MEASUREMENTS OF THE EFFECTIVE THERMAL CONDUCTIVITIES OF GRANULAR
SOLIDS AND OXIDE MELTS UNDER SHEAR STRAIN BY A NEW TECHNIQUE

by

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Heat transfer is fundamental to pyrometallurgy processing and the viability of new or existing processes often hinges on achieving high rates of heat transfer into the material being processed. Slags are a feature of extractive and refining processes and heat transfer through liquid slag is an important part of many operations, for example utilizing energy from port-combination in the EAF for scrap melting. However, relatively little work has been directed at characterizing heat transfer through non-stagnant slag layers. Heat transfer through the slag includes conduction, radiation and, with stirring, advection due to flow within the slag. The combined effect of all three mechanisms is convection but it is often more convenient to combine all three as an effective thermal conductivity.

A review of methods currently employed to measure the conductivity of stagnant oxide melts, which involve numerical solution of the transient conduction problem for interpretation of the data, did not indicate that any method was readily adaptable to stirred systems nor were they broadly applicable to a wider range of materials such as liquid metals or granular solids. Based on these factors the objectives of the work were to develop a broadly applicable steady-state method for directly measuring the effective thermal conductivity of a variety of materials ranging from stationary granular solids to liquid oxides and metals (with or without stirring) for temperatures covering the spectrum encountered in extractive metallurgy, and to use the technique to determine the extent to which the effective thermal conductivity of liquid oxides might be increased by flow-induced stirring. The methodology covers a range of issues related to the design of the apparatus, interpretation of the raw data and general validation of the technique.
For stagnant systems the technique was validated at low temperature against existing conductivity data for both water and silicon oil and good agreement was obtained. For intermediate temperatures trials were also carried out using several granular materials at temperatures up to about 800°C and results were shown to be within the range indicated by some common models for predicting conductivity of packed beds. As expected, the measured conductivity of packed beds increases with temperature due to the radiative contribution to heat transfer. For the limited range of sites tested no clear link between particle site and conductivity was observed. High temperature validation was obtained against existing data for oxide melts (40%CaO – 40%SiO$_2$ – 20%Al$_2$O$_3$) and again good agreement was shown.

For non-stagnant systems, of the liquids tested, water, silicon oil and oxide melts, only the former showed the rotation-induced flow to have any significant effect on effective thermal conductivity. This was explained by the calculated peripheral Reynolds and Taylor numbers for the systems that indicted laminar flow for oil and oxide melts and turbulent flow for water. For water with turbulent flow and at shear rates up to 0.55 sec$^{-1}$, the effective conductivity increased by a factor up to ~2.7 which is well short of the order of magnitude increase deemed desirable for heat transfer through the slag layer in the rotary scrap-melting furnace. For silicon oil with paddle mixers and at shear rates up to 0.55 sec$^{-1}$, the effective conductivity showed only a small increase.
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<th>Description</th>
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<tbody>
<tr>
<td>a</td>
<td>mathematical coefficient; locally defined</td>
</tr>
<tr>
<td>$\text{Area}_{\text{annular}}$</td>
<td>annular area of the annulus gap between concentric cylinders; m$^2$</td>
</tr>
<tr>
<td>b</td>
<td>mathematical coefficient; locally defined</td>
</tr>
<tr>
<td>B</td>
<td>lime/silica ratios; (-)</td>
</tr>
<tr>
<td>c</td>
<td>mathematical coefficient; locally defined</td>
</tr>
<tr>
<td>$C_p$</td>
<td>specific heat at constant pressure; J/kg.K</td>
</tr>
<tr>
<td>$C_v$</td>
<td>specific heat at constant volume; J/kg.K</td>
</tr>
<tr>
<td>d</td>
<td>the particle size; m</td>
</tr>
<tr>
<td>$D_h$</td>
<td>hydraulic diameter; m</td>
</tr>
<tr>
<td>$D_p$</td>
<td>diameter of particle; m</td>
</tr>
<tr>
<td>$f_v$</td>
<td>the particle volume fraction; (-)</td>
</tr>
<tr>
<td>$F_{\text{c-w}}$</td>
<td>the shape factor for the thermocouple and the wall; (-)</td>
</tr>
<tr>
<td>Gr</td>
<td>Grashof number; (-)</td>
</tr>
<tr>
<td>h</td>
<td>heat transfer coefficient; W/ m$^2$.K</td>
</tr>
<tr>
<td>$h_r$</td>
<td>heat transfer coefficient for radiation; W/ m$^2$.K</td>
</tr>
<tr>
<td>H</td>
<td>the depth of the material filled within the annulus gap; m</td>
</tr>
<tr>
<td>$k_b$</td>
<td>thermal conductivity of the packed bed; W/ m.K</td>
</tr>
<tr>
<td>$k_{\text{eff}}$</td>
<td>the effective thermal conductivity; W/ m.K</td>
</tr>
<tr>
<td>$k_g$</td>
<td>apparent thermal conductivity of the gas phase; W/ m.K</td>
</tr>
<tr>
<td>$k_g^0$</td>
<td>original thermal conductivity of the gas phase; W/ m.K</td>
</tr>
<tr>
<td>$k_p$</td>
<td>contact thermal conductivity between particles; W/ m.K</td>
</tr>
<tr>
<td>$k_r$</td>
<td>radiation contribution to thermal conductivity; W/ m.K</td>
</tr>
<tr>
<td>$k_r^0$</td>
<td>= $h_rD_p$; W/ m.K</td>
</tr>
<tr>
<td>$k_s$</td>
<td>thermal conductivity of the solid phase; W/ m.K</td>
</tr>
<tr>
<td>$k_\lambda$</td>
<td>the absorption coefficient; m$^{-1}$</td>
</tr>
<tr>
<td>$K_{\text{er}}$</td>
<td>= $\frac{\alpha_m}{\eta^2}$</td>
</tr>
<tr>
<td>l</td>
<td>the mean free path of phonons; m</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>mass flow rate; kg/s</td>
</tr>
<tr>
<td>L</td>
<td>characteristic physical dimension; m</td>
</tr>
<tr>
<td>P</td>
<td>pressure; N/ m$^2$</td>
</tr>
<tr>
<td>$P_b$</td>
<td>break-away pressure; N/ m$^2$</td>
</tr>
<tr>
<td>q</td>
<td>rate of heat transfer; W</td>
</tr>
<tr>
<td>$q_c$</td>
<td>rate of heat transfer of the conduction; W</td>
</tr>
<tr>
<td>$q_r$</td>
<td>rate of heat transfer of the radiation; W</td>
</tr>
<tr>
<td>$q_m$</td>
<td>total rate of heat transfer of the radiation and conduction; W</td>
</tr>
<tr>
<td>$Q_{\text{s.rs}}$</td>
<td>efficiency (Absorption, Extinction, Scattering); (-)</td>
</tr>
<tr>
<td>Q</td>
<td>rate of heat transfer; W</td>
</tr>
</tbody>
</table>
\( r_i \) radius of inner crucible; m
\( r_o \) radius of outer crucible; m
\( \text{Re} \) Reynolds number; (-)
\( \text{Ri} \) Richardson number; (-)
\( T \) temperature; °C
\( T_{Ta} \) Taylor number; (-)
\( T_{TC} \) the temperature of the thermocouple; °C
\( T_{g} \) the gas temperature; °C
\( T_{i} \) the temperature at inner interface; °C
\( T_{o} \) the temperature at outer interface; °C
\( T_{w} \) the temperature of the wall; °C
\( U_T \) tangential velocity; m/s
\( v \) the velocity of sound; m/s
\( v_{gas} \) velocity of the gas flow; m/s

**Greek**

\( \alpha_m \) the mean absorption coefficient; m\(^{-1}\)
\( \delta \) void fraction; (-)
\( \varepsilon \) emissivity; (-)
\( \eta \) the refractive index; (-)
\( \kappa_{eff}^1 \) effective thermal diffusivity; m\(^2\)/s
\( \lambda \) the heat conductivity; W/m.K
\( \lambda_p \) photon mean free path; m
\( \mu_0 \) the original viscosity; N.s/ m\(^2\)
\( \mu_g \) viscosity of the gas; N.s/ m\(^2\)
\( \mu_{liq} \) viscosity of liquid; N.s/ m\(^2\)
\( \mu_m \) the viscosity at the melting point; N.s/ m\(^2\)
\( \rho \) density; kg/ m\(^3\)
\( \sigma \) Boltzmann Constant; W. m\(^2\)/ K\(^4\)
\( \tau_{0\lambda} \) monochromatic optical thickness of the medium; m
\( \nu \) the kinematic viscosity of the liquid; m\(^2\)/s
\( \psi \) network parameter; (-)
\( \omega \) the shear rate; 1/s

**Subscripts**

a Absorption
e Extinction
s Scattering
ACKNOWLEDGEMENTS

The author wishes to express his gratitude and thanks to his research supervisors, Dr. P. V. Barr and Dr. T. R. Meadowcroft for their advice, encouragement, expertise and patience during the course of this project. Thanks are also extended to the faculty, staff and fellow students. Financial assistance from NSERC is gratefully acknowledged.
Heat transfer is fundamental to pyrometallurgy processing and the viability of new or existing processes often hinges on achieving high rates of heat transfer into the material being processed. Slags are a feature of extractive and refining processes and heat transfer through liquid slag is an important part of many operations, for example utilizing energy from port-combination in the EAF for scrap melting. However, relatively little work has been directed at characterizing heat transfer through non-stagnant slag layers.

Recent work\(^1\) has proposed the rotary melting furnace shown in Fig. 1 as an alternative to the EAF. As shown in Fig. 2, the rotation of the refractory includes regenerative heat transfer between the metal bath and the refractory. This has two benefits; (i) the area for heat transfer to the bath is more than doubled and (ii) the refractory is effectively cooled by the metal bath.
Heat transfer in the transverse plane of the rotary furnace in Fig. 2 involves:

1. Radiation from the freeboard gas to the exposed bath surface.
2. Radiation from the freeboard gas to the exposed refractory surface.
3. Radiation from the exposed refractory to the exposed bath surface.
4. Regenerative heat transfer from the covered refractory to the bath.
5. Convection and conduction through the slag layer.
6. Conduction through the refractory wall.
Computer modeling\(^1\) of the proposed rotary melting furnace suggests that \(~60\%\) of heat transfer to the bath is to the exposed liquid surface and, to be viable, the thermal resistance of the slag layer must be relatively low. Stagnant liquid slag is known to have relatively low thermal conductivity \(\sim 1 \text{ to } 5 \text{ W/m-K}\), yet the modeling work indicated that values for effective conductivity across the slag layer should be about one order of magnitude greater if acceptably high melting rates are to be achieved in the furnace.

