MATHEMATICAL MODELING OF MICROPOROSITY
IN A356 ALUMINUM ALLOY

by

PHUONG VO

B.A.Sc. The University of British Columbia, 1999

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

in

THE FACULTY OF GRADUATE STUDIES

(Department of Metals and Materials Engineering)

We accept this thesis as conforming
to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

December 2001

© Phuong Vo, 2001
In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of **METALS AND MATERIALS ENGINEERING**

The University of British Columbia
Vancouver, Canada

Date **DECEMBER 5, 2001**
Abstract

Die cast aluminum wheels are one of the most difficult automotive castings to produce because of stringent cast surface and internal quality requirements. As part of a collaborative research agreement between researchers at the University of British Columbia and Canadian Autoparts Toyota Inc., work has been underway to predict heat transport and porosity formation in die cast A356 wheels. Preliminary work has focused on assessing a number of criteria functions previously proposed in the literature. Model results in the form of temperature and criteria function predictions are compared with experimentally measured temperature and porosity data obtained from a directionally chilled A356 aluminum alloy solidified under conditions resembling those found in an industrial die-casting operation. The results suggest that the Niyama function is best suited to qualitatively predict porosity of the four criteria functions examined. However, all criteria functions, including the Niyama, do not appear to be well suited to predict the amount of porosity quantitatively as they fail to include the effect of varying hydrogen and inclusion content. A 2-D axisymmetric mathematical model incorporating flow through the interdendritic network and the thermodynamics of hydrogen solubility has been developed. This model has been successfully applied to the prediction of the amount of porosity in a series of test castings with varying hydrogen content.
# Table of Contents

Abstract.................................................................................................................. ii

List of Tables ........................................................................................................... v

List of Figures .......................................................................................................... vi

List of Symbols ......................................................................................................... ix

Acknowledgements ................................................................................................. xii

1 Introduction ........................................................................................................... 1

2 Literature Review ................................................................................................ 5

2.1 Formation of Microporosity ............................................................................. 5

2.1.1 Mechanisms .............................................................................................. 5

2.1.2 Factors ....................................................................................................... 8

2.2 Criteria Functions ........................................................................................... 11

2.2.1 Development of Criteria Functions ............................................................ 11

2.2.2 Evaluation of Criteria Functions ................................................................. 15

2.2.3 Discussion .................................................................................................. 17

2.3 Hydrogen-Shrinkage Models ......................................................................... 19

2.3.1 Fang and Granger ..................................................................................... 20

2.3.2 Huang, Mori, and Conley ......................................................................... 26

2.3.3 Discussion .................................................................................................. 30

2.4 Discussion ......................................................................................................... 33

3 Scope and Objectives ........................................................................................... 35

4 Experimental Work .............................................................................................. 37

4.1 Setup ................................................................................................................ 37

4.2 Results .............................................................................................................. 42

5 Mathematical Model ............................................................................................. 54

5.1 Heat Transfer Model ....................................................................................... 54

5.2 Criteria Function Model .................................................................................. 60

5.3 Hydrogen-Shrinkage Model .......................................................................... 62

5.4 Sensitivity Analysis ......................................................................................... 68

6 Discussion ............................................................................................................ 74

6.1 Criteria Function Analysis ............................................................................. 75

6.2 Microporosity Analysis ................................................................................... 84
List of Tables

Table 1.1: Nominal composition of A356 alloy................................................................. 2
Table 2.1: Relationship between grain size and cooling rate in the grain refined castings
       of A356 in the experiment......................................................................................... 25
Table 2.2: Comparison of experimental and calculated area fraction porosity.................. 30
Table 4.1: Summary of casting conditions in Trials 1 and 2............................................. 40
Table 5.1: Material properties used in the model............................................................... 56
Table 5.2: Values of heat transfer boundary condition variables...................................... 60
Table 5.3: Criteria functions predicted by model.............................................................. 60
Table 5.4: Properties of A356 and constants used in microporosity calculations.............. 68
Table 5.5: Sensitivity of model to selected parameters varied by +/- 20% (High/Low)........ 73
Table B.1: Lower limits of conductive heat transfer for test castings.............................. 104
List of Figures

Figure 1.1: Schematics of low-pressure (a) and vacuum casting (b) units used with permanent moulds .......................................................... 4
Figure 2.1: Solubility of hydrogen in aluminum and aluminum alloys. (C is the dissolved concentration and $C_{\text{unit}}$ is defined as 1 cm3/100g at 0°C and 1 atm) .......................................................... 6
Figure 2.2: Schematic representation of pore formation in an equiaxed grain casting produced by directional solidification with (a), (b), and (c) representing sequentially larger magnification in area .......................................................... 21
Figure 2.3: The calculated pore diameters as a function of grain size at a hydrogen content of 0.31 cc/100g in A356 alloy ........................................................................................................ 25
Figure 2.4: The calculated pore diameters as a function of hydrogen content at a cooling rate of 6 C/s ......................................................................................................................... 26
Figure 2.5: Schematic for the cellular automation used to simulate gas pore growth .................................................................................. 29
Figure 2.6: Area fraction of porosity as a function of solid fraction for different hydrogen contents ..................................................................................................................... 29
Figure 4.1: Schematic of experimental setup .................................................................................................................................................. 38
Figure 4.2: Sectioned test casting ........................................................................................................................................................................ 41
Figure 4.3: Measurement of porosity using raster .............................................................................................................................................. 42
Figure 4.4: Cooling curve for test casting. (Trial 1 – 1 min degas) .............................................................................................................................................. 43
Figure 4.5: Cooling curve for test casting. (Trial 2 – 2.5 min degas) .............................................................................................................................................. 44
Figure 4.6: Measured hydrogen in test castings .................................................................................................................................................. 45
Figure 4.7: Measured inclusion content in test castings .......................................................................................................................................... 46
Figure 4.8: Typical etched samples from (a) Trial 1 and (b) Trial 2 displaying a predominantly equiaxed microstructure ........................................................................................................ 48
Figure 4.9: Typical pores in test casting ................................................................................................................................................................. 49
Figure 4.10: Polished sample displaying fine microporosity and isolated areas of macroporosity. (Trial 1 – No degas) ........................................................................................................ 49
Figure 4.11: Polished sample displaying a high level of macroporosity near the top of the casting. (Trial 2 – 1 min degas) ........................................................................................................ 49
Figure 4.12: Variation of porosity with distance from the chill. (Trial 1 – 3 min degas) ......................................................................................................................... 50
Figure 4.13: Average porosity in test castings from Trial 1 ................................................................................................................................ 51
Figure 4.14: Average porosity in test castings from Trial 2 ................................................................................................................................ 52
Figure 5.1: Evolution of solid fraction with temperature ................................................................................................................................. 55
Figure 5.2: FEM mesh of model ........................................................................................................................................................................ 57
Figure 5.3: Variation of critical local pressure and hydrogen gas pressure with fraction solid for test casting. (Trial 1 – 1 min degas - 45 mm from chill) ........................................................................................................ 66
Figure 5.4: Sensitivity of local solidification time to $h_{\text{cond,cc}}$ and $\Delta T_{\text{gap}}$ ......................................................................................................................... 69
Figure 5.5: Sensitivity of fraction porosity to $h_{\text{cond,cc}}$ and $\Delta T_{\text{gap}}$ ........................................................................................................................................ 70
Figure 5.6: Sensitivity of fraction porosity to microporosity model parameters ........................................................................................................ 71
Figure 5.7: Variation in fraction porosity with distance from the chill with +/-20% change in microporosity model parameters ........................................................................................................ 72
Figure 6.1: Contour plot of temperatures (°C) in test casting at 250 sec. (Trial 1 - 1 min degas) ........................................................................................................................................ 74
Figure 6.2: Predicted cooling curves and experimental data. (Trial 1 – 1 min degas) ......................................................................................................................... 75
Figure 6.3: Profile of thermal parameters of criteria functions of a typical casting. (Trial 1 – 1 min degas) ................................................................. 77
Figure 6.4: Profile of criteria function values along the vertical centreline of a typical casting. (Trial 1 – 1 min degas) ................................................................. 79
Figure 6.5: Profile of criteria function values along the vertical centreline of Trial 2 – 1 min degas sample ................................................................. 80
Figure 6.6: Profile of thermal parameters of criteria functions of Trial 2 – 1 min degas sample ....................................................................................... 81
Figure 6.7: Niyama function calculations for castings. (Trial 1) ................................................................. 83
Figure 6.8: Comparison of predicted and measured porosity in (a) high hydrogen and (b) low hydrogen level test castings for Trial 1 ................................................................. 85
Figure 6.9: Comparison of predicted and measured porosity in (a) high hydrogen level and (b) low hydrogen level test castings for Trial 2 ................................................................. 86
Figure A.1: (a) Polished and (b) etched surface of Trial 1 – 0 min degas sample. ................................................................. 98
Figure A.2: (a) Polished and (b) etched surface of Trial 1 – 1 min degas sample. ................................................................. 99
Figure A.3: (a) Polished and (b) etched surface of Trial 1 – 2 min degas sample. ................................................................. 99
Figure A.4: (a) Polished and (b) etched surface of Trial 1 – 3 min degas sample. ................................................................. 100
Figure A.5: (a) Polished and (b) etched surface of Trial 1 – 4 min degas sample. ................................................................. 100
Figure A.6: (a) Polished and (b) etched surface of Trial 2 – 0.5 min degas sample. ................................................................. 101
Figure A.7: (a) Polished and (b) etched surface of Trial 2 – 1 min degas sample. ................................................................. 101
Figure A.8: (a) Polished and (b) etched surface of Trial 2 – 1.5 min degas sample. ................................................................. 102
Figure A.9: (a) Polished and (b) etched surface of Trial 2 – 2 min degas sample. ................................................................. 102
Figure A.10: (a) Polished and (b) etched surface of Trial 2 – 2.5 min degas sample. ................................................................. 103
Figure A.11: (a) Polished and (b) etched surface of Trial 2 – 3 min degas sample. ................................................................. 103
Figure C.1: Predicted and experimental temperature vs time of Trial 1 – 0 min degas sample. ................................................................. 106
Figure C.2: Predicted and experimental porosity vs distance from the chill of Trial 1 – 0 min degas sample. ................................................................. 106
Figure C.3: Predicted and experimental temperature vs time of Trial 1 – 1 min degas sample. ................................................................. 107
Figure C.4: Predicted and experimental porosity vs distance from the chill of Trial 1 – 1 min degas sample. ................................................................. 107
Figure C.5: Predicted and experimental temperature vs time of Trial 1 – 2 min degas sample. ................................................................. 108
Figure C.6: Predicted and experimental porosity vs distance from the chill of Trial 1 – 2 min degas sample. ................................................................. 108
Figure C.7: Predicted and experimental temperature vs time of Trial 1 – 3 min degas sample. ................................................................. 109
Figure C.8: Predicted and experimental porosity vs distance from the chill of Trial 1 – 3 min degas sample. ................................................................. 109
Figure C.9: Predicted and experimental temperature vs time of Trial 1 – 4 min degas sample. ................................................................. 110
Figure C.10: Predicted and experimental porosity vs distance from the chill of Trial 1 – 4 min degas sample. ................................................................. 110
Figure C.11: Predicted and experimental temperature vs time of Trial 1 – 0.5 min degas sample. ................................................................. 111
Figure C.12: Predicted and experimental porosity vs distance from the chill of Trial 1 – 0.5 min degas sample. ................................................................. 111
Figure C.13: Predicted and experimental temperature vs time of Trial 1 – 1 min degas sample. ........................................................................................................ 112
Figure C.14: Predicted and experimental porosity vs distance from the chill of Trial 1 – 1 min degas sample. ................................................................. 112
Figure C.15: Predicted and experimental temperature vs time of Trial 1 – 1.5 min degas sample. ........................................................................................................ 113
Figure C.16: Predicted and experimental porosity vs distance from the chill of Trial 1 – 1.5 min degas sample. ................................................................. 113
Figure C.17: Predicted and experimental temperature vs time of Trial 1 – 2 min degas sample. ........................................................................................................ 114
Figure C.18: Predicted and experimental porosity vs distance from the chill of Trial 1 – 2 min degas sample. ................................................................. 114
Figure C.19: Predicted and experimental temperature vs time of Trial 1 – 2.5 min degas sample. ........................................................................................................ 115
Figure C.20: Predicted and experimental porosity vs distance from the chill of Trial 1 – 2.5 min degas sample. ................................................................. 115
Figure C.21: Predicted and experimental temperature vs time of Trial 1 – 3 min degas sample. ........................................................................................................ 116
Figure C.22: Predicted and experimental porosity vs distance from the chill of Trial 1 – 3 min degas sample. ................................................................. 116
# List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1, A_2$</td>
<td>Areas of solid-gas and liquid-gas interfaces</td>
<td>($m^2$)</td>
</tr>
<tr>
<td>$C_0$</td>
<td>Feeding resistance number</td>
<td></td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat</td>
<td>($J/kg \cdot ^\circ C$)</td>
</tr>
<tr>
<td>$c$</td>
<td>Geometric factor</td>
<td></td>
</tr>
<tr>
<td>$D_0$</td>
<td>Equiaxed grain size</td>
<td>($m$)</td>
</tr>
<tr>
<td>$F_{drag}$</td>
<td>Friction drag on interdendritic liquid</td>
<td></td>
</tr>
<tr>
<td>$f_{cond}$</td>
<td>Fraction of conductive heat transfer</td>
<td></td>
</tr>
<tr>
<td>$f_l$</td>
<td>Fraction liquid</td>
<td></td>
</tr>
<tr>
<td>$f_{lim}$</td>
<td>Lower limit to the fraction of conductive heat transfer</td>
<td></td>
</tr>
<tr>
<td>$f_v$</td>
<td>Volume fraction porosity</td>
<td>($m/s^2$)</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration due to gravity</td>
<td></td>
</tr>
<tr>
<td>$g_e$</td>
<td>As-cast eutectic volume fraction</td>
<td></td>
</tr>
<tr>
<td>$G$</td>
<td>Thermal Gradient</td>
<td>($^\circ C/m$)</td>
</tr>
<tr>
<td>$G_{crit}$</td>
<td>Thermal gradient at critical temperature</td>
<td>($^\circ C/m$)</td>
</tr>
<tr>
<td>$G_T$</td>
<td>Thermal gradient at temperature $T$</td>
<td>($^\circ C/m$)</td>
</tr>
<tr>
<td>$[H_0]$</td>
<td>Initial hydrogen content</td>
<td>($cm^3/100g$)</td>
</tr>
<tr>
<td>$[H_s], [H_l]$</td>
<td>Hydrogen concentration in solid and liquid</td>
<td>($cm^3/100g$)</td>
</tr>
<tr>
<td>$h$</td>
<td>Height of metallostatic head</td>
<td>($m$)</td>
</tr>
<tr>
<td>$h_{eff}$</td>
<td>Effective heat transfer coefficient</td>
<td>($W/m^2 \cdot ^\circ C$)</td>
</tr>
<tr>
<td>$h_{cond}$</td>
<td>Conductive heat transfer coefficient</td>
<td>($W/m^2 \cdot ^\circ C$)</td>
</tr>
<tr>
<td>$h_{conv}$</td>
<td>Convective heat transfer coefficient</td>
<td>($W/m^2 \cdot ^\circ C$)</td>
</tr>
<tr>
<td>$h_{cond,cc}, h_{cond,cm}, h_{cond,cw}$</td>
<td>Conductive heat transfer coefficients for casting/chill, casting/mould, and chill/water interfaces</td>
<td>($W/m^2 \cdot ^\circ C$)</td>
</tr>
<tr>
<td>$h_{rad}$</td>
<td>Radiative heat transfer coefficient</td>
<td>($W/m^2 \cdot ^\circ C$)</td>
</tr>
<tr>
<td>$K$</td>
<td>Permeability</td>
<td>($cm^3/100g \cdot atm^{1/2}$)</td>
</tr>
<tr>
<td>$K_s, K_l$</td>
<td>Sievert’s constants for solid and liquid</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>Thermal conductivity</td>
<td>($W/m \cdot ^\circ C$)</td>
</tr>
<tr>
<td>$L$</td>
<td>Latent heat</td>
<td>($J/m^3$)</td>
</tr>
<tr>
<td>$L_m$</td>
<td>Mushy zone length</td>
<td>($m$)</td>
</tr>
<tr>
<td>$n$</td>
<td>Amount of hydrogen gas inside pore</td>
<td>(mol)</td>
</tr>
<tr>
<td>$n_c$</td>
<td>Number of interdendritic channels</td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>Density of hydrogen gas inside pore</td>
<td>($kg/m^3$)</td>
</tr>
<tr>
<td>$P_a$</td>
<td>Atmospheric pressure</td>
<td>($N/m^2$)</td>
</tr>
<tr>
<td>$P_c$</td>
<td>Critical pressure</td>
<td>($N/m^2$)</td>
</tr>
<tr>
<td>$P_g$</td>
<td>Gas pressure</td>
<td>($N/m^2$)</td>
</tr>
<tr>
<td>$P_l$</td>
<td>Local liquid pressure</td>
<td>($N/m^2$)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>$P_{\text{met}}$</td>
<td>Metallostatic pressure</td>
<td>(N/m$^2$)</td>
</tr>
<tr>
<td>$P_s$</td>
<td>Shrinkage pressure</td>
<td>(N/m$^2$)</td>
</tr>
<tr>
<td>$P_{\text{surf}}$</td>
<td>Surface tension pressure</td>
<td>(N/m$^2$)</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Total pressure</td>
<td>(N/m$^2$)</td>
</tr>
<tr>
<td>$P_c$</td>
<td>Critical pressure</td>
<td>(N/m$^2$)</td>
</tr>
<tr>
<td>$\dot{Q}$</td>
<td>Latent heat released due to liquid-to-solid transformation</td>
<td>(W/m$^3$)</td>
</tr>
<tr>
<td>$q^*$</td>
<td>Heat flux</td>
<td>(W/m$^2$)</td>
</tr>
<tr>
<td>$q^*_{\text{conv}}$</td>
<td>Convective heat flux</td>
<td>(W/m$^2$)</td>
</tr>
<tr>
<td>$q_{\text{rad}}$</td>
<td>Radiative heat flux</td>
<td>(W/m$^2$)</td>
</tr>
<tr>
<td>$R$</td>
<td>Gas constant</td>
<td>(J/mol·K)</td>
</tr>
<tr>
<td>$R_c$</td>
<td>Cooling Rate</td>
<td>(°C/s)</td>
</tr>
<tr>
<td>$R_{\text{crit}}$</td>
<td>Cooling rate at critical temperature</td>
<td>(°C/s)</td>
</tr>
<tr>
<td>$R_{SL}$</td>
<td>Radius of solid/liquid interface</td>
<td>(m)</td>
</tr>
<tr>
<td>$R_T$</td>
<td>Cooling rate at temperature $T$</td>
<td>(°C/s)</td>
</tr>
<tr>
<td>$r$</td>
<td>Distance in radial direction</td>
<td>(m)</td>
</tr>
<tr>
<td>$r_p$</td>
<td>Pore radius</td>
<td>(m)</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
<td>(K)</td>
</tr>
<tr>
<td>$T_{i-1}, T_i$</td>
<td>Temperature at the previous and current time increments</td>
<td>(°C)</td>
</tr>
<tr>
<td>$T_\infty$</td>
<td>Ambient temperature</td>
<td>(°C)</td>
</tr>
<tr>
<td>$T_{\text{cast}}, T_{\text{mould}}$</td>
<td>Temperature of casting and mould</td>
<td>(°C)</td>
</tr>
<tr>
<td>$T_{\text{crit}}$</td>
<td>Critical temperature</td>
<td>(°C)</td>
</tr>
<tr>
<td>$T_{\text{eutectic}}$</td>
<td>Eutectic temperature</td>
<td>(°C)</td>
</tr>
<tr>
<td>$T_{\text{max}}, T_{\text{min}}$</td>
<td>Upper and lower temperatures of interval during which gap formation occurs</td>
<td>(°C)</td>
</tr>
<tr>
<td>$T_{\text{solidus}}$</td>
<td>Solidus temperature</td>
<td>(°C)</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>(s)</td>
</tr>
<tr>
<td>$t_p$</td>
<td>Unit vector along direction of gravity</td>
<td></td>
</tr>
<tr>
<td>$t_{\text{pore}}$</td>
<td>Time for pore growth</td>
<td>(s)</td>
</tr>
<tr>
<td>$t_s$</td>
<td>Local solidification time</td>
<td>(min)</td>
</tr>
<tr>
<td>$\mathbf{u}$</td>
<td>Interdendritic flow velocity</td>
<td>(m/s)</td>
</tr>
<tr>
<td>$V$</td>
<td>Volume of porosity</td>
<td>(m$^3$)</td>
</tr>
<tr>
<td>$V'$</td>
<td>Dimensionless volume average interdendritic liquid velocity</td>
<td></td>
</tr>
<tr>
<td>$v$</td>
<td>Solidification rate</td>
<td>(m/s)</td>
</tr>
<tr>
<td>$v_{\text{crit}}$</td>
<td>Isotherm velocity at critical temperature</td>
<td>(m/s)</td>
</tr>
<tr>
<td>$v_s$</td>
<td>Solidus velocity</td>
<td>(m/s)</td>
</tr>
<tr>
<td>$v_T$</td>
<td>Isotherm velocity at temperature $T$</td>
<td>(m/s)</td>
</tr>
<tr>
<td>$z$</td>
<td>Distance in axial direction</td>
<td>(m)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Gas conversion factor</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Solidification shrinkage</td>
<td></td>
</tr>
<tr>
<td>$\Delta G$</td>
<td>Free energy change</td>
<td>(J)</td>
</tr>
<tr>
<td>$\Delta P$</td>
<td>Difference in gas and critical pressure</td>
<td>(atm)</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>Freezing range</td>
<td>(°C)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>$\Delta T_{gap}$</td>
<td>Temperature interval over which gap formation occurs</td>
<td>$^{\circ}$C</td>
</tr>
<tr>
<td>$\Delta t_i$</td>
<td>Time increment</td>
<td>s</td>
</tr>
<tr>
<td>$\varepsilon_{eff}$</td>
<td>Effective emissivity</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{cast}$, $\varepsilon_{mould}$, $\varepsilon_{chill}$</td>
<td>Emissivity of the casting, mould, and chill</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{surf}$</td>
<td>Surface emissivity</td>
<td></td>
</tr>
<tr>
<td>$\phi_e$</td>
<td>Integrated pressure</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Permeability constant</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>Secondary dendrite arm spacing</td>
<td>m</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Viscosity</td>
<td>Pa·s</td>
</tr>
<tr>
<td>$\mu_e$</td>
<td>Empirical constant</td>
<td>1/atm·s</td>
</tr>
<tr>
<td>$\rho_l$</td>
<td>Liquid density</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Solid density</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan-Boltzmann constant</td>
<td>W/m$^2$·K$^4$</td>
</tr>
<tr>
<td>$\sigma_{SG}$, $\sigma_{LG}$, $\sigma_{SL}$</td>
<td>Solid-gas, liquid-gas, and solid-liquid surface tension.</td>
<td>N/m</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Normal angle of contact</td>
<td>$^{\circ}$</td>
</tr>
</tbody>
</table>
Acknowledgements

