THE ESL STUDENT IN THE MATHEMATICS CLASSROOM: 
STUDENT QUESTIONS AS A MODE OF ACCESS TO KNOWLEDGE 

By 

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Abstract

Over the past decade, a sizeable body of research has addressed issues in metacognition, the way in which the learner plans, implements and monitors cognitive behavior (Garofalo and Lester, 1985). This type of consideration is of interest to studies which try to build models of human cognitive process for such applications as artificial intelligence and/or curriculum development.

To form one’s own mental map of a body of knowledge is to discover a structure of, or to impose a structure on, that body of knowledge. In the case of secondary school mathematics curricula, the student is typically discovering structure which is to some degree made explicit in the presentation of the material. However, when the language of instruction is not the student’s first language, when the student is unaccustomed to many of the communication conventions of the language of instruction and of the subject register as well, fewer assumptions can be made about how the student is navigating around the body of knowledge.

In this study, the relatively scarce questions asked by ESL (English as a Second Language) students in a secondary school English-speaking mathematics classroom were observed over time. The data provide some
evidence of the natural manner in which the students attempt to form a mental map of the body of knowledge under exploration.

The body of research on classroom questions (e.g. Sinclair and Coulthard, 1975) has focused almost entirely on questions asked by the teacher. Questions asked by students differ in both form and intention from questions asked by teachers, however; as a result the methods of analysis employed in studies of teacher questions are inappropriate for the analysis of student questions. A more appropriate method of analysis for this study's examination of student questions about a body of knowledge was found to be an ethnographic one which regarded questions as a means of eliciting aspects of a structured knowledge domain. Mohan's (1986) knowledge framework, which embodies a structured taxonomy of topics and tasks, is used here to categorize the data according to the type of knowledge sought through each student question.

Observed differences between the surface content of student questions and the context-apparent intention of these questions provide some insight into how students may be assisted to better ask the questions which they use to seek help in their navigation of bodies of knowledge. Published teaching materials intended for ESL students of secondary mathematics are examined here for relevance to the students' need to develop help-seeking strategies; suggestions for more effective accommodation of this need are made. Computer software developed by the researcher for exploration of possibilities in computer aided instruction in question formation is described.
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Chapter I
The ESL Student in the Mathematics Classroom: An Overview

1.1 The ESL Mathematics Student as Ethnographer

Ethnographers attempt cultural description from contact experience with members of a culture, or better, from experience of living within a culture. Working from Stenhouse's (1967, p.16) definition of culture as "a complex of shared understandings which serves as a medium through which individual human minds interact in communication with one another", Bishop (1988) argues that mathematics is a culture, given that it is just such a complex of understandings.

The ESL student arrives in the mathematics class in much the same position with regard to information as the ethnographer in a community: language is an obstacle to communication, there is information (encoded culturally within the scenario) which dictates behavior, and elements of the scenario may be similar to elements in the observer's home culture. The goals of the two cases of observation/participation are strikingly different: the ethnographer is hoping to translate the observations of culture into a description in language appropriate to the home culture, while the ESL math student is trying to form a mental map of the body of knowledge (mathematics) so as to be able to enter the culture on a full-membership basis. Another pertinent difference lies in the fact that the ethnographer is able to decide which aspects of the culture to investigate, and in which order; in most of today's mathematics instruction this is not the case (Bishop, 1988). The teacher and the curriculum
and to some extent the design of the textbook/activities/materials will decide which mathematical topics will be revealed to the learner, and how. As well, the mode of exploration is to a great extent imposed on the learner.

Despite the external control of topic and view of topic, the learner nevertheless has considerable autonomy in the matter of interpreting and structuring the information received. Werner and Schoepfle (1987, p.30) make this point in their comparison of ethnographers and human learners:

Ethnoscience analytical skill consists of the ability to break down the sequential, linear order of the texts into the non-linear, network-like semantic structures that underlie these texts. This process is analogous to the operations human beings perform when they reduce linear speech to nonlinear, multidimensional knowledge in permanent memory.

1.2 The Need for a Knowledge Framework

Werner and Schoepfle (1987) go on to postulate the need for a knowledge framework in order to examine any body of knowledge, and propose the following knowledge typology (which goes beyond the usual descriptive and classificatory anthropological activities of eliciting folk definitions and building taxonomies thereof):

1. Structural analysis: elicit surface structure in the form of folk definitions and apply it to learning the use of simple cognitive structures.
2. Using tree graphs or similar devices, develop taxonomies, and an inventory of taxonomies, of objects, events, and other cultural symbols.

3. Taxonomies speak to relations; transform texts into verbal action plans embodying these relations.

4. Observe informants deciding between competing action plans; using the implicit causal relations, produce multilevel tree structures (flowcharts) which delineate the decision process.

5. Values are taxonomically themes; evaluation is probably the most fundamental capacity of all living organisms. (Until recently in anthropology, themes and values were not thought amenable to formal analysis.)

Werner and Schoepfle (1987) maintain that as far as structuring the input one receives from informants, the efficient ethnographer alternates text elicitation, for the identification of important clues about the culture of the natives, with structural questioning. The result of classroom mathematics instruction in the textbook-driven, instructor-driven model is that the learner is generally not required to elicit information. Bishop (1988) deprecates the exclusive use of this form of instruction, claiming that it deprives the learner of the opportunity for autonomous enculturation.

Of course the native speaker of English also approaches mathematics as a new culture (Bishop, 1988), but language-related problems are subtler: the mathematics register in English Canada is rooted in English for the most part, and communication obstacles can be dealt with by student and informant (teacher) in casual language which is comfortable for both. The ESL student of
mathematics, however, faces one more obscuring layer of linguistic interference: the observer and the informant do not share much external language to which they can resort in the interests of repairing meaning. The ESL student is obliged to learn the language of the informant in its conversational form, and as well must learn the specific elements of language unique to the informant's specialized vocation/avocation (here, teacher/user of mathematics).

1.3 Meeting the Needs of the ESL Learner of Mathematics

Research towards development of instructional materials for ESL students of mathematics, as exemplified by Dale and Cuevas (1987) and Spanos, Rhodes, Dale and Crandall (1988), has for the most part aimed to extract elements of existing mathematics curricula and to build activities and tasks which support the language demands involved in performing within the mathematics context. The present approach attempts to go further, to account for the students' problem processing experiences and for individual differences in students' approaches to forming a mental map of the body of knowledge under consideration.

In the mathematics instruction of an individual learner, once the teacher has introduced a specific concept or skill, the next phase of instruction may be seen as a collection of structured teacher reactions (refinement and/or remediation of learner behavior) to learner performance and to learner utterance (see fig.1.). A number of sources (e.g. Pimm, 1987) hold that most mathematics instruction at the present time fails to consider learner performance adequately. Moreover, learner utterances are seldom analysed as
Figure 1. A model of possible teacher responses to learner utterance.
representations of their interpretations of knowledge or their socialization into the specific register of mathematical language. Learners' utterances are a potentially rich source of insight into learners' needs concerning both language development and development of a personal mental map of the body of knowledge being studied.

1.4 This Study

This study focuses on one behavior (question formation by ESL students) and its semantic content (intention), rather than on the entire range of behaviors and values examined in most ethnographic work. The phenomenon under observation is the verbal behavior of the subjects as they work in an ethnographic manner towards grasping the content, function and structure of a specific body of school knowledge. This study is an exercise in participant observation, yet the data recorded are narrow: students' questions of the teacher. These student utterances are one of very few sources of evidence of how a student navigates around a body of knowledge. Other potential sources, such as students' notes and self-directed (think-aloud) utterances, are not easy to collect in a revealing form without altering students' naturally-occurring behavior in the classroom.

This study was carried out in the researcher's own ESL mathematics class at a secondary school in Vancouver. The subjects, who are referred to by pseudonym throughout, were the students enrolled in the class. Approximately 15 in number, the students were being prepared for entry into regular mathematics classes at the grade 8, 9 and 10 levels.
Longitudinal observation of ESL students in Vancouver secondary schools is difficult. It is not uncommon that in his first two years in Vancouver, the ESL secondary school student may transfer from school to school as many as four times, the moves typically prompted by initial lack of space in schools near the home, family relocation and eventual enrolment in higher status schools. As well, the ESL student will often be promoted from level to level in some courses (especially mathematics and science) during the school year.

ESL students come from a variety of home country situations, ranging from affluence to poverty, and the students' educational backgrounds reflect this. Within one ESL math class there will typically be some students from Hong Kong who have strong mathematics background and are in the class for a few months to develop some communicative competence in English, and at the other extreme, some students from strife-torn countries such as El Salvador who have had interrupted education and may take quite some time to develop basic classroom behaviors, and years to qualify for entry into secondary level mathematics courses.

The approach taken by Crandall et al (1987) in English for Algebra, explicitly teaching the language underlying various mathematical topics and activities, was incorporated into the teaching methodology in the ESL math class under examination. In addition, some effort was made to extend the content of that source to incorporate instruction in help-seeking questioning (HSQ) with the thought in mind that perhaps teaching the range of questions a student might ask about a given problem might not only provide the student with model language, but might also implicitly demonstrate to the student the structure of generic problem solving technique. The content of the instruction in
HSQ is based on the recorded observations of the natural-language questions which arise in ESL math classes.

Research consisted of several connected activities: observation and recording of student questioning, analysis of recorded questions according to the type of information sought, articulation of student needs with regard to help seeking, examination of relevant instructional materials, and exploration of application of the findings to hypermedia instruction. These activities are discussed individually below.

1.4.1 Observation: student production of help-seeking questions.

The technique employed in the collection of data for this study was essentially the audio tape recording of discourse in the process of ordinary mathematics lessons, with an additional feature: the mathematical tasks assigned to the students were often given an "information gap" aspect. All questions asked by students in the course of ordinary classroom instruction and activity were recorded with a hand-held microcassette recorder. Student curiosity led to the practice of replaying collections of questions, initially for the students' amusement and eventually for class discussions of language found in the recordings. The distinction between instructional approach and data collection technique is in danger of becoming blurred in such a situation, but student questions were left to arise naturally, and teacher requests for rephrasing or clarification of student questions may be seen as ordinary teacher behavior. The situation of the observer-teacher here is analogous to the ordinary teacher's situation when simultaneously responding to a student's question, assessing the state of the student's knowledge and competence, and guiding the student toward more effective performance.
Two aspects distinguish the observation technique employed in this study from the "secret tape recorder" technique: (1) each time the teacher judges a student utterance to be linguistically deficient, the student is asked to improve the utterance (this leads to a focus of student attention on the form of questions), and (2) the rather abrupt introduction of the recorder into each student-teacher exchange initially influences the casual nature of the exchange (within one or two sessions students no longer gave any indication of being distracted by the procedure).

Some written notes accompany the transcripts in an attempt to account for the non-verbal and symbolic elements of discourse which complement the oral aspect of communication. A small number of case studies are also included, in the interest of exploring student variability and individual growth with regard to questioning style.

1.4.2 Analysis of data

Given the transient nature of the class population, the question asked by the learner, rather than the individual human subject over a long period of time, was selected as the unit of analysis. Given the extreme variability of the subjects, analysis of the data was qualitative rather than quantitative, although high-frequency help-seeking questions were noted. Qualitative analysis shows that students' questions fall into the six categories of Mohan's (1986) knowledge framework, as well as the categories outlined by Werner and
Schoepfle (1987) as comprehensive for the examination of the body of knowledge that is a culture. The difference between the two is that Werner and Schoepfle (1987) merely present the categories as elements of a knowledge structure, whereas Mohan (1988) arranges them in a theory/practice framework of six major types of knowledge into which topics or content can be divided (see fig. 2).

The intended content of each question was deduced from ensuing context, and was assigned a place in Mohan's (1986) knowledge framework. This was done in the interest of examining the extent to which students' questions are appropriately framed with regard to the information sought. Such an examination of student performance is a potential mode of teacher access to student navigation, and provides some insight into patterns of student questioning and knowledge acquisition.

As well, during the course of the observation in the present study, there were two different mathematical topics being studied: probability (predominantly word problems) and then algebra (mostly symbol manipulation). The distributions of question type over Mohan's (1986) categories of knowledge during the two periods of instruction are compared in section 6.3 to tentatively explore whether students ask different kinds of questions about different mathematical topics.

Coding was carried out by assigning each item of data to one category of Mohan's (1986) six-category knowledge framework or to a seventh category, grand tour questions. This coding was repeated twice by the researcher as a simple check on reliability.
General, theoretical (background knowledge)

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Specific, practical (action situation)

Figure 2. Knowledge framework of Mohan (1986)
This method of coding posed some unique problems, because the questions were so highly dependent on the context in which they were asked. Specifying the coding criteria was therefore difficult because the received evidence was just the question itself. A more elaborate study might incorporate video recording, and have a consistency check using a second coder, perhaps an experienced mathematics teacher.

1.4.3 Interpretation: examination of instructional needs.

From this emergent question-based description of learners' questioning about mathematical knowledge, a set of instructional needs are postulated to guide development of new instructional/learning materials for the development of ESL students' proficiency in question formation. In view of this description of the questions which students do ask, some further questions arise, questions such as:

1. How may students better ask these questions (in terms of language)?
2. What other questions or types of questions might students ask to advantage in the interests of mathematical learning?
3. How can the development of the use of such questions be fostered by learning experiences provided by the teacher?

It is not only the intent of this study to begin to provide answers to questions such as these, but also to present a framework for the analysis of the underlying parameters.
1.4.4 Application: examination of the link between the nature of student questions (in the abstract) to questions in the context of problem solving.

In light of the above observations and suggested instructional needs, selected materials from Crandall et al's (1987) *English Skills for Algebra* are examined for relevance to the subjects' learning needs: do these published materials address the question-formation needs of the students observed in the present study? This examination is intended only to suggest an application of the analytical approach outlined here.