Heat transfer through the slag includes conduction, radiation and, because of the stirring induced by furnace rotation, advection due to flow within the slag. The combined effect of all three mechanisms is convection but it is often more convenient to combine all three as an 'effective' thermal conductivity.

In general the conductivity of stagnant liquid (or solid) slag is the result of both molecular interaction (true conduction) and, at elevated temperature, thermal radiation. Although some work has been directed at measuring the conductivity of stagnant slags and glasses, the effect of stirring, in particular rotation-induced stirring such as will occur in the rotary melting furnace, is largely unknown.

The initial motivation for the current work was to determine the extent to which conductivity (effective basis) might be enhanced by induced stirring. Because no suitable existing experimental technique was found, a new method was developed and the scope was expanded to include measurements of thermal conductivity for stationary packed beds, stagnant fluids and fluids with induced stirring.
2.1 Measurements of the thermal properties of high temperature melts

One technique for determining the thermal conductivity of molten slag is the laser flash method carried out by Ohta et al.\(^2\) A schematic diagram of the technique is shown in Fig. 3.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.png}
\caption{Model of two layer sample in the laser flash method}
\end{figure}

For this configuration, the heat transport equations, initial and boundary conditions are given by equations (2.1) to (2.6), respectively.

\begin{equation}
- \rho_d C_{pd} \frac{d}{dt} \left( \frac{\partial T_d}{\partial t} \right) = -\lambda_s \frac{\partial T_s}{\partial x} \bigg|_{x=0} + RT_d(t) .
\end{equation}
For liquid layer:

$$C_p \rho \frac{\partial T_s(x,t)}{\partial t} = \lambda_s \frac{\partial^2 T_s(x,t)}{\partial x^2},$$

(2.2)

Initial conditions:

$$T_d = T_0, \ T_s = 0 \ (x = 0),$$

(2.3-2.4)

Boundary conditions:

$$T_s(\infty, t) = 0, \ T_s(0, t) = T_d.$$  

(2.5-2.6)

The system can be solved using either numerical methods or analytical solutions. Radiative heat transfer, which depends upon the temperature of the plate and refractive index of the melt, was assumed as linear function with respect to temperature difference because the increase of the temperature is typically only in the order of 10 K, so the relationship between thermal conductivity and the temperature rise of metal plate can consequently be established. The thermal conductivity can be calculated by fitting the measured temperatures with calculated ones by the least squares method.

The capability of the laser flash method for measuring thermal conductivity, and numerical solution of the heat transfer equation, including heat radiation, were presented by Ohta et al.\textsuperscript{2} with respect to a thin platinum plate in contact with a semi-infinite column of melt. Except for a short period of time after a laser pulse was just flashed, the temperature response of the platinum plate, detected by thermocouples, was found to be well reproduced by the numerical solution and characterized by the three parameters: the initial temperature rise of the platinum plate, the radiant coefficient, and conductive heat transfer coefficient.

Using this method, Ohta et al.\textsuperscript{2} determined the thermal conductivity of alkaline metal silicates in the molten and supercooled states in the range 900 – 1400 K from measurements of specific heat
capacity and conductive heat transfer coefficient obtained in separate runs. The effect of radiation emitted from the platinum plate through the transparent melt was discussed, and the presented data processing technique was confirmed to be a possible way of separating radiative heat flow from conductive flow.

Kishimoto et al.\textsuperscript{3} applied the technique to measure the thermal conductivity and specific heat of metallurgical slags over the temperature range $25^\circ C \sim 1500^\circ C$. Expressions for the radiant heat transfer were derived and incorporated into the analytical solution of the heat transfer equation. Results for a typical blast furnace slag ($40\%\text{SiO}_2, 40\%\text{CaO}, 20\%\text{Al}_2\text{O}_3$) are shown in Fig. 4.

![Figure 4. Results of Kishimoto, for 40SiO2-40CaO-20Al2O3 Slag\textsuperscript{3}](image)

The radiation effect in thermal diffusivity measurements using the laser-flash method was investigated by Srinivasan et al.\textsuperscript{4} A theoretical analysis based on the three-layer technique (metal-melt-metal, as shown in Fig. 5) was developed which included multiple reflections between the two metal layers. The system was solved numerically using a finite difference
scheme. It was concluded that, at higher temperatures, radiation correction is necessary for the laser-flash method.

Based on a similar three-layer laser-flash method, using a Pt-crucible layer instead of the above platinum layer, Eriksson et al.\textsuperscript{5} measured the effective thermal diffusivity of synthetic $\text{SiO}_2 - \text{CaO} - \text{Al}_2\text{O}_3$ slags. The effective thermal diffusivity combined the phonon and photon heat-transfer mechanisms. The results showed that the effective thermal diffusivities of all the studied slag compositions increased with increasing temperature. It was also found that, at a $\text{CaO}/\text{Al}_2\text{O}_3$ molar ratio of 4.42, an increase of $\text{SiO}_2$ content had very little effect on the effective thermal diffusivity values. On the other hand, addition of $\text{SiO}_2$ to a slag with the $\text{CaO}/\text{Al}_2\text{O}_3$ molar ratio of 2.59 resulted in a significant increase in the effective thermal diffusivity. The addition of $\text{Al}_2\text{O}_3$ to slags with a constant $\text{CaO}/\text{Al}_2\text{O}_3$ molar ratio resulted in a marked increase in the effective thermal diffusivity.
From the above analysis and experimental results, it is evident that the laser flash method is a useful technique for measuring thermal properties of liquid slag. The effective thermal conductivity of liquid slag, composed of conduction and radiation effect, could be measured by the laser flash method. Nevertheless, by this method, the radiation effect could be separated from conduction by appropriate assumptions and numerical processing. However, the method is not practical for stirred melts because of the limitation of experimental assembly and the difficulty of performing transient numerical analysis for this chaotic experimental state.

As an alternative of the laser flash method, thermal conductivity of liquid slags can also be measured using a hot-wire technique. Nagata et al. used this method to measure the thermal conductivity of Na₂O– and CaO- based slags over the range of 25°C to 1500°C.
A schematic of the employed hot-wire method is shown in Fig. 7. As can be seen, the hot wire, "heater", is dipped vertically into the slag. When a constant electric current is supplied to the heating wire, the rate of temperature rise of the heater is controlled by the rate of heat transfer to the slag. Assuming i) zero initial temperature throughout, and ii) constant heat input per unit time to the heating wire, the radial heat flow equation for an infinite medium can be solved to give conductivity as a function of temperature rise:

\[ \lambda = \frac{q}{4\pi \Delta \ln t} \Delta T \quad (2.7) \]
The workers found that conductivity increased up to about 700°C and then decreased with increasing temperature in both the solid glassy state and the liquid state. The absolute magnitude of the conductivity ranged from 0.1 to 2.0 W/m.K.

The relationship between the mean free path of phonons and conductivity was discussed as well. An equation was presented as:

$$\lambda = \frac{1}{3}C_v \ast L \ast v,$$

where $\lambda$ is the heat conductivity, $C_v$ is the heat capacity, $L$ is the mean free path of phonons, and $v$ was the velocity of sound. The calculated mean free paths were discussed in relation to the probable ionic structures of the slags.

Because the hot wire method was designed to measure the thermal conductivity of insulators, for an electrically conductive material such as slag at high temperature, problems occurred due to current leakage. The experiment and data processing technique was designed to exclude radiative effects, but it is impractical to introduced stirring into the system.

The role of microstructure on the thermal conductivity of molten silicates was investigated by Mills.\textsuperscript{7,28} It was found that the thermal conductivity of molten silicates contained a substantial contribution from the structure of the slag, and this contribution increased with increasing chain length of the silicate anions, i.e. increasing silica content. A method was proposed for predicting thermal conductivity by (i) attributing thermal resistances to the ionic and valence bonds; and (ii) introducing the structural aspects of $(\text{Si}_n\text{O}_{2n+3})^{6-}$ anion rings. The method was shown to be in good agreement with previous experimental data.\textsuperscript{2,3,6}
Thermal conductivities of complex, molten slags have also been estimated by Mills from a knowledge of the chemical composition and melting point of the slag, by use of the relationship between thermal conductivity and the network parameter $\psi$, which could be derived by

$$\psi = \log[\mu_m / \mu_o]^2,$$

where $\mu_m$ was the viscosity at the melting point, which could be calculated according to chemical composition.

It was proposed that the change in slope of the thermal conductivity – temperature relationship was due to the change in thermal expansion coefficient occurring at the glass transition. Mills et al also suggested that the thermal resistance associated with the movement along the silicate chain or ring is relatively small when compared with that associated with the movement of phonons from chain to chain. It was proposed that the thermal resistance would be expected to increase as the melt becomes progressively depolymerised. The thermal conductivity of a silicate melt is a linear function of the nonbonded oxygen in the melt, which was called $\text{(NBO/T)}$ ratio as well.

However, Mills did not account the large contribution from both convection and radiation into the thermal conductivity. And only values produced by using transient techniques were adopted to establish the relationship.

The effective thermal diffusivity, defined as a combined effect of conduction and radiation, in stagnant liquid glass which is similar in composition to metallurgical slags was also measured by Fine et al. Using new ‘periodic steady-state’ technique, glasses containing between 10.9 and 26.8 pct FeO and having lime/silica ratios of 1.0 and 1.5 were examined at temperatures up to
1500°C. To determine the monochromatic absorption coefficients for the visible and infrared spectra, measurements of radiative transmissivity at room temperature were also carried on glasses containing 0 to 12.6 pct FeO.

The research provided a novel and effective method to measure the effective thermal diffusivity under high temperature. A periodically varying temperature was produced at one surface of the glass sample by heating it with a molybdenum wire carrying a periodically varying current. The temperature at the other surface of the glass was also measured. The effective thermal diffusivity was then determined from the difference in phase of the two periodically varying temperatures. The analytical solution for the periodical steady problem was carried out based on heat transfer functions.

![Figure 8. Specimen and crucible assembly for measurements of thermal diffusivity](image)

The results indicate strong radiative absorption by Fe$^{++}$ ions throughout the visible and infrared spectra from the glasses containing 6.3 pct and 12.6 pct FeO. And reasonably, higher absorption coefficients were discovered when more FeO were presented. The results of experiments performed on glasses containing between 10.9 and 26.8 pct FeO, with lime/silica ratios, B, of 1.0
and 1.5, and at temperatures ranging from the liquidus temperatures of the glasses to 1750K are represented by

\[
\kappa_{\text{eff}}^1 = 0.001 \times (1.5 - 0.5B) + 0.018 \frac{(T/1500)^3}{(\text{pctFeO})^{0.8}}; \text{(cm}^2/\text{s}). \tag{2.10}
\]

Because of the design and the dimension of the apparatus, only the effective thermal diffusivity of stagnant state samples could be tested, which did not accord with the motivation of introducing stirring effect into consideration. The research's high accuracy of data acquisition and experimental control was also difficult to realize.

