HEAT TRANSFER AND CRACK FORMATION IN WATER-COOLED ZINC FUMING FURNACE JACKETS

by

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We accept this thesis as conforming to the required standard

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Abstract

In the zinc slag fuming process, zinc is extracted from lead blast furnace slag by reduction with a coal/air mixture injected into the slag through submerged tuyeres. The furnace is constructed of water-cooled jackets to contain the molten bath and freeze a protective slag layer. The slag layer greatly reduces vessel wear caused by the corrosive and violently agitated bath. However, the jackets are known to develop cracks in the working face panel that initiate on the slag face and propagate towards the water cavity. If the cracks reach the water cavity explosions may result should the molten slag come into contact with the water.

In this study an analysis of heat transfer in the jacket has been carried out using in-plant measurements and mathematical modelling. The working face of a water jacket was instrumented with thermocouples and positioned in a fuming furnace at the Trail smelter of Cominco Ltd. Measurements revealed the presence of large thermal transients or temperature "spikes" in the panel approximately 20 cm above the tuyeres. The transients were observed during charging and tapping of the furnace and are likely associated with slag fall-off due to surface wave action and gas injection effects when the bath level is low. Temperatures at the mid-thickness were seen to rise by as much as 180°C above the steady-state level. Under these conditions large compressive stresses are produced in the panel that are sufficient to cause yielding. Over time, the transients lead to low-cycle fatigue of the working face panel with crack formation initiating at pre-existing surface flaws.

A mathematical modelling analysis of the transient freezing phenomena has been carried out using the finite element method. The results indicate that the temperature spikes are associated with the sudden removal of patches of slag and molten slag.
coming into direct contact with the jacket. The temperature spikes are large enough to
generate compressive stresses that cause yielding of the material in the exposed area. In
order to reduce the damage caused by the removal of the slag shell an increased
number of anchoring studs should be used in critical areas and a higher water
circulation velocity should be employed to increase the size of the frozen slag layer and
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Nomenclature

\{a\} \quad \text{displacement vector from virtual work theory}

A \quad \text{area of 2-dimensional finite element (m}^2\text{)}

[B] \quad \text{strain-displacement matrix}

Bi \quad \text{Biot number}

C_{pi} \quad \text{specific heat of species } i \ (\text{J/kg}-\text{°C})

C' \quad \text{enhanced specific heat (J/kg-°C)}

C_{pi} \quad \text{specific heat of liquid (water) (J/kg-°C)}

C_{sf} \quad \text{liquid/surface constant required for empirical boiling equation}

[C] \quad \text{capacitance matrix}

[D] \quad \text{elasticity matrix}

D \quad \text{the domain of an element}

E \quad \text{elastic modulus (MPa)}

Fo \quad \text{Fourier number}

g \quad \text{gravitational acceleration (m/s}^2\text{)}

h_b \quad \text{boiling heat transfer coefficient (W/m}^2\text{-°C)}

\bar{h}_c \quad \text{forced convection heat transfer coefficient calculated from Reynolds analogy (W/m}^2\text{-°C)}

h_{int} \quad \text{effective slag-steel interfacial heat transfer coefficient (W/m}^2\text{-°C)}

h_{int,l} \quad \text{liquid slag-steel interfacial heat transfer coefficient (W/m}^2\text{-°C)}

h_{int,s} \quad \text{solidified slag-steel interfacial heat transfer coefficient (W/m}^2\text{-°C)}

h_w \quad \text{water-steel heat transfer coefficient (W/m}^2\text{-°C)}

H_{fg} \quad \text{heat of vaporization of water (J/kg)}

H_s \quad \text{latent heat of solidification of slag (J/kg)}

[J] \quad \text{Jacobian matrix}

k_i \quad \text{thermal conductivity of species } i \ (\text{W/m}-\text{°C)}

[K] \quad \text{stiffness matrix}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[K_c]$</td>
<td>conduction component of the finite element stiffness matrix</td>
</tr>
<tr>
<td>$[K_v]$</td>
<td>convection component of the finite element stiffness matrix</td>
</tr>
<tr>
<td>$\bar{K}$</td>
<td>finite element stiffness matrix including time stepping algorithm</td>
</tr>
<tr>
<td>$L$</td>
<td>water flow length along the cold face of the panel (m)</td>
</tr>
<tr>
<td>$n_x, n_y$</td>
<td>normals to the edge of an element</td>
</tr>
<tr>
<td>$N_i$</td>
<td>finite element shape function</td>
</tr>
<tr>
<td>${P}$</td>
<td>finite element load vector (stress calculations)</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>$q_{bath}$</td>
<td>heat flux from the combustion in the bath (W/m$^2$)</td>
</tr>
<tr>
<td>$q_b$</td>
<td>heat flux from boiling (W/m$^2$)</td>
</tr>
<tr>
<td>$Re_L$</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>${R}$</td>
<td>finite element load vector (thermal calculations)</td>
</tr>
<tr>
<td>${R_T}$</td>
<td>isothermal boundary condition component of the finite element load vector (thermal calculations)</td>
</tr>
<tr>
<td>${R_q}$</td>
<td>heat flux boundary condition component of the finite element load vector (thermal calculations)</td>
</tr>
<tr>
<td>${R_h}$</td>
<td>convective boundary condition of the finite element load vector (thermal calculations)</td>
</tr>
<tr>
<td>${R}$</td>
<td>finite element load vector (thermal calculations) following time stepping algorithm</td>
</tr>
<tr>
<td>$t$</td>
<td>time (s)</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature (°C)</td>
</tr>
<tr>
<td>$T_1, T_2$</td>
<td>Analytical solution boundary condition temperatures (°C)</td>
</tr>
<tr>
<td>$T_{liq}$</td>
<td>liquidus temperature (°C)</td>
</tr>
<tr>
<td>$T_{mp}$</td>
<td>melting temperature (°C)</td>
</tr>
<tr>
<td>$T_o$</td>
<td>stress/strain free state temperature (°C)</td>
</tr>
<tr>
<td>$T_s$</td>
<td>steel temperature at the water-face (°C)</td>
</tr>
<tr>
<td>$T_{sat}$</td>
<td>water saturation temperature for boiling (°C)</td>
</tr>
<tr>
<td>$T_{set}$</td>
<td>set temperature where contraction of the slag shell has stopped (°C)</td>
</tr>
<tr>
<td>$T_{slag}$</td>
<td>slag temperature at the panel interface (°C)</td>
</tr>
</tbody>
</table>
\( T_{so} \)  
Temperature at the chill face used in the analytical solution of solidification (°C)

\( T_{sol} \)  
solidus temperature (°C)

\( T_{steel} \)  
steel temperature at the slag face (°C)

\( T_w \)  
water temperature (°C)

\( T_\omega \)  
ambient temperature (°C)

\( x \)  
coordinate (m)

\( u \)  
displacement in \( x \) coordinate (m)

\( v \)  
displacement in \( y \) coordinate (m)

\( W_i \)  
internal work (J)

\( y \)  
coordinate (m)

\( \alpha \)  
thermal expansion coefficient (1/°C)

\( \{\delta\} \)  
displacement vector (m)

\( \{\delta\} \)  
virtual displacement vector (m)

\( \{\epsilon\} \)  
virtual strain vector

\( \{\epsilon_T\} \)  
thermal strain vector

\( \varepsilon_{eff} \)  
effective strain

\( \eta \)  
local coordinate in the isoparametric finite element formulation

\( \Gamma \)  
the surface of an element

\( \lambda \)  
solidification parameter employed in the analytical solution

\( \phi \)  
field variable

\( \rho_i \)  
density of species \( i \) (kg/m\(^3\))

\( \rho_v \)  
density of vapour (kg/m\(^3\))

\( \rho_l \)  
density of liquid (water) (kg/m\(^3\))

\( \mu_i \)  
viscosity of liquid (water) (N-s/m\(^2\))

\( \nu \)  
Poisson ratio

\( \sigma \)  
surface tension of water/steam interface (N/m)

\( \sigma_{eff} \)  
effective stress (MPa)

\( \sigma_T \)  
thermal stress (MPa)
\( \sigma_x \): stress in x coordinate (MPa)
\( \sigma_y \): stress in y coordinate (MPa)
\( \sigma_z \): stress in z coordinate (MPa)
\( \sigma_{I,II} \): principal stresses (MPa)
\( \tau_{xy} \): shearing stress component (MPa)
\( \theta \): two-level time stepping algorithm factor
\( \xi \): local coordinate in the isoparametric finite element formulation

Subscripts and Superscripts

- \( b \): boiling
- \( (e) \): element
- \( i,j \): finite element node positions
- \( l \): liquid
- \( s \): solid
- \( t \): time
- \( t + \Delta t \): time at next time step
- \( T \): transposed matrix or vector
- \( v \): vapour
- \( w \): water
- \( x,y \): x and y coordinate
Acknowledgement

I would like to express my sincerest appreciation to Professors G.G. Richards and I.V. Samarasekera for their guidance and support during this project. I would like to thank them for the opportunities they have given me and for their patience. I am grateful for the interest and funding of this project by Cominco Ltd. and in particular I would like to thank Dr. G.W. Toop for the support he has given. I wish to thank Dr. S.L. Cockroft for his advice relating to the mathematical modelling. Finally, I would like to express my gratitude to my parents and Ms. Brenna Leong for their understanding, patience and support.
In a number of pyrometallurgical operations water cooling is employed to extend the service lives of components that come into contact with molten baths. An example in the non-ferrous industry where water cooling is utilized is the zinc slag fuming furnace where the walls and floor are water cooled. The service lives of these components depend on many factors including the temperatures the component is subjected to when it comes into contact with the melt, the corrosive nature of the melt and the intensity of erosion inherent in the process.

At Cominco’s smelter in Trail, British Columbia, lead blast furnace slag is charged into a rectangular furnace constructed of water-jacketed walls and a water-cooled floor. At operating temperatures in the range of 1150-1300°C, the molten slag is corrosive and, when combined with the intense mixing conditions that are present in the furnace, refractory linings have had unacceptably short service lives. In order to protect the jackets from the bath a thin layer of slag approximately 10 mm in thickness is frozen onto the jacket. The freezing of the slag shell is possible because of the heat extracted by the cooling water.

During operation of the zinc fuming furnaces Cominco personnel have observed that the fire face panel of the jacket develops extensive cracking. These cracks initiate on the slag face of the panel and with time propagate towards the water cavity. If the cracks are permitted to reach the water face there is a possibility that violent explosions may occur when the molten slag contacts the water. The economic costs associated with the failure of these components include the inspection and replacement of the jackets and the smelter also suffers economic penalties because of the furnace down-time and lost production.
This study which is a joint project between Cominco Ltd. and the Centre for Metallurgical Process Engineering is concerned with investigating the thermal history of a typical water jacket and to elucidate a mechanism for jacket failure.
Chapter 2. Zinc Slag Fuming

The ore that is mined for the treatment in Trail is a mixed lead-zinc sulphide. It is crushed and treated at the mine where the ore is separated into lead and zinc concentrates. The lead concentrate is sintered at the smelter and fed into blast furnaces producing lead bullion and slag. The slag primarily consists of gangue materials but it also contains up to 18% zinc making its removal an economically viable process. The molten slag is tapped out of the blast furnace into several nine tonne steel pots and transferred by crane to a storage area until the slag can be processed in an available fuming furnace.

The first of two slag fuming furnaces constructed by the Consolidated Mining and Smelting Company in Trail, British Columbia, was put into service in 1930. Murray\(^1\) described the initial construction of the furnace and the first few years of operation were reviewed by McNaughton\(^2\). More recently, Yurko\(^3\) provides an updated overview of the lead smelter operation indicating that the fuming furnaces have remained virtually unchanged for over fifty years.

The fuming furnaces are rectangular vessels that have water-cooled walls and a water-cooled floor (see Figure 2.1). Situated above the furnace is a waste heat boiler and a cooling flue that leads to a baghouse where the re-oxidized zinc fume is collected. The dimensions of the No. 1 furnace are 3 m by 7.3 m with a height of 3 m and the size of the No. 2 furnace is the same except that it is 7.9 m long. The floor of both furnaces consists of 3 m by 0.61 m wide cast iron jackets that have 25.4 mm diameter pipes cast into them.

The walls of the furnace are constructed of two tiers of water-cooled jackets with the lower tier of jackets failing during service. A schematic diagram of a lower tier water-
cooled jacket is presented in Figure 2.2. The jackets are fabricated from ASTM A516 Grade 70 pressure plate steel of 12.7 mm (1/2") thickness. The chemical composition of this steel is 0.31\%(max) C, 0.85-1.20\% Mn, 0.035\%(max) P, 0.04\%(max) S and 0.15-0.30\% Si, indicating a low degree of alloying. The lower tier jackets are 0.61 m in width and 1.83 m in height. The back wall of the jacket is vertical and the fire face of the jacket is sloped giving the jacket a depth of 0.32 m at the bottom and 0.10 m at the top. The sloped fire face presumably aids in supporting the frozen slag shell and also improves the circulation of the slag. To further anchor the shell onto the jacket, 12.7 mm sections of 20 mm diameter pipe are welded onto the slag face on 75 mm centres. The slag freezes around the studs and this additional support helps to resist the erosive effects of the turbulent bath. Also indicated in this figure are the locations of the thermocouples that were used to measure the heat flow out of the jackets. The setup and results of the experimental trials are discussed later in Chapter 5.

Each of the lower jackets are equipped with three tuyere openings located 0.18 m from the furnace floor. The injection system is of a double inlet design through which primary air containing coal and secondary air can be injected into the molten bath. Combustion of the coal provides thermal energy to the bath with the excess coal serving as a reducing agent for the zinc oxide.

Located above the jackets is a cylindrical tank that runs parallel to the jackets (see Figure 2.3). The water flows by gravity from this tank into the jackets and subsequently removes heat from the fire face panel of the jacket. The water recirculates back to the tank because of the convection driven flow of the warmer water and the hydrostatic head of the incoming water. The water temperature is measured by a thermocouple and regulated in the tank through the controlled addition of colder inlet water. It can be seen in Figure 2.2
that there are two ports located above the tuyere line where the water enters the jacket. The ports are angled in this manner to ensure that the plumbing will not interfere with the tuyere assembly. Water exits the jacket from two ports located near the top of the jacket and returns to the tank. The inlet temperature ranges from 40-60°C with a bulk temperature increase of 10-20°C upon exiting the jacket.

The feed for charging consists primarily of molten slag from the lead blast furnaces. The slag is charged into the furnace through a water cooled duct located at one end of the furnace. Depending on the production schedule of the blast furnace during the charging process, a pot may be taken from the blast furnace and charged directly into the fuming furnace. Usually, however, there is a delay before a particular pot is charged to an available furnace and slag may begin to freeze in a pot producing a "pot shell" which can be up to 25% of the total volume of slag. After a pot is emptied, the pot shell is broken up by slamming the pot against a block located in the crane aisle and charged into the furnace. A typical charge consists of about 50 tonnes and may require 5 pots of hot slag. If the size of the hot slag charge is too small it is augmented with granulated blast furnace slag that is kept in stock. The granulated slag is fed into the furnace from feed screws located above the waste heat boilers. It has been observed that if the charge becomes wet, fairly violent explosions can take place in the furnace and efforts are made to keep the charge dry.

The temperature profile during a typical fuming cycle is shown schematically in Figure 2.4. The fuming cycle is approximately 150-180 minutes in duration with charging of the hot slag requiring about 15-20 minutes and tapping approximately 10-15 minutes. During the initial stages of fuming a heating stage is conducted during which the coal rate is reduced to a level that is close to stoichiometric and this condition generates the maximum amount of heat. When the bath has reached approximately 1300°C, the coal rate is
increased to encourage greater coal entrainment into the bath and a higher rate of fuming. If the higher coal rate is maintained during the entire fuming cycle the slag temperature begins to cool as heat is absorbed by the endothermic reduction of the zinc oxide. With time the zinc oxide is expended and the rate of cooling of the slag temperature decreases. Towards the end of the fuming cycle the slag temperature has dropped significantly and it is necessary to again reduce the coal rate in order to increase the bath temperature. A higher slag bath temperature gives it a higher fluidity which makes it easier to tap out of the furnace.

At the end of the fuming cycle, slag is tapped out of the furnace through two holes 140 mm in diameter located at the opposite end of the furnace from the charge spout. The tap holes are set in a cast iron block 64 mm below the tuyere line. The slag is tapped into a receiver which splits the slag stream allowing for a spray of water to quickly cool and thus granulate the slag. When the furnace is emptied the holes are closed using clay dobbies and a water-cooled keeper in behind. Owing to the height of these holes, the slag level drops below the tuyere level, but a heel of slag remains into the next fuming cycle.
Figure 2.1: Schematic diagram of a zinc slag fuming furnace.
Figure 2.2: Schematic diagram of a water-cooled jacket showing (a) the rear view, (b) the side view, and (c) the approximate location of the six thermocouple groups on the slag face.
Figure 2.3: Schematic diagram of the water circuit used for cooling the water-jackets.
Figure 2.4: Slag temperature profile during a typical fuming cycle.
3.1 Jacket failure

The failure of the lower tier water-cooled fuming furnace jackets was a significant problem during the initial years of furnace operation at Trail, accounting for a majority of the furnace down time. Initially it was believed that the jackets would become worn due to the agitation of the slag and therefore a 12.7 mm (1/2") thick steel plate was used during fabrication. However, observations of a damaged jacket indicated that failure was due to embrittlement likely caused by the continued "reversal" of stresses on the hot face of the jacket. The mechanism that caused the generation of these stresses was not precisely known but the stresses were attributed to the physical and chemical action of the molten bath and the injected coal/air mixture. Usually observed in the same location directly above the tuyeres, the failures were identified by vertical cracks measuring 75 to 300 mm in length. Measures taken in an attempt to minimize the effect of the thermal stresses included constructing the jacket of 6.35 mm (1/4") plate and cross-ribbing the fire face of the jacket using strips of steel. The jacket was divided into two jackets resulting in the current dimensions and tests were carried out on copper jackets and jackets fabricated from a more heat-resistant steel. The thinner fire face plate resulted in only minor improvement in the life of the jacket and because there was some concern regarding its strength a 12.7 mm (1/2") thick plate is currently used. The jackets constructed from copper and heat-resisting steel were successful but they were too costly to justify full scale implementation.
3.1 Jacket failure

The water-jackets used in fuming furnaces at Hudson Bay Mining and Smelting Company’s smelter in Flin Flon, have been reported by Mast and Kent to also experience significant damage. At this smelter, copper converter slag is treated to recover zinc. The damage to the jackets was attributed to the removal of the protective slag layer during tapping of the furnace and the welding of studs onto the fire face helped to minimize this damage. The authors also noted that owing to a high silica concentration in the slag the bath has been known to become very viscous at low temperatures causing the furnace to vibrate.

Over the last twenty-five years, a trend in slag fuming has been to convert the water-jackets of comparable design to those in this study to jackets constructed of vertical tubes. Sundstrom discusses the operation of a tubular membrane furnace that was used for fuming zinc from an electric copper smelter slag. Transverse cracks located near the bottoms of the tubes were observed in the initial years of furnace operation and these were attributed to variations in length of the tubes and the changing volume of the bath as its temperature changed.

During the development of a tin fuming furnace at Capper Pass, England, Halsall observed the presence of cracks in the region above the tuyeres. The jackets had studs welded onto them and similar attempts using a thinner fire face panel proved to be only partially successful. Surface examination of the fire face panel indicated that the fire face surface reached temperatures in excess of 800 °C while the bath temperatures were approximately 1300 °C. He attributed failure of the jackets to thermal fatigue generated by the shear forces from the bath that remove the frozen shell and prevent a new shell
from adhering securely to the jacket. Halsall commented that extending the tuyeres into the bath did not work very well because they would eventually erode back to become flush with the jacket.

3.2 Heat Flow

Recent research has focussed on evaluating the amount of heat that flows from the slag bath into the jackets. Woodside has estimated that 44% of the heat lost from the furnace is conducted through the jackets while the majority of the remaining heat is carried away by the combustion products. It is meaningful to measure the rate of heat flow through the jackets since this rate determines the efficiency of the furnace and the thickness of the frozen slag shell that solidifies onto the jackets. In cases where the jackets are failing in service the heat flux through the jackets also helps to establish the thermal history of the wall material.

