A STUDY OF THE FLUID-DYNAMIC CHARACTERISTICS OF TURBULENT GAS-LIQUID BUBBLE PLUMES

by

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ABSTRACT

The physical behaviour of air-water plumes during upward injection in ladle-shaped vessels has been investigated. The study involved the experimental determination of the spatial distribution of the flow parameters characterizing the behaviour of the gas phase: gas fraction, bubble frequency, mean bubble velocity and pierced length, and the spectrum of the bubble velocity and pierced length.

A computer-aided electroresistivity sensor was developed to determine simultaneously the several local parameters. In particular the accurate measurement of the bubble rise velocity and bubble pierced length, under the turbulent conditions studied, necessitated an instrument capable of ensuring that sequential voltage pulses originating at the probe tips corresponded to a single bubble travelling axially and undisturbed between the tip contacts. To achieve this, special electronic instrumentation and software was built to analyse, in real time, the signals produced by the contact of the bubbles with the sensor. The extensive evaluation of the measuring system indicated a high accuracy and reproducibility of the results.

The plumes were investigated under various conditions of air flow rate, orifice diameter and bath depth. The measurements indicate that the radial gas fraction profiles, at different
axial positions in the plume, exhibit similarity. The reduced gas fraction profiles can be approximated by a single Gaussian distribution for all the conditions studied. Thus a full description of the spatial distribution of gas could be obtained through the correlation of the axial gas fraction and half-value radius with the modified Froude number. The development of flow in gas-liquid plumes is evidenced by changes in the bubble frequency, mean bubble velocity and mean pierced length distributions. In the region close to the injection point, there is a steep change radially in bubble velocity and the motion of the bubbles is strongly affected by the gas injection velocity. Measurements of bubble frequency and pierced length indicate that bubble break-up occurs in this zone before a dynamic process of break-up and coalescence establishes a nearly constant bubble size distribution. In the region of fully developed flow in the plume, the mean bubble velocity and the standard deviation of the bubble velocity spectrum exhibit relatively flat radial profiles and the bubbles affect the flow only through buoyancy. The spectra of bubble pierced length and diameter in this zone can be fitted to a log-normal distribution. Injection conditions have only a slight influence in determining the size of the bubbles in this region. Close to the bath surface a third zone is identified in which bubble velocity decreases more rapidly as liquid begins to flow radially outward from the plume. A mathematical model proposed in the literature for bubble plumes has been used for comparison with the experimental results.
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<td>$A, A_p$</td>
<td>Area; cross-sectional area of the plume, mm².</td>
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<td>Lower tip of electroresistivity sensor, coefficient in Eq. (2.30).</td>
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<td>Nominal plume width with respect to gas fraction; nominal plume width with respect to liquid velocity, mm.</td>
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<td>$C$</td>
<td>Delay pulse channel.</td>
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<td>$C_a$</td>
<td>Added mass coefficient.</td>
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<td>$C_D$</td>
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<td>Local bubble diameter; geometric mean local bubble diameter, mm.</td>
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<td>Vertical separation between lower and upper contacts of the probe, mm.</td>
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<td>Orifice diameter, mm.</td>
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<td>$e$</td>
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<td>$f_b$</td>
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<td>$f_d$</td>
<td>Doppler frequency, s⁻¹.</td>
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<tr>
<td>$f_k$</td>
<td>Field variable associated with phase k.</td>
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<td>$F_D$</td>
<td>Drag force, N.</td>
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<tr>
<td>$Fr$</td>
<td>Modified Froude number $= \frac{Q_o^2 \rho_{go}/gd_o^5}{\left(\rho_1-\rho_{go}\right)}$.</td>
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<tr>
<td>$g$</td>
<td>Acceleration due to gravity, (9.81 m/s²).</td>
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<td>$h_a, h_b$</td>
<td>Atmospheric pressure; Bath depth, mm.</td>
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<td>$k$</td>
<td>Phase index.</td>
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<td>$l_b$, $\bar{l}<em>b$, $\bar{l}</em>{bg}$</td>
<td>Local bubble pierced length; arithmetic mean of local pierced length; geometric mean of local pierced length, mm.</td>
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<td>$m_b$</td>
<td>Mass of liquid bath, kg.</td>
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<td>$m_v$</td>
<td>Slope of falling edge of signals, v s$^{-1}$.</td>
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<td>$M$</td>
<td>Subindex indicating model.</td>
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<td>$n$</td>
<td>Space dimension, exponent in Eq. (2.30), subindex in Eq. (5.15) indicating class interval with the largest central value in bubble velocity histogram.</td>
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<td>$N_{lb}(i)$</td>
<td>Number of pierced lengths in histogram interval $i$.</td>
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<td>$N_u$</td>
<td>Number of bubble velocities collected in the experiment.</td>
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<td>$N_v(i)$</td>
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<td>$P$</td>
<td>Subindex indicating prototype.</td>
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<td>$P_a$</td>
<td>Atmospheric pressure, Pa.</td>
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<td>$q_k$, $\bar{q}_k$</td>
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<td>$Q$, $Q_o$</td>
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<td>$r_o$, $r_{bf}$</td>
<td>Internal radius of orifice; radius of bubble at detachment, mm.</td>
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<td>$R$</td>
<td>Radius of the vessel, mm.</td>
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<tr>
<td>$\dot{R}$</td>
<td>Velocity of expansion of bubble surface, m s$^{-1}$.</td>
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<td>$R_{12}$</td>
<td>Cross-correlation coefficient.</td>
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<td>$Re$</td>
<td>Reynolds number = $4Q_o \rho_o / \pi d_o U$.</td>
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<td>$s$</td>
<td>Vertical distance of centre of bubble from centre of orifice, mm.</td>
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<td>$S_{Ub}$</td>
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<td>Bubble terminal velocity; bubble transport velocity, m/s$^{-1}$.</td>
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<td>Volume of bubble; volume of bubble at detachment, mm$^3$.</td>
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<td>$We$</td>
<td>Weber number = $\rho L U^2/\sigma$.</td>
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<td>$x$</td>
<td>Position.</td>
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<td>Phase density function for phase k, dependent on position and time.</td>
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<td>$\beta$</td>
<td>Bubbling factor.</td>
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<td>$\varepsilon$</td>
<td>Entrainment coefficient.</td>
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<td>$\varepsilon_b, \varepsilon_K$</td>
<td>Specific buoyancy power; specific kinetic power, Watt Kg$^{-1}$.</td>
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Δ

Length of histogram interval.

ΔP

Pressure drop, N/m².

λ

Wavelength of light, m, constant defined in Eq. V.6

θ, θ_c

Angle between incident and reflective beam, angle defined in Fig. 1.1; jet cone angle.

γ

Angle defined in Fig. 1.1.

ρ_l, ρ_go

Density of liquid; density of gas at orifice, g/cm³.

ρ_M

Mean density of the two phase mixture, kg m⁻³.

μ

Viscosity of air, cP.

ν

Kinematic viscosity, cm² s⁻¹.

τ_M

Time interval at which R₁₂ is a maximum, s.

−

Time averaging operator over T.

−X

Time averaging operator over T_k.

< >

Space averaging operator over entire space domain.

< >

Space averaging operator over domain of phase k.
ACKNOWLEDGEMENTS

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CHAPTER 1

INTRODUCTION

1.1 Submerged Gas Jets in Metallurgical Processes

Submerged gas injection techniques are ubiquitous in the smelting and refining of metals both in the ferrous and nonferrous industries. The injection of a reactive gas has been used for a very long time to remove impurities through chemical reactions occurring at gas/melt interfaces. More recently, the injection of an inert gas has been used to stir up the melt to produce homogeneous conditions within the bath and to promote slag-metal reactions. Variations in submerged gas injection techniques involve differences in: gas blowing rates (0.01 to 5 Nm³/min tonne), direction of injection (bottom, top, lateral and combined injection), injection method (tuyere, porous plug and lance) and materials injected (reactive gas, inert gas and gas-powder mixtures).

In the ferrous industry there has been a very rapid growth in the utilization of submerged gas injection. The Q-BOP¹,² process for the refining of pig iron revived bottom blown injection in steelmaking. This process uses concentric tuyeres, placed at the bottom of the converter, to inject oxygen, lime and hydrocarbons. The Q-BOP offers improved interaction between slag and metal and higher rates of decarburization than the BOF process. Thus recently, a number of processes have emerged that retain the BOF top blown configuration and advantages e.g. early
slag formation and high scrap to metal ratio utilization, but also allow for bottom injection of inert (LD-KG, LBE) or oxidizing (LD-OB, LD-OTB) gases.\textsuperscript{3-6} The AOD\textsuperscript{7-8} process for the production of stainless steel utilizes concentric tuyeres, set at the sides of the reactor, to inject argon and oxygen.

Secondary steelmaking has developed into a wide variety of ladle metallurgical treatments.\textsuperscript{9-11} The treatments extend from simple practices to improve deoxidation and alloy addition, to sophisticated ladle processes which can supply heat, inject materials and vacuum degas.

The non-ferrous industry also makes extensive use of submerged gas jets. Today, converting of copper mattes is almost universally carried out in Pierce-Smith converters. The Pierce-Smith converter is a horizontal cylindrical vessel with a row of horizontally directed tuyeres that blow air into matte to produce blister copper.\textsuperscript{12} The resulting blister copper is further refined by the injection of oxidizing and reducing gases in an anode furnace. The slag-fuming furnace is another process that utilizes submerged injection of air and carbon to recover zinc and tin from their respective slags. The zinc fuming furnace is a rectangular shaped vessel with two rows of horizontally directed tuyeres, one on either side of the vessel.\textsuperscript{13}

This review of industrial submerged processes has been necessarily brief but serves to outline the variations that exist and their widespread application in the metal processing
industry. A more complete description may be found in the papers by Felski et al\textsuperscript{14}, Upadhya\textsuperscript{15} and Themelis et al.\textsuperscript{16}.

Submerged gas injection techniques have been adopted widely for the following reasons: efficient mixing of the liquid bulk, large surface area produced for reaction, easy control of process variables such as gas flow rate and rate of addition, relative simplicity and low capital costs. However, a number of problems also can be found such as: severe bath slopping, poor utilization of certain additions, tuyere erosion and blockage and refractory wear. The origin of most of the advantages and problems of gas injection systems can be traced back to their fluid flow behaviour induced by the buoyancy of the discharging gas and to the behaviour of the jet itself, as Szekely\textsuperscript{17} and Brimacombe\textsuperscript{18} have pointed out.

In a simplified representation, a gas-stirred melt can be considered to consist of two regions: a liquid recirculating zone and a gas dispersed-liquid zone. A review of the literature shows that most of the theoretical and experimental work has focussed on the liquid zone and on the zone of gas-liquid close to the injection point. The gas-liquid bubble zone has received less attention due to the analytical and experimental difficulties that the study of two-phase flow systems entails\textsuperscript{17,19}.

Two-phase flow is not yet an area in which theoretical prediction of flow parameters is generally possible. Thus, the role of experiments and measurements of flow parameters is
particularly important. Measurement of time-averaged and space-
averaged void fraction and velocity of the phases is required to 
confirm current modeling efforts. Measurements are also required 
to verify hypotheses regarding the shape and interrelation of the 
profiles of void fraction, bubble passage frequency and velocity 
profiles. Measurements also may provide useful dimensionless 
correlations expressing the variation of void fraction, velocity 
and turbulence with energy input. The experimental information 
produced in this work represents an effort in these directions.

The general aims of this investigation were to develop 
signal interpretation techniques that allow reliable measurement 
of two-phase flow parameters in gas-liquid bubble jets and to 
provide information of the kind mentioned in the previous 
paragraph. The parameters measured were local time-averaged 
quantities: gas fraction, bubble frequency, bubble velocity and 
pierced length, as well as the spectrum of bubble velocity and 
pierced length. The specific objectives of this work are 
presented in Chapter 3.
2.1 **Bubbling and Jetting Phenomena in Submerged Gas Injection**

The gas injected into a molten bath may discharge in the form of discrete bubbles or in the form of a jet depending on the kinetic energy of the gas and on the physical properties of the gas and liquid. In the metal industry, both regimes of discharge are important. In steelmaking processes, using tuyeres, gas is injected at velocities of 200-300 m/s and discharges in the jetting regime, e.g. Q-BOP\textsuperscript{18}, STB\textsuperscript{16}, K-BOP\textsuperscript{16}, AOD\textsuperscript{20}. On the other hand, processes using porous plugs for injection have gas discharge velocities of only 1m/s and operate in the bubbling regime, e.g. LBE\textsuperscript{16}, LF\textsuperscript{10}, VOD\textsuperscript{21}. In the non-ferrous industry, processes normally involve injection in the bubbling regime e.g. Pierce-Smith converter\textsuperscript{18}, slag fuming furnace\textsuperscript{13} and anode furnace\textsuperscript{20}. Thus any successful comprehension of these processes requires an understanding of the mechanism of bubble formation and its transition into long gas envelopes or jets. As a consequence, much information has been generated on the phenomena occurring close to the point of injection.

Three different flow regimes have been identified as a function of gas flow rate\textsuperscript{22-27}. At very low gas flow rates (Re < 500), the process of bubble formation is a static one. The bubble grows until the buoyancy exceeds the surface tension force
holding it to the nozzle. The bubbling frequency is proportional to the gas flow rate and the bubble size is almost constant. At higher gas flow rates (500 < Re < 2100) the process of bubble formation becomes dynamic. The size of the bubbles is determined by buoyancy and inertial forces that result from the displacement of the surrounding fluid. The bubble volume increases with gas flow rate while the frequency of formation remains almost constant. With increasing gas flow rate, the forming bubble becomes affected by the wake of the preceding bubble and large and small bubbles start to form in pairs. As the flow rate further increases, coalescence takes place more frequently and closer to the orifice. Finally at very high flow rates the jetting regime appears in which a continuous gas channel is formed close to the nozzle.

During bubble formation, the pressure inside the bubble decreases due to the upward movement of its centroid and therefore the gas flow rate may vary with time. If there is a restriction that produce a high pressure drop between the gas reservoir and the orifice, the pressure fluctuations due to forming bubbles will not have an effect on the pressure drop. Then, the gas flow rate during bubble formation can be considered as constant. If the volume of the reservoir is large compared with the volume of the bubbles being formed, the pressure in the chamber will not significantly change. This corresponds to the case of bubble formation under constant pressure conditions.
A large number of models for predicting bubble formation under different conditions have evolved. They have been reviewed by Clift, Grace and Weber\textsuperscript{31} and Kumar and Kuloor\textsuperscript{32}. All the models are mechanistic and depend on a force balance to describe the evolution of bubble growth.

The transition from bubbling to jetting has been investigated by Hoefele and Brimacombe\textsuperscript{18}. They studied the discharge of gas through a horizontal nozzle into water, a zinc chloride solution and mercury. Based on the interpretation of tuyere pressure traces and high speed films, they were able to represent the behaviour of the discharging gas in a flow pattern map. The flow regimes - bubbling, transition and jetting - were determined by the modified Froude number and the ratio of the density of the gas to that of the liquid. The jet regime was defined as that characterized by the continuous presence of a short stable gas envelope at the nozzle tip. For systems with a low gas-liquid density ratio, choked flow in the tuyere is necessary for steady jetting; for systems with a greater gas-liquid density ratio, steady jetting can be achieved under conditions involving smaller gas velocities in the tuyere. Mori et al.\textsuperscript{33,34} found that the transition from bubbling to jetting was achieved once the velocity of the gas at the nozzle exceeded the sonic velocity irrespective of the physical properties of the liquid and orifice diameter. McNallan and King\textsuperscript{35} have later proposed that the jet regime is established when the mass flux of the gas is greater than 40 g/cm\textsuperscript{2}s.
As will be described later, shortly after injection, both jets and large bubbles break up into a multitude of small bubbles. These bubbles entrain liquid and create a two-phase bubble flow which rises to the surface, to produce a recirculating motion in the bath.

2.2 Size of Gas Bubbles at Detachment

Many models have been proposed to predict the size of the bubbles that detach from a gas source. The first fully theoretical models, for a viscous liquid and later for an inviscid liquid, were developed by Davidson and Schuler\textsuperscript{36,37}. In one of their models, which has received wide attention, they assumed constant flow, an inviscid liquid, and that the bubble behaves as a rigid expanding sphere growing from a point source. The vertical motion of the bubble was considered to be determined by a balance between the buoyancy force and the acceleration of the fluid around the bubble; the mass of this liquid swept by the moving bubble is considered through an added mass coefficient. Neglecting the mass of the bubble, the balance may be written as:

\[
\rho_l V_b g = \frac{d}{dt} \left( C_a \rho_l V_b \frac{ds}{dt} \right)
\]  

(2.1)

Considering that detachment occurs when the bubble centre has travelled a distance equal to its radius, i.e. \( s = r_{b,f} \), integration of Equation (2.1) produces a final bubble volume of
\[ V_{b,f} = \beta g^{-3/5} Q_o^{6/5} \quad (2.2) \]

For a spherical bubble forming at an orifice in a flat plate, the added mass coefficient has a value of 11/16; then the bubbling coefficient in Equation (2.2) becomes 1.378\textsuperscript{36,37}. This value produced good agreement with experiments in aqueous systems with injection rates in the range 1.5 to 20 cm\textsuperscript{3}/s. However, the agreement becomes increasingly poor for higher flow rates as the observed bubble volumes are smaller than predicted. This was interpreted as being due to the upward current induced by previous bubbles and to the deformation of the bubble forming at the orifice. An empirical expression similar to Equation (2.2) with \( \beta = 1.725 \) had been previously developed by Van Krevelen and Hoftijzer\textsuperscript{38}.

The Davidson-and-Schuler model\textsuperscript{37} was adapted by Walters and Davidson\textsuperscript{39} to represent bubble formation in a free-standing nozzle. This was achieved by using an added mass coefficient of 1/2, which corresponds to the motion of a sphere in an infinite ideal fluid. For this case Equation (2.2) with \( \beta = 1.138 \) gave good agreement with experiments, for flow rates in the range of 10 to 10000 cm\textsuperscript{3}/s.

Wraith\textsuperscript{40} studied bubble formation during vertical downward injection. He developed a model which reduced to that of Walters and Davidson\textsuperscript{39} when the radius of the lance is small or the gas flow rate is large. Wraith\textsuperscript{40} pointed out that as the gas flow rate increases, the bubble continues to grow after its base has left the gas source. The additional growth is due to a
connecting stem of gas. The final volume is therefore larger than the value predicted by Equation (2.2) with $\beta = 1.138$. The condition of gas cut-off to a forming bubble was further investigated by Wraith and Kakutani $^{41}$. They found that the severing of the connecting stem occurred after the bubble has risen to a certain distance and the pressure behind the bubble has risen sharply. They incorporated these observations into a simple one-stage model and found $\beta = 1.39$ for bubble formation in a free-standing nozzle and $\beta = 1.54$ for formation at a plate orifice. Wraith $^{42}$ and Kumar and Kuloor $^{32}$ also have proposed two-stage models for bubble formation at plate orifices. The models consist of a first stage dominated by inertial forces and a second stage dominated by buoyancy. The considerations regarding bubble shape evolution are different in the two models. The Wraith model $^{42}$ yields $\beta = 1.09$ and the Kumar and Kuloor model $^{32}$ gives, after some simplifications, $\beta = 0.976$. Hoefele and Brimacombe $^{18}$, working on a one tuyere model of a copper converter, indicated that bubble formation can be adequately described by Equation (2.2) with $\beta = 1.57$ or 0.88 in mercury and aqueous baths respectively. It is interesting to observe that the various equations mentioned previously differ only in the value of the coefficient, $\beta$, despite the fact that they have been based on different mechanisms or that they have been adjusted to apply to systems with different physical properties.

A factor that becomes important at high injection velocities is the inertia of the gas issuing from the nozzle. Davidson and Schuler $^{36}$ proposed that the net effect of the gas momentum was to
exert an additional force for bubble detachment. Nilmani and Robertson\textsuperscript{43} included the gas momentum in the equation of motion given in Equation (2.1). They found that their model agreed with the Davidson-and-Schuler model for a small gas-liquid density ratio. However, for an air-water system the predictions differed considerably. McCann and Prince\textsuperscript{29} have pointed out that at high flow rates, wake effects and bulk circulation affect bubble growth; the bubble that forms is elongated and grows so rapidly that it coalesces with the previous bubble at some distance from the orifice. As the flow rate increases the coalescence point moves closer to the orifice.

Equation (2.2) contains no dependence upon orifice diameter since the gas was assumed to issue from a point source. The effect of a finite orifice diameter was considered by Davidson and Schuler\textsuperscript{36}. In their model, they assumed that a residual sphere of gas was left after the release of a bubble and that the detachment occurred when \( s = r_{b,f} + r_o \). In this case the model has no simple analytic solution although it does predict that the bubble volume will increase with increasing orifice diameter.

Davidson and Amick\textsuperscript{22} proposed the following empirical relation for bubble size

\[
V_{b,f} = 0.11 \left( Q_0 / r_o \right)^{0.5} 0.867
\]  

(2.3)

for flow rates up to 250 cm\textsuperscript{3}/s in water. Sano et al.\textsuperscript{44}, during an investigation of bubble formation in mercury and liquid silver,
found that Equation (2.3) was applicable if the internal orifice radius was replaced by the external radius. For an aqueous system, where the liquid wets the nozzle, the bubble forms around the edge of the orifice whilst in a metallic non-wetting system, the bubbles grow beyond the edge of the orifice. Irons and Guthrie\textsuperscript{24,25} also observed this phenomenon and further pointed out the pronounced effect that the antechamber volume has on bubble size, particularly at low gas flow rates.

Andreini et al.\textsuperscript{45} discharged argon at low flow rates into copper, lead and tin baths. They found that the bubble size depended on the gas flow rate, orifice diameter, liquid surface tension, and gas density. Their results could be correlated as a function of the orifice Froude and Weber numbers; therefore they concluded that empirical correlations cannot be extrapolated from aqueous solutions with a very different surface tension. However, as in aqueous systems, a constant volume and a constant frequency range are identified. Similar results have been reported by Berdnikov et al.\textsuperscript{46}.

Guthrie\textsuperscript{47} developed a mathematical model to predict the shape and size of bubbles growing at an orifice. The model was applied to bubble formation in water and molten iron. It predicted that argon bubbles forming in iron are about three times larger than air bubbles forming in water for the same gas flow rate and orifice diameter. Another numerical model\textsuperscript{48} was developed to predict bubble formation in a copper converter. The model considers the effect of heat transfer, chemical reaction
and bath motion on bubble formation. It was found that heat transfer acts to decrease bubble frequency while bath circulation increases it.

2.3 Size of Gas Bubbles after Detachment

It was previously mentioned that shortly after injection both jets and large bubbles break up into a multitude of small bubbles. These bubbles will, with continuous coalescence and disintegration, tend to establish a dynamic range of stable sizes. Walters and Davidson\textsuperscript{39} have investigated, theoretically and experimentally, the changing shape of growing and accelerating free bubbles. Their work dealt with the analysis of the initial motion of a spherical bubble starting from rest. Their theory predicted that the pressure field generated around the accelerating bubble is accompanied by the distortion of the bubble into the form of a mushroom. As this happens the base of the bubble is depressed to develop ultimately into a rising tongue of liquid which reaches into the bubble interior and results in its disintegration. The initial motion theory was extended to deal with the problem of a growing bubble. Walters and Davidson\textsuperscript{39} found that the expansion of the bubble causes the change in shape to occur much more slowly. Wraith et al\textsuperscript{40,42,49}, in agreement with this result, have observed that growing bubbles accelerate and deform more slowly than detached bubbles. Their observations in vertical upward and downward injection indicate that bubble disintegration is retarded until the bubble is severed from the source.
Leibson et al.\textsuperscript{23} reported, in their study with water, that an increase in the Reynolds number produces an increasingly wrinkly appearance of the bubble surface. Portions of the air bubble surface are pinched or torn off to form very minute bubbles that remain oscillating near the gas source. These investigators also have pointed out that the formation of bubbles having nonuniform size was greatly accelerated by the explosion of large irregular bubbles at a point approximately 10 cm above the orifice. The bubbles thus produced had a size distribution that could be fitted by a log-normal probability distribution. The volume-surface mean diameter was approximately 4 mm in the range of $10000 < \text{Re} < 50000$; the volume surface mean diameter is the diameter of a hypothetical bubble whose ratio of volume to surface area is equivalent to that of the entire bubble size distribution.

Oryall and Brimacombe\textsuperscript{50} and Fruehan et al.\textsuperscript{51} used an electroresistivity probe to study gas-liquid jets. They found a very large bubble frequency, relative to that of formation, within 0.5 cm of the gas source. This indicates that the gas stream discharging from the tuyere disintegrates into bubbles very rapidly. Fruehan\textsuperscript{52} reported that the volume of argon bubbles emerging at the surface of copper and silver baths were much smaller than the expected volumes for a frequency of formation of $22 \text{ s}^{-1}$. Therefore, it was obvious that the large bubbles formed at the orifice were breaking up into smaller bubbles. The gas flow rates were relatively low ~19 cm$^3$/s.
Grace et al.\textsuperscript{53} have reported, on the basis of perturbation theory, that it is possible to set a lower limit on the maximum stable bubble diameter. Their predicted maximum stable diameter for air bubbles in water and mercury are 4.9 cm and 3.4 cm respectively\textsuperscript{31}. Other models have been proposed to describe bubble break up. They have been reviewed comprehensively by Clift, Grace and Weber\textsuperscript{31}. Some of these models have been applied by Sano et al.\textsuperscript{30,54} to obtain an estimation of bubble size in bubble swarms. Sano et al.\textsuperscript{54} used the distribution of gas pulse length, generated with an electroresistivity probe, as an estimation of the bubble size distribution. They injected nitrogen upward in mercury through a nozzle, and found that the gas pulse lengths were widely distributed and independent of the radial position and nozzle diameter. The frequency of large wave lengths increased with gas flow rates. Pierced lengths were calculated assuming an average bubble rise velocity of 0.66 m/s and were in the range of 7 to 75 mm. In contrast to Leibson et al.\textsuperscript{23}, Sano et al.\textsuperscript{30} found that the volume-area mean diameter increased with Reynolds number. The discrepancy was attributed to the method of measurement and to the presence of coalescence. Kawakami et al.\textsuperscript{55} found similar results in their study of nitrogen and argon injection into liquid iron.

In a recent investigation, Tacke et al.\textsuperscript{56} defined two quantities related to the size of the bubbles after detachment. These size parameters indicated a decrease in bubble size toward the periphery of the jet and also a tendency for the bubble size to decrease with the orifice Reynolds number. In this
investigation, however, no direct measurement of bubble size was undertaken.

2.4 Shape and Distribution of Flow Parameters in Gas-Liquid Bubble Jets

Some general similarities exist in the behaviour of homogeneous and heterogeneous two-phase jets with respect to the shape and distribution of flow parameters. The theory describing the flow of a turbulent jet into a stagnant bath of the same fluid (homogeneous jet) has been treated in detail by Abramovich\textsuperscript{57} and Schlichting\textsuperscript{58}. Figure 2.1 illustrates the situation in which a circular jet of fluid discharges, with uniform velocity, into a large body of the same fluid at rest. The velocity difference produces a turbulent axisymmetric shear layer around the constant velocity core. The thickening of the boundary layer due to turbulence leads to the eventual extinction of this core. This first region is called the flow development region. At some distance downstream from this region the jet appears to issue from a point source. The jet continues to spread as it entrains the surrounding fluid and the maximum axial velocity, occurring on the axis of the jet, decreases continuously. In this new region the flow is said to be fully developed.

Abramovich\textsuperscript{57} has pointed out some important properties of homogeneous jets:
(a) The pressure is constant throughout the jet. This implies that the axial momentum flow is constant.

(b) The jet grows rather slowly, that is, the thickness of any section is small compared to its distance from the nozzle.

(c) The transverse velocity component is much smaller than the axial velocity component.

(d) The radial profiles of the axial velocity at different sections of the fully developed region have the same shape. Furthermore, the profiles can be non-dimensionalized so thus they are described by one general curve. Thus it is said that similarity exists at all sections perpendicular to the main flow direction. The distributions of concentration and temperature exhibit this property as well.

Figure 2.1 Schematic diagram of homogeneous jet showing constant velocity core and velocity profiles at various distances from the orifice.
The jet expansion angle is a vital parameter in characterising the behaviour of a submerged jet, since it determines the degree to which the jet interacts with the surrounding fluid. The jet angle is defined as the angle between the outer edges of the turbulent boundary layer in the region of developed flow. Laats\textsuperscript{59}, Hetsroni et al.\textsuperscript{60} and Popper et al.\textsuperscript{61} found that a circular jet of air discharging into air, both at the same temperature, expanded with a cone angle of 18° while Corrsin and Uberoi\textsuperscript{62} determined, by blowing hot air into cool air, that a reduction in the density of the jet relative to the receiving medium increases the rate of spread. They reported an angle of 21°. Binnie\textsuperscript{63} found that a circular water jet in water expanded with a cone angle of 14°. Hetsroni et al.\textsuperscript{60} and Popper et al.\textsuperscript{61} measured a cone angle of 16° for an air-droplets jet with a small loading ratio. Laats\textsuperscript{59} measured an angle of 16° for a dusty air jet, with a small loading ratio. Engh\textsuperscript{64} reported a cone angle of 3° for a polyethylene-loaded air jet injected into air.

Donald and Singer\textsuperscript{65} performed a series of experiments with homogeneous and nonhomogeneous jets and proposed the following empirical relationship

\[
\tan \left( \theta_c / 2 \right) = 0.238 \, u^{0.133} \quad (2.4)
\]

This relationship implies that the expansion of the jet is determined solely by the physical properties of the injected fluid. Further confirmation was provided by Themelis et al.\textsuperscript{66} who
measured a cone angle of 20° for air jets in water. Oryall and Brimacombe injected air horizontally into mercury and measured the jet trajectory using an electroresistivity probe. Their measurements revealed that the jets expanded extremely rapidly upon discharge from the nozzle with an initial expansion angle of 155°. This value indicates that the physical properties of the liquid exert considerable influence on the behaviour of gas jets in liquids. Fruehan et al. also used an electroresistivity probe to investigate the shape of air jets injected vertically upward and horizontally into water and glycerol. They found that the jets exhibit a rapid expansion near the tuyere and then rise almost vertically. They also reported that the expansion of the jets increased with the gas flow rate, as well as with the density and viscosity of the liquid. In a later investigation Hoefele and Brimacombe showed that for the nozzle velocities used by Oryall and Brimacombe the gas emerged as bubbles, implying that the concept of cone angle was not strictly applicable.