The effective thermal conductivity of liquid slags was examined by Mikrovas et al.\textsuperscript{9} The work focused on the SiO\textsubscript{2} – CaO – Al\textsubscript{2}O\textsubscript{3} system, chiefly in the composition ranges typical of the steelmaking ladle and tundish as well as mould powder for continuous casting. The results were reported as effective thermal conductivity \(k_{\text{eff}}\); i.e. combination of conduction, radiation and convection.
The research provided a technique for measuring the effective thermal conductivity of molten slags. The technique involved melting of a cylindrical metallic specimen within a constant temperature slag bath, the temperature profile of the specimen was measured as a function of time. A mathematical model was built based on the experimental setup and relative initial and boundary conditions. The effective thermal conductivity could be expressed as a function of temperature and time, and be deduced after solving the function by numerical analysis.

The results indicated $k_{\text{eff}}$ increased with increasing temperature, and decreased with titania addition. However, there was no difference in the amount of heat entering the immersed specimen for the case of "low degree" forced convection and natural convection. Under constant super heat, the effective thermal conductivity varied between slags of different chemical composition, but no pattern simply on the basis of chemical composition was found.
A further discussion and evaluation of results generated from previous literatures was needed. A summary of literature results for 40SiO2-40CaO-20Al2O3 Slag was shown in Fig. 10. Results were obtained by several techniques; i.e. hot wire method (Nagata et al.), coaxial-cylinders method (Ogino et al.\textsuperscript{10}), laser flash method (Kato et al.\textsuperscript{11} and Kishimoto et al.\textsuperscript{3}), and unsteady state melting (Mikrovas et al.\textsuperscript{9}).

![Graph showing thermal conductivity vs. temperature for 40SiO2-40CaO-20Al2O3 Slag](image)

**Figure 10. Thermal conductivity vs. temperature for 40SiO2-40CaO-20Al2O3 Slag**

The distribution of results given by various techniques could be deduced from Fig. 10. Thermal conductivities measured by using laser method and hot wire method may only contain small contributions from radiation, particularly at high temperatures. However, contributions from radiation in the glassy slags may be dominant at high temperatures because the glassy slags have similar optical properties to glasses. The effective thermal conductivity measured by Mikrovas et al.\textsuperscript{9} increases dramatically with increasing superheat. The reason is because radiant conductivity is proportional to the third power of temperature. The radiant conductivity would be an order of magnitude greater than the phonon conductivity at very high temperature.
2.2 Heat transfer in Packed Beds and Porous Media

Heat and mass transfer through porous media such as packed beds of granular particles is of wide ranging importance in materials processing. Due to its importance, a large body of experimental and/or modeling work exists in the literature.

As shown in Fig. 11, porous media is a heterogeneous system composed of a solid matrix whose void is filled with fluids. Transport phenomenon in it can be treated by properly accounting for the role of each phase in transport through the system of phases. In general, the conductivity of a loose packing, or packed bed, is a function of pressure, size of the particles, thermal conductivities of the gas filled in the pores, the solid, the void fraction in the packed bed, and the temperature, because, as the temperature increases, the particle-to-particle thermal radiation increases as well.

The model developed by Schotte\textsuperscript{12} to predict the thermal conductivity of packed beds for various conditions of pressure, temperature, and particle size has been widely adopted. The model took into account the reduction in thermal conductivity of the gas phase at lower pressures when the
distance over which the conduction took place became of the same order as the mean free path of the gas molecules. The $k_g$ value could be obtained from the Fig. 12.

Deissler and Eian's correlation\textsuperscript{13} for the break-away pressure $P_b$ in N·m$^{-2}$

\begin{equation}
P_b = 4.32 \times 10^{-21} \frac{T}{D_p \sigma^2},
\end{equation}

was adopted to determine whether pressure, temperature and/or particle size will affect the thermal conductivity of the gas phase. When the actual pressure was greater than $P_b$, no correction to the value of $k_g$ was needed, but if the actual pressure was less than $P_b$, then the thermal conductivity of the gas phase should be calculated by using the equation below:
\[ k_b = \frac{k_g^0}{1 + 4.95 \times 10^{-22} \left[ \frac{C_p}{C_v} \frac{1 - \delta}{\delta} \right] \left[ \frac{T_k^0}{PD_p d^2 C_p \mu_g} \right]} \]  

(2.12)

The correlation of the effect of porosity on packed bed thermal conductivity from Deissler and Eian, shown in Fig. 13, was used to predict the bed conductivity \( k_b \) for the limit of \( 1 < \frac{k_s}{k_g} < 6000 \).

![Figure 13. Effect of porosity on packed bed thermal conductivity. (From Deissler and Eian, Ibid.) Curves outside the range 0.2–0.6 have not been tested as extensively as the others](image)

For very large values of \( \frac{k_s}{k_g} \), solid conduction at contact points became significant. Wilhelm\(^{14}\) presented a correlation for this contact conductivity:

\[ \log k_p = -1.76 + 0.0129 \frac{k_s}{\delta} \]  

(2.13)
Schotte pointed out a deficiency in Wilhelm's correlation in that it does not account for the particle size, so the predicted values could be an order of magnitude larger than experimental values for very small particles. However, a new correlation for the contact conductivity will not be available until more new data become available.

If the temperature is above 200°C we must add another term to \( k_b \), in order to obtain the true effective thermal conductivity of the packed bed. At high temperature, there is a contribution to the effective thermal of the bed by radiation heat-transfer. This radiation is important for big size particles (~1mm) even at low temperatures (~400°C).

Schotte also included a model to predict the effect of radiation (Details were shown in Appendix B), which assumed spherical particles as the elements. The thermal-conductivity correction for radiation heat transfer rate across the total area could be written as

\[
q = -k_r \frac{\pi}{4} \frac{D}{x} \left( \frac{1}{1 - \delta} \right) \left( \frac{dt}{dx} \right) .
\]  

(2.14)

The radiation contribution could be found as

\[
k_r = \frac{1 - \delta}{1 + \frac{1}{k_s + k_r^0}}.
\]  

(2.15)

The angle factor was unity and the emissivity factor was close to the emissivity of the particle, since the particle was surrounded by the irregular surface of the packed bed, it could be approximated that

\[
k_r^0 = 22.9 \le D_p \frac{T^3}{10^8},
\]  

(2.16)
and finally, by adding $k_r$ and $k_b$, it got $k_{\text{eff}} = k_r + k_b$.

Schotte’s derivation was limited to opaque particles, which are large compared with the wavelength of radiation. Transparency of the particles would increase the rate of heat transfer and therefore the thermal conductivity of the bed. In the book written by Sparrow et al.\textsuperscript{15}, the monochromatic optical thickness of the medium was defined as

$$\tau_{0\lambda} = k_\lambda L,$$

where $k_\lambda$ is the absorption coefficient, and $L$ is a characteristic physical dimension.

The photon mean free path was $\lambda_p = 1/k_\lambda$, the optical thickness may be rephrased as

$$\tau_{0\lambda} = \frac{L}{1/k_\lambda} = \frac{L}{\lambda_p}.$$

The case that $\tau_{0\lambda} > 2-3$ refers to the optically thick limit; on the contrary, the case that $\tau_{0\lambda} < 0.1$ refers to the optically thin limit; the case involves intermediate values of $\tau_{0\lambda}$ refers to a more practical but also more complicated situation.

When the results from the model were compared to experimental results, reasonable agreement was observed. However, Schotte mentioned in his paper that the accuracy of the correlation of Deissler et al.\textsuperscript{13}, or those of other workers, was difficult to ascertain. For example, it appeared that data for the system glass spheres-air at nearly the same temperature and with almost the same void fraction differed widely. The data differed by as much as a factor of 2.4, and as a result some data appeared to be greatly in error were not considered in his study.

Kaviany\textsuperscript{16} collected the available closed-form solutions for the isotropic effective thermal conductivity for beds of spherical particles. And the results were plotted in Fig. 14 for $\varepsilon = 0.38$.
(near that of random packing of spheres), and were compared with experimental results given by Nozad et al. The differences between various predictions and experimental results varied.

![Figure 14. Effective thermal conductivity of beds of spherical particles predicted by various analyses. Also shown are the experimental results given by Nozad et al.](image)

By assuming that the temperature field within a particle is unaffected by the presence of other particles—called Maxwell's lower bound, for dilute particle concentration ($\delta \approx 1$), it had

$$\frac{k_b}{k_g} = \frac{2\delta + (3-\delta)k_s / k_g}{3 - \delta + \delta k_s / k_g}; \quad (2.19)$$

by using geometric mean approach, it had

$$\frac{k_b}{k_g} = \left(\frac{k_s}{k_g}\right)^{1-\delta}; \quad (2.20)$$

when $k_s / k_g \geq 1$, symmetry in unit cell, by using variational formulation plus structural statistics of Miller's lower bound, for spherical cell shape, it had
by using weighted average of the Maxwell upper bound with an expression obtained by
introduction of an adjustable function into a weighted averaged expression, introduced by
Hadley, it had

\[
\frac{k_c}{k_g} = \frac{k_s}{k_g} \left[ \frac{k_s}{k_g} - (1 - \delta)(\frac{k_s}{k_g} - 1) - \frac{4}{3} \frac{(\frac{k_s}{k_g} - 1)^2 (1 - \delta)\delta}{1 + \frac{k_c}{k_g} + (1 - 2\delta)(\frac{k_s}{k_g} - 1)/3} \right]^{-1}; \tag{2.21}
\]

\[
\frac{k_c}{k_g} = (1 - \alpha_0) \frac{\delta f_0 + k_s / k_g (1 - \delta f_0)}{1 - \delta (1 - f_0) + k_s / k_g (1 - f_0)} + \alpha_0 \frac{2(k_s / k_g)^2 (1 - \delta) + (1 + 2\delta)k_s / k_g}{(2 + \delta)k_s / k_g + 1 - \delta}, \tag{2.22}
\]

where

\[
f_0 = 0.8 + 0.1\delta, \tag{2.23}
\]

and

\[
\log \alpha_0 = -4.898\varepsilon, \quad 0 \leq \varepsilon \leq 0.0827
\]

\[
\log \alpha_0 = -0.405 - 3.154(\varepsilon - 0.0827), \quad 0.0827 \leq \varepsilon \leq 0.298
\]

\[
\log \alpha_0 = -1.084 - 6.778(\varepsilon - 0.298), \quad 0.298 \leq \varepsilon \leq 0.580 \tag{2.24}
\]

Among the predictions, Hadley's Weighted Average, Geometric Mean, and Miller's lower bound were closer to the experimental results. Due to this reason, these models were adopted in order to calculate the effective thermal conductivity from other points of view, and to check how close the data from various models to the data from the experiment was. The mathematical expressions of these models were written and could be found as equations (2.19) ~ (2.24). Nevertheless, if the temperature is above 200°C, the true thermal conductivity will be the sum of the effective thermal conductivity predicted from those models and the correspondent radiant conductivity. The radiant heat transfer could be estimated by using a few methods, which included Two-Flux Approximations, Diffusion Approximations, Spherical Harmonics-Moment (P-N) Approximation, Discrete-Ordinates (S-N) Approximation and Noncontinuum Treatments:
Monte Carlo Simulation. However, the study of those methods would need solutions of Maxwell Equations and different scattering and boundary conditions. That would be complicated and beyond the purpose of this study.