Kiselev et al discuss several technical factors that were considered in building a zinc slag fuming complex. They observed that the design of the jacket plays an important role in terms of the replacement costs associated with jacket failure and the economic viability of a furnace. The jackets that were used were constructed of tubes welded onto a protective fire face plate and secured with horizontal ribbing. This design was believed to be superior to the natural convection cooling design since in the latter case regions of the plate may not be sufficiently cooled. The authors suggested that the tube design ensured that the fire face plate would experience low temperature gradients and would therefore not deform during operation. Their experience was that the older jackets installed at the tuyere level lasted for only 3 months while the upper tier jackets lasted 1-2 years.
3.2 Heat Flow

The new jackets were initially cooled with recirculating water but conversion to evaporative-cooling allowed for the heat energy to be reused. Their calculations based on the increase in temperature of the steam estimated the heat flow through the jacket to be 98.9 kW/m². The new jackets in the tuyere row lasted for over two years while the upper tier jackets remained in operation for over four years.

Rafalovich describes how the heat flow through a jacket should be calculated when a refractory lining is used. If the cooling agent is unable to extract enough heat to encourage the solidification of a protective shell onto a refractory, the refractory would deteriorate due to both erosion and corrosion by the slag bath. The heat flow calculations assumed perfect contact existed at slag layer/refractory and refractory/steel interfaces. He used this analysis to predict the performance of a refractory/jacket system under different cooling conditions: air, water sprays, and internal water cooling. The tests were carried out on a three-electrode furnace used for "stripping" converter slags at approximately 1300°C. The heat flow through the water-cooled jackets was estimated to be 70 kW/m² based on the increase in water temperature and his calculations predicted that the refractory would remain intact. However, a breakdown of the refractory was observed indicating that steady-state conditions were not always present in the refractory and almost the entire refractory reached the temperature of the melt. He further commented on the importance of preventing physical damage to the lining during charging and avoiding thermal shocks to the refractory.

Evaporative cooling at a particular lead-zinc smelter has been examined by Tunitskii. At this smelter the conversion to evaporative-cooling from water-cooling was desirable because of the poor quality of the available water. It was observed during the operation of the furnace that the water-cooled jackets developed scale deposits due to
boiling of the cooling water. The deposits inhibit good heat transfer and reduce the cooling efficiency of the jacket. The author notes that the water-cooled jackets developed cracks on the fire face that radiated from the tuyere port. An examination of these cracks showed evidence of erosion on the fire face suggesting that the molten slag had come into contact with the jackets. It was noted that the cracks formed after approximately two years of operation using evaporative-cooling while the water-cooled jackets developed cracks after only six months. As well, there was a greater number of cracks near the charging end of the furnace and it was suspected that this was due to sudden fluctuations in temperature that occurred during charging. Further tests were carried out using copper jackets and these proved to be successful for over three years but their cost was inhibitive.

The use of solidified-shell linings in pyrometallurgical units has been examined in great detail by Ermakov\textsuperscript{14} and Ermakov and Osipov\textsuperscript{15}. Because of the erosive and corrosive nature of the melt, a solidified shell was preferred for protecting furnaces that are periodically filled with a high temperature melt and where the bath is in vigorous motion. Through the use of forced cooling a shell can be formed on the jacket walls but the authors stated that the adhesion between the shell and the jacket may be weak if the shell solidifies on a film of oxides which form when the jacket surface begins to corrode. Ermakov and Osipov postulated that a rammed refractory of chrome-magnesite that contained cooling tubes and supporting fins may adequately cool the refractory and thus maintain its integrity. In their tests on this particular jacket design which employed steam cooling heat fluxes of 140-174 kW/m\textsuperscript{2} were measured. They reported that these fluxes were approximately 2.5 times larger than those obtained for similar baths when still. However, during tapping the jacket lost its protective slag shell and the heat flux rose to 630 kW/m\textsuperscript{2}.
3.3 Mixing Conditions

A thorough study on the relationship between the type of molten bath and the rate of heat flow through the furnace walls was carried out by Grechko. In his study, he analyzed eleven different industrial pyrometallurgical installations processing a wide variety of melts under a range of blowing conditions and cooling agents. One of the units studied was a slag fuming furnace with a water-cooled firebox design. He found that the heat flow generally increased 6-8 times when going from a still bath to a well mixed bath and as the temperature of the bath increased, the heat flux also increased. From these findings he grouped the two types of mixing conditions and produced the following empirical correlations:

\[ q_{\text{mixing}} = 3.96 \times 10^{-0.24} T_b^{9.26} \text{ W/m}^2 \text{°C} \]  \hspace{1cm} (3.1a)

\[ q_{\text{quiet}} = 2.99 \times 10^{-33} T_b^{18.32} \text{ W/m}^2 \text{°C} \]  \hspace{1cm} (3.1b)

The slag fuming process was considered to be a well mixed process and using a typical slag temperature of 1250°C a heat flux of 188 kW/m²°C can be calculated. It will be shown in Chapter 5 that the heat flux measured in Trail is approximately half this value.

3.3 Mixing Conditions

The degree of mixing in the molten slag bath is an important factor in characterizing the amount of heat transferred to the water-jackets. A highly mixed bath transports thermal energy from the bath to the jacket at a much greater rate than if the bath were still. Also, it has been suspected by Cominco personnel that the mixing action can be so severe that it contributes to the failure of the jackets.
3.3 Mixing Conditions

In a study by Intykbaev et al the ability to intensify the fuming process through temperature control was investigated.\textsuperscript{17} Thermocouples with protective refractory tips were used since chrome-alumel thermocouples would burn out at higher bath temperatures. During periods when solid slag is charged into the furnace it was observed that pieces of slag would sometimes destroy the thermocouple tips. The authors carried out tests using a water-cooled U-tube to measure the bath temperature but the pieces of slag destroyed the U-tube as well. These observations suggest that it is possible that during charging pieces of solid slag may be directed into the jackets and thus break away the protective slag shell. If the slag shell is removed the fire face panel would experience a rapid rate of heating.

Richards et al examined the kinetics of fuming zinc-bearing lead blast furnace slag.\textsuperscript{4} They concluded that the furnace can be divided into two zones (see Figure 3.1): a reduction zone and an oxidation zone. The reduction zone is located in the bulk of the bath where zinc oxide is reduced by entrained coal from the injected coal/air mixture. The oxidation zone is located next to the jackets and in this zone the coal/air mixture combusts supplying thermal energy to the bath. However, owing to this zone's close proximity to the jackets, a significant amount of thermal energy is lost directly through the jackets. Bustos et al observed from tuyere back pressure measurements carried out on a fuming furnace that the bubbles formed at the tuyeres do not coalesce.\textsuperscript{18} As a result the oxidation zone would be well mixed increasing the rate of heat transfer through the jackets. The recirculation velocity of the slag is not precisely known but values of 1 to 3 m/s are proposed by Richards et al who used these values in their studies on the kinetics of the zinc fuming process.
3.3 Mixing Conditions

Figure 3.1: Mixing conditions in a fuming furnace.
Chapter 4. Objectives and Scope

The information provided in the literature gives evidence that failure of slag fuming furnace water-cooled jackets is a common industrial problem. Failure is known to occur in the region above the tuyeres in the form of cracks that initiate on the fire face of the jacket and propagate into the jacket with time. In the literature several authors have suggested possible mechanisms for the formation of the cracks but a conclusive analysis has yet to be presented. This thesis is concerned with an investigation into the thermal history of the water-cooled jackets with the objective of elucidating the mechanisms leading to the formation of the cracks and developing measures to minimize their occurrence.

The study consists of two parts. First, a typical jacket had been instrumented with thermocouples and placed into service. The data that was obtained was used to establish typical steady-state fuming behaviour including the rate of heat flow into the jackets. During the plant trials furnace personnel maintained an operating log so that the effect of different process variables could be examined. The data indicates that the heat flow through the jacket would significantly deviate from steady-state behaviour and it has been possible to suggest mechanisms for the failure of the jackets. In an attempt to support these observations a mathematical modelling analysis was employed in an attempt to simulate the measured thermal history of the jackets. The model incorporated thermal stress calculations in order to determine the magnitude of the thermal stresses produced and whether they would lead to failure of the jackets. Based on an understanding of the fuming process and the modelling results, recommendations can be made for improving the service life of the jackets.
5.1 Setup of Plant Trials

Chapter 5. Experimental

5.1 Setup of Plant Trials

The experimental work was concerned with measuring temperatures in the fire face panel of a fuming furnace jacket in order to characterize the thermal history of the jacket while in service. Since the panel was known to develop cracks in the vicinity of the tuyeres particular attention was paid to this area. In 1984, Cominco personnel instrumented a lower tier jacket with 34 chromel-alumel thermocouples under the direction of Dr. Samarasekera and Dr. Richards at the University of British Columbia. The thermocouples were divided into six groups and placed in different locations as illustrated by Figure 2.3c. They were positioned 25.4 mm (1") apart and at alternating depths of 3.175 mm (1/8") and 6.35 mm (1/4") from the water face. These depths represent the quarter depth and the half depth in the 12.7 mm thick fire face panel.

The thermocouples were positioned at alternating depths in order to facilitate the calculation of heat fluxes based on the thermal gradient between two successive thermocouples assuming one-dimensional heat flow. Studies that involve measuring the heat flow through the jacket based on the increase in cooling water temperature are not capable of locating regions where the heat flux becomes extreme or where conditions deviate from steady-state. The method used in this study enables the detection of these changes and also gives an indication of how the heat flux varies along the height of the jacket.

In order to measure the increase in water temperature, a thermocouple was installed at one of the water inlet ports and another at one of the water outlet ports. A third ther-
5.1 Setup of Plant Trials

mocouple recorded the molten slag temperature and was inserted into the furnace through a tuyere on a nearby jacket. All 37 thermocouples were connected to a Fluke 2280A data logger and readings were taken every 5 seconds. During the period from Sept. 28 to Oct. 4, 1984, 12 trial runs each approximately 70 minutes in length were performed. At the University of British Columbia, the data in the form of millivolt readings was transferred from the data logger to an AST 286 personal computer and converted to temperatures.

After the plant trials were completed the jacket was taken out of service and the back plate was removed. Holes were drilled through the stud centres to the water face in order to determine the locations of the thermocouples with respect to the studs. From this procedure it was determined that thermocouple Group F in Figure 2.32b was positioned about 23 mm above a horizontal row of studs. The studs themselves were positioned on 75 mm centres. The remaining groups of thermocouples were found to lie closer to the midline between adjacent rows of studs.
5.2 Results

The thermocouple measurements obtained during the plant trials revealed a significant amount of information on the relationship between the slag fuming process and the flow of heat through a typical water jacket. The runs were performed continuously in order to examine the effect of different process variables during entire fuming cycles. Unfortunately, the thermocouples that were installed in the jacket had a fairly high failure rate as indicated in Table 5.1. In total, ten of the thirty-five thermocouples in the jacket ultimately failed and because these thermocouples measured temperatures comparable to the water temperature it is possible that they were not adequately sealed from the water. The twelve runs that were successfully recorded are presented in Table 5.2 with a brief classification defining the runs as either steady-state in nature or as containing thermal transients. These thermal transients will be discussed in Section 5.2.2, but it is first important to develop a good understanding of the heat flow through the jacket under "quasi" steady-state conditions.
## 5.2 Results

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<td></td>
<td></td>
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</tr>
<tr>
<td></td>
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<td>6.35</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>35</td>
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<tr>
<td></td>
<td>36</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>37</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1: Thermocouple depth and location with respect to the groups shown in Figure 2.3c.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Length (min)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct 1a</td>
<td>69</td>
<td>Steady-state with a transient near the end.</td>
</tr>
<tr>
<td>Oct 3a</td>
<td>70</td>
<td>Steady-state.</td>
</tr>
<tr>
<td>Oct 3g</td>
<td>69</td>
<td>Thermal transients.</td>
</tr>
<tr>
<td>Oct 4a</td>
<td>66</td>
<td>Thermal transients, thermocouple #9 failed.</td>
</tr>
<tr>
<td>Oct 4b</td>
<td>66</td>
<td>Thermal transient at the start.</td>
</tr>
<tr>
<td>Oct 4c</td>
<td>69</td>
<td>Thermal transients.</td>
</tr>
<tr>
<td>Oct 4e</td>
<td>64</td>
<td>Steady-state.</td>
</tr>
<tr>
<td>Oct 4f</td>
<td>69</td>
<td>Thermal transient at the end.</td>
</tr>
<tr>
<td>Oct 5a</td>
<td>68</td>
<td>Thermal transient at the start.</td>
</tr>
<tr>
<td>Oct 5b</td>
<td>66</td>
<td>Steady-state.</td>
</tr>
<tr>
<td>Oct 5c</td>
<td>69</td>
<td>Thermal transients.</td>
</tr>
<tr>
<td>Oct 5d</td>
<td>68</td>
<td>Steady-state, slag thermocouple burned out.</td>
</tr>
</tbody>
</table>

Table 5.2: Summary of plant trial runs successfully completed.
5.2 Results

5.2.1 Quasi Steady-State

The slag fuming process is a batch process. During the fuming cycle the zinc concentration is reduced from about 18% to under 2% and, owing to the endothermic nature of zinc oxide reduction, the slag temperature generally is expected to decrease. In a reduction period of about two hours the bath temperature may decrease by up to 100°C. This rate of temperature change is relatively small and hence during this period the panel is under quasi steady-state. Fluctuations in other fuming parameters such as water inlet temperature, water flow rate, coal combustion and tuyere bubble formation can have a greater effect on the rate of heat flow through the jacket. However, it is not believed that these fluctuations on their own are large enough to cause the damage that has been observed in the water jackets.

Of the runs completed, there are five runs that can be characterized as quasi steady-state and owing to their similarity three typical runs are presented here. These runs are Oct 3a, Oct 4e, and Oct 5b and the operating data for these runs is presented in Table 5.3. In this table it can be seen that the coal rate was recorded as percentages by the furnace operators. The conversion to kg/s is included in Table 5.4 but should only be considered as an estimate.

Oct 3a

The operating data in Table 5.3 indicates that the only change in furnace operation during this run was an increase in the coal rate from 60 to 70%, 15 minutes from the start of the run. Ten minutes prior to this run the furnace was charged with 51 tons of hot slag. The initial slag temperature recorded for this run was 1302°C as shown in Figure 5.1a and
with such a high temperature it is likely that a large portion of the charge came directly from the blast furnaces. Near the 15th minute the heating stage was completed and the coal rate was increased to enhance fuming. In the figure the slag temperature can be seen to decrease during the entire run to approximately 1215°C at the end. A higher coal rate is used during the fuming cycle to encourage the reduction of the zinc oxide and this reaction absorbs heat from the bath resulting in the decreasing slag temperature. As the reduction rate of zinc oxide declines during the run, the slag temperature can be seen to decrease at a slower rate.

The water temperature profiles for run Oct 3a are also presented in Figure 5.1a. From the figure it can be seen that the inlet water temperature goes through a 1.5°C oscillation about every seven minutes. This oscillation is produced by the thermosyphon vapourizer which regulates the water inlet temperature. The effects of this cyclic behaviour are dampened in the water outlet temperature profile because the water had become heated and well mixed as it flowed through the jacket. The increase in water temperature for this run was about 15-17°C and it is this value that is commonly used to calculate the heat flow into the jacket from the bath. Towards the end of the run the water inlet and outlet temperatures decreased in a similar manner to the decrease in slag bath temperature.

Figures 5.1b-5.1i contain the panel temperatures recorded during run Oct 3a. In Figure 5.1b the Group A thermocouples 2 and 4 can be seen to have similar profiles which is expected since they are positioned at the same depth from the water face (3.175 mm). For a given run such as this there were small differences in temperature among thermocouples at the same depth within the same thermocouple group. However, between runs the magnitude of this difference also varies and may be due to a combination of three factors. First, there is some error associated with the exact depth of each thermocouple
well. For the case of Group F which was located directly behind a row of studs, the studs could act as fins funnelling heat into the jacket and locally producing higher temperatures in the panel. Finally, because of the constant interaction with the bath, the frozen slag layer thickness and porosity across the surface of the jacket could also change. Because the slag layer acts as a refractory, small variations in the slag layer can lead to significant differences in the measured temperature response. The variation in panel temperatures will become more apparent when the remaining steady-state runs are presented.

It is evident from the temperature profiles of Group A and the remaining profiles that the water surface temperature of the fire face panel did not exceed 100°C during the entire run. Scale deposits have been observed on the water face but this data indicates that under normal operating conditions boiling does not occur. The temperature profiles in the Group B thermocouples are shown in Figure 5.1c and they were similar to Group A except that the relative differences in temperature from the 3.175 mm depth to the 6.35 mm depth were about 4-5°C and 7-10°C respectively. The remaining thermocouples were positioned lower in the furnace and the responses contain more erratic fluctuations with decreasing height. For example the fluctuation in the response of thermocouple 24 is about 2°C and this behaviour may indicate some degree of interaction with the injected coal/air mixture.

If heat flow is assumed to be one-dimensional through the fire face panel, heat fluxes can be calculated for those pairs of thermocouples that were not in the vicinity of studs. The thermocouple pairs that met this criteria were 11-12, 19-20, 21-22, 22-23, and 23-24. Heat fluxes calculated from these thermocouple pairs are presented in Figure 5.2. Since
the heat flux is calculated from the difference in measured temperatures the fluctuations in the thermocouple response are often magnified in the calculated heat flux profile. To reduce this effect the calculated heat fluxes are averaged over twenty seconds.

For this run the heat flux is greatest in the thermocouple pair 19-20 with an initial value of approximately 140 kW/m²-°C decreasing to about 120 kW/m²-°C in response to the decreasing slag temperature. These values are in good agreement with those seen in the literature as mentioned previously. The heat flux calculated from the lower thermocouple pairs can also be seen to decrease with time. This suggests that there is a relationship between the slag temperature and the rate of heat flow out of the jacket. This observation has been recognized in the literature and will be discussed in greater detail in Section 5.3.

For this run the heat flux is greatest in the thermocouple pair 19-20 with an initial value of approximately 140 kW/m²-°C decreasing to about 120 kW/m²-°C in response to the decreasing slag temperature. These values are in good agreement with those seen in the literature as mentioned previously. The heat flux calculated from the lower thermocouple pairs can also be seen to decrease with time. This suggests that there is a relationship between the slag temperature and the rate of heat flow out of the jacket. This observation has been recognized in the literature and will be discussed in greater detail in Section 5.3.

Oct 4e

The runs performed on October 4, 1984, were the most successful during the plant trials in terms of operation of the equipment. It is apparent from Table 5.3 that changes were not made to the operation of the furnace during the run. The furnace was charged prior to this run although an exact time and the size of the charge was not noted in the operating log. In Figure 5.3a it can be seen that the charged slag was much colder (1235-1240°C) than in run Oct 3a. The colder charge required that a lower coal rate of 40% had to be maintained throughout the entire fuming stage.

The lower panel temperatures observed in this run were more sensitive to the oscillatory nature of the inlet water temperature than in the previous run. Because the thermocouple responses were similar to the previous run, only the thermocouple profiles of Groups D and E are presented in Figure 5.3b-d and in order to compare this run to the other two steady-state runs the initial and final measured temperatures for all of the
thermocouples are presented in Table 5.5. From this table it is interesting to again note that some of the thermocouples that are at the same depth not only record different temperatures with respect to each other but this relative difference has changed from run to run.

Generally, the temperatures are lower during this run than for the previous run and there is a smaller difference in temperature within the thermocouple groups. The heat flux through the jacket is presented in Figure 5.4 and it can be seen to be lower than in the previous run with values of about 85-105 kW/m²°C for the middle pair of thermocouples (19-20). It is apparent from the figure that the heat flux profiles follow the slag temperature profile, starting at a high value and levelling off at a steady value when the slag temperature also levels out.