Szekely et al. injected argon at the bottom of pilot and industrial scale ladles containing steel. They observed that the disturbed region on the surface of the melt increased with the depth of the bath and the gas flow rate. This could give an indication that the jet has a conical shape. Hsiao et al., on the basis of their experiments with argon injection in steel, have found that the gas-liquid flow zone could be represented adequately as a buoyant plume having a conical shape.
Themelis et al. applied an integral-profile method to develop a model for predicting the trajectory of an air jet injected horizontally into water and liquid metals. Consideration of the conservation equations of mass as well as of vertical and horizontal momentum resulted in an equation for the jet trajectory. They assumed that the jet expansion was proportional to the horizontal distance from the origin of the cone, with an angle of 20° for all systems. The gas concentration and velocity distribution transverse to the jet axis were assumed constant and to be a function only of axial distance from the orifice; the influence of buoyancy was then applied. The model gave very good agreement with experiments on air-water systems but failed to represent jet trajectories in gas-metal systems. Engh et al. introduced some modifications to the model of Themelis et al. but the predictions from both models were similar. The model of Themelis et al. produced good results for metallic systems when an angle of expansion of 155° was considered. However, McKelliget et al. demonstrated that the prediction was fortuitous and that the model breaks down when the radius of curvature of the jet axis becomes equal to the radius of the jet.

Oryall and Brimacombe generated detailed contour maps of the gas fraction and bubble frequency distribution of horizontally injected air jets in mercury. Their measurements indicated that close to the nozzle the jet consisted of a core of high gas concentration which gradually decreased toward the edge of the jet. They found that the distribution of gas is affected by the Froude number and the nozzle diameter; both
factors have the effect of increasing the forward penetration of the jet and also the gas fraction in the core of the jet. However, they do not alter the columnar shape of the plume nor its back penetration. Fruehan et al.\textsuperscript{51} reported that the size of the gas-rich zone increases with the gas flow rate and that strong radial variations in gas concentration are present.

Recently Tacke et al.\textsuperscript{56} used an electroresistivity probe to measure the radial profiles of gas concentration and bubble frequency during upward vertical injection. They found that the radial profiles of gas fraction and bubble frequency are similar and could be represented by a single Gaussian curve. Using the modified Froude number they presented correlations for the axial gas fraction and half-value radius to describe the distribution of gas within the plume, in water and mercury systems. Kawakami et al.\textsuperscript{55}, injecting argon into pig iron, found that the profiles of bubble frequency and gas fraction exhibited different shapes - single peak, double peak and combination of the two - depending on the position along the jet. Single-peak profiles could be represented by a Gaussian curve.

Several researchers have developed mathematical models to represent fluid flow in turbulent bubble-driven recirculating flows. In the more recent models\textsuperscript{70-74} the gas-liquid plume has been included. For the purpose of modelling, the plume has been treated as a homogeneous medium of variable density. McKelliget et al.\textsuperscript{71} have reported the axial profiles of the gas fraction for two different exit gas velocities. The model predicts that the
gas fraction decays at a steady rate which is virtually the same for both jets.

2.5 Fluid Motion in the Gas-Liquid Region

The gas-liquid region formed during submerged injection is characterized by a suspension of bubbles moving in a continuous liquid. Owing to its reduced density, this gas-liquid plume provides a recirculating flow pattern, with upward flow close to it and downward flow near the walls of the bath containing vessel. In regard to fluid motion in bubble-driven recirculating flows, it has been found that the system is strongly non-uniform. The region of high velocities and high values of turbulent kinetic energy is confined to the jet plume and to the vicinity of the free surface, whilst the remainder of the system is relatively quiescent. Fluid motion in the gas-liquid zone is highly important as the determining factor of turbulent mixing.

The motion of the gas envelope produced by a point source of gas located in a body of ideal liquid of large size has been much discussed by many investigators as was seen in Sections 2.1 and 2.2. In the Davidson and Schuler\textsuperscript{37} model, for the growth of bubble in an inviscid liquid and under constant flow, a gas-filled envelope, assumed rigid, expands radially from the source and translates upward under buoyancy. As the bubble grows and the buoyancy force begins to act strongly, pushing the center of the bubble upward, the bottom surface of the bubble is brought to rest when its downward velocity relative to the centre of the
bubble is equal to that of the bubble center. Thereafter the bubble base has a net upward velocity and detachment of the bubble occurs when the base reaches the point source. With an effective inertia of \(1/2\rho l V_b\), the theory of Davidson and Schuler predicts the initial rise velocity of the bubble at detachment to be

\[
\frac{ds}{dt} = 1.138 g^{2/5} Q_0^{1/5}
\]

(2.5)

Wraith and Kakutani\(^{41}\) have observed that fully formed bubbles detach from the orifice to form a lengthening stem and then continue to grow as they rise. The initial theoretical rise velocity of this bubble when the stem is severed is \(1.6(Q_0 g^2)^{1/5}\) and its acceleration is about \(1.8g\). This large severed gas bubble disintegrates to produce a turbulent column of smaller bubbles.

Andreini et al.\(^{45}\) measured the rise velocity of gas bubbles in liquid metals under laminar flow conditions at the orifice. The velocities were calculated by the ratio of the bath depth to the residence time of the bubbles in the melt. They found that bubble rise velocities were greater than those corresponding to the rising of single bubbles. This was believed to be caused by the proximity and wake effects of the bubbles as they rise under dynamic conditions. The velocity was observed to increase with increasing bubble size and decreasing bubble proximity.

Fruehan et al.\(^{51}\) determined the average gas velocities at different levels of an air jet in water and water-glycerol solutions. The average velocities of the gas were calculated from
integration of the gas fraction over a small volume and from the known gas flow rate. The results of their calculations show that the gas velocity decreased very rapidly from the superficial velocity at the orifice to a limiting value of about 1.2 m/s at a distance of 5 cm above the tuyere, for all the conditions studied. Kawakami et al. used a double-contact electro-resistivity sensor to measure directly the rise velocity of nitrogen bubbles in pig iron. Their results indicated that at low flow rates the radial bubble velocity profiles were very flat and did not change with vertical position, the profiles became steeper with increasing gas flow rate. The average bubble rise velocities were in the range of 0.5 to 3.5 m/s. Haida and Brimacombe used an electrochemical technique to measure the liquid velocity in nitrogen and helium-water plumes. They reported that increasing gas kinetic energy results in higher liquid velocities in the vicinity of the tuyere tip. However, at higher levels above the tuyere, channeling, due to high gas kinetic energy reduces the liquid velocity. They concluded that the liquid velocity in the plume decreases with vertical distance from the tuyere when the gas kinetic energy exceeds the buoyancy energy by a factor of about 10.

Hsiao et al. studied gas injection into a water model and in ladles, 6 and 60 tonnes, stirred with argon. They carried out measurements of the liquid velocity in the gas-liquid plume with the aid of a drag probe. Their measurements show that the radial velocity profiles could be represented by a Gaussian curve. The
centreline velocity varies with the gas flow rate to the power of approximately 0.23 and is almost independent of the height in the plume, except in the initial region. This region was found to be about one tenth of the total height of the bath. They concluded that the bubble-liquid flow above the orifice is driven by buoyancy except in the initial region, and that a plume model is more adequate than a jet model to represent the gas-liquid region.

Figueira and Szekely recently reported measurements of fluid flow and turbulence in a water model of an AOD vessel. Laser Doppler velocimetry was used to determine time-smoothed velocities and rms of the fluctuating velocity components inside the gas-liquid zone. The radial velocity profiles were normalized with respect to the maximum velocity and the total thickness of the jet. The profiles showed reasonable similarity, with the maximum displaced toward the nozzle. The relative rms of the fluctuating velocity indicated that turbulence outside the jet is fairly isotropic and high. A comparative study concluded that for the AOD model, both the velocity field and the spatial distribution of turbulent kinetic energy seem reasonably uniform outside the two-phase region, in contrast to the model of the argon-stirred ladle, where the velocities and turbulence levels are very high near the jet cone and in the free surface but are much lower elsewhere.

Recently several macroscopic models based on the conservation of mass, energy and momentum have been proposed to represent the fluid motion in the plume. This
research has been motivated by the need to derive some simple mathematical expressions that relate liquid circulating flow rate and mixing time to vessel dimensions, gas flow rates and plume characteristics.

Sano et al.\textsuperscript{79} have proposed a simple model to characterize fluid flow in a liquid bath agitated by bubbles. The model postulates that the bath consists of two zones: a central bubble plume moving upward and a liquid annular zone moving downward. The analysis is based on a steady-state energy balance for the liquid plume, which establishes that the rate of energy dissipation due to liquid circulation is equal to the rate of energy input. These authors assumed that the rate of energy dissipation is equal to the difference between the rate of kinetic energy associated with the liquid moving upward and that of the liquid moving downward plus an energy dissipation due to the bubble slip. The equations thus derived allow the calculation of the liquid velocity in the plume zone, the liquid circulation flow rate and the mixing time as a function of gas flow rate, liquid depth and cross sectional area of the plume. The model predicts that the liquid velocity in the plume is directly proportional to the gas flow rate to the power of one-third.

Sahai and Guthrie\textsuperscript{78} also have proposed a model to analyze the interaction of a submerged gas jet with liquids to cause stirring. Like Sano et al.\textsuperscript{79} they considered vertical upward injection at the center of a cylindrical vessel and also conducted an energy balance. The balance establishes that once
steady state is reached in the bath the rate of energy input by the rising bubbles counter-balances the rate of turbulent energy dissipation within the bulk of the liquid. The equation resulting from the analysis predicts that

\[ U_p \alpha \left( \frac{Q_o^{1/3} h_b^{1/4}}{R^{1/3}} \right) \]  

(2.6)

In this model it is assumed that gas and liquid in the plume move at the same velocity. More recently the model was modified to represent vertical downward injection with a lance.

Tacke et al. \textsuperscript{56} applied an integral-profile technique to study vertical gas injection in water and mercury. This involved consideration of the conservation equations of mass for the gas and liquid together with the equation of conservation for the vertical momentum in integrated form. The profiles of gas concentration and liquid velocity were taken to be Gaussian and the velocity difference between the rising bubbles and the liquid was introduced. The group of differential equations that resulted from the analysis was solved for a region separated from the nozzle where the maximum gas fraction had dropped to fifty percent, since in this region the assumptions of the model were considered to be closely approached. Reasonable predictions of axial bubble velocity profiles were obtained. Most of the work on integral-profile methods applied to gas-liquid plumes has been directed to environmental problems, such as inhibition of ice formation, barriers against intrusion of salt water to locks and water aeration. \textsuperscript{81-84} Here large quantities of gas are injected into large volumes of water, and whilst these conditions do not
relate directly to bath stirred in vessels, some common behaviour can be observed. For example the velocity and gas concentration profiles are Gaussian and the mean rising speed of the bubble stream increases with the gas flow rate to the 1/6 power.

In recent years major advances have been made in the development of mathematical models for turbulent recirculating buoyancy-driven flows. In these models the researchers have recognized the importance of the plume in generating buoyancy flow and have proposed computational schemes wherein the gas-liquid mixture contained in the jet region is represented by a fluid of variable density. Reasonably good agreement between computed and observed flow patterns outside the gas-liquid zone have been achieved. However, computed results on the plume characteristics (voidage, velocity of the phases and so forth) have scarcely been verified or reported. McKelliget et al. have studied the influence of the gas injection velocity on the centerline velocity of the plume. They found that the relative effect of buoyancy on the induced plume motion was inversely proportional to the square of the injection velocity. For an injection velocity of 10 m/s the axial velocity along the plume fell continuously as the surrounding fluid was entrained, while for lower superficial velocities the axial velocity profile exhibited a lower region where the plume was accelerated by a comparatively strong buoyancy force until it reached a maximum velocity. The velocity then decreased as the effect of buoyancy decreased as more liquid was being entrained and the gas hold-up diminished.
2.6 Techniques for Measuring Local Gas-Liquid Bubble Flow Parameters

The realization that two-phase flows in general are not homogeneous and that more attention has to be paid to the detailed discrete structure of these flows, has resulted in the development of many experimental devices and techniques. Two-phase flow instrumentation has most of the problems and characteristics found in single-phase instrumentation, plus additional problems inherent to the presence of a second phase. Hinze\(^8\) has listed the requirements that a measuring instrument must satisfy before local measurements can be undertaken in turbulent one-phase flows. Jones and Delhaye\(^8\) have listed the difficulties encountered in two-phase flow measurements, due to the presence of the second phase.

In both single and two-phase flow systems an instrument must have the following characteristics to produce reliable results:

(a) The detecting element introduced into the flowing field must have an adequate shape and be so small that it causes only a minimum admissible disturbance of the flow pattern and of the passing interfaces.

(b) The detecting element must be smaller than the dimensions of the flow element to be detected.

(c) The inertia of the instrument must be low, so that it can respond to fluctuations in the measured parameter.

(d) The instrument must have sensitivity to small variations in the measured parameter.
(e) The instrument must be physically and chemically stable, so that no noticeable changes in response occur to fixed conditions.

(f) The instrument must be sufficiently strong and rigid to withstand vibrations or motions caused by the turbulent field.

The additional characteristics required in two-phase instrumentation are:

(a) The instrument must exhibit a substantially different response to the passing phases.

(b) The instrument must have an adequate discrimination level to be able to separate the contribution of each phase to the signal.

(c) The instrument must be provided with a suitable signal interpretation technique to produce information related to each phase.

Some of the instruments used to measure local parameters in two-phase bubble flows are:

(a) Local void fraction
   - electroresistivity probes
   - optical probes
   - isokinetic probes
   - hot wire anemometers
   - phase detecting microthermocouples
(b) Gas velocity
- double resistivity probes
- double optical probes

(c) Liquid velocity
- hot-wire anemometers
- hot-film anemometers
- isokinetic probes
- impact probes
- laser Doppler anemometers

(d) Temperature of each phase
- phase detecting microthermocouples.

In this review these methods are described, a few of which have been applied to the study of gas-liquid jets in injection metallurgy. Excellent reviews on the general subject of two-phase bubble flow instrumentation are given in the works of Jones and Delhaye, Hewitt and in the work edited by Hetsroni and by LeTourneau and Bergles.

2.6.1 Definition of Some Fundamental Quantities Describing Two-Phase Flows

Given the fluctuating character of two-phase flows, averaging operators have to be introduced. The definition and application of these operators have been reviewed by Delhaye and Achard, Delhaye and Ishii. Some of the operator
definitions will be presented here owing to their relevance to the present work.

(a) Phase density function

The existance of a phase k at a given point, x, and time, t, can be expressed by a phase density function \( X_k(x,t) \),

\[
X_k(x,t) = \begin{cases} 
1 & \text{if point } x \text{ at time } t \text{ pertains to } k \\
0 & \text{if point } x \text{ at time } t \text{ does not pertain to } k 
\end{cases}
\]  

(b) Local time-averaging

If we consider a given point, x, in a two-phase flow and phase k passes this point intermittently, then a field variable \( f_k(x,t) \) associated with phase k will be a piece-wise continuous function. Two time-averaging operators can be defined for the phase density function

\[
\frac{-X}{T_k(x,t)} = \frac{1}{T} \int_T dt
\]  

which averages over the whole time interval T, and

\[
\frac{-X}{T_k(x,t)} = \frac{1}{T_k(x,t)} \int_{T_k(x,t)} dt
\]  

which averages over the cumulative residence time of phase k, \( T_k(x,t) \).

The local time-averaged k-fraction, \( \alpha_k \), is defined as the average over T of the phase density function \( X_k(x,t) \)
\[ \alpha_k(x,t) = \frac{X_k(x,t)}{T_k} = \frac{1}{T} \int_T X_k \, dt = \frac{1}{T_k} \int_{T_k} \, dt \quad (2.10) \]

\[ \alpha_k = \frac{T_k}{T} \quad (2.11) \]

(c) Instantaneous space averaging

An instantaneous field variable associated with phase \( k \) may be averaged over a space \( M \) of dimension \( n \). Two space-averaging operators can be defined

\[ \langle \rangle = \frac{1}{M} \int_M dM \quad (2.12) \]

over the whole flow space, and

\[ \langle \rangle = \frac{1}{M_k} \int_{M_k} dM \quad (2.13) \]

over the space pertaining to phase \( k \).

The instantaneous space-averaged \( k \)-fraction, \( R_{kn} \), is defined as the average over \( M \) of the phase density function \( X_R(x,t) \)

\[ R_k = \langle X_k(x,t) \rangle = \frac{1}{M} \int_M X_k \, dM = \frac{1}{M_k} \int_{M_k} dM \quad (2.14) \]

\[ R_k = \frac{M_k}{M} \quad (2.15) \]

(d) Commutativity of averaging operators
Considering the time-averaging of a space-averaged function, the following can be written

\[ \langle X_k f_k \rangle = \frac{1}{T} \int_T \left[ \frac{1}{M} \int_M X_k f_k \, dM \right] \, dt \]  

(2.16)

and since both intervals of integration are finite, they can be commutated such that

\[ \langle X_k f_k \rangle = \frac{1}{M} \int_M \left[ \frac{1}{T} \int_T X_k f_k \, dt \right] \, dM \]  

(2.17)

\[ \langle X_k f_k \rangle = \langle X_k f_k \rangle \]  

(2.18)

For the special case where \( f_k = 1 \)

\[ \bar{R}_{kn} = \langle \alpha_k \rangle_n \]  

(2.19)

That is, the time-averaged and space-averaged void fractions are related through the commutativity of the averaging operators.

(e) Time and space-averaged volume flux

The instantaneous volume flux rate of phase \( k \) through an area \( A \), may be given as

\[ q_k = \frac{1}{A} \int_A X_k U_kz \, dA \]  

(2.20)

Averaging Equation (2.20) with respect to time and commuting operators
\[ \bar{q}_k = \frac{1}{A} \int_A \left[ \frac{1}{T} \int_{t_k}^{t} X_k U_{kz} \, dt \right] \, dA \]  
\[ (2.21) \]

\[ \bar{q}_k = \frac{1}{A} \int_A \left[ \frac{1}{T} \int_{t_k}^{t} U_{kz} \, dt \right] \, dA = \frac{1}{A} \int_A \frac{T_k}{T} \bar{U}_{kz} \, dA \]  
\[ (2.22) \]

\[ \bar{q}_k = \langle \alpha_k \bar{U}_{kz} \rangle \]  
\[ (2.23) \]

Application of Reynolds rules\(^{90}\) leads to \( \bar{f} \bar{g} = \bar{f} \bar{g} \). Thus Equation (2.23) can be written as

\[ \bar{q}_k = \langle \alpha_k \rangle \langle \bar{U}_{kz} \rangle \]  
\[ (2.24) \]

(f) Transit velocity of a moving interface

If a fluctuation travels with the flow at the flow velocity, the fluctuation can be treated as a tracer. For example, when a probe is used to sense a local parameter such as void fraction, and another similar probe is placed downstream, two fluctuating curves will result. The curves will be similar but will exhibit time shift. The time shift represents the time required for the fluctuation to travel from one probe location to the other.

Double-contact probes have been used to determine local bubble migration velocities by two methods: two-state signal method and cross-correlation method. In the two-state signal method, an individual pair of consecutive signals is considered and their time difference is measured. For a defined and accurately known probe separation
\[ U_b = \frac{d_p}{t_C} \] (2.25)

This method of measuring gives the mean bubble velocity together with the bubble transport velocity spectrum. This technique presents the difficulty of having to relate sequential signals in the upper and lower channels to the individual events occurring at the probe tips.

In the cross-correlation method the whole signal spectrum, obtained over a long time, is used to calculate a cross-correlation coefficient

\[ R_{12}(\tau) = \frac{1}{T} \int_T f_1(t) f_2(t+\tau) \, dt \] (2.26)

where \( f_1(t) \) is the measured quantity at one point as a function of time and \( f_2(t+\tau) \) is the quantity measured at a downstream point at time \( t + \tau \). The value of \( R_{12}(\tau) \) reaches a maximum at \( \tau = \tau_M \), where "\( \tau_M \)" is the most probable transit time of the disturbance between the probes. The most probable velocity of the bubbles is then given by

\[ \bar{U}_b = \frac{d_p}{\tau_M} \] (2.27)

One major problem with this technique is that it measures only one velocity. Jones and Delhaye\(^{86}\) have pointed out that calculating bubble size from the velocity obtained by cross-
correlation may neglect the correlative effects of bubble size on velocity.

Delhaye et al. have pointed out that by using a double-contact probe, a bubble velocity can be measured but one has to be careful when assigning a physical significance to this bubble velocity. For a more detailed discussion on this subject see Appendix I.

2.6.2 Electroresistivity Probes

The electroresistivity probe detects the presence of a phase based on the difference in electrical conductivity between the phases. The difference in electrical conductivity must be large and the conductive phase must be continuous. The probe array consists of one small measuring electrode, made up of a wire insulated except at its tip, and a reference electrode with a much larger surface area. The reference electrode may be the metallic protecting case of the probe, a second wire adjacent to the active probe and welded to its case or the general ground of the test section. The two electrodes are generally connected by a generator and a resistance in series. Current flows whilst the measuring electrode is resident in the conducting phase but it is interrupted as the gas surrounds the electrode, and thus develops a voltage pulses across the load resistor.

Suitable processing of the signal allows the measurement of the local void fraction and the arrival frequency of the bubbles at a given location. The analysis of the signals
may result in statistical information characterizing the flow pattern. Bubble migration velocities and characteristic bubble dimensions have been measured through the use of double-contact sensors.

The main feature that differentiates electroresistivity circuits is the type of power supply. Direct-current supply systems require low voltages to minimize electrochemical phenomena at the sensor, when used in an ionic liquid conductor. In addition resultant electronics may prove troublesome as amplification of the signals may be required. Some authors have reported problems when working with a direct-current supply in aqueous media, whilst others have not. Direct-current supply has been widely used in liquid metallic systems such as mercury and liquid iron.

Alternating-current supply has been used by several investigators to suppress problems associated with electrochemical phenomena at the sensor tip. The current frequency has to be substantially different from the frequency of the phenomena measured. Frequencies higher and lower in relation to that of the phenomena measured have been used. Lower frequencies provide pseudo-direct current operation.

Depending on the manner in which the sensor is energized, the ideal output signal of a resistive probe is either a binary wave sequence, or a sequence of bursts of constant amplitude oscillations separated by zero voltage areas. In actual practice, however, the response of the electroresistivity probes and other
needle contact probes e.g. optical probes, is misshapen with respect to the ideal signals. This is due to hysteresis in the contact of the phases with the probe. Delhaye et al.\textsuperscript{102} have conducted a detailed investigation of the response of resistivity probes to local void fraction fluctuations. The time lapse for the removal of the liquid film from the probe tip has been found to be affected by the geometry and size of the probe tip, the velocity of the bubbles\textsuperscript{98,102} and the type of liquid.\textsuperscript{56,95} The probe performance is improved by using small sharp tipped probes at high flow rates and in non-wetting liquids. Gas-liquid voltage transitions occur more rapidly than liquid-gas voltage transitions\textsuperscript{99,103}.

The true signal is generally transformed to a sequence of square pulses with the help of a single threshold level near the liquid voltage. It has been found, that if the local void fraction is plotted versus the trigger level, an S-shaped curve is obtained with a slowly changing plateau over a certain trigger level range. Herringe et al.\textsuperscript{94} and Iida et al.\textsuperscript{100} chose a trigger level corresponding to this plateau. The level adjustment also has been based upon comparison of the line-averaged gas fraction with the line void fraction measured directly with $\gamma$-ray absorption\textsuperscript{104}. In other cases the threshold level simply has been selected close to the liquid voltage level.\textsuperscript{56,97,99}

Galaup\textsuperscript{105} found that the optical, the anemometer and the electroresistivity probe give comparable results regarding the measurement of local gas fraction. In another study Herringe et
al. compared the quality of response of hot-wire and hot-film anemometer probes, electroresistivity probes and infrared absorption probes. They found that the most suitable system for phase detection was the electroresistivity probe, followed by the hot-wire anemometer.

Double contact probes have been used to determine local bubble velocity by two methods: two-state signal method and cross-correlation method.

Calderbank et al. developed a tridimensional resistivity probe, consisting of five measuring contacts, to determine the velocity, size and shape of bubbles rising in a vertical tube. The arrangement of the contacts and the signal analysis performed by a computer allowed resolution of the location at which the bubbles were struck by the probe relative to their respective centers. The technique eliminated most of the uncertainties associated with the varying and random positions and sequences at which the bubble contacted the probe. The technique produced very reliable information. However, the probe arrangement was bulky and the strict discriminating logic could render the technique difficult to use in two-phase systems having a highly turbulent and concentrated dispersion of bubbles. Other multiple-electrode systems have been reported.

Serizawa et al. used a double-contact sensor to study air-water bubble flow in a vertical column. They measured the bubble transport velocity using the two-state signal method and the cross-correlation method. The mean and the most probable bubble
velocity, obtained from each method compared reasonably well. Uncertain time delays, which appear during the use of the two-state signal method e.g. due to the arrival of different bubbles at the contacts or to the bubble transverse motion, were considered to be homogeneously distributed and to have a small probability of occurrence. Hence, the average bubble velocity was obtained by simply neglecting the velocities associated with those times. A more rigorous discriminating analysis of the signals carried out by Lewis et al.\textsuperscript{106} indicated that typically 40 per cent of the bubbles intercepted could not be considered in the measurement of the velocity, since these bubbles did not produce consecutive signals having good correlation. Gunn et al.\textsuperscript{107} in their study of bubbles flows in fluidized bed considered as physically possible only those bubbles velocities within the range 0.05 to 2 m/s. Velocities outside this range were considered as outliers and were neglected during the processing of the data.

Herringe et al.\textsuperscript{98} used a double-contact electroresistivity probe to measure the migration velocity of bubbles in air-water mixtures flowing vertically. They used the cross-correlation method to measure the bubble velocity. The chord lengths of the bubbles were calculated from the residence time of the bubbles at the tip and the velocity obtained from cross-correlation of the signals. Statistical analysis allowed them to calculate the bubble size distribution from the chord length distribution.
Tacke et al. measured gas fraction and bubble frequency distributions in submerged gas jets in water and mercury. They also measured a reduced number of bubble transit velocities using a double-contact sensor. The transit velocities were measured on an oscilloscope for only those bubbles that produced consecutive similar signals. Kawakami et al. reported, in a recent investigation, the measurement of gas fraction, bubble frequency and bubble transport velocity in a nitrogen jet in pig iron. They used a probe made of a graphite rod of 1.5 mm in diameter connected to a nickel wire. The graphite rod was electrically insulated with a mullite tube encased in a silica and a stainless steel tube. The cross-correlation method was used to measure the bubble transport velocity.

2.6.3 Optical Probes

An optical probe is a device sensitive to the change in the refractive index of the surrounding medium and is thus responsive to the passage of interfaces. This enables the measurement of local void fraction and interface passage frequency. The use of two sensors can allow the measurement of the bubble transit velocity.

The optical probe consists of a two-way glass light guide "joined" to a right-angled prism which reflects light back if surrounded by air but not if surrounded by water. The light is provided by a lamp at one end of the light guide and it is detected at the other end by a phototransistor. For a given
angle, temperature, refractive index of the probe material and wave length of light, the amount of refraction depends on the refractive index of the surrounding phase. Air-water bubble flows can be studied adequately using a glass fiber probe having a tip with a half angle of $45^\circ$. A U-shaped fiber probe allows a greater degree of miniaturization than glass-rod or fiber-bundle probes.

A thorough investigation of the response of the optical probe for local void fraction measurement has been reported by Jones et al. They found that the dynamic response of the probe is affected by the adherence of liquid to the probe tip. Thus the output departs from the ideal binary form. Jones et al. found that the performance of the probe decreased with an increase in bubble velocity. This behaviour was explained by proposing that the water film thickness left on the probe increases with the bubble velocity. This response of the optical probe to the velocity of the bubble seems to be opposite to the response of the electrical probes described in Section 2.6.2.

Signal discrimination in optical probes has been accomplished through the use of two adjustable threshold levels, which enable the transformation of the true signal into a square wave signal. The threshold voltage selection presents the same problems as described for resistivity probes, and the researchers decisions for the selection have also been the same.

Herringe et al. reported another type of optical probe, an infrared absorption probe. The system was comprised of an
emitting and a detecting diode approximately 1.5 mm apart. An indication of the presence or absence of liquid between the two elements relies upon the strong absorption of infrared radiation by liquids in comparison with gases. Signal interpretation in this case was carried out by analysis of the amplitude probability density distribution of the signals. The data produced by the probe had a wide scatter because it was subjected to reflection and refraction of infrared radiation by small bubbles.

2.6.4 Isokinetic, Differential Pressure and Impact Probes

Isokinetic probes belong to the class of flow separation methods. Here a two-phase sample is withdrawn through a probe at such a rate that the pressure just inside the probe orifice is equal to the local static pressure in the stream. That is to say, the flow into the probe would ideally take place at the velocity prior to the insertion of the probe. Where there is slip between the two phases, the term "velocity" is ambiguous. Shires et al., in an extensive study of two-phase bubble flow, have demonstrated theoretically that for gas-liquid mixtures where $\frac{\rho_g}{\rho_l}$ is small, the superficial velocity of the liquid on entering the probe is equal to the local value in the stream, i.e. the probe samples isokinetically in terms of the superficial water velocity, and the voidage of the sample is equal to the local voidage in the stream. Both characteristics were confirmed experimentally in air-water flows in vertical tubes.
If the sampling line of the isokinetic probe is closed, the probe will act as a Pitot probe measuring the local impact pressure, but again, the interpretation of the results is somewhat difficult when the two phases move at different velocity. One approach which commonly has been used is to assume that the two-phase flow behaves as a homogeneous mixture with no slip velocity. Under these conditions the velocity measured by the probe is given by

\[ U = \left( \frac{2 \Delta P}{\rho_M} \right)^{1/2} \]  

where the homogeneous density must be known. This approach has proven to be effective in considering high mass velocity flows and drop-laden flows in a gas continuum. Sahai and Guthrie have used a Pitot tube to determine mean (homogeneous) plume velocities, in gas-liquid plumes. Hsiao et al. measured the impact pressure along the axis of various gas-water jets.