In summary, thermal conductivity of packed beds can be predicted by utilizing mathematical models quoted above. If the temperature is high, radiative effect need to be added and can be derived by using equations (2.15) and (2.16), whose applicability needs to be verified by checking the optical thickness of the medium.
2.3 The Viscosity of liquids and melts

Investigation of the viscosity of liquids and melts is necessary because of one of the project’s motivations of measuring thermal properties of slag with introduced stirring effect. Viscosity will influence the flow behavior, which will consequently affect the effective thermal conductivity of the liquid.

One study for investigating viscosity of Fe₂O₃ – CaO – SiO₂ melts at elevated MgO content was performed by Drissen et al. The rotary viscometer technique, shown in Fig. 15, was used to measure the viscosity.

![Figure 15. Schematic view of rotary viscometer technique](image)
In the study, both the basicity \((\text{CaO} + \text{MgO})/\text{SiO}_2\) and \(\text{Fe}_2\text{O}_3\) content of the slags were varied. A remarkably larger increase of viscosity is observed when basicity of \(\text{Fe}_2\text{O}_3\) content is decreased in the molten slag.

**Figure 16. Compositions of the investigated \(\text{CaO} – \text{Fe}_2\text{O}_3 – \text{SiO}_2\) slags**

**Figure 17. Viscosity – \(1/T\) plot for slags at a molar basicity of 1.35**
Figure 18. Viscosity – 1/T plot for slags at a molar basicity of 1.07

Figure 19. Viscosity – 1/T plot for slags at a molar basicity of 0.72
McCauley et al. investigated temperature dependence of the viscosity of liquids. Andrade-Arrhenius equation, absolute reaction rate theory and Clausius-Clapeyron equation were discussed and regression analysis was applied to fit data to the viscosity models for several oxide
melts. A relationship to better express the temperature dependence of viscosity of oxide melts is derived from the Clausius-Clapeyron equation in the form of

\[ \ln \mu_{\text{liq}} = a + b/T + c \ln T. \]  \hspace{1cm} (2.25)

It was concluded that this relation worked well for all systems, including silicate melts, liquid melts, fused salts, and organic liquids.

In addition, Machin et al.\textsuperscript{22,23} gave information about the viscosity for CaO – Al\textsubscript{2}O\textsubscript{3} – SiO\textsubscript{2} – MgO melts. The results are summarized in Fig. 22 ~ 23.

![Figure 22. Isoviscosity coefficients in poise of CaO – Al\textsubscript{2}O\textsubscript{3} – SiO\textsubscript{2} melts at 1500 °C](image)
Barrett et al.\textsuperscript{24} presented the densities of \( \text{CaO} - \text{Al}_2\text{O}_3 - \text{SiO}_2 \) melts for various temperatures (shown in Fig. 24), the results could be combined with above viscosity used to estimated the kinematic viscosity for oxides melts.

<table>
<thead>
<tr>
<th>Composition, Weight Per Cent</th>
<th>Density at t°C (g/cm(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>CaO</td>
<td>( \text{Al}_2\text{O}_3 )</td>
</tr>
<tr>
<td>35</td>
<td>10</td>
</tr>
<tr>
<td>34</td>
<td>30</td>
</tr>
<tr>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td>29</td>
<td>40</td>
</tr>
<tr>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>25</td>
<td>10</td>
</tr>
<tr>
<td>23</td>
<td>15</td>
</tr>
<tr>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

Figure 24. Densities of \( \text{CaO} - \text{Al}_2\text{O}_3 - \text{SiO}_2 \) melts for the temperature range 1350°C - 1650°C
2.5 Measurements of thermal conductivity for shearing flow

Some experimental setups for measuring thermal properties of liquid slag were introduced in section 2.1. However, they were originally designed to measure thermal properties of the slag under stagnant state. To accord with the motivation of investigating the effective thermal conductivity of liquid slag under stirring effect, a study of available apparatus for measuring thermal conductivity of shearing flow is carried out in order to obtain ideas of designing the apparatus for this project.

An apparatus for measuring the thermal conductivity and viscosity of polymers under shear strain was designed by Tavman. By using the apparatus, thermal conductivity could be measured at temperatures approaching 220 °C and at strain rates up to 300 s⁻¹. The viscosity was measured by using the similar theory of traditional Couette viscometer. The thermal conductivity was measured by a transient plane heat source method which consists of generating heat at a constant rate by using the nickel wire wrapped around the periphery of the inner cylinder and recording the temperature rise of the wire as a function of time. The thermal conductivity and viscosity of a linear low-density polyethylene, LLDPE HX 5631, were studied and results were shown in the paper.
An instrument with computerized data acquisition was designed by Kostic\textsuperscript{26} to investigate the thermal conductivity of a non-Newtonian fluid while it is subjected to shearing flow, and to determine its dependence, if any, on shearing itself. The measurement and control were accomplished by using a computerized data acquisition system and a comprehensive virtual instrument, developed by using the LabVIEW application software. It was found that this apparatus would satisfy the need of providing interactive and accurate data for the experiment. And it was easy to be enhanced by modification of software program.
The apparatus introduced above was useful and effective, however, it is unavailable to be used directly for the purposes of measuring thermal conductivity at high temperatures and meeting the specific requirement of oxides melts. The methods of data acquisition and data analysis would also be unavailable due to current facilities. Therefore, a modification and enhancement is necessary for the new apparatus used in the present study.
3.0 SCOPE & OBJECTIVES

Although recent modeling work\(^1\) has indicated that an oxy-fuel-fired rotary furnace might offer significant advantages over the EAF for scrap melting the study also indicated that the thermal resistance of the protective slag layer would have to be relatively low for the process to be viable. Model results suggested that rotation-induced stirring of the slag would need to decrease the thermal resistance by roughly an order of magnitude relative to that of a stagnant oxide melt. Thus an initial goal for the current work was to investigate experimentally the degree to which flow-induced stirring might increase the effective thermal conductivity of an oxide melt. However, a review of the methods currently employed for measuring the conductivity of stagnant oxide melts, which involve numerical solution of the transient conduction problem for interpretation of the data, did not indicate that any was readily adaptable to stirred systems nor were they broadly applicable to a wider range of materials such as liquid metals or granular solids. In addition, the methods employed were impractical for stirred melts because of the limitation of experimental assembly and the difficulty of performing transient numerical analysis for unsteady experimental state.

Based on these factors the objectives of the work were:

1. To develop a broadly applicable steady-state method for directly measuring the thermal conductivity of a variety of materials ranging from stationary granular solids to liquid oxides and metals (with or without stirring) for temperatures covering the spectrum encountered in extractive metallurgy.
2. Use the technique to determine the extent to which the effective thermal conductivity of liquid oxides might be increased by flow-induced stirring.

The methodology would cover a range of issues related to the design of the apparatus, interpretation of the raw data and general validation of the technique.
4.0 METHODOLOGY

4.1 Overview and Design Issues

In order to avoid the difficulties associated with transient methods, the conductivity would be determined by measuring steady-state heat transfer across the annulus formed by two concentric cylinders, as shown schematically in Fig. 27:

As shown in the figure, the material to be tested is contained within the annulus and heat transfer is in the radial direction from the high temperature source to the low temperature coolant flowing...
through the inner cylinder. By simultaneously measuring the rate of heat transfer and temperature difference across the annulus under steady state conditions, the effective thermal conductivity of the material within the annulus can be calculated by rearranging the solution for 1-Dimensional cylindrical conduction as:

\[
K_{\text{eff}} = \frac{Q \ln\left(\frac{r_o}{r_i}\right)}{(T_o - T_i)2\pi L},
\]

(4.1)

where \( r_o \) and \( r_i \) are radius of inner crucible and outer crucible separately, \( L \) is the axial length. \( T_o \) and \( T_i \) were measured using thermocouples positioned at the hot and cold interfaces, while \( Q \) was obtained from the coolant:

\[
\dot{Q} = (mC_p\Delta T)_{\text{coolant}}.
\]

(4.2)

For liquid materials, flow would be induced by rotating the inner cylinder.
4.2 Design of The Apparatus

For the design of the apparatus, two basic factors were considered:

1. The temperature drop across the annulus should be large enough that small errors in the thermocouple data would not result in significant error in the results.

2. The temperature rise for the coolant should be relatively small to ensure reasonably uniform heat transfer condition along the axial length of the test section.

Assume $\Delta T_a$ for annulus, $\Delta T_c$ for coolant. From Eq. 4.1, $\Delta T_a$ is related to $\dot{Q}$ and $K_{eff}$ by:

$$\Delta T_a = \frac{\dot{Q}}{K_{eff}} \frac{\ln\left(\frac{r_0}{r_i}\right)}{2\pi L}.$$  \hspace{1cm} (4.3)

Thus $\Delta T_a$ increases with the rate of heat extraction by the coolant which is a function of coolant flow. Therefore one aspect of the apparatus design was to analyze heat transfer to the coolant:

$$\dot{Q} = m_c C_{p,c} \Delta T_c = h(2\pi r_i L)(T_i - T_c),$$  \hspace{1cm} (4.4)

to ensure that the heat transfer coefficient was large enough to ensure a significant temperature drop across the annulus.