Oct 5b

An interesting feature of this run is that it reports some of the lowest slag bath temperatures observed during the plant trials. As in run Oct 4e, Table 5.3 shows that changes were not made in the furnace operating parameters during this run. Initially, the bath temperature was about 1210°C and decreased to approximately 1165°C near the end of the run (see Figure 5.5a). As shown in Table 5.5 the relative difference between thermocouples at different depths had increased as compared with the run Oct 4e even though the bath is colder. This implies that a greater fraction of heat generated during combustion flows through the jackets instead of heating the bath. The response of the thermocouples in Group E presented in Figure 5.5c-d can be seen to be much more erratic. As the molten bath becomes colder and approaches the liquidus temperature of the slag, the viscosity of
5.2 Results

the bath would increase. A higher viscosity would in turn reduce the extent of penetration of the injected gas stream. This would likely result in more violent bubbling in closer proximity to the jackets and produce the erratic behaviour seen in the measured response.

The heat flux profiles for this run are presented in Figure 5.6 and they reflect the erratic nature of the thermocouple responses. Since the relative difference in temperature between thermocouples had increased the heat flux through the jacket also had increased. From the thermocouples in Group E the heat flux varies from 120-150 kW/m\(^2\)-°C at the start of the run to 100-125 kW/m\(^2\)-°C towards the end. Again, it can be noted that the heat flux profile follows the slag temperature as it decreases during the run.

A number of points can be made in summary. For each of these runs there was a general correlation between slag temperature and heat flux through the jacket. For a given run as the slag temperature decreased the heat flux through the jacket correspondingly decreased. However, as run Oct 5b demonstrated, a bath with a much lower slag temperature produced higher heat fluxes through the jacket than a run which had hotter slag, Oct 4e. This phenomena will be discussed in greater detail following an analysis of the thermal transients. The heat fluxes through the jacket are of the order of 80-120 kW/m\(^2\)-°C which are similar to those observed in the literature.\(^{16}\) It can also be seen from these measurements that the panel temperatures at the water face remain below 100°C during each of the runs and therefore boiling does not normally occur in the jacket. This is an important point as shown in the literature\(^{13}\) since boiling leads to the formation of scale deposits which have also been observed in the fuming furnace jackets at Trail.\(^{19}\)
### Table 5.3: Operating data for the steady-state runs

<table>
<thead>
<tr>
<th>Time from start of run (min)</th>
<th>Run Oct 3a</th>
<th>Coal rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Start</td>
<td>60</td>
</tr>
<tr>
<td>15</td>
<td>Raised coal 10%</td>
<td>70</td>
</tr>
<tr>
<td>72</td>
<td>End</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run Oct 4e</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Start. Coal increased 10%</td>
<td>40</td>
</tr>
<tr>
<td>69</td>
<td>End</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run Oct 5b</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Start</td>
<td>65</td>
</tr>
<tr>
<td>69</td>
<td>End</td>
<td></td>
</tr>
</tbody>
</table>

### Table 5.4: Coal rate conversion table as calculated by Cominco personnel during the plant trials.

<table>
<thead>
<tr>
<th>Recorded Percentage (%)</th>
<th>Approximate Coal rate (kg/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.40</td>
</tr>
<tr>
<td>45</td>
<td>0.46</td>
</tr>
<tr>
<td>50</td>
<td>0.54</td>
</tr>
<tr>
<td>55</td>
<td>0.64</td>
</tr>
<tr>
<td>60</td>
<td>0.82</td>
</tr>
<tr>
<td>70</td>
<td>0.97</td>
</tr>
</tbody>
</table>
Table 5.5: Initial and final temperatures for each thermocouple during the steady-state runs.
5.2 Results

Figure 5.1: Thermocouple temperature profiles during run Oct 3a: (a) water and slag temperatures and (b) Group A.
Figure 5.1: Thermocouple temperature profiles during run Oct 3a: (c) Group B and (d) Group C.
5.2 Results

Figure 5.1: Thermocouple temperature profiles during run Oct 3a: (e) Group D and (f) Group E.
Figure 5.1: Thermocouple temperature profiles during run Oct 3a: (g) Group E and (h) Group F.
Figure 5.1: Thermocouple temperature profiles during run Oct 3a: (i) Group F.
5.2 Results

Figure 5.2: Heat flux profiles during run Oct 3a: (a) Group B and D and (b) Group E. Numbers refer to thermocouple pairs which were used to calculate the heat flux.
Figure 5.3: Thermocouple temperature profiles during run Oct 4e: (a) water and slag temperatures and (b) Group D.
5.2 Results

Figure 5.3: Thermocouple temperature profiles during run Oct 4e: (c) and (d) Group E.
Figure 5.4: Heat flux profiles during run Oct 4e: (a) Group B and D and (b) Group E. Numbers refer to thermocouple pairs which were used to calculate the heat flux.
Figure 5.5: Thermocouple temperature profiles during run Oct 5b: (a) water and slag temperatures and (b) Group D.
Figure 5.5: Thermocouple temperature profiles during run Oct 5b: (c) and (d) Group E.
Figure 5.6: Heat flux profiles during run Oct 5b: (a) Group B and D and (b) Group E. Numbers refer to thermocouple pairs which were used to calculate the heat flux.
5.2 Results

5.2.2 Thermal Transients

Many of the runs summarized in Table 5.2 are identified as having "thermal transients". The steady-state runs that were analyzed in the previous section contain small fluctuations of under 2°C in the measured panel temperatures and these fluctuations are fairly regular. The fluctuations that are presented in this section are much greater reaching temperatures in excess of 200°C in the mid-plane thermocouples and they do not occur in the same regular manner. The transients only occur in the Group E thermocouples which are located approximately 20 cm above the centre tuyere. Owing to their size, it is believed that these transients are responsible for the failure of the jackets and it is therefore important to establish how they are generated.

The operating data for the runs in which thermal transient were observed are included in Table 5.6. A summary of the thermal transients that were observed is given in Table 5.7 and it is evident from this table that the thermal transients are associated with charging and tapping of the furnace as well as changes in the coal rate. This relationship is discussed in Section 5.2.3 following the presentation of the thermal transients.

The first run presented in this section is Oct 4c. Previously, run Oct 3a was shown as a typical quasi steady-state case and all of the thermocouple profiles recorded during this run were included. Owing to limitations in space only the thermal transient run of Oct 4c is presented in this detail. This run was selected because it offers a look into the thermal history of the jacket on a separate day than the steady-state run. Several thermal transients were recorded on Oct. 4, 1984, and the thermal transients observed in this run
are less complicated than those in the other runs and thus are more easily described. For the remaining runs the discussion will be primarily restricted to the temperature profiles of Group E since the thermal transients only occur in this group.

Oct 4c

This run contains an example of the charging and heating stages that begin a typical fuming cycle. The slag bath temperature profile is presented in Figure 5.7a and it can be seen that the bath heats up to about 1280 °C and then drops to 1260 °C following the charging of a pot of slag. The effect of charging subsequent pots can be seen to lead to further drops in the slag temperature profile. The final pot was charged at about the 27th minute but its effect on the bath temperature was small because the furnace was nearly full. At a coal rate of 30% the slag can be seen to cool initially from about 1255 °C to about 1230 °C and then starts to heat up. Although the slag temperature was increasing near the end of the run the water temperature gradually decreases throughout the run. This indicates that as the slag temperature was increasing the heat flux through the jacket was decreasing.

The temperature profiles in the panel are presented in Figures 5.7b-i and in particular the thermocouples in Group E are given in Figures 5.7f and g. It can be immediately seen that the temperature profiles in this group of thermocouples exhibit some of the thermal transients that are believed to initiate damage to the jackets. Thermocouple 23, which is located at the midplane of the fire face panel, experiences three larger transients during charging of the furnace that reach peak temperatures of 235 °C, 215 °C and 261 °C respectively. The profiles of thermocouple 21, 22 and 24 also experience thermal transients at the same time although the magnitudes of these spikes are somewhat smaller. The peak temperature of thermocouple 21 during the largest transient was approximately 200 °C.
while thermocouple 22 and 24 reach about 167°C and 173°C respectively. Since thermocouple 21 is at the midplane of the panel its behaviour would be expected to be much closer to thermocouple 23 but clearly it was significantly colder. The spike of the largest transient lasts for about one minute indicating that a significant amount of time passes before the panel returns to steady-state. It will be shown in the Summary (Section 5.2.3) that this time interval, or plateau, is an important component of the mechanism for failure that is proposed in this study.

Further up the jacket in Group D (Figure 5.7e) the response of thermocouples 16, 19 and 20 experiences an inverse thermal transient of about 3°C during the largest spike. Other than smaller inverse thermal spikes seen in thermocouples 2, 4 and 5 and elsewhere there is little indication that other regions of the jacket had endured the thermal transients seen in the region of thermocouple Group E. In thermocouple Group F, which is the group closest to Group E, only thermocouple 28 showed any indication of abnormal behaviour although not to the extent seen in Group E.

During the thermal transients the surface temperature of the panel behind thermocouple Group E likely becomes hot enough to induce nucleate boiling. This agitates the cooling water in close proximity to the panel enhancing heat transfer in regions higher up the jacket. The thermocouples in Group D would then experience a cooling trend, or inverse thermal transients, as shown in the temperature profiles of thermocouples 16, 19 and 20. It is important to note that the water outlet temperature does not show any evidence of the thermal transients. It is likely design criteria for the jackets based on this parameter would overlook the thermal transients which are likely causing failure of the jackets.
5.2 Results

In Figure 5.8 the heat flux that is calculated from the larger thermal transients is shown to be between 1.5 MW/m$^2$-°C and 1.7 MW/m$^2$-°C for the thermocouple pairings of 23-22 and 23-24. However, the thermocouple pair 21-22 produces a heat flux profile that is inverted and negative during the thermal transients. This behaviour indicates that thermocouple 21 does not experience the same rate of heating as do the lower thermocouples. Since thermocouple 26 also does not show evidence of the transient it suggests that the extent of this heating phenomena is approximately the distance between thermocouples 21 and 24 or 76 mm (3").

Oct 3g

The slag bath thermocouple from this run depicts the last 30 minutes of a fuming cycle followed by charging and tapping (see Figure 5.9a). During the first 25 minutes the slag bath was cooling at a decreasing rate until about the 30th minute when the coal was cut back from 70% to 60%. The coal rates used on this day are much higher than the coal rates used during the runs on Oct. 4, 1984. Tapping was started at about the 40th minute and during tapping the slag temperature can be seen to rise to approximately 1275°C. Charging was started near the 49th minute and with each successive charge it can be seen that the bath temperature progressively became colder. In Figures 5.9c and d, four large thermal transients were recorded in thermocouple group E at the 17th, 31st, 42nd and 59th minutes. At these times thermocouple 23 reached peak temperatures of 220°C, 218°C, 215°C and 218°C which are virtually identical. Because of this similarity it is likely that the event leading to the generation of the thermal transients occurs during each of the spikes. For each thermal transient there was a plateau in the thermocouple response with the longest plateau of 30 seconds occurring in the third large spike.
5.2 Results

In this run thermal transients occurred much more frequently than in the previous run and the remaining runs. The first two transients were associated with changes in the coal rate while the remaining two were related to tapping and charging respectively. The temperature profiles for the thermocouples in Group D, Figure 5.9b, exhibit the inverse transients that were observed previously.

Since the thermal transients in this run were similar with respect to each other the heat flux through the jacket would also be expected to be similar for each transient. The heat flux profiles are presented in Figure 5.10 with the largest heat flux being obtained from the thermocouple pairs of 23-22 and 23-24. The peak heat flux is 1.6 MW/m\(^2\)-\(^\circ\)C during the first two transients and about 1.5 MW/m\(^2\)-\(^\circ\)C during the fourth transient. As before, the thermocouple pair of 21-22 leads to a negative heat flux during the transients except for the fourth transient where a positive heat flux of approximately 650 kW/m\(^2\)-\(^\circ\)C is calculated. This suggests that the event leading to the generation of the transients tends to slightly vary in position on the jacket.

Oct 4a

This run shows a transition between the heating stage that follows charging and the onset of the fuming stage. Approximately 15 minutes into the run the coal rate was increased from 30% to 40% to encourage a higher fuming rate and its effect on the slag temperature can be seen in Figure 5.11a. Initially the slag temperature increased from 1195 to 1218\(^\circ\)C during the first stage and then decreases to about 1190\(^\circ\)C towards the end of the run. The associated water temperature profiles do not show the same behaviour as both decreased gradually throughout the run. In Figures 5.11c and d, a thermal transient is shown to have occurred at about the 12th minute. In this case the greatest temperature
was found in the temperature profile of thermocouple 21 which reaches about 187°C. The fact that the peak temperature of thermocouple 21 was much higher than the peak temperature of thermocouple 23 suggests that the event leading to the formation of the thermal transients contains a random element in terms of exact location above the tuyere. Compared to the previous run, the thermocouple temperature profiles in the Group E thermocouples had a greater degree of erratic behaviour.

The heat fluxes for thermocouple pairs 11-12 and 19-20 are erratic but generally fall in the range of 95-105 kW/m²°C as shown in Figure 5.12. The heat flux profiles in Group E begin at typical values of 50-120 kW/m²°C but near the 15th minute, the thermocouple pair 22-23 reaches a heat flux of approximately 525 kW/m²°C. In this case, however, the thermocouple pair 22-23 leads to a negative heat flux of about -300 kW/m²°C. The thermocouple pair 21-22 exhibits even more erratic behaviour.

Oct 4b

As a continuation of the previous run, this data set contains the end of the fuming cycle and the tap out of the furnace. In Figure 5.13a the end of the fuming cycle is shown by a decreasing slag temperature up to the 35th minute. An increase in the coal rate from 40% to 45% occurred near the 10th minute which caused the bath temperature to decrease at a higher rate. At the 35th minute the coal rate was cut by 10% and the slag began to heat up in preparation for tapping.

Concentrating on Group E, Figure 5.13c shows that the erratic nature of thermocouples 21, 22 and 23 had continued from the previous run. Of particular interest is the magnitude of the peaks seen by thermocouple 23. Four of the peaks reached temperature of approximately 150°C and appear to be quite consistent. The fifth large
peak was significantly different as it reached a peak temperature of 187°C near the 55th minute. The first peak occurred during a change in the coal rate near the 10th minute and the final four transients occurred during the tapping stage of the fuming cycle. The final transient possesses a plateau in its response with its duration being approximately 35 seconds.

As before, the water temperature profiles do not show any evidence of the thermal transients (Figure 5.13a). In Figure 5.13b the inverse thermal transients were again present in the Group D thermocouples although to a lesser degree since the magnitudes of the thermal transients were relatively small.

The heat flux profiles presented in Figure 5.14 show a close agreement in the responses of thermocouple pairs 23-22 and 23-24 with a peak heat flux of about 1.1 MW/m²•°C during the largest transient. As before, the heat flux profile of 21-22 is inverted and sometimes becomes negative indicating that thermocouple 22 is heated at a higher rate than thermocouple 21.

Oct 4f

Following the mid-cycle steady-state run of Oct 4e, which was presented in Section 5.2.1, the run Oct 4f contained the end of the fuming cycle and the furnace tap. The slag bath temperature profile given in Figure 5.15a shows the start of a slag heating stage around the 35th minute and tapping near the 57th minute. The response of thermocouple 23 in Figure 5.15c contains large thermal transients in the 62nd and 67th minute with peak temperatures of 247°C and 250°C respectively. These spikes occur during the charging stage of the next fuming cycle. In a similar fashion as before the first thermal transient possesses a plateau in its response approximately 40 seconds in duration.
5.2 Results

The response of the thermocouples during this run were not as erratic as the previous runs of Oct 4a and Oct 4b. Noting that the furnace was processing a different charge the smaller transients of less than 20°C found throughout runs Oct 4a and Oct 4b may be associated with many factors. These factors may include the bath temperature, blowing conditions into the bath and the bath’s chemistry which may affect the viscosity of the bath, however, there is not enough information here to fully characterize these effects. The thermocouples in Group D experienced inverse thermal transients that were seen in the previous runs and the outlet water temperature profile again does not show any evidence that the thermal transients have occurred.

The heat flux profiles in Figure 5.16 show the pairings 23-22 and 23-24 acting in a similar fashion while the pair 21-22 has an inverted heat flux profile. The peak heat fluxes occur in the thermocouple pair of 23-22 and they are very close at a value of approximately 1.8 MW/m²·°C.

Oct 5c

In the previous runs it has been shown that there is a definite relationship between the occurrence of the thermal transients and either charging or tapping and changes in the coal rate. Although this is true for the majority of cases this run is included to demonstrate a particular case where the furnace was tapped out and charged during which thermal transients were not observed. Any proposed mechanism leading to the failure of the jackets would have to account for the fact that the thermal transients do not occur during every occurrence of charging and tapping.
5.2 Results

This run followed a steady-state run that was discussed in Section 5.2.1, run Oct 5b. The slag temperature profile for this run is given in Figure 5.17a and the slag can be seen to have been tapped from the 15th to the 22nd minute. Charging began near the 24th minute and ended near the 36th minute after which the slag was heated up to about 1175°C.

Given that charging has occurred it might be expected that the thermocouples in Group E would exhibit thermal transients. However, it can be seen from Figures 5.17c and d that this is not the case. The mid-plane panel temperatures of thermocouples 21 and 23 did not exceed 108°C and the heat flux profiles in Figure 18 are generally below 140 kW/m²°C. The operating log indicates that 35 tons of slag were charged into the furnace which is 70% of a typical charge and the smaller charge may account for the absence of the transients. Further, it is possible that a partial tap out was performed where a larger than normal heel of slag is carried over from one cycle to the next. In the next section a mechanism is formulated for the generation of the thermal transients which can account for the results seen in this run.
### Table 5.6: Operating data for the plant trials in which thermal transients were observed.