Obviously the main problem with the use of Pitot probes to measure local velocity is that they can be applied only in situations were the flow is likely to be homogeneous with no slip between the gas and the liquid. Other difficulties arise in controlling the kind of fluid present in the tapping lines, which is generally difficult in two-phase flow measurement.
Drag body probes of different shapes have been used to measured the momentum flow of the fluid. In a single liquid-phase flow the velocity of the flow can be calculated as

\[
U = \left( \frac{2 F_D}{\rho_1 C_D A} \right)^{1/2}
\]

(2.29)

Drag form probes have been applied widely in two-phase flow studies, but the interpretation of the recorded drag force is difficult and various models have been adopted to interpret the results. Hsiao et al. used a drag body probe to measure radial profiles of an average plume velocity. The formula given above was used with a calibrated drag coefficient.

2.6.5 Hot Film / Wire Anemometry

Anemometer measurements are based on the response of the electrical resistance of a detecting element to the flow velocity of the fluid passing the sensor. The detecting element in a hot-wire anemometer is a very short metal wire. Hot-film anemometry uses an inert substrate, such as glass, plated with a thin metallic film. The geometry of the probe may be cylindrical or conical. The resistance of the metal is a function of its temperature; as the sensor is cooled by the flowing fluid, its electrical resistance diminishes. Wire cooling is controlled by forced convection.
Hot-wire and hot-film anemometers can operate in the constant current mode and in the constant temperature mode. Today the latter mode is the most commonly used. The response of hot-wire and hot-film probes unfortunately is very non-linear. The calibration curve for either probe type usually can be fitted over workable velocity ranges by a relation of the type

$$e = A + B U^n$$

(2.30)

Owing to the difficulties associated with the nonlinearity of the hot-wire / -film probe calibration, linearizers are available to give an output voltage proportional to velocity over a narrow range. Hinze\textsuperscript{85} has given a detailed discussion of the principles of hot-wire and hot-film anemometry.

Hot-wire and hot-film anemometers have been used widely for measuring velocities and turbulent fluctuations in gases over large velocity ranges. The application to liquids has been more recent and restricted to low velocities. Various researchers have used these instruments in two-phase air-water bubble flows to determine the instantaneous velocity and turbulent intensity of the water phase\textsuperscript{97,119} and also to measure the local gas fraction and arrival frequency of the bubbles\textsuperscript{110,120}

Delhaye\textsuperscript{110}, in his study of air-water bubble flows, used a conical quartz coated film probe working in the constant-temperature mode. He found that the conical geometry had the advantages of almost completely eliminating the deposition of small particles on the sensor and of having little influence on
the trajectory and shape of the bubbles. The constant temperature anemometer, with a small superheat, also avoided degassing on the sensor. In order to determine the void fraction and velocity of the liquid, the gas and liquid signals had to be separated. Delhaye did this by obtaining the amplitude probability density distribution of the output signal. This distribution indicates the probability that the voltage is near a given level (gas or liquid), and it distinctly shows high probabilities of the voltage signal near the air and water level. To a first approximation the local void fraction was calculated as the ratio of the area associated to the gas phase to the total area below the probability density curve. To verify the signal analysis procedure, the local gas fraction was averaged over the diameter of the test section and compared with the averaged gas-fraction on the same diameter obtained with a γ-ray absorption method. The liquid time-averaged velocity and the liquid turbulent intensity were calculated from the liquid-related area of the amplitude probability density distribution and the calibration curve of the probe immersed in liquid.

Jones and Zuber$^{120}$ used a cylindrical hot-film anemometer to measure void fraction and velocity profiles in a vertical channel. The sensor experienced problems at velocities much greater than 1.5 m/s. The problems were zero drift, calibration change, and degassing on the sensor and originated from the cracking of the quartz coating due to fatigue and overstress in moderate and high velocity water flow. They used voltage level discriminators to produce a binary signal and to measure void
fraction and bubble frequency. Errors in the void fraction were encountered when calibrated against X-ray void measurements. The errors were attributed to finite dewetting times. The local volume flux also was measured; this property is defined as the time average of the signal which is equal to the phase velocity when the phase is at a point, and equal to zero when the phase is not at a point. The liquid velocity then was obtained by pointwise division of the measured liquid flux by the measured gas fraction. This procedure follows from the definition of the averaging operators given in Section 2.6.1.

Serizawa et al.\textsuperscript{97} used the method and signal procedure recommended by Delhaye\textsuperscript{110} to measure water velocity and turbulent intensity in bubble and slug flow in a vertical tube. The liquid flow rate calculated from integration of the profiles of the product of local liquid velocity and local liquid fraction was in good agreement with that measured directly by a turbine flowmeter. Thus, the authors concluded that the calibration of the anemometer obtained in water flows is valid also for bubble flow as long as the temperature of the fluid and the overheat ratio of the probe are kept constant.

2.6.6 Laser-Doppler Anemometry

The laser-Doppler anemometer (LDA) makes use of the frequency shift of a wave which occurs when the wave is transmitted by a moving source or received by a moving receiver. The LDA uses a laser to provide incident light, which is
scattered by moving particles in the fluid under study. The particles scatter the light and therefore can be considered as moving transmitters. The relationship between the frequency shift of the light i.e. Doppler frequency, and the fluid velocity is

$$U = \frac{\lambda f_d}{2 \sin(\theta/2)}$$

(2.31)

The LDA technique is ideal to measure velocity and turbulence in transparent fluids because it is non-intrusive, its response is linear, and it is calibration free. The laser-Doppler anemometer consists of the laser, a beam splitter, lenses, photodetector and electronics which may be arranged in several forms of operation. Two of the most common arrangements are: the reference-beam mode and the fringe mode.

Recently some researchers have used LDA to study two-phase flow systems. Ohba$^{121}$ employed a laser-Doppler anemometer in the reference-beam mode to measure void fraction and liquid velocity and turbulence in a rectangular duct. Sullivan et al.$^{122}$ used a laser-Doppler anemometer in the fringe mode to measure liquid velocity and turbulence in a bubble flow contained in a vertical cylindrical pipe. Single bubble experiments were done to verify that the light scattered by bubble interfaces was discriminated and not interpreted as a valid liquid velocity measurement. The axial turbulent intensities from the LDA were approximately twice the values obtained by Serizawa et al.$^{97}$ using a hot-film anemometer. The reason for this discrepancy is not known.
Figueira and Szekely\textsuperscript{77} used a LDA operating in the fringe mode, with analysis of the forward-scattered light, to measure the velocity and turbulence of the water in an air-water jet. The problems of signal interpretation produced by the refraction of the laser by the bubbles were overcome by filtering the measurements using a microcomputer.

Recently Boerner et al.\textsuperscript{119} explored the relative merits of laser-Doppler and hot-film anemometry techniques in a study of buoyancy-induced bubble two-phase flow, in a rectangular column. The study involved the determination of the local mean liquid velocities and the root-mean-square values of the liquid velocity fluctuations. The gas component of the signal, in both techniques, was identified and discriminated. The authors found that in the case of mean velocities, the two experimental techniques agreed very well with one another. On the other hand, the rms-values of the liquid velocity fluctuations were slightly larger with the laser-Doppler than with the hot-film anemometer. This difference was explained as a tendency of laser-Doppler velocimeters to statistically overemphasize fast motions. The result of the investigation also showed that the sampling time required to obtain statistically meaningful averages increased with both the width of the flow field and the void fraction, limiting the practical utility of LDV in the instance of a 120 mm wide bubble column, to a void fraction $< 2\%$. This problem was associated with the frequent disruption of the measuring volume by bubbles.
2.6.7 Miscellaneous Methods

Chang et al.\textsuperscript{123} have employed an ultrasonic technique to measure liquid film thickness and to characterize flow regimes in gas-water and gas-liquid metal flows. The use of this method to obtain void information is difficult in geometries where reflections occur or at moderate and high void levels, > 20\%.

Davis et al.\textsuperscript{124} used X-ray cinematography to make observations of bubble formation in liquid metals. The results confirmed that bubbles in liquid metals are formed at the outer circumference of a nozzle.

Microthermocouples have been used to investigate the temperature and phase distribution in two-phase systems of interest in nuclear engineering. The analysis of the temperature histograms has allowed the separation of the temperature of the liquid phase from the temperature of the gas phase. This instrument gives statistical properties of the temperature in each phase as well as the local void fraction\textsuperscript{125}. 
CHAPTER 3

OBJECTIVES OF THE PRESENT WORK

3.1 Summary of Previous Works

As described in the last chapter, certain aspects of the fluid dynamic behaviour of gas injection systems have received considerable attention in the metallurgical literature. These aspects are the process of bubble formation, the regimes of discharge of the gas and the motion of liquid that surrounds the gas-liquid plume. The forces that determine the bubble volume at detachment, under different bubbling regimes, have been identified. Also models have been proposed to predict, with sufficient accuracy, the bubble volume for gas injection into liquid metals, particularly under low gas flow rates and non-reactive conditions. Regarding the behaviour of the discharging gas, bubbling and jetting regimes have been identified depending on the driving pressure of the gas. Several transition criteria, which allow the prediction of the discharge regime under different situations, have been presented based on visual assessment of the flow. The difficulties in obtaining a general transition criterion undoubtedly indicate the need for more research into the nature of the two-phase bubble plume.

Extensive experimental and theoretical work has been carried out on the velocity and turbulence fields in the liquid surrounding gas-liquid plumes; and a fairly good understanding
of mixing and reaction kinetics in gas-stirred ladle systems has resulted. A major factor which complicates the study of the overall fluid dynamic behaviour of these systems is the gas-liquid plume which introduces additional variables, in addition to the problems of turbulent motion which are common to both single-phase and two-phase flows. These extra variables include the distribution of the phases, the velocity of the two phases, the existence of interfacial and gravitational forces, the expansion of the gas phase and the geometry of the boundary between the plume and the surrounding liquid. Very little experimental evidence exists on the role of these variables as a function of the gas inlet conditions and the physical properties of the system.

Work has been reported on the influence of the liquid physical properties ($\rho_l, \mu, \sigma$) and the orifice Reynolds number on the mean size of bubbles rising in gas-liquid dispersions. Generally, investigators have concluded that an 'equilibrium' bubble size is established after the bubbles have travelled a certain distance from the nozzle. However very little experimental evidence of the existence of such an 'equilibrium' condition is available. There is only scattered information on the size distribution of the bubbles and the distance over which the disintegration of the initial gas envelopes occurs in systems of metallurgical interest. The size of bubbles after detachment has been determined mainly by direct photography or electro-resistivity probe techniques. The photographic technique, despite the tedious effort involved, usually does not provide very high
accuracy. Similarly, the determination of bubble size by cross-correlation of the signals from the electroresistivity probes has severe limitations because it assumes that all the bubbles move at the same velocity.

The radial profiles of gas concentration and bubble frequency in gas-liquid bubble jets have been measured using resistivity probes. It has been found that these profiles can be represented well by a Gaussian function and that they exhibit good similarity along the jet. The gas distribution and the geometry of the jet have been reported to depend on the modified Froude number and the physical properties of the liquid.

The liquid velocities in the plume have been measured using different techniques. It has been reported that these velocities decrease only slightly with position along the jet and that their radial profiles are Gaussian. A few measurements of the mean bubble rise velocity have been done in gas-liquid plumes, using a double-contact electroresistivity sensor. The measurements have involved the cross-correlation of recorded signals or the direct measurement from an oscilloscope. The natural limitations of these techniques precludes determination of the bubble velocity fluctuation. There has been no attempt to assess the validity of the experimental techniques under the conditions in which they have been applied. Several macroscopic hydrodynamic models of submerged gas injection have been proposed to represent the gas-liquid plume and its effect on liquid stirring.
Numerous instruments have been developed to obtain information of gas-liquid bubble flows. While no single instrument can give a complete diagnosis of the flow structure, the electrical probes have been the most widely used since they can be adapted to measure the greatest number of parameters under different conditions. Studies of various phase detection techniques have indicated that the resistivity probe gives comparable results to optical and anemometer probes and in some cases has proven to be the most suitable method for measuring local instantaneous phase changes in gas-liquid systems. The measurement of bubble velocity by a phase detecting probe, e.g. double-contact electroresistivity sensor, has commonly involved the use of the cross-correlation technique. Severe limitations of this technique are that the particular events occurring at the probe tips are ignored. Under the best conditions this technique allows the measurement of only the most probable bubble velocity, providing no direct information on the probability distribution of the velocities and hence of their fluctuations. In more sophisticated multiple electrode systems, efforts have been made to measure the spectrum of bubble velocity and pierced length through the individual analysis of sequential voltage pulses. However, the large size of the probes used in these works means that only bubbles larger than \( \sim 3 \text{ mm} \) can be detected, which may under some conditions cause a serious bias in the measuring process. These investigations have shown that in turbulent gas-liquid mixtures a successful bubble encounter with the probe is a relatively rare event.
3.2 Objectives

In view of the literature findings just mentioned, the present work was undertaken to increase our understanding of the gas-liquid bubble plumes formed under upward gas injection into vertical cylindrical vessels. In striving toward this ultimate goal, this research project pursued the following specific objectives:

(a) To develop, construct and verify an accurate computer-aided electroresistivity sensor for measuring the properties of the structure of turbulent gas-liquid plumes.

(b) To determine the structural development of air-water bubble plumes by detailed measurement of the local properties characterizing the behaviour of the gas phase, including: void fraction, bubble frequency, mean bubble velocity and pierced length, and the spectra of bubble velocity and pierced length.

(c) To clarify the effect that injection conditions have on the spatial development of gas-liquid bubble plumes.

(d) To compare the predictions of some of the mathematical models, proposed in the literature, representing gas-liquid plumes to the experimental results.
CHAPTER 4

EXPERIMENTAL APPARATUS AND CONDITIONS

As discussed earlier, it was essential in this work to develop detailed information on the structural development of vertical gas-liquid plumes. To accomplish this objective a microcomputer controlled electroresistivity probe system was developed to measure simultaneously two-phase flow parameters under a wide range of injection conditions. The following sections describe the experimental system under investigation and the measuring instrument.

4.1 Experimental Apparatus

The apparatus employed in the study of air-water plumes is illustrated schematically in Figure 4.1. It consisted of a ladle-shaped vessel containing de-ionized water which was slightly acidified, an air delivery system to supply clean air to the nozzle and the newly developed measuring instrument.

The measuring system was based on a double-contact electroresistivity sensor coupled to a microcomputer through a conditioning and logic circuit and a counter-timer parallel input/output interface. The system was able to analyse in real time the electrical signals originating from the interaction of the bubbles with the sensor tips. Thus, it produced the data required to determine the local gas fraction, bubble frequency,
bubble velocity and pierced length as well as the spectrum of the last two quantities. An oscilloscope, a digital analyzer, a function generator and a high-speed camera were used as supporting equipment.

Figure 4.1 Schematic diagram of experimental facility
4.1.1 The Physical Model

Physical models commonly have been used to measure velocities and turbulence fields because of the inherent complications that these measurements present particularly in gas-liquid bubble flows. In physical modelling work on gas injection systems, the liquid metal usually is replaced by water or by other liquids which are easier to handle and allow a more detailed investigation of the flow. However, it is necessary to recognize the implications of this substitution.

A major difficulty encountered in the physical modelling of a gas-liquid metal system lies in the number of physical parameters involved in the process. In order that the fluid flow in the physical model does represent that in the actual system, certain quite strict conditions have to be observed, namely:\[126\]: geometric, dynamic, kinematic, and thermal similarities. Thermal similarity requires that the dimensionless numbers involving heat transfer or thermally induced flow be equal in the model and the prototype. Thermal similarity appears to be unimportant in modelling the overall fluid motion in gas stirred ladle systems since the contribution of thermal gradients to this motion is small compared to that of other forces. Close to the injection point, however, thermal effects are important since they determine the expansion of the discharging gas and the growth of accretions. Kinematic similarity is ensured in a model that conforms to geometric and dynamic similarity.
In the case of ladle metallurgical processes, geometric similarity is easily and perfectly obtained in the water model. For dynamic similarity the principal forces to be considered in two-phase systems are: inertial, gravitational, viscous and surface tension forces. Thus, in principle, similarity would be possible through simultaneous adjustment of the modified Froude number, Reynolds number, and Weber number which relate inertial and gravitational forces, inertial and viscous forces and inertial and surface tension forces respectively. However, in practice, this is quite difficult owing to major differences in surface tension and density between aqueous and molten metal system.

Regarding bubble formation at the orifice, surface tension dominates at low gas flow rates. At higher gas flow rates and for liquids of low viscosity, inertial and buoyancy forces play the dominant role in determining bubble size. Also it has been found\textsuperscript{18} that during gas injection into a liquid the transition from bubbling to jetting depends on the modified Froude number as well as on the gas-liquid density ratio. In model experiments designed to simulate the bath motion during submerged injection in a teeming ladle\textsuperscript{127}, useful results concerning mixing have been obtained by choosing modelling conditions according to the modified Froude number. In a recent investigation by Bustos\textsuperscript{128} concerning bath movement and slopping in copper converters, the rationalization of the observations according to the modified Froude number proved to be successful in transferring laboratory information to industrial systems.
Therefore similarity based on the modified Froude number and on gas-liquid density ratio is essential to simulate gas injection processes. This implies

\[
\frac{\rho \cdot F}{\rho_1} P = \frac{\rho \cdot F}{\rho_1} M
\]  

(4.1)

and

\[
U_{oM} = U_{oP} \left( \frac{d_o M}{d_o P} \right)^{1/2}
\]  

(4.2)

Then useful information concerning the behaviour of the discharged gas in a gas-metal system can be obtained by injecting helium in water. Unfortunately economic considerations owing to the high consumption of gas required in this study made the use of helium prohibitive. Therefore it was decided to inject air through the orifice of an isothermal water model and investigate the behaviour of gas-liquid jets under a wide range of experimental conditions.

4.1.2 The Ladle-Shaped Vessel

A 1/6th scale model of a 150 tonne teeming ladle was built. Gas was injected through a nozzle located at the center of the bottom of the vessel which contained deionized water that was slightly acidified with nitric acid, approximately 0.01 % by volume. The vessel was made from 9.5 mm Plexiglass plate and had an internal diameter of 500 mm and a height 900 mm.

A baffle consisting of a Plexiglass ring with an external diameter of 495 mm and an internal diameter of 300 mm was mounted
near the bath surface to minimize slopping. The baffle was held 3 mm above the quiescent bath surface. This arrangement allowed the free rising and release of bubbles from the bath while providing a damping action of the surface waves. Excessive surface waves produced instability of the initially vertical rising path of the plume which resulted in a periodic swaying motion relative to the vessel axis. It is possible that this kind of motion of the jet produced the double peaks that Kawakami et al. observed in the profiles of bubble frequency and gas hold-up. The damping action of the baffle on the waves corrected the jet trajectory and permitted reproducible experiments. Figure 4.2 shows a photograph of the ladle-shaped vessel employed in this work.

4.1.3 Nozzle and Gas Delivery System

The nozzles used were constructed from straight-bore Plexiglass pipe having internal diameters 4.10 mm and 6.35 mm. The nozzles were located in a nozzle-holder such that the gas discharge was level with the bottom of the vessel, Figure 4.3. The lower end of the nozzle-holder, made of stainless steel, was connected to the gas supply through a needle valve used to control the flow. This was done to achieve constant flow rate to the nozzle by means of a high pressure drop across the needle valve and the small sub-nozzle volume. The pressure drop across the nozzle was measured with a Bourdon-type pressure gauge connected close to the nozzle.
Figure 4.2 Photograph of the ladle-shaped model and ancillary apparatus used in the tests.
Air was supplied to the nozzle by an air compressor, with a capacity of 0.02 m³/s (47 CFM) at a gauge pressure of 103.4 kPa (15 psig). The gas flow rate was monitored using a rotameter with a spherical float of tungsten and calibrated with a dry gas meter. The rotameter was flanked by a needle valve at the top and a Bourdon-type pressure gauge at the bottom. The pressure inside the rotameter was held constant at 62.05 kPa (9 psig) in all the experiments. The gas flow rate at the orifice was calculated...
considering both the atmospheric pressure and the static head of water. An auxiliary rotameter having smaller capacity was used to prevent liquid from draining into the nozzle while the measuring system was being prepared. The switch from one rotameter to the other was made through the action of two solenoid valves, as shown in Figure 4.1.

4.1.4 The Electroresistivity Probe

A double-contact electroresistivity probe was used since in conjunction with a suitable signal analysis procedure it allows the simultaneous measurement of a large number of fluid flow parameters. The electroresistivity probe used in this work is illustrated in Figure 4.4. The probe sensing elements were stainless steel needles soldered to a length of insulated copper. The lower needle was carefully bent to be in vertical alignment with the upper needle. The entire surface of the needles was coated with a thin layer of red Glyptol (a General Electric insulating finish) and baked at 400 K for 2 hours; this procedure was repeated several times until a hard coherent coating developed. The Glyptol coating was dissolved from the needle tips using Strip-X (a solvent marketed by G.C. Electronics) leaving a bare tip ~0.130 mm long and ~0.100 mm at its largest diameter. The wires were then passed through the holes of a ceramic insulating tube which was housed in a stainless steel support tube. The top and bottom of the stainless steel case were sealed with silicone sealant to prevent any
Figure 4.4 Details of electroresistivity probe for simultaneous gas fraction and velocity measurement, dimensions in mm.
penetration of water. The wires emerging from the top of the tube were encased in heat-shrink tubing to secure complete insulation.

The geometry of the needle tips was found to be critical as they affected the quality of the signals and consequently the value of the measured parameters. A travelling microscope was used to measure the size of the tips, the vertical separation between the tips and the lateral separation between the needles. These distances were measured once the probe was assembled. The probes used in the experiments were selected from several constructed on the basis of these distances and their performance as follows:

(a) The size of the two tips was kept in the range of 0.130 ± 0.015 mm. As discussed in Section 5.3.1, both tips must be small and have a narrow size difference to ensure reliable measurements.

(b) The vertical separation between the contacts was kept in the range 1.9 ± 0.2 mm in all the probes used. The exact distance for a particular probe was measured to an accuracy of microns. This separation had to be small to ensure local velocity measurements.

(c) The closest distance between the needles laterally was 1.5 mm, Figure 4.4. It was found that closer spacing produced a relatively stable water bridge between the needles which had a very detrimental effect on the signal quality of the upper contact. Signals from the
upper contact tended to have a very long falling time, particularly at low flows rates and at positions close to the jet edge.

The streamlining and small size of the sensors prevented their interference with the trajectory of the bubbles as was verified by filming the bubbles passing the probe under the experimental injection conditions. Also frequent verification of the tip dimensions indicated that they did not change during weeks of experiments; and microscopic examination of their surfaces showed no alteration.

The reference electrode common to both tips consisted of a 450 X 100 mm stainless steel sheet 0.2 mm thick, that was soldered to a copper wire. The wire was passed through a ceramic tube which was then encased into a stainless steel tube for support. The electrode was curved to fit snugly against the vessel wall. The reference electrode was placed on one side of the vessel parallel to the radial motion of the measuring probe as shown in Figure 4.2.

All the objects in contact with the acidified water were made of Plexiglas or stainless steel to avoid solution contamination that would affect the probe performance.

4.1.5 Traversing Mechanism

A traversing mechanism was built to locate the probe tips accurately at different positions within the jet, Figure 4.2. To
take fuller advantage of the symmetry of the flows studied, the traversing mechanism was basically two-dimensional. The position and alignment of the probe tips was determined using scales and levels which were placed on the vertical and horizontal arms of the traversing mechanism. The position was measured with respect to the vessel centerline and bottom.

Wire braces were used to make the long probe sufficiently strong and rigid to exclude vibrations or motions caused by the turbulent flow and to allow its accurate placement. The wire braces worked very well and permitted the use of a small probe case.

4.1.6 Conditioning and Logic Circuit

The conditioning and logic circuit that was designed and built for the present purposes consisted basically of three parts:

(a) Voltage detection circuit
(b) Level detection circuit
(c) Pulse delay generation circuit

The detailed circuitry is shown in Figures 4.5(a) and 4.5(b).

The voltage detection circuit reacted to the changes in phase around the measuring electrode. The change in resistance between the measuring and the reference electrode was associated with the voltage across the load resistor, Figure 4.6. The voltage was maximum when the needle tips were in the liquid phase.
Figure 4.5(a) Wiring diagram of electronic circuit for electroresistivity probe measuring system.
and approached zero when the tips were surrounded by the gas phase. For the aqueous solution used, the voltage detection circuit produced signals with an amplitude of 3.3 volts. A voltage amplifier with variable gain was used to compensate for the effects that differences in tip size had on the amplitude of the signals from contact A and B, Figure 4.6. In all the
Figure 4.6 Schematic diagram of bubble sensor and signals obtained.
experiments carried out in this work the voltage amplitude after amplification was $3.52 \pm 0.01$ volts. Close examination of the signals on an oscilloscope verified that the amplification did not distort the width of the pulses.

The phase detection circuit consisted of a comparator to which a reference voltage was applied. When the incoming signal was above the threshold voltage it was considered that the probe was in the liquid phase and vice versa, Figure 4.6. The threshold voltage was fixed, after examining its effect on the measurements, at $2.84 \pm 0.01$ volts, i.e. 20% below the liquid level voltage. The binary signal resulting from this step was in a suitable form to be analyzed by the logic circuit.

The delay pulse generating circuit provided a first step in the signal analysis procedure. This circuit consisted of a set of Boolean devices that satisfied specified input/output relations necessary to determine the velocity of the bubbles. The logic of the circuit produced the truth table given in Table 4.1.

Table 4.1 Truth table for logic circuit

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
The truth table indicates that when at a certain instant a transition from 0 to 1 occurs at the lower contact i.e. bubble surrounds tip A, and at that same instant tip B is in the liquid phase, the logic circuit starts to produce a delay pulse C. Pulse C remains high for as long as A is high and B is low. As shown in Figure 4.6, the width of pulse C corresponds to the time spent by the bubble travelling from contact A to B. This time is inversely proportional to the bubble rise velocity if the bubble travels colinear to the probe.

4.1.7 Computer, Counter/Timer and Parallel Input/Output Interface

The computer system was a 16-Bit Proteus microcomputer from Innovative Electronics Technology. The computer was programmed in Z-8000 assembler language\textsuperscript{129} to reset, initialize and configure the I/O board to drive the signal analysis and data acquisition in real time.

The counter/timer and parallel input/output interface was also built by Innovative Electronic Technology for its microcomputer. The board consisted of direct memory access, interrupt controllers and counter/timers and parallel input/output devices. The counter/timer and parallel I/O devices were a pair of Z8036 Z-CIO chips\textsuperscript{130} (a registered trade mark of Zilog Corporation). These devices allowed the precise timing and counting of the signals generated by the probe.
The ports in the Z-CIO chips were configured as bit ports for external input access and were enabled for pattern recognition. Times $t_A^g$ and $t_C^g$ were measured by the two timers in one of the chips and time $t_B^g$ by a timer in the second chip. The timers had a resolution of 1μs. A counter in the first chip was used to register the number of signals coming from contact A.

4.1.8 Miscellaneous Equipment

Ancillary equipment consisted of:

(a) Tektronic 564 storage oscilloscope
(b) Sony/Tektronic 308 data analyzer
(c) Exact 123A function generator
(d) Hycam K2054E high-speed camera.

The first three instruments helped in debugging and optimizing the probe, the conditioning and logic circuit and the assembler program. The high-speed camera was used at 400 frames per second to measure the velocity of individual spherical cap bubbles used to assess the performance of the probe and at speeds of 1000 to 3000 frames per second to record the events under actual experimental conditions.

4.2 Conditions for the Tests and General Procedure

The following parameters were studied in the air-water experiments:

(a) Gas flow rate
Table 4.2 summarizes the conditions of the experiments. Appendix II lists the modified Froude numbers, Reynolds numbers and specific kinetic and buoyancy powers corresponding to each of the tests. Detailed investigation of the flow structure of the gas-liquid jet was carried out in each of the tests.

Table 4.2 Experimental conditions of the study

<table>
<thead>
<tr>
<th>Orifice diameter $d_0$ (mm)</th>
<th>Gas flow rate $Q$ (Ncm$^3$/s)</th>
<th>Bath depth $h_b$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>371 876 1257</td>
<td></td>
</tr>
<tr>
<td>6.35</td>
<td>○ ▽ ◇</td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>▽</td>
<td>600</td>
</tr>
<tr>
<td>4.10</td>
<td>□ △</td>
<td>400</td>
</tr>
</tbody>
</table>

The procedure outlined below was followed throughout preliminary and final experiments.

(a) The electroresistivity probe was placed on the center of the nozzle to establish the reference position in the traversing mechanism and to make connections between the electrodes and the measuring system. The sensor was then moved to the center of the selected cross-section where measurements were to start.
The compressor was turned on and air injection was commenced to deliver the desired air discharge. Time was allowed for flow establishment and preparation of the measuring equipment. This preparation consisted of setting the proper threshold voltage and signal amplification. The liquid-voltage level was measured inside and outside the jet and the gain of the amplifier was adjusted until the level was stable and independent of position. The signals were monitored on an oscilloscope to detect and correct any signal distortion due to amplification or other operational problem.

Measurements started with the execution of program GLJET700 which carried out signal analysis and data acquisition. The sampling time could take from ~90 to 5400 seconds depending on the location of the sensor and the gas flow rate. At the end of the measuring period the computer had collected times $t_A^G$ and $t_C^G$, the number of bubbles that passed contact A and the time of duration of the experiment. Program DATSTRDK transferred this information together with the experimental conditions from computer memory to floppy disk for later data reduction and analysis. Programs GLJET700 and DATSTRDK are discussed in Section 5.2.

The measurements were repeated at symmetric (-r and +r) radial positions. Before starting any experiment at new locations, the signals were observed on the oscilloscope.
for control, e.g. verify the constancy of the liquid voltage level. The locations of the measuring stations are discussed further in Section 5.2.5.

(d) After the desired traverse was completed, program DATPRC was used to carry out data reduction to obtain values of the measured flow parameters. Program DATPRC is discussed in Section 5.2.

The entire preparation, measurement and data reduction for a selected level lasted about 8 to 10 hours. For each experimental condition 9 levels were selected involving approximately 170 measurements.

The aqueous solution in the tank was periodically changed. Proper control of the cleanliness of the solution and of the air as well as adequate probe construction permitted maintenance of the amplitude of the signals at 3.52 ± 0.01 volts and the threshold level at 2.84 ± 0.01 volts in all the experiments.
This chapter describes the signal analysis procedure developed to interpret the voltage pulses produced by the interaction of the bubbles with the sensor. The consideration given to the characteristics of the electroresistivity sensor and the conditioning circuit, to ensure their quick and reproducible response to the passing of bubbles is also discussed. At the end of the chapter it is demonstrated that careful attention to all the aspects mentioned produced highly reliable measurements.

