consistently higher than
those taken perpendicular to the casting direction for all the alloys studied. This difference is especially acute for the AA6111 material and definitely warrants further study.

Typical experimental variation was \(\sim 5\%\) sample to sample for the same test conditions. An example of the typical error in one condition is given in Figure A.1 in Appendix A. The difference in Parallel vs. Perpendicular comparisons is more than 10\% hence it is probably a "real" difference and is worth pursuing further.

The reason behind this difference is believed to be due to some retained strain in this direction. Another study [68] noted a 17\% elongation in the grain structure in this direction. As a result of these differences, the majority of the tests measured were taken in the perpendicular direction and used to develop the constitutive behaviour model. The measured test results for the parallel specimens are given in Table A.1, Table A.3, and Table A.6.
Figure A.3 - Comparison of the stress-strain response of test specimens taken parallel and perpendicular to the casting direction showing: (a) AA3104 (25°C, strain rate ~ $1 \times 10^{-3}$ s$^{-1}$) (b) AA5182 (150°C, strain rate ~ $1 \times 10^{-3}$ s$^{-1}$) (c) AA6111 (100 °C, strain rate ~ $1 \times 10^{-4}$ s$^{-1}$)
(b) AA5182

(e) AA6111
A.5 Correlations for parameters $K$, $n$, and $m$ determined for alloys AA3104, AA5182, and AA6111

Table A.9 - Parameter-temperature correlations for AA3104

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Temperature Range (°C)</th>
<th>Temperature – parameter correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>$25 \leq T \leq 144$</td>
<td>$K = -0.2705 \times T + 344.82$</td>
</tr>
<tr>
<td></td>
<td>$144 &lt; T \leq 500$</td>
<td>$K = -0.7339 \times T + 411.64$</td>
</tr>
<tr>
<td>$n$</td>
<td>$25 \leq T \leq 148$</td>
<td>$n = -0.00002 \times T + 0.179$</td>
</tr>
<tr>
<td></td>
<td>$148 &lt; T \leq 443$</td>
<td>$n = -0.0006 \times T + 0.266$</td>
</tr>
<tr>
<td></td>
<td>$443 &lt; T \leq 500$</td>
<td>$n = 0$</td>
</tr>
<tr>
<td>$m$</td>
<td>$25 \leq T \leq 196$</td>
<td>$m = 0$</td>
</tr>
<tr>
<td></td>
<td>$196 &lt; T \leq 450$</td>
<td>$m = 0.0006 \times T - 0.117$</td>
</tr>
<tr>
<td></td>
<td>$450 &lt; T \leq 500$</td>
<td>$m = 0.153$</td>
</tr>
<tr>
<td>Yield Stress</td>
<td>$25 &lt; T \leq 194$</td>
<td>$\sigma_{YS} = -0.0385 \times T + 122.57$</td>
</tr>
<tr>
<td></td>
<td>$194 &lt; T \leq 500$</td>
<td>$\sigma_{YS} = K \times 0.002^n \times \dot{\varepsilon}_p^m$</td>
</tr>
</tbody>
</table>

Table A.10 - Parameter-temperature correlations for AA5182

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Temperature Range (°C)</th>
<th>Temperature – parameter correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$ (MPa)</td>
<td>$25 \leq T &lt; 331$</td>
<td>$K = -0.3409 \times T + 361.83$</td>
</tr>
<tr>
<td></td>
<td>$331 \leq T \leq 500$</td>
<td>$K = -1.1015 \times T + 613.59$</td>
</tr>
<tr>
<td>$n$</td>
<td>$25 \leq T &lt; 206$</td>
<td>$n = -0.0003 \times T + 0.170$</td>
</tr>
<tr>
<td></td>
<td>$206 \leq T &lt; 361$</td>
<td>$n = -0.0007 \times T + 0.252$</td>
</tr>
<tr>
<td></td>
<td>$361 \leq T \leq 500$</td>
<td>$n = 0$</td>
</tr>
<tr>
<td>$m$</td>
<td>$25 \leq T &lt; 183$</td>
<td>$m = 0$</td>
</tr>
<tr>
<td></td>
<td>$183 \leq T &lt; 361$</td>
<td>$m = 0.001 \times T - 0.183$</td>
</tr>
<tr>
<td></td>
<td>$361 \leq T \leq 500$</td>
<td>$m = 0.0003 \times T + 0.069$</td>
</tr>
<tr>
<td>Yield Stress (YS)</td>
<td>$25 &lt; T \leq 183$</td>
<td>$\sigma_{YS} = -0.0862 \times T + 145.65$</td>
</tr>
<tr>
<td></td>
<td>$183 &lt; T \leq 500$</td>
<td>$\sigma_{YS} = K \times 0.002^n \times \dot{\varepsilon}_p^m$</td>
</tr>
</tbody>
</table>
Table A.11 - Parameter-temperature correlations for AA6111