I would like to thank my advisors, Dr. Steve Cockcroft and Dr. Daan Maijer, for their guidance and support during my research over these past two years. I would also like to thank my fellow graduate students and all of the faculty and staff of the Department of Metals and Materials Engineering. I am grateful to Mr. Neill McDonald and Ms. Ka-Hay Law for their assistance in the experimental work. I would also like to acknowledge Mr. Chris Hermesmann, of Canadian Autoparts Toyota Inc., for the technical assistance he provided. The financial support of Canadian Autoparts Toyota Inc. and the Science Council of British Columbia was also greatly appreciated.

I am especially thankful for the support and encouragement from my family (Minh, Anh, and Chinh) and friends, without which this would not have been possible.
1 Introduction

In the auto industry, the use of cast aluminum alloy wheels over traditional stamped steel parts is growing due to advantages gained in aesthetic appeal and design flexibility. The number of new North American cars equipped with cast wheels grew from less than 15% in 1985 to almost 50% a decade later.\(^1\) Due to stringent surface finish requirements, however, cast aluminum wheels are some of the most difficult automotive castings to make and rejection rates are high relative to other aluminum castings which have less demanding surface finish requirements.\(^2\)

Processing efficiency during start up and production can be increased through the use of computer modeling techniques to predict solidification behaviour. Savings in cost and production lead-time can be realized with significantly less experimentation than required by conventional methods of trial and error. In particular, the ability to isolate potential problems in die or wheel design prior to die manufacture and prototyping is of significant importance to the manufacturer as die costs are expensive (approximately 100,000$US per die).\(^2\)

Microporosity, which involves the formation of small voids within the finished product, is the predominant defect associated with cast aluminum products.\(^2\) As opposed to the formation of larger scale macroporosity, which is caused by feeding problems associated with a lack of directional solidification, the mechanism of microporosity formation is complex and dependent upon an interaction of hydrogen evolution and interdendritic shrinkage during solidification. In the wheel manufacturing process, castings are rejected
when machined surfaces intersect areas of microporosity to mar wheel appearance or when microporosity inter-connects within the casting to cause a thru-thickness path for air leakage. The later defects, often difficult to detect visually, are identified through a rim leak test, which is conducted by pressurizing the wheel with air while submerged in water. These defects are extremely costly, as considerable value has been added to the wheel during the various manufacturing stages prior to rejection.

In collaboration with Canadian Autoparts Toyota Inc. (CAPTIN), a leading manufacturer of die-cast aluminum wheels, a mathematical model is being developed to quantitatively describe the thermal behaviour in the casting processes at the CAPTIN facility. The aim of the present work is to develop a microporosity model which may be coupled to the overall thermal model. Experimental test castings of A356 alloy were produced to evaluate alternative methods of predicting porosity (criteria functions) and to develop a mathematical model of porosity formation using a fundamentally based methodology.

A356 alloys are commonly used in the automotive industry due to their good castability, high strength-to-weight ratio, and good corrosion resistance.\(^3\) Comparing their castability characteristics to other aluminum alloys, they have very good resistance to hot cracking, good pressure tightness, fluidity, and shrinkage tendencies.\(^4\) The nominal composition of the alloy is shown in Table 1.1.

**Table 1.1: Nominal composition of A356 alloy.**\(^4\)

<table>
<thead>
<tr>
<th>Element</th>
<th>Composition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al</td>
<td>Balance</td>
</tr>
<tr>
<td>Si (wt%)</td>
<td>6.5-7.5</td>
</tr>
<tr>
<td>Mg</td>
<td>0.25-0.45</td>
</tr>
<tr>
<td>Fe</td>
<td>0.20</td>
</tr>
<tr>
<td>Cu</td>
<td>0.20</td>
</tr>
<tr>
<td>Mn</td>
<td>0.10</td>
</tr>
<tr>
<td>Zn</td>
<td>0.10</td>
</tr>
<tr>
<td>Ti</td>
<td>0.20</td>
</tr>
</tbody>
</table>
CAPTIN’s manufacturing facility employs two types of casting processes to supply approximately 40-50% of Toyota’s North American demand for aluminum wheels. The first process used at CAPTIN is the low pressure process, which is the dominant method for the production of aluminum wheels. The dominance of the low pressure die casting process results from its ability to cost-effectively produce high quality wheels. The second process is a variant on the vacuum casting method, called the Toyota New Differential Casting Process or TDP process. It presently accounts for approximately 40-45% of production. Competing technologies include gravity die-casting, counter-pressure die-casting and squeeze casting.

Low-pressure die casting and vacuum die casting are very similar techniques that involve repeatedly casting parts using permanent metal moulds. They are suitable for high volume production as they are easily automated and yet still offer good dimensional accuracy and surface finish. In low-pressure die casting, the mould is situated above a holding furnace and melt is introduced via a connecting fill tube, as shown in Figure 1.1. Metal is forced up the tube by pressurizing the chamber containing the melt at pressures of 170 kPa or less. Vacuum die casting is differentiated from low-pressure die casting in that a vacuum is applied within the mould to pull melt into the die cavity from the holding furnace.
Low-pressure die casting and vacuum die casting have a number of advantages over other casting processes. They offer a high metal yield compared with other casting processes such as gravity die casting. As well, the introduction of the metal via the bottom of the mould aids in maintaining a smooth flow minimizing turbulence. In addition, these processes are usually associated with lower mould temperatures and shorter cycle times than gravity die casting.
2 Literature Review

Attempts have been made to model porosity through criteria functions, statistical models, and hydrogen-shrinkage models, which account for the combined effects of hydrogen solubility and interdendritic shrinkage. A number of studies concerning these models will be reviewed in order to illustrate some of the work that has been accomplished to date. Initially, however, the mechanisms of porosity formation are detailed.

2.1 Formation of Microporosity

During solidification, microporosity results from gas evolution, interdendritic shrinkage, and/or a combination of the two. In aluminum alloys, gas evolution focuses on hydrogen, as it is the only gas with measurable solubility.\(^9\) The characteristics of porosity, such as amount, size, and distribution, in the final cast structure are due to a number of material and process parameters.

2.1.1 Mechanisms

The hydrogen evolution process is summarized well by Shivkumar \textit{et al.}\(^{10}\) who separated the process of hydrogen evolution into five distinct stages:

\begin{itemize}
  \item[I.] Enrichment of liquid in hydrogen
  \item[II.] Nucleation of pore
  \item[III.] Instantaneous growth of pore
  \item[IV.] Gradual increase in pore size
  \item[V.] End of solidification
\end{itemize}
During solidification, hydrogen gas is rejected from growing dendrites into the surrounding interdendritic liquid due to a significant decrease in solubility. The solubility limit of a gas in metal is a function of temperature, with higher solubility limits possible at higher temperatures in most metals.\textsuperscript{11} The solubility of hydrogen as a function of temperature is shown in Figure 2.1.

![Figure 2.1: Solubility of hydrogen in aluminum and aluminum alloys. (C is the dissolved concentration and C\textsubscript{unit} is defined as 1 cm\textsuperscript{3}/100g at 0°C and 1 atm)](image)

The balance of hydrogen pressure between the solid, liquid, and gas pore during solidification serves as a fundamental basis for porosity formation. Pores are nucleated when the gas pressure in the liquid is large enough to overcome the local pressure and
surface tension forces. Thermodynamically speaking, this condition is achieved when the free energy change of formation ($\Delta G$) is zero as shown:\(^\text{12}\)

$$\Delta G = V(P_g - P_l) + A_1\sigma_{SG} + A_2\sigma_{LG} - A_f\sigma_{SL} = 0$$  \(1\)

where $V$ is the volume of porosity, $P_g$ and $P_l$ are gas and local liquid pressure, $A_1$ and $A_2$ are the areas of solid-gas and liquid-gas interfaces, and $\sigma_{SG}$, $\sigma_{LG}$, and $\sigma_{SL}$ are the solid-gas, liquid-gas, and solid-liquid interfacial energies, respectively.