5.1 Signal Analysis and Definition of Measured Parameters

Figure 5.1 shows typical traces of the digital form of the signals generated by the contact of the bubbles with the sensor under actual experimental conditions. The voltage traces reveal that the bubbles arrive at the contacts at close intervals and in random fashion. This makes it necessary to examine the state of the contacts continuously by a computer to extract information on the bubble behaviour in gas-liquid plumes, as described below. Digital processing requires conversion of the analogue signal prior to signal analysis. The conversion may be either an on-line or a delayed process. In this investigation the digitization of the response of the electroresistivity probe and the digital
Figure 5.1 Voltage traces showing the modified digital signals generated by bubbles intercepting the sensor contacts.
analysis of the signals, based on the definition of the measured parameters, were carried out in real time.

5.1.1 Local Gas Fraction

The local gas fraction is defined as the probability that a point, probe tip A in Figure 4.6, is in the gas phase; from Equation (2.11) the local time-averaged gas fraction for steady flow is given as

\[ \alpha = \frac{\sum T_A}{T} \]  

(5.1)

As shown in Section 4.1.6 and Figure 5.1 the response from the probe system consisted of a two-state signal indicating the presence of gas or liquid at probe tip A. Then to measure the local time-averaged gas fraction, the computer was enabled to follow every voltage transition occurring at contact A and a timer recorded each individual bubble residence time over the time of the experiment. The area-averaged gas fractions for transverse sections of the plume could be obtained from integration of the local gas-fraction profiles over the areas, Appendix IV.

5.1.2 Local Bubble Frequency

The local bubble frequency also was measured utilizing only the signals from the lower tip. The number of modified digital signals, produced by bubbles arriving at Contact A, Figure 4.6, for a given duration of the measuring period, was determined by a
digital counter. The sampling time depended on the time required to acquire a predetermined number of delay pulses as explained below. These times were sufficiently long to ensure good statistical significance of the local gas fraction and bubble frequency measurements.

5.1.3 Local Bubble Rise Velocity

From what has been said it is obvious that the signal analysis involved in the measurement of the local gas fraction and bubble frequency requires only the recognition, on the part of the computer, of the state (low or high) of the signals generated at the lower contact. The computer must record the required data unconditionally and independently of the events occurring at the upper contact. On the other hand, the measurement of the bubble velocity and pierced length requires the simultaneous analysis of the state of the signals from both the lower and the upper contacts. Figure 4.6 shows a schematic diagram of the voltage pulse sequence which is generated when single bubbles approach the assembly of contacts. Under such rather ideal conditions, the consecutive pulses at the lower and upper contact and the corresponding delay pulses, Channels A, B and C respectively, are uniquely associated with a single undisturbed bubble travelling axially from the lower to the upper contact of the sensor, such that

\[ t_A^g = t_B^g \]  

(5.2)
for a defined and accurately known probe separation, the bubble rise velocity, Equation (2.25), is given by

\[ U_b = \frac{d_p}{t_g^C} \quad (5.3) \]

Under actual injection conditions the passage of bubbles through the sensor does not always generate consecutive signals of similar width, Figure 5.1(b) and 5.1(c). This results from the fact that under turbulent conditions the bubble motion through the sensor present several possibilities, Figure 5.2:

(a) Bubbles moving parallel to the probe axis
(b) Bubbles moving obliquely to the probe axis
(c) Different bubbles arriving at the contacts
(d) Bubble coalescence occurring during the probe intercept period
(e) Bubble break-up occurring during the probe intercept period
(f) Bubbles travelling very close together

The variety of these events and the randomness and speed at which they occur make it necessary to examine the state of channels A, B and C continuously by computer. The purpose is to ensure that the time \( t_g^C \) used to determine the bubble rise velocity is uniquely related to a signal sequence corresponding to a single bubble travelling undisturbed in a direction parallel to the probe. The approach followed to determine the velocity of individual bubbles was to eliminate the uncertainty in the origin
of time $t_g^C$ by using pattern recognition methods. The object of pattern recognition is to identify patterns that share some common properties and assign them to well-defined classes. In this study these methods allowed the sequence and duration of the signals to be related to the most probable bubble behaviour at the probe tips.

Figure 5.3 shows all the possible pattern classes that can be produced when the bubbles interact with the sensor tips. The logic used by the computer to analyse the signals, in order to measure the bubble rise velocity and the pierced length in turbulent gas-liquid plumes, was based on this figure. In the experiments the signals from the probes are treated by the
Figure 5.3 Timing diagram of the different signal patterns generated by the interaction of bubbles with the sensor under turbulent conditions.
computer as follows. The computer, initially in the ready state, waits until

\[ V^A(t) = 1 \]  \hspace{1cm} (5.4)

which indicates that the approaching bubble interface has enveloped tip A. At the instant that this condition occurs, the computer examines the state of the delay pulse channel. If the condition

\[ V^C(t) = 0 \]  \hspace{1cm} (5.5)

is met at the instant Equation (5.4) is satisfied, then the upper channel is already in the gas phase when the lower probe was struck by the bubble. It is difficult under these conditions to determine the bubble velocity since the signal sequence was generated by a bubble moving in a direction not parallel to the probe or by bubbles which were travelling very close together. For this case, pattern class (3) in Figure 5.3, the logic in the conditioning circuit does not produce a delay pulse and the analysis to determine bubble velocity is abandoned immediately. The computer waits until the pulse from the lower contact ends to measure its duration, before returning to the ready state.

If condition

\[ V^B(t) = 0 \]  \hspace{1cm} (5.6)
is satisfied at the instant Condition (5.4) occurs, then the logic in the conditioning circuit starts generating the delay pulse,

\[ V_c(t) = 1 \]  

(5.7)

Condition (5.7) lasts for as long as the bubble remains in contact with the lower tip and the liquid stays in the upper tip. This condition is represented by pattern class (4) in Figure 5.3. Depending on the relation among times \( t_A^g \), \( t_B^g \), and \( t_C^g \), pattern class (4) may or may not be related to a bubble travelling parallel and undisturbed from the lower to the upper contact.

Pattern class (c), Figure 5.3, is such that

\[ t_A^g = t_C^g \]  

(5.8)

This pattern presents interpretation problems for the determination of the bubble velocity, since it is generated by bubbles travelling obliquely to the sensor or by bubbles smaller than the separation between the probe contacts. For this case the computer accepts time \( t_A^g \) and rejects time \( t_C^g \), before returning to the ready state.

Pattern class (b), Figure 5.3, represents bubble sequences where delay pulses are generated but the transition of \( v_B^d(t) \) from high to low level occurs before that of \( v_A^d(t) \). The delay pulse generated under these conditions is not satisfactory for the
determination of the bubble velocity since it can be the result of bubbles moving laterally, of bubbles breaking up or of the encounter of the probe with different bubbles. The time $t_g^A$ is measured and stored in memory while the accompanying time $t_g^C$ is rejected. The computer then returns to ready-state.

Pattern class (a), Figure 5.3, consists of sequences where the voltage from contact B remains high after the accompanying previous pulse from contact A has dropped to the low level. This pattern gives rise to a series of classes: (a1), (a2a), (a2b), (a3a) and (a3b). The categorization of patterns into these classes is based on the relative duration of the voltage pulses from channels A and B and on whether or not a second bubble arrives at the lower probe before the pulse from contact B drops. The final decision on the acceptance of time $t_g^C$ depends on how similar time $t_g^A$ and $t_g^B$ are. For an undisturbed bubble travelling parallel to the probe axis,

$$t^* = \left| \frac{t_g^A - t_g^B}{t_g^A} \right| = 0 \quad (5.9)$$

However, in a turbulent dispersion a perfect hit is a rare event and it is better to define limiting conditions for the time difference ratio to achieve a satisfactory sampling time. In the present study $t^*$ was allowed to fall in the range

$$t^* \leq 0.1 \quad (5.10)$$
The effect that a different choice of $t^*$ has on the measurements is discussed later in the chapter. Pattern classes (a2a) and (a3a) represent the cases where the conditions for the acceptability criterion, Condition (5.10), for $t_g^C$ is met, while pattern classes (a1), (a2b) and (a3b) represent situations where it is not possible to determine the velocity of the pulses unequivocally since they could have been generated by bubbles travelling obliquely to the sensor, by bubbles undergoing coalescence or break-up during their interception by the probe, or by the encounter of the probe tips with different bubbles.

It should be stressed that Figure 5.3 shows all the possible combinations of signal patterns that may occur when bubbles in a turbulent gas-liquid dispersion interact with the electro-resistivity probe tips. To confirm that all these patterns actually exist, under the conditions of the present experiments, an extensive investigation of the signal sequences was carried out with a digital analyzer. Figure 5.4 shows the digital patterns observed indicating their correspondance to Figure 5.3.

The velocity of the bubbles producing successful encounters with the probe was calculated from Equation (5.3). It is assumed in measuring the characteristic local velocity distribution that the bubbles intercept the probe in a random fashion and that the measurements are therefore characteristic of an unbiased sampling. In this work the ensemble average of the bubble rise velocity measurements was given by the arithmetic mean.
Figure 5.4 Digital signal patterns generated under actual injection conditions, showing the correspondence to the classes given in Figure 5.3.
\[ \bar{U}_b = \frac{\sum U_b}{N_U} \quad (5.11) \]

and the standard deviation of the bubble rise velocity spectrum, i.e. the r.m.s. of the fluctuating component of the bubble velocity, was

\[ S_{U_b} = \left( \frac{\sum (U_b - \bar{U}_b)^2}{N_U} \right)^{1/2} \quad (5.12) \]

5.1.4 Pierced Length of Bubbles

For each bubble accepted by the discriminating logic discussed in the previous section, the pierced length of the bubbles was calculated as

\[ l_b = U_b t_g^A \quad (5.13) \]

The arithmetic mean was used as the best estimator of the central tendency of the pierced length spectrum,

\[ \bar{l}_b = \frac{\sum l_b}{N_U} \quad (5.14) \]

The values of this statistic are compared with its geometric equivalent in Section 6.4.

Provided that the flow of the bubbles is locally homogeneous, the sensor has equal probability of piercing the
bubbles at any point of their projected frontal area, so that the measuring pierced length can vary from zero to the bubble diameter. Thus it is possible, under certain conditions, as discussed in Section 6.4.1, to obtain information on the local distribution of bubble diameters from the distribution of pierced lengths.

5.2. Data Acquisition and Data Reduction

The processes of data acquisition and data reduction consisted of the recognition and classification of the signal patterns according to the criteria and definitions discussed previously. Figure 5.5 shows the flowchart representation of these processes. All the programs required for signal analysis and data acquisition were written in assembler Z8000 language since it offered the flexibility and speed required to carry out these processes in real time. In addition to this software, other programs were written in Basic to carry out data reduction and analysis of the information. Figure 5.6 shows a flowsheet of the data acquisition and data reduction programs.

An important factor in any data acquisition process is the determination of the amount of data needed to represent the phenomena studied. Inadequate amounts of data can lead to inaccurate results, but the cost of the data gathering and processing forces one to limit the amount to a minimum which will assure statistically meaningful results. In this study two
aspects had to be considered: i) the size of the signal sample required for measurement of the air-phase flow parameters and, ii) the number of test points and their location.
Figure 5.6 Flow diagram of computer program for data acquisition and data reduction processes.
5.2.1. Program for Data Acquisition (GLJET700)

Program GLJET700 was written in assembler Z8000 language to carry out the pattern recognition process accurately in real time. The program reset, initialized and configured the different components of the I/O interface. The Z8036 (Z-CIO) devices were configured to recognize the state of the ports to which channels A, B and C were connected. The pattern recognition logic of these devices was activated whenever the voltage level in channel A suffered a transition from 0 to 1. In this active condition the computer continuously checked the state of channels A, B and C to measure the times $t_A$ and for the acceptable situation, time $t_C$ also. After completing these measurements the computer went back to the ready-state where it waited for the arrival of another bubble at the lower contact. In the final experiments, the process of data acquisition stopped once 800 values of $t_C$ were collected. When this condition was met, the computer had stored in memory all the values of $t_A$ generated, the predetermined number of $t_C$ times, the number of bubbles passing contact A, the time duration of the experiment, and the location of the measurement, Figure 5.6. A program, DATSTRDK, written in Basic transferred these data from computer memory to floppy disk. Programs GLJET700 and DATSTRDK are listed in detail in Appendix III.

To ensure that program GLJET700 worked adequately in counting and timing the events occurring at channels A, B and C, an extensive testing of the program was undertaken using the
digital analyzer. This instrument provided an independent form of assessing program performance in regard to signal analysis logic and configuration of the I/O board. In the form that program GLJET700 was written, it could lose information only if consecutive pulses at channel A occurred during a time interval smaller than 11 µs, and this was confirmed using a function generator. This condition occurred very rarely in this work.

5.2.1.1 Frequency of Occurrence of the Accepted Pattern Classes

During the many experiments that were conducted, it was found that typically 25 to 35 % of the bubbles intercepted by contact A were accepted, for extracting information concerning bubble rise velocity, although this proportion was smaller at certain locations. As discussed previously, the accepted bubbles belonged to pattern classes (a2a) and (a3a) for which \( t^* \leq 0.1 \).

Experiments carried out by the release of individual spherical cap bubbles of different size showed that under these flow conditions all the bubbles intercepted by the probe were accepted. The rejection of bubbles under actual experimental conditions then was clearly associated with the irregular and random motion of the bubbles around the sensor.

A version of program GLJET700 was written to count the frequency of occurrence of the different pattern classes shown in Figure 5.3. Some results of these tests are shown in the form of histograms in Figure 5.7. It can be seen that the accepted
Figure 5.7 Histograms of the pattern classes generated by the bubbles at different locations in the plume and for different gas flow rate conditions.
pattern classes (a2a and a3a) have a frequency of occurrence that is, in general, larger than or equal to the other classes. This indicates that a successful bubble-probe encounter had a satisfactory probability of occurrence. Patterns (3) and (c) are also present in relatively high frequency. From these and other results at different gas flow rates and vertical positions, it is possible to say that pattern class (3) was the result of the close proximity of the bubbles, while class (c) was most probably the result of the oblique motion of some bubbles with respect to the probe axis or of bubbles that were smaller than the tip separation.

The relaxation on the criterion of acceptability based on $t^*$, i.e. enlarging the range of acceptability on $t^*$, had the effect of increasing the average and standard deviation of the bubble velocity and consequently of the pierced length. This was most probably due to the increased acceptance of bubbles which strike the probe tips obliquely. As discussed in Appendix I, these kinds of encounters between the bubbles and the sensor generate large "velocities". A time difference ratio $t^* \leq 0.1$ was adopted since it produced the best measurements, while it kept the duration of the acquisition process within reasonable limits.

5.2.2 Program for Data Processing and Reduction (DATPRC)

The program DATPRC consisting of a main program and several subroutines, used the data measured by GLJET700 to calculate
individual bubble velocities and pierced lengths as well as the local gas fraction, bubble frequency and the mean and standard deviation of the bubble velocities and pierced length, Figure 5.6. These quantities were calculated according to the definitions given in Section 5.1. All the information was organized into files containing: frequency distribution of bubble velocity, frequency distribution of pierced length, and reduced information of all the measured parameters. A detailed listing of DATPRC is given in Appendix III. As discussed below, the values of time \( t_g \) collected by GLJET700 had to be filtered by consideration of physically possible and therefore acceptable pierced length. Program DATPRC contained instructions to carry out this filtering process.

5.2.2.1 Discrimination of Time-Delay \( (t_g^C) \) According to Pierced Length

The filtering of the signals based on the criteria discussed in Section 5.1.3. eliminates most of the uncertainties associated with the diverse origin of the delay pulses. However, uncertainty still exists owing to the impossibility of accurately determining the direction of movement of the bubble with respect to the probe using a double-contact sensor. This problem manifested itself as the presence of a small number of unreasonably high velocities in the bubble velocity spectrum, as is shown in Figures 5.8(a) and 5.8(b). Despite the very small frequency of occurrence of these apparent velocities, their large magnitude had an excessive
Figure 5.8(a) Typical bubble velocity distributions indicating the velocities associated to pierced lengths larger than the maximum accepted.
Figure 5.8(b) Typical bubble velocity distributions indicating the velocities associated to pierced lengths larger than the maximum accepted.
weight on the value of the standard deviation of the bubble velocity spectrum.

The first step in the filtering of the bubble velocities consisted of neglecting the velocities which exceeded the velocity of the gas at the orifice. The number of these events was very small and occurred mainly at large gas flow rates and at positions close to the nozzle where conditions are very turbulent.

A second rational form of neglecting the large "velocities" was found by examining their associated pierced length. Visual observation of the bubbles from recorded films with a high-speed camera permitted the determination of reasonable upper limits for the pierced length. These limits then were used as the acceptable maximum in the filtering of data velocities. Figure 5.9 shows some examples of the large bubbles that existed in the plumes. Depending on the location inside the jet two maximum acceptable pierced lengths were selected: one of 150 mm for positions below 100 mm from the nozzle and the other of 70 mm at higher positions. These two limits were fixed in the treatment of all the data collected. Figures 5.8(a) and 5.8(b) show that the large bubble "velocities" measured were in general related to pierced lengths larger than those observed and that their probability of occurrence was very small. Then it was justifiable to neglect these velocities without unduly introducing bias in the data treatment.
Figure 5.9 Large bubbles occurring at different positions in the plume, \( Q = 876 \text{ Ncm}^3/\text{s}, h = 400 \text{ mm}, d = 6.35 \text{ mm}, \) probe at centreline \( z = 110 \text{ mm} \) for (a) and (b), \( z = 200 \text{ mm} \) for (c) and (d) photographs.
As Figures 5.8(a) and 5.8(b) show, a few velocity values remained that were markedly different from the bulk of the data. Johnson and Leone\textsuperscript{132} have discussed a number of tests of significance to decide whether isolated observations, called outliers, can be regarded as being too doubtful to be accepted as part of a sample. They indicated that such tests are mainly heuristic and mostly relate to samples of normal population. In this work the following statistic given by Dixon and Massey\textsuperscript{133} for large samples was applied,

\[ r = \frac{U_{b\, n} - U_{b\, n-3}}{U_{b\, n-2} - U_{b\, 1}} \quad (5.15) \]

The outlier was neglected, i.e., removed from the sample, if the test criterion was significant. In this work, from the table of critical values for testing outliers given by Dixon and Massey\textsuperscript{133}, it can be said that the test was significant to 95\% if the value for, $r$, exceeded 0.40. This value adopted was only approximate. As Johnson and Leone\textsuperscript{132} have recommended, the testing procedure was applied sequentially removing one outlier at a time until no further significant values of, $r$, resulted. It is considered that this test gives enough information to eliminate, by this objective method, the outliers. It is important to note that the considerations described in this section, regarding the acceptance of individual bubble velocity data, had their largest influence on the value of the calculated standard deviation and had a much smaller influence on the values
of the mean bubble velocity. This is reflected in the typical results for $\bar{U}_b$ and $S_{U_b}$ shown in Figures 5.8(a) and 5.8(b).

The average percent of accepted time delays, with respect to the total collected by GLJET700, was typically 97%. Figure 5.10 shows how this percentage behaved in the experiments. It is seen that the maximum rejection occurred in the region close to the nozzle and at the jet edge, most probably due to the strong radial motion of the bubbles in this region.

5.2.3 Program for Data Acquisition (GLJET100)

Program GLJET100 was written to collect times $t^A_g$ and $t^C_g$ without applying any of the discriminating criteria based on pattern recognition. With this program, the computer accepted and timed all the delay pulses that arrived from the conditioning circuit. A series of experiments using this program was performed to demonstrate the need for the rigorous signal analysis based on the recognition of the patterns shown in Figure 5.3. The mean and the standard deviation of the bubble velocity obtained from such tests were compared with those obtained from GLJET700. The comparison showed that the data acquired by GLJET100 were consistently much higher than those obtained from GLJET700. The acquired sets of data from both programs were treated in the same form by DATPRC. The results from the tests are shown in Table 5.1. It is obvious from these results that in order to obtain meaningful information on the velocities of bubbles in turbulent gas-liquid plumes it is necessary to carry out the
signal analysis procedure discussed in Section 5.1.3.

![Graph showing percentage of accepted time delays as a function of vertical position, with data points for different flow rates and boundary locations.](image)

**Figure 5.10** Percentage of accepted time delays as function of position; empty and full symbols correspond to centreline and plume boundary locations, respectively.

5.2.4 **Sample Size**

Determination of mean values of stationary random processes requires analysis of samples which are sufficiently large to ensure their statistical significance. To determine the appropriate averaging sample size, preliminary experiments were done under conditions chosen to represent the final experiments. Since the number of delay pulses collected by the computer over the duration of the experiment was smaller than the total number
Table 5.1 Comparison of mean bubble velocity and standard deviation obtained by GLJET700 and GLJET100.

<table>
<thead>
<tr>
<th>z (mm)</th>
<th>$Q_o = 371 \text{ Ncm}^3/\text{s}$</th>
<th>$Q_o = 1257 \text{ Ncm}^3/\text{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GLJET100</td>
<td>GLJET700</td>
</tr>
<tr>
<td></td>
<td>$U_b$</td>
<td>$S_{Ub}$</td>
</tr>
<tr>
<td>20</td>
<td>1.87</td>
<td>1.58</td>
</tr>
<tr>
<td>65</td>
<td>1.65</td>
<td>1.08</td>
</tr>
<tr>
<td>240</td>
<td>1.38</td>
<td>1.25</td>
</tr>
</tbody>
</table>

$h_b = 400 \text{ mm} \quad d_b = 6.35 \text{ mm}$
of signals generated by contact A, it was decided to base the sample size on the number of the delay pulses necessary to ensure the convergence of the measured quantities. Results of these tests for gas fraction, bubble frequency and mean bubble velocity are summarized in Figure 5.11. The final electroresistivity probe measurements were based on data samples containing 800 time delays. This large number required measuring times between 90 to 5400 seconds depending on the location in the plume, but ensured reproducible results.

5.2.5 Measurement Locations

The accuracy in the determination of the spatial distribution of flow parameters depends upon the degree of spatial uniformity and improves as the number of measurement points is increased. Considering that changes in the measured quantities in the direction of the main flow are slower than those in the transverse direction, together with the effort required in data handling and the advantage of the symmetry in the flows, the measurement locations given in Figures 6.1 to 6.6 were adopted. The locations of the measurements were referenced with respect to the nozzle center and the flow width was estimated based on experience from preliminary experiments; moving outward radially measurements were taken until the bubble frequency was between 1 and 3 s$^{-1}$ for a particular cross-section. Together with the signal analysis, the detail of the measurements allowed to determine an integral picture of the flow behaviour of the plumes for the first time.
Figure 5.11 Establishment of sample size for statistically meaningful measurements by electroresistivity probe in turbulent gas-liquid plumes.

5.3 Characteristics of the Response of the Sensor and the Conditioning Circuit

This section presents the results of preliminary experiments carried out especially to examine the effect of the probe and circuit characteristics on the quality of the signals generated and on the measurements.
5.3.1 Effect of Probe Characteristics on the Measurements

Probe streamlining and control of vertical separation of the tips, as well as of the lateral separation between the needles of the sensor and the size of the probe tips, Section 4.1.4, were necessary to ensure a fast and matched response from both elements. Figure 5.12 compares the signals from two different probes. It is seen that with close needle separation, liquid bridging results in a very slow response from the upper contact. This phenomenon was particularly noticeable at low gas flow rate and at positions close to the plume edge. This problem was corrected for the final experiments; the tips were kept vertically aligned to measure the bubble velocity correctly and at the same time liquid bridging was avoided by controlling the lateral separation between the needles, Section 4.1.4.

Table 5.2 shows the effect of probe tip length on gas fraction and bubble frequency. The smaller values of these quantities, recorded by the probe with the longer tip, were related to the longer times required for the voltage transitions to occur. This effect is particularly detrimental at high bubble frequencies because the falling and rising times of the signals are in many cases longer than the time between the arrival of bubbles. For the final experiments the size of the sensor tips was kept very small, Section 4.1.4. The sensor tip size was approximately a twentieth of the smallest pierced length measured.
Figure 5.12 Comparison of signals produced (a) in the presence and (b) absence of liquid bridging between the sensor contacts.

Table 5.2 Variation of indicated properties with probe tip A length.

<table>
<thead>
<tr>
<th>z (mm)</th>
<th>Exposed Tip Length (mm)</th>
<th>( Q_o = 1257 \text{ Ncm}^3/\text{s} )</th>
<th>( h_b = 400 \text{ mm} )</th>
<th>( d_b = 6.35 \text{ mm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha (%) )</td>
<td>( f_b (\text{s}^{-1}) )</td>
<td>( \alpha (%) )</td>
<td>( f_b (\text{s}^{-1}) )</td>
</tr>
<tr>
<td>65</td>
<td>72.6</td>
<td>103.6</td>
<td>61.3</td>
<td>58.6</td>
</tr>
<tr>
<td>290</td>
<td>14.4</td>
<td>25</td>
<td>11.3</td>
<td>21.4</td>
</tr>
</tbody>
</table>
Another important factor concerning the characteristics of the probe is the difference in size between the upper and lower tips, since it results in differences in the amplitude of the signals from the two contacts. Under these conditions, the definition of a single threshold voltage becomes ambiguous. The elimination of amplitude differences is also important owing to their effects on the measured time delay, as given by

\[ t^C_g = t^C_{go} - \frac{1}{m_v} (V^B_1 - V^A_1) \]  

(5.16)

assuming that the falling edges of the lower and upper signals have the same slope. Figure 5.13 shows how the measured time delay \( t^C_g \) depends on the relative amplitude of signals A and B, when the respective falling voltage edges are parallel. In this

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.13.png}
\caption{Figure 5.13 Influence of voltage signal amplitude difference on the detected time delay, \( t^C_g \).}
\end{figure}
work the problems associated with differences in signal amplitudes were initially present but were corrected by fine adjustment using amplifiers with variable gain in the signal conditioning circuit.

Figure 5.14 shows examples of analogue signals generated during the final experiments. The signals exhibit a very short falling time of the order of approximately 600 μs; the signals from both contacts also have the same amplitude and their falling edges were closely parallel. Therefore both probes had very similar detection times and consequently the bubble velocity measurements were virtually independent of the threshold voltage.

5.3.2 Threshold Level

The digital identification of bubbles requires specification of the threshold level $V_t$. Thus, there is a need for a consistent selection of this level owing to its effects on the measured gas-phase parameters. As mentioned earlier, the liquid voltage level was independent of the experimental conditions (2.53 ± 0.01 volts) during the whole study. Then, the threshold levels could be selected as fractions of the peak voltage to study their effect on the measured gas phase characteristics. In the several tests carried out it was found that the values of the measured parameters varied only slightly in the range of threshold voltages between $0.7V_l \leq V_t \leq 0.9V_l$. The threshold level selected in this work was $V_t = 0.8V_l = 2.84 \pm 0.01$ volts. This value was adequate since:
Figure 5.14 Oscillographs showing signals produced by bubbles intercepting the lower and upper contacts of the probe, (Q = 876 Ncm/s, z = 65 mm, r = 0 mm).
(a) the time required to reach the threshold level was of the order of 200 μs. or shorter, allowing for an accurate determination of the bubble residence time.
(b) most of the falling and rising edges of the signal reached this level thereby allowing the identification of individual bubbles.
(c) as mentioned earlier, around this voltage level, the falling edges of signals A and B were parallel.

Figure 5.15 shows the influence of the threshold level on the value of the measured parameters. As discussed in the next section, the computation of the gas flow rate from the cross-sectional distribution of mean bubble velocities and gas fraction support the adequacy of the selected threshold level.

5.4 Evaluation of Measuring System

The system described in this work was designed to carry out signal analysis in real time and generate information on the greatest number of gas-phase parameters in bubble plumes. In this section, an attempt will be made to evaluate the reliability of the system for measuring void fraction and bubble velocity.

5.4.1 Measurement of Rise Velocity of Individual Spherical-Cap Bubbles

One method employed to test the performance of the sensor was to measure the velocity of single bubbles rising vertically in water. Single spherical cap bubbles were generated using a
Figure 5.15 Variation of indicated flow parameters with threshold level for double-contact electroresistivity sensor.
stainless steel scoop to which the desired volume of gas was injected from a syringe. Bubble release was accomplished by rotating the scoop. The velocity of the bubbles was measured by the probe and from films recorded by a high-speed camera. Figure 5.16 shows the results of the tests for bubbles of different volume. The measurements are compared with values calculated from the Davies-Taylor\textsuperscript{134} relationship. Considering that this formula has been tested extensively, the results obtained by the probe show very high accuracy. The small discrepancy for the largest bubbles was due to the slight break-up that these bubbles suffered during their release.

![Graph showing bubble velocities](image)

**Figure 5.16** Bubble velocities reported by the probe (○) for single spherical cap bubbles, compared with those measured from high-speed film (□) and calculated from Davies-Taylor\textsuperscript{134} relationship.
5.4.2 *Gas Phase Volume Balance*

Simultaneous measurement of the void fraction and bubble velocity distributions permits another form of evaluation of the reliability of the resistivity probe for measuring these parameters. From Equation (2.23) equating the mean local gas phase velocity to the measured mean bubble rise velocity, the gas volume flow rate at a given cross-section can be calculated as

\[ Q_o = \int_{A_p} \alpha \bar{U_b} \, dA \]  

(5.17)

This integration was carried out over all the cross-sections and conditions studied. The integrated gas flow rates were corrected for the total static pressure. Details of the integration procedure are given in Appendix IV. The values obtained from integration of the local flow properties are given in Figure 5.17 in the form of discrepancies from the input air volume flow rate,

\[ \text{Discrepancy} = \frac{\text{integrated air rate} - \text{input air rate}}{\text{input air rate}} \times 100 \]  

(5.18)

Most of the calculated gas volume flow rates are within ±10 % of the values obtained from flow rate measurements and are generally lower. In all cases the discrepancy is less than 17 % except for those cross-sections at 20 mm from the nozzle, which were close to 25 %. From the scatter of the data appearing in the figure, a clear trend is apparent from positive scatter in the region near the nozzle to negative scatter at higher positions. The negative
Figure 5.17 Discrepancies between Input Air Rates and Air Rates obtained from Equation (5.17).

discrepancy can be explained partially by the use of a threshold level and the possible deflection of the smallest bubbles. The positive discrepancies are more difficult to explain but it is possible that they occur because the calculation of the gas volume flow rate did not consider dynamic pressure effects likely to be important close to the nozzle where the bubbles accelerate. McCann et al.\textsuperscript{29}, in their theory of "doublet" formation,
suggested that the pressure in the wake of preceding bubbles can cause up to 20% increase in the volume of following bubbles.

