BED BEHAVIOUR IN ROTARY CYLINDERS
WITH APPLICATIONS TO ROTARY KILNS

by

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ABSTRACT

Two modes of transverse solids motion, slumping and rolling, in rotary kiln operation have been experimentally characterized and mathematically modelled in this study. Other modes of bed behaviour encountered in rotary cylinders; slipping, cascading, cataracting and centrifuging have been formulated mathematically. The models have been verified using experimental observations reported in the literature.

An experimental study of those conditions under which the bed changed from slumping to rolling was undertaken and the characteristics of these modes of motion quantified using different types of solids in three horizontal rotary cylinders and a small pilot kiln. A Bed-Behaviour Diagram which is a plot of bed depth versus rotational speed was developed to delineate the various areas of dominance of slumping and rolling and it was shown using this Diagram that bed behaviour observations made on batch cylinders were representative of solids motion in a continuous kiln operation. The effects of bed depth, particle size, particle shape and cylinder diameter on the position of the slumping-rolling boundary were also experimentally investigated. The quantitative characterization of slumping and rolling indicated that a new interpretation of the change in bed motion from slumping to rolling was required.
A study of segregation in the bed revealed that while the presence of fines affected the slumping-rolling boundary, they had little effect on the slumping frequency, the shear angle, the static and dynamic angles of repose and the active layer thickness of the mixtures when compared to those for the parent materials. This indicated that for the bulk solids tested, segregation occurred by the percolation and not by the flow mechanism. Sampling of the bed revealed two segregation cores whose formation and effect on kiln operations is discussed. A mathematical model is also presented to predict the composition and size of the central segregation zone.

A semi-empirical mathematical model of the slumping-rolling boundary was developed and the effects of operating, material and cylinder variables were illustrated. Scale-up criteria were found to be the fill ratio, the Froude number, and the minimum shear wedge. For materials having the same shape but different size, this latter criterion may be replaced by the cylinder diameter to particle size ratio. Slipping, cascading and cataracting were also modelled and their boundaries illustrated on the Bed-Behaviour Diagram. Observations by other workers of these modes of bed behaviour are compared to the model predictions and the appropriate scale-up criteria are presented.
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<td>a</td>
<td>initial acceleration of shear wedge</td>
<td>m/sec²</td>
</tr>
<tr>
<td>dₚ</td>
<td>particle size</td>
<td>m</td>
</tr>
<tr>
<td>g</td>
<td>gravitational acceleration constant</td>
<td>m/sec²</td>
</tr>
<tr>
<td>n</td>
<td>rotational speed of the cylinder</td>
<td>r/min</td>
</tr>
<tr>
<td>n_c</td>
<td>critical rotational speed of the cylinder</td>
<td>r/min</td>
</tr>
<tr>
<td>s</td>
<td>average distance travelled by bulk solids in the shear wedge in its chordal trajectory</td>
<td>m</td>
</tr>
<tr>
<td>s₀</td>
<td>initial distance travelled by the shear wedge in the direction of the average chordal trajectory</td>
<td>m</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
<td>seconds</td>
</tr>
<tr>
<td>t₀</td>
<td>time required to traverse the limiting shear wedge</td>
<td>seconds</td>
</tr>
<tr>
<td>t₁</td>
<td>time from minimum to maximum bed inclination</td>
<td>seconds</td>
</tr>
<tr>
<td>t₂</td>
<td>time from maximum to minimum bed inclination</td>
<td>seconds</td>
</tr>
<tr>
<td>tₛ</td>
<td>slumping time</td>
<td>secs</td>
</tr>
<tr>
<td>tₜ</td>
<td>total time per slump</td>
<td>seconds/slump</td>
</tr>
<tr>
<td>v</td>
<td>linear velocity</td>
<td>m/sec</td>
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<tr>
<td>v₀</td>
<td>initial granule velocity in its parabolic trajectory</td>
<td>m/sec</td>
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<tr>
<td>v₀T</td>
<td>initial velocity of the shear wedge in the direction of the average chordal trajectory</td>
<td>m/sec</td>
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<tr>
<td>wₖ</td>
<td>width of central segregated core</td>
<td>m</td>
</tr>
<tr>
<td>x</td>
<td>weight fraction of the fine component in the slice</td>
<td>dimensionless</td>
</tr>
<tr>
<td>x_ABC</td>
<td>abscissa of the centroid of the upper wedge</td>
<td>m</td>
</tr>
<tr>
<td>x_DEC</td>
<td>abscissa of the centroid of triangle DEC</td>
<td>m</td>
</tr>
<tr>
<td>x_OAB</td>
<td>abscissa of the centroid of sector OAB</td>
<td>m</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
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<td>( \bar{x}_{PSC} )</td>
<td>abscissa of the centroid of the lower wedge</td>
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<td>( y )</td>
<td>weight fraction of the coarse component in the central segregated core</td>
<td>dimensionless</td>
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<td>m</td>
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<tr>
<td>( \bar{y}_{DEC} )</td>
<td>ordinate of centroid of triangle DEC</td>
<td>m</td>
</tr>
<tr>
<td>( \bar{y}_{OAB} )</td>
<td>ordinate of centroid of sector OAB</td>
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<td>ordinate of centroid of triangle ODB</td>
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<tr>
<td>( \bar{y}_{ODE} )</td>
<td>ordinate of centroid of triangle ODE</td>
<td>m</td>
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<tr>
<td>( \bar{y}_{OEA} )</td>
<td>ordinate of centroid of triangle OEA</td>
<td>m</td>
</tr>
<tr>
<td>( \bar{y}_{PSC} )</td>
<td>ordinate of the centroid of the upper wedge</td>
<td>m</td>
</tr>
<tr>
<td>( A_{ABC} )</td>
<td>area of shear wedge ABC</td>
<td>m²</td>
</tr>
<tr>
<td>( A_{DEC} )</td>
<td>area of triangle DEC</td>
<td>m²</td>
</tr>
<tr>
<td>( A_{OAB} )</td>
<td>area of sector OAB</td>
<td>m²</td>
</tr>
<tr>
<td>( A_{ODB} )</td>
<td>area of triangle ODB</td>
<td>m²</td>
</tr>
<tr>
<td>( A_{ODE} )</td>
<td>area of triangle ODE</td>
<td>m²</td>
</tr>
<tr>
<td>( A_{OEA} )</td>
<td>area of triangle OEA</td>
<td>m²</td>
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<td>( C )</td>
<td>number of components in the Phase Rule</td>
<td>dimensionless</td>
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<td>regression coefficient from curve fit on ( t_1 ) data</td>
<td>seconds</td>
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<td>( C_2 )</td>
<td>regression coefficient from curve fit on ( t_2 ) data</td>
<td>seconds</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>regression coefficient from curve fit on ( t_2 ) data</td>
<td>60 ( \frac{s^2}{rev} )</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>regression coefficient from curve fit on slumping frequency data</td>
<td>seconds</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
<td>Unit</td>
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<td>$C_5$</td>
<td>regression coefficient from curve fit on slumping frequency data</td>
<td>$60 \frac{s^2}{rev}$</td>
</tr>
<tr>
<td>D</td>
<td>inside diameter of the cylinder</td>
<td>m</td>
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<tr>
<td>F</td>
<td>number of degrees of freedom in the Phase Rule</td>
<td>dimensionless</td>
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<td>$F_C$</td>
<td>centrifugal force</td>
<td>N</td>
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<td>frictional force</td>
<td>N</td>
</tr>
<tr>
<td>$F_G$</td>
<td>gravity force</td>
<td>N</td>
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<tr>
<td>$F_N$</td>
<td>normal force</td>
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<tr>
<td>$F_R$</td>
<td>resultant force</td>
<td>N</td>
</tr>
<tr>
<td>$F_R'$</td>
<td>reaction force</td>
<td>N</td>
</tr>
<tr>
<td>Fr</td>
<td>Froude number ($\frac{\omega^2 R}{g}$)</td>
<td>dimensionless</td>
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<td>H</td>
<td>depth of the bulk solids in the rotary cylinder or rotary kiln</td>
<td>m</td>
</tr>
<tr>
<td>$H_K$</td>
<td>depth of the central core of segregation</td>
<td>m</td>
</tr>
<tr>
<td>J</td>
<td>degree of fill</td>
<td>per cent</td>
</tr>
<tr>
<td>K</td>
<td>constant in equation (7.6) by Oyama</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>length of rotary cylinder or rotary kiln</td>
<td>m</td>
</tr>
<tr>
<td>M</td>
<td>total mass of bulk solids in the rotary cylinder</td>
<td>kg</td>
</tr>
<tr>
<td>P</td>
<td>number of phases in the Phase Rule</td>
<td>dimensionless</td>
</tr>
<tr>
<td>R</td>
<td>inside radius of the cylinder</td>
<td>m</td>
</tr>
<tr>
<td>$R_{ABC}$</td>
<td>distance between the centroid of the shear wedge from the centre of rotation of the cylinder</td>
<td>m</td>
</tr>
<tr>
<td>$R_B$</td>
<td>radius of rotation of the centre of gravity of the bulk solids</td>
<td>m</td>
</tr>
<tr>
<td>$R_K$</td>
<td>outer radius of rotation of the central core of segregation</td>
<td>m</td>
</tr>
<tr>
<td>S</td>
<td>slumping frequency</td>
<td>slumps/min</td>
</tr>
<tr>
<td>$V_K$</td>
<td>volume occupied by the bulk solids that is associated with the central core of segregation</td>
<td>$m^3$</td>
</tr>
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</table>
\( W_k \) total weight of bulk solids associated with the central core of segregation kg

\( W_T \) total weight of bulk solids in the bed kg

\( X \) abscissa of parabolic trajectory of granules m

\( X_e \) abscissa of the mid-point of the bed surface m

\( Y \) ordinate of parabolic trajectory of granules m

\( Y_e \) ordinate of the mid-point of the bed surface m

\( \alpha \) angular measure of the centroid of lower wedge from abscissa, measured in clockwise direction degrees

\( \Delta \alpha \) difference between the maximum and minimum bed inclinations for a slipping bed radians

\( \beta \) angle relating the friction force and the normal force in a force balance on a slipping bed degrees

\( \gamma \) shear wedge degrees

\( \gamma_0 \) limiting shear wedge degrees

\( \gamma_1 \) shear wedge degrees

\( \gamma_{01} \) regression constant from curve fit on \( t_1 \) data degrees

\( \gamma_{02} \) regression constant from curve fit on \( t_2 \) data degrees

\( \gamma^* \) angle traversed by rotary cylinder in time \( t \) degrees

\( \delta \) angular measure of degree of fill degrees

\( \varepsilon \) void fraction of the bulk solids dimensionless

\( \Delta \varepsilon_{\text{max}} \) maximum bed contraction dimensionless

\( \eta \) slope of average chordal trajectory of the shear wedge degrees

\( \theta \) angle between the apex of the bed and the abscissa degrees

\( \lambda \) angular measure of the degree of fill degrees

\( \mu_D \) dynamic coefficient of friction of bulk solids dimensionless

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<td>$\mu_L$</td>
<td>shearing coefficient of friction of bulk solids</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$\mu_{W/S}$</td>
<td>coefficient of friction between the cylinder wall and the bulk solids</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>bulk density of bulk solids</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>particle density</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\rho_{B_K}$</td>
<td>bulk density of central core of segregation</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\phi'$</td>
<td>resultant angle of bed inclination for the force balance of a slipping bed</td>
<td>degrees</td>
</tr>
<tr>
<td>$\phi_d$</td>
<td>dynamic angle of repose of bulk solids</td>
<td>degrees</td>
</tr>
<tr>
<td>$\phi_L$</td>
<td>shear angle</td>
<td>degrees</td>
</tr>
<tr>
<td>$\phi_R$</td>
<td>static angle of repose of the bulk solids</td>
<td>degrees</td>
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<tr>
<td>$\phi_s$</td>
<td>angle of slipping friction</td>
<td>degrees</td>
</tr>
<tr>
<td>$\phi_u$</td>
<td>upper angle of repose</td>
<td>degrees</td>
</tr>
<tr>
<td>$\phi'd$</td>
<td>resultant angle of bed inclination for the force balance of a cascading bed</td>
<td>degrees</td>
</tr>
<tr>
<td>$\psi$</td>
<td>angular measure of the sector OAB from the abscissa</td>
<td>degrees</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular velocity of the rotary cylinder</td>
<td>radians/sec</td>
</tr>
<tr>
<td>$\omega_{R/C}$</td>
<td>rotational speed at the rolling-cascading boundary</td>
<td>radians/sec</td>
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Subscripts

- P indicates a prototype
- M indicates a model
"Man, God created to be a witness and grateful interpreter of His works."

St. Antony the Great
ACKNOWLEDGMENTS

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Chapter 1

INTRODUCTION

1.1 Introduction

This work is concerned with an analysis of the different modes of transverse solids motion in rotary kilns. The principles derived and models presented will also find application in other industrial processes where rotary cylinders are used. Other applications will include the effect of the physical properties of bulk solids on their flow characteristics. However, since the rotary kiln was the main impetus of this project, reference will be made primarily to its application.

1.2 Description of a Rotary Kiln

A rotary kiln is a horizontally inclined steel cylinder lined with refractory. The burden is continuously charged at the upper end and moves slowly along the furnace by virtue of the inclination and rotation of the kiln. In most cases, the depth of the moving bed is a small fraction of the kiln diameter. Dams are often placed at the solids discharge end and also at selected points along the length of the kiln to
increase its hold-up.* The length to diameter ratio of typical rotary kilns varies from 15 to 40.\textsuperscript{1,2}

As the object of these reactors is to effect a chemical or physical change to the burden by raising its temperature, a burner for fuel combustion is generally located at the lower end of the kiln allowing the gases to flow countercurrent to the charge. To reduce heat losses through the kiln walls the refractory lining may be from 0.15 m to 0.30 m thick. The cylinder rests on a set of trunions with rotation being effected by a variable speed D.C. motor.\textsuperscript{3,4} Auxiliary equipment commonly found in rotary kiln plants include gas handling and dust collecting installations as well as heat recuperators for the hot exit gases and discharge solids. Some common rotary kiln operating conditions are listed in Table I.

### 1.3 Rotary Kiln Applications

Due to the mechanical simplicity of the rotary kiln\textsuperscript{3} it is not surprising therefore, that it is widely used for continuous processing in the chemical and metallurgical

*The ratio between the volume occupied by the solids and the total volume of the kiln (also called the per cent fill or the fill ratio).
TABLE I  Sample Details of Process Kilns

<table>
<thead>
<tr>
<th>Material</th>
<th>Particle Size</th>
<th>Kiln</th>
<th>Inner Shell Dia. (m)</th>
<th>Rotational Speed (r/min)</th>
<th>% Fill</th>
<th>Slope (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pebble lime</td>
<td>-63 mm</td>
<td>3.5</td>
<td>1.5</td>
<td>10-12</td>
<td>4.2</td>
<td></td>
</tr>
<tr>
<td>Wet Sludge lime</td>
<td>-</td>
<td>3.8</td>
<td>1.0</td>
<td>4</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Magnesite</td>
<td>-13 mm</td>
<td>3.0</td>
<td>1.0</td>
<td>6-10</td>
<td>4.2</td>
<td></td>
</tr>
<tr>
<td>SL/RN</td>
<td>13 mm</td>
<td>6.0</td>
<td>0.4</td>
<td>13</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Petroleum Coke</td>
<td>40% + 10 mesh</td>
<td>4.0</td>
<td>1.0</td>
<td>5-8</td>
<td>5.2</td>
<td></td>
</tr>
<tr>
<td>Wet Process Cement</td>
<td>-</td>
<td>4.5-5.6</td>
<td>1.0</td>
<td>7-10</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
industries. In the production of cement, rotary kilns are used for calcination. Calcined dolomite, lime mud (in the Kraft Pulping process) and limestone (for desulphurizing in the steel industry) are other examples of rotary kiln applications in the chemical industry. Several processes have been proposed for direct reduction of iron in a rotary kiln,\textsuperscript{6-11} while in the non-ferrous industry the kiln is used in treating zinc oxide and pyrrhotite ores. The calcination of hydrated alumina and of petroleum coke in the aluminum industry should also be cited, as well as the application of the rotary kiln as a preheater and a dryer in the production of high grade ferro-chrome and titanium respectively. In short, the rotary kiln is among the most common gas-solid reactors used in the primary industries.

1.4 Some Intricacies of the Reactor

Although the rotary kiln is a very simple reactor to operate, the processes occurring in it are quite complex. For example, different temperatures, hold-up and wall roughness may prevail at different positions of the kiln, while the charge may undergo physiochemical changes affecting its motion. Often the kiln feed consists of a mixture of materials, each having its own respective size and shape distribution. These, as well as other complicating factors, have rendered many operating improvements directly applicable
only to the kiln tested, even when considering the same process. In addition, rotary kilns of the same design for the same application will require different operating strategies. This is due to differences in the physical properties of the material feed (e.g. density, grain size and shape), particularly when the charge is composed of several materials differing in physical as well as chemical properties.

Improper balance of fuel and feed can lead to accretion build-up on the kiln walls, which may require continual adjustment of the operating conditions. Left to accumulate, these accretions will decrease the open cross-sectional area of the kiln and result in significant down time for clean out. In view of current trends towards larger diameter reactors, these potential difficulties must be dealt with in the scale-up and design stages of an installation. The result is very costly if design expectations are not met. This was recently seen in Canada when a rotary kiln facility failed after an investment of $65 million. Fundamental research in this area is therefore essential for the efficient operation and sound design of these kilns.

1.5 The Approach of Research to Date

The aim of a process research program is usually to
enhance our understanding of a process and to provide criteria for efficient operation, and sound design, given the range of operating conditions expected. In a rotary kiln, efficient operation means developing a processing strategy that will maintain the lowest energy costs for a given production demand. In designing rotary kiln plants, it is advantageous to utilize the minimum required plant site area while properly balancing the capital cost and the capacity of the kiln with that of the auxiliary equipment required. This latter may account for 60% of the capital investment of the plant.18

With these factors in mind, research on rotary kilns has been carried out in two main areas: heat flow analyses and residence time studies. The former is concerned with the analysis of the heat flow steps in a kiln to obtain the minimum time required for a chemical or physical change to occur. The latter includes theoretical and experimental studies estimating the time required for the feed to physically move from the charge end to the discharge end. Clearly, for a given application these two times must correspond.19 Better still, the two times must correspond for each zone in the kiln. For example; in the Strategic-Udy process for the reduction of iron ore, over one-half the kiln length is used for drying.6 If this can be
reduced to one-third, a greater portion of the kiln can then be utilized for the more heat intensive reduction zone. An added benefit to this approach would be the potential increase in production.

1.5.1 Heat Flow Analyses

Several mathematical models of the heat flow steps in a rotary kiln have been formulated. Heat flow between the gas and the solids is the only mode of heat transfer of concern in this discussion. It has been recognized to take place in two stages. Firstly, heat is transferred from the gases to the solids on the bed surface by radiation and convection. Secondly, heat is exchanged from the hotter to the colder solids when the former are mixed into the bed. This second step is enhanced by the mixing of the solids when they re-surface from the bulk of the bed. In all mathematical models cited, the first of these two steps was assumed to be rate limiting (i.e. 'well-mixed' bed assumption) and the mechanisms of heat transfer formulated were those of radiation and convection to a flat planar surface.

On the basis of visual observations in industrial kilns and experimental evidence on pilot facilities, some
investigators$^{5,24,26,29,30}$ have concluded that the 'well-mixed' assumption is not always valid. It has been recognized,$^{5,14,24-26,31}$ as will be discussed extensively in chapter 2, that the solids in a kiln can have any one of three possible types of transverse motion: slipping, slumping and rolling. Each of these modes would present to the hot gases different surfaces for varying periods of time and their transverse mixing behaviours would differ greatly. Furthermore, based on heat flow measurements in a pilot kiln,$^5$ it has been concluded that the treatment of the solids surface as a flat plane is inadequate to characterize the heat flow steps from the hot gases to the solids. It is therefore clear that the transverse bed motion encountered in rotary kiln operation must be fully defined in order to understand the degree of its effect on the heat transfer processes to the solids in rotary kilns.

1.5.2 Residence Time Studies

The solids residence time in a rotary kiln is affected by kiln inclination and rotation. Since the temperature and composition of the gas will also vary along the kiln length, the physical properties of the charge may change and hence also affect the residence time of the solids. Gases generated in the bed will further complicate the
motion of the charge not to mention the effect of dams, accretion build-up, varying kiln diameter and feed size and shape distributions. Many investigators have therefore applied radioactive tracer techniques to residence time studies in industrial kilns,\textsuperscript{1,12,13,32,33} Their results are limited to the reactors tested over the range of variables investigated.

Other fundamental approaches\textsuperscript{1,3,14,19,34-44} have identified four mechanisms by which the charge moves along the kiln length. These are: movement radially and axially by convection and diffusion. Although these studies provide quantitatively the magnitude of each of these mechanisms, they have been applied only to a rolling or cascading bed. A systematic investigation of all reactor and material variables, in the various types of transverse bed motion, over the full operating range of the kiln has not yet been carried out.

It has been recognized that each type of transverse bed motion (slipping, slumping and rolling) will affect the residence time of the burden.\textsuperscript{44-48} Hence the complete characterization of transverse bed motion is lacking even though it forms one of the basic building blocks of rotary kiln operation.
1.6 The Objectives of this Work

The first objective of this work is to identify those rotary kiln operating and material variables that affect the different modes of transverse bed behaviour: slipping, slumping and rolling. These operating variables include kiln bed depth, diameter, rotational speed and wall roughness; the material variables are particle shape, particle size, size distribution, and density.

The second objective is to interpret and apply the results thus obtained to rotary kiln operating conditions and to develop scale-up criteria for kiln design based on bed behaviour similarity.
Chapter 2

LITERATURE REVIEW OF BED BEHAVIOUR

2.1 Introduction

The aim of this chapter is firstly to propose a consistent terminology by which the varying types of bed motion will be addressed throughout the text of this thesis. In the literature the same type of solids motion has been identified by different terminology; while different types of solids motion have been addressed by the same term. The second aim will be to review the descriptions of the varied bed behaviours encountered in rotary kiln operations. Thirdly, the different methods of experimentally characterizing these various bed behaviours will be presented. Three methods will then be developed for modelling solids flow in rotating cylinders and finally the scale-up criteria that have been proposed in the literature for bed behaviour similitude will be described.

2.2 The Characterization of the Modes of Bed Behaviour

In a cylinder of a given diameter, filled with varying amounts of granular solids and rotated over a wide range of rotational speeds, the following six modes of bed
behaviour have been observed: slipping, slumping, rolling, cascading, cataracting and centrifuging. Only the first four of these will be discussed in this section, since cataracting and centrifuging are not normally found in rotary kiln operation.

2.2.1 Slipping

2.2.1.1 Description and Transverse Mixing

Three types of slipping have been observed. The first is an oscillatory type or pendulum-like motion and is described as follows. If a smooth-walled circular cylinder is filled with a given quantity of bulk solids and slowly rotated, the solids bed will move with the wall of the cylinder until it reaches a bed inclination lower than the angle of repose of the bulk solids. At this point, the whole bed will slip against the cylinder wall as a solid mass and come to rest at an even lower angle of bed inclination. The solids bed will then start rotating with the cylinder wall once more and will again slip when it reaches approximately the same maximum inclination as before. Thus, with the continuous rotation of the cylinder, a pendulum-like or oscillatory type of bed motion results. Rutgers states that this type of slipping action has a definite frequency and amplitude; however, no supporting
experimental results have been presented. A schematic representation of this type of a slipping bed is shown in Figure 2.1. A second type of slipping action has been reported by Ronco. In this case, the solids rise with the cylinder wall but with a smaller rotational speed. This behaviour is due to the layer of solids in contact with the wall acting as bearings between the burden and the cylinder wall. The solids reaching the apex (the upper portion of the bed surface) move slowly down the inclination of the bed towards its lower extremity, the chordal base. Finally, within a certain range of rotational speeds and degrees of fill, the solids bed may take up a stationary condition. This phenomenon constitutes a third type of slipping action where the bed is seen to remain motionless within the rotating walls.

All of the investigators who have observed the slipping mode report that little or no transverse solids mixing results. This is in agreement with the experimental findings of Rose.

2.2.1.2 Effect of Variables

In the range of operating conditions for rotary kilns, no rigorous experimental
Figure 2.1 Schematic diagram of slipping.

Figure 2.2 Schematic diagram of slumping.
investigation of the variables affecting slipping has been carried out. Some qualitative observations, however, have been recorded while other modes of bed behaviour were being investigated. These are summarized below.

Within the wide range of process applications and operating conditions of rotating cylinders, low degrees of fill have been commonly observed to enhance slipping. For example, at the high rotational speeds of ball milling, slipping can occur at degrees of fill as high as 30 and 40 per cent; while at 55 per cent, slipping may be eliminated. Smooth walls and high internal burden friction are also believed to promote slipping.

Rose and Blunt are the only investigators who have attempted to interpret their experimental observations of slipping in ball mills. Their dimensional analysis yielded the following non-dimensional groups:

\[ \Delta \alpha = f \left( \frac{D}{d_p}, \frac{u_{Ws}}{J}, \frac{d}{D} \right) \] (2.1)

The dimensionless group \( (D/d_p) \) was given to maintain the geometrical similarity of the number of contact points at the wall-bed interface. The functional relationship of
equation (2.1) was graphically presented by Rose and Blunt, but has not received wide application. Thus, four variables have been experimentally identified to affect the slipping mode of bed behaviour: an operating variable, the degree of fill; a material variable, the internal angle of friction; and two material/cylinder variables, the bed-wall friction coefficient and the ratio of cylinder diameter to the average particle size.

2.2.2 Slumping

Slumping was briefly described by Davis as early as 1920. Most investigators since have carried out their experimental work either on smooth walled cylinders or at rotational speeds higher than is required for a bed of bulk solids to slump. Hence, the slumping bed has received very little attention despite its industrial importance in rotary kilns. A description of the slumping bed as recorded by Zablotny, Reuter, Schnabel and Wahlster et al., is given below.

2.2.2.1 Description and Transverse Mixing

If a rotary cylinder, filled with bulk solids (Figure 2.2), is rotated at a low speed and if there is no relative motion between the bulk solids and the cylinder wall, the surface inclination of the bed will increase. When it reaches the static angle of repose, $\phi_R$, 
of the solids, a segment of the bulk solids will detach from the remaining solids, away from the cylinder wall, and will slump towards the lower extremity of the bed surface. The final bed inclination will be lower than the static angle of repose and will be given by position BP (Figure 2.2). Since the cylinder is continuously rotated, the inclination of the bed will start to increase again until it reaches the static angle of repose at which point another slump will occur. The segment of solids slumping, ABC, forms an angle called the shearing angle, $\gamma$, which has been reported to be $12^\circ - 15^\circ$ at 1-2 r/min.\textsuperscript{1}

Wahlster et al.\textsuperscript{57} have carried out a small number of tracer experiments in the slumping mode to determine the flow pattern of the particles in the solids bed. Although their primary objective was to study segregation patterns in the burden, some of their interpretations are relevant to a discussion on transverse mixing in slumping beds and will be presented below.

Since there is no relative motion between the solids bed and the cylinder wall, the solids in the bed have a fixed circular trajectory whose centre coincides with the centre of rotation of the cylinder. Therefore, no mixing takes place in the bed as its inclination
increases. When a slump occurs, however, the solids moving down the bed surface are seen to change circular trajectory positions. The final radius of rotation of the solids in the bed could either be greater or less than the one from which they had emerged. This mixing process is believed to be slow and for solids with constant physical properties, it is also thought to be random; however, no quantitative analysis has been reported.

2.2.2.2 Effect of Variables

The first method used by investigators to characterize slumping is the measurement of the slumping frequency of the bulk solids. Zablotny\textsuperscript{1} believed that it increased with rotational speed and was dependent on the physical properties of the bulk solids. The observations of slumping frequency made by Pearce on several industrial kilns\textsuperscript{24} can be presented as follows:

\[ S = 60 + \frac{24}{t_S} \frac{1}{n} \]  

Wahlster et al.\textsuperscript{57} on the other hand, measured the slumping frequency of coke breeze, silica, and iron ore and found no dependence on particle size, shape or density. More recently, Reuter\textsuperscript{27} analyzed the slumping frequency
of beds with varying particle size and mixtures of coal and iron ore. Three frequency groups were identified:

(i) \( 0 < \frac{n}{n_C} < 0.04 \)  
(ii) \( 0.04 < \frac{n}{n_C} < 0.10 \)  
(iii) \( 0.10 < \frac{n}{n_C} < 0.20 \)

The first group was noted for a steep increase in slumping frequency for relatively smaller increases in rotational speed. It was believed to be characteristic of the boundary between slipping and slumping. In the second group of rotational speeds, smaller increases in slumping frequency were observed. Reuter also believed that the particle density determined the type of particle motion in this second zone. For \( 0.1 < \frac{n}{n_C} < 0.20 \), a change in bed mode was seen and attributed to the increased effect of the centrifugal force. Renewed, large increases in

*The critical rotational speed, \( n_C \), is the cylinder rotational speed at which a particle on the cylinder wall centrifuges. For a small particle size relative to the cylinder diameter, \( n_C \), is given by:

\[
n_C = \frac{30}{\pi} \sqrt{\frac{2g}{D}}
\]
slumping frequency result which equal the rate of arrival of particles to the bed surface when the change in bed behaviour occurs. In the studies of Reuter, Pearce and Wahlster et al. there appears to be no consistent bed behaviour pattern of what variables affect the slumping frequency. Their observations will again be discussed in the light of results from the current investigation.

Other dependent variables that would enable a characterization of a slumping bed to be carried out are the angle of repose and the surface particle residence time. Wahlster et al.\(^\text{57}\) reported no variation in the angle of repose with increasing rotational speed for the materials tested. They also noted that the ratio of surface residence time to bulk circulation time was in the range of one-twelfth (1/12) to one-fifth (1/5). These were not compared with those of other bed modes, and no description was given of the experimental conditions and techniques used.

In conclusion, it can be stated that while some of the parameters required to characterize a slumping bed are known, their dependence on material, cylinder and operating variables is largely unknown.
2.2.3 Rolling and Cascading

There is little differentiation made in the literature between rolling and cascading. In fact, the terms have often been used interchangeably. Although, in this thesis, new criteria will be developed to distinguish them, they will be discussed together in this section.

2.2.3.1 Description and Transverse Mixing

At higher rotational speeds, the slumping motion of the bed is replaced by rolling.\textsuperscript{27,57} This rolling bed is characterized by the continuous motion of a layer of solids over the bed surface. This zone is continually fed with solids from the bulk of the burden, which reach the upper part of the bed by means of the rotation of the cylinder. The remainder of the charge remains below the moving layer. As there is no relative motion between the cylinder wall and the solids in the bulk of the bed, these bulk particles therefore travel in fixed trajectories.

With the cylinder rotating at low rotational speeds, the rolling bed assumes a constant angle of inclination and is seen to have a flat planar surface. With increasing speeds the solids at the upper part of the cross section
of the bed, the apex, ride higher up the wall before detaching themselves from it. The cross section of the bed then assumes a more 'crescent' or 'kidney' shape. This has been termed cascading.

Three models describing the flow patterns of the solids have been proposed. The first by Rutgers\(^2\) is presented in Figure 2.3, and applies at the higher rotational speeds where cascading dominates. The moving layer of solids on the bed surface is relatively thin and is responsible for all the transverse mixing which occurs by convection and diffusion. Below this layer of particles is a stagnant core. When the degree of fill is greater than 50 per cent, this core becomes a dead spot. Although Hogg\(^58\) has shown photographically the presence of this dead spot, further experimental work by Hogg as well as the results of other workers\(^{27,36,59,60}\) have shown that the treatment by Rutgers of the moving layer of solids to be limited in application.

In the second flow visualization model (Figure 2.4), solids are believed to travel on fixed paths both in the bulk of the bed as well as on the surface moving layer.\(^{46,51,61,62}\) In this representation, particles do not
Figure 2.3 Rolling or cascading according to Rutgers.\textsuperscript{2}

Figure 2.4 Second flow visualization model of rolling or cascading\textsuperscript{46,51,61,62}
interchange trajectory paths. This is an incorrect representation because recent experimental evidence has indicated that transverse mixing does indeed occur. 27,36,37,52,59,63-66

In fact, when cascading, the bed can reach a steady state of mixedness* within five (5) cylinder revolutions.