1.4.5 Development: exploration of application of findings to hypermedia instruction.

If computer-assisted instruction (CAI) is to move away from "electronic workbook" status towards active/creative/personal use of the computer, then there is a need to build in functions (as in Macintosh computer's HyperCard application) which extend the student's senses and impulses. There is a considerable volume of recent research in this area. Swallow, Scardamalia and Olivier (1988), for example, explore the impact of software support designed to aid students in directing and monitoring their peer partners' extraction and expansion of information gained from reading. The use of such software support for students' cognitive and metacognitive activities may be seen to use interactive text ("hypertext", [Nelson, 1987]) applications to create a new sort of learning environment.
Browsing in a HyperCard stack may be seen as analogous to asking questions in the math classroom. If the HyperCard stack design models the navigation through a math problem, then students are prompted to engage in one problem solving approach relevant to their learning in this domain. If the HyperCard buttons (linking devices) have the visual form of help-seeking questions which model central expressions and vocabulary, then the student will be at least exposed through representative language to the underlying cognitive and metacognitive considerations which arise in the problem's solution.

This study's findings about high-frequency student questions and deficiencies in these questions have been applied here to the development of such HyperCard stacks (interactive documents). These stacks are intended to expose the student to patterns of questioning in the course of following the steps in the application of algorithms to sample mathematics problems. Eventually the student would be expected to build similar stacks and supply appropriate questions for user access to the information in the stack.
Chapter II
Literature Review

2.1 Introduction

Students for whom English is a second or additional language experience language-related difficulties in the course of their mathematics studies with English as a medium of instruction (Mestre, 1988). To date, relatively few studies have examined mathematical performance by bilingual students, although there has been a recent increase in the number of such studies, reflecting contemporary social developments. In North America, England and Australia, large numbers of immigrants have changed the makeup of classroom populations; in many third world countries the predominance of English in world communications has led educational planners to use English language materials. As well, large bilingual populations in Canada and the United States have recently drawn considerable attention (and funding).

The scope of this literature review will include language-related issues in mathematics learning and performance, applied research, and some research in related fields which either bears on the topic or suggests useful research approaches. The literature search was conducted by tracing bibliographical references from prominent journals in the fields of mathematics education research, English as a second language instruction, and cognitive psychology. Other sources of references pursued were ERIC searches and personal
communications with six presenters in the ethno-mathematics section of the Sixth International Council of Mathematics Educators in Budapest in July of 1988.

In this review, an attempt is made to articulate language related issues in ESL mathematics acquisition and performance as found in recent and earlier publications; then directions of contemporary research, as suggested by recent developments in psychology and linguistics, are outlined.

2.2 Language Related Issues in Mathematics

Research in the broad area of mathematical learning and performance by ESL students began long before the topic was named; in some ways the following subtopics and the order in which they are presented show the history of the development of this field of interest.

2.2.1 Effects of Language on Mathematical Performance

The effects of language factors on performance in mathematics have long been recognized (Monroe & Engelhart, 1931), but as late as the 1960s and 1970s discussion and research were generally confined to studies concerning readability of text, reading ability of students, and teacher-student verbal exchange. Almost never did ESL come under consideration with regard to mathematical performance (Austin & Howson, 1979).
2.2.2 The Language of Mathematics and the Mathematics Register

When mathematicians (and mathematics teachers) speak about mathematical topics, they use special rules and definitions. However, this phenomenon has only gradually been elucidated in the research literature. Some earlier studies did focus specifically on the language used in talking about mathematical concepts, processes and decisions. For instance, Munroe (1963) spoke of "Mathese", the language of mathematics, but abandoned attempts to standardize it, finding it too varied and inconsistent for prescriptive generalization. Ausubel and Robinson (1969) drew a parallel between learning algebraic symbols and syntax and learning a second language, but found the many-to-one correspondence of algebraic symbols to numerical symbols too difficult to include in their psycho-educational model. Primary studies reported by Rimoldi, Aghi & Burder (1968) and Proctor and Wright (1961) used think-aloud protocols to examine problem solving and tactical thinking, and found variation by individual in encoding and translation procedures. Studies such as the above had some bearing on, but did not directly address, the situation of the ESL student of mathematics.

Interest in the problems of mathematics learners whose first language differs from the language of instruction found its first major expression at a UNESCO conference in Nairobi in 1974. The focus of the conference was linguistic aspects of mathematics education in Anglophone developing countries. Perhaps the most enduring presentation was made by Halliday (1974), who addressed language difference (or 'distance') as an instructional obstacle and developed a "register of mathematics" which to this day is considered definitive in discussions about language in mathematics.
2.2.3 Languages and Logic

The issue of whether or not logic, an underpinning of mathematical behavior, is language-governed was raised by Whorf (1956), and the ensuing debate raged for years. Gay and Cole (1967), Bloom (1981), Zepp (1982), and Zepp, Monin and Lei (1987) conducted correlational studies in this area; none found empirical support for Whorf's much-bandied hypothesis that logic is to some extent language-governed. What was observed in the statistical evidence, however, was that knowledge of the mathematical and English usages of logical connectors significantly influences problem solving performance (see, e.g., Zepp, 1982). This has proven to be a pivotal notion for addressing pedagogical concerns regarding the mathematics register and mathematics instruction for ESL students, as may be seen in Orr's (1987) extensive study of the disabling effect of black American student's first dialect-based misunderstandings of logical connectors.

2.2.4 Semantic Patterns in Mathematics Discourse

As seen in the case of logical connectors, students' proficiency with the syntactic patterns of natural English is no guarantee that the learners have acquired the semantic patterns which constitute mathematical discourse. Mathematics register-specific vocabulary is an obvious example: Spanos, Rhodes, Dale and Crandall (1988) point as well to the multiple ways language can be used to signal the same mathematical operation, and the perplexing array of signals of reference. Lochhead (1980) reports findings that more than 40% of high school mathematics teachers and college mathematics faculty
sampled made errors when translating from natural language to mathematical register.

2.3 Second Language Issues in Mathematics

2.3.1 Bilinguals and Problem Solving

The considerable efforts in the last decade to develop frameworks, instruments and procedures for the study of language and cognition in the bilingual, particularly the child, are well represented by the work of Jim Cummins. Cummins' "developmental interdependence" hypothesis (Cummins, 1978) concerning the interrelation of the development of L1- and L2-based cognitive skills, and his "threshold" hypothesis (Cummins, 1981), concerning the necessity of some minimal L1 development for L2 cognitive development, are benchmarks. Cummins' cognitive academic language proficiency (CALP) refers to some proficiency in the context-reduced language related to cognitive tasks. Dawe (1984) modifies Cummins' CALP to (the narrower) cognitive, analytical mathematical proficiency (CAMP) which he shows to be a factor in bilinguals' development of mathematical reasoning ability in his thorough statistical treatment of data concerning performance of large groups of bilingual and monolingual children (Dawe, 1985).

Burns, Gerace, Mestre, and Robinson (1983) similarly expanded on Cummins' linguistic threshold notion, proposing a technical threshold, a minimum level of language skill and cognitive skill necessary for solution of technical problems. Their research findings agree with those of Dawe (1983; 1984): minority students were much more likely than non-minority students to
misinterpret word problems, and slower reading speed and comprehension caused minority students to leave more problems incomplete.

Bilingual problem solving studies in Ireland (Macnamara, 1967), Quebec (d'Anglejean, Gagnon, Hafez, Tucker, and Winsberg, 1970) and the United States (Mestre, 1984; Duran and Enright, 1983) show bilinguals to be slower in L2 than in L1 in solving syllogisms. Possible explanations given were limited familiarity with L2, and insufficient time for subjects to complete the instrument. Duran (1988) showed through statistical analysis that other factors correlated with the observed effects on bilinguals' logical reasoning: parental education, socioeconomic level, parent-child interaction style, and home-school compatibility. From an ethnographic study of mathematical problem solving by Ute Indian students, Leap (1988) comes to similar conclusions.

De Avila's (1988) research criticizes the long-standing argument that bilinguals' poor academic performance in mathematics is attributable to bilingualism. De Avila proposes that motivation, interest, opportunity and access are significant factors. This leads to the author's claim that if classroom organization provides language minority students with access to home language support, peer consultation, manipulation and other physical contexts, the students will acquire English language proficiency and basic skills as easily as do mainstream students.

The ESL student in the mathematics classroom encounters a tangle of languages: L1 as the most likely mode for cognitive processing, L2 interlanguage as the mode of self-expression, and a varying combination of native L2, classroom procedural language, and mathematics register as input.
Stone (1988) characterizes the problem facing the teacher in such a situation as being to establish the lexical and syntactic character of instructional utterances so that these utterances will become conventionally associated with the intended effect. Mestre (1986), examining issues such as technical threshold and CAMP, suggests that mathematics instruction for ESL students, or perhaps for all students, should be based on an approach that integrates mathematical skills and language skills.

2.3.2 Discourse Features in Problem Solving

Written mathematical discourse is difficult for native speakers as well as for ESL students (Dale and Cuevas, 1987); it does not allow for embellishment or alternate phrasing and differs in many ways from ordinary written English (Bye, 1975). Context is often either opaque or unrelated to the student's experience. For this reason Pimm (1987) suggests there has been an augmenting of the mathematics register by means of metaphors which transfer terms from ordinary language (extra-mathematical metaphor). A second type of metaphor, the structural metaphor, associates a mathematical operation (e.g., subtraction) with a real-world process (e.g., taking away); a barrier to understanding even for native speakers, the structural metaphor must taken into account in mathematics-related language instruction.

Pimm's (1987) work reflects the influence of the findings of discourse analysis studies of classroom communication conducted by Stubbs (1983). Discourse analysis (defined as the study of spoken communication) has been used for quite some time in the examination of problem solving learning and instruction (Proctor and Wright, 1961; Schoenfeld, 1985). Spanos, Rhodes, Dale and Crandall (1988) analyzed classroom dialogue, using Morris' (1955)
syntactics/semantics/pragmatics semiotic framework to examine the difficulties faced by ESL students attempting to acquire the mathematics register; the authors state that the results served as a basis for their proposed teaching approach (Crandall, Dale, Rhodes, and Spanos, 1987) built around interactive exercises and a variety of language activities aimed at developing student understanding of, and competence in using, mathematics language.

2.3.3 Integration of Instruction: Mathematics and ESL

Mestre (1986) suggests that mathematics instruction for ESL students and perhaps for all students, be based on an approach that integrates mathematical skills and language skills. This proposal is timely as research on problem solving in mathematics and science education has been extensive in the 1980s, and language related concerns are quite well represented in that research. Cuevas (1981) drew on the work of Halliday (1974) and Cummins (1979, 1981) and diverse elements such as the notional-functional syllabus to formulate his second language approach to mathematics skills (SLAMS) model for integrating language skills in the mathematics classroom. Using this approach, Crandall, Dale, Rhodes, and Spanos (1987) developed a pioneering set of instructional materials to assist ESL college students in basic algebra.

In counterpoint to SLAMS, Dale and Cuevas (1987) propose the use of mathematics materials in the ESL classroom, specifically in problem solving activities, thus bringing attention to a long-standing and widely-held belief that mathematics requires minimal language proficiency. Mohan (1986) also makes a (more general) case for this integration of instruction in language and in subject area content, and suggests extensive use of visual/tactile means of
clarification and reinforcement of language-mediated mathematical meaning. Cazden (1979) notes that mathematics tends to be a silent, individual activity; this also suggests that teachers must make provisions in their instructional planning both for quality input of mathematics related language and for student talk.

2.4 Discourse in the (Mathematics) Classroom

2.4.1 Studies of the Structure of Classroom Discourse

The studies of Bellack, Kliebard, Hyman and Smith (1966), Sinclair and Coulthard (1975) and Mehan (1979) were all concerned with discovering the structure of classroom discourse. The dominant concern in all three was the content and intention of teachers' utterances.

Bellack et al. examined pedagogical concerns through the analysis of the linguistic behavior of teachers and students in the classroom; the unit of analysis was the pedagogical move, which constituted a dimension of meaning in teachers' utterances.

Sinclair and Coulthard (1975) characterized classroom discourse as for the most part following a pattern described as initiation-response-feedback (I - R - F): the teacher asks a question, the student answers, the teacher evaluates the answer. This research was important for its attempt to address the analysis of extended spoken discourse, rather than isolated sentences. The 'act' was regarded as the central concept in their treatment of meaning.
Mehan (1979), in his exploration of structure and structuring of classroom events, adopted Sinclair and Coulthard's (1975) I - R - F characterization of typical classroom interactions. Mehan's work may be seen as a "constitutive ethnography", more concerned with portraying how students are socialized into classroom (lesson) conduct than with content related concerns about classroom discourse.

2.4.2 Student Talk in the Mathematics Classroom

Pimm's (1987) study of communication in mathematics classrooms also sees the largest part of classroom discourse as following Sinclair and Coulthard's (1975) I-R-F pattern; Kress (1985) views this as a manifestation of the teacher's role as key agent in the reproduction of culture, managing the construction of classroom meanings. Students do initiate interactions with the teacher, though, and some studies have examined interactions initiated by student questions; there is some evidence that teachers typically convert student questions into occasions for teacher questions (Mishler, 1975).

Pimm (1987) characterizes student talk in secondary mathematics classrooms as being of three types:

1. Talking for others (to communicate).
2. Talking for oneself (aloud, to organize thoughts)
3. Talking so that the teacher may gain access to and insight into the ways of thinking of the students. This extends beyond Barnes' (1976) explanatory and exploratory forms of talk.