Pores can be transported to the bulk liquid where rapid growth occurs, which decreases the hydrogen level in the liquid. The lowered hydrogen content in the liquid decreases the rate of pore growth allowing a gradual increase in pore size until the end of solidification. The decrease in growth rate can also be attributed to the decrease in local temperature and the pore surface/volume ratio.\(^\text{13}\)

Aluminum alloys undergoing solidification experience shrinkage effects (approximately 6% for A356)\(^\text{14}\) that must be checked by additional liquid metal feeding. Substantial metal flow can result from the pressure gradients formed as a result of any shrinkage effects.\(^\text{10}\) Campbell\(^\text{15}\) describes a number of feeding mechanisms in relation to pore nucleation, among them being restricted interdendritic feeding as a possible cause for microporosity. For the A356 alloy, which exhibits a dendritic structure with a long freezing range, the metal flow in the interdendritic region is similar to flow through a porous medium. If the feeding pressure is insufficient to enable flow through the mushy zone, shrinkage-caused microporosity can result.
2.1.2 Factors

There are several factors that are important in controlling microporosity in the final cast structure. The amount, size, and distribution of porosity can be affected by a number of parameters including:

- Alloy chemistry
- Hydrogen content
- Processing variables
- Grain refinement
- Eutectic modification
- Inclusion content

Alloy chemistry can be a significant factor in porosity formation. The feeding capability of Al-Si alloys is altered by varying the amounts of silicon and copper, as well as impurities such as iron and manganese. Calcium increases the hydrogen solubility in the melt. A356 aluminum alloys, in particular, display a number of thermal physical properties, such as a relatively long freezing range and high thermal conductivity, which promote microporosity formation. A large mushy zone, due to the long freezing range, hampers interdendritic feeding while high thermal conductivity minimizes steep temperature gradients.

Alloys with high initial hydrogen content will form a large amount of hydrogen-based porosity because hydrogen content is the predominant factor in the amount of porosity. A number of studies have recognized a threshold hydrogen content below which
porosity is effectively eliminated. Shivkumar et al.\textsuperscript{10} estimate a threshold value of below 0.05 to 0.08 cm\textsuperscript{3}/100g for A356 alloy.

Casting variables, such as thermal gradient, cooling rate, and solidification time can have significant effects on porosity. The length of interdendritic feeding channels is inversely proportional to thermal gradient.\textsuperscript{20} As a result, lower porosity levels are observed at higher thermal gradients due to improved liquid feeding through a reduced channel length. Higher cooling rates increase the demand for liquid while producing a finer secondary dendrite arm spacing, which increases the resistance to interdendritic feeding.\textsuperscript{3} However, permeability at high cooling rates is increased by an increased number of fluid flow channels, a narrow mushy zone, and a corresponding short interdendritic channel length. In addition, a higher solidification rate limits the amount of time available for hydrogen to diffuse into the interdendritic region.\textsuperscript{21} Regarding local solidification time, Lee et al.\textsuperscript{20} observed two distinct regimes of behaviour, with the first characterized by an increasing porosity level with time followed by a second regime demonstrating decreasing porosity at longer solidification times. The behaviour in the first regime can be attributed to the effects of higher cooling rates characteristic of shorter solidification times while behaviour in the second is likely due to the high permeability of the coarser dendritic structure formed at longer times.

The grain structure can affect microporosity formation by influencing the flow of liquid in the interdendritic region. Grain refinement has been experimentally shown to reduce the amount of porosity in samples cast with a similar cooling rate and hydrogen content.\textsuperscript{22}
In addition, pores are more evenly dispersed in the final cast structure. Common grain refiners used in foundries are Al-Ti-B alloys. TiAl₃ particles increase the number of sites for the nucleation of pro-eutectic alpha aluminum while B serves as nucleating agents or stabilizers of the TiAl₃ particles.

Eutectic modification of Al-Si alloys is carried out using additions of Na or Sr, which change the eutectic silicon shape from acicular to fibrous with a resulting enhancement in mechanical properties. The addition of these elements has been observed to increase both the amount and size of porosity, although the mechanism by which this occurs is unclear. An increase in hydrogen absorption or feeding difficulty has been suggested. However, Emadi et al. did not observe any increase in hydrogen pickup and Iwahori et al. found that Na improves feeding capability. Other proposed effects of modification include increased inclusion content, reduced surface tension, and/or increased volumetric shrinkage.

Inclusion content must be controlled as its presence can significantly increase the amount of porosity in the casting. Gaps, cracks, and concave interfaces present on inclusions are generally believed to act as sites for heterogeneous pore nucleation. Homogenous nucleation in a metallic melt is considered unlikely to occur as an extremely high gas pressure or low local pressure is required. Mohanty et al. also describe two additional potential mechanisms of pore nucleation due to inclusions. Interactions between the solid/liquid interface and inclusions, which are concentrated and pushed ahead of the interface, can enhance gas segregation and the viscous pressure drop.
in the local region. In addition, inclusions may also become mechanically trapped in the interdendritic space and restrict fluid flow in the region.

### 2.2 Criteria Functions

In practical terms, criteria functions are attractive as they are relatively simple to apply to determine optimal casting conditions for control of thermal parameters such as thermal gradient, cooling rate, and solidification time. In general, criteria functions use thermal parameters to determine the difficulty of liquid feeding through the interdendritic region. Experimentally measured values of porosity amount and distribution are used to correlate the amount of porosity to criteria function values (e.g. higher criteria function value means higher porosity value) and/or to determine a critical criteria function threshold value (e.g. porosity is likely to form above/below a certain value). The development and assessment of four criteria functions is described in this review.

#### 2.2.1 Development of Criteria Functions

Niyama et al.\(^{32,33}\) studied centerline shrinkage in steels and developed their widely referenced criteria function. By analyzing measured porosity with thermal gradient, local solidification time, and cooling rate, Niyama et al.\(^{33}\) concluded that the most reliable parameter for shrinkage prediction was:

\[
Niyama = \frac{G}{\sqrt{R_c}}
\]  

(2)

where \(G (°C/cm)\) is the temperature gradient and \(R_c (°C/min)\) is the cooling rate at the end of the solidification (\(T = T_{solidus}\)) at each point in the casting. Observations of
experimental castings revealed that most of the shrinkage defects were enclosed by a contour of \( G/\sqrt{R} = 1 \, ^{\circ}C^{1/2} \text{min}^{1/2}/\text{cm} \).

As a theoretical basis for their work, they presented a simplified model assuming shrinkage porosity would form at the root of dendrites due to a critical pressure drop. Darcy’s law describing flow through porous media was applied to interdendritic fluid flow and can be represented in its general form as:  

\[
u = -\frac{K}{\mu f_i} (\nabla P_s + \rho_i g t) \]  

where \( \nu \) (m/s) is the interdendritic flow velocity, \( K \) (m\(^2\)) is the permeability, \( \mu \) (Pa-s) is the viscosity, \( f_i \) is the volume fraction liquid, \( P_s \) (Pa) is the shrinkage pressure, \( \rho_i \) (kg/m\(^3\)) is the density of the liquid, \( g \) (m/s\(^2\)) is the acceleration due to gravity and \( t \) is the unit vector along the direction of gravity. Employing a number of simplifying assumptions, the pressure drop was represented as a function of material and thermal variables. The material variables were combined into a single material constant and the remaining thermal variables are represented by the Niyama function.

The use of Darcy’s law is common in many studies involving aluminum alloys including the works of Lee et al.\(^{20}\) (Al-7Si-0.3Mg) and Kao and Chang\(^{34,35}\) (A356). Alternatively, Suri et al.\(^{36}\) (A356) attempted to determine the extent of friction drag on liquid in the interdendritic region. Other works have described the level of feeding by establishing a limiting solid fraction of feeding\(^{37}\), a critical feeding range\(^{38}\), or a critical feeding angle\(^{39}\). Tynelius et al.\(^{40}\) (A356) and others\(^{41,42}\) employed strictly empirical approaches to determine the importance of various parameters.
Lee et al.\textsuperscript{20} developed their criteria function by relating the final volume of porosity to the local pressure at the interdendritic channel via the ideal gas law. The local pressure consisted of atmospheric and metallostatic (from riser) pressures, which were assumed constant, as well as the pressure drop due to increased difficulty in interdendritic feeding. Darcy’s law was employed to model the pressure drop in a manner similar to Lecomte-Beckers\textsuperscript{43}, which differs from the treatment of Niyama et al. in their assumptions of the variation of the volume fraction liquid in the mushy zone and the calculation of permeability. As a result, the pressure drop calculated by Lee et al. included the local solidification time as an additional thermal parameter. The thermal parameters were employed as the criteria function, which was defined as:

\begin{equation}
Lee = \frac{G \cdot t_s^{1/2}}{v_s} \tag{4}
\end{equation}

where \(G\) (°C/cm) is the thermal gradient, \(t_s\) (min) is the local solidification time, and \(v_s\) (cm/min) is the solidus velocity.

Using a method comparable to Lee et al., Kao and Chang\textsuperscript{34,35} also developed a criteria function relating the amount of porosity to the local interdendritic pressure. The key difference between the two criteria functions is in the treatment of secondary dendrite arm spacing (\(\lambda_2\)), which is one of the variables used in their calculation of permeability in the mushy region. While both researchers related \(\lambda_2\) to local solidification time, Kao and Chang represented local solidification time as a function of freezing range, thermal gradient, and solidus velocity. As a result, their criteria function does not include solidification time and is defined as:
\[ KC = \frac{G^{0.38}}{v_s^{1.62}} \]  

(5)

where \( G \, (^{\circ}C/m) \) is the thermal gradient and \( v_s \, (m/s) \) is the solidus velocity. A further difference in the work of Kao and Chang\(^{34}\) is in the presence of an additional surface tension pressure term in their calculation of local interdendritic pressure.

Although the use of Darcy’s law in approximating flow through the mushy zone is fairly common, Suri et al.\(^{36}\), attempting to avoid the difficulties associated with determining accurate permeability values, proposed that friction drag on the liquid would be an effective measure of the resistance to interdendritic feeding. In using this method, the effects of microstructure and grain size, which are often ignored in conventional criteria functions, were included in the model. Both factors affect the surface area of the solid in the interdendritic region. For equiaxed dendrites, the friction drag can be calculated using:

\[
F_{\text{drag}} = \frac{216 \mu \Delta T}{\rho_l G v_s \beta D_0^2} \left( \frac{V'}{f_1} \right) = C_0 \frac{V'}{f_1}
\]  

(6)

where \( V' \) is the dimensionless volume average interdendritic liquid velocity, \( \Delta T \, (^{\circ}C) \) is the freezing range, \( \mu \, (Pa\cdot s) \) is the viscosity, \( \beta \) is the solidification shrinkage ratio, \( \rho_l \, (kg/m^3) \) is the liquid density, \( G \, (^{\circ}C/m) \) is the thermal gradient, \( v_s \, (m/s) \) is the solid front growth velocity, and \( D_0 \, (m) \) is the equiaxed grain size. The dimensionless parameter \( C_0 \) is termed the Feeding Resistance Number (FRN) and the probability of the porosity formation increases with increasing FRN. Lower values of FRN are indicative of better feeding conditions whereas higher values are indicative of the opposite.
With the goal of accurate porosity prediction, Tynelius et al. developed an empirical model using statistical reduction techniques on experimental data. Advantages of this approach include the ability to handle the effects of multiple parameters, to examine the effects of their interactions, and to assess the magnitude of these effects. A large number of parameters, including microstructural, alloy, and thermal variables, were studied using two experimental moulds in an attempt to separate any mold/geometry effects from the results. They concluded that the % porosity (area) was best predicted by hydrogen content, modifier (Sr) content, solidification time, solidus velocity, and grain size.

2.2.2 Evaluation of Criteria Functions

A significant amount of research has been performed in determining the applicability of the criteria functions to A356 alloys. A representative sample of the papers evaluating only the criteria functions described earlier is given below in chronological order.

Laurent and Rigaut cast A356 alloy using two separate moulds to independently examine 1-D unidirectional cooling and 3-D cooling conditions. Experimental castings were measured for grain refinement, modification, inclusion content, and hydrogen content. A statistical analysis was carried out that concluded that porosity was predicted best using the thermal gradient and solidification rate, as well as variables to account for the effects of hydrogen and riser size. In comparing their findings with the Niyama function, they found that only relative values of the Niyama function could be used as indicators of porosity formation. Absolute values were found to be geometry dependent while melt variables, such as gas content, can increase porosity formation without affecting the Niyama function values.
Suri, Huang, Berry, and Hill\textsuperscript{31} attempted to determine the viability of the Niyama and Lee criteria functions using a validated 3-D solidification heat transfer model. Experimental casting porosity data, obtained from samples cast with and without the presence of degassing, end chill, and/or riser, was related to criteria function values using a power law relation. Predicted porosity values for the two criteria functions were found to be very similar and it was concluded that either criteria function could be used as a qualitative measure of porosity. Huang and Berry\textsuperscript{44} also evaluated the validity of the Niyama and Lee criteria functions by casting A356 alloy in both laboratory and industrial environments. Although an acceptable correlation between porosity and the Niyama and Lee functions was found, the correlation levels were similar to that of thermal gradient alone. Instead, a generalized criterion including gradient, cooling rate, and local solidification time was found to be slightly more accurate.

Citing drawbacks in the work by Suri \textit{et al.}\textsuperscript{31}, Spittle \textit{et al.}\textsuperscript{45} attempted to verify the Niyama function by duplicating the work of Niyama \textit{et al.}\textsuperscript{33} with an LM25 (Al7SiMg) aluminum alloy. In reproducing the results of the steel sample used by Niyama \textit{et al.}, a good agreement was found. However, a correlation between the Niyama function and porosity was not found in the LM25 alloy. It was concluded that porosity in aluminum castings alloys is likely to be dominated by the hydrogen content and thus, the Niyama function is not applicable to aluminum alloys.
The statistical model developed by Tynelius et al.\textsuperscript{40} was not directly compared with any criteria functions. However, the results displayed a correlation factor ($r^2$) of 0.84 for \% porosity (area), which is a measure of the amount of variance in the data that can be accounted for by the model.

In developing the FRN, Suri et al.\textsuperscript{36} compared it against the Niyama and Lee functions by casting A356 alloy plates under a variety of degassing, chill, and riser conditions. A statistical analysis revealed that the Niyama and Lee functions have nearly identical correlation factors ($r^2$) of 0.71 and 0.72, respectively, while the FRN possessed a superior factor of 0.84.

A statistical comparison of the criteria function of Kao and Chang with other criteria functions has not been performed. Consequently, the relative effectiveness of this criteria function has yet to be determined. However, in experiments involving different mould preheat temperatures, Kao and Chang\textsuperscript{35} found that only one correlation of porosity to the criteria function was necessary. In contrast, the Niyama function displayed a different porosity correlation at each preheat temperature.

2.2.3 Discussion

A number of researchers have investigated the use of several criteria functions for predicting microporosity in A356 aluminum alloys. The FRN and KC functions have shown good correlations to porosity levels, however, these criteria functions have not been evaluated through independent research, as has been the case with the Niyama and Lee functions. Although the relative effectiveness of the KC function has not been
assessed, its predictive capability is assumed to be similar to that of the Lee function considering the comparable theory used in development. The statistical model of Tynelius et al. has also not been independently assessed, although the study does quote a correlation factor equivalent to that found by Suri et al. for the FRN. As the exact relationship used in the model was not indicated, independent review is not possible. In any case, statistical models are not as useful in determining the theoretical mechanisms of microporosity formation.