The third model of solids motion, which will now be presented, has been largely developed based on observations made on cascading beds. Photographic and tracer techniques have shown that the bulk solids bed is essentially composed of two regions: active and passive (Figure 2.5). 27,36,52,58,59,63-65, 67 The majority of the solids are in the passive region. Here, they have no relative motion with one another nor with the cylinder wall. They are therefore on fixed trajectories rotating with the same angular velocity as the cylinder. Solids enter the active region at AB (Figure 2.5) in the upper half of the bed. Once they cross the mid-point of the surface, OB, they enter the passive zone along CB. Their point of entry into the passive region is not equidistant from their point of exit. Thus solids motion in the active zone is fairly random, and therefore all mixing is achieved in the active region.

*Mixedness is defined as "the state in which there is the same probability of a particle selected at a given point being of a certain type, for all points in the mixture." 37
Figure 2.5  Third flow visualization model of rolling or cascading. 36,59,60,65

Figure 2.6  Schematic representation of the velocity profile of the solids in the active and passive regions. 51,52,67
Some workers\textsuperscript{51, 52, 67} also suggest that there exists a velocity profile of the solids in the active region (Figure 2.6). Those solids travelling on the bed surface have the highest velocity down the inclination of the bed. In moving radially into the bed the velocity of the solids decreases until it reaches zero at the active-passive boundary which is fixed in space. The solids below it have no relative velocity with each other nor with the cylinder wall; while those above it are in the active zone and their velocity down the bed surface (from the apex to the chordal base) increases as the surface of the bed is approached.

\section*{2.2.3.2 Effect of Variables}

A rolling or a cascading bed is more difficult to characterize than a slumping bed. To date this has been attempted by measuring the bed inclination, the residence time of solids on the surface, the active layer thickness and the conditions at which a bed will change into the rolling mode.

When rolling, the angle of inclination of the bed is believed to be the dynamic angle of repose of the bulk solids. Wahlster et al.\textsuperscript{57} report no change in this angle with rotational speed. Wes et al.,\textsuperscript{68} on the other hand,
have found it to decrease with increasing cylinder rotational speed. This is not consistent with the fact that the solids ride higher on the cylinder wall as the rotational speed increases. The results of Wes et al. were probably due to the presence of lifters in their cylinders. Rutgers, Reuter and Ronco have compared the dynamic angle of repose to the static and report the former to be higher. On the basis of their own experimental results, Franklin and Johanson have reported the opposite to be the case, which is in line with the basic principles of mechanics.

The measurement of surface residence times has been carried out by Reuter and Oyama. Unfortunately, their data is not in a useable form for bed characterization and their experimental techniques are not described.

The active layer measurements of Reuter, Figure 2.7, indicate an increase in the active layer depth with increasing rotational speed. A smaller increase in the ratio of the active layer thickness to the bed depth with increasing degree of fill is also illustrated. Although no further analysis of this data is presented by Reuter, a more thorough treatment of it will be made in this thesis.

Several observers have recorded the operating
Figure 2.7 Effect of drum speed on the relative active layer thickness of a mixture of pellets in a mixing drum 0.8 D x 0.5 m.

Figure 2.8 Relation between flow limit and particle size.
1. Particle size
2. Kiln rotation speed for the flow limit
3. Degree of filling
4. Kiruna-D-Ore
5. Bicorite
6. Coke
conditions under which a bed will change from slumping to rolling. Davis\textsuperscript{56} stated that this change was due to the inertia of the particles as well as to the centrifugal forces acting on them. Rutgers,\textsuperscript{2} however, reports that the change in bed behaviour occurs at a rotational speed one-tenth that of the critical ($0.1 \, n_c$). Carley-Macauly and Donald\textsuperscript{63} claim that this change in bed behaviour occurs at $0.056 \, n_c$. This would correspond to a ratio of centrifugal to gravitational forces of 0.01 and 0.003 respectively. Clearly the centrifugal force is negligible.

The most thorough study of the change in the bed behaviour from slumping to rolling is that of Wahlster et al\textsuperscript{57} however, their results, shown in Figure 2.8, are somewhat confusing. The ordinate of the graph as well as the text of their paper state that the rotational speed is plotted\textsuperscript{71} but the axis indicates a linear speed. Owing to this confusion only a qualitative analysis of their results is possible. The speed at which the bed changes from slumping to rolling, 'the flow limit' is plotted against the particle size for varying fill ratios for three materials tested. It is evident that as the fill ratio increases, the 'flow limit' decreases. Also for small particle sizes, the 'flow limit' increases until a maximum is reached at a particle size of 2-4 mm. With further increases in particle size,
the 'flow limit' decreases. Furthermore, the type of material was not observed to affect this 'flow limit'. While the concept of a 'flow limit' appears to be useful in characterizing rolling beds, the conclusions and the results of Wahlster et al. have limited usage.

In conclusion, of the techniques described in this section, the active layer thickness and the 'flow limit' measurements appear to be the two most fruitful methods of characterizing the rolling bed and the variables affecting it.

2.3 Bed Behaviour Modelling

There have been three main approaches to the modelling of solids flow in rotating cylinders based respectively on the active layer thickness, transverse mixing and the mechanics of a static rigid body. Each of these will be discussed in the following sections.

2.3.1 Active Layer Thickness

The object of this first approach was to predict the active layer thickness of a rolling or a cascading bed based on first principles. This approach has not been very successful in predicting real systems because of
the very complex nature of particle kinematics.

2.3.2 Transverse Mixing

The primary aim of the mixing studies in rotary cylinders reported in the literature\textsuperscript{37,72-77} was to develop, using statistical techniques or diffusion analogies, a coefficient of mixedness by which the mixed or un-mixed state of the granular solids could be evaluated. A host of these coefficients have been proposed, but no single one has found wide application.

The work of Carley-Macauly and Donald\textsuperscript{37,63} and of Hogg\textsuperscript{58} bears directly on the transverse mixing of granular cascading solids in circular cylinders. The former applied a statistical modelling approach while the latter used a diffusion analogy. Both show that the transverse mixing process is at least two orders of magnitude faster than axial mixing. Their models were used to study the kinetics of the mixing process as well as the effects of the operational parameters on the state of mixedness. Their results indicate that the mixing kinetics satisfy a first-order rate equation.

This finding was also corroborated by the work of
Lehmberg et al.\textsuperscript{36} using a unique approach. Heated solid particles were added axially to colder bulk solids rotating in a cylinder. A thermocouple placed in the active layer measured the temperature of the solids and its transient response was successfully fitted to a first-order rate equation.

The effects of material and cylinder variables were not investigated by Carley-Macauly and Donald,\textsuperscript{37,63} Hogg\textsuperscript{58,64} or by Lehmberg et al.\textsuperscript{36} Their experimental investigations were also restricted to the cascading mode of bed behaviour. A more important drawback to these approaches though is that little information is provided about the underlying mechanisms of the mixing process. Hence, while the mixing speed and the mixing kinetics may be adequately expressed mathematically, no insight is gained on the mechanisms involved or on the interaction of the many variables of the system.

2.3.3 The Mechanics of Rigid Bodies

In applying the principles of solid mechanics, the granular bed is treated as a rigid body and all the forces acting on it are accounted for. The resultant force or the configuration of the bed for which the force polygon is closed (i.e. equilibrium) is hence
calculated. This would yield, for example, the likelihood of a bed of granular solids slipping against the wall. However, the assumption of treating the granular solids as a rigid body is not totally valid and will be further discussed in Chapters 6 and 7. In this section, only a review of those models based on this method will be presented.

The solid mechanics approach has only been attempted for the case of a slipping bed. While the forces acting on the solids bed can be easily identified as gravitational and frictional, many different points of application and magnitudes have been proposed. In fact, there is still not a satisfactory solution for the slipping bed.

In 1930, Uggla set up a free body diagram of the solids bed with the gravity force acting through the centre of gravity of the bed, and the centrifugal force through the centre of rotation of the cylinder (in ball mill application the centrifugal force is significant). He then calculated the frictional force required to balance the resultant force due to gravity and centrifuging. This analysis is incomplete, as it does not take into account the fact that not all forces are coincident, and hence a moment balance is also necessary. Subsequent attempts at analyzing the forces acting on the bed have not advanced Uggla's work, and
have either fallen short of his treatment or have reproduced it. More recently, Reuter\textsuperscript{27} and Cross\textsuperscript{14,31} have attempted to account for the moment of the gravitational force about the centre of rotation of the cylinder. Their respective mathematical expressions, which describe the conditions under which slipping would result, do not agree with one another and on closer examination were found to contain derivational errors.

In summary therefore, while the solids mechanics approach has the potential of elucidating the interdependence of the modes of bed behaviour, no satisfactory analysis has been adequately carried out for any of the modes of bed behaviour.

2.4 Scale-up of Solids Flow in Rotating Cylinders

In the absence of mechanistic models, the dimensional analysis technique has been applied to describe and predict the occurrence of the various modes of bed behaviour. The dimensional analysis of Rose and Bull (Section 2.2.1.2) suggested the following scale-up factors: the degree of fill, the coefficient of friction for the solids-wall interface, and the ratio of cylinder diameter to particle size. Subsequent application to industrial ball mills showed good agreement with their laboratory trials.
For slumping and rolling, no scale-up factors are specifically suggested. Instead, bed behaviour similitude has been generally tied to the scale-up of the cylinder diameter, of which there are two schools of thought. The first is to apply equal peripheral speeds.\textsuperscript{46,73} This scaling criterion will obviously not hold at centrifuging speeds and is thus not reliable. A more widely accepted criterion is the Froude number, a ratio of inertial to gravitational forces\textsuperscript{2,44,61,76,77,82} which is given by:

\[
Fr = \frac{\omega^2 R}{g} \quad (2.7)
\]

or

\[
Fr = \frac{\pi^2}{900} \left[ \frac{n^2 R}{g} \right] \quad (2.8)
\]

The justification of the Froude number criterion is based on the centrifuging mode of bed behaviour where the gravitational force is equal to the centrifugal force. Thus:

\[
Fr_c = 1 \quad (2.9)
\]

For Froude number similarity:

\[
(Fr)_M = (Fr)_p \quad (2.10)
\]

Dividing equation (2.10) by (2.9), and substituting equations (2.6) and (2.8) then simplifying:
\[
\begin{pmatrix}
\frac{n}{n_c} \\
\frac{n_c^R}{n_c^R}
\end{pmatrix}_M = \begin{pmatrix}
\frac{n}{n_c} \\
\frac{n_c^R}{n_c^R}
\end{pmatrix}_P \tag{2.11}
\]

Multiplying both sides by one:

\[
\begin{pmatrix}
\frac{nR}{n_c^R} \\
\frac{n_c^R}{n_c^R}
\end{pmatrix}_M = \begin{pmatrix}
\frac{nR}{n_c^R} \\
\frac{n_c^R}{n_c^R}
\end{pmatrix}_P \tag{2.12}
\]

Hence, at equal fraction of critical speed or at equal fraction of peripheral critical speed, the bed behaviour would be expected to be equal. Equations (2.11) and (2.12) are other forms of the Froude number criterion which also contradicts the peripheral speed criterion. It is important to re-emphasize that this criterion has not been tested for diameter effects for any of the modes of bed behaviour, and in the light of this work has been found not to be sufficient for bed behaviour similitude.

2.5 Summary

A description of slipping, slumping and rolling has been presented and a review of the varied attempts at characterizing the bed motion has been made. The degree of fill and some of the physical properties of the material: static and dynamic angles of repose coefficient of friction of the solids-wall interface and particle size, have been found to affect transverse bed motion. While a comprehensive
description of slipping, slumping and rolling can be deduced from the literature, little is understood about their occurrence and interrelationship.

There is little agreement in the literature on the conditions under which a bed changes from one type of bed motion to another. Three modelling approaches of bed behaviour that have been used are: the active layer thickness, the transverse mixing and the mechanics of rigid bodies. These approaches have so far yielded little fundamental understanding of the effect of material, cylinder or operating variables on the transverse flow characteristics of bulk solids in rotary kilns.

The Froude number has been suggested as a scale-up criterion for slumping and rolling beds. It has been justified on the basis of centrifuging calculations but has not been verified. For slipping, scale-up criteria have been proposed to be the coefficient of friction between the bulk solids and the cylinder wall and the degree of fill.

An in depth experimental and mathematical analysis of slipping, slumping and rolling will therefore be carried out in this study. The relative contribution of the material, cylinder and operating variables will be investigated in an
attempt to enhance the current level of understanding of the mechanisms in effect. This would also provide more useful techniques of characterizing the flow of bulk solids in rotary kilns and other process applications.
Chapter 3

PARTICLE CHARACTERIZATION AND
DESCRIPTION OF APPARATUS

3.1 Introduction

A description of the materials and apparatus used for the study of bed behaviour in rotary kilns will be presented in this chapter. A total of ten materials were characterized. The bed behaviour studies of these materials were carried out in the UBC pilot kiln as well as in three batch rotating cylinders. The characterization of the bed-wall static friction conditions is also presented.

3.2 Particle Characterization

The particle characterization was carried out by measuring or identifying the following properties: particle size, particle shape, void fraction and static angle of repose. The techniques applied will be presented for each respective property.

3.2.1 Particle Size

The particle size analysis was undertaken in
standard wire cloth sieves using the US Standard Sieve Series. The sampling and sieving methods used were in accord with ASTM proposed procedures.

At least three samples were riffled from industrially bagged materials. The samples were sieved in standard 203 mm test sieves and placed in a Ro-Tap test sieve shaker for 20 - 25 minutes. The weight retained on each screen was recorded and an average size distribution was calculated on the basis of all the samples sieved. Graphical representations of the distributions of all materials tested are shown in Figure 3.1 and the detailed sieve analysis results for each material are listed in Tables A.1 to A.10 (Appendix A). The average particle size for each material which was calculated using the weighted arithmetic mean method is also listed in Table II. Finally, the screen analysis data was plotted on a log probability plot, as shown in Figure 3.2.

With the exception of sand C, all the materials used had very narrow particle size distributions. (Figure 3.1). Sand C was also observed to have a multimodal distribution as may be seen in Figure 3.2. Limestone D also seems to have a slight multimodal distribution. Both nickel oxide and limestone F appear to have log-normal
Figure 3.1 Particle size distributions of materials tested.
<table>
<thead>
<tr>
<th>Material</th>
<th>Average Size (mm)</th>
<th>Particle Shape</th>
<th>Particle Density (kg/m³)</th>
<th>Loose Bulk Density (kg/m³)</th>
<th>Dense Bulk Density (kg/m³)</th>
<th>Loose Void Fraction</th>
<th>Dense Void Fraction</th>
<th>Static Angle of Repose (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel</td>
<td>3.0</td>
<td>Angular</td>
<td>2,870</td>
<td>1,560</td>
<td>1,690</td>
<td>0.46</td>
<td>0.41</td>
<td>40.7</td>
</tr>
<tr>
<td>Iron Oxide</td>
<td>11.6</td>
<td>Spherical</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>31.5</td>
</tr>
<tr>
<td>Limestone B</td>
<td>4.3</td>
<td>Irregular</td>
<td>2,700</td>
<td>1,450</td>
<td>1,610</td>
<td>0.46</td>
<td>0.40</td>
<td>40.3</td>
</tr>
<tr>
<td>Limestone C</td>
<td>1.5</td>
<td>Irregular</td>
<td>2,690</td>
<td>1,520</td>
<td>1,600</td>
<td>0.43</td>
<td>0.40</td>
<td>37.8</td>
</tr>
<tr>
<td>Limestone D</td>
<td>0.58</td>
<td>Irregular</td>
<td>2,680</td>
<td>1,490</td>
<td>1,570</td>
<td>0.44</td>
<td>0.41</td>
<td>35.6</td>
</tr>
<tr>
<td>Limestone E</td>
<td>0.54</td>
<td>Equi-dimensional</td>
<td>2,670</td>
<td>1,680</td>
<td>1,860</td>
<td>0.37</td>
<td>0.30</td>
<td>38.6</td>
</tr>
<tr>
<td>Limestone F</td>
<td>8.1</td>
<td>Angular</td>
<td>2,690</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>42.8</td>
</tr>
<tr>
<td>Nickel Oxide</td>
<td>4.9</td>
<td>Spherical</td>
<td>-</td>
<td>870</td>
<td>900</td>
<td>-</td>
<td>-</td>
<td>32.5</td>
</tr>
<tr>
<td>Sand B</td>
<td>0.50</td>
<td>Nodular</td>
<td>2,660</td>
<td>1,640</td>
<td>1,740</td>
<td>0.38</td>
<td>0.35</td>
<td>33.4</td>
</tr>
<tr>
<td>Sand C</td>
<td>0.23</td>
<td>Nodular</td>
<td>2,730</td>
<td>1,710</td>
<td>1,810</td>
<td>0.37</td>
<td>0.34</td>
<td>32.2</td>
</tr>
</tbody>
</table>

TABLE II Summary of Results for the Particle Characterization of the Materials Tested
Figure 3.2 Log probability plot of screen analysis results of materials tested.
distributions as indicated by their straight line plots in Figure 3.2. The log-normal distribution plots of the other materials tested indicate that they result from a coarse size separation process. This conclusion was confirmed for the limestones and the gravel by the respective suppliers.

3.2.2 Particle Shape

The most time consuming and difficult parameter of a particle to be quantitatively measured is the particle shape. Several investigators have attempted to develop theoretical models for describing it. The complexity of the expressions and their lack of immediate practicality in quantifying the particle shape has prevented other investigators from applying them. Empirical techniques have also been used to characterize particle shapes. These have included flow property measurements from bins, sieve analysis and the ratio of particle sizes determined by two different sizing methods. Finally, shape measurements made directly on the particles in question have generally been preferred, particularly the shape factors termed "sphericity" and the "Heywood" ratios.

The "sphericity", which has been applied where the
surface area of the particle is of importance, may be measured several ways. The easiest involves measurement of the porosity of a randomly packed bed of uniformly sized particles. However, since the materials used in this study have size distributions, their sphericity cannot be measured using this technique. Another technique involves particle surface area and volume measurements which are very tedious and time consuming. The measurement of the "Heywood ratios" would also be as involved as the latter measurement of "sphericity", particularly when dealing with industrial products. Notwithstanding, it has yet to be firmly established in the literature whether either the "sphericity" or the "Heywood ratios" reflect the effect of particle shape on the flowability of particles, although the results of Sutherland and Neale seem to support the use of "sphericity". The quantitative characterization of particle shape was not pursued in this study, however the treatment of particle shape will be further discussed in Chapters 4 and 7.

The qualitative description of particle shape of the materials used was carried out in accordance with the definitions of particle shape presented in the British Standard 2955. They were more applicable to the materials used in this study than the classification
proposed by Hausner for powdered metals. The classification of the materials used is listed in Table II, while photographs of representative samples are shown in Figures 3.3 to 3.12. Observations of the particles before and after tumbling in the rotating cylinders showed no detectable degradation or change in particle shape.

3.2.3 Void Fraction

Whenever a quantity of granular material is placed in a given volume, the resulting density of the material is less than the apparent particle density, and is termed the bulk density. The closer the value of the bulk density is to that of the apparent particle density, the denser the packing characteristics of the particles and the smaller the interparticle voidage. Many attempts have been made to theoretically calculate the void fraction for a given material or for a mixture of materials. However, for industrial products with particle size distributions, the direct experimental approach is still the most reliable. The void fraction can be calculated from experimental results as follows:

\[ \varepsilon = 1 - \left( \frac{\rho_B}{\rho_p} \right) \]  

(3.1)

The bulk density of the material can be measured using
Figure 3.3  Angular gravel with 3 mm average particle size (1 division = 1 mm)

Figure 3.4  Spherical iron oxide with 11.6 mm average particle size (1 division = 1 mm)
Figure 3.5  Irregular limestone B with 4.3 mm average particle size (1 division = 1 mm)

Figure 3.6  Irregular limestone C with 1.5 mm average particle size (1 division = 1 mm)
Figure 3.7  Irregular limestone D with 0.58 mm average particle size (1 division = 1 mm)

Figure 3.8  Equi-dimensional limestone E with 0.54 mm average particle size (1 division = 1 mm)
Figure 3.9  Angular limestone F with 8.1 mm average particle size (1 division = 1 mm)

Figure 3.10  Spherical nickel oxide with 4.9 mm average particle size (1 division = 1 mm)
Figure 3.11  Nodular sand B with 0.50 mm average particle size (1 division = 1 mm)

Figure 3.12  Nodular sand C with 0.23 mm average particle size (1 division = $10^{-2}$ mm)
the method proposed by Eastwood et al.,\textsuperscript{107} who suggest the use of measuring containers with a diameter 50 to 100 times that of the average particle size of the solids to be measured. Hence to minimize wall effects,\textsuperscript{107,108} two 0.285 m diameter containers were used. The iron oxide and limestone F were not tested as their wall effects would have been significant. Initially, the volume of one of the containers was measured with water and was used as a reference. To measure the bulk density of a material, excess solids were placed in one of the containers. It was covered by the second container and both were inverted several times into each other. The contents of the reference container were levelled, weighed and the random loose bulk density calculated. The random dense bulk density was obtained in a similar manner except rather than inverting the container, it was tapped up to ten times on the floor. Several measurements of each material were taken for the random loose and random dense bulk densities. The averaged results are listed in Table II.

The apparent particle densities were measured using a pycnometric technique in a 1 L volumetric flask. The results, along with the calculated void fractions, are also listed in Table II.

In order to fully characterize the void fraction of
the materials to be used in the segregation studies, the void fraction as a function of composition was measured for sands B and C and limestones B and E. The apparent particle density of each mixture was calculated using the weighted apparent particle densities of the individual components for a given mixture composition. Only the random loose void fraction was measured for the mixtures. The results are shown in Figure 3.13.

The particle shapes of the materials used in this study differed from the glass spheres of Eastwood et al. Also the industrial materials had particle size distributions, while in the study of Eastwood et al., mixtures of two equally sized particles had been tested. Despite these differences the observations of the void fraction-composition diagram obtained in this study agree well with those of Eastwood et al. Namely, as the average particle size of the two components increase, the maximum bed contraction, $\Delta \varepsilon_{\text{max}}$, of the resulting mixture increases. A coarse to fine average size ratio of 8.0 for the limestones yielded a $\Delta \varepsilon_{\text{max}}$ of 0.114, while for the sands $\Delta \varepsilon_{\text{max}}$ was 0.054 for a coarse to fine average particle size ratio of 2.2. The second conclusion of Eastwood et al. was that the maximum bed contraction occurs at a mixture composition of 50-70% of the larger component. From Figure 3.13 it appears that this conclusion is in agreement with the
Figure 3.13 Void fraction composition diagram for limestone B and E and sands B and C.
limestone results but not with those for the sands. On closer examination though it is found that at a mixture composition of 40% fines, the bed contraction is 0.046; while at 60% fines composition, it is 0.054. This difference in $\Delta \varepsilon_{\text{max}}$ is within the measured weighing accuracy of ±1%. The void fraction of mixtures of granular solids will again be discussed in Chapter 5 with reference to the segregation studies.

3.2.4 Angle of Repose

Several types of angles of repose have been observed: the drained, the piled and the dynamic angles of repose. In this section, the measurements reported correspond to the piled or static angle of repose. There are three methods of measuring the static angle of repose: the fixed cone, the tilting box and the rotating cylinder methods (Figure 3.14). 70,91,92,106,108-110

In the fixed-cone method, material is passed through a funnel or pipe and piled on to a flat surface. The discharge point of solids must be relatively close to the height of the pile in order that the particles do not have too great a momentum when piled. The angle with the horizontal is measured when a significant quantity of material has been poured.
Figure 3.14 Three methods for measuring the static angle of repose.
In the tilted-box method, material is placed in a horizontally-positioned box. The surface of the material is also horizontal, hence parallel to the base of the box. One end of the box is tilted upwards very slowly by means of a winch until a quantity of material slumps to the bottom of the box. At this point, the winch is turned off and the angle of inclination of the box is measured. Since the surface of the bottom of the box is parallel to the material surface at the start of the test, the measured angle of the box is the maximum angle of repose the material had prior to its slumping (i.e. the static angle of repose).

The third method is similar to the inclined box technique. Solids are placed inside a cylinder with a plexiglas end plate which is slowly rotated by an electric motor. The inclination of the surface of the bed is followed manually with a long arm protractor measuring the inclination of the bed through the plexiglas end plate. The maximum inclination of the bed is thus recorded as the static angle of repose of the material.

The measured results for the testing methods described above are listed in Table III for some of the materials used. All data was measured to an accuracy of 0.5°. It is seen that all three methods yield results that are comparable.
<table>
<thead>
<tr>
<th>Material</th>
<th>Method</th>
<th>Angle of Repose $\phi_R$ (degrees)</th>
<th>Standard Deviation (degrees)</th>
<th>Number of Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel</td>
<td>Cone</td>
<td>40.7</td>
<td>0.8</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Inclined Box</td>
<td>40.7</td>
<td>0.7</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Rotary Cylinder (D = 1m)</td>
<td>40.7</td>
<td>0.3</td>
<td>5</td>
</tr>
<tr>
<td>Limestone B</td>
<td>Cone</td>
<td>40.3</td>
<td>1.5</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Inclined Box</td>
<td>41.7</td>
<td>0.9</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Rotary Cylinder (D = 0.4m)</td>
<td>41.6</td>
<td>1.2</td>
<td>11</td>
</tr>
<tr>
<td>Limestone C</td>
<td>Cone</td>
<td>37.8</td>
<td>1.3</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>Rotary Cylinder (D = 0.4m)</td>
<td>38.1</td>
<td>0.6</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>Rotary Cylinder (D = 1m)</td>
<td>37.6</td>
<td>0.5</td>
<td>8</td>
</tr>
<tr>
<td>Nickel Oxide</td>
<td>Cone</td>
<td>32.5</td>
<td>1.6</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Inclined Box</td>
<td>33.3</td>
<td>0.6</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Rotary Cylinder (D = 0.4m)</td>
<td>31.9</td>
<td>1.2</td>
<td>6</td>
</tr>
</tbody>
</table>
However, in a thorough discussion of these three methods, Richards and Brown favored the cone method because of dimensional constraints offered by the vessels of the other two techniques. The three-dimensional nature of the force distribution in the cone as opposed to the two-dimensional force distribution in the other two methods was also cited as an advantage. Despite Richards and Brown's discussion, little is understood of the differences between the three measuring techniques. Due to its wide application, the cone method was used in this study for all materials. The results are listed in Table II.

3.3 Descriptions of the Rotary Kiln and the Rotary Cylinders

3.3.1 The Pilot Rotary Kiln

The rotary kiln used for the bed behaviour observations has been described in detail by Brimacombe and Watkinson and is shown in Figure 3.15. It is sufficient here to state that the kiln has an internal diameter of 0.4 m and is 5.5 m in length with variable inclination and a castable refractory lining. Castable dams of variable geometry can be placed at the solids discharge end. The rotational speed as well as the solids feed rate are also variable. A ruler welded perpendicularly to a long
Figure 3.15  The U.B.C. pilot kiln.
horizontal pipe was introduced axially into the kiln to manually measure the bed depth along the kiln length.

3.3.2 Rotary Cylinders

Three batch rotating cylinders were constructed for the detailed bed behaviour observations. Their dimensions are listed in Table IV. Both cylinders A and B were constructed with the same internal diameter as the pilot kiln. This allowed data obtained on the rotary cylinders to be related to a continuous operation and will be further discussed in Chapter 4.

Cylinder A was constructed from 9.5 mm thick mild steel plate, whereas cylinders B and C were constructed from Perma Tube,* a dimensionally stable cardboard. To ensure a high frictional coefficient between the cylinder wall and the solids, as well as to ensure similar wall conditions in all three cylinders, a 24-3 grit type E silicon carbide abrasive paper was glued to the inside walls of the three cylinders. One of the two cylinder end plates was made of plexiglas and was placed on the front end of the cylinder thus enabling the cross section of the bed to be

*Perma Tubes Ltd., 4751 Vanguard, Richmond, B.C., Canada.
<table>
<thead>
<tr>
<th>Cylinder</th>
<th>Inside Diameter (m)</th>
<th>Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.4</td>
<td>0.46</td>
</tr>
<tr>
<td>B</td>
<td>0.4</td>
<td>0.86</td>
</tr>
<tr>
<td>C</td>
<td>1.0</td>
<td>0.41</td>
</tr>
</tbody>
</table>
viewed and photographed. A hole was cut out in the centre of the end plates to allow easy access to the solids bed for charging and discharging.

The cylinders were supported on two frictional rolls, of which one was the drive roll. These rolls were bolted to a support stand on which two thrust bearings were also attached. At the viewing end of the cylinder, a small variable height platform was attached to allow the measurements of the inclination of the bed. The horizontal position of this platform was adjustable to ensure that it was horizontally level whenever angular measurements were made. The cylinders were rotated by a 1/6 HP GE electric motor attached to a Zero-Max Drive Power Block, Model 22, with a speed range from 0 to 400 revolutions per minute. The speed was stepped down by means of a pulley arrangement attached to the drive roller. The maximum rotational speed imparted to the rotating cylinder was about seven revolutions per minute for cylinders A and B. The rotational speed of the cylinder was measured directly off the cylinder outer circumference using a tachometer with an accuracy of 0.05 revolutions per minute and a maximum range of 100 revolutions per minute. A photograph of the equipment is shown in Figure 3.16.
Figure 3.16  Rotary cylinder A (0.4 m ID x 0.46 m L) set on rolls.
3.4 Solids-Wall Friction Angle

Several methods have been suggested for measuring the coefficient of friction of an abrasive surface. \(^{81,112-116}\) For example, Halbart and Freymann\(^{81}\) suggest a friction coefficient based on the packing density of the material in contact with the wall. However each of these methods characterizes only the surface of the wall or solids, while the combination of the material and wall surfaces would not be accounted for. Other surface roughness measuring techniques must therefore be pursued.

To measure the solids-wall friction coefficient, Rose and Blunt\(^{54}\) developed their own experimental method. A flat surface was prepared replicating the cylinder wall surface. A bottomless box was placed on top of this surface in a horizontal position and was filled with the material of interest. A spring load was used to push the box until the latter slipped against the flat surface. The wall-solids coefficient of friction was then calculated using a force balance.

Conrad et al.\(^{117}\) used a different technique. A given quantity of the material to be studied was sprinkled onto a glass plate, which was slowly rotated from a horizontal position. The angle of inclination of the plane was recorded.
when all of the material slid off it simultaneously. The particle size and the quantity of solids placed on the glass plate were varied. Their results revealed that for a material having a particle size greater than 0.25 mm, the wall-solids friction coefficient was independent of the mass placed on the flat surface, and that the inter-particle cohesive force was negligible. Therefore, the coefficient of friction for the wall-solids surface was simply given by:

\[ \mu_{W/S} = \tan \phi_s \]  (3.2)

The method of Conrad et al. is much simpler than that of Rose and Blunt and was adapted for characterizing the wall-solids friction coefficient in this study. The experimental set up used, shown schematically in Figure 3.17, consisted of a flat wooden board to which the abrasive paper used for the inside cylinder surface was secured. The granular materials of interest were epoxied onto cardboard backings which were glued to a small weight to ensure good contact between the solids and the abrasive paper. The board was raised by a variable speed electric motor (see Section 4.2.2) at a maximum lifting rate of 0.17 m/min, until the granular solids slipped. The angle of inclination of the board was measured to within 0.5°. The average slipping angles for the materials tested are listed in Table V. These values are greater than the static angles of repose, given in Table II, for each respective material.
Figure 3.17 Schematic diagram of the apparatus used for measuring the solids-wall friction coefficient.
<table>
<thead>
<tr>
<th>Material</th>
<th>Angle of Slip (degrees)</th>
<th>Standard Deviation (degrees)</th>
<th>Number of Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel</td>
<td>48.8</td>
<td>1.2</td>
<td>5</td>
</tr>
<tr>
<td>Iron Oxide</td>
<td>38.9</td>
<td>2.9</td>
<td>8</td>
</tr>
<tr>
<td>Limestone B</td>
<td>43.9</td>
<td>1.6</td>
<td>8</td>
</tr>
<tr>
<td>Limestone C</td>
<td>43.2</td>
<td>2.4</td>
<td>8</td>
</tr>
<tr>
<td>Limestone D</td>
<td>40.9</td>
<td>1.3</td>
<td>8</td>
</tr>
<tr>
<td>Limestone F</td>
<td>43.3</td>
<td>1.9</td>
<td>8</td>
</tr>
<tr>
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Chapter 4

EXPERIMENTAL RESULTS AND ANALYSIS

4.1 Introduction

The observations and the experimental characterization of slumping and rolling will be presented in this chapter. The experiments were mostly carried out on batch rotating cylinders. Since rotary-kiln application is the primary thrust of this research, a method will be presented in Section 4.2 to relate the results obtained on the batch rotary cylinders to a continuous rotary kiln. This will be followed by a presentation of the transverse bed motion observations on a Bed-Behaviour Diagram which was developed to delineate the various modes of bed behaviour encountered: slipping, slumping and rolling.