Students do ask the teacher questions in the mathematics classroom, and many are of the "referential" type (genuinely seeking information) rather than the
2.5 Metacognition

2.5.1 Metacognition and ESL Problem Solving

Studies of metacognition (taken here to refer to the active monitoring, regulation, and control of one's own cognitive processes) provide insights into and models of problem solving. Polya's (1957) four-phase description of problem solving activity serves as a framework for identifying heuristic processes. Basing his work on Polya's framework, Schoenfeld (1983) has devised a scheme for parsing think-aloud protocols into episodes in order to analyze problem solving "moves". Garofalo and Lester (1985) extended Polya's framework itself, distinguishing cognition (doing math) from metacognition (choosing, planning and monitoring action).

One of the first studies of ESL problem solving to use the frameworks developed in metacognition research, Kessler, Quinn, and Hayes (1985), shows that ESL children's mathematics-based L2 development will be inhibited (a) if the children believe that math is hard or that they will not perform well on mathematical tasks, or (b) if they are limited to rote strategy usage.
2.5.2 Ethnographic Studies of Mathematics Learning/Teaching

Mathematical learning and performance depend heavily on conceptual competence, with computational competence in a secondary position, especially with the advent of the calculator and the personal computer. Central to discussion of the conceptual bases of mathematics learning must be a notion of structure, and central to the teaching of mathematical structures must be some consideration of learner's intellectual capacity. Historically there have been three theoretical perspectives on the psychological structure of mathematics, found in the theories of the gestalt movement, of Piaget, and of cognitive psychology (Resnick and Ford, 1981). Gestalt theory sees perception and thinking as organized into functional wholes. Piaget postulated that logical structures in the human mind (developed through active interaction with the environment) determine the individual's understanding of mathematical events and acts. Radically different with its emphasis on descriptive and predictive models of learner behavior, cognitive psychology is strong concerned with thinking processes (Keane, 1988; Kintsch, 1988).

Research on ESL students' mathematics performance and learning seldom articulates its psychological underpinnings, but for the most part such research rests loosely on a Piagetian base, despite the fact that research on first language aspects of mathematics problem solving has been peppered with predictive models (in the form of computer programs) based on think-aloud studies for over a decade (Greeno, 1978). Certainly such recent studies as Kintsch (1988) and Kintsch and Greeno (1985), which develop models of humans' arithmetic word problem solving, are at the intersection of language
and mathematics performance; certainly the design of such studies is applicable to the ESL case.

Moving away from the intrusive think-aloud method of observation, Spanos, Rhodes, Dale, and Crandall (1988) examine teacher-student discourse in order to determine the linguistic needs of ESL students of algebra. Such discourse data may provide some phenomenological impression of the students' problem processing experience. Eisenhart (1988) argues for an exchange of wisdom between the research approaches of mathematics education research (which she broadly describes as aimed at descriptively and prescriptively answering the question: How can mathematics education be improved?) and educational anthropology (which she characterizes as concerned with acquiring a holistic understanding of an educational scenario).

One study which suggests a way to access the contextual perspective is Campbell (1986), which combined discourse analysis, ethnographic description of settings and events, and extensive reflection to examine the teacher-student interaction context of Philippine students' L2 acquisition of an element of the mathematics register. Campbell's detailed examination of a small amount of interaction, and his subsequent attempt to build a formulation of the ebb and flow of verbal interaction, show the value of focusing on what is said and how that relates to the context in which it is said.
3.1 The Setting and the Subjects

In Vancouver public schools in the 1988/89 school year, approximately 25,000 of the 52,000 students enrolled speak English as a second language (Vancouver Board of School Trustees, 1989). Of that 25,000, approximately 13,000 are receiving some form of assistance with English. Generally, Vancouver ESL students receive special language support for 3 years before they are fully integrated into a regular program. Recent research (e.g. Cummins, 1981) suggests that it would take from 4 to 8 years for ESL students to reach grade-level norms in the regular school program.

In the case of secondary ESL students, mathematics is very often the first academic course into which the student is integrated, given the relatively low language content of a number of types of mathematics activities, and the fact that many students come to Canada with a relatively strong background in mathematics. It is difficult, however, to generalize about the mathematical background or ability of ESL students: many ESL students come as refugees from strife-torn countries, and may have had little or very interrupted schooling for some years. The focus of this study is narrowed, then, to those secondary ESL students who have sufficient arithmetic knowledge and skills with whole numbers, fractions, and decimals to be considered candidates for integration
(within one school year) into a regular secondary mathematics course, and those ESL students who have already been integrated into such a course.

Even in a secondary school with a student population of 2000, 750 of whom were designated as ESL students, the diversity of students within a single ESL mathematics class proved to be considerable. The students differed one from another in at least the following relevant dimensions:

A. Mathematical skills/exposure to concepts (in English and in L1)
   - arithmetic
   - number theory
   - algebra
   - word problems
   - Whorfian differences between L1 math and English math

B. English competence
   - oral/aural
   - reading/writing
   - first language (L1)
   - classroom management register
   - mathematics register (as defined by Halliday [1975])

C. Social background (as defined by, e.g. Duran [1988])
   - value placed on schooling
   - value placed on mathematics acquisition
   - attitude towards mathematics
   - family attitude towards the above (especially for girls)
   - home encouragement
   - study facilities/time at home
   - student awareness of curriculum structure
   - curiosity/creativity/L1 view of teacher/student role

At the school where the present study took place, if the students in a group are deficient in arithmetic concept knowledge and skills, their instruction
will lean more towards mathematics (and incidental use of descriptive English and the classroom management register). If students are found to be competent in all pre-secondary mathematics skills, their instruction focuses on the language used in instruction, in problems, in teacher-student negotiation: these ESL-math classes aim at integration of the student into mainstream math classes of appropriate level within one school year. This type of class is multi-level, perhaps giving the equivalent of Mathematics 8 to some students, and a one- to six-month language preparation to students destined for higher level mathematics courses.

3.1.1 Serving the Needs of the ESL Student of Mathematics

A variety of pedagogical approaches are possible in the mathematics instruction of ESL students with secondary level mathematics background. At one end of the spectrum of possible approaches would be immediate placement of such students into mainstream mathematics classes; this approach has been taken in some Vancouver secondary schools, with reports of success with many students. In some cases, though, ESL students with secondary mathematics background do not fare well in regular mathematics classes and are returned to ESL math classes; and even when the ESL student survives the immersion into regular mathematics, it is not clear if the student is functioning optimally (see 7.1.1: Edward). At the other extreme would be instruction of language aspects of classroom communication, textbook reading, and subject-specific vocabulary and idiom, with little attention to the actual conduct or content of mainstream mathematics instruction until the student is integrated into regular mathematics class. This approach has been taken in some schools as well.
Unfortunately no measurement or comparison of the effect of such diverse methods has been made. This is understandable, given such factors as the high variability of student educational, language, and socio-cultural background, the transient nature of the student population (some students may transfer from school to school as many as four times in one year), between-teacher differences, and lack of established measures (or even parameters) of proficiency in mathematics-related language and communication skills.

No matter which pedagogical approach is chosen, a characterization of the language-related obstacles facing the ESL student of mathematics would be of considerable value. What must the ESL student do in order to successfully undertake mathematics learning through a second language, English?

3.2 On the Desirability of Developing Articulate Metacognition

3.2.1 Intention and Convention

Stone's (1988) examination of the (common) difficulties experienced by deaf students and ESL students in mathematics classrooms focuses on the difference between computational ability and the ability to apply basic arithmetic and algebraic operations and concepts appropriately. Stone points out that beyond the obvious compounding of the student's difficulties caused by differences between the language of instruction and the student's everyday language, much difficulty arises from the fact that mathematical convention often diverges from English language convention, and that both may diverge somewhat from the intention of the speaker (writer) of the problem.
Stone presents the case where students have studied the Pythagorean theorem in several contexts: for practice in working with irrational numbers; as part of a series of theorems about similar triangles; in trigonometry to establish identities. Later the students are expected to "see" the practical application of this basic theorem in simple surveying problems, but despite the position held in concept formation literature, the concept of the Pythagorean theorem is not found to "transfer" to the engineering classroom. Stone maintains that the language of instruction is not meaningful to the students outside of the context of the mathematics classroom. Clearly something is missing in the instructional approach which exposes students to a collection of concepts and skills and then only later introduces the application of the material in an "oh, by the way" manner. If the long-range intention (here, application) of an instructional sequence is not made explicit at the outset by a teacher, the students are placed at an understandable disadvantage when called upon to perform.

There are of course many layers to the issue of intention and convention. Stone (1989) gives a much simpler example than the above: students have mastered the exercise/worksheet form of whole number addition, yet find great difficulty in applying that concept/skill to a word problem such as: "One can of tuna fish costs $1.05. How much do two cans of tuna fish cost?" There are several potential sources of confusion here. First, language convention suggests that the problem may be subtly complex; the possibility of a discount on two cans of fish may make addition seem an overly simple resort. Second, mathematical convention is not precisely determined by the problem; multiplication may be used as well as addition. Students who search for linguistic clues such as the words and and of to the choice of operation may
see the word of in the problem to suggest the use of multiplication. As well, if the context of the problem is a page of problems which all call for addition, the student may select the desired operation for a rather unfortunately chosen contextual reason.

The occasion of student inquiry is a key opportunity for revealing issues in intention and convention to the student. For example, if student question formation is found to be overly general in form, careful attention must be given to providing the student with a model for the analysis of his/her felt difficulty.

3.2.2 A Holistic View of Problem Solving: "How to Cond It?"

One frequently heard question in the ESL math class is unique to Hong Kong students: "How to cond it?" The origin of this expression is uncertain; some informants suggest that "cond" derives from "count", and that this usage is a misnomer stemming from direct translation of the Cantonese verb equivalent of the English "to calculate".

A typical scenario: the Cantonese speaking student holds out his/her textbook, pointing to a problem. The teacher asks the student to ask an articulate question. The student: "How to cond it?" This is a formulaic utterance, and suggests a simple communication/help-seeking strategy; rather than specify some obstacle within the solving process, ask the teacher to go through the entire solving process and wait for the specific sought-for insight to arise. Several factors may account for this rather ineffective pattern of help-seeking:

1. This may be the accepted mode of communication in the student's home culture. There are many cultures in which students learn from
observing the teacher at work, then mimicking the teacher's performance (Lancy, 1983); in such cultures there is usually no step by step analysis, and student performance is refined through repetition and correction in this apprenticeship mode.

2. This mode of questioning may accurately indicate a rather holistic view of problem solving, much as a song is never sung in parts, but always from start to finish. The rote study techniques of many students from Hong Kong (i.e. do lots of problems of each kind, developing a rather automatic response to each type of problem) lends credence to this suggestion.

3. This mode of questioning may be maximally articulate for the student's degree of L2 development.

3.2.3 The Need for Metacognitive Skills

Help-seeking by use of very general questions such as "How to cond it?" is clearly not time efficient. As well, recent research in mathematics education (e.g. Schoenfeld, 1987) would suggest that the use of general, rather than specific, questions has a much more undesirable aspect: it restricts the student's development of problem solving ability in non-rote situations.

In his "semitheoretical commentary" on metacognitive issues in mathematics education, Schoenfeld (1987) makes a detailed case for the need for metacognitive skill in mathematical problem solving. In his apprenticeship model of mathematical training, Schoenfeld is attempting to prevent students from acquiring the incorrect and counterproductive beliefs which may systematically arise in the application of rote technique. For example, the
metacognitive skill of appraising a theory-based solution in the light of the pragmatic context of the original problem prevents the type of erroneous solution found in the following example:

Question: If 75 students must be transported to the football game, and if each bus can carry 20 students, how many buses are required?
Solution: 75 students divided by 20 students per bus equals 3.75 buses.

Schoenfeld's instructional design incorporates a number of activities:
1. Students watch videos of other students solving problems in a group setting and analyze the performance of the subjects.
2. Teacher demonstrates metacognitive behavior in problem solving (Schoenfeld sees this as artificial and to be used sparingly).
3. The whole class brainstorms solutions to problems: teacher serves only as a moderator.
4. Students work in groups of 3 or 4 to solve problems: teacher circulates as a resource, answering questions and offering advice (this accounts for over 30% of instructional time).

Schoenfeld asks his students three central questions when he acts as resource:
1. What (exactly) are you doing? (Can you describe it precisely?)
2. Why are you doing it? (How does it fit into the solution?)
3. How does it help you? (What will you do with the outcome?)
Citing Vygotsky (1962, 1978), Schoenfeld argues that all higher order cognitive skills originate in, and develop by the internalization of individuals' interaction with others. If this point of view is accepted, then the teacher in the ESL mathematics classroom has two types of interactions which he/she can observe in order to monitor the student's use of language in the interests of metacognition: student-student interactions and student-teacher interaction.

Observation of student-teacher interactions is a suitable, albeit predictably narrow, source of data both for research and for pedagogical planning, given the teacher's ability to control and direct student-teacher questioning (especially in a whole-class discussion scenario). Although more numerous, student-student interactions are much more difficult to observe exhaustively, especially where students often revert to first language use.

3.2.4 Navigation Within a Body of Knowledge

Teachers of mathematics to ESL students would be better equipped to serve those students if they had some sense of how students approach the task of "navigation" through the body of knowledge under study in the mathematics class, or if they had some analytical technique by which to detect and describe the student's approach to such navigation. An ethnographer approaches much investigation of a body of knowledge through language, asking questions either in the abstract in interviews, or asking questions related to observed events, objects, processes, and people (Spradley, 1979). Questioning is a technique available to the ESL mathematics student as well, although contact with the informant (the teacher) is heavily restricted. In the present circumstances the
pupil-teacher ratio is as high as 20, time is heavily constrained, and the students' limited language proficiency makes questioning a cumbersome tool.