A possible cause of error in the evaluation of the criteria functions is the use of data obtained from a wide variety of casting conditions, which could affect porosity levels. Suri et al.\textsuperscript{31} included data from castings with and without a chill as well as with and without a riser in the same data group for their correlations. Benefits could result if the data is divided into more specific categories as was done by Huang and Berry\textsuperscript{44}, who obtained a higher correlation for castings with an end chill.

Another source of variation is in the selection of the critical temperature at which the thermal parameters are calculated. Thermal parameters are generally calculated at or near the eutectic temperature in order to describe the conditions when the dendritic network is well established and interdendritic feeding problems are likely to occur.\textsuperscript{44} However, the exact critical value is unclear and past studies have employed a variety of critical temperatures.\textsuperscript{31,44}
In assessing these works, it is important to examine the conditions in which each criteria function would be applicable. One condition that is clearly shown by the studies is the necessity to minimize hydrogen concentration in the melt. The study by Spittle et al., which concluded that the Niyama function is not applicable to aluminum alloys, did not employ a degassing treatment and displayed high levels of hydrogen. As such, the effect of hydrogen likely overshadowed interdendritic shrinkage effects.

Care should also be taken to ensure reliable results, as reproducibility of porosity data can be a problem as observed by Suri et al. They found that porosity levels could change despite similar casting conditions and degassing treatments, but sampled from separate heats. The variability was attributed to the reabsorption of hydrogen prior to pouring or an increase in inclusion content.

While there has been some success at modeling microporosity using criteria functions to determine critical parameters and/or feeding conditions, these models do not adequately account for the significant effect of hydrogen evolution. As a result, the practical application of criteria functions is somewhat limited. Thus, it is clear that the effect of hydrogen must be included to accurately predict porosity.

### 2.3 Hydrogen-Shrinkage Models

The early work by Fang and Granger provided a good basis for the modeling of A356 aluminum alloy by accounting for the effects of hydrogen evolution and interdendritic shrinkage. The recent work by Huang et al., who also modeled the A356 alloy, is representative of the method used in many other studies. Many components of the model
are similar to the early works by Kubo and Pelhke\textsuperscript{12} (1985) and Poirier \textit{et al.}\textsuperscript{46,47} (1987), who both modeled Al-4.5Cu alloys, and Shivkumar \textit{et al.}\textsuperscript{10} (1990), who described A356 alloys. Others\textsuperscript{48,49,50} later developed similar models for Al-Cu alloys.

2.3.1 Fang and Granger

Fang and Granger\textsuperscript{13,22} developed a mathematical model to predict the size of interdendritic pores as past work has indicated that it is has the largest impact on pore volume fraction. Focusing on the interdendritic region between equiaxed grains, the pore formation process was represented schematically as shown in Figure 2.2. For modeling purposes, the geometry was simplified and the pore, liquid and solid regions were idealized as concentric spheres.

During solidification, the local structure becomes isolated as channels connecting the interdendritic liquid to the feeding liquid freeze and local hydrogen concentration increases. Once the hydrogen solubility limit is reached, nucleation was assumed to occur immediately with an initial pore size equal to 1 \textmu m. Initial pore size was assumed to be similar to pores formed near the end of solidification. Further saturation of hydrogen in the region contributed to the growth of the pore.
Figure 2.2: Schematic representation of pore formation in an equiaxed grain casting produced by directional solidification with (a), (b), and (c) representing sequentially larger magnification in area.\textsuperscript{13}

Pore growth was modeled using a hydrogen mass balance between the hydrogen absorbed into the pore and that rejected at the solidification front. The growth of the pore was divided into three stages:
I. Growth between the liquidus and eutectic temperature

II. Isothermal growth at the eutectic temperature

III. Growth at the end of local isothermal solidification

Based on a hydrogen mass balance, Stage I growth was modeled as:

\[ \frac{d}{dt} \left( \frac{4}{3} \pi r_p^3 \cdot N \right) = 4 \pi R_{SL}^2 \left( \frac{dR_{SL}}{dt} \right) \left( \rho_l [H_l] - \rho_s [H_s] \right) \]  

(7)

where \( \frac{4}{3} \pi r_p^3 \) (m^3) is the volume of the pore (assumed spherical); \( r_p \) (m) is the instantaneous pore radius; \( N \) (kg/m^3) is the density of hydrogen gas inside the pore; \( R_{SL} \) (m) is the radius of the instantaneous solid/liquid interface; \( \rho_l \) (kg/m^3) and \( \rho_s \) (kg/m^3) are the melt and solid densities, respectively; and \([H_l]\) (cm^3/100g) and \([H_s]\) (cm^3/100g) are the hydrogen concentrations in the liquid and solid, respectively. The left-hand side of the equation represents the hydrogen inside the growing pore while the right-hand side represents the hydrogen rejected at the solidification front.

Assuming local equilibrium, the density of hydrogen gas in the pore was calculated using the ideal gas law relating local pressure and temperature as:

\[ P_l V = nRT \]  

(8)

where \( P_l \) (N/m^2) is the total pressure inside the pore, \( V \) (m^3) is the volume of the pore, \( n \) (mol) is the amount of hydrogen, \( R \) (J/mol·K) is the gas constant, and \( T \) (K) is the local temperature. The total pressure was determined using:
\[ P_i = P_a + 2 \frac{\sigma_{LG}}{r_p} + \rho_i gh + P_s \]  

(9)

where \( P_a \) (N/m\(^2\)) is the external (atmospheric) pressure, \( \sigma_{LG} \) (N/m) is the surface tension, \( g \) (m/s\(^2\)) is the acceleration due to gravity, \( h \) (m) is the metallostatic head, and \( P_s \) (N/m\(^2\)) is the shrinkage pressure. Shrinkage pressure was calculated using the equation:

\[ P_s = \frac{\beta}{1 - \beta} \cdot \mu v L_m \cdot \ln \left( \frac{g_e}{1 - g_e} \right) \]  

(10)

where \( \beta \) is the volume shrinkage factor, \( \mu \) (Pa\(\cdot\)s) is the viscosity, \( v \) (m/s) is the solidification rate in the heat flow direction, \( L_m \) (m) is the mushy zone length, \( g_e \) is the as-cast eutectic volume fraction, and \( \gamma \) (m\(^2\)) is a measure of the permeability of the mushy zone. Combination and rearrangement of the equations, with a number of simplifying assumptions including constant external pressure, surface tension, and metallostatic head, results in an equation that relates the rate of change in pore size with temperature, shrinkage pressure, and hydrogen release.

Once the eutectic temperature is reached, Stage II growth prevails in which pore growth is isothermal. During this stage, it is assumed that the bulk liquid is still feeding the local liquid and thus, the hydrogen released contributes to pore growth. During Stage III, however, the local liquid is completely isolated and any pore growth is assumed to result solely from shrinkage effects. Stage III growth is assumed to be active during the solidification of the last 5% eutectic liquid before complete solidification.

Regarding the nucleation event, Fang and Granger note that it is possible to achieve 'zero' porosity if the hydrogen concentration in the liquid does not reach the solubility
limit before cooling to the eutectic temperature. Consequently, a threshold hydrogen content below which there would be no porosity formation was defined. However, some porosity was observed to exist even at low hydrogen levels. Therefore, definition of the threshold hydrogen content at a small finite porosity level was considered more appropriate than extrapolating to a zero porosity level. Theoretically, another method of avoiding porosity formation would be to increase the hydrogen solubility via an increase in the external pressure.

Extensive experimental validation of the model, involving directionally solidifying A356 alloy castings, was quantitatively performed on pore volume fraction and pore size. Regarding pore volume fraction, the results revealed a decrease with an increase in cooling rate and a decrease hydrogen content, which agrees well with literature. Pore size was also observed to decrease under the same conditions. However, as pore number density was found to decrease only slightly, it was concluded that increases in pore volume fraction are due mainly to increases in pore size.

The pore size as a function of grain size predicted by the model, displayed in Figure 2.3, shows relatively good agreement with experimental data. As grain size was related to cooling rate, as shown in Table 2.1, cooling rate could also be used to predict pore size.
Figure 2.3: The calculated pore diameters as a function of grain size at a hydrogen content of 0.31 cc/100g in A356 alloy.22

Table 2.1: Relationship between grain size and cooling rate in the grain refined castings of A356 in the experiment.22

<table>
<thead>
<tr>
<th>Cooling Rate, °C/s</th>
<th>25</th>
<th>6</th>
<th>2</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grain Size, mm</td>
<td>260</td>
<td>450</td>
<td>570</td>
<td>790</td>
</tr>
</tbody>
</table>

Figure 2.4 shows a relatively poor quantitative correlation of hydrogen content to pore size. This is a concern due to the large effect of hydrogen on microporosity. This suggests that while the effects of thermal parameters are relatively well modeled, improvements can be made in the treatment of the hydrogen gas.
Figure 2.4: The calculated pore diameters as a function of hydrogen content at a cooling rate of 6 °C/s.²²

2.3.2 Huang, Mori, and Conley

Huang, Mori, and Conley¹⁶,⁵¹ developed a comprehensive 2-D model, coupling hydrogen and shrinkage pore formation mechanisms with a grain growth simulation model, to predict pore size, morphology, and location. The gas pore evolution model employed is based on the gas pressure in the melt exceeding a critical local pressure, defined as:
\[
P_g \geq P_c
\]
(11)

\[
P_c = P_a + \rho_gh + P_s + P_{surf}
\]
(12)

where \( P_a \) (N/m\(^2\)) is the ambient pressure, \( \rho_gh \) (N/m\(^2\)) is the metallostatic pressure, \( P_s \) (N/m\(^2\)) is the shrinkage pressure, and \( P_{surf} \) (N/m\(^2\)) is the surface tension per unit area between gas and liquid. The ambient and metallostatic pressures were assumed to be constant while the surface tension per unit area was calculated using:

\[
P_{surf} = \frac{2\sigma_{LG}}{r_p}
\]
(13)

where \( \sigma_{LG} \) (N/m) is the surface tension between the gas and liquid and \( r_p \) (m) is the radius of the pore.

The computation of gas pressure was accomplished using a hydrogen mass balance under equilibrium solidification conditions. The hydrogen balance before the formation of a pore was represented using:

\[
[H_0] = [H_s]f_s + [H_l](1 - f_s)
\]
(14)

where \([H_0]\) (cm\(^3\)/100g) is the initial melt hydrogen content, \([H_s]\) and \([H_l]\) (cm\(^3\)/100g) are the hydrogen content in the solid and liquid, respectively, and \(f_s\) is the volume fraction solid. The hydrogen mass balance after the formation of the pore was expressed as:

\[
[H_0]\rho_t = [H_s]\rho_s f_s + [H_l]\rho_l (1 - f_s - f_v) + \alpha f_v P_g
\]
(15)

where \(\rho_t\) (kg/m\(^3\)) and \(\rho_s\) (kg/m\(^3\)) are the densities of the liquid and solid metal, \(f_v\) is the volume fraction porosity, \(\alpha\) is the gas conversion factor, \(P_g\) (N/m\(^2\)) is the gas pressure and
$T$ (K) is the temperature. The third term represents the amount of hydrogen trapped in the gas pore.

As previously mentioned by Fang and Granger, a critical hydrogen content below which porosity does not form was observed. The initial hydrogen content can be determined through the substitution of the gas pressure into the pore nucleation criterion. Rearrangement of this equation can also yield the critical solid fraction, which is another parameter that is cited to describe the nucleation event.

The shrinkage pressure was estimated by employing the mass conservation equation represented as:

$$\left( \frac{\rho_s}{\rho_l} - 1 \right) \frac{\partial f_x}{\partial t} - \frac{\partial f_y}{\partial t} + \text{div}(f_u) = 0$$

(16)

where $t$ (s) is the time; and $u$ (m/s) is the interdendritic flow velocity vector. From left to right, the equation represents solidification shrinkage, porosity growth, and liquid feeding, respectively. Darcy's law was applied to solve the interdendritic flow velocity.

During the implementation of the model, if the gas pressure was sufficient to overcome the local pressure, a stable pore was assumed to form and a liquid cell was randomly selected to become a gas pore. It was assumed that the initial pore nucleus size was half of the size of a cell, or approximately 5-10 μm. At this point, the pore was assumed to grow given sufficient dissolved hydrogen pressure, as shown in Figure 2.5. The conditions were evaluated for each time step and the process repeated until the end of solidification.
Figure 2.5: Schematic for the cellular automation used to simulate gas pore growth.\textsuperscript{16}

Experimental validation of the model was carried out using A356 alloy platelike castings with varying levels of modifier and hydrogen content. The model predicts the expected increase in porosity with increasing hydrogen content, as shown in Figure 2.6, and displays the five stages of pore nucleation and growth described by Shivkumar \textit{et al.}\textsuperscript{10}

Figure 2.6: Area fraction of porosity as a function of solid fraction for different hydrogen contents.\textsuperscript{16}
A comparison of porosity, displayed in Table 2.2, reveals a relatively good agreement between model and experimental values. However, quantitative comparisons were not reported for samples with varying hydrogen content.

Table 2.2: Comparison of experimental and calculated area fraction porosity.\textsuperscript{16}

<table>
<thead>
<tr>
<th></th>
<th>% Porosity (Image Analysis)</th>
<th>% Porosity (Calculated)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unmodified ([H]=0.092 mL/100 g)</td>
<td>1.23</td>
<td>1.19</td>
</tr>
<tr>
<td>Modified ([H]=0.087 mL/100 g)</td>
<td>2.32</td>
<td>1.92</td>
</tr>
</tbody>
</table>

2.3.3 Discussion

The models developed by Fang and Granger and Huang \textit{et al.} are indicative of the general form of microporosity models that attempt to account for the effects of both interdendritic shrinkage and hydrogen gas evolution. Regarding the nucleation event, the use of a critical pressure criterion similar to Huang \textit{et al.} is common. However, the model by Fang and Granger assumes nucleation upon supersaturation of hydrogen. In general, the local pressure in the interdendritic region can be expressed as

\[ P_t = P_a + P_{\text{met}} + P_{\text{surf}} + P_s \]

(17)

where \( P_t \) is the total pressure in the pore, \( P_a \) is the atmospheric or ambient pressure, \( P_{\text{met}} \) is the metallostatic pressure, \( P_{\text{surf}} \) is the surface tension pressure, and \( P_s \) is the shrinkage pressure.
The ambient and metallostatic pressures are usually assumed constant, but in some cases, are neglected in the pressure equation. The surface tension effect is calculated by similar methods and is generally given in a form similar to that used by Huang et al. However, the initial pore size used by each of the models varies. Studies generally use a constant value or a fraction of the secondary dendrite arm spacing.

The calculation of the gas pressure is generally determined using a hydrogen gas balance which employs Sievert's law to relate the mass concentrations of hydrogen in the liquid and solid to the gas pressure. Sievert's law is expressed as:

\[ [H_s] = K_s P_g^{1/2} \]  \hspace{1cm} (18)
\[ [H_l] = K_l P_g^{1/2} \]  \hspace{1cm} (19)

where \([H_s]\) and \([H_l]\) (cm\(^3\)/100g) are the concentration of hydrogen in the solid and liquid, respectively, \(K_s\) and \(K_l\) (cm\(^3\)/100g atm\(^{1/2}\)) are the equilibrium constants for the solid and liquid, respectively, and \(P_g\) (atm) is the gas pressure.