An instrumented technique for the determination of bed behaviour will be applied to an investigation of the effect of cylinder and material variables on the slumping-rolling boundary. The variables studied are cylinder wall effects, particle size, particle shape, combined effects of size and shape, the static angle of repose, the degree of fill, the rotational speed and the cylinder diameter.
Finally, an attempt at quantitatively characterizing slumping and rolling will be presented in Section 4.5. For the slumping bed, the upper angle of repose, the shear angle and the slumping frequency were measured; while the dynamic angle of repose and the active layer thickness were reported for the rolling bed. Whenever applicable, these variables will be related to the material variables and hence to the Bed-Behaviour Diagrams. This analysis will show that the theories already put forward by previous workers (see Chapter 2) do not provide a satisfactory interpretation of the change in bed behaviour from slumping to rolling and that the quantitative characterization methods of the bed motion do not identify the conditions for this change.

4.2 Bed Behaviour: Continuous versus Batch Operations

A study of bed behaviour in rotary kilns naturally implies making direct observations of various types of bed motion in a continuous process. Due to the large number of variables in continuous kiln operation, and due to the time required for the solids discharge to reach steady state (3 to 7 hours, depending on the operating conditions), it would be advantageous to be able to carry out experiments on a batch cylinder and relate the results to continuous operation. The kiln variables studied were the feed rate,
the rotational speed, the inclination of the kiln, the height of dam and the axial bed depth profile. The effects of temperature and of kiln internals will be briefly discussed in Chapters 5 and 7.

4.2.1 Reduction of Kiln Variables

It has been observed in industrial kilns that changes in operating variables result in an increase or a decrease in the retained volume of the kiln (i.e. varying bed depths along the kiln length). Since there is no existing comprehensive fundamental equation to relate the bed depth profile to the geometry and operating conditions of a kiln, it was decided to experimentally investigate the interrelationship of these variables, using the UBC pilot kiln. Thus, for the first stage of the investigation, subsequent observations could be directly related to the operating variables of the pilot kiln.

The dependence of the bed profile on the pilot kiln geometry and its operating variables was therefore investigated. Only a synopsis of the results, which illustrate the observed trends, will be presented. The experiments consisted of feeding the charge at a given rate and rotational speed under varying kiln configurations (i.e. inclination and
discharge dam). Once the solids discharge rate had reached steady state, the bed profile in the kiln was measured, using the ruler described in Section 3.3.1 which was manually inserted into the bed of solids at a given axial position and the depth recorded. Thus for any set of operating conditions, the bed depth profile along the kiln length could be directly measured.

The limestone bed profiles (Figures 4.1 and 4.2) were measured subsequent to calcination experiments, while the sand experiments (Figure 4.3) were carried out in a cold test. Early in this experimental campaign, several bed profile measurements were performed before and after the calcination of limestones A and B. No difference in bed profiles was observed.

The effect of kiln inclination is seen in Figure 4.1. As the kiln inclination increases, the bed depth at the charge end decreases, while at the discharge end the decrease in bed depth is not as pronounced. Also, the resultant bed profile contours are markedly different. Illustrated in Figure 4.2 is the effect of kiln rotational speed. In the operating range chosen for limestone B, a linear bed profile resulted for the three rotational speeds tested. The only observed effects are that at a given axial location the
Bed depth profile for Limestone A in the UBC pilot kiln, illustrating the effect of kiln inclination.
Figure 4.2 Bed depth profile for limestone B in the UBC pilot kiln, illustrating the effect of kiln rotational speed.
Figure 4.3  Bed depth profile for sand A in the UBC pilot kiln, illustrating the effect of kiln dams and solids feed rate.
bed depth decreases as the rotational speed increases and that the slope of the bed profiles increases with increasing rotational speed. Finally, the effects of feed rate and discharge dam height are illustrated in Figure 4.3 using sand A. An increase in bed depth is observed as a result of increases in both feed rate and dam height.

Therefore, changes in the kiln geometry or in its operating variables result in a change in the bed depth profile of the kiln. Hence, it would be acceptable to report subsequent bed behaviour observations as a function of the bed depth profile. The number of kiln variables to be investigated in a study of bed behaviour may thus be greatly reduced.

4.2.2 Bed Behaviour in a Continuous Operation

While measuring the bed depth profiles described in Section 4.2.1, the various modes of bed behaviour encountered were also recorded. Four types were observed: slipping, slumping, rolling and transitional, which exhibited mixed characteristics of slumping and rolling. The corresponding kiln operating variables were also recorded; some of the results for sand A are presented in Table VI from which it is clear that the bed depth and rotational speed suffice to identify the mode of bed behaviour. That is, for quite
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<th>Feed Rate (kg/h)</th>
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<td>Rolling</td>
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different feed rates and dam heights, the bed behaviour of sand A was the same when the resulting bed depths and rotational speeds were equal. The same conclusion was also reached using limestone A when the angle of inclination of the kiln was varied. Therefore, it appears that there are no downstream effects of bed behaviour in the kiln and that only the bed depth and rotational speed need be specified.

To further test this conclusion, all of the bed behaviour observations for sand A have been presented in Figure 4.4 which is a plot of bed depth versus rotational speed and is called a Bed-Behaviour Diagram. Since the Froude number has been proposed by other workers (see Chapter 2) as the scale-up criterion of bed behaviour, it is also shown on the diagram. The bed depth measurements recorded in the bed profile results (e.g. Figure 4.3) were replotted in Figure 4.4 at their respective rotational speeds. Slipping, slumping, transition and rolling modes of bed behaviour were labeled accordingly.

While making these observations, it was not unusual to observe two or three types of bed modes occurring at different axial positions. In such cases, the bed depths were plotted, properly coded for the type of bed behaviour that had been observed at the respective axial positions. It is
evident from Figure 4.4 that there are regions on the diagram where a particular type of bed mode dominates. For example, over the range of rotational speeds tested, the slipping mode occurs whenever the bed depth was less than 0.03 m. At rotational speeds less than 0.3 r/min and at bed depths greater than 0.03 m, the solids bed was always observed to slump. As the rotational speed increases, slumping is favoured at the lower bed depths; while at the higher bed depths the observed behaviour is the transition mode. No slumping is seen to occur at rotational speeds greater than 1.0 r/min. Lines demarcating the boundaries between the various bed modes were visually drawn and delineate a bed depth dependency for the slumping-transition-rolling boundaries (Figure 4.4). Therefore for sand A tested in the pilot kiln, the solids bed behaviour is solely a function of the bed depth and the rotational speed. This conclusion though needs to be further tested by comparing the bed behaviour in a continuous inclined operation with that in a batch horizontal cylinder.

4.2.3 Continuous versus Batch Bed Behaviour

In this section, the possibility of simplifying the continuous apparatus to a batch operation will be investigated. The latter would offer the added advantage of experimentally treating the bed depth and the rotational
Figure 4.4 Bed-Behaviour Diagram of sand A visually determined in the UBC pilot kiln.

Figure 4.5 Bed-Behaviour Diagram visually determined for sand A in cylinder A (0.4 m ID x 0.46 m L) and compared to the boundaries obtained in the pilot kiln.
speed as two independent variables. This was not always possible in the continuous operation, as a change in the rotational speed had always resulted in an accompanying change in the bed depth.

A Bed-Behaviour Diagram was experimentally determined in cylinder A using sand A. Various quantities of the granular material were placed in the cylinder which was tested in the horizontal position. For each solids bed depth the rotational speed of the cylinder was varied over the slumping-rolling range of the material. The run number of each of these traverses is listed in Appendix B. In its turn, the bed depth was varied over the range of 3 to 20% fill which would be found in rotary kiln operation. The bed behaviour observations were made visually and were recorded in Figure 4.5. Also shown in Figure 4.5 are the slumping-transition-rolling boundaries determined on the pilot kiln. No slipping was observed in cylinder A with sand A, as its wall was much rougher than that of the pilot kiln. Hence, the slipping boundary for the pilot kiln was not included in Figure 4.5.

At first inspection, the agreement between the slumping-transition-rolling boundaries does not appear to be very good. However, in the pilot kiln the protruding
thermocouple sheaths were continuously disturbing the solids. This resulted, as observed, in the bed rolling at lower rotational speeds than would otherwise have been the case in the absence of these internal devices. The slumping-transition boundary shows very good agreement between the continuous and the batch operations. It is therefore felt that bed behaviour observations of slumping and rolling, made on batch cylinders are representative of those made in continuous operations. Expressed differently, given a set of kiln operating variables and the resulting bed profile, the observed bed modes, slumping and rolling, at each axial position would be the same as those occurring in a batch cylinder of the same internal diameter operated under equal conditions of bed depth and rotational speed. Subsequent Bed-Behaviour Diagrams were therefore determined on batch rotary cylinders.

4.3 Instrumentation of Bed Behaviour Observations

4.3.1 Construction and Application

The visual determination of bed behaviour presents two disadvantages: firstly, near the bed behaviour boundaries, the judgement of the observer lacks absolute objectivity and reproducibility and secondly, a permanent record of the bed motion is not obtained. An instrumented
A photographic method was first attempted. While it provided a permanent record of the bed motion, the objectivity in the judgement of the bed mode identification was still lacking. A Light Reflection Distance Gauge (henceforth referred to as an IR sensor or just simply as the sensor) was therefore developed. It consisted of an Infra Red Emitter/Sensor Array (Fairchild Model FPA 104) which emitted Infra Red Signals and detected those reflected from the rebounding surface. The current output from the sensor was proportional to the distance between the sensor and the rebounding surface. Thus, a slumping bed characterized by a constantly changing bed inclination would result in a variable signal while a rolling bed would yield a more constant signal.

The sensor was mounted in an acrylic tube in order to suspend it above the solids bed surface inside the rotating cylinder; thereby not interfering with the solids motion. The axial and radial position of the sensor was variable. The output signal obtained was amplified and fed to a Heathkit recorder, model EU-20B, with a 250 mv full range.
The roughness of the bed surface (due to the individual particles) was only seen to adversely affect the output sensor signal when larger sized particles were tested, namely limestone F and the iron oxide. For all other materials, the effects of surface roughness were less than the changes in bed inclination resulting from a slump. The positioning of the sensor is shown in Figure 4.6, while a sample chart recorder output from the sensor for both slumping and rolling is shown in Figure 4.7. The transition mode was identified by a sensor signal showing characteristics of both slumping and rolling signals. Hence, the sensor overcame the disadvantages of the visual technique. As will be shown in Section 4.4.3 and in Chapter 6, further application of the sensors output data allowed the successful quantitative characterization of a slumping bed as well as the development of a semi-empirical mathematical model predicting the slumping-rolling boundary.

4.3.2 Correspondence of Instrumented and Visual Observations

Bed-Behaviour Diagrams were determined for limestone B and nickel oxide in cylinder A using both the visual technique (Figures 4.8 and 4.9) and the instrumented technique (Figures 4.10 and 4.11). The boundaries obtained
Figure 4.6  Photograph illustrating the position of the sensor in the cylinder.
Figure 4.7 Sample output of sensor for slumping and rolling.
Figure 4.8 Bed-Behaviour Diagram of limestone B in cylinder A (0.4 m ID x 0.46 m L), visually determined.

Figure 4.9 Bed-Behaviour Diagram of nickel oxide in cylinder A (0.4 m ID x 0.46 m L) determined visually.
Figure 4.10 Bed-Behaviour Diagram of limestone B in cylinder A (0.4 m ID x 0.46 m L) determined by instrumentation and compared to the visual determination.

Figure 4.11 Bed-Behaviour Diagram of nickel oxide in cylinder A (0.4 m ID x 0.46 m L) determined by instrumentation and compared with the visual determination.
using both techniques are in excellent agreement. The largest difference in the boundaries was obtained with the transition-rolling boundary of the nickel oxide particles, which was only of the order of 0.2 r/min. Since good correspondence is obtained between the instrument and visual observations and since the sensors proved useful in quantifying other parameters (Sections 4.4.3 and 6.2) all subsequent Bed-Behaviour Diagrams were derived using the sensors and are illustrated in Figures 4.10 to 4.17.

4.4 Effect of Variables on Bed Behaviour

4.4.1 Wall Effects

Before investigating the effect of material and cylinder variables on the slumping-rolling boundary, the interaction of the cylinder wall with the solids motion when slumping and rolling must first be elucidated. To accomplish this, the sensor position inside the rotary cylinder was varied radially and axially and these tests were carried out using limestone B in cylinders A and B (Figures 4.10 and 4.12, respectively). For the first part of this investigation, two sensors were positioned axially in the centre of the cylinder with one of them pointed at the apex of the bed and the other at the chordal base. At equal bed depths, both sensors indicated a change of bed behaviour,
Figure 4.12 Bed-Behaviour Diagram of limestone A comparing the results from cylinder A (0.4 m ID x 0.46 m L) and cylinder B (0.4 m ID x 0.86 m L).

Figure 4.13 Bed-Behaviour Diagram of limestone D in cylinder A (0.4 m ID x 0.46 m L).
whether slumping, transition or rolling, at the same cylinder rotational speed and at the same moment. Hence, the slumping-rolling boundary is independent of the radial position of the sensors. As a slump is initiated at the top half of the bed surface, in subsequent runs the sensor was always pointed at the apex of the bed.

With both sensors positioned at the apex but at different cylinder axial positions, a difference in the loci of the bed behaviour boundaries was observed. The solids adjacent to the end plates tended to roll at lower rotational speeds than those located at the axial mid-point. This is due to the lifting action of the end walls on the particles. Hence, the axial centre of the cylinder was preferred for making bed behaviour observations.

In order to further confirm the influence of the end walls on the bed behaviour results, cylinder B was constructed with a length to diameter ratio of 2:1 as compared to 1:1 for cylinder A. With a sensor positioned at the axial centre of each of the cylinders, a Bed-Behaviour Diagram was experimentally determined with limestone B (Figure 4.12). At degrees of fill greater than 6%, the transition zone of the longer cylinder B was smaller than that of cylinder A, which confirms the presence of the wall effects discussed
earlier. However, the difference in the position of the bed behaviour boundaries is not significant. Therefore, cylinder A was subsequently used for all Bed-Behaviour Diagram determinations with a sensor positioned at the apex of the bed in the axial centre of the cylinder.

In Figures 4.10, 4.11 and 4.12 it is observed that as the bed depth decreases, a higher rotational speed is required for the bed to display transitional behaviour. At low bed depths, some materials display a reversal of this trend which is accompanied by a widening of the transition zone (Figure 4.12). The following explanation is suggested as the cause of this phenomenon.

When a material slumps in a cylinder, the entire bed along the cylinder length does not necessarily slump simultaneously. At sufficiently low bed depths (≤5% fill) the slumping sequence of two adjacent regions of the bed of some materials could result in their interface having a constant inclination. If this interface occurs below the sensor, the output would indicate a rolling bed, since the bed surface is a relatively constant distance away from the sensor. Furthermore, this interface does not always occur at the same cylinder axial position and at times does not exist, as the whole bed could be slumping simultaneously.
Therefore, the output of a sensor at a fixed axial position would indicate a bed in the transition mode, while visually it was observed to be slumping with no evidence of any rolling action. For these minor number of cases, both visual and instrumented observations were noted and will be identified accordingly, for example Figure 4.13. This visually recorded bed depth dependency for the slumping-transition boundary, always indicated increasing rotational speeds for decreasing bed depths (i.e. the same trend observed at higher fill ratios).

The transition-rolling boundary was also bed depth dependent, but could not be attributed to either circumferential or end-plate wall effects.

4.4.2 Particle Shape

While the average particle sizes of nickel oxide and of limestone B are about equal, 4.9 mm and 4.3 mm respectively, their Bed-Behaviour Diagrams differ markedly (Figures 4.10 and 4.11). The slumping-transition-rolling boundaries for the nickel oxide occur at much lower rotational speeds than for limestone B. This difference is attributable to differences of particle shapes between the materials. The one aspect which is common to both diagrams
is the bed depth dependency of the boundaries.

The Bed-Behaviour Diagrams for limestone D and sand B are shown in Figures 4.13 and 4.14 respectively. These two materials also have approximately equal average particle size - 0.58 mm for limestone D and 0.50 mm for sand B - but different particle shapes. The nodular sand is seen to have slumping-transition-rolling boundaries at lower rotational speeds than the irregular limestone; again, both their boundaries are observed to be bed depth dependent.

It may therefore be concluded from this analysis that the Bed-Behaviour Diagram does show the effect of particle shape, with spherical and nodular shaped particles rolling more easily than irregularly shaped particles. This is in agreement with general observations made on the flow-ability of granular materials in bins and hoppers. 90, 97, 108

4.4.3 Particle Size

Figures 4.10, 4.13 and 4.15 illustrate the effect of particle size on the bed behaviour boundaries. The particle shape of both limestones B and D is irregular although there is almost an order of magnitude difference between their average particle sizes, 4.3 mm and 0.58 mm
Figure 4.14 Bed-Behaviour Diagram of sand B in cylinder A (0.4 m ID x 0.46 m L).
Figure 4.15 Bed-Behaviour Diagram of limestone C in cylinder A (0.4 m ID x 0.46 m L).
respectively. Despite the large experimental transition zone obtained for limestone D, it can be seen that smaller particles roll more easily than coarser ones. To further illustrate this point, a Bed-Behaviour Diagram was experimentally determined for limestone C which has the same particle shape as limestones B and D but an intermediate average particle size of 1.5 mm. The resultant boundaries shown in Figure 4.15 are at rotational speeds which fall between those for limestone B (Figure 4.10) and limestone D (Figure 4.13). The boundaries for limestone C also show a bed depth dependency which is in agreement with the observations of Wahlster et al. 57

4.4.4 Combined Particle Size and Shape Effects

The aim of this section is to study the combined effects of particle size and shape and to observe which of the two variables is dominant in determining the flow characteristics of a material.

Figures 4.11 and 4.14 will first be used to illustrate these effects. Here, a coarse spherical nickel oxide is compared to a finer nodular sand. Had both materials the same particle size, the difference in their shape would suggest that the boundaries of sand B lie to the right of those for the nickel oxide. On the other hand, if the shapes were
the same, the difference in size would indicate that the relative positions of the boundaries should be reversed, i.e. the boundaries for sand B should be to the left of those for nickel oxide. When comparing the experimentally determined boundaries shown in Figures 4.11 and 4.14, it is seen that the transition-rolling boundaries of both materials nearly coincide, but the slumping-transition boundary of sand B is to the left of that for nickel oxide. Therefore, it appears that in this case the particle size has partially dominated.

The results of the angular gravel (Figure 4.16) are now analysed with respect to those for limestones B (Figure 4.10) and C (Figure 4.15). With a coarser particle size of 3.0 mm, as compared to the 1.5 mm of limestone C, and with an angular particle shape rather than an irregular one, it would be expected that the bed behaviour boundaries of the gravel would lie to the right of those for limestone C. This result was indeed obtained when comparing Figures 4.15 and 4.16. In comparing gravel and limestone B based on the particle shape difference, the bed behaviour boundaries for gravel should lie to the right of those for limestone B (Figure 4.10). On the other hand, the particle size difference favours the opposite effect. The experimental results suggest that these effects have cancelled each other out since the boundaries are coincident.
Figure 4.16  Bed-Behaviour Diagram of gravel in cylinder A (0.4 m ID x 0.4 m L).

Figure 4.17  Bed-Behaviour Diagram of limestone B in cylinder C (1.06 m ID x 0.4 m L).
These analyses show that while the respective qualitative effects of particle size and shape on bed behaviour are clearly illustrated on a Bed-Behaviour Diagram, their combined effect is not easily predicted. These experimental results would, thus, not provide a predictive method of determining whether the effect of size or shape would dominate. A mathematical model which predicts the slumping and rolling boundary would be required. The model developed in this thesis is presented and verified in Chapters 6 and 7.

4.4.5 Static Angle of Repose

The static angle of repose has been considered to be an empirical measure of the flow properties of granular materials.\textsuperscript{90,91} If the static angle of repose data listed in Table II are compared with the bed behaviour boundaries experimentally derived a trend can be seen. The smaller the static angle of repose the more the bed behaviour boundaries are shifted towards lower rotational speeds, i.e. the easier it is to change from slumping to rolling. This trend does not at first appear to hold for sand B and the nickel oxide spheres. Although the order of their bed behaviour boundaries was not in accord with the respective magnitudes of their static angles of repose, the differences are within the experimental errors. Therefore, while appearing to be a simplistic measurement, the angle of repose
provides an easy and quick method of classifying the relative position of the slumping-rolling boundaries for materials with combined particle size and shape effects.

4.4.6 Rotary Cylinder Diameter

A Bed-Behaviour Diagram was determined for limestone B in cylinder C. It has already been suggested in the literature (Section 2.4) that bed behaviour similarity is obtained at equal Froude numbers based on calculations for centrifuging. However, comparing Figures 4.10 and 4.17 it is evident that the Froude criteria is insufficient. Firstly, in both diagrams the bed behaviour boundaries show a bed depth dependency; hence the degree of fill must in some way appear in the scale-up criteria. Secondly, at equal degrees of fill, the boundaries in Figure 4.10 occur at higher Froude numbers than those in Figure 4.17. Therefore, the development of scale-up criteria directly based on an analysis of the slumping-rolling bed modes is imperative, since the criterion based on centrifuging is not reliable. This will be achieved by the quantitative characterization of the slumping and rolling beds and by the mathematical description of the slumping-rolling boundary.
4.5 Characterization of Slumping and Rolling Beds

4.5.1 Slumping

For a slumping bed, the parameters investigated were: the upper angle of repose of the bed, the shear angle and the slumping frequency.

The maximum angle of bed inclination (the upper angle of repose) was measured on a slumping bed for various materials in the manner described in Section 3.3.2. The results for gravel are shown in Figure 4.18 and indicate a slight dependence of the upper angle of repose on the cylinder rotational speed. A linear least-square fit was carried out on this data and the following equation was obtained:

\[ \phi_u = 0.596 \, \omega + 40.6 \]  \hspace{1cm} (4.1)

Although the multiple correlation coefficient was equal to 0.342, indicating a poor fit, the asymptotic value of \( \phi_u \), 40.6°, is in good agreement with the measured static angle of repose (\( \phi_R = 40.7^\circ \)). It is observed in Figure 4.18 that there is a greater scatter in the data at higher than at lower rotational speeds, which would contribute to the poor fit.

The effect of bed depth on the upper angle of repose
Figure 4.18 Upper angle of repose as a function of rotational speed for gravel in cylinder A (0.4 m ID x 0.46 m L).

Figure 4.19 Upper angle of repose as a function of rotational speed measured at several bed depths in cylinder A (0.4 m ID x 0.46 m L) using limestone C.
was investigated using limestone C (Figure 4.19). No dependence was observed. The linear regression, applied to all the data, yielded the following equation:

$$
\phi_U = 1.01 n + 37.7
$$

(4.2)

Again, the multiple correlation coefficient with a value of 0.599 was poor but the ordinate intercept of $\phi_U$ is in good agreement with the measured static angle of repose ($\phi_R = 37.8^\circ$). For this material, the scatter in the experimental results was about $1.5^\circ$ and $2.5^\circ$ for the low and high rotational speeds respectively.

The large scatter in the results has been attributed to the following factors. Firstly, the protractor used for these measurements could only be read to the nearest $0.5^\circ$. Secondly, the surface of the bed was not always perfectly planar due to the bridging of particles at the end-wall. Thirdly, over the range of rotational speeds, the maximum recorded increase in the upper angle of repose was only $3^\circ$. In view of the accuracy of measurement discussed above it is not surprising that poor fits were obtained. The last contributing factor lies in the difficulty of locating the precise position of the upper angle of repose, due to the continuous cylinder rotation. This difficulty, of course, increases with increasing rotational speed.
In conclusion, over the slumping range of a material, the upper angle of repose increases with increasing rotational speed. Unfortunately, the measuring technique used was not sensitive enough to reliably quantify this dependence. The upper angle of repose was also not observed to be a function of the quantity of material in the cylinder. In general, the measurement of upper angle of repose is not suitable for studying the flow properties of materials.

The characterization of a slumping bed using the shear angle will now be described. Subsequent to a material reaching its upper angle of repose, a slump occurs. The shear wedge of solids in the slump moves from the apex to the chordal base of the bed while the cylinder is continuously rotating. The plane in the bed dividing the particles moving with the slump and those moving with the cylinder wall is called the shear plane. Its position has been visually observed to be fixed in space during a slump and to form the final bed inclination once the solids in the slump have come to rest. The measurement of the shear angle is made in this latter position, using the technique described in Section 3.2.2.

The shear angle was measured for all materials and in the three cylinders. The results are presented in Table VII.
where it can be seen that in a given cylinder the shear angle was found to be independent of the rotational speed and of the bed depth. On closer examination of the results, the dependence of the shear angle on the cylinder diameter is apparent, particularly for limestone B. In general, it was observed that the shear angle was lower in the larger diameter cylinder. This concurs with the results for the static angle of repose obtained by other workers which Richards and Brown have attributed to wall effects.

When comparing the shear angles of all materials tested in the same cylinder, the effects of particle size and shape are evident. For example, the iron oxide and the nickel oxide have lower shear angles than limestones B, C and D which all have irregular shapes. Within each of these two groups of particle shapes, greater particle sizes result in greater resultant shear angles. Although the shear angle may be used to characterize a material, it reveals no insight into the flow properties of materials. However, its lack of dependence on bed depth and rotational speed in a given cylinder proved invaluable in the development of the bed behaviour mathematical model.

It has been shown that the upper angle of repose has not been useful to adequately quantify the slumping bed
<table>
<thead>
<tr>
<th>Material</th>
<th>Cylinder Diameter (m)</th>
<th>Shear Angle (degrees)</th>
<th>Standard Deviation (degrees)</th>
<th>Number of Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel</td>
<td>0.40</td>
<td>34.7</td>
<td>0.8</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>1.06</td>
<td>34.4</td>
<td>0.6</td>
<td>26</td>
</tr>
<tr>
<td>Iron Oxide</td>
<td>0.40</td>
<td>33.3</td>
<td>0.4</td>
<td>7</td>
</tr>
<tr>
<td>Limestone B</td>
<td>0.40</td>
<td>37.7</td>
<td>1.1</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>1.06</td>
<td>34.5</td>
<td>0.7</td>
<td>39</td>
</tr>
<tr>
<td>Limestone C</td>
<td>0.40</td>
<td>33.6</td>
<td>0.5</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>1.06</td>
<td>32.5</td>
<td>0.8</td>
<td>16</td>
</tr>
<tr>
<td>Limestone D</td>
<td>0.40</td>
<td>33.5</td>
<td>0.3</td>
<td>51</td>
</tr>
<tr>
<td>Limestone F</td>
<td>1.06</td>
<td>38.5</td>
<td>1.2</td>
<td>6</td>
</tr>
<tr>
<td>Nickel Oxide</td>
<td>0.40</td>
<td>29.9</td>
<td>0.7</td>
<td>56</td>
</tr>
<tr>
<td>Sand A</td>
<td>0.40</td>
<td>32.4</td>
<td>0.7</td>
<td>5</td>
</tr>
<tr>
<td>Sand B</td>
<td>0.40</td>
<td>32.2</td>
<td>0.5</td>
<td>44</td>
</tr>
<tr>
<td>Sand C</td>
<td>0.40</td>
<td>33.0</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>
motion, to characterize the material or to elucidate the causes for the change in bed motion from slumping to rolling, while the shear angle has only achieved the second of these objectives; The slumping frequency measurements to be described will fulfil the first two objectives and will indicate that a new interpretation of the reason materials change from a slumping to a rolling bed is required.

As was mentioned in Section 2.2.2.2, the easiest and most obvious method of characterizing a slumping bed is by measuring its slumping frequency. A sample of a sensor output was displayed in Figure 4.7 where each peak seen on the slumping chart represents one slump of the material. Thus the slumping frequency could be obtained from the record of the sensor output. The slumping frequency was measured from a continuous sensor output ranging usually from 5 to 30 minutes. Only during the transition mode was the continuous measuring period below 5 minutes. In these circumstances, the total measuring time of at least 5 minutes incorporated several individual slumping periods; each having a minimum of one minute duration.

The effect of sensor positioning on the measurement of slumping frequency was first investigated. The positions tested in cylinder A varied axially and radially. For the
same testing conditions, all sensor positions recorded the same slumping frequency. The sensor was therefore positioned axially in the centre of the cylinder at the apex of the bed for all measurements. In fact, the slumping frequency measurements were made from the same sensor position as the bed behaviour observations. For the sake of clarity, only a fraction of the results have been presented in Figures 4.20 to 4.23.

The effect of cylinder length and bed depth on the slumping frequency is shown in Figures 4.20 and 4.21. Neither variable is seen to affect the slumping frequency. In contrast, the slumping frequency is dependent on the cylinder diameter. This is illustrated in Figure 4.21 using limestone B tested in cylinders A and C. These findings were further confirmed by the results of the remaining materials tested, as is discussed below.

The general characteristics of these slumping frequency curves are as follows. At low rotational speeds, small increases in rotational speed result in large increases in slumping frequency. As the rotational speed increases, the slumping frequency increases by smaller increments. The largest slumping frequencies are obtained for the lowest bed depths. This is consistent with the finding that at
Figure 4.20 Slumping frequency as a function of rotational speed for limestone B tested in cylinders A (0.4 m ID x 0.46 m L) and B (0.4 m ID x 0.86 m L).
Figure 4.21 Slumping frequency as a function of rotational speed for limestone B showing the independence of bed depth and the dependence on cylinder diameter.
smaller bed depths the slumping zone occurs at higher rotational speeds than it does at higher bed depths. Therefore, the bed changes from slumping to rolling without a rapid increase in slumping frequency. This observation is contrary to that of Reuter in his categorization of slumping frequency into three parts. 27

At low rotational speeds, a material displays the same slumping frequency in cylinders of varying diameters (Figures 4.21 and 4.22). With increasing rotational speeds, the frequencies in the larger diameter cylinder are lower than those in the smaller diameter cylinder. This effect of cylinder diameter is evident for cylinders A and C at rotational speeds above 0.3 r/min for limestone B, above 0.75 r/min for limestone C and above 0.65 r/min for gravel. This effect of diameter is not eliminated by plotting the slumping frequency as a function of the Froude number.

The following conclusions can be drawn from the effects of particle size and shape on the slumping frequency as illustrated in Figure 4.23. Firstly, materials having the same particle shape will exhibit increased slumping frequencies with decreasing average particle size. Secondly, materials having the same particle size will exhibit increased
Figure 4.22 Slumping frequency as a function of rotational speed, showing the effect of cylinder diameter.
Figure 4.23 Slumping frequency as a function of rotational speed for several materials in cylinder A (0.4 m ID x 0.46 m L) showing the effect of particle shape and size.
slumping frequencies as the particle shape becomes more nearly spherical. Therefore, the slumping frequency of a material is a function of its physical properties as suggested by Zablotny. Furthermore, the combined effect of particle size and shape are accounted for by the relative position of the slumping frequency curves. For example, note the results of gravel and limestone B and C. Also, there does appear to be a relationship between the order of slumping frequency curves in Figure 4.23 and the relative position of the slumping-rolling boundaries for the materials tested. The higher the slumping frequency curve, the further the bed behaviour boundary will be shifted to the left on the Bed-Behaviour Diagram. This criterion seems to be more consistent than that observed for the static angle of repose described in Section 4.3.5. The slumping frequency will be further discussed in Chapter 6 in connection with the mathematical modelling of bed behaviour.

4.5.2 Rolling

The rolling bed was characterized by its dynamic angle of repose and the thickness of the active bed layer. The effect of the system variables were investigated and the active layer thickness measurements of Reuter were compared to those obtained in this study.
The dynamic angle of repose was measured using the same technique described for the shear angle and the upper angle of repose. The cylinder rotational speed was varied between the slumping-rolling boundary of a material and approximately 3 r/min. The results are summarized in Table VIII and do not reflect a rotational speed or a bed depth dependency; while the effects of particle size and shape are the same as those observed for the shear angle and the static angle of repose. Combined particle shape and size effects as well as the effect of cylinder diameter on the dynamic angle of repose are also similar to those for the shear angle. The magnitude of the dynamic angle of repose was always higher than the shear angle, but less than the static angle of repose. The relative magnitude of the dynamic angles of repose of various materials does not reflect their flow properties, as shown on their respective Bed-Behaviour Diagrams.