But ESL students do ask questions in mathematics class, more than were intuitively anticipated by some mathematics teachers ("You're studying ESL kids' questions? If they ask any!"). Further, if the information and tasks presented to these students are carefully composed so as to necessitate help seeking, and if question asking by students is encouraged and guided, the students will ask a great number of questions.

It is the intent of this study to examine the questions asked by a small number of ESL students in the context of mathematics working/learning in one classroom over a three month period, in the interests of determining what the students are trying to discover about the body of knowledge before them, and how they try to discover it.

3.2.5 A Lens on Process: Help-seeking Questions

The logistics of group management to some extent must constrain the interactions between the teacher and the individual student in any classroom: the teacher's strategy for instruction rarely allows for full exploration of individual cases, and the students have likely learned to adjust their communication style (or to limit it) to the rather restricted time and discourse frames of classroom instruction. A clearer view of the ESL student's plight in English-based mathematics instruction might be had through an examination of
the help-seeking questions asked by the student, where for a moment the student has the full attention of the teacher.

Instruction may be seen as a process of exposing students to a body of knowledge. To some extent each learner will have a unique mental map of that body of knowledge; this map develops during (and after) the exposure to the new material. The learner can gain access to the body of knowledge through questioning. In addition to the many existing views of questioning, questioning can also be regarded as an action by means of which the learner navigates around the body of knowledge under consideration. In the interest of characterizing the way in which learners work towards their personal "maps" of a body of knowledge, it is interesting to regard the body of knowledge as a hypertext (Nelson, 1983), and to think of the learner's questions as a means of linking points (or of discovering links) in a body of knowledge, rather than thinking about questioning in terms of typologies of question.
As stated earlier, the unit of analysis in the present study is the question asked by the learner, rather than the individual subject over a period of time, given the transient nature of the class population. It is important to find an appropriate framework for the examination of the questions asked by the subjects.

The fairly extensive body of research on questioning in the classroom (e.g. Pica and Long, 1986; Mehan, 1979) has for the most part concentrated on question formation by teachers; reference to student questions is scant (Gerot, 1989; Lemke, 1982). In the *Encyclopedia of Educational Research* (Mitzel et al., 1982), only one index entry for questions is to be found, and that refers to questioning as teaching method in social studies. In her encyclopedic treatment of classroom discourse, Cazden (1986) devotes considerable attention to teacher questions as an element of teaching method, but in the section dealing with student initiatives and help-seeking, no specific mention is made of students' questions.

A number of ERIC on-line searches were carried out as part of this study in an attempt to verify this impression of imbalance in research attention between teacher question formation and student question formation. Although the somewhat subjective, perhaps inconsistent way in which ERIC descriptors are attached to documents makes general statements dangerous, searches
such as the one described below do afford some impression of the relative number of the two types of studies.

One of the ERIC searches carried out called first for all the studies with descriptors "Discourse Analysis or Communication Skills or Sociolinguistics or Questioning Techniques or Classroom Communication or Metacognition". 3,423 such documents were located in the ERIC files. When this body of studies was restricted by the addition of "and Teaching Methods", the number of studies found decreased to 532. However, when "and Student Participation" was added to the original string, only 59 studies were located. Abstracts of these 59 studies were inspected, and topics of discussion included teacher talk, class talk, metacognition, and question use in reading. No studies were found which dealt with student questions directed towards the teacher. Although a search such as this cannot assure an accurate description of the state of the literature on a given topic, some sense of the relative size of bodies of research may be gained.

It is not unreasonable to question whether or not the frameworks used to examine teacher question formation are appropriate for the examination of student questions; a summary of the findings of research on teacher questions is described below.

Cazden (1986) summarizes current findings concerning teacher questions as follows:

1. Teacher questions occur with high frequency.
2. Teacher questions generally have pedagogical intent.
3. Teacher questions exert control over classroom talk, and thereby over the enacted curriculum.
Cazden (1986) lists more specific findings concerning teacher talk, and a number of these are mentioned in the literature review in this study. Briefly, some of the relevant attributes of teacher questioning are:

1. Teacher questions are predominantly of the display, rather than the referential variety, and questioning patterns are usually of the short-cycle variety.

2. Teacher questions are articulate and well-formed even though they may not be easily understood by students.

3. Teacher questions are often in the context of one of two interactive strategies, preformulation (where the teacher prefaces the question with one or two orienting utterances) and reformulation (where the initial answer is wrong and the teacher asks further enabling questions).


4.1 Teacher Questions vs. Student Questions

It is intuitively acceptable that student questions may differ qualitatively from teacher questions with regard to form, pattern, intention and scenario of occurrence, considering the point of view of the information-seeking or help-seeking student. Teacher questions, as described above, are uttered in the context of pedagogy and institutional management, with regard to both monitoring of performance and to control of student comportment. Student questions are usually genuine, personally initiated queries (except when the
student uses the question as a display of his/her knowledge). Given the possibility of considerable qualitative differences between student questions and teacher questions, some doubt arises as to whether those analytic frameworks used in the literature for the examination of teacher questions may be appropriate for the study of student questions.

4.2 Frameworks of Knowledge

The search for a framework for analysis of the verbal behavior which takes place in the course of exploration of a body of knowledge leads quite naturally to the examination of studies of ethnography, where there is a substantial body of research examining frameworks of knowledge and the exploration of bodies of knowledge.

Spradley's (1979) examination of ethnographic interviewing sets out a typology of questions which may be asked in the course of exploring a body of knowledge when the questions and the answers come from two different cultural meaning systems. Spradley assumes that the question-answer sequence is a single element in human thinking; he posits that statements of any kind always imply questions. In ethnographic exploration, both questions and answers must be discovered from the informant. Spradley's (1979) developmental research sequence proposes a question hierarchy consisting of three main categories: descriptive questions, contrast questions, and structural questions.

Descriptive questions aim to elicit utterances which describe a selected cultural scene, and aim at the hypothesization of folk categories (domains) of
knowledge. Spradley (1979) categorizes descriptive questions as grand tour questions, example questions, experience questions, and native language questions. In grand tour questions the investigator asks the informant to give a global description of a place, an event or the performance of a task. Example questions may be asked in isolation, but more commonly they are woven into larger chunks of discourse involving responses of a "grand tour" nature.

Structural questions go beyond the exploration of referential and denotational meaning of cultural terms and symbols to explore relationships between symbols in the interest of forming hypothetical frameworks such as taxonomies. Structural questions predominantly use the language of classification. The contrast question, which is employed to more deeply explore local considerations within a folk taxonomy, makes distinctions between key dyads and triads within a domain.

Werner's (1987) characterization of ethnographic behavior is in much the same tradition as that of Spradley (1979), but it sets out a much greater number of categories of information typically sought in the course of investigating a body of knowledge. Beyond Spradley's (1979) structural analysis framework, which focuses on terms and symbols, their meaning and the relationship between them, Werner (1987) directly incorporates the domain of activity and the cultural knowledge underlying such activity, adding four new main categories of cultural information: action plans and the notion of temporal order, decisions and decision criteria, principles such as causality, and values and evaluation.

Strikingly similar in content is Mohan's (1986) Knowledge Framework, a theoretical framework which structures the integration of the teaching of
language and the teaching of subject-area knowledge. The six major categories within the Knowledge Framework are the same as the six areas of knowledge listed by Werner and Schoepfle (1987), but Mohan (1986) presents the categories as a schematic model with a strong interrelation between categories. The knowledge types in Mohan's (1986) framework are arranged as follows: (1) theoretical or general knowledge, which includes (a) classification, (b) principles, and (c) evaluation or values; and (2) specific practical knowledge, which includes (a) description, (b) sequence, and (c) choice or decision-making (see figure 2). The close connection between Mohan's (1986) framework and the current typologies of knowledge in Spradley (1979) and Werner (1987) recommend Mohan's (1986) framework as a way of categorizing the questions asked by students as they explore a body of knowledge.

4.3 Some Views of Students' Questions

For the purposes of this study, it is not assumed that students ask what they want to know, since clearly some ESL students have insufficient language to communicate at the level which would serve their cognitive background or social intentions. Further, a well-phrased question in a second language may actually be an erroneous form of an entirely different question, perhaps correct in form as a result of a fortuitous combination of diverse language components rather than a speaker's intentions. It is tautologous to say that a second language student's ability to negotiate meaning will not fully serve his/her needs in terms of cognitive development; yet observed over time, the questions asked by a given student do grow in complexity, specificity and variety (see case studies of Patrick and Juan).
Students' help-seeking questions in an ESL mathematics class may be viewed in several different ways:

1. As indicators of students' immediate, felt needs. Questions may give a false impression in this matter, for a number of reasons:
   (a) as mentioned above, the question may be of the wrong form.
   (b) a student's L2 language development may be insufficient to represent the student's cognitive level or social intentions
   (c) the student may be concerned that his/her intended question might be socially inappropriate or in danger of misunderstanding, and ask a "safer" question.

2. As indicators of what language students are interested in trying out linguistically to learn the language, i.e., in the case where the mathematical material under discussion is already within the students' grasp. This is particularly notable when the student's language learning style includes risk-taking (Rubin, 1975) and negotiation (Long, 1983). It is of interest to note that not asking a question is also significant; this behavior could have at least the following very different interpretations:
   (a) The student feels confidence with the instructional material.
   (b) The student is confronting an obstacle but feels inadequate to attempt the linguistic or social demands of asking a question.
   (c) Questioning is not in the student's repertoire of problem resolution techniques.
Similarly, the behavior of asking a question, getting an answer and returning to individual activity is open to at least these interpretations:

(a) The student understood the information and is now going to apply it.

(b) The student did not obtain helpful information or insight, has given up on language-based tactics and is going to try individual effort again.

(c) The student did not obtain helpful information or insight, and has given up on the task altogether.

3. As indicators of students' individual knowledge exploration techniques, giving insights into the student's approach to the task of forming a functional impression of the body of knowledge under examination. It is this view of student questions which will be employed in this study.

4.4 Recognition Criteria: Questions

In addition to syntactically interrogative utterances, there are other forms of utterances, such as indirect questions (Brown and Yule, 1983), whose intended function is also help-seeking. If contextual circumstances or subsequent actions or discourse indicate that a student's utterance is intended as a request for information or action (Garvey, 1984) which might support the student's exploration of the body of mathematical knowledge, that utterance will be accepted as a question for the purposes of this study.
Chapter V
Patterns of Questioning

The context of the ESL student eliciting knowledge about the body of mathematical knowledge as presented in the English-speaking mathematics classroom is highly variable. The mathematical concepts and/or skills under investigation by the student may be familiar or unfamiliar, given the variety of students' educational backgrounds, but the language of instruction is always an interacting factor. English as the student's second language repeatedly rises up as an obstacle to comprehension of input or task and to the student's expression of concern or difficulty, or as a threat to the student's confidence in his/her (first language) mathematical background. This occurs in both classroom discourse (be it the language used in instructions, definitions, explanations, demonstrations or discussions) and the written language found in textbooks or other materials. The student is not only working on mathematical tasks, but as well may be seen to be acquiring the linguistic and behavioral patterns appropriate to English-based mathematics learning.

5.1 The Data

Naturally arising requests directed to the teacher by the students, along with the ensuing related discourse, were audio tape recorded and subsequently transcribed over a three month period. The data thus obtained are relatively few in number; embedded in 25 hours of recorded discourse, 104 student questions (distinct in surface structure) directed to the teacher were identified as pertaining specifically to mathematical issues. In fact, only in 11 of the 25
sessions documented did students ask questions. This may be accounted for by the fact that some classes were teacher-centered, involving demonstration and teacher-led activities designed to provide unambiguous, structured input, while others were testing sessions. The relative paucity of questions is not unexpected considering that questions were not controlled, but were left to arise from student need rather than from teacher solicitation.

The students did ask questions, and this provided the teacher with an ongoing source of information about the students' process of acquisition of knowledge about the body of mathematical knowledge under study. Questions by students were not a continuous phenomena by which the teacher could monitor student development; rather the questions proved to be revealing as to student perceptions on an immediate basis, giving insight into what the student was thinking at a given moment. Observed over time, questions asked by some individuals did show some shift in the student's question formation strategy (see case studies).

Over the three-month observation period, the following questions (relating to math) were asked in varying forms by the subjects. In many cases the students' inquiry was in declarative syntactic form, but constituted a request within the immediate context (see Garvey, 1984: request for action). Fourteen question types were documented:

1. (No utterance: subject points at a piece of text).
2. How to do it?
3. What does _____ mean?
4. I don't have enough data to proceed.
5. What should I do next?
6. What does this problem ask me to do?
   - I have a vocabulary problem.
   - I don't understand a math question idiom.
   - Some other question in mistaken form.

7. Is this method correct?

8. My answer is different from the book's answer.
   (multiple interpretations)

9. Are these two different forms the same?
   - Is the subject's answer correct?
   - A question of mathematical equivalency.
   - A question of the acceptability of a variation.

10. How do we say that?
    - Pronunciation
    - Linguistic form (e.g. three over two)

11. Is my answer right?
    - Is the linguistic form okay?
    - Is the numerical value/mathematical form okay?

12. What is this symbol?
    - Language: how to read it.
    - Math: how to expand or interpret it.

13. How can I generalize my solution to this (simple yet representative) problem? (see, e.g., Jordan, June 21)

14. How can I encode this English text in mathematical symbols?

Other questions were asked pertaining to social or management issues in the classroom, questions such as whether another class had already had a test which the class was writing, whether a student might rewrite an exam, or whether the class was too full for a tutorial student to sit at the back of the room. These data were not analyzed as the questions fall outside this study's interest in student exploration of mathematical knowledge.
Nevertheless one class of such non-mathematical questions does merit mention. These are questions which reveal a student's concern about his/her status in the class, questions which in model form might be expressed as:

1. Am I in the wrong class?
2. Do I have enough ability to do this material?
3. Don't I need more help?

Although such questions are tangentially within the context of mathematics learning, they are concerned with issues other than the comprehension of a body of knowledge and as such were not analyzed in this study.