An important indication of the accuracy of any measurement technique is the reproducibility of the results obtained. Figure 5.18 compares the bubble velocity and pierced length distribution taken nine days apart under identical conditions. There is very good agreement between the distributions. The results reported in Chapter 6 reflect also the high degree of reproducibility obtained in all the parameters measured.

The very good agreement that in general existed between integrated and input air rates give support to the measuring instrument and to the considerations adopted in the signal analysis procedure, and indicate that the results obtained in this work present a correct physical picture of the behaviour of the bubbles in turbulent gas-liquid plumes.
Figure 5.18 Bubble rise velocity and pierced length distributions measured nine days apart.
CHAPTER 6
PRESENTATION AND DISCUSSION OF RESULTS

The previous chapter contains an assessment of the computer controlled experimental technique applied to measure local properties of gas-liquid plumes in vertically injected air jets in water. This chapter reports and discusses the following: the results of the experimental measurements of flow parameters in such plumes, the influence of different injection conditions on the flow behaviour of the bubbles within the plume, the plume geometry, the results of some correlations for the distribution of gas inside the plume and a comparison of the model predictions of Tacke et al.\textsuperscript{56} with some of the experimental results.

Measurements were performed under conditions involving different gas flow rates, bath depths and nozzle diameters. In effect six different injection conditions were examined as shown in Table 4.2. The data obtained in all the tests consist of: the time-averaged gas fraction and bubble frequency, the mean bubble velocity and pierced length and the spectrum of bubble velocity and pierced length, measured at selected points.

6.1 Profiles of Void Fraction

The radial gas fraction distribution, at nine levels in the plume, was measured for each of the six experimental conditions studied. The results are plotted in Figures 6.1 to 6.6. The gas
Figure 6.1 Gas fraction profiles in different cross-sections of an air-water plume.

Q = 371 N cm⁻³/s
h_b = 400 mm
d_0 = 6.35

Z, mm \( \times \alpha \), %

- 20 26.03
- 40 19.61
- 65 12.49
- 100 7.87
- 140 6.11
- 190 4.76
- 240 3.84
- 290 2.99
- 350 1.89

Radial position r, mm

Local gas fraction \( \alpha \), %
Figure 6.2 Gas fraction profiles in different cross-sections of an air-water plume.
Figure 6.3 Gas fraction profiles in different cross-sections of an air-water plume.
Figure 6.4 Gas fraction profiles in different cross-sections of an air-water plume.
Figure 6.5 Gas fraction profiles in different cross-sections of an air-water plume.
Figure 6.6 Gas fraction profiles in different cross-sections of an air-water plume.

$Q = 876\, \text{Nm}^3/\text{s}$

$h_b = 400\, \text{mm}$

$d_0 = 4.10\, "$

$z, \text{mm}\quad \alpha, \%$

- $\nabla$ 20 21.68
- $\bigcirc$ 40 22.50
- $\triangle$ 65 20.89
- $\bigcirc$ 100 13.12
- $\bigcirc$ 140 9.59
- $\bigcirc$ 190 6.40
- $\bigcirc$ 240 5.53
- $\nabla$ 290 5.08
- $\bigtriangleup$ 350 3.59
fraction profiles are seen to be symmetrical and each exhibits a similar bell-shape. The measurements reveal the continuous spread of the gas within the plume, as liquid is entrained by the rising bubbles; the profiles become flatter and wider downstream from the injection point. Viewed in terms of area-averaged, the gas fraction seen in the figures decreases with distance from the orifice.

The similarity of the gas fraction profiles in successive sections of the jet becomes evident when the profiles are normalized by division of the local gas fraction by $\alpha_{\max}$ and are related to the dimensionless distances obtained with a length scale $r_{\alpha_{\max}/2}$. This distance called the half-value radius is the distance from the axis of symmetry at which the gas fraction is half the maximum value. The normalized gas fraction profiles at the nine levels considered are shown in Figures 6.1a to 6.6a. The similarity of the cross-sectional profiles is observed over the entire plume length. Under all the conditions studied, the experimental reduced gas fraction distributions can be approximated by the following Gaussian curve

$$\frac{\alpha}{\alpha_{\max}} = \exp\left(-0.7 \left(\frac{r}{r_{\alpha_{\max}/2}}\right)^2\right)$$ (6.1)

This equation produces very good agreement with the experimental data practically across the entire plume. However, values that are slightly below the measurements are obtained near the apex of
Figure 6.1(a) Dimensionless radial gas fraction profiles at different axial distances from the nozzle in an air-water plume; symbols belong to the transverse sections in the corresponding previous figure.
Figure 6.2(a) Dimensionless radial gas fraction profiles at different axial distances from the nozzle in an air-water plume; symbols belong to the transverse sections in the corresponding previous figure.

\[ Q = 876 \text{ N cm}^3\text{s}^{-1} \]
\[ h_b = 400 \text{ mm} \]
\[ d_0 = 6.35 \text{ mm} \]
Figure 6.3(a) Dimensionless radial gas fraction profiles at different axial distances from the nozzle in an air-water plume; symbols belong to the transverse sections in the corresponding previous figure.
Figure 6.4(a) Dimensionless radial gas fraction profiles at different axial distances from the nozzle in an air-water plume; symbols belong to the transverse sections in the corresponding previous figure.

\[ Q = 876 \text{Ncm}^3 \text{s}^{-1} \]
\[ h_b = 600 \text{mm} \]
\[ d_0 = 6.35 \text{mm} \]
Figure 6.5(a) Dimensionless radial gas fraction profiles at different axial distances from the nozzle in an air-water plume; symbols belong to the transverse sections in the corresponding previous figure.

Q = 371 N cm$^3$ s$^{-1}$

$h_b = 400$ mm

$d_0 = 4.10$ mm
Figure 6.6(a) Dimensionless radial gas fraction profiles at different axial distances from the nozzle in an air-water plume; symbols belong to the transverse sections in the corresponding previous figure.
the gas fraction distribution; at the edge of the plume the values from Equation (6.1) are slightly too high.

Figures 6.7(a) to 6.7(c) show maps of the void fraction in a vertical plane passing through the centreline of the plume. The maps consist of isovoid fraction curves which are numbered to indicate the gas fraction expressed as a percentage. From these figures it is clear that in an axisymmetrical air-jet issuing vertically upward, the gas-liquid plume has the form of a regular cone with its origin some distance upstream of the nozzle mouth. The angle subtended by the cone varied between 18° and 22° and increased with the gas flow rate.

6.1.1 Correlations for the Axial Gas Fraction and the Half-Value Radius

Since the laboratory work revealed similarity in the distributions of gas fraction, it was desirable to obtain correlations for $\alpha_{\text{max}}$ and $r_{\alpha_{\text{max}}/2}$ as a function of position and system variables, so that combined with Equation (6.1) the gas distribution throughout the plume could be characterized.

Figures 6.8 and 6.9 show the log-log plots of the axial gas fraction and dimensionless half-value radius as a function of dimensionless vertical position, respectively. It is observed that the fitted lines for the axial gas fraction are almost parallel when $\alpha_{\text{max}}$ is below approximately 70 % and that the dimensionless half-value radius lines are closely parallel over the entire plume length. The fitted equations have correlation
Figure 6.7(a) Gas fraction maps for different air-water bubble plumes.
Figure 6.7(b) Gas fraction maps for different air-water bubble plumes.
Figure 6.7(c) Gas fraction maps for different air-water bubble plumes.
coefficients in the range 0.96 - 0.99 and are expressed as

\[ \alpha_{\text{max}} = C_\alpha \left( \frac{z}{d_0} \right)^{0.993} \tag{6.2} \]

\[ r\alpha_{\text{max}/2} = C_r \left( \frac{z}{d_0} \right)^{0.48} \tag{6.3} \]

The intercepts of the logarithmic lines in Figures 6.8 and 6.9 at \( z/d_0 = 1 \), denoted by \( C_\alpha \) and \( C_r \) respectively, correlated well with
the orifice modified Froude number, as can be seen in Figures 6.10 and 6.11. Thus the following expressions for the axial gas fraction and half-value radius were obtained

\[
\alpha_{max} = 293.77 \left[ \left( \frac{g d_0}{Q_o \rho_g} \right)^{\frac{5}{2}} \left( \frac{\rho_1 - \rho_g}{\rho_g} \right)^{0.269} \right]^{\frac{z}{0.993 - 1}} \frac{z}{d_0} \quad \text{for} \quad N \geq 4 \quad (6.4)
\]

Figure 6.9 Variation of dimensionless half-value radius with dimensionless distance from the nozzle for different air-water plumes.
Figure 6.10 Intercepts of the parallel portion of the lines in Figure 6.8 as function of the modified Froude number.

\[ Fr = \frac{Q_0^2 \rho_g - Q_0^2 \rho_{go}}{g d_0^5 (\rho_1 - \rho_{go})} \]

\[ \gamma_{\text{max}}/2 \left( \frac{g}{Q_0^2} \right) = 0.243 \left[ \frac{g d_0^5 (\rho_1 - \rho_{go})}{Q_0^2 \rho_{go}} \right]^{0.184} \]

Equations (6.4) and (6.5) are plotted in Figures (6.12) and (6.13) together with the experimental data; a close fit with little scatter in the data is observed. The variable \( N \) defining the range of applicability of Equation (6.4) is defined as
For the region of the plume where $\alpha_{\text{max}} > 70\%$ the following expression holds for the variation of the axial gas fraction with position and modified Froude number

$$\alpha_{\text{max}} = 100 \left[ \left( \frac{g d_0^5 (\rho_1 - \rho_{\text{go}})}{Q_0^2 \rho_{\text{go}}} \right)^{0.269} \right] \left( \frac{z}{d_0} \right)^{0.993 - 0.22}$$

$N < 4$ (6.7)
This equation is also shown in Figure (6.12). Equations (6.4) and (6.5) are similar in form to expressions reported recently by Tacke et al.\(^{56}\), but the values of the coefficients and exponents are different. This dissimilarity may arise from differences in the Froude numbers employed and in the speed of response of the measuring system to liquid-gas transitions. Tacke et al.\(^{56}\) worked with modified Froude numbers above 200 and reported transition
times of 2 ms, while in this study the modified Froude numbers were below 100 and the transition times of the signals were in the order of 600 μs; the times to reach the threshold level, indicating the presence of gas at the probe contacts, were even shorter. Equations (6.1), (6.4), (6.5) and (6.7) describe the gas distribution in plumes, under the conditions investigated, very well.
6.2 Profiles of the Bubble Frequency

The local bubble frequency, like the gas fraction, was measured by means of the lower contact of the electroresistivity probe. The radial profiles of bubble frequency at different axial distances from the nozzles are shown in Figures 6.14 to 6.19, for the different experimental conditions investigated. Similar to the void fraction distributions, the bubble frequency distributions are closely symmetrical and exhibit a bell-shape. However, an important difference resides in the fact that the continuous flattening observed in the gas fraction distribution curves, with distance from the injection point, does not exist in the distribution of bubble frequency. Instead, what is observed is an increase in the bubble frequency near the nozzle, i.e. over the transverse sections at axial distances below 100 mm from the orifice. Over this region of the plume the bubble frequency profiles become generally steeper as distance from the nozzle increases; this tendency is particularly strong at higher gas flow rates, Figures 6.15 and 6.16. As will be seen later, these results are consistent with the occurrence of breakup, in the vicinity of the nozzle, of the bubbles forming at the orifice. The figures suggest that the sudden increase in the number of bubbles is not compensated by the expansion undergone by the plume as result of liquid entrainment and hence the local bubble frequency increases. However, once a stable bubble size distribution is established beyond 100 mm, the continued expansion of the jet produces bubble frequency profiles which
Figure 6.14  Bubble frequency profiles in different cross-sections of an air-water plume.
Figure 6.15 Bubble frequency profiles in different cross-sections of an air-water plume.
Figure 6.16 Bubble frequency profiles in different cross-sections of an air-water plume.

\[ Q = 1257 \text{ N cm}^{-3} \text{s}^{-1} \]
\[ h_b = 400 \text{ mm} \]
\[ d = 6.35 \text{ mm} \]
Figure 6.17 Bubble frequency profiles in different cross-sections of an air-water plume.
Figure 6.18  Bubble frequency profiles in different cross-sections of an air-water plume.
Figure 6.19  Bubble frequency profiles in different cross-sections of an air-water plume.
become lower and wider with increasing distance from the beginning of the plume.

6.3 Bubble Rise Velocity and its Spectrum

The local bubble velocity spectrum was measured simultaneously with the other parameters, through the analysis of the signals from the lower and upper contacts of the electro-resistivity probe. Figure 6.20 shows typical bubble velocity spectra for several axial positions along the centreline of a plume. Thus it is seen that the shape of the bubble velocity spectra changes with position. The spectra become less skewed to the large velocity values i.e. the distribution becomes more symmetric, with increasing distance from the nozzle. It is interesting to note that larger velocities, along the centreline, become more probable in moving from $z = 40 \text{ mm}$ to $z = 100 \text{ mm}$; beyond this position the tendency reverses to decreasing velocities. This behaviour can be explained by noting that close to the nozzle the probe is measuring, to a large degree, the upward velocity of bubbles at detachment and the speed of displacement of the growing interfaces. Considering to a first approximation Equation (2.7) given by Walters and Davidson\textsuperscript{39} for the bubble velocity at detachment, a value of 0.70 m/s is calculated for the conditions reported in Figure 6.20. This velocity corresponds reasonably well to the first peak of the velocity spectrum at $z = 40 \text{ mm}$. The largest velocities of the spectrum would correspond to those bubbles accelerating in the wake of the leading bubbles. Beyond the region where gas
Figure 6.20 Bubble velocity spectra at the plume centreline
discharge has an influence, the bubble velocities decrease as liquid is entrained and the bubbles dissipate their energy. Changes in the motion of the bubbles with distance from the injection point are further discussed in the next section. The spread of the bubble velocity spectra clearly reveals the inadequacy of the assumption made by some investigators, when calculating the pierced length of the bubbles, that all the bubbles move at the same velocity.

The local mean bubble velocity was evaluated from the bubble velocity spectrum according to Equation (5.11). Figures 6.21 to 6.26 show the radial profiles of the mean bubble rise velocity at different levels in the gas-liquid plumes. It is evident that the jet dissipates its momentum almost immediately upon discharge due to the sudden expansion of the gas. Close to the injection point, steep velocity profiles are observed and these become gradually less pronounced downstream as more liquid is entrained in the plume. Similar to the radial velocity profiles in homogeneous jets, the bubble rise velocity distributions can be approximated by a Gaussian curve. The maximum bubble rise velocities are located along the centreline of the jet as were the maximum void fraction and bubble frequency.

As mentioned previously, the gas fraction and the mean bubble rise velocity distributions appearing in Figures 6.1 to 6.6 and Figures 6.21 to 6.26, respectively, were used to assess the reliability of the measurements. Since the gas volume flow rates, obtained by integration of the product of the gas fraction
Figure 6.21 Mean bubble velocity profiles in different cross-sections of an air-water plume.
Figure 6.22 Mean bubble velocity profiles in different cross-sections of an air-water plume.
Figure 6.23 Mean bubble velocity profiles in different cross-sections of an air-water plume.
**Figure 6.24** Mean bubble velocity profiles in different cross-sections of an air-water plume.
Figure 6.25 Mean bubble velocity profiles in different cross-sections of an air-water plume.
Figure 6.26 Mean bubble velocity profiles in different cross-sections of an air-water plume.
and mean bubble velocity over the flow cross-section, were generally within ±10% of the injected gas volume flow, it appears that the measured bubble velocity spectra adequately represent the motion of the bubbles within the plume.

The standard deviation of the bubble velocity spectrum or intensity of turbulence were calculated according to Equation (5.12). The radial variation of the standard deviation for different axial positions is plotted in Figure 6.27, under those conditions involving the lowest and highest gas flow rates studied. The figure shows that the standard deviation of the bubble velocity spectrum has a fairly uniform distribution over most of the flow areas except close to the injection point where it decreases sharply toward the plume edge. This characteristic of the profiles close to the nozzle may be explained by the suppression of the fluctuating motions of the bubbles due to the inertia of a relatively quiescent liquid at the bottom of the vessel near the plume boundary. Also from the same figure it is seen that the standard deviation of the bubble velocity spectrum increases with the gas flow rate. For the low gas flow rate it appears that the dissipation of turbulent motion occurs over a short distance after which the turbulence in the direction of flow becomes nearly homogeneous. For the high gas flow rate the decay of the turbulent motion of the bubbles is slower. This may be due to the interaction of the bubbles with a liquid which is more thoroughly mixed by a more buoyant plume and possibly by a more effective interaction among the bubbles themselves.
Figure 6.27  Local standard deviation of bubble velocity spectrum for different gas flow rate conditions.
6.3.1 Axial Profiles of Bubble Rise Velocity - Influence of Injection Conditions

Some other interesting characteristics in the development of gas-liquid plumes, and the effect that injection variables have on the bubble motion, can be observed from the axial profiles of the mean bubble rise velocity along the plume centreline and the plume edge. The profiles are shown in Figures 6.28 to 6.30 for the various injection conditions investigated. The axial bubble velocity distributions reveal three characteristic flow regions. In the first zone of developing flow the shape of the centreline profile is strongly affected by the injection velocity. The bubble velocity at the jet axis increases with height for low gas flow rates; this increase becomes smaller as the injection velocity is raised until above ~41.2 m/s (Q = 1257 Ncm$^3$/s, $d_o = 6.35$ mm). Figure 6.29, the bubble velocity begins to show a decrease with height. These characteristics of the velocity profiles seem to be intimately related to the phenomena of bubble formation at the orifice. At low gas flow rates, individual bubbles form and during their initial rise exert strong wake effects on trailing bubbles, thus increasing the velocity of the latter. With increasing injection velocity, severance of the bubbles from the orifice becomes less successful and elongated gas envelopes result to form irregular and discontinuous jets which lose their momentum as they penetrate into the liquid. In this region of the plume, the bubble rise velocity at the plume boundary decreases to an approximately constant value, for given injection conditions.
Figure 6.28 Mean bubble velocity profiles at the centreline and boundary of plumes to illustrate the effect of gas injection velocity at two gas flow rates.
Figure 6.29 Mean bubble velocity profiles at the centreline and boundary of plumes formed under different gas flow rate conditions.

Figure 6.30 Mean bubble velocity profiles at the centreline and boundary of plumes for two bath depth conditions.
In the second region of developed flow, the centreline profiles show a slow decrease in the bubble rise velocity as the bubbles continue to dissipate their energy and more liquid is entrained into the plume. In this region of the plume the bubbles affect the flow mainly (or only) through the buoyancy force they induce. As mentioned previously, the bubble velocity at the edge of the plume in this region remains appreciably constant with height. This can be explained if the bubble velocity depends on both liquid velocity and terminal bubble velocity as

$$\bar{U}_b = \bar{U}_l + U_{bt}$$

(6.8)

The increase in the velocity of the liquid with height, along the plume boundary, would compensate for the decrease in the bubble terminal velocity associated with the slight decrease in the size of the bubbles as they rise.

Finally in the third region, the bath surface comes into play and causes the bubble rise velocity to decrease more rapidly as the direction of liquid circulation changes from upward to radially outward.

Figure 6.28 illustrates the effect of injection velocity on the axial mean bubble velocity distribution. From the plots it is seen that the velocity of the bubbles in the region of developed flow is independent of the injection velocity. Thus the plumes in this region affect the motion of the surrounding liquid mainly through their buoyancy. In agreement with Abramovich the results indicate that the kinetic energy of the injected gas has
only a localized effect upon the motion of the bubbles since this energy is dissipated rapidly upon injection. The results also agree with the mixing studies of Haida and Brimacombe\textsuperscript{135} who found that the kinetic power of the injected gas has only a small influence on mixing as compared to its buoyancy power.

In Figure 6.29 the axial bubble rise velocity profiles are plotted for different gas flow rates. It is seen that the mean velocity of the bubbles in the plume increases with the gas flow rate owing primarily to an increase in the specific buoyancy power of the injected gas. The bath depth, on the other hand, did not have an effect on the rise velocity of the bubbles, as shown in Figure 6.30, likely because the power input per unit mass for the two baths under consideration was very similar, Appendix II. In addition, the gas distribution and the plume spread in the two baths were nearly the same; thus from the gas mass conservation equation it should be expected that the bubble velocity distribution in both cases should be closely similar.

6.3.2 Comparison of the Experimental Results with the Predictions of the Model of Tacke et al.\textsuperscript{56}

Tacke et al.\textsuperscript{56} recently proposed a model to estimate the axial variation of gas fraction, half-value radius and bubble rise velocity in gas-liquid plumes. The model was of the integral-profile type and consisted of the formulation of the continuity equations for the gas and the liquid in the plume and the conservation of vertical momentum of the plume. The radial
profiles of gas fraction and liquid velocity were assumed Gaussian. These profiles were related through the ratio of their width, while the liquid and gas velocities were connected by the slip velocity of the bubbles which was assumed equal to the bubble terminal velocity. The model equations and assumptions adopted by the authors are presented in Appendix V, together with the method of solution employed in this work.

Tacke et al. defined the initial conditions for the model, i.e. the conditions stating the start of the developed buoyant region of the plume, to be the position corresponding to a centreline gas fraction of 50%. However, in the present work it was found that this gas fraction, particularly at low gas flow rates, is within the region of developing flow close to the nozzle. Therefore the condition \( \alpha_{\text{max}} = 50\% \) does not specify the start of the fully developed buoyant region of the plume for which the model was proposed. The initial conditions used in this investigation corresponded to the position at which the second zone, the fully developed buoyant zone, commences based on the measured centreline velocity profiles, as discussed in the previous section. The experimental gas fraction and half-value radius at this position were used to start the model.

Figure 6.31 gives the predictions of the model for a given set of experimental conditions. The plots show the sensitivity of the solution to the entrainment coefficient, which for practical purposes can be considered a fitting parameter. To calculate the bubble rise velocity, the entrainment coefficient, \( \varepsilon \), was
Figure 6.31 Comparison between experimental and predicted variation of axial gas fraction, half-value radius and bubble velocity with distance from the injection point. Predictions from model of Tacke et al.
adjusted to produce the best fit of the axial gas fraction and half-value radius. From the figure it is seen that the calculated gas fraction and half-value radius vary in opposite directions with the entrainment coefficient, and therefore cannot be fully adjusted.

Figures 6.32 and 6.33 show the results of the calculations together with the experimental data for the other two gas flow rates studied. It may be considered that the predictions of the model are semiquantitatively satisfactory, but again it should be remembered that the model predictions depend on the entrainment coefficient to fit the results.

6.4 Distribution of Bubble Pierced Length

As discussed in Section 5.1.4, the residence time of the bubbles at the lower tip of the probe can be used in conjunction with their individual transit velocities to determine the pierced length distribution of the detected bubbles. A probe assembly of the type used here will Pierce the bubble at any point of the projected frontal area of the bubble, and therefore any calculation of the bubble diameter distribution requires the adoption of certain assumptions regarding bubble shape.

In Figure 6.34, the arithmetic mean of the pierced length of the bubbles along the centreline is plotted as function of distance from the nozzle. From this figure it is clear that the injection conditions, in particular gas flow rate and nozzle diameter, initially produce different-sized bubbles; but moving
Figure 6.32 Comparison between experimental and predicted variation of axial gas fraction, half-value radius and bubble velocity with distance from the injection point. Predictions from model of Tacke et al.
Figure 6.33 Comparison between experimental and predicted variation of axial gas fraction, half-value radius and bubble velocity with distance from the injection point. Predictions from model of Tacke et al.
upward from the nozzle, flow development is accompanied by a reduction in bubble size until a stable distribution of small-sized bubbles is established. The "equilibrium" pierced-length is practically independent of injection conditions. From the measurements of the bubble pierced-length close to the nozzle it is clear that the effect of the gas momentum is to distort the gas envelopes in the vertical direction, giving the bubbles a jet-like appearance although irregular and discontinuous.

Observations of the bubbles were made from high speed motion pictures taken at different injection conditions and locations in

Figure 6.34 Variation of mean pierced length of bubbles with distance from the nozzle along the plume centre-line, for different injection conditions.
the jet. Figure 6.35 shows a sequence of events occurring close to the nozzle under conditions of low gas flow rate. Under these conditions the breakup of the bubbles was most clearly observed. The growing bubble, Figure 6.35(a), developed a depression in its base as inflowing gas penetrated the bubble dragging a column of liquid that reached the top surface of the bubble. The change in shape of the bubble and the distortion of its surface became more rapid as the bubble detached from the orifice, Figure 6.35(b), and the basal penetration became more pronounced. The final disruption and disintegration of the leading bubble was brought about as result of binary coalescence, i.e whereby the growing bubble becomes elongated and coalesces with the previously released bubble reestablishing the flow of gas into this bubble, Figure 6.35(c). The outcome of all these processes was a cloud of different sized-bubbles travelling closely packed as shown in Figure 6.35(d). The sequence of events observed in these photographs agrees very well with the trends revealed by the bubble frequency and bubble pierced length measurements, in the region of developing flow. Figure 6.36 illustrates the situation that exists higher in the bath, where it is seen that bubbles of different sizes are randomly distributed in the plume.

In Figure 6.37 the probability distribution of pierced lengths, for the region where the arithmetic mean of pierced lengths is constant, is seen to follow a log-normal distribution. The geometric mean, $\overline{l}_{bg}$, shown in Figure 6.34, is thus a better estimator of the central tendency of the distribution and as expected this quantity is smaller than the arithmetic mean. The
Figure 6.35 Fast speed pictures taken at 1500 frames/s of the breakup of bubbles in the vicinity of the nozzle, \( Q = 371 \text{Ncm}^3/\text{s}, h = 400 \text{ mm}, d = 6.35 \text{ mm}, \) probe at centreline \( z = 110 \text{ mm} \).
Figure 6.36 Fast speed pictures of bubbles in the region of developed buoyant plume, \((Q = 371 \text{Nm}^3/\text{s}, h_b = 400 \text{ mm}, d_0 = 6.35 \text{ mm}, \text{probe at centreline } z = 200 \text{ mm})\).
distributions of pierced length, as shown in Figure 6.37, are very close for different axial positions but show that smaller bubbles become slightly more frequent downstream in the plume.

The different results concerning axial variations of gas fraction, bubble frequency, mean bubble velocity and pierced length show characteristic changes with position. In particular, it should be noticed that the regions of developing flow and bubble break-up extend over the same distance of the plume and that the region of developed flow is characterized, apart from being mostly a buoyant flow region, by a practically constant bubble size distribution.

Provided that the flow is locally homogeneous, the probe has equal probability of piercing the bubbles at any point on the projected frontal area and the measured chord length may vary from zero to the largest vertical dimension of the bubble, e.g. diameter for a spherical bubble. Considering this to be the case in the region of developed flow, the distribution of bubble diameters can be obtained from the distribution of pierced lengths.

6.4.1 Distribution of Bubble Diameters

Different methods are found in the literature\textsuperscript{137} that can be used to determine the bubble size distribution from the distribution of pierced lengths of the detected bubbles. The most commonly applied methods require that: 1) all bubbles have spherical shape, 2) spheres of various sizes are randomly
Figure 6.37 Log-normal probability plots of bubble pierced length for different centreline positions in the region of developed flow of air-water plumes.
distributed over all the region under study and 3) the size of the sample of pierced lengths is statistically meaningful. For the bubble swarms studied in this work the first two requirements would be most closely approximated in the region where the "equilibrium" bubble size distribution has been established, while the third condition is met as demonstrated in Section 5.2.4. Although the bubbles are more generally not spherical, the use of a method involving this assumption may be justified as a first approximation since many different bubble shapes exist in turbulent gas-liquid plumes, and only one bubble size parameter has been measured i.e. bubble pierced length.

The method of Spektor which is discussed by Underwood has been applied in this work to determine the distributions of bubble diameters from the probability density function of the pierced lengths of detected bubbles. This technique is based upon consideration of the penetration of a polydisperse system of spheres randomly distributed in space. Then for a discrete distribution of pierced lengths, the number of bubbles with a diameter equal to the pierced length characterizing an interval of the pierced length histogram is given as

\[ N_v (i) = \frac{4}{\pi \Delta^2} \left[ \frac{N_{lb} (i)}{2i - 1} - \frac{N_{lb} (i+1)}{2i + 1} \right] \] (6.9)

The derivation and use of this expression is explained in detail by Underwood. To overcome the problem that Equation (6.9) can predict negative numbers, the measured distributions of pierced
length were arranged in intervals of 5 mm. This equation magnifies differences in the distributions of pierced length.

Figure 6.38 shows the distributions of bubble diameter corresponding to the distributions of pierced length, along the centreline of the plume, given in Figures 6.37. The lines representing the distribution of bubble diameters were calculated from the distributions of pierced length denoted by the lines in Figures 6.37, which hold reasonably well along the entire plume centreline. The data points on the bubble diameter distribution plots correspond specifically to the axis of the plume 190 mm downstream of the nozzle. It is evident that the bubble size distribution is well represented by a log-normal distribution. Figure 6.39 shows the geometric mean of the bubbles travelling along the centreline of plumes for all the conditions studied. It is seen that in general the bubble size decreases with an increase in the orifice Reynolds number; the geometric mean diameter ranged between 5 and 8 mm. It is important to note that Figure 6.39 presents the geometric mean diameter of the bubbles and not the volume-surface mean diameter which was the quantity reported by Leibson et al. Figure 6.40 shows an example of the distribution of bubble diameter across several sections of the plume. It is clear that, as previously mentioned, the differences in bubble size over the plume radius are small, indicating that bubbles of different size are practically homogeneously distributed in the region of developed flow.
Figure 6.38 Log-normal probability plots of bubble diameter at the centreline of the region of developed flow of air-water plumes.
Figure 6.39 Geometric mean bubble diameter at the centreline of the region of developed flow versus Reynolds number.

Figure 6.40 Local geometric mean bubble diameter in the region of developed flow of an air-water plume.
The present work has sought to shed new light on the complex behaviour of turbulent gas-liquid bubble plumes, in vertically injected jets, through the experimental determination of the fluid-dynamic characteristics of the gas phase: gas fraction, bubble frequency, mean bubble velocity and pierced length, and the spectra of the bubble velocity and pierced length. This has required the development of a unique measuring system consisting of a two-element probe to sense bubbles in the plume and a new signal analysis procedure. The latter has involved the assembly of hardware and software to implement pattern recognition logic in real time. The analysis ensures that the measured delay times of signals from the two probe contacts are uniquely related to the transit of bubbles travelling axially from the lower to the upper electrode. This feature is vital for the effective determination of the bubble rise velocity and pierced length. The measuring system also allows simultaneous acquisition of the data necessary to evaluate all the parameters mentioned previously.