To aid in the design of the apparatus a computer model was developed to include the various design parameters. Figure 28 below provides details of the algorithm procedure and the model developed for design of the apparatus.
Figure 28. Flow chart for designing the apparatus
Heat transfer to the coolant was calculated using the correlation for laminar and turbulent forced convection through circular tubes with uniform temperatures. The applicability of the correlation for turbulent forced convection was extended to the region of laminar turbulent transition.

\[ Nu = \frac{hD}{k} = 3.656, \quad Re < 2100, \]  
\[ Nu = 0.023 Re^{0.8} Pr^{0.33}, \quad 2100 < Re < 10^7. \]

\[ (4.5) \]
\[ (4.6) \]

The viscosity and thermal conductivity of the nitrogen coolant were calculated from the fits suggested by the reference:

\[ k_g = (5.0 \times 10^{-3}) + (7.5 \times 10^{-5})T - (1.3 \times 10^{-8})T^2, \]
\[ \mu_g = (-1.4 \times 10^{-5}) + (8.0 \times 10^{-8})T + (5.2 \times 10^{-11})T^2. \]

\[ (4.7) \]
\[ (4.8) \]

For circular concentric tubes in the experiment, it had

\[ \text{Area}_{\text{annular}} = \pi(r_o^2 - r_i^2), \]
\[ \text{Area}_{\text{circular}} = 2\pi r_o L, \]
\[ D_h = \frac{4\text{Area}_{\text{annular}}}{2\pi(r_o + r_i)}. \]

\[ (4.9) \]
\[ (4.10) \]
\[ (4.11) \]

So that the gas velocity could be estimated as

\[ v_{\text{gas}} = \frac{\dot{m}_g}{\rho_g \text{Area}_{\text{annular}}}. \]

\[ (4.12) \]

Because the methodology assumed steady-state, the design model also included a module to calculate the approximate time required to reach steady-state. This response time of our
apparatus depends on numerous factors, such as the thermal diffusivity of the media within the annulus as well as the thermal inertia of the various components.

Assume $T=T(r,t)$, the basic equation to be solved is 1-D transient conduction

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + Q = \rho C_p \frac{\partial T}{\partial t}, \quad (4.13)$$

Initial Condition : $T(r,0) = T_0, \quad (4.14)$

Boundary Conditions 1 : at $r = r_o$, $q_r = \text{constant}, \quad (4.15)$

Boundary Conditions 2 : at $r = r_i$, $q_r = h(T_i - T_c). \quad (4.16)$

Standard implicit finite difference methods were used to solve the differential equation. An initial temperature is given, and the boundary conditions are assumed to be constant heat for both the inner and outer surfaces. The code for the numerical analysis is listed in Appendix A.

Based on the objective and scope of this experiment, a series of estimations for the thermal properties and boundary conditions were made for the model. Results of the numerical analysis show that the response time for the apparatus is generally fast enough (<5min) for practical consideration. Another advantage of this model is that the temperature difference between hot interface and cold interface could be predicted. It is especially helpful for liquid slag system, because the temperature difference between two interfaces cannot be too high to result in solidification or too low to be tested by thermocouples.
4.3 Detailed description of the apparatus

Details of the apparatus that evolved from the design process are shown in Figs. 29 to 32.

Figure 29. Schematic view of the apparatus for measuring effective thermal conductivity
Figure 30. Details of the apparatus showing the copper induction coils surrounding the unit as well as the variable speed motor (left) used for rotation of the inner cylinder.
Figure 31. Details showing the chain device for the rotation of the inner cylinder.
Figure 32. Details of the control panel of the induction heater
As shown in Fig. 29 ~ 32, induction heating was employed using a graphite susceptor. This heating element combined with the magnesia ring and the alumina ring, separated the furnace into two chambers. The inner chamber inside the graphite bushing was used as the working space for the crucible assembly. The outer chamber provided insulation and protection for the graphite bushing. To protect the bath and graphite bushing from oxidation, argon was introduced into the inner chamber through three jets located on the bottom fiber plate and one jet in the top refractory cover. The flow rate of the argon was measured by rotameters. The apparatus would be used to test the effective thermal conductivities of materials at high temperatures and materials that would probably have erosive properties. Based on its high melting point and good mechanical thermal expansion properties, alumina was used for the crucibles. Nevertheless, alumina will not be easily eroded by slag with the compositions of this study. The inner crucible was single bore cast alumina 99.8% tube (38.1mm OD, 31.75mm ID). The outer one was single bore cast alumina 99.8% tube (69.85mm OD, 63.5mm ID).

Temperatures were measured using AWG gauge 30 (0.010 inch) Type S thermocouples within an alumina tube to provide corrosion protection. Thermocouple signals were routed via a rotary selector switch to an i/32 Temperature Process Monitor. During some of the initial high temperature trials the thermocouple being used to measure the coolant temperature at exit from the test section received significant thermal radiation from the relatively hot surface of the inner crucible. In this case the thermocouple junction temperature would be significantly higher than the true gas temperature required for determining $\dot{Q}$. In order to avoid the necessarily uncertain correction for radiation the thermocouple was moved further up into the much cooler upper section of the furnace to effectively minimize the view factor for any hot surface to a negligibly small value.
4.4 Materials Tested

A summary of all materials tested is given in Table 1. Water and silicon oil were included as base cases for validation of the experimental technique. For packed beds, SiO$_2$ and sodium chloride were selected because they were generally stable from room temperature and the intrinsic properties were satisfactory for the study. For the oxide melts, the CaO-SiO$_2$-Al$_2$O$_3$ system was selected because these are main components of typical blast furnace slag. In some cases MgO and FeO were also added into the mixes. One factor in choosing the mix was to ensure a reasonable low melting point. The phase diagrams of relevance are shown in Fig. 33 ~ 34. The basicity of the oxides was also a factor to ensure minimal interaction with the crucibles being used.

<table>
<thead>
<tr>
<th>Fluids</th>
<th>Boiling Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon Oil 200</td>
<td>~250°C</td>
</tr>
<tr>
<td>Water</td>
<td>100°C</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Packed Beds</th>
<th>Particle size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SiO$_2$</td>
<td>0.8</td>
</tr>
<tr>
<td>NaCl</td>
<td>0.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High Temperature Melt</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixture</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CaO</td>
<td>40.00%</td>
<td>36.60%</td>
<td>25.60%</td>
</tr>
<tr>
<td>SiO$_2$</td>
<td>40.00%</td>
<td>36.60%</td>
<td>25.60%</td>
</tr>
<tr>
<td>Al$_2$O$_3$</td>
<td>20.00%</td>
<td>14.60%</td>
<td>10.20%</td>
</tr>
<tr>
<td>MgO</td>
<td>0.00%</td>
<td>12.20%</td>
<td>8.60%</td>
</tr>
<tr>
<td>FeO</td>
<td>0.00%</td>
<td>0.00%</td>
<td>30%</td>
</tr>
<tr>
<td>Melting point (°C)</td>
<td>~1266</td>
<td>~1050</td>
<td>~1220</td>
</tr>
</tbody>
</table>

Table 1. Materials used for the experiment
Temperatures up to approximately 1550°C are on the Geophysical Laboratory Scale; those above 1550°C are on the 1948 International Scale.

Figure 33. Phase diagram of CaO-SiO2-Al2O3
Figure 34. Phase diagram of CaO-SiO2-FeO
4.5 Experimental Procedure

The experimental procedures can be summarized as follows:

1. 350ml ~ 400ml of the material to be tested was placed into the annulus chamber within concentric cylindrical crucibles.
2. Induction heating was then initiated.
3. Once the temperature in the furnace reached 500°C, the argon gas flow was started to protect the carbon graphite ring susceptor.
4. As temperature approached the target levels for the test, the N₂ cooling flow was set to the predetermined value.
5. If a stirring effect was required, rotation of the inner crucible was started and maintained constant.
6. Furnace power and coolant flow was then maintained constant until the furnace reached steady-state; i.e. the thermocouple readings stabilized. The temperature data, gas flow and power settings were then recorded.
7. If the test involved multiple temperatures, the power settings were changed and conditions maintained constant until steady-state was again achieved.
5.0 RESULTS AND DISCUSSION

5.1 Effective Thermal Conductivity of Water and Silicon Oil 200

Because the thermal properties of these fluids are well known, the initial work involved measuring effective thermal conductivity under stagnant conditions; i.e. without rotation of the inner crucible. Following this, stirring was introduced in order to examine this influence of shear rate on effective conductivity.

Because boiling, or even incipient boiling was to be avoided, the temperature of the system was sufficiently low that any radiation effects could be neglected. The tested temperature was ~50°C for water, and ~100°C for silicon oil 200.

For both water and silicon oil, several trials were carried out under stagnant state (shear rate = 0) in order to check the accuracy and the repeatability of the experiment. The results are summarized in Fig. 35 and 36.
The values obtained from different trials are close, and the error factor was estimated to be approximately 10% according to the difference between experimental results and standard values. The values of the effective thermal conductivities obtained from stagnant state were close to the accepted values (0.50W/m.K for water, 0.15W/m.K for silicon oil). Thus giving confidence in the technique at low temperatures.
### Table 2. Comparison of experimental and typical results

<table>
<thead>
<tr>
<th>Fluids</th>
<th>K (W/m.K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>0.50</td>
</tr>
<tr>
<td>Experimental Results</td>
<td>0.43</td>
</tr>
</tbody>
</table>

In addition, a series of experiments were carried out with induced stirring. The shear rate was used to characterize the intensity of the induced stirring. The shear rate equals to the tangential velocity of rotation divided by the radial separation between two concentric cylinders.

\[
\tau = \frac{U_T}{(r_o - r_i)},
\]

where \( \tau \) is the shear rate, and \( U_T = r_i \omega \) is the tangential velocity of the inner crucible surface.

The results for water and silicon oil 200 under various shear rates are summarized in Fig. 37.
As can be seen in Fig. 39, the response to shearing flow was quite different for the water and silicon oil. This can be explained by examining the peripheral Reynolds number (also called rotational Reynolds number)

\[ Re = \frac{4\tau r^2}{\nu}, \]  

(5.2)

where \( \tau \) is the shear rate, and \( \nu \) is the kinematic viscosity of the fluid. For \( Re < 10 \), the flow will be laminar.

According to Burmeister\textsuperscript{27}, for concentric cylinders with inner cylinder at rotation and the outer cylinder at rest, there are three flow regimes described by the Taylor number:

\[ Ta = \frac{U_T (r_o - r_i) (r_o - r_i)^{\frac{1}{2}}}{\nu r_i} \]  

(5.3)

where:

\[ Ta < 41.3 \quad \text{laminar Couette flow} \]
\[ 41.3 < Ta < 400 \quad \text{laminar flow with Taylor vortices} \]
\[ Ta > 400 \quad \text{turbulent flow} \]

The kinematic viscosities of water and silicon oil are summarized in Table 3, and test conditions are summarized in Table 4, which shows both Re and Ta for the rotation rates employed.

<table>
<thead>
<tr>
<th>Kinematic Viscosity(m2/s)</th>
<th>Water</th>
<th>Silicon Oil</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0E-06</td>
<td>2.0E-04</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Kinematic Viscosity of fluids to be tested
### Table 4. Applied shear rates and their corresponding Re, Ta number

<table>
<thead>
<tr>
<th>Rotation Rate (rpm)</th>
<th>Shear Rate (1/s)</th>
<th>Water Re</th>
<th>Water Ta</th>
<th>Silicon Oil Re</th>
<th>Silicon Oil Ta</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.1</td>
<td>135.5</td>
<td>12.3</td>
<td>0.7</td>
<td>0.1</td>
</tr>
<tr>
<td>2.0</td>
<td>0.3</td>
<td>456.0</td>
<td>41.4</td>
<td>2.3</td>
<td>0.2</td>
</tr>
<tr>
<td>3.5</td>
<td>0.6</td>
<td>804.8</td>
<td>73.0</td>
<td>4.0</td>
<td>0.4</td>
</tr>
</tbody>
</table>

For annular flow between concentric rotating cylinders, the radial velocity profile was also obtained using empirical relationships between radial position and velocity. The results are summarized in Fig. 38, where $R_1$, $R_2$ represent for $r_1$, $r_0$ respectively.