As viewed from the plexiglass end plate, the circulation pattern of the granular solids was in agreement with that proposed by Lehmberg et al. 36 (Figure 2.5) as well as with the particle velocity profiles shown in Figure 2.6. The active layer depth was manually measured using a millimeter scale at the maximum thickness of the active layer, namely at the closest distance of the bed to the centre of
<table>
<thead>
<tr>
<th>Material</th>
<th>Cylinder Diameter (m)</th>
<th>Dynamic Angle (degrees)</th>
<th>Standard Deviation (degrees)</th>
<th>Number of Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel</td>
<td>0.40</td>
<td>37.5</td>
<td>0.4</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>1.06</td>
<td>37.0</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>Iron Oxide</td>
<td>0.40</td>
<td>35.2</td>
<td>0.3</td>
<td>3</td>
</tr>
<tr>
<td>Limestone B</td>
<td>0.40</td>
<td>39.6</td>
<td>1.3</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>1.06</td>
<td>36.5</td>
<td>1.2</td>
<td>12</td>
</tr>
<tr>
<td>Limestone C</td>
<td>0.40</td>
<td>36.0</td>
<td>0.6</td>
<td>21</td>
</tr>
<tr>
<td>Limestone D</td>
<td>0.40</td>
<td>34.9</td>
<td>0.3</td>
<td>31</td>
</tr>
<tr>
<td>Limestone F</td>
<td>1.06</td>
<td>41.5</td>
<td>0.5</td>
<td>3</td>
</tr>
<tr>
<td>Nickel Oxide</td>
<td>0.40</td>
<td>30.2</td>
<td>0.5</td>
<td>6</td>
</tr>
<tr>
<td>Sand A</td>
<td>0.40</td>
<td>33.8</td>
<td>0.9</td>
<td>5</td>
</tr>
<tr>
<td>Sand B</td>
<td>0.40</td>
<td>33.6</td>
<td>0.8</td>
<td>41</td>
</tr>
<tr>
<td>Sand C</td>
<td>0.40</td>
<td>34.0</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>
rotation of the cylinder. The results plotted as a function of the bed depth for various materials tested are shown in Figure 4.24.

The following conclusions can be drawn: firstly, at equal bed depths, the smaller the particle size of the material, the thinner is the active layer. Secondly, for a given material, the higher the rotational speed the thicker is the active layer, and thirdly, as the bed depth increases the active layer thickness of all materials also increases.

An attempt was made to clarify the effect of material variables on the active layer depth; a dimensionless plot of the maximum number of particles in the active layer versus the maximum number in the bed depth is shown in Figure 4.25. The data of Reuter\textsuperscript{27} (Figure 2.7) have also been plotted on this diagram. The effect of particle size and cylinder rotational speeds are clearly evident; with increasing rotational speeds and decreasing particle size, the number of particles in the active layer thickness increases. On the other hand, the particle shape does not seem to have an effect on the active layer thickness.

Figure 4.26 is a plot of the relative distribution of the particles between the active and passive zones. It
Figure 4.24  Active layer thickness as a function of bed depth, measured for several materials.
**Figure 4.25** Dimensionless plot of active layer thickness as a function of bed depth.

**Figure 4.26** Dimensionless plot of the relative size of the active layer to the bed depth.
is seen that as the number of particles in the bed depth increases, the number of particles in the active layer decreases until the $H/d_p$ ratio is equal to 50. Greater increases in this ratio result in the relative volume occupied by the active and passive layers becoming insensitive to changes in the particle size. The effect of rotational speed is also apparent. The higher the rotational speed, the greater the amount of particles in the active layer relative to those in the passive layer. If extended to the higher rotational speeds where cascading and cataractation occur, this trend would be found to be in agreement with the experimental observations made in these modes by other workers. 4, 53, 55, 56, 61, 120. Finally, no relationship is observed between the active layer thickness and the slumping-rolling boundary on a Bed-Behaviour Diagram.

4.6 Summary

Three modes of bed behaviour: slipping, slumping and rolling, were represented on a Bed-Behaviour Diagram which consists of a plot of bed depth versus rotational speed. It was shown that for a rotary kiln where the relationship between the bed profile, the feed rate and the rotational speed is known, then the bed behaviour could be determined from the Bed-Behaviour Diagram of the same material determined in a batch horizontal rotary cylinder of the same internal diameter.
An instrumental technique for identifying and recording bed behaviour has been developed. The range of operating conditions in which transition is observed was increased by circumferencial and end-wall effects. Using these sensors Bed-Behaviour Diagrams were determined for several materials. Particle size and shape were both found to affect the locus of the slumping-rolling boundary. Spherical shape and smaller particle sizes favoured a boundary at lower rotational speeds; while angular and coarser particles rolled at higher rotational speeds. The static angle of response of the materials proved effective in categorizing the relative position of the bed behaviour boundaries. It was most useful when coupled effects of particle size and shape were encountered. The slumping-rolling boundary consistently displayed a bed depth dependency which was not explained by any of the experimental results and will be explored later in the thesis. Furthermore, the Froude number has proved to be an insufficient scale-up criteria for bed behaviour similitude.

Several flow parameters of granular solids were measured for the slumping and rolling modes. Slumping was characterized using the upper angle of repose, the shear angle and the slumping frequency. The dynamic angle of
repose and the active layer thickness were measured for rolling beds.

The upper angle of repose increased with increasing rotational speed. However, the measuring technique used was not accurate enough to reliably detect this increase. The shear angle and the dynamic angle of repose were shown to be independent of the rotational speed and the bed depths of the cylinder over the range tested. They were, however, dependent on the cylinder diameter. Larger angles were obtained for smaller diameter cylinders. When comparing both angles in the same cylinder, though, the effects of particle size and shape were observed. The larger the particle size and the greater the deviation from spherical shape, the greater the shear angle of the material. The dynamic angle of repose was always higher than the shear angle. A slumping bed was characterized by its slumping frequency which increased with rotational speed and was independent of the bed depth. A decrease in cylinder diameter, average particle size or a change in particle shape from angular to spherical resulted in an increase in slumping frequency at a given rotational speed. The relative order of the slumping frequencies of a group of materials tested in a cylinder is the same as the relative position of the slumping-rolling boundaries of these materials on a
Bed-Behaviour Diagram. The scale-up criteria of the Froude number did not apply for the slumping frequencies of varying cylinder diameters.

Finally, the active layer thickness was shown to be independent of particle shape, but dependent on particle size at ratios of $H/d_p$ less than 50. An increase in rotational speed resulted in an increase in the active layer thickness.

The characteristics of slumping and rolling have thus been quantified. The basic cause of the bed motion changing from slumping to rolling will be pursued in Chapters 6 and 7 where a semi-empirical mathematical model of bed motion will be presented.
Chapter 5

SEGREGATION

5.1 Introduction

An analysis of the effect of material properties on the flow of granular solids would not be complete without investigating the role played by the particle size distribution. Previous work has shown that materials having a size distribution will de-mix (or segregate) when subjected to flow. That is, the composition of the material will not remain uniform throughout its total volume. This state of segregation can be affected by differences in material density and shape, and can seriously hamper industrial operations.

In rotary kilns, for example; the residence time of coarser sized or less dense particles has been reported to be less than the residence time of the finer sized or the denser particles. 12,32,44,45,118,121,122 This difference in residence time, can result in the production of an unacceptable product. 32,123 For example, in the direct reduction of iron ore using a rotary kiln, coal has been seen to segregate to the surface of the bed, leaving the iron oxide in the core. 27,57,124 This results in either
unreduced iron discharging from the kiln or an excessively high coal consumption. A similar problem can arise in limestone calcination due to size rather than density segregation. Smaller particle sizes have been observed to require a longer time to calcine than the coarser fraction.\(^\text{47,125}\)

Finally, industrial observations on lime rotary kilns have revealed that slipping has been associated with the presence of significant quantities of fines.\(^\text{47}\)

These findings have provided the impetus for research in segregation. In the following section a brief presentation will be made outlining the current state of knowledge in this area. Those areas requiring greater attention will then be more closely examined to clearly identify segregation patterns and mechanisms in rotary cylinders.

The experimental techniques, already developed and presented in chapter 4, will be applied in this study and the effects of segregation on the slumping-rolling boundary in a Bed-Behaviour Diagram will be studied. Changes in the static angle of repose, the shear angle, dynamic angle of repose, slumping frequency and the active layer thickness will also be reported. A theoretical and an experimental analysis of the segregated core in the solids bed will
follow, and will clearly identify the segregation mechanism. In this study, more emphasis will be placed on size segregation as it is the most common. However, the principles derived from this current analysis may also be applied to other systems where density and shape differences or coupled effects are encountered. The industrial process implications of these findings will also be discussed.

5.2 Previous Work

In this section, only those investigations on the segregation of bulk solids carried out primarily in rotary cylinders will be discussed.

5.2.1 Types of Segregation

Three types of segregation patterns have been observed in horizontal rotary devices. A schematic representation of these is illustrated in Figure 5.1. Broadly they can be categorized as radial and axial or longitudinal segregation zones. Axial segregation may in turn be subdivided into banding and end-longitudinal. In radial segregation, smaller or denser particles are observed to form a horizontal core in the bed, whereas in banding segregation, alternate bands of coarse and fine or light and heavy particles form down the length.
Figure 5.1 Types of segregation encountered in horizontal rotary cylinders. 

- (i) Radial
- (ii) Banding longitudinal
- (iii) End-longitudinal

- Coarser or lighter solids
- Finer or denser solids

\[59,67\]
of the rotary cylinder. \(^{59,66,67,72}\) Finally, the end-
longitudinal segregation is such that the finer or denser particles form two end-bands at the cylinder walls with the coarse or lighter particles in the centre of the cylinder axis. \(^{37,59,66,67,129}\)

The most comprehensive analysis of these types of segregation has been made by Donald and Roseman. \(^{67,130,131}\) Their findings showed that radial segregation always occurs while the type of axial segregation, if it occurs, may be predicted, using the static angles of repose of the components of the mixture. When the smaller sized particles have a static angle of repose greater than the coarse, the banding type of axial segregation results. The end-
longitudinal segregation occurs when the static angle of repose of the smaller particles is less than that of the coarser particles. These conclusions were based on an experimental program involving over 30 binary mixtures which were tested for a rolling bed in horizontally rotating cylinders.

5.2.2 Segregation Kinetics

Segregation like mixing is a first order process. Radial segregation always occurs much faster than axial segregation. \(^{66,130}\) Rogers and Clements\(^ {66}\) show radial
segregation to be well established following 10 cylinder revolutions. Reuter\(^{27}\) reports it to be complete after only 2 revolutions. These results have been recently confirmed in tests carried out on continuous rotary kilns.\(^{132}\) Axial segregation, on the other hand, only reaches steady state after 500 to 10,000 revolutions.\(^{37,66,129,130}\) All investigations were carried out with the bed rolling or cascading.

Reuter has successfully described the radial segregation process by a first order rate equation.\(^{27}\) However, no attempt has yet been made to mathematically describe the kinetics of axial segregation.

### 5.2.3 Mechanisms of Segregation

There are three mechanisms by which granular materials segregate: percolation, flow and vibration.\(^{67,90,130,133}\)

Percolation segregation is favoured when the size of the voids between coarser particles is large enough to allow smaller particles to pass through them. It occurs in the region of an inclined shear plane. The smaller particles in motion can have their trajectory abruptly changed by falling into a void on the stationary side of the shear plane. This type of segregation is therefore a function of the packing characteristics of granular solids,
the particle shape and the particle size.

Segregation by flow also occurs when granular solids are set in motion over an inclined surface. It arises from coarse particles travelling further than smaller particles down an inclined surface. Unfortunately, there are very few experimental results reported in the literature to illustrate this mechanism. Those available have been carried out by pouring binary mixtures into heaps. This does not provide conclusive evidence however, since segregation by precolation would also occur. Nevertheless, based on the slumping frequency results shown in Figures 4.20 to 4.23 it can be confirmed that there are differences between the flow characteristics of coarse and fine, spherical and angular particles. Furthermore it was observed that slumps of the smaller sized materials, initiating at the apex of the bed, came to rest on the bed surface at some distance from the chordal base. On the other hand, slumps from coarse sized materials reached the chordal base of the bed, and in some cases certain particles even impacted the cylinder wall. Hence, segregation by flow may occur in rotary cylinders.

Segregation by vibration is the third mechanism. If a container is filled with a mixture of particle sizes and
vibrated, it would be observed that the larger particles would segregate to the top of the container and the smaller ones to the bottom. Since in rotary kilns, no vibration is imparted to the system, segregation by vibration is unlikely and will not be discussed further.

5.2.4 Effect of Variables on Segregation

Segregation in rotary cylinders is enhanced by large differences in particle size, shape and density.\textsuperscript{27,52,57,66,124,126-130} The effects of these material variables should be reflected in the flow properties and packing characteristics of granular solids in rotary cylinders. For example, for any given material the slumping frequency and angle of repose are believed to change with the addition of finer sized solids.\textsuperscript{25,110} However, no conclusive experimental evidence of this has been found. Observations on the flow of granular solids from bins and hoppers indicate that the addition of fines to a coarser grade of material does affect the flow characteristics.\textsuperscript{134} by enhancing or abating the discharge of the solids. Similar effects have not yet been reported for solids flow in rotary cylinders.

No rigorous study of the packing characteristics of the segregated core (commonly called the 'kidney' in
industry) in a rotary cylinder has been carried out. Some visual observations have been reported, although many of them are contradictory. Most investigators believe that the composition of the segregated core is made-up of the finer, or denser material, surrounded by the coarser, or lighter particles. However, the experimental observations of several investigators indicate that this core is comprised of both fine and coarse, dense and light particles.

In conclusion, it is clear that more experimental measurements on segregated materials in rotary cylinders are required. Based on current knowledge, the mechanisms of segregation in these devices cannot be fully elucidated.

5.3 Material Preparation

Four granular solids mixtures were used in this investigation, two were prepared from sands B and C and two from limestone B and E. Their compositions are listed in Table IX. Seventy mesh and 20 mesh sieves were used to measure the fines content in the sand and limestone mixtures respectively. Both identify the fines end of the size distribution for sand B and limestone B. Sand B contained 1.1% - 70 mesh and limestone B, 0.7% - 20 mesh.
<table>
<thead>
<tr>
<th>Mixture</th>
<th>Coarser Component</th>
<th>Weight % of Coarser Component</th>
<th>Finer Component</th>
<th>Weight % of Finer Component</th>
<th>Overall Mixture Composition</th>
<th>$d_p$ coarse</th>
<th>$d_p$ fine</th>
</tr>
</thead>
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<tr>
<td>Sand:</td>
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<td></td>
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<tr>
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<td>Sand C</td>
<td>27.4</td>
<td>16.7% - 70 mesh</td>
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<td></td>
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<td>78.8</td>
<td>Limestone E</td>
<td>21.2</td>
<td>19.6% - 20 mesh</td>
<td>8.0</td>
<td></td>
</tr>
</tbody>
</table>
The following method of material preparation was followed for the first stage of the study. A sufficient quantity of material was weighed and mixed to fill 20% of the volume of cylinders A (0.4 m ID x 0.4 m L) and B (0.4 m ID x 0.86 m L). Subsequently, the total prepared mixture was placed in the appropriate cylinder and the required observations and data were recorded for that bed depth. After the 20% fill run, further removal of solids was made by manually mixing the cylinder contents and randomly removing a certain portion of the mixture, from different locations in the cylinder.

For the investigation of the composition and size of the segregated core, the second stage of the study, only one bed depth was tested in both cylinders A and B. The mixture was reused for the slumping and rolling experiments performed in the same cylinder.

5.4 Segregation and Modes of Bed Behaviour

It has already been shown in Section 5.2 that segregation due to size differences occurs in the solids bed. It is not known, however, whether this segregation will affect the position of the slumping-rolling boundary on a Bed-Behaviour Diagram. Using the sensors, Bed-Behaviour Diagrams were therefore experimentally determined for sand
mix A and B and for limestone mix A. The results are shown in Figures 5.2 to 5.4. and the run numbers corresponding to each of the bed depths tested are listed in Table B.2 (Appendix B). In these tests the solids bed was manually mixed at regular intervals and each bed behaviour observation was made over a relatively short period of time (from 5-30 minutes). Therefore, on the basis of the kinetics of segregation, the measured bed behaviour should only be affected by radial segregation. That radial segregation had occurred was confirmed by manually displacing layers of particles from the bed surface. The particles on the surface were always the coarsest components of the mixture and the finer components were seen only after several particle layers had been removed.

For the sand mixtures, no appreciable effect of fines was apparent with mix A which contained 3.6% - 70 mesh (Figure 5.2), although the boundaries did shift slightly to the left. At a fines content of 16.7% - 70 mesh, a significant shift in the bed behaviour boundaries resulted (Figure 5.3). Since the slumping-transition boundary of sand mix B (16.7% - 70 mesh) occurred at under 0.1 r/min, it was not recorded; but clearly transition-rolling boundary had also shifted to lower rotational speeds. Note that all boundaries still show a bed depth dependency.
Figure 5.2 Bed-Behaviour Diagram for sand mix A (3.6%-70 mesh) in cylinder A (0.4 m ID x 0.46 m L) experimentally determined and compared to the Bed-Behaviour Diagram of sand B (1.1%-70 mesh).

Figure 5.3 The transition-rolling boundary for sand mix B (16.7%-70 mesh) in cylinder A (0.4 m ID x 0.46 m L) experimentally determined and compared to those for sand B (1.1%-70 mesh) and sand mix A (3.6%-70 mesh).
Figure 5.4  Bed-Behaviour Diagram of limestone mix A (8.7%-20 mesh) in cylinder A (0.4 m ID x 0.46 m L) experimentally determined and compared to the Bed-Behaviour Diagram of limestone B (0.7%-20 mesh).
Quite a different result was obtained for the limestone mix A which contained 8.7% - 20 mesh. Here, the bed behaviour boundaries have significantly shifted to the right. The bed depth dependency has also been greatly enhanced. In conclusion, fines do affect the bed behaviour of granular materials and the Bed-Behaviour Diagram adequately reflects it. However, the cause of the shift in the slumping-rolling boundary is not clear at this point.

5.5 Segregation and Bed-Behaviour Characterization

The effect of fines on the characterization variables of slumping and rolling will now be investigated and will show whether flow segregation is one of the mechanisms responsible for changes in bed behaviour. The variables measured are the static and dynamic angles of repose, the shear angle, the slumping frequency and the active layer thickness.

5.5.1 Slumping Characteristics

The static angles of repose, measured in the rotating cylinder and the shear angles of the components and mixes used in this study are listed in Table X. The static angle of repose was not affected by the fines; on the other hand, a slight trend is apparent in the shear


<table>
<thead>
<tr>
<th>Type</th>
<th>Composition</th>
<th>Static Angle of Repose (degrees)</th>
<th>Standard Deviation (degrees)</th>
<th>Number of Observations</th>
<th>Shear Angle (degrees)</th>
<th>Standard Deviation (degrees)</th>
<th>Number of Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand:</td>
<td>1.1</td>
<td>34.2</td>
<td>0.6</td>
<td>7</td>
<td>32.2</td>
<td>0.5</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>Mix A</td>
<td>3.6</td>
<td>0.7</td>
<td>6</td>
<td>32.8</td>
<td>0.3</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Mix B</td>
<td>16.7</td>
<td>0.3</td>
<td>5</td>
<td>33.0</td>
<td>0.5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>61.4</td>
<td>-</td>
<td>1</td>
<td>33.0</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>Limestone:</td>
<td>% -20 mesh</td>
<td>B</td>
<td>42.0</td>
<td>1.1</td>
<td>37.7</td>
<td>1.1</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>Mix A</td>
<td>8.7</td>
<td>0.8</td>
<td>23</td>
<td>36.9</td>
<td>0.7</td>
<td>23</td>
</tr>
</tbody>
</table>

* - measured from cylinder A (0.4 mID x 0.46 mL)
angle. With increasing fines content, the shear angle seems to increase for the sand and decrease for the limestone. This trend though, is not statistically significant.

The slumping frequency results are presented in Figures 5.5 and 5.6 and show that the slumping frequencies of the mixes were the same as those of their respective coarse components. Attempts to measure the slumping frequency of the finer components of the sand and limestone mixes proved unsuccessful. Both sand C and limestone E changed from slumping to rolling at such low rotational speeds, that sufficient data over a wide enough range of speeds could not be obtained. It can be deduced however, from the results in Section 4.5.1, that the slumping frequencies would have been higher than those for their respective components.

The measurements of the static angles, shear angles and the slumping frequencies indicate that for the systems tested, the flow characteristics of slumping beds were not significantly affected by the presence of fines. The flow mechanism, therefore, did not contribute to the segregation of these mixes when slumping.
**Figure 5.5** Slumping frequency versus rotational speed for sand B (1.1%-70 mesh) and sand mix A (3.6%-70 mesh) and B (16.7%-70 mesh) in cylinder A (0.4 m ID x 0.46 m L).
Figure 5.6 Slumping frequency versus rotational speed for limestone B (0.7%-20 mesh) and limestone mix A (8.7%-20 mesh) in cylinder A (0.4 m ID x 0.46 m L).
5.5.2 Rolling Characterization

The dynamic angle of repose and the active layer thickness were measured for the mixes under study for a rolling bed. It is seen from the results listed in Table XI that the dynamic angle of repose was not affected by the fines. The active layer thickness measurements shown in Figures 5.7 and 5.8 for the sands and limestones also show no indication of the presence of fines. These results were then made non-dimensional with respect to the coarse particle size, and the results compared with those presented in Figures 4.25 and 4.26 (Figures 5.9 and 5.10). Again, the presence of fines is not detected. It is therefore clear that for the mixes used, the flow mechanism was responsible for neither the change in bed behaviour reported in Section 5.4 nor for the resulting radial segregation.

5.6 The Segregation Cores

There remains, therefore, only one mechanism by which segregation could have occurred: the percolation mechanism. Since none of the investigators citing the presence of the segregation core have explained the process by which segregation occurred, an attempt will be made to describe it in this section. Prior to that discussion, however, it is important to understand the final configuration and composition of the
TABLE XI  Effect of Fines on the Dynamic Angle of Repose for the Mixture Tested

<table>
<thead>
<tr>
<th>Type</th>
<th>Composition</th>
<th>Dynamic Angle of Repose (degrees)</th>
<th>Standard Deviation (degrees)</th>
<th>Number of Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>%-70 mesh</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1.1</td>
<td>33.6</td>
<td>0.8</td>
<td>41</td>
</tr>
<tr>
<td>Mix A</td>
<td>3.6</td>
<td>33.6</td>
<td>0.4</td>
<td>16</td>
</tr>
<tr>
<td>Mix B</td>
<td>16.7</td>
<td>33.8</td>
<td>0.3</td>
<td>12</td>
</tr>
<tr>
<td>C</td>
<td>61.4</td>
<td>34.0</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>Limestone:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>%-20 mesh</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.7</td>
<td>39.6</td>
<td>1.3</td>
<td>19</td>
</tr>
<tr>
<td>Mix A</td>
<td>8.7</td>
<td>39.2</td>
<td>1.0</td>
<td>10</td>
</tr>
</tbody>
</table>
Figure 5.7  Active layer thickness versus bed depth for sand B (1.1%-70 mesh) and sand mix A (3.6%-70 mesh) and B (16.7%-70 mesh) in cylinder A (0.4 m ID x 0.46 m L).

Figure 5.8  Active layer thickness versus bed depth for limestone B (0.7%-20 mesh) and limestone mix A (8.7%-20 mesh) in cylinder A (0.4 m ID x 0.46 m L).
Figure 5.9 Dimensionless Plot of active layer thickness versus bed depth. Comparison of segregation results with those from Section 4.5.2.

Figure 5.10 Relative thickness of active layer with respect to the dimensionless bed depth. Comparison of segregation results with those from Section 4.5.2.
core, as it could well shed light on this segregation process.

The first step in this analysis is to determine whether the first criterion of the percolation mechanism is met by the materials used in this study. This criterion states that the voids between the coarser particles must be of a larger size than the finer particles. Donald and Roseman\textsuperscript{67,130} have claimed that if the average coarse-to-fine size ratio is greater than 1.2, radial segregation will occur since the size of voids will be greater than that of the fines. The experimental results of Reuter also verify this finding.\textsuperscript{27}

As shown in Table IX, both sets of components used in this study had size ratios greater than 1.2. That the segregation criterion of void size has been met was further confirmed by the void fraction versus composition plot of Figure 3.13. Note that the limestone mixture has a larger coarse to fine size ratio than the sand mixture and it also experiences a greater maximum bed contraction, $\Delta \varepsilon_{\text{max}}$. Hence, the first criterion of the percolation mechanism is satisfied.

The second stage of this analysis would be to determine the location and composition of the segregated core. For this part of the study, limestones B and E were chosen as they displayed the higher component size ratio, as well as the largest $\Delta \varepsilon_{\text{max}}$. Hence, they show the greater segregation tendency. The most direct method of locating the core
and identifying its composition is by careful removal or sampling of solid granules both in the bed depth and along the bed surface. For the system used, however, the removal of layers of solids in the bed depth direction implies that each sample layer collected would only be a few coarse sized particles deep. The results would not be meaningful. Instead, portions of the bed were removed from its surface to the inner cylinder wall and analyzed for the fines component. A theoretical model was also independently developed to calculate the dimensions of the core. The model predictions were then compared with the experimental results.

5.6.1 The Segregated Core Model

To develop the model of the segregated core, its shape must first be assumed. Several configurations are suggested by Vaillant, ranging from the elliptical and circular to the rectangular. None of these, though, are in agreement with the circulation patterns of the solids discussed in Sections 2.2.3 and 4.5.2. The core is expected to therefore have a shape similar to that of the solids bed but recessed from the bed surface as shown in Figure 5.11a (recall that fines were not visible on the bed surface in the Bed-Behaviour Diagram determination). Thus the core will be symmetrical about the perpendicular OA (Figure 5.11a).
Figure 5.11  Schematic representation of the central segregation core
(a) segregation core recessed from the surface of the bed
(b) surface of core at the bed surface.
size of the core will of course be dependent on its composition and on the total fines content.

Assuming that all the fines are segregated in the core and that longitudinal segregation is negligible, the weight fraction of the finer component in the core is given by:

\[ 1 - y = \frac{xW_T}{W_K} \]  \hspace{1cm} (5.1)

The volume \( V_K \), occupied by the core is therefore:

\[ V_K = \frac{xW_T}{(1-y)\rho_B K} \]  \hspace{1cm} (5.2)

Since, in the experimental phase of this study, the solids bed was not sampled radially, the distance the core is recessed from the bed surface cannot be verified. Only its width can therefore be compared to the experimental results. For the purposes of the model calculations, the surface of the core was considered to be at the surface of the bed in order to simplify the derived formulae (Figure 5.11b). This does not appreciably affect the magnitudes of the calculated width of the core volume. The radius of the outer core boundary, \( R_K \), is therefore given by:

\[ R_K = \left\{ (R-H)^2 + \left[ \frac{W_K}{2} \right]^2 \right\}^{1/2} \]  \hspace{1cm} (5.3)

and the core volume is then calculated as follows:
Finally, the depth of the core, $H_K$, may be obtained from:

$$H_K = R_K - (R - H)$$  \hspace{1cm} (5.5)

Equations (5.2) to (5.4) are now solved by trial and error for $w_K$. Given a core composition, its corresponding bulk density may be measured as described in Section 3.2.3. The fines content is given by the mixture composition and the total weight, both of which are measured. Using Equations (5.3) and (5.4), an appropriate value of $w_K$ is chosen to yield a $V_K$ equal to that calculated from Equation (5.2).

Two core compositions were assumed for the model predictions. For the first, the core was considered to consist of only the finer component of the limestone mix B. For the second, it was assumed that the core composition was equal to that for which maximum bed contraction, $\Delta \varepsilon_{\text{max}}$, occurred in Figure 3.13. The core width predictions from both these cases were compared to the experimental results obtained and described in the following section.

5.6.2 Experimental Procedure, Sampling and Analysis

The segregation tests were carried out at only
one bed depth, 0.08 m or 14% degree of fill, using limestone mix B (19.6% - 20 mesh) in both cylinders A and B (0.4 m ID x 0.46 m L and 0.86 m L respectively). The required weights of each of the mixture components, limestones B and E, were placed in the cylinders and manually mixed. To test the segregation pattern for slumping, and rolling, two rotational speeds were chosen for the trials, 0.6 and 3.1 r/min. The duration of each run was at least 4 hours. Longitudinal segregation was observed to occur for both the slumping and rolling beds and was visually determined to be complete within the 4 hours of cylinder rotation.

Following the completion of a run, material had to be sampled for analysis. To accomplish this, several techniques reported in the literature were attempted but were not found adaptable to the current system. A different approach was therefore developed. Four steel plates each with a measuring grid, glued in place, were cut to the shape of the inside circumference of the rotating cylinder (Figure 5.12) and were suspended above the solids bed inside the cylinder. At the end of the run, these plates were lowered into the solids bed by holding them against the cylinder wall opposite the solids bed and rotating the cylinder through half a revolution. Thus, the whole length of the bed was divided axially into five slices. The middle
Figure 5.12  Slicer made of 1.6 mm thick steel sheet covered with poster board.

Figure 5.13  Slicers introduced into the charge, in preparation for solids removal.
three slices, in cylinder A, were sampled, each having 76 mm thickness (Figure 5.13). In cylinder B, their thicknesses were increased to 144 mm in order to equal the same thickness to cylinder length ratio as in cylinder A.

Rather than removing small fractions of solids from several locations of the bed and analyzing the results statistically, as has been commonly done by other investigators, the total contents of each of the middle three slices were removed for analysis. The former method would have required more time to implement and to analyze the data. Furthermore, using the latter approach, the analysis would be free of any of the statistical uncertainties usually associated with sampling techniques. The solids bed was therefore left in its inclined position and the solids in the bed were removed in several portions starting at the apex and working down to the chordal base. Each portion sampled represented the solids from a given increment of the bed surface. They were collected using a vacuum cleaner equipped with a cloth bag starting at the bed surface and working towards the circumferential wall. This allowed the selective removal of the bulk solids. The collected portions of the bed were analyzed for their - 20 mesh content. The screen analysis results are reported as
a weight per cent - 20 mesh with respect to the total weight of the bed portion screened.

The location of each of these portions had also to be identified. By means of both a mirror and the grids on the steel plate slicers, the co-ordinates of the solids left in the cylinder were recorded. These were then re-plotted on graph paper using a 1:1 scale. Firstly, the area of the bed was measured using a Compensating Planimeter (Koizumi Type KP-27), and the bed depth of the slice calculated by trial and error using the following equation:

$$A = R^2 \cos^{-1}\left(-\frac{R-H}{R}\right) - (R-H)(2RH-H^2)^{\frac{1}{2}}$$  \hspace{1cm} (5.6)

The distance from the apex of each collected portion was also measured from the graphed grid plot, and was reported as a percentage of the total chord length, for ease in comparing the results from various trials. The experimental results are presented for each slice on a bar graph as shown in Figures 5.14 to 5.17.

5.6.3 Discussion

5.6.3.1 Radial Segregation

From the experimental results shown in Figures 5.14 to 5.17, it is seen that radial segregation
Figure 5.14 Experimental results and core width model predictions for slumping in cylinder A (0.4 m ID x 0.46 m L).
Figure 5.15 Experimental results and core width model predictions for rolling in cylinder A (0.4 m ID x 0.46 m L).
Figure 5.16 Experimental results and core width model predictions for slumping in cylinder B (0.4 m ID x 0.86 m L).
Figure 5.17 Experimental results and core width model predictions for rolling in cylinder B (0.4 m ID x 0.86 m L).
occurred for both slumping and rolling beds. Since longitudinal segregation also occurred, the core width model was applied to each slice rather than to the total cylinder length. The predictions of the core width model for each of the slices are also shown in Figures 5.14 to 5.17. It is quite evident that the model based on a core composition of only fines does not show good agreement with the experimental results. Irrespective of bed behaviour, slumping or rolling, as well as cylinder length to diameter ratio, coarse particles must form a part of the core in its steady state configuration. Excellent agreement with the experimental results was obtained for all the predicted core widths using the $\Delta \varepsilon_{\text{max}}$ composition. This implies that the solids always took on a configuration that radially minimized the total volume of the system.

5.6.3.1 Validation of Assumptions and the Second Core

Two of the assumptions of the core width model are that all fines are associated with the core and that it lies symmetrically about the perpendicular to the bed surface from the centre of rotation of the cylinder. The experimental results reveal that these two assumptions are not strictly adhered to in the system. Regarding the first assumption, a more detailed analysis of the results reveals
that the percent of -20 mesh in a slice lying outside the predicted core ranges from 3 to 13% in the apex and from 0.9 to 3.0% at the chordal base. This is because the bulk of the bed volume lies between 30% and 70% of the chord length. Hence, the first assumption is valid.