Calculation of the shrinkage pressure is generally the most difficult and is treated using a number of methods. A common characteristic of each approach, however, is the use of Darcy's law to approximate interdendritic fluid flow in the mushy region as flow through a porous medium. One approach has been to determine shrinkage by solving a metal mass balance, which accounts for shrinkage, pore growth, and interdendritic liquid flow. This approach requires the simultaneous solution of the mass balance and Darcy's law for the shrinkage pressure. Fang and Granger employed a method developed by Flemings and coworkers, which calculated the pressure drop along a bundle of
simple capillary tubes to approximate the flow in interdendritic channels. This method of shrinkage pressure calculation was often employed to develop criteria functions.\textsuperscript{20,43,35}

There is a wide variation in the methods of determining the permeability in the interdendritic mushy zone. In some studies,\textsuperscript{16,12,55} permeability is calculated using the Carman-Kozeny equation, which is expressed as:

$$K = \frac{f_i^3 \lambda_2^2}{180(1 - f_i)^3}$$

(20)

where $K$ (m$^2$) is the permeability, $f_i$ is the volume fraction liquid, and $\lambda_2$ (m) is the secondary dendrite arm spacing. The approach of Flemings and coworkers\textsuperscript{52,54} determined that permeability could be calculated using:

$$K = \gamma f_i^2$$

(21)

where $\gamma$ (m$^2$) is a constant dependent on the number and tortuosity of the interdendritic channels. Tortuosity is a geometric factor which accounts for the fact that flow channels are neither straight nor parallel.\textsuperscript{56} In another approach, Shivkumar \textit{et al.}\textsuperscript{10} use permeability values obtained from work done with a multilinear regression analysis.\textsuperscript{57}

Another area of uncertainty is the measurement of surface tension effects. Emadi \textit{et al.}\textsuperscript{14} characterized the effect of modification on surface tension for A356 alloys. However, their results agree relatively poorly with other researchers' results describing other aluminum alloys. Although this may simply be due to the differences in composition of the alloys studied, further work is necessary to determine accurate measures of surface tension.
In evaluating the models developed in past studies, it is necessary to examine the methods used in validating the individual models. In the two models described in detail by Huang et al. and Fang and Granger, experiments were performed to provide quantitative validation. In a number of other studies, however, models were validated using past experimental results by other researchers. Consequently, although these models are very useful to further characterize microporosity formation, their actual application is limited due to a lack of quantitative verification. The model by Huang et al. appears promising but its agreement with experimental results has only been confirmed at low hydrogen levels (≤0.092 cm³/100g). It remains to be determined whether the model can adequately describe porosity at varying hydrogen levels and for different casting conditions (e.g. grain refinement, inclusion content, etc.).

2.4 Discussion

A number of criteria functions have been developed which describe microporosity relatively well. The FRN and KC functions have been shown to potentially improve on the predictions of the Niyama and Lee functions. Independent verification of these criteria functions would be worthwhile. Although criteria functions offer the tremendous advantage of simplicity in microporosity prediction, their application to commercial practice is difficult. The main drawback of the criteria functions is their inability to handle the significant effect of hydrogen on porosity. In addition, they also ignore the effects of modification, inclusions, and grain refinement. All of the criteria functions reviewed suffered this disadvantage.
Recent work in the area of coupled hydrogen evolution and interdendritic shrinkage models have shown that microporosity in A356 aluminum alloys can be modeled effectively. Despite being only qualitative in nature, the work by Kubo and Pehlke, and Poirier et al. on Al-4.5%Cu alloys and Fang and Granger on A356 alloys provided a solid basis for this type of model. The recent model by Huang et al.\textsuperscript{16} is a good example of this type of coupled model and its treatment of gas evolution and interdendritic shrinkage synthesizes earlier work in this area. Improvements in the accuracy of the properties used in the models (permeability, surface tension, etc.) must still be continually made in order to increase the effectiveness of the models. Although the modeling of microporosity seems promising, the complexity of this technique could potentially limit its application.
3 Scope and Objectives

Researchers have predicted microporosity developed in A356 castings qualitatively using criteria functions as well as quantitatively using hydrogen-shrinkage models, with varying degrees of success. Four criteria functions were described in detail in the literature review. Various attempts have been made to compare and evaluate the four criteria functions, though experiments were usually performed at relatively low and constant hydrogen levels and with a number of different experimental setups. Most work, however, has considered porosity measured in experimental test castings, with little in the way of industrial cast products. By performing experiments in an industrial setting, test conditions will more closely resemble actual conditions in terms of melt composition, operating temperature, and hydrogen content.

Although hydrogen-shrinkage models contain many elements common to each other, there is still a significant degree of variation in their handling of gas and shrinkage pressure and in their overall complexity. In general, these models are quite complex and resource (computing power) intensive. A synthesis of the various theoretical treatments employed to date coupled with an accurate thermal model has the potential to approach the efficiency (computing time) of criteria functions but with the ability to account for hydrogen evolution and interdendritic shrinkage.

In light of the above discussion, the objectives of the research include:

- Evaluation of four criteria functions found in the literature using a common experimental setup and test conditions.
• Development of a microporosity model that can account for the combined effects of hydrogen evolution and interdendritic shrinkage in A356 alloy.

• Validation of the microporosity model using a series of experiments involving directionally solidified test castings of A356 alloy with varying hydrogen content.
4 Experimental Work

Experimental work was performed in an industrial environment to duplicate the operating conditions in commercial die-casting practice, with respect to initial melt temperature, alloy composition, melt treatment, etc. Two trials, consisting of 5 and 9 test castings, respectively, were carried out. Data from the first trial was employed in model development while data from the second trial was used for model validation.

4.1 Setup

The experimental work involved casting test samples of A356 alloy in a chilled cylindrical mould, shown in Figure 4.1. This setup, with a chilled bottom and insulated sides, offers the advantage of one-dimensional cooling and thus, progressive solidification in the vertical direction. In addition to the relatively simple cooling conditions achieved, the symmetry of the setup can be capitalized upon during modeling to save computing time.

The copper chill located at the bottom of the mold was cooled with water at a flow rate of approximately 12 liters/min. The sides of the casting consisted of a bottomless fireclay crucible (7 mm thick at the bottom tapering to 3 mm at the top, not shown) surrounded by Fiberfrax insulation to minimize heat loss out the sides. A tapered crucible was used to avoid hang-up in the mould and ensure good contact between the ingot base and the chill.
Figure 4.1: Schematic of experimental setup

The setup was instrumented with six thermocouples: five at varying heights in the casting and one within the chill near the bottom of the casting. Thermocouple locations for the two trials differed slightly as test castings from the second trial were slightly smaller in height. Thermocouple locations for the two trials were at heights of approximately 5, 15, 35, 55, and 75 mm from the chill and at 5, 15, 30, 50, 70 mm, respectively. The temperatures during solidification were recorded using a computer data acquisition system at a sampling frequency of 1 Hz. Temperature data was collected for approximately 10 min for each casting.
The A356 alloy used to produce the castings was taken from a series of ladles, each of which had been degassed for a different time to vary the amount of hydrogen present in the melt. The transfer ladles were degassed with dry argon using a rotor degassing system. Samples for hydrogen content and inclusion content analysis were also taken at this time using Ransley and Prefil moulds, respectively. It proved necessary to take the sample from different ladles to allow sufficient time for the casting and its removal from the mold prior to the next casting. Melt samples for the first trial were taken at 1 min intervals of degas time while melt samples for the second trial were sampled at 30 second intervals.

The average initial temperature of the melt was 679°C and 654°C (standard deviation of 12°C) for the first and second trials, respectively. Temperature variations were present due to variations in plant operating conditions at the time of the trials. The initial temperatures of the chill and mold were typically 16 and 38°C, respectively, for the first trial and 13 and 37°C, respectively, for the second trial. As a thermocouple was not located within the mold, the initial temperature of the mold was assumed to equal the thermocouple readings within the casting cavity prior to pouring. The ambient temperature was assumed to be 25°C. A summary of the conditions in the two trials is shown in Table 4.1.

Alcoa Intalco Works, Ferndale, WA, quantified the amount of hydrogen from Ransley samples using a Leco RH-402 machine. The repeatability of the tests is ±0.012 cm³/100 g Al. The Leco method is an offline technique in which a solid sample is melted in a
flowing stream of nitrogen gas.59 Hydrogen content is determined through measurement of thermal conductivity of the gas stream.

Table 4.1: Summary of casting conditions in Trials 1 and 2

<table>
<thead>
<tr>
<th></th>
<th>Trial 1</th>
<th>Trial 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of castings</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>Degas times (min)</td>
<td>0, 1, 2, 3, and 4</td>
<td>0, 0.5, 1, 1.5, 2, 2.5, and 3</td>
</tr>
<tr>
<td>Water flow rate (liters/min)</td>
<td>11.4</td>
<td>12.2</td>
</tr>
<tr>
<td>Thermocouple locations (mm)</td>
<td>5, 15, 35, 55, and 75</td>
<td>5, 15, 30, 50, and 70</td>
</tr>
<tr>
<td>Average Initial Temperature (°C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Melt</td>
<td>679</td>
<td>654</td>
</tr>
<tr>
<td>Chill</td>
<td>38</td>
<td>37</td>
</tr>
<tr>
<td>Mould</td>
<td>16</td>
<td>13</td>
</tr>
</tbody>
</table>

The inclusion content was measured by ABB Bomem Inc., Quebec City, Que., via Prefil samples. The repeatability of the analysis is ±16% for an inclusion content greater than 1.25 mm²/kg. However, an increase in metal cleanliness will reduce accuracy, with repeatability deteriorating to ±40% for an inclusion content of less than 0.07 mm²/kg.60 The Prefil method is a pressure filtration test where inclusions built up in a ceramic foam filter are quantitatively examined using metallographic analysis.61

The solidified castings were cut along the vertical direction into several sections for analysis, as shown in Figure 4.2, with one quarter used for microporosity measurement and one quarter for microstructure analysis. Tucker's reagent62 was employed to etch the sample for study of the solidification structure.
Figure 4.2: Sectioned test casting.

The amount of porosity (% area) was determined using image analysis of a polished sample. A complete examination of the entire polished surface was impractical due to time and data collection restraints. Consequently, porosity was measured over an area covering the length of the casting and approximately 7.2 mm of the width. A raster was employed wherein porosity was measured in increments of 0.536 mm in height and 0.402 mm in width. A schematic of the process is shown in Figure 4.3. At each location, porosity was measured over an area of 0.22 mm$^2$, with a lower limit of 0.5 μm$^2$ set for the porosity size.
4.2 Results

Typical cooling curves for the test castings are shown in Figure 4.4. The curve for the thermocouple located furthest from the chill clearly shows an initial change of slope at approximately 610°C consistent with primary solidification followed by a change near 568°C associated with eutectic formation. The cooling curves measured by thermocouples closer to the chill exhibit relatively minor or no changes in slope at the liquidus and eutectic temperatures due to more rapid cooling near the chill.
In a number of castings, an abrupt change was observed in the slope of cooling curves at temperatures not associated with the liquidus or eutectic temperature for locations near the chill, as shown in Figure 4.5 (e.g. at 15 sec for 8 mm TC). This sudden increase in cooling rate implies a significant variation in the expected level of contact between the chill and the sample. Generally, the chill-sample contact is expected to degrade due to shrinkage during solidification. However in this case, contact remains high throughout solidification. This behaviour is potentially the result of leakage occurring during testing.
In a number of test castings, a varying amount of material leaked due to an imperfect fit between the crucible bottom and the chill. This may have caused the sample to 'adhere' more tightly to the chill and thus maintain a higher level of contact than observed in the cases where the material did not leak.

Figure 4.5: Cooling curve for test casting. (Trial 2 – 2.5 min degas)

Although cooling conditions were not intentionally varied, cooling rates varied significantly from casting to casting. A comparison of the cooling curves in Figure 4.4 and Figure 4.5 clearly illustrate this phenomenon. The difference can be attributed to the
level of contact maintained during solidification between the casting and the chill, with good contact resulting in more rapid cooling. In addition, the area of contact between the test casting and the chill is affected by the amount of material leaked out of the bottom of the crucible.

The hydrogen level decreases with increasing degas times, as shown in Figure 4.6. However, the effect of degassing for periods greater than ~2.5 min was not significant as similar hydrogen contents were obtained. The results also show a clear difference in hydrogen content between the two trials, with the first displaying consistently higher hydrogen levels at equivalent degas times. As the trials were performed ten months apart, it is unknown whether the difference is due to shifts in the material or processes at the industrial facility or simply to variations in the test conditions (e.g. degree of humidity).

![Figure 4.6: Measured hydrogen in test castings.](image-url)
Inclusion content data is only available for the first trial. The results, shown in Figure 4.7, reveal significantly higher amounts of inclusions and oxide films in the 2 and 3 min degas samples. Inclusions consisted mainly of Al₄C₃ particles less than 3 μm in size, along with small amounts of Al₄C₃ greater than 3 μm (0 and 3 min) and MgO (0, 2, 3, and 4 min) in a number of samples. Trace amounts of Al₄C₃ greater than 3 μm (1, 2, and 4 min) and MgAl₂O₄ (0, 1, 2, and 4 min) were also found in the test castings.

![Graph showing inclusion content in test castings.](image)

**Figure 4.7:** Measured inclusion content in test castings.

Typical results of the etched samples are shown in Figure 4.8. Images of all etched samples can be found in Appendix A. Initially, the aluminum solidifies with a columnar structure in proximity to the chill, which transitions to an equiaxed structure within
approximately 10 mm. In Trial 1, the equiaxed grain size varied from approximately 1-2 mm near the chill to approximately 4-5 mm at the top end of the casting, as shown in Figure 4.8(a). In Trial 2, the grain size was generally finer and more uniform throughout the casting, as shown in Figure 4.8(b). The grain size is less than 1 mm except for a small portion at the top end of the casting, which shows grains of approximately 3-4 mm. However, two castings from Trial 2 displayed grain sizes that were more consistent with the larger grains observed Trial 1. Cooling conditions for these two samples were similar to the remaining samples from Trial 2, thus it would appear that there were a potentially smaller number of nucleation sites available in these samples.

Examination of the porosity generally showed pores similar to those shown in Figure 4.9. Visual observations of the polished castings revealed finely distributed porosity, which tended to increase in amount toward the top of the casting, as shown in Figure 4.10. Images of all polished castings can be found in Appendix A. In general, a limited number of macro-shrinkage pores were observed at the top of castings. However, two samples (1 and 1.5 min degas – Trial 2) exhibited significantly greater levels of microporosity, as shown in Figure 4.11 (1 min degas). This is potentially the result of contact between the top of the casting and a metal jig designed to hold the thermocouples. During solidification, the contact increased cooling at the top of the sample and thus, interfered with directional solidification.
Figure 4.8: Typical etched samples from (a) Trial 1 and (b) Trial 2 displaying a predominantly equiaxed microstructure.
Figure 4.9: Typical pores in test casting.

Figure 4.10: Polished sample displaying fine microporosity and isolated areas of macroporosity. (Trial 1 – No degas)

Figure 4.11: Polished sample displaying a high level of microporosity near the top of the casting. (Trial 2 – 1 min degas)
To facilitate comparison of porosity between castings, porosity values across the width of the sample area were averaged to represent the porosity level at selected heights in the casting. The average porosity as a function of distance from the chill is shown in Figure 4.12, with error bars representing one standard deviation. Although data represented in this form exhibited a high degree of scatter, a clear trend to increasing porosity content with distance from the chill was observed.

Figure 4.12: Variation of porosity with distance from the chill. (Trial 1 – 3 min degas)

Quadratic correlations fitted to the data from each casting in Trial 1 and Trial 2 are shown in Figure 4.13 and Figure 4.14, respectively (note: different scales are used). Areas of
significant macroporosity near the top of the 1 and 1.5 min degas samples from Trial 2 were not included in the data. A comparison of porosity in castings from Trial 1 shows a clear and significant drop in porosity with decreasing hydrogen content. Castings from Trial 2 show the same general behaviour, with the 1 and 1.5 min degas samples displaying similar porosity levels for comparable hydrogen contents. When all castings are considered together, the 2 min degas sample from Trial 1 displays a lower porosity level than a similar hydrogen content sample from Trial 2. However, as the two trials were performed ten months apart, it is difficult to pinpoint a specific cause. In addition, inclusion content data for the second trial is not available.