![Radial Velocity Profile](image)

**Figure 38. Radial velocity vs. Radial position**

Considering the above, we could deduce that it was not always laminar flow for the fluid of water but it was laminar flow of silicon oil 200 with the specific shear rate applied on the system. Whereas laminar flow is orderly, turbulent flow is chaotic. The laminar transport of energy (and...
momentum) is by molecular interaction. However, turbulent flow is characterized by much stronger 'eddy' transport, which explains why water responds to shear rate in such way.

In order to understand the problem better, another set of experiment was carried out on the silicon oil with the paddle system shown in Fig. 39 and 40. The paddles were made from sheet metal.

Figure 39. The part for connection between metal stirring paddles and apparatus
The results of this set were compared with previous results for silicon oil, and they were both shown in Fig. 41 below.

![Figure 41. Thermal conductivity vs. shear rate for silicon oil 200 with/without paddles](image-url)
Under the same induced shear rate, the effective thermal conductivity of silicon oil 200 with paddles appeared to be higher than the one without paddles. This is mainly due to the impact of these paddles. The paddles will act as barriers for fluids to pass over them; the original path of the laminar flow was 'destroyed', and there will be 'eddy' current near these paddles. Therefore it improved the mass transport and hence the energy transport of the fluid.
5.2 Effective Thermal Conductivity of Packed Beds

A series of trials were carried out for stationary packed beds of SiO$_2$ ($D_p = 0.8$mm and $D_p = 0.3$mm) for temperatures between $-100$ °C and $-800$ °C, as well as sodium chloride ($D_p = 0.4$mm) for temperatures between $-100$ °C and $-700$ °C. The average particle size $D_p$ was obtained by selective screening in order to achieve a tight size from literature particle distribution. Results obtained from the experiments of packed beds were compared with models for effective thermal conductivity.

One of the models was that proposed by Schotte$^{12}$. The details of the model were introduced in Section (2.2). For the SiO$_2$ used in the trials the particle size was Tyler mesh +20(0.841mm) and −25(0.707mm) for $D_p = 0.8$mm. The gas phase filled in the porosity was dry air, and the porosity was measured at 30%.

As noted earlier, the model applies an additional term to account for radiative transfer. However, this is only applicable to optically thick media, optical thickness being the nondimensional path length in units of mean free photon paths, given by Eq. (1.11) as $\tau_{vA} = \frac{L}{\lambda_p}$. The photon mean free path could be calculated as

$$\lambda_p = \frac{1}{K_e}, \quad (5.4)$$

where $K_e$ is the extinction coefficient, for a monodispersion of spherical particles,

$$K_{s,s,e} = \frac{1.5Q_{s,s,e} f_v}{d}, \quad (5.5)$$
where $Q$ is efficiency, $f_v$ is the void fraction, $d$ is the particle size. $K$ means coefficient, subscripts $s$, $a$, $e$ stand for scattering, absorption and extinction separately.

The void fraction of the tested packed beds was measured as $0.3$, and for particles much larger than the characteristic wavelength of radiation ($\lambda \sim 2900 \mu m \cdot K/1000 K$), which was the case of the particles in tested packed bed, the extinction coefficient is $1$. Therefore the extinction coefficient is approximately

$$K_{s,a,e} = \frac{1.5(1)(0.3)}{d} = \frac{0.45}{d}, \quad (5.6)$$

and the photon mean free path is approximately

$$\lambda_p = \frac{1}{K_e} = \frac{d}{0.45}, \quad (5.7)$$

and the optical thickness is approximately

$$\tau_{oa} = \frac{L}{\lambda_p} = \frac{0.45L}{d} \quad \cdot \quad (5.8)$$

The ratio of the characteristic length, which is the width of the annulus between the concentric cylindrical setup in this case, to the particle size in tested packed beds is big enough ($\tau_{oa} > 3$) to be deemed as optically thick media. Hence, the radiant conductivity could be calculated by using equations (2.15) and (2.16). The results from the experiment and the predictions from models are summarized in Fig. 42 below. According to the tolerance of adopted packed beds models, the error bar was generated in the range of $\pm10\%$ of the original values, and the trend line was generated using linear function according to the apparent trend of those models. As can be observed there is significant variation among results from the various models. In general the
experimental results are in reasonable agreement with the models, providing further confidence in the technique.

In order to examine the effect of particle size, trials were also carried for SiO$_2$ using Tyler mesh +45(0.354) and -50(0.297mm) for $D_p=0.3$mm. The void fraction was again measured at 30%. Results for the fine SiO$_2$ are summarized in Fig. 43. Again, the error bar was generated in the range of ±10% of the original values, and the trend line was generated using linear function.
The experimental results for both particle sizes are shown in Fig. 44.

Figure 44. Comparison of Results of SiO2 packed bed of size 0.3mm and 0.8mm
As can be seen, with fixed porosity and particle size, the effective thermal conductivity of SiO$_2$ packed bed is increased with increasing temperature. And the results are similar if the void fractions are the same and the particle sizes were in the same order of magnitude, unless the temperature is so high that the radiation effect would be very sensitive to small difference in size.

Experiments were also carried out on sodium chloride particles. Results for the tests on sodium chloride particles ($D_p=0.4\text{mm}$) are shown in Fig. 45. The error bar was generated in the range of $\pm 10\%$ of the original values, and the trend line was generated using linear function.

![Graph](image)

**Figure 45. Results for NaCl packed bed of size 0.4mm**

As can be seen in Fig. 45 the measured values for NaCl are in reasonable agreement with the various models.
5.3 Effective Thermal Conductivity of High Temperature Liquid Melts

The third series of tests were performed on liquid oxide melts, both stagnant and with shear flow. The effective thermal conductivities under different shear rate and temperature settings were compared in order to find out the relationships between them.

Tests were done on typical blast furnace slag with composition of mixture A. The effective thermal conductivities under certain temperatures were measured. Since Mikrovas et al.\(^9\) carried out the study of effective thermal conductivity for the same composition before. Their experimental results were also quoted here. The results are summarized in Fig. 46.

\[\begin{array}{c}
\text{Temperature (°C)} \\
1280 & 1290 & 1300 & 1310 & 1320 & 1330 & 1340 & 1350 & 1360 \\
\hline
k (W/m.K) & 5.0 & 4.5 & 4.0 & 3.5 & 3.0 & 2.5 & 2.0 & 1.5 & 1.0 & 0.5 & 0.0 \\
\end{array}\]

Figure 46. Results for stagnant slag with the composition of mixture A
The experimental results for mixture B are summarized in Fig. 47, which indicates that effective thermal conductivity tends to increase with temperature, primarily due to the increasing radiative transport contribution.

![Figure 47. Results of Thermal Conductivity at various shear rate for mixture B](image1)

Similar tests were done on oxide mixture C. At ~1260°C, its effective thermal conductivities at various shear rate were measured.

![Figure 48. Results of Thermal Conductivity at various shear rate for mixture C at ~1260°C](image2)
As shown in Fig. 47 and 58, shear stress was also applied to this system and the effective thermal conductivity appeared to be different. However, the change for the effective thermal conductivities didn’t reach an order of magnitude for the shear rate tested. The results were related with the viscosity of molten slag, which was influenced by its basicity and FeO content. The molar basicities \((\text{CaO + MgO})/\text{SiO}_2\) of the above mixtures were calculated and listed in Table 5 below.

<table>
<thead>
<tr>
<th>Mixture</th>
<th>Molar basicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.07</td>
</tr>
<tr>
<td>B</td>
<td>1.57</td>
</tr>
<tr>
<td>C</td>
<td>1.57</td>
</tr>
</tbody>
</table>

Table 5. Molar basicities of the mixtures to be tested

As discovered by Drissen et al.\(^\text{20}\), viscosity will increase sharply when basicity or iron oxide content is decreased in the molten slag. As shown in Table 5, all the oxides tested were basic and for mixture C, the 30% FeO would ensure low viscosity.

According to the literature review section (2.3), a quantitative analysis for peripheral Reynolds number and Ta number was carried out and the results were shown in Table 6 and 7.

<table>
<thead>
<tr>
<th>Rotation Rate (rpm)</th>
<th>Shear Rate (1/s)</th>
<th>Mix. A</th>
<th>Mix. B</th>
<th>Mix. C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Re</td>
<td>Ta</td>
<td>Re</td>
</tr>
<tr>
<td>0.60</td>
<td>0.10</td>
<td>0.47</td>
<td>0.04</td>
<td>0.87</td>
</tr>
<tr>
<td>2.00</td>
<td>0.30</td>
<td>1.59</td>
<td>0.14</td>
<td>2.95</td>
</tr>
<tr>
<td>3.50</td>
<td>0.60</td>
<td>2.81</td>
<td>0.26</td>
<td>5.20</td>
</tr>
</tbody>
</table>

Table 6. Kinematic Viscosity of oxides mixtures to be tested

<table>
<thead>
<tr>
<th>Rotation Rate (rpm)</th>
<th>Shear Rate (1/s)</th>
<th>Mix. A</th>
<th>Mix. B</th>
<th>Mix. C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.60</td>
<td>0.10</td>
<td>0.47</td>
<td>0.04</td>
<td>0.87</td>
</tr>
<tr>
<td>2.00</td>
<td>0.30</td>
<td>1.59</td>
<td>0.14</td>
<td>2.95</td>
</tr>
<tr>
<td>3.50</td>
<td>0.60</td>
<td>2.81</td>
<td>0.26</td>
<td>5.20</td>
</tr>
</tbody>
</table>

Table 7. Applied shear rates and their corresponding Re, Ta number
As can be seen from Table 7, the Re and Ta numbers are not high. According to the analysis in section 5.1, this means the shear rate applied on the system was not high enough to cause turbulent flow. Therefore the effective conductivity of the slag appeared to have no dramatic change with the applied shear rate.

In the above experiments, the effective conductivity always increased with increasing temperature. This is due to strong radiative effect under high temperature. In order to understand the problem better, an analysis for the relative radiative effect was given.