<table>
<thead>
<tr>
<th>Time from start of run (min)</th>
<th>Run Oct 3g</th>
<th>Coal rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Start</td>
<td>70</td>
</tr>
<tr>
<td>26</td>
<td>Cut coal 10%</td>
<td>60</td>
</tr>
<tr>
<td>37</td>
<td>Open tap holes</td>
<td>50</td>
</tr>
<tr>
<td>45</td>
<td>Closed tap holes</td>
<td>50</td>
</tr>
<tr>
<td>51</td>
<td>Cut coal 10%</td>
<td>50</td>
</tr>
<tr>
<td>73</td>
<td>Finish charging</td>
<td>50</td>
</tr>
<tr>
<td>81</td>
<td>End of test</td>
<td>50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run Oct 4a</th>
<th>0</th>
<th>Start</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15</td>
<td>Finished charging (44 tons hot slag)</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>72</td>
<td>End</td>
<td>40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run Oct 4b</th>
<th>0</th>
<th>Start</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>Increased coal 5%</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>Cut coal 1%</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>39</td>
<td>Open tap holes</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>49</td>
<td>Closed tap holes</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>55</td>
<td>Cut coal 5%, Finish charging hot slag (37 tons)</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>57</td>
<td>End</td>
<td>30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run Oct 4c</th>
<th>0</th>
<th>Start</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>75</td>
<td>End. Coal increased 10%</td>
<td>40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run Oct 4f</th>
<th>0</th>
<th>Start</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>35</td>
<td>Reduced coal 10%</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>38</td>
<td>Open tap holes</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>48</td>
<td>Closed tap holes</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>56</td>
<td>Charging furnace, coal cut 5%</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>72</td>
<td>End</td>
<td>25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run Oct 5c</th>
<th>0</th>
<th>Start</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
<td>Open tap holes, oxygen on</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>21</td>
<td>Closed tap holes</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>29</td>
<td>Cut coal 15%</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>46</td>
<td>Finish charging hot slag (35 tons)</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>54</td>
<td>Finish cold charge (5 ton granulated slag)</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>77</td>
<td>End</td>
<td>50</td>
</tr>
<tr>
<td>Event No.</td>
<td>Run</td>
<td>Time from start of run (min.)</td>
<td>Maximum temperature of thermocouple #23 (°C)</td>
</tr>
<tr>
<td>----------</td>
<td>-------</td>
<td>-------------------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>Oct 3g</td>
<td>15</td>
<td>220</td>
</tr>
<tr>
<td>2</td>
<td>Oct 3g</td>
<td>31</td>
<td>218</td>
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<td>58</td>
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<td>Oct 3g</td>
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<td>6</td>
<td>Oct 4a</td>
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<tr>
<td>7</td>
<td>Oct 4b</td>
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<td>152</td>
</tr>
<tr>
<td>8</td>
<td>Oct 4b</td>
<td>45</td>
<td>152</td>
</tr>
<tr>
<td>9</td>
<td>Oct 4b</td>
<td>51</td>
<td>150</td>
</tr>
<tr>
<td>10</td>
<td>Oct 4b</td>
<td>54</td>
<td>150</td>
</tr>
<tr>
<td>11</td>
<td>Oct 4b</td>
<td>56</td>
<td>185</td>
</tr>
<tr>
<td>12</td>
<td>Oct 4c</td>
<td>3</td>
<td>235</td>
</tr>
<tr>
<td>13</td>
<td>Oct 4c</td>
<td>5</td>
<td>215</td>
</tr>
<tr>
<td>14</td>
<td>Oct 4c</td>
<td>10</td>
<td>261</td>
</tr>
<tr>
<td>15</td>
<td>Oct 4f</td>
<td>63</td>
<td>247</td>
</tr>
<tr>
<td>16</td>
<td>Oct 4f</td>
<td>68</td>
<td>242</td>
</tr>
</tbody>
</table>

Table 5.7: A summary of the thermal transients that were observed during the plant trials.
Figure 5.7: Thermocouple temperature profiles during run Oct 4c: (a) water and slag temperatures and (b) Group A.
Figure 5.7: Thermocouple temperature profiles during run Oct 4c: (c) Group B and (d) Group C.
5.2 Results

Figure 5.7: Thermocouple temperature profiles during run Oct 4c: (e) Group D and (f) Group E.
Figure 5.7: Thermocouple temperature profiles during run Oct 4c: (g) Group E and (h) Group F.
Figure 5.7: Thermocouple temperature profiles during run Oct 4c: (i) Group F.
Figure 5.8: Heat flux profiles during run Oct 4c: (a) Group B and D and (b) Group E. Numbers refer to thermocouple pairs which were used to calculate the heat flux.
5.2 Results

Figure 5.9: Thermocouple temperature profiles during run Oct 3g: (a) water and slag temperatures and (b) Group D.
5.2 Results

Figure 5.9: Thermocouple temperature profiles during run Oct 3g: (c) and (d) Group E.
Figure 5.10: Heat flux profiles during run Oct 3g: Group E. Numbers refer to thermocouple pairs which were used to calculate the heat flux.
Figure 5.11: Thermocouple temperature profiles during run Oct 4a: (a) water and slag temperatures and (b) Group D.
Figure 5.11: Thermocouple temperature profiles during run Oct 4a: (c) and (d) Group E.
5.2 Results

Figure 5.12: Heat flux profiles during run Oct 4a: (a) Group B and D and (b) Group E. Numbers refer to thermocouple pairs which were used to calculate the heat flux.
5.2 Results

Figure 5.13: Thermocouple temperature profiles during run Oct 4b: (a) water and slag temperatures and (b) Group D.
5.2 Results

Figure 5.13: Thermocouple temperature profiles during run Oct 4b: (c) and (d) Group E.
Figure 5.14: Heat flux profiles during run Oct 4b: (a) Group B and D and (b) Group E. Numbers refer to thermocouple pairs which were used to calculate the heat flux.
Figure 5.15: Thermocouple temperature profiles during run Oct 4f: (a) water and slag temperatures and (b) Group D.
5.2 Results

Figure 5.15: Thermocouple temperature profiles during run Oct 4f: (c) and (d) Group E.
5.2 Results

Figure 5.16: Heat flux profiles during run Oct 4f: (a) Group B and D and (b) Group E. Numbers refer to thermocouple pairs which were used to calculate the heat flux.
5.2 Results

Figure 5.17: Thermocouple temperature profiles during run Oct 5c: (a) water and slag temperatures and (b) Group D.
5.2 Results

Figure 5.17: Thermocouple temperature profiles during run Oct 5c: (c) and (d) Group E.
5.2 Results

Figure 5.18: Heat flux profiles during run Oct 5c: (a) Group B and D and (b) Group E. Numbers refer to thermocouple pairs which were used to calculate the heat flux.
5.2 Results

5.2.3 Summary

From the analysis performed in the previous section, it is evident that the event leading to the generation of the thermal transients has to satisfy specific criteria. First, the thermal transients occur during either changes in the coal rate or during charging and tapping of the furnace. However, as was the case in run Oct 5c, transients are not observed during every charging and tapping period. The thermal transients were shown to occur in the Group E thermocouples and not in the other groups of thermocouples. The Group F thermocouples which are close to the Group E thermocouples but in close proximity to a row of studs also did not experience the thermal transients. At the same time the thermal transients occurred in the Group E thermocouples, inverse thermal transients were observed in the Group D thermocouples. Finally, the duration of the peak, or "plateau", varied among the transients and in one instance it was approximately one minute in length.

The mechanism for the generation of the thermal transients is believed to be the complete removal of the slag layer over a relatively small section of the jacket and it is likely that the slag fall-off is a result of intense mixing. When the furnace is fully charged, the slag bath that is in close proximity to the jackets is violently agitated because of the coal/air injection and combustion. As the coal rate is changed the injection dynamics and combustion would also be affected and this would alter the mixing conditions near the tuyeres. During tapping the level of the bath falls to a level that is at the same height that the Group E thermocouples were positioned (20 cm above the tuyere line). At this height the tuyeres are still submerged and the tuyere bubbles generate significant surface wave action capable of removing the slag layer. This effect is further enhanced when molten slag is charged into a near empty furnace. During changes in the coal rate or charging and
tapping, the intensified mixing washes off a section of the slag layer that had previously formed and stabilized during steady-state fuming. It is believed that the plateau behaviour that has been observed in many of the transients is an indication of the time required for a new slag layer to successfully adhere to the jacket. The duration of the plateau period is likely a function of mixing conditions in the bath and the size of the slag patch that is removed. The exposed area is of the order of the distance covered by the Group E thermocouples 21 to 24, 76 mm, although its size would likely vary among the thermal transients. It is interesting to note that this distance is also the distance between stud centres.

The analysis of the thermal transients carried out in the previous section tended to focus on the larger thermal transients which reached mid-panel temperatures of 220 to 260°C. In many of the runs it was observed that some of the transients were much smaller with mid-plane temperatures of 140 to 160°C at thermocouple 23. Examples of these smaller transients can be seen in the runs Oct 4c (Figure 5.7f - 33rd minute), Oct 3g (Figure 5.9c - 66th minute), and Oct 4b (Figure 5.13c - 9th, 45th, 51st and 54th minutes). It is possible that these transients are the result of slag motion that presses the slag shell onto the jacket. This reduces the contact resistance between the shell and the panel and increases the rate of heat transfer to the jacket. With higher rates of heat extraction because of lower contact resistance, the slag layer initially becomes thicker. As the contact resistance returns to its original state the slag layer melts back to its steady-state thickness. Because of this dynamic equilibrium the plateaus that were observed in the larger transients are not found in the smaller transients. An attempt to simulate this behaviour with the mathematical model will be carried out in a later section.
5.2 Results

It was mentioned earlier that the Group F thermocouples were in close proximity to a row of studs. The studs serve to anchor the slag shell onto the jacket so that even under intense mixing conditions that are capable of removing the slag layer elsewhere on the jacket the studded region remains protected. This is compatible with the proposed mechanism for the formation of the thermal transients as it accounts for the absence of thermal transients in the temperature profiles of the Group F thermocouples.

It is believed that the larger thermal transients generate significant thermal stresses in the panel which will eventually lead to failure. As a small exposed area of the panel heats up, the portion of the panel on the slag face is restricted from expansion by the surrounding colder material. This leads to the generation of compressive stresses in this small section and eventually, if the compressive stresses are large enough, plastic deformation would result. As the thermal load is removed, the elastic region around the plastically deformed section would want to contract. Since the plastically deformed section is unable to return to its original shape the plastic zone remains in tension. When the panel is heated up by another thermal transient, the process is repeated and the plastic strain accumulates. In this way the panel can eventually fail by low-cycle fatigue with minute cracks developing at pre-existing surface flaws when the hot face of the panel is in tension. Thus, a minimum of surface flaws on the panel would be desirable and care is practiced at Cominco during the fabrication of the jackets.²⁰ A mathematical model of thermal stresses in the jacket, developed later in the thesis, shows that the panel would yield. However, an analysis of the question of low-cycle fatigue in the jackets is beyond the scope of this thesis.
5.3 Characterization of Cracks

In an effort to characterize the nature of cracking on the working face of the water-jacket, a section of the panel located above a tuyere was removed. Existing cracks were opened up by cooling the section in liquid nitrogen and inducing final fractures. The opened face consists of the interior surface of the crack and a distinguishable region where brittle fracture occurred. Photographs were taken using a Zeiss light microscope and photomicrographs were obtained using a Hitachi S-570 scanning electron microscope.

The working face of the panel is shown in Figure 5.19. At the bottom of the specimen is a weld toe formed where the tuyere was secured to the jacket. A stud that was welded onto the jacket to aid in supporting the frozen slag shell is located near the centre of the specimen. The stud is partially eroded indicating it was in contact with the molten bath. The cracks on the working face have a generally random orientation with perhaps a bias towards the vertical. The random nature of the cracks indicates that a single major flaw does not lead to fracture, but several minor surface flaws which act as initiation sites generate the characteristic pattern. This suggests that failure is due to thermal fatigue on the working face of the panel. The lips of the cracks are eroded as a result of molten slag coming into contact with the panel. The intensity of cracking decreases in the vicinity of the stud indicating that the protective slag shell had remained intact locally.

As molten slag contacts the jacket in the area of the panel located above the tuyere, the panel locally heats up and may deform plastically in compression. As the section cools, the material surrounding the deformed region is placed in tension as it returns to its original state. This tension may have induced failure at the weld toe. The slight bias in crack
orientation towards the vertical indicates that the stresses in the horizontal plane of the jacket were greater than in the vertical direction. The failure at the weld toe allows for some expansion in the vertical direction.

When a crack was opened up and viewed under the light microscope the existence of three distinct layers became apparent (Figure 5.20). The bottom layer is a result of brittle fracture when the liquid nitrogen was used to help induce final fracture. The region near the slag face appears dull and heavily corroded as a result of contact with molten slag. The extent of the slag penetration into the crack appears to have a fairly defined limit being approximately three millimeters in thickness with an overall crack length of about eight millimeters. A secondary crack running through the corrosion layer into the interior of the larger crack was observed with the scanning electron microscope and is presented in Figure 5.21. The crack length is approximately three millimeters in length and also gives evidence of the eroded nature of the lips of the crack.

At the crack tip striations were observed that were absent in regions remote from the crack tip (Figure 5.22). The striations are spaced out fairly evenly about every five micrometers indicating thermal cycling. The absence of such features farther away from the crack tip suggest that the major crack closed as a result of cyclic thermal loading. Also near the tip of the crack there is further indication of thermal fatigue as shown by the random nature of the cracking in Figure 5.23. A concluding observation is the presence of nodules at the crack tip (Figure 5.24). Noting that water vapour is a product of the combustion process the nodules are likely formed from corrosion of the steel at the crack tip.
5.3 Characterization of Cracks

Figure 5.19: Photograph of the working face of a section taken from a water-cooled jacket.
5.3 Characterization of Cracks

Figure 5.20: Interior of a crack after final fracture (specimen thickness 12.7 mm).
Figure 5.21: Secondary crack on the interior surface of the crack in Figure 5.20.
Figure 5.22: Striations observed at the crack tip indicating load cycling.
Figure 5.23: Interior surface failures at the crack tip indicative of thermal fatigue.
Figure 5.24: Nodule formation at the crack tip resulting from corrosion.
In the literature review (Section 3.2) most of the studies involving heat flow through the water-cooled jackets suggested a direct relationship between the heat flux through the jacket and the slag bath temperature. In these studies the heat flux was calculated from the temperature increase of the cooling water. The steady-state calculations performed in this section employ measurements in the panel using the thermocouple pairing, 19-20. This thermocouple pair was located near the centre of the jacket and was not located near a row of anchoring studs. Since there were only a few runs that did not contain thermal transients it was necessary to extract the steady-state segments from some of the thermal transient runs.

The observations made by Grechko\textsuperscript{16} indicated there was a relationship between the heat flux through the jackets and the slag temperature. The results from this study are presented in Figure 5.25 and it appears that a relationship between the heat flux and the slag temperature exists although there is some interaction with the coal rate. The run that best exhibits this behaviour is Oct 3a since it covers a wide range of slag temperatures. From the figure it can be seen that for a given slag temperature a higher coal rate produces a higher heat flux through the panel. The higher heat flux likely results from an increased rate of combustion as more coal is available. Future work would be required to focus on this relationship in greater detail with experimental runs covering a wide range of slag temperatures being performed in a similar fashion to run Oct 3a.

To develop an understanding of how the heat flow through the jackets is related to height up the jacket the same steady-state data is used to produce Figure 5.26. In these calculations the thermocouple pairs employed were 11-12, 19-20 and an average of the
Group E pairs. The heat flux was then averaged over the steady-state portions of the runs in order to produce Figure 5.26. It can be seen in a majority of the cases that the heat flux tends to decrease with height. This implies that coal combustion and liquid stirring are more prominent just above the tuyereline compared to higher up the jacket. In runs where this trend is not closely followed, it is likely that varying blowing conditions or slag chemistry have altered the nature of fluid flow in the vicinity of the tuyeres and the rate of heat flow across the face of the jacket. This phenomena is extremely complicated to analyze and is not discussed further in this study.

As stated earlier the proposed mechanism for the formation of the thermal transients is a partial removal of the slag layer. If this is the case then there would be a relationship between the panel temperature and the slag temperature. In Figure 5.27 it can be seen that this is generally true although there is a large degree of deviation. It will be shown in the mathematical modelling analysis that the rate of heat transfer between the fire face panel and the liquid slag is more sensitive to the contact resistance at this interface than the temperature of the slag.

The thermal transients that have been observed in this study have come about from either a change in the coal rate or as a result of tapping and charging. In either case these events may change or intensify the mixing in the vicinity of the tuyeres and eventually lead to the washing-off of a small patch of slag. Perhaps the simplest measure that can be taken to improve the performance of a jacket is to increase the stud density directly above the tuyeres. The thermocouple measurements have shown that the panel temperatures in the vicinity of the studs (Group F) do not experience thermal transients and this indicates that the slag layer locally remains intact.
5.4 Discussion of Results

In terms of changes in the coal rate, the thermal transients may be reduced if this change is made gradually rather than in a single step. This would provide a less dramatic change in the mixing conditions in the vicinity of the tuyeres. The operation of the furnace does not necessarily require that the coal rate be changed instantaneously and if the change could be made over about 5 minutes the fuming cycle would not be greatly affected. Another possibility would be to insert the tuyeres deeper into the bath by employing water-cooling. The injected coal/air mixture would not agitate the slag bath in the vicinity of the jackets to the same degree and there would not be additional forces exerted on the slag shell. This approach may also be more energy efficient since a larger fraction of the combustion energy would be retained in the bath. However, water-cooled tuyeres in the bath present certain design problems and operating hazards which may make them impractical. The solution may then be to "jet" the coal/air mixture into the bath using air under higher pressures.
Figure 5.25: Relationship between the heat flow through thermocouple pair 19-20 and the slag bath temperature for steady state operation. Percentages refer to the coal rate.
Figure 5.26: Variation in heat flux through the water jacket with respect to height above the furnace floor.
Figure 5.27: Relationship between the increase in temperature of the thermal transients and the slag bath temperature.
6.1 Thermal Resistances to Heat Flow

In the literature it has been shown by Grechko that the heat flux is proportional to the temperature of the molten bath. The thermocouple measurements carried out in this study have demonstrated that this is true but the heat flow is also a function of other conditions in the bath such as combustion and bath mixing. As conditions in the bath change the heat flux from the bath to the jacket would be expected to change correspondingly. Such changes would alter the temperatures of the hot face panel of the jacket and, more importantly, they would affect the thickness of the frozen slag layer. For example, if the heat flux from the bath increases dramatically the slag layer becomes thinner as it approaches a new steady-state thickness. Consequently, the jacket/slagn layer system may be considered to be in a dynamic equilibrium where changes in the heat flux from the bath may change the slag temperature, slag layer thickness and the fire face panel temperatures. A mathematical model would have to include both the slag layer and the jacket in order to properly represent these relationships.

It should be pointed out that an undesirable condition can develop where a higher heat flux, usually resulting from more intense mixing conditions in the bath, is coupled with a thinner slag layer that is more susceptible to breaking away. It is therefore necessary to model the quasi steady-state behaviour of the bath prior to the large thermal transients since these conditions may have a bearing on the occurrence of these transients.
6.1 Thermal Resistances to Heat Flow

The mathematical model consists of a panel domain coupled to a slag domain. The heat flow into the slag domain is characterized by a heat flux from the bath and the heat flow into the jacket is governed by a heat transfer coefficient across the panel/slag interface. Heat is extracted at the water/panel interface and is also governed by a heat transfer coefficient. However, under steady-state conditions and for a given heat flux, the temperatures in the panel are determined by the temperature of the cooling water and the water/panel heat transfer coefficient. Under these conditions the panel temperatures are independent of the panel/slag heat transfer coefficient. In order to fully understand the conditions at the panel/slag interface it is therefore necessary to model the cooling portion of the thermal transient profiles. As will be discussed later, this portion of the curve represents the solidification of a new slag layer onto a small region of the jacket.

Since solidification is also considered in the model, the solidified layer has to grow into the slag domain. This approach introduces three important thermal resistances to heat flow: the conductance across the panel/slag layer interface, the slag layer thickness, and the resistance of the remaining molten slag. With the bath being well mixed the latter resistance would be expected to be comparatively small since the mixing would reduce the thermal gradients in the molten slag and the size of the thermal boundary layer ahead of the freezing interface. The characterization of heat flow at this interface was difficult and required the inflation of the thermal conductivity in the liquid slag as will be discussed later.
6.2 Heat Flow Model

The heat flow model was designed to simulate three stages of the slag fall-off phenomena and these stages are schematically shown in Figure 6.1. The first stage is the region of quasi steady-state that exists before the slag layer is removed. This stage is comprised of only minor changes in bath conditions that can be considered to be relatively constant immediately preceding the transients. Next, the slag shell is removed so that molten slag comes directly into contact with the jacket. Finally, the third stage consists of a new slag shell solidifying to a steady-state thickness. To facilitate these calculations the following simplifying assumptions were made:

1) The heat flow through the jacket is probably three-dimensional over the entire jacket due to such factors as variations in the bath mixing intensity along the height of the jacket. During the second stage of the model where there is slag fall-off, the heat flow through the thickness of the panel will be much greater than that in the plane of the panel suggesting a one dimensional model. However, in order to couple the temperatures to a thermal stress analysis, a two-dimensional finite element heat flow model was developed.

2) Since the model is concerned with simulating the removal of a small portion of the slag layer conditions within the bath, and in particular the heat flux from the bath, were assumed to be constant for the duration of the thermal transient. It should be noted that the length of time during which the slag layer is removed and renewed is small compared to the total fuming cycle time.
3) The thermophysical properties of the slag are assumed to be temperature independent. Since the slag would quickly freeze onto the jacket it would have a porosity similar to that of the liquid slag. Therefore, in a similar approach adopted by Richards and Brimacombe\textsuperscript{21} the thermophysical properties of the shell were taken to be those of the bath and these are given in Table 6.1.

A two-dimensional plane through the cross-section of the jacket is shown in Figure 6.2. The model incorporates two separate domains: the jacket domain and the slag domain. It is assumed that the temperature field is symmetric about the centre half plane of the jacket and that there is no flow of heat across the centreline, boundary BI in Figure 6.2b. Similarly, there is symmetry at \( x=300 \) with the neighbouring jacket and the heat flux is zero across the boundary defined by AH. Finally, the heat flux out of the back of the jacket, boundary HI, was assumed to be zero. A flow chart of the fuming furnace jacket model is presented in Figure 6.3.