Tests on the accuracy and reproducibility of the measuring system have revealed the following:

(1) The dimensions, geometry and alignment of the probe contacts are critical to the reliability and reproducibility of the measurements and therefore must be carefully controlled.
Probe effects are manifested in the speed of the signal transitions and in the degree of parallelism between the falling edges of signals from the two contacts.

(2) Reliable bubble velocity measurements require that the signals from both contacts have a common liquid voltage level. This permits the use of a single threshold level and ensures that both signals have the same detection time.

(3) Under the turbulent conditions of the plumes studied, typically about 25 to 35 per cent of the bubbles intercepted could be accepted to extract information on bubble velocity based on the pattern recognition logic. There is strong evidence that the signal analysis does not unduly bias the results. The reproducibility of the results, their internal consistency, and the fact that the bulk of data produced integrated air flow rates within ±10 per cent of the input rates are indications of the suitability of this technique and the validity of the results.

With this measuring system, the following results were obtained:

(1) The radial gas fraction distributions across the plume are symmetric and can be approximated by Gaussian curves. It has been found that the profiles exhibit similarity along the entire plume length.

(2) A set of experimental correlations have been formulated to express the axial variation of the gas fraction and half value radius with injection conditions, as represented by
the modified Froude number. A very good representation of the gas dispersion was obtained under the conditions studied.

(3) During vertical upward injection, air-water plumes expand as a cone with an angle between 18° and 22°; the angle increases with the gas flow rate.

(4) For the first time, bubble frequency measurements have revealed the increase in the number of bubbles that occurs over the region of the plume within 100 mm from the nozzle. These results show that smaller bubbles are continuously produced by the shattering of large bubbles over this region. Downstream of this zone the bubble frequency profiles become increasingly flatter and wider.

(5) It has been found that the bubble velocity spectra are skewed. The measurements also indicate that the standard deviation of the bubble velocity spectrum increases with the gas flow rate, thus revealing the increased turbulent nature of the flow.

(6) The radial mean bubble velocity profiles are symmetric and can be approximated by a Gaussian curve. The mean bubble velocity gradients are large close to the injection point.

(7) Axial bubble velocity profiles clearly reveal three regions of bubble motion behaviour: a region of nozzle influence or developing flow, a region of fully developed buoyant flow and a region of surface influence.
(8) The axial mean bubble velocity profiles in the region of developing flow have different shapes depending on the injection velocity. At low injection velocities the profiles reveal an increase in the mean bubble velocity downstream from the injection point, while at injection velocities above ~42 m/s the profiles exhibit a decrease with height. High speed film observations suggest that this effect of the injection velocity is related to the nature of gas discharge, i.e. if the gas discharge produces single bubbles or short jets.

(9) The mathematical model of Tacke et al.\textsuperscript{56} proposed to represent the behaviour of the developed buoyant region of the plume compares reasonably well with the experimental results. However, the usefulness of the model to make predictions on plume behaviour is limited since it depends sensitively on an entrainment coefficient to fit the results.

(10) The pierced length measurements in the region of developing flow, in agreement with the bubble frequency measurements, indicate that the large bubbles forming at the orifice become rapidly unstable after detachment from the orifice. The bubbles break continuously until an "equilibrium" bubble size distribution is established.

(11) In the region of developed flow the dynamically stable bubble size distribution is maintained reasonably constant. The spectra of bubble pierced length and bubble diameters
are represented by a log-normal distribution. Injection conditions have only a slight effect on the size of the bubbles existing in this region.

Suggestions for Further Work

It is believed that the present thesis has extended the knowledge on many of the aspects concerning the physical characteristics of gas-liquid plumes of interest in ladle metallurgy. However much work remains to be done. The following emerge as desirable extensions of the present work:

1. In the laboratory work it is necessary to use the experimental techniques developed here to conduct additional experiments to cover more fully the effect of several injection variables on plume behaviour. This must be pursued through the study of larger scale water models and non-isothermal liquid metal systems. New experimental data will be particularly valuable for scaling bubble velocities, bubble sizes and void fraction distributions to industrial injection reactors and determine how plume behaviour could be controlled to affect heat and mass transfer rates.

2. Another area is the investigation of the mathematical representation of gas-liquid plume behaviour with available models or, if necessary, modified models. This will prove beneficial in the interpretation and extension of the experimental results.
REFERENCES


Speed of Displacement of a Rising Spherical Bubble

Consider a spherical bubble and let $U_c$ be the velocity of its center and $\dot{R}$ be the velocity of expansion of the surface. Then the velocity, $U_n$, perpendicular to a surface element $dS$, Figure I.1, is

$$U_n = U_c \cos \gamma + \dot{R}$$  \hspace{1cm} (I.1)

Fig. I.1 Sketch of a bubble moving toward a double-contact sensor.
From the figure it is seen also that the radial displacement, $dr$, of a point on the surface is associated to a vertical displacement, $dz$, of the interface, given approximately by

$$dz = \frac{dr}{\cos \theta}$$  \hspace{1cm} (I.2)

The radial displacement can be expressed as

$$dr = U dt$$ \hspace{1cm} (I.3)

Thus the transport velocity, $U_t$, detected by a vertically aligned sensor, is given as

$$U_t = \frac{U_c \cos \gamma + R}{\cos \theta}$$  \hspace{1cm} (I.4)

This equation indicates that if $R = 0$ and the bubble rises along the vertical axis, i.e. $\gamma = \theta$, then

$$U_t = U_c$$  \hspace{1cm} (I.5)

in which case the velocity measured by the sensor will be equal to the velocity of the gas. The transit velocity, $U_t$, can be interpreted as measuring the speed of displacement of a vertically rising and expanding bubble only if $\theta = 0^\circ$. Equation (I.4) also show that a sensor with a small tip separation will register an increasingly larger transport velocity, of a laterally moving and/or expanding bubble, as the
intersection with the bubble occurs closer to the equatorial plane.
APPENDIX II

Conditions of the Experiments

The experimental conditions defined in terms of

\[ Fr = \frac{Q_o^2 \rho_{go}}{g d_o^5 (\rho_1 - \rho_{go})} \]  \hspace{1cm} (II.1)

\[ Re = \frac{4 Q_o \rho_{go}}{\pi d_o \mu / g} \]  \hspace{1cm} (II.2)

\[ \varepsilon_b = \frac{2 Q_o \rho_a}{m_b \ln \left( \frac{P_a - \rho_1 g h_b}{P_a} \right)} \]  \hspace{1cm} (II.3)

\[ \varepsilon_K = \frac{1/2 \rho U^2 Q_o}{m_b} \]  \hspace{1cm} (II.4)

are given in Table II.1, where symbols correspond to those conditions in Table 4.2.

Table II.1 Experimental conditions of the study

<table>
<thead>
<tr>
<th>Experiments</th>
<th>Fr</th>
<th>Re</th>
<th>(\varepsilon_b \times 10^2)</th>
<th>(\varepsilon_K \times 10^4)</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Watt Kg(^{-1})</td>
<td>Watt Kg(^{-1})</td>
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<td>O</td>
<td>1.8</td>
<td>5304</td>
<td>3.8</td>
<td>4.5</td>
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<td>▽</td>
<td>10.1</td>
<td>12542</td>
<td>8.9</td>
<td>59.0</td>
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<tr>
<td>◇</td>
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<td>18003</td>
<td>12.8</td>
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<tr>
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<tr>
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<td>3.8</td>
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<tr>
<td>△</td>
<td>90.2</td>
<td>19425</td>
<td>8.9</td>
<td>339.6</td>
</tr>
</tbody>
</table>
APPENDIX III

Data Acquisition and Data Reduction Programs
ORG "X88000000

**Chip's addresses**

BASE_9519 EQU "X79 19519 ADDR
CIO1_BASE EQU "X7B 1STCIO ADDR
CIO2_BASE EQU "X7F 2NDCIO ADDR
DMA_BASE EQU "X7E DMA PGM 8237 ADDR

**System's calls**

S_GETTIME EQU "X05 ！RTRN TIME FROM REAL TIME CLOCK
S_PUTCH EQU "X29 ！PRNT CHARACTER MESS TO SCREEN
S_PUTS EQU "X2B ！PRNT STRING MESS TO SCREEN
S_PUTN EQU "X2C ！PRNT NUMBER MESS TO SCREEN

**Ending condition**

N_GDBUBL EQU "X0320 ！PGRM END AFTER 0320(16) GB

**Offset 0 -- initialize chips**

CALR INIT_CHIP
LDAR RR2.MESS_INDN ！LOAD ADDR OF INIT DONE MESS
CALR PUTMESS
DW "X7F00

CONTINUE:

**Offset A -- enables counter/timers carrying out data acquisition**

LDAR RR6,STARTTIME ！PNT TO START TIME BUF ADDR
SC #S_GETTIME
LD R6,"X8A00 ！ISTCIO STR DATA IN SEG A ... 
LD R7,"X0000 ！STARTING AT OFFSET 0, GB
SUB R12,R12 ！CALCULATE DATA...
LD R13,#N_GDBUBL ！OFFSET...
MULT RR12,"X4 ！FOR NGB
LD R12,"X8A00 ！ISTCIO STR DATA IN SEG A ... 
LD R11,"XFFFF ！STARTING AT OFFSET N_GDBUBL*4, NGB
SUBB RL4,RL4 ！CLR FLAG
SUB R9,R9 ！CLR GB CNTR
SUBL RR10,RR10 ！CLR DIVIDEND
LD R11,"XFFFF ！65535(10) IN R11
LDB RL1,"XF4 ！NBL PA, PB, C/T1, C/T2, C/T3 By ...
OUTB ^X7B03,RL1 !F4(16) TO MCCR 1STC10
LDB RL1,^X06 !6(16) PUT 1 IN ...
OUTB ^X7B19,RL1 !TCB, GCB IN CSR C/T3 ...
OUTB ^X7B17,RL1 !C/T2 ...
OUTB ^X7B15,RL1 !C/T1

WAIT_START: INB RL1,^X7B1B !READ PA'sDR
BITB RL1,#0 !CH (PA 0), BUBBLE IN LWR CNTCT?
JR Z,WAIT_START !NO - WAIT UNTIL IT COMES
INB RL1,^X7B13 !YES - READ PB CSR, SEE IF A GB
BITB RL1,#5 !IS IPB SET? (E1. PATTERN MATCHED?)
JR NZ,PARNBL_7F !YES - GO APPARENTLY A GB
CHPA_IN_LC1: INB RL1,^X7B1B !NO - CH (PA 0)
BITB RL1,#0 !BUBL PASSED LWR CNTCT?
JR NZ,CHPA_IN_LC1 !NO - WAIT UNTIL IT PASS
READ_T2: INB RH2,^X7B25 !YES - READ C/T2 CCR - MSB
INB RL2,^X7B27 !... - LSB
LD @RR12,R2
INC R13,#2
SUB R2,R2
LD @RR12,R2
JR DBL_JR_3 !1ST JUMP
PARNBL_7F: LDB RL1,^XA4 !NBL PB, PA, C/T2 BY ...
OUTB ^X7F03,RL1 !A4(16) TO MCCR 2NDC10
LDB RL1,^X26 !LOAD 26(16) TO ...
OUTB ^X7F17,RL1 !CLR IPB & IUSB, 1 TO TCB GCB C/T2 CSR
CHPA_IN_UC1: INB RL1,^X7B1B !READ PA'sDR
BITB RL1,#1 !CH (PA 1), BUBL ARRIVED UP CNTCT?
JR NZ,CHPA_IN_UC1 !NO - WAIT UNTIL IT ARRIVES
BITB RL1,#0 !YES - BUBL PASSED LWR CNTCT TOO?
JR Z, DBL_JR_1 !YES - 1ST JUMP
CHPA_IN_LC2: INB RL1,^X7F13 !NO- READ PB CSR SEE IF A GB
BITB RL1,#5 !IS IPB SET? (E1. PATTERN MATCHED?)
JR NZ, DBL_JR_4 !YES - 1ST JUMP
INB RL1,^X7B1B !NO - READ PA'sDR
BITB RL1,#0 !CH (PA 0), HAS BUBL PASSED LWR CNTCT?
JR NZ,CHPA_IN_LC2 !NO - WAIT UNTIL IT PASS
READ_T2_T1: INB RH2,^X7B25 !YES - READ C/T2 CCR - MSB
INB RL2,^X7B27 !... - LSB

204
LD  R3, R2  !KEEP C/T2 CLK_CNTS 1STC10
LD  R5, R2  !...
INB  RH2, ^X7B21  !READ C/T1 CCR - MSB
INB  RL2, ^X7B23  !... - LSB
LD  R8, R2  !KEEP C/T1 CLK_CNTS 1STC10
RDY_TMR5:
LD  RL1, ^X20  !LOAD 20(16) TO ...
OUTB  ^X7B13, RL1  !CLR IPB & IUSB PB CSR
LDB  RL1, ^X26  !LOAD 26(16) TO ...
OUTB  ^X7B17, RL1  !CLR IPB & IUSB, 1 TO TCB GCB C/T2 CSR
OUTB  ^X7B15, RL1  !... C/T1 CSR
SUB  R11, R3  !LOAD DIVIDEND IN RR10
DIV  RR10, ^X00A  !/ 1/10 OF C/T2 CLK_CNTS IN R11

CHPAIN_ULC1:
INB  RL1, ^X7B1B  !READ PA'sDR
BITB  RL1, #0  !CH (PA 0), BUBL IN LWR CNTCT?
JR  NZ, CHPAIN_ULC2  !YES - GO TO TAKE FURTHER DESITIONS
INB  RL1, ^X7F13  !NO - READ PB CSR 2NDClO
BITB  RL1, #5  !IS IPB SET? (EI. PATTERN MATCHED?)
JR  Z, CHPAIN_ULC1  !NO - KEEP CH
INB  RH2, ^X7F25  !YES - READ C/T2 CCR - MSB
INB  RL2, ^X7F27  !... - LSB
JR  DBL_7F_1  !JUMP, WHILE CH UPR NO BUBL LWR CNTCT

CHPAIN_ULC2:
INB  RL1, ^X7B1B  !READ PA'sDR
BITB  RL1, #0  !CH (PA 0), BUBL STILL IN LWR CNTCT?
JR  Z, DBL_7F_2  !NO - IT PASSED NGLCT C/T1 CLK_CNTS
INB  RL1, ^X7F13  !YES - READ PB CSR 2NDClO
BITB  RL1, #5  !IS IPB SET? (EI. PATTERN MATCHED?)
JR  Z, CHPAIN_ULC2  !NO - KEEP CH
INB  RH2, ^X7F25  !YES - READ C/T2 CCR - MSB
INB  RL2, ^X7F27  !... - LSB
LDB  RL4, #1  !SET FLAG

DBL_7F_1:
LDB  RL1, ^X00  !LOAD 00(16) TO ...
OUTB  ^X7F03, RL1  !CLR MCR 2NDClO
LDB  RL1, ^X20  !LOAD 20(16) TO ...
OUTB  ^X7F13, RL1  !CLR IPB & IUSB PB CSR
CP  R3, R2  !C/T2 CLK_CNTS 1STC10 < 2NDClO?
JR  UBT, DIF2  !YES - GO DIF2

DIF1:
SUB  R3, R2  !NO - C/T2 CLK_CNTS 1STC10 - ... 2NDClO?
CP  R3, R11  !IS DIF >= QUTNT?
JR  UGE, N1_1  !YES - NGLCT C/T1 CLK_CNTS
LD @RR6,R5
INC R7,#2
LD @RR6,R8
INC R9,#1
JR CH_FLAG
JR DBL_7F_4
DBL_JR_1:
JR DBL_JR_4
DBL_JR_2:
SUBB RL4,RL4
SUBL RR10,RR10
LD R11,#"FFFFF"
JR WAIT_START
DBL_JR_3:
INC R13,#2
JR CH_CT3
DBL_JR_4:
INC R13,#2
JR DBL_7F_3
DIF2:
SUB R2,R3
CP R2,R11
JR UGE,N1_1
LD @RR6,R5
INC R7,#2
LD @RR6,R8
INC R9,#1
JR CH_FLAG
JR DBL_7F_3
N1_1:
LD @RR12,R5
INC R13,#2
SUB R8,R8
LD @RR12,R8
INC R13,#2
BITB RL4,#0
JR Z,CH_CT3_S
JR CHPA_IN_LC3
CH_FLAG:
INC R7,#2
BITB RL4,#0
JR Z,CH_CT3_S
CHPA_IN_LC3:
INB RL1,~X7B1B
BITB RL1,#0
JR NZ,CHPA_IN_LC3
INB RH2,~X7B25
INB RL2,~X7B27
LD @RR12,R2
INC R13,#2
SUB R2, R2
LD @RR12, R2
INC R13, #2
JR CH_CT3

DBL_7F_2:
LDB RL1, #X00
OUTB ^X7F03, RL1
LDB RL1, #X20
OUTB ^X7F13, RL1
INB RH2, ^X7B25
INB RL2, ^X7B27

N1_2:
LD @RR12, R5
INC R13, #2
SUB R8, R8
LD @RR12, R8
INC R13, #2
LD @RR12, R2
INC R13, #2
LD @RR12, R2
INC R13, #2
JR CH_CT3

DBL_7F_3:
LDB RL1, #X00
OUTB ^X7F03, RL1
LDB RL1, #X20
OUTB ^X7F13, RL1
INB RL1, ^X7B1B
BITB RL1, #0
JR NZ, CHPA_IN_LC4
JR READ_T2_N1

CHPA_IN_LC4:
INB RL1, ^X7B1B
BITB RL1, #0
OUTB ^X7F03, RL1
LDB RL1, #X20
OUTB ^X7F13, RL1
INB RH2, ^X7B25
INB RL2, ^X7B27

READ_T2_N1:
INB RH2, ^X7B21
INB RL2, ^X7B23
SUB R2, R2
LD @RR12, R2
INC R13, #2

CH_CT3:
LDB RL1, ^X20
OUTB ^X7B13, RL1
LDB RL1, ^X26
OUTB ^X7B17, RL1
OUTB ^X7B15, RL1

CH_CT3_S:
INB RL1, ^X7B19
BITB RL1, #5
JR NZ, END_ACQ
CP R9, #N_GDBUBL
JR NZ, DBL_JR_2
LDB RL1, ^X08
OUTB ^X7B19, RL1

END_ACQ:
LD R9, R13
LDAR RR6, ENDTIME
SC #S_GETTIME

PRINF:
LDAR RR2, MESS_FIN
CALR PUTMESS
LDAR RR2, MESS_ST
CALR PUTMESS
LDAR RR12, STARTTIME
CALR PRINTIME
LDAR RR2, MESS_NDT
CALR PUTMESS
LDAR RR12, ENDTIME
CALR PRINTIME
LDL RR6, ^STRING1
SC #S_PUTS
SUB R6, R6
LD R7, R9
DIV RR6, ^X4
LD R9, R7
LD R5, ^D10
LD R4, ^D4
LD R3, "'
SC #S_PUTN
LDL RR6, ^STRING3
SC #S_PUTS
XOR R6,R6 !FAST CLR OF R6
LD R7,#XFFFF
INB RH2,#X7B29 !READ C/T3 CCR - MSB
INB RL2,#X7B2B !... - LSB
SUB R7,R2 !CALCULATE No. OF BUBL THAT PASSED ...
ADD R7,#X0001 !LWR CNTCT
LD R5,#D10
LD R4,#D4
LD R3,#'
SC #S.PUTN
LDL RR6,#STRING3
SC #S_PUTS
CALR CNTS_TIME
CALR GAS_TIME
LDL RR6,#STRING4 !PRNT ...
SC #S_PUTS
LDL RR6,RR4
LD R5,#D10
LD R4,#D4
LD R3,#'
SC #S_PUTN !SUM.TIME SZ
LDL RR6,#STRING3
SC #S_PUTS
CALR SORT_TVEL
LDAR RR2,MESS_NDPRC
CALR PUTMESS
LDL RR6,#STRING5 !PRNT ...
SC #S_PUTS
SUB R6,R6
LD R7,R9
MULT RR6,#X4
SUB R7,#X1
LD R5,#D16
LD R4,#D4
LD R3,#'
SC #S_PUTN !DATA OFFSET IN SEG A
LDL RR6,#STRING3
SC #S_PUTS
SC #0
INIT_CHIP:

****Initialization of the 9519, 1STCIO, 2NDCIO and DMA chips****

LDB RH1,#BASE_9519
LD R2,#INIT_SIZE_9519
LDAR RR4,TB_9519
CALR INIT

LDB RH1,#CI01_BASE
LD R2,#CI01_INIT_SIZE
LDAR RR4,CI01_TB
CALR INIT

LDB RH1,#CI02_BASE
LD R2,#CI02_INIT_SIZE
LDAR RR4,CI02_TB
CALR INIT

LDB RH1,#DMA_BASE
LD R2,#DMA_INIT_SIZE
LDAR RR4,DMA_TB
INIT

RL1,@RR4
RL1,R5
Z,SKIP
OUTIB @R1,@RR4,R2
JR NOV,INIT
RET

INIT:

LDB RL1,@RR4
INC R5,#1
ORB RL1,RL1
JR Z,SKIP
OUTIB @R1,@RR4,R2
JR NOV,INIT
RET

SKIP:

INC R5,#1
DEC R2,#1
JR NZ,INIT
RET

TB_9519:

DW ^X0380
DW ^X03B0
DW ^X01FF
C101_TB:

DW ~X0340
DW ~X03C0
DW ~X0100
DW ~X03A9
DW ~X0000

!LOW) CHANGE TO 01DF TO TURN ON
!CLR 9519
!PRESELECT AUTO CLR REGISTER
!NO AUTO CLR
!CHIP ARMED, IREQ REG

C1O1_TB:

DW ~X0101
DW ~X0100
DW ~X390C
DW ~X1504
DW ~X2D00
DW ~X2F00
DW ~X3B0C
DW ~X1704
DW ~X3100
DW ~X3300
DW ~X5102
DW ~X57FF
DW ~X5B88
DW ~X5D80
DW ~X5F88
DW ~X3D20
DW ~X35FF
DW ~X37FF
DW ~X0D0F
DW ~X4140
DW ~X47FF
DW ~X0000

!RESET C101
!CLR RESET
!C/T1: EGEB, REB MSR
!C/T1: GCB CSR
!C/T1: INIT TCR - MSB
!... - LSB
!C/T2: EGEB, REB MSR
!C/T2: GCB CSR
!C/T2: INIT TCR - MSB
!... - LSB
!PB BTPORT PMSB's (AND) MSR
!PB - IN DIRECTION DDR
!PB PPR 10001000 1 BIT3
!PB PTR 10000000 AND
!PB PMR 10001000 0-->1 BIT7
!C/T3: EGEB (CNTR) MSR
!C/T3: 65535(EI. FFFF) TO ...
!C/T3: TCR
!PC - 0..3 IN DIRECTION
!PA - INPUT PORT
!PA - IN DIRECTION DDR

C102_TB:

DW ~X0101
DW ~X0100
DW ~X3B0C
DW ~X1704
DW ~X3100

!RESET C102
!CLR RESET
!C/T2: EGEB, REB MSR
!C/T2: GCB CSR
!C/T2: INIT TCR - MSB
DW ^X3300 !... - LSB
DW ^X5102 !PB BTPORT PMSB's (AND) MSR
DW ^X57FF !PB - IN DIRECTION DDR
DW ^X5B00 !PB PPR 00000000 1--->0 BIT3
DW ^X5D08 !PB PTR 00001000 AND
DW ^X5F88 !PB PMR 10001000 0 BIT7
DW ^X4140 !PA - INPUT PORT
DW ^X47FF !PA - IN DIRECTION DDR
DW ^X0000
DW ^X0000

DMA_TB:

DW ^X1B00 !MASTER CLR
DW ^X1900 !CLR BYTE PTR OFF
DW ^X1140 !CMD REG. FIXED PRNT. DABL DMA
DW ^X1715 !PRGRM MODE REGISTER
DW ^X0500 !DMA WRITE ADDR - INIT AT 2000
DW ^X0520
DW ^X07FF !DMA TC INIT AT 1K
DW ^X0703 !(EI. IK-1= 3FF BYTES
DW ^X0000
DW ^X0000

END_DMA:

DS 0

INIT_SIZE_9519 EQU (CI01_TB-TB_9519)/2
CI01_INIT_SIZE EQU (CI02_TB-CI01_TB)/2
CI02_INIT_SIZE EQU (DMA_TB-CI02_TB)/2
DMA_INIT_SIZE EQU (END_DMA-DMA_TB)/2

PUTMESS:

***Print messages on the screen***

NEXT_CHAR:
LDB RL1,@RR2 !RL1 HAS ONE CHARACTER
ORB RL1,RL1 !CH FOR END OF MESS
RET Z !IF END RTRN
SC #S_PUTCH !IF NO END - PRNT CHARACTER
INC R3,#1 !MOVE TO NEXT CHARACTER
JR NEXT_CHAR !LOOP BACK

MESS INDN: DB "r\n INIT DONE ... GO TO 08.000A TO CONTINUE",13,10,0
MESS_FIN: DB "r\n ACQUISITION ENDED ... DOWN COUNTER = 0",13,10,0
MESS_ST: DB "\r\nTIME AT WHICH EXPERIMENT STARTED :",13,10,0
MESS_NDT: DB "\r\nTIME AT WHICH EXPERIMENT ENDED :",13,10,0
MESS_NDPRC: DB "\r\nCONVERSION, SUM AND SORTING FINISHED",13,10,0
.EVEN
PRINTTIME:

***Print start and end times of the experiment***

SUBL RR6,RR6 !CLR RR6
LDB RL7,@RR12 !RR6 CONTAIN TIME UNIT TO PRNT
LD R5,#~D16 !REMEMBER TIME UNIT IS IN BCD
LD R4,#~D2 !R4 AND R3 TAKE CARE OF ...
LD R3,'#' !PRINTING FORMAT
SC #S_PUTN
LDL RR6,#STRING2 !PNT TO STRING2
SC #S_PUTS
INC R13,#1 !PNT TO NEXT TIME UNIT - M
LD R5,#~D16
LD R4,#~D2
LD R3,'#'
SUBL RR6,RR6
LDB RL7,@RR12
SC #S_PUTN
LDL RR6,#STRING2
SC #S_PUTS
INC R13,#1 !PNT TO NEXT TIME UNIT - DM
LD R5,#~D16
LD R4,#~D2
LD R3,'#'
SUBL RR6,RR6
LDB RL7,@RR12
SC #S_PUTN
LDL RR6,#STRING2
SC #S_PUTS
INC R13,#1 !PNT TO NEXT TIME UNIT - DW
LD R5,#~D16
LD R4,#~D2
LD R3,'#'
SUBL RR6,RR6
LDB RL7,@RR12
SC #S_PUTN
LDL RR6,#STRING2
SC #S_PUTS
INC R13,#1
LD R5,#"D16
LD R4,#"D2
LD R3,#'
SUBL RR6,RR6
LDB RL7,@RR12
SC #S_PUTN
LDL RR6,#STRING2
SC #S_PUTS
INC R13,#1
LD R5,#"D16
LD R4,#"D2
LD R3,#'
SUBL RR6,RR6
LDB RL7,@RR12
SC #S_PUTN
LDL RR6,#STRING2
SC #S_PUTS
INC R13,#1
LD R5,#"D16
LD R4,#"D2
LD R3,#'
SUBL RR6,RR6
LDB RL7,@RR12
SC #S_PUTN
LDL RR6,#STRING3
SC #S_PUTS
RET

STARTTIME: DS 7
ENDTIME: DS 7
STRING1: DB "\n\rNUMBER OF TIME-SIZES COLLECTED :\n\r0"
STRING2: DB "\ / \0"
STRING3: DB "\n\r0"
STRING4: DB "\n\rSUM TIME SIZE :\n\r0"
STRING5: DB "\n\rDATA OFFSET IN SEGMENT A : \n\r\0"
.EVEN

CNTS_TIME:

****Convert from clock-counts to microseconds****
LD R6,#X8A00
LD R7,#X0000
LD R1,R9
INC R1
NEXT_DATA:
DEC R1
JR Z,DONE_CONV
LD R2,#XFFFF
SUB R2,@RR6
LD @RR6,R2
INC R7,#2
TEST @RR6
JR Z,UPDT
LD R2,#XFFFF
SUB R2,@RR6
LD @RR6,R2
UPDT:
INC R7,#2
JR NEXT_DATA
DONE_CONV:
RET

GAS_TIME:

****add times corresponding to gas presence at lwr cntct****
LD R6,#X8A00
LD R7,#X0000
CLR R2
CLR R5
CLR R4
LD R1,R9
SUM32:
LD R3,@RR6
ADDL RR4,RR2
INC R7,#4
DJNZ R1,SUM32
RET

SORT_TVEL:
! ****sort time vel in decreasing order keeping corresponding time size**** !
CLR R0 !CLR INTERCHANGE FLAG BEFORE PASS!
LD R1,#N_GDBUBL !LENGTH OF CNTS SZ LIST IN R1!
DEC R1 !ONE LESS PAIR THAN ELEMENTS!
JR LE,DONE_SORT !CATCH 0 OR 1 ELEMENTS CASES!
LD R4,#^X8A00
LD R5,#^X0000
LD R6,#^X8A00
LD R7,#^X0002

PASS:
LD R3,@RR6 !TAKE ARRAY TIME VEL ELEMENT!
LD R2,@RR4 !TAKE ARRAY TIME SZ ELEMENT!
INC R7,#4
INC R5,#4
CP R3,@RR6 !IS 1ST LESS THAN 2ND IN PAIR?
JR UGE,STEP !NO - NO INTERCHANGE NECESSARY!
LDK R0,#1 !YES - SET INTERCHANGE FLAG
EX R3,@RR6 !INTERCHANGE TIME VEL DATAS
DEC R7,#4
EX R3,@RR6 !INTERCHANGE TIME SZ DATAS
INC R7,#4
EX R2,@RR4
DEC R5,#4
EX R2,@RR4
INC R5,#4