For the above experiments, in the absence of convection, the general equation of heat transfer for combined radiation and conduction \(^{26}\) is

\[
\rho c_p \frac{\partial T}{\partial t} + \nabla \cdot (q_c + q_r) = q_m ,
\]

(5.9)

where

\[
\nabla \cdot q_c = \nabla \cdot (-k_c \nabla T) ,
\]

(5.10)

and

\[
\nabla \cdot q_r = \int_0^\infty \alpha_\lambda \left[ 4e_{b\lambda}(\lambda, T) - \int_{\alpha=0}^{4\pi} i_\lambda(\lambda, W) dW \right] d\lambda .
\]

(5.11)

When the steady state is reached, according to Equation (5.10), there will be

\[
\nabla \cdot (q_c + q_r) = 0 .
\]

(5.12)

From the dimension of the apparatus and general monochromatic absorption coefficients of slags, it could be derived that the system under investigation belonged to optically thick system. For optically thick systems, the diffusion approximation can be used to represent the radiative flux.
q_r = -k_r(T)\nabla T, \hspace{1cm} (5.13)

a radiative conductivity can be defined for optically thick media using the Rosseland mean extinction coefficient

\[ k_r(T) = \frac{16\sigma T^3}{3K_{er}}, \hspace{1cm} (5.14) \]

and

\[ K_{er} = \frac{\alpha_m}{\eta^2}, \hspace{1cm} (5.15) \]

where \( \alpha_m \) is the mean absorption coefficient, \( \eta \) is the refractive index, \( \sigma \) is the Stefan-Boltzmann constant.

Equation (5.14) could be written as

\[ \nabla q_r = \nabla \cdot (-k_r \nabla T). \hspace{1cm} (5.16) \]

Combining equations (5.10), (5.11) and (5.16), it gave

\[ \nabla \cdot (k_r \nabla T + k_c \nabla T) = 0. \hspace{1cm} (5.17) \]

The two coefficients could be combined to give the effective thermal conductivity

\[ k_{eff} = k_r + k_c. \hspace{1cm} (5.18) \]

Applying equation (5.18) in (5.17) and integrating, the total heat flux due to combined radiation and conduction could be written as

\[ q_{total} = -k_{eff} \nabla T. \hspace{1cm} (5.19) \]
A quantitative analysis is performed to evaluate the effect of temperature on the radiant conductivity based on the study of Fine et al. The minimum absorption coefficients, in the range of significant thermal energy, were calculated. It was also indicated that the absorption coefficient measured at room temperature gave approximate values at 1300°C to 1500°C. By using the absorption coefficient of glasses that had similar composition with the system in this study. Applying equations (5.14) and (5.15), the radiant conductivity under certain temperatures could be obtained.

![Graph](image)

**Figure 49. Results of Radiant conductivity vs. Temperature, based on the monochromatic absorption coefficient from the study of Fine et al.**

The discussion above was for the situation without or with very little convection effects. When convection effects were large and were involved, the problem became different and complicated. The relative equations and theories were introduced in section 5.1, and empirical equations describing the flow behavior were needed in order to further analyze the problem. The factors needed to be considered were velocity gradients, turbulent intensity, intrinsic properties of fluid and positions as well.
6.0 CONCLUSIONS

As indicated in the Scope and Objectives a new steady-state method was developed for measuring the effective thermal conductivity of granular solids and liquids over the range of temperature of interest in extractive metallurgy. In contrast to existing methods that involve numerical solution of the transient conduction equation to interpret the data, the technique allows calculation of effective thermal conductivity by simultaneously measuring the steady-state temperature drop and heat transfer rate through the material contained within the annulus between concentric cylinders. In order to meet the specific needs of a parallel program for the development of a rotary scrap-melting furnace, by introducing rotation for the inner cylinder the method allows for measurement of effective conductivity in non-stagnant liquids with rotation-induced flow.

For stagnant systems the technique was validated at low temperature against existing conductivity data for both water and silicon oil and good agreement was obtained. For intermediate temperatures trials were also carried out using several granular materials at temperatures up to about 800°C and results were shown to be within the range indicated by some common models for predicting conductivity of packed beds. The results show that effective thermal conductivity of packed beds increases with increasing temperature due to the radiative contribution to heat transfer. For the same kind of particle with fixed void fraction, minor size differences will not significantly influence the effective thermal conductivity. High temperature validation was obtained against existing data for oxide melts (40%CaO – 40%SiO₂ – 20%Al₂O₃) and again good agreement was observed.
For non-stagnant systems, of the liquids tested, water, silicon oil and oxide melts, only the former showed the rotation-induced flow to have any significant effect on effective thermal conductivity. This was explained by the calculated peripheral Reynolds and Taylor numbers for the systems that indicted laminar flow for oil and oxide melts and turbulent flow for water. For water with turbulent flow and at shear rates up to $0.55 \text{ sec}^{-1}$, the effective conductivity increased by a factor up to $\approx 2.7$ which is well short of the order of magnitude increase deemed desirable for heat transfer through the slag layer in the rotary scrap-melting furnace. For silicon oil with paddle mixers and at shear rates up to $0.55 \text{ sec}^{-1}$, the effective conductivity showed only a small increase.
7.0 FUTURE WORK

Following from the work, several theoretical and practical issues warrant further investigation.

1. The limited number of synthetic oxides used in the present work gave insufficient information about the relationship between composition and effective thermal conductivity. It would be of interest to perform experiments for a series of melts with a wide range of composition, such as varied iron oxide content. Such a sample range would allow for the assessment of the impact of varying composition on the effective thermal conductivity.

2. The shear rate covered in this study was limited by the experimental setup. Therefore, it would be helpful to utilize improved experimental setup, which was based on the same theory as in the present work, to perform experiments with a wide range of the intensity of shear flow. This type of investigation would help to give deeper understanding of the relationships between shear flow and the effective thermal conductivity.

3. Because the induction furnace placed constraints on both the temperature range and the heating rate, either the heating efficiency and/or the unit might be increased, or an alternative such as resistance heating might be considered.
REFERENCES


Appendix A. Code for Thermal Inertia Analysis

Dim Tcrucible(10)
Dim bb(100), T(100)
Dim aa(100, 100)

Sub GaussSeidel()
Dim acof(100)
I = Worksheets("inner_crucible").Cells(8, 8)
Do
    flag = 0
    For m = 1 To I
        Told = T(m)
        bb(m) = Worksheets("inner_crucible").Cells(25 + m, 12)
        Sum = bb(m)
        For k = 1 To I
            acof(k) = Worksheets("inner_crucible").Cells(25 + m, k)
            If k <> m Then
                Sum = Sum - acof(k) * T(k)
            End If
        Next k
        T(m) = Sum / acof(m)
        If (T(m) - Told) / T(m) >= 0.00001 Then
            flag = 1
        End If
    Next m
Loop While flag = 1
End Sub

Sub implicitmatrixa()
area = Worksheets("inner_crucible").Cells(11, 8)
h = Worksheets("inner_crucible").Cells(3, 4)
I = Worksheets("inner_crucible").Cells(8, 8)
For ni = 1 To 100
    For nj = 1 To 100
        aa(ni, nj) = 0#
    Next nj
Next ni
n = 1
Do
    xup = Worksheets("inner_crucible").Cells(10 + n, 12)
xdown = Worksheets("inner_crucible").Cells(10 + n, 11)
xv = Worksheets("inner_crucible").Cells(10 + n, 16)
    If n = 1 Then
        aa(n, 1) = -xv - h * areain - xup
        aa(n, 2) = xup
    End If
Next n
Next m
Loop While n <= 100
End Sub
Worksheets("inner_crucible").Cells(25 + n, 1) = aa(n, 1)
Worksheets("inner_crucible").Cells(25 + n, 2) = aa(n, 2)
ElseIf n = I Then
aa(n, n - 1) = xdown
aa(n, n) = -xv - xdown
Else
aa(n, n - 1) = xdown
aa(n, n) = -xup - xdown - xv
aa(n, n + 1) = xup
Worksheets("inner_crucible").Cells(25 + n, n - 1) = aa(n, n - 1)
Worksheets("inner_crucible").Cells(25 + n, n) = aa(n, n)
Worksheets("inner_crucible").Cells(25 + n, n + 1) = aa(n, n + 1)
End If
n = n + 1
Loop While n <= I