The governing equation for two-dimensional heat flow is given by:

\[
\frac{\partial}{\partial x} \left( k_{x,i} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_{y,i} \frac{\partial T}{\partial y} \right) - \rho_i C_{p,i} \frac{\partial T}{\partial t} = 0
\]  

(6.1)

where \( i \) refers to either the slag or the jacket. This equation is used in each of the three stages, however, towards the end of stages 1 and 3, quasi steady-state is being approached and the heat accumulation term tends to zero. The derivation of element matrices for this equation subject to the following boundary conditions is included in the Appendix. The
thermophysical properties of the steel used, ASTM A516, are not well documented and
the properties of a similar steel were employed. These properties are given in Table 6.1
along with other important model parameters.

<table>
<thead>
<tr>
<th>STEEL: SAE 1042 STEEL USED FOR A516</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Thickness</strong></td>
</tr>
<tr>
<td><strong>Specific heat</strong></td>
</tr>
<tr>
<td><strong>Steel-water steady state heat transfer coefficient</strong></td>
</tr>
<tr>
<td><strong>Thermal conductivity</strong></td>
</tr>
<tr>
<td><strong>Density</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SLAG:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specific heat</strong></td>
</tr>
<tr>
<td><strong>Thermal conductivity</strong></td>
</tr>
<tr>
<td><strong>Density</strong></td>
</tr>
<tr>
<td><strong>Latent heat of solidification</strong></td>
</tr>
<tr>
<td><strong>Solidus temperature</strong></td>
</tr>
<tr>
<td><strong>Liquidus temperature</strong></td>
</tr>
<tr>
<td><strong>Molten slag bath conductivity enhancement factor</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WATER:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Estimated mean film temperature</strong></td>
</tr>
<tr>
<td><strong>Saturation temperature</strong></td>
</tr>
<tr>
<td><strong>Specific heat</strong></td>
</tr>
<tr>
<td><strong>Thermal conductivity</strong></td>
</tr>
<tr>
<td><strong>Density</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MODEL PARAMETERS:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time increment during transient peak (boiling)</strong></td>
</tr>
<tr>
<td><strong>Slag zone depth</strong></td>
</tr>
<tr>
<td><strong>Mean film temperature</strong></td>
</tr>
<tr>
<td><strong>Interfacial heat transfer coefficient (fitted)</strong></td>
</tr>
<tr>
<td>molten slag - steel</td>
</tr>
<tr>
<td>solidified slag - steel</td>
</tr>
</tbody>
</table>

Table 6.1: Thermophysical data (from Richards and Brimacombe$^{21}$) and
model parameters used in the solidification model.
Figure 6.1: Schematic diagram of the thermocouple temperature profiles showing the quasi steady-state and thermal transient stages for modeling.
Figure 6.2: Cross-section of one half of the jacket: (a) giving dimensions and (b) used to defining boundary conditions.
6.2 Heat Flow Model

Figure 6.3: Flow chart of the zinc-fuming furnace jacket model.
6.2 Heat Flow Model

6.2.1 Water-Face Boundary Condition

a) Forced and Free Convection

The flow of water through the jacket cavity is complicated because of the angle of the inlet port and the slope of the fire face panel. A further complication results in the fact that the water located near the fire face panel will be heated and its velocity will be greater than the cooler water. Therefore, it was expected that the heat transfer at this interface is a combination of forced and free convection and it is incorporated in the model through the use of an effective cooling-water heat transfer coefficient, $h_w$.

$$k_{steel} \frac{\partial T}{\partial y} = -h_w(T_s - T_w)$$

(6.2)

This heat transfer coefficient is applied to the boundaries EF, EG and GJ in Figure 6.2b. Under quasi steady-state conditions, adjacent thermocouples at alternating depths were used to back calculate the panel surface temperature, $T_s$. When the jacket is under quasi steady-state during stage 1 the panel temperatures at the water face are approximately 80°C. The water temperature at this interface was taken to be the average of the inlet water temperature, about 40°C, and the panel surface temperature, approximately 80°C. This averaged temperature is referred to as the mean film temperature and it was calculated to be about 60°C. These temperatures combined with the heat flux calculated from the thermocouples are used in Equation 6.2 to calculate the heat transfer coefficient, $h_w$. From an analysis of several quasi steady-state runs a value of approximately 5.2 kW/m²·°C was obtained.
6.2 Heat Flow Model

To assess its accuracy, it is possible to estimate a value of $h_w$ from empirical correlations. Assuming pure forced convection, Reynolds analogy for turbulent flow over a flat plate is given as

$$\frac{h_L}{k} = 0.036Pr^{1/3}Re_L^{0.8}$$

(6.3)

10,000 $< Re_L < 120,000$

0.7 $< Pr < 120$

In the region above the tuyeres, the water is not sufficiently hot to generate significant buoyancy effects and therefore are not necessary for this calculation. The properties of the water were evaluated at the mean film temperature of approximately 60°C. The water flow rate through a jacket has been measured by Cominco personnel to be about 120 l/min and assuming plug flow in the region of the jacket behind thermocouple group E, a water velocity of 0.5 m/s is calculated. This value when used in Equation 6.3 produces a heat transfer coefficient of 3.1 kW/m$^2$-°C and is therefore somewhat less than the value found experimentally. This then may be an indication of how the water flow deviates from plug flow conditions because of mixing and convective heating. The effect of higher velocities are shown in Table 6.2 and a velocity of 1.0 m/s is required to produce a heat transfer coefficient of 5.2 kW/m$^2$-°C. This is well within the expected uncertainty of the calculation.
b) Boiling

When the slag layer is removed, stage 2, the experimental data indicates that nucleate boiling occurs inside the jacket. To incorporate nucleate boiling into the model the following correlation developed by Rosenhow was used.\(^{22}\)

\[
C_{pl} \frac{(T_s - T_{sat})}{H_f \rho_f Pr_{l1.7}} = C_{cf} \left[ \frac{\dot{q}}{\mu_l H_f \left( \frac{\sigma}{g (\rho_l - \rho_v)} \right)} \right]^{0.53} \left[ \frac{C_p H_l}{k_l} \right]^{1.0} \quad (6.4)
\]

The water pressure through the jackets is a function of the hydrostatic head in the system and this pressure affects the saturation temperature. A value of 110°C was obtained corresponding to an absolute system pressure of 145 kPa which is an estimate of the hydrostatic head in the jacket. When the temperature of the steel surface exceeds \(T_{sat}\) a boiling heat flux is calculated from Equation 6.4. A boiling heat transfer coefficient is then calculated where

\[
h_b = \frac{\dot{q}}{T_s - T_w} \quad (6.5)
\]

The value of \(h_b\) is added to the forced convection heat transfer coefficient, \(h_w\), and this becomes the effective heat transfer coefficient to characterize heat flow to the water. It has been observed from the one-dimensional analysis that \(h_b\) exceeds \(h_w\) by an order of magnitude.
6.2 Heat Flow Model

<table>
<thead>
<tr>
<th>Water Velocity (m/s)</th>
<th>Heat Transfer Coeff. (W/m²°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>3.1</td>
</tr>
<tr>
<td>0.75</td>
<td>4.2</td>
</tr>
<tr>
<td>1.00</td>
<td>5.2</td>
</tr>
<tr>
<td>1.25</td>
<td>6.4</td>
</tr>
</tbody>
</table>

Table 6.2: Heat transfer coefficients for different water velocities using Reynolds analogy for turbulent flow over a plate.

6.2.2 Steel/Slag Interfacial Boundary Condition

In a similar fashion to the water/steel interface, a heat transfer coefficient was used to characterize heat flow across the steel/slag interface defined by boundary CD in Figure 6.2b. The heat flow is given by

\[
k_{\text{slag}} \left( \frac{\partial T}{\partial y} \right)_{\text{slag}} = -h_{\text{int}} (T_{\text{slag}} - T_{\text{steel}})
\]

with the equivalence of the heat flux on each side of the interface

\[
k_{\text{steel}} \left( \frac{\partial T}{\partial y} \right)_{\text{steel}} = k_{\text{slag}} \left( \frac{\partial T}{\partial y} \right)_{\text{slag}}
\]

The value of \( h_{\text{int}} \) will depend on conditions in the slag as follows. In the first and third stages a solid slag shell is in contact with the steel panel. Coal may be present between the shell and the panel and the steel interface may also be partially oxidized. These factors make it is difficult to characterize the heat flow at this interface and calculate the appropriate heat transfer coefficient. In the second stage molten slag is in contact with the panel and a higher heat transfer coefficient results. Here, mixing conditions in a bubbling bath
also make it difficult to determine a heat transfer coefficient. As will be discussed in the Results, these values become fitting parameters in the model by comparing the calculated temperatures to the measured response for a given run.

6.2.3 Liquid slag boundary node

In the previous analysis of heat resistances it was noted that there is a high degree of mixing in front of the solidifying interface. It is then desirable that the heat be quickly transferred from the bath to the slag layer since this would simulate a reduced thermal boundary layer. In the model formulation this can be carried out for steady-state problems by applying a heat transfer coefficient at the slag layer/molten slag interface with the temperature gradient established by the difference between the bath temperature and the liquidus temperature. However, for transient solidification problems the slag layer thickness is changing and the position of this interface at which to apply this heat transfer coefficient is moving. This is difficult to incorporate in the model formulation and so the technique used in this study requires extending the slag domain beyond the steady-state slag layer thickness and into the bath. This creates the situation that a region of molten slag is always present in front of the slag layer.

The high degree of mixing that is in front of the jackets is mainly due to large volumes of gas and the combustion of the coal/air mixture. Combustion is occurring in the vicinity of the jackets and to simulate this phenomena would be difficult. To simplify the model, the heat flux from the bath was taken to be a constant value for a given transient. This value would then represent an averaged heat flux that can be calculated from steady-state
conditions of the measured thermocouple response just prior to the thermal transients. The bath heat flux is applied at the molten slag boundary defined by boundary AB in Figure 6.2b where

\[ -k_{\text{slag}} \frac{\partial T}{\partial y} = \dot{q} \quad (6.8) \]

The modelling of convection in the bulk of the liquid has been examined by Flood and Hunt who developed a model of columnar growth of an alloy.\(^{23}\) The effect of mixing reduces the temperature gradients outside a boundary layer that is formed at the solidifying surface. The greater the degree of mixing, the smaller the temperature gradients and size of the boundary layer.

In the case of the solidifying slag shell, the mixing is driven by both convection and the injected gas which further reduces the gradients in the bulk liquid slag. To account for bath mixing in the model the transfer of heat from the molten slag boundary to the slag layer had to be enhanced and thus reduce the temperature gradients. This is accomplished by increasing the thermal conductivity of the liquid slag. This approach has also been employed in continuous casting to represent mixing conditions in the mould pool.\(^{24}\) The absence of this parameter would represent the solidification of a slag layer in a stagnant bath and would not recognize the higher rates of heat transfer that were observed by Grechko as discussed earlier.
6.3 Solidification

The release of latent heat during solidification of the slag shell is a complex problem to solve numerically. This question has been explored by Thomas et al.\textsuperscript{25} where two techniques which employ the specific-heat method were examined. The first method is based on a temperature-dependent effective specific heat which is given by

\[ C_p' = C_p + \frac{H_s}{T_{\text{liq}} - T_{\text{sol}}} \]  \hspace{1cm} (6.9)

The specific heat curve becomes a step function and this method requires very small time steps or iterations within a time step to maintain stability. It also requires that each node passes through the mushy zone upon solidification and that no node bypasses the mushy zone altogether. Because of its simplicity this method is often used in finite difference formulations.

The second specific-heat method is termed the enthalpy method and it lends itself well to finite element techniques. The methods examined by Thomas are those developed by Lemmon and Del-Giudice et al.\textsuperscript{25} The Lemmon method is used in this analysis and is represented by

\[ C_p = \left[ \left( \frac{\partial h}{\partial x} \right)^2 + \left( \frac{\partial h}{\partial y} \right)^2 \right]^{1/2} \]  \hspace{1cm} (6.10)

Thomas concluded that this method was superior in representing solidification problems.\textsuperscript{25}
6.4 Time Stepping

It was also established by Thomas that a three level solution algorithm is preferable to a two level scheme in problems that involve solidification. He looked at two three-level schemes: the Dupont technique and the Lees technique. He determined that the Dupont method gave better accuracy and it was therefore used in this study. The Dupont technique is expressed as:

\[
K_i \frac{3T_{i+\Delta t} + T_{i-\Delta t}}{4} + C_i \frac{(T_i - T_{i-\Delta t})}{(\Delta t)} = R_i
\]  

(6.11)

Thomas investigated the effect of using different averaging techniques to determine the temperature that is used to calculate the thermophysical properties in the thermal conductivity matrix, \(K\), and the capacitance matrix, \(C\). He concluded that the best results were produced when these matrices are calculated at time \(t\) and therefore the thermophysical properties were calculated at time \(t\) for this study.

The Dupont three-level scheme is not self starting and a two level scheme, the implicit "\(\Theta\)" numerical integration algorithm, was employed for the first iteration. Using this algorithm the differential equations are converted to

\[
[\bar{K}] [T]_{i+\Delta t} = \{ \bar{R} \}_{i+\Delta t}
\]  

(6.12)

where

\[
[\bar{K}] = \Theta [K] + \frac{[C]}{\Delta t}
\]  

(6.13a)
and

\[
\{\bar{R}\}_{t+\Delta t} = \left[ -(1-\Theta)[K] + \frac{[C]}{\Delta t}\right]\{T\}_t + (1-\Theta)\{R\}_t + \Theta\{R\}_{t+\Delta t}
\]

(6.13b)

The value of theta determines different algorithms and for this work a value of \(\Theta = 2/3\) was used. Even though these algorithms are unconditionally stable they may produce oscillatory tendencies especially when thermophysical properties change rapidly or when the boundary conditions also change rapidly and therefore the three-level schemes are more desirable.

### 6.5 Thermal Stress Model

The experimental measurements reveal that the fire face panel endures large heating transients and these are likely due to the removal of the protective slag layer. This heating would be greatest at the hot face of the panel and would lead to compressive stresses at this region. If the magnitude of these compressive stresses is large enough to cause yielding, then as the panel cools a stress reversal would result when it tries to return to its original state.

The jacket was modelled as though it was a tube noting that the fire face panel and the jacket sides are constructed from one sheet and formed to shape. Inside of the jacket stiffeners are welded onto the slag-face panel (see Figure 2.1). A back plate is then placed on the stiffeners and welded to the sides of the jacket. Tuyere casings are inserted through the jacket and welded to the fire face panel on the slag face of the jacket and to the outside
of the back plate. Because of the stiffeners and the tuyere casing, thermal expansion in the vertical direction is expected to be relatively small. Further, the slag fall-off phenomena is expected to occur in small patches of the order of the distance between the studs (75 mm) and, since one-dimensional heat flow is dominant, the regions above and below this patch would remain cold. To support this observation, the thermocouple measurements indicated that the Group D thermocouples remained cold while the section of the panel at Group E heated up. The model has therefore been defined as a state of strain bounded by plane strain and generalized plane strain but closer to the former. Plane strain occurs in long tubes with no axial temperature variation and with the ends completely restrained. Generalized plane strain is similar but with the ends free of any constraint.

The model was simplified by ignoring the effects of plastic deformation and any occurrences of strain hardening. The stress-strain curve is assumed to be linearly elastic and if regions of the jacket experience stresses that exceed the yield point of the steel then they were considered to have surpassed the desired design criteria. The physical properties of the steel used in this study are included in Table 6.3. Because the steel used in this study is not very common, approximated values were used.

The derivation of the finite element thermal stress equations to be solved for is briefly carried out in the Appendix. The derivation results in the following equations that are to be solved for under conditions of plane strain,

$$ [K] \{\delta\} = \{P\} \quad (6.14) $$

where
6.5 Thermal Stress Model

\[ [K] = \int_{D} [B]^T [D] [B] dA \] (6.15a)

is the stiffness matrix and

\[ \{ P \} = \int_{D} [B] [D] \{ \varepsilon \} dA \] (6.15b)

is the load vector. After the displacements have been solved for, stresses and strains in the jacket can be calculated. Because the problem is formulated for plane strain, stresses in the x-direction and y-direction are obtained. These are converted to principle stresses using the relationship

\[ \sigma_{I,II} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \] (6.16)

The stress in the z-direction is related to the stresses in the x-y plane using the following equation

\[ \sigma_z = \nu(\sigma_I + \sigma_{II}) - E\alpha \Delta T \] (6.17)

which assumes the steel is restricted from expanding in the vertical direction. Using Von Mises criteria, an effective stress and an effective strain are calculated as follows

\[ \sigma_{\text{eff}} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_z)^2 + (\sigma_z - \sigma_I)^2} \] (6.17)

\[ \varepsilon_{\text{eff}} = \sqrt{\frac{2}{3}} \sqrt{(\varepsilon_I - \varepsilon_{II})^2 + (\varepsilon_{II} - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_I)^2} \] (6.17)
Yielding occurs when the effective stress exceeds the yield point of the steel.

6.5.1 Boundary Conditions

In order to calculate thermal stresses throughout the cross-section of the jacket it is necessary to locate a reference point where the stresses can be calculated from. When the furnace is constructed the jackets are not flush together. It is expected that the space between the jackets would be closed off by frozen slag during operation. The side of the jackets are therefore capable of moving laterally to a small degree. Owing to the symmetry of the jacket and noting that the back plate of the jacket remains cold the reference point was located at the back of the jacket along the centreline as described by point I in Figure 6.2b. This reference point is assigned to have zero displacement in both the x and y directions. Points along the boundary defined by DF are free to move in the y direction but are fixed in the x-direction because of the symmetry of the jacket.

<table>
<thead>
<tr>
<th>STEEL PHYSICAL PARAMETERS:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus</td>
<td>200 GPa</td>
</tr>
<tr>
<td>Coefficient of Thermal Expansion</td>
<td>$11.7 \times 10^{-6} , ^\circ\text{C}^{-1}$</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Yield Stress</td>
<td>260 MPa</td>
</tr>
</tbody>
</table>

Table 6.3: Physical properties used in the thermal stress model.
Chapter 7. Results

The mathematical modelling analysis was divided into two parts. First, a preliminary one-dimensional heat flow analysis was carried out in order to determine the interfacial heat transfer coefficients at the panel/slag interface and to develop a basic understanding of the nature of heat transfer during the slag fall-off phenomena. This was followed by the two-dimensional problem of the cross-section of the jacket whose domain is shown in Figure 6.2. The one-dimensional problem is smaller than the two-dimensional problem and consequently offered substantial savings in terms of computation time. The heat flow, solidification and the thermal stress models were validated using analytical solutions and the fitting of interfacial heat transfer coefficients was supported using the experimental data.

7.1 Model Validation

Analytical solutions have been employed to establish the accuracy of the computer program under the diverse conditions that the model is to simulate. These solutions require thermophysical properties that are not temperature dependent and in the case of convective cooling the heat transfer coefficient is also constant. An effort was made to use thermophysical properties similar to those used in the modelling of the jacket whenever possible. The analytical solutions verify the program in terms of the thermal boundary conditions that are used in the model, solidification of the slag layer and the calculation of the thermal stresses.
7.1 Model Validation

Convective cooling of a semi-infinite slab

The temperature profile of a semi-infinite slab with convective heat losses is given by

\[
\frac{T(x,t) - T_1}{T_2 - T_1} = 1 - \text{erf}(\zeta) - [1 - \text{erf}(\zeta + \sqrt{\eta})]e^{(Bi + \eta)}
\]

(7.1)

where

\[
Fo = \frac{\alpha}{x^2}, \quad Bi = \frac{hx}{k}, \quad \zeta = \frac{1}{2\sqrt{Fo}}, \quad \eta = Bi^2Fo
\]

The initial uniform temperature of the slab is given by \( T_1 \) and the cooling media temperature is at \( T_2 \). The error function was evaluated using the following Taylor series

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{N} \frac{(-1)^n x^{2n+1}}{(2n+1)(n!)}
\]

(7.2)

The calculation of the Taylor series and the error function was carried out using a separate computer program. To generate results that would be similar to conditions in the jacket the following properties were used: \( k=50 \text{ W/m-}^\circ\text{C}, \quad C_p=475 \text{ J/kg-}^\circ\text{C}, \quad \rho=7844 \text{ kg/m}^3, \) and \( h=500 \text{ W/m}^2\cdot^\circ\text{C}. \) The initial temperature was 500°C and the cooling media temperature was 60°C. The grid used for these calculations is shown in Figure 7.1a. A comparison of the results from the finite element program and the analytical solution is given in Figure 7.1b and it can be seen that there is very good agreement. Convective boundaries are used to characterize the water/panel and panel/slag interfaces and these results confirm that the heat transfer coefficients are properly applied.
Solidification of a semi-infinite slab

For a molten material initially with a uniform temperature at the melting point, the temperature profile in the slab is given by

\[
\frac{T_{(x,t)} - T_1}{T_{mp} - T_1} = \frac{\text{erf}\left(\frac{1}{2\sqrt{\lambda t}}\right)}{\text{erf}(\lambda)}
\]  

(7.3)

where

\[
\lambda e^{x^2} \text{erf}(\lambda) = \frac{-C_p,\lambda(T_{mp} - T_{so})}{\pi^{1/2}\Delta H}
\]  

(7.4)

and \(\lambda\) is found through successive iterations.