The second assumption, however, that of symmetry, presents a more interesting problem. The lack of symmetry is clearly apparent in all slices, regardless of bed behaviour and cylinder length. The core model had implied that the fines are surrounded by the coarser particles and hence should have no contact with the cylinder wall. During the experimental trials, though, it was observed that the abrasive lining in the cylinder became blinded with fines shortly after the start of runs where care had been taken to clean the wall. Therefore, there must be a second concentration of fines at the top of the bed near the wall. This was further corroborated by Sunnergren based on his observations of a similar system and using similar sampling techniques (Figure 5.18). It is believed that this second core consists of the finest particles which do not move down the bed surface when they reach the apex. They are so small that when set in motion, they are able to percolate through the coarser sizes even when the latter are stationary. These fine particles finally come to rest at the cylinder wall where they accumulate, forming this second segregation core.
Figure 5.18  Photograph of second segregation core at the top of the bed adjacent to the cylinder wall[^1].
The fine particles in this core could easily fill wall imperfections and subsequently be compacted into place by the weight of the bed. In rotary kiln operations, this would result in a significant smoothening of the wall that could then result in a change in bed behaviour occurring from slumping or rolling to slipping. This phenomenon was observed in cylinder A when determining a Bed-Behaviour Diagram for iron-oxide pellets. Since their static angle of repose and angle of slipping were 31.5° and 38.9° respectively, no slipping had been expected. However slipping occurred partway through the test, when the walls had been smoothened and covered by the very fine iron oxide particles. Hence, the formation of this second segregation core seems to be one of the factors responsible for the onset of slipping which occurs in industrial kilns.

5.6.3.1-B The Percolation Segregation Process

The percolation segregation process will be presented in this section with reference to the rolling mode rather than the slumping mode, both for the sake of brevity and because it will be easier to describe. The discussion, though, applies equally to both.

The path of the fines will be described for an initially well mixed solids bed composed of a coarse and a
due to the rotation of the cylinder fine particles in position 1 (Figure 5.19) will follow a trajectory with the cylinder wall in the passive region until it crosses the shear plane AB and enters the active region at 2. Initially, they would travel down the surface bed inclination but would soon find as the active layer increases in thickness that there were coarser particles in motion below it. When the voids between these coarser particles are larger than the fine particle size, the latter would fall into these voids. Furthermore, if these fine particles are sufficiently fine, they could easily fall past the active layer and trickle through the voids until they reach the cylinder wall, position 3. This would result in the formation of the segregation zone of very fine particles as earlier described. On the other hand, if the fine particles are not able to percolate through the shear plane AB, they must remain in the active region and will seek the first opportunity to fall into the passive zone. These fine particles will therefore travel adjacent to the shear plane, between coarse particles, until line OB is crossed. Here particles may begin to re-enter the passive zone. The first one to do so, of course, are those particles, fines and coarse, travelling just above the shear plane. Once they have entered the passive zone, they must travel with the
Figure 5.19  Schematic of the percolation segregation process for both cores.
cylinder wall and reappear in the active layer equidistant from their previous point of entry into the passive region.

Subsequent fines crossing OB and travelling near the slip plane would find the zone around B saturated with fines according to the $\Delta \varepsilon_{\text{max}}$ composition, and would be forced to travel further down the slip plane BC until the next favourable site is encountered. As the fines content increases therefore, it is clear that the central core will increase in size. The fine particles will therefore not be visible at any time on the bed surface except at very high fines content which agrees with the experimental findings. Fines were only visible on the bed surface when their content in a given slice was between 40 and 50% by weight. It therefore follows that, at lower fines content, the fines active layer, will be smaller than the total active layer, and the core will be recessed from the bed surface.

In industrial kilns those fines associated with the central core will not be exposed to gas in the freeboard. They will therefore be colder than the coarse particles and may reach the discharge of the kiln only partly reacted. The fines associated with the second segregation core may receive sufficient heat from their contact with the wall. However, high local wall temperatures could result in these fines sticking to the wall and enhancing accretion build-up, or glazing the wall enhancing slipping. These temperature
effects are, of course, most likely to occur at the solids discharge. Other operating effects of segregation have already been discussed in the previous section.

To overcome these many problems, recent industrial research has suggested the introduction of specially designed protrusions to continually expose the fines of both segregated zones to the gas freeboard. The results of the industrial trials are reported to be very favourable.

5.6.3.2 Longitudinal Segregation

From the results obtained from cylinder A (Figure 5.14), it appears that when the bed is slumping, fines tend to segregate in the centre of the cylinder axis. For a rolling bed (Figure 5.15), the fines segregated to the cylinder end walls which is in agreement with the investigations of other workers described earlier. The tests of slumping and rolling beds were repeated under identical conditions in cylinder B having a length to diameter ratio of 2:1. Similar segregation patterns resulted with areas of longitudinal fines segregation being visible on the surface. To recheck the results obtained, the bed was often manually remixed in the cylinder and the test restarted. The resulting segregation differed only in the case of a rolling bed. Here, fines were seen to segregate either to both end-walls
of the cylinder or to either end-wall. No reason was found for this behaviour. A final trial was made in which a well mixed bed was allowed to slump at 0.6 r/min for a minimum of four hours. When the fines had segregated to the centre of the cylinder axis, the rotational speed was increased to 3 r/min and left for at least another four hours. The fines then moved to either or both end walls. The rotational speed was again reduced to 0.6 r/min and the fines were seen to move back to the centre of the cylinder axis. The bed was remixed and the sequence of trials was repeated; this time starting with a rolling bed. The same results were observed. The slumping frequency was measured at various times during the slumping tests and was not found to vary over the testing period.

According to the analysis of Donald and Roseman, the static angles of repose of limestones B and E (Table II) indicate that, for a rolling bed, end-longitudinal segregation should have taken place; as was indeed the case. They provided the following explanation for its occurrence. The radial velocity of particles falling down the bed inclination varies along the cylinder length with a maximum occurring some short distance away from the cylinder end-wall. A fine particle in a band adjacent to this zone of maximum velocity would see more voids passing it on one side
than on the other. Hence, the fine particle would be favoured to move axially to the zone of maximum velocity. Since the static angle of repose of the fines is less than that for the coarse, the latter was believed to be radially more mobile; therefore the fines would only be able to form end-wall bands.

It was observed that when slumping, the fines axially segregated to the centre of the cylinder axis. Therefore it appears, following the argument of Donald and Roseman, that for a slumping bed the radial velocity at the centre of the axis in a horizontally rotating cylinder is a maximum and that the radial velocity profile along a cylinder axis is dependent on bed behaviour. This was not confirmed, however, using the slumping frequency measurements along the cylinder axis.

In an inclined cylinder, it would be expected that the fines accumulation would occur at the lower end of the cylinder irrespective of bed behaviour. The problem of axial segregation reported in this study may therefore not directly apply to industrial rotary kiln practice. It would be of interest, however, in industries concerned with the mixing of granular materials, where it is important to obtain a product of consistent composition, one that will
show minimum segregation tendencies upon subsequent handling. Many of these materials are mixed in batch horizontal rotary cylinders. Based on the results of this analysis, an operating strategy could be developed for these applications.

If a given mixture of solids is placed in a rotary cylinder and is allowed to segregate radially and axially, the resulting longitudinally segregated zone would have the composition corresponding to that mixture's $\Delta \varepsilon_{\text{max}}$. If it can be separated from the remaining contents of the cylinder, it can be handled without further segregation. It would also be the densest volume of solids attainable for that mixture; a property much sought after in pharmaceutical, powder metallurgical and ceramic processing, for example. The only processing difficulty envisaged with this strategy lies in the recycling of the remaining contents of the mixer. In some instances, this could be very costly. The advantages and disadvantages of this approach must therefore be analyzed for each specific application in the context of local plant economics.

5.7 Summary

The effect of the particle size distribution on the flow properties of granular solids has been investigated. Two types of segregation that occur in horizontal rotary
cylinders were illustrated: radial, end-longitudinal and band longitudinal.

Bed-Behaviour Diagrams, slumping and rolling characteristics were determined for two sand and one limestone mixtures. While the fines resulted in significant changes in bed behaviour, no effects were observed in either static or dynamic angles of repose, shear angles, slumping frequencies or active layer thicknesses. It was therefore concluded that the flow mechanism of segregation did not affect the systems studied. Hence, the visually observed radial segregation must have occurred by means of the percolation mechanism.

A theoretical model was developed to predict the width of the central segregated core, the 'kidney', which was assumed to have a shape similar to that of the solids bed. Its dimensions were dependent on its composition and on the total fines content. The predicted core widths were shown to agree with the experimental results when the core composition was taken to be equal to that for the $\Delta e_{\text{max}}$ relationship.

The rotary cylinder bed was sampled from the apex to the chordal base. The experimental results indicated the
existence of a second segregation zone at the top of the bed. This was confirmed when these results were compared to those obtained recently by Sunnergren on a similar system using a similar sampling technique. Using the experimental and theoretical results of this chapter as well as the solids circulation model for a rolling bed presented in Sections 2.2.3 and 4.5.2, the percolation segregation mechanism process for both segregation zones was also described.

Industrial implications of segregation include poor reactivity of the particles in the central core, due to the coarse particles shielding them from the hot gases in the freeboard. The second segregation zone is believed to be responsible for the onset of slipping from slumping or rolling beds, which is detrimental to rotary kiln operation.
6.1 Introduction

A detailed analysis of the slumping process will be presented based on an experimental and a statistical analysis using photographic and instrumental techniques. The data will be statistically analysed and will be used in the derivation of the mathematical model of the slumping-rolling boundary to be described in this chapter. Slipping, cascading and cataractating will also be mathematically modelled. The dimensionless form of the equations will be used to reveal those variables affecting the various boundaries of bed behaviour on a Bed-Behaviour Diagram.

6.2 The Slumping-Rolling Boundary

The mathematical model of the slumping-rolling boundary is based on an analysis of a slumping bed. An attempt will be made to describe the model in terms of the three general principles of bulk solids flow: mobilization of friction, dilatancy and minimum energy. The first two of these will be discussed in the following section; while the minimum energy principle will be considered in
Section 6.2.2. The criterion of the slumping-rolling boundary will then be developed.

6.2.1 Mobilization of Friction and Dilatancy

The mobilization of friction is concerned with the magnitude of the friction force between the granules which could vary between zero and some limiting value. Due to the nature of bulk solids, the internal stress distribution is not uniform in a bed and is indeterminate until the limiting condition of friction is attained when a shear plane is formed. The bulk solids lying above the shear plane will be termed the shear wedge. Prior to this condition, the process of dilatancy takes place wherein bulk solids increase or decrease the volume they occupy. In analyzing a slump, in terms of these two principles, both the position of the shear plane and the size of the shear wedge must be identified. Since the state of knowledge of the flow of bulk solids, however, is not well enough advanced to predict mechanistically the onset of slumping, an empirical approach was therefore adopted.

6.2.1.1 The Slumping Process

The first stage in the development of this empirical model was to examine the various processes
associated with the occurrence of a slump. This examination was undertaken in three phases. Firstly, traces of the bed profile were superimposed on each other to observe the changes in bed inclination during slumping. Secondly, the bed inclinations of the same slump were then digitized. The data thus collected were incorporated with the time measurements of bulk solids motion and no motion as was observed from the bed surface. Since a photographic record of slumping was required for all three analyses the bed of bulk solids was filmed through the plexiglass end-plate using a HYCAM motion picture camera operated at 40 frames/s. Cylinder B (0.4 m ID x 0.86 m L) was filled with limestone B at 5.7% fill for these trials. Horizontal and vertical measuring grids were also photographed for a digitizing reference, and direct object lighting was used for the filming. The bed inclination was subsequently traced from the screen of an electronic film digitizing unit and was measured using the angle digitizing feature.

Typical results of the traced inclinations are shown in Figures 6.1 and 6.2 for one slump when the rotational speed of the cylinder was 0.85 r/min. The starting point chosen for this sequence, position 1 (Figure 6.1), is the lowest inclination that the bed attained following the occurrence of a previous slump. Position 1 will be used as
**Figure 6.1** Traced bed inclinations for the first part of a slump of limestone B in cylinder B (0.4 m ID x 0.86 m L) rotated at 0.85 r/min.

**Figure 6.2** Traced bed inclinations for the latter part of a slump of limestone B in cylinder B (0.4 m ID x 0.86 m L) rotated at 0.85 r/min.
a reference for these bed profile traces. Position 2, which
occurs 0.7 seconds after position 1, has a higher bed in-
clination than the reference position. Further increases
in time (positions 3 and 4) reveal that the apex of the bed
does not rise significantly. Although, the chordal base
of the bed seems to have greatly shifted between positions
2 and 3, the recorded basal position of frame number 28,
position 2, is not reliable due to the presence of loose
granules. Therefore, in equal time increments, 0.7 seconds,
larger increases in bed inclination occur from position 1
to 2 as compared to position 2 to 4. The further lapse of
time is illustrated in Figure 6.2 when the apex and the
chordal base approach the starting inclination of the bed.
The total duration of this slump is 2.3 seconds and the
maximum bed inclination takes place between 1.4 and 1.7
seconds, which is over twice as long as the time of de-
creasing inclination.

The digitized bed inclinations for this same slump,
the second phase of this analysis, is shown in Figure 6.3,
and the positions of the traced bed inclinations of Figures
6.1 and 6.2 are clearly indicated. The observations made
from the traced results, described above, are in agreement
with the digitized data. In Figure 6.3, there is a steady
increase in bed inclination until about 0.7 to 1.1 seconds
Figure 6.3  Digitized bed inclinations for the same slump shown in Figures 6.1 and 6.2. The times of bed motion and no motion as recorded from the bed surface are also shown.
where this increase slows down considerably. The bed then attains the maximum bed inclination after 1.6 seconds from the reference frame. The continued rotation of the cylinder results in a sudden decrease in inclination until the initial position is again reached, thereby completing the slumping cycle.

For the final phase of this analysis of the same slump, time durations were recorded for the bulk solids, when stationary and in relative motion to the cylinder wall. These measurements were made with the aid of an inclined mirror placed inside the cylinder which reflected the bed surface and was then simultaneously photographed with the end wall and reference grids. Starting from the reference frame, the frame numbers at which motion on the bed surface started and stopped were recorded. The results, when superimposed on the digitized data in Figure 6.3 and compared to Figures 6.1 and 6.2, reveal the following chronology of events for a slump. The bulk solids rotate with the cylinder wall from time zero to 0.9 seconds. Subsequently, some motion is seen at the apex of the bed, but the inclination of the bed increases at a lower rate until at 1.6 seconds the maximum bed inclination is attained. Hence the process of dilatancy occurs between 0.9 and 1.6 seconds and is characterized by a shrinkage in volume as seen from the traced and digitized
inclinations of Figures 6.1 and 6.3. This shrinkage has been associated with the dilatancy of loosely packed bulk solids. Friction is seen to have reached a limiting value at the point of maximum inclination which by definition is the upper angle of repose. The shear plane is formed at this point and those solids in the shear wedge that had undergone dilatancy will now fall towards the chordal base of the bed; hence the drastic decrease in bed inclination past the 1.6 second mark. When motion ceases, the bed inclination is equal to its initial starting position, which is believed to be the shear angle.

6.2.1.2 The Application of the IR Sensor

Recall that the aim of this empirical approach is to identify the position of the shear plane and the size of the shear wedge. It was shown in Section 4.5.1 that the upper angle of repose is a function of rotational speed. Hence, from the analysis in the previous section, the size of the shear wedge will also be a function of the rotational speed and could be calculated from,

\[ \gamma = \phi_U - \phi_L \]  

(6.1)

However, due to the poor accuracy of the method used in measuring \( \phi_U \) (Section 4.5.1), the shear wedge was not calculated using Equation (6.1). Another approach that was
considered, was to obtain the data photographically, as seen in Figure 6.3. This method is very tedious and time consuming, therefore the feasibility of applying the sensor to this task was investigated.

One of the features of the sensor, discussed in Section 4.3, is that its millivoltage output is a function of the distance from the reflecting surface. Therefore, for a slump, the change in the magnitude of the sensor output is related to the size of the shear wedge. The application of the sensor in this manner presents two difficulties; namely, that the sensor output is also related to the nature of the reflecting surface and the initial distance from which it is placed. However, even if this effect was eliminated by standardization techniques, a second and more serious drawback would still be present i.e., the non-linear relationship between the sensor output and the reflecting distance. The conversion of the sensor output to the shear wedge size would thus be further complicated, so that an indirect approach appeared necessary. The times to maximum, and minimum inclinations, $t_1$ and $t_2$ respectively, were measured from the sensor output, and are compared in Table XII to the times to maximum bed inclination which were measured photographically. Excellent agreement is observed. The data using the sensor were obtained while recording the
slumping frequency by increasing the chart speed on the chart recorder to about 0.2 s/mm (5 s/in). Subsequently, the distance on the chart output between the maxima and minima of these expanded slumping signals was measured directly and converted to times. From the response characteristics of the sensor already discussed above and from the experimental verification (Table XII), it is concluded that the maxima and minima of the slumping signals represent the upper angle of repose and the shear angle of the bed respectively. The size of the shear wedge could thus be easily calculated from,

\[ \gamma_1 = 6 \cdot t_1 \cdot n \]  

where the 6 is a conversion factor. The size and position of the shear wedge were identified using the sensor and the shear angle measurements respectively. Therefore, the mathematical expression relating the shear wedge to the system variables remains to be derived for subsequent modelling usage.

6.2.1.3 Times to Maximum and Minimum Bed Inclinations

The times to maximum and minimum bed inclinations, \( t_1 \) and \( t_2 \) respectively, were measured for different bed depths and rotational speeds for the various
<table>
<thead>
<tr>
<th>Rotational Speed (r/min)</th>
<th>Method of Measurement</th>
<th></th>
<th></th>
<th></th>
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<tr>
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<td>Sensor</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>0.54</td>
<td>Average Time (s)</td>
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<td>0.18</td>
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<td>1.92</td>
<td>0.18</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.85</td>
<td>Average Time (s)</td>
<td>1.59</td>
<td>0.28</td>
<td>9</td>
<td>1.60</td>
<td>0.13</td>
<td>16</td>
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<td></td>
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<tr>
<td></td>
<td>Standard Deviation (s)</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>0.96</td>
<td>Average Time (s)</td>
<td>1.19</td>
<td>0.18</td>
<td>22</td>
<td>1.33</td>
<td>0.14</td>
<td>27</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Standard Deviation (s)</td>
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<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.82</td>
<td>Average Time (s)</td>
<td>0.86</td>
<td>0.14</td>
<td>3</td>
<td>0.86</td>
<td>0.08</td>
<td>27</td>
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<tr>
<td></td>
<td>Standard Deviation (s)</td>
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<td></td>
<td></td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>
materials used in this study. A typical set of results is presented graphically for gravel in Figure 6.4. Not all the experimental data are included on the diagram for the sake of clarity in presentation. Furthermore, each of the points plotted represents the average of at least 25 measurements. The results of the other materials tested displayed the same characteristics as the gravel and are shown in Appendix C. Visual inspection of the results reveals that the times $t_1$ and $t_2$ are both independent of bed depth. At low and high rotational speeds, they appear to be asymptotic to the time axis and to a line parallel to the rotational speed axis respectively. The time $t_1$ is generally greater than time $t_2$ and their difference decreases with increasing rotational speeds.

The form of the equation used for the regression of the $t_1$ data was,

$$t_1 = \frac{\theta_{01} + C_1}{6.n}$$  

(6.3)

The form of this equation reflects the principles of dilation and of the mobilization of friction as well as adequately providing their interpretation in the slumping process. This is evident when both sides of Equation (6.3) are multiplied by $(6.n)$ yielding,
Figure 6.4 The $t_1$ and $t_2$ measured results as well as the regression curves for gravel in cylinder A (0.4 m ID x 0.46 m L).
\[ \gamma_1 = \gamma_{01} + 6.\,C_1 \cdot n \]  

(6.4)

The size of the shear wedge is therefore equal to a constant, the minimum shear wedge, \( \gamma_{01} \), plus the rotational speed times a time constant, \( C_1 \), when dilatancy is believed to occur and will be referred to as the dilatancy coefficient. Referring again to the photographic results in Figure (6.3), \( \gamma_{01} \) would represent the period of time during which no motion was observed on the bed surface, while \( C_1 \) would represent the time increment between the start of motion on the bed and the point at which the upper angle of repose is attained.

For time \( t_2 \), the following equation was used:

\[ t_2 = \frac{\gamma_{02}}{6.\,n} + C_2 + C_3 n \]  

(6.5)

Although, it is in agreement with the experimental results, the first term in equation (6.5) has no physical significance with respect to the motion of the shear wedge. To illustrate this, consider a cylinder rotating at a very low rotational speed with a slump about to occur. The bulk solids will take a finite time to slump from the apex to the chordal base of the bed. Meanwhile, the cylinder has rotated through a finite angle. As the rotational speed approaches zero, the slumping time will still be finite.
but the angle travelled by the cylinder will approach zero. Therefore, the first term in Equation (6.5) should be zero. The asymptotic behaviour experimentally observed for all the $t_2$ data at the low rotational speeds has therefore been attributed to the following two effects. At low rotational speeds, when times $t_1$ and $t_2$ were measured from the recording chart, it was more difficult to pinpoint the precise location of the maxima and minima of the sensor output. This was due to the sensitivity of the sensor at both positions of bed inclinations. A consistent method of locating the maxima and minima of the sensor output was therefore adopted. The second and more important cause was that the system has a longer response time at the low rotational speeds, hence more time for the internal distribution of stress. However, since these bulk solids are not composed of ideally uniform shapes and sizes, a scatter in the measured limiting shear wedge would thus be expected. In fact the experimental measurements of the times $t_1$ and $t_2$ had higher standard deviations at low rotational speeds than at the higher ones; thus the first term in Equation (6.5) is due to both a measuring and a random experimental error. The minimum shear wedge should therefore be equal to,

$$\gamma_0 = \gamma_{01} + \gamma_{02} \quad (6.6)$$

and the time to the limiting friction condition is given by,

$$t_1 = \gamma_0 + C_1 \quad (6.7)$$
The third term in Equation (6.5) is included because it would be expected that the time of bed collapse should be a function of rotational speed. In fact it should decrease with increasing speed, thus $C_3$ will be negative.

Now adding Equation (6.3) and (6.5) yields,

$$t_T = \frac{\gamma_{01} + \gamma_{02} + (C_1 + C_2) + C_3 n}{6n} \quad (6.8)$$

But, the slumping frequency is given by,

$$S = \frac{60}{t_T} \quad (6.9)$$

Therefore,

$$1 = \frac{1}{S} \left( \frac{60}{t_T} \right) \left[ \frac{\gamma_{01} + \gamma_{02} + (C_1 + C_2) + C_3 n}{6n} \right] \quad (6.10)$$

To verify the regression coefficients obtained from the $t_1$ and $t_2$ data, the slumping frequency data already presented in Section 4.5.1. were regressed using Equation (6.10) thus yielding an independent set of coefficients.

A simple regression (SIMREG) was performed on the $t_1$ data using Equation (6.3) and a standard multiple regression (STPREG) on the $t_2$ and the slumping frequency data using Equations (6.5) and (6.10) respectively. The UBC Triangular Regression Package\(^{143}\) was used to obtain these curve fits.
on the UBC Amdahl 470 V6 Model II computer. The regression coefficients obtained for each of the sets of data for the various materials on hand are listed in Tables XIII to XV along with the standard errors for each of the coefficients, based on a 95% confidence limit. The squared multiple correlation coefficients are in the majority of cases greater than 0.95 indicating a good fit. This is further illustrated in Figures 6.4 and C.1 to C.10 where the fitted equations of $t_1$ and $t_2$ are compared with the experimental data. The coefficients from the $t_1$ and $t_2$ data are now compared in Table XVI with those from the slumping frequency data. Very good agreement is observed. Hence, the limiting shear wedge size and the shear wedge size as a function of rotational speed are mathematically identified in a form that allows their application to study the chordal trajectory of the slump.

6.2.2 The Chordal Trajectory of a Slump

When the friction force is mobilized to its maximum value, the granular solids content of the shear wedge is unstable at the apex and must fall to the chordal base of the bed. The chordal trajectory of each of the granules in the shear wedge (wedge ABC, Figure 6.5) would be given by the third principle of solids flow, the minimization of energy. However, since there is no information on the manner
TABLE XIII  Regression Results for the $t_1$ Measurements ($t_1 = \gamma_{01} + C_1$)

<table>
<thead>
<tr>
<th>Material</th>
<th>Cylinder Diameter (m)</th>
<th>$\gamma_{01}$ (degrees)</th>
<th>Standard Error on $\gamma_{01}$ (degrees)</th>
<th>$C_1$ (s)</th>
<th>Standard Error on $C_1$ (s)</th>
<th>Squared Multiple Correlation Coefficient $R^2$</th>
<th>Number of Observations*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel</td>
<td>0.4</td>
<td>3.67</td>
<td>0.0463</td>
<td>0.536</td>
<td>0.0219</td>
<td>0.9911</td>
<td>58</td>
</tr>
<tr>
<td>Gravel</td>
<td>1.0</td>
<td>3.75</td>
<td>0.0371</td>
<td>0.699</td>
<td>0.0251</td>
<td>0.9970</td>
<td>33</td>
</tr>
<tr>
<td>Iron Oxide</td>
<td>0.4</td>
<td>4.13</td>
<td>0.147</td>
<td>0.722</td>
<td>0.0941</td>
<td>0.9839</td>
<td>15</td>
</tr>
<tr>
<td>Limestone B</td>
<td>0.4</td>
<td>4.41</td>
<td>0.0624</td>
<td>0.491</td>
<td>0.0410</td>
<td>0.9974</td>
<td>15</td>
</tr>
<tr>
<td>Limestone B</td>
<td>1.0</td>
<td>4.79</td>
<td>0.0588</td>
<td>0.487</td>
<td>0.0624</td>
<td>0.9955</td>
<td>32</td>
</tr>
<tr>
<td>Limestone C</td>
<td>0.4</td>
<td>5.53</td>
<td>0.124</td>
<td>0.158</td>
<td>0.0830</td>
<td>0.9793</td>
<td>44</td>
</tr>
<tr>
<td>Limestone C</td>
<td>1.0</td>
<td>4.49</td>
<td>0.0807</td>
<td>0.153</td>
<td>0.0716</td>
<td>0.9917</td>
<td>28</td>
</tr>
<tr>
<td>Limestone D</td>
<td>0.4</td>
<td>4.30</td>
<td>0.0661</td>
<td>0.103</td>
<td>0.0445</td>
<td>0.9858</td>
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</tr>
<tr>
<td>Limestone F</td>
<td>1.0</td>
<td>4.48</td>
<td>0.105</td>
<td>0.770</td>
<td>0.175</td>
<td>0.9882</td>
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</tr>
<tr>
<td>Nickel Oxide</td>
<td>0.4</td>
<td>2.70</td>
<td>0.0345</td>
<td>0.614</td>
<td>0.0699</td>
<td>0.9955</td>
<td>30</td>
</tr>
<tr>
<td>Sand B</td>
<td>0.4</td>
<td>1.78</td>
<td>0.0764</td>
<td>0.364</td>
<td>0.111</td>
<td>0.9347</td>
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</table>

* - each observation is an average of at least 25 readings.
### TABLE XIV  Regression Results for the $t_2$ Measurements ($t_2 = \frac{\gamma_{02}}{e_n} + C_2 + C_3 n$)

<table>
<thead>
<tr>
<th>Material</th>
<th>Cylinder Diameter (m)</th>
<th>$\gamma_{02}$ (degrees)</th>
<th>Standard Error on $\gamma_{02}$ (degrees)</th>
<th>$C_2$ (s)</th>
<th>Standard Error on $C_2$ (s)</th>
<th>$C_3$ (60 $s^2_{rev}$)</th>
<th>Standard Error on $C_3$ (60 $s^2_{rev}$)</th>
<th>Squared Multiple Regression Coefficient $R^2$</th>
<th>Number of Observations*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel</td>
<td>0.4</td>
<td>1.33</td>
<td>0.0543</td>
<td>1.11</td>
<td>0.0502</td>
<td>-0.243</td>
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<td>0.9712</td>
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</tr>
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<td>Gravel</td>
<td>1.0</td>
<td>0.88</td>
<td>0.0385</td>
<td>1.35</td>
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<td>-0.387</td>
<td>0.0430</td>
<td>0.9835</td>
<td>33</td>
</tr>
<tr>
<td>Iron Oxide</td>
<td>0.4</td>
<td>0.60</td>
<td>0.062</td>
<td>2.25</td>
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<td>1.26</td>
<td>0.6650</td>
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<tr>
<td>Limestone B</td>
<td>0.4</td>
<td>0.78</td>
<td>0.049</td>
<td>1.07</td>
<td>0.0652</td>
<td>-0.220</td>
<td>0.0404</td>
<td>0.9886</td>
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<tr>
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<td>0.88</td>
<td>0.066</td>
<td>1.66</td>
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<td>-0.561</td>
<td>0.185</td>
<td>0.9434</td>
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<tr>
<td>Limestone C</td>
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<td>0.94</td>
<td>0.042</td>
<td>1.09</td>
<td>0.0489</td>
<td>-0.231</td>
<td>0.0353</td>
<td>0.9695</td>
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<tr>
<td>Limestone C</td>
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<td>1.09</td>
<td>0.0316</td>
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<td>Limestone D</td>
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<td>0.38</td>
<td>0.043</td>
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<td>0.053</td>
<td>-0.353</td>
<td>0.0690</td>
<td>0.8699</td>
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</tr>
<tr>
<td>Limestone F</td>
<td>1.0</td>
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<tr>
<td>Nickel Oxide</td>
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<td>0.024</td>
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<td>0.100</td>
<td>-2.49</td>
<td>0.322</td>
<td>0.9484</td>
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</tr>
<tr>
<td>Sand B</td>
<td>0.4</td>
<td>0.38</td>
<td>0.030</td>
<td>1.44</td>
<td>0.0699</td>
<td>-1.96</td>
<td>0.295</td>
<td>0.9711</td>
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* - Each observation is an average of at least 25 readings.
<table>
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<th>Cylinder Diameter (m)</th>
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<th>Standard Error on $\gamma_0$ (degrees)</th>
<th>$C_4$ (s)</th>
<th>Standard Error on $C_4$ (s)</th>
<th>$C_5$ (60. s$^2$/rev)</th>
<th>Standard Error on $C_5$ (60. s$^2$/rev)</th>
<th>Squared Multiple Regression Coefficient $R^2$</th>
<th>Number of Observations$^*$</th>
</tr>
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<tbody>
<tr>
<td>Gravel</td>
<td>0.4</td>
<td>4.89</td>
<td>0.0729</td>
<td>1.80</td>
<td>0.0639</td>
<td>-0.364</td>
<td>0.0396</td>
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<td>4.95</td>
<td>0.0753</td>
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<td>0.0665</td>
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<td>0.491</td>
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<td>-1.49</td>
<td>0.931</td>
<td>0.9775</td>
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<td>0.157</td>
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<td>0.109</td>
<td>1.24</td>
<td>0.147</td>
<td>-0.211</td>
<td>0.209</td>
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<td>2.76</td>
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<td>1.64</td>
<td>0.9588</td>
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</tbody>
</table>

* - Each observation was taken over a period of at least 5 minutes.
TABLE XVI  Comparison of Regression Coefficients Obtained from $t_1$, $t_2$ Data and from the Slumping Frequency Data

<table>
<thead>
<tr>
<th>Material</th>
<th>Cylinder Diameter (m)</th>
<th>Data from $t_1$, $t_2$ Regressions</th>
<th>Data from Slumping Frequency Regressions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\gamma_0$ (deg)</td>
<td>$C_1 + C_2$ (s)</td>
</tr>
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<td>Gravel</td>
<td>0.4</td>
<td>5.00</td>
<td>1.64</td>
</tr>
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<td>Limestone B</td>
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<td>5.67</td>
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</tr>
<tr>
<td>Limestone C</td>
<td>0.4</td>
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<td>1.25</td>
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<tr>
<td>Sand B</td>
<td>0.4</td>
<td>2.16</td>
<td>1.80</td>
</tr>
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</table>
in which the potential energy of the solids is dissipated, a trajectory model will be developed based on the conversion of the potential energy of the granules in the shear wedge solely to kinetic energy. Furthermore, it will be assumed that the granules below the shear wedge will not be disturbed by the trajectory of those in it; hence the granules in the shear wedge will fall into wedge PSC (Figure 6.5). Their average trajectory will be given by the straight line joining the centroids of the two wedges and the resultant force on the shear wedge will act through its centroid and will be given by,

\[ M_a = F_G \sin \eta - F_F \]  \hspace{1cm} (6.11)

where the gravitational force is given by,

\[ F_G = Mg \]  \hspace{1cm} (6.12)

and the frictional force by,

\[ F_F = \mu_L F_N \]  \hspace{1cm} (6.13)

The force balance normal to the line of the average chordal trajectory yields,

\[ F_N = Mg \cos \eta \]  \hspace{1cm} (6.14)

Now substituting Equations (6.12) to (6.14) into (6.11),

\[ a = g (\sin \eta - \mu_L \cos \eta) \]  \hspace{1cm} (6.15)
Figure 6.5  Force balance on a shear wedge.
Since the limiting friction condition occurs at the shear angle, it follows that,

\[ u_L = \tan \phi_L \]  \hspace{1cm} (6.16)

Integrating Equation (6.15) with the initial conditions,

(i) \[ t = 0 \quad v_{0T} = 0 \]  \hspace{1cm} (6.17)

and

(ii) \[ t = 0 \quad s_0 = 0 \]  \hspace{1cm} (6.18)

and substituting equation (6.16), yields,

\[ s = \frac{1}{2} gt^2 \left( \sin \eta - \tan \phi_L \cos \eta \right) \]  \hspace{1cm} (6.19)

It would have been more precise to express the initial condition in (6.17) as follows,

when \( t = 0 \) \[ v_{0T} = \omega (y_{ABC} \cos \eta + x_{ABC} \sin \eta) \]  \hspace{1cm} (6.20)

However, due to the small angular velocities of slumping beds the contribution of the initial condition (6.20) would have a negligible effect on the average trajectory time. Hence, initial condition (6.17) was used in the model.