5.2 The Place of Each Question Type in the Knowledge Structure

The recorded data, the questions asked by the subjects in the course of their ESL mathematics classes, were classified first according to the surface content of each question and then according to the nature of the intended request. The intended connotation was deduced from context and from the outcome which in each case appeared to satisfy the felt need of the student. For example, if a student asked the teacher how to do a word problem, and eventually identified one step in calculation far into a grand tour as the source of his interest, it would be inferred that the student's question in fact concerned sequence. This approach is highly inferential, but perhaps is to some extent representative of the explorative, intuitive nature of teacher response to student request.

Each category of question has been tentatively analyzed as to its place in Mohan's (1986) knowledge framework; this assignment of framework placement was repeated twice by the researcher in the interest of accuracy and
consistency. The modified version of Mohan's (1986) six-part structure used in this study is outlined below:

a. **Description.** Here questions relate to how language (or symbols) are assigned in a naming or descriptive mode, as in the requests "What is 'the occurrence'?", where the student is asking for a definition of a word in a textbook problem, and "I don't understand what this thing is.", where the student is asking for the meaning of a combination of symbols.

b. **Sequence.** Here questions relate to the carrying out of a sequence which has already been chosen according to some principle. For instance, when a student asks, "What should I do next?" it is assumed that the student has chosen an algorithm of some sort but is unsure of the step by step performance of that algorithm.

c. **Choice.** Here the student is asking for assistance in choosing between some alternatives, be it a choice of manner of representation of quantities symbolically, a choice of algorithm (where several appear appropriate), or any other mathematics acquisition/performance related choice.

d. **Classification.** The student is asking where a symbol, a concept, a procedure or some other aspect of mathematics knowledge/procedure fits in a typology of similar entities.
e. Principle. A question regarding the principles governing behavior in a given situation.

f. Evaluation. A request for assigning, or a strategy for assigning, some discriminatory value to an object (includes symbols, words, expressions) an event, or a situation, particularly one involving choice.

In addition, one further category, a global one, is necessary:

g. Grand tour requests. As characterized earlier, this type of request is employed when the student wishes the teacher to display a full range of language and behavior which constitute an appropriate response to a given situation or task.
Chapter VI
Findings

6.1 Navigational moves: Typology of Question Content and Intention

An item of recorded data, a student's question of the teacher, may be taken as a surface indicator of what aspect of the body of knowledge the student is focusing on at one point in time. Taken as a whole, given sufficient number of subjects in a large enough number of scenarios dealing with a large enough number of mathematical topics, the data may indicate whether ESL students in general probe certain aspects of a body of knowledge while ignoring others. The following sections constitute an exploratory analysis of the data collected in this study, examining where student attention appears to be focused in terms of the knowledge framework proposed by Mohan (1986) (see fig. 2). The capitalized terms in parentheses in this section refer to the categories of Mohan's (1986) knowledge framework. At times the topic of reference of a question is discussed as well. This varies in importance. For example, the (Principle/Sequence) question "How do I do it?" is relatively articulate if the student is pointing to a decimal division question; the topic of the question is quite specific. But if the same question is referring to a four-sentence word problem about interest on investments at varying interest rates, the question is imprecise indeed.
6.1.1 The "grand tour" question.

Spradley (1979) coined the phrase "grand tour question" to refer to a request by an ethnographer that an informant demonstrate what does or should take place in a given situation, or describe the physical attributes of a given situation. The present subjects were found to frequently resort to grand tour questions when seeking the teacher's help. Two general forms of this question were observed.

(a) 'How to do it?' and equivalent utterances

Here the student uses language to frame his question, though perhaps not accurately. The utterance "How to do it?" is at face value a request for information about procedure (comprising (a) the relationship between a language-based task and mathematical encoding (Principle/Description), (b) the correct mathematical procedure once the mathematical encoding has been carried out (Sequence), and (c) translation back from mathematical symbols to English (Principle/Description)). This class of question is often too general to achieve the student's communicative intent, as may be seen in the following cases:

Edward: Oh, I want to find the answer.
Teacher: Can you read it?
E: Yuh.
T: Do you know all the words?
E: Yuh.
T: And then what? What do you have to do in the question?
E: Solve it. Solve the cost.
T: Find the cost. The cost is a number, right? You have to find a number.
As the conversation proceeded, it turned out that Edward wanted to know the definition of 'cost' and 'principle' (Description), and a formula for cost in terms of principle and interest (Principle). Two much more specific questions could have led the teacher directly to serving Edward's needs. It is possible that Edward had not been clear about the nature of his own request until the interaction with the teacher took place. In the next example, however, the student had a very specific question from the onset, yet chose to initiate her request with a very general question:

Lucia: (Points) In this line. (In a table: students are to convert a decimal fraction to a fraction and to a percent)
Teacher: If you've got the decimal, the percent is easy.
L: How you do that?
T: You move the decimal point two places...
L: Two places on the left?

Rather than immediately asking if the decimal point is moved to the left or the right (Principle/Sequence), which she subsequently admitted was her intended question, the student asked the teacher to go through a more complete demonstration. The specific (Principle) question, which might take the form "When I convert from a decimal to a percent, which way do I move the decimal point?", is decidedly complex for a language learner; furthermore, Lucia is not a risk taker in her learning style, and often expresses a preference for extensive review rather than brief periods where students ask the teacher to demonstrate selected tasks.
Incomplete questions may have more than one interpretation. In the interaction below, Tom asks a question which upon hearing appears to be of the grand tour type, but Tom's pointing to his page reveals his consternation over a typographical error:

Tom: (points at a mistyped question which asks for the evaluation of $2n = 3$ when $n$ is 3) Sir, how?
Teacher: It should be two $n$ plus three.

Here Tom is asking about what he perceives to be a discontinuity in the relationship between the principle being studied and the data within a problem. His question "How?" is not a clear communication of his (correct) perception of the inappropriateness of the form of the question.

At other times this class of question may be too specific, as seen in the following:

Tom: I don't know the first question.
Teacher: Yeah, what's your question?
Tom: I don't know this....fractions.
Teacher: (Looks at the question in the book) I don't know how to make a fraction into a percent.

Here Tom has asked a question about a specific task; the teacher begins to try to have Tom narrow the question, on the assumption that the obstacle to task performance is some specific part of a process (Sequence). It turns out, however, that in fact what Tom is seeking is very general information (Principle/Sequence) about a whole class of problems.
(b) No utterance: silent pointing.

The student silently points to a task or problem on the textbook page. The teacher has several choices of response:

1. Ask the student for verbal clarification of the non-verbal query.

2. Perform the task in its entirety as a demonstration of the desired behavior, perhaps talking through it in order to model language for future requests.

3. Guide the student through the steps of problem solution, with teacher utterances serving both as commands and as comprehensible input for language acquisition. Here, responses 2 and 3 follow the guided tour strategy mentioned earlier. However, the student's metacognitive development is only served indirectly by these responses. In the interest of metacognitive development, the teacher is behooved to:

4. Negotiate with the student whether indeed a grand tour is being requested, or whether one specific step in problem solution is blocking the student: is it understanding the vocabulary or the intent of the problem as stated (Description)? is it the underlying Principle of connection between language and mathematical encoding? is it a decision as to suitable procedure (Choice)? is it one particular step in a procedure which the student has already selected (Sequence)? or is it knowing if an answer already reached by the student is correct (Evaluation) or appropriate (Principle)? These possibilities are derived from the assumption that the problem solving approach found in such works as Polya (1957) is in fact
central and comprehensive, and is suitable for all learners with their varied learning styles.

6.1.2 'What does _______mean?' and equivalent utterances.

The obvious intent of such a question is to ask for the definition of a word or phrase (Description), and in most of the observed interactions, this was indeed the case. Such questions as "What's true or false?", "What's an arse, sir?", and even the inarticulate "Dai...vi...de...." (intended to ask for both the pronunciation and definition of the verb 'divide') are clearly simple requests for lexical definition in the contexts observed.

Questions about the representational value (Description/Principle) of mathematical symbols have the same or similar linguistic form as questions about English terms, but often the representational power of the symbol allows the student to use non-verbal moves in place of language, as seen in the case of Patrick transforming two pairs of brackets \(-3)(2x)\ by inserting a times sign \((-3)x(2x)\) and asking if the first meant the second.

6.1.3 'I don't have enough data.'

Help-seeking questions of this type indicate that the student has selected (Choice) some overall strategy (Principle/Sequence) for solution of the problem, and has found (Evaluation) that the problem data somehow do not fit the actions (Sequence) to be carried out. In some cases the student is correct, as in the case of Tom and the misprinted question described in section 6.1.1 (a).

In other cases, however, the student has underestimated the information content of the problem:
Albert: Excuse me, I don't know how many [playing cards in a deck] is red.
T: Oh, yes you do.... do you guys know anything about cards?
All: Not really, sir.

The ensuing discussion revealed that in fact the students did know the necessary facts (Classification) about the cards in a deck, but had failed to connect the words in the problem to the cards they play with every lunch hour.

6.1.4 'What should I do next?'

Only two utterances with this intended (Sequence) question were recorded. For the most part students used much more general forms such as "How to do it?" or "This one." Only very verbal students such as Juan identified the nature of their difficulties specifically as knowledge of a step in a sequence: this suggests proficiency in metacognitive and social skills as well as language alone. When Juan asked, "First you simplify, but after you simplify, but then, when you simplify, what you do?" he may have been asking for information at the level of rule of operation (Principle) rather than order of operation (Sequence). When the teacher informed Juan that there was no further operation required, Juan related the question to the practical level of the question on which he was working. Juan's answer was a fraction and the textbook's answer was in decimal form; Juan had assumed that converting the fraction from one form (rational) to another (decimal) was part of the algorithm for the solution of the problem. That Juan had viewed his knowledge deficiency as lying in the sequence domain rather than the descriptive reveals his lack of awareness of the equivalence of symbolic forms (13/25 and 0.52); that he chose
to begin his help-seeking at the generic level (Principle) and then resort to repair of communication at the specific level (Description) sets him quite apart from his classmates. This is not to suggest a higher level of mathematical proficiency, but rather a structurally different style of negotiating a body of knowledge.

6.1.5 'What does this problem ask me to do?'
- I have a vocabulary problem
- I don't understand a math problem idiom

Such requests as "Want to know what they want to ask, right?" are slightly different from "How to answer?" and "How to do this one?". The more specific questions of this group have narrowed at least the intent of the request to interpretation of the text of a problem, although the ensuing discourse may reveal that the request was for a full display of the behavior called for by the problem. In other cases, questions which appear to be seeking help in problem interpretation may prove to be much more specific:

Patrick: (reads a probability question) How to cond it?
Teacher: How to?
P: Cond it? How to do it?
T: Okay, what's your question, Peter?
P: (reads)'What is the occur...occur...occur...'
T: Occurrence?
P: Yeah.
T: What does occurrences mean?
P: Yeah.
6.1.6 Is this method (which I used) correct?

Only one recorded interaction contained a question of this type, Juan's "Is this method correct?" However, it was never determined that Juan was in fact seeking an evaluation of his choice of algorithm and not an evaluation of his carrying out of the sequence of actions called for by the algorithm used.

6.1.7 My answer is different from the book's answer.

This question is one of several instances of Spradley's (1979) contrast question type, and is aimed at obtaining more than verification of the contrast by the teacher: what is indirectly requested here is an evaluation of the student's solution to a problem, either in terms of (a) correctness of the (alternate) symbolic form used to express the solution or of (b) correctness of numerical value of the student's solution. Spradley suggests that contrast questions are aimed towards classification of elements of a "folk taxonomy" (collection of symbols of a culture); here we see a contrast question being used to ask the informant about the true/false or the acceptable form/unacceptable form classifications of a problem solution.

6.1.8 Are these two different forms the same? (a question of information: mathematical equivalence)

Question types 7 and 8 may appear to be two similarly intended variations of contrast question, but in most cases of type 7, evaluation of student performance rather than symbol comparison (Principle/Description) was at
issue. If the two expressions were different in appearance, students never considered symbol equivalence as the root of the disparity; rather they tended to question algorithm choice or (more often) performance of the operations within an algorithmic sequence.

Although a considerable body of research (e.g. Cummins, 1984) characterizes discourse about such academic topics as mathematics as context-reduced, it should be noted that in the case of mathematics, the richly symbolic written form itself is a very explicit discourse context. Often the difference between two symbolic expressions or statements is the source of learner interest or wonder, as in the conversation below.

Teacher: Okay. Show me one that you're not sure of.
Lucia: This one. It's different (points to two sample problems in the textbook).
T: Different answer?
L: No, different in here (points to the textbook's working of the two sample problems).
T: Oh, yes, because here 7 is bigger than 2, so you don't have to borrow. But here we have to borrow, because (points) 4 minus 9, we can't do it (demonstrates borrowing).

Lucia's statement, "It's different." constitutes a question; she is asking for some kind of help. To her, the two problems are identical in structure, yet the procedure for their solution is different. If analysis in terms of Mohan's (1986) knowledge framework is carried out here, we have Lucia asking about a variation in Sequence. Her implicit question is why the two procedures are different. This probably reflects Lucia's concern that there is a governing Principle of which she is unaware. The question is linguistically inarticulate: she
could ask, "Why does the book do these two problems in different ways?" In terms of mathematical knowledge, though, Lucia is applying her previously acquired Principle (perhaps erroneous) that there is usually a "best way" to do an operation, and another more subtle Principle, that if a textbook is going to use different methods, it should announce that in the surrounding text.