The temperature, \(T_1\), is the temperature at \(x=0\) for \(t>0\) which represents an isothermal boundary condition. The properties used for this analysis are similar to those used for the slag in the finite element model: \(k=1.5\) W/m-°C, \(C_p=870\) J/kg-°C, and \(\rho=3600\) kg/m\(^3\). In the model the latent heat of solidification is 340 kJ/kg over a solidification range of 1150°C to 1100°C. The analytical solution assumes that a freezing range does not exist and the freezing range in the model had to be minimized in order to simulate this condition. In the model the latent heat release over the mushy zone was 340 kJ/kg for a freezing range of 50°C or 6.8 kJ/kg-°C. A similar rate of heat release was employed in the analytical solution using a latent heat of 6.8 kJ/kg over a 1°C freezing range. The isothermal boundary was set at 1100°C.
Using these values the right hand side of Equation 7.4 can be calculated and $\lambda$ was found to be 1.1303. The finite element grid that was used for this test is given in Figure 7.2a and the results are presented in Figure 7.2b. The results demonstrate that there is a good agreement between the analytical solution and the finite element model. At a depth of 0.005 m there is a small error of about 1°C during the first 50 seconds but this error is significantly reduced with time. The error is likely due to a combination of the isothermal boundary condition and the requirement of a freezing range in the model. The isothermal boundary condition sets the nodes at $x=0$ to a temperature that is below the freezing range and in the model the latent heat of solidification produced at these nodes is lost. It is expected that a finer grid or a smaller time step would produce better results, however, the error shown here is expected to be within the error associated with process variables used in the model.

*Isothermal expansion*

The thermal stress calculations do not incorporate any dynamic effects with respect to time and they do not include plastic deformation calculations. The temperature profile at a given time is used to calculate the effective thermal stress profile. These simplifications allow for a predetermined temperature profile to be directly imported into the model in order to calculate the extent of thermal expansion. Using a grid that covers the entire jacket, a uniform temperature was assigned to the jacket and it was allowed to expand while unconstrained. The zero displacement reference temperature was 25°C and the uniform temperature field was 125°C. The difference between the model results and the analytical solutions was found to be less than 0.01%.
7.1 Model Validation

**Heating of a Rod Segment**

Under certain conditions it is possible to calculate thermal stresses in a rod assuming plane stress and a fixed temperature profile. The plane stress formulation involves a small modification to the plane strain formulation used to model the jacket. This analysis is used here to validate the computer code relating to the calculation of thermal stresses. The grid used for these calculations is given in Figure 7.4 and the analytical solution for this problem as presented by Timoshenko\(^{28}\) is

\[
\begin{align*}
    u_r &= (1 - v)\alpha \frac{ar^3}{3} + (1 - v)\alpha \frac{ab}{3} r \\
    \sigma_r &= \frac{1}{3} \alpha E a (b - r) \\
    \sigma_\theta &= \frac{1}{3} \alpha E a (b - 2r)
\end{align*}
\]  

(7.5a)  

(7.5b)  

(7.5c)

For this grid the constants, \(a\), the temperature difference divided by the radius, and \(b\), the radius of the cylinder, are 100 °C/m and 2 m respectively. The thermophysical constants used are \(v=0.3\), \(E=200\) MPa and \(\alpha=11.7(10^{-6})\). The displacements and stresses that were calculated from the analytical solution and the results from the finite element model are presented in Table 7.1. The displacements and stresses calculated from the program are in very good agreement with the analytical solutions considering the fact that the grid is coarse. It should be noted that at \(r=2\) m the stress in the radial direction should be zero but a value of 3.134 MPa was calculated in the model. This value represents a boundary condition error which when divided by the stress in the theta direction produces an error of about 2%. This error would be smaller if a finer grid was used.
Figure 7.1: Validation of the heat transfer coefficient boundary condition: (a) finite element grid representing a semi-infinite slab, (b) analytical solution and finite element results.
Figure 7.2: Validation of the isothermal boundary condition and the release of latent heat: (a) finite element grid representing a semi-infinite slab, (b) analytical solution and finite element results.
Figure 7.3: Finite element grid representing a cylinder under plane stress used to verify the thermal stress calculations.

<table>
<thead>
<tr>
<th></th>
<th>Analytical Solution</th>
<th>Finite Element Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_0)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(u_{1,0})</td>
<td>1.053 mm</td>
<td>1.051 mm</td>
</tr>
<tr>
<td>(u_{2,0})</td>
<td>3.120 mm</td>
<td>3.129 mm</td>
</tr>
<tr>
<td>(\sigma_{rr=1})</td>
<td>78 MPa</td>
<td>78.6 MPa</td>
</tr>
<tr>
<td>(\sigma_{rr=2})</td>
<td>0</td>
<td>3.134 MPa</td>
</tr>
<tr>
<td>(\sigma_{\theta r=0})</td>
<td>156 MPa</td>
<td>156.7 MPa</td>
</tr>
<tr>
<td>(\sigma_{\theta r=1})</td>
<td>0</td>
<td>3.24 MPa</td>
</tr>
<tr>
<td>(\sigma_{\theta r=2})</td>
<td>-156 MPa</td>
<td>-156.7 MPa</td>
</tr>
</tbody>
</table>

Table 7.1: Comparison of thermal stress calculations from an analytical solution and the finite element model.
7.2 Heat Flow Analysis

In Chapter 5, the thermocouple measurements indicated that the heat flow through the jacket occasionally deviated from steady-state producing large thermal transients. It was also shown that the spikes were associated with charging and tapping of the furnace as well as changes in the coal rate. The magnitude of the temperature increase was similar among transients that occurred during the same run suggesting that the phenomena producing the thermal transients was reproducible. However, among separate runs there were observed differences in the magnitude of the thermal transients. These differences may be dependent on many factors including slag viscosity and composition and fluid flow effects in the vicinity of the tuyere that would be extremely difficult to simulate. Since the thermal transients are fairly reproducible within a given run the approach taken in this study has been to use a typical experimental run and determine how well the observed temperature profiles during the thermal transients could be simulated.

The experimental run chosen for this simulation was the run Oct 4f which was presented earlier in Figure 5.15a-d. In this run the thermocouple response exhibited steady-state conditions for about the first 60 minutes of the run followed by two thermal transients that reach midpanel temperatures of about 250°C in both cases. The period of steady-state enables the calculation of the water heat transfer coefficient and the steady-state bath heat flux prior to the removal of the slag layer. The water heat transfer coefficient was calculated in a manner described earlier in Section 6.2.1 giving a value of 5.2 kW/m²-°C. The steady-state heat flux from the bath was calculated to be 105 kW/m² just prior to the thermal transients and this heat flux is applied at the liquid slag boundary nodes during the entire run.
7.2 Heat Flow Analysis

The thermal transients are depicted on a finer time scale in Figure 7.4. Noting that each data point was taken every 5 seconds, it can be seen from this figure that the peak midpanel temperature of both transients (thermocouple 23) was reached in 15-20 seconds. Following the heating stage, the peak temperature was maintained for about 20 seconds producing a plateau in the temperature profiles. This behaviour was more prevalent in the first transient and the plateau represents the inability of a new slag layer to successfully adhere to jacket. This phenomena was incorporated into the model by maintaining the slag in front of the jacket at the bath temperature for the duration of the plateau after which time the slag layer was permitted to form. Although the duration of the plateau is somewhat random a value of 20 seconds was used for both peaks.

To demonstrate the similarities among the different transients, transients from runs Oct 3g and Oct 4c are presented in Figure 7.5. The transient in the Oct 3g run can be seen to reach the peak temperature in approximately 10 seconds followed by a plateau of about 20 seconds. The thermal transient has a total length of about 2 minutes which is similar to the length of the transients observed in the Oct 4f run. The thermal transients from the Oct 4c run are shown in Figure 7.5b and it can be seen that the largest transient reaches peak temperatures of over 250°C in a similar fashion to the Oct 4f transient. The peak temperature is reached in about 15 seconds and the plateau is nearly 60 seconds in length. The thermal transients are very similar among runs Oct 4f, Oct 3g and Oct 4c but it would be difficult to model the latter two cases. The Oct 4f run contains a long period of quasi steady-state before the thermal transients enabling the calculation of steady-state properties including the bath heat flux.
Figure 7.4: Temperature profiles of thermocouples 22, 23, and 24 showing greater detail of the thermal transients observed in run Oct 4f.
Figure 7.5: Temperature profiles of thermocouples 22, 23, and 24 showing greater detail of thermal transients observed in runs (a) Oct 3g and (b) Oct 4c.
7.2 Heat Flow Analysis

7.2.1 Panel/Slag Heat Transfer Coefficients

Based on the assumption that the thermal transients were generated when the slag layer was washed off, the heating portion of the thermal transients is then associated with molten slag coming into contact with the panel. The heat transfer coefficient that characterizes heat flow into the jacket has to be sufficiently large to simulate this heating and also maintain the temperatures at a plateau. A comparison of different panel/liquid slag interfacial heat transfer coefficients is presented in Figure 7.6 and it can be seen that this is a critical parameter in the model. The best fit was estimated to be a heat transfer coefficient of 1000 W/m²·°C. With this value the model underestimates the midpanel temperatures by about 10°C in the first peak and 5°C in the second peak. At the quarter depth the model overestimates the temperatures by about 25°C in the first peak and 5°C in the second peak. This response suggests that a higher amount of heat flow in the jacket may be required and this point will be discussed in greater detail later in this section.

Since the heat transfer coefficient is a fitted parameter it is worthwhile to obtain an estimate of its validity using the Reynolds analogy which was also used to estimate the water heat transfer coefficient in the previous chapter. The Prandtl number for this slag system is approximately 420 and exceeds the upper limit of 120 that is acceptable for this correlation. As a result the Reynolds analogy is used here only to provide an estimate of the fitted heat transfer coefficient.

From an analysis of the zinc slag fuming process Richards and Brimacombe estimated the slag circulation velocity to be 1.0 m/s which was a fitted parameter in a fuming process model. In a similar fashion Cockroft et al estimated this velocity to be 3.0 m/s. Using the Reynolds analogy to calculate a slag recirculation velocity, a heat transfer
7.2 Heat Flow Analysis

coefficient of 1000 W/m²·°C is obtained using a slag recirculation velocity of 1.9 m/s. This value is within the velocities mentioned above and thus suggests that the fitted heat transfer coefficient is an acceptable value.

Between different runs the amount of air injected through the tuyeres may vary. This in turn may affect the slag recirculation velocity and the heat transfer coefficient calculated from the Reynolds analogy would change. Further, as the slag temperature and composition vary from run to run the viscosity of the slag would change and this would also affect the calculated heat transfer coefficient. These relationships are beyond the scope of this study but they could offer an explanation of the relationship between the slag temperature and the magnitude of the thermal transients especially since it will be shown that the predicted temperature profile is more sensitive to the panel/liquid slag heat transfer coefficient than the slag temperature.

The panel/solidified slag shell heat transfer coefficient was fitted using the cooling the portion of the thermal transients. The cooling trend represents the successful attachment of the slag shell to the jacket and its subsequent growth. Figure 7.7 shows a comparison of several heat transfer coefficients with a value of 250 W/m²·°C appearing to give the best fit towards the end of the first transient. In Figure 7.8 the influence of this heat transfer coefficient on the steady-state slag shell thickness can be examined. Taking the thickness of the slag layer at the solidus temperature (1100°C) a change of 100 W/m²·°C in the heat transfer coefficient leads to about a 2 mm change in thickness of the slag layer. The magnitude of this heat transfer coefficient, or interfacial conductance, suggests that the slag shell is not in perfect contact with the jacket. Coal and air gaps have been observed between the slag shell and the jacket. Secondly, the slag shell may contract away from the
panel upon freezing and thus increase the thermal resistance at this interface. And finally, during periods when molten slag comes into contact with the jackets the surface may become slightly corroded and thus prevent good contact when a new slag layer is formed.

Given the value of panel/slag heat transfer coefficients discussed above a comparison of thermal resistances throughout the domain of the model can be made and these results are given in Table 7.2. In the slag fall-off case the largest thermal resistance is at the panel/slag interface. As the slag shell freezes the solid shell begins to contribute significantly to the overall resistance in the system and at steady-state the solid shell becomes the major thermal resistance.

These calculations were repeated for other runs containing thermal transients. Particular attention was paid to fitting the panel/liquid slag heat transfer coefficient since it was the most sensitive parameter. However, just as conditions in the bath change this heat transfer coefficient varies. The heat transfer coefficient can depend on slag temperature, viscosity and chemistry as well as the slag recirculation velocity. The value presented here is therefore a fit for this particular run. Future work could be directed towards the study of this relationship under more controlled conditions. This study is concerned with the high rates of heat transfer that result when the liquid slag comes into contact with the panel and the failure of the jacket that results.

It can be seen from the fitting of the heat transfer coefficients that the rate of cooling calculated by the model was greater than that observed in the data. This suggested that the transition between the panel/liquid slag and panel/solid slag heat transfer coefficients is not instantaneous at the solidus temperature of 1100 °C. As mentioned above the frozen slag shell may initially be in good contact with the jacket but with further cooling the shell
contracts away from the jacket to a final steady-state condition. To model this behaviour a sinusoidal curve was used to model the transition between the heat transfer coefficients as given by the following equation

\[ h_{int} = h_{int,i} - (h_{int,i} - h_{int,e}) \left( 1 - \cos \left[ \frac{T_{liq} - T}{T_{liq} - T_{set}} \pi \right] \right) \]  

The value \( T_{set} \) represents the temperature at which the slag layer has stopped contracting away from the panel and is a fitted parameter with a value of 700 °C. Figure 7.9 presents a comparison of this method to the previous case and the figure shows that there was only a small improvement in the fitting to the experimental data.

It was mentioned earlier that the panel/liquid slag heat transfer coefficient of 1000 W/m\(^2\)-°C underestimated the temperature difference between the half depth and the quarter depth. In order to increase the temperature difference between the two depths a higher heat flux from the bath would be required. The heat flow into the jacket can be increased by raising the panel/liquid slag heat transfer coefficient but this would increase the temperatures throughout the panel and make the fit at the quarter depth even worse.

The discrepancies between the experimental data and modelling results may be due a discrepancy between the assumed and the actual position of the thermocouples. The objective was to position the thermocouples 3.175 mm (1/8") and 6.35 mm (1/4") from the water face but the actual depths of the wells were not measuread and recorded after drilling. There may be some error in the calculation of heat fluxes since it was assumed
7.2 Heat Flow Analysis

the thermocouples are 3.175 mm apart in terms of distance from the water face.
Figure 7.6: Effect of panel/liquid slag heat transfer coefficient on the predicted thermal transient profiles. Data points from run Oct 4f.
Figure 7.7: Effect of panel/solid slag heat transfer coefficient on the predicted thermal transient profiles. Data points from run Oct 4f.
Figure 7.8: Comparison of different panel/solid slag heat transfer coefficients on the temperature distribution in the slag.
Figure 7.9: Calculated temperature profiles incorporating a sinusoidal change in the panel/slag heat transfer coefficient.
### 7.2 Heat Flow Analysis

#### 7.2.1 Thermal Resistance

<table>
<thead>
<tr>
<th>Physical Region</th>
<th>Steady-state</th>
<th>Thermal Transient</th>
</tr>
</thead>
<tbody>
<tr>
<td>water/panel interface</td>
<td>0.00019 (1.8%)</td>
<td>0.00019 (13.2%)</td>
</tr>
<tr>
<td>panel</td>
<td>0.00025 (2.4%)</td>
<td>0.00025 (17.4%)</td>
</tr>
<tr>
<td>panel/slag interface</td>
<td>0.00333 (31.7%)</td>
<td>0.0010 (69.4%)</td>
</tr>
<tr>
<td>solid slag</td>
<td>0.00507 (48.3%)</td>
<td>N/A</td>
</tr>
<tr>
<td>liquid slag</td>
<td>0.00165 (15.7%)</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 7.2: Thermal resistances in the model during steady-state and during slag fall off.

### 7.2.2 Discretization and Time Stepping

The approach taken in this study to discretize the slag domain has been to ensure that at least one node remains in the mushy zone during solidification. The mushy zone is defined here as the region of slag bounded by the solidus (1100°C) and the liquidus (1150°C) temperatures. Since a slag domain of 150 mm was used, at least 10 elements were required to satisfy this criteria. In the case of the fire face panel, the elements were chosen to coincide with the placement of the thermocouples which were located at the quarter and half depths from the water/panel interface with 8 elements being used.

The heat flow through the panel is predominantly through the thickness of the panel. Because the gradients through the thickness of the panel are relatively high the effect of the number of elements used through the thickness has to be examined. To test the discretization sensitivity a one-dimensional problem was used to model the panel and slag domains. Three cases were examined: 1) 8 fire face panel elements and 10 slag domain elements 2) 12 fire face panel elements and 10 slag domain elements (Grid A), and 3) 12 fire
face panel elements and 20 slag domain elements (Grid B). The results indicated that the
temperature profiles at the quarter and half depths were in good agreement with peak
temperatures varying by only 1.1 °C.

During the thermal transient the time step used in the model is governed by con­
ditions at the water/panel interface. An oscillation in surface temperature occurs as a
result of boiling at the water/panel interface during slag fall off. For example a 0.025
second time step during the thermal transient produced a temperature oscillation of 1.9 °C
while a 0.05 second time step had an oscillation of 8.3 °C. At the quarter depth the difference
in temperature between the two runs was only 2.1 °C and oscillations were not observed.
Since the larger time step, 0.05 s, reduced the solution time and did not appear to greatly
compromise the accuracy of the panel temperatures, it was chosen.

As the slag shell begins to solidify, the fire face panel cools and boiling ceases to
occur. The time step is then governed by the stability of the solidifying interface and the
time step was increased as solidification progressed when it was felt that it did not com­
promise the results. Because the slag has a low thermal conductivity its rate of solidification
is slower than for steel under the same conditions. The change in time step was never
increased by more than a factor of 4 and the maximum time step never exceeded
1.0 seconds. The smallest time step (0.05 seconds) was used for the first 10 seconds of
solidification in an attempt to account for the latent heat during the initial stages of
solidification.
7.2.3 Sensitivity of Process Variables

In Section 7.2.1, the sensitivity of the fitted panel/slag interfacial heat transfer coefficients were examined. From this analysis it was suggested that the most critical parameter related to the rapid heating of the panel is the panel/liquid slag heat transfer coefficient. In this section a discussion is presented on the sensitivity of the thermal transients to process variables including panel thickness and material. An emphasis is placed on those variables that can have a significant impact on rate of heat transfer through the domain in comparison to the interfacial heat transfer coefficients. These parameters are:

1) Panel thickness
2) Material type
3) Water temperature and heat transfer coefficient
4) Slag temperature
5) Steady-state bath heat flux
6) Mixing thermal conductivity enhancement factor

1) Panel thickness

It was discussed in the literature review that Cominco had examined the effect of the fire face panel thickness on the service life of the jacket and their experience demonstrated that a thinner fire face panel performs better. Calculations were carried out using panel thicknesses of 10 mm and 20 mm in addition to the standard case of 12.7 mm. Figure 7.10 includes the temperature profiles through the panel during steady-state and at the peak of the thermal transients 30 seconds after the removal of the slag layer. The
latter case is well within the plateau portion of the thermal transient. The steady-state profiles overlap for the three thicknesses because the surface temperature at the water-panel interface is determined by the water temperature, heat transfer coefficient and bath heat flux of 105 kW/m² which is the same for all three cases. The peak temperatures during the thermal transients are 310°C for a 10 mm panel, 351°C for the 12.7 mm panel and 419°C for the 20 mm panel. With a thicker panel the temperature at the hot face increases, however, this is offset by a lower heat flux into the jacket. As the hot face temperature increases the difference in temperature between this temperature and the bath temperature decreases and the heat flow into the jacket decreases. Even with this offset the thinner panel experiences lower temperatures and the resulting thermal stresses would be expected to be smaller. This will be discussed in greater detail in the next section.