Finally, since the shear wedge is bounded by the upper and lower angles of repose the distance between the wedge centroids, ABC and PSC, and the slope of the average chordal trajectory may be calculated from,

\[ s = \left( (\bar{x}_{ABC} - \bar{x}_{PSC})^2 + (\bar{y}_{ABC} - \bar{y}_{PSC})^2 \right)^{\frac{1}{2}} \]  \hspace{1cm} (6.21)
and \( \eta = \tan^{-1} \left[ \frac{y_{ABC} - y_{PSC}}{x_{ABC} - x_{PSC}} \right] \)  \( (6.22) \)

where the calculation of the centroid co-ordinates \((x_{ABC}, y_{ABC})\) and \((x_{PSC}, y_{PSC})\) is presented in Appendix D. Hence, the average slumping time of the contents of the shear wedge may be calculated from Equation (6.19).

### 6.2.3 The Criterion of the Slumping-Rolling Boundary

As a shear wedge fails and its contents fall into the chordal base of the bed more granules cross the shear plane due to the continuous rotation of the cylinder. Thus in the slumping time of the shear wedge, calculated from Equation (6.19), the angular rotation of the cylinder (i.e. the quantity of solids crossing the shear plane) may be calculated from,

\[ \gamma^* = 6. n. t \]  \( (6.23) \)

If the angle travelled by the cylinder, \( \gamma^* \), during the slumping time is less than the minimum shear wedge (\( \gamma^* < \gamma_0 \)) then it is proposed that the bed will continue to slump. If the angle travelled by the cylinder is greater than the minimum shear wedge (\( \gamma^* > \gamma_0 \)) then the bed will roll. It follows that the condition when the angle travelled equals the minimum shear wedge (\( \gamma^* = \gamma_0 \)) represents the slumping-rolling boundary and that the chordal trajectory time is
given by,

\[ t_0 = \frac{30}{\pi} \left( \frac{y_0}{\omega} \right) \]  \hspace{1cm} (6.24)

or

\[ t_0 = \frac{1}{6} \left( \frac{y_0}{\nu} \right) \]  \hspace{1cm} (6.25)

Substituting Equation (6.24) in (6.19), yields,

\[ s = 450 \frac{g}{\pi^2} \left( \frac{y_0}{\omega} \right)^2 (\sin \eta - \tan \phi_L \cos \eta) \]  \hspace{1cm} (6.26)

Also substituting,

\[ \omega = \frac{v}{R} \]  \hspace{1cm} (6.27)

and dividing by \( s \), Equation (6.26) becomes,

\[ 1 - \left( \frac{450 g R}{\pi^2 v^2} \right) \left( \frac{R}{s} \right) \left( \frac{y_0}{\omega} \right)^2 (\sin \eta - \tan \phi_L \cos \eta) = 0 \]  \hspace{1cm} (6.28)

This equation is therefore the dimensionless form of the criterion for the slumping-rolling boundary and presents the scale-up criteria for the material, the cylinder and the operating variables. The first dimensionless parameter \( gR \) is recognized as the inverse of the Froude number which is a ratio of the inertial to gravitational forces. The second scale-up factor, \( (R/s) \) reflects the effect of filling degree as well as the size of the shear wedge, since the calculation of \( s \) is based on the location of the wedge.
centroids. Thus it is an operating and a material scale-up factor. The third, $\gamma_0$, represents a material variable, the minimum shear wedge; while the last one ($\sin \eta - \tan \phi L \cos \eta$) is dependent on the fill ratio, the size of the shear wedge, the shear angle and the cylinder diameter. For a given material and cylinder, only the first two dimensionless parameters ($\gamma R/v^2$) and $(R/\delta)$ could vary, and their value would be solely a function of rotational speed and bed depth. Hence, their use is mathematically justified for the axes of the Bed-Behaviour Diagram.

In conclusion the mathematical model for predicting the slumping-rolling boundary of a bulk solid requires the following input variables; the first three of which must be experimentally determined:

1) the minimum shear wedge,
2) the shear wedge as a function of rotational speed,
3) the shear angle,
and 4) the cylinder inner radius.

The equation in Table D.1 to D.3 (Appendix D) and Equation (6.28) were solved by trial and error on the UBC Amdahl computer. The source listing of the program is included in Appendix E and the model predictions will be presented in Chapter 7.
6.3 The Slipping Model

A model will now be presented to predict those conditions when slipping rather than slumping, rolling or cascading will occur. The model is based on the principles of mechanics for rigid bodies. Assume that the bed of bulk solids in Figure 6.6 is rotated with the cylinder wall and has the angle of inclination $\phi'$. The gravity force and the centrifugal force will act through the centre of gravity of the bed whose location is given by,

$$R_B = \frac{2R^3 \sin^3 \lambda}{3A} \quad (6.29)$$

where the area of the bed may be calculated using Equation (5.6) or,

$$A = \frac{R^2}{2} (2 \lambda - \sin 2\lambda) \quad (6.30)$$

The gravity force is given by Equation (6.12) and the centrifugal force by,

$$F_C = M \omega^2 R_B \quad (6.31)$$

The resultant force is therefore,

$$F_R = (F_C^2 + 2F_C F_G \cos \phi' + F_G^2)^{1/2} \quad (6.32)$$

The moment balance about the centre of rotation of the cylinder yields,

$$F_F = \frac{R_B}{R} F_G \sin \phi' \quad (6.33)$$
Figure 6.6 Force balance on the bed of bulk solids for the slipping model.
In order for the forces to be in equilibrium (i.e. no slipping at the wall) the normal and frictional forces must also have a resultant, \( F_R^i \), which is equal in magnitude but opposite in direction to \( F_R \). Therefore,

\[
F_F = F_R^i \sin \beta \tag{6.34}
\]

and

\[
F_N = F_R^i \cos \beta \tag{6.35}
\]

It is known that the maximum frictional force is given by,

\[
F_F = \mu_{W/S} F_N \tag{6.36}
\]

and that,

\[
\mu_{W/S} = \tan \phi_S \tag{6.37}
\]

Therefore substituting Equations (6.32) to (6.35) and (6.37) in (6.36) yields,

\[
\tan \phi_S = \frac{R_B^2 F_G \sin \phi'}{R} \left( \frac{F_C^2 + 2F_C F_G \cos \phi' + F_G^2 \cos \left( \sin^{-1} \left[ \frac{R_B^2 F_G \sin \phi'}{R} \right] \right)}{(F_C^2 + 2F_C F_G \cos \phi' + F_G^2)^{1/2}} \right)
\]

Equation (6.38) was solved by trial and error for the resultant bed inclination, \( \phi' \), using the UBC Amdahl computer (the program listing is shown in Appendix F). It was then compared to the upper angle of repose, \( \phi_U \), and the
resultant dynamic angle of inclination \( \phi_D^i \), to be described in the following section). Three situations may arise,

(i) \( \phi^i > \phi_U > \phi_D^i \)
(ii) \( \phi_U > \phi^i > \phi_D^i \)
and (iii) \( \phi_U > \phi_D^i > \phi^i \)

In the first situation, no slipping would occur; while in the second slipping would occur rather than slumping. Finally, slipping would replace all other modes of bed behaviours in the last condition. For rolling beds the analysis could be simplified by comparing \( \phi^i \) to \( \phi_D \). Furthermore, when the centrifugal force is negligible, as in rotary kilns for example, Equation (6.38) simplifies to:

\[
\tan \phi^i = 3 \tan \phi_S \left( \lambda - \cos \lambda \sin \lambda \right) \frac{1}{2 \sin^3 \lambda} \tag{6.39}
\]

Slipping in rotary kilns is therefore a function of the slip angle \( \phi_S \) and the degree of fill. The results of the slipping model could therefore be plotted on a Bed-Behaviour Diagram and will be presented in Chapter 7.

6.4 The Cascading and Cataracting Models

The calculation of the onset of cascading and cataracting have thus far received little attention in the
literature. Cascading has been identified by the crescent shape of the bed; while cataracting has been characterized by a significant portion of granular solids being projected into the free space of the cylinder. These solids shower back into the bed. Several attempts have been made to mathematically analyze a cataracting bed, but most of them are concerned with the trajectory of the solids above the bed. Only two investigators have attempted to calculate the angle of detachment of particles from the solids bed: Davis⁵⁶ and Uggla.⁷⁹ Their calculation of the onset of cataracting was based on the force balance on a single particle resting on the surface of the bed. When the cylinder has zero rotational speed, Davis placed his particle at the cylinder wall, in line with the centre of rotation of the cylinder. This corresponds to a fill ratio of 0.5 and a horizontal bed surface. Davis also did not take the friction force into account. Uggla on the other hand included it and carried out a force balance on a single particle with the friction force applied as an interparticle friction on the bed surface. Improved estimates of the angle of detachment were obtained.

The static angle of friction, \( \phi_R \), applied by Uggla as an interparticle friction on the bed surface is an empirical measure of the static friction coefficient of the
bed but the dynamic angle of friction, $\phi_D$, should have been applied. Therefore, for the current model the force balance was carried out at the centroid of the bed with the angle of kinetic friction equal to the dynamic angle of repose of the bulk solids. The force polygon is now solved for a given bed depth, cylinder rotational speed and material-dynamic angle of friction (Figure 6.7a). The force balance yields,

$$F_G \sin \phi_D - F_F = 0 \quad (6.40)$$

$$F_N - F_C - F_G \cos \phi_D = 0 \quad (6.41)$$

Finally, 

$$F_F = \mu_D F_N \quad (6.42)$$

Substituting for the appropriate terms and simplifying,

$$(\sin \phi_D - \mu_D \cos \phi_D) = \frac{\mu_D \omega^2 R_D}{g} \quad (6.43)$$

where,

$$\mu_D = \tan \phi_D \quad (6.44)$$

Hence, the bed inclination, for which the forces acting on the bed are in equilibrium, can be calculated by trial and error using Equation (6.43) which can also be simplified to,

$$\sin (\phi_D - \phi) = \frac{\omega^2 R_B}{g} \sin \phi_D \quad (6.45)$$

or

$$\sin (\phi_D - \phi) = \frac{\omega^2 R}{g} \left[ \frac{2 \sin \lambda}{3(\lambda - \cos \lambda \sin \lambda)} \right] \sin \phi_D \quad (6.46)$$
Figure 6.7
(a) Force balance on the bed of bulk solids for the cascading model with the apex in the first quadrant.
(b) Bed configuration with the apex in the fourth quadrant.
(c) Bed configuration defining the rolling-cascading boundary.
The resultant bed inclination is therefore a function of the Froude number, the degree of fill and the dynamic angle of repose.

When the resultant bed inclination, \( \phi_D \), places the apex of the bed in the fourth quadrant, the outermost particle at the cylinder wall will have an initial velocity into the wall (Figure 6.7b) and the angle \( \theta \) will be negative. However, once the resultant bed inclination places the apex in the first quadrant, the velocity of the outermost particle will be away from the wall and will be projected into the open space of the cylinder (Figure 6.7a). This has been defined as cascading. Note that in this configuration the angle \( \theta \) is positive. The rolling-cascading boundary is therefore defined by the rotational speed which yields a resultant bed inclination \( \phi_D' \) which places the apex of the bed on the abscissa; as illustrated in Figure 6.7c. In this configuration,

\[
\theta = 0 \quad (6.47)
\]

and

\[
\lambda = 90 - \phi_D' \quad (6.48)
\]

Substituting Equation (6.48) into (6.46) and simplifying yields,

\[
\frac{\omega_{R/C}^2 R}{g} = 3 \left( \frac{2 \tan \phi_D' - 1}{\mu_D} \right) \left\{ \frac{\pi}{2} - \phi_D' - \tan \phi_D' \right\} \left( \cos^2 \phi_D' \right) \quad (6.49)
\]
Equation (6.49) reveals that the rolling-cascading boundary is a function of the cylinder size, the dynamic angle of friction of the material and the degree of fill. Hence, the rolling-cascading boundary may be described on a Bed-Behaviour Diagram. The predicted boundaries using this model will be presented in Chapter 7.

Once a bed is cascading its surface will no longer be planar due to the solids at the apex being projected into the cylinder free space. In this analysis, a parabolic trajectory will be applied to a single particle at the apex of the bed. No interactions with other particle trajectories will be accounted for as it would greatly complicate the solution of the problem. While this treatment may seem unrealistic, it is a good approximation and will illustrate the basis of using the Bed-Behaviour Diagram to describe cataracting. The cascading-cataracting boundary has been defined as the locus of rotational speeds at various bed depths for which the parabolic trajectory of a particle projected from the apex will intersect the midpoint of the bed surface inclined at the resultant angle of inclination. This angle is calculated using Equation (6.46). The parabolic trajectory of the uppermost particle on the bed surface is given by,
\[ Y = R \sin \theta - \frac{(R \cos \theta - X) + g (R \cos \theta - X)^2}{\tan \theta \, 2 \frac{v_o^2 \sin^2 \theta}{v_o^2 + \sqrt{Rg \sin \theta - \sin^2 \theta}}} \]  \hspace{1cm} (6.50) \\

where

\[ X = R \cos \theta - \left[ \frac{2v_o^2 \sin \theta}{g} \right] \left[ \cos \theta + \sqrt{\frac{Rg \sin \theta - \sin^2 \theta}{v_o^2}} \right] \]  \hspace{1cm} (6.51) \\

and \[ v_o = \omega R \]  \hspace{1cm} (6.52) \\

The co-ordinate of the mid-point of the bed surface (Figure 6.7a) is given by,

\[ X_E = (R-H) \cos (\theta - \lambda) \]  \hspace{1cm} (6.53) \\
and \[ Y_E = (R-H) \sin (\theta - \lambda) \]  \hspace{1cm} (6.54) \\

From the above equations it is seen that as with the rolling-cascading boundary, the cascading-cataracting boundary is dependent on the cylinder diameter, the dynamic angle of repose of the material and the fill ratio. A computer program was written to predict a complete Bed-Behaviour Diagram for a given material, showing the various areas of predominance of slipping, slumping, rolling, cascading, cataracting and centrifuging. The program, listed in Appendix G requires the following input variables:

1. the radius of the cylinder, \( R \),
2. the angle containing the shear wedge, \( \gamma_o \),
3. the shear wedge as a function of rotational speed, $\gamma_0$ and $C_1$,
4. the shear angle, $\phi_L$,
5. the dynamic angle of repose, $\phi_D$; and
6. the angle of slipping friction, $\phi_S$.

The predictions of the model will be presented in the following chapter for the materials used in this study.
Chapter 7

MODEL PREDICTIONS AND DISCUSSION

7.1 Introduction

The predicted boundaries for the various modes of bed behaviour will be presented in this chapter and will be compared both to the experimental results discussed in chapter 4 and to experimental and industrial observations reported in the literature. The scale-up criteria suggested by the model equations and by the experimental results will also be tested and the results will be applied to the industrial operations of rotary kilns. The effect of temperature on the bed behaviour of bulk solids will also be briefly discussed, along with some applications of the Bed-Behaviour Diagram in other situations. A comparison between the Phase Diagram of fluid-solid systems and the Bed-Behaviour Diagram will then be made. A derivation of the internal angle of friction of cohesionless bulk solids will be proposed based on an analysis of slumping.

7.2 The Slumping-Rolling Boundary

Model predictions showing the effects of bed depth, particle size, particle shape, combined effects of size and
shape and of cylinder diameter will be discussed. The scale-up criteria of the slumping-rolling boundary will be investigated using experimental results to verify the model predictions.

### 7.2.1 Predicted versus Measured Results

Model predictions of the slumping-rolling boundary are presented in Figures 7.1 to 7.6. The predicted and experimental boundaries for limestone B tested in cylinder A are shown in Figure 7.1. Using the standard error on each of the regression coefficients (Table 6.2 and 6.3), the maximum and minimum values of \( \gamma_0 \) and \( C_1 \) were calculated for a 95% confidence limit and were used to obtain the upper and lower bounds of the slumping-rolling boundary, also plotted in Figure 7.1. For limestone B, the variations in \( \gamma_0 \) of \( \pm 0.3 \) degrees and in \( C_1 \) of \( \pm 0.1 \) seconds result in a change in the predicted boundary of about \( \pm 0.15 \) r/min and \( \pm 0.25 \) r/min for the upper and lower confidence limits of the coefficients respectively. This is of the same order as the width of the experimentally determined transition zone. Good agreement is observed between predicted and experimental boundaries, particularly with respect to the bed depth dependency.

Comparing the predicted and the experimental bed
**Figure 7.1** Predicted and experimental boundaries for limestone B tested in cylinders A and B (0.4 m ID x 0.46 m L and 0.86 m L respectively).

**Figure 7.2** Predicted and experimental boundaries for nickel oxide tested in cylinder A (0.4 m ID x 0.46 m L).
behaviour boundaries for nickel oxide (Figure 7.2), it is observed that the form of the predicted bed depth dependency is similar to that for the measured slumping-transition boundary, indicating that at low degrees of fill the enhanced curvature of the transition-rolling boundary may be due to circumferential wall effects. However, this could not be experimentally confirmed. The 95% confidence limit on the regression coefficients has yielded a much smaller range in the predicted boundary than had been experimentally obtained. The model predictions for other materials tested are also in good agreement with the experimental results and are shown in Figures 7.3 to 7.6.

7.2.2 The Bed Depth

The effect of bed depth on the boundary was not revealed using any of the slumping or rolling characterization techniques described in Section 4.5.1 and will now be explained in the light of the mathematical model developed.

It was shown in Section 6.2.1.3 that the times to maximum and minimum inclinations are independent of bed depth, as is the size of the shear wedge. Recall that the shear angle was also independent of bed depth. Thus for varying bed depths of a given material, the shear planes and bed surfaces will be parallel in a given cylinder. Hence,
the angles defining the geometry of the system will be similar; however, the distance between the upper and lower centroids will increase with increasing bed depth. Since the contents of the shear wedge have a longer distance to travel at the higher bed depths, their average travelling time will correspondingly increase. At equal rotational speeds, the angular distance travelled by the cylinder during the collapse of the shear wedge will thus be greater for the higher bed depths. Since the minimum shear wedge for all bed depths is a constant, the deeper beds would require smaller rotational speeds to satisfy the slumping-rolling boundary criterion, hence the bed depth dependency of the boundary.

This can also be mathematically illustrated using Equation (6.28) whose ratio for two bed depths yields,

\[ \omega_M = \left( \frac{S_p}{S_M} \right)^{\frac{1}{2}} \omega_P \]  

(7.1)

If \( H_P < H_M \) from geometry it follows that,

\[ S_p < S_M \]

In order to maintain the equality of Equation (7.1), the angular velocities must have the following relationship,

\[ \omega_P > \omega_M \]
Therefore, the slumping-rolling boundary model correctly predicts the bed depth dependency observed in the Bed-Behaviour Diagrams.

7.2.3 Particle Shape

Comparing now the results of Figures 7.1 and 7.2, it is clear that the model has accounted for the difference in particle shape between the irregular limestone B and the spherical nickel oxide. A survey of model inputs (Tables VII, XIII and XIV) for both materials reveals that the shear angle and the minimum shear wedge are smaller for the spherical solids; while their dilatancy coefficient is larger. The shear angles are reported to be 37.7° and 29.9° for limestone B and nickel oxide respectively, while the respective minimum shear wedges are 5.2° and 2.9°. Although the dilatancy coefficient for limestone B, 0.49 seconds, is lower than that for nickel oxide, 0.61 seconds, the shear wedge of the nickel oxide will be smaller until a rotational speed of 3.2 r/min, where their shear wedges would be equal. Hence, over the speeds of interest the slope of the average chordal trajectory will be steeper for limestone B than for nickel oxide. This, combined with a smaller minimum shear wedge results in a slumping-rolling boundary occurring at lower rotational speeds for the spherical bulk solids (Figures 7.1 and 7.2). The boundary for the irregular
limestone is also more bed-depth dependent than the spherical nickel oxide.

The effect of particle shape on bed behaviour is also illustrated in Figures 7.3 and 7.4, for the irregular limestone D and the nodular sand B. A comparison of the model inputs for both materials reveals that although the shear angles of both these bulk solids are nearly equal, 33.5° for limestone D and 32.2° for sand B, the minimum shear wedge for limestone D is at least double that for sand B, 4.7° and 2.2° respectively. On the other hand, the dilatancy coefficient of the irregular solids is lower than that of the nodular solids, 0.10 and 0.36 seconds, but, as in the previous illustration with limestone B and nickel oxide, this difference is not sufficient to yield a steeper trajectory for sand B. Hence, the slumping-rolling boundary for sand B occurs at lower rotational speeds than for limestone D, which also displays a more accentuated bed depth dependency.

The effect of particle shape, can be mathematically expressed from Equation (6.28) as follows,

\[
\omega_M = \frac{[\gamma_0 (\sin \eta - \cos \eta \tan \phi_L)^{\frac{1}{2}}]_M}{[\gamma_0 (\sin \eta - \cos \eta \tan \phi_L)^{\frac{1}{2}}]_P} \omega_p
\] (7.2)
Figure 7.3 Predicted and experimental boundaries for limestone D tested in cylinder A (0.4 m ID x 0.46 m L).

Figure 7.4 Predicted and experimental boundaries for sand B tested in cylinder A (0.4 m ID x 0.46 m L).
and will be further discussed in the following sections.

7.2.4 Particle Size

A comparison of the predicted boundaries for limestones B and D (Figure 7.1 and 7.3) reveals that rolling sets in at lower rotational speeds for the smaller sized bulk solids. The respective model inputs for limestones B and D are: the shear angle, 37.7° and 33.5°, the minimum shear wedge, 5.2° and 4.7° and the dilatancy coefficient, 0.49 and 0.10 seconds; while their respective average particle sizes are 4.3 mm and 0.58 mm. Thus, as the size decreases the slope of the chordal trajectory decreases and the minimum shear wedge decreases. Both these factors will result, as observed, in the slumping-rolling boundary occurring at lower rotational speeds. This is further confirmed in Figure 7.5 for limestone C whose model inputs are: $\phi_L = 33.6^\circ$, $\gamma_0 = 6.5^\circ$ and $C_1 = 0.16$ seconds. Except for the minimum shear wedge, the magnitude of the remaining input variables lie between those for limestones B and D. The deviation of $\gamma_0$ for limestone C is attributed to a greater experimental scatter in the maximum and minimum bed inclination times which is evident by the large 95% confidence range of the predicted boundaries. Hence, the effect of particle size is reflected in Equation (6.28) in the same manner as particle shape and may also be described by Equation (7.2).
Figure 7.5 Predicted and experimental boundaries for limestone C tested in cylinder A (0.4 m ID x 0.46 m L).

Figure 7.6 Predicted and experimental boundaries for limestone B tested in cylinder C (1.06 m ID x 0.4 m L).
7.2.5 Combined Effect of Particle Size and Shape

It was shown in the previous sections that both particle size and shape affect the slope of the chordal trajectory of the solids as well as the size of the minimum shear wedge, and that Equation (7.2) presents the scale-up criteria for each of these material variables. It therefore follows that when comparing two bulk solids having different average sizes and shapes, Equation (7.2) should still apply in predicting the boundary of one of the materials from that of the other. Furthermore, these differences in particle size and shape would also be reflected in the input variables of the model for both materials as will be illustrated for the spherical nickel oxide and the nodular sand B (recall that their particle sizes are 4.9 mm and 0.50 mm respectively). Their respective model inputs are: the shear angle, 29.9° and 32.2°, the minimum shear wedge, 2.9° and 2.2°, and the dilatancy coefficient 0.61 and 0.36 seconds. As it was difficult in Chapter 4 to estimate qualitatively whether the particle size or the particle shape would dominate, so it is equally difficult at this point to estimate from the above input data which variable will have a dominant effect on the model prediction of the boundaries. The shear angle indicates that the boundary for nickel oxide will be to the left of that for sand B; while the minimum shear wedge and the dilatancy coefficient indicate
the reverse. The model predictions clearly show the latter variables to have dominated (Figures 7.2 and 7.4). Hence, the model does indeed account for the effects of both particle size and shape through appropriate changes in the shear angle, \( \phi_L \), the average slope of the chordal trajectory, \( \eta \), and the minimum shear wedge, \( \gamma_0 \). Furthermore, in view of the results presented thus far, the minimum shear wedge appears to be the dominant material variable in determining the relative position of the boundary.

### 7.2.6 Cylinder Diameter

The minimum shear wedge, the dilatancy and the shear angle were determined for limestone B in cylinder C (1.06 m ID) to be 5.7°, 0.49 seconds and 34.5° respectively. Using this data the slumping-rolling boundary was predicted and compared in Figure 7.6 to the experimental results. At the higher degrees of fill the model overpredicted the boundary by about 0.3 r/min and the experimental boundary displays a more accentuated bed depth dependency. Nevertheless, in view of the very narrow experimental transition zone and the many simplifying assumptions of the model, the agreement is reasonably good.

The input data for this same material, tested in cylinders A and B, reveals that only the cylinder diameter
and the shear angle differed from those listed above for the larger cylinder. That is, the minimum shear wedge and the dilatancy coefficient were equal. Therefore, the ratio of the chordal trajectory equation for both cylinders reduces to,

\[
\frac{s}{R_M} = \frac{(Fr)_p (\sin n - \cos n \tan \phi_L)_M}{(Fr)_M (\sin n - \cos n \tan \phi_L)_p} \left( \frac{s}{R_p} \right)
\]  

(7.3)

In order to identify the relative contribution of the angular function in Equation (7.3), the shear angle for cylinders A and B was used to predict a boundary for cylinder C; thus ensuring that the angular relations in both cylinders were equal. Negligible difference in the boundary resulted, indicating that Equation (7.3) may be further simplified to,

\[
\frac{s}{R_M} = \frac{(Fr)_p}{(Fr)_M} \left( \frac{s}{R_p} \right)
\]  

(7.4)

Equation (7.4) therefore points to two dimensionless variables for the scale-up of the slumping-rolling boundary with respect to the cylinder diameter, the degree of fill and the Froude number.

**7.2.7 Scale-up**

An attempt will be made in this section to consolidate and simplify the scale-up criteria that have been
presented for material and cylinder variables. The analysis will be aimed at relating the regression coefficients to material and cylinder variables.

The minimum shear wedges and the dilatancy coefficients for all materials and cylinders used in this study are plotted as a function of particle size in Figures 7.7 and 7.8. The bars on these graphs represent the 95% confidence limits for the respective coefficients. The effects of particle shape, particle size and cylinder diameter are evident. Spherical and nodular solids have lower minimum shear wedges than irregular and angular solids. Furthermore, a dependency of particle size on \( \gamma_0 \) is apparent for spherical and nodular solids whereas for the irregular and angular solids the minimum shear wedge appears to be independent of particle size. However, excluding the results for limestone C, whose data exhibited a greater scatter, the minimum shear wedge does appear to display a slight dependency on particle size. The cylinder diameter is observed to have negligible effect on both \( \gamma_0 \) and \( C_1 \). Since the minimum shear wedge and the shear angle are related by the equation

\[
\phi_R = \gamma_0 + \phi_L
\]  

and since lower values of the static angle of repose indicate
Figure 7.7  Minimum shear wedge versus average particle size for all materials tested.
Figure 7.8  Dilatancy coefficient versus average particle size for all materials tested.
easier flowability of solids, therefore the lower the values of $\gamma_0$ the easier the flowability of the bulk solid. Hence, the lower the rotational speed at which a bulk solid will roll, as has been experimentally observed and mathematically predicted. Finally, the dilatancy coefficients shown in Figure 7.8 seem to be only a function of particle size. Therefore, it is apparent that over the range of variables investigated, the minimum shear wedge is the most important material variable which is affected by shape and size and which reflects the location of the slumping-rolling boundary.

The material scale-up criteria could therefore be simplified in terms of $\gamma_0$ and could be expressed in terms of more primary material variables, namely the particle size and shape. Since the latter was not quantified in this study and is inherently very difficult to quantify, the former was used in the following analysis. The experimental boundaries for limestone B and C determined in cylinders A and B are presented on a dimensionless plot of % fill versus Froude number in Figure 7.9a. No agreement in the boundaries is observed. If the Froude numbers of the limestone C boundaries are modified as follows with respect to particle size but for equal cylinder diameter,

$$[Fr]_M = \frac{d_{p_M}}{d_{p_p}} \left(\frac{Fr}_p \right)^{\frac{1}{2}}$$  \hspace{1cm} (7.6)
Figure 7.9 Dimensionless Plot of the experimental boundaries of limestones B and C tested in cylinders A and B (0.4 m ID x 0.46 m L and 0.86 m L respectively).

(a) using Froude number and % Fill
(b) using Froude number, % Fill and size ratio
where the subscript \( M \) represents the model, limestone \( B \), and the subscript \( P \) the prototype, limestone \( C \), the transition zones of both materials overlap as shown in Figure 7.9b. Therefore, even though the particle size is not explicitly expressed in Equation (6.28) it is implicitly accounted for in the minimum shear wedge size, \( \gamma_0 \) (Figure 7.7) and in the shear angle, \( \phi_L \) (Table VII). That it cannot be directly introduced in the original trajectory equation is one of the main drawbacks in applying the principles of mechanics for rigid bodies to bulk solids which has been encountered in other models of bulk solids flow.\(^{108}\) For example, in modelling the discharge rate of bulk solids from bins and hoppers, the particle size does not appear in the equations of kinematics but is known to affect the discharge rate. In fact, it was found in those applications that the ratio of particle diameter to orifice diameter is a scale-up factor.\(^{108}\) In solid-fluid reactors such as spouted beds and fluidized beds the \((D/d_p)\) ratio has also been found to apply as one of the scaling criteria.\(^{100,149-151}\) Rose and Blunt\(^{54}\) have experimentally determined that \((D/d_p)\) is one of the scale-up criteria for slipping in rotary cylinders and indicate that its purpose is to maintain the equality of the number of contact points between the bulk solids and the cylinder wall. Applying this observation to the slumping-rolling boundary and the particle size relationship
in Equation (7.6) would indicate that the functional relationship of this scale-up factor is \((D/d_p)^{1/2}\). Therefore at equal degrees of fill the scale-up criterion would be given by,

\[
[Fr]_M = \left[ \frac{d_p}{D} \right]^{1/2} \left[ \frac{D}{d_p} \right]^{1/2} [Fr]_p \tag{7.7}
\]

The experimental boundaries of limestones B, C and D in cylinders A and B were scaled-up to the particle size of limestone B in cylinder C using Equation 7.7 and were compared to the experimental results of the latter, referred to as the model. The results, plotted in Figure 7.10, indicate that the transition zones of all three scaled limestones overlap. The transition zone of limestone D also overlaps with the model results. The discrepancy between the boundaries of limestones B and C prototypes and those of the model is very small, of the order of 0.3 r/min in cylinders A and B (0.4 m ID) and 0.15 r/min in cylinder C (1.06 m D). Therefore for bulk solids of equal shape the scale-up factors of the slumping-rolling boundary may be simplified to: the degree of fill and the Froude number multiplied by \((D/d)^{1/2}\) as given by Equation (7.7).

As has been discussed in Section 3.2.2, the particle shape of the materials used in this study has not been quantified, hence a scaling criterion based on particle shape could not be developed but is recommended for future
Figure 7.10 Dimensionless Plot of experimental boundaries of limestones B, C and D tested in cylinders A and B, scale-up, using the Froude number, the % Fill and the particle size to cylinder diameter ratio, and compared to the boundaries of limestone B tested in cylinder C.
work. Such an investigation, carried out along the same lines as the current one, would also identify the shape factor which influences the flowability of bulk solids (the sphericity or the Heywood ratios). Since several investigators have suggested that the number of interparticle contact points, the co-ordination number, is related to solids mixing and flowability\textsuperscript{108,152} and since it is a more fundamental property of bulk solids, it would therefore be fruitful to also quantify it. A study of its relationship to the shape factor, particle size and minimum shear wedge, with the aid of Bed-Behaviour Diagrams, should be concurrently carried out using ideally sized and shaped materials.