6.1.9 How do we say that?
- pronunciation
- linguistic form of a symbol (e.g. three over two)

Nowhere in the data was a student question sufficiently articulate to discriminate between these two requests. Repair always consisted of refusal of an erroneous teacher response, followed by an "I mean..." utterance, as below:

Charles: (points to symbol 7/4) How do we say, seven fourths?
Teacher: Yes, you can say seven fourths, or seven quarters,...or you can say seven over four.
C: No, is my, I say correct?
T: Oh, yes, your pronunciation is good. Seven fourths.

6.1.10 Is my answer right?

These are numerous clear-cut requests for evaluation of student performance. In every case the request was for evaluation of the final outcome of the problem solving, rather than for evaluation of the choice of algorithm or
performance of the sequence of operations and decisions prescribed by the algorithm chosen.

6.1.11 What is this symbol?
- language: what's it called/how to read it?
- mathematics: how to expand or interpret it?

Almost all of the recorded utterances by students seeking information about a symbol failed to discriminate between linguistic and mathematical aspects of the symbol (e.g., Charles' "What is this?"). This is not to suggest that the students were unclear as to which aspect of the symbol was unknown to them.

6.1.12 How can I generalize my solution to this simple yet representative problem?

Only one student utterance was found with such an intended request. Juan showed the teacher his solution for a simple equation, \( x + 17 = 32 \). Juan wrote below: \( 15 + 17 = 32 \), and waited for a response from the teacher.

T: Now you have to write (teacher writes) \( x \) equals fifteen. So that's okay for an easy one, what's your method for a difficult one?
J: I don't know. That's what I want to know.

The teacher had established a pattern over a period of months wherein a new topic was treated at first in an informal way where intuitive solutions were sufficient, and only in subsequent classes were generalization and algorithm formation discussed. Juan had induced this pattern and rather than waiting for
the eventual move from specific to generic discussion, was asking for generalization (Principle and resultant Sequence).

6.1.13 How can I write this (text) expression in mathematical symbols?

Although usually asked in guided tour form such as "How to do it?", this type of request was occasionally asked specifically, especially when the student had been exposed to exercises which focus directly on encoding. In the following interaction, a student has identified encoding as a necessary step towards problem solution:

Sam: How to write tree less dan eight times a number?
Teacher: (explains).

6.2 Classification of Student Questions:

Surface Content and Intention

It is of interest, particularly in the ESL case, to examine the distribution of both surface meaning and intended meaning of student questions over question type. What kind of questions do these students ask? What kind of questions do they intend to ask? What kinds of questions are lacking in students' help-seeking behavior? The data in the present study are too few in number to answer these questions in a definitive way, but may indicate some directions for future research. Table I shows the frequency distribution of surface content and intended content of student questions according to question type.
Table I. Frequency distribution of questions by surface content and intended content.

<table>
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<th>QUESTION TYPE</th>
<th>SURFACE CONTENT</th>
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<td>15</td>
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<td>EVALUATION</td>
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<td>22</td>
</tr>
<tr>
<td>GRAND TOUR</td>
<td>31</td>
<td>22</td>
</tr>
</tbody>
</table>

Table II shows the classification of the data according to which of Mohan's (1986) knowledge structures is being referred to by the surface content of the question, and secondarily by the intended nature of the question, as determined by subsequent events and speech acts. This gives a sense of how the student questions recorded here have been inappropriately framed with regard to knowledge type.
Table II. Frequency distribution of questions by surface and intended content

<table>
<thead>
<tr>
<th>SURFACE CONTENT</th>
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(continued)
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<td>Grand tour</td>
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6.3 Variation of Question Type According to Topic Type

Pimm (1987) claims that the language used in discourse about mathematical matters differs from problem class to problem class, e.g. from geometric measurement to equation creation and solution. This suggests an interesting question: do students ask different types of questions while studying one mathematical topic than while studying another? A preliminary look at this issue may be gained from the data in this study: two distinct types of topics were given instructional attention during the period of observation, symbol manipulation (algebra and fractions) and problem solving requiring format shifts from English to symbols and back (probability problems).

In table III the questions asked during the study of these two different types of material are classified according to question type. Considerable similarity may be seen between the two sets of data. One quite striking difference may be noted, however: there was a much higher frequency during the algebra classes of occurrence of questions whose surface content was related to matters of Principle. However, only three of the fourteen Principle questions asked during algebra classes were intended as Principle questions. No conclusions may be drawn from such small samples, but this type of comparison of question formation over different classes of mathematical knowledge and skill might prove useful in a more extensive study.
Table III. Frequency distribution of questions by surface and intended content  
(topics: probability and algebra)

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Table III (continued)

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<tr>
<td>Grand tour</td>
<td>16/15</td>
<td>Description 4/0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sequence 0/1</td>
</tr>
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<td></td>
<td></td>
<td>Choice 0/0</td>
</tr>
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<td></td>
<td></td>
<td>Classification 0/0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Principle 0/0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Evaluation 1/3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Grand tour 11/11</td>
</tr>
</tbody>
</table>
6.4 Types of Question Formation Errors

Brown and Yule (1983) use the term thematisation to denote the manner in which topics are announced and negotiated. Below, student errors in question formation are examined with regard to how initial questions (surface content) differ from intended questions (intention).

6.4.1 Specific questions with general intent

Mohan’s (1986) knowledge framework is divided into two major categories, specific knowledge (description, sequence and choice) and general knowledge (classification, principle and evaluation). In more than 15% of questions recorded in the data, the initial question asked was of a specific nature, yet the student was found to have a general question in mind. In only a few cases did the student have a help-seeking strategy which accounted for this discrepancy (see Case Studies: Juan). Occurrences of general content/specific intention accounted for only about 3% of questions asked.

Most notable were questions intended to probe the principles governing a situation: 60% of these were pitched at the specific level (Mohan’s (1986) description/sequence/choice); 67% of these over-specific questions concerning principles were sequence questions. This suggests that the student was thinking about a rule of operation but was either lacking in metacognitive knowledge/skill (did not discriminate between the principle underlying an algorithm and an algorithm as a sequence of operations) or did not to ask the intended abstract question competently in English. As well, the usual help seeking scenario has the student working on a set of tasks; it is easy to imagine
the student stuck on one task, even stuck in mid-algorithm, asking a specific question of the teacher, and only after some negotiation realizing that the obstacle is indeed lack of knowledge at a more general (abstract) level.

6.4.2 Questions with appropriate specificity, inappropriate category

Approximately 8% of all questions asked were appropriately pitched concerning specificity/generality yet fell into an inappropriate category of Mohan's (1986) framework. In a number of cases, these category discrepancies could be seen as possibly intentional. For example, a number of students asked the teacher to compare their solution and the textbook's solution; usually a classification question was asked (e.g. "I got this, but the book says this.") whereas in fact evaluation ("Am I right or wrong?") was being sought.

6.5 Teacher Questions vs. Student Questions

In section 4.1, it was suggested that the question of the appropriateness of teacher question frameworks to the study of student questions is of some interest. The following is a comparison of teacher questions (as characterized in the current research literature) and student questions as evident in the present study's data. The student questions observed in this study may be seen to differ from teacher questions with regard to:

1. Frequency of occurrence. Unlike teacher questions, which may be characterized as lined up for rapid answer and evaluation, ESL student questions are relatively infrequent (104 student questions were observed in 25 hours of classroom time in this study).
2. Specificity of intent. Whereas teacher questions are specific and structured in sequence, ESL students' questions are exploratory, and sequenced for the most part in reaction to the content of responses received. In a number of cases, students ask what appears to be a grand tour question, but is revealed by negotiation to be a much more specific question, as seen in the interaction below:

Herman: What's...what's this one?
Teacher: Probability of getting a spade?
H: I want to get this one.
T: (looks at Herman's work in his notebook) Yeah, that's correct.
H: How, how? How it's thirty?

Herman had apparently copied the answer from the back of the textbook, and then found he could not understand how the answer was determined: his apparent grand tour question was in fact a principle-sequence question.

3. Clarity of formation. Teacher questions are typically clearly formulated, whereas ESL students' questions, often vague in form and more general than the intended query, place considerable demand on the teacher for interpretation. Initially, the teacher who has been asked an inarticulate question by a student must distinguish between two possibilities: (a) the student has a specific question in mind, but cannot command suitable language to communicate the question, and (b) the student has only a vague conception of the obstacle he/she has encountered (and only a vague conception of what he/she is doing). These two possibilities demand very different strategies for response. For example, when Tom points to $\sqrt{9}$ in his textbook and asks the
teacher, "What's this equal?" it is not clear if Tom is asking for numerical information, a definition of the function of a symbol, or linguistic information. In this case it turned out that Tom wanted linguistic information: how to read the symbols orally. In striking contrast, on the same day, when Charles pointed to the same symbol, the following interaction ensued:

Charles: (mumbling) How do I say this?
Teacher: How do I say this...(reads Charles' writing) 'Nine divided by the square root.' Did you study it in Costa Rica?
C: No, we are going to study it in grade 8.
T: Then you didn't get it. I'll give you some review pages from the math 8 book, and you can look at them on the weekend.

Charles' initial question was well-formed linguistically, but this was misleading: in fact Charles was using a question which had been practiced recently in the class, and using the question carelessly.

4. Display vs. referential questions. Whereas teachers ask predominantly display questions (Cazden, 1986), the subjects in this study asked mostly referential questions. Typically the student does not know the answer to his/her question; he/she is in search of missing pieces, either local or global, of his/her mental map of a body of knowledge for which the teacher has a complete, if somewhat differently constructed, mental map. As a result, if the student does not receive a satisfactory response to his/her question, the question will be rephrased, pursued or negotiated. Consider the two bits of discourse below where students initiate interactions which are sustained (long cycle):
Lucia: I'm not happy about divide. I don't know how to do it.
Teacher: It's like this (does a sample question).
L: (points to answer, a proper fraction) Can we divide?
T: But it's proper. We don't need to simplify....well, we can but
it's not going to be simple (demonstrates). But we don't need
to do that. This is quite fine. It's only when it's improper that
we want to divide. (Looks through Linda's pages of work) It's
no problem to do these?
L: Ehh, substracting.
T: Okay, show me one you're not sure of.
L: This one. It's different.
T: Different answer?
L: No, different in here (points to the working of two sample
problems).
T: Oh, yes, because here 7 is bigger than 2, so you don't have
to borrow. But here we have to borrow, because (points) 4
minus 9, we can't do it (demonstrates borrowing).

Albert: ...how come this one is positive, right?
Teacher: (explains Albert's faulty operation)
A: (points to the second term in the expression C - 18) Then
how about if we count this one first?
T: If we what?
A: Count that one, write that one first?
T: If we say negative 18 plus C? Correct. There are three ways
to write it. You can say C minus 18, you can say C plus
negative 18, and you can say negative 18 plus C. Three ways
to write it, they're all the same.
A: The answer's the same?
T: The value's the same.
Sam: (points) This one's not the same.
T: Why is that... it's the same.
Albert: Oh, I see.
5. Long cycle vs. short cycle. As mentioned earlier, recent studies (e.g. Cazden, 1986) hold that teachers typically use questions in short cycles of interaction. The students observed here, however, often asked questions in succession (long cycle) as may be seen in the two examples above.

6. Evaluation of responses. It is also important to note that unlike the teacher-led question discourse, interactions stemming from student questions do not follow the I-R-F pattern: in all the interactions observed here, there was no evaluation (F) in the strict sense of the term. Rather the question-asking student in some verbal or non-verbal way signalled an acceptance of the teacher's response (R). This acceptance either marked the end of the interaction or led to further questions.

The considerable qualitative and quantitative differences between teacher questions and student questions cast doubt on the appropriateness of the approach (which Dunkin and Biddle (1974) refer to as the process-product approach) employed in the majority of recent studies of teaching process, for the examination of student navigation of a body of knowledge by means of student questions. In the last few years, exploratory studies such as Lindholm (1987) have attempted to provide a more appropriate framework for the study of the process of question development in ESL students. Lindholm’s (1987) study of English question use by Spanish-speaking ESL children employed Morris’ (1955) syntactic/semantic/pragmatic framework to examine the development of various functions in children’s questions. Such an analysis goes beyond the examination of isolated language elements of utterances to include cognitive and metacognitive considerations. Even so, studies such as Lindholm (1987),
using theories of second language acquisition to examine the second language learner's questions as an aspect of second language development, cannot provide an appropriate framework for the conduct of studies such as this one, which aims at using theories of knowledge acquisition to examine student questions as evidence for the student's role in the classroom.
Chapter VII
Case Studies

The purpose of this chapter is to provide some contextual perspective in counterpoint to the quantitative generalizations suggested by the data in chapter 6. The same question asked by two different students may represent two very different inquiries, or may be employed in very different strategies of exploration of a body of knowledge.

7.1 Student Variability

Individual students manifest very different combinations of English language proficiency, mathematics knowledge and skill, learning style and question asking strategy. The following case studies are presented to give some sense of the extent of this variability.

7.1.1 Edward

Edward is 15 years old, three months out of Hong Kong. His study skills and cognitive background are strong. He has mastered most of the concepts and skills covered in Math 11, and he is functioning at a "B" level in Math 10, but he has requested that he be allowed to spend some time in ESL Math 1 in order to acquire math-related English. As yet Edward's help-seeking is quite inarticulate. His often repeated utterance "How to do this one?" may in different cases be interpreted as:
I don't understand the question.

or Show me the range of behaviors required here.

or I can't get one step of the solution.

Typically the teacher will demonstrate the approach to and solution of the problem, looking for Edward's specific block. As the demonstration proceeds, it usually turns out that the third option is in fact the case, and that Edward's help-seeking is in a very crude form. Edward is using the same utterance for a variety of purposes, expecting an intuitive hearing and pushing his resource person through unneeded steps until the needed information arises.