2) Material type

Cominco personnel have performed tests using jackets fabricated from copper. Owing to its high thermal conductivity, the thermal resistance of the copper is smaller and lower panel temperatures would be expected under the same operating conditions. Using a jacket constructed of copper, the slag fall off phenomena was modelled and the results are given in Figure 7.11. After a slag removal time of 30 seconds the hot face of the copper jacket is only 158°C compared to 351°C for the steel jacket. With the significantly lower temperatures the copper jacket would be expected to perform better while in service. These results were observed at Cominco but the cost of this material relative to steel made its implementation uneconomic.
3) Water temperature and heat transfer coefficient

The inlet temperature of the water was in the range of 35°C to 50°C during the majority of the experimental runs. The 15°C variance would be incorporated into the model using a heat transfer coefficient of 5.2 kW/m²·°C that was calculated from the experimental data. The heat withdrawn associated with the 15°C temperature change is 78 kW/m². Although this can have a large effect on the steady-state temperature profile in the panel its contribution to the heat withdrawn during boiling is small. The heat withdrawn from boiling is about 0.95 MW/m²·°C depending on the panel/liquid slag interfacial heat transfer coefficient that is employed. Similarly, small changes in the water/panel heat transfer coefficient would only have a small effect on the rate of heat flow during boiling conditions. From an operational point of view a higher steady-state water/panel heat transfer coefficient would increase the thickness of the slag shell. A thicker shell could be less prone to being washed off.

4) Slag temperature

The slag temperature used in this analysis, 1280°C, is the approximate temperature of the bath before the first thermal transient. From the experimental data the slag temperatures were seen to range from 1160°C to just above 1300°C making the value used here in the upper limits of this range. The liquid heat transfer coefficient is approximately 1000 W/m²·°C as shown in the previous section. With a panel temperature of about 350°C the flux through the jacket is 1000(1280-350) or approximately 950 kW/m². A decrease in slag temperature to 1200°C reduces the heat flux by only 80 kW/m² or 8.4%. Therefore, the slag bath temperature is not as important as the interfacial liquid heat transfer coef-
7.2 Heat Flow Analysis

Ficient but its indirect effects cannot be ignored. For example the panel/liquid slag heat transfer coefficient can vary due to changes in bath viscosity associated with a cooler slag and in this way the slag temperature may play a greater role.

5) Steady-state bath heat flux

The typical range of steady-state heat fluxes through the Group E thermocouples was shown to be 80 to 120 kW/m² and a value of 105 kW/m² was used in the model. The heat flux calculated from panel/liquid slag contact is 950 kW/m². The effect of a small change in the steady-state bath heat flux can be demonstrated by changing the heat flux ±20 kW/m². This change represents a 20% change in the steady-state bath heat flux and would produce a corresponding change in the steady-state temperature profiles. However, it is only 2.1% of the heat flux experienced during slag fall-off. The heat flux into the jacket during the thermal transients is largely a function of the panel/liquid slag contact and not affected significantly by the initial steady-state temperature profile.

6) Mixing thermal conductivity enhancement factor

During the formulation of the model particular attention was focussed on the effect mixing would have on the rate of heat transfer in the liquid slag. A mixing enhancement factor was incorporated into the model and its effect is shown in Figure 7.12 where values of 1 (no mixing), 5 (standard case) and 10 were used. There is some difference in the profiles between the case without mixing and the other two, however, it can be seen that the difference between enhancement factors of 5 and 10 is small. With the enhanced mixing the slag in front of the solidifying interface is hotter because the heat applied at the slag boundary node is more quickly transferred to the interface. Without mixing the region in front of the slag interface would cool more rapidly. The effect the mixing enhancement
factor has on the slag temperature profiles with respect to distance from the panel/slag interface is shown in Figure 7.13. As this factor is increased the temperature profile in the liquid slag (above 1150 °C) shows a decreasing slope. Again it can be seen that the difference between factors of 5 and 10 is relatively small.
7.2 Heat Flow Analysis

Figure 7.10: The influence of fire face panel thickness on the temperature profiles in the panel for steady-state conditions and at the peak of the thermal transients.
Figure 7.11: A comparison of temperature profiles in the fire face panel of a jacket made from ASTM A516 steel and a jacket made from copper.
Figure 7.12: The influence of the mixing enhancement factor on the temperature profiles during the thermal transients.
Figure 7.13: The influence of the mixing enhancement factor on the temperature profiles through the fire face panel for steady-state conditions.
7.3 Thermal Stress Analysis

The finite element program that was used for the preliminary one-dimensional heat flow analysis was also used for the two-dimensional heat flow and stress analysis over the entire jacket. The domain used for this analysis is that given in Figure 6.2. Previously, it was demonstrated in the one-dimensional heat flow analysis that 8 elements through the thickness of the fire face panel and 10 elements through the slag domain were adequate in determining the rate of heat transfer through the domain. The same number of elements were used in the two-dimensional analysis through the thicknesses of the panel and slag domains respectively. For the finite element method a general guideline for the discretization of a domain is that the element’s aspect ratio, or length to width ratio, should approach unity. This guideline becomes increasingly important in cases where the local field variable gradients become extreme, however, it is possible to deviate from this criterion if the changes in the field variable become small.

At the height of the jacket where the thermocouple Group E was located the distance between the edge of the jacket and the centreline is approximately 309 mm. For simplicity in calculating the finite element mesh this distance was taken to be 300 mm. The fire face panel was defined by 8 elements through the thickness and 100 elements along the length to give an aspect ratio of 8/12.7:100/300 or 1.89. Even though this value is greater than unity it was deemed acceptable because the temperature and stress gradients are severe through the thickness of the panel but not along the length.

A separate program was written to evaluate the elements and the nodal connectivity in both the jacket and the slag domain. Since the temperature gradients are less severe
down the sides of the jacket, 50 by 2 elements were used along both the side and back. In order to link the elements along the panel/slag boundary, 100 elements were used along the width of the slag domain matching the fire face panel discretization.

From the thermocouple measurements the extent of the slag fall off was found to be approximately the distance between adjacent studs or 75 mm. Since this occurred at the centreline of the jacket, the model would have to represent half of this patch giving a 32.5 mm radius. This was simulated by removing the slag layer over 10 elements which is about 30 mm of slag. From the experimental results it appears that this is well within the expected variance.

In the following sections the contour plots of temperature and effective stress contour are presented. The side and back of the jacket are not shown because no significant gradients were observed in these regions. The slag domain isotherms were also plotted to demonstrate the effect of removing the slag shell.

7.3.1 Standard Case

The standard case consists of the 12.7 mm thick panel and the jacket dimensions currently employed in the furnaces (Figure 6.2). The size of the slag patch that is removed is 30 mm in radius. The interfacial heat transfer coefficients are those determined to have given the best fit from Section 7.2.1. In Figure 7.14a-c, the temperature and stress contours are shown for the standard case just prior to the removal of the slag layer. The temperature contours of the slag domain are shown in Figure 7.14a. The temperature of the frozen shell is 546°C at the jacket face. The solidus (1100°C) is 7.6 mm from the jacket face and
the liquidus (1150°C) is approximately 8.2 mm from the panel/slag interface. The effect of the increasing thermal conductivity of the slag can be seen beyond the liquidus as the distance between the isotherms becomes larger.

The temperature profiles of the fire face panel shown in Figure 7.14b indicate that the water face of the panel is 81.8°C, while the slag face reaches approximately 108.9°C. At the jacket sides the temperatures are slightly higher due to the somewhat larger resistance to heat flow. Beyond 50 mm from the edge of the jacket the heat flow is essentially one-dimensional and the isotherms have become parallel to the water/panel and panel/slag interfaces.

The effective stress contours of the thermal field shown in Figure 7.14b are presented in Figure 7.14c. The stress is a maximum at the hot face reaching values of 207 MPa at the centreline and about 220 MPa at the jacket edge. The values in the figure are presented as being positive, however, these stresses are all compressive and the stresses in the jacket do not exceed the yield stress of 260 MPa under steady-state conditions. On the cold face the stresses are smaller with a value of 155 MPa at the centreline.
7.3 Thermal Stress Analysis

Slag fall off time of 5 seconds

The contour plots presented in Figure 7.15a-c represent conditions in the jacket and the slag domain five seconds after the removal of the slag layer. The slag temperature contours shown in Figure 7.15a demonstrate the effect of removing a portion of the slag layer. The temperatures from the centreline to 30 cm from the centreline indicate that this region consists of molten slag.

The panel temperatures presented in Figure 7.15b indicate that the panel heats up in conjunction with the slag fall-off. After only 5 seconds the hot face of the panel reaches 300°C and the cold face reaches 120°C indicating that boiling has occurred at this interface. It is interesting to note that even after this short time the heat flow along the panel centreline has become essentially one-dimensional. This lends support to the preliminary one-dimensional analysis made earlier.

In Figure 7.15c, the stress contours appear very similar to the temperature contours. The maximum compressive stress occurs at the hot face with a value of 675 MPa. It should be restated that this analysis is purely elastic and does not account for plastic deformation. However, it is clear that with a yield stress of 260 MPa, the material under these conditions would experience large amounts of deformation. Interestingly, the cold face of the panel reaches compressive stresses of about 247 MPa which are close to yielding as well. In a similar fashion to the isotherms it can be seen from the figure that the stresses are confined to the region of the jacket associated with the removal of the slag layer. The localization of these stresses suggests that changes in the dimensions of the jacket such as length and depth would not give significant improvement in its service life. The effect of the fire face panel thickness will be shown later.
7.3 Thermal Stress Analysis

**Slag fall off time of 10 seconds**

The slag domain isotherms in Figure 7.16a are similar to those after 5 seconds. Again about a 30 mm radius slag patch is prevented from solidifying onto the jacket. The maximum fire face panel temperature at the centreline has risen to 352°C and the cold face reaches 124°C (Figure 7.16b). As before the heat flow can be seen to be one-dimensional at the centreline. The maximum effective stress is 810 MPa and compressive as shown in Figure 7.16c and the higher stresses are concentrated near the centreline.

**Slag fall off time of 30 seconds**

In the one-dimensional analysis the temperature plateau was shown to be reached in approximately 15 seconds. Therefore, the results at 30 seconds would represent the maximum temperatures and stresses seen in the panel. In Figure 7.17a, the slag domain isotherms have changed very little and the panel isotherms, Figure 7.17b, indicate that the maximum temperature at the centreline is 374°C compared to 352°C after 10 seconds. At this location the compressive stress is 867 MPa at the hot face and 281 MPa at the cold face, Figure 7.17c. Clearly the yielding could be expected to occur through the thickness of the panel with the most damage occurring at the hot face.

Because of the large magnitude of the thermal stresses, plastic deformation would occur in the region of the panel where the slag layer was removed. As the slag layer adheres and begins to grow to its steady-state thickness the panel begins to cool and the elastic regions adjacent to the deformed material begin to contract. The deformed region
is unable to return to its original form and the elastic regions are placed in tension. With the deformation induced from successive thermal transients the process is repeated and significant amounts of plastic strain begin to accumulate. Eventually, the panel experiences low cycle fatigue and fails with pre-existing surface flaws serving a crack initiation sites.
Figure 7.14a: Slag domain temperature isotherms (°C) prior to the partial removal of the slag layer.
Figure 7.14b: Fire face panel temperature isotherms (°C) prior to the partial removal of the slag layer.
Figure 7.14c: Fire face panel effective stress contours (MPa) prior to the partial removal of the slag layer.
Figure 7.15a: Slag domain temperature isotherms (°C) 5 seconds after the partial removal of the slag layer.
Figure 7.15b: Fire face panel temperature isotherms (°C) 5 seconds after the partial removal of the slag layer.
Figure 7.15c: Fire face panel effective stress contours (MPa) 5 seconds after the partial removal of the slag layer.
Figure 7.16a: Slag domain temperature isotherms (°C) 10 seconds after the partial removal of the slag layer.
Figure 7.16b: Fire face panel temperature isotherms (°C) 10 seconds after the partial removal of the slag layer.
Figure 7.16c: Fire face panel effective stress contours (MPa) 10 seconds after the partial removal of the slag layer.
Figure 7.17a: Slag domain temperature isotherms (°C) 30 seconds after the partial removal of the slag layer.
Cooling water

Figure 7.17b: Fire face panel temperature isotherms (°C) 30 seconds after the partial removal of the slag layer.
Figure 7.17c: Fire face panel effective stress contours (MPa) 30 seconds after the partial removal of the slag layer.
7.3.2 Influence of Panel Thickness

In the literature a thinner fire face panel was observed to give a better service life.\(^1\) Calculations were made assuming the jacket was constructed of 10 mm and 20 mm steel plate as compared to the 12.7 mm plate currently used. The analysis is restricted to the results at 30 seconds since it was shown for the standard case that the peak stresses occurred during the plateau.

The temperature and stress contours for the 10 mm and 20 mm panel are presented in Figures 7.18 and Figures 7.19 respectively. The slag temperature contours in Figure 7.18a are similar to those run for the standard case where it can be seen that 30 mm of slag is removed at the centreline. From the temperature profiles in the panel a maximum temperature of 333 °C is reached at the centreline compared to 374 °C in the standard case. At this location a maximum stress of 766 MPa was generated as opposed to 867 MPa seen previously. In a similar fashion the temperature and stress profiles after 30 seconds for the 20 mm panel indicate a higher maximum temperature of 449 °C and a maximum stress of 1048 MPa. These results agree with in-plant experience where thinner panels have been seen to perform better. However, a thinner panel should not be recommended until other factors, such as hydrodynamic loadings from the bath, are considered.

The maximum temperatures and maximum effective stress have been shown to be located at the point where the centreline of the jacket meets the hot face of the fire face panel. When these parameters are plotted as a function of time for panels of varying thickness, as in Figure 20, it is apparent that the peak stress the panel reaches decreases with panel thickness. This is due to lower overall panel temperatures reducing the tendency for thermal expansion. However, the decrease in peak stress is smaller as the panel

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becomes thinner. A thinner panel will experience a larger temperature gradient, $\Delta T/\Delta y$, and this increases the thermal strain in the panel because adjacent sections of the panel remain relatively cold. The stress that is generated by this increase in thermal strain offsets partially the lower stresses as a result of lower overall panel temperatures.
Figure 7.18a: Slag domain temperature isotherms (°C) 30 seconds after the partial removal of the slag layer for a fire face panel thickness of 10 mm.
Figure 7.18b: Fire face panel temperature isotherms (°C) 30 seconds after the partial removal of the slag layer for a fire face panel thickness of 10 mm.
Cooling water

Figure 7.18c: Fire face panel effective stress contours (MPa) 30 seconds after the partial removal of the slag layer for a fire face panel thickness of 10 mm.
Figure 7.19a: Slag domain temperature isotherms (°C) 30 seconds after the partial removal of the slag layer for a fire face panel thickness of 20 mm.
Figure 7.19b: Fire face panel temperature isotherms (°C) 30 seconds after the partial removal of the slag layer for a fire face panel thickness of 20 mm.
Figure 7.19c: Fire face panel effective stress contours (MPa) 30 seconds after the partial removal of the slag layer for a fire face panel thickness of 20 mm.
Figure 7.20: Maximum temperatures (a) and effective stresses (b) for panels of different thicknesses during the removal of a patch of slag.
7.3.3 Slag Patch Size and Location

In Figures 7.21a-c the extent of slag fall off has been reduced to 3 elements giving a slag patch radius of 9 mm. The effect of the smaller patch being removed is shown in the slag domain isotherms of Figure 7.21a. The peak panel temperature occurs at the centreline with a temperature of about 320°C (Figure 7.21b) with a maximum effective stress of approximately 720 MPa. Based on these results the size of the slag patch that is removed should be minimized although it is evident from the calculations that even the removal of a small patch of slag is sufficient enough to cause the steel at the hot face to yield.

Finally, it is possible that the slag layer may fail above one of the side tuyeres and this case is depicted in Figures 7.22a-c. From Figure 7.22a the effect of removing 30 mm of slag can be seen in the slag domain isotherms. The maximum panel temperature is about 350°C (Figure 7.22b), while the stress at this point is nearly 800 MPa. Again, the stresses are confined to a narrow band with the stresses in the bulk of the jacket being less than 250 MPa. Clearly, wherever slag fall off occurs the rate of heat transfer from the bath is so large that yielding would result.
Figure 7.21a: Slag domain temperature isotherms (°C) 30 seconds after the partial removal of the slag layer with a slag patch radius of 9 mm.
Figure 7.21b: Fire face panel temperature isotherms (°C) 30 seconds after the partial removal of the slag layer with a slag patch radius of 9 mm.
Figure 7.21c: Fire face panel effective stress contours (MPa) 30 seconds after the partial removal of the slag layer with a slag patch radius of 9 mm.
Figure 7.22a: Slag domain temperature isotherms (°C) 30 seconds after the partial removal of the slag layer over a side tuyere.
Figure 7.22b: Fire face panel temperature isotherms (°C) 30 seconds after the partial removal of the slag layer over a side tuyere.
Figure 7.22c: Fire face panel effective stress contours (MPa) 30 seconds after the partial removal of the slag layer over a side tuyere.
Chapter 8. Recommendations

Panel Design

Since slag fall-off appears to be the most likely mechanism for the larger thermal transients in the panel, greater attention must be paid to securing the slag layer to the jacket. One solution may be to weld a larger number of studs onto the jacket, particularly in the region above the tuyere line. This procedure is relatively inexpensive and the studs provide a better base in which to secure the slag layer, especially in the region where the slag layer appears to wash off readily. In the worst case the size of the slag patch that would be removed would be expected to be smaller with a higher stud density. It may also be possible to design new anchors which the frozen shell can better adhere to. The anchors would have to be smaller than the steady-state thickness of the slag shell to limit erosion by the bath and they would have to be designed so that they are not stress raisers. A wire grid tacked onto the working face may provide a means for securing the shell in regions remote from the anchors.

Many of the fuming furnaces discussed in the literature employ furnace jackets constructed from tubes. The tubes offer lower requirements on the amount of water needed and therefore a higher water velocity can be used. This extracts more heat from the slag bath producing a thicker slag shell which may be capable of remaining intact during operation of the furnace. The use of steam cooled tubular jackets enables greater amounts of heat to be removed without the possibility of boiling and scale deposit formation observed with water cooling. Although these solutions improve the performance of the jackets, higher rates of heat extraction would require a higher rate of coal consumption. Therefore the savings gained in longer jacket life may be offset by a higher coal demand.
The mathematical modelling studies indicated that the jacket would perform better if a higher conductivity material were used, such as copper. It is difficult to recommend this material since other factors including the thermal expansion of the copper and the hydrodynamic loadings of the bath have not been considered. However, tests carried out by Cominco personnel did not indicate that these would be problems and it was the cost of the sheet copper that inhibited the use of copper jackets. The modelling also recommended the use of a jacket fabricated from thinner plate but again this ignores the effects of hydrodynamic loadings from the bath. Ideally, a three-dimensional analysis could be performed which would include these factors.