<table>
<thead>
<tr>
<th>Degas time (min)</th>
<th>( [H_0] ) (cm(^3)/100g)</th>
<th>Inclusion (mm(^2)/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.137</td>
<td>0.141</td>
</tr>
<tr>
<td>1</td>
<td>0.100</td>
<td>0.131</td>
</tr>
<tr>
<td>2</td>
<td>0.075</td>
<td>0.252</td>
</tr>
<tr>
<td>3</td>
<td>0.051</td>
<td>0.224</td>
</tr>
<tr>
<td>4</td>
<td>0.053</td>
<td>0.131</td>
</tr>
</tbody>
</table>

Figure 4.13: Average porosity in test castings from Trial 1.
To some extent, the amount of porosity observed in Trial 1 in the 3 min degas sample, which has virtually the same hydrogen content as the 4 min degas sample, is unexpectedly high. Referring to the analysis of inclusion content presented in Figure 4.7, presumably, the difference in behavior of the 3 min degas sample can be attributed to its higher \( \text{Al}_4\text{C}_3 \) content or oxide film content. Porosity nucleation may be aided by inclusions, which act as heterogeneous nucleation sites for hydrogen.\textsuperscript{27} The effect would be to produce more porosity overall, which is observed.

---

**Figure 4.14:** Average porosity in test castings from Trial 2.
In general, the experimental results agree relatively well with past work of other researchers. Two clear trends of increasing porosity with increasing distance from the chill and with increasing hydrogen content were observed. Both trends are consistent with those reported in literature. Inclusion content was also observed to affect porosity levels although measurements were limited to the first trial only.
5 Mathematical Model

A 2-D axisymmetric mathematical model was developed using the commercial finite element package ABAQUS™*. The model is capable of predicting the cooling behaviour, criteria function values, and microporosity levels of test castings.

5.1 Heat Transfer Model

The thermal behaviour during solidification was modeled by considering heat transport by diffusion only. The effects of convection in the liquid, while present, are not treated in detail. Instead, its effects are approximated by an increase in conductivity in the liquid melt. Assuming that the solid and liquid can be treated as a single domain, with a 2-D axisymmetric geometry, heat diffusion is governed by the following equation:

\[
\frac{k}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + k \frac{\partial^2 T}{\partial z^2} + \dot{Q} = \rho C_p \frac{\partial T}{\partial t} \quad \text{with} \quad \dot{Q} = L \frac{\partial f_s}{\partial t}
\]

where \( T \) (°C) is the temperature, \( k \) (W/m·°C) is the conductivity, \( \rho \) (kg/m\(^3\)) is the density, \( C_p \) (J/kg·°C) is the specific heat, \( r \) and \( z \) (m) are the radial and axial distance, respectively, \( t \) (s) is the time, \( L \) (J/m\(^3\)) is the latent heat, \( f_s \) is the solid fraction, and \( \dot{Q} \) (W/m\(^3\)) is the latent heat released due to the liquid-to-solid phase transformation.

The thermal conductivity employed in the model was assumed to vary with temperature. The conductivity in the liquid has been increased (from approximately 95 W/m·°C) to account for convection in the liquid. The solid fraction was related to temperature using five linear segments, as shown in Figure 5.1, which were determined

* ABAQUS is a trademark of Hibbit, Karlsson & Sorenson, Inc., Pawtucket, RI.
from reported temperature-solid fraction experimental data. The release of latent heat of A356 (397.5 kJ/m$^3$) in the model was assumed to parallel the increase in solid fraction. The properties of the materials used in the heat transfer model are shown in Table 5.1.

Figure 5.1: Evolution of solid fraction with temperature.
Table 5.1: Material properties used in the model.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density $\rho$ (kg/m$^3$)</th>
<th>Specific Heat $C_p$ (J/kg·°C)</th>
<th>Conductivity $T$ (°C)</th>
<th>Conductivity $k$ (W/m·°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A356</td>
<td>2685</td>
<td>963</td>
<td>610</td>
<td>145</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>557</td>
<td>166.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>393</td>
<td>166.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>316</td>
<td>167.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>204</td>
<td>166.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>159</td>
<td>154.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>147</td>
<td>153.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>134</td>
<td>146.54</td>
</tr>
<tr>
<td>Fireclay</td>
<td>2300</td>
<td>960</td>
<td>1100</td>
<td>1.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>800</td>
<td>1.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>500</td>
<td>1.28</td>
</tr>
<tr>
<td>Fiberfrax</td>
<td>244</td>
<td>1130</td>
<td>760</td>
<td>0.403</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>649</td>
<td>0.331</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>538</td>
<td>0.272</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>427</td>
<td>0.218</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>316</td>
<td>0.168</td>
</tr>
<tr>
<td>Copper</td>
<td>8940</td>
<td>386</td>
<td>398</td>
<td></td>
</tr>
</tbody>
</table>

The casting was discretized with 360 8-noded quadratic elements (for a total of 1165 nodes) with the mesh size biased toward the bottom of the casting, as shown in Figure 5.2. Along the centerline, elements varied from 1.68 mm in height adjacent to the chill to 4.57 mm at the top of the casting. In order to save computation time, the chill and the mold were modeled with a coarser mesh of 4-noded linear elements approximately 5 mm in size. Thus, the chill and mold consisted of 208 and 260 elements, respectively.
The initial temperature of each part was assumed to be uniform and equal to measured thermocouple values. At the casting interfaces (casting/chill and casting/mould), heat transfer is assumed to be high initially and decrease as a gap forms due to shrinkage of the casting during solidification. The gap conductance boundary condition employs an effective heat transfer coefficient, which combines contact conductive and radiative heat transfer as shown:

$$ q^* = h_{	ext{eff}} (T_{\text{cast}} - T_{\text{mould}}) $$

(23)

where $q^*$ ($\text{W/m}^2$) is the heat flux, $h_{\text{eff}}$ ($\text{W/m}^2 \cdot ^\circ \text{C}$) is the effective heat transfer coefficient, and $T_{\text{cast}}$ and $T_{\text{mould}}$ ($\text{K}$) are the temperatures of the casting and the mould, respectively.
Above a maximum temperature \((T_{\text{cast}}>T_{\text{max}})\), the contact between the interfaces is assumed ‘perfect’ and heat transfer is due solely to conduction:

\[
h_{\text{eff}} = h_{\text{cond}}
\]

where \(h_{\text{cond}} (\text{W/m}^2\cdot\text{°C})\) is the conductive heat transfer coefficient. As the gap increases with decreasing temperature, the gap heat transfer is decreased linearly to a lower limit while radiation heat transfer is increased using an equation of the form:

\[
h_{\text{eff}} = f_{\text{cond}}h_{\text{cond}} + (1 - f_{\text{cond}})h_{\text{rad}}
\]

where \(f_{\text{cond}}\) is the fraction of conductive heat transfer and \(h_{\text{rad}} (\text{W/m}^2\cdot\text{°C})\) is the radiative heat transfer coefficient. \(f_{\text{cond}}\) is calculated using:

\[
f_{\text{cond}} = \frac{T_{\text{cast}} - T_{\text{min}}}{T_{\text{max}} - T_{\text{min}}} (1 - f_{\text{lim}}) + f_{\text{lim}}
\]

where \(f_{\text{lim}}\) is the lower limit to gap conduction heat transfer and \(T_{\text{max}}\) and \(T_{\text{min}}\) (K) characterize the temperature interval over which the gap forms. \(h_{\text{rad}}\) is determined using:

\[
h_{\text{rad}} = \sigma \varepsilon_{\text{eff}} \cdot \left( (T_{\text{cast}}^2 + T_{\text{mould}}^2) \cdot (T_{\text{cast}} + T_{\text{mould}}) \right)
\]

where \(\sigma (\text{W/m}^2\cdot\text{K}^4)\) is the Stefan-Boltzmann constant and \(\varepsilon_{\text{eff}}\) is the effective emissivity. \(\varepsilon_{\text{eff}}\) is calculated using the equation:

\[
\varepsilon_{\text{eff}} = \frac{1}{\frac{1}{\varepsilon_{\text{cast}}} + \frac{1}{\varepsilon_{\text{mould}}} - 1}
\]

where \(\varepsilon_{\text{cast}}\) and \(\varepsilon_{\text{mould}}\) are the emissivities of the casting and mould, respectively. At temperatures lower than a specified minimum \((T_{\text{cast}}<T_{\text{min}})\), a lower limit to the gap conduction \(f_{\text{lim}}\) is reached and the effective heat transfer coefficient is calculated using:

\[
h_{\text{eff}} = f_{\text{lim}}h_{\text{cond}} + (1 - f_{\text{lim}})h_{\text{rad}}
\]
The lower limit varied significantly between castings (0.07-0.88) due to the differences in the final level of contact that each casting maintained with the chill. In addition, two values of $f_{lim}$ were often necessary due to the sample leakage out of the bottom of the crucible (described earlier in Section 4.2). The first value was used to describe the initial gap formation while the second value was applied to account for the higher level of contact observed in the cases where material leaked. The time at which the second value was applied was arbitrarily chosen to fit the thermocouple data. The values and times at which $f_{lim}$ were applied for each casting are shown in Appendix B.

Heat transfer from the top of the casting to the surroundings was modeled using free convection and far-field radiation as shown:

\[ q_{\text{conv}} = h_{\text{conv}} (T_{\text{surf}} - T_\infty) \]  
\[ q_{\text{rad}} = \varepsilon_{\text{surf}} (T_{\text{surf}}^4 - T_\infty^4) \]

where $q_{\text{conv}}$ and $q_{\text{rad}}$ (W/m$^2$) are the convective and radiative heat fluxes, $h_{\text{conv}}$ (W/m$^2$°C) is the convective heat transfer coefficient, $\varepsilon_{\text{surf}}$ is the surface emissivity, and $T_{\text{surf}}$ and $T_\infty$ (K) are the surface and ambient temperatures, respectively. Heat transfer from the chill to the chill water was incorporated using a heat transfer coefficient of 8000 W/m$^2$°C. Heat loss from the chill and mould to the surroundings and heat transfer at the chill/mould interface were assumed to be negligible. The values of the variables used in the boundary condition calculations are shown in Table 5.2.
Table 5.2: Values of heat transfer boundary condition variables.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{max}}$ (°C)</td>
<td>610</td>
</tr>
<tr>
<td>$T_{\text{min}}$ (°C)</td>
<td>550</td>
</tr>
<tr>
<td>$h_{\text{cond}, \text{casting/chill}}$ (W/m²°C)</td>
<td>7000</td>
</tr>
<tr>
<td>$h_{\text{cond}, \text{casting/mould}}$ (W/m²°C)</td>
<td>1000</td>
</tr>
<tr>
<td>$h_{\text{conv}, \text{casting/air}}$ (W/m²°C)</td>
<td>10</td>
</tr>
<tr>
<td>$\varepsilon_{\text{cast}}$</td>
<td>0.31</td>
</tr>
<tr>
<td>$\varepsilon_{\text{mould}}$</td>
<td>0.75</td>
</tr>
<tr>
<td>$\varepsilon_{\text{chill}}$</td>
<td>0.78</td>
</tr>
</tbody>
</table>

5.2 Criteria Function Model

The four criteria functions evaluated within the model are summarized in Table 5.3. The method of calculation of the thermal parameters employed in the criteria functions has a considerable impact on the value of criteria functions. The criteria functions reviewed contain four thermal parameters: thermal gradient ($G$), cooling rate ($R_c$), solidus velocity ($v_s$), and local solidification time ($t_s$).

Table 5.3: Criteria functions predicted by model.

<table>
<thead>
<tr>
<th>Investigators</th>
<th>Abbreviation</th>
<th>Criteria Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Niyama et al.</td>
<td>Niyama</td>
<td>$G/\sqrt{R_c}$</td>
</tr>
<tr>
<td>Lee et al.</td>
<td>Lee</td>
<td>$G \cdot t_s^{2/3}/v_s$</td>
</tr>
<tr>
<td>Kao and Chang</td>
<td>KC</td>
<td>$G^{0.38}/v_s^{1.62}$</td>
</tr>
<tr>
<td>Suri et al.</td>
<td>FRN</td>
<td>$(216 \mu \Delta T)/(\rho L v_s \beta D_0^2)$</td>
</tr>
</tbody>
</table>
Thermal gradient was calculated using the following equation:

\[ q = k \left( \frac{\partial T}{\partial r} + \frac{\partial T}{\partial z} \right) \]  \hspace{1cm} (32)

where \( q \) (W/m\(^2\)) is the heat flux, \( k \) (W/m °C) is the conductivity, \( T \) (°C) is the temperature, and \( r \) and \( z \) (m) are the radial and axial distances, respectively. However, due to the strong directional cooling represented by the chill, heat flow in other directions was considered negligible and the gradient, \( G \) (°C/m), was calculated using:

\[ q = k \frac{\partial T}{\partial z} \Rightarrow \frac{\partial T}{\partial z} = G = \frac{q}{k} \]  \hspace{1cm} (33)

The cooling rate was determined using an equation of the form:

\[ R = \frac{T_{i-1} - T_i}{\Delta t_i} \]  \hspace{1cm} (34)

where \( R \) (°C/s) is the cooling rate, \( T_{i-1} \) and \( T_i \) (°C) are the temperatures at the previous and current time increments, respectively, and \( \Delta t_i \) (s) is the time increment. The velocity of an arbitrary isotherm temperature, \( v_T \) (m/s), was calculated as a function of the cooling rate and thermal gradient in the following manner:

\[ v_T = \frac{R_T}{G_T} \]  \hspace{1cm} (35)

where \( R_T \) and \( G_T \) are evaluated at the isotherm temperature – i.e. evaluating these parameters at the solidus temperature would give the solidus velocity, \( v_s \). Local solidification time, \( t_s \) (min), was the time required for the local point to cool from the liquidus temperature to the solidus temperature.
Parameters such as thermal gradient can vary significantly depending on the temperature at which they are determined. Most studies in the literature have defined a critical temperature at which these variables are calculated. The specific temperature varies with different researchers; however, it is generally set near the eutectic temperature, where a well-established dendritic network has already formed.\textsuperscript{31} At this temperature, the predicted cooling behaviour is sensitive to the release of latent heat associated with eutectic formation. Thus, the method employed to model the latent heat effect can significantly affect the value of the thermal parameters. In the model, thermal parameters were calculated at a critical temperature of 573°C ($T_{\text{crit}}=T_{\text{eutectic}}+5^\circ\text{C}$) to avoid abrupt oscillations seen for critical temperatures closer to the eutectic temperature (i.e. $G=G_{\text{crit}}$, $R=R_{\text{crit}}$, and $v_s=v_{\text{crit}}$).

The criteria function values were determined in a ‘user subroutine’ within ABAQUS\textsuperscript{TM} called ‘usdfld.f’. Parameter values at each integration point were calculated at the critical temperature and stored in solution-dependent state variables (STATEV), which can be updated at every time increment. Criteria functions were calculated after local solidification was complete.