Edward does know the other question forms to which he could resort; he is an aggressive learner who does identify with the need for language in mathematics. Yet somehow when he is thinking hard about math, he does not readily implement his knowledge about question asking. Even so, Edward's tactics work relatively efficiently. Some other students, even those with stronger math skills than Edward, rely almost entirely on L1 whispering to their neighbors.

A preliminary attempt to have the student look at the problem from a metacognitive perspective often draws a blank:

Teacher: Do you have any questions about this problem?
Student: No.

The student's response here can mean

I don't know how to ask.

I can't identify the obstacle to my solving the problem.

I don't know what you just said.
This suggests a need for (a) student knowledge of the sequenced components of successful problem solving and (b) student knowledge of and competence in the language of help-seeking. In fact, study of the language of help-seeking may well reveal the structure of problem solving knowledge to the student. Sample problems and exercises designed to show the knowledge structure of problem solving may well also reveal aspects of problem solving technique itself.

7.1.2 Patrick

Patrick is a 13 year old, four months out of Hong Kong. His math skills are typical of other students from Hong Kong public schools; Patrick is of grade 8 age but he is capable of starting in British Columbia’s math 10. However, Patrick's English is weak: his use of tenses other than the present is heavily flawed, his knowledge of basic conversation patterns is minimal, and he does not know the words for many everyday items and actions. When it comes to math, Patrick is lazy with regard to homework, partly because he is working on material drawn from the grade 9 curriculum (which he studied the previous year in Hong Kong), and partly as a reflection of his approach to studying. For four months, Patrick has expressed his difficulties with textbook comprehension to the teacher in the form of mumbled statements and requests, placing a heavy burden of interpretation and strategy formation on the teacher.

One day, though, Patrick showed his first innovation in help-seeking. He came to the front of the class with a piece of paper on which he had written \((-3)(2x)\). Patrick showed the symbols on the page to the teacher and said, "This here mean this one?" As he said, "....this one?" Patrick wrote a tiny
multiplication sign between the two pairs of parentheses: (-3)x(2x). The teacher, overjoyed to find Patrick scheming a communication that was both time-efficient and unambiguous, answered, "Yes, Patrick, the two brackets together mean multiply." This was intended as a kernel of English teaching; but was Patrick perhaps already somewhere else in his mind now that much of his math text was instantly quite a bit clearer? From that day on, Patrick became freer with his verbal approaches to the teacher.

During the observation period Patrick did ask a number of different types of questions. Noteworthy among these were three instances of contrast questions being used to indirectly request information regarding some principle, or to indirectly request an evaluation of his working of a problem. For the most part, though, Patrick relied on the ubiquitous Hong Kong English "How to cond it?" (see the discussion of this utterance in section 3.2.2.

7.1.3 Jack

Jack is 18, one year out of Viet Nam. Since September Jack has been attending regular classes of Math 10 and Science 10. Suddenly at the beginning of January, Jack was quietly returned to ESL Math and ESL Science classes by his mathematics and science teachers, with no explanation of the reasons for his return to ESL status. Teamed with Edward, who has just been integrated into Math 10 and who attends ESL math class for language support, Jack continued to do the work of the regular math 10 class as reported by Edward. This was a weak strategy as far as remedying Jack's difficulties with regular math class, but the teacher was looking for evidence of what Jack did experience in his regular math class. Jack worked steadily, and did exercises
at home. Jack was often observed to ask Edward for help when he did not understand how to do a question, but it was usually not possible to record these questions without interrupting the conduct of the class. Edward typically answered Jack's questions by doing the problem step by step for Jack to watch; verbalization was minimal and usually consisted of general directives such as, "Then you have to do like this." In his second day in class, Jack approached the teacher for help with a problem. Jack had filled two pages with properly sequenced subordinate operations and had come to a point where synthesis of all the accumulated elements was called for. Jack pointed to the question in the book and said to the teacher, "This is too hard one." The teacher looked in Jack's notebook and found the problem partly completed, asked Edward what he had discussed with Jack, and found that Edward was operating on a know-how-don't-know-why basis. The one type of problem where Jack consistently used the wrong question type for his purposes was in questions regarding sequence; he invariably requested principle-related information when he in fact only wanted help with one step in an algorithm.

A month later Jack stopped working on the math 10 assignments Edward reported to him; Jack appeared despondent, yet when approached by the teacher, Jack claimed he was fine, that math was no problem. As in Kubler-Ross' (1969) stages of dying, Jack was experiencing the death of his dream of attending regular classes in math and science. At first Jack experienced denial, continuing with the math 10 work as if it were appropriate; later he stopped work and his sullen silence was evidence of the anger stage.

Another month later, Jack was watching a new group of students in the class being sent to the book room for math 9 texts. Jack quietly suggested that
this might be a good time for him to get a math 9 textbook too. Almost three months after his demotion, Jack accepted the wisdom and necessity of the move and took on the responsibilities of a math 9 student.

At the time of his acceptance of his new lower status, Jack also stopped asking questions entirely. This appeared to be due at least in part to the fact that the class was working on manipulation of algebraic expressions and equations at that time: Jack was capable of the symbolic manipulations and calculations at the math 9 level; his difficulty in the mathematics class was with language.

7.1.4 Chi Ming

Chi Ming is a 13 year-old Taiwanese girl who came to Vancouver after 6 years in Costa Rica. She had received her elementary schooling in Spanish, and casual informants said that she spoke Mandarin rather strangely. Chi Ming asked quite a few questions, but the utterances were usually inarticulate, indirect requests:

Chi Ming: I take this one (points to her working of a problem) but it's this (points to the answer key). I bad.

Expressed here, but difficult to convey is Chi Ming's air of low self-esteem. The topics covered in the ESL Math class were within her grasp, but minor setbacks such as getting one question wrong at times led to her stopping work, appearing to be on the verge of tears. An interaction which took place between Chi Ming and the teacher almost every week focused on this feeling of despair, as in the interaction recorded below:
CM: This math too hard.
T: Would you like to go back to Mr. L's class? It would be easier for you.
CM: No, I like this class too much. This math good.

Chi Ming never did resolve the difficulties suggested by her question asking pattern; an influx of new immigrant students into the school led to revision of most ESL students' timetables and Chi Ming went to a lower level class. It appears that she was more successful there, but no happier.

7.2 Evidence of Complex Question Formation Strategy:

Juan and Henry

Juan, age 14, arrived in Canada from Argentina six months ago. He demonstrated a fascination with vocabulary from the outset in his ESL English classes. He is an avid talker, and loves to ask questions which separate out similar concepts, sequential elements or structural levels. What is noteworthy about Juan's question formation is that he never needed to repair his communication; if he did extend a query, it was to further narrow his question, or to follow a strategy which gave evidence of premeditation (see 6.1.12). Another example:

Juan: (points to his work) Zero, right? Mr. Hunter? This won't be okay, right? That won't be okay.
Teacher: (looks) No.
J: I have to divide by, right?
T: Well, you can write it as a fraction... how is it as a fraction?
J: Three...
T: ... and something (converts fraction part of 3.2 to 1/5).
Though Juan had gaps in his mathematics knowledge skills (no fraction division, little awareness of percentage, conceptual but not procedural knowledge of algebra), he always gave evidence of knowing the structure of the knowledge of a subtopic even when he had not acquired all the concepts and skills. Even in English class, Juan’s questions were indicative of strategic intent; he would seek information about principles (“When do we say ____?”) rather than specific information (“How do you spell ____?”). It would be of interest to conduct a longitudinal study of a student like Juan: does such a predisposition towards asking articulate, strategic questions correlate with success in dimensions of acquisition and application of mathematical concepts and procedures?

Sequenced questions asked by a student are not always an indication of complex question formation strategy. Henry, an 18 year old Vietnamese male in Canada for two years, is a case in point. Henry frequently expressed frustration with his placement in the ESL math class. His often heard utterance, "We done this before." tells that he has studied a topic years ago in Viet Nam; however, Henry seldom did his homework and seldom reached even 50% on a test. This utterance reveals considerably more about Henry’s relationship with mathematics: it suggests the viewpoint that if a topic in mathematics has been studied at one time at one level of difficulty, that topic is somehow complete, is no longer of interest, and cannot be extended or deepened.

This perception of a limited range of knowledge and activities around a body of knowledge may be seen in the types of questions Henry asks in math class. Over a 3 month period, Henry asked 4 grand tour questions and 5
description questions; the absence of other question types, along with Henry's poor performance on tasks, suggests that Henry does not have an articulate mental map of the process of problem solving.

In contrast, over the same period, and often on the same topics, Juan asked 1 description, 3 sequence, 3 principle, 2 evaluation, and 2 grand tour questions. Furthermore, Juan's questions were often in sequences which suggested premeditated strategy, progressing from one question type to another (see Juan's interaction above, where an evaluation question, "This won't be okay, right?" leads directly to a principle question, "I have to divide, right?").

7.3 The Importance of Individual Consideration of Student Behavior

As may be seen in general from the cases discussed above, students have quite individual approaches to knowledge acquisition and problem solving and the related use of language. Competence and/or confidence in the realm of mathematics does not imply strength in communication about mathematics, and vice versa. As well, competence or knowledge in one area of mathematics does not necessarily indicate continuing development or exploration. It behooves the mathematics teacher to formulate tentative, frequently revised impressions of each student's relationship to mathematical knowledge and performance, and to form some notion of how the language surrounding these considerations facilitates or obstructs that student's growth.
Chapter VIII
Implications/Applications

For the mathematics teacher of ESL students, a central implication of the findings of this study is this: ESL students do ask questions, albeit not frequently in the present study. In the midst of the motion and complexity of conducting a mathematics class, these questions ought not be dismissed as prohibitively flawed in form for interpretation, nor as too burdensome to negotiate meaning from. Rather the intention of the question should be sought out for three purposes:

1. aiding the learner's immediate quest for information/guidance;
2. discovering how the intended question, found in ensuing context, is related to the initial surface utterance (and hence how the student may be led to greater articulation); and
3. gaining insight into the way in which the individual student approaches the mapping of a body of knowledge.

8.1 Developing Articulate Help-seeking

Consideration of issues in ESL mathematics acquisition at the secondary school level is strongly tempered by time constraints: students who begin their English-based schooling during their secondary school years are attempting to step onto a rapidly moving vehicle. Their successful graduation before reaching maximum public school age calls for rapid acquisition of English and integration into regular classes.
In the opinion of many teachers, math is an ideal starting area for integration because so much of the content transfers from culture to culture, especially since most immigrants to Canada come from cultures where mathematical perceptions are equivalent to or more advanced than those of the English-based culture. Integrating a student into a mathematics class whose level is slightly low for the student (i.e. one grade level low) allows at least some opportunity for the student to observe and experience cultural and linguistic differences which impinge on the math classroom.

Given this "jump on the merry-go-round" situation, how then may the student and teacher monitor the student's acquisition of math-related English concepts and skills? Certainly, given (a) the already time- compressed experience of the ESL student trying to catch up, and (b) the diversity of students' educational backgrounds, English competence, mathematics ability, and social backgrounds, there is little time for full treatment of the four points seen by Presmeg (1988) as necessary for mutual understanding when distinct cultures come together in a math instruction scenario:

1. Children need the stability of their cultural heritage, especially during periods of rapid social change.
2. The mathematics curriculum should incorporate elements of the cultural histories of all the people of the region.
3. The mathematics curriculum should be experienced as "real" by all children, and should resonate, as far as possible, with diverse home cultures.
4. The mathematics curriculum should be seen by pupils as relevant to their future lives.
Rather, pedagogical deliberations reduce to a different pragmatic level. As students in the ESL mathematics class await with varying degrees of patience and eagerness their integration into regular mathematics classes, the ESL math teacher is striving to afford the students experiences and insights which will enable the students to more easily begin to take part in the English-based mathematics instruction of the regular mathematics classroom. Certainly the teacher is paying attention to both language/communicative skills and mathematics skill and concept acquisition. With regard to the communicative aspect of language in the mathematics classroom, the teacher can be the predominant informant, a source who may be accessed in variable, negotiable ways. The fact that there is only one teacher for as many as 20 students in the ESL math classroom, however, places the onus on the student to monitor his/her own difficulties and seek help in efficient, articulate ways. On the basis of the present study, articulate help-seeking may be seen as having three components:

1. Being able to identify the obstacle to one's progress in tackling a problem or acquiring a new concept;
2. Being able to ask others in the class (peers, teacher, tutor) questions about that obstacle; and
3. Being able to guide a discussion (via specific questions and other negotiative signals) towards a satisfactory resolution of one's difficulties.

One key area for the development of articulate help-seeking is help-seeking questioning (HSQ) as a student skill to be modeled, taught, and
monitored by the teacher. Two central questions arise in this area and fall within the scope of this study:

1. What kinds of questions do ESL students ask?
2. How can ESL students be helped to develop mathematical questioning skills in English?

The first question has been answered in a preliminary way in the findings section of this study. The second may be examined from the point of view of materials and methods. Below is an examination of how one central publication addresses (or fails to address) the dimension of student question formation.

Crandall et al's (1987) workbook, *English Skills for Algebra* is perhaps the only current publication available designed specifically to address the math-related language needs of the ESL learner of mathematics. In the section "Talking about solving equations", for instance, the step by step solution of an equation is shown, and each step is accompanied by a relevant sample of English discourse:

\[
\begin{align*}
7a - 2 &= 3a + 9 \\
7a - 2 + (-3a) &= 3a - 9 - (-3a) \\
4a - 2 &= 9 \\
4a - 2 + 2 &= 9 + 2 \\
\end{align*}
\]

The given problem. Add -3a to both sides to get the variable and its coefficient on one side only. Combine like terms. Add 2 to both sides so that only the variable and its coefficient are on one side.

detcetera.
This approach is uniform throughout most of Crandall et al. (1987). There is a section devoted to the language aspects of word problems which focuses on the skills involved rather than on the communicative language skills required to discuss a problem or seek help with its solution. The knowledge of descriptive language as modelled and drilled in this workbook is essential. The teaching of question asking or negotiation is not within the scope of Crandall et al. (1987). Recorded classroom discourse documented in the present study indicates a need for such teaching. Below are some examples of utterances which show the need for such specific instruction:

1. Patrick: We can do dis? (points to a line in his solution)
2. Albert: This right? (holds out his notebook)
3. Edward: This (points to 3(2x =1) in his notebook) means (writes a multiplication sign after the 3) this?