Sub implicitmatrixb()
qin = Worksheets("inner_crucible").Cells(3, 5)
areain = Worksheets("inner_crucible").Cells(11, 8)
qout = Worksheets("inner_crucible").Cells(3, 6)
areaout = Worksheets("inner_crucible").Cells(19, 8)
h = Worksheets("inner_crucible").Cells(3, 4)
l = Worksheets("inner_crucible").Cells(8, 8)
Tf = Worksheets("inner_crucible").Cells(3, 7)
Ti = Worksheets("inner_crucible").Cells(3, 8)
deltaj = Worksheets("inner_crucible").Cells(8, 10)
For m = 1 To I
Worksheets("inner_crucible").Cells(40, m + 1) = m
Next m
n = 1
Do
Tsum = 0
volume = 0
Worksheets("inner_crucible").Cells(40 + n, 1) = (n - 1) * deltax
If n = 1 Then
For m = 1 To I
xv = Worksheets("inner_crucible").Cells(10 + m, 16)
T(m) = Ti
Tsum = Tsum + T(m) * xv
volume = volume + xv
Worksheets("inner_crucible").Cells(40 + n, m + 1) = T(m)
Next m
Else
GaussSeidel
For m = 1 To I
xv = Worksheets("inner_crucible").Cells(10 + m, 16)
Tsum = Tsum + T(m) * xv
Next m
End If
n = n + 1
Loop While n <= I
End Sub
```vbnet
volume = volume + xv
Worksheets("inner_crucible").Cells(40 + n, m + 1) = T(m)
Next m
End If
For m = 1 To I
xv = Worksheets("inner_crucible").Cells(10 + m, 16)
If m = 1 Then
    bb(m) = -xv * T(m) - h * areain * Tf
ElseIf m = I Then
    bb(m) = -xv * T(m) - qout * areaout
Else
    bb(m) = -xv * T(m)
End If
Worksheets("inner_crucible").Cells(25 + m, 12) = bb(m)
Next m
Tavg = Tsum / volume
Worksheets("inner_crucible").Cells(40 + n, I + 2) = Tavg
Tdif = T(1) - T(I)
Worksheets("inner_crucible").Cells(40 + n, I + 3) = Tdif
n = n + 1
Loop While (n - 1) * delta_t <= 600
End Sub

Sub inner()
    implicitmatrixa
    implicitmatrixb
End Sub```
### Appendix B. Typical Experimental Data

#### Table C-1 Experimental Results for Water (Trial 1)

| $t_{i\text{n}}$-in(°C) | $t_{i\text{out}}$-out(°C) | flowrate(m3/s) | $q$(W) | $r_{in}$(m) | $r_{out}$(m) | $l$ (m) | $T_{in}$(°C) | $T_{out}$(°C) | $\Delta T$(°C) | $k$(W/m.K) | $\tau$ (1/s) |
|------------------------|------------------------|----------------|--------|-------------|-------------|--------|-------------|-------------|-------------|-------------|-------------|-------------|
| 20.3                   | 29.8                   | 9.44E-04       | 11.66  | 0.02        | 0.03        | 0.20   | 42.2        | 53.3        | 11.1        | 0.43        | 0           |
| 21.0                   | 36.9                   | 9.44E-04       | 19.51  | 0.02        | 0.03        | 0.20   | 44.3        | 57.1        | 12.8        | 0.62        | 0.093       |
| 21.0                   | 38.2                   | 9.44E-04       | 21.11  | 0.02        | 0.03        | 0.20   | 51.3        | 59.1        | 7.8         | 1.10        | 0.314       |
| 21.0                   | 38.0                   | 9.44E-04       | 20.86  | 0.02        | 0.03        | 0.20   | 54.1        | 61.3        | 7.2         | 1.18        | 0.554       |

#### Table C-2 Experimental Results for Water (Trial 2)

| $t_{i\text{n}}$-in(°C) | $t_{i\text{out}}$-out(°C) | flowrate(m3/s) | $q$(W) | $r_{in}$(m) | $r_{out}$(m) | $l$ (m) | $T_{in}$(°C) | $T_{out}$(°C) | $\Delta T$(°C) | $k$(W/m.K) | $\tau$ (1/s) |
|------------------------|------------------------|----------------|--------|-------------|-------------|--------|-------------|-------------|-------------|-------------|-------------|-------------|
| 21.3                   | 31.5                   | 9.44E-04       | 12.52  | 0.02        | 0.03        | 0.20   | 40.90       | 53.30       | 12.40       | 0.41        | 0           |
| 21.3                   | 36.9                   | 9.44E-04       | 19.14  | 0.02        | 0.03        | 0.20   | 45.70       | 57.10       | 11.40       | 0.68        | 0.093       |
| 21.3                   | 38.2                   | 9.44E-04       | 20.74  | 0.02        | 0.03        | 0.20   | 51.30       | 59.10       | 7.80        | 1.08        | 0.314       |
| 21.3                   | 39.0                   | 9.44E-04       | 21.72  | 0.02        | 0.03        | 0.20   | 52.90       | 61.00       | 8.10        | 1.09        | 0.554       |

#### Table C-3 Experimental Results for Silicon Oil 200 (Trial 1)

| $t_{i\text{n}}$-in(°C) | $t_{i\text{out}}$-out(°C) | flowrate(m3/s) | $q$(W) | $r_{in}$(m) | $r_{out}$(m) | $l$ (m) | $T_{in}$(°C) | $T_{out}$(°C) | $\Delta T$(°C) | $k$(W/m.K) | $\tau$ (1/s) |
|------------------------|------------------------|----------------|--------|-------------|-------------|--------|-------------|-------------|-------------|-------------|-------------|-------------|
| 19.0                   | 28.9                   | 9.44E-04       | 12.15  | 0.02        | 0.03        | 0.15   | 57.7        | 101.9       | 44.2        | 0.151       | 0           |
| 19.0                   | 29.5                   | 9.44E-04       | 12.88  | 0.02        | 0.03        | 0.15   | 58.2        | 102.1       | 43.9        | 0.161       | 0.093       |
| 19.0                   | 29.5                   | 9.44E-04       | 12.88  | 0.02        | 0.03        | 0.15   | 59.3        | 102.2       | 42.9        | 0.165       | 0.314       |
| 19.0                   | 29.3                   | 9.44E-04       | 12.64  | 0.02        | 0.03        | 0.15   | 60.8        | 102.3       | 41.5        | 0.167       | 0.554       |

#### Table C-4 Experimental Results for Silicon Oil 200 (Trial 2)

| $t_{i\text{n}}$-in(°C) | $t_{i\text{out}}$-out(°C) | flowrate(m3/s) | $q$(W) | $r_{in}$(m) | $r_{out}$(m) | $l$ (m) | $T_{in}$(°C) | $T_{out}$(°C) | $\Delta T$(°C) | $k$(W/m.K) | $\tau$ (1/s) |
|------------------------|------------------------|----------------|--------|-------------|-------------|--------|-------------|-------------|-------------|-------------|-------------|-------------|
| 16.5                   | 31.0                   | 9.44E-04       | 17.79  | 0.02        | 0.03        | 0.15   | 40.90       | 107.50      | 66.60       | 0.147       | 0           |
| 16.5                   | 31.5                   | 9.44E-04       | 18.41  | 0.02        | 0.03        | 0.15   | 44.50       | 107.80      | 63.30       | 0.160       | 0.093       |
| 16.5                   | 31.8                   | 9.44E-04       | 18.77  | 0.02        | 0.03        | 0.15   | 48.90       | 108.80      | 59.90       | 0.172       | 0.314       |
| 16.5                   | 31.8                   | 9.44E-04       | 18.77  | 0.02        | 0.03        | 0.15   | 52.40       | 108.50      | 56.10       | 0.184       | 0.554       |

#### Table C-5 Experimental Results for Silicon Oil 200 (With Paddle)

| $t_{i\text{n}}$-in(°C) | $t_{i\text{out}}$-out(°C) | flowrate(m3/s) | $q$(W) | $r_{in}$(m) | $r_{out}$(m) | $l$ (m) | $T_{in}$(°C) | $T_{out}$(°C) | $\Delta T$(°C) | $k$(W/m.K) | $\tau$ (1/s) |
|------------------------|------------------------|----------------|--------|-------------|-------------|--------|-------------|-------------|-------------|-------------|-------------|-------------|
| 24.0                   | 26.6                   | 9.44E-04       | 3.19   | 0.02        | 0.03        | 0.15   | 44.00       | 55.10       | 11.10       | 0.158       | 0           |
| 24.0                   | 28.6                   | 9.44E-04       | 5.64   | 0.02        | 0.03        | 0.15   | 42.10       | 59.30       | 17.20       | 0.180       | 0.093       |
| 24.0                   | 29.8                   | 9.44E-04       | 7.12   | 0.02        | 0.03        | 0.15   | 42.50       | 61.20       | 18.70       | 0.209       | 0.314       |
| 24.0                   | 30.3                   | 9.44E-04       | 7.73   | 0.02        | 0.03        | 0.15   | 43.70       | 62.60       | 18.90       | 0.225       | 0.554       |
Table C-6 Experimental Results for SiO$_2$ ($D_p=0.8$mm)

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<th>$r_1$ (m)</th>
<th>$r_2$ (m)</th>
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Table C-7 Experimental Results for SiO$_2$ ($D_p=0.3$mm)

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<th>$r_2$ (m)</th>
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<th>$T_{1\text{out}}$ (°C)</th>
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Table C-8 Experimental Results for NaCl ($D_p=0.4$mm)

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<th>$q$ (W)</th>
<th>$r_1$ (m)</th>
<th>$r_2$ (m)</th>
<th>L (m)</th>
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<th>$T_{1\text{out}}$ (°C)</th>
<th>$\Delta T$ (°C)</th>
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Table C-9 Experimental Results for Oxide Mixture A

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Table C-10 Experimental Results for Oxide Mixture B

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Table C-11 Experimental Results for Oxide Mixture C

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<th>$t_q$-in(${}^\circ$C)</th>
<th>$t_q$-out(${}^\circ$C)</th>
<th>flowrate(m3/s)</th>
<th>$q$ (w)</th>
<th>$r_1$ (m)</th>
<th>$r_2$ (m)</th>
<th>$L$ (m)</th>
<th>$T_{in}$(${}^\circ$C)</th>
<th>$T_{out}$(${}^\circ$C)</th>
<th>$\Delta T$(${}^\circ$C)</th>
<th>$k$ (W/m.K)</th>
<th>$\tau$ (1/s)</th>
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Appendix C. Schotte’s Model for Radiation Effect

A brief introduction is given here to introduce Schotte’s model, which was adopted in this study for predicting the radiation effect in a packed bed. A simplified analysis was made for spherical particles, but Schotte declared experimental data indicated that the final correlation would apply also to other shapes.

When one considered the radiation from a plane located on one side of a particle to a plane located on the other side of the particle, there are two parts to be considered. There is radiation across the void space, which could be written as

\[ q = -h_1 \frac{\pi D_p^2}{4} \left( \frac{\delta}{1 - \delta} \right) (D_p \frac{dt}{dx}), \]  

(C.1)

and there is radiation from the particle in series with conduction through the particle. For an element of the particle, shown in Fig. 52, the rate of heat transfer from left to right would be

\[ q = -k_s (2\pi dz) \frac{t_2 - t_1}{2x} = h_r (2\pi dz)(t_2 - t_g) = h_r (2\pi z) \left( \frac{rdz}{x} \right)(t_2 - t_g). \]  

(C.2)

So there is

\[ t_2 = \frac{k_s t_1 + 2h_r t_g}{k_s + 2h_r r}. \]  

(C.3)

and there was an assumption that the temperature gradient, for the solid-gas system (packed bed) is constant for a distance of 1 particle diameter. Then

\[ t_1 - t_g = -2x \left( \frac{dt}{dx} \right). \]  

(C.4)

And equation (B.2) becomes

\[ q = -\frac{\pi}{4} D_p^2 \left( \frac{k_s h_r D_p}{k_s + h_r D_p} \right) \left( \frac{dt}{dx} \right). \]  

(C.5)
Adding these two parts of radiation together, one gets

\[ q = -\frac{\pi}{4} D_p^2 \left(\frac{k_s h_r D_p}{k_s + h_r D_p} \right) \frac{dt}{dx} - h_r \left(\frac{\pi}{4} D_p^2 \frac{\delta}{1 - \delta} \right) D_p \frac{dt}{dx}. \]  

(C.6)

The rate of heat transfer across the total area can be written as

\[ q = -k_r \frac{\pi}{4} D_p^2 \left(\frac{1}{1 - \delta} \right) \frac{dt}{dx}, \]  

(C.7)

by equating the right-hand sides of equations of (C.6) and (C.7), one gets
\[ k_r = \frac{1 - \delta}{\frac{1}{k_s} + \frac{1}{k_r^0}} + \delta k_r^0, \quad (C.8) \]

and there is

\[ k_r^0 = h_r D_p, \quad (C.9) \]

and the radiation heat transfer coefficient

\[ h_r = 0.692 \varepsilon \frac{T^3}{10^8}, \quad (C.10) \]

then

\[ k_r^0 = 0.692 \varepsilon D_p \frac{T^3}{10^8}. \quad (C.11) \]

By combining equations (C.11) and (C.8), one would get the radiation contribution \( k_r \).