### 5.3 Hydrogen-Shrinkage Model

The hydrogen-shrinkage model employs a local pressure criterion to determine the likelihood of porosity formation and predicts the amount of porosity if conditions are favourable. This model was also calculated within the ‘usdfld.f’ subroutine. Porosity
was assumed to form when the gas pressure \( P_g \) meets or exceeds a critical pressure \( P_c \) consisting of the local liquid metal pressure \( P_l \) and surface tension \( P_{surf} \):\(^{16}

\[
P_g \geq P_c
\]

\[
P_c = P_l + P_{surf}
\]

\[
P_l = P_a + P_{met} + P_s
\]

where \( P_a \), \( P_{met} \), and \( P_s \) (N/m\(^2\)) are the ambient, metallostatic, and shrinkage pressures, respectively. The ambient pressure was assumed constant while the metallostatic pressure was assumed to be a function of height and calculated as:

\[
P_{met} = \rho_l gh
\]

where \( \rho_l \) (kg/m\(^3\)) is the liquid density, \( g \) (m/s\(^2\)) is gravity, and \( h \) (m) is the height of the metal head. Assuming heterogeneous nucleation of a spherical pore, the surface tension per unit area was calculated using:\(^{14}\)

\[
P_{surf} = 1.12 \left( \frac{2\sigma_{LG}}{r_p} \right) f(\theta) \frac{1}{2}
\]

where:

\[
f(\theta) = \sqrt{\frac{2 + 3 \cos \theta - \cos^3 \theta}{2}}
\]

where \( \sigma_{LG} \) (N/m) is the surface tension between gas and liquid, \( r_p \) (m) is the radius of the pore, and \( \theta \) (°) is the normal angle of contact (e.g. measured by a sessile drop technique). The term \( 2\sigma_{LG}/r_p \) represents the surface tension in the case of homogenous nucleation. During solidification, \( P_{surf} \) was assumed constant\(^{22}\) with a pore radius equal to one-quarter of the secondary dendrite arm spacing\(^{10}\) (e.g. \( r = \frac{1}{4} \lambda_2 \)).
A shrinkage pressure term was employed to account for solidification shrinkage effects using the method by Flemings and coworkers. By approximating flow through the mushy zone as flow through a simple bundle of capillary tubes, the flow velocity through each tube can be related to pressure gradient. Assuming the pressure gradient occurs over the length of the mushy zone, the shrinkage pressure was calculated by:

$$P_s = -\frac{\beta}{(1 - \beta)} \cdot \frac{\mu v L_m}{\gamma \cdot f_l}$$

where $\beta$ is the volume shrinkage, $\mu$ (Pa·s) is the kinematic viscosity, $v$ (m/s) is the solidification rate, $L_m$ (m) is the mushy zone length, $\gamma$ (m$^2$) is a measure of the resistance of the dendrites to fluid flow, and $f_l$ is the fraction liquid.

The flow resistance term $\gamma$ was calculated using the equation:

$$\gamma = \frac{1}{24n_c \pi c^3}$$

where $n_c$ is the number of interdendritic channels and $c$ is a geometric factor. The geometric factor is a constant included to account for fact that flow channels are neither straight nor parallel and is often referred to as the tortuosity. Values have reported to vary between 1 and 4.6, with larger values indicating more convoluted channels. Assuming one channel per dendrite arm, the number of interdendritic channels was determined using:

$$n_c = \frac{1}{\lambda_2^2}$$

where $\lambda_2$ (m) is the secondary dendrite arm spacing. $\lambda_2$ was related to the cooling rate using an equation similar in form to that of Emadi et al. The constants have been
changed to better correlate the predicted values to measured $\lambda_2$ values and the following equation was used:

$$\log \frac{dT}{dt} = -\frac{\log(\lambda_2) - 2.20}{0.34}$$  \hspace{1cm} (45)

where $dT/dt$ (°C/min) is the cooling rate.

The local gas pressure in the liquid was determined using the following hydrogen mass balance:\textsuperscript{12}

$$[H_0] = f_s[H_s] + (1 - f_s - f_c)[H_l] + \alpha \frac{f_c P_g}{T}$$  \hspace{1cm} (46)

where $[H_0]$ (cm\(^3\)/100g) is the initial hydrogen concentration, $\alpha$ is the gas conversion factor, $T$ (K) is the temperature, and $[H_s]$ and $[H_l]$ (cm\(^3\)/100g) are the hydrogen concentrations in the solid and liquid metal, respectively. The amount of hydrogen in the pore was considered negligible and the third term on the right hand side of the equation has been neglected. Using Sievert’s law, the hydrogen concentration in the solid and liquid metal was expressed as:

$$[H_s] = K_s P_g^{\frac{1}{2}}$$  \hspace{1cm} (47)

$$[H_l] = K_l P_g^{\frac{1}{2}}$$  \hspace{1cm} (48)

where $K_s$ and $K_l$ (cm\(^3\)/100g-atm\(^{1/2}\)) are equilibrium constants for the solid and liquid, respectively.

The evolution of local critical pressure and gas pressure for a test casting is shown in Figure 5.3. As solidification proceeds, the shrinkage pressure decreases as interdendritic
feeding is increasingly hampered. The critical local pressure required for porosity formation experiences a corresponding decrease in value. In contrast, hydrogen gas pressure increases due to the difference in solubility between the solid and liquid phases. Once the condition for porosity formation \((P_g \geq P_c)\) is met, porosity can form. Continued solidification will further increase feeding difficulty and increase hydrogen gas levels. The increased likelihood of porosity formation at increase volume fractions of solid is reflected by a larger difference between the gas and critical pressures. The slight fluctuation in the critical pressure curve at approximately \(f_s = 0.96\) is a numerical result associated with a large fluctuation in solidification rate.

![Graph](image)

**Figure 5.3:** Variation of critical local pressure and hydrogen gas pressure with fraction solid for test casting. (Trial 1 – 1 min degas - 45 mm from chill)
While a rigorous approach to porosity development would track the nucleation and growth of individual pores, a relatively simple approach is proposed where the total time ($t_{pore}$) for pore growth is determined and is used as a measure of pore fraction. Pore growth is assumed to start when $P_g = P_c$ and stop at the end of solidification, assumed at $f_s = 0.99$ or $f_i = 0$ (note: $f_i = 1 - f_s - f_v$), whichever occurs first. These times are shown in Figure 5.3.

The pressure difference and pore growth time have been incorporated into a new criteria to predict microporosity growth. The unconstrained growth of porosity is tracked with integrated pressure, $\phi_e$, which is calculated as:

$$\phi_e = \mu_e \int_0^{t_{pore}} \Delta P \, dt$$  \hspace{1cm} (49)

where $\Delta P$ (atm) is the difference in gas pressure and critical pressure, $t$ (s) is the time and $\mu_e$ (1/atm-s) is an empirical constant. The increment in volume fraction porosity, $d\phi_e$, has been corrected for impingement with evolving solid fraction using a modified form of the Avrami correction factor as shown:

$$df_v = (1 - f_s - f_v) d\phi_e$$  \hspace{1cm} (50)

to yield an estimate of the actual volume fraction, $f_v$. The values of the constants used in the previous calculations are listed in Table 5.4.
Table 5.4: Properties of A356 and constants used in microporosity calculations.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface tension</td>
<td>$\sigma_{LG}$ (N/m)</td>
<td>0.79</td>
<td>14</td>
</tr>
<tr>
<td>Normal angle of contact</td>
<td>$\theta$ (°)</td>
<td>135</td>
<td>14</td>
</tr>
<tr>
<td>Volume shrinkage</td>
<td>$\beta$</td>
<td>0.057</td>
<td>14</td>
</tr>
<tr>
<td>Viscosity</td>
<td>$\mu$ (Pa·s)</td>
<td>0.0013</td>
<td>14</td>
</tr>
<tr>
<td>Equilibrium constants</td>
<td>$K_r$ (cm³/100g-atm¹/²)</td>
<td>0.04</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>$K_i$ (cm³/100g-atm¹/²)</td>
<td>0.7</td>
<td>9</td>
</tr>
<tr>
<td>Empirical constant</td>
<td>$\mu_e$ (1/atm·s)</td>
<td>0.015</td>
<td></td>
</tr>
</tbody>
</table>

### 5.4 Sensitivity Analysis

A sensitivity analysis was performed to determine the effect of key variables on the thermal and microporosity models. The parameters with uncertain or poor defined values were chosen and included variables dealing with gap formation between casting interfaces and difficult-to-characterize melt properties. Thermophysical properties such as conductivity and latent heat were considered accurate and were not included in the sensitivity analysis. The thermal model variables selected were the emissivity ($\epsilon_{cast}$, $\epsilon_{mould}$, and $\epsilon_{chill}$), the temperature interval over which gap formation occurs ($\Delta T_{gap}$=$T_{max}$-$T_{min}$, varying $T_{min}$), and the conductive heat transfer coefficients for the casting/chill ($h_{cond,cc}$), casting/mould ($h_{cond,cm}$), and chill/water ($h_{cond,cw}$) interfaces. Microporosity model variables chosen included hydrogen solubility constants ($K_r$ and $K_i$), permeability constant ($\gamma$), surface tension ($\sigma_{LG}$), pore radius ($r_p$), and contact angle between the pore and substrate ($\theta$). Local solidification time ($t_s$) and fraction porosity ($f_v$) were chosen as indicators of model sensitivity to varying each parameter by ±20%.
The sensitivity of the indicators \( t_s \) and \( f_v \) to changes in the thermal model parameters \( \Delta T_{\text{gap}} \) and \( h_{\text{cond,cc}} \) are shown in Figure 5.4 and Figure 5.5, respectively, as a function of distance from the chill. The variation in \( h_{\text{cond,cc}} \) changed the predicted values of \( t_s \) and \( f_v \) by as much as 20% and 26%, respectively, over the bulk of the casting (>25 mm from chill). The region near the chill displayed a larger percentage change, however, this was largely a numerical phenomenon as values near the chill were relatively small. By comparison, the variation in \( \Delta T_{\text{gap}} \) caused less than 7% and 11% change in \( t_s \) and \( f_v \), respectively, over the same region.

![Figure 5.4: Sensitivity of local solidification time to \( h_{\text{cond,cc}} \) and \( \Delta T_{\text{gap}} \).](image-url)
Variables from the microporosity model do not impact the thermal model. Thus, only $f_v$ is evaluated in the sensitivity analysis. The sensitivity of $f_v$ to the microporosity model parameters $K_s$, $K_i$, and $\theta$ are shown in Figure 5.6 and Figure 5.7, as a function of distance from the chill. The model displayed significant sensitivity to all three parameters, in particular $K_s$, which caused 30-40% change in $f_v$ over the bulk of the casting. The larger percentage increases were again a numerical phenomenon.
Figure 5.6: Sensitivity of fraction porosity to microporosity model parameters.
Figure 5.7: Variation in fraction porosity with distance from the chill with +/-20% change in microporosity model parameters.

The results of varying the model parameters on the thermal and microporosity indicators are summarized in Table 5.5. With respect to the thermal model variables, local solidification time and fraction porosity were the most sensitive to changes in $\Delta T_{gap}$ and $h_{cond,cc}$. This sensitivity seems reasonable considering that most heat transfer during solidification is through the casting/chill interface. The fraction porosity is also clearly sensitive to a number of parameters in the microporosity model, in particular, $K_s$, $K_i$, and $\theta$. The sensitivity of $K_s$ and $K_i$ underscores the critical role that hydrogen evolution plays...
in the formation of porosity while $\theta$ illustrates the significant effect of heterogeneous nucleation. However, despite the sensitivity of the model to a number of variables, the predicted porosity trends with distance from the chill in each case are still relatively similar. Thus, the ability of the model to predict areas of likely porosity formation is maintained even if all variables are not known with complete certainty.

Table 5.5: Sensitivity of model to selected parameters varied by +/- 20% (High/Low).

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\Delta s$ (%)</th>
<th>$\Delta f_v$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emissivity $\varepsilon_{\text{casting}}, \varepsilon_{\text{mould}}, \text{and } \varepsilon_{\text{chill}}$</td>
<td>1.7/0.8</td>
<td>0.8/1.1</td>
</tr>
<tr>
<td>Gap Temp Interval $\Delta T_{\text{gap}}$ (°C)</td>
<td>2.0/4.9</td>
<td>0.4/3.1</td>
</tr>
<tr>
<td>Casting/chill HTC $h_{\text{cond},cc}$ (W/m²·°C)</td>
<td>12.6/17.9</td>
<td>14.2/21.5</td>
</tr>
<tr>
<td>Casting/mould HTC $h_{\text{cond},cm}$ (W/m²·°C)</td>
<td>0.5/0.5</td>
<td>0.4/0.4</td>
</tr>
<tr>
<td>Chill/water HTC $h_{\text{cond},cw}$ (W/m²·°C)</td>
<td>2.2/0.3</td>
<td>0.8/0.6</td>
</tr>
<tr>
<td>Equilibrium constants $K_i$ (cm³/100g·atm⁰.⁵)</td>
<td>26.6/26.6</td>
<td></td>
</tr>
<tr>
<td>$K_i$ (cm³/100g·atm⁰.⁵)</td>
<td>14.5/19.6</td>
<td></td>
</tr>
<tr>
<td>Permeability $\gamma$ (m²)</td>
<td>4.2/3.9</td>
<td></td>
</tr>
<tr>
<td>Surface tension $\sigma_{\text{LG}}$ (N/m)</td>
<td>3.8/3.9</td>
<td></td>
</tr>
<tr>
<td>Pore radius $r_p$ (m)</td>
<td>3.3/4.7</td>
<td></td>
</tr>
<tr>
<td>Contact angle $\theta$ (°)</td>
<td>17.4/20.6</td>
<td></td>
</tr>
</tbody>
</table>

* Values are reported for a point at 42 mm from chill.
6 Discussion

The mathematical model was run on a Silicon Graphics Inc. (SGI™) Origin 200 machine with a 64-bit MIPS™ RISC R10000 180 MHz microprocessor. A casting simulation of 5 minutes required approximately 1 hour of execution time. A contour plot of the temperature distribution in a test casting is shown in Figure 6.1. Cooling is generally in the vertical direction with a small degree of variation in temperature across the width of the casting.

![Contour plot of temperatures (°C) in test casting at 250 sec. (Trial 1 - 1 min degas)](image)

Figure 6.1: Contour plot of temperatures (°C) in test casting at 250 sec. (Trial 1 - 1 min degas)

Typical predicted cooling curves (for nodes at the centerline) and experimental data are shown in Figure 6.2. The cooling curves for all of the test castings are included in
Appendix C. There was a relatively minor deviation observed near the top of the casting in the area of the uppermost thermocouple. The simplifying assumption of a level surface at the top of the casting is a potential cause of this discrepancy, as the surface geometry was in reality, irregular and varied. In general, however, the predicted and experimental cooling curves agree reasonably well.