ESL students resort heavily to gesture accompanied by vague and ill-formed questions. A sensitive teacher can very often intuit their needs and help them, but this may be a disservice in the long term. Consistent, explicit teaching of the language of negotiation is essential.

The present study has examined student questions and the utterances and behaviors which constitute the ensuing context. It is interesting also to consider the context from which a question arises: can we detect contextual clues which indicate that a question, or even a certain question, will arise? This could an area for future research.
8.2 Techniques of Teacher Elicitation of Student Questions

The information gap aspect of the mathematical tasks assigned to the students appeared to give rise to a new pattern of discourse, one quite different from the I-R-F pattern of teacher initiated QUESTION - student RESPONSE - teacher EVALUATION which Pimm (1987) sees as forming the bulk of communication in the typical mathematics classroom. The new pattern observed here might be characterized as follows:

Initiation: A (poorly formed or strategy-weak) question is asked by the student.
Re-initiation: Teacher asks student to restructure or rephrase the question.
Response: Student asks a refined version of the original question.

This is followed by either

Response: Teacher answers student question.
Acknowledgement: Student acknowledges teacher response.

or

Re-initiation: Teacher asks student to restructure or rephrase the question.

In this discourse pattern the student's question about the body of knowledge is held in abeyance until linguistic and strategic considerations about question formation are addressed. Pimm (1987) warns of a danger in this type of approach: over-control by the teacher can interfere with the development of student talk, especially if the teacher is too concerned with form, at the expense of meaning: but students must become aware of the characteristics of what Pimm calls "disembodied speech". Concern for form can be difficult to
serve at any rate; it is difficult to draw students' attention to either language rule formation or language usage after the solution to a math problem has been found.

The pattern of discourse outlined above allows for some attention to form and to strategy since the student is waiting for goal-related information and is to some degree a captive audience. This observed pattern of discourse suggests a possible instructional strategy for an ESL (or regular) math class: initial teacher-centered instruction which purposely does not discuss some elements of the material, followed by a question time where student questions about the unclear elements of the lesson are elicited, guided, structured as to strategy, and taught with regard to English (format, idiom, pattern). Audio-visual aids, such as wall-mounted posters (see figure 3) bearing sequences or clusters of model help seeking questions and accompanying sample problems as context, can provide supporting input of a more passive but continuously accessible nature.
MATHEMATICS QUESTIONS WHICH YOU CAN ASK

1. What does _______ mean?
2. What does this symbol mean?
3. How do you pronounce this? How do you say this?
4. Are these two expressions the same? Do they have the same value?
5. How can I make an equation from this word problem?
6. What does this problem ask me to do?
7. My answer is different from the book's answer. Are both correct?
8. What method should I use to solve this equation?
9. I did these steps, but now I don't know what to do next.
10. Is this answer in the correct form?
11. Is it okay to do this problem this way?
12. Why does the book do these two problems differently?
13. Do I need to simplify my answer? Is it wrong if I don't simplify?

Figure 3. Contents of a wall-mounted ESL math poster
8.3 Articulating Metacognitive Issues for the Student

With regard to the knowledge structures which underlie metacognitive behavior, the sequenced topics presented in Crandall et al.'s (1987) teaching materials do implicitly suggest a model of metacognitive behavior to the student. For example, in the section dealing with the distributive property in algebra, skills which support mastery of the concept and the language used to talk about it are listed as:

a. listening to and reading theorems and examples
b. demonstrating comprehension by answering multiple choice questions
c. demonstrating comprehension by answering true-false questions
d. demonstrating comprehension by answering questions orally
e. demonstrating comprehension by identifying paraphrases and inferences
f. rewriting theorems.

This sequence of activities provides a rich source of opportunities for discussion of, and instruction in, help-seeking strategy and language. The additional activity of asking for information or help could be added to the sequence.

The problems presented for solution in mathematics classes are a rich form of context for the development of language. As the student carries out his/her ethnographic study of the culture of mathematics, folk definitions are being created all the while. The student facing difficulty with a problem, and
asking the teacher for help, is an ethnographer inspecting an object with an informant. And the object, the mathematical problem, is profoundly important: Werner and Schoepfle (1987, p.74) say of it,

*The morphology of the object... imposes an order on the interview. An imposition of order is a step towards systematization. The analyst's task is to translate the linear order of answers to questions into systematically related sets of terms.*

Mathematics curricula are intended to facilitate the students' mastery of the body of knowledge that is mathematics or a subclass thereof. This facilitation is seldom made explicit: students are left to follow in a point-to-point manner the exploratory path imagined by the curriculum designers and are not usually empowered to explore elements in a non-linear manner.

### 8.4 A Computer Application: ESL Math Helper

Rising out of the present study is a preliminary project in the school's ESL-dedicated Macintosh computer lab. The format of the project is in flux; at present the plan is for students to seek information in, and eventually help to create, HyperCard stacks (interactive electronic documents) about mathematical topics and about the English required for communication about those topics (some sample monitor displays from one stack are shown in the Appendix). Typically, confronted with a representative math problem on the screen, the student will have access to various forms of help, available by way of "buttons" which appear on the screen in the form of help-seeking questions. The student may thus be exposed to more and more articulate questioning patterns simply through the nature of the information access.
Granted, the sample document presents a typical linear solution pattern from secondary school mathematics, but its primary intent is to model language in known context. Also, as a prototype it may be used repeatedly with many types of sample problems and many different exploratory approaches. A number of questions suggest areas for further investigation, among them:

1. Are help-seeking questions better regarded as objects of direct study, or in context, as means to learning ends?
2. Which patterns of questions, at what degree of challenge in language, are most appropriate to ESL students' needs?
3. How may the effect of incidental exposure to question language, as in ESL Math Helper, be measured? How may effectiveness be assessed?
4. What is the value of student autonomy with regard to the choice of path of knowledge exploration?
5. To what extent is the external (pedagogical) imposition of structure on student knowledge acquisition (as opposed to student autonomy in exploring interactive hypertext) of value?

The more general of these questions have begun to be addressed recently in a number of similar, related studies. Swallow et al. (1988) describe two studies where software is used to support paired peer interaction by providing data storage as well as metacognitive prompts for the member of each pair who has been assigned a directive role in the pair's activities. These studies are part of a larger project, CSILE (Computer Supported Intentional Learning Environments), described in Scardamalia et al. (1989). Scardamalia et al. (1989) work from recent cognitive research to propose a set of principles for the
design of computer environments which support purposeful and mature processing of information. There is a need for the development of similar principles governing the software support of language acquisition in knowledge contexts.
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The following figures show the computer screen contents (called cards) as the student moves/is led through the interactive document (stack) ESL Math Helper. The student must click the mouse pointer on active areas on the screen (buttons) in order to obtain access to information. The function of each button is shown by the text appearing within the button box; the text in each case is a question in a form which a student might be expected to produce in the course of seeking help in the mathematics classroom.

In figure 4a, the problem under discussion appears in a field at the top of the card. The problem has been selected by the instructor so as to have relevant content with regard to grade level and class of problem, and so as to be pitched at the appropriate level of language difficulty. The text in the field is locked so that the browsing student cannot alter the form of the problem.

The button "What does ___ mean?" allows the student to obtain definitions (Descriptive) of words encountered in the reading of the problem. Clicking on this button causes a message box to appear; when the student types the word or phrase into the message box and strikes the return key, the glossary (box below) is searched for occurrences of the word. Upon each search, the word searched is listed in the field in the upper right of the screen. This is intended to afford the instructor information as to what efforts the student has made to explore the vocabulary in the problem. This glossary function is
made available on every card in the stack, since there may be new vocabulary encountered at a number of stages of stack exploration; further, it is intended to indicate to the student that such description issues are in question at every step of the way in a problem solving process.

Still at card one, when the student feels sure that at least the surface language in the problem is understood, the button "I understand all the words in the question" will take the student to the second card (figure 4b). One new button which appears on this screen reads: "How do you find the probability of something?" Clicking on this button brings up the field below the button, as seen in figure 4b. This field contains some form of text and/or visual information which will answer the (Principle/Sequence) question. The information in this field is not intended to be new to the student in the initial use of the ESL Math Helper stacks; rather it is intended to present in a condensed form information which the student is already familiar with as a result of classroom experiences. Once this little review has been read, perhaps absorbed, a second button presents the question "So how do I do this problem?" In response to this (Sequence) question, card three is displayed (see figure 4c), presenting the student with a number of questions which the student would be well advised to be able to produce when seeking information about sequences.

Clicking each of the three new buttons on card three brings up a small field (see figure 4d) which tersely answers the question on that button. Again, at the initial stage of use of this stack, the answers to the questions should not be news to the student, but rather should be review material. Clicking the final button in the sequence brings up a button as well as a field; this new button shows the student language with which to "ask" if the sequence is complete.
Clicking on this button results in movement to the fourth card, which rather dramatically signals the successful completion of the problem solving sequence (see figure 4e) and demonstrates the application of the principle given earlier to the results of performing the sequence given on the previous card. A new button, "How can I check this answer?", provides the student with the language to ask for assistance with the checking (Evaluation) of the solution to the problem.

The choice of evaluation techniques presented in card five (see figure 4f) is presented without any questions about the choice itself. This could be altered if the instructor felt that students would or do in fact ask this kind of question in a situation of this type.

Having explored the functions of the buttons and the contents of the fields, the student will have been implicitly exposed to the underlying structure of the problem solving technique employed:

1. Analyse the meaning of the language of the problem. (Description)
2. Determine the quantity sought in the problem.
3. Find the principle(s) which operate in the solution in the class of problems to which this specific problem belongs.
4. Apply the sequence of operations called for by the principles above.
5. State the solution to the problem in the appropriate form.
6. Verify the correctness/appropriateness of the solution.
As well, the student will have been incidentally exposed (it is not necessary to read the buttons in order to click on them) to the text form of utterances which may be used to seek help with the performance of elements of that structure.

The intention is that the student be given a number of such stacks for browsing. Eventually the student would be given a template stack (all buttons and fields blank, as shown in fig. 4i) in which a problem, chosen by either the teacher or the student from a class of problems, is to be entered, a glossary built, and the problem solving steps and relevant help-seeking questions entered into the buttons and fields provided. This maintains the strategic structure of one problem solving approach.

Further extension into other problem solving approach structures might be explored through student creation of new stacks using only the presentation elements used in ESL Math Helper, or through presentation by the teacher to the student of new stacks built around alternate problem solving approaches, for example de Bono's (1978) lateral thinking.
Problem: Find the probability that a telephone number selected at random from the Vancouver phone book will end in a 7.

*clear the list:

(what does _____ mean?) {I understand all the words in the question}

random: without plan or pattern.

Figure 4a. Initial card of ESL Math Helper stack.
Problem: find the probability that a telephone number selected at random from the Vancouver phone book will end in a 7.

How do you find the probability of something?

The probability of an event is calculated by dividing the number of ways that event can happen by the number of things that can happen. The probability of a die coming up 5 is

\[ P(5) = \frac{1}{6} \text{ or } \frac{1}{6} \text{ or } 0.1666 \]

what does ______ mean?

So how do I do this problem?

Figure 4b. Principle card of Math Helper stack.
Problem: find the probability that a telephone number selected at random from the Vancouver phone book will end in a 7.

What should I do first?

What should I do next?

Then what should I do?

What does ______ mean?

---

Figure 4c. Sequence questions on card 3 of Math Helper stack.
Problem: find the probability that a telephone number selected at random from the Vancouver phone book will end in a 7.

What should I do first?
Find out the number of ways a seven can happen. There is only one way a seven can come up: 7.

What should I do next?
Find the number of different things that can happen: this is the digits from 0 to 9: 10 things.

Then what should I do?
Divide the number of successes (1) by the total number of outcomes (10). \(1 + 10 = 1/10 = 0.1\)

That's it?

What does _____ mean?

Figure 4d. Responses to sequence questions, card 3.
Problem: find the probability that a telephone number selected at random from the Vancouver phone book will end in a 7.

Yes! So the answer is 0.1
There is a probability of 0.1 or 1/10 or 1 in 10 that a telephone number selected at random will end in a 7.

Figure 4e. Statement of solution accompanied by evaluation question, card 4.
Problem: find the probability that a telephone number selected at random from the Vancouver phone book will end in a 7.

A probability calculation may be checked in two ways:

1. By a trial
2. By theory, a diagram

Figure 4f. Statement of principle of evaluation, card 5.
Problem: find the probability that a telephone number selected at random from the Vancouver phone book will end in a 7.

A TRIAL is a way to check a probability. The trial will not give the same result as the calculation, but it should give similar results.

Here we can count the telephone numbers on one page of the phone book and then count the number of numbers which end in 7. Try it and see if your probability is close to 0.1

...and the other way?

what does ______ mean?

Figure 4g. Demonstration of evaluation by trial, card 6.
Problem: find the probability that a telephone number selected at random from the Vancouver phone book will end in a 7.

We can draw a tree diagram to check our calculation:

Here there are ten possible outcomes and only one of them is a success: 7. So the theoretical probability is one in ten, or 0.1.

Figure 4h. Demonstration of evaluation by diagram, card 7.
Figure 4i. Sample card of template stack.
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Figure 4j. Waive of copyright.