**Furnace operation**

The contact between the slag shell and the jacket is believed to be poor leading to limited bonding and a high risk of slag fall-off. The results from model fitting of the panel/solid slag interfacial heat transfer coefficient suggests that an air gap separates the two. At the start of charging molten slag is dumped from pots into an almost empty furnace and the kinetic energy of the charged slag likely creates a highly agitated bath. The resulting surface waves would wash against the furnace walls and in a near empty furnace this action could remove the slag layer near the bottom of the furnace. As the furnace volume increases the slag bath is more capable of absorbing the kinetic energy of the charged slag. Therefore the slag layer higher up the jacket would be expected to remain intact. This may explain in part why the jackets fail just above the tuyere line and why the jackets near the charging end have to be replaced more frequently. If this is the mechanism for jacket failure then conversion to continuous fuming would eliminate these problems.
Tuyere design and changes in the coal rate

Since fluid flow in the vicinity of the tuyeres is a determining factor in the degree to which molten slag will remove the slag layer from the jacket, efforts should be made to minimize the effects of the tuyere blast. During normal furnace operation bubbles rising in close proximity to the walls could interfere with the slag freezing process, thus preventing good contact with the jacket. These problems could be reduced by jetting the coal-air mixture into the bath. This would also move the combustion zone away from the walls and thereby reduce the heat flux into the jackets. A more stagnant bath in front of the jackets may also reduce the time required for a new slag layer to freeze to the jacket and with a shorter slag renewal time the jacket could experience less damage.

It was noted in discussion of the experimental data that some of the thermal transients were associated with changes in the coal rate. It was suggested that the coal rate should be changed gradually as opposed to the sudden changes that are currently employed. Further research into the effect of the injected coal/air mixture should be performed. In particular, a fluid flow model in the vicinity of the tuyere region may reveal the hydrodynamic forces that are acting on the jacket causing the slag layer to be initially washed off.
Chapter 9. Conclusions

Industrial measurements of a zinc slag fuming furnace water jacket have been analyzed. The steady-state heat flux through the jacket is 80-120 kW/m² and primarily depends on the coal rate and the temperature of the molten bath. Thermal transients were observed approximately 20 cm above the centre tuyere with the midpanel temperatures reaching as high as 260°C. The generation of the thermal transients is associated with charging and tapping of the furnace and changes in the coal rate. The heat flux into the panel during the thermal transients can become as high as approximately 1.6 MW/m². The mechanism for the generation of the thermal transients is the complete removal of a patch of the frozen slag.

Mathematical modelling has been employed to simulate the thermal transients by removing a portion of the slag shell. The parameter that is critical in determining the magnitude of the thermal transients was found to be the panel/liquid slag heat transfer coefficient which may vary depending on slag properties and temperature. Thermal stress calculations indicate that the jacket yields throughout its thickness in the area where the slag patch is removed.

Recommendations in jacket design and furnace operation have been made in an effort to increase the service life of the jacket. These include increasing the stud density 20 cm above the tuyeres and jetting the coal air mixture into the bath. Future work is required in the area of fluid flow in the vicinity of the tuyeres and in determining the relationship between conditions in the molten slag and the panel/liquid slag heat transfer coefficient.
Chapter 10. References

19. Dr. G. Toop, Cominco, private communication.
31. Dr. G. Toop, Cominco, private communication.
Appendix - Finite Element Derivations

1.1. Heat Transfer

The governing equation for heat transfer in two dimensions can be expressed as:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) - \rho C_p \frac{\partial T}{\partial t} = 0 \quad (A.1.1)$$

In this form heat generation terms have been ignored since they are not relevant to this study. If the domain, $D$, is discretized into $m$ elements with $n$ nodes per element, the temperature distribution in a given element may be approximated by:

$$T^{(e)}_{(x,y)} = \sum_{i=1}^{n} N_i(x,y)T_i = \{N\}^T \{T\}^{(e)} \quad (A.1.2)$$

The terms $\{N\}$ are referred to as shape functions or interpolation functions. They approximate the behaviour of a field variable, for example temperature, over the domain of an element. Further, they are functions of position with the property that $N_i$ is unity at node $i$ and zero at all other nodes and this produces the correct temperature at each node.

Equation (A.1.2) is then substituted into Equation (A.1.1). Since Equation (A.1.2) is only an approximation, a residual error is produced upon substitution and this error is given by

$$R = \frac{\partial}{\partial x}\left(k_x \frac{\partial T^{(e)}}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T^{(e)}}{\partial y}\right) - \rho C_p \frac{\partial T^{(e)}}{\partial t} \quad (A.1.3)$$
where $R$ is termed the residual. To minimize the residual and thus choosing a set of $T^{(e)}$ to best approximate Equation (A.1.1), Galerkin's criterion minimizes the residual by using weighting factors which are the shape functions themselves. When these weighting functions are used the following integral is produced

$$\int_{D^{(e)}} N_i R dD^{(e)} = 0 \quad (A.1.4)$$

For a two dimensional heat transfer analysis, Galerkin's technique is expanded to give

$$\int \int_{D^{(e)}} N_i \left[ \frac{\partial}{\partial x} \left( k_x \frac{\partial T^{(e)}}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial T^{(e)}}{\partial y} \right) - \rho C_p \frac{\partial T^{(e)}}{\partial t} \right] dx dy \quad i = 1, 2, \ldots, n \quad (A.1.5)$$

To simplify the evaluation of the integral the order of the derivative on $T^{(e)}$ is reduced by integrating by parts the first two terms. In two dimensions this is performed using Green's theorem where

$$\int \int \phi \frac{\partial \psi}{\partial x} dx dy = - \int \int \frac{\partial \phi}{\partial x} \psi dx dy + \int_{\Gamma} \phi \psi n \cdot d\Gamma \quad (A.1.6)$$

The first term then becomes

$$- \int \int_{D^{(e)}} N_i k_x \frac{\partial T^{(e)}}{\partial x} dx dy + \int_{\Gamma^{(e)}} N_i k_x \frac{\partial T^{(e)}}{\partial x} n_x d\Gamma^{(e)} \quad (A.1.7)$$
where $\Gamma^{(e)}$ denotes the boundary of the element.

Insertion of Equation (A.1.2) into (A.1.6) leads to

\[- \int \int_{D^{(e)}} \frac{\partial N_i}{\partial x} k_x \frac{\partial N_j}{\partial x} \, dx \, dy \{T\}^{(e)} + \int \int_{\Gamma^{(e)}} N_i \frac{\partial N_i}{\partial x} n_x d\Gamma^{(e)} \{T\}^{(e)} = j = 1, 2, \ldots, n \]

(A.1.8a)

Similarly, the second term becomes:

\[- \int \int_{D^{(e)}} \frac{\partial N_i}{\partial y} k_y \frac{\partial N_j}{\partial y} \, dx \, dy \{T\}^{(e)} + \int \int_{\Gamma^{(e)}} N_i \frac{\partial N_i}{\partial y} n_y d\Gamma^{(e)} \{T\}^{(e)} = j = 1, 2, \ldots, n \]

(A.1.8b)

In Equation (A.1.8) the second terms represent the boundary residual noting that

\[k_x \frac{\partial T^{(e)}}{\partial x} n_x + k_y \frac{\partial T^{(e)}}{\partial y} n_y = -q^{(e)} - h(T^{(e)} - T_\infty) \]

(A.1.9)

Equations (A.1.7), (A.1.8) and (A.1.9) are combined to transform Equation (A.1.5) to:

\[- \{ \int \int_{D^{(e)}} \left( \frac{\partial N_i}{\partial x} k_x \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} k_y \frac{\partial N_j}{\partial y} \right) dx \, dy \} \{T\}^{(e)} + \int \int_{\Gamma^{(e)}} N_i q + h\{N\}^T \{T\}^{(e)} - hT_\infty d\Sigma^{(e)} + \int \int_{D^{(e)}} N_i \rho C_p N_j \, dx \, dy \left\{ \frac{dT}{dt} \right\}^{(e)} = 0 \]

(A.1.10)

again noting that
1.1 Heat Transfer

\[ T^{(e)} = \{N\}^T \{T\}^{(e)} \]

and

\[ \frac{\partial T^{(e)}}{\partial t} = \{N\} \left\{ \frac{dT}{dt} \right\}^{(e)} \]

Simplifications can be made by letting

\[ \left[ B \right] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \cdots & \frac{\partial N_s}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \cdots & \frac{\partial N_s}{\partial y} \end{bmatrix} \]

and for an isotropic material

\[ k = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \]

The matrix \([B]\) is the thermal gradient matrix and \([k]\) is the thermal conductivity matrix. The latter matrix has been reduced to only the diagonal terms since the influence of the thermal conductivity in one direction on the thermal conductivity in the other direction is not considered for an isotropic material.

The boundary terms that apply to the domain as a whole are referred to as fixed boundary conditions and they are introduced into the system of equations following the assembly of all the elements. The governing equation for heat transfer in two directions then transforms to:
1.1 Heat Transfer

\[
[C] \frac{dT}{dt} + ([K_c] + [K_h]) \{T\} = \{R_T\} + \{R_q\} + \{R_h\}
\]

(A.1.12)

where

i) \[ [C] = \int_D \int \rho C_p \{N\} \{N\}^T dx dy \] the capacitance matrix

ii) \[ [K_c] = \int_D \int [B]^T [k] [B] dx dy \] the thermal stiffness matrix

iii) \[ [K_h] = \int_{\Sigma_1} h \{N\} \{N\}^T d\Gamma \] the convection influence stiffness matrix

iv) \[ \{R_T\} = - \int_{\Sigma_1} (q.n) \{N\} d\Gamma \] isothermal load vector

v) \[ \{R_q\} = \int_{\Sigma_2} q_T \{N\} d\Gamma \] heat flux load vector

vi) \[ \{R_h\} = \int_{\Sigma_3} h T_w \{N\} d\Gamma \] convection load vector
1.2. Thermal Stresses

For a given two dimensional element the displacements at each node may be defined as:

\[
\{\delta\} = \begin{bmatrix}
\delta_1 \\
\delta_2 \\
\cdots \\
\delta_n
\end{bmatrix}
\]

(A.2.1)

In a similar manner to the temperature field above, the displacement field in an element may be expressed as

\[
u = \sum_{i=1}^{n} N_i(x, y) u_i \quad ; \quad \nu = \sum_{i=1}^{n} N_i(x, y) \nu_i
\]

(A.2.2)

where \(N_i\) are the same shape functions that were used in the heat flow model. The relationship between strain and displacement in two dimensions is given by

\[
\{\varepsilon\} = \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial u}{\partial x} \\
\frac{\partial v}{\partial y} \\
\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}
\end{bmatrix}
\]

(A.2.3)

and after including the shape functions
1.2 Thermal Stresses

\[ \sigma = [D][B]\{\delta\} \]  

(A.2.4)

The matrix \([B]\) is defined as the strain-displacement matrix. With this equation, the stress within the element is then given by

\[ \{\sigma\} = [D][B]\{\delta\} \]  

(A.2.5)

The matrix \([D]\) is the elasticity matrix given by

\[ [D] = \frac{E}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix} 1 - \nu & \nu & 0 \\ \nu & 1 - \nu & 0 \\ 0 & 0 & \frac{1 - 2\nu}{2} \end{bmatrix} \]  

for plane strain, \(E\) is Young’s modulus and \(\nu\) is Poisson’s ratio.

If the domain is heated or cooled from a stress free temperature field, \(T_0\), to a new temperature field, \(T(x,y)\), the thermal strains in an unconstrained domain governed by plane strain are expressed as,
1.2 Thermal Stresses

\[
\{\varepsilon_T\} = \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \alpha(1 + \nu)(T - T_0) \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}
\]

(A.2.6)

where \( \alpha \) is the thermal expansion coefficient. The total strain in the domain is governed by two components that interact: thermal strains result from the thermal expansion of the material leading and these strains are opposed by the rigidity of the structure. The thermal strains can be suppressed to zero by super imposing a thermal stress field over the domain,

\[
\{\sigma_T\} = -[D]\{\varepsilon_T\}
\]

(A.2.7)

The total stress field in the domain can then be expressed as

\[
\{\sigma\} = \{\sigma_r\} + \{\sigma_T\} = [D][B]\{\delta_r\} - [D]\{\varepsilon_T\}
\]

(A.2.9)

The principle of virtual work is similar to the residual mentioned earlier. For any domain under equilibrium, the internal work in the system should equal the external work applied to the system. In a model based on thermal stresses as in this study the external work is zero and the sum of the internal work is also zero. Therefore, energy leading changes in the thermal expansion of the material produce corresponding changes in the internal energy of the structure. If a small deviation in strain is introduced into the system the internal work can be expressed as

\[
W_i = \int_D \{\sigma\} \{\varepsilon\} dA
\]

(A.2.10)
where \( \{ \bar{e} \} \) is the virtual strain, noting that

\[
\{ \bar{e} \} = \{ B \} \{ \bar{\delta} \}
\]

Substituting in the expression for the total stress of the system produces

\[
W_i = \int_D \left( [D] [B] \{ \delta_s \} - [D] \{ \varepsilon_T \} \{ B \} \{ \delta \} \right) dA
\]

\[
= \int_D \{ \delta_s \}^T [B]^T [D] [B] \{ \delta \} dA - \int_D \{ \varepsilon_T \} [B] \{ \delta \} dA
\]

\begin{equation}
(A.2.11)
\end{equation}

The equation then takes the familiar form of

\[
[K] \{ a \} = \{ P \}
\]

\begin{equation}
(A.2.12)
\end{equation}

where

\[
[K] = \int_D [B]^T [D] [B] dA
\]

\begin{equation}
(A.2.13a)
\end{equation}

is the structural stiffness matrix and

\[
\{ P \} = \int_D [B] [D] \{ \varepsilon_T \} dA
\]

\begin{equation}
(A.2.13b)
\end{equation}

is the load vector resulting from the thermal expansion.
1.3. The Eight-noded Isoparametric Element

The use of isoparametric elements makes it possible to better estimate a domain with curved boundaries. The element is first solved in a local coordinate system in $\zeta - \eta$ space, as in Figure (A.3.1), and then mapped into the global coordinate system. It is necessary that when the element is mapped, any point in the global coordinate system has a unique point in the local coordinate system. An eight-noded isoparametric element is shown in Figure (A.3.1). Cockroft\(^1\) [thesis?, private communication] examined several different elements and determined that an eight-noded isoparametric element gave an acceptable stability for a given number of nodes especially when solidification problems are being considered.

Figure (A.3.1): Eight-noded isoparametric element in (a) Cartesian space and (b) local coordinates in $\zeta - \eta$ space.
1.3 The Eight-noded Isoparametric Element

The eight-noded quadrilateral element has three nodes defining each boundary. Therefore, in order to map the element quadrilateral shape functions are required to estimate domain properties at any point in the domain. The shape functions are defined in a local coordinate system defined in $\zeta-\eta$ space and converted to x-y Cartesian space by an appropriate transformation. These shape functions in $\zeta-\eta$ space are calculated at each node using the following equations:

a) for nodes at $\zeta = \pm 1, \ \eta = \pm 1$

$$N_i(\zeta, \eta) = \frac{1}{4}(1 + \zeta \zeta_i)(1 + \eta \eta_i)(\zeta \zeta_i + \eta \eta_i - 1) \quad (A.3.1a)$$

b) for nodes at $\zeta = 0, \ \eta = \pm 1$

$$N_i(\zeta, \eta) = \frac{1}{2}(1 - \zeta^2)(1 + \eta \eta_i) \quad (A.3.1b)$$

c) for nodes at $\zeta = \pm 1, \ \eta = 0$

$$N_i(\zeta, \eta) = \frac{1}{2}(1 + \zeta \zeta_i)(1 - \eta^2) \quad (A.3.1c)$$

These functions have the characteristic that when $\zeta = \zeta_i$ and $\eta = \eta_i$ the value is unity but at any of the other nodes $\zeta \neq \zeta_i$ and $\eta \neq \eta_i$ the value is zero. Therefore, using a simplified notation, nodal properties are defined as:
1.3 The Eight-noded Isoparametric Element

\[ x = \sum_{i=1}^{8} N_i(\zeta, \eta)x_i \]  
\[ y = \sum_{i=1}^{8} N_i(\zeta, \eta)y_i \]  
\[ T = \sum_{i=1}^{8} N_i(\zeta, \eta)T_i \]
1.4 Evaluation of Element Matrices

Again concentrating on an eight-noded element, a given field property, $\phi$, can be expressed as

$$\phi = \sum_{i=1}^{8} N_i \phi_i$$  \hspace{1cm} (A.4.1)

where $N_i$ are the shape functions mentioned in the previous section. If the shape functions were defined in $x$-$y$ space their derivatives can be expressed as:

$$\frac{\partial \phi}{\partial x} = \sum_{i=1}^{8} \frac{\partial N_i}{\partial x} \phi_i$$  \hspace{1cm} (A.4.2a)

$$\frac{\partial \phi}{\partial y} = \sum_{i=1}^{8} \frac{\partial N_i}{\partial y} \phi_i$$  \hspace{1cm} (A.4.2b)

The shape functions are then expressed in local coordinates, $\zeta-\eta$ space, using the chain rule,

$$\frac{\partial N_i}{\partial \zeta} = \frac{\partial N_i}{\partial x} \frac{\partial x}{\partial \zeta} + \frac{\partial N_i}{\partial y} \frac{\partial y}{\partial \zeta}$$  \hspace{1cm} (A.4.3a)

$$\frac{\partial N_i}{\partial \eta} = \frac{\partial N_i}{\partial x} \frac{\partial x}{\partial \eta} + \frac{\partial N_i}{\partial y} \frac{\partial y}{\partial \eta}$$  \hspace{1cm} (A.4.3b)

and in matrix form they become,
1.4 Evaluation of Element Matrices

\[
\begin{align*}
\left\{ \frac{\partial N_i}{\partial \zeta} \right\} &= \left[ \begin{array}{cc} \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{array} \right] \left\{ \frac{\partial N_i}{\partial x} \right\} \\
&= [J] \left[ \begin{array}{c} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{array} \right] \quad \text{(A.4.4)}
\end{align*}
\]

The matrix \([J]\) is termed the Jacobian matrix and is evaluated for an eight-noded element in \(\zeta-\eta\) space by substituting Equation (A.3.2):

\[
[J(\zeta, \eta)] = \left[ \begin{array}{c} \sum_{i=1}^{8} \frac{\partial N_i}{\partial \zeta} (\zeta, \eta)x_i \\ \sum_{i=1}^{8} \frac{\partial N_i}{\partial \eta} (\zeta, \eta) \end{array} \right] \quad \text{(A.4.5)}
\]

To convert the results to the global coordinate system the Jacobian matrix is inverted to change Equation (A.4.4) to:

\[
\left\{ \frac{\partial N_i}{\partial x} \right\} = [J]^{-1} \left[ \begin{array}{c} \frac{\partial N_i}{\partial \zeta} \\ \frac{\partial N_i}{\partial \eta} \end{array} \right], \quad i = 1, \ldots, 8 \quad \text{(A.4.6)}
\]

The derivatives of the field variable may now be defined as
1.4 Evaluation of Element Matrices

\[
\begin{bmatrix}
\frac{\partial \phi}{\partial x} \\
\frac{\partial \phi}{\partial y}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \cdots & \frac{\partial N_8}{\partial x} \\
\frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \cdots & \frac{\partial N_8}{\partial y}
\end{bmatrix} \begin{bmatrix}
\phi_1 \\
\phi_2 \\
\vdots \\
\phi_8
\end{bmatrix}
\]

\[= [J]^{-1} \begin{bmatrix}
\frac{\partial \zeta}{\partial \xi} & \frac{\partial \zeta}{\partial \xi} & \cdots & \frac{\partial \zeta}{\partial \xi} \\
\frac{\partial \eta}{\partial \xi} & \frac{\partial \eta}{\partial \xi} & \cdots & \frac{\partial \eta}{\partial \xi}
\end{bmatrix} \begin{bmatrix}
\phi_1 \\
\phi_2 \\
\vdots \\
\phi_8
\end{bmatrix}
\]

(A.4.7)

The transformation enables the element integrals to be solved by Gauss-Legendre quadrature over the unit square element defined in \(\xi - \eta\) space noting that

\[dxdy = |J| d\zeta d\eta\]