7.2.8 Gas Evolution and the Temperature of the Solids

In a continuously operated rotary kiln the evolution of gases from the bed of the solids and the temperature of the solids may enhance or retard the solids motion. Some of these aspects have already been addressed in Chapter 5 in connection with segregation. Two effects of temperature and of gas evolution that reflect on the predicted slumping-rolling boundary are of interest; the first is concerned with normal kiln operation and the second touches on certain aspects of overheating the solids.
In some rotary kiln operations, such as limestone calcination gases may be generated in the bed. In rising through the charge to the freeboard, they exert a drag force on the bulk solids which also undergo changes in their physical and chemical nature as they travel through the kiln. The drag force exerted on the bed by the rising gases may be accounted for in the slumping-rolling model by quantifying the mass of gas evolving at a given location along the kiln and modifying the gravity force of the shear wedge accordingly. This would result in a slumping-rolling boundary which will occur at lower rotational speeds than in the absence of these gases. To confirm that these gases have an effect on bed behaviour, the slumping frequency of limestone C, measured in the horizontal cylinder, is compared to the measurements made during cold and hot runs in the UBC pilot kiln (Figure 7.11). The difference between the slumping frequency of the cold run and those for the horizontal cylinder has been attributed to the axial slope of the kiln. The results from the hot run are higher than those of the cold run and are up to 4 slumps/min greater than the frequencies in the horizontal cylinder. In the hot run, the solids attained a temperature of 1107°K at 0.4 m from the solids discharge end. Calcination tests carried out by Brimacombe and Watkinson have showed that at 1107°K, there is a 2.5% conversion to lime and that on complete
Figure 7.11 Slumping frequencies of limestone C in cylinder A (0.4 m ID x 0.46 m L) compared with those measured in the pilot kiln during hot and cold runs.
calcination the change in particle size and the fines generation were minimal. Visual inspection of the stone indicated that no change resulted in particle shape and that calcination had started, hence the carbon dioxide which evolved from the charge may have been responsible for the increase in slumping frequency. As discussed in Chapter 4, a higher slumping frequency curve is indicative of a slumping-rolling boundary occurring at lower rotational speeds. To further verify this conclusion, the contribution of the drag force was estimated. If all the gases generated from the calcination reaction in the bed were evolved through the shear plane, the gravity force of the shear wedge would have been reduced by approximately 10% for the system described above. From Equation (6.28) this would result in the slumping-rolling boundary occurring at a lower rotational speed and hence the evolution of gases would affect bed behaviour.

The physical changes undergone by the solids, will affect their shape and size as they travel through the kiln. For example, observations of industrial calcining kilns have revealed that angular limestone fed to a rotary kiln will discharge as irregular solids. Coal lumps fed with iron oxide pellets will be quickly ground to a powder and will be gasified. The iron oxide pellets in turn may undergo
swelling at the early stages of reduction, then prior to discharging they may decrease again in size. Under these conditions it is not possible to provide a single Bed-Behaviour Diagram to describe the transverse motion of solids over the entire length of the kiln. Rather, several diagrams could be determined for each characteristic zone in the furnace which could be used to establish effective operating strategies and which could be incorporated into future kiln designs.

Overheating the solids could have a serious detrimental effect on bed behaviour due to the agglomeration of solids in the bed. This would increase the internal friction of the bulk solids, provide poor gas-solid contact as well as poor solids mixing. To anticipate this occurrence a series of Bed Behaviour Diagrams could be experimentally determined over a range of operating temperatures. The shift in the slumping-rolling boundary, corresponding to various operating solids temperatures, could be determined and the conditions of solids overheating identified. This information could be useful in developing an operating practice for industrial kilns as well as in providing useful information to indicate some design and operating constraints.
7.3 Slipping

The predictions of the slipping model, presented in Chapter 6, are shown in Figure 7.12 as a plot of the resultant bed inclination, $\phi'$, which is calculated for a given fraction of critical speed and degree of fill versus the slipping angle. These results were obtained for three kiln diameters: 0.4 m, 1.0 m and 2.7 m; as well as for three Froude numbers: 0, 0.02 and 0.075. For equal slip angles and Froude numbers the model predictions for the three kiln diameters are identical at equal fill ratios. It is further observed in Figure 7.12 that the effect of rotational speed is negligible. For example, for a slipping angle of 40° and a 10% fill the equilibrium angle of bed inclination for a Froude number of 0 and 0.075 are 52.5° and 55° respectively. The negligible effect of rotational speed is further substantiated by industrial observations of ball mills where changes in rotational speed have had a negligible effect in eliminating slipping.\textsuperscript{2,4,55} This will be further discussed in the following section. Therefore the equilibrium angle of bed inclination need only be compared to the static and dynamic angles of repose of a material rather than $\phi_U$ and $\phi_D$ as described in Section 6.3.

The application of Figure 7.12 for predicting the occurrence of slipping will now be illustrated using a
Figure 7.12  Predictions of the slipping model for kiln diameters of 0.4 m, 1.0 m and 2.7 m.
bed/wall angle of slip of 30° and static and dynamic angles of repose of 40° and 35° respectively. When the bed is normally slumping the slipping angle of equilibrium is compared to the static angle of repose. For example, locate in Figure 7.12 the co-ordinates (30°, 40°). If the degree of fill in the kiln is 5%, then the equilibrium bed inclination for the slipping force and moment balance is 35°. Hence, the bed will slip prior to reaching the slumping angle of 40°. If on the other hand the degree of fill is 20%, the equilibrium angle of slip is then 45° and the bed will slump before reaching the equilibrium bed inclination of slip. Therefore, it appears in this case that all degrees of fill less than 12.5% will result in the bed slipping while those greater than 12.5% will result in the occurrence of slumping. If the bed were rolling then the slipping boundary would occur at the 10% fill. Hence, the upper left of Figure 7.12 represents the no slip situation and the bottom right, the slipping situation.

From this analysis it is clear that to avoid slipping in kilns, only two variables are of interest: the bed/wall friction angle and the degree of fill. In kiln operations, the former may be increased by roughening the wall which may be effected in a number of ways. In the lime industry the kiln wall may be roughened by feeding salt into the kiln;
however, this has the accompanying detrimental effect of decreasing the reactivity of the lime subsequently produced and is therefore not widely applied. Alternately, operators are faced with a total production shut down of up to one week to manually roughen the refractory wall. The placement of kiln internals has also the effect of increasing the solids/wall friction angle and is reported to be successfully applied. Other methods for avoiding slipping include increasing the feed rate to the kiln, thus increasing the fill ratio. Caution would be advised in taking this approach as an unacceptable product could be obtained due to the possible increase in solids residence time at temperature which may be compensated for by a corresponding increase in rotational speed, for example. A more novel approach would be to control the accretion build-up on the kiln wall thus roughening the wall or increasing the hold-up of the kiln. Again, to ensure that the solids residence time is not excessively increased due to the presence of accretion dams the rotational speed of the kiln may also be increased.

A further complication in characterizing slipping in kilns is that the wall roughness and the physical properties of the solids may vary along the kiln length. Under these conditions, the solids and the solids/wall friction can be
characterized using the methods described in this thesis (Section 3.4) and an operating strategy developed for each section or zone in the kiln. This approach could also be used to help identify those operating conditions when slipping might be encountered and appropriate preventative action taken.

7.4 The Complete Bed-Behaviour Diagram

All six predicted modes of bed behaviour are plotted in Figure 7.13 for gravel in cylinder A. Other predictions are shown in Figures 7.14 to 7.16. The solid lines on the diagrams denote the boundaries which were calculated using the respective models presented in Chapter 6. In the case of gravel (Figure 7.13) since the bed/wall friction angle, $\phi_s$, for this case was 48.8° and the static and dynamic angles of repose were 40.7° and 37.5° respectively, no slipping took place. Had the cylinder wall been smoother, however, with a $\phi_s = 35°$; the slipping boundary would have been given as indicated in Figure 7.13, by the $\phi_s = 35°$ lines, below which only slipping would have occurred. As the bed/wall friction angle decreases (i.e. smoother wall) the area on the diagram dominated by slipping increases. As shown in the previous section the rotational speed has a negligible effect on the slipping boundary. Furthermore, for the same $\phi_s$, the slipping boundary in the slumping zone
Figure 7.13  Complete Bed-Behaviour Diagram of gravel in cylinder A (0.4 m ID x 0.46 m L).

Figure 7.14  Complete Bed-Behaviour Diagram of nickel oxide in cylinder A (0.4 m ID x 0.46 m L).
occurs at a higher degree of fill than that in the rolling or cascading zones. This is due to the dynamic angle of repose being lower than the static. Hence, the slipping boundary is affected by the bed/wall friction angle, $\phi_s$, and the degree of fill, as earlier discussed. It is believed that the $(D/d_p)$ scale-up ratio proposed by Rose and Blunt is incorporated in the measurement.

The criteria proposed in this thesis for slipping are in agreement with the experimental and industrial observations of other workers. The experiments of Rose and Blunt on ball mills showed that the Froude number was not an important scaling criterion for slipping. Although their results are presented on a plot similar to the Bed Behaviour Diagram, they cannot be verified using the current model as the $\phi_R$ and $\phi_D$ of the materials used were not reported. The negligible effect of rotational speed on slipping was also observed by Korotich using a cylinder, 0.3 m D, rotating between 10-65 r/min (Froude numbers from 0.02 to 0.71). The charge in the cylinder always slipped over that range of rotational speeds at all degrees of fill up to 20% when slipping ceased. The observations of Gow et al. and Duda on ball mills are also similar. In rotary kilns, Ronco observed slipping to cease with increasing fill ratio. Finally the observations of Lehmberg et al.
seem at first sight to contradict the observations of all the above workers with respect to the effect of rotational speed on slipping. On closer examination their results could be easily explained using the Bed Behaviour Diagram and the slipping model. They had observed slipping at 2 r/min; by increasing the rotational speed to 5 r/min, slipping ceased and rolling set in. With a rougher cylinder wall or a higher static angle of repose of the charge, these workers may have otherwise observed slumping at 2 r/min. By increasing the rotational speed to 5 r/min the (slumping)-rolling boundary was crossed and the solids were rolling, since the rolling-slipping boundary probably occurred at lower fill ratios. Therefore, the slipping model predictions are in agreement with the observations reported in the literature.

At the higher degrees of fill, in Figure 7.13, rolling is absent and slumping changes directly to cascading. This follows from the definition of cascading given in Chapter 6 and results in the occurrence of a triple point between slumping, rolling and cascading which will be a function of the dynamic angle of repose, $\phi_D$. The lower $\phi_D$, the higher the degree of fill required for this triple point which is illustrated for nickel oxide ($\phi_D = 30.2^\circ$) in Figure 7.14. Comparing the cascading-cataracting boundaries of both
Figures 7.13 and 7.14, it is evident that the lower $\phi_D$, the higher the Froude number required for cataracting at equal degrees of fill. The full cataracting line on both these diagrams represents the rotational speed for which the parabolic trajectory of the outermost granule on the apex intersects the cylinder wall at the chordal base of the charge. As expected, it behaves similarly to the cascading-­cataracting boundary. It is also observed that with increasing bed depth the rotational speed required to get cataracting decreases. This is due to the solids at the apex being projected into the freeboard at lower rotational speeds. Hence, cataracting is really an extension of cascading.

Finally, the effect of cylinder diameter is shown in Figures 7.15 and 7.16. The dynamic angle of repose of limestone C in cylinder A is 36°; while that for limestone B in cylinder C is 36.5°. Both materials nearly have equal $(D/d_p)$ ratios as shown earlier in this chapter. Comparing their complete Bed-Behaviour Diagrams, it is observed that the boundaries for slumping, rolling, cascading, cataracting and centrifuging occur at equal Froude numbers and degree of fills. The minor deviations observed are due to the small differences in $(D/d_p)$ and $\phi_D$. Hence, the scale-up criteria for cascading and cataracting are the dynamic
Figure 7.15  Complete Bed-Behaviour Diagram of limestone C in cylinder A (0.4 m ID x 0.46 m L).

Figure 7.16  Complete Bed-Behaviour Diagram of limestone B in cylinder C (1.06 m ID x 0.4 m L).
angle of repose, the Froude number and the degree of fill rather than the angle of detachment which has often been suggested in the literature.\textsuperscript{56,79}

It is very difficult to directly compare experimental observations of cascading and cataracting reported in the literature with the model predictions because the majority of the former were made at degrees of fill of 30\% and greater. Hence, only trends in the observations could be compared. Cataracting has been observed to occur at Froude numbers in the range of 0.25 to 0.72\textsuperscript{2,61,154} which is in agreement with the predicted results. The speed of the particles falling down the slope of the bed is reported by Müller\textsuperscript{52} to be a function of the slope of the bed, hence cascading is a function of the dynamic angle of friction as shown in this study. Finally, Oyama\textsuperscript{72} presents the following empirical relationship for predicting the cascading-ctataracting and the full cataracting boundaries,

\begin{equation}
\frac{\pi}{n} = K \frac{D^{0.47}}{\theta^{0.14}}
\end{equation}

where $K$ is a constant equal to 54 for the cascading-ctataracting boundary and 72 for the full cataracting condition. Although Oyama has not accounted for the dynamic angle of repose, the predicted boundaries using Equation (7.8) agree within $\pm 15\%$ of the rotational speeds predicted
with the models presented in Chapter 6. The cascading and cataracting predictions are therefore reliable.

7.5 Other Aspects

7.5.1 Internal Friction

Jenicke and Johanson have developed a shear cell for measuring the internal friction of cohesive powders which has not been widely applied to cohesionless bulk solids. Since their testing method and data analysis is fully described and discussed in the literature, only its relevance to the Bed-Behaviour Diagram will be discussed. In a rotary cylinder, when the limiting friction force is reached the bulk solids fail at the shear plane, which represents the maximum shear stress. Its magnitude may be estimated from the component of the gravitational force on the shear wedge parallel to the shear plane. The normal stress associated with it would be given by the stress normal to the shear plane in the solids bed. Mohr's circle may therefore be drawn for each type of bulk solid in each cylinder tested. Since cohesionless bulk solids have their yield locus passing through the origin of the stress axes, the internal angle of friction may thus be identified. Such plots have been derived by Jenicke and Johanson and are called Yield Loci Diagrams and as described above may relate
the internal angle of friction of a bulk solid to the shear angle and the minimum shear wedge size. Since the Yield Locii Diagrams have been applied to predict the conditions of flow and no flow in bins and hoppers and since it may also be used to describe a slump, its relationship to the Bed-Behaviour Diagram should be thoroughly investigated in future research programs.

7.5.2 The Phase Rule

From the discussion in the above section, the variables affecting the slumping-rolling boundary may be reduced to four: the degree of fill, the internal angle of friction, the gravity and the centrifugal forces. For the remaining boundaries on a Bed Behaviour Diagram the internal angle of friction may be replaced by the dynamic angle of repose or the bed-wall friction angle. Hence, for each equilibrium state of solids motion, slipping, slumping etc., four independent parameters define a system: a material variable, the frictional behaviour of the solids; a system variable, the degree of fill; and two independent external forces acting on the system, the gravity and the centrifugal forces. The latter two may be regarded as force components, C, which are required to describe an equilibrium state of motion, P. If the only independent parameter held constant is the gravity force, the system will have three
degrees of freedom. This corresponds to only one equili-
brium state of solids motion, as seen on a Bed-Behaviour
Diagram, for example slumping, i.e. $P = 1$. If two in-
dependent parameters are held constant, for example the
gravity force and the material, then only two degrees of
freedom exist, $F = 2$, the centrifugal force and the degree
of fill. This condition corresponds to a boundary between
two modes of bed behaviour, $P = 2$. For the case when three
independent parameters are held constant, only one degree
of freedom exists, $F = 1$. It follows that the equilibrium
states of motion will be three, $P = 3$, a triple point.
These equilibrium states of motion may be regarded as phases
in a context similar to that used to describe Solid-Fluid
Phase Diagrams.\textsuperscript{149-151, 159} The relationship of the para-
meters described above could then be termed a phase rule
and may be expressed as follows,

$$P + F = C + 2 \tag{7.9}$$

where the 2 represents the degree of fill and the frictional
properties of a bulk solid.

This same approach when applied to the phase diagrams
of fluid-solid systems\textsuperscript{149-151, 157} would result in the super-
ficial velocity, plotted on the abscissa, to be indicative
of the ratio of the buoyancy force to the gravity force
by means of a modified Froude number and is therefore a basically similar representation to the Bed-Behaviour Diagram. It follows, therefore, that when gases are evolved from rotary kiln beds due to the occurrence of chemical reactions or drying there would be three components in the system, the gravity force, the centrifugal force and the buoyancy force. Hence, a ternary Bed-Behaviour Diagram or a modified binary diagram could be developed.

Since the Bed-Behaviour Diagram, the Solids-Fluids Phase Diagram and the Yield Locii Diagram each represent an equilibrium state of the bulk solids and since it appears that they may be related as described above, it is proposed that future work be directed to study their interrelationships and to investigate whether, as has been suggested, a thermodynamic basis does exist for bulk solids motion. It was not feasible in this study to develop these aspects of the Bed-Behaviour Diagram and it is recommended that they be pursued in future studies.
Chapter 8

SUMMARY AND CONCLUSIONS

8.1 Summary and Conclusions

The various modes of bed behaviour encountered in rotary cylinders have been qualitatively described based on the observations reported in the literature and in this study. A Bed-Behaviour Diagram, which is a plot of bed depth versus rotational speed, was proposed to illustrate the regions of dominance of each of these modes. An in-depth experimental analysis of the slumping-rolling boundary was carried out to illustrate the effects of material, cylinder and operating variables. It was shown that a decrease in particle size and a change in particle shape from angular to spherical resulted in the boundary occurring at lower rotational speeds; increasing the diameter of the cylinder also displayed the same effect. Slumping beds were experimentally characterized using the static and the upper angles of repose, the shear angle, the slumping frequency; while the dynamic angle of repose and the active layer thickness were used to characterize rolling.

With the aid of the Bed-Behaviour Diagram, the effect of the addition of fine particles on the flow characteristics
and segregation patterns of bulk solids was investigated. When compared to the coarser materials, the presence of fines did not affect the angles of repose, the slumping frequencies, the shear angles or the active layer thicknesses of the three mixtures tested. Slumping was observed to be enhanced for the limestone mixture and rolling for the two sand mixtures. The mechanism of transverse segregation of fine solids in rotary cylinders and the composition and location of the central segregated core, the 'kidney', were also identified. It was found that the fines segregated in the charge by the percolation mechanism and occupied a core of the same shape as the bed. Its composition corresponded to the maximum bed contraction, $\Delta \varepsilon_{\text{max}}$, of the components mixed. A second segregation core was also identified to be in the top portion of the bed near the wall. It occurred for slumping and rolling beds and is believed to be responsible for slipping in industrial rotary kilns. Longitudinal segregation was also observed to take place and was found to be dependent on bed behaviour.

A semi-empirical model was developed to predict the slumping-rolling boundary of bulk solids. The model was successfully applied to predict the effects of bed depth, particle size, particle shape, combined effects of size and shape and of cylinder diameter. Based on the
experimental and theoretical analyses, scale-up criteria have been proposed for material, cylinder and operating variables. These were the Froude number, the degree of fill and the minimum shear wedge or the \( (D/d_p) \) ratio for solids of equal shape.

Slipping, cascading and cataracting have also been mathematically modelled in this study and their predictions were in agreement with experimental observations reported in the literature. A comprehensive analysis of the scale-up criteria has been carried out for each of the bed behaviours and has been related to the Bed-Behaviour Diagram. These are, for slipping, the bed/wall slip angle and the degree of fill, and for cascading and cataracting, the Froude number, the degree of fill and the dynamic angle of repose.

8.2 Recommendations for Future Work

It is recommended that future studies be carried out in the following areas:

1. A more detailed verification of the width of the central segregation core model should be undertaken for varying coarse to fine size ratios, various bed depths, particle shapes, densities, and cylinder diameters. The model should also be refined to
predict the distance the surface of the cores is recessed from the bed surface.

2. The cause of the shift in the slumping-rolling boundary with fines addition should be investigated.

3. The size of particles associated with the second segregation zone should be determined and a size criterion developed.

4. The effects of segregation on inclined continuous kiln operation should also be investigated.

5. Using equally sized solids of ideal shapes, the applicability of the "Heywood ratios" or the "sphericity" to quantify particle shape and its effect on bed behaviour could be established.

6. The characterization of slumping using the slumping frequency and of rolling using the active layer thickness allows a new approach to modelling the residence time of solids in rotary kilns. Particularly of interest would be those cases where two of three types of bed behaviour occur in a given kiln.

7. This characterization of the solids motion may also be applied to formulate a more accurate description of the gas-solids heat flow step in a kiln.
8. Another area recommended for future work is the quantitative study of the effect of gas evolution from the charge on its motion.

9. Finally, research in bulk solids flow properties as described by Fluid-Solids Phase Diagrams, Bed-Behaviour Diagrams and Yield Locii Diagrams will prove most useful in helping to increase our fundamental understanding of the flow of bulk solids and of their mixing characteristics.
BIBLIOGRAPHY


16 Zimmer, W., K.V.S. Corporation, Private Communications.


18 Whitlock, E., Lime Division, Domtar Chemicals Ltd., Private Communications.


29 Pearce, K.W., Investigations into Heat Transfer from Rotary Kiln Flames, Chapter IV, Source: Brimacombe, J.K., University of British Columbia, Canada.

30 Cross M., Private Communications.


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Zengler, R., Modellversuche über den Materialtransport in Drehrohröfen [Examination of Material Transport in Rotary Kilns Using a Model Installation], Doctoral Thesis, Technischen Universität Clausthal, July 1974, [Translated by L. Miller, University of British Columbia, Canada].


Ronco, J.J., "Tecnología de las Operaciones y los Procesos de la Industria Química" [Technology of Operations and Processes in the Chemical Industry], Industria y Química, vol. 20, 1960, pp. 605-614, [Translated by E. Hoefele, Petro Canada Research, Canada].


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<td>59</td>
<td>Yamaguchi, K.</td>
<td>&quot;Mixing of Solids and its Recent Progress&quot;</td>
<td>Seramikkusu, vol. 6, no. 1, 1971, pp. 47-54 [Translated by M. Hori, University of British Columbia, Canada].</td>
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Miller, L.L., Germanic Studies Department, University of British Columbia, Canada, Private Communications.


Khodorov, E.I., The Movement of Material in Rotary Kilns, Moscow, 1957 [Translated by Gill, W., University of British Columbia, Canada].


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Conrad, Von F., Cremer, E. and Kraus, Th., "Über das Haften von Magnesitpulvern auf fester Unterlage (I)" [The Adhesion of Magnesite Powders to a Solids Base (I)], Radex-Rundschau, 1951, Heft 6, pp. 227-233 [Translated by Bell, L., University of British Columbia, Canada].


132 Sunnergren, E., Bethlehem Steel Corp., Unpublished research.


Appendix A

Results of Screen Analyses
TABLE A.1  Screen Analysis Results for Gravel

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<td>1.5</td>
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Average Particle Size (mm) 3.0
TABLE A.2  Screen Analysis Results for Iron Oxide

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</tr>
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<tr>
<td><strong>Average Particle Size (mm)</strong></td>
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<td><strong>Total</strong></td>
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**Average Particle Size (mm)**

| Average Particle Size (mm) | 0.58 |
### Screen Analysis Results for Limestone E

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**Average Particle Size (mm):** 0.54
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<td>- 1/4 in + 3 1/2 in</td>
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<tr>
<td>- 3 1/2</td>
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<td>Total</td>
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**Average Particle Size (mm):** 4.9
TABLE A.9 Screen Analysis Results for Sand B

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Average Particle Size (mm) 0.50
## TABLE A.10  
**Screen Analysis Results for Sand C**

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| Total                                   | 99.8                        |

| Average Particle Size (mm)              | 0.23                        |
APPENDIX B

Identification of the Run Numbers Corresponding to the Bed Behaviour Observations.
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<td>30.5</td>
<td>182</td>
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</tbody>
</table>
TABLE B.1
Identification of the Run Numbers
Corresponding to the Bed Behaviour Observations

<table>
<thead>
<tr>
<th>Figure</th>
<th>Material</th>
<th>Cylinder</th>
<th>Bed Depth (mm)</th>
<th>Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.13</td>
<td>Limestone D</td>
<td>A</td>
<td>87.5</td>
<td>199</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>71.0</td>
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<td>56.0</td>
<td>201</td>
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<td>202</td>
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<td></td>
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<td>31.0</td>
<td>203</td>
</tr>
<tr>
<td>4.14</td>
<td>Sand B</td>
<td>A</td>
<td>92.0</td>
<td>194</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>73.0</td>
<td>195</td>
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<td>4.16</td>
<td>Gravel</td>
<td>A</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>65.0</td>
<td>205</td>
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<td>84.0</td>
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<td>C</td>
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<td>106.0</td>
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<td>165.0</td>
<td>219</td>
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</table>
TABLE B.2  Identification of the Run Numbers
Corresponding to the Bed Behaviour Observations
(Segregation Tests)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Material</th>
<th>Cylinder</th>
<th>Bed Depth (mm)</th>
<th>Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.2</td>
<td>Sand Mix A</td>
<td>A</td>
<td>92.5</td>
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<td></td>
<td></td>
<td>72.</td>
<td>122</td>
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<tr>
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<td></td>
<td></td>
<td>54.</td>
<td>123</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>34.</td>
<td>124</td>
</tr>
<tr>
<td>5.3</td>
<td>Sand Mix B</td>
<td>A</td>
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<td></td>
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<td>62.</td>
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<td>128</td>
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<td>77.5</td>
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<td></td>
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<td>52.</td>
<td>130</td>
</tr>
<tr>
<td>5.4</td>
<td>Limestone Mix A</td>
<td>A</td>
<td>93.</td>
<td>132</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>71.5</td>
<td>133</td>
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<td></td>
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<td>53.</td>
<td>134</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>38.5</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>52.</td>
<td>145</td>
</tr>
</tbody>
</table>
Appendix C

Experimental and Regression Results of the Times to Maximum and Minimum Bed Inclinations.
**Figure C.1** The $t_1$ and $t_2$ measured results as well as the regression curves for gravel in cylinder C (1.06 m ID x 0.4 m L).

**Figure C.2** The $t_1$ and $t_2$ measured results as well as the regression curves for iron oxide in cylinder A (0.4 m ID x 0.46 m L).
Figure C.3 The $t_1$ and $t_2$ measured results as well as the regression curves for limestone B in cylinder B (0.4 m ID x 0.86 m L).

Figure C.4 The $t_1$ and $t_2$ measured results as well as the regression curves for limestone B in cylinder C (1.06 m ID x 0.4 m L).
### Limestone C

<table>
<thead>
<tr>
<th>$t_1$ (m)</th>
<th>$t_2$ (m)</th>
<th>Bed depth (m)</th>
<th>% Fill</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0039</td>
<td>0.029</td>
<td>0.06</td>
<td>3.2</td>
</tr>
<tr>
<td>0.0052</td>
<td>0.029</td>
<td>0.08</td>
<td>1.1</td>
</tr>
<tr>
<td>0.0068</td>
<td>0.029</td>
<td>0.09</td>
<td>1.8</td>
</tr>
<tr>
<td>0.014</td>
<td>0.039</td>
<td>0.14</td>
<td>4.9</td>
</tr>
<tr>
<td>0.030</td>
<td>0.039</td>
<td>0.8</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Figure C.5 The $t_1$ and $t_2$ measured results as well as the regression curves for limestone C in cylinder A (0.4 m ID x 0.46 m L).

### Limestone C

<table>
<thead>
<tr>
<th>$t_1$ (m)</th>
<th>$t_2$ (m)</th>
<th>Bed depth (m)</th>
<th>% Fill</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0030</td>
<td>0.029</td>
<td>0.06</td>
<td>3.2</td>
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</tbody>
</table>

Figure C.6 The $t_1$ and $t_2$ measured results as well as the regression curves for limestone C in cylinder C (1.06 m ID x 0.4 mL).
Figure C.7 The $t_1$ and $t_2$ measured results as well as the regression curves for Limestone D in cylinder A (0.4 m ID x 0.46 m L).
Figure C.8 The $t_1$ and $t_2$ measured results as well as the regression curves for limestone F in cylinder C (1.06 m ID x 0.4 m L).
**Figure C.9** The $t_1$ and $t_2$ measured results as well as the regression curves for nickel oxide in cylinder A (0.4 m ID x 0.46 m L).
Figure C.10 The $t_1$ and $t_2$ measured results as well as the regression curves for sand B in cylinder A (0.4 m ID x 0.46 m L).
Appendix D

The Co-Ordinates of the Wedge Centroids
The calculation of the centroids of the upper and lower wedges of a slumping bed will be presented in this appendix. These wedges are bounded by the initial ($\phi_U$) and final ($\phi_L$) surface inclinations for a slumping bed of unit cylinder length (Figure D.1). Applying the method of composite shapes to the upper wedge ABC yields the following equations for the co-ordinates of the centroid ABC,

$$x_{ABC} = x_{OAB}^A + x_{ODB}^A - x_{OEA}^A - x_{ODE}^A - x_{DEC}^A$$ \hspace{1cm} \text{(D.1)}

and,

$$y_{ABC} = y_{OAB}^A + y_{ODB}^A - y_{OEA}^A - y_{ODE}^A - y_{DEC}^A$$ \hspace{1cm} \text{(D.2)}

where,

$$A_{ABC} = A_{OAB} + A_{ODB} - A_{OEA} - A_{ODE} - A_{DEC}$$ \hspace{1cm} \text{(D.3)}

The centroid of wedge PCS will be calculated using the geometrical symmetry of the system. Before calculating the terms in equations (D.1) to (D.3) the geometry of the system must first be defined.

Assume that the apexes of the bed before and after slumping lie in the first quadrant (Figure D.1). Therefore,
Figure D.1 Geometrical construction of the bed for the apex in the first quadrant for both the $\phi_U$ and $\phi_L$ bed inclinations.
$\angle ATU = \angle OGC = \phi_U$

$\angle BQR = \angle GHC = \phi_L$

$\angle ACB = \gamma = \phi_U - \phi_L \quad (D.4)$

Construct,

$OE \perp AS$

and, $OD \perp BP$

In $\triangle OEA$ and $\triangle ODB$,

$\begin{align*}
\text{given} & \quad OA = OB = R \\
\angle OEA = \angle ODB &= 90^\circ \\
OE &= OD = (R-H) \\
\therefore \triangle OEA \equiv \triangle ODB \ (\text{sas})
\end{align*}$

$\angle OAE = \angle OBC = \delta$

and, $\delta = \sin^{-1} \left( \frac{R-H}{R} \right) \quad (D.5)$

From $\triangle OGA$,

$\angle AGH = \angle GOA + \angle OAG$

$\therefore \phi_U = \theta + \delta \quad (D.6)$

Also in $\triangle OBH$,

$\angle BHI = \angle BOH + \angle OBH$

$\therefore \phi_L = (\theta - \angle AOB) + \delta$

Substituting equation $(D.4)$ and $(D.6)$ and simplifying yields,

$\angle AOB = \gamma \quad (D.7)$
Now,
\[ \angle AOE = \angle AOB + \angle BOG + \angle GOE \]

and,
\[ \angle BOD = \angle BOG + \angle GOE + \angle EOD \]

\[ \therefore \triangle OEA \cong \triangle ODB \]
\[ \angle AOE = \angle BOD \]

\[ \therefore \angle AOB = \angle EOD = \gamma \] \hspace{1cm} (D.8)

Furthermore, in \triangle EOC and \triangle COD,

OC is common

OE = OD

and \[ \angle OEC = \angle ODC = 90^\circ \]

\[ \therefore \triangle EOC \cong \triangle COD \text{ (sas)} \]
\[ \angle EOC = \angle COD = \frac{\gamma}{2} \] \hspace{1cm} (D.9)

and \[ \angle ECO = \angle DCO = 90 - \frac{\gamma}{2} \]

It follows that,

\[ \triangle EOF \cong \triangle ODF \text{ (sas)} \]

and \[ \triangle EFC \cong \triangle DFC \text{ (sas)} \]

\[ \therefore \ DF = EF = (R-H) \sin \frac{\gamma}{2} \]

Hence, the geometry of wedge ABC is defined \ QEF.
The centroids and areas of the components listed in equations (D.1) to (D.3) may now be calculated. They consist of two basic geometrical shapes: the circular sector and the triangle. The centroid of the sector is calculated from (see Figure D.2),

$$
\bar{x}_{OAB} = \frac{\int x \, dA'}{\int dA'} \quad (D.10)
$$

where

$$
x = \frac{2}{3} R \cos \psi \quad (D.11)
$$

and

$$
dA' = \frac{1}{2} R^2 \, d\psi \quad (D.12)
$$

Substituting equation (D.11) and (D.12) into (D.10), integrating and simplifying yields,

$$
\bar{x}_{OAB} = \frac{120}{\pi \gamma} R \left[ \sin \theta - \sin (\theta - \gamma) \right] \quad (D.13)
$$

Similarly,

$$
\bar{y}_{OAB} = \frac{120}{\pi \gamma} R \left[ \cos (\theta - \gamma) - \cos \theta \right] \quad (D.14)
$$

and

$$
A_{OAB} = \frac{\pi}{360} R^2 \gamma \quad (D.15)
$$
Figure D.2 Construction illustrating the derivation of the centroid of sector OAB.
The centroid of a triangle is given by the point of intersection of the lines connecting the vertices and the midpoints of their opposite sides. It will be sufficient to solve for this point of intersection using two straight lines. The required co-ordinates of the vertices of the triangles are listed in Table D.1 while in Table D.2 to D.3 the derived centroid co-ordinates and areas of the required triangles are listed. Substituting the areas from Table D.3 in equation (D.3) yields,

\[ A_{ABC} = \frac{\pi}{360} R^2 \gamma - (R-H)^2 \tan \frac{\gamma}{2} \]  \hspace{1cm} (D.16)

Finally, the centroid of wedge ABC may be calculated. If either or both apexes, A and B, lie in the fourth quadrant the above equations will still hold if the definitions of \(\phi_U\), \(\phi_L\), \(\gamma\), \(\delta\) and \(\theta\) are maintained as shown in Figure D.3 and D.4.