STEP:
DJNZ R1,PASS !WAS THERE ANY INTERCHANGE?
DEC R0 !YES - GO THROUGH IT AGAIN
JR Z,SORT_TVEL !NO - SORTING FINISHED

DONE_SORT:
RET

END
208 PRINT USING "########" ; EXDT!
211   K = K + 2
214 IF K < 128 GOTO 202
217 IF LOC(2) <= NRMEM GOTO 193
220 RETURN
223 '
226 PRINT : PRINT
229 PRINT TAB(18) ; "TYPE Y OR y IF YES ANYTHING ELSE IF NOT"
232 ANSWER$ = INPUT$(1)
235 RETURN
238 '
241 NEX% = CVI(NE$)
244 EDM% = CVI(DM$) : EDD% = CVI(DD$) : EDY% = CVI(DY$)
247 TEX! = CVS(TE$) : PSEX! = CVS(PS$) : NBEX! = CVS(NB$) : NVEX! = CVS(NV$)
250 STSEX! = CVS(STS$) : DPEX! = CVS(DP$)
253 PPZ! = CVS(PZ$) : PPR! = CVS(PR$) : PPA! = CVS(PA$)
256 ROEX! = CVS(RO$) : QVGEX! = CVS(QVG$)
259 PRINT CHR$(26) : PRINT : PRINT : PRINT
262 PRINT "EXPERIMENT'S NUMBER" ; TAB(45) ; NEX%
265 PRINT "EXPERIMENT'S DATE" ; TAB(45) ; EDM% ; CHR$(47) ; EDD% ; CHR$(47) ; EDY%
268 PRINT "DURATION OF THE EXPERIMENT" ; TAB(45) ; TEX! ; "sec"
271 PRINT "DISTANCE BETWEEN THE CONTACTS" ; TAB(45) ; PSEX! ; "m"
274 PRINT "No. OF BUBBLES THAT PASSED LOWER CONTACT" ; TAB(45) ; NBEX!
277 PRINT "No. OF MEASURED VELOCITIES" ; TAB(45) ; NVEX!
280 PRINT "SUM TIME SIZE" ; TAB(45) ; STSEX! ; "sec"
283 PRINT "DEPTH OF THE BATH" ; TAB(45) ; DPEX! ; "m"
286 PRINT "POSITION OF THE PROBE" ; TAB(45) ; PPZ! ; "m" ; SPC(3)
289 PRINT PPR! ; "m" ; SPC(3) ; PPA! ; "deg"
292 PRINT "ORIFE RADIUS" ; TAB(45) ; ROEX! ; "m"
295 PRINT "GAS VOLUME FLOW AT ORIFICE CONDITIONS" ; TAB(45) ; QVGEX! ; "m3/sec"
298 PRINT : PRINT : PRINT
301 RETURN
304 '
307 '
310 '
313 LSET NE$ = MKI$(NEX%)
316 LSET DM$ = MKI$(EDM%) : LSET DD$ = MKI$(EDD%) : LSET DY$ = MKI$(EDY%)
LSET TE$= MKS$(TEX!)  
LSET PS$= MKS$(PSEX!)  
LSET NB$= MKS$(NBEX!) : LSET NV$= MKS$(NVEX!)  
LSET STS$= MKS$(STSEX!)  
LSET DP$= MKS$(DPEX!)  
LSET PZ$= MKS$(PPZ!) : LSET PR$= MKS$(PPR!) : LSET PA$= MKS$(PPA!)  
LSET RO$= MKS$(ROEX!) : LSET QVG$= MKS$(QVGEX!)  
RETURN  
'  
'  
PRINT CHR$(26) : PRINT : PRINT : PRINT  
INPUT "EXPERIMENT'S NUMBER ", NEX%  
INPUT "EXPERIMENT'S MONTH ", EDM%  
INPUT "EXPERIMENT'S DAY ", EDD%  
INPUT "EXPERIMENT'S YEAR ", EDY%  
INPUT "DURATION OF THE EXPERIMENT ", TEX!  
INPUT "DISTANCE BETWEEN CONTACTS ", PSEX!  
INPUT "NO. OF BUBBLES THAT PASSED LOWER CONTACT ", NBEX!  
INPUT "NO. OF MEASURED VELOCITIES ", NVEX!  
INPUT "SUM TIME SIZE ", STSEX!  
INPUT "DEPTH OF THE BATH ", DPEX!  
INPUT "PROBE'S AXIAL POSITION ", PPZ!  
INPUT "PROBE'S RADIAL POSITION ", PPR!  
INPUT "PROBE'S ANGULAR POSITION ", PPA!  
INPUT "ORIFICE'S RADIUS ", ROEX!  
INPUT "GAS VOLUME FLOW AT ORIFICE CONDITIONS ", QVGEX!  
RETURN
`100 'THIS PROGRAM PROCESS THE DATA DISK FILE TO PRODUCE FILES
103 'OF THE VELOCITIES AND SIZES OF BUBBLES AND OF THE
106 'REDUCED INFORMATION
109 '
112 OPEN "R", #2, "EXPDATA100", 128
115 FIELD #2, 2 AS NE$, 2 AS DM$, 2 AS DD$, 2 AS DY$, 4 AS TE$, 4 AS PS$, 4 AS NB$, 4
119 @ AS NV$, 4 AS STS$, 4 AS DP$, 4 AS PZ$, 4 AS PR$, 4 AS PA$, 4 AS RO$, 4 AS QVG$
118 FIELD #2, 128 AS ED$
121 J = 1 : GET #2, J
124 GOSUB 1231
127 OPEN "R", #3, "EXPVELSZ100", 64
130 FIELD #3, 2 AS NE$, 2 AS DM$, 2 AS DD$, 2 AS DY$, 4 AS TE$, 4 AS PS$, 4 AS NB$, 4
134 @ AS NV$, 4 AS STS$, 4 AS DP$, 4 AS PZ$, 4 AS PR$, 4 AS PA$, 4 AS RO$, 4 AS QVG$
133 FIELD #3, 4 AS VL$, 4 AS S1$, 4 AS V2$, 4 AS S2$, 4 AS V3$, 4 AS S3$, 4 AS V4$, 4
137 @ AS S4$, 4 AS V5$, 4 AS S5$, 4 AS V6$, 4 AS S6$, 4 AS V7$, 4 AS S7$, 4 AS V8$, 4 AS S8
136 GOSUB 1306
139 PUT #3, 1
142 OPEN "R", #4, "EXPSZ100", 64
145 FIELD #4, 2 AS NE$, 2 AS DM$, 2 AS DD$, 2 AS DY$, 4 AS TE$, 4 AS PS$, 4 AS NB$, 4
149 @ AS NV$, 4 AS STS$, 4 AS DP$, 4 AS PZ$, 4 AS PR$, 4 AS PA$, 4 AS RO$, 4 AS QVG$
148 FIELD #4, 4 AS Z1$, 4 AS Z2$, 4 AS Z3$, 4 AS Z4$, 4 AS Z5$, 4 AS Z6$, 4 AS Z7$, 4
152 @ AS Z8$, 4 AS Z9$, 4 AS Z10$, 4 AS Z11$, 4 AS Z12$, 4 AS Z13$, 4 AS Z14$, 4 AS Z15$, 4
158 @ AS Z16$
151 GOSUB 1306
154 PUT #4, 1
157 FLAG = 0
163 INPUT "MAXIMUM ALLOWED SIZE ", SNR!
169 PRINT TAB(12); "DO YOU WANT TO SUPPLY THE MAXIMUM ALLOWED VELOCITY, TOO?"
172 GOSUB 1216
175 IF ANSWER$ = "Y" OR ANSWER$ = "y" THEN FLAG = 1
178 OPEN "R", #5, "EXPR1100", 64
181 FIELD #5, 2 AS NE$, 2 AS DM$, 2 AS DD$, 2 AS DY$, 4 AS TE$, 4 AS PS$, 4 AS NB$, 4
185 @ AS NV$, 4 AS STS$, 4 AS DP$, 4 AS PZ$, 4 AS PR$, 4 AS PA$, 4 AS RO$, 4 AS QVG$
184 FIELD #5, 4 AS BF$, 4 AS BHLPS$, 4 AS AVSZ$, 4 AS SDS$, 4 AS AVV$, 4
162 AS SDV$, 4 AS NBVS$, 4 AS NBVSC$, 4 AS V$, 4 AS SL$, 4 AS VO$
187 ON FLAG+1 GOTO 190, 199`
190 FIELD #5, 4 AS AVSZS$, 4 AS SDSS$, 4 AS AVVS$, 4 AS SDVS$  
193 FIELD #5, 4 AS NS$, 4 AS VS$  
196 GOTO 211  
199 FIELD #5, 4 AS AVSZS$, 4 AS SDSS$, 4 AS AVSZSC$, 4 AS SDSSC$, 4 AS AVVS$, ^@4 AS SDVS$, 4 AS AVVSC$, 4 AS SDVSC$  
202 FIELD #5, 4 AS NS$, 4 AS VS$, 4 AS NSC$, 4 AS VSC$  
208 INPUT "MAXIMUM ALLOWED VELOCITY ": VNR!  
211 GOSUB 1306  
214 PUT #5, 1  
217 NRMEM= INT(NBEX! * 4/128) + 1: NRMEM1= INT(NVEX! * 4/128) + 1  
220 NRCLR= NRMEM + 1  
223 GOSUB 829  
226 ARY.SIZE= 850  
229 DIM BS(ARY.SIZE), BV(ARY.SIZE), BSS(ARY.SIZE), BVS(ARY.SIZE), S9(30,2)  
232 SUM.EXBS!= 0: SUM.EXBV!= 0: SUM.SQS!= 0: SUM.SQV!= 0  
235 I= 0: J= 1  
238 PSEXMI= PSEX! * 1E+06  
241 J= J+1: K= 1  
244 GET #2, NRCLR  
247 GET #2, J  
250 EXBTS!= CVI(MID$(EDS,K,2))  
253 IF EXBTS! < 0 THEN EXBTS!= 65536! + EXBTS!  
256 K= K+2  
259 EXBTV!= CVI(MID$(EDS,K,2))  
262 IF EXBTV! < 0 THEN EXBTV!= 65536! + EXBTV!  
265 IF EXBTV! <> 0 GOTO 274  
268 K= K+2  
271 IF K<128 THEN GOTO 250 ELSE GOTO 295  
274 EXBV!= (PSEXMI/EXBTV!)  
277 EXBS!= EXBTS! * EXBV! * .000001  
280 I= I+1  
283 BS(I)= EXBS!  
286 BV(I)= EXBV!  
289 K= K+2  
292 IF K<128 THEN GOTO 250  
295 IF LOC(2) <= NRMEM1 THEN GOTO 241  
298 N!= I  
301 GOSUB 379
304 GOSUB 640
307 BFRC! = NBEX! / TEX!
310 GHUP! = (STSEX! / TEX!)
313 LSET BF$ = MKS$(BFRC!) : LSET BHP$ = MKS$(GHUP!)
316 LSET AVSZ$ = MKS$(ABSZ!) : LSET SDS$ = MKS$(SIGMA.S!)
319 LSET AVV$ = MKS$(ABV!) : LSET SDV$ = MKS$(SIGMA.V!)
322 LSET NBVS$ = MKS$(N!) : LSET NBVSC$ = MKS$(NC!)
325 LSET VS$ = MKS$(VL!) : LSET SL$ = MKS$(SNR!) : LSET VO$ = MKS$(VOE!)
328 PUT #5, 2
331 LSET AVSZS$ = MKS$(ABSZS1!) : LSET SDSS$ = MKS$(SIGMA.SS1!)
334 IF FLAG= 0 GOTO 340
337 LSET AVSZSC$ = MKS$(ABSZS2!) : LSET SDSSC$ = MKS$(SIGMA.SS2!)
340 LSET AVVS$ = MKS$(ABVS1!) : LSET SDVS$ = MKS$(SIGMA.VS1!)
343 IF FLAG = 0 GOTO 349
346 LSET AVVSC$ = MKS$(ABVS2!) : LSET SDVSC$ = MKS$(SIGMA.VS2!)
349 PUT #5, 3
352 LSET NS$ = MKS$(NS1!) : LSET VS$ = MKS$(VS1!)
355 IF FLAG= 0 GOTO 361
358 LSET NSC$ = MKS$(NS2!) : LSET VSC$ = MKS$(VS2!)
361 PUT #5, 4
364 GOSUB 1009
367 END
370 ' 373 ' 376 '
379 ARO = 3.1416 * ROEX!*2 : VOE! = QVGEX!/ARO! : I= N
382 IF BV(I) < VOE! GOTO 388
385 I = I - 1: GOTO 382
388 NC= I : VL= BV(NC)
391 J = 0
394 FOR I = 1 TO NC
397 IF BS(I) > SNR! THEN GOTO 403 ELSE J = J + 1
400 BSS(J) = BS(I) : BVS(J) = BV(I)
403 NEXT I
406 NS1 = J : VS1 = BV(S(NS1)
409 IF FLAG = 0 GOTO 430
412 I = NS1
415 IF BVS(I) < VNR GOTO 421
418 I = I - 1 : GOTO 415
A = I : B = I - 1 : DIFV = BVS(A) - BVS(B)

IF DIFV <= .6 THEN NS2 = A ELSE NS2 = B

VS2 = BVS(NS2)

FOR I = 1 TO NC
    SUM.EXBS! = SUM.EXBS! + BS(I)
    SUM.EXBV! = SUM.EXBV! + BV(I)
    SQS! = BS(I) * BS(I)
    SUM.SQS! = SUM.SQS! + SQS!
    SQV! = BV(I) * BV(I)
    SUM.SQV! = SUM.SQV! + SQV!
NEXT I

ABSZ! = SUM.EXBS!/NC! : ABV! = SUM.EXBV!/NC!

SIGMA.S! = SQR((SUM.SQS - (NC * ABSZ!^2))/NC)

SIGMA.V! = SQR((SUM.SQV - (NC * ABV!^2))/NC)

SUM.EXBS! = 0 : SUM.EXBV! = 0 : SUM.SQS! = 0 : SUM.SQV! = 0

FOR I = 1 TO NS1
    SUM.EXBS! = SUM.EXBS! + BSS(I)
    SUM.EXBV! = SUM.EXBV! + BVS(I)
    SQS! = BSS(I) * BSS(I)
    SUM.SQS! = SUM.SQS! + SQS!
    SQV! = BVS(I) * BVS(I)
    SUM.SQV! = SUM.SQV! + SQV!
NEXT I

ABSZS1! = SUM.EXBS!/NS1 : ABVS1! = SUM.EXBV!/NS1!

SIGMA.SS1! = SQR((SUM.SQS - (NS1 * ABSZS1!^2))/NS1)

SIGMA.VS1! = SQR((SUM.SQV - (NS1 * ABVS1!^2))/NS1)

IF FLAG = 0 GOTO 538

SUM.EXBS! = 0 : SUM.EXBV! = 0 : SUM.SQS! = 0 : SUM.SQV! = 0

FOR I = 1 TO NS2
    SUM.EXBS! = SUM.EXBS! + BSS(I)
    SUM.EXBV! = SUM.EXBV! + BVS(I)
    SQS! = BSS(I) * BSS(I)
    SUM.SQS! = SUM.SQS! + SQS!
    SQV! = BVS(I) * BVS(I)
    SUM.SQV! = SUM.SQV! + SQV!
NEXT I

ABSZS2! = SUM.EXBS!/NS2! : ABVS2! = SUM.EXBV!/NS2!

SIGMA.SS2! = SQR((SUM.SQS - (NS2 * ABSZS2!^2))/NS2)

SIGMA.VS2! = SQR((SUM.SQV - (NS2 * ABVS2!^2))/NS2)
538 \( J = 1 \) : \( I = 0 \) : \( I = I + 1 \)
541 \( M1 = 2*N*4 \) : \( M01 = M1 \mod 64 \)
544 IF \( M01 = 0 \) THEN \( NR3 = M1/64 \) ELSE \( NR3 = \text{INT}((M1/64) + 1) \)
547 \( J = J + 1 \)
550 \( \text{VEL1}! = \text{BV}(I) \) : \( \text{SZ1}! = \text{BS}(I) \)
553 \( I = I + 1 \) : IF \( I > N \) THEN GOTO 577 ELSE \( \text{VEL2}! = \text{BV}(I) \) : \( \text{SZ2}! = \text{BS}(I) \)
556 \( I = I + 1 \) : IF \( I > N \) THEN GOTO 580 ELSE \( \text{VEL3}! = \text{BV}(I) \) : \( \text{SZ3}! = \text{BS}(I) \)
559 \( I = I + 1 \) : IF \( I > N \) THEN GOTO 583 ELSE \( \text{VEL4}! = \text{BV}(I) \) : \( \text{SZ4}! = \text{BS}(I) \)
562 \( I = I + 1 \) : IF \( I > N \) THEN GOTO 586 ELSE \( \text{VEL5}! = \text{BV}(I) \) : \( \text{SZ5}! = \text{BS}(I) \)
565 \( I = I + 1 \) : IF \( I > N \) THEN GOTO 589 ELSE \( \text{VEL6}! = \text{BV}(I) \) : \( \text{SZ6}! = \text{BS}(I) \)
568 \( I = I + 1 \) : IF \( I > N \) THEN GOTO 592 ELSE \( \text{VEL7}! = \text{BV}(I) \) : \( \text{SZ7}! = \text{BS}(I) \)
571 \( I = I + 1 \) : IF \( I > N \) THEN GOTO 595 ELSE \( \text{VEL8}! = \text{BV}(I) \) : \( \text{SZ8}! = \text{BS}(I) \)
574 GOTO 598
577 \( \text{VEL2}! = 0 \) : \( \text{SZ2}! = 0 \)
580 \( \text{VEL3}! = 0 \) : \( \text{SZ3}! = 0 \)
583 \( \text{VEL4}! = 0 \) : \( \text{SZ4}! = 0 \)
586 \( \text{VEL5}! = 0 \) : \( \text{SZ5}! = 0 \)
589 \( \text{VEL6}! = 0 \) : \( \text{SZ6}! = 0 \)
592 \( \text{VEL7}! = 0 \) : \( \text{SZ7}! = 0 \)
595 \( \text{VEL8}! = 0 \) : \( \text{SZ8}! = 0 \)
598 LSET \( \text{V1}! = \text{MKS}$(\text{VEL1}!) \) : LSET \( \text{S1}! = \text{MKS}$(\text{SZ1}!) \)
601 LSET \( \text{V2}! = \text{MKS}$(\text{VEL2}!) \) : LSET \( \text{S2}! = \text{MKS}$(\text{SZ2}!) \)
604 LSET \( \text{V3}! = \text{MKS}$(\text{VEL3}!) \) : LSET \( \text{S3}! = \text{MKS}$(\text{SZ3}!) \)
607 LSET \( \text{V4}! = \text{MKS}$(\text{VEL4}!) \) : LSET \( \text{S4}! = \text{MKS}$(\text{SZ4}!) \)
610 LSET \( \text{V5}! = \text{MKS}$(\text{VEL5}!) \) : LSET \( \text{S5}! = \text{MKS}$(\text{SZ5}!) \)
613 LSET \( \text{V6}! = \text{MKS}$(\text{VEL6}!) \) : LSET \( \text{S6}! = \text{MKS}$(\text{SZ6}!) \)
616 LSET \( \text{V7}! = \text{MKS}$(\text{VEL7}!) \) : LSET \( \text{S7}! = \text{MKS}$(\text{SZ7}!) \)
619 LSET \( \text{V8}! = \text{MKS}$(\text{VEL8}!) \) : LSET \( \text{S8}! = \text{MKS}$(\text{SZ8}!) \)
622 \text{PUT} #3, J : \( I = I + 1 \) : IF \( I <= N \) THEN GOTO 547
625 \text{GOSUB} 883
628 \text{RETURN}
631 '
634 '
637 '
640 \( I1 = 1 \) : \( J1 = NC \)
643 \( I = I1 \) : \( J = J1 \) : \( S = -1 \)
646 IF \( \text{BS}(I) <= \text{BS}(J) \) THEN 655
649 \( T = \text{BS}(I) \) : \( \text{BS}(I) = \text{BS}(J) \) : \( \text{BS}(J) = T \)
652 \( S = \text{SGN}(-S) \)
655 IF S = 1 THEN I = I+1 ELSE J = J-1
658 IF I<J THEN 646
661 IF I+1>=J1 THEN 667
664 P = P+1 : S9(P,1) = I+1 : S9(P,2) = J1
667 J1 = I-1
670 IF I<J1 THEN 643
673 IF P = 0 THEN 682
676 I1 = S9(P,1) : J1 = S9(P,2) : P = P-1
679 GOTO 643
682 J = 1 : I = 0 : I = I+1
685 M2 = NC*4 : M02 = M2 MOD 64
688 IF M02 = 0 THEN NR4 = M2/64 ELSE NR4 = INT((M2/64) + 1)
691 J = J+1
694 SZ1 = BS(I)
697 I = I+1 : IF I>NC THEN GOTO 745 ELSE S22! = BS(I)
700 I = I+1 : IF I>NC THEN GOTO 748 ELSE S3! = BS(I)
703 I = I+1 : IF I>NC THEN GOTO 751 ELSE S4! = BS(I)
706 I = I+1 : IF I>NC THEN GOTO 754 ELSE S5! = BS(I)
709 I = I+1 : IF I>NC THEN GOTO 757 ELSE S6! = BS(I)
712 I = I+1 : IF I>NC THEN GOTO 760 ELSE S7! = BS(I)
715 I = I+1 : IF I>NC THEN GOTO 763 ELSE S8! = BS(I)
718 I = I+1 : IF I>NC THEN GOTO 766 ELSE S9! = BS(I)
721 I = I+1 : IF I>NC THEN GOTO 769 ELSE S10! = BS(I)
724 I = I+1 : IF I>NC THEN GOTO 772 ELSE S11! = BS(I)
727 I = I+1 : IF I>NC THEN GOTO 775 ELSE S12! = BS(I)
730 I = I+1 : IF I>NC THEN GOTO 778 ELSE S13! = BS(I)
733 I = I+1 : IF I>NC THEN GOTO 781 ELSE S14! = BS(I)
736 I = I+1 : IF I>NC THEN GOTO 784 ELSE S15! = BS(I)
739 I = I+1 : IF I>NC THEN GOTO 787 ELSE S16! = BS(I)
742 GOTO 790
745 S22! = 0
748 S3! = 0
751 S4! = 0
754 S5! = 0
757 S6! = 0
760 S7! = 0
763 S8! = 0
766 S9! = 0
769 S10! = 0
772  SZ11! = 0
775  SZ12! = 0
778  SZ13! = 0
781  SZ14! = 0
784  SZ15! = 0
787  SZ16! = 0
790  LSET Z1$ = MKS$(SZ1!) : LSET Z2$ = MKS$(SZ2!)
793  LSET Z3$ = MKS$(SZ3!) : LSET Z4$ = MKS$(SZ4!)
796  LSET Z5$ = MKS$(SZ5!) : LSET Z6$ = MKS$(SZ6!)
799  LSET Z7$ = MKS$(SZ7!) : LSET Z8$ = MKS$(SZ8!)
802  LSET Z9$ = MKS$(SZ9!) : LSET Z10$ = MKS$(SZ10!)
805  LSET Z11$ = MKS$(SZ11!) : LSET Z12$ = MKS$(SZ12!)
808  LSET Z13$ = MKS$(SZ13!) : LSET Z14$ = MKS$(SZ14!)
811  LSET Z15$ = MKS$(SZ15!) : LSET Z16$ = MKS$(SZ16!)
814  PUT #4, J : I= I+1 : IF I<=NC THEN GOTO 691
817  GOSUB 946
820  RETURN
823
826
832  PRINT TAB(19): "DO YOU WANT TO SEE THE ACQUIRED DATA FILE?"
835  GOSUB 1216
838  IF ANSWER$= "Y" OR ANSWER$= "y" THEN GOTO 841 ELSE GOTO 874
841  J= 1
844  GOSUB 1252
847  J= J+1
850  K= 1
853  GET #2, J
856  EXDT! = CVI(MID$(ED$,K,2))
859  IF EXDT! < 0 THEN EXDT! = 65536! + EXDT!
862  PRINT USING "###########"; EXDT!;
865  K= K+2
868  IF K<128 GOTO 856
871  IF LOC(2) <= NRMEM GOTO 847
874  RETURN
877
880
886  PRINT TAB(17): "DO YOU WANT TO SEE THE VELOCITY AND SIZE FILE?"
GOSUB 1216

IF ANSWER$ = "Y" OR ANSWER$ = "y" THEN GOTO 895 ELSE GOTO 937

J = 1 : GET #3, J

GOSUB 1231

GOSUB 1252

J = J + 1

GET #3, J

VEL1! = CVS(V1$) : SZ1! = CVS(Z1$) : VEL2! = CVS(V2$) : SZ2! = CVS(Z2$)

VEL3! = CVS(V3$) : SZ3! = CVS(Z3$) : VEL4! = CVS(V4$) : SZ4! = CVS(Z4$)

VEL5! = CVS(V5$) : SZ5! = CVS(Z5$) : VEL6! = CVS(V6$) : SZ6! = CVS(Z6$)

VEL7! = CVS(V7$) : SZ7! = CVS(Z7$) : VEL8! = CVS(V8$) : SZ8! = CVS(Z8$)

PRINT USING "##.##" ; VEL1! ; SZ1! ; VEL2! ; SZ2!

PRINT USING "##.##" ; VEL3! ; SZ3! ; VEL4! ; SZ4!

PRINT USING "##.##" ; VEL5! ; SZ5! ; VEL6! ; SZ6!

PRINT USING "##.##" ; VEL7! ; SZ7! ; VEL8! ; SZ8!

IF LOC(3) <= NR3 THEN GOTO 904

RETURN


PRINT TAB(24) ; "DO YOU WANT TO SEE THE SIZE FILE?"

GOSUB 1216

IF ANSWER$ = "Y" OR ANSWER$ = "y" THEN GOTO 958 ELSE GOTO 1000

J = 1 : GET #4, J

GOSUB 1231

GOSUB 1252

J = J + 1

GET #4, J

SZ1! = CVS(Z1$) : SZ2! = CVS(Z2$) : SZ3! = CVS(Z3$) : SZ4! = CVS(Z4$)

SZ5! = CVS(Z5$) : SZ6! = CVS(Z6$) : SZ7! = CVS(Z7$) : SZ8! = CVS(Z8$)

SZ9! = CVS(Z9$) : SZ10! = CVS(Z10$) : SZ11! = CVS(Z11$) : SZ12! = CVS(Z12$)

SZ13! = CVS(Z13$) : SZ14! = CVS(Z14$) : SZ15! = CVS(Z15$) : SZ16! = CVS(Z16$)

PRINT USING "##.##" ; SZ1! ; SZ2! ; SZ3! ; SZ4!

PRINT USING "##.##" ; SZ5! ; SZ6! ; SZ7! ; SZ8!

PRINT USING "##.##" ; SZ9! ; SZ10! ; SZ11! ; SZ12!

PRINT USING "##.##" ; SZ13! ; SZ14! ; SZ15! ; SZ16!