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Temperature Range (°C)</th>
<th>Temperature – parameter correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$ (MPa)</td>
<td>25 &lt; $T$ ≤ 186</td>
<td>$K = -0.2228 \times T + 381.92$</td>
</tr>
<tr>
<td></td>
<td>186 &lt; $T$ ≤ 365</td>
<td>$K = -1.6281 \times T + 642.65$</td>
</tr>
<tr>
<td></td>
<td>365 &lt; $T$ ≤ 500</td>
<td>$K = -0.0552 \times T + 68.73$</td>
</tr>
<tr>
<td>$n$</td>
<td>25 &lt; $T$ ≤ 208</td>
<td>$n = -0.0002 \times T + 0.164$</td>
</tr>
<tr>
<td></td>
<td>208 &lt; $T$ ≤ 344</td>
<td>$n = -0.0009 \times T + 0.309$</td>
</tr>
<tr>
<td></td>
<td>344 &lt; $T$ ≤ 500</td>
<td>$n = 0$</td>
</tr>
<tr>
<td>$m$</td>
<td>25 ≤ $T$ ≤ 32</td>
<td>$m = 0$</td>
</tr>
<tr>
<td></td>
<td>32 &lt; $T$ ≤ 500</td>
<td>$m = 0.0003 \times T - 0.009$</td>
</tr>
<tr>
<td>Yield Stress (YS)</td>
<td>25 ≤ $T$ ≤ 500</td>
<td>$\sigma_{YS} = K \times 0.002^n \times \dot{\varepsilon}_p^m$</td>
</tr>
</tbody>
</table>
B.1 Comparison of measured and predicted stress-strain responses for strain rate jump tests.

Figure B.1 - Comparison of predicted and measured stress-strain response during strain rate jump tests (a) AA3104 (b) AA5182 (c) AA6111
APPENDIX C

C.1 Calculation of different strain components during a continuous cooling compression test

LE11: Logarithmic strain component in 11 (X) direction. See Figure C.1 below for various directions.
EE11: Elastic strain component in 11 direction
PE11: Plastic strain component in 11 direction
THE11: Thermal strain component in 11 direction
NE22: Nominal strain component in 22 (Y) direction
EE22: Elastic strain component in 22 direction
PE22: Plastic strain component in 22 direction
THE22: Thermal strain component in 22 direction
PEEQ: Equivalent strain (Von-Mises equivalent of plastic strain in uniaxial compression)

Table C.1 - Formulation of various strain components.

<table>
<thead>
<tr>
<th>Strain Component</th>
<th>Formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>LE11</td>
<td>( \ln \left( \frac{AD}{D_{at}} + 1 \right) )</td>
</tr>
<tr>
<td>THE11 (Logarithmic)</td>
<td>( \alpha' \Delta T )</td>
</tr>
<tr>
<td>EE11</td>
<td>( -\nu \frac{\sigma_{zz}}{E} )</td>
</tr>
<tr>
<td>PE11</td>
<td>= LE11-THE11-EE11</td>
</tr>
</tbody>
</table>
\begin{align*}
\text{THE22} & \quad \text{THE11} \\
\text{EE22} & \quad \frac{\sigma_{22}}{E} \\
\text{PE22} & \quad = -2 \ \text{PE11} \\
\text{PEEQ} & \quad = \text{PE22} \ (\text{Uniaxial compression}) \\
\text{LE22} & \quad = \text{PE22} + \frac{\sigma_{22}}{E} + \text{THE22}
\end{align*}

Length at time \( t \): \( L(t) \)

\[ L(t) = L_0 \times \exp(LE22) \]

Figure C.1 - Different directions considered for formulation of strain components
APPENDIX D