The centroid of the lower wedge PSC must now be calculated (Figure D.5). In \(\triangle OCA\) and \(\triangle OCP\),

- \(OC\) is common
- \(OA = OP\)
- \(PC = AC\)

\[ \therefore \triangle OCA \equiv \triangle OCP \]
\[ \angle AOC = \angle POC \]
Figure D.3  Geometrical construction of the bed for the apex in the first and fourth quadrants at both the $\phi_U$ and $\phi_L$ bed inclinations respectively.
Figure D.4 Geometrical construction of the bed for the apex in the fourth quadrant at both the $\phi_U$ and $\phi_L$ bed inclinations.
Figure D.5 Geometrical construction of the bed illustrating the calculation of the centroid of PSC.
### TABLE D.1  Vertex Co-Ordinates

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Co-Ordinates ((x,y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>((0,0))</td>
</tr>
<tr>
<td>A</td>
<td>((R \cos \theta, R \sin \theta))</td>
</tr>
<tr>
<td>B</td>
<td>((R \cos (\theta - \gamma), -R \sin (\theta - \gamma)))</td>
</tr>
<tr>
<td>D</td>
<td>(((R-H) \sin \phi_L, -(R-H) \cos \phi_L))</td>
</tr>
<tr>
<td>E</td>
<td>(((R-H) \sin \phi_U, -(R-H) \cos \phi_U))</td>
</tr>
</tbody>
</table>

### TABLE D.2  Centroid Co-Ordinate of Component Triangle Areas

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Abscissa ((x))</th>
<th>Ordinate ((y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>ODB</td>
<td>(\frac{1}{3} (R \cos (\theta - \gamma) + (R-H) \sin \phi_L))</td>
<td>(\frac{1}{3} (R \sin (\theta - \gamma) - (R-H) \cos \phi_L))</td>
</tr>
<tr>
<td>OEA</td>
<td>(\frac{1}{3} ((R-H) \sin \phi_U + R \cos \theta))</td>
<td>(\frac{1}{3} (R \sin \theta - (R-H) \cos \phi_U))</td>
</tr>
<tr>
<td>ODE</td>
<td>(\frac{2}{3} (R-H) \cos \frac{\gamma}{2} \sin \frac{(\phi_U - \gamma)}{2})</td>
<td>(-\frac{2}{3} (R-H) \cos \frac{\gamma}{2} \cos \frac{(\phi_U - \gamma)}{2})</td>
</tr>
<tr>
<td>DEC</td>
<td>((R-H) \sin \frac{\phi_U - \gamma}{2})</td>
<td>(-\frac{1}{2} (R-H) \cos \frac{(\phi_U - \gamma)}{2})</td>
</tr>
<tr>
<td></td>
<td>((\cos \frac{\gamma}{2} + \frac{1}{3} \sin \frac{\gamma}{2} \tan \frac{\gamma}{2}))</td>
<td>((\cos \frac{\gamma}{2} + \frac{1}{3} \sin \frac{\gamma}{2} \tan \frac{\gamma}{2}))</td>
</tr>
</tbody>
</table>
### TABLE D.3 Areas of Triangular Components

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>ODB</td>
<td>$\frac{1}{2} (R^2 - (R-H)^2)^{\frac{1}{2}} (R-H)$</td>
</tr>
<tr>
<td>OEA</td>
<td>$\frac{1}{2} (R^2 - (R-H)^2)^{\frac{1}{2}} (R-H)$</td>
</tr>
<tr>
<td>ODE</td>
<td>$(R-H)^2 \cos \frac{\gamma}{2} \sin \frac{\gamma}{2}$</td>
</tr>
<tr>
<td>DEC</td>
<td>$(R-H) \sin \frac{\gamma}{2})^2 \tan \frac{\gamma}{2}$</td>
</tr>
</tbody>
</table>
and OC is a line of symmetry of the upper and lower wedges.

In \( \triangle HOD \),

\[
\angle HOD = 90 - \phi_L
\]

Using equation (D.9),

\[
\angle HOC = 90 - \phi_L - \frac{\gamma}{2}
\]

\[
\therefore \angle JOC = 90 - \phi_L - \gamma + \tan^{-1} \frac{y_{ABC}}{x_{ABC}}
\]

\[
\angle KOC = \angle JOC
\]

and

\[
\angle HOK = \alpha = 180 - 2\phi_L - \gamma + \tan^{-1} \frac{y_{ABC}}{x_{ABC}}
\]

Substituting equation (D.4) into (D.17),

\[
\alpha = 180 - 2\phi_U + \gamma + \tan^{-1} \frac{y_{ABC}}{x_{ABC}}
\]

also

\[
R_{ABC} = (x_{ABC}^2 + y_{ABC}^2)^{\frac{1}{2}}
\]

\[
\therefore x_{PSC} = R_{ABC} \cos \alpha
\]

and

\[
y_{PSC} = R_{ABC} \sin \alpha
\]

Therefore the required equations to calculate the co-ordinates of the centroids of the upper and lower wedges have been derived.
APPENDIX E

Program Listing for Slumping-Rolling Boundary.
This program calculates the rotational speed at which a given bed of solids will change from a slumping to a rolling mode of bed behavior.

The inputs to this model are:
- \( R \) - radius of rotary kiln (cm).
- \( \Delta \theta_1 \) - angular change in bed inclination between max. and min. bed inclination as rotational speed approaches zero for a bed moving from the min. to max. bed inclination (degrees).
- \( \Delta \theta_2 \) - same as \( \Delta \theta_1 \) but for a bed going from max. to min. bed inclination (degrees).
- \( C_2 \) - slope of \( \Delta \theta_1 \) or \( \Delta \theta_2 \) vs RPM curves (secs).
- \( \Phi_\text{IR} \) - also called lamca in log - lowest bed inclination (degrees).

REAL \( K, R, GMA01, GMA02, C2, \Phi_\text{IR} \)
DIMENSION \( DF(100), DFC1(100), H(100), \Delta E(100), \Delta T(100) \)
INTEGER CHARGE(20)
READ(5,1)CHARGE
FORMAT(20A1)
R=50.0,GMA01,GMA02,C2,PHIR
FORMAT(F7.2,2X,F5.2,2X,F5.2,2X,F6.3,2X,F5.1)
WRITE(6,21)R,GMA01,GMA02,C2,PHIR
FORMAT(A7,2X,'Bed behaviour diagram for:
A7,2X,'Bed behaviour model of slumping/rolling boundary

CROSS SECTIONAL AREA OF KILN.
\( A=3.141593*(R^2) \)

CALCULATION OF BED DEPTHS.

\[ DF(I)=1.0 \]
\[ H(I)=1.0 \]
\[ R2=ACOS(R1/R) \]
\[ R3=SCRT((R1**2)-(R2**2)) \]
\[ DFC(I)=(((R**2)-(R2**2)-(R3**2))/A)*100.0 \]
\[ IF(DFC(I),EC,DFC(I))/EC TE 5 \]
NOTE: WHEN ANY ANGULAR VARIABLE ENDS WITH AN R (E.G. CELTAR) THAT ANGLE IS IN RADIANS (EXCEPT PHIR = PHIRR IN RADIANS).

DELTA - FUNCTION OF DEGREE OF FILL (DEGREES).

THETA - DEFINES UPPER POSITION OF BED SURFACE (DEGREES).

ANGLE IS IN RADIANS (EXCEPT PHIR = PHIRR IN RADIANS).

FINISH INITIALIZING ALL OTHER VARIABLES:

DELTA - FUNCTION OF DEGREE OF FILL (DEGREES).

THETA - DEFINES UPPER POSITION OF THE BED SURFACE (DEGREES).

NOTE: WHEN ANY ANGULAR VARIABLE ENDS WITH AN R (E.G. CELTAR) THAT ANGLE IS IN RADIANS (EXCEPT PHIR = PHIRR IN RADIANS).

CALCULATE CENTRIFID OF UPPER WEDGE - XABC, YABC.

XABC = ((1.0*GMA1R/2.0)**2 + (1.0*GMA1R/2.0)**2 + (1.0*GMA1R/2.0)**2)

YABC = ((1.0*GMA1R/2.0)**2 + (1.0*GMA1R/2.0)**2 + (1.0*GMA1R/2.0)**2)

X0AB = (2.0*(R-H(I)**2 - ((R-H(I))**2)) + (R-H(I))**2)

Y0AB = (2.0*(R-H(I))**2 - ((R-H(I))**2)) + (R-H(I))**2)

X0OE = (2.0*(R-H(I))**2 - ((R-H(I))**2)) + (R-H(I))**2)

Y0OE = (2.0*(R-H(I))**2 - ((R-H(I))**2)) + (R-H(I))**2)

X09C = ((X0AB-X0A3)*(XCCB-A0CB) - (XOEA*AOE) - (XCD-F*A30EI) - (X0EC*ACFC))

Y09C = ((X0AB-X0A3)*(XCCB-A0CB) - (XOEA*AOE) - (XCD-F*A30EI) - (X0EC*ACFC))
OPTIONS IN EFFECT = DCD,ERGIC,SOURCE,NCLASS,NODES,LOAD,APC

OPTIONS = NAME = MAIN , LNFONT = 60

*STATISTICS* SOURCE STATEMENTS = 98, PROGRAM SIZE = 6014

*ERRORS* NO ERRORS GENERATED

EXECUTION TERMINATED 10:32:54 TO 0.524 RC = 0.29
APPENDIX F

Program Listing for Slipping.
C GENERALIZED SLIPPING BED BEHAVIOR.

C GENERALIZED PLOT OF EQUILIBRIUM BED INCLINATION FOR A GIVEN BED/WALL

C FRICTION ANGLE AT A GIVEN KILN RPM & BED DEPTH.

C

C INPUT INNER RADIUS OF KILN - R IN CM.

C

0001 DIMENSION H(251),DF(251),CF(251)

0002 R=50.12

0003 IF(14,15,16)

0004 WRITE(6,14)

0005 14 FORMAT(251,E2)

0006 WRITE(6,15)

0007 15 FORMAT(251,E2),*GIVEN BED/WALL FRICTION ANGLE AT A GIVEN KILN RPM & BED DEPTH.

0008 WRITE(6,16)

0009 16 FORMAT(251,E2)

0010 WRITE(6,17)

0011 17 FORMAT(251,E2),*BED DEPTHS.

0012 WRITE(6,19)

0013 19 FORMAT(501,E2)

C CROSS SECTİONAL AREA OF KILN.

0014 D1*=3.141592

0015 5*(DIF*(R**2))

C CALCULATE BED DEPTHS.

0016 I=1

0017 DF(I)=1.0

0018 H(I)=1.0

0019 R=R+RCCS(94/3)

0020 P=SQRT((P**2)-((E**2)))

0021 DF(I)=((1.0**2)-P2)-(94**2)/4)*100.0

0022 IF(DFC(I),FC,DF(I))GC TO 5

0023 IF(6(5)+P(I))GC TO 5

0024 IF(DFC(I)+P(I))GC TO 5

0025 IF(DFC(I)-P(I)+1.0)GC TO 5

0026 H(I)=H(I)+0.02

0027 GC TO 4

0028 3 H(I)=H(I)+1.0

0029 GC TO 4

0030 5 IF(1.0>25.0)GC TO 2

0031 I=I+1

0032 DF(I)=CF(I)+1.0

0033 H(I)=H(I)+1.0

0034 GC TO 4

C CALCULATE CENTRIFUGING RPM.

0035 2 RPM=((30.0*P(1))*SQRT(986.1/C(1)))

C INITIALIZE VARIABLES.

C
C

14

6

ALPHA+ALPHAR-((C,1+F1)/180.C)

GO TO 10

15

5

LS-1

10

S1=SORT(CBED**2)+12,0*CBED*WED*COS(ALPHAR)+13**2+2111

S2=RI*WED**2*SIN(ALPHAR))/R

S3=((S2)/S1)*COS(AR(S1/S2/S1))

S4=S**3

16

5

IFLS.EQ.0 GO TO 7

17

5

IF S4.LE.1.0 GO TO 9

18

15

(IF(ALPHAR.GE.(PIE/2)).GO TO 20

19

5

S1=1

ALPHA=ALPHAR-((C,1+F1)/180.C)

GO TO 10

20

6

FORMAT(* 1,7X,F5,1,3X,F6,1,6X,F5,1,18X,F,1)

21

6

PHI=PI**2.0

GO TO 13

22

6

IF(PHIS.EQ.0.11)GO TO 24

23

6

PHIS=PHIS**5.0

GO TO 19

24

6

IF (J.EQ.0)GO TO 23

25

6

J=J+1

GO TO 6

26

6

IF (J.EQ.25)GO TO 23

27

6

WRITE(*6,21)MU,J,PHI,PHIS

28

6

IF (J.EQ.25)GO TO 23

29

6

GO TO 22

30

6

STOP

31

6

END
APPENDIX G

Program Listing for Predicting
Full Bed-Behaviour Diagram.
Bed Behaviour Model

The program calculates the rotational speed at which a given bed of solids will change from:

1. Slumping to Rolling
2. Slipping
3. Rolling (Cascading) to Catastrophing
4. Catastrophing to Centrifuging

The inputs to this model are:

- R - RADIUS OF ROTARY CYLINDER [CM]
- GM01 - ANGULAR CHANGE IN BED INCLINATION BETWEEN MAX. & MIN. BED INCLINATION AS ROTATIONAL SPEED APPROACHES ZERO FOR A BED MOVING FROM THE MIN. TO MAX. BED INCLINATION (DEGREES)
- GM02 - SAME AS GM01 BUT FOR A BED GOING FROM MAX. TO MIN. BED INCLINATION (DEGREES)
- C1 - SLOPE OF GM01 OR GM02 VS RPM CURVES (SECS)
- PHi3 - ALSO CALLED LMF3 IN LOG - LOWEST BED INCLINATION (DEGREES)
- DAVG - AVERAGE PARTICLE SIZE OF MATERIAL [CM]
- PHi5 - BEC/WALL FRICTION ANGLE (DEGREES)
- PHi6 - DYNAMIC ANGLE OF REPOSE OF CYLINDER CHARGE (DEGREES)

REAL N,NR
DIMENSION OFF(100),PGC(100),GM4(100),DELTA(100),DELTA(100),CM4S(61),
RPMCS(61,1CC), RPKS(100)
INTEGER CHARGE(20)
READ(1)ICHRG
READ(2)RFAC(5,2),GM01,GM02,C1,PHi5,DM4,GHi5,PHI3
FORMAT(2X,15X,'Bed Behaviour Diagram For: **.2X,204
WRITE(6,15)ICHRG
15 FORMAT('Bed Behaviour Diagram For: **.2X,204
WRITE(6,16)RFAC(5,2),GM01,GM02,C1,PHi5,DM4,GHi5,PHI3
16 FORMAT('Bed Behaviour Diagram For: **.2X,204
WRITE(6,17)RFAC(5,2),GM01,GM02,C1,PHi5,DM4,GHi5,PHI3
17 FORMAT('Bed Behaviour Diagram For: **.2X,204
WRITE(6,18)RFAC(5,2),GM01,GM02,C1,PHi5,DM4,GHi5,PHI3
18 FORMAT('Bed Behaviour Diagram For: **.2X,204
WRITE(6,19)RFAC(5,2),GM01,GM02,C1,PHi5,DM4,GHi5,PHI3
19 FORMAT('Bed Behaviour Diagram For: **.2X,204
WRITE(6,20)RFAC(5,2),GM01,GM02,C1,PHi5,DM4,GHi5,PHI3
20 FORMAT('Bed Behaviour Diagram For: **.2X,204
WRITE(6,21)RFAC(5,2),GM01,GM02,C1,PHi5,DM4,GHi5,PHI3
21 FORMAT('Bed Behaviour Diagram For: **.2X,204
WRITE(6,22)RFAC(5,2),GM01,GM02,C1,PHi5,DM4,GHi5,PHI3
22 FORMAT('Bed Behaviour Diagram For: **.2X,204
WRITE(6,23)RFAC(5,2),GM01,GM02,C1,PHi5,DM4,GHi5,PHI3
23 FORMAT('Bed Behaviour Diagram For: **.2X,204
WRITE(6,24)RFAC(5,2),GM01,GM02,C1,PHi5,DM4,GHi5,PHI3
24 FORMAT('Bed Behaviour Diagram For: **.2X,204
WRITE(6,25)RFAC(5,2),GM01,GM02,C1,PHi5,DM4,GHi5,PHI3
25 FORMAT('Bed Behaviour Diagram For: **.2X,204
WRITE(6,26)RFAC(5,2),GM01,GM02,C1,PHi5,DM4,GHi5,PHI3
26 FORMAT('Bed Behaviour Diagram For: **.2X,204
WRITE(6,27)RFAC(5,2),GM01,GM02,C1,PHi5,DM4,GHi5,PHI3
27 FORMAT('Bed Behaviour Diagram For: **.2X,204

Calculate Bed Depths to be used for model calculations.
Bed Depths will be obtained corresponding to 25, 15, 10, 5 per cent fill.

C CROSS SECTINAL AREA OF KILA.
C
A=3.141593*(R**2)
C CALCULATION OF BED DEPTHS.
C
0023 1=1
0024 CF(I)=1.0
0025 H(I)=1.0
0026 4 R1=R-I(I)
0027 R2=SQRT((R**2)-(R1**2))
0028 FC(I)=((1.0*21+21)-R1*31)/A1*100.O
0030 IF(FC(I).EQ.DF(I))GO TO 5
0031 IF(FC(I).LT.DF(I))GC TO 3
0032 IF(DFC(I).GT.DF(I).LE.0.1)GC TO 5
0033 H(I)=H(I)-0.02
0034 GC TO 4
0035 3 H(I)=H(I)+1.0
0036 GC TO 4
0037 5 IF(FC.C25)GO TO 6
0038 I=I+1
0039 CF(I)=CF(I-1)+1.0
0040 H(I)=H(I-1)
0041 GC TO 4
C FINISH INITIALIZING ALL OTHER VARIABLES:
C DELTA - FUNCTION OF DEGREE OF FILL (DEGREES).
C THETA = DEFINES UPPER POSITION OF THE BED SURFACE (DEGREES).
C NOTE: WHEN ANY ANGULAR VARIABLE ENDS WITH AN R (EG DELTAR) THAT
C ANGLE IS IN RADIANS (EXCEPT PHIR - PHIR IN RADIANS).
C
0042 6 I=1
0043 8 DELTAR(I)=ASIN((R-1(I))/R)
0044 DELTAR(I)=DELTAR(I)*(180.0/3.1415931)
0045 IF(FC.C25)GO TO 19
0046 I=I+1
0047 GC TO 8
0048 19 I=1
0049 7 JNC
0050 RP=NC.10
0051 11 GMA1=GM401+(C1*6.0*RPV)*GM402
0052 ALPH=PHIR+GMA1
0053 THETA=ALPHA+DELTA(I)
0054 GMAR1=(GMA1*3.1415931)/16C.C
0055 ALPH=ALPH-(3.1415931)/180.C
0056 THETA=THETA-(3.1415931)/180.C
0057 PHIR=(PHIR*3.1415931)/180.C
0058 GMAR1=GMAR1*GMAR2
0059 GMAR1=GMAR1*3.1415931/180.O
C CALCUALTION OF SUMPING/ROLLING BOUNDARY.
C
C CALCULATE CENTROID OF UPPER WEDGE - XABC, YABC.
C
XABC=((2.0*PI)/(3.0*GMA1))*(SIN(THETA1)-SIN(THETA2-GMA1))
$X_{OB} = (R \cos \Theta_{OB} - GM_{OB}) + ((R - H(1)) \sin(\Phi_{OB}))/3.0$

$X_{OB} = (R \sin \Theta_{OB} - GM_{OB}) + ((R - H(1)) \cos(\Phi_{OB}))/3.0$

$C_{OB} = TAN(GM_{OB}/2.0)/2.0$

$Y_{OB} = (2.0R)/(3.0GM_{OB}) + (R-H(1))/(R-H(1))/3.0$

$Y_{OB} = (R \sin \Theta_{OB} - GM_{OB}) + ((R - H(1)) \cos(\Phi_{OB}))/3.0$

$C_{OB} = TAN(GM_{OB}/2.0)/2.0$

$X_{OB} = (R \cos \Theta_{OB} - GM_{OB}) + ((R - H(1)) \sin(\Phi_{OB}))/3.0$

$X_{OB} = (R \sin \Theta_{OB} - GM_{OB}) + ((R - H(1)) \cos(\Phi_{OB}))/3.0$

$C_{OB} = TAN(GM_{OB}/2.0)/2.0$

$Y_{OB} = (2.0R)/(3.0GM_{OB}) + (R-H(1))/(R-H(1))/3.0$

$Y_{OB} = (R \sin \Theta_{OB} - GM_{OB}) + ((R - H(1)) \cos(\Phi_{OB}))/3.0$

$C_{OB} = TAN(GM_{OB}/2.0)/2.0$

$X_{OB} = (R \cos \Theta_{OB} - GM_{OB}) + ((R - H(1)) \sin(\Phi_{OB}))/3.0$

$X_{OB} = (R \sin \Theta_{OB} - GM_{OB}) + ((R - H(1)) \cos(\Phi_{OB}))/3.0$

$C_{OB} = TAN(GM_{OB}/2.0)/2.0$

$X_{OB} = (R \cos \Theta_{OB} - GM_{OB}) + ((R - H(1)) \sin(\Phi_{OB}))/3.0$

$X_{OB} = (R \sin \Theta_{OB} - GM_{OB}) + ((R - H(1)) \cos(\Phi_{OB}))/3.0$

$C_{OB} = TAN(GM_{OB}/2.0)/2.0$

$X_{OB} = (R \cos \Theta_{OB} - GM_{OB}) + ((R - H(1)) \sin(\Phi_{OB}))/3.0$

$X_{OB} = (R \sin \Theta_{OB} - GM_{OB}) + ((R - H(1)) \cos(\Phi_{OB}))/3.0$

$C_{OB} = TAN(GM_{OB}/2.0)/2.0$

$X_{OB} = (R \cos \Theta_{OB} - GM_{OB}) + ((R - H(1)) \sin(\Phi_{OB}))/3.0$

$X_{OB} = (R \sin \Theta_{OB} - GM_{OB}) + ((R - H(1)) \cos(\Phi_{OB}))/3.0$

$C_{OB} = TAN(GM_{OB}/2.0)/2.0$

$X_{OB} = (R \cos \Theta_{OB} - GM_{OB}) + ((R - H(1)) \sin(\Phi_{OB}))/3.0$

$X_{OB} = (R \sin \Theta_{OB} - GM_{OB}) + ((R - H(1)) \cos(\Phi_{OB}))/3.0$

$C_{OB} = TAN(GM_{OB}/2.0)/2.0$

$X_{OB} = (R \cos \Theta_{OB} - GM_{OB}) + ((R - H(1)) \sin(\Phi_{OB}))/3.0$

$X_{OB} = (R \sin \Theta_{OB} - GM_{OB}) + ((R - H(1)) \cos(\Phi_{OB}))/3.0$

$C_{OB} = TAN(GM_{OB}/2.0)/2.0$

$X_{OB} = (R \cos \Theta_{OB} - GM_{OB}) + ((R - H(1)) \sin(\Phi_{OB}))/3.0$

$X_{OB} = (R \sin \Theta_{OB} - GM_{OB}) + ((R - H(1)) \cos(\Phi_{OB}))/3.0$

$C_{OB} = TAN(GM_{OB}/2.0)/2.0$
Michigan Terminal System FORTRAN G

322

10:32:57 12-19-80

0100  \text{RH} = (80.0 \times 900.0) / (10.0 \times DAVG / 2.0) \times (3.141593 \times 21)

0101  RPM = \text{SORT}(F \times P)

0102  IF(K.EQ.2) P = 0.05

0103  IF(K.EQ.3) P = 0.10

0104  IF(K.EQ.4) P = 0.25

0105  IF(K.EQ.5) P = 0.50

0106  IF(K.EQ.6) P = 0.75

0107  \text{GC TC} = 23

0108  \text{CASCADING & CATARACTING}

0109  \text{FIN P BED INCLINATION FOR WHICH FORCE BALANCE ON BED IS IN EQUILIBRIUM}

0110  \text{FO FOR SOME GIVEN RPM}

0111  22 RPMCAT = RPM

0112  LA = C

0113  LR = 0

0114  \text{GC TC} = 23

0115  \text{IF}(34 \leq 6.0) \text{GO TO} 29

0116  \text{IF}(T1R \geq 0) \text{GO TO} 30

0117  31 \text{CB} = 0.003141593 \times 180.0

0118  32 B1 = C \times E1

0119  33 B2 = B1 \times E2

0120  34 B3 = B2 \times E3

0121  \text{FIN P BED INCLINATION FOR THE ROLLING/CASCADING BOUNDARY}

0122  \text{FOR SOME GIVEN RPM}

0123  26 RPMCAT = RPM

0124  LA = C

0125  LR = 0

0126  GC TC = 23

0127  \text{IF}(34 \leq 6.0) \text{GO TO} 29

0128  \text{IF}(T1R \geq 0) \text{GO TO} 30

0129  \text{IF}(T1R \leq 0.0) \text{GO TO} 32

0130  \text{FIN P BED INCLINATION FOR THE ROLLING/CASCADING BOUNDARY FOR SOME GIVEN RPM}

0131  25 TIR = T1R + 31 \times \text{E1}

0132  \text{IF}(TIR \leq 0.0) \text{AND} (RPMCAT \times E4 \times 5.0) \text{GO TO} 30

0133  \text{IF}(TIR \leq 0.0) \text{GO TO} 30

0134  \text{IF}(TIR \leq 0.0) \text{GO TO} 32

0135  \text{IF}(TIR \leq 0.0) \text{GO TO} 30

0136  \text{IF}(TIR \leq 0.0) \text{GO TO} 32

0137  \text{IF}(TIR \leq 0.0) \text{AND} (RPMCAT \times E4 \times 5.0) \text{GO TO} 30

0138  \text{IF}(TIR \leq 0.0) \text{GO TO} 30

0139  \text{IF}(TIR \leq 0.0) \text{GO TO} 32

0140  \text{IF}(TIR \leq 0.0) \text{GO TO} 30

0141  \text{IF}(TIR \leq 0.0) \text{GO TO} 32

0142  \text{IF}(TIR \leq 0.0) \text{GO TO} 30

0143  \text{IF}(TIR \leq 0.0) \text{GO TO} 32

0144  \text{IF}(TIR \leq 0.0) \text{GO TO} 30

0145  \text{IF}(TIR \leq 0.0) \text{GO TO} 32

0146  \text{IF}(TIR \leq 0.0) \text{GO TO} 30

0147  \text{IF}(TIR \leq 0.0) \text{GO TO} 32
C FIND RPM FOR THE CASCADING/CATARACTING BOUNDARY & FOR FULL
C CATERACTING.
C
C IF(L4.EQ.0)  RPM=RPMCAT
C IF(L4.EQ.1)  RPM=RPMB
C IF((L4.LT.0) OR (L4.GT.1))  L4=0
C IF(L4.EQ.0)  RPM=RPMCAT
C IF(L4.EQ.1)  RPM=RPMB
C IF((L4.LT.0) OR (L4.GT.1))  L4=0
C
C IF(T1R.LT.0.0)  T1R=(PNCR*L/RM*COS(T1R)*(PI/180.0))
C IF(T1R.LT.0.0)  T1R=(PNCR*L/RM*COS(T1R)*(PI/180.0))
C
LAA=1

C CENTRIFUGING.

C-----------------------------------------------------------------
C
C OUTPUT.
C
C RPM=PM(RPM+0.1)
C RPM=PM(RPM+0.1)
C
C PROCEED TO THE NEXT BED DEPTH (FOR A MAXIMUM OF 25).
C
C IF(L4.EQ.25)  GO TO 50
C IF(L4.EQ.25)  GO TO 50
C
C SLIPPING.
C
C-----------------------------------------------------------------
C
C WRITE((6,55))
C WRITE((6,55))
C
C FOR SLIPPERY, SLIPPING BOUNDARY:
C FOR SLIPPERY, SLIPPING BOUNDARY:
C
C FORMAT('','BED DEPTH (CM), SLUMP ZONE, INIT. BOUND. ZONE, OUTER BOUND.
C FORMAT('','BED DEPTH (CM), SLUMP ZONE, INIT. BOUND. ZONE, OUTER BOUND.
C
C 0
C 0
WRITE(6,57)

57 FORMAT('*','25X,'(REV/MIN)',21X,'(REV/MIN)',19X,'(REV/MIN)')

CC 51 I=1,25

CALL SLIPHIS,WFD,R,RPCS,RPHR,GMAO1,GMAO2,G1,RPMK(I1,4(I)

51 CONTINUE

C FINAL OUTPUT.

C

14 WRITE(6,25)

25 FORMAT(*11,'RPMS AT WHICH THE CENTRIFUGING FORCE IS 1X, 5X, 10X, 2

C5X, 50X & 75X OF THE GRAVITY FORCE.**

WRITE(6,24)

WRITE(6,34)

WRITE(6,35)

WRITE(6,36)

WRITE(6,37)

WRITE(6,38)

WRITE(6,39)

WRITE(6,40)

STOP

END
3049 IF(L.EQ.0) GO TO 29
3050 IF(B.4.GE.1.0) GO TO 29
3051 GO TO 29
3052 23 IF(B.4.LE.1.0) GO TO 29
3053 RC=RC+IC(IR=3.141592/180.0)
3054 GO TO 24
3055 28 L=1
3056 RC=RC+(O.3141593/180.0)
3057 GO TO 24
3058 20 IF(B.6.GT.LSPLR(1)1) GO TO 50
3059 IF(RPM.CE.RPM1)RPMT2=EPMC
3060 IF(RPM.CE.RPM2)GE.0.1)RPMT3=RPMT
3061 IF(RPM.CE.RPM3)25.0.1)GO TO 52
3062 T=1+1
3063 RPM=RPMT+0.1
3064 LDC
3065 GO TO 31
3066 50 IF(RPM.CE.RPM4) GO TO 51
3067 RPMT2=RPMT
3068 GO TO 52
3069 51 RPMT2=C.0
3070 RPMT3=C.0
3071 52 WRITE(6,541),EPMT1,EPMT2,EPMT3
3072 54 FORMAT(' 1.4X,F5.1,18X,F5.1,23X,F5.1,19X,F5.11,19X,F5.11
3073 53 RETURN
3074 END

OPTIONS IN EFFECT  IC,ESDCG,source,NOLIST,NOECK,LOAD,NOMAP
*OPTIONS IN EFFECT  NAME = SLIP , LINTER = 60
*STATISTICS* SOURCE STATMENTS = 74, PROGRAM SIZE = 6216
*STATISTICS* UC DIAGNOSTICS GENERATED

0 STATEMENTS FLAGGED IN THE ABOVE COMPILATIONS.

LAST NUMBER OF ERRORS/WARNINGS SEVERITY

SLIP  0  0

EXECUTION TERMINATED 19122159 T=1.203 RGC=  5.71