IF LOC(4) <= NR4 THEN GOTO 967

RETURN

1000
1006 ' 1009 PRINT CHR$(26) : PRINT : PRINT : PRINT : PRINT : PRINT : PRINT : PRINT : PRINT 1012 PRINT TAB(16); "DO YOU WANT TO SEE THE REDUCED INFORMATION FILE?" 1015 GOSUB 1216 1018 IF ANSWER$ = "Y" OR ANSWER$ = "y" THEN GOTO 1021 ELSE GOTO 1210 1021 GET #5, 1 1024 GOSUB 1231 1027 GOSUB 1252 1030 GET #5, 2 1033 BFRC! = CVS(BF$) : GHUP! = CVS(BHLP$) 1036 ABSZ! = CVS(AVSZ$) : SIGMA.S! = CVS(SDS$) 1039 ABV! = CVS(AVV$) : SIGMA.V! = CVS(SDV$) : N! = CVS(NBVS$) 1042 NC! = CVS(NBVSC$) : VL! = CVS(V$) : SNR! = CVS(SL$) : VOE! = CVS(VO$) 1045 GHUP! = GHUP! * 100 1048 GET #5, 3 1051 ABSZS1! = CVS(AVSZS$) : SIGMA.SS1! = CVS(SDSS$) 1054 IF FLAG = 0 GOTO 1060 1057 ABSZS2! = CVS(AVSZSC$) : SIGMA.SS2= CVS(SDSSC$) 1060 ABVS1! = CVS(AVVS$) : SIGMA.VS1!= CVS(SDV$) 1063 IF FLAG = 0 GOTO 1069 1066 ABVS2!= CVS(AVVSC$) : SIGMA.VS2!= CVS(SDVSC$) 1069 GET #5, 4 1072 NS1! = CVS(NS$) : VS1!= CVS(VS$) 1075 IF FLAG = 0 GOTO 1081 1078 NS2!= CVS(NSC$) : VS2!= CVS(VSC$) 1081 LPRINT "EXPERIMENT'S No."; TAB(30); 1084 LPRINT USING "###"; NEX% 1087 LPRINT "GAS HOLD-UP"; TAB(30); 1090 LPRINT USING "###.###"; GHUP! 1093 LPRINT "BUBL FREC (bb1/sec)"; TAB(30) 1096 LPRINT USING "###.###"; BFRC! 1099 LPRINT "No. OF MEASURED V & S"; TAB(30); 1102 LPRINT USING "###"; N! 1105 LPRINT "No. OF V-S DATA USED IN IR"; TAB(30) 1108 IF FLAG = 1 GOTO 1117 1111 LPRINT USING "###"; NC!, NS1! 1114 GOTO 1120 1117 LPRINT USING "###"; NC!, NS1!, NS2! 1120 LPRINT "AVRG BUBL S (m)"; TAB(30);
IF FLAG = 1 GOTO 1132
1126 LPRINT USING "###.### " ; ABSZ!, ABSZS1!
1129 GOTO 1135
1132 LPRINT USING "###.### " ; ABSZ!, ABSZS1!, ABSZS2!
1135 LPRINT "SD OF BUBBLE SIZE (m)" ; TAB(30);
1138 IF FLAG= 1 GOTO 1147
1141 LPRINT USING "###.### " ; SIGMA.S!, SIGMA.SS1!
1144 GOTO 1150
1147 LPRINT USING "###.### " ; SIGMA.S!, SIGMA.SS1!, SIGMA.SS2!
1150 LPRINT "AVRG BUBL V (m/sec)" ; TAB(30);
1153 IF FLAG= 1 GOTO 1162
1156 LPRINT USING "###.### " ; ABV!, ABVS1!
1159 GOTO 1165
1162 LPRINT USING "###.### " ; ABV!, ABVS1!, ABVS2!
1165 LPRINT "I OF TURB (m/sec)" ; TAB(30);
1168 IF FLAG= 1 GOTO 1177
1171 LPRINT USING "###.### " ; SIGMA.V!, SIGMA.VS1!
1174 GOTO 1180
1177 LPRINT USING "###.### " ; SIGMA.V!, SIGMA.VS1!, SIGMA.VS2!
1180 LPRINT "LRGST V CNSDR (m/sec)" ; TAB(30);
1183 IF FLAG= 1 GOTO 1192
1186 LPRINT USING "###.### " ; VL!, VS1!
1189 GOTO 1201
1192 LPRINT USING "###.### " ; VL!, VS1!, VS2!
1195 LPRINT "LRGST S CNSDR (m)" ; TAB(30);
1198 LPRINT USING "###.### " ; SNR!
1201 LPRINT "GVOC (m/sec)" ; TAB(30);
1204 LPRINT USING "###.### " ; VOE!
1207 LPRINT : LPRINT : LPRINT
1210 RETURN
1213`
1216 PRINT : PRINT
1219 PRINT TAB(18) ; "TYPE Y OR y IF YES ANYTHING ELSE IF NOT"
1222 ANSWER$= INPUT$(1)
1225 RETURN
1228`
1231 NEX%= CVI(NE$)
1234 EDM%= CVI(DM$) ; EDD%= CVI(DD$) ; EDY%= CVI(DY$)
1237 TEX!= CVS(TE$) ; PSEX!= CVS(PS$) ; NBEX!= CVS(NB$) ; NVEX!= CVS(NV$)
1240 STSEX! = CVS(STS$) : DPEX! = CVS(DP$)
1243 PPZ! = CVS(PZ$) : PPR! = CVS(PR$) : PPA! = CVS(PA$)
1246 ROEX! = CVS(RO$) : QVGEX! = CVS(QVG$)
1249 RETURN
1252 PRINT CHR$(26) : PRINT : PRINT : PRINT
1255 PRINT "EXPERIMENT'S NUMBER"; TAB(48); NEX%
1258 PRINT "EXPERIMENT'S DATE"; TAB(48); EDM%; CHR$(47); EDD%; CHR$(47); EDY%
1261 PRINT "DURATION OF THE EXPERIMENT"; TAB(48); TEX!; "sec"
1264 PRINT "DISTANCE BETWEEN THE CONTACTS"; TAB(48); PSEX!; "m"
1267 PRINT "No. OF BUBBLES THAT PASSED LOWER CONTACT"; TAB(48); NBEX!
1270 PRINT "No. OF MEASURED VELOCITIES"; TAB(48); NVEX!
1273 PRINT "SUM TIME SIZE"; TAB(48); STSEX!; "sec"
1276 PRINT "DEPTH OF THE BATH"; TAB(48); DPEX!; "m"
1279 PRINT "POSITION OF THE PROBE"; TAB(48); PPZ!; "m"; SPC(3);
1282 PRINT PPR!; "m"; SPC(3); PPA!; "deg"
1285 PRINT "ORIFICE RADIOUS"; TAB(48); ROEX!; "m"
1288 PRINT "GAS VOLUME FLOW AT ORIFICE CONDITIONS"; TAB(48); QVGEX!; "m3/sec"
1291 PRINT : PRINT : PRINT
1294 RETURN
1297 ,
1300 ,
1303 ,
1306 LSET NE$ = MKI$(NEX%)
1309 LSET DM$ = MKI$(EDM%) : LSET DD$ = MKI$(EDD%) : LSET DV$ = MKI$(EDY%)
1312 LSET TE$ = MKS$(TEX!)
1315 LSET PS$ = MKS$(PSEX!)
1318 LSET NB$ = MKS$(NBEX!) : LSET NV$ = MKS$(NVEX!)
1321 LSET STS$ = MKS$(STSEX!)
1324 LSET DP$ = MKS$(DPEX!)
1327 LSET PZ$ = MKS$(PPZ!) : LSET PR$ = MKS$(PPR!) : LSET PA$ = MKS$(PPA!)
1330 LSET RO$ = MKS$(ROEX!) : LSET QVG$ = MKS$(QVGEX!)
1333 RETURN
APPENDIX IV

Calculation of Gas Volume Flow Rate and Area Averaged Gas Fraction

By equating the local gas phase velocity to the local mean bubble rise velocity, the gas volume flow rate across transverse sections of the plume were obtained by numerical integration as

\[ Q = \sum A_n \frac{U_{bi} \alpha_i}{A_i} \]  \hspace{1cm} (IV.1)

where \( A_1, A_2, A_3, \ldots, A_n \) are the areas of a circle with 5 mm radius and that of rings with 5 - 10, 10 - 15, ..., \( r_{p-1} - r_p \) radius. The values of \( U_{bi} \) and \( \alpha_i \) are the local values corresponding to the radial positions 0.25, 0.75, 1.25, ..., \( r_{p-1} + 0.25 \) mm. The radius of the plume, \( r_p \), corresponded to a gas fraction of 0%. The integration results are plotted in Figure 5.17 in the form of discrepancies from the input air rate. The input gas flow rate used in the comparison was that for a temperature of 20°C and a pressure \( h_a + (h_b - z) \).

The area-averaged gas fraction was calculated from the local values as

\[ \langle \alpha \rangle = \frac{1}{A} \sum \frac{\alpha_i A_i}{A} \]  \hspace{1cm} (IV.2)
The integration interval was subdivided in the form mentioned previously. Results of the integration are presented in Figures 6.1 to 6.6.
APPENDIX V

Model of Tacke et al.\textsuperscript{56} for the Zone of Fully Developed Buoyant Flow

In order to calculate the axial variation of the gas fraction, half-value radius and velocity of the bubbles in the region of developed buoyant flow, in turbulent gas-liquid plumes, Tacke et al. formulated a model based on the conservation of mass of gas and liquid in the plume and in the conservation of vertical momentum, given respectively as

\[
Q_T \frac{h_a}{h_a + h_b - z} = 2 \int_0^\infty a U_b r \, dr \tag{V.1}
\]

\[
\frac{d}{dz} \left[ 2 \int_0^\infty (1 - \alpha) U_1 r \, dr \right] = 2 \pi b \epsilon U_{l \text{max}} (1 - \alpha) \tag{V.2}
\]

\[
\frac{d}{dz} \left[ 2 \pi \rho_1 \int_0^\infty (1 - \alpha) U_1^2 r \, dr \right] = 2 \pi \int_0^\infty g (\rho_1 - \rho_g) \alpha r \, dr \tag{V.3}
\]

The profiles of gas fraction and liquid velocity were assumed Gaussian and represented as

\[
\alpha = \alpha_{\text{max}} \exp \left( - \frac{r^2}{b_\alpha^2} \right) \tag{V.4}
\]

\[
U_1 = \frac{U_{l \text{max}}}{U_{l \text{max}}} \exp \left( - \frac{r^2}{b_u^2} \right) \tag{V.5}
\]
where \( b_u \) and \( b_\alpha \) (\( b = r_{1/2} / \sqrt{\ln 2} \) for Gaussian distributions) are the nominal plume widths at \( \frac{U_1}{U_{\text{max}}} = e^{-1} \), and \( \frac{\alpha}{\alpha_{\text{max}}} = e^{-1} \), respectively, and were related by a constant width ratio

\[
\lambda = \frac{b_\alpha}{b_u} \tag{V.6}
\]

The velocity of the bubbles in the plume was given as

\[
\bar{U}_b = \frac{U_1 + U_{bt}}{2} \tag{V.7}
\]

Integrating Equations (V.1) to (V.3) considering Equations (V.4) to (V.7), the following expressions are obtained

\[
Q_T \frac{h_a}{h_a + h_b - z} = \pi \alpha_{\text{max}} b_\alpha^2 \left[ \frac{U_{\text{max}}}{1 + \lambda^2} + U_{bt} \right] \tag{V.8}
\]

\[
\frac{dY(2)}{dz} = \frac{d}{dz} \left[ U_{\text{max}} b_\alpha^2 \left( 1 - \frac{\lambda^2 \alpha_{\text{max}}}{1 + \lambda^2} \right) \right] = 2\varepsilon \lambda b_\alpha U_{\text{max}} \left( 1 - \frac{\alpha_{\text{max}}}{e^{1/\lambda^2}} \right) \tag{V.9}
\]

\[
\frac{dY(1)}{dz} = \frac{d}{dz} \left[ b_\alpha^2 U_{\text{max}}^2 \left( \frac{1}{2 \lambda^2} - \frac{\alpha_{\text{max}}}{1 + 2 \lambda^2} \right) \right] = g_{\alpha_{\text{max}}} b_\alpha^2 \tag{V.10}
\]

The initial conditions required for the solution of the above system of equations were selected at the position indicating the
start of the developed buoyant plume as given by the experimental axial mean bubble velocity profiles. This specification of the initial conditions differs from that given by Tacke et al.\textsuperscript{56}. In that study the initial conditions corresponded to a position at which $a_{\text{max}} = 50\%$, but as seen in this investigation this location is in no form related to the onset of fully buoyant flow for which the model was proposed. Values\textsuperscript{56} of $\lambda = 0.7$ and $U_{bt} = 0.25\, \text{m/s}$ were used in the solution of the equations.

To calculate the variation of $a_{\text{max}}$, $r_{\text{max}}/2$ and $U_{\text{bmax}}$ with position, Equation (V.9) and (V.10) were solved by a fourth-order Runge-Kutta method\textsuperscript{138} for $Y(1)$ and $Y(2)$. The system of non-linear algebraic equations that resulted

\begin{equation}
\pi a_{\text{max}} b_{ai} \left( \frac{U_{\text{lmax}} i + U_{bt}}{1 + \lambda^2} \right) - Q_T \frac{h_a}{h_a + h_b - z_i} = 0 \tag{V.11}
\end{equation}

\begin{equation}
U_{\text{lmax}} i b_{ai} \left( \frac{1}{1 + \lambda^2} - \frac{\lambda^2 a_{\text{max}} i}{1 + \lambda^2} \right) - Y(2)_i = 0 \tag{V.12}
\end{equation}

\begin{equation}
b_{ai} U_{\text{lmax}} i \left( \frac{1}{2 \lambda^2} - \frac{a_{\text{max}} i}{1 + 2 \lambda^2} \right) - Y(1)_i = 0 \tag{V.13}
\end{equation}

was then solved using the Newton-Raphson method\textsuperscript{138} to obtain the values of the desired parameters at position $i$. This procedure was repeated to cover the entire length of the plume. Detailed listing of the program constructed for the solution of the model
is given at the end of the appendix. It is important to note that the results of the model needed to be fitted to the measured quantities by selecting values of the entrainment coefficient, $\varepsilon$. 
C THIS PROGRAM SOLVES TACKE ET AL. MODEL. IT IS IDENTICAL TO VEL2.F77 EXCEPT THAT IT IS LINKED USING 80287...

IMPLICIT REAL*8 (A-H,O-Z)
IMPLICIT INTEGER (I, J, K, L, M, N)
INTEGER RUNGE
REAL*8 LAMBDA
REAL*8 F, Y, PHI, SAVEY, XOLD, XINC, A
REAL*8 DUX, F1, Q, HA, H, Z, PI, G
REAL*8 ZO, HD, ALPHA, EPS1, EPS2
REAL*8 BE, BEPS0, EMAX0, UMAX0, UGMAX0, R10
REAL*8 BEPS, EMAX, UMAX, ZMAX, UGMAX, R1
REAL*8 RD1, RD2, RD3
REAL*8 DEXP, DATAN, DLOG, DSQRT
DIMENSION F(2), Y(2), PHI(50), SAVEY(50), XOLD(21)
DIMENSION XINC(21), A(21,21)
COMMON LAMBDA, DUX, F1, Q, HA, H, Z, PI, G, Y

C READ AND PRINT DATA ...
OPEN(UNIT=10,FILE='NUM2')
READ (10,100) ZO, R1, H, Q, HD, ALPHA
OPEN(UNIT=11,FILE='NUM3')
READ(11,101) ITMAX, IPRINT, N, EPS1, EPS2
OPEN (UNIT=12,FILE='DAV')
PI = 4.0*DATAN(DBLE(1.0))
G = 9.810
BE = DLOG(DBLE(2.))
BEPS0 = (R1/DSQRT(BE))
EMAX0 = 0.50283
LAMBDA = 0.7
DUX = 0.250
F1 = 1.
HA = 1.02347E1
WRITE(6,200) ZO, R1, H, Q, HD, ALPHA
WRITE (6,201) ITMAX, IPRINT, N, EPS1, EPS2
WRITE ( 12,300) Q, H, ALPHA, HD, EPS1, EPS2

C INITIALIZE Z, Y(1), Y(2) AND UMAX ...
BEPS = BEPS0
EMAX = EMAX0
ZMAX = H
Z = Z0
ULMAX0 = (1. + LAMBDA**2)*( Q/(PI*BEPS**2*EMAX)*
+ (HA/(HA + H - Z0)) - DUX*F1)
ULMAX = UMAX0
UGMAX0 = UMAX0 + 0.25
R10 = BEPS0 * DSQRT(BE)
WRITE(12,301) Z0, EMAX0, R10, UMAX0
WRITE(12,302) Y(1) = BEPS**2*ULMAX**2*(1./(2.*LAMBDA**2) - EMAX/
+ (1.+2.*LAMBDA**2))
Y(2) = BEPS**2*ULMAX*(1. - LAMBDA**2*EMAX/
GO TO 4
WRITE(6,202)
READ(5,102) ICONT
IF (ICONT .EQ. 0) STOP
WRITE(6,203) Z, Y(1), Y(2), ULMAX
GO TO 11
K = RUNGE(2,Y,F,Z,HD)
IF (K.NE.1) WRITE (6,203) Z, Y(1), Y(2), ULMAX
WHENEVER K=1, COMPUTE DERIVATIVE VALUES...
GO TO 13
F(1) = BEPS**2*EMAX*G
F(2) = 2.*ALPHA*LAMBDA*BEPS*ULMAX*(1.-EMAX/
+ DEXP(DBLE(1./LAMBDA**2)))/
GO TO 11
IF Z EXCEEDS ZMAX, TERMINATES INTEGRATION...
IF (Z.LE.ZMAX) GO TO 1
STOP
CALCULATE ESTIMATES FOR XOLD(I), I=1,...,3.
XOLD(1)= DEXP(DBLE(-2.5565 + 0.5480*DLOG(Z)))/DSQRT(BE)
XOLD(2)= DEXP(DBLE(-3.3224 - 1.1307*DLOG(Z)))/DSQRT(BE)
XOLD(3)= DEXP(DBLE(-0.1821 - 253.6236*DLOG(Z) + 126.600*
+ DLOG(Z**2)))/
T1= XOLD(1)
T2= XOLD(2)
T3= XOLD(3)
CALL SIMNLE (ITMAX, IPRINT, N, EPS1, EPS2, XOLD, ICONV, ITER)
IF (ICONV.EQ.0) GO TO 2
WRITE(6,204) ITER, (XOLD(I), I=1,N)
BEPS = XOLD(1)
EMAX = XOLD(2)
ULMAX = XOLD(3)
UGMAX = ULMAX + 0.25
R1= BEPS * DSQRT(BE)
RD1= -(XOLD(1)**2*XOLD(3)**2*(1./(2.*LAMBDA**2) - Y(1))
+ XOLD(2)/(1. + 2.*LAMBDA**2)) - Y(1))
RD2= -(XOLD(1)**2*XOLD(3)*(1. - (LAMBDA**2*XOLD(2))/
+ (1. + LAMBDA**2)) - Y(2))
RD3= -(PI*XOLD(1)**2*XOLD(2)*(XOLD(3)/(1. + LAMBDA**2)
+ + DUX*F1) - Q*HA/(HA+H-Z))
WRITE(12,303) T1, T2, T3
WRITE(12,304) RD1, RD2, RD3
WRITE(12,305) Z, R1, EMAX, UGMAX
UPDATE RUNGE'S FUNCTIONS...
Y(1) = BEPS**2*ULMAX**2*(1./(2.*LAMBDA**2) - EMAX/
+ (1. + 2.*LAMBDA**2))
Y(2) = BEPS**2*ULMAX*(1. - LAMBDA**2*EMAX/
+ (1. + LAMBDA**2))
GO TO 4
100 FORMAT (9X, 3(F9.6,9X) / 9X, 3(F9.6,9X))
101 FORMAT (10X,I3,17X,I1,19X,I3 / 10X,E7.1,13X,E7.1)
102 FORMAT (5X,I2)
200 FORMAT (9X, 3(F9.6,9X) / 9X, 3(F9.6,9X))
201 FORMAT (10X,I3,17X,I1,19X,I3 / 10X,E7.1,13X,E7.1)
202 FORMAT (5X, 'TO STOP PROGRAM INPUT 0' / 8H0 ** / )
203 FORMAT (5X, 'Z, Y(1), Y(2), ULMAX :' / 10X,F10.5,5X,2F18.8 / + 2X, F18.8)
204 FORMAT ( 24H0SUCCESSFUL CONVERGENCE / 10HOITER = , I3 / + 3(2X,F9.6))
300 FORMAT (5X, 4H Q= , 4X, F9.6, 5H M3/S, 3X, 4H H= , 3X, F9.6, + 2H M / 5X, 8H ALPHA= , F9.6, 8X, 5H HD= , 2X, F9.6, 2H M / 5X. + 7H EPS1= , 1X, E9.2, 8X, 7H EPS2= , E9.2 / )
301 FORMAT (5X, 5H Z0= , 3X, F9.6, 2H M / 5X, 8H EMAX0= , F9.6, + 8X, 6H R10= , 1X, F9.6, 2H M. 5X, 9H UGMAX0= , F9.6, + 4H M/S / )
302 FORMAT (5X, 42H ALTERNATED ROWS OF ESTIMATED AND SOLUTION / + 5X, 35H PARAMETERS AS FUNCTION OF POSITION //)
303 FORMAT (19X, 3(5X,F9.6))
304 FORMAT (19X, 3(5X,F9.6))
305 FORMAT (5X, 4(5X,F9.6))
STOP
END

C

FUNCTION RUNGE(N,Y,F,X,HD)
C
.. THE FUNCTION RUNGE USES THE 4TH ORDER RUNGE-KUTTA METHOD ..
IMPLICIT REAL*8(A-H, O-Z)
REAL*8 Y, F, PHI, SAVEY
REAL*8 X, HD
INTEGER RUNGE
DIMENSION PHI(50), SAVEY(50), Y(N), F(N)
DATA M/0/
C
M = M + 1
GO TO (1.2,3.4.5, ) M
C
...... PASS 1 ......
1 RUNGE = 1
RETURN
C
...... PASS 2 ......
2 DO 22 J = 1, N
SAVEY(J) = Y(J)
PHI(J) = F(J)
Y(J) = SAVEY(J) + 0.5*HD*F(J)
CONTINUE
X = X + 0.5*HD
RUNGE = 1
RETURN
C
C .... PASS 3 ....
3 DO 33 J = 1, N
PHI(J) = PHI(J) + 2.0*F(J)
Y(J) = SAVEY(J) + 0.5*HD*F(J)
33 CONTINUE
RUNGE = 1
RETURN
C
C .... PASS 4 ....
4 DO 44 J = 1, N
PHI(J) = PHI(J) + 2.0*F(J)
Y(J) = SAVEY(J) + HD*F(J)
44 CONTINUE
X = X + 0.5*HD
RUNGE = 1
RETURN
C
C .... PASS 5 ....
5 DO 55 J = 1, N
Y(J) = SAVEY(J) + (PHI(J) + F(J))*HD/6.0
55 CONTINUE
M = 0
RUNGE = 0
RETURN
C
END
C
C SUBROUTINE SIMNLE(ITMAX,IPRINT,N,EPS1,EPS2,XOLD,ICONV,ITER)
C THIS PROGRAM SOLVES N SIMULTANEOUS NON-LINEAR EQUATIONS
C IN N UNKNOWNS BY THE NEWTON-RAPHSON ITERATIVE PROCEDURE.
C INITIAL GUESSES FOR VALUES OF THE UNKNOWNS ARE READ INTO
C XOLD(1) ... XOLD(N). THE PROGRAM FIRST CALLS ON THE
C CALCN TO COMPUTE THE ELEMENTS OF A, THE AUGMENTED MATRIX
C OF PARTIAL DERIVATIVES, THEN ON FUNCTION SIMUL TO SOLVE
C THE GENERATED SET OF LINEAR EQUATIONS FOR THE CHANGES IN
C THE SOLUTION VALUES XINC(1) ... XINC(N). DETER IS THE
C JACOBIAN COMPUTED BY SIMUL. THE SOLUTION ARE UPDATED
C AND THE PROCESS CONTINUED UNTIL ITER, THE NUMBER OF
C ITERATIONS, EXCEEDS ITMAX OR UNTIL THE CHANGE IN EACH
C OF THE N VARIABLES IS SMALLER IN MAGNITUDE THAN EPS2
C (ITCON = 1 UNDER THESE CONDITIONS). EPS1 IS THE MINIMUM
C PIVOT MAGNITUDE PERMITTED IN SIMUL. WHEN IPRINT = 1,
C INTERMEDIATE SOLUTION VALUES ARE PRINTED AFTER EACH
C ITERATION.
C
IMPLICIT INTEGER(I, J, K, L, M, N)
IMPLICIT REAL*8(A-H, O-Z)
REAL*8 XOLD, XINC, A
REAL*8 EPS2, EPS1, DETER
DIMENSION XOLD(21), XINC(21), A(21,21)
L= 0

..... NEWTON-RAPHSON ITERATION .....  
DO 9  ITER  = 1, ITMAX

...... CALL ON CALCN TO SET UP A MATRIX ..... 
CALL CALCN( XOLD, A, 21 )

.. CALL SIMUL TO COMPUTE JACOBIAN AND CORRECTIONS IN XINC ....
DETER = SIMUL(N, A, XINC, EPS1, 1, 21 )
L= L+1
IF (L.LT.5) GO TO 84
WRITE (6,*) (XINC(I), I=1,N)
WRITE (6,83)
83 FORMAT (' THIS ARE XINC(I) VALUES ')

84 IF ( DETER.NE.0. ) GO TO 3
WRITE (6,201)
ICONV = 0
RETURN

C
C ..... CHECK FOR CONVERGENCE AND UPDATE XOLD VALUES .....  
3 ITCON = 1
ICONV = 0
DO 5 I = 1  , N
IF (ABS(XINC(I)).GT.EPS2 ) ITCON = 0
XOLD(I) = XOLD(I) + XINC(I)
5 CONTINUE
IF (L.LT.5) GO TO 85
IF ( IPRINT.EQ.1 ) WRITE (6,202) ITER,DETER,(XOLD(I),I=1,N)
L= 0

85 IF ( ITCON.EQ.0 ) GO TO 9
ICONV = 1
WRITE (6,203) ITER,(XOLD(I),I=1,N)
RETURN

9 CONTINUE

C

WRITE (6,204)
RETURN

C
C ..... FORMATS FOR INPUT AND OUTPUT STATEMENTS ..... 
C
201 FORMAT (38H MATRIX IS ILL-CONDITIONED OR SINGULAR )
202 FORMAT (10HOITER = ,18/10H DETER = , E18.5 /
+ 4(2X, E15.8) / 3(2X, E15.8))
203 FORMAT (24H SUCCESSFUL CONVERGENCE / 10HOITER = ,I3 /
+ 4(2X, E10.4) / 3(2X, E10.4))
C
FUNCTION SIMUL( N, A, X, EPS, INDIC, NRC )
IMPLICIT REAL*8(A-H, O-Z)
IMPLICIT INTEGER(I, J, K, L, M, N)
REAL*8 Y, A, X
REAL*8 EPS, SIMUL, DABS, PIVOT, DETER, AIJCK
DIMENSION IROW(50), JCOL(50), JORD(50), Y(50), A(NRC,NRC)
DIMENSION X(N)
C
MAX = N
IF ( INDIC.GE.0 ) MAX = N + 1
C
..... IS N LARGER THAN 50 ..... 
IF ( N.LE.50 ) GO TO 5
WRITE (6,200)
SIMUL = 0.
RETURN
C
..... BEGIN ELIMINATION PROCEDURE ..... 
5 DETER = 1.
DO 18 K = 1, N
KM1 = K - 1
C
..... SEARCH FOR THE PIVOT ELEMENT ..... 
PIVOT = 0.
DO 11 I = 1, N
DO 11 J = 1, N
C
.. SCAN IROW AND JCOL ARRAYS FOR INVALID PIVOT SUBSCRIPTS ..
IF ( K.EQ.1 ) GO TO 9
DO 8 ISCAN = 1, KM1
DO 8 JSCAN = 1, KM1
IF ( I.EQ.IROW(ISCAN)) GO TO 11
IF ( J.EQ.JCOL(JSCAN)) GO TO 11
8 CONTINUE
9 IF ( DABS(A(I,J)).LE.DABS(PIVOT)) GO TO 11
PIVOT = A(I,J)
IROW(K) = I
JCOL(K) = J
11 CONTINUE
C
..... INSURE THAT SELECTED PIVOT IS LARGER THAN EPS ..... 
IF ( DABS(PIVOT).GT.EPS ) GO TO 13
SIMUL = 0.
RETURN
C
..... UPDATE THE DETERMINANT VALUE .....
13  IROWK = IROW(K)  
    JCOLK = JCOL(K)  
    DETER = DETER*PIVOT  
C  
C  ...... NORMALIZE PIVOT ROW ELEMENTS ......  
    DO 14  J = 1, MAX  
    A(IROWK,J) = A(IROWK,J)/PIVOT  
    CONTINUE  
C  
C  ...... CARRY OUT ELIMINATION AND DEVELOP INVERSE ......  
    A(IROWK,JCOLK) = 1./PIVOT  
    DO 18  I = 1, N  
      AIJCK = A(I,JCOLK)  
      IF ( I.EQ.IROWK ) GO TO 18  
      A(I,JCOLK) = -AIJCK/PIVOT  
    CONTINUE  
    J = 1, MAX  
    IF ( J.NE.JCOLK ) A(I,J) = A(I,J) - AIJCK*A(IROWK,J)  
17  CONTINUE  
18  CONTINUE  
C  
C  ...... ORDER SOLUTION VALUES (IF ANY) AND CREATE JORD ARRAY ......  
    DO 20  I = 1, N  
      IROWI= IROW(I)  
      JCOLI= JCOL(I)  
      JORD(IROWI) = JCOLI  
      IF ( INDIC.GE.0 ) X(JCOLI) = A(IROWI,MAX)  
20  CONTINUE  
C  
C  ...... ADJUST SIGN OF DETERMINANT ......  
    INTCH = 0  
    NM1 = N - 1  
    DO 22  I = 1, NM1  
      IP1 = I + 1  
      DO 2  2  J= I, N  
        IF ( JORD(J).GE.JORD(I)) GO TO 22  
      JTEMP = JORD(J)  
      JORD(J) = JORD(I)  
      JORD(I) = JTEMP  
      INTCH = INTCH + 1  
22  CONTINUE  
    IF ( INTCH/2*2.NE.INTCH ) DETER = -DETER  
C  
C  ...... IF INDIC IS POSITIVE RETURN WITH RESULTS ......  
    IF ( INDIC.LE.0 ) GO TO 26  
    SIMUL = DETER  
    RETURN  
C  
C  ...... IF INDIC IS NEGATIVE OR ZERO, UNSCRAMBLE THE INVERSE FIRST BY ROWS ....  
26  DO 28  J = 1, N
DO 27 I = 1, N
IROWI = IROW(I)
JCOLI = JCOL(I)
Y(JCOLI) = A(IROWI,J)
27 CONTINUE
DO 28 I = 1, N
A(I,J) = Y(I)
28 CONTINUE
C ..... THEN BY COLUMNS .....  
DO 30 I = 1, N
DO 29 J = 1, N
IROWJ = IROW(J)
JCOLJ = JCOL(J)
Y(IROWJ) = A(I,JCOLJ)
29 CONTINUE
DO 30 J = 1, N
A(I,J) = Y(J)
30 CONTINUE
C ..... RETURN FOR INDIC NEGATIVE OR ZERO .....  
SIMUL = DETER
RETURN
C ..... FORMAT FOR OUTPUT STATEMENT .....  
200 FORMAT(1OHON TOO BIG )
END
C
C SUBROUTINE CALCN( DXOLD, A, NRC )
IMPLICIT REAL*8(A-H, O-Z)
IMPLICIT INTEGER(I, J, K, L, M, N)
REAL*8 LAMBDA
REAL*8 Y
REAL*8 DUX, F1, Q, HA, H, Z, PI, G
DIMENSION XOLD(20), DXOLD(NRC), A(NRC,NRC), Y(2)
COMMON LAMBDA, DUX, F1, Q, HA, H, Z, PI, G, Y
C ..... SHIFT ELEMENTS OF DXOLD TO XOLD AND CLEAR A ARRAY .....  
DO 1 I = 1, 3
XOLD(I) = DXOLD(I)
DO 1 J = 1, 4
A(I,J) = 0.
1 CONTINUE
C ..... COMPUTE NON-ZERO ELEMENTS OF A .....  
A(1,1) = 2.*XOLD(3)**2*(1./(2.*LAMBDA**2) - XOLD(2)/
(1. + 2.*LAMBDA**2)) + XOLD(1)**2*XOLD(3)**2/( 1. + 2.*LAMBDA**2)
A(1,2) = - XOLD(1)**2*XOLD(3)**2/( 1. + 2.*LAMBDA**2)
A(1,3) = 2.*XOLD(1)**2*(1./(2.*LAMBDA**2) - 1. + 2.*LAMBDA**2)

C
A(1, 4) = -(XOLD(1)**2*XOLD(3)**2*(1./(2.*LAMBDA**2)) - XOLD(2)/(1. + 2.*LAMBDA**2)) - Y(1))
A(2, 1) = 2. * XOLD(3)*(1. - (LAMBDA**2*XOLD(2))/((1. + LAMBDA**2)*XOLD(1)))
A(2, 2) = - (XOLD(1)**2*XOLD(3)*LAMBDA**2)/(1. + LAMBDA**2)
A(2, 3) = XOLD(1)**2*(1. - (LAMBDA**2*XOLD(2))/((1. + LAMBDA**2)))
A(2, 4) = -(XOLD(1)**2*XOLD(3)*(1. - (LAMBDA**2*XOLD(2))/((1. + LAMBDA**2)) - Y(2))
A(3, 1) = 2*PI*XOLD(2)*(XOLD(3)/(1. + LAMBDA**2) + DUX*F1)
A(3, 2) = PI*XOLD(1)**2*(XOLD(3)/(1. + LAMBDA**2) + DUX*F1) + PI*XOLD(1)**2*XOLD(2)/(1. + LAMBDA**2)
A(3, 4) = -(PI*XOLD(1)**2*XOLD(2)*(XOLD(3)/(1. + LAMBDA**2) + DUX*F1) - Q*HA/(HA + H - Z))
RETURN
END