D.1 Variation of strain rate as a function of time and temperature during continuous cooling compression tests

![Graph showing variation of strain rate as a function of time and temperature during continuous cooling compression tests.](image)

(a) AA3104

Figure D.1 - Variation of strain rate as a function of time and temperature during continuous cooling compression tests. (a) AA3104 (b) AA5182 (c) AA6111
(b) AA5182

(c) AA6111
D.2 % Difference between measured and predicted stress during continuous cooling tests

Figure D.2 - % Difference between measured and predicted stress during continuous cooling tests of the three alloys. (a) AA3104 (b) AA5182 (c) AA6111
(d) AA5182

(e) AA6111
D.3 Formulation of creep strain as employed for the continuous cooling tests.

The formulation of the amount of “creep strain” as given in Chapter VI is applicable for each time step (temperature step). We assume that in our continuous cooling tests, for every time increment the temperature remains constant. So the incremental creep strain can be written as shown below in Equation D.1.

\[
\Delta \varepsilon'_{\text{creep}} = \Delta \varepsilon'_{\text{plastic}} \left( 1 - \frac{n}{n_{RT}} \right)
\]  

(D.1)

\(\Delta \varepsilon'_{\text{creep}}\) is the creep strain for \(\Delta \varepsilon'_{\text{plastic}}\) increment of plastic strain during current (t s) time step. \(n\) and \(n_{RT}\) are the strain hardening parameters at current temperature and at room temperature respectively. Incremental plastic strain \(\Delta \varepsilon'_{\text{plastic}}\) is calculated as given in Equation D.2.

\[
\Delta \varepsilon'_{\text{plastic}} = \varepsilon'_{\text{plastic}} - \varepsilon'_{\text{plastic}}^{t-1}
\]  

(D.2)

\(\varepsilon'_{\text{plastic}}\) is the total accumulated plastic strain up to current time step, and \(\varepsilon'_{\text{plastic}}^{t-1}\) is total accumulated plastic strain up to previous time step. Therefore the total accumulated plastic strain will be given by:

\[
\varepsilon'_{\text{creep}} = \sum_{i=0}^{t=N} \Delta \varepsilon'_{\text{creep}}
\]  

(D.3)

Where \(\varepsilon'_{\text{creep}}\) is the total accumulated creep strain up to current time step. Based on Equation D.3, plastic strain considering recovery can be calculated as follows:

\[
\varepsilon'_{\text{plastic}} = \varepsilon'_{\text{plastic}} - \varepsilon'_{\text{creep}}
\]  

(D.4)
D.4 Amount of % recovery at each temperature calculated using Equation 6.2 in Chapter VI

Figure D.3 - Amount of % recovery at each temperature calculated using Equation 6.2 in Chapter VI for AA3104 (a) AA3104 (b) AA5182 (c) AA6111
(b) AA5182

(c) AA6111
E.1 Material parameters used for creep law used in Chapter VI

Table E.1 - Material parameters used for creep law used in Chapter VI. (a) AA3103, (b) AA5182

(a) AA3103 Ref. [11]

\[ \dot{\varepsilon} = A \left[ \sinh \left( \frac{\sigma_s}{\sigma_o} \right) \right]^{\nu_o} \exp \left( \frac{-Q}{RT} \right) \]

<table>
<thead>
<tr>
<th>Alloy</th>
<th>( A )</th>
<th>( Q/R )</th>
<th>( \sigma_o )</th>
<th>( n_H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA3103</td>
<td>1.33 x 10^{16} s^{-1}</td>
<td>29012 K</td>
<td>31.4 MPa</td>
<td>7.94</td>
</tr>
</tbody>
</table>

(b) AA5182 Ref. [6]

\[ \dot{\varepsilon} = A \left[ \sinh (a\sigma_s) \right]^{\nu_H} \exp \left( \frac{-Q}{RT} \right) \]

<table>
<thead>
<tr>
<th>Alloy</th>
<th>( A )</th>
<th>( Q )</th>
<th>( R )</th>
<th>( \alpha )</th>
<th>( n_H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA5182</td>
<td>2.91 x 10^{11} s^{-1}</td>
<td>126000</td>
<td>8.314</td>
<td>0.005</td>
<td>6.74</td>
</tr>
</tbody>
</table>

J.mole^{-1} J.mole^{-1}.K (MPa^{-1})