![Temperature vs. Time Graph](image)

**Figure 6.2**: Predicted cooling curves and experimental data. (Trial 1 – 1 min degas)

### 6.1 Criteria Function Analysis

The thermal parameters, $G$, $R_c$, $v_s$, and $t_s$, calculated by the model that are needed for evaluation of the various criteria functions are shown in Figure 6.3. Both thermal gradient and cooling rate were seen to rapidly decrease with increasing distance from the chill. In contrast to the smooth curve predicted for the variation in gradient however, the curve for predicted cooling rate was comparatively irregular near the chill (<15 mm from
The relatively abrupt changes in slope at 6 and 13 mm were due to the heat transfer boundary condition at the casting/chill interface. The calculation of the thermal parameters for these positions coincided with the times at which the lower limits of conductive heat transfer ($f_{lim}$) were reached, which caused a sudden change in the effective heat transfer coefficient, $h_{eff}$. The uneven decrease in cooling rate also produced some associated fluctuation in the isotherm velocity in this region. In general, isotherm velocity decreased steadily with increasing distance from the chill except near the top of the casting. The relatively large increase seen at the top of the casting is likely a numerical result associated with the behaviour of the gradient in this area. Slight changes in the relatively small value of the gradient are magnified during the calculation of the isotherm velocity. Finally, solidification time increases steadily with increasing distance from the chill.
Profiles of the various criteria functions along the vertical centreline of a typical casting are shown in Figure 6.4. All of the criteria functions displayed some oscillation in value near the chill as well as an inflection point at 13 mm from the chill. This behaviour was directly linked to the abrupt changes observed in cooling rate and by extension, isotherm velocity in the region. The predicted Niyama function displayed a trend of decreasing magnitude (increasing porosity) with increasing distance from the chill near the bottom.
(<13 mm) and top (>50 mm) of the casting. However, a relatively constant magnitude was predicted in the middle region of the casting. In contrast to the Niyama function, the predicted trend of the Lee function at the bottom of the casting (<13 mm) exhibited a brief increase (decrease in porosity) in the area immediately adjacent to the chill before it decreased in magnitude with increased distance from the chill. This was followed by a trend of increasing magnitude with distance before decreasing again near the top of the casting (>50 mm). The KC function followed a similar trend with distance to the Lee function although it displayed more fluctuation near the chill. The last of the four functions examined, the FRN, displayed an increase in magnitude (increase in porosity) with increasing distance from the chill throughout most of the casting. In addition to the drop in magnitude seen at 13 mm however, the FRN also displays a significant drop at the top of the casting.
Figure 6.4: Profile of criteria function values along the vertical centreline of a typical casting. (Trial 1 – 1 min degas)

All of the castings displayed criteria function profiles similar to those shown in Figure 6.4 with the exception of the Trial 2 – 1 min degas sample. The profiles of the criteria functions for this casting are shown in Figure 6.5. The Niyama function displayed a continuous decrease in magnitude with increasing distance from the chill. With the exception of some fluctuation near the chill, the Lee function and the KC function behaved in a similar manner. On the other hand, the FRN increases in magnitude with
distance from the chill at the bottom of the casting (<20 mm) before leveling off and decreasing over the rest of the casting.

![Graph showing criteria functions and porosity percentage](image)

**Figure 6.5:** Profile of criteria function values along the vertical centreline of Trial 2 - 1 min degas sample.

The thermal parameters for the Trial 2 - 1 min degas casting are shown in Figure 6.6. The cooling rate, which was significantly lower than for any other casting, decreased with increasing distance at the bottom of the casting (<20 mm) and then remained relatively constant over the bulk of the casting. Meanwhile, the predicted thermal
gradient continuously decreased with increasing distance from the chill. As a result, the
Niyama function displayed a trend of continuous decrease. The Lee and KC functions
displayed behaviour similar behaviour except for a slight discrepancy near the chill. This
predicted behaviour is more consistent with past studies in the literature\textsuperscript{31,44}, which have
shown that the trend of the Lee function follows the Niyama function.

Figure 6.6: Profile of thermal parameters of criteria functions of Trial 2 – 1 min
degas sample.
Due to the good agreement of the thermal model predictions with experimental data, the calculated thermal parameters were considered to accurately reflect the conditions within the test casting. Thus, the trends of the criteria functions with distance from the chill were predicted with confidence. However, the criteria functions investigated generally exhibited behaviour inconsistent with measured porosity in the castings, with only one sample displaying the proper trend with distance from the chill for the Niyama function and to a lesser extent, the Lee and KC functions. The cause of this behaviour was unclear although the solidification time for this sample was significantly longer than for other castings.

Overall, the behaviour of the Niyama function exhibited the greatest level of consistency. It would appear that the other criteria functions displayed a greater sensitivity to the cooling rate whereas the Niyama function gives a heavier weight to the thermal gradient. Although the other criteria functions are presented in slightly different forms, each can be effectively reduced to a function based on thermal gradient and cooling rate (and solidification time in the case of the Lee function). The additional variables, aside from the equiaxed grain size, in the FRN were considered constant during calculation. The grain size was assumed to be a function of thermal gradient and solidus velocity and was calculated according to the relationship given by Suri et al.\textsuperscript{36}

A common drawback to all of the criteria functions, however, was their inability to account for the large effect of hydrogen on porosity levels. Figure 6.7 shows a comparison between the Niyama parameter calculated for the 0, 1, 2, 3, and 4 min degas
castings from Trial 1. The changes in magnitude of the criteria functions between these castings were not significant. However, as mentioned earlier, the varying hydrogen content dramatically affected the porosity levels in these castings. Secondarily, the results from these test castings also indicated that the inclusion content significantly influences porosity, which is also not addressed in the Niyama function. Thus, regardless of whether Niyama criteria functions can provide a good qualitative assessment of the likelihood of porosity formation, quantitative predictions of porosity levels require a model incorporating the effect of hydrogen and inclusions.

Figure 6.7: Niyama function calculations for castings. (Trial 1)
6.2 Microporosity Analysis

A comparison of the microporosity predictions (at the vertical centerline) from the hydrogen-shrinkage model to the quadratic correlations previously fitted to measured porosity for Trial 1 and Trial 2 is shown in Figure 6.8 and Figure 6.9, respectively. In general, the model predicted an increasing amount of porosity with increasing distance from the chill. In a number of castings (e.g. 0 and 1 min degas – Trial 1 and 0.5, 1, and 1.5 min degas – Trial 2) however, the model predictions displayed a small drop in porosity at certain locations in the casting (e.g. at 25 mm for 1 min degas – Trial 1 sample) instead of a continuous increase. This irregularity was caused by the abrupt increase in cooling rate (as described in the previous section) associated with the casting/chill boundary condition.

In Trial 1, the predicted values for the 0 and 1 min degas samples were within 0.2% (absolute difference) of the measured amount in the bulk of the casting, although the difference was greater near the top. This prediction was deemed acceptable considering the large scatter in the data for their measured correlations (see Figure C.2 and Figure C.4 in Appendix C). Although the prediction for the 2 min degas sample was also generally within one standard deviation of the measured values (see Figure C.6 in Appendix C), it was consistently higher throughout the casting. A potential factor contributing to this difference is the slight error associated with the thermal predictions for this casting, which predicted cooling rates slightly slower than measured (see Figure C.5 in Appendix C). As discussed in Section 2.1.2 of the literature review, slower cooling rates can lead to higher porosity.
Figure 6.8: Comparison of predicted and measured porosity in (a) high hydrogen and (b) low hydrogen level test castings for Trial 1.
Figure 6.9: Comparison of predicted and measured porosity in (a) high hydrogen level and (b) low hydrogen level test castings for Trial 2.
The porosity predictions for the 3 and 4 min degas samples were nearly identical, which was expected given their comparable hydrogen contents and cooling conditions. While the 4 min degas sample displayed close agreement with measured porosity however, the 3 min degas sample deviated from the measured amount in the upper portion of the casting. Presumably, this discrepancy is related to the inclusion content, which was significantly higher than in the 4 min degas sample. As stated earlier in Section 2.1.2, inclusions can act as sites for heterogeneous nucleation. In particular, Al₂O₃ and MgO have been shown to promote porosity formation.²⁷ It should be noted however, that the 2 min degas sample also contained an inclusion content similar to the 3 min degas sample, but displayed less porosity than predicted. Thus, the actual effect of inclusion content was not clear.

In Trial 2, the porosity predictions were lower than the measured amount for all castings except for the 1 min degas sample. The errors associated with the predictions in these samples increased with increasing distance from the chill although they were again generally within one standard deviation of the measured values (see Appendix C). However, the model prediction for the 1 min degas sample disagreed significantly with the measured value. In comparing this sample with the other test castings, two clear differences were observed. First, the cooling conditions for this casting were relatively slow (see Figure C.13 in Appendix C). Second, the distribution of porosity in this casting, shown in Figure 4.10, was markedly different than all of the other castings (except Trial 2 – 1.5 min degas), which generally displayed a continuous increase in
porosity with increasing distance from the chill, as shown in Figure 4.11. The effect of
the large concentrated amount of porosity near the top of this casting on the development
of porosity in other areas of the casting is unclear.

In general, model results compare favorably with the measured values (within one
standard deviation) in ten of the eleven test castings. A possible cause for some of the
error in the predictions is the potential effect of inclusion content, which was seen to
affect the 3 min degas sample in Trial 1. However, this effect was not seen in the 2 min
degas sample in Trial 1 and was not measured in Trial 2. Overall, the model predicted
two trends of decreasing porosity with decreasing hydrogen content and increasing
porosity with increasing distance from the chill. Both trends are consistent with the
measured data from the current experimental work and with past work in literature.
7 Conclusion

The aim of the present work was to characterize the formation of microporosity in A356 aluminum alloy. In support of this, a literature review was conducted to establish past methods developed by researchers for characterization of microporosity and to identify the critical factors involved. The final outcome of this work was the development of a validated microporosity model, which is capable predicting microporosity levels in test castings based upon a relatively straightforward treatment of the effects of hydrogen evolution and interdendritic shrinkage in A356 alloy.

Regarding the objectives detailed in Chapter 3, the following conclusions were reached:

- Of the four criteria functions examined, the Niyama function and to a lesser extent, the Lee and KC functions, correctly predicted the trend with respect to distance from the chill in porosity formation in only one test casting. Overall, the behaviour of the Niyama function exhibited the greatest level of consistency due to the greater sensitivity of the other criteria functions to the cooling rate. However the Niyama function fails to consider the influence of hydrogen concentration and inclusion content, both of which were found to have a significant impact on the amount of porosity observed.

- Microporosity can be successfully predicted using a model incorporating flow through the interdendritic network and the thermodynamics of hydrogen solubility. A 2-D axisymmetric mathematical model was developed using the commercial finite element package ABAQUS™.
In general, the model adequately predicted the amount of porosity throughout the test castings. However, the model fails to account for the effect of inclusion content. The results show trends of decreasing porosity with decreasing hydrogen level and increasing porosity with increasing distance from the chill. Both trends are in agreement with past results reported in literature.
8 Future Work

The effort to date has delivered a mathematical model capable of predicting porosity in test castings with relatively simple one-directional cooling conditions. A logical extension of this work is its application to a more complicated geometry (e.g. a wheel). However, due to the increased complexity of the associated cooling behaviour, further refinement to the model is potentially necessary.

A number of improvements to the model which are warranted include:

- Development of a microstructure model
- Improvement of the treatment of flow through the interdendritic network by considering a more fundamental fluid flow mass conservation equation incorporating global mass transport.
- Further characterization of inclusion content as results for the limited inclusion data indicated a potentially significant effect.
- Characterization of modification effects.
References


5 C. Hermesmann. Personal Communication, Canadian Autoparts Toyota Inc.


ProCAST Materials Database, Version 3.1, UES Software Inc., Annapolis MD.


Appendix A

Experimental Results

Polished and Etched Surfaces

Figure A.1: (a) Polished and (b) etched surface of Trial 1 – 0 min degas sample.
Figure A.2: (a) Polished and (b) etched surface of Trial 1 – 1 min degas sample.

Figure A.3: (a) Polished and (b) etched surface of Trial 1 – 2 min degas sample.
Figure A.4: (a) Polished and (b) etched surface of Trial 1 – 3 min degas sample.

Figure A.5: (a) Polished and (b) etched surface of Trial 1 – 4 min degas sample.
Figure A.6: (a) Polished and (b) etched surface of Trial 2 – 0.5 min degas sample.

Figure A.7: (a) Polished and (b) etched surface of Trial 2 – 1 min degas sample.
Figure A.8: (a) Polished and (b) etched surface of Trial 2 – 1.5 min degas sample.

Figure A.9: (a) Polished and (b) etched surface of Trial 2 – 2 min degas sample.
Figure A.10: (a) Polished and (b) etched surface of Trial 2 – 2.5 min degas sample.

Figure A.11: (a) Polished and (b) etched surface of Trial 2 – 3 min degas sample.
Appendix B

Lower Limits of Conductive Heat Transfer

Table B.1: Lower limits of conductive heat transfer for test castings.

<table>
<thead>
<tr>
<th>Casting</th>
<th>Trial</th>
<th>Degas Time (min)</th>
<th>Casting/Chill Interface $f_{lim,1}$</th>
<th>Casting/Chill Interface $f_{lim,2}$</th>
<th>Casting/Mould Interface Time $f_{lim,2}$ Applied (s)</th>
<th>Casting/Mould Interface $f_{lim}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.35</td>
<td>0.42</td>
<td>30</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.22</td>
<td>0.26</td>
<td></td>
<td>75</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>0.40</td>
<td>0.54</td>
<td></td>
<td>15</td>
<td>0.2</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>0.35</td>
<td>0.42</td>
<td></td>
<td>30</td>
<td>0.1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>0.5</td>
<td>0.15</td>
<td>0.32</td>
<td>37</td>
<td>0.2</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>0.07</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.2</td>
</tr>
<tr>
<td>8</td>
<td>1.5</td>
<td>0.16</td>
<td>0.24</td>
<td></td>
<td>50</td>
<td>0.2</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>0.28</td>
<td>0.51</td>
<td></td>
<td>22</td>
<td>0.1</td>
</tr>
<tr>
<td>10</td>
<td>2.5</td>
<td>0.40</td>
<td>0.88</td>
<td></td>
<td>15</td>
<td>0.2</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>0.35</td>
<td>0.88</td>
<td></td>
<td>17</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Appendix C

Comparison of Predicted and Experimental Results
Temperature and Porosity
Figure C.1: Predicted and experimental temperature vs time of Trial 1 − 0 min degas sample.

Figure C.2: Predicted and experimental porosity vs distance from the chill of Trial 1 − 0 min degas sample.
Figure C.3: Predicted and experimental temperature vs time of Trial 1 – 1 min degas sample.

Figure C.4: Predicted and experimental porosity vs distance from the chill of Trial 1 – 1 min degas sample.
Figure C.5: Predicted and experimental temperature vs time of Trial 1 – 2 min degas sample.

Figure C.6: Predicted and experimental porosity vs distance from the chill of Trial 1 – 2 min degas sample.
Figure C.7: Predicted and experimental temperature vs time of Trial 1 – 3 min degas sample.

Figure C.8: Predicted and experimental porosity vs distance from the chill of Trial 1 – 3 min degas sample.
Figure C.9: Predicted and experimental temperature vs time of Trial 1 – 4 min degas sample.

Figure C.10: Predicted and experimental porosity vs distance from the chill of Trial 1 – 4 min degas sample.
Figure C.11: Predicted and experimental temperature vs time of Trial 2 – 0.5 min degas sample.

Figure C.12: Predicted and experimental porosity vs distance from the chill of Trial 2 – 0.5 min degas sample.
Figure C.13: Predicted and experimental temperature vs time of Trial 2 – 1 min degas sample.

Figure C.14: Predicted and experimental porosity vs distance from the chill of Trial 2 – 1 min degas sample.
Figure C.15: Predicted and experimental temperature vs time of Trial 2 – 1.5 min degas sample.

Figure C.16: Predicted and experimental porosity vs distance from the chill of Trial 2 – 1.5 min degas sample.
Figure C.17: Predicted and experimental temperature vs time of Trial 2 – 2 min degas sample.

Figure C.18: Predicted and experimental porosity vs distance from the chill of Trial 2 – 2 min degas sample.
Figure C.19: Predicted and experimental temperature vs time of Trial 2 – 2.5 min degas sample.

Figure C.20: Predicted and experimental porosity vs distance from the chill of Trial 2 – 2.5 min degas sample.
Figure C.21: Predicted and experimental temperature vs time of Trial 2 – 3 min degas sample.

Figure C.22: Predicted and experimental porosity vs distance from the chill of Trial 2 – 3 